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Incremental and Innovative Approaches to Professional Development for Mathematics Teachers

Edited by
Samuel Otten, Amber G. Candela and Zandra de Araujo

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Editorial

The Benefits of Modesty: Considering Incremental Professional Development for Mathematics Teachers

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Abstract: Professional development (PD) for mathematics teachers often emphasizes transformative instructional change. However, a more modest, incremental approach may offer a higher likelihood of success in ways that complement transformational efforts. This Editorial discusses the potential advantages of incremental PD where teachers make small but meaningful improvements to their practice over time. We explore the differences between transformational PD and incremental PD as evidenced by the articles in this Special Issue. We also distinguish between varieties of incremental PD projects, from those that focus on a small piece of a larger instructional vision (piece-and-support incremental PD), those that set out to achieve a modest goal without appeal to a broader instructional transformation (modest-goal-with-support incremental PD), to a more radically modest approach that involves offering small, optional instructional suggestions without investments of time or training (nudge-based incremental PD). Ultimately, this Editorial, and this Special Issue as a whole, advocates for new conversations and investigations into how PD is structured and how the promoted instructional changes might spread among mathematics teachers.

Keywords: professional development; instructional change; mathematics teacher education; mathematics teaching

1. Introduction

We, individually and now collaboratively, have been active in mathematics teacher professional development (PD) for more than 10 years (e.g., Candela, 2019; de Araujo et al., 2018; Herbel-Eisenmann et al., 2017; Otten, 2012), but recently we have stepped back to take a wider look at the field. Particularly in regard to the PD efforts aimed at influencing mathematics instruction, we share the view of Arbaugh (2025) and most of the other authors in this Special Issue that decades of dedicated efforts and high-quality work by brilliant and tireless scholars have failed to garner widespread change in mathematics instruction. Taking the United States as one example, there remains a large discrepancy between the type of mathematics instruction promoted by professional developers or scholars and the instruction commonly observed in classrooms across the country, the latter remaining largely procedural in focus and teacher-directed in style. Of course, the situation in other countries is different (e.g., Hansson, 2010; Kaur & Leong, 2021; Szendrei, 2007), but our own analysis of hundreds of secondary mathematics lessons across many states in the US (Otten et al., 2023, 2024a) and our experience with and inspection of research in elementary classrooms (Schweig et al., 2020) lead us to fully face the discrepancy. As Arbaugh (2025) puts it, "we have certainly developed pockets of teaching for mathematical

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understanding across the U.S. But we have yet to have widespread influence on the teaching of mathematics" (p. 4).

Based on these observations, we began wondering if it might be worthwhile for at least some of us in the mathematics education scholarly realm to consider an approach complementary to *transformational PD*, a term we use to refer to PD seeking large, transformational changes in instruction for the participating teachers (Otten et al., 2024c). This wondering led us to pursue *incremental PD* (Otten et al., 2022), which is our attempt to operationalize the notion that Star (2016) put forward about focusing on small improvements in mathematics teaching. By incremental PD, we mean PD that seeks only modest or small-scale changes in the participating teachers' instructional practice, not a difficult or profound shift in their teaching. Note that incremental PD is defined by the scope of the PD and also the characteristics of the teachers involved (i.e., what is a modest change for one teacher may not be for another, and what may seem profound and out of reach for one teacher may be quite doable for another). Moreover, incremental PD refers to the approach taken within the PD; it does not specify the content of the PD (there are many small things one could focus on) nor the ultimate aim (i.e., what the PD is incrementing toward). But for us, these realizations spurred us to assemble a team of collaborators for our Practice-Driven PD project (www.PracticeDrivenPD.com), described below. We were also motivated to learn from other scholars who have been similarly discouraged by the lack of widespread success of transformational PD and who have made their own attempts to innovate their approach to PD or have attempted something akin to incremental PD.

Thus, in this Special Issue, we collect a variety of articles about innovative, and especially incremental, PD with mathematics teachers. With the remainder of this Editorial, we prepare readers for this Special Issue by (A) making some further comments about PD that has sought transformational changes to instruction, (B) pointing out some key differences between transformational PD and incremental PD while also identifying some varieties of incremental PD, and (C) describing our Practice-Driven PD project, which we argue is substantially different even from the other incremental PD efforts in this Special Issue.

2. A Broad Description of Transformational PD (With Concerns)

This section contains a broad overview of PD (Figure 1), admittedly missing many important elements and nuances, but we have attempted to represent the key elements that are contrasted in the subsequent sections. With that disclaimer in mind, we can say that our review of PD literature in mathematics education and our general involvement in the field has revealed that many PD efforts begin with the developers identifying a specific goal or focus for the PD. This often occurs before the PD even has teachers identified or signed on to the PD. Because we are especially interested in PD focused on mathematics instruction, examples of orienting PD goals or aims would be promoting a certain form of mathematical discourse (Herbel-Eisenmann et al., 2017; Smith & Stein, 2011), the implementation of certain types of mathematical tasks (Santagata et al., 2010), or reasoning routines (Kelemanik et al., 2016). In the case of Leshin et al. (2024) in this Special Issue, the PD goal is the enactment of Mindset Mathematics, which includes student-centered mathematical activity around challenging mathematical tasks and also a growth mindset stance in the classroom. We feel that these aims would be transformational in nature for the vast majority of teachers, at least in the US, and we believe most of the cited authors would agree with that characterization because they openly acknowledge the profundity of what they are pursuing.

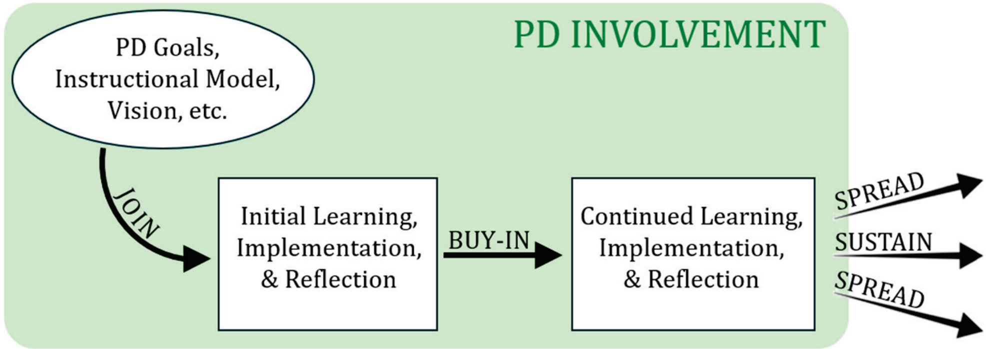

Figure 1. A depiction of transformational PD for mathematics teachers.

We are not opposed to transformational goals or visions for instruction per se, but we are somewhat pessimistic about the likelihood of success at scale when they are made the starting point and the focus of PD efforts. One fundamental concern is that the goals of transformational PD are, by definition, quite distant from current, conventional mathematics teaching practice. This distance not only means that the PD-promoted changes will be difficult to enact but also that, if enacted, the changes may trigger backlash that falls upon the teacher because the instruction is noticeably different from the expectations of students, other teachers, administrators, parents, community members, etc. (Otten et al., 2024c). This distance might also lead to a feeling of separation between the "reality-based" teachers and the "idealistic" developers or scholars (hence the depiction in Figure 1 of the PD goals of transformational PD as spatially above the realm of implementation). Arbaugh (2025) expressed similar concerns about transformational PD goals, especially with regard to feasibility for many teachers. For example, she is supportive of the Five Practices (Smith & Stein, 2011) as an excellent approach to orchestrating mathematical discussions, but she has come to view it as too ambitious for many preservice teachers or even some in-service teachers to enact, and so she proposed narrowing in on just one aspect of the Five Practices, such as spending a semester on a certain pedagogical skill incorporated into the Five Practices and then proceeding incrementally from there.

But of course, the goals of transformational PD are shared by and inspiring to some teachers who want to develop those sorts of visions or forms of instruction. They would hopefully have opportunities to join a transformational PD project, most of which do typically involve teachers joining or becoming actively involved in the PD in some manner. This joining can take many forms (e.g., enrolling in a course, attending a workshop or summer institute, joining a book study, participating in a year-long or multi-year university-sponsored project, etc.) and it is often voluntary, but also sometimes compulsory. Even with this variety in formats and opportunities to join, we have a substantial concern that many mathematics teachers will be left out. It may be a capacity limitation because transformational PD is often intensive and expensive, so perhaps there is not room to include enough teachers to reach critical mass for widespread change, and there are also limitations on the teacher side because they may not have the time, energy, interest, or support necessary to join the transformational effort. In short, even if transformational PD was 100% successful in achieving instructional change with the teachers who join the project, it would still fail to reach the many teachers who cannot or do not participate.

Unfortunately, transformational PD is not 100% successful in bringing about the intended instructional change with its participants because, even after teachers join, there remains important and challenging work to do. There is usually some form of initial learning such as a PD kick-off or an introduction to the key ideas and rationales for the work and, although there is wide variation in PD models and activities, there might be some sort of initial implementation or attempt in practice which then spurs reflection, with the reflection often guided by the PD itself. These connections to practice and implementations are represented by the square boxes lying horizontally in the visual model (Figure 1). The hope is that teachers fully buy-in to the aims and practices promoted by the PD, but because the PD is transformational and thus pursuing things distal from conventional practice, this buy-in is not assured. As mentioned above, some teachers might have been on board from the start when they joined the PD, knowing full well its goals and intentions. Other teachers may be only curious (or externally compelled) to join the PD and so it may be later through the initial learning opportunities and the initial implementations that they become convinced of the merits of the PD. But as many scholars and developers state (e.g., Kelemanik et al., 2016; Lampert et al., 2013), transformational changes are often difficult to make, and many of the ambitious practices promoted may not go well at the beginning. For these and other reasons (e.g., a sincerely different vision of mathematics instruction or a different opinion about the problems and remedies in mathematics education held by the teacher than by the developer), we feel that buy-in levels for transformational PD are not uniformly high.

Nevertheless, there are many "pockets" of success, as Arbaugh (2025) refers to them, and to achieve that success, transformational PD often involves extended interactions for teachers within the PD and multiple opportunities for implementation and reflection. Indeed, duration and ongoing professional collaboration are often criteria for effective PD (Blank & de las Alas, 2009; Desimone, 2011), or effective transformational PD, as we would amend it. Through all of this, the PD is directly involved with teachers in various ways, as represented by the green coloration that surrounds the PD goals, the initial learning and implementations, and the continued learning and implementations of the promoted instruction. PD involvement can take many different forms, such as the presentation of content or examples of ambitious instruction, the analysis of artifacts from practice through the lens of the PD goals, facilitation of discussions around the PD aims and rationales, creation of opportunities for teachers to view one another's teaching or to view and reflect upon one's own teaching, a sequence of specific trainings for certain pedagogical skills that are assembled into the broader instructional transformation, or many other PD offerings and forms of involvement. Some of our concerns in this area are that substantial and lengthy PD involvement is costly on the PD side both in terms of resources as well as time and effort. It is also challenging on the teacher side because they are often already busy or overburdened with other responsibilities. In short, transformational PD often includes many components to try to promote the difficult (though important) changes in instruction it is pursuing. And when the changes do not occur, we have noticed that many PD leaders tend to view the solution as needing more resources, more involvement between the PD and the teachers, more buy-in from school leaders, students, and parents (in addition to buy-in from the teachers), or more alignment with regard to curriculum materials or assessments. But seeking or requiring more of these things would, in our view, possibly increase the local likelihood of success but would simultaneously reduce the likelihood of the transformational PD actually leading to widespread change.

Widespread change is the final element of our model (Figure 1). The hope of transformational PD is that it will sustain with the teachers involved, despite challenging contextual realities such as teacher turnover, administrator changes, curriculum adoption cycles, shifts in district priorities, and more. But this hope is not always realized (Karsenty & Brodie, 2023; Tirosh et al., 2015). There is also a hope, perhaps even more fleeting, that the profound instructional changes so hard-won through the transformational PD will also spread to other teachers and other schools. People have written about how this spread can occur (e.g., Bartell & Aguirre, 2019; Clarke & Hollingsworth, 2002; Coburn, 2003; Timperley et al., 2014), but it does not seem to have occurred yet, even after decades of our field's best efforts. There are some important insights to be had, however, through the existing literature and through the work of Leshin et al. (2024) in this Special Issue, who explicitly studied the aspects of a transformational PD experience that teachers took up and sustained versus those aspects of the PD that were not sustained. Their PD was rooted in an intensive summer camp wherein teachers tried an ambitious model of instruction centered on Mindset Mathematics, and the authors then followed the teachers back to their regular school classrooms the next academic year to gather data. They found that teachers did, after the intensive work of the PD, take up and sustain certain instructional practices such as posing the open tasks supplied by the PD, giving collaboration time to students, and asking questions to push student reasoning. But other promoted practices were not sustained, such as explicitly including mindset messages in their teaching or making mathematical mistakes a focus of instruction. This kind of research, examining what is taken up from PD and why, is extremely valuable as we collectively reflect on the efficacy of transformational PD efforts and might help to alleviate our concern that those arrows on the right side of Figure 1, representing transformational changes to instructions that will sustain and spread, are merely hopeful rather than probable.

In summary, although we do not claim that the basic model presented here captures all instances of transformational PD, we do believe that many instances of transformational PD involve a goal or vision for instruction that is substantially different from conventional mathematics instruction and that the PD seeks teachers who will join the effort to try to enact those instructional changes. Subsequently, the teacher becomes involved with the PD for an extended period, requiring investments of time and energy to engage in various undertakings such as learning activities, the implementation of PD-promoted instruction, reflection, and more. This work is difficult, but the hope is that the changes become deep-seated and will therefore sustain for the teacher, possibly within a professional community that has formed via the PD, and then somehow the PD (or its goals and vision) will also spread to other teachers. But, as stated above and in many of the articles in this Special Issue, it is challenging to (A) reach a majority of teachers, (B) achieve the transformational changes with participating teachers, and (C) for such changes to spread beyond those directly involved in the PD. Thus, many scholars are investigating incremental approaches to PD that may avoid some of these concerns.

3. An Overview of Incremental PD Efforts Contained in This Special Issue (With Two Subtypes Identified)

Incremental PD, by definition, is more modest than transformational PD in the instructional shifts it expects from teachers. But as some of the articles in this Special Issue demonstrate, there can be a great deal of overlap between the incremental PD model and the transformational PD model, and there can be differences in the ways scholars have attempted to make their PD more incremental. Starting as in the previous section with the PD goals, we have identified two subtypes of incremental PD. The first subtype involves building PD around one piece of a larger goal or vision for mathematics instruction. In a sense, this form of PD takes a transformational goal and breaks it down into something more manageable and thus makes the PD incremental for teachers. The forms of initial learning and continued learning are similar to those discussed for transformational PD, but now the focus is more modest. We refer to this subtype as piece-and-support incremental PD because it centers on a piece of a larger (transformational or ambitious) instructional goal and provides support through robust PD involvement (see Figure 2).

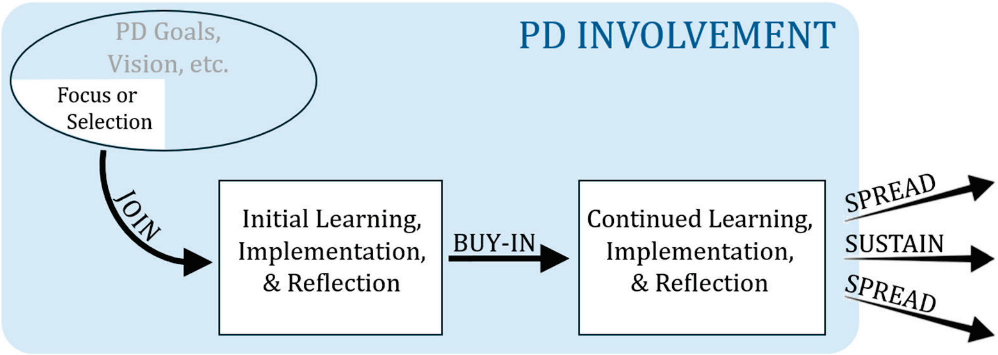

Figure 2. A depiction of piece-and-support incremental PD for mathematics teachers.

Arbaugh's (2025) example already mentioned, of spending a semester on just one aspect or precursor of Five Practices discussion orchestration, would fit in this piece-and-support incremental PD subtype. Several other detailed examples can also be found in this Special Issue. Bondurant (2024) and Litke et al. (2025) both identify equitable mathematics instruction as the ambitious, complex teaching phenomenon that is the ultimate goal of their PD efforts, but they allow the teachers in their PD to make a selection so that just one (or a small number) of simpler phenomena can be the focus of the PD. Bondurant had teachers read a book containing complex and challenging (and worthwhile!) ideas about mathematics pedagogy, but then to make their implementation and reflection more manageable, they identified just one aspect or idea from the book to focus on. Similarly, Litke et al. provided teachers with many different rubrics that elucidated various aspects of equitable instruction, and although all the rubrics are worthwhile, the PD consisted of coaches working with teachers to choose just one rubric to focus on for their own practice. In both cases, this element of teacher choice was hypothesized to increase teachers' buy-in to the PD, which may be a tangible advantage that this form of incremental PD has over transformational PD while still sharing the same end goals as transformational PD.

The two examples just cited involved equitable instruction at the secondary school level. Marzocchi et al. (2024) worked instead at the university level, but they focused on a similar overarching goal, which they called equity-minded active instruction. The PD providers offered many different specific strategies and resources related to equity-minded active instruction, but the university mathematics instructors were free to select whichever they wanted to implement. This freedom for the instructors also included the option of choosing none of the strategies or resources; PD involvement was not obligatory. Similarly, Lee et al. (2024) offered free choice to teachers about what they wanted to access from online PD resources. In this case, the broad goal of the PD was the teaching and learning of statistics and data science, with a wide range of content-based material for teachers' own knowledge enhancement as well as instructional material and insights into student tasks and thinking. Even though all teachers completed at least 20 h of module materials, there was variability in the order in which they completed modules, with fewer teachers completing a module on "Discourse" than on "What is statistics and data science?", and even then tending to complete the "Discourse" module near the end of their learning experience. As with Leshin et al.'s (2024) study, we find it interesting to look closely at what teachers take up from PD and what they shy away from, though with Lee et al.'s (2024) PD, they were allowing teachers to take just one piece of a larger offering.

Within piece-and-support incremental PD, it is not always the teachers who make the selection. For Quebec Fuentes (2025), the broad goal of the PD is to facilitate mathematically rich small-group discourse, but recognizing that this is a challenging undertaking for most teachers, Quebec Fuentes, as the designer of the S^3D approach to PD, made the choice to focus the teacher's attention and efforts onto just one small group. In this way, after building awareness of the nuances of group dynamics and also finding success with the mathematical interactions in that single group, the teacher could increment this in the future to incorporate more groups. Elliott and Roberts (2024), with the stated endpoint of robust instructional routines for student interaction and mathematical thinking, also describe their careful process of breaking down the routines into component parts and trying to design the PD in such a way that the teachers would incrementally approach the full implementation of the routines.

This work is all meritorious and we feel that those interested in transformational PD and incremental PD alike will be interested in the results of these efforts to parse transformational goals for the purpose of promoting instructional change. We also expect that some scholars will have criticisms because they will argue that some of the complex and ambitious goals can only truly be approached by embracing their entirety and their complexity, that something is lost when the goals are sliced or partitioned. These points are well worth considering, but they should also be placed into dialog with the need for higher uptake and wider spread of the instruction being promoted. And there is another discussion to be had as well, wherein a large, transformational instructional goal does not enter the model at all.

This leads us to the second subtype of incremental PD. In this version, there is no lofty goal at all; instead, the instructional foci or the suggestions for practice from the PD are modest and close to teachers' existing practice (see the oval in Figure 3, which is smaller and placed lower than the goal ovals in Figures 1 and 2). One might refer to this as modest-goal-with-support incremental PD, and it is similar to what we have advocated for in the past (Otten et al., 2022), drawing inspiration not only from Star's (2016) work but also Cortina and Višňovská's (2023) research about teaching suggestions that will actually spread, as well as from curriculum research, where shifts in curriculum need to be not-too-far from teachers' comfort zones (Choppin et al., 2018).

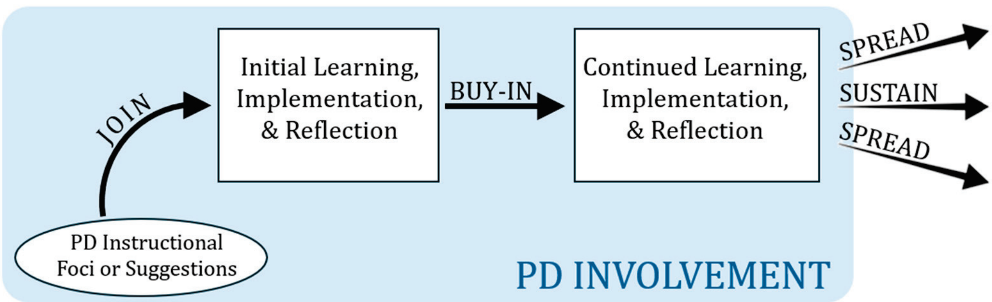

Figure 3. A depiction of modest-goal-with-support incremental PD for mathematics teachers.

Within this Special Issue, Lesseig and Hoppe (2024) made their PD modest and manageable by encouraging teachers to focus on just one facet of instruction stemming from a specific question or aspect of practice they were interested in examining (e.g., using an open task, pursuing differentiation through student choice). Although these instructional changes might be built upon in future PD sessions, such expansion was not the stated goal of the PD. In that way, each PD session could stand on its own, with teachers free to investigate different aspects of practice and individualize their learning. Similarly, teachers who experienced PD from Jansen et al. (2024) were exposed in some manner to ideas of Rough Draft Math (Jansen, 2020), and they could try just one small suggestion in their own teaching. A teacher taking up even a single instructional move of allowing students to revise their mathematical thinking would be regarded as a PD success; the instructional change was not required or expected to be part of a broader vision of mathematics instruction. Admittedly, Jansen et al.'s (2024) article in this Special Issue encompasses several different studies and thus several different versions of PD from a single post shared on social media to a single conference session or a more extensive book study. The full notions of Rough Draft Math as articulated in the book (Jansen, 2020) might be fairly transformative for many teachers, and thus some of the incremental PD may be interpreted as the piece-and-support subtype, but in other instances where a specific, small-scale rough draft strategy is being shared and there is no reference to or expectation of teachers engaging with the broader instructional notions, we view it as an example of modest-goal-with-support incremental PD. (If a teacher simply grabs a suggestion off of social media and does not have any further interactions with the developers or scholars behind the suggestion, then the lack of follow-up support may actually make it an instance of the nudge-based incremental PD described in the next section.)

Thus far, we have discussed the foci of incremental PD, which can sometimes be a purposeful portion of a larger, transformational instructional goal (piece-with-support incremental PD), or it can be a modest, small-scale instructional suggestion made on its own merits without appeal to a more transformational goal (modest-goal-with-support incremental PD). A theme that emerged is the potential role of teacher choice in establishing the foci of incremental PD. In the next section, we describe our own Practice-Driven PD model which relies on teacher choice to an even greater extent, but first we must comment on the extent and types of PD involvement in the incremental PD projects reported in this Special Issue (represented by the blue coloration in Figures 2 and 3). Readers may notice that transformational PD and the two subtypes of incremental PD already discussed (both "with-support") have similar levels of PD involvement.

Other than portions of Jansen et al.'s (2024) work, all of the PD projects reported in this Special Issue involve teachers who join the PD in some manner. A few allow for minimal participation (i.e., Marzocchi et al., 2024; Lee et al., 2024), but the teachers nevertheless receive access to the PD materials and are visibly included in some group of teachers or instructors to whom the PD is offered. The other PD models all have some substantial form of expected participation. What is that involvement and what does the PD provide to the participating teachers? For Marzocchi et al. (2024) and Lee et al. (2024), the involvement is, as just alluded to, the opportunity to explore and consume PD resources and engage in interactions facilitated within the PD platform. For Elliott and Roberts (2024) and Lesseig and Hoppe (2024), the teachers are expected to actively participate in multiple cycles of the Mathematics Studio PD format, which is a structured approach to professional collaboration and reflection that draws inspiration from Japanese Lesson Study. In other cases, the PD offering was a graduate-level course that functioned as a professional learning community led by the university scholar (Bondurant, 2024), a coaching relationship established between the teachers and the scholars to engage together in video- and rubric-based reflection cycles (Litke et al., 2025), or a collaborative series of phases of setting instructional goals, planning instruction, and reflecting on implementation (Quebec Fuentes, 2025).

Just as the form of the PD involvement varies, so too does the duration. Within several of the incremental PD efforts, the duration was entirely up to the teachers and could be very short if they wished (Jansen et al., 2024; Marzocchi et al., 2024; Lee et al., 2024), but it could also be months or longer. Commitments of one semester or multiple academic quarters were entailed in the PD of Bondurant (2024) and Quebec Fuentes (2025), and other projects lasted for a year (Litke et al., 2025) or even multiple years (Lesseig & Hoppe, 2024; Elliott & Roberts, 2024). So, it is not the forms of support nor the duration of involvement that distinguishes incremental PD from transformational PD; both approaches can have substantial levels of involvement, and therefore the concerns about the resources necessary to enact the PD or the burdens placed on already-busy teachers are not alleviated by these types of incremental PD. But it is possible that, by having more modest aims, it might make the promoted instructional changes more likely to occur within incremental PD than in transformational PD. For similar reasons, the modest aims might make it easier for teachers to sustain and subsequently spread the instructional changes to others.

This leads us to the question of what happens after the PD involvement ends. As discussed above and attested by many of the authors in this Special Issue, there is substantial evidence pointing to the lack of sustainment and lack of spread for transformational PD. But would incremental PD fare any better? Given that most of the studies in this Special Issue were reporting detailed investigations of the PD that took place, it was beyond their scope to also study the sustainment and spread. But Leshin et al. (2024) did collect some of these data following their transformational PD effort, and it is worthwhile to consider why certain practices were taken up and sustained more readily than others. We hypothesize that the promoted practices more proximate to conventional instruction (or those that offer direct replacements of things in conventional instruction) had higher uptake. For example, the open tasks provided by the PD project could be directly inserted in place of lower-level tasks already in the standard curriculum, and this might be part of the reason it had relatively strong uptake. Similarly, the question types that push student reasoning can be directly inserted in place of other questions that teachers typically ask students as they work. But mindset messages are meta-textual in a manner that might not be usual for many teachers, and dedicating classroom attention to mathematical errors might be a substantial deviation from conventional instruction that focuses exclusively on correct procedures and concepts. Lee et al. (2024) also provided some data around notions of uptake by the teachers, even if they could not study the spread to other teachers. They found that modules recommended to users were accessed by more teachers and that a few modules were accessed by noticeably fewer teachers. This finding may be a simple matter of teacher interest, time constraints, and navigational layout in the PD platform. We noticed, however, that one of the least-accessed modules of "Discourse" happens to be related to what we described above as a transformational instructional aim. Moreover, "Comparing Distributions" addresses important statistical content that many math teachers need to be prepared to teach, but science and social studies teachers in the study may have found it less important to them, so it may be a matter of providing options that resonate with the specific teachers involved.

We have more thoughts on this matter, but we invite readers to consider this phenomenon independently, free from our persuasion. Overall, we are genuinely curious about whether the sustainment and spread of incremental PD might be different than transformational PD. We have collected this and other empirical questions to be explored about incremental PD in Appendix A. Next, we share a brief description of our own approach to incremental PD, which, like modest-goal-with-support incremental PD, has modest goals as its foundation but differs in several important regards.

4. A Summary of Practice-Driven PD (With an Explanation of Its Differences from Other Incremental PD)

Building off of initial attempts to support teachers with brief videos as part of the Two-Minute Teacher's Guide (de Araujo & Otten, 2017), we have been working for several years with a team of mathematics educators to develop and study Practice-Driven Professional Development (www.PracticeDrivenPD.com). Our formal data collection involves distributing PD materials and studying classroom practice specifically with algebra teachers in the U.S., but our approach to the PD in general can be used for any grade level or mathematical content area (Otten et al., 2022). Figure 4 displays a simple representation of

Practice-Driven PD for the purpose of contrasting it with the transformational PD and also the other styles of incremental PD discussed above.

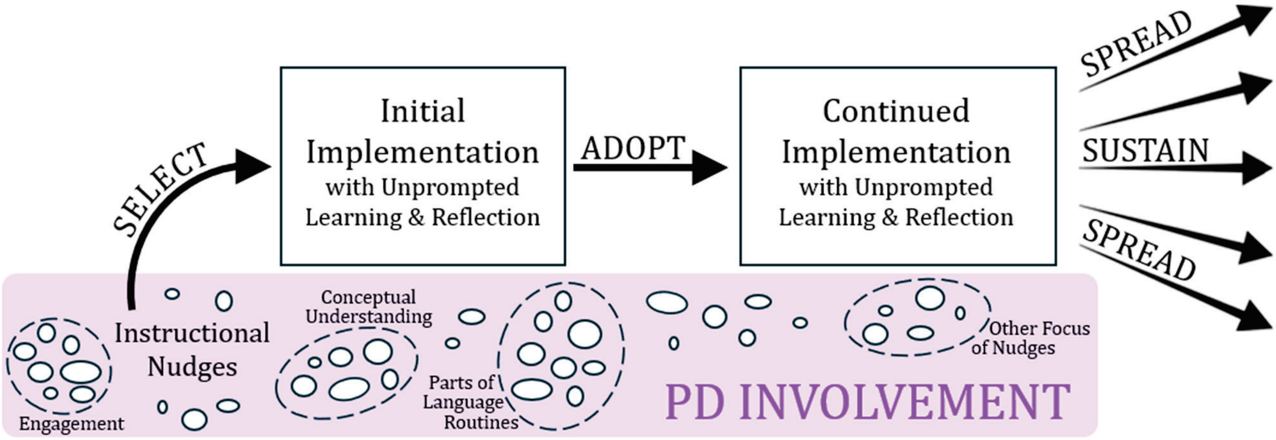

Figure 4. A depiction of the Practice-Driven PD approach to incremental PD for math teachers.

With regard to the starting point of the PD, we have no instructional vision or larger goal at play (at least not explicitly named or visible to teachers). Instead, PDPD consists of offering a wide variety of standalone instructional nudges. By instructional nudge, we mean a modest suggestion for teachers that is designed for easy implementation in their current practice (see www.PracticeDrivenPD.com for many examples, all of which can be represented on a single-page document or a short video). The nudges are designed to be (A) immediately comprehensible to teachers so that minimal time investment is needed to attempt the nudge (Cortina & Višňovská, 2023; Otten et al., 2022), (B) modest and in close proximity to conventional instructional practices so that teachers can easily identify where to implement it in their lessons and it works in a wide variety of contexts, and (C) to have a relatively high likelihood of success upon first attempt because this is influential in them continuing to employ the nudge and in potentially trying additional nudges. Although the instructional nudges can each stand alone, we have designed a few of them around (implicit) goals such as increased student engagement, opportunities for the development of conceptual understanding, and the implementation of incremental aspects of mathematical language routines (Zwiers et al., 2017). These (implicit) goals are represented by the dashed ovals in Figure 4 demarking subsets of instructional nudges, but overall, the PDPD approach involves simply making a wide variety of nudges available to teachers. We do not promote certain nudges over others and do not predetermine the "best" or "worst" nudges but instead trust the teachers to choose whichever nudges will work best for them, and we are actively studying the uptake of nudges, as revealed by those teacher choices. Thus, in PDPD, it is teacher selection that occurs as the link between the PD offering and the teachers' actual practice, not a teacher joining the PD effort as in transformational PD and also most of the incremental PD examples in this Special Issue.

In this way, the PD involvement within the PDPD approach is minimal and, as professional developers, we never actually enter into the realm of practice (thus, in Figure 4, the purple coloration is only around the instructional nudges that were designed by the PD team, whereas Figures 1–3 show PD involvement with the teachers in relation to learning, reflection, and practice). This is part of what we mean with our project title of Practice-Driven PD. The teachers are the sole drivers of their implementations of the instructional nudges, the learning or reflection that they have in relation to those nudges, and in the decision to continue implementing said nudges in practice. We do not need to pursue buy-in during extended project involvement because the only threshold to overcome is their act of selecting a nudge that interests them. We do not train them on how to implement the nudge "correctly" beyond what is on the one-page nudge document or in the brief nudge video; just as we trust the teachers with selection, we also trust the teachers with implementation and allow them freedom and agency in this regard. The absence of PD involvement, however, does not mean the role of the PD is minimal; it is still substantial and important, but it takes a completely different form than the extended supportive role, as seen in the PD models above. With PDPD, there are years of work carefully designing the instructional nudges to be clear, concise, worthwhile, and appealing to teachers (and especially appealing to teachers who are unlikely to be reached by more intensive forms of PD; see our web article (Otten et al., 2024b) for more on this important point). We also work to be responsive to teachers, so if we find that some instructional nudges are not working or are not being selected at all, we have to be willing to modify them or abandon them and create new nudges that will have higher uptake.

So, there is a great deal of effort and expertise that goes into the creation and distribution of instructional nudges, but once high-uptake nudges are created and available to teachers, the fact that PDPD involves no further investments of resources, time, or energy leads us to hypothesize that the sustainment and spread might be qualitatively and quantitatively different than the sustainment and spread of more intensive instructional changes. Because the nudges are close to conventional practice (and hence close to the current practice of many mathematics teachers) and they are compact and modest, this makes them not only easy for a teacher to implement but also easy for that teacher to share with others in their building, in their social network, or beyond. To be clear, we do not think that all of the available nudges will be taken up and spread, but we think it is plausible that some of them will. This instructional "survival of the fittest" (or Practical Selection, to extend the evolutionary analogy for incremental changes) together with inevitable tweaks and adaptations along the way can gradually lead to nudges that spread and improve mathematics instruction in modest but meaningful ways. Our hope would be that decades of this type of work would lead to more change than has occurred over the past few decades of PD efforts. And we acknowledge that teachers are already spreading small teaching ideas in this organic manner (see the popularity of Teachers Pay Teachers, or now TPT, for example), but in our view, the scholarly community of university-based mathematics educators and professional developers has not yet made concerted efforts in this way.

5. Conclusions

In this Editorial, we have drawn a broad contrast between transformational PD that pursues substantial changes in mathematics instruction and incremental PD that has more modest aims. We have also attempted to articulate some differences among the varieties of incremental PD, from piece-and-support incremental PD that focuses on a small, manageable part of what might eventually be a transformational change to modest-goal-and-support incremental PD that focuses on smaller instructional phenomena without explicit reference to a broader transformational change. We also briefly described our own Practice-Driven PD project, which has modest goals but does not entail extended PD support and so may be described as a particular attempt at nudge-based incremental PD. Because our project lacks the kind of PD support that is entailed in all the other forms of PD, it is possible some people may feel that Practice-Driven PD is not really PD at all, but we nevertheless have hypothesized that it is precisely the lower threshold for entry and the reduced investment of time and resources that might give nudge-based approaches the potential for sustainment and spread in ways that have not occurred from intensive transformational efforts.

Of course, in these pages, we have only given a brief overview of some of the dynamics at play with incremental and innovative attempts at PD for mathematics teachers. The remaining articles in this Special Issue feature a variety of starting points for PD and a variety of forms of PD support and involvement. The authors also share data and interpretations that show substantial promise for incremental approaches as something that can be successful with teachers while being mindful of all the constraints and burdens that teachers face. As you read the articles in this Special Issue, you can attend to the empirical questions the authors answer as well as additional questions that should be considered by the field. We have collected a variety of such questions in Appendix A, but we are sure there are many more that are worthwhile as we reflect on the state of PD in our field and discern a way forward. We also acknowledge a broad question: when we increment mathematics instruction, what are we incrementing toward? Transformational PD has a strength in its clarity of purpose, and at least the piece-and-support type of incremental PD may share this strength. But those engaging in modest-goal and nudge-based PD would be wise to consider their longer-term trajectory, even if specific PD efforts are purposefully small-scale. As Arbaugh (2025) cautions, there should still be orienting theories at work even for the most modest PD endeavors.

We close by thanking all of the scholars for their contributions to this Special Issue. We are truly honored to have helped bring this work to fruition and hope it spurs further reflection on PD for mathematics teachers and on the continued efforts to promote instructional improvements in mathematics classrooms. Even modest insights and small steps in the direction of improved understanding of the possibilities (and pitfalls) of incremental PD will have been worthwhile.

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Abbreviations

The following abbreviations are used in this manuscript:

PD	Professional Development
PDPD	Practice-Driven Professional Development
S^3D	Small-group, Student-to-Student Discourse

Appendix A. Empirical Questions Related to Incremental PD

Table A1. A set of questions to ponder or investigate, organized by the phase of the PD models as depicted in this article.

PD Offering and Uptake	Initial Implementation	Continued Implementation	Spread
1. What are the affordances and constraints of PD that allow broad teacher choice versus PD with a determinative focus or pre-scribed implementation? 2. What is the relationship between uptake* rates of PD-promoted actions and: a. Features of the actions? b. Characteristics of the individual teachers? c. Facets of the teachers' school context? 3. How transformative can the PD-promoted actions** be while still maintaining high levels of uptake and success for teachers?	4. What is the relationship between the level of support the PD provides for implementation and the nature of the implementation of PD-promoted actions? 5. Even when the PD-promoted actions are small and incremental, what is the variation in instructional implementation? 6. What are teachers (and students) initial reactions to implementations of the PD-promoted actions? How do the reactions of incremental changes compare to reactions at the first attempts of transformational changes?	7. What is the relationship between the level of support the PD provides over time and the nature of the sustained implementation of PD-promoted actions? 8. Which PD-promoted actions are sustained in teachers' practice and why? 9. What is the variation, refinement, or adaptation of the implementation over time? 10. What is the relationship between sustained incremental changes in instruction and various student outcomes?	11. To what extent does the PD project need to be involved (and resourced) to achieve spread of the PD-promoted actions to teachers beyond the project? 12. What PD-promoted actions (or which aspects of the PD) do teachers organically spread to others and through what forms/mediums do they spread them? 13. To what extent are teachers open to further incrementing their instructional practice? Are they open to future transformational changes in their practice?

* Uptake when the PD involves choice from a wide array of options likely consists of at least two phases, teacher initial attention (what are they immediately drawn to?) and teacher selection (what do they decide to focus on and potentially implement?). Uptake within PD that is more prescribed may involve the way in which the teacher understands the PD-promoted actions (which may not be the same as the PD leaders' understanding; Braseth, 2022) and the level of buy-in the teacher has for the PD-promoted actions. ** If the PD-promoted actions are very small, close to existing practice, and designed for immediate success, we have termed them "instructional nudges" (Otten et al., 2022). This is based on the broad psychological notion that people are more likely to do things that are easy (compared to things that are difficult) and more likely to continue doing things with which they find success (compared to those that begin unsuccessfully). Also, if the nudges are targeted toward ubiquitous practices, the teachers can make them routine (Kelemanik et al., 2016), and the value of the nudge and how to implement it will likely be readily recognized by other teachers as well, and the small scope makes them shareable. But we do not wish to make the PD-promoted actions unnecessarily small or modest; if a transformative suggestion can garner high uptake, then it is certainly worthwhile.

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Commentary

Commentary: Personal Transformations and the Possibilities of Incremental Progress in Mathematics Teacher Professional Development

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Abstract: In this commentary, I challenge the field to take seriously the idea of incremental professional development (PD) for teachers of mathematics. I begin by briefly describing the history of PD in the United States since the publication of NCTM's Curriculum and Evaluation Standards, what we have learned about effective PD through research, and then pose set of dilemmas we face in having a wide impact on the teaching of mathematics in the United States. Through reflecting on my story of incremental and transformative change as a high school mathematics teacher and 28 years as a mathematics teacher educator and scholar, I argue that the concept of incremental PD is worthy of exploration.

Keywords: professional development; teacher education; mathematics education

1. Introduction

For the entirety of my academic career, both as a graduate student and a university mathematics teacher educator, I have been driven by a passion to understand professional learning opportunities for mathematics teachers. I intentionally seek to link research and practice, engaging in what Boyer (1997) called "use-inspired" research. In my scholarship, I have identified problems of practice—which manifest themselves in the everyday work of teacher education and are inherently localized, contextual, and idiosyncratic—and designed research projects to develop and test possible solutions.

When I read the call for papers for this special issue, I was intrigued by the premise of the editors:

> We are motivated by a desire to understand how to help support teachers within their lived constraints (not some idealized goal for math pedagogy) and to investigate PD efforts that have a high likelihood for **immediate success**, even if it's **modest in scope** or **incremental in spirit** (as opposed to transformational PD efforts that require large amounts of effort or which only reach a select group of teachers).
>
> (Email communication from Samuel Otten, 30 January 2024; emphasis added)

During my 28-year tenure as a university-based mathematics teacher educator, I have often reflected on my own growth as a high school mathematics teacher (1984–1996) and subsequent work with preservice and in-service mathematics teachers (1996–2024). I have mused on what prompted me to shift from a 100% "chalk and talk" high school mathematics teacher to one whose students worked together solving problems—the type of problems that I would come to know can be classified as problems that require a high level of cognitive demand (Stein & Smith, 1998). How I started having them explore mathematical

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relationships, talk to each other about their mathematical thinking, and make mathematical connections across different representations. After reading the call, I asked myself: (a) Was my own transition as a mathematics teacher transformational or incremental (or both)? and (b) In my facilitation of PD, has my focus on transformational PD been "effective?" Did my approaches really lead to teachers transforming their mathematics instruction?

I use the word transformational here deliberately. According to the Merriam-Webster Dictionary (www.Merriam-Webster.com accessed on 10 December 2024), the definition of transformative is "causing or able to cause an important and lasting change in someone or something." To say that someone has transformed their instructional practices is to indicate that they have modified prior instructional practices by adopting or adapting them in important ways. In the fields' current policy and practice recommendations, "important ways" suggest moving away from a teacher demonstrate/student practice instructional mode to an instructional mode that supports inquiry, problem solving, and sense making (e.g., National Council of Teachers of Mathematics, 2014).

Before I explore these questions in this commentary, I want to lay some common ground by briefly describing a history of the last 3.5 decades on what we know from mathematics teacher PD research, starting with what I argue was the impetus of modern PD practices for teachers of mathematics in the U.S.

2. A Brief History of Where We Have Been

In the U.S., the publication of Curriculum and Evaluation Standards for School Mathematics (National Council of Teachers of Mathematics, 1989) changed how many mathematics educators thought about teaching and learning mathematics. Building on the problem-solving research that began in the 1970s (see Lester, 1994 for commentary on this research), the *Standards* authors' focus on mathematics as problem solving, mathematics as communication, mathematics as reasoning, and mathematical connections challenged the field to think deeply about what it means to know mathematics and teach mathematics. NCTM has subsequently published a number of standards documents (see https://www.nctm.org/standardspositions/, accessed on 20 November 2024) that have contained updated recommendations regarding the teaching, learning, and assessing of mathematical knowledge and skills. As noted by the contents of journals such as *Journal of Mathematics Teacher Education*, changes in the ways that the field thinks about PD is not unique to the U.S.

Since the publication of the 1989 *Standards*, our understanding of what constitutes effective PD for mathematics teachers has both widened and deepened (Sowder, 2007; Sztajn et al., 2017). Researchers have documented PD's impact on the growth of teacher knowledge (see Alamri et al., 2018 for a recent literature review), changes in teacher beliefs (Philipp, 2007) and visions (e.g., Munter, 2014), and changes in teaching practices (e.g., Gore et al., 2017). Fewer studies have sought to connect PD with K-12 student outcomes, although some do exist (e.g., Jacob et al., 2017).

In the U.S., funding for providing and studying mathematics teacher PD also blossomed during this time period. The National Science Foundation (NSF) began funding programs specifically targeted at supporting and studying PD, such as the Local Systemic Change (LSC) program from the decade of the 1990s. The current NSF program titled Discovery Research PreK-12 (DRK-12) program has a Teaching Strand, which

> invites proposals that advance current understanding of pre- and in-service teachers' knowledge, beliefs, and practices related to STEM content and that demonstrably enhance teaching practice. The overarching goal of the Teaching Strand is to contribute to the development of a science of teaching that addresses the complexity of how people facilitate other people's STEM learning.

(National Science Foundation, 2023, pp. 23–596)

The U.S. Department of Education has also developed programs to fund PD efforts, such as the Teacher Quality Partnership (TQP) Grants program, which aims to:

> improve student achievement, and improve the quality of prospective and new teachers by improving the preparation of prospective teachers and enhancing professional development activities for new teachers.
>
> (U. S. Department of Education, 2024)

In addition, PD programs have been funded by philanthropic foundations, such as the QUASAR (Quantitative Understandings: Amplifying Student Achievement and Reasoning) project (Silver & Stein, 1996) funded in the 1990s by a grant from the Ford Foundation. Similar increases in funding for mathematics teacher PD have also occurred across the world. For example, the European Commission's European Education Area program has several avenues of grant funding for working with all levels of education.

The field of mathematics education has invested copious time and money into providing and studying PD for teachers of mathematics. We have gained important insights into teacher learning opportunities from both small-scale PD projects (e.g., Scroggins et al., 2017) and large-scale PD projects that take a systemic approach to systemically improving teaching and learning of mathematics in a large district (e.g., Cobb & Smith, 2008) by working with various stakeholder groups. We have gained valuable tools to support quality PD. For example: (a) the Levels of Cognitive Demand framework (Stein & Smith, 1998) developed in the QUASAR project; (b) NCTM's *Principles to* Action (2014) and subsequent *Taking Action* book series which draws from the field's research findings; (c) *Five Practices for Orchestrating Productive Mathematical Discussions* (Smith & Stein, 2018); (d) materials for PD about teacher discourse moves (e.g., Herbel-Eisenmann et al., 2013); and (e) materials to support teachers to enhance reasoning and proving in their instruction (e.g., Arbaugh et al., 2018) (These are just a few of the plethora of materials available for continuing the education of a mathematics teacher across their career and does not imply any rating of materials by inclusion. A comprehensive list is, of course, out of the scope of this commentary). We have also gained valuable research tools; for example, the Visions of High-Quality Mathematics Instruction framework (VMQMI; Munter, 2014), which supports researchers' documentation of changes in teachers' instructional visions over time and emerged from work conducted in the Middle School Mathematics and the Institutional Setting of Teaching (MIST; Cobb & Smith, 2008) project. Over time, research on effective mathematics PD has also become more theoretical in nature, taking into account theories of learning, teacher learning, and change.

In the 1990s, an additional support for teacher learning arose around the development of "*Standards*-based" mathematics curricula in U.S. During that time, the NSF funded 3–4 curriculum development projects at three grade band levels (K-5, 6–8, and 9–12). The resulting curricular materials were based on the NCTM *Standards* (1989) and supported students of mathematics as they built conceptual knowledge of mathematics prior to developing procedural knowledge and skills. They focused on developing deep mathematical knowledge through problem solving, communication, reasoning, and connections. Many of the curriculum materials included well-developed teacher materials that contained implementation support for teachers. An area of research emerged from the use of these materials—mathematics curricula as educative for teachers (e.g., Drake et al., 2014).

My point here is that despite what we know about effective PD for mathematics teachers, the overabundance of PD-focused funding, and the plethora of materials for deepening their understandings of mathematics and mathematics teaching and learning, the instructional mode of teacher demonstrate/students practice remains prolific in U.S.

schools (Horizon Research, Inc., 2019). I am not at all arguing that we have not done good work in PD—we have certainly developed pockets of teaching for mathematical understanding across the U.S. But we have yet to have wide-spread influence on the teaching of mathematics.

The 2011 TIMSS (Trends in International Mathematics and Science Study) indicated that teaching of mathematics across 50 countries reflects instructional modes common to the U.S. Teachers reported that students engage in the following activities in every or almost every lesson (percentages are the international averages for fourth and eighth grade): (a) work problems (individually or with peers) with teacher guidance (55%, 55%); (b) work problems together in the whole class with direct teacher guidance (45%, 48%); (c) work problems (individually or with peers) while teacher is occupied by other tasks (16%, 14%); (d) memorizing of rules, procedures, and facts (37%, 45%); and (e) explaining their answers (62%, 60%). Additionally, eighth grade teachers report that 49% of students engage in applying facts, concepts, and procedures (Mullis et al., 2012, pp. 398–401).

A number of hypothetical and anecdotal explanations for this dilemma exist in our current discourse, and include but are not limited to the following conditions (in no order):

(a) Changing mathematical instruction practices is difficult and takes a lot of time and effort on the part of teachers and PD providers;
(b) Scaling effective PD is challenging;
(c) Much funding for PD is typically based on the development of innovative models for PD rather than expanding the impact of existing PD (we keep re-inventing the wheel);
(d) Influencing systems and cultures in which the learning of mathematics resides is especially difficult;
(e) Teacher attrition and turnover in U.S. schools causes a dilemma of continuity.

Kennedy (2016) argued that this phenomenon can also be attributed to the state of knowledge in our field: "There is little consensus about how PD works, that is, about what happens in PD, how it fosters teacher learning, and how it is expected to alter teaching practice" (p. 945). If, as I have argued thus far, we have invested much time, effort, and money on PD opportunities for mathematics teachers and yet have not been able to impact the teaching of mathematics widely in the U.S., what do we do? Where do we go from here?

3. A Brief Journey Through My Transformation as a Mathematics Teacher

Please indulge me as I share some of my story of transforming my mathematics teaching practices. I have come to understand that my transformation as a teacher started with one pivotal PD experience. Around 1990, I participated in a Woodrow Wilson mathematics PD institute. It was in that week-long institute that I learned about the TI-81 graphing calculator and subsequently began to more deeply understand the (symbolic) mathematics that I had been teaching to my algebra students. I also began to learn how to engage students in learning activities using a graphing calculator. This one experience formed the seed for my development of thinking about the nature of mathematics, learning mathematics, and my skills in teaching mathematics. I began to have my students explore mathematical relationships using a graphing calculator; for example, I asked my Algebra 2 students to link symbols, tables, and graphs in an exploration of parameters in quadratic equations—how the changes in the symbolic representation affect changes in the graphs and tables. In reflection, I consider the mode of how I was changing as incremental. I still did plenty of lecturing in my Algebra classes, but I was beginning to inch down the road to teaching mathematics for understanding. A more transformative experience happened as a result of enrolling in a mathematics masters of education (MEd) program, which occurred in my last four years of teaching high school (1992–1996). In that program, I was pushed even further about my personal mathematical knowledge, what it means to know mathematics, and how to teach so that students learned mathematics both conceptually and procedurally.

Over the four years of my MEd program, my teaching changed a little at a time (incrementally), culminating in a transformation of my methods of teaching. Each new tidbit of knowledge I was building, on top of prior knowledge, came to fruition in my classroom in ways that I could see mattered. I had drunk the Kool-Aid (so to speak) and changes in my teaching began to "snowball"—each change building on top of previous changes. A bonus was that I began to understand the mathematics I was teaching more deeply and conceptually as my students and I worked through activities that I often found in NCTM's Mathematics Teacher journal. In the summers, I was reading all I could about our burgeoning understandings about mathematics teaching and learning. My entire transformation occurred across approximately six years.

The deepening of my students' mathematical understandings was cemented for me during a pencil/paper test about working with algebraic fractions in my Algebra 2 course. Mid-test, one of my students called me over and asked me this question: "If I put the original fraction into Y1 and my simplified version into Y2 and they make the same graph, does that not show that I did the simplifying right?" I was blown away by this question. While we had studied linear and non-linear functions by connecting different representations using graphing calculators, we had not done what this student was suggesting during our study of algebraic fractions. I had to think for a minute before I answered "yes"—this was the first time I had considered graphs of algebraic fractions as tools for algebraic symbol manipulations.

These changes in my thinking, understandings, and teaching propelled me into a doctoral program in mathematics education, which I began at Indiana University Bloomington in 1996. Based on my own pivotal experiences in PD and graduate study, I knew that I wanted to provide other teachers with similar experiences and study those experiences in my upcoming career as a mathematics educator. For my dissertation study, I worked with a group of high school geometry teachers for a year in a relatively new form of PD in mathematics—study groups. I orchestrated the study groups, supported them in their classrooms, and sought to document what the teachers learned from their participation (Arbaugh, 2003; Arbaugh & Brown, 2005).

Since that time, I have facilitated a good number of PD opportunities for mathematics teachers. I have faithfully followed recommendations stemming from the research (my own and others) about effective PD. I have designed and implemented PD experiences that were long-term, practice-based, and had in-school support components. I was hoping to see major changes in the teaching of my PD participants, but for the majority of the teachers in my projects, I did not see major transformations in their teaching. On reflection, I now wonder if I expected too much too soon. I had not considered how I changed my teaching a little at a time; for PD that I was designing and implementing, I was innovating and adhering to the research.

All of this is to say, the premise of this special journal is very intriguing to me. I can envision outcomes that are more realistic and PD that is more incremental in nature. I can see this kind of PD reaching many more teachers than the field has worked with to date, particularly if good educational materials for teachers result from work in this area. I am connecting the work of these researchers about incremental change to concepts that stem from the work around pedagogies of practice and the influence of those ideas on my teaching of preservice teachers. I have documented preservice teachers' (PSTs') growth (Arbaugh et al., 2018) from engaging them in teacher education activities that utilize decompositions, representations, and approximations of practice (Grossman et al., 2009). Decompositions of Practice are components of instruction that can provide a single focus of study for preservice teachers. In mathematics education, some of those decompositions could be lesson planning, asking high-level questions, and grading equitably. Representations of Practice are the materials we use when engaging teachers in learning opportunities—videos, exemplar lesson plans, written cases, and samples of student work. Approximations of Practice are teaching opportunities for PSTs prior to them having their own classrooms—e.g., peer teaching, working with small groups of students, and supervised teaching in classrooms. I have found power in focusing more of my instruction around pedagogies of practice.

For the past 10 or so years, I have realized that educating PSTs for transformational change is just too much. I have come to believe that they cannot, for example, become proficient with Smith and Stein's (2018) Five Practices in one semester. I needed to focus on much smaller decompositions of practice. Thus, in recent years, I have focused my secondary mathematics methods students' learning on the decomposition of eliciting and responding to students' thinking (National Council of Teachers of Mathematics, 2014) for the entirety of their semester with me. I thought, "If I could just get them to understand the power of responding to student questions with 'tell me how you are thinking about this' then I could feel that I have impacted their teaching in important ways." (Of course, there was other work in the methods course, but this decomposition of practice was always front and center.) It has been exciting to see my PSTs grow over the semester in their abilities to elicit and respond to student thinking, and I believe that I have incrementally gotten them started on what could be a transformational journey towards impactful mathematics teaching.

4. A Caution and Then a Challenge for Researchers

Before I end, I have one caution for the PD facilitators and researchers who engage in this work. Those of us who have "been around a while" in this field may be thinking that this approach to PD is similar to process–product work from the 1970s. Medley (1977), in a literature review, sought to connect teacher actions (process) to student outcomes (product). He identified eight teaching actions that correlated positively with student gains: (1) working with groups; (2) classroom management; (3) time allotment; (4) questioning techniques; (5) teacher reactions; (6) attending to behavior problems; (7) teaching techniques; and (8) working with individual pupils When I was teaching high school, I was trained to use the Madeline Hunter model of instruction (Hunter, 2004), which was based in process–product work and was generic in nature (not mathematics specific), and thus was able to be implemented across the content areas.

T. Good et al. (1983) conducted perhaps the most well-known process–product research in mathematics education, which they compiled into a book titled *Active Mathematics Teaching*. In a journal article, T. L. Good and Grouws (1979) described a two-phase process–product research study. The first phase was an observation study of 40 fourth-grade teachers' actions while providing mathematics instruction. Post observations they identified,

> a group of teachers who were consistent and relatively effective or ineffective in obtaining student achievement results. To estimate teachers' effectiveness, residual gain scores were computed for their students using their pretest and post-test scores on a standardized achievement test (the student's pretest score was used as a covariate in a linear model). (p. 355)

They then sought to find correlations between teachers' actions and student outcomes for each group of teachers. Full results of this correlational study can be found in T. Good and Grouws (1975).

The second phase of the project (T. L. Good & Grouws, 1979) was to provide PD based on the six clusters of teacher actions found effective in the prior correlational study. They randomly assigned the same teachers (and some principals) from the prior study to two groups: treatment and control. The provided PD for the treatment group, gave them a comprehensive dictionary of the six clusters of teacher actions, and returned two weeks later to these teachers' classrooms to collect observational data. They then analyzed data from their pre-treatment observations vs. the post-treatment observations to see if the treatment "worked"; they also compared the student test scores across treatment and experimental groups. Full results can be found in T. L. Good and Grouws (1979) and T. Good et al. (1983).

I see two major differences in the process–product work of the 1970s and the PD work described in this special issue. First, the field's growing understanding and development of theoretical frameworks about teacher learning allow us deeper understandings of what prompts teacher learning. Second, the field has greatly expanded the development of PD that is mathematics specific. We are using teacher education materials and curricula that are meant for teachers of mathematics—materials that are different from materials in, say, PD for science teachers. I recommend that researchers in this area become familiar with the process–product work in our field and caution them to be careful not to fall into the non-theoretical nature of that work.

In closing, I am excited to see researchers who are investigating a different kind of PD for mathematics teachers. I think that this idea has "legs." I feel that this kind of PD has the potential to mitigate the research-to-practice dilemmas I described above and I look forward to seeing where this research agenda goes in the next decade. I can foresee research-based evidence of which incremental PD topics can be used for three groups of teachers on a continuum. Those who are: (a) beginning their journey of instructional change; (b) are midway through their journey of instructional change; and (c) those who have undergone transformative change over years in time. To be able to do this, researchers will need to talk (and listen) to teachers about where they are on this continuum and base their PD on where the teachers currently sit. We know about the importance of eliciting and responding to students' thinking (National Council of Teachers of Mathematics, 2014) to meet students where they are in their understandings of mathematics and help them grow from those understandings. This research agenda has the potential to meet teachers where they are in terms of their instructional understandings and practices and help them grow in their instructional practices. How exciting!! Keep calm and carry on.

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Article

Teachers' Mixed Implementation of Mindset Mathematics Practices During and After a Novel Approach to Teacher Learning

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Abstract: Supporting teachers to transfer their learnings from innovative professional development settings to the classroom is challenging. In this paper, we investigate a novel approach to teacher learning, in which teachers from seven US districts taught in a mathematics summer camp using a research-based curriculum centered on student reasoning and mindset messages. We examined the practices teachers did and did not implement in their camp and school-year classrooms, as well as the possible reasons for the greater or lesser changes in practice. Through analysis of classroom video, teacher artifacts, and teacher interviews, we found that teachers implemented several important practices in both camp and school-year classrooms, namely posing open tasks, giving students ample time to collaborate, and asking questions that pushed students to reason. Interview analysis revealed that the act of centering students' reasoning and witnessing their subsequent engagement seemed to motivate teacher uptake of these practices. At the same time, however, teachers less frequently integrated mindset messaging directly into their teaching and gave space for the exploration of students' mistakes and struggles. These findings suggest implications for innovative professional development efforts outside of the school year, as well as incremental approaches.

Keywords: teacher learning; mindset; mathematical reasoning; equity

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1. Introduction

Decades of research in mathematics education has demonstrated the need to support all students to engage in mathematics, as opportunities to learn are not equitably distributed across subgroups of students [1], and many students do not have opportunities to actively engage in mathematics [2,3]. Narratives about mathematical ability as limited to the gifted few [4,5] and disproportionately held by white and Asian males [6] permeate our society and our schools. Problematically, many teachers have lower expectations for students who are from racially marginalized backgrounds [7], have lower socioeconomic statuses [8], and/or have learning differences [9].

Researchers have found that teachers tend to teach in line with these expectations. Low income and marginalized students are less likely to be taught in ways that encourage student agency, thinking, and collaboration [2,10] or to have choice in lessons [11]. Additionally, some teachers believe that rich mathematical tasks are only appropriate for high-achieving children and limit which students have access to inquiry-based mathematics [12]. This is especially concerning given that the use of narrow questions rather than rich tasks has been shown to contribute to student underachievement, thus perpetuating this inequality [13–16].

A related area of concern in mathematics education is the over-emphasis on procedures and answer-finding rather than the development of deep and connected mathematical understanding through inquiry-based mathematics. Decades of research have communicated the importance of supporting students' development of mathematical reasoning [17–21]. In 2000, the National Council of Teachers of Mathematics (NCTM) released the *Principles and Standards for School Mathematics*, calling for an emphasis on not only procedural fluency but on conceptual understanding [22]. In 2013, the Common Core State Standards for Mathematics reinforced this message with the release of the Standards for Mathematical Practice [23].

Boaler and Greeno (2000) argued that supporting students' development of conceptual understanding goes hand in hand with supporting their agency in their learning [24]. Drawing on Holland and colleagues' (1998) notion of the "space of authoring", Boaler and Greeno (2000) noted that different mathematics classroom environments create different possibilities for student engagement, which may range from authoring their own ideas as agentic actors to passively receiving content [24,25]. Importantly, the authors found that when students are given opportunities to create, explore, and reason about their own ideas—as well as those of their peers—they can construct and own their mathematical understandings [24]. Multiple studies across grade levels have found that when students generate their own ideas and engage with those of their peers, this can lead to powerful individual and collective learning [26–28].

While there is a strong consensus from research that students need opportunities to reason, engage actively, and use their agency, less is known about effective ways of supporting teachers to make and sustain these shifts in practice. Studies of professional development in student-centered and justice-oriented pedagogies have shown the efficacy of the following structures: providing concrete lesson analysis tools [29], anchoring equity discussions in school data [30], looking closely at students' work [31], watching classroom video [32], and engaging in mathematics tasks [33]. These structures appear to support teachers to engage in rich discussions about students' thinking and about equity, and to begin to enact more student-centered teaching approaches, to some extent.

At the same time, other work has revealed that supporting teachers to instantiate (and sustain) these ideas into their practice remains a challenge [34,35] and may be impeded by teachers' ideas about student engagement and mathematics [36]. More broadly, transformational professional development, which has ambitious instructional goals and requires an intensive commitment from participating teachers, often does not gain traction beyond a small group of teachers [37], especially given the practice-based constraints [38,39] and discourses about mathematics [35,40] with which teachers must contend.

The challenge of designing effective learning opportunities for teachers, along with the need to center students' reasoning and to change ideas of who can be successful in mathematics, motivated the initiative and study that is the focus of this paper. In this paper, we share findings from our study of a novel approach to teacher learning, in which teachers from seven US school districts taught in a mathematics summer camp, the format of which seemed to encourage more teacher learning than may be typical, while leaving some important practices unlearned or unenacted.

2. Conceptual Framework: Teacher Learning Approaches

To examine teachers' learning, we draw on several approaches, all of which recognize that teaching is a complex practice, as expert teachers know how to respond, in the moment, to issues that take place at the intersection of students, content, pedagogy, and community [41,42]. The situated and relational nature of the knowledge teachers need to be effective [42–44] has led researchers to propose and study different models of teacher learning that go beyond learning from books and university courses, to learning in and from the practice of teaching [45].

Cochran-Smith and Lytle (1999) helpfully offered a framework for understanding teacher learning, proposing three forms of knowledge that can be developed in different models of teacher learning [45]. "Knowledge for practice" describes the type of knowledge that comes from sites outside of teaching, such as university departments, often used with an assumption that teachers can learn about practice outside of the act of teaching. This idea was famously challenged by Ball and Cohen (1999), who proposed that learning to teach outside of classrooms is analogous to learning to swim on a sidewalk [42].

Ball and Cohen (1999) proposed instead that teachers, even in university courses, learn in and from records of practice, such as student work, videotapes of lessons, curriculum materials, and teachers' notes [42]. This is described by Cochran-Smith and Lytle's (1999) second form of knowing, "knowledge in practice", which is the knowledge teachers learn from experience, the practical knowledge that is embedded in teaching, and teachers' reflections on that knowledge [45]. Whereas knowledge for practice centers upon knowledge shared by others, knowledge in practice is usually assumed to be knowledge that a teacher develops, sometimes referred to as the "craft knowledge" of teaching. Cochran-Smith and Lytle's (1999) third conception of knowledge, "knowledge of practice", blurs the distinction between the first two. It is knowledge that is often constructed collectively, with those involved holding critical views of education and the power entailed by it. Knowledge of practice, similar to knowledge in practice, is learned as a pedagogical act, relevant to immediate use, but, different to knowledge in practice, it is also learned through a process of theorizing. In this way, the knowledge is not bound by the situation in which it develops, and it helps to shape the "conceptual and interpretive frameworks teachers develop to make judgements, theorize practice, and connect their efforts to larger intellectual, social and political issues" [45] (p. 273).

In the field of teacher learning, there is widespread consensus that teachers should learn in and of practice. Researchers have worked to increase opportunities for such learning, including lesson study [46], video clubs [32], and teacher groups studying artifacts of students' work [31,47]. Franke and colleagues (2001) found that years after participating in a professional development program focused on records of students' mathematical thinking, teachers continued to engage in "generative growth" [48]. They found that the act of teaching—combined with opportunities for collaboration and reflection—became an ongoing learning opportunity that supported teachers to shift their practice, even after the professional development program had ended. Franke and colleagues' (2001) findings are particularly striking given the potential of environmental factors—such as lack of teacher prep time and administrative support—to limit, rather than encourage, teachers' transfer of their learnings from professional development to the classroom [38,39]. All these initiatives, as well as valuing the opportunity for learning of practice, are based on the theoretical assumption that teacher learning, like all learning, is situated within particular contexts [44], co-constructed in community with others [31,32], and accomplished through engagement with tools and artifacts, especially mathematics tasks [33].

Relatedly, researchers have reached some consensus on the content of teacher learning that is important, particularly the types of mathematics knowledge that support teaching and that help teachers to understand and formatively assess student thinking [49–52]. Additionally, studies have shown that teachers need to engage students actively, giving them opportunities to think, problem-solve, and reason, rather than passively receive knowledge [21,24,26]. The extent of the field's belief in the importance of mathematics knowledge and active student engagement is revealed by the number of times this list of effective components of teacher learning has been reproduced:

> (a) focus on specific content, (b) engage teachers in active learning and inquiry, (c) facilitate interaction and collaboration among teachers, and (d) be aligned with state standards and school goals [53,54].

This set of characteristics has been repeated often, though with variation [55]. Increasingly, mathematics educators have paid more attention to the need to support teachers in implementing culturally relevant and equitable pedagogies [29,34,56,57], which may be worth adding to this frequently cited list of effective components of teacher learning.

Although researchers have coalesced around the importance of infusing mathematics content and active learning pedagogies into professional development, there is some disagreement about whether teacher beliefs or practices should be the focus of a particular effort, as it is unclear whether beliefs influence practices or practices influence beliefs. Research has shown, however, that these two concepts are interconnected [36,58,59]. For example, Maher and colleagues (2023) found that teachers whose beliefs about mathematics education aligned with the NCTM standards were more successful in recognizing students' mathematical reasoning than those whose beliefs were less aligned [60]. The connection between teacher beliefs and pedagogy suggests that professional development efforts must attend to both teachers' practices and their perceptions of students and their engagement with mathematics.

As such, we designed, implemented, and studied a novel approach to teacher learning that sought to support teachers in shifting both their practices and their perceptions of students. Through the act of teaching in a mathematics summer camp with a research-based curriculum, teachers had opportunities to develop "knowledge of practice" [45]. In the next section, we describe the mindset mathematics pedagogy used in the camp and our approach to studying teachers' learning of it.

3. The Mindset Mathematics Approach

The "mindset mathematics" approach [5,15] has similarities with other initiatives in mathematics education, such as "ambitious math instruction", "active learning", "inquiry-based learning", and "student-centered learning." All of these pedagogies involve students engaging in cognitively challenging mathematics tasks and generating their own mathematical ideas, such that teachers can then draw on that thinking to support the class to develop new understandings. Variations of these approaches have been shown by different studies across grade levels to be effective [61–63]. What the mindset mathematics approach adds to previous initiatives is the sharing of ideas about growth mindset and the importance of times of struggle.

