

Special Issue Reprint

Advances in Multi-GNSS Technology and Applications

Edited by
Bobin Cui and Jungang Wang

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Advances in Multi-GNSS Technology and Applications

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Guest Editors

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About the Editors

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Bobin Cui is an Associate Professor at Chang'an University. He earned his doctoral degree from the Technical University of Berlin and the GFZ German Research Centre for Geosciences. His core research interests lie in the field of Global Navigation Satellite System (GNSS) real-time kinematic Precise Point Positioning (PPP-RTK), with a specific focus on atmospheric modeling, precise positioning, and clock offset estimation techniques. He has presided over multiple national and provincial/ministerial research projects, including the National Natural Science Foundation and the Key Research and Development Program. He acts as a peer reviewer for more than 20 SCI-indexed journals and has published over 50 academic papers. He contributes actively to the scientific community and has been recognized as a "Outstanding Reviewer" by the journal *Measurement Science and Technology*.

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Jungang Wang is a research scientist in space geodesy at Technische Universität Berlin. His research focuses on the integrated analysis of space geodetic techniques, including Global Navigation Satellite Systems (GNSS), Very Long Baseline Interferometry (VLBI), and Satellite Laser Ranging (SLR). His work contributes to the realization and improvement of terrestrial and celestial reference frames, the determination of Earth orientation parameters, high-precision orbit determination, and geoscientific applications of GNSSs. His scientific interests include advanced analysis strategies for space geodetic observations, innovative approaches for multi-technique combination, and improved modeling of tropospheric delays. He has extensive expertise in both post-processing and real-time GNSS data analysis and has been actively involved in international collaborations related to multi-technique integration and space-time reference systems. He contributes actively to the scientific community and has been recognized as a "Top Reviewer" by the journal *Advances in Space Research*.

Preface

This Reprint presents a curated collection of 13 original research papers from the Special Issue entitled “Advances in Multi-GNSS Technology and Applications”, published in the open access journal *Remote Sensing*. As the Guest Editors of this Special Issue, we are pleased to introduce this Reprint to the global research community, detailing the core focus, research boundaries, foundational goals, driving motivations, and intended readership of the work in this collection.

At its core, this Reprint centers on theoretical breakthroughs, technological advancements, and cross-domain application expansion of multi-constellation Global Navigation Satellite System (multi-GNSS) technology. Against the backdrop of integrated development across four core global navigation systems (GPS, GLONASS, Galileo, and BDS), this body of work addresses the key technical bottlenecks and real-world application challenges that multi-GNSS faces when delivering high-precision, high-reliability, and high-integrity Positioning, Navigation, and Timing (PNT) services in complex and challenging environments.

Spanning the full technical pipeline of multi-GNSS research from fundamental theoretical modeling to real-world engineering validation, the work compiled in this Reprint covers six interconnected research pillars aligned with the original Special Issue’s vision. These pillars include: high-precision multi-GNSS positioning and time transfer algorithms, such as LEO constellation-enhanced PPP time transfer, refined clock model-optimized time transfer, and high-precision satellite clock offset estimation; navigation signal processing and integrity monitoring, including arbitrary-component GNSS meta-signal construction, SBAS positioning integrity risk mitigation, BDSBAS ionospheric degradation parameter optimization, and multi-GNSS multipath effect suppression; atmospheric and space environment modeling tailored for GNSS applications, covering tropospheric delay estimation in PPP and ionospheric anomaly response for satellite-based augmentation systems; intelligent algorithm empowerment for GNSS performance enhancement, including deep learning-based positioning error estimation and urban navigation scene recognition for low-cost GNSS receivers; marine and geoscience applications of GNSS technology, such as wave parameter retrieval from GNSS buoy measurements and fault creep quantification using dense continuous GNSS arrays; and geological hazard monitoring and early warning applications, including GNSS-driven deformation prediction for expansive soil landslides.

Our primary aim in curating this Reprint is to systematically compile and disseminate the latest peer-reviewed advances in multi-GNSS research, closing the gap between theoretical innovation and real-world engineering implementation across the satellite navigation sector. We pursue this goal with three core purposes in mind: first, to showcase cutting-edge research outcomes that solve the core technical challenges of multi-GNSS in harsh and complex scenarios, from urban canyons and open ocean surfaces to seismically active zones; second, to foster academic exchange and interdisciplinary collaboration across satellite navigation, geodesy, geoscience, artificial intelligence, and transportation engineering, while advancing the establishment of unified technical standards and international cooperation in the multi-GNSS field; third, to deliver a systematic, authoritative technical reference for the industrialization and large-scale deployment of high-precision multi-GNSS technology in safety-critical fields including civil aviation, autonomous driving, and geological disaster prevention and mitigation.

This Special Issue and its accompanying Reprint were born from a critical inflection point in the global multi-GNSS field, where unprecedented opportunities for innovation coincide with pressing technical and application challenges. The integration of multi-constellation navigation systems has

dramatically improved the geometric configuration, observation redundancy, and service coverage of satellite navigation, while also placing more rigorous demands on algorithm design, signal processing, error modeling, and scenario adaptation. Meanwhile, the rapid growth of emerging industries, including intelligent connected vehicles, the low-altitude economy, and smart ocean management, has created urgent demand for PNT services with higher precision, stronger reliability, and better environmental robustness. Even so, notable technical bottlenecks persist in current multi-GNSS research: slow convergence of precise positioning in complex environments, insufficient integrity assurance for safety-critical applications, limited adaptability of low-cost receivers in harsh scenarios, and a need for deeper integration between GNSS technology and cross-disciplinary fields. It was against this backdrop that we launched this Special Issue to gather global research efforts to tackle these challenges, and compiled this Reprint to preserve and widely disseminate these valuable peer-reviewed research outcomes, with the long-term goal of driving continued innovative development of multi-GNSS technology and its cross-domain applications.

This Reprint is crafted to serve a diverse, global audience spanning academia, industry, and policy within the navigation and geospatial sectors. It is primarily intended for researchers, scholars, and graduate students specializing in global navigation satellite systems, geodesy and geomatics, remote sensing, and space geodesy. It also serves as a key resource for engineering and technical personnel engaged in the research, development, and application of PNT technology across civil aviation, intelligent transportation, marine engineering, geological hazard monitoring, and geographic information services. In addition, this work provides valuable insights for policy makers and standard setters working in the satellite navigation and geospatial information industry.

We would like to express our sincere gratitude to all authors who contributed their high-quality research to this Special Issue, to the peer reviewers for their rigorous and professional comments that enhanced the quality of the included papers, and to the editorial team of *Remote Sensing* for their strong support and meticulous work throughout the publication process of the Special Issue and the compilation of this Reprint. We hope this Reprint will serve as a valuable resource and a solid foundation for future research and application innovation in the multi-GNSS field.

Bobin Cui and Jungang Wang

Guest Editors

Article

LEO-Enhanced Multi-GNSS Real-Time PPP Time Transfer

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Highlights

What are the main findings?

- Integrating GNSS and LEO observations can significantly reduce the convergence time and improve the precision of time links.
- Using 1 s observations, the time link errors can converge to 1 ns within 1 min.

What are the implications of the main findings?

- With LEO augmentation, time links can achieve high precision with less time.
- Not only fast, but also high-performance time links can be achieved with LEO augmentation.

Abstract

GNSS Precise Point Positioning (PPP) technology has been applied to the time transfer for a long time, enabling time synchronization between two arbitrary stations on a global scale. Over the past decade, Low Earth Orbit (LEO) satellite constellations have been developed to enhance GNSS, offering rapid geometry configuration variations that can accelerate PPP convergence and enhance the time link performance. In this contribution, LEO observations are integrated into GNSS to enhance the real-time PPP time transfer. Simulated LEO constellations with varying numbers of satellites are used to assess their impact on real-time PPP time transfer performance. One week of observation data from 11 globally distributed stations is used to generate 10 time links, and five experimental schemes are designed: (1) GPS/BDS-3/Galileo solution (GCE), (2) GCE with 120 LEO satellites (GCE+120L), (3) GCE with 180 LEO satellites (GCE+180L), (4) GCE with 240 LEO satellites (GCE+240L), and (5) GCE with 300 LEO satellites (GCE+300L). Results showed that compared to the GCE solution, integrating 120, 180, 240, and 300 LEO satellites increases the average number of observed satellites from 23.4 to 30.6, 34.1, 37.7, and 41.3, respectively, while reducing Time Dilution of Precision (TDOP) values from 0.547 to 0.424, 0.391, 0.363, and 0.342, respectively. Using 30 s observations, the average convergence time to STD of time link errors better than 0.1 ns is reduced from 7.95 to 5.94, 4.83, 4.46, and 4.45 min in static mode, with improvements of 25.3%, 39.2%, 43.9%, and 44.0%, respectively, and from 8.75 to 6.18, 5.17, 4.89, and 4.72 min in kinematic mode, with improvements of 29.3%, 40.8%, 44.1%, and 46.0%, respectively. Using 1 s observations, Scenarios GCE+120L, GCE+180L, GCE+240L, and GCE+300L can achieve 1 ns convergence within 1 min. The time link precision was also found to be significantly improved, i.e., from 0.337 to 0.243 ns in static mode with improvements of 27.9%, and from 0.377 to 0.253 ns in kinematic mode with improvements of 32.9%. The time link stability is significantly enhanced for averaging times between 60 and 20,000 s in both static and kinematic modes, with a

maximum improvement of nearly 50%. These results have demonstrated that integrating LEO satellites can significantly enhance real-time PPP time transfer performance.

Keywords: LEO; PPP; time transfer; convergence time; time link stability

1. Introduction

Time transfer is the transmission of high-precision time information between two or more locations, enabling clocks in different sites to achieve time synchronization [1]. Precise time transfer is essential for various fields, such as communication networks and power transmission [2].

GPS Precise Point Positioning (PPP) technology has been utilized by the Bureau International des Poids et Mesures (BIPM) for International Atomic Time (TAI) comparison since 2008 [3] due to its high precision, low cost, and global scale coverage. Multi-GNSS observations were afterward incorporated into time transfer to enhance redundancy and robustness [4,5]. Despite the convenience brought by PPP time transfer, the large altitude difference (about 20,000 km or more) between the GNSS Medium Earth Orbit (MEO) satellites and the ground stations results in slow satellite geometry changes and strong temporal correlations in estimated parameters. Consequently, PPP time transfer requires nearly 10 min (or more) to converge [6], during which the time link stability is degraded. This is especially crucial for kinematic PPP time transfer.

In recent years, Low Earth Orbit (LEO) navigation satellite constellations have been widely developed [7], such as Xona Space [8], Centispace [9], and Geely [10]. Some LEO satellites will broadcast GNSS-like navigation signals to ground users, enabling LEO-enhanced GNSS ground Positioning, Navigation, and Timing (PNT) services. The LEO satellite orbit altitude is about 400–1500 km [11]. The lower altitudes compared to those of GNSS satellites imply faster motion and geometry change, which also helps to decrease the correlation among the estimated parameters between adjacent epochs and shorten the convergence time.

Simulated LEO constellations have been investigated for augmenting Real-Time Kinematic (RTK) [12], GNSS PPP [13], PPP-Ambiguity Resolution (AR) [14], and PPP-RTK [15]. Compared to the multi-GNSS solutions, the convergence time, the positioning accuracy, and the Time To First Fix (TTFF) in AR are remarkably improved after using LEO-enhanced GNSS. Real LEO observations from CENTISPACETM LEO mission [16,17] have been used for LEO-enhanced multi-GNSS PPP, confirming both the improved convergence time and positioning performance.

In the PPP time transfer, the receiver clocks are simultaneously estimated with the station coordinates. The LEO-enhanced GPS PPP time transfer was investigated by Liu et al. [6], in which one-day observation data with 160 LEO satellites in fixed station mode was used, showing that the short-term stability and the convergence time can be improved. Nowadays, the fixed station coordinates mode is not sufficient considering applications in kinematic scenarios, such as autonomous driving. The LEO enhancement in time transfer is worthwhile to investigate in multi-GNSS cases under both static and kinematic scenarios across multiple days, with varying LEO constellation sizes, considering the different phases of the constellation development. The performance of LEO-enhanced PPP time transfer under different observation sampling intervals, e.g., 30 s and 1 s, can also be compared.

This contribution focuses on LEO-enhanced multi-GNSS real-time PPP time transfer. One week of observation data from 11 Multi-GNSS Experiment (MGEX) stations [18] were selected to generate 10 time links, and the LEO constellations with 120, 180, 240, and

300 satellites were simulated and used to conduct the experiments. After this introduction, the LEO-enhanced multi-GNSS real-time PPP time transfer methods are introduced. Next, the LEO constellation simulation, data and processing strategies are presented. Thereafter, the performance of LEO-enhanced multi-GNSS PPP time transfer is evaluated in terms of the number of satellites and Time Dilution of Precision (TDOP), convergence time, time link precision and stability in both the static and kinematic scenarios. Finally, the conclusion is provided.

2. Methods

This section first introduces the time transfer model based on multi-GNSS PPP. The LEO-enhanced multi-GNSS PPP model is then presented. The time transfer method is given at the end.

2.1. Multi-GNSS-Based PPP Model

The dual-frequency Ionospheric-Free (IF) observation equations from a GNSS satellite s to receiver r for system g (GPS, BDS-3, Galileo) can be expressed as:

$$\delta_{r,IF}^{s,g} = \rho_r^{s,g} + c(t_{r,IF}^g - t_{IF}^{s,g}) + T_r^{s,g} + \epsilon_{r,pc}^{s,g} \quad (1)$$

$$\varphi_{r,IF}^{s,g} = \rho_r^{s,g} + c(t_{r,IF}^g - t_{IF}^{s,g}) + \lambda_{IF}^g N_{r,IF}^{s,g} + T_r^{s,g} + e_{r,lc}^{s,g} \quad (2)$$

where δ and φ represents the code and carrier phase IF observations, respectively. $\rho_r^{s,g}$ is the geometric distance between the GNSS satellite and the receiver. c denotes the speed of light. $t_{r,IF}^g$ and $t_{IF}^{s,g}$ are the receiver and satellite clocks containing the corresponding IF code biases [19], respectively. $T_r^{s,g}$ is the slant troposphere delay, and λ_{IF}^g is the IF wavelength. $N_{r,IF}^{s,g}$ is the IF combined ambiguity in cycles, the carrier phase and code delay on the satellite and receiver end were absorbed by ambiguity [20]. $\epsilon_{r,pc}^{s,g}$ and $e_{r,lc}^{s,g}$ represent the measurement noise for code and carrier phase observations, respectively. The antenna Phase Center Offsets (PCOs) and Variations (PCVs) on the satellite and receiver end, the relativistic effects, the tidal loadings, and the phase windup were corrected using the existing models in Equations (1) and (2) [21]. Note that in addition to the traditional IF form, the PPP can also be formed in the Undifferenced and Uncombined (UDUC) form [22].

The observation equations can be transformed into error equations for PPP, expressed as:

$$v_{r,pc}^{s,g} = \mu \cdot \hat{X} + c \cdot \hat{t}_{r,IF}^g + m_r^{s,g} \hat{Z}_w - l_{r,pc}^{s,g} \quad (3)$$

$$v_{r,lc}^{s,g} = \mu \cdot \hat{X} + c \cdot \hat{t}_{r,IF}^g + \lambda_{IF}^g \hat{N}_{r,IF}^{s,g} + m_r^{s,g} \hat{Z}_w - l_{r,lc}^{s,g} \quad (4)$$

where pc and lc indicate the code and phase IF combination, respectively. v and l represent the posterior and the Observed-Minus-Computed (OMC) terms, respectively. μ is the unit direction vector from the satellite to the receiver. $\hat{X}$ is the vector of the coordinate increments relative to their a priori positions. $\hat{t}_{r,IF}^g$ is the estimated receiver clock, $m_r^{s,g}$ and $\hat{Z}_w$ are the tropospheric mapping function, and the estimated Zenith Wet Delay (ZWD) of the troposphere.

By integrating GPS, BDS-3, and Galileo observations, the multi-GNSS-based PPP models can be formed and expressed as:

$$\begin{cases} v_{r,pc}^{s,G} = \mu \cdot \hat{X} + c \cdot \hat{t}_{r,IF}^G + m_r^{s,G} \hat{Z}_w - l_{r,pc}^{s,G} \\ v_{r,pc}^{s,C} = \mu \cdot \hat{X} + c \cdot \hat{t}_{r,IF}^G + I \hat{S} B_r^{CG} + m_r^{s,C} \hat{Z}_w - l_{r,pc}^{s,C} \\ v_{r,pc}^{s,E} = \mu \cdot \hat{X} + c \cdot \hat{t}_{r,IF}^G + I \hat{S} B_r^{EG} + m_r^{s,E} \hat{Z}_w - l_{r,pc}^{s,E} \end{cases} \quad (5)$$

$$\begin{cases} v_{r,lc}^{s,G} = \mu \cdot \hat{X} + c \cdot \hat{t}_{r,IF}^G + \lambda_{IF}^G \hat{N}_{r,IF}^{s,G} + m_r^{s,G} \hat{Z}_w - l_{r,lc}^{s,G} \\ v_{r,lc}^{s,C} = \mu \cdot \hat{X} + c \cdot \hat{t}_{r,IF}^G + \lambda_{IF}^C \hat{N}_{r,IF}^{s,C} + I\hat{S}B_r^{CG} + m_r^{s,C} \hat{Z}_w - l_{r,lc}^{s,C} \\ v_{r,lc}^{s,E} = \mu \cdot \hat{X} + c \cdot \hat{t}_{r,IF}^G + \lambda_{IF}^E \hat{N}_{r,IF}^{s,E} + I\hat{S}B_r^{EG} + m_r^{s,E} \hat{Z}_w - l_{r,lc}^{s,E} \end{cases} \quad (6)$$

where $I\hat{S}B_r^{CG}$ and $I\hat{S}B_r^{EG}$ stand for the Inter-System Biases (ISB) between GPS and BDS, and between GPS and Galileo, respectively.

2.2. LEO-Enhanced Multi-GNSS PPP Model

The LEO observations are integrated into the multi-GNSS observations to enhance the PPP time transfer, which can be expressed as:

$$\begin{cases} v_{r,pc}^{s,G} = \mu \cdot \hat{X} + c \cdot \hat{t}_{r,IF}^G + m_r^{s,G} \hat{Z}_w - l_{r,pc}^{s,G} \\ v_{r,pc}^{s,C} = \mu \cdot \hat{X} + c \cdot \hat{t}_{r,IF}^G + I\hat{S}B_r^{CG} + m_r^{s,C} \hat{Z}_w - l_{r,pc}^{s,C} \\ v_{r,pc}^{s,E} = \mu \cdot \hat{X} + c \cdot \hat{t}_{r,IF}^G + I\hat{S}B_r^{EG} + m_r^{s,E} \hat{Z}_w - l_{r,pc}^{s,E} \\ v_{r,pc}^{s,L} = \mu \cdot \hat{X} + c \cdot \hat{t}_{r,IF}^G + I\hat{S}B_r^{LG} + m_r^{s,L} \hat{Z}_w - l_{r,pc}^{s,L} \end{cases} \quad (7)$$

$$\begin{cases} v_{r,lc}^{s,G} = \mu \cdot \hat{X} + c \cdot \hat{t}_{r,IF}^G + \lambda_{IF}^G \hat{N}_{r,IF}^{s,G} + m_r^{s,G} \hat{Z}_w - l_{r,lc}^{s,G} \\ v_{r,lc}^{s,C} = \mu \cdot \hat{X} + c \cdot \hat{t}_{r,IF}^G + \lambda_{IF}^C \hat{N}_{r,IF}^{s,C} + I\hat{S}B_r^{CG} + m_r^{s,C} \hat{Z}_w - l_{r,lc}^{s,C} \\ v_{r,lc}^{s,E} = \mu \cdot \hat{X} + c \cdot \hat{t}_{r,IF}^G + \lambda_{IF}^E \hat{N}_{r,IF}^{s,E} + I\hat{S}B_r^{EG} + m_r^{s,E} \hat{Z}_w - l_{r,lc}^{s,E} \\ v_{r,lc}^{s,L} = \mu \cdot \hat{X} + c \cdot \hat{t}_{r,IF}^G + \lambda_{IF}^L \hat{N}_{r,IF}^{s,L} + I\hat{S}B_r^{LG} + m_r^{s,L} \hat{Z}_w - l_{r,lc}^{s,L} \end{cases} \quad (8)$$

where L indicates the observations from LEO satellites. $I\hat{S}B_r^{LG}$ is the estimated receiver ISB between GPS and LEO signals. The estimated parameters can be expressed:

$$Y = [\hat{X}, \hat{t}_{r,IF}^G, I\hat{S}B_r^{CG}, I\hat{S}B_r^{EG}, I\hat{S}B_r^{LG}, \hat{Z}_w, \hat{N}_{r,IF}^{s,G}, \hat{N}_{r,IF}^{s,C}, \hat{N}_{r,IF}^{s,E}, \hat{N}_{r,IF}^{s,L}] \quad (9)$$

2.3. Time Transfer Method

The estimated receiver clock $\hat{t}_{r,IF}^G$ is used to conduct precise time transfer. The time difference between the receiver clocks of two stations can be expressed as [23]:

$$\Delta dt = (\hat{t}_A - t_{ref}) - (\hat{t}_B - t_{ref}) = \hat{t}_A - \hat{t}_B \quad (10)$$

where $\hat{t}_A$ and $\hat{t}_B$ are the estimated receiver clocks at stations A and B, and t_{ref} represents the time reference, which depends on the GNSS satellite clocks used in the PPP. The time reference of $\hat{t}_A$ and $\hat{t}_B$ is the same because the same satellite clocks were applied when estimating the receiver clocks.

3. Data and Processing Strategies

This section first introduces the simulation of the LEO satellite constellations to be used in the processing. The experimental settings are then briefly explained.

3.1. LEO Satellite Constellations

Due to the limited availability of public LEO satellite navigation signals, navigation signals from LEO satellites are simulated for the PPP processing. The Walker constellation is applied in this contribution due to the uniform coverage of its geometric distribution to the Earth [24,25]. As shown in Figure 1, the Walker constellations 120/12/1, 180/12/1, 240/12/1, and 300/12/1 are to be used, with the number of LEO satellites amounting to 120, 180, 240, and 300, respectively. The satellite orbits have an altitude of 1175 km and an inclination of 85° [24,25]. In this study, only the near-polar orbit configuration is employed to simulate a relatively conservative situation at the beginning phase of the LEO

constellation development, while adding more LEO satellites with inclined orbits should further improve the overall geometry for the LEO PNT services.

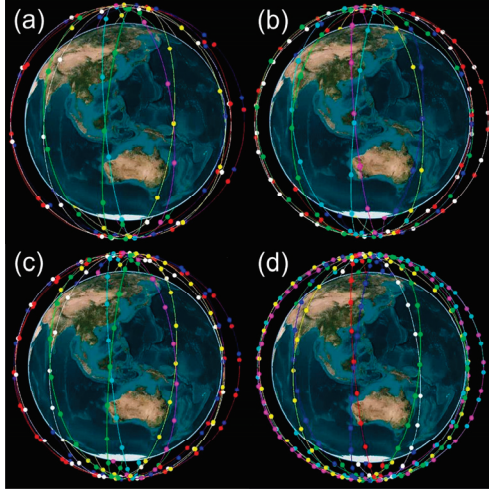


Figure 1. Simulated LEO satellite constellations with 120 (a), 180 (b), 240 (c), and 300 satellites (d).

The initial orbits of the LEO satellites are calculated using the six Keplerian parameters, and the satellite orbits can then be numerically integrated to the desired time points. Because the variations in GNSS satellite clocks present nearly linear characteristics [26], the LEO satellite clocks are simulated using a linear model and random noise [27], which can be expressed as:

$$clk_i^s = a_0^s + a_1^s(t_i - t_0) + \varepsilon_i^s \quad (11)$$

where clk_i^s is the LEO satellite clock at the epoch i for satellite s . a_0^s , and a_1^s represent the clock bias and clock drift for satellite s , respectively. t_i and t_0 indicate the time at epoch i and reference epoch, respectively. ε_i^s is the random noise term with the Standard Deviation (STD) of 0.001 ns [28]. The real LEO satellite clocks contain other systematic effects as described in [29–32]. For LEO-enhanced PPP, however, only the differences between the simulated and the used ones (in the PPP), i.e., the real-time LEO satellite orbital/clock estimation errors, are of concern. Their influences on the PPP are investigated in detail in our other work [33], and are not further studied in this contribution.

For the observation simulations, the downlink signal frequencies are assumed to be the same as the BDS B1I (1575.42 MHz) and B3I (1268.52 MHz) [24,25], with a sampling of 30 s. The observation noise is set to 3 mm for the carrier phase and 0.3 m for the code observations. The relativistic effects [34,35], the phase windups [36], and the Earth rotation corrections are simulated using the existing models and are assumed to be corrected in the PPP processing. The PCOs and PCVs of the LEO satellite are set to 0 considering they are known and corrected. The ionospheric delay is simulated using the Global Ionosphere Map (GIM) provided by the Center for Orbit Determination in Europe (CODE) [37]. The slant tropospheric delay consists of hydrostatic and wet components [38], both calculated using the Saastamoinen model [39] and the Global Mapping Function (GMF) [40]. The station coordinates are obtained from International GNSS Services (IGS) weekly solutions, and the receiver PCO and PCV for LEO signals are also set to 0. The receiver clocks are applied using values from European Space Agency Multi-GNSS (ESM) final products [41], with the ISBs between LEO and GPS signals set to zeros.

3.2. Experimental Settings

A total of 11 MGEX stations [18] were selected to perform the LEO-enhanced multi-GNSS real-time PPP time transfer experiments, as shown in Figure 2. The details of these

stations are listed in Table 1, in which all stations are equipped with high-performance atomic clocks. The station CEBR, equipped with an external H-Maser, is selected as the central node (red dot in Figure 2), while the other stations (blue triangles in Figure 2) are used to conduct the time transfer experiments. The time link distance ranges from 1741.1 to 12,484.1 km, i.e., long to very long baselines.

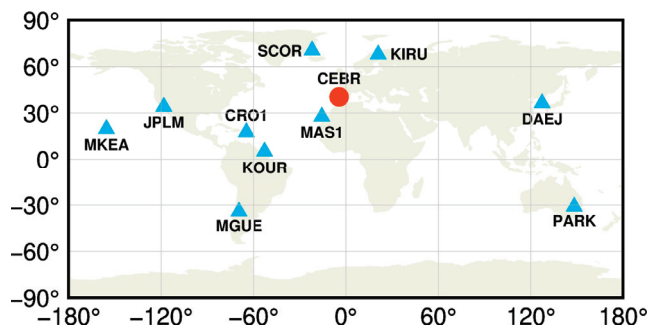


Figure 2. Station distribution of the 11 MGEX stations used for the time transfer experiments.

Table 1. Station information used for LEO-enhanced multi-GNSS PPP real-time transfer.

Station	Receiver Type	Clock	Location	Distance (km)
CEBR	SEPT POLARX5TR	External H-Maser	Spain	/
CRO1	SEPT POLARX5TR	External H-Maser	USA	5998.0
DAEJ	TRIMBLE NETR9	External Cesium	South Korea	9134.5
JPLM	SEPT POLARX5	External Rubidium	USA	8517.1
KIRU	SEPT POLARX5TR	External Cesium	Sweden	3373.9
KOUR	SEPT POLARX5TR	External H-Maser	France	5959.4
MAS1	SEPT POLARX5	External Cesium	Spain	1741.1
MGUE	SEPT POLARX5TR	External H-Maser	Argentina	9511.0
MKEA	SEPT POLARX5	External H-Maser	USA	10,709.9
PARK	SEPT POLARX5TR	External H-Maser	Australia	12,484.1
SCOR	SEPT POLARX5	External Rubidium	Denmark	3449.0

Real GNSS and simulated LEO observations from DOY 119 to 125, 2024, were used. The dual-frequency signals from GPS L1/L2, BDS-3 B1I/B3I, Galileo E1/E5a, and LEO B1I/B3I, sampled at 30 s, were employed to generate the IF combination. Real-time GNSS satellite orbits and clocks released from the Centre national d'études spatiales (CNES) [42], and simulated LEO satellite orbits and clocks were applied and fixed. The GNSS satellite PCOs and PCVs were corrected using values from IGS antenna files, while the LEO satellite PCOs and PCVs were not corrected, as they are set to zeros in LEO signal simulations. Relativistic effects [34], phase-windup [36], and Earth rotation are calculated by applying the existing models as mentioned before. The tropospheric slant hydrostatic delay is corrected using the Saastamoinen model [39] and the GMF [40]. Receiver PCOs and PCVs were only corrected for GNSS observations, as those for LEO signals are simulated to be zeros. The Kalman filter [43,44] was used for estimating the parameters epoch-by-epoch, and the station coordinates were estimated as constants in the static mode [45] and white noise in the kinematic mode [45]. Both the receiver clocks and ISBs are estimated as white noise [46]. The variance setting is $60 \times 60 \text{ m}^2$ for both kinematic positioning and ISBs parameters. The ZWD is estimated using a random walk model with the process noise standard deviation of $(1 \times 10^{-4} \times \sqrt{30}) \text{ m}$ [45]. Ambiguities were treated as constants unless a cycle slip or loss of lock occurred. The simulated LEO satellite orbits and clocks are directly used in the PPP, with their errors not considered in this study.

Five schemes were designed with different numbers of LEO satellites: (1) GPS+BDS-3+Galileo solution (GCE), and (2) GPS+BDS-3+Galileo solution with 120 (GCE+120L), (3) 180 (GCE+180L), (4) 240 (GCE+240L), and (5) 300 LEO satellites (GCE+300L). The program was initialized every 3 h to collect more samples to investigate the performance of the convergence time and the time link precision. Additionally, daily solutions were applied to evaluate the time link stability.

4. Experimental Results

The experimental results were evaluated in terms of the number of satellites and TDOP, the convergence time, the time link precision, and the time link stability in this section.

4.1. Number of Satellites and TDOP

The performance of the PPP time transfer is directly influenced by the number of observed satellites and the TDOP values. Figure 3 presents the time series of satellite visibility and TDOP variations using different schemes over 7 days for three representative stations at different latitudes: KOUR (low latitude), CEBR (mid-latitude), and KIRU (high latitude). Under the GCE solution (blue lines), the number of visible satellites ranges from 20 to 32 for KOUR, 17 to 29 for CEBR, and 21 to 34 for KIRU. The integration of LEO constellations (120, 180, 240, and 300 satellites) significantly enhances satellite visibility. When incorporating 300 LEO satellites (magenta lines), the number of observed satellites can be remarkably increased to 28–46 for KOUR, 30–45 for CEBR, and 57–76 for KIRU. Notably, high-latitude stations exhibit significantly more satellite visibility compared to those of low- and mid-latitude counterparts. This spatial disparity is caused by two factors: First, the near-polar orbital configuration of the used Walker constellations results in satellites frequently passing over high-latitude regions. Second, high-latitude areas serve as intersection zones for multiple orbital planes, thereby increasing satellite visibility.

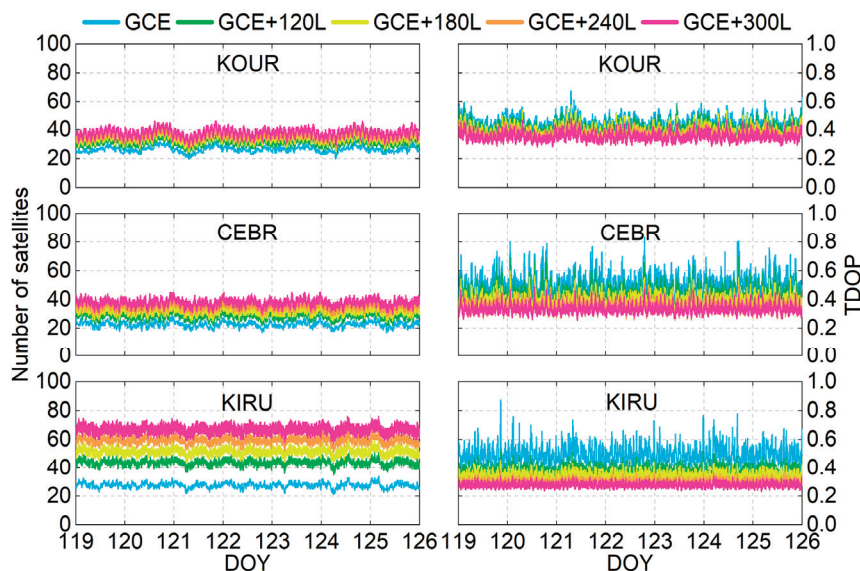


Figure 3. Number of observed satellites and TDOP time series for stations KOUR, CEBR, and KIRU from DOY 119 to 125, 2024.

For the TDOP time series (see the right panel of Figure 3), the values range from 0.354 to 0.677 for KOUR, 0.341 to 0.830 for CEBR, and 0.355 to 0.875 for KIRU under the GCE solution. These values exhibit a significant reduction after integrating LEO observations. Notably, distinct spikes in the TDOP time series are observed in the GCE

solution, particularly at the mid- and high-latitude stations (CEBR and KIRU). However, these phenomena are mitigated when integrating LEO observations. A clear inverse relationship is observed between satellite visibility and TDOP values, i.e., the larger the number of observed satellites, the smaller the TDOPs. A higher number of observed satellites can improve the geometric configuration and enhance the observation model. These factors collectively reduce uncertainties in receiver clock estimation, thereby lowering the TDOP values. The integration of LEO satellites into GNSS observations is expected to significantly enhance time transfer performance based on the model strength improvement.

The average number of observed satellites and the TDOP values across all stations are presented in Figure 4. For the GCE solution (blue bars), the number of observed satellites ranges from 13.7 to 27.8 for different stations, while the TDOP values vary between 0.459 and 0.865. After integrating 120, 180, 240, and 360 LEO satellites, the number of observed satellites increases to 19.5–43.5, 22.4–51.4, 25.3–59.3, and 28.2–67.2, respectively. Correspondingly, TDOP values can significantly decrease to 0.360–0.541, 0.325–0.481, 0.296–0.433, and 0.275–0.400, owing to the increased number of observed satellites and the enhanced observation model strength. The two high-latitude stations, KIRU and SCOR, exhibit significantly higher satellite visibility compared to stations located at mid- and low latitudes. Notably, the number of observed satellites at station DAEJ is lower than that of other stations, with only 13.7 satellites being observed for GCE solutions, resulting in a higher TDOP value of 0.865. By checking observation files, the BDS-3 satellite Pseudorange Random Noise (PRN) larger than C32 cannot be received by this station, which may be related to hardware or software updates to be performed. However, its TDOP is reduced to 0.400 when integrating 300 LEO satellites, demonstrating the effectiveness of LEO-enhanced GNSS observations in improving time transfer performance, especially for stations not tracking enough GNSS observations, or in single- or dual-system situations.

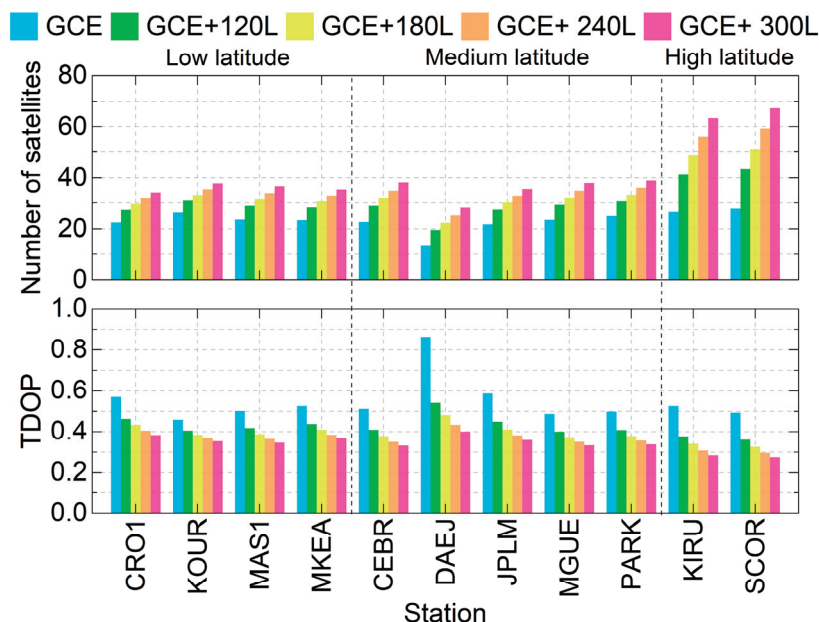


Figure 4. Average numbers of satellites (**top**) and TDOPs (**bottom**) for ground stations.

The average number of observed satellites and the TDOPs of all stations are listed in Table 2. Under the GCE solutions, the average number of satellites amounts to 23.4, with a corresponding TDOP of 0.547. After integrating 120, 180, 240, and 300 LEO satellites, the number of observed satellites is increased to 30.6, 34.1, 37.7, and 41.3, with improvements of 30.8%, 45.7%, 61.1%, and 76.5%, respectively. Meanwhile, the TDOP values

decrease to 0.424, 0.391, 0.363, and 0.342, with a reduction rate of 22.5%, 28.5%, 33.6%, and 37.5%, respectively.

Table 2. Average numbers of satellites and TDOPs for all stations from DOY 119 to 125, 2024.

	GCE	GCE+120L	GCE+180L	GCE+240L	GCE+300L
Number of satellites	23.4	30.6	34.1	37.7	41.3
TDOP	0.547	0.424	0.391	0.363	0.342

4.2. Convergence Time

The time link error is defined as the time link difference between the estimated values and those based on the European Space Agency Multi-GNSS (ESM) final products. Figure 5 presents the time series of time link errors for three time links using the five schemes without or with different LEO constellations over the first three hours on DOY 122, 2024. The top and bottom panels represent the static and kinematic modes, respectively. The time series of the time link errors for the GCE solutions (blue lines) exhibits larger fluctuations than the other schemes in both the static and kinematic modes, particularly for the CEBR-CRO1 and CEBR-JPLM time links during the convergence period. After incorporating LEO observations, these fluctuations are significantly reduced or sometimes even eliminated. The improvements are especially significant for the kinematic mode (bottom panels) with weaker observation models.

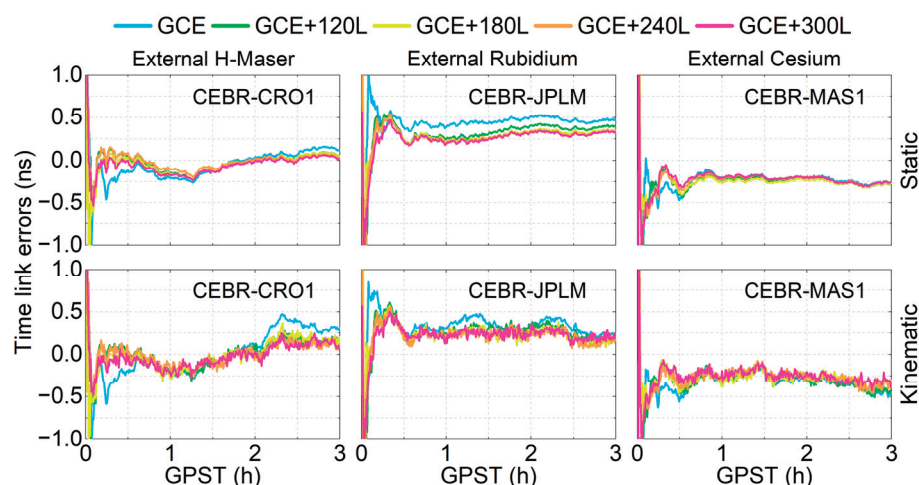


Figure 5. Time link errors of three time links in static (**top**) and kinematic modes (**bottom**) for the first 3 h on DOY 122, 2024.

Also observed in Figure 5 is the accelerated convergence process after integrating LEO observations. When the time link errors STD between the current and the next 10 epochs can be kept within 0.1 ns (approximately 3 cm), the time elapsed from the start of data processing to the current epoch is regarded as the convergence time [47]. To evaluate the convergence performances, the program is restarted every 3 h, resulting in a total of 560 solution pieces. The percentage distribution of the convergence times at 2 min intervals for all pieces is presented in Figure 6 for both the static and kinematic modes. For the GCE solutions, the percentage of the convergence times within 6 min amounts to 31.1% for the static mode, and 23.4% for the kinematic mode. After integrating 120, 180, 240, and 300 LEO satellites, it can be improved to 57.9%, 78.2%, 82.9%, and 82.9% in the static mode, and 53.9%, 72.3%, 77.5%, and 79.5% in the kinematic mode, respectively. For convergence times within 8 min, the percentage increases from 59.3% (static) and 49.6% (kinematic) in the GCE solutions to over 90% after integrating 180, 240, and 300 LEO satellites in both

modes. Furthermore, the percentage of solutions with convergence times exceeding 10 min is 18.6% for the static mode and 27.0% for the kinematic mode in the GCE solutions. They are reduced to below 4% after integrating LEO observations. The convergence performance in the static mode is overall better than that of the kinematic mode, adding or not adding the LEO observations.

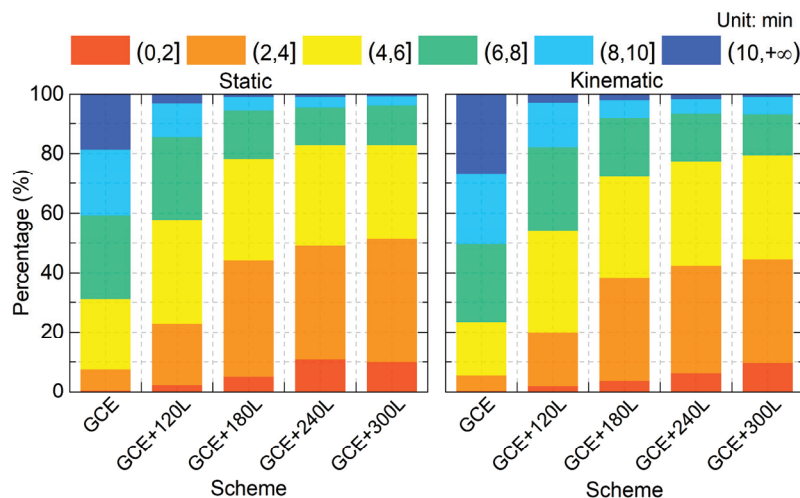


Figure 6. Percentage of the convergence times across each 2 min interval for different schemes.

Figure 7 presents the average convergence times for each time link in both the static and kinematic modes across different schemes. It can be observed that the GCE solutions require at least 7 min to achieve convergence in both modes, with the longest convergence time observed for the CEBR-DAEJ time link, which may be attributed to the lower number of observed satellites and their poor TDOPs (see Figure 4). After integrating LEO observations, the convergence time is significantly reduced for all time links in both the static and kinematic modes. Specifically, the average convergence time decreases to approximately 5 min when utilizing a LEO constellation with at least 180 satellites. As the number of LEO satellites further increases, the improvement becomes marginal. The convergence time differences among the GCE+180L, GCE+240L, and GCE+300L schemes are negligible in both static and kinematic modes.

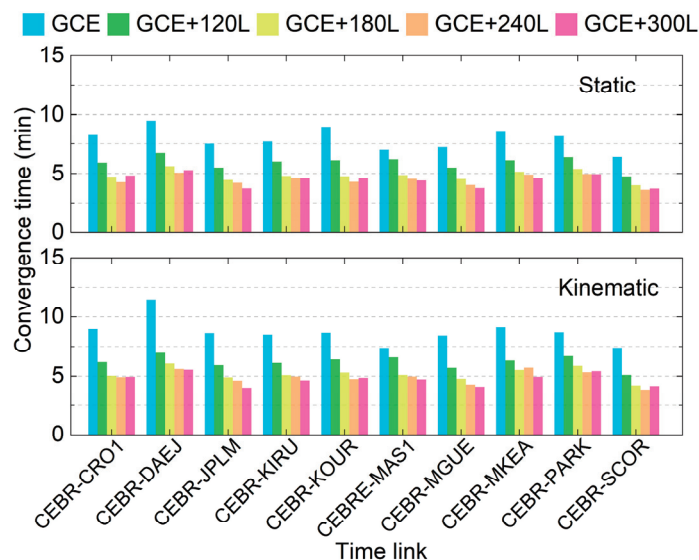


Figure 7. Average convergence times of different time links applying the five schemes.

The average convergence times of all time links are listed in Table 3. When using the GCE solutions, the average convergence time amounts to 7.95 min in static and 8.74 min in kinematic mode. By integrating 120, 180, 240, and 300 LEO satellites, the convergence time is reduced to 5.94, 4.83, 4.46, and 4.45 min in the static mode, and to 6.18, 5.17, 4.89, and 4.72 min in the kinematic mode, with improvements of 25.3%, 39.2%, 43.9%, and 44.0% in static mode, and 29.3%, 40.8%, 44.1%, and 46.0% in kinematic mode, respectively.

Table 3. Average convergence times of different schemes for static and kinematic mode (unit: min).

	GCE	GCE+120L	GCE+180L	GCE+240L	GCE+300L
Static	7.95	5.94	4.83	4.46	4.45
Kinematic	8.74	6.18	5.17	4.89	4.72

The convergence times presented in this section were derived from observations with a 30 s sampling interval. Further reducing the sampling interval, i.e., to 1 s, should shorten the convergence time for all schemes. This will be discussed in Section 5.

4.3. Time Link Precision

The time link precision is evaluated using the Standard Deviation (STD) of the time link errors (including the convergence period). Consistent with the convergence time analysis, a total of 560 solution instances were examined. Figure 8 presents the percentage distribution of the STD values in 0.1 ns increments from 0 to 0.5 ns, and those exceeding 0.5 ns. The percentage of solutions within the 0–0.4 ns range is 73.2%, 82.9%, 89.8%, 92.1%, and 92.3% for the static GCE, GCE+120L, GCE+180L, GCE+240L, and GCE+300L solutions, respectively. In the kinematic mode, the corresponding percentages are 64.8%, 81.4%, 88.64%, 90.5%, and 91.1%. The improvement in STD percentages confirms that integrating LEO observations can significantly enhance the time link precision.

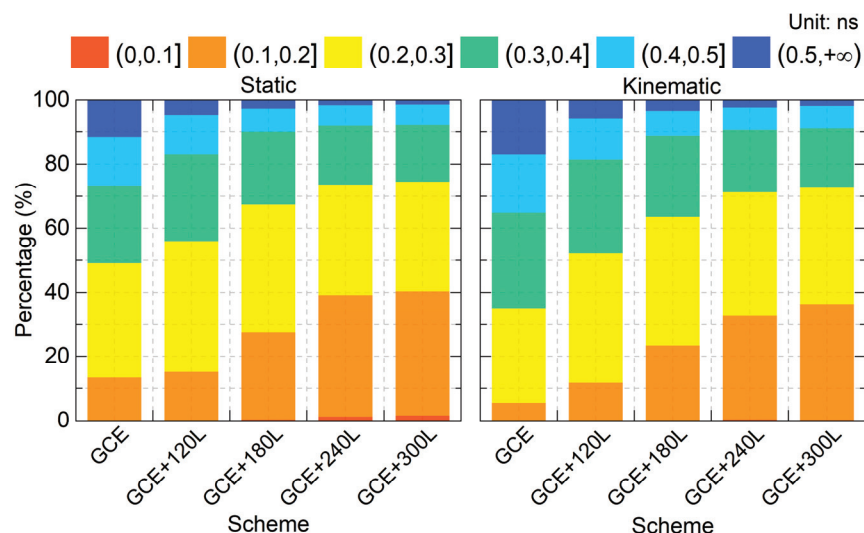


Figure 8. STD percentage across each 0.1 ns interval for the static (left) and the kinematic mode (right).

Figure 9 presents the STD of each time link under different schemes for both the static and kinematic modes. Compared to the GCE solutions, the integration of LEO observations can significantly improve the time link STD, particularly for links with sub-optimal precision, e.g., the CEBR-DAEJ time link. The difference in time link STD between the GCE+240L and GCE+300L solutions is marginal.

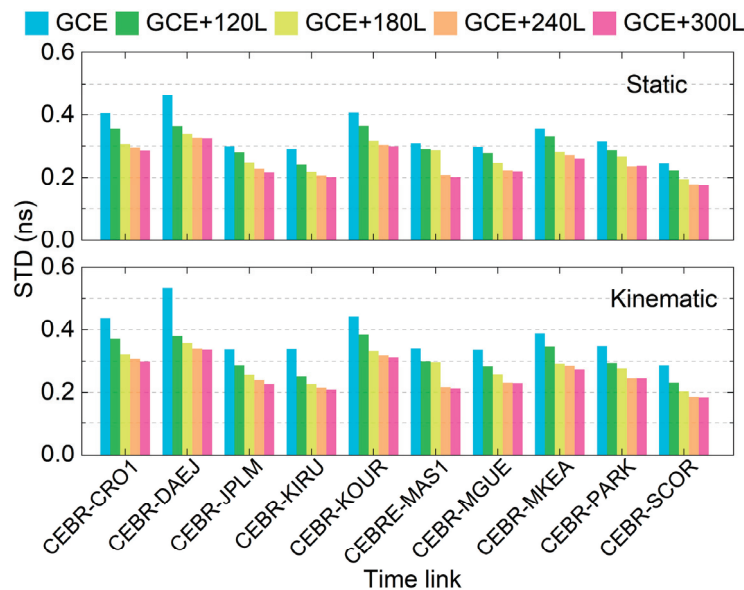


Figure 9. STD of each time link across different schemes for the static and kinematic modes.

The average time link precision of each scheme for the static and kinematic modes is listed in Table 4, including and excluding the first hour. Having the convergence period included, the STDs of the static and kinematic GCE solutions are 0.337 and 0.377 ns. After integrating 120, 180, 240, and 300 LEO satellites, the STD is reduced to 0.302, 0.266, 0.248, and 0.243 ns in the static mode, and 0.313, 0.277, 0.259, and 0.253 ns in the kinematic mode. This implies improvements of 10.4%, 21.1%, 26.4%, and 27.9% in the static mode, and 17.0%, 26.5%, 31.3%, and 32.9% in the kinematic mode. After removing the first hour of the time link errors, i.e., excluding the convergence period, the time link STDs amount to 0.064 ns for the GCE solution in the static mode, and are improved to 0.058, 0.055, 0.055, and 0.054 ns after integrating 120, 180, 240, and 300 LEO satellites, with improvements of 9.4% to 15.6%. For the kinematic mode, the STDs amount to 0.092, 0.084, 0.081, 0.080, and 0.079 ns for the five schemes, with the improvements amounting to 8.7% to 14.1%. This demonstrated that even after convergence, the improvements are still significant after integrating LEO satellites in the multi-GNSS time link performances.

Table 4. Average time link precision across each scheme for the static and kinematic modes, including and excluding the first processing hour (unit: ns).

	GCE	GCE+120L	GCE+180L	GCE+240L	GCE+300L
Static	0.337	0.302	0.266	0.248	0.243
Static (excl. 0–1 h)	0.064	0.058	0.055	0.055	0.054
Kinematic	0.377	0.313	0.277	0.259	0.253
Kinematic (excl. 0–1 h)	0.092	0.084	0.081	0.080	0.079

4.4. Time Link Stability

To further evaluate the performance of the estimated time links, the Modified Allan Deviation (MDEV) [48] is applied to evaluate the time link stability. The average time link stability (considering also the convergence period) of three time links equipped with different types of clocks is presented in Figure 10 for both the static and kinematic modes. The LEO-enhanced solutions exhibit poorer short-term stabilities, i.e., at an averaging time of 30 s in the static mode and 30–60 s in the kinematic mode. This can be attributed to the

increased number of estimated parameters after integrating LEO observations, leading to higher short-term noise. However, a notable improvement was observed for averaging times between 60 and approximately 1000 s in the static mode. For the kinematic mode, improvements are observed for averaging times ranging from 90 to 10,000 s, with the most significant enhancements occurring between 1000 and 10,000 s. Since the kinematic PPP model strength is weaker than its static counterpart, a longer convergence time is required. As a result, the improvements in the 1000–10,000 s range are more pronounced.

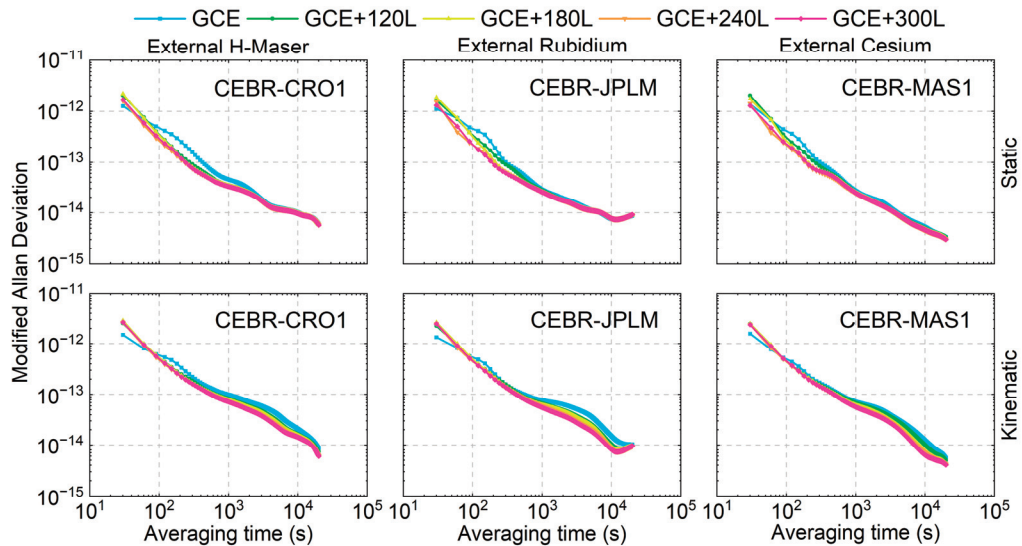


Figure 10. Average MDEVs of three time links in the static (**top**) and kinematic modes (**bottom**).

The average improvements in the MDEV for all time links at different averaging times are presented in Figure 11 for both the static and kinematic modes. Similar to Figure 10, the time link stability improvement is not observed for averaging times of 30 s in static mode and 30–60 s in the kinematic mode after integrating LEO-enhanced multi-GNSS observations. However, positive improvements are evident for averaging times longer than 90 s. The maximum improvement occurs at an averaging time of approximately 210 s, with improvements of 35.3%, 45.0%, 49.8%, and 48.6% for the GCE+120L, GCE+180L, GCE+240L, and GCE+300L solutions, respectively. For the kinematic mode, the maximum improvement is observed at an averaging time of approximately 5600 s, with improvements of 27.0%, 36.7%, 42.8%, and 46.8% for GCE+120L, GCE+180L, GCE+240L, and GCE+300L schemes, respectively. The time link stability improvements achieved with GCE+240L and GCE+300L solutions are superior to those of GCE+120L and GCE+180L. Similar to the time link STD results, the difference between the GCE+240L and GCE+300L solutions is marginal in both the static and kinematic modes.

The time link stabilities were also evaluated using the time link errors after removing the first convergence hour. The time link stability improvements in the LEO-enhanced multi-GNSS solutions compared to those of GNSS-only solutions are presented in Figure 12. It can be observed that the improvements are less than 5% and can be ignored for the static mode. For the kinematic mode, the improvements can be observed when the averaging time becomes longer than 480 s. The maximum improvement is about 45% for the GCE+300L solutions.

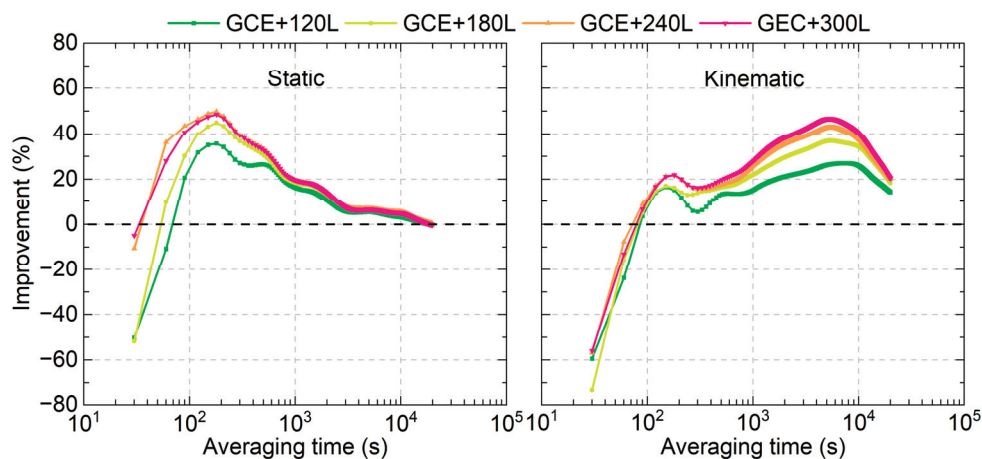


Figure 11. Average improvements in the time link stabilities after integrating LEO observations in multi-GNSS PPP time transfer.

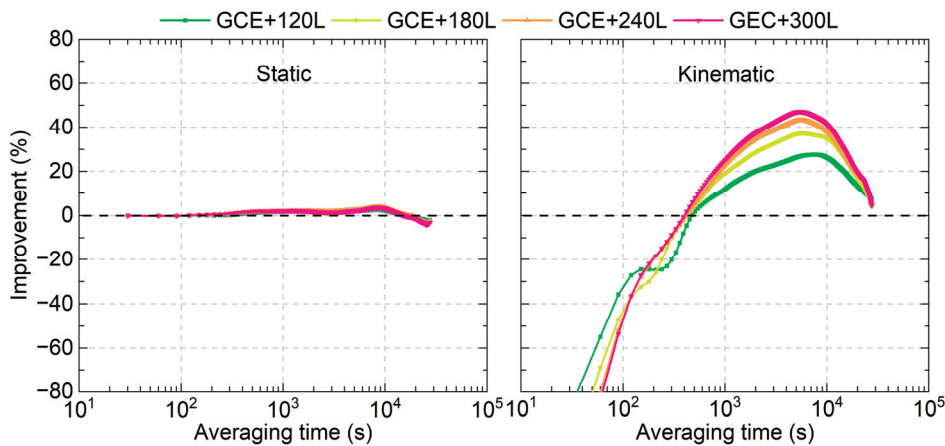


Figure 12. Average time link stability improvements in the LEO-enhanced multi-GNSS real-time PPP time transfer compared to those of multi-GNSS solutions.

5. Discussions

In this study, the sampling interval was set to 30 s, as often used in GNSS-based PPP [1,2]. Compared to using 30 s observations, it was reported that the PPP convergence can be shortened from 7.9, 5.3, and 6.9 min to 4.4, 3.1 and 3.7 min in the east, north and up coordinates, respectively, after using 1 s observations in LEO-enhanced GPS/BDS PPP [49], which concluded that a higher sampling rate is beneficial for further shorten the convergence time. Therefore, in this contribution, we have additionally conducted LEO-enhanced multi-GNSS PPP time transfer using 1 s observations to evaluate the impacts. One-day observations on DOY 124, 2024, were processed with the program restarted every 30 min, resulting in 480 PPP pieces for the assessment. The continuous network solutions from DOY 118 to 125, 2024, were processed using the ESM final GPS satellite orbits, in which the receiver clocks on DOY 124 were used as references. The detailed clock estimation strategy can be found in [50–52].

With the mean offsets removed, the 50%, 68%, and 95% percentile lines of the time link errors are shown in Figure 13. The GCE solutions require 68, 108 and 159 s to converge to 1 ns in the static mode for the three percentile lines, and 74, 119 and 162 s for the kinematic mode. After integrating 120, 180, 240 and 300 LEO satellites, the required times are shortened to 13–26, 18–46, 19–47 s for the static mode, and 13–26, 18–47, and 19–46 s in the kinematic mode. The results show that the 1-sigma (68%) percentile line of the time

link errors can converge to 1 ns within 1 min in using LEO-enhanced multi-GNSS real-time PPP time transfer.

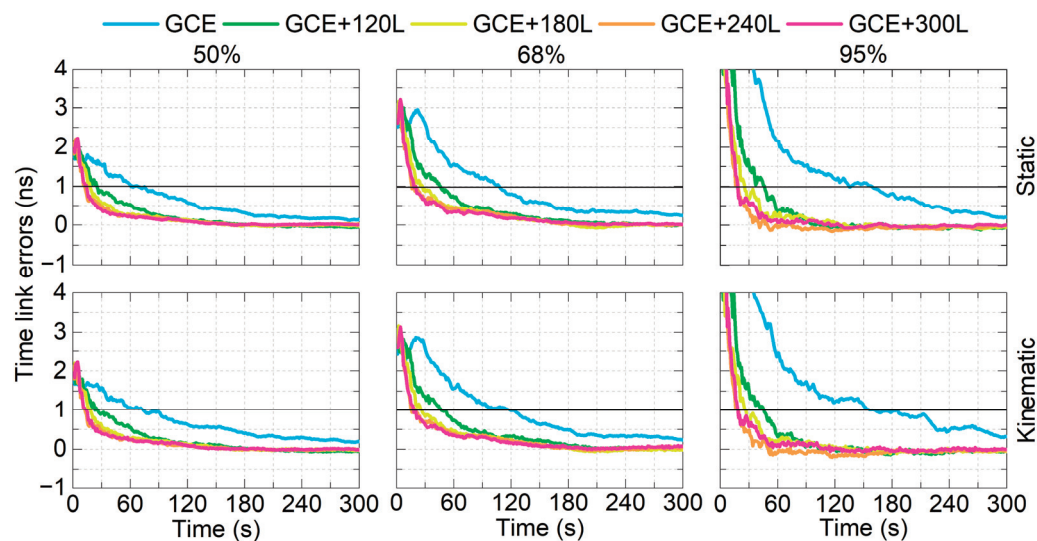


Figure 13. Percentile lines of time link errors of LEO-enhanced multi-GNSS PPP time transfer with different numbers of LEO satellites for the static (**top**) and kinematic mode (**bottom**).

6. Conclusions

GNSS PPP time transfer suffers from a long initialization time due to the slow geometry change in GNSS satellites. The fast motion of LEO satellites helps accelerate changes in satellite geometry, thereby improving the convergence speed. This contribution investigates the performance of LEO-enhanced multi-GNSS real-time PPP time transfer. Simulated LEO constellations consisting of different numbers of LEO satellites (120, 180, 240, and 300) are integrated into the multi-GNSS scenario to perform time transfer in both the static and kinematic modes. The analysis is based on one week of observations from 11 globally distributed stations. 3 h solutions are used to assess the convergence times and the time link precision, while the MDEVs are evaluated using a daily based solution.

Compared to the GCE solutions, the integration of 120, 180, 240, and 300 LEO satellites can increase the average number of observed satellites from 23.4 to 30.6, 34.1, 37.7, and 41.3, respectively, while reducing the TDOP values from 0.547 to 0.424, 0.391, 0.363, and 0.342. Using 30 s observations, the average convergence times for all time links are shortened from 7.95 to 5.94, 4.83, 4.46, and 4.45 min for the static mode, and from 8.75 to 6.18, 5.17, 4.89, and 4.72 min in the kinematic mode. Additionally, the time link precision in the static mode is improved from 0.337 to 0.302, 0.266, 0.248, and 0.243 ns, while it is from 0.377 to 0.313, 0.277, 0.259, and 0.253 for the kinematic mode. The time link stabilities (MDEVs) are enhanced in both the static and kinematic modes compared to the GCE solutions. Using 1 s observations, the 1-sigma percentile line of the time link errors can converge to 1 ns within 1 min after using LEO-enhanced multi-GNSS real-time PPP time transfer.

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Data Availability Statement: The observations can be found at <ftp://igs.gnsswhu.cn/> (accessed on 25 September 2025). The DCB files can be found at <https://data.bdsmart.cn/pub/product/bias/> (accessed on 25 September 2025). The GNSS satellite orbits and clocks can be found at http://www.ppp-wizard.net/products/REAL_TIME/ (accessed on 25 September 2025). The receive clocks can be found at the <http://navigation-office.esa.int/products/gnss-products/> (accessed on 25 September 2025).

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Conflicts of Interest: The authors declare no conflicts of interest.

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Article

Mitigating Integrity Risk in SBAS Positioning Using Enhanced IGG III Robust Estimation

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Highlights

What are the main findings?

- The improved IGG III robust estimation algorithm significantly enhances the user-end positioning performance of SBAS. In various scenarios, it greatly improves the positioning accuracy in both the horizontal and vertical directions, reduces the integrity risk, and enhances the availability.
- This method effectively suppresses the influence of outliers in the observed data, avoids overly conservative protection level (PL) estimation, and obtains more reasonable and reliable protection level values without affecting real-time performance.

What is the implication of the main finding?

- This research offers a powerful and computationally efficient solution that can enhance the service reliability of SBAS in challenging environments such as urban dynamics and UAV maneuvering flight, making it highly suitable for safety-critical applications in aviation and intelligent transportation.
- This algorithm has strong adaptability and stability in multiple practical scenarios, providing an important reference for the performance optimization and integrity guarantee of future SBAS user-end under poor data quality conditions.

Abstract

To address the limitations in positioning accuracy and the risk of integrity degradation in Satellite-Based Augmentation Systems (SBAS) user-end after applying augmentation information, this study proposes a positioning algorithm integrating an improved IGG III robust estimation method. By using integrity information from SBAS, this method improves protection level calculations and better adjusts observed weights by adding new factors to the weight function model. This improvement allows for better discrimination between reliable and anomalous measurements, thereby enhancing positioning accuracy, reducing integrity risks, and improving availability. Experimental results show that, compared to conventional SBAS user positioning, the proposed method achieves notable performance improvements across various scenarios. In static environments, it reduces horizontal integrity risk by up to 6.7%, increases availability by up to 6.6%, and improves positioning accuracy by up to 71.3%. In urban vehicular environments, horizontal integrity risk is reduced by 0.5%, availability is increased by 0.5%, and accuracy

improves by up to 58.7%. In Unmanned Aerial Vehicle flight scenarios, horizontal integrity risk is reduced by 2.8%, availability increases by 2.8%, and accuracy improves by up to 50.38%. In all scenarios, vertical integrity risk is completely eliminated and availability improves slightly. Additionally, compared to the conventional IGG III estimator, the improved method offers more effective control over weight adjustment during solution estimation, thereby avoiding excessive down-weighting and mitigating over-bounding of protection levels. These results demonstrate the potential of the proposed method to improve the performance and reliability of SBAS user-end under both static and dynamic conditions.

Keywords: robust estimation; SBAS; positioning; protection level; integrity

1. Introduction

Satellite-Based Augmentation Systems (SBAS) have been widely implemented to enhance the accuracy, integrity, and availability of Global Navigation Satellite Systems (GNSS) positioning, particularly for safety-critical applications such as aviation and intelligent transportation [1]. Despite the provision of correction and integrity information, SBAS user-end may still experience degraded positioning performance in challenging environments [2], where observation quality is uneven and outliers are more frequent. In such cases, ensuring robust positioning and reliable Protection Level (PL) estimation becomes critical to maintain user trust and service availability. This motivates the development of improved positioning algorithms that can effectively handle observation anomalies while preserving integrity assurance.

At present, with the continuous development of SBAS technology, its user-end positioning method has become more mature and can show better positioning results and integrity performance when the data quality is better. However, in further expanding the application field of SBAS, the complexity of the environment has become more prominent, and the unstable quality of observation data has become more frequent, which leads to poor positioning results and integrity risks despite the use of enhancement information in messages to improve positioning accuracy. Therefore, the SBAS user-end positioning method still needs to be further improved in scenarios with more complex environments and poor data quality.

In recent years, there have been studies from the aspect of multi-system fusion and weighted model optimisation in SBAS user-end positioning. To improve the accuracy of Unmanned Aerial Vehicle (UAV) positioning, Krasuski et al. integrated the augmented data from multiple European Geostationary Navigation Overlay Service (EGNOS) satellites through an average weighted model to improve the accuracy of positioning [3]. Meanwhile, a proposed weighting algorithm based on dual GNSS receivers optimises the weighting model by taking the number of satellite tracking as a weighting factor, which effectively improves the positioning accuracy of System for Differential Correction and Monitoring (SDCM) and EGNOS in air navigation [4]. Thereafter, in order to better apply SBAS to UAV, a positioning accuracy improvement algorithm based on the joint solution of multiple augmentation systems is proposed, which integrates the ionospheric correction data provided by multiple SBAS through the average weighting model to enhance the positioning accuracy of UAV [5]. SBAS in urban environments where Geostationary Earth Orbit (GEO) visibility is poor and SBAS signals are obscured; Yoon et al. proposed online transmission of SBAS correction information through the communication link of the ground control station, which achieves the improvement of positioning accuracy in complex environments [6].

Liang Li et al. used SBAS enhancement parameters for real-time single-frequency Precise Point Positioning (PPP), which significantly improved the positioning accuracy compared to the traditional SBAS method [7]. Park et al. proposed an efficient multi-GNSS and multi-SBAS positioning method by independently calculating the positioning results of each GNSS and each SBAS combination, and then determining the final position by weighted summing, which ultimately improves the positioning accuracy [8]. To address the problem that single-frequency SBAS is affected by ionospheric storms and scintillation, Li et al. proposed a new dual-frequency user positioning algorithm, which improves the positioning accuracy and integrity [9]. Based on the concept of flexible Positioning Navigation Timing (PNT) [10], Zhang et al. proposed an integration framework without ground network, which achieves multi-system error correction by flexibly integrating B1C, B2a and broadcast ephemeris messages, and improves the positioning stability of BeiDou Satellite-Based Augmentation System (BDSBAS) in complex environments [11]. To address the problem that the overlapping areas between different SBAS systems cannot be effectively fused, Wang et al. proposed a fusion positioning algorithm for BDSBAS and Multi-functional Satellite Augmentation System (MSAS) based on fully connected neural networks, which can provide more stable and high-precision augmented navigation services [12]. Wang et al. proposed a novel service weighting model for road transport SBAS Dual-Frequency Multi-Constellation (DFMC), which integrates the Carrier-to-Noise Ratio (CNR) and the smoothing time factor to dynamically adjust the weights of the observed data, which effectively reduces the protection level and improves the availability [13]. Zheng et al. proposed two improved integrity monitoring approaches: one based on covariance analysis and another based on pseudorange residuals combined with satellite-station geometry [14,15]. These methods significantly enhanced system integrity and usability. Wang et al. employed a Kalman filter to amplify the covariance matrix of observation noise, thereby reducing the observation weight when the number of stations decreased. This adjustment improved both the accuracy and continuity of the system [16]. Shao et al. introduced a Dual-Frequency Range Error (DFRE) estimation technique based on the projection method, which establishes an envelope for the maximum correction error with a specified probability [17]. In addition, a dual-frequency SBAS integrity optimization algorithm and an improved over-limit pairing method have been developed [18,19]. These approaches enhance system availability by optimizing weight assignments and modeling error characteristics more precisely. More recently, with the rapid increase in Low Earth Orbit (LEO) satellites and their global coverage, new opportunities have emerged. Xin et al. improved monitoring performance and service coverage by optimizing the observation link, effectively mitigating the impact of satellite entry and exit, and substantially enhancing integrity performance [20]. In summary, there are fewer current improved SBAS user-end positioning methods, and most of them still use traditional user-end positioning methods. In addition, the existing weighted model optimisation has unspecified positioning performance in more complex environments with poorer data quality, or the weighted model is complex and dependent on smoothing time. And although multi-system fusion can improve the positioning performance of SBAS users, it cannot reduce the effect of coarseness in the observations when the data quality is poor, and it is not applicable to single-system users, which has certain limitations.

To address these limitations, this study introduces an improved robust estimation algorithm into the SBAS user positioning process, based on the IGG III framework. The proposed method is designed with consideration of both the weighting behavior of the user algorithm and the PL computation. Through extensive experiments in static, urban vehicular, and UAV flight scenarios, the method is shown to significantly reduce integrity risk, improve availability, and enhance positioning accuracy compared to traditional ap-

proaches. The results also demonstrate that the improved estimator avoids overbounding by maintaining controlled weight variation compared to the traditional IGG III robust estimation, thereby providing a more balanced and effective solution for SBAS user-end positioning.

The remainder of this paper is organized as follows. Method presents the improved IGG III-based robust estimation algorithm, including the design of the weight function and its integration with PL computation. The results describe the experimental setup and test scenarios, covering static, urban, and UAV environments and further discusses the experimental results, focusing on positioning accuracy, integrity risk, and availability performance. Finally, we conclude the study and outline potential directions for future work.

2. Method

2.1. IGG III Variance Expansion Function

The IGG III variance expansion function divides the observed data into three categories according to quality standards: effective information, available information and harmful information. Based on the principle of robust estimation, differentiates information of different quality, that is, effective information retains the original weight value, the available information is estimated with a down-weighting, and the harmful information is given a zero-weighting [21].

The IGG III weight function can be expressed as follows [22]:

$$\bar{P}_i = \begin{cases} P_i & |\tilde{v}_i| \leq k_0 \\ P_i \frac{k_0}{|\tilde{v}_i|} \left(\frac{k_1 - |\tilde{v}_i|}{k_1 - k_0} \right)^2 & k_0 < |\tilde{v}_i| \leq k_1 \\ 0 & |\tilde{v}_i| > k_1 \end{cases} \quad (1)$$

where P_i is the weight of the i -th satellite, k_0 and k_1 are harmonic coefficients that partition the standardized residual $|\tilde{v}_i|$ into three intervals, corresponding, respectively, to effective information, available information, and harmful information. In practical applications, k_0 is typically set in the range of 1.0 to 2.0, k_1 is chosen between 2.5 and 3.0 and $\tilde{v}_i$ is the standardised residuals.

It can be seen from the weight function that the selection of harmonic coefficients directly influences whether observation weights are appropriately reassigned, thereby enabling the effective utilization of reliable observations, the cautious handling of suspicious data, and the exclusion of outliers. Assuming that the observation residuals v_i follow a Gaussian distribution, the corresponding test statistic can be expressed as follows:

$$\tilde{v}_i = \frac{v_i - \mu}{\sigma_0 \sqrt{q_{v_i}}} \sim N(0, 1) \quad (2)$$

where q_{v_i} denotes the weight of the i -th observation, σ_0 is the standard deviation of the residuals, and μ is the mean of the residuals from all observations. Based on the statistical principle of three-sigma σ outlier detection, the IGG III weight function is applied with harmonic coefficients set as $k_1 = 3\sigma$ and $k_0 = 2\sigma$, where σ is the standard deviation of the residuals. Under this configuration, the standardized residuals are divided into three intervals: outliers $(-\infty, -3\sigma) \cup (3\sigma, +\infty)$, accounting for approximately 0.3% of the data; suspicious values $(-3\sigma, -2\sigma) \cup (2\sigma, 3\sigma)$, accounting for about 4.3%; and valid values $(-2\sigma, 2\sigma)$, covering roughly 95.4% of the total. This classification effectively distinguishes between outliers, questionable data, and reliable observations, enhancing the robustness of the estimation process.

2.2. SBAS User-End Positioning and Protection Level Calculation

The SBAS user-end consists primarily of two components: the augmented positioning solution and the PL. The augmented positioning solution utilizes GNSS observation data, broadcast ephemerides, and SBAS correction messages to estimate the user position and receiver clock offset using a least squares estimation approach. The PL computation relies on the integrity information provided by SBAS. If PL exceeds a predefined Alarm Limit (AL), an alert is triggered to prevent potential hazards during navigation. The relationship among PE, PL, and AL, along with the corresponding system service status, is summarized in Table 1. The integrity risk rate refers to the probability that the location information error provided by the navigation system exceeds the specified tolerance range within a given service time, but the system fails to issue an alarm to the user. Availability rate is the ability of a navigation system to provide the required functionality and performance within the expected service time, usually expressed in percentage.

Table 1. Relationship among PE, PL, and AL.

Relationship	Service State	Affect
$PE \leq PL \leq AL$	Available	None
$PL < PE \leq AL$	Misleading Information (MI)	Integrity risk
$AL < PL < PE$	MI	Integrity risk
$PL < AL < PE$	Hazardous MI (HMI)	Integrity risk (missing detections)
$PE < AL < PL$	Unavailable	Alarm

(1) Enhanced positioning solution

SBAS user can improve satellite orbit and satellite clock accuracy using satellite-based augmentation corrections. The pseudorange single-point positioning observation equation (dual-frequency ionosphere-free combination) using the satellite-based enhancement corrections can be expressed as follows [23]:

$$P_j^i(IF) = \rho_j^i + c * (\Delta t_j - \Delta t^i) + T_j^i + \varepsilon_j^i + cor^i \quad (3)$$

where ρ_j^i is the geometric distance from the station to the satellite, c is speed of light, Δt_j and Δt^i are the receiver and satellite clock offsets, respectively, T_j^i is tropospheric slant delay, ε_j^i is the pseudorange noise, and cor^i is the correction using SBAS corrections.

The specific process is as follows: first, the unit direction vector between the receiver and each satellite is computed; then, the satellite orbit corrections are projected onto the line-of-sight vector from the receiver to the satellite; finally, these are combined with the satellite clock corrections to form the total range correction. Unlike conventional single-point positioning, SBAS user positioning incorporates integrity information to determine the variance of the spatial signal error, which is used in the weighting of observations.

$$P = \begin{bmatrix} \frac{1}{\sigma_1^2} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \frac{1}{\sigma_n^2} \end{bmatrix} \quad (4)$$

where $\sigma_i^2 = \sigma_{i,DFC}^2 + \sigma_{i,trop}^2 + \sigma_{i,air_DF}^2 + \sigma_{i,iono}^2$, where $\sigma_{i,DFC}^2$ is the variance of the spatial signal error (specifically, the variance of the orbital clock corrections mapped in the line-of-sight direction), $\sigma_{i,trop}^2$ is the variance of tropospheric residual error, σ_{i,air_DF}^2 is the variance of the dual-frequency measurement noise, multipath, and antenna group delay errors, and

$\sigma_{i,iono}^2$ is the variance of the residuals in the ionosphere from the dual-frequency ionosphere combination, and they can be expressed as

$$\begin{cases} \sigma_{i,DFC}^2 = (\sigma_{DFRE} * \delta_{DFRE})^2 + \varepsilon_{corr}^2 + \varepsilon_{er}^2 \\ \sigma_{i,trop}^2 = 0.12 * \frac{1.001}{\sqrt{0.002001 + \sin(El e)^2}} \\ \sigma_{i,air_DF}^2 = 0.36 + 0.4 * e^{-\frac{El e}{14}} \\ \sigma_{i,iono}^2 = \frac{40.0}{261.0 + (el e_{deg})^2} + 0.018 \end{cases} \quad (5)$$

where σ_{DFRE} is the sigma value of Dual Frequency Range Error (DFRE), which can be retrieved from the correspondence table of DFRE and Dual Frequency Range Error Indicator (DFREI) [24], ε_{corr}^2 and ε_{er}^2 are both degradation parameters, and δ_{DFRE} can be computed from the user positional information and the covariance array information of the satellite.

(2) Protection level solution

The calculation of the PL must balance both integrity and availability requirements. As such, the PL should neither be overly conservative (loose) nor overly optimistic (tight). The PL computation formula adopted in this study is given as follows [25]:

$$\begin{cases} HPL = K_H * d_{major} \\ VPL = K_V * d_U \end{cases} \quad (6)$$

where K_H and K_V are constants. For non-precision approach, $K_H = 6.18$ and no PL is solved in vertical direction. For precision approach, $K_H = 6.0$ and $K_V = 5.33$ [26]; d_{major} and d_U are obtained from the elements in the covariance matrix $invG$, and they can be expressed as

$$\begin{cases} d_{major} = \sqrt{\frac{d_{east}^2 + d_{north}^2}{2} + \sqrt{\left(\frac{d_{east}^2 - d_{north}^2}{2}\right)^2 + d_{EN}^2}} \\ d_{east}^2 = invG(1,1) \\ d_{north}^2 = invG(2,2) \\ d_U = invG(3,3) \end{cases} \quad (7)$$

The covariance matrix can be expressed as follows:

$$invG = (G^T W G)^{-1} \quad (8)$$

where G is the spatial geometric matrix and W is the weight matrix for position solving as follows [27]:

$$G = \begin{bmatrix} -\cos(El e^1) * \sin(Azl^1) & -\cos(El e^1) * \cos(Azl^1) & -\sin(El e^1) & 1 \\ \vdots & \vdots & \vdots & \vdots \\ -\cos(El e^n) * \sin(Azl^n) & -\cos(El e^n) * \cos(Azl^n) & -\sin(El e^n) & 1 \end{bmatrix} \quad (9)$$

$$W = \begin{bmatrix} \frac{1}{\sigma_{1,DFC}^2 + \sigma_{1,trop}^2 + \sigma_{1,air_DF}^2 + \sigma_{1,iono}^2} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \frac{1}{\sigma_{n,DFC}^2 + \sigma_{n,trop}^2 + \sigma_{n,air_DF}^2 + \sigma_{n,iono}^2} \end{bmatrix} \quad (10)$$

where $El e$ and Azl are the elevation angle and azimuth angle of the satellite relative to the user, n is the number of satellites.

2.3. Improved SBAS User Positioning Algorithm Combined with IGG III Robust Estimation

In SBAS user positioning, the observation weight is generally composed of three parts: spatial signal influence, signal propagation process and receiver-related effects. In addition to position estimation, the PL must also be calculated using the same observation weight applied during positioning. When the standardized residual exceeds the threshold, although the traditional IGG III method can reduce the weight of unreliable data and improve positioning accuracy by increasing the observation variance, it adjusts the above three parts at the same time, which has the risk of excessive weight changes, which may lead to the calculated PL overbounding issue. Therefore, directly applying the traditional IGG III robust estimation method is not suitable for SBAS user positioning. To solve this problem, this study uses the spatial signal error variance accurately calculated from integrity information to divide the total variance into two parts: (1) the spatial signal influence, expressed as $\sigma_{i,DFC}^2$; (2) signal propagation, and the comprehensive influence of the receiver-related factors is expressed as $\sigma_{i,trop}^2 + \sigma_{i,air_DF}^2 + \sigma_{i,iono}^2$. The former has higher accuracy, while the latter is mainly related to the height angle. In robust estimation, the two components are adjusted according to the variance of spatial signal error.

To facilitate the selection of appropriate harmonic coefficient values for robust estimation, it is necessary to further analyze the distribution characteristics of the spatial signal error variance, $\sigma_{i,DFC}^2$. In this study, statistical analysis was performed on the values of $\sigma_{i,DFC}^2$ for all GPS satellites on Day of Year (DOY) 159 in 2023. The Cumulative Distribution Function (CDF) of these values is presented in Figure 1. It shows the probability that a random variable takes a value less than or equal to a certain value. The horizontal axis represents the value range of the random variable, and the vertical axis represents the cumulative probability, that is, the probability that the value of the random variable is less than or equal to the corresponding value on the horizontal axis. It can be observed that in 90% of the cases, $\sigma_{i,DFC}^2$ remains below 0.25, with only a small proportion of values exceeding this threshold. The rapid rise in the curve indicates that $\sigma_{i,DFC}^2$ is highly concentrated near a specific value, which further highlights the characteristics of $\sigma_{i,DFC}^2$ distribution: most of the values are closely clustered in a concentrated area, while only a few values are scattered far away from the concentrated area.

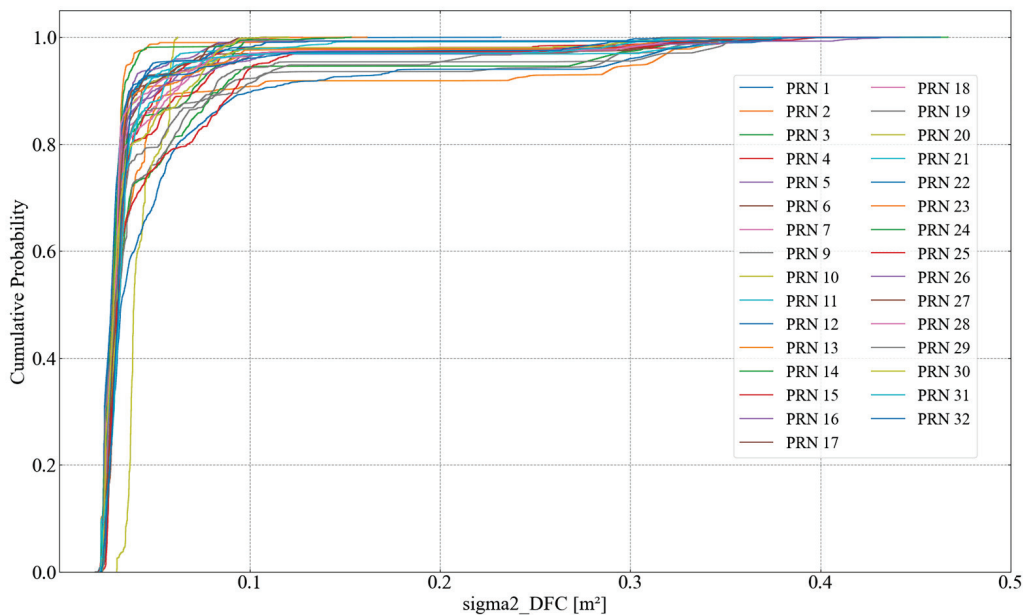


Figure 1. Cumulative distribution function of $\sigma_{i,DFC}^2$ for different GPS.

Meanwhile, $\sigma_{i,DFC}^2$ reflects the correction accuracy of the satellite orbit and clock information. When the correction accuracy is low, the value of $\sigma_{i,DFC}^2$ becomes larger, indicating that the residual ephemeris errors have a more significant impact on user positioning accuracy. Therefore, when the pseudorange residual satisfies the robust estimation condition, the magnitude of $\sigma_{i,DFC}^2$ can be used to selectively adjust the observation variance, thereby controlling the extent of deweighting. This approach helps to avoid excessive suppression of valid measurements. Based on this idea, the improved weight function for SBAS user positioning is formulated as follows:

$$\bar{P}_i = \begin{cases} P_i & |\tilde{v}_i| \leq k_0 \\ \frac{P_i}{\frac{\sigma_{i,DFC}^2}{|\tilde{v}_i|} \left(\frac{k_1 - |\tilde{v}_i|}{k_1 - k_0} \right)^2 + \sigma_{i,trop}^2 + \sigma_{i,air_DF}^2 + \sigma_{i,iono}^2} & k_0 < |\tilde{v}_i| \leq k_1, \tilde{\sigma}_{i,DFC}^2 \geq k_2 \\ \frac{1}{\sigma_{i,DFC}^2 + (\sigma_{i,trop}^2 + \sigma_{i,air_DF}^2 + \sigma_{i,iono}^2) / \left(\frac{k_0}{|\tilde{v}_i|} \left(\frac{k_1 - |\tilde{v}_i|}{k_1 - k_0} \right)^2 \right)} & k_0 < |\tilde{v}_i| \leq k_1, \tilde{\sigma}_{i,DFC}^2 < k_2 \\ 0 & k_1 < |\tilde{v}_i| \end{cases} \quad (11)$$

For ease of representation, the spatial signal error variance $\sigma_{i,DFC}^2$ is standardized in this paper and denoted as $\tilde{\sigma}_{i,DFC}^2$. The threshold for normalization is defined as twice the median value of the error. The harmonic coefficients k_0 and k_1 are retained as in the traditional IGG III formulation, where k_0 is typically chosen between 1.0 and 2.0, and k_1 is set between 2.5 and 3.0. Here, $\tilde{v}_i$ denotes the standardized residual, P_i is the weight of the i -th satellite. $\sigma_{i,trop}^2$, σ_{i,air_DF}^2 and $\sigma_{i,iono}^2$ have explained in Equation (4). In addition, it can be seen from the calculation formula of the protection level in Equations (6)–(10) that the covariance matrix of the positioning robust estimation output will be used to calculate the protection level. Therefore, the improved IGG III robust estimation also has corresponding optimization effects on the protection level calculation while finely adjusting the weight.

The flowchart of the improved robust algorithm for SBAS user positioning is shown in Figure 2. First, an initial positioning solution is obtained, and convergence is checked. If the solution has not yet converged, the pseudorange residuals are computed and standardized, while the spatial signal error variances are also statistically analyzed and standardized. Next, a decision is made on whether to apply down-weighting based on whether the standardized residual exceeds a predefined threshold. If down-weighting is required, the observation variance is selectively adjusted according to the magnitude of the standardized spatial signal error variance. The updated weights are then used in the augmented positioning solution to improve positioning accuracy. If no weight adjustment is needed, the original weights are retained. After convergence is reached through iterative updates, the PL is computed accordingly.

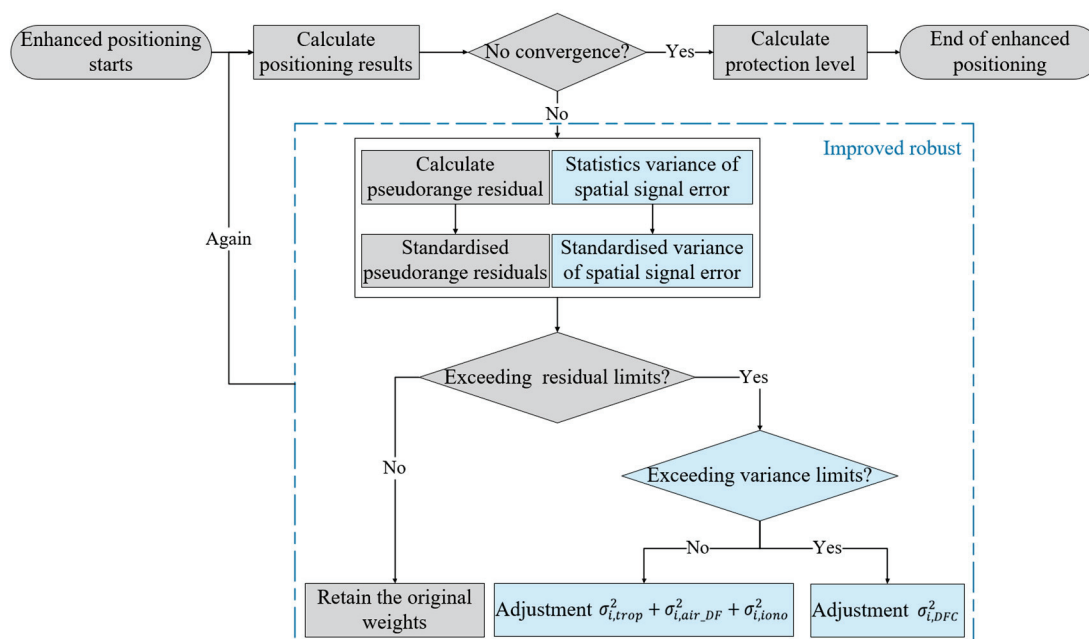


Figure 2. The flowchart of improved SBAS user positioning robust algorithm.

3. Experiment and Result Analysis

3.1. Processing Strategy and Data Quality Analysis

This study conducts experimental analysis through three scenarios: static, urban environment, and UAV maneuvering flight to verify the effectiveness of the algorithm in terms of accuracy, availability, integrity, and PL. Among them, the static experimental data selects the DOY 159–165 in 2023 actual measured data of three monitoring stations in China (green) CSH2, FUZ2, and KAS2. The sampling rate is 1 s. There are 21 enhanced stations (red). They are used to calculate the correction information of SBAS and serve the user-end positioning. The station distribution is shown in Figure 3.

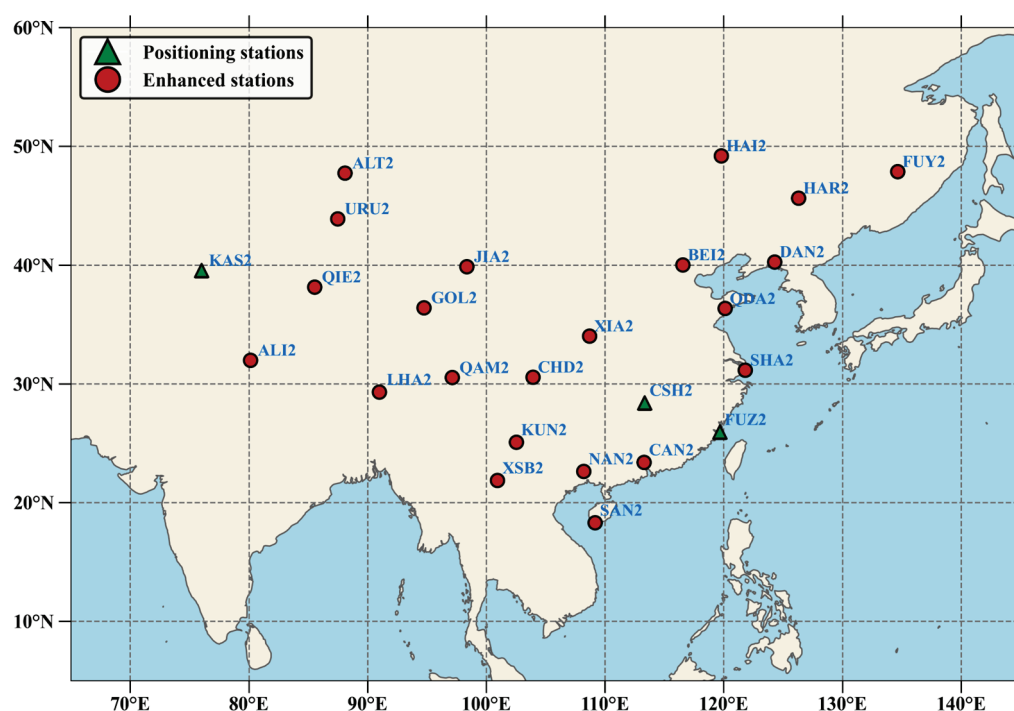


Figure 3. Distribution of stations.

To verify that the corrective values employed in this study are neither anomalous nor in violation of SBAS end-user requirements, data from the enhanced stations illustrated in Figure 3 were used to compute the corrections. The methodology and procedural flow are outlined in Figure 4, and the key steps are summarized as follows: First, dual-frequency ionosphere-free combinations of pseudorange observations were generated to eliminate the majority of ionospheric delays. The combined observations were subsequently smoothed, after which the effects of satellite clock errors and tropospheric delays were removed, yielding pseudorange residuals that primarily contained errors from satellite orbit, satellite clock, and receiver clock. Following clock synchronization, the receiver clock error was eliminated. Finally, the post-synchronization residuals were utilized to estimate the satellite orbit and clock corrections. It should be emphasized that the corrections derived in this study are directly integrated into the user positioning algorithm rather than being broadcast. In most practical applications, such corrections are typically transmitted to users through data messages; however, this delivery mechanism does not compromise the accuracy of the estimated parameters.

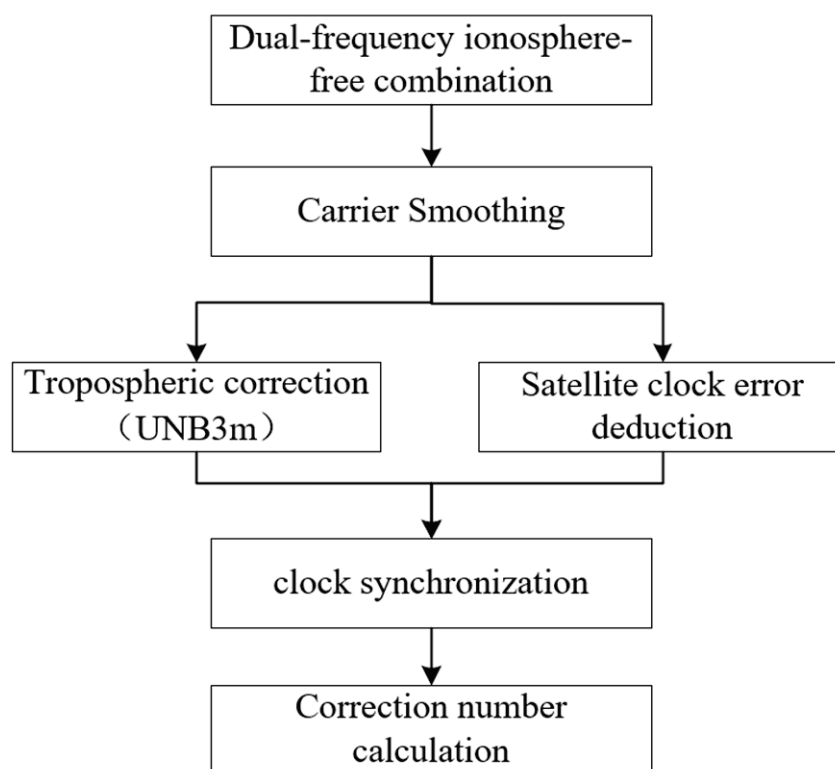


Figure 4. The process of correcting the number calculation.

Figure 5 counts the pseudorange residuals before and after enhancement of all GPS satellites over a total of 159–165 days, and uses the pseudorange residuals before and after enhancement as the evaluation criteria. The specific calculation is as follows: first calculate the average value of the pseudorange residuals before enhancement from all enhancement stations for a given satellite, and then calculate the average value of the pseudorange residuals after enhancement using the correction number, repeating the process until all satellites are evaluated. Compared with the values before enhancement, the pseudorange residuals after enhancement for all satellites are reduced to varying degrees, and the values of the pseudorange residuals after enhancement fall between 0.1 m and 0.3 m, indicating that the corrections used in this study are not abnormal and the accuracy is in line with the current level.

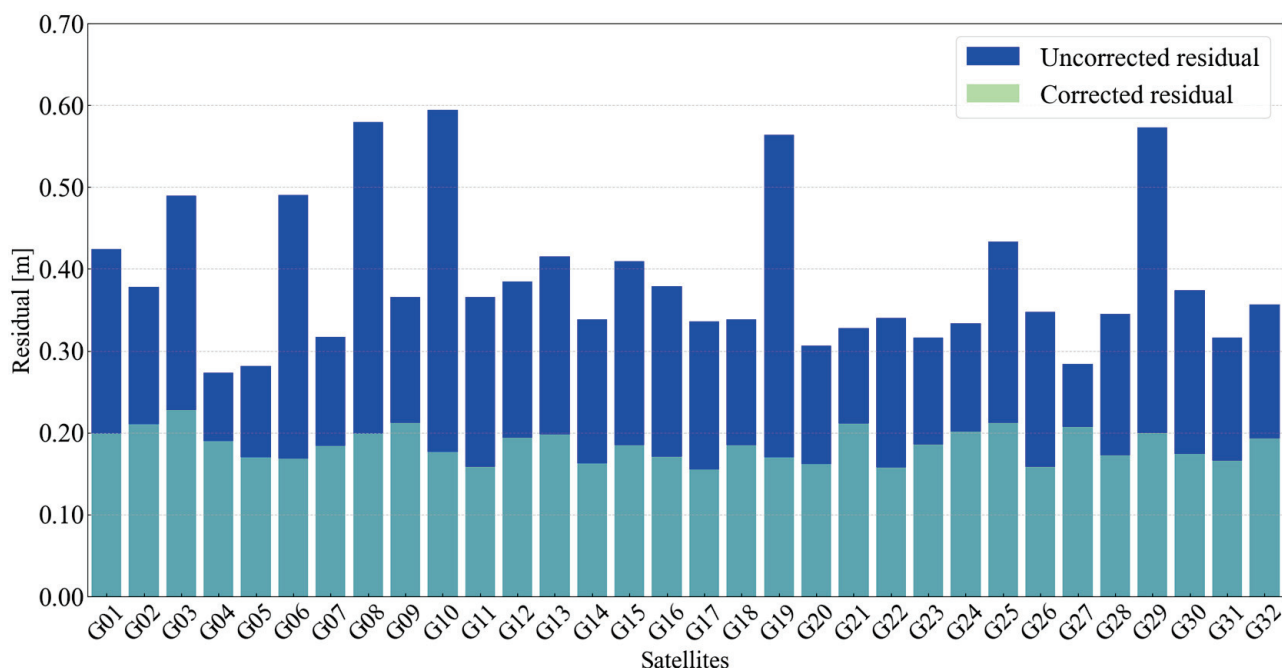


Figure 5. DOY 159–165 pseudorange residuals before and after enhancement of GPS satellites.

To demonstrate that the improved IGG III robust estimation method proposed in this study does not introduce substantial computational overhead for SBAS users, Figure 6 presents a comparison of the computation delays of SBAS-enhanced positioning with and without the improved algorithm. The results indicate that the average delays are 28.3 ms and 32.69 ms, respectively. The overall difference is minimal, corresponding to an average increase of 4.4 ms. Such an increment is negligible relative to current hardware capabilities and therefore does not compromise real-time applicability. Moreover, when deployed on high-performance servers, the delay difference is expected to be further reduced.

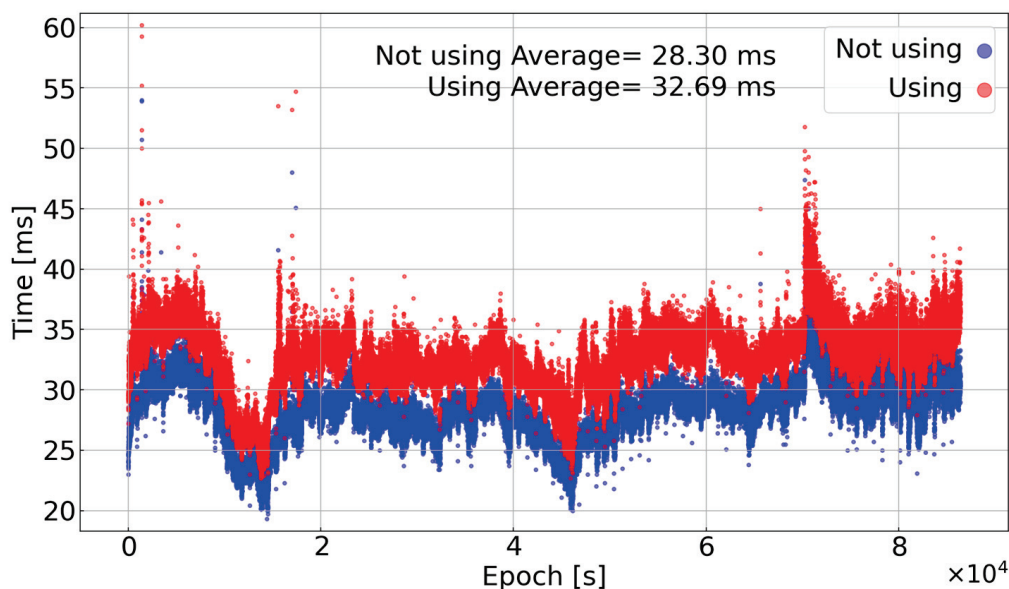


Figure 6. The computational delay of not using the improved algorithm and using the improved algorithm.

The vehicle dataset in the urban environment is collected using a passenger car driving through city streets. The data acquisition lasts for approximately one hour, with a sampling interval of 1 s. The collection environment and trajectory are illustrated in Figure 7. The

UAV maneuvering flight dataset is obtained from a flight conducted in a relatively open environment, lasting around 20 min, also with a sampling rate of 1 s. It is worth noting that only GPS data are used in this experiment, with no additional onboard sensors. Therefore, the data collection environment is relatively unobstructed and does not include areas with severe signal occlusion. The experimental equipment in the two scenarios is shown in Figure 8.



Figure 7. Scenario and real tracks.



Figure 8. Vehicle-mounted and UAV collection equipment.

A comparison of the positioning strategies and solutions for dual-frequency SBAS user is summarized in Table 2. Solution 1 represents enhanced positioning without the application of robust estimation, while Solution 2 incorporates the improved robust algorithm into the positioning process. In both cases, the PL is calculated using the observation weights derived from their respective enhanced positioning strategies.

Table 2. Solution and enhanced positioning strategy.

Item	Solution 1	Solution 2
Signal frequency	GPS L1&L2	GPS L1&L2
Orbital correction	Broadcast ephemeris + DFMC SBAS	Broadcast ephemeris + DFMC SBAS
Clock correction	Broadcast ephemeris + DFMC SBAS	Broadcast ephemeris + DFMC SBAS
Ionospheric correction	Dual-frequency combination	Dual-frequency combination
Tropospheric correction	UNB3m	UNB3m
Elevation cut-off	10°	10°
Robust	No	Yes

In order to show that the experimental data quality in the three scenarios meets the premise of poor data quality, the GPS (L1 and L2) observation data in the three scenarios are analyzed in terms of Signal-to-Noise Ratio (SNR), multipath error, and Position Dilution of Precision (PDOP) value. This study uses a box plot to count different values. The upper and lower boundaries of the box represent the first quartile (Q1) and the third quartile (Q3), respectively. A line in the box represents the median (Q2) of the data. The values beyond the whiskers are considered outliers, which are values other than 2.7 times the median value of the error. The data quality results are illustrated in Figure 9. Experimental data are affected to varying degrees by environmental factors, which cause outliers to appear in different indicators. In the static scenario, CSH2, KAS2, and FUZ2 stations show large outliers in PDOP values and multipath errors. Although there are few outliers in the SNR (L2), the dispersion is relatively high. In the on-board scenario of urban environments, there are large outliers in the SNR (L1), PDOP value, and multipath error, among which the outliers in the SNR are the most significant. In the drone maneuvering flight scenario, outliers appear in the SNR, PDOP value, and multipath error, and the range is very large in the SNR (L1). In addition, this study also counts the average values of each indicator under different scenarios in Table 3. It can be seen that except for some outliers, the overall quality of observation data in each scenario can meet the positioning requirements, and the overall quality of observation data in the drone maneuvering flight scenario is the worst. In summary, the data quality in the three scenarios is poor, and many outliers appear, which eventually have a great impact on the positioning results and integrity. This can be used as experimental data in this paper to effectively verify the effectiveness of the algorithm.

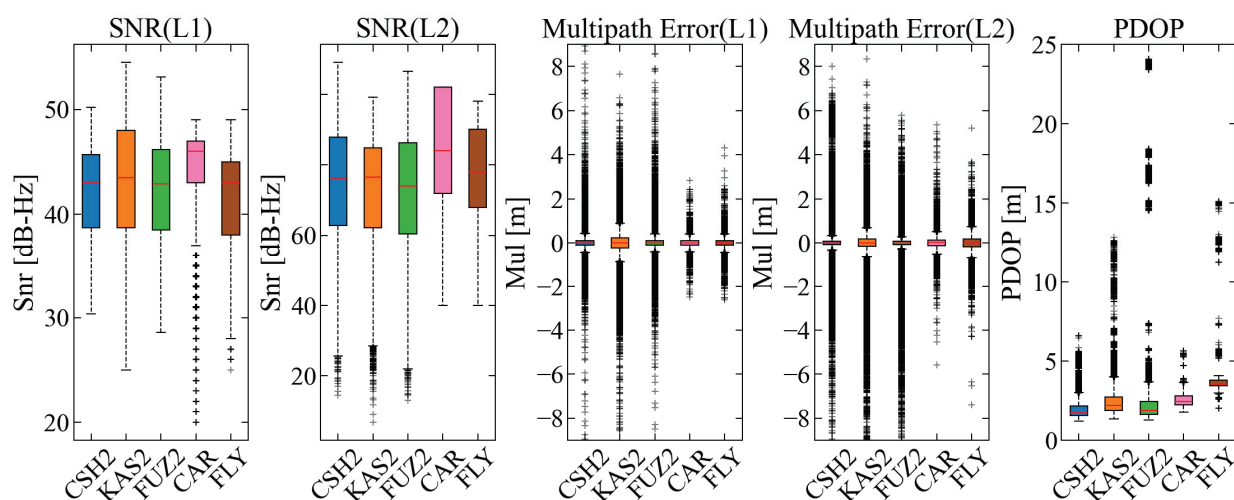
**Figure 9.** GPS (L1, L2) data quality analysis chart in different scenarios.

Table 3. Average of data quality analysis indicators in different scenarios.

Station ID	SNR (L1)/dB-Hz	SNR (L2)/dB-Hz	Multipath Error (L1)/m	Multipath Error (L2)/m	PDOP/m
CSH2	42.16	37.25	0.17	0.14	1.98
KAS2	43.29	36.34	0.35	0.25	2.44
FUZ2	42.42	36.47	0.16	0.14	2.2
CAR	44.15	41.00	0.13	0.17	2.51
FLY	41.59	38.83	0.17	0.31	4.13

3.2. Static Scenario

The three stations used two methods of solution 1 and solution 2 for positioning and protection level calculation, and compared with the APV-I standard (HAL is 40 m, VAL is 50 m), the percentage of availability improvement and the percentage of integrity risk reduction in solution 2 is DOY 159–165 compared with solution 1, as shown in Figure 10. The CSH2, KAS2 and FUZ2 stations used the solution 2 method to reduce the percentage of horizontal integrity risk by 6.5, 11.15 and 0.66, respectively, and the percentage of horizontal availability by 6.5, 11.12 and 0.66, respectively, and the percentage of vertical integrity risk by 5.18, 2.45 and 0.28, respectively, and the percentage of vertical availability by 5.08, 2.09 and 0.18, respectively. It shows that in static scenarios, adding an improved robust algorithm can effectively reduce the risk of integrity and improve availability.

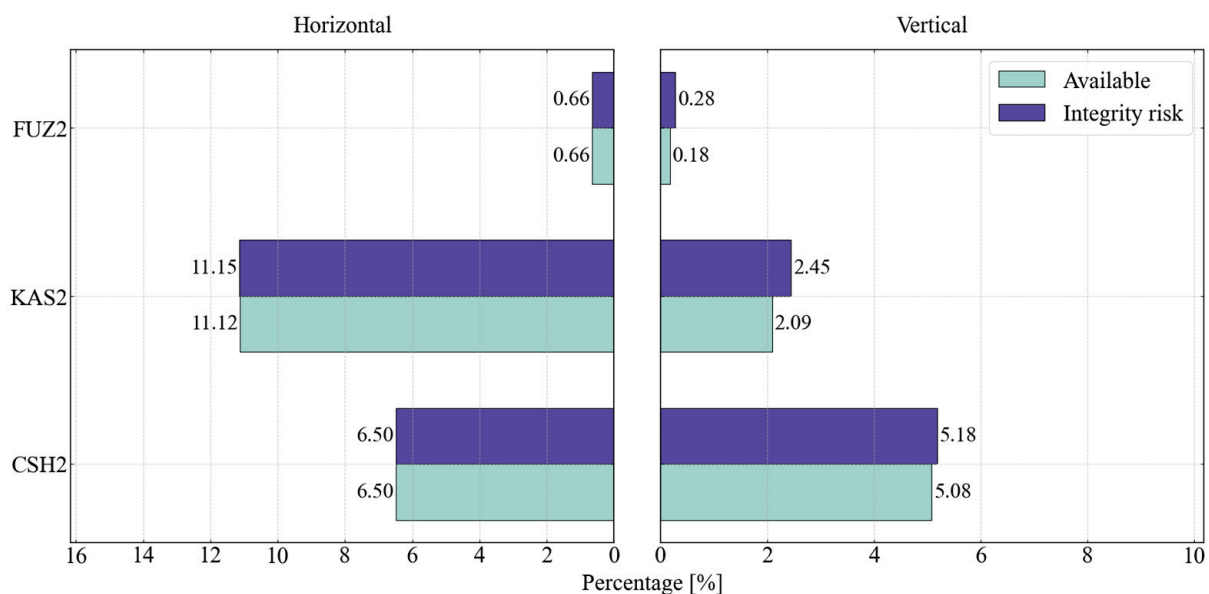


Figure 10. Percentage of availability improvement and integrity risk reduction for three stations using improved robust algorithm.

