

Special Issue Reprint

New Insights into High-Energy Astrophysics, Galaxies, and Cosmology

Celebrating the 10th Anniversary of the
Re-establishment of the Department of Astronomy
at Xiamen University (2012–2022)

Edited by
Taotao Fang, Weimin Gu, Tong Liu and Beibei Liu

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Guest Editors

Taotao Fang

Weimin Gu

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Contents

About the Editors	vii	
 Soumen Kumar Bera, Tapan K. Sasmal, Soumen Mondal, Taotao Fang and Xuelel Chen Strange and Odd Morphology Extragalactic Radio Sources (STROMERSs): A Faint Images of the Radio Sky at Twenty-Centimeters (FIRST) Look at the Strange and Odd Radio Sources Reprinted from: <i>Universe</i> 2024 , 10, 347, https://doi.org/10.3390/universe10090347		1
 Zhenyi Cai The Composite Spectral Energy Distribution of Quasars Is Surprisingly Universal Since Cosmic Noon Reprinted from: <i>Universe</i> 2024 , 10, 431, https://doi.org/10.3390/universe10110431		15
 Tian-Wen Cao, Zi-Jian Li, Pei-Bin Chen, Venu M. Kalari, Cheng Cheng, Gaspar Galaz, et al. The Properties of an Edge-On Low Surface Brightness Galaxies Sample Reprinted from: <i>Universe</i> 2024 , 10, 432, https://doi.org/10.3390/universe10110432		37
 Bao-Quan Huang and Tong Liu Jet Precession in Gamma-Ray Bursts Reprinted from: <i>Universe</i> 2024 , 10, 438, https://doi.org/10.3390/universe10120438		54
 Shan-Shan Weng and Long Ji X-Ray Views of Galactic Accreting Pulsars in High-Mass X-Ray Binaries Reprinted from: <i>Universe</i> 2024 , 10, 453, https://doi.org/10.3390/universe10120453		72
 Hening Wu and Ya-Ping Li Effects of Thermodynamics on the Concurrent Accretion and Migration of Gas Giants in Protoplanetary Disks Reprinted from: <i>Universe</i> 2025 , 11, 431, https://doi.org/10.3390/universe11010001		103
 Li Xue, Cheng-Liang Jiao and Li-Xin Zhang A Scenario for Origin of Global 4 mHz Oscillations in Solar Corona Reprinted from: <i>Universe</i> 2025 , 11, 14, https://doi.org/10.3390/universe11010014		120
 Hao-Yan Chen Reconciling the Waiting Time Peaks Variations of Repeating FRBs with an Eccentric Neutron Star–White Dwarf Binary Reprinted from: <i>Universe</i> 2025 , 11, 133, https://doi.org/10.3390/universe11050133		141
 Bing-Hang Chen, Jun-Jie Zhao, Hao-Ran Yu, Yu Liu, Jian-Hua He and Yipeng Jing Cosmological Simulations with Massive Neutrinos: Efficiency and Accuracy Reprinted from: <i>Universe</i> 2025 , 11, 212, https://doi.org/10.3390/universe11070212		159

About the Editors

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Article

Strange and Odd Morphology Extragalactic Radio Sources (STROMERSs): A Faint Images of the Radio Sky at Twenty-Centimeters (FIRST) Look at the Strange and Odd Radio Sources

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Abstract: We report the identification of an extremely rare and peculiar set of irregular radio sources, termed “STROMERSs” (STRange and Odd Morphology Extragalactic Radio Sources). The irregular radio sources with very anomalous morphologies that make them exceptionally different from all the known classes and subclasses of irregular radio sources are detected as STROMERSs. A thorough search for this class of sources from the Very Large Array (VLA) Faint Images of the Radio Sky at Twenty-Centimeters (FIRST) gave a total of nine such candidates. We checked the corresponding morphology of the identified sources in other frequency surveys. We found a detectable radio emission for all of the nine sources in the NRAO VLA Sky Survey (NVSS) at 1.4 GHz and in the TIFR GMRT Sky Survey (TGSS) at 150 MHz, while the same was found for only three sources in the Westerbork Northern Sky Survey (WENSS) at 625 MHz. However, the strange morphology was not found in all of those other survey images. We also characterized the sources with their corresponding physical parameters like optical counterpart, size, spectral index, and radio luminosity. The estimated spectral values of the sources indicated that the STROMERSs were most likely radio galaxies. The presence of any nearby galaxy clusters for the STROMERSs was also checked.

Keywords: active galaxies; galaxies; galaxy jets; galaxy morphology; irregular radio continuum; radio galaxies; radio survey

1. Introduction

Radio sources are one of the strong and energetic astronomical objects with significant radio emission. The radio emission from these objects is primarily non-thermal and originates from the synchrotron radiation produced by relativistic electrons spiraling in the strong magnetic fields present in the jets and lobes of radio galaxies. A classical double-lobed radio galaxy is characterized by a pair of extended lobes in the opposite direction with a bright compact radio core at the center.

Apart from the typical radio galaxy, there are some irregular radio galaxies. The irregular radio galaxies are a type of active galactic nuclei (AGNs) that exhibit complex, non-standard radio morphologies that do not conform to the typical double-lobed structure. In the set of the irregular class of galaxies, there are some subsets; for example, winged radio galaxies (WRGs; these include both X-shaped radio galaxies (XRGs) and Z-shaped radio galaxies (ZRGs) [1–5]), double double radio galaxies (DDRGs [6,7]), bet-tail (BT) radio galaxies (which comprise the wide angle tail (WAT) and narrow angle tail (NAT) [8–13]), and recently identified odd radio circles (ORCs [14,15]). In addition, there are some individual mentions of peculiar radio sources in the literature, e.g., [16–18]. Very recently, Sasmal et al. [19] reported the findings of four miscellaneous radio sources from the LOFAR Two-meter Sky Survey First Data Release (LoTSS DR1) [20] at 144 MHz.

As extensively documented in the literature and clearly illustrated by the classic case of BL Lacs [21], valuable insights into the “central engine” of radio galaxies can be obtained by examining their rare specimens that exhibit highly unusual traits, such as a distinctive radio morphology, spectrum, polarization, or time variability. These “odd entities” can challenge and refine both theoretical models of double radio sources and their numerical simulations. HYMORSs (HYbrid MORphology Radio Sources) are one such example, where the two lobes of this type of radio sources exhibit different FR morphology on the other sides of AGNs [22]. Though they are less populated (<1%) radio galaxies, they strongly constrain the notion that the jets in radio galaxies belonging to the two FR classes have different energy carriers [23]. In retrospect, we conclude that seemingly unusual radio sources exhibiting uncommon characteristics are far from regular types of radio sources and that casual annotations such as “freaks” [24], “curiosities” [25] or simply “miscellaneous” [19] do not sufficient capture their astrophysical potential. A modification is thus necessary, commensurate with the true potential of such peculiar types of radio galaxies. Motivated by this, we suggest a novel annotation for extended radio galaxies and quasars with radio morphologies that are unique from one another, or as “erroneous”; that is, in a way that differs significantly from the recognized standard morphological types mentioned in the most recent review paper by Baldi [26]. The present work focuses on searching for radio sources that fall within the broad category of “STROMERSs” (STRange and Odd Morphology Extragalactic Radio Sources), which is the term we propose for such radio morphological outliers.

Aside from very rare kinds of sources, the study of STROMERSs may give some prominent or crucial information on those specific sources and radio galaxies in general. The complex radio morphologies might be an indication of the interactions between the AGN jets or outflows and the host galaxy’s interstellar or intergalactic medium. The irregular radio galaxies are frequently associated with merging or interacting galaxies, and it could also be a similar scenario for the STROMERSs. Thus, the study of such intriguing sources may ensure the understanding of the connection between irregular radio morphologies and galaxy mergers or how those interactions are crucial for models of AGN triggering and co-evolution with the host galaxy. The sources may also represent an important transitional phase in the life cycle of radio-loud AGNs. Mapping the detailed radio structures of these sources can reveal signatures of shocks, backflows, and turbulence induced by the AGN activity. Hence, studying such exceptional radio structures of these galaxies hopefully provide important insights into the physics of AGN feedback and its impact on galaxy evolution.

The recently improved telescopes and their cutting-edge technology allow us to observe more and in detail the fine structures of distant radio sources. Here, we present the search result for this kind of sources from the NRAO Very Large Array (VLA) Faint Images of the Radio Sky at Twenty-Centimeters (FIRST) at 1.4 GHz [27].

In this paper, Section 2 describes our identification procedure, which includes the data and source selection, cross-reference of the sources in other frequency surveys, and corresponding optical counterpart identification. Sections 3 and 4 present the results and the summary and discussion portions, respectively. In the results section, we include a short note on each of the individual sources and the presence of nearby galaxy clusters for the STROMERS candidates. We used the following cosmology parameters in this paper: $H_0 = 67.4 \text{ km s}^{-1} \text{ Mpc}^{-1}$, $\Omega_m = 0.315$, and $\Omega_{vac} = 0.685$ [28]. In the required parameter calculations, we used the cosmological calculator for the flat universe by Nick Gnedin¹. The spectral index (α) was defined as $S_\nu \propto \nu^\alpha$, where S_ν is the flux density at the particular frequency ν . The contour map drawing, overlaying the radio map of the sources on their respective optical image, and drawing of the presence of nearby galaxy clusters were performed using the Common Astronomy Software Applications (CASA)². We also used the Astronomical Image Processing System (AIPS)³ [29] for measuring the flux density, source size, etc. Every position presentation used the J2000 coordinate system.

2. Identification of the STROMERS Candidates

2.1. Data Selection: The VLA FIRST Survey Data

Sasmal et al. [19] presented the search result for miscellaneous sources from the low-frequency LoTSS DR1. The LoTSS DR1 has a small sky coverage of 424 square degrees [20]. In this work, we shifted our focus to the high-frequency survey data, preferentially those that had a comparatively larger sky-coverage area. The available high-frequency surveys that have a relatively large sky coverage are mainly the NRAO Very Large Array Sky Survey (NVSS) at 1.4 GHz [30] and the VLA FIRST at 1.4 GHz. Now, the VLA FIRST outperforms the other with its better-resolving power, which is needed to observe fine morphological details.

The FIRST survey uses the NRAO VLA B configuration, and it covers 10,575 square degrees of sky area, which is 25% of the total sky. Among the total sky coverage, it has 80% sky coverage in the north, and 20% in the south galactic cap [27]. Here, we used the latest data release of VLA FIRST, i.e., the FIRST data of 14 December 2017 (14Dec17 version). The 14Dec17 version covers the sky from R.A. = 07.0 hr to 17.5 hr, Decl. = -08.0 deg to $+57.6$ deg in the northern sky. In the southern sky, the survey covers from R.A. = 20.4 hr to 4.0 hr and Decl. = -11.5 deg to $+15.4$ deg. The typical rms of the survey is 0.15 mJy [27].

2.2. Selection of the STROMERS Candidates

The 14Dec17 data release of FIRST has a total of 946,432 sources. We filtered the total sources and created our investigating sample as the sources with an angular size higher than or equal to $10''$. We set the angular size selection criterion as twice the convolution beam size, in order to spot the sufficiently large sources. Additionally, to find bright sources, we considered only the sources whose total flux density given in the catalog was greater than 10 mJy at 1400 MHz. After this filtering, a sample size of 95,243 targets was obtained. Then, for each of those 95,243 sources, we conducted a manual visual search (MVS). The MVS included checking for their grayscale images and contour images. Radio sources with strange morphologies that do not resemble any known classes and subclasses of irregular radio galaxies are identified as STROMERSs. Hence, in selecting such candidates, we first spotted the odd morphology sources from their grayscale radio images, then we drew the respective contour plots for each of those sources. After scrutinizing the peculiar morphology from the contour image, we finally selected the particular source as a STROMERS candidate. To ensure a reliable morphology, the minimum contour levels were drawn at least from three times the level up to the rms level. We aimed to find the irregular radio galaxies from the FIRST survey. From the above selection criterion, first, we reported 296 “winged” radio galaxies Bera et al. [1], 717 bent-tail radio galaxies Sasmal et al. [13], and in this paper, we reported nine peculiar radio morphologies named “STROMERSs”.

2.3. Cross-Referencing the Sources with Other Survey Data

We cross-checked the identified STROMERSs and their respective morphology in other available prominent surveys at different frequencies like NVSS at 1.4 GHz, the Westerbork Northern Sky Survey (WENSS) at 625 MHz [31], and the first alternative data release of the TIFR GMRT Sky Survey (TGSS ADR1) at 150 MHz [32]. The angular resolution and rms of these three surveys (NVSS, WENSS, and TGSS ADR1) are inferior to that of the FIRST. Thus, we may not have any strange morphology as seen in FIRST; however, this cross-referencing is mainly important for the source characterization (e.g., galaxy, nebulae, or H-I cloud).

2.4. Optical Counterpart Identification

Often, we found an optical host galaxy, typically located near the central core of a radio source. That specific optical host galaxy was taken as the optical counterpart of that corresponding radio source. A visual assessment allowed us to select the optical counterparts for the STROMERSs, based on their relative position in the radio map. We used the Sloan Digital Sky Survey (SDSS) [33] data catalog to look for the optical counterparts. Here, the

SDSS data release 13 (SDSS DR13) [34] and the SDSS-i band images were used. The optical sources near to the radio core were found using the NASA/IPAC Extragalactic Database (NED)⁴. We used the associated infrared source (IR counterpart), such as the 2MASS (Two Micron All Sky Survey) [35] or WISE (Wide-field Infrared Survey Explorer) [36,37], when the optical counterpart was not available.

3. Results

Our exhaustive search gave a total of nine STROMERS candidates. In Table 1, we present the sources with their basic parameters. The contour images of the sources are presented in Figure 1. We found a redshift (z) for seven sources. Here, we measured the angular (θ) and linear size (l), when the redshift information of the radio sources was available. The two-point spectral index (α_{1400}^{150}) was calculated at frequencies between 1400 MHz and 150 MHz. We used the NVSS flux for the 1400 MHz flux, and for the 150 MHz flux, we used the TGSS flux. Here, we used the NVSS data for the 1400 MHz flux instead of FIRST because of the configuration of these two survey systems. Due to the high resolution and the absence of closely spaced antennas, the FIRST data could miss some flux. On the other hand, the NVSS could compromise the accuracy of a source position, as it measures more exact flux values. The spectral index was calculated for all nine sources. The optical counterpart was found for five sources and the IR counterpart was identified for three sources, when no optical or IR counterpart was found for the STROMERS J1452+0212. Using the redshift and flux information (F_{1400}), we calculated the luminosity value ($L_{1400\text{MHz}}$) for the possible seven sources. We calculated the radio luminosity (L_{rad}) using the formula (adapted from O’Dea & Owen [38]):

$$L_{\text{rad}} = 1.2 \times 10^{27} D_{\text{Mpc}}^2 S_0 \nu_0^{-\alpha} (1+z)^{-(1+\alpha)} \times (\nu_u^{(1+\alpha)} - \nu_l^{(1+\alpha)}) (1+\alpha)^{-1} \text{erg s}^{-1} \quad (1)$$

In this equation, the following notations are used: D_{Mpc} is the luminosity distance to the source (Mpc), S_0 is the flux density (Jy) at a given frequency ν_0 (Hz), z is the redshift of the radio galaxy, α is the spectral index, ν_u (Hz) is the cutoff frequency, and ν_l (Hz) is the lower cutoff frequency. In this luminosity calculation, we used values of 15 GHz and 100 MHz for the upper and lower cutoff frequencies, respectively.

Table 1. List of candidates for the identified STROMERSs.

Sl. No. (1)	Name (2)	R.A. (J2000.0) (3)	Decl. (J2000.0) (4)	Ref. (5)	z (6)	θ (arcsec) (7)	l (Mpc) (8)	F_{1400} (mJy) (9)	F_{150} (mJy) (10)	α_{150}^{1400} (11)	$L_{1400\text{MHz}}$ (erg/s) ($\times 10^{41}$) (12)
1	J0707+4327	07 07 11.57	+43 27 10.8	WISEA	—	35	—	22.5	85.5	−0.60	—
2	J0708+4114	07 08 06.78	+41 14 06.2	2MASS	0.124	61	0.178	55.6	298.7	−0.75	1.70
3	J0739+3947	07 39 31.43	+39 47 20.0	SDSS	0.098	63	0.144	43.7	181.4	−0.64	0.86
4	J0803+1050	08 03 37.68	+10 50 42.4	SDSS	0.142	112	0.379	85.2	663.1	−0.92	3.44
5	J0858−0107	08 58 28.92	−01 07 17.3	2MASS	0.197	75	0.363	101.9	448.0	−0.66	8.68
6	J1440+1125	14 40 20.78	+11 25 07.1	SDSS	0.311	68	0.553	119.9	578.7	−0.70	27.60
7	J1452+0212	14 52 24.61	+02 12 00.9	—	—	63	—	201.8	970.9	−0.70	—
8	J1453+2210	14 53 17.40	+22 10 55.8	SDSS	0.128	85	0.258	96.2	495.1	−0.73	3.19
9	J2155+0846	21 55 26.16	+08 46 48.4	SDSS	0.150	58	0.208	125.5	776.7	−0.82	5.71

The basic parameters of the STROMERSs are mentioned in this table. The sources are cataloged in the table in the ascending order of Right Ascension (RA). The table contains the following columns: column 1: catalog number, column 2: host name, column 3: RA (J2000.0), column 4: declination (J2000.0), column 5: position reference, column 6: redshift (z), column 7: angular size (θ) in arc-second (major axis), column 8: linear size (l) in Mpc, column 9: flux density at 1400 MHz in mJy (F_{1400}), column 10: flux density in TGSS at 150 MHz in mJy (F_{150}), column 11: spectral index (α_{150}^{1400}), column 12: luminosity in erg/s (L). References—WISEA: Wide-field Infrared Survey Explorer—ALL-SKY [36,37]; 2MASS: 2 Micron All Sky Survey Point Source Catalog—Final Release [35]; SDSS: Sloan Digital Sky Survey [34,39,40].

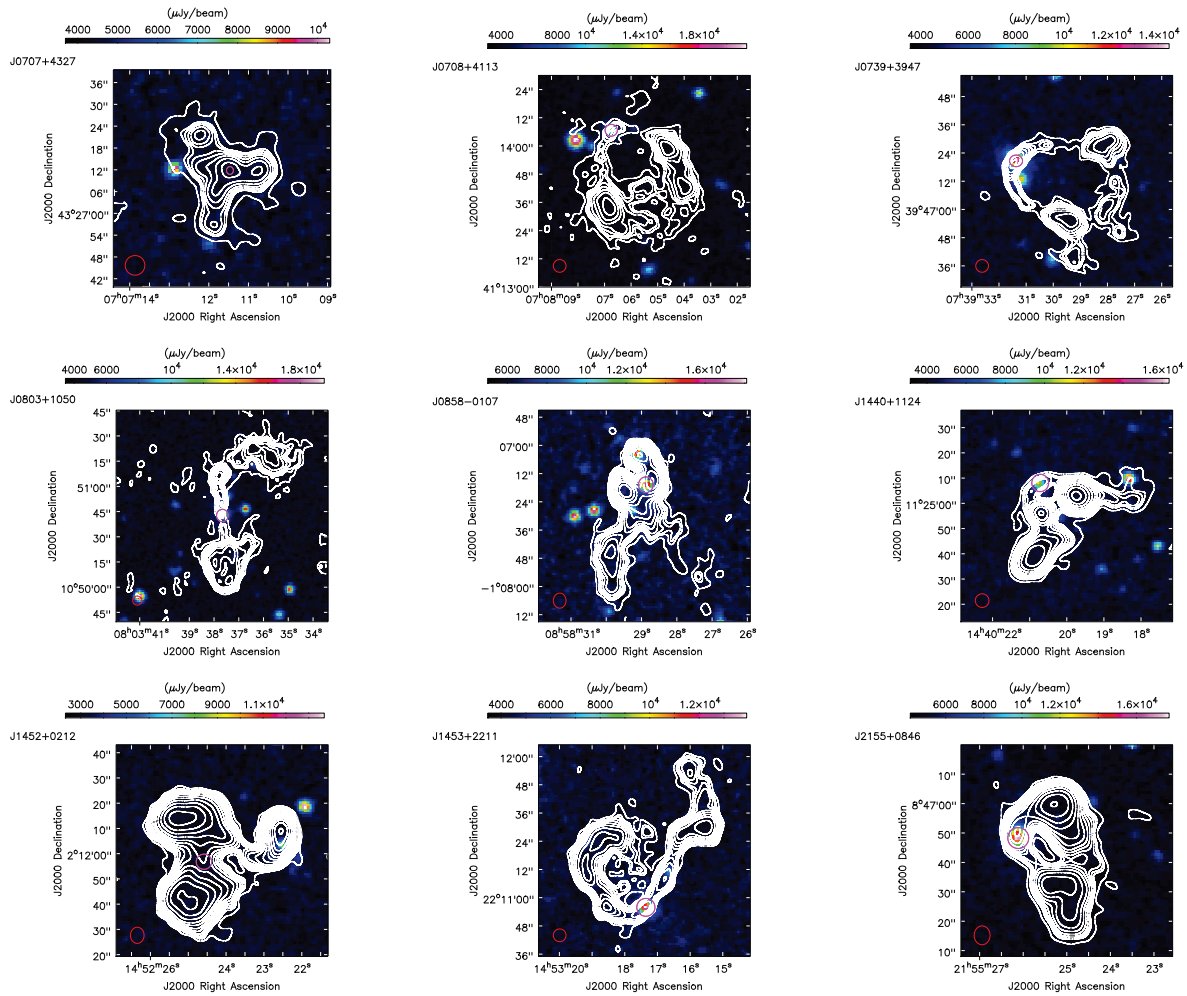


Figure 1. The FIRST image of the STROMERS candidates (contours) overlaid on the optical Digitized Sky Survey 2 Red (DSS2r) image. Kindly note that the DSS2r data were used for imaging purposes only. The identified optical or alternatively IR counterparts are marked by small pink circles for each of the sources. For the source J1452+0212, no optical or IR counterpart was found, and for this case, the pink circle indicates the location of the source.

3.1. Notes on Individual Sources

In this section, we present a few details of the individual sources and their respective morphological structure, overall appearance, and nature.

3.1.1. ingJ0707+4327

The FIRST radio map of the source at 1.4 GHz showed that the source had three lobes, which were quite symmetrically ejected from the central core. The overall radio morphology gave the radio source a triangular “spinnerlike” shape. The lobes were directed towards the north-east (NE), west (W), and south (S) directions. Concerning our viewing plane, the angles in between the lobes were measured as 110 degrees for the W–S lobes and 125 degrees for both the S–NE and NE–W lobes, respectively. At 1.4 GHz, we found that the west lobe had a slightly greater flux density of 6.9 mJy, whereas the NE and south lobes had almost similar flux densities of 5.7 and 5.8 mJy, respectively. The core region had a radio flux density of 4.0 mJy. No optical or infrared (IR) counterpart was found for the source. The radio source was identified as NVSS J070711+432710 [30] and 7C 0703+4331 [41] in the NVSS and the seventh Cambridge Catalogue of Radio Sources (7C) [41]. We cross-checked this peculiar morphology in other frequencies, but due to inadequate resolution, the source appeared to be a point source in both TGSS and WENSS.

3.1.2. J0708+4114

The radio source had a circular morphology. The angular size of the source was ~ 1 arcmin. The source is known as NVSS J070805+411342 [30] in the NVSS catalog. However, the poor resolution does not show this fascinating circular shape. This ring morphology is also absent in the TGSS and WENSS maps. The source 2MASS J07080679+4114063 [35] may be the possible IR counterpart of this source. The source had a redshift of 0.12410 [42]. The radio emission along the circumference was not smoothly distributed. Some region showed a compact and collimated emission while in some regions, it had a diffuse emission. The radio map showed a more compact emission on both sides of the possible host and a diffuse emission on the opposite side of the host.

3.1.3. J0739+3947

Similar to the previous source, this source also had a circular morphology. The angular size was also quite similar at ~ 1 arcmin. However, this source showed a more compact ring-shaped structure with respect to the earlier one. The only small diffuse emission was observed on the opposite side of the optical core. The optical host of this radio source was identified as SDSS J073931.42+394719.9 [34] with a redshift value of 0.09766 [34]. The circular morphology was unavailable in the NVSS, TGSS, and WENSS data.

3.1.4. J0803+1050

For this radio source, the lobes on the opposite side of the central host showed different kinds of morphologies with each other. The lobe in the north side traveled straight and then arced towards the west direction. This north-side lobe seemed to have a double-lobe structure. The outer lobe was diffuse whereas the inner one was more compact, which is the characteristic of a DDRG. However, the south lobe did not show a similar trend. The lobe on the south side showed a compact radio lobe emission. The source was quite peculiar as a possible recurrent activity may have happened on the north side but the same was not seen on the south side. The source had an optical host SDSS J080337.68+105042.4 [39] with a redshift of 0.142356 [34]. The galaxy cluster WHL J080337.7+105042 [43] was found to be located near the optical host with an angular separation of less than $0.1''$. The closeness implied that the galaxy cluster may be associated with the source. The approximate angular dimension of the source was $112''$, which corresponds to a linear size as 0.38 Mpc. We found no radio counterpart in WENSS for the source, whereas the TGSS data showed just two compact lobes directed in the opposite direction. No such peculiar kind of morphology was seen in that radio map. In the NVSS data, a pointlike structure was found for the source.

3.1.5. J0858–0107

This peculiar source had a “C”-type morphology [13] on the north side. On the south side, the source had a diffused tail. A weak diffused-tailed emission was also observed in the southwest direction. The source had a total of three prominent optical sources inside the radio extension. However, 2MASS J08582891-0107169 [35] is supposed to be the host of the galaxy. The source had a redshift of 0.197057 [42]. We found the galaxy cluster WHL J085829.2–010704 [43] was located at an angular distance of $0.24'$ from the proposed host. The radio source was cataloged as NVSS J085829-010720 [30] in the NVSS data. Due to the inferior resolution, the source looked like a point source both in the NVSS and TGSS data. However, no radio data were found in WENSS.

3.1.6. J1440+1125

The radio image of the source showed a “C”-shaped structure, with its primary lobes directed towards the same southwest direction. However, in addition to this, the primary lobe in the lower side (with respect to our image plane) was further enhanced towards the southeast direction. On the other hand, the upper-side lobe showed an extra diffuse emission in the west direction. The combination of these further extensions of the lobes resulted

in an overall strange morphology. The optical galaxy SDSS J144020.78+112507.1 [39] was identified as the optical counterpart of this source with a redshift value of 0.310929 [34]. The galaxy cluster WHL J144020.8+112507 [43] was located at an angular distance of $0.24''$ from the optical host. The source was unresolved in both the NVSS and TGSS data, while there was no radio counterpart found in the WENSS data.

3.1.7. J1452+0212

This peculiar radio source had an overall triangular shape, a bit similar to the first STROMERS in this list (STROMERS J0707+4327). The three lobes were directed towards the northeast (NE), southeast (SE), and west directions. However, in contradiction to the STROMERS J0707+4327, the lobes of this source were not linearly directed and equispaced in the respective directions. The radio map showed that the west lobe was further bent towards the north direction and was more compact with respect to the other two lobes. In an anticlockwise direction, the angles between the west, NE, and SE lobes were 110, 105, and 145 degrees, respectively. The measured flux densities were 61.6 mJy, 64.1 mJy, and 76.3 mJy for the SE, west, and NE lobes, respectively. There was no optical host found at the central core of the radio map. However, the optical galaxy SDSS J145222.56+021203.4 [34] with a redshift value of 0.30932 [34] was located in the west lobe and it might be the possible optical counterpart of that source. The source is also known as PMN J1452+0212 [44] and 87 GB 144952.0+022404 [45] in other respective radio survey catalogs. Neither NVSS nor TGSS showed such an integral structure in their respective radio data, whereas the WENSS had no radio data for this source.

3.1.8. J1453+2210

The jets of this radio source showed some unusual bending. When seen from the central core, the lobes were initially directed towards the northeast (NE) and northwest (NW) directions. The edge of the NE lobe was further diffused and bent towards the west direction. On the other hand, the NW lobe first bent in the west direction and then pointed to the north direction. SDSS J145317.40+221055.8 [39] was identified as the optical host of this radio source. The source had a redshift of 0.12816 [34]. Though an extended structure was found in the TGSS data, the poor resolution was unable to give a detailed morphological structure. No radio counterpart was found in the WENSS data.

3.1.9. J2155+0846

The radio lobe of the source was primarily bent towards the northwest (NW) and southwest (SW) directions. Both lobes were then further extended towards the south and gave an overall unique morphology (looking like a mirror image of the question mark “?”) and made the source a STROMERS candidate. The optical counterpart of the source was SDSS J215526.16+084648.4 [34], which had a redshift of 0.150143 [34]. The galaxy cluster RM J215526.2+084648.4 [46] was associated with this source. The TGSS data showed the source as a pointlike source, whereas no radio data were found in WENSS.

3.2. The Presence of Nearby Galaxy Clusters

A STROMERS is a very new kind of identification; hence, as a first step toward understanding this kind of source, we tried to see if there was any association with any nearby cluster for this class of sources. In this context, we tried to check for the presence of any galaxy cluster in the vicinity of such sources. The main purpose of finding the relationship between STROMERSs and associated galaxy clusters was to obtain a better understanding of their strange and odd morphologies. However, there are several other purposes for the study of the environment of the sources: (a) the determination of the physical condition of the radio jets, (b) the triggering of AGN activity, (c) the determination of the particular modes of AGN feedback, (d) testing models of source dynamics and environmental impact, and (e) finding the galaxy cluster at high redshift using the source. Here, we considered a spherical region of radius 1 Mpc from the source as the nearby

environment of the respective source. We used the search radius of 1 Mpc at the redshift of each source. We assumed that the STROMERSs and clusters of galaxies were associated if $z = |z - z_{cl}| \leq 0.05$, where z_{cl} is the redshift of the galaxy cluster. By keeping the optical host at the center, we drew a circle of radius 1 Mpc as shown in Figure 2. In Figure 2, the position of the identified nearby galaxy clusters is presented and marked as square boxes.