3.1. Mindset Messaging Through Open Tasks, Encouraging Struggle and Formative Assessment

In the mindset mathematics approach, teachers first engage students in open tasks that enable them to reason, problem-solve, and struggle. In particular, Yankelewitz and colleagues (2010) have shown that the design of the mathematics task itself can shape the forms of mathematical reasoning in which students engage [64]. As students work on the tasks, teachers highlight that students can learn and grow, and that struggle is important for learning. As such, teachers communicate mindset messages both implicitly, by providing time for students to collaborate on and discuss open tasks, and explicitly, by sharing that all students are capable and that being challenged is good, as it provides time for an important 'brain workout'. The encouragement of times of struggle, drawing from multiple research studies [65–67] and policies in mathematics education [22], is a relatively novel aspect of the mindset mathematics approach, which was emphasized in the professional development that accompanied the camp curriculum.

Steuer et al. (2018) conceptualized the importance of struggle and what they call 'positive error climates' in mathematics classrooms through eight interrelated features [65]. They propose the following:

1. Error tolerance by the teacher;
2. Irrelevance of errors in assessment;
3. Teacher support following errors;
4. Absence of negative teacher reactions;
5. Absence of negative classmate reactions;
6. Taking error risks;
7. Analysis of errors;
8. Functionality of errors for learning.

The most significant of these features for this study of teacher learning are the seventh and eighth features, which describe classrooms in which teachers spend time analyzing mistakes and errors (7) and center mistakes as starting points for learning (8). These practices can be challenging for teachers to implement, given dominant views of mathematics that emphasize right answers [35]. In their study of pre-service teachers' skills around eliciting students' thinking in response to an incorrect answer, Shaughnessy and colleagues (2020) found that teachers were more likely to elicit the students' process rather than their conceptual understanding and more likely to elicit their revised ideas rather than explore why the mistake had been made [66]. Analyzing errors and centering mistakes, though challenging, can support student learning [65,66].

The mindset mathematics approach also recommends changes to assessment, as traditional assessment practices of grading and testing often give fixed messages to students, suggesting that students are defined by a particular number or grade—undermining growth mindset messages [15]. The recommended alternative is for teachers to assess formatively, during a course, consistent with Black and Wiliam's (1998) proposed "assessment for learning" approach [68]. When assessing formatively, teachers observe and make use of information on students' understanding gained from classroom activities and discussions, to guide their teaching decisions [69]. Consequently, research shows that when formative assessment is integrated into professional development through teachers' engagement with formative assessment routines and classroom artifacts, this may lead to enhanced teacher reflection [70] and increases in student achievement [52]. With this research in mind, formative assessment routines were incorporated into the training for teachers of the summer camp, which we explain in more detail in the next section. Teachers engaged in formative assessment throughout the camp, as these routines were embedded in the summer camp curriculum, and the only test given was an end-of-intervention assessment administered by the researchers.

3.2. Studying Teachers' Learning of the Mindset Mathematics Approach

This study of teachers' learning during and after a mindset mathematics summer camp drew from the wisdom of the teacher learning studies preceding it but departed from previous work in two important ways. First, the study focuses upon a novel approach to teacher learning, by centering learning from teaching, through a curriculum that was designed by university researchers, to empower teachers to envision what is possible. In engaging with this research-based curriculum and attending trainings on it (which we describe below), teachers had the opportunity to build their understanding of the mindset mathematics approach, developing "knowledge for practice" [45]. In teaching the curriculum themselves, teachers had the opportunity to learn directly from the act of teaching, developing "knowledge in practice" [45]. Taken together, the project is captured by Cochran-Smith and Lytle's third conception, "knowledge of practice", which blurs this distinction, as the teachers learned research-based pedagogical practices through collective implementation with their colleagues, with an eye to challenging inequitable outcomes [45]. The approach was also novel in that the teaching took place outside of the regular school year, such that teachers were not constrained by district tests, mandated curricula, or pacing guides, and instead had the freedom to focus on student learning.

Second, our initiative departed from the well-circulated list of teacher learning characteristics by working to offer meaningful learning opportunities to students who may have experienced marginalization in mathematics, such as those from racially and linguistically marginalized communities [2,3] and those with special education needs [9,71,72]. The curriculum took seriously the recognition that many mathematics teachers (and students and parents) subscribe to dominant discourses about mathematics and mathematical ability [35], which dictate that students either have a math brain or they do not [4,5] and that mathematical excellence is available only to some students, particularly white or Asian males [6]. To combat these discourses, the curriculum was specifically designed to enable student thinking to be visible to teachers, to share representations of mathematicians of color, and to offer opportunities for students to see themselves as mathematicians.

In this study, we examined teachers' learning of transferable practices through the act of teaching in a mindset mathematics summer camp, which was centered around a research-based curriculum. Through analysis of classroom video, artifacts, and interviews, we explored which practices from the approach teachers did and did not implement both during and after the camp. Further, we examined teacher interview data to understand how teachers made sense of their learning, uncovering the mechanisms of the novel learning experience of teaching a summer camp. In particular, we ask the following research questions:

1. Which practices from the mindset mathematics approach did teachers implement during and after the summer camp? Which practices did they not implement?
2. How did teachers make sense of their learning of mindset mathematics practices during the summer camp?

4. Methods

4.1. Origins of the Camp

Prior to this study, researchers developed and taught a "mindset mathematics" four-week summer camp in 2015, which was designed to actively engage middle school students, with prompts and tasks that encouraged them to ask their own questions, come up with conjectures, and investigate ideas focused upon algebraic reasoning [73]. Mindset messages were not only shared with students—centralizing the ideas that all students can achieve at high levels, that struggle is an important goal, and that speed is not valued—they were infused into the teaching of content [74]. This meant that students were given tasks that gave them the opportunity to struggle, and when they found the work difficult, teachers shared that it was because their brains were working so hard. They were also given tasks that encouraged brain connections [75], as students connected numbers with visuals, physical objects, and movement. The students took a pre- and post-assessment of algebraic thinking before and after camp, which revealed that they had improved their performance by an average of 50 percent across the students, with an effect size of 0.91 standard deviation, equivalent to 2.8 years of growth [73].

As a follow-up to the 2015 study, teachers from ten school districts were invited to teach the same summer camp in their local areas in 2019, and they were given access to a range of professional development opportunities, described below. A different study investigated the student learning and achievement from the ten camps that were conducted across the US, finding that students who attended the camps significantly increased their achievement at the end of camp, as measured by MAC/MARS performance tasks, and achieved significantly higher math GPAs in their school districts in the following school year [76]. This paper reports upon the learning opportunities that the teachers received, through the act of teaching in the camp, to learn knowledge of practice in new ways [45], and to learn new teaching practices that could be used in their regular school year.

4.2. Student and Teacher Demographics

In the summer of 2019, ten districts in five US states implemented the summer camp. The districts recruited students who were diverse in terms of ethnicity, gender, and socioeconomic status, focusing on students who are Black, Latine, and/or experiencing poverty to ensure that camp attendees reflected these groups. This study shares findings from 20 teachers, across seven of those ten districts, who provided multiple sources of data. Student data for these seven districts are shown in Table 1 and teacher data for these seven districts are shown in Table 2. Teachers' experience from these seven districts ranged from first-year teachers to veteran teachers with more than 15 years of experience.

Table 1. Student demographics of participating school districts.

School District	State	Urban/Rural Classification	Total Students	% Black	% Latine	% Free/Reduced Lunch
District 1	Michigan	Suburb: large	3000	24%	3%	48%
District 2	Alaska	City: small	15,000	5%	9%	31%

Table 1. *Cont.*

School District	State	Urban/Rural Classification	Total Students	% Black	% Latine	% Free/ Reduced Lunch
District 3	Illinois	Suburb: large	20,000	5%	38%	41%
District 4	Illinois	Suburb: large	40,000	7%	54%	59%
District 5	California	City: small	5000	1%	38%	41%
District 6	California	Suburb: large	1500	1%	61%	53%
District 7	New Mexico	Rural: fringe	2000	0%	79%	62%

Table 2. Teacher demographics of participating school districts.

School District	Number of Camp Teachers	Gender		Years of Teaching Experience						
		Male	Female	0–2	3–5	6–8	9–11	12–15	15+	N/A [1]
District 1	19	6	13	3	2	2	1	3	3	5
District 2	4	1	3						4	
District 3	10		10	1	3	1		2	3	
District 4	4		4	1	1				2	
District 5	2	1	1			1				1
District 6	5	2	3							
District 7	5	1	4							

[1] N/A refers to teachers who did not report their years of teaching experience.

4.3. Support for Teachers

Different forms of support were offered to participating teachers before and during the camp (see Table 3). In terms of classroom resources, teachers were given a curriculum, mindset videos, and physical manipulatives. The curriculum included a flexible lesson plan for each day and focused on several big ideas: number sense, algebra as a tool for problem solving, generalization, and mathematics as pattern seeking [77]. A lesson plan included a "number talk" to build number flexibility and a short video with growth mindset messages. The remaining time was dedicated to instruction organized into big ideas [78], with students participating in open-ended mathematics tasks that encouraged them to engage with agency and authority [79,80], time to work in groups, and often a whole-class discussion. The tasks all had the feature of being "low floor and high ceiling"—the low floor meant that all students could access the tasks with multiple entry points, and the high ceiling meant that the tasks extended to high levels and in multiple directions/areas of mathematics [74,81,82]. In conjunction with these open tasks, the curriculum emphasized whole-class discussions in which students shared their reasoning from the task with their classmates. Various structures for discussion were offered, including students presenting their ideas to peers via group posters, or the approach of "convince yourself, convince a friend, convince a skeptic" in which students practiced justifying their thinking to peers taking on a "skeptic" perspective.

Table 3. Forms of support for teachers.

Type of Support	Specific Offerings
Classroom resources	Summer camp curriculum Mindset videos to share with students Manipulatives for math tasks
Teacher learning opportunities	Three one-hour webinars Online Mindset Mathematics class Reading material

Consequently, the summer camp curriculum did not prescribe ways to teach but instead offered teachers multiple structures for collaboration and discussion from which they could choose, and shared rich tasks that students could take to different levels. This flexibility invited teachers to act with agency and to actively iterate on the curriculum in the moment, based on their students' thinking [77]. As such, teachers had opportunities to develop their "knowledge in practice" by engaging with, implementing, and continually revising the curriculum, learning directly from their teaching [45]. Further, because many teachers co-taught their camp classes with colleagues (resulting in lower student-to-teacher ratios), they also had opportunities to collectively make sense of the mindset mathematics approach, which is critical to the co-construction of "knowledge of practice" [45].

In addition to the curriculum, teachers were given videos to share with students on mindset messaging. Physical materials and manipulatives (e.g., sugar cubes, Cuisenaire rods) were also provided to (1) remove barriers teachers might face in accessing supplies and (2) encourage students to explore different ways of representing their thinking.

To build teachers' understanding of the mindset mathematics approach, several trainings were offered. In the spring of 2019, all participating teachers were required to take part in three one-hour webinars and were offered additional learning opportunities, including reading materials and an online class. These webinars were hosted by teacher educators and involved supporting the attendees to (1) understand the mindset mathematics approach and (2) engage with some of the curriculum that would be taught during the camp. Teachers learned about growth mindset and mindset messages, the value of a struggle and mistakes-friendly culture [65], and the concept of open tasks that enable students to reason, problem-solve, and struggle. Teachers were also encouraged to take their time with the curriculum and be flexible with timing (i.e., take more time on tasks that students seem particularly excited or intrigued about). The online class offering (in addition to the webinars) showed videos of these mindset mathematics practices in action during the original offering of the summer camp and encouraged teachers to reflect on how these practices impact student reasoning, mindset, and learning. These learning opportunities not only trained teachers on how to use the curriculum but also introduced teachers to research on the mindset mathematics approach. In engaging directly with research and then enacting a research-based curriculum, teachers were encouraged to build both theoretical and practical understandings of the mindset mathematics approach. As Cochran-Smith and Lytle note, this "knowledge of practice" can help shape "conceptual and interpretive frameworks" to guide teachers' instructional decisions [45] (p. 273).

4.4. Data Sources

We investigated teachers' learning through three data sources: videos of their teaching, teacher timeline artifacts, and teacher interviews. To consider which practices of the approach teachers implemented during camp, seven classroom videos across four sites were analyzed. All teachers had been asked to record and submit a classroom video of the same task, "Painted Cube" (Figure 1). Across all districts, nine classroom videos were submitted and seven were determined to be of sufficient length and audio quality for analysis. The two excluded videos each consisted of several short clips: the first included eight clips (ranging from eight seconds to two minutes and 45 seconds) and the second included three clips (ranging from 20 seconds to two minutes). Both videos were excluded for analysis because (1) it was impossible to contextualize the individual clips in relation to each other and the broader lesson, (2) the total time recorded across the clips was less than half of the duration of each lesson, and (3) audio quality was an issue in several of the clips. For these reasons, the research team agreed that we did not have enough context or audio input to accurately code these two videos.

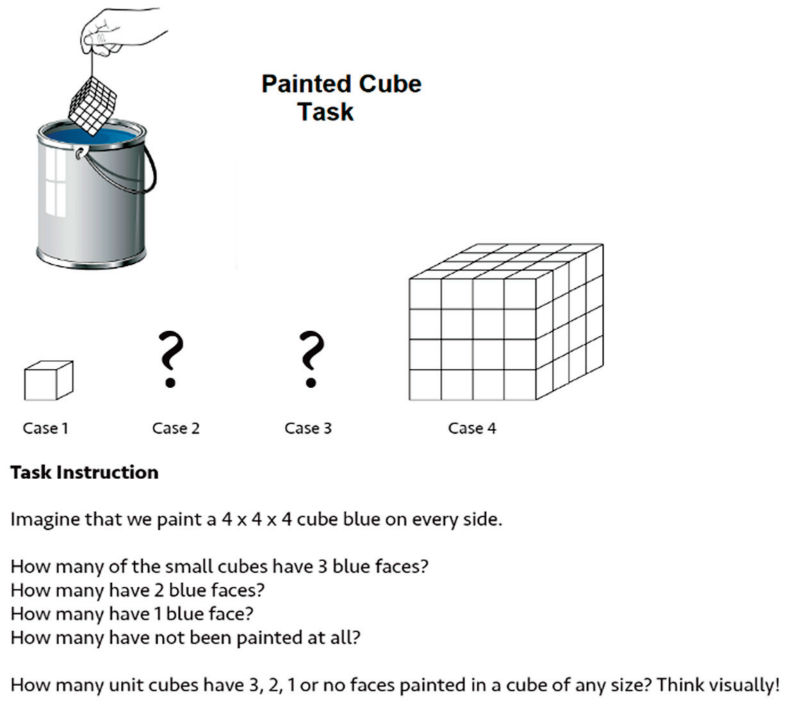


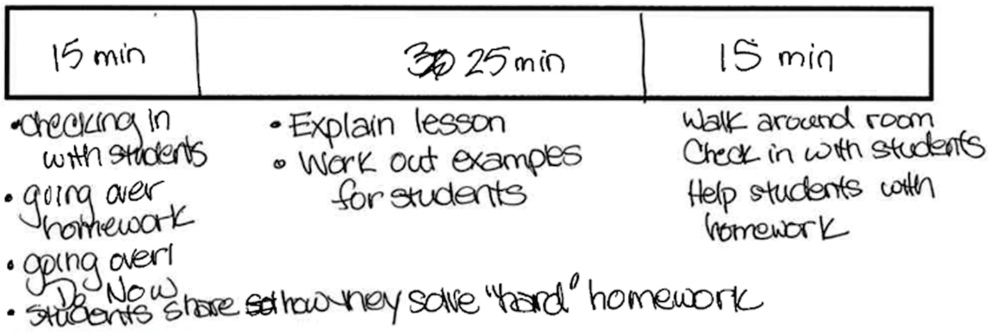
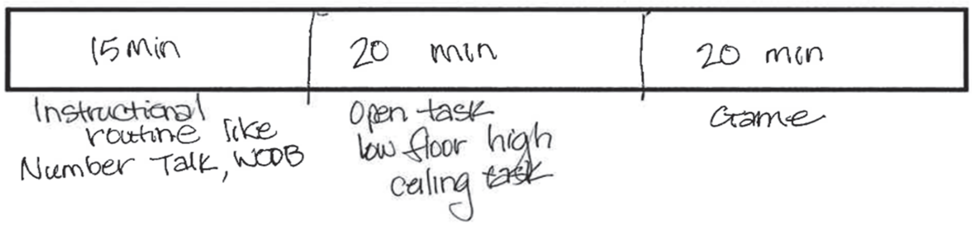
Figure 1. Painted Cube task.

The research team did, however, watch both excluded videos, noting the potential presence of several mindset mathematics teaching practices in the first one and the appearance of several less-aligned practices in the second one. While excluding these videos from our analysis may have impacted our findings, the alternative of attempting to code these videos without proper context or audio would have been problematic, as it would have required coders to make inferences beyond what was directly observable in the clips. Further, because one eliminated video likely contained mindset mathematics teaching practices and the other seemed to contain less-aligned practices, we argue that these two videos were likely to be similar to the other seven videos in terms of which practices were implemented. We expand on these limitations in our discussion section.

To consider teachers' shifts in their practice from before camp to after camp, teachers submitted timelines or logs of the typical set of practices and routines they would conduct during class time. The goal of the timelines was to understand how teachers structured their class time and what kinds of activities they engaged in with students, both before and after teaching in the summer camp. An example of a pre-and post-timeline (from the same teacher) is shown in Table 4. Pre-timelines were collected in spring 2019 and post-timelines were collected in fall 2019. Thirty-eight teachers completed at least one timeline, with 28 teachers completing both pre- and post-timelines.

To dive more deeply into and make sense of teachers' uptake of practices, researchers conducted semi-structured interviews [83] with teachers during the 2019–2020 school year, following the implementation of the camp. All participating teachers were invited for interviews. The 20 teachers who accepted the invitation to be interviewed came from seven districts, representing a range of camps that differed in their achievement impact, based on MARS effect sizes [76]. During the interview, teachers were asked about their experiences teaching in the summer camp (i.e., how they used the curriculum, how their students engaged in it) and their experiences teaching in the current school year (i.e., which practices, if any, they were currently using and how they were using them). These interviews lasted approximately 45 min, were conducted and recorded via Zoom, and were transcribed.

Table 4. Pre- and post-timeline example.

Pre-Timeline

Post-Timeline


4.5. Analytic Approach

4.5.1. Video Analysis

The videos were analyzed in two different ways. Researchers created content logs [84] of approximately seven hours of video, outlining the events on each video and conducting a time analysis of how many minutes were spent on each segment of the lesson (i.e., task launch, work time, and whole-class discussion) for each teacher and across the teachers. One of the seven videos was excluded from the time analysis because it only captured one segment of the lesson.

In the second form of analysis, researchers examined the seven videos using the Mindset Mathematics Teaching Guide (MMTG) (https://www.youcubed.org/mathematical-mindset-teaching-guide-teaching-video-and-additional-resources/) as a tool for coding classroom practice. An initial video was selected to consider in depth, based on the teacher's implementation of several mindset mathematics teaching practices, which was determined during the content logging process. A team of three researchers then re-watched this video, independently identifying 7–10 "critical moments" in which one of the mindset mathematics teaching practice categories (Growth Mindset Culture, Nature of Mathematics, Challenge and Struggle, Connections and Collaboration) was enacted. The fifth category, Assessment, was excluded because it was not feasible to identify the teacher's range of assessment practices in one lesson. After discussing these moments and the extent to which each aligned with the proposed practice category, the combined critical moments were developed into a list of indicators for each dimension of each practice category. This initial draft was then tested on two contrasting cases from two different sites, for which researchers identified evidence that either validated an indicator or suggested a need to refine an indicator (e.g., re-wording, clarifying, adding). These pieces of evidence were discussed until consensus was reached, which led to refinement of the indicators. These revised indicators (Table 5) were then used to code the remainder of the data set, after which the team constructed visuals summarizing the distribution of practices across teachers. While "indicator" was a useful term in the context of our analysis, we refer to the indicators as "micro practices" throughout our findings to emphasize the ways in which these micro teaching moves are nested within the broader teaching practices reported in timelines and interviews.

Table 5. Video analysis tool.

Practice Category	MMTG Dimensions	Indicators (Micro Practices)
1.Growth Mindset Culture	1A. Mindset messages	Teacher gives explicit messages about brain growth, challenges, struggle, etc.
	1B. Praising the learning process	Teacher provides space and time for students to (i) grapple with the task on their own or with peers and (ii) to share their thinking with each other. Teacher elicits, engages with, and praises student thinking and ideas.
	1C. Students' mindsets	(i) Student shares their thinking even if it differs from the rest of the class; (ii) multiple students volunteer to share their answers; (iii) students come up to the board to physically model their ideas; (iv) students feel the freedom and ownership of the physical space to share their ideas (e.g., going to the board or standing up, without asking for permission).
2. Nature of Mathematics	2A. Open task (task as written)	Does the task allow for multiple approaches?
	2B. Reasoning and multiple perspectives	(i) Students approach the task in multiple ways (e.g., student work shows different approaches, student participation shares different ideas); (ii) these various approaches are given attention, valued, and explored by the teacher (who makes space for students to do so too); (iii) students are given access to a range of resources and materials that afford multiple approaches; (iv) students are expected and explicitly invited to bring multiple ideas to the task and to justify or reason through their ideas (in writing and/or verbally).
	2C. Depth over speed	(i) Teacher encourages and makes time for students to be curious about the task and their peers' ideas; (ii) teacher encourages the class to come to a consensus via justification (convincing the class), rather than focus on the answer; (iii) consensus comes from the students, not the teacher.
3. Challenge & Struggle	3A. Mistakes	Space and time are provided for students' thinking to evolve, without the undue pressure of a finished correct answer. Mistakes are valued and explored.
	3B. Struggle and persistence	Teacher provides space and time for students to (i) grapple with the task on their own or with peers and (ii) to share their thinking with each other. Teacher elicits, recognizes, and celebrates students' struggles.
	3C. Questioning	At various times in the lesson, teacher asks deep thinking questions that (i) center students' current thinking, such as, "What do you notice?" "What do you wonder?" "What are you trying to figure out?", before pushing them in new directions; (ii) encourage multiple ways of thinking and require justification, such as, "Do you agree with _____?" "Does anyone else think it's something different than ___?" "How can you prove it?" "How do you know?"
4. Connections & Collaborations	4A. Mathematical connections	(i) Students have the resources and time to try out different methods; (ii) in class discussion or small groups, there are connections being made between methods and representations.
	4B. Connecting in small groups	(i) Students collaborate and build off each other's ideas in small groups; (ii) all students within a group are involved in the task.
	4C. Connecting as a whole class	Students talk directly to each other in math discussions, instead of relying on the teacher to mediate.

4.5.2. Teacher Timeline Analysis

To consider potential shifts in teachers' practice from before camp to after camp, timelines from the 38 teachers who had returned at least one timeline were analyzed. Four researchers first open coded a subset of timelines and generated a list of reported practices, which the team then consolidated into a codebook of practices (see Table 6) and applied to all the timeline data. Practices were chosen as the unit of analysis, which resulted in a total count of n = 126 individual reports of practice in the pre-timelines and n = 104 individual reports of practice in the post-timelines. Researchers then constructed visuals to compare the distribution of reported practices in the pre-timelines to those in the post-timelines.

Table 6. Descriptions of practices reported in teacher timelines.

Practice	Description
	Practices Less Aligned with MM
Review/assign homework	Teacher goes over answers to students' homework from the night before or assigns students to work on their homework during class time.
Direct instruction	Teacher introduces and/or explains new content via direct instruction or lecture with limited or no student discourse.
Practice	Teacher assigns students to work on practice problems. Sometimes referred to as the last part of the "I do, we do, you do" style of instruction, in which students independently work on sets of short problems at their desks.
	Practices More Aligned with MM
Mindset messages	Teacher explicitly communicates that all students can learn and grow and that struggle is key to learning and brain growth.
Classroom discussion	Teacher facilitates a whole-class discussion, during which multiple students share their mathematical reasoning and ideas with the class.
Pose an open task	Teacher poses an open, low-floor, high-ceiling task for students to investigate during class. Sometimes referred to as the first part of the "launch, explore, summarize" style of instruction, in which the teacher launches an open task for students to then explore.
Number talk	Teacher facilitates a number talk, a classroom routine in which teachers pose a problem for students to solve mentally and then facilitate a discussion in which multiple students share their strategies while the teacher scribes it on the whiteboard. Because of their relatively short length (approximately 15–20 min), number talks are often used as warm-ups.

4.5.3. Teacher Interview Analysis

To analyze the interview data, a team of three researchers open coded the 20 transcripts [85], constructed a codebook based on emergent themes and a priori codes [86] from the MMTG, and refined and validated the codebook on a subset of interviews through a process of adjudication. Researchers applied the revised codebook to the entirety of the data set and then analyzed code application and co-occurrence across the coded data, selecting the four highest codes for deeper theme analysis. These themes were considered alongside the timeline and video analyses, which enabled triangulation of several key findings.

5. Findings

Multiple forms of data showed that teachers implemented several practices of the mindset mathematics approach in both their summer camp and school-year classrooms. While video and timeline data revealed teachers' implementation of these practices, interview data triangulated this finding and clarified how teachers made sense of these shifts themselves, highlighting the importance of centering students' reasoning and witnessing their high engagement. At the same time, however, other practices proved challenging for teachers to take up in both settings, as the following sections will share.

5.1. Implementing Practices of the Mindset Mathematics Approach

5.1.1. Practices Implemented During Camp

Applying the video analysis tool to the seven videos revealed that teachers implemented some of the micro practices at high levels and others at lower levels. The task that was the subject of analysis (see Figure 1, Painted Cube) prompted students to consider the faces of the small $1 \times 1 \times 1$ cubes that comprise a $3 \times 3 \times 3$ cube. The task is "low-floor" in that all students can build with cubes and think about patterns and is also "high-ceiling", as the upper ends of the task involve forming different expressions, linear, quadratic, and cubic. Figure 2 shows teachers' observation scores using the video analysis tool.

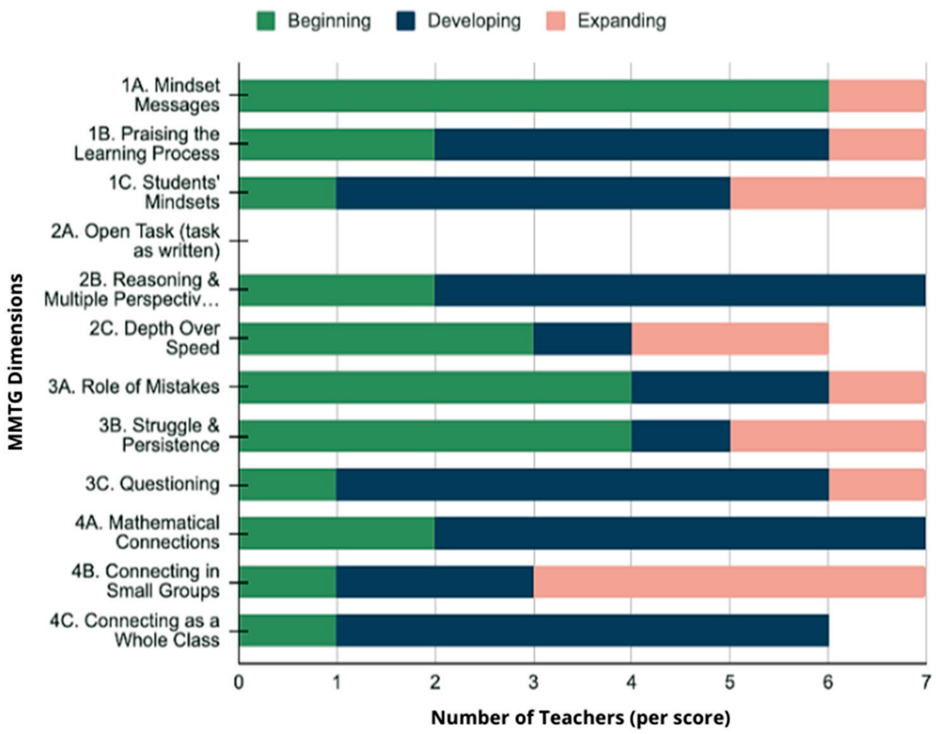


Figure 2. Observation scores of summer camp teachers for Painted Cube task ($n = 7$). *Note*: Dimension 2A was not assessed, given that all teachers were implementing the same task. Dimensions 2C and 4C were not possible to assess in one video due to its length.

As shown in Figure 2, the majority of teachers scored "developing" or "expanding" on the following dimensions: praising the learning process (1B), students' mindsets (1C), reasoning and multiple perspectives (2B), teachers' questioning (3C), making mathematical connections (4C), connecting in small groups (4B), and connecting as a whole class (4C). Below, we discuss each of these dimensions with evidence from the videos.

Across the videos, teachers explicitly encouraged multiple perspectives (2B) in several ways. First, all teachers launched the task by inviting students to build different-sized cubes using sugar cubes—beginning with a $3 \times 3 \times 3$ cube and then extending to a $4 \times 4 \times 4$ cube—and to engage in three-dimensional visualization and drawing. To help them draw different-sized cubes, students were provided notebooks with squared paper. These resources enabled students to experience math physically, see it visually, and think about generalization. Throughout the lesson, teachers offered students opportunities to experience these ideas in multidimensional ways: they saw a 2-D representation of the cube, built a 3-D model, drew different sized cubes, collected and recorded patterns, organized their thinking, discussed ideas with each other, and considered generalization of different-sized cubes.

To praise the learning process (1B), teachers not only made explicit comments like, “Thank you for persevering with us”, but put that statement into action by giving students significant time to grapple with the task in their groups. Time analysis showed that teachers launched the task for approximately five minutes on average and then allowed students to grapple with the task in groups of two to four students—building cubes, collaborating with peers, and recording in their journals or on chart paper—for an average of 53 min. Nearly an hour is a significant amount of time for students to work in small groups, talking with each other and physically constructing different-sized cubes using the sugar cubes (4B). As students worked, teachers supported students to connect in small groups (4B) by circulating and asking questions (3C) like, “Do you agree with your partner?” and “Can you prove it to her? Show her how you know it is eight”.

To support students in finding, extending, and reasoning about patterns (2B), teachers in six out of seven classrooms created a table on the whiteboard to document the number of cubes within each type of cube that would have each amount of their faces shaded. This table, as written in a student’s journal, is shown in Figure 3.

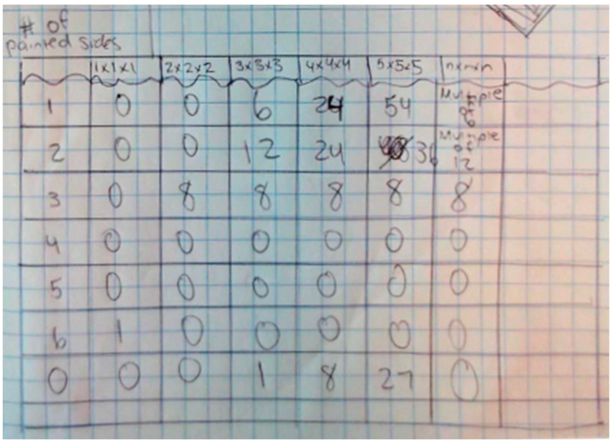


Figure 3. Student’s journal during Painted Cube task.

After most students had moved beyond the original question to work on the $4 \times 4 \times 4$ cube or generalized even further, teachers facilitated whole-class discussions (4C) to synthesize and connect the ideas from student groups (4B). These discussions were noted in five of the seven videos and lasted approximately nine minutes on average. In these discussions, teachers asked questions that pushed students to justify their thinking (3C), such as, “How do you know it is one?” and “Does anyone else think it is something different than. . .?” Teachers also asked questions to support students to make sense of and connect each other’s ideas (4C), such as asking students to predict what a $5 \times 5 \times 5$ cube would look like based on the patterns discussed for $3 \times 3 \times 3$ cubes and $4 \times 4 \times 4$ cubes.

Across the videos, students displayed positive mindsets (1C) by sharing their thinking even if it differed from previous answers given and coming up to the whiteboard to physically model their ideas. For example, in one lesson, four different students came up to the board to draw their cube and write and explain their thinking. These actions suggest a sense of student agency and a comfort with sharing their ideas with each other.

At the same time, as shown in Figure 2, teachers implemented some of the micro practices at a lower level: mindset messages (1A), role of mistakes (3A), and supporting struggle and persistence (3B). We return to these practices later in this section.

5.1.2. Practices Implemented After Camp

Timeline and interview data revealed that not only did teachers implement many mindset mathematics practices during the camp, but that they reported using many of these practices in their school-year classrooms after teaching in the camp.

Figure 4 shows the practices teachers reported using in their school classrooms before they taught the summer camp (pre-camp) and the practices they reported using in their school classrooms after teaching the camp (post-camp). Note that the unit of analysis displayed in this figure is practice, rather than teacher, such that 126 counts of reported practice were recorded in the set of 32 teachers' pre-camp timelines and 104 counts of reported practice were recorded in the set of 34 teachers' post-camp timelines. To allow for comparison of pre-camp and post-camp reports of practice, each practice is displayed as a percentage of the whole set of reported practices for that timeline.

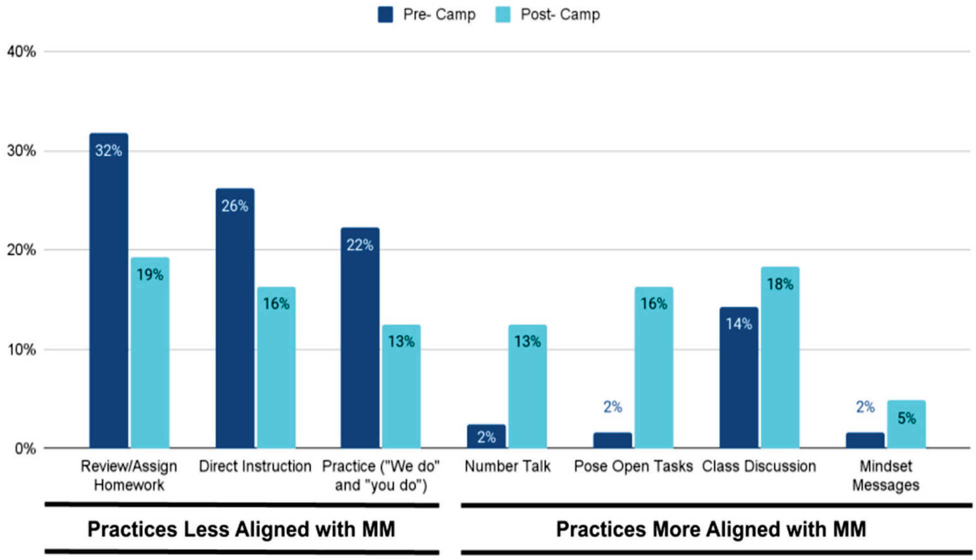


Figure 4. Percentage of reported practices in pre-camp and post-camp timelines. *Note*: The unit of analysis displayed in this figure is practice, rather than teacher, such that 126 counts of reported practice were recorded in the set of 32 teachers' pre-camp timelines and 104 counts of reported practice were recorded in the set of 34 teachers' post-camp timelines.

As shown in Figure 4, the teachers reported that they used more open tasks, more number talks, more classroom discussions, and shared more mindset messages in their post-camp timelines. Consequently, their reports showed a decrease in reported time spent reviewing homework, introducing content through direct instruction, and engaging students in individual practice time. Figure 4 suggests that the teachers took up new practices during their teaching of the summer camp that they continued to use once they returned to their regular classroom settings.

Interview data supported the timeline findings, as teachers shared more details of how they implemented a mindset mathematics approach once they returned to their regular classrooms. Of the 20 teachers interviewed, 15 returned to elementary and/or secondary mathematics classrooms in the academic year following the camp, while the remaining five took on roles as coaches (three people) or taught other subjects (two people). In this section, we focus on the 15 educators who taught mathematics in the following school year, though it is worth noting that coaches discussed working with teachers on the mindset mathematics practices and non-mathematics teachers reflected on using growth mindset messages in their classrooms.

All 15 of these teachers discussed implementing one or more mindset mathematics practices in their school-year classrooms. The practice with the highest uptake was posing open tasks, as 10 of the 15 teachers shared that they had implemented open tasks from the summer curriculum (and beyond) during the school year. For example, Laura (all teacher names are pseudonyms) noted that she regularly implements open tasks during a specific portion of her math block:

> We have a half hour "WIN" time, which is "What I Need", and that's where they absolutely have to differentiate if they haven't already in their math time. So that was a great place to incorporate some of these tasks, especially since, yes, they're getting the practice underneath, but when you challenge them with "Find the Pattern" or "Find the Relationships", they keep going and they keep going back and back and back until they figure it out. (Laura)

In this quote, Laura noted the affordances of open tasks in offering students an opportunity to practice discrete skills, to experience intellectual challenge, and to build persistence, simultaneously.

While Laura discussed implementing open tasks in the elementary grades, Richard shared that he used open tasks with his high school students:

> I think it was called "[Curved] Shapes Task", but it's basically giving them...You give them different regions and they have to try to figure out a way to find the area under a curve, which is the intro to Riemann sums, second semester calculus. So it was cool because they kinda came up with their own ways to approximate the area under the curve before we actually did Riemann sums and now we're starting to do some calculus methods with anti-derivatives. But it was really powerful and it was trying to do some of the stuff that I did over the summer. (Richard)

Here, Richard appreciated the way that this open task supported his students to construct their own methods for approximating the area under a curve, eliciting student-generated ideas from which he could then build.

In addition to using open tasks across grade levels, teachers described implementing student-centered participation structures that are embedded in the mindset mathematics approach. For example, nine teachers reported their use of classroom discussions and/or small-group collaboration, which created opportunities for students to interact with each other and to explain their reasoning. The same teachers reported that their students were engaging in productive struggle and generating their own ways of solving problems, as Laura and Richard both described above. Nate explained the shift in his pedagogical approach:

> So we take that conversation, we take those underlying messages in math, and then we teach the content, and we find that the kids, I've been finding anyway, using it quite a bit, that the kids are more prepared for those curve balls. They're thinking more about what's happening, 'cause they have some just genuine understanding of that content versus, we're just gonna go over this worksheet and we're gonna do several example problems. . .So it's been really, really kind of a wonderful experience this year. (Nate)

Here, Nate noted that his shift away from direct instruction and worksheets to tasks and discussions supported students to construct their own knowledge and to think critically.

Finally, teachers noted implementing instructional strategies and messaging from camp in their school-year classrooms. Eight of the teachers shared that they were using manipulatives and games from the summer in their classrooms. Additionally, six of the teachers noted that they were implementing specific routines from camp, such as number talks (as Nate mentioned above), "Notice and Wonder", and "Which One Doesn't Belong?". Further, ten teachers discussed sharing messages about growth mindset, mistakes, and struggle with their students, through watching videos, prompting students to reflect in writing, or facilitating discussions. Taken together, analysis of video, interview, and timeline data revealed that teachers implemented many mindset mathematics practices during and after the summer camp.

5.2. Making Sense: Centering Students' Reasoning and Witnessing Engagement

When sharing their implementation of open tasks, student-centered pedagogies, instructional routines, and/or mindset messages in their school-year classrooms, many teachers also self-reflected on their journey into making these shifts. Specifically, teachers discussed the ways in which implementing these practices enabled them to elicit and draw on students' reasoning in ways that promoted high student engagement. Witnessing this engagement supported teachers to see their students in new ways, motivating them to continue to try out these new approaches.

5.2.1. Centering Students' Reasoning

One of the most frequent codes that emerged from the interviews was that of shifting the cognitive load to students. Sixteen of the 20 teachers interviewed discussed students' reasoning, and they described making a pedagogical shift from guiding the students to mathematical answers to asking them questions to further their thinking, as Nate noted:

> A lot of the time, traditionally, we teach, and then have them [students] learn our method, and then explain back to us what our method was. This takes a little bit of a non-traditional approach, a better approach in my opinion... [to allow] the kids to form meaning, and from that meaning, we can kind of guide their learning. Because they might be noticing things that are true, but maybe not traditional, or they might be noticing things in a different way that might impact their education better. (Nate)

Here Nate explained that by supporting students to generate their own noticings, rather than memorize traditional approaches to problems, students could construct their own understandings. Nate also contrasted direct instruction with the approach of the summer camp, which was to engage students in tasks before teaching methods, allowing new strategies and ideas to emerge from students and from whole-class discussions.

5.2.2. Witnessing Engagement

Analysis of interviews revealed that as teachers shifted the cognitive load to students, they witnessed high engagement. Seventeen of the 20 teachers shared that they were excited to see students from various subgroups engaged, excelling, and developing confidence in themselves during the camp. These teachers reported that the students' motivation came from the tasks in which they were deeply engaged, as Laura shared:

> They want to do it, they don't want to stop. And some of the smaller activities that you gave us were "Close to 100" or "Tic Tac Toe Products." I've never seen fifth graders wanna practice multiplication...To see the kids say, "No, we don't wanna go to recess. Can we keep playing?" Like, "We'll play after lunch, okay?" That's super exciting. Because when they have that drive, they're going to succeed. (Laura)

Laura's surprise at students' engagement when they were able to practice multiplication facts through a conceptual game, rather than worksheets, was shared by many teachers.

A subset of teachers specifically expressed surprise at seeing students typically labeled as "low-achieving" reasoning at high-levels and making mathematical connections during camp. Teachers spoke of the "incredible" engagement of students who had not been previously successful in classrooms, as Theresa noted:

> We're drawing from kids on an IEP, native, migrant, homeless...And so, I would say kids a lot of times maybe get labeled as not always being the greatest math students ...But I think we've seen the biggest gain in our kids that were on—it's usually labeled as not great math students, on an IEP for math, and then they are able to do grade level math and make these connections and realize anybody is capable, right? (Theresa)

Here, Theresa shared that students who had been labeled as not "the greatest math students"—via IEP labels and other marginalized identities—were able to not only complete the mathematics but also, importantly, realize their own capabilities.

Similarly, other teachers described the learning of students who had previously been unsuccessful, which they witnessed in the post-assessment, as Nate shared:

> One of the most powerful things for me was during our summer program, I had a student who really, really struggled. We had the entrance exam for her in the [research center] program and she did not do very well at all, I mean very close to zero. What I found was by the end of the summer, she was more confident to answer those questions, she had no fear about this test, she had no remorse about this test, she put answers on paper, she thought about nontraditional ways, she put a lot more time and energy into it and she did exceptional on it regarding her first score. She went from zero to a passing score, which for somebody like that is a really, really important thing, it builds that confidence huge. So I can just... I'll never forget this one student who really had no way to access that information when she came into the camp, but only five weeks later, she could develop that into some really, really solid thinking. And she wasn't necessarily always right, but you could see her thinking progress, and that was a beautiful thing. (Nate)

In this quote, Nate noted the increased confidence and achievement of a previously low-achieving student through the camp's emphasis on thinking and reasoning.

Some teachers expressed surprise that previously high-achieving students who were usually bored in class became highly motivated. Richard shared that one of his students he considered to be "gifted" was usually bored and uninterested in his regular math class, yet his engagement in the summer camp was so noteworthy that he contacted the student's parents to share his changed motivation:

> And there was another kid too. So, so unbelievably gifted in math and he would...Same thing, I think he was bored, I think his thing was more that he was bored with math, and when they got together in groups and they were doing the activities, like, he was really shining. It was incredible. I emailed his parents, and I told them how impressed I was and they were shocked by how engaged he was and how big of an asset he was to the summer program. Yeah, I could keep going, there's so many students that really shined with that. (Richard)

Like Richard, many teachers reported that students from varied academic backgrounds improved their achievement significantly, which was confirmed by student data [76]. Indeed, the success of the summer teaching to mixed-achievement groups of students disrupts a widespread belief that students have to be divided into tracks in order to be appropriately challenged or supported [87].

Importantly, teachers' excitement about students' engagement and capabilities appeared to be linked to their uptake of these practices in the following school year. Eight of the 15 math teachers reported that not only did seeing their students' high engagement during the summer camp "surprise" them, but also that seeing the approach in action in their own teaching shifted their sense of what was possible in their teaching moving forward. It was the witnessing of engagement and learning for such a broad range of students—which was not typical and therefore a surprise for some—that seemed to prompt teachers' uptake of these practices in their school-year classrooms.

5.3. Struggling with Struggle: Practices Implemented Less Frequently

In interviews, one of the practices that teachers discussed implementing was that of sharing growth mindset messages with students. Video analysis, however, revealed a complicated picture. As shown in Figure 2, the following micro practices were implemented at lower levels, with the majority of teachers falling in the "beginning" level: mindset messages (1A), role of mistakes (3A), and supporting struggle and persistence (3B). Although teachers were likely sharing growth mindset messages in conjunction with showing mindset videos and prompting students to reflect on them (as described in interviews), these messages appeared to be rarely integrated directly into mathematics tasks, as was the case in the Painted Cube lessons.

As noted earlier, Steuer et al. (2018) conceptualized positive error climates in mathematics classrooms as the interrelation of eight classroom features [65]. Although many of the features (e.g., error tolerance, teacher support following errors, absence of negative teacher reactions) were observed in the analyzed videos, two features were notably absent: analyzing student errors and using them as a starting point for learning. Despite positive overall messaging to students, when working inside mathematics problems most teachers in the videos steered students to correct answers if they were incorrect, rather than facilitating collective sense-making of mistakes.

Video analysis showed that mistakes (3A) were rarely explored or valued during small group work-time and whole-class discussion. For example, one teacher said to a student as they were circulating to their small group that they should fix their mistake so "it doesn't throw you off". Similarly, although students were given ample time to explore the task, at times teachers' interventions when circulating to small groups interfered with a potentially productive struggle (3B). For example, one teacher directly instructed students who were working in a group how to paint the bottom of the sugar cube and then counted the cubes for the students. To consider the challenges inherent in giving space for struggle and cultivating respect for mistakes, we discuss one classroom in detail below.

5.3.1. An Example: Adriana's Lesson

Adriana's lesson offers a representative example from the video data set of a teacher implementing some of the micro practices, but not others. In her launch of the task, Adriana encouraged students to take the time to think before sharing, saying: "I want you to think first, visualize in your mind, hands down" (1B). While students were working on the task, Adriana circulated, telling one group who shared an answer with her that they needed to "show it to me, prove it to me" and asking another group probing questions, such as, "what did you use?" and "tell me your strategy" (2B). For their part, some students in the class physically built $3 \times 3 \times 3$ cubes with their partners during the work time and eagerly raised their hands at the beginning of the whole-class discussion (1C, 4B, 4C). A poster in the classroom read, "Mistakes Allow Teaching to Happen" (3A). As such, Adriana scored "developing" on these indicators, offering students opportunities to reason throughout the lesson.

In other moments in the lesson, however, Adriana steered students to the correct answer, rather than allowing time and space to explore mistakes (3A). When students called out multiple answers during the discussion, Adriana called on the student who was correct and asked them to explain their thinking, without addressing or returning to those who shared incorrect answers. Adriana then validated that student's answer, saying, "okay it's eight", rather than asking other students in the class what they thought of the shared answer. Later in the discussion, Adriana rephrased the thinking of the student who was correct and praised them with "good job". Multiple times throughout the discussion, Adriana asked students with the right answers to explain their thinking and wrote down their answers on the whiteboard as they shared and did neither when students shared incorrect answers.

Similarly, while circulating, Adriana intervened as students worked in ways that may have limited a potentially productive struggle (3B). With one group of students, she offered explicit scaffolding, telling them to "color it" and pointing to which faces of the small cubes should be colored. While this kind of direct instruction may be useful for some students at some point during a task, this directive was given four minutes into the lesson and only a few minutes into group work. With another group, when Adriana verbalized the task directions to them (perhaps because they were building a rectangle, rather than a cube) and they looked at her with potentially confused looks, she responded by saying, "here's what you're going to do, you're going to color it. . ." Throughout the work time, Adriana interacted with groups that were obtaining correct answers by asking them to explain their reasoning (as noted above) but interacted with groups that were obtaining incorrect answers and/or were confused by telling them what to do and asking yes or no follow-up questions.

Adriana did implement several mindset mathematics practices in this lesson (e.g., praising the learning process, reasoning and multiple perspectives, supporting students to connect in small groups, etc.); however, these practices may have been undermined by a focus on right answers and correct reasoning, rather than an encouragement of struggle or exploration of mistakes. Analysis of all Painted Cube videos similarly revealed that giving students space to struggle without teacher intervention and exploring students' mistakes continued to be a challenge for six out of seven teachers in that lesson.

At the same time, all teachers took up some mindset mathematics practices in their videos, which begs the question of why some practices might have been "easier" than others for teachers to implement. Additionally, nearly all teachers implemented mindset mathematics practices alongside traditional practices, such as focusing on right answers. For example, analysis of videos frequently showed a teacher asking a probing question in one moment and then a narrow question in the next. The latter seemed to undermine the former for some teachers in some moments. We return to this point in the discussion.

5.3.2. Teacher Reflections on Struggling with Struggle

In interviews, teachers similarly noted that supporting their students to struggle productively on the tasks was a challenge for them at times. Several teachers specifically pointed to the Painted Cube task, as Theresa noted: "We really struggled with the Painted Cube activity. I think...Partly I think we probably should have just really done fully from start to end, done it ourselves, all the way through". Theresa and others shared that they found the Painted Cube task to be challenging mathematically, in contrast to the other tasks. Consequently, it is possible that discomfort with the task itself may have hindered teachers' capacity to explore students' mistakes or encourage their struggles in this lesson.

Regardless of the task, however, several teachers noted that their students sometimes experienced frustration while engaging with the mathematics and that they grappled with how to respond to that frustration. Dolores reflected:

> So I think the only thing I will say that I struggled with is for some of those activities they would lose stamina. Some of them just would get frustrated. And trying to keep them working on... I had some that had no problem pulling that out. They wanted to keep working on that, but others that quickly just felt defeated like, "I can't figure this out." But I think overall, their perception changed.

Here, Dolores explained that her students experienced the tasks differently, with some enjoying the challenge and others feeling defeated by it. Given that direct instruction is common in most classrooms, it is likely that students may have been used to teachers intervening to offer a hint or a procedure when they are struggling (as was observed in Adriana's video), such that being prompted to figure things out for themselves or with their peers, even when stuck, may have felt frustrating. At the same time, Dolores also noted that over time, her students' perception of struggle and how to engage with it productively may have changed during their experience at the camp.

5.4. Summarizing Teachers' Learning

When teachers returned to school, under the typical constraints and pacing pressures, it is highly possible that the practices they reported implementing in their school-year classrooms morphed or became infrequent. Here, we discuss the barriers to implementation that teachers experienced upon returning to their classrooms and then reflect on which aspects of the camp's design seemed to support the learning that did happen.

5.4.1. Barriers to Implementation

In interviews, when discussing challenges to implementing the mindset mathematics approach in their school-year classrooms, teachers coalesced around several specific barriers (Figure 5). Pressure to cover their mandated district curriculum emerged as the most frequently reported barrier to implementing these practices as fully or as regularly as they had at camp, as reported by six teachers. Melanie noted:

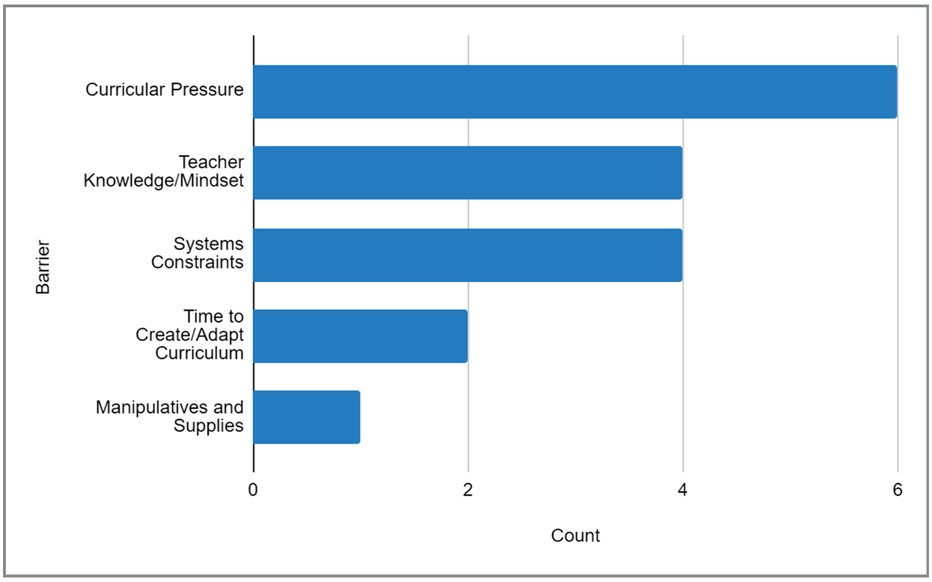


Figure 5. Barriers to implementing mindset mathematics practices (n = 20 interviews).

> So I've been struggling this year just because I have 30 minutes with the kids, but yet I'm expected to cover all of this curriculum. So if we get into a good discussion and my lesson goes on to the next day, I get kind of sunk... Sometimes it's hard to let those discussions happen just because I'm under the crunch of getting through curriculum and I only get them for 30 minutes. (Melanie)

Here, Melanie explained that her short math block, combined with curriculum pressures, made it challenging for her to extend a "good discussion" beyond its allotted time, as this would mean also extending her lesson into the following day.

Relatedly, four teachers reported system-level constraints, consisting of the following: (1) district grading practices that contradicted the mindset mathematics emphasis on mistakes as learning opportunities, (2) convincing administrators to allow teachers to use open tasks, and (3) teaching in isolation, as compared to co-teaching during the camp, which allowed for observation and support of students, as well as collaboration among teachers. In terms of being allowed to use open tasks, Laura reflected: "But a question that a lot of us took to our math director is how do we get more of these [tasks] in our pacing guides". In sharing these concerns, teachers seemed to imply that district curricula, pacing guides, and grading practices had different philosophies than the mindset mathematics approach and the camp curriculum, such that integrating the latter within the former was a challenge.

Additionally, teachers cited their colleagues' lack of knowledge of this approach as a barrier. Isaac shared: "The lack of understanding by admin, principals, colleagues, grade-level friends. It's really hard, especially if you're not supported". Similarly, Melanie shared: "And I think there's a lot of teachers who are stuck on fact fluency and time tests and procedures. And they have to teach the procedures so they can get through the curriculum". Specifically, four teachers expressed a need for more training for their colleagues in how to implement open tasks, how to continue to hold a growth mindset, and how to avoid the trap of low expectations of their students. Additional barriers included the lack of time to create open activities or adapt open activities to their grade level (two teachers) and lack of access to manipulatives (one teacher).

Despite these challenges, teachers expressed a desire to implement mindset mathematics practices when possible:

> The biggest struggle I have is balancing the standards that I have to cover and the approach that I wanna take, and that's the approach that we did in the summer. So, for me, it's tough because I wanna always have them explore. I wanna have them take ownership in the mathematics we're doing, but at the same time I feel like it's tough because I'm told I have to cover all this content. (Richard)

Here, Richard summarized the tension between what he *wanted* to do in his classroom and what he felt he *had* to do. Despite these challenges, our analysis showed that teachers did implement some features of the approach during the school year, which suggests that some aspects of the camp were still useful to their learning.

5.4.2. Design Features of the Camp

As teachers described implementing mindset mathematics practices during and after camp, as well as the ways in which these practices supported their students' reasoning and engagement, they commented on the collective impact these elements had on their own professional growth. Theresa explained:

> The power of working at the summer math camp and seeing that in action when it can be messy and we don't have these bell schedules and we don't feel that we have all this curriculum we have to cover, right? We can just be like, "Oh, we didn't get there today. Oh, well. It doesn't matter." That kind of freedom that the math camp kind of gives in that, I think that it would really be beneficial for other teachers to be able to come in and see that and experience that, right? Look, when you let kids have a little bit of freedom, they'll surprise you. (Theresa)

In one sense, Theresa explicitly commented on the "freedom" that the summer camp gave students—the freedom to generate their own thinking and to engage with mathematics in new ways. In another sense, however, Theresa implicitly commented on the "freedom" that the summer camp gave teachers—the time that she and the other teachers had to explore students' ideas, given the lack of pressure to cover content standards.

Theresa's suggestion that other teachers might shift to more exploratory work if they saw and experienced it themselves, as that would lead to surprise at students' engagement and reasoning, is important to consider. Throughout interviews, teachers spoke about the ways in which longer lessons, smaller class sizes, and lack of curricular and district pressures supported them to use exploratory tasks and to elicit and focus on their students' thinking. About half of the teachers noted that co-teaching or collaborating with colleagues during camp also supported their learning and eventual uptake of the practices. These design features of camp—combined with the task-based curriculum—seemed to support teachers to implement the mindset mathematics approach in the summer, experience students' wonder and engagement for themselves, and then bring some of these practices into their school-year classrooms.

Taken together, our analysis revealed one potential mechanism for this learning: teachers' experiencing the mindset mathematics approach themselves in camp and witnessing their students' reasoning. We discuss the implications of this mechanism below, especially how it might be applied to both transformational and incremental PD efforts.

6. Discussion

Mathematics education researchers and policymakers have long called for a focus on problem solving, reasoning, and student thinking [18–23]. Yet, the mathematics curricula that are used in most classrooms are filled with short questions that do not invite student reasoning or problem solving. The summer camp was designed as an antidote to this broader context. The teachers in this study engaged in an unusual model of teacher learning that involved teaching a task-based curriculum designed by researchers, in an unusual setting of a summer camp, free from pressures of the regular school year.

Through analysis of classroom video, timelines, and interviews, we found that teachers implemented several important practices in both their camp and school-year classrooms, namely, posing open tasks, giving students ample time to collaborate, and asking questions that pushed students to reason. Further, interview data suggested that it was teachers' witnessing of students' reasoning—rather than engaging in a coaching cycle or in a professional development session—that motivated these shifts. Teachers reported that they believed other teachers would shift their practice if they had the opportunity to teach in these ways and to observe students' high engagement and reasoning during open tasks. Teachers' learning appeared to come from building their "knowledge of practice" [45], as they connected research ideas to their own expertise through a task-based curriculum that left space for them to make their own decisions.

Although we found that several mindset mathematics practices were taken up, we also found that others were not. Teachers less frequently integrated mindset messaging directly into their teaching and less frequently gave space for the exploration of students' mistakes and struggles. We hypothesize that it may have been easier for teachers to share growth mindset messages during the separate activity of showing a mindset video than to integrate these messages into their teaching, which requires shifts in potentially ingrained pedagogical habits, such as asking narrow questions (as observed in Adriana's video). In Louie's (2017) study of teachers working to implement inclusive pedagogies, she found that although teachers employed the inclusive pedagogies that they had learned, they did so alongside the traditional ones that they had been accustomed to using, such as centering procedures [35]. In other words, the teachers in Louie's study may have had an easier time *adding* new pedagogies to their practice, rather than *subtracting* ingrained pedagogies [35]. In sharing mindset messages through isolated activities but less frequently exploring mistakes, we wonder if the teachers in our study similarly added in a new practice without changing prior practices that emphasized right answers.

Further, we hypothesize that exploring mistakes and struggles may have been less familiar to teachers than practices like encouraging groupwork and emphasizing multiple perspectives, which are present in the Common Core State Standards [23] and in CCSS-aligned curricula. It may have been easier for teachers to implement these practices during camp, if they were already somewhat familiar with them, and to bring these practices into their school-year classrooms, if they were also reinforced by their schools' curricula. Research has similarly shown that teachers are more likely to transfer their learning to the classroom when their schools' systems and philosophies are aligned with those of their PD experience [38,39]. Additionally, even if a particular practice was not present in teachers' curricula, it may have still been more feasible for teachers to occasionally pose open tasks, integrate collaboration into their lesson plan, and ask probing questions, while still following their mandated curriculum and pacing guides, rather than offering students extended time to productively struggle or engage in a lengthy class discussion.

6.1. Limitations

Given that the interview and timeline data are self-reported, it is possible that teachers over-stated their use of these practices. As noted previously, teachers cited specific barriers to implementing the mindset mathematics approach in their school-year classrooms. On the one hand, the existence of these barriers necessitates continued discussion in the field of how to create the conditions in schools and districts that enable teachers to sustain their learnings from PD efforts. On the other hand, however, as teachers shared these barriers in their interviews, they simultaneously discussed ways that they *were* implementing this approach despite the barriers, which might bolster these findings.

Another limitation relates to our sample and setting. It is possible that teachers who consented to participate in interviews and to turn in timeline data had a higher rate of uptake than the entire summer camp teacher population. Similarly, it is also possible that those who turned in video recordings of the Painted Cube lesson had a higher rate of uptake than the entire summer camp teacher population. Relatedly, it is possible that among the nine teachers who turned in video recordings, the seven teachers who turned in recordings of sufficient length and audio quality to be analyzed had a higher rate of uptake than the two teachers who turned in sets of short clips. For example, if these two sets of short clips contained minimal implementation of the mindset mathematics teaching practices, then eliminating them from our analysis would have skewed our findings. Based on our watching of these two sets of clips, however, it appeared that several mindset mathematics practices were implemented in one of the videos and several less-aligned practices were implemented in the other video. Given the mixed implementation that we found in our analysis of the seven videos, we contend that these two excluded clips were not significantly dissimilar from the rest of the data set. While there is always a possibility for bias when excluding parts of a data set, the alternative of attempting to code videos with limited context and poor audio quality would have been problematic.

More broadly, it is also possible that teachers who choose to teach in a math summer camp in the first place comprise a small, unique sample that is not representative of the broader teaching force. Consequently, we cannot argue that our findings are generalizable to the broader teaching force or that they are generalizable to the school-year context, as opposed to the unique context of a summer camp. Instead, we contend that this is a first result that reveals the potential efficacy of a novel approach to teacher learning with a small group of teachers. Given the low transfer rate of many teacher learning initiatives [38,39], however, teachers' potential transfer of practice from the summer camp to their teaching in the regular school year is particularly noteworthy.

6.2. Implications for Research and Practice

Our findings illuminate several implications for the field of teacher learning. By teaching a research-based curriculum and engaging in associated professional learning opportunities, teachers in this study were able to integrate new research ideas with prior expertise to construct "knowledge of practice" in and through their teaching [45]. This blending of theory and practice is an important form of knowledge, as a focus only on theory or only on practice is not sufficient. As researchers and practitioners design professional learning experiences that support teachers to learn in and of their practice, they might provide teachers with opportunities to draw on and experiment with research-based approaches in classroom environments free from the typical constraints. While other teacher learning initiatives have also supported teachers to experiment with research-based approaches of some kind [31,32,55], here we offer a novel approach to professional development in that teachers could engage in and focus on this experimentation without having to simultaneously juggle testing, grading, and curricular pressures. Although this is a transformational rather than incremental approach, our study shows that it has efficacy in supporting some teachers to take on some practices.

Building on these results, future studies might investigate teachers' learning in specialized settings at a larger scale and with a particular focus on longer-term impacts. Researchers might closely follow teachers during their teaching of the camp, collecting video recordings of multiple lessons, teacher reflections, and student work artifacts, to more extensively capture the implementation of mindset mathematics practices and to do so among a larger group of teachers. Similarly, researchers might follow teachers for several years after participating in the camp, observing teachers' school-year classrooms to investigate which mindset mathematics practices they are continuing to implement, which they are not, how they might be adapting the practices for their school-year contexts, and what external barriers impede implementation. Given the limitations mentioned previously, observing teachers' school-year classrooms might triangulate shifts in practice that teachers self-report in an interview. Additionally, studying the learning of cohorts of teachers from the same schools may also illuminate specific site-based barriers and supports that they experience when transferring these practices to their schools.

Further, scaling the intervention such that teachers participate in the camp for multiple summers could allow for multiple rounds of data collection across both camp and school contexts. Researchers could analyze how teachers continue to grow their practice across several years through multiple iterations of teaching in the camp and multiple opportunities to try out the practices in their school-year classrooms. For example, what might teaching in the camp for a second year look like? And how might teachers' implementation of these practices in their regular classrooms inform how they approach a second year at camp? Teachers' learning of mindset mathematics practices would likely travel back and forth between these two sites of practices in specific ways that could inform future interventions. Moreover, a multi-year, multi-site study may afford opportunities for collaboration between camps and schools, which may begin to ameliorate the disconnect that many teachers experienced upon returning to their schools.

At the same time, our finding that a subset of practices did not seem to be transferred to classrooms raises important questions about the efficacy of the transformational teacher learning design embedded in this study. An argument might be made that the mindset mathematics approach involves too many components to learn at once and/or that it is too challenging to implement in a school-year setting with mandated curricula and assessments that may limit teachers' time and capacity to try out new approaches. It is possible that an incremental, practice-based approach [37] might more effectively nudge teachers toward integrating mindset messaging into mathematics tasks directly or responding to incorrect answers with probing questions. In particular, the practice of supporting students to struggle, which requires teachers to actively resist reducing the cognitive challenge, may be an especially challenging practice that takes time for teachers to develop, perhaps after engaging in multiple incremental shifts over time.

Our finding about teachers' observations of students' engagement suggests that an incremental approach [37] may be fruitful for learning mindset mathematics practices. As noted previously, implementing open tasks that made space for students to author their own ideas enabled teachers to witness engagement from students of all achievement levels and to appreciate students' mathematical ideas. Consequently, teachers' implementation of student-centered pedagogies that made students' ideas (and by extension, their competence) visible to themselves and to their teachers emerged as a critical component of teacher learning [53–55]. If teachers' witnessing of students' engagement can motivate uptake of student-centered practices after a summer camp, then teachers' witnessing of students' engagement and reasoning when trying out new practices in their regular classroom may motivate them to sustain that practice and try out similar ones. For example, imagine a teacher is working on responding to incorrect answers with probing questions rather than leading students to the right answer. In asking probing questions, they will likely elicit students' reasoning, potentially making students' thinking and competence visible to them. As we found in our study, observing students' engagement may motivate this teacher to continue to make small shifts. As such, the notion that small tweaks in practice may yield student engagement and reasoning, which then may motivate continued shifts, may be critical to teachers' learning of mindset mathematics practices.

We contend that both incremental and transformational approaches to teacher learning are needed to improve mathematics education for students in the US. The shifting of teachers to a mindset mathematics approach through an immersive learning experience is only one piece of this challenge. Many factors, outside of teachers' control, work against change. It is our hope that this study pushes the field to think more expansively about teacher learning and to consider professional learning opportunities beyond the typical school setting.

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education sciences

Article

Incremental Growth through Professional Learning Communities of Math Teachers Engaged in Action Research Projects

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Abstract: This study investigates the experiences of a professional learning community (PLC) composed of six secondary math teachers enrolled in a graduate math methods course. Through the discussion of educational texts and collaborative inquiry, the teachers identified classroom challenges they aimed to address through individual action research projects. The PLC provided a supportive environment for teachers to share their processes, receive peer feedback, and collectively reflect. This study underscores the value of action research and PLCs in driving educational improvement. By engaging in structured inquiry within a collaborative setting, teachers gained insights into pedagogical issues, developed targeted incremental interventions, and contributed to the broader discourse on math education pedagogy. The collaborative PLC model facilitated reflective practice, challenged assumptions, and empowered teachers as agents of change. Implications for teacher professional development, instructional practices, and future research directions are discussed.

Keywords: professional learning community; action research; mathematics education

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1. Introduction

Continuous improvement is a hallmark of effective teaching, and engaging in action research allows teachers to critically examine their practices, identify areas for growth, and implement targeted incremental interventions. In this study, I, a mathematics teacher educator, explore the experiences of a professional learning community (PLC) composed of six secondary math teachers enrolled in a graduate-level online math methods course that I taught. Through collaborative inquiry and the discussion of educational texts, the teachers selected classroom challenges they aimed to address through action research projects. The PLC provided a supportive environment for the teachers to share their processes, receive feedback from peers, and collectively reflect on their findings.

For the first eight weeks of the course, teachers read and discussed four books. These works introduced methods for fostering equity in math classrooms [1], encouraging iterative mathematical thinking [2], orchestrating productive mathematics discussions [3,4], and cultivating environments that promote student thinking [5]. I wanted teachers to be exposed to the math pedagogies in all the books, but I knew assigning four books would not be realistic. Therefore, each teacher chose one book to read (with two books being chosen by two teachers). Each teacher was responsible for providing a weekly summary of the book they were reading, and all teachers were responsible for reading their peers' weekly summaries.