Taking the annual accumulation date of 159 as an example, we will analyze the performance of solution 2 in three aspects: integrity, positioning accuracy and protection level. In this study, Stanford diagrams were used to calculate its integrity performance according to the APV-I standard, as shown in Figures 11–13. The horizontal axis is the positioning error, the vertical axis is the protection level, and the red line is the alarm limit. The figure is divided into 5 sub-graphs according to the size relationship between them, representing different navigation states (Table 1). The area in the upper left corner of the figure indicates that the system is unavailable when the position error exceeds the protection level. The area in the lower left corner indicates that the positioning information provided by the system is available within the protection level. The orange area in the lower right corner indicates that the position error is below the protection level, but the

positioning information provided by the system may mislead the user. The red area in the lower right corner indicates that the position error exceeds the protection level. The positioning information provided by the system will not only mislead the user, but may also pose danger.

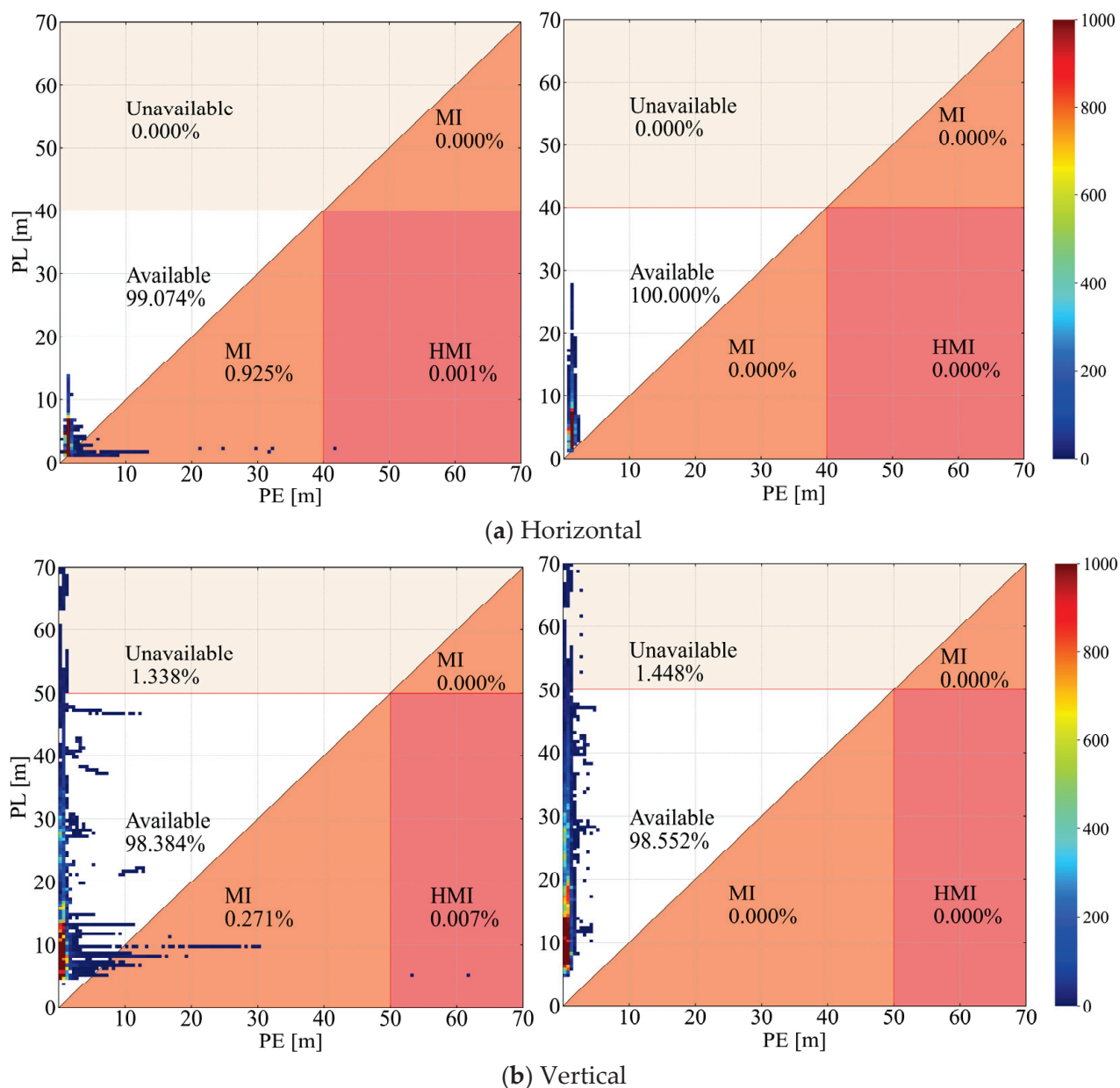


Figure 11. Stanford diagram of CSH2 station pre-robust (left) and post-robust (right).

Solution 1: Although the three stations use SBAS correction information, the positioning error (PE) still large, and there are still occurring MI and HMI. Solution 2: The PE of the three stations is significantly improved, reducing the percentage of MI and HMI, and the availability rate is also improved. Details are outlined in Table 4. Specifically, in the horizontal direction, MI rates dropped from 0.931%, 0.863%, and 10.441% to 0%, 0.002%, and 3.72%—decreasing by approximately 0.93%, 0.85%, and 6.72%, respectively—while HMI was eliminated entirely. Availability correspondingly rose from 99.07%, 98.82%, and 89.49% to 100%, 99.66%, and 96.12%, an improvement of 0.93%, 0.84%, and 6.62%. Vertically, MI and HMI were completely eliminated, and availability increased modestly from 98.38% to 98.55%, from 98.13% to 98.27%, and from 95.91% to 96.06%, gains of 0.17%, 0.14%, and

0.15%. The results show that the improved robust algorithm can further reduce the missing detections rate and integrity risk rate while weakening the impact of the coarse errors, so as to achieve effective alarms, and to improve the availability rate.

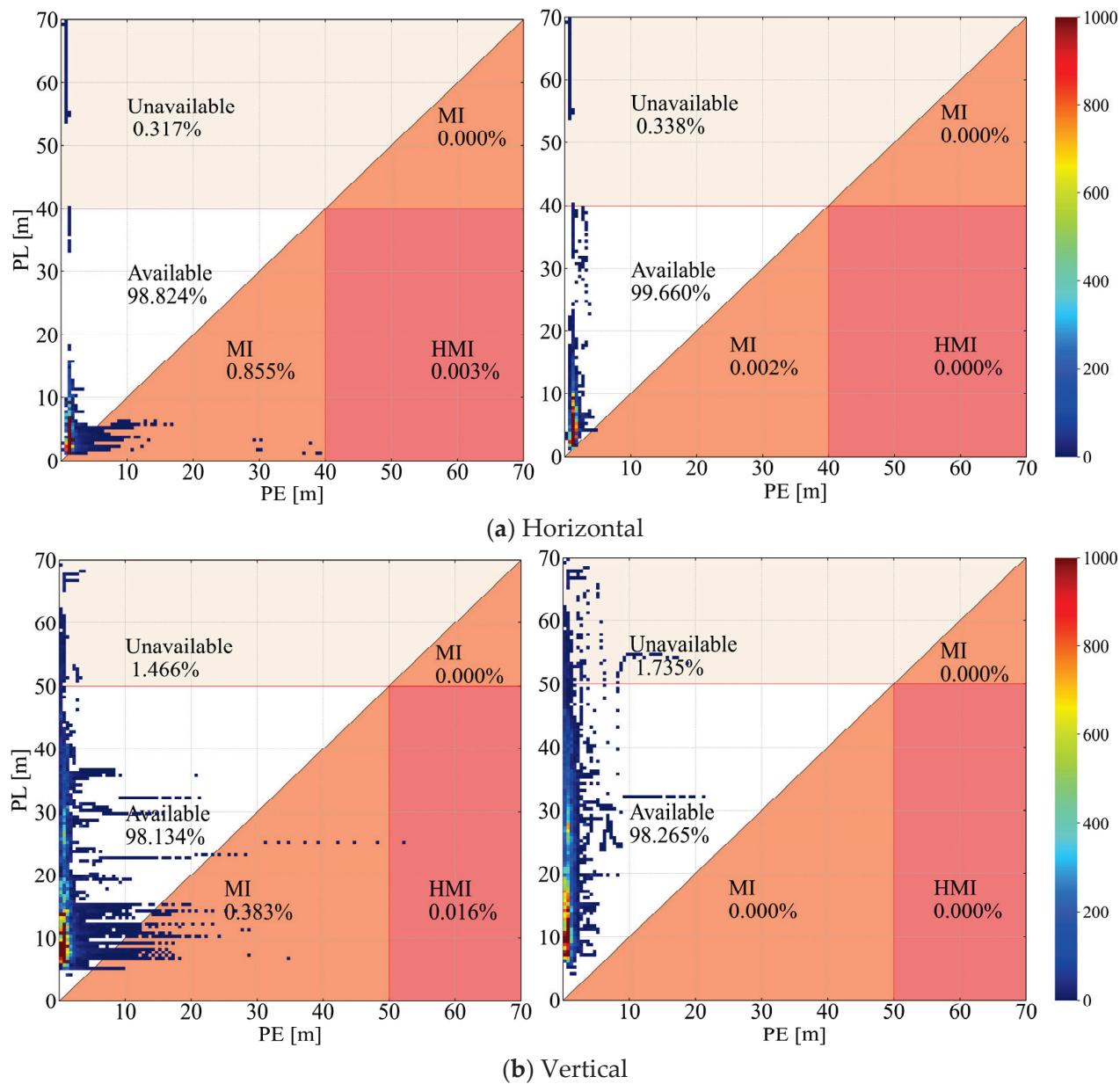


Figure 12. Stanford diagram of FUZ2 station pre-robust (left) and post-robust (right).

Table 4. Statistics of integrity monitoring performance of three stations.

ID	Solution	Horizontal			Vertical		
		Availability	HMI	MI	Availability	HMI	MI
CSH2	Solution 1	99.07%	0.001%	0.93%	98.38%	0.007%	0.27%
	Solution 2	100%	0%	0%	98.55%	0%	0%
FUZ2	Solution 1	98.82%	0.003%	0.86%	98.13%	0.02%	0.38%
	Solution 2	99.66%	0%	0.002%	98.27%	0%	0%
KAS2	Solution 1	89.49%	0.001%	10.44%	95.91%	0.002%	0.48%
	Solution 2	96.12%	0%	3.72%	96.06%	0%	0%

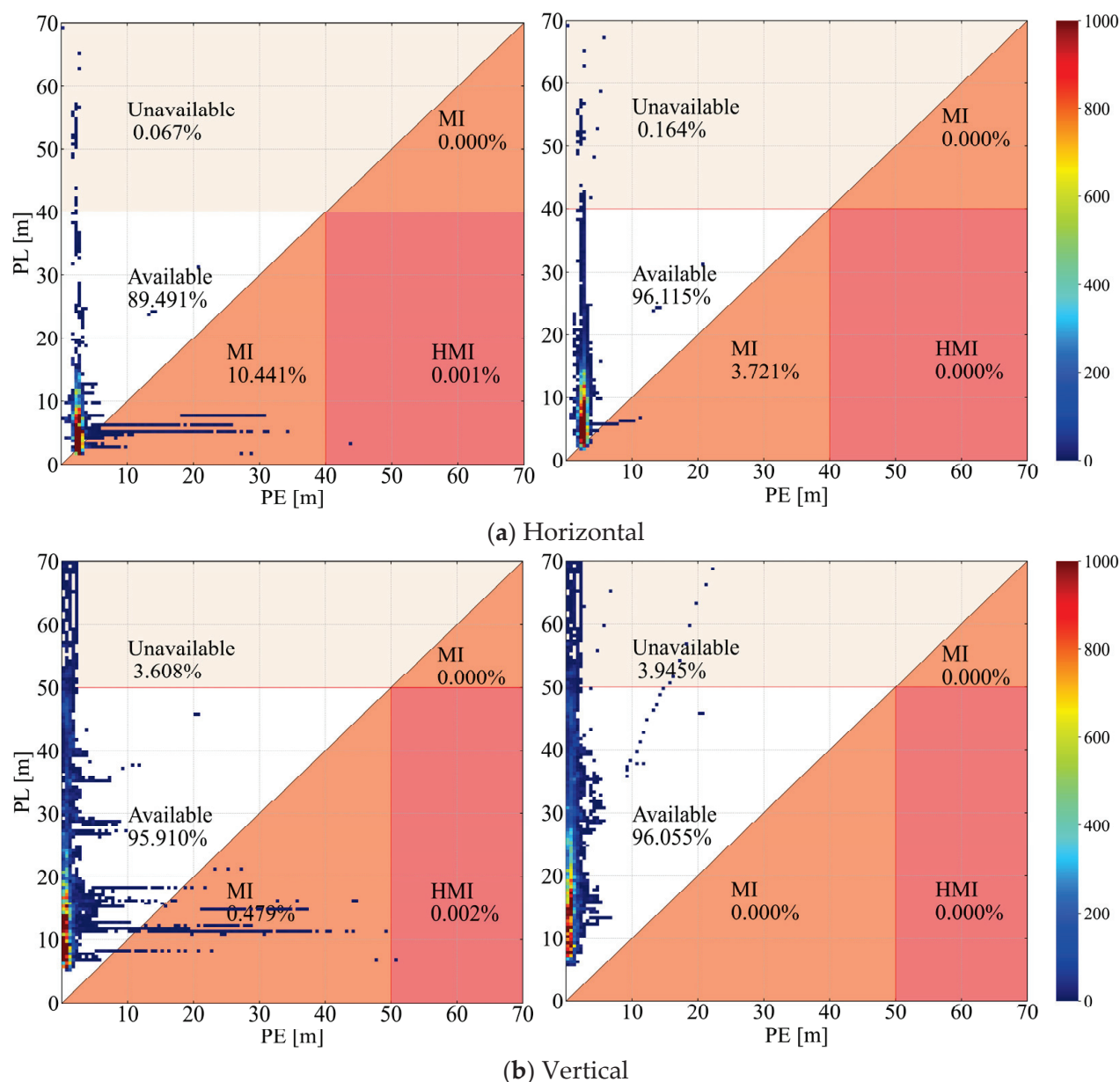


Figure 13. Stanford diagram of KAS2 station pre-robust (left) and post-robust (right).

Through the presence of outliers in the data quality indicators, it can be observed that under the influence of the surrounding environment, the quality of certain observations is significantly degraded, leading to substantial errors. These inaccuracies are likely attributable to factors such as receiver quality, short-term adverse atmospheric conditions, signal reflections from buildings in urban settings, and temporary unfavorable satellite geometric configurations. In order to more clearly analyze the improvement effect of the improved robust algorithm on the positioning results, we also compared and analyzed with the M-estimators method, and the PE sequence diagrams of the horizontal and vertical directions of the three stations are drawn and their internal and external consistency accuracy are counted, as shown in Figures 14–16 and Table 5. The positioning results of Solution 2 are smoother than those of Solution 1, which effectively weakens the influence of coarse errors. As shown in Table 5, the Standard Deviation (STD) and Root Mean Square Error (RMSE) of horizontal and vertical directions of the three stations in Solution 2 are reduced compared with that of Solution 1. Among them, the horizontal and vertical internal consistency accuracy of the KAS2 station increased from 1.22 to 0.35 and from 1.87 to 0.62,

respectively, which increased by 71.31% and 66.84%, respectively; the external consistency accuracy increased from 2.73 to 2.4, and from 1.99 to 0.82, respectively, which increased by 12.09% and 58.79%, respectively. The horizontal and vertical internal consistency accuracy of the FUZ2 station increased from 0.52 to 0.2 and from 1.28 to 0.62, respectively, which increased by 61.54% and 51.56%, respectively; the external consistency accuracy increased from 1.13 to 1.01 and from 1.48 to 0.82, which increased by 10.62% and 44.59%, respectively. The horizontal and vertical internal consistency accuracy of the CSH2 stations increased from 0.42 to 0.14 and from 0.95 to 0.3, respectively, which increased by 66.67% and 68.42%, respectively; the external consistency accuracy increased from 1.08 to 0.99 and from 1.04 to 0.43, which increased by 8.33% and 58.65%, respectively. The results show that when the data quality is poor and the PE is still large even with the addition of SBAS correction information, adding the improved robust algorithm to the SBAS user positioning can effectively weaken the effect of coarse errors and improve the positioning accuracy. The M-estimators method can also effectively reduce the impact of coarse errors, but compared with the improved IGG III robust estimation algorithm, the improvement effect is weaker. After the robust estimation, a very small number of elements still have large errors, and this study considers that there are two reasons: firstly, the number of satellites participating in the positioning of this element is relatively small, and no robust treatment; secondly, this element has too many satellites involved in robust estimation, which makes the robust estimation ineffective.

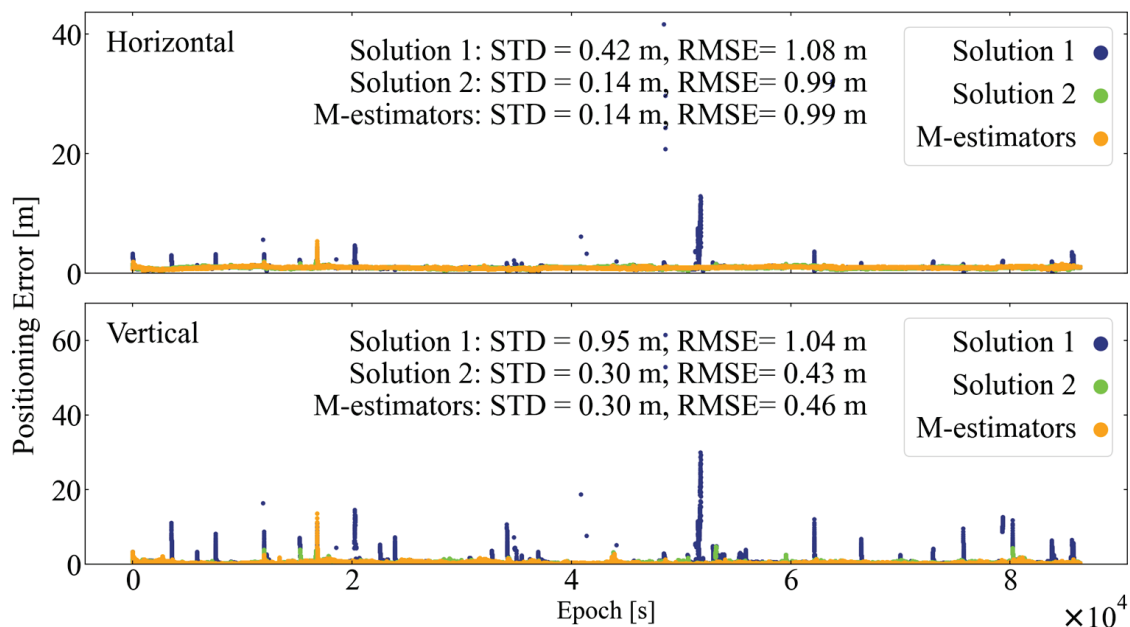


Figure 14. Comparison of positioning errors of CSH2.

Table 5. Statistics of positioning errors of three stations.

Station ID	Solution 1	Horizontal		Vertical	
		STD/m	RMSE/m	STD/m	RMSE/m
KAS2	Solution 1	1.22	2.73	1.87	1.99
	Solution 2	0.35	2.4	0.62	0.82
	M-estimators	0.85	2.64	1.43	1.61
FUZ2	Solution 1	0.52	1.13	1.28	1.48
	Solution 2	0.2	1.01	0.62	0.82
	M-estimators	0.29	1.04	0.89	1.07

Table 5. Cont.

Station ID	Solution 1	Horizontal		Vertical	
		STD/m	RMSE/m	STD/m	RMSE/m
CSH2	Solution 1	0.42	1.08	0.95	1.04
	Solution 2	0.14	0.99	0.3	0.43
	M-estimators	0.14	0.99	0.3	0.46

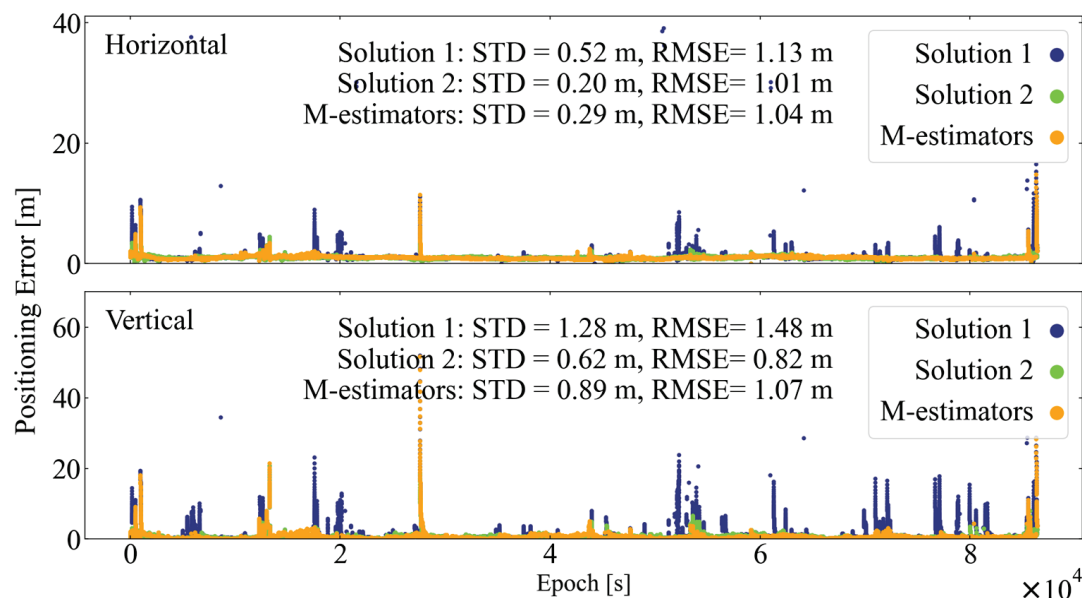


Figure 15. Comparison of positioning errors of FUZ2.

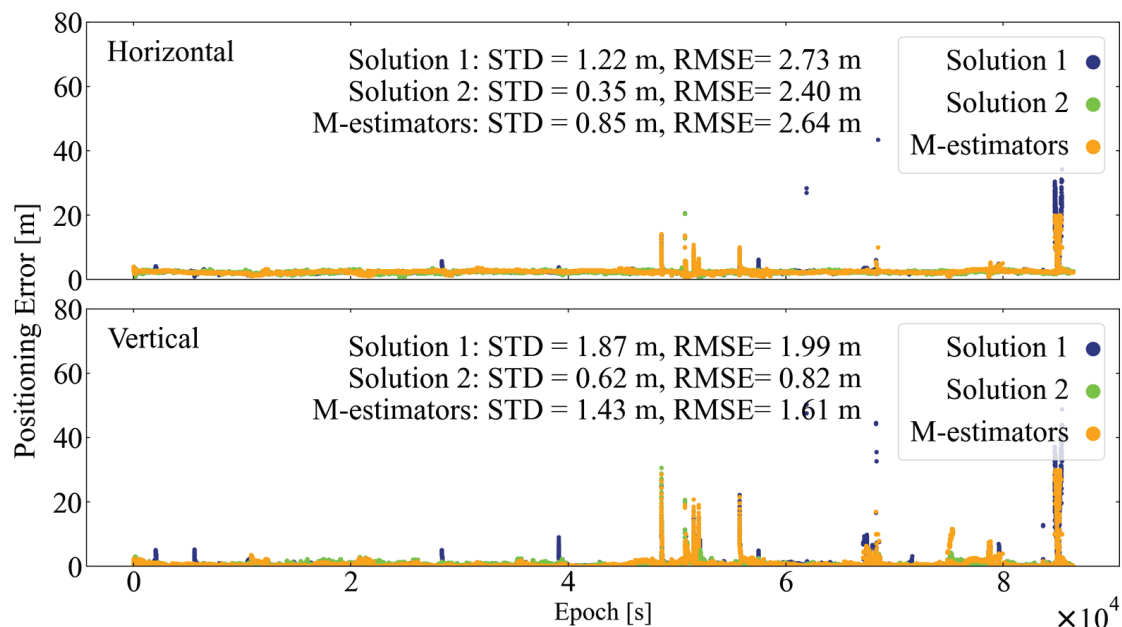


Figure 16. Comparison of positioning errors of KAS2.

To verify that the proposed improved robust algorithm offers better control over weight adjustment in SBAS user positioning compared to the traditional IGG III method, and to prevent excessive conservatism in protection level (PL) calculation caused by robustness strategies, a comparative analysis was also conducted with the PL results derived from the M-estimators method. The protection levels calculated using the traditional IGG III method, the improved robust algorithm, and the M-estimators approach are presented

in Figures 17–19. The three stations use the Horizontal Protection Level (HPL) and Vertical Protection Level (VPL) of the improved robust algorithm in part of the epoch is smaller than the traditional IGG III robust algorithm, which indicates that the improved robust algorithm will not be too ‘loose’ on the PL solving under the premise of meeting the integrity and positioning accuracy. The PL calculation under the M-estimators method is the most conservative, further explaining the superiority of the improved algorithm. In other epochs, the PL difference between the traditional IGG III robust algorithm and the improved IGG III robust algorithm is not obvious, because when the enhancement information is better, the variance of the corrections is smaller compared with the variance of other errors, and at this time, the variance of the corrections has less influence in the deweighting, which makes the PL calculated by the two algorithms comparable.

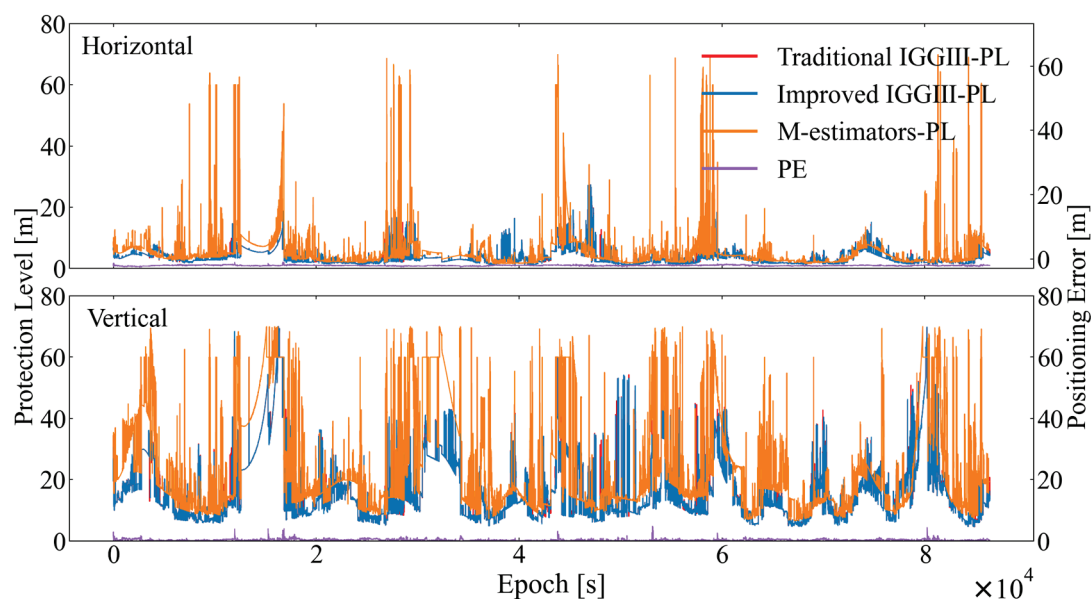


Figure 17. Comparison of protection level of CSH2 (Red is the protection level calculated by traditional IGG III robust algorithm, blue is the protection level calculated by the improved robust algorithm in such pictures, orange is the protection level calculated by the M-estimators method in such pictures, and purple is the positioning error).

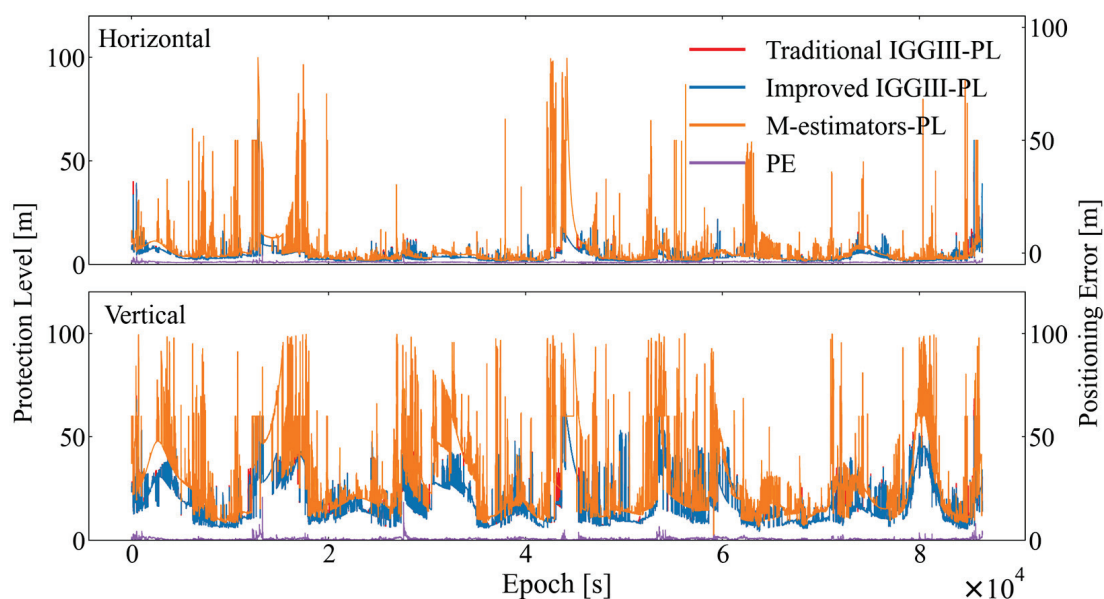


Figure 18. Comparison of protection level of FUZ2.

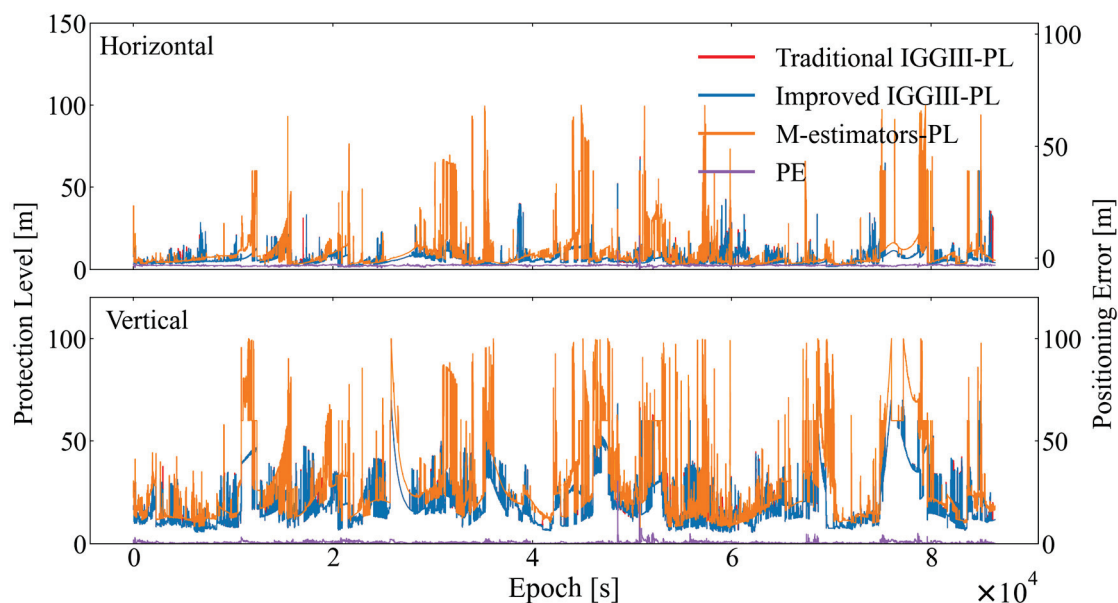


Figure 19. Comparison of protection level of KAS2.

3.3. Urban Environment Vehicle Scenario

The data in the urban environment are used to perform positioning and protection-level solutions using two methods: Solution 1 and Solution 2, respectively. The RTK result is used as the true value, and the APV-I standard is selected to count its integrity performance. The results are shown in Figure 20. In Solution 1, in the urban environment, which is susceptible to environmental interference, using SBAS correction information, the PE still exists in a large situation, and the MI and HMI occurs; in Solution 2, the horizontal and vertical PE is improved, the percentage of MI and HMI are reduced, and the availability rate is improved. The percentage of MI in the horizontal direction decreased from 0.6% to 0.08%, a decrease of 0.52%, and the availability increased from 99.39% to 99.91%, an increase of 0.52%. The vertical percentages of MI and HMI both decreased to 0%, and the availability increased from 98.73% to 98.79%, an increase of 0.06%. The results show that the improved robust algorithm can reduce the missing detections rate and integrity risk by weakening the effect of the coarse errors in the SBAS-enhanced urban environment vehicle scenario, achieve effective alarms, and improve the availability rate.

In order to more clearly analyze the improvement effect of the improved robust algorithm on the positioning results, the PE sequence and three-dimensional motion trajectory in the horizontal and vertical directions, which are illustrated in Figure 21. Compared with Solution 1, the positioning results of Solution 2 are more stable, eliminating larger error jumps. In addition, the STD and RMSE in Solution 2 are both lower than Solution 1. Significant improvements were observed across all accuracy metrics. The horizontal internal consistency accuracy improved by 23.28%, increasing from 0.73 to 0.56. The horizontal external consistency accuracy showed a 5.4% enhancement, rising from 1.48 to 1.4. In the vertical direction, the internal consistency accuracy increased by 58.77%, from 2.28 to 0.94, while the external consistency accuracy improved by 49.8%, from 2.63 to 1.32. The M-estimators method effectively improves positioning accuracy, but the improvement effect is not as good as Solution 2. Figure 22 further demonstrates the effect of the improved robust algorithm in the motion trajectory, with a smoother elevation direction. The results show that the use of improved robust algorithms in the positioning of the SBAS-enhanced urban environment vehicle scenario can effectively weaken the influence of coarse errors and improve positioning accuracy.

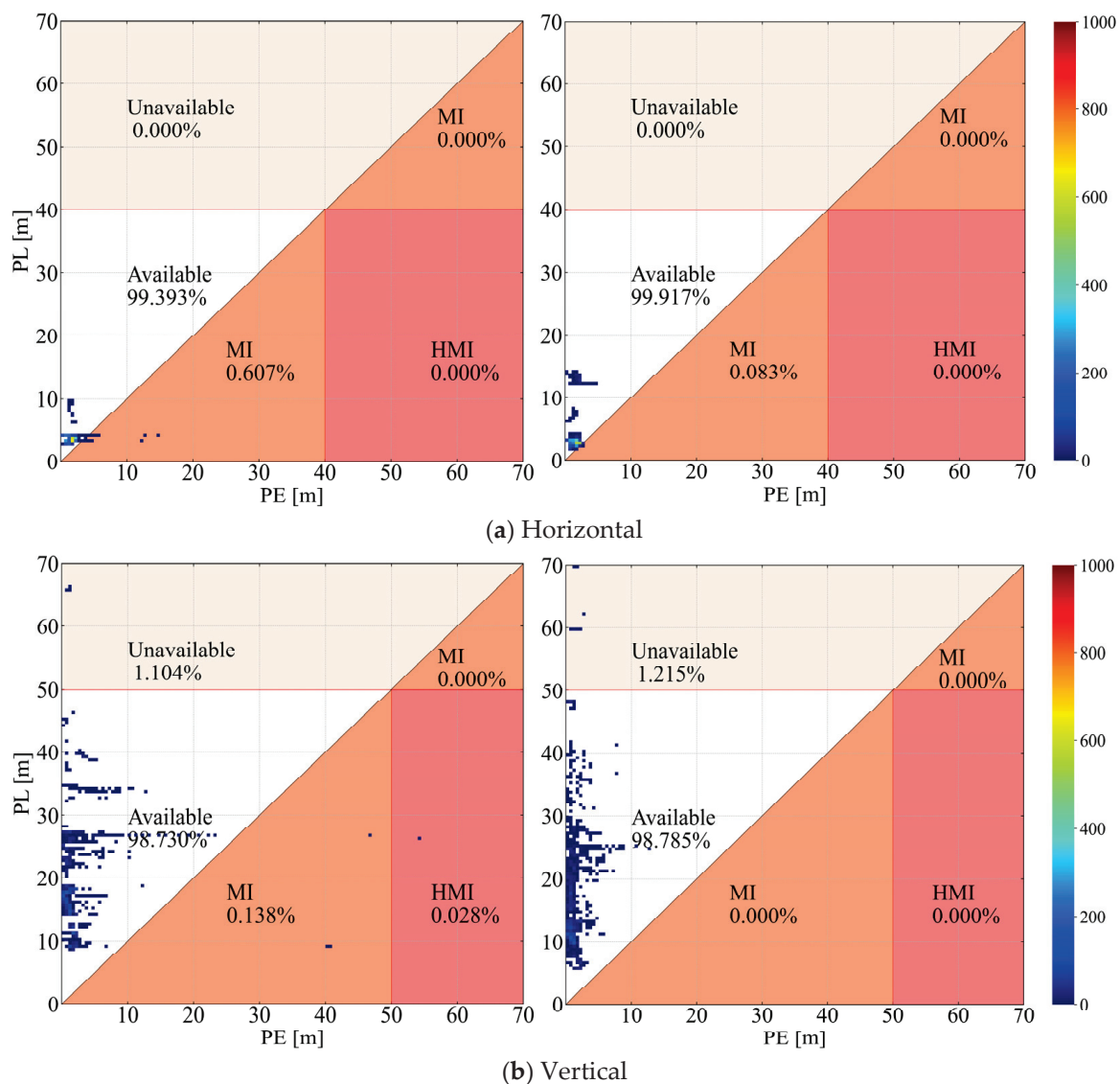


Figure 20. Stanford diagram of CAR pre-robust (left) and post-robust (right).

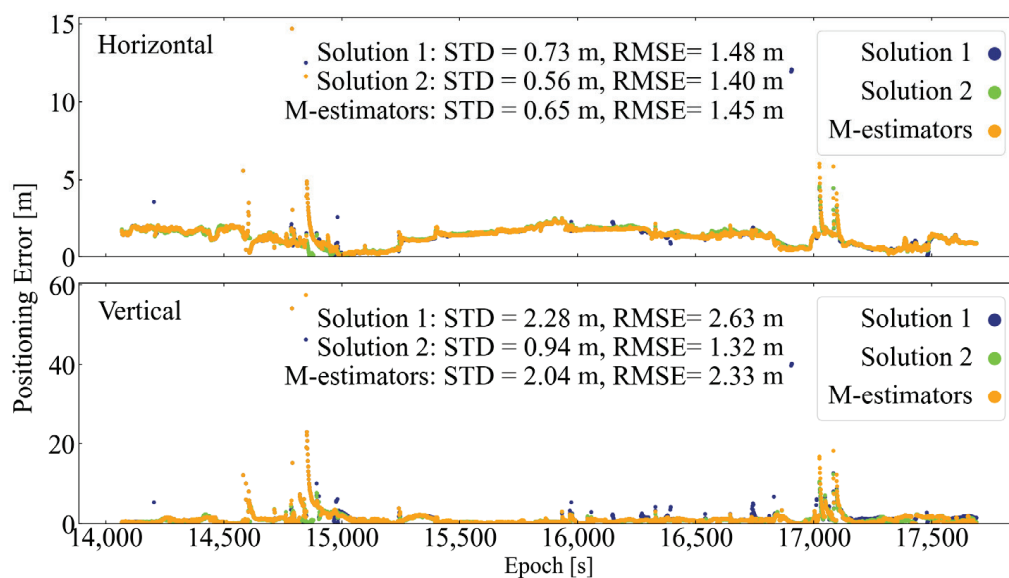


Figure 21. Comparison of positioning errors of CAR.

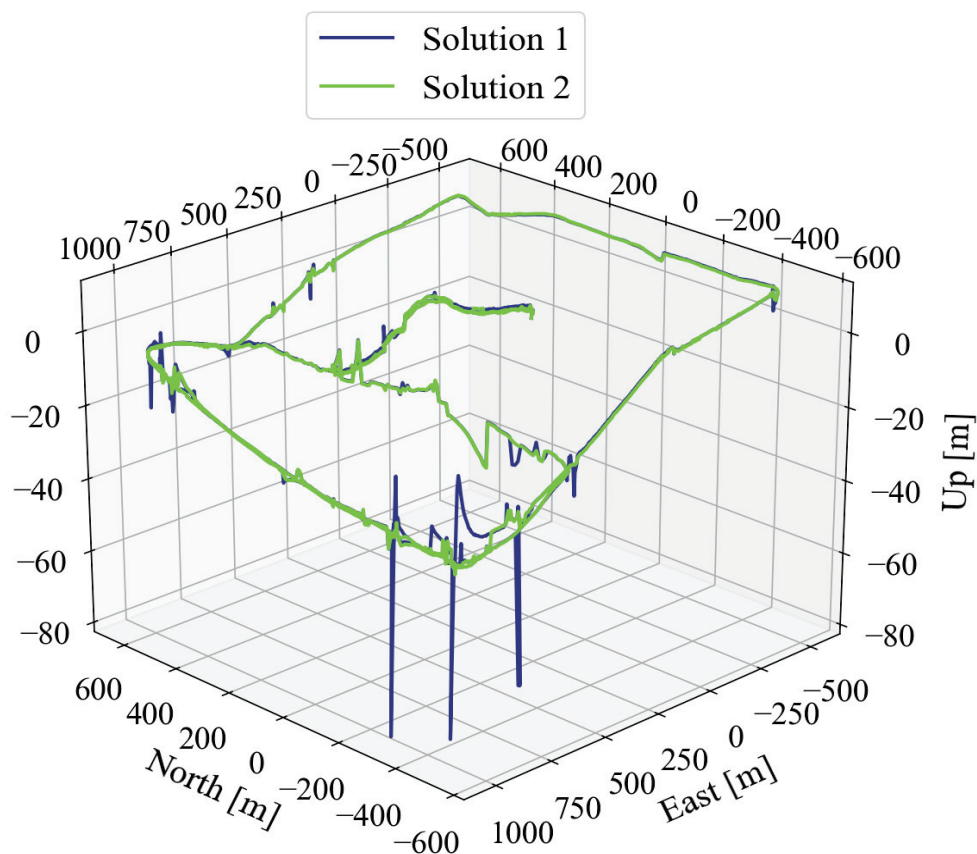


Figure 22. Three-dimensional trajectory of CAR.

The PL results calculated by the positioning weights after robust estimation using the traditional IGG III robust algorithm, the improved robust algorithm and the M-estimators method are shown in Figure 23. In the SBAS-enhanced Urban environment vehicle scenario, compared with traditional IGG III robust algorithm, HPL and VPL using improved robust algorithms have been reduced in most epochs. The result show that the improved robust algorithm is better in the calculation of PL under the premise of integrity and positioning accuracy. The M-estimators method can envelop positioning errors, but the results are too conservative and will reduce usability.

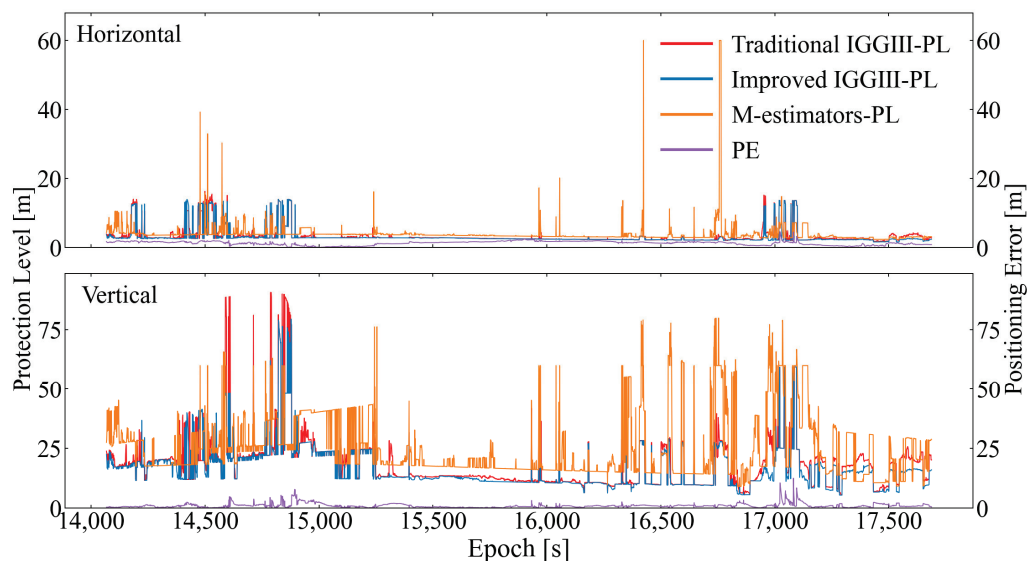


Figure 23. Comparison of protection level of CAR.

3.4. UAV Maneuvering Flight Scenario

Using the data in the UAV maneuvering flight scenario to perform positioning and PL solutions using the two methods of Solution 1 and Solution 2, respectively, the RTK results are taken as the true values and the APV-I standard are chosen to count the integrity, and the result are shown in Figure 24. In Solution 1, using SBAS correction information, the PE is still large and there are MI and HMI. In Solution 2, the horizontal and vertical PE are improved, the percentage of MI and HMI are reduced, and the availability rate is improved. The horizontal percentage of MI is reduced by 2.76%, and the availability rate is improved by 2.76%; the vertical percentages of MI are reduced by 1.71%, and the availability rate is improved by 1.65%. The results show that the improved robust algorithm can reduce the missing detections rate and integrity risk by weakening the effect of the coarse errors in the SBAS-enhanced UAV maneuvering flight scenario, achieve effective alarms, and improve the availability rate.

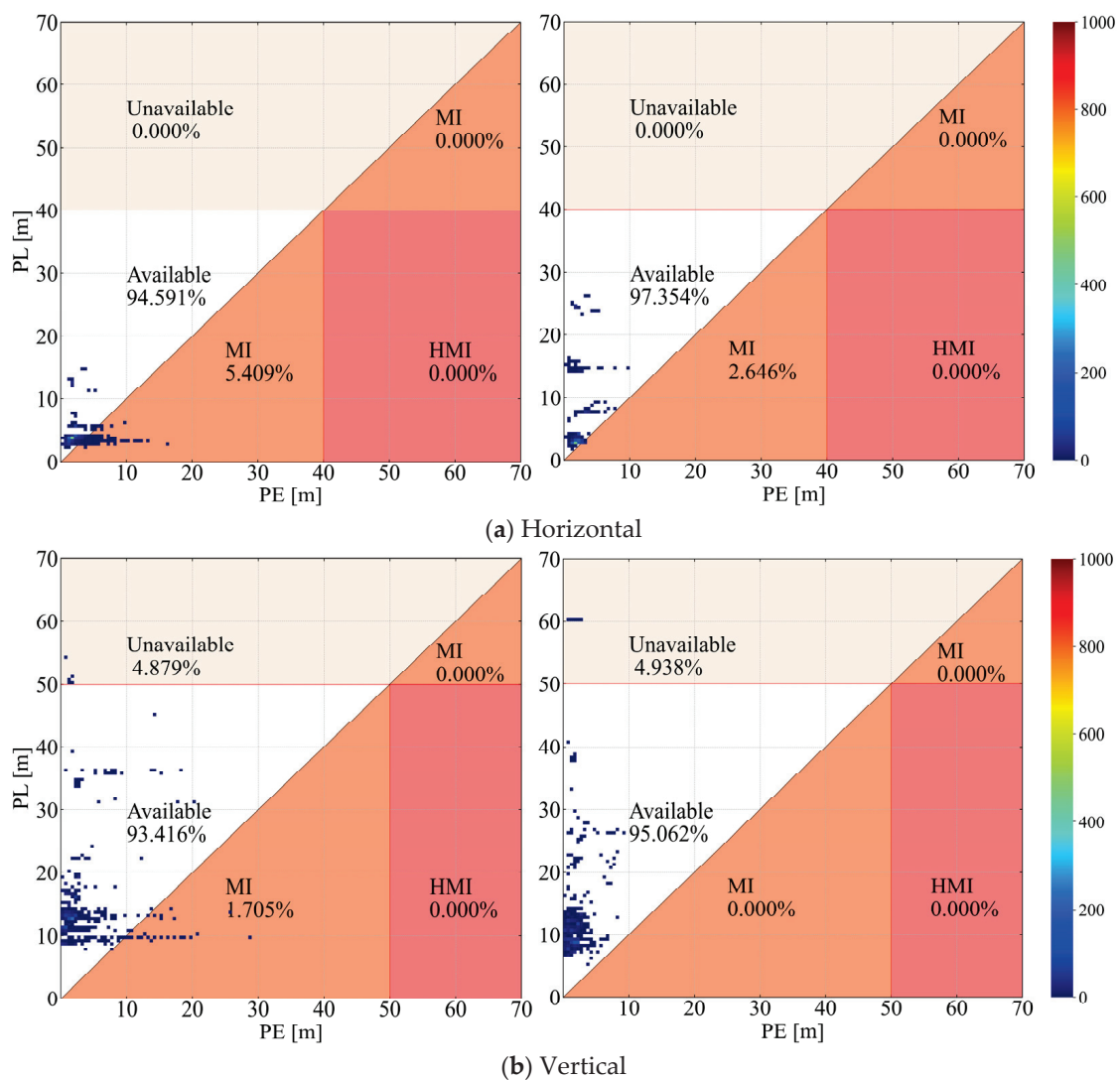


Figure 24. Stanford diagram of FLY pre-robust (left) and post-robust (right).

The positioning error sequences in the horizontal and vertical directions are illustrated in Figure 25, respectively, showing in detail the improved effect of the improved robust algorithm and M-estimators methods on the positioning results. Compared with Solution 1, the positioning results of Solution 2 are smoother and the larger error jump points are eliminated; the positioning accuracy of the M-estimators method has been improved, but

the improvement effect is not as good as that of Solution 2. In addition, the STD and RMSE of Solution 2 in the horizontal and vertical directions are reduced compared with that of Solution 1, of which the internal and external consistency accuracies in the horizontal direction can be improved by 47.1% and 16.45%, the internal and external consistency accuracies in the vertical direction can be improved by 50.38% and 38.15%, respectively. The results show that in the SBAS-enhanced UAV maneuvering flight scenario, the use of the improved robust algorithm in the user-end positioning can effectively weaken the influence of the coarse errors and improve the positioning accuracy.

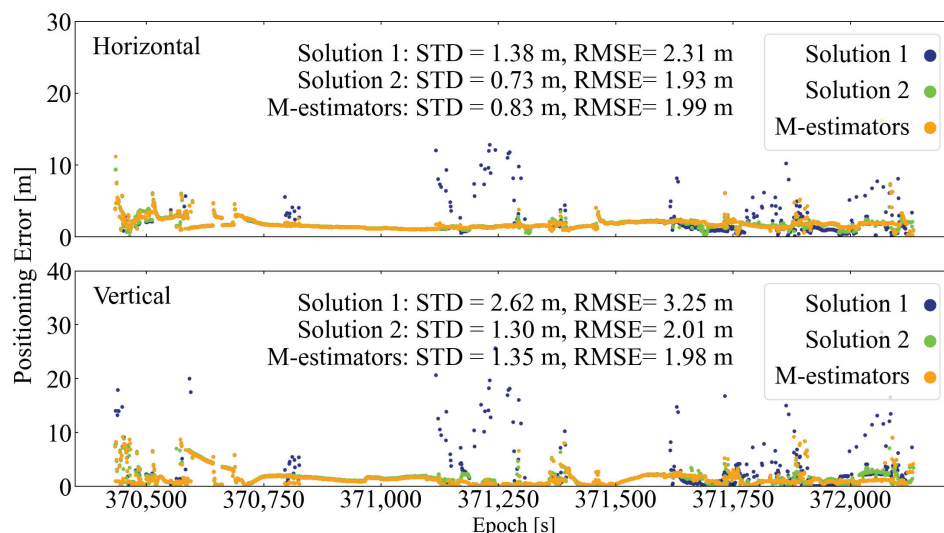


Figure 25. Comparison of positioning errors of FLY.

The results of the PL calculations using the traditional IGG III robust algorithm, the improved robust algorithm and M-estimators method in the SBAS-enhanced UAV maneuvering flight scenario are shown in Figure 26. In SBAS-enhanced UAV maneuvering flight scenario, compared with traditional IGG III robust algorithm, HPL and VPL using improved robust algorithms are reduced most of epochs. The result show that, while ensuring integrity and positioning accuracy, the improved robust algorithm is better than the traditional IGG III robust algorithm when calculating the PL. In addition, the M-estimators method is too conservative in PL calculations, which will affect the availability of positioning users, further explaining the advantages of improved robust algorithms in PL calculations.

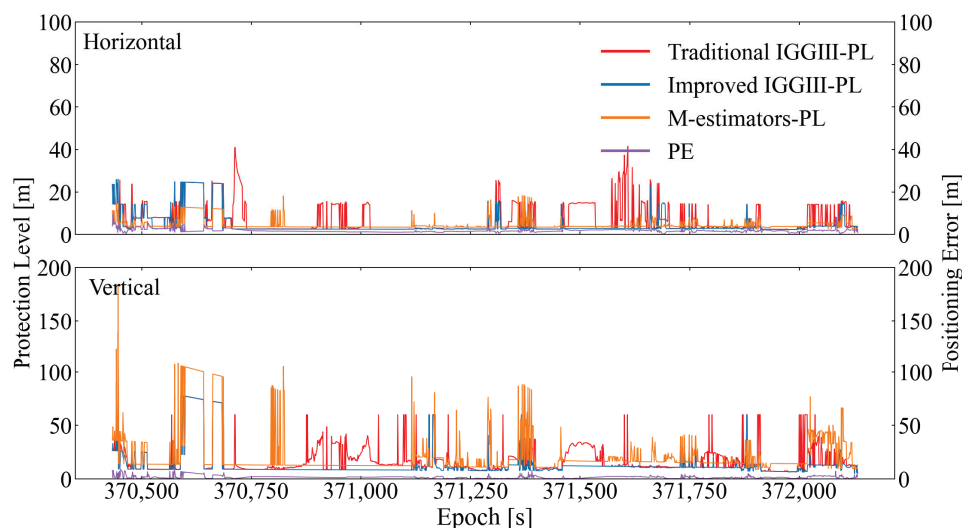


Figure 26. Comparison of protection level of FLY.

3.5. Statistical Test

This study conducted a statistical analysis (*t*-test) on the experimental results of the above three scenarios. The differences between the positioning errors and protection level results of Solution 1 and Solution 2 are shown in Table 6. The *t*-test results show that, except for the horizontal positioning error results in the urban scenario, there are significant differences in the experimental results of all other scenarios, and the degree of effect is above moderate. The differences in the results of horizontal positioning errors in urban scenarios did not reach statistical significance, but a moderate effect was observed, indicating that there might be differences of practical significance.

Table 6. *t*-test of the experimental results in three scenarios.

Item	T	<i>p</i>	D
CAR-vertical-errors	10.509	<0.001	2.151
CAR-vertical-PL	18.525	<0.001	5.916
CAR-horizontal-errors	0.477	0.633	0.542
CAR-horizontal-PL	21.942	<0.001	0.551
FLY-vertical-errors	6.705	<0.001	2.466
FLY-vertical-PL	26.699	<0.001	0.931
FLY-horizontal-errors	2.084	<0.037	1.244
FLY-horizontal-PL	16.346	<0.001	2.083
FUZ2-vertical-errors	51.247	<0.001	1.145
FUZ2-vertical-PL	22.317	<0.001	1.920
FUZ2-horizontal-errors	5.282	<0.001	0.498
FUZ2-horizontal-PL	−1.860	0.063	0.689
KAS2-vertical-errors	22.743	<0.001	1.811
KAS2-vertical-PL	5.642	<0.001	1.552
KAS2-horizontal-errors	16.181	<0.001	1.18
KAS2-horizontal-PL	11.656	<0.001	1.087
CSH2-vertical-errors	35.629	<0.001	0.933
CSH2-vertical-PL	17.022	<0.001	1.56
CSH2-horizontal-errors	11.257	<0.001	0.408
CSH2-horizontal-PL	4.91	<0.001	0.454

($p > 0.05$: Not Significant; $p \leq 0.05$: Significant; $p \leq 0.01$: Highly Significant; $p \leq 0.001$: Extremely Significant).

4. Discussion

This study addresses the issues of degraded positioning performance, increased integrity risk, and higher rates of missed detection events under poor data quality conditions in SBAS user-end positioning by proposing an improved IGG III robust estimation positioning algorithm. Through validation using real-world data from three scenarios—static, urban vehicular, and UAV maneuvering flight—several noteworthy observations were made.

In terms of system performance, the improved robust algorithm significantly enhances the availability of SBAS and effectively reduces integrity risks and missed detection rates in both horizontal and vertical directions. This improvement markedly strengthens system stability in complex environments. The influence of coarse errors in observation data on positioning accuracy is also effectively suppressed by the improved robust estimation, leading to substantial improvements in both horizontal and vertical positioning accuracy. Particularly noteworthy is the outstanding performance of the improved algorithm in protection level computation, avoiding the common issue of overbounding seen in traditional methods and resulting in more reasonable and reliable protection level estimates.

Comparative analysis with the M-estimators method further demonstrates the unique advantages of the proposed improved IGG III robust estimation for SBAS user-end applica-

tions. It not only effectively addresses the limitations of traditional methods in suppressing coarse errors but also avoids potential overconservatism in protection level computation.

From the perspective of current research trends, previous improvements in SBAS performance have predominantly focused on the system end, such as data preprocessing, time synchronization, and model optimization. While these efforts have enhanced overall system performance, there has been relatively little attention paid to user-end performance under challenging conditions, especially when performance remains unsatisfactory even after enhancements. Although various methods exist to suppress coarse errors and improve positioning accuracy, few are well-suited for SBAS user-end applications. This study focuses on this practical issue and offers a new approach to addressing positioning problems at the user end in degraded environments.

It should be noted that this study has certain limitations regarding datasets. Due to constraints in data acquisition, the current research is validated only based on data from China. This geographical limitation may affect the assessment of the algorithm's applicability in other regions, and future work will expand the study area to further verify the method's universality.

5. Conclusions

This study demonstrates that the enhanced IGG III robust estimation-based SBAS user-end positioning algorithm effectively improves positioning performance under challenging environmental conditions. The proposed method not only preserves strong robustness but also leads to notable enhancements in system availability, positioning accuracy, and the reliability of protection level calculations. It offers a practical and efficient solution for mitigating positioning challenges in degraded operational environments. Future work will focus on extending the validation to broader geographical regions and more diverse scenarios to further assess the algorithm's adaptability and robustness.

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Article

Retrieval of Wave Parameters from GNSS Buoy Measurements Using Spectrum Analysis: A Case Study in the Huanghai Sea

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Abstract

Global Navigation Satellite System (GNSS) buoys are widely used to retrieve wave parameters such as significant wave heights (SWHs) and dominant wave periods. In addition to the statistical methods employed to estimate wave parameters, spectral-analysis-based approaches are also frequently utilized to analyze them. This study presents statistical and spectral methods for retrieving wave parameters at GNSS buoy positioning resolution in the Huanghai Sea area. To verify the method's effectiveness, the zero-crossing method and three spectral analysis techniques (periodogram, autocorrelation function, and autoregressive model methods) were used to estimate wave height and period for comparison. The vertical positioning resolution was decomposed into low-frequency ocean-tide level information and high-frequency wave height and period information with the Complete Ensemble Empirical Mode Decomposition (CEEMD) method and moving average filtering. The horizontal positioning results and velocity parameters were used to determine the wave direction using directional spectrum analysis. The results show that the three spectral methods yield consistent effective wave heights, with a maximum difference of 0.02 s in the wave period. Compared with the zero-crossing method results, the wave height and period obtained through spectral analysis differ by 0.05 m and 0.79 s, respectively, while the average wave height and period differ by 0.09 m and 0.08 s, respectively. The GNSS-derived wave heights also closely match tidal gauge observations, confirming the method's validity. Directional spectrum analysis indicates that wave energy is concentrated in the 0.2–0.25 Hz frequency band and within a directional range of $0^\circ \pm 30^\circ$, with a dominant northward propagation trend. These findings demonstrate that the proposed approach can provide high accuracy and physical consistency for GNSS-based wave monitoring under complex sea conditions.

Keywords: GNSS buoy; RTK technology; wave inversion; spectral analysis; directional spectrum analysis

1. Introduction

Ocean waves, primarily driven by wind and tidal forces, are dynamic surface phenomena with significant variability across time and space scales. They play a crucial role in shaping marine ecosystems and designing coastal infrastructure, and they hold vast poten-

tial for renewable energy [1–3]. Accurate wave measurement and analysis are essential for ensuring maritime safety and supporting ocean energy exploitation [4].

Various techniques are used for wave measurement, each with distinct advantages and limitations. A wave staff, a straightforward tool fixed vertically to the seabed or platform, measures wave height via calibrated markings but is limited to stationary point measurements [5]. Pressure sensors, which determine wave height by measuring seabed pressure variations, are suitable for deep-water zones but exhibit reduced accuracy in shallow waters [6]. Acoustic wave gauges utilize sound wave reflections, making them suitable for various water depths, but they are costly and sensitive to changes in water temperature and salinity [7]. Optical wave measurement systems employ lasers or cameras to capture spatial wave profiles and require appropriate lighting conditions [8]. Shipborne wave measurement systems estimate wave parameters by analyzing vessel motion, allowing for real-time observations during navigation, though they are less accurate [9]. Radar systems provide all-weather, large-area wave monitoring capabilities but involve expensive equipment and complex data processing [10]. Satellite remote sensing provides global wave data coverage, but its temporal resolution is limited [11]. Currently, no single method achieves an optimal balance between high accuracy, low cost, and real-time measurement capabilities for comprehensive wave monitoring.

Wave buoys enable real-time measurement of key wave parameters, such as wave height, period, and direction, and are widely used for ocean wave monitoring. The first generation of laboratory wave buoys was developed in 1953 by the UK National Oceanography Centre to measure wave height and period [12]. In the 1970s, Datawell Corporation introduced the Waverider series of wave buoys, which provided high precision, real-time data transmission, and multi-parameter measurements, fulfilling diverse scientific research needs in ocean wave studies [13,14]. With continuous technological advancements, wave buoys have evolved from single-parameter measurement devices to multifunctional instruments capable of simultaneously determining wave direction, spectral characteristics, and marine environmental parameters [15]. In addition to high measurement accuracy, these buoys offer operational flexibility, allowing for their deployment in both open ocean and coastal areas and thus providing continuous wave data records [16].

In recent years, GNSS-equipped wave buoys have gained significant attention as cost-effective and accurate tools for ocean wave monitoring [17]. By integrating GNSS receivers into buoy systems, three-dimensional position and velocity data can be acquired through Precise Point Positioning (PPP) or Real-Time Kinematic (RTK) techniques, enabling effective wave inversion [18,19]. GNSS buoys offer advantages such as low cost, high precision, all-weather capability, and ease of deployment, making them particularly suitable for large-scale ocean surface wave monitoring [20]. Lutsenko et al. employed GNSS signals for wave monitoring and analyzed the feasibility of monitoring surface disturbances [21]. Yang et al. evaluated the feasibility and accuracy of using the single-frequency time-differential carrier phase (TDCP) to determine wave height and period, achieving high-precision and low-cost wave monitoring [22]. Using GNSS technology for wave measurement and directional wave spectrum analysis, Gu et al. demonstrated that GNSS buoys, through carrier phase and Doppler frequency shift data, can accurately extract parameters such as wave height, period, and direction, generating high-resolution directional spectra [23]. Liang et al. deployed and tested GNSS buoys in the open ocean; the results verified the ability of GNSS buoys to continuously measure sea level and waves in the open ocean [24].

In addition to their standalone capabilities, GNSS buoys have been increasingly integrated with complementary technologies to enhance measurement accuracy, spatial resolution, and functionality in ocean wave monitoring. GNSS Reflectometry (GNSS-R), which analyzes signals reflected from the sea surface, has proven effective in retrieving

wave height and wavelength [25,26]; Ruffini et al. further demonstrated its centimeter-level accuracy in low-altitude aerial experiments [27]. GNSS–acoustic positioning combines GNSS signals with underwater acoustics for high-precision monitoring of seabed displacement and wave dynamics [28], with Tomita et al. improving vertical accuracy by addressing uncertainties in sound speed profiles [29]. Shore-based GNSS reference networks, which deploy high-precision GNSS receivers along coastlines, are widely applied in marine observation and deformation monitoring [30,31]. Virtual Base Station Real-Time Kinematic (VBS-RTK) technology, which uses virtual reference stations derived from surrounding continuously operating GNSS stations to achieve precise real-time positioning, was validated by Lin et al. in the Su’ao waters of Taiwan—demonstrating agreement with both tide gauge data and ATC (accelerometer–inclinometer–compass) sensor results [32]. Shipborne GNSS platforms provide flexible real-time observation solutions for complex sea states; for example, Wang et al. successfully applied shipborne GNSS-R during Antarctic expeditions to invert wave height and wind speed from reflected signals [33,34]. Through the integration of these technologies, GNSS buoys have evolved into more comprehensive and versatile observation platforms for high-resolution, multi-parameter ocean wave monitoring under diverse conditions.

Accurate inversion of ocean wave parameters is essential for understanding sea state dynamics, supporting marine engineering, and ensuring navigational safety. With the growing availability of high-resolution measurement platforms such as GNSS buoys, robust and precise wave inversion algorithms have become a research focus. These algorithms typically extract and interpret wave signals using two primary analytical frameworks: statistical techniques and spectral analysis [35]. Statistical methods describe wave motion characteristics under simplified conditions but are limited to time and spatial domains [36,37]. Waves can be modeled as the superposition of cosine components with different amplitudes, frequencies, orientations, and phases, with the wave energy distribution depending deterministically on wave frequencies [38,39]. Wave spectrum analysis and directional spectrum analysis are widely applied to overcome the limitations of statistical methods [40,41]. Spectral analysis decomposes waves into frequency components and provides wave energy distribution versus frequency, enabling derivation of statistical parameters such as wave height and period via spectral moments [42]. Besides the commonly used Fast Fourier Transform (FFT) and Welch method, other techniques, such as the periodogram (a direct Fourier-based method), autocorrelation function (based on signal correlation), and autoregressive (AR) model (a parametric method using linear prediction), have also been applied to wave analysis [43–45]. While spectral shapes may vary depending on the method, the derived statistical features are generally consistent [46]. Directional spectrum analysis extends this method by estimating wave energy distribution in terms of both frequency and direction, offering insights into wave propagation [47]. Key approaches include the direct Fourier transform, parametric modeling, maximum likelihood and entropy methods, wavelet-based techniques, Bayesian estimation, and extended eigenvector methods [48,49].

This study proposes a multi-parameter wave inversion framework based on GNSS buoys that integrates time-domain statistical and frequency-domain spectral techniques to extract key wave parameters—including height, period, and direction—and characterize the two-dimensional wave energy distribution. GNSS buoys effectively combine accuracy and cost efficiency: by acquiring the absolute position and velocity of the buoy in real time, they avoid introducing systematic bias during wave measurement. As GNSS receiver technology becomes more affordable, such buoys enable low-cost, high-density deployments over wide marine areas, significantly expanding observational coverage and improving monitoring efficiency. Field tests in the Yellow Sea demonstrate the method’s stability and reliability for large-scale, real-time monitoring. The results show the enhanced accuracy of

wave parameter inversion, offering reliable data support for applications such as marine disaster mitigation and resource development.

In this study, a 10 m GNSS buoy—originally developed for long-term marine meteorological observation and satellite altimetry calibration—was used as the experimental platform. Given its proven operational stability and deployment history, the platform provides a practical basis for validating the proposed GNSS-based wave monitoring approach. Rather than conducting simulation-based assessments of the buoy’s wave-following characteristics, we focused on evaluating its real-world measurement performance under actual sea conditions. This design reflects the study’s emphasis on feasibility verification under actual field constraints.

2. Methods and Principles

2.1. Statistical Inversion of Wave Period and Wave Height

Statistical analysis is used to characterize and examine the statistical properties of ocean waves. These methods provide quantitative representations of wave data, facilitating a deeper understanding of wave variability and temporal patterns. The zero-crossing method is a statistical approach commonly employed in wave analysis, primarily to derive wave periods and heights from time series data. This technique relies on zero-crossing points within wave signals to capture wave periodicity and height characteristics [35].

First, the wave signal is analyzed to detect zero-crossing points, which occur when the signal transitions from negative to positive, indicating the start or end of a wave cycle. By measuring the time interval between consecutive zero-crossing points, the wave period can be determined. The duration between two successive zero-crossings represents one complete wave period, and averaging multiple periods offers insights into the wave’s periodic characteristics. Subsequently, wave height is estimated using the zero-crossing method. After identifying the wave’s crests (highest points) and troughs (lowest points), the vertical distance between them is calculated to represent the wave height. This method effectively determines wave height by analyzing the amplitude variations between successive crests and troughs.

To obtain the wave height sequence $H_i (i = 1, 2, \dots, N)$, the wave heights are arranged in descending order, which represents the ranking of wave height magnitudes. Correspondingly, the wave period sequence $T_i (i = 1, 2, \dots, N)$ can be derived, though it should be noted that the wave period sequence is not ordered by period magnitude but rather follows the ranking obtained for the wave height sequence. Table 1 provides the commonly used characteristic values in wave statistical analysis [36].

Table 1. Commonly used characteristic values in wave statistical analysis.

Characteristic Value	Calculation Formula	Characteristic Value	Calculation Formula
Maximum wave height	$H_{\max} = H_1$	Maximum wave period	$T_{\max} = T_1$
Highest one-tenth wave height	$H_{1/10} = \frac{10}{N} \sum_{i=1}^N H_i$	Mean period of the highest one-tenth waves	$T_{1/10} = \frac{10}{N} \sum_{i=1}^N T_i$
Significant wave height	$H_{1/3} = \frac{3}{N} \sum_{i=1}^N H_i$	Significant wave period	$T_{1/3} = \frac{3}{N} \sum_{i=1}^N T_i$
Mean wave height	$\bar{H} = \frac{1}{N} \sum_{i=1}^N H_i$	Mean wave period	$\bar{T} = \frac{1}{N} \sum_{i=1}^N T_i$

The highest one-third wave height ($H_{1/3}$) and highest one-third wave period ($T_{1/3}$) are commonly referred to as the significant wave height (H_s) and significant wave period (T_s), respectively, serving as generalized representations of sea state characteristics.

2.2. Wave Period and Wave Height Inversion Based on Spectral Analysis

Wave spectrum analysis is used to investigate the frequency composition of ocean waves, specifically how wave energy is distributed across different frequencies. Through spectral analysis, it is possible to identify the dominant frequency components within a wave signal and characterize the periodic nature of the waves. The wave spectrum not only provides the energy distribution as a function of frequency but also enables the derivation of various first-order statistical wave parameters, such as wave height and wave period, from the spectral moments [38].

2.2.1. Theoretical Basis of Wave Spectrum Analysis

Wave spectrum analysis is based on random wave theory, the core of which is the treatment of sea surface fluctuations as a random process composed of simple harmonic waves of different frequencies, amplitudes, and phases. According to Longuet-Higgins' linear wave theory, the vertical displacement of the sea surface can be expressed as follows:

$$\zeta(t) = \sum_{n=1}^{\infty} a_n \cos(\omega_n t + \varepsilon_n) \quad (1)$$

where a_n represents the wave amplitude, ω_n denotes the angular frequency, and ε_n is defined as a uniformly distributed random phase. According to random wave theory, the total energy of a wave in the frequency interval $(\omega, \omega + \Delta\omega)$ is as follows:

$$\sum_{\omega}^{\omega+\Delta\omega} \frac{1}{2} a_n^2 = S(\omega) \Delta\omega \quad (2)$$

where $S(\omega)$ is the wave power spectral density, which characterizes the distribution of wave energy in different frequency components [42].

This theoretical framework lays a solid foundation for wave spectrum analysis and provides a theoretical basis for subsequent spectral estimation methods.

2.2.2. Wave Spectrum Estimation Method

The process of deriving wave spectra from measured wave data using various estimation techniques is referred to as wave spectrum estimation. The wave spectrum primarily characterizes the relationship between wave energy and frequency. The corresponding spectral estimation methods can be categorized into classical spectral estimation and modern spectral estimation.

Classical spectral estimation methods are based on the Fourier transform and are suitable for analyzing stationary random processes. These methods obtain the power spectral density by calculating the autocorrelation function and subsequently performing a Fourier transform. Commonly used classical spectral estimation methods include the autocorrelation function, periodogram, Bartlett, and Welch methods [43,50].

Modern spectral estimation methods, on the other hand, introduce parametric models to describe the signal generation process and obtain the power spectral density by estimating the model parameters. These methods are typically applied to process short data records and non-stationary signals. In modern spectral estimation, the most commonly used models include the AR (Autoregressive) model, MA (Moving Average) model, and ARMA (Autoregressive Moving Average) model. Among these, the AR model is particularly effective in providing high-resolution spectrum estimates with relatively simple computational implementation, making it suitable for rapid application and analysis, especially for wave data with short time series [45].

(1) Classical Spectral Estimation Methods

Classical spectral estimation methods are based on the Fourier transform. These methods feature straightforward computation and clear physical interpretation. Commonly used classical spectral estimation techniques include the autocorrelation function and periodogram methods.

1. Autocorrelation Function Method

The autocorrelation function method uses indirect processing. First, the wavefront time series is processed to obtain its autocorrelation function. Then, the autocorrelation function is processed using the Fourier transform to obtain the estimated value of the coarse spectrum, and finally, the obtained coarse spectrum is smoothed. The formula of the autocorrelation function is as follows:

$$R(\tau) = \frac{1}{N-v} \sum_{n=1}^{N-v} \zeta(t_n) \zeta(t_n + \tau), v = 1, 2, \dots, m \quad (3)$$

where N represents the number of samples, τ denotes the time delay at which the autocorrelation is computed, v is an auxiliary variable that adjusts the summation range to ensure that $t_n + \tau$ does not exceed the signal length N , and m is the maximum lag order, which defines the upper limit of v and τ .

$$S(\omega) = \frac{2}{\pi} \int_0^\infty R(\tau) \cos(\omega\tau) d\tau \quad (4)$$

The spectral value can be obtained by substituting Equation (3) into the above equation and integrating the corresponding values, with L_n used to represent the rough spectral estimate at frequency f_n . The trapezoidal formula is used in the numerical integration, and the spectral value is as follows:

$$L_n = \frac{2}{\pi} \left[\frac{1}{2} R(0) + \sum_{v=1}^{m-1} R(v\Delta t) \cos(2\pi f_n v\Delta t) + \frac{1}{2} R(m\Delta t) \cos(2\pi f_n m\Delta t) \right] \quad (5)$$

$n = 0, 1, 2, \dots, m$

The frequency interval used in the above expression is as follows:

$$\begin{cases} \Delta f = f_N / m = \frac{1}{2m\Delta t} \\ f_n = n\Delta f = \frac{n}{m} \frac{1}{2\Delta t} \end{cases} \quad (6)$$

Substituting the above equation into Equation (5) yields the following:

$$L_n = \frac{2\Delta t}{\pi} \left[\frac{1}{2} R(0) + \sum_{v=1}^{m-1} R(v\Delta t) \cos \frac{\pi v n}{m} + \frac{1}{2} R(m\Delta t) \cos \pi n \right] \quad (7)$$

$n = 0, 1, 2, \dots, m$

The spectral values obtained from the above estimation may exhibit local fluctuations due to the finite sample length and numerical effects. To suppress these variations and enhance spectral smoothness, a three-point weighted moving average filter is applied. The weights used are 0.23, 0.54, and 0.23, assigned to the current point and its two immediate neighbors.

This specific set of weights is not arbitrary; it is chosen to satisfy the following desirable properties: (1) symmetry, which ensures zero phase distortion; (2) normalization, preserving the amplitude scale of the spectrum; (3) a larger central weight that emphasizes the main frequency component while still smoothing out local fluctuations.

Among possible choices that meet these criteria, the values {0.23, 0.54, 0.23} represent a practical and balanced solution and are widely adopted in engineering practice for smoothing tasks [51,52]. The smoothing operation is expressed as follows:

$$S(f_n) = 0.23L_{N-1} + 0.54L_N + 0.23L_{N+1} \quad (8)$$

2. Periodogram Method

The periodogram method, also known as the direct method, is a classical spectral estimation technique. The basic idea is to directly apply the Fourier transform to the wave surface displacement sequence to obtain the spectral density. Specifically, this method first calculates the autocorrelation function of the wave surface displacement and then applies the Fourier transform to obtain the spectrum. During this process, the periodogram method effectively estimates the spectral distribution of the signal [43].

Assuming that the waves are a stationary process with statistical stationarity, observing the waves allows for the acquisition of a wave surface height sequence $\zeta(t)$. The spectral energy density function of this sequence can be expressed as follows:

$$\hat{S}(\omega) = \lim_{T \rightarrow \infty} \frac{1}{2\pi T} |X(\omega)|^2 \quad (9)$$

where ω is the angular frequency, $\hat{S}(\omega)$ is the spectral density, T is the total observation period, and $X(\omega)$ is the Fourier transform of $\zeta(t)$. For the discrete value of $\zeta(t)$, the following formula is obtained:

$$X(\omega) = \sum_{n=1}^N \zeta_n e^{-jn\omega\Delta t} \Delta t \quad (10)$$

where ζ_n represents the sample values of $\zeta(t)$ for $n = 1, 2, \dots, N$; Δt is the sampling interval; j is the imaginary unit; and $T = N\Delta t$. Substituting Equation (10) into Equation (9) yields the following formula:

$$\hat{S}(\omega) = \frac{\Delta t}{2\pi N} \left| \sum_{n=1}^N \zeta_n e^{jn\omega} \right|^2, |\omega| < \pi / \Delta t \quad (11)$$

When $\Delta t = 1$, the above formula can be changed to the following formula:

$$\hat{S}_1(\omega) = \frac{1}{2\pi N} \left| \sum_{n=1}^N \zeta_n e^{jn\omega} \right|^2, |\omega| < \pi \quad (12)$$

where ζ_n is the wavefront height obtained by sampling per unit time. The above two formulas can be used to obtain the following formula:

$$\hat{S}(\omega) = \Delta t \hat{S}_1(\omega \Delta t) \quad (13)$$

Under the condition of $\Delta t \neq 1$, the initial computation may be performed using Equation (12), with subsequent conversion to $\hat{S}_1(\omega)$ via Equation (13). To enable discrete-value computation, the following formula is introduced:

$$\omega'_r = 2\pi r / N \quad (14)$$

where r is an integer, and $0 \leq r \leq \frac{N}{2}$. Thus, the equation becomes the following:

$$\hat{S}_1\left(\frac{2\pi r}{N}\right) = \frac{1}{2\pi N} \left| \sum_{n=1}^N \zeta_n e^{j\frac{2\pi r}{N}(n-1)} e^{j\frac{2\pi}{N}} \right|^2 = \frac{1}{2\pi N} \left| \sum_{n=1}^N \zeta_n e^{j\frac{2\pi r}{N}(n-1)} \right|^2 \quad (15)$$

To align with the functional formulation of the Fourier transform, we adjust the index of ζ_n by defining $\tilde{\zeta}_n = \zeta_{n-1}$, yielding the following:

$$\hat{S}_1\left(\frac{2\pi r}{N}\right) = \frac{1}{2\pi N} \left| \sum_{n=1}^N \tilde{\zeta}_{n-1} e^{j\frac{2\pi r}{N}(n-1)} \right|^2 = \frac{1}{2\pi N} |A_r|^2, r = 0, 1, 2, \dots, N/2 \quad (16)$$

The expanded form of A_r is $A_r = \sum_{k=1}^N \zeta_k e^{j\frac{2\pi r}{N}k}$; the spectrum of the above formula is the value of $\omega \in (0, \pi/\Delta t)$, and the value of $\omega \in (-\pi/\Delta t, 0)$ can be obtained from the symmetry.

Consequently, the problem of power spectral estimation reduces to the computation of A_r . As with the autocorrelation function method, the raw spectrum obtained using this approach may exhibit local fluctuations due to numerical artifacts. To mitigate these effects and improve spectral continuity, a three-point weighted moving average filter is applied to smooth the spectrum. This post-processing step enhances the stability and accuracy of the spectral estimates.

(2) Modern Spectral Estimation Methods

Modern spectral estimation methods are based on stochastic process modeling, where sampled data are used to construct parametric models that provide improved spectral resolution. Common models include the Autoregressive (AR), Moving Average (MA), and Autoregressive Moving Average (ARMA) models. For power spectral estimation, the MA model generally provides lower resolution, while the ARMA model involves more complex and cumbersome parameter estimation. Therefore, the AR model is typically preferred in modern spectral analysis due to its computational efficiency and relatively simple implementation [45].

In the AR parametric method, the input sequence $u(n)$ is assumed to be a white noise series with zero mean and variance σ^2 . This sequence drives a linear system with the transfer function $H(z)$, producing the output sequence $x(n)$. The system parameters can be estimated from either $x(n)$ or its autocorrelation function $r_x(m)$, enabling power spectral estimation. The input–output relationship is as follows:

$$\zeta_n = -\sum_{k=1}^p a_k \zeta(n-k) + \sum_{k=0}^q b_k u(n-k), b_0 = 1 \quad (17)$$

The transfer function and power spectrum of the AR model are as follows:

$$\zeta_n = -\sum_{k=1}^p a_k \zeta(n-k) + u(n) \quad (18)$$

In the above equation, the white noise sequence $u(n)$ has a zero mean and a variance of σ^2 , where p denotes the model order, and a_k represents the model parameters. The transfer function and power spectrum of this model are, respectively, given by

$$H(z) = \frac{1}{\left| 1 + \sum_{k=1}^p a_k e^{-j2\pi f k \Delta t} \right|^2} \quad (19)$$

$$P_x(f) = \frac{\sigma^2 \Delta t}{\left| 1 + \sum_{k=1}^p a_k e^{-j2\pi f k \Delta t} \right|^2} \quad (20)$$

The autocorrelation function $r_x(m)$ and AR coefficients a_k satisfy the following:

$$r_x(m) = \begin{cases} -\sum_{k=1}^p a_k r_x(m-k) & m \geq 1 \\ -\sum_{k=1}^p a_k r_x(k) + \sigma^2 & m = 0 \end{cases} \quad (21)$$

which can be expressed in matrix form (using the even symmetry of $r_x(m)$) as follows:

$$\begin{bmatrix} r_x(0) & r_x(1) & r_x(2) & \cdots & r_x(p) \\ r_x(1) & r_x(0) & r_x(1) & \cdots & r_x(p-1) \\ r_x(2) & r_x(0) & r_x(0) & \cdots & r_x(p-2) \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ r_x(p) & r_x(p-1) & r_x(p-2) & \cdots & r_x(0) \end{bmatrix} \begin{bmatrix} 1 \\ a_1 \\ a_2 \\ \vdots \\ 0 \end{bmatrix} = \begin{bmatrix} \sigma^2 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \quad (22)$$

This is the Yule–Walker equation, the cornerstone of AR parameter estimation.

In practical AR spectral estimation, the model order p is a critical parameter influencing the accuracy and stability of the spectral results. In this study, the AR model order was empirically determined using the Burg algorithm, with a relatively high order (order = 100) adopted. Sensitivity analyses were conducted by comparing spectral estimations with AR orders ranging from 10 to 100. These analyses indicated that the spectral shapes and derived wave parameters stabilized for AR orders above approximately 50, while orders below 20 produced noticeable spectral distortions. Therefore, an order of 100 was selected as a practical trade-off between spectral resolution and computational complexity.

2.2.3. Calculation of Wave Characteristic Elements

The wave spectrum not only describes the distribution of wave energy in different frequencies but also enables the derivation of first-order statistical wave parameters through its spectral moments [22,23]. The relationships between these statistical parameters and the spectral moments m_n are as follows:

$$\begin{cases} \bar{H} = \sqrt{2\pi m_0} \\ H_s = H_{1/3} = 4\sqrt{m_0} \\ \bar{T} = 1.2T_{0,2} = 2.4\pi\sqrt{m_0/m_2} \\ T_s = T_{H_{1/3}} = 0.937T_p \end{cases} \quad (23)$$

where $\bar{H}$ is the mean wave height, H_s is the significant wave height, $\bar{T}$ is the mean wave period, T_s is the significant wave period, T_p is the peak period (corresponding to maximum spectral density), and $T_{0,2}$ is the mean period calculated from the zeroth and second spectral moments [37] as follows:

$$T_{0,2} = 2\pi\sqrt{m_0/m_2} \quad (24)$$

where m_n ($n = 0, 1, 2, \dots$) is the n -th-order spectral moment of the wave spectrum $S(\omega)$, and its expression is as follows:

$$m_n = \int_0^\infty \omega^n S(\omega) d\omega \quad (25)$$

where ω^n represents the n -th power of angular frequency ω , and other symbols maintain their previously defined meanings.

2.3. Wave Direction Analysis Based on Directional Spectrum

Wave spectrum analysis provides the one-dimensional distribution of wave energy with respect to frequency. However, to obtain the two-dimensional distribution of wave energy in terms of both frequency and direction, directional spectrum analysis must be employed. Observing and analyzing directional spectra typically require synchronous measurements of wave characteristics from multiple observation points, making the process significantly more complex than frequency spectrum analysis, with considerably higher observation costs. When the wave frequency spectrum is already known, the directional spectrum can be estimated by utilizing the relationship between the one-dimensional frequency spectrum and the two-dimensional directional spectrum [16].

The directional spectrum can be expressed in terms of the wave frequency spectrum $S(f)$ and the directional spreading function $G(f, \theta)$ as follows:

$$S(f, \theta) = S(f) \cdot G(f, \theta) \quad (26)$$

$$\int_{-\pi}^{\pi} G(f, \theta) d\theta = 1 \quad (27)$$

where $S(f)$ is the spectrum; $G(f, \theta)$ is the directional distribution function, which cannot be negative, and the integral from $-\pi$ to π is 1; and the variables f and θ correspond to wave frequency and direction, respectively. It is evident that once $S(f)$ is determined, $G(f, \theta)$ becomes the key to solving for the directional spectrum.

The directional spreading function $G(f, \theta)$ can be obtained through cross-spectral analysis of the wave displacement time series measured in two horizontal directions. The function may be expressed as follows:

$$G(f, \theta) = \frac{1}{\pi} \left[\frac{1}{2} + a_1 \cos(\theta) + b_1 \sin(\theta) + a_2 \cos(2\theta) + b_2 \sin(2\theta) + \dots \right] \quad (28)$$

This formulation can be derived from wave cross-spectra.

Wave cross-spectra represent frequency-domain characteristics of wave signals. In wave analysis, cross-spectra not only reveal frequency coherence between two time series but also indicate the directional distribution of wave energy, making them essential tools for directional spectral estimation.

The cross-spectrum between any two wave signals is given by the following [23]:

$$S_{p,q}(f) = \frac{1}{T} \bar{F}_p(f) F_q(f) = C_{pq}(f) - iQ_{pq}(f), p, q = 1, 2, 3, \dots \quad (29)$$

where $S_{p,q}(f)$ denotes the cross-spectrum between wave measurements p and q . Here, $F_q(f)$ is the Fourier transform of signal q , while $\bar{F}_p(f)$ represents the complex conjugate of the Fourier transform of signal p . The terms $C_{pq}(f)$ and $Q_{pq}(f)$ correspond to the co-spectrum and quadrature spectrum, respectively.

The wave surface elevation sequence and the displacement time series (u, w, n) in the east–west and north–south directions are Fourier transformed to obtain the real and imaginary parts of their frequency-domain Fourier coefficients as follows:

$$F_{uf} = \alpha_{uf} + i\beta_{uf}, F_{wf} = \alpha_{wf} + i\beta_{wf}, F_{nf} = \alpha_{nf} + i\beta_{nf} \quad (30)$$

The isotropic spectrum $C_{pq}(f)$ and anisotropic spectrum $Q_{pq}(f)$ can be derived as follows:

$$\begin{cases} C_{nu} = \bar{A}_{nf} \times \bar{A}_{uf} = \alpha_{nf}\alpha_{uf} + \beta_{nf}\beta_{uf} \\ Q_{nu} = \bar{A}_{nf} \times \bar{A}_{uf} = \alpha_{nf}\alpha_{uf} - \beta_{nf}\beta_{uf} \end{cases} \quad (31)$$

From the definition of the anisotropic spectrum, we know that $Q_{uu} = Q_{ww} = Q_{nn} = 0$, and from the physical properties of waves, we know that $Q_{nw} = Q_{wn} = 0$; thus, we can obtain the 3×3 matrices of isotropic and anisotropic spectra as follows:

$$\begin{bmatrix} C_{ww} & C_{wn} & C_{wu} \\ C_{nw} & C_{nn} & C_{nu} \\ C_{uw} & C_{un} & C_{uu} \end{bmatrix} \text{ and } \begin{bmatrix} 0 & 0 & Q_{wu} \\ 0 & 0 & Q_{nu} \\ Q_{uw} & Q_{un} & 0 \end{bmatrix} \quad (32)$$

Based on the joint analysis of isotropic and anisotropic spectra, the directional distribution characteristics of wave energy are determined by substituting the calculated Fourier coefficients a_1 , a_2 , b_1 , and b_2 into the directional distribution function (28). The peak direction of the directional distribution function $G(f, \theta)$ is the dominant propagation direction of the frequency component.

3. Wave Data Acquisition and Signal Extraction

3.1. GNSS-Buoy-Based Data Collection and Processing

The experimental data used in this study were acquired from a GNSS buoy deployed in the coastal waters of Qingdao, China, on 27 April 2023. The location of the field experiment is illustrated in Figure 1.

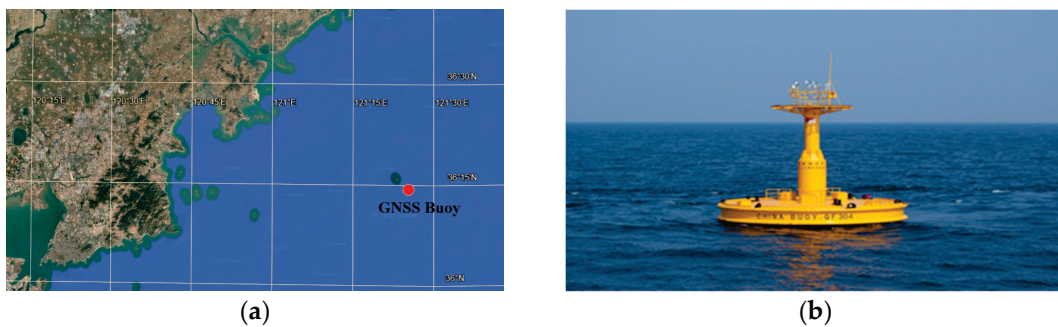


Figure 1. Experimental site and setup. (a) GNSS buoy test area; (b) GNSS buoy.

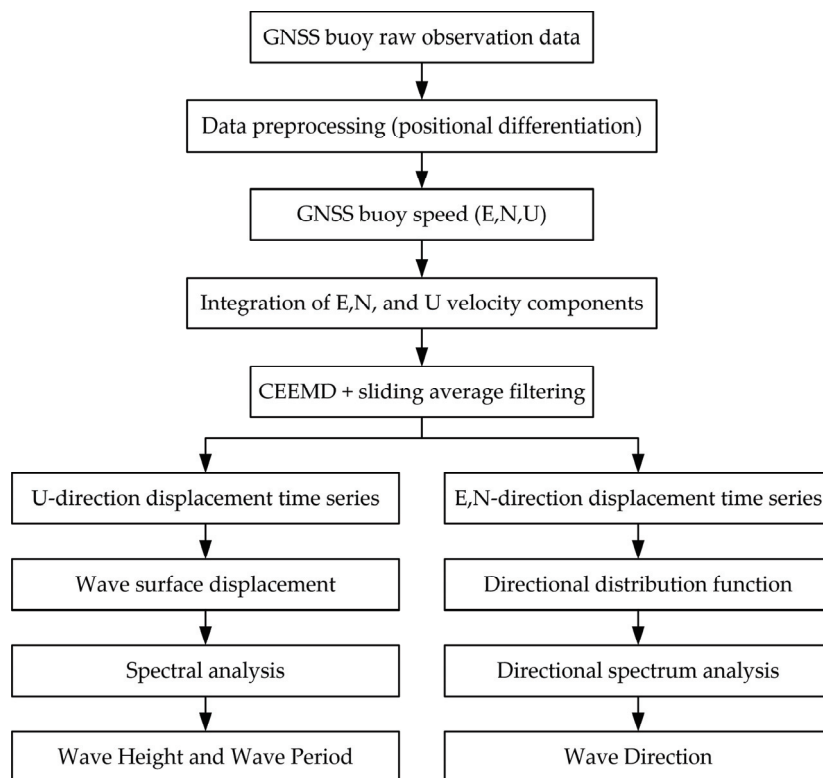
The experiment employed a large GNSS buoy (10 m buoy body) deployed 3 km from the reference station. The buoy was equipped with a Hi-Target Vnet8 receiver (Hi-Target, Shenzhen, China) as the GNSS module and a high-precision HZZACF-S808 GNSS antenna (Hi-Target, Shenzhen, China), with data sampled at 1 Hz. To validate the effectiveness of the proposed wave measurement method, wave monitoring results obtained from other sensors installed on the buoy were used as reference data.

The GNSS buoy data were processed using Real-Time Kinematic (RTK) technology to derive coordinate time series. Data were processed using RTKLIB (version 2.4.3), an open-source GNSS positioning software developed by the Tokyo University of Marine Science and Technology. Key parameter settings used for data processing are summarized in Table 2 (unlisted parameters retained their default values).

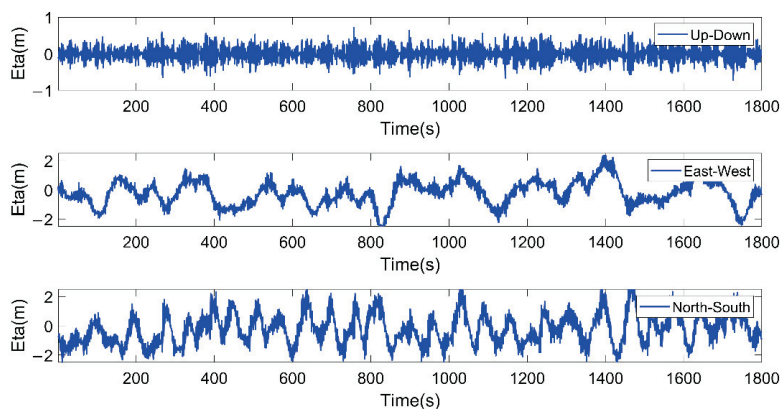
The raw GNSS buoy observations were processed using RTK positioning to obtain displacement time series in the North–East–Up coordinate system, as shown in Figure 2. Through position differencing and coordinate transformation, velocity information in the NEU directions was derived. Subsequent integration of these velocity components, followed by Complete Ensemble Empirical Mode Decomposition (CEEMD) filtering—a data-driven signal decomposition technique effective for extracting non-stationary and nonlinear wave components—and moving average filtering, yielded the final wave surface displacement measurements from the GNSS buoy.

Table 2. Parameter configuration.

Parameter	Configuration
Position Mode	Kinematic
Frequencies	L1
Solution Type	Forward
Elevation Mask	15.0 deg
Ephemeris Data	Broadcast
Navigation Systems	GPS/GLONASS/Galileo/BDS
Ambiguity Resolution	Instantaneous
Integer Validation Threshold	3.0

**Figure 2.** Diagram of ocean wave measurement process using GNSS buoy.

The three-dimensional wave surface displacement time series of the GNSS buoy were obtained by integrating the velocity components in the NEU (North–East–Up) directions, as shown in Figure 3.

**Figure 3.** Three-dimensional wavefront displacement time series from GNSS buoy.

Spectral analysis of the vertical (U-direction) displacement time series yields key wave parameters, including wave height and period. By applying Fourier transforms to both vertical and horizontal displacement components, cross-spectral analysis can be performed to derive the directional spreading function. Combined with the frequency spectrum results, this function enables the determination of directional wave spectrum characteristics.

3.2. GNSS-Based Ocean Wave Signal Data Processing

3.2.1. Ocean Wave Signal Separation and Extraction

By isolating the vertical (U-direction) displacement time series, sea surface fluctuations can be obtained. Figure 4 presents a 30 min U-direction wave surface displacement sequence from 11:30 to 12:00 UTC on 27 April 2023. The time series exhibits distinct low-frequency components, primarily attributable to tidal signals. Wave and tidal signals differ in their periodicities, so appropriate filtering methods are used to effectively separate these components. Commonly employed approaches include time-domain filtering and spatial filtering techniques.

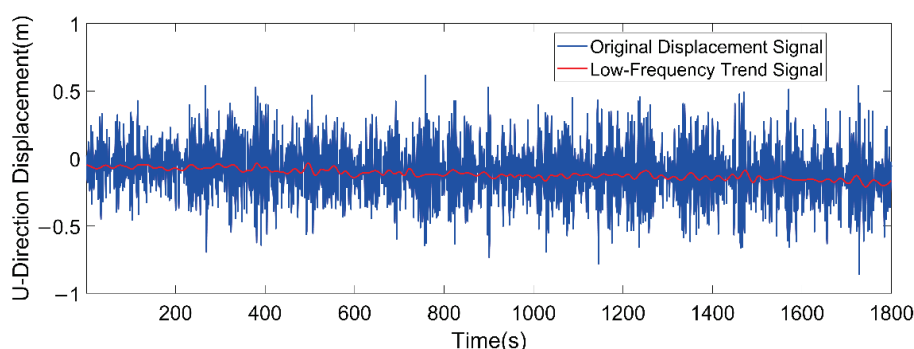


Figure 4. U-direction wavefront displacement sequence (sampling rate 1 Hz).

Figure 5 demonstrates the filtering results of the original wave surface displacement using two methods: CEEMD filtering combined with moving average filtering (time domain) and Butterworth high-pass filtering (frequency domain). Both approaches effectively extract the wave-induced fluctuations from the original signal.

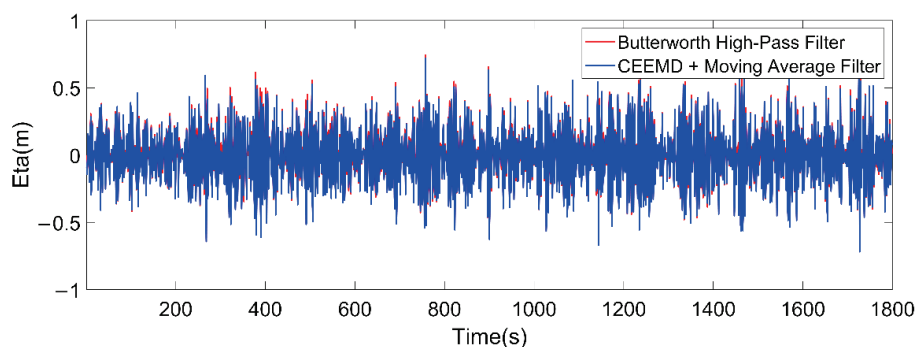


Figure 5. Wavefront displacement sequence filtering (time domain and frequency domain).

The combination of CEEMD and the moving average filtering method leverages the adaptive decomposition capability of CEEMD and the smoothing effect of moving average filtering, enabling precise analysis of trend and periodic characteristics in the signal. This approach achieves effective separation of wave and tidal signals, and subsequent wave analysis will be based on the wave–tide separation results obtained through this combined filtering method.

3.2.2. Correction of Directional Spreading Function

The cross-spectra obtained through direct Fourier transform yield preliminary estimates, resulting in inaccuracies in the calculated parameters (a_1 , a_2 , b_1 , and b_2) and, consequently, an imprecise directional spreading function. Therefore, necessary corrections must be applied to the directional spreading function.

As shown in Figure 6, small negative values appear in the estimated directional spreading function due to numerical artifacts introduced during spectral estimation. These negative values are non-physical, as the directional spreading function is theoretically non-negative and must integrate to unity. To correct this while minimizing the impact on directional accuracy, we apply the following procedure [23]:

- (1) Set all negative values in the directional spreading function to zero.
- (2) Redistribute the integrated area of the negative-valued portions to the positive-valued regions proportionally based on their respective weights.
- (3) Verify that the corrected directional spreading function satisfies the normalization condition (integral equals unity).

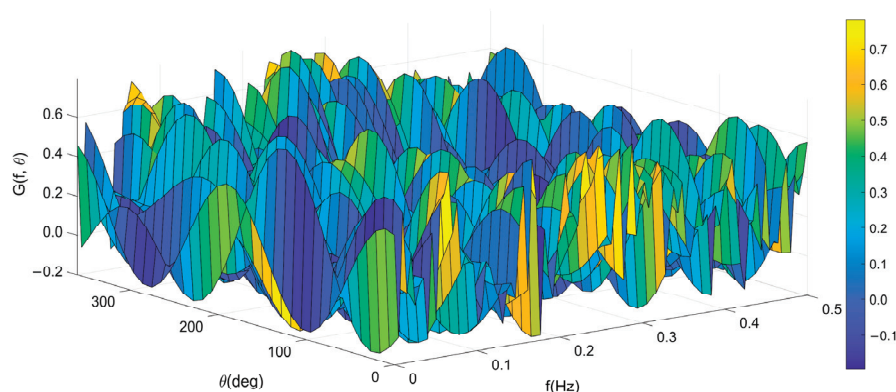


Figure 6. Direction distribution function before correction.

The corrected directional spreading function, as shown in Figure 7, now ensures both non-negativity and normalization properties, making it suitable as input for ocean wave directional spectrum calculations.

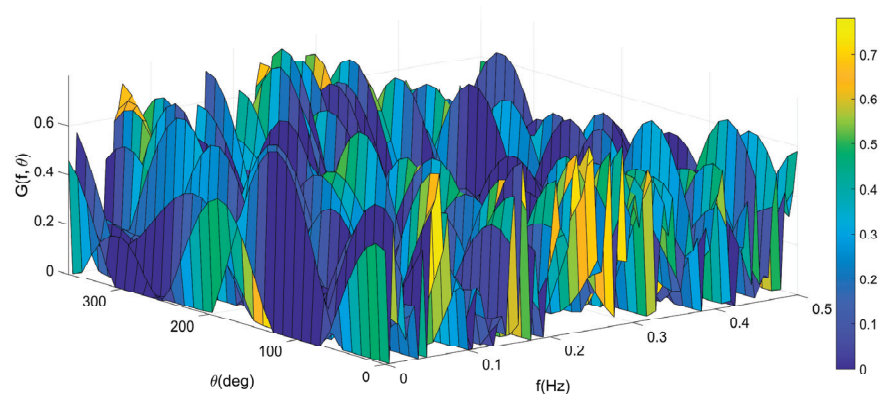


Figure 7. Revised directional distribution function.

4. Analysis of Measured Wave Results

4.1. Wave Parameter Retrieval Based on Statistical Analysis

Ocean wave statistical analysis primarily employs statistical and spectral methods. Statistical approaches characterize wave properties through quantitative descriptors, elucidating wave variability and trends. The zero-upcrossing method, a statistical technique for

wave analysis, estimates wave periods and heights by detecting zero-crossing points in time series data [50].

Wave Period and Height Estimation via Zero-Upcrossing Method

To evaluate the feasibility of retrieving wave parameters from GNSS buoy data, measurements were conducted from 08:00 to 12:00 on 27 April 2023, divided into eight 30 min intervals. Figure 8 displays the first 8 min of wave surface elevation data for each interval, demonstrating clear wave height variations recorded by the GNSS buoy. Throughout the observation period, the buoy data exhibited high stability.

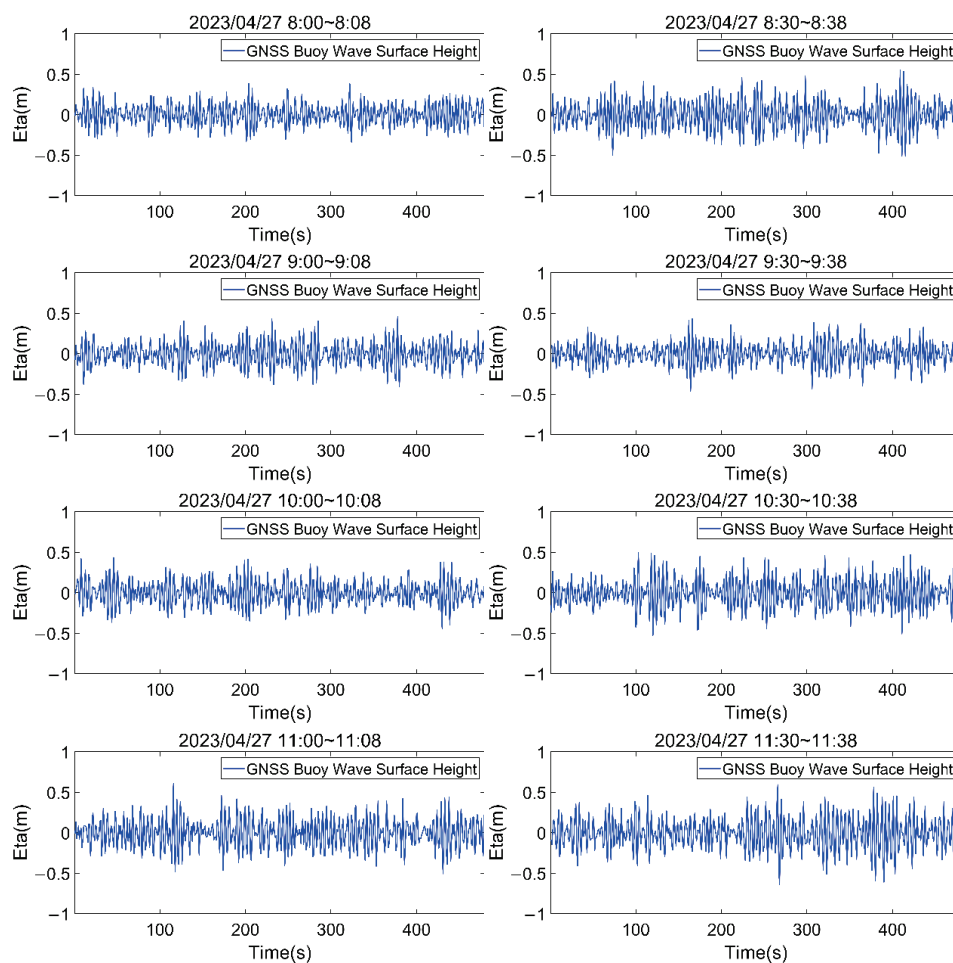


Figure 8. GNSS buoy wavefront height variation (data duration: 8 min).

For each time interval, wave statistical analysis was conducted using approximately 100 consecutive waves as a standard sample. The following figure (Figure 9), which consists of eight subplots labeled a to h, presents the wave statistical results obtained from the GNSS buoy in the eight different time periods. Each subplot for the time interval consists of three parts: the first part shows the crest and trough heights of each wave arranged chronologically; the second part displays the wave height distribution, with all waves sorted in descending order, highlighting the top one-third of the wave sequence using a pink dashed line to represent the significant waves; and the third part, based on the sorting in the second part, presents the corresponding wave periods using asterisks to denote all individual periods, while the significant wave period is explicitly identified in the legend.

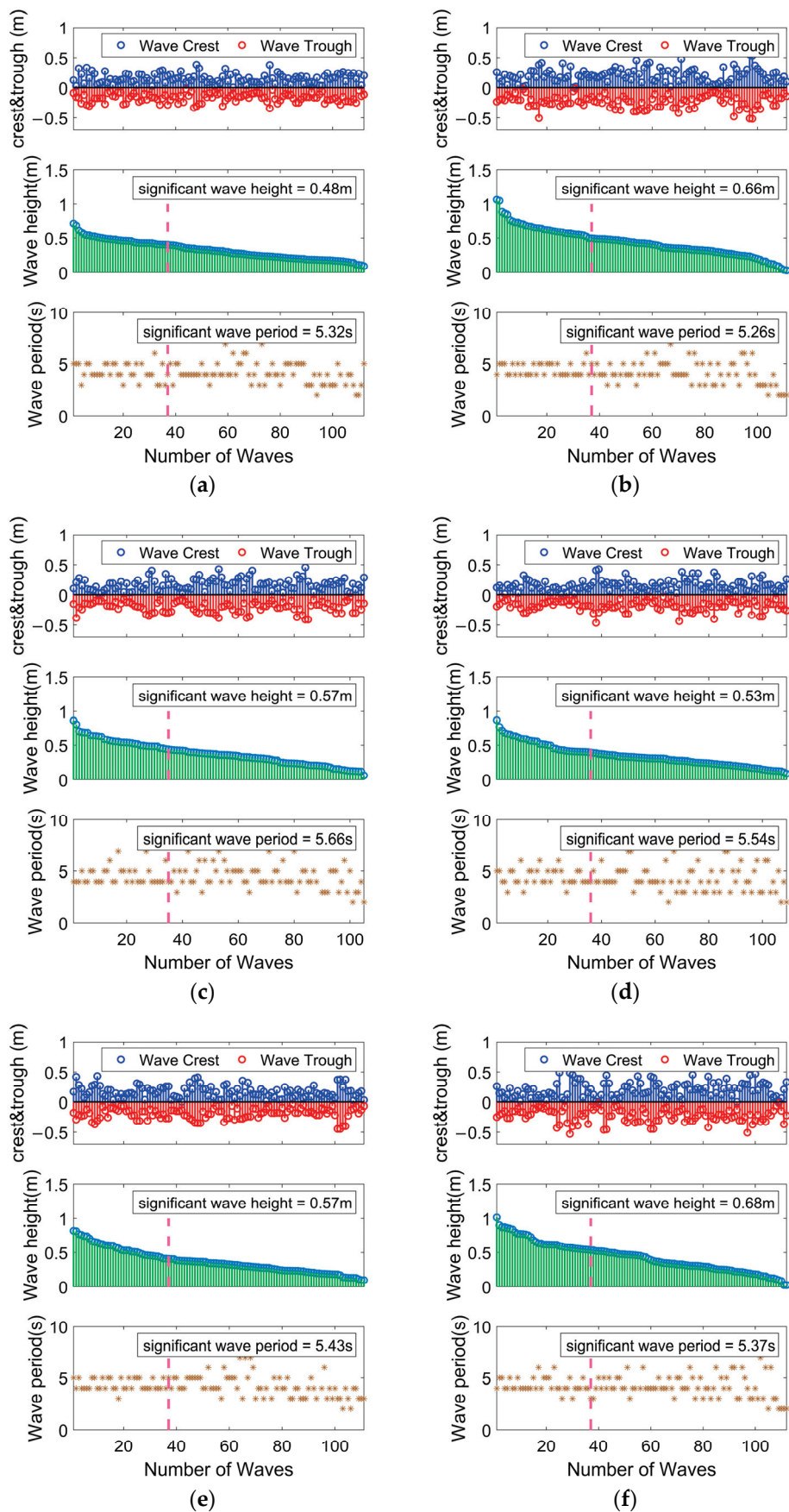


Figure 9. Cont.