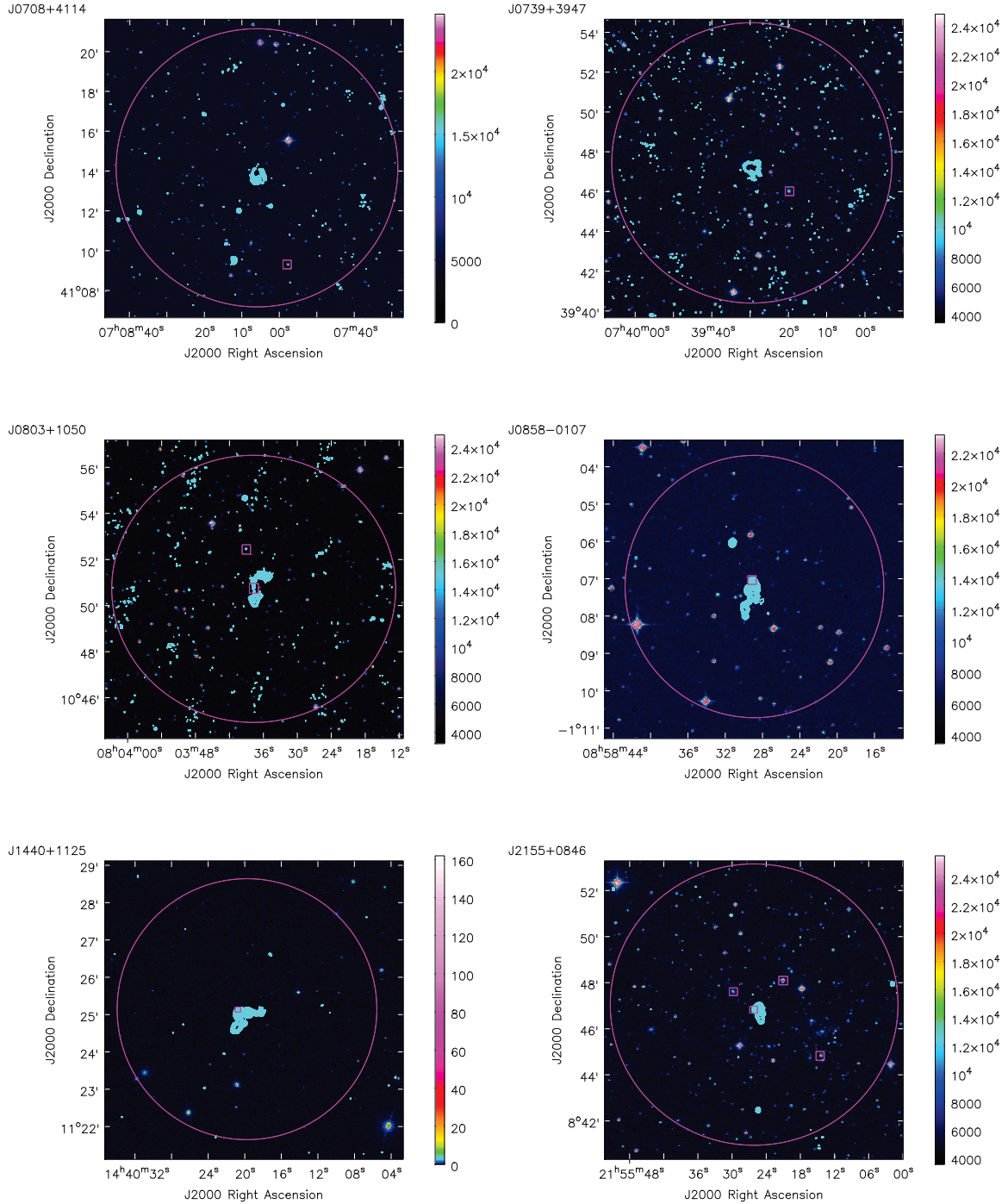


Figure 2. The images show the presence of nearby known galaxy clusters for our identified STROMERSs. The identified nearby clusters are indicated by boxes, whereas the respective circles denote the region within 1 Mpc from the source center.

From Table 2, we see that a total of six out of seven (86%) STROMERSs with known redshift were associated with at least one galaxy cluster. For each of the STROMERSs,

we found at least one galaxy cluster within the size of the STROMERS, except for the STROMERSs J0708+4114 and J0739+3947. STROMERS J2155+0846 was associated with four galaxy clusters within a 1 Mpc radius, whereas the STROMERS J0803+1050 consisted of two galaxy clusters. Thus, the final outcome of our search was that at least one cluster was associated inside the 1 Mpc radius for each STROMERS. Since our number of samples was too small to draw any conclusions from any of the aforementioned activities, we need to study a large number of such STROMERS samples in order to draw any further viable and more conclusive statement on the association of nearby galaxy clusters with STROMERSs.

Table 2. List of nearby galaxy clusters for the identified STROMERSs.

STROMERS Candidate	Nearby Galaxy Cluster	S_θ In arcmin	z	S_l In kpc	m_r	r_{500}	R_{L*500}	M_{500} $\times 10^{14} M_\odot$	N_{500}
STROMERS J0708+4114	RXGCC 258	6.35	0.104	879.19					
STROMERS J0739+3947	MSPM 06508	2.60	0.098	353.36					
STROMERS J0803+1050	WHL J080337.7+105042	0.01	0.142 *	1.55	16.98	0.70	20.43	1.11	14
	MSPM 09531	1.78	0.142 *	276.47					
STROMERS J0858–0107	WHL J085829.2–010704	0.24	0.245 *	48.65	17.07	0.85	41.99	2.41	15
STROMERS J1440+1125	WHL J144020.8+112507	0.005	0.311 *	1.42	17.58	0.79	26.14	1.45	15
STROMERS J2155+0846	RM J215526.2+084648.4	0.003	0.153	0.48					
	WHL J215529.8+084736	1.21	0.154 *	196.76	15.82	1.06	65.28	3.89	29
	NSC J215521+084647	1.27	0.114	206.52					
	ABELL 2395	2.86	0.151	465.07					

In this table, we have listed the galaxy clusters in the vicinity of the identified STROMERSs. The nearby galaxy clusters are listed in column 2. The measured angular separation (S_θ in arcmin) and linear separation (S_l in kpc) between the source and their respective clusters are presented in columns 3 and 5, respectively. The redshift (z) of the galaxy clusters is cataloged in column 4. The “*”-marked redshifts indicate that these z values are spectroscopic redshift (SPEC), otherwise, the redshifts are photometric (PHOT). The value m_r is the r -filter magnitude of the galaxy clusters. In column 7, we cataloged r_{500} , which is the cluster radius for optical luminosity. Column 8 presents the cluster richness, R_{L*500} . The cluster mass (M_{500} in the unit of $\times 10^{14} M_\odot$) and the number of all member galaxies (N_{500}) within the cluster radius (r_{500}) are listed in columns 9 and 10, respectively. References—RXGCC: ROSAT All-Sky Survey X-ray Galaxy Cluster Catalog [47]; MSPM: MultiScale Probability Mapping [48]; WHL: Wen+Han+Liu [43]; RM: RedMapper Cluster [46]; NSC: Northern Sky optical Cluster [49–51]; ABELL: Abell Clusters of Galaxies [52,53].

4. Summary and Discussion

In this paper, we presented nine newly identified radio galaxies with a strange and rare kind of radio morphology, which did not match with other known classes and subclasses of radio galaxies; we named them as STROMERSs. Considering our sample size of 95,243, the proportion of such sources is nearly 0.01%. However, this is not an appropriate estimation, and additional similar identifications would allow us to give an accurate proportion value of these kinds of sources concerning all radio sources. With reference to our previous identification of such interesting and typical sources from LoTSS DR1 in Sasmal et al. [19], we can consider this as an extended list of the same but from the VLA FIRST at 1.4 GHz. Besides the fascinating morphology, the study of these sources may be helpful in understanding the surrounding medium and the galaxy’s evolution itself.

Here, we checked the basic properties of these sources like the available redshift (z), angular size, linear size (for the available redshift sources), flux densities, spectral index, and luminosity. We noticed that the identified STROMERSs had a moderately low redshift value. The redshift values lay in the range between 0.098 to 0.311. This range is well under the value of our previous identification in Sasmal et al. [19]. In our previous identification, we found the highest redshift for the source J1428+4556. However, we should remember that the probability of finding higher redshifted sources in VLA FIRST was lower than the same for the LoTSS DR1, and that might be a reason for the relatively low redshift values. The angular sizes of the sources were found in the span of 35 arcseconds to 112 arcseconds. Now, if we compare the angular sizes with the four sources from Sasmal et al. [19], we can see that the sizes of the sources from Sasmal et al. [19] are way

larger than the sources identified in this paper. In this paper, the source with the largest size (STROMERS J0803+1050 with a size of ~ 2 arcmin) was nearly 2.5 times smaller than the smallest source in the Sasmal et al. [19] list (J1323+5059 with a size of ~ 5 arcmin). However, this is expected as FIRST is designed to observe more fine structures and sources, and hence, it has a tendency to miss the large-scale structures and sources. The calculated linear size showed that STROMERS J1440+1125 was the largest source with a size of ~ 550 kpc, while STROMERS J0739+3947 was the smallest with a projected size of ~ 150 kpc. Due to the unavailability of redshifts, we could not estimate the linear size for the two sources (STROMERS J0707+4327 and STROMERS J1452+0212). The previously identified sources from Sasmal et al. [19] had quite large sizes which led to a possibility of misidentification. The misidentification could be due to the overlaps of more than one source in our line of sight [19]. However, in this list, the probability of such misidentification was much less. The calculated two-point spectral index (α_{150}^{1400}) had a range of -0.60 to -0.92 . This spectral index value is typical for radio galaxies [54–58]. However, the steep spectrum ($\alpha_{150}^{1400} \leq -0.50$) for all the sources indicated that these were lobe-dominated sources. This may indicate that there is a possibility of the sources being relic candidates [59]. However, the spectral index was less steep than that of the previously identified sources. The radio luminosity ($L_{1400\text{MHz}}$) of the sources was found to be on the order of $\times 10^{41}$ erg sec $^{-1}$, except for the sources STROMERS J0739+3947 and STROMERS J1440+1125. The STROMERS J0739+3947 has the lowest luminosity of 8.6×10^{40} erg sec $^{-1}$ and the STROMERS J1440+1125 has the highest luminosity of 2.76×10^{42} erg sec $^{-1}$. The luminosity value was quite similar to previous sources in Sasmal et al. [19] and also similar to the average from Nilsson et al. [60]. Here, we also cross-identified the sources with other existing radio catalogs. The other catalogs' identification was important to identify the radio sources' character. We identified the radio sources as radio galaxies, and hence we could also treat them as such. However, in other frequencies, the sources did not show any such strange morphologies. We also checked the environment by identifying the presence of known galaxy clusters, and we found that there was at least one galaxy cluster inside 1 Mpc of the source for most cases. We also checked the same fact for other classes of radio sources from FIRST on a random basis and found that the presence of a nearby cluster was less frequent than for the STROMERSs. For the typical double-lobed radio galaxies and winged radio galaxies, the cluster association was less or equal to 10%, whereas for the BT sources and the HYMORSs, the values were 70% and 60%. The high value of the cluster association for BT sources was expected for the generation of ram pressure; however, the quantification of that association is still unknown for the HYMORSs and our STROMERSs. Thus, we can assert that there might be a possibility of some kind of association between the STROMERSs and their respective nearby galaxy clusters. However, the small number of samples restricted us from concluding and establishing any of the above activities.

In the absence of more detailed data and study, if we try to scrutinize only the radio morphology, then we may consider different possibilities for different sources, regarding the mechanism behind such morphology. If we consider each of the sources individually, the three-lobe structure of the STROMERS1 (J0707+4327) indicates a possible change in one of the jet directions. Now, the change in the jet direction may occur due to the reorientation caused by a spin-flip or the precision of jets or may be due to a merger. A spectral age analysis may give information about the aging of the individual lobes. The overall morphology of the seventh source (J1452+0212) in our list has a similarity with STROMERS1; however, the lobes and their extensions of the source are not as symmetrical as for STROMERS1. The further bend of the west lobe indicates that there might be some other attribution or other factor/s besides the jet reorientation or precision. The compactness of the west lobe may also be due to the fact that this source is relatively young (or active), and the other ones are relic ones. This may be confirmed by a spectral analysis. The second and third STROMERSs (J0708+4114 and J0739+3947) both have an overall similar circular morphology, although the detailed nature of the two is different. The sources have a similar one arcmin diameter as the ORC in [14]. However, these might

not be the ORC candidates, as the sources have a ring-shaped emission instead of the circular diffused emission as in the case of ORCs. A possible explanation may be that the source is an extreme case of a bent-tail source. Initially, the source may have been bent due to “ram pressure” [61,62]; however, due to some external, environmental effect, the diffused tails were further bent to give such a circular shape. These two sources may also be a possible case of imaging artefact, a supernova remnant (SNR), ring galaxy, Einstein ring, etc. However, the possibility of an SNR is lower, as there are some radio hotspots in the ring of the sources, which is not conventional for an SNR. The fourth source J0803+1050 may be a special case of a DDRG as it has a pair of lobes on one side. However, there is a single lobe on the other side. Moreover, the diffuse lobe on the pair side has a bent structure, while the single-lobe side shows an arced structure. A recurrent activity is a possible case for the pair lobe side; however, the same is not true for the single arced lobe side. The “f”-like structure of STROMERS J0858–0107 has a mix of both, a bent feature and an extended emission. The three sources (STROMERS J1440+1124, STROMERS J1453+2211, and STROMERS J2155+0846) have an arced structure at the host region, but the orientation and extension of the lobe towards the edges are different, and these features make each of the sources a STROMERS candidate. The arced portion may be caused due to some “ram pressure” or bouncy effect, but we have to consider some alternative mechanism or environmental effect for the edges.

If we look individually at all of the nine STROMERSs, we see that the STROMERSs exhibit a diverse range of morphologies. We think this diversity is likely driven by a combination of factors, such as the properties of the central black hole, the dynamics of the accretion flow, and the environment in which the radio galaxy resides. Moreover, the combination might be different for different cases. However, there is also a possibility of imaging artefacts. A more extensive study of these sources is needed to determine the particular origin. For now, we conclude that the most likely explanation for the STROMERSs is that there is no particular scenario, mechanism, or event which is responsible for these peculiar kinds of morphologies. We think there is a possibility that more than one reason or event is responsible for this extremely rare kind of morphology. We also believe that the study of such source morphologies offers a unique window into the complex interplay between supermassive black hole activity and galaxy evolution, making them an important area of ongoing research in extragalactic astrophysics. Hence, we plan to extend our search for these STROMERSs with other available high-resolution data surveys and obtain an adequate number of such sources. We also plan to conduct observations on the individual sources and conduct some multi-wavelength studies. A larger number of such sources and further detailed studies of the sources will be helpful in understanding this strange and odd kind of morphology.

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Notes

- ¹ <https://astro.uchicago.edu/gnedin/cc/> (accessed on 1 June 2024).
- ² <https://casa.nrao.edu/> (accessed on 1 June 2024).
- ³ <http://www.aips.nrao.edu/index.shtml> (accessed on 1 June 2024).
- ⁴ <https://ned.ipac.caltech.edu/> (accessed on 1 June 2024).

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Article

The Composite Spectral Energy Distribution of Quasars Is Surprisingly Universal Since Cosmic Noon

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Abstract: Leveraging the photometric data of the Sloan Digital Sky Survey and the Galaxy Evolution Explorer (GALEX), we construct mean/median spectral energy distributions (SEDs) for unique bright quasars in redshift bins of 0.2 and up to $z \simeq 3$, after taking the GALEX non-detection into account. Further correcting for the absorption of the intergalactic medium, these mean/median quasar SEDs constitute a surprisingly redshift-independent mean/median composite SED from the rest-frame optical down to $\simeq 500 \text{ \AA}$ for quasars with bolometric luminosity brighter than $10^{45.5} \text{ ergs}^{-1}$. Moreover, the mean/median composite quasar SED is plausibly also independent of black hole mass and Eddington ratio, and suggests similar properties of dust and gas in the quasar host galaxies since cosmic noon. Both the mean and median composite SEDs are nicely consistent with previous mean composite quasar spectra at wavelengths beyond $\simeq 1000 \text{ \AA}$, but at shorter wavelengths, are redder, indicating, on average, less ionizing radiation than previously expected. Through comparing the model-predicted to the observed composite quasar SEDs, we favor a simply truncated disk model, rather than a standard thin disk model, for the quasar central engine, though we request more sophisticated disk models. Future deep ultraviolet facilities, such as the China Space Station Telescope and the Ultraviolet Explorer, would prompt revolutions in many aspects, including the quasar central engine, production of the broad emission lines in quasars, and cosmic reionization.

Keywords: galaxies: quasars; quasars: general; ultraviolet: galaxies

1. Introduction

A quasar, or luminous active galactic nucleus (AGN), is a supermassive black hole (BH) actively accreting in a tiny region of a galactic center. The enormous and long-lasting luminosity radiated by a quasar makes it outshine its host galaxy and become the brightest persistent object glowing in the universe. No sooner had quasars been discovered about 60 years ago [1,2] than the basic central engine for their powerful emissions was proposed [3–5]. The quasar central engine consists of a supermassive BH surrounded by an accretion flow/disk.

While the exact details of the accretion flow are still being studied, the major part of the accretion disk is likely cold and optically thick [6,7], producing the thermal continuum emission from the rest-frame optical to ultraviolet (UV) wavelengths [8–10], whereas the innermost region of the accretion flow could be an optically thin hot corona [11–13], accounting for the non-thermal X-ray emission of quasars [14–16]. Meanwhile, a warm corona between the cold disk and the hot corona (or atop the cold disk) could be responsible for the soft X-ray excess ([17–21]; however, see [22,23] for the absorption and reflection explanations, respectively). Up to now, investigating the multi-wavelength spectral energy distribution (SED) of quasars has been one of the important avenues for uncovering the disk-corona coupling mystery of their central engine [24].

In terms of the available spectroscopic or photometric data, there are many composite spectra [25–37] or broadband SEDs [38–40] for quasars, respectively. The average radiative properties of quasars have been extensively explored in a broad wavelength range spanning

from the near-infrared to X-ray, except the extreme UV (EUV; the rest-frame $\sim 1000 \text{ \AA}$ – $100 \text{ \AA}$). This is because EUV observation is very difficult owing to the significant absorption of the neutral hydrogen in our Milky Way as well as in the intergalactic medium (IGM) along the line of sight (LOS) to quasars. To circumvent the problem of the hydrogen absorption, one needs the rest-frame EUV-involved spectra of individual quasars (only ~ 500 from local to $z \sim 2.5$ according to [31,32,35,36]; see also Figure 1) in order to carefully correct for the IGM absorption before building composite spectra. It has long been suggested that the EUV-involved composite spectra of quasars with different UV luminosity are rather similar in shape at wavelengths beyond $\simeq 1000 \text{ \AA}$ (optical-to-FUV), but exhibit a luminosity dependence in the EUV: the EUV shape is on average redder/softer for more luminous quasars. Later broadband analyses based on the photometric data of the Sloan Digital Sky Survey (SDSS) and the Galaxy Evolution Explorer (GALEX) confirmed the luminosity dependence in the EUV [39,40].

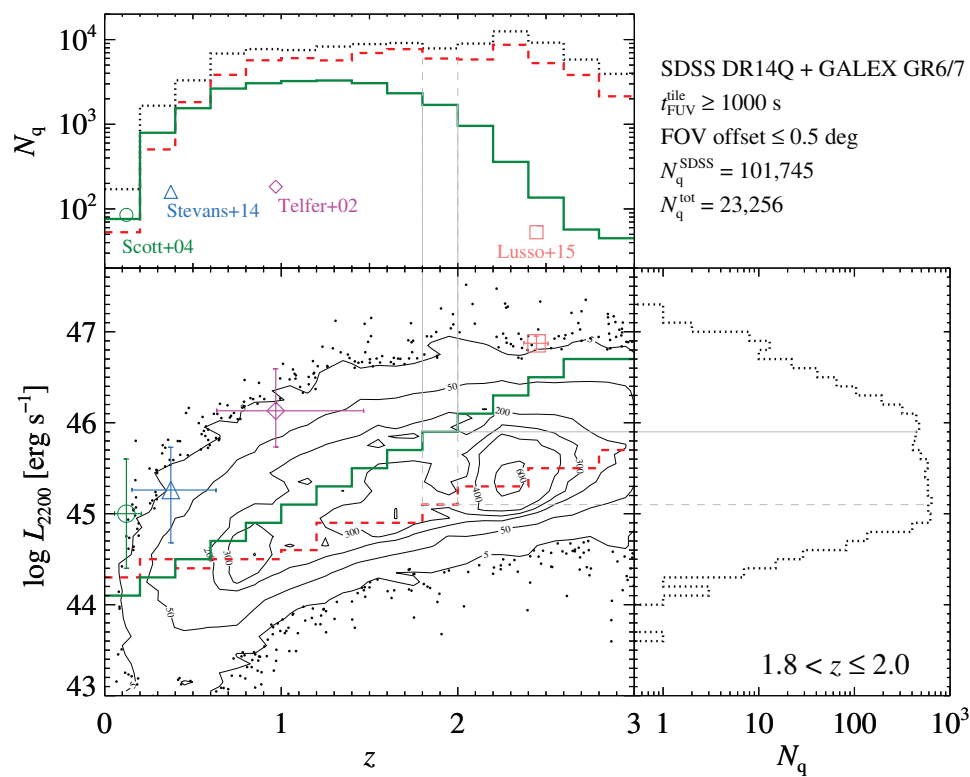


Figure 1. The **(main)** panel presents distributions of our parent quasars (101,745; contours surrounded by sparse dots) and unique bright quasars (23,256; brighter than the green solid stepwise curve) in the luminosity-redshift space, while the **(top)** panel shows their redshift distributions, i.e., the black dotted histogram and the green solid histogram, respectively. The red dashed stepwise curve in the **(main)** panel indicates the peak of the luminosity distribution of quasars as a function of redshift, e.g., the peak of the dotted histogram for quasars in $1.8 < z \leq 2.0$ in the **(right)** panel. The red dashed histogram in the **(top)** panel is the redshift distribution of quasars brighter than the red dashed stepwise curve. For comparison, the **(main)** panel includes the medians and 25–75th percentile ranges of redshift and UV luminosity of four representative quasar samples (i.e., a circle for Scott et al. [32], a triangle for Stevans et al. [35], a diamond for Telfer et al. [31], and a square for Lusso et al. [36]), while the **(top)** panel contains the corresponding quasar numbers.

However, both previous spectral and photometric analyses of the quasar EUV SEDs only consider quasars detected in the EUV and thus are subject to a prominent selection bias: the more luminous quasars with redder EUV shapes are more likely detected. Consequently, the reported luminosity dependence of the quasar EUV SEDs could be naturally expected

given such a bias. Actually, Vanden Berk et al. [41] have demonstrated that the EUV color of quasars is substantially redder than expected once taking this bias into account.

Recently, after controlling for this bias, Cai and Wang [42] unveiled that the average quasar SEDs down to the rest-frame $\simeq 500$ Å are nearly luminosity-independent for $z \sim 2$ quasars with the rest-frame 2200 Å monochromatic luminosity brighter than $10^{45} \text{ ergs}^{-1}$, i.e., $\log L_{2200} \geq 45$, where $L_{2200} \equiv \nu L_\nu(\lambda_{\text{rest}} = 2200 \text{ Å})$. Furthermore, after considering the EUV non-detection and correcting for the IGM absorption for a unique bright sub-sample of $z \sim 2$ quasars, i.e., with $\log L_{2200} \geq 46$, the unveiled intrinsic average quasar SED is redder in the EUV than previous quasar composite spectra. The new intrinsic average quasar SED does not match predictions of the standard thin disk model in which the inner disk boundary extends to the innermost stable circular orbit. Instead, it prefers a simply truncated disk model, such as the windy disk proposed by Laor and Davis [43], where a microscopic atomic origin is responsible for the disk truncation such that the resultant SEDs are weakly dependent on the quasar physical properties, e.g., the BH mass, M_{BH} , and the Eddington ratio, λ_{Edd} .

Here, we extend our previous analysis of $z \sim 2$ quasars to quasars at $z \leq 3$ or since cosmic noon. Utilizing the photometric data of the 14th release of the SDSS quasar catalog (DR14Q; [44]) and the associated GALEX legacy data [45], Section 2 presents our selection criteria for the unique bright quasars in different redshift bins and the resultant exciting luminosity- and redshift-independent composite quasar SED after considering the GALEX non-detection and correcting for the IGM absorption. Discussion and implications on the universal composite quasar SED are given in Section 3, and Section 4 briefly summarizes our conclusions. Throughout, we assume a flat cosmology with Hubble constant $H_0 = 70 \text{ kms}^{-1} \text{ Mpc}^{-1}$ and dark energy density $\Omega_\Lambda = 0.7$.

2. Quasar Sample and the Universal Mean/Median Composite Quasar SED

2.1. SDSS Quasars

The SDSS DR14Q catalog contains 526,356 quasars [44]. Although $\simeq 92\%$ of the DR14Q quasars are covered by the GALEX survey, the inhomogeneity of the GALEX survey, with very diverse exposure times ranging from $\simeq 30$ s to $\sim 10^5$ s, makes the construction of a meaningful quasar SED very complex. The analysis of quasars covered by the GALEX tiles with long exposure times (e.g., $\geq 10^3$ s; [42]) should, to some extent, alleviate the problem.

In total, there are 45,195 GALEX tiles¹. Dropping 311 tiles dedicated to the spectroscopic surveys, we consider 44,884 tiles for the imaging surveys. Nearly all (99.8%) imaging tiles have positive exposure times in the GALEX near-UV (NUV) band, i.e., $t_{\text{NUV}}^{\text{tile}} > 1$ s, while 76.7% have $t_{\text{FUV}}^{\text{tile}} > 1$ s in the GALEX far-UV (FUV) band. We require each GALEX tile to have positive exposure times in both GALEX bands since we need the exposure times to determine flux upper limits for GALEX undetected quasars. Then, we focus on 4688 imaging tiles with $t_{\text{FUV}}^{\text{tile}} \geq 10^3$ s, a trade-off between high GALEX detection and large sample size [42].

Further requiring that the positions of the SDSS quasars are within the central 0.5° radius of the field of view (FOV) of any one GALEX tile (that is, FOV offset $\leq 0.5^\circ$), we find that 107,504 SDSS quasars with positive flux densities and errors in all five SDSS bands are covered by 3453 GALEX tiles with $t_{\text{FUV}}^{\text{tile}} \geq 10^3$ s in the northern sky. As illustrated by the contours surrounded by sparse dots in the main panel of Figure 1, we consider the 101,745 SDSS quasars at $z \leq 3$ (N_q^{SDSS} ; hereafter the parent quasar sample), beyond which both the GALEX detection is very low and the IGM absorption is rather significant (see below).

Separating the parent quasar sample in redshift bins of 0.2 from $z = 0$ to $z = 3$, the dotted histogram in the top panel of Figure 1 shows the quasar number as a function of redshift, while the dotted histogram in the right panel of Figure 1 shows the quasar number as a function of $\log L_{2200}$ for quasars in $1.8 < z \leq 2.0$ as an example. For each redshift bin, there is a specific value of $\log L_{2200}$ corresponding to the peak of the luminosity distribution of quasars (the right panel of Figure 1). These specific values of $\log L_{2200}$ for different

redshifts constitute a red dashed stepwise curve in the main panel of Figure 1. Beneath the red dashed stepwise curve, the parent quasar sample is likely subject to significant sample incompleteness. Our unique bright quasar sample is selected well above the red dashed stepwise curve at $z > 0.5$, thus may not heavily suffer from the sample incompleteness.

2.2. GALEX Detections

Searching the GALEX counterparts in the GALEX_GR6Plus7 context² for our parent quasars, we find 126,665 GALEX counterparts with positive exposure times in both GALEX bands within an SDSS/GALEX offset of ≤ 2.6 arcsec [39]. Of the 101,745 parent quasars, $\simeq 22.6\%$ have a unique counterpart, and $\simeq 43.4\%$ have more than one counterpart, while the remaining $\simeq 34.0\%$ do not have a counterpart. Following Cai and Wang [42], a unique counterpart is selected for each quasar with multiple GALEX counterparts, while a 3σ GALEX NUV/FUV detection limit is assigned to each quasar either without a counterpart or with a unique counterpart but solely detected in the GALEX NUV (or FUV) band.

In the right panel of Figure 2, the GALEX NUV- and FUV-detected fractions (i.e., f_{NUV} and f_{FUV}) of our parent quasars in $1.8 < z \leq 2.0$ and brighter than a given minimal $2200 \text{ \AA}$ luminosity, $\log L_{2200}^{\text{min}}$, are shown as the orange dotted and blue solid histograms, respectively. It is clear that including fainter quasars gives rise to a larger sample size, but the GALEX detection fractions become lower. Therefore, for each redshift bin, it is critical to choose a specific luminosity limit such that both the quasar sample size and the GALEX detection fractions are not too small.

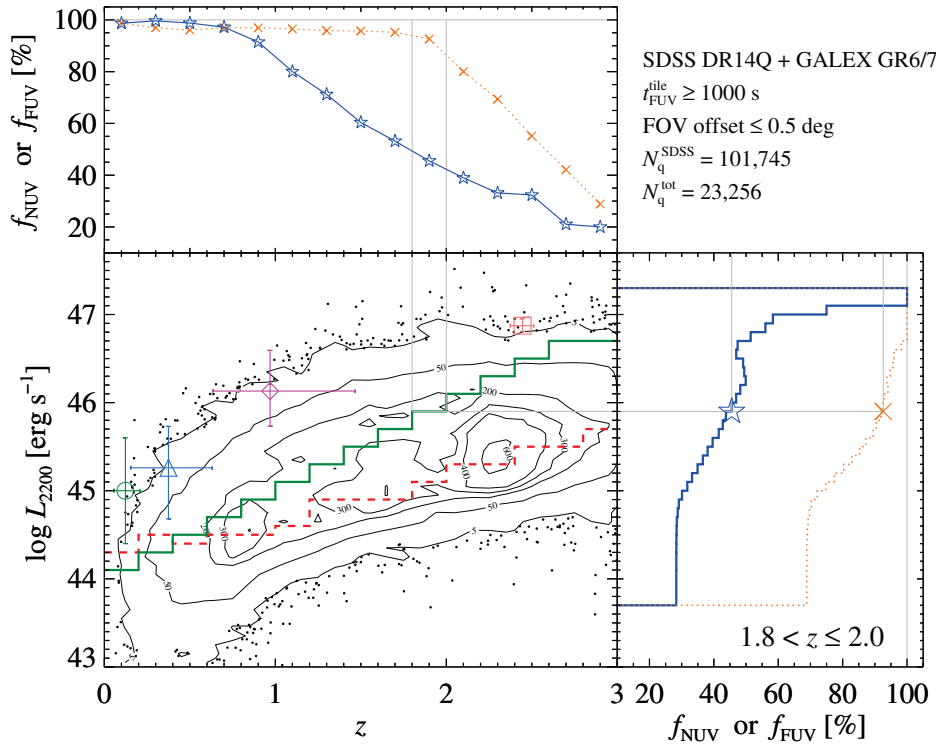


Figure 2. The (main) panel is the same as that of Figure 1. The orange crosses linked by dotted lines (the blue stars linked by solid lines) in the (top) panel are the GALEX NUV (FUV) detection fractions, f_{NUV} (f_{FUV}), for our unique bright quasars, while the orange dotted (blue solid) histogram in the (right) panel is the GALEX NUV (FUV) detection fraction as a function of luminosity for quasars brighter than a given minimal $2200 \text{ \AA}$ luminosity and in $1.8 < z \leq 2.0$ as an example.