I used Project Zero's Connect, Extend, Challenge routine to promote teachers' reflections and discussions about the readings [6]. The goals of the discussions were to help teachers link the new methods that they were learning about in the books to their existing knowledge and prompt them to reflect on how their thinking had evolved through their learning and experiences. I posed the following prompts in a weekly online discussion board to initiate the discussions:

1. How are the ideas and information connected to what you already know?

2. What new ideas did you get that broadened your thinking or extended it in different directions?
3. What challenges or puzzles emerge for you?

Every week, and after each teacher participated, I created a post aimed at synthesizing the discussion. I also interacted with each teacher using private comments in the grading page of the learning management system. My intention was to use inviting, suggesting, explaining, describing, and evaluating discursive moves to assess and advance teachers' knowledge of the methods presented in the texts [7].

Next, each teacher identified a specific problem of the practice they were currently encountering in their classroom that they thought a method that they learned about in the text(s) could maybe help solve. I thought this teacher-driven, bottom-up approach could promote incremental improvements in teachers' practices [8]. I intentionally chose this approach with the goal of disrupting typical professional development approaches, which often fail to have a direct, long-term, or scalable impact [8]. I guided each teacher in their selection of a high-uptake practice [9]. According to practicality theory, attention to instrumentality, congruence, and cost can facilitate teachers' acceptance of change [10]. Instrumentality is accomplished when the teacher's actions are clearly articulated. Congruence involves alignment of the practice with the teacher's current instructional approach and context. The cost involves the expected benefit from implementing the practice compared to the effort and resources required to enact it. Teachers shared diverse problems of practice and chose unique teaching interventions for their action research projects. The diversity of teachers' projects underscores the multifaceted nature of effective math instruction. Additionally, given the range of topics that the teachers chose, I was glad that I did not attempt to prescriptively tell teachers what to focus on.

For the next eight weeks of the course, teachers engaged in a Plan-Do-Study-Act (PDSA) action research project cycle [11]. Working together as a PLC, the teachers reflected on and discussed each stage of their PDSA cycles in online discussion boards. Through this collaborative process, the PLC aimed to generate insights that could inform the teachers' future teaching practices and contribute to the broader discourse on math education pedagogy.

2. Theoretical Framework

The theoretical foundation for this study is grounded in the principles of action research projects and PLCs. PLCs provide a supportive environment for teachers to engage in collaborative inquiry, share knowledge and experiences, and offer constructive feedback [12,13]. Whether virtual or in-person [13], referred to as pods [14], pacts [15], or communities of practice [13], effective PLCs share common features.

These communities have a common purpose, shared goals, and a commitment to student success. Teachers work collaboratively to examine student data and instructional practices, deciding on strategies to enhance teaching and learning [16]. PLCs are evidence-based, focusing on ongoing growth and learning, and they continuously experiment to improve practices. They foster cooperation, emotional support, personal growth, and synergy of efforts.

Deliberative leadership, safe working environments, and essential practices contribute to the success of PLCs [17]. By collaborating, sharing knowledge, and learning together, teachers create a culture of continuous improvement. The collective efforts within PLCs lead to better outcomes for students [16,17].

Creating a productive PLC involves aligning these features, fostering collaboration, and valuing professional growth. By promoting a sense of collective responsibility and a shared commitment to student success, PLCs can positively impact instructional practices and drive continuous improvement [16].

Action research, as defined by Mertler, is a cyclical process where teachers identify an area of focus, implement interventions, collect and analyze data, and reflect on the outcomes, all aimed at improving their practice and enhancing student learning [18]. This iterative process allows teachers to continuously refine their practices based on evidence and reflection.

Action research PLCs involve educators working collaboratively to refine their practices, with teachers acting as integral members of the research process, making it both collaborative and participative. In these PLCs, teachers work together to identify areas for improvement, design and implement interventions, collect and analyze data from their classrooms, and engage in critical reflection on the outcomes.

Action research can provide teachers with practical and relevant insights about how incremental changes can lead to improvements in their practices. Since teachers actively participate in all aspects of the research, the values and voices of the affected communities are genuinely incorporated, and deficit perspectives can be challenged [19]. This approach ensures that the research is grounded in the realities of the classroom and the lived experiences of teachers and students, rather than imposing external perspectives or assumptions.

Through action research projects, teachers can inquire about and critically reflect on their practices in a systematic and rigorous manner. This process of inquiry and reflection can help teachers develop a deeper understanding of their teaching strategies, classroom dynamics, and student learning needs. Action research can empower teachers to become reflective practitioners and active agents in their professional growth [20].

By engaging in action research PLCs, teachers can collaborate with their colleagues, share their findings, and learn from one another's experiences. This collaborative approach fosters a culture of continuous learning and improvement, where teachers can support and challenge each other in a safe and supportive environment. Collectively, teachers can identify and address common challenges, explore innovative pedagogical approaches, and contribute to the broader knowledge base of effective teaching practices.

Overall, action research PLCs provide a powerful framework for teachers to engage in ongoing professional development, enhance their teaching practices, and ultimately improve student learning outcomes. Through a cyclical process of inquiry, intervention, data collection, and reflection, teachers can become researchers of their own practice, empowering them to make informed decisions and drive positive change in their classrooms and schools.

The synergy between action research and PLCs has the potential to catalyze incremental improvements [21–23]. The collaborative nature of PLCs can enhance the rigor and validity of action research. When teachers work together, they can draw upon diverse perspectives, challenge assumptions, and refine their interventions, ultimately leading to more robust and meaningful findings.

Research supports the potential efficacy of PLC-supported action research projects with teacher-selected foci and incremental goals. Otten et al. found that bottom-up approaches can lead to significant improvements in teaching practices [8]. Additionally, de Araujo et al. emphasized the importance of high-uptake practices [9]. Moreover, as described in the Introduction, Doyle and Ponder highlighted the role of practicality theory in facilitating teacher acceptance of change [10].

3. Methods

This qualitative study employed a case study approach to examine six secondary math teachers' perceptions of the impact of their PLC-supported action research projects. The goal of the investigation was to understand teachers' perceptions of the outcomes of their PLC-supported action research projects, which had teacher-selected foci and incremental goals. This case study spanned sixteen weeks. The data included teachers' action research project final papers. In this section, I discuss the research setting and participants, data, and the analytic approach that guided this study.

3.1. Research Setting and Participants

This study was conducted within a graduate math methods course at a large research institution in the southern United States. Table 1 provides information about the six secondary mathematics teachers. One of the six teachers obtained her teaching license through an undergraduate secondary mathematics education teacher preparation program. This teacher was in her fourth year of teaching secondary math. The five other teachers earned bachelor's degrees in kinesiology, information technology, physics, business administration, and meteorology. These five teachers had 1–2 years of teaching experience and had obtained their secondary mathematics teaching licenses through an alternate route graduate degree program. All six teachers were White, four were men, and two were women. While all taught at rural public schools, none taught at the same school.

Table 1. Secondary Mathematics Teacher Participants.

Pseudonym	Years of Experience	Age	Bachelor's Degree	Course
MC	4	27	Secondary Math Education	7th Grade
BB	1	25	Meteorology	Algebra II
AW	1	26	Kinesiology	8th Grade
CK	1	28	Physics	8th Grade
RH	2	34	Information Technology	6th Grade
AN	1	39	Business Administration	8th Grade

3.2. Data

Throughout the 16 weeks of this study, teachers participated in online discussions. During the initial eight-week period, teachers read and discussed four books. These literary works introduced methodologies for promoting equity within math classrooms [1], fostering iterative mathematical thinking [2], facilitating productive mathematical discourse [3,4], and cultivating environments conducive to student thinking [5]. I used the Connect, Extend, Challenge routine from Project Zero to stimulate reflective discussions about the readings [6]. After each teacher contributed, I created a weekly synthesis post, summarizing the discussions. Additionally, I used private comments to individually interact with the teachers. I purposefully used inviting, suggesting, explaining, describing, and evaluating discursive moves to assess and advance teachers' knowledge of the methods presented in the texts [7].

During the next eight weeks of the course, teachers discussed their PDSA action research projects [11]. The discussion board topics were selected based on the sections that teachers would need to include in their final action research project papers. The discussion topics included the following: Identify Your Problem, Write Research Question, Identify Connections to Each Book, Finalize Your Plan, Data You Collected, Data Analysis, Interpretation of Results, Limitations, Implications, Contributions to the Field, and Future Teaching and Research Directions. Each discussion functioned like a PLC, as teachers discussed their progress, challenges, and provided each other with constructive feedback aimed at helping each other refine their action research projects. The teachers' final reports summarized each stage of their action research projects. For this study, the data analysis focused on teachers' final reports, because the final reports represented a polished version of teachers' perceptions of the impacts of their PLC-supported action research projects.

3.3. Data Analysis

To assess teachers' perceptions of the impacts of the PLC-supported action research projects, I analyzed the six teachers' action research project final reports. I began by carefully examining each report to identify the specific problem of practice or challenge the teacher aimed to address through their action research project. Then, I analyzed the intervention chosen by the teacher as a potential solution, considering how it was justified based on the course readings. I noted patterns and themes across the different problems and interventions. To assess the teachers' perceptions of the impacts of the PLC-supported action research projects, I closely read teachers' reflections on and evaluations of the impacts of their chosen interventions. I analyzed evidence such as data from student surveys, observations, and assessments to understand how teachers interpreted the intervention's successes, challenges, and outcomes in relation to their identified problems of practice.

I used thematic analysis to systematically code the reports and identify recurring themes related to problems, interventions, and perceptions [24]. I examined the language, tone, and underlying assumptions reflected in the teachers' writing. To develop a comprehensive understanding, I triangulated findings with teachers' discussion board posts and individual messages. Through synthesis across the six papers, I aimed to gauge teachers' perceptions of whether incremental improvements resulted from their PLC-supported action research projects.

3.4. Limitations

While this qualitative study offers valuable insights into PLC-supported action research projects through an in-depth analysis of teachers' reports, several limitations must be acknowledged. Firstly, the small sample size of only six reports restricts the generalizability of the findings to broader populations of teachers or educational contexts. Additionally, the teachers involved in this study were a convenience sample, which has more potential to be biased than if I had used random or systematically sampling methods to select teachers. Moreover, thematic analysis is inherently subjective, and my own biases and theoretical lenses could have influenced my interpretation of the data. Also, the action research reports themselves may not provide a comprehensive or nuanced account of the teachers' experiences, perceptions, and reflections. Since the data were self-reported by the teachers, it may have been subject to biases such as social desirability or selective memory. I could have conducted classroom observations and interviews to triangulate the data. These findings may also be context-specific, limiting their transferability to other grade levels, subject areas, or school settings. Finally, framing my analysis with PLC-supported action research lenses could potentially overlook alternative explanations or perspectives.

4. Findings

I begin with the set of problems and changes the teachers made. Then, I point out some of the themes and noteworthy phenomena from within the teachers' choices. Table 2 provides an overview of the incremental action research projects the teachers chose to carry out within the PLC. MC and BB both reported feeling overwhelmed by the quantity of questions their students asked during class time when they were supposed to be working independently or in small groups. MC and BB investigated the nature of the questions their students were asking but took different approaches. MC surveyed her 22 seventh-grade math students, and BB video recorded her 25 students during their Algebra II class to find out if their questions were stop thinking, proximity, or keep thinking questions [5]. MC found that most students self-reported asking stop thinking and proximity questions. BB watched one week of video recordings of her class and found that most stop thinking questions occurred after the lunch break in the middle of the class. Additionally, BB noticed some gender differences, with males asking the most stop thinking questions and females asking the most proximity questions. Both MC and BB talked with their students, and they developed a shared agreement that they would only answer keep thinking questions. Both MC and BB reported feeling that their incremental action research projects led to their students thinking more and both felt less overwhelmed.

Table 2. Secondary mathematics teachers' action research projects.

Pseudonym	Students in Sample	Problem	Intervention(s)	Instrumentation	Results
MC	22	many students asking many questions	only answer keep thinking questions	student survey	mostly proximity or stop thinking questions
BB	25	many students asking many questions	only answer keep thinking questions	observation (video recording)	most proximity or stop thinking questions
AW	100	students not enjoying math and not thinking	group work, non-curricular tasks	student survey	promoted enjoyment and thinking
CK	35	students not enjoying math and not thinking	incentivize test corrections	student survey	promoted enjoyment and thinking
RH	90	ineffective note-taking	write notes to future forgetful selves, open notes test	content test	improved test performance
AN	130	classroom management	co-construct norms	student survey	improved behavior

The problems that AW and CK aimed to address with their incremental action research projects were students not enjoying class and not thinking. To address the problem, AW tried two interventions with his 100 eighth-grade students, working in random groups of three, and using non-curricular tasks [5]. Inspired by Jansen, CK tried three different incentives to motivate student completion of test corrections, a daily grade, a prize, and an improved test grade [2]. Both AW and CK surveyed their students to uncover their self-reported enjoyment and thinking. On average, all of AW's students reported enjoying and thinking during both the group work and the non-curricular tasks. More specifically, on average, AW's students reported group work promoting the most thinking and the non-curricular tasks promoting the most enjoyment. AW stated that although he "found success with this research study, in the future I could improve on this study by adding more detailed questions in my student survey and trying out different activities." Similarly, on average, CK's students reported that the incentives motivated them to complete test corrections. However, on average, students associated low thinking and low enjoyment with the daily grade, high enjoyment and low thinking with the prize, and high thinking and low enjoyment with the improved test grade. CK created the graphs in Figure 1 to summarize his results. From left to right, students' ratings for the daily grade, prize, and increased test score incentives. Blue represents students' ratings for thinking or effort. Orange represents students' ratings for enjoyment. Likert scale ratings from 1 to 5 were used, with 1 representing "not at all" and 5 representing "very much."

RH's problem was that the notes his 90 sixth-grade students were mindlessly taking were meaningless notes during class. RH stated that his issue was "conveying all the information in a way that students can digest. What I find is students don't seem to translate what they learn to assessments". Instead of copying all of his notes throughout class, RH's incremental change involved having students write brief notes, in their own words, only at the end of class. RH also allowed students to use their notes on their test. RH found that, compared to their performance on previous tests, overall, students performed better on the test when they could use the notes they wrote to their future forgetful selves [5]. RH shared that he "learned that having students write brief notes in their own words at the end of class greatly improves their ability to recall and ultimately perform better on assessments".

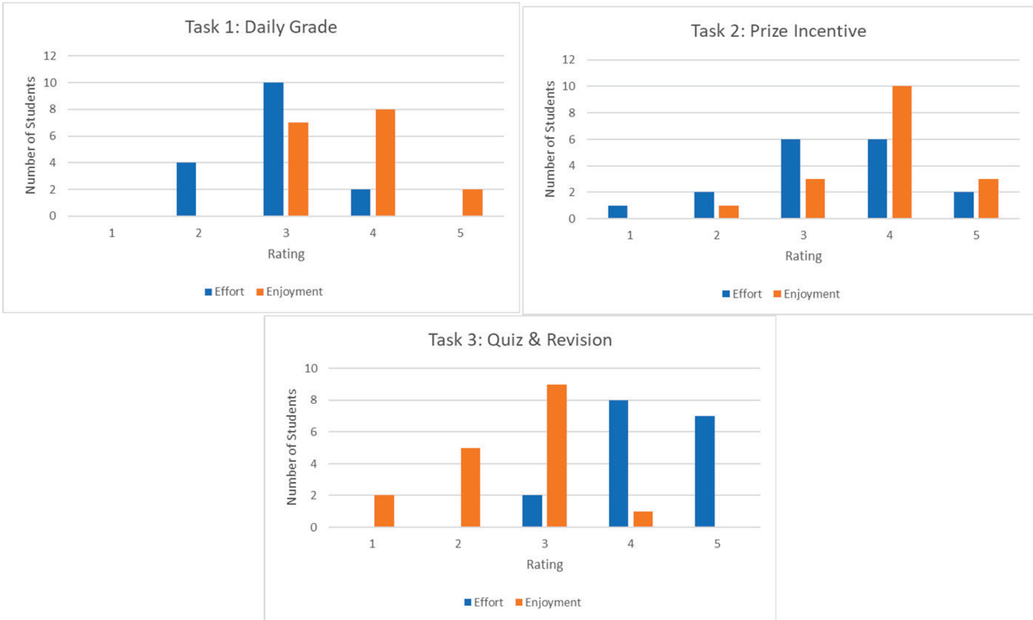

Figure 1. Results from CK's student survey.

Management of his classes of 130 eighth-grade students was the problem that AN aimed to address with his incremental action research project. Inspired by Jansen and Seda and Brown, AN co-constructed new class norms with his students [1,2]. During class, AN and his students brainstormed class norms. Then, AN surveyed his students about the rights and responsibilities they discussed [2]. Overall, students appreciated being included in the establishment of class norms. Although AN's former norms of being quiet, starting class when the bell rings, and not using their cell phones during class were not popular, students considered it important for the class to be conducive to learning. AN stated, "I was surprised at how much the students wanted structure. I think knowing the norms removed the anxiety caused by the former conditions of disorder. Once the norms were established I requested that the students sign a copy stating they agreed and understood. Common discipline issues declined so I felt very confident of my research".

The thematic analysis of the teachers' identified problems of practice revealed several overarching themes. The six teachers identified various classroom challenges. MC and BB were overwhelmed by the quantity of student questions during independent or small-group work. AW and CK faced issues with students not enjoying class and lacking cognitive engagement. RH noticed his sixth-grade students were taking meaningless notes. Finally, AN struggled with managing his classes and ensuring appropriate student behavior.

One common theme across the six action research projects was the teachers' use of participatory design. Through participatory approaches, historically marginalized communities, including students, are engaged as co-researchers and are empowered to design and implement a custom intervention. By taking a participatory design approach, the six teachers fostered collaborative decision making and mutual ownership of the research process [19]. MC surveyed her seventh-grade math students to categorize their questions, while BB used video recordings to analyze question types in her Algebra II class. MC stated that the survey results, which revealed that most questions were stop thinking or proximity questions, "were not surprising to me because I was expecting very similar results". On the other hand, BB was surprised by the "particularly a high concentration of questions following the mid-class lunch break and a gender disparity in the types of questions asked. The results suggest that the scheduled interruptions of the classroom significantly influence student engagement and questioning behavior. Moreover, the unexpected gender differences in questioning types highlighted potential areas for further investigation into socialization practices and gender-specific educational strategies". After sharing the results with their students, both MC and BB co-constructed agreements with their students. Because they were involved in the research, MC's and BB's students understood why it was important for their teachers to only answer their keep thinking questions. AW's and CK's students helped brainstorm possible interventions and shared how much they enjoyed and how deeply they thought as a result of each intervention. AW introduced group work in random groups and non-curricular tasks to his eighth-grade students, and CK used three incentives (a daily grade, a prize, and an improved test grade) to motivate students to complete test corrections. RH had a discussion with his students about their current note-taking predicament and what notes to future forgetful selves are [5]. Together, RH and his students agreed to postpone their note-taking until the end of class and to write brief notes that were in their own words. They also collaboratively decided that students would be allowed to use their notes on tests. Finally, AN co-constructed class norms with his eighth-grade students to improve behavior and classroom management.

The results shed light on teachers' perceptions of the impacts of their PLC-supported action research projects. Overall, all six teachers perceived incremental improvements from their interventions. MC and BB felt their interventions led to students asking less proximity and stop thinking questions, which led to MC and BB feeling less overwhelmed. AW and CK observed increased student enjoyment and cognitive engagement. AW's students reported that group work promoted more thinking, while non-curricular tasks were more enjoyable. CK's students found the incentives effective for completing test corrections, although each incentive had different impacts on enjoyment and thinking. RH's intervention of personalized notetaking resulted in better test performance, indicating that the notes were more meaningful and helpful for students. AN's co-construction of class norms led to improved behavior, even though students did not value all the norms.

5. Discussion

5.1. Incremental Instructional Changes and Teacher Success

The findings from this study demonstrate that improvements can result from incremental instructional changes made through PLC-supported action research projects. All six teachers perceived improvements in the conditions that inspired their action research projects. These results align with the theoretical foundations of continuous improvement and the principles of PLCs and action research, which emphasize collaborative inquiry, targeted interventions, and reflective practice [12,13]. Incremental improvements, as described by Otten et al., are critical in creating sustainable change in educational practices by promoting gradual, manageable adjustments that lead to significant long-term benefits [8]. I consider each component of the action research projects important, including the support of the PLC, the teacher-selected foci, and the incremental nature of the teachers' goals critical to the teachers' success. While I am unable to tease out the extent to which each component may have contributed to teachers' success, I conjecture that teachers may have been less likely to experience success if they had transformational goals, because such ambitious changes may have been overwhelming and difficult to achieve within the time constraints and support structures available.

5.2. Tailored Interventions and Diverse Classroom Challenges

The diversity of problems addressed by the teachers—ranging from managing student questions and engagement to improving note-taking and classroom behavior—highlights the multifaceted nature of effective math instruction. The PLC-supported action research projects were tailored to meet each teacher's needs because each teacher had the autonomy to identify a specific classroom challenge that they wanted to focus on and with the support of their peers, design and implement a custom intervention. The teachers exemplified the practical applicability of action research in fostering incremental instructional improvements. This reflects the notion that action research allows teachers to critically examine their practices, identify areas for growth, and implement targeted changes [18]. Moreover, this approach aligns with de Araujo et al. , who emphasize that high-uptake practices are most effective when they are directly relevant to teachers' existing contexts and needs [9].

MC and BB's focus on managing student questions through categorizing and selectively answering keep thinking questions illustrates how targeted incremental instructional changes can promote deeper student thinking and help teachers feel less overwhelmed. Their strategy of developing an agreement with students to only answer keep thinking questions fostered an environment where students were encouraged to think critically and independently, leading to improved classroom dynamics and student engagement. This approach demonstrates the importance of aligning new methods with existing knowledge, a key aspect of effective professional development [16]. AW and CK tried a variety of incremental instructional changes. AW's students worked in random groups of three and completed non-curricular tasks [5]. In an attempt to foster a rough draft philosophy in his math class [2], CK incentivized completing test corrections with a daily grade, a prize, and an improved test score. On average, all of AW's students reported enjoying and thinking during both the group work and the non-curricular tasks. More specifically, AW's students felt that group work promoted the most thinking, while the non-curricular tasks were the most enjoyable. Similarly, CK's students indicated that incentives motivated them to complete test corrections. However, students generally associated the daily grade with low thinking and low enjoyment, the prize with high enjoyment but low thinking, and the improved test grade with high thinking but low enjoyment. The teachers' incremental changes underscore the role of innovative pedagogical strategies in addressing classroom challenges. Their findings suggest that different strategies can have varied impacts on student engagement and thinking, reinforcing the need for context-specific solutions that promote incremental progress [8].

RH's approach to notetaking, which involved students writing notes to their future forgetful selves [5] at the end of class, led to improved test performance. This intervention highlights the effectiveness of practical, student-centered strategies in enhancing meaningful learning. Allowing students to use their personalized notes on tests not only made the notes more relevant and useful but also encouraged students to engage more deeply with the material. This incremental change in note-taking practices resulted in significant improvements in student outcomes, supporting the idea that small, manageable adjustments can lead to substantial educational gains [8]. AN's co-construction of class norms with his students resulted in better classroom behavior, demonstrating the value of involving students in the creation of rights and responsibilities of members of the mathematics learning community [1,2]. This approach aligns with the principles of practicality theory, which emphasize the importance of instrumentality, congruence, and cost in facilitating teachers' acceptance of change [10]. By ensuring that the new norms were practical, aligned with existing practices, and perceived as beneficial by students, AN was able to foster a more positive and well-managed classroom environment.

5.3. The Role of Reflection, PLCs, and Enjoyment in Teacher Growth

The PLC-supported action research projects also led to shifts in teachers' ideologies regarding the root causes of the problems they were experiencing, the key principles of effective interventions, and the profound potential impact of incremental changes in their practices. Initially, the teachers framed the problems as inherent in their students, and therefore very difficult to solve. As a result of the discussions and reflections in the PLC, teachers began to realize the roles they played in the problems of practice they identified. BB shared that the PLC-supported action research project led her to "reassess my educational practices to ensure I am providing an environment that is conducive to increasing student engagement". Regarding the nature of their interventions, all teachers discovered the importance of taking a participatory design approach by involving their students in developing effective interventions [19]. Through their PLC-supported action research projects, teachers experienced and reflected on how incremental changes in their practices could solve the problems they were experiencing. The PLC-supported action research projects empowered teachers to realize the profound impact they have in solving their own problems of practice.

The supportive environment provided by the PLC, characterized by collaborative inquiry, the sharing of knowledge and experiences, and constructive feedback, was instrumental in the success of the action research projects. The PLC facilitated the teachers' ability to reflect on their practices, receive peer feedback, and collectively refine their interventions, which is consistent with the evidence-based, continuous improvement focus of effective PLCs [16,17]. This collaborative effort not only enhanced the rigor and validity of the action research projects but also promoted a culture of continuous learning and incremental improvement, where teachers could draw on each other's strengths and insights to refine their practices [21].

Additionally, the structured reflection prompts (Connect, Extend, Challenge) used in the PLC discussions helped teachers link the new methods they learned about in the four books they collectively read to their existing knowledge, broaden their thinking, and address emerging challenges. This reflective practice is crucial for promoting deeper understanding and ongoing professional growth [6]. By engaging in these reflective activities, teachers were able to connect new pedagogical strategies to their classroom experiences, extend their thinking by exploring new ideas, and address any challenges that arose during the implementation of their action research projects. This iterative process of reflection and adjustment is key to achieving incremental improvements in teaching practices [8].

Teachers also reported enjoying conducting their PLC-supported action research projects. They liked trying different incremental changes, assessing their effectiveness, experiencing positive changes, and receiving feedback from the PLC. Moreover, teachers reported enjoying learning about each other' studies and providing feedback. AN stated, "I enjoyed keeping up with my classmates' research. I know this class improved my teaching skills. I came away with new ideas and established research". It seems the grain size of the incremental changes contributed to teachers' success and enjoyment. I joked with the teachers that they had been bitten by the research bug. Although the teachers' enjoyment was not the focus of this study, it is important because it increases the likelihood of the teachers seeing themselves as researchers and conducting more incremental action research in the future. The sustained cumulative impact of ongoing action research could be profound. The veteran of the group, MC stated, "I have been teaching for four years. However, I know that no matter how long I teach, there is always something new that I can learn".

5.4. Implications for Teachers and Teacher Educators

Prior to engaging in their PLC-supported action research projects, half of the course was spent reading and discussing four books. Based on the discussions, teachers enjoyed and benefited from learning about the pedagogies in all of the books. However, looking across the action research studies, I noticed trends in which of the course readings teachers relied on. One trend I immediately noticed was that the practices from *Building Thinking Classrooms* resonated with the teachers and were the right grain size for their action research projects. MC's and BB's projects, which focused on incremental changes in the types of questions they answer, drew inspiration from *Building Thinking Classrooms.* AW's incremental changes in using random groups of three and non-curricular tasks were also guided by *Building Thinking Classrooms.* Moreover, *Building Thinking Classrooms* motivated RH's incremental change in having students make notes to their future forgetful selves. *Rough Draft Math* also inspired several teachers and offered incremental strategies. *Rough Draft Math* inspired CK to make an incremental change in his assessment policies by allowing and incentivizing test corrections. *Rough Draft Math* and *Choosing to See* motivated AN to co-construct norms with his students. I conjecture that although the teachers valued the practices shared in the *Five Practices in Practice*, they may not have considered it a feasible undertaking to learn to implement the five practices.

The findings from this study have several implications for math teachers, teacher educators, and researchers. The PLC model proved invaluable in supporting the action research process by offering a platform for teachers to share experiences, receive feedback, and engage in collective reflection. This collaborative environment fostered a sense of community, encouraged open dialogue, and facilitated the exchange of diverse perspectives, leading to more robust and well-informed interventions. By focusing on incremental changes, teachers can make manageable adjustments that lead to significant long-term improvements, ultimately benefiting both educators and students [8–10].

6. Conclusions

In conclusion, this article makes valuable contributions by sharing cases of how incremental approaches to professional development can integrate well with PLC-supported action research projects. Teachers were in the driver's seat, selecting the focus of their action research projects. However, these incremental changes were inspired by worthwhile ideas from math education scholars. Some may argue that the teachers did not implement the pedagogies with "full fidelity", and others may critique the teachers for not setting transformational goals. However, the cases underscore the significant potential of incremental instructional changes implemented through PLC-supported action research projects to improve teaching and learning. The collaborative and reflective nature of PLCs, combined with the iterative process of action research, provides a powerful framework for continuous professional development and effective teaching. Through structured inquiry within a supportive, collaborative environment, math teachers effectively identified and addressed pressing challenges in their classrooms, providing valuable insights into specific pedagogical issues and highlighting the interconnectedness of various factors that contribute to effective math instruction [8–10].

This study contributes to the broader discourse on math education pedagogy by demonstrating the practical benefits of integrating PLCs and action research in fostering ongoing improvement and innovation in teaching practices. By focusing on incremental changes, teachers can make manageable adjustments that lead to significant long-term improvements, ultimately benefiting both educators and students. This article provides important insights about the forms of support that can surround and promote the success of incremental improvements in teachers' practices. PLC discussions and reflections led to shifts in teachers' ideologies. Teachers realized the roles they played in causing the challenges that they were experiencing and therefore felt empowered to enact changes in their practices. Teachers also recognized the importance of taking a participatory approach by involving their students in developing interventions. The PLC model facilitated a culture of continuous learning, where teachers could draw upon the collective expertise of their peers, challenge assumptions, and refine their approaches based on evidence and their shared experiences.

Moving forward, it is crucial for educational institutions and policymakers to recognize the value of action research and PLCs as catalysts for pedagogical innovation and ongoing professional development. By providing structured support and dedicated time for collaborative inquiry, schools can empower teachers to become agents of change, continuously adapting and improving their practices to meet the evolving needs of their students. Future research could explore the long-term impact of sustained participation in PLCs on teacher efficacy, student outcomes, and school culture. Additionally, investigating the potential synergies between action research, PLCs, and other professional development approaches, such as coaching or mentoring programs, could yield valuable insights into creating comprehensive and holistic systems for teacher growth.

Ultimately, this study serves as a testament to the dedication and commitment of teachers to continually enhance their craft and create environments conducive to student success. By embracing a culture of inquiry, collaboration, and ongoing learning, educators can address immediate challenges and contribute to the broader advancement of math education pedagogy. The sustained, cumulative impact of ongoing action research and

PLC participation could be profound, leading to a continuous cycle of improvement that benefits both teachers and students alike.

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Article

Supporting Mathematics Instructors' Transition to Equity-Minded Active Instruction Using a Community of Practice Framework

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Abstract: With evidence mounting on the benefits of equity-minded and active mathematics instruction, increasing numbers of mathematics faculty members are seeking to transform their instruction. Yet, many lack the skills and/or confidence to make the transition. To support faculty in meaningful instructional improvement, the authors of this paper facilitate frequent and innovative professional development (PD) guided by a community of practice framework. PD is intentionally designed to be incremental and supportive. Using one-on-one interview data from ten faculty participants who participated in PD on equity-minded and active mathematics instruction, we report on three crucial characteristics of a community of practice: the domain, the community, and the practice. Findings have implications for mathematics departments that aspire to support instructors to transform their teaching. Incremental PD guided by a community of practice framework could support faculty through the challenges of instructional transformation.

Keywords: community of practice; professional development; undergraduate mathematics education; active learning; equity-minded instruction; departmental change

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1. Introduction

With increased publications pointing to the benefits of active learning over lecturing in mathematics and other science, technology, engineering, and mathematics (STEM) disciplines [1–7], as well as the importance of implementing equity-minded teaching practices [2,8–11], many mathematics faculty are motivated to improve their instruction [12–14]. An NSF-funded report on the state of instructional reforms across STEM disciplines stated:

> It appears there is a growing consensus . . . that educational reform is needed in undergraduate mathematics. There is a call for more active classroom engagement that attends to the education research literature and acknowledges equity issues [15]. (p. 117)

The report found an increase in faculty requests for professional development (PD) [15]. However, being convinced it is time to change is not the same as being equipped with the skills to change [16]. The field has a pressing need for innovative PD to improve instruction [12].

The aforementioned NSF report indicated that faculty awareness and use of evidence-based instructional strategies has increased but there is not yet widespread implementation of the strategies [15]. STEM faculty receive varying degrees of teacher preparation [17] and many default to the lecture-based mode of instruction that they experienced as learners [2,9]. Pedagogical innovation is further stifled by institutions that value research productivity over effective teaching [9]. As a result, instructional transformation has been slow [5,18] and lecture remains the predominant mode of mathematics teaching [19].

Further, recent publications increasingly point to the importance of attending to diversity, equity, inclusion, and belonging in mathematics teaching [2,8–11,20]. Equity-minded instruction could be new territory for faculty who were educated under outdated and problematic notions of colorblindness in mathematics and lack the knowledge to be critically race-conscious instructors. Thus, faculty who desire to transform their instruction need support to make the transition [21]. As stated by Rasmussen et al. [14], "as the use of active-learning approaches increases, so does the need for professional development" (p. 107). When designing this PD, we must consider that decades of PD for instructors have not led to widespread change. We must innovate.

1.1. A Need to Focus on Equity-Minded Instruction

Initially, research pointed to active learning as a means to address longstanding opportunity gaps in mathematics and other STEM disciplines [1–7]. However, as researchers continued to investigate the impact of active learning on course outcomes across student demographic categories, it soon became clear that active learning alone is not sufficient [2,8–11].

Reinholz et al. [20], who compared performance outcomes and participation of different genders in inquiry-oriented and non-inquiry-oriented college mathematics classes, noted that gender inequities could actually be exacerbated if inquiry-oriented instruction is not implemented "with explicit attention to gender equity in student participation" (p. 1). These researchers did not advocate against inquiry-oriented instruction but instead cautioned against overstating the benefits of active learning in closing opportunity gaps. They noted that a way to address this is by offering instructors PD "with an *explicit* focus on equity" (p. 36, emphasis in original).

Research in science classrooms has yielded similar results. Dewsbury et al. [2] compared lecture-based introductory biology courses to those taught with inclusive and active pedagogies. Although their research revealed positive outcomes in the promise of active learning to close opportunity gaps, they cautioned that implementing learning-centered pedagogy "without appropriate consideration of context" may lead to harmful outcomes (p. 2). Harmful outcomes could include maintaining or widening opportunity gaps.

Schmid et al. [17] pushed us to think outside the walls of our classroom and to consider systemic barriers that contribute to opportunity gaps. Institutional barriers could include the overall campus climate around student diversity, faculty diversity, course size and access to the major, instructional approaches, and the types of academic support in place for students. From these past studies, we note that faculty must be supported to make positive changes for student success.

1.2. Equity-Focused Professional Development to Support Instructional Transformation

Researchers have found value in PD for supporting faculty to transform their instruction [12,14,18]. A paramount challenge faced by institutions today is addressing opportunity gaps that recognize that systemic injustices provide some students with greater opportunities than others. Ching and Roberts [8] argued that opportunity gaps will only close once we embrace race-conscious teaching. However, many faculty may not yet be aware of the need to explicitly address issues of inequity in mathematics [8].

Perhaps more concerning, opportunity gaps can still emerge even in classrooms where faculty are using research-based teaching approaches and express a commitment to equity; thus, it is important that faculty be taught to recognize and address biases, to reflect on issues of privilege, and to increase their multicultural knowledge [17,22]. Increasingly, institutions are implementing PD programs, particularly those that focus on equity-minded and inclusive teaching practices [8,17].

1.3. Faculty Experience in Equity-Focused Professional Development

Research on the experience of faculty members who aspire to change is sparse [16,18], making our site ripe for this investigation. Noted exceptions include Reinholz et al. [22]

and Schmid et al. [17]. Reinholz et al. [22] presented results from a learning community of three university mathematics instructors who used a classroom analytic tool to track participation patterns among their students and found that the supportive community was a crucial aspect of the faculty learning experience. Schmid et al. [17], who described Madison Teaching and Learning Excellence that supports early career tenure-track faculty to teach more effectively, found that PD on inclusive teaching raised instructor awareness and increased their use of inclusive teaching practices.

Since 2018, authors Marzocchi and Soto have facilitated a PD program focused on instructional improvement within our mathematics department at California State University, Fullerton (CSUF), a large, comprehensive public university with designations as a Hispanic-Serving Institution and an Asian American and Native American Pacific Islander-Serving Institution. Faculty members have been invited to participate in a variety of PD activities ranging from low stakes to high stakes [23,24].

Our model of PD is intentionally designed to be incremental and supportive. The original focus of our activities was on supporting faculty to transition to active learning. In 2021, we began to explicitly emphasize equity-minded instruction rather than assuming equity was an implicit consequence of active learning. Throughout the tenure of our project, we have engaged dozens of faculty members in PD offerings while offering small stipends as tokens of appreciation for their time. The overall goal of this work is to support faculty as they incorporate more equity-minded active learning into their pedagogical practices. With this in mind, we use the community of practice framework to answer the following research question: how do mathematics faculty describe the domain, the community, and the practice after participating in an incremental professional development on equity-minded active learning pedagogy guided by a community of practice framework?

2. Materials and Methods

Context

The authors, who are faculty at CSUF, are the principal investigators of National Science Foundation-Funded META: Mathematics Equity through Teaching Actively Grant (Grant number 2142122), which strives to support faculty in the transition to equity-minded active mathematics instruction. The primary mechanism we use to support instructional transformation is faculty PD. Our PD offerings use innovative approaches to support incremental change. We strive to offer a variety of lower-stakes and higher-stakes PD options. Lower-stakes options generally involve less of a time commitment and/or less accountability. Higher-stakes options generally involve a greater time commitment and more accountability. The following PD options were offered to participants during the time of this research:

- Learning management system: Faculty joined a learning management system (Canvas) course to access resources, ask questions, and contribute to discussions related to the PD.
- Strategy of the month: Each month, a teaching practice was selected and shared with faculty through a department-wide email. The teaching practices included an explicit way to support equity-minded active instruction along with several strategies to enact the teaching practice in the classroom. As the year progressed, voluntary participants were elevated to leadership roles and tasked with selecting the subsequent strategy of the month.
- Brown bag lunches: Faculty attended monthly brown bag lunches to discuss targeted topics related to equity-minded active instruction. During a brown bag lunch, participants brought their own lunch and had informal but semistructured conversations around successes and challenges. Ideas were shared as participants learned in a supportive community environment. As the year progressed, voluntary participants were elevated to leadership roles and tasked with leading the brown bag lunches.
- Workshops: Participants were invited to attend a workshop each semester. The workshops were run actively so that the participants were themselves experiencing

the equity-minded active teaching practices that we encouraged them to implement in their classrooms. One workshop was led by our student researchers, allowing participants to learn from students about how to better support student learning.

The PD offerings are described in detail elsewhere [25].

To date, fifty mathematics faculty from our department have attended at least one of our PD offerings. Faculty participants represented every rank (teaching assistants, part-time lecturers, full-time lecturers, pretenure tenure-track faculty, and tenured faculty) and four mathematics subdisciplines (applied, pure, statistics, and teaching) in our department.

3. Participants and Data Collection

Ten faculty participants volunteered to participate in an end-of-semester interview, nine in spring 2022 and one in fall 2022. Among the ten interview participants, ranks included part-time lecturer, full-time lecturer, pretenure tenure-track faculty, and tenured faculty. Participants' mathematics subdisciplines included pure, statistics, and teaching. Participants' aggregated self-identified demographic characteristics included:

- Gender: women and men
- Race/ethnicity: Asian and white
- Nation of origin: foreign born and United States born
- Linguistic repertoire: multilingual and monolingual English speakers
- College lineage: first-generation college as well as those whose parents attended college.

Interviews were led by a principal investigator, lasted approximately an hour, and were audio recorded and transcribed. Interview questions included questions about the participant's background experience in mathematics, identity components, conceptualization of and experience with equity-minded instruction, conceptualization of and experience with active instruction, and experiences with PD within and outside of META.

Community of Practice Framework and Data Analysis

We apply a community of practice [26,27] framework to our PD design and qualitative research methods. Wenger-Trayner and Wenger-Trayner [27] describe communities of practice as "groups of people who share a concern or a passion for something they do and learn how to do it better as they interact regularly" (p. 2). The learning that occurs in a community of practice is authentic and informal. We employ qualitative research methodologies using interview transcripts as the primary data sources and a community of practice framework for data analysis.

Communities of practice have three crucial characteristics: (1) the domain, (2) the community, and (3) the practice [27]. The three crucial characteristics served as the primary sorting categories in our data analysis, as described below. The *domain* is the shared interest of the community members. Members have a shared competence and shared commitment to the domain. The *community* includes the members, their relationships with each other, and their engagement in joint activities and discussions. Members learn from each other and care about each other's learning. The *practice* refers to the learning products that emerge within the community such as the "experiences, stories, tools, [and] ways of addressing recurring problems" (p. 2). The repertoire of resources emerges from the experiences and expertise within the community. Members can have diverse experience levels and knowledge; this diversity can enhance the group dynamic and provide learning opportunities. Overall, a community of practice gives members the collective responsibility for "managing the knowledge they need, [and] recognizing that, given the proper structure, they are in the best position to do this" (p. 5).

Oliver and Olkin [28] reported success using a community of practice model to shift department instruction toward active learning. In fact, after just one semester of launching their PD work, they saw opportunity gaps close in targeted courses. The community of practice model also helped to shift department culture from individual preparation to one that involved regular sharing of ideas about pedagogy, content, and active learning. Schmid et al. [17] also used a community of practice model to ground their work in the

Madison Teaching and Learning Excellence Program, which supports early career tenure-track faculty to teach more effectively. Under their model, participants are encouraged to use the experiences of others in their cohort, as well as peer feedback, to inform their own teaching practice.

In our qualitative analysis, we used the three characteristics of a community of practice (the domain, the community, and the practice) to parse interview transcript data. Relevant excerpts were sorted into one or more of these primary characteristics. A fourth sorting category was included for nonexamples. Nonexamples captured interview excerpts that did not align with the crucial characteristics of communities of practice. After the primary sort, sorted excerpts were analyzed within crucial characteristics for themes and exemplars. We present our findings in the following section.

4. Results

As described above, Wenger-Trayner and Wenger-Trayner [27] indicate three crucial characteristics of a community of practice: the domain, the community, and the practice. We use interview data to exemplify the three characteristics in the context of mathematics faculty members working together, during incremental and supportive PD, to learn and support each other as they transform their instruction to be more equity minded and active. For each characteristic, we describe the characteristic in our setting, then report examples and nonexamples (when applicable) of how participants did or did not experience the characteristic.

4.1. The Domain

According to Wenger-Trayner and Wenger-Trayner [27], the domain is the shared interest of the community members. In the case of our META PD, faculty participants were interested in shifting their instruction to become more equity minded and active. Participants expressed motivation to join the community because they believed that equity-minded instruction was important and thought META would provide time, space, and support for them to explore this interest. Participants said things like, "I know [equity is] the big hot topic right now, which it should be", "[equity-minded instruction] was definitely something I wanted to have conversations about and felt like I was not at a level that I wanted to be", or "[equity] is something that the entire university is figuring out right now".

However, some participants felt that university-level initiatives "don't feel like something I can use in the classroom". Instead, several commented that META is "the only space where I feel like [equity] has been a focus" or that META "is specific, and it's colleagues, and I can do this in calculus, I can do this in a statistics class, I can do this in my classroom". Essentially, participants reported a shared interest in equity-minded instruction and felt that the university and discipline were encouraging a shift to more equity-minded instruction; several reported that META provided the space to explore this domain.

4.2. Examples

Although participants reported a shared interest in increased implementation of equity-minded instruction, their specific motivations varied. One participant, who has been teaching for several decades, reported that they "need to know more" because when they were going through their education "a long time ago, a lot of these conversations [about equity] weren't taking place". They appreciated the META events for providing the opportunity to learn from "people who are a little younger and maybe had more exposure [to conversations around equity]". Another participant, who identifies as having several historically dominant identities, noted that "[equity] is something where I do feel ill-equipped" and that they appreciated learning from "the experiences from other people in our department who have different backgrounds and different experiences". They particularly appreciated the opportunity to recognize when to "check [their] privilege". Yet another participant reported that they attended META PD because they "wanted to get better at teaching" due to their unfavorable student evaluations of teaching. They feel that their teaching is a bit more "old school" and that META can provide the support for them to shift to more active teaching. Finally, another participant, who has been participating in META since its inception, reported:

> I don't mean to brag, but I feel like I'm okay with active learning now . . . but when I see the word equitable, then it's like, 'pause', what the heck was that? Is there even such a thing? So I wanted to learn in that aspect.

This participant felt that their teaching practice was strong in terms of active learning but that they could improve to be more equity minded. The participants quoted above are in different stages of their careers and have varying levels of skills and confidence to implement equity-minded active instruction. By this, we mean that some of our participants have been learning about and implementing classroom strategies for equity-minded active instruction for many years, whereas others are only just beginning to engage with these ideas. Thus, some participants arrive to our workshops with a greater foundation of knowledge and greater confidence than others. Yet, they participate in META PD because of a shared commitment to the domain.

4.3. Nonexamples

As a nonexample to the domain described above of commitment to improving instruction to be more equity minded and active, one participant reported that they were not committed to the domain. They stated:

> it would take more work to convince somebody like me that these things are, number one important in the classroom, and number two doable in the classroom . . . I don't have that lived experience and I don't feel like I have necessarily the credibility to pull off [equity-minded teaching].

Interestingly, this participant continued their involvement in META events, even though event participation was voluntary. An analysis of the next crucial characteristic, the community, reveals the rationale for this participant's continued involvement. They explained that "the social ties that I've had with [the leaders] and the other META participants, even though this isn't something I'm passionate about, makes it easier to get the motivation to show up". The benefits this participant reported from the community outweighed their lack of commitment to the domain. The next section will share how the other participants reflected on the community characteristic.

4.4. The Community

According to Wenger-Trayner and Wenger-Trayner [27], the community includes the members, their relationships with each other, and their engagement in joint activities and discussions. In the case of our META PD, participants represent every faculty rank and four mathematics subdisciplines. As described above, they also possess varying levels of skills and confidence to implement equity-minded active instruction. META provides time, space, and support for faculty to learn from each other through participation in a variety of incremental PD activities. One participant described the community as a "kind of grassroots support" that involves "just having good colleagues". Another described the value of the shared expertise in the room, stating "the people in that room are impressive to me, they're good role models . . . and it slowly changes your view on teaching and what your goal should be in the classroom".

Other participants noted value in META's approach, which invites participants to candidly discuss both their successes and their challenges. One participant reported that they benefit from "having my colleagues around where I feel like it's safe to share my struggles especially . . . and it's just great to be in the same room with the people who are willing to try new things and are not afraid of failing". Similarly, another participant shared that "it is nice to have the chance to talk with other instructors who are trying the same things" and to know that "other people are trying and it's not smooth sailing for everybody as well". They describe this in contrast to going "to a workshop to sit there and listen to different strategies". In the case of META, a community is formed for faculty to learn from each other, as opposed to a top-down workshop approach.

An additional reported benefit of the community-centered approach is that learning extends informally, beyond the scheduled PD activities. One participant shared that:

> just talking to other instructors that I know are in the META group has been very helpful . . . even just popping by . . . there were a couple times that I would just run by [a META leader's] office . . . we were all very supportive of one another, I never felt like my ideas were stupid, and we were all able to listen to one another.

Once the community was established, participants knew they had a set of colleagues, united under a shared domain, whom they could consult when issues and questions arose.

4.5. Examples

Participants described a variety of ways that the community supported their learning. One of these ways was through healthy peer pressure. A participant shared, "there is, in fact, some pressure . . . just having colleagues raise the bar in terms of teaching leads me to raise my own bar a little bit". Another participant felt that their peers helped to increase their confidence and skills. They shared:

> I think that if you want to talk about active learning, engaging with other people is a really important aspect of that and that's part of what META is all about, that community that you're building . . . and the more I learn the more confident I feel, the more I think I can do it, the more support I feel.

This stands in contrast to the historical culture of university teaching behind a closed door, with little opportunity to consult with colleagues.

Another participant, who has been a longtime participant in META PD, noted the value of growing into leadership roles in the community. They shared:

> all of my past experiences with META have been so positive that I knew I would . . . continue to develop as an instructor and, also, I knew I could contribute as well, because . . . [from] the start . . . I did a lot of the activities and professional development . . . So I knew I could learn more as well as contribute. I've been a part of [META from the beginning], so it's like, 'Hey I can help too.'

In this case, a participant with advanced skills and confidence, through their years-long participation in our project and years of implementing the teaching strategies we encourage, was able to participate in the community nonetheless by taking on leadership roles.

Finally, we are compelled to report that two of the participants who were part-time lecturers reported on the value of being invited to the community. One shared:

> to be invited [to META events] was pretty cool. I think as a part-time faculty member we're a little bit isolated sometimes, there's a lot of stuff like this that goes on that we're not always invited to, so it's just nice to feel included.

They went on to explain that:

> as a part-time faculty member . . . sometimes it feels like there's not a lot of incentive to do your job better and the ways to defeat that are interactions and speaking with [colleagues] around you and realizing these people are doing a lot and going out of their way to really try and improve their classes and I think I should be doing the same thing.

Another participant who was a part-time faculty member explained, "I was forever grateful when [a META leader] reached out to me when I was just a part-time lecturer". This participant shared that several years ago they:

> actually suggested that [META leadership] needs to reach out to the part-time faculty because they work at multiple campuses and they have more experi-

> ence learning a little bit from here, from there, and they have richer experience compared to us full-timers where we just work at one school.

This participant reported that now they "appreciate the diversity of the faculty members this time around because now, I started seeing more part-timers getting on board". The above examples show how the community characteristic of a community of practice supports faculty by providing healthy social pressure, building confidence, providing leadership opportunities, and increasing a sense of belonging for part-time faculty members.

Our dataset did not include a nonexample for the community characteristic.

4.6. The Practice

According to Wenger-Trayner and Wenger-Trayner [27], the practice refers to the learning products that emerge within the community. In the case of our META PD, we focus on incremental improvement where faculty set their own goals and reflect on their own instruction after actively experiencing active learning themselves within their communities. Incremental improvement means that faculty have the agency to select something that allows them to grow but also feels manageable for their classroom, implementing their selection, and continuously reflecting and tinkering for continuous growth in their teaching practice. Faculty come together to discuss successes and challenges. One participant noted the benefit of gradual improvement in a low-stakes environment, stating:

> I can go to a [META] workshop and increase my teaching effectiveness by epsilon ... I'm not going to go and turn my class upside down but I'm going to go and make these small changes that little by little add to my toolkit and that's less intimidating, but I also feel benefit coming from it ... just knowing that that something is going to be relevant to me and often something I can put into practice pretty quickly.

Another participant noted the metalearning that occurs in META by explaining, "you share the ideas [at META PD], but we also ourselves do the ideas, so you use the teaching strategy to teach us the teaching strategy".

4.7. Examples

In terms of specific products emerging from the community, participants reported that they incorporated numerous new equity-minded and/or active teaching practices. These included board work, interest surveys, group work, grouping strategies, jigsaw, notice and wonder, think-pair-share, wait time, and worksheets in disguise. For example, one participant noted how their strategy for creating student groups changed after participation in META. They stated that after a META PD event, "I was encouraging the students to get into groups and then work with each other, whereas before, I kind of hoped that they would naturally form". As another example, a participant noted the effectiveness of the notice and wonder strategy in their classroom, stating:

> the notice and wonder was my very first time doing it and I was very pleased ... [I used it with a notoriously challenging class and] they surprised me ... they gave me really, really insightful answers ... and they asked very powerful questions ... And I didn't have to tell them anything, they just figured it out by themselves ... So I like that method a lot, like a lot.

The notice and wonder strategy was shared by one community member during a META workshop and was quickly implemented by many other community members who were delighted by the results.

In addition to naming new teaching practices they learned, participants were also able to name strategies they planned to try in the future which, for some participants, included notice and wonder, Padlet, or worksheets in disguise. This points to the community's culture of continuous improvement.

Not only did participants report learning new teaching practices but some also reported philosophical shifts after participation in the community. One participant described a powerful shift in how they thought about equity in their classroom:

> The realization that I've had about equity in more recent years is that you need to assume that it does not exist when you enter your classroom. I think I used to just take the approach of treating all my students the same and that'll be fair. Until you really think about it, and some students have had a lot of advantages and everything's worked out well for them. And other students may have had terrible high school teachers or they might be the first one in their family [to go to college], all kinds of disadvantages that you can have. So just being more mindful of what you're saying and what you're doing to try and help as many students as possible. Make sure that the way that you're running your classroom is working for as many students as possible and not just the students who already had everything work out for them. I think if everyone does that hopefully we can fill in some of those equity gaps and bring everyone a little bit closer together, without dropping a lot of people as well. Just raise more people up.

This participant provides an exemplar of a faculty member transitioning from the pervasive colorblind approach of mathematics teaching to an approach that is more equity minded by attending to individual students

Other participants discussed philosophical shifts in how they cultivated their classroom learning community. One participant explained:

> Something that I've learned is that creating the community so that students want to engage with each other is really important, it's almost more important than what I do in the classroom. And this is something that's definitely different from when I was a student is how to develop that community amongst the students and also develop that relationship with the students that's different than just a lecturer-lecturee role.

Another participant who experienced a similar philosophical shift around the classroom community explained that:

> I started coming up with a question of the day . . . just kind of looking for ways to build a little bit more community. I think in the past stuff like this, I viewed as not important or not part of the mathematical content, kind of not my job. But I think I've been convinced more that it's worth it, it's only a couple minutes at the beginning of class and if you can get more people interested . . . or enjoying it, I think that's worth it and you're going to get more benefits than what you lose for wasting those three minutes.

The above examples show that participants' teaching practices were expanded through classroom strategies and philosophical shifts.

4.8. Nonexamples

As a nonexample to the above participants who benefited from the learning products that emerged from the community, one participant reported feeling that the learning within the community was not advanced enough for them. They reported:

> I've tried most if not all the techniques. [So I made a suggestion to META leadership that] maybe in the future the META team could organize a different type of workshop for people who have done it many many times . . . For me, personally, I was looking for more . . . If it's possible to have a different type of workshop for advanced [strategies], I think it would work out a lot more.

This participant did not feel that they benefitted as much from the practice of the community as they could have if a more advanced community existed.

5. Discussion

PD is an important mechanism to support the increasing number of mathematics faculty members who aspire to transform their instruction [12–14]. The findings above exemplify the benefit of guiding PD with a community of practice framework. Under this model, PD facilitators provide time, space, and support to faculty. From there, as evidenced above, faculty can learn from each other's expertise. The community of practice framework includes the crucial characteristics of the domain, the community, and the practice. Our findings revealed that participants shared a domain of commitment to increased use of equity-minded instruction. Participants described a supportive community to reflect on their own learning together, including both successes and challenges. The resulting practice was a toolkit of new strategies as well as new conceptualizations of the classroom learning environment.

META was designed to support faculty who desire to transform their instruction to be more equity minded and active. Oliver and Olkin [28] used a community of practice model to increase faculty implementation of active learning. They found that "the strength of the community of practice, paired with a consistent message of support, has resulted in instructors who are increasingly less afraid to try out a new activity in their classes" and that "no coercion or heavy-handed coordination of curricular material has been necessary to increase active learning in the classroom" (p. 266). According to Reinholz et al. [20], this support is even more crucial when faculty aspire to use equity-minded teaching practices. They claim that "unless instructors receive sustained, ongoing PD of this type [that explicitly addresses inequities], we imagine that it will be difficult for most instructors to address the surmountable inequities in mathematics education" [20] (p. 36).

Yet, despite PD participants collaborating under a community of practice model, our incremental approach to PD allowed them to individually engage in a way that aligned with their own skills, confidence, and goals. We offer an incremental approach both in the PD offerings of META as well as in the equity-minded active strategies we explore with participants. With a variety of low-stakes to high-stakes PD opportunities to choose from, faculty are given multiple opportunities to engage in incremental improvement of their teaching practice. Similarly, at the classroom level, each participant could set their own goals from a variety of teaching strategies, allowing them to incrementally improve their instruction to be more equity minded and active. For instance, a participant who largely teaches through lecture could opt for a simpler strategy that keeps the lecture intact but swaps out the questions they ask of their students. A more experienced participant who regularly engages their students in collaborative learning could opt for a more complex strategy that changes the way students interact with each other. Even when working in a community together, each individual faculty participant had the autonomy to incrementally improve their instruction at their own pace.

As is often true with PD initiatives, time is a limiting factor. Despite our efforts to provide time and space for faculty to learn together in a community, two participants reported feeling that there was not adequate time allocated for collaboration. One stated that sometimes they felt the conversations "were a little rushed . . . some of the conversations had to be cut short because we're all trying to cram so much stuff in there . . . [If there was more time], I could have had some really rich discussions with other faculty". A second participant noted an issue with limited time, stating:

> it's just a matter of having the time to have those discussions with colleagues [to ask things like], 'how did you fit this in' or 'how did you work this out'. I think that is probably a challenge for all of us, finding the time.

Despite these time constraints, as evidenced above, faculty were able to unite under a shared domain to support each other in a community as they worked to improve their practice.

At a time when there is a pressing need for faculty to shift to equity-minded and active mathematics instruction, our META PD provides a case for using a community of practice model to support faculty in carrying out the challenging work of transforming their instruction. The findings reported above indicate that faculty can come together under a common goal and draw from the expertise of the community to work toward incremental but meaningful instructional improvement. Participants of varying academic rank, from varying mathematics subdisciplines, and with varying confidence and skills around equity-minded active instruction were able to support each other in carrying out the difficult work of instructional transformation.

5.1. Limitations

Participation in META PD was voluntary. Thus, the participants of this research represent a sample of mathematics faculty who were aspiring to improve their instruction and are willing to devote time to this endeavor. The results of our research may not generalize to a context where PD participation is obligatory.

We also note that we were able to provide small participation stipends to participants. When replicating our project, other institutions should consider compensating participants. If compensation is not possible, other institutions should consider whether this will impact participation.

5.2. Future Research

The next phase of our research will investigate the scalability of our project to different contexts. We will collaborate with a mathematics department at a different institution type as well as a science department in our institution. This will allow us to research the affordances and limitations of our PD model outside of our department. An opportunity that could arise from this research is an improved PD model that can broadly benefit STEM departments of varying disciplines and institution types. An anticipated challenge is building our knowledge base on how best to support the needs of departments outside our own, with varying disciplines and institution types. We will use the results of this research to refine and strengthen our model to benefit other departments broadly.

5.3. Conclusions

Many participants reported ways in which the META PD aligned with crucial characteristics of communities of practice: the domain, the community, and the practice. Although there were some nonexamples, participants' interview responses paint an overall picture of a community of faculty motivated by a common interest who learned from each other's expertise to create products to improve their teaching. The community of practice model may be enticing to departments; rather than a costly high-stakes venture, effective PD can be local and grown by the faculty members themselves.

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Examining Teachers' Professional Learning in an Online Asynchronous System: Personalized Supports for Growth and Engagement in Learning to Teach Statistics and Data Science

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Abstract: Teachers' professional learning often includes online components. This study examined how a case of 37 teachers utilized a specific online asynchronous professional learning platform designed to support teachers' growth in learning to teach statistics and data science in secondary schools in the United States. The platform's features and learning materials were designed based on effective online learning designs, supports for self-guided learning, and research on the teaching and learning of statistics and data science. We paid particular attention to the features we designed into the platform to support self-regulation and personalizing the experiences to meet their preferred learning goals such as allowing for free choice of learning materials, flexibility of when and how long to engage, providing personal recommendations based on user input, internal systems to track progress, and generating certificates of completion. In this study, we used a case study with both quantitative and qualitative data to examine whether teachers had gains in meeting learning goals related to their development in teaching statistics and data science, had sustained engagement, and found the features for personalization supportive for their learning. Results showed, overall, positive growth towards meeting learning goals and making small changes towards improved classroom practice. Most teachers were generally engaged in sustained ways across the study period, though we found six different patterns of completion that highlight ways in which teachers' goal-directed and self-regulated learning occurred within the busy schedules of educators. Several personalized features, especially the recommendations and tracking system, were highly utilized and perceived as supportive of teachers' learning.

Keywords: teacher learning; statistics education; e-learning; recommendation systems; self-regulated learning; online professional development

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1. Introduction

Teachers tend to be lifelong learners, motivated to pursue professional learning that is meaningful to their particular needs. In 2013, Marrongelle and colleagues [1] noted "it is incumbent on the field to capitalize on emerging technologies in the design and delivery of effective professional development" [p. 208]. While the past decade has seen an increase in the development of opportunities for online learning for teachers [2–4] and specifically for personalized learning for mathematics teachers in online spaces [5], more work is needed to provide additional research-based opportunities for educators and to understand the impact of online teacher learning. Our research contributes to how teachers can be supported in self-directed online educational environments in which their learning and changes in teaching practices may be incremental in nature.

The Invigorating Statistics and Data Science Teaching through Professional Learning [InSTEP] professional learning platform aims to support grades 6–12 teachers' professional learning in teaching statistics and data science concepts through personalized online learning. Statistics and data analysis are included in standards for both mathematics and science [6,7], and many states across the U.S. have re-envisioned pathways that include a heavier emphasis on statistics and stand-alone courses on data science [8]. In the U.S., there has been national-level support for this increased focus on statistics and data science through position statements from organizations such as the National Council of Teachers of Mathematics and American Statistical Association [9–11], and reports such as the Statistical Education of Teachers [12] and the PreK-12 Guidelines for Assessment and Instruction for Statistics Education II [13].

Our approach recognizes the busy life of teachers and assumes that learning new ideas and approaches can happen in incremental and personalized ways. This can result in small shifts in preparedness to teach and small changes to teaching practices that have the potential to impact students' learning opportunities related to statistics and data science. Our three research questions are as follows:

RQ1. How does participation in professional learning in the InSTEP online asynchronous platform contribute to teachers' confidence, knowledge of statistics, and professional growth towards learning to teach S&DS?

RQ2. In what ways do participants engage with the learning activities in the InSTEP Platform?

RQ3. In what ways did personalized features support their professional learning in the InSTEP platform?

2. Background and Framing

To aid us in considering how professional learning experiences may have an impact on teachers' beliefs and perspectives, understandings, and practices, we draw upon Mezirow's [14] theory of transformational learning in adult education, which is consistent with constructivist assumptions about learning. Mezirow [14] describes how meaning schemes--comprising of knowledge, expectations, beliefs and perspectives, and feelings --are used by an individual to interpret their experiences, and through reflection on these experiences, may transform their understandings. Peters [15] illustrated how this theory could be used to understand statistics teachers' development of understanding statistics concepts. For example, a teacher might transform her meaning scheme for teaching statistical variation by rejecting a conception that variation is only a measure computed to indicate spread. Transforming meaning schemes often begins with a stimulus, a disorienting dilemma, which requires one to question understandings and beliefs formed from experiences [14]. We intend for materials in the InSTEP learning opportunities to trigger disorienting dilemmas for teachers that can lead to changes in their meaning schemes.

Because the InSTEP platform provides asynchronous learning opportunities for teachers to complete on their own time, self-regulation learning theory is applicable to our work. The process of self-regulated learning involves cognitive, affective, and behavioral components in a continuous process of achievement striving, monitoring, and evaluation that successful learners engage in over time [16]. Adult learners with a stronger sense of self-regulation in their learning are more successful in online learning (e.g., [17]). After an episode of poor performance (e.g., rating themselves low in confidence to teach a statistical idea or encountering a statistics question they are unsure how to answer), a teacher may re-evaluate their learning goals. In contrast, good performance is a sign that progress is being made and their strategy is effective, and this may lead to further goal setting. Specifically in learning to teach statistics, researchers have shown that teachers are motivated to pursue professional learning related to improving their own statistical understandings and confidence to teach as well as learning new teaching strategies for teaching statistics [18]. Others have shown that feedback and positive experiences support teachers' motivation, improved self-efficacy, and are central to maintaining persistence in professional learning related to statistics [19,20].

2.1. Effective Online Professional Learning for Educators

The past 20 years of development of online learning environments has drawn upon the foundational work of Mayer and Moreno [21] about the importance of using multimedia resources to support active engagement. Evidence from past studies specific to teachers' professional learning in online settings shows that focusing on the development of teachers' content understandings and pedagogical content knowledge provides support for learning, promotes active engagement, and addresses varied needs and abilities of participants, which can be effective in changing teachers' instructional practice (e.g., [2,3,22]). Findings from Qian and colleagues [23] led to three recommendations for effective online professional learning: use activities that match teachers' background knowledge and experiences, align activities with curricula, and use motivational design to enhance teachers' engagement. Six design features that emerged from the work of Powell and Bodur [24] include the following: relevancy, authenticity, usefulness, collaboration and interaction, reflection, and context. They also emphasized the importance of learning being job-embedded, meaning teachers should be able to use resources in their job, and that learning opportunities utilize aspects of a teachers' job (e.g., understanding content they need to teach, planning lessons, making sense of students' work, implementing tasks and reflecting on learners' experiences). This aligns with four design principles that informed the design of a collection of Massive Open Online Courses (MOOCs) for Educators offered from an institution in the U.S. that support the following: (a) self-directed learning, (b) peer-supported learning, (c) job-connected learning, and (d) learning from multiple voices [25].

Some members of the author team (Lee and Mojica) designed and implemented three educator-specific MOOCs aimed at developing expertise in teaching statistics using rich data-enabled experiences. Teachers from around the world engaged in these courses and have reported changes in their statistics teaching practices to include larger real datasets and using an investigation process [26,27]. Unequivocally, teachers' confidence to teach statistics drastically increases after engaging in the courses [27]. Many educators did not complete an entire course but instead only engaged in the first 1–2 units (of 5) of a course. Reasons for this pattern of engagement were often that they did not have time to complete a course or were looking for more resources to help learn statistics themselves. Some teachers indicated in follow-up surveys that although they did not finish a course, they learned how teaching statistics involves a cycle and habits of mind and found resources for their classroom that met their needs [28]. Deng and colleagues [29] analyzed patterns of engagement across many MOOCs and classified users in three general categories of engagement: (1) individually engaged users typically enrolled in MOOCs of shorter duration and were highly engaged with completing only a small portion of a course; (2) least-engaged users who also only completed a subset of the course but with mid to low levels of engagement over time; and (3) wholly engaged users who were motivated by the course goals, were highly engaged, and had a high rate of completion. These three patterns are similar to those found by Wiebe and colleagues [30] in MOOCs specifically designed for educators. In their review of MOOCs, Davis and colleagues [4] noted that the designs of typical MOOCs do not account for a learner's past behavior in delivering personalized content, and they suggested that developing and implementing adaptive, personalized systems in MOOCs could make them more adaptable and able to cater instruction based on individual learners. Results such as these led our team to envision a different online professional learning experience for teachers, which led to the InSTEP platform.

2.2. Statistics Teacher Practices, Knowledge, and Beliefs

We know that most mathematics teachers are underprepared to teach statistics and data science concepts and often feel less confident to teach such concepts as compared to other areas of their curriculum (e.g., [31–33]). For over a decade, many researchers have been encouraging opportunities for students to actively engage in real data investigations (e.g., [34,35]). Current recommendations to investigate large data require the use of technology tools throughout an investigative process, such as in preparing, collecting, exploring and visualizing, and summarizing data [36]. While researchers have shown that grades 6–12 students can successfully manage, wrangle, visualize, and model big data sets (e.g., [37,38]), teachers often only utilize tidy, small data sets with students [35], even in AP Statistics courses [39].