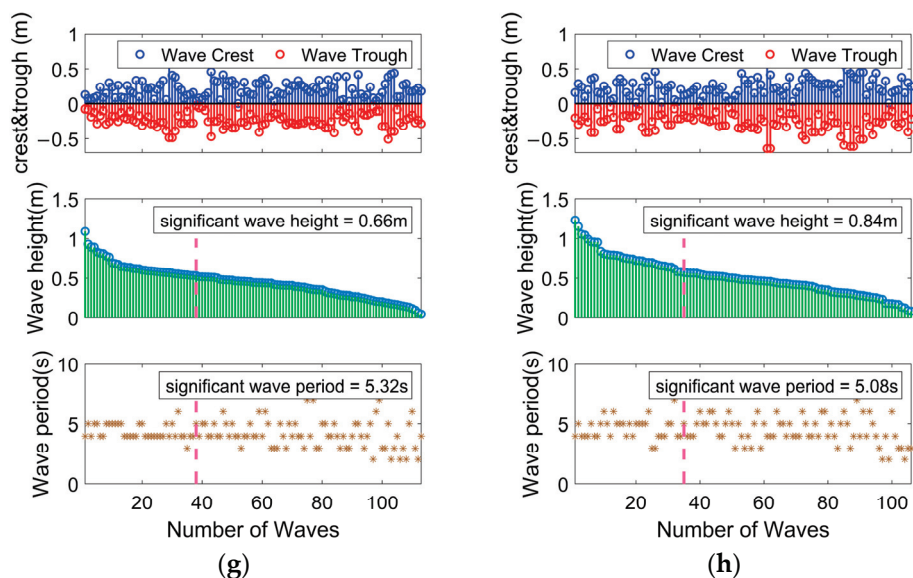


Figure 9. GNSS buoy wave crest, trough, height, and period statistics. (a) 27 April 2023 8:00~8:08; (b) 27 April 2023 8:30~8:38; (c) 27 April 2023 9:00~9:08; (d) 27 April 2023 9:30~9:38; (e) 27 April 2023 10:00~10:08; (f) 27 April 2023 10:30~10:38; (g) 27 April 2023 11:00~11:08; (h) 27 April 2023 11:30~11:38. The asterisks (*) in figure represent the sorted wave periods (in seconds) corresponding to wave heights arranged in descending order.

Based on the statistical wave characteristics presented in the figure above, the characteristic wave heights and corresponding wave periods were calculated for each dataset.

For each data segment, wave sequences were statistically analyzed using the zero-upcrossing method. The maximum wave height represents the absolute crest value within the sequence, while the highest one-tenth wave height denotes the average of the top 10% largest waves—both exhibiting relatively strong randomness. In contrast, the significant wave height (highest one-third of the waves' average) and mean wave height (average of all waves) better reflect the statistical characteristics of the wave field [23]. The mean wave parameters particularly demonstrate a more stable representation of the prevailing sea state.

Theoretical analysis indicates that shorter wave samples tend to yield larger mean wave heights and periods. For instance, the 09:00–09:08 segment containing 105 waves showed greater mean values compared to the 111-wave (08:30–09:08) and 110-wave (09:30–09:38) segments. These results confirm that GNSS buoys can effectively capture wave height and period parameters through zero-upcrossing analysis and reliably record oceanic wave details.

It is worth noting that the 10 m GNSS buoy used in this study, while offering excellent attitude stability and robustness for GNSS positioning, is not specifically optimized for wave-following performance, especially in low-sea-state conditions. This inherent design characteristic may reduce its sensitivity to small-amplitude wave fluctuations, leading to limited correlation between significant wave height and significant wave period, as observed in Tables 3 and 4. This phenomenon is consistent with the expected behavior of large-diameter buoys and reflects a trade-off between positional accuracy and wave-tracking responsiveness. Therefore, the results discussed here should be interpreted within the context of the buoy's physical limitations. This highlights the need for further comparative studies involving different buoy configurations and sea conditions.

Table 3. Statistical values of characteristic wave height and number of waves.

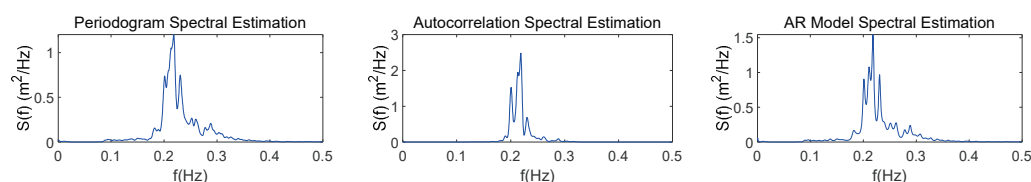
Data Period	Statistical Wave Height Parameters (m)				Number of Waves
	$H_{\max}$	$H_{1/10}$	$H_{1/3}$	H_{mean}	
08:00–08:08	0.72	0.57	0.48	0.32	112
08:30–08:38	1.06	0.82	0.66	0.44	111
09:00–09:08	0.87	0.70	0.57	0.38	105
09:30–09:38	0.87	0.68	0.53	0.35	110
10:00–10:08	0.82	0.72	0.57	0.37	111
10:30–10:38	1.01	0.84	0.68	0.44	113
11:00–11:08	1.09	0.82	0.66	0.45	113
11:30–11:38	1.23	0.98	0.84	0.54	106

Table 4. Statistical value of characteristic wave period.

Data Period	Statistical Wave Period Parameters (s)			
	$T_{\max}$	$T_{1/10}$	$T_{1/3}$	T_{mean}
08:00–08:08	8.00	6.00	5.32	4.19
08:30–08:38	7.00	5.83	5.26	4.22
09:00–09:08	8.00	6.73	5.66	4.52
09:30–09:38	8.00	6.36	5.54	4.34
10:00–10:08	7.00	6.33	5.43	4.30
10:30–10:38	8.00	6.25	5.37	4.22
11:00–11:08	8.00	6.00	5.32	4.22
11:30–11:38	7.00	6.18	5.08	4.33

4.2. Wave Period and Height Retrieval Based on Spectral Analysis

Section 4.1 analyzed GNSS buoy measurements from 08:00 to 12:00 on 27 April 2023, using the zero-upcrossing method. This section focuses on the 11:30–12:00 dataset, employing spectral analysis to calculate and examine wave parameters. The following presents spectral analysis results of U-direction displacement time series obtained through three methods: the periodogram, autocorrelation function, and AR model approaches, as shown in Figure 10.

**Figure 10.** Spectral estimation diagram of measured data.

To better visualize the differences and similarities between the three spectral estimation methods, we normalized the power spectral densities to eliminate amplitude variations, enabling direct comparison on a unified scale. The normalized spectra clearly show the frequency-domain characteristics of each method, facilitating identification of their respective strengths and limitations. By plotting the spectral results from all three methods together, we can intuitively compare their spectral shapes and distribution patterns, thoroughly analyzing their performance in different frequency bands and potential biases, as shown in Figure 11.

Comparison of the power spectral density obtained using the periodogram, autocorrelation function, and AR model methods reveals nearly identical spectral shapes. While minor differences exist in certain areas due to variations in wave data processing methods, they do not affect the calculation of first-order wave characteristics. Table 5 presents wave parameters obtained from the three spectral analysis methods, showing identical values for significant and mean wave heights across methods, with only minor differences in wave

periods: 0.02 s for the significant period and 0.01 s for the mean period. Compared to parameters obtained using the zero-upcrossing method, the spectral analysis results show differences of 0.05 m in significant wave height, 0.79 s in significant wave period, 0.09 m in mean wave height, and 0.08 s in mean wave period.

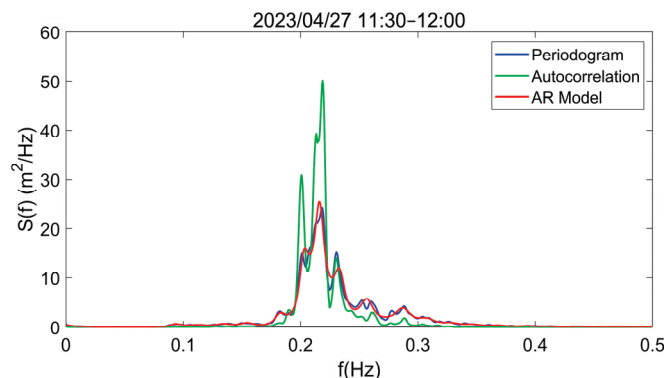


Figure 11. Comparative analysis of power spectral density using periodogram, autocorrelation function, and AR model methods.

Table 5. Comparison of wave characteristic element results using periodogram, autocorrelation function, and AR model methods.

Wave Parameters	Periodogram	Autocorrelation	AR Model	Zero-Crossing
Significant Wave Height (m)	0.89	0.89	0.89	0.84
Significant Wave Period (s)	4.27	4.27	4.29	5.08
Mean Wave Height (m)	0.56	0.56	0.56	0.54
Mean Wave Period (s)	4.25	4.24	4.25	4.33

The comparison between the reference significant wave height and GNSS-buoy-derived wave heights is shown in Figures 12 and 13.

To quantitatively assess agreement between GNSS-derived results and the reference measurements, we calculated the Root Mean Square Error (RMSE) and Pearson correlation coefficient. For significant wave height, the RMSE was 0.0788 m with a correlation of 0.9298. For significant wave period, the RMSE was 0.1926 s with a correlation of 0.9381. These metrics confirm that the GNSS-based inversion method yields reliable and accurate wave parameters [23].

Comprehensive statistical and spectral analyses of the same GNSS buoy data further demonstrate consistency across methods, fully validating the effectiveness and applicability of the proposed inversion framework.

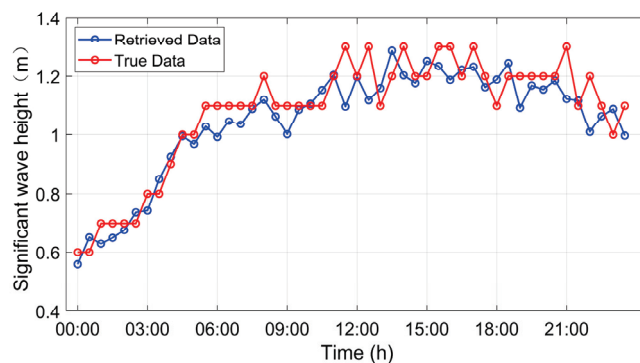


Figure 12. Comparison chart of inverted sea surface wave height and true sea surface wave height.

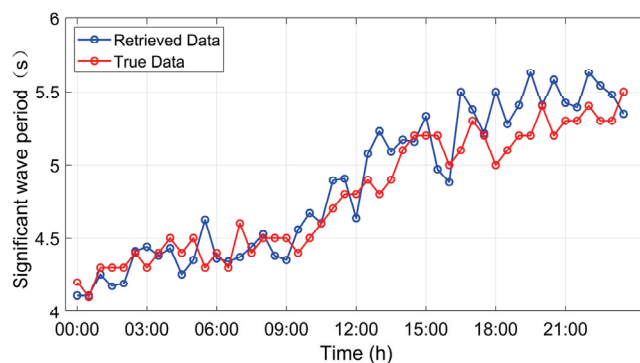


Figure 13. Comparison chart of inverted period and real period.

4.3. Wave Direction Retrieval Based on Directional Spectrum Analysis

Directional spectrum analysis provides the two-dimensional distribution of wave energy with respect to both frequency and direction, offering deeper insight into wave propagation characteristics. In this study, we applied cross-spectral analysis using the horizontal displacement time series of the GNSS buoy to estimate the directional spreading function. This enabled the construction of the directional wave spectrum and the identification of dominant propagation trends under actual sea conditions.

4.3.1. Three-Dimensional Wave Directional Spectrum Analysis

Based on the wave frequency spectrum and directional spreading function, the directional wave spectrum for the corresponding time period is presented below, as shown in Figure 14.

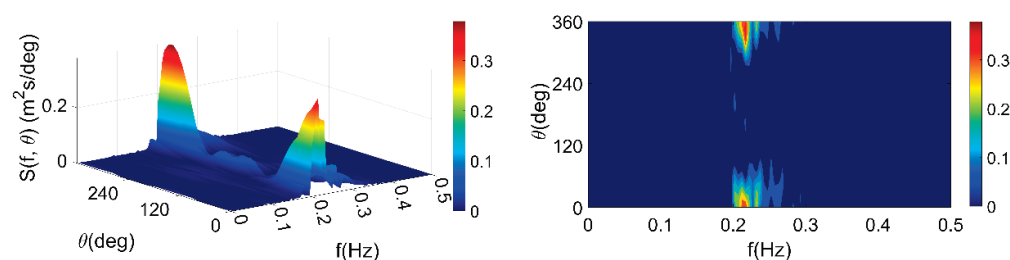


Figure 14. Directional spectrum: 3D and plan views (27 April 2023 11:30–12:00).

During the test, the wind speed and direction were stable, and the overall wave changes were not significant in the short term. The wave direction in the figure is given in the form of wave direction azimuth (0° to the north and clockwise to the positive direction).

The following is a three-dimensional diagram of the directional spectrum from 8:00 to 12:00 on 27 April, with the directional spectrum given every half hour, as shown in Figure 15.

The horizontal axis in the figure is frequency, ranging from 0 Hz to 0.5 Hz, indicating the frequency distribution of waves; the vertical axis is direction, ranging from 0° to 360° , indicating the propagation direction of waves; and the vertical axis is spectral density, indicating the wave energy density.

In the figure, the peak of the spectral density is mainly concentrated in the frequency range of 0.2 Hz to 0.25 Hz, and the direction ranges from 0° to 60° and 300° to 360° , which indicates the frequency band where the wave energy is mainly concentrated; the main propagation direction of the wave is northward. In addition to the dominant spectral density peak, a secondary peak with lower amplitude is also consistently present. The main peak is generally caused by the main wind direction in the sea area, while the secondary

peak is generally the result of the combined effect of the main wind direction of the sea area and the long-distance swell [53].

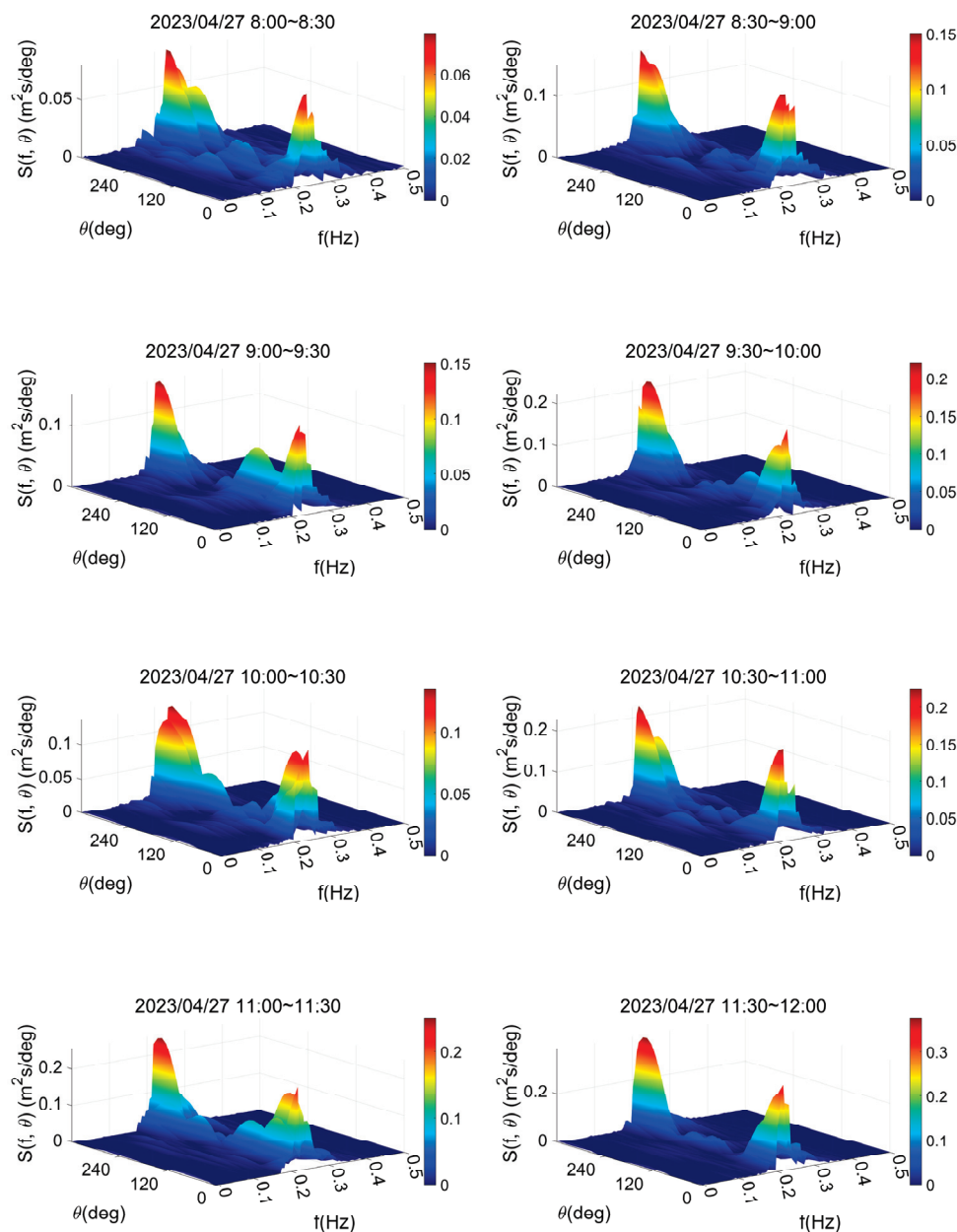
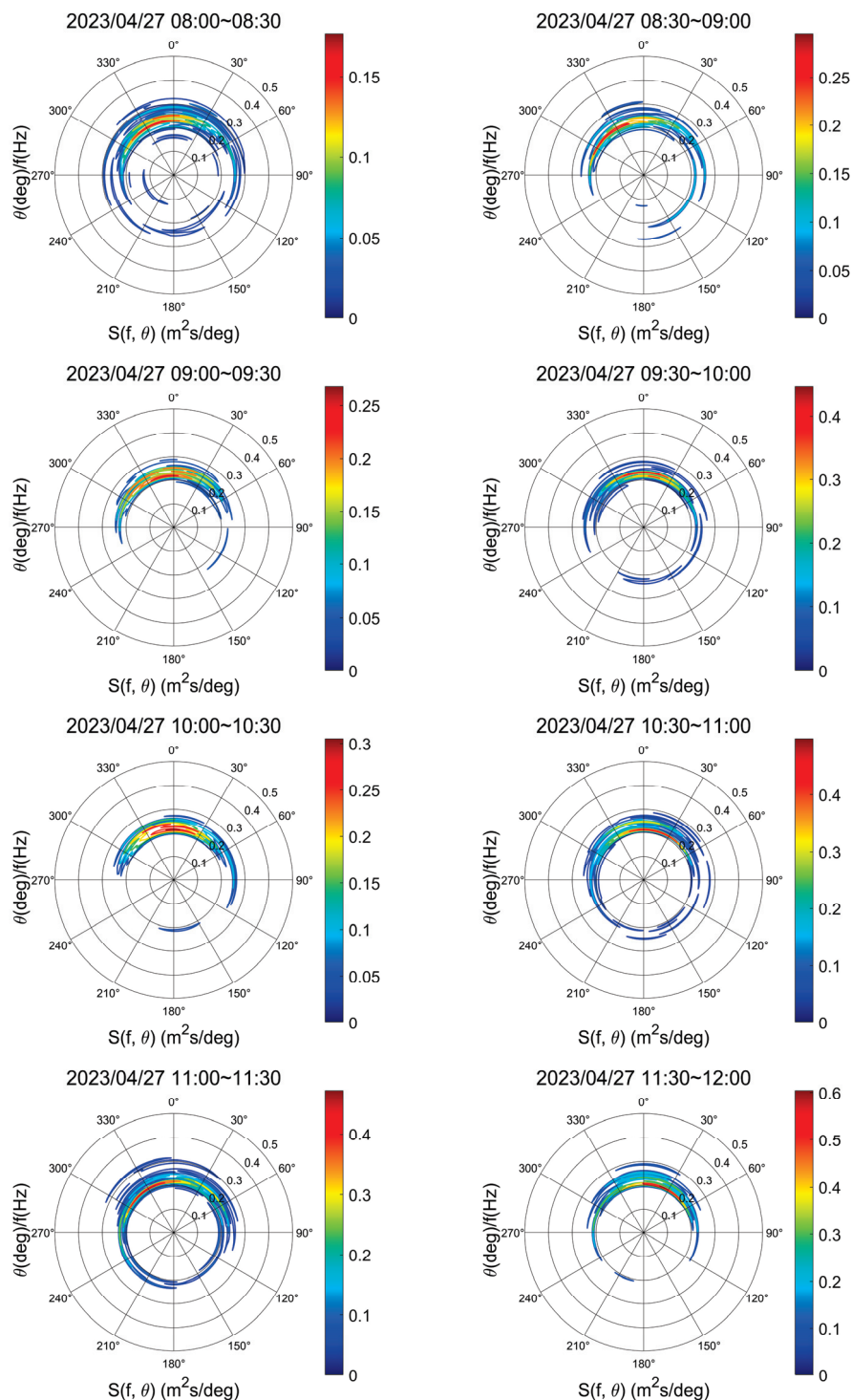


Figure 15. Three-dimensional wave directional spectrum.

4.3.2. Analysis of Wave Direction–Height Relationship

The three-dimensional graph intuitively illustrates the spatial distribution of wave energy; however, limitations in resolution and viewing angle may affect the clarity and interpretability of the visualization. The polar coordinate graph can simply and clearly display multi-dimensional information, making it easier to extract and analyze key information. The following is the corresponding polar coordinate graph of the directional spectrum, as shown in Figure 16.

The figure illustrates the directional distribution of waves over a 24 h period, showing predominant wave directions clustered near 0° (true north). This indicates that wave energy was primarily concentrated in northerly directions throughout the day. Notably, wave heights at 0° were both significantly larger and more concentrated, further confirming the strong directional focusing of wave energy toward the north.



tion of energy in specific frequency bands and revealed a predominantly northward wave propagation trend, validating the method's applicability under real sea conditions.

As an observational platform, the GNSS buoy continuously records three-dimensional motion trajectories. Compared to traditional wave gauges and radar-based instruments, GNSS buoys offer advantages such as low cost, easy deployment, and long-term stability, making them highly suitable for long-duration and wide-area marine monitoring.

To overcome limitations in traditional wave signal processing and directional spectral estimation, this study employed a combination of statistical and spectral approaches. The zero-crossing method and three spectral techniques—the periodogram, autocorrelation function, and autoregressive model—were applied to estimate wave height and period, providing a robust cross-validation framework. Horizontal displacement and velocity data were used to retrieve wave propagation characteristics in two dimensions for directional spectral estimation.

Analysis of data collected on 27 April 2023 confirmed the stability and performance of the proposed method. The three spectral approaches produced wave period estimates with a maximum discrepancy of only 0.02 s. Compared with the zero-crossing method, the spectral analysis results show differences of 0.05 m and 0.79 s in significant wave height and period, respectively, with average differences of 0.09 m and 0.08 s. These results demonstrate strong internal consistency and validate the accuracy of the GNSS-based approach. Directional spectrum analysis showed that energy was concentrated in the 0.2–0.25 Hz frequency band and within a directional range of $0^\circ \pm 30^\circ$, reflecting a clear northward propagation pattern, consistent across all observation periods.

Taken together, the results demonstrate that the proposed GNSS-buoy-based wave inversion framework is accurate, consistent, and robust under complex sea conditions. This study highlights the practical potential of GNSS buoys as cost-effective tools for real-time ocean wave monitoring, with broad applications in marine engineering, disaster prevention, and oceanographic research.

Nevertheless, several limitations should be acknowledged. The large size (10 m) of the buoy reduces its wave-following sensitivity, especially for low sea states, potentially affecting accuracy. In rough sea conditions, multipath interference, buoy instability, and antenna tilt may further degrade GNSS positioning. Additional stabilizing structures or more rigid antenna mountings may be required to ensure reliable operation. Moreover, the reliance on uninterrupted GNSS signals and post-processing may limit real-time performance in extreme environments.

Future research will focus on enhancing signal robustness under variable sea conditions, improving hardware stability, and extending validation across diverse platforms and environmental scenarios. These efforts aim to support the development of a scalable, all-weather GNSS-based wave monitoring solution.

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Data Availability Statement: The data presented in this study are available on request from the corresponding author.

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Article

GPS and Galileo Precise Point Positioning Performance with Tropospheric Estimation Using Different Products: BRDM, RTS, HAS, and MGEX

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Abstract: The performance of Precise Point Positioning (PPP) using different Global Navigation Satellite System (GNSS) product sets, including broadcast ephemerides, International GNSS Service Real-Time Service (IGS-RTS) corrections, Galileo High Accuracy Service (HAS) corrections, and precise products from the Center for Orbit Determination in Europe (CODE) Multi-GNSS Experiment (MGEX), has been evaluated. The availability of solutions, convergence time, position accuracy and Zenith Tropospheric Delay (ZTD) estimation across these products were analyzed using simulated real-time and postprocessing static modes, using data from globally distributed stations with a 1 s observation interval. The results indicate that precise products from the MGEX provide the highest accuracy, achieving centimeter-level precision in post-processed mode. Real-time simulated solutions, such as HAS and IGS-RTS, deliver promising results, with Galileo HAS meeting its target accuracy of 20 cm horizontally and 40 cm vertically and a convergence time under 5 min. However, Global Positioning System (GPS) performance within HAS is limited by a significantly lower correction availability—around 67% on average compared to over 95% for Galileo—which negatively impacts PPP performance. ZTD estimation results show that real-time services (HAS, IGS-RTS) achieved errors within 1–3 cm, sufficient for meteorological applications. This study highlights the growing importance of HAS in real-time positioning applications and suggests further improvements in GPS for enhanced performance.

Keywords: HAS; RTS; PPP; GPS; Galileo; ZTD; convergence time

1. Introduction

Over the past three decades, the concept of precise absolute positioning has evolved significantly since its initial formulation [1–5]. Today, the Precise Point Positioning (PPP) method is widely applied across a broad spectrum of scientific and engineering domains [6], including seismology [7], reference frame determination [8], atmospheric sounding [9,10], natural hazard monitoring [11], and the estimation of drone trajectories [12]. PPP can achieve a millimeter- to centimeter-level accuracy in static, post-processed applications and a decimeter-level accuracy in kinematic and real-time scenarios. However, the primary limitation of PPP remains its relatively long convergence time [13].

In response to this limitation, substantial research efforts in recent years have focused on enhancing the efficiency and reliability of absolute positioning. Notable advancements include the development of PPP with Ambiguity Resolution (PPP-AR) [14,15], PPP-Real-Time Kinematic (PPP-RTK) [16,17], different/combined products that are available [18–20], and the use of multi-GNSS and multi-frequency measurements [21–24]. In parallel, interest

has grown in single-frequency solutions and low-cost GNSS receivers, particularly for emerging technologies like smartwatches, drones, and smartphones [25,26].

In the coming years, the adoption of PPP is expected to expand further, particularly in real-time applications. This trend is driven by ongoing technological developments, including the increasing use of unmanned aerial systems, the advancement in smart city infrastructures, and the rising demand for reliable, high-precision positioning in autonomous systems. These use cases require independent, globally available, and highly accurate localization solutions.

Recent advancements in GNSS infrastructure have further accelerated the evolution of precise positioning techniques. Next-generation satellites—such as the GPS Block III, which enables additional observations on the L5 frequency [27]—are gradually being deployed. Concurrently, the number of operational Galileo satellites continues to increase [28]. As the Galileo constellation expands, further improvements in positioning accuracy are anticipated, particularly when leveraging onboard ephemeris products [29–31]. Notably, while GPS ephemeris data are updated every two hours, Galileo satellites provide more frequent updates, ranging from every 10 to 60 min.

To facilitate high-accuracy positioning in real time, the International GNSS Service (IGS) initiated the Real-Time Pilot Project in 2007 and officially launched the Real-Time Service (RTS) in 2013 [32,33]. The RTS supports real-time applications by providing access to precise GNSS products, including orbit and clock corrections, as well as code and phase biases [34]. These corrections serve as a viable alternative to ultra-rapid products, thereby enabling global real-time PPP, time synchronization, and natural hazard monitoring.

Currently, IGS RTS offers multi-GNSS combination products—SSRA02IGS1/SSRC02IGS1, which support GPS, GLONASS, and Galileo, and SSRA03IGS1/SSRC03IGS1, which also include BeiDou corrections. These corrections are applied to broadcast data but require an internet connection and compatible software such as the BKG NTRIP Client (BNC) [35]. In addition to the official IGS RTS solutions, several individual Analysis Centers (ACs) provide their own real-time products [36]. A study by Li et al. [37] comparing official combined RTS corrections and individual AC products showed that the highest accuracy was achieved using corrections from CNES and WHU, with CNES offering superior availability.

Despite their utility, the dependence on internet connectivity limits the use of these products in many regions. Recently, new correction services broadcast directly via navigation systems have emerged, offering enhanced accessibility. These include the BeiDou B2b service, Japan's Centimeter-Level Augmentation Service (CLAS) for QZSS [38], and Europe's Galileo High Accuracy Service (HAS) [39]. While B2b and CLAS are broadcast via geostationary satellites and thus limited to the Asia-Pacific region, Galileo HAS data are transmitted by a subset of satellites—currently up to 20—from the overall constellation, enabling global coverage with varying levels of availability [40].

The Galileo HAS will operate at two levels [41]. Level 1 provides global coverage with a target horizontal accuracy below 20 cm and vertical accuracy below 40 cm, with convergence times under 300 s. Level 2, limited to Europe, aims to achieve the same accuracy with a faster convergence time (under 100 s) by incorporating regional atmospheric corrections. HAS's development comprises three phases: Phase 1 (testing and experimentation, 2020–2022); Phase 2 (Initial Service), launched on 24 January 2023; and Phase 3 (Full Service), which will deliver Level 2 capabilities [40].

As a central element of the European GNSS ecosystem, Galileo HAS offers free access to PPP corrections globally, broadcast via the E6-B signal or through the internet. Although the service is relatively new, its potential is already evident. Open-source tools such as HASlib [42], HAS PPP [43], and Python-based decoding toolboxes have been developed to enable the real-time reception and processing of HAS corrections [44]. Preliminary studies

using test signals have demonstrated a promising PPP performance [45–47]. Furthermore, integration with tools like RTKLIB [48] highlights the potential of open-source platforms to broaden access to high-precision GNSS positioning. Nevertheless, further research is required to enhance the service’s reliability and broaden its applicability.

One of the key advantages of PPP is its capability to estimate tropospheric delay parameters, particularly the Zenith Tropospheric Delay (ZTD), in both post-processed and real-time modes. These estimates are essential for atmospheric studies, as they can be further converted into Precipitable Water Vapor (PWV) and assimilated into Numerical Weather Prediction (NWP) models to improve short-term weather forecasts. Several studies have highlighted the sensitivity of ZTD estimations to the quality of input products and modeling strategies [49]. For real-time applications, particularly those supporting NWP data assimilation, accuracy thresholds of 0.6 cm (target) and 3 cm (maximum) are typically required [10]. Recent advances in real-time GNSS processing, including PPP based on broadcast ephemerides [46,49] and the integration of multiple GNSS constellations [50,51], have shown the potential to meet these requirements. Additionally, current research continues to investigate how spatial resolution and data assimilation methods affect the accuracy of humidity fields derived from GNSS-based PWV products [52], reinforcing the importance of reliable real-time ZTD estimations in operational meteorology.

Currently, several studies on PPP positioning using HAS corrections have been carried out. However, they differ in terms of the time periods, processing software, and models used, and they often do not compare the obtained results with other products and corrections.

The aim of this study is to analyze PPP performance using different types of products and corrections: broadcast ephemerides, CNES RTS corrections, Galileo HAS corrections, and final precise products from the Center for Orbit Determination in Europe (CODE) within the Multi-GNSS Experiment (MGEX). The analysis is carried out using a uniform PPP processing scheme: all test cases use identical 1 s interval observations from globally distributed stations, the same processing software, and consistent modeling assumptions. This setup enables an objective and direct comparison of PPP performance across the different products.

The key performance indicators evaluated include solution availability, convergence time, positioning accuracy, and ZTD estimation for GPS, Galileo, and combined GPS+Galileo solutions. Importantly, HAS data were collected in real time, enabling a realistic simulation of near-real-time PPP performance. BRDM and CNES RTS corrections were also used in post-processing, representing additional simulated real-time scenarios. The final CODE MGEX products serve as the most precise reference solution and are used as a benchmark for the highest achievable PPP accuracy.

This study also examines the availability of HAS products and PPP solutions, highlighting practical challenges in real-time applications. In addition, 1 s observation intervals were employed with final CODE products—an uncommon approach in post-processing that enables a more detailed analysis of short-term convergence behavior.

Following the introduction, Section 2 describes the data, product sets, and processing methodology. Section 3 presents the results of the conducted analyses. Section 4 provides a discussion, and Section 5 offers a summary of the key findings.

2. Methodology and Data

This analysis utilized data from 12 globally distributed MGEX stations (Figure 1 and Table 1), which collected multi-GNSS observations at 1 s intervals over a six-day period, from Day of Year (DoY) 223 to DoY 228 in 2024. These days were selected because real-time HAS product data were successfully collected for them. For each day, 8 three-hour sessions

were analyzed, resulting in a total of 48 sessions per station across the six-day span. The computations were performed using the raPPPid software (version 3.1), which employs an Extended Kalman Filter (EKF), in combination with the conventional PPP model [53,54].

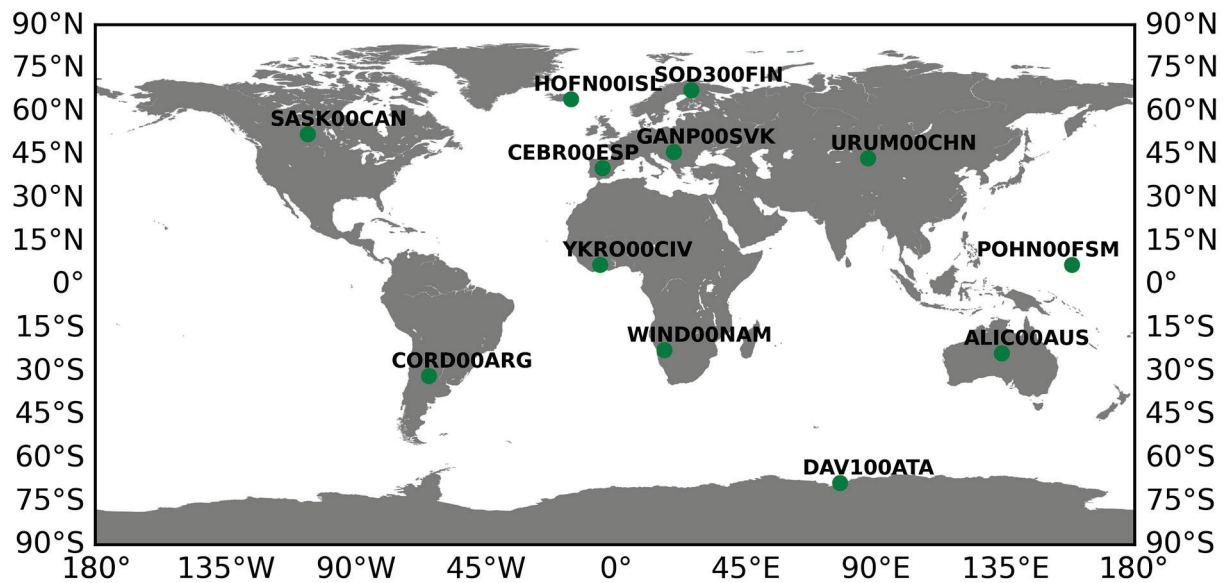


Figure 1. Location of test stations.

Table 1. Equipment of the used IGS stations.

Station	Receiver	Antenna
ALIC00AUS	SEPT POLARX5	TWIVC6050 NONE
CEBR00ESP	SEPT POLARX5TR	SEPCHOKE_B3E6 NONE
CORD00ARG	SEPT POLARX5	TPSCR.G3 NONE
DAV100ATA	SEPT POLARX5	LEIAR25.R3 LEIT
GANP00SVK	TRIMBLE ALLOY	TRM59800.00 SCIS
HOFN00ISL	LEICA GR50	LEIAR25.R4 LEIT
POHN00FSM	SEPT POLARX5	JAVRINGANT_DM NONE
SASK00CAN	JAVAD TRE_G3TH DELTA	NOV750.R4 NOVS
SOD300FIN	JAVAD TRE_3 DELTA	JAVRINGANT_DM SCIS
URUM00CHN	JAVAD TRE_3	JAVRINGANT_G5T NONE
WIND00NAM	JAVAD TRE_3	JAVRINGANT_G5T NONE
YKRO00CIV	JAVAD TRE_3 DELTA	ASH701945C_M NONE

The undifferenced ionosphere-free linear combinations for GPS and Galileo code and carrier-phase observations (in units of length) are expressed as

$$\begin{aligned}
 PR_{IF,G} &= \rho_r^G + c(\delta_{r, \text{clock}, G} - \delta_{\text{clock}}^G) + T_r^G + \epsilon_{IF} \\
 \Phi_{IF,G} &= \rho_r^G + c(\delta_{r, \text{clock}, G} - \delta_{\text{clock}}^G) + T_r^G + \lambda_{IF}^G N_{IF}^G + \epsilon_{IF} \\
 PR_{IF,E} &= \rho_r^E + c(\delta_{r, \text{clock}, E} - \delta_{\text{clock}}^E) + T_r^E + \epsilon_{IF} \\
 \Phi_{IF,E} &= \rho_r^E + c(\delta_{r, \text{clock}, E} - \delta_{\text{clock}}^E) + T_r^E + \lambda_{IF}^E N_{IF}^E + \epsilon_{IF}
 \end{aligned} \tag{1}$$

where

- G, E denote the GPS and Galileo systems, respectively;
- PR_{IF} and Φ_{IF} are ionosphere-free linear combination for code and carrier-phase observations, respectively [m];

- ρ_r^S is the true geometric range between the satellite in the emission time and the receiver in the reception time [m];
- c is the speed of light [m/s];
- $\delta_{r, \text{clock}, G}$, $\delta_{r, \text{clock}, E}$ represent the receiver clock offsets in the GPS and Galileo system time, respectively [s]
- δ_{clock}^G , δ_{clock}^E denote the satellite clock offsets for GPS and Galileo satellites, respectively [s];
- T_r^S is the slant tropospheric delay [m];
- λ_{IF} is the wavelength for the ionosphere-free linear combination [m];
- N_{IF} is the real value of the ambiguity ionosphere-free linear combination [cycles];
- ε_{IF} , ϵ_{IF} are other errors, for example noise and multipath [m].

This model also requires the inclusion of errors such as receiver and satellite phase center corrections, relativistic correction for code and carrier-phase observations of the ionosphere-free linear combinations and carrier-phase wind-up for phase observations, and the use of site displacement effects, which are defined in the IERS conventions [55].

In this way, the station coordinates, clock correction of the receiver for each system, wet component of the tropospheric delay, and real ambiguity values were estimated. A list of the used products, models, and methods are shown in Tables 2 and 3.

Table 2. Solutions names and input data sources.

Solution Name	Products
BRDM	broadcast products BRD400DLR; GPS LNAV and Galileo INAV
HAS	broadcast products BCEP00BKG0 with HAS corrections SSRA00EUH0; GPS LNAV and Galileo INAV; collected in real-time with BNC software (https://igs.bkg.bund.de/ntrip/bnc (accessed on 14 June 2025))
CNES	broadcast products with IGS RTS CNES corrections converted to daily CNES archive products as SP3, CLK, and BIA (http://www.ppp-wizard.net/products/REAL_TIME/ (accessed on 14 June 2025))
CODE	precise CODE MGEX FINAL products: SP3 with 5 min interval, CLK with 30 s interval

Table 3. Methods and models used. I think the table should be on one page.

Items	Models/Methods
Positioning mode	static mode
PPP model	conventional PPP model using undifferenced dual-frequency code and phase ionosphere-free linear combination
Sessions	eight three-hour sessions per day (48 sessions for each station from six days)
Signals	BRDM, CNES, and CODE solutions: L1/L2 for GPS and E1/E5a for Galileo HAS solution: L1/L2 for GPS and E1/E5b for Galileo
Stochastic modeling	$\sin^2 ele$
Constellation	GPS (G), Galileo (E), GPS+Galileo (GE)
Standard deviation of raw observations	code = 0.300 m, phase = 0.002 m
Cut-off elevation angle	10°
Interval estimation	1-s
Periods	six days: from 223 DoY to 228 DoY of 2024
Reference frame	IGS20

Table 3. Cont.

Items	Models/Methods
Reference coordinates	daily IGS SINEX files
PCO and PCV for satellite antenna	igs20.atx for CNES and CODE solutions; for BRDM and HAS the products were referred to APC
PCO and PCV for receiver antenna	igs20.atx
Ionospheric delay	ionosphere-free linear combination a priori value: GPT3 (VMF3 for CODE solution)
Tropospheric delay	estimated: wet component mapping function: GPT3 (VMF3 for CODE solution)
Solid tides, ocean loading, Shapiro effects, phase wind-up	gradients: estimated IERS convention 2010
Ambiguities	float
Receiver clock correction	estimated separately for each system

Four different solutions were computed for GPS, Galileo, and GPS+Galileo constellations:

- **‘BRDM’ solution:** computed using only broadcast products, with the BRD400DLR broadcast ephemerides downloaded;
- **‘HAS’ solution:** computed using broadcast ephemerides combined with HAS corrections. The products were collected in real-time using BNC software;
- **‘CNES’ solution:** computed using CNES archive products. The broadcast ephemerides and real-time CNES corrections were converted to SP3 and CLK files;
- **‘CODE’ solution:** computed using the final CODE MGEX products. These products were downloaded as SP3 and CLK with 5 min and 30 s intervals, respectively.

The analysis was conducted using a 10° elevation angle cut-off, with the $\sin^2 ele$ function applied for stochastic modeling. The reference coordinates were based on the daily IGS20 SINEX reference frame. The estimated tropospheric delay was compared with the official IGS value.

To estimate the ZTD for the BRDM, HAS, and CNES solutions, the GPT3 model was applied, while the VMF3 model was used for the CODE solution [56]. Additionally, for the CODE solution, the CODE MGEX satellite biases were included as observable-specific signal bias (OSB) files [14]. For the HAS and CNES solutions, bias corrections were obtained from the respective correction streams. In the case of the CNES solution, phase bias corrections were also applied, while for the HAS solution, only code bias corrections were used. For the BRDM solution, the Time Group Delay (TGD) data, as transmitted in the navigation messages, were utilized [57].

3. Results

The analysis focused on four key evaluation criteria: solution availability, convergence time, accuracy, and the precision of ZTD estimation. The following subsections present the results for each of these aspects.

3.1. Solution Availability

Solution availability was defined as the ratio of epochs in which position solutions were successfully derived to the total number of epochs for which position solutions were obtained using the CODE products. The analysis included only epochs where both GNSS observations and the corresponding orbit and clock products (or corrections) were available, which allowed the computation of positions and enabled an objective comparison of the performance of different solutions.

Table 4 presents the results of this analysis, including the average number of satellites used for each solution.

Table 4. Availability of obtained solutions and used satellites. Averaged across all analyzed stations.

Solution	GNSS	Number of Used Satellites	Availability [%]
BRDM	G	8.9	100.0
	E	6.8	99.8
	GE	15.6	100.0
HAS	G	5.0	74.0
	E	6.9	99.9
	GE	10.6	99.1
CNES	G	9.0	100.0
	E	6.5	99.7
	GE	15.5	100.0
CODE	G	9.3	-
	E	8.1	-
	GE	17.4	-

For both the BRDM and CNES solutions, a 100% availability was achieved for the GPS and combined GPS+Galileo constellations, with an average of approximately 9 GPS satellites and approximately 15.5 satellites, respectively, for the combined systems. For Galileo-only solutions, the availability was slightly lower—99.8% for BRDM and 99.7% for CNES—with a slightly reduced number of satellites: 6.8 and 6.5 on average, respectively. This is due to the fact that fewer Galileo satellites were available in BRDM and CNES products; for CNES products, about 20–21 Galileo satellites were available.

Different results were observed for the HAS products. For Galileo, solution availability reached 99.9% with about 6.9 satellites used, while for GPS it was significantly lower—only 74%, with an average of 5 satellites used. The HAS service is not yet fully operational and is primarily focused on the Galileo system, which may explain the lower availability and accuracy of the GPS-based solutions [44]. This is further illustrated in Figure 2, which shows the average availability of HAS orbit and clock corrections per satellite for both the Galileo and GPS constellations, averaged over the entire observation period. The dashed line marks the 90% availability threshold, highlighting that Galileo consistently exceeds this level with average availabilities of 95.49% for orbit corrections and 95.95% for clock corrections. In contrast, GPS average availabilities are much lower, at 67.11% for orbit and 67.64% for clock corrections. Notably, 10 GPS satellites show an availability below 50%, with 4 of them even below 20%—G07 (IIR-M), G13 (IIR), G15 (IIR-M), and G30 (IIF)—which negatively impacts the overall GPS performance in HAS products. While these satellites come from different blocks—IIR, IIR-M, and IIF—there is no clear dependency between satellite block type and correction availability. This indicates that other factors, such as satellite-specific conditions, operational status, or current HAS configuration, may be more influential in determining the performance of individual satellites.

Combining GPS and Galileo for HAS products slightly reduced the overall solution availability to 99.1%, with an average of 10.6 satellites used. This is likely due to the lower accuracy of the HAS products for GPS satellites, negatively affecting the overall quality of position solutions.

The highest number of satellites used was recorded for the CODE solution: 9.3 GPS satellites, 8.1 Galileo satellites, and 17.4 satellites for the combined GPS+Galileo constellation.

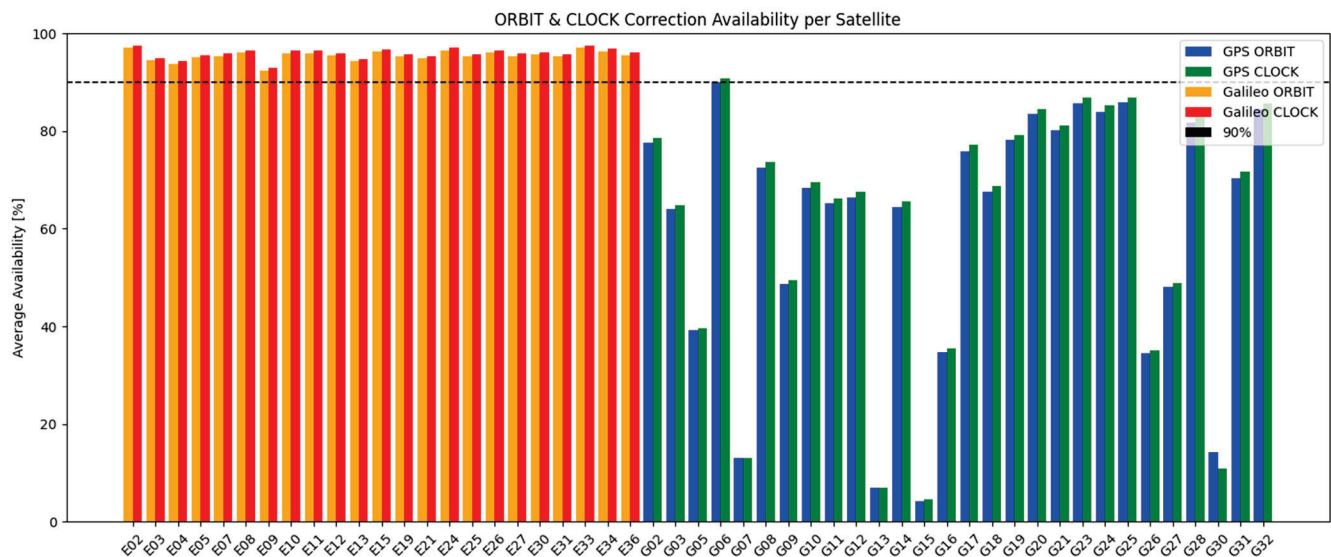


Figure 2. Average availability of HAS ORBIT and CLOCK corrections per satellite for the Galileo and GPS constellations. The dashed line marks the 90% availability threshold.

3.2. Convergence Time

This subsection analyzes the convergence times relative to the required accuracy levels. Additionally, the number of observational sessions meeting the specified thresholds was determined. Convergence time was defined as the moment when the errors reached the specified threshold and remained within it for at least the next 30 min. The thresholds were defined as 10 cm for both 2D and 3D for the CNES and CODE solutions. For the BRDM and HAS solutions, the thresholds were set at 20 cm horizontally (2D) and 40 cm vertically (U), which is the main objective of the HAS service; additionally, a 40 cm threshold was applied for spatial (3D). Identical accuracy thresholds were used for both BRDM and HAS to ensure a fair comparison, as the main goal is to highlight the impact of HAS corrections relative to broadcast-only data.

Figure 3 presents the results, and detailed data are provided in Appendix A. Table 5 summarizes the percentage of sessions meeting the required accuracy levels.

Table 5. Percentage of sessions in which the convergence time met the defined accuracy thresholds for all solutions and constellations.

Solution	GNSS	U	2D	3D
BRDM	G	80.7	27.4	35.9
	E	92.0	64.6	70.8
	GE	93.2	62.8	74.0
HAS	G	45.1	22.2	26.0
	E	75.0	70.7	73.3
	GE	75.0	71.0	74.0
CNES	G	99.1	99.5	97.6
	E	96.2	96.7	91.7
	GE	99.6	99.6	99.0
CODE	G	100.0	99.5	99.5
	E	99.8	99.6	98.8
	GE	100.0	99.8	99.6

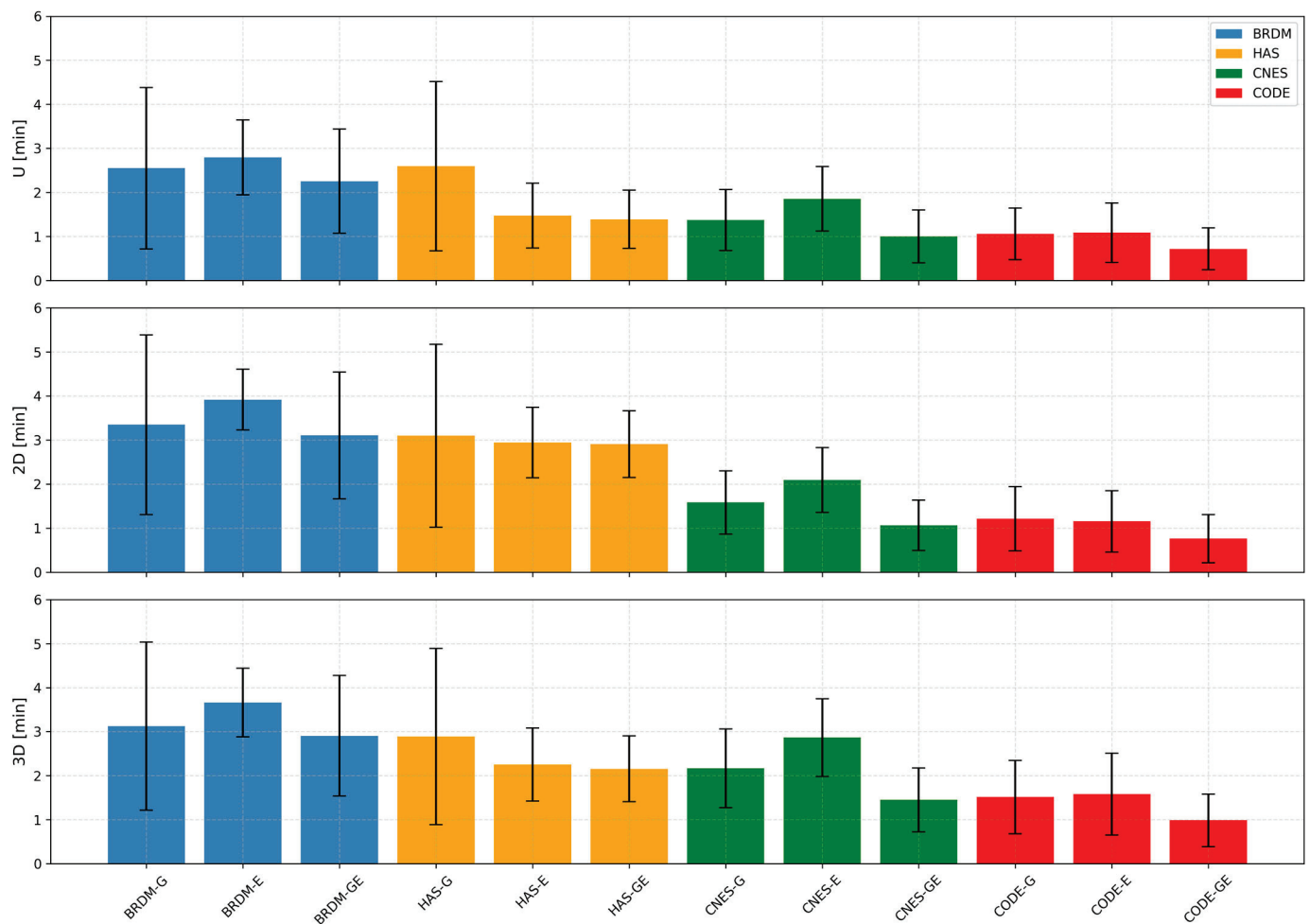


Figure 3. Mean convergence time with standard deviation for all solutions and constellations for U, 2D, and 3D.

Solutions based on CNES and CODE products demonstrated a consistently outstanding convergence performance, with success rates approaching or exceeding 99% in the most favorable configurations. For the CODE solution, convergence times were remarkably short, averaging approximately 1.2 min in 2D and 1.5 min in 3D when using GPS-only signals, with a slightly better performance observed for Galileo signals in 2D. The combined GPS+Galileo (GE) configuration yielded the best results, achieving convergence in less than 1 min in 2D and approximately 1 min in 3D, with success rates reaching up to 99.6%.

The CNES solution, simulating real-time operational conditions, also demonstrated a high percentage of sessions that successfully met the defined convergence criteria, indicating a reliable convergence performance, although convergence times were slightly longer compared to CODE. For GPS-only observations, convergence thresholds were achieved in 99.5% of sessions for 2D and 97.6% for 3D positioning. However, when using Galileo-only signals, the percentage of sessions that successfully reached convergence dropped to 96.7% for 2D and 91.7% for 3D, with average convergence times increasing to 2.09 min in 2D and 2.87 min in 3D. Nevertheless, when combining GPS and Galileo signals, the CNES solution achieved convergence within approximately 1.1 min in 2D and 1.5 min in 3D, with the percentage of sessions successfully reaching convergence closely matching that of the CODE solution—up to 99.6% for 2D and 99.0% for 3D.

In contrast, products based on BRDM and HAS corrections exhibited significantly longer convergence times and a lower percentage of sessions successfully reaching convergence. It should be noted that different convergence thresholds were applied for BRDM

and HAS compared to CODE and CNES solutions. For the BRDM solution using GPS-only observations, the average convergence time was approximately 3.17 min for the vertical component (U), 3.35 min in 2D, and 3.13 min in 3D, with a relatively high variability between sessions (standard deviations of ± 2.04 min for 2D, ± 1.91 min for 3D, and ± 2.04 min for U). The percentage of sessions achieving the required thresholds was noticeably lower, amounting to only 80.7% for U, 27.4% for 2D, and 35.9% for 3D positioning. Using Galileo-only signals resulted in slightly worse average times—3.51 min for U, 3.92 min in 2D, and 3.66 min in 3D—but the variability was considerably smaller (± 0.69 min for 2D, ± 0.78 min for 3D, and ± 0.69 min for U), and the percentage of sessions successfully reaching convergence improved to 92.0% for U, 64.6% for 2D, and 70.8% for 3D. A moderate improvement was observed when combining GPS and Galileo signals, with average convergence times of 2.86 min for U, 3.10 min in 2D, and 2.91 min in 3D, although the percentage of sessions successfully converged remained relatively modest.

The HAS solution, benefiting from additional correction data, generally outperformed BRDM, although the differences were not uniform across all configurations. When using GPS-only observations, the convergence performance was noticeably worse, with average times of 2.59 min for U, 3.10 min in 2D, and 2.89 min in 3D, accompanied by a large variability between sessions of ± 2.07 min for U, ± 2.07 min for 2D, and ± 2.00 min for 3D. In this case, the percentage of sessions achieving the required convergence thresholds was very low, amounting to only 45.1% for the vertical component (U), 22.2% for 2D positioning, and 26.05% for 3D. A clear improvement was observed when using Galileo-only signals, where the convergence times decreased to 1.47 min for U, 2.94 min for 2D, and 2.25 min for 3D, with a relatively low variability (± 0.80 and ± 0.83 min for 2D and 3D and ± 0.80 min for U), and success rates exceeded 70%. The best performance for the HAS solution was again obtained when combining GPS and Galileo signals, achieving average convergence times of 1.39 min for U, 2.91 min in 2D, and 2.16 min in 3D, with a relatively small variability (± 0.76 and ± 0.75 min for 2D and 3D and ± 0.76 min for U) and with over 70% of sessions meeting the required accuracy thresholds.

These results indicate that although HAS corrections significantly enhance the positioning performance compared to relying on broadcast data alone (BRDM), the convergence is still slower and less stable than what can be achieved with post-processing (CODE) or with real-time precise products (CNES).

Detailed convergence results for each coordinate component are available in Appendix A.

3.3. Accuracy

The positioning accuracy was evaluated by comparing the coordinates obtained in this study with the daily reference coordinates provided in the IGS20 SINEX files. To mitigate the influence of the initial convergence period, the first 10 min of data were excluded from the analysis. Mean errors and standard deviations are summarized in Tables 6 and 7.

The results obtained for the BRDM, HAS, CNES, and CODE products show significant differences in positioning accuracy, particularly depending on the GNSS signals used. For the BRDM solution, the results indicate a relatively low positioning accuracy, particularly for GPS signals. The mean errors in the N, E, and U components were -1.92 cm, 1.43 cm, and -6.14 cm, respectively, resulting in 2D and 3D accuracies of 64.83 cm and 93.18 cm. Despite relatively small mean errors, the significant standard deviations (± 11.38 cm in N, ± 24.98 cm in E, and ± 31.78 cm in U) indicate considerable variability. For Galileo signals, although the mean errors in N, E, and U slightly increased, the 2D and 3D accuracies improved significantly, reaching 50.33 cm and 71.34 cm, respectively, which was notably better than for GPS. The use of combined GPS and Galileo (GE) signals slightly improved

the results, with mean errors of 0.99 cm (N), -1.50 cm (E), and -0.87 cm (U), resulting in 2D and 3D accuracies of 49.57 cm and 69.40 cm, with the standard deviations remained high (± 15.66 cm in N, ± 26.63 cm in E, ± 24.07 cm in U).

Table 6. Mean coordinate difference to reference coordinates of all stations for all stations for all analyzed solutions. Values in cm.

Solution	GNSS	N	E	U	2D	3D
BRDM	G	−1.9	1.4	−6.1	64.8	93.2
	E	−3.4	3.56	−0.7	50.3	71.3
	GE	1.0	−1.5	−0.9	49.6	69.4
HAS	G	0.4	−7.9	−2.5	86.2	128.5
	E	0.2	0.1	1.5	18.9	28.8
	GE	−0.2	0.1	0.4	19.9	29.5
CNES	G	−0.5	−1.2	0.0	5.7	8.0
	E	−1.0	−1.4	−0.2	7.2	11.2
	GE	−0.6	−1.0	0.0	4.0	5.7
CODE	G	0.3	−0.7	−0.2	3.8	5.4
	E	0.1	0.0	−0.4	3.2	4.9
	GE	0.2	−0.2	−0.2	2.2	3.3

Table 7. Standard deviation of coordinate differences to reference coordinates of all stations for all analyzed solutions. Values in cm.

Solution	GNSS	N	E	U	2D	3D
BRDM	G	11.4	25.0	31.8	17.6	21.7
	E	6.8	19.6	14.9	13.1	16.8
	GE	15.7	26.6	24.1	28.5	35.8
HAS	G	21.8	41.8	47.0	38.9	47.4
	E	3.2	6.1	9.1	6.1	10.2
	GE	3.4	7.4	8.6	6.7	10.0
CNES	G	1.7	3.1	2.2	3.0	4.2
	E	1.2	2.6	3.2	2.8	4.7
	GE	1.5	2.2	2.1	2.0	2.8
CODE	G	1.5	2.1	1.4	2.9	4.1
	E	0.6	1.1	1.3	1.9	3.0
	GE	1.2	1.1	0.8	1.8	2.5

For the HAS solution, the results show an intense variability for GPS. The mean errors for GPS were 0.40 cm in N, -7.87 cm in E, and -2.52 cm in U, corresponding to 2D and 3D accuracies of 86.15 cm and 128.50 cm, which are even worse than those for BRDM. GPS results were characterized by particularly large standard deviations (± 21.81 cm in N, ± 41.82 cm in E, and ± 46.98 cm in U). A substantial improvement was observed with Galileo signals, where the mean errors decreased to 0.15 cm (N), 0.06 cm (E), and 1.47 cm (U), achieving 2D and 3D accuracies of 18.91 cm and 28.81 cm. For the GE combination, the mean errors were -0.23 cm in N, 0.09 cm in E, and 0.42 cm in U, with accuracies of 19.93 cm (2D) and 29.50 cm (3D). These results meet the requirements of the HAS service, except for GPS-only. This variability is consistent with the significantly lower availability of HAS corrections for GPS satellites, averaging around 67%, compared to over 95% for Galileo (Section 3.1). The HAS service primarily focuses on Galileo, and several GPS satellites have an availability below 50%, which negatively impacts the GPS positioning accuracy.

The CNES solution, based on real-time precise corrections, achieved a markedly better performance. For GPS, the mean errors were -0.48 cm (N), -1.25 cm (E), and -0.02 cm (U), resulting in 2D and 3D accuracies of 5.66 cm and 7.98 cm. For Galileo, there were mean errors of -0.95 cm (N), -1.39 cm (E), and -0.17 cm (U), leading to accuracies of 7.21 cm (2D) and 11.17 cm (3D), which were not as good as for GPS. The GE combination provided the best performance, with mean errors of -0.64 cm (N), -1.05 cm (E), and -0.04 cm (U) and accuracies of 3.95 cm (2D) and 5.73 cm (3D). Standard deviations were exceptionally low (e.g., ± 1.74 cm in N, ± 3.11 cm in E, and ± 2.21 cm in U for GPS), confirming the high precision and stability of the CNES solution.

The CODE solution, based on precise products, demonstrated the best performance. For GPS, the mean errors were 0.26 cm (N), -0.72 cm (E), and -0.15 cm (U), corresponding to 2D and 3D accuracies of 3.80 cm and 5.39 cm. For Galileo, the mean errors were minimal (0.06 cm in N, 0.00 cm in E, and -0.36 cm in U), with 2D and 3D accuracies of 3.24 cm and 4.88 cm. The GE combination further improved the results, with mean errors of 0.16 cm (N), -0.23 cm (E), and -0.24 cm (U), and 2D and 3D accuracies of 2.21 cm and 3.26 cm, respectively. Standard deviations remained very low across all components, confirming the superior precision and consistency of the CODE solution.

It should be noted that the 2D and 3D values represent the mean absolute differences from the reference coordinates. Therefore, these values may appear large, which is consistent with the large standard deviations.

In summary, worse results were obtained for GPS in both BRDM and HAS solutions. The inclusion of Galileo significantly improved the achieved accuracy. The best results for each of the products were obtained by combining GPS and Galileo signals, which provided the best convergence times and the highest measurement accuracy.

3.4. ZTD (Zenith Tropospheric Delay)

The ZTD analysis was performed by comparing the estimated zenith delays with the reference values provided by the IGS. The results are illustrated in Figure 4. The figure also highlights the 6 mm (target) and 3 cm (maximum acceptable) thresholds commonly required for real-time applications [10].

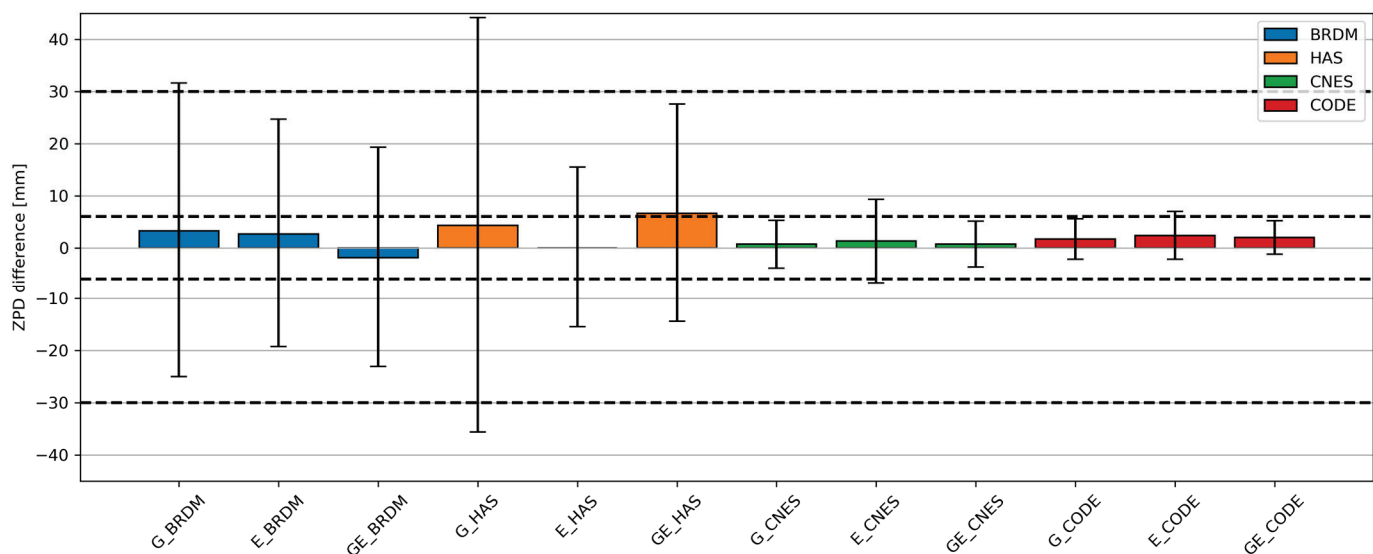


Figure 4. Mean difference and STD of the ZTD estimates at all stations with respect to the IGS reference for all analyzed solutions.

The BRDM solution, based solely on broadcast ephemerides, demonstrated a somewhat lower performance, particularly for GPS signals. The mean ZTD errors reached

3.2 mm for GPS, 2.7 mm for Galileo, and 1.5 mm for the combined GPS+Galileo solution. Although these mean values are within the 6 mm target, the associated standard deviations were relatively high: ± 30.6 mm for GPS, ± 21.9 mm for Galileo, and ± 27.6 mm for GPS+Galileo. Notably, the standard deviation for GPS slightly exceeded the 30 mm threshold, while for Galileo and GPS+Galileo it remained within acceptable limits. This indicates that while the average ZTD estimation was satisfactory, the variability of the results, particularly for GPS, could impact reliability in real-time applications.

In the case of the HAS solution, a larger variability was observed, especially for GPS signals. The mean ZTD errors were 5.0 mm for GPS, -0.2 mm for Galileo, and 7.5 mm for the combined GPS+Galileo solution. The corresponding standard deviations were ± 43.1 mm for GPS, ± 15.4 mm for Galileo, and ± 19.3 mm for GPS+Galileo. These results indicate that the use of Galileo signals, either alone or in combination with GPS, provides a noticeably better stability and precision compared to using GPS signals alone when applying HAS corrections.

The CNES solution, based on real-time precise corrections, demonstrated outstanding performance in ZTD estimation. The mean ZTD errors were 0.7 mm for GPS, 1.3 mm for Galileo, and 0.7 mm for GPS+Galileo. The associated standard deviations were also exceptionally low: ± 4.6 mm for GPS, ± 8.0 mm for Galileo, and ± 4.4 mm for GPS+Galileo.

Similarly, the CODE solution, based on post-processed precise products, yielded very high-quality results. The mean ZTD errors were 1.7 mm for GPS, 2.4 mm for Galileo, and 2.0 mm for GPS+Galileo. The standard deviations were also the lowest among all the solutions analyzed: ± 3.9 mm for GPS, ± 4.6 mm for Galileo, and ± 3.2 mm for GPS+Galileo.

Overall, while BRDM and HAS demonstrated acceptable mean errors, their standard deviations—particularly for GPS signals—slightly exceeded the thresholds, which could pose challenges for high-reliability real-time applications. However, despite these slight exceedances, both solutions remain highly useful and can still be considered reliable for many applications. In contrast, the CNES and CODE solutions clearly outperformed the others, achieving both target mean errors and maintaining a low variability, ensuring robust and accurate ZTD estimations suitable for operational NWP systems. Additionally, as the Galileo HAS service continues to evolve, improvements in its performance are expected, which could further enhance its reliability and precision in the near future.

4. Discussion

In the conducted research, various GNSS solutions were compared under simulated real-time conditions using 1 s interval observations for static positioning. The analysis revealed significant differences in performance, particularly regarding solution availability, convergence time, accuracy, and ZTD estimation. Notable differences were observed between solutions based on broadcast ephemerides (BRDM), real-time precise corrections (CNES RTS), post-processed precise products (CODE MGEX), and the HAS service.

Convergence time was one of the key factors differentiating GNSS solutions based on PPP positioning. For the BRDM and HAS services, more lenient accuracy thresholds were applied (20 cm horizontally (2D) and 40 cm vertically (U) and spatially (3D)), whereas stricter thresholds (10 cm both horizontally and spatially) were used for CNES and CODE. BRDM and HAS exhibited longer and more variable convergence times compared to CNES and CODE, especially when using GPS-only data. However, integrating signals from Galileo (or combined GPS+Galileo) improved performance, although convergence times remained longer. These differences stem from the limitations associated with broadcast ephemerides in BRDM solutions and the focus of the HAS service on the Galileo system.

In contrast, for HAS solutions, the use of Galileo signals significantly improved availability and accuracy, whereas relying solely on GPS limited these parameters. The

reduced availability of HAS orbit and clock corrections for GPS satellites indicates the need for the further development of this service. Similarly, Zhao et al. (2024) [44] demonstrated that HAS products for GPS are of a lower accuracy and that the positioning results for GPS were worse compared to those for Galileo. In the case of CNES, slightly worse results were obtained for Galileo signals compared to GPS. However, the combined use of both signals (GPS+Galileo) provided better results, both in terms of positioning accuracy and ZTD estimation. This clearly demonstrates that employing both systems together substantially enhances the overall solution performance.

The ZTD analysis showed that BRDM solutions had a poorer ZTD estimation quality, especially for GPS, where large standard deviations were observed. In comparison, integrating GPS with Galileo improved the results by reducing ZTD variability. A similar dependency was reported by Yang et al. (2024) [49], confirming that the addition of Galileo signals enhances the stability and precision of ZTD retrieval. Moreover, this study also demonstrated that the use of broadcast ephemeris products allows for a ZTD estimation with sufficient accuracy, especially when using the Galileo signal. HAS further showed improvements when using Galileo signals, with standard deviations of ZTD being significantly lower than for GPS. However, the CNES and CODE solutions, which utilize precise corrections and products, proved to be the most accurate and stable in ZTD estimation, making them suitable for high-reliability applications such as weather forecasting.

5. Summary

The research shows that GNSS solutions based on broadcast ephemerides (BRDM) and correction services like HAS exhibit a lower performance in terms of convergence time, accuracy, and ZTD estimation, especially when using GPS-only data. The HAS service demonstrated significant improvement when integrated with Galileo, enhancing solution availability and accuracy. In real-time applications, Galileo consistently provides better results than GPS alone, partly because of more frequent updates of broadcast ephemerides. Nevertheless, solutions such as CNES and CODE, offering real-time precise corrections and post-processed precise products, continue to deliver a higher performance for high-precision applications.

The shortened convergence times observed in this study (1–2 min for CODE and CNES, below 5 min for HAS) were achieved using 1 s interval observations, demonstrating the potential for rapid solution stabilization, which is essential for real-time GNSS applications. While HAS meets its accuracy and convergence time requirements, further development of the service, particularly regarding GPS support, could further enhance its performance.

The presented results are based on real-time/near real-time simulation conditions under ideal circumstances and compared with the best possible results obtained using postprocessed precise CODE MGEX products. It should also be emphasized that the conditions assumed in the study were ideal, and deviations from them could occur in real-world real-time applications, for example, interruptions in receiving corrections or signal outages.

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Abbreviations

The following abbreviations are used in this manuscript:

AC	Analysis Center
APC	Antenna Phase Center
AR	Ambiguity Resolution
BNC	BKG NTRIP Client
CLAS	Centimeter-Level Augmentation Service
CNES	Centre National d'Études Spatiales
CODE	Center for Orbit Determination in Europe
DoY	Day of Year
EKF	Extended Kalman filter
GNSS	Global Navigation Satellite System
GPT	Global Pressure and Temperature
GPS	Global Positioning System
HAS	High Accuracy Service
IERS	International Earth Rotation and Reference Systems Service
IGS	International GNSS Service
NWP	Numerical Weather Prediction
OSB	Observable-specific Signal Bias
PCO	Phase Center Offset
PCV	Phase Center Variations
PPP	Precise Point Positioning
PWV	Precipitable Water Vapor
QZSS	Quasi-Zenith Satellite System
RTK	Real-Time Kinematic
RTS	Real-Time Service
VMF	Vienna Mapping Functions
TGD	Time Group Delay
ZTD	Zenith Tropospheric Delay

Appendix A

Results of the convergence time analysis presented in tabular form.

Table A1. Mean convergence time of all stations for all analyzed solutions. Values in minutes.

Solution	GNSS	N	E	U	2D	D3
BRDM	G	2.24	3.17	2.55	3.35	3.13
	E	2.34	3.51	2.80	3.92	3.66
	GE	1.82	2.86	2.25	3.10	2.91
HAS	G	2.07	2.82	2.59	3.10	2.89
	E	1.14	2.66	1.47	2.94	2.25
	GE	1.20	2.61	1.39	2.91	2.16
CNES	G	0.61	1.50	1.37	1.58	2.17
	E	0.88	1.83	1.85	2.09	2.87
	GE	0.52	0.98	1.00	1.06	1.45
CODE	G	0.54	1.14	1.06	1.21	1.52
	E	0.50	1.02	1.08	1.15	1.58
	GE	0.40	0.69	0.72	0.76	0.98

Table A2. Standard deviation in convergence time of all stations for all analyzed solutions. Values in minutes.

Solution	GNSS	N	E	U	2D	D3
BRDM	G	1.62	1.97	1.83	2.04	1.91
	E	0.72	0.69	0.85	0.69	0.78
	GE	1.03	1.45	1.18	1.44	1.37
HAS	G	1.64	1.97	1.92	2.07	2.00
	E	0.62	0.82	0.73	0.80	0.83
	GE	0.62	0.74	0.66	0.76	0.75
CNES	G	0.39	0.70	0.70	0.72	0.89
	E	0.41	0.72	0.73	0.74	0.88
	GE	0.41	0.59	0.60	0.57	0.73
CODE	G	0.51	0.69	0.59	0.73	0.83
	E	0.43	0.67	0.67	0.70	0.93
	GE	0.42	0.53	0.48	0.55	0.59

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Article

Performance Assessment of Undifferenced GPS/Galileo Precise Time Transfer with a Refined Clock Model

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Abstract: Conventional Global Navigation Satellite System (GNSS) time transfer algorithms typically model receiver clock offsets as white noise for estimation, neglecting the physical characteristics of atomic clocks, which consequently limits the performance of GNSS time transfer. To overcome this limitation, this study proposes an undifferenced GPS/Galileo combined Precise Point Positioning (PPP) time transfer model, incorporating both one-state (only clock offset parameter) and two-state (both clock offset and frequency offset parameters) refined clock models with clock instantaneous re-initialization (CIR) strategy at the day boundary epoch. Using observations from International GNSS Service (IGS) Multi-GNSS Experiment (MGEX) stations equipped with external hydrogen masers, precise time transfer performance under refined clock models was evaluated based on undifferenced GPS/Galileo combined PPP float solutions and PPP ambiguity resolutions. Experimental results demonstrate that, compared to traditional models, the refined clock models improve time transfer accuracy and frequency stability by an average of 6.7% and 25.8%, respectively. The improvement is most significant for short term frequency stability, with a maximum enhancement exceeding 85%. As the averaging time increases, the improvement in long term frequency stability gradually diminishes. Notably, the two-state refined clock model slightly outperforms the one-state model in time transfer performance, with the two-state refined clock model improving short-, medium-, and long term frequency stability by 11.5%, 8.0%, and 0.2%, respectively, compared to the one-state refined clock model. These findings strongly advocate adopting the two-state refined clock model to optimize both time transfer precision and short term stability in high-accuracy applications.

Keywords: GNSS; PPP; time transfer; refined clock model; ambiguity resolution

1. Introduction

Time is one of the seven fundamental physical quantities in the International System of Units (SI). Its precise transfer is increasingly critical in communication, satellite navigation and positioning, timing, and other fields. Since Allan and Weiss [1] proposed the Global Navigation Satellite System (GNSS) common-view time transfer method in 1980, GNSS has gradually become a vital tool for high-precision remote time transfer [2]. Notably, the advent of GNSS carrier-phase Precise Point Positioning (PPP) technology over the past two decades has achieved a quantum leap in time transfer accuracy [3], advancing from the initial microsecond level to the sub-nanosecond regime [4]. This technological

breakthrough led to the formal adoption of PPP by the International Bureau of Weights and Measures (BIPM) in 2009 as a standard methodology for International Atomic Time (TAI) computation and Coordinated Universal Time (UTC) maintenance [5].

The scientific community has extensively investigated multi-GNSS PPP time transfer capabilities. Defraigne and Baire [6] conducted GPS/GLONASS combined PPP time transfer experiments, demonstrating that the combination enhances robustness. Zhang et al. [7,8], Tu et al. [9] and Lyu et al. [10] studied multi-GNSS combined PPP time transfer models, showing that multi-GNSS integration improves accuracy and frequency stability. Ge et al. [11], Lyu et al. [12], Wang et al. [13], and Xu et al. [14] further validated these advantages in real-time processing scenarios. Researchers have proposed various undifferenced ambiguity resolution methods to recover the integer nature of undifferenced ambiguities in PPP [15,16]. Subsequently, PPP ambiguity resolution time transfers were implemented, proving their superiority over traditional float solutions [17,18]. Petit et al. [19–22] demonstrate that ambiguity resolution time transfer can achieve sub-nanosecond, with frequency stability reaching 1×10^{-16} @5 days and sub 10^{-17} @10 days. Studies by Xu et al. [23], Ren et al. [24], Qin et al. [25], and Zhang et al. [26] indicated that undifferenced ambiguity resolutions marginally enhance long term stability but show limited improvement in short term stability. In addition, with the development of Galileo and BDS multi-frequency signals in recent years, Su et al. [27], Zhang et al. [28], Ge et al. [29], and Xu et al. [30] have tested the models and accuracy of PPP time transfer using Galileo and BDS-3 triple-frequency, quad-frequency, and penta-frequency signals, demonstrating the advantages of multi-frequency signals in improving short term frequency stability.

In the aforementioned GNSS PPP time transfer studies, the receiver clock offset is commonly estimated as high-frequency white noise without considering the correlation between adjacent epochs or the physical noise characteristics of high-precision atomic clocks. Currently, with the development trend of atomic clocks toward low cost, lightweight, and high precision, over 8% of the stations in the International GNSS Service (IGS) Multi-GNSS Experiment (MGEX) network are equipped with cesium atomic masers, and approximately 15% are equipped with hydrogen masers, creating new opportunities for refined clock offset modeling. In response to these developments, Ge et al. [31–33] investigated the performance of post-processing and real-time BDS PPP time transfer under a one-state refined clock model. Lyu et al. [34] proposed a refined two-state clock model incorporating clock offset and frequency offset parameters, demonstrating its significant improvement in short term frequency stability. Qin et al. [35,36] used the refined clock model to predict missing satellite clock offsets in real-time PPP time transfer and used it for mitigating day boundary jumps. Mikoś et al. [37] tested the Galileo and multi-GNSS PPP positioning and time transfer performance with a priori clock offset parameters as constraints, verifying the advantages of the proposed method. Han et al. [38] studied a PPP time transfer model with an adaptive refined clock method.

While previous studies have developed and evaluated refined clock models for PPP time transfer, a critical research gap persists: the comparative analysis of one-state refined clock models (accounting solely for clock offset parameter) versus two-state refined clock models (incorporating both clock offset and drift parameters) and their respective impacts on time transfer accuracy remains unexplored. Moreover, conventional refined clock models necessitate daily initialization of receiver clock parameters by treating them as white noise, inevitably introducing discontinuities in time transfer results. To address this limitation, this study introduces an instantaneous re-initialization strategy based on a refined clock model, which mitigates day-boundary jumps by resetting ambiguity parameters instead of clock parameters, enabling seamless time transfer. Furthermore, with the emergence of undifferenced PPP ambiguity resolution and its demonstrated advantages

in high-precision time transfer, there is an urgent need to assess the applicability of these one-state and two-state refined clock models in PPP ambiguity resolution time transfer.

The structure of this research includes the following: Firstly, the GPS/Galileo combined PPP time transfer model and refined receiver clock method are studied. Secondly, three time transfer links with atomic clocks are selected to compare and evaluate the performance of the PPP time transfer with refined clock models. Finally, the research work is discussed, and the conclusion is given.

2. Methodology

2.1. GPS/Galileo Combined PPP Time Transfer Model

The original GPS/Galileo combined code and carrier phase observations can be expressed as:

$$\begin{cases} P_{r,j}^{G,i} = \rho_r^{G,i} + c(dt_r^G - dt^{G,i}) + M_r^{G,i} \cdot Z_r + \gamma_j^G \cdot I_{r,1}^{G,i} + d_{r,j}^G - d_j^{G,i} + \zeta_{r,j}^{G,i} \\ P_{r,j}^{E,i} = \rho_r^{E,i} + c(dt_r^E - dt^{E,i}) + M_r^{E,i} \cdot Z_r + \gamma_j^E \cdot I_{r,1}^{E,i} + d_{r,j}^E - d_j^{E,i} + \zeta_{r,j}^{E,i} \\ L_{r,j}^{G,i} = \rho_r^{G,i} + c(dt_r^G - dt^{G,i}) + M_r^{G,i} \cdot Z_r - \gamma_j^G \cdot I_{r,1}^{G,i} + \lambda_j^{G,i} (N_{r,j}^{G,i} + b_{r,j}^G - b_j^{G,i}) + \xi_{r,j}^{G,i} \\ L_{r,j}^{E,i} = \rho_r^{E,i} + c(dt_r^E - dt^{E,i}) + M_r^{E,i} \cdot Z_r - \gamma_j^E \cdot I_{r,1}^{E,i} + \lambda_j^{E,i} (N_{r,j}^{G/E,i} + b_{r,j}^E - b_j^{E,i}) + \xi_{r,j}^{E,i} \end{cases} \quad (1)$$

where P and L represent the code and carrier phase observations in meters, respectively; superscript i denotes the i -th satellite; superscripts G and E denote the GPS and Galileo system; subscripts r and j denote the GNSS receivers and frequency identifiers, respectively; λ is the wavelength corresponding to the frequency f_j ; ρ represents the geometric distance between the satellite and the receiver at the time of satellite signal emission instant; c represents the speed of light in a vacuum; dt_r and dt^s denote the offsets of the receiver clock offset and satellite clock referred to the system time in s, respectively; M denotes the wet mapping function; Z denotes the zenith wet delay; I represents the slant ionosphere delay; γ denotes the frequency-dependent ionosphere delay amplification factor; N is carrier phase integer ambiguity; d denotes the uncalibrated code delay (UCD) in meters; b denotes the uncalibrated phase delay (UPD) in cycles; and ζ and ξ represent the code and carrier phase observation noises, respectively.

After using precise products and eliminating the first-order ionosphere delay by using a dual-frequency IF combination, the linearized GPS/Galileo combined PPP observation equations can be expressed as follows:

$$\begin{cases} p_{r,IF}^{G,i} = \mathbf{u}_r^{G,i} \cdot \mathbf{x} + c\tilde{dt}_r^G + M_r^{G,i} \cdot Z_r + \zeta_{r,IF}^{G,i} \\ p_{r,IF}^{E,i} = \mathbf{u}_r^{E,i} \cdot \mathbf{x} + c\tilde{dt}_r^G + M_r^{E,i} \cdot Z_r + ISB^E + \zeta_{r,IF}^{E,i} \\ l_{r,IF}^{G,i} = \mathbf{u}_r^{G,i} \cdot \mathbf{x} + c\tilde{dt}_r^G + M_r^{G,i} \cdot Z_r + \lambda_{IF}^{G,i} \cdot \tilde{N}_{r,IF}^{G,i} + \xi_{r,IF}^{G,i} \\ l_{r,IF}^{E,i} = \mathbf{u}_r^{E,i} \cdot \mathbf{x} + c\tilde{dt}_r^G + M_r^{E,i} \cdot Z_r + \lambda_{IF}^{E,i} \cdot \tilde{N}_{r,IF}^{E,i} + \xi_{r,IF}^{E,i} \end{cases} \quad (2)$$

with

$$\begin{cases} c\tilde{dt}_r^G = cdt_r^G + d_{r,IF}^G \\ ISB^E = -d_{r,IF}^G + d_{r,IF}^E \\ \tilde{N}_{r,IF}^{G/E,i} = (N_{r,IF}^{G/E,i} + b_{r,IF}^{G/E} - b_{IF}^{G/E,i}) - d_{r,IF}^G / \lambda_{IF}^{G/E,i} \end{cases} \quad (3)$$

where p and l are code and carrier phase observed minus computed values in meters, respectively; subscript “IF” denotes ionospheric-free combination; $\mathbf{u}$ denotes the unit vector of the component from the receiver to the satellite; $\mathbf{x}$ denotes the vector of the receiver position increments; and the symbol “ $\sim$ ” denotes the re-parameterized estimate. ISB denotes inter-system bias between the GPS and Galileo systems, which only exists in Galileo’s code observation equation.

The satellite UPD in Formula (3) can be corrected using the integer recovery clock and wide-lane satellite bias products provided by Centre National d'Etudes Spatiales (CNES). Meanwhile, the single difference between satellites eliminates the UPD at the receiver end. Subsequently, the integer characteristic of the undifferenced ambiguity is restored, thereby achieving PPP ambiguity resolution.

The CNES integer recovery clock $dt_I^{G/E,i}$ and wide lane satellite bias $\bar{\mu}_{WL}^{G/E,i}$ products can be expressed as:

$$\begin{cases} \bar{\mu}_{WL}^{G/E,i} = \lambda_{WL}^{G/E} (b_1^{G/E,i} - b_2^{G/E,i}) - \lambda_{NL}^{G/E} (d_1^{G/E,i} / \lambda_1 + d_2^{G/E,i} / \lambda_2) \\ dt_I^{G/E,i} = dt^{G/E,i} + \lambda_{IF}^{G/E} b_{IF}^{G/E,i} \end{cases} \quad (4)$$

When the CNES integer recovery clock is employed to substitute the satellite clock in the code equation, the linearized GPS/Galileo combined PPP time transfer model can be expressed as:

$$\begin{cases} p_{r,IF}^{E,i} = \mathbf{u}_r^{G,i} \mathbf{x} + c\tilde{dt}_r^G + M_r^{G,i} \cdot Z_r + e_{L-P}^{G,i} + \zeta_{p,IF}^{G,i} \\ p_{r,IF}^{E,i} = \mathbf{u}_r^{E,i} \mathbf{x} + c\tilde{dt}_r^G + M_r^{E,i} \cdot Z_r + ISB^E + e_{L-P}^{E,i} + \zeta_{p,IF}^{E,i} \\ l_{r,IF}^{G,i} = \mathbf{u}_r^{G,i} \mathbf{x} + c\tilde{dt}_r^G + M_r^{G,i} \cdot Z_r + \lambda_{IF}^{G/E} \widehat{N}_{r,IF}^{G,i} + \zeta_{r,IF}^{G,i} \\ l_{r,IF}^{E,i} = \mathbf{u}_r^{E,i} \mathbf{x} + c\tilde{dt}_r^G + M_r^{E,i} \cdot Z_r + \lambda_{IF}^{E/E} \widehat{N}_{r,IF}^{E,i} + \zeta_{r,IF}^{E,i} \end{cases} \quad (5)$$

with

$$\begin{cases} e_{L-P}^{G/E,i} = d_{r,IF}^{G/E,i} - \lambda_{IF}^{G/E,i} b_{r,IF}^{G/E} - d_{IF}^{G/E} + \lambda_{IF}^{G/E,i} b_{IF}^{G/E,i} \\ \widehat{N}_{r,IF}^{G/E,i} = N_{r,IF}^{G/E,i} - d_{IF}^{G/E,i} / \lambda_{IF}^{G/E} \end{cases} \quad (6)$$

where e is the difference between the satellite code clock and integer recovery clock products, which code equation residuals will absorb.

2.2. Refined Clock Model with the Characteristics of an Atomic Clock

The noise of a precision atomic clock is typically composed of five types of superimposed noise: Random Walk Frequency Modulation (RWFM), Flicker Frequency Modulation (FFM), White Frequency Modulation (WFM), Flicker Phase Modulation (FPM), and White Phase Modulation (WPM) [39]. The specific expression is as follows [40]:

$$\begin{cases} z(t) = z_{-2}(t) + z_{-1}(t) + z_0(t) + z_1(t) + z_2(t) = \sum_{\alpha=-2}^2 z_{\alpha}(t) \\ s_y(f) = h_{-2}f^{-2} + h_{-1}f^{-1} + h_0f^0 + h_1f^1 + h_2f^2 = \sum_{\alpha=-2}^2 h_{\alpha}f^{\alpha} \end{cases} \quad (7)$$

where $z(t)$ represents the total clock noise; $z_{\alpha}(t)$ ($\alpha = -2, -1, 0, 1, 2$) corresponds to RWFM, FFM, WFM, FPM, and WPM, respectively; $s_y(f)$ is the power spectral density of the clock frequency (unit: 1/Hz); f is the Fourier frequency, h_{α} ($\alpha = -2, -1, \dots, 2$) are the intensity coefficients of the five types of clock noise. WFM and RWFM noise dominate precision atomic clocks (such as hydrogen masers), and the influence of linear frequency drift can be neglected [41].

Introducing clock offset and frequency offset into the Kalman filter means estimating the clock frequency offset parameter simultaneously with the receiver clock offset parameter. This paper refers to it as a two-state refined clock model (RCM2) for ease of expression. The Kalman filter observation equation considering only the clock offset and frequency offset parameters can be simplified as:

$$\mathbf{L}_k = \mathbf{A}_k \mathbf{X}_k + \mathbf{V}_k \quad (8)$$

where L_k denotes the observations, X_k denotes the two-state vector of clock offset and frequency offset, A_k is the observation matrix, and V_k represents the observation noise. Assuming the time interval between epoch t_k and t_{k-1} is Δt , the state transition matrix $F_{k,k-1}$ is given by [42]:

$$F_{k,k-1} = \begin{bmatrix} 1 & \Delta t \\ 0 & 1 \end{bmatrix} \quad (9)$$

The corresponding state equation can be expressed as:

$$X_k = F_{k,k-1}X_{k-1} + W_k \quad (10)$$

Among them, W_k represents the two-state process noise vector.

Since phase measurement noise is relatively small and does not accumulate over time, the noise covariance matrix primarily considers the phase noise generated by the accumulation of frequency modulation noise. In this case, the prior noise covariance matrix Q_W corresponding to W_k is given by [43]:

$$Q_W = \begin{bmatrix} q_1\Delta t + \frac{q_2\Delta t^3}{3} & \frac{q_2\Delta t^2}{2} \\ \frac{q_2\Delta t^2}{2} & q_2\Delta t \end{bmatrix} \quad (11)$$

where q_1 and q_2 represent the coefficients of WFM noise and RWFM noise, respectively.

Research by Yang et al. [44] demonstrates that the relationship between Allan variance and the noise coefficients is given by:

$$\begin{aligned} \delta_y^2(\tau) &= (\sqrt{3q_0}/\tau)^2 + (\sqrt{q_1/\tau})^2 + (\sqrt{q_2\tau/3})^2 \\ &= \left(\delta_y^{WPM}(\tau)\right)^2 + \left(\delta_y^{WFM}(\tau)\right)^2 + \left(\delta_y^{RWFM}(\tau)\right)^2 \end{aligned} \quad (12)$$

where $\delta_y^2(\tau)$ represents the Allan variance at averaging time τ , q_0 denotes the measurement noise coefficient for WPM, $\delta_y^{WPM}(\tau)$, $\delta_y^{WFM}(\tau)$ and $\delta_y^{RWFM}(\tau)$ represent the Allan Deviation (ADEV) for WPM, WFM, and RWFM at averaging time τ , respectively.

In practical applications, the clock offset can first be obtained from IGS final clock products or PPP solutions. Then, the Allan variance of the station clock offset at different averaging times τ is computed. Finally, a least-squares piecewise fitting method is applied to the Allan variance to derive the noise coefficients q_0 , q_1 , and q_2 .

When considering only the one-state clock offset parameter (RCM1), the corresponding prior noise covariance Q_W can be expressed as [45]:

$$Q_W = \left(\delta_y^2(\Delta t) \cdot \Delta t^2\right) = 3q_0 + q_1\Delta t + \frac{1}{3}q_2\Delta t^3 \quad (13)$$

Given that the magnitudes of q_0 and q_1 are comparable, while q_2 is substantially smaller than both, the third term on the right-hand side of Equation (13) becomes negligible relative to the first two terms for a small sampling interval Δt . After ignoring the influence of the third term, Equation (13) can be approximately regarded as the one-state representation of Equation (10). Compared to Equation (10), this approximation amplifies the effect of WPM. It should be noted that for high-precision clocks without manual intervention or major maintenance/resets, the noise coefficients of the refined clock model and their corresponding variance-covariance matrices do not need to be recalculated across different periods. Their stability allows for consistent application over extended durations unless significant hardware changes occur.

2.3. Clock Instantaneous Re-Initialization Strategy

Due to discontinuities in precise satellite products at day boundaries, receiver clock offsets estimated by traditional models tend to absorb part of these jumps. However, since observation conditions vary across stations, the amount of jump absorbed by each receiver clock differs, making it impossible to eliminate day-boundary jumps through between-station differencing fully.

Conventional studies on GNSS PPP time transfer with RCM, required initializing clock offset parameters at day boundaries and re-estimating receiver clock offsets as white noise. This approach caused part satellite clock offset jumps to be absorbed into time transfer results, leading to discontinuities at day boundaries and degrading time transfer accuracy. This study employs a Clock Instantaneous Re-initialization (CIR) strategy based on an RCM to address this issue. The strategy leverages the strong correlation between receiver clock offset and ambiguity parameters to absorb day-boundary jumps in clock offset estimations via ambiguity parameters, thereby restoring the continuity of time transfer.

Implementing the CIR strategy involves the following steps: First, reset all ambiguity parameters at the day boundary epoch (k) and assign them relatively large variances ($10^4 \text{ m}^2/\text{s}^2$). Subsequently, reinitialize the RCM using the method researched by Stein et al. [46]. The prior clock offset dt_k , the initial frequency offset df_k , and the covariance Q_w for initializing the RCM are as follows [46]:

$$dt_k = dt_{k-1} \quad (14)$$

$$df_k = \frac{\tau_1 + \tau_2}{\tau_1} \left[\frac{dt_{k-1} - dt_{k-2}}{\tau_2} \right] - \frac{\tau_2}{\tau_1} \left[\frac{dt_{k-2} - dt_{k-3}}{\tau_1 + \tau_2} \right] \quad (15)$$

$$Q_w = \begin{bmatrix} q_0 & \frac{\tau_1 + 2\tau_2}{\tau_2(\tau_1 + \tau_2)} q_0 \\ \frac{\tau_1 + 2\tau_2}{\tau_2(\tau_1 + \tau_2)} q_0 & \frac{2\tau_1^4 + 8\tau_1\tau_2 + 10\tau_1^2\tau_2^2 + 4\tau_1\tau_2^3 + 2\tau_2^2}{\tau_1^2\tau_2^2(\tau_1/\tau_2)^2} q_0 \\ & + \frac{\tau_1^3 + 4\tau_1^2\tau_2 + 4\tau_1\tau_2^2 + \tau_2^3}{\tau_1\tau_2(\tau_1 + \tau_2)} q_1 + \frac{\tau_2}{3} q_2 \end{bmatrix} \quad (16)$$

where dt_{k-3} , dt_{k-2} , and dt_{k-1} are the estimated receiver clock offsets at the day boundary of the previous three consecutive epochs, where τ_1 represents the time interval between dt_{k-3} and dt_{k-2} , and τ_2 represents the time interval between dt_{k-2} and dt_{k-1} . Apart from the state parameters of the RCM and the ambiguity parameters, the initial values and prior variances of all other parameter estimates can be inherited from the epoch immediately preceding the day boundary.

The receiver clock offset parameter can achieve rapid and accurate convergence using the CIR strategy, thanks to the high-precision prior information. Meanwhile, the reset ambiguity parameters, characterized by relatively large variances, can absorb the discontinuities in satellite clock products at day boundaries. This approach ultimately realizes the objective of continuous receiver clock offset estimation.

3. Data Selection and Processing Strategies

Four stations (BRUX, PTBB, SPT0, and USN7) are selected from the IGS MGEX network to conduct the PPP time transfer experiment with the refined clock model. These stations were chosen primarily for their high-precision hydrogen atomic clocks, critical roles as key nodes in international time transfer networks, and consistently excellent observation data quality and time transfer accuracy. Specifically, BRUX, PTBB, SPT0, and USN7 are stations of the UTC time-keeping laboratories ORB (Observatoire Royal de Belgique), PTB (Physikalisch-Technische Bundesanstalt), RISE (Research Institutes of Sweden AB), and USNO (United States Naval Observatory), respectively. Taking USN7 as the centre, three time and frequency links, namely USN7-BRUX, USN7-PTBB, and USN7-SPT0, are formed

to verify the time transfer performance of the refined clock model. The observation period is 8 days (DOYs 279–286 in 2021), and the observation sampling interval is 30 s. The CNES GRG precise clock and orbit products are used for data processing. The geographical distribution of these stations and the basic information are shown in Figure 1 and Table 1.

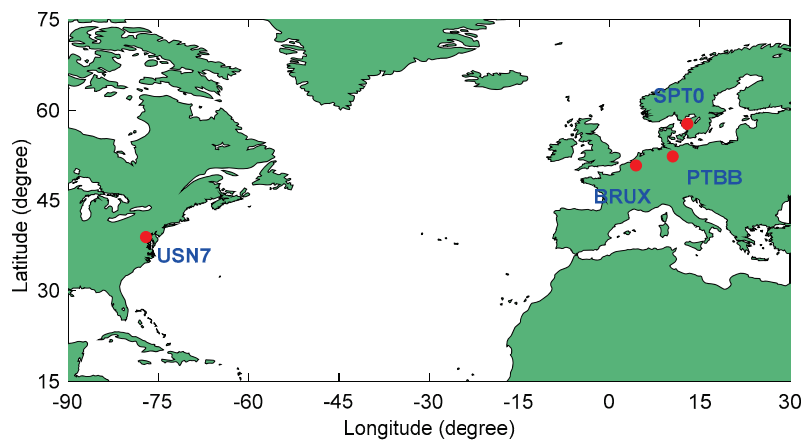


Figure 1. Geographical distribution of experimental stations.

Table 1. Basic information on the experimental stations.

Station	Receiver	Antenna	Country
BRUX	SEPT POLARX5TR	JAVRINGANT_DM	Belgium
PTBB	SEPT POLARX5TR	LEIAR25.R4	Germany
SPT0	SEPT POLARX5TR	JNSCR_C145-22-1	Sweden
USN7	SEPT POLARX5TR	TPSCR.G5	America

To verify the time transfer performance of the refined clock model, this research conducts experiments of PPP using the model without the refined clock model (receiver clock offset estimated as white noise), within the one-state refined clock model and the two-state refined clock model, respectively, which are denoted as RCM0, RCM1, and RCM2. By performing piecewise least squares fitting on the Allan variance of the station clock offsets provided by the IGS, the noise coefficients q_0 , q_1 , and q_2 of the atomic clocks at the selected stations are calculated. The Allan deviations (ADEV) corresponding to the observation sampling interval of 30 s are obtained from the fitting coefficients, and the specific values are shown in Table 2.

Table 2. Atomic clock noise coefficients and Allan deviations of the selected stations.

Station	q_0	q_1	q_2	ADEV
BRUX	1.67×10^{-24}	2.37×10^{-25}	6.96×10^{-35}	1.16×10^{-13}
PTBB	1.59×10^{-24}	2.97×10^{-25}	1.16×10^{-35}	1.23×10^{-13}
SPT0	1.01×10^{-23}	1.77×10^{-25}	9.32×10^{-35}	2.06×10^{-13}
USN7	9.50×10^{-23}	6.33×10^{-25}	1.03×10^{-33}	2.30×10^{-13}

GPS and Galileo raw code and carrier phase observation noises are set to 0.3 m and 3.0 mm, respectively; elevation-dependent weighting for the observations was applied. Since the IGS antenna correction product does not provide receiver-side antenna phase center offset (PCO) and phase center variation (PCV) correction parameters for Galileo, and considering the similar or identical signal frequencies between GPS and Galileo, this study herein employs GPS receiver-side antenna PCO and PCV correction parameters to

approximate those for Galileo’s receiver-side antenna. The detailed processing strategies are summarized in Table 3.

Table 3. Details of the PPP time transfer processing strategy.

Item	Strategy
Observations	GPS/Galileo combination; GPS: L1/L2; Galileo: E1/E5a
Elevation cutoff	7°
Satellite orbit & clock	CNES GRG final precise products
Satellite DCB	Corrected by Chinese Academy of Sciences product [47]
Earth’s rotation	International Earth Rotation Service (IERS) convention [48]
Relativistic effect	Corrected: elliptical orbit and Shapiro effect [49,50]
Phase windup effect	Corrected by model [51]
Tide effect	Corrected [48,52]
Antenna PCO and PCV	Corrected igs14.atx file
Inter-system bias	Estimated as white noise ($10^4 \text{ m}^2/\text{s}^2$)
Inter-frequency bias	Estimated as constants
Station coordinate	Static, estimated as constant
Receiver clock	RCM0 (estimated as white noise), RCM1 (refined one-state clock model), and RCM2 (refined two-state clock model)
Ionospheric delay	First-order eliminated by IF combination
Tropospheric delay	Saastamoinen model [53] and estimated as a random walk ($10^{-8} \text{ m}^2/\text{s}^2$), global mapping function (GMF) [54]
Ambiguity	Estimated as constant, float, and fixed solutions

4. Results

4.1. Quality Assessment of Observation

As shown in Figure 2, the number of satellites at the experimental stations is relatively similar. The number of satellites in the GPS/Galileo combination can reach about 16, and the number of GPS satellites is slightly higher than that of Galileo satellites. The average number of satellites in the GPS/Galileo combination at the stations of BRUX, PTBB, SPT0, and USN7 is 16.3, 16.2, 16.9, and 15.9, respectively, with SPT0 having marginally the highest number and USN7 slightly fewer. The Time Dilution of Precision (TDOP) values across all stations are comparable, predominantly clustered around 0.6. The mean TDOP values for the GPS/Galileo combination at BRUX, PTBB, SPT0, and USN7 stations are 0.61, 0.63, 0.61, and 0.65, respectively. USN7 exhibits a slightly higher TDOP, corresponding to its marginally lower average satellite number than the other three stations.

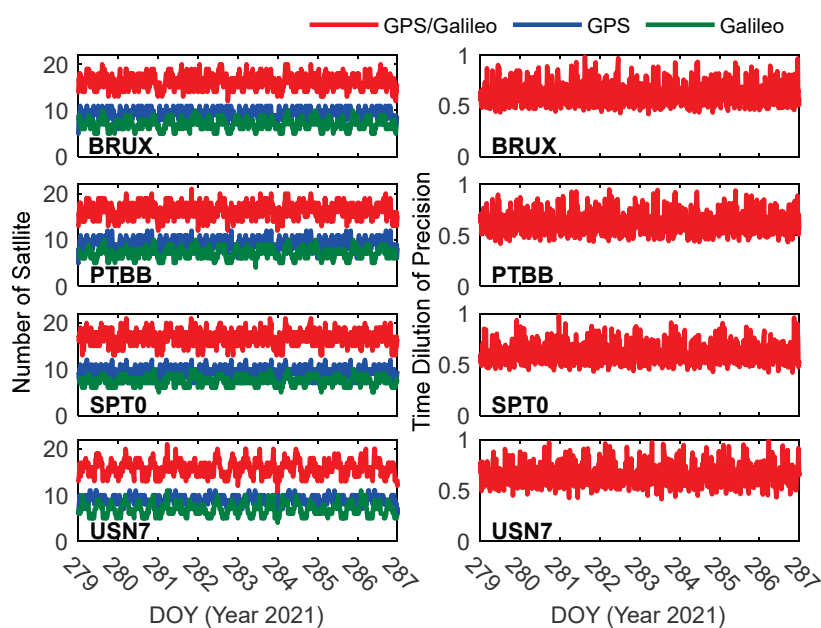


Figure 2. The number of satellites and the TDOP of the experimental station.

As shown in Figure 3, the magnitudes of the MPC (multi-path combination) noise at each station are relatively close, all within 0.5 m. The mean RMS values for GPS L1, L2, and Galileo E1 and E5a are 0.381 m, 0.327 m, 0.291 m, and 0.214 m, respectively. The MPC noise at the L1 frequency is the largest, while that at the E5a frequency is the smallest. The mean MPC noises for BRUX, PTBB, SPT0, and USN7 are 0.253 m, 0.325 m, 0.270 m, and 0.365 m, respectively. Among them, the MPC noise at BRUX is slightly smaller, and the noise at USN7 is slightly larger.

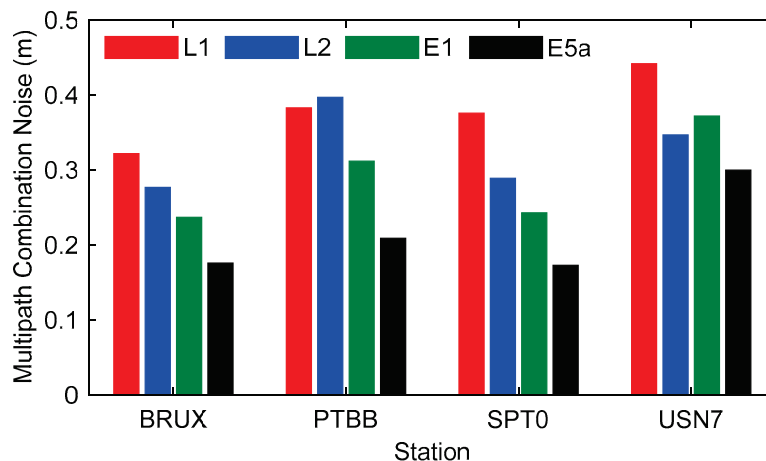


Figure 3. The root mean square (RMS) value of multi-path combination (MPC) noise at the BRUX, PTBB, SPT0, and USN7 stations.

4.2. PPP Time Transfer with Refined Clock Model

Due to space limitations, Figure 4 only shows the receiver clock offset sequences of the BRUX and PTBB stations without and within the CIR strategy. It can be found that the traditional algorithm (without CIR) demonstrates a jump ranging from -0.2 ns to 0.4 ns at the daily boundary. In contrast, the CIR strategy successfully eradicates this jump at the daily boundary, rendering the clock offsets more continuous. This phenomenon validates the efficacy of the CIR strategy in eliminating the daily boundary jump. When the CIR strategy is implemented, the standard deviations (STDs) following epoch-to-epoch differencing at the BRUX and PTBB stations are 0.6 ps and 1.3 ps, respectively. Compared to the STDs of the traditional algorithm, these values are decreased by 88.6% and 77.2% , respectively, which indicates that the CIR strategy reduces the noise in time transfer and enhances the accuracy. Considering the advantage of the CIR strategy in attenuating the jump at the daily boundary, the CIR strategy will be adopted in the refined model in the subsequent experiment.

Figure 5 depicts the clock offset sequences of the three links after eliminating the corresponding trend terms from the initial clock offsets. The clock offset trends in RCM0, RCM1, and RCM2 are generally consistent. Nevertheless, the outcomes of RCM1 and RCM2 exhibit greater smoothness and stability. For instance, around DOY 284.5, there are some abrupt jumps in the data of RCM0, whereas RCM1 and RCM2 have effectively suppressed such sudden changes. Moreover, when juxtaposed with RCM0, the clock offset sequence of RCM1 demonstrates enhanced continuity. This distinctive feature gives rise to an overall divergence between RCM0 on the one hand and RCM1 and RCM2 on the other hand during specific time intervals.

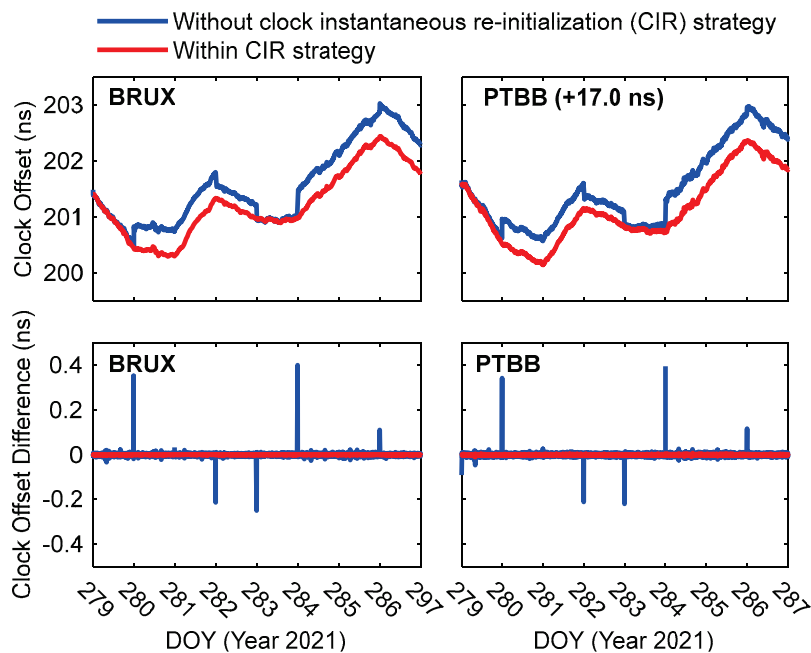


Figure 4. The receiver clock offset sequences of BRUX and PTBB stations.