Guided by the fact that quasars are typically brighter at higher redshift (Figure 2) and a luminosity limit of $\log L_{2200}^{\text{min}} = 46$ has been used to construct a unique bright quasar sample at $z \sim 2$ by Cai and Wang [42], we adopt redshift-dependent luminosity limits (the green solid stepwise curve in the main panel of Figures 1 and 2), which increase from

$\log L_{2200}^{\min} = 44.1$ for $0 < z \leq 0.2$ to $\log L_{2200}^{\min} = 46.7$ for $2.6 < z \leq 2.8$ in steps of 0.2 dex, except for the highest redshift bin where $\log L_{2200}^{\min} = 46.7$ is adopted to avoid including too few quasars. The top panel of Figure 1 shows the number of quasars brighter than $\log L_{2200}^{\min}$ as a function of redshift (green solid histogram), while the top panel of Figure 2 illustrates the corresponding f_{NUV} (blue open stars) and f_{FUV} (orange crosses) as a function of redshift. In the following, our mean/median composite quasar SED would be constructed using 23,256 quasars brighter than $\log L_{2200}^{\min}$ (hereafter, the unique bright quasar sample). We note that utilizing other quasar samples selected with larger or smaller $\log L_{2200}^{\min}$, e.g., by ± 0.2 dex in all redshift bins, than our reference values of $\log L_{2200}^{\min}(z)$, confirms our conclusion on the universality of the quasar SED. However, the EUV portion of the resultant SED is more uncertain owing to either smaller sample size for larger $\log L_{2200}^{\min}$ or lower GALEX detection fraction for smaller $\log L_{2200}^{\min}$ (Figure 3 and Section 3.1).

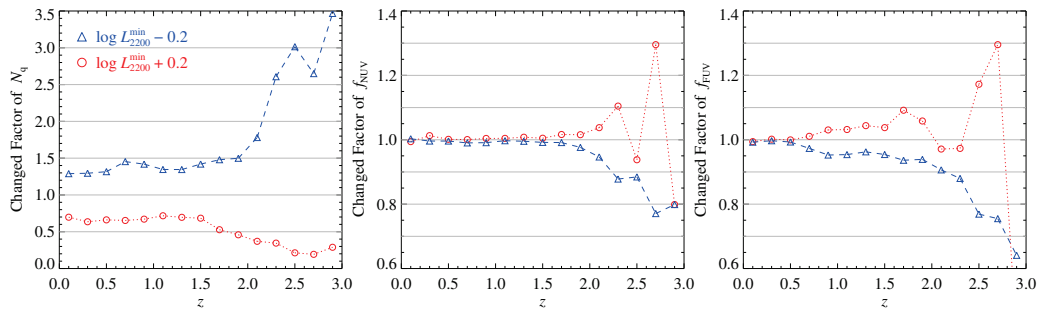


Figure 3. Changed factors of N_q , f_{NUV} , and f_{FUV} as a function of redshift for quasar samples selected with larger (circles linked by dotted lines) or smaller (triangles linked by dashed lines) $\log L_{2200}^{\min}$, i.e., by ± 0.2 dex in all redshift bins, than our reference values of $\log L_{2200}^{\min}(z)$. Please note that the smaller $\log L_{2200}^{\min}$, the larger the sample size but the lower the GALEX detection.

2.3. Bias-Free Mean/Median Quasar SEDs in Different Redshift Bins

After considering the GALEX non-detections, we construct the bias-free average (i.e., both mean and median) SEDs for quasars in different redshift bins and brighter than the corresponding $\log L_{2200}^{\min}$. A brief introduction to the method is given here, while readers are referred to Cai and Wang [42] for all details.

For each quasar in a redshift bin, its rest-frame SED consists of de-redshifted SDSS (*ugriz*) and GALEX (FUV and NUV) photometry, where the GALEX non-detections are replaced by 3σ upper detection limits. Each quasar SED is normalized at the rest-frame 2200 Å before taking the mean/median value of all normalized SEDs. The mean/median quasar SED is easily reachable in the de-redshifted SDSS bands, and the 1σ statistical errors are estimated as the standard deviation of all normalized SEDs divided by the square root of the quasar number in each redshift bin [39]. Instead, since there are non-detections in the de-redshifted GALEX bands, an exponentially modified Gaussian function [41] is fit to the distribution of $\log(L_w/L_{2200})$, where L_w is the rest-frame w Å monochromatic luminosity. Here, the subscript w indicates the rest-frame wavelength corresponding to the effective wavelength of the GALEX NUV/FUV band divided by $1 + z$, where z is the median redshift of quasars in each redshift bin. Then, the mean/median quasar SED at w Å is inferred from the best-fit exponentially modified Gaussian function, while the 1σ uncertainty is estimated by bootstrapping 1000 times the distribution of $\log(L_w/L_{2200})$, including both GALEX detections and non-detections.

Figure 4 shows the resultant mean (top panel) and median (bottom panel) SEDs for quasars in redshift bins with central redshifts increasing from $z = 0.1$ to $z = 2.9$ in steps of 0.2. Both the mean and median SEDs at the optical-to-FUV wavelengths beyond $\simeq 1000$ Å are in good agreement with previous mean composite spectra, such as Vanden Berk et al. [29] and Telfer et al. [31]. The consistency between the mean and median optical-to-FUV SEDs indicates that distributions of the optical-to-FUV luminosities are symmetric. Instead, both the mean and median quasar EUV SEDs seemingly become redder

with increasing redshift. Since the IGM absorption is also more significant with increasing redshift, these bias-free SEDs of quasars at different redshifts, without corrections for the IGM absorption, cannot be compared to each other and to the EUV portion of the mean composite spectrum of Telfer et al. [31], to which corrections for the IGM absorption has been applied.

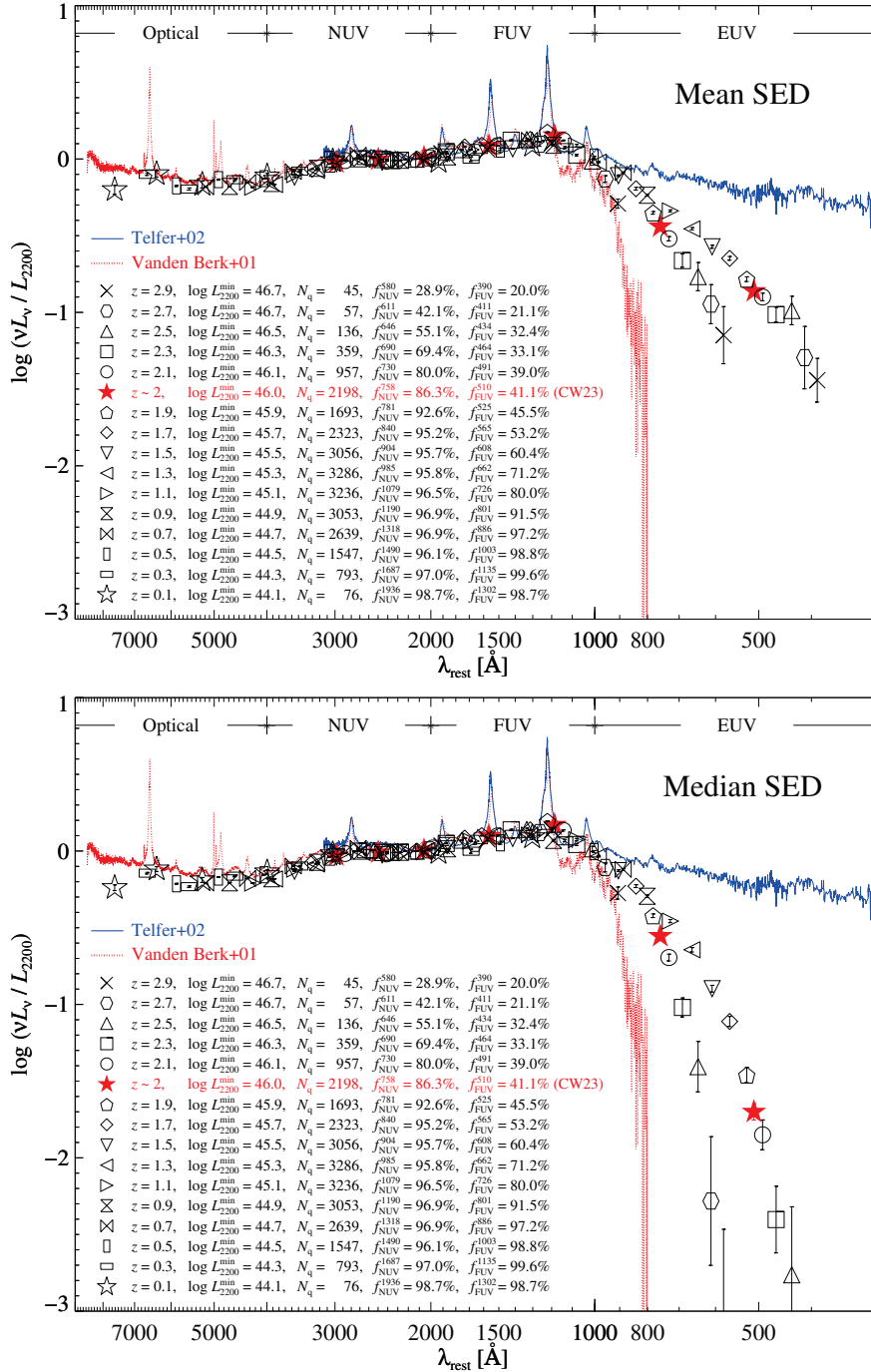


Figure 4. The (top) panel: the bias-free mean SED for N_q quasars brighter than $\log L_{2200}^{\min}$ in each redshift bin. Each legend contains the GALEX NUV- and FUV-detected fractions, i.e., f_{NUV}^w and f_{FUV}^w , where the superscript, w , is the de-redshifted wavelength corresponding to the effective wavelength of the GALEX NUV/FUV band. We also show the mean quasar SED constructed by ([42], CW23) as well as the mean composite quasar spectra from ([29], without correction for the IGM absorption; red dotted curve) and ([31], with correction for the IGM absorption; blue solid curve) for comparison. The (bottom) panel: same as the (top) one, but for the bias-free median quasar SEDs.

2.4. Monte Carlo Simulation for the IGM Absorption and Correction

Details on the Monte Carlo simulation for the IGM transmission curves are documented in Cai and Wang [42]. Adopting the Faucher-Giguère [46] distribution of absorbers with the HI column density of $12 \leq \log N_{\text{HI}} \leq 22$ and the Doppler broadening parameter of $b = 30 \text{ km s}^{-1}$, Figure 5 illustrates the average IGM transmission curves for 1000 quasars at four different redshifts. Globally, the IGM absorption is more significant with increasing redshift. At all redshifts, the mean IGM transmission curve is very different from the median IGM transmission curve. At low redshifts, e.g., $z = 0.5$, although less than 25% quasars are heavily absorbed owing to the rare encounter with Lyman limit systems along the LOS to quasars, the mean IGM transmission curve is somewhat lower than the median IGM transmission curve. Instead, at high redshifts, e.g., $z = 2.9$, there are still $\sim 25\%$ quasars that are not heavily absorbed such that the mean IGM transmission at $<700 \text{ \AA}$ remains about ~ 0.2 when the median IGM transmission is almost zero.

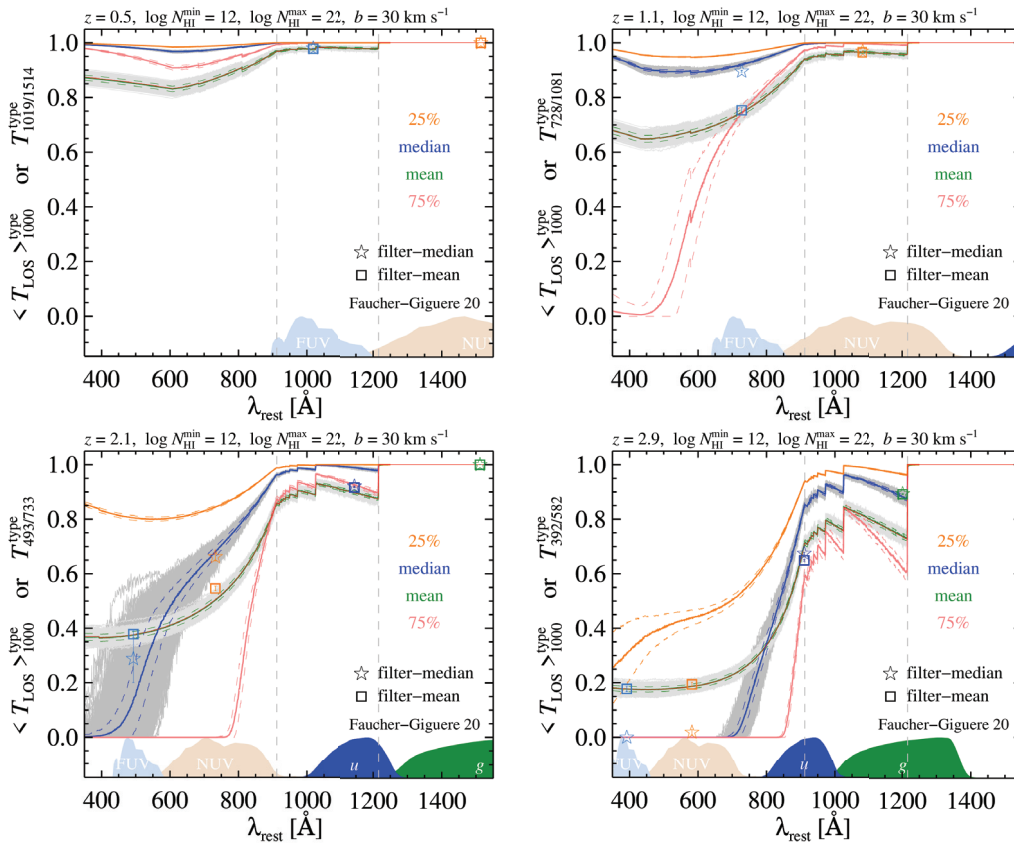


Figure 5. The IGM transmission curves as a function of the rest-frame wavelength for $z = 0.5$ (top-left panel), $z = 1.1$ (top-right panel), $z = 2.1$ (bottom-left panel), and $z = 2.9$ (bottom-right panel). Averaging transmissions in 1000 LOS (equivalent to 1000 quasars) randomly selected from 10^5 simulated LOS gives rise to the mean transmission and three upper transmission quantiles (i.e., 25%, median, and 75%), $\langle T_{\text{LOS}} \rangle_{1000}^{\text{type}}$. For each “type” of transmission, the solid and dashed curves are the mean and 1σ dispersion of 1000 different realizations of $\langle T_{\text{LOS}} \rangle_{1000}^{\text{type}}$ (cf. the gray and light-gray curves for 1000 realizations of $\langle T_{\text{LOS}} \rangle_{1000}^{\text{median}}$ and $\langle T_{\text{LOS}} \rangle_{1000}^{\text{mean}}$, respectively). At the bottom of each panel, there are de-redshifted FUV-, NUV-, u -, and g -band transmission curves from left to right, respectively. In each panel and each band, the square and star, superimposed by 1σ uncertainties, indicate the filter-weighted broadband mean and median IGM transmissions, respectively.

In practice, the filter-weighted broadband mean and median IGM transmissions are used to correct the IGM-absorbed EUV portion of the bias-free mean and median SEDs, respectively. If there are N_q quasars in a redshift bin with a central redshift of z , the mean/median broadband IGM transmission is estimated assuming all N_q quasars are at z , regardless of the slight differences in redshift among quasars. We obtain a value for the mean/median IGM transmission by averaging N_q LOS randomly selected from 10^5 LOS, repeat the random selection 1000 times, and take the standard deviation of the 1000 values as the 1σ uncertainty for the mean/median broadband IGM transmission given N_q quasars. Figure 6 illustrates the mean/median broadband IGM transmissions as well as the 1σ uncertainties for 1000 quasars at each redshift as an example. Finally, the applied 1σ uncertainty of the mean/median broadband IGM correction is estimated according to the real quasar number in each redshift bin, and is propagated to the EUV portion of the intrinsic mean/median quasar SED.

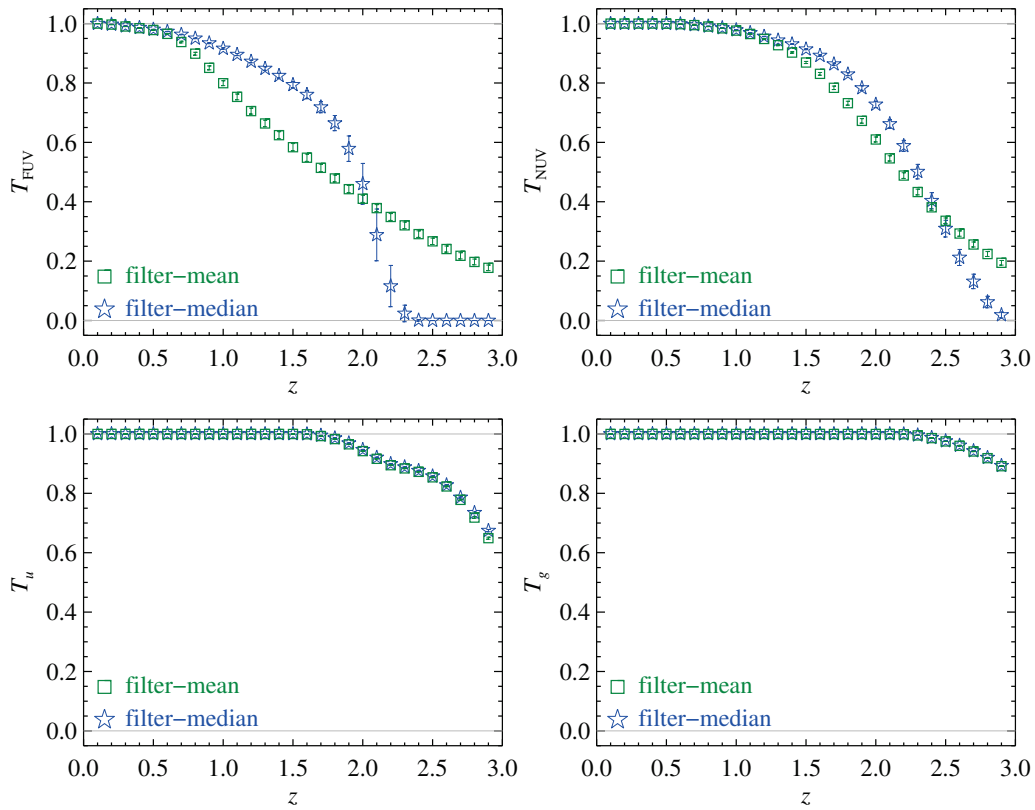


Figure 6. By averaging 1000 LOS, squares and stars show the filter-weighted broadband mean and median IGM transmission as a function of redshift for four bands, i.e., T_{FUV} (top-left), T_{NUV} (top-right), T_u (bottom-left), and T_g (bottom-right). The superimposed 1σ uncertainties are associated with 1000 LOS. Please note that averaging more (less) LOS gives smaller (larger) 1σ uncertainties.

2.5. An Intrinsic Mean/Median Composite SED for Quasars Since Cosmic Noon

After applying corrections for the IGM absorption, Figure 7 illustrates the intrinsic mean (top panel) and median (bottom panel) SEDs for quasars at different redshifts. In contrast to the bias-free quasar SEDs in the EUV (Figure 4), the EUV portions of these intrinsic quasar SEDs are strikingly consistent with each other. The EUV portions of the intrinsic quasar SEDs are contributed by quasars at $z \gtrsim 0.5$ up to $z \sim 2.9$ and with typical 2200 Å luminosities increasing from $\log L_{2200}^{\text{med}} = 44.8$ to $\log L_{2200}^{\text{med}} = 46.8$. Consequently, these intrinsic mean/median quasar SEDs strongly suggest an intrinsic mean/median composite SED for quasars since cosmic noon.

The intrinsic median quasar SED at the rest-frame $\lambda_{\text{rest}} < 500 \text{ \AA}$ is very uncertain because of both the low GALEX detection fractions, smaller than 35% (Figures 2 and 4), and the significantly uncertain corrections for the broadband median IGM transmissions (top panels of Figure 6). However, the intrinsic mean quasar SED at $\lambda_{\text{rest}} < 500 \text{ \AA}$ is plausible since (1) the mean value is not sensitive to those small values corresponding to the upper limits assigned for the GALEX non-detections and (2) the rather reliable corrections for the broadband mean IGM transmissions (top panels of Figure 6).

The intrinsic mean/median composite quasar SED is fit by a smoothly broken power law in the form of

$$\nu L_\nu \propto \left[\left(\frac{\lambda}{\lambda_b} \right)^{S(1+\alpha_{\text{OPT-FUV}})C} + \left(\frac{\lambda}{\lambda_b} \right)^{S(1+\alpha_{\text{EUV}})C} \right]^{S/C}, \quad (1)$$

where λ_b is the break wavelength, $\alpha_{\text{OPT-FUV}}$ is the optical-to-FUV spectral index at $\lambda \gg \lambda_b$, α_{EUV} is the EUV spectral index at $\lambda \ll \lambda_b$, C is the curvature parameter, and S is the sign of $(\alpha_{\text{EUV}} - \alpha_{\text{OPT-FUV}})$. We consider the following line-free windows for the continuum fitting: 480–1000, 1060–1140, 1350–1450, 1700–1800, 2000–2300, 3800–4200, and 5100–6000 $\text{\AA}$ (gray thick bars in Figure 7). A minimum χ^2 approach performed using the routine MPFIT³ gives the best-fit parameters and the associated 1σ uncertainties. For the intrinsic mean composite quasar SED, we find that $\alpha_{\text{OPT-FUV}} = -0.535 \pm 0.003$, $\alpha_{\text{EUV}} = -2.73 \pm 0.03$, $\lambda_b = 1144.0 \pm 6.2 \text{ \AA}$, and $C = 9.34 \pm 1.73$. For the intrinsic median composite quasar SED, we find that $\alpha_{\text{OPT-FUV}} = -0.468 \pm 0.003$, $\alpha_{\text{EUV}} = -6.78 \pm 0.21$, $\lambda_b = 906.0 \pm 11.4 \text{ \AA}$, and $C = 1.08 \pm 0.07$.

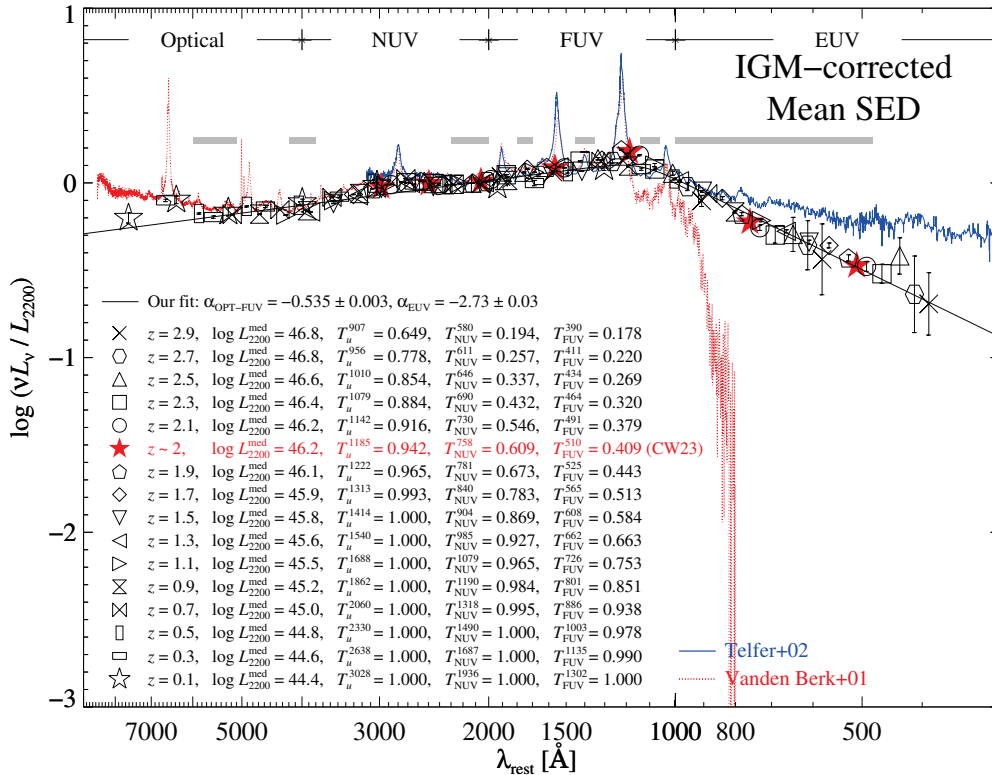


Figure 7. Cont.

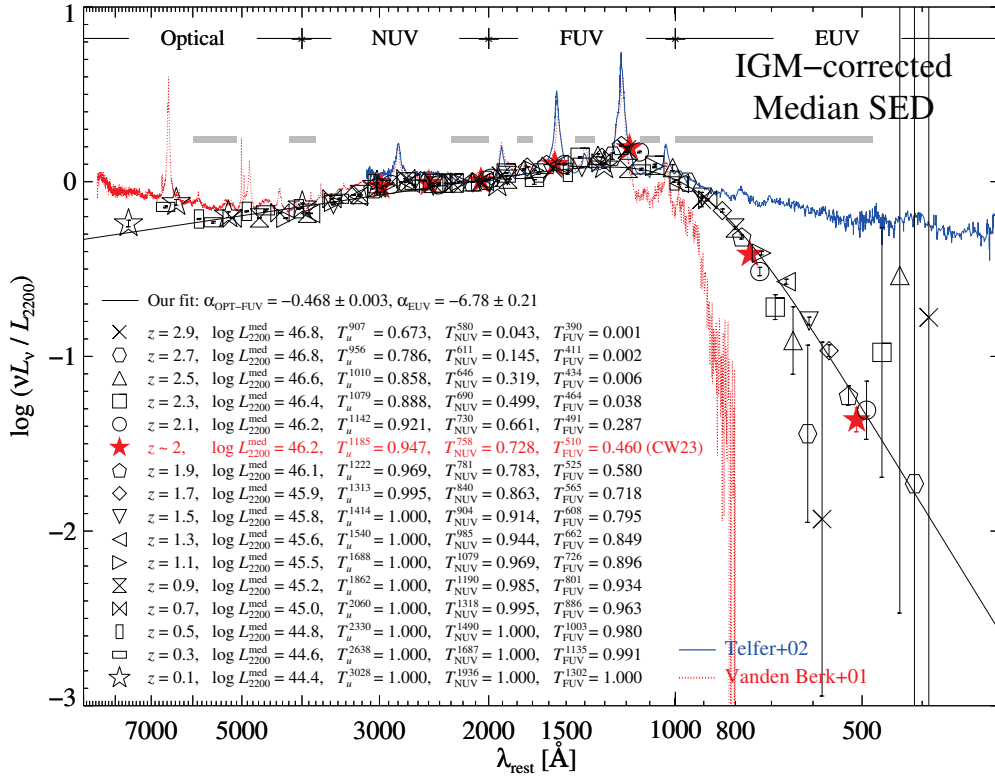


Figure 7. Same as Figure 4, but for the intrinsic mean (top panel) and median (bottom panel) SEDs, whose EUV portion has been corrected for the IGM absorption. For each redshift bin with a central redshift z , the legend includes the median 2200 Å luminosity of quasars, $\log L_{2200}^{\text{med}}$, and the filter-weighted broadband IGM transmissions, T_b^w , where b indicates the band name. After correcting for the IGM absorption, the intrinsic mean/median SEDs of quasars at different redshifts become strikingly consistent in the EUV and together form a mean/median composite SED. A smoothly broken power law with optical-to-FUV ($\alpha_{\text{OPT-FUV}}$) and EUV (α_{EUV}) spectral indices for $L_\nu \propto \nu^\alpha$ is fit to the mean/median composite SED in several line-free windows (thick gray bars).

3. Discussion and Implications

3.1. Universality of the Mean/Median Composite Quasar SED

After controlling the EUV detection incompleteness bias for a quasar sample at $z \sim 2$, Cai and Wang [42] suggested that there is a universal luminosity-independent average SED for quasars at least for $\log L_{2200} > 45$. The claim was made in terms of a small number of quasars at $z \sim 2$, and more critically, only $\simeq 3\%$ (up to 20%) of them were used in order to compare the same fraction of the brightest quasars among different luminosity bins. Here, the consistency among the intrinsic average SEDs of quasars at different redshifts strongly strengthens our previous claim on the luminosity independence of the average quasar SED.

For $z > 0.5$ quasars, capable of providing EUV measurements with the help of GALEX, the median 2200 Å luminosity increases from $\log L_{2200}^{\text{med}} = 44.8$ for $z \sim 0.5$ quasars to $\log L_{2200}^{\text{med}} = 46.8$ for $z \sim 2.9$ quasars. Regardless of the uncertainties in the conversion of the monochromatic optical/UV luminosity into the bolometric luminosity by adopting a redshift-dependent bolometric correction factor [47], the median bolometric luminosity increases $\log L_{\text{bol}} \simeq 45.5$ for $z \sim 0.5$ quasars to $\log L_{\text{bol}} \simeq 47.3$ for $z \sim 2.9$ quasars (top-left panel of Figure 8). Therefore, over two orders of magnitude in luminosity and since cosmic noon, our universal mean/median composite quasar SED demonstrates not only the luminosity independence but also the redshift independence of the average optical-to-EUV radiation properties of quasars.

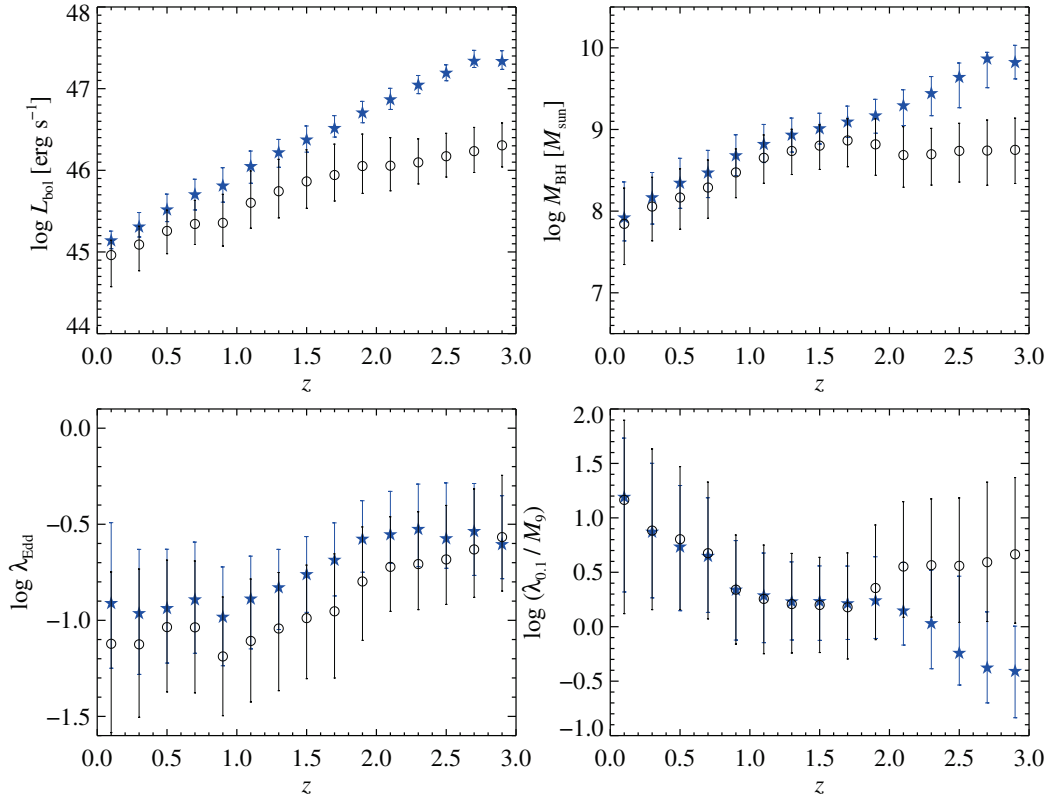


Figure 8. Physical properties, i.e., bolometric luminosity (L_{bol} ; **top-left** panel), BH mass (M_{BH} ; **top-right** panel), Eddington ratio (λ_{Edd} ; **bottom-left** panel), and $\lambda_{\text{Edd}}/M_{\text{BH}}$ (**bottom-right** panel; note $\lambda_{0.1} = \lambda_{\text{Edd}}/0.1$ and $M_9 = M_{\text{BH}}/10^9 M_{\odot}$), as a function of redshift for our parent quasar sample (black open circles; Section 2.1) and our unique bright quasar sample (blue filled stars; Section 2.2). At each redshift, symbols are median values, while vertical bars are the 25–75th percentile ranges. Nearly all quasars have measurements on L_{bol} , M_{BH} , and λ_{Edd} , provided by Rakshit et al. [47], but the large uncertainties on, and even systematic offsets of, these measurements must be taken seriously (see discussion in Sun [48] for example).