Affective constructs such as teachers' beliefs and perspectives about statistics are an important component to building effective statistics and data science teaching practices. These include a teacher's ideas about the nature of statistics, about oneself as a learner of statistics, and about the classroom context and goals for students' learning statistics [20,40]. Experiences learning statistics with a focus on tools and computations may lead teachers to believe statistics is about a set of procedures to produce results or graphs. However, teachers may also feel that reasoning with context-rich data and uncertainty in statistical claims can make statistics difficult to learn and teach (e.g., [31,41]). Confidence to teach statistics is influenced by a teacher's beliefs, their experiences in learning and teaching statistics, and statistical understandings [31,42]. Eichler [40] posited that the focus of teachers' intended curriculum in statistics can be considered on a continuum from traditionalists (focused on procedures) to those wanting students to be prepared to use statistics in everyday life (focused on an investigative process tightly connected to contexts of real data). A goal in statistics teachers' professional learning is to move teachers along this continuum, which requires impacting teachers' beliefs about the nature of statistics and learning goals for students. In prior work, Lee and colleagues [27] showed that engagement in online professional learning for teaching statistics can shift teachers' perspectives about the nature of statistics and use of real-world investigations in their teaching and significantly increases their confidence to teach statistics.

To support students in developing productive statistical thinking, teachers need to carefully reconsider the use of procedurally oriented, teacher-centered learning environments. "Learning statistics is not about passively acquiring a set of facts and procedures but rather about actively constructing meaning and understandings of big ideas, ways of reasoning, and articulating arguments" [43] (p. 475). Ben-Zvi and colleagues [43] suggest that focusing on interrelated aspects of instructional design such as the tasks used and ways to orchestrate discussions about trends in data are needed to impact the way statistics is taught. Building from this, we argue that effective instructional design for supporting students' learning of statistics and data science should focus on seven interrelated dimensions and how each operates in relation to others: (1) developing students' thinking and practices of doing statistics in authentic ways; (2) focusing on central statistical ideas such as variability and uncertainty; (3) using well-designed tasks; (4) using real multivariate datasets; (5) supporting discourse and argumentation about claims with data; (6) integrating technological tools to support data processing and visualization; and (7) making sense of students' thinking in written, verbal, and technological work with data.

3. Design of the Online Platform to Impact Teachers' Learning

The design, development, and implementation of the InSTEP online personalized professional learning platform aims to support teachers' growth in knowledge and confidence are needed to create effective statistics and data learning environments. We hypothesized that personalized learning experiences focused on statistics content and pedagogy can effectively provide sustained engagement that results in motivating teachers to engage in learning opportunities aligned with their interests and goals, increasing teachers' confidence in teaching statistics, meeting teachers' professional growth goals, advancing their ability to create meaningful statistical learning environments for their students, and perceiving the personalized supports and learning materials as effective in supporting their learning goals.

The InSTEP platform is unique in several ways. Building from self-regulation learning theory, the design of incremental professional learning opportunities should help a teacher identify their goals and desired outcomes, and support the tracking of progress to achieve desired goals. A well-designed self-guided learning experience can also help learners develop self-regulatory abilities [17,44]. Features in the InSTEP platform are based on research-based design principles of effective online learning and include being self-guided and ongoing, contextualized and job-embedded, and learning from multiple voices that include experts, students' voices, and other teachers. The InSTEP platform also contains features to assist teachers in setting and monitoring goals, finding learning opportunities related to their interests, providing feedback, and tracking their progress. First, the platform provides opportunities for teachers to build skills in data investigations and innovative teaching approaches based on practices of data professionals and research on students' learning with data. Second, we personalize learning to meet teachers' professional needs through customized recommendations, allowing teachers to select learning activities based on these recommendations or other goals and interests [45].

3.1. Structure of Learning Experiences

A major goal of InSTEP is to support teachers' growth in knowledge and confidence to create effective statistical and data learning environments where all students are learning about important statistical and data ideas and engaging in key practices and processes to make sense of data. Central to our approach is building teachers' expertise in understanding interrelated dimensions of statistics and data learning environments that support students' reasoning about statistics and data [43]. Teachers have opportunities to engage with materials through two primary opportunities: (1) engaging in a data investigation module structured to help teachers experience the six phases of a data investigation process [46,47] through investigating a real-world phenomenon or issue with a larger multivariate dataset using an online data tool, and (2) learning with multimedia (text, video, images) materials organized in modules that include active engagement and opportunities to reflect.

The Learning Hub provides a visual interface of the organization of learning experiences that can also be used to navigate to specific experiences (Figure 1). The first type of learning experiences, data investigations, are organized by the Data Investigation Process (see top of Figure 1). This process helps guide teachers to investigate a problem or phenomenon using real data and a technology tool and provides opportunities for teachers to engage with data. Teachers are guided in exploration using the different data investigation phases: framing the problem, considering and gathering data, processing data, exploring and visualizing data, considering models, and finally communicating and proposing action as a result of making sense with data.

Modules, the second type of learning experience, are organized by seven dimensions (Figure 1) to support teachers in using well-designed tasks to support statistical thinking by engaging students in key data and statistical practices and processes to develop central statistical ideas about statistics and data. This approach involves integrating many opportunities for teachers to use real data to engage in investigations using technology tools that afford statistical and data practices. Our approach helps teachers learn to establish practices that promote productive argumentation and discourse, which include making and supporting data-based arguments, and the use of assessments of students' thinking about statistics and data to inform instructional decisions (not visible in Figure 1). Learning activities within modules are organized by essential resources, foundational materials to understand key ideas that are needed for module completion, and extended resources, which are materials that go beyond and may include classroom-ready resources.

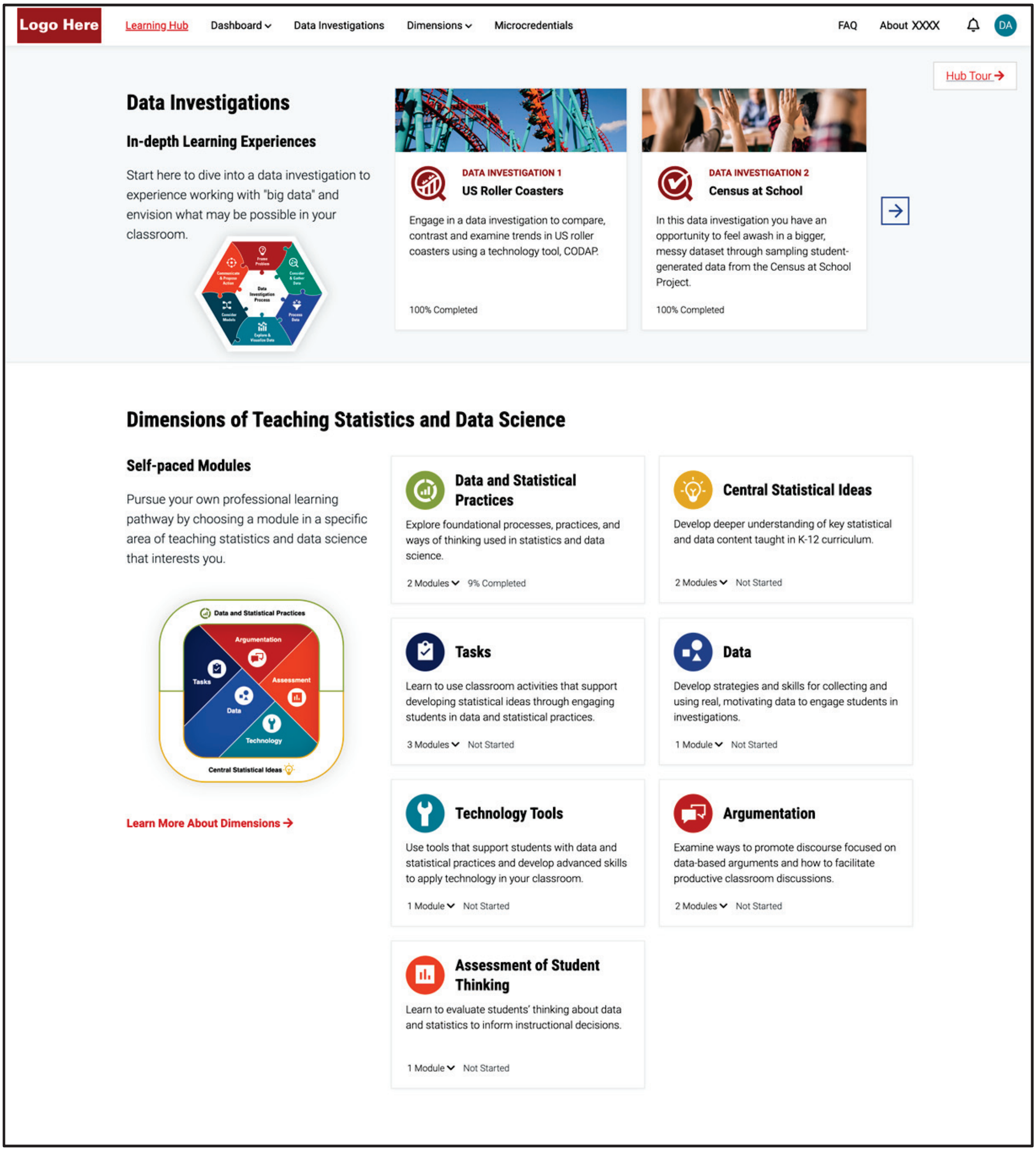

Figure 1. Learning Hub that organizes learning experiences by Data Investigation Modules and Self-paced Modules.

Table 1 includes the breakdown of all learning activities [30.2 h] available during the Fall 2022 field test study, the focus of this paper. Each page in a data investigation or module has time estimates assigned to help users guide their learning when they log in with limited time available for learning that day (see sidebar in Figure 2).

Table 1. Summary of learning experiences and time estimates for completion available during the Fall 2022 field test.

Learning Experiences and Hours Needed for Completion	Brief Title and Description
Data Investigations Module 1 Essential Resources: 9 pages, 1.9 h Extended Resources Hours: 1 page, 3.8 h (these extended resources are mostly part of other modules below)	Roller Coasters [RoCo]. A data investigation to compare, contrast, and examine trends in U.S. roller coasters using a large multivariate dataset of 635 cases.
Data and Statistical Practices Module 1 Essential Resources: 6 pages, 1.8 h Extended Resources: 8 pages, 3.1 h	What is Statistics and Data Science [S&DS]? Learn about the big ideas, habits of mind, and dispositions of statistics and data science.
Data and Statistical Practices Module 2 Essential Resources: 5 pages, 1 h Extended Resources: 8 pages, 2.2 h	Investigation Process [InvP]. Introduction to a 6-phase data investigation process that incorporates processes and practices of data scientists.
Central Statistics Ideas Module 1 Essential Resources: 5 pages, 1.6 h Extended Resources: 11 pages, 3.75 h	Big Ideas in Statistics [BIiS]. Introduction to the key ideas in statistics and how they are foundational for learning statistics.
Central Statistics Ideas Module 2 Essential Resources: 5 pages, 1.4 h Extended Resources: 10 pages, 4.7 h	Comparing Distributions [CDist]. Consider important concepts related to comparing distributions and their important role within statistics.
Tasks Module 1 Essential Resources: 6 pages, 1.9 h Extended Resources: 3 pages, 0.5 h	Worthwhile Tasks [WwT]. Explore what it means to identify and select worthwhile statistical and data tasks.
Data Module 1 Essential Resources: 6 pages, 2.1 h Extended Resources: 8 pages, 2.4 h	Data for Classrooms [DforC]. Learn strategies and how to get started in collecting and using real, motivating data to engage students in data investigations.
Discourse and Argumentation Module 1 Essential Resources: 6 pages, 2.9 h Extended Resources: 6 pages, 1.5 h	Discourse [Disc]. Introduction to components of effective classroom discourse and different ways to promote and support discourse while teaching statistics or data.

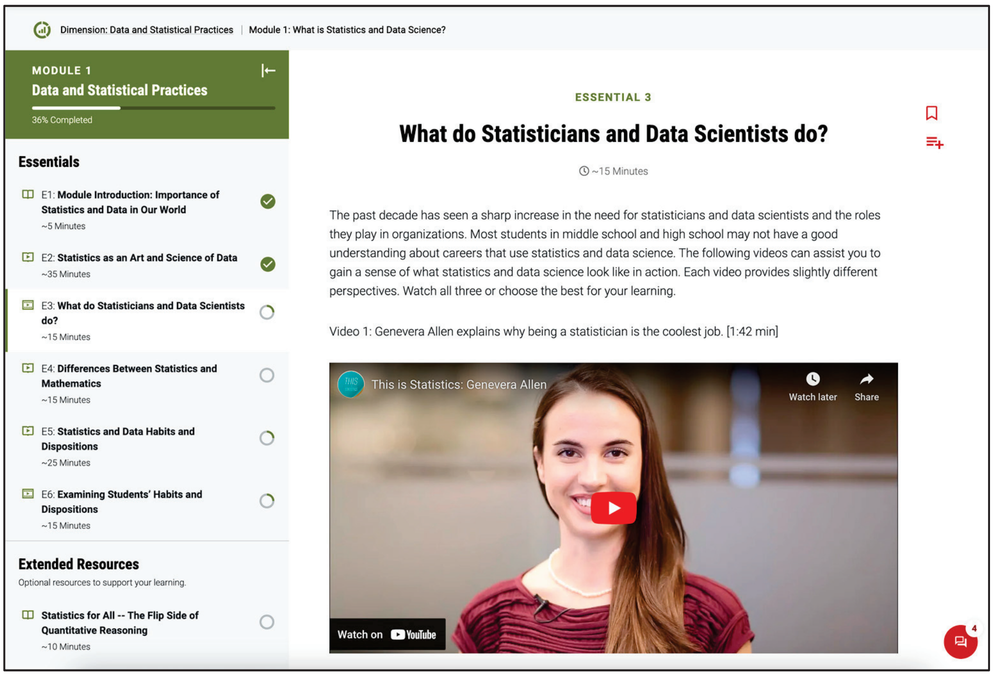

Figure 2. Example module with time estimates and tracking progress shown.

3.2. Supporting Personalized Learning

The platform is designed such that users can choose to engage in any learning experience that interests them in whatever order they choose. Thus, flexibility in when to engage and what to engage with is a primary way for the learning experiences to be personalized to meet a teacher's needs. We have four other design features that can assist teachers in making informed decisions for their learning and to keep track of what they have already completed.

The first design towards personalization is to provide recommendations to a participant. The use of recommendations in educational platforms is not a new endeavor (e.g., [48,49]). The purpose of recommendation systems in learning contexts is to assist users in making decisions and to help them feel comfortable to begin to navigate a free choice system. The participant is provided three recommendations at a time on the dashboard in a place to draw their attention and a statement describing how the recommendations were generated. This is a good design practice used for displaying recommendations in other educational systems (e.g., [49]). As one recommendation is completed, another recommendation replaces it based on a logic model that uses data collected from surveys our participants completed. Participants were given up to 6 total recommendations, and these are visible on the dashboard and profile page. More details on the recommendation logic model is in the Methods section.

The second design for personalization is based on recommendations from Kizilcec and colleagues [17] on strategies to assist learners in self-regulation behavior such as strategic planning and time management. We designed an internal tracking system and user dashboard that keep track of progress made on investigations, modules, and microcredentials (not examined in this study). Within a learning module, each page is given a time estimate, and when a user completes the activities on a page, they indicate completion, and the system keeps track of their completed work (Figure 2). The dashboard (Figure 3) provides a central place where a user can see all their progress and provides an easy way to resume their learning in a module or data investigation that they started but have not yet completed. With regard to tracking, users get credit for completing a module if they finish (and mark as complete) all essential resources within a module (i.e., extended resources contribute to hours earned but not module completion). Also, the dashboard displays their top three personalized recommendations and provides easy access to their progress towards meeting the necessary hours for each 10 h certificate, saved resources, playlists, and discussions (see right side of Figure 3).

The third feature for supporting teachers' personal learning experiences is the ability to save a resource and to collect specific resources into a "playlist" that they can name. These lists are then accessible from the users' dashboard (see right side of Figure 3) so that a user can quickly return to a specific resource from within a module without having to click through a module to remember where to find it. The fourth design to support personalization is a profile page where users see a report of survey results that includes details about the personal learning goals they chose in a survey, scores on a confidence survey and content assessment (described in Methods), and their top three recommendations for a suggested learning pathway (Figure 4). These displayed survey results can provide an opportunity for self-reflection and goal setting.

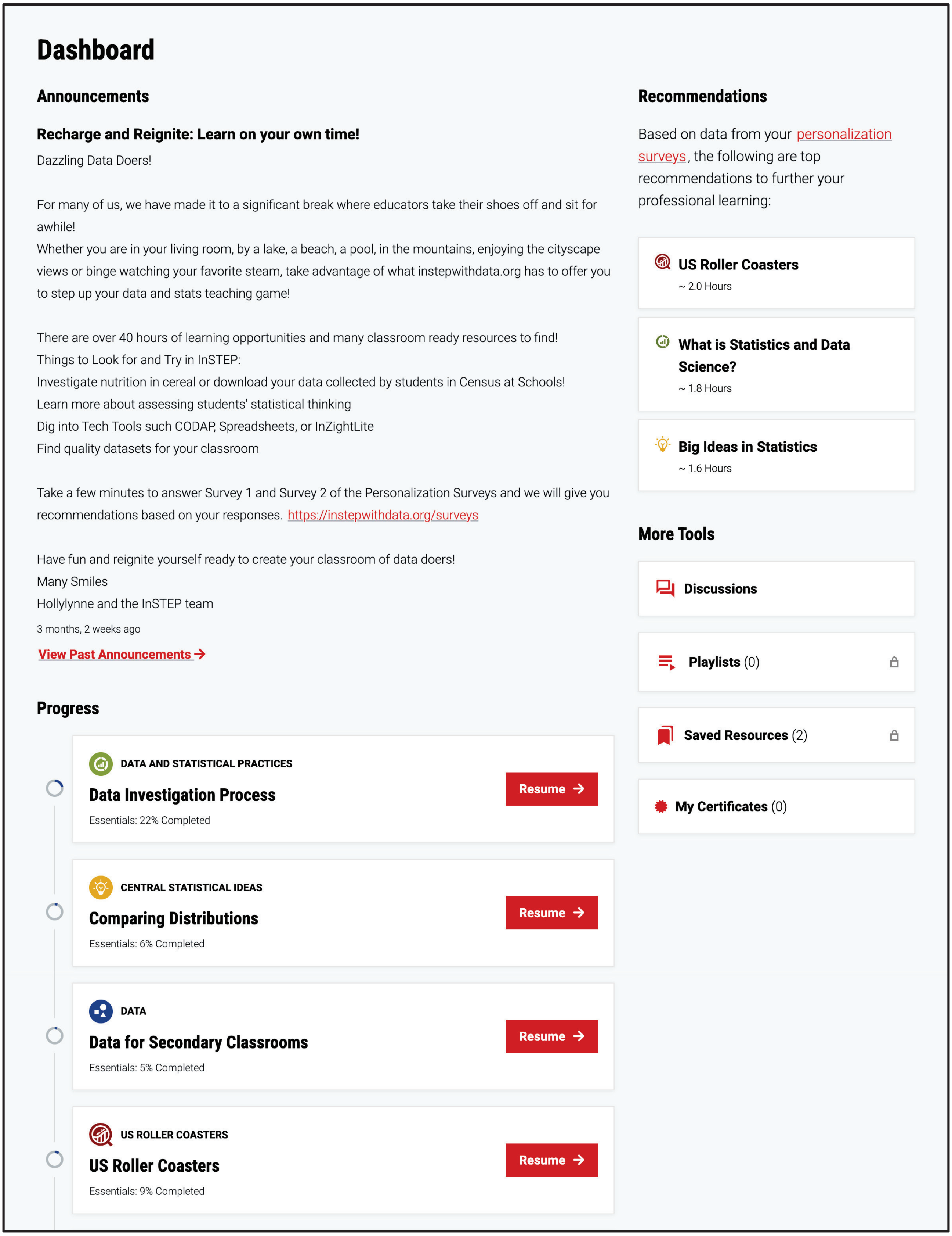

Figure 3. A user's dashboard showing recommendations, announcements, progress in completing the learning material, and access to saved resources and playlists.

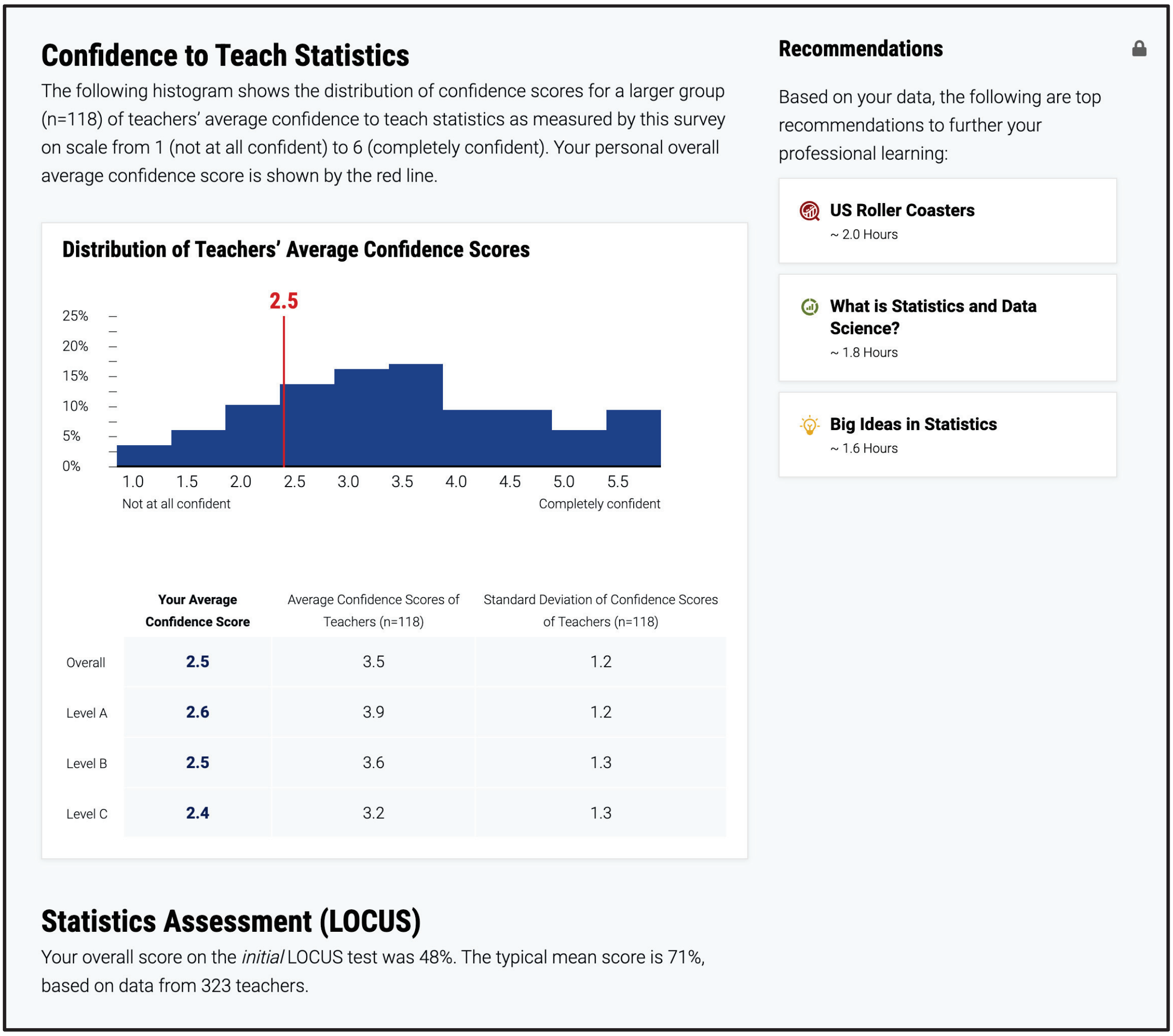

Figure 4. User's profile page with survey results and recommendations.

4. Methods

This research used a case study design [50]. Case studies are particularly a valuable methodology to explore how a group of people experience a contemporary event while using multiple sources of evidence. In this case, we explored ways in which teachers utilized different features and supports to personalize their learning in the InSTEP platform, and the platform's impact on the teachers' confidence to teach and classroom practices. Our case study, grounded in Mezirow's [14] theory of transformational learning in adult education and self-regulation theory [16], relied on a combination of qualitative and quantitative data to allow for "analytic generalizations" [50] that would support better understanding of how teachers engaged and were supported within the InSTEP platform.

4.1. Defining the Case

In Fall 2022, we recruited broadly (e.g., social media, listservs, personal contacts in school districts, and state supervisors), with a goal of recruiting 75–100 participants. Ultimately, 82 teachers chose to participate in a field test of the InSTEP platform. In this paper, we are focusing on a subset of these teachers to serve as a case study. Prior research on participation in online learning courses such as MOOCs has identified clusters of participants based on their engagement patterns and has always identified a subset of participants who have high engagement and are motivated to learn the content, complete courses, and earn certificates of completion (e.g., [29,30,51]). Thus, we wanted to examine more closely a subset of high-completion users to better understand the ways in which they engaged with learning experiences and how they used features in the platform designed to support their learning. Full participation in the larger field test included earning a certificate for completing 20 h of learning material. To be included in the case study for this paper, a participant had to meet the following criteria so that we would have consistent data across measures. Participants were paid incrementally for their participation for each of the following completed: (1) four pre-surveys with responses that led to personalized recommendations, (2) at least 20 h of professional learning (tracked activity with data logs), and (3) the post-experience survey. Thirty-seven participants met this criteria.

Our case consisted primarily of female identifying participants (78%) and included 27 mathematics/statistics teachers (16 high school, 9 middle school), 2 district-level math coaches, 7 science teachers (5 high school, 2 middle school), 2 middle school math and science teachers, and 1 middle school social studies teacher. These educators were highly experienced with a mean of 17.6 years (range of 5–31 years) in teaching/coaching and were employed in four states: California (n = 7), Iowa (n = 2), Maryland (n = 8), or North Carolina (n = 19).

4.2. Data Sources

To answer the three research questions, multiple surveys and assessments, as well as data logs, were collected and analyzed. Data included data logs and teachers' responses to the following: Goals and Background Survey, Self-Efficacy for Teaching Statistics (SETS), Levels of Conceptual Understanding of Statistics assessment (LOCUS), post-experience survey, and interviews. All surveys and assessments were conducted within the InSTEP platform. Below, we describe the data sources and indicate the research question each source is used to answer.

4.2.1. Instruments

At the beginning of the study, teachers responded to the Goals and Background Survey to collect data related to teachers' prior experiences and their professional goals. This survey also included demographic questions used to describe the participants. More details about how their professional goals were used are included in Section 4.2.3 describing the recommendation model, one of the personalization features we investigate in RQ3.

Prior to teachers' engagement with the InSTEP platform and, again, at the conclusion of the study, teachers responded to the SETS survey [52], a 44-item survey measuring teachers' confidence to teach students the skills necessary to complete specific statistical topics/tasks (e.g., use boxplots to compare the characteristics of two groups such as boxplots of test scores for males and females). Teachers rated their confidence using a 6-point Likert scale (ranging from 1—not at all confident to 6—completely confident). They also took the LOCUS assessment, a 23-item multiple choice statistics content assessment [53], which includes statistics content that is typically taught in grades 6–12 in the U.S. and is very similar to topics on the SETS instrument. Both instruments are used to answer RQ1.

At the end of study, teachers completed the post-experience survey so we could assess their overall professional growth from their learning experiences (RQ1) and their experiences engaging with personalization features of the InSTEP platform (RQ3). This survey consisted of 13 Likert scale questions using a 6-point or 7-point scale (very ineffective to very effective or strongly disagree to strongly agree), with multiple items for each question, and 4 open-ended questions. Frequencies and percentages for each rating were calculated for items focusing on the effectiveness of platform features. To analyze open-ended responses, we utilized open coding and constant comparative methods to identify emergent themes [54,55].

After 14 November 2022, if a participant earned a 20 h certificate, they were sent the post surveys to complete. After 28 November 2022, all participants were sent a reminder to complete their desired learning experiences and the post surveys, as we were closing the study on 4 December 2022. We had several participants in the larger study (n = 82) ask for extensions and, thus, we allowed a few to continue work and performed our last data download on 16 January 2023.

At the conclusion of the study, 7 of the 37 case study teachers participated in an interview focusing on teachers' experiences using the InSTEP platform. In the interviews, teachers were asked about how they used different features of the platform and to describe the typical ways in which they engaged over the study period. They were also asked about how what they had learned had impacted their practices. Since a structured interview protocol was administered, teachers' responses were summarized for each interview question, providing additional qualitative data describing teachers' experiences with various personalization features of the platform. Data from the interviews were used to better understand and situate other data sources for answering each research question. Teachers' responses to interviews and open-ended questions on the post-experience survey were used to provide examples and make sense of trends we saw from the quantitative analysis of datalogs, surveys, and instruments.

4.2.2. Data Logs

Data logs of teachers' engagement in the InSTEP platform captured how users were navigating through the platform and engaging with the different learning experiences. These data logs were essential in answering RQ2. Data logs captured individual views on all learning experience pages within the platform with the date and time. Additionally, data logs captured the date and times users marked a page as completed as well as the users' recommendations. The model for generating and displaying recommendations for a user is described next.

4.2.3. Recommendation Model

Recommendations are a critical aspect of the personalization features we designed into the platform and are used to investigate RQ3. Our approach to designing the recommendation system aligns with others who used ontology-based recommendation systems, specifically in platforms designed to support learning (e.g., [56,57]). "Most knowledge based e-learning recommender systems use ontologies to represent knowledge about the learner and learning resources. In such a case, ontology is used to establish the relationship between learners and their preferences about the learning resources" [57] (pp. 30–31). Our recommendation system is based on user inputs, including both psychological aspects (ranked goals and confidence to teach statistics) with cognitive aspects of a user's knowledge of statistics content typically taught in secondary schools. As designers of the learning experiences and experts in research on teachers' professional learning in statistics and data science, we created a mapping system to align certain goals, levels of confidence, and statistics knowledge to specific learning experiences in the platform.

While users can choose to engage with any of the learning experiences in any order they wish, we designed the online platform to support users' decision-making with recommendations based on the Goals and Background survey, LOCUS assessment, and SETS survey. Users receive up to six recommendations based on their responses to these surveys. Three recommendations for learning experiences were provided at a time and appeared on both the dashboard (Figure 3) and profile page (Figure 4) as stacked recommendations. To help avoid the "cold start" problem of users not knowing a place to start from or feeling overwhelmed in a system with many options to choose from [48,57], every user received the Roller Coaster and What is S&DS? learning experiences, even if they did not complete any surveys. The first two recommendations were given to allow users to experience one of each of the primary means of learning—a data investigation and a learning module since many users had limited experiences investigating real, large data as learners themselves and had limited experiences with key practices related to S&DS. The recommendation for What is S&DS? was based on material found in other studies to be highly influential in impacting teachers' beliefs and practices in teaching statistics through data [27]. In this way, the recommendation model uses a fixed system for all users to nudge them towards these important foundational learning experiences.

To determine the third initial recommendation, a question was asked within the Goals and Background survey to have users rank order at least five of nine learning goals according to what they would like to prioritize in their learning. The goal and the corresponding learning experience that most closely align from Table 1 are as follows:

It is important for my professional learning that I. . .

- Strengthen my understanding of key statistics and data concepts and skills. (BIiS)
- Strengthen my understanding of how to engage students in the practices related to statistics and data science. (IniP)
- Engage in real world data investigations with large data using technology. (RoCo)
- Deepen my ability to help students use data to make evidence-based claims. (IniP)
- Improve my ability to lead productive discussions about important ideas related to data and statistics. (Disc)
- Improve my ability to design, modify, and implement tasks to promote deeper understanding of ideas related to data and statistics. (WwT)
- Improve my ability to make sense of students' thinking through assessing their work, including written, verbal and technological. (Assessment 1, not available at time of study)
- Improve my ability to collect and use real-world data to support student's learning in statistics and data science. (DforC)
- Improve my ability to use technology tools to collect, process, visualize, and analyze data. (Technology Tools 1, not available at time of study)

Recommendation four and five were determined by users' scores on SETS and their statistics understanding as measured through the LOCUS assessment, respectively. The sixth recommendation was based on users' highest-ranked goal from their prioritized list that had not already been completed or given as a prior recommendation. The recommendation model did not include the Comparing Distributions module.

Recommendations are not explicitly numbered, but order is implied by placing them in a stacked appearance. As learning experience recommendations are completed, any remaining recommendations are added to the bottom of the recommendation stack. When a user completes enough experiences where there are less than three recommendations remaining, no new suggestions appear, and the ordered list continues to deplete with the same logic.

5. Analysis and Results

The primary aim of the InSTEP platform is to enhance teachers' expertise in teaching and learning statistics and data science by providing features that can support personalized, sustained engagement. First, we describe how we answered RQ1 regarding users' professional growth to teach statistics and data science through participation in the platform. After establishing the perceived effect of teachers' participation in learning with this platform, we investigate RQ2 by a close examination of participants' engagement within the platform through page views and module completion paths. Finally, we tackle RQ3 by analyzing participants' experiences with the features designed to support personalization. The analytic techniques used for each data source are described along with the results.

5.1. Growth in Expertise in Teaching and Learning of S&DS

This section aims to answer RQ1 regarding how participation in professional learning in the InSTEP online asynchronous platform contributes to professional growth in the teaching and learning of S&DS. Three data sources were used to examine the effects of the platform on teachers' perceived growth in the seven dimensions of the teaching and learning of S&DS (post-experience survey), their confidence to teach statistics (SETS), statistical content knowledge (LOCUS), and evidence of any incremental changes to classroom practices from the post-experience survey and interviews.

5.1.1. Perceived Growth in Dimensions of Teaching and Learning Statistics and Data Science

This platform frames expertise in teaching and learning using the Seven Dimensions framework. To help evaluate users' perceived growth within these seven dimensions, the post-experience survey asked about teachers' growth related to these dimensions. Users (n = 37) reported that overall, the InSTEP professional learning was effective (51%) or very effective (35%) in supporting their growth within the Seven Dimensions. There were three participants who indicated the professional learning experiences were very ineffective (n = 2) or ineffective (n = 1), though one of those participants rated everything else on the survey as positive and left a positive comment that, "I truly think the product is great!"; thus, it is not clear whether their low effective rating is valid. The two other participants had varied ratings across survey items and did not leave comments, so it seems those two participants likely had an overall ineffective learning experience.

Users also indicated their agreement for how well the professional learning experience helped them make progress in the nine personal learning goals that were listed on the initial Goals and Background survey (see Table 2), which align with the Seven Dimensions framework. For each item, there was a vast majority of users who agreed or strongly agreed that they were able to make progress towards developing their own knowledge and skills with statistics as well as their ability to enact effective strategies that align with the design of the modules and data investigations. There were three areas in which users reported the most agreement (agreed or strongly agreed) related to their progress: (1) strengthening their understanding of how to engage students in the practices related to statistics and data science (89.2%), (2) how to help students use data to make evidence-based claims (83.8%), and (3) their ability to lead productive discussions about important ideas related to data and statistics (81.1%).

Table 2. Distribution of agreement that participation in InSTEP professional learning helped them make progress towards specific learning goals.

n = 37	Percent of Teachers					
	Strongly Disagree	**Disagree**	**Somewhat Disagree**	**Somewhat Agree**	**Agree**	**Strongly Agree**
Strengthening my understanding of key statistics and data concepts and skills. (BIiS)	0%	5.4%	0%	18.9%	48.6%	27.0%
Engage in real world data investigations myself with large data using technology. (RoCo)	0%	2.7%	8.1%	10.8%	43.2%	35.1%
Strengthening my understanding of how to engage students in practices related to statistics and data science. (IniP)	0%	0%	2.7%	8.1%	51.4%	37.8%
Deepening my ability to help students use data to make evidence-based claims. (IniP)	0%	2.7%	2.7%	10.8%	51.4%	32.4%
Improving my ability to lead productive discussions about important ideas related to data and statistics. (Disc)	0%	2.7%	2.7%	13.5%	56.8%	24.3%
Improving my ability to design, modify, and implement tasks to promote deeper understanding of ideas related to data and statistics. (WwT)	0%	2.7%	2.7%	27.0%	45.9%	21.6%
Improving my ability to make sense of students' thinking through assessing their work, including written, verbal and technological. (Assessment 1, not available at the time of the study)	0%	0%	2.7	32.4%	51.4%	13.5%

Table 2. *Cont.*

n = 37	Percent of Teachers					
	Strongly Disagree	Disagree	Somewhat Disagree	Somewhat Agree	Agree	Strongly Agree
Improving my ability to collect and use real-world data to support students' learning in statistics and data science. (DforC)	2.7%	0%	2.7%	24.3%	51.4%	18.9%
Improve my ability to use technology tools to collect, process, visualize, and analyze data. (Technology Tools 1, not available at the time of the study)	0%	5.4%	2.7%	18.9%	54.1%	18.9%

Two areas that users reported making less progress on were their ability to design, modify, and implement tasks (67.6%) and to make sense of students' thinking through assessing their work, including written, verbal, and technological (64.9%). At the time of the study, the platform did not include a module dedicated to assessment, so this makes sense (a module was added in 2023). However, a few videos in other modules included an opportunity to make sense of students' work using technology as well as examples of student work (e.g., examining posters of students' data investigations).

5.1.2. Examining Confidence to Teach Statistics

Of the 37 users, 36 of them completed both the pre- and post-SETS survey to assess their confidence to teach statistics. Recall that this survey measures confidence on a 6-point scale. We hypothesized that the post-confidence levels would be greater than the pre-confidence levels. The results of a paired-t test indicated that there was a significant large difference between their confidence before the professional learning experience (mean = 3.3, Stdev = 1.1) and their confidence measured after they received a 20 h certificate (mean = 4.5, Stdev = 1), $t(35) = 8.8$, $p < 0.0001$. The 1.2 mean increase in confidence score, small p-value, and Cohen's d effect size of 1.47 indicate that the magnitude of the difference between the average of the differences and the expected average of the differences is large.

5.1.3. Examining Statistics Content Knowledge

There were 32 of 37 users who completed both the pre- and-post statistics content assessment (LOCUS). Scores are the percentage correct out of 23 items. The results of the paired-t test indicated that there was a non-significant, very small difference between Pre-LOCUS (Mean = 72.4, Stdev = 18) and Post-LOCUS (Mean = 73.6, Stdev = 17.8), $t(31) = 0.5$, $p = 0.643$. While some teachers had large gains in their content score (e.g., +34%, +21%, +17%), others had decreases (e.g., −21%, −43%, −8%) or almost no change. Thus, as a collective, the professional learning experience did not appear to improve statistical content understanding. The LOCUS assessment was the last instrument users completed at the end of the study. Though they could complete these instruments in their own time over the span of a few weeks, we do wonder if assessment fatigue may have contributed to the ways in which users approached the post-LOCUS assessment. Additionally, with the focus of the LOCUS being on statistics content, only two modules would support growth on this assessment.

5.1.4. Self-Reported Classroom Changes

As in many professional learning projects with teachers, we did not have an opportunity to follow participants into their classroom to observe their classroom practices. Thus, we relied on a small amount of self-reported qualitative data. In the post-experience survey ($n = 37$) and interviews ($n = 7$), users were asked to comment on how their teaching practices had been impacted by learning experiences. Not all participants responded in ways that connected what they were learning to their classroom.

In the survey, 10 participants commented explicitly about how learning about and engaging in data investigation learning experiences (e.g., Roller Coaster, Investigative Process) were impactful because they “helped me model similar activities in my class” and “kept me the most engaged and allowed me to easily see how to bring it into the classroom”. A few specifically mentioned a plan to use the roller coaster data with students. As an example, one participant commented, “The rollercoaster activity ties in with a project our math and science classes do at the end of the school year. The data on rollercoasters was a great resource”. One teacher told us about a more general direct classroom impact related to implementing data investigations:

> “I have given my students two big data assignments as a direct result of [InSTEP]. One, I helped guide a little bit with questions and the second one I didn’t. The students said it was interesting and fun”.

Other teachers felt they learned a lot about pedagogical practices related to teaching statistics and data science that “gave me a new perspective about how to approach teaching statistics in middle grades science classes” and that they learned “the importance of using real and messy data to encourage true discussion”.

Some users expressed a desire for more classroom-ready lesson plans about specific topics or grade levels that they could implement in their classroom. One teacher reflected on their experience and noted how they really needed to gain more statistical knowledge to be able to impact their classroom practices.

> “Overall, I enjoyed the experience. I think for me personally since I am so weak when it comes to statistics, I need to build more background knowledge before diving more into the things on here. A lot of the things talked about are ideas that I wouldn’t know how to bring into my current classroom but am sure I could eventually get there”.

Another teacher specifically discussed the timing and coverage of statistics in their curriculum as a potential barrier for classroom change, though they indicated an intent for making a small change.

> “I thought the supports and resources were helpful and wonderful. My struggle is that my math curriculum does not include a lot of statistics and those topics are taught towards the end of the year. Although it is only touch on, I plan to give students more opportunities to collect and analyze their own data”.

5.2. Engagement in Professional Learning

Overall, our findings for RQ1 show evidence that the learning experiences had a positive impact on users’ growth in teaching statistics and data science. We next pursue RQ2 to examine in more detail the ways in which teachers engaged in learning experiences. This is examined through data logs that track views per day of learning experiences in the platform and tracking completion of modules.

5.2.1. Viewing Learning Experiences Across Study Period

Figure 5 shows the total number of page views in modules and the data investigation per day by all 37 case users. The view count does not include page views of when users were looking at the Learning Hub, their dashboard, playlists, profile, or taking surveys. To put these numbers in perspective, a typical module had 5–6 essential pages with 8–10 extended pages (not required for module completion) and the data investigation had 9 pages. Thus, on days in which there were 50 or less views, there could have been only a very small number of users engaging in learning experiences, while days with over 100 views likely consisted of page views of a large number of users. Throughout the study period, there were only 12 days when none of the 37 users were viewing a learning experience. Thus, as a group, they had sustained engagement, with August and November being particularly high.

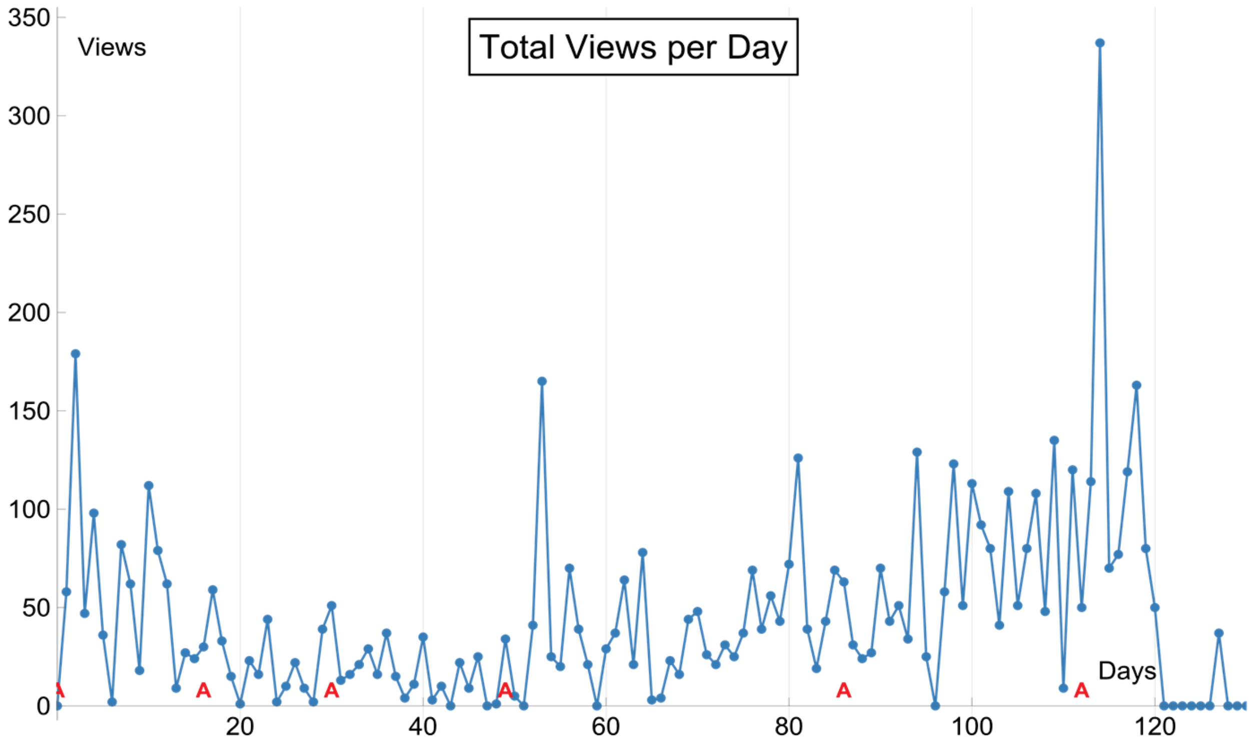

Figure 5. Time series of total views by 37 users of learning experiences (modules and data investigation) during the study period. "A" indicates when an announcement was sent to all study users.

The three major peaks (8/10, 9/28, and 11/28) in the time series all correspond with dates (indicated by an A) that users received emails about the project and reminders of expectations. The largest peak occurred shortly after an email on 11/28 announcing the official end of the field test (4 December 2022). There are also two main valleys in the time series. The first period of low engagement in September is most likely explained by users being busy with the new school year. The flatline at the end of the graph shows that while users still had access to the InSTEP platform, there were only two days of engagement by users (note: others in the larger field study group were active in trying to complete study requirements).

5.2.2. Patterns in Completion of Learning Experiences

The platform tracks views on pages with date and time as well as when a user indicates completion by pressing the Complete button at the bottom of a page. The two learning experiences with the most views (essential and extended resource pages) were What is S&DS? and Big Ideas in Statistics. Three modules had very similar view counts on the low end: Comparing Distributions, Worthwhile Tasks, and Discourse. Most users (59%) completed the Roller Coasters investigation first, with an additional 14% completing it second (see Table 3). There were two high school science teachers that did not complete Roller Coasters. What is S&DS? was the first or second experience completed for 73% of users, with all 37 users eventually completing this module. There were two additional modules that all users completed: Investigation Process and Big Ideas in Statistics. Only 26 users completed the Discourse module and 30 completed Comparing Distribution, even though a few more users viewed at least one page in those modules but chose not to complete them.

Table 3. Views and completions for each learning experience.

	RoCo	S&DS	InvP	WwT	BIiS	DforC	Disc	CDist
Total Views	704	1063	620	450	952	598	440	472
Users Viewed	36	37	37	37	37	34	31	32
Users Completed	35	37	37	35	37	32	26	30
Completion Order								
1st	22	8	5	0	1	0	0	0
2nd	5	19	7	2	3	2	0	0
3rd	1	6	17	4	7	0	1	1
4th	2	2	2	5	15	5	1	5
5th	1	2	3	10	4	4	6	7
6th	4	0	1	7	5	10	2	8
7th	0	0	1	5	2	7	6	8
8th	0	0	1	2	0	4	10	1

To further investigate patterns related to when users completed learning activities, we used the data logs of when a user clicked "complete" for each of the essential pages in the eight learning activities available to them. A module is considered complete once a user completes all essential pages in a module and does not consider their completion of any of the extended resource pages. For each user, we created a visualization of their learning experience completion over the period of the study. By examining all 37 visualizations, we looked for patterns in when and how quickly they completed modules. This process resulted in groups of users with similar completion behaviors. Through this process, multiple members of the research team proposed different groups of users that had similar patterns, and these were discussed and negotiated until the final six groups were made. The six groups are visualized in Figure 6a–f. The graphs show which learning experience they completed (vertical axis) on which day of the study (from day 0 to day 120). The red A markings indicate the days on which participants received announcements about the study. The six completion pattern groups are as follows:

A. Sustained Long Term: A user was engaged for at least 60 days with completions spread out over time. (n = 5)
B. Sustained with Sprints: A user was engaged for at least 60 days with completions spread out over time with at least two time periods where they completed two or more experiences within a day or two of each other. (n = 6)
C. Early Completion: A user started early in the study period and finished all their intended learning experiences within 60 days, some more quickly. (n = 6)
D. Late Start: A user did not complete any learning experiences until 40 days into the study period. (n = 8)
E. Deadline Motivated: A user started early in the period but completed several learning experiences within 3 days of the deadline for closing the study. (n = 5)
F. Super Sprint: A user completed most learning experiences in a 1–2 day time span. (n = 7)

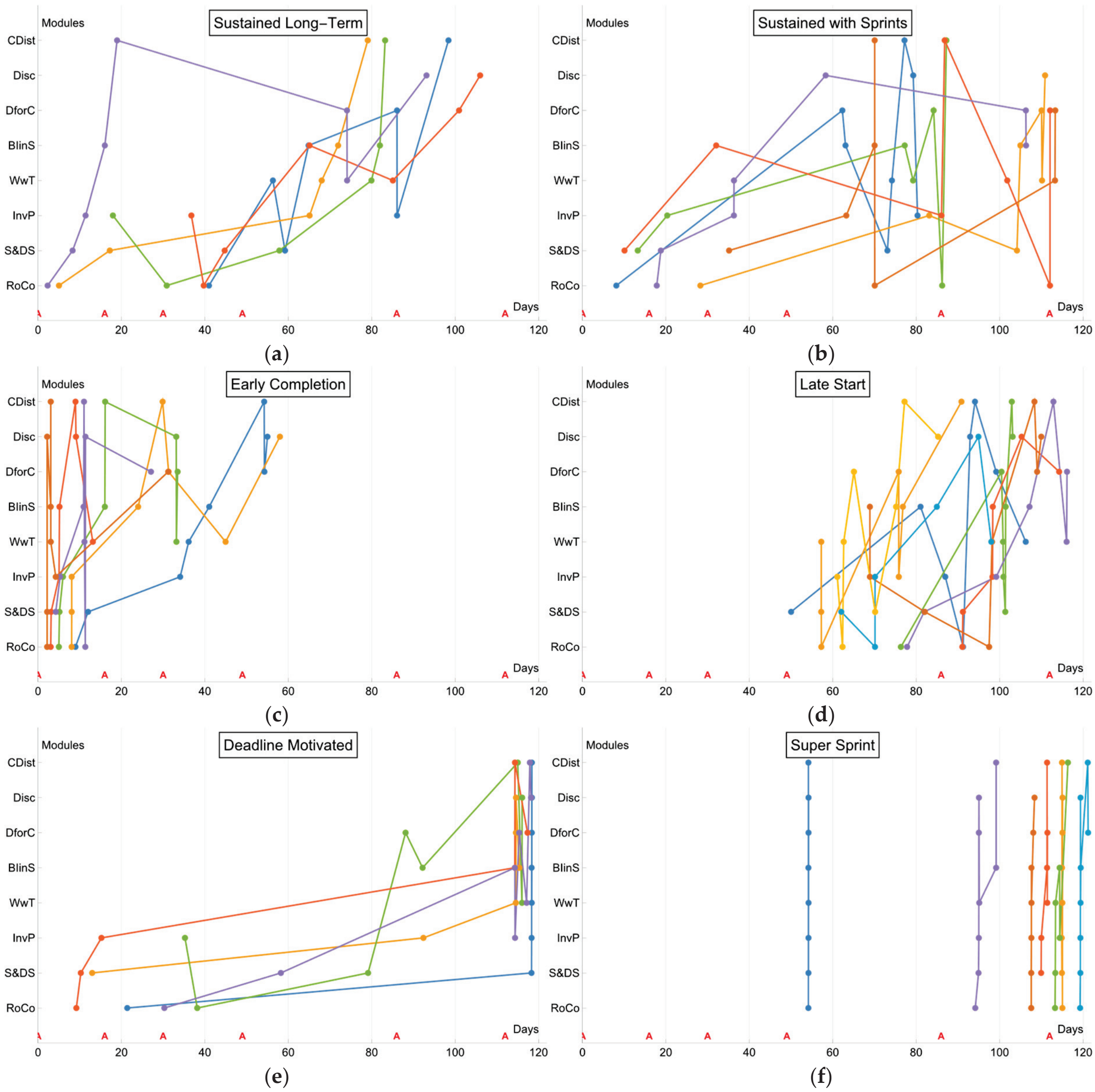

Figure 6. Six different completion patterns by users earning a 20 h certificate of completion: (**a**) Sustained Long Term, (**b**) Sustained with Sprints, (**c**) Early Completion, (**d**) Late Start, (**e**) Deadline Motivated, and (**f**) Super Sprint.

Though as a whole group it appears they had sustained engagement (Figure 5), there are distinct differences in the ways users individually completed the learning experiences. For the most part, the majority of users had sustained engagement within different time periods of the study, with only the Super Sprint group showing a very short completion time period. The completion pathways indicated that within the two dimensions that contained two modules, many users completed both modules in a linear order: specifically, 19 participants completed the two modules in Data and Statistics Practices in order and 21 did so within Central Statistical Ideas. The existence of different patterns of completion suggests that personalization and flexibility in when and how users could engage was supportive of self-guided learning towards completion and earning a certificate.

5.3. Participants' Use and Perception of Support for Personalized Features

To answer RQ3, we focus on a few of the features designed to support the ways a user could personalize the experience for themselves: recommendations, progress tracking, saving and grouping resources into playlists, and the profile page. We first examine what learning experiences users were recommended and how those compare with their completion pathways. We then examine users' perceived effectiveness of several other personalization features using the post-user survey responses and interviews.

5.3.1. Recommended Versus Completed Learning Experiences

At the time of the study, the recommendation logic model (described in Section 4.2.3) only provided a maximum of six recommendations matching six of the eight learning experiences that were available (Table 1). Recall that all users were recommended the Roller Coaster data investigation and the What is S & DS? Module, and the logic model did not include Comparing Distributions as a possible recommendation. Most users had either 5 (n = 15) or 6 (n = 21) recommended learning experiences, with one user receiving 4 recommendations. Table 4 shows the total times a module was recommended as well as the number of users that received a recommendation for a module in each of the positions. There were two learning experiences recommended to all users, and two modules (Worthwhile Tasks and Big Ideas in Statistics) recommended somewhere in their list to 35 (of 37) participants. Even though the Investigation Process module was recommended most in the third position, it appeared almost as often as the second replacement recommendation in the list for 9 users. Discourse was only recommended to 12 users, and its position in their lists was most often 3rd or 6th.

Table 4. Number of users given a recommendation for a learning experience.

	RoCo	S&DS	InvP	WwT	BIiS	DforC	Disc	CDist
Initial 3 recommendations	37	37	10	7	7	8	5	0
Replacement recommendation 1	0	0	2	28	6	0	1	0
Replacement recommendation 2	0	0	9	0	22	3	2	0
Replacement recommendation 3	0	0	7	0	0	10	4	0
Total	37	37	28	35	35	21	12	0

It is interesting to compare whether a user viewed at least one page or completed a learning experience (Table 3) and if that learning experience was recommended to them at all (Table 4). For four of the eight learning experiences, these three numbers were almost identical (Roller Coasters, What is S & DS?, Worthwhile Tasks, Big Ideas in Statistics). However, for the four other modules, even if a user was not recommended a module, many viewed or completed it. This indicates that users were choosing to engage with modules outside their recommendation list, which is indicative of the flexibility within the platform and users' choice making for what to learn more about. It is important to remember that the users in this case study were selected because they earned a 20 h certificate. Thus, completing modules is a highly popular way to earn these hours. Recall that users can earn time towards certificates by engaging with extended resources as well, which do not count towards module completion.

To examine how supportive recommendations, as a personalization feature, were for teachers' learning (part of RQ3), we analyzed each users' completion path and compared it to what was visible in their personal list of recommendations (three or less visible). For this analysis, we considered the first module that everyone completed as the first round of completions, the second completed module as the second round, and so forth. For each round of completion, we tallied the number of users who made a module completion and tallied those that diverged from their visible recommended pathway (top 3 or less, Table 5). For example, every user completed some first module, and for those 37 choices, 3 of them diverged from a users' visible recommendation list. As another example, only 31 users completed a sixth module while having non-empty recommendations lists, with all but 9 users following their recommendation.

Table 5. Actual and expected divergent choices from recommendations and probabilities of divergence.

	Round of Completion							
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**
Number of completions by users	37	37	37	37	37	31	15	5
Number of actual divergent choices by users	3	8	8	8	14	9	2	0
Expected number of divergent choices under uniformly random assumption	23.1	21.1	18.5	15.4	12.8	11	7	0
Probability of actual divergent choices occurring	7×10^{-12}	1×10^{-5}	4×10^{-4}	0.008	0.7	0.3	0.02	1

To examine how often users diverged from their recommendation list, we applied probability principles to compute an expected number of divergent choices for each completion round. Then, we computed the probability of achieving the actual number of divergent choices participants made, all assuming that users made a uniformly random choice for which module to complete. Table 5 shows the actual and expected number of divergent choices as well as the probabilities for the actual number of divergent choices occurring in each round. The details of all expected values and probability computations are in Appendix A.

For the first four completed modules, the computed probabilities shown in Table 5 indicate it is highly unlikely that participants chose a module to complete in a random way considering the probability of each occurring is less than 0.01. These results are encouraging as they suggest a strong correlation between the visible recommendation list and the choices that participants made about which learning experience to complete and in what order, especially in the first 3–4 modules completed. It is possible that there were other contributing factors that informed their choices, such as the order of the modules within dimensions as shown on the Learning Hub page (see Figure 1 with Roller Coaster at the top of the page), users seemingly sprinting through the modules by clicking them all as completed (Figure 6f), and the fact that one module (Comparing Distributions) was not recommended to any user but was available to all. Starting at round five, the number of divergent choices recorded, 14 and 9, respectively, are not all that dissimilar to the expected divergent choices of 12.8 and 11 (Table 5). This could suggest that the correlation between the visible recommendation list and user choice of learning activity disappears after four recommendations. Some possible explanations could be that their experiences within the platform impacted their goals after the recommendations were generated, or that with only eight learning experiences, users' choice patterns changed.

5.3.2. Perceptions on Effectiveness of Design of Platform and Learning Experiences

The post-experience survey and interviews gave an opportunity for users to reflect on and share how different features in the platform supported their learning. Table 6 shows the distribution of perceived effectiveness for these features for these 37 users.

Table 6. Perception of effectiveness of personalization features to support professional learning.

***n* = 37**	**Percent of Teachers**						
How effective were the following features in supporting your professional learning?	Never Used	Very Ineffective	Ineffective	Somewhat Ineffective	Somewhat Effective	Effective	Very Effective
Recommendations	8.1%	2.7%	0.0%	0.0%	10.8%	29.7%	48.7%
Progress Tracking on Dashboard	5.4%	0.0%	2.7%	2.7%	0.00%	21.6%	67.6%
Progress Tracking within a Module	5.4%	0.0%	0.0%	2.7%	0.0%	13.5%	78.4%
Saved Resources	13.5%	0.0%	0.0%	5.4%	10.8%	27.0%	43.2%
Private Playlists	56.8%	0.0%	0.0%	5.4%	8.1%	16.2%	13.5%
Profile Page	10.8%	0.0%	2.7%	5.4%	24.3%	29.7%	27.0%
Data from Surveys on Profile Page	16.2%	0.0%	2.7%	0.0%	13.5%	32.4%	35.1%

Users were quite positive that the recommendations effectively (29.7%) or very effectively (48.7%) supported their professional learning, with only 8.1% (n = 3) reporting not ever using the recommendations. In interviews and in open-ended questions on the survey, users expressed appreciation for having recommendations that were based on their survey results and indicated they typically used them to inform their choices. This corresponds with our analysis showing that, in general, the users did not deviate much from their visible recommendation list. For example, one user commented:

> This program was excellent. The personalized plan based on my survey and the route it took me at first was a little confusing, but I eventually got that 'aha' moment. That could also have been because there were gaps between my being able to get online and work on this. (Sustained with Sprints participant)

However, one interviewee, a Super Sprint user, reported not using the recommendations and exploring the modules in an order of interest to them, typically navigating through menus at the top of the platform rather than the layout on the Learning Hub (see Figure 1). Examining this user's pathway of completion, though, showed only one divergent choice from the visible top three recommendations in their list. Thus, even though they reported not using recommendations, their choices and pathways indicate that the recommended list generated from their survey results may have been appropriate for their interests.

Almost all users reported that the tracking capabilities on the dashboard and within modules were effective or very effective in supporting their learning (Table 6). In interviews, teachers told us they used the tracking features to pick up where they left off when they logged back into the platform. Specifically, many users noted that within modules, they progressed linearly through material and used the left side-bar tracking (see Figure 3) to confirm "Yes, you did all of these things". For example, one user in the Early Completion group reported "It took me awhile to understand that I got to pick and choose [their own activities]", but that "the progress things helped a lot". One user from the Super Sprint group told us "I enjoyed the fluidity of the sessions where you had time to complete the assignments at your own pace and often I felt I rushed through some lessons. I used the self-checklist". Another teacher explained in an interview how tracking features helped them re-engage where they last left off since they often completed learning experiences in 20–30 min sessions during lunch or planning time and spent more extended times during the weekends (Sustained with Sprints user).

Most users took advantage of the feature to save resources, which were then easily accessible from their dashboard, and most thought this feature was an effective or very effective way to support their learning. However, many users (56.8%) did not use the private playlist feature where they could group saved resources into specific lists that had a personal meaning to them (e.g., "stuff for my classroom"). Those that did use playlists indicated it was an effective support for them, with one user commenting "The saved resource area for my playlist was VERY useful for things I wanted to return to". Others discussed in the interview that they would return to their saved resources or playlists to quickly find resources, and some mentioned they would print a PDF of the resource. One suggestion for improvement came from a user who thought it would be useful to be able to search the saved resources and playlists because sometimes they forgot where a resource was saved.

A little more than half of the users indicated the profile page, in general, was effective or very effective, and they were slightly more positive about the effectiveness of the data from surveys that were displayed on the profile page (see Figure 4) in supporting their learning and professional growth. Participants did not bring up the profile page or data from surveys as a source of reflection for supporting their learning in the open-ended questions on the post-experience survey. In the interviews, a few teachers noted that results from pre-surveys (SETS and LOCUS) "gave me an idea of what I was low on". A middle school math and science teacher explicitly discussed that they used results of the pre-surveys (SETS and LOCUS) to reflect on areas for growth and they tried to find learning experiences in the platform to help them improve but that there was not much they could find to help them with more advanced statistics content they wanted to grow in. Thus, we only have limited evidence for how a few users used the profile page and data from surveys to help them make choices and navigate the platform.

6. Discussion

We sought to answer three research questions using a case study of 37 secondary teachers (78% taught mathematics/statistics) who volunteered for a field test of the InSTEP professional learning platform. Related to RQ1, we saw evidence of incremental growth for our participants related to learning to teach statistics and data science. Recall that the learning experiences on the platform are structured and organized according to a framework of seven interrelated dimensions for teaching and learning statistics and data science [43]. Our participants perceived the InSTEP learning experiences as overall supportive of their growth in meeting the learning goals aligned with this framework, including a goal of engaging in data investigations themselves as learners. This aligns with prior research from Lee, Mojica, and colleagues about similar growth after learning experiences about teaching statistics in a MOOC format [26–28]. One learning goal that was rated lower in the current study involved designing, selecting, and implementing worthwhile tasks. While 35 participants completed a module on Worthwhile Tasks, the question on the survey also included implementation of tasks; thus, many teachers may not have had opportunities during the study period to focus on task selection, modification, and implementation if they were not currently teaching a course or unit that included statistics and data ideas.

Participants were likely motivated to join the research study and engage in professional learning because their personal professional learning goals aligned in some way with what they perceived as the goals of the learning material in the platform, based on advertising material and the website. Results from Barker and colleagues' [18] analysis of teachers' motivation for taking an online MOOC about teaching statistics through data investigations indicate that most teachers' motivation to enroll aligned with the course goals or a general goal of improving their knowledge and confidence to teach statistical topics.

We saw strong evidence for a growth in confidence to teach statistics, but little to no change in statistics content understanding for this group. The gains in confidence were unsurprising, and they align with prior findings that relatively short online asynchronous courses focused on teaching statistics improve teachers' confidence [27]. This finding makes sense since our platform focuses more strongly on pedagogy than further developing teachers' content understandings. The only modules that were directly aimed at strengthening teachers' understanding of specific statistics concepts were the two modules in the Central Statistical Ideas dimension. All 37 participants completed the Big Ideas in Statistics module and only 30 completed the Comparing Distributions module. The LOCUS assessment includes several items with advanced statistical content related to understanding *p*-values, confidence intervals, and linear regression. None of these concepts were included explicitly in modules in InSTEP at the time of the study. In a study of preservice teachers, Lovett and Lee [32] found that many of their participants' estimation of their confidence to teach statistics as measured through SETS was often relatively high, even while scores on the LOCUS assessment were low to moderate. Thus, having growth in confidence in being able to teach a topic in statistics is not only about one's statistics content knowledge but likely involves one's broader understanding of pedagogical strategies for supporting students' learning of statistics and having access to, or knowing where to find, good materials to use in the classroom.

Our second research question (RQ2) focused on ways in which participants engaged with learning materials. Through analyses of page views and completion patterns, there was evidence that our case of users had overall sustained engagement in the InSTEP personalization platform. Over the study's period, about 120 days, the teachers were engaged in viewing learning materials. We used a qualitative approach to classify users into different groups using their time series graph of when entire modules were marked as complete. We identified six engagement patterns that help better understand how participants can engage with a personalized online professional learning platform. We consider most of these patterns as representing sustained engagement just over different time periods (see Figure 6). The Super Sprint users were not considered to have sustained engagement as they completed all their modules within a few days. As many researchers have suggested, sustained engagement is considered a key feature of effective professional learning, whether online or in person [58–61]. Within research on MOOCs for educators, methods such as latent profile analysis have shown that teachers' engagement patterns over time are typically classified as consistent high engagers; mid-level engagement, which drops off near the end of a course's time period; and sharp drop offs for those that start a course and quickly have low to no engagement (e.g., [30,51]). The variety of completion patterns from this case study suggests that users utilized the personalized nature of the platform to engage when they wanted and for the length of time that worked best for them. Having a flexible format in an online professional learning environment where teachers can choose their learning pathways and have extended time to complete materials has been linked to a higher sustained engagement and completion certificates for teachers [62].

The third research question (RQ3) examined how personalization features supported users' professional learning within the platform. The personalization features that were considered most supportive were recommendations and tracking capabilities on the dashboard and within modules. These features are specific to supporting users in their navigation and choice of learning activities and seem to allow teachers to dive deeper into materials like extended resources that match their personal interests and needs. Situated in self-regulation theory, these features seem to support the user's ability to self-monitor their progress and quickly re-engage after extended time away from the platform, ultimately supporting their professional learning experience [16,44]. Kizilcec [17] and colleagues similarly found that goal setting and strategic planning supported online learners in their study.

Features that supported users to personalize learning through organizing materials they already engaged with (saved resources and playlists) or displaying results to surveys on the profile page had more mixed results. In fact, many users reported not using a playlist, and some users never used the profile page and saved resources. We suspect it is possible that users already had systems in place for curating resources for their teaching (e.g., Google Drive to save pdfs, bookmarking resources within their own browsers) and may have continued to use these instead of the ones built into the platform.

Our analysis comparing completion of modules to a user's recommendations provides additional evidence that the recommendation logic model generated a set of recommended learning experiences that were supportive to users through the first four recommendations provided. This finding provides support that a user input, ontology-based recommendation system that includes psychological aspects and cognitive aspects of a user's knowledge can support teachers' initial learning in an online setting. However, this finding also suggests that there could be value in incorporating a recommendation system that generates recommendations based on user behavior, such as patterns and profiles of similar users (near neighbors) [48,49]; such a recommendation system could support teachers' sustained engagement over time, not only through four recommendations.

Our case study with a small number of users has shown that using an asynchronous approach to online professional learning for teachers can be effective in helping them meet professional growth goals that appear to lead to incremental changes in their confidence and intent to implement new strategies or resources within their classroom. Now that the InSTEP platform is available to the public, we are very interested in expanding our research on teachers' engagement patterns and use of the recommendations with users who self-initiate their personal learning by voluntarily creating an account and choosing their own pathway.