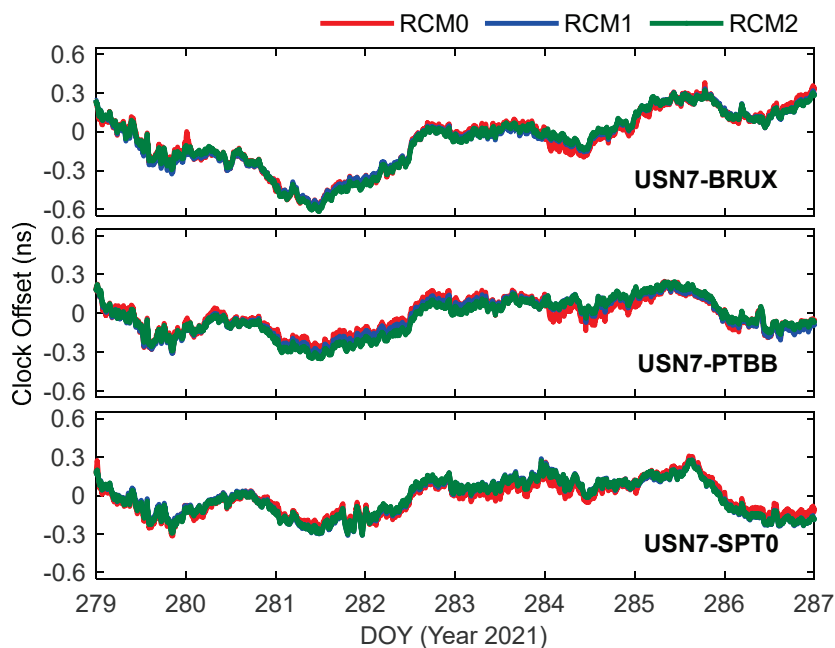


Figure 5. The PPP time transfer clock offset within the refined clock model.

Table 4 presents the STDs of the discrepancies between each scheme's time transfer outcomes and the IGS clock offsets. Evidently, relative to RCM0, the STDs of RCM1 and RCM2 exhibit a downward trend. Specifically, the reduction rates are 7.4% for RCM1 and 10.3% for RCM2 on average. Notably, the STD of RCM2 is marginally higher than that of RCM1. This deviation may be attributed to the accuracy of the fitting coefficients for clock offset noise in RCM2. After implementing the refined clock model, significant decreases are observed in the average STD values. Specifically, the reduction rates are 5.6%, 15.1%, and 5.4%, respectively, in USN7-BRUX, USN7-PTBB, and USN7-SPT0 links.

Table 4. Standard deviations (STD) of the PPP time transfer with the refined clock model (ps).

Link	RCM0	RCM1	RCM2
USN7-BRUX	32.04	30.72	29.80
USN7-PTBB	36.27	31.21	30.36
USN7-SPT0	34.13	32.92	31.69
Mean	34.14	31.62	30.62

As shown in Figure 6, the frequency stability of the time transfer after applying the refined clock model has been significantly improved, especially in terms of short term frequency stability. Taking the USN7-BRUX link as an example, the frequency stabilities of RCM0 at 30 s, 60 s, and 120 s are 2.26×10^{-13} , 1.30×10^{-13} , and 7.67×10^{-14} , respectively. In contrast, the frequency stabilities of RCM1 at the corresponding moments are 4.82×10^{-14} , 4.04×10^{-14} , and 3.43×10^{-14} , respectively, and those of RCM2 at the corresponding moments are 2.78×10^{-14} , 2.44×10^{-14} , and 2.34×10^{-14} , respectively. Compared with RCM0, the frequency stabilities of RCM1 and RCM2 have been improved by more than half an order of magnitude in the short term. The improvement effect of RCM2 on short term frequency stability is better than that of RCM1, which may be because RCM2 adds constraints to the state transition matrix, and its constraints are stronger than those of RCM1. As time increases, the differences in frequency stability among RCM0, RCM1, and RCM2 gradually decrease, and their frequency stabilities at 120,000 s are the same.

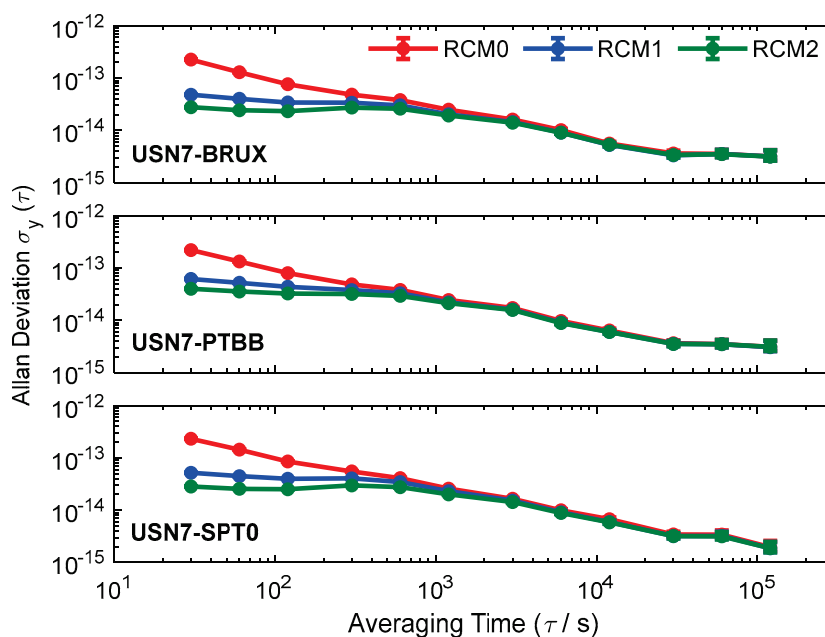
**Figure 6.** Frequency stability of the PPP time transfer with the refined clock model.

Figure 7 illustrates the mean values of the stability gain at different moments. The average magnitudes of the stability improvement for RCM1 and RCM2 are 23.7% and 30.2%, respectively. Both RCM1 and RCM2 exhibit the most significant improvement in short term stability. For RCM1, the improvement magnitudes at 30 s, 60 s, 120 s, and 240 s are 76.2%, 66.3%, 51.3%, and 26.2%, respectively. Correspondingly, for RCM2, the magnitudes at these moments are 85.8%, 79.0%, 66.5%, and 41.6%, respectively. However, the refined clock model has a relatively minor improvement in long term stability. The improvement magnitudes of RCM1 and RCM2 at 60,000 s and 120,000 s are around 2.0%. Although the improvement magnitude of RCM2 is higher than that of RCM1 before 3000 s, the two

become quite close in terms of the improvement magnitude after 12,000 s. Additionally, the improvement magnitudes of RCM2 at 60,000 s are slightly smaller than those of RCM1.

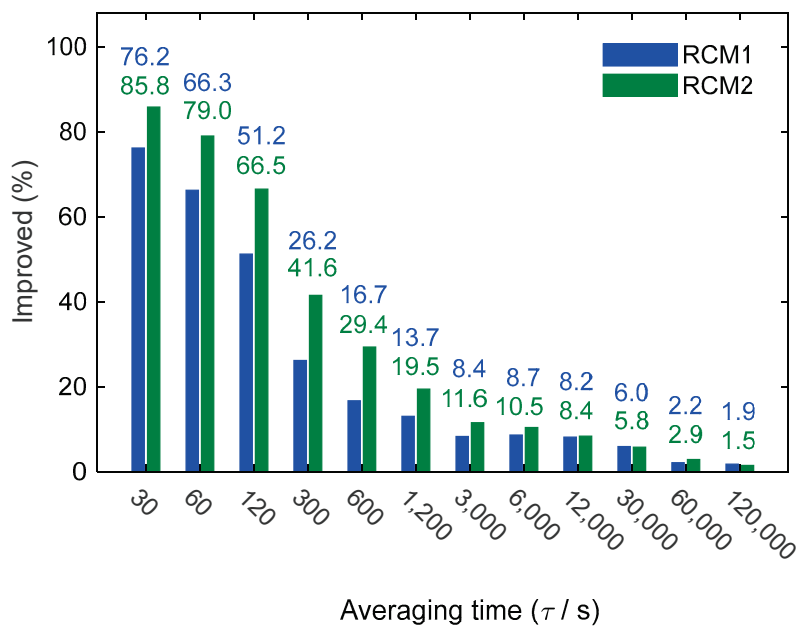


Figure 7. Gain of PPP time transfer stability for refined clock model.

4.3. PPP Time Transfer with Refined Clock Model and Ambiguity Resolution

This study utilizes the wide-lane Satellite Bias and Integer-Recovered Clock products provided by CNES to conduct time transfer experiments using undifferenced ambiguity resolution within and without refined clock models in a GPS/Galileo combined system.

As shown in Figure 8, after adopting the refined clock model, the RMS value of the narrow-lane ambiguity residuals shows a slight downward trend compared with the unrefined scheme, with a decline of approximately 0.001 cycles. Meanwhile, the proportion of residuals distributed within the range of ± 0.15 cycles has increased by about 0.1%. This result indicates that the refined clock model plays a positive role in accurately separating ambiguity parameters, significantly improving the calculation accuracy of ambiguity parameters. In addition, since the CIR strategy resets the ambiguity parameters at the daily boundary, the ambiguity parameters absorb the jumps of the satellite clock products, which may lead to the inability of the ambiguity parameters to be quickly fixed at the epoch of the daily boundary. Furthermore, this situation results in the ambiguity fixing rate when using the refined clock model being slightly lower than that of the conventional scheme.

Figure 9 and Table 5 present the comparative results between the time transfer solutions and the IGS products. Since the IGS only provided the clock offset of the PTBB station on DOY 294, Figure 9 shows the clock offsets of the USN7-PTBB link for a single day. To display the results of each scheme, Figure 9 performs a global shift on the results of the RCM1 and RCM2 schemes. The results reveal that each scheme exhibits highly consistent trends in the variations of clock offsets. Notably, the ambiguity resolution integrated with the refined clock model demonstrates enhanced robustness and reliability compared to the conventional solution. This superiority is particularly evident around DOY 284 for the USN7-BRUX link and around DOY 285 for the USN7-SPT0 link.

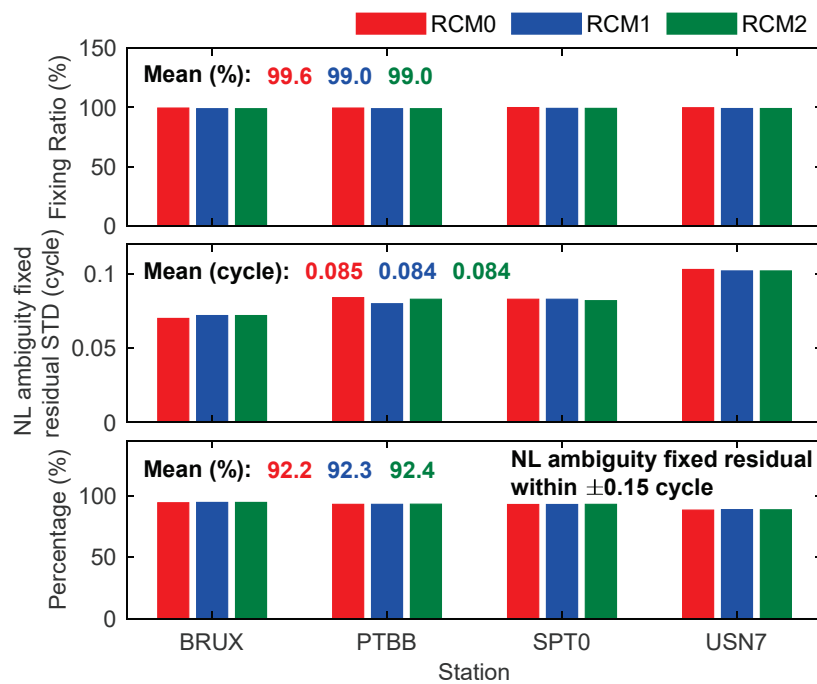


Figure 8. RMS of the narrow-lane ambiguity fractional-cycle residual and fixing rate.

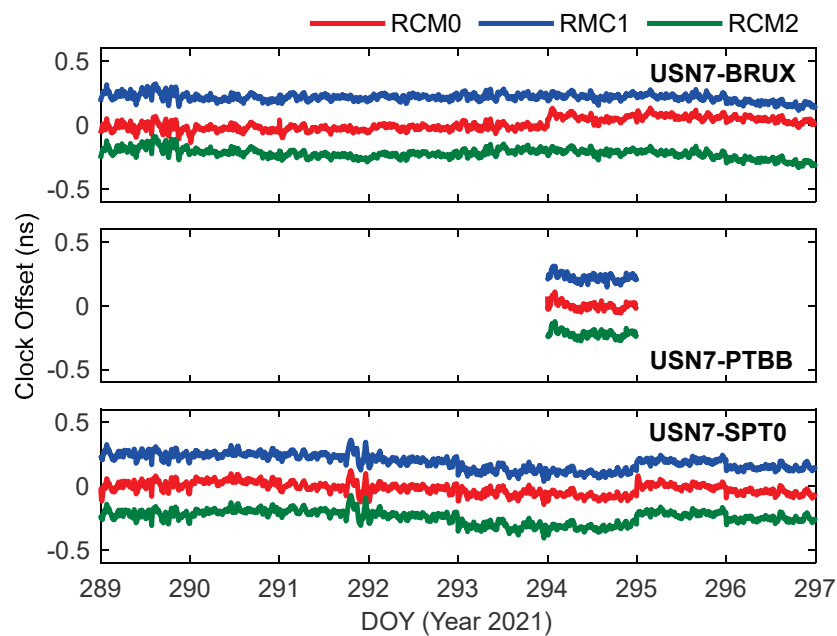


Figure 9. Clock offset sequences of PPP time transfer stability for refined clock model with ambiguity resolution.

Table 5. STDs of PPP time transfer with the refined clock model and ambiguity resolution (ps).

Link	RCM0	RCM1	RCM2
USN7-BRUX	22.32	21.59	20.67
USN7-PTBB	30.52	28.17	29.29
USN7-SPT0	24.47	23.87	23.53
Mean	25.77	24.54	24.49

The statistical results show that the STD of the RCM1 and RCM2 schemes is slightly lower than that of the traditional scheme. Specifically, compared with the RCM0, the STD

of RCM1 and RCM2 decreases by approximately 4.7% and 4.9% on average, respectively. Among them, in the BRUX-PTBB link, the improvement effect of RCM1 on the accuracy is the most remarkable. These findings comprehensively confirm the technical advantages of the refined clock model in the PPP ambiguity resolution time transfer. Moreover, through a comparison of Tables 4 and 5, the accuracy of the ambiguity resolution is 22.4% higher than that of the float solution on average, which attests to the superiority of the ambiguity resolution in time transfer applications.

As shown in Figure 10, the frequency stability of the RCM1 and RCM2 has been significantly improved compared with that of the RCM0, especially the improvement effect in the short term, which is the most obvious. For instance, at the USN7-BRUX link, during the averaging time of 30 s to 300 s, the ADEV corresponding to the refined clock model scheme is distinctly lower than that of the traditional scheme. This phenomenon could be attributed to the refined clock model suppressing the observation noise, significantly boosting the short term stability. With the increase of the averaging time, the improvement range of the frequency stability of the refined clock scheme compared with the unrefined clock scheme shows a gradually decreasing trend, and the long term frequency stabilities of the two schemes are the same. Additionally, the frequency stability of RCM2 is superior to that of RCM1, which is generally consistent with the results from PPP float time transfer. This consistency can be attributed to the fact that RCM2 employs a more rigorous model than RCM1.

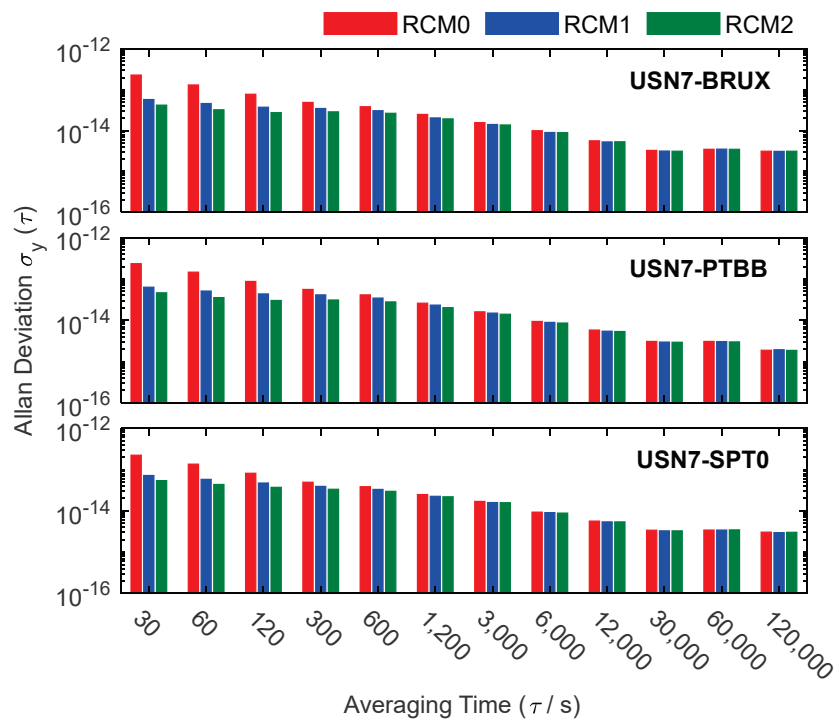


Figure 10. Frequency stability of PPP time transfer with the refined clock model and ambiguity resolution.

Table 6 shows the mean frequency stability characteristics for the three links USN7-BRUX, USN7-PTBB, and USN7-SPT0 in time transfer. It can be observed that when the averaging time is less than 3000 s, the frequency stability improves most rapidly (e.g., the ADEV of RCM0 decreases from 2.2×10^{-13} to 1.6×10^{-14}). When the averaging time exceeds 30,000 s, the frequency stabilities of the RCM0, RCM1, and RCM2 schemes approach each other, with their variation ranges narrowing to extremely small values and gradually tending to a stable state. Statistically, compared with the RCM0 scheme, the mean frequency stabilities of RCM1 and

RCM2 improved by 21.7% and 27.5%, respectively. The improvement is most significant in the short term (averaging time < 1000 s), reaching 60.7% and 70.2%, respectively; in the medium term ($1000 \text{ s} \leq \text{averaging time} \leq 10,000 \text{ s}$), the improvements are 13.6% and 21.7%; while in the long term (averaging time $> 10,000 \text{ s}$), the improvements are only 2.6% and 2.7%. This result indicates that the RCM strategy mainly optimizes the frequency stability in the short term (with the maximum improvement exceeding 70%). In contrast, its effect on long term stability is relatively limited. Compared to the PPP ambiguity float time transfer solutions, the ambiguity fixed solutions of RCM1 and RCM2 show slightly inferior performance in improving frequency stability, with the stability improvement being approximately 2.5% lower than that of the PPP float solutions. This phenomenon may be related to the ambiguity fixing rate not reaching 100% and, second, to the fact that the receiver clock offsets parameters absorbing the receiver phase biases after ambiguity resolution. In addition, the refined clock model is highly sensitive to prior variance, which may lead to over-constraint problems and affect time transfer performance. Therefore, a reasonable method for determining prior variance still needs further exploration.

Table 6. Mean PPP time transfer frequency stability values with the refined clock model and ambiguity resolution ($\times 10^{-15}$).

Tau/s	RCM0	RCM1	RMC2
30	229.513	64.346	47.524
60	137.617	51.783	37.202
120	82.093	42.716	31.649
300	51.495	38.429	31.080
600	39.307	32.808	27.977
1200	25.264	22.068	20.465
3000	16.210	15.003	14.503
6000	9.554	8.980	8.801
12,000	5.695	5.370	5.359
30,000	3.250	3.133	3.118
60,000	3.339	3.337	3.323
120,000	2.697	2.672	2.682

5. Discussion

This study establishes a time transfer model based on undifferenced GPS/Galileo combined PPP ambiguity float and fixed solutions, comparing the time transfer accuracy and frequency stability between traditional white-noise receiver clock offset estimation and refined first-order (one-state) versus second-order (two-state) clock error models. The findings reveal that refined clock models exhibit significant sensitivity to prior noise parameters covariance configuration. Suboptimal selection of excessively small prior noises may induce over-constraint issues, leading to systematic biases in time transfer. Furthermore, the refined clock model demonstrates applicability only under stable clock performance conditions. Any clock reset or adjustment necessitates a recalibration of prior noise coefficients, constraining the model's operational versatility. Consequently, developing adaptive prior noise coefficient determination methodologies remains a critical research frontier.

Additionally, while this investigation focuses on two primary clock noise components (WFM and RWFM), comprehensive modeling incorporating other clock noise types warrants further exploration. Analysis of time transfer outcomes indicates that although the two-state model offers theoretical rigor—yielding marginally superior time transfer accuracy and frequency stability compared to the one-state model, particularly in short term stability enhancement—it introduces increased computational complexity. In multi-GNSS

integration scenarios, the two-state model necessitates additional ISB parameters. Consequently, developing refined ISB parameterization strategies compatible with advanced clock models is a pivotal challenge for future research.

6. Conclusions

Leveraging the physical noise characteristics of atomic clocks, this study established an undifferenced GPS/Galileo PPP time transfer model incorporating one-state and two-state refined clock methods with the CIR method. By selecting three time transfer links composed of four MGEX stations (BRUX, PTBB, SPT0, and USN7), the performance of precise time transfer under refined clock models was evaluated using PPP ambiguity float solutions and ambiguity resolutions. Experimental results reveal that the refined clock strategies effectively mitigate receiver clock offset discontinuities at daily boundaries induced by satellite products, achieving average improvements of 6.7% in time transfer accuracy and 25.8% in frequency stability compared to conventional models. The enhancement is most significant for short term frequency stability, with stability improvements reaching 78.3%, 70.2%, 56.8%, and 33.2% at 30 s, 60 s, 120 s, and 300 s averaging times, respectively. The improvement gradually diminishes as the averaging time increases, showing less than 2% enhancement at 3000 s and 120,000 s averaging times. Additionally, the two-state refined clock model slightly outperforms the one-state model, demonstrating 11.5%, 8.0%, and 0.2% better frequency stability in short term, medium-term, and long term periods, respectively. Consequently, the two-state refined clock model is recommended for applications prioritizing high-precision time transfer and enhanced short term stability performance.

With the continuous advancement of atomic clocks towards miniaturization and cost reduction, equipping many IGS stations with high-precision hydrogen clocks is becoming increasingly feasible. Against this backdrop, the RCM based on clock characteristics is set to see broader applications in high-precision time transfer and positioning. Compared with traditional algorithms, RCM can enhance time transfer accuracy and frequency stability, thereby directly improving time-frequency signals' synchronization and timing capabilities. This holds critical significance for 5G communication synchronization and UTC maintenance fields.

Additionally, in recent years, with the growing research interest in using GNSS time-frequency signals to determine geopotential and orthometric height, the importance of high-precision time-frequency transfer has become increasingly prominent. Theoretical research [55] indicates that the enhancement in time-frequency transfer stability brought by RCM will equivalently improve the accuracy of determining geopotential and orthometric height using time-frequency signals. The undifferenced time transfer technology incorporating RCM is expected to provide critical technical support for achieving centimeter-level cross-sea height transfer and promoting the future unification of global elevation benchmarks.

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Data Availability Statement: The GPS and Galileo observations are available at <ftp://igs.gnsswhu.cn/pub/gps/data/daily/> (accessed on 1 April 2025).

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Article

High-Precision Satellite Clock Offset Estimated by SRIF Based on Epoch-Wise Updated Orbit

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Abstract: High-precision clock offset products directly affect the performance and reliability of precise point positioning (PPP) applications. Currently, real-time clock offset products offered by institutions such as the Centre national d'études spatiales (CNES) rely on ultra-rapid predicted orbits. However, these orbits have limited accuracy and exhibit jumps during updates, constraining the accuracy of real-time clock estimation. To address this issue, we propose an undifferenced ambiguity resolution (UD AR) technique for clock offset estimation based on epoch-wise updated orbits. Clock estimation experiments were performed using both predicted and epoch-wise updated orbits, with square root information filtering (SRIF) applied in three schemes: double-differenced (DD), UD, and float solutions. Compared with predicted orbits, epoch-wise updated orbits provided smoother sequences with higher accuracy, significantly improving clock offset estimation accuracy in all schemes. Moreover, the UD AR solution significantly enhanced clock offset estimation accuracy, and the high-precision epoch-wise updated orbit products increased the narrow-lane fixing rate of the UD solutions. The clock accuracies of BDS-3, Galileo, and GPS reached 0.032 ns, 0.023 ns, and 0.026 ns, respectively, representing improvements of 36%, 34%, and 41% compared with the float solutions and 41%, 30%, 26% compared with the UD solution based on 1 h predicted orbits. Finally, the positioning performance of the proposed method was validated via PPP using 25 stations, showing improvements of 50%, 48%, and 41% in the north, east, and up directions compared with CNES products. Therefore, by combining epoch-wise updated orbit products with the UD AR to improve clock accuracy, this method provides a new approach to generating high-precision clock products, significantly contributing to enhancing PPP services.

Keywords: clock estimation; SRIF; epoch-wise updated orbit; integer ambiguity resolution

1. Introduction

Global Navigation Satellite System (GNSS) technology plays a vital role in high-precision positioning, navigation, and timing (PNT) applications [1]. As core elements of GNSS technology, real-time satellite clock offset products directly determine the quality of real-time applications such as precise point positioning (PPP) and PPP-RTK [2,3]. With the development of GNSS technology, the demand for real-time and high-precision applications has steadily increased, particularly in fields such as autonomous driving, precise

agriculture, and geological hazard monitoring [4–8]. Consequently, real-time precise clock offset products have become a significant research focus in the GNSS field.

Currently, real-time clock offset products are primarily generated through two methods: prediction and estimation. Due to the relatively low precision of the prediction method, high-precision clock products predominantly rely on real-time clock estimation [9]. Over the past few years, numerous scholars have extensively researched satellite clock offset estimation. In terms of quality control, Fu et al. proposed an improved quality control algorithm based on the Sequential Least Squares (SLSQ) method, significantly enhancing clock offset accuracy [10]. Meanwhile, Bock et al. effectively eliminated outliers in observation data through post-residual analysis [11]. Regarding computational efficiency, studies have primarily focused on multi-threaded parallel processing, Kalman filtering, and square root information filtering (SRIF), all of which have demonstrated excellent performance in multi-system satellite clock offset estimation [12–14]. Furthermore, numerous studies have shown that ambiguity resolution (AR) techniques can substantially enhance the precision of satellite clock offset estimation [15–18], although fixing rates and computational efficiency require further enhancement.

In real-time clock estimation processing, satellite orbit products are typically considered known information, providing precise satellite positions at specific epochs [19]. As a result, the precision of real-time satellite orbit products directly determines the accuracy and reliability of real-time clock offset estimation [20]. Some institutes, such as the International GNSS Service (IGS), provide ultra-rapid predicted orbit products (e.g., IGU products), but these face challenges, such as discontinuities at update epochs and limited accuracy. These issues constrain further improvement in real-time clock estimation precision and, as a result, impact the performance of real-time PPP [21].

To address these challenges, this study proposes a clock estimation method based on epoch-wise estimated orbit products. Compared with ultra-rapid predicted orbit products, real-time epoch-wise estimated orbit products can effectively avoid discontinuities at update epochs and offer higher orbit accuracy, providing a foundation for high-precision real-time clock offset estimation. Notably, the Shanghai Astronomical Observatory (SHAO) has been releasing epoch-wise, updated, real-time estimated orbit products since November 2024, presenting a significant opportunity for advancing clock offset estimation research. Furthermore, to overcome the limitations of existing clock offset estimation methods, this study incorporates square root information filtering (SRIF) for parameter estimation and ambiguity resolution techniques to enhance the precision of real-time satellite clock offset estimation. The ultimate goal is to obtain high-precision products that support improvements in the service quality of real-time PPP.

This paper is organized as follows: Section 2 details the clock offset estimation model, ambiguity resolution technique, SRIF parameter estimation method, and data processing procedures. Then, the predicted and epoch-wise estimated orbits are analyzed in Section 3.1. Sections 3.2 and 3.3 present an analysis of ambiguity resolution and a comparison of clock estimation based on different orbits. The results are then validated in Section 3.4 using kinematic PPP. Finally, Section 4 summarizes the conclusions.

2. Methods

This section introduces the mathematical models of clock estimation, undifferenced (UD) AR, double-differenced (DD), and real-time satellite clock estimation by SRIF. The sources of orbit data and our clock processing strategy are also illustrated.

2.1. Real-Time Clock Estimation Model

The ionospheric-free (IF) combination is an effective method for mitigating the impact of first-order ionospheric delays in observation data, commonly used in satellite clock offset estimation. For the dual-frequency IF combination observation from satellite (*s*) to receiver (*r*), the mathematical expression can be formulated as follows:

$$P_{r,IF}^s = \rho_r^s + c(\bar{t}_{r,IF} - \bar{t}_{IF}^s) + m_r^s T_r + \varepsilon_{r,pc}^s \quad (1)$$

$$L_{r,IF}^s = \rho_r^s + c(\bar{t}_{r,IF} - \bar{t}_{IF}^s) + \lambda_{IF} \bar{N}_{r,IF}^s + m_r^s T_r + \varepsilon_{r,lc}^s \quad (2)$$

where the $P_{r,IF}^s$ and $L_{r,IF}^s$ are the IF observations of the code and phase, respectively. ρ_r^s is the distance between the satellite (*s*) to receiver (*r*). $\bar{t}_{r,IF}$ and $\bar{t}_{IF}^s$ represent clock offsets of the receiver and satellite after the IF combination, respectively, which include the uncalibrated hardware delays from the receiver $d_{r,IF}$ and satellite d_{IF}^s [22]. These can be expressed as follows: $\bar{t}_{r,IF} = t_{r,IF} + d_{r,IF}$ and $\bar{t}_{IF}^s = t_{IF}^s + d_{IF}^s$. m_r^s and T_r are the mapping function and tropospheric zenith wet delay (ZWD). λ_{IF} is the wavelength. $\bar{N}_{r,IF}^s$ is the re-parameterized phase ambiguity of the IF combination. $\varepsilon_{r,pc}^s$ and $\varepsilon_{r,lc}^s$ are the code and phase measurement noise, and c denotes the speed of light.

2.2. Ambiguity Resolution

GNSS ambiguity resolution methods are primarily divided into DD and UD. Ambiguities lose integer characteristics after IF combinations and can be split into wide-lane (WL) and narrow-lane (NL). Since ambiguity absorbs the hardware delay at both the receiver and the satellite [22], it can be expressed as follows [16]:

$$\lambda_{IF} \cdot \bar{N}_{r,IF}^s = \frac{cf_2}{f_1^2 - f_2^2} \bar{N}_{r,WL}^s + \frac{c}{f_1 + f_2} \bar{N}_{r,NL}^s \quad (3)$$

After the IF combination, the WL and NL ambiguities can be expressed as follows:

$$\bar{N}_{r,WL}^s = N_{r,WL}^s + b_{r,WL} - b_{WL}^s \quad (4)$$

$$\bar{N}_{r,NL}^s = N_{r,NL}^s + b_{r,NL} - b_{NL}^s \quad (5)$$

where $N_{r,WL}^s$ and $N_{r,NL}^s$ denote the integer WL and NL ambiguities. $b_{r,WL}$ and b_{WL}^s are the WL uncalibrated phase delay (UPD) on the receiver and satellite sides, respectively. $b_{r,NL}$ and b_{NL}^s are the NL UPD on the receiver and satellite sides, respectively.

The DD AR primarily eliminates the UPD of the satellite and receiver sides by differencing between two satellites, s_1 and s_2 , and two receivers, r_1 and r_2 . The specific principle is as follows:

$$\Delta \bar{N}_{r_1 r_2, WL}^{s_1 s_2} = N_{r_1, WL}^{s_1} - N_{r_1, WL}^{s_2} - N_{r_2, WL}^{s_1} + N_{r_2, WL}^{s_2} \quad (6)$$

$$\Delta \bar{N}_{r_1 r_2, NL}^{s_1 s_2} = N_{r_1, NL}^{s_1} - N_{r_1, NL}^{s_2} - N_{r_2, NL}^{s_1} + N_{r_2, NL}^{s_2} \quad (7)$$

Substituting the above expression into Equation (3), the DD ambiguity can be expressed as follows:

$$\lambda_{IF} \cdot \Delta \bar{N}_{r_1 r_2, IF}^{s_1 s_2} = \frac{cf_2}{f_1^2 - f_2^2} \Delta \bar{N}_{r_1 r_2, WL}^{s_1 s_2} + \frac{c}{f_1 + f_2} \Delta \bar{N}_{r_1 r_2, NL}^{s_1 s_2} \quad (8)$$

The MW (Melbourne–Wübbena) combination is commonly employed to compute float WL ambiguity. The UPD of the receiver and satellite sides can be eliminated by differencing between the satellites and the stations, enabling the determination of the DD WL ambiguity. If the fractional part of the rounded value of $\Delta \bar{N}_{r_1 r_2, WL}^{s_1 s_2}$ is smaller than the

threshold, the ambiguity is successfully fixed. Then, DD NL ambiguity can be calculated using Equation (8), and the fractional component of the rounded value of the float DD NL ambiguity can be compared with the threshold. If the fractional part is smaller than the threshold, the ambiguity is successfully fixed; otherwise, it is not. Finally, the fixed and float DD ambiguities are combined to form the constraint equations. These are incorporated into the original equations for updating, improving the accuracy of the unknown parameters.

In terms of UD AR, the integer values of the UD WL ambiguities can be determined based on the MW combined observations [23,24], leaving the fractional parts, thus representing the WL UPD observation $UPD_{r,WL}^s$ as follows:

$$UPD_{r,WL}^s = b_{r,WL} - b_{WL}^s + \varepsilon_{WL} \quad (9)$$

where ε_{WL} is the noise of the observation. Assuming there are rn stations capable of observing sm satellites, the observation equation is given by the following:

$$\begin{bmatrix} UPD_{r1,WL}^{s1} \\ \vdots \\ UPD_{rn,WL}^{sm} \end{bmatrix} = \begin{bmatrix} -1 & \cdots & 0 & 1 & \cdots & 0 \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & -10 & \cdots & \cdots & 1 \end{bmatrix} \begin{bmatrix} b_{WL}^{s1} \\ \vdots \\ b_{rn,WL} \end{bmatrix} \quad (10)$$

There is a linear relationship between the receiver clock and the satellite clocks, leading to a rank deficiency of 1 in Equation (10). Consequently, one satellite clock is chosen as the reference and set to 0. The least-square adjustment is then applied to estimate the UPD for the stations and satellites. Based on the WL UPD and IF ambiguities, the same processing procedure then can be employed to estimate the NL UPD.

If the WL and NL UPD products are obtained, the UD AR solution can be applied. Initially, the WL ambiguities are fixed. The criterion for determining if the WL ambiguities are fixed is based on whether the fractional part falls below the predefined threshold after accounting for the receiver and satellite UPD estimates. If this value is lower than the threshold, the ambiguities are fixed. In this study, the threshold for fixing UD WL ambiguity is 0.25 cycles. Subsequently, based on Equation (9), UD NL ambiguities can be fixed using the floating ambiguity variance and NL UPD. Once both the wide-lane and narrow-lane ambiguities are fixed, the IF ambiguity with integer characteristics can be resolved. Finally, the fixed ambiguities are combined with the float ambiguities to form a strong constraint added to the original equation for parameter updating, improving the precision of clock estimation.

2.3. Real-Time Clock Estimation by SRIF

SRIF is a state estimation algorithm that enhances numerical stability and computational efficiency by decomposing the square root of the information matrix [25]. Assuming there are m observations and n unknown parameters x at the epoch i , then the observation equation is given by the following:

$$l_i = H_i x_i + \varepsilon_i \quad (11)$$

$\begin{matrix} m \times 1 & m \times n & n \times 1 & m \times 1 \end{matrix}$

where l_i is the observation series, H_i represents the design matrix, and ε_i indicates the measurement noise. The state parameters can be divided into time-varying parameters ΔX_i and deterministic parameters y . The state equation for the time-varying parameters is given by the following:

$$\Delta X_i = \Phi_{i,i-1} \Delta X_{i-1} + w_i \quad (12)$$

where $\Phi_{i,i-1}$ represents the state transition matrix and w_i denotes the processing noise with $E(w_i) = 0$, $D(w_i) = D_{w_i}$. Given the initial values of the state parameters, the corresponding initial constraint equation is given by the following:

$$x = x_0 + w_0 \quad (13)$$

Since the SRIF requires that the noises must follow a normal distribution $N(0,1)$, the aforementioned equations need to be standardized. The standardized equations are as follows:

$$\begin{cases} z_i = A_i x_i + v_i \\ R_{w_i} \Delta X_i = R_{w_i} \Phi_{i,i-1} \Delta X_{i-1} + R_{w_i} w_i \\ z_0 = R_{w_0} x + v_0 \end{cases} \quad (14)$$

where $z_i = R_{v_i} l_i$, $A_i = R_{v_i} H_i$, $v_i = R_{v_i}^{-1} \varepsilon_i$, $z_0 = R_{w_0} x_0$, and $v_0 = R_{w_0} w_0$.

In SRIF, the observation equations of the estimated parameters are as follows:

$$\begin{bmatrix} R_0 \\ A \end{bmatrix} x = \begin{bmatrix} z_0 \\ z \end{bmatrix} - \begin{bmatrix} v_0 \\ v \end{bmatrix} \quad (15)$$

The SRIF estimates the parameters epoch by epoch, and its time update relies on the information and parameter states from the previous epoch. The above equation can be expressed after the measurement update:

$$\begin{bmatrix} \hat{R}_{\Delta X_{i-1}} & \hat{R}_{\Delta X_{i-1}y} \\ 0 & \hat{R}_y \end{bmatrix} \begin{bmatrix} \Delta X_{i-1} \\ y \end{bmatrix} = \begin{bmatrix} \hat{z}_{\Delta X_{i-1}} \\ \hat{z}_y \end{bmatrix} \quad (16)$$

Combining the state equation, the matrix after the time update is given by the following expression:

$$\begin{bmatrix} -R_{w_i} \Phi_{i,i-1} & R_{w_i} & 0 \\ \hat{R}_{\Delta X_{i-1}} & 0 & \hat{R}_{\Delta X_{i-1}y} \\ 0 & 0 & \hat{R}_y \end{bmatrix} \begin{bmatrix} \Delta X_{i-1} \\ \Delta X_i \\ y \end{bmatrix} = \begin{bmatrix} \hat{z}_{w_i} \\ \hat{z}_{\Delta X_{i-1}} \\ \hat{z}_y \end{bmatrix} \quad (17)$$

After the Householder orthogonal transformation, the above equation can be converted into an upper triangular matrix of the following form [13]:

$$\begin{bmatrix} \tilde{R}_{\Delta X_{i-1}} & \tilde{R}_{\Delta X_{i-1}\Delta X_i} & 0 \\ 0 & \tilde{R}_{\Delta X_i} & \tilde{R}_{\Delta X_i y} \\ 0 & 0 & \tilde{R}_y \end{bmatrix} \begin{bmatrix} \Delta X_{i-1} \\ \Delta X_i \\ y \end{bmatrix} = \begin{bmatrix} \tilde{z}_{w_i} \\ \tilde{z}_{\Delta X_i} \\ \tilde{z}_y \end{bmatrix} \quad (18)$$

From the above equation, it is evident that the parameter ΔX_{i-1} can be eliminated. Consequently, the state parameters can be predicted by this equation:

$$\begin{bmatrix} \tilde{R}_{\Delta X_i} & \tilde{R}_{\Delta X_i y} \\ 0 & \tilde{R}_y \end{bmatrix} \begin{bmatrix} \Delta X_i \\ y \end{bmatrix} = \begin{bmatrix} \tilde{z}_{\Delta X_i} \\ \tilde{z}_y \end{bmatrix} \quad (19)$$

Following the time update, the state parameters for the current epoch are integrated into the equation. This updated equation is then used as the initial input for the measurement update, which can be represented as follows:

$$\begin{bmatrix} \tilde{R}_i \\ A_i \end{bmatrix} x_i = \begin{bmatrix} \tilde{z}_i \\ z_i \end{bmatrix} \quad (20)$$

By reapplying the Householder orthogonal transformation and updating the observations with the residuals, the equation is transformed into the following:

$$\begin{bmatrix} \hat{R}_i \\ 0 \end{bmatrix} x_i = \begin{bmatrix} \hat{z}_i \\ e_i \end{bmatrix} \quad (21)$$

During the real-time satellite clock estimation, the observation residuals and the variance of unit weight are obtained from the covariance matrix. The residuals are stored in vector e_i and $e_i \sim N(0, I)$. The variance of unit weight is given by the following:

$$\sigma_0 = \sqrt{\frac{V^T V}{m}} = \sqrt{\frac{e_i^T e_i}{m}} \quad (22)$$

By employing hypothesis testing to detect and identify outliers [26–28], and through steps such as the Householder orthogonal transformation, the following equation can be derived [13]:

$$\begin{bmatrix} \hat{R}_i & \hat{R}_{\Delta,i} \\ 0 & \hat{R}_{\Delta} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ \Delta \end{bmatrix} = \begin{bmatrix} \hat{z}_i \\ z_{\Delta} \\ \hat{e}_i \end{bmatrix} \quad (23)$$

The final solutions for the estimated parameters and covariance matrix are as follows:

$$x_i = \hat{R}_i^{-1} \hat{z}_i \quad (24)$$

$$D_{x_i} = \hat{R}_i^{-1} \hat{R}_i^{-T} \quad (25)$$

The overall process of real-time clock estimation based on SRIF is shown in Figure 1, which provides the main steps and corresponding formulas.

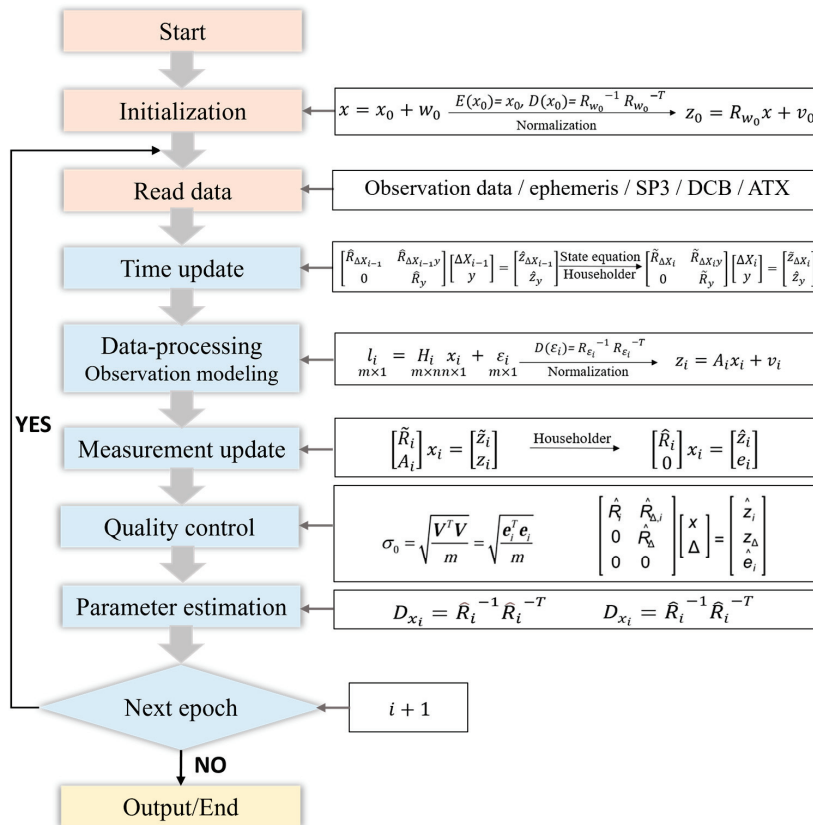


Figure 1. Data processing flowchart.

2.4. Data Collection and Processing Strategy

To evaluate the precision of clock estimation and the effectiveness of ambiguity resolution based on different orbit products, epoch-wise updated and predicted orbit products were collected for the experiment. Epoch-wise updated orbits for 1–31 October 2024 (DOY 275–305), were estimated using the same processing strategy as the real-time orbits recorded by the SHAO. Since the ultra-rapid orbit from IGS is updated every 6 h and lacks BDS data, the predicted orbits were obtained from the Wuhan University IGS analysis center, which generates ultra-rapid predicted orbits that are updated every hour based on observations (https://igs.org/products/#orbits_clocks-2024.09.17, accessed on 21 February 2025). Three sets of predicted orbits were then integrated into daily files with update intervals of 1 h, 3 h, and 6 h. Specifically, these updated orbit products are constructed by extracting segments from each ultra-rapid file starting from its update time. The orbits and clocks are compared to the precise products of the CODE (Center for Orbit Determination in Europe), ensuring the reliability and accuracy of the experimental results.

The station distribution of the clock estimation experiments is illustrated in Figure 2. The orange dots represent 100 stations estimating clocks, and the green stars represent 25 stations validating positioning results.

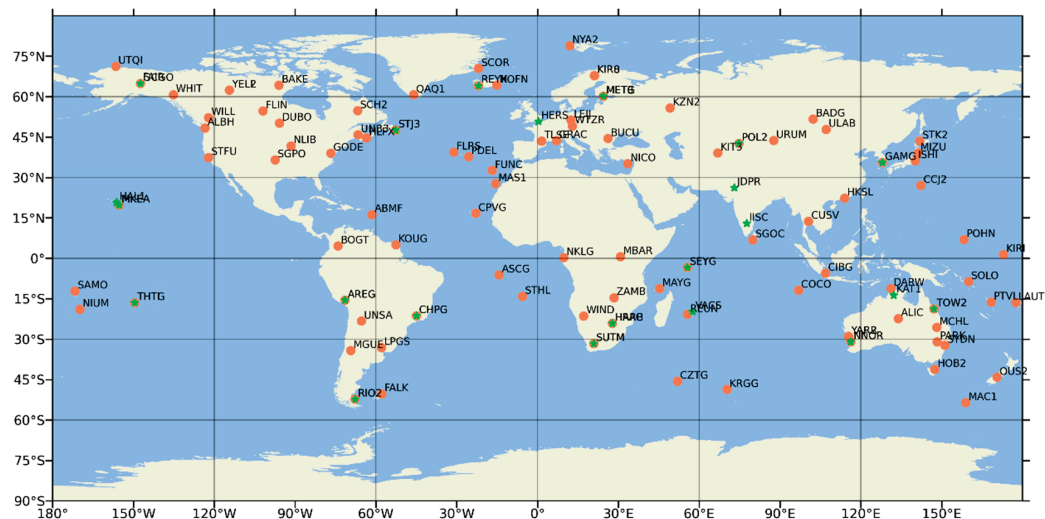


Figure 2. Stations distribution of the experiment.

Table 1 outlines the specific strategy for real-time clock estimation. All experiments were conducted using the PANDA software. The clock estimation experiments for GPS, Galileo, and BDS-3 were conducted in three modes: UD, DD, and float. Due to the characteristics of the GLONASS, only the float solution was implemented.

Table 1. Clock Offset Estimation Strategy.

Item	Strategy
Software	Modified PANDA
Stations	100 IGS/MGEX tracking stations [29,30]
Time interval	DOY 275–DOY 305, 2024
Observation	Undifferenced IF combination observations
Sampling interval	GPS: L1/L2, BDS-3: B1C/B2a, E: E1/E5a, R: G1/G2
Cutoff angle	30 s
Satellite antenna error	7°
Receiver antenna error	PCO and PCV corrections from the igs20.atx [31]
Phase wind-up	PCO corrections from the igs20.atx [31]
	Model correction [32]

Table 1. Cont.

Item	Strategy
Station coordinate	Fix to IGS weekly solution
Station displacement	Solid earth tide, pole tide and ocean tide: IERS Convention 2010 [33,34]
Satellite orbit and ERP	Real-time and predicted orbit products
Satellite and receiver clock	Estimated as white noise
ISB	Estimated as white noise
Estimator	SRIF
Troposphere delay	Saastamoinen model, GMF mapping function, piecewise constants [35,36]
Ambiguity	Estimated to be constant in the absence of cycle slips
Round-off criterion	0.25 cycles for WL, 0.15 cycles for NL
Differential code bias	CODE P1C1 bias product

3. Results

In this study, the accuracies of the predicted and epoch-wise updated orbit products were analyzed. Then, clock offset estimation experiments were conducted using SRIF based on these two products, and a discussion on the effects of UD and DD ambiguities fixing in clock offset estimation is presented. The clock offset accuracies were compared to precise clock products of the CODE. Finally, kinematic PPP was used for validation.

3.1. Analysis of Predicted and Estimated Satellite Orbit Products

To compare the differences between ultra-rapid predicted and real-time estimated satellite orbit products, precise orbits from the CODE were used as a reference to derive orbit residuals and accuracy. Taking BDS-3 as an example, the ultra-rapid orbit residuals at update intervals of 1 h, 3 h, and 6 h in the along (A), cross (C), and radial (R) directions are illustrated in Figure 3. Evidently, regardless of the update interval, significant jumps occurred in orbit residuals at each update epoch, indicating discontinuity in the residual sequences at these epochs. Additionally, the orbit accuracy of the 1 h update interval is superior to that of the 3 h and 6 h intervals, with the 6 h update exhibiting larger residual fluctuations and a wider distribution range. Out of all directions, the radial residuals are the smallest, while the along and cross residuals show larger variability.

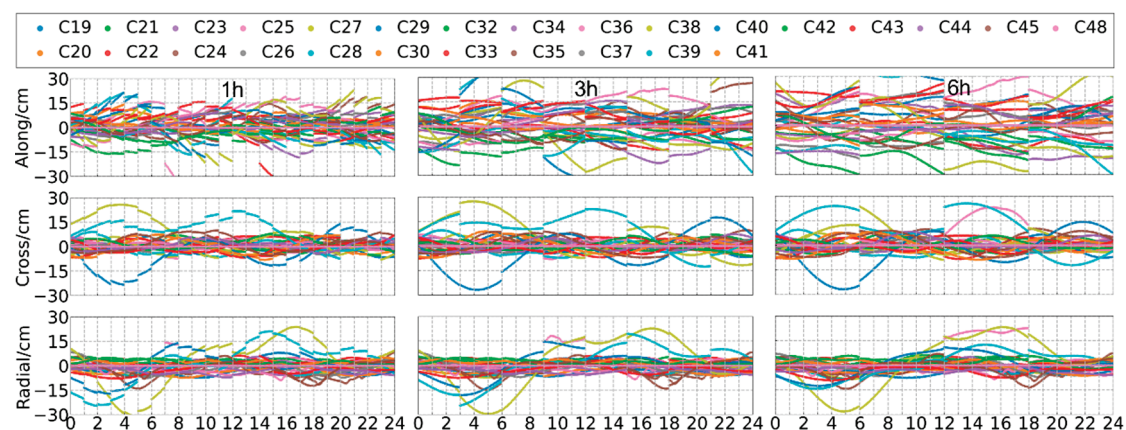


Figure 3. The ultra-rapid orbit residual series for BDS-3 (DOY: 283).

Figure 4 shows the same day residuals of the real-time orbits estimated epoch by epoch for BDS-3. Compared with the predicted orbit residuals, these orbits demonstrate higher accuracy and better continuity. The satellites with larger residuals in the figure are C38, C39, and C40, which are IGSO satellites and exhibit lower orbit accuracy compared

with MEO satellites. Excluding these three, the residuals of the remaining satellites in the A, C, and R directions fluctuate within 10 cm, indicating excellent overall performance.

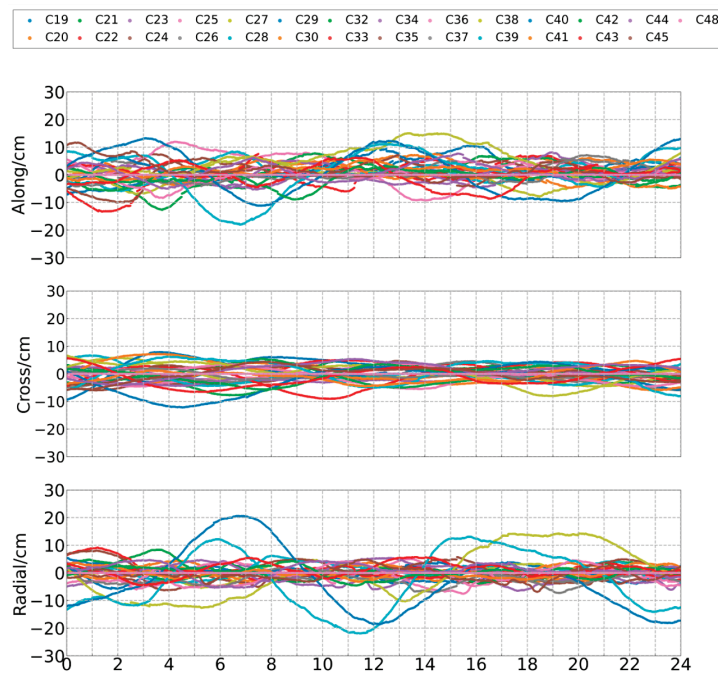


Figure 4. The estimated orbit residual series for BDS-3 (DOY: 283).

Figure 5 shows the average accuracy of the ultra-rapid predicted and epoch-wise updated orbits, where C, E, G, and R represent BDS-3, Galileo, GPS, and GLONASS, respectively. The labels 1 h, 3 h, 6 h, and SRIF represent predicted orbits with update frequencies of 1 h, 3 h, and 6 h, as well as epoch-wise updated orbits derived using SRIF. The accuracy of the along component is the worst, while the radial direction achieves the best accuracy, likely because a portion of radial errors are absorbed into the clock offset. For predicted orbits, the accuracies in the along and cross directions for a 1 h update frequency are significantly better than those with 3 h and 6 h update intervals, while the difference in the radial direction is minimal. The epoch-wise estimated orbit accuracy is clearly superior to that of the predicted orbit, with a particularly notable improvement in the along component. Additionally, the orbit accuracies of GPS and Galileo demonstrate superior performance, while BDS-3 and GLONASS exhibit comparable results. Overall, epoch-wise estimated orbits demonstrate a significant accuracy advantage, with the most noticeable improvement in the along direction.

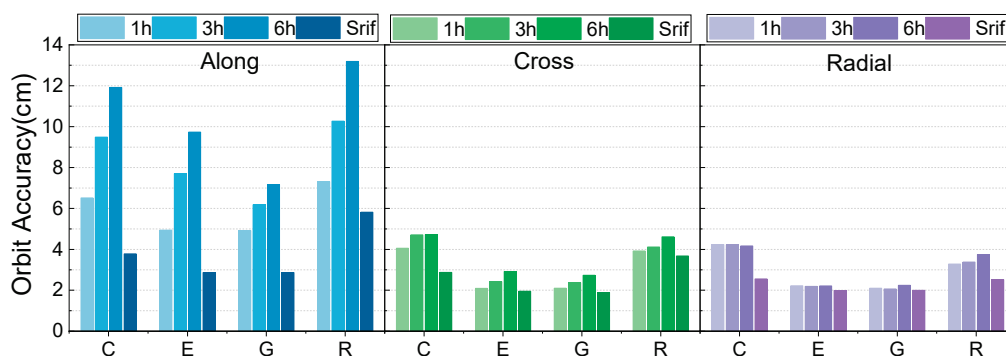


Figure 5. Average accuracy of predicted and epoch-wise updated orbits for BDS-3, Galileo, GPS, and GLONASS.

The average 1D-RMS errors of the GNSS orbits are summarized in Table 2. Clearly, the orbit precision of Galileo and GPS is generally superior to those of BDS-3 and GLONASS. As the update frequency decreases, the accuracy of the predicted orbit becomes worse. Compared with predicted orbits, epoch-wise updated orbits demonstrate significant advantages in both accuracy and continuity.

Table 2. The average 1D-RMS of different orbit products (Unit: cm).

Scheme	BDS-3	Galileo	GPS	GLONASS
1 h	5.23	3.41	3.43	5.28
3 h	6.75	4.88	4.15	6.12
6 h	7.97	6.07	4.78	8.53
Epoch-wise	3.17	2.46	2.45	4.33

3.2. Analysis of Ambiguity Resolution

In real-time clock estimation, orbit accuracy significantly impacts AR. In this section, UD, DD AR, and float solutions in clock estimation are applied to BDS-3, Galileo, and GPS based on predicted and epoch-wise updated orbit products. Then, the UD and DD ambiguity fixing rates of BDS-3, Galileo, and GPS are discussed.

The WL and NL of the UD and DD ambiguity fixing percentages are displayed in Figure 6 based on epoch-wise updated orbits recorded over 10 days. The WL fixing rate of DD is 100% for all systems, while the NL fixing rate significantly decreases. This is because the observation data were preprocessed according to the elevation angle and quality control methods during the DD AR process. For UD, the WL fixing rate is nearly 99%, and the NL fixing rate slightly decreases, with fluctuations of around 95%. Compared with DD, the WL fixing rate is almost similar, while the NL fixing rate is notably high.

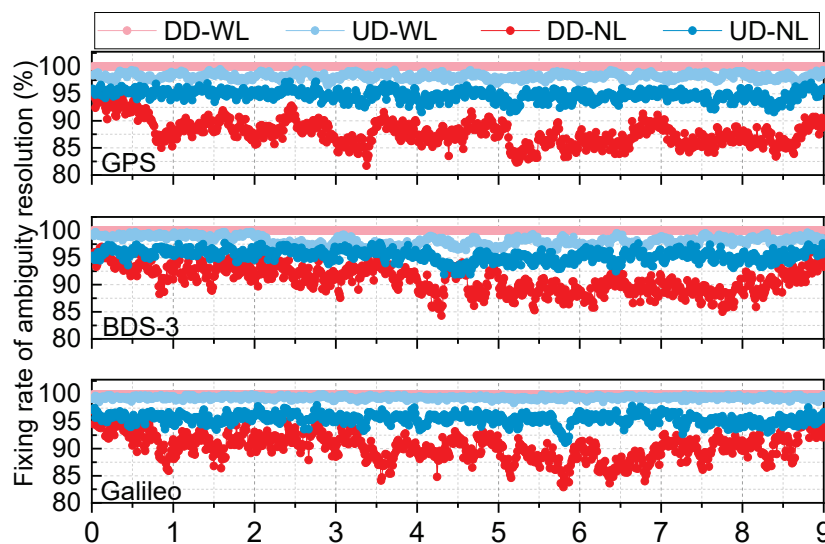


Figure 6. UD and DD ambiguity fixing percentages in clock estimation based on epoch-wise updated orbit.

To explore the impact of different satellite orbits on the DD AR of clock offset estimation, Figure 7 presents DD ambiguity fixing rates in clock estimation based on 1 h, 3 h, and 6 h predicted orbit, as well as epoch-wise updated orbits (marked as SRIF). It can be seen that all the WL fixing rates for BDS-3, Galileo, and GPS based on different orbits reach 100%. Conversely, the average NL fixing rate for DD ambiguities ranges from approximately 84.7% to 90.1%. Epoch-wise updated orbits obviously outperform predicted orbits in NL

fixing, and the NL fixing rate slightly decreases as the orbit update frequency decreases, indicating that orbit precision affects NL ambiguity fixing.

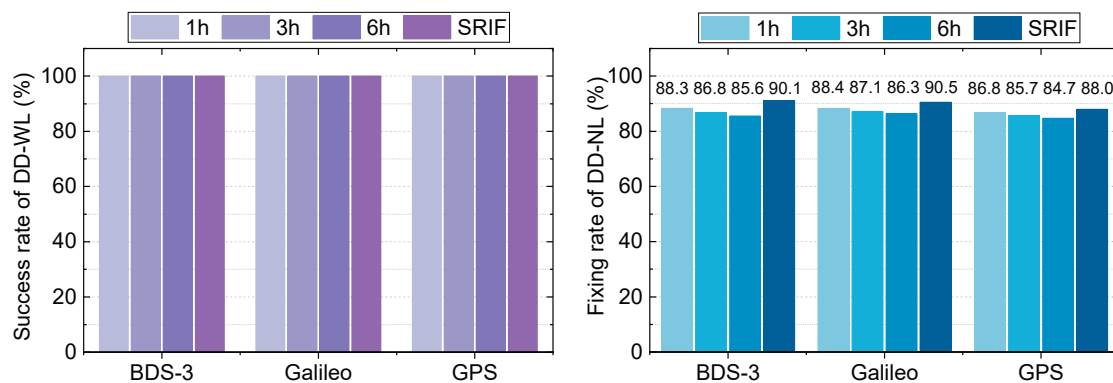


Figure 7. DD ambiguity fixing rate in clock estimation based on different orbits.

The UD ambiguity fixing rates of the predicted orbit are shown in Figure 8. The WL ambiguity fixing rates demonstrated excellent performance, with the rates for BDS-3, Galileo, and GPS all exceeding 98%. The results for the 1 h, 3 h, and 6 h predicted orbits, as well as for the epoch-wise updated orbits, show minimal variation. In terms of NL ambiguity fixing, predicted orbits with a 1 h update frequency exhibited significantly higher fixing rates than those of the 3 h and 6 h orbits. Furthermore, epoch-wise estimated orbits demonstrated a notable improvement over predicted orbits, particularly for BDS-3, where the NL fixing rate improved by 5%, 10%, and 19% compared with the 1 h, 3 h, and 6 h predicted orbits, respectively. Therefore, the quality and precision of orbit products have a remarkable impact on UD NL fixing, with higher-precision orbits leading to better fixing rates.

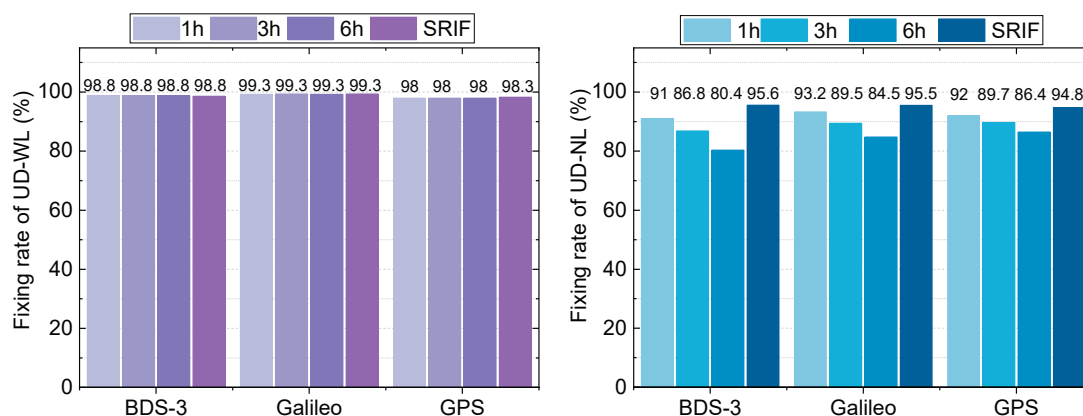


Figure 8. UD ambiguity fixing percentages in clock estimation based on different orbits.

3.3. Comparison of Clock Estimation Results

Real-time clock offset estimation depends on accurate real-time orbit products as known information. Orbit products with high update frequency can promptly capture dynamic variations, thus reducing error accumulation. Therefore, the results for clock offset estimation based on predicted and epoch-wise estimated orbit products are compared below.

Figure 9 displays the average accuracy of satellite clocks estimated on basis of 1 h, 3 h, and 6 h predicted orbit products. The STDs of the clock offsets are calculated by first taking the mean of the single differences between all satellite clocks within the same GNSS system as a virtual reference and then computing the second differences relative to this mean. The standard deviation of these second differences is used as the clock STD.

All results are better than 0.1 ns, except for the float solution with 6 h predicted orbits for BDS-3. The precision of satellite clock offsets significantly decreases as the predicted orbit update frequency reduces. Furthermore, the results for the DD and float solutions are slightly worse than those of the UD solution, indicating that the accuracy of satellite clock estimation can be improved by UD AR to some extent. It is notable that the clock offset accuracies of Galileo and GPS are slightly higher than those of BDS-3, which can be attributed to the more precise and stable ultra-rapid orbit products provided for Galileo and GPS. Therefore, both the accuracy and update frequency of satellite orbit products significantly impact the performance of real-time clock offset estimation.

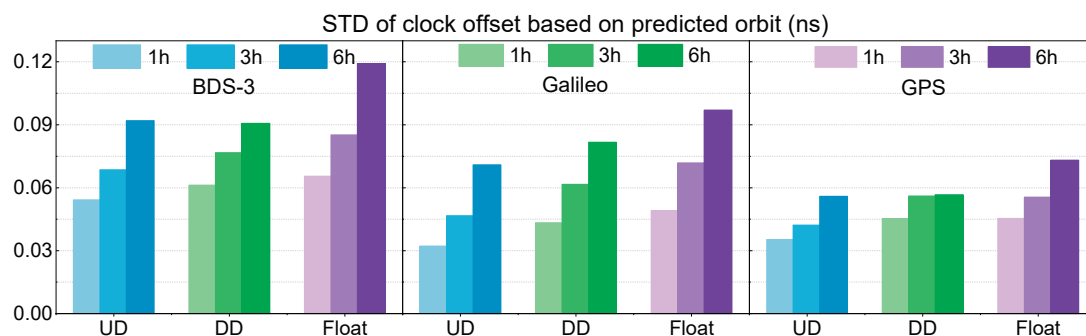


Figure 9. Averaged accuracy of clock offsets based on predicted orbit.

Figure 10 presents the clock estimation results for BDS-3, Galileo, and GPS with three schemes (UD, DD, and float), based on the epoch-wise estimated orbit, to further investigate the impact of orbit accuracy on clock offset estimation. The DD AR and float solutions are quite similar, and the UD solutions exhibit the best accuracy. Of the three systems, Galileo demonstrates the best performance, with a UD solution accuracy for all satellites within 0.03 ns. The GPS results are slightly better than those of BDS-3, with the most significant improvement observed after applying the UD AR. For BDS-3, the clock accuracy of the DD AR solution is worse than that of the float solution. Overall, the UD AR solution demonstrates a significant advantage in improving clock offset estimation accuracy.

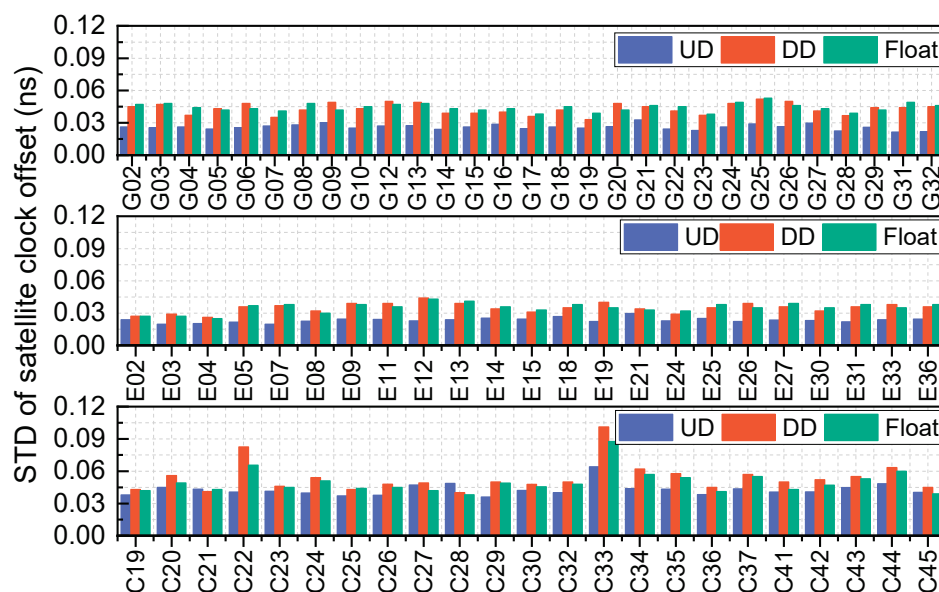


Figure 10. Clock accuracy of each satellite based on the epoch-wise updated orbit.

From Figures 9 and 10, it can be observed that both ambiguity resolution and high-quality orbit products significantly contribute to improving satellite clock offset estimation.

Specifically, with float solutions, the epoch-wise updated orbits consistently outperform ultra-rapid orbit products in clock estimation accuracy across all solution types—float, DD, and UD. This highlights the advantage of the proposed orbits in real-time clock estimation. Moreover, regardless of the orbit used, the UD solution yields better clock estimation performance than the float and DD solutions. When using epoch-wise updated orbits, the NL fixing rate of UD is significantly higher, resulting in more accurate and stable clock estimates. This further demonstrates the effectiveness of the proposed method.

Table 3 shows the mean accuracy of clock offset estimation using SRIF based on estimated orbits. The DD solution clearly does not enhance the precision of real-time satellite clock offset estimation, as its results are nearly identical to those of the float solution. By contrast, the UD solutions provide the best performance. The BDS-3, Galileo, and GPS clock offset accuracies reach 0.032 ns, 0.023 ns, and 0.026 ns, respectively, representing improvements of 36%, 34%, and 41% compared with the float solution. In contrast to the results based on predicted orbits, all the UD, DD, and float solutions exhibit notable improvements. In conclusion, high-precision orbit products and UD AR significantly enhance real-time clock offset estimation.

Table 3. The average accuracy of clock offset based on epoch-wise updated orbit (Unit: ns).

Scheme	BDS-3	Galileo	GPS
Float	0.050	0.035	0.044
DD	0.053	0.035	0.043
UD	0.032	0.023	0.026

3.4. PPP Validation

To further validate the performance of the optimal clock estimation method, kinematic PPP experiments with BDS-3, Galileo, and GPS were conducted using epoch-wise estimated orbit and the optimal clock products over 30 days. The PPP experiment involved 25 stations, as indicated in Figure 2. The IF combination with the same frequency as the clock offset estimation was used in the experiment. Furthermore, CNES real-time satellite orbit and clock products were used for comparison.

Positioning errors in the north (N), east (E), and up (U) directions for station TOW2 from DOY 287 to 293 are shown in Figure 11. The orange dots represent the positioning results using CNES real-time orbit and clock products, and the blue dots represent those of the estimated orbit and clock products combined with UD AR using SRIF. Both sets of results exhibit rapid convergence in all three directions, with errors in the N and E directions stabilizing after convergence. In comparison, the SRIF product provides more stable positioning results with higher accuracy in all three NEU directions, outperforming the CNES results.

Figure 12 presents the PPP positioning results for all stations using SRIF and CNES products. The red bars on the left represent the average convergence time in the N, E, and U directions, and the green bars on the right represent the accuracy in these directions. The convergence time of SRIF is slightly shorter than that of the CNES products for most stations in the N and U components. However, in the E component, some stations show a slower convergence when using SRIF products. Regarding positioning accuracy, the SRIF products outperform the CNES product in the N, E, and U components, demonstrating higher precision overall.

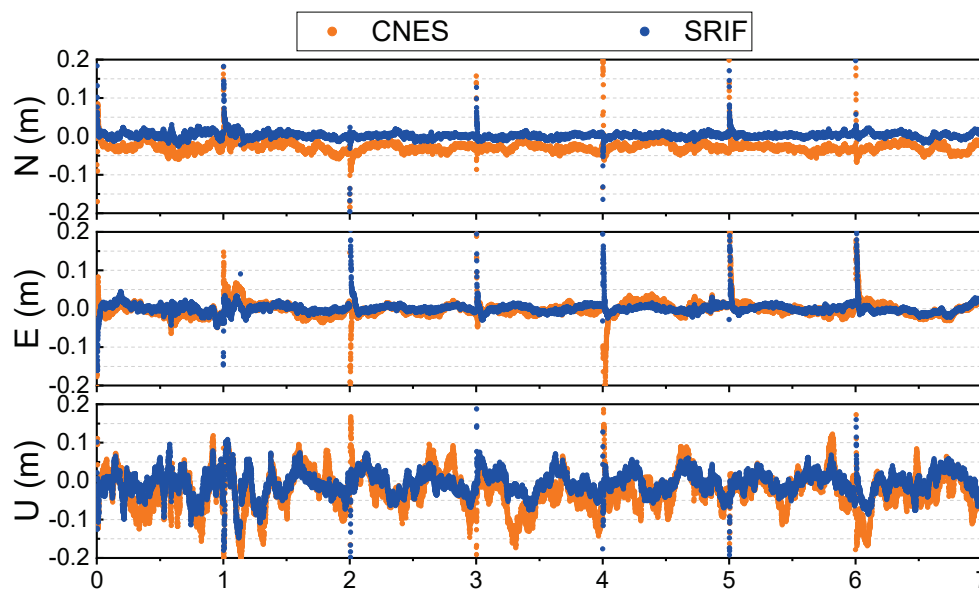


Figure 11. Positioning errors of station TOW2 using CNES and SRIF solution products.

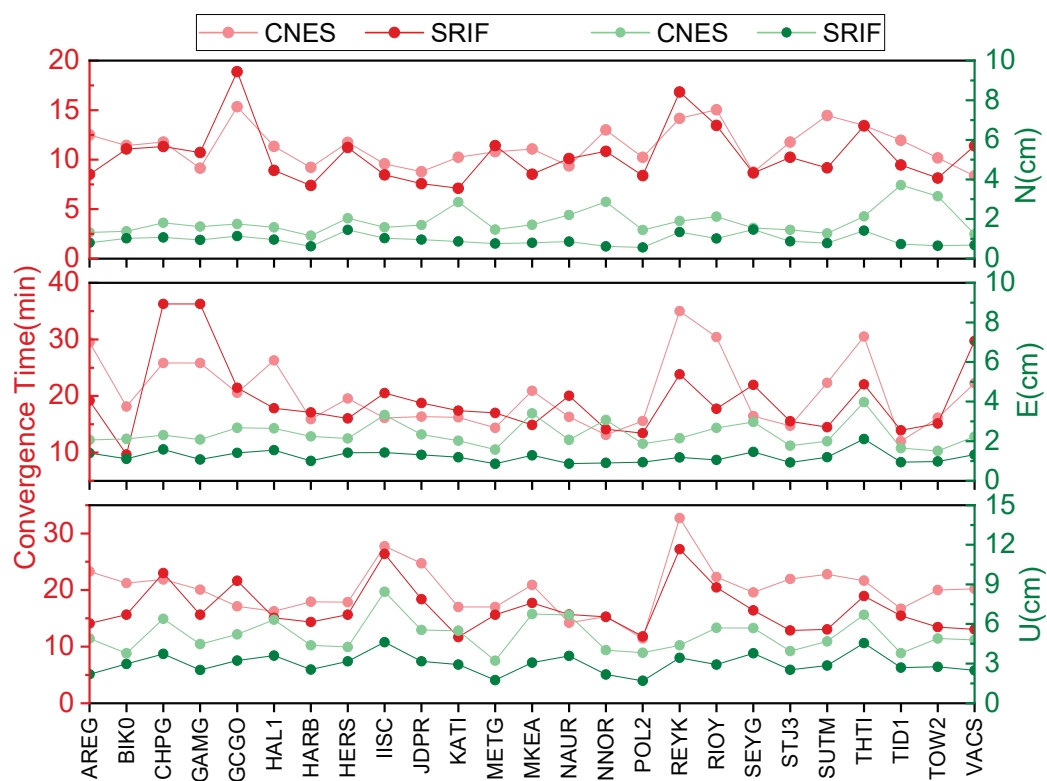


Figure 12. Positioning results and convergence times of SRIF and CNES products for each station.

Table 4 summarizes the average positioning precision and convergence time for the 25 stations. The PPP results demonstrate that the average convergence time using SRIF products is 15.57 min, comparable to the 17.28 min for the CNES products. Regarding positioning accuracy, the SRIF products achieve accuracies of 0.94 cm, 1.22 cm, and 3.01 cm in the N, E, and U directions, respectively, representing improvements of 50%, 48%, and 41% compared with CNES product. Consequently, the epoch-wise estimated orbit and UD AR satellite clock products using SRIF offer faster convergence and significantly enhance positioning accuracy for PPP.

Table 4. Average positioning errors and convergence time for SRIF and CNES.

	N (cm)	E (cm)	U (cm)	Convergence Time (min)
SRIF	0.94	1.22	3.01	15.57
CNES	1.88	2.36	5.13	17.28

4. Conclusions

Real-time clock offset products are among the core components of PPP services. Currently, real-time clock offset products provided by institutions such as the CNES rely on ultra-rapid predicted orbits. However, these predicted orbits have limited accuracy and discontinuous updates, which constrain the precision of real-time clock offset estimation. Therefore, this study proposes a UD AR clock offset estimation method implemented by SRIF based on real-time estimated orbits.

First, a comparison between the predicted orbits and the epoch-wise updated orbits highlighted the significant advantages of the latter in terms of both accuracy and continuity. Following this, three clock offset estimation schemes (UD, DD, and float solutions) were designed. The results show that the clock estimation accuracy and AR fixing rate can be significantly improved with high-precision epoch-wise updated orbits. In addition, the UD AR technique enhances clock offset estimation accuracy, while the DD AR does not significantly improve clock offset estimation. High-precision epoch-wise updated orbit products notably enhance the NL fixing rate of UD, substantially improving clock offset estimation accuracy. Specifically, the clock offset accuracies for BDS-3, Galileo, and GPS reach 0.032 ns, 0.023 ns, and 0.026 ns, respectively. Finally, the effectiveness of the proposed method was validated through PPP, and it was determined that this method outperforms real-time clock offset products based on predicted orbits of CNES. In conclusion, combining epoch-wise updated orbit products with the UD AR method can effectively improve the accuracy of real-time clock offset products, thereby enhancing the service quality of PPP.

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Article

BeiDou Satellite-Based Augmentation System Algorithm Optimization and Performance Validation of Ionospheric Degradation Parameters with RTCA Protocol

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Abstract: The BeiDou Satellite-Based Augmentation System (BDSBAS), based on the Radio Technical Commission for Aeronautics (RTCA) protocol, aims to provide high-precision, single-frequency positioning with integrity assurance for civil aviation users in China and surrounding regions. Given the anticipated high solar activity between 2023 and 2025, ionospheric anomalies may degrade positioning accuracy and significantly impact BDSBAS integrity performance. To enhance BDSBAS integrity, this study evaluates and analyzes the system's ionospheric degradation parameters for 2023. The results indicate that during the active ionospheric period in 2023, the rate of ionospheric grid delay changes exceeding the limits of the currently broadcasted parameters increased by 0.86%, posing potential integrity risks compared to 2022. To address this issue, we propose a novel algorithm for ionospheric degradation parameters and assess its applicability, stability, and effectiveness using BDSBAS single-frequency service message data from IGS monitoring stations in China. Statistical analysis in the localization domain demonstrates that the new method reduces the rate of ionospheric degradation parameters exceeding the threshold by 1.10% in 2023–2024. This approach significantly enhances BDSBAS integrity service capabilities, supporting its performance improvement and official deployment.

Keywords: BeiDou satellite-based augmentation system (BDSBAS); integrity performance; degradation parameters; service availability

1. Introduction

With the widespread adoption of Global Navigation Satellite Systems (GNSS), many countries and regions have established satellite-based augmentation systems (SBAS) to meet higher demands for positioning accuracy and integrity [1,2]. Currently, ten SBAS are either operational or under construction worldwide, including, as follows: the Wide Area Augmentation System (WAAS) in the United States; the European Geostationary Navigation Overlay System (EGNOS) in Europe; the Multi-functional Transport Satellite-based Augmentation System (MSAS) in Japan; the GPS Aided Geo Augmented Navigation (GAGAN) in India; the System for Differential Correction and Monitoring (SDCM) in Russia; the BeiDou Satellite-Based Augmentation System (BDSBAS) in China; the SBAS for Africa and the Indian Ocean (A-SBAS); the Algerian Satellite-Based Augmentation System (AL-SBAS); the Southern Positioning Augmentation Network (SPAN) in Australia and New Zealand; and the Korea Augmentation Satellite System (KASS) in Korea [3]. Among these,

WAAS, EGNOS, MSAS, and GAGAN have officially launched RTCA single-frequency services, while BDSBAS, SDCM, A-SBAS, AL-SBAS, SPAN, and KASS are currently in testing or trial operations.

BDSBAS broadcasts comprehensive correction information, including differential corrections for satellite orbits and clocks, ionospheric grid corrections, and integrity data such as User Differential Range Error (UDRE), Grid Ionospheric Vertical Error (GIVE), and degradation parameters [4,5]. These parameters ensure the timeliness and accuracy of differential corrections in both spatial and temporal domains, particularly benefiting users experiencing correction delays or who are operating at the service area's edge [6]. This information is transmitted via three geosynchronous pseudo-random noise satellites (PRN 130, PRN 143, and PRN 144) to augment GPS signals and provide APV-I level services in China and the surrounding regions, with service indicators detailed in Table 1 [7,8].

Table 1. Signal-in-space performance requirements for SBAS on APV-I.

Flight Operation	Horizontal/Vertical Accuracy (95%)	Integrity Level	HPL/VPL	Alarm Time	Continuity	Availability
APV-I	16 m/20 m	2×10^{-7} /approach	40 m/50 m	10 s	8×10^{-6} /15 s	0.99~0.99999

BDSBAS message type 10 retains the spatiotemporal characteristics of the most recent effective differential information, minimizing service degradation even if differential correction data are lost. Table 2 provides a detailed description of the efficacy degradation parameters [9], which are categorized by function into the following four groups: (1) fast-correction degradation parameters, encompassing category B_{rrc} ; (2) long-term correction degradation parameters, including variation limit C_{ltc_v0} and update period I_{ltc_v0} , etc.; (3) geosynchronous orbit (GEO) message degradation parameters, consisting of categories update period I_{ltc_v0} , variation limit C_{geo_lsb} and variable speed limit C_{geo_v} ; and (4) ionospheric degradation parameters are divided into variation limit C_{iono_step} , update period I_{iono} , and variable speed limit C_{iono_ramp} . In addition, RSS_{iono} and RSS_{UDRE} are, respectively, the root sum square flag and user differential range error flag, and $C_{covariance}$ is equivalent to the residual variance.

Table 2. Message type 10 detailed content.

Parameter	Effective Range	Units	Parameter	Effective Range	Units
B_{rrc}	0–2.046	m	I_{geo}	0–511	s
C_{ltc_lsb}	0–2.046	m	C_{er}	0–31.5	m
C_{ltc_v1}	0–0.05115	m/s	C_{iono_step}	0–1.023	m
I_{ltc_v1}	0–511	s	I_{iono}	0–511	s
C_{ltc_v0}	0–2.046	m	C_{iono_ramp}	0–0.005155	m/s
I_{ltc_v0}	0–511	s	RSS_{UDRE}	0–1	unitless
C_{geo_lsb}	0–0.5115	m	RSS_{iono}	0–1	unitless
C_{geo_v}	0–0.05115	m/s	$C_{covariance}$	0–12.7	unitless

Ionospheric degradation parameters define the change limits of vertical delays at Ionospheric Grid Points (IGPs) broadcast via message type 26 and are disseminated through message type 10. The parameters are as follows: C_{iono_step} denotes the limit on the variation between successive ionospheric grid delay values; I_{iono} indicates the interval between updates to ionospheric delay corrections; and C_{iono_ramp} defines the rate limit for changes

in successive ionospheric grid delay values. RSS_{iono} serves as a root sum square flag for the ionospheric degradation parameters. Currently, different SBAS broadcast ionospheric degradation parameters are constant values, as shown in Table 3.

Table 3. Global SBAS ionospheric degradation parameters.

SYSTEM	C_{iono_step}	I_{iono}	C_{iono_ramp}	RSS_{iono}
BDSBAS	0.952	240	0.000000	0
WAAS	0.836	300	0.000000	0
EGNOS	0.657	300	0.000000	0
SDCM	0	300	0.000740	1
MSAS	0.836	300	0.000000	0
GAGAN	0.952	300	0.000000	0
KASS	0.657	300	0.000000	0
SPAN	0.657	300	0.000000	0
ASBAS	0.657	300	0.000000	0

SBAS services aim to enhance the accuracy, integrity, availability, and continuity of GNSS positioning [10]. Integrity performance is a key reliability indicator, evaluated using Horizontal Protection Levels (HPL) and Vertical Protection Levels (VPL) derived from SBAS integrity messages. These protection levels quantify error bounds in positioning, ensuring navigation system integrity and safety.

Degradation parameters contribute to protection level calculations. If the computed HPL and VPL remain below the required Horizontal and Vertical Alert Limits (HAL and VAL) for a given service level, the service is deemed available; otherwise, it is considered unavailable. Additionally, if the user's protection level falls below actual Horizontal and vertical Positioning Errors (HPE and VPE), an integrity risk arises. Thus, ionospheric degradation parameters are crucial for ensuring SBAS service reliability and availability [11,12].

In BDSBAS studies, service performance and integrity have received significant attention [13–15]; however, degradation parameters have been less explored. Ionospheric correction plays a vital role in achieving high-accuracy, single-frequency positioning [6,16,17]. Therefore, this study focuses on ionospheric degradation parameters from message type 10. Section 2.1 analyzes cases where BDSBAS ionospheric degradation parameters failed to provide effective envelopment during peak ionospheric activity, evaluating their impact on service performance. Section 2.2 identifies limitations in BDSBAS ionospheric degradation representation during high solar activity and proposes an improved algorithm. Section 3 details the experimental results validation of this new method and Section 4 discusses the impact of the new degradation parameters on positioning services.

2. Materials and Methods

2.1. Performance Analysis of Ionospheric Degradation Parameters

Ionospheric degradation parameters, defined as an integrity threshold, are designed to encapsulate and mitigate ionospheric delay errors. This capability ensures compliance with APV-I's performance criteria for integrity services. The research presented in this paper indicates that with BDSBAS degradation parameters C_{iono_step} set at 0.952 and C_{iono_ramp} at 0, the probability of exceeding the threshold for BDSBAS remained relatively low in 2021. However, as ionospheric activity has increased in recent years, the probability of integrity parameter leakage has risen annually. The frequency of ionospheric variations ΔIGP_{delay} exceeding the threshold I_{limit} has also increased. The threshold of ionospheric delay change is determined by the ionospheric degradation parameter in Equation (1), as follows:

$$I_{\text{limit}} = C_{\text{iono_step}} + C_{\text{iono_ramp}} * I_{\text{iono}} \quad (1)$$

The parameters $C_{\text{iono_step}}$, $C_{\text{iono_ramp}}$ and I_{iono} in Equation (1) have different fixed values in different systems. The specific values are shown in Table 1. In the previous paragraph, $\Delta IGPdelay$ represents the difference in the same grid points delays between the two adjacent epochs, as follows:

$$\Delta IGPdelay = IGPdelay_t - IGPdelay_{t-1} \quad (2)$$

The probability of ionospheric grid delay variations exceeding the threshold can be calculated by Equation (3), as follows:

$$P(\Delta IGPdelay > I_{\text{limit}}) = \frac{\sum_{t=1}^T \sum_{j=1}^J (\Delta IGPdelay_{t,j} > I_{\text{limit}})}{T * J} \quad (3)$$

In Equation (3), T is the total number of the epoch and J is the total number of ionospheric grid points. $\Delta IGPdelay_{t,j}$ is the variation in ionospheric delay at the grid point j on the epoch t .

This study selects a grid point at 15°N latitude and 100°E longitude, a region of high ionospheric activity, for detailed analysis. The ionospheric delay variation envelope for the first day of each year from 2021 to 2024 in BDSBAS is shown in Figure 1. The horizontal axis is in hours and the vertical axis is in meters, the blue dots represent the ionospheric delay variation between adjacent epochs at this grid point from 2021 to 2024, and the red line indicates the variation threshold I_{limit} . It is equal to $C_{\text{iono_step}}$ because $C_{\text{iono_ramp}}$ is the fixed value 0.

In 2021, the degradation parameter threshold nearly encompassed all ionospheric delay variations, as shown in Figure 1a. However, as ionospheric activity intensified in 2022, Figure 1b shows that the degradation parameter could no longer fully envelop the actual variation in ionospheric delay; the probability of exceeding the threshold increased further in 2023 and 2024, as shown in Figure 1c,d.

Table 4 provides more detailed data on the annual probability of exceeding the limits calculated by Equation (3) from 2021 to 2024 for BDSBAS, EGNOS, MSAS, and WAAS. The probability of exceeding the threshold for BDSBAS and MSAS has increased greatly, whereas EGNOS and WAAS have shown no significant rise. This discrepancy is likely due to the higher proportion of low-latitude regions in China, where ionospheric anomalies have a greater impact [18–20]. In contrast, EGNOS and WAAS operate in regions with fewer ionospheric anomalies [21].

Table 4. The annual probability of exceeding the threshold probability in 2021–2024.

	2021	2022	2023	2024
BDSBAS	0.12%	0.78%	1.64%	1.96%
MSAS	0.54%	1.44%	2.42%	3.24%
EGNOS	0.15%	0.36%	0.38%	0.40%
WASS	0.02%	0.07%	0.16%	0.23%

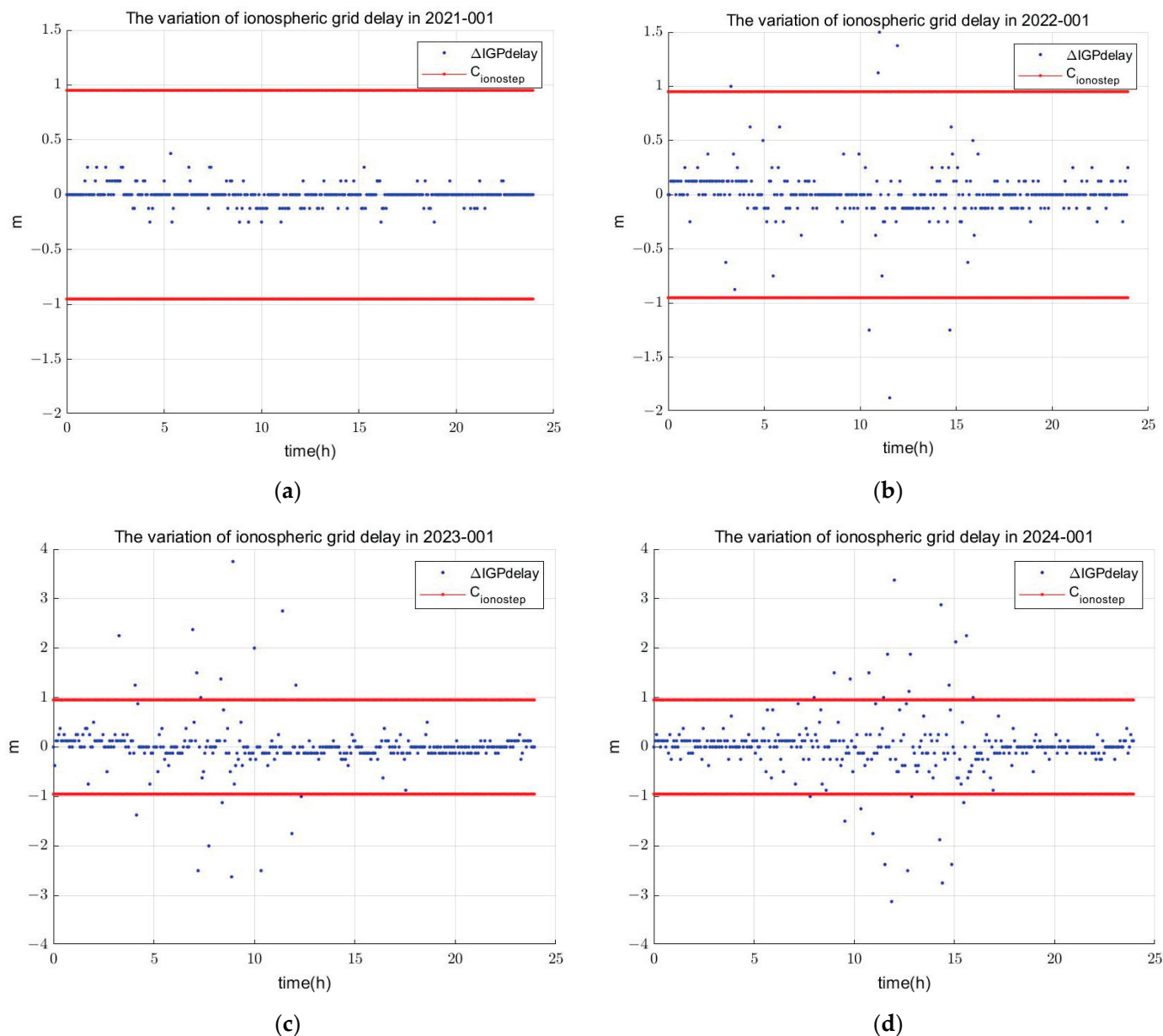


Figure 1. (a) Variation in ionospheric delay in the 17th grid point in the 7th band on the first day of 2021; (b) variation in ionospheric delay in the 17th grid point in the 7th band on the first day of 2022; (c) variation in ionospheric delay in the 17th grid point in the 7th band on the first day of 2023; and (d) variation in ionospheric delay in the 17th grid point in the 7th band on the first day of 2024.

The daily probability of exceeding the threshold for BDSBAS and MSAS from 2021 to 2024 is shown in Figure 2. The vertical axis represents the probability of exceeding the threshold, while the horizontal axis indicates the number of days. The exceeding threshold rate of MSAS is generally higher than that of BDSBAS, with both showing synchronized variations. The probability of ionospheric changes in both BDSBAS and MSAS exceeding the degradation parameter threshold increases over time. This trend reflects the growing impact of increased solar activity on China and the surrounding region.

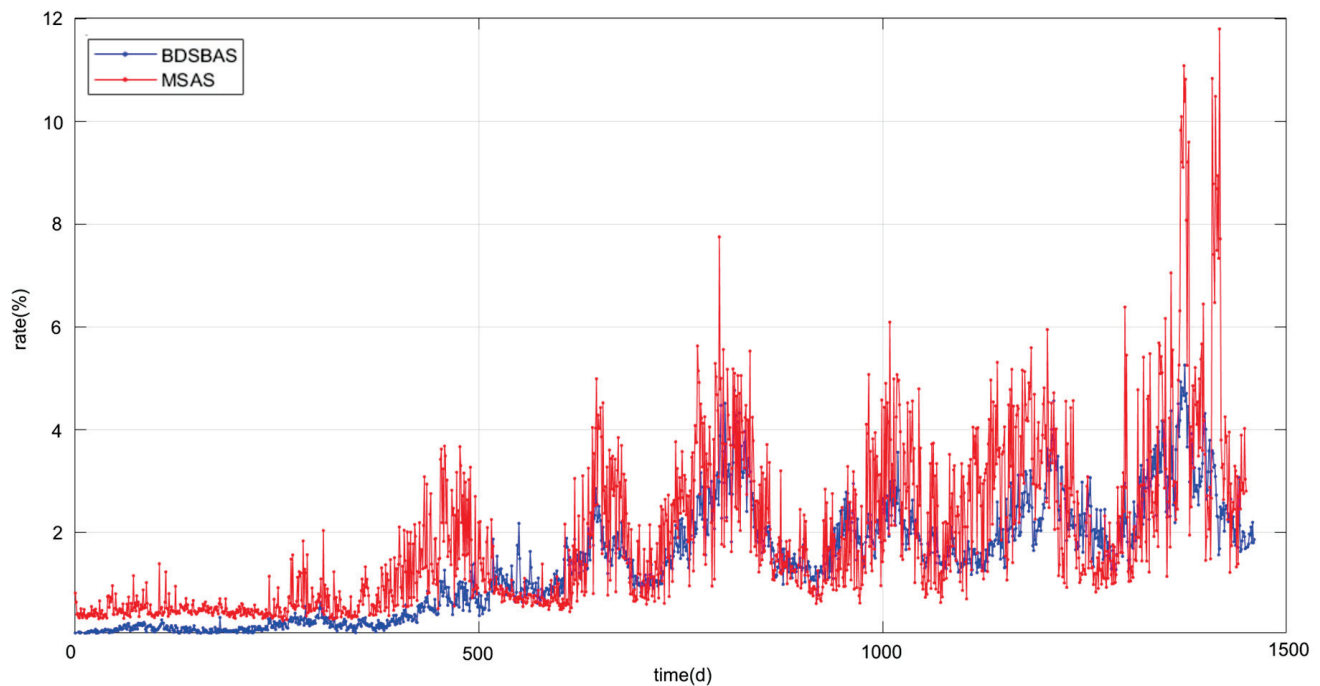


Figure 2. Probability of exceeding the threshold of BDSBAS and MSAS from the years 2021 to 2024.

To further investigate the relationship between ionospheric anomalies and the exceeding threshold rate, as well as their connection to solar activity, a time-series analysis of the BDSBAS exceeding threshold rate and sunspot number (SSN) from 2021 to 2024 was conducted. In Figure 3, the daily exceeding threshold rate is plotted on the left vertical axis, and SSN is plotted on the right vertical axis, with the horizontal axis representing the number of days. SSN, derived from sunspot counts, is a widely used index for quantifying solar activity [22]. It is strongly correlated with most solar activity indicators and serves as a key driver of ionospheric variations [23].

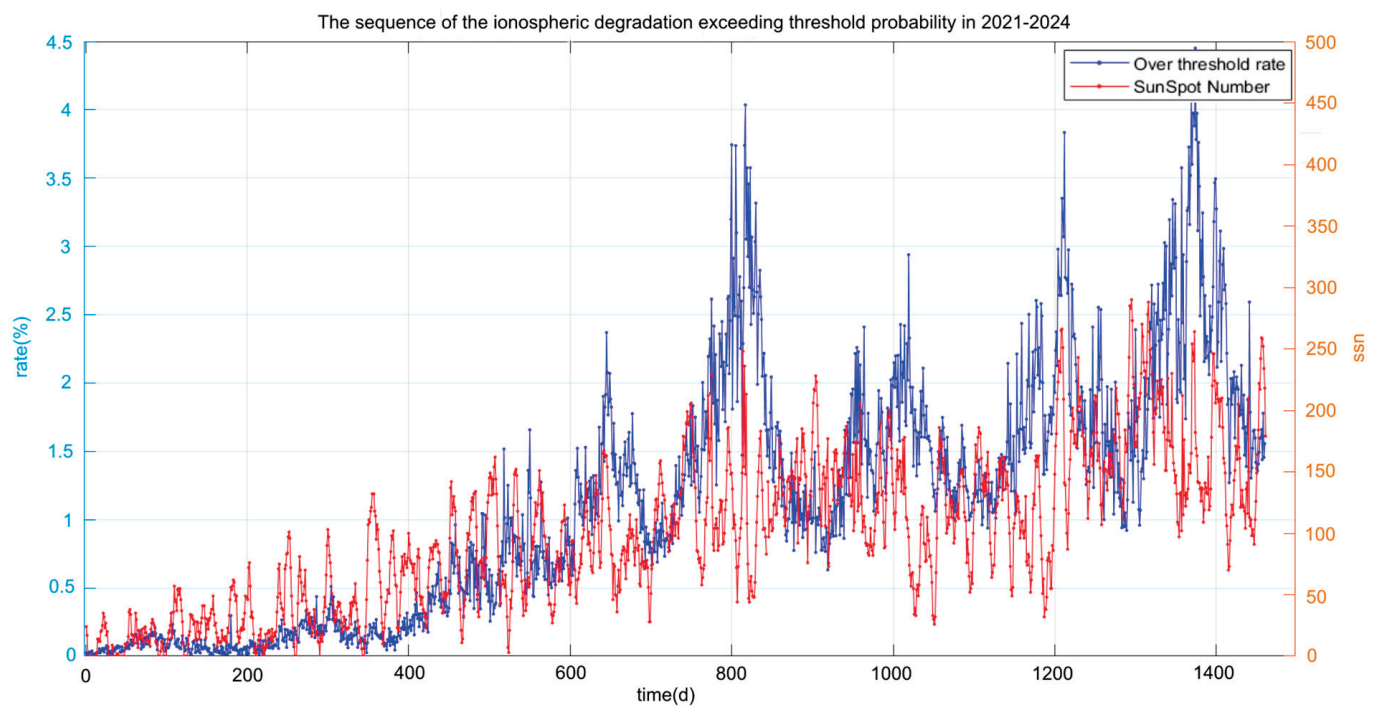


Figure 3. Probability of exceeding the threshold from the years 2021 to 2024 (horizontal axis: days).

The proportion of ionospheric delay variations exceeding the threshold has risen annually, as shown in Figure 3. The exceeding threshold rate of ionospheric degradation parameters correlates with ionospheric activity, which in turn is influenced by solar activity. While SSN and total electron content (TEC) exhibit a strong long-term correlation, their short-term correlation is weaker. Both SSN and the over-limit rate exhibit oscillatory increases over four years; however, local correlations vary due to factors such as geomagnetic variations [21,24]. Additionally, the solar activity index reveals hysteresis and saturation effects in TEC, where TEC values during the ascending and descending phases of the solar cycle differ despite similar levels of solar activity. Saturation implies that beyond a certain threshold, increasing solar activity does not proportionally increase TEC [25,26]. Overall, solar activity and ionospheric changes exhibit a strong correlation.

A statistical analysis of ionospheric grid points in the BDSBAS service area was conducted to examine the exceeding threshold probability across different latitude bands. Figure 4 illustrates the changes in exceeding the threshold daily probability from 2021 to 2024 for latitudes between 5° and 55° . The horizontal axis is in days, and the vertical axis shows the probability of exceeding the threshold. The changes in exceeding the threshold probability vary across latitude bands, with an overall cyclical upward trend displaying seasonal periodicity; it is especially conspicuous in low-latitude areas.

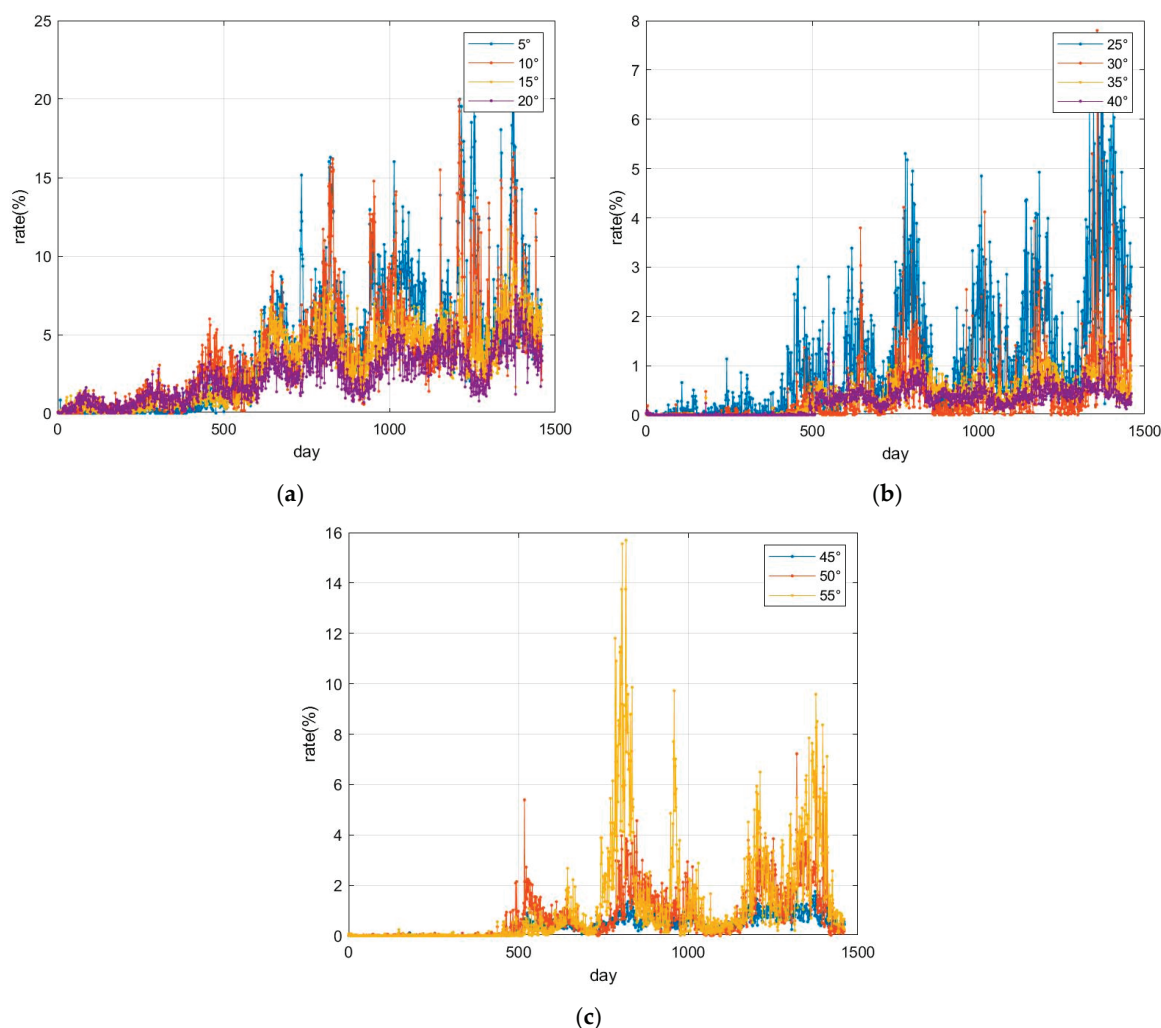


Figure 4. (a) Probability of exceeding the threshold in the 5° to 20° latitude band; (b) probability of exceeding the threshold in the 25° to 40° latitude band; and (c) probability of exceeding the threshold in the 45° to 55° latitude band.

Figure 5 summarizes the relationship between latitude and the probability of exceeding the threshold. The horizontal axis is in latitudes and the vertical axis shows the probability of exceeding the threshold. The lowest probability occurs at the 40° latitude and increases toward the edges, with more significant issues in low-latitude areas. The ionosphere is particularly active below the 35° latitude, directly affecting ionospheric degradation parameters and, consequently, service availability and integrity. Therefore, during periods of ionospheric activity, its impact on the integrity in southern China is significantly heightened.

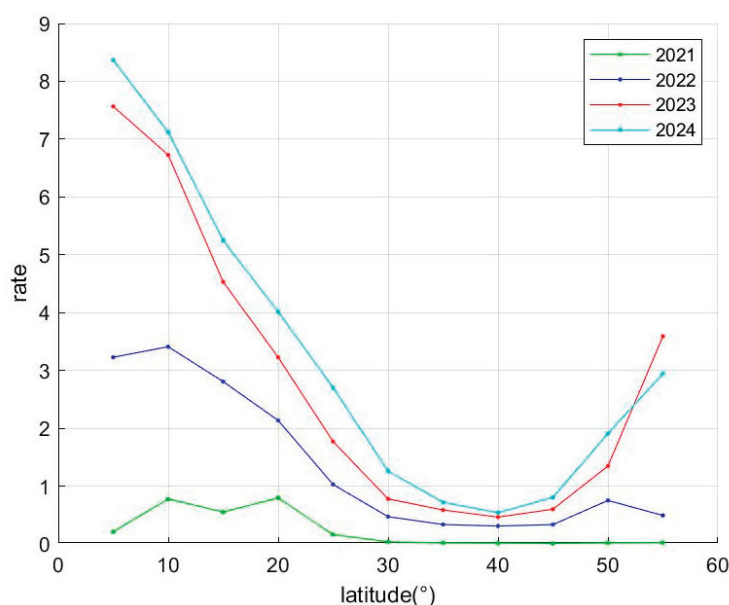


Figure 5. Annual probability of exceeding the threshold in the different latitude from the years 2021 to 2024.

To assess the impact of ionospheric activity on service performance, BDSBAS service data from 2022 and 2023 were analyzed, focusing on enhanced positioning, protection levels, and service availability. The positioning performance and protection levels of selected IGS stations were evaluated using degradation parameters. The algorithm of the protection level refers to the RTCA [27]. Present service availability statistics for the WUH2 station from DOY (day of the year) 001–003 in 2022 are shown in Figure 6, showing the service availability of WUH2 station from DOY 001–003 in 2023, where blue represents the Horizontal Protection Level, yellow denotes the Horizontal Position Error, and red marks the Protection Level threshold, with the horizontal axis in days and the vertical axis in meters. The figures indicate that while positioning errors remained stable, the probability of integrity and misleading information events increases. In addition, the increased protection level can lead to reduced availability in areas with larger protection levels.

The service availability maps in the China region for DOY 001–015 in 2022 and 2023 are shown in Figure 7. The color axis denotes service availability; the horizontal axis is the latitude, and the vertical axis is precision. This shows a decline in single-frequency enhancement service performance in 2023. This decline is particularly pronounced in lower-latitude regions, where ionospheric activity is higher.

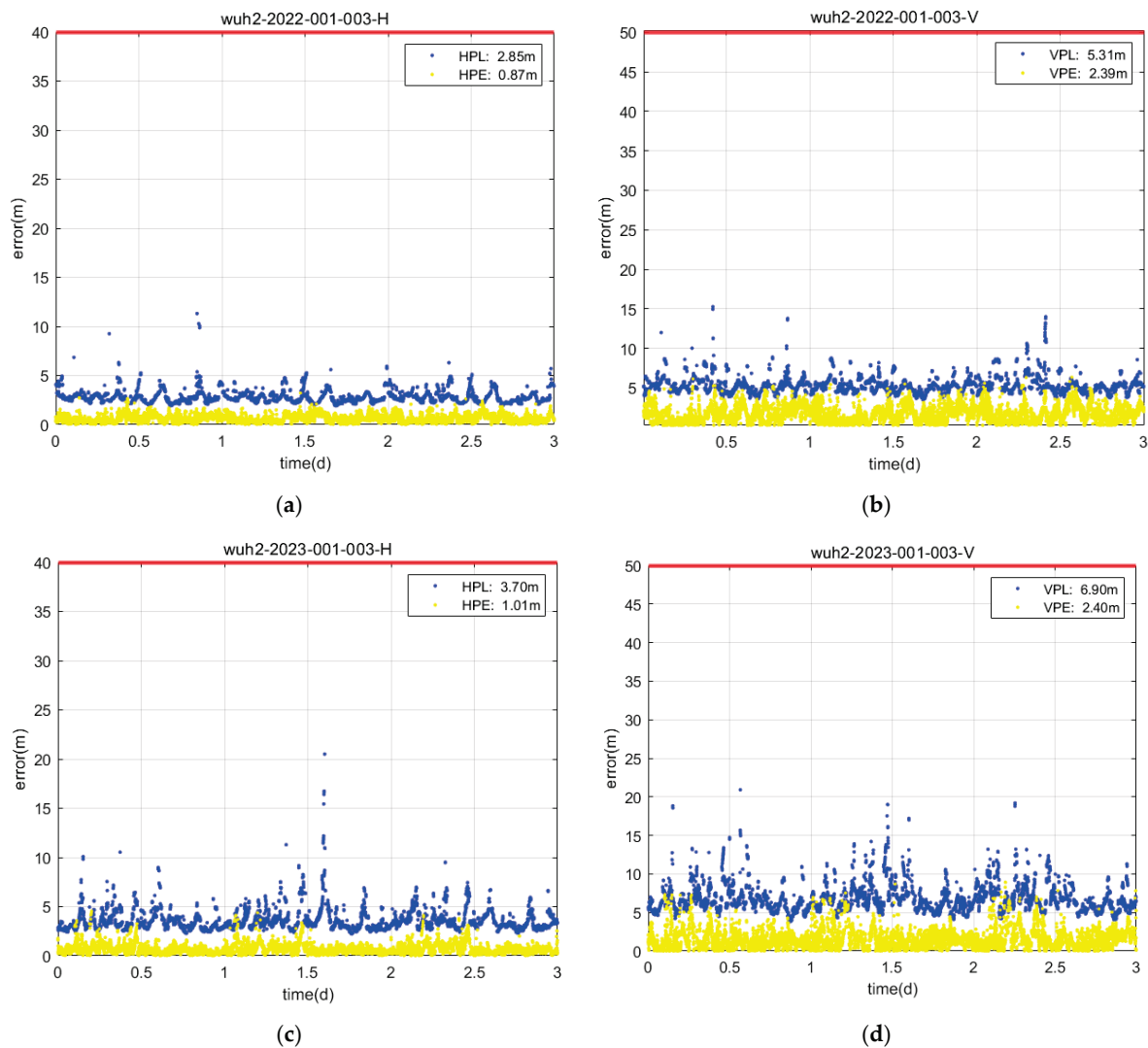


Figure 6. (a) HPL and HPE of WUH2 in 2022; (b) VPL and VPE of WUH2 in 2022; (c) HPL and HPE of WUH2 in 2023; and (d) VPL and VPE of WUH2 in 2023.