Moreover, since both M_{BH} and λ_{Edd} of our unique bright quasar sample used to construct the mean/median composite quasar SED are dependent on redshift, the universality of our composite quasar SED further implies an independence on both BH mass and Eddington ratio, at least for quasars with $M_{\text{BH}} \gtrsim 10^8 M_{\odot}$ and $\lambda_{\text{Edd}} \gtrsim 0.1$ (Figure 8). However, for low-luminosity AGN with $\lambda_{\text{Edd}} \sim 0.01$, the universality may break down, given a potential transition of the accretion modes [49–51].

As introduced in Section 2.2, we adopt redshift-dependent luminosity cutoffs to select the unique bright quasar sample, guided by the trade-off between high GALEX detection and large sample size (Figure 3). One may consider other cutoffs, but it is hard to quantify which one is “the best”. To convince the readers that the adopted cutoffs are indeed reasonable and that perturbations in some cutoffs will not lead to significantly different conclusions, we present in Figure 9 the intrinsic mean/median quasar SEDs of another two quasar samples selected with larger or smaller $\log L_{2200}^{\text{min}}$, i.e., by ± 0.2 dex in all redshift bins, than our reference values (Section 2.2). It is interesting to find that the universality of the intrinsic quasar SED preserves for changing the luminosity cutoffs across 0.4 dex. Although an even larger range for the luminosity cutoffs can be explored, the uncertainties induced by either a small sample size or low GALEX detection should be taken carefully.

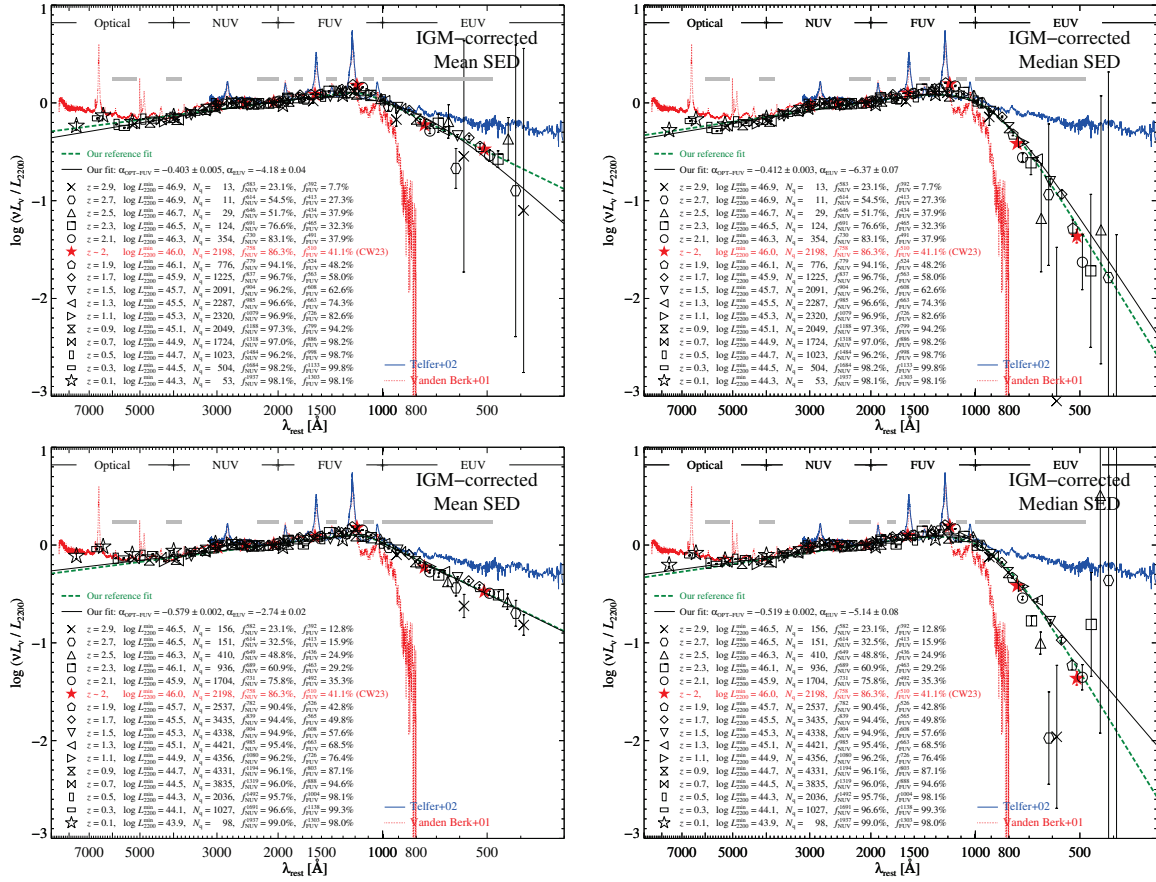


Figure 9. Panels in each row are the same as Figure 7, but for the intrinsic quasar SEDs constructed from another two quasar samples selected with larger (top panels) or smaller (bottom panels) $\log L_{\text{min}}^{\text{min}}$, i.e., by ± 0.2 dex in all redshift bins, than our reference values. In each panel, the green dashed curve is the best-fit smoothly broken power law taken from the corresponding panel of Figure 7.

3.2. How Can the Model-Predicted SEDs Be Properly Compared with the Observed Composite SEDs?

In each redshift bin, there are many quasars with different physical properties, i.e., distinct M_{BH} and λ_{Edd} . Previously, we generated a single model-predicted SED for a quasar sample using its median value⁴ of $\lambda_{\text{Edd}}/M_{\text{BH}}$ and compared to the observed median quasar SED [42]. But what about the mean SED? Here we construct the model-predicted mean/median composite SED in the same way as the observed mean/median composite one. Utilizing measurements on M_{BH} and λ_{Edd} from Rakshit et al. [47], we generate a model-predicted SED for every quasar in a redshift bin and normalized all SEDs at 2200 Å before obtaining the mean/median SED.

Same as Cai and Wang [42], we consider two typical disk models. One is the standard thin disk model [7], whereas the other is the simply truncated disk model [43], in which the truncation radius is linked to a maximum temperature of $T_{\text{max}} \simeq 3.6 \times 10^4 [(\lambda_{0.1}/M_9)/0.02]^{0.07} \text{K}$, where $\lambda_{0.1} = \lambda_{\text{Edd}}/0.1$ and $M_9 = M_{\text{BH}}/10^9 M_{\odot}$. Please note that in both thin and truncated disk models adopted here, the model-predicted SED for a quasar is fixed as long as its M_{BH} and λ_{Edd} are given. In other words, there is no free parameter in these two models. Other models such as the slim disk [52], the disk evaporation model [53,54], the condensation of the corona [55–57], and the collapse of the disk [50] are deferred to future works.

Figure 10 shows SEDs predicted by both the thin and truncated disk models for quasars in $1.8 < z \leq 2.0$ as an example. For both disk models, we find that the SED implied by the median $\lambda_{\text{Edd}}/M_{\text{BH}}$ of a quasar sample is almost the same as the predicted median composite SED. The difference in the optical-to-FUV between the predicted mean

and median composite SEDs is small. However, a large EUV difference between them is implied by the thin disk model but not by the truncated disk model. We confirm that these claims are also valid for quasar samples in other redshift bins. Consequently, directly comparing the model-predicted composite SED constructed in the same way as the observed composite SED is necessary, especially for the mean SED.

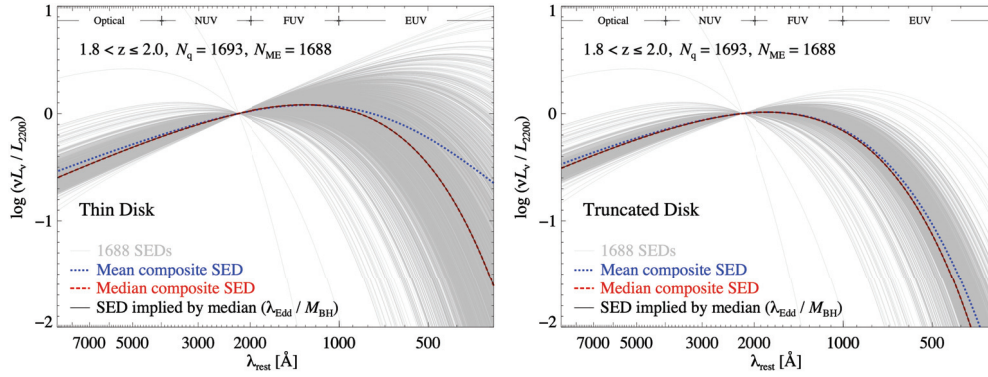


Figure 10. SEDs predicted by the thin disk (left panel) and truncated disk (right panel) models for N_{ME} quasars, with measurements on both M_{BH} and λ_{Edd} , of N_q quasars in $1.8 < z \leq 2.0$ as an example. Each gray thin solid curve is an SED of a quasar, while the blue dotted and red dashed curves are the resultant model-predicted mean and median composite SEDs for quasars in that redshift bin, respectively. For comparison, a black solid curve superimposed on the red dashed curve represents a specific SED implied by the median λ_{Edd} / M_{BH} of the N_{ME} quasars.

3.3. The Central Engine at Work for Quasars Remains an Enigma

Although the much steeper EUV decrease of the median composite SED compared to the mean SED predicted by the thin disk model for quasars in each redshift bin (the left panel of Figure 10) is similar to the observed difference between the mean and median composite quasar SEDs (Figure 7), the left panels of Figure 11 demonstrate that both the mean and median composite SEDs predicted by the thin disk model are strongly dependent on redshift and thus in conflict with the redshift independence of both the mean and median composite quasar SEDs observed.

Instead, the mean and median composite SEDs predicted by the truncated disk model are comparable among redshift bins (the right panel of Figure 10) and are only somewhat dependent on redshift (the right panels of Figure 11), much weaker than predictions of the thin disk model. In the truncated disk model, it is the similar maximum temperature, which is attributed to a microscopic atomic origin and thus is weakly dependent on both M_{BH} and λ_{Edd} [43], such that the truncated disk model can give rise to relatively universal SEDs for quasars at different redshifts, where their typical M_{BH} and λ_{Edd} are distinct (Figure 8). For quasars at different redshifts, the median composite SEDs predicted by the truncated disk model are very close to the observed median composite quasar SED, though there are some discrepancies (the bottom-right panel of Figure 11). More significant discrepancies can be found in comparing the mean composite SEDs where the truncated disk model predicts EUV-redder mean SEDs with increasing redshift (the bottom-left panel of Figure 11).

Since the physical properties of quasars are significantly dependent on redshift (Figure 8), the universal mean/median composite quasar SED could not be the result of the classic thin disk model (the left panels of Figure 11). Instead, the truncated disk model provides a possibility for the quasar central engine, though clear discrepancies still exist (the right panels of Figure 11). Please note that the two models considered here cannot account for the corona emission and soft X-ray excess as well as the ubiquitous AGN variability.

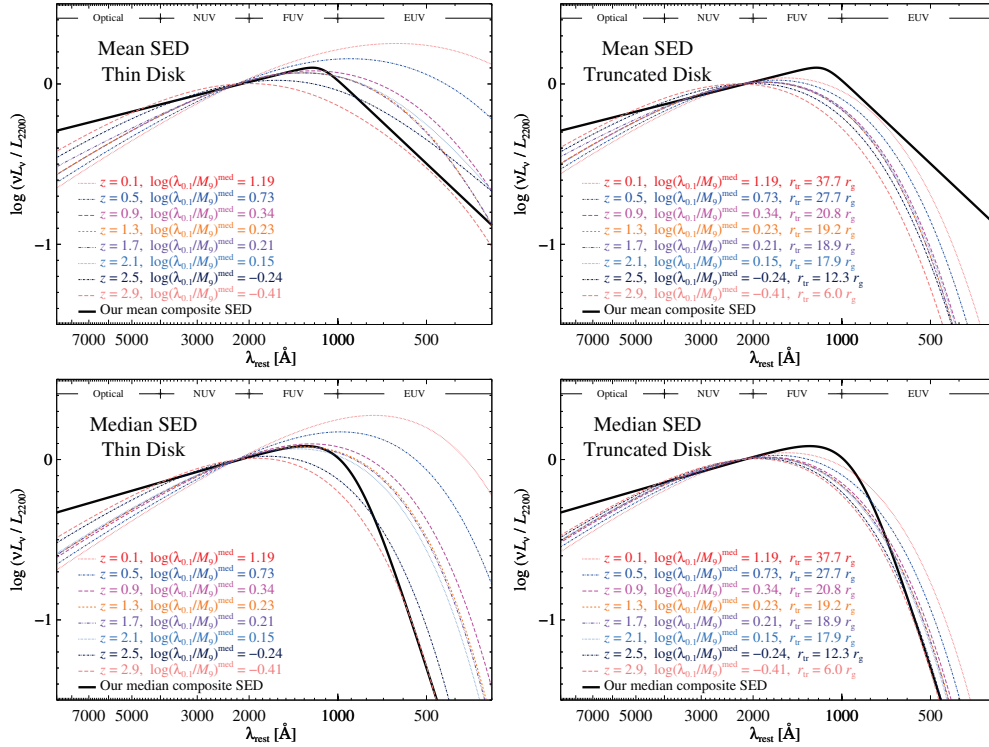


Figure 11. (The **top** panels): comparing the model-predicted mean composite SEDs for quasars in different redshift bins (colored thin curves) to the observed mean composite SED (black thick solid curve), the truncated disk model (**top-right** panel) performs better than the thin disk model (**top-left** panel). The median values of z and $\lambda_{\text{Edd}}/M_{\text{BH}}$ for quasars in each redshift bin are nominated in the legend. In the right panel for the truncated disk model, the legend contains a typical truncation radius, r_{tr} (in gravitational radius r_g), corresponding to a maximum disk temperature determined by the median $\lambda_{\text{Edd}}/M_{\text{BH}}$ of quasars. (The **bottom** panels): same as the top ones, but for comparing the median composite SEDs.

We note that models including X-ray emission may not be easily responsible for the universality of our composite quasar SED, which is independent of many physical properties of quasars. For example, in the X-ray model proposed by Kubota and Done [21], the warm corona mainly contributes the EUV emission, and parameters relevant to the warm corona are all free regardless of M_{BH} and λ_{Edd} . A tentative fit using this model indeed finds that achieving model-predicted SEDs comparable to our universal composite quasar SED is possible for different M_{BH} and λ_{Edd} but in a fine-tuning way, and that the derived dependences of parameters on M_{BH} and λ_{Edd} are obscure. Another interesting X-ray model to be considered is the disk evaporation model by ([49], see their Equation (6)) where the truncation radius does not depend on M_{BH} and is weakly dependent on the viscosity parameter, but strongly depends on λ_{Edd} and the ratio of gas pressure to the total pressure.

Extending our SED analyses from EUV to (soft/hard) X-ray is essential for understanding the quasar central engine. Unfortunately, the X-ray detections of the SDSS-selected quasars are exceedingly low so far. For example, even for our unique bright quasar sample (Figure 1), only $\simeq 7\%$ have soft/hard X-ray measurements by *XMM-Newton* according to the multi-wavelength cross-matched catalog of Pâris et al. [44]. Since the *XMM-Newton* observations do not cover most SDSS quasars, the real *XMM-Newton* detection fraction should be larger than 7% but is hard to assess. The situation in the soft X-ray is much better thanks to the extended ROentgen Survey with an Imaging Telescope Array (eROSITA) instrument aboard the Spectrum-Roentgen-Gamma (SRG) mission [58]. Considering ~ 9600 quasars in our unique bright quasar sample within the German footprints of the first six months of survey operations (eRASS1; [59]) conducted by eROSITA, $\simeq 38\%$ have counterparts in the eRASS1

main catalog within a matching radius of 15 arcsec. Excitingly, a continuing increase in the eROSITA detection can be expected once subsequent deeper eROSITA data releases are available. At the moment, we caution again that extending the SED analyses to X-ray by simply dropping the X-ray non-detections may easily result in improper conclusions.

3.4. Properties of Dust and Gas in the Quasar Host Galaxies

When constructing the intrinsic mean/median SEDs for quasars at different redshifts, the dust attenuation has been neglected. More uncertain than the amount of dust in the quasar host galaxies is the EUV shape of the dust-attenuation curve. Actually, no measurement on the dust-attenuation curve has been available at wavelengths shorter than the rest-frame $\sim 1000 \text{ \AA}$ until recently. Gaskell et al. [60] derived, for the first time, a mean attenuation curve down to the rest-frame $\sim 400 \text{ \AA}$ by (1) comparing the SED difference between normal quasars and quasars with broad absorption lines (BAL) and (2) associating the SED difference with an extra dust absorption along the LOS to BAL quasars. The derived mean attenuation curve in the optical-to-FUV is very similar to the attenuation curve of the Small Magellanic Cloud (SMC; [61]) and not to the relatively flat mean attenuation curve of normal quasars [62], suggesting that the dust in the host galaxy along the LOS to normal quasars has distinct properties from the dust likely in clouds outflowing from the broad line region (BLR) along the LOS to BAL quasars.

Although the mean attenuation curve derived by Gaskell et al. [60] has a large dispersion in the EUV, it suggests a more significant attenuation over the whole EUV and an attenuation peak at a wavelength shorter than predicted by Weingartner and Draine [61]. If the mean attenuation curve for our quasars has a peak in the EUV, likely around $\sim 700\text{--}500 \text{ \AA}$ as predicted by Weingartner and Draine [61] and measured by Gaskell et al. [60], there would be a resultant dip (or concavity) in the composite mean/median quasar SED, which is however not observed. Therefore, the universality and the smoothness of our mean/median composite quasar SED suggests that (1) the amount of dust is little in the LOS of most normal quasars, such as our quasars brighter than $\log L_{\text{bol}} \simeq 45.5$ (see also Figure 5 of Gaskell et al. [63]), and/or (2) the AGN attenuation curve is very flat extending from the optical-to-FUV [62] to the EUV for normal quasars.

The EUV ionizing continuum is also very sensitive to the amount of neutral hydrogen. Similarly, the universality of our mean/median composite quasar SED suggests that the interstellar and circumgalactic mediums along the LOS to quasars should be gas-free (or highly ionized). This is possible since bright quasars are more frequently found in massive early-type galaxies, which generally have a low gas content. And it is enough that the LOS to a quasar is gas-free, rather than the whole host galaxy of the quasar, given the general anisotropic radiation of the accretion disk [64] and the existence of the dusty torus and the ionization cone [65]. When the LOS of a quasar host galaxy has an abundant interstellar medium, the “quasar” is obscured and may be undetected or too faint to be identified as a quasar.

If there were a redshift dependence of the dust attenuation and/or host-related hydrogen absorption, a fine-tuning of the dust attenuation and/or hydrogen absorption as a function of redshift would be unavoidable. Otherwise, it is hard to understand why the observed quasar SEDs at different redshifts are nicely consistent with each other after dust attenuation and/or hydrogen absorption. Instead, the universality of our mean/median composite quasar SED may imply that properties of dust attenuation and/or hydrogen absorption, if any, in the quasar host galaxies at $z \lesssim 3$ should be similar on average, independent of redshift and luminosity. In future, it would be interesting to, after taking the dust attenuation and/or hydrogen absorption into account, directly compare the model-predicted distribution of $\log(L_w/L_{2200})$ in the EUV to the observed asymmetric one, indicated by the huge difference between the mean and median composite quasar SEDs in the EUV (Figure 7).

3.5. Evolutionary of Quasars Since Cosmic Noon

The decline in the maximum luminosity of quasars since $z \sim 3$ (Figure 1) intuitively suggests an evolutionary where the most violent BH accretion shifts from quasars at $z \sim 2-3$ with $L_{\text{bol}} \sim 10^{47} \text{ ergs}^{-1}$, $M_{\text{BH}} \sim 10^{9.5} M_{\odot}$ and $\lambda_{\text{Edd}} \sim 0.3$ to those at $z \sim 0.1$ with $L_{\text{bol}} \sim 10^{45} \text{ ergs}^{-1}$, $M_{\text{BH}} \sim 10^8 M_{\odot}$, and $\lambda \sim 0.1$ (Figure 8). Why are there few low-redshift quasars as bright as those at cosmic noon? Our universal quasar SED yields new constraints on the evolution of quasars. As hinted by the simple truncated disk model, the accretion disk of a quasar with a less massive BH likely has a larger truncation radius (the right panels of Figure 11). If the disk wind indeed plays a role in determining the truncation radius, it would also result in a lower radiative efficiency for the accretion to a less massive BH [43], and in the meantime, slow down the BH growth. Therefore, when quantitatively comparing the model to data, including the cosmological evolution of the quasar luminosity function and the BH mass function, it is worthy of consideration of a more realistic accretion model with a mass-dependent radiative efficiency rather than a simple model with a conventional constant radiative efficiency of $\simeq 0.1$.

3.6. Implications of a Deficit of the EUV Ionizing Radiation

As illustrated in Figure 7, our mean composite quasar SED at the rest-frame wavelengths beyond $\sim 800 \text{ \AA}$ is nicely consistent with but redder at wavelengths short of $\sim 800 \text{ \AA}$ than, the mean composite spectrum of Telfer et al. [31], which is the EUV-reddest one among previous mean composite quasar spectra [32,35,36]. Our median composite quasar SED is even redder in the EUV than our mean composite quasar SED. Both our mean and median composite quasar SEDs indicate a deficit of the EUV ionizing continuum. Relative to the mean composite spectrum of Telfer et al. [31], the amount of EUV ionizing continuum, i.e., the integral from $912 \text{ \AA}$ to $300 \text{ \AA}$, implied by our best-fit smoothly broken power law for the mean (median) composite quasar SED is smaller by a factor of $\simeq 1.6$ ($\simeq 3.6$). Therefore, the deficit of the EUV ionizing continuum has important implications for, e.g., the production of broad emission lines and cosmic reionization.

3.6.1. Production of Broad Emission Lines

Broad emission lines, such as $\text{Ly}\alpha$, $\text{CIV } 1549$, $\text{MgII } 2798$, $\text{H}\beta$, and $\text{H}\alpha$, are prominent characteristics of quasars, and are generally discussed within three cloud models: magnetic pressure confined clouds [66], locally optimally emitting clouds [67], and radiation pressure-confined clouds [68]. Regardless of the different physics and conditions involved in these cloud models and the fact that, hitherto, no such models have reproduced all known line properties [69,70], photoionization owing to the incident ionizing radiation of the central quasar is a key ingredient for generating these emission lines.

Less ionizing radiation usually results in weaker line emission. Combining the radiation pressure-confined clouds with a theoretical SED that has relatively weak EUV emission (cf. the AD2 SED in Figure 1 of Netzer [69] and the black dotted curve in Figure 12), Netzer [69] showed in his Figure 3 that the AD2 SED fails to produce large enough line luminosities, including $\text{CIV } 1549$, $\text{HeII } 1640$, and $\text{H}\beta$. As illustrated in Figure 12, our mean and median composite quasar SEDs have even weaker EUV emissions than the AD2 SED and thus would give rise to smaller line luminosities according to the model of Netzer [69]. Our median composite quasar SED implies that more than 50% of quasars with broad emission lines have a strong deficit of the EUV emission.

Given the very low EUV emission, why the emission lines are there is an interesting question to be addressed. Although detailed calculations on the line production are demanded, our EUV-deficit composite SEDs hint at that (1) the cloud density may be smaller and/or (2) the role of collision excitation or the other energy sources, such as the magnetic energy associated with the magnetic pressure-confined clouds, may be more important than expected. Another possibility for the large EUV deficit found in the median composite quasar SED is that the EUV emission seen by the BLR clouds is not attenuated by dust within the BLR and thus is different from that observed by us, which is attenuated

by dust in the quasar host galaxy. On the one hand, examining differences in the broad emission lines among sub-samples of our unique bright quasars divided by L_w/L_{2200} , where w is shorter than ~ 1000 Å, can tell us whether the EUV emission seen by the BLR clouds is attenuated: no or little difference indicates unattenuated EUV emission for clouds. On the other hand, examining the relationship between L_w/L_{2200} and L_d/L_{2200} , where L_d is the dust luminosity at wavelengths likely spanning from near- to far-infrared, could suggest whether the EUV emission is attenuated by dust in the quasar host galaxy: a negative correlation between them may indicate dust attenuation up to the host scale.

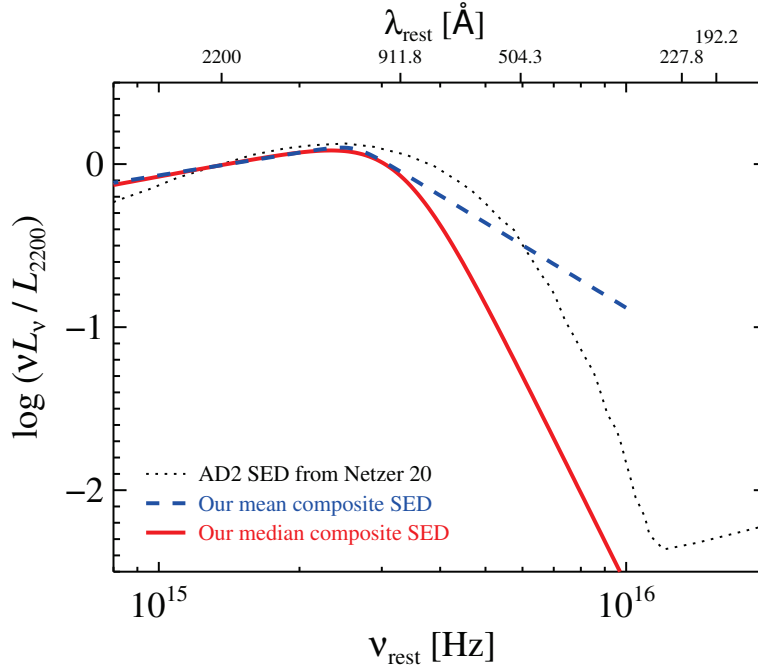


Figure 12. The blue dashed (red solid) curve is our mean (median) composite quasar SED. The black dotted curve is the AD2 SED of Netzer [69], whose ionizing continuum is weakest among the four SEDs adopted by Netzer [69] and has difficulties in producing large enough line luminosities, including CIV 1549, HeII 1640, and H β . On the top axis, there are marked specific wavelengths corresponding to the ionization potential energies of H I (911.8 Å), He I (504.3 Å), He II (227.8 Å), and C IV (192.2 Å).

3.6.2. Cosmic Reionization

Two important episodes in cosmic history are the reionization of neutral hydrogen and helium. The relatively well-understood helium reionization completing at $z \sim 3$ is mainly contributed by quasars [71], whereas the exact process of hydrogen reionization, though likely ending around $z \sim 5$ – 6 , and the major ionizing contributor, e.g., star-forming galaxies [72–80] or quasars/AGN [81–86], are still under debate, especially after the discovery of a large population of moderate-luminosity type 1 AGN at $4 \lesssim z \lesssim 13$ by the James Webb Space Telescope [87].

The cosmic reionization is more relevant to the mean ionizing radiation and, thus, to the mean composite quasar SED. Here, our finding of an EUV-deficit mean quasar SED may imply a smaller AGN contribution to the hydrogen reionization. Note that properties of dust and gas in the host galaxies of quasars at $z > 5$ are not necessarily similar to those of the quasar host galaxies at $z \lesssim 3$ (see Brooks et al. [88] for a potential sample of heavily dust-attenuated AGN at $z \gtrsim 3.5$ owing to dust in the BLR but not in the host galaxy) and neither does the mean composite quasar SED. Directly extending our SED analysis to quasars at higher redshifts with the help of future deep ultraviolet facilities would provide important constraints on the mean escape fraction of the Lyman continuum radiation of quasars. Moreover, more consequences of our EUV-deficit mean composite quasar SED on

the helium reionization, the cosmic UV background, and the thermal history of the IGM are worthy of investigation.

4. Conclusions

We consider a unique bright quasar sample with 23,256 sources selected from the SDSS DR14Q catalog and covered by the GALEX tiles with FUV exposure times longer than 1000 s and FOV offsets smaller than 0.5° . Combining the SDSS and GALEX photometry, bias-free mean and median SEDs of quasars in 15 redshift bins, i.e., from $z = 0$ to $z = 3$ in bins of 0.2, are obtained after taking the GALEX non-detection into account. These bias-free SEDs of quasars at different redshifts are very similar and are consistent with previous mean composite quasar spectra at wavelengths beyond $\simeq 1000 \text{ \AA}$. Instead, at wavelengths shorter than $\simeq 1000 \text{ \AA}$, these bias-free SEDs become redder in the EUV with increasing redshift, in line with the increasing significance of the IGM absorption.

After correcting for the IGM absorption, these intrinsic mean/median SEDs of quasars at different redshifts are surprisingly similar, i.e., independent of redshift, and form a universal mean/median composite SED for quasars. Our mean composite quasar SED is redder in the EUV than the previous EUV-reddest mean composite quasar spectrum, whereas our median composite quasar SED is even redder in the EUV. Furthermore, our mean/median composite quasar SED is plausibly independent of both the BH mass and the Eddington ratio, which clearly depends on redshift.

Considering two typical disk models, i.e., the standard thin disk model and the simply truncated disk model, we demonstrate that directly comparing the model-predicted composite SED constructed in the same way as the observed composite SED is necessary, especially for the mean one. Thus, we show that our universal mean/median composite quasar SED prefers the simply truncated disk model rather than the standard thin disk model, though there are discrepancies requiring more sophisticated models.

Regardless of the uncertain physics of the quasar central engine, the universality of our mean/median composite quasar SED strongly suggests that properties of dust and gas in the quasar host galaxies at $z \lesssim 3$ are similar. Otherwise, fine-tuning for both the dust content and the gas ionization state as a function of redshift is unavoidable. Furthermore, given our EUV-deficit mean/median composite quasar SED, implications for the production of broad emission lines of quasars and the cosmic reionization by quasars are discussed.