7. Limitations

Case studies serve as an effective research method for examining multifaceted topics and yielding in-depth, contextual insights. However, this approach comes with certain limitations. The results of case studies are closely tied to the specific context in which they occur, making it difficult to generalize the findings to broader populations or other situations. This study took place with a group of teachers from the United States on a platform written in English. The socio-cultural context of our participants could have implications for our findings. In addition, we recognize that teachers' background knowledge and experiences in statistics likely varied due to their primary teaching responsibilities (math, statistics, science, or social studies) and influenced the ways they approached their professional learning in the platform and how they envisioned using their new knowledge, skills, and resources in their teaching practices. With such a small sample, we did not investigate any differences in engagement or professional growth through disaggregating by background or subjects taught. Such a nuanced investigation may be useful in future studies with a larger number of participants.

Additionally, we had a few limitations that could cause a lack of analytic generalizability. Both limitations involve conditions that may not accurately represent a broader setting. We had a small sample of volunteer teachers who were paid to participate in the field test study. While we suspect their engagement patterns may be representative of the ways in which teachers would engage on their own during a busy school year, the fact that they were paid to complete learning experiences could have influenced their engagement patterns. Additionally, the field test version of the platform only had eight modules for users to complete. Considering users were asked to engage in 20 h of learning, and these materials only covered 30.2 h, users may have completed learning experiences that they would not have chosen to complete without that requirement. However, users still had a free choice of order for those experiences.

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Appendix A. Computations to Support Analysis Comparing Users' Recommendations to Their Completed Modules

To compare a user's recommendations to their completed modules, we are only counting completions made while there was at least one recommendation visible to a user. There were 18 users that completed an 8th module (completed a module in the 8th round), but only 5 of them did so with a non-empty recommendation list. Since there are only eight modules, these five users had no choice but to complete the recommended modules, thus there were 0 divergent choices (see last column in Table 5). For the first round of module completions, each participant had eight modules to choose from, with three of them being recommended. If each of the 37 participants makes a uniformly random choice of their first module to complete, the expected number of divergent choices is 37x* (5/8) = 23.125 modules. Using the associated binomial distribution (shown below), we can compute the probability of having 3 or less divergent choices in round 1.

$$\binom{37}{0}\left(\frac{5}{8}\right)^{0}\left(\frac{3}{8}\right)^{37}+\binom{37}{1}\left(\frac{5}{8}\right)^{1}\left(\frac{3}{8}\right)^{36}+\binom{37}{2}\left(\frac{5}{8}\right)^{2}\left(\frac{3}{8}\right)^{35}+\binom{37}{3}\left(\frac{5}{8}\right)^{3}\left(\frac{3}{8}\right)^{34}\approx 7\times 10^{-12}$$

For the second round of module completions, each participant had seven modules to choose from, with three of them being recommended. If each of the 37 participants makes a uniformly random choice of their second module to complete, the expected number of divergent choices is 21.1429 modules. Again, using the associated binomial distribution, the probability of having eight or less divergent choices is approximately 1×10^{-5}.

When choosing their third module to complete, there was one participant that only had two modules on their recommendation list, as their recommendation list had only four modules, two of which were the ones completed by this participant. However, the remaining 36 participants still had three modules in their list of recommendations, and all participants completed a third module from a remaining list of six available modules. Thus, it is very reasonable to consider the third round using the same binomial distribution approach. If each of the 37 participants makes a uniformly random choice of their third module to complete, the expected number of divergent choices is 18.5 modules. Using the associated binomial distribution, the probability of having eight or less divergent choices is approximately 4×10^{-4}.

As we look at the cases for four or more completed modules, the number of recommendations seen by some participants does begin to drop below 3, making a binomial distribution approach inaccurate. Instead, we used a Monte Carlo approach to build approximate distributions for the remaining rounds of module completions and to estimate the expected value and probabilities of the actual number of divergent choices occurring.

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Article

Studio as a Catalyst for Incremental and Ambitious Teacher Learning

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Abstract: This article explores how the practice-focused Studio professional learning (PL) model can provide incremental and ambitious teacher learning opportunities. We argue that when the model's structures and practices are grounded in ambitious and equitable teaching, they catalyze incremental teacher learning. Studio, like lesson study, supports teachers in considering the entailments of lessons, focusing on the live shared enactment to strengthen teaching and learning through collaborative analysis and reflection. To build our argument, we drew from two Studio projects that had shared structures of cycles of learning and routines, as well as shared practices of using rich representations and collective interpretations of teaching. While both projects' structures and practices take up ambitious and equitable teaching, they use different routines and attend to different features of equitable teaching. Building on a history of PL models, such as lesson study, which use similar structures and practices as powerful catalysts of teacher learning, we argue that Studio's structures and practices can catalyze teachers' incremental learning of ambitious and equitable teaching. We discuss the implications for future research based on this argument and for those leading PL.

Keywords: mathematics education; professional learning; studio; multilingual learners; data literacy; pedagogical reasoning

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1. Introduction

The editors of this Special Issue identified professional learning (PL) focused on ambitious teaching as different from PL that attended to incremental instructional improvement [1]. They suggested that the aims of these two types of PL emphasize different learning units, but both seek to improve practice and thus engage those involved in teacher learning. In this conceptual paper, we build on a history of research on the PL models that are ongoing, grounded in teaching, and engaged in collective interpretations of teaching to support incremental teacher learning [2]. We draw on two such PL projects, Beachside and Mesa, which used an adaptive practice-focused model of Studio PL (herein called Studio) to provide opportunities for both incremental and ambitious teacher learning [2,3].

Studio is situated in teaching involving cycles of activities used across a school year to improve instruction collectively [4,5]. Like lesson study [6], Studio cycles support teachers in considering the entailments of lessons, focusing on learning from a shared enactment and strengthening the teaching and learning process through collaborative analysis and reflection [4]. Studio differs from lesson study in that it does not focus on just a research lesson. Instead, it focuses on the lesson and the live lesson enactment with students, where a Studio teacher enacts a lesson, and colleagues closely observe students' participation. Analysis and reflection follow, where teachers make sense of the mathematics, lesson design, and pedagogical choices and how these are linked to the documented observations of students' participation. Through a debrief process, all teachers consider their future teaching.

In this conceptual paper, we offer examples from two Studio projects to argue that Studio structures and practices catalyze incremental learning like other similar PL models. Our paper extends the previous arguments and provides insights into how these structures and practices can support ambitious and equitable teaching [7]. Catalysts, by definition, are the people or events that cause change [8]. We aim to explore how Studio structures and practices cause incremental change. Lave called on us to pay attention to the mechanisms or the "ways by which learning comes about" [9] (p. 156). Over a decade ago, Jackson and Cobb [2] challenged the field to understand how districts and school structures support ongoing teacher learning of ambitious and equitable teaching. Given this Special Issue's focus, we see this call as still relevant. We draw on the work of Borko et al. [10] and Borko et al. [11] to compare these two Studio enactments. We pose what Borko et al. [11] called "an important question" (p. 260), contributing a conceptual argument for the field to consider how Studio structures and practices catalyze incremental teacher learning of ambitious and equitable teaching. As an existence proof, rather than an empirically-based argument, we explore intentionally chosen rich images of Studio structures and practices and how they foster teacher learning. Our argument contributes analytic insights on framing Studio's teacher learning opportunities. We provide a chain of evidence based on our intentionally-selected illustrations of two instances of Studio to address our important question. We next provide an overview of our important question. We then discuss the perspectives that guide our thinking on our important question for Studio structures. We share the contexts of the two Studio structures before delving into comparing our two Studio enactments. We finish with discussions of our conceptual argument, including the implications for the field.

1.1. Important Question for the Field

In this paper, we build an argument for the following question: How do Studio's structures and practices act as catalysts for the incremental learning of ambitious and equitable teaching? We address this question by discussing intentionally-selected examples from two Studio settings with similar structures and practices. Beachside and Mesa used a repeated activity cycle across Studios for the teachers' observation, analysis, and reflection. They also used routines, the sequences of teacher instructional moves guided by student mathematics and participation goals. Routines were the focal "lessons" in Beachside and Mesa Studios [12,13]. Beachside participants used Mathematics Language Routines (MLRs) to meet the multilingual learners' language and mathematics learning needs [14]. At Mesa, a facilitator-designed data literacy routine was used to serve the school's detracked mathematics courses, integrating new and innovative content. Although they used different routines, they both involved ambitious and equitable pedagogies. Both routines provided students with opportunities to reason, explain their thinking, and solve mathematics tasks in ways that supported all students to use authentic mathematical tools [14]. Both routines attended to language and mathematics but were designed to support different aims. MLRs specifically supported the language and content needs of multilingual learners in the Beachside community [14]. The data literacy routine supported the language of data literacy for heterogeneous groups of students in the Mesa community. Beachside and Mesa also utilized a similar set of practices, rich public representations of teaching that offered an object for collective interpretations of teaching [15,16]. Studio's public representations of teaching were rehearsals and live enactments of routines. Teaching representations became objects for analysis and reflection through facilitators' and teachers' collective interpretation of teaching. In this paper, we build evidence to argue that the repeated Studio structures and practices enabled facilitators to focus on incrementally supporting teachers' ambitious and equitable teaching [4,17].

We draw upon pedagogical reasoning to consider teachers' incremental learning of ambitious and equitable teaching. Greeno and Engeström framed learning as participation in activity systems and, from this perspective, we considered teachers' understanding of ambitious and equitable teaching as participation in pedagogical reasoning and how changes in that reasoning were catalyzed by Studio structures and practices [18–20]. Pedagogical reasoning, which we expand on below, entails the Studio facilitators' and faculty's discourse on ambitious and equitable instruction. Our views of ambitious and equitable teaching, which we expand on below, follow the current calls for centering students' discourse, focusing on reasoning versus attending to answer-seeking and (re)positioning the students often marginalized within educational systems [2,21,22]. In the next section, we provide details on the Studio contexts drawn upon for this paper.

1.2. Contexts

While the two Studio studies drawn upon to illustrate our conceptual argument involved similar structures and practices, they had different features, as shown in Table 1. The first key difference was Studio facilitation. In the Beachside Studio, the research team facilitated. In the Mesa Studio, two teacher leaders (herein called facilitators) facilitated.

Table 1. Key differences between Beachside and Mesa Studio.

	Beachside	**Mesa**
Routine	Mathematics Language Routines (MLRs)	Data Literacy
Grade Level	Junior High School	High School
Facilitator	Researcher Team	Two Teacher Leaders
Cycle	Pre-Studio Day, Studio Day, Post-Studio Day	Pre Studio, Studio Day

A second key difference between the projects was related to the routines they used. While each project centered Studio on routine use, Beachside utilized MLRs [14], and Mesa took up a facilitator-designed routine for data literacy. While these routines each supported teacher and student interactions toward a focus on mathematics and participation goals, including students' mathematical reasoning, they contrasted in unique ways that we will discuss in Section 2.3.2.

The final key difference between these projects was adapting the Studio cycle to the local needs of teacher availability. Beachside's Studio cycle took place across three meetings, with only the Studio Day occurring during the school day; the Pre-Studio Days and Post-Studio Days took place after school. In Mesa, the facilitators planned during the Pre-Studio Day, with the Studio teacher involved for a portion of that time. Mesa's Studio took place on a single day and included activities similar to those across Beachside's cycle of three meetings.

2. Perspectives

In this paper, we make a conceptual argument that Beachside's and Mesa's Studio structures, their cycles and routines, and their practices, the rich public representations of teaching involving collective interpretations of teaching, served as catalysts for the incremental learning of ambitious and equitable teaching. Our paper unfolds in the following manner. Because Studio seeks to change teachers' practices and knowledge, we conceptualize teacher learning as an act of pedagogical reasoning. Pedagogical reasoning is a lens through which to consider the Beachside and Mesa Studio structures and practices as catalysts for the incremental learning of ambitious and equitable teaching. Within our Perspectives section, we review research on pedagogical reasoning and a definition of ambitious and equitable teaching. We then turn to the slim but compelling body of Studio research for fostering pedagogical reasoning. We conclude this section by describing Beachside's and Mesa's Studios' structures and practices.

2.1. Pedagogical Reasoning

At the center of the Studio structures and practices are teaching and teacher conversations directed toward teacher learning about teaching and the knowledge of teaching. Researchers of teacher conversations as learning opportunities frame this as pedagogical reasoning [23–26]. Loughran and colleagues [26], in their comprehensive review of the evolution of pedagogical reasoning, offered nuanced details on this construct beyond this paper's scope. However, their work and that of others have highlighted the key features of pedagogical reasoning that we find compelling. Our conceptualization of pedagogical reasoning involves teachers collaboratively focused on generative teaching dilemmas. These dilemmas are rich in detail, allowing for the elaboration of evidence, explanation, and rationale to interpret classroom events. Further, as teachers engage in pedagogical reasoning, they offer differing analyses and justifications linked to principles of teaching and learning. These conversations lead to action and future teaching [16,26]. Lefstein et al. [25] offered additional insights into our conceptualization of pedagogical reasoning with what they called pedagogical productive talk. In their case-study research, teacher conversations are productive when they fulfil the following: (1) focus on problems of classroom practice; (2) use evidence, explanations, and reasons to interpret events and analyze and justify the courses of action; (3) are anchored in rich representations of practice; (4) involve different perspectives; (5) support a generative stance on the instructional system (content, students, teaching/learning and contexts) and problems of practice; and (6) foster trust, collegiality, and critical inquiry. Loughran [20] and Horn [26] remind us that teachers see their knowledge of teaching through a lens of practice and therefore do not always make explicit their rationale, or what Loughran calls the 'why' of their teaching [20]. Moreover, this type of reasoning is often not built into the typical teaching workday.

Horn et al.'s study [16], which built a taxonomy of low- and high-depth teacher conversations, empirically affirmed Loughran's [20] and Horn's [26] claim that pedagogically productive, or what Horn and colleagues [16] called high-depth, conversations were rare. Horn et al.'s [16] taxonomy detailed the complexity of features of pedagogically productive talk and our working conception of pedagogical reasoning. Here, they documented how rich representations of teaching, a feature from Lefstein et al. [24] and our working conception, are necessary but insufficient to ensure high-depth conversations. For example, when rich representations were univocally discussed, they were low depth because a teaching event would unfold with little questioning or press for explanation and rationale. In contrast, high-depth conversations involved what Horn and colleagues called a collective interpretation of teaching linked to future teaching [16]. These conversations included a rich representation of teaching. They dialogically elaborated justifications involving mathematics, students, and pedagogy, with multiple participants connecting representations of teaching to teaching and learning principles and future teaching. These conversations focused on the social and analytic scaffolding imperative for ambitious teaching [15,27]. Collective interpretation of teaching is the discursive practice entailed in constructing pedagogical reasoning.

We draw upon the construct of pedagogical reasoning, synthesized from our literature review, as a lens to examine Studio structures and practices as catalysts for the incremental learning of ambitious and equitable teaching. Pedagogical reasoning helps us analyze how Studio cycles and routines engage facilitators and teachers in predictable and repeated opportunities to use rich representations of teaching and the collective interpretation of teaching. This paper illustrates how Studio cycles afforded teachers opportunities to make teaching public. Routines were the shared rich representation of teaching that was both doable and embodied ambitious and equitable teaching principles. Moreover, these structures afforded space for the collective interpretation of teaching, such as taking up dilemmas, pressing for elaboration, examining alternatives, and considering future teaching. Next, we review the research on ambitious and equitable teaching to share our working definition.

2.2. Ambitious and Equitable Instruction

Ambitious teaching aims to ensure that all learners develop conceptual understanding and procedural fluency, engage in mathematical argumentation and reasoning, and communicate that reasoning using different representations [2]. Students' equitable mathematics learning means that teaching must mitigate learning barriers that shape lesson structures and the discipline of mathematics while also attending to students' multidimensional identities, status, and teachers' biases [28]. We additionally argue for attention to the structures of tracking and so-called ability grouping policies and practices that often shape the instruction of students of color and multilingual learners [29–31].

Because multilingual learners, students of lower socioeconomic status, and students of color have often been provided with a less than ambitious pedagogy, it is essential to provide teachers with opportunities to learn about equity-based pedagogies [21,32,33]. Teachers must trace linkages between practice and students' participation and self-reflect on bias [34]. When teachers teach in detracked systems and work with multilingual learners, PL must offer opportunities to engage in pedagogical reasoning to reframe the students' strengths with asset-based language [35], to discuss ways for the students to experience grade-level mathematics [36], and to provide multiple opportunities to share and refine ideas; e.g., rough drafts, [37], before formalizing mathematics [38–40].

Studio structures and practices provided teachers space for these pedagogical conversations that explicitly attend to what it means to have ambitious goals and work toward equitable practice. In the Beachside and Mesa Studios, teachers enacted and reasoned about the ambitious goals of MLRs and a data literacy routine (what we call a Studio structure). Because each Studio cycle included a common teaching experience, time for collective analysis, and consideration of future teaching, Beachside Studio members explicitly considered how to equitably support all the multilingual learners' mathematics and language production [14]. Mesa Studio members considered how heterogeneous student groups in their detracked classes worked with the new and often perceived "advanced" mathematics of data literacy. As illustrated in the selection of the Studio data shared, teachers' practices and pedagogical reasoning changed across cycles, thus advancing our argument that the structures and practices catalyzed the incremental learning of ambitious and equitable teaching.

2.3. Studio Research

A slim but compelling body of mathematics Studio research has emerged documenting mathematics teachers' learning [4,17,41]. We compare the Studio research to similar PL models published in approximately the past decade, which all build on lesson study in Table 2. All these PL models leverage the importance of planning, viewing teaching, and reflecting on teaching in cycles. The models differ in how teaching representations are accessed, either through a live enactment [4,42–47] or through viewing video-recorded enactments and artifacts from enactments [6,48–51]. Further, differences include who teaches lessons and who facilitates the learning. Finally, another difference is the lesson content in these models. Lesson study models are aimed at building a "research lesson" based on children's thinking [6,43–45]. Studio studies vary their content focus across science and mathematics, with most making reference to ambitious teaching. We note that across all these models, teachers work with rich representations of practice that anchor their conversations on practice [19]. Many of the PL models provide opportunities for teachers' incremental learning of ambitious teaching, with some specifically aimed at disrupting classroom inequities [6,10,43,44]. These models build on the lesson study's structure and create opportunities for teachers' sensemaking through the observations of teaching and engagement in a community of other practitioners.

We now delve into the science [45] and mathematics Studio research [4,42,46,47]. Thompson et al.'s [45] study found that the science Studio structures and tools that teachers adapted across Studios were critical to the development of teachers' ambitious science teaching. The researchers focused on teachers' sense-making conversations based on problems of practice they faced and their improvement of ambitious teaching practices and tools as they engaged in Studio cycles. Kim et al. [46] examined elementary mathematics teachers' pedagogical reasoning and the PL opportunities made available by Studio's structure. They found that teachers' pedagogical reasoning about children's thinking shifted and deepened across Studio cycles with the support of the Studio facilitator. Lai et al. [47] in a study of first grade mathematics Studio focused on teachers' pedagogical reasoning found that Studio structures allowed teachers to examine the reasons and purpose of decisions leading to building knowledge of and for teaching. In a unique mathematics leader Studio study, Carlson and colleagues [42] found that they needed to revise their initial goals and prompts to shift the participants' reasoning to support coaches and principals.

The researcher-facilitators leveraged the Studio's iterative cycle to realize their goals. By formatively tracking participants' discourse, the authors found that participants focused on students' content learning even though they were responsible for teacher development. Additionally, changes in facilitation across the cycles shifted the participants' focus to mathematics for teachers and support of K–12 students. While these authors did not use the construct of pedagogical reasoning, we see the nature of the changes they found aligned with this construct. In another Studio study, Lesseig [4] focused on the teachers' reasoning with the content and pedagogical knowledge of mathematical conjecturing, generalizing, and justifying. Lesseig's middle-level mathematics study examined the leader and participant discourse practices, skilled facilitation, and norms of interaction. Studio practices cultivated participants' inquiry stance, their use of evidence to support claims, their press for others to do so, and their elicitation of alternative interpretations. Again, this author did not use the specific construct of pedagogical reasoning, but the teachers' ways of reasoning built in Studio align well with our conception of pedagogical reasoning described above [16,26].

Table 2. A Comparison of Studio, Lesson Study, and Lesson Lab.

Professional Learning Model	General Structure	Who Teaches the Lesson	Who Facilitates the Model	Grade Level and Content Focus	Examples of Scholars Who Have Used This Model
Studio	Collaboratively Plan, Gather Data During Live Enactment, Reflect on Observation, Implement Revisions	Teacher with Coaches	Teacher Educator	Elementary, Math	Carlson, et al. [42]
		Teacher (with researchers and other teachers monitoring student progress)	Research Team	Middle School, Science	Thompson et al. [45]
		Teacher	Teacher Educator	Kindergarten, Math	Kim et al. [46]
		Teacher	Teacher Educator	First Grade, Math	Lai et al. [47]
		Teacher	Math Coach	High School Math	Lesseig [4]
Lesson Study	Curriculum Study, Lesson Planning, Observation of Lesson, Debrief	Teacher	Outside Expert	Variety	Morris and Hiebert [48]
		Teachers, Teacher Leaders	Teacher Leader and University-Based Mentor	K-8, Math	Ebby et al. [49]
		Teachers	Teachers	K-8, Math	Fernandez [6]
		Teachers	University-Based Outside Experts	Middle and High School, Math	Lewis [50]
Teaching Lab	Examine Standards and Research, Co-Plan Lesson, Co-Enact Lesson, Debrief	Facilitator	Facilitator in Participant Classroom	Middle School, Math	Amador et al. [51]
		Teacher Educator with Inservice Teachers	Teacher Educator	Elementary, Math	Gibbons et al. [43] Kazemi et al. [44]

These studies suggest Studio cycles enhanced the participants' pedagogical reasoning and allowed them to move beyond quick solutions to focus on instruction and student learning "problems". Next, we describe the Studio structures of cycles and routines.

2.3.1. Studio Cycles as a Structure

We illuminate the critical aspects of Studio structure and practices to situate our work as catalysts for incremental learning, beginning with the vital elements of the Studio cycle. Each Studio involved opportunities for an initial collective investigation for a Studio lesson (e.g., the MLRs at Beachside and data literacy routine at Mesa), Studio Lesson enactment with students, where the resident teachers (herein called residents) observed and took notes. The enactment debriefs involved collective descriptive conversations amongst all the residents to take up new instructional actions based on the discussions [4]. Each Studio cycle included these elements, and Studio is generally conducted for multiple cycles in a year. The Studio cycle provided a myriad of opportunities to engage in the collective interpretation of teaching [16] within a cycle, starting with the initial segment in what Beachside called the "pre-brief" before the Studio lesson and what Mesa called the "rehearsal" and the "rehearsal debrief." The Studio enactment phase then asked the residents to listen carefully to the students' participation and take notes (a form of interpretation) to support the third opportunity for collective interpretation of teaching, the enactment "debrief" of the Studio lesson. Studio cycles offered structural coherence from cycle to cycle, using the same activity sequence [52]. There were also opportunities across cycles for the collective interpretations of teaching supported by a coherent set of mathematical and instructional goals that connected one cycle to the next. In the Studio studies highlighted here, conceptual coherence was established and maintained using the same routine within each site [52].

2.3.2. Studio Routine as a Structure

We highlight routines as a structure within Studio that catalyzed incremental teacher learning and their pedagogical reasoning. As other authors have noted, routines are "patterned sequences of interactions that teachers and students jointly enact to organize opportunities for student learning in classrooms" [12] (pp. 1–2). A routine's sequence offers stable predictability via the actions guiding teachers and students. However, the routine does not dictate lockstep decisions; it provides opportunities for responsiveness to teachers' localized needs and students' contributions [53]. Examples of routines abound, such as creating a "five practices" structure for holding a mathematical discussion to confer with students on their reasoning [54–57]. As a bounded representation of practice, with a definite start and a closing set of actions, routines can be "sized" to fit within the classroom curricular materials. Indeed, routines have become more prominent in curricula, such as those developed to support the Common Core State Standards [58] mathematical practices [13] and those embedded in curricula to support the needs of emergent multilingual students [59].

Researchers studying routines suggest that they offer ambitious opportunities for teachers to (re)position learners as agentic and accountable to shared mathematical reasoning, drawing upon the epistemically diverse ways of knowing to solve complex and authentic problems relevant to learners [21,55]. For example, MLRs are instructional routines that expand the diversity, inclusion, and equity of the students' language and mathematics development [14]. The data literacy routine (described below) designed by Mesa facilitators borrowed structures from Kelemanik and colleagues [13], with embedded participation structures [60] and reflection prompts, but shifted to a goal of fostering students' data literacy.

The two routines explicitly intended to disrupt inequities that have long shaped school mathematics. The MLRs are specifically designed to challenge the "persistent assumptions about how to support and develop students' disciplinary language. . . to meet the needs of linguistically and culturally diverse students who are simultaneously learning mathematics while acquiring English" [14] (p. 3). These MLRs provide a pathway through the systemic and systematic barriers that multilingual learners face in mathematics classrooms to tasks, materials, and educators' expectations [14]. The data literacy routine pressed heterogeneous groups of students to hone their awareness of data and use descriptive language to make sense of data representations widely used in "real life" but not standard to U.S. high school textbooks and curricula, directly disrupting the practices that only advanced students or "math people" work with complex data representations [61]. Routines were the repeated representations of teaching within Studio, which centered on collective investigation, enactment, and debriefing within each cycle. Teachers learned about these routines through the repeated Studio cycle elements and across cycles as the routine was enacted multiple times. Routines were units of analysis for investigating collective learning. Given the similar ambitious goals of MLRs and data literacy routine and their specific aims to disrupt classroom inequities, they were structures for fostering Studio faculty's incremental learning of ambitious and equitable instruction [2].

2.3.3. Using Rich Representations of Teaching as a Practice within Studio

We examined the Studio practice of using rich representations of teaching to catalyze incremental learning and pedagogical reasoning. Facilitators, Studio teachers, and residents were encouraged to foster norms by making teaching actions and rationales public and available for discussion as a part of the Studio [23,27]. The Studio teachers, residents, and facilitators constructed a rich representation of teaching by dwelling on classroom events. Studio participants replayed these events by quoting students' and teacher's contributions, viewing lesson video clips, examining student work, or previewing a lesson plan [25]. At Beachside, rich representations of practice included a "pre-brief" discussion of the targeted MLR, observation of the MLR with students in the Studio enactment, a debrief discussion, and the viewing of video clips from the Studio lessons during the Post-Studio Day. Mesa's Studio included multiple opportunities to build insights from rich representations of teaching, starting with the routine rehearsal and debrief and ending with the routine enactment and debrief. Both projects grounded debrief conversations in residents' observational notes.

Through the rich representations of teaching, Studio teachers and residents deprivatized their practice, pressed each other for instructional specificity, unpacked the entailments of the routine, and allowed it to be viewed from a lens of benefits and challenges [24]. Beachside provided residents with an observation sheet to complete during the Studio lessons, prompting them to record what they saw students and Studio teachers doing in various stages of the MLR (i.e., Compare and connect stages of the Compare and Connect MLR), as well as how the teacher attended to other critical goals related to the MLR, such as how the MLR was integrated into the lesson. The observation sheet in Beachside also provided overall reflection questions to start the initial discussions of the lesson debrief, asking Studio teachers and residents to reflect on their strengths, struggles, and how students engaged with the MLR. At Mesa, facilitators framed each observation of teaching (rehearsals and Studio enactment) with a set of directions. The teachers were directed to record their observations linked to the goal and collect direct quotes from the students and the teacher. Each time the facilitators opened the rehearsal and enactment debrief conversations, they reminded the residents to use their notes, saying something similar each time, such as Georgia's quote from Studio 1: "When you start, we want you to say, 'I noticed. . .' then add the student thinking quotes, or 'Oliver said this, and I think.'" Notably, both projects asked the residents to "replay" or cite verbatim the students' or the Studio teacher's quotes in debrief conversations [62]. Observation tools and the consistent reminders for observing and debriefing shaped the norms for participating in Studio, namely the members' collective interpretations of how they made sense of instructions [4,63,64].

2.3.4. Using Collective Interpretations of Teaching as a Practice Within Studio

The second practice within Studio that we examined was using collective interpretations of teaching, which were the discursive practices of pedagogical reasoning. We highlight here what was required to support collective interpretation teaching as a Studio practice. Studio's cycle elements reviewed above offered Studio teachers and residents opportunities for focused discussions of the representations of teaching. Shaped by social norms of mutual trust, the debrief conversations following the enactments provided the Studio teacher and residents opportunities to share their perspectives on the routine, where the facilitator could press everyone to use evidence, explain their reasoning, and build on each other's ideas. For example, in the Beachside Studio, facilitators supported the Studio teacher in identifying a focal lesson goal for the cycle and paying attention to the MLR-centered discussion of the Studio enactment.

Further, the facilitators provided time for the residents to complete their observation handouts and overall reflection questions before beginning the debrief. By doing so, the facilitators prompted the Studio teacher and residents to ground the discussions in the routine and focal goal of the lesson [65]. As a result, Studio members could use evidence, unpack critical features of the MLR to enhance students' mathematical and linguistic learning, and consider their future teaching [14]. In Mesa's Studio, the collective interpretation of teaching was supported in several ways. The facilitators moved to lift and press on the routine's goals in debrief conversations [66]. The facilitators, Studio teachers, and residents upheld normative ways of interacting, including the goal-directed observation of students' and Studio teachers' contributions and cultivating trust while engaging in critical inquiry [67]. Studio members and facilitators acknowledged the group's distributed expertise, providing insights that beget further inquiry for collective learning about content, pedagogy, and students [24]. We note that these structures and practices within Studio can co-occur and reinforce each other to support Studio members' pedagogical reasoning about ambitious and equitable teaching. We next discuss our materials and methods.

3. Materials and Methods

3.1. Beachside Studio and Participants

Beachside, situated in the Beachside School District (pseudonyms are used for all proper nouns), is a high-needs K–12 school district in California with approximately 15,000 students. At the time of our study, approximately 25% of Beachside's students were classified as multilingual learners, and 48% qualified for free or reduced-price lunch. The district demographics were reported as 60% Latino/a, 31% white, 3% Asian/Pacific Islander, and the remainder of minoritized students.

Beachside occurred as part of a funded research project, which supported Beachside's Studio cycles for three years. Teachers engaged in Studio Day Cycles, as shown in Figure 1. In the data shared in this paper, the Studio teachers (those teaching during Studio lessons) and residents (those who did not teach during Studio lessons) were in the second of three cycles in the first of these three years during Fall 2023. They used the Compare and Connect MLR [14]. The project was focused on working with Studio members to simultaneously attend to language and mathematics through MLRs [14] and multilingual learner mathematics core practices [65]. In these cycles, the primary facilitator of the Studio Day Cycles was the PI of the grant (Roberts), who collaborated with a district mathematics instructional support specialist to implement these cycles. Six teachers who taught at the district's four junior high schools participated in the project during the year, as shown in Table 3. Beachside had a history of Studio (with Roberts) and previously participated in a two-year high school project with Math I teachers in 2019–2021. The two secondary Studio projects focused on using MLRs, which supported the district's interest in providing mathematics-specific PL that attended to multilingual learners. The MLRs were selected to meet the district's goals of accessing text and communicating reasoning. All teachers who participated in Studio consented to participate in the study. Two teachers were identified as bilingual, one bilingual in Russian, and the second teacher was semi-fluent in Spanish; all held a secondary teaching license in mathematics. During the Studio we examine in this paper, Ms. Taylor did not participate, as she left the project after Fall 2023 for personal reasons, and Mr. Valle had not joined the project yet, as he joined the project in Spring 2024.

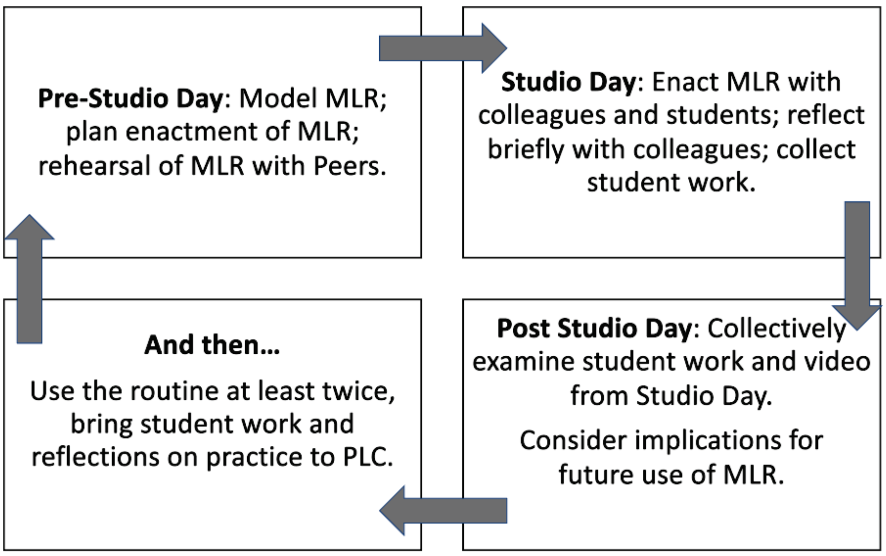


Figure 1. Beachside's Studio Cycle.

Table 3. Pseudonyms, Roles, and History of Beachside Teachers in Studio.

Name (Pseudonym)	Grade Level	Race/Ethnicity (Self-Described)	Years Teaching Mathematics	Studio Role
Ms. Ruth	7th Grade	White	26	Resident
Ms. Severn	7th Grade	Caucasian	4	Studio Teacher/Resident
Ms. Taylor	7th Grade	White	34	Resident
Ms. Foster	8th Grade	White	3	Resident/Studio Teacher
Ms. Penny	8th Grade	White	2	Resident
Mr. Valle	8th Grade	White/Caucasian	5	Resident
Ms. Hope	Instructional Support Specialist	Caucasian	25	Instructional Support Specialist

3.2. Mesa Studio and Participants

Mountainside School District, the site of Mesa Studio, was a K–12 district on the West Coast of the United States in a mid-size city serving approximately 17,400 students. At the time of the study, approximately 7% of students were designated as ever English language learners, and 36% qualified for free/reduced-priced lunch. The district was 80% white, and 20% were minoritized youth, with the schools segregated by socioeconomic background. Mesa HS students were majority white (87% across the district; 1 October 2019 data) and dominant English speakers (93%), with no school-level data available on free or reduced-price lunch because of COVID-19.

Elliott's funded project supported Mesa's two Studio cycles during 2020–2021, the facilitator planning and the associated Studio costs. In Fall 2020, Studio 1 was online due to the COVID-19 remote teaching restrictions, with nine teachers and two facilitators. In Spring 2021, Studio 2 was conducted on Mesa's campus, with seven teachers and the same two facilitators (see Table 2). Two teachers from Studio 1 opted out of Studio 2 because of upcoming job changes. Both Studio cycles focused on enacting the data literacy routine. After being introduced in the Fall 2020 Studio cycle, facilitators asked all teachers if they would commit to regularly using the data literacy routine across all courses. In the past, Mesa teachers had developed routines supporting mathematical practices, and they used their well-established Studio model to create a shared vision to improve instruction. Mesa's mathematics department had engaged in Studio for ten years, and the Mountainside district sponsored Studio in elementary and middle schools. For the past five years, including the study year, Mesa Studio was facilitated by two teacher-leaders. All the teachers and facilitators in Studio consented to participate in the study.

Eleven educators (Table 4; five presenting female and six presenting male) participated in Studio 1, and nine educators participated in Studio 2. The educators reported having taught 12 years on average, and all held a mathematics teaching license. Studio facilitators Georgia and Jasper taught high school mathematics for over a decade, teaching nearly all their careers at Mesa. They had participated in Studio since they were novice teachers. Six of the 11 Mesa teachers in Studio 1 had participated in Studio for nearly ten years, and only one teacher, Monte, new to the district, was new to Studio.

Table 4. Pseudonyms, Roles, and History of Mesa HS Teachers in Studio 1 and 2.

Participant Pseudonym	Studio Role	Years Teaching Math
Benson	Studio 2 Teacher	8
Brooke	Resident	15
Kay	Resident	26
Georgia	Studio Facilitators	14
Jasper	Studio Facilitators	13
Justine	Resident	13
Oliver	Studio 1 Teacher	1
Tyson	Resident	7
Zandra	Resident	10
Monte	Resident (Studio 1 Only)	Not Reported
Henry	Resident (Studio 1 Only)	Not Reported

3.3. Studio Models

3.3.1. Beachside Studio Cycle

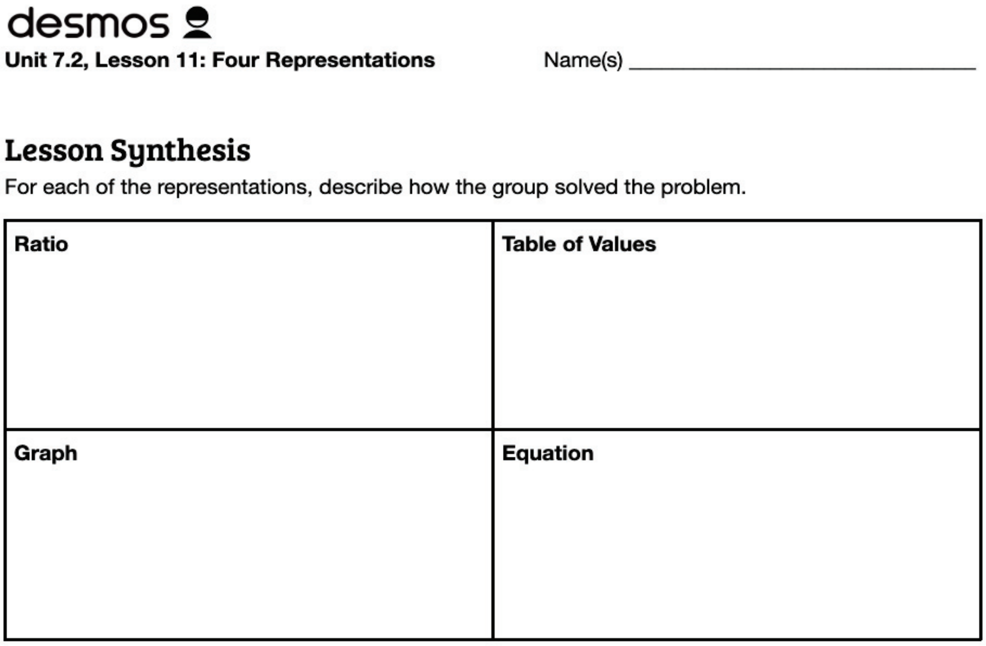
Beachside held three Studio cycles during 2023–2024. The data shared in this paper came from the second cycle, hereafter called Studio 2. Each Studio cycle includes the following four parts: three professional development meetings and considerations for future implementation (see Figure 1). In the Pre-Studio Day meeting, the research team introduced Studio members to the focal MLR through a sample lesson enactment. For example, in Pre-Studio 2, the group participated in a lesson on comparing the average change in sea ice thickness using the Compare and Connect MLR. Compare and Connect engages students in comparing and contrasting the different mathematical approaches by examining different mathematical representations, approaches, examples, or language [14]. The group discussed considerations for their lessons, including one teacher asking about the possibility of using a gallery walk to enact the Compare and Connect, an idea raised during the sample lesson. Residents collaborated during the Pre-Studio Day with residents from their school sites and others across the district. Two residents volunteered to be the Studio teachers and teach the MLR during the Studio Day in two different lessons. We sought consent from parents/guardians and assent from students to study the Studio enactment.

On Studio Day, the two Studio teachers enacted an MLR-focused lesson in their classrooms, and the residents observed it. Teacher observations included engaging with students, asking questions, asking students to share, and sometimes sharing students' responses during small group and whole class discussions when appropriate. The teachers noted their observations and reflections during the lesson, specifically on the MLR. The research team then facilitated a debrief after each implemented lesson. The Studio Teachers and residents identified how they might modify their instruction related to the MLR and their work with their multilingual learners going forward.

During the Post-Studio Day, approximately a week after the Studio Day, teachers came together to reflect on their and their students' work completed during their Studio Day lesson. The Studio members discussed the MLR enactments, analyzed the student's work, viewed video clips from the studio clip enactments, shared challenges and successes, and considered the additional implications for their future practice. Selected video clips from Studio Day were a means to elaborate on the teaching event. As the final component of the Studio Day Cycle, all Studio members were encouraged to implement the MLR at least two more times during the school year. They were also encouraged to bring student work and reflections from their routine implementation to other PL opportunities, such as their professional learning communities (PLCs).

We focus on illustrative examples from Studio 2, which used the Compare and Connect routine. This MLR had a math and language goal of comparing and contrasting different mathematical approaches by examining the different mathematical representations, approaches, examples, or language [14]. While two teachers, Ms. Severn and Ms. Foster, taught Studio lessons in cycle 2, we focus on Ms. Severn's lesson in this manuscript. Ms. Severn noted that they had not performed this MLR previously. She explained that she would use the gallery walk.

Further, part of her goal in enacting the Studio lesson was as follows: How could she get the students to talk to each other about mathematics because the class was "really hesitant to talk at all?" While the gallery walk was an idea that a facilitator had shared during the Pre-Studio Day, Ms. Severn planned her lesson independently. Ms. Severn shared during the pre-brief of the Studio lesson that students would complete the following problem from Desmos Lesson 7.2, Lesson 11: "The two quantities are: Five friends shared three pizzas. The total came to $36" [59] using a single representation (e.g., ratio, table of values, graph, or equation), which included additional prompts for their given representations (e.g., "Write the equation here."). Students were to post their solutions on poster boards to share them with the class. Ms. Severn planned to have the students rotate around the room and examine their peers' posters. She provided students with a graphic organizer (Figure 2), which asked them to record a ratio, table of values, graph, and equation [59] from their peers' solutions during the gallery walk. Students had four minutes to look at how their peers solved the problem and to record this work in the corresponding box on the graphic organizer.


desmos

Unit 7.2, Lesson 11: Four Representations Name(s) ______________________________

Lesson Synthesis

For each of the representations, describe how the group solved the problem.

Ratio	**Table of Values**
Graph	**Equation**

Figure 2. Ms. Severn's Graphic Organizer for Compare and Connect. Note: Used with permission. Image taken from https://teacher.desmos.com (accessed on 15 May 2024).

3.3.2. Mesa Studio Cycle

In 2020–2021, the Mesa teachers used the state's draft standards to infuse statistics and data reasoning into all their courses. In their work, the Mesa teachers drew on the expertise of the following colleagues who had strong statistics backgrounds: Tyson, the AP statistics teacher, who supported all the Math 10 teachers in developing units, and Benson, who volunteered to teach a data science course for third-year HS students in 2021–2022, the year following the study. Moreover, the Mesa Studio had transitioned over the years to embed innovative pedagogies, such as the *Five Practices for Orchestrating Productive Mathematical Discussions* [57], *Routines for Reasoning* [13], and routines that Mesa teachers designed to leverage mathematical modeling [68,69].

Similar to the Beachside's Studio cycle described above, Mesa's Studio cycle (See Figure 3) provided a structure that was a predictable set of elements centered on preparing for the live enactment by the Studio teacher with students and debriefing that event such that everyone had opportunities to take away insights. Before the Studio Day, Jasper and Georgia planned the Studio sequence of activities. They selected a Studio teacher based on the teacher's and students' willingness (parent/guardian consent and student assent) and the mathematical topic. The Studio teacher shifted based on who had teaching responsibilities during a time that allowed for Steps 1 and 2 in the cycle to be completed before the live enactment and whose students would profit from the content goals that could be leveraged in the Studio cycle.

In our results, our Mesa illustrative examples will focus on the two Studio cycles from 2020–2021. Three Studio cycles were initially scheduled for the year. However, two cycles were completed because of COVID-19 and the complexity of shifting from distance learning to opening schools for in-person teaching. Jasper and Georgia engaged in extensive Studio planning that was observed and recorded (Studio 1, three hours; Studio 2, four hours) before each Studio Day. Here, the facilitators designed the data literacy routine (described in the next paragraph) used in the two Studio cycles. Planning for Studio 2 included interactions with the Studio teacher to co-plan. Studio 1 created opportunities for the Studio teacher to plan during the Studio Day.

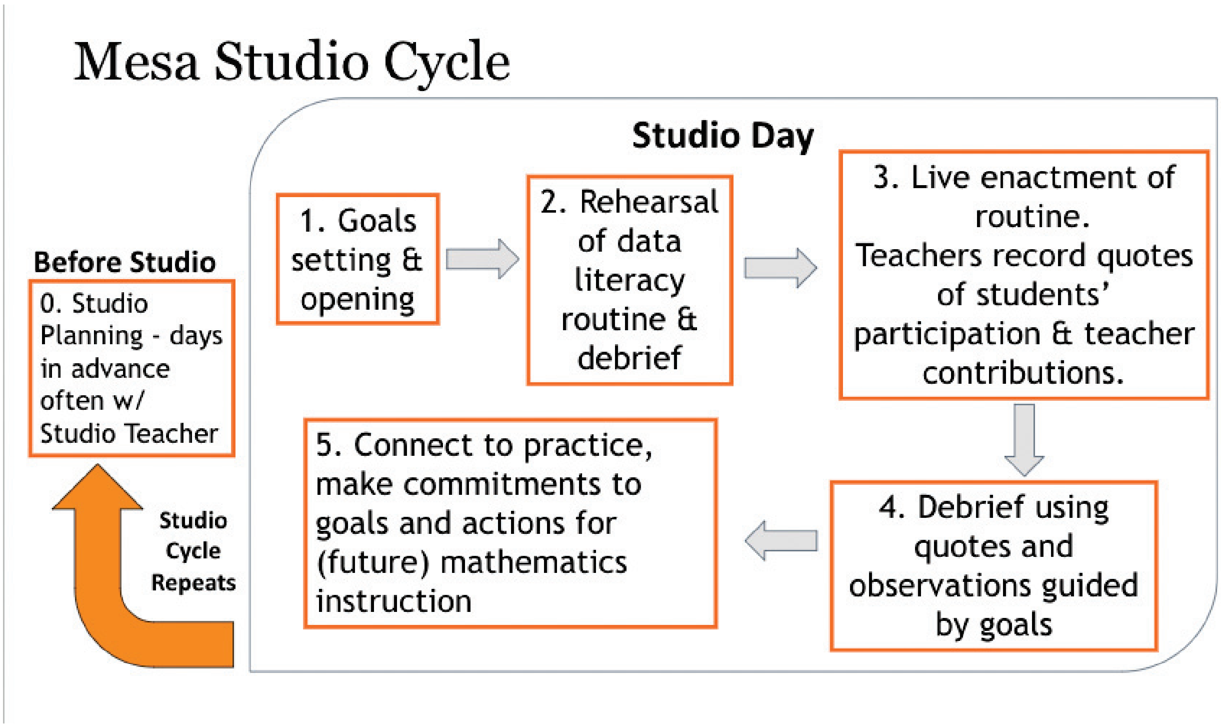


Figure 3. Mesa's Studio Cycle.

The Studio opened with a presentation of goals and an agenda for the day, followed by some introductory discussion or activity leading to a routine rehearsal (two rehearsals in Studio 1 and one in Studio 2). The rehearsal allowed all the residents to experience the routine as participants and to debrief it to engage in the collective interpretation of teaching. These debriefing discussions were goal-focused and unpacked instructional decisions, considered rationales of decisions, anticipated students' thinking, contemplated alternatives, and supported the Studio teacher in refining how to anticipate and lead the routine's important mathematics moments. The Studio teacher then enacted the routine with high school students and residents closely observed, noting students' participation, and guided by norms for recording specific contributions of students' mathematical participation and teacher contributions.

Facilitators guided the notetaking and the debriefing, assuring that the goal of recording quotes was not to critique the Studio teacher. For example, Georgia noted, "We are not trying to critique, specifically not trying to make an exemplar." These verbal reminders, which happened at the start of every debrief, framed the conversations as a space for critical inquiry and a way to build trust [67]. The opening of Studio 2's enactment debrief also started with a statement from Georgia, "Teacher moves create the student moves", and she provided reminders not to critique. This framing led us to look for patterns in our data and to uncover what we called a principle that, while unspoken, guided Studio members' actions and responses. Their principle was that students' ideas are sensible, and it was the teachers' responsibility for all students to show their brilliance, exemplified by how they interpreted students' contributions and critiqued lesson designs and prompts [15,16]. We offer more detail on this idea in the Results section and focus on it as part of Mesa's collective interpretation of teaching.

The Mesa Studio enactment debriefs focused on the Studio members' collective interpretation of teaching, which included attention to the students' mathematical contributions and pedagogical moves concerning the routine's learning goal. Facilitators were observed pressing the Studio members to consider how the routine supported students' data literacy.

The enactment debrief was a second opportunity for the collective interpretation of teaching, with the routine typically happening in two phases. The first was an opportunity to build a shared understanding of the enactment and to provide feedback to the Studio teacher. The second phase shifted to connecting the routine to everyone's practice by discussing what "tweaks" the Studio teacher or residents would make to the routine. This pressed everyone to consider their teaching and collective commitments to enact the routine in the future. The data literacy routine was used in Studio 1 (November 2020, online, 4 h), where facilitator Georgia and Studio teacher Oliver each rehearsed, and Oliver enacted the routine online. The Studio teacher, Benson, used the data literacy routine in Studio 2 for in-person teaching (June 2021, in person, 5 h). As stated by the facilitators, the goal of the routine was to "support students' data literacy and support teachers in routine development that focuses on data literacy." At the time of this study, data literacy was a new K–12 content area rapidly emerging in education [61]. The Mesa facilitators were invested in data literacy and data science because the state was moving to rewrite standards for data reasoning (citation blinded for review). Between Studio 1 and 2, the Mesa faculty attended an online data science workshop from Youcubed [70]. There was an additional goal of considering the sociopolitical climate, which Jasper noted as he opened Studio 1's first rehearsal debrief, explaining the goal and their intention to the Studio members.

> *"We're trying to look at something twice and change your thinking. And, we felt like there's an urgency for that right now, especially if we talk about Coronavirus. If you show people a graph, are people comfortable being calm? So, it seems like there's an urgency for that, which is part of the reason for the routine. And, the second part was about having students be able to recognize the variables that they're paying attention to . . . are they able to make a connection between those things?"*

Jasper referenced the United States' sociopolitical climate; when COVID-19 rates were skyrocketing, there was a contentious presidential election, and there was no COVID-19 vaccine. The need to understand data was a shared sentiment across the Studio members, evidenced in their buy-in to plan a routine and commitment to use it with students.

The data literacy routine involved using the Desmos platform [71] to support Studio members with teacher notes and a template that everyone could use for any data representation. The routine created three opportunities to examine a data representation (The routine was similar to slow reveal graphs: see [72]; however, the facilitators were not aware of this site at the time.) by slowly revealing the full representation to see and make sense of it. Prompts organized the teacher and students' interactions with the representation, asking students to "Notice and Wonder" and consider how their thinking had changed as the activity revealed the complete representation. The routine concluded with a set of sentence stems for reflection. In Studio 2, facilitators added a goal of "using the mathematical practices to pass the baton," meaning teachers would pay attention to when students were "meta-cognitive" that they were using mathematical reasoning. For example, when students changed their interpretation of a representation based on their data literacy, the residents' goal was to make them aware of this. Across the two Studio cycles, the Studio members participated in five routine enactments, three via rehearsals and two in classrooms with students.

4. Examples from Beachside and Mesa Studio

In this section of the manuscript, we build our conceptual argument by using a chain of evidence to answer our important question. We share examples of how Beachside's and Mesa's Studio cycles and routines catalyzed incremental learning. We use the Beachside example to bring to the forefront how the first practice, using rich representations of teaching, catalyzed incremental learning. We use the Mesa example to put the second practice, collective interpretations of teaching, at the forefront. In both examples, we examined the Studio structure of routines to catalyze incremental change; however, we focused on a single cycle in Beachside and multiple cycles at Mesa, exploring different types of routines at the sites. We share these examples to argue that Studio structures and practices catalyzed incremental learning of ambitious and equitable teaching [7] and advanced pedagogical reasoning.

4.1. Beachside Studio Within a Cycle Highlighted a Rich Representation of Teaching

To illustrate Beachside's incremental learning, we share episodes from Studio 2—two from the Studio Day debrief and one from the Post-Studio Day. These episodes provide illustrations of the Studio teacher and the residents engaged with MLRs, with an explicit focus on attending to ambitious and equitable instruction. In this instance, the teachers considered the Compare and Connect MLR such that students could examine and discuss multiple representations. As such, the MLR allowed Studio members to examine ambitious and equitable instruction by focusing on instruction that could meet the needs of linguistically diverse students by providing avenues for rich language opportunities and cognitively demanding mathematics content [12]. We focused on different aspects of the gallery walk related to Ms. Severn's implementation of the MLR, with the Method section providing an initial preview of Ms. Severn's Studio lesson and this gallery walk. Teachers engaged with rich representations of teaching and collective interpretations of teaching during the Studio cycle, and the cycle and Compare and Connect routine were catalysts to foster the incremental learning of ambitious and equitable teaching. We begin with Ms. Severn's debrief of the Studio as an illustrative example of the structures and routines of the studio and the practice of using rich representations of practice.

Once the residents completed their observation form with questions and reflections (introduced in the method section), the debrief of the Studio lesson was further structured with questions from the research team posted on a PowerPoint. Ms. Severn began her debrief first. Some of the questions from the PowerPoint for Ms. Severn were as follows: How do you feel about the lesson? How, if at all, would you consider multilingual learners during the lesson? How did you adapt the MLR in real-time? And how might you modify your Compare and Connect going forward? Ms. Severn explained the following:

> *"I feel like I spent more time on the actual process [of the gallery walk] than I did on the conversation at the end. And so, I was trying to think of either, 'How can I cut down on some of the process of getting the posters made?' Or, 'How can I incentivize kids to speak a little, to be willing to share a little faster,' because that conversation took a long time to get voices going. And a lot of it was just me being like, 'Are you sure you don't want to share?' So yeah, I'm trying to think of how I can get them talking more? Or, how can I leave more room for that at the end, so that if it does take a long time, we still get to the rich conversation?"*
>
> *(Debrief, 105–112)*

Ms. Severn shared a teaching dilemma with her peers about how she might move her process of using the routine of Compare and Connect more quickly to support students to get to the mathematical conversations that she wanted students to have and how she might make more space for rich conversations as part of using the MLR with her students. She also returned to her original question posed in the pre-brief, which we noted in the Method section, of wanting to get the students to talk to each other about mathematics because the class was "really hesitant to talk at all".

In this Studio debrief, we began by noting some of the roles of structures to catalyze discussion. First, the Studio structure made space for debriefing the lesson and returning to ideas from earlier in the day, such as the goal Ms. Severn had stated in her pre-brief. Second, the routine structure became a focal component of Ms. Severn's reflection, related to how she might move forward with the routine more quickly and create rich mathematical conversations. Further, the structure of the routine grounded the conversation, with Ms. Severn considering the affordances and constraints of the MLR and how she might use the MLR to get students in the class talking while also using it more efficiently. Finally, regarding using rich representations of practice, Ms. Severn started the conversation with the rest of the participants by drawing on her representation of practice. She referred to her

Studio lesson, noting how she felt she spent more time on the process of the routine than on engaging students with the mathematical content of the routine. This initial debrief created a space for incremental learning through pedagogical reasoning for Ms. Severn because she felt comfortable with her peers; Ms. Severn was in a space of mutual trust to attend to this teaching dilemma and could honestly share her thoughts on her teaching from the Studio lesson. The Studio norms created space for Ms. Severn to share her thoughts about the lesson before others did so, honoring the expertise of the teacher who taught the Studio lesson. This norm positively positioned the Studio teacher and their public display of teaching, lifting the teacher's voice.

Following Ms. Severn's sharing, the facilitators opened the reflection to the residents, allowing the group to unpack the lesson. The group tackled Ms. Severn's question of how she could get students to talk more by discussing the value of having students examine multiple mathematical representations and using a mathematics task that allowed students to use multiple modes of communication [73] within their work with their peers. The group then discussed their role during Studio in visiting small groups, including the types of questioning they used with students to support reasoning and increase student dialogue about mathematics, such as by asking students to identify specific points on the graph. The group next attended to the time spent on the MLR routine. They discussed that Compare and Connect could be a short routine or used for an entire class period, as Ms. Severn had enacted. Ms. Hope (the district math specialist) initiated the following conversation.

> *Ms. Hope: "What I saw is that you took this MLR that some people might think of as small, but it was a lesson structure. And so, you went to the selection of the task, to the way that you work with teams, to the gallery walk, to the debrief. If that lesson structure became a routine, then some of this would speed up for you."*
>
> *Ms. Severn: "That's true, getting them used to it."*
>
> *Ms. Hope: "Right, so, that's where then the routine becomes routine. But this evidence of providing students opportunities to use multiple means of communication. . .. And so, you created a lesson where you set up this time to, yes, time to listen and share, write, listen, and speak about what they had seen and written, right, and the written part of the representations. There's that lesson [that] captured a bunch of modes of communication, from reading, writing, speaking, listening, whatever. And so, that was the design of the whole lesson. And it was like, I didn't even realize that until I was writing this down. I'm like, oh, just look at how you hit all of those modalities. And if you did that again, and again, students will get better at it."*
>
> *Ms. Severn: "Yeah, that's a good point. If we keep repeating this structure at least once per unit, they'll get used to it for sure."*
>
> *Ms. Ruth: "The trick is finding the problem that allows for that structure. But it lent itself nicely because you did have these multiple—no, not entry points so much, but multiple representations or multiple ways of solving it."*

Ms. Hope explained that the MLR could be a fast routine or a whole lesson structure, including working with groups, completing a gallery walk, and debriefing the work in the Compare and Connect. The group strategized how teachers could draw upon students' language and mathematical ideas in the MLR. They also considered ways to use the MLR as a complete lesson or a brief routine. Ms. Hope highlighted that in teaching a lesson with Compare and Connect, students would have the opportunity to engage with their multiple modes of communication to address Ms. Severn's concern about students communicating in class. Using this repeated structure within a lesson over time, students could develop familiarity with the structure and routine, supporting their work with the routine, the communication, and the mathematics. Ms. Ruth also raised the challenge of finding suitable tasks to complete this work [74].

The Studio structures of cycles and routines catalyzed learning during the debrief. The Studio structure of a cycle created a space for residents to revisit Ms. Severn's teaching dilemma from earlier in the Studio cycle. Additionally, the gallery walk initially posed on Pre-Studio Day was a source of continued reflection and a rich representation of practice. During the lesson debrief, Ms. Severn and the other teachers considered it and the MLR. The debrief created opportunities for discussion around practice and supported unpacking the structure and role of Compare and Connect as an MLR, including the gallery walk. The routine was a catalyst for learning in multiple ways. In particular, the group highlighted that Compare and Connect helped illuminate multiple modes of communication [73], and they considered whether the routine might be helpful as a shorter or a whole-period routine, as Ms. Severn had enacted. Drawing on Ms. Severn's lesson as a rich representation of practice, for example, when Ms. Hope noted, "What I saw is that you took this MLR," the residents were taken back to Ms. Severn's classroom enactment during the debrief. In their conversations, there continued to be a mutual trust to unpack ideas around teaching practice. The residents worked together to consider a course of action for moving forward with the MLR—to make it a routine. The debrief asked residents to draw on evidence, including using rich representations of teaching in the Studio lesson and their observation protocols, to consider their observations, to interpret these events, to unpack teaching dilemmas, to develop collective understandings, and to consider a future course of action.

The final episode of the Beachside Compare and Connect Studio came from the Post-Studio Day, a few weeks following Ms. Severn's Studio lesson. At the beginning of the Post-Studio Day, the facilitator asked what the Studio teachers and residents had been thinking about related to the Studio cycle and Compare and Connect since Studio Day. Ms. Severn began by noting how much she enjoyed the routine. However, she was still wondering whether she should "just stick to doing routines that are shorter" (Post Studio, 88–89) because she did not "feel like I have time to do [Compare and Connect as a full day] every unit and throughout a whole day" (Post Studio, 86–87). Ms. Ruth followed by sharing that she had used Ms. Severn's enactment in her class before Thanksgiving.

> *"I did Ms. Severn's lesson on Thursday and Friday. It was, uh, supposed to be an easy day before Thanksgiving, right? We were just gonna do something that was super easy and didn't require a lot of prep. And, I, you know, I was like, I can do this. And, all I did is create a ton of work [before Thanksgiving]. . . And they were engaged. . .. And, it was just really cool. So, thank you Ms. Severn for showing me that I can do it. And, I just took her lesson and implemented it in two days. And it was great. . .. And, and well, it was thanks to this whole process. It was, like, 'I can do this!' And, it was really good. Nobody else [in my grade level] did it. I was the one who did it, because I was willing to put in the work."*
>
> *(Post Studio, 113–143)*

Ms. Ruth drew on Ms. Severn's lesson. She highlighted being able to see the lesson in the Studio lesson and how that implementation and seeing it during the Studio allowed her to confidently use Ms. Severn's lesson and the Studio Compare and Connect MLR in her classroom with her students. However, she also highlighted that this work was challenging, particularly before the holiday. Ms. Ruth's sentiments, perhaps, illuminate that the incremental work of ambitious, equitable pedagogy can be demanding. Even so, Ms. Ruth also reflected that "it was really good" and "thank you, Ms. Severn, for showing me that I can do it", illustrating that, sometimes, it can be helpful to have a model or a rich representation of practice to create an image of a new practice. The lesson engaged her students, which was a positive experience for Ms. Ruth. While she noted that it was some work to create the lesson, particularly before Thanksgiving and without any support at her site, she successfully tried something new through the Studio process. The Studio group provided a place of encouragement and support beyond Ms. Ruth's site-based team, where she could try out and debrief ambitious, equitable instruction. Ms. Ruth saw Ms. Severn develop a lesson that included the MLR Compare and Connect, try out the MLR, and publicly debrief the MLR. In this process, Ms. Ruth, with the confidence she gained through the Studio process, developed her version of a lesson that used Compare and

Connect, making it her own, and shared her practice with her peers on Post-Studio Day. In this example, Ms. Ruth exemplified incremental learning during Studio.

On the Post-Studio Day, the structure of the Studio allowed the teachers to return to the routine structure, Compare and Connect. The structure of Studio allowed the group to return to Ms. Severn's use of the gallery walk once more, this time, with Ms. Severn continuing to think about how she could make the routine a routine. They also heard about Ms. Ruth's enactment of Compare and Connect, using Ms. Severn's model. The structure of the routine and the practice of using rich representations of practice provided a shared focus across the conversations. In their work together, the Studio team used prompts from the facilitators to begin their discussions to focus on the Studio lesson, specifically around the Compare and Connect MLR. The group used the structure of the Studio and the rich representations of Ms. Severn's and Ms. Ruth's teaching to make sense of the Compare and Connect MLR. Ms. Ruth, drawing on Ms. Severn's prompt of having time for the Compare and Connect within a unit, shared her experience enacting Ms. Severn's lesson with her students and having such a positive experience, even if it was challenging at times. Ms. Ruth built on Ms. Severn's ideas and those developed during the Studio Day to plan her lesson and enact it with her students, interpreting the classroom enactment she witnessed and debriefed with her peers and developing a course of action. Ms. Ruth then shared her initial impressions of completing her Compare and Connect lesson, describing her representation of practice with her peers. Following Ms. Ruth's sharing, Ms. Severn, Ms. Ruth, and Ms. Hope further discussed how the gallery walk might be enacted aside from using the trifold boards that both Ms. Ruth and Ms. Severn had used, such as using portable whiteboards. Through his process of debriefing in the Post-Studio Day, we saw multiple opportunities for incremental learning, with Ms. Severn considering how to continue to develop her use of the MLR in her teaching with the support of her Studio team and with Ms. Ruth developing, enacting, and reflecting on her Compare and Connect lesson.

The space of the Studio allowed the teacher participants to unpack the seen and unseen aspects of practice in both Ms. Ruth's and Ms. Severn's practice, thereby informing the incremental learning of the group through their pedagogical reasoning [18]. Engaging in discussions during the debrief, guided by discussion prompts, provided a norm during the MLR Studio to guide the pedagogical reasoning, where the teacher participants were able to provide supportive feedback and to push on future practices, such as how to make Compare and Connect a routine and whether to make a gallery walk part of this practice. Teachers drew on their experiences in the Studio, using representations of practice to ground their conversations. The Compare and Connect routine, as a structure, helped to guide conversations. The routine was the lesson's focus in the Studio classroom, and the residents brought their discussions back to what had occurred in the classroom during both the Studio debriefs and the Post-Studio Day by attending to the rich representations of teaching. Teachers noted how they might make incremental changes, such as trying out the routine, as Ms. Ruth had, or using a different method for implementing the gallery walk, such as portable whiteboards.

4.2. Mesa Studio Across Cycles Highlighted the Collective Interpretation of Teaching

The Mesa Studio site highlighted how incremental learning happened across cycles via facilitators' press on Studio members to attend to the mathematics of data literacy, students' reasoning, and teachers' pedagogical moves. They did so by leveraging Studio's practices, building multiple rich representations of teaching using the same data literacy routine and collective interpretation of teaching to analyze the use of the routine. The Mesa site forefronts the Studio members' collective interpretation of teaching driven by their commitment to the principle that students' ideas are sensible, and it is the teachers' responsibility for all students to be able to use their brilliance. The data literacy routine explicitly took up this asset-based principle for student learning by asking teachers to recognize how students used descriptive data literacy language and narrate it for the whole group in ways that would make it accessible to others. We organized the examples in the following manner. Episode one is Studio 1's discussions from three debriefs of the two rehearsals and the Studio lesson. Episode two is Studio 2's discussions from two debriefs of the rehearsal and the Studio lesson.

4.2.1. Episode 1

In Studio 1, we saw a sequence of facilitator moves and resident contributions aimed at the Studio members learning to lift students' contributions and narrate their data literacy. In this cycle, facilitator Georgia's rehearsal was the Studio members' first opportunity to experience the routine described in the methods. During the rehearsal, Jasper interjected, "I'm noticing that people are thinking about a couple different variables . . . [and] the relationship between those . . . I just wanted to appreciate the attempt to connect some of the variables that are happening in this [graph]." Jasper's comment lifted the goal for the residents to hear and link to ideas they were generating. The routine debrief opened with Jasper telling residents, "I popped in a couple of times to, [say] we're making connections between the variables that are happening there, and I want to make that transparent".

Recall Studio 1 had two rehearsals as described in the methods. The Studio teacher's rehearsal debrief also called Jasper into action driven by the Studio goals, "I'm going to pause this, [let's] talk more about what [Studio teacher] Oliver was saying to help kids focus on data literacy." Jasper, here, pressed the Studio members to consider the pedagogical moves that lifted the students' contributions and narrated their data literacy. However, we saw a limited uptake of Jasper's press, albeit the debrief was only 12 min long. Instead, the residents focused on the students' engagement, followed by the facilitators summarizing the mathematical and pedagogical ideas based on Oliver's strengths.

The routine rehearsal allowed Studio members to learn more about the routine's ambitious and equitable goals for student learning. Facilitators could formatively attend to the Studio teacher's and residents' learning as they heard and experienced the routine's goals in action. The routine's structure was introduced and unpacked during the rehearsal and debrief, which, similar to Beachside, ground the conversations in the students' brilliance by lifting the students' contributions. Finally, the collective interpretation of teaching in the rehearsal debriefs provided a space for incremental learning through building on teachers' ideas, such as Jasper's role in the debrief of lifting the instructional goal and lifting teachers' contributions, focusing on the collective understanding of the group.

In Studio 1's lesson debrief, residents replayed Oliver's responses such as, "Could you expand on . . ., and "Wow, I love your analysis." Residents focused mostly on how Oliver socially scaffolded students' contributions [27]. Numerous Studio members commented on the students' brilliance, demonstrating their principled commitment to the students' ideas are sensible and that it is the teachers' responsibility for all students to show their brilliance. For example, when one student in the enactment conjectured about data beyond the representation, Tyson noted, "I thought that was awesome." Georgia commented on that student's contribution and the brilliance of other's contributions with exuberance, "The students are f**ing rad. How blown away were you guys by how well they interpreted that graph"?

Studio members replayed the students' contributions as assets; however, they did not specifically address how the ideas related to data literacy and routine goals were related. Zandra's comment linked the students' brilliance to the routine design, "They did amazing things that probably wouldn't have happened had they only seen that [one] graph," remarking on the benefits of the routine to reveal the representation slowly. Jasper moved to connect to the routine goal at one point and used a resident's question that pressed Oliver for a rationale to narrate how Oliver emphasized changes in the students' data interpretations, one feature of the routine's goal stressed at the start of Studio 1.