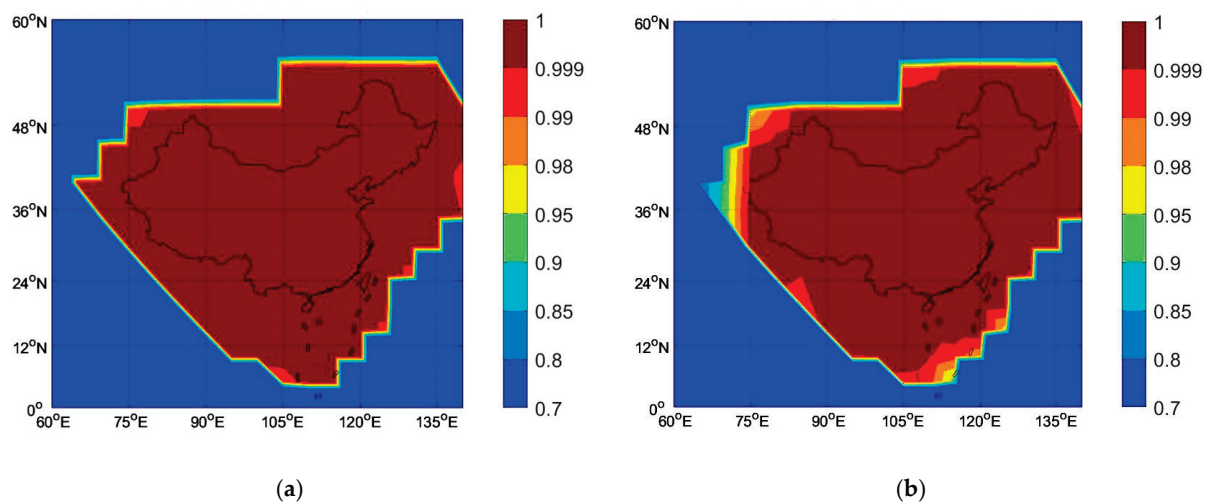


Figure 7. (a) Service availability in the China region in 2022; and (b) service availability in the China region in 2023.

The data in Table 5 compare BDSBAS service metrics between 2022 and 2023 when broadcasting a fixed degradation parameter value. Performance metrics include enhanced positioning, protection levels, service availability, and integrity risk, using select International GNSS Service observation stations as examples. The table presents differences between the 2023 and 2022 results, with positive values indicating increases and negative values indicating decreases. Each statistical period starts on January 1st, May 1st, and September 1st, covering 15 days.

Table 5. Service performance change table for fixed value degradation parameters in 2023 (unit: meters).

STATION	DATE	HPE	VPE	HPL	VPL	Availability	PMI	PHMI
JFNG	1-1	0.08	−0.23	0.54	1.04	0.00%	0.28%	0.000%
	5-1	−0.10	−0.11	0.06	0.19	0.00%	0.57%	0.000%
	9-1	−0.23	−0.66	−0.02	0.12	0.00%	0.17%	0.000%
URUM	1-1	0.03	−0.15	0.39	0.99	0.00%	0.20%	0.000%
	5-1	−0.04	−0.07	−0.48	−0.24	0.00%	0.12%	0.000%
	9-1	−0.12	−0.36	−0.59	−0.28	0.00%	0.25%	0.000%
WUH2	1-1	0.14	0.01	0.57	1.11	0.00%	0.20%	0.000%
	5-1	−0.03	0.10	0.04	0.20	0.00%	0.52%	0.000%
	9-1	−0.16	−0.38	−0.03	0.11	0.00%	0.39%	0.000%
HKSL	1-1	0.20	0.47	0.66	1.38	−0.01%	0.87%	0.000%
	5-1	0.14	0.34	0.03	0.24	0.00%	1.57%	0.000%
	9-1	0.04	0.18	−0.11	0.06	0.00%	0.64%	0.000%
HKWS	1-1	0.20	0.44	0.65	1.37	−0.01%	0.91%	0.000%
	5-1	0.14	0.35	0.02	0.23	0.00%	1.57%	0.000%
	9-1	0.04	0.19	−0.10	0.07	0.00%	0.98%	0.000%

Table 5 shows the decline in service availability located in edge-service areas in 2023 compared to 2022. Additionally, some misleading information (MI) events are present and the probability of hazardous misleading information (HMI) events remains consistently zero in 2023, particularly in the vertical direction, due to ionospheric influences.

2.2. New Calculation Method of Ionospheric Degradation Parameters

The current method for BDSBAS broadcasting ionospheric degradation parameters is as follows: Set C_{iono_step} as a fixed value of 0.952; set I_{iono} as a fixed value of 240; set C_{iono_ramp} as a fixed value of 0; and set RSS_{iono} as 0. Due to the current ionospheric activity, the predefined threshold for ionospheric variations in the degradation parameter fails to fully encompass actual ionospheric delay changes. This paper proposes a new algorithm for determining the broadcast value of the ionospheric degradation parameter.

Real-time calculation of ionospheric grid point vertical delay values is possible through observations from evenly distributed monitoring stations utilizing a dual-frequency approach to correct ionospheric errors. Consequently, this method enables fitting vertical delay parameters at grid points, relying on ionospheric delay measurements at piercing points.

Initially, dual-frequency analysis is used to extract high-precision ionospheric delay values at piercing points, as follows:

$$IGPdelay = \frac{f_2^2}{f_1^2 - f_2^2} (P_2 - P_1) \quad (4)$$

In Equation (4), f denotes frequency and P represents corrected pseudo-range observations.

Ionospheric delays are determined for piercing points within four quadrants around grid points, based on their distances from the grid points. If a minimum of three piercing points exist within a 5° radius ($\varphi \pm 5^\circ, \lambda \pm 5^\circ$) of a grid point (φ, λ), an inverse-distance-weighted fitting for zenith ionospheric delay is performed, considering the spherical geometry, as follows:

$$IGPdelay_j = \frac{\sum_{i=1}^n \frac{I_j}{I_i} \cdot IPPdelay_i \cdot W_{ij}}{\sum_{i=1}^n W_{ij}} \quad (5)$$

In Equation (5), $IGPdelay_j$ is the vertical delay of the ionosphere grid point j , $IPPdelay_i$ is the vertical delay of the piercing point i . I_j and I_i are the ionospheric delay values for the grid point j and ionospheric piercing point i . W_{ij} and W_{kj} are the weights assigned to piercing points i and k in relation to grid point j .

W_{ij} is defined as the reciprocal of the distance from the piercing point i to the grid point j , as follows:

$$W_{ij} = \frac{1}{R \cdot \arccos(\sin(\varphi_i) \sin(\varphi_j)) + \cos(\varphi_i) \cos(\varphi_j) \cos(\lambda_i - \lambda_j)} \quad (6)$$

In Equation (6), R denotes the Earth's radius, (φ_i, λ_i) refers to the latitude and longitude of the piercing point, and (φ_j, λ_j) indicates the latitude and longitude of the grid point.

According to the change in ionospheric grid delay between the adjacent epoch $\Delta IGPdelay$ by Equation (2), the new degradation parameters are calculated according to Equation (7), as follows:

$$\begin{cases} C_{iono_step} = \Delta IGPdelay_{MAX} \\ C_{iono_ramp} = 0 \end{cases} \quad \text{if } \Delta IGPdelay_{MAX} \leq (C_{iono_step})_{LIMIT} \quad (7)$$

$$\begin{cases} C_{iono_step} = (C_{iono_step})_{LIMIT} \\ C_{iono_ramp} = (\Delta IGPdelay_{MAX} - C_{iono_step}) / I_{iono} \end{cases} \quad \text{if } \Delta IGPdelay_{MAX} > (C_{iono_step})_{LIMIT}$$

$$C_{iono_ramp} = (C_{iono_ramp})_{LIMIT} \quad \text{when } C_{iono_ramp} > (C_{iono_ramp})_{LIMIT}$$

In Equation (7), the $\Delta IGPdelay_{MAX}$ is the maximum ionospheric grid delay increments at all valid ionospheric grid points determined for each epoch. The $(C_{iono_step})_{LIMIT}$ and $(C_{iono_ramp})_{LIMIT}$ are specified by the RTCA [27], as follows:

$$\begin{aligned} \Delta IGPdelay_{MAX} &= \text{MAX}(|\Delta IGPdelay_{1,2,\dots,j}|) \\ (C_{iono_step})_{LIMIT} &= 1.023 \\ (C_{iono_ramp})_{LIMIT} &= 0.005155 \end{aligned} \quad (8)$$

The C_{iono_step} is determined by the maximum ionospheric grid delay increment $\Delta IGPdelay_{MAX}$ of the delay values from all real-time valid ionospheric grid points. If the $\Delta IGPdelay_{MAX}$ is less than the maximum $(C_{iono_step})_{LIMIT}$ specified by the RTCA protocol, then $\Delta IGPdelay_{MAX}$ is broadcast and the C_{iono_ramp} is set to 0. If ΔI_{max} exceeds the maximum $(C_{iono_step})_{LIMIT}$ specified by the RTCA protocol, the $(C_{iono_step})_{LIMIT}$ is broadcast directly, and C_{iono_ramp} is broadcast within the maximum range $(C_{iono_ramp})_{LIMIT}$ specified by the RTCA protocol to envelop $\Delta IGPdelay_{MAX}$. Additionally, I_{iono} is set to the actual update cycle of the ionospheric grid, which is 240 s, and the ionospheric square root and factor parameter RSS_{iono} is set to 1. The degradation parameters are synchronized with the ionospheric delay update time, which is also 240 s.

In recent years, ionospheric activity has led to frequent large variations in the ionospheric grid delay between consecutive epochs. Directly setting the maximum ionospheric delay change to meet the envelope requirement is therefore unreasonable. In

this study, large ionospheric grid delay jumps were excluded to ensure service integrity. However, frequent meter-level ionospheric delay jumps may reduce availability while maintaining integrity. Despite this, integrity remains ensured with the application of degradation parameters.

3. Results

The comparative analysis of the features between the newly calculated parameter and the existing parameter in the first week of 2024, as shown in Figure 8, is derived from Equation (7). The green line represents the current fixed parameter, while the red dot denotes the new parameter computed in this study. The horizontal axis is in days and the vertical axis is in meters. Since the values of the majority of the new C_{iono_ramp} are zero for the most part, the current C_{iono_ramp} is almost entirely covered in the figure at present. This approach updates degradation parameters in real time rather than using fixed values, allowing for greater flexibility to accommodate significant and occasional ionospheric changes to reduce the probability of exceeding the threshold.

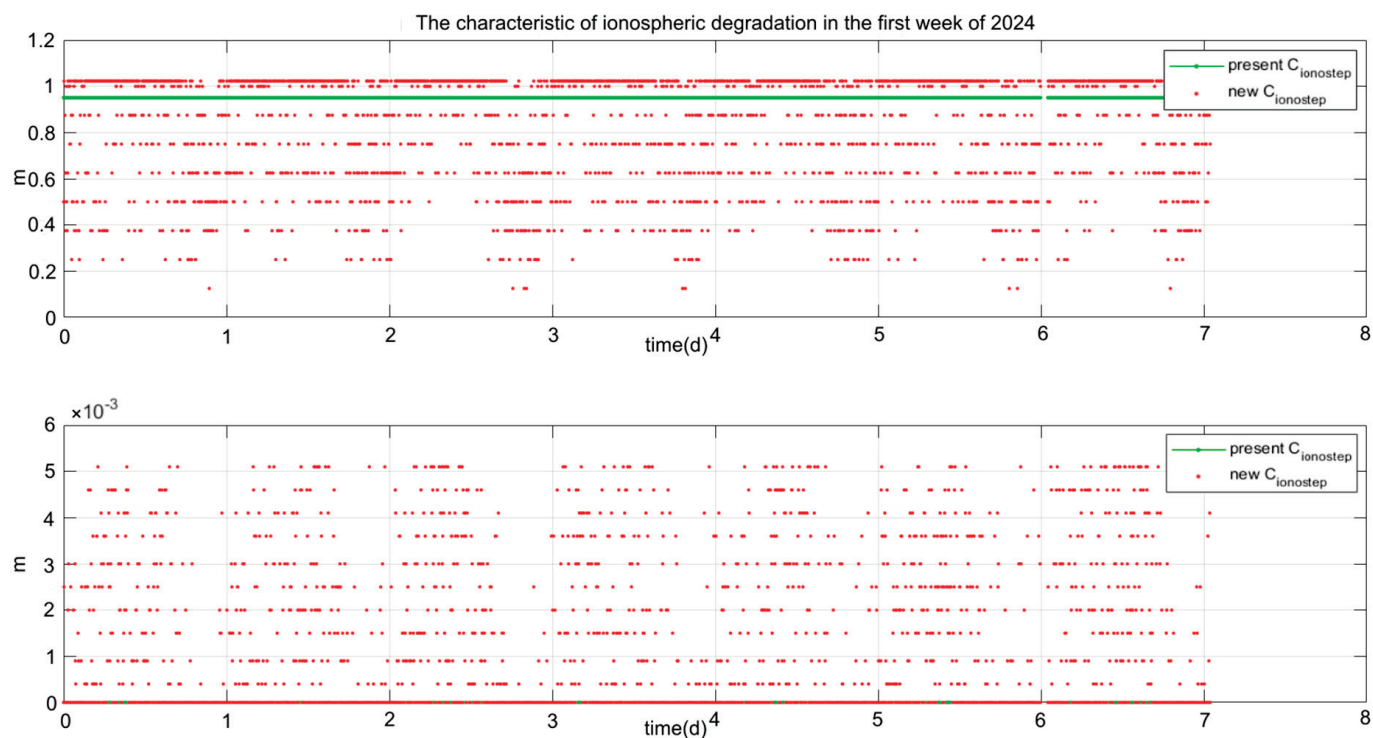


Figure 8. The feature of exceeding threshold probability under different ionospheric degradation parameters in 2024-001.

Real-time calculation of ionospheric degradation parameters using the proposed method significantly improves the envelope rate. As shown in Figure 9 below, the probability of exceeding the threshold at grid points is illustrated for the first day of 2023 and 2024. The blue dots represent ionospheric delay changes. The green line represents the current fixed envelope threshold; the horizontal axis is in hours and the vertical axis is in meters, while the red line illustrates the proposed real-time varying envelope threshold derived from Equation (1), aiming for more effective envelopment. This approach adapts flexibly to ionospheric delay variations.

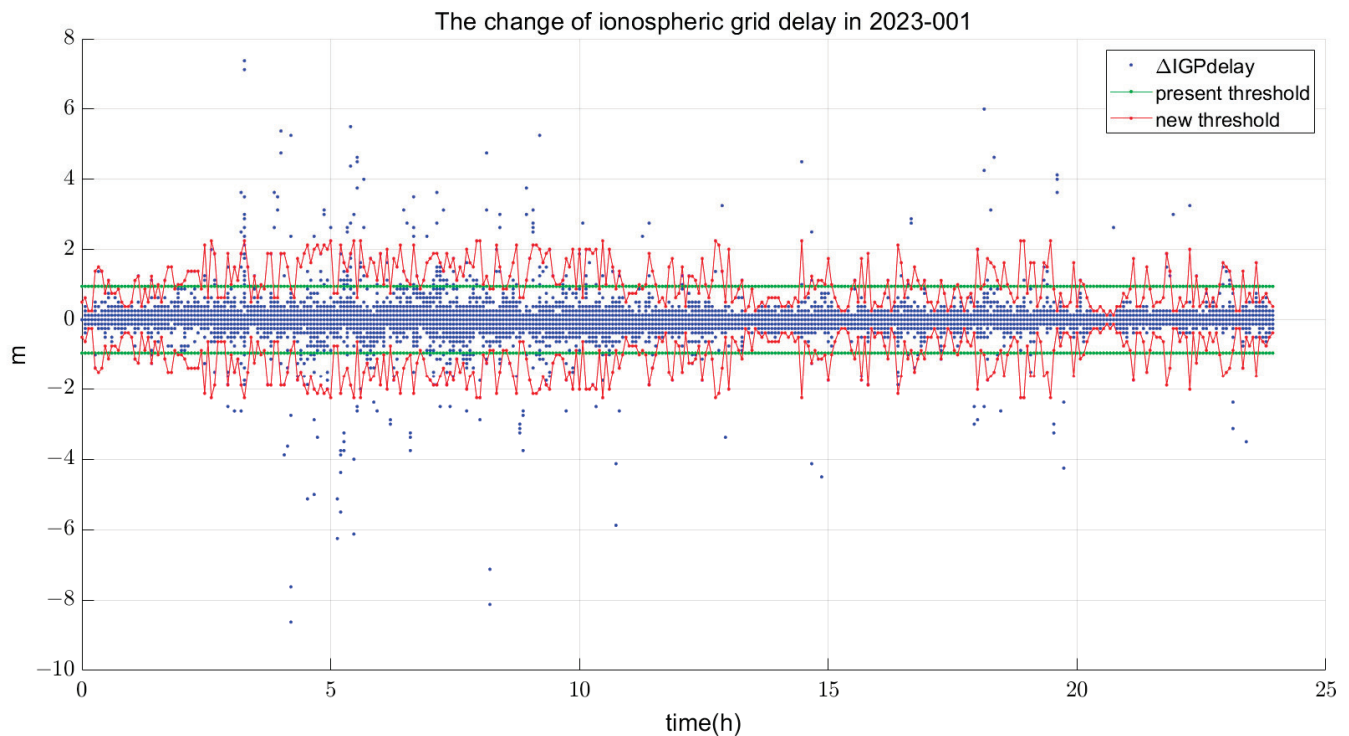


Figure 9. The rate of exceeding threshold probability under different ionospheric degradation parameters in 2023-001.

Applying the new method to the real-time calculation of ionospheric degradation parameters for BDSBAS and MSAS significantly reduces the rate of ionospheric grid delay changes exceeding the threshold. This reduction is especially noticeable in the more active ionospheric years of 2023 and 2024. Table 6 provides the more specific annual probability of exceeding the threshold for the ionospheric grid during the highly active ionospheric period from 2022 to 2024, the BDSBAS probability of exceeding the threshold dropped from 1.46% to 0.58%, while the MSAS probability decreased from 2.37% to 0.4%.

Table 6. The probability of exceeding the threshold of grid ionospheric delay variations under different ionospheric degradation parameters.

	Degradation	2021	2022	2023	2024
BDSBAS	present	0.12%	0.78%	1.64%	1.96%
	new	0.05%	0.33%	0.67%	0.74%
MSAS	present	0.54%	1.44%	2.42%	3.24%
	new	0.05%	0.2%	0.41%	0.59%

The changes in the probability of exceeding the threshold for ionospheric grid delay calculated by Equation (3) from 2021 to 2024, for both BDSBAS and MSAS, are shown in Figure 10, after implementing the new parameters. The horizontal axis represents time in days, while the vertical axis shows the probability of exceeding the threshold. The blue line corresponds to the probability of exceeding the threshold with the new degradation parameter, and the red line represents the probability of exceeding the threshold with the current degradation parameter.

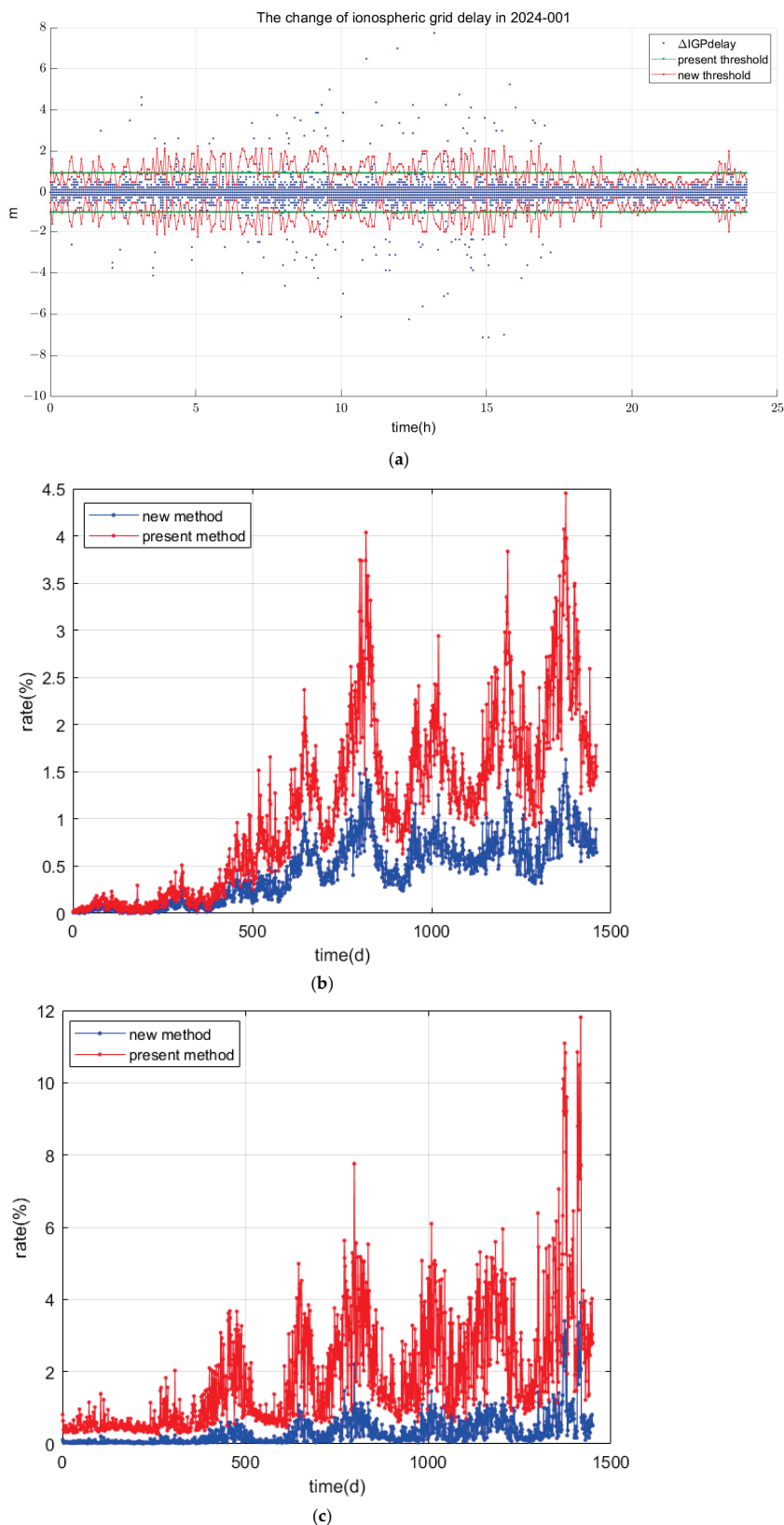


Figure 10. (a) The rate of exceeding the threshold probability under different ionospheric degradation parameters in 2024-001; (b) The rate of grid ionospheric delay variations exceeding the threshold under different ionospheric degradation parameters for BDSBAS in 2021–2024; (c) The rate of grid ionospheric delay variations exceeding the threshold under different ionospheric degradation parameters for MSAS in 2021–2024.

The probability of grid ionospheric delay variations exceeding the threshold decreases overall when the new parameter is used. A higher exceeding threshold rate corresponds to a larger reduction amplitude, with a more significant decrease in the rate of exceeding the threshold for MSAS after the new degradation parameter is applied. Specifically, the maximum rate for exceeding the threshold for MSAS drops from 11.8% to 3.9%, indicating that the proposed new degradation parameter strongly inhibits the growth of the rate for exceeding the threshold during ionospheric activity, effectively buffering the impact of ionospheric anomalies.

4. Discussion

To assess the impact of service availability after updating the efficiency reduction parameter algorithm, this paper employs the BDSBAS single-frequency user performance positioning algorithm and protection level calculation method to evaluate BDSBAS service performance across various dates in 2023. The new ionospheric degradation parameter proposed in this study was applied. The location performance, protection levels, probability of misleading information events, and probability of hazardous misleading information events for several IGS stations were calculated and compared with the results obtained using a fixed degradation parameter.

The data in Table 7 reflect the performance indicators of five IGS stations, including JFNG, URUM, WUH2, HKSL and HKWS, as well as five regional monitoring stations located in Beijing, Chengdu, Sanya, Kashi and Shantou in the same order as the table, during the selected time periods. Table 7 summarizes the positioning accuracy of ten stations across three time periods, highlighting changes in protection levels, availability, and integrity following the implementation of new degradation parameters. Positive values indicate improvements, such as an increase in protection levels, whereas negative values denote a decline, including misleading integrity information. Each statistical period spans 15 days. As shown in the table, during the high-activity ionospheric year of 2023, the updated degradation parameter algorithm enhanced protection levels. However, this improvement led to reduced service availability for stations located in edge-service areas. Despite this, misleading information significantly decreased, and integrity was markedly improved.

The vertical integrity Stanford diagrams for the URUM station on DOY 135 in 2023 using different ionospheric degradation parameters, as shown in Figure 11a–d, display the vertical integrity Stanford diagrams for the WUH2 station on DOY 225 as a more concrete example. The horizontal axis represents the VPL, while the vertical axis denotes the VPE. The Alert Limit is depicted as a horizontal line at 50 m. Each dot represents a specific (VPE, VPL) pair, with its color, based on a logarithmic scale, indicating the frequency of occurrences. The diagrams clearly illustrate a reduction in misleading information. However, during periods of highly active ionospheric conditions, the increased protection level results in fewer available epochs.

The new degradation parameters improve integrity more significantly in regions with higher ionospheric activity, similar to their effect on reducing the ionospheric grid delay exceedance rate. The reduction in misleading integrity information is particularly effective in southern and edge regions of China. In contrast, the probability of misleading integrity events in northern regions is inherently low or even zero, with availability generally remaining stable at 100%. Furthermore, these parameters do not reduce the availability of stations in northern regions. Broadcasting the new degradation parameters decreases the exceedance rate and enhances integrity during periods of high ionospheric activity, while having no adverse effects during low-activity periods. In ionospheric active regions, such as MSAS and BDSBAS, broadcasting these parameters effectively reduces the exceedance

rate. Likewise, in ionospheric inactive regions, SBAS systems, such as EGNOS and WAAS, can also feasibly implement this method for broadcasting degradation parameters.

Table 7. Service performance improvement table under new ionospheric degradation parameters (unit: meters).

STATION	DATE	HPE	VPE	HPL	VPL	Availability	PMI	PHMI
JFNG	1-1	0.98	2.11	0.26	0.56	0	−0.19%	0.000%
	5-1	1.14	3.07	0.29	0.58	0	−1.21%	0.000%
	9-1	1.02	2.51	0.29	0.62	0	−0.43%	0.000%
URUM	1-1	0.68	1.36	0.28	0.64	0	−0.01%	0.000%
	5-1	0.85	1.86	0.28	0.60	0	−0.07%	0.000%
	9-1	0.77	1.57	0.29	0.68	0	−0.08%	0.000%
WUH2	1-1	1.01	2.40	0.26	0.57	0	−0.55%	0.000%
	5-1	1.21	3.29	0.29	0.58	0	−1.66%	0.000%
	9-1	1.08	2.81	0.30	0.64	0	−0.86%	0.000%
HKSL	1-1	0.97	2.98	0.25	0.57	−0.01%	−0.68%	0.000%
	5-1	1.29	4.22	0.26	0.59	0	−2.42%	0.000%
	9-1	1.19	4.06	0.27	0.60	0	−2.18%	0.000%
HKWS	1-1	0.97	2.94	0.25	0.57	−0.01%	−0.74%	0.000%
	5-1	1.29	4.17	0.26	0.59	0	−2.34%	0.000%
	9-1	1.19	4.02	0.27	0.60	0	−2.13%	0.000%
BJ01	1-1	0.55	0.98	0.25	0.57	0	0.00%	0.000%
	5-1	0.74	1.36	0.32	0.77	0	0.00%	0.000%
	9-1	0.53	1.36	0.27	0.59	0	0.00%	0.000%
CD01	1-1	0.83	1.90	0.24	0.52	0	−0.13%	0.000%
	5-1	1.32	2.17	0.32	0.74	0	−0.50%	0.000%
	9-1	0.90	2.07	0.25	0.57	0	−0.16%	0.000%
SY01	1-1	1.23	2.23	0.27	0.61	−0.01%	−0.27%	0.000%
	5-1	1.71	2.99	0.36	0.90	−0.01%	−1.29%	0.000%
	9-1	1.69	3.03	0.24	0.60	0	−1.05%	0.000%
KS01	1-1	0.69	1.13	0.10	0.26	−0.01%	0.00%	0.000%
	5-1	0.80	1.12	0.42	1.04	0	0.00%	0.000%
	9-1	0.71	0.94	0.30	0.71	0	0.00%	0.000%
ST01	1-1	2.78	3.40	0.34	0.80	−0.03%	−1.90%	0.000%
	5-1	2.04	2.89	0.30	0.78	−0.01%	−2.21%	0.000%
	9-1	1.96	2.53	0.23	0.53	0	−1.91%	0.000%

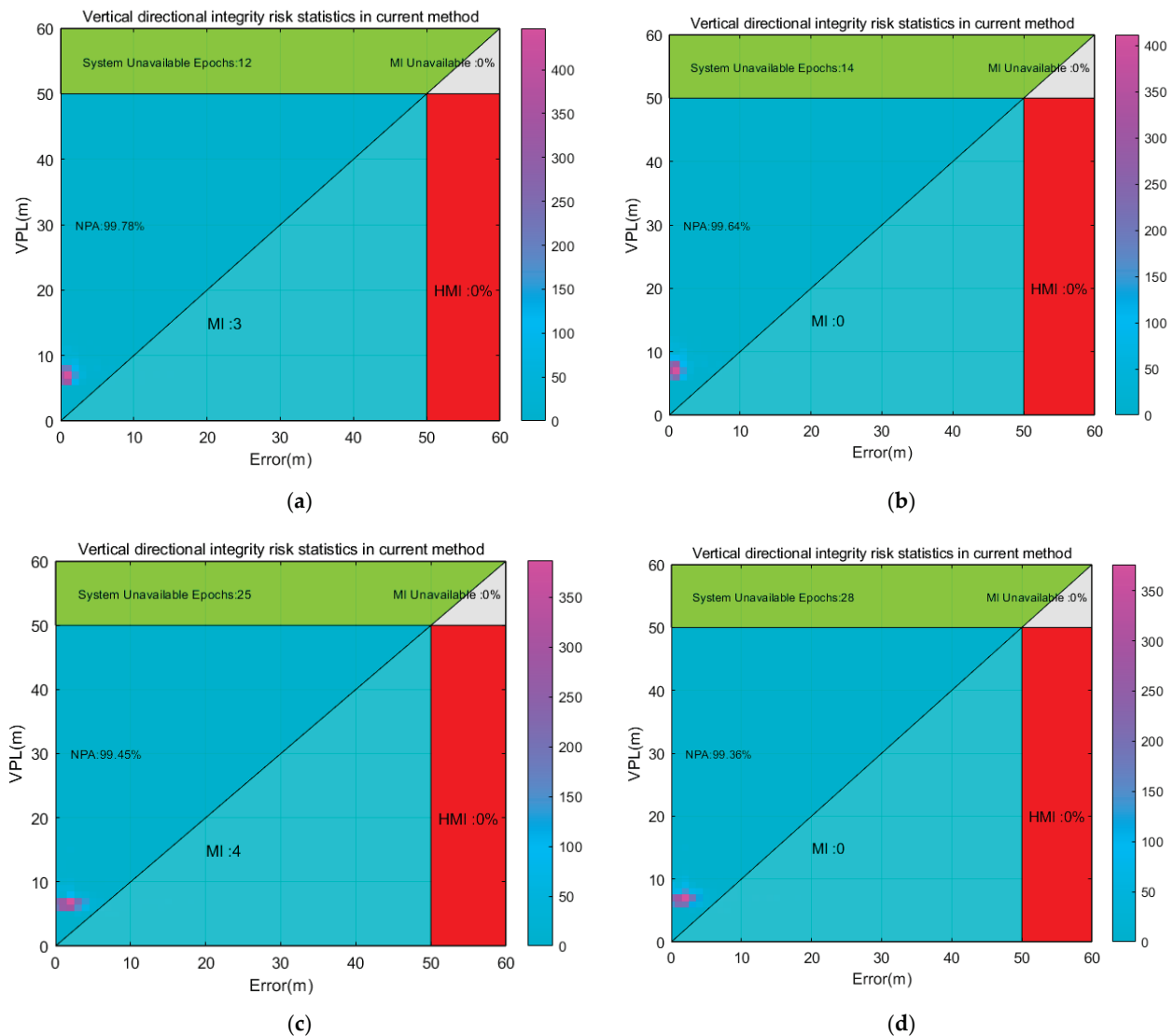


Figure 11. (a) Statistical diagram of vertical integrity of URUM on present ionospheric degradation parameters on DOY 135; (b) statistical diagram of vertical integrity of URUM on new ionospheric degradation parameters on DOY 135; (c) statistical diagram of vertical integrity of WUH2 on present ionospheric degradation parameters on DOY 225; and (d) statistical diagram of vertical integrity of WUH2 on new ionospheric degradation parameters on DOY 225.

5. Conclusions

This study evaluates the impact of ionospheric delay variations in grid-based ionospheric models and assesses the effect of ionospheric degradation parameters on service availability and integrity by statistically analyzing the packet loss rate of BDSBAS from 2021 to 2024. The results indicate that in 2024, when ionospheric activity was more intense, the rate of grid ionospheric delay variation exceeding the threshold increased by 1.85%, leading to decreased service availability and the emergence of misleading information compared to 2021.

To address the ineffective envelopment of grid-based ionospheric delay variations and its impact on BDSBAS service integrity caused by fixed broadcast values of ionospheric degradation parameters during periods of high ionospheric activity, this study proposes a new method for ionospheric degradation parameters. The proposed method was verified and analyzed using IGS data. The results show that after implementing the new ionospheric degradation parameters, the probability of exceeding the threshold of

grid-based ionospheric delay variations during peak ionospheric activity decreased by 1.10%. Misleading information events were decreased by 1.04%, ensuring service integrity.

In summary, the new BDSBAS degradation parameters proposed in this study effectively reduce the exceeding threshold probability of grid-based ionospheric delay variations during high ionospheric activity periods and improve service integrity for users.

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Conflicts of Interest: The authors declare no conflicts of interest.

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Article

Global Navigation Satellite System Meta-Signals with an Arbitrary Number of Components

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Abstract: Global Navigation Satellite System (GNSS) meta-signals are obtained when components from different frequencies are jointly processed as a single entity. While most research work has focused on GNSS meta-signals made of two side-band components, meta-signal theory has been recently extended to the case where the number of components is a power of two. This condition was dictated by the use of multicomplex numbers for the representation of GNSS meta-signals. Multicomplex numbers are multi-dimensional extensions of complex numbers whose dimension is a power of two. In this paper, the theory is further extended and a procedure for the construction of GNSS meta-signals with an arbitrary number of components is provided. Also in this case, multicomplex numbers are used to effectively represent a GNSS meta-signal. From this representation, multi-dimensional Cross Ambiguity Functions (CAFs) are obtained and used to derive acquisition and tracking algorithms suitable for the joint processing of components from different frequencies. The specific case with three components is analysed. Theoretical results are supported by experimental findings obtained by jointly processing Galileo E5a, E5b and E6 signals collected using three synchronized Software-Defined Radio (SDR) HackRF One front-ends. Experimental results confirm the validity of the developed theory.

Keywords: acquisition; GNSS; meta-signal; multi-frequency; tracking

1. Introduction

Modern Global Navigation Satellite System (GNSS) signal modulations, such as the Binary Offset Carrier (BOC), adopt a subcarrier to split the signal energy away from the signal centre frequency [1]. For this reason, the spectra of BOC modulations are characterized by two main lobes symmetrically placed around the signal Radio Frequency (RF). BOC acquisition and tracking algorithms try to recover all the signal energy by combining the contributions from the BOC side-lobes [2]. BOC modulations have been progressively generalized by considering side-band components carrying different information [3] and with different spectral shapes [4–6]. The concept of the GNSS meta-signal was originally introduced to describe an advanced modulation obtained by combining two signals broadcast on two different frequencies [7]. A meta-signal made of two components can be seen as a generalized BOC modulation where the side-band signals are split on two different RFs using a complex subcarrier [8]. Jointly processing signals from different frequencies allows one to recover and coherently combine the energy from the different components. Moreover, a GNSS meta-signal is characterized by a wide Gabor bandwidth, which can lead to high-accuracy pseudorange measurements [7,9]. Several algorithms have explored approaches for coherently combining signals from different side-bands and GNSS meta-signal approaches have been successfully applied to the Galileo Alternative Binary Offset Carrier (AltBOC) [9], the Beidou Asymmetric Constant-Envelope Binary

Offset Carrier (ACE-BOC) and B1 modulations [4–6]. Combinations involving the Galileo E6 signal have also been considered [9].

A convenient representation of a GNSS meta-signal with two components can be obtained using bicomplex numbers [8], which are two-dimensional extensions of complex numbers [10]. Using bicomplex numbers, it is possible to jointly represent the side-band components and explicitly express the subcarrier as a complex exponential. Bicomplex representation also allows one to derive optimal, in the Maximum Likelihood (ML) sense, acquisition and tracking algorithms [8], effectively combining signal energy from different frequencies.

Meta-signal theory has been recently generalized to the case where the number of components is a power of two [11]. The generalization is based on multicomplex numbers [12,13], which are multi-dimensional extensions of complex and bicomplex numbers. In this case, a set of 2^{M-1} signals, with M as an integer, is jointly represented as a single entity modulated by $2^{M-1} - 1$ subcarriers, which split the signal components on 2^{M-1} centre frequencies. While GNSS meta-signals with 2^{M-1} components can be effectively considered as a form of multi-channel modulation [14], they significantly differ from multi-carrier schemes such as the Orthogonal Frequency Division Multiplexing (OFDM) [15,16]. GNSS meta-signals combine a limited number of components irregularly spaced in frequency and with potentially very different spectral characteristics. This is not the case for OFDM modulations, which combine hundreds to thousands of orthogonal components overlapping in frequency.

At the receiver side, the recovery of the meta-signal subcarriers allows one to align in phase the different signal components and effectively combine their energies. The constraint of having a number of components equal to a power of two was dictated by the structure of multicomplex numbers whose dimension is indeed a power of two.

In this paper, the GNSS meta-signal theory is further extended and a procedure for the construction of GNSS meta-signals with an arbitrary number of components is provided. Also in this case, multicomplex numbers are used for the meta-signal construction and representation. The procedure allows one to build a GNSS meta-signal with K components and identify $K - 1$ subcarriers that split the different signals on K RFs.

The obtained representations are then used to derive a K -dimensional Cross Ambiguity Function (CAF), which, in turn, is used as basis for the design of multi-dimensional acquisition and tracking algorithms. These algorithms, effectively combining components from several frequencies, are detailed in Section 2.4. Similar to the case with a number of components equal to a power of two, acquisition and tracking algorithms are restated in terms of standard elements from single-component processing. The obtained acquisition approach is a form of weighted linear detector [17], whereas a multi-loop tracking architecture is found. A single Delay Lock Loop (DLL) is used to track a delay assumed to be common to all components, a Phase Lock Loop (PLL) is adopted to track the common meta-signal carrier phase and $K - 1$ Subcarrier Phase Lock Loops (SPLLs) are used to recover the subcarrier parameters. In this way, a new class of Multi-Tracking Loop (MTL) approaches [18,19] is found. MTLs from the literature exploit the relationships between the Doppler frequencies of the different signal carriers [18–20]. A Kalman filter is usually adopted to track common carrier variations and estimate residual differences. Alternatively, inter-frequency carrier aiding is adopted [18]. A different approach is considered here: instead of tracking the original carrier components, Doppler shifts and carrier phases are projected at first in a transformed domain where they are expressed in terms of a common meta-signal carrier and subcarriers. In this domain, all the signal components contribute to the estimation of common Doppler and phase parameters. Also in this case, the relation-

ships between the different Doppler frequencies can be exploited [21]. The analysis of this option is, however, left for future work.

Theoretical findings have been supported by experiments involving the Galileo E6, E5b and E5a components, giving emphasis to the case of a GNSS meta-signal with three components. Signals have been processed with a custom Python (version 3.9) Software-Defined Radio (SDR) receiver implementing the developed acquisition and tracking algorithms. Experimental results confirm the validity of the developed theory.

The remainder of this paper is organized as follows: Section 2 describes the methods developed in the paper and the materials used for their experimental validation. These include a brief summary on multicomplex numbers, the extension of meta-signal theory to an arbitrary number of components and the derivation of multi-component acquisition and tracking algorithms. Section 3 describes sample experimental results and Section 4 provides a discussion on the main findings and on future research directions. Finally, Section 5 concludes the paper.

2. Materials and Methods

2.1. Multicomplex Numbers and Multicomplex Signal Representation

Multicomplex numbers are multi-dimensional extensions of complex numbers [12,22] characterized by commutative products. A set of multicomplex numbers is usually denoted as $\mathbb{C}_M$ [12], where M is an integer defining the dimension of the space. $\mathbb{C}_1$ is the set of complex numbers whereas $\mathbb{C}_2$ is the bicomplex number space [10]. A multicomplex number is characterized by 2^M real components or equivalently by 2^{M-1} complex components. It features one real unit, 2^{M-1} imaginary units squaring to -1 and $2^{M-1} - 1$ hyperbolic units [23], which square to 1. A multicomplex number can be written as [24]

$$z_M = \mathbf{k}_M^T \mathbf{a}_M + \mathbf{j}_M^T \mathbf{b}_M = \mathbf{k}_M^T \mathbf{z}_M, \quad (1)$$

where $\mathbf{a}_M$ and $\mathbf{b}_M$ are two real vectors of size 2^{M-1} . $\mathbf{k}_M$ and $\mathbf{j}_M$ are the vectors with the hyperbolic and imaginary units, respectively:

$$\begin{aligned} \mathbf{k}_M &= [1, k_1, \dots, k_{2^{M-1}-1}] \\ \mathbf{j}_M &= [j_0, j_1, \dots, j_{2^{M-1}-1}]. \end{aligned} \quad (2)$$

$\mathbf{z}_M$ is a complex vector obtained as

$$\mathbf{z}_M = \mathbf{a}_M + j_0 \mathbf{D}_M \mathbf{b}_M \quad (3)$$

where $\mathbf{D}_M$ is a square diagonal matrix of size $2^{M-1} \times 2^{M-1}$. The diagonal elements of $\mathbf{D}_M$ are equal to either 1 or to -1 . The construction of $\mathbf{D}_M$ can be found in [24]. j_0 is the first imaginary unit common to all multicomplex number spaces.

Since multicomplex numbers feature several imaginary and hyperbolic units, several conjugations can be defined. Only the star-conjugation is considered in the following. It is obtained by negating the sign of the imaginary components. The star-conjugation of (1) is given by

$$z_M^* = \mathbf{k}_M^T \mathbf{a}_M - \mathbf{j}_M^T \mathbf{b}_M = \mathbf{k}_M^T \mathbf{z}_M^* \quad (4)$$

where $\mathbf{z}_M^*$ is the vector whose elements are the complex conjugates of the elements of $\mathbf{z}_M$. Using the star-conjugation, it is possible to define the hyperbolic modulus of a multicomplex number [10,24]:

$$|z_M|_k = \sqrt{z_M \cdot z_M^*} = \mathbf{k}_M^T \mathbf{r}_M, \quad (5)$$

where $\mathbf{r}_M$ is a vector with 2^{M-1} real components.

A multicomplex number can always be represented in terms of orthogonal components

$$z_M = \mathbf{e}_M^T \mathbf{z}_M^O = \sum_{i=0}^{2^{M-1}-1} z_{O,i} e_{M,i} \quad (6)$$

where $\mathbf{e}_M$ is the vector with idempotent orthogonal units, $e_{M,i}$. $z_{O,i}$ are complex coefficients arranged in the complex vector, $\mathbf{z}_M^O$. The idempotent units are characterized by the following property:

$$e_{M,i} e_{M,j} = \begin{cases} e_{M,i} & \text{for } i = j \\ 0 & \text{for } i \neq j \end{cases}. \quad (7)$$

They can be constructed according to the procedure outlined in [12,24]. Vectors $\mathbf{e}_M$ and $\mathbf{k}_M$ are related through the following transformation [24]:

$$\mathbf{e}_M = \frac{1}{2^{M-1}} (\mathbf{D}_M \mathbf{H}_{2^{M-1}})^T \mathbf{k}_M \quad (8)$$

where $\mathbf{H}_{2^{M-1}}$ is a Hadamard matrix of size 2^{M-1} [25] obtained through the recursive procedure outlined in [24]. Equation (8) provides a simple way to compute $\mathbf{z}_M^O$ from $\mathbf{z}_M$:

$$\mathbf{z}_M^O = (\mathbf{D}_M \mathbf{H}_{2^{M-1}})^T \mathbf{z}_M. \quad (9)$$

Orthogonal decomposition (6) allows one to perform component-wise operations when manipulating multicomplex numbers. For instance, the orthogonal components of the product of two multicomplex numbers are given by the products of the orthogonal components of the factors. In a similar way, the orthogonal components of hyperbolic modulus (5) are obtained as the (complex) moduli of the orthogonal components of z_M .

Finally, multicomplex numbers can be expressed in polar form as [24]:

$$z_M = \mathbf{k}_M^T \mathbf{r}_M \exp\{\mathbf{j}_M^T \Phi_M\} \quad (10)$$

where $\mathbf{r}_M$ and Φ_M are real vectors of size 2^{M-1} containing modulus and phase information, respectively. $\mathbf{r}_M$ has been introduced in (5) and Φ_M can be computed from the orthogonal components as

$$\Phi_M = \frac{1}{2^{M-1}} \mathbf{H}_{2^{M-1}} \Phi_M^O, \quad (11)$$

where

$$\Phi_{2^{M-1}}^O = \begin{bmatrix} \angle z_{O,0} \\ \angle z_{O,1} \\ \vdots \\ \angle z_{O,2^{M-1}-1} \end{bmatrix}. \quad (12)$$

2.2. Multicomplex Signal Representation

Multicomplex numbers can be used to provide a joint representation of 2^{M-1} RF signals [11,24]. In particular, it is possible to show

$$\begin{aligned} z_{2^{M-1},RF}(t) &= \sum_{i=0}^{2^{M-1}-1} x_{i,RF}(t) = \sum_{i=0}^{2^{M-1}-1} \Re\{x_i(t) \exp\{j_0 2\pi f_{X,i} t\}\} \\ &= \Re\{z_{2^{M-1}}(t) \exp\{2\pi \mathbf{j}_M^T \mathbf{f}_{2^{M-1}} t\}\} \end{aligned} \quad (13)$$

where $x_{i,RF}(t)$ is the i th RF signal obtained by modulating the i th baseband component, $x_i(t)$, at frequency $f_{X,i}$. $z_{2^{M-1}}(t)$ is a multicomplex signal obtained as

$$z_{2^{M-1}}(t) = 2^{M-1} \sum_{i=0}^{2^{M-1}-1} x_i(t) e_{M,i}. \quad (14)$$

Equation (14) implies that the orthogonal components of $z_{2^{M-1}}(t)$ are proportional to the baseband signals, $x_i(t)$. In (13), $\mathbf{f}_{2^{M-1}}$ is a real vector obtained from the original RFs as

$$\mathbf{f}_{2^{M-1}} = \frac{1}{2^{M-1}} \mathbf{H}_{2^{M-1}} \begin{bmatrix} f_{X,0} \\ f_{X,1} \\ \vdots \\ f_{X,2^{M-1}-1} \end{bmatrix}. \quad (15)$$

The key point arising from (13) is that a set of 2^{M-1} RF components can be represented as a single baseband multicomplex signal modulated by a set of carrier/subcarriers with frequencies common to all components. The multicomplex exponential in the second line of (13) can be expanded as the product of exponentials, each with a single imaginary unit at exponent. The exponential with respect to the j_0 unit is the carrier, whereas the remaining terms are the subcarriers. The product between the multicomplex signal, $z_{2^{M-1}}(t)$, and the subcarriers defines the GNSS meta-signal with 2^{M-1} components. At the reception side, these carrier/subcarriers are affected by Doppler frequencies and phases introduced by the communication channel. These parameters need to be estimated. In the meta-signal approach, instead of independently tracking 2^{M-1} carriers, the exponential terms in (13) are jointly considered. Since all the signal components in (13) contribute to the estimation of the carrier/subcarrier Doppler frequencies and phases, improved performance is obtained with respect to the individual tracking approach. In order to generalize the theory of GNSS meta-signals, (13) has to be extended to the case where an arbitrary number of components is considered.

2.3. Meta-Signals with an Arbitrary Number of Components

In this section, a simple procedure for the construction of a meta-signal with an arbitrary number of components is described. A representation similar to (14) is provided where the missing baseband components required to obtain a number of elements equal to a power of two are set to zero. The process is described in detail for the case with three components. The procedure is then generalized to the case with an arbitrary number of components.

2.3.1. Meta-Signals with Three Components

A meta-signal with three components can be obtained by considering the space of tricomplex numbers, $\mathbb{C}_3$ [12], which can jointly represents up to four (2^{M-1} , with $M = 3$) complex components. The case with four components was illustrated in [11] for the Galileo signal landscape. The space of tricomplex numbers features one real unit, four imaginary units, j_0, j_1, j_2 and j_3 , and three hyperbolic units k_1, k_2 and k_3 . Moreover, tricomplex numbers are characterized by four orthogonal idempotent units:

$$\begin{aligned} e_{3,0} &= \frac{1-k_2}{2} \frac{1-k_1}{2}, & e_{3,1} &= \frac{1-k_2}{2} \frac{1+k_1}{2} \\ e_{3,2} &= \frac{1+k_2}{2} \frac{1-k_1}{2}, & e_{3,3} &= \frac{1+k_2}{2} \frac{1+k_1}{2}. \end{aligned} \quad (16)$$

When four RF signals are present, (13) becomes:

$$\begin{aligned} z_{4,RF}(t) &= \sum_{i=0}^3 x_{i,RF}(t) = \sum_{i=0}^3 \Re\{x_i(t) \exp\{j_0 2\pi f_{X,i} t\}\} \\ &= \Re\{z_4(t) \exp\{2\pi j_3^T \mathbf{f}_4 t\}\} \end{aligned} \quad (17)$$

where

$$z_4(t) = 4 \sum_{i=0}^3 x_i(t) e_{3,i} \quad (18)$$

is the tricomplex baseband signal obtained by combining the different baseband signals, $x_i(t)$. $\mathbf{j}_3^T = [j_0, j_1, j_2, j_3]$ is the vector with the imaginary units and

$$\mathbf{f}_4 = \begin{bmatrix} f_0 \\ f_1 \\ f_2 \\ f_3 \end{bmatrix} = \frac{1}{4} \mathbf{H}_4 \mathbf{f}_{X,4} = \frac{1}{4} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ -1 & 1 & 1 & -1 \end{bmatrix} \begin{bmatrix} f_{X,0} \\ f_{X,1} \\ f_{X,2} \\ f_{X,3} \end{bmatrix}, \quad (19)$$

where $\mathbf{H}_4$ is the Hadamard matrix [25] of size 4. Product $\mathbf{j}_3^T \mathbf{f}_4$ can be explicitly written as a summation of pure imaginary terms. Moreover, exponential $\exp\{2\pi j_3^T \mathbf{f}_4 t\}$ can be written as the product of a carrier, with nominal frequency f_0 , and three subcarriers.

When three components are present, a representation similar to (18) can be obtained by simply setting one of the orthogonal components of the multicomplex signal to zero. While different choices are possible, $x_3(t) = 0$ is assumed here. By doing so, the last RF, $f_{X,3}$, remains undetermined and can be selected arbitrarily without compromising the validity of (17). This fact can be proved by considering the exponential, $\exp\{2\pi j_3^T \mathbf{f}_4 t\}$. From the properties of the imaginary vector, $\mathbf{j}_M$, it follows that [24]

$$\mathbf{j}_M = j_0 \mathbf{H}_{2^{M-1}} \mathbf{e}_M. \quad (20)$$

Thus,

$$\exp\{2\pi j_3^T \mathbf{f}_4 t\} = \exp\{2\pi j_0 \mathbf{e}_3^T \mathbf{H}_4^T \mathbf{f}_4 t\} = \exp\{2\pi j_0 \mathbf{e}_3^T \mathbf{f}_{X,4} t\}, \quad (21)$$

where condition $\mathbf{f}_{X,4} = \mathbf{H}_4^T \mathbf{f}_4$ directly derives from (19) and the fact that the inverse of a Hadamard matrix is equal to its transpose normalized by the matrix dimension [25]. Since the exponential function admits an absolutely convergent Taylor series expansion, it is possible to show that [13]

$$\exp\left\{\sum_{i=0}^{2^{M-1}-1} a_i e_{M,i}\right\} = \sum_{i=0}^{2^{M-1}-1} \exp\{a_i\} e_{M,i}, \quad (22)$$

where a_i are complex numbers expressed with respect to the j_0 unit. Using (22) and (21) becomes:

$$\exp\{2\pi j_3^T \mathbf{f}_4 t\} = \sum_{i=0}^3 \exp\{j_0 2\pi f_{X,i} t\} e_{3,i}. \quad (23)$$

Expansion (23) shows that the multiplication with the multicomplex exponential in (17) is equivalent to multiplying each orthogonal component of $z_4(t)$ by the corresponding complex carrier. In (23), the only term depending on $f_{X,3}$ multiplies $e_{3,3}$, which is now multiplied by zero in (18). Thus, the product $z_4(t) \exp\{2\pi j_3^T \mathbf{f}_4 t\}$ does not depend on $f_{X,3}$, which can be selected arbitrarily. A convenient solution is to fix $f_{X,3}$ such that the last

component of $\mathbf{f}_4$ (any other component of $\mathbf{f}_4$ would lead to a similar result) is zero. Using this approach, it follows that

$$\mathbf{f}_{X,4} = \mathbf{H}_4^T \mathbf{f}_4 = \begin{bmatrix} 1 & 1 & 1 & -1 \\ 1 & -1 & 1 & 1 \\ 1 & 1 & -1 & 1 \\ 1 & -1 & -1 & -1 \end{bmatrix} \begin{bmatrix} f_0 \\ f_1 \\ f_2 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \\ 1 & -1 & -1 \end{bmatrix} \begin{bmatrix} f_0 \\ f_1 \\ f_2 \end{bmatrix}. \quad (24)$$

Since the actual value of $f_{X,3}$ is not of interest, transformation (24) can be written as

$$\mathbf{f}_{X,3} = \begin{bmatrix} f_{X,0} \\ f_{X,1} \\ f_{X,2} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{bmatrix} \begin{bmatrix} f_0 \\ f_1 \\ f_2 \end{bmatrix} = \mathbf{H}_3^{-1} \mathbf{f}_3, \quad (25)$$

where $\mathbf{H}_3^{-1}$ denotes the matrix obtained by removing the last column and row from $\mathbf{H}_4^T$ and $\mathbf{f}_3$ is the vector with frequencies f_i , for $i = 0, 1$ and 2 . In this way, a meta-signal with three components is obtained. Representation (17) becomes

$$z_{3,RF}(t) = \Re \left\{ z_3(t) \exp \{ 2\pi \mathbf{j}_{3,3}^T \mathbf{f}_3 t \} \right\} \quad (26)$$

where $\mathbf{j}_{3,3}$ is the vector with the first 3 imaginary units of $\mathbb{C}_3$ and

$$z_3(t) = 4 \sum_{i=0}^2 x_i(t) e_{3,i}. \quad (27)$$

The exponential in (26) defines one carrier and two subcarriers. $\mathbf{H}_3^{-1}$ has been obtained by removing the last column and the last row of $\mathbf{H}_4^T$ and is no longer an orthogonal matrix. Its inverse

$$\mathbf{H}_3 = \frac{1}{2} \begin{bmatrix} 0 & 1 & 1 \\ 1 & -1 & 0 \\ 1 & 0 & -1 \end{bmatrix} \quad (28)$$

defines how the meta-signal carrier and subcarrier frequencies are obtained. In this case, the carrier nominal frequency is directly proportional to the narrow-lane combination of the last two signal RFs. The subcarrier frequencies correspond to the wide-lane linear combinations of the other two possible signal pairs.

The result of this construction is illustrated for the Galileo E5a, E5b and E6 signals in Figure 1. The actual allocation of the different components on the leaves of the binary tree displayed in Figure 1 depends on the ordering of the different components and their pairing with the orthogonal elements in (27). In this case, the E6 signal is found in isolation whereas the E5a and E5b components are on the same branch and form the AltBOC modulation.

In order to illustrate the fact that alternative triple-component meta-signals can be obtained using a different signal ordering, a second configuration for the Galileo E5a, E5b and E6 signals is provided in Figure 2. In this case, a carrier frequency equal to the Galileo AltBOC RF is found. Moreover, the nominal frequency of the second subcarrier is increased from 15.345 MHz to 35.805 MHz.

In addition to different ordering, alternative triple-component meta-signals can be built using different construction approaches. For instance, a component could be split and and paired with two different orthogonal units in (18). These options are outside the scope of this work.

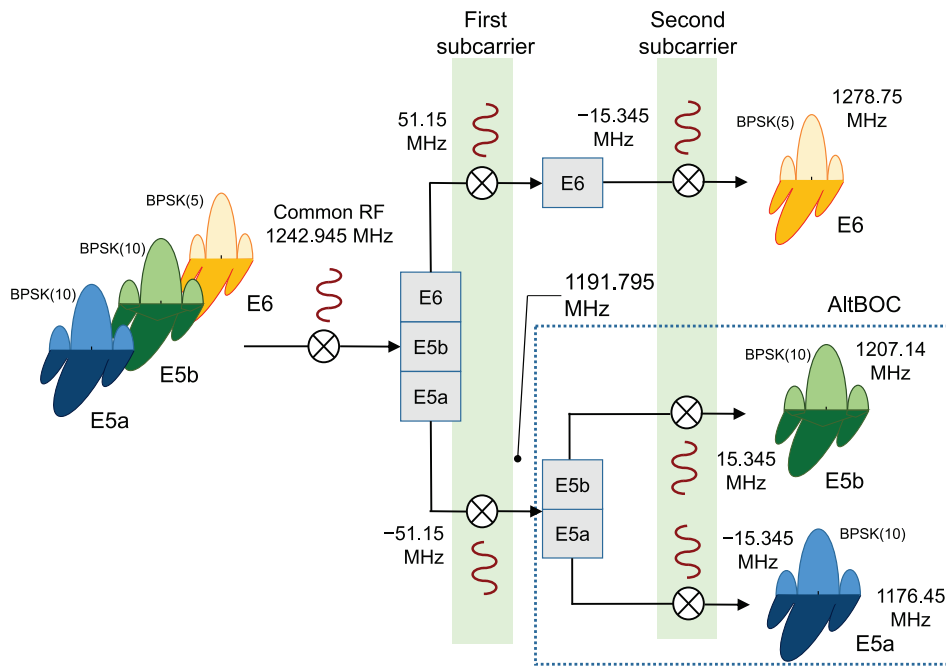


Figure 1. Construction of a GNSS meta-signal with three components. The Galileo E5a, E5b and E6 signals are combined using two subcarriers. With the ordering adopted, the AltBOC modulation is found on the bottom branch of the tree.

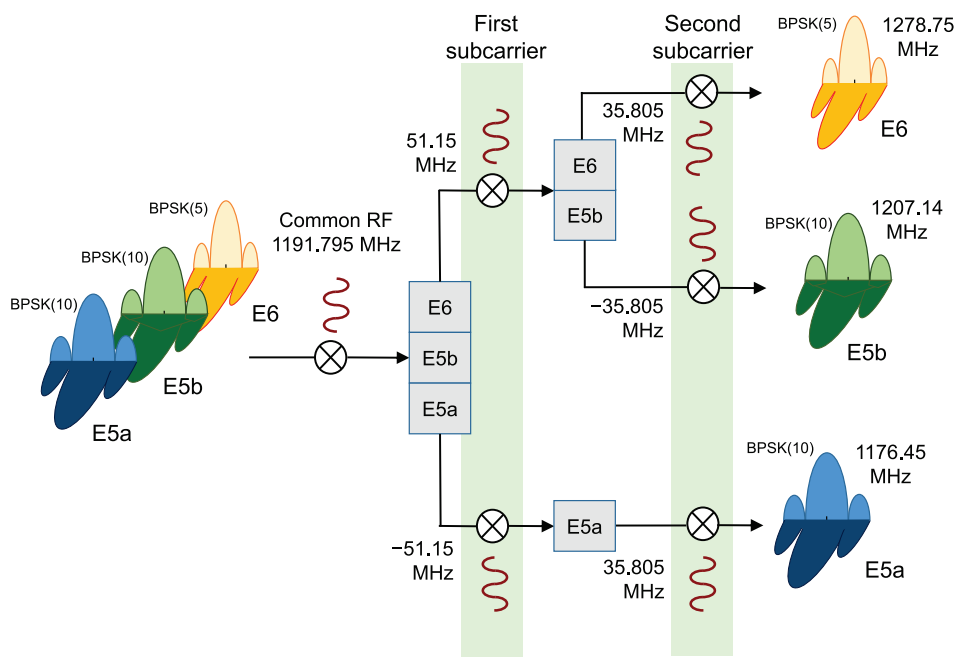


Figure 2. Alternative option for the construction of a GNSS meta-signal based on the combination of the Galileo E5a, E5b and E6 signals. The AltBOC RF is now the nominal meta-signal carrier frequency.

2.3.2. General Case

In the previous section, a procedure for constructing a GNSS meta-signal with three components was provided. This procedure can be easily generalized to an arbitrary number of components by following the steps summarized in Figure 3. Given K RF components, they can be combined using $K - 1$ subcarriers and expressed as a single meta-signal using the appropriate multicomplex space. The first step consists in the identification of $\mathbb{C}_M$, a multicomplex space sufficiently large to contain the different components.

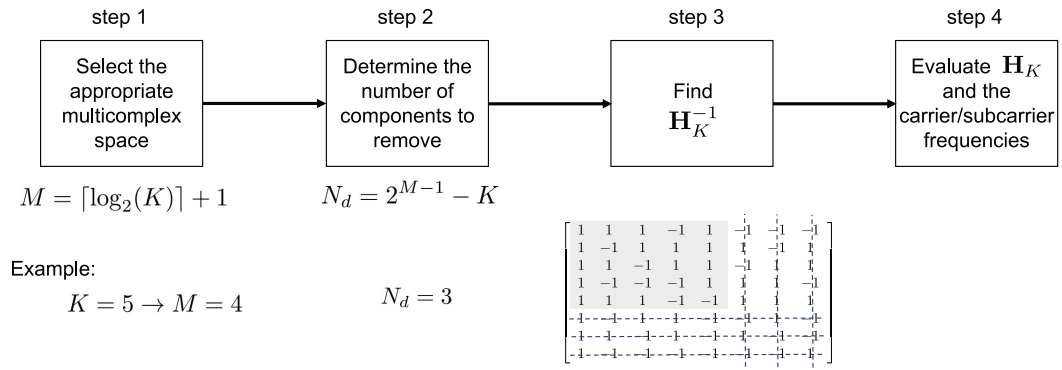


Figure 3. General procedure for the construction of GNSS meta-signals with an arbitrary number of components. K is the number of components considered.

If K is the number of RF signals, then M is found as

$$M = \lceil \log_2(K) \rceil + 1 \quad (29)$$

where $\lceil \cdot \rceil$ denotes the ceiling operator. In Figure 3, the case with $K = 5$ components is considered as example. When five RF signals are present, $M = 4$ and $\mathbb{C}_4$ are considered. In this space, up to eight components can be combined to form a single meta-signal. The second step reported in Figure 3 consists of determining the number of components to remove from the full meta-signal with 2^{M-1} elements. This number, denoted here as N_d , is given by

$$N_d = 2^{M-1} - K. \quad (30)$$

In the example of Figure 3, $N_d = 3$. After these preliminary operations, it is finally possible to determine the matrix $\mathbf{H}_K^{-1}$ connecting the different RFs to the meta-signal carrier and subcarrier frequencies. $\mathbf{H}_K^{-1}$ is obtained by removing the last N_d columns and rows from $\mathbf{H}_{2^{M-1}}^T$, where $\mathbf{H}_{2^{M-1}}$ is the Hadamard matrix characterizing the $\mathbb{C}_M$ space and obtained according to the procedure outlined in [24]. For $K = 5$,

$$\mathbf{H}_5^{-1} = \begin{bmatrix} 1 & 1 & 1 & -1 & 1 \\ 1 & -1 & 1 & 1 & 1 \\ 1 & 1 & -1 & 1 & 1 \\ 1 & -1 & -1 & -1 & 1 \\ 1 & 1 & 1 & -1 & -1 \end{bmatrix}. \quad (31)$$

$\mathbf{H}_K^{-1}$ is finally inverted and the meta-signal carrier and subcarrier nominal frequencies are computed as

$$\mathbf{f}_K = \mathbf{H}_K \mathbf{f}_{X,K} \quad (32)$$

where $\mathbf{f}_{X,K}$ is the vector with the RFs of the original signal components. The resulting multicomplex baseband signal is given by

$$z_K(t) = 2^{M-1} \sum_{i=0}^{K-1} x_i(t) e_{M,i}, \quad (33)$$

and the corresponding RF signal can be represented as

$$z_{K,RF}(t) = \Re \left\{ z_K(t) \exp \left\{ 2\pi \mathbf{j}_{M,K}^T \mathbf{f}_K t \right\} \right\}, \quad (34)$$

where $\mathbf{j}_{M,K}$ is the vector with the first K imaginary units of $\mathbb{C}_M$.

For $K = 5$,

$$\mathbf{H}_5 = \frac{1}{4} \begin{bmatrix} -1 & 1 & 1 & 1 & 2 \\ 1 & -1 & 1 & -1 & 0 \\ 1 & 1 & -1 & -1 & 0 \\ -1 & 1 & 1 & -1 & 0 \\ 2 & -0 & 0 & 0 & -2 \end{bmatrix}, \quad (35)$$

and it involves quad-frequency linear combinations derived from the Hadamard matrix of order 4 and narrow /wide-lane combinations. This case is illustrated in Figure 4, where all the five Beidou Navigation Satellite System (BDS) III signals [26] are combined using four subcarriers.

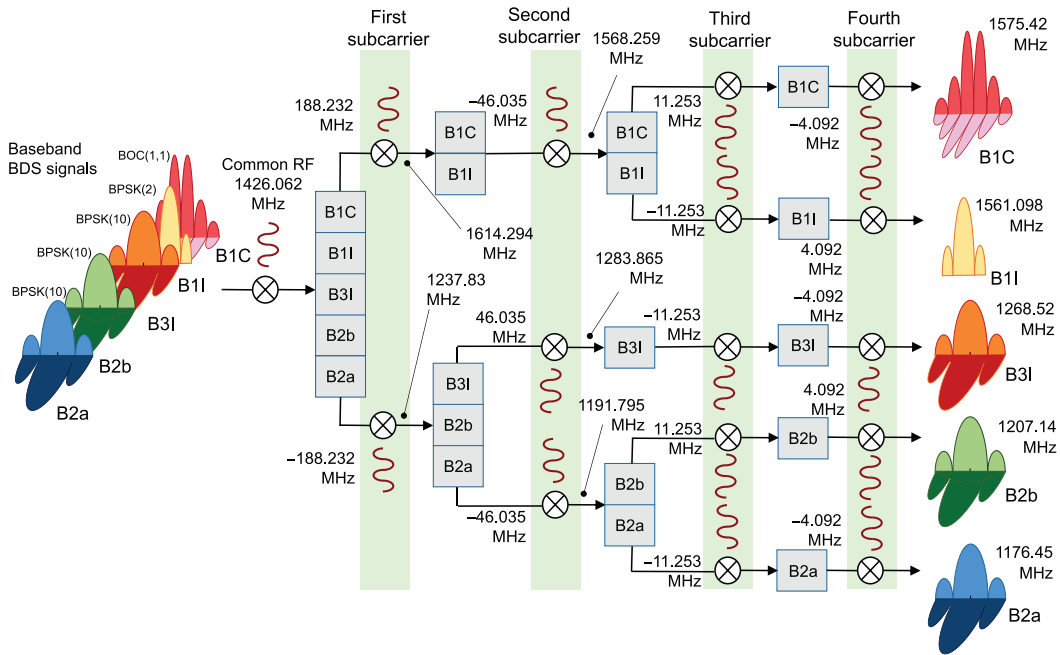


Figure 4. Construction of a GNSS meta-signal with five components. All the five BDS III signals are combined using four subcarriers.

2.4. Baseband Processing

In the previous section, it was shown that a set of K RF signals can be represented as a single meta-signal with a single carrier up-converting all the components to a common RF. $K - 1$ subcarriers are then used to split the different components on their final RFs. K can be an arbitrary integer value. In this section, the obtained signal representation is used to derive meta-signal acquisition and tracking algorithms.

2.4.1. Received Signals

The different RF components, $x_{RF,i}(t)$, are transmitted by a single GNSS satellite and pass through a common communication channel. When received on the ground, a multi-frequency GNSS receiver is used to recover the different components. The receiver front-ends amplify, filter and down-convert the different components. In this way, baseband versions of the received signals are obtained. A simple communication channel is assumed [27]. In particular, each received component is modelled as

$$x_{rx,i}(t) = \alpha_i x_i(t - \tau_0) \exp\{j_0 2\pi f_{X0,i} t + j_0 \varphi_{X0,i}\} + \eta_i(t), \quad (36)$$

where the definitions are as follows:

- τ_0 is a delay assumed common to all signals. While biases among the different components can be present, for instance due to hardware imperfections at both satellite and receiver side, these biases are neglected here. The calibration procedure developed in [11] is adopted to compensate for possible Differential Code Biases (DCBs),
- α_i is the attenuation introduced by the communication channel on the i th signal component,
- $f_{X0,i}$ and $\varphi_{X0,i}$ are the Doppler frequency and carrier phase introduced on the i th signal component,
- $\eta_i(t)$ is Additive White Gaussian Noise (AWGN) with independent real and imaginary parts. Noise terms from different frequencies are assumed independent.

The receiver front-ends digitize the different $x_{rx,i}(t)$, producing the samples

$$x_{rx,i}[n] = x_{rx,i}(nT_s) = \alpha_i x_i(nT_s - \tau_0) \exp\{j_0 2\pi f_{X0,i} nT_s + j_0 \varphi_{X0,i}\} + \eta_i(nT_s), \quad (37)$$

where T_s is the sampling interval assumed common to all the receiver front-ends and n is the time index. The inverse of T_s , $f_s = \frac{1}{T_s}$ defines the sampling frequency. The noise components, $\eta_i[n] = \eta_i(nT_s)$, are independent and identically distributed (i.i.d.) random processes with independent real and imaginary parts. The same variance, σ_η^2 , is assumed for the real and imaginary part of $\eta_i[n]$.

The received baseband multicomplex signals can be reconstructed by assigning a different component (37) to the orthogonal elements of $\mathbb{C}_M$ as in (33)

$$\begin{aligned} z_{rx,K}[n] &= 2^{M-1} \sum_{i=0}^{K-1} x_{rx,i}[n] e_{M,i} \\ &= Az_K(nT_s - \tau_0) \exp\left\{2\pi j_{M,K}^T \mathbf{f}_{K,0} nT_s + j_{M,K}^T \mathbf{P}_{K,0}\right\} + \eta[n] \end{aligned} \quad (38)$$

where

$$A = \sum_{i=0}^{K-1} \alpha_i e_{M,i} \quad (39)$$

is the multicomplex attenuation obtained by combining the α_i terms introduced above. $\mathbf{f}_{K,0}$ and $\mathbf{P}_{K,0}$ are the vectors with the meta-signal carrier and subcarrier Doppler frequencies and phases. They are obtained through the transformation defined by (32):

$$\mathbf{f}_{K,0} = \mathbf{H}_K \begin{bmatrix} f_{X0,0} \\ f_{X0,1} \\ \vdots \\ f_{X0,K-1} \end{bmatrix} \quad \mathbf{P}_{K,0} = \mathbf{H}_K \begin{bmatrix} \varphi_{X0,0} \\ \varphi_{X0,1} \\ \vdots \\ \varphi_{X0,K-1} \end{bmatrix}. \quad (40)$$

Finally, $\eta[n]$ is a multicomplex noise term obtained as

$$\eta[n] = 2^{M-1} \sum_{i=0}^{K-1} \eta_i[n] e_{M,i}. \quad (41)$$

Equation (38) is analogous to the signal model obtained in [11] for the case with 2^{M-1} components. The main difference here is that the summation in the previous equations now extend to K instead of 2^{M-1} . Moreover, the transformations connecting the parameters of the individual received components to those of the resulting meta-signal are no longer defined by a Hadamard matrix and the procedure outlined in Section 2.3.2 has to be followed. This analogy is exploited for the derivation of acquisition and tracking algorithms jointly processing several components from different frequencies.

In the following, a standard model for the GNSS signals, $x_i(t)$, is assumed. In particular, $x_i(t)$ is modelled as [27]

$$x_i(t) = d_i(t)c_i(t), \quad (42)$$

where $c_i(t)$ is a spreading sequence from a family of quasi-orthogonal codes and $d_i(t)$ represents the navigation message. Note that $x_i(t)$ may include several components such as a data and pilot channel. This case is not considered here and a single component is assumed. The presence of both data and pilot channels can be addressed with an approach similar to that developed in [28]. $d_i(t)$ may be absent if pilot channels are considered. The codes from the different components can be combined into a single multicomplex term

$$\tilde{c}(t) = \sum_{i=0}^{K-1} \tilde{\alpha}_i c_i[n] e_{M,i} \quad (43)$$

and used for the computation of the multicomplex CAF as better described in the next section. In (43), $\tilde{\alpha}_i$ are estimates of the α_i coefficients introduced in (36). Since each signal is received with a different amplitude, the local codes also have to be weighted differently. $\tilde{\alpha}_i$ can be fixed according to previous knowledge on received signals or using a calibration process.

2.4.2. Multicomplex CAF

The goal of the signal processing stage of a GNSS receiver is to estimate the received signal parameters. In this case, these parameters are τ_0 , $\mathbf{f}_{K,0}$ and $\mathbf{P}_{K,0}$. When 2^{M-1} components are considered, this is performed through the maximization of a multicomplex CAF [11]. This approach can also be adopted for the general case with K components. In particular, estimates of τ_0 , $\mathbf{f}_{K,0}$ and $\mathbf{P}_{K,0}$ are found as

$$\tilde{\tau}, \tilde{\mathbf{f}}_K, \tilde{\mathbf{P}}_K = \arg \max_{\tau, \mathbf{f}_d, \mathbf{P}} \Re \left\{ C_z(\tau, \mathbf{f}_d) \exp \left\{ -\mathbf{j}_{M,K}^T \mathbf{P} \right\} \right\}, \quad (44)$$

where

$$C_z(\tau, \mathbf{f}_d) = \sum_{n=0}^{N-1} z_{rx,K}[n] \tilde{c}^*(nT_s - \tau) \exp \left\{ -2\pi \mathbf{j}_{M,K}^T \mathbf{f}_d nT_s \right\} \quad (45)$$

is a multicomplex CAF obtained by correlating $z_{rx,K}[n]$ with local code (43) delayed by τ and modulated by the frequency vector $\mathbf{f}_d$. In (45), $*$ denotes the star-conjugation introduced in Section 2.1. CAF (45) can be derived following a procedure similar to that developed in [11] for the case with 2^{M-1} components. Following the same approach used in [11], solution (44) can be rewritten as

$$\begin{aligned} \tilde{\tau}, \tilde{\mathbf{f}}_K &= \arg \max_{\tau, \mathbf{f}_d} \Re \{ |C_z(\tau, \mathbf{f}_d)|_k \} \\ \tilde{\mathbf{P}}_K &= \mathbf{P}_{CAF} \end{aligned} \quad (46)$$

where $|C_z(\tau, \mathbf{f}_d)|_k$ is the hyperbolic modulus of the CAF and $\mathbf{P}_{CAF}$ is the vector with the phases of the CAF evaluated for $\tau = \tilde{\tau}$ and $\mathbf{f}_d = \tilde{\mathbf{f}}_K$.

2.4.3. Multicomplex Acquisition

Maximization of (46) is performed through a two-step process: acquisition and tracking. Acquisition maximizes $\Re \{ |C_z(\tau, \mathbf{f}_d)|_k \}$ through a search on a finite grid of Doppler frequencies and delay values. The process is illustrated in Figure 5 and it is analogous to the case with 2^{M-1} components. The incoming multicomplex signal is multiplied by local code (43) and by local replicas of the carrier and subcarriers. The resulting multicomplex signal is coherently integrated on N samples. $T_c = NT_s$ is the coherent integration time. In

this way, the CAF is computed. Different Doppler frequencies and code delays are tested and $\Re\{|C_z(\tau, \mathbf{f}_d)|_k\}$ is finally maximized.

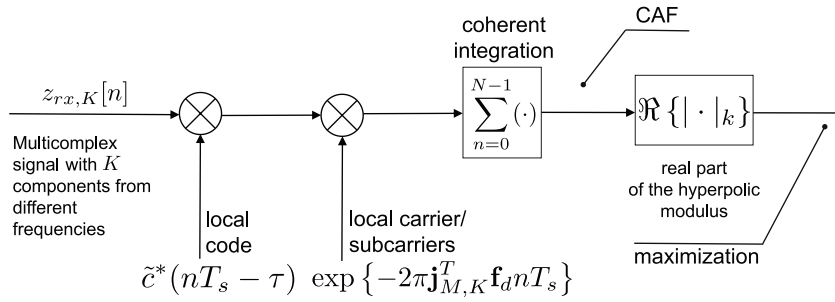


Figure 5. General representation of the joint acquisition process of K components from different frequencies using multicomplex numbers.

Note that while different carrier/subcarriers are present, relationships between the different Doppler frequencies can be exploited to reduce the dimensionality of the search process. For instance, only the carrier Doppler can be considered as a search parameter. The Doppler frequencies of the subcarriers can be obtained by scaling the carrier Doppler by the corresponding nominal frequencies.

As for the case with 2^{M-1} components, $\Re\{|C_z(\tau, \mathbf{f}_d)|_k\}$ can be evaluated in terms of single-frequency signals. Since the proof is analogous to that provided in [11] for the case with 2^{M-1} components, only the final result is reported here. In particular, it is possible to show that

$$\Re\{|C_z(\tau, \mathbf{f}_d)|_k\} = \sum_{i=0}^{K-1} \tilde{\alpha}_i |C_{x,i}(\tau, f_{h,i})| \quad (47)$$

where $C_{x,i}(\tau, f_{h,i}) = \sum_{n=0}^{N-1} x_{rx,i}[n] c_i^*(nT_s - \tau) \exp\{-j_0 2\pi f_{h,i} n T_s\}$ are the standard single-frequency CAFs computed for the different signal components, $x_{rx,i}[n]$. $f_{h,i}$ is the Doppler frequency tested on the i th component. These frequencies are obtained as

$$\mathbf{f}_h = \mathbf{H}_K^{-1} \mathbf{f}_d. \quad (48)$$

As for the case with 2^{M-1} , a form of weighted linear detector is found [17]. The joint acquisition of K components requires a common integration time, T_c . This can be implemented for the Galileo case considering the E5a, E5b and E6 components.

2.4.4. Multicomplex Tracking

A multicomplex tracking scheme can be obtained by adapting the architecture described in [11] to the case where K components are present. The tracking scheme obtained is schematically represented in Figure 6: a single DLL, a single PLL and $K - 1$ SPLs are used to track the code, carrier and subcarrier components present in signal representation (38).

The incoming multicomplex signal, $z_{rx,K}[n]$, is first multiplied by the exponentials generated by the carrier and subcarrier Numerically Controlled Oscillators (NCOs). Three copies of local code (43) are also generated with different delays. The prompt code replica is generated considering the best code delay estimate available at the current processing epoch. Early and late code replicas are obtained by advancing and delaying the prompt replica by fixed amounts, which are the early-minus-late spacings. The same considerations made in [11] on the use of different code spacings for different frequency components apply and are not repeated here. The three code replicas are multiplied by the multicomplex signal obtained after removing the carrier and subcarrier components and finally integrated coherently considering N samples. In this way, three multicomplex correlators P , L and E , are obtained. Code and phase residual errors are extracted using appropriated discrimina-

tors operating on the multicomplex correlators. In particular, a generalized non-coherent early-minus-late envelope discriminator [27] can be adopted for the code error:

$$D_c = \frac{\Re\{|E|_k\} - \Re\{|L|_k\}}{\Re\{|E|_k\} + \Re\{|L|_k\}}, \quad (49)$$

where $|\cdot|_k$ denotes the hyperbolic modulus and D_c is the code discriminator output. Carrier and subcarrier phase error estimates are obtained by considering the polar representation of the prompt correlator output that is a multicomplex number:

$$P = |P|_k \exp\left\{\mathbf{j}_{M,K}^T \mathbf{P}_{p,K}\right\} = |P|_k \exp\{j_0 \Delta \varphi_0\} \prod_{i=1}^{K-1} \exp\{j_i \Delta \varphi_i\}. \quad (50)$$

Since P has been obtained from $z_{rx,K}[n]$, it inherits several of its properties. For instance, P has only K non-zero orthogonal components. Moreover, owing to the construction proposed in Section 2.3.2, only its first K phases expressed with respect to the imaginary units, j_i , are different from zero. For this reason, polar representation (50) involves $\mathbf{j}_{M,K}$, which is made of K components. $\mathbf{P}_{p,K}$ is the vector with the K non-zero phases of P . These phases are the residual errors that the different carrier and subcarrier loops are trying to minimize. These phases can be obtained from the polar representation of P . A procedure for extracting them using real operations is detailed in the next section.

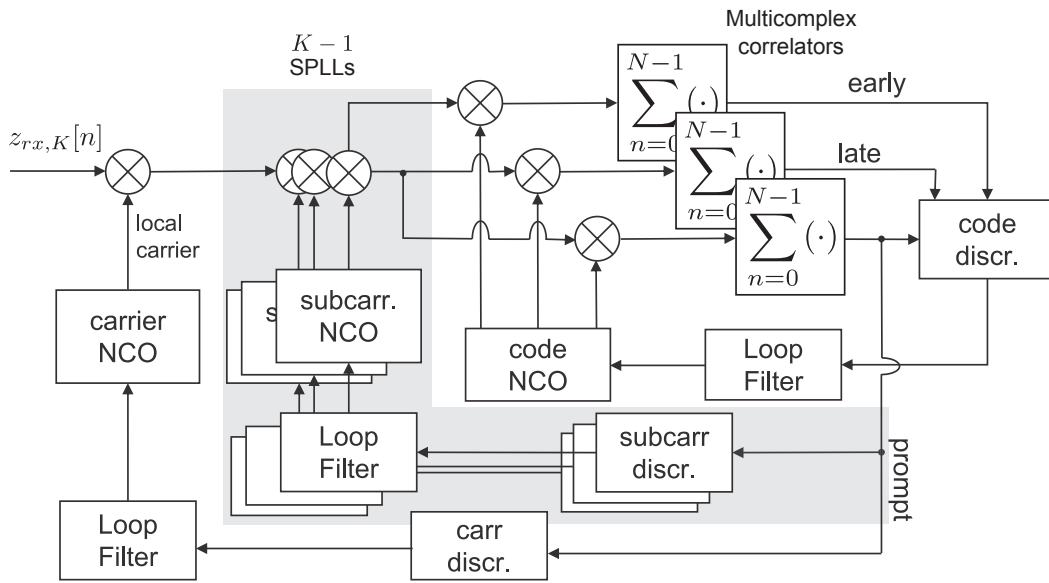


Figure 6. Schematic representation of a multicomplex tracking architecture for jointly processing K signal components from different frequencies. Multicomplex operations and $K - 1$ SPLs are used.

The code, carrier and subcarrier discriminator outputs are filtered using standard loop filters. The filter outputs are estimates of the Doppler frequencies affecting the different components and are used by the NCOs to generate the code, carrier and subcarrier local replicas. In this way, the different loops are closed.

2.4.5. Implementation in Terms of Standard Components

The multicomplex tracking scheme described above can be implemented using the standard components available in single-frequency tracking loops. This is possible using the orthogonal decomposition and the polar representation of multicomplex numbers. In

the DLL case, the multicomplex early, late and prompt correlators can be decomposed as follows:

$$\begin{aligned} P &= 2^{M-1} \sum_{i=0}^{K-1} \tilde{\alpha}_i P_{x,i} e_{M,i} \\ E &= 2^{M-1} \sum_{i=0}^{K-1} \tilde{\alpha}_i E_{x,i} e_{M,i} \\ L &= 2^{M-1} \sum_{i=0}^{K-1} \tilde{\alpha}_i L_{x,i} e_{M,i} \end{aligned} \quad (51)$$

where $P_{x,i}$, $E_{x,i}$ and $L_{x,i}$ are the complex prompt, early and late correlators computed for the i th signal component, $x_{rx,i}[n]$. Using (51), it follows that

$$\begin{aligned} \Re\{|E|_k\} &= \sum_{i=0}^{K-1} \tilde{\alpha}_i |E_{x,i}| \\ \Re\{|L|_k\} &= \sum_{i=0}^{K-1} \tilde{\alpha}_i |L_{x,i}|, \end{aligned} \quad (52)$$

which is analogous of (47) for the early and late correlators. Using (52), it is finally possible to compute (49) in terms of single-component correlator outputs:

$$D_c = \frac{\sum_{i=0}^{K-1} \tilde{\alpha}_i |E_{x,i}| - \sum_{i=0}^{K-1} \tilde{\alpha}_i |L_{x,i}|}{\sum_{i=0}^{K-1} \tilde{\alpha}_i |E_{x,i}| + \sum_{i=0}^{K-1} \tilde{\alpha}_i |L_{x,i}|}. \quad (53)$$

Thus, the multi-component DLL can be implemented using standard loop elements. The single-frequency correlators, $\{P_{x,i}, E_{x,i}, L_{x,i}\}_{i=0}^{K-1}$, are at first computed using the code generated by single-component NCOs. These correlators are used for the evaluation of discriminator output (53), which is filtered using a standard loop filter. In this way, a common code Doppler estimate is obtained. Finally, the loop filter output is used to drive the code generation from all the K single-component NCOs. This architecture is similar to that described in [11] for the case with 2^{M-1} components.

For the evaluation of the different correlators, single-component local carriers are needed. These are provided by the multicomplex PLL/SPLLs, which can be also implemented in terms of standard components.

This is illustrated in Figure 7, which shows a joint implementation of the different PLL/SPLLs. The prompt correlators, $P_{x,i}$, are computed using the local codes from the multicomplex DLL and the local carriers generated by single-component carrier NCOs. These correlators are passed to standard carrier discriminators, which provide estimates of the phases of the different $P_{x,i}$. The discriminator outputs are the phases of the orthogonal components of P and are processed using a carrier/subcarrier joint filter. A schematic representation of this joint loop filter is provided in Figure 8: the carrier discriminators are grouped in a vector and multiplied by $\mathbf{H}_K$, the matrix obtained according to the procedure outlined in Section 2.3.2. In this way, the phase vector $\mathbf{P}_{p,K}$ is obtained:

$$\mathbf{P}_{p,K} = \mathbf{H}_K \begin{bmatrix} \Delta\varphi_{x,0} \\ \Delta\varphi_{x,1} \\ \vdots \\ \Delta\varphi_{x,K-1} \end{bmatrix}, \quad (54)$$

where $\Delta\varphi_{x,i}$ is the discriminator output from the i th signal component.

The first element of $\mathbf{P}_{p,K}$ is the phase error associated to the meta-signal common carrier, whereas the other elements are the phase errors affecting the different subcarriers. These phase errors are processed using standard loop filters, which produce estimates of the Doppler shifts on the different meta-signal carrier and subcarrier components. Doppler estimates are arranged in a vector and multiplied by the inverse transform defined by $\mathbf{H}_K^{-1}$. In this way, estimates of the Doppler frequencies affecting the individual components, $x_{rx,i}[n]$, are found. These Doppler estimates are finally used to drive the carrier NCOs in Figure 7. This closes the loops in the PLL/SPLLs implementation using standard components.

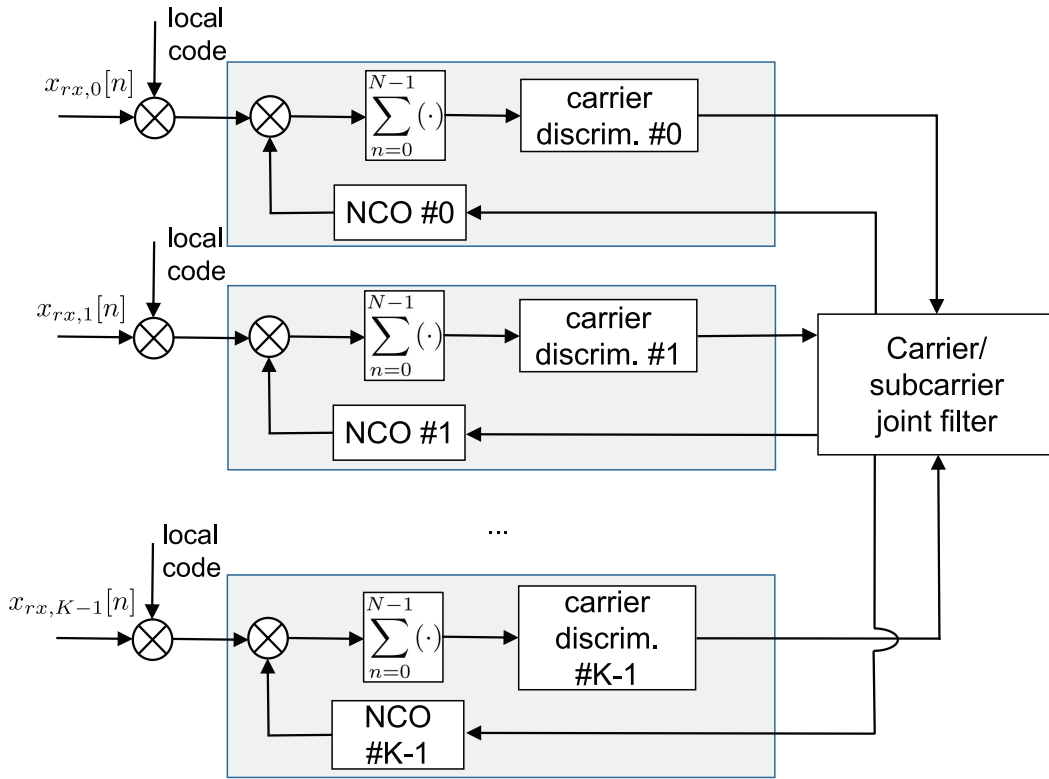


Figure 7. Schematic representation of a possible implementation of multicomplex PLL/SPLLs using standard components. The multicomplex PLL/SPLLs can be implemented using single-frequency standard PLLs sharing a joint multi-dimensional loop filter.

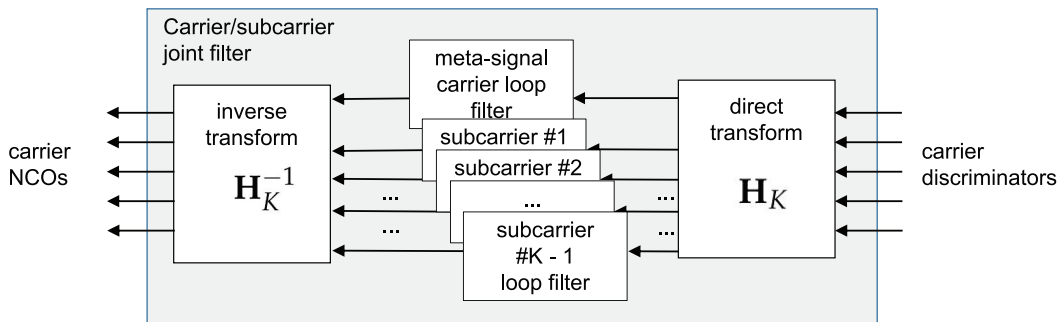


Figure 8. Schematic representation of the multi-dimensional joint loop filter adopted for the PLL/SPLL implementation.

2.5. Experimental Setup

In order to demonstrate the feasibility of the meta-signal approach developed for the joint processing of an arbitrary number of signals, the baseband data used in [11] have been reprocessed considering three components, namely the Galileo E6, E5b and E5a signals. Since a complete description of the experimental setup adopted for the data collection can

be found in [11], only a brief summary is provided here. The second part of this section describes the actual implementation in a Python (version 3.9) SDR receiver of the acquisition and tracking algorithms introduced in Section 2.4 considering three components.

The data recording system used in [11] consisted of four HackRF One front-ends (Great Scott Gadgets, Lakewood, CO, USA) [29] sharing the same clock and synchronized according to the procedure detailed in [30]. While four front-ends were used to collect baseband samples from four Galileo frequencies, only three channels are considered in this work. In particular, the In-Phase Quadrature (I/Q) samples from the three front-ends tuned on the E6, E5b and E5a frequencies have been jointly processed. The parameters used for configuring these front-ends are summarized in Table 1.

Table 1. Parameters related to the three data streams from the HackRF One front-ends used for the experimental analysis.

Parameter	Value
Sampling frequency, f_s	20 MHz
Centre frequency, E6 front-end, f_{E6}	1279 MHz
Residual IF, second front-end, $f_{E6,IF}$	0.25 MHz
Centre frequency, E5b front-end, f_{E5b}	1207 MHz
Residual IF, third E5b, $f_{E5b,IF}$	0.14 MHz
Centre frequency, E5a front-end, f_{E5a}	1176 MHz
Residual IF, E5a front-end, $f_{E5a,IF}$	0.45 MHz
Sampling type	Complex I/Q
No. of bits	8

All the front-ends were characterized by the same sampling frequency, $f_s = 20$ MHz. Moreover, each front-end was tuned to a different centre frequency as reported in Table 1. Note that, as analysed in [31], the fractional PLLs used by HackRF One front-ends to synthesize the local carrier for the signal downconversion, are usually not able to completely match the parameters set by the user. This effect leaves small differences in the residual signal Intermediate Frequency (IF), which have been accounted for through a calibration procedure.

The I/Q samples collected using the three HackRF One front-ends have been processed using the custom Python SDR receiver implementing the acquisition and tracking stages described in the previous section. The Galileo E6, E5a and E5b signals are characterized by the same primary code duration, which is equal to 1 ms. Thus, meta-signal acquisition can be implemented directly through (47): individual CAFs are computed on the different components and linearly combined. Appropriate scaling of the different Doppler frequencies has been implemented.

In tracking, the architecture illustrated in Section 2.4.5 has been adopted. The signal configuration depicted in Figure 1 has been considered. This choice led to the implementation of a PLL and two SPLs. A single DLL was adopted for tracking a common code delay. The tracking loop parameters provided in Table 2 were adopted for the experiments described in Section 2.5.

The Python SDR receiver features several tracking stages, such as the adoption at of Frequency Lock Loop (FLL)/Subcarrier Frequency Lock Loops (SPLLs) for achieving frequency lock conditions. Secondary code synchronization is also implemented on the pilot channels of the three Galileo signals. After secondary code synchronization, the coherent integration time is increased from 1 to 5 ms and pure pilot processing with four-quadrant phase discriminators is used [27]. This type of processing is possible when using the architecture described in Section 2.4.5.

As for the case with four components considered in [11], sub-sample offsets can be present among the data streams collected from the different channels. For this reason and similarly to [11], a calibration procedure has been implemented as part of the DLL operations. For a configurable period of time, the DLL operates in calibration mode using a single component while the other channels are processed in a passive way. The correlators from the passive channels are used to estimate sub-sample delays between components. After estimating such delays, the DLL moves to joint processing using the information from all the channels. In particular, discriminator (53) is used. The DLL also estimates the combining weights, $\tilde{\alpha}_i$. This is performed using an exponential filter on the amplitude of the different prompt correlators.

Table 2. Parameters of the Python SDR receiver used to demonstrate multicomplex tracking using the Galileo E6, E5a and E5b components. The same parameters have been used for the two SPLLS.

Parameter	Value
DLL order	2
DLL bandwidth	2 Hz
PLL order	3
PLL bandwidth	15 Hz
SPLL order	2
SPLL bandwidth	3 Hz
Integration time before secondary code synch.	1 ms
Integration time after secondary code synch.	5 ms
Early-minus-late chip spacing, E6	1 BPSK(10) chip
Early-minus-late chip spacing, E5a and E5b	0.5 BPSK(10) chips

Finally, the computation of the early and late correlators requires the specification of the early-minus-late chip spacings for the different components. These spacings are reported in Table 2 in terms of Binary Phase Shifting Keying (BPSK)(10) chips. This is the modulation of the E5a and E5b signals. The values reported in Table 2 have been selected in order to have a constant product between chip spacing and slope of the correlation function of the corresponding signal.

3. Results

Sample results obtained by processing the data collected using the experimental setup described in Section 2.5 are briefly reported in this section. These results support the feasibility of the approach derived in the paper.

3.1. Acquisition

Since the Galileo E6, E5b and E5a signals share the same primary code duration, they can be directly combined using linear detector (47). Figure 9 shows the four CAFs computed using individual signals (plots (a), (b) and (c)) and by jointly considering the three components. From the figure, it emerges that the three Galileo components are characterized by different Signal-to-Noise Ratio (SNR) conditions. The E6 signal is the noisiest and the corresponding CAF peak is barely visible in Figure 9. In contrast, the E5b component is characterized by the highest SNR. The different signal qualities are due to the different transmit power levels and by different antenna/front-end gains provided at different frequencies.

All the CAFs have been computed using a 1 ms coherent integration time and normalized with respect to the CAF maximum. Non-coherent integrations were not implemented. In acquisition, the different signal gains, $\tilde{a}_i$, are not available. For this reason, all the single-component CAFs have been weighted equally. Moreover, sub-sample delays between components have not been accounted for. While these choices are not optimal, a clear gain is visible in Figure 9. The noise floor of the triple-component CAF in Figure 9d is clearly reduced with respect to the E6 one: the stronger components aid the acquisition of the weakest one. Also, a limited noise floor reduction is observed when comparing the triple component CAF with that obtained using the E5b signal. This is expected given the high quality of the E5b signal.

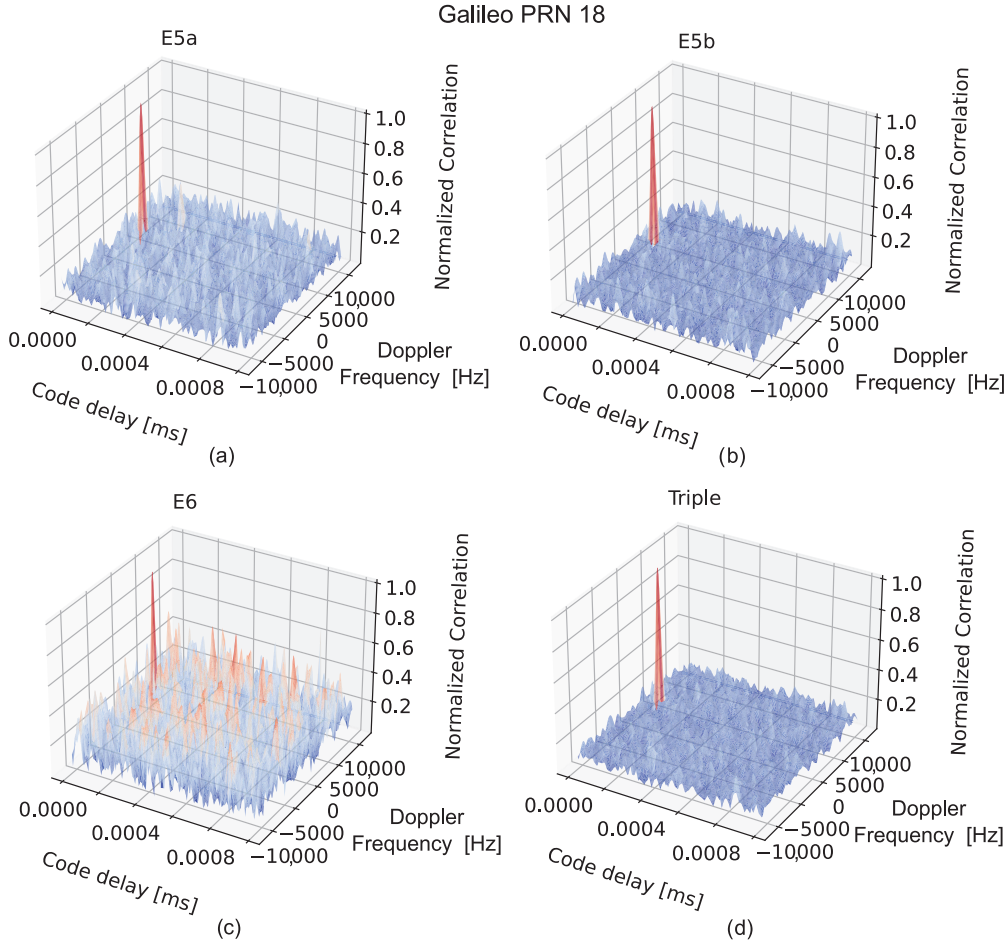


Figure 9. CAFs computed considering the Galileo E6, E5b and E5a components. (a) E5a component. (b) E5b component. (c) E6 component. (d) Combined triple-component CAF.

The analysis provided in Figure 9 is qualitative and aims at showing the feasibility of the approach derived in Section 2.4.3. A more thorough analysis of the weighted linear detectors resulting from the meta-signal theory developed in this paper is left for future work.

3.2. Tracking

Results obtained by jointly tracking the E6, E5b and E5a components are provided in the following. The signal representation shown in Figure 1 has been considered with a common carrier, with nominal frequency equal to 1242.945 MHz and two subcarriers with frequencies equal to 51.15 and 15.345 MHz, respectively. Figure 10 shows the single-component prompt pilot correlators obtained using the quad-loop architecture. These are

the orthogonal components of the multicomplex prompt correlator obtained by considering the E6, E5b and E5a signals.

Both I/Q branches are provided in Figure 10: after an initial transient period, the meta-signal PLL and SPLs are able to correctly align all the useful power in the in-phase correlator parts. From the figure, the different operating modes of the implemented tracking architecture are also visible. After less than two seconds, the secondary codes of the pilot components are recovered and removed from the prompt correlators. For this reason, after secondary code removal, only positive values are visible in the in-phase parts of the prompt correlators.

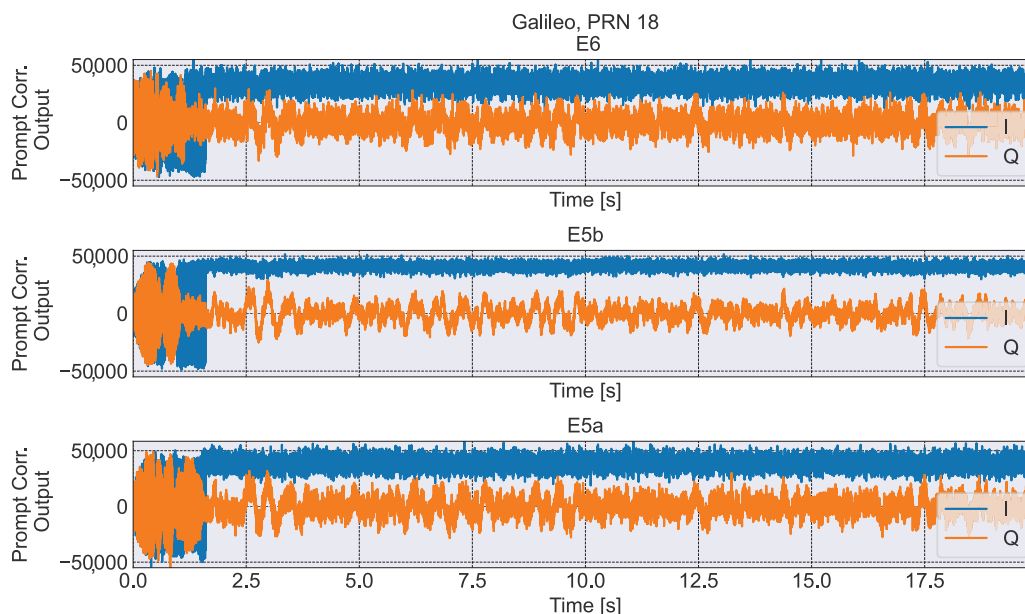


Figure 10. Pilot prompt correlator outputs obtained using the quad-loop architecture jointly tracking the E6 (upper plot), E5b (middle plot) and E5a (lower plot) components.

The proper functioning of the implemented quad-loop tracking architecture is also supported by Figure 11, which shows the normalized code, carrier and subcarrier Doppler estimates obtained at the output of the filters of the DLL, PLL and two SPLs. Since the architecture depicted in Figure 7 has been implemented, the loop filter outputs correspond to the values obtained before the inverse transform shown in Figure 8. The different Doppler estimates have been normalized by the corresponding nominal frequencies. After normalization, the different components practically overlap, indicating the possibility of aiding between different components. After secondary code removal, the integration time is increased from 1 to 5 ms. In this way, the noise on the different components is reduced. Code estimates are also improved when moving from calibration to standard processing mode. After calibration, all the signal components contribute the estimation of a common delay, thus reducing the associated noise variance.

The right part of Figure 11 provides a zoomed-in section of the Doppler estimates associated to the three carrier/subcarrier components. The curve related to subcarrier C is the noisiest. This is expected since it is characterized by a nominal frequency equal to 15.345 MHz, which is the lowest among the three components. The normalizations used in Figure 11 also scale the noise variations. Noise performance can be improved by reducing the SPLL bandwidth. The transition from 1 to 5 ms is also visible in the right part of Figure 11.

Finally, Figure 12 compares the Carrier-To-Noise Power Spectral Density Ratio (C/N_0) estimates obtained using the meta-signal architecture with three components and single-

component tracking loops. As already emerged from the discussion in Section 3.1, the three Galileo components are characterized by significantly different C/N_0 values. The E5b component is about 8 times stronger than the E6 signal. While this significant difference reduces the overall gain of the joint architecture when evaluated with respect to the E5b component, the implemented architecture is still able to effectively combine the different signals. The resulting C/N_0 is close to the maximum theoretical C/N_0 (52.4 dB-Hz), corresponding to the direct sum of the signal energy from the individual channels.

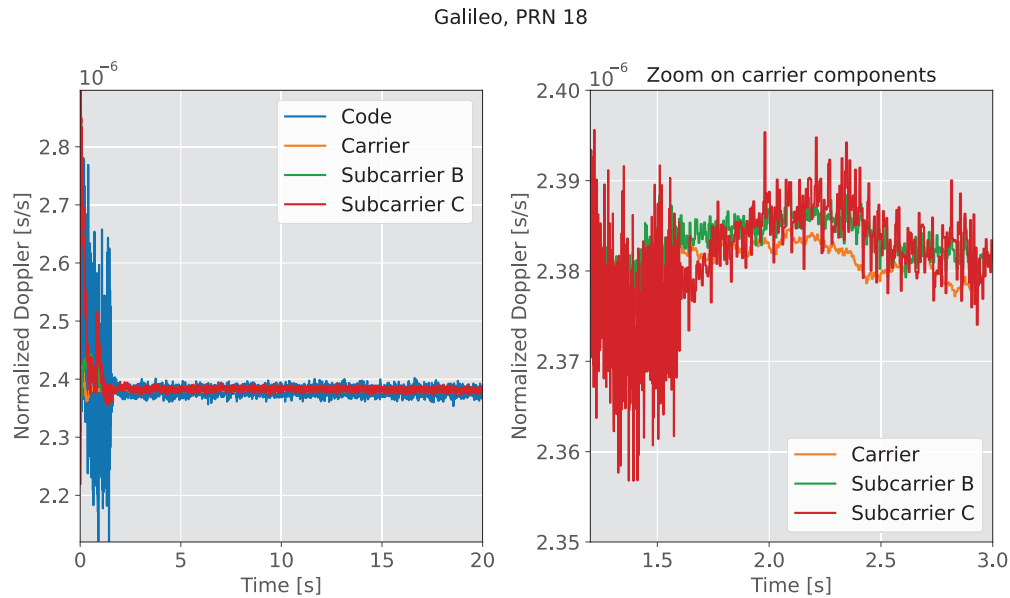


Figure 11. (Left) Normalized code, carrier and subcarrier Doppler estimates from the DLL, PLL and two SPLs used to jointly track the E6, E5b and E5a components. (Right) Zoom of the carrier and subcarrier Doppler estimates.