Complemented by the all-sky X-ray surveys, such as eROSITA, and with the help of future deep ultraviolet facilities, such as the China Space Station Telescope [89] and the Ultraviolet Explorer [90], revolutions in our understanding of the quasar central engine as well as many related aspects are promising. Therefore, exploring the discovery capability of these UV and X-ray facilities deserves further comprehensive work.

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Data Availability Statement: Data for the quasar samples, the filter-weighted broadband IGM transmissions, and the universal intrinsic mean/median composite quasar SED are available at http://staff.ustc.edu.cn/~zca/agnsed_c24/ (accessed on 19 Nov 2024), while the other data underlying this article will be shared upon request to the corresponding author.

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Conflicts of Interest: The author declares no conflicts of interest.

Abbreviations

The following abbreviations are used in this manuscript:

AGN	Active Galactic Nucleus/Nuclei
BAL	Broad Absorption Line
BH	Black Hole
BLR	Broad Line Region
DR14Q	Data Release 14 Quasar
eROSITA	extended ROentgen Survey with an Imaging Telescope Array
EUV	Extreme Ultraviolet
FOV	Field of View
FUV	Far Ultraviolet
GALEX	Galaxy Evolution Explorer
IGM	Intergalactic Medium
LOS	Line of Sight
NUV	Near-ultraviolet
SDSS	Sloan Digital Sky Survey
SED	Spectral Energy Distribution
SMC	Small Magellanic Cloud
SRG	Spectrum-Roentgen-Gamma
UV	Ultraviolet

Notes

- ¹ <https://galex.stsci.edu/Doc/gr67.txt>
- ² <https://galex.stsci.edu/casjobs/default.aspx>
- ³ <https://cow.physics.wisc.edu/~craigm/idl/idl.html>
- ⁴ Nominated in the legend of Figure 4 of Cai and Wang [42], the median value of $\lambda_{\text{Edd}}/M_{\text{BH}}$ for the quasar sample in each luminosity bin was not correctly typed, although the resultant model-predicted SED is correct. Therein, the correct values for $\log(\lambda_{\text{Edd}}/M_{\text{BH}})^{\text{med}}$ should be added by two.

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Article

The Properties of an Edge-On Low Surface Brightness Galaxies Sample

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Abstract: We analyzed the properties of a sample of edge-on low-surface brightness galaxies, which are referred to as Cao23 ELSBGs. Cao23 ELSBGs exhibit a wide range of luminosities ($-22 < M_r < -13$) with a mean scale length of 3.19 ± 1.48 kpc. Compared to HI-rich dwarf ELSBGs, Cao23 ELSBGs display more extended disk structures and redder (g-r) colors. They are also, on average, more massive than HI-rich dwarf ELSBGs. Star formation rates (SFRs) were calculated using WISE 12 μ m luminosity conversions and spectral energy distribution (SED) fitting methods, respectively. Cao23 ELSBGs fall below the main sequence with specific star formation rates (sSFRs) primarily in the range of $0.01\text{--}0.1 \text{ Gyr}^{-1}$. More massive Cao23 LSBGs tend to have lower sSFRs. Additionally, we derived the non-parametric star formation histories (SFHs) of Cao23 ELSBGs by SED fitting, dividing the SFHs into seven look back time bins with constant SFRs assumed for each bin. Our analysis indicates that high-mass ($M_* > 10^{9.0} M_\odot$) Cao23 ELSBGs assembled their mass earlier than their lower-mass counterparts, supporting a downsizing trend for LSBGs.

Keywords: low surface brightness galaxies; star formation; star formation history

1. Introduction

Low surface brightness galaxies (LSBGs) are characterized by their extremely faint structure with a central surface brightness (μ_0) of the disk lower than $22.0\text{--}23.0 \text{ mag arcsec}^{-2}$ in the B-band [1–9]. LSBGs encompass various galaxy types, including dwarf LSBGs [10], ultra-diffused galaxies (UDGs) characterized by the effective surface brightness $\mu_e \geq 24 \text{ mag arcsec}^{-2}$ and the effective radius $R_e \geq 1.5 \text{ kpc}$ [11,12], and giant LSBGs (GLSBGs) with radii up to 130 kpc and stellar masses of the order of $10^{11} M_\odot$, as exemplified by Malin 1 [13–18]. The stellar masses of LSBGs exhibit a wide range, spanning from dwarf LSBGs to GLSBGs. The faint features of LSBGs are related to their low-surface stellar mass densities [19]. LSBGs are metal-poor [20–22]. In the mass-metallicity ($M_* - Z$) relation, LSBGs show a flatter trend compared to star-forming galaxies, suggesting inefficient oxygen abundance enhancement [23]. Furthermore, LSBGs display lower star formation rates (SFRs) than the main sequence galaxies [24–26].

LSBGs typically exhibit lower dust content compared to high surface brightness galaxies [27], and their far-infrared emissions are weaker than those of normal star-forming galaxies [28]. Among five GLSBGs, only UGC 6614 and UGC 6151 show 160 μ m (cool) dust emission in [28] with the dust mass of UGC 6614 measured at $10^{8.42} M_\odot$ [27]. Ref. [29] have discussed the higher dust content in some extreme GLSBGs. Additionally,

LSBGs are characterized by a shortage of molecular gas [30,31]. Ref. [32] estimated the molecular gas surface density in Malin 1 to range from approximately 2 to $0.8 M_{\odot} pc^{-2}$ with a very low molecular-to-atomic gas mass ratio. Parts of LSBGs are HI-rich [33], exhibiting higher HI gas mass fractions ($f_{HI} = M_{HI}/M_*$) than those in the HI-selected spiral galaxies in the HI Parkes All Sky-Survey Catalogue (HICAT) and the Wide-field Infrared Survey Explorer (WISE) sample from [34]. The spiral galaxies in [34] are high surface brightness systems. Many LSBGs are late-type (Sc and later) spiral galaxies with amorphous or fragmentary and faint spiral patterns, and some are similar to irregular galaxies [33]. Some GLSBGs generally exhibit clearly defined spiral structures [35]. The differences between gas-rich and gas-poor LSBGs are not clear. Our current understanding of various LSBG properties remains limited. The origins and evolutions of these LSBGs are under debate [36–39]. The typical formation scenario is that LSBGs arise in haloes with high angular momentum under the Λ CDM framework [40,41]. Other scenarios also considered in recent research, such as ram-pressure stripping [42], extreme feedback processes [43], and mergers [22,44–47].

The star formation history (SFH) is an essential parameter tracing the stellar assembly history and thereby revealing the evolutionary pathway of galaxies. The specific star formation rate (sSFR) at different redshifts and the observed spectral energy distributions (SEDs) can provide valuable insights into SFH. The sSFR, defined as the ratio of SFR to stellar mass (M_*) [48], reflects the rate ratio of new stars to the assembled galaxy mass [49]. Estimating the sSFR at different redshifts [50,51] offers a deeper understanding of the galaxy's star formation history. The sSFR–stellar mass relation exhibits a steeper trend at high redshift [52] for main sequence galaxies. It indicates that the massive galaxies have assembled their stellar mass at the early time, which is a concept known as downsizing [53]. Very few studies have explored LSBGs at high redshifts [54,55]. The topic of SFH of LSBGs is a rarely mentioned area [10,25], and the relevant discussion is based on the HI-gas depletion time ($t_{dep} = M_{HI}/SFR$, [56]). Ref. [25] demonstrated that the current SFRs of their HI-detected LSBGs are close to average SFRs in Hubble time by the HI depletion time t_{dep} -sSFR relation, suggesting that most LSBGs are stable systems.

Ref. [23] presented a sample of 330 blue edge-on low-surface brightness galaxies (ELSBGs) with SDSS fiber spectra and analyzed the chemical evolution of this sample. The ELSBGs in [23] have distance information, which is often unavailable in most optically selected LSBG samples. Additionally, the ELSBGs show higher projected surface stellar mass densities than face-on LSBGs, making it possible to obtain high-quality integral field spectroscopy (IFS) data. This enables detailed investigations into their perpendicular disk structure providing insights into the formation and evolution of LSBGs through dynamical studies. For instance, ref. [22] presented IFS observations of an ELSBG (AGC 102004) from [23] and discussed evidence for a minor or mini-merger event in AGC 102004.

In this study, we investigate properties of the ELSBGs from [23], focusing on SFR, sSFR, t_{dep} , and non-parametric SFH. We give a review of ELSBGs from [23] in Section 2 and compare ELSBGs with previous ELSBG samples (HI-rich dwarf ELSBGs and super-thin galaxies) in Section 3. In Section 4, we examine the SFR of ELSBGs based on multi-band data, and in Section 5, we assess their HI gas depletion times. The SFH of ELSBGs inferred from the non-parametric model using the SED fitting code PROSPECTOR is represented in Section 6. We summarize our findings in Section 7. Throughout this paper, we utilize AB magnitudes for all measurements. We adopt a Λ CDM cosmology with $\Omega_m = 0.3$, $\Omega_{\Lambda} = 0.7$, $H_0 = 70 \text{ km s}^{-1} \text{ Mpc}^{-1}$. In this paper, the stellar masses of ELSBGs are from the MPA-JHU catalog [57], and the redshifts of ELSBGs are SDSS spectroscopic redshifts. We use the Chabrier universal initial mass function (IMF, [58]) throughout the article. We convert Salpeter IMF [59] and Kroupa IMF [60] with the following conversion factors: $SFR^{cha} = SFR^{sal}/1.41$ [61–63], $SFR^{cha} = SFR^{kro} \times 0.943$ [64].

2. Cao23 ELSBGs

The ELSBGs' sample is drawn from [23], including 330 blue ELSBGs with SDSS spectra. The initial edge-on galaxies sample is from [65]. The ELSBGs sample are selected based on their face-on mean surface brightness in $3''$ radius (μ_{3-face}) in a g-band lower than $25 \text{ mag arcsec}^{-2}$ and the projected (u-r) color in the center $3''$ aperture region below the green valley upper boundary [66]. The details of conversion from edge-on to face-on surface brightness estimation are given in [23]. The final 330 ELSBGs show the face-on center surface brightness (μ_{0-face}) ranges from 22.34 to 23.75 mag arcsec^{-2} and redshifts from 0.0026 to 0.14. Hereafter, we refer to the ELSBGs from [23] as “Cao23 ELSBGs”.

We separate Cao23 ELSBGs into five bins according to their inclination angle and test how the inclination angle affects the surface brightness correction for edge-on galaxies with the method from [67–70] due to deviations from a 90-degree inclination angle. We show the relationship between the mean value of the scale height to scale length ratio ($-2.5 \times \log(h_s/r_s)$) and the inclination angle of Cao23 ELSBGs in Figure 1. The h_s and r_s are from the GALFIT edge-on disk model fitting [23], and the inclination angle is from [65]. It shows a weak trend that the $-2.5 \times \log(h_s/r_s)$ increases with the inclination angle increasing and $-2.5 \times \log(h_s/r_s)$ does not change too much with respect to Cao23 ELSBGs. It suggests that the inclination angle does influence the surface brightness correction, but the influence is not significant for inclination angle $> 81^\circ$. The inclination angle $> 81^\circ$ contributes 86% of Cao23 ELSBGs which does not significantly affect the main results of the sample.

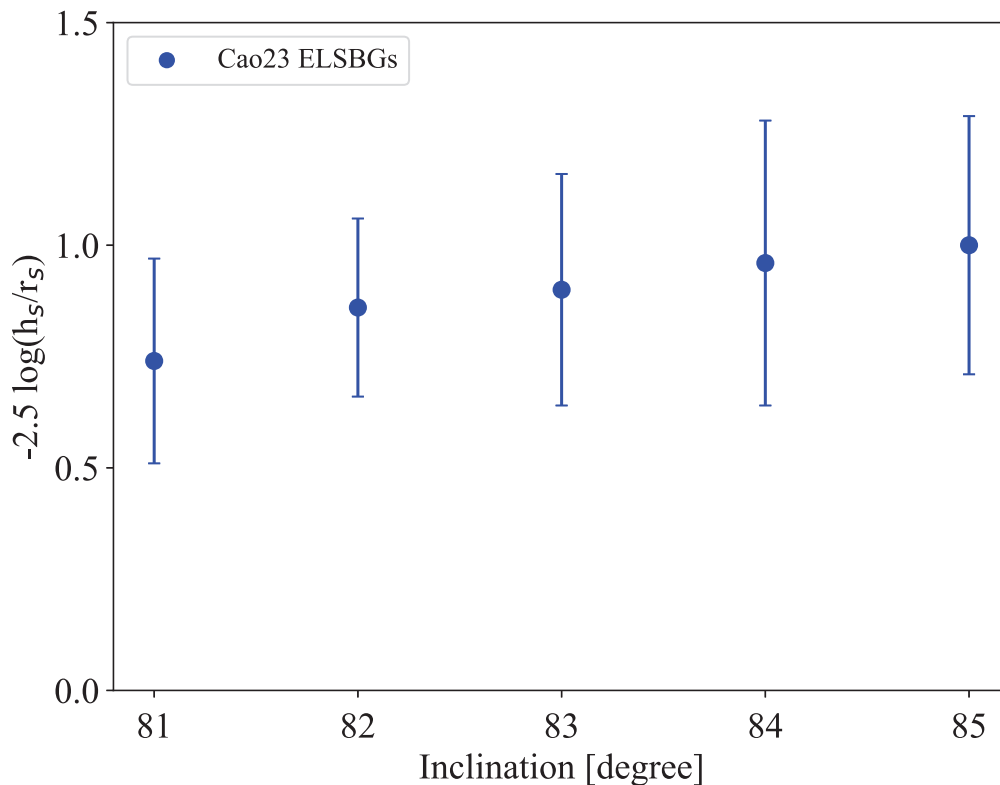


Figure 1. The relationship between the mean value of the scale height to scale length ratio ($-2.5 \times \log(h_s/r_s)$) and inclination. We note the $-2.5 \times \log(h_s/r_s)$ is the correction factor from the observed center surface brightness to face-on center surface brightness for edge-on cases [70].

The selection of the LSBG sample usually excludes those high ellipticity ($e = (1 - b/a) \geq 0.7$, a and b are the semi-major axis and semi-minor axis, respectively) targets to avoid the edge-on galaxies, high redshift, spurious artifacts, and galaxy blends [33,71,72]. It is worth noting that the surface brightness cutting sometimes does not account for inclination angle correction [71,72], which may result in parts of ELSBGs being missed in this step.

Recent studies [70] have shown that a significant portion of ELSBGs would be overlooked if inclination effects are not considered. Furthermore, the number of highly inclined galaxies is less than in other cases. These factors collectively contribute to the rarity of ELSBGs in LSBGs. This limitation may introduce biases in studies of the intrinsic morphology of LSBGs.

3. Comparison with Other ELSBG Samples

We compare the physical parameters of Cao23 ELSBGs with two other ELSBGs samples: HI-rich dwarf ELSBGs from [70] and super-thin galaxies from [73]. Both studies provide comparisons between edge-on ELSBGs and other LSBGs, concluding that ELSBGs exhibit similar properties to other LSBGs.

3.1. HI-Rich Dwarf ELSBG Sample

Ref. [70] present 281 HI-rich dwarf ELSBGs from the crossmatch between Sloan Digital Sky Survey Data Release 7 and the 40% ALFALFA catalog. They require the $b/a \leq 0.3$ in the SDSS g-band or r-band to select edge-on galaxies. They compare their ELSBGs with face-on HI-rich dwarf LSBGs from [33] and conclude that the differences between edge-on and face-on LSBGs tend to be much smaller.

We compare the physical parameters of Cao23 ELSBGs with these HI-rich dwarf ELSBGs. We use the same method with [70] to obtain $\mu_{0_{face}}$ in Figure 2a. Cao23 ELSBGs show slightly fainter $\mu_{0_{face}}$ (with the mean $\mu_{0_{face}}$ value 22.89 ± 0.51 mag arcsec⁻²) than HI-rich dwarf ELSBGs (with the mean $\mu_{0_{face}}$ value 22.67 ± 0.47 mag arcsec⁻²). The mean value of b/a of Cao23 ELSBGs and HI-rich dwarf ELSBGs is 0.20 ± 0.06 and 0.18 ± 0.08 , respectively.

The g-band absolute magnitude of HI-rich dwarf ELSBGs is much fainter than that of the Cao23 ELSBGs in Figure 2c. The mean value of M_g for Cao23 ELSBGs and HI-rich dwarf ELSBGs is -19.36 ± 0.85 mag and -16.63 ± 1.12 , respectively. Cao23 ELSBGs are selected with SDSS fiber spectra, which leads to faint targets being missed. The difference in luminosity reflects the varying mass distributions between the two samples. These HI-rich dwarf LSBGs are mainly occupied by low-mass galaxies ($<10^9 M_\odot$, [35]), while the Cao23 ELSBGs are, on average, more massive.

The scale length (r_s) of Cao23 ELSBGs is larger than that of HI-rich dwarf ELSBGs. The mean value of r_s of Cao23 ELSBGs is 3.19 ± 1.48 kpc, while for HI-rich dwarf ELSBGs, it is 1.84 ± 0.89 kpc. HI-rich LSBGs are primarily composed of dwarf LSBGs [33,70], indicating that they are likely small galaxies.

We compare the observed (g-r) color for two ELSBG samples in Figure 2e. Cao23 ELSBGs are redder in (g-r) color than HI-rich dwarf ELSBGs. The mean value of (g-r) of Cao23 ELSBGs and HI-rich dwarf ELSBGs is 0.46 ± 0.16 and 0.34 ± 0.11 , respectively.

Overall, Cao23 ELSBGs show slightly fainter surface brightness, more extended disk structure, and redder color than HI-rich dwarf ELSBGs from [70]. The redder color and larger r_s of Cao23 ELSBGs are consistent with Cao23 ELSBGs being on average more massive.

3.2. Super-Thin Galaxies Sample

Ref. [73] present a sample of super-thin (ST) galaxies with b/a of 0.05–0.1 and no bulge component (the center surface brightness in B-band (μ_B) > 23.0). A comparison LSBG sample with $\mu_B > 23.0$ also is included in [73]. Ref. [73] compares the SFR of STs and LSBGs by different methods and conclude that STs and LSBs have equal intrinsic SFR, which possibly implies STs are just the LSBGs seen edge-on.

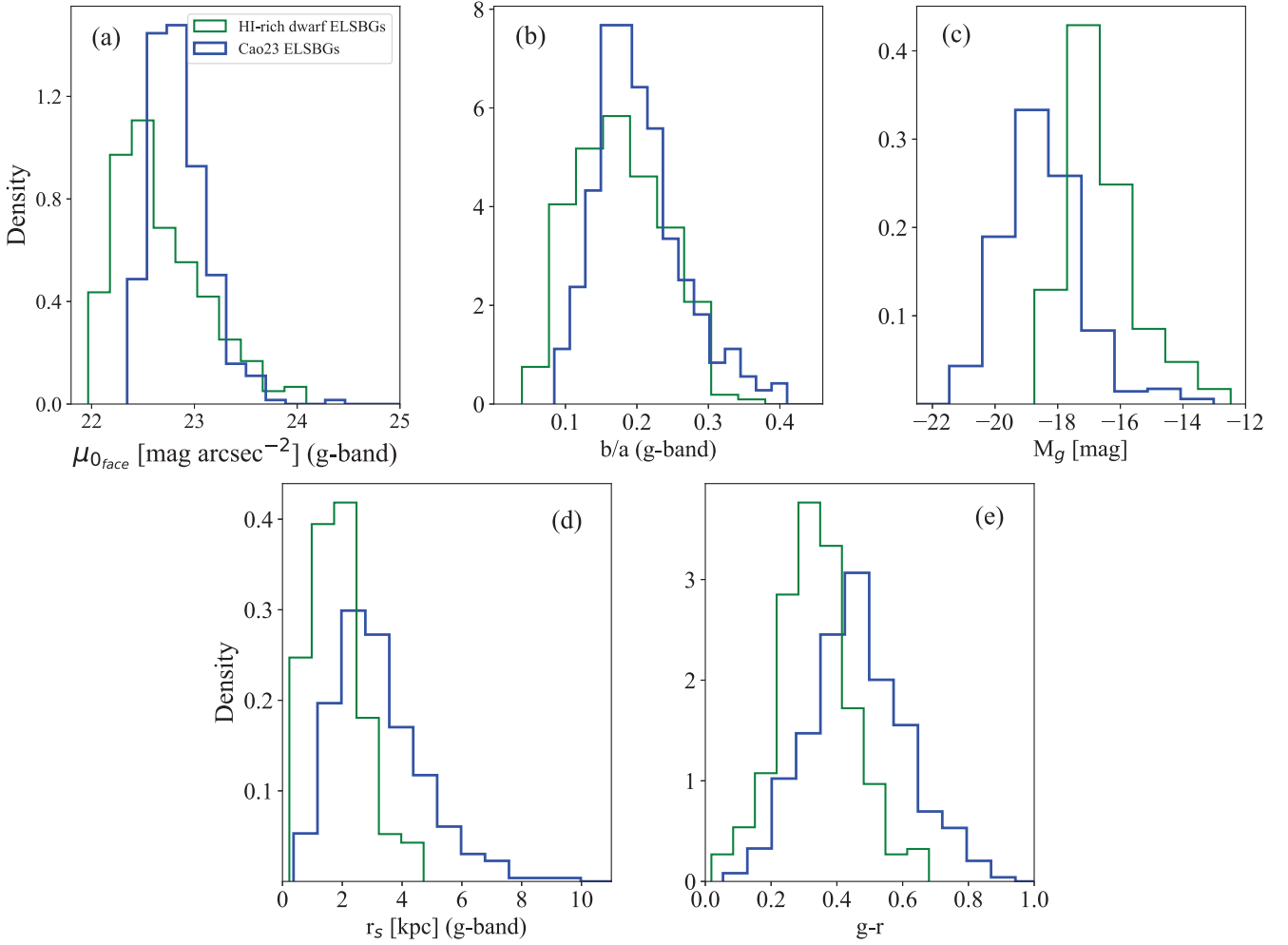


Figure 2. Panel (a): The distribution of μ_{0_face} in g-band. Panel (b): The distribution of axis ratio (b/a) in g-band. Panel (c): The distribution of g-band absolute magnitude. Panel (d): The distribution of scale length (r_s) of g-band in the unit of kpc. Panel (e): The distribution of (g-r) color. The blue and green line represents Cao23 ELSBGs and HI-rich dwarf ELSBGs from [70] in all panels, respectively.

Cao23 ELSBGs show a comparable distribution of M_r with the STs and LSBGs' sample for $M_r < -17$ mag in Figure 3a.

We compare the SFR of Cao23 ELSBGs with those of STs and LSBGs. We apply the same code (Multi-wavelength Analysis of Galaxy Physical Properties, MAGPHYS [74]) to obtain SFR as [73]. Ref. [73] used ten bands of photometric data from 0.15 to 2.2 μm for SED fitting. We require that Cao23 ELSBGs have observation data in GALEX: FUV, NUV, SDSS: u,g,r,i,z, and WISE: W1, W2, and a signal-to-noise ratio (SNR) larger than five in W2, and they also keep consistent the similar wavelength coverage of [73]. Overall, 192 Cao23 ELSBGs meet the requirement. We use the model magnitude associated with de Vaucouleurs and exponential model fits in each band to avoid pollution from the nearby targets. The SEDs cover the rest-frame wavelength ranges from FUV to NIR (from 0.15 to 4.6 μm) for Cao23 ELSBGs. This subsample is also used in Sections 4 and 6.

Stellar population synthesis is used to predict the observed SED [75] and model the spectral evolution of the stellar population with an IMF. The IMF is assumed [58]. The IMF and the dust attenuation of the stellar spectrum are modeled using the two-component model of [76] in MAGPHYS code. The exponentially declining SFH model is parametrized as $\text{SFR}(t) \propto \exp(-\gamma t)$, which is characterized by an age t_{gal} of the galaxy and star formation time-scale γ^{-1} , and random bursts are set to occur with equal probability at all times [77].

We show the Stellar Mass-SFR (M_* -SFR) relation in Figure 3b of 75 Cao23 ELSBGs with reduced χ^2 values smaller than five. The $\text{SFR}_{\text{MAGPHYS}}$ distribution of Cao23 ELSBGs with a median value of 0.22 ± 0.55 is consistent with that of STs ($0.2^{+0.9}_{-0.2}$). The LSBGs in Figure 3b show a wide range and have larger $\text{SFR}_{\text{MAGPHYS}}$ (with median value $0.4^{+2.2}_{-0.3}$) than that of Cao23 ELSBGs and STs on average. The range of $\text{sSFR}_{\text{MAGPHYS}}$ for Cao23 ELSBGs is from 0.01 to 0.1 Gyr^{-1} .

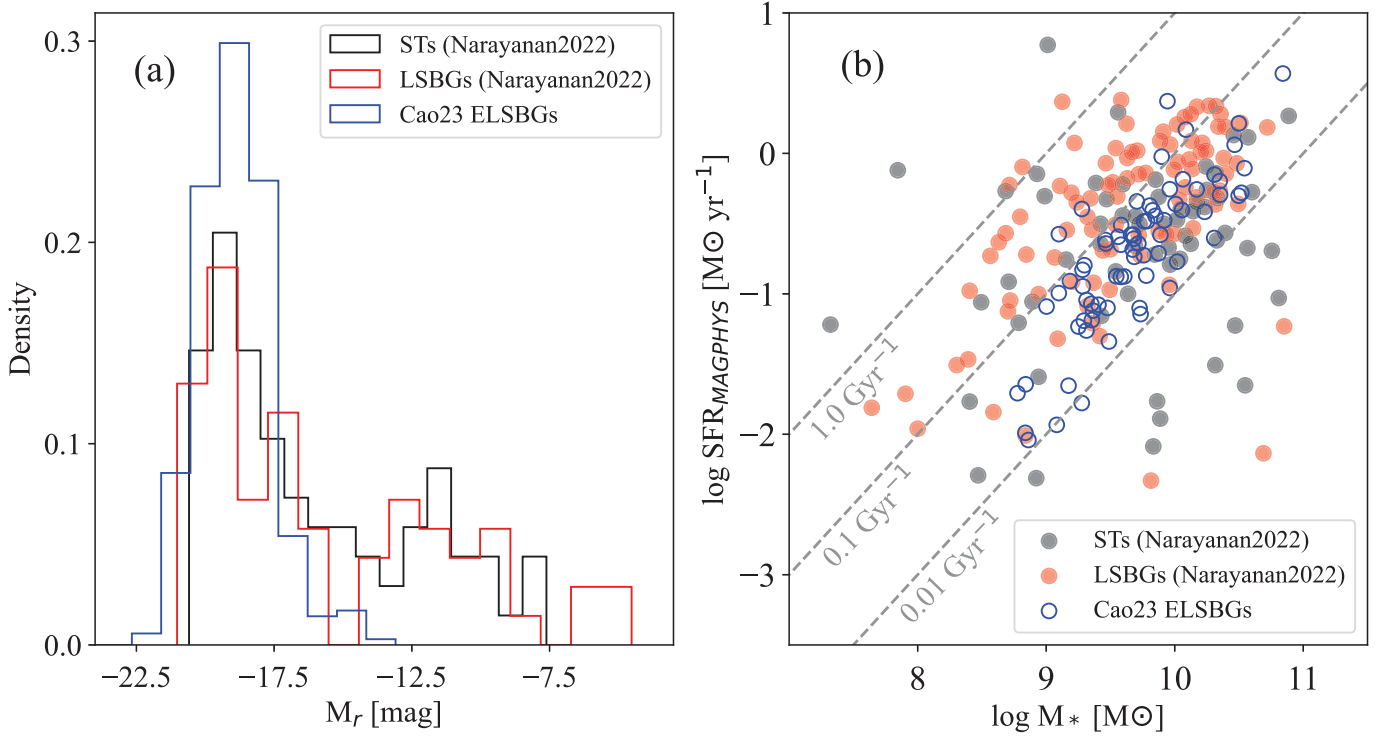


Figure 3. Panel (a): The distribution of M_r of Cao23 ELSBGs (blue line), STs (black line) [73], and LSBGs (red line) [73]. Panel (b): The M_* -SFR relation of Cao23 ELSBGs (open blue circles), STs (gray dots) [73], and LSBGs (red dots) [73]. The gray dashed lines, arranged from bottom to top, correspond to specific star formation rates (sSFRs) of 0.01 Gyr^{-1} , 0.1 Gyr^{-1} , and 1 Gyr^{-1} .

4. SFR of Cao23 ELSBGs

We use the infrared luminosity of WISE W3 12 μm and the SED fitting method to estimate the SFR for Cao23 ELSBGs. The W3 luminosity traces the emission from polycyclic aromatic hydrocarbon (PAHs), which is well related with the star formation rate [78]. And W3 luminosity–SFR conversion is not affected by dust attenuation. However, since only one third of our sample has high SNR W3 photometries, we also obtain the SFRs through composite stellar population synthesis.

4.1. SFR Derived from W3 Luminosity

The 131 Cao23 ELSBGs show WISE W3 12 μm detection with SNR values larger than three. The redshifts of Cao23 ELSBGs are smaller than 0.14, and we have ignored the k-correction. For most of our object, our treatment of observed 12 μm flux as an SFR estimator will induce less than 10% error, which is small compared with the photometric error. We derive SFR_{W3} following the formula form [79,80]:

$$\log \text{SFR}_{\text{W3}} = 0.66 \times (\log L_{\nu}(\text{W3}) - 22.5) + 0.16 \quad (1)$$

with $L_{\nu}(\text{W3})$ in W Hz^{-1} and [58] IMF.

Figure 4 shows the SFR_{W3} and sSFR_{W3} of 131 Cao23 ELSBGs. Overall, Cao23 ELSBGs deviate from the main sequence (MS) and around the 50% level of the MS line. The MS line

in Figure 4 is according to SFR_{W3} . We note that the MaNGA edge-on sample was selected based on two considerations: edge-on inclination and no clear evidence for ongoing major merging interaction [80]. The SFR_{W3} and sSFR_{W3} of the MaNGA edge-on sample show a wider range than that of the Cao23 ELSBGs in Figure 4. The typical error is the mean value of the SFR_{W3} error, which is derived from the W3 band magnitude error.