At this point in the Studio cycle, noticeably absent in the conversations were linkages between the pedagogical moves centering students' voices and the teachers' capacities to lift and narrate students' data literacy. Facilitators and Studio members grappled with their data literacy understanding catalyzed by the Studio's routine and practices. This was confirmed by Georgia's remark reflecting on Studio 1, "I was hoping for more about what it means to be data literate. I'm not sure of that myself or if others really know what it means to be data literature. It's difficult to teach kids how to be data literate when we ourselves don't necessarily know." Jasper lamented, "I wanted to talk more about how to promote teachers to build on talk about [variables] with their kids, but we didn't quite get there". The collective interpretation of teaching involved generative attention to core features of pedagogical reasoning but fell short of advancing the routine goals.

4.2.2. Episode 2

In Studio 2, facilitators and residents focused on the same routine to build a rich representation of teaching and to collectively interpret it. The goal was to support students in data literacy and to support teachers in routine development on data literacy. Jasper and Georgia capitalized on the resident Justine's feedback from Studio 1 to add a goal of using the mathematical practices to pass the baton. This was a teacher learning goal asking the Studio members to make evident in their teaching and their reflections on teaching when the students were using mathematical practices. This included attention to data literacy. Studio 2's rehearsal debrief opened with Jasper saying, "I think we're still missing the moments when we are telling kids when they're getting better at data literacy." Studio teacher Benson agreed and presented a dilemma focused on students' access to the data representation. As a resident started to talk about her data literacy insights from the routine, Jasper quickly remarked, "I'm hearing good data scientists right now . . . but I think we got to be more meta with the kids," as though to counter Benson's dilemma that there was an accessibility issue.

With an invitation to all the Studio members to generate ways to lift students' contribution and to narrate their data literacy in the rehearsal debrief, a multivocal discussion unfolded in which everyone but the Studio teacher narrated when their data literacy ideas emerged and built off these examples to offer alternative "tweaks" to the routine. The collective debated various revisions, clarified student goals aligned to those revisions, anticipated students' responses to revised prompting, and rehearsed ways everyone might lift and narrate students' viable arguments for data literacy (i.e., a mathematical practice linked to data literacy). This discussion differed notably from Studio 1 because Studio members offered generative pedagogical alternatives linked to students' mathematical and contextual knowledge, pressed for specific goals, and analyzed the routine design connected to data literacy opportunities—their collective interpretations of teaching advanced teachers' pedagogical reasoning [15].

After nine minutes of Studio members' collective interpretations of teaching, Benson stepped back into the conversation. "I agree, there wasn't much depth. . .with the first image, we weren't getting very far. What I'm concerned about right now is this thing gonna stall out dead, and there's not gonna be a lot happening?" Benson was concerned that students would not have access to the ideas, and the conversations would fall flat. As the debrief was winding down, the following transcript provides an example of the group's collective interpretation of teaching regarding their future teaching. The principle, students' ideas are sensible, and it is the teachers' responsibility for all students to show their brilliance, underpinned how the Studio members grappled with the routine's design and goals.

1. *Benson: The time's coming up [for class]. I'm just trying to think about how to tweak this.*
2. *Georgia: What would we say is the purpose of the routine?*
3. *Zandra: For me, [it's] that your first moment of looking isn't enough. Keep looking.*
4. *Benson: Yeah, [the routine] draws your attention to what's missing.*
5. *Jasper: Which is making sense and persevering.*
6. *Zandra: And iteration.*
7. *Georgia: We want them to become data literate. So, if that's what the baton is, can we see the baton in action?*
8. *Tyson: If data literacy is the baton, and not the practice, then I feel like, you want to give them everything and see if they will make that connection. If the baton is the practice, critique*

and debate the same and different. Some kids are going to look at that and still not be able to make sense of it, especially if they are not [aware of] sports [the context of the second use of the routine]. Other kids are gonna look at it in two seconds and know what's going on.

9. *Benson: Why are we so concerned about passing three batons [math practice 1, 3, and data literacy] in 20 min? When we don't even do data science in the first place!*
10. *Brooke: We just need to know what the goal is though!*
11. *Jasper: All I'm saying is, I naturally tend to, and I think we all tend to, leave the data literacy baton off.*
12. *Benson: Which I am fine with.*
13. *Georgia: Why don't we try?*
14. *Jasper: Right, that is why we are here, to try it, because it doesn't have to be an exemplar.*
15. *Benson: Just tell me before the bell! [the group erupts in laughter]*

In this excerpt, Studio members were engaged in the collective interpretation of teaching with future teaching imminent (lines 1 and 15). Based on his previous remarks, Benson framed his dilemma as a design "tweak" to ensure students' access (line 1). The facilitators reframed the dilemma to be about clarifying goals, enacting the foundational principle, students' ideas are sensible, and it is the teachers' responsibility for all students to show their brilliance [14]. The residents offered ideas (line 6) and unpacked alternative goals (line 8), and the facilitators pressed (lines 2, 7, 11, 13, 14) Benson on a data literacy goal. They reminded Benson and the residents that the Studio was a place to try new ideas. Benson specified his concern (line 9) that the routine had too many goals and, in a cheeky way that elicited laughter, he asked for direction (line 15). As the debrief wrapped up, residents and Benson reviewed the pedagogical moves to support the students' mathematical ideas. In response, Benson proposed a strategy to attend to data literacy, which he thought he could accomplish.

Studio 2's lesson debrief demonstrated the teachers' incremental learning as multiple residents evaluated where "the baton" was passed and the students' data literacy brilliance shined. This was a shift from Studio 1, when only the facilitators offered this evaluation. Justine said, "The moment that it happened was when the reflection prompt changed into doing. All of a sudden, they all started talking about the things we would want them to do every time they look at graphs." Residents and facilitators effusively noted the students' brilliance across Benson's teaching. After discussing revising the routine, Benson commented, "I agree with Brooke; I would try [next time] to talk about the variables in each graph. I think that is the inroad on it, and that didn't come out." Benson showed a change in his understanding of the goal of the routine that echoed many of his colleagues' collective interpretations of teaching. Benson acknowledged the value of lifting the students' data literacy contributions and narrating them. Studio 2 debrief documented Benson's and the residents' shifting pedagogical reasoning associated with the routine [26]. In Studio 1, residents focused on student engagement and pedagogy without explicitly finding ways to leverage the students' data literacy. In Studio 2, Studio members concentrated on strengthening the routine for future teaching so that the students' brilliance was evident every time.

We noted a difference in the collective interpretation of teaching in Studio 1 and 2 debriefing. We hypothesized that this may be due to several factors. Studio teacher Oliver was a first-year teacher, and Benson was at the end of his ninth year. There were also differences in the level of the facilitators' and residents' press for specificity and rationale [16]. In Studio 1, the facilitators primarily took on the press, interpreting the routine goals and learning opportunities. In Studio 2, the press interpreting the routine goals was shared by facilitators and residents. Studio cycles that used the same routine catalyzed the members' incremental learning. Through Studio members' collective interpretation of teaching, we saw their principled commitment to the students' ideas are sensible, and it is the teachers' responsibility for all students to show their brilliance underpin the ways debriefs evolved across the Studio cycles. In Studio 1, the Studio members noted the students' brilliance, and the facilitators invited, pivoted, and reminded residents of the routine's goal to lift students' contributions and leverage their data literacy [28,38]. Residents took seriously their responsibility to center students' brilliance by elevating their voices and appreciating their contributions [35]. In Studio 2, we saw incremental shifts in how residents took up their teacher responsibilities, and brilliance was linked to explicit pedagogical work they needed to do in the routine to deepen students' data literacy [15,21,61].

5. Discussion and Conclusions

We aimed to offer a conceptual argument on how Studio structures and practices catalyze the incremental learning of ambitious and equitable teaching. We offer these ideas building on the research documenting how practice-focused PL models, such as lesson study and teaching cycles reviewed above, are opportunities for the incremental learning of ambitious teaching. In particular, these PL models employ cycles of teacher learning focused on rich representations of teaching and collective interpretation of teaching, structures, and practices also employed by Studio. We also suggest an additional element is necessary, a routine aimed explicitly to disrupt inequities that we posit is essential for incremental learning of ambitious and equitable teaching. The following sections discuss how the illustrative Studio structures and practices catalyzed incremental learning in the Beachside and Mesa Studios. We share insights for research and practice connected to the themes of the Special Issue and the limitations of our ideas.

5.1. Studio Structures for Incremental Learning

Beachside Studio, examined through a single cycle, and Mesa Studio, examined across multiple cycles, were critical structures for the Studio teachers and residents to collaborate on a shared bounded instructional routine. Cycle elements were repeated structures that the residents could depend on to support them in trying new ideas within a trustworthy yet generative space for learning. Consistent with previous research, trying new ideas was not an isolated task but, like at Beachside, meant that residents would support the Studio teacher's experiment, and the innovation could spread to other classrooms, where there was an opportunity to then learn from each other because of the collective experience [24]. Further, the dilemmas of teaching, which drove the MLR's innovative uses, were opportunities for collective support and sensemaking.

Similarly, at Mesa, the Studio teacher's public display of practice meant that residents could take up dilemmas that arose to design more robust opportunities for student learning in the Studio enactment. The Studio teacher was not on display, and teaching was not to be critiqued; their teaching and the classroom was a working space for learning about students' reasoning, mathematics teaching, pedagogical moves, and creating opportunities to "try something new." While the examples we highlighted were similar to previous research on Studio teacher conversations linked to future practice [4,41], we argue that Studio structures and practices catalyzed incremental learning, as evidenced in the spread of ambitious pedagogies and changes in teachers' pedagogical reasoning [9,15,26,44,45].

Although the Beachside and Mesa routines differ, there were commonalities—the MLRs and data literacy routine shaped teacher and student interactions during the studio lessons, and they both attended to students' mathematical language production. At Beachside, integrating the gallery walk into the Compare and Connect routine was a strategy for the students to share their solutions and record solutions to support student learning. Ms. Severn aimed for the gallery walk to facilitate whole group discussion and support students' willingness to speak. While challenges arose, the opportunity to use the MLR and gallery walk uncovered for the Studio members the complexity of supporting students' mathematical and linguistic learning and the interactions among the students [35,73]. Because multiple Studio members enacted the same MLR and used the gallery walk, there were opportunities to unpack that complexity. With the lack of Studio research in the field, Beachsides' illustrative examples offer existence proof to show how Studio structures and practices are opportunities for incremental learning that advances ambitious and equitable mathematics teaching for multilingual learners [34]. Our argument offers insights into how MLRs were used by practicing teachers [5].

At Mesa, Studio members experienced five representations of the data literacy routine to interpret and unpack these repeated opportunities across the routine rehearsals and enactments. These were structural catalysts for the Studio members' shifting interpretations of teaching and their incremental learning [55,56]. The data literacy routine's ambitious goals catalyzed Studio members to build capacity incrementally; they learned how to lift students' data literacy contributions and narrate them in detracked classrooms, thereby taking up ambitious and equitable teaching. Beachside's and Mesa's cycles and routines catalyzed incremental learning, aligning with the limited body of routine research documenting teacher learning [53–56]. Our illustrative Beachside and Mesa examples suggest that not only are cycles critical for the incremental learning of ambitious teaching but routines explicitly focused on disrupting inequities are essential elements of classroom-embedded PL models to catalyze teacher learning of ambitious and equitable teaching [38]. While this may be logically obvious, our illustrative examples provide a proof of concept.

5.2. Studio Practices for Incremental Learning

Studio practices were featured in both Studio sites. Beachside's examples of incremental learning within the cycle with the MLRs demonstrated how the rich representations of teaching catalyzed the Studio teacher's and residents' ambitious and equitable teaching. Ms. Severn's lesson with the MLR offered a rich representation of teaching as a foundation to which others could reason and add based on their experiences and expertise [62]. The Mesa Studio examples of incremental learning across cycles demonstrated how their collective interpretation of teaching was grounded in their principle of students' brilliance and catalyzed the Studio members' ambitious and equitable teaching. Across cycles, Mesa Studio members shifted their focus from social scaffolds supporting students' contributions [27] to deeply engaging in how heterogeneous groups of students might show their data literacy, and teachers might make that evident [38,39]. Our argument that Studio practices catalyze incremental teacher learning of ambitious and equitable teaching aligns with research that asks teachers to disrupt inequities for the most minoritized students and to disrupt systems that create barriers, such as tracking, to access robust mathematics [19,20,28–32]. We framed the Studio practice of collective interpretation of teaching as a form of pedagogical reasoning that involved the Studio members collaboratively focusing on generative teaching dilemmas [20]. These dilemmas were rich in detail, linked to the shared representations of teaching, and fostered the elaboration of evidence, explanation, and rationale to interpret classroom events. Similar to the previous research on teacher conversations, our examples document how the Studio practices led to action and future teaching [16,20,26]. The MLR guided the Beachside Studio's teaching dilemma and, as they worked on it, it also honed the MLR's development. Across elements of one cycle, Studio members engaged in pedagogical reasoning by interpreting classroom events, building on each other's ideas, unpacking dilemmas, and considering future courses of action [16,25,26] to create opportunities to support multilingual learner students. Studio members fostered collegiality, leveraged students' strengths, and attended to language and mathematical goals [14,35,73]. These incremental changes allowed teachers to enact more ambitious and equitable teaching. Mesa Studio's illustrative examples from across cycles on the same routine shifted the nature of Studio members' pedagogical reasoning, honing how they made sense of students' data literacy contributions and how they could lift and narrate students' data literacy. Their pedagogical reasoning led them to consider routine revisions to ensure the students could show their data literacy brilliance using explanations and evidence, building on each other's ideas to develop a collective understanding that would affect future practice [4,16,26,44,74].

We offered two illustrations of our data and examined the underpinnings of Studio members' pedagogical reasoning. Both examples showed how the Studio cycles and the use of a routine supported the Studio members learning from one another through the shared rich representations of teaching, collective interpretations of teaching, and subsequent opportunities to plan and teach using the routine. Further, by arguing that ambitious and equitable teaching happens through incremental teacher learning catalyzed by Studio structures and practices, we offer an existence proof that ambitious teaching and incremental learning are not mutually exclusive constructs. Instead, incremental learning is a means for taking up ambitious and equitable teaching.

5.3. Connections to the Special Issue and Implications for Research and Practice

This paper contributes to the following three areas highlighted in the Special Issue of Educational Sciences: professional development for secondary mathematics instruction, innovative models of professional development, and incremental professional development models. Studio is an innovative PL model supporting secondary mathematics instruction using structures that have been successful for incremental learning [4,23,42]. The Studio model, like lesson study and other practice-focused models that center on the live enactment of teaching [75,76], is widely used in the U.S. The slim yet compelling body of small studies embodies the dimensions of high-quality PL identified in larger-scale studies [77]. As Studio studies are still emerging, it is understandable that small studies are the norm. Drawing from the two Studio study sites, our conceptual argument contributes to unpacking Studio mechanisms that enhance teacher learning [9].

We built a chain of evidence to conceptually argue that Studio structures and practices can catalyze incremental teacher learning of ambitious and equitable teaching. We highlight two research implications from the ideas we raised. First, research that investigates Studio and other practice-focused PL models would need complex and nuanced methodologies to trace the residue of teachers' incremental learning of ambitious and equitable teaching, which often takes the form of changes in pedagogical reasoning and incremental change in instruction across different models [78]. Tracing pedagogical reasoning into practice, including attention to how instruction mitigates inequitable practices (e.g., minoritized students' mathematical learning and participation), is an emerging research area. There are many ways to frame disrupting inequities and define equity [22,39,41]. Such a design would, by necessity, be longitudinal and highly contextualized because disrupting classroom inequities is tied to the specific social characteristics of students [34]. A second research implication from our conceptual argument links to future Studio research. Future research should examine cross-case analyses of Studio projects that could empirically support or refute how structures and practices catalyze the incremental learning of ambitious and equitable teaching. Further, Studio studies across sites could build an empirical case for how Studio models differ and what infrastructures are necessary to sustain Studio.

Practically, we have provided the implications for the facilitators of practice-focused models of PL, like Studio. Our paper highlighted the pivotal role of repeated and predictable structures, such as the Studio cycle and routines, in supporting teacher learning. It also provided images of the practices needed to attend to teachers' pedagogical reasoning, including creating space for rich representations of teaching and collective interpretations of teaching. Selecting a routine with goals to disrupt inequities intentionally is critical to supporting incremental teacher learning of ambitious and equitable teaching. For school-based coaches who may lead professional learning communities of teachers, adaptations of these structures and practices could include identifying a shared routine to disrupt a pressing inequity that all teachers might use across a year. Observations and recording rich representations of teaching could then serve as a foundation to facilitate the collective interpretation of teaching. We argue that the Studio's routines and practices we put forward are necessary elements to support the incremental learning of ambitious and equitable teaching. However, we recognize that they may still be insufficient without the critical role of the skilled facilitator in guiding PL goal-directed activities and facilitating the teachers' pedagogical reasoning while being responsive to the teachers' incremental learning [75,76,79–81].

The limitation of this conceptual manuscript is that our examples are based on limited images from the two Studio sites. As a result, while our examples are rich, we do not claim that every Studio PL community will support incremental learning. Nor do we suggest that the structures and practices we examined, common to other practice-focused PL models, such as lesson study [6,43–45] or Math Labs [46,47,51], would necessarily catalyze incremental learning. We also did not explore all of Studio's structures and practices to determine how they might catalyze incremental learning. For example, we did not foreground practices such as norm-setting that would guide the teachers' generative and collective interpretations of teaching rather than the critique of teaching [64,75]. As noted in our practical implications, we did not highlight the critical role skilled facilitation plays in fostering incremental learning of ambitious and equitable teaching nor track how teachers' pedagogical reasoning translated into long-term use of classroom practices [25,68,76,79–81]. Future research provides opportunities for exploring teachers' incremental learning of ambitious and equitable teaching within the context of Studio in the ways we have suggested above.

In this paper, we argue that Studio structures and practices catalyze learning. Like other practice-focused PL models, Beachside's and Mesa's teachers had the opportunity to improve their pedagogical reasoning, including pedagogical innovations and ambitious and equitable mathematical goals in the routines they were taking up as part of their practice. PL communities that deprivatize practice through rich representations of teaching and the collective interpretation of learning focused on content, student learning, pedagogy, and future practice are more likely to improve teaching knowledge and practice and enhance student learning in both large-scale studies [77,79,82] and case studies [76], similar to the Beachside and Mesa studies drawn upon for this manuscript.

Lave [9] called us to explore the mechanisms for learning, and the field has built compelling frameworks and investigated PL models, taking up her call [6,7,15,44–46,50,51]. Our paper builds on what Borko et al. [10,11] have called important questions for the field, that uncover constructs for further investigation. Studio's structures and practices are mechanisms for incremental teacher learning and ambitious and equitable teaching. They offer another PL model for further investigation.

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Article

Beyond Traditional Lesson Study: How Mathematics Studio Supports Generative Teacher Learning

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Abstract: In this paper, we describe the interworking of a group of middle school mathematics teachers who engaged in Mathematics Studio, an adaptation of lesson study, across three years. We use this case to illuminate specific structures, protocols, and norms of interaction within Studio that create conditions for teacher learning and incremental changes in teachers' instruction. Our analysis revealed several discourse practices, including the adoption of a research lens, a shared language of affordances and constraints, and an orientation toward student learning that supported a culture of inquiry as teachers investigated genuine questions they had about instruction. In this paper, we elaborate on these practices and share examples of pedagogically productive talk. We claim Mathematics Studio has the potential to circumvent previously identified challenges to support generative learning and the ongoing growth in teachers' instructional practice.

Keywords: professional development; discourse practices; secondary mathematics teachers; generative learning; lesson study

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1. Introduction

Despite substantial resources devoted to teacher professional development (PD), there is little evidence that investment in PD has contributed to significant changes in mathematics instruction or student learning. Many have attributed the limited effectiveness of PD to the persistent disconnect between research and practice [1,2]. Factors contributing to this disconnect include inattention to teachers' actual instructional problems, ignorance of the grain size of information teachers need in order to improve their practice, an insufficient understanding of the influence of local contexts, and a culture of professional development that perpetuates a narrow view of teacher and researcher roles [3,4]. Mathematics Studio, a local adaptation of lesson study, has the potential to bridge the research-to-practice divide by positioning teachers as researchers of their own practice. [2]. In this way, Studio supports incremental change in teachers' instruction by more closely aligning the proposed changes to teachers' current practice [5,6] and providing teachers with a common vision of the new or revised instructional practice in their own (or a colleague's) classroom [7].

This paper was motivated by our work with a group of middle school teachers who participated in Mathematics Studio across three years. As researchers, we were struck by the degree of teacher ownership and the number of times participants reported Studio was "the best professional learning experience" they had ever encountered. Thus, our broad research goal was to better understand what made Studio work, in the sense that it promoted long-term engagement and learning. In pursuit of this goal, we identified discourse practices within Mathematics Studio that supported pedagogically productive teacher discussions, opening the door for generative learning [8]. In this paper, we detail these practices and, thus, contribute to the limited knowledge base on aspects of teacher-driven professional development that foster incremental instructional change [6].

We begin by outlining our theoretical perspectives on teacher learning, with a focus on how learning is enhanced within a discourse-rich community. Next, we summarize difficulties researchers have identified regarding implementing lesson study in US contexts. Together, these sections provide a backdrop for our description of the Mathematics Studio model. We claim the adaptations within Studio may combat some of the difficulties encountered in lesson study, and contribute to collaborative teacher learning more generally.

2. Theoretical Perspectives and Literature Review

2.1. Perspectives on Teacher Learning

Our research is grounded in theoretical perspectives that highlight the importance of situating teachers' professional learning in the practice of teaching [9,10], and which also acknowledge the learning that takes place within teacher communities [11]. From a situative perspective, professional knowledge is intertwined with the activities and aspects of the setting in which it is developed and later deployed [10]. Learning is enhanced when teaching becomes an object of study, with teachers' practical experiences serving as a basis for inquiry and reflective debate about instructional practices [9,11–13]. Evidence from mathematics teacher communities specifically reveals how learning opportunities are shaped by the ways in which teachers collectively represent and explore instructional issues [14,15]. The key to knowledge development is the extent to which teacher discourse around specific instances of teaching practices is linked to abstract principles and provides teachers with resources for dealing with future problems of practice [16].

Unfortunately, idealized goals for teacher learning within professional communities are not easily realized [17]. Norms of privatization, tendencies to avoid conflict, and pressures of accountability often derail opportunities for teachers to interrogate current practices and engage in productive talk [11,14]. Conversations within teacher groups often devolve into evaluation or advice sharing that promotes a "best practice" or "correct" solution to a teaching dilemma. In contrast, Lefstein, Vedder-Weiss, and Segal [18] propose a framework for pedagogically productive talk. According to their framework, pedagogically productive talk is characterized by the following: (1) a focus on issues or concerns that arise in teachers' classrooms; (2) the use of pedagogical reasoning to interpret classroom events and justify instructional actions; (3) rich representations of practice; (4) attention to different perspectives; (5) generative orientations toward students, teaching, and problems of practice; and (6) a balance of support and critique that fosters trust and collegiality while maintaining a critical inquiry stance.

2.2. The Promise and Challenge of Lesson Study

Research both internationally and in the US has demonstrated how lesson study contributes to growth in teacher knowledge and beliefs [19–21]. Importantly, lesson study has also been shown to promote the generative, evidence-based dialogue about teaching and learning characterized above [22]. By its very nature, lesson study makes teaching practice public and provides a clear structure for teachers to focus on student learning while investigating teaching [22–24]. Through joint lesson planning and observation, teachers have opportunities to explore the effectiveness of new teaching materials and instructional practices in a supportive setting [21,25,26].

Despite this promise, researchers have enumerated several challenges associated with implementing lesson study in the US. These challenges include structural barriers and differing norms around professional learning, as well as an incomplete understanding of lesson study purposes and processes [27–29]. For example, Yoshida [29] describes how lesson study, as adapted within US contexts, is often misunderstood as a means of developing best practices or producing exemplary lessons, rather than as a process to help teachers become life-long learners. The result is often a superficial adoption of lesson study structures without sufficient attention to observing students and collecting data on student learning during the lesson [29,30]. An inability to adopt student and researcher lenses during planning may also result in a lesson that does not make student thinking visible, or an insufficient data collection plan [27,28,30]. Relatedly, teacher learning may be compromised in the post-lesson discussion due to poor data quality or to a culture of evaluation or politeness. Such challenges have led many to claim that lesson study in the US will not reach its full potential until teachers learn how to craft researchable questions about their practice and design lessons that can elicit concrete evidence to shed light on those questions.

We claim that, as a shorter, more teacher-driven form of lesson study, Mathematics Studio has the potential to circumvent many of these challenges and foster collaborative learning that can lead to incremental, but sustainable, changes to instruction. In the following section, we provide a brief description of Mathematics Studio, including some of the ways it differs from traditional lesson study.

2.3. Mathematics Studio Model

The basic activities and structures that comprise a Mathematics Studio are outlined in Table 1 below. This one-day cycle includes the following: (1) establishing a research question aligned with an overarching vision or goal for instruction the week prior; (2) beginning the day with a review of Studio protocols; (3) engaging teachers in the mathematics of the lesson; (4) reviewing and modifying a lesson from the teachers' current curriculum; (5) collecting data while observing the lesson implemented in the Studio teacher's classroom; (6) debriefing the observation; and (7) reflecting on the research question in light of the day's activities and articulating individual commitments. The table illustrates alignment, as well as points of contrast, to the four-step cycle of lesson study outlined by Lewis and colleagues [28]. The most notable adaptation is the fact that, unlike traditional lesson study, the full Mathematics Studio cycle is often completed in a single day. By keeping the grain size small and situating the work in teachers' current curriculum and context, Mathematics Studio opens classrooms as spaces for teacher learning without getting caught up in larger-scale pacing and planning conversations that sometimes undermine teacher learning within professional communities [14].

Table 1. Mathematics Studio components.

Studio Activity	Activity Description	Purpose	How Studio Differs from Traditional Lesson Study
Determine research question	Prior to Studio, the Studio teacher (often in consultation with grade-level PLC) chooses question to explore based on a current problem of practice.	Ground the Studio in something teachers want to learn/improve about their practice.	The school team identifies a general goal/vision for the year, but each Studio stands alone. There is no in-depth study of the topic prior to the Studio.
Review Studio norms	Facilitator begins every Studio with a review of the purpose and norms for Studio.	Establish that Studio is a safe place to investigate genuine questions about instruction.	Participants may vary slightly from one Studio to the next depending on the session focus and teacher availability.
Do the math	Participants solve task for themselves, anticipate student approaches, and identify concepts or procedures needed to access the task.	Specify learning goal to determine success measures; lay foundation for lesson planning (e.g., what resources or scaffolds are needed).	Group does not research student thinking about the specific math topic prior to Studio.
Plan the lesson	Modify lesson to incorporate new instructional strategies or, otherwise, address the research question.	Make instructional decisions visible.	Group starts with the lesson provided in school's adopted curriculum.

Table 1. *Cont.*

Studio Activity	Activity Description	Purpose	How Studio Differs from Traditional Lesson Study
Observe the lesson	Studio teacher teaches while others observe and collect detailed data on what students do and say during the lesson.	Gather data to inform research question—build habit and skill of careful observation of students; enhance quality and impact of conversations by grounding them in shared observation.	Depending on the research question, more than one lesson may be observed. Specific data collection processes and tools are often created in-the-moment.
Debrief the lesson observation	Participants first share student data collected during the lesson before discussing aspects of instruction that impacted student learning and engagement.	Hone teachers' focus on student thinking and support explicit connections between instructional decisions and student learning.	While some adjustments to the lesson may be suggested, the focus is on how instruction impacted students, not on revising the lesson plan.
Reflect	Discuss data in relation to the Studio research question before final individual reflection: what did you learn today and what will you do as a result?	Make new learnings available to the group and support teachers in making small changes to their future instruction.	There is no expectation for teachers to implement the focus lesson or report to broader community.

3. Methods

3.1. The Mathematics Studio Group

The core Mathematics Studio group consisted of four middle school teachers (Nick, Sam, Kelly, and Randy), a university researcher, and an instructional coach (Haley) who served as a facilitator. Given their academic status, the researcher (first author) was sometimes positioned as an expert, but, for the most part, avoided any formal role as knowledgeable other [31]. Instead, they served as a participant-observer, offering ideas from their own experiences as a former teacher and current mathematics teacher educator as appropriate. The university researcher had prior experience with Mathematics Studio alongside Haley, the instructional coach [32] and thus also played a small role in establishing the purpose of Studio and modeling productive norms. This core group attended every Studio session. Other professionals (i.e., middle school colleagues, elementary or high school teachers from the same district, pre-service teachers, paraeducators, and school administrators) attended select Mathematics Studios based on their availability and the Studio topic.

The group conducted thirteen Mathematics Studios across three academic years. Each Studio was designed around a research question chosen by the Studio teacher, or developed by a grade level team, prior to the Studio session and followed the general process outlined in Table 1. After a brief introduction (or review) of Studio norms and purposes, teachers worked on the mathematics task and anticipated how students might go about solving it. The group then offered suggestions or modifications to the lesson plan based on the goals of the lesson and determined what student data should be gathered to address the Studio research question. Depending on the research question, the group observed and collected data from a single lesson, or across multiple classes. During the debrief, teachers were first asked to share what they observed students say and do during the lesson, and what that implied about student learning. Second, teachers were asked to consider how components of the lesson contributed to the mathematics students accomplished and how this informs the Studio research question. Each Studio session ended with a final reflection wherein individuals each shared what they learned and what they planned to do differently based on their learning.

3.2. Data Collection and Analysis

Primary data consisted of video recordings and artifacts (e.g., Studio agendas, lesson planning documents, and student work) from a representative sample of 6 of the 13 total Studio sessions. The six Studio sessions spanned the entirety of the project, were situated in various classrooms with different teachers serving as the Studio teacher, and illustrated the range of research questions the group explored. The format of the Studios varied, with some sessions incorporating multiple classroom observations and others focusing on only one classroom observation. It was important to include a variety of research foci and Studio formats as these differences not only impacted the amount of time teachers spent planning and debriefing, but also influenced the dialogic interactions throughout the Studio. Table 2 provides background details on the structure and focus of these Studio sessions.

Table 2. Mathematics Studio descriptions.

Studio # Teacher	Date	Research Focus	Studio Structure
1—Nick	Winter Year 1	How much scaffolding is needed? What are the affordances and constraints to presenting tasks so that they are more open-ended?	Observation of two Grade 8 classes. Same underlying task, but the lesson launch and amount of scaffolding provided differed.
2—Sam	Spring Year 1	Improving student discourse. What is it about fractions, or about division, that is difficult for students?	Observation of two Grade 7 classes. Adjustments to the second lesson were made on the fly based on evidence of student learning during the first observation.
3—Kelly	Spring Year 1	MENU as a differentiation strategy. Does giving students a choice in tasks improve engagement and/or learning?	Observation of one Grade 8 class. Students were given a choice of three tasks with differing complexities related to the base angle theorem.
4—Randy	Fall Year 2	What are the affordances and constraints of students working in groups of four vs. pairs?	Observation of two Grade 6 classes. Students worked on same task in each lesson but were grouped differently.
5—Sam	Fall Year 3	Differentiation—where is it inherent in task design and where are there opportunities to add it in?	Observation of one Grade 7 class.
6—multiple teachers	Spring Year 3	How do students at different grade levels think about a similar task?	Observation of four classes including: Grade 1, 2, 5, and 7. All lessons used the same video launch with students exploring similar (but grade-appropriate) questions.

Given our research goal and theoretical perspective on teacher learning as occurring and being evidenced through interaction in context, we focused our analysis on the video data from each Studio, utilizing the agendas and Studio artifacts as supplementary data. The two authors viewed the video recordings independently, with the second author creating detailed event summaries of each Mathematics Studio session that the first author then reviewed. Like other forms of field notes, these summaries included organizational information (e.g., who participated, the guiding research question, and grade level of the observed classroom lesson or lessons), as well as objective accounts of central topics discussed during each Studio activity [33]. The researchers then met to discuss initial themes and develop codes (i.e., research lens, not-evaluative, common vision, affordance/constraints, student focus, and ownership) to guide subsequent analysis.

With these initial themes in mind, we engaged in several rounds of individually re-watching Studio video, conferring, and memoing [33]. We purposefully zoomed in on dialogic exchanges in which teachers challenged ideas, engaged in evidence-based discussions, or made pedagogical rationales explicit. Based on the literature, we considered these to be particularly noteworthy as they open opportunities for teachers to debate and consider ideas from a new perspective [14,32]. These dialogic exchanges were transcribed and color-coded to illuminate commonalities within and across the six Studio sessions. Through this process, we identified Studio features and norms that contributed to potential knowledge- or perspective-building opportunities. This focused analysis also led us to hone our initial themes into three characteristics of teachers' discourse we think are key to generative professional learning and to motivating incremental instructional change.

4. Results

4.1. Mathematics Studio Discourse Practices

The goal of our study was to determine what made Mathematics Studio "work". Our analysis revealed how structures and norms established within Mathematics Studio supported three key discourse practices critical to the success of Studio, namely, (1) the adoption of a researcher lens, (2) a language of affordances and constraints, and (3) a foregrounding of students. In the sections that follow, we discuss each of these in turn. Throughout those descriptions, we offer extended excerpts from teacher discussions to illustrate pedagogically productive talk [16], adding to our argument that Studio provided opportunities for generative learning in support of incremental instructional change.

4.1.1. Adopting a Research Lens

As in lesson study more generally, the development of a researcher lens is a critical aspect of Mathematics Studio. Each Mathematics Studio is framed by a research question driven by a local problem of practice. With this research question in mind, the group designs a lesson that will help them answer the question or further their thinking around common pedagogical dilemmas. Together, teachers decide what to look and listen for as evidence of student learning and determine what role each will play in the data collection process. The group often utilizes specific tools, such as discourse observation protocols, for clarity and consistency in data collection.

The discussion from Studio 3 illustrates how teachers took on this researcher role to consider the type of data needed to inform their research question. As captured in Kelly's statement below, Studio 3 was driven by a desire to improve student engagement, and, hence, student learning, through differentiation. Kelly, Grade 8, was the Studio teacher:

Kelly: How do you get students to engage in the lesson enough to do the hard work that's necessary to really learn the content? Because that's the thing, the biggest reason I think that students don't learn content is that they're not actively engaged in the content. So that's been kind of an ongoing exploration Nick [the other Grade 8 teacher] and I have been working on for some time now. What can we do to get more engagement? And then Nick brought up the idea of differentiation—that differentiation is a really effective way to help students meet the proper level of rigor for them individually because if you differentiate, the task will more likely be at a level they can actually engage in.

Based on these ideas, the group went on to collectively create, or revise, three tasks to offer the students. Each task covered similar content, but, in the teachers' opinion, were of differing rigor.

Just prior to the classroom lesson, Haley, the instructional coach, prompted the group to consider what data they would be collecting during the observation. Lisa, Randy, and Sam teach Grades 5, 6, and 7, and Kristin (first author) is a participant-researcher.

Haley: What is it that we are going to be looking for? What data will help us know?
Nick: Appropriate rigor—
Haley: What would that look/sound like?
Randy: If they are using proper vocabulary for the task at hand.
Lisa: Should we be listening to questions Kelly asks? Because the level of her questions will tell us about the level of their—
Kristin: (nods in agreement) And the questions students ask.

Nick: They (students) are making progress but still have to discuss. I mean if they are questioning themselves as they're going but they aren't just stuck. That would be appropriate rigor.
Kelly: And questioning each other. That's what I am really hoping. If they are engaged and really talking about content, they should be asking each other questions like "how do you know that" or "are you sure about that" or they might be arguing with each other saying things like "no you can't put that down because that would mean blank and we need to do blank". Those are the kind of statements I would hope for because that would mean they are actually engaging in argumentation.
Randy: And that would be another check for appropriate vocabulary that was used and the understanding of the vocabulary.
Kelly: Yea, and precision.
Sam: Would it be reasonable to measure engagement every five minutes? Have one of us go through the room every five minutes and count the number of students who are actively engaged at the moment? We don't have anything to compare it to—a day when we're not doing menu, but that has been one of the hypotheses.
Kelly: Does someone feel comfortable doing that?
Sam: I will.
Kelly: It will be interesting because that seems kind of ambiguous a little bit. How do you count whether they are engaged or not? I mean it will be interesting to see what observations you make.

Multiple teachers responded to Haley's prompt and offered potential indicators of "appropriate rigor", such as students using precise vocabulary or questioning each other. Sam then suggested a more systematic way of monitoring engagement, although he acknowledged that they lack comparable data from a non-menu lesson. Kelly agrees the information could be "interesting", but also responds with some tentativeness and wondering about what might count as engagement.

This healthy skepticism, further evidence of teachers taking on a research lens, often emerged during post-observation discussions when teachers considered linkages between student behaviors and the instructional aspects of the lesson. Within those discussions, teachers speculated about the impact other lesson adjustments might have and generated additional research questions. For example, in Studio 4, teachers were curious about the affordances and constraints of having students work in groups of four versus pairs. To investigate this, the same task was implemented in two Grade 6 classes, with students in one class working in table groups of four and the other class working in pairs. Teachers' observations of very diverse patterns of interaction among the student groups left them with new questions such as "how do power dynamics related to gender and/or status influence student learning" and "how do teachers best mitigate issues related to status?" This type of generative teacher discussion was common across the Studio sessions as teachers continued to hone their research skills, and this manner of inquiry became second nature.

4.1.2. Utilizing a Language of Affordances and Constraints

The continual consideration of the affordances and constraints of pedagogical decisions is perhaps the most salient feature of Mathematics Studio. Mathematics Studio is grounded in the stance that teaching is about decision-making; there is not one right answer or "best move"; rather, each decision provides affordances and constraints for student learning. The language of affordances and constraints was modeled early and quickly adopted by the teachers. Sometimes, the framing research question itself was stated in terms of affordances and constraints, as in Studios 1 and 4 (see Table 2), in which teachers gathered evidence related to the impacts of different ways of launching tasks or grouping students.

Other times, this language emerged naturally as teachers considered the relationships between the observed student actions and the instructional decisions. In Studio 2, for example, teachers considered how the differing levels of scaffolding provided in two observed lessons supported students' understanding of fractions. Teachers analyzed the evidence from the lessons to uncover "what helped and what got in the way?" When planning lessons, teachers made deliberate decisions about how to phrase the directions, what order to present tasks, which numbers to use, and what tools they should provide to students. In defending their choices, teachers consistently weighed affordances and constraints, which led them to grapple with several pedagogical issues such as accessibility, student agency, short-term versus long-term student learning goals, and the cognitive demand of tasks.

The excerpt below, taken from the post-observation discussion during Studio 5, provides a snapshot of how those discussions progressed. The research focus was again framed by questions teachers had around differentiation. These included, "What features of the task naturally allow for differentiation? What strategies would allow for more differentiation? How do these differentiation strategies support or hinder student learning?" During the classroom lesson, teachers observed how many students were slow to begin or struggled to understand the task. Based on this evidence, the group wondered whether some of the students' difficulties could have been avoided by launching the task with some form of joint reading.

Haley introduced the idea of a joint reading as a tool that might reduce the need for differentiation. Sam (the Studio teacher) then opened this up for group consideration.

Sam: So, what's the thinking on that? I mean I don't do that very much, joint reading. I make them read it and interpret it.
Haley: I don't know, what do we think about that?

As teachers began to debate the affordances and constraints of the strategy, they drew on both their previous experiential knowledge base, and the observational data from this specific lesson. After a few minutes of discussion, Haley summarized the ideas that had been suggested and encouraged further debate.

Haley: So, I'm going to go back to Sam's question because I see there are three ways you could do it. One is you don't read it to them you just say here is the task. The other is that you ask the kids to read it together in a group with some kind of structure. The third would be that you read it as a whole class. So, I would like to think about the affordances of those and the constraints to those.
Kelly: In the long run we would like them to be able to read it independently and figure it out on their own. So, I think the danger is if you always read it as a whole class, they never have the opportunity to practice the skill of reading it themselves and making sense out of it individually so that would be a real constraint if you did it all the time.
Sam: Right.
Nick: So, if you put in the structure, I am kind of thinking if we were to balance it, maybe the structure of just okay, stop and read individually, don't share, just read individually, just so they, okay, I have to read. Now, begin. Just to stop them to make them do it, but then they still have to be able to reason. But they may, or may not, be able to stop themselves and read it right? That is also something we would want to. . .
Kelly: But maybe it would develop a habit.
Nick: . . .yeah
Kelly: I mean the advantage of reading it out loud is then everybody knows what they are supposed to do at the beginning and so they probably will make more efficient use of their time. Maybe.
Sam: I wonder if that's less effective than making everybody read it quietly to themselves?
Kelly: Yeah.
Sam: Everybody read it, now turn to your partner and say, what you think it said. Because when I read, I don't think of any of those things half the time.
Kelly: That's a good point.
Haley: Can all of them read at grade level? I mean would that be another thing. So, if you are asking them to read, and they can't even get the idea of what it is they are trying to accomplish because they can't understand what is being asked of them.

As the teachers weighed the affordances and constraints of different joint reading structures, they considered issues of access and equity (e.g., can all students read at grade level?), as well as a combination of short- and long-term goals they have for students. Other aspects of the discussion worth noting include the use of "we" throughout, indicating joint responsibility and ownership, as well as the tentative stance taken. Teachers couched their suggestions with "probably", or "maybe", again reinforcing that there is no one "best" decision or teacher move. As illustrated here and in the previous vignette from Studio 3, teachers' adoption of a research lens and the considerations of affordances and constraints always centered on how specific aspects of instruction might affect student learning, leading to the third discourse practice.

4.1.3. Maintaining a Student Focus

The research focus of every Mathematics Studio centered on how particular pedagogical moves, structures, or in-the-moment teacher decisions affect student learning and engagement. Kelly summarized this student-oriented purpose in Studio 6 when she provided new attendees with a brief overview of Mathematics Studio:

Studio is about when I do this, this is what the kids do. When I do that, this is what the kids do. What do I want to elicit from the students? In this particular lesson, what are we really trying to get the students to do? Because that should guide the decisions that I make as a teacher about how I proceed and what kind of teacher actions I have.

Several features of Mathematics Studio supported this orientation. First, Studio lessons were purposefully designed to promote reasoning and problem solving and engage students in mathematical discourse. Thus, student thinking was generally visible during the lessons. Relatedly, there were multiple observers listening in on small group conversations, making student thinking even more accessible. Prior to the observations, teachers anticipated student thinking while working through the mathematics and discussed protocols they would use to record specific student words and actions during the lesson. The first debrief prompt was purposefully designed to start with the student data and teachers were continually reminded to avoid making broad claims or interpretations that did not explicitly tie back to student talk or actions evidenced during the lesson.

These final excerpts, from the initial debrief discussions of each of the two lessons observed in Studio 4, illustrate that push for evidence. The research question guiding this Studio was "What are the affordances and constraints of students working in groups of four vs. pairs?" After the first lesson, when students worked in pairs, Sam opened the debrief by sharing how he attended to access and participation when students moved to the second part of the task, which was to create a poster illustrating their work on the card sort. Haley (the instructional coach) quickly interrupted Sam and pressed him for details around his data collection methods.

Sam: When the poster making began, I began studying access—was it equal access or not? Every four minutes I walked around and did a tally. I found that when it was two girls paired together, they were more likely to have equal access. The second most likely to have equal access was—

Haley: (interrupting) What's your data?

Sam: You want all my data? There's way too much.

Haley: I want enough of it to have a sense of what you did.

Sam: As I walked around, I made three T-charts. One of them has two girls on the T-chart and underneath that was equal or not. And then I did a mixed group T-chart and I tracked whether the girl had the access, the boy had the access, or if it was equal. And then finally I did a two-boy group and measured the same.

Haley: And how did you determine equal or not equal?

Sam: I did it based on who had the poster in front of them and furthermore if, while I was watching, the person without the poster reached over and wrote—I. . .

Haley: Okay, so now I have a picture of how you did it, so now you can tell me what you inferred from that.

Sam: What I inferred was a pair of girls was most likely to have equal access. Second most likely was a pair of boys to have equal access. In the mixed group, it was uniformly the girls that had the access. Only one time out of four samples did a group have equal access when it was a mixed pair.

Nick: Was there any observation of if one was writing and one was talking, like if they had defined roles that way? How often did you find the person with the poster was doing both and one person was just watching?

Before Sam was allowed to share his inferences about different student groupings, Haley insisted that he detail his data collection decisions. This reinforced both the research lens and the focus on gathering evidence directly from students. Specifically, Sam was pressed to define what student actions he counted as "having access". As the discussion continued, teachers were pressed (by Haley, as well as other teachers) to describe exactly what the students they observed were saying and doing, and how those actions indicated whether students had equal access or took ownership of the mathematics.

This consistent press to ground inferences in direct student words and actions was also present during the debrief of the second lesson in which students worked on the same task, only this time in groups of four. Liz, the first to share, and Mary who later adds on to Liz's observation, were both preservice teachers.

Liz: I had a group of four boys. They were interesting. They didn't get much math done—I called it a power struggle. As soon as they got the cards, Max (student pseudonym) took them and split them up so they didn't know what to do with them. They were very confused by what to do math-wise. So, they messed around with the paper, totally off task in that way.

Haley: What was the confusion about? What did the math look like?

Liz: They weren't sure how to sort it, or they weren't sure how to put in ratios, because even after they sorted it into three and four (piles), they didn't put the pieces together that it was the same ratio.

Randy: Didn't I come over there and help them sort it once?

Liz: I think you did, but even at the very end they labeled, this is a match, this is 1, and they wrote down the ratios with 3 boxes per truck, but I don't think they had a complete connection. And especially with the 1½ trucks, they kind of all shrugged. As soon as they were silent Jason (student) had a chance to put in a word. "Oh 4 boxes for the complete truck and 2 for the half a truck". And as soon as he said that Max just took over and Jason got shoved to the side again. . .

Sam: Over and over and over he would try to ask a question and he was shut out.

Haley: What did being shut out look like?

Mary: I actually wrote it down when Jason came up with that idea, the one and one-third ratio, and Max was like, "Oh that's a good idea" and then took his pen and wrote it down. Jason had a look on his face like, "But I wanted to write that". The paper was only on Max's desk, and he just took over. He was very in control of if they wrote anything on the paper.

Again, the purpose of the debrief was to provide substantive details regarding what students said and did during the lesson. Only with sufficient evidence could teachers make judgements regarding student learning and engagement. When the descriptions were too general, Haley was quick to ask teachers to provide more detail (e.g., what did being shut out look like?). Thus, this excerpt also illustrates norms that had been established regarding the level of specificity with which teachers were expected to record student data.

5. Discussion

This study extends prior research on lesson study adaptations and adds to current theory on professional learning designed to support teachers' incremental change in instruction [6]. Mathematics Studio, a local adaptation of lesson study, promoted pedagogically productive talk [18] that we claim supported inquiry, curiosity, and a willingness to make instructional changes. As teachers collectively grappled with problems of practice arising in their own classrooms, they drew on data from the shared observation to weigh the affordances and constraints of pedagogical decisions. The overall tone of teacher discussions (e.g., acknowledging the complexities of teaching, and considering multiple reasons for students' difficulties or seeming lack of engagement) was indicative of the generative orientation the group took toward their students and toward teaching dilemmas.

Through our analysis, we identified three interrelated discourse practices that were characteristic of pedagogically productive talk and a key to the success of Mathematics Studio. These practices included taking up a research lens, the language of affordances and constraints, and an orientation toward student learning. Below, we first review the significance of these discourse practices as related to the literature on incremental change before discussing how specific Studio adaptations created a more manageable professional learning experience that can lead to ongoing instructional improvement. We end with a short discussion of how Mathematics Studio serves a more long-term goal as a catalyst for generative teacher learning.

5.1. Discourse Practices Arising in Mathematics Studio

The adoption of a research lens, coupled with the continual foregrounding of direct evidence from students, allowed teachers to dig into authentic problems of practice and investigate varying effects of instructional choices. These attributes have been called out as critical to the success of lesson study, but are also difficult for US teachers to achieve [30,34]. In each Studio, teachers played a primary role in not only generating the research question, but also devising a method to test their hypotheses. Specific protocols, coupled with explicit expectations and group norms, kept the post-observation discussions grounded in data. This close study of practice helped teachers see how small changes make a difference (e.g., when the teacher provided less scaffolding, students were more dependent on peers for support; and different group dynamics emerged when students worked in pairs vs. groups of four). Most importantly, these investigations emerged from teachers' own context and illustrated accessible, practical changes that could be made—key characteristics of incremental instructional improvement [7,35].

The language of affordances and constraints modeled by the facilitator and taken up by teachers ensured that post-lesson discussions avoided quick interpretation and evaluation—qualities that often shut down learning opportunities [36]. Instead, the Studio promoted the perspective that teaching was a complex endeavor with no one "right" or "wrong" way to teach. Teachers were inspired to slow down and carefully consider their goals for student learning and engagement to make more informed instructional decisions. Important to incremental change [5], discussions about the affordances and constraints were grounded in a shared vision of mathematics instruction aligned with research-based effective mathematics teaching practices (e.g., [37]). In other words, the lesson modifications that teachers investigated, while consistent with teachers' current practice, were designed to create a richer learning environment for students [5,6].

5.2. Mathematics Studio as a Feasible Pathway to Instructional Change

Lesson study in the US has been criticized for giving insufficient time to the lesson planning process [38] and prior study of the mathematical content to be addressed in the lesson [39]. Admittedly, Mathematics Studio, as conceived by this group of middle school teachers, is open to similar criticisms. While teachers, sometimes in consultation with the instructional coach, do spend time considering curricular resources related to the lesson in advance, the full Studio cycle of planning, implementing, and reflecting occurs in a single day, limiting the depth of that study. However, Mathematics Studio still adheres to lesson study's underlying principles with the goal to promote teacher learning [29,40].

Further, we contend that many of the structural characteristics of Mathematics Studio that deviate from traditional lesson study are the very ones that make this form of professional learning not only feasible in a US context but also more immediately impactful.

The fact that the cycle takes place in one day creates a sense of urgency to focus on the investigation at hand, and subsequently reduces the chances that conversations stray toward logistics or pacing issues which may detract from teacher learning [14]. Teachers do not need to commit to long-term curricular study prior to the Studio, nor do they create a formal research report at its conclusion. However, the Studio still includes time to engage in mathematics together, allowing teachers to anticipate student approaches or trouble spots, expand teachers' knowledge of standards and expectations across grade bands, and enhance their ability to support student access to tasks [41]. Together, teachers plan one or more parts of the lesson together, often co-creating questions to encourage more student discourse, or making intentional decisions about how to group students or what resources they need to provide. This co-planning not only creates a sense of ownership in the lesson, but also offers tangible instructional adjustments that teachers may then implement in their own classroom. Indeed, each Studio ends with individual teachers identifying a change they will make based on their learning. Often, these are modest modifications such as starting the next day's lesson with a Notice and Wonder activity [42] or being more explicit with students about group roles.

5.3. Mathematics Studio as a Catalyst for Generative Teacher Learning

Generative teacher learning requires a disposition toward learning in and from practice [15,43] characterized by teachers' ability to continue to learn from their students' thinking [44]. The non-negotiable component of Mathematics Studio, and what distinguishes lesson study from many other forms of professional development, is the shared observation of a live lesson. Rather than merely speculate on how students will engage with a particular task or react to a teacher question, the group gets immediate feedback and can explore classroom interactions in their full complexity. This joint observation does not necessarily guarantee that conversations stay grounded in the data. However, the likelihood of pedagogically productive talk is dramatically increased.

As illustrated in the excerpts above, discussions were generative in the sense that teachers saw their classrooms as places to try out new instructional strategies and build principled knowledge to support instructional decision-making [44]. Mathematics Studio structures and norms demand that teaching decisions, and the rationale behind them, be made explicit [32]. This explication was aided by a continual press to consider affordances and constraints of instructional moves. Situating teacher learning directly within classrooms that are familiar maximizes the chances teachers can apply their new learning in future teaching situations [10]. This is especially true when the instructional innovation is recognizable by teachers and is consistent with the contextual constraints within which they work [35].

6. Conclusions

Successfully bringing research closer to teaching will require a cultural shift in roles [2,4]. Researchers need to become more accountable for solving specific problems in teachers' classrooms and teachers need to be willing to experiment with different instructional approaches. Our study revealed how Mathematics Studio facilitated this shift and supported teachers' commitments to incremental change by starting with often ubiquitous instructional practices teachers recognized and supported [6,7]. Teachers were empowered to examine genuine problems of practice and take responsibility for making instructional shifts to improve student learning and engagement. We began this study encouraged by teachers' long-term participation in and enthusiasm for Mathematics Studio. Our hope is that this work provides a starting point for mathematics educators and professional development providers to design similar learning opportunities with teachers, for teachers [2], that support continuous, incremental instructional change.

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Article

Rough Draft Math as an Evolving Practice: Incremental Changes in Mathematics Teachers' Thinking and Instruction

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Abstract: This paper presents exploratory findings suggesting that mathematics teachers can implement Rough Draft Math (RDM) by making small, incremental changes that align with their current practices and local contexts, including curriculum materials, with minimal support. Following a conference presentation and/or reading a book about pedagogy, teachers reported shifts in their thinking that facilitated their interest in enacting RDM and small changes they made to their teaching. The flexibility of RDM, as a general concept rather than a set of prescribed practices, allowed teachers to incorporate RDM to meet their own teaching goals. We propose that this adaptability enables teachers to incorporate RDM into their classrooms incrementally, reflecting their existing objectives for their students.

Keywords: mathematics teaching; engagement; motivation; teaching practice

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1. Introduction

A persistent challenge for mathematics teaching is motivating and engaging students to participate actively in their learning. Teachers have reported a need to put forth additional efforts to increase students' motivation to learn since the COVID-19 pandemic, e.g., [1]. However, rather than locating this problem within students, we view students' engagement and motivation as a dilemma for teaching. It is important to identify teaching practices that encourage more students to participate in mathematics class and that support students with learning mathematics content.

One persistent reason why students shut down in mathematics classrooms is that, in many cases, they have experienced that being good in mathematics means that they must obtain correct answers quickly [2]. Mathematics educators have challenged this perspective. Researchers have been advocating for learning environments that engage students in productive struggle [3] and perseverance [4] in the face of worthy challenges.

An approach to teaching that appears to have potential in engaging students in mathematics learning is Rough Draft Math [RDM] [5,6]. "Rough draft thinking happens when students share their unfinished, in-progress ideas and remain open to revising those ideas" [5] (p. 3). Teaching that incorporates RDM involves treating all students' work and thinking as a rough draft and explicitly incorporates opportunities for students to revise their work and thinking. There is not one specific way to teach with RDM. Rather, RDM is a generative concept that teachers can enact in ways that fit into their contexts. If teachers treat students' thinking as a potentially revisable draft and incorporate revising into their teaching, they are teaching with RDM.

We conjecture that RDM can promote students' engagement in learning mathematics. If all draft ideas are welcome, students may feel safer to participate, particularly if strengths in students' drafts are recognized (see [7]). When drafted ideas are treated as valuable, and then workshopped to build on those strengths in order to revise them, it is possible that students will be less likely to avoid taking intellectual risks and not as likely to avoid participating to save face (lower performance avoidance goals) [8]. Additionally, honoring the strengths in students' draft thinking can support students with seeing themselves as mathematically competent, which can potentially increase their self-efficacy [9]. When students have opportunities to revise their thinking regularly in mathematics classrooms, students may be able to develop a growth mindset [10] such that they recognize that their efforts to persevere can lead to greater learning. This process could lead to students developing higher mastery goals [11] or participating to learn rather than to appear smart. Additionally, if students experience that they can learn from their peers' draft thinking and if students experience that their peers can learn from their drafts, then students may build a stronger sense of community in the classroom as they come to value learning from each other (stronger sense of belonging in mathematics) [12].

Our claim in this paper is that Rough Draft Math can be viewed as an incremental change to support students' engagement in learning mathematics that, with minimal support, can be understood by teachers and feasibly taken up in the contexts of teachers' lived constraints. First, we will elaborate on the construct of RDM and argue that it can be an incremental change in teaching mathematics that could lead to supporting students' engagement. Then, we will describe forms of relatively minimal support for teachers (e.g., conference presentations, reading a book alone or in a book club).

Next, in the rest of the paper, we describe results from a series of studies. Study 1 describes shifts in teachers' thinking after attending a conference presentation. Study 2 summarizes variations in teachers' enactment of RDM after reading a book. Study 3 is a case of teachers' curricular noticing, using RDM as a frame for noticing curriculum materials, which is also an example of incremental change that is possible after reading a book. Study 4 illustrates a case of a teacher who enacted RDM because it aligned with her current motivations for teaching, which helps to explain why RDM could be an incremental change. This set of studies, taken together, provides insight into the ways that teachers can incorporate RDM in their teaching without extensive professional support.

1.1. Rough Draft Math as an Incremental Change in Teaching

RDM can be viewed as an incremental change in teaching. Teachers could enact RDM without buying new curriculum materials or without revamping their pedagogical approach. For instance, some teachers teach mathematics by first, directly demonstrating how to solve a problem, giving students opportunities for guided practice, and then, giving students time for independent practice. With this direct instruction approach [13,14], teachers can invite students to share their draft ideas or document their first thinking, and they can ask students to revise a problem that they have initially attempted after guided practice. Alternatively, other teachers may teach mathematics by launching a task that students can begin solving with their prior knowledge and develop new knowledge through making progress on that task. With this teaching through problem-solving approach [15,16], teachers can treat students' initial attempts to solve the problem as drafts, ask students to compare and contrast their solution strategies, and then, invite students to revise their first attempts. In any teaching approach, teachers are likely to close the lesson by consolidating ways of thinking about mathematics during a lesson closure, either promoting connections across strategies or emphasizing the important ideas from the lesson. A lesson closure could be an opportunity to invite students to reflect upon how their thinking changed, or how they revised their thinking, during the lesson. As we detail below, drafting and revising can be woven into how teachers already approach teaching mathematics.

1.1.1. Tag Students' Talk as Rough Drafts

One example of an incremental change associated with RDM is explicitly tagging students' talk about their thinking as "rough drafts" [6]. This involves telling students that it is okay to share their thinking in rough draft form or telling students that in-progress or incomplete thinking is valuable to share with the class. This tag, "share your rough draft thinking", has the potential to elicit more students' thinking. If students hear that rough drafts are welcome, they may feel less pressure to be correct and more welcome to share whatever they are currently thinking.

Tagging students' talk as a rough draft is an example of an incremental change because teachers are likely already working on strategies to elicit their students' thinking; inviting students to share their rough drafts is a quick move to integrate. Students often adopt the language, too, saying, "I don't know, this is just my rough draft thinking, but. . ." and then go on to explain their thinking. In this way, the tag of "rough draft" can be used as a face-saving caveat so that students share their thinking when they are not sure about whether or not it is correct.

1.1.2. Document How Thinking Changed

Additionally, another incremental change related to RDM is explicitly asking students to document how they revised their thinking at a moment in a lesson, perhaps after collaborating with peers or at the end of a whole class discussion. Teachers often ask their students to share their strategies with one another. This might be through a turn and talk [17] or small group activity. Discussion of students' strategies also can occur during a whole class discussion. Regarding whole class discussions, teachers are encouraged to intentionally select students' solutions to be discussed in class, and sequence them for the discussion in ways that lead to mathematical connections [18].

After the interactions (in partners, small groups, or whole class discussions), teachers could ask students to write down any new ideas they had after listening to others' thinking. This allows students to record any revisions they made to their thinking while listening to peers or their teacher and promotes reflection. Documenting revisions normalizes that we are constantly changing our thinking, promotes listening to one another, and allows students to record what they learned. Asking students to document how they revised their thinking during a class discussion or small group discussion is a minimal change that can support students to learn more from interacting in mathematics class.

1.1.3. Ask Students to Reflect on How Their Thinking Changed over Time

Another example of an incremental change in the spirit of RDM is incorporating opportunities for students to reflect on how they revised their thinking across a lesson, unit, marking period, or semester. At the end of any class discussion or as an exit ticket at the end of a lesson, teachers could promote metacognitive reflection [19] by asking students to write down how they revised or changed their thinking. This sort of reflection could be a part of an end of unit assessment or an end of marking period assessment. Incorporating brief moments of reflection is a small change that helps students become more aware of their learning.

1.2. Minimal Supports Provided to Teachers to Enact Rough Draft Math

One way to think about incremental changes in teaching is whether the change can occur with relatively minimal support. For instance, teachers have learned about RDM by attending a single presentation at a conference, such as a keynote talk or a breakout session. When the first author facilitates these single-session professional learning opportunities, she engages the participants in thinking about the value of drafting and revising in mathematics classrooms, and she facilitates opportunities for the participants to draft and revise their thinking as they do mathematics together. A single conference presentation could be viewed as a relatively minimal support because it is a short period of time to learn about an idea, particularly if no systematic follow up is in place to support teachers' learning beyond that session.

Another form of minimal support for professional learning is reading a book. Teachers might read a professional book alone or in a study group of other teachers, without an external facilitator or the time and resource commitments of a professional development leader. If teachers adjust their teaching after reading a book, this is a relatively low cost way to support teachers. The costs include the price of a book, the time spent reading and, potentially, time spent on reflecting and discussing the book. If teachers want to learn about RDM, they could read *Rough Draft Math: Revising to Learn* [5].

Certainly, more extensive support could support teachers' learning and uptake of RDM. For example, RDM has been promoted in school districts through multi-session professional learning opportunities, both remotely and in person. However, it is worth considering the potential impacts of more minimal levels of support for RDM. Below, we will explore the impacts of two minimal supports: (a) Study 1 illustrates the impacts on teachers' understandings of mathematics teaching with RDM after attending a single professional learning presentation and (b) the results from Studies 2–4 illustrate the impacts on teachers' enactments of mathematics teaching with RDM after reading a book.

2. Methods

To illustrate incremental changes in teachers' thinking and practice, as informed by RDM, we present findings from four studies. To understand incremental changes in teachers' thinking after a relatively minimal intervention of a single conference presentation, we present an analysis of participants' reflections after the presentation (Study 1). To understand incremental changes in teaching practices after a relatively minimal intervention of reading a book, we present three studies. We summarize findings from Study 2, which has already been published [20]; these findings describe variations in how teachers enacted rough drafting and revising. Study 3 illustrates a case of how a teacher noticed opportunities to engage their students in drafting and revising in their curriculum materials. Study 4 describes a case of how a teacher saw RDM as helping her achieve her goals for mathematics teaching. Studies 2–4 demonstrate the feasibility of incorporating RDM into current teaching practice by making relatively small changes.

For all studies, the sampling of participants was conducted by soliciting volunteers. Thus, our process for recruiting participants for all studies was not random, but we used convenience sampling practices. For Study 1, participants attended conference sessions about RDM voluntarily, and it was their choice whether or not to complete the Google Form at the end of the session. Participants in Study 2 and 4 were recruited by soliciting volunteers from those who participated in book studies (referred to the first author by leaders of those book studies). Participants for Study 3 were recruited via social media (e.g., Twitter/X).

Data analyses for all studies were conducted using an inductive approach [21] to conduct a thematic analysis [22] of the Google Form responses and interviews. For the Google Forms in Study 1, we sought to identify trends in the sample. For the interviews in Study 2, 3, and 4, we used the teaching practices in the book about RDM [5] as a starting point to identify themes that were described by the participants. We selected quotations to present in this paper that were representative of the sample.

3. Results

3.1. Study 1: Teachers' Shifts in Thinking About Mathematics Teaching with RDM After Attending One Professional Learning Presentation

This study addressed the following research question: *After attending a conference presentation about RDM, what do participants report about how their thinking about teaching changed?* At the end of conference presentations for teachers about RDM, to model inviting learners to reflect on how their thinking changed, the first author asked participants to respond to a reflection prompt on a Google Form. The following sentence starter was used as a quick way to prompt reflection: "I used to think... and now I think..." (This prompt is from the Visible Thinking project from Harvard's Project Zero [23].) Typically, approximately 25–33% of attendees at these presentations completed these exit questions on the Google Forms.

To understand ways that teachers' thinking shifted incrementally after experiencing a conference presentation about RDM, we analyzed 262 participants' responses to this prompt from seven different single professional learning sessions conducted by the first author. These presentations took place at events from February 2023–February 2024 in six states and the District of Columbia. Attendees were classroom teachers and mathematics teacher leaders (e.g., coaches, curriculum supervisors). We identified common themes across responses. These themes represent incremental changes in teachers' thinking that could occur from one single professional learning session. Figure 1 summarizes the themes for Study 1.

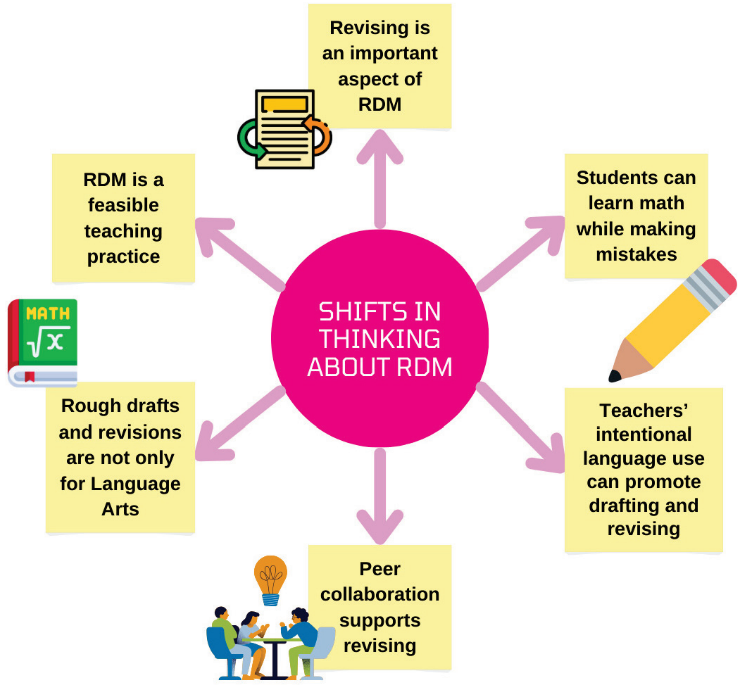

Figure 1. Changes in thinking after attending a conference presentation about Rough Draft Math. Note: Attendees responded to the following prompt: "I used to think... and now I think...".

3.1.1. Revising Is an Important Aspect of RDM

Teachers reported developing greater appreciation for the value of revising in mathematics (27.1% of responses). A reflection that demonstrated this incremental change in thinking was the following,

> I used to think more about the importance of the 'rough draft' portion and less about the benefits of revision. The activity we did to write and revise our story about the graph reminded me of the benefits of revising to clarify, make connections more clear, and think of our audience.

Some teachers may be initially attracted to RDM because they want to welcome students' drafts to invite more students to participate and share their in-progress thinking. However, the role of revising is essential for moving students' drafts forward. During a professional learning session, teachers can begin to recognize the importance of revising in mathematics.

Additionally, teachers could become more open to what should or could be revised when learning mathematics. A teacher wrote, "I used to think that revising and editing was more limited to students correcting their work, but now I think it is really about learning other perspectives, looking at things differently than we do initially". Revising certainly can involve analyzing errors and correcting mistakes, but it can be so much more. We can revise our thinking by expanding our points of view and developing alternative solution strategies or ways of viewing a situation. We can revise by making new connections between strategies. We can create a new representation to illustrate a mathematical relationship. We can revise a justification to make it more precise, concise, or more illuminating, even if it is already correct. Shifting understandings about what it means to revise is another incremental change in teachers' thinking that can occur during a single professional learning session.