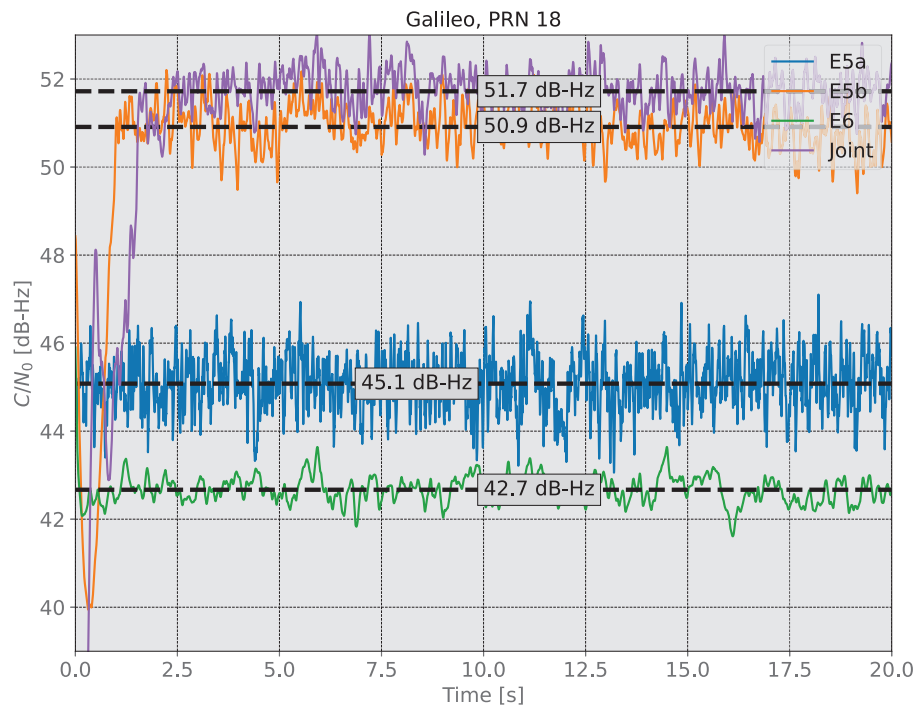


Figure 12. Comparison between C/N_0 estimates obtained using the meta-signal tracking architecture with three components and single-component tracking loops.

4. Discussion

The results illustrated in the previous section confirm the fact that in the proposed acquisition/tracking architectures all signals contribute to the estimation of common parameters. Moreover, stronger components help the processing of the weakest ones. In this case, the E6 signal is the one benefiting the most from meta-signal approaches.

A key result of this paper is the possibility to restate the algorithms derived using multicomplex numbers in terms of standard components. As a result, the cost function optimized at the acquisition stage is a form of weighted linear detector combining the single-frequency CAFs. A single DLL combining all the single-frequency correlators was also designed. Finally, the meta-signal PLL/SPLLs were obtained by modifying standard single-frequency PLLs. The discriminator outputs from different single-frequency PLLs are brought into a transformed domain and processed using carrier and subcarrier loop filters. In this way, the estimates of the carrier/subcarrier Doppler frequencies are found. An inverse transform is finally used to compute the Doppler shifts on the different signal components. The transformation from individual carriers to the meta-signal carrier/subcarriers was obtained from the structure of multicomplex numbers as a truncated Hadamard matrix. The truncation, implemented by removing rows and columns from a square Hadamard matrix of size equal to a power of two, makes the resulting transformation not orthogonal. Additional work is required to better understand the implications of the loss of orthogonality. Moreover, while the use of truncated Hadamard matrices has some appealing properties, future work should consider the adoption of different transformations obtained according to different criteria.

While this paper provides the theoretical foundations for the design of multi-dimensional acquisition/tracking algorithms with an arbitrary number of components, additional tests are required to fully determine the benefits and limitations of meta-signal approaches. The algorithms proposed can also be easily extended by introducing constraints among the different carrier and subcarrier components. For instance, a Kalman filter can be integrated within the multi-dimensional loop filter introduced in the Section 2.4.5.

Finally, the theory developed in this paper has been supported by experiments conducted using the Galileo E5a, E5b and E6 signals. However, the results presented are general and can be applied to signals of opportunity, such those broadcast by Low Earth Orbit (LEO) satellites [14]. Future research directions may include, for instance, the analysis of meta-signal approaches for tracking a limited number of OFDM components, such as pilot tones, sparsely located in an OFDM symbol.

5. Conclusions

In this paper, the general theory for representing and processing a GNSS meta-signal with an arbitrary number of components was derived. The theory, which uses multicomplex numbers, extends previous results, which required a number of components equal to a power of two. It is shown that a general meta-signal can be obtained as the product of a multicomplex baseband term multiplied by $K - 1$ subcarriers, which split the meta-signal components on K RFs. Using multicomplex numbers, it was possible to derive a generalized CAF computed using signals from K different RFs. In turn, the CAF was used to derive acquisition and tracking algorithms for the processing of K frequency components, which are jointly used to estimate common parameters such as a single delay and common carrier and subcarrier parameters. While multicomplex numbers proved beneficial for algorithm design and for properly representing the different signal components, the possibility of restating the different processing strategies in terms of standard components allows their efficient implementation. This fact has been proved through the implementation of the derived algorithms in an SDR receiver. Experiments involving the processing of Galileo

signals collected using three synchronized SDR front-ends support the theoretical findings derived in this paper.

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Article

Multi-Layer Perceptron Model Integrating Multi-Head Attention and Gating Mechanism for Global Navigation Satellite System Positioning Error Estimation

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Abstract: To better understand and evaluate the GNSS positioning performance, it is convenient to adopt corresponding measures to reduce the impact of errors on positioning. A GNSS positioning error estimation scheme based on an improved multi-layer perceptron model is proposed. The multi-head attention mechanism and gating operation are integrated into the multi-layer perceptron model to adaptively select and filter features, enhancing the model's ability to understand input features. First, the original positioning error of the satellite is obtained through the Kalman filter positioning method. The data are then preprocessed to extract available features. Finally, the features are input into the constructed model for training and testing to obtain the estimated positioning error value. Two types of comparative experiments were completed. The performance of the presented model is evaluated by the root mean square error. Experimental results show that the proposed method performs well in terms of performance indicators, and has obvious advantages over other state-of-the-art methods. In particular, the root mean square error of the presented method in the first dataset is 0.239 m, which is 39.2% and 17% lower than the current state-of-the-art long short-term memory network and convolutional neural network, respectively. The presented method can provide higher-precision estimated values for studying the GNSS positioning error estimation problem.

Keywords: global navigation satellite system; positioning error estimation; machine learning; multi-layer perceptron model

1. Introduction

Global navigation satellite system (GNSS) positioning technology has a wide range of applications in aviation, navigation, vehicle navigation, and other fields. GNSS positioning technology is necessary for achieving precise flight trajectory control and target navigation in aviation and navigation [1]. Real-time positioning is required to support autonomous driving and intelligent transportation systems in vehicle navigation [2]. Accurate positioning data are needed to study crustal movement and atmospheric changes in seismic monitoring and climate research [3]. The demand for high-precision and high-reliability positioning is increasing with the continuous development and transformation of GNSS technology. Previously, scholars focused on the research of GNSS high-precision positioning, and less exploration was undertaken on the problem of GNSS positioning error estimation. However, GNSS positioning and GNSS positioning error estimation are equally important [4]. Positioning errors may lead to the inaccuracy and unreliability of GNSS positioning, which has a serious impact on practical applications. Therefore, accurately

estimating and controlling the positioning error are the keys to ensuring the accuracy and reliability of GNSS positioning.

GNSS positioning is affected by errors that directly lead to inaccurate positioning results. The error sources in the GNSS positioning process mainly include satellite clock errors, ionospheric delay, tropospheric delay, multipath effect, receiver error, etc. [5]. Early research on GNSS positioning error estimation mainly focused on the estimation of a certain positioning error [6]. To minimize the impact of various factors on positioning results through modeling and analysis, many scholars have explored the problem of single error term estimation [7–9]. Han et al. designed a scheme to achieve high-precision positioning by estimating satellite clock errors [10]. This method has achieved excellent positioning results in both static and dynamic positioning. To address the problem that the dual-frequency ionospheric correction method ignores the high-order ionospheric error, Hoque et al. investigated a different approximation formula to estimate and correct the error caused by the excess path length [11]. Due to the lack of joint estimation of other error terms, the positioning error estimation method for a single error term has the problems of low accuracy and poor robustness. Subsequently, scholars have begun to focus on more refined positioning error modeling methods, and GNSS positioning accuracy has been improved to a certain extent [12]. The focus of related research is gradually shifting to error handling and precision improvement in complex environments [13–15]. Lesouple et al. studied the GNSS positioning problem in a restricted environment, and applied the sparse estimation theory to estimate the GNSS positioning error caused by different environments [16]. Jiang et al. explored the GNSS positioning problem in an environment with severe occlusion and interference, and presented an assisted joint positioning algorithm that exploits near-Earth Doppler and other measurement data to track the target position [17]. Although the research has promoted the advancement of GNSS positioning error estimation methods in complex environments, it has not solved the problem of limited estimation accuracy caused by estimating only a single error term. Nowadays, multisystem fusion has become a popular research topic [18]. Researchers have begun to study how to apply multisystem satellite observation data for error estimation and correction to improve the accuracy and reliability of positioning. Researchers have gradually shifted from estimating individual errors to estimating the overall error in the GNSS positioning process [19–21]. Marais et al. expatiated a GNSS positioning algorithm based on pseudorange error modeling and adaptive filtering [22]. Biswas et al. explored the influence of satellite spatial distribution on GNSS position estimation, and analyzed the GNSS positioning error estimation performance of four Kalman filters in detail [23]. Although estimating the overall error in the positioning process has been proven to be able to effectively mitigate the impact of error factors on positioning results, the method of estimating the overall error has the defect of relying on the assumptions of GNSS observation noise and motion model.

The application of GNSS positioning has introduced many complex and highly dynamic situations. How to solve the problems of error handling in complex environments and reliable positioning in highly dynamic environments urgently needs to be considered. Research on positioning error estimation methods can reduce or eliminate positioning deviations caused by additional positioning error factors. Traditional positioning error estimation methods for single error terms have the problems of low estimation accuracy and the need to make assumptions about GNSS observation noise. Although traditional methods for estimating the overall error can achieve higher estimation accuracy, they also require assumptions about GNSS observation noise. Therefore, a new GNSS positioning error estimation method is urgently needed. In recent years, the GNSS positioning error estimation field has been further developed. Scholars have applied artificial intelligence methods to address the GNSS positioning error estimation problem and obtained good

results. The GNSS positioning error estimation method based on artificial intelligence does not rely on prior assumptions, and can exploit GNSS observation data to model complex positioning error factors, providing a new idea for improving the positioning accuracy and robustness of GNSS [24,25]. Closas et al. utilized an iterative approach to deal with multivariate optimization problems, and obtained the maximum likelihood estimator of the position under GNSS [26]. Kuratomi et al. explored a GNSS positioning error estimation method based on the random forest algorithm, and obtained excellent error estimation results [27]. Considering that the least squares method ignores the fact that velocity information is time-dependent, Jiang et al. investigated and implemented a GNSS position estimation method based on graph optimization, using historical states and measured values to estimate the current state [28]. By aiming at the problem that the accuracy and stability of multisource fusion navigation still cannot meet the needs of current practical applications, Bao et al. studied a GNSS position error estimation algorithm based on multitask learning [29]. To obtain more accurate GPS positioning accuracy than existing statistical and linear machine learning methods, Yang et al. utilized the long short-term memory network (LSTM) to dynamically model the factors affecting GPS positioning accuracy within a certain coherent time window and then predict the positioning error in the next second to a few seconds. The effectiveness of the presented method was verified through experiments. Implementing the presented LSTM model on hardware requires further considerations [30]. Liu et al. studied an LSTM model with infinite impulse response (IIR) filtering by constructing a dynamic adaptive filter to remove outliers and interference. This innovative solution enhances the robustness of the LSTM network when processing noisy data and achieves better experimental results [31]. Shen et al. studied the unmodeled error using historical GNSS observation data and proposed a positioning error estimation method based on convolutional neural network (CNN). The effectiveness of the presented method was verified through experiments, especially in the low-frequency component [32]. The research is of great significance to error elimination and reliability estimation in the GNSS positioning process.

Traditional GNSS positioning error estimation methods are mostly based on linear assumptions, and cannot effectively capture complex nonlinear relationships, which limits the accuracy of error estimation. GNSS positioning error estimation methods based on deep learning can comprehensively consider multiple error factors, and have powerful nonlinear modeling capabilities, which can effectively improve the accuracy of error estimation. Therefore, a GNSS positioning error estimation method is studied based on an improved multi-layer perceptron (MLP) model. The multi-head attention (MHA) and gating operation are exploited to selectively weight and transform input features, enabling adaptive feature selection and information filtering. Unlike traditional GNSS positioning error estimation methods that rely on precise mathematical models and assumptions, the proposed method is a data-driven deep learning model. The relationship between error factors is learned through historical GNSS data. The proposed method does not require assumptions about error factors, and can adaptively extract features from GNSS data, thereby providing more accurate error estimates. The main contributions of the paper are as follows:

- (1) A MLP model integrating MHA and gating operation is proposed for GNSS positioning error estimation. To our knowledge, no scholars have utilized this method to address the problem of GNSS positioning estimation.
- (2) Traditional GNSS positioning error estimation methods are usually based on linear models or empirical rules. The presented method can model nonlinear relationships more flexibly, thereby achieving higher estimation accuracy.

- (3) Two types of comparative experiments are performed. The experimental results show that the presented method has higher estimation accuracy than other GNSS positioning error estimation methods.

The remainder of the article is structured as follows. In Section 2, the GNSS positioning error estimation problem is formulated. Section 3 focuses on the presented improved MLP model. In Section 4, the performance of the improved MLP model is evaluated, and the experimental results are analyzed in detail. In Section 5, the proposed method is discussed. Finally, a brief summary is presented in Section 6.

2. Problem Formulation

GNSS positioning adopts the principle of trilateration to determine the position of a receiver by measuring the distance between the receiver and multiple satellites. However, the GNSS positioning process may be affected by various error factors. Therefore, accurately estimating the GNSS positioning error is challenging.

In an Earth-centered, Earth-fixed (ECEF) coordinate system, assuming that there are n satellites, the coordinates of the satellites are $S_i = (x_i, y_i, z_i)$, where the value of i ranges from 1 to n . The coordinates of the receiver are $P = (x, y, z)$. Different positioning methods are utilized in the GNSS positioning process, and different observation equations are obtained. The most common GNSS positioning method is pseudorange measurement positioning, which is achieved by measuring the geometric distance from the receiver to the satellite and various error factors. If each error factor is regarded as an independent random variable, the observation equation of the pseudorange measurement model is

$$P_i = \|P - S_i\| + c\Delta t + T_i + I_i + M_i + \varepsilon_i \quad (1)$$

where $\|P - S_i\|$ denotes the geometric distance from the receiver to satellite i , Δt expresses the receiver clock error, T_i means the tropospheric delay, I_i represents the ionospheric delay, M_i denotes the multipath error, and ε_i expresses the measurement error. Notably, due to space reasons, only several important GNSS positioning error factors are shown in (1). In the GNSS positioning, there are other methods, such as carrier phase measurements, which can be applied to satellite positioning.

An observation equation for each satellite is applied to describe the positional relationship between the receiver and the satellite during the GNSS positioning process. Multiple observation equations are obtained by observing multiple satellites at the same time. Then, by solving the system of simultaneous equations from multiple observation equations, the position coordinates $P = (x, y, z)$ of the receiver in the ECEF coordinate system can be solved. The above process can be realized by utilizing the least squares method or other positioning algorithms.

Assume that the true position of the receiver is P_{true} . Then, the positioning error is $\Delta P = P_{\text{true}} - P$. Therefore, the GNSS positioning error estimation problem is

$$P_{\text{true}} = f(D) + \Delta P_{\text{est}} \quad (2)$$

where $f(D)$ expresses the positioning algorithm based on the observation data D and ΔP_{est} denotes the estimated value of the positioning error.

As shown in Figure 1, the presented method aims to estimate the positioning error ΔP as accurately as possible, and make the difference between ΔP_{est} and ΔP as small as possible, to obtain better GNSS positioning error estimation performance.

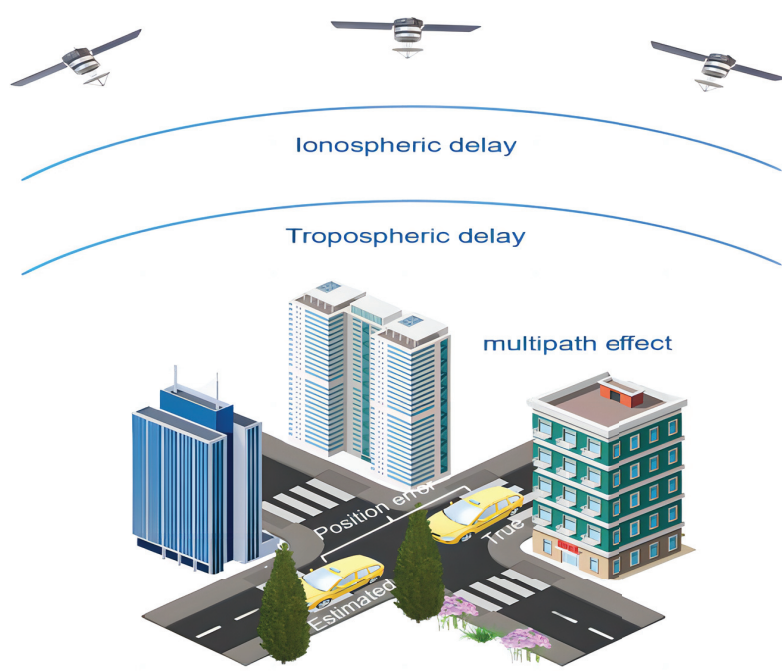


Figure 1. Schematic diagram of positioning error estimation.

The main application scenario of the proposed method is the positioning needs of daily electronic devices, such as smartphones. Although the positioning accuracy of code observation is lower than that of carrier phase observation, the positioning time required for code observation is shorter than that of carrier phase observation, and it is less affected by environmental changes. Therefore, code observation is more suitable for the application scenario of smartphone positioning.

3. Proposed Method

The solution to the GNSS positioning error estimation problem depends on the analysis and processing of historical data. Currently, the common methods of GNSS positioning error estimation tend to ignore the relationships among different types of data, resulting in unsatisfactory estimation accuracy. With the rapid development of artificial intelligence methods, the excellent ability of artificial intelligence methods in data processing and feature learning has been proved. Therefore, after exploring and experimenting with various artificial intelligence methods, a multi-head attention gating unit (MAGU) that combines the MHA and gating operation was designed, and then the MAGU and MLP models were integrated to address the problem of GNSS positioning error estimation. The MHA is beneficial to improving the feature extraction ability of the model. The gating operation can enhance the model's dynamic information selection capability.

The traditional MLP model is a feedforward neural network model. It consists of multiple neural layers. Each MLP neuron is connected to all neurons of the previous layer, and receives input from the previous layer. Weighting calculations and nonlinear transformations are then performed on the input and, finally, the result is passed to the next layer. The explored model represents an enhancement of the traditional MLP model, incorporating a gating operation and a MHA, and updating the network structure and parameters.

According to Figure 2, the overall process of the designed scheme is to first extract relevant parameters from GNSS data and adopt them as feature variables. Next, the positioning error is obtained through the positioning solution, and the positioning error is selected as the target variable. Then, the feature variables and target variables are input

into the improved MLP model for multiple iterations of training. Finally, the estimated value of the positioning error is output.

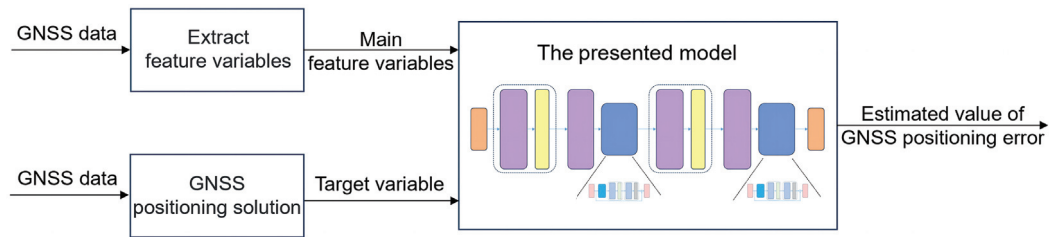


Figure 2. Overall flow chart of the presented scheme.

3.1. Improved MLP Model

Although the traditional MLP model can be used to perform the nonlinear transformation on all input features in each hidden layer, it cannot selectively process different features. Therefore, the presented method integrates the MAGU module into the network structure of the traditional MLP model. This module can dynamically select the input features so that the model can better adapt to the importance and correlation of different features, thus enhancing the feature selection ability of the model.

3.1.1. Model Network Structure

The improved MLP model mainly consists of an input layer, four fully connected layers, two MAGU modules, an output layer, and some activation functions. Four fully connected layers and two MAGU modules constitute the hidden layer of the improved MLP model. The hidden layer mainly performs a series of linear transformations and feature extractions on the input data. The sigmoid activation function is applied in the MAGU module. The fully connected layer uses the rectified linear unit (ReLU) activation function, which can improve the nonlinear ability of the model and speed up its training.

In Figure 3, assuming that the input of the presented model is x , its output after passing through the first fully connected layer and the first ReLU activation function is

$$x_1 = F_1(W_1x + b_1) \quad (3)$$

where W_1 and b_1 denote the weight matrix and bias vector of the first linear layer, x means the input data, and F_1 expresses the ReLU activation function. The ReLU activation function [33] is

$$F_1(x) = \begin{cases} x & \text{if } x \geq 0 \\ 0 & \text{if } x < 0 \end{cases} \quad (4)$$

The output of the second linear layer can be expressed as

$$x_2 = W_2x_1 + b_2 \quad (5)$$

After inputting x_2 into the first MAGU, the output is

$$x_3 = g_1 \odot u_1 + (1 - g_1) \odot x_2 \quad (6)$$

where $\odot$ means elementwise multiplication, g_1 represents the output of the gating operation in MAGU, and u_1 denotes the output of the MHA after linear transformation in MAGU.

The output of x_3 after the third linear layer and the second ReLU activation function can be expressed as

$$x_4 = F_1(W_3x_3 + b_3) \quad (7)$$

The output of the data after the fourth linear layer is

$$x_5 = W_4 x_4 + b_4 \quad (8)$$

After taking x_5 as the input of the second MAGU module, the output can be expressed as

$$x_6 = g_2 \odot u_2 + (1 - g_2) \odot x_5 \quad (9)$$

The final output of the proposed model is

$$y = F_2(W_5 x_6 + b_5) \quad (10)$$

where F_2 denotes the linear activation function. The linear activation function is adopted in the output layer of the improved MLP model. The final estimation result is obtained through the identity mapping of the linear activation function.

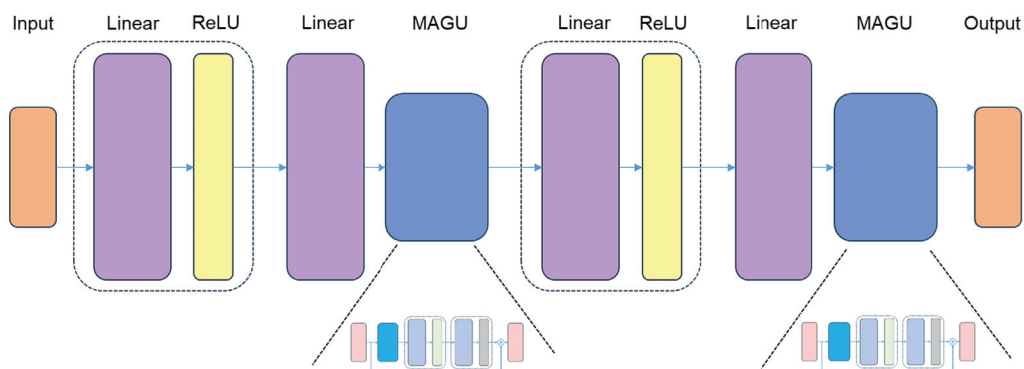


Figure 3. The overall network structure of the improved MLP model.

The number of neurons in the input layer of the presented model is set to be consistent with the number of feature vectors in the dataset. The first and second fully connected layers have 128 neurons. The number of fully connected layer neurons of the first MAGU module is 128. The third and fourth fully connected layers have 64 neurons. The number of fully connected layer neurons of the second MAGU module is 64. The manuscript is studying a regression problem, and the output is a scalar. Therefore, the number of neurons in the output layer is 1.

3.1.2. Model Training and Parameter Setting

The design process of the improved MLP model mainly includes steps such as data preprocessing, model initialization, training parameter setting, loss function definition, and model performance evaluation. First, the input data are standardized and each feature is scaled to a range with a mean of 0 and a standard deviation of 1. The training data and test data in the dataset are divided at a ratio of 7:3. Then, an initialized model, which includes an input layer, a fully connected layer, an MAGU module, and an output layer, is built. Finally, the relevant parameters of the model are set and the positioning error estimation accuracy of the model is evaluated.

The inputs of the presented model in the training phase are feature variables and target variables. Feature variables include carrier-to-noise ratio, satellite clock deviation, etc. The target variable is the positioning accuracy obtained through high-precision positioning solution technology, which is regarded as the true value of the positioning error. The output of the presented model during the training phase is estimated value of the positioning error.

The adaptive moment estimation (Adam) optimizer and L_2 regularization are adopted in the improved MLP model. Gradients are calculated by the backpropagation algorithm,

and the parameters of the model are updated by adopting the optimizer during training. The Adam optimizer combines the characteristics of the momentum method and the adaptive learning rate, effectively updating the parameters and adaptively adjusting the learning rate. $L2$ regularization penalizes model weights, making the weight values tend to a smaller range. It is mainly employed to control model complexity and prevent overfitting to enhance the generalizability of the model. $L2$ regularization is

$$L2_{reg} = L + \lambda \sum_{i=1}^n w_i^2 \quad (11)$$

where L denotes the original loss function, λ means the weight decay coefficient, w expresses the weight, n represents the number of data samples, and i is the index.

The improved MLP model adopts the mean square error (MSE) as the loss function, and the MSE is

$$\text{MSE} = L(y, \hat{y}) = \frac{1}{n} \sum_{i=1}^n (y_i - \hat{y}_i)^2 \quad (12)$$

where n means the number of samples, y_i and $\hat{y}_i$ represent the true value and its estimated one. The smaller the MSE value, the smaller the difference between estimated value and true value.

3.2. Multi-Head Attention Gating Unit

One defect of the traditional MLP model is that it ignores the interdependencies between different time steps in the sequence data when processing sequence data. The presented model aims to solve this defect by adding MAGU to the MLP model. MAGU mainly consists of three parts, namely MHA, gating operation, and linear transformation. The MHA can extract features from different subspaces in parallel and better capture the dependencies between time series data. The gating operation can selectively control the information flow by ignoring irrelevant information or amplifying important information, enabling the improved MLP model to better process sequence data. A linear transformation can map input data into a new representation space, which usually includes matrix multiplication operations and offset addition operations.

The MAGU network structure in Figure 4 mainly includes an input layer, a multi-head attention layer, a gating subunit, a linear transformation operation, and an output layer. The gating subunit contains a linear layer and a sigmoid activation function, which can generate gating weights through the sigmoid function to achieve gating operations. The linear transformation layer includes a linear transformation layer and a linear activation function. Linear transformation refers to linear transformation of the input data and the hidden state of the previous time step, and then mapping them to a new feature space.

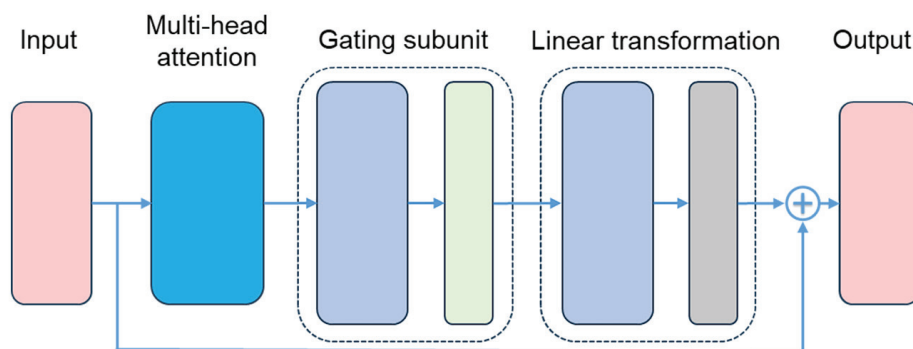


Figure 4. Network structure diagram of the multi-head attention gating unit.

During the forward propagation, the model works as follows: First, an input is received. Then, it is passed to the multi-head attention layer. Subsequently, it is passed to the gating subunit. In the gating subunit, the gating signal is calculated by the sigmoid activation function, and after the calculation is completed, the input is passed to the linear transformation layer. Afterward, the input is linearly transformed in the linear transformation layer. The final output is the result of multiplying the gating signal by the transformation result and adding the input residual.

MAGU is a module that combines MHA and gating operation to enhance the model's ability to process time series data. MHA can capture complex dependencies in the input sequence. Gating operation can improve the nonlinear expression ability of the model by dynamically adjusting the flow of information. Next, the two important components of the MAGU module, the MHA and the gating operation, will be introduced in detail.

3.2.1. Multi-Head Attention Mechanism

The MHA mechanism can be utilized to capture long-term dependencies in the input sequence [34]. Its principle is to adopt multiple attention heads in parallel to capture the features of different subspaces in the input sequence. First, each attention head independently performs a linear transformation on the input. Then, the attention weights are calculated. Finally, the outputs of all heads are concatenated and linearly transformed to obtain the final output.

According to Figure 5, the MHA is mainly composed of four linear layers, a scaled dot product attention layer, and a connection layer. The scaled dot product attention layer includes dot product operation, scaling calculation and softmax operation. The execution process of the MHA is to first pass the input vector through three different linear layers to generate the query matrix, key matrix, and value matrix, respectively. Afterward, the attention score matrix is obtained by calculating the dot product of the query matrix and the key matrix for each attention head. Then, the obtained attention score matrix is divided by a scaling factor to prevent the gradient from disappearing or exploding. Subsequently, a softmax operation is performed on the scaled attention score matrix to obtain the attention weight matrix. Then, the attention weight matrix is multiplied by the value matrix to obtain the weighted value matrix. Repeat the above steps for each attention head. Finally, the outputs of all attention heads are connected and passed through a linear layer to obtain the final output of the MHA mechanism.

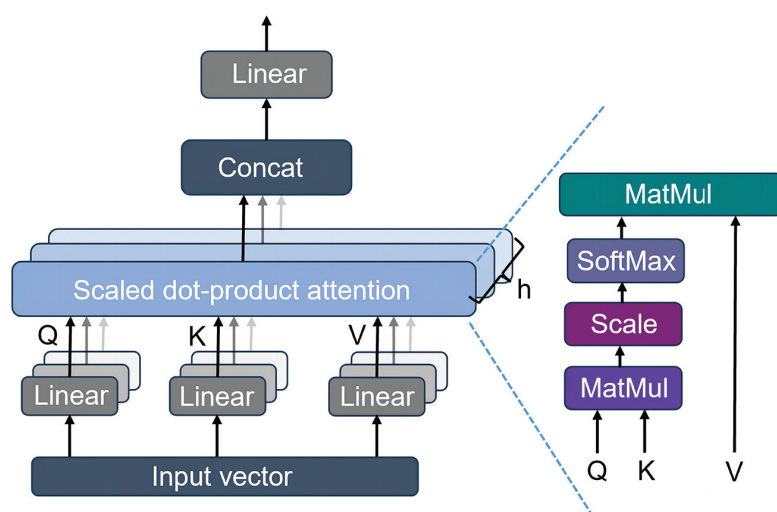


Figure 5. Structure diagram of the multi-head attention mechanism. Q, K, V represent the query, key, and value of the attention head, respectively.

Assume that the input vector passes through a linear layer to obtain the query, key, and value of each attention head.

$$Q_i = xW_i^Q, K_i = xW_i^K, V_i = xW_i^V \quad (13)$$

where W expresses the weight matrix and the subscript i represents the i -th attention head.

The output of each attention head can be expressed as

$$H_i = \text{softmax}\left(\frac{Q_i K_i^T}{\sqrt{d_k}}\right) V_i \quad (14)$$

where d_k represents the dimension of the key vector.

Then, the outputs of all attention heads are concatenated and passed through another linear transformation to obtain the final output.

$$M(Q, K, V) = \text{Concat}(h_1, \dots, h_n) W^O \quad (15)$$

where W^O means the weight of the last linear transformation layer, and h denotes the total number of attention heads.

The presented model can focus on information at different positions in the input sequence through the MHA, which improves the proposed model's ability to understand and express the input data. It is worth noting that traditional recurrent neural networks, which are commonly used for time series feature extraction, cannot be computed in parallel, while the MHA can be computed in parallel.

3.2.2. Gating Operation

The gating mechanism regulates the flow of information by introducing control signals. The gating weights are generated by the sigmoid function, and the input and the transformed results are weighted and summed to achieve feature screening. Since the sigmoid function has a smooth curve characteristic, it can perform a nonlinear transformation on the input and help the model adapt to nonlinear relationships to learn more complex features.

The gating operation consists of a linear layer and a sigmoid activation function. Its execution process is to first receive the input from the MHA. Then, the input is linearly transformed through the linear layer. Subsequently, the gating weight is generated through the sigmoid activation function. Finally, the gating weight is passed to the next layer of the network.

The output after the gating operation can be expressed as

$$g_i = \sigma(W_g H_i + b_g) \quad (16)$$

where W_g denotes the weight matrix of the gating operation, b_g is the bias vector of the gating unit, H_i expresses the output of the MHA in the corresponding layer, and σ represents the sigmoid activation function.

The sigmoid activation function [35] is

$$f(x) = \frac{1}{1 + e^{-x}} \quad (17)$$

where e means the basis of the natural logarithm. The sigmoid function can map the input x into a continuous value between 0 and 1.

Since the activation function in the gating operation is differentiable, this facilitates the use of the backpropagation algorithm to calculate the gradient and adopt the optimizer to update the model parameters during training to minimize the loss function. The gating

operation can promote the stable propagation of gradients, which can alleviate the problem of gradient vanishing or gradient exploding to a certain extent.

The MHA helps the model focus on important features. The gating operation can further filter the features. The adoption of MHA and gating operation enables the MLP model to effectively capture long-term dependencies in the input data, thereby improving the performance of the model.

4. Experimental Process and Result Analysis

To clearly demonstrate the performance of the proposed method, three types of experiments are conducted. First, the estimation accuracy of the presented method is comprehensively demonstrated. Then, the proposed method is compared with the most commonly used methods and the current state-of-the-art methods. Finally, an ablation experiment is performed to verify the effectiveness of the MAGU module.

Two datasets were adopted in the experiments. The datasets were collected on tree-lined streets and roads with wide views. The data were collected dynamically with a collection interval of 1 s. The true position information of the receiver was obtained from the NovAtel SPAN system. The adopted dataset contains code measurements and does not contain carrier phase measurements. The test environment is Python 3.9. The hardware is supported by RTX 3090 GPU, whose total memory is 24 GB. The first dataset was collected and publicly released by Fu et al., which consists of GNSS positioning data collected by multiple low-cost receivers in the San Francisco area of the United States [36]. The second dataset was collected and publicly released by Kuratomi et al. [27]. Both datasets were used for comparative experiments and ablation experiments. Applying the same dataset for comparison can improve the persuasiveness of the comparison experiment to a certain extent.

The experimental process of employing the first dataset is mainly as follows. First, the dataset collected and publicly released by Fu et al. was obtained. Secondly, the original positioning error of the receiver is obtained by the Kalman filter (KF) method. Then, the parameters that may affect the positioning error are extracted from the dataset as feature variables, and the obtained positioning error is adopted as the target variable. Finally, the feature variables and target variables are input into the improved MLP model together, and the estimated results of the positioning error are output after multiple iterations of training. Next, each step will be introduced in detail, and corresponding diagrams will be given.

4.1. Obtain the Original Positioning Error

The KF method is used to obtain the original positioning error in the first dataset. The positioning process not only includes a series of processing and analysis of satellite observation data and navigation message data, but also includes the correction and elimination of various error factors such as satellite clock deviation and ionospheric delay. The second dataset includes original positioning error data. Therefore, the step of calculating the original satellite positioning error can be ignored during the experiment using this dataset, and the subsequent steps can be directly carried out.

According to Figure 6, the calculated positioning error fluctuates greatly, with a minimum error of about 0.1 m and a maximum error of more than 6 m. The average positioning error is calculated to be 2.840 m. Notably, the purpose of the designed method is mainly to explore positioning error estimation, which does not depend too much on the accuracy of the original positioning error. Therefore, not only the KF method can be used to obtain the value of the positioning error, but also other GNSS positioning methods can be utilized to obtain the value of the positioning error.

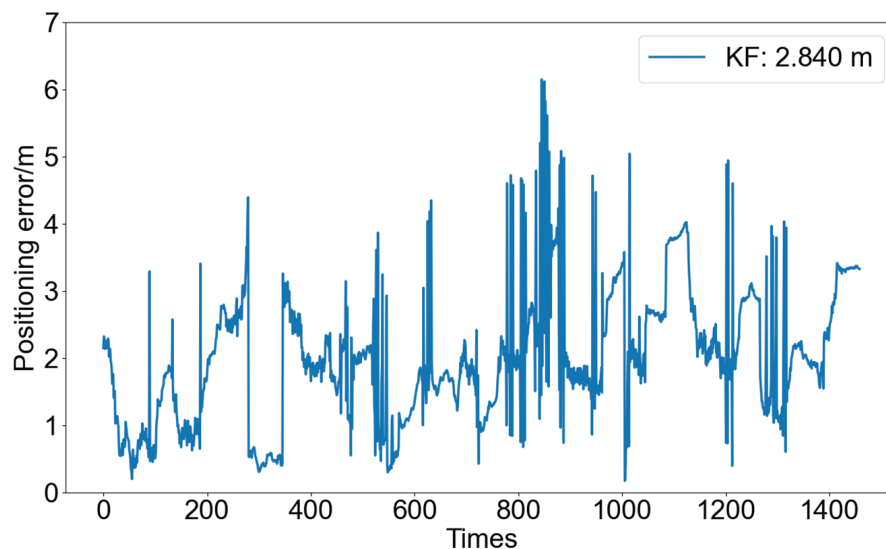


Figure 6. Positioning error results are obtained through the Kalman filter method.

4.2. Preprocessing the Experimental Dataset

The original satellite observation and navigation message files contain many parameters. However, not all of these parameters are directly related to the positioning error estimation. Therefore, the data in these files need to be preprocessed. Data preprocessing includes unifying the data value range and preliminary feature screening. Since the value ranges of different data are different, it is necessary to standardize the data. The standardization operation is to subtract the mean of the same type of data from the data to be standardized and divide it by the standard deviation, so as to achieve the purpose of scaling different data in proportion.

The main features in the dataset and their corresponding descriptions are listed in Table 1. These features are utilized in the training and testing processes of the improved MLP model. The quality of the feature selection strategy will directly affect the GNSS positioning error estimation accuracy of the presented method. The improved MLP model mainly selects features through MHA and gating operation. Then, the weights are updated according to the gradient, thereby gradually optimizing the performance of the model.

Table 1. The main features in the dataset and their corresponding descriptions.

Feature	Description
Satellite clock difference	The difference in time between an atomic clock on a satellite and a reference clock on Earth.
Satellite clock deviation	The offset of an atomic clock on a satellite from its theoretically precise time.
Satellite elevation	The angle between the satellite and the horizon where the receiver is located.
Ionospheric delay	The ionospheric delay means that when a satellite signal passes through the ionosphere, the free electrons in the ionosphere will scatter and reflect the satellite signal, thereby changing the propagation path of the satellite signal. The ionospheric delay is obtained by the Klobuchar model.
Satellite azimuth	The angle between the line from the ground observation point to the satellite and the due north direction.
Tropospheric delay	The tropospheric delay means that when a satellite signal passes through the atmosphere, the water vapor in it will cause the satellite signal to refract, thereby changing the propagation path of the satellite signal. The tropospheric delay is obtained by the Saastamoinen model.
Carrier-to-noise ratio	The ratio of the carrier power of the received signal to the noise power in the surrounding environment.
Satellite coordinate information	The satellite position information in the ECEF coordinate system is calculated using the satellite broadcast ephemeris.

4.3. Performance Analysis of the Improved MLP Model

Next, the improved MLP model is trained and tested with the two datasets. To demonstrate the performance of the presented model more intuitively, the root mean square error (RMSE) is adopted in the experiments as the evaluation index of the estimation accuracy of the model. The smaller the RMSE value is, the closer the estimated value is to the true value, and the higher the accuracy of the model. The formula for calculating the RMSE [37] is

$$\text{RMSE} = \sqrt{\frac{1}{n} \sum_{i=1}^n (y_i - \hat{y}_i)^2} \quad (18)$$

where n denotes the number of samples, y_i represents the true value, and $\hat{y}_i$ represents the estimated value.

The x -axis and y -axis in Figure 7 are the training epochs and loss values of the model during the training process, respectively. A total of 500 epochs are iterated during the model training process. In accordance with Figure 7, the loss of the model is continuously reduced during the training process and is ultimately maintained at a low value, demonstrating that the model has converged normally and achieved better performance.

The general idea of GNSS positioning is adopted, and the output results of the model are divided into three components: East, North, and Up. Figure 8 shows the experimental results of the proposed model on the three components of East, North, and Up. It can be seen from Figure 8 that the trained and converged model has a high estimation accuracy.

After the two datasets are input into the model separately, the model gradually converges after several iterations of training. The presented method obtains RMSE values of 0.239 m and 0.029 m for the first and second datasets, respectively. The random forest (RF) algorithm is widely adopted in the field of GNSS positioning error estimation, and it has high estimation accuracy. Therefore, to demonstrate the performance of the presented method more clearly, the experimental results of the presented method and the RF algorithm will be displayed and comparatively analyzed next.

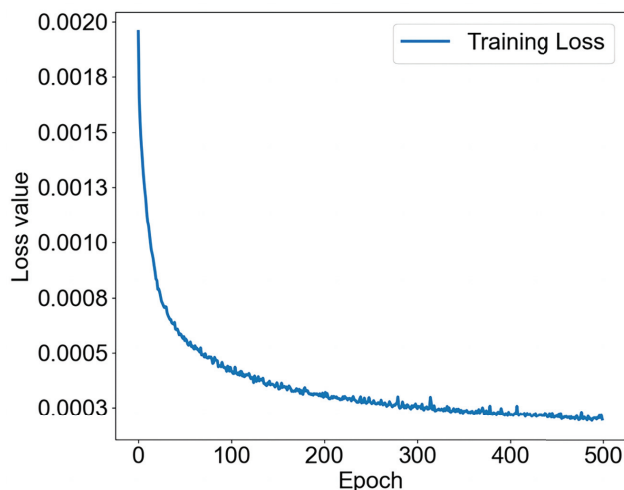


Figure 7. Image of the loss function of the presented model during training.

The gray portion in Figure 9 indicates the confidence interval for the positioning error estimate. The confidence interval is denoted by the area of the estimated value minus two standard deviations and the estimated value plus two standard deviations. The confidence interval is a concept from statistics, which means that the true value has a certain probability of falling within a certain interval around the estimated value. The confidence interval is an interval range and is often used to indicate the credibility of the estimated value. Applying confidence intervals to the problem of positioning error

estimation is mainly used to describe the uncertainty range of positioning error estimation, which is conducive to evaluating the reliability and stability of the proposed model.

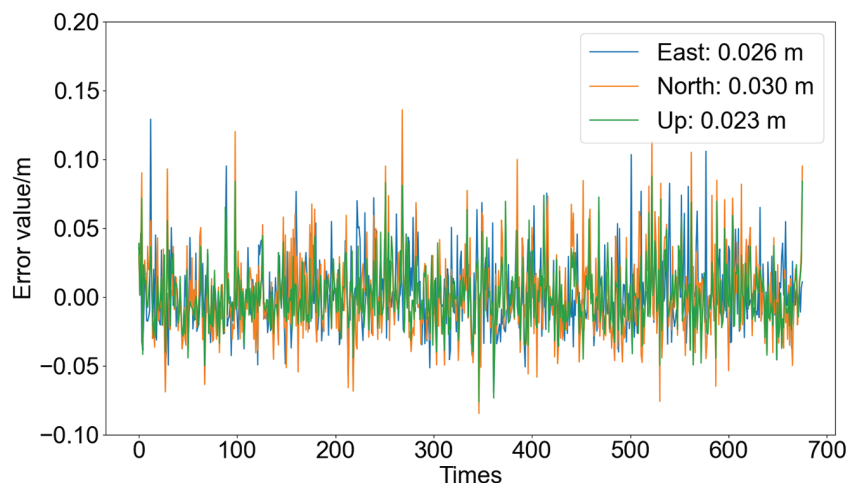


Figure 8. Experimental results of the proposed model on the three components of East, North, and Up.

As shown in Figure 9, the estimated values of the presented method have a large repetition range with respect to the true values, and they are basically within the confidence interval, which shows that the estimation results of the positioning errors of the presented method have high accuracy. However, the RF algorithm is sensitive to noise and outliers in the data, which affects the estimation accuracy of the algorithm. On the whole, the presented method has a larger repetition range of estimated values and true values compared to the RF algorithm. The estimation accuracy of the presented method is significantly better than the RF algorithm.

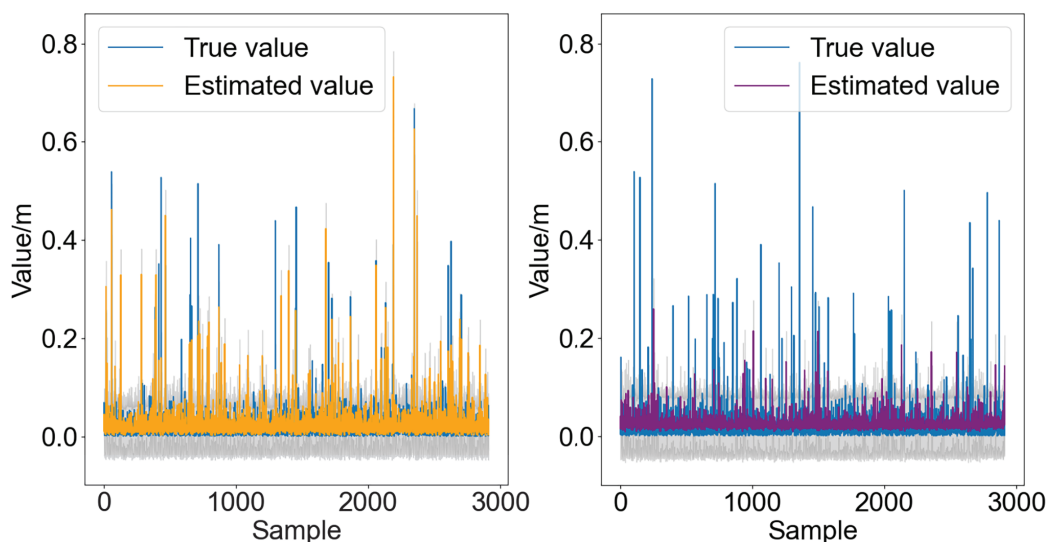


Figure 9. Comparison of the estimated and true values of the presented method (left) and the random forest algorithm (right).

It can be seen from Figure 10 that most of the estimation errors of the presented method are approximately distributed around 0, and the difference does not generally exceed 0.1. This shows that the estimated values obtained by the presented method are more accurate and have better performance. The RF algorithm is a model based on decision trees that cannot handle the interdependence in time series data well. Compared with the

RF algorithm, the presented method has a more concentrated error distribution and the overall error estimate is smaller.

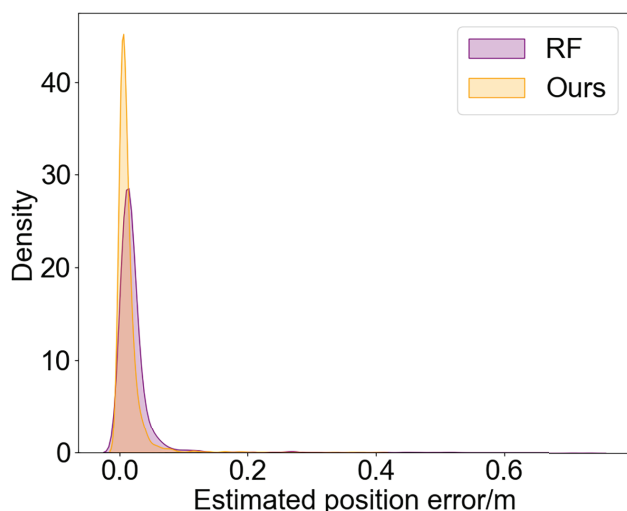


Figure 10. Comparison of the estimation error distributions between the presented method and the RF algorithm.

The Stanford diagram can show how well the estimated positioning error matches the original positioning error. The closer to the original positioning error the estimated positioning error is, the closer the data points are to the diagonal. According to Figure 11, most of the data points of the presented method are in the lower left corner of the graph, and only a very small number of data points are distributed on either side of the diagonal. This shows that the presented method obtains more accurate estimates. However, part of the data in the positioning error estimate obtained by the RF algorithm is biased, and there are a large number of data points where the positioning error estimate is less than the true value of the positioning error.

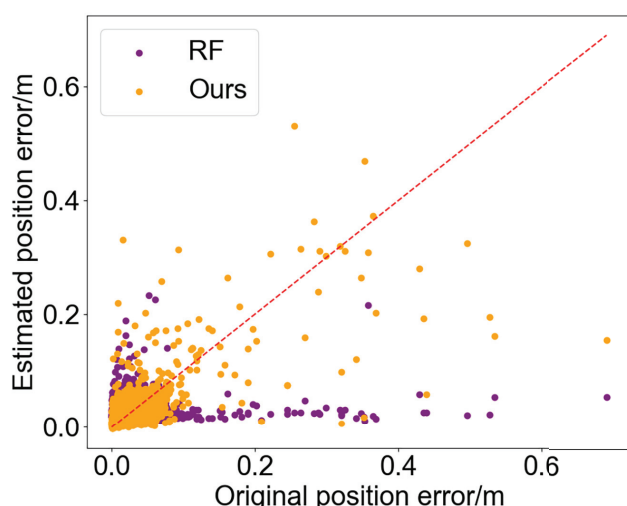


Figure 11. Stanford diagram with the presented method and the RF algorithm. The red dotted line indicates the diagonal line.

A residual is defined as the difference between the true value and an estimated value. The residual histogram clearly shows the residual distribution of the proposed method. The x -axis and y -axis in Figure 12 are the size and frequency of the residual value, respectively. Based on Figure 12, the residual histogram of the presented method has a higher kurtosis than the residual histogram of the RF algorithm, and the peak value is closer to 0. This

shows that the residual distribution of the presented method is relatively concentrated, the overall value is small, and it has better estimation performance.

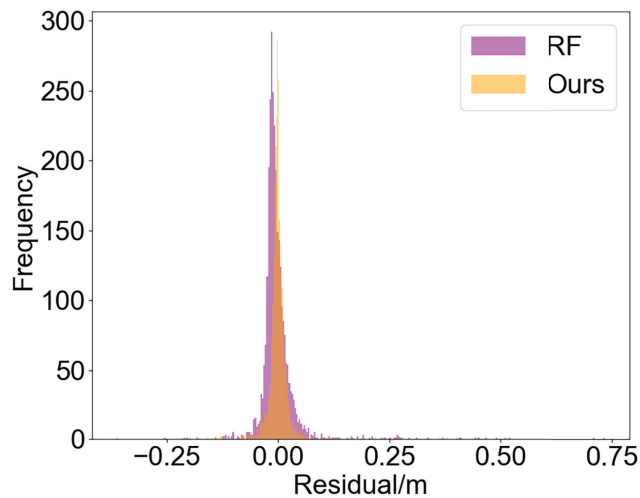


Figure 12. Residual histogram with the presented method and the RF algorithm.

Table 2 records the quantitative analysis results of the proposed method and the random forest algorithm. RMSE, mean absolute error (MAE), and standard deviation are used as evaluation indicators. It can be seen from Table 2 that the performance of the proposed method is better than that of the random forest algorithm in all three evaluation indicators.

Table 2. Quantitative analysis experimental results of the proposed method and random forest algorithm (unit: m).

Methods	RMSE Value	MAE Value	Standard Deviation
RF	0.047	0.022	0.016
Ours	0.029	0.014	0.010

4.4. Comparative Experiments

To better demonstrate the performance of the presented method, two comparative experiments are carried out. The first comparative experiment is between the presented method and other common methods, and the second comparative experiment is between the presented method and methods studied by other scholars.

4.4.1. Comparison of the Presented Method with Other Common Methods

Common methods in the GNSS positioning error estimation research mainly include the extended Kalman filter (EKF), linear regression (LR), support vector machine (SVM), decision tree (DT), and RF. These five common methods and the presented method are compared in the first comparative experiment.

By incorporating the MAGU module into the MLP model, the presented method minimizes the influence of unimportant features on the model, enabling the model to achieve better performance. It can be seen from Table 3 that the presented method obtains smaller RMSE values for both datasets and has obvious advantages over other common methods. This shows that the estimation accuracy of the presented method is higher.

Table 3. Comparative experimental results of the proposed method and five common methods (unit: m).

Methods	RMSE Value on the First, Dataset	RMSE Value on the Second, Dataset
EKF	1.324	0.118
LR	0.864	0.071
SVM	0.618	0.065
DT	0.527	0.060
RF	0.473	0.047
Ours	0.239	0.029

4.4.2. Comparison of the Presented Method with the Current State-of-the-Art Method

To our knowledge, the current state-of-the-art methods are the long short-term memory network model studied by Yang et al. [30] and the convolutional neural network model designed by Shen et al. [32] when dealing with the GNSS positioning error estimation problem. Therefore, the proposed method is compared with two state-of-the-art GNSS positioning error estimation methods to show the superiority of the presented method in the second comparative experiment.

The LSTM model is a method commonly applied to predict time series data. It usually consists of input gate, forget gate, output gate, and cell state. Compared with the MLP model, it requires additional gating units and loop connections. The CNN models are mostly used to process image data, and are not suitable for processing data containing geographic information. It can be seen from Table 4 that the RMSE value of the presented method is significantly smaller than that of the other two methods, especially compared with the LSTM method. The proposed method can adaptively choose which features to retain based on the importance of the input features. This characteristic can improve the representation ability and generalization performance of the model to a certain extent. Since the experiment was conducted with the same dataset, the results of the comparative experiment are convincing to a certain extent.

Table 4. Comparative experimental results of the proposed method and two state-of-the-art methods (unit: m).

Methods	LSTM [30]	CNN [32]	Ours
RMSE value on the first dataset	0.393	0.288	0.239
RMSE value on the second dataset	0.039	0.033	0.029

4.5. Ablation Experiment

To verify the effectiveness of the MHA mechanism and gating operation, ablation experiments are performed. The ablation experiment consists of four groups of experimental objects. The control variable method was adopted in the ablation experiment. Four groups of experimental subjects were tested separately under the premise that other parameters remained unchanged.

The MLP model without the MHA mechanism and the gating operation has difficulty effectively processing different features in the data due to the lack of feature importance screening ability. Therefore, it can be seen from Table 5 that the RMSE values obtained by the MLP model with the MHA mechanism and the gating operation are smaller than those of the MLP model without the MHA mechanism and the gating operation on both datasets. This shows that the proposed model gives full play to the advantages of the MHA mechanism and the gating operation, and can better learn and extract features in the data and obtain more accurate estimation accuracy. In addition, from the experimental results of the MLP model with only the gating operation and the MLP model with only the MHA

mechanism, it can be seen that the MHA mechanism improves the model performance more than the gating operation.

Table 5. Ablation experiment results of the proposed model (unit: m).

Gating Operation	MHA	RMSE on the First, Dataset	RMSE on the Second, Dataset
		0.449	0.042
✓		0.427	0.041
	✓	0.398	0.038
✓	✓	0.239	0.029

5. Discussion

In applications that require high positioning accuracy, if the errors of the GNSS positioning system are not promptly warned or corrected, serious consequences may occur. The purpose of studying GNSS positioning error estimation methods is to evaluate and correct errors, ensure that GNSS can still provide high-quality positioning services in complex environments, and avoid large deviations in positioning results. As can be seen from Table 3, the proposed method achieves a lower RMSE value than the traditional method. This is because the traditional GNSS positioning error estimation method has the following shortcomings:

- (1) Traditional positioning error estimation methods rely on precise assumptions, and have difficulty accurately reflecting the error size in complex environments. Therefore, the estimation accuracy is usually low.
- (2) There are large differences in orbital errors, clock biases, and other factors between different satellite systems when multiple GNSS systems are used together. Traditional GNSS positioning error estimation methods have difficulty compensating for errors between multiple systems at the same time, and are prone to cause accumulation and cross-interference of errors between multiple systems.

However, the GNSS positioning error estimation method based on deep learning has powerful nonlinear modeling capabilities. It can learn the trend of error changes from historical data to effectively handle complex error factors, and has the potential to further improve accuracy with the support of large-scale data. Therefore, a new multi-layer perceptron model is proposed for GNSS positioning error estimation. Multi-head attention mechanisms and gating operations are designed to improve the standard MLP model. The improved MLP model can better capture the relationship between input features by combining multi-head attention mechanism and gating mechanism. It can be seen from the two types of comparative experiments that the proposed model not only has better performance than other commonly used methods, but also has obvious advantages over the current state-of-the-art methods.

Our study aims to demonstrate the potential of the presented method in dealing with real-time position error estimation problems. However, in actual applications, factors such as data transmission delay, data reliability, and application performance requirements may need to be considered. In the face of building occlusion in urban environments and signal interference in natural environments, additional measures need to be taken to deal with data uncertainty to improve the stability and reliability of position estimation.

6. Conclusions

Traditional GNSS positioning error estimation methods rely on prior assumptions about observation noise and motion models, and have difficulty handling the complex

nonlinear relationships between different error factors. However, the GNSS positioning error estimation method based on deep learning does not require prior assumptions. It can extract effective information from a large amount of historical data, and capture the complex nonlinear relationship between different error factors, thereby improving the accuracy of positioning error estimation. Therefore, a GNSS positioning error estimation method based on deep learning is explored. The nonlinear modeling ability and feature selection ability of a MLP model are improved by introducing a multi-head attention mechanism and gating operations into the MLP model. Experimental results show that the presented method has better performance than that of the extended Kalman filter, linear regression, decision tree, support vector machine, and other methods for two datasets, and the presented method is also better than the current state-of-the-art long short-term memory network and convolutional neural network considering comparative experiments. This shows that the presented method can reduce the influence of errors on the positioning results to a certain extent, and has more accurate positioning error estimation capabilities. Currently, the proposed model only estimates the positioning error based on GNSS data. However, other sensors, such as IMU and visual sensors, also provide valuable information in practical applications. In the future, we will explore how to fuse data from different sensors to further improve the error estimation accuracy of the model in complex environments.

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Article

Analysis of Multi-GNSS Multipath for Parameter-Unified Autocorrelation-Based Mitigation and the Impact of Constellation Shifts

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Abstract: Multipath effects can significantly reduce the accuracy of GNSS precise positioning. Traditional methods, such as sidereal filtering and grid-based approaches, attempt to model and mitigate these errors by leveraging the spatial autocorrelation of multipath based on residuals. However, these methods can only approximately handle spatial autocorrelation data, limiting their effectiveness. This study investigates the spatial cross-correlation of residuals between various GNSS frequency bands, analyzes their covariance function parameters, and evaluates the impact of constellation shifts on long-term multipath mitigation. Based on this, a simplified autocorrelation-based approach utilizing unified covariance parameters for multipath mitigation is proposed, with its efficacy assessed for both short- and long-term applications. The study demonstrates the correlation of multipath effects across various GPS and Galileo frequencies, including GPS L1/L2/L5 and Galileo E1/E5a/E5b/E5ab/E6, by analyzing correlation coefficients. A strong correlation (greater than 0.8) is observed between residuals of closely spaced frequencies, such as E5b and E5ab, despite their frequency differences. Additionally, the covariance parameters of the residuals are found to be consistent across all frequencies for a baseline, suggesting that unified parameters can be applied effectively for spatial autocorrelation-based multipath mitigation without sacrificing performance. The orbit shifts of certain GPS satellites, particularly G02, G20, and G21, result in significant changes in orbital parameters and satellite tracks, reducing the effectiveness of long-term multipath mitigation. However, the impact of GPS orbit shifts can be minimized through periodic model updates or by integrating GPS and Galileo modeling. In experiments, the LSC correction strategy based on a GPS/Galileo combination, utilizing unified parameters, outperforms the grid method based on the GPS/Galileo combination, improving the mean residual variance elimination rate by 11.3% for GPS L1 and 11.4% for Galileo E1. These improvements remain consistent, with rates of 11.3% and 15.7%, respectively, even on DOY 365, which is 327 days after the modeling data were collected.

Keywords: multi-GNSS; multipath; least-squares collocation; orbit shift; spatiotemporal correlation

1. Introduction

The signal received by the GNSS receiver antenna is a combination of the direct line-of-sight (LOS) signal and signals reflected and/or diffracted from nearby objects. The distortion of the desired LOS signal, caused by undesired non-LOS signals, results in tracking errors in the receiver and subsequent code and phase measurement errors, with the maximum theoretical magnitude extending to 1/2 of the ranging code chip and 1/4 of the carrier wavelength [1]. While notable advancements have been achieved in mitigating various systematic errors in high-precision GNSS positioning, multipath propagation continues to persist as a prominent error source due to its inherent challenges in parameterization and its resistance to differencing techniques.

The most straightforward way to mitigate multipath effects is by selecting observation environments that minimize reflections near the receiver antenna. However, this approach is often impractical, especially in complex environments. As a result, researchers have explored various methods to address multipath errors, including both hardware enhancements and software processing techniques. Hardware solutions typically focus on improving antenna designs to reduce multipath, such as using right-handed circular polarization antennas [2], choke ring antennas [3], helix antennas [4], and narrow correlator delay-locked loops [5]. While these approaches can lessen the impact of multipath, they are only partially effective and often come with significant costs. Software-based methods involve developing analytical or simulated models for calibration. These approaches include weighting GNSS observation data based on the signal-to-noise ratio [6,7] or carrier-to-noise-power-density ratio [1,8]. Additionally, multipath effects can be extracted or suppressed in the frequency domain using filtering techniques. Common methods include the Vondrak filter [9], wavelet analysis [10,11], and the Kalman filter [12].

Additionally, two methods based on the spatiotemporal repeatability of multipath are commonly used for real-time mitigation in static or quasi-static environments. The first, known as the sidereal filtering (SF) method, exploits the near-daily repetition of GNSS satellite constellations. This repetition leads to recurring multipath effects on almost a daily basis, allowing residuals from post-processing to be applied to correct current measurements [13–15]. Essentially, SF uses constellation repetition to identify the closest residuals from post-processing and applies them to mitigate the multipath error in the current observation. The second method is the grid-based technique, which mitigates multipath effects by stacking residuals within azimuth and elevation grids, usually $1^\circ \times 1^\circ$ [16–20]. Both methods aim to model and calibrate multipath errors by leveraging spatial autocorrelation.

However, these methods handle spatial autocorrelation data only approximately, limiting their effectiveness. Actually, spatial autocorrelation can be more accurately modeled using covariance functions, applied through techniques such as Least-Squares Collocation (LSC). These functions can be homogeneous and isotropic, depending only on distance and exhibiting rotational symmetry, or anisotropic, where spatial orientation affects their values [21–24]. This approach significantly improves the precision of spatial autocorrelation modeling compared to traditional sidereal and grid methods.

Moreover, the growing number of satellites and the increasing frequency overlap among GNSS systems have made signal compatibility and interoperability key trends in high-precision data processing for multi-GNSS applications. These advancements offer new perspectives on improving multipath correction. Research by Lu et al. [25] highlights a strong correlation in the residuals of overlapping frequencies across GPS, BDS-3, and Galileo at the same spatial location. Similarly, Geng et al. [26] demonstrate that the interoperability of GPS, Galileo, and BDS-3 enhances spatial resolution, improves the efficiency of multipath modeling, and boosts correction performance. These findings suggest that using overlapping frequencies can significantly improve multipath mitigation. However, there remains considerable interest in exploring the common multipath characteristics across various GNSS frequencies, including non-overlapping ones. Understanding these characteristics could further enhance the effectiveness of spatial autocorrelation-based multipath mitigation strategies.

Additionally, due to various perturbation factors, all satellite orbits experience shifts [27–29], which result in the residual values of satellites after an orbit drift no longer accurately representing the multipath errors of their pre-drift positions. Consequently, multipath modeling must be reconstructed, and new residuals from post-processing are required to account for these shifts. In contrast, if the constellations remain stable and the surrounding environment of the station remains unchanged, a single model can be used consistently over time without the need for updates. However, it remains unclear whether all satellite constellations drift far enough to significantly impact multipath mitigation. Therefore, investigating the extent of constellation drifts within GNSS systems and their potential effects on multipath mitigation is of particular interest.

To overcome the limitations of traditional multipath mitigation methods and better leverage the spatial autocorrelation of GNSS data, this study proposes a simplified autocorrelation-based multipath mitigation method utilizing unified covariance parameters. Specifically, this method investigates the spatial cross-correlation of residuals across different GPS and Galileo frequencies, analyzing the relationship between the covariance function parameters of the residuals and evaluating the impact of constellation shifts on long-term multipath mitigation. Based on this analysis, the method applies multi-GNSS data for multipath mitigation, simplifying the modeling process while maintaining the performance, thereby improving the short- and long-term positioning accuracy.

This study introduces the multipath modeling techniques and the mathematical tools used for analysis in Section 2. Section 3 explores the cross-correlation of residuals across different GNSS frequencies, the spatial autocorrelation parameters of these residuals, and an analysis of orbit shifts. The study concludes with a summary of the findings in Section 4.

2. Multipath Modelling and Mathematic Tools for Analysis

2.1. Multipath Models

The equation for GNSS carrier phase observations is given by

$$\phi_{r,i}^s = \rho_r^s + c \cdot (\delta t_r - \delta t^s) - I_{r,i}^s + T_r^s + \lambda_i N_{r,i}^s + b_{r,\phi_i} - b_{\phi_i}^s + M_{r,\phi_i}^s + \varepsilon_{r,\phi_i}^s \quad (1)$$

where $\phi_{r,i}^s$ is the carrier phase observation value on frequency point i ; the superscript s represents the GNSS satellite PRN (Pseudo-Random Noise) number; the subscript r represents the receiver identifier; ρ_r^s is the geometric distance between the satellite and the receiver; c is the speed of light in a vacuum; δt_r and δt^s , respectively, represent the receiver clock errors and satellite clock errors; $I_{r,i}^s$ is the ionospheric delay; T_r^s denotes the tropospheric delay; λ_i is the carrier wavelength and $N_{r,i}^s$ is the ambiguity; b_{r,ϕ_i} and $b_{\phi_i}^s$ are the receiver phase bias and satellite phase bias, respectively; M_{r,ϕ_i}^s is the multipath errors; and ε_{r,ϕ_i}^s is the noise term and other unmodeled errors.

Double-difference is a commonly adopted strategy in short baseline positioning, which effectively mitigates correlated errors at both the receiver and satellite ends. Consequently, the carrier phase observation equation is transformed as follows.

$$\nabla \Delta \phi_{r_1,r_2,i}^{s_1,s_2} = \nabla \Delta \rho_{r_1,r_2}^{s_1,s_2} + \lambda_i \nabla \Delta N_{r_1,r_2,i}^{s_1,s_2} + \nabla \Delta M_{r_1,r_2,\phi_i}^{s_1,s_2} + \nabla \Delta \varepsilon_{r_1,r_2,\phi_i}^{s_1,s_2} \quad (2)$$

where $\nabla \Delta$ is the double-difference operator, s_1 and s_2 , respectively, represent any two satellites from the same GNSS system, and r_1 and r_2 are the two receivers located at the endpoints of the baseline. If the station coordinates are known with high precision, the baseline vector and integer ambiguity can be solved, leaving only the residual term primarily composed of the multipath effect and measurement noise.

$$\nabla \Delta I_{r_1,r_2}^{s_1,s_2} = \nabla \Delta M_{r_1,r_2,\phi_i}^{s_1,s_2} + \nabla \Delta \varepsilon_{r_1,r_2,\phi_i}^{s_1,s_2} \quad (3)$$

To avoid the impact of reference satellite changes on the residuals, we applied a “zero-mean” constraint to obtain the single-difference carrier-phase residuals from double-difference residuals [26]. We therefore have

$$\begin{bmatrix} w_1 & w_2 & w_3 & \cdots & w_n \\ 1 & -1 & 0 & \cdots & 0 \\ 1 & 0 & -1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & 0 & 0 & \cdots & -1 \end{bmatrix} \begin{bmatrix} \Delta I_{r_1,r_2}^{s_1} \\ \Delta I_{r_1,r_2}^{s_2} \\ \Delta I_{r_1,r_2}^{s_3} \\ \vdots \\ \Delta I_{r_1,r_2}^{s_n} \end{bmatrix} = \begin{bmatrix} 0 \\ \nabla \Delta I_{r_1,r_2}^{s_1,s_2} \\ \nabla \Delta I_{r_1,r_2}^{s_1,s_3} \\ \vdots \\ \nabla \Delta I_{r_1,r_2}^{s_1,s_n} \end{bmatrix} \quad (4)$$

where $w_1, w_2, \dots, w_n$ refer to the weight of the satellite; and $\Delta I_{r_1,r_2}^{s_1}, \Delta I_{r_1,r_2}^{s_2}, \dots, \Delta I_{r_1,r_2}^{s_n}$ are the single-difference residuals which will be used to construct the multipath models.

2.2. Interpolation with Grid Method

The grid method for GNSS multipath mitigation models the spatial distribution of multipath errors by partitioning the sky into a grid structure based on azimuth and elevation angles. The method assumes that multipath errors exhibit spatial repeatability, meaning that errors observed at a particular position in the sky can be used to estimate future errors in the same or nearby locations. This approach allows for more precise error correction over time. Mathematically, the grid method can be described as follows.

First, the sky is divided into grids with resolutions determined by azimuth angle θ and elevation angle ϕ . For example, a $1^\circ \times 1^\circ$ grid resolution means that the sky is divided into cells of size $\Delta\theta = 1^\circ$ and $\Delta\phi = 1^\circ$, resulting in $N_{az} \times N_{el}$ total grid cells, where N_{az} and N_{el} are the number of divisions in azimuth and elevation, respectively.

Then, over multiple observation days, the post-processing residuals $l(t, \theta, \phi)$, where t represents time, are collected for each grid cell. These residuals capture the differences between the observed and true GNSS signals, with the assumption that multipath errors dominate these residuals. For each grid cell, we compute the average residual:

$$\bar{l}(\theta_i, \phi_i) = \frac{1}{n} \sum_{k=1}^n l(t_k, \theta_i, \phi_i) \quad (5)$$

where $l(t_k, \theta_i, \phi_i)$ is the residual at time t_k for the grid cell defined by azimuth θ_i and elevation ϕ_i , and n is the number of observations for that grid.

During subsequent observations, the estimated multipath error for each satellite position is corrected by subtracting the corresponding average residual from the raw GNSS observation. For a satellite positioned at azimuth θ_i and elevation ϕ_i , the corrected observation O_{corr} is

$$O_{corr}(t) = O_{obs}(t) - \bar{l}(\theta_i, \phi_i) \quad (6)$$

where $O_{obs}(t)$ is the raw observation at time t .

In summary, the grid method mitigates GNSS multipath errors by constructing a spatial model of the errors through grid partitioning and residual averaging. This mathematical framework helps to reduce multipath-induced inaccuracies in GNSS observations by using past data to predict and correct future signal distortions.

2.3. Interpolation with Moving-Neighborhood LSC

LSC, widely used in physical geodesy, is a statistical estimation technique employed to separate systematic trends, spatially correlated signals, and random noise in observation data. Originally developed for geoid determination, LSC is based on the spatial autocorrelations of observables, such as gravity, which are described by a covariance function [21–23]. This function can be homogeneous and isotropic [30], meaning its value depends solely on distance and exhibits rotational symmetry, or it can be anisotropic. The primary goal of LSC is to predict unknown values at unmeasured locations (or times) using known observations, accounting for both spatial correlation and random noise. By leveraging the covariance characteristics of the data, LSC provides an optimal estimation of the underlying signal, making it a powerful tool for improving accuracy in geodetic applications.

It decomposes the observed values into three components: the trend, signal, and noise [31]. The observation equation, represented by a vector matrix, can be expressed as

$$l = AX + t + e \quad (7)$$

where l is the measurement vector of the samples, A denotes the design matrix of the trend model, X represents the non-random parameters, t stands for the autocorrelated random parameters at the observation points, and e is the random noise.

Assuming that the other signals at the prediction points are vectors s correlated with the signal t , despite their potential spatial differences, the solution for the unknown signal vector s is derived based on the minimum principle of LSC given as [23]

$$\text{Min} \left(e^T C_{ee}^{-1} e + s^T C_{ss}^{-1} s \right) \quad (8)$$

where C_{ee} is the variance-covariance (VC) matrix of the noise e , and C_{ss} is the VC matrix of the signal s .

If the trend is neglected or has been calibrated, the estimate of the unknown signal s is expressed by [23]

$$\hat{s} = C_{st} C_{ll}^{-1} l \quad (9)$$

where C_{ll} is the VC matrix of the measurements l and has $C_{ll} = C_{tt} + C_{ee}$, and C_{st} refers to the covariance matrix between the signal s at the prediction point and the signal t at the measurement point. Basically, the estimated $\hat{s}$ is predicted based on the spatial correlation between the observed and predicted points. The elements in the matrices C_{ll} and C_{st} are computed using the covariance function, which is determined based on the distances between the sample points and the current satellite. Specifically, the covariance function describes how the spatial correlation between measurements decreases with increasing distance.

LSC considers both random and non-random components in observations. It can be applied when observations are spatially correlated, which is often the case in geodetic and GNSS applications. LSC allows for accurate interpolation of values at unmeasured points by leveraging spatial autocorrelation.

2.4. Covariance Function

The covariance function is a method used to describe the spatial correlation between data points, where the covariance between any two measured values can be represented as the function $C(d)$, and d represents the distance between the two points. Assuming there are two points, x_i and x_j , in the spatial domain with a distance of d_{ij} , their covariance can be expressed as

$$C(x_i, x_j) = C(d_{ij}) \quad (10)$$

The function $C(d)$ decreases as the distance d increases, attaining its maximum value at zero distance and approaching zero as the distance tends to infinity. Commonly used covariance functions that satisfy these conditions include the Gaussian function, Markov's function, the Hirvonen function, and so on [31]. Markov's function can be expressed by

$$C(d) = C_0 e^{-d/d_0} \quad (11)$$

where C_0 represents the covariance at zero distance, d_0 is the coefficient of the covariance function, and d is the distance between two points.

Furthermore, when the observed values are numerical data, the empirical covariance function (ECVF) can be computed and expressed numerically [24], which is commonly employed in practice for LSC. It can be written as

$$\begin{cases} C_0 = \frac{1}{m} \sum_{i=1}^m l_i l_i \\ C(d_k) = \frac{1}{m_k} \sum_{i < j} l_i l_j \end{cases} \quad (12)$$

where C_0 is the variance when distance d equals zero, d_k is the distance between two points i and j , m is the number of point pairs at distance d_k , and the terms l_i and l_j represent the multipath values at points i and j , respectively.

In this study, we computed the covariance at various distance intervals using the ECVF. The procedure for obtaining the ECVF can be referenced from Tian et al. [24]. Next, we employed Markov's function to fit the covariance coefficients, and the parameters C_0 and d_0 in Equation (11) are derived.

2.5. Correlation

The Pearson correlation coefficient (ρ) is employed to analyze the correlation between residual sequences of any two frequencies. The formula for ρ is given by [32]

$$\rho_{XY} = \frac{\text{cov}(X, Y)}{\sigma_X \sigma_Y} \quad (13)$$

where X and Y represent two sets of residual sequences; and cov and σ denote the covariance and standard deviation operators, respectively. Typically, a ρ value between 0.8 and 1.0 indicates a very strong correlation, 0.6 to 0.8 represents strong correlation, 0.4 to 0.6 is moderate, 0.2 to 0.4 is weak, and 0.0 to 0.2 reflects very weak or no correlation.

Calculating the correlation between residuals for frequencies within the same system is straightforward, as the satellite positions remain consistent across all observations. However, for different systems, this process becomes more complex due to the non-overlapping satellite trajectories.

To solve this, we first generate uniformly distributed positions close to the trajectories of both systems, ensuring a maximum distance of 0.02 rad from each trajectory. Next, the multipath values at these positions are interpolated for each system's residuals using a spatial autocorrelation-based method. Once the multipath values are obtained for both systems at the uniformly distributed positions, the correlation coefficients are calculated.

The positions are illustrated in Figure 1, where the blue and green solid lines represent the satellite trajectories from different systems, and the red-dotted triangles indicate the uniformly distributed sample points located within 0.02 rad of both trajectories.

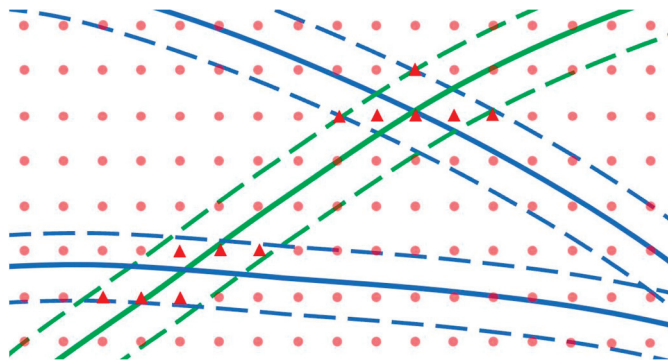


Figure 1. Schematic diagram of the data locations used to calculate the correlation between residuals from different frequencies. The blue and green solid lines represent satellite tracks from two distinct systems, while the blue and green dashed lines indicate the 0.02 radians boundaries to the satellite tracks. All red points, including the red circles and red triangles, represent uniformly distributed samples generated for the correlation analysis. The red triangles are the points that fall within the 0.02 radians overlapping region around the satellite tracks of both systems, i.e., the region where the dashed lines overlap. The red circles represent points that fall outside the overlapping region and are therefore excluded from the analysis.

The spatial correlation characteristics of multipath are analyzed to enhance its mitigation in multi-GNSS, multi-frequency observations. This involves calculating the correlation coefficient between different GNSS frequencies, examining the spatial autocorrelation of residuals, and identifying consistent modeling parameters for the covariance function. Based on these findings, a spatial autocorrelation-based method utilizing a unified covariance function is proposed. Additionally, the impact of orbit shifts on long-term multipath error mitigation is evaluated, and strategies to reduce this impact are assessed. The flowchart of the procedure is plotted in Figure 2.

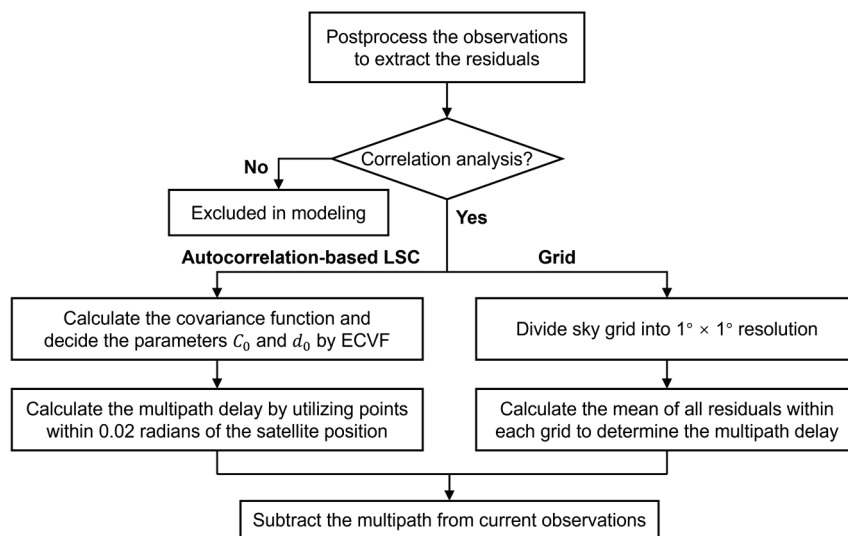


Figure 2. Flowchart of multipath analysis and mitigation.

3. Computation and Analysis

3.1. Employed Data

The observations from the short baselines MATE_MAT1 and KERG_KRGG, provided by the International GNSS Service, are utilized to analyze and validate methods for mitigating multipath effects. The lengths of these baselines are 10.7 m and 8.6 m, respectively, with a sampling interval of 30 s. The antenna photographs of the four stations are shown in Figure 3. Figure 4 illustrates the multipath distribution for both baselines, revealing that the errors are predominantly concentrated in low-elevation areas. The presence of a large metallic structure near station MAT1 leads to significant multipath errors in the MATE_MAT1 baseline, particularly around an elevation angle of 20° toward the southeast. Additionally, the KERG_KRGG baseline shows higher-frequency multipath errors in low-elevation regions, primarily due to the influence of ancillary equipment, such as antenna mounts [33].

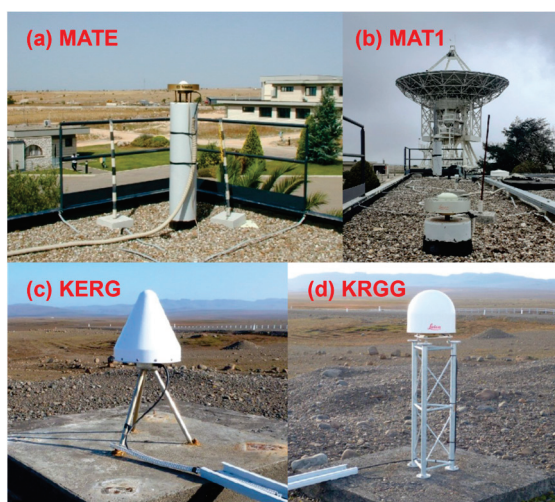


Figure 3. Antenna photographs of MATE_MAT1 (a,b) and KERG_KRGG (c,d). Photographs are from www.sonel.org (accessed on 14 October 2024).

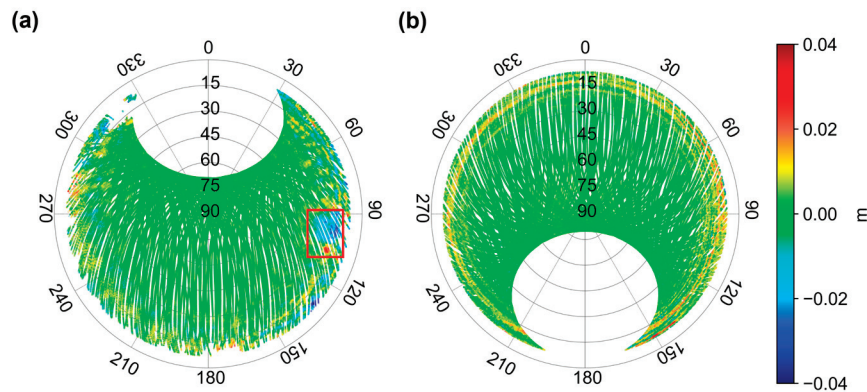


Figure 4. Multipath distributions for the baselines MATE_MAT1 (a) and KERG_KRGG (b). The red boxed area in (a) represents the significant multipath errors of the MATE_MAT1 baseline around an elevation angle of 20° toward the southeast.

Based on the satellite orbit repeat period of GPS and Galileo [13,34,35], we utilize 10 days of observation data from Days of the Year (DOY) 029–038 in 2023 for analysis and modeling, which are then applied to mitigate the multipath effects on DOY 039 and subsequent days. During this period, the GPS and Galileo frequencies available for each baseline are listed in Table 1, with specific frequency values. During data processing, a satellite elevation cutoff angle of 7° is applied, and the igs20_2309.atx antenna file is used to correct the phase center offsets (PCOs) and phase center variations (PCVs) for both satellites and receivers. For the KERG_KRGG baseline, due to the lack of Galileo PCO/PCV correction values, GPS PCO/PCV correction values are used as substitutes.

Table 1. The frequency values of the available open service signals transmitted by GPS and Galileo for the two baselines. “√” means that the baseline has this frequency, and “×” means that the baseline does not.

Frequency (MHz)	Signal		Baselines	
	GPS	Galileo	MATE_MAT1	KERG_KRGG
1575.420	L1	E1	√	√
1278.750		E6	×	√
1227.600	L2		√	√
1207.140		E5b	√	√
1191.795		E5ab *	√	√
1176.450	L5	E5a	√	√

* i.e., E5(E5a + E5b).

3.2. Correlation Analysis Between GPS/Galileo Frequencies

The carrier phase SD residual distribution for all frequencies of the MATE_MAT1 and KERG_KRGG baselines over the 10-day period from DOY 029–038 in 2023 is presented in Figures 5 and 6. The SD residuals for satellites at low elevation angles (below 30°) are relatively large, a common feature associated with the heightened multipath effects in this region [36]. Additionally, it is noted that the number of satellites observed at the E5a frequency for the KERG_KRGG baseline is relatively low when the elevation angle is below 15°. This reduction is primarily due to the lack of low-elevation satellite data at the KERG station for this frequency.

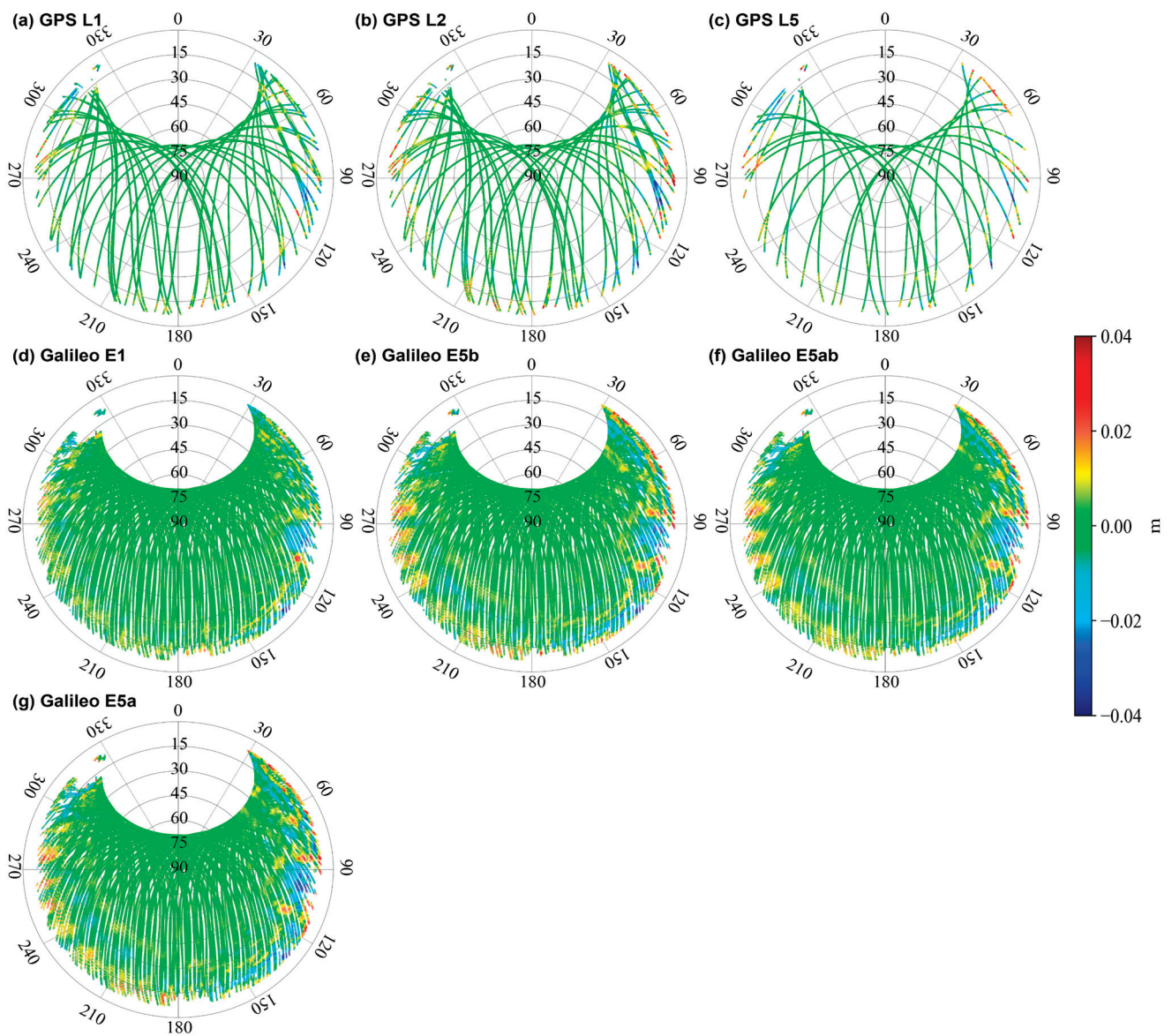


Figure 5. Satellite tracks and SD residuals for all frequencies of the baseline MATE_MAT1 on DOY 029–038 of 2023.

The residuals for the GPS L1/L2/L5 and Galileo E1/E5b/E5ab frequencies over a 10-day period, as the satellites traversed a designated square window, are presented in Figure 7, offering a comparative view of the signal variations across different GNSS frequencies. The windows for the MATE_MAT1 baseline cover an elevation angle of 7° to 17° and an azimuth angle of 90° to 100° , while for the KERK_KRGG baseline, the range is 7° to 17° in elevation and 140° to 150° in azimuth. A consistent pattern is observed for different frequencies at the same spatial locations. Notably, for overlapping or closely spaced frequencies, the satellite trajectory colors appear similar or nearly identical. For instance, this applies to overlapping frequencies such as GPS L1 and Galileo E1, closely spaced frequencies like GPS L2 and Galileo E5b, as well as GPS L5 and Galileo E5ab, all of which follow the same pattern. In contrast, for frequencies with larger differences, such as GPS L1 and Galileo E5b/E5ab, the trajectory colors vary significantly, and in some instances, even exhibit opposite sign values.

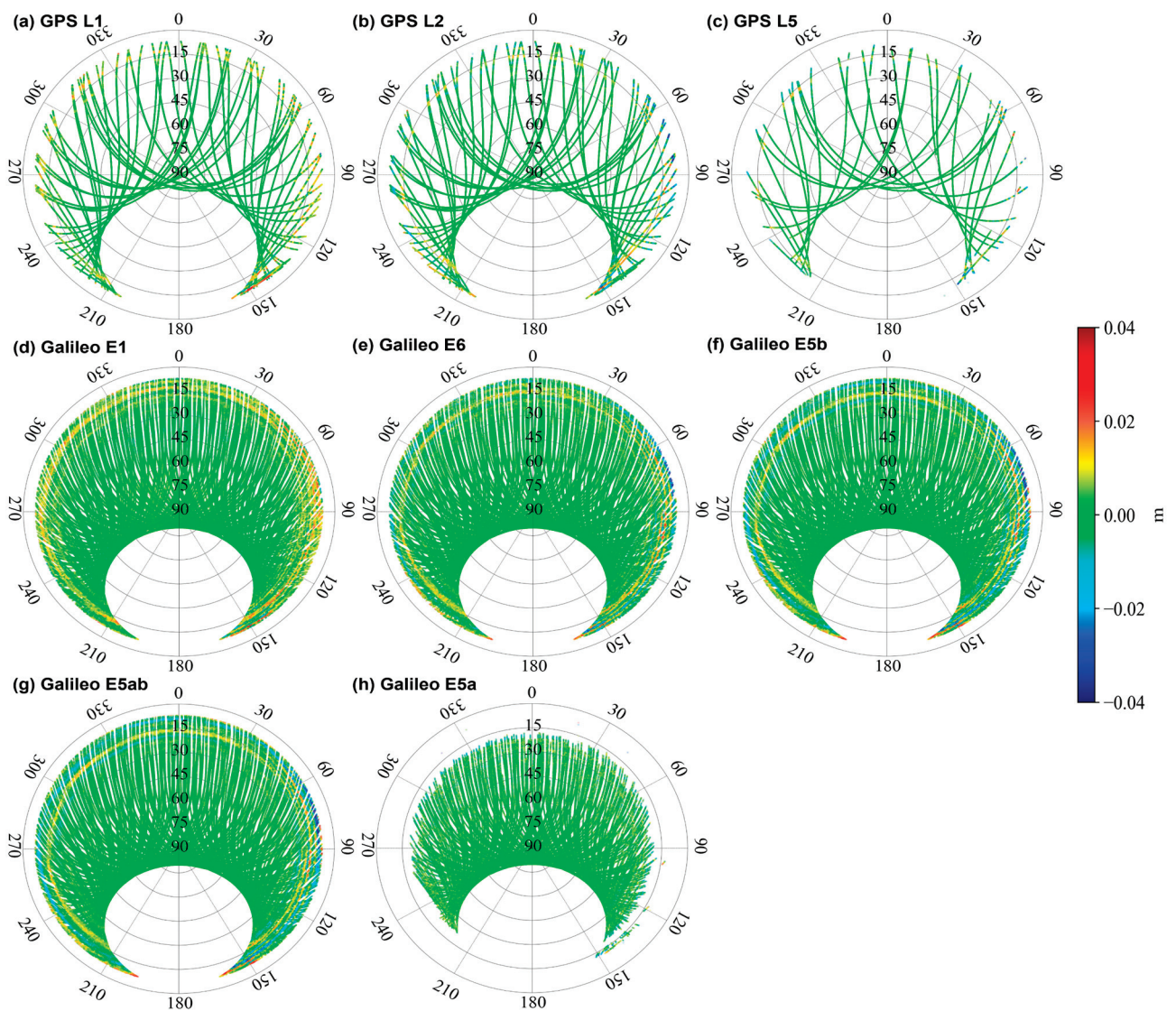


Figure 6. Satellite tracks and SD residuals for all frequencies of the baseline KERG_KRGG on DOY 029–038 of 2023.

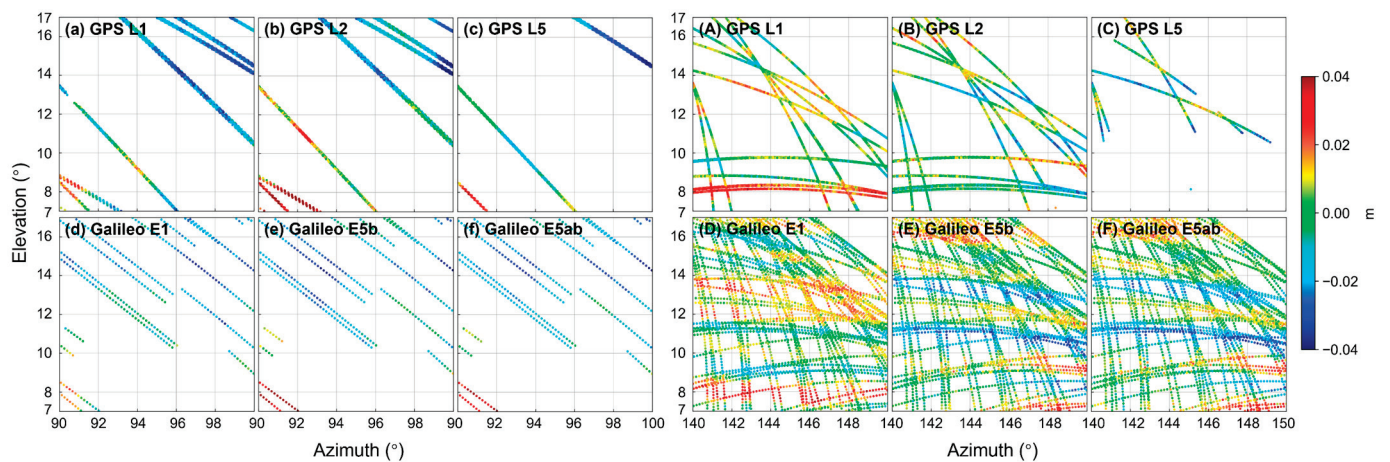


Figure 7. SD residuals for different frequencies of the MATE_MAT1 baseline ((a–f), 7°–17° in elevation and 90°–100° in azimuth) and the KERG_KRGG baseline ((A–F), 7°–17° in elevation and 140°–150° in azimuth).

The Pearson correlation coefficient is used to evaluate the correlation between the residual sequences of any two frequencies. The absolute values of the resulting correlation coefficients are displayed in Figure 8, showing diagonal symmetry along the value of 1. To better visualize the strength of multipath error correlations between frequency pairs, a color gradient is applied, ranging from dark (strong correlation) to light (weak correlation). It is clear from the figure that correlations exist between multipath errors for all frequency pairs, though the strength of this correlation decreases as the frequency differences increase. Additionally, the relationship between frequency difference and the absolute value of the correlation coefficient is illustrated in Figure 9, offering a more intuitive visualization of how frequency differences affect correlation strength.

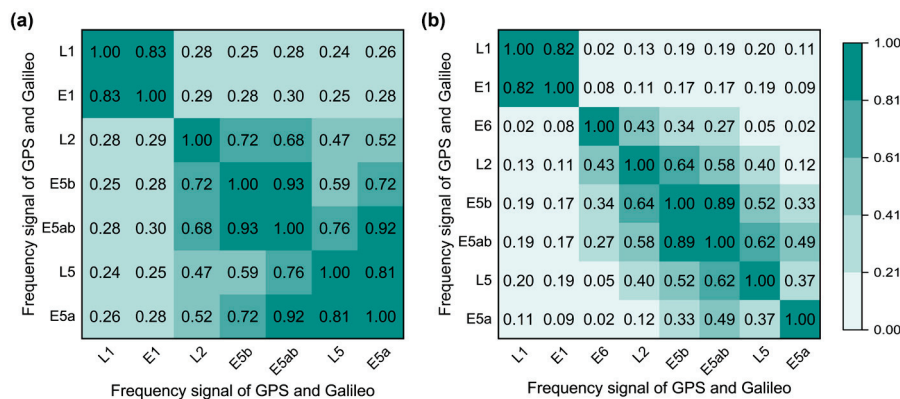


Figure 8. Absolute values of correlation coefficients between all frequencies of the baselines MATE_MAT1 (a) and KERG_KRGG (b). Utilizing a color gradient ranging from dark to light to represent the hierarchical levels of correlation strength, spanning from very strong to very weak or no correlation.

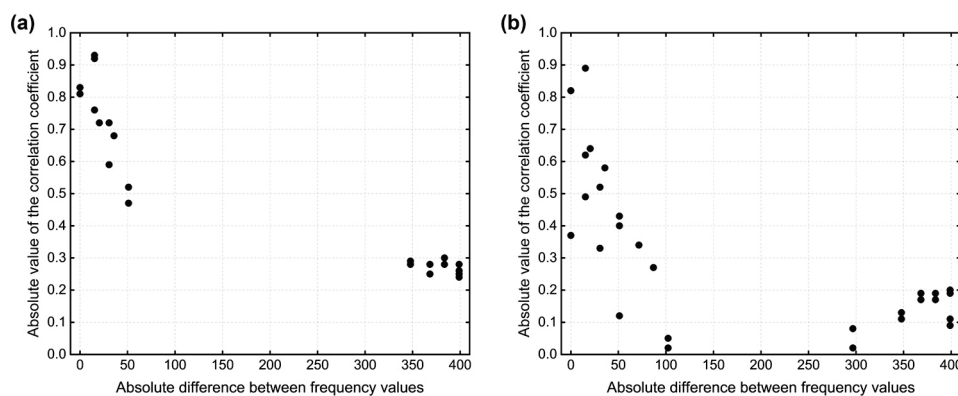


Figure 9. Variation of all correlation coefficients with frequency differences for the baselines MATE_MAT1 (a) and KERG_KRGG (b).

Due to the lack of low-elevation observations at the Galileo E5a frequency from the KERG station, the low correlations between E5a and the other frequencies in the KERG_KRGG baseline may not accurately reflect the true correlations. Apart from this, the residuals from frequency pairs, such as overlapping ones like L1/E1 and L5/E5a between GPS and Galileo, as well as non-overlapping frequencies like L2 and E5b, and the Galileo frequencies E5a, E5b, and E5ab, exhibit relatively large correlation coefficients, indicating strong correlations. Other frequencies also demonstrate strong to moderate correlations between narrow-spaced frequencies. These strong correlations suggest that their residuals contain similar multipath errors, supporting the potential for combining frequencies to enhance multipath error correction.

3.3. Parameters of Covariance Function

The covariance function used in this study is Markov's function by Equation (11), where C_0 and d_0 determine the function. These parameters for different frequencies are initially calculated by fitting the theoretical function to the empirical covariance function and then compared across the frequencies.

The computed and fitted ECVF curves for the GPS L1 and Galileo E1 frequencies, from the MATE_MAT1 and KERG_KRGG baselines on DOY 038 in 2023, are shown in Figure 10 as an example. The curves of other frequencies are similar to these two frequency signals. Figure 11 presents the computed values of C_0 and d_0 over a 10-day modeling period for both the GPS and Galileo frequencies for the two baselines.

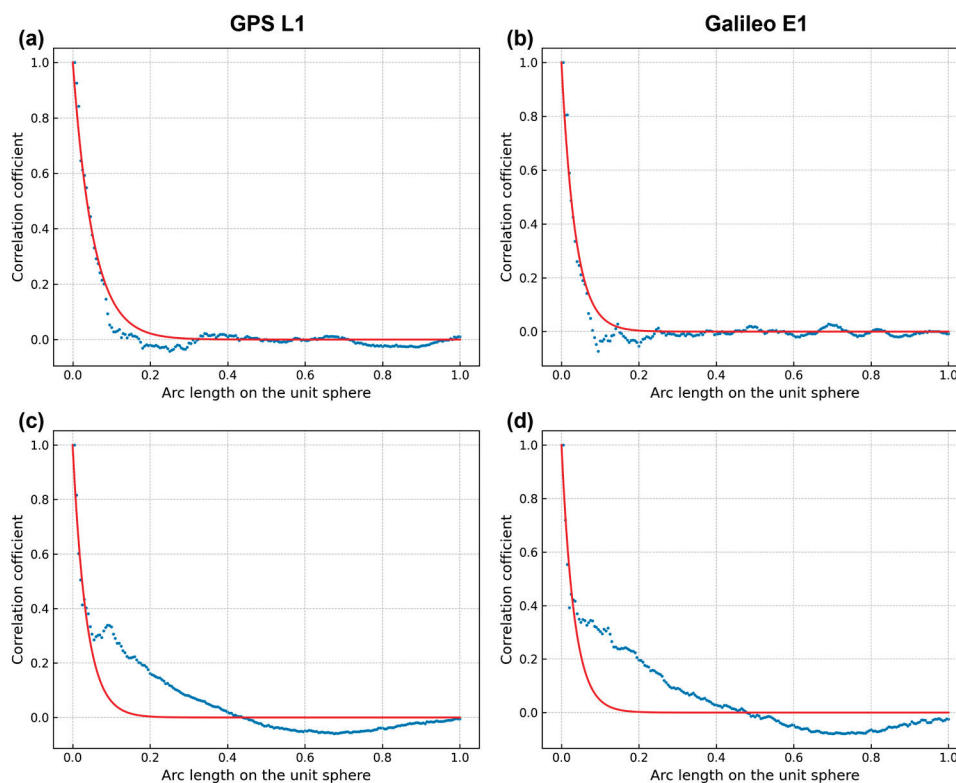


Figure 10. Theoretical covariance functions and ECVFs of GPS L1 and Galileo E1 for MATE_MAT1 (a,b) and KERG_KRGG (c,d) on DOY 038 of 2023. The blue dot curve is the ECVF derived with the GPS L1 (a,c) and Galileo E1 (b,d) SD residuals. The red curve is the fitted curve.

The C_0 values for the MATE_MAT1 baseline are slightly larger than those for the KERG_KRGG baseline, though the ranges for both baselines overlap significantly. In contrast, the d_0 parameter for the MATE_MAT1 baseline is notably larger than for KERG_KRGG, which is expected due to the higher spatial frequency of multipath observed in the KERG_KRGG baseline.

Additionally, the deviations of parameters relative to the averages of each single baseline and both baselines combined are computed and shown in Figure 12. Due to the absence of low-elevation data with large residuals at the E5a frequency for the KERG_KRGG baseline, the calculated covariance parameters are not representative of the multipath errors, leading to the exclusion of this frequency from the analysis. As anticipated, the differences between the original parameter values and the averages for each baseline are relatively small (Figure 12), while the deviations from the combined averages of both baselines are slightly larger. Nonetheless, the differences in C_0 for this case remain small, around 1×10^{-5} , and the d_0 differences stay within 0.02.

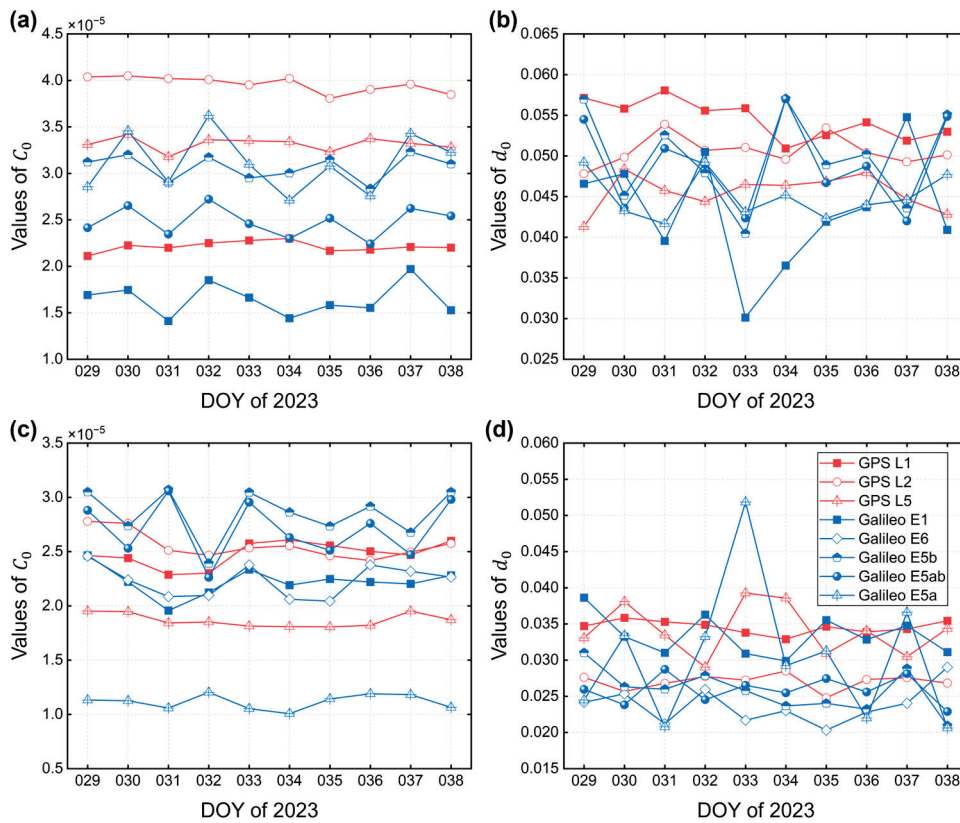


Figure 11. Daily C_0 and d_0 parameters for MATE_MAT1 (a,b) and KERG_KRGG (c,d) on DOY 028–039 of 2023.

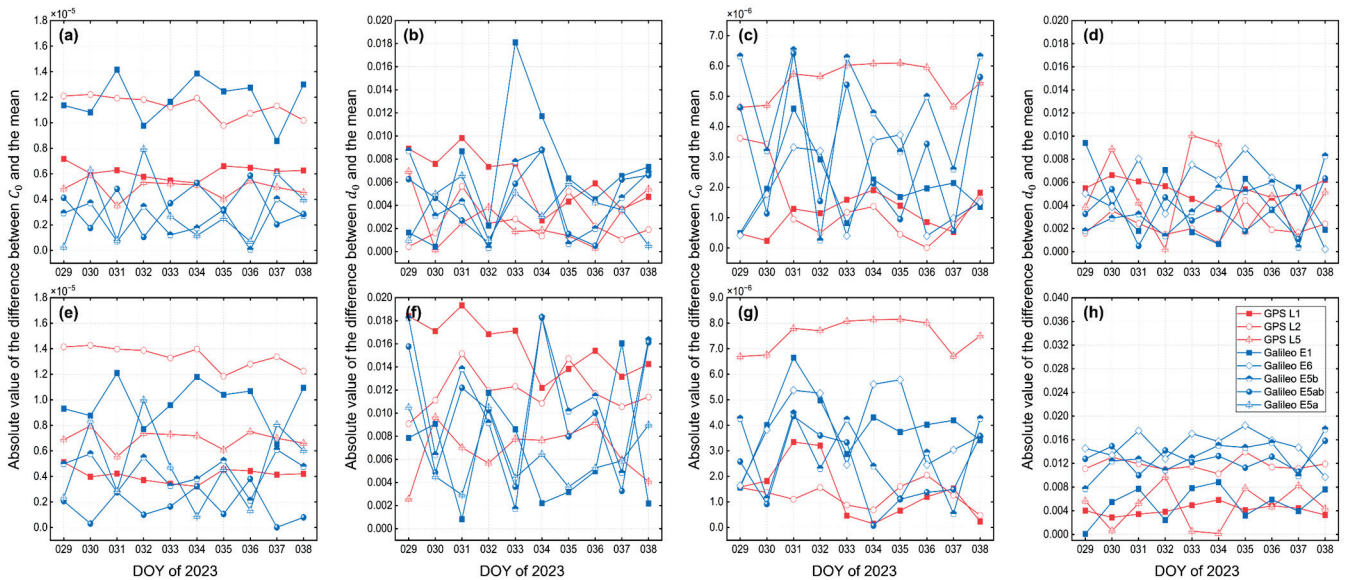


Figure 12. Deviations of the parameters respective to the average of each single baseline (a–d) and both baselines (e–f) are computed, separately, for MATE_MAT1 (a,b,e,f) and KERG_KRGG (c,d,g,h).

The parameters C_0 and d_0 of all frequency residuals are unified by averaging all frequency parameters across two baselines. These unified parameters are then applied in a spatial autocorrelation-based method for multipath mitigation. To evaluate performance, GPS L1 frequency observations from both baselines are analyzed. The residual variance reduction rates after applying the unified parameters are displayed in Figure 13, alongside rates using average parameters for each baseline and for each frequency individually.