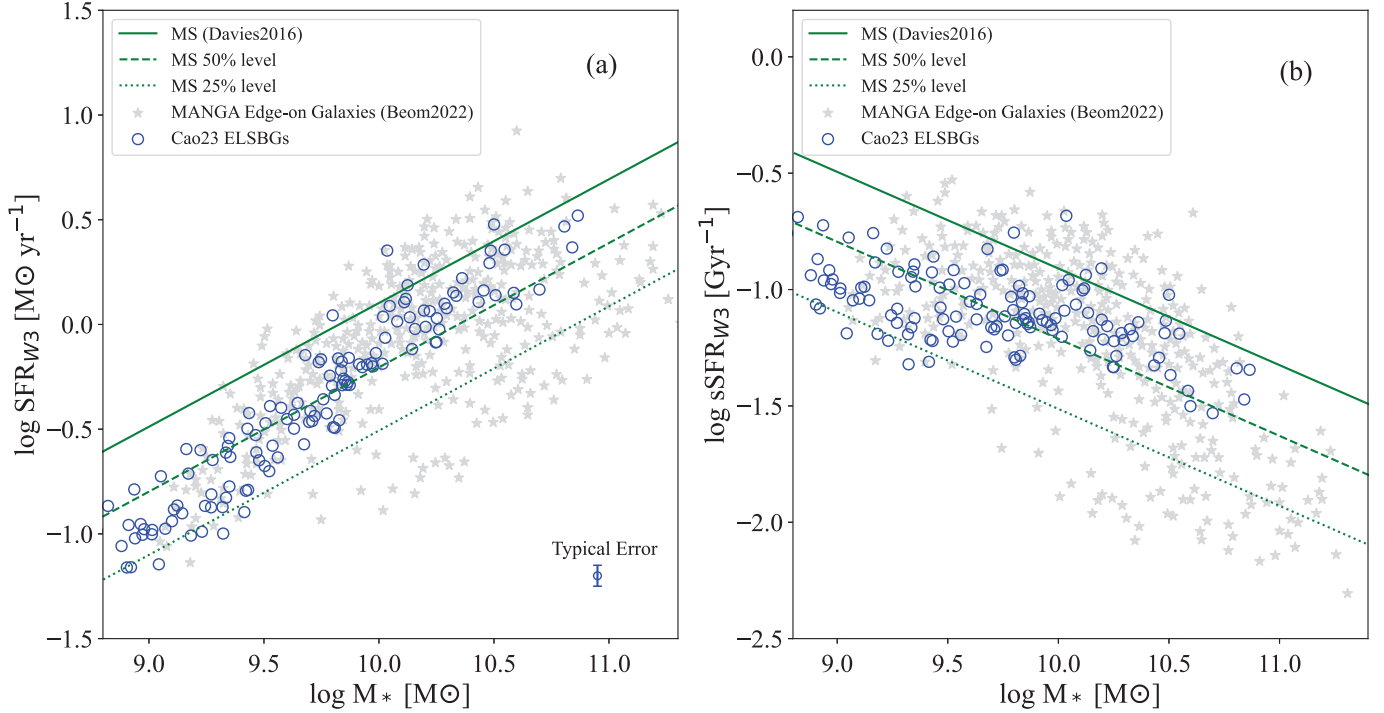


Figure 4. Panel (a): SFR_{W3} versus M_* of 131 Cao23 ELSBGs (open blue circles) and MaNGA 472 edge-on galaxies (gray stars) from [80]. The typical error is the mean value of the SFR uncertainty derived from the W3 magnitude error. Panel (b): sSFR_{W3} versus M_* of 131 Cao23 ELSBGs (open blue circles) and MaNGA 472 edge-on galaxies (gray stars) from [80]. The green line represents the main sequence line for star-forming nearby galaxies with SFR derived from W3 luminosity [79,80]. The dashed and dotted lines indicate 50% and 25% levels of the main sequence lines from [80].

4.2. SFR Derived from CIGALE

We apply the Code Investigating GALaxy Emission (CIGALE, [81,82]) to perform the SED fitting and estimate SFRs of 192 Cao23 ELSBGs (multi-band data from GALEX (FUV, NUV), SDSS (u, g, r, i, z), and WISE (W1, W2). The SEDs cover the rest-frame wavelength from 0.15 to 4.6 μm).

The stellar population synthesis models are from [75] with a [58] IMF. We set the stellar metallicities of 0.0001, 0.0004, 0.004, and 0.008. We employed the delayed star formation history (SFH) model consistent with [83]. The dust emission model is from [84] and the dust attenuation is modified [85] attenuation law. The details of the input parameters set are listed in Appendix A Table A1.

We use the likelihood-weighted mean SFR from the CIGALE results in Figure 5. The typical error is the mean value of the $\text{SFR}_{\text{cigale}}$ error from CIGALE outputs. Cao23 ELSBGs show a distribution of $\text{sSFR}_{\text{cigale}}$ in the range of 0.01–0.1 Gyr^{-1} in Figure 5. Different types of LSBGs (UDGs, HI-rich UDGs, dwarf LSBGs and GLSBGs) are also shown in Figure 5. It shows a large scatter in the M_* -SFR relation for those LSBGs with $M_* < 10^{8.5} M_\odot$. Part of the HI-rich dwarf LSBGs from [33] shown in [35] exhibit sSFRs larger than 0.1 Gyr^{-1} and show a larger SFR than Cao23 ELSBGs overall, which means those HI-rich dwarf LSBGs tend to have a larger SFR value. We note the SFR derived from different methods, and sample selection bias would induce the scatter in the M_* -SFR relation.

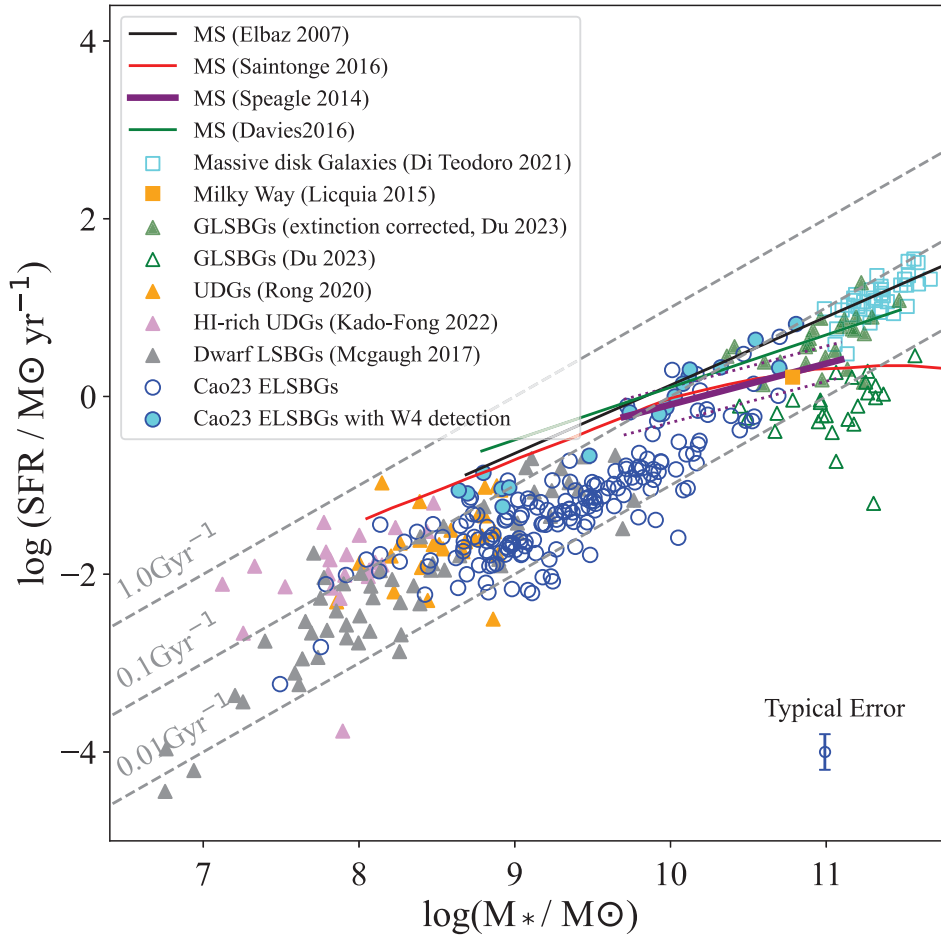


Figure 5. The M_* -SFR relation. Blue open circles denote Cao23 ELBSGs and blue open circles with filled cyan color are those Cao23 ELBSGs with W4 detection. Gray triangles represent dwarf LSBGs from [10], orange triangles distinguish UDGs from [86], and purple triangles are HI-rich UDGs from [83]. Green filled and unfilled triangles are GLSBGs from [35] with dust extinction correction and none correction, respectively. The orange and open cyan squares are the Milky Way [35] and Massive disk galaxies [87], respectively. The purple line, black line, and red line represent the main sequence from [61,88,89]. The green line represents the main sequence line for star-forming nearby galaxies with the SFR derived from W3 luminosity [79,80]. The gray dashed lines, arranged from bottom to top, correspond to specific star formation rates (sSFRs) of 0.01 Gyr^{-1} , 0.1 Gyr^{-1} , and 1 Gyr^{-1} . The typical error is the mean value of the SFR error from CIGALE. The IMF has been corrected to Chabrier IMF for all data points.

The variable integrated galactic initial mass function (IGIMF) has been mentioned in [56,90,91]. The IGIMF systematically varies with the global SFR and metallicity and also affects the MS [92]. Ref. [92] shows that estimates of the SFRs for local star-forming galaxies differ by factors of approximately 10 between the IGIMF theory to the canonical universal IMF, particularly at the low-mass end. Cao23 ELBSGs primarily include massive galaxies ($M_* > 10^{9.0} M_\odot$), and the difference for those massive ones are small in [92]. Additionally, the lack of far-infrared data in the SED fitting may impact results, as noted by [93]. Ref. [94] estimated that using only stellar SEDs (sampling starlight from 0.4 to $2.2 \mu\text{m}$) introduces an uncertainty of about 0.3 dex. However, this does not affect the main results in Figure 5, which show that Cao23 ELBSGs lie below the MS.

5. HI-Gas Content and t_{dep}

We compare the HI-gas content and depletion time t_{dep} , which characterizes the evolutionary status of star formation for HI-rich and normal dwarf LSBGs in Figure 6.

Figure 6a shows the relation between HI gas mass (M_{HI}) and M_* . The HI-rich UDGs display higher M_{HI} values than the dwarf LSBGs at similar stellar masses. Among the Cao23 ELBSGs, 75 have HI detections [23] and belong to HI-rich ones. Figure 6b presents the relation between t_{dep} and sSFR. The SFRs of both dwarf LSBGs [10] and HI-rich dwarf LSBGs [25] are derived from $H\alpha$ luminosities, and both share a similar stellar mass range. The parent sample of HI-rich dwarf LSBGs is from [33]. The HI-rich dwarf LSBGs exhibit higher sSFR and shorter t_{dep} than low-HI-fraction dwarf LSBGs, indicating that a higher HI gas content correlates with a higher SFR. We also show that 35 Cao23 ELBSGs with HI detection also show in Figure 6b. Additionally, 35 Cao23 ELBSGs with HI detection are located close to dwarf LSBGs but are, on average, more massive. This trend is evident in Figure 7.

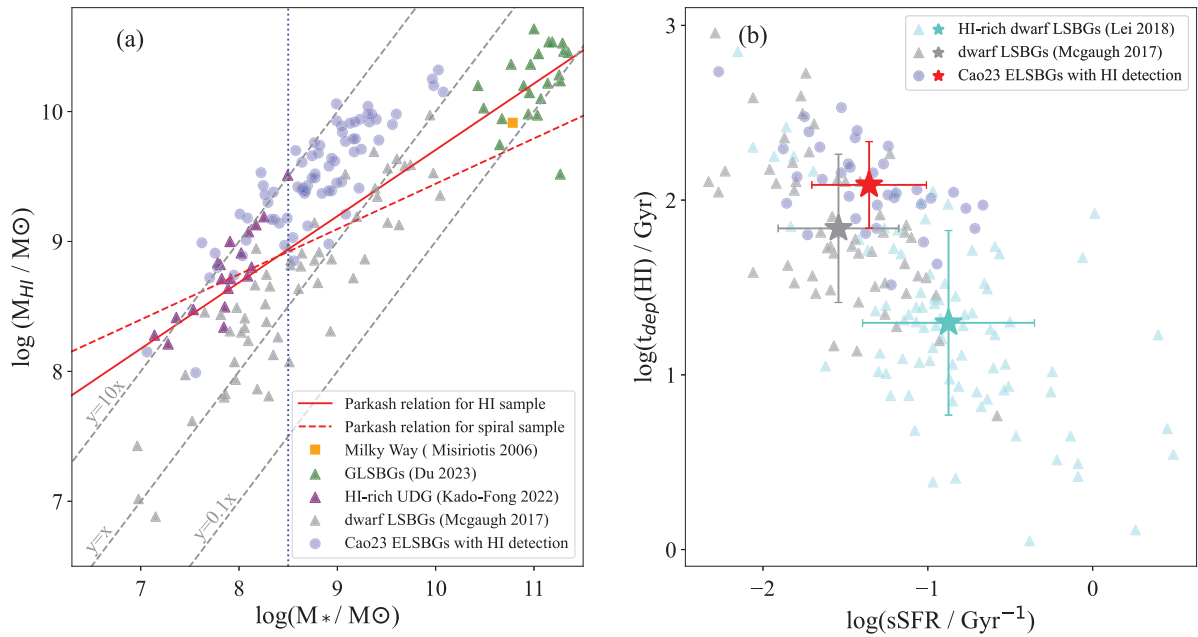


Figure 6. Panel (a): The M_{HI} - M_* relation for LSBGs. Blue filled circles denote Cao23 ELBSGs with HI detection, gray triangles represent dwarf LSBGs from [10], and purple triangles distinguish UDGs from [83]. Green filled triangles are GLSBGs from [35]. The orange square represents the Milky Way. The red solid line represents the relation followed by the HI-selected sample of galaxies with a 1σ scatter of 0.5 dex, and the red dashed line shows the relation followed by the spiral galaxies out of the M_* -selected sample with a 1σ scatter of 0.4 dex from [34]. Panel (b): The t_{dep} (HI) vs. sSFR diagram of LSBGs. Blue filled circles denote Cao23 ELBSGs with HI detection, gray triangles denote dwarf LSBGs from [10], and cyan triangles indicate HI-rich dwarf LSBGs from [25]. The red star symbol is the mean value of Cao23 ELBSGs, while the gray and cyan stars represent the mean values for dwarf LSBGs and HI-rich dwarf LSBGs, respectively.

We show the sSFR and M_* (sSFR- M_*) relation in Figure 7. The sSFR values of Cao23 ELBSGs derived from SFR_{cigale} . The sSFR values for the majority of Cao23 ELBSGs fall in the range of 0.1 to 0.01 Gyr^{-1} . Notably, a subset of ELBSGs exhibits higher sSFR values exceeding 0.1 Gyr^{-1} . Thirty-five Cao23 ELBSGs with HI detection show average higher sSFR values to other Cao23 ELBSGs. The sSFR shows a large scatter below $10^{8.5} M_{\odot}$ in Figure 7. The sSFR values of dwarf LSBGs [10] show a systematically lower sSFR than those of HI-rich UDGs [83] when considering stellar masses below $10^{8.5} M_{\odot}$. Specifically, the mean sSFR for dwarf LSBGs and HI-rich UDGs with $M_* < 10^{8.5} M_{\odot}$ stands at 0.04 and 0.16 Gyr^{-1} , respectively. GLSBGs and/or massive Cao23 ELBSGs on average have lower sSFR values in Figure 7.

HI-rich dwarf LSBGs exhibit higher sSFRs than the low-HI-gas fractions dwarf LSBGs at the low-mass end. The authors of [31] explored the molecular gas content in nine

HI-rich LSBGs and found that these galaxies may have higher star formation efficiency (SFE; defined as SFR/M_{H_2}) compared to typical star-forming galaxies. This suggests that HI-rich dwarf LSBGs may contain more molecular gas than low-HI-fraction dwarf LSBGs, contributing to their elevated sSFR.

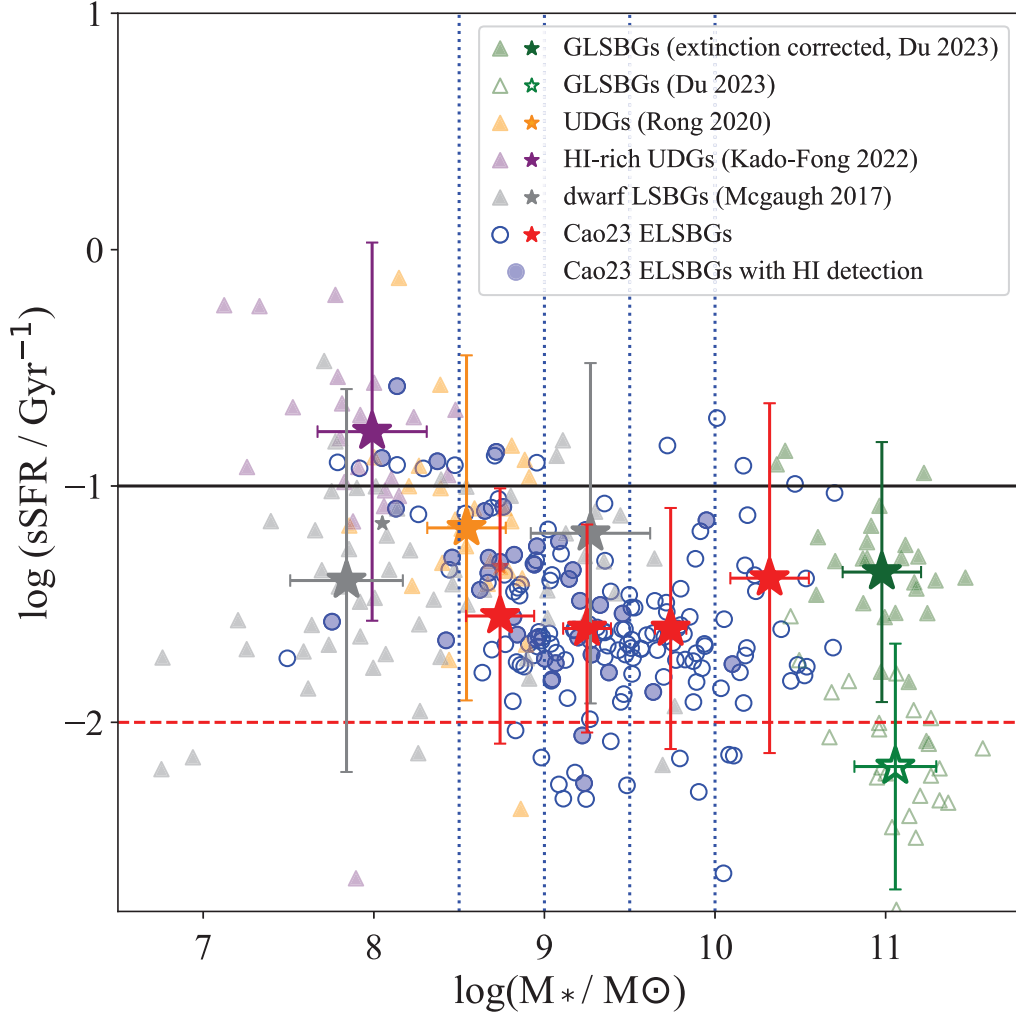


Figure 7. The sSFR- M_* relation for LSBGs. Blue open circles denote Cao23 ELSBGs, gray triangles represent dwarf LSBGs from [10], and orange triangles and plum triangles distinguish UDGs from [83,86], respectively. The blue dotted lines demarcate the boundaries of each stellar bin, with intervals of 0.5 dex in $\log(M_*/M_\odot)$ spanning from 8.5 to 10.5. Stellar masses falling below 8.5 and exceeding 10.5 are grouped into single bins, respectively. The red star symbols signify the mean values within each stellar bin. The black line is at $\log(\text{sSFR}/\text{Gyr}^{-1}) = -1.0$. Furthermore, a red dashed line at $\log(\text{sSFR}/\text{Gyr}^{-1}) = -2.0$ provides sSFR thresholds with passive galaxies typically falling below the red line. Purple and gray star symbols represent the mean values of HI-gas UDGs from [83] and dwarf LSBGs from [10] with $\log(M_*/M_\odot) < 8.5$. The IMF has been corrected to Chabrier IMF for all data points.

6. Non-Parametric Star-Formation History of Cao23 ELSBGs

Parametric models are widely employed for inferring the SFH of galaxies through the SED fitting technique [95]. However, this method relies on prior physical parameters [96,97]. The non-parametric SFH [98] assumes a constant SFR in each interval of a cosmic time grid instead of an exponential form of the $\text{SFR}(t)$ [99–101].

We use the SED-fitting code PROSPECTOR [102] to fit the photometric data from an FUV to an NIR of 192 Cao23 ELSBGs. We adopt the non-parametric fixed-bin SFH models

preferred by [98]. The non-parametric fixed-bin SFH is described by a piecewise function where the SFR is a constant in each of the N time bins, and the edges of each time bin are fixed. The SFR in each fixed time bin is determined using the “continuity” prior, which places a Student-t prior on the log of the ratio of the SFR in adjacent bins ($\log \text{SFR}_{\text{ratio}}$). We use the same $N = 7$ time bins (the edges are 0 Gyr, 0.03 Gyr, 0.1 Gyr, 0.33 Gyr, 1.1 Gyr, 3.6 Gyr, 11.6 Gyr, 13.7 Gyr) as [98] in our non-parametric SFH and apply [58] IMF. The dust type is chosen as the default power-law curve. The free parameters of dust attenuation and stellar metallicity apply flat priors over $0 < \tau_V < 3$ and $-2 < (\log Z/Z_\odot) < 0.19$. The redshift is fixed to SDSS spectroscopic redshift. We choose an ensemble Markov chain Monte Carlo (EMCMC, [102]) sampler to fit the model.

We compare the stellar mass assembly histories in different stellar bins. We show the on-average cumulative SFH in Figure 8. The low-mass ELSBGs ($10^{7.8} \leq M_* \leq 10^{9.0} M_\odot$) exhibit a slower stellar assembly history on average compared with high-mass ELSBGs ($M_* > 10^{9.0} M_\odot$). The low-mass galaxies build 80% of their stellar mass at look back time 1.4 Gyr, while the high-mass galaxies do so at look back time 3.7 Gyr. Low-mass ELSBGs assembled their 50% and 90% stellar masses at look back time 4.0 Gyr and 0.7 Gyr, respectively. High-mass ELSBGs assembled their 50% and 90% stellar masses at look back time 6.6 Gyr and 1.9 Gyr, respectively.

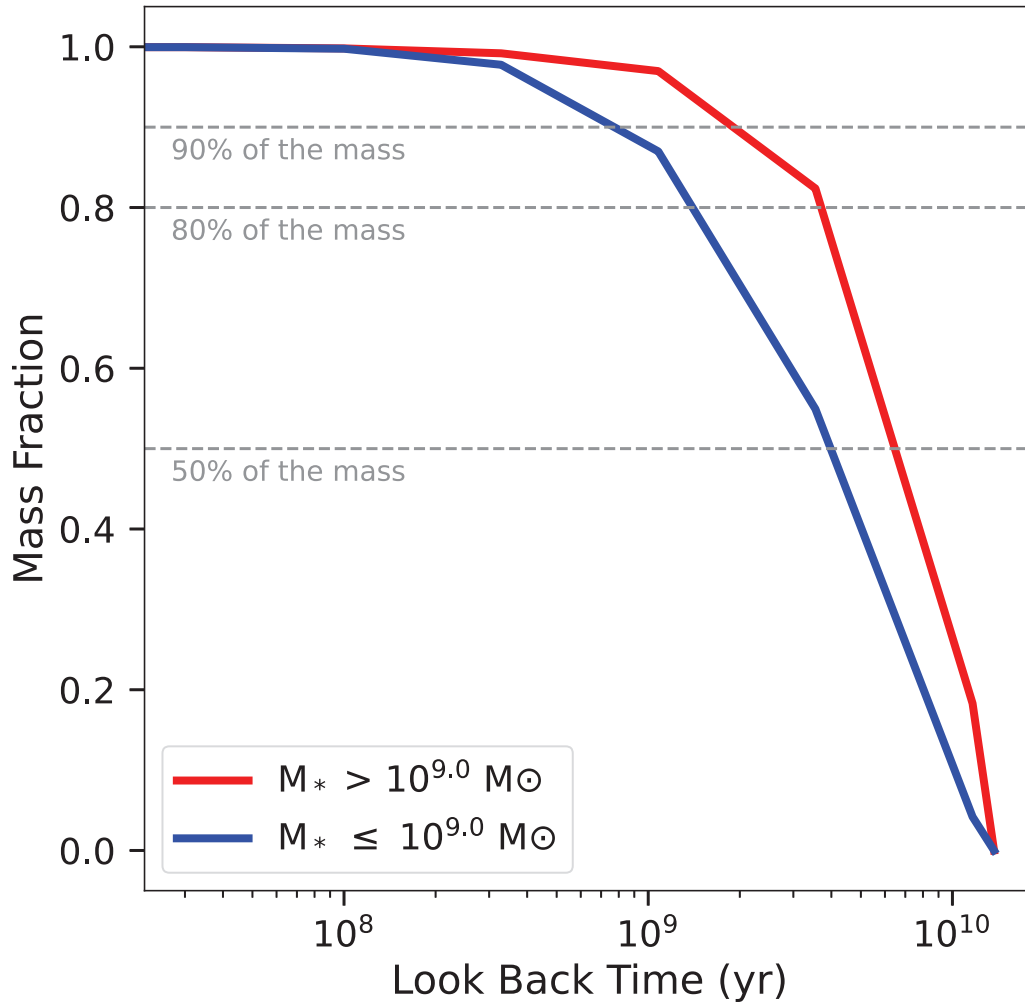


Figure 8. The mass growth curve through the look back times. The red line represents the mean value for those Cao23 ELSBGs with $M_* > 10^{9.0} M_\odot$, and the blue line represents the mean value for those Cao23 ELSBGs with $M_* \leq 10^{9.0} M_\odot$.

Figure 8 illustrates the downsizing paradigm in LSBGs. The authors of [23] analyzed the stellar population of Cao23 ELSBGs using SDSS fiber spectra, finding that more massive Cao23 ELSBGs are dominated by older stellar populations, supporting evidence for downsizing [103]. Ref. [104] noted an inverse correlation between galaxy mass and star formation duration (i.e., downsizing) by examining the $[\alpha/\text{Fe}]$ abundance with variable galaxy-wide IMF. Ref. [23] also show an α -enhancement in massive Cao23 ELSBGs, which is consistent with the downsizing trend [103,104]. Additionally, ref. [92] discussed the lower star formation efficiencies in more massive galaxies, and massive disk galaxies initiate star formation earlier than their low-mass counterparts with variable galaxy-wide IMF. The findings from Cao23 ELSBGs align with prior studies on normal spiral and elliptical galaxies [92,103–105]: massive LSBGs tend to have lower sSFRs, while low-mass ELSBGs form later than their massive counterparts. These results suggest that LSBGs share similar stellar build-up histories with normal galaxies.

7. Conclusions

This paper investigates the star formation rate and star formation history of ELSBGs from [23]. The principal outcomes are summarized as follows:

1. We compare Cao23 ELSBGs with HI-rich dwarf ELSBGs from [70] and super-thin (ST) galaxies from [73]. These ELSBGs have wide range luminosities ($-22 < M_r < -13$). The mean value of r_s for Cao23 ELSBGs is 3.19 ± 1.48 kpc, which is larger than that of HI-rich dwarf ELSBGs. Cao23 ELSBGs also show a redder (g-r) color than that of HI-rich dwarf ELSBGs. Cao23 ELSBGs are on average more massive ones compared to HI-rich dwarf LSBGs. We use the same method as [73] regarding the derived $\text{SFR}_{\text{MAGPHYS}}$. Cao23 ELSBGs and STs share a similar position in the M_* -SFR relation. The comparison results suggest that Cao23 ELSBGs share similar properties with other LSBGs.
2. We estimated the SFR by WISE W3 luminosity and SED fitting methods. SFRs of Cao23 ELSBGs are below the main sequence in the M_* -SFR relation. The sSFR of ELSBGs are mainly in the range of $0.01\text{--}0.1 \text{ Gyr}^{-1}$.
3. The HI-rich dwarf LSBGs show larger sSFR and shorter t_{dep} values than low-HI gas fraction dwarf LSBGs, which means higher HI-gas content dwarf LSBGs tend to have higher SFRs. HI-rich dwarf LSBGs likely contain more molecular gas than their low HI-gas fraction counterparts, contributing to their higher sSFR. In contrast, GLSBGs and more massive Cao23 ELSBGs tend to have lower sSFR values on the sSFR- M_* relation.
4. We acquire the non-parametric SFHs of ELSBGs by SED fitting. The SFHs are divided into seven bins of look back time, and the SFR in each bin is assumed to be constant. We find that high-mass ($M_* > 10^{9.0} M_\odot$) ELSBGs assembled mass at an earlier time than that of low-mass ELSBGs. Combined with the results from the fiber spectra in [23], we conclude that our results support a downsizing trend for LSBGs.

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Data Availability Statement: The derived data generated in this research will be shared on reasonable request to the corresponding author.

Conflicts of Interest: The authors declare no conflicts of interest.

Appendix A

We list all of the input parameters settings of CIGALE used in Section 4.2.

Table A1. Parameters setting of each module in code CIGALE.

Modules	Parameters	Values
sfhdelayedbq	tau_main	500, 750, 1100, 1700, 2600 3900, 5800, 8800, 11000 (Myr)
	age_main	2000, 4500, 7000, 9500, 12000 (Myr)
	age_bq	0.0 (Myr)
	r_sfr	0.01, 0.03, 0.1, 0.3, 1.0, 3.0, 10
	sfr_A	1.0 ($M_{\odot}\text{yr}^{-1}$)
bc03	imf	Chabrier
	metallicity	0.0001, 0.0004, 0.004, 0.008. ($M_{\odot}$)
	separation_age	10 (Myr)
nebular	logU	−2.0
distant_modified_starburst	E_BV_lines	0.0, 0.005, 0.0075, 0.011, 0.017, 0.026, 0.038, 0.058, 0.13, 0.2, 0.44, 0.66, 1.0 (mag)
	E_BV_factor	0.25, 0.50, 0.75
	Amplitude of the UV bump	0.0
	Powerlaw_slope	0.0
	Ext_low_emission_lines	Milky Way
	Rv	3.1
themis	qhac	0.02, 0.06, 0.10, 0.17, 0.24, 0.28
	umin	0.1, 0.15, 0.3, 0.35, 0.4 0.5, 0.6, 0.7, 0.8, 1.2, 2.0
	alpha	2.0
	gamma	0.0, 0.001, 0.002, 0.004, 0.008 0.016, 0.031, 0.063, 0.13, 0.25, 0.5

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Jet Precession in Gamma-Ray Bursts

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Abstract: Jet precession is thought to be a ubiquitous phenomenon in astronomical events of various scales, including gamma-ray bursts (GRBs). If GRB jets undergo precession, periodic features might be introduced into their light curves. Detecting these periodic signals is therefore crucial for confirming the properties of GRBs' central engines. However, periodic signals are always missing from observed GRB light curves. Against this backdrop, the broader effects of jet precession on GRBs have been widely studied. In this review, we summarize recent research progress on jet precession in GRBs. The main content focuses on four aspects of the effects of jet precession on GRBs: light curves, jet structures, polarization, and gravitational waves.

Keywords: gamma-ray bursts; gravitational waves; polarimetry; relativistic jets; shocks

1. Introduction

In high-energy astrophysics, jets refer to narrow beams of matter and energy with velocities close to the speed of light, and they are ubiquitous in astrophysical systems of various scales, such as active galactic nuclei (AGNs), X-ray binaries, and gamma-ray bursts (GRBs). Precession phenomena are expected to occur when there is misalignment between the direction of the launched jets and the angular momenta of part of these systems.