3.1.2. RDM Could Be a Feasible Teaching Practice

One shift that teachers reported is that, after experiencing multiple moments of drafting and revising in a single PL session, they could see that this approach to teaching is feasible to enact, even in their own professional contexts. This theme was evident in 18.7% of the responses. A teacher wrote, "I used to think that this would be difficult to bring into the classroom but it makes sense on how it would be so useful and not so difficult to incorporate into a class". One teacher responded, "I can change tasks already created to implement this strategy". Another teacher wrote that they used to think "this concept was more time consuming to implement and now I think there are quick ways to incorporate this as a daily practice and shift in classroom practice". If teachers can understand how RDM fits into their current teaching practices, they may be more likely to put it into practice. It is promising that some teachers became more aware of the feasibility of RDM from one professional learning session

3.1.3. Rough Drafts and Revisions Are Not Only for Language Arts

Teachers recognized that they could leverage practices from literacy, rough drafting and revising, and use them to support mathematics learning (13.4% of the sample). A teacher wrote that they used to think "revision was primarily for ELAR [English Language Arts and Reading]" and now they think that "revision happens everyday and is an essential component of problem solving, learning, and thinking". Revision can be viewed as any changes in how we think; so, really, revising is learning. Teachers shifted to see how revising can occur for any academic content, including math, even after one single professional learning session.

3.1.4. Peer Collaboration Supports Revising

After attending a single professional learning session, some teachers reported that they see collaborative learning as a process where students could gain inspiration from their peers about how they could revise their thinking (12.6% of the sample). This teacher wrote about shifting from viewing revising as an individual activity to a collaborative one:

> I used to think about revision as looking at only your work and making it better. Now I see the power of looking at another's work, understanding their strategy, and then revising my work using their method. Sometimes it isn't 'comfortable', but it has the power to open a new avenue of thinking or solidify that the method I'm using is the one that works best for me.

This teacher recognized that comparing and contrasting one's own work with another person's work and trying to understand another person's thinking can provide ideas for revising or improving one's own thinking. Additionally, a teacher wrote, "I used to think that rough draft thinking just meant having students fix mistakes. I now think it's about an initial thought followed by learning from others and then expanding your initial thinking based on what you learned". This response captures the idea that revising can involve expanding your perspective on what strategies are valid for a problem, and this kind of expansion can occur through collaboration. If teachers revised their thinking after collaborating with a colleague on a task during a professional learning session, they may begin to see the value of peer collaboration among students to support revising to learn mathematics.

3.1.5. Intentional Language Use Can Promote Drafting and Revising

Teachers (10.7% of the sample) noticed that they could make small changes in their language use that would support students' engagement in rough drafting and revising. One teacher wrote, "By just adding the words/question 'what's your rough draft thinking on this?' creates a safer space for students to express themselves". Another teacher wrote, "I used to ask, 'is anyone brave enough to share something they did wrong?' Now I plan on asking, 'Can anyone share something they revised in their thinking?'" Teachers reported that they became aware of small shifts in their language that they could use with students. After one professional learning session, some teachers recognized that such intentional uses of language could help students feel safe to share their thinking.

3.1.6. Students Can Learn Mathematics While Making Mistakes

Some teachers (8.8% of the sample) reported coming to value what students can learn from being incorrect. One teacher wrote that they used to think "I had to focus on just the right answer. However, I now think of the power of just working on the answer. I love the feeling of a rough draft because you know you can fix it later". This teacher appeared to shift away from thinking that their job was getting all students to correct answers. This reflection suggests that they shifted to recognize that students are learning as they work toward the correct answer. Another teacher wrote, "I used to think teachers were meant to discourage mistakes and now I think that mistakes are simply a path to accomplishment". If teachers can make this kind of incremental shift from one professional learning session, it is exciting that teachers could shift to be less likely to view students' thinking through a deficit lens if they make mistakes while engaging in mathematics, and instead they can come to appreciate that students are learning through the process of working toward a correct answer.

3.1.7. Conclusions from Study 1

Overall, we found that this prompt ("I used to think... and now I think...") has been a helpful tool for capturing incremental changes in teachers' thinking. From this analysis, we are more aware of possible incremental changes in teachers' thinking from a single opportunity to learn about RDM. Naming and labeling small changes in teachers' thinking provides insight about the kinds of learning that might take place when teachers attend one professional learning session.

The two most prominent themes in these responses were that teachers came to appreciate the role of revising in RDM and the feasibility of RDM. Although the role of drafting matters in RDM, revising is the unique contribution of RDM. Without RDM, many mathematics teachers already attempt to create safe spaces for students to participate. Although framing the activity as sharing one's "rough draft thinking" does contribute to creating that safe space, incorporating revising is something that teachers may not otherwise do in mathematics classrooms without being introduced to the idea of RDM. Additionally, if teachers can recognize feasible ways to incorporate RDM after one conference presentation, that is a powerful finding.

3.2. Study 2: Variations in Enactments of RDM After Reading

Another way of investigating how a minimal intervention could prompt an incremental change in practice is by exploring small changes teachers made in their teaching after reading a book. After all, when a book is published, authors are curious about what readers take away from it. Facilitators of book studies of *Rough Draft Math: Revising to Learn* [5] reached out to the first author. They asked for reflection questions to support the book studies, invited the first author to attend one of the book study meetings, or generally informed the first author that the book studies were taking place. The first author then asked facilitators for names of book study participants who might be willing to share how they enacted RDM after reading.

In a previously published study [20], we documented variations of teachers' self-reported enactments of RDM, and we provide a short summary of the results from this paper below. Our research questions were: *When teachers described enacting rough draft math, which teaching practices were salient and feasible? Among these salient and feasible enactments of rough draft math, how did teachers' descriptions vary and in what ways could these variations be viewed as potentially having productive and/or powerful impacts?* Participants in the study were recruited in two ways: invited to participate from a list of names of book club members provided by facilitators or invited to participate via social media if teachers read the book on their own.

These findings of feasible and salient incremental changes in teaching were identified from self-reports from 32 teachers in eight states in the U.S.A. Teachers were interviewed remotely. Prior to the interview, teachers sent a digital artifact that represented how they integrated RDM into their teaching practices. Artifacts included student work samples, Desmos activities or other student-facing tasks, and short video clips or photographs of their students interacting and problem solving. During the interviews, teachers were asked to describe how their artifact was an example of RDM, what RDM meant to them, why they enacted this approach to teaching, and how much RDM was a part of their regular mathematics teaching practices. Findings were first identified by classifying teachers' practices by categories that aligned with the book's contents (i.e., building and sustaining a culture in support of RDM, task selection and enactment, revising practices, incorporating reflection on changes in thinking), and then, these themes became refined through further passes through the data.

There were two enactments that were the most salient and feasible for teachers in this sample: (a) incorporating explicit revising opportunities into mathematics lessons and (b) selecting and implementing mathematics tasks purposefully to invite rough drafts and revising. Salient enactments were those described repeatedly in the interviews with extensive detail. We considered enactments that were described by a high percentage of teachers in the sample to be the most feasible for these teachers. These salient and feasible enactments can be viewed as incremental changes because they are relatively small changes teachers were able to make to their teaching after reading a book. Below, we briefly summarize variations in these enactments, as more detailed findings were published previously [20].

3.2.1. Variations in Revising

Teachers described that incorporating explicit opportunities for students to revise their thinking was one incremental change that they made to their teaching after reading. However, teachers engaged their students in revising in different ways. One variation was whether or not teachers incorporated structured or unstructured opportunities for revision into their lessons. An example of a structured revision opportunity was when a teacher would ask the students to draw a line across or down a piece of paper (or fold the paper) and told the students that they would have opportunities to make more than one attempt to solve a problem or write an explanation. An unstructured revision opportunity was when students were told that they could look back at their work at any point and make changes to their work using a different color of a writing instrument, but they were not necessarily given directions at a particular moment to write a new revised solution.

Another variation in enacting revising was related to assessments. Some teachers invited students to revise by making corrections to their tests or quizzes, which involved revising their work. Other teachers invited students to instead revise their thinking by incorporating self-assessments. Students wrote reflections on how their thinking grew and changed by looking across their work over time and drew upon their previous work as artifacts to justify how their thinking grew and changed. This self-assessment approach sometimes looked like a portfolio assessment as the end of a unit.

3.2.2. Variations in Selecting and Implementing Tasks to Promote Drafts and Revisions

Another incremental change that teachers reported enacting after reading was their approach for selecting and implementing tasks. Some teachers described selecting tasks from their textbook and modifying the tasks to invite more reasoning and sense making. An example was implementing a task by showing students only part of a prompt, which included removing the question that students were expected to address in the task. Then, they asked students what they noticed and wondered. This process invites students to share drafts of what they are beginning to notice about the task. Another example was changing the prompt of the problem to invite multiple solution strategies, which would invite revisions after students compared and contrasted their strategies. Revising, then, is a process of expanding students' repertoires of strategies.

Additionally, teachers described implementing instructional routines in a way that more intentionally centered rough drafts and revising. For example, one possible routine that a teacher could implement is a dot talk. This is a routine where students share how they chunked or segmented dots in an image in order to count them. (The focus here is not on the total dots, but on sharing different ways of seeing structure in the image.) One intentional language change was naming all possible ways of initially chunking the image as "first thinking" or a "draft". Also, an intentional implementation shift often involved asking students to share how they revised their thinking after hearing other ways of chunking the image. They would ask students to turn and talk and reflect. They might invite students to complete a sentence starter like, "A way of chunking the image that I didn't think about at first was..." The purpose of the dot talk routine is, indeed, to support students with seeing structure in a variety of ways, and the concept of RDM supported teachers with enacting this routine in ways that made this purpose more explicit to students.

In these ways, teachers found ideas in the book *Rough Draft Math: Revising to Learn* [5] to be salient and feasible, which suggests that incremental changes are possible for teachers to enact after reading a book. It is promising that mathematics teachers reported enacting revising and enacting their tasks in ways that invited drafting and revising after reading, and without extensive coaching or extended time in professional learning sessions. Additional examples and greater details about these variations in enacting RDM can be found in [20].

3.3. Study 3: Incorporating RDM While Using Curriculum Materials

Teachers reported that it was feasible to enact RDM in their teaching, as indicated above in their reflections after a conference presentation ("I used to think... and now I think..."). We interpreted this to mean that some teachers saw it possible to enact RDM using their own textbooks or curriculum materials. To seek to understand more about using RDM with their own tasks, we conducted a study to investigate what teachers noticed in their curriculum materials regarding opportunities to enact RDM. We addressed this research question: *When teachers view their curriculum materials using rough drafts and revising as a lens, what do teachers report about opportunities to engage students in rough drafting and revising while enacting their curriculum materials?*

Research on teachers' noticing suggests that what teachers notice is shaped by the frame that they use while noticing. Teachers do not simply observe; they make sense out of what they notice, and their frame for noticing is their lens for interpreting what they notice [24]. Louie and colleagues [25] explored teachers' frames for noticing in terms of the degree to which teachers' frames emphasized deficit perspectives on students' thinking and learning. If teachers hold an interpretative frame of mathematics learning as absorbing a fixed body of knowledge, then they might attend to accuracy and correctness in students' thinking, interpret mathematical work in relation to whether it is correct or incorrect, and respond to students' thinking by correcting errors or praising correctness. Alternatively, according to Louie and colleagues [25], if teachers view mathematics learning through a frame of creatively exploring ideas, then they may attend to students' diverse ways of making sense of mathematics, interpret students' work as sensible and a valuable resource for their classmates' learning, and respond by giving students opportunities to develop their own ideas and leverage those ideas to advance their learning. This alternative approach is an anti-deficit frame because students' thinking is assumed to be viable and to have strengths worth building upon.

3.3.1. Curricular Noticing

We were interested in how teachers engaged in curricular noticing for enacting RDM. Curricular noticing is a concept informed by research on teachers' noticing of students' thinking [26,27]. Skills for curricular noticing are ways that a teacher makes sense of the opportunities provided in mathematics curriculum materials [28]. An assumption underlying curricular noticing is that all teachers participate with curriculum materials [29] as they read, adapt, and evaluate their materials to plan and enact lessons. Curricular noticing skills are attending, interpreting, and responding to curriculum materials.

Curricular attending "describes the skills involved in viewing information within curriculum material to inform the teaching and learning of mathematics" [28] (p. 525). For the purposes of our study, we considered what teachers attended to in curriculum materials. In other words, *what* in their curriculum materials did teachers notice when asked how they saw opportunities to engage students in drafting and revising?

Curricular interpreting "refers to the skills used by teachers to make sense of that to which they attended" [28] (p. 536). In our study, this meant that we were interested in how teachers interpreted what they attended to in the materials regarding opportunities to incorporate drafting and revising into mathematics teaching. Essentially, *how* could teachers use what they noticed (through their attention) to invite students to share their rough draft thinking or revise their thinking?

Curricular responding "describes the skills involved in making curricular decisions based on the interpretation of curricular materials" [28] (p. 526). Responding to materials is a process of enactment. To investigate curricular responding, researchers could ask teachers to describe how they would enact the materials in practice.

3.3.2. A Case of Curricular Noticing with RDM as a Frame

Below, we provide evidence from a curricular noticing study. We intentionally asked teachers to use RDM as a frame to notice their materials. We assumed that asking teachers about the opportunities that they see in their curriculum materials to enact drafting and revising would mean that teachers would use RDM as a frame for attending and interpreting their materials.

We conducted 12 interviews with mathematics teachers and teacher leaders (e.g., coaches who support mathematics teachers with curriculum materials) to investigate what they noticed regarding opportunities to enact RDM while using their curriculum materials. All interviewees were solicited via social media (Twitter/X), with the caveat that participants had to be familiar with RDM. All participating teachers reported that they had read *Rough Draft Math: Revising to Learn*. For these interviews, which were conducted remotely, we asked participants to choose a single lesson of their choice and to send us lesson artifacts from their curriculum materials, including the teachers' guide and student facing materials for that lesson. We asked the teachers about the opportunities they saw in that lesson specifically, and in the materials more generally, to invite students to engage in drafting and revising.

This analysis is important for considering the ways in which RDM could be viewed as an incremental change in teaching. If teachers can use their curriculum materials to enact RDM, then RDM is feasible to enact in the teachers' current contexts. Below, we report what one teacher attended to in his materials and how he interpreted what he noticed as potentially affording an enactment of RDM. We focus here on what he saw as possible in the materials as they were written, not on modifications to materials to enact RDM.

Mr. Louis Johnson (pseudonym) shared about his work implementing *Illustrative Math* in a middle school classroom. He self-identified as a Hispanic man. He has been both a teacher leader and a classroom teacher in the Mid-Atlantic region of the United States. He reported having 11 years of classroom teaching experience, and 24 years in all working in the field of education. He said that he learned about RDM through reading the book and attending a presentation by the first author. We selected this teacher as the case for this paper because his interview represented some common themes across the interviews.

He found *Illustrative Mathematics* [*IM*] to align well with enacting RDM. He said the following when asked about enacting RDM with these curriculum materials:

> So, the way that I think about it is that there are opportunities for students throughout a lesson to revise some of their initial thinking. And that can happen

> in multiple ways. That can happen from teacher feedback. That can happen from student-to-student feedback. So that's what I think of with rough draft math—the idea of revising student thinking or student work within a certain space and time.

He focused on the role of revising in RDM when reflecting on his curriculum materials more than the role of drafting. We summarized what Mr. Johnson noticed in his curriculum materials in Table 1 and described his noticings in more detail below.

Table 1. Mr. Johnson's curricular noticing using rough drafts and revising as a frame.

Features of Curriculum Materials	Work of Teaching	Connections to Rough Drafting and Revising
Curriculum materials as source of rich tasks	Select and enact tasks that promote reasoning and sense making that align with the central lesson goal.	If the task asks students to generate a representation (or a strategy or a justification), then, there can be opportunities for students to draft and revise.
Curriculum materials' recommendations for student collaboration	Foster collaboration from students so they can learn from one another and develop productive dispositions.	Intentionally encourage revising by asking students to come to a consensus.
Curriculum materials as source of instructional routines	Opens access for students for opportunities to engage in reasoning.	Revising is often built into an instructional routine, or the routine can be enacted more intentionally to incorporate revising.
Structure of lessons in curriculum materials	Provide students with collaborative problem-solving experiences. Then make connections explicit to support achievement of the learning goal of the lesson.	Identified that the lesson synthesis invited students to compare and contrast their thinking and resolve disagreements, which is a process of revising.

3.3.3. Role of the Task

When teachers noticed rigorous tasks in their lessons, they saw RDM as an incremental change that could be helpful to engage students in solving them. For example, Mr. Johnson reported that he selected a particular lesson to reflect upon its potential for RDM because of the tasks in the lesson: students were asked to create tape diagrams to represent story problems. His rationale for selecting this lesson was as follows: "If students are cre-ating something, that's a space where they are able to revise something". He interpreted tasks in this lesson to be amenable to drafting and revising, because students were asked to create a representation, and then they could compare and contrast their representations to better represent the story. Teachers in these interviews regularly mentioned that RDM could support their students to persevere while solving challenging tasks, such as these tasks involving creating tape diagrams.

3.3.4. Intentionally Enact Group Work by Explicitly Encouraging Drafts and Revisions

Teachers in these interviews described how RDM provided them with insights about small changes they could make to support students in learning through collaborating. Mr. Johnson noticed an activity in *IM* that provided specific instructions for collaboration (see Figure 2):

Mr. Johnson interpreted the potential to enact this activity with language that could intentionally connect with RDM:

> I would have a little bit more intentionality of each person doing their own draft. There are three stories. And so, maybe what I would do is, if you're in a group of three, assign one person; you do the rough draft for number one, number two, number three. Then you get the feedback from the others that they agree or disagree with the way that you drew it. And then, you would come back together. So, each group would have a consensus.

Getting feedback from peers and obtaining a consensus as a group could lead to students revising their drafts. Mr. Johnson noticed the potential to respond to his curriculum materials by making small changes in how he talked with students about how to engage; he could use the activity as written, otherwise, to enact RDM.

Let's use tape diagrams to make sense of different kinds of stories.

In this activity, you will work in groups of 2–3.

You will take turns speaking and listening to your group members as you analyze how tape diagrams represent different situations.

You and your group will have two things to do with the diagram: explain why it represents the story and also figure out any unknown values in the story.

We will end this activity with a whole-class discussion.

Figure 2. Instructions for collaboration on Mr. Johnson's math activity. Note: This activity is from Illustrative Mathematics.

3.3.5. Routines Written into Curriculum Materials Promote RDM

Some of these curriculum materials incorporated instructional routines, such as mathematics language routines [30], directly into lessons. Mr. Johnson observed that *IM* contained routines throughout its units. He reported that, if teachers implement lessons as written, including these routines, students may be engaging in RDM without even realizing it. He attended to a routine called Stronger and Clearer Each Time [31]. He described this routine as follows: "...where someone writes a response, they have a partner read it, they get feedback from the partner, they revise their response, they give it to a second partner, that second partner reads it and gives feedback again". He interpreted that the routine has drafting and revising through collaboration built into it. He said, "When that routine comes up, and it comes up fairly regularly in the lessons... you're essentially implementing a version of Rough Draft Math when doing that routine". If teachers do not skip over these routines, they are engaging students in RDM even if they do not know that they are doing so. With an awareness of RDM, teachers may even respond to the materials by enacting the routine using the language of RDM to incorporate greater intentionality (e.g., "now you will write your first draft... after you read your partner's draft, revise your first draft...").

3.3.6. Lesson Structure Aligned with RDM

Curriculum materials may have lesson structures that promote drafting and revising. Mr. Johnson reported that the *IM* lessons tended to have an opening routine, collaborative learning experiences with challenging tasks, and a synthesis discussion. Mr. Johnson noticed the directions for the lesson synthesis, shown in Figure 3.

Mr. Johnson interpreted these question as supporting students with revising their thinking:

> So, how are they alike or different to me helped unpack some of the thinking and potential revisement. It's the thinking. How are they like or different? Then, do you have any disagreements? That, to me, is a space where, some of these rough drafts can be; that's where the revision can happen. And, so, how are your disagreements resolved? Could be a space where I shared, you know, or Mandy

> shared her thinking about my diagram, and I realized that her thinking was right. And, so, that led me to want to alter it or change it or something like that.

The synthesis prompt elicited students to compare and contrast their thinking, and through these comparisons, they could have new ideas about how to revise. Resolving disagreements also supported revising.

Our analysis of these interviews is ongoing; for the purposes of this paper, we only reported this single case. Our findings illustrate not only other ways that curriculum materials, as written, could align with RDM, but also, teachers reported small modifications to promote RDM that teachers made to materials. Across the interviews, we have heard how these teachers viewed RDM as an incremental enhancement to achieve their goals for students' learning while implementing their curriculum materials.

Let's discuss our diagrams:

How are the group diagrams alike and different?

Did you have any disagreements in your groups? If so, how were they re-

solved?

Figure 3. Instructions for lesson synthesis used by Mr. Johnson. Note: These instructions are from Illustrative Mathematics.

3.4. Study 4: A Teacher's Motivations to Enact RDM

We have a conjecture about why teachers can integrate RDM into their thinking and instructional practices with minor supports: RDM can be woven into teachers' current practices when RDM aligns with teachers' current motivations for mathematics teaching or their goals for what they are trying to achieve with their students. In other words, teachers are drawn to enact RDM if it fits into what they are already attempting to accomplish in their classrooms.

The concept of Professional Working Theory [PWT] [32] refers to understandings that develop when teachers reflect upon and interweave their professional knowledge, practical experiences, and ethical and moral principles or beliefs. As teachers learn about new ideas (professional knowledge), they consider how those ideas might fit into the rest of their knowledge, experiences, and beliefs. They may integrate the new ideas into their PWTs or they may reject them.

Integrating new knowledge into a PWT can be viewed as weaving. Jónsdóttir and Gísladóttir [33] are teacher educators who conducted a self-study of how they supported teachers with the development of their professional working theories. As teachers weave what they learn from professional learning opportunities (e.g., conference presentations, reading a professional book), they craft coherence in their larger PWT about their teaching practice [34] through their efforts to continuously improve as professionals. In Jónsdóttir and Gísladóttir's study [33], one teacher, Hanna, wrote a metaphor of weaving a tapestry:

> My professional working theory—who I am in my work and what I want to stand for—consists of many influences from different sources. These threads of influence weave together into the tapestry of my professional working theory. Each thread is important but individually fragile. When woven together with the others, each thread is strengthened, can bear more strain, and progresses towards its fullest potential. (p. 154)

Previous research suggests that teachers are more willing to take up a practice if they view these practices as being able to be implemented immediately [35]. Our work suggests that some mathematics teachers view RDM as a practice that advances what they are already trying to achieve and as a practice they are able to integrate into their current teaching practice through incremental changes.

A Case of How RDM Can Be Woven into a Teacher's Motivations

To illustrate how RDM fits into a teachers' current motivations, we present reflections from a sixth-grade teacher in the Pacific Northwest. Ms. Alderman self-identified as a white woman. At the time of the study, she had been a mathematics teacher for eight years. She read the book about RDM and had participated in a multi-session professional development experience in her school district led by the first author. In an interview, Ms. Alderman reported enacting both revising and implementing math tasks to invite drafts and revisions with salience. (For more on her enactment of RDM, see [20]. She was a participant in that study, but we did not analyze teachers' motivations for that publication.)

Ms. Alderman shared how RDM helped her achieve goals that she held for her students, such as supporting their mathematics learning and growing students' positive dispositions toward learning mathematics. For Ms. Alderman, RDM helped her focus students on the process of doing mathematics and de-emphasized a focus on getting a correct answer quickly. As she enacted RDM, she observed, ". . .it was like, immediately, the pressure of being correct, the pressure of having the right answer was, like—we didn't have to worry about that anymore". If she could reduce the pressure of being correct quickly for her students, they were more likely to take intellectual risks. Ms. Alderman found RDM to be particularly useful with engaging her students after coming back to face-to-face learning after the period of remote instruction during the initial phase of the COVID-19 pandemic.

> I wanted students talking. They weren't talking to each other and they weren't talking about math, and it was really hard to get them talking. There was a lot of fear, and I think more than just the typical, like, math anxiety, there was a lot of, like, we've been behind a computer for a year and how do I do this? It was shocking [after introducing RDM]. . . it was just, immediately, kids started owning their right [to share what made sense to them]. Like, 'well, I'm gonna share my thinking, and it's not done, but that's okay'.

Regarding her work with RDM, Ms. Alderman said,

> I don't think I could teach math another way now. <laugh> I really don't think I could. . . hearing students just so excited. And they're like, oh, can we come back to this activity tomorrow? Or could we, can we look at this more? Could we, could we do this more? Or students are like, can I take this home and think about this more? <laugh> like, well, yeah, I didn't assign homework, but yes, you can definitely think about it more. . .

She reported that enacting RDM supported students with developing a desire to continue learning and persevering. She wanted students to engage in actively making sense of mathematics through discourse, to build a classroom culture that focuses on understanding the process of doing mathematics, and to support her students' confidence. She found that RDM helped her work toward these goals. Overall, we conjecture, based on our interviews with teachers, that teachers who are drawn to enact RDM see it as a teaching practice that helps them achieve what they are already striving to enact.

4. Discussion

The results of Studies 2–4 provide evidence that RDM can be enacted with incremental changes to a teacher's practice and, in the cases of these participants, with relatively minimal support. Additionally, the results of Study 1 show that even attending a single conference talk can provide opportunities for teachers to shift their thinking about their teaching. We are hopeful that readers will potentially be inspired to make incremental changes to their own teaching (or their thinking about their teaching) after reading about what was possible among these participants.

Across these studies, it appears that revising was possible for teachers to integrate into their mathematics teaching practice; they found revising to be valuable, and revising could be conducted in a variety of ways in a mathematics classroom. Teachers considered ways to invite students to revise assessments. They enacted tasks in ways that gave students more than one attempt at solving them, and they enacted instructional routines and collaborative work in a manner that made opportunities to draft and revise explicit to their students. Drafting and revising also could help teachers achieve some of their goals, such as engaging more students in discourse so that they persevere to make sense of mathematics.

Results across these analyses demonstrate the potential for RDM to be a feasible incremental change for mathematics teachers to enact in their current contexts. However, we acknowledge that our participants are those who volunteered to reflect upon RDM. Above, when we asked participants in a conference session to share how their thinking changed, not all participants at conference sessions completed the Google Forms at the end of the sessions. On average, about one third of attendees at a conference session respond to the invitation to complete a Google Form and share how their thinking changed. We conjecture that participants whose thinking was not impacted or participants who did not enjoy the presentation also did not complete the Google Forms. This means we are not aware of how or why the sessions did not impact some teachers' thinking.

In the interviews we conducted about teachers' enactments of RDM after participating in book studies, we spoke with teachers who were interested in sharing their thinking about RDM with us. We did not systematically investigate the thinking of all teachers who participated in the book studies. This means that we did not learn about how the book did not impact some teachers' practice.

Similarly, we solicited volunteer participants for our study about enacting RDM with curriculum materials. Teachers who participated in interviews were interested in talking about how they saw opportunities for enacting RDM while using their current textbooks. This means we did not learn from teachers who found it more challenging to integrate RDM with their curriculum materials.

Although we do not yet have a strong understanding about teachers' resistance to enacting RDM, we also have not faced challenges with finding participants for our research on RDM. Teachers and teacher leaders around the country have been willing and interested to share how they enact RDM. Findings from those who are willing to enact RDM provide existence proofs of what is possible. The ease with which we have been able to solicit participants around the country suggests that there are teachers who are invested in this instructional approach. We hope that what we have learned from these participants will allow us to support more teachers with enacting RDM.

Additional Supports Needed to Enact RDM

In this article, we have shared a range of ways that teachers reported being able to enact RDM with minimal levels of support, which suggests that RDM is a feasible incremental change in mathematics teaching. However, it is possible that RDM could be enacted even more powerfully with more intensive support. Mathematics teacher educators could build on the momentum of incremental changes by providing additional opportunities for teachers to grow their practice in enacting RDM.

As an example, we conjecture that for students to feel safe to share their draft thinking, teachers must recognize and highlight strengths in students' drafts. This involves believing that students' thinking makes sense to them and is viable for making progress in their learning. Teachers can monitor which students participate to assess if teachers appear to be operating out of implicit biases [36]. With support, teachers can learn to recognize strengths in students' work [37,38]. Engaging in identifying implicit biases and learning to see strengths in students' thinking is an intensive, but important, endeavor that goes beyond incremental changes in teaching.

5. Conclusions

In this paper, we have shared exploratory findings to illustrate that mathematics teachers can enact RDM by making incremental changes that align with their current teaching practices and local contextual expectations, including curriculum materials, with relatively minimal support. After experiencing a conference presentation, teachers could articulate small changes in their thinking that support an interest in enacting RDM. After reading *Rough Draft Math: Revising to Learn* [5], teachers could take up and enact practices from the book, often while using their current curriculum materials. We conjecture that this is possible, in part, due to RDM being a general concept, not a set of prescribed practices, that teachers can enact in ways that make sense to them. We also conjecture that teachers were able to make incremental changes to enact RDM if they saw RDM as supporting the goals that they have for their students.

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education sciences

Article

S^3D Approach: Incremental Professional Development for Fostering Small-Group Discourse

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Abstract: The value of discourse in the mathematics classroom is evident across standards, policy documents, and research. The quality of discourse is associated with students' mathematical understanding and achievement. Productive mathematical discourse includes students providing explanations and critically listening to and evaluating the ideas of others. Such high-level discourse takes time to develop and is impacted by how teachers structure lessons and interact with students (e.g., talk moves). The present case study reports on an incremental professional development, S^3D Approach, that guides teachers through a two-phase process to enhance small-group, student-to-student discourse. One middle school mathematics teacher implemented the incremental stages of the S^3D Approach with support from a mathematics teacher educator, who conducted weekly observations and debrief meetings. Qualitative analyses revealed that, despite initial challenges, the S^3D Approach became integrated into the teacher's practice, enabling the teacher to identify and build upon incremental improvements in the small-group discourse. Overall, the findings demonstrate how through a reframing of starting points and the definition of success, incremental PD supports sustainable changes in a teacher's practice and student engagement in productive mathematical discourse.

Keywords: mathematical discourse; small groups; talk moves; incremental professional development

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1. Introduction

To be responsible citizens with the ability to engage in lifelong learning, students need to develop more than just content knowledge (DeSeCo, 2024). Under this premise, the OECD initiated the Definition and Selection of Competencies [DeSeCo] (2016) Program to create a framework of key competencies (Organisation for Economic Co-operation and Development [OECD], 2005). The framework consists of three categories, one of which is to interact in heterogenous groups. Similarly, Ananiadou and Claro (2009) developed a three-dimensional framework for 21st century skills with one of the dimensions being communication. "Communication plays an important role in the preparation of students to be not only lifelong learners, but also members of a larger community with voice and a sense of responsibility to others. Young people need to have the ability to communicate, exchange, criticize, and present information and ideas" (Ananiadou & Claro, 2009, p. 10).

In the context of the learning and teaching of mathematics, opportunities to collaborate and interact in heterogenous groups contribute to the development of students' reasoning capabilities and conceptual understanding of mathematics (National Council of Teachers of Mathematics [NCTM], 2014; National Research Council [NRC], 2012; Organisation for Economic Co-operation and Development [OECD], 2021, 2023). Bishop (2021) claims, "Learning occurs in interaction. And one of the primary means of interaction in classrooms is discourse" (p. 467). The value of discourse in the mathematics classroom is evident across standards, policy documents, and research. The nature and quality of discourse impacts "what students learn about mathematics as well as how they learn it" (National Council of Teachers of Mathematics [NCTM], 1991, p. 54). In the United States, communication is one of the National Council of Teachers of Mathematics (National Council of Teachers of Mathematics [NCTM], 2000) process standards and is reflected in one of the Common Core State Standards for Mathematical Practice (SMP), Construct viable arguments and critique the reasoning of others (SMP 3) (National Governors Association Center for Best Practices and Council of Chief State School Officers [NGA & CCSS], 2010). The standards and curricula of other countries also address the importance of mathematical discourse (Ing et al., 2015) (e.g., Standards for Excellence in Teaching Mathematics in Australian Schools (Australian Association of Mathematics Teachers [AAMT], 2006)).

Helping students develop their ability to interact with others enhances their level of participation (Mercer & Sams, 2006). In turn, the level of student participation is positively associated with the development of student understanding and achievement (Warner, 2008; Webb et al., 2014). For example, when teachers restated the ideas of students, encouraged students to share explanations, and questioned student reasoning, the students provided more detailed explanations and scored higher on assessments of their reasoning (Gillies & Haynes, 2011). In fact, discourse is a mediating factor between teachers fostering such discourse and student achievement (Bishop, 2021; Ing et al., 2015).

Discourse is "the spoken and written words, representations, and gestures people use to communicate, interact, and act" (Bishop, 2021, p. 468). The primary focus of the incremental professional development (PD) shared herein is on developing student-to-student talk in small groups. "Students must talk with one another . . . ideas and knowledge are developed collaboratively, revealing mathematics as constructed by human beings within an intellectual community" (National Council of Teachers of Mathematics [NCTM], 1991, p. 34). Student actions that involve high levels of intellectual work (Bishop, 2021) include making conjectures, reasoning logically, and constructing viable arguments (Bishop, 2021; National Council of Teachers of Mathematics [NCTM], 1991; National Governors Association Center for Best Practices and Council of Chief State School Officers [NGA & CCSS], 2010); knowing if others understand their arguments (National Council of Teachers of Mathematics [NCTM], 2000); listening to, attempting to understand, and evaluating the arguments of others (National Governors Association Center for Best Practices and Council of Chief State School Officers [NGA & CCSS], 2010; Ing et al., 2015; National Council of Teachers of Mathematics [NCTM], 1991, 2000, 2014); trying and building upon the strategies of others (National Council of Teachers of Mathematics [NCTM], 2000, 2014); and comparing strategies and making connections between ideas and various representations (Bishop, 2021; Franke et al., 2015; National Council of Teachers of Mathematics [NCTM], 1991, 2000, 2014).

Further, "discourse should be focused on making sense of mathematical ideas" (National Council of Teachers of Mathematics [NCTM], 1991, p. 45). Therefore, students must explain their ideas and reasoning (Bishop, 2021; National Governors Association Center for Best Practices and Council of Chief State School Officers [NGA & CCSS], 2010; Franke et al., 2015; Ing et al., 2015; National Council of Teachers of Mathematics [NCTM], 1991, 2000, 2014); ask clarifying questions (National Governors Association Center for Best Practices and Council of Chief State School Officers [NGA & CCSS], 2010; Ing et al., 2015; National Council of Teachers of Mathematics [NCTM], 1991, 2000, 2014); provide critiques of others' correct and incorrect thinking with justifications (Bishop, 2021; National Council of Teachers of Mathematics [NCTM], 1991, 2014); reply to these critiques (National Governors Association Center for Best Practices and Council of Chief State School Officers [NGA & CCSS], 2010); describe the strategies of others and propose strategies (National Council of Teachers of Mathematics [NCTM], 1991, 2000, 2014); and ask classmates to engage in the aforementioned actions (Bishop, 2021). For productive or meaningful mathematical discourse to occur (National Council of Teachers of Mathematics [NCTM], 1991, 2000, 2014), the conversation builds on what has been said and done (Barron, 2000).

A communication-rich mathematics classroom (National Council of Teachers of Mathematics [NCTM], 2000) with students engaged in the aforementioned actions and interactions develops over time (National Council of Teachers of Mathematics [NCTM], 1991). Hufferd-Ackles et al. (2004) describe levels through which teachers and students progress as they develop a discourse community. Students transition from providing closed answers to teacher questions to sharing their reasoning. Students progress to engaging in student-to-student talk with teacher support and then independent of the teacher (Hufford-Ackles et al., 2014). Webb et al. (2014) differentiate between low, medium, and high levels of student engagement with others' ideas. Low-level engagement occurs when a student's reference to another's idea does not include any mathematical details, and medium-level engagement involves the reiteration of another student's idea without elaboration. In contrast, when students engage at a high level, they build upon correct conceptions, address misconceptions, or collaborate. The level of engagement with others' reasoning is positively correlated with achievement.

Teachers help students move toward these higher levels of engagement. One of the eight effective teaching practices in NCTM's *Principles to Actions* is to facilitate meaningful mathematical discourse: "Effective teaching of mathematics facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments" (National Council of Teachers of Mathematics [NCTM], 2014, p. 29). The teacher orchestrates discourse (National Council of Teachers of Mathematics [NCTM], 1991, p. 34) in the mathematics classroom using a two-pronged approach (Steele, 2019). First, teachers must set goals, choose tasks, and structure lessons to provide a foundation for high levels of engagement (e.g., Smith & Stein, 2018). Second, teachers need to interact with students when they engage with tasks and share their approaches with others.

Researchers name or categorize various teacher interactions in different ways. B. A. Herbel-Eisenmann et al. (2013) describe six Teacher Discourse Moves: waiting, inviting, revoicing, asking students to revoice, probing a student's thinking, and creating (opportunities to engage with another's reasoning) (p. 183). Candela et al. (2020) differentiate between two categories of Discourse Actions. Teacher press actions prompt students to explain and justify their reasoning, and teacher linking actions encourage students to attend to others' thinking. Franke et al. (2015) distinguish between invitation moves that initiate engagement with each other's ideas and support moves that enhance the engagement. To encourage student-to-student talk in small groups, Quebec Fuentes (2020) identifies specific talk moves to address various group dynamics.

Chapin et al. (2013) define talk moves as, "strategic ways of asking questions and inviting participation in classroom conversations" (p. 11). A common feature across all categorizations of teacher interactions, or talk moves, is the goal for students to critically listen and respond to the thinking of peers. To meet this goal, Chapin et al. articulate four steps to classify the intention of talk moves:

- Step 1: Helping Individual Students Clarify and Share Their Own Thoughts (e.g., wait time, turn and talk, say more, revoicing)
- Step 2: Helping Students Orient to the Thinking of Others (e.g., Who can put that into their own words?)
- Step 3: Helping Students Deepen Their Own Reasoning (e.g., Why do you think that?)

- Step 4: Helping Students Engage with the Reasoning of Others (e.g., Do you agree or disagree . . . and why? Who can add on?) (p. 10)

Bishop (2021) found a significant and positive relationship between teacher interactions that helped students to engage with classmates' ideas and student learning. However, such interactions did not happen often.

"Engaging every student in the discourse of the class requires considerable skill" (National Council of Teachers of Mathematics [NCTM], 1991, p. 34). Teachers must establish both general norms around the role of the teacher and students and a culture of collaboration (Manouchehri & Enderson, 1999) and sociomathematical norms (Yackel & Cobb, 1996). Stein (2007) uses the term "motivational discourse" to build norms around student involvement in discourse, such as viewing mistakes as learning opportunities and engaging in productive struggle (National Council of Teachers of Mathematics [NCTM], 2014). To transition to such norms, teachers must alter the classroom talk from being focused on teacher thinking (e.g., univocal or funneling) to being centered on the reasoning of students (e.g., dialogic or focusing) (Knuth & Peressini, 2001; Wood, 1998). Small-group discourse presents additional challenges, as a teacher cannot be present with all groups at all times (Quebec Fuentes, 2020). PD can support teachers in this large-scale undertaking. Examples of such ambitious PD includes the yearlong experience of high school teachers who learned about and then conducted action research centered on classroom discourse (B. Herbel-Eisenmann & Cirillo, 2009) and the two-year Mathematics Discourse in Secondary Classrooms PD that involves 40 to 50 h for participation in a study group followed by a capstone project (B. A. Herbel-Eisenmann et al., 2013, 2017).

As an alternative to ambitious PD, Otten et al. (2022) propose incremental PD, which involves smaller changes to familiar teacher practices. Quebec Fuentes (2013), as a secondary mathematics teacher, conducted action research to study her own practice with respect to improving small-group discourse. Through repeated action research cycles, she discovered that she could not haphazardly utilize talk moves. Instead, she first needed to assess the dynamics of a group, the nature and quality of student-to-student talk within the group, and her instinctual ways of interacting with the group. With this information, she was able to purposefully choose talk moves that addressed a particular group's dynamics to effectively enhance the student-to-student discourse. Based on these findings, Quebec Fuentes (2020, 2022) developed and implemented three iterations of an incremental PD, called the S^3D Approach, to support teachers who want to foster Small-group, Student-to-Student Discourse. Although the overarching goal of developing productive small-group discourse may be considered ambitious, the S^3D Approach is broken down so that teachers attend to various aspects of small-group discourse individually and over time. In particular, the S^3D Approach consists of two phases. In Phase 1, the teacher assesses the current discourse of small groups through three lenses (Table 1).

Teachers use what they learn from the three lenses of Phase 1 as they move on to Phase 2. They set goals for groups, plan Process Help talk moves that align with each group's dynamics, implement the talk moves, and reflect on the impact of the talk moves. Teachers are encouraged to maintain the same groups for a period of time (e.g., 10 weeks) and start small by initially implementing the S^3D Approach with one to two focus groups. When teachers change groups, they repeat both phases with the new groups. Additionally, tools support teachers' assessment at each phase/lens. Refer to Appendix A for each of the tools that contain detailed information about each phase/lens.

The purpose of the present study is to examine the implementation of the S^3D Approach from the perspective of incremental PD. The study focuses on the most recent implementation of the S^3D Approach with one teacher; therefore, a qualitative case study design was used (Merriam & Tisdell, 2016). Specifically, the following research questions were addressed:

1. What is the teacher's experience when engaging in incremental PD?
2. How do the tools support the teacher as she progresses through the incremental PD?
3. How does the teacher's practice change over time?

Table 1. Descriptions of the three lenses involved in Phase 1of the S^3D Approach (Quebec Fuentes, 2020, 2022).

Lens	Description
Group Dynamics	The Group Dynamics are ways in which students behave in small groups that inhibit productive mathematical discourse. The 10 dynamics (e.g., Non-Participatory Student, Dominant Student, Rush to Complete Task) reflect ways that groups initially interact and provide a starting point for transitioning toward more productive discourse.
Discourse Quality	The Discourse Quality is the nature and quality of the student-to-student talk. Students' questions and their peers' responses are categorized according to eight types of question–response (QR) pairs that increase in their level of cognitive demand (e.g., questions focused on logistics, answers, explanations, or critiques).
Teacher Support	Teacher Support refers to how a teacher interacts with a group, differentiating between Product Help and Process Help (Dekker & Elshout-Mohr, 2004). Product Help focuses on the mathematics content and often results in a conversation between the teacher and only one student (e.g., funneling). Process Help focuses on helping the students communicate with each other about the mathematics.

2. Materials and Methods

2.1. Participants and Setting

The present article reports on the third iteration of the PD, which was conducted with one middle school mathematics teacher, Arden (pseudonym), the primary participant of the study. Arden had been a student of the author in her graduate studies and volunteered to work on fostering small-group discourse with the author, a mathematics teacher educator (MTE) who both conducted the professional development and the research surrounding its implementation. Arden was in her second year of teaching after completing an undergraduate degree in middle school mathematics education and a master's degree in mathematics education. She taught in a classical online charter school in the Southwest United States. Classes at the school are taught synchronously via Zoom. The class periods were 45 min in length. Arden's lessons typically involved a whole-class lesson launch, small-group work on a task, and then a whole-class summary of connections between the groups' solution strategies and mathematical concepts underlying the tasks (Smith & Stein, 2018). Arden used Google Jamboard as a platform to facilitate all three phases of the lesson. Each group had a shared Jamboard slide; one group member shared their screen in the Zoom breakout room. During small-group work, Arden could see each group's work on Jamboard and visited the various groups as they collaborated in the breakout rooms.

As part of the professional development, Arden chose to center her work on fostering small-group discourse in her two seventh grade mathematics classes. Each class had 15 to 25 students (11 to 12 years of age), who were divided into groups consisting of 2 to 4 students. In the first quarter at the beginning of the year, the students were randomly assigned to groups. In the subsequent quarters, Arden purposefully grouped students based on what she had learned about how they interacted in groups using a variety of reasons (e.g., grouping a shy student with more collaborative and supportive peers). Students were physically located in the same state, but classes took place at their respective homes. Since the school was in its second year of operation, many of the students came from other charter and public in-person schools, and several were previously homeschooled. The student body comprised individuals from various ethnic backgrounds, including

Asian, African American, Hispanic, and White. In Arden's seventh-grade pre-algebra class, much of the year was spent delving into proportionality. Specifically, the class addressed scale factors as percentages, coordinate rules for the enlargement and reduction of shapes, proportional relationships to make predictions, graphs and tables, and equations. Additionally, students learned preliminary data analysis, constructing and measuring, and function foundations. The school adopted Carnegie curriculum materials, and the teacher supplemented it with Connected Mathematics Project Grade 7 throughout the year.

2.2. Procedure

Arden participated in the PD each quarter, for which she created new groups that remained together for the 10 weeks. She chose to focus on two groups, one from each seventh-grade class, during each quarter. To identify the focus groups, Arden used the Group Dynamics tool to identify groups manifesting multiple dynamics inhibiting productive discourse. The PD was cyclical and broken into incremental phases according to the S^3D Approach (Figure 1). In Phase 1, Arden assessed the current discourse of the focus groups through the three lenses: Group Dynamics, Discourse Quality, and Teacher Support. For each lens (left side of Figure 1), Arden learned about that component of small-group discourse and the associated tool. She used the tool during and after her lessons to document her observations and then participated in a weekly debrief session during which she shared what she learned about the discourse of the focus groups with respect to that particular lens. She repeated this cycle for the two other lenses in Phase 1. Arden then progressed to Phase 2 (right side of Figure 1). She used all of her reflections from the three lenses of Phase 1 to set goals for each focus group and plan the talk moves she would use, implemented the talk moves in practice, reflected on the impact of the talk moves on the student-to-student discourse, and shared her reflections in the weekly debrief session. This Phase 2 cycle was repeated for the remainder of the quarter. Arden repeated the entire process each quarter, restarting with the assessment of the Group Dynamics of the new groups (i.e., first lens of Phase 1).

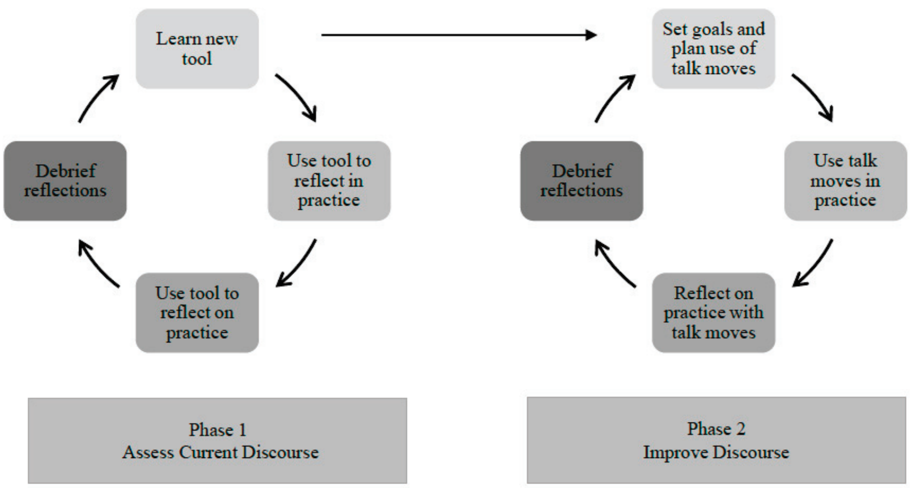

Figure 1. Cyclical implementation of the PD (Quebec Fuentes, 2020).

The MTE observed each seventh-grade class one time per week. She observed the entirety of the lesson (whole-group launch, small-group work, and whole-group summary). During the small-group work, the MTE observed the focus group in their breakout room. The MTE took fieldnotes (i.e., a script of the lesson interactions) during the observations and documented the nature of the small-group discourse using the four tools associated with the S^3D Approach (Group Dynamics, Discourse Quality, Teacher Support, and Talk Moves) after the observation. At the end of each week, the MTE debriefed with Arden. The debrief meetings had several purposes. During each conversation, Arden shared her reflections on the small-group discourse with respect to the current phase/lens and her experience engaging with the S^{3}D Approach, and the MTE provided feedback as well. At the close of each session, Arden and the MTE decided whether to continue with the current cycle or transition to the next cycle. In the latter case, the MTE introduced Arden to the new phase or lens and the associated tool. The MTE video-recorded each debrief session and took fieldnotes.

2.3. Data Analysis

The data sources included the observation fieldnotes and the videos and fieldnotes from the debrief meetings. The MTE used the observation fieldnotes to analyze the four aspects of the small-group, student-to-student discourse. First, the MTE coded the observation fieldnotes with respect to Group Dynamics (10 group dynamics), Discourse Quality (eight question–response pairs), Teacher Support (Product Help and Process Help), and Talk Moves (type of Process Help talk move). For each observation, the MTE then completed the Group Dynamics, Discourse Quality, Teacher Support, and Talk Moves tools. For each group dynamic coded, the MTE provided a description of how the dynamic manifested. For the Discourse Quality, the MTE tallied each question–response pair and wrote a summary of the nature of the student-to-student talk. For Teacher Support, the MTE indicated whether Arden used Product Help and/or Process Help. The MTE also documented the specific Process Help talk moves used and the impact of those talk moves using the Talk Moves tool. After each 10-week quarter was completed, the MTE synthesized the information documented in the four tools from all observations across the quarter to create a profile of each group, showing the evolution of the student-to-student talk as related to the impact of the teacher Process Help talk moves.

The transcribed videos and fieldnotes from the debrief meetings provided evidence of the teacher's experience engaging with the S^{3}D Approach. The MTE analyzed the fieldnotes using the constant comparative method (Glaser & Strauss, 1967). First, the MTE read through the fieldnotes and openly coded them using the comments feature in Word. The codes were then refined and categorized into five categories: teacher practice as problem solving (e.g., formal reflection, unanticipated outcomes), nature and implications of incremental PD (e.g., choose pace, change in norms), use of tools to enhance teacher practice (e.g., reveal intricacies of small-group discourse, modification of tools), teacher challenges (e.g., managing multiple groups, impatience), and incremental change over time (e.g., not recognizing and recognizing improvement).

3. Results

The present study reports on Arden's experiences during the first two quarters of the academic year. The results are organized according to the five themes (teacher practice as problem solving, nature and implications of incremental PD, use of tools to enhance teacher practice, and teacher challenges and incremental change over time). Codes within each theme are in italics.

3.1. Teacher Practice as Problem Solving

The S^{3}D Approach centers on a problem of practice. In particular, the approach involves understanding the current state of small-group, student-to-student talk and moving toward enhancing the quality of that discourse. The MTE explained the foundations of the process to Arden: "You are actively taking this [problem of practice] on, you are purposefully thinking about it, collecting data, and making decisions off of that data. That is how you are going to see improvement. It will not be tomorrow, but you will have it" (Debrief Meeting 5). The tools associated with the S^{3}D Approach support data collection and a transition from informal to more formal reflection. Arden explained, "It is interesting as we move through the tools . . ., [the informal] observations are vague until I can look at the tool and see these very specific [aspects of the discourse]" (Debrief Meeting 3). The formal reflection sometimes revealed unexpected information. For instance, when Arden changed groups for the second quarter, she had anticipated that one of the students in a focus group would be non-participatory. Arden expressed her disbelief:

> I noticed . . . when I went into their groups a couple times to observe that Milton was speaking more than I'd ever seen him speak in a group. . . . Lillie is bringing him out a little bit and that was a completely unexpected dynamic. In the beginning you [said] you can predict these dynamics, but it might surprise you. And I'm surprised at how much he's talking right now. (Debrief Meeting 11)

With such targeted reflection, Arden was able to see intricacies of the small-group discourse including *unanticipated outcomes*.

3.2. Nature and Implications of Incremental PD

One of the goals of the S^3D Approach is for small groups "to have sustainable conversations when [the teacher] is not there" (MTE, Debrief Meeting 5). Arden expressed the enormity of this goal, "it is a lot of work. . . . I hope it pays off" (Debrief Meeting 8). In response, the MTE explained that there are various factors that influence the nature of small-group talk, "There are so many intricacies and that probably feels overwhelming from the teacher perspective: 'I have so much going on and now I have to be thinking about the intricacies of how they interact.'" (Debrief Meeting 8). To address these *intricacies* and make the process more manageable, the S^3D Approach is broken down into two incremental phases, as explained previously. Arden described the approach: "It is completely a science to get them to learn how to talk to each other. And then not just talk, . . . really speak mathematically and then speak meaningfully" (Debrief Meeting 9).

To further make the approach accessible, Arden was able to *choose the pace* at which she progressed through the phases and lenses each quarter. For example, the following conversation about whether to move on to the next Phase 1 lens occurred during Debrief Meeting 5:

> MTE Where do you want to go as you delve into next week?
>
> Arden Well, I still think I need to live in the Discourse Quality. I need more time in that . . . with juggling the other groups . . . especially since I only gathered two days [of data] this week.

Similar conversations about goals and talk moves occurred during Phase 2.

Arden progressed through the two phases each quarter. In the first quarter, *getting acquainted with the phases, lenses, and associated tools* took time. For instance, Arden shared her experience with implementing talk moves for the first time:

> I'm not taking as many notes as I thought I would. . . . I would reference the talk moves . . . to refresh myself before going to the group. I would find myself doing those moves and then realizing I'm accomplishing a lot by doing this move. . . . [But] I forget to track it; I'm just so in the weeds right now. (Debrief Meeting 9)

By progressing through the two phases during the first quarter, Arden became more familiar with the intricacies of the approach. During Debrief Meeting 13, she described her familiarity with the talk moves: "I still think about them in the back of my head." As a result, she was able to move through the phases more quickly with new groups during the second quarter. Table 2 shows how much time she spent on each aspect of the S^3D Approach for the first and second quarters. In the first quarter, Arden started the S^3D Approach during the third week. She spent most of the quarter in Phase 1, implementing the talk moves of Phase 2 for the last two weeks. In the second quarter, Arden quickly progressed to Phase 2 after spending only three weeks in Phase 1.

Table 2. Time spent on each phase/lens in Quarters 1 and 2.

	Phase/Lens	
Week	**Quarter 1**	**Quarter 2**
1		Phase 1—Group Dynamics
2		Phase 1—Group Dynamics
3	Phase 1—Group Dynamics Choose Quarter 1 Focus Groups	Phase 1—Discourse Quality
4	Phase 1—Group Dynamics	Phase 2—Talk Moves
5	Phase 1—Group Dynamics Phase 1—Discourse Quality	Phase 2—Talk Moves
6	Phase 1—Discourse Quality	Phase 2—Talk Moves
7	Phase 1—Teacher Support	Phase 2—Talk Moves
8	Phase 1—Teacher Support	Phase 2—Talk Moves
9	Phase 2—Talk Moves	Phase 2—Talk Moves Choose Quarter 3 Focus Groups
10	Phase 2—Talk Moves Choose Quarter 2 Focus Groups	

The MTE discussed this shift when transitioning to the second cycle: “You could jump [to Phase 2]. I know you’re already [implementing talk moves] because I have it in my notes and my data. . . . You’re hitting this point where . . . you can go through Phase 1 much faster” (Debrief Meeting 13). The nature of the student-to-student talk also improved more quickly in the second quarter. The MTE shared her observations during the first couple of weeks of the quarter: “There was definitely a lot of student talk going on; whereas, when I started observing at the beginning of the year there wasn’t” (Debrief Meeting 9) and “They’re still fairly new groups. So, you’re going to get a lot of question-response pair 1 [QR1; lower cognitive demand]. . . . [However] we’re only two weeks in, and . . . they’re [transitioning to QR pairs of higher cognitive demand] pretty fast” (Debrief Meeting 12). Even though the students were in new groups, the benefits of Arden going through the S^3D Approach in Quarter 1 were evidenced in Quarter 2.

Another way the process was incremental is that Arden *chose one group in each class as her focus group*. She implemented the S^3D Approach with the focus groups. When Arden was trying to choose between two groups for one of the Quarter 1 focus groups, the MTE explained:

> This is not an absolute decision. You are going to be working with both of these groups on the regular. Just one of them you are going to be taking that step to collect some data. Eventually everything you will be learning with your focus groups will be integrated into other groups as well.(Debrief Meeting 3)

Working with the focus groups allowed Arden to become acquainted with the process and then consider ways to *extend what she learned to other groups*. Arden and the MTE discussed the act of “juggling” the different groups during Debrief Meeting 5:

> Arden What I want to try and do is when I eventually start isolating some of the things I notice help some of the groups, I want to be able to recognize quickly that I can use that in another group for a different situation. So, that is my goal hopefully by just having these focus groups. . . .
>
> MTE What happens is the tools are going to start getting ingrained in your brain and it’s going to carry over to the other groups.

Even though Arden had her focus groups, she still needed to attend to other groups. Arden was able to extend what she learned with the focus groups to other groups. In Quarter 2, Arden explained:

> I feel like I'm at that stage where I check [tools] every once in a while, but I'm pretty familiar with these dynamics. . . . It's almost inherent in what I'm doing, because I don't really reference them. But, I noticed I'm carrying it over to the eighth graders as well. (Debrief Meeting 14/15)

As the various aspects of the S^3D Approach became integrated into her practice, Arden was able to attend to more seventh-grade groups as well as groups in her other classes in a purposeful way.

Just as the process was incremental, so too was the *change in classroom norms*. A highly effective mathematical discourse community is not going to be established immediately. Arden made this point after watching a video with ideal student-to-student interactions:

> Arden [The video showed] the perfect image of what student-to-student collaboration looks like. . . . What I notice is that this has been developed over a long time. . . .
>
> MTE That is a really important point. When we show those videos to teachers, we need to let them know that this does not happen instantaneously and then we need to support them with strategies in how you move them there. (Debrief Meeting 7)

The S^3D Approach involves redefining engagement in the learning of mathematics. Arden explicitly articulated to her students what success means in her class: "I discussed that our success is growth. It is not about getting the best grade possible; it is about communicating and working together" (Debrief Meeting 7). An overarching norm is the establishment of a culture of respectful collaboration:

> MTE The student-to-student interactions are very important; how we foster those with the talk moves [is] very important. But, there's also this norm above it all about just speaking respectfully.
>
> Arden That's a really big point to make. What we're talking about . . . humanizing what we're doing.

Arden also talked about reinforcing with students that they should support each other: "I've been keeping up the norm that we're a team and we help each other" (Debrief Meeting 17). However, students who have not had previous experiences communicating mathematically do not understand the various aspects of productive discourse. Arden talked about such a student in Debrief Meeting 3:

> Arden There is a level of she does not know how to communicate her thinking because this is something she has done on her own for so long.
>
> MTE . . . You are being pretty explicit—these are the norms of the classroom. That's really one important piece. But, we can say we want you to do this, but they don't necessarily know how to do it. . . . That's where this whole process comes in where the way that you are interacting with them and your actions implicitly will reinforce . . . the type of interactions that you want between the students. Again, it's just going to take time.

Through the implementation of the S^3D Approach, students learned to explain their ideas, listen to the ideas of others, and evaluate those ideas. Arden observed change with respect to student explanations: "[I've been] going through a lot of my groups lately and finding them saying, 'I think we need to explain this. I think we need to stop and talk about it'" (Debrief Meeting 16). Arden also observed the impact of the talk moves during the whole-class discussions:

> I've also been doing that teacher move [restate in your own words]. . . . In the main session, they're starting to expect to have to reiterate what someone said or say it in their own words. . . . I've actually also been [asking]: "Can you compare what this group did to what you did?" . . . I was really impressed. [One group said]: "In comparison to group five we did this, but similar, we did that." (Debrief Meeting 9)

Both the act of restating another person's ideas in one's own words and comparing approaches requires students to critically listen to one another. In turn, students start to critique the thinking of others. During Debrief Meeting 11, Arden and the MTE discussed reframing norms around the idea of critique:

> MTE You're basically establishing this norm that critiquing doesn't have to be negative. It's how we learn. . . .
>
> Arden I tried to publicly give them that feedback of it's okay, that we make mistakes here. . . . In the beginning, especially early norm setting, it was just drilled into them that right answers mean very little to me. It's what we learned from making our mistakes. That means more.

The students were learning that critiquing the work of others and making mistakes are an integral part of the learning process.

3.3. Use of Tools to Enhance Teacher Practice

Since the S^{3}D Approach is incremental, Arden *did not use the talk moves immediately*. During Debrief Meeting 3, Arden noticed the difference between the S^{3}D Approach and professional development that has more immediate products:

> [In] PDs, you want it to be make and take, . . . you want to open a book and find a page of a whole list of talk moves that you can do. But, that is not how [the S^{3}D Approach] works . . . because I have to consider everything that is involved with the individuals. . . . We can't just spout out a bunch of talk moves that may or may not be applicable to them.

Arden further explained how the tools associated with each phase and lens of the S^{3}D Approach *helped her identify the aforementioned norms*: "As you go through the tools, there's norms we're developing or that I realize I need to develop and maintain by doing these tools" (Debrief Meeting 5).

The tools also *reveal the intricacies of and influences upon the development of small-group-student-to-student discourse*. The tools moved Arden from intuitions about a problem of practice to understanding how to identify and address the various aspects of talk in small groups.

> There's this overwhelming feeling that I've had that [the work on small-group discourse] needs to go further, but I don't know where or how. And the tools guide me along the way exactly how [the conversations] need to go further, . . . how I can explicitly and implicitly support these conversations. (Debrief Meeting 5)

The tools were critical to Arden's engagement with the S^{3}D Approach. They allowed her to see the nature of the small-group discourse as well as how to bring about change.

In fact, as Arden progressed through the phases and lenses, each tool played a role in the development of her practice. Arden used the Group Dynamics tool to assess the dynamics of each group in her class and choose a focus group for the quarter.

> The first [class], I identified a group that I think really needs the extra support and really needs this attention. And, I even have here on my Group Dynamics tool that there was a sense of Learned Helplessness. Two of the three were Non-participatory. All of them, when they came across an Obstacle, they just sat there,

> and they would sit with their microphones off during that obstacle until there was Teacher as Authority situation where I did not just give them the answer. . . . So, that's four check marks for just this one group. (Debrief Meeting 3)

The Discourse Quality tool helped Arden identify the types of student-to-student interactions, their quality, and areas for growth. The MTE shared her observations with the Discourse Quality tool for both of the focus groups.

> MTE Both of the groups this week were not falling in that first set of dynamics where people were not participating. . . . There was full on participation, talk the majority of the time that I was in there. Once you start using the [Discourse Quality] tool . . . it's not just is this question-response occurring, it's also what's the nature of it and is it high quality? Because you can have . . . a lot of talking going on which is . . . a good first step. But, the cool part is this is showing all the areas where there can be improvement.
>
> Arden It is like I am laying it all out there. These are my flaws let's fix them.
>
> MTE But, it is also not personal. They are not your flaws The view of it is: This is where my students are currently in their discourse quality, this is giving me a baseline and allowing me to establish some goals for growth. (Debrief Meeting 5)

By using the Teacher Support tool, Arden was able to interrogate how she interacted with the groups and the implications of using Process Help rather than Product Help.

> I find myself, as I am tracking, shying away from giving the Product Help. . . . [The Teacher Support tool] is really revealing. . . . The reflective part of teaching is right there in that tool. . . . How much am I holding my scholars' hands? How much of this is their words versus my words? And that is the part that I really don't want. It should never be my words; it should always be theirs. (Debrief Meeting 8)

Following Phase 1 and the three associated tools, Arden used the Talk Moves tool to identify Process Help talk moves to use with her focus groups. After an extensive conversation during which Arden reviewed the predominant dynamics of each of her focus groups and identified which aligned talk moves she was going to use, the MTE summarized how she would document their ideas: "I'll do exactly what you said. I'll take the Talk Moves tool and jot down the notes about what we said, what the purposes [of the talk moves] are. . . . Then, there is less of a cognitive load on you . . . planning them is helpful" (Debrief Meeting 8). By using the Talk Moves tool as a planning document, Arden did not have to identify talk moves in the moment and was better prepared to implement preplanned talk moves in response to the group dynamics.

As Arden engaged with the tools, she also problem-solved how to use them efficiently. For example, Arden noticed how she used the Discourse Quality tool in a different way than the MTE:

> I was just thinking that I noticed how you use your Discourse Quality [tool], and I use it differently, where I would jot down a brief note inside the square. . . . I noticed that there's obviously a bunch of question-response pairs in one observation. But, I was only documenting one because I only had time to type out notes for that one. So, I'm going to . . . go back and jot down some notes after I've done the tick marks. (Debrief Meeting 10)

Arden actively considered how to *effectively integrate the tools into her daily practice*. In another instance, when using the Discourse Quality tool, Arden discovered a student-to-student interaction not represented in the tool.

MTE Probably 50% of those [QR1] are low-level like: What page are we on? . . . But the other 50% are [interactions] like: Dittika, do you want to plot the point? So, it's still that . . . practical question. But, it's totally different. It's inviting people to participate or orienting each other to the work. . . . It's a more important type of question.

Arden If you look at my [Discourse Quality tool from] today, I was thinking the same thing. And I said the word: Olive invited Dittika into the conversation. . . . In some way, I almost feel that's a lower tier [i.e., higher cognitive demand in Discourse Quality tool] for them to have to invite each other into the conversation, especially without prompting.

MTE . . . These [tools are] a working document. So, . . . if you're noticing other question-response pairs, if you're noticing other dynamics, add them to your toolbox. (Debrief Meeting 12)

Similarly, Arden's observations of group dynamics led to an expansion of the interpretation of the Obstacle dynamic: "[The obstacle] doesn't prevent the work, it's preventing them in how they think about [the mathematics] . . . They could communicate incorrectly" (Debrief Meeting 14/15). Overall, the tools not only served as a guide through the S^3D Approach, they also allowed Arden to take ownership of the process with respect to the logistics of employing the tools, use of the tools to set goals and make plans, and the autonomy to *expand upon the tools*.

3.4. Teacher Challenges and Incremental Change over Time

Even though the S^3D Approach is broken down into incremental steps, Arden initially experienced *challenges incorporating each phase/lens into her daily practice*. She described her experience using the Discourse Quality tool: "[Using the Discourse Quality tool] was not pretty. . . . So, that is the part that's difficult, just needing to do it and try and think about it while you are teaching. It is a lot, but you end up doing it" (Debrief Meeting 5). Due to the novelty of the tools, Arden viewed them as an additional responsibility on top of her typical teaching responsibilities rather than an integral part of her practice. For each new phase/lens, the MTE provided information and advice about how to use the tool. In Debrief Meeting 6, Arden was progressing to the Teacher Support lens/tool:

MTE Just interact with the kids intuitively and then think back: What were my natural inclinations? Where are you falling [Product Help or Process Help]? You could be all one, you could be all the other, you could be a mix.

Arden My hypothesis is I probably do both. I try to lead with Process Help and then ends up becoming Product Help. That's my hunch.

MTE Just like with Discourse Quality, it's letting us look at [the small-group discourse] more deeply. So, when we move on to Phase 2, . . . we know what we are working with, what are the goals for ourself, and our goals for our students.

The tools enabled targeted observations culminating in a baseline of the three lenses from which goals for growth were established for Phase 2.

Phase 2 introduced additional challenges, since Arden had to learn to navigate the Talk Moves tool as well as assess the impact of the talk moves on the small-group discourse (using the three tools from Phase 1). The MTE shared ideas to address this challenge of *managing multiple tools at once*: "In Phase 2, you don't necessarily need [the Teacher Support tool] because you're documenting that with your Talk Moves tool. . . . When you're in the Talk Moves tool, there's . . . a focus on Process [Help]" (Debrief Meeting 10). The Talk Moves tool not only reinforced Process Help interactions but was also organized according to the 10 dynamics in the Group Dynamics tool. Later in the same debrief meeting, the MTE continued: "It's overwhelming to have all the tools. What tool should I focus on? In

Phase 2, I would say the Discourse Quality tool. . . . The goal is that we want the quality of their discourse to improve. That's the point of the talk moves" (Debrief Meeting 10). Both the Talk Moves and Discourse Quality tools incorporated the Group Dynamics and Teacher Support Tools. Focusing on the former two tools helped Arden manage Phase 2.

Arden progressed through Phases 1 and 2 collecting information on her two focus groups. The S^{3}D Approach allowed her to incrementally change her practice; however, steps to make integrating the process more feasible simultaneously introduced two additional challenges. First, Arden still needed to conduct her lessons and interact with multiple small groups while slowly learning how to enhance the small-group discourse.