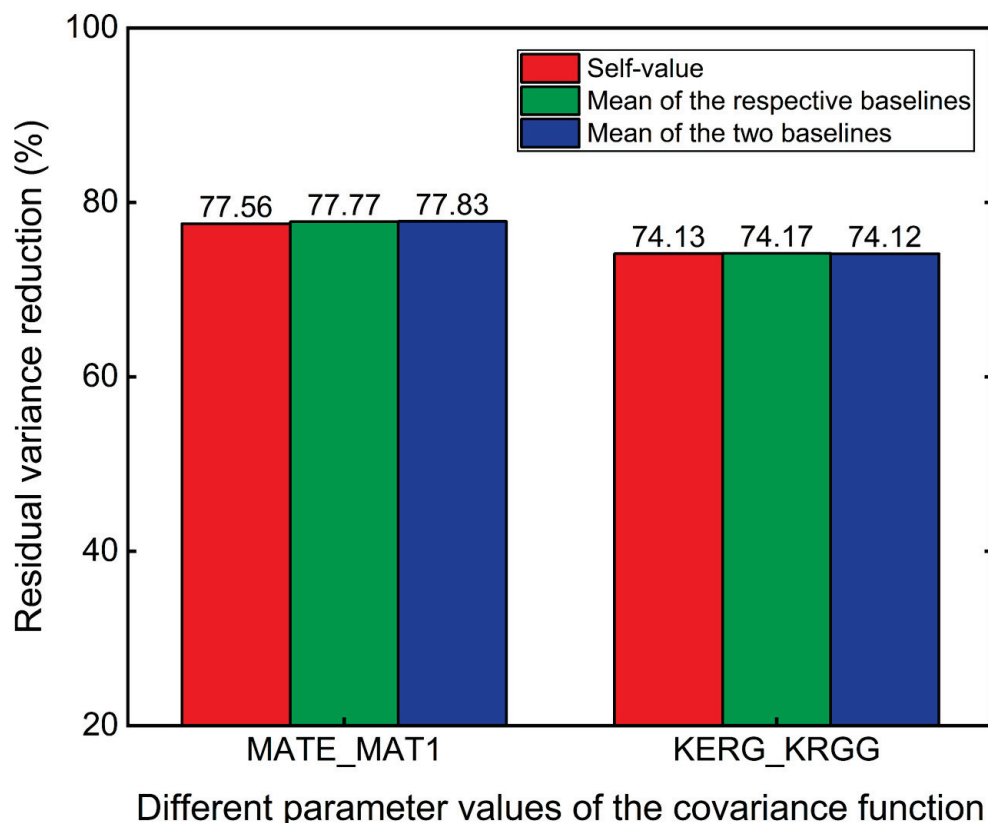


Figure 13. Variance reduction rates of SD residuals after multipath correction using different parameter values. The results of the parameters C_0 and d_0 by averaging all frequency parameters across two baselines (mean of the two baselines), averaging parameters for each baseline (mean of the respective baselines), and for each frequency individually (self-value) are compared.

The results demonstrate that the variance reduction rates achieved with the unified parameters are comparable to those obtained with baseline-specific averages. This suggests that using unified covariance function parameters for different baselines and frequency signals is effective, simplifying the computational process by eliminating the need to estimate the parameters C_0 and d_0 individually. Moving forward, these unified parameters will be employed in subsequent research efforts.

3.4. Orbit Shift over a Long Time Span

Due to various perturbations, satellite orbits exhibit drift, with variations across different satellites and systems. Figures 14 and 15 depict the satellite tracks of all GPS and Galileo satellites observable at the MATE station over a four-year period, from 1 January 2020 to 31 December 2023. In these figures, the satellite tracks for 2020 are shown in red, 2021 in blue, 2022 in orange, and 2023 in purple, visually representing the orbit drift across the years.

Within the Galileo system, satellites E14 and E18 show more pronounced orbit drift compared to others, indicating less stable orbits. Conversely, many GPS satellites exhibit significant orbit drift, notably satellite G02 in 2022 and 2023, G20 in 2021, and G21 in 2020, with their drift far exceeding that observed in the Galileo system. Specifically, substantial orbital drift was observed for G02 during July–November 2022 and April–June 2023, G20 during March–July 2021, and G21 during May–November 2020. Figure 16 illustrates the variation trends of selected orbital parameters—namely eccentricity, Right Ascension of the Ascending Node (RAAN), orbital inclination, argument of perigee, and semi-major axis—for these three satellites during the noted periods. Significant adjustments in these orbital parameters occurred during times of pronounced deviations in satellite trajectories.

The significant changes in the semi-major axis correspond to a substantial variation in the satellite's orbit repeat period. Consequently, if substantial changes occur in the orbital parameters such as the semi-major axis, the residual data from previous periods may not accurately represent the current multipath errors due to significant spatial discrepancies between the expected and actual satellite positions, leading to inaccuracies in multipath error correction.

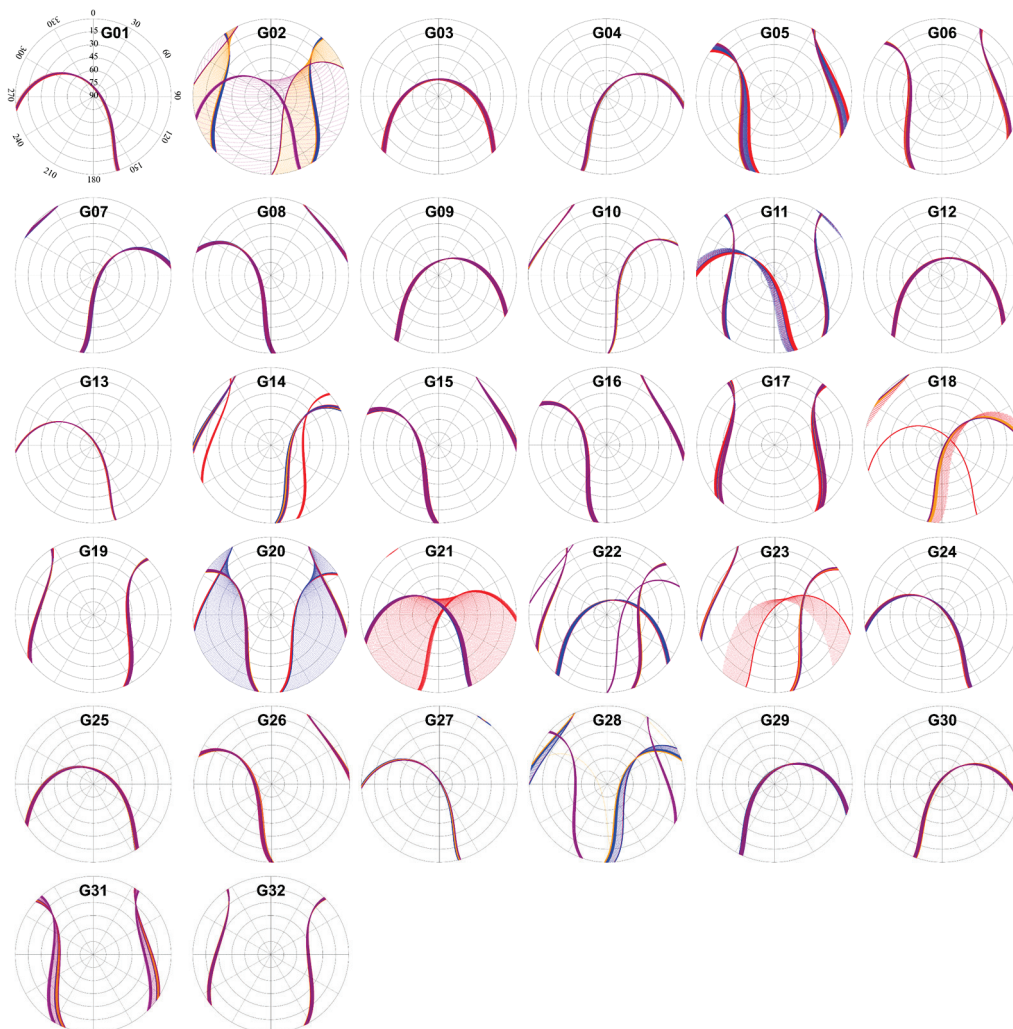


Figure 14. Satellite tracks of all GPS satellites observable at the MATE location over the four-year period from 2020 to 2023. The tracks are color-coded by year: 2020 in red, 2021 in blue, 2022 in orange, and 2023 in purple.

Utilizing orbital data from DOY 029–038 of 2023 as reference orbits for both the GPS and Galileo systems, the average unit spherical distance from the positions of satellites G01–G32 and E01–E36 during DOY 039–365 of 2023 to their nearest reference orbit was calculated. Figure 17a,b demonstrates these deviations. The average deviation distance for Galileo satellites remains relatively stable, peaking at only 0.004 rad, while that for GPS satellites shows a marked increase, with satellite G02 reaching a maximum deviation of 0.029 rad. Interestingly, around DOY 249, the average deviation for GPS satellites stabilizes.

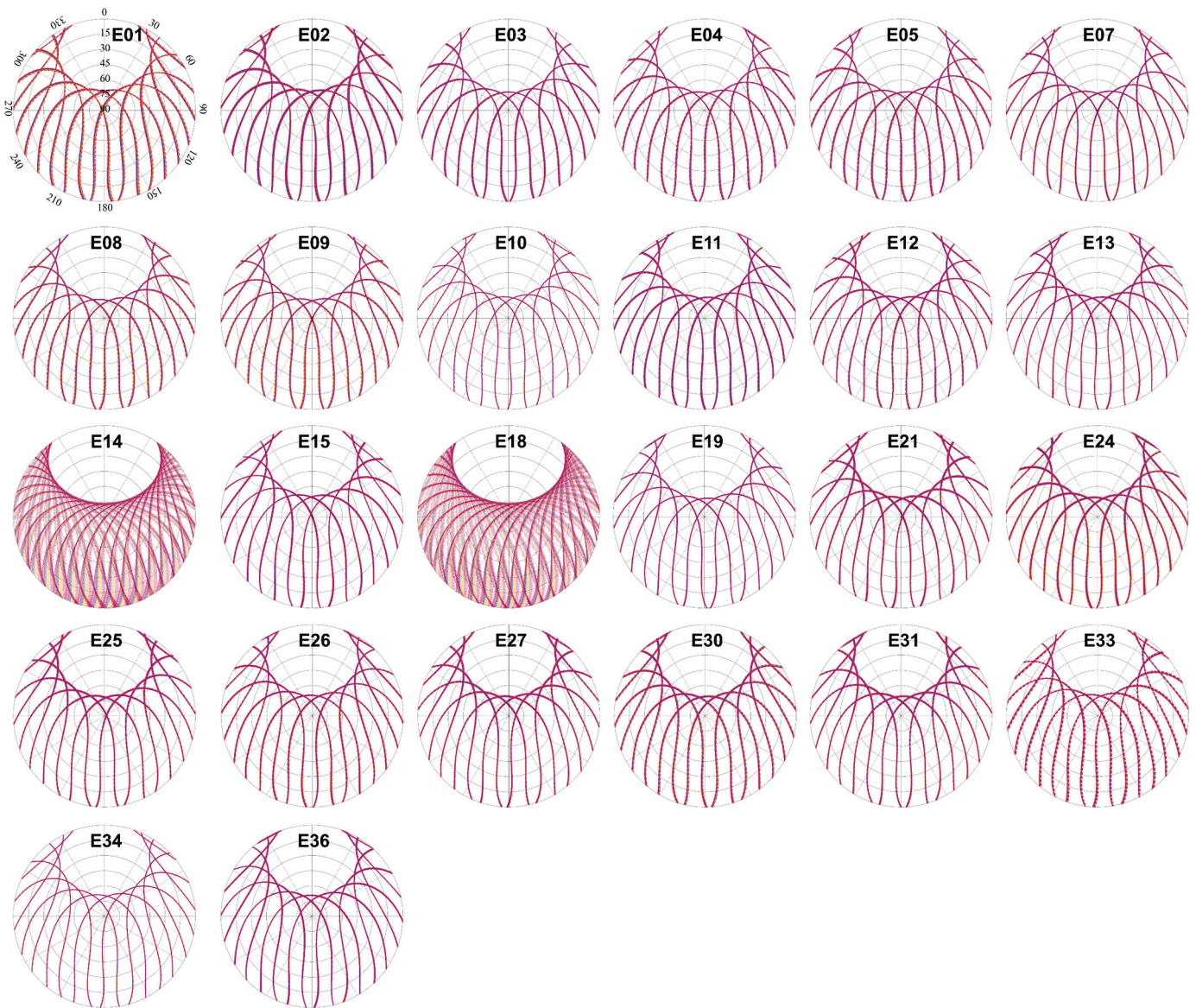


Figure 15. Satellite tracks of all Galileo satellites observable at the MATE location over the four-year period from 2020 to 2023. The tracks are color-coded by year: 2020 in red, 2021 in blue, 2022 in orange, and 2023 in purple.

By recalculating the average unit spherical distance using orbital data from DOY 239–248 as a new reference for the period DOY 249–365, the results, shown in Figure 18, reveal a reduction in the maximum average deviation for GPS satellites to 0.017 rad, which is significantly lower than earlier measurements. This adjustment indicates that using more recent data as reference orbits leads to more accurate multipath error correction, suggesting that outdated historical data may not effectively correct for current GPS multipath errors.

Based on the analysis in Section 3.2, there is a notable correlation in multipath errors between the GPS and Galileo systems, with overlapping or narrowly-spaced frequencies showing very strong correlation. Consequently, integrating these two systems could enhance the density of orbit trajectories, potentially mitigating the negative impacts of GPS orbit shifts on long-term multipath correction.

Using orbital data from both GPS and Galileo systems from DOY 029–038 of 2023, a combined reference orbit was established. We calculated the average unit spherical distance from the positions of satellites G01–G32 and E01–E36 to this combined orbit for the same period. The results, shown in Figure 17c,d, reveal that while the average deviation distance

for Galileo satellites from the combined orbit remains stable at 0.004 rad, the deviation for GPS satellites shows a significant reduction to 0.003 rad. Additionally, these distances do not exhibit significant long-term fluctuations.

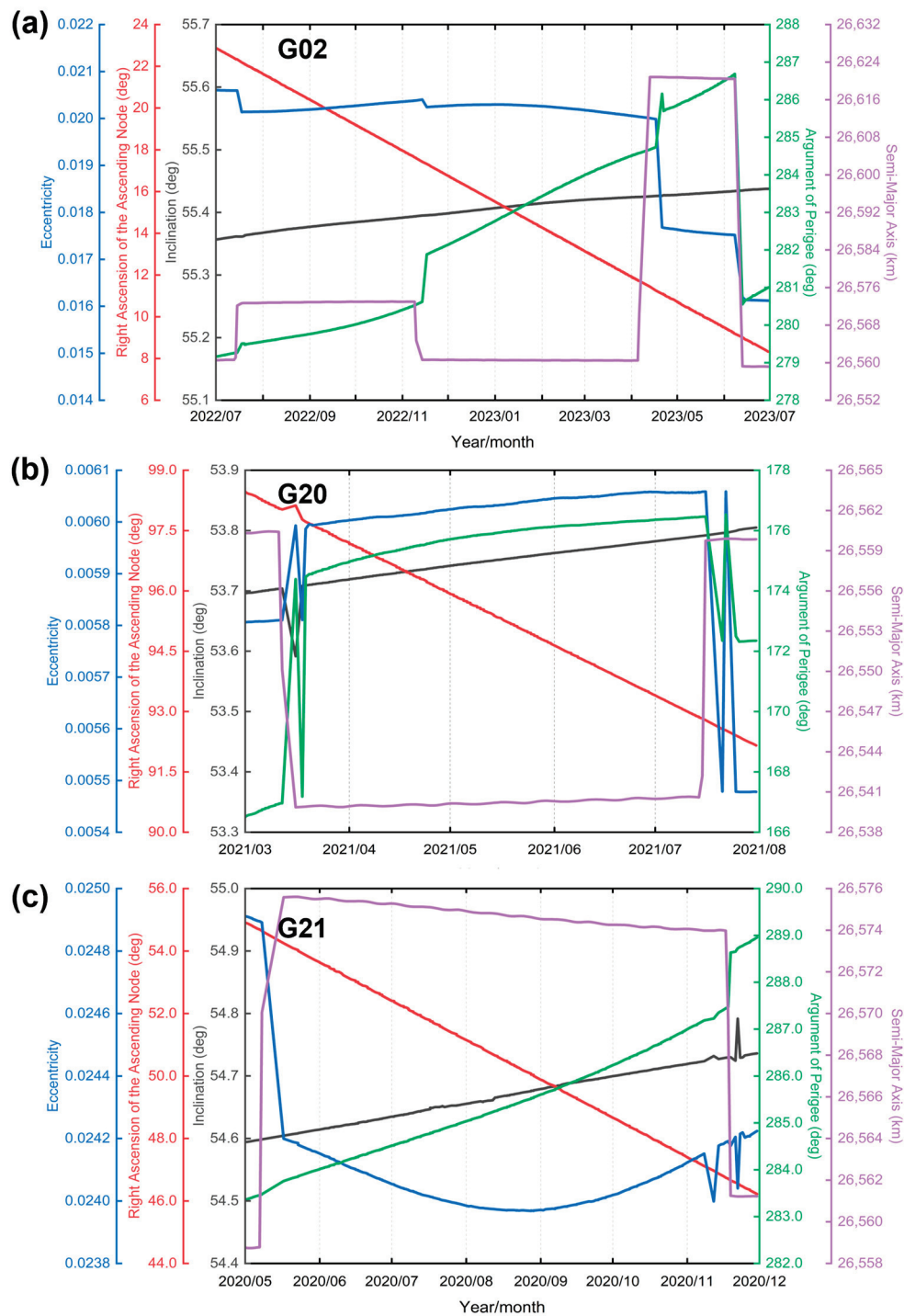


Figure 16. Eccentricity (the blue line), Right Ascension of the Ascending Node (the red line), orbital inclination (the black line), argument of perigee (the green line), and semi-major axis (the purple line) for satellites G02 (a), G20 (b) and G21 (c).

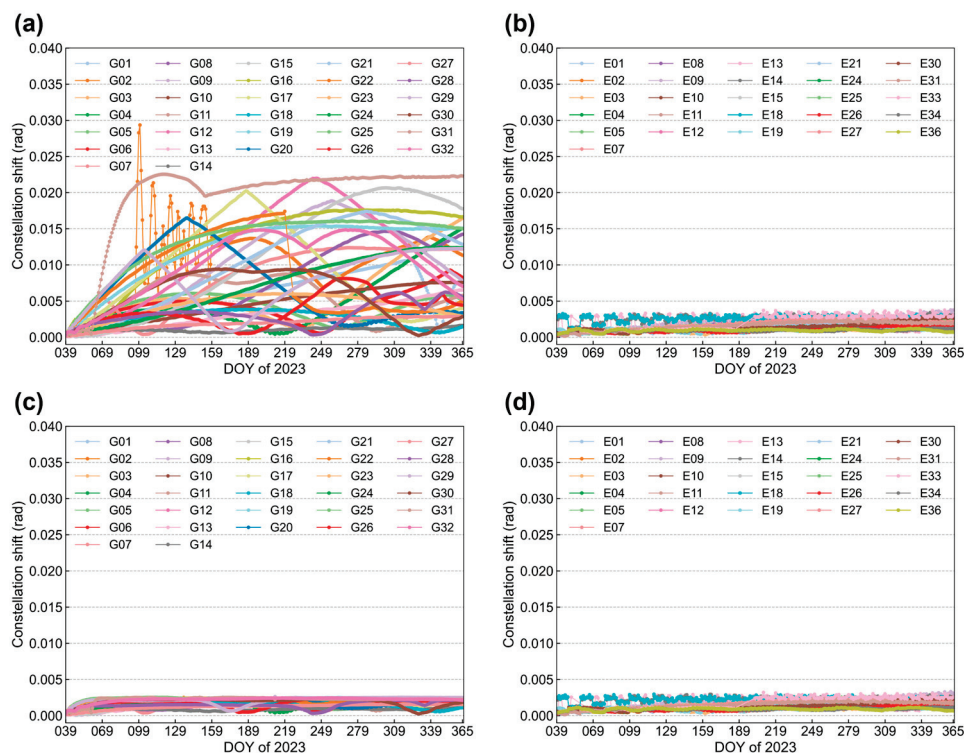


Figure 17. Average unit spherical distances of the GPS and Galileo satellites from reference orbits before and after the GPS/Galileo combination. Panel (a) shows the distances of GPS satellites from the GPS reference orbit, panel (b) shows the distances of Galileo satellites from the Galileo reference orbit, panel (c) shows the distances of GPS satellites from the GPS/Galileo combined reference orbit, and panel (d) shows the distances of Galileo satellites from the GPS/Galileo combined reference orbit.

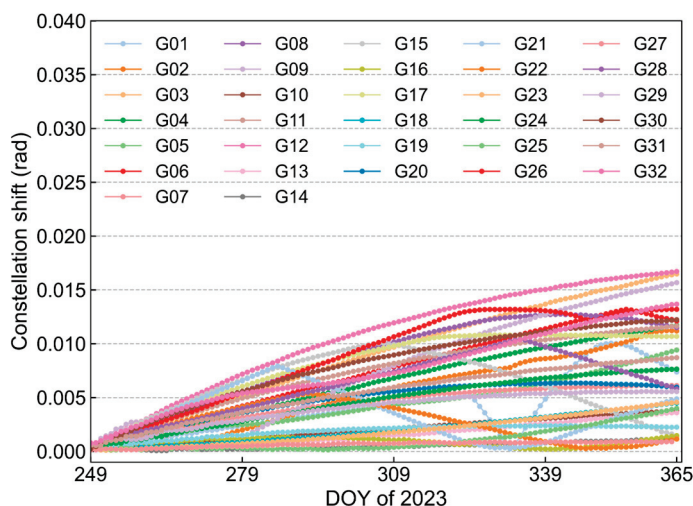


Figure 18. Average spherical distance of GPS satellites from the reference orbit after using the GPS satellite orbits from DOY 239–248 as the new reference.

In Figure 17, the average deviation distances for both GPS and Galileo satellite orbits generally stay within 0.02 rad. Using this threshold, satellite coordinates within this distance from the reference orbit on a unit sphere are considered consistent, indicating an orbit repeat. Orbit repeatability for both systems is then calculated daily from DOY 039 to 365 in 2023 and is presented in Figure 19, with reference to both individual constellations and the combined constellation. When using the GPS satellite orbits from DOY 239 to 248 as the reference, the GPS orbit repeatability gradually declines, dropping to 84.5% after DOY 072.

This reduction negatively impacts the accuracy and stability of the multipath error model. However, when GPS and Galileo satellite orbits are combined, the GPS orbit repeatability remains stable at 100%. Galileo satellites consistently show 100% repeatability throughout the observation period, thanks to their inherent orbital stability. Thus, integrating GPS and Galileo systems for multipath modeling and mitigation is expected to improve correction accuracy and ensure more stable results, particularly for GPS.

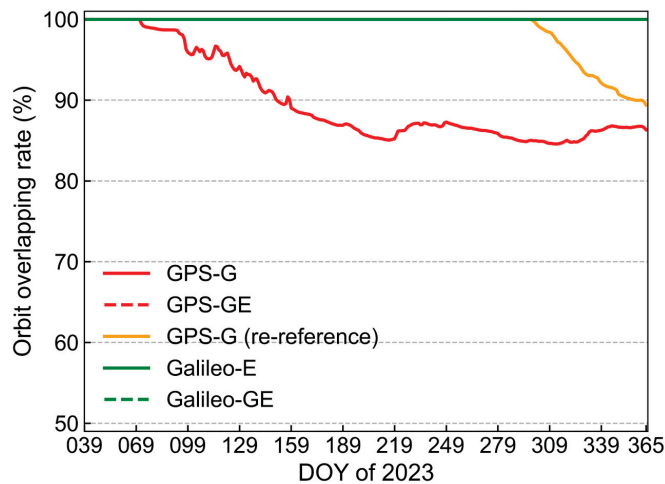


Figure 19. Repetition rate of GPS and Galileo satellite orbits between DOY 039–365 in 2023 before and after the GPS/Galileo combination.

3.5. Unified Covariance Function Parameters for Short and Long-Term Multipath Correction Analysis

Using the MATE_MAT1 baseline’s GPS L1 and Galileo E1 frequencies as examples, we applied both the grid method and the autocorrelation-based LSC using unified covariance function parameters in the multipath mitigation. The calibration, carried out from DOY 039 to 365 in 2023, employs a 30-day interval. The moving neighborhood parameter is set at 0.02 rad, and the covariance function parameters C_0 and d_0 are averaged across all frequencies from both baselines. The specific strategies for multipath correction are detailed in Table 2.

Table 2. Multipath correction strategies in this study.

Frequencies to be Corrected	Method	Modeling Date	Frequencies Used for Correction	Abbreviations
GPS L1	Grid	2023 DOY 029–038	GPS L1	Grid_G for L1
		2023 DOY 239–248	GPS L1/Galileo E1	Grid_GE for L1
	Autocorrelation-based LSC	2023 DOY 029–038	GPS L1	Grid_G for L1 (remodel)
		2023 DOY 029–038	GPS L1/Galileo E1	Grid_GE for L1 (remodel)
		2023 DOY 239–248	GPS L1	LSC_G for L1
		2023 DOY 239–248	GPS L1/Galileo E1	LSC_GE for L1
Galileo E1	Grid	2023 DOY 029–038	GPS L1	LSC_G for L1 (remodel)
		2023 DOY 029–038	GPS L1/Galileo E1	LSC_GE for L1 (remodel)
		2023 DOY 239–248	GPS L1	LSC_G for L1 (remodel)
		2023 DOY 239–248	GPS L1/Galileo E1	LSC_GE for L1 (remodel)
	Autocorrelation-based LSC	2023 DOY 029–038	Galileo E1	Grid_E for E1
		2023 DOY 029–038	Galileo E1	Grid_E for E1
		2023 DOY 239–248	Galileo E1	Grid_E for E1 (remodel)
		2023 DOY 239–248	Galileo E1	Grid_E for E1 (remodel)

The standard deviations of the SD residuals before and after applying the multipath correction strategies are detailed in Tables 3 and 4. On DOY 039 of 2023, the performance of the grid methods, labeled Grid_G and Grid_GE, are initially comparable. However, Grid_G’s performance declines more rapidly over time. In contrast, the LSC method demonstrates robust initial performance, maintaining superiority over Grid_G and showing slightly lesser performance only compared to the combined GPS/Galileo grid method after 120 days. The LSC_GE strategy, specifically modeled on DOY 029–038,

consistently offers superior performance compared to all other strategies modeled on later dates (DOY 239–248).

Table 3. STDs of the SD residuals before and after multipath calibration for GPS L1 (unit: mm).

	DOY of 2023											
	039	069	099	129	159	189	219	249	279	309	339	365
Uncalibrated	5.14	5.38	5.31	5.12	5.12	5.17	5.76	5.63	5.75	5.59	5.85	5.66
Grid_G	2.97	3.67	3.99	4.08	4.08	4.15	4.71	4.48	4.61	4.47	4.62	4.66
Grid_GE	2.96	3.34	3.53	3.52	3.55	3.52	4.18	3.91	4.05	3.83	3.88	3.97
LSC_G	2.43	3.02	3.46	3.41	3.54	3.72	4.26	3.99	4.15	3.99	4.07	4.09
LSC_GE	2.43	2.86	3.05	2.99	3.11	3.04	3.70	3.43	3.63	3.31	3.33	3.49
Grid_G (remodel)								3.37	4.05	4.42	4.61	4.59
Grid_GE (remodel)								3.37	3.81	3.77	3.84	3.82
LSC_G (remodel)								2.86	3.50	3.67	4.03	4.14
LSC_GE (remodel)								2.85	3.31	3.18	3.26	3.34

Table 4. STDs of the SD residuals before and after multipath calibration for Galileo E1 (unit: mm).

	DOY of 2023											
	039	069	099	129	159	189	219	249	279	309	339	365
Uncalibrated	4.12	4.10	4.46	4.20	4.09	4.44	4.56	4.45	4.65	4.65	4.89	5.09
Grid_E	2.46	2.45	2.66	2.66	2.51	2.71	2.95	2.76	2.86	2.74	3.04	3.56
Grid_GE	2.49	2.46	2.68	2.64	2.56	2.71	2.95	2.70	2.87	2.75	3.03	3.59
LSC_E	2.05	2.07	2.23	2.28	2.18	2.30	2.59	2.37	2.49	2.33	2.63	3.04
LSC_GE	2.02	2.05	2.18	2.24	2.17	2.27	2.57	2.32	2.45	2.29	2.52	2.97
Grid_E (remodel)								2.51	2.71	2.60	2.96	3.58
Grid_GE (remodel)								2.54	2.74	2.64	2.98	3.51
LSC_E (remodel)								2.07	2.31	2.16	2.48	3.16
LSC_GE (remodel)								2.04	2.27	2.14	2.47	3.00

Regarding the Galileo E1 frequency, the combined GPS/Galileo correction strategy proves to be as effective or slightly more so than the Galileo-only approach. This effectiveness persists even after 210 days of modeling, where the LSC_GE strategy consistently surpasses the grid method's performance. This trend is maintained irrespective of whether the correction strategy involves only Galileo or a GPS/Galileo combination, with the combined LSC method achieving the best results. Thus, the study confirms the advantage of employing unified covariance function parameters for diverse baselines and frequency signals, which simplifies the computational process and enhances the effectiveness of multipath error mitigation.

Then, the residual variance reduction rate calculated by $(1 - \sigma_1^2 / \sigma_0^2) \times 100\%$, where σ_1^2 is the remaining SD residual variance after multipath correction and σ_0^2 denotes the uncorrected SD residual variance, is shown in Figure 20. For GPS L1, the variance reduction rate on DOY 039 of 2023 using the LSC_GE strategy shows improvements of 0.13%, 11.05%, and 11.16% over LSC_G, Grid_GE, and Grid_G, respectively. By DOY 365 of 2023, these improvements markedly increase to 14.24%, 11.25%, and 29.59%, respectively. This suggests that the LSC_GE strategy not only enhances multipath correction immediately after model application but maintains superior correction capabilities over extended periods. Additionally, the LSC_GE's variance reduction consistently exceeds that achieved by the other strategies, even following repeated modeling. For the Galileo E1 frequency, the LSC_GE strategy improves the variance reduction rate by 0.75%, 12.27%, and 11.58% over LSC_E, Grid_GE, and Grid_E respectively on the first day. By DOY 365, these improvements rise to 1.5%, 15.69%, and 14.96%, respectively. Notably, the grid method's performance post-re-modeling remains inferior to the pre-re-modeling performance of the LSC method. On DOY 339, the re-modeled LSC_E strategy matches the pre-re-modeling performance of

the LSC_GE strategy, but it falls short after this period. The re-modeled LSC_GE strategy maintains similar performance to the pre-re-modeling performance of the LSC_GE.

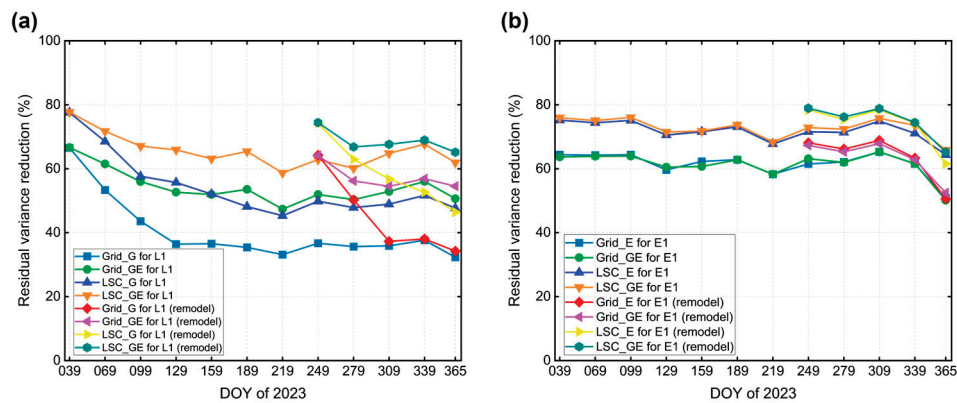


Figure 20. SD residual variance reduction rates of the baseline MAT1_MATE for GPS L1 (a) and Galileo E1 (b) by the grid method and the autocorrelation-based LSC method.

Kinematic mode processing of GPS L1 and Galileo E1 data was conducted using the outlined strategies, with the results presented in Figure 21. The LAMBDA method was employed for ambiguity estimation in the processing, and the ambiguity fixing rates were over 99%. This figure displays the root mean square (RMS) values for the North, East, and Up components before and after multipath calibration. The combined GPS/Galileo solution for GPS L1 provided better results than the GPS-only solution. For Galileo E1, the combined solution was on par with the Galileo-only solution. Across both frequencies, the autocorrelation-based LSC using unified covariance function parameters yielded lower RMS errors and enhanced positioning accuracy compared to the grid method, indicating its efficacy whether applied to combined or single-system data.

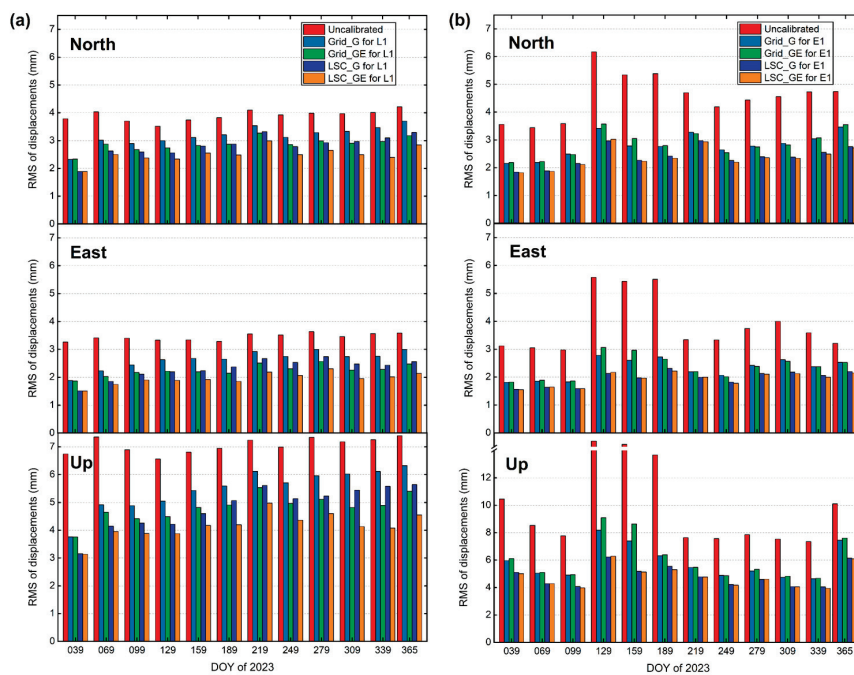


Figure 21. RMS of displacements of the North, East, and Up components of the baseline MAT1_MATE for GPS L1 (a) and Galileo E1 (b) using different correction strategies.

4. Results

Multipath error significantly affects the accuracy of GNSS precise positioning but can be effectively mitigated using spatial autocorrelation-based methods in static and quasi-static environments. Traditional techniques like sidereal filtering and grid-based approaches only approximate the handling of spatial autocorrelation data. However, the spatiotemporal correlations of multipath can be more accurately modeled by utilizing the covariance function. This study investigates the spatial cross-correlation between different frequency bands across various GNSS systems, along with spatial autocorrelation function parameters and the effects of constellation shifts, to simplify autocorrelation-based methods like Least-Squares Collocation and enhance long-term performance.

The results reveal that overlapping frequencies, such as L1/E1 and L5/E5a, show very strong correlations, while non-overlapping frequencies, such as L2 and E5b, also exhibit strong correlations, with coefficients exceeding 0.8. The covariance function parameters, typically including the residual variance and the range parameter that control the rate of correlation decay with distance, remain relatively consistent across frequencies. This consistency allows autocorrelation-based methods using unified covariance function parameters to achieve a variance reduction rate (VRR) comparable to methods that rely on frequency-specific covariance function parameters. Additionally, the GPS satellites G02, G20, and G21 exhibit noticeable orbit shifts, leading to significant changes in orbital parameters and satellite tracks, which diminishes the effectiveness of long-term multipath mitigation. These satellites display larger average deviations, reaching a maximum deviation of 0.029 rad. In contrast, the Galileo satellites demonstrate more stable orbits, with fluctuations confined to 0.004 rad. When combining GPS and Galileo reference orbits, the maximum deviation for GPS satellites reduces to 0.003 rad, and the orbital repeat rate stabilizes at 100%.

Multipath mitigation experiments using both the grid method and the autocorrelation-based LSC with a unified covariance function for GPS L1 and Galileo E1 frequencies on the MATE_MAT1 baseline were conducted. This analysis considers the GPS, Galileo, and combined GPS/Galileo data separately. For GPS L1, the LSC_GE strategy improves the residual variance reduction rate by 0.13%, 11.05%, and 11.16% compared to LSC_G, Grid_GE, and Grid_G, respectively, on DOY 039, the first day after modeling. These improvements increase to 14.24%, 11.25%, and 29.59% on DOY 365, 327 days from the modeling data. Similarly, for Galileo E1, the LSC_GE strategy shows improvements of 0.75%, 12.27%, and 11.58% on the first day following modeling, rising to 1.5%, 15.69%, and 14.96% by DOY 365. As a result, the accuracy of kinematic positioning is improved.

Overall, the unified covariance function proves effective for autocorrelation-based multipath mitigation, outperforming conventional grid methods. Although GPS orbit drift negatively affects long-term mitigation, integrating GPS and Galileo data consistently produces lower RMS values and more accurate baseline solutions, significantly mitigating the impact of orbit drift. However, the method is based on spatial autocorrelation. Therefore, in the case that the spatial autocorrelation is low, the performance will be limited. This may happen in complex multipath environments. However, this needs to be tested when complex multipath environments are encountered in practice.

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Conflicts of Interest: The authors declare no conflicts of interest.

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Article

Quantifying Creep on the Laohushan Fault Using Dense Continuous GNSS

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Abstract: The interseismic behavior of faults (whether they are locked or creeping) and their quantitative kinematic constraints are critical for assessing the seismic hazards of faults and their surrounding areas. Currently, the creep of the eastern segment of the Laohushan Fault in the Haiyuan Fault Zone at the northeastern margin of the Tibetan Plateau, as revealed by InSAR observations, lacks confirmation from other observational methods, particularly high-precision GNSS studies. In this study, we utilized nearly seven years of observation data from a dense GNSS continuous monitoring profile (with a minimum station spacing of 2 km) that crosses the eastern segment of the Laohushan Fault. This dataset was integrated with GNSS data from regional continuous stations, such as those from the Crustal Movement Observation Network of China, and multiple campaign measurements to calculate GNSS baseline change time series across the Laohushan Fault and to obtain a high spatial resolution horizontal crustal velocity field for the region. A comprehensive analysis of this primary dataset indicates that the Laohushan Fault is currently experiencing left-lateral creep, characterized by a partially locked shallow segment and a deeper locked segment. The fault creep is predominantly concentrated in the shallow crustal region, within a depth range of $0\text{--}5.7 \pm 3.4$ km, exhibiting a creep rate of 1.5 ± 0.7 mm/yr. Conversely, at depths of 5.7 ± 3.4 km to 16.8 ± 4.2 km, the fault remains locked, with a loading rate of 3.9 ± 1.1 mm/yr. The shallow creep is primarily confined within 3 km on either side of the fault. Over the nearly seven-year observation period, the creep movement within approximately 5 km of the fault's near field has shown no significant time-dependent variation, instead demonstrating a steady-state behavior. This steady-state creep appears unaffected by postseismic effects from historical large earthquakes in the adjacent region, although the deeper (far-field) tectonic deformation of the Laohushan Fault may have been influenced by the postseismic effects of the 1920 Haiyuan M8.5 earthquake.

Keywords: northeastern margin of the Tibetan Plateau; Laohushan fault; fault creep; GNSS measurement; present crustal deformation

1. Introduction

In contrast to interseismic locked coupling (continuous strain energy accumulation) and coseismic stick-slip (rapid destabilizing sliding) on earthquake faults, fault creep-slip motions indicate that the fault plane is open/uncoupled, and thus there is no significant accumulation of strain energy or risk of strong earthquakes [1,2]. Research has demonstrated that large earthquakes often nucleate at the transition zones between the locked and creeping segments of a fault [3], and that creeping segments can serve as barriers to the propagation of large ruptures [4]. The effectiveness of these barriers is influenced by the size of the creeping segments and the frictional properties of the fault plane [5,6]. In addition, fault creep may also trigger local small earthquake activity [7]. Therefore, understanding and characterizing fault creep is essential for constraining the seismic hazard

assessments of the fault itself, evaluating the potential for cascading ruptures based on fault segmentation, and elucidating the mechanisms driving regional seismicity [8]. To date, numerous major strike–slip faults worldwide, such as the Xianshuihe Fault on the eastern margin of the Tibetan Plateau [9], the San Andreas Fault [2,10], and the North Anatolian Fault [11,12], are believed to exhibit fault creep behavior. With advancements in geodetic technology, GNSS and InSAR observations have become widely employed for characterizing fault creep deformation. However, due to the limited distribution of GNSS stations, studies focusing exclusively on fault creep motion using GNSS data remain relatively scarce. At present, GNSS-based investigations have primarily focused on the Atotsugawa Fault [13], the Alto Tiberina Fault [14], and the Dead Sea Fault [15].

The Haiyuan Fault, located at the northeastern margin of the Tibetan Plateau, is a large left-lateral strike–slip fault zone composed of multiple nearly E–W-trending fault segments (shown as thick black solid lines in Figure 1). Since the Holocene, it has been rapidly slipping at a rate of 3–5 mm/yr [13–18]. Significant seismic events, including the 1920 Haiyuan M8.5 earthquake and the 1927 Gulang M8.3 earthquake, have occurred along this fault (Figure 1). Recent InSAR monitoring studies have revealed a ~35 km long segment of fault creep on the Laohushan Fault within the Haiyuan Fault Zone, with an estimated left-lateral creep rate of 5 ± 1 mm/yr [19]. This rate closely aligns with GNSS-derived far-field displacement rates across the fault [20,21], which is consistent with a velocity-strengthening friction mechanism. This mechanism suggests that the creeping segment may serve as a permanent barrier to seismic rupture propagation and could also potentially trigger moderate seismic activity in the surrounding region [7,19,22]. While recent InSAR results support the presence of creep along the Laohushan segment of the Haiyuan Fault, there remains debate regarding the exact rate of this creep and its associated spatial deformation characteristics [23–26].

It is important to note that while InSAR is a valuable technique for measuring line-of-sight surface deformation, it is not capable of directly and accurately capturing horizontal fault movements. Moreover, the complex vertical crustal deformation present in this region [27] complicates the effective removal of vertical components from the InSAR deformation field [22], potentially leading to an overestimation of the creep rate [23]. Although some studies have incorporated published GNSS horizontal velocity fields as a priori constraints to enhance the accuracy of InSAR-derived deformation results, there remains a lack of GNSS constraints in critical near-field fault zones (<5 km) [23,24]. Accurate near-field fault deformation is crucial for correctly determining fault creep characteristics (such as creep depth, rate, and other kinematic parameters) and understanding its deformation mechanisms [28,29]. This study addresses these challenges by utilizing nearly seven years of observation data from a dense GNSS continuous monitoring profile (with a minimum station spacing of 2 km) across the eastern segment of the Laohushan Fault. Supplemented by data from regional land–state networks, additional GNSS continuous stations, and multiple GNSS survey campaigns (Figure 1), we have calculated GNSS baseline change time series and have generated high-resolution regional crustal horizontal velocity fields across the Laohushan Fault. This comprehensive dataset reveals the dynamic deformation characteristics of both near- and far-field crustal deformation across the fault. It provides precise constraints for determining the current creep rate, its temporal variations, and the depth distribution of creeping and locking segments, thereby offering valuable insights into the fault’s deformation mechanisms and contributing to regional seismic hazard assessments.

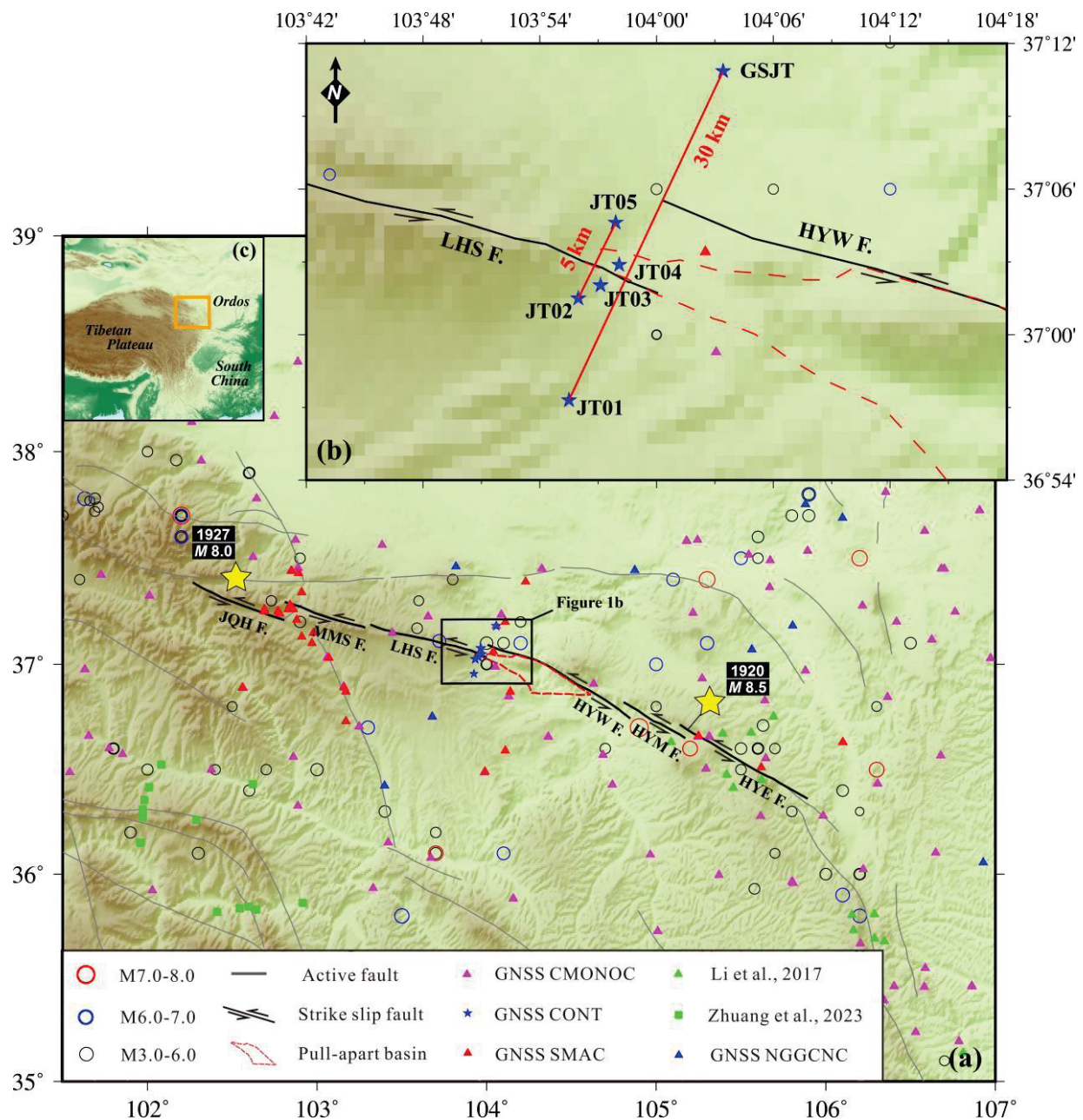


Figure 1. Regional active tectonics map. (a) Gray lines represent major active faults [30]. The red dashed line area indicates the Pull-Apart Basin [31]. Hollow circles represent historical earthquakes [32]. Blue pentagons mark the distribution of GNSS continuous observation profile sites established by the Second Monitoring Center of the China Earthquake Administration across the Laohushan Fault. Red triangles represent GNSS survey stations from the Fifteen Digital Seismic Network Construction Project. The pink triangles denote observation stations from the Crustal Movement Observation Network of China. The blue triangles indicate stations from the National GNSS Geodetic Control Network of China. Green triangles and green squares represent published GNSS observation results. Major fault segments of the Haiyuan Fault are shown as thick solid lines, with the following abbreviations: JQH F., the Jinqianghe Fault; MMS F., the Maomaoshan Fault; LHS F., the Laohushan Fault; HYW F., the western segment of the Haiyuan Fault; HYM F., the central segment of the Haiyuan Fault; HYE F., the eastern segment of the Haiyuan Fault. (b) Details of the solid black line in (a). (c) The orange rectangle in (c) shows our study area.

2. GNSS Data and Processing

2.1. GNSS Observations and Data Collection

The GNSS continuous observation profile across the eastern segment of the Laohushan Fault in the Haiyuan Fault Zone is shown in Figure 1b. This profile includes a total of 5 GNSS continuous stations. Among them, stations JT02, JT03, JT04, and JT05 are located within approximately 5 km of the fault, while JT01 and GSJT are 30 km apart, forming a combined near- and far-field GNSS continuous deformation observation system consisting of a total of 6 GNSS continuous stations (Figure 1b). We obtained continuous observation data from the above-mentioned GNSS continuous observation system (6 stations) from August 2018 to May 2021. All stations meet the requirements for continuous deformation field studies, with an unobstructed view of more than 10° , a data availability rate of over 90%, and multipath errors MP1 and MP2 both less than 0.5 m. With the exception of JT04, all stations have more than 2.5 years of observation data, which can ensure meeting the need to obtain stable and reliable observation results. In addition, a one-time GNSS survey campaign was conducted at the site in March 2024, with a continuous observation duration of 72 h. The survey met the requirements for unobstructed views of more than 10° , a data availability rate of over 90%, and multipath errors MP1 and MP2 both less than 0.5 m.

This study also collected GNSS survey data from other projects, including the following: (1). Data from the Gulang–Yongdeng GNSS profile and the Jingtai–Baiyin GNSS profile established through key platform projects supported by the Ministry of Science and Technology, National Natural Science Foundation projects, and Sino-French cooperation projects. This includes data from surveys conducted in 1999, 2004, 2005, and 2011. (2). GNSS survey data from the Huazangsi and Shagouhe profiles on the Maomaoshan Fault, established by the Second Monitoring Center of the China Earthquake Administration, with observations in 2005, 2006, 2007, and 2011. (3). Observation stations from the Crustal Movement Observation Network of China from 1999 to 2024. (4). Observation data from the National GNSS Geodetic Control Network of China (NGGCNC), which conducted their first survey in 2005 and 2014; surveys were conducted again twice by the Second Monitoring and Application Center (SMAC) and the China Earthquake Administration in 2012 and 2018–2019, respectively.

2.2. GNSS Data Processing

2.2.1. The Baseline Time Series of GNSS Profile

According to the principles of GNSS relative positioning, short-baseline double-difference solutions are particularly effective in reducing and eliminating regional biases introduced by ionospheric and tropospheric disturbances, as well as inaccuracies in geophysical models, satellite orbit errors, and reference frame instability. With fixed ambiguities, these solutions exhibit rapid convergence [33]. Additionally, noise time series from adjacent continuous observation stations tend to have similar spectral characteristics [34], allowing short-baseline double-difference solutions to cancel out or suppress such noise effectively. Consequently, GNSS short-baseline double-difference solutions offer superior accuracy and reliability in crustal deformation monitoring compared to long-baseline solutions [33,35].

The short-baseline GNSS time series data were processed using the GAMIT/GLOBK software [36,37], applying a free-network solution with fixed satellite orbits (precise ephemerides). To reduce first-order ionospheric effects, a dual-frequency ionosphere-free combination was utilized, and second-order ionospheric corrections were applied based on the Total Electron Content (TEC) data from the IONEX product, provided at two-hour intervals by the Center for Orbit Determination in Europe (CODE). Tropospheric zenith delays were corrected using the GPT2 model, with one zenith delay parameter estimated per station per hour, employing the VMF1 model as the mapping function. Corrections for the Earth's gravity field, solid Earth tides, and pole tides were made in accordance with the latest IERS standards (IERS2003) [38]. Crustal deformation due to ocean tidal loading was accounted for using the FES2004 global ocean tide model, and additional adjustments were

made for Earth's center of mass variations induced by ocean tides [39]. Given that this study focuses on monitoring the differential movements across the Laohushan Fault by assessing relative displacement characteristics on either side of the fault, it is unnecessary to constrain the coordinates of the GNSS stations (as shown in Figure 1) within a global reference frame. Instead, a free-baseline solution mode utilizing fixed satellite orbits (precise ephemeris) is employed to obtain GNSS short-baseline time series. This approach mitigates potential errors that could be introduced during the reference frame implementation process.

The GNSS stations selected for calculating the baseline time series in this study include JT01, JT02, JT03, JT04, JT05, and GSJT (Figure 1). The maximum inter-station distance is less than 30 km, with the shortest spacing being approximately 2 km. Due to a receiver malfunction, station JT04 recorded less than one year of intermittent observation data. Additionally, JT04 is located on the periphery of the Laolongwan Pull-Apart Basin (Figure 1a,b), making its observations potentially susceptible to influences from basin activity and non-tectonic factors. Therefore, this station was excluded from the subsequent GNSS baseline and velocity field analyses. Using the aforementioned data processing methods, baseline horizontal vector time series were computed by pairing the five remaining GNSS continuous stations across the Laohushan Fault, resulting in 10 baseline horizontal vector sets (Figure 2). These horizontal vectors were projected both along the fault strike direction ($SE102^\circ$) and perpendicular to the fault strike ($NE12^\circ$), with the results presented in Figure 3a,c, respectively.

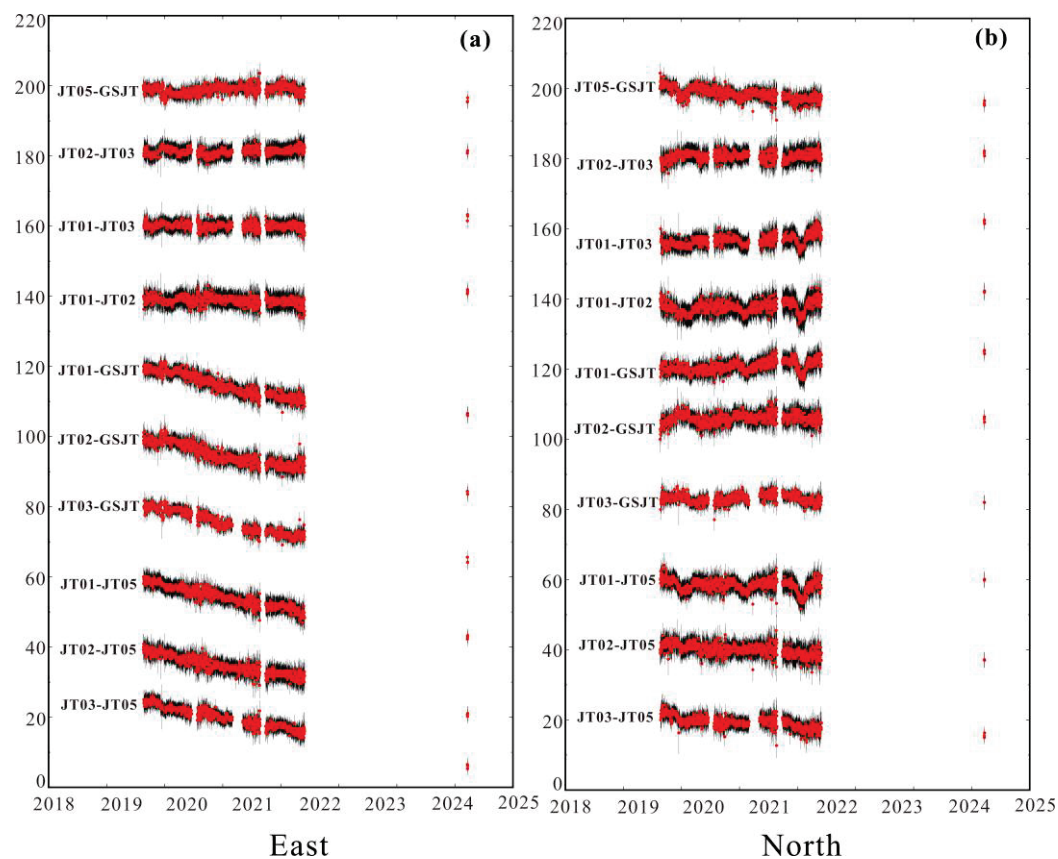


Figure 2. Time series of GNSS short-baseline horizontal vectors. (a) Eastward horizontal baseline vector; (b) northward horizontal baseline vector.

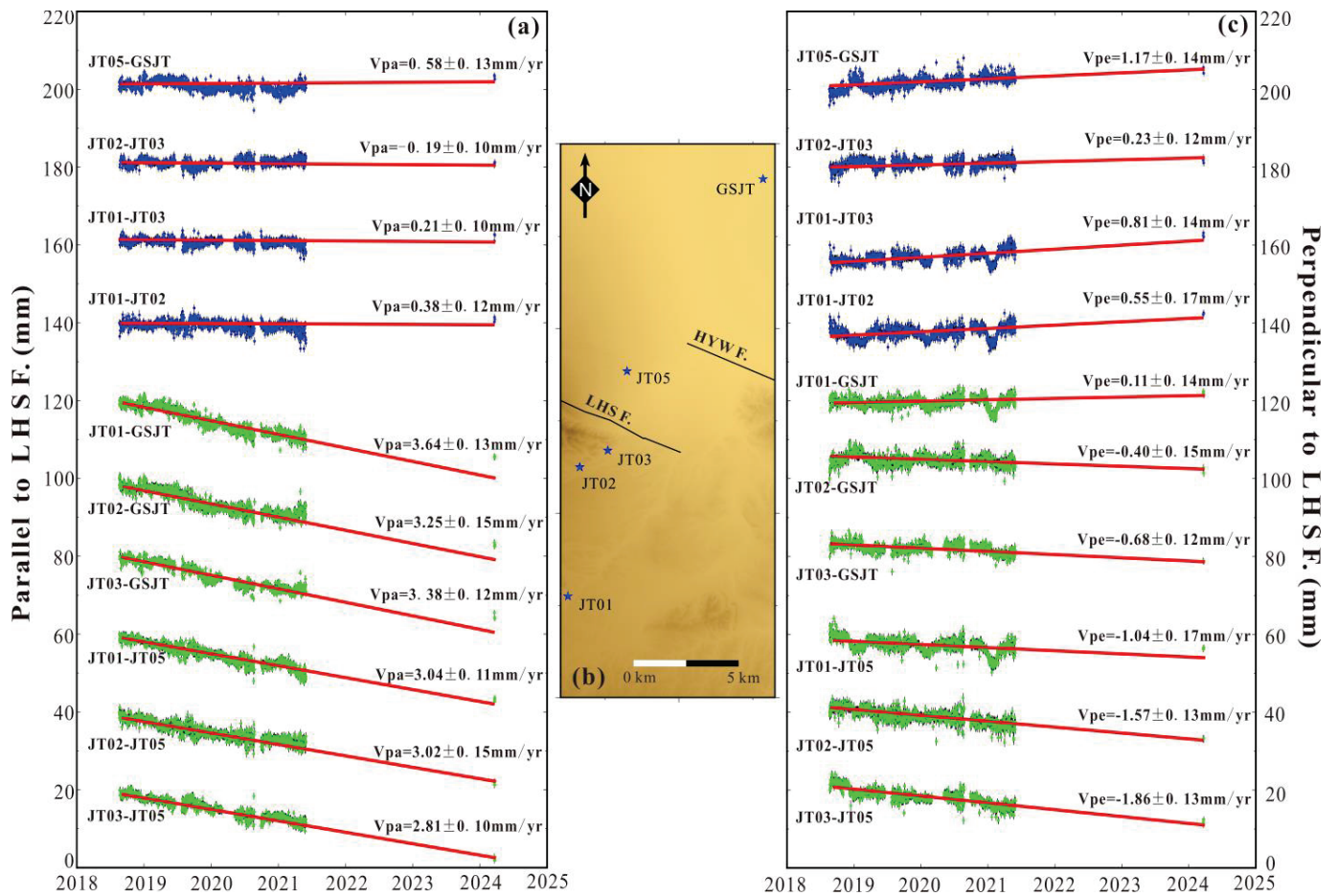


Figure 3. Baseline component variation time series for the Laohushan Fault; (a) time series of baseline changes parallel to the Laohushan Fault, where V_{pa} represents the variation rate of the baseline component parallel to the fault; (b) distribution of GNSS stations; (c) time series of baseline changes perpendicular to the Laohushan Fault, where V_{pe} represents the variation rate of the baseline component perpendicular to the fault. Green points represent the baseline time series across the fault. Blue points represent the baseline time series on the same side of the fault. Red solid lines represent the linear rate of best fit from the least-squares method.

2.2.2. Regional GNSS Horizontal Velocity Field

The GAMIT/GLOBK software suite [36,37] was used to process double-differenced carrier phase observations (Figure 1), and daily loosely constrained solutions were obtained. A dual-frequency ionosphere-free combination was employed to mitigate first-order ionospheric effects, while higher-order ionospheric effects were corrected using the 2 h interval Total Electron Content (TEC) IONEX products provided by the Center for Orbit Determination in Europe (CODE). The geophysical models and parameters applied in the processing, such as the FES2004 tidal loading model, the GPT2 global pressure and temperature model, and the VMF1 mapping function, were consistent with those used by [35].

Instead of incorporating global daily solutions provided by the IGS analysis centers, we processed GNSS observation data from 70 core ITRF reference stations globally using the GAMIT package with identical modeling parameters. Daily coordinates and associated uncertainties were estimated by constraining the daily regional solutions with global solutions, and the daily free network solutions were transformed into the ITRF2014 reference frame [40] using the GLOBK package [37]. For campaign GNSS sites, linear velocities relative to ITRF2014 were computed using a weighted least-squares adjustment. Finally,

the GNSS horizontal velocity field was transformed into the Eurasian reference frame using the Euler rotation vector for the Eurasian plate provided by [40], as illustrated in Figure 4.

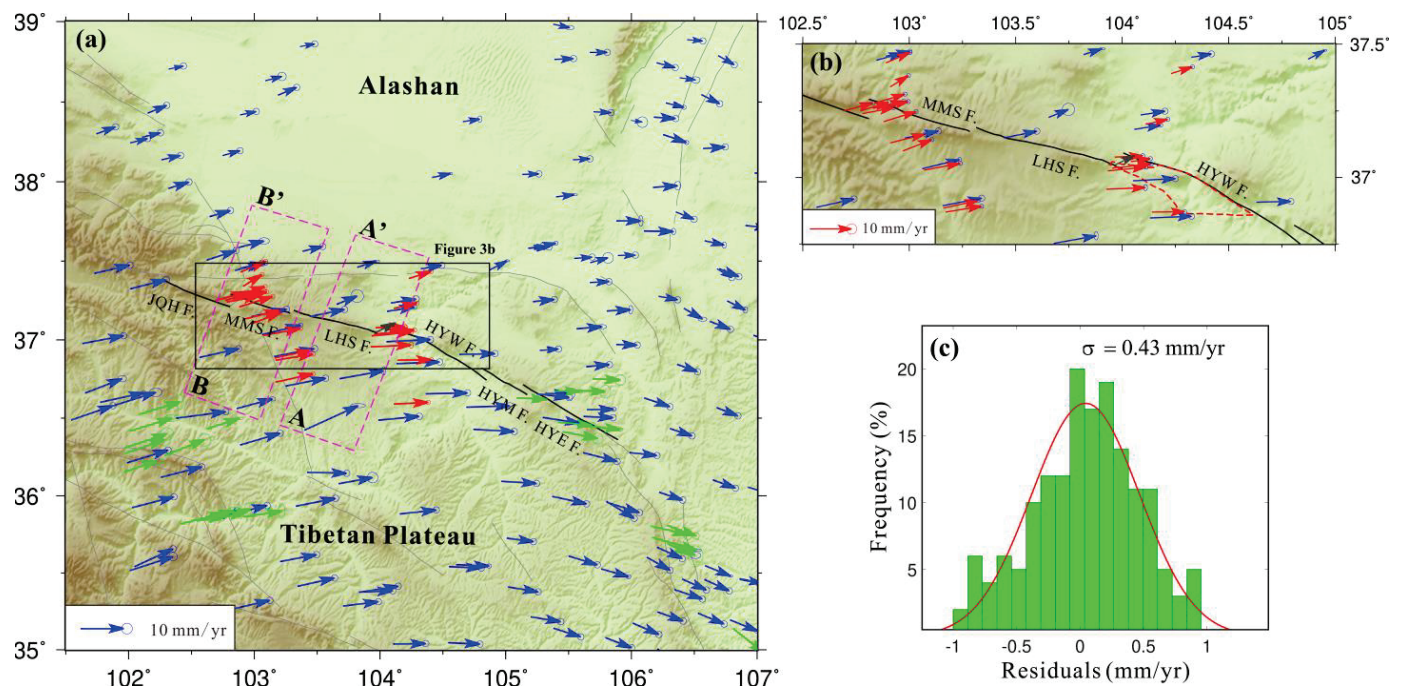


Figure 4. GNSS horizontal velocity field of the Laohushan Fault and surrounding area (relative to the Eurasian reference frame); (a) the pink box indicates the location of the GNSS profile shown in Figure 5; (b) an enlarged view showing the denser GNSS velocity field around the Maomaoshan Fault and Laohushan Fault; (c) the errors associated with common points used in the 200 integration of the published velocity field.

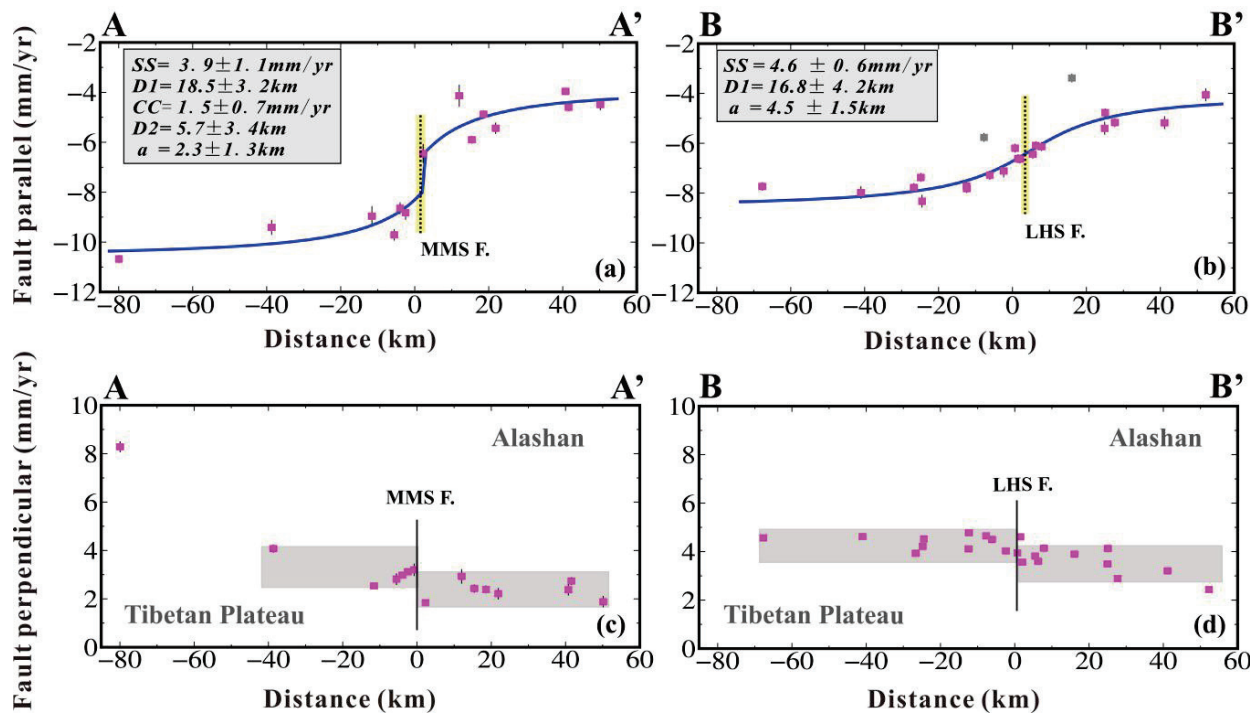


Figure 5. (a,b) display the GNSS velocity components parallel to the fault, with pink squares indicating the data and gray squares representing outliers (which were not used in the inversion). (c,d) show the GNSS velocity components perpendicular to the fault, with similar color coding: pink

squares for data and gray squares for outliers. The black dashed lines in (a–d) indicate the position of the fault as determined by inversion, with yellow shading representing the error margin. The 0 km mark denotes the actual fault position. The blue solid lines in (a,b) represent the inverse tangent curves calculated from the fault slip model, where SS denotes the slip rate, D1 indicates the locking depth, CC represents the shallow creep rate, and D2 denotes the creep depth. The gray shading in (c,d) represents the differential motion perpendicular to the fault on both sides. The black solid line shows the actual fault position.

To accurately analyze differential crustal motion between tectonic blocks, we utilized a GNSS velocity field referenced to the stable Eurasian plate [41]. In addition, to precisely model the tectonic characteristics of the Laohushan Fault and enhance our understanding of regional crustal horizontal deformation, we incorporated published high-density GNSS results [42,43], as indicated by the green arrows in Figure 4a. The Helmert parameter transformation method was employed to integrate these published results with the GNSS velocity field derived in this study, ensuring that the residual error at common points in the merged dataset was less than 1 mm/yr (Figure 4c).

3. GNSS Baseline Time Series

Using the GNSS baseline component time series (Figure 3), we applied the least-squares method to fit the rates of change for the baseline components both parallel and perpendicular to the Laohushan Fault. In this analysis, the southernmost station of each baseline served as the reference point, with other stations considered positive if they moved left relative to the reference point and negative if they moved right (as depicted in Figure 3 and Table 1). The results indicate that the baseline components parallel to the Laohushan Fault consistently exhibit a left-lateral strike-slip motion, while the components perpendicular to the fault primarily indicate relatively weak compressional shortening.

Table 1. Fault-parallel and fault-perpendicular movement rates and associated error estimates derived from linear least-squares fitting.

Category	Baseline Name	Parallel Fault Rate (mm/yr)	Parallel Fault Rate Error (mm/yr)	Perpendicular Fault Rate (mm/yr)	Perpendicular Fault Rate Error (mm/yr)
Cross-Fault Baselines	JT03-JT05	2.81	0.10	−1.86	0.13
	JT02-JT05	3.02	0.15	−1.57	0.13
	JT01-JT05	3.04	0.11	−1.04	0.17
	JT03-GSJT	3.38	0.12	−0.68	0.12
	JT02-GSJT	3.25	0.15	−0.40	0.15
	JT01-GSJT	3.64	0.13	0.11	0.14
Same-Side Baselines	JT01-JT02	0.38	0.12	0.55	0.17
	JT01-JT03	0.21	0.10	0.81	0.14
	JT02-JT03	−0.19	0.10	0.23	0.12
	JT05-GSJT	0.58	0.13	1.17	0.14

Notably, all baselines crossing the Laohushan Fault (JT03-JT05, JT02-JT05, JT01-JT05, JT03-GSJT, JT02-GSJT, and JT01-GSJT) display significant variations in fitted parallel fault slip rates (>2.8 mm/yr). Specifically, within a narrow span of approximately 5 km across the fault (JT03-JT05), a left-lateral strike-slip rate of 2.81 ± 0.10 mm/yr is observed (Figure 2), accounting for 77% of the slip rate revealed by the longest baseline, JT01-GSJT (approximately 30 km), which measures 3.64 ± 0.13 mm/yr. In contrast, the baselines on the same side of the fault (not crossing the Laohushan Fault) (JT01-JT02, JT01-JT03, JT02-JT03, and

JT05-GSJT) show minimal variation in their fitted parallel fault slip rates (<0.6 mm/yr) (Table 1). These deformation characteristics collectively suggest that within a 30 km span across the Laohushan Fault, nearly all strike-slip shear deformation is concentrated within a narrow zone of less than 3 km on either side of the fault, with the differential strike-slip motion across the fault inducing negligible deformation on the same side of the fault.

Additionally, the least-squares analysis of GNSS horizontal movement rates perpendicular to the fault strike reveals a degree of horizontal extensional deformation (0.5–1.2 mm/yr) on the same side of the fault (Table 1). Conversely, across the fault, there is consistent but relatively minor horizontal compressional deformation (-0.4 to -1.9 mm/yr) (Figure 3b). These findings suggest that neglecting deformation perpendicular to the fault could introduce uncertainties in accurately estimating the kinematic parameters of the Laohushan Fault's creeping motion when using InSAR data.

4. GNSS Profiles and Inversions

The third section of this paper explores the strain distribution characteristics in the near-field region (within approximately 15 km on either side) of the Laohushan Fault, as revealed by the GNSS short-baseline fault motion components. The analysis identifies significant left-lateral creep along the Laohushan Fault. To further elucidate the detailed variations in strain distribution across both the near- and far-field regions of the fault, and to enhance our understanding of the constraints on fault movement characteristics at different depths, this study utilizes the regional GNSS horizontal velocity field. This field, which includes GNSS creep observation sites along the Laohushan Fault, was used to construct the GNSS velocity profile A–A' across the fault (Figure 4b).

For a comparative analysis of the transverse differences in interseismic fault motion within the region, an additional GNSS velocity profile, B–B', was constructed across the Maomaoshan Fault, located approximately 100 km west of the Laohushan Fault, using dense GNSS data along the Maomaoshan Fault (Figure 4b). The GNSS horizontal velocity components within the scope of these profiles were projected along directions parallel and perpendicular to the strikes of the Laohushan Fault ($NE102^\circ$) and the Maomaoshan Fault ($NE103^\circ$), respectively, to generate the GNSS velocity profiles for these two faults (Figure 5). A two-dimensional dislocation model incorporating fault creep motion was subsequently employed to invert and determine the geometric parameters (creep depth/locking depth) and kinematic parameters (creep rate/deep loading rate) associated with fault creep and locking.

4.1. Methods and Solution

We assume that plate motion occurs along one or more nearly vertical (upright) strike-slip faults, which are either locked or creeping from the surface to a certain depth. We use a combination of screw dislocation models [44–48] to perform inversion calculations for the Laohushan Fault and the Maomaoshan Fault. This model can simultaneously fit the deformation signals caused by both fault creep and deep locking, which represent two different physical mechanisms. The mathematical expression for this model can be described as follows:

$$V(x) = \frac{S}{\pi} \arctan\left(\frac{x}{d_1}\right) + C \left[\frac{1}{\pi} \arctan\left(\frac{x}{d_2}\right) - H(x) \right] + a \quad (1)$$

V represents the velocity parallel to the fault, x represents the distance perpendicular to the fault, S and d_1 represent the deep slip rate and locking depth of the fault, respectively, C and d_2 represent the shallow creep rate along the fault strike and the creep depth, respectively, $H(x)$ is the Heaviside step function, and a represents the offset between the data and the model. In the actual inversion process, reasonable constraint ranges for the parameters to be estimated were provided based on previous research results [23–25,42]: $0 < S < 20$ mm/yr, $0 < d_1 < 20$ km, $0 < C < 10$ mm/yr, and $0 < d_2 < 10$ km, $-20 < a < 20$ km. These parameters were assumed to follow a uniform prior distribution within the specified

ranges, with an additional constraint $d_2 < d_1$. After eliminating some obvious outliers and results with large errors (Figure 5), we used the Levenberg–Marquardt (L-M) method to linearize the nonlinear equations through a Taylor series expansion. The best-fit values and uncertainties for the parameters in the model were then determined through iterative calculations (Figure 5).

4.2. Results of GNSS Profile Inversion

The inversion results show (Figure 5a) that the Laohushan Fault currently exhibits a coupled motion characteristic with shallow creep and deep locking (the fault is not fully open). Specifically, the shallow left-lateral creeping rate is 1.5 ± 0.7 mm/yr, with a creep depth of 5.7 ± 3.4 km. The left-lateral elastic loading rate due to deep tectonic loading is 3.9 ± 1.1 mm/yr, and the elastic locking depth ranges from 5.7 ± 3.4 km to 16.8 ± 3.2 km. In contrast, the same inversion model applied to the Maomaoshan Fault profile (Figure 5b) does not reveal evidence of shallow fault creep. The left-lateral slip rate for the Maomaoshan Fault is determined to be 4.6 ± 0.6 mm/yr, with a locking depth of 16.8 ± 4.2 km, indicating a higher degree of strain accumulation compared to the Laohushan Fault.

Additionally, the horizontal offsets, estimated as free parameters, are 2.3 ± 1.3 km for the Laohushan Fault and 4.5 ± 1.5 km for the Maomaoshan Fault. These offsets correspond closely with the geological fault baselines, providing indirect validation of the observational data and inversion results. Furthermore, the horizontal shortening/extension rates across the faults, inferred from differential motion, suggest varying degrees of crustal shortening at both the Laohushan and Maomaoshan faults, with magnitudes not exceeding 1 mm/yr (Figure 5c,d).

5. Discussion

5.1. Comparison with Existing InSAR Results

In calculating the fault creep rate using InSAR observations, the effects of regional vertical deformation and deformation perpendicular to the fault cannot be neglected [23,25,26]. To account for these effects, we collected precise leveling data for the region from 1970 to 2012. By applying the dynamic least-squares adjustment method [49], we mitigated systematic errors by incorporating the vertical motion rates from continuous GNSS stations as a priori information. This approach enabled us to derive the vertical crustal deformation field for the study area (Figure 6). As depicted in Figure 6, the Haiyuan region exhibits complex vertical deformation, with vertical displacement rates of 1–2 mm/yr observed near the eastern segment of the Laohushan Fault.

We analyzed GNSS profiles and fault dislocation model inversion results for the Laohushan Fault (Figure 5) and found that its left-lateral creep occurs at shallow depths, above 5.7 ± 3.4 km, with a creep rate of 1.5 ± 0.7 mm/yr. These findings were compared with previously published estimates of the Laohushan Fault's creep rate based on InSAR data (Table 2). As shown in Table 2, the GNSS-derived creep rate is lower than the values obtained using InSAR. Even within studies utilizing the same InSAR dataset (InSAR + GNSS (far-field)), discrepancies persist. Specifically, creep rates derived from combined ascending and descending InSAR observations [23,25,26] are lower than those obtained from single-track InSAR data [19,24].

When calculating fault deformation using InSAR technology, single-track InSAR observations require neglecting both vertical deformation and deformation perpendicular to the fault. In contrast, combined ascending and descending track InSAR data only necessitate ignoring deformation perpendicular to the fault [23,50,51]. Existing leveling observations (Figure 6) suggest a vertical deformation of 1–2 mm/yr near the eastern segment of the Laohushan Fault, while near-field GNSS continuous data from this study (Figures 3 and 5) show 1 mm/yr of deformation perpendicular to the fault. In prior InSAR studies, whether using single-track or combined ascending and descending track observations, the inability to accurately separate vertical crustal deformation from fault-perpendicular deformation

has likely led to overestimations of the creep rate of the Laohushan Fault. Therefore, to reliably calculate fault deformation characteristics from InSAR data, it is crucial to fully account for both vertical crustal deformation and deformation perpendicular to the fault.

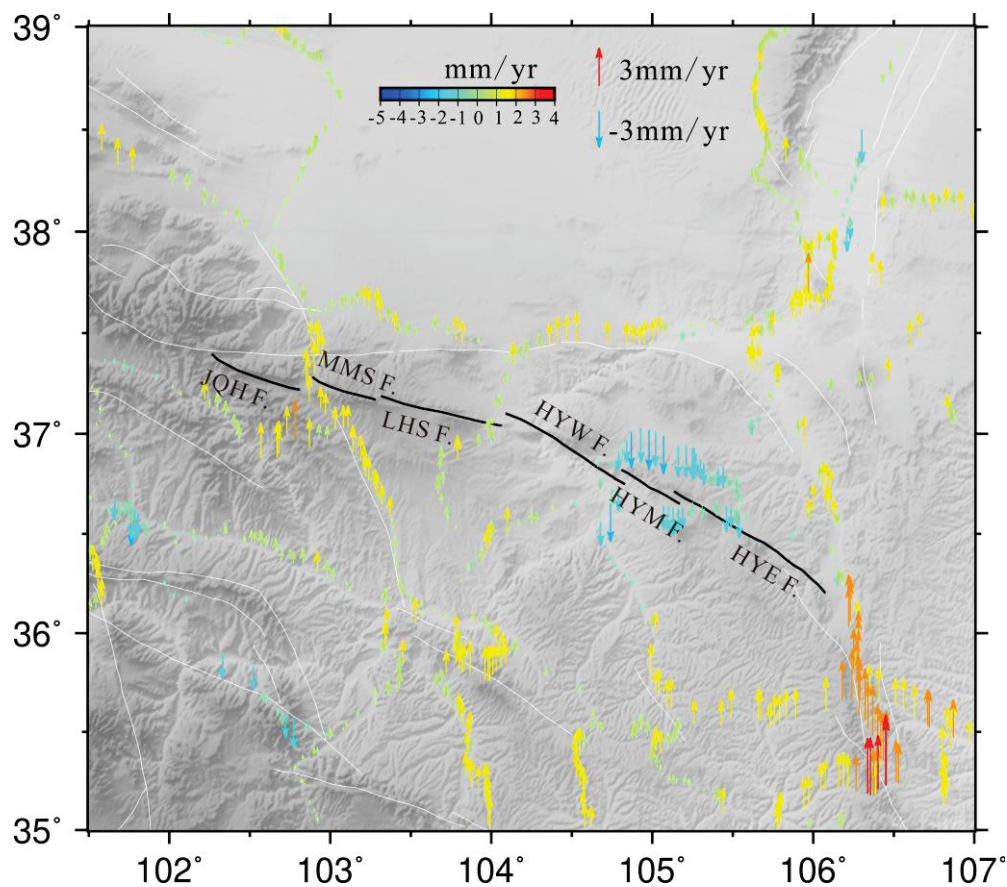


Figure 6. Leveling vertical land motion rates in the Haiyuan Fault and its surrounding regions. Scale of the arrows indicates velocity magnitude, underlined by color. Positive (negative) values denote crustal uplift (subsidence).

Table 2. Summary of creep rates along the Laohushan Fault Zone. (D: descending track InSAR observations; A/D: combined ascending and descending track InSAR observations).

Fault Name	Data Detection	Creep Rate (mm/yr)	Creep Depth (km)	References
LaoHuShan Fault	InSAR (D)	5 ± 1	-	Jolivet et al., 2013 [19]
	InSAR (A/D) + GNSS (Far-field)	2.5 ± 0.4	0~4–8	Li et al., 2021 [23]
	InSAR (D) + GNSS (Far-field)	4.5~4.8	$0 \sim 0.7 \pm 1.3$	Qiao et al., 2021 [24]
	InSAR (A/D) + GNSS (Far-field)	2.9 ± 0.4	$0 \sim 16.9 \pm 8.3$	Huang et al., 2022 [25]
	InSAR (A/D) + GNSS (Far-field)	2.1 ± 0.1	$0 \sim 0.3 \pm 0.06$	Guo et al., 2023 [26]
	GNSS (Near-field)	1.5 ± 0.7	$0 \sim 5.7 \pm 3.4$	Our Results

5.2. Temporal and Spatial Variation in Creep Motion on the Laohushan Fault

Considering the lack of observations between June 2021 and March 2024 (Figure 2), and to avoid crustal deformation caused by seasonal changes, we selected observation data from the same season in different years. The slip rates for the two cross-fault GNSS baselines (JT03–JT05 and JT02–JT05) within the near-field of the Laohushan Fault (approximately

5 km) were calculated in segments (Figure 7a). The results show that the creep rate in the near-field of the fault does not exhibit significant changes over different time periods, indicating that the creep motion in the fault's near-field is steady.

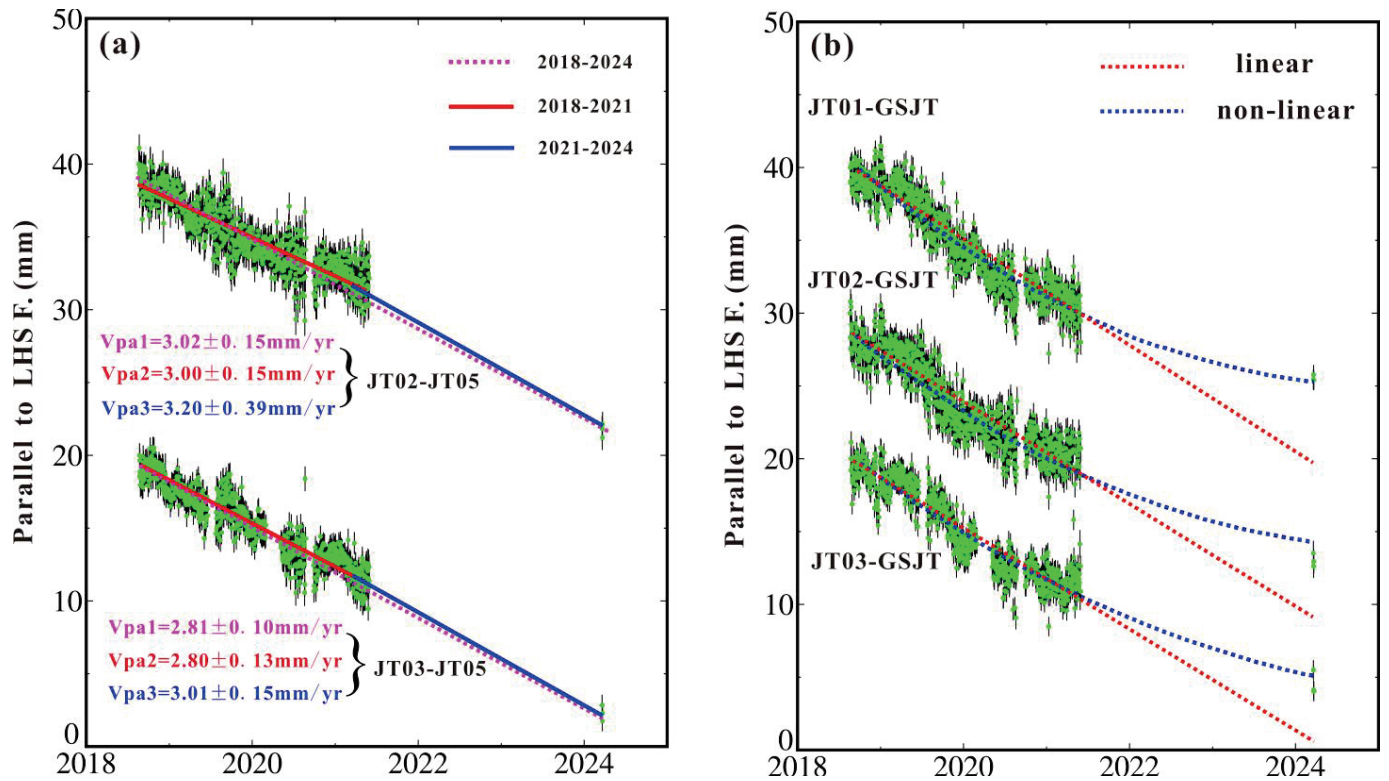


Figure 7. (a) Rate fitting of the baseline component parallel to the fault in the near-field across the fault; (b) rate fitting of the baseline component parallel to the fault in the far-field across the fault. The red dashed line represents the least-squares linear fit, and the blue dashed line represents the result of the fitting using Equation (2).

In contrast, Figure 3a shows that the cross-fault baseline components parallel to the fault, composed of observation points farther from the fault (greater than 20 km), such as JT03-GSJT, JT02-GSJT, and JT01-GSJT, cannot be accurately fitted with a least-squares linear method for the final observation in 2024. There appears to be significant nonlinear deformation, and the discrepancies in the least-squares linear fit become more pronounced with longer baseline distances. Therefore, we attempt to fit the time series of these three parallel fault baseline components using a postseismic viscoelastic relaxation model (Equation (2)).

$$y = v \times dt + D \times \log(1 + dt/\tau) \quad (2)$$

In this context, y represents the baseline component parallel to the fault, dt denotes the epoch observation time, v indicates the linear rate of change of the baseline component parallel to the fault, and $D \times \log(1 + dt/\tau)$ represents a commonly used postseismic time series fitting function [52–54].

The fitting results presented in this paper (Figure 7b, blue dashed line) demonstrate that the postseismic relaxation model not only closely matches the continuous observations from 2018 to 2021 but also aligns well with the final observations in 2024. Notably, these nonlinear deformation features are absent in the fitting results of the baseline components perpendicular to the fault (Figure 3c). This suggests that if these effects are indeed related to postseismic relaxation from a nearby earthquake, the event in question was likely a left-lateral strike-slip earthquake.

In addition, the nonlinear deformation features observed in the far-field baselines parallel to the fault may be related to postseismic effects from the 1920 Haiyuan earthquake. To explore this possibility, we projected and decomposed the regional high-density GNSS velocity field obtained in this paper (Figure 3) along the direction parallel to the Haiyuan Fault (Figure 8). The results reveal a significant zone of rapid activity south of the 1920 Haiyuan earthquake rupture zone, overlapping with the far-field region of the Laohushan Fault. Additionally, GNSS velocity profiles (Figure 5) indicate that the fault-parallel component on the southern side of the Laohushan Fault is approximately 2 mm/yr faster than that of the Maomaoshan Fault. This observational analysis suggests that the far-field deformation of the Laohushan Fault is probably influenced by postseismic deformation from the 1920 Haiyuan earthquake.

On the other hand, baseline time series and dislocation model inversion results (Figures 3, 5 and 7) indicate that the near-field creep of the Laohushan Fault (within approximately 5 km) does not exhibit the postseismic exponential decay deformation pattern observed in the far-field long baselines. Instead, it shows a stable creep motion. Therefore, the stable shallow creep of the Laohushan Fault is unlikely to be significantly affected by the postseismic deformation of the 1920 Haiyuan earthquake.

5.3. Regional Fault Movement and Earthquake Hazard

Based on the GNSS profiles and dislocation model inversion results for the Laohushan Fault (Figure 5), the left-lateral creep is observed to occur at shallow depths above 5.7 ± 3.4 km, with a rate of 1.5 ± 0.7 mm/yr. This is comparable to the 2–3 mm/yr rate obtained from InSAR data, which accounts for vertical deformation effects [23,25]. Additionally, the results of this study do not support the claim by [22] that the Laohushan Fault is currently experiencing fully open creep, as also suggested by InSAR-based studies [24,42]. Instead, the Laohushan Fault is found to be locked within a depth range of 5.7 ± 3.4 to 18.5 ± 3.2 km (Figure 5).

The characteristics of shallow creep and deep locking observed on the Laohushan Fault are supported by regional seismic activity results [55]. Previous studies indicate that there have been almost no earthquakes of magnitude 3 or greater within the 0–5 km depth range of the Laohushan Fault, which is attributed to the presence of a loose, thick fault gouge in the shallow part of the fault [55]. The collected regional aftershock data from 2008 to 2019 also show that earthquakes of magnitude 3 or greater are primarily concentrated in the 5–20 km depth range of the Laohushan Fault (Figure 9b,c), which aligns closely with the deep fault locking layer position inferred in this study. Additionally, paleoseismic trench studies reveal that the Laohushan Fault has recorded more than two prehistoric earthquakes of magnitude 7.0 or greater since the Holocene [56]. Based on these findings, we conclude that the creep on the Laohushan Fault represents shallow fault activity, while the deep part of the fault is currently locked and experiencing significant strain accumulation, indicating that the fault's potential for a strong earthquake warrants attention.

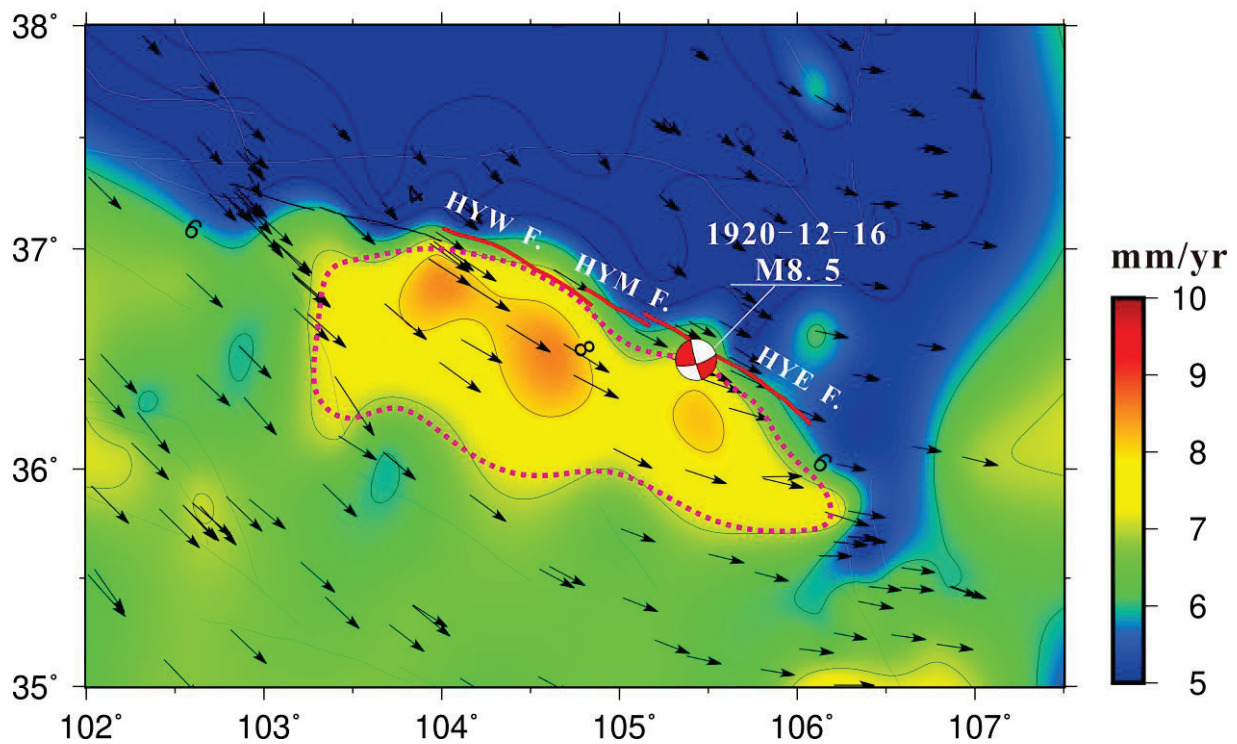


Figure 8. GNSS velocity field parallel to the Haiyuan Fault. The colors in this Figure represent the magnitude of the velocity, while the direction of the arrows indicates the velocity direction. The red beach ball represents the focal mechanism solution [57].

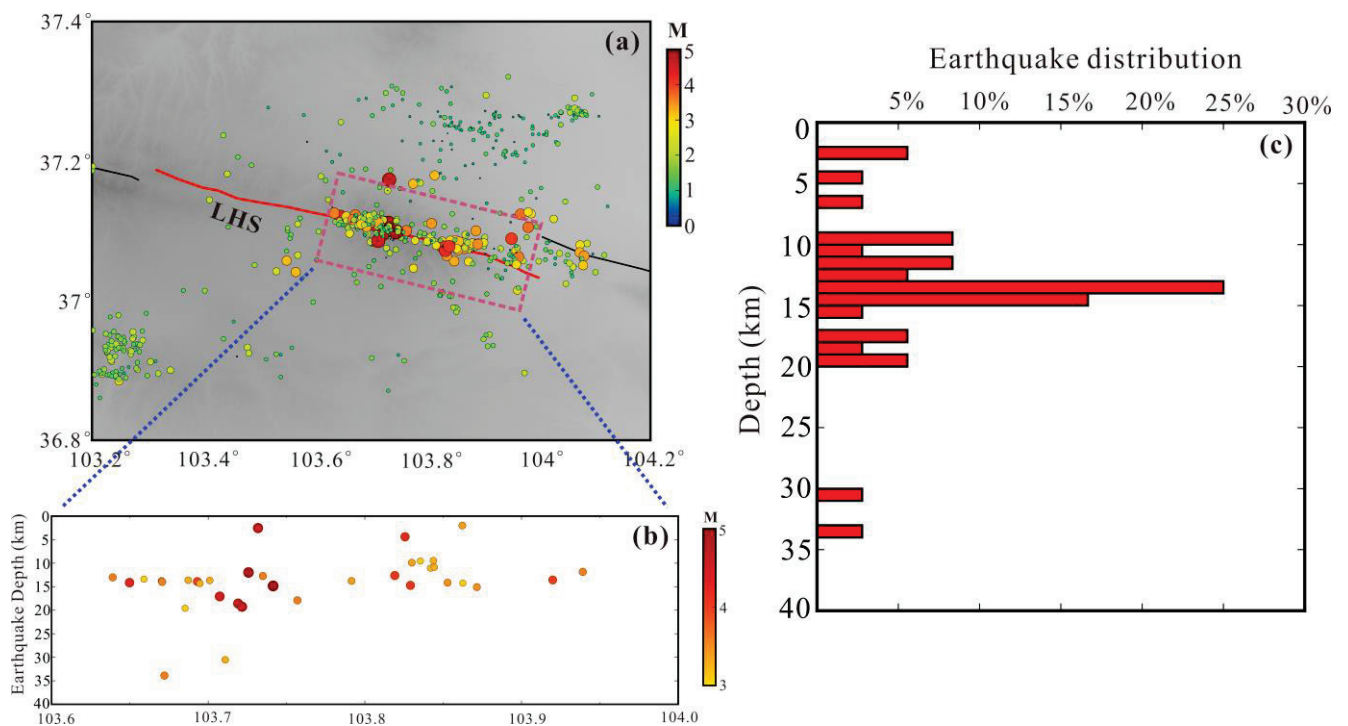


Figure 9. The distribution of small earthquake activity on the Laohushan Fault: (a) The spatial distribution of small earthquake activity on the Laohushan Fault. (b) Red circles represent the projections of small earthquake activity along the depth of the Laohushan Fault, totaling 36 events. (c) The statistical distribution of earthquakes of magnitude 3 or greater with respect to the fault depth.

We compared the inversion results for the Laohushan and Maomaoshan Faults (Figure 4). The Maomaoshan Fault does not exhibit abrupt fault creep signals across the fault but rather shows typical interseismic locking characteristics. The inversion indicates a locking depth of 16.8 ± 4.2 km, which aligns with the 15 km estimate by [42]. The left-lateral slip rate of the Maomaoshan Fault, as determined in this study, is 4.6 ± 0.6 mm/yr, which is consistent with previous geological estimates [13–18] and geodetic estimates of 3 to 5 mm/yr [20,21,58]. Therefore, with its higher slip rate and locking depth, the Maomaoshan Fault currently poses a higher background risk for strong earthquakes.

It is worth noting that the results for the near-field baselines across the Laohushan Fault (approximately 5 km), JT03-JT05 and JT02-JT05 (Figure 3), show a left-lateral slip rate difference across the fault of 2.8–3.0 mm/yr. This is higher than the 1.5 ± 0.7 mm/yr creep rate estimated from the GNSS velocity profile (Figure 5b). This discrepancy may be due to the creep on the Laohushan Fault being concentrated in a smaller region on either side of the fault [23,25], which is smaller than the spacing of the existing GNSS near-field measurements. Therefore, future research should consider integrating dense GNSS and InSAR data to more accurately determine the creep parameters of the Laohushan Fault.

6. Conclusions

We used the latest densely spaced GNSS continuous observations across the Laohushan Fault and regional multi-source GNSS data as constraints to process and obtain precise baseline time series across the Laohushan Fault, as well as a densely monitored regional GNSS horizontal velocity field. The comprehensive analysis of these data reveals that the Laohushan Fault currently exhibits a left-lateral creep motion with characteristics of shallow creep and deep locking. The current fault creep mainly occurs in the shallow crust at depths of 0 to 5.8 ± 3.4 km at a rate of 1.5 ± 0.7 mm/yr. Below the creeping layer, down to a depth range of 16.8 ± 4.2 km, the fault is in a locked state with a locking rate of 3.9 ± 1.1 mm/yr. Additionally, the near-field creep motion (approximately 5 km) of the Laohushan Fault appears stable and seems unaffected by the postseismic effects of historical strong earthquakes in the adjacent area. However, the deep (far-field) tectonic deformation of the Laohushan Fault may be influenced by the postseismic effects of the 1920 Haiyuan M8.5 earthquake.

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Article

A Deep-Learning Based GNSS Scene Recognition Method for Detailed Urban Static Positioning Task via Low-Cost Receivers

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Abstract: Global Navigation Satellite Systems (GNSS)-based position service is widely applied in cities, but the precision varies significantly in different obstruction scenes. Scene recognition is critical for developing scene-adaptive GNSS algorithms. However, the complexity of urban environments and the unevenness of received signal especially in low-cost receivers limit the performance of GNSS-based scene recognition models. Therefore, our study aims to construct a scene recognition model suitable for urban static positioning and low-cost GNSS receivers. Firstly, we divide the scenes into five categories according to application requirements, including open area, high urban canyon, unilateral urban canyon, shade of tree and low urban canyon. We then construct feature vectors from original observation data and consider the geometric relationships between satellites and receivers. The different sensitivity to different scenes is discovered through an analysis of the performance of each feature vector in recognition. Therefore, a GNSS positioning scene recognition model based on multi-channel LSTM (MC-LSTM) is proposed. The results of experiments show that an accuracy of 99.14% can be achieved by our model. Meanwhile, only 0.75 s and 1.95 ms are required in model training per epoch and model prediction per data on a CPU, which presents a significant improvement of over 90% compared with existing works. Furthermore, our model can be transferred into different time periods quickly and can maintain robustness in situations where one or two types of observation data are missed. A maximum accuracy of 81.13% can be achieved when two channels are missed, while 96.06% is attainable when one channel is missed. Therefore, our model has the potential for real applications in complex urban environments.

Keywords: GNSS scene recognition; static urban positioning; deep learning; long short-term memory; transfer learning

1. Introduction

Global Navigation Satellite Systems (GNSS) have undergone rapid development and progress in past decades. With the successive establishment of global services such as GPS, BDS, GLONASS, Galileo and regional services such as QZSS, IRNSS, GNSS-based applications have also widely been implemented. Bike-sharing is a typical example among them. Due to its convenience for short-length travel and last-mile connectivity of public transportation, bike-sharing services have expanded to over 300 cities in mainland China and gained widespread popularity especially in super cities [1]. However, the prosperous growth of bike-sharing has brought a series of problems. Over-deployment by operators, false-parking by users and poor scheduling by public management have caused mass disarray of public sharing-bikes, which has become a hot and troubling issue among citizens [2].

A practical solution to this problem is to use electronic fence technology to regulate users' parking behaviors, which means the bike-sharing service is made unable to deactivate unless they are parked in the range of virtual "parking zones". Unfortunately, shared bikes

often need to be parked in complex environments, which means GNSS signals could be easily blocked by street trees or affected by strong reflections from buildings on one or both sides [3]. The deteriorated signal quality leads to a reduction in positioning precision, which limits the performance of electronic fence. To address this issue, adjusting the threshold of electronic fence according to the positioning precision is a feasible approach. However, the precision of positioning significantly varies in different obstruction conditions [4], which makes it important to recognize the obstruction scenes before adjusting. On the other hand, improving the precision of GNSS positioning in complex urban environments is also considered as an effective approach and several studies have been proposed in recent years, including anti-multipath [5,6], array signal processing [7,8], high-sensitivity tracking algorithms [9,10] and so on [11]. However, all these scene-adaptive methods require rapid and accurate recognition, or they may cause unexpected errors when scenes are mismatched.

In order to recognize the positioning scenes, researchers have proposed a series of methods, which can be mainly categorized into two technical approaches: multi-sensor fusion and GNSS signal-based methods. From the perspective of multi-sensor fusion, camera, LiDAR [12–14], and signal sensors such as accelerometers and gyroscopes are put into use to detect the environmental features surrounding GNSS receivers in order to recognize contexts [15,16]. Although accurate context recognition could be accessible by multi-sensor fusion, the high computational power and device costs make it economically unappealing for bike-sharing operators. From the perspective of GNSS signal-based methods, different observation data and data-constructed features are selected to recognize GNSS positioning contexts. In the field of indoor and outdoor detection, C/N_0 and the number of visible satellites are widely used as classification features [17–19]. Recently, more and more research has focused on detailed contexts segmentation and the performances of machine learning in this field. Lai et al. [20] use support vector machine (SVM) to divide the environment into open outdoors, occluded outdoors and indoors, which has reached a recognition accuracy of 90.3%. Dai et al. [21] change the GNSS observation data into sky plots and compare the performances of CNN and Conv-LSTM in recognition of open-outdoor, semi-outdoor, shallow indoor and deep indoor. The results show that CNN can reach an accuracy of 98.82% while Conv-LSTM can reach 99.2%. Zhu et al. [22] conducted an analysis of 196 variables related to visible satellite, satellite distribution, signal strength and multipath effect based on statistical methods to find the most important feature elements. They then used those selected features to recognize contexts and compare the performances of eight machine learning models. The results show that LSTM can reach an accuracy of 95.37% in vehicle-mounted dynamic environmental contexts (open area, urban canyon, boulevard, under viaduct and tunnel) awareness.

The above scene recognition methods show that excellent results can be achieved using GNSS observations to recognize scenes in the condition of lacking multi-sensors, which proves that GNSS-based methods are suitable for shared bikes' application. However, there are still several unsolved issues:

- (a) Many studies only consider large-scale scenes. With regard to urban static positioning tasks such as shared-bike parking, more detailed scenes should be taken into consideration.
- (b) Research studies about machine learning in GNSS scenes recognition still remain in the stage of directly using existing neural networks or constructing feature inputs that meet the requirements of these networks. Those space-transfer strategies may lead to the degradation of original data features. Being able to make full use of observation data in scene recognition remains a challenging problem.
- (c) Multi-systems and complex deep learning networks are used to improve the accuracy of recognition. However, the multi-system observations and computation consumption may not be available in low-cost receivers and microprocessors equipped on public devices such as shared bikes. Moreover, there are relatively few studies on

transfer learning in GNSS scenes recognition. It is crucial to develop methods that can be quickly transferred into different time periods.

Therefore, summarizing the shortcomings of existing works and considering the characteristics of urban static positioning (shared bike parking) application scenes, we propose a deep-learning-based scene recognition method. Our contributions can be summarized as follows:

- A more detailed set of scenes that is suitable for urban static positioning is proposed, including open area, shade of tree, high urban canyon, low urban canyon and unilateral urban canyon. A dataset of **15,000 epochs (3000 s for each scene)** is collected for research.
- A spatio-temporal correlated method for constructing raw data features is used to analyze the importance of different observation data in recognition based on machine learning. A multi-channel Long Short-Term Memory (MC-LSTM) network for GNSS scene recognition is proposed. The result shows that our method can achieve an accuracy of **99.14%** under the condition of using low-cost GNSS receivers to observe **single satellite navigation system**.
- In order to manage the degradation in performances between different time periods, we conduct the transfer learning test of our method's model. The result shows that our pre-trained model can be fine-tuned with a small number of epochs to adapt to different time periods at the same location, which is cost-acceptable for bike-sharing operators.

The remainder of this article is organized as follows: Section 2 introduces the methodology of our proposed model. Section 3 presents the experiments and an analysis of the results. Section 5 provides our conclusions and directions for future works.

2. Methodology

In this section, we first present the feature analysis and construction. Then we propose a deep-learning-based scene recognition model. The overview of our model is shown in Figure 1. The observation data are firstly constructed into different feature vectors. Then those feature vectors are fed into a multi-channel long short-term memory network (MC-LSTM) to identify five scenes (open area, high urban canyon, unilateral urban canyon, shade of tree, low urban canyon).

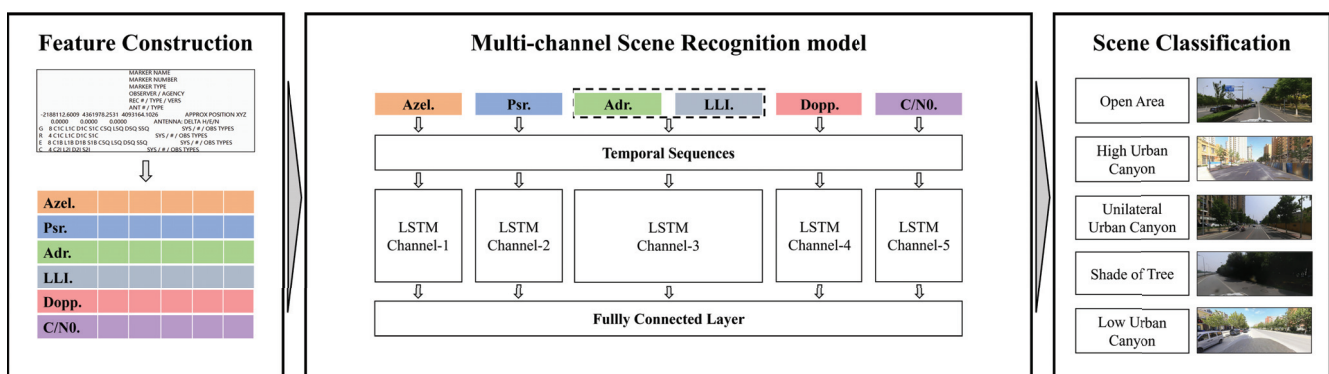


Figure 1. The overview of our proposed model.

2.1. Feature Analysis and Constructions

In this subsection, we first present the observation data and their hidden information. Then we introduce the elevation and azimuth, which shows the geometric relationship between navigation satellites and receivers. Finally, we propose the feature vector definition for our model's input.

2.1.1. GNSS Observations

Global Navigation Satellite Systems (GNSS) typically consist of three segments: the space (space navigation constellation and satellites), the ground control (master control

station, monitoring stations and uploading stations), and the user terminals. Users can receive carrier signals transmitted by the space satellites by various of terminals (receivers). The carrier signals are modulated with ranging codes and navigation messages which can be processed by receivers into different observation data, including the Pseudo range [23], Carrier Phase [24], Doppler frequency [25] and C/N_0 [26].

Pseudo range: The pseudo range observation is an absolute measurement of the distance from the satellite to the receiver, obtained by measuring the signal propagation time delay, and is expressed in meters. The pseudo range includes atmospheric delays and clock biases, and its accuracy is typically at the meter level.

Carrier Phase: Carrier phase observations measure the phase difference between the satellite carrier signal and the reference carrier signal produced by the receiver's oscillator, making it a relative observation. It is expressed in cycles and includes atmospheric delays, clock biases and ambiguity of whole cycles.

Doppler frequency: The frequency of the GNSS carrier signal received by the receiver differs from the actual frequency transmitted by the satellite. This difference is known as the Doppler shift, and its magnitude is related to the rate of change in distance between the receiver and the satellite. Specifically, the Doppler observation equation for GNSS is as follows:

$$\lambda D_r^s = \dot{\rho}_r^s - (\delta \dot{t}_r - \delta \dot{t}^s)c - \delta \dot{t}_{ion}^s + \delta \dot{t}_{trop}^s \quad (1)$$

where D_r^s is the Doppler frequency and it is expressed in Hz. λ is the wavelength of the carrier wave. $\dot{\rho}_r^s$ is the rate of change of the distance between the receiver (subscript r denotes the receiver) and the satellite (superscript denotes the satellite). $\delta \dot{t}^s$ and $\delta \dot{t}^s$ is the clock drift of the receiver and the satellite. $\delta \dot{t}_{ion}^s$ and $\delta \dot{t}_{trop}^s$ is the time derivatives of ionospheric and tropospheric delays.

C/N_0 : The carrier-to-noise density ratio (or CN_0 in more straightforward terms) refers to the ratio of the signal power to the noise power per unit bandwidth in the received GNSS signal. It is expressed in dB and reflects the quality of the RF signal received by the receiver.

Then we consider the characteristics of above observations in urban static positioning scene. Under the conditions of no signal occlusion, the distance between the receiver and the satellite changes in a predictable manner due to the receiver being stationary and the satellite moving in a fixed orbit. In different obstruction scenes, the nature of the obstructions and the characteristics of multipath reflections cause varying rates of change in the distance between the receiver and the satellite. Therefore, we believe that the temporal characteristics of the observations can serve as a basis for scene recognition.