Jet precession in AGNs, including blazars [1–6], quasars [7–9], and radio galaxies [10,11], has been extensively studied over several decades. Observational evidence for the precession of AGN jets includes periodic signals in optical or radio light curves [12,13], periodic variations in the position angle of outflow components [14–18], and the S- or Z-shaped structures of large-scale radio jets [19–23], such as the clear S-shaped structure in the kpc-scale jets of the blazar 5BZU J1345+5332 [24]. There are several prevalent mechanisms driving jet precession in AGNs, including spin-induced precession [25–27], disk-driven precession [28,29], and Newtonian-driven precession [30–33]. Spin-induced and disk-driven precession are caused by the angular momenta of supermassive black holes (BHs) and their accretion disks, respectively, and both are mainly considered in single supermassive BH scenarios. Newtonian-driven precession arises from the rotation of an accompanying supermassive BH, and it is considered in binary supermassive BH scenarios. Spin-induced precession, also known as the Lense–Thirring precession effect [25], means that the frame drag caused by a rapidly rotating BH, such as a Kerr BH, can cause a particle to precess around the BH with its orbital plane inclined with respect to the equatorial plane of the BH. Thus, if the angular momentum directions of the BH and the disk are misaligned, the jets will precess. In addition, the Bardeen–Petterson effect is an extension of the Lense–Thirring effect [26] involving a viscous accretion disk, where regions of the disk corresponding to different radii are affected differently by the Lense–Thirring effect. Thus, under the Bardeen–Petterson effect, the disk is warped, resulting in three distinct regions: the inner, transition, and outer regions. The rotation axis of the inner region is aligned with the spin axis of the BH, while the outer region retains the original rotation direction. Continuous precession occurs in the transition region, since the rotation axis is misaligned with the spin axis of the BH. Regarding disk-driven precession, the precession of a jet ejected from the innermost disk can be driven by the massive outer disk because the angular momentum of

the massive outer disk greatly exceeds the total angular momentum of the inner disk and the central supermassive BH [28,29]. In Newtonian-driven precession, because the accretion disk around the primary BH is inclined to the orbital plane of the binary, the orbital motion of the companion BH can cause precession of the disk through dynamic torque, resulting in precession of the jet launched by the disk [30,31]. In such cases, the period of jet precession can reach thousands of years, which is usually used to explain the precession of large-scale (Kpc or Mpc) jets. However, the specific precession mechanism eventually needed to explain the precession phenomena in AGNs is determined not only by the precession period but also by multidimensional information.

In X-ray binaries, the strongest observational evidence for jet precession comes from the well-known SS433, whose jets change their orientation in space over a period of 162 days [34–36]. In addition, quasi-periodic oscillations (QPOs) observed in the emission are also well explained by jet precession [37,38], as seen in MAXI J1820+070 [39].

The observational and theoretical studies of AGNs and X-ray binaries have been explored in detail. Both involve BH accretion systems. Whether in AGNs or X-ray binaries, within the framework of BH accretion, jet precession mainly depends on the BH mass and spin, the accretion rate, and the type of accretion disk. Therefore, the mechanisms driving jet precession in these phenomena are intrinsically identical. Since GRBs can be induced by BH accretion systems, it is widely believed that GRB jets should also exhibit precession, with precession mechanisms analogous to those in AGN and X-ray binaries.

In GRBs, observational images of jet precession, like those of radio jets in AGNs, cannot be obtained due to the large distance, small scale, short duration, and precise period signals of jet precession, as observed in the X-ray binary SS433, have yet to be detected. Although convincing evidence has been consistently absent, GRB jet precession is universally expected.

GRBs are extremely energetic events in the Universe, characterized by a sudden and intense flash of gamma rays, releasing as much energy in a few seconds as the Sun will release in its entire 10 billion year lifetime, with a typical isotropic energy of about 10^{50} – 10^{52} erg [40,41]. Based on the statistics of GRBs collected by the Burst and Transient Source Experiment (BATSE), GRBs are classified into short and long GRBs depending on whether or not their duration is greater than 2 s [42–44]. For a more physics-driven classification, GRBs can also be divided into Type I and II GRBs [45]. In general, short GRBs are thought to originate from the merging of compact objects [46], and the joint observation of GW170817/GRB 170817A has confirmed that binary neutron star (NS) mergers are the origin of short GRBs [47,48]. Long GRBs are thought to originate from the collapse of massive stars, and some have been found to be associated with supernova explosions [49]. Independent of the progenitors, there are two types of central engines: a rapidly rotating stellar-mass BH surrounded by an accretion disk [50] and a millisecond magnetar [51–53]. In a BH hyperaccretion scenario, the central engines can launch ultrarelativistic jets through neutrino annihilation [54] or Blandford–Znajek (BZ) [55] mechanisms. The emission of GRBs includes both prompt emission and afterglows. Prompt emission is generally thought to be caused by energy dissipation from ultrarelativistic jets. Prevalent models of energy dispersal include internal shock models [56–58], photosphere models [59,60], and magnetic reconnection models [61]. Afterglows are believed to originate from the interaction between the jets and the circumburst medium. For both, the dominant radiation mechanisms are considered to be leptonic models, i.e., synchrotron emission, synchrotron self-Compton emission, and the external Compton emission of accelerated electrons.

Jet precession can, in varying degrees, contribute to many aspects of GRBs, such as prompt (afterglow) light curves, jet structures, polarization, and gravitational waves (GWs). In this review, we summarize previous efforts to study jet precession in GRBs. In Section 2, the effects of jet precession on GRB light curves are presented. In Section 3, possible ejecta structures under jet precession are discussed. In Section 4, the properties of the polarization signal with jet precession are provided. In Section 5, the properties of GW signals under jet precession are presented. In Section 6, we give a brief summary.

2. Light Curves with Jet Precession in GRBs

2.1. Prompt Light Curves with Jet Precession

The light curves of GRB prompt emission are highly variable. Some show a single pulse with a rapid rise and exponential decay, while others show multiple pulses or complex multi-episode patterns separated by quiescent time. In addition, certain light curves exhibit a weak precursor component or extended emission. Periodic signals are considered convincing evidence of jet precession. However, early studies failed to identify high-quality periodic signals in prompt light curves [62–64]. This result implies the difficulty encountered by GRB jet precession in producing clear periodic signals, leading researchers to turn their attention to the search for QPOs. In recent years, several studies have been conducted to search for QPOs in the prompt emissions [65–68]. Chirenti et al. (2023) discovered kilohertz QPOs in the prompt emissions of short GRBs 910711 and 931101B using data from the BATSE archive [69]. Xiao et al. (2024) identified a potential QPO signal of approximately 22 Hz in the precursor emission of GRB 211211A [70], and a strong 19.5 Hz flux oscillation was also observed in this burst [71].

It is widely believed that the prompt emission of GRBs originates from the inner regions of the jets, and that jet precession may cause variations to be imprinted in the prompt light curves. Consequently, various jet precession models have been proposed to explain the diverse profiles observed in prompt light curves. Jet precession should essentially be driven by misalignment between the direction of the angular momentum of a BH and accretion matter. The accretion materials are derived from the companion star in short GRBs and from the envelopes of massive collapsars in long GRBs.

Regarding the scenario of binary systems or BH-NS mergers, Blackman et al. (1996) were the first to study a relativistic blob-emitting jet with precession primarily driven by Newtonian tidal torque in GRBs [72]. In their model, the precessing jet originates from a binary pulsar system, where the pulsars may be formed by the collapse of white dwarfs due to accretion. They used this precessing jet model to explain the short durations observed in GRBs, finding that the duration is mainly determined by the timescale during which the jet sweeps across the line of sight. Portegies Zwart et al. (1999) proposed that in a close NS-BH binary system, when the NS fills the Roche lobe of the BH, mass transfer from the NS to the BH occurs, leading to the formation of an accretion disk around the BH [73,74]. Misalignment between the binary angular momentum axis and the BH spin axis can result in a precessing jet, produced via the BZ mechanism. Their model also incorporated the nutation effect, which they applied in an attempt to reproduce the complex shapes of GRB prompt light curves. However, they later found that it was still difficult to explain some observed GRBs with asymmetric light curve shapes when only considering a precessing jet. As a result, they revised the model [75], proposing that a relativistic precessing jet interacts with the interstellar medium, producing a shock wave. The cooling of shock-accelerated electrons is then considered the cause of the asymmetric shapes in the prompt light curves, as well as of exponential decay in the emission flux. Using this modified model, they were able to fit the prompt light curves of several GRBs with reasonable precision. Stone et al. (2013) investigated jet precession in the context of short GRBs, focusing on a scenario where the accretion disk forms following an NS-BH merger [76]. They considered 10 different cases to explore how various disk parameters influence the precession period and number of precession cycles. They found that the precession period typically falls within the range of ~ 0.01 to 0.1 s, while the precession angle is approximately 10° . Additionally, they found that the number of precession cycles is significantly influenced by disk viscosity, where higher viscosity leads to fewer precession cycles, which can further obscure the features of observed light curves. They suggested that these distinct precession characteristics could help differentiate between NS-NS mergers and NS-BH mergers as progenitors of short GRBs. Li et al. (2023) proposed a jet precession model within the framework of NS-BH mergers to predict possible QPOs in the prompt light curves of short GRBs [77]. In their model, the predicted QPOs primarily depend on several angular parameters, including the jet opening angle, viewing angle, and precession angle, with the periods ranging from

0.01 to 0.1 s. Additionally, due to the time required for the disk winds to align the jet with the disk, the QPOs may appear with a delay relative to the trigger time. They suggested that the delayed QPOs are a characteristic feature of NS–BH mergers with misaligned orbital planes.

Regarding BH accretion systems in which progenitors are not considered, Reynoso et al. (2006) investigated spin-induced precession and nutation occurring in a neutrino-cooled disk around a stellar-mass Kerr BH in the framework of GRBs [78]. In their model, the precession and nutation of the disk are transferred to relativistic jets launched via the BZ mechanism. The combination of these effects can explain the oscillating microstructures of the prompt light curves, as well as the shape of a slow rise and fast decay observed in some GRBs. Lei et al. (2007) proposed a precessing jet model that incorporates magnetic coupling processes, accounting for the evolution of the half-opening angle of the magnetic flux on the BH horizon [79]. They found that the model can produce a light curve profile characterized by a fast rise and slow decay. Liu et al. (2010) proposed a model of jet precession driven by a neutrino-dominated accretion disk to explain the prompt light curves of GRBs [80]. In their model, a critical radius is defined where the angular momentum of the BH equals that of the disk. Within this radius, the rotation of the inner disk aligns with the spin direction of the BH, while beyond this radius, the outer disk retains its original rotation. Since the angular momentum of the outer disk is significantly larger than the combined angular momentum of the inner disk and the BH, the outer disk drives the precession of both the inner disk and the BH, resulting in jet precession. The characteristics of the precessing jet depend on the BH mass M_{BH} , accretion rate $\dot{M}$, and spin parameter a . This model not only provides a potential explanation for the temporal evolution of GRB pulses but also demonstrates that the spectral peak energy E_p tracks the flux evolution under jet precession.

These jet precession models were used to explain the characteristics of the prompt light curves of certain GRBs. Wang et al. (2022) proposed that GRB 200826A should be a long GRB resulting from the collapse of a massive star, with the prompt emission consisting of both precursor and main emissions [81]. They interpreted the observed prompt emission, lasting approximately 1 s, as being the precursor emission, and concluded that the main emission was not detected. They attributed this absence to jet precession, which likely caused the radiation region to shift away from the line of sight of the observers. Gao et al. (2023) suggested that the repeating episodes with similar temporal profiles observed in the prompt light curves of some GRBs, such as GRB 950830, could be explained by a precessing jet [82]. Li et al. (2023) attributed the delayed QPO behavior in the prompt light curve of short GRB 130310A to the precession jet from an NS–BH merger [77]. Zhang et al. (2023) used the jet precession and nutation model in [73] to explain three emission episodes in the prompt light curve of GRB 220408B, which exhibited similar temporal and spectral characteristics [83]. They obtained values for the precession and nutation periods of GRB 220408B, 18.4 ± 0.2 s and 11.1 ± 0.2 s.

Overall, although the jet precession model has been employed to explain the complex behavior of prompt light curves, convincingly observational evidence supporting GRB jet precession, namely significantly periodic signals, is always missing, and QPOs are an alternative being considered. A possible reason for the absence of significant periodic signals is that most observed GRBs are viewed nearly on axis, and the precession angle is quite small when their jets are precessing. As a result, the effects of jet precession are weakly reflected in the prompt light curves. Of course, precession should occur in GRBs during their violent evolution. GRBs are catastrophic events, and during the early phases, the central engines might experience instabilities due to irregular accretion, leading to significant and irregular variations in both the precession period and angle as well as in jet luminosity.

2.2. Afterglow Light Curves with Jet Precession

In general, prompt gamma-ray emission is followed by long-lasting (hours to days) multi-band afterglow emission, ranging from radio to X-ray [84], sometimes also accompanied by high-energy and very high-energy photons (>100 GeV), as seen in GRB 221009A, with detected photon energies of up to 5 TeV [85]. Since the first afterglow event was discovered in 1997 (GRB 970228) [86], GRB afterglow has been extensively studied. Multi-wavelength afterglow is believed to be produced by electrons accelerated by the shock waves generated as ultrarelativistic jets sweep up the circumburst medium, according to the external shock model [84,87]. In the Swift era, a large number of X-ray afterglow events are observed, and the shapes of the X-ray light curves exhibit diversity. These can be categorized into five segments: rapid decay, flat decay or plateau, normal decay, jet break, and one or more flares [88]. These five phases are thought to have different physical origins, with fast-decaying phases having the same origin as the prompt emission. Flat decay (plateaus) may be due to energy injection from the central engines. Normal decay with a power-law index is in agreement with the prediction of the standard external shock model. Jet break occurs when the visible region is larger than the physical region of the jets. Some flares are thought to be likely associated with the activation of the central engines. A proposition of current relevance, regarding the shape of the X-ray light curve, is the idea that most shallow decay (plateaus) and flares share the same physical origin [89], though the origin of the plateaus is still under debate.

Jet precession can affect not only prompt light curves but also afterglow light curves. One possible effect is the emergence of QPO behavior in the afterglow light curves. Currently, significant effort has been dedicated to searching and studying the behavior. One view is that QPOs in the afterglow light curves originated from precessing millisecond magnetars. In cases where the central engine of a GRB is a rapidly rotating and highly elliptical millisecond magnetar, Suvorov and Kokkotas (2020) proposed that its early precession could cause QPOs of spin-down luminosity, which should manifest as QPO signals in the plateau phase of X-ray afterglow light curves [90]. To test this, they analyzed two GRBs with plausible QPOs in X-ray plateaus likely driven by millisecond magnetars: long GRB 080602 and short GRB 090510. They fitted the X-ray afterglow light curves of these bursts using two models: one for a non-precessing millisecond magnetar and another for a precessing one. Using the Akaike Information Criterion, they found that the precessing millisecond magnetar model provided a better fit to the data. As a result, they suggested that QPOs in the X-ray afterglow plateaus of GRBs 080602 and 090510 could be evidence of precession in a newly formed millisecond magnetar. They further explored whether this precession is a common phenomenon among newborn millisecond magnetars by expanding the number of sample GRBs, selecting 25 short GRBs. They applied the same method to fit these GRBs, finding that 16 out of the 25 GRBs favored the precessing millisecond magnetar model, representing 64% of the sample [91]. Zou et al. (2021) analyzed GRB 101225A and found plausible QPOs in its X-ray afterglow light curve between 4900 s and 7500 s, with periods of 488 s and 250–300 s, at a confidence level of 90% [92]. They applied a magnetar precession model to explain these QPOs and found that, when accounting for spin-down dominated by GW radiation, the signals can be well explained. In addition to GRB 101225A, they also found candidate QPOs in GRB 180620A with a period of approximately 650 s [93] and in GRB 210514A with a period of approximately 11 s [94], which they explained using the magnetar precession model. Besides the scenario of precessing millisecond magnetars, a rapidly spinning and precessing BH could also serve as another scenario for QPOs. Zheng et al. (2024) proposed that QPOs observed during plateau and subsequent sharp decay phases can be produced by a precessing BZ jet [95]. Using this model, they explained the oscillations appearing in the plateau and steep decay phases of X-ray afterglow light curves of GRB 050904. They suggested that the simultaneous emergence of QPOs in plateau and steep decay phases could serve as a potential clue for the central engines as a BH, rather than a millisecond magnetar.

where E_{inj} represents the energy injected as the sub-jets catch up with the forward shock waves, M_0 denotes the initial mass, R is the distance from the central source, u refers to the internal energy, m is the sweep-up mass from the circumburst medium, t is the time measured in the observer frame, c denotes the speed of light, and $\hat{\gamma}$ is the adiabatic index. In addition, energy injection term dE_{inj}/dt is written as [99]

$$\frac{dE_{\text{inj}}}{dt} = \begin{cases} P_{\text{jet}}, & T_{\text{start}} < t < T_{\text{end}}, \\ 0, & \text{others,} \end{cases} \quad (2)$$

where $T_{\text{start}} = ((i-1)\Delta t + j\tau)$, $T_{\text{end}} = (i\Delta t + j\tau)$, $i (= 1, 2, \dots, k)$ refers to the sequence number of sub-jets within each precession period, $j (= 1, 2, \dots, t_{\text{end}}/\tau - 1)$ represents the serial number of precession cycles, τ is the precession period, and $\Delta t = \tau/k$. P_{jet} represents the jet power.

In their model, synchrotron radiation is produced by shock-accelerated electrons. To accurately compute the radiation flux from the forward shock waves, each shock wave generated by the sub-jets interacting with the circumburst medium during the first precession cycle is divided into 300×1000 emitters along the θ' and ϕ' directions in the spherical coordinates comoving with the precessing jet, while the corresponding comoving Cartesian coordinates are denoted as (x', y', z') , with the z' axis aligned along the jet axis (as shown in Figure 1). In addition, due to the line-of-sight effect, radiation photons from regions moving in directions not directly aligned with the line of sight will experience a time delay, compared with photons emitted from regions moving directly toward the line of sight. This can be expressed as $t_{\text{de}} = R(1 - \cos \theta_{\text{em}})/c$, where θ_{em} represents the angle between the radiation emitters and the line of sight. Finally, the total observed flux from a precessing GRB jet is obtained by summing the flux from radiation emitters that have the same observation time.

First, the case of the precession jet without power evolution was explored. Figure 2 presents the X-ray afterglow light curves for a precessing jet with steady power. It is evident that plateaus appear in a large portion of the light curves, some of which display slight positive slopes. In this figure, the influence of various parameters on the plateaus is examined, including the precession period τ , precession angle θ_{pre} , jet power P_{jet} , and viewing angle θ_{obs} . The left and right columns of the figure show the results for $\tau = 10$ s and 100 s, respectively. Clearly, plateaus with shorter precession periods appear earlier (left column). Additionally, there is little difference between plateaus at $\theta_{\text{pre}} = 5^\circ$ and 10° , but the fluxes of the plateaus with $P_{\text{jet}} = 10^{49} \text{ erg s}^{-1}$ are approximately an order of magnitude lower than those with $P_{\text{jet}} = 10^{50} \text{ erg s}^{-1}$. The viewing angle also significantly affects the plateaus. When the line of sight is outside the precession path, the plateaus vanish, and a bump-like feature with a relatively lower flux emerges at a later time. Mainstream models for the origin of the plateaus involve energy injection. The findings imply that jet precession could be a new mechanism for energy injection.

Overall, QPOs seem to appear more easily in the X-ray afterglow light curves (several hundred to thousands of seconds) than in the prompt light curves. However, the potential QPO signals in the X-ray afterglow light curves have relatively poor quality. Hopefully, future instrumentation will detect more QPOs of higher quality, such as the Transient High-Energy Sky and Early Universe Surveyor (THESEUS). On the other hand, jet precession could also leave traces in the X-ray afterglow light curves through other features, such as a plateau structure, whose origin remains controversial. Additionally, if the late-time flares are caused by jet precession, they should be preceded by similar flares with higher peak fluxes, and evidence of approximately regular evolution should be observed in the time intervals between adjacent flares. In this context, evidence from multiple wavebands should be considered when testing for jet precession by GRB afterglow light curves.

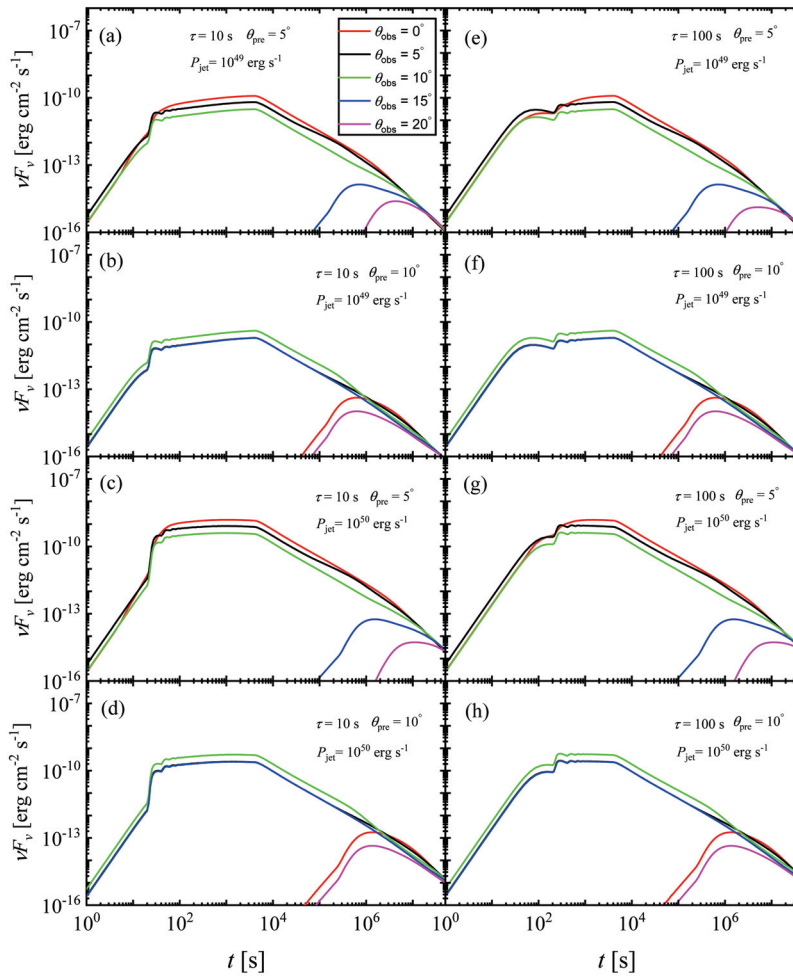


Figure 2. X-ray afterglow light curves of a precessing jet with steady power. The precession periods $\tau = 10$ s, 100 s, the precession angles $\theta_{\text{pre}} = 5^\circ, 10^\circ$, and the initial jet powers $P_{\text{jet}} = 10^{49} \text{ erg s}^{-1}, 10^{50} \text{ erg s}^{-1}$ are adopted. The red, black, green, blue, and magenta lines represent the light curves for $\theta_{\text{obs}} = 0^\circ, 5^\circ, 10^\circ, 15^\circ$, and 20° , respectively (adapted from [99]).

3. Ejecta Structures with Jet Precession in GRBs

The structure of a jet might be either uniform [101] or structured [102]. In a uniform jet, often referred to as a “top-hat” jet, the energy per unit solid angle remains constant regardless of the angular distance from the jet axis, exhibiting sharp cutoffs at the edges. In contrast, structured jets exhibit variations in both energy density and Lorentz factors with angular distance. The most common models of structured jets include power-law [103–105] and Gaussian [106,107] structured jets. Although structured jets have been proposed for a long time and used to explain certain light curve features, such as double jet breaks and X-ray flares [106–109], their physical origin remains unclear. For a long time, research on structured jets was limited because the top-hat jet model was sufficient for explaining most observed GRBs. This situation changed with successful detection of the multi-wavelength afterglow of GRB 170817A, associated with GW170817 [47,48,110], which indicated that its jet was consistent with a structured jet [111,112]. The physical origin of structured jets has gained renewed attention. The structures of GRB jets determine the morphology of light curves and provide rich information about their formation and propagation. These structures are influenced by several factors, including the properties of the central compact object, the characteristics of the accretion disk, and the nature of the stellar envelope (in the case of massive star collapse) or ejecta (in the case of binary compact object mergers). Numerical simulations suggest that as a jet propagates through a dense envelope, its structure can be reshaped by interaction with the medium and thus differ significantly

from its initial structure after breakout [113–115]. Additionally, as the jet passes through the envelope, it can heat the surrounding material, forming a hot cocoon [116]. If this cocoon breaks out alongside the jet, it can create a composite structure, which may be one possible origin of structured jets. Structured jets may be either inherent from the central engines or gradually shaped by external factors. Although the structured jets formed by cocoons are fundamentally different from those caused by jet precession, it is difficult to distinguish them through existing observations. The cocoon characteristics critically depend on the interaction between the jets and the progenitor's envelope or the ejecta, and even depend on the circumstances. If the structures of the envelope or the ejecta can be constrained using other observations, then the characteristics of the cocoon may be narrowed down and the origin of the structured jet can be identified.

Huang et al. (2019) investigated the structure of a short-duration precessing jet during the afterglow phase and found that jet precession causes a top-hat jet to exhibit structured profiles [117]. In their calculations, they divided the precession duration of the top-hat jet into multiple equal and sufficiently short time intervals. Thus, the energy emitted by the jet during each interval is the product of the jet power and the interval duration. The ejecta during each interval were discretized into a large number of cells, each carrying energy equal to the total energy emitted in that interval divided by the number of cells. They further divided the plane perpendicular to the jet precession axis into fine grids and projected all cells emitted in each time interval onto this plane. By tracking the number of cells within each grid and calculating the total energy carried by them, they derived the energy distribution of the jet on the plane perpendicular to the precession axis, as illustrated in Figure 3.

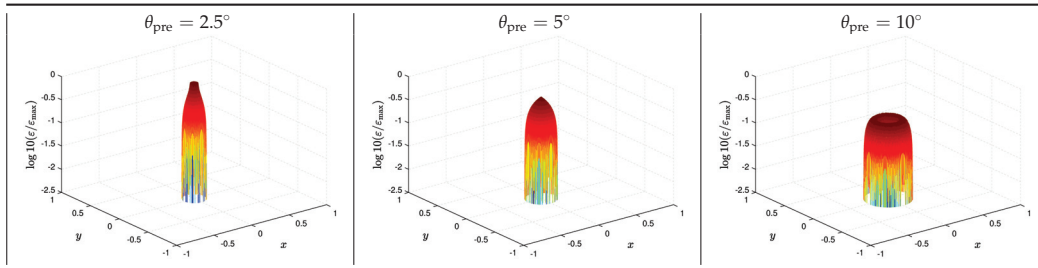


Figure 3. Distribution of energy density of a precessing GRB jet with a constant power, considering precession angles $\theta_{\text{pre}} = 2.5^\circ, 5^\circ$, and 10° (adapted from [117]).

Figure 3 shows the distribution of energy density ε with different precession angles, where the left, middle, and right panels represent the results for the precession angles $\theta_{\text{pre}} = 2.5^\circ, 5^\circ$, and 10° , respectively. The distribution of energy density is projected in the x-y plane with $x = \sin \theta \cos \phi$ and $y = \sin \theta \sin \phi$. Since the half-opening angle of the jet is set, $\theta_{\text{jet}} = 5^\circ$, then $\theta_{\text{pre}} = 2.5^\circ, 5^\circ$, and 10° correspond to cases of $\theta_{\text{pre}} < \theta_{\text{jet}}$ (case I), $\theta_{\text{pre}} = \theta_{\text{jet}}$ (case II), and $\theta_{\text{pre}} > \theta_{\text{jet}}$ (case III), respectively. In case I, when $\theta < \theta_{\text{jet}} - \theta_{\text{pre}}$, ε remains constant over θ ; however, when $\theta > \theta_{\text{jet}} - \theta_{\text{pre}}$, ε exhibits power-law decay with a sharp cutoff tail. This structure can be described as [104,117]

$$\frac{\varepsilon(\theta)}{\varepsilon_{\text{max}}} = \begin{cases} 1, & \theta \leq \theta_c, \\ \left(\frac{\theta}{\theta_c}\right)^{-k_1}, & \theta_c < \theta \leq \theta_m, \\ \left(\frac{\theta_m}{\theta_c}\right)^{-k_1} \exp\left[-\frac{(\theta - \theta_0)^2}{2\theta_g^2}\right], & \theta_m < \theta < \theta_{\text{pre}} + \theta_{\text{jet}}, \end{cases} \quad (3)$$

where $\theta_c = 2.5^\circ$, $\theta_m = 5^\circ$, $k_1 = 1.05$, $\theta_0 = 4.8^\circ$, and $\theta_g = 1.6^\circ$. In case II, ε can be well described by the Gaussian function [106,117]

$$\varepsilon(\theta) \propto \exp[-(\theta - \theta_l)^2 / 2\theta_n^2], \quad (4)$$

where $\theta_l = 1.6^\circ$ and $\theta_n = 5.0^\circ$. In case III, both power-law and Gaussian function cannot adequately describe ε evolving over θ . In this case, the jet in the afterglow phase presents

a ring shape. The results from Figure 3 indicate that the jet structures are significantly dependent on the jet precession angle under jet precession. The jet structures will be significantly more complex if the power evolution of the precessing jet is considered, as shown in Figure 4 of the paper.

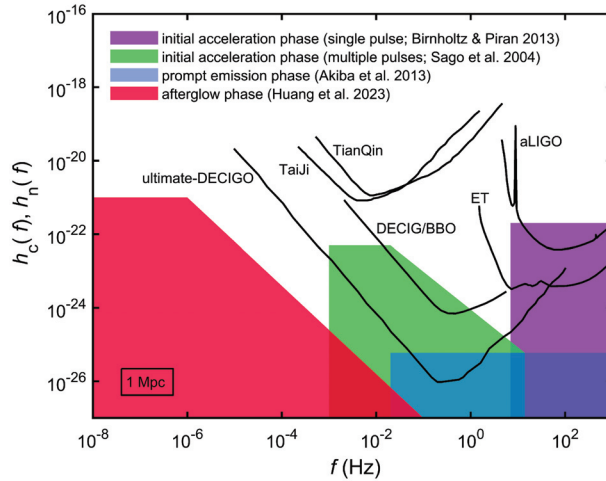


Figure 4. The characteristic amplitude ranges of GW signals from GRB jets at various stages, assuming a distance of 1 Mpc. The shaded areas in purple, green, blue, and red indicate the initial acceleration phase (single pulse) [118], initial acceleration phase (multiple pulses) [119], prompt emission phase [120], and afterglow phase [121], respectively. The black curves represent the sensitivity thresholds of various detectors, including aLIGO, ET, TianQin, Taiji, DECIGO/BBO, and ultimate-DECIGO (adapted from [121]).