MTE Even though you are [in] Phase 1, you are still doing the interactions. So, even though we will formally address the interactions down the road, right now it doesn't mean you don't think about them.

Arden I do them, the ones that I can think about on the spot (What do you think about this? Can you explain in your own words what this group just said?), just the basic teacher moves that we all inherently do. That's all that I can do in those moments.

MTE . . . You are still going on with your class and practices, but we are inserting some of these purposeful data collection pieces as we go through [the process]. (Debrief Meeting 3)

Second, as Arden learned more about the process, she was able to see what she learned (e.g., identification of group dynamics) in groups other than her two focus groups. She was then overwhelmed with *thinking about multiple groups*:

Arden What makes me so fearful is that I feel where do I go from here. I see what's happening now. What's worse is I am seeing it happen in other groups where there are still people not communicating at all—so I feel that I am juggling all of these dynamics, and it is so challenging.

MTE . . . That is what happens. There is this: "Wait, I have five, six groups, you are telling me to focus on two?" . . . The tools are going to start getting ingrained in your brain. It is going to carry over to the other groups. (Debrief Meeting 5)

Ironically, the strategies to simplify the process (e.g., choosing focus groups, learning one phase/lens at a time) at times made the process overwhelming. As Arden gained a deeper understanding of the intricacies of small-group discourse, she began observing multiple aspects to address across all groups.

Arden felt responsible for bringing about change in the small-group discourse and was nervous that this change would not happen. For instance, when she initially used the Discourse Quality tool, many of the student-to-student interactions were of lower cognitive demand:

> They are asking practical questions about what they need to be doing next to answer the question. . . . It is really . . . humbling me. I really want them to have deeper discussions . . .. I feel . . . almost fearful that I won't be able to get them to a place where they are having more discussion [beyond] just answering the question. (Debrief Meeting 5)

Arden also felt *frustration* with particular students, who were repeatedly not showing any improvement: "I feel so lost on what to do with them [two non-participatory students]. . . . I just don't know what else to do. . . . I know what you're saying and I am going to use that [talk move]. I guess I feel so defeated by them" (Debrief Meeting 7).

Being embedded in the process and managing all of her teacher roles and responsibilities, Arden expressed *insecurity* at times. Early in the process, Arden was *not able to see improvement* in the small-group discourse. She felt frustrated and *impatient*: "[It] feels frustrating. . . . I feel like I am not making a difference because I know their dynamics haven't changed at all. And I don't know what to do about that" (Debrief Meeting 9). The MTE was able to see the incremental growth and used these examples to encourage Arden. For instance, in one of Arden's focus groups, Erin was a Non-Participatory Student. The Group Dynamics tool was used to show both the identification and improvement in the dynamics. The MTE wrote:

> Erin is the non-participatory student. However, the group has demonstrated progress in this area. As noted in the Discourse Quality Tool, Lila regularly checked in with Erin to see if she was keeping up with the group. Additionally, Erin asked logistical questions about completing the task to the group. (Observation 9, Group Dynamics tool)

The MTE also showed Arden the observation data in the Discourse Quality tool (Figure 2): "This is really powerful. What I was seeing before was so much QR Pair 1 and maybe a sporadic [tally below] and now they're even moving down here [QR Pairs 3 through 7]" (Debrief Meeting 9). Figure 2 shows a greater array of higher cognitive demand student-to-student interactions in both of the focus groups.

At the end of Cycle 1, Arden started to *notice improvement*: "I've spent a little bit more time in the other groups who still have non-participatory dynamics, and I think there's been a bit of improvement" (Debrief Meeting 10) and "I felt like their discourse quality has improved" (Debrief Meeting 10). In Cycle 2, Arden observed and articulated growth. For example, Arden described the student-to-student interactions between Maria and Olive during a discussion about similar figures:

> I was actually really excited. ... They were down in [the QR pairs of higher cognitive demand]; they were asking some good questions. Olive was criticizing whatever Maria was documenting. ... Olive would say, "Well, I disagree because it doesn't mean that they have the same scale factor. It just means that all of the sides changed by the same number." ... And then Maria would ask her to explain that. (Debrief Meeting 12)

Not only was there improvement in the group dynamics and discourse quality early in Cycle 2 with the new groups, but Arden was also able to recognize the change herself and provide details about the nature of the improvement (guided by the tools). Further, in the latter debrief sessions of the second quarter, the conversations between the MTE and Arden predominantly focused on details of the small-group discourse and the implementation of the talk moves.

> MTE The place where there can be some improvement ... [is] these three QR fives were all Blindly Accept. Lillie said ..., "I think distance is the independent variable." Selma [agreed]. And then Lillie said, "I think time is the independent variable," and Selma [agreed].
>
> Arden At least Selma is agreeing.
>
> MTE Yes, ... normally that's a soliloquy by Lillie reasoning through [the task]. So, the positive side is that you're getting Selma inserted, Selma inserted.
>
> Arden That is a major improvement.
>
> MTE In the talk moves tool, [we planned] when Lillie is having a soliloquy, insert yourself at that point and ask the other students to respond to the question.
>
> Arden I did it. I did it.
>
> MTE ... That's exactly what that they needed. You [said], "I'm stopping you, Lillie. You two [Selma and Milton], I want you to respond to this. (Debrief Meeting 18)

Arden saw the impact of the talk moves on incrementally enhancing the student-to-student talk. Becoming more familiar with the S^{3}D Approach resulted in the debrief sessions transitioning from conversations about the tools to conversations about what she was learning from the tools.

Question	Response	Focus Group 1	Focus Group 2
1. *A* asks *B* a practical question	*B* answers *A*'s question	*III*	*IIIIII*
2. *A* asks *B* a question about previously learned content	*B* answers *A*'s question	*I*	
3. *A* asks *B* to show work	*B* shows own work	*II*	
4. *A* asks *B* to explain work	*B* explains own work	*III*	*II*
5. *A* asks *B* to evaluate work	*B* evaluates *A*'s work	*I*	*I*
6. *A* criticizes *B*'s work	*B* justifies own work	*II (non-math) I*	*III*
7. *A* rejects *B*'s justification	*B* reconstructs own work		*I*
8. *A* suggests a strategy to the group	The group tries the strategy		

Figure 2. The Discourse Quality tool with data from Observation 9.

4. Discussion

Otten et al. (2022) posit that incremental PD requires "redefining success, rethinking the starting points for PD, and new mechanisms for sustainability and scaling" (p. 1447). The S^3D Approach itself and Arden's experiences engaging with it reflect several of these characteristics, reveal the implications of incremental change, and open up further questions for study. The S^3D Approach centers on changes to a teacher's regular practice (Otten et al., 2022) as long as the teacher, like Arden, regularly incorporates small-group work into instruction. Further, the S^3D Approach is designed to be accessible to teachers in several ways.

First, the approach has progressive *starting points*. In particular, it involves two phases, assessing the current discourse of groups and then using talk moves to enhance the discourse. The first phase is broken down into three different ways of examining small-group discourse (Group Dynamics, Discourse Quality, and Teacher Support). Arden focused on each of the lenses of the first phase one at a time. She then used this multifaceted assessment to purposefully plan Process Help talk moves that aligned with each group's dynamics to improve the discourse quality. Another component of *rethinking starting points* is the identification of focus groups. Instead of attempting the approach with all of her groups in all of her classes, Arden identified one group in each of her two seventh grade mathematics classes. Implementing the approach with just two focus groups allowed Arden to become familiar with the process on a much smaller scale.

Second, the S^3D Approach includes tools associated with each phase/lens (National Academy of Education, 1999). Each of the tools played a role in helping Arden understand the intricacies of small-group discourse. As Steele states, "Shifting to a more discourse-centered classroom certainly means changing our interactions with students during the lesson. Those interactions, however, are more effective when we use tools to plan for discourse" (Steele, 2019, p. 355). The Group Dynamics tool helped Arden recognize dynamics that could inhibit productive mathematical discourse (e.g., Non-participatory Student) and then use this information to identify focus groups. With the Discourse Quality tool, Arden was able to categorize and assess the quality of student-to-student interactions within groups and, based on this evaluation, set goals to move the students to interactions of higher cognitive demand. The Teacher Support tool provided a framework to distinguish between her teacher moves that fostered student-to-student talk and those that did not. The Talk Moves tool served as a planning document for the Process Help talk moves that Arden would use to address group dynamics and enhance discourse within the small groups. "Identifying and naming these moves, and then carefully planning to use them, supports teachers in thinking more strategically about mathematics classroom discourse" (Cirillo et al., 2014, p. 141). The tools also helped Arden develop general norms around collaboration, including speaking respectfully, helping peers, critically listening to each other, and ensuring that all members understand the work of the group (Manouchehri & Enderson, 1999). Additionally, Arden established sociomathematical norms, specifically "what counts as an acceptable mathematical explanation and justification" (Yackel & Cobb, 1996, p. 461). Overall, the tools had multiple roles. They provided information about different aspects of small-group discourse and associated norms, served as a means of documentation during lessons, and enabled more formalized reflection as well as guided conversations about and evaluations of the small-group discourse in Arden's classes (Cirillo et al., 2014).

Third, the S^{3}D Approach allowed Arden a level of autonomy, or a feeling of ownership of the process (Power & Goodnough, 2019). Arden made decisions about at what pace she moved through the lenses and phases. For instance, in the first cycle, she spent more time with the Phase 1 lenses. In comparison, in the second cycle, she felt more familiar with the tools and chose to move into Phase 2 more quickly. By using the tools and reflecting on the information about the small-group discourse, Arden was able establish goals for growth. In particular, the Discourse Quality tool indicated directions for higher quality student-to-student talk and, as previously mentioned, the Talk Moves tool provided guidance with respect to talk moves that would help students move in that direction. Arden was also reminded to consider each of the tools as a working document. In fact, she identified a student-to-student interaction (inviting peers to participate) that was not present in the Discourse Quality tool. She also expanded the meaning of some of the Group Dynamics and identified different ways to frame the Talk Moves. Regardless of the S^{3}D Approach being incremental, incorporating the tools into Arden's practice was difficult, especially early in the process. In response to this challenge, Arden considered ways to more effectively and efficiently employ the tools. In fact, she was instrumental in transitioning the tools from a paper-and-pencil format (Appendix A) to an intermediary pilot of an electronic format to the formal development of an app. Throughout the PD, Arden was actively involved in making logistical and data-based pedagogical decisions.

Fourth, the S^{3}D Approach involves repeated cycles of going through Phase 1 and then Phase 2. Each quarter, Arden created new groups, identified focus groups, progressed through the three lenses of Phase 1, and then planned and implemented talk moves in Phase 2. The present study focuses on the first two cycles. The process was quicker moving from the first to the second cycle. Arden spent the majority of the first quarter in Phase 1. In comparison, she spent the majority of the second quarter in Phase 2. In the second cycle, Arden was more familiar with the various lenses of Phase 1 and the use of the associated tools. Therefore, she was able to move quickly through Phase 1 and into Phase 2. The process and tools became integrated into her practice, evidence of the *sustainability* of the approach. Otten et al. (2022) similarly described how incremental changes "can carry forward for the teacher by becoming their new instructional 'habits'" (p. 1447).

Arden's increased familiarity with the process changed the nature of the conversations during the debrief sessions. Initial discussions centered on learning the details of the phases and lenses and the structures of and strategies for utilizing the tools. Arden also shared various challenges that she was experiencing, such as using the tools in addition to all of her other responsibilities as a teacher. Even though Arden used the tools with just two focus groups, she struggled with managing all of her groups. The incremental approach contributed to this challenge and, in the long term, its resolution. At first, Arden interacted with all of her groups and knew that they needed her support. However, she felt that, with her developing understanding, she was not able to provide the necessary and appropriate attention. Over time, as her understanding of the process developed and became part of her *instructional habits*, she was able to implement the approach with groups other than the focus groups, further supporting the *sustainability* as well as the *scaling* of the process.

During the early conversations, Arden also expressed impatience and insecurity. She wanted to see greater improvement and, when she did not, she was unsure of her implementation of the approach. In fact, on two occasions, she asked MTE, "Did I do something wrong?" and "Maybe I'm not doing something right?" (Debrief Meetings 9 and 13). Further, she sometimes felt that the tools were revealing her flaws. Through engaging in incremental PD, Arden needed to *redefine success*. She needed to be reminded that with persistence, her practice and the small-group discourse would change over time. Rather than exposing faults, the tools served as a baseline for growth and a means of seeing minor incremental change, especially with students that posed the greatest challenges.

Since Arden was engrossed in learning and enacting the approach as well as implementing her lessons, she was initially not able to see improvement. The MTE, who was observing focus groups for an entire class period with that as her only focus, did see growth in the Group Dynamics and enhancement in the Discourse Quality. The MTE shared this information with Arden, providing positive reinforcement of her work and ways to recognize the incremental change. Toward the end of the first cycle, Arden started to recognize and share improvement on her own. In the second cycle, the substance of the discussions became more focused on the intricacies of the small-group discourse based on Arden's observations and the complexities of the strategies to bring about incremental change. These findings support the hypothesis of Otten et al. (2022): "It seems plausible that when teachers enact small improvements, they may gain confidence and feelings of success, which may lead them towards the addition of more small improvements" (p. 1447). Even with the change in groups (and the introduction of new group dynamics) in the second quarter, improvement from the first quarter was *sustained*, allowing Arden to build upon the progress in the second quarter. In other words, the growth was cumulative.

The MTE played a critical role in providing guidance about the S^3D Approach, conducting weekly observations of the focus groups, and debriefing with Arden about her experiences each week. Further, the MTE encouraged Arden to persist through the incremental process, especially when she felt impatient, frustrated, and insecure. With this perseverance, Arden was eventually able to realize improvement in the small-group discourse. A major limitation of the current iteration of the S^3D Approach is that it was conducted on a small scale. Just one teacher worked one-on-one with the MTE, therefore raising questions about the *scaling* of the process. The S^3D Approach could be conducted with a larger group of teachers either led by an MTE or independent of an MTE. In the first case, MTEs (including university mathematics teacher educators, mathematics coaches, or department chairs) could facilitate the incremental PD (Kohen & Borko, 2022). Quebec Fuentes (2020) designed a Professional Development Guide for structuring and implementing a sequence of sessions. In the latter case, teachers could collaboratively engage with the process, observe each other's classes (Chen & Chan, 2022), and serve as a support through challenges. During Debrief Meeting 5, Arden explained the importance of working with other teachers: "It's really helpful when using the tools to have someone to collaborate with on it. So, maybe as another piece of advice to someone who would be using [the S^{3}D Approach] is find someone else in the school who would be willing to also do this."

The infrastructure exists for the approach to be scaled up whether led by an MTE or not. A variety of resources that meet the needs and capacities of different teachers exist. For example, Quebec Fuentes (2020) designed a PD resource, including a breakdown of the phases and lenses, examples of teachers using the approach, and prompts and worksheets to support teachers as they engage in and reflect upon the approach. However, this may be overwhelming for teachers. As an alternative, Quebec Fuentes has created a series of social media posts across various outlets (Instagram [s3dmathed], X [@s3d_math], and TikTok [@s3dmathed]). The posts include information and videos about each of the incremental aspects of the S^{3}D Approach and their implementation with the tools. The tools are available in both paper-and-pencil format (linktr.ee/s3dmathed) or embedded within an app. "The freedom to deliver specific, actionable practices via on-demand formats, such as videos, downloadable documents, or social media suggestions will allow for the scaling of those efforts by the PD creators and practitioners themselves" (Otten et al., 2022, p. 1447).

Several questions remain and can serve as the focus of future research. First, how can the S^{3}D Approach be exposed to a greater number of practitioners? In other words, with the aforementioned mechanisms in place, how does one support the expansion of the reach of the approach? Second, how is the approach being implemented independent of the MTE? Future studies can examine how communities of practice with the shared goals of fostering small-group discourse (Quebec Fuentes & Spice, 2017) work with the incremental PD.

5. Conclusions

Bishop (2021) described the implications of her study that examined the relationship between teachers' attending to student thinking and learning outcomes:

> Analyses indicate that relatively small increases of highly responsive teacher moves may have significant effects on achievement. In terms of instructional implications, these kinds of smaller, incremental changes may be more realistic for teachers, teacher educators, and researchers who hope to increase the responsiveness of classroom discourse—changing a small number of turns of talk from low to high levels of responsiveness around a preplanned task or concept might be a productive starting point for shifting classroom discourse. (p. 502)

The S^{3}D Approach is purposefully designed to support incremental change in a teacher's practice to foster small-group, student-to-student discourse. The current study demonstrates how, through a reframing of starting points and the definition of success, incremental PD supports sustainable changes in a teacher's practice and student engagement in productive mathematical discourse.

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Appendix A

	Dynamic										
	Participation				Momentum			Definition of Success			
Group	**Learned Helplessness** The group members need you, or a student that typically leads the group, to initiate work on a task. There is either no communication or off-task conversations.	**Help-Leave-Silence** You initiate communication about a task between group members. The conversation terminates after you depart.	**Own Zones** The group members individually work on a task.	**Non-participatory Student** One or two group members are not partaking in the conversation about a task between the other group members.	**Obstacle** The group members have worked on a task and ask you for help. To respond appropriately, you need to know what work has been done.	**Unsuccessful Help** Despite one group member trying to help another, the group member receiving assistance remains confused often not admitting to his or her remaining questions.	**Dominant Student** One group member dictates the conversation often minimizing or completely overlooking the ideas of the other group members.	**Rush to Complete Task** The group members' priority is to complete a task with little to no assessment of the approach and/or solution.	**Teacher as Authority** The group members perceive you as the sole person who can provide help.	**Blindly Accept Work of Others** One or more group members concur with erroneous, incomplete, or incomprehensible reasoning.	Notes
1											
2											
3											
4											
5											
6											
7											
8											

Figure A1. The Group Dynamics tool.

Question-response Pair		Groups							
Question	Response	Group 1 (Focus)	Group 2 (Focus)	Group 3	Group 4	Group 5	Group 6	Group 7	Group 8
1. *A* asks *B* a practical question	*B* answers *A*'s question								
2. *A* asks *B* a question about previously learned content	*B* answers *A*'s question								
3. *A* asks *B* to show work	*B* shows own work								
4. *A* asks *B* to explain work	*B* explains own work								
5. *A* asks *B* to evaluate work	*B* evaluates *A*'s work								
6. *A* criticizes *B*'s work	*B* justifies own work								
7. *A* rejects *B*'s justification	*B* reconstructs own work								
8. *A* suggests a strategy to the group	The group tries the strategy								

Figure A2. The Discourse Quality tool.

Group	Type of Help: Product	Type of Help: Process	Notes
1			
2			
3			
4			

Guiding Question	Indicators and Examples: Product Help	Indicators and Examples: Process Help
What is the focus of teacher questions or prompts?	Mathematics Content	Helping Students Communicate with One Another about the Mathematics
What is the communication pattern?	Teacher-Student-Teacher-Student	Teacher-Student-Student-Student
How many students are participating in the conversation?	One	All
Who is leading the conversation?	Teacher	Students
Who is doing the majority of the talking and thinking?	Teacher	Students
	Mr. Warren Yes, what's wrong? *Gina* How do you know how much to take out? That always confuses me. *Helen* I'm stuck on that one, too. *Mr. Warren* Let's look at it systematically. So, we're going to look just at the coefficients. What is the largest number that divides 12 and 15? *Gina* 3. *Mr. Warren* So, that's what you factor out. So, now look at the variables. What's the largest factor? *Gina* *x*. *Mr. Warren* Now, what's left over after you factor out 3*x*? *Gina* 4*x*. *Mr. Warren* Now, do the second term. Three times what gives you –15? *Gina* –5. *Mr. Warren* So, in parentheses, you have 4*x* – 5. Now, repeat the process for the next two terms. What's the largest number that divides both? *Gina* 2. *Mr. Warren* Now, factor out the 2. *Gina* 4*x*. *Mr. Warren* And? *Gina* –5. *Mr. Warren* Yes, and notice that 4*x* – 5 is the same. So, those are your two factors.	*Brooke* Right here, it's negative five. So, would that make it positive five? *Mr. Warren* Did you ask Paige? *Brooke* No. *Paige* Yes. It would turn positive. *Mr. Warren* Ask Zoe as well. See what she thinks. *Zoe* I agree because –5 is negative and –*b* is negative. A negative and negative makes a positive. *Brooke* I'll do that. Five plus or minus the square root of … *Paige* I just got confused at one point because I ended up with square root of one over four. *Brooke* Well, wait. Let me do it. [Brooke, Paige, and Zoe continue to discuss the problem.]

Figure A3. The Teacher Support tool.

Element/Dynamic	Description	Talk Moves	Example Talk Moves	Notes
PARTICIPATION				
Learned Helplessness	The group members need the teacher, or a student that typically leads the group, to initiate work on a task. There is either no communication or off-task conversations.	Clarify questions Redirect questions to group Direct explanations to group members Refer to other resources	Can you ask me a specific question? Teacher restates student question directed at teacher to the rest of the group. Explain that to your group members. Where can you find the answer to your question?	
Help-Leave-Silence	The teacher initiates communication about a task between group members. The conversation terminates after the teacher departs.	Leave group with a task Follow-up on progress	Each write down your explanation and then compare it with those of your groupmates. Did you compare your explanations? What are the similarities and differences?	
Own Zones	The group members are individually work on a task.	Redirect questions Individual work and then compare strategies	Could you answer her question? Explain your strategies to each other and then compare them.	
Non-participatory Student	One or two group members are not partaking in the conversation about a task between the other group members.	Explain what has been done Another student explains Restate in own words Answer another student's question	Can you explain what they were just discussing? Can you explain your strategy to him? To person being helped: Now what are you going to do to figure out …? Could you help her with her question?	

Element/Dynamic	Description	Teacher Interactions	Example Teacher Interactions	Notes
MOMEMNTUM				
Obstacle	The group members have worked on a task and ask the teacher for help. To respond appropriately, the teacher needs to know what work has been done.	Ask for explanation – Push for clarity Focus on Errors	What did you do to figure out …? Without directly identifying error, the teacher models the process of evaluating work. Do you agree with what he did? Why not? What is the difference between your strategies?	
Unsuccessful Help	Despite one group member trying to help another, the group member receiving assistance remains confused often not admitting to her remaining questions.	Restate in own words Agree with restatement	Can you explain what she is saying/doing? Do you agree with how he described your strategy/reasoning? Why or why not?	
Dominant Student	One group member dictates the conversation often minimizing or completely overlooking the ideas of the other group members.	Restate in own words Highlight overlooked idea of another student	Can you explain what they just said? How did you (ignored student) solve the problem?	
DEFINTION OF SUCCESS				
Rush to Complete Task	The group members' priority is to complete a task with little to no assessment of the approach and/or solution.	Compare strategies Evaluate work of others	Compare your answers/strategies. Can you determine which answers/strategies are correct and which are incorrect?	
Teacher as Authority	The group members perceive the teacher as the mathematical authority.	Redirect questions to group Ask student to redirect question to group Explain work to others Ask others to evaluate work	A student asks the teacher a question. The teacher restates the question to the group. Can you ask your question to them (other group members)? Explain to the group what you did and maybe they can come up with a suggestion. Do you agree with what he said? Why or why not?	
Blindly Accept Work of Others	One or more group members concur with erroneous reasoning.	Restate in own words Evaluate student's ideas	Can you explain it now in your own words? Does that explanation make sense? Why?	

Figure A4. The Talk Moves tool.

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Article

Equity-Focused, Rubric-Based Coaching: An Incremental Improvement Approach to Supporting Teachers to Shift Toward More Equitable Mathematics Instruction

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Abstract: Historically, inequities in mathematics education have resulted in mathematics classrooms that do not support all students, and particularly students from marginalized backgrounds. Efforts to transform mathematics classrooms to be culturally responsive, sustaining, and justice-oriented have met limited success at scale. It may be that supporting teachers to develop more equitable teaching practices would benefit from a more incremental improvement approach. This article considers how school-based mathematics coaches can support teachers to make incremental shifts toward more equitable instruction. We describe a coaching model designed to include elements of incremental improvement, in which coaches and teachers analyze video against a set of rubrics that delineate equitable teaching practices. Using established routines and structures, coaches and teachers work together to identify and enact small, actionable changes that build toward more ambitious equity-oriented practices. Drawing on pilot data, we articulate how the coaching model both reflects and builds on an improvement approach to professional learning. We argue that while incremental shifts may be insufficient to fully address systemic inequities, they can serve as a meaningful bridge toward larger changes. We conclude with considerations for engaging in equity-oriented incremental improvement work.

Keywords: mathematics education; equity; teacher professional development; mathematics coaching; instructional improvement; teacher learning

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1. Introduction

Mathematics classrooms can function as both supportive and marginalizing spaces for students. Supportive mathematics classrooms are equitable, responsive, and culturally and mathematically affirming and are characterized by strong relationships, high expectations and responsive teaching (Abdulrahim & Orosco, 2020; Comstock et al., 2023). Mathematically, they provide students with access to rigorous mathematics, support students' mathematical thinking, and leverage and affirm students' cultural and mathematical identities (Comstock et al., 2023; Gutiérrez, 2012; Ladson-Billings, 1997). Yet evidence suggests that classrooms such as these are not the norm, especially for Black and Latine students (Boston & Wilhelm, 2017). Students from minoritized racial and ethnic backgrounds experience weaker teacher–student relationships (Bottiani et al., 2016) as well as lowered expectations, reduced cognitive challenge, and teachers' racial bias in instructional decision-making (Boston & Wilhelm, 2017; Coppersmith et al., 2024; Harber et al., 2012; TNTP, 2018).

Even when mathematics instruction incorporates ambitious teaching practices, it is not always equitable (Jackson & Cobb, 2010).

Calls for more equitable mathematics education have emphasized how teachers might enact culturally relevant or sustaining teaching practices or integrate math with social justice (Comstock et al., 2023; Kokka, 2019). Classrooms in which historically marginalized students excel affirm students' cultural and mathematical identities, make expectations for mathematical activity both high and explicit, and provide mathematical and social support (Wilson et al., 2019). However, evidence suggests that this type of teaching represents a significant departure from current practice and mathematics teachers may need support to deepen their instruction along these lines (Hill et al., 2018; Wilson et al., 2019). While impediments to adoption may include accountability pressures or other contextual factors, such teaching may also require large shifts in teachers' dispositions and beliefs, as well as their skills and knowledge.

One way to support teachers to enact more equitable mathematics teaching practices may be to take a more incremental approach to improving instruction (Litke, 2020a; Otten et al., 2022; Star, 2016). Incremental approaches target smaller shifts in practice that build and bridge to more ambitious changes. Incremental improvement approaches begin with teachers' current practice, incorporate modest and actionable improvement goals, consider teachers' classroom context, focus on specific teaching practices that may not be widely enacted, and advocate for gradual changes. In theory, it is the accumulation of these smaller changes that work to shift practice. In the context of equitable mathematics teaching, such incremental changes may be insufficient for disrupting long-standing inequities in mathematics education (e.g., Martin, 2019). However, while the field must continue to push for culturally responsive and sustaining mathematics teaching, given evidence that such pedagogies are not widely enacted (Comstock et al., 2023), we argue that *in addition*, we can support teachers to make smaller shifts that move practice toward more equitable teaching.

Instructional coaching may be particularly suited to incremental improvement; coaches work directly and regularly with teachers, have content and contextual knowledge, and can provide targeted support around finer-grained practices in ways that are grounded in teachers' current practice and context. Individualized and sustained coaching around improving aspects of mathematics instruction can positively impact both instructional quality and student achievement (Campbell & Malkus, 2011; Kraft et al., 2018; Kraft & Hill, 2020). Reviews of equity-focused professional learning suggest that these efforts may lack instructional or subject-matter specificity (Bottiani et al., 2016; Parkhouse et al., 2019); coaching around mathematics-focused equitable teaching practices may address these issues and thus allow teachers to more easily translate what they learn into practice.

Our goal in this article is to illustrate how one model of mathematics coaching focused on more equitable mathematics teaching reflects an incremental approach. In doing so, we consider the ways in which lessons from the incremental change literature can inform program design and highlight how the model's enactment expands on and provides specificity to elements of incremental change. We describe Honoring Equity and Access: Rubrics for Mathematics Instruction (HEAR-MI) Coaching, a coaching model leverages video analysis using the Equity and Access Rubrics for Mathematics Instruction (EAR-MI), a set of empirically grounded classroom observation rubrics (Wilson, 2022) alongside coaching routines and structures. In HEAR-MI Coaching, coaches and teachers work collaboratively through cycles of goal setting, video analysis, and reflection. The model (a) engages teachers in setting their own improvement goals, (b) introduces teachers to research-aligned practices that serve as a shared language and external standard, (c) includes routines and structures designed to support teacher learning of specific aspects of equitable teaching practices, and (d) allows teachers to build on current practice to integrate this new learning through smaller adjustments.

In what follows, we first describe our conceptualization of incremental improvement approaches, identify the teaching practices underlying HEAR-MI Coaching, and outline how coaching can support incremental improvement efforts. Next, we describe the model itself. We draw on the reflections of teachers who participated in a pilot study of HEAR-MI Coaching to delineate the ways in which the model's components reflect incremental improvement and to further our understanding of such approaches. By articulating how HEAR-MI Coaching both builds from and contributes to the incremental improvement literature, we seek to articulate design elements that support others engaged in this work. We conclude with a discussion of how incremental improvement approaches can support teacher learning around equitable teaching practices and raise considerations for the field around engaging in equity-oriented incremental improvement.

2. Perspectives

2.1. Features of Incremental Approaches to Improving Mathematics Teaching

Despite long-standing calls for mathematics instruction to shift toward student-centered, ambitious, and conceptually oriented approaches, large-scale investigations into teaching practices have shown that teacher-directed, didactic, and procedurally focused instruction persists (Hiebert et al., 2005; Kane & Staiger, 2012; Litke, 2020b; Wilson et al., 2019). This is particularly true in classrooms serving large percentages of minoritized students (Boston & Wilhelm, 2017). Professional development efforts have shown limited success at supporting teachers to make meaningful and lasting shifts toward ambitious teaching, with teachers often grafting reform ideas onto existing practice, limiting positive impacts for students (Cohen, 1990; Otten et al., 2022).

As a result, scholars have begun to consider whether professional learning might more effectively support teachers to make incremental, rather than wholesale, changes to instruction (Janssen et al., 2015; Litke, 2020a; Otten et al., 2022; Star, 2016). Key considerations for designing such professional learning include determining both the focus of the professional learning (e.g., what instructional practices could be best leveraged through an incremental approach) and the activities that support teachers to build on their practice in incremental ways. To achieve this, it is necessary to understand the principles behind incremental improvement approaches.

One key feature of incremental improvement approaches is that they center smaller, finer-grained practices that support student learning, but may not be widely enacted or may not be enacted deeply (Star, 2016). These practices can be embedded in existing instructional modalities (Litke, 2020a; Star, 2016), facilitating uptake. For example, Litke (2020a) identified specific teaching practices that support students' learning opportunities in algebra (e.g., supporting procedural flexibility or connecting across mathematical representations), demonstrating how they might be leveraged to promote more student voice and discussion in teacher-centered classrooms. A focus on finer grained features of instruction can "open a door within teachers' practice for more ambitious instructional changes in the future" (Star, 2016, p. 59). Practices that are likely to be taken up by teachers are congruent with existing practice, include clear articulations of teacher actions, and are those in which teachers see value and benefit (de Araujo et al., 2022; Janssen et al., 2015). We suggest that a focus on fine-grained, high-uptake practices that support more equitable mathematics classrooms may serve as a bridge toward more "ambitious" equitable teaching practices, allowing teachers to build their skills and knowledge for engaging in this work.

A second feature of incremental improvement approaches is a focus on small adjustments rather than wholesale changes to instruction (Otten et al., 2022; Star, 2016). For example, some advocate for small suggestions or nudges (Otten et al., 2022), providing bite-size changes that are easy to take up and integrate into existing practice. A focus on smaller changes can support teachers to see improvement efforts as practical and actionable (de Araujo et al., 2022; Janssen et al., 2013), increasing the likelihood they are adopted. The accumulation of multiple smaller shifts can also serve as a bridge from current to sought-after practices (Janssen et al., 2013; Litke, 2020a; Otten et al., 2022).

Finally, such approaches are grounded in teachers' current practice. Incremental approaches that reflect more modest (and realistic) goals are attuned to teachers' contexts and constraints (Otten et al., 2022). For example, in higher education settings, rather than asking instructors to abandon traditional lecture for active learning, professional development focused on "active lecture" reflect less of a departure from current practices and may support instructors to develop skills and dispositions needed to shift toward more student-centered approaches (He, 2021). Furthermore, adapting or building on existing practice through incremental steps meets teachers where they are, supporting buy-in and enactment (Davis et al., 2016; Janssen et al., 2015; Litke, 2020a; Otten et al., 2022). Given evidence that mathematics classrooms—and particularly those with high percentages of students from historically marginalized backgrounds—may not widely feature equitable teaching practices (e.g., Comstock et al., 2023; Wilhelm et al., 2023), understanding and building on teachers' extant practice with smaller adjustments focused on fine-grained teaching practices may be a promising approach.

2.2. Descriptions of Equity-Focused Instructional Practice in Mathematics

Mathematics education research has focused on supporting teachers to enact ambitious teaching practices (Lampert et al., 2010, 2013). However, while ambitious teaching can support equitable outcomes and opportunities (Kang, 2022), it can also be enacted in ways that reinforce or worsen inequities (Nasir et al., 2008). Researchers focused specifically on equitable teaching have identified teaching principles that support students who have been historically marginalized in school. These include building students' academic and social-emotional skills, affirming students' social and cultural histories, and preparing students to recognize, analyze, and address inequality. In such classrooms, teachers hold high expectations for students, express commitment to students' learning, and demonstrate respect for students' knowledge and agency (Ladson-Billings, 1995; Milner, 2011; Siddle Walker, 1993; Tate, 1995; Ware, 2006). In mathematics, such teaching incorporates a greater focus on students' cultural backgrounds, identities, and funds of knowledge, supports students to use mathematics to address and respond to social injustice, and encourages students to question whose mathematics matters (Abdulrahim & Orosco, 2020). However, evidence suggests that such critical or culturally sustaining pedagogies are not the norm in mathematics classrooms, are challenging for teachers to enact, and come with barriers to implementation (Comstock et al., 2023; Cruz et al., 2020).

It may be fruitful to identify finer-grained teaching practices that can, through smaller adjustments, build on teachers' existing practice and potentially bridge to the more critical practices described above. To achieve this, we draw on a set of mathematics-focused teaching practices that research has shown to support equitable outcomes for historically marginalized students (Wilson et al., 2019). Drawing on data from the Middle School Mathematics and the Institutional Setting of Teaching study, researchers identified classrooms that successfully supported African American students' mathematics achievement. Analyzing video recordings of mathematics lessons and scores from the Instructional Quality Assessment, the authors held constant conceptually oriented teaching and identified equity-focused instructional practices that characterized these classrooms. Notably, these practices were not as present in conceptually oriented classrooms in which African

American students did not thrive. While the full set of practices are described in depth in Wilson et al. (2019), for the purposes of this article, we focus on a sub-set of these practices (see Figure 1) that formed the basis of our coaching model.

EAR-MI Practice	Practice Description
Proactively Building Relationships and Productive Classroom Culture	Teacher builds and/or maintains a productive and nurturing learning environment by attempting to connect with students and/or by reinforcing classroom values.
Reactively Responding to Apparent Off-Task Behavior	Teacher builds and/or maintains a productive learning environment by responding to what appears to be "off-task student behavior" by determining students' needs, attempting to provide help, and reengaging students in the math task.
Positioning Students as Competent	Teacher makes explicit statements that students are capable of participating in mathematical activity and/or are making important contributions to the learning community by offering affirmations specifying what students did that was productive and providing a rationale emphasizing specific productive practices.
Explicit Mathematical Expectations	Teacher provides clear statements about what is expected of students *mathematically* by providing clear instructions, an image of productive participation, and a rationale that indicates why these expectations are important.
Explicit Social Expectations	Teacher provides clear statements about what is expected of students *socially* (how they should work together) by providing clear instructions, an image of productive co-participation, and a rationale that indicates why these expectations are important.
Attributing Responsibility to Students in Response to Student Requests for Assistance	Teacher shifts mathematical authority back to students when they ask for assistance by "pushing back" in response to students' requests for help and providing additional support, encouraging students to rely on resources, peers, or themselves.

Figure 1. Description of selected teaching practices that promote access and equity in mathematics classrooms (Wilson et al., 2019).

In these more equitable mathematics classrooms, teachers attended to classroom community, explicitly focusing on building and maintaining productive learning environments (Wilson et al., 2019). Teachers achieved this by *proactively building relationships and productive classroom culture* through connecting with students and reinforcing classroom values. Teachers also attended to classroom environment in how they *reactively responded to apparent off-task behavior*, leading with curiosity and determining students' needs while also re-engaging students in mathematics. In addition, teachers *positioned students as competent*, making public statements that indicated that they valued students' mathematical thinking and contributions to the learning community. Rather than making blanket statements about the class's ability level or competence, teachers carefully and intentionally identified individual students' contributions, noting what about that contribution was mathematically important and making visible to the class how the contribution supported mathematical learning. Teachers also set clear, *explicit expectations* for students' social and mathematical behavior. In doing so, teachers carefully articulated what they expected of students mathematically in relationship to a mathematical task and what they expected socially in terms of how they expected students to work together on these tasks. Teachers thus made visible the often-unwritten rules of the mathematics classroom. Finally, these teachers *attributed mathematical responsibility to students in response to requests for assistance*, transitioning authority for mathematical problem-solving and decision-making away from themselves and onto students. Specifically, teachers pushed back on students' requests for help, directing them to consult with peers or leverage other resources at their disposal (e.g., their notes, anchor charts, etc.). In doing so, they explicitly encouraged students to persist in solving problems and normalized productive mathematical struggle as part of the learning process, fostering student thinking and independence.

The practices described above reflect only a subset of those that the field has suggested comprise equitable mathematics teaching. We do not argue that these are the only—or even the most effective—equitable teaching practices in mathematics, but rather highlight them because they are a relatively small grain-size, have been shown to support student learning and success, and lend themselves to the smaller adjustments to practice advocated for in incremental approaches. In addition, these practices have been developed into a classroom observation tool, the EAR-MI (Wilson, 2022), which provides guidance for what "good, better, and best" enactments might entail. Observation rubrics have been used in coaching models to support teachers to shift practice (Kraft & Hill, 2020) and the EAR-MI is a central component of HEAR-MI Coaching.

2.3. Coaching as a Lever for Incremental Improvement

Instructional coaching is a form of job-embedded professional learning that consists of regular contact between teachers and coaches knowledgeable in content and pedagogy for the purpose of improving instruction (Mangin & Dunsmore, 2015). We see mathematics coaching as a particularly promising mechanism for incremental improvement approaches to professional learning. Most U.S. schools have access to an instructional coach (National Center for Education Statistics, 2020), making it possible to integrate coaching into the regular work of teachers in schools. Furthermore, mathematics coaching has the potential to support teachers to enact equitable teaching practices through coaches' situated knowledge of school contexts and students' backgrounds and strengths (Marshall & Buenrostro, 2021).

Coaching activities vary in form and structure (Campbell & Malkus, 2011). In mathematics, individualized, intensive, and sustained coaching work with teachers has shown positive benefits on both instructional practice (Kraft et al., 2018) and student achievement (Campbell & Malkus, 2011). Specifically, coaching that centers in-depth, focused conversations about mathematics content, pedagogical practice, and student learning can support teachers' growth and development (Russell et al., 2020). Effective coaching conversations feature specificity, depth, and a connection between teacher, student, and mathematical content (Russell et al., 2020). Gibbons and Cobb (2016) highlight the importance of goal setting, analysis of current practice within a trajectory of pedagogical development, and the co-construction of action steps for improvement.

One increasingly common activity in mathematics coaching is to ground coaching conversations in the analysis of video recordings of classroom instruction. Video serves as an artifact of classroom instruction, can be paused, rewatched, and decomposed, and clips can be selected to focus on a particular practice or purpose (Borko et al., 2008). Video analysis can foster productive discussions focused on teaching and learning. Video has been used in professional learning as a tool to support teachers' professional noticing (Amador et al., 2023), facilitating teachers to identify and reflect on critical instructional moments (van Es & Sherin, 2021) and attend to student thinking (Walkoe, 2015). Studies of coaching models that incorporate video analysis find that such activities can support teachers to notice both positive enactments of and opportunities for growth around specific instructional practices (Suh et al., 2021). When teachers analyze video from their own classroom, they can situate pedagogical reflection in their own context, supporting a close connection to practice. However, some teachers may be hesitant to engage in this self-reflection; video analysis of instruction from unknown teachers can promote participation and engagement in instructional improvement conversations (Beisiegel et al., 2018). Video from unknown teachers may de-personalize coaching conversations, allowing coaches or facilitators to foreground specific aspects of practice for discussion.

Some coaching models use classroom observation rubrics as a lens through which to engage in video analysis (e.g., Beisiegel et al., 2018; Kraft & Hill, 2020). Grounding video analysis in an observation rubric can support teachers to both learn aspects of the effective teaching practices highlighted by the tool and strengthen enactment of these practices (Kraft & Hill, 2020).

2.4. The Current Paper

In this article, we describe HEAR-MI Coaching, a model that was developed as part of a National Science Foundation-funded project with the goal of supporting middle school teachers to enact more equitable mathematics instruction. HEAR-MI Coaching integrates conversational routines and activities from an effective, video-based coaching model (Kraft & Hill, 2020) with a set of empirically developed, validated classroom observation rubrics focused on equitable mathematics teaching practices (EAR-MI; Wilson, 2022). The theory of action for HEAR-MI Coaching posits that if teachers work directly with coaches to learn to analyze instruction using EAR-MI, they will deepen their instruction in ways that align with EAR-MI practices and better support students' learning and success, particularly for those students who have been historically underserved in mathematics classrooms. We designed HEAR-MI Coaching to take an incremental approach, using video analysis to ground coaching conversations in teachers' existing practice and leveraging the EAR-MI to support teachers to make small, manageable shifts to their teaching. Drawing from teachers' experiences in the pilot year of HEAR-MI Coaching, we seek to illustrate features of the model that both reflect and build on elements of incremental improvement approaches. Specifically, we ask the following question: *How does an equity-focused coaching model instantiate and elaborate guidance about incremental improvement?*

3. Materials and Methods

We piloted HEAR-MI Coaching with four mathematics coaches from four different schools in a very large, urban school district in the western part of the U.S. The district serves students who are majority Hispanic or Latine, and the participating schools largely mirrored the district demographics. Participating coaches worked full-time in middle schools, supporting teachers around mathematics instruction specifically. To recruit the participating coaches, the research team collaborated with an external organization that provided support to district schools. The coaches had 4–8 years of coaching experience, and all had taught mathematics prior to becoming coaches. Each coach selected 2–3 teachers in their school to pilot HEAR-MI Coaching (10 teachers total). Participating teachers taught middle school math (6th, 7th, or 8th grade) and ranged from novice (first year teacher) to experienced (8 years).

In the summer prior to the pilot year, coaches participated in a four-day training session in which project staff supported coaches to learn the EAR-MI, the model, and the coaching routines. In this training, coaches used EAR-MI to engage in video analysis of video clips from our project library. They learned the HEAR-MI Coaching cycle (described below) and engaged in role plays of coaching conversations. Through these activities, they participated in discussions around issues that might arise during HEAR-MI Coaching. During the school year, coaches and teachers worked together to enact multiple coaching cycles. Coaches received ongoing support from project staff through monthly webinars and one-on-one check-ins as needed. Webinars included analysis of coaching conversations, clarification around EAR-MI rubrics, and discussions of common issues that arose during coaching.

One goal of the pilot was to understand how coaches and teachers perceived and experienced HEAR-MI Coaching. We conducted two 45–60 min semi-structured interviews with all participating teachers at the beginning and end of the school year. End of year interviews asked teachers to reflect on their experiences with HEAR-MI Coaching, with the specific EAR-MI rubric they had selected as a focus for coaching, and how HEAR-MI Coaching influenced their thinking about their teaching. Data analysis is ongoing; however, we draw on teacher interviews to illustrate the ways in which the model takes an incremental approach to improving teaching and to identify additional elements that support such an approach. To achieve this, we mapped the elements of HEAR-MI Coaching to the features of incremental improvement. We then analyzed interview transcripts, identifying when teachers themselves noted incremental improvement elements in the coaching model and identifying additional aspects that could inform incremental improvement. In what follows, we first describe the HEAR-MI Coaching model as it was designed. Next, we draw on the interview data to illustrate how HEAR-MI Coaching both reflects an incremental approach to supporting teacher learning and adds to our understanding of what incremental improvement approaches may entail.

4. Findings

4.1. The HEAR-MI Coaching Model: Designed for Incrementnal Improvement

In HEAR-MI Coaching, teachers work with instructional coaches knowledgeable about equity and instruction. Prior to beginning coaching cycles, the teacher and coach have an introductory meeting in which the coach shares an overview of each of the EAR-MI rubrics and the teacher articulates a problem of practice or improvement goal of interest to them. Working together, the teacher selects one EAR-MI rubric that they perceive to be aligned with their improvement goals as the focus of the first coaching cycle. For example, a teacher may share with their coach that one of their goals is for their students to become more independent and less reliant on the teacher for guidance. Through discussion, the coach and teacher may collaboratively decide to work with *Attributing Responsibility to Students in Response to Student Requests for Assistance*, an EAR-MI rubric that focuses on supporting teachers to shift mathematical authority and agency onto students, while holding students accountable for their learning. Coaches and teachers then follow a five-step cyclical process of analysis and reflection (see Figure 2). We describe this process in detail below.

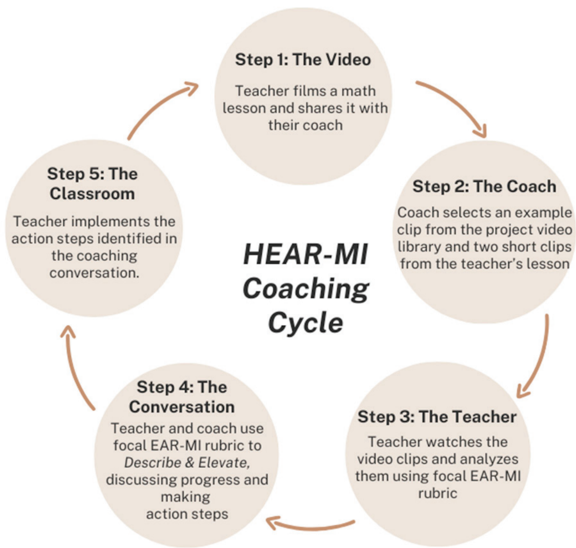

Figure 2. Five Steps of the HEAR-MI Coaching Cycle.

First, the teacher video records a lesson from their own classroom and shares the recording with their coach (Step 1). The coach reviews the lesson video recorded by the teacher and selects one to two short video clips from that lesson to use in the coaching conversation (Step 2). In addition, the coach selects one to two video clips from the project's video library that support analysis and reflection around the focal EAR-MI rubric. The use of clips from our video library allows coaches and teachers to deepen their understanding of teaching practices, while video clips from the teachers' own classroom allow for a direct connection between video analysis and teacher's context and practice.

Next, the teacher watches the selected clips, considering each in light of the focal EAR-MI rubric (Step 3). The coach and teacher engage in a coaching conversation using a coaching routine we call *describe and elevate* (Step 4). The purpose of this two-part routine is to support the teacher to better understand and operationalize EAR-MI practices and calibrate their analysis of teaching to the rubric standards. The routine provides structure to discussions of teaching, encourages self-reflection, and seeds actionable steps for the teacher's own practice related to their improvement goal. The coach and teacher enact the routine first using video clips from the project's video library. This both depersonalizes the instructional analysis and allows the teacher an opportunity to better familiarize themselves with the components of the EAR-MI practice before turning attention to video from their own classroom. After analyzing one to two clips from our project's video library, the coach and teacher repeat the process with video clips from the teacher's classroom.

During the *describe* portion of the routine, the teacher and coach watch and discuss the video clip through the lens of the focal EAR-MI rubric. They do this by identifying instructional moments in the clip that feature instantiations of the focal practice, using the rubric's indicators and evidence from the video transcript to guide the discussion and support their analysis. Specifically, the coach and teacher draw on the rubric's definition and anchoring questions to identify and mark transcript excerpts that illustrate opportunities for the focal EAR-MI practice. Then, they use the rubric indicators to consider where the clip might fall along a continuum for the given practice, focusing on finer grained aspects of the practice associated with varying levels of enactment.

For example, in a coaching conversation focused on the *Positioning Students as Competent*, the coach and teacher might first consider how the rubric defines the practice (e.g., providing a specific affirmation for what student did mathematically and providing a rationale for why the contribution was valuable). The coached teacher might next identify instances in which the teacher in the video marks a student's contribution as contributing to the learning community. The coach might then highlight that portion of the lesson transcript. Drawing on the rubric, the coach may ask the teacher to consider the extent to which the video clip showed evidence that the highlighted affirmation was specific and included a clear rationale for why the student's contribution was productive. The coach and teacher might then describe the depth which this practice was enacted in the video, using indicators from the rubric. For example, they might consider whether the teacher simply praised a student contribution with a statement such as, "Good job" (lower-level enactment) or whether the enactment moved beyond marking to provide specificity about *what* was productive about the student contribution and/or a rationale for *why* the contribution was productive and connects to disciplinary practices in mathematics (higher level enactment).

The conversation then shifts to the *elevate* portion of the routine, in which the coach and teacher discuss ways to deepen the highlighted transcript interactions, grounded in the rubric indicators. This discussion focuses on how the instruction in the clip might, through small adjustments related to rubric indicators, shift toward more in-depth enactments of the practice. This portion of the conversation is framed around the idea that there is no such thing as "perfect" instruction, acknowledging that teaching is a continual process of reflection and improvement. Thus, the focus is not on improving the teacher's "score" on the rubric, but rather on specific, actionable elevations that improve learning opportunities for students. Importantly, the coach and teacher do not revamp the lesson entirely, rather they begin from the instruction *as is*, revising and adding on to teacher and student interactions.

As part of this discussion, the coach and teacher collaboratively mark the transcript, inserting additions, alternative phrasing, or other possibilities as they discuss them. For example, in the elevate portion of a discussion focusing on the *Positioning* rubric, the coach and teacher might look at a transcript segment in which the teacher marked a student's contribution with a broad affirmation such as, "Good job with the triangles" and consider what the teacher might add to this affirmation to provide specificity and a rationale (e.g., "Good job—I like how you broke the rectangle in half to show that it was made of two triangles as a way to prove that the area of the triangle was half the area of the rectangle. That kind of modeling is important because it allows us to make sense of formulas").

Finally, after analyzing the video clips and engaging in two to three *describe and elevate* discussions, the teacher reflects on the conversation, identifying specific goals for their next cycle and developing action steps for their own teaching (Step 5). These action steps are determined through discussion between the teacher and the coach, and the teacher commits to a plan for their enactment. Examples of goals and action steps include intentional planning for ways to surface the focal EAR-MI practice, goals related to a particular part of the lesson (e.g., enacting a practice during the launch of a mathematical task or the whole group discussion following work on a task), or goals related to specific aspects of the focal rubric (e.g., providing specificity or a rationale). The action steps also serve to seed a new coaching cycle, with the teacher planning to record a new video for the next discussion.

Several aspects of HEAR-MI Coaching were designed to align with an incremental improvement approach to professional learning. First, the model focuses on specific, research-backed teaching practices that do not feature widely in classrooms. The EAR-MI rubrics support coaches and teachers to decompose the practices into their component parts. As such, they provide specificity for instructional shifts, provide coaches and teachers with common language, and calibrate discussions of teaching to an external standard. Selecting a single EAR-MI rubric for a coaching cycle allows teachers and coaches to focus their instructional improvement work around deepening one practice at a time. Second, the model was designed to emphasize small adjustments to practice. Through the *describe and elevate* routine, coaches and teachers annotate the transcript with additions and suggestions but do not redesign the lesson. Similarly, teachers' goal setting and action steps are focused on smaller adjustments rather than wholesale reform. The coaching conversations are grounded in moving from "good" to "better" rather than targeting a "best" level of practice. Finally, HEAR-MI Coaching is designed to build from and deepen teachers' current practice. Teachers begin by identifying an improvement goal stemming from their practice. By integrating video clips from teachers' own classrooms into the *describe and elevate* routine, instructional improvement conversations are grounded in teachers' existing contexts. Similarly, teachers articulate action steps for their own planning and instruction based on the coaching conversations.

4.2. Teachers' Experiences with HEAR-MI Coaching: Developing Our Understanding of Incremental Improvement Approaches

We were interested in what aspects of HEAR-MI Coaching participating teachers identified as facilitating incremental changes to teaching practice. In interviews, we asked teachers to describe how HEAR-MI Coaching supported them to reflect on and improve their teaching. When teachers' responses identified specific features of incremental improvement (e.g., fine-grained practices, small adjustments, building from current practice), we sought to understand what aspects of the coaching model they found salient. Our goal in doing so was to build our understanding of design elements that can support incremental improvement. Teachers highlighted how grounding coaching conversations in the EAR-MI facilitated them to build on their current practice. By decomposing the focal practices into component parts, the rubrics provided specificity that made it possible for them to enact concrete instructional changes. Teachers also noted how the use of structured coaching conversations—specifically the analysis of video clips using the *describe and elevate* routine—supported them to make small adjustments to their own teaching. Finally, teachers emphasized how choice both in setting goals and in rubric selection, allowed the coaching to connect to and build upon their current context and practice.

In designing HEAR-MI Coaching, we centered EAR-MI because the rubrics reflect research-aligned equitable teaching practices. The practices included in the rubrics were also of an appropriate grain-size for a focus on smaller adjustments to practice rather than wholesale reform. Interviews with teachers suggest that the use of rubrics and their structure were an important feature of the model. Teacher participants noted that the rubrics provided a clear structure and guidance for instructional change, allowing them to make connections between their current approaches and clearly articulated adjustments they might make. Furthermore, the rubrics supported them to see these as practical and actionable. For example, one teacher stated, "seeing like examples that were provided in the rubric was kind of like, oh, okay, these are some things that I've—I'm already doing, but I could enhance it by adding more and enhance it by providing that rationale".

In addition, participants noted that the EAR-MI rubrics supported shifts in teaching practice because they decomposed the focal teaching practices into their component parts and provided concrete guidance for enactment. For example, one teacher noted the specificity the rubrics provided around aspects of the teaching practices, sharing, "the rubrics [are helpful] because they, because of the examples of like what the teacher would say on a given level, I think all the times many rubrics are like, 'teacher is supposed to give clear instructions', but what is the clear instructions going to sound like?" Another teacher reflected on how understanding the specific aspects of the practice supported them to integrate the practice into their teaching:

> When I was doing *Math Expectations*, there was a third part that talked about–what was it? I think it was like—[Interviewer: Rationale?] Right, right. Telling kids why that is important. A lot of the times when you when—when I'm teaching, I tend to forget that part, right? And that's super important for the kids to know. So, so again, that rooted me back, like gave me that focus and that goal to try to meet it as much as I could. So that's good. I think I did it—I did it, and I and I felt like—I felt that I—it was successful, and–and because I had that time to talk about it and, and practice it and hear my coach's like feedback, I—I feel—I feel better now and more confident that I'm gonna use it next year more naturally, right, than I did this year.

In describing what they found useful about working their focal rubric (*Math Expectations*) one teacher described how the specific indicators of the practice (providing clear instructions, providing an image of productive participation, and providing a rationale for why the expectations are important) influenced an awareness that led to changes in their own practice:

> I go day in, day out, teach these lessons, but I don't always know if the students know, like what the punchline or what the goal of the day was, so the rubric, especially for the clear instructions, I think it made me very much aware that I need to at some point tell the students like why it's important. . . So, being able to give them that at one point during our lesson, and then being able to give them those clear instructions and solidify everything at the end like this is what we did today, this is how it's going to connect to tomorrow, or this is how it connected to yesterday, so that they get kinda like that flow going and seeing how every

> single lesson is not just, I'm going to learn it and forget it, because I got tested on it—that they are going to come back, and you know they all connect to each other.

A different teacher described how decomposing *Math Expectations* into its component parts and then discussing these with their coach supported them to reflect on and enhance existing practice:

> Usually, I'm able to state the clear expectation of what I want them to do, and then, when it comes to the different ways of showing example or modeling of successful work, she gives me a lot of ideas on how I can enhance, maybe what I'm already doing. Let's say I'm modeling, she could enhance it even more by showing like an anchor chart, or pointing to where the resource are, and just like simple things like that, but providing them with that choice of access point, and then the rationale, the rationale is where I struggle with the most, and being able to talk about it through every translation with her during the pre—in the beginning of the cycle, was really helpful.

Another teacher reflected that the small, meaningful changes suggested through aspects of one rubric practice supported her to consider incorporating other ways to make her instruction more equitable:

> So, I actually had talked to my coach about, possibly like, now that we've talked about the clear instructions, maybe starting to think about where else we can include one of the other rubrics here on out. . . So now it's like, how can you make [lessons] better each year? And how can you make those small changes to make the lessons just more impactful for the students?"

In this way, seeing the impact of small shifts from working on one of the rubrics opened up the possibility for additional changes tied to other rubrics.

Another element of HEAR-MI Coaching that supported incremental improvement was the use of predictable, structured routines in coaching conversations. The *describe and elevate* routine was designed to provide structure and accountability, focusing coaching conversations around specific teaching practices rather than allowing them to drift to other topics. Teachers identified aspects of the routine itself as facilitating adjustments to their teaching practice. For example, one teacher reflected on how video analysis—and specifically the use of the rubric and transcript annotations—supported her to envision small adjustments to her own practice:

> That's when it started getting a little, I guess interesting, seeing once we started the actual conversation, how I would place things specifically for the evidence. Like for the evidence I found myself kind of all over the place the first time, not really knowing well does it count for this? . . . Where would I place it? But it also made me aware of, like, you know when we—we added, like certain dialogue that you could have changed, like if we could have gone back and edit ourselves, how just one sentence could have made a huge difference in the rubric. . . I was able to comment directly on the transcript and then we went over that together. One of the things that I really liked is seeing how we could have edited the transcript, or essentially what I could have changed to get that to the next level. So, I really did like that because I felt like it made me realize this—how small changes could really essentially make the lesson better—like a lot better.

A different teacher reflected that using the rubrics to analyze video from our project library facilitated connections to their current practice, noting that "we'd look at like what they [the teacher in the video] did well, what could have been improved, and then see how that maybe connects to what I'm currently doing in the classroom". For these teachers, using the transcript alongside the rubric during the *elevate* portion of the discussion was key to understanding how to deepen the rubric practice.

A key feature of incremental improvement approaches is that they build from teachers' current practice. HEAR-MI Coaching was designed to incorporate self-directed learning and teachers identified choice and agency around goal-setting and rubric selection as supportive features for this building work. As part of the model, teachers determine their own goal for each coaching cycle and, with support from their coach, select a focal EAR-MI rubric that they feel aligns with their goals. Teachers described the selection of the focal rubric as a collaborative conversation with their coach rooted in their broader goals for their teaching. For example, according to one teacher,

> It was a conversation. We kind of sat down, she had me read through all of them [the EAR-MI rubrics] and think of my own teaching and kind of like just my goals for the year. One of my goals is always just to talk less and have the students do more and so that just was aligned to that rubric [that I chose].

Another teacher aligned their choice of rubric to both their goals for their students and to their school's broader goals:

> I did have a very large [English Language Learner] group. . . So, I wanted to know how I could just—I wanted this, you know, feel for how I can easily make [math] accessible to the students. . . So [my coach]—she agreed that that was something that we needed to do. Our school is moving to more of a student-centered environment either way, so we felt that [*Math Expectations*] was one that aligned with our schools goals as well as my own.

The choice and agency afforded teachers to direct the focus of the coaching cycles allowed them to take ownership of the process. For example, one teacher reflected on how this feature supported buy-in:

> [My coach] showed me the rubrics, and she said, you know, depending on—like depending on your own goals as a teacher, here's some rubrics that we can look at and work on and so it was mainly my choice. So I got to choose the rubrics and the times that we switched rubrics was because I felt that—that I was already strong in that. . . I really like that she—she allowed me—she gave me a lot of choice, and because she gave me a lot of choice, I was more dedicated to meeting the rubric.

For this teacher, the ability to select rubrics, both initially and throughout the year, supported their enactment of the EAR-MI practices in their own teaching. Because the model allowed teachers to focus on one practice at a time, teachers were also able to determine when they felt ready to move on to another focal practice.

5. Discussion

Despite calls for mathematics instruction to be more ambitious, student-centered, and meaningful for students, large shifts in teaching practice have not taken root (Hiebert et al., 2005; Litke, 2020b). Similarly, we see minimal evidence that mathematics classrooms at scale support and affirm students from historically marginalized backgrounds and include equitable teaching practices (Boston & Wilhelm, 2017; Wilson et al., 2019). Professional learning that aims to shift teaching in ways that reflect large shifts from current practice may encounter challenges if teachers do not see those changes as practical, congruent with their context, or when perceived benefits outweigh the costs of reform efforts (Janssen et al., 2015). In contrast, incremental approaches to instructional improvement begin with current practice and seek to bridge toward larger shifts through the accumulation of smaller, meaningful changes to finer-grained aspects of teaching (Janssen et al., 2015;

Litke, 2020a; Otten et al., 2022; Star, 2016). In this article, we shared how one professional learning model—HEAR-MI Coaching—was designed to encompass these features. Through understanding teachers' experiences with the coaching model, we also identified additional features that supported teachers to shift their teaching toward practices that reflect more equitable mathematics classrooms.

Through its use of the EAR-MI, HEAR-MI Coaching focuses on a sub-set of research-aligned, equitable teaching practices (e.g., considering how to respond to students' requests for assistance in ways that support mathematical agency) that are at a grain size suitable for small adjustments (Star, 2016). More ambitious or critical conceptualizations of equitable teaching (e.g., lessons that integrate math with social justice) may still be an end goal; however, they require extensive planning and knowledge to implement. Teaching practices such as those delineated by the EAR-MI may be less dependent on teachers developing specific critical or cultural knowledge or understanding, can be integrated into existing curricular expectations, and may require less buy-in from administration or district leaders. As such, they may have lower barriers to enactment and may be more immediately actionable.

While incremental improvement approaches to teacher professional learning may not always center frameworks such as EAR-MI, our experience suggests benefits to providing teachers with clear descriptors of teaching practices, delineating aspects of those practices, and providing guidance on enactment. Teachers who participated in HEAR-MI Coaching identified that the rubrics provided a level of specificity that supported them to enact small changes to their instruction. They highlighted how the rubrics' descriptors of the components of each practice provided concrete guidance that facilitated uptake (de Araujo et al., 2022; Janssen et al., 2015). Furthermore, by articulating "good, better, and best" indicators of each practice (Kraft & Hill, 2020; Wilson et al., 2019), the rubrics also provided teachers with an image of successful enactment. Rubric indicators provided specificity that allowed teachers to envision small but meaningful shifts that they could then implement in their own classroom. In this way, the rubrics supported reflection on teaching by providing coaches and teachers both a shared language around equitable teaching and a focus for video analysis (Borko et al., 2008; Suh et al., 2021).

In addition, the routines and structures in HEAR-MI Coaching (e.g., goal setting, *describe and elevate*, action steps) provide a productive framework for coaches and teachers to engage in targeted discussions around teaching improvements geared toward enacting changes to practice. In particular, the predictable and repeated structure provided by the *describe and elevate* routine allowed for coaching conversations to go into greater depth (Horn & Little, 2010) and held coaches and teachers accountable for discussing specific teaching practices, rather than allowing conversations to drift to other topics. The use of video clips from our project library supported coaches and teachers to discuss a range of practices, depersonalize the analysis, and develop a shared understanding of what the EAR-MI practices look like. The use of video from the teacher's classroom allowed for the application of the EAR-MI to teachers' own context, supporting both self-reflection and instructional shifts. Teachers identified ways that the routine supported them to enact small adjustments to teaching (e.g., using transcript annotations to visualize what enactment could entail). These adjustments were then relatively easy to implement. Such modest instructional changes hold the potential to be sustainable and scalable (Otten et al., 2022). Those interested in developing professional learning focused on incremental improvement might consider how routines such as this can support teachers to envision, and ultimately enact, instructional shifts.

A key feature of incremental improvement approaches is that they meet teachers where they are, building on current practice. In HEAR-MI Coaching, while improvement goals and action steps are guided by the EAR-MI, they are chosen by the teacher to reflect their needs, contexts, and classroom realities. As adult learners, teachers benefit from a problem-centered approach to learning that leverages intrinsic motivation and is both applicable to their current context and immediately actionable (Knowles et al., 2012). Participating teachers noted the importance of setting their own improvement goals and choosing a focal rubric that aligned with those goals. Building teacher choice and agency into HEAR-MI Coaching therefore supported teachers to connect to and build from their current practice. Other approaches to professional learning focused on incremental improvement might consider how to foreground similar self-directed learning, as doing so has the potential to support buy-in and seed sustainability (Otten et al., 2022; Star, 2016).

We close by noting that coaching models such as HEAR-MI Coaching may be particularly suited for incremental approaches to developing more equitable teaching. The model leverages mathematics coaches as a catalyst for professional learning and incremental instructional change. Coaching is a form of job-embedded professional learning and coaches bring to their work with teachers situated, contextualized knowledge of teachers and schools (Marshall & Buenrostro, 2021). Coaches can provide individualized, sustained, and content-focused support that is job-embedded and rooted in teachers' contexts (Kraft et al., 2018). Content-focused coaches can support teachers to develop their mathematics teaching through goal setting, evaluating teaching practice, locating teaching quality along a continuum, and identifying action steps (Gibbons & Cobb, 2016). In addition, coaching cycles leverage a pre-existing structure in schools (Otten et al., 2022). HEAR-MI Coaching cycles occurred during the time in which coaches and teachers already met with one another, shaping *how* that time was used but not layering on additional requirements. While organizational structures in schools and districts can obstruct large-scale instructional change (Star, 2016), coaches may be able to work within existing structures to support shifts in practice (Woulfin & Spitzer, 2024). Furthermore, coaching offers a potentially promising way to support teachers to enact more equitable teaching specifically, as coaches can leverage knowledge of equitable practices and relationships with teachers to encourage justice-oriented approaches to teaching (Marshall & Buenrostro, 2021).

It is important to consider whether incremental improvement approaches are appropriate for equity-oriented work. Otten et al. (2022) rightly ask, "Can deep systemic issues of inequity and injustice in mathematics education be meaningfully addressed in an incremental manner?" (p. 1448). However, we argue that smaller adjustments focused on equitable teaching practices may be a necessary first step in bridging between current practice and the types of teaching that will affirm students' mathematical and cultural knowledge and identities. Star (2016) cautions that the focus of incremental improvement efforts should be chosen "for their potential to positively affect student learning while opening a door within teachers' practice for more ambitious instructional changes in the future" (p. 59). Whether the accumulation of smaller-scale equity-focused practices will lead to more equitable classrooms remains an open question. Additional research is needed to understand whether and how models such as HEAR-MI Coaching can shift teachers' beliefs and practice and support student outcomes. As part of our larger project, we aim to address questions such as this. Through an experimental study of HEAR-MI Coaching in two additional school districts, future research will explore the extent to which participation supported shifts in teachers' beliefs and practice, student outcomes, and students' sense of belonging in mathematics class. Further research is also needed to understand *how* to support teachers to bridge from the types of practices articulated in the EAR-MI to more "ambitious" teaching rooted in culturally sustaining or justice-oriented practices.

We do not advocate that professional development approaches abandon efforts at large scale changes to mathematics instruction focused on equitable, culturally sustaining and affirming, and justice-oriented teaching. However, given the complexity and difficulty of enacting large changes to teaching practice (Cohen, 2011), coaching efforts that focus on small shifts building on teachers' extant practice to include facets of more equitable teaching may be an important start. Such efforts have the potential to impact how students experience mathematics in meaningful ways. Coaching models that aim to shift toward more equitable instruction in these ways should be intentionally designed to incorporate key features of incremental improvement that support teacher learning and practice.

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