All these observation data can be read directly from Receiver Independent Exchange Format (RINEX format) files or the receivers' output data stream without additional calculations. This is favorable for our application scene which only supports edge computing power. In order to unify the data scale, we normalize the raw observations as follows:

$$d_{norm} = \frac{d - d_{min}}{d_{max} - d_{min}} \quad (2)$$

where d_{max} and d_{min} are the maximum and minimum values of a specific type of observation data within a time step, and d is the original observation data. The normalization only requires simple arithmetic operations, but can ensure network convergence [27,28].

2.1.2. Satellite Elevation and Azimuth

The satellite elevation angle refers to the angle between the vector from the user's location to the satellite's position and its projection onto the Earth ellipsoid tangent plane passing through the user's location. The satellite azimuth angle refers to the angle between this projection and the true North coordinate axis on the tangent plane, with counter-clockwise direction considered positive. The satellite elevation angle and azimuth angle are both related to the position of the user's receiver [29]. They reflect certain aspects of satellite signals, ranging accuracy, and multipath effects. As shown in Figure 2, visible

satellites often exhibit different geometric distributions under various obstruction scenes. The calculation methods for satellite elevation angle and azimuth angle are as follows [30]:

$$\begin{bmatrix} N_{zs} \\ E_{zs} \\ U_{zs} \end{bmatrix} = H \times \begin{bmatrix} X_{es} - X_p \\ Y_{es} - Y_p \\ Z_{es} - Z_p \end{bmatrix}, H = \begin{bmatrix} -\sin B_p \cos L_p & -\sin B_p \sin L_p & \cos B_p \\ -\sin L_p & \cos L_p & 0 \\ \cos B_p \cos L_p & \cos B_p \sin L_p & \sin B_p \end{bmatrix} \quad (3)$$

where N_{zs} , E_{zs} , U_{zs} represent the satellite position in the local topocentric coordinate system (ENU). X_{es} , Y_{es} , Z_{es} and X_p , Y_p , Z_p represent the satellite position (calculated from the satellite ephemeris) and station position in the Earth-Centered Earth-Fixed coordinate system (ECEF), respectively. H is the transformation matrix between ENU system and the ECEF system. B_p and L_p are the geodetic latitude and longitude of the receiver. Therefore, the elevation (El) and azimuth (Az) angle is calculated as follows:

$$Az = \arctan\left(\frac{E_{zs}}{N_{zs}}\right), El = \arcsin\left(\frac{U_{zs}}{\sqrt{E_{zs}^2 + N_{zs}^2 + U_{zs}^2}}\right) \quad (4)$$

From the physical definitions of the satellite elevation angle and azimuth angle, it can be observed that both the elevation and azimuth angles have upper and lower bounds. The elevation angle ranges from 0 to 90 degrees, and the azimuth angle ranges from -180 degrees to 180 degrees. Similarly to the normalization method used for raw observations, we normalize the satellite elevation and azimuth angles as follows:

$$Az_{norm} = \frac{Az - (-180)}{180 - (-180)}, El_{norm} = \frac{El - 0}{90 - 0} \quad (5)$$

where the elevation and azimuth angles are both expressed in degree.

2.1.3. Feature Vector Definition

The feature vector is used as the input of recognition model. In this subsection, we are going to introduce our feature vector definition, which considers both original observation data and geometric relationships between satellites and receivers. We use fixed-length sequential vectors to represent the constellation of a specific satellite navigation system. For example, a 32-dimensional vector is used to store the normalized pseudo range observations of GPS because the pseudo-random noise (PRN) code of satellites is in range of 1 to 32 (G01~G32).

Additionally, we regard the satellite elevation and azimuth angles as a combined feature, which represents the geometric distribution of satellites that the receiver can use. Instead of changing visible satellites into sky plots like Figure 2, we use a $2N$ -dimensional vector to store the normalized angles (where N is the max PRN of the navigation satellite system). Actually, the different dimensions of observation data is the basic of establishment of our multi-channel model, which will be introduced in Section 3 in detail. Our feature definition can be summarized as Table 1. It is worth noting that these vectors of different dimensions can be used both individually and in combination as the input of the multi-channel model.

Table 1. Brief Feature Definition List.

Feature Information	Feature Dimension
Elevation and azimuth angles	$2N$ ¹
Pseudo range	N
Carrier phase	N
Carrier phase and LLI	$2N$
Doppler frequency	N
C/N_0	N

¹ Where N is the max number of PRN for the navigation satellite system. For example, if the satellite system is GPS, $N = 32$.

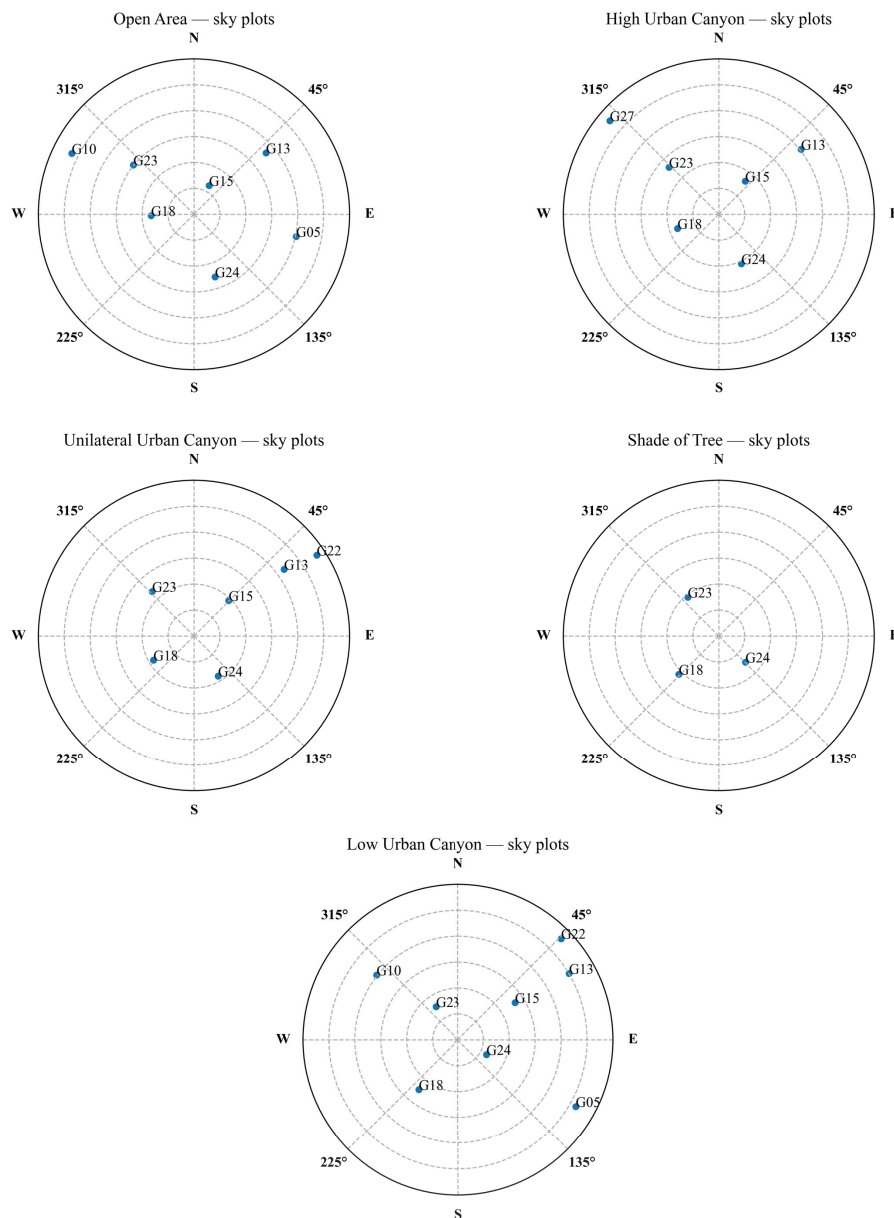


Figure 2. The sky plots of GNSS satellites (GPS system only) in different scenes. This figure shows the elevation and azimuth angles of navigation satellites.

2.2. A Multi-Channel Model for Scene Recognition

In this subsection, we first introduce the Long Short-Term Memory (LSTM) model, which is served as the fundamental building block for the construction of our model. After comparing the performances of different channels (different feature vectors), we propose our multi-channel model to integrate information from different channels. Finally, we consider the transferability of the model and introduce the transfer learning strategy we use in the temporal dimension.

2.2.1. LSTM and Single-Channel Network

Long short-term memory (LSTM) networks are primarily used for learning and predicting temporal features in data sequences. Actually, in the field of GNSS positioning scene recognition, LSTM has been proven to be the most effective network model among numerous machine learning models [21,22,31]. LSTM is composed of numerous LSTM cells Figure 3, which are used to determine whether information is useful. There are three

gates in one cell, including a input gate, a forget gate and an output gate. Additionally, a candidate memory cell is set in the LSTM cell in order to process the chosen memory. A hidden state h_{t-1} and the memory passed to the next cell C_t are output by each cell. The key expression of the LSTM cell is as follows [32–34]:

$$I_t = \sigma(X_t W_{xi} + h_{t-1} W_{hi} + b_i) \quad (6)$$

$$F_t = \sigma(X_t W_{xf} + h_{t-1} W_{hf} + b_f) \quad (7)$$

$$O_t = \sigma(X_t W_{xo} + h_{t-1} W_{ho} + b_o) \quad (8)$$

$$\hat{C}_t = \tanh(X_t W_{xc} + h_{t-1} W_{hc} + b_c) \quad (9)$$

$$C_t = F_t \odot C_{t-1} + I_t \odot \hat{C}_t; \quad (10)$$

where $\odot$ represents the dot product and σ represents the sigmoid activation function. I_t , F_t , O_t are the outputs of the input gate, forget gate and output gate. W_{xi} , W_{hi} ; W_{xf} , W_{hf} ; W_{xo} , W_{ho} are the weight matrices of input, forget, output gates. W_{xc} , W_{hc} are the weight matrices of the candidate memory cell. b_i , b_f , b_o , b_c are the biases. The hidden state h_t is calculated as follows:

$$h_t = O_t \odot \tanh(C_t) \quad (11)$$

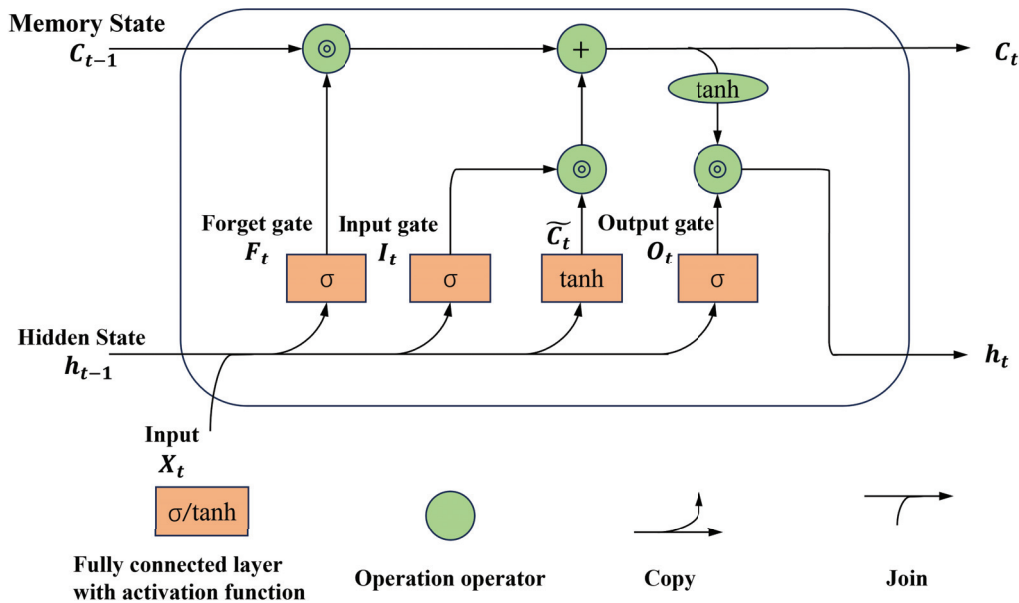


Figure 3. The structure of LSTM cell, including the input gate, forget gate, output gate and the candidate memory cell. The LSTM cell is used to determine whether information is useful.

To change our feature vector to the input of LSTM, the original feature vector sequences should be transmitted into time segments with a sliding window. It means we combine data vectors within a certain time step into a sequence in chronological order and make it the input of single channel. Take pseudo range feature for example, if a N -dimensional vector represents the normalized pseudo range of the time t_0 , the input of single LSTM channel is in dimension of (t_s, N) , where t_s is the time step of sequence. The overview of our single channel LSTM is shown in Figure 4.

To ensure the real-time performance of the algorithm and consider the practical application requirements, we add only 3 LSTM layers in a single channel which is used to process a single type of feature vector. Meanwhile, the time step is set in 10 s. The sliding window is set in 1 s so that 2990 sequences of each scene can be collected. Similar to the

majority of machine learning classification tasks, we use the cross-entropy loss function and optimize the parameters of the network with Adam. The network is trained on training and validation datasets for approximately 500 epochs. The batch size is set to 32 and the learning rate is set to 0.00001.

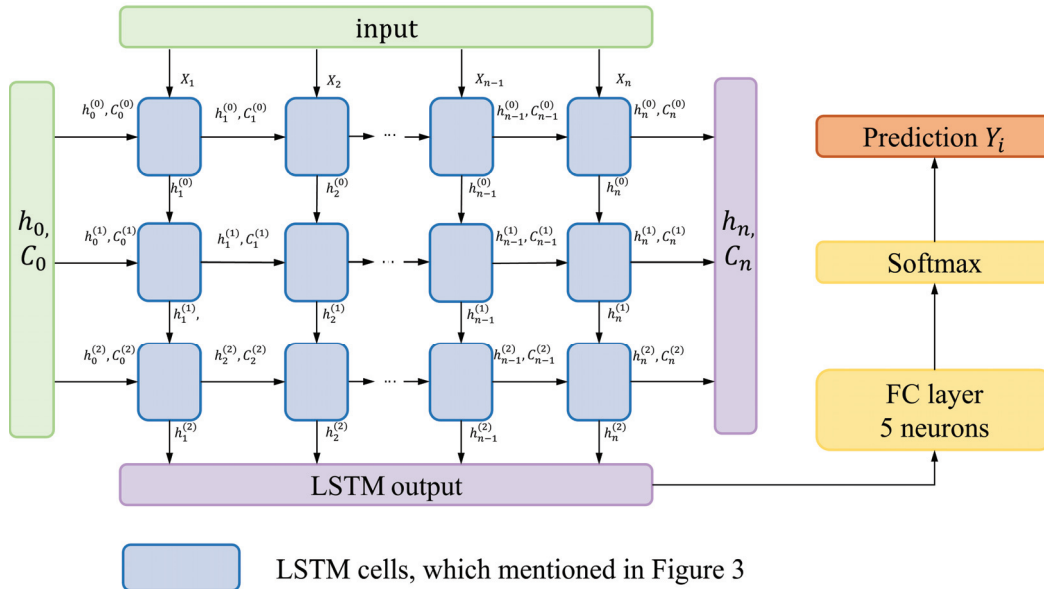


Figure 4. The structure of LSTM network and the full structure of our single channel recognition model. The blue blocks in this figure represent the LSTM cells mentioned in Figure 3.

2.2.2. Scene Categories and Single Channel LSTM Performances

Usually, GNSS positioning scenes are divided into two main parts: outdoor and indoor [17–19]. Within urban environments, urban canyons and boulevard are also considered in scene recognition [22,35]. Given the specificity of our application scene (static positioning for shared bikes' parking), we have subdivided urban canyons into high urban canyon, low urban canyon and unilateral urban canyon. So we divide the positioning scenes into five categories: **open area**, **shade of tree**, **high urban canyon**, **low urban canyon** and **unilateral urban canyon**. Due to the bike parking rule, we no longer consider the indoor scenes, which are easily identifiable based on the presence of satellite signals. Figure 5 shows the real-world locations and the street view where we collected data, and Figure 2 shows the distribution of satellites in various scenes.

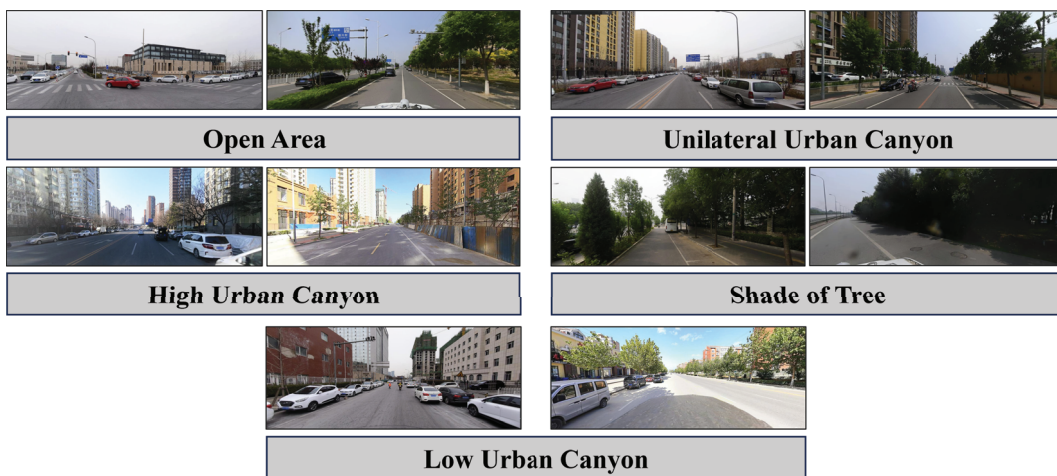


Figure 5. The street view of real-world locations where we collected data, including open area, high/low/unilateral urban canyon and shade of tree.

In our proposed algorithm, we encode the scene categories as one-hot vectors as Table 2. So the index of the max number of output vector represents the highest possibility of the scene. For example, if the model outputs a vector of $[0.1, 0.1, 0.5, 0.2, 0.1]$, the prediction result of scene recognition is the unilateral urban canyon. Under the training setting mentioned in subsection, we have trained and validated the single channel model and the performances are shown in Figure 6.

Table 2. Scene Categories and Encoding.

Categories	Encoding
open area	$[1, 0, 0, 0, 0]$
high urban canyon	$[0, 1, 0, 0, 0]$
unilateral urban canyon	$[0, 0, 1, 0, 0]$
shade of tree	$[0, 0, 0, 1, 0]$
low urban canyon	$[0, 0, 0, 0, 1]$

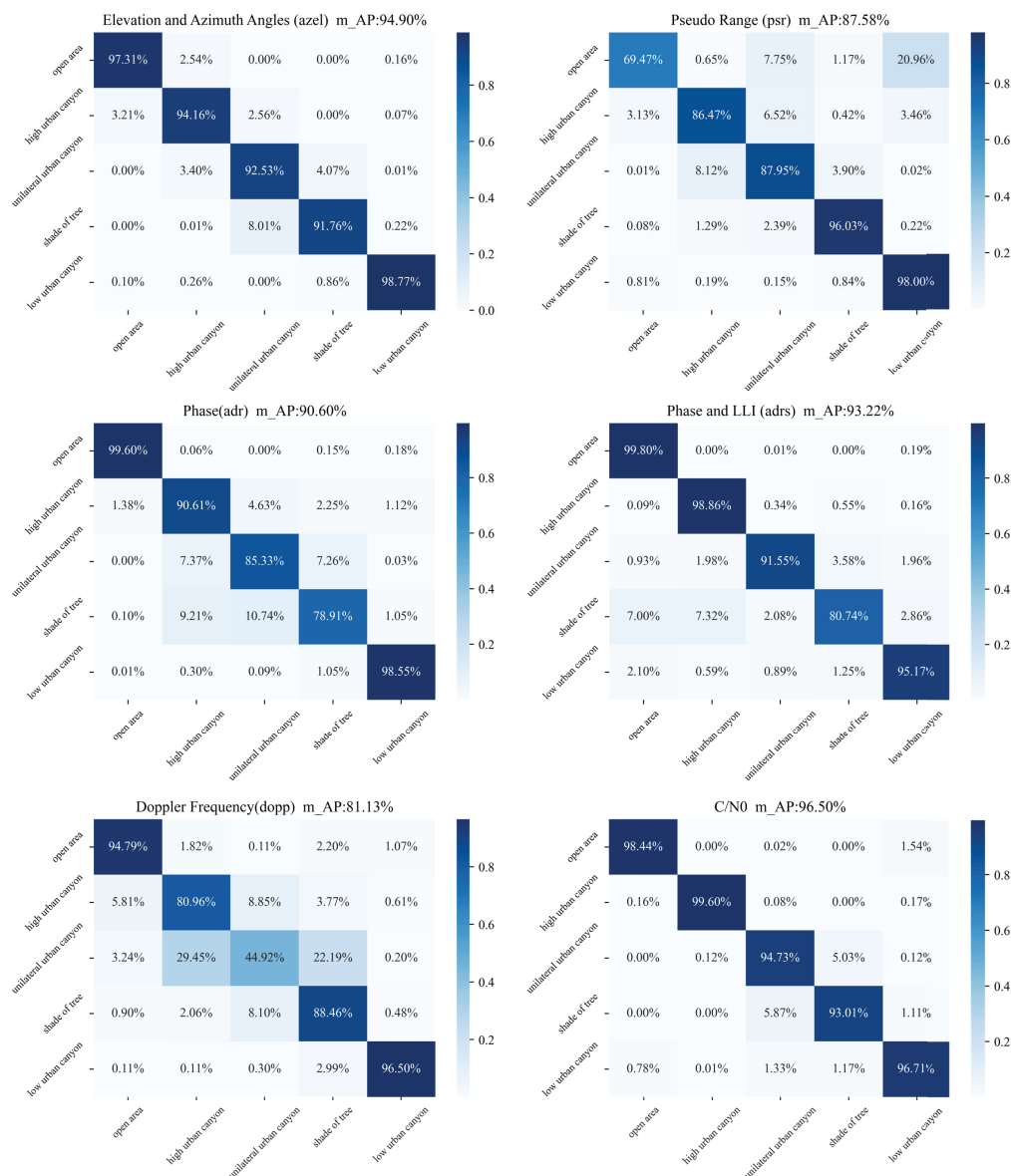


Figure 6. The confusion matrices of different feature vector, which shows the performances of different channels in recognition.

From the confusion matrices shown in Figure 6, we can find that different feature data have varying sensitivities to the recognition of different scenes. The feature of az/el and C/N_0 performs the best in all categories while other feature data performs in specific category (pseudo range in shade of tree, phase + LLI in high urban canyon). Meanwhile, the convergence speeds of the individual single-channel models also show significant differences, as shown in Figure 7. The az/el has the fast convergence speed while the phase has the slowest. Moreover, the performance of phase + LLI is better than single phase no matter the accuracy or the convergence speed, which prompts us to combine the carrier phase and LLI into one feature consideration in the subsequent research.

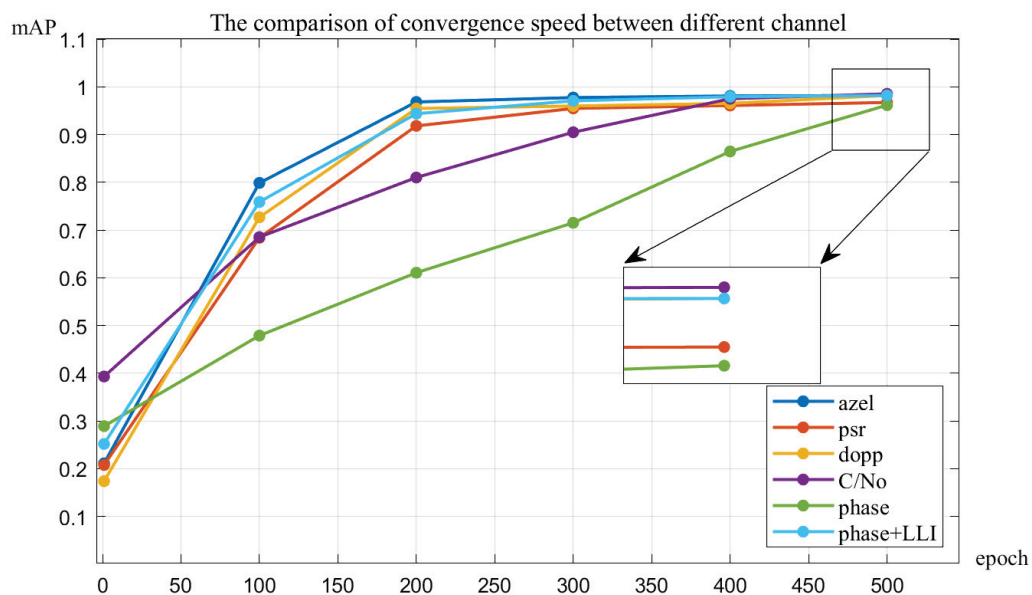


Figure 7. The comparison of convergence speed between different channel, including elevation and azimuth angles (azel), pseudo range (psr), doppler frequency(dopp), C/N_0 , phase and phase + LLI. The horizontal axis represents the training epochs, and the vertical axis represents the average accuracy of the model on the validation set.

2.2.3. MC-LSTM Design

In the last subsection, we analysed the performances of different feature vectors. To improve recognition accuracy and convergence speed of our model across all scenes, we integrate information from different feature vectors through a multi-channel parallel design. The overview of our model is shown in Figure 8. The model consists of channel layers and a fusion classification layer. Different feature vectors are input into corresponding LSTM channels. After LSTM processes and outputs hidden states, these are passed through fully connected layers to merge and classify scene recognition results. It is important to note that the dropout layers are used to introduce non-linearity to the model, which helps prevent overfitting and enhances the model's ability to generalize [36,37].

In addition, our proposed multi-channel model allows for flexible combinations of channels based on scenes and requirements. If the receiver cannot output a certain type of observation, the corresponding data vector channel can be excluded.

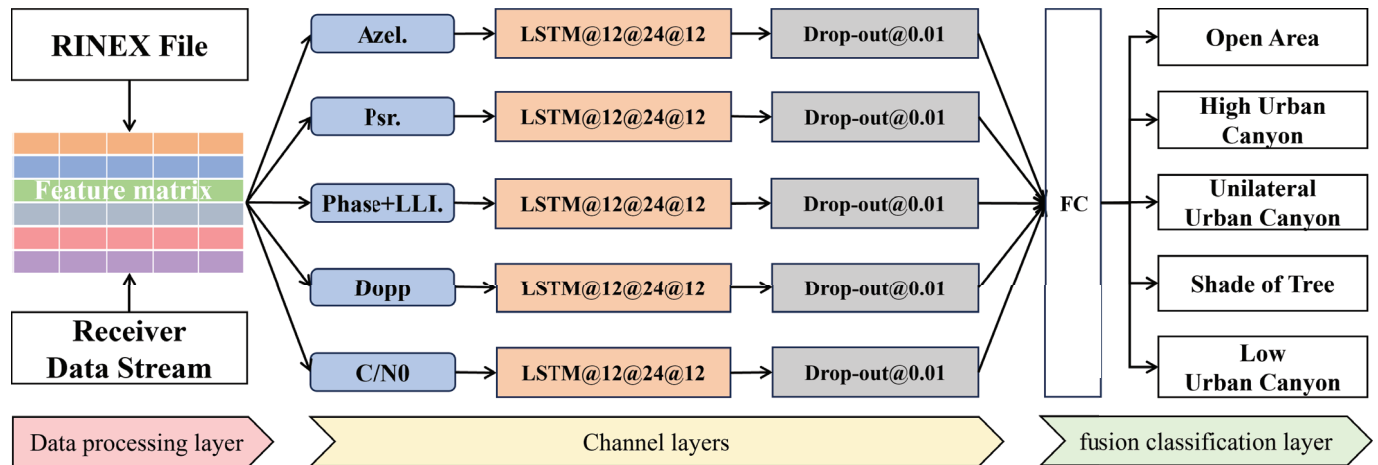


Figure 8. The overview of our proposed multi-channel LSTM model for GNSS scene recognition.

2.2.4. Transfer Learning Settings

A trained model can achieve a high accuracy in training and validation dataset but usually does not perform well in another dataset. Therefore, classification models need to be retrained when changes in the feature space or the feature distribution occur [38]. In our application scene for recognizing shared bicycle parking locations, the absolute position used is generally fixed, but the time when users park is random. It makes the cost exceptionally high for collecting data from all time periods for training. Therefore, the quicker we can execute the transfer learning between time periods, the stronger our model's generalization is. This means the operator of shared bikes can quickly deploy the pre-trained model with simple fine-tuning to different time periods, significantly reducing the time required for model training. To test the temporal transfer performance of our model, we collected data from different time periods and performed full-layer transfer. The distribution of the data over time is shown in Table 3.

Table 3. Transfer Learning Data.

Dataset	Scene	Time of First Observation	Lasting Time
Dataset-1	open area	10:31/13:04/14:16/15:21/16:26	10 min for each
	high urban canyon	10:43/13:16/14:28/15:32/16:38	10 min for each
	unilateral urban canyon	10:57/13:33/14:40/15:45/16:51	10 min for each
	shade of tree	11:16/13:44/14:55/16:00/17:15	10 min for each
	low urban canyon	11:34/14:02/15:07/16:13/17:28	10 min for each
Dataset-2 *	open area	11:09	10 min
	high urban canyon	11:21	10 min
	unilateral urban canyon	11:33	10 min
	shade of tree	11:44	10 min
	low urban canyon	11:57	10 min

* The dataset which is used as the target of transfer learning.

3. Results

In this section, we first introduce the device we used and the dataset we collected. We then analyze the performance of our model and compare it with those from other studies. Finally, we evaluate the real-time performance of our proposed model and present the transfer learning results.

3.1. Device and Dataset

The experimental data were collected using a receiver based on HD8020 series chipset (Allystar Technology, Shenzhen, China), which is a low-cost GNSS receiver chip solution widely used in shared bikes. The appearance of the data collection device is shown in

Figure 9. In order to closely replicate the shared bicycle parking scene, we chose not to alter the structure of the bike's positioning system (GNSS module + antenna). Instead, we directly connected the bike's built-in module to a PC via a serial port to obtain data.

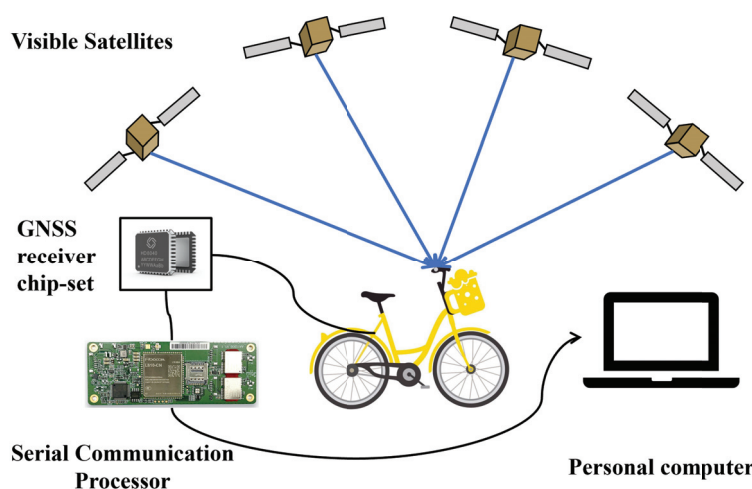


Figure 9. The data collection platform.

To ensure the generalization performance of our proposed model, During each time periods, we collected 10 min's data for each positioning scene (open area, high urban canyon, unilateral urban canyon, shade of tree and low urban canyon). Five such time periods are taken into consideration. Moreover, We shuffled the collected data samples before training and used a cross-validation strategy (70% for training while 30% for validation). As for the dataset for time-transfer-learning, we collected the same distribution data a few days later than training data. The total distribution of the dataset is shown in Table 3.

3.2. Multi-Channel LSTM Model Results

In this subsection, we analyze the performances of our proposed model. First, we introduce the recognition accuracy of our multi-channel LSTM model by confusion matrices and show the changes of accuracy and loss through training epochs by figure. We then compare the results (including accuracy and convergence speeds) of multi-channel model and single channel LSTM mentioned in Section 3. Finally, we compare the performances of our model and other proposed algorithms in recent years.

The confusion matrices and loss and accuracy changes in training epochs are shown in Figure 10. Our model can reach a mean accuracy of 99.14% in all categories and an accuracy of at least 97.5% for each individual category on validation set. From the loss & accuracy changes we can easily find that after only a few training epochs (about 50 epochs) our model is already reaching an accuracy over 90%, which shows it performs well both in recognition as well as training consumption. We observed that the training and validation accuracy of the model initially dropped once during the training process and then quickly increased. This indicates that adding dropout layers in the single channel helps prevent the model from getting stuck in local optima, which aligns with our understanding of nonlinear models [39].

Now we are going to take multi-channel and single-channel into consideration. Compared with the single-channel model mentioned in Section 3, the multi-channel can reach higher accuracy in scene recognition, including mean accuracy of all categories as well as the specific category. The detailed comparison is shown in Table 4.

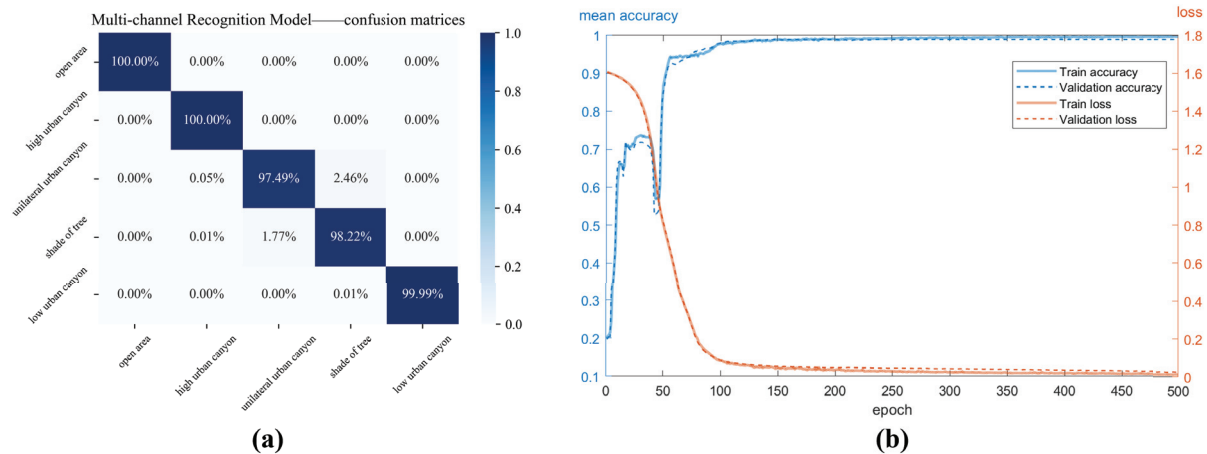


Figure 10. The training results of our proposed multi-channel LSTM GNSS scenes recognition model. (a) is the confusion matrices while (b) shows the changes in loss and accuracy.

Table 4. Comparison between Multi-Channel and Single-Channel.

Channel *	Accuracy in Scenes					Mean Accuracy
	Open Area	High Urban Canyon	Unilateral Urban Canyon	Shade of Tree	Low Urban Canyon	
azel	97.31%	94.16%	92.53%	91.76%	98.77%	94.90%
psr	69.47%	86.47%	87.95%	96.03%	98.00%	87.58%
adr	99.60%	90.61%	85.33%	78.91%	98.55%	90.60%
adrs	99.80%	98.86%	91.55%	80.74%	95.71%	93.22%
dopp	94.79%	80.96%	44.92%	88.46%	96.50%	81.13%
C/N_0	98.44%	99.60%	94.73%	93.01%	96.71%	96.50%
Multi-channel	100.00%	100.00%	97.49%	98.22%	99.99%	99.14%

* Where “azel” represents the azimuth and elevation angles, “psr” represents the pseudo range, “adr” represents the carrier phase, “adrs” represents the carrier phase plus LLI (loss of lock indicator), “dopp” represents the doppler frequency. “multi-channel” represents our proposed recognition model: MC-LSTM.

Despite the accuracy, convergence speed is also an important metric for evaluating model performance. Compared with the single channel model, be similar to accuracy, multi-channel model can also converge faster. Take azimuth and elevation angles channel (the fastest channel in convergence speed) for example, just like Figure 11 shows, the multi-channel model is faster than any other single channel in convergence speed. All results above indicate that our multi-channel design significantly enhances the performance of LSTM in recognizing GNSS positioning scenes, in terms of both accuracy and convergence speed. This demonstrates the benefits of our strategy to integrate multiple channels. It is worth noting that instead directly using existing network, our proposed model design strategy is one of the few specialized network structures specifically designed for GNSS positioning scenes in current related research.

After analyzing the accuracy and convergence speeds, we have already substantiated that our model exhibits superior performance in GNSS positioning scenes recognition. Table 5 shows the comparison between our model and other proposed algorithms in recent years, including accuracy and scene granularity. Our model is the only algorithm which can achieve an accuracy of 99% across five scenes (the scene no GNSS signal received is not included). Indeed, we are also the only researchers who take detailed urban scenes (high, low and unilateral) into consideration.

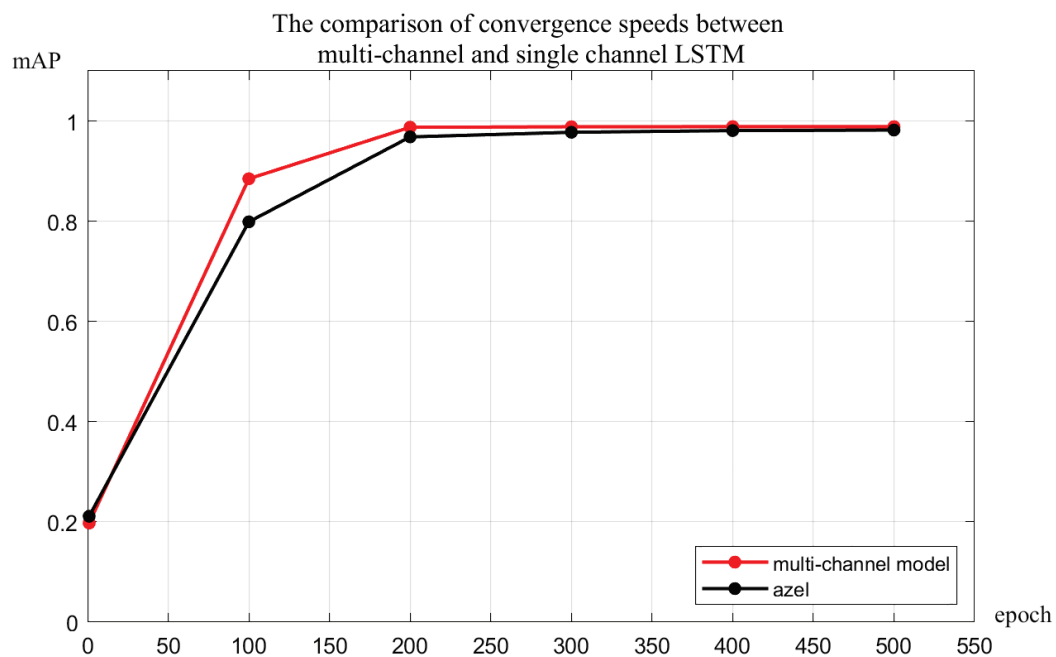


Figure 11. Comparison of convergence speeds between multi-channel and single-channel LSTM.

Table 5. Scene Categories and Encoding.

Authors	Years	Methods	Scenes *	Accuracy
Chen et al. [17]	2017	Threshold judgment	I/O	85.6%
Wang et al. [35]	2019	SVM & temporal filtering	5 urban dynamic scenes	89.30%
Lai et al. [20]	2021	SVM	3 scenes of I/O	90.3%
Dai et al. [21]	2022	CNN & conv-LSTM	4 scenes in I/O	98.82% & 99.92%
Zhu et al. [22]	2024	LSTM	4 urban dynamic scenes	95.39%
Ours	2024	MC-LSTM	5 urban static scenes	99.14%

* Where the scenes that GNSS signals can not be received is ignored in this table. I/O means the indoor and outdoor scenes.

3.3. Potential Real-Time Ability

In this subsection, we analyze the real-time ability of our proposed model, including the consumption of offline model-training and online model-prediction. We randomly selected 3000 s (2950 time sequences are constructed) of data from the training dataset to evaluate the real-time performance of our model. It is noteworthy that we chose a CPU (Intel (R) Core (TM) i5-12500H, 2.5 GHz) but not a GPU as the computing platform for our real-time performance test and still achieved excellent results.

To evaluate the time consumption of model-training and model-prediction, we first introduce the definition of the metrics: training time per epoch (TPE) and prediction time per data (PPD). Our TPE and PPD is shown in Figure 12. 0.75 s is cost the most in training a dataset of 2950 sequences for one epoch, which makes it possible for our model to do the online fine-tune according to the real-time data. Moreover, only 1.95 ms is needed to perform a full prediction, which makes it possible for our model to be deployed in real-time scenes without any noticeable additional time delay. Compared with existing convolution blocks-relied works, our specially designed multichannel model has a 60% improvement in TPE and 90% in PDD [21,22]. Only 40,817 parameters are included in our model.

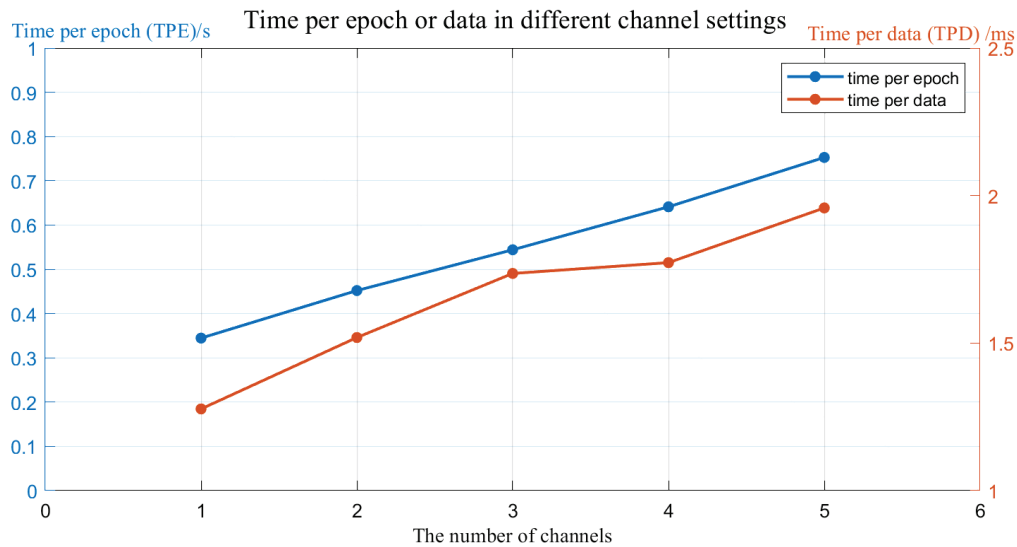


Figure 12. The training time per epoch and prediction time per data of our model. X-axis shows the number of channels. Left y-axis shows the training time per epoch and expresses in seconds. Right y-axis shows the prediction time per data and expresses in milliseconds.

3.4. Time-Transfer Ability

The high precision a pre-trained model can achieve requires that the training data and the test data have similar distributions in feature spaces. Therefore, classification models need to be retrained when changes in the feature space or the feature distribution occur [38]. In our application scenes, we always face that the cost of collecting data of all time periods is unacceptable. Meanwhile, the temporal dimension is a decisive factor in evaluating the generalization capability of our model. So the transfer ability between time periods is taken into consideration in our model. Table 6 shows the details of transfer learning and we can find that our pre-trained model can be fine-tuned with a small number of epochs to adapt to different time periods at the same location. Moreover, the transfer learning performs better than non-transfer (directly) learning in most scenes and training stages. The mean accuracy of transfer learning at every check points (0, 10, 20, 30 and 100 epochs) is higher than non-transfer learning.

Table 6. Comparison between transfer and non-transfer learning.

Scenes *	Accuracy through Epochs									
	0 Epoch		10 Epochs		20 Epochs		30 Epochs		100 Epochs	
	T.	Non-T.	T.	Non-T.	T.	Non-T.	T.	Non-T.	T.	Non-T.
open a.	98.31%	19.84%	100.00%	0.00%	100.00%	85.59%	100.00%	0.00%	100.00%	99.83%
high u.c.	3.39%	19.85%	90.85%	55.76%	94.92%	98.64%	97.63%	100.00%	99.32%	100.00%
unilateral	0.00%	19.68%	98.81%	0.00%	100.00%	0.00%	96.27%	3.90%	96.27%	88.81%
s.o.t.	0.00%	19.97%	0.00%	19.32%	6.61%	88.81%	100.00%	91.53%	100.00%	100.00%
low u.c.	85.93%	20.58%	88.47%	100.00%	95.76%	100.00%	95.93%	99.66%	100.00%	98.64%
mean	37.52%	19.98%	75.62%	35.02%	79.46%	74.61%	97.97%	59.02%	99.12%	97.46%

* Where “open a.” represents the open area, “high u.c.” represents the high urban canyon, “unilateral” represents the unilateral urban canyon, “s.o.t.” represents the shade of tree, “low u.c.” represents the low urban canyon. Columns named “T.” shows the transfer learning results while “Non-T.” shows the non-transfer learning.

3.5. Data-Loss Robustness

Due to the limitations in the number of observation channels of low-cost receivers, the application of our model always faces the situation that some observation data are missing. Therefore, we decide to do an additional experiment to evaluate the robustness of our model in such data-missing situations. As the azimuth and elevation angles are calculated

by satellite ephemeris and receiver's position, the pseudo range is the least requirements of calculating the azimuth and elevation angles. So we divide the channels into four groups to test our model's robustness: pseudo range and azimuth & elevation angles (G1), carrier phase and LLI (G2), Doppler frequency (G3), C/N_0 (G4). As the results show in Table 7, our model can achieve an accuracy of over 80% even in the situation that one or two channels' missing. For example, if the carrier phase data and C/N_0 data are missing, our model can also reach an accuracy of 81.13% in scenes recognition.

Table 7. Channel-missing Robustness.

Combinations	Mean Accuracy
G1 & G2	54.80%
G1 & G3	81.13%
G1 & G4	54.24 %
G1 & G2 & G3	90.28%
G1 & G2 & G4	57.17%
G1 & G3 & G4	96.06%

4. Discussion

4.1. Detailed Scenes Recognition in Urban Static GNSS Positioning

Accurate GNSS scenes recognition in urban environments plays an important role in high-precision urban positioning. All the scene-adaptive methods to improve the precision of GNSS positioning require efficient scenes recognition. As shown in Table 5, existing works mainly focus on Indoor and Outdoor recognition [17,20,21] or only consider coarse-grained outdoor scenes [22,35]. Especially for the urban canyon, most research has ignored the different directions of signal blockage. Our proposed model considers the real requirements of urban static positioning application (shared bike parking) and divides the urban canyon scene into high, low and unilateral categories. It achieves an accuracy of over 99% in scenes recognition and is specially designed in multi-channel structure according to the characteristics of GNSS tasks, which represents great progress compared with directly using the existing networks in this field. Detailed scenes and multi-channel network structure design provide a new perspective of GNSS scenes recognition and has the possibility of freely adapting to different tasks according to environments.

4.2. Potential in Real GNSS Scenes Recognition Application

The complexity of urban environment and the limitation of computing power of equipment have always been the main obstacles to the practical application of GNSS scene recognition method based on deep learning. The lightweight network parameter scale, real-time ability and robustness to signal reception failure should be taken into consideration in the real GNSS scenes recognition application. Existing works achieve high precision of recognition by using convolution blocks, which makes the scale of network parameters unacceptable for low-cost computing devices. Our proposed model lightened the hidden layers according to the characteristics of GNSS features and only requires milliseconds of extra time in recognition. Surprisingly, our multi-channel model can also achieve high recognition accuracy even in cases where some observations are lost. All the above findings show that our proposed multi-channel LSTM network structure has a wide potential in real GNSS scenes recognition applications.

5. Conclusions

In this study, a scenes recognition model for urban static GNSS positioning is proposed. The proposed model is able to provide detailed scene information only using a single satellite system's observations via low-cost receivers equipped on the shared bikes. It organizes the original observation data into five feature vectors with different dimensions but without any additional calculation except normalization. After analyzing the performances of different feature vectors, a specially designed multi-channel LSTM model

is proposed to further improve the recognition accuracy in different scenes. Experiments are designed to evaluate the proposed model in terms of accuracy, time consumption and robustness. The results show that our proposed model can achieve an average accuracy of 99.14% in all scenes and at least 97.5% in an individual scene. The time delays of model training and prediction on a CPU are 0.75 s per epoch and 1.95 ms per data, which makes it possible for our model to be deployed in real-time applications. Furthermore, our model can be transferred into different time periods with only a few epochs training and maintain a high accuracy even in the presence of missing channel data (96.06% for one channel missed and 81.13% for two channels missed), which shows the robustness of our model in real-world applications. In future works, more detailed and mixed scenes should be taken into consideration. In addition, the contributions from different satellite systems and frequencies require further research.

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Article

Prediction of Deformation in Expansive Soil Landslides Utilizing AMPSO-SVR

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Abstract: A non-periodic “step-like” variation in displacement is exhibited owing to the repeated instability of expansive soil landslides. The dynamic prediction of deformation for expansive soil landslides has become a challenge in actual engineering for disaster prevention and mitigation. Therefore, a support vector regression prediction (AMPSO-SVR) model based on adaptive mutation particle swarm optimization is proposed, which is suitable for small samples of data. The shallow displacement is decomposed into a trend component and fluctuating component by complete ensemble empirical mode decomposition with adaptive noise (CEEMDAN), and the trend displacement is predicted by cubic polynomial fitting. In this paper, the multiple disaster-inducing factors of expansive landslides and the time hysteresis effect between displacement and its influencing factors are fully considered, and the crucial influencing factors which eliminate the time lag effect and state factors are input into the model to predict the fluctuation displacement. Monitoring data in the Ningming area of China are employed for the model validation. The predicted results are compared with those of the traditional model. The model performance is evaluated through indicators such as the goodness of fit R^2 and root mean square error RMSE. The results show that the prediction RMSE of the new model for three monitoring stations can reach 2.6 mm, 6.6 mm, and 2.5 mm, respectively. Compared with the common Grid search support vector regression (GS-SVR), the Particle Swarm Optimization Support Vector Regression (PSO-SVR) and Back Propagation Neural Network (BPNN) models have average improvements of 58.4%, 38.1%, and 25.2% respectively. The goodness of fit R^2 is superior to 0.99 in the new method. The proposed model can effectively be deployed for the displacement prediction of non-periodic stepped expansive soil landslides driven by multiple influencing factors, providing a reference idea for the deformation prediction of expansive soil landslides.

Keywords: expansive soil landslide; displacement prediction; multiple driven factors; AMPSO-SVR; time lag effect

1. Introduction

Expansive soil landslides are a complex clay geological hazard distributed in more than 20 provinces in China, where nearly 400 million people live. Uneven settlements of buildings, instability of engineering slopes, and other detriments are easily attributed to the unstable deformation characteristics of expansive soil due to water absorption expansion deformation and water loss shrinkage cracking [1].

To effectively prevent expansive soil landslide hazards, it is crucial to accurately predict future displacements by utilizing historical landslide data. Expansive soil landslides are unstable nonlinear systems with complex instability mechanisms and special engineering characteristics of repeated expansion and contraction; these special “step-like” characteristics are presented in displacement curves [2]. Compared with traditional loess

landslides, the step-like rise in the displacement of expansive soil landslides does not follow strict rainfall periodicity and is often accompanied by strong burstiness [3–8]. This is because expansive soil landslides are jointly affected by multiple internal and external factors. In addition to precipitation, landslide stability is also significantly impacted by earth pressure and soil moisture content [9–12]. Therefore, the dynamic prediction of the displacement of step-like expansive soil landslides driven by multiple factors is an important technical issue that needs to be solved urgently in the current research on landslide prediction.

Solutions for the prediction of step-like landslide displacement have been researched by numerous scholars so far. For example, Zhu Xing proposed an extreme learning machine (ELM) to predict the displacement of the Baijiabao landslide in the Three Gorges [13], Gao Wei fused a grey system (GM) with an evolutionary neural network (ENN) to predict two landslides in the Three Gorges Reservoir Area [14], and Yang Beibei constructed a long short-term memory network (LSTM) for the displacement prediction of the Baishui river landslide [15]. Nevertheless, single factors such as the reservoir water level and rainfall were mainly considered in the above methods, but they are more suitable for landslides with more obvious deformation cycles and a single influencing factor. The internal characteristics of landslides, such as non-periodic soil stress or the internal moisture field of slopes, have not been fully considered. The above neural-network-based prediction models have complex algorithm structures and require high-quality feature data and sample sizes, which is not suitable for predicting the displacement of non-periodic expansive soil landslides under the joint influence of multiple internal and external factors. A strong generalization ability is exhibited by Support Vector Regression (SVR) [16], and the nonlinear relationship between multiple external factors and target displacement can be handled effectively. It does not require complex operation for preprocessing and feature selection in an input dataset, and the model has an excellent robustness. It is very suitable for predicting the deformation of step-like landslides dominated by multiple factors. However, it is easy for the traditional SVR model to fall into local optimal, which can be solved effectively by adaptive mutation particle swarm optimization (AMPSO) [17,18].

Therefore, a machine learning prediction algorithm based on AMPSO-SVR is proposed to predict the displacement of expansive soil landslides. The main idea includes: (1) The total displacement being decomposed into trend and fluctuation terms, respectively, by complete ensemble empirical mode decomposition with adaptive noise (CEEMDAN). (2) The trend displacement being predicted by a fitting method and the relationship between the fluctuation term and the influencing factors being analyzed. (3) The time delay effect between the displacement and multi-factor being considered, the multi-factor wave term component being extracted by CEEMDAN and input into the AMPSO-SVR model to predict the wave term displacement, and the prediction results being compared with grid search support vector regression (GS-SVR), and particle swarm optimization support vector regression (PSO-SVR), verifying that the “step-like” non-periodic displacement can be accurately and dynamically predicted, which has a certain feasibility and scientificity. (4) The total predicted displacement sequence being obtained by superimposing the predicted values of the trend and fluctuation terms.

2. Displacement Prediction Model of Expansive Soil Landslides

2.1. Displacement Time Series Theory

The displacement of expansive soil exhibits nonlinear characteristics under the influence of crack evolution and wet–dry cycles. The total displacement is decomposed into a trend term, wave term displacement, and random term displacement according to the time series theory [19,20].

$$S(t) = T(t) + P(t) + R(t) \quad (1)$$

where $S(t)$ is the total displacement value, $T(t)$ is the trend term displacement, and $P(t)$ is the fluctuation term displacement. The magnitude of $R(t)$ is small and negligible, and the prediction of $T(t)$ and $P(t)$ is mainly focused on in this paper.

2.2. Complete Ensemble Empirical Mode Decomposition with Adaptive Noise

Moving average, wavelet decomposition, and EMD are commonly applied to landslide displacement decomposition [21]. EMD decomposition is a conventional method for decomposing non-stationary data. However, anomalies in displacement decomposition may be caused by the existence of mode aliasing, and the displacement trend term and the fluctuation term can be extracted inaccurately. Moving average has a poor adaptability to noise, and wavelet decomposition has challenges in processing non-stationary signals such as landslide displacement curves. The above problems can be effectively overcome by CEEMDAN decomposition. Adaptive noise is added to landslide displacement curves, which improves the robustness of decomposition and is more suitable for processing non-stationary signals. These decomposed displacement components are more consistent with the actual change trends of landslides. The CEEMDAN decomposition flow is designed as follows in this paper:

(1) The landslide displacement $S(t)$ is added with Gaussian white noise, whose K -th mean is 0 to construct the K sequence $S_i(t)$ to be decomposed. $i = 1, 2, 3, \dots, k$, ε is the signal-to-noise ratio, and $\delta_i(t)$ is the white noise sequence added for the i -th time.

$$S_i(t) = S(t) + \varepsilon \delta_i(t) \quad (2)$$

(2) The EMD decomposition of $S_i(t)$ is performed, the first modal component (IMF) is decomposed, and its mean is taken as the first modal component obtained by CEEMDAN decomposition. Each modal component is a different frequency component of the original displacement. $r_1(t)$ is the first residual component, which represents the non-extractable aperiodic component of the displacement sequence.

$$IMF_1(t) = \frac{1}{K} \sum_{i=1}^K IMF_1^i(t) \quad (3)$$

$$r_1(t) = S(t) - IMF_1(t) \quad (4)$$

(3) The EMD decomposition is continued after noise is added to the residual component $r_j(t)$ of phase j .

$$IMF_j(t) = \frac{1}{K} \sum_{i=1}^K E_1(r_{j-1}(t) + \varepsilon_{j-1} E_{j-1}(\delta_i(t))) \quad (5)$$

$$r_j(t) = r_{j-1}(t) - IMF_j(t) \quad (6)$$

(4) If the EMD stop condition is satisfied and the displacement residual component $r_n(t)$ of the n -th decomposition is close to a monotone signal, then the CEEMDAN algorithm ends its decomposition.

2.3. Support Vector Regression (SVR)

Expansive soil landslides are driven by various correlation factors [22]. The relationship between displacement and external factors is not a simple linear form, but a more complex curve shape, such as a polynomial or periodic relationship, etc., which can be solved effectively by Support Vector Regression (SVR). SVR began with the research of Drucker et al. (1997) [23]. The principle is to find a nonlinear mapping from the input space to the output space, and the feature data x are mapped to a high-dimensional feature space, and the regression plane with the optimal fitting performance in the high-dimensional space is selected. Its mathematical model is as follows:

$$P_{pre}(x) = \omega^T \varphi(x) + b \quad (7)$$

where $P_{pre}(x)$ is the output value of the model, namely, the predicted displacement value of the landslide fluctuation term corresponding to the predicted value of item $P(t)$

in Equation (1), ω is the inertia weight, b is the offset term, and $\varphi(x)$ is the mapping function between the landslide correlation factor x and predicted displacement, which includes the input values of the model, namely, the data of multiple influence factors that affect expansive soil landslides and the historical displacement data. The data of multiple influence factors are mapped to a high-dimensional feature space for linear analysis in that space. The nonlinear relationship between displacement and multi-factor features can be modeled accurately by utilizing the kernel function in the SVR model. The key to SVR machine learning is to find the optimal nonlinear mapping relationship so that the output displacement $f(x)$ is closer to the actual value.

2.4. Adaptive Mutation Particle Swarm Optimization (AMPSO)

The particle swarm optimization algorithm (PSO) is an evolutionary algorithm inspired by bird foraging [24]. The solution is regarded as a particle, the particle quality is measured by the particle fitness, and the optimal solution is obtained through multiple iterations. However, the particles are likely to quickly gather near a local optimum in the early stages of the search process, and the search space is not fully explored. It is easy to fall into the local optimum, which makes it difficult to obtain the global optimal solution. The adaptive mutation particle swarm optimization (AMPSO) is an improvement of the standard PSO, and it adjusts the search range of the solution space by dynamically adjusting the weight and adaptive mutation mechanism so that the algorithm has a large global search ability, which can address this issue. The strategy adopted by the algorithm is to adjust the inertia weight adaptively. The larger weight is beneficial for improving the global optimization ability of the particle that can make it skip the local optimum. Therefore, a better optimal position on a large scale can be explored by taking a larger value of ω initially, while a smaller weight will improve the local optimal ability of particles so that the search process can be gradually converged. Later, the value of ω will decrease to improve the convergence accuracy. The weight linear attenuation method is adopted in this paper according to the iterative process, and its mathematical model is as follows:

$$\begin{aligned} V_{id}^{k+1} &= \omega V_{id}^k + c_1 r_1 (P_{id}^k - W_{id}^k) + c_2 r_2 (P_{gd}^k - W_{gd}^k) \\ W_{id}^{k+1} &= W_{id}^k + V_{id}^{k+1} \end{aligned} \quad (8)$$

$$\omega_{iner} = \omega_{start} - \frac{(\omega_{start} - \omega_{end}) \cdot I}{N} \quad (9)$$

where i is the number of particles, d is the dimension of the prediction space, k is the number of iterations, v is the particle velocity, W is the particle position, ω is the inertia weight, Equation (9) is the updating method for weight ω in Equation (8), ω_{iner} is the inertial weight of adaptive change, ω_{start} is the maximum weight (value is 0.9), and ω_{end} is the minimum weight value (value is 0.4). The inertia weight in Equation (8) can be linearly decreased in this way, and the tradeoff between global search and local search can be balanced in the iterative process. This strategy of decreasing weights is aimed at giving the algorithm a greater exploration ability in the initial stage, and gradually increasing the focus on local search in subsequent iterations. Oscillation near the global optimal solution in the late particle search is effectively avoided. I is the current number of iterations, N is the total number of iterations, c_1 is the learning factor 1, and c_2 is the learning factor 2. r_1 and r_2 are random floating point numbers ranging from 0 to 1, P is the optimal position of the particle individual, and g is the global optimal position of the particle swarm.

Simultaneously, random particle mutation should be added to the particle position and velocity search in the iterative process to increase the diversity of the population. The mutation method selected is as follows:

$$\text{if } R < C_m, \begin{cases} \text{ceil}(2 \times R) = 1, v_c = R \times (C_{max} - C_{min}) + C_{min} \\ \text{ceil}(2 \times R) = 2, v_g = R \times (g_{max} - g_{min}) + g_{min} \end{cases} \quad (10)$$

In the formula, R is the random number of distributions $[0, 1]$, C_m is the preset mutation probability (constant, set as 0.8 in this paper), v_c is the particle motion velocity of hyperparameter c , v_g is the velocity of the particle with the hyperparameter gamma, and C_{min} , C_{max} , g_{min} , and g_{max} are the search boundaries of the parameters. The adaptive mutation search for the optimal particles is implemented by this method, which enables the SVR model to jump out of the local optimum and find the best parameters c and g . These parameters enable the model to find the best mapping function $\varphi(x)$. The risk of easily falling into overfitting is reduced. The adaptive mutation particle swarm optimization utilizes adaptive optimization to enable the SVR to select parameters more robustly, reducing the problem of model performance degradation caused by improper parameter selection. This is particularly significant for small sample data, as they are more sensitive to parameter selection. The prediction accuracy is further improved.

2.5. The Flow of Prediction Method

The process of the proposed prediction model is shown in Figure 1. The steps are as follows: (1) GNSS (Global Navigation Satellite System) displacement and earth pressure, precipitation, soil moisture content, and other data are collected by multi-source sensors. Among them, the displacement is obtained by self-developed GNSS monitoring equipment with a Ublox ZED-F9P board, and the measurement accuracy is better than 5 mm in both the horizontal and vertical directions. The soil moisture content is obtained by soil moisture meters, with a measurement accuracy of $\pm 3\%$. The earth pressure is measured by earth pressure boxes, with a nominal measurement accuracy of $\pm 0.5\%$ F.S. The precipitation is obtained by funnel-type rain gauges, with an accuracy of measurement of ± 0.2 mm. The multi-source data are processed on a self-built expansive soil landslide monitoring cloud platform; the GNSS displacement is calculated by real-time relative positioning with a solution frequency of 1 s and the GNSS receiver is mainly deployed at the leading edge and the middle of the slope, where the stability of expansive soil landslides is poor. A soil hygrometer determines the soil moisture content by measuring the change in the soil dielectric constant; its measurement frequency is 4 h and it should be deployed 30 cm below the surface of the expansive soil. The earth pressure box converts the mechanical pressure into an electrical signal and converts the electrical signal into the actual pressure value through calibration, with a measurement frequency of 4 h. The box is laid in the middle of the soilbag and reinforced with steel bars. The rain gauge collects precipitation through a tipping bucket structure. Its measurement frequency is 6 h and it is deployed in a stable and open location. (2) The landslide displacement is decomposed into a trend term and wave term, respectively, with CEEMDAN. The trend term displacement represents the long-term trend of landslide development and is predicted by the polynomial fitting method. (3) The correlation and the influence lag time between the fluctuation displacement and external factors are determined. The wave term component of multiple factors is the key part that affects changes in the fluctuation term displacement, which is similarly extracted by CEEMDAN and will be set as the optimal correlation factor to predict the step-like displacement of expansive soil landslides. (4) The AMPSO-SVR model is established by considering the state factors of landslide development. (5) Finally, the performance of the prediction model is evaluated with indicators like R^2 , MAPE, and RMSE.

2.6. Evaluation Indicators for Displacement Prediction

Three evaluation indicators were used to evaluate the prediction accuracy of the proposed model R^2 (goodness of fit), $RMSE$ (root mean square error), and $MAPE$ (mean absolute percentage error). Their mathematical expressions are shown in Equations (11)–(13):

$$R^2 = 1 - \frac{\sum_{i=1}^n (y_i - \hat{y}_i)^2}{\sum_{i=1}^n (y_i - \bar{y})^2} \quad (11)$$

$$RMSE = \sqrt{\frac{1}{n} \sum_{i=1}^n (y_i - \hat{y}_i)^2} \quad (12)$$

$$MAPE = \frac{1}{n} \sum_{i=1}^n \left| \frac{\hat{y}_i - y_i}{y_i} \right| \quad (13)$$

where y_i is the measured displacement, $\hat{y}_i$ is the predicted displacement, $\bar{y}$ is the average displacement, which corresponds to the fluctuation term displacement, and i is the monitoring period.

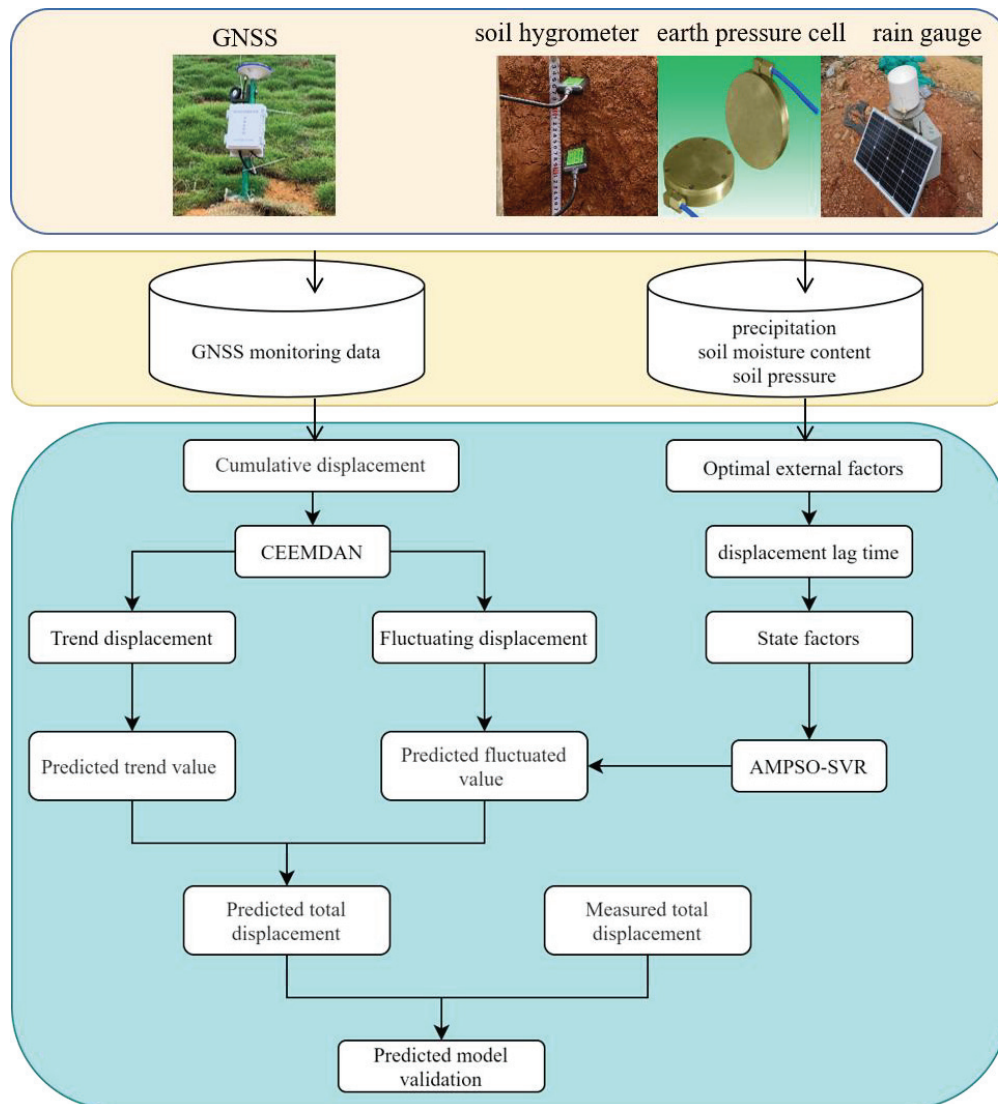


Figure 1. Multi-factor support vector regression displacement prediction flow of expansive soil.

3. Experimental Analysis—Expansive Soil Landslide on Chongai Highway in Ningming

3.1. Landslide Overview

The Ningming expansive soil landslide is located in Ningming Basin, Guangxi Province, China, which has the largest expansive soil deposit in the world and is a typical expansive soil research area. The expansive soil in this area belongs to the type of residual expansive soil [25]. It is a medium expansive soil with a free expansion coefficient of about 58%. The geographical location and overview of the slope are shown in Figure 2. The specific location of the landslide is about 5.2 km on the north bank of the Mingjiang River, a tributary of the Pearl River, 1 km west of Wayao Village, and near the Chongzuo–Aidian highway. Mainly

in the shape of a long and narrow fan, it is a highway cutting landslide with a slope of about 1:1.5.

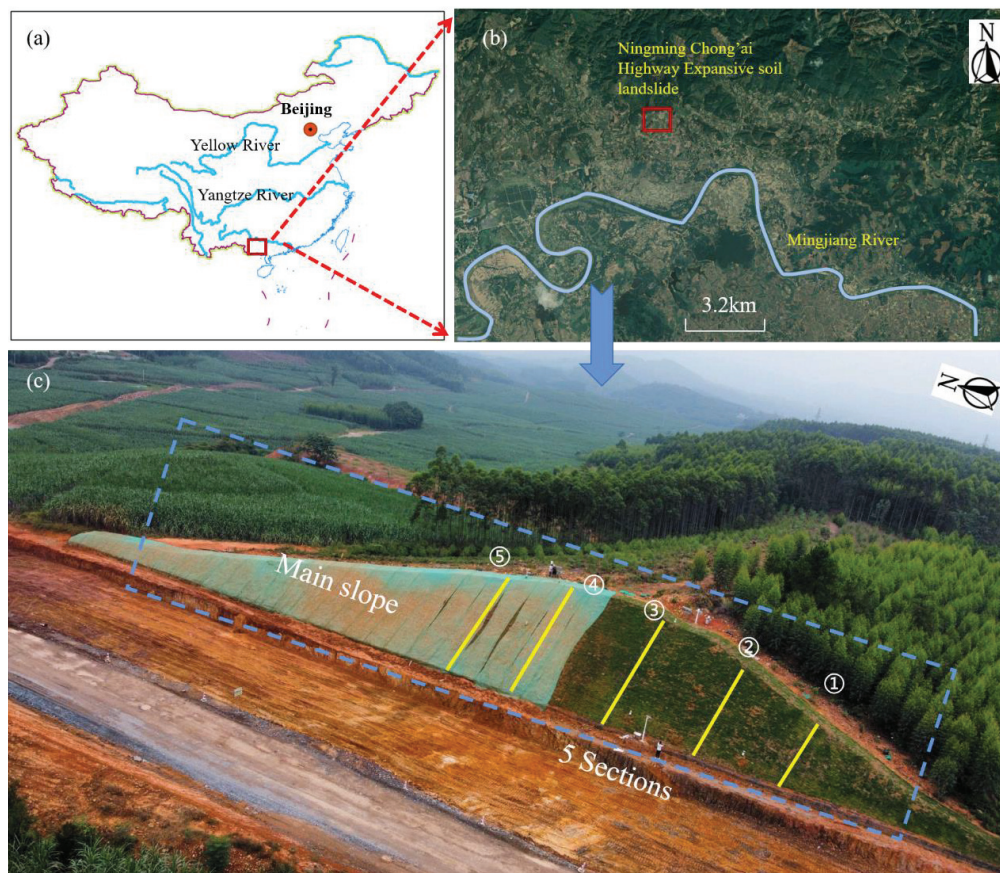


Figure 2. Landslide location and geological overview map. (a,b) the specific location of the landslide; (c) aerial figure of the expansive soil landslide, which is divided into five sections ①–⑤.

The region has a tropical marine monsoon climate with an average annual precipitation of 1200 mm and with spring and summer from April to October and autumn and winter from November to March. A total of eight GNSS monitoring points (NN01–NN08) were deployed to establish a real-time displacement-monitoring system with a monitoring accuracy better than 5 mm due to the obvious displacement of slope unloading. In addition, sensors such as those for earth pressure and soil moisture content were installed to strengthen the monitoring system, and the data transmission interval was 4 h. A rain gauge was arranged at the rear edge, and the data transmission interval was 1 h.

3.2. Deformation Characteristics

The GNSS cumulative displacement curve of the Ningming expansive soil slope is shown in Figure 3, which was calculated by the real-time GNSS landslide cloud platform. Due to the low stability at the three monitoring points on the top position of the slope, there was a significant increase in displacement compared to other points, hence, these three points were selected as the experimental dataset. The red, green, and blue curves represent the displacements of the NN06, NN07, and NN08 monitoring points, showing a step-like characteristic. They are located in sections ④–⑤ of the slope, where the risk of instability is high as a result of excavation unloading, and the slide direction is 78° north by west. The interstitial rise in displacement was mainly due to the occurrence of heavy rainfall events around 25 d and 155 d in the figure. The fissure of expansive soil provided an excellent channel for rainwater infiltration, resulting in a rapid increase in soil water content and a decrease in matrix suction as a result of the decrease in slope shear

strength with an increase in saturation. The whole process time was relatively short, so the cumulative displacement increased rapidly with a strong mutability. There was no significant change in displacement during the drying period.

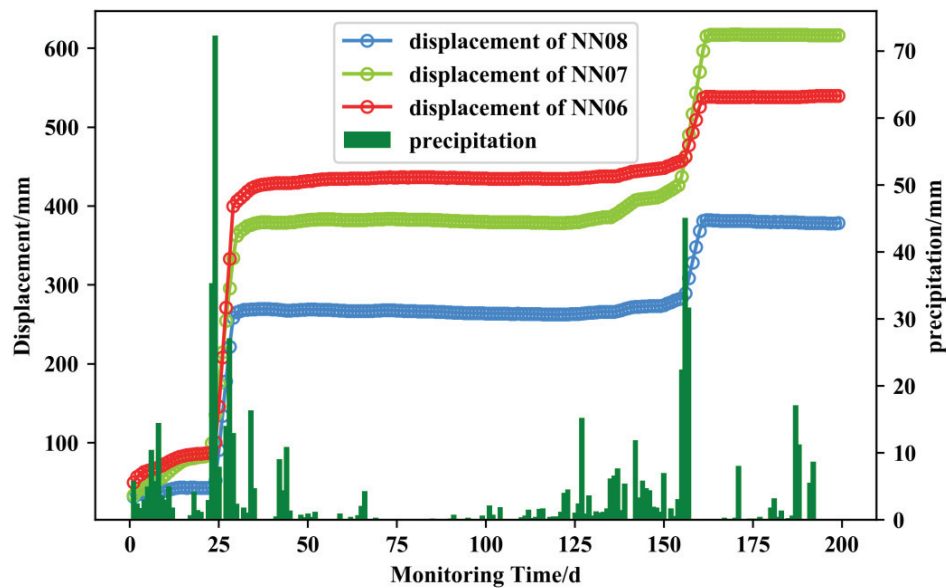


Figure 3. Displacement curve of landslide monitoring points.

The on-site failure form of the slope is shown in Figure 4. The soil near the GNSS monitoring equipment was significantly cracked, and fish-scale shallow failure characteristics appeared after two instability events. Many cracks developed obviously, and these cracks gradually closed and deepened with an increase in instability frequency, finally collapsing in the form of a traction landslide.



Figure 4. Shallow instability characteristics of Ningming expansive soil. (a–d) The visible cracks near the monitoring station.

4. Displacement Prediction

A multi-factor support vector regression prediction model was established with the GNSS cumulative displacement, soil moisture content, earth pressure, precipitation, and other data of monitoring points NN06–NN08 for about 200 days. The first 150 days of data were selected as the training set, the last 50 days were selected as the prediction set. The 75% training set ratio helped the model to better learn the pattern of the data, leaving a smaller proportion (25%) of sufficient samples to evaluate the generalization ability of the model, with the problem of overfitting of the training set being effectively avoided. Furthermore, the displacement of expansive soil experienced irregular changes such as a smooth and steep rise in the first 150 days, including the dynamic changes of expansive soil landslides from the safety stage to the instability stage. Hence, the selection of training and prediction sets in this way was more representative.

4.1. Extraction and Prediction of Trend Displacement

The trend term and fluctuation term of the displacement time series were extracted by CEEMDAN decomposition. The decomposition results are shown in Figure 5, from which it can be seen that the trend displacement component was nearly monotonically increasing, while the fluctuation term displacement was characterized by intermittent oscillation.

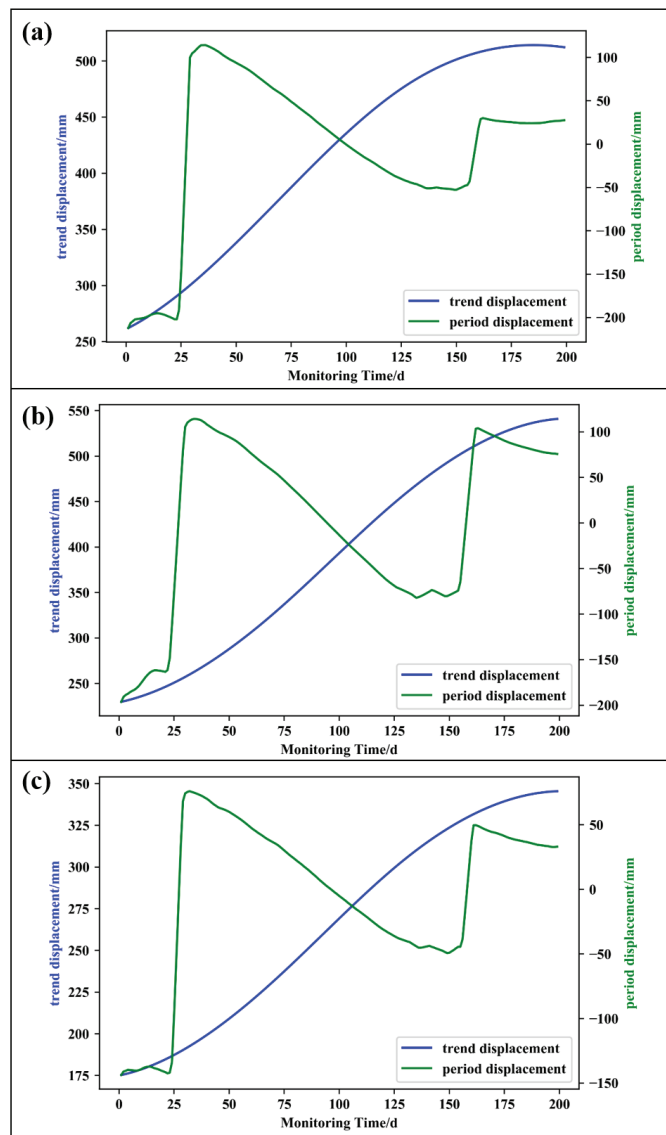


Figure 5. GNSS displacement decomposition results ((a) NN06; (b) NN07; (c) NN08).

The trend displacement was fitted by the least squares cubic polynomial according to the nonlinear curve characteristics of the trend displacement at the monitoring points NN06, NN07, and NN08. The fitting expression is $s(t) = at^3 + bt^2 + ct + d$, where $s(t)$ is the trend term displacement, t is the monitoring time, and a , b , c , and d are the coefficients. The prediction results and accuracy of the trend term displacement are shown in Figure 6. The fitting accuracy RMSEs of the NN06, NN07, and NN08 monitoring points were 0.68 mm, 0.51 mm, and 0.46 mm, respectively, and the goodness of fit R^2 were 0.9989, 0.9959, and 0.9976, respectively.

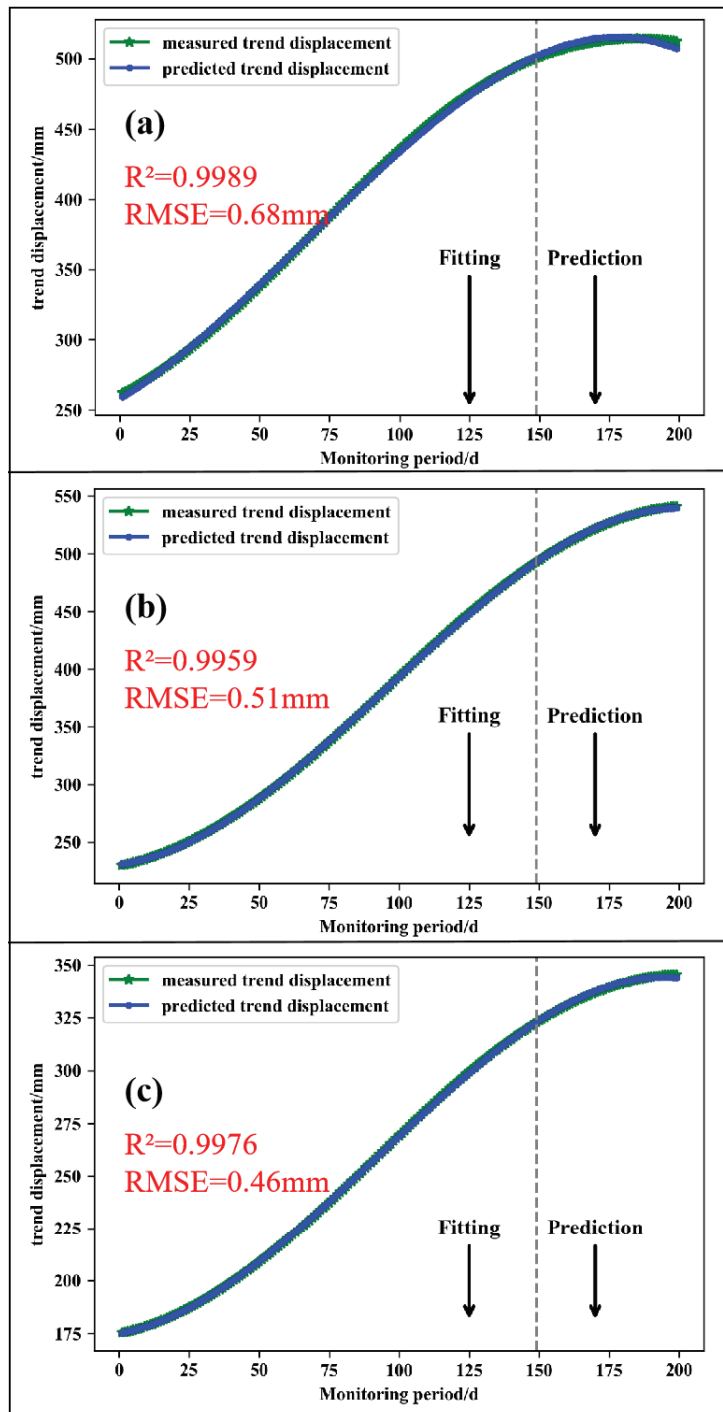


Figure 6. Fitting results of trend terms at monitoring points ((a) NN06; (b) NN07; (c) NN08).

4.2. Extraction and Prediction of Fluctuation Term Displacement

The fluctuation term displacement was obtained by subtracting the trend term displacement from the total displacement, which reflects the landslide displacement changes under the influence of external factors. In this paper, a multi-factor machine learning dynamic prediction model was established for the GNSS fluctuation term displacement at NN06, NN07, and NN08 for 200 d. The displacement data of the first 150 d were selected as the training set, and the last 50 d were selected as the test set.

4.2.1. Determination of Key Disaster-Inducing Factors

The prediction performance of the landslide displacement of expansive soil is directly affected by the selection of disaster-inducing factors [26]. Strength attenuation and swelling deformation are caused by unsaturated seepage under rainfall conditions, which has a significant impact on landslide stability. Moreover, the infiltration effect of rainwater is enhanced by the existence of cracks in expansive soil, and the effective water content of the soil is increased, which leads to the hygroscopic expansion of soil. The volume deformation of expansive soil under the dry–wet cycle has a significant impact on the pressure distribution of the retaining wall, and the lateral earth pressure increases significantly under the action of infiltrating rainwater and the mass of the overlying soil [27–29]. Therefore, precipitation, soil moisture content, and earth pressure are regarded as key factors for displacement prediction.

4.2.2. Time Lag Correlation

During the deformation process of an expansive soil landslide induced by rainfall, the soil moisture content and earth pressure first begin to increase, there is a lag time in the corresponding displacement changes, and the lag time is different at several monitoring points. To find the best correlation factor of the landslide displacement for the model training and avoid obvious deviations in the prediction results, the displacement response lag times of each monitoring point were determined by time delay correlation analysis (TDCA). The results are shown in Figure 7 (Unit: d): for the NN06, NN07, and NN08 monitoring points, the time lag parameters of the fluctuation term displacement and soil moisture content, earth pressure, and cumulative rainfall were (3 d, 1 d, 3 d), (2 d, 0 d, 3 d), and (3 d, 1 d, 4 d), respectively.

4.2.3. Fluctuation Term Displacement Prediction

Compared with the neural network prediction model, the prediction problem of multi-dimensional features can be solved by SVR more efficiently, and the prediction model is built by combining the AMPPO optimization algorithm. Since each non-periodic step change of the expansive soil slope displacement is greatly impacted by the fluctuation term of the external multi-source factor, the fluctuation term of the influencing factor is extracted by CEEMDAN. The mutation information of the influencing factors can be reflected by the fluctuation term component of the multi-factor, which is strongly related to the mutation of the fluctuation term displacement. Hence, it is more suitable for predicting the wave term displacement. As shown in Figure 8, the blue, green, orange, and red circled curves represent the fluctuation term displacement, fluctuation term cumulative precipitation, fluctuation term soil moisture content, and fluctuation term earth pressure at the monitoring points. The correlation between the wave term displacement and the influence factors sequence was more notable when the time lag was eliminated. It can be seen that the fluctuation term cumulative precipitation, soil moisture content, and fluctuation term periodic earth pressure started to increase, and the fluctuation term displacement showed a sudden increase simultaneously. If the cumulative precipitation, soil moisture content, and earth pressure of the fluctuation items continued to decrease overall, although there was a small local rebound, the corresponding fluctuation term displacement decreased synchronously and rose synchronously again during the next instability. Therefore, the

fluctuation term displacement was more consistent with the fluctuation term change trend of external influence factors.

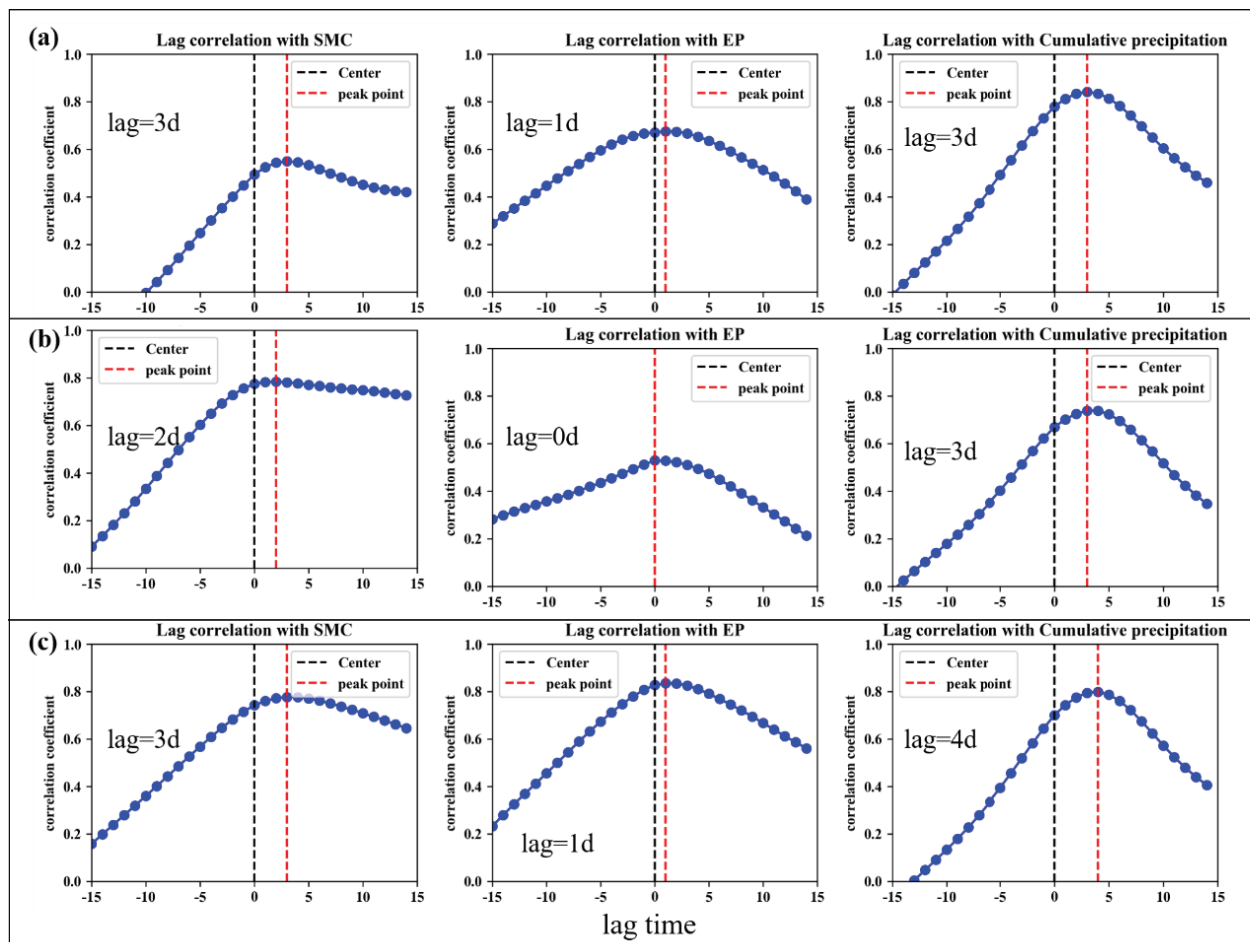


Figure 7. Time lag parameters of fluctuation term displacement and influencing factors ((a) NN06; (b) NN07; (c) NN08).

A self-learning AMPSO-SVR displacement prediction model was established based on the libsvm toolkit in matlab2021b software by normalizing the multi-factor association data and displacement data which eliminated the time lag effect. The model in this section was trained on a Hewlett-Packard computer with an Intel core i7-11800H processor (clocked at 2.3 GHz, eight cores, and sixteen threads), an NVIDIA RTX3050 GPU, and 16 G of memory. The fitness function is defined as the mean square error between the predicted and measured displacement values.

Taking into account the different lag times of the displacements for each monitoring point, multi-factor data that eliminated the time lag effect were added to the training model. The multi-factor data to eliminate the delay effect were added to the training model while considering the different lag times of the displacements at each monitoring point. For the NN06 monitoring point, six types of data, including the fluctuation item soil moisture content (SMC), fluctuation item cumulative precipitation 3 days ago, fluctuation item earth pressure (EP) 1 day ago, and historical displacements in the past 1 d, 2 d, and 3 d, were taken as model inputs. Similarly, for the NN07 monitoring point, the fluctuation term soil moisture content 2 days ago, fluctuation term cumulative precipitation 3 days ago, and fluctuation term earth pressure 1 day ago were combined. For the NN08 point, the fluctuation term soil moisture content 3 days ago, cumulative precipitation of the fluctuation term 4 days ago, and earth pressure data of the fluctuation term 2 days ago were input into the model for data training.

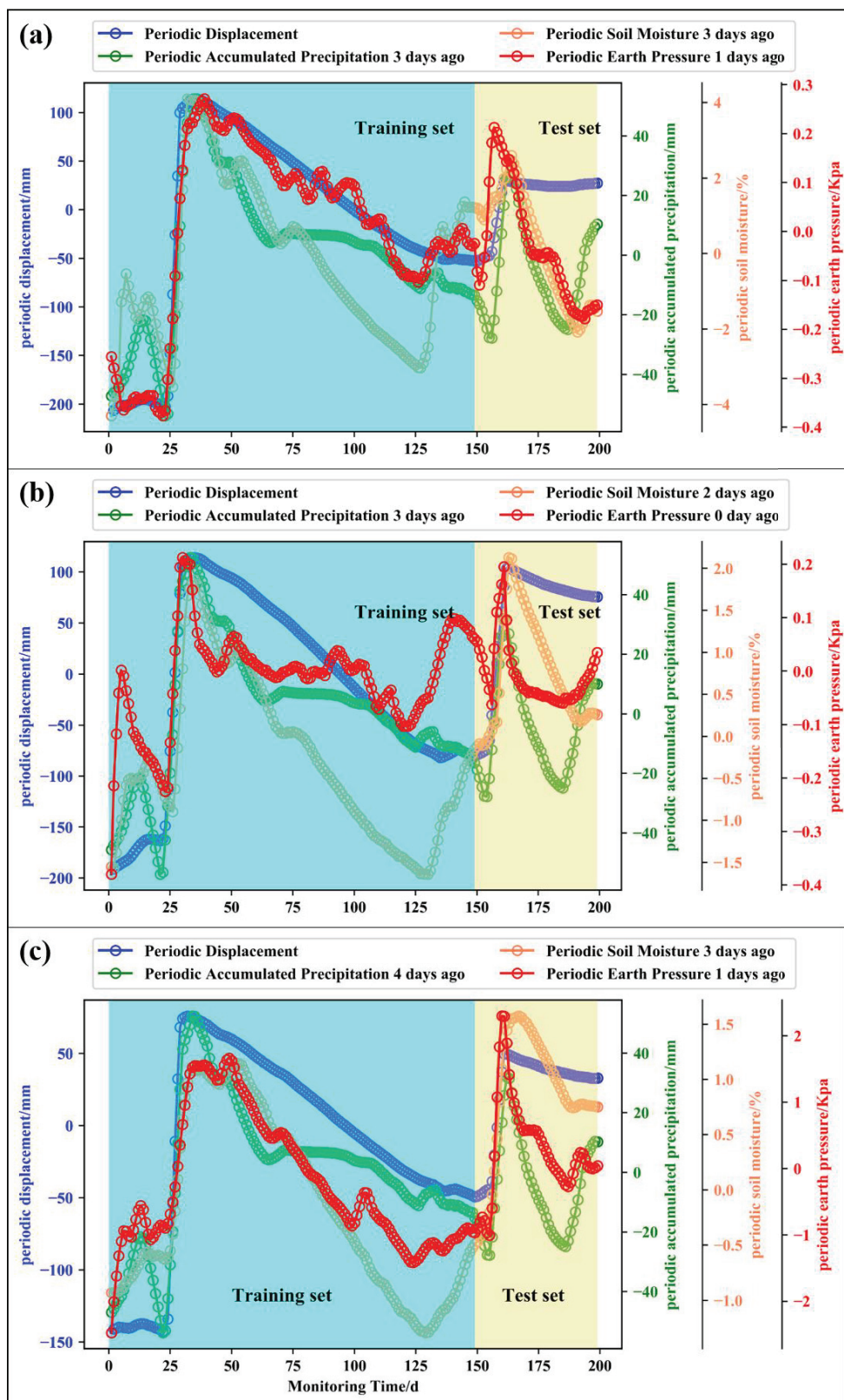


Figure 8. Fluctuation displacement and fluctuation term multi-factor sequence at monitoring point ((a) NN06; (b) NN07; (c) NN08).

However, it is not reliable to predict the displacement by only considering the influencing factors and ignoring the development state of the landslide itself. The historical displacements before 3 d, 2 d, and 1 d were added into the model as predicted state factors.

A total of six sequences were taken as inputs to the AMPSO-SVR model (Figure 9). After 150 iterations of parameter optimization, the final displacement prediction value was obtained.

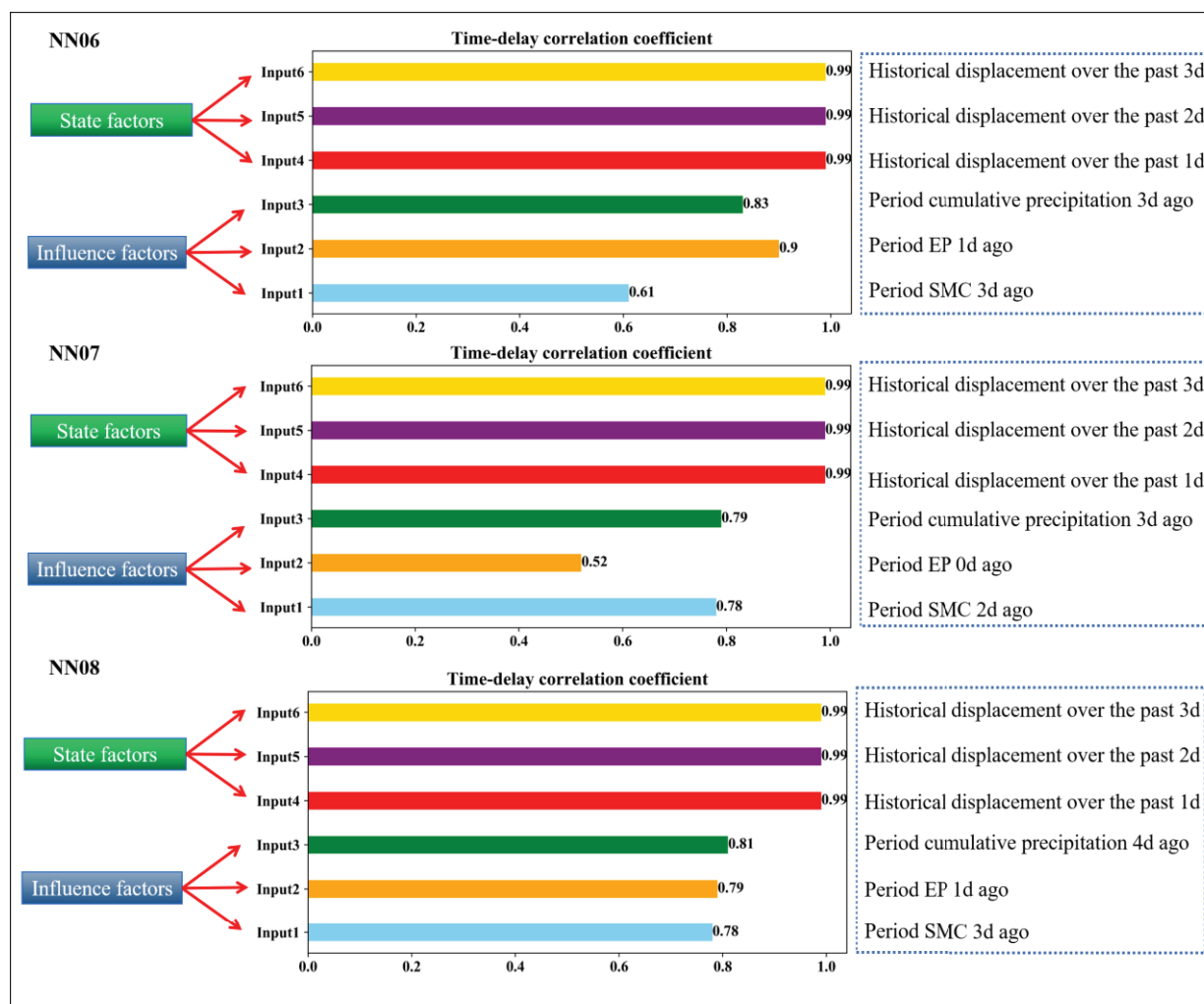


Figure 9. Input items of CEEMDAN-AMPSO-SVR prediction model.

To evaluate the actual performance of the prediction model, the results were compared with the commonly used GS-SVR, PSO-SVR, and BPNN models (see Figure 10). The goodness of fit R^2 , root mean square error RMSE, and mean absolute percentage error MAPE were calculated. The calculation formula is as shown in Equation (10). The calculation results of related indicators are shown in Table 1. As can be seen from the prediction results in Figure 10, the green circle curve is the measured displacement data of the landslide, the blue circle curve is the prediction result of the GS-SVR model, the yellow square curve is the prediction result of the PSO-SVR model, the purple triangle curve is the prediction result of the BPNN model, and the red diamond curve is the prediction result of the AMPSO-SVR model. Among them, the predicted results of the GS-SVR model at the monitoring points NN06 and NN08 during the instability period were significantly greater than the measured displacement, and the predicted displacement was not stable in the short time after the slope failure, which is not consistent with the characteristics of the displacement tending to be stable after the actual landslide failure. The goodness of fit and root mean square error at point NN06 were 0.9524 and 5.9 mm, respectively. The goodness of fit and root mean square error at point NN07 were 0.9459 and 13.2 mm, respectively. The goodness of fit and root mean square error at point NN08 were 0.9239 and 8.5 mm, respectively. The prediction error was larger among the four prediction models.

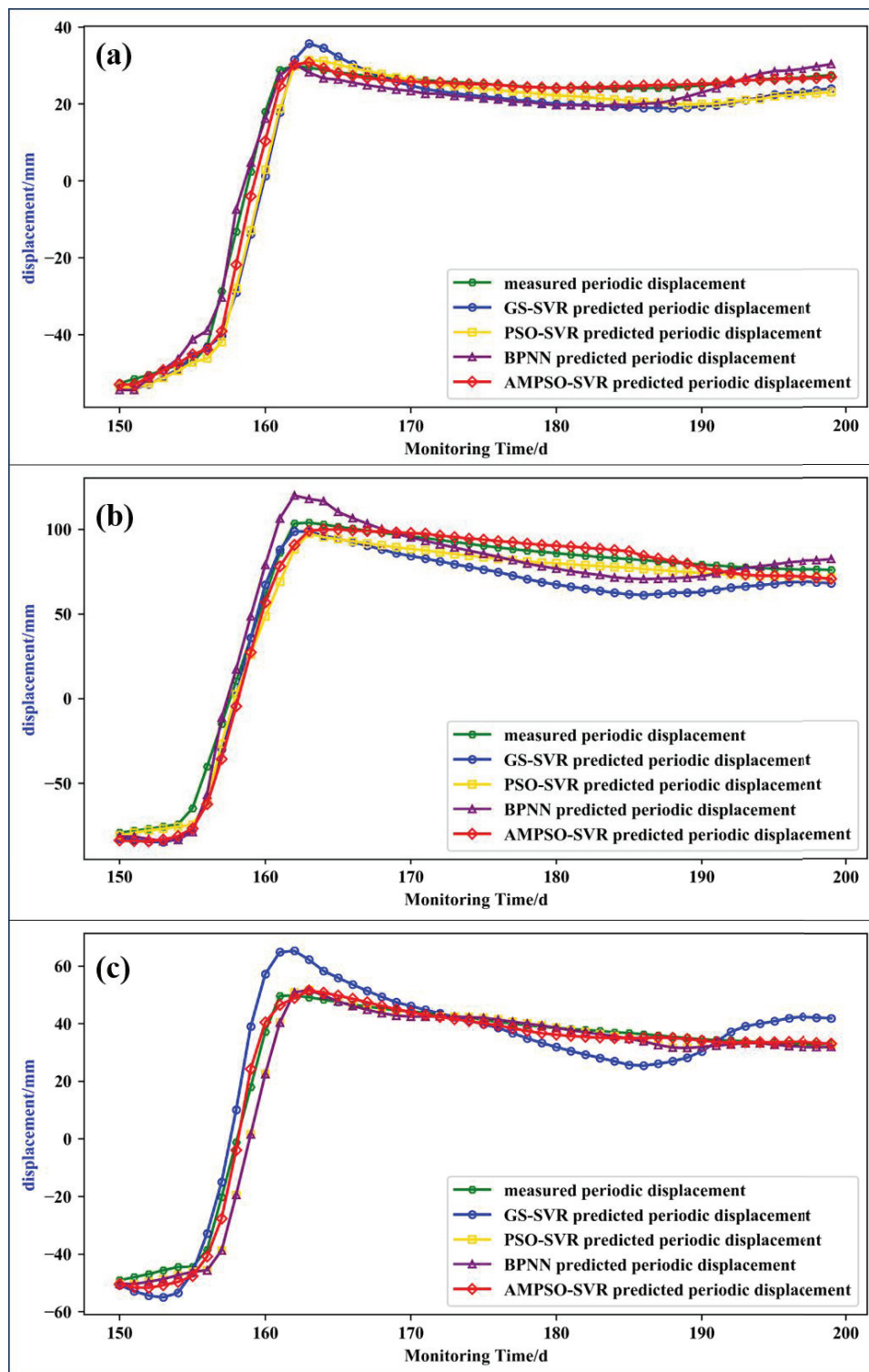


Figure 10. Comparison of prediction performances of different models ((a) NN06; (b) NN07; (c) NN08).

The predicted results of PSO-SVR were similar to the measured results, but the goodness of fit and root mean square error were inferior to the results of the AMPSO-SVR model. The BPNN prediction results showed a good performance at points NN06 and NN08, but a poor performance at point NN07. It can be seen that the predicted value at point NN07 was significantly higher than the measured value during the sliding stage, which had significant errors. Therefore, the generalization ability of this model was insufficient.

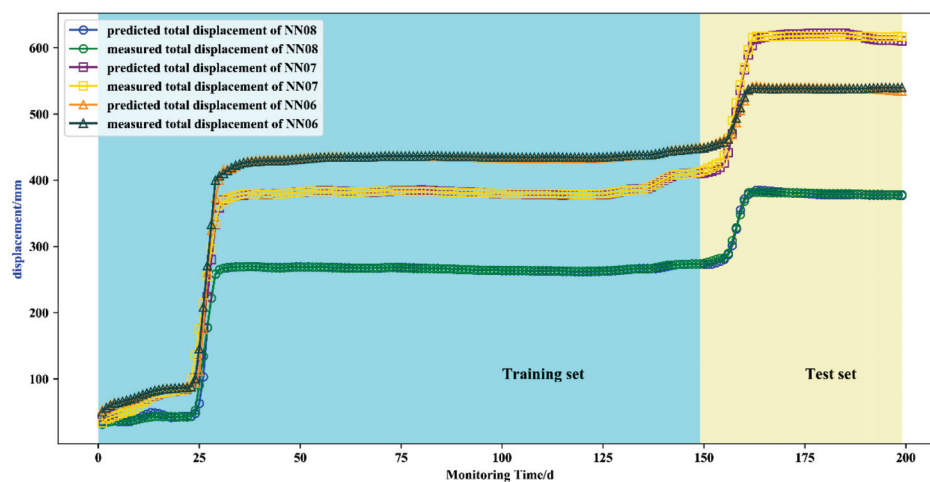
Table 1. Evaluation indicators of different prediction models.

Monitoring Points	R^2	RMSE (mm)	MAPE	Model
NN06	0.9524	5.9	0.3131	GS-SVR
	0.9626	5.2	0.2661	PSO-SVR
	0.9864	3.1	0.1273	BPNN
	0.9907	2.6	0.1017	AMPSO-SVR
NN07	0.9459	13.2	0.1793	GS-SVR
	0.9821	7.6	0.1124	PSO-SVR
	0.9776	8.9	0.1254	BPNN
	0.9857	6.6	0.1181	AMPSO-SVR
NN08	0.9239	8.5	0.3766	GS-SVR
	0.9697	5.3	0.3871	PSO-SVR
	0.9872	3.8	0.2135	BPNN
	0.9934	2.5	0.0964	AMPSO-SVR

The AMPSO-SVR prediction results were most consistent with the measured displacements at the three monitoring points. The goodness of fit and root mean square error at point NN06 were 0.9907 and 2.6 mm, respectively. The goodness of fit and root mean square error at point NN07 were 0.9857 and 6.6 mm, respectively. The goodness of fit and root mean square error at point NN08 were 0.9934 and 2.5 mm, respectively. With the maximum goodness of fit and minimum root mean square error, the comprehensive generalization ability of the model was better than that of the other three models. Hence, the prediction performance of the AMPSO-SVR model was superior to other models.

4.3. Total Displacement Prediction and Accuracy Analysis

The predicted value of the total displacement was obtained by superpositioning the predicted value of the displacement of the fluctuation term and the trend term, which was compared with the measurements. The prediction result curves are shown in Figure 11. The goodness of fit R^2 and RMSE of the predicted and measured displacements of the NN06, NN07, and NN08 monitoring points were 0.99, 2.6 mm, 0.99, 6.6 mm, and 0.99, 2.5 mm, respectively. These prediction results were consistent with the actual measurements, and the accuracy was in the order of millimeters. This indicated that the AMPSO-SVR prediction model can be well applied to multi-factor-driven non-periodic step-like expansive soil landslide displacement prediction.

**Figure 11.** Total displacement prediction results of NN06/NN07/NN08.

5. Conclusions

- (1) The characteristics of the “ladder-type” deformation of expansive soil landslides due to their non-periodic repeated instability were recorded and analyzed. The important relationship between key influencing factors such as earth pressure, soil moisture content, and cumulative precipitation with this step displacement were also revealed. Meanwhile, the lag response time of landslide displacement and influencing factors was determined. The GNSS displacement of monitoring points at different parts of the Ningming expansive soil slope was different from the lag time of the influencing factors. The average GNSS displacement lagged behind rainfall, soil moisture content, and earth pressure at 3 d, 2 d, and 1 d, respectively. The GNSS displacement sequence corrected by the lag period was in good agreement with the multi-source influence factor sequence.
- (2) The displacements of the trend term and fluctuation term were obtained by CEEM-DAN decomposition. The displacements of the trend term were predicted by cubic polynomial fitting. Taking into account the non-periodic step of the fluctuation displacement of the expansive soil landslide that was affected by multiple external factors, a dynamic prediction model driven by multi-factors was established to predict the displacement of the fluctuation term. The prediction results were in good agreement with the obtained measurements. The average RMSE predicted by AMPSO-SVR was 3.94 mm, compared with the results of the GS-SVR, PSO-SVR, and BPNN models, it was increased by 58.3%, 38.1%, and 25.2%, respectively.
- (3) The proposed model was feasible and reliable in the prediction of step-like and non-periodic expansive soil landslides, and the stepped deformation and external multiple factors could be modeled efficiently, which gives it the potential to be applied to other expansive soil landslide deformation predictions.

In general, the proposed method had a good performance in predicting the step-wise displacement of expansive soil with a strong suddenness, especially expansive soil landslides driven by multiple internal and external factors. However, it is mainly aimed at the instability of expansive soil landslides with relatively short monitoring periods, and further research is needed for the displacement prediction process of expansive soil landslides with longer terms (e.g., more than 5 years). In addition, more complex prediction algorithms such as deep learning models will be tested and evaluated in subsequent research.

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