4. Polarization Signal with Jet Precession in GRBs

The polarization signals of GRB emission primarily arise from the ordered nature of magnetic fields, which offer insights into the magnetic field characteristics of the emission region. Therefore, polarization detection is regarded as a unique tool for studying the characteristics of magnetic fields in GRB jets. In polarization measurements of the prompt emission phase, the detected signals typically exhibit high polarization degrees (PDs > 50%) [122–125]. Since prompt emission arises from internal energy dissipation within the jet, high PDs are believed to result from large-scale ordered magnetic fields associated with GRB central engines. Additionally, a 90° reversal of polarization angles (PAs) within a single pulse has also been observed [126]. In afterglow polarization detection, the focus primarily lies in the optical band, which is divided into early and late optical afterglow polarization measurements. Early optical afterglow polarization measurements typically exhibit PDs mainly in the range of 3–30% [127–134], whereas those in late optical afterglows generally remain below 3% [135–142]. The polarization levels measured in late optical afterglow are consistent with the expectations for a forward shock origin. However, most PDs measured in early optical afterglows do not match the high polarization levels predicted for reverse shock waves with large-scale ordered magnetic fields [143], falling significantly below theoretical expectations.

The polarization behavior of GRB emission may be affected by GRB jet precession. Lan et al. (2019) studied the polarization characteristics of several astrophysical phenomena, including GRBs, in the context of jet precession [144]. Their research demonstrated that GRB jet precession can result in periodic signals in the polarization curves of prompt gamma-ray emission, with the polarization angle undergoing sudden 90-degree flips. They suggested that these polarization features could potentially serve as a key factor in identifying jet precession. Huang et al. (2022) investigated the impact of long-lasting jet precession on the polarization signals of early optical afterglow and found that jet precession can lead to a certain degree of depolarization [145]. To model the precession and calculate the polarization signals of the early optical afterglow, they adopted the jet

dissociation model proposed by [99]. In the model, the interaction of each sub-jet with the circumburst medium during the first precession cycle gives rise to the formation of a pair of shocks: a forward shock wave moving outward through the circumburst medium and a reverse shock wave propagating inward through the jet material, with the dynamics of both shock waves determined by solving a set of differential equations. In addition, large-scale ordered magnetic fields are considered within the jet, with two configurations explored: a toroidal magnetic field and an aligned magnetic field. Additionally, they assumed that no large-scale ordered magnetic fields exist in the circumburst medium. Thus, only randomly amplified magnetic fields are present downstream of the forward shock waves, and high polarization signals in the early optical afterglow are primarily generated by radiation from the reverse shock waves. Further, to quantify the overall polarization effect, they summed the Stokes parameters from each forward–reverse shock system at the same observation time. However, before summing, it was necessary to transform the Stokes parameters from the coordinate system comoving with jets to a global coordinate system [144].

Figure 5 shows the light curves and polarization signals of early optical afterglows dominated by reverse shock waves under jet precession, with the left and right panels corresponding to the field configurations being aligned and toroidally magnetic, respectively. Different colored lines represent the results for different θ_{obs} , with θ_{obs} being the angle between the line of sight and the precession axis. The top, middle, and bottom panels show the light curves and the evolution of PDs and PAs over time, respectively. In the top panel, the light curves are seen to broadly remain consistent across different magnetic field configurations, implying that the magnetic field configurations do not significantly affect the light curves. In the middle panel, the PD in $\theta_{\text{obs}} = 0^\circ$ is almost zero, indicating that the high PDs produced in the reverse shock waves with large-scale magnetic fields can almost be canceled out by the precession process. However, this is strongly dependent on θ_{obs} , and as θ_{obs} increases, the depolarization effect caused by jet precession becomes insignificant. In other word, it becomes evident only when the line of sight is around the precession axis. In the bottom panel, the evolution of PAs over time exhibits several features, including periods of basic constancy, gradual decay, and abrupt changes. The PA characteristics are determined by the magnetic field directions in the visible core. For example, the near-constant PA with time in $\theta_{\text{obs}} = 7^\circ$ implies that the ordered magnetic field in the visible core has a stable direction. However, when the Lorentz factor of the jets decreases, the observational region increases, and the sub-jets moving at different positions become observable such that the direction of magnetic fields in the observable core may abruptly or gradually change, as reflected in the PA evolution.

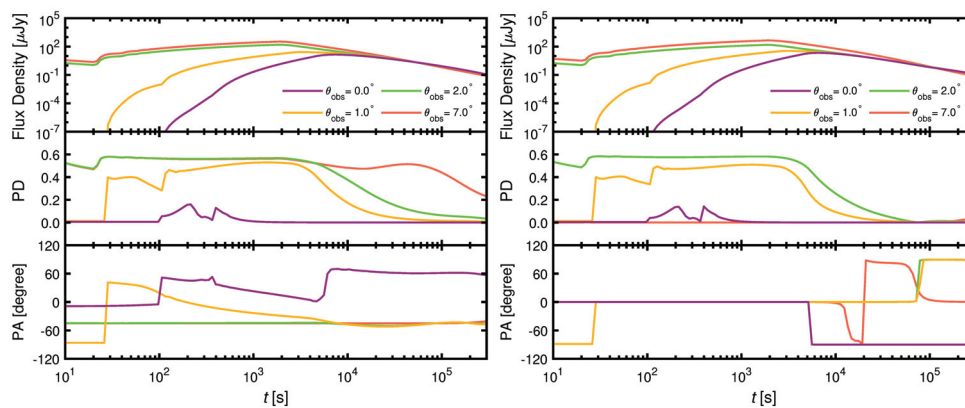


Figure 5. Light curves and polarization evolution of early optical afterglow dominated by reverse shock waves. The left and right panels are the results of magnetic field configurations in the reverse shock waves being aligned and toroidal, respectively. The top, middle, and bottom panels show the light curves and the time evolutions of polarization degrees and polarization angles, respectively. The violet, yellow, green, and red lines represent the results with observation angle $\theta_{\text{obs}} = 0^\circ, 1^\circ, 2^\circ$, and 7° , respectively (adapted from [145]).

In order to test the possibility that this model could explain the generally low PDs measured in the early optical afterglows, they collected GRBs with PD measurements within the early optical afterglow phase ($<10^3$ s), and a sample of 16 GRBs was obtained where seven GRBs (GRBs 090102, 091208B, 101112A, 110205A, 120308A, 141220A, and 190114C) had a definite PD value and the others only had an upper limit of PD. They matched the PDs of these GRBs with the theoretical results for different θ_{obs} , finding that the measured PDs significantly support a small angle of θ_{obs} . Then, they examined the effects of other relative parameters on the PDs, including the precession period of the jets, duration of the jets, initial Lorentz factor, jet luminosity, magnetization parameter, fraction of internal energy density downstream of the reverse shock waves shared by the electrons, and redshift. It was found that these parameters do not significantly impact the PDs compared to θ_{obs} . This indicates that jet precession is a potential origin of the low PDs observed in early optical afterglow. However, due to the lack of direct observational evidence for jet precession, it remains impossible to attribute the low polarization degree of any GRB in the sample to jet precession.

5. GW Signal with Jet Precession in GRBs

The successful detection of GW150914 in 2015 marked the beginning of a new era in GW detection [146]. In 2017, the joint observation of GW170817 and GRB 170817A signaled the start of a new era in multimessenger astronomy [47,48]. In addition to the mergers between binary compact objects, known as strong GW sources, the ultrarelativistic jets of GRBs could also be potential sources of strong GWs. The GW signals from GRB jets have been studied extensively and are produced by the acceleration of the jets [118–120,147], independent of the energy form, known as the GW memory. If the jet interacts with a dense stellar envelope or ejecta, this GW memory can be modulated [148]. Similarly, the GW memory can be affected during the afterglow phase if energy is injected from reactivation of the central engines [121]. Figure 4 illustrates the GWs of GRB jets at different stages. The shaded areas in purple, green, blue, and red indicate the initial acceleration phase (single pulse) [118], initial acceleration phase (multiple pulses) [119], prompt emission phase [120], and afterglow phase [121], respectively. In the initial acceleration phase, the amplitude and frequency range of GWs are larger and narrower, respectively, in the case of a single-pulse scenario than in a multiple-pulse scenario. In the prompt emission phase, the frequency of GWs is slightly lower than that in the initial acceleration phase. When the source position is 1 Mpc, they have the potential to be detected by the Ultimate Decihertz Interferometer Gravitational Wave Observatory (Ultimate-DECIGO). Furthermore, the signal from the afterglow phase exhibits a markedly lower frequency range in comparison to that in the acceleration and prompt phases. Romero et al. (2010) studied the GWs produced by a precession system that includes a neutrino-cooled accretion disk in GRBs [149]. They assumed that the precession is driven by a spin-induced mechanism and discussed scenarios of both long and short GRBs. In both cases, they studied the effect of the accretion rate on the GW amplitude and found that a higher accretion rate results in a larger GW amplitude. They also found that the frequency of the GW signal falls within the range of tens to hundreds of hertz, with the amplitude gradually decreasing as the frequency increases. Due to the requirement for extremely high accretion rates, Romero and colleagues concluded that GW signals from precessing accretion disks would likely only be detectable in the case of long GRBs. Based on the model of [80], Sun et al. (2012) calculated the GW signals from GRBs in the context of neutrino-dominated disk with disk-driven precession [150]. They found that the frequency of the resulting GW signals ranged from a few tenths of a hertz to tens of hertz, which was slightly lower than that predicted in the model proposed by [149]. Additionally, they found that the amplitude increased as the frequency rose, in contrast to the findings presented in [149]. For long GRBs occurring in local galaxies, the GWs could potentially be detected by GW detectors like DECIGO and the Big Bang Observer (BBO). Detecting GWs from GRBs would provide crucial insights into the dynamics of relativistic jets and the physics of their central engines.

6. Summary

Direct observational evidence for the jet precession of GRBs, including clearly periodic signals and QPOs in the prompt and afterglow light curves, has been actively sought. However, clearly periodic signals are always missing, while QPOs are rarely observed and are generally of poor quality. Therefore, it is suggested that in many cases, information in the GRB light curves related to jet precession could be encoded in a different way. Unfortunately, since several mechanisms can account for these non-periodic structures, it is not possible to definitively determine whether they are a consequence of jet precession. Beyond the morphology of the light curves, jet precession can also affect the ejecta structures, polarization signals, and GW signals. Jet precession can result in structured jets and depolarization of early optical afterglow polarization, and systems with precessing jets are capable of producing GW signals with prospects for their detection. Currently, although the effects of jet precession on various aspects of GRBs have been roughly understood theoretically, confirmation of the existence of precession in GRB jets remains a significant challenge due to the lack of direct or joint observations. In this scenario, simultaneous verification across multiple time phases (prompt and afterglow phases), multiple wavelengths (from radio to gamma rays), and even multiple aspects (e.g., jet structures, polarization, and GWs) is necessary. Fortunately, next-generation instruments show promise for providing additional clues in the search for evidence of GRB jet precession, for example, the Space-based multi-band astronomical Variable Objects Monitor (SVOM) with its capability for simultaneous multi-wavelength observations, including optical, X-ray, and gamma-ray bands, plus its ability to respond quickly, which should be able to detect afterglows much earlier, and high-quality QPOs caused by jet precession will be easier to detect, which is expected to assist in confirming the existence of jet precession in GRBs.

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Review

X-Ray Views of Galactic Accreting Pulsars in High-Mass X-Ray Binaries

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Abstract: Accreting X-ray pulsars, located in X-ray binaries, are neutron stars with magnetic fields as strong as $B \sim 10^{12-13}$ G. This review offers a concise overview of the accretion and radiation processes of X-ray pulsars and summarizes their rich observational features, particularly focusing on complex and variable temporal phenomena, spectral properties, and evolution, the new window for X-ray polarimetry and multi-wavelength advances. We also briefly discuss other related systems, i.e., gamma-ray binaries and pulsating ultraluminous X-ray sources.

Keywords: X-ray pulsars; neutron stars; high-mass X-ray binaries; accretion; accretion disks; strong magnetic fields

1. Introduction

As one of the brightest extra-solar X-ray sources in the sky, the X-ray binary Sco X-1 was first detected on 18 June 1962 [1]. This groundbreaking discovery marked the beginning of X-ray astronomy. X-ray binaries (XRBs) are systems consisting of a compact object (such as a neutron star or black hole) and a companion star. Their bright X-ray emissions originate from the compact object accreting material from the donor star. XRBs are classified based on the type of compact object into black hole (BH) and neutron star (NS) XRBs [2]. Alternatively, they can be grouped by the mass of the donor star (M_{co}): low-mass X-ray binaries (LMXBs) with $M_{\text{co}} < 1 M_{\odot}$, intermediate-mass X-ray binaries (IMXBs) with $1 M_{\odot} < M_{\text{co}} < 8 M_{\odot}$, and high-mass X-ray binaries (HMXBs) with $M_{\text{co}} > 8 M_{\odot}$ [3].

LMXBs are typically the old population ($\sim 10^9$ – 10^{10} yr), in which the compact objects accrete matter from late-type main-sequence stars, A-type stars, F-G-type subgiants and white dwarfs by Roche-lobe overflow (RLO). Most LMXBs are located in the Galactic center, bulge, and disk; a substantial fraction of LMXBs are revealed in globular clusters [4,5]. The NSs in LMXBs generally have low magnetic fields ($B \sim 10^8$ – 10^9 G), where the magnetosphere plays a relatively minor role in accretion. Consequently, the states of BH-LMXBs and NS-LMXBs are similar and primarily governed by the accretion rate [6–9].

HMXBs, in contrast, represent a younger population ($\sim 10^5$ – 10^7 years). Some HMXBs, such as SXP 1062 [10,11], SXP 1323 [12], and XMMU J050722.1-684758 [13], have been associated with supernova remnants, highlighting their status as some of the youngest XRBs, still near their birthplaces. Over 169 Galactic HMXBs are concentrated along the Galactic plane, reflecting the star formation rate and tracing the Milky Way's structure [14,15]. Meanwhile, hundreds of HMXBs have been detected in nearby galaxies, especially in Small Magellanic Clouds (SMCs) and Large Magellanic Clouds (LMCs).¹ Alternatively, the majority of ultra-luminous X-ray sources (ULXs) are thought to be the brightest end of the HXMBs in outer galaxies [16–18]. The spatial distribution and scaling relationships between the number

and collective X-ray luminosity of HMXBs and the star formation rate further support their role as tracers of recent star formation history in their host galaxies [19,20].

HMXBs are conventionally divided into supergiant X-ray binaries (SGXBs) and Be X-ray binaries (BeXBs) based on their optical companions. Generally, the former is fed directly from stellar winds (although very few of them are suggested to be disk-fed via the RLO, e.g., LMC X-4, Cen X-3, SMC X-1). BeXBs, on the other hand, are characterized by their Be star companions with circumstellar disks, which are formed by material ejected from Be stars. The compact object in these systems accretes material from both the stellar wind and the Be disk, leading to dramatic variations in their X-ray emission.

To date, only about ten compact objects in HMXBs are black holes and candidates, including several wind-fed systems Cyg X-1, LMC X-1, M33 X-7, Cyg X-3, IC 10 X-1, NGC 300 X-1, HD96670 [21], and the famous micro-quasar super-Eddington accretor SS 433. Most other HMXBs harbor young NSs with magnetic fields of $B \sim 10^{11}–10^{13}$ G, identified through their X-ray pulsations. Different types of accreting X-ray pulsars (AXRPs, e.g., disk-fed, wind-fed SGXBs, and BeXBs) occupy distinct regions in the spin period (P_{spin}) versus the orbital period (P_{orb}) diagram (a.k.a. the Corbet diagram [22], Figure 1).

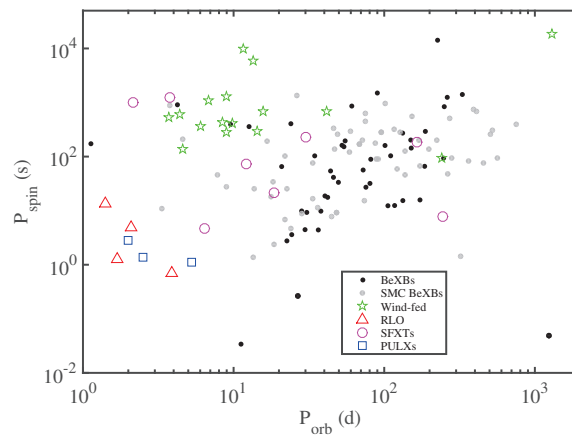


Figure 1. Corbet diagram. BeXBs in the Galaxy and Magellanic Clouds are marked with black and grey circles. The wind–fed SGXBs, disk–fed SGXBs, SEXTs, PULXs are presented with green, red, magenta, and blue symbols, respectively. Data were adopted from [18,23–27] and the references therein.

Unlike low-magnetic-field NS and BH accreting systems, the accreting processes in AXRPs are influenced not only by the accretion rate but also by the strong magnetic fields of NSs. This interplay results in distinct X-ray properties and more complex evolutionary patterns. Since the launch of X-ray telescopes in the 1970s, the accretion physics of AXRPs have been extensively studied. Several excellent reviews with different emphases on HMXBs have been published over the past decade [28–34]. For the convenience of reference, we also summarize the reviews and landmark papers on various related topics in Table 1.

Table 1. Reviews and landmark papers on HMXBs.

Description	Reference
Recommended reviews	[28–35]
Catalogue of HMXBs	[14,15,27,36,37]
Achievements of <i>BATSE</i> , <i>INTEGRAL</i> , <i>Fermi</i> /GBM	[30,38,39]
Formation and evolution of binaries	[19,40–44]
Mass transfer in binary stars	[45–47]
Fundamental physics in strong magnetic fields	[48]
Accretion and radiation near polar caps	[49–53]
Cyclotron resonant scattering features	[31,54]
Accretion torques	[55–59]
X-ray polarimetry	[60–62]
Multi-wavelength information	[28,63–69]

In this paper, we provide a general review on the X-ray observational advances and accretion physics of AXRPs, with a particular focus on the recent developments achieved by the ongoing X-ray missions, *Insight-HXMT*, *NICER*, *Swift*, *AstroSAT*, and *IXPE* [70]. We introduce basic physical processes in Section 2. The temporal, spectral, and polarization characteristics of AXRPs are summarized in Sections 3–5. A brief comparison between AXRPs and related binary systems, e.g., gamma-ray binaries and pulsating ultraluminous X-ray sources (PULXs), is provided in Section 6. In Section 7, we discuss key advances in multi-wavelength observations that provide a more comprehensive picture.

2. Fundamental Physics in X-Ray Pulsars

The X-ray behavior of HMXBs is primarily determined by the characteristics of their optical companions, interactions between the accreting matter and the NS's magnetosphere as well as accretion and radiation processes occurring in the vicinity of the NS. This section provides a brief overview of the relevant core concepts and simplified physical processes underlying these systems. We encourage readers to explore more detailed and comprehensive review papers [33,34].

2.1. Mass Transfer in Binary Stars

In most HMXBs, the optical donor star is either a supergiant O/B star or a Be star. For the former, the mass transfer is through accretion from the stellar wind (or the RLO in a few sources). The wind is expected to be line-driven due to the scattering of a large ensemble of spectral lines [45], and the velocity profile of the wind can be approximately described as

$$v_w(r) = v_\infty \left(1 - \frac{R_\star}{r}\right)^\beta, \quad (1)$$

where v_∞ is the terminal wind velocity, $R_\star$ is the radius of the supergiant, and $\beta \sim 0.5$ – 1 is the velocity gradient parameter [71]. We note that the actual scenario is considerably more complex, as the wind is not perfectly smooth due to the clumpiness induced by the line-deshadowing instability [72,73], as well as the presence of accretion and photoionization wakes [74]. In BeXBs, the donor star has a circumstellar decretion disk, formed by material ejected from the star's equatorial region [75]. This disk operates as an inverse version of the Shakura–Sunyaev disk, transferring angular momentum outward. Accretion occurs when the NS approaches or passes through the Be disk if the Be disk is large enough. This interaction may happen near periastron quasi-periodically, leading to the so-called Type-I outbursts [46,76]. Conversely, the mechanism behind Type-II outbursts (Section 3.1.1) remains unclear but is thought to involve highly misaligned and eccentric decretion disks [77].

The matter can be captured by the NS only if its kinetic energy is less than the potential energy. This condition applies to inflowing matter at impact distances smaller than an accretion radius (R_a), defined as

$$R_a = \frac{2GM}{v_{\text{rel}}^2} \approx \frac{2GM}{v_w^2}, \quad (2)$$

where M is the NS mass, and $v_{\text{rel}} \approx v_w$ is the relative velocity of the wind to the NS when the binary orbital velocity is $v_{\text{orb}} \ll v_w$. In addition, another crucial factor is the magnetic field (B) of the NS, which significantly affects the behavior of the accreted matter within the magnetospheric radius (R_m). The magnetospheric radius can be estimated by balancing the magnetic pressure and the ram pressure as

$$R_m = k \left(\frac{\mu^4}{2GM\dot{M}^2} \right)^{\frac{1}{7}}, \quad (3)$$

where μ is the magnetic dipole moment, $\dot{M}$ is the accretion rate, and k is a model-dependent dimensionless parameter. Typically, k is assumed to be $k \sim 0.5$ for a disk accretion and $k \sim 1$ for a spherical accretion [56]. Within R_m , the accreted matter is expected to be coupled with magnetic field lines and to corotate with the NS. If $R_m < R_a$, the accretion process can proceed effectively with the accretion rate given by $\dot{M} \approx \pi R_a^2 \rho v_w$ when $v_{\text{orb}} \ll v_w$, where ρ is the wind mass density. Considering the stellar wind mass-loss rate ($\dot{M}_w$) of the donor is $\dot{M}_w \approx 4\pi D^2 v_w \rho$, where D is the binary separation, the accretion rate can be described as

$$\dot{M} \approx \frac{R_a^2}{4D^2} \dot{M}_w. \quad (4)$$

When $v_{\text{orb}} \sim v_w$, the orbital motion should be taken into account; the accretion rate is

$$\dot{M} \approx \dot{M}_w q^2 (1+q)^2 \frac{\xi \tan^4 \beta}{\pi (1 + \tan^2 \beta)^{3/2}}, \quad (5)$$

where $q = M/M_{\text{donor}}$ is the binary mass ratio, $\xi \sim 1$, and $\tan \beta = v_{\text{orb}}/v_w$ [33,55]. On the other hand, when $R_m > R_a$, the accretion is significantly suppressed due to the magnetic inhibition (for details, see [47]).

2.2. Interactions Between the Magnetosphere and Accretion Flows

If the captured matter from the wind possesses sufficient specific angular momentum, an accretion disk is expected to form outside the magnetosphere when R_m is smaller than the circularization radius (R_{circ}), a characteristic radius within which an accretion disk exists. The circularization radius is defined as

$$R_{\text{circ}} = \frac{\xi^2 \omega_{\text{orb}}^2 R_a^4}{GM}, \quad (6)$$

where ω_{orb} is the orbital angular velocity of the NS, and $\xi \sim 0.2$ accounts for the angular momentum dissipation due to the density and velocity gradients in the accreting non-magnetized gas [78,79]. This scenario typically applies to BeXBs and may also occur in certain wind mass-loss systems. Otherwise, when $R_{\text{circ}} < R_m$, the matter accretes spherically, which is more commonly observed in supergiant systems.

The fundamental nature of accretion is the transfer of angular momentum. Within R_m , the accreted matter is forced to co-rotate with the NS due to its coupling with the magnetic field. Efficient accretion occurs only when the NS's spin frequency is less than the local Keplerian frequency at R_m . In other words, R_m must be smaller than the co-rotation radius (R_{co}), which is defined as

$$R_{\text{co}} = \left(\frac{GMP_{\text{spin}}^2}{4\pi^2} \right)^{\frac{1}{3}}, \quad (7)$$

where P_{spin} is the spin period of the NS (for details about both aligned and oblique dipole cases, see Lyutikov [80]). On the other hand, when $R_{\text{co}} < R_m$, the centrifugal barrier prevents the accretion, and the system enters into the so-called supersonic “propeller” regime, which is typically characterized by a low luminosity and the lack of pulsations [81,82].

The interaction between the magnetosphere and the accretion flow exerts torques on the NS. The total torque (N) comprises three components: the accretion torque (N_m), the magnetic torque (N_B) and the radiation torque (N_{rad}). In the disk accretion scenario, the accretion torque is given by $N_m = \dot{M} \sqrt{GMR_m}$, which arises from the conservation of angular momentum of the accreted material. This term is dominant during BeXRB outbursts when the accretion rate is relatively high. The radiation torque can be approximately expressed as $N_{\text{rad}} \sim \frac{2}{3c^3} (\mu \sin i)^2 \Omega^3$, where i is the angle between the magnetic and rotational axes, and $\Omega = \frac{2\pi}{P_{\text{spin}}}$ is the NS's angular frequency. This torque plays a significant role in the spinning down during quiescent states. The N_B term is more intricate, depending on how the magnetic field interacts with the accreting matter.

In the disk accretion scenario, it is generally thought that the NS's magnetic field partially penetrates into the accretion disk, generating a toroidal magnetic field and inducing torque due to the differential motion between the disk and the NS. Assuming different prescriptions for the toroidal magnetic field, several magnetically threaded accretion models have been proposed and discussed by different authors [56,83–89]. In addition, the interaction between the magnetosphere and the accretion disk is influenced by the disk's structure, which can result in observable phenomena in both temporal and spectral studies. According to the accretion disk theory proposed by Shakura and Sunyaev [90], the disk is thin if the opacity is primarily due to electron scattering when the pressure is dominated by the gas pressure. On the other hand, the inner region of the disk might be thick at high accretion rates as the radiation pressure takes over as the dominant pressure [91,92]. The disk thickness at R_m affects the size of polar caps on the NS surface, which in turn influences the observed pulse profiles (see Figure 2) and spectra [93–95]. Furthermore, different accretion structures cause different variabilities in the time domain [93,96,97].

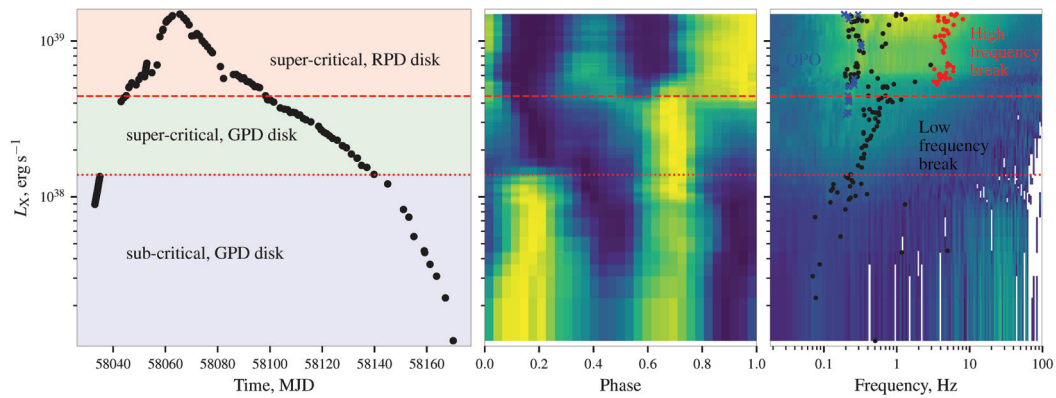


Figure 2. (Left) long-term lightcurve of Swift J0243.6+6124 observed with *Insight*-HXMT, where color shadows present different accretion states. Middle: The evolution of pulse profiles, where transitions from the sub- to supercritical regimes and from the gas-pressure-dominated (GPD) to radiation-pressure-dominated (RPD) states are marked with red dashed lines. (Right) The color shows the evolution of power spectra with luminosity. The black and red points demonstrate the break and QPO frequencies measured in the power spectra. This figure is adopted from Doroshenko et al. [93].

In the case of spherical accretion in wind-fed systems, the situation is more complex and less well understood. When the accretion rate exceeds $\dot{M} \gtrsim 4 \times 10^{16} \text{ g s}^{-1}$, direct Bondi–Hoyle–Littleton accretion can take place, resulting in the formation of a shock above the magnetosphere. The shocked matter cools down efficiently and enters the magnetosphere via Rayleigh–Taylor instability [98,99]. On the other hand, when $\dot{M} \lesssim 4 \times 10^{16} \text{ g s}^{-1}$, the infalling matter subsonically settles down on to the rotating magnetosphere, forming an extended quasi-static shell, and this shell mediates the angular momentum transfer to the NS. In this scenario, the accretion is less efficient and in literature it is known as the subsonic propeller regime [47,57]. It is expected to exist within certain parameter spaces of the wind velocity, the spin and the magnetic field of the NS (for details, see Figure 2 in Bozzo et al. [47]). In this case, the radial velocity of the accreting matter is slower than the free-fall velocity and is determined by the Compton or radiation cooling rate near the magnetosphere. Within this settling accretion regime, either spinning-up or spinning-down of the NS is possible (for details, see [58]).

2.3. Physics in the Vicinity of Magnetic Poles

Within the magnetosphere, the accreted matter is guided along the dipole magnetic field lines onto the magnetic poles of the NS, where the majority of the observed X-rays are emitted. The resulting footprint is a filled hot spot for a spherical accretion (or a hollow circle for a disk accretion), corresponding to different structures at the base of the accretion

channel [100,101]. In theory, despite the complex coupling between matter and radiation, several approximate analytic models and simulations have been conducted by different authors [49–51,53,102–105].

It is believed that the accretion process near magnetic poles and its observable properties mainly depend on the luminosity (L) and the magnetic field (B). When the luminosity exceeds a critical value,²

$$L > L_{\text{crit}} \sim 1.5 \times 10^{37} \left(\frac{B}{10^{12}} \right)^{\frac{16}{15}} \text{ erg s}^{-1} \quad (8)$$

In [51], a radiation-dominated shock was found to form above the NS's surface, where the kinetic energy of the infalling matter is significantly reduced. Below the shock, there is a sinking zone (referred to as the accretion column), which is an extended and high-energy-density region. The column height is expected to be positively correlated with luminosity. In this case, most of the X-rays escape from the system through the “wall” of the accretion column, resulting in a “fan” beam pattern (Figure 3). The radiation from the accretion column is expected to be highly polarized because the Compton scattering cross-section is energy and polarization dependent for the energy band below the cyclotron line energy, as well as the vacuum polarization effect during propagation [61]. However, this theoretical prediction is not fully consistent with observations (see Section 5).

When the luminosity is below L_{crit} , the radiation pressure is insufficient to fully decelerate the infalling matter. As a result, the matter eventually comes to rest on the NS surface via the Coulomb braking or a collisionless gas-mediated shock. It is possible that the latter is more important when $L < L_{\text{coul}} \sim 1.2 \times 10^{37} \left(\frac{B}{10^{12}} \right)^{1/3} \text{ erg s}^{-1}$ [51,106]. In this regime, most of the radiation is emitted near the NS surface, where a dense accretion mound is formed, resulting in a “pencil” beam with the radiation emitted predominantly along the magnetic field lines. At very low accretion rates, the thermal emission from the accretion mound may serve as the primary source of seed photons for the Comptonization (see Figure 9 in [53]). In addition, the overheated atmosphere, excited by the collision of infalling matter, may play a crucial role in modifying the observed spectra. Theoretical models have been developed for luminosities in the range of $L \lesssim 10^{35} \text{ erg s}^{-1}$, considering polarization-dependent radiative transfer using different methods (for details, see [107,108]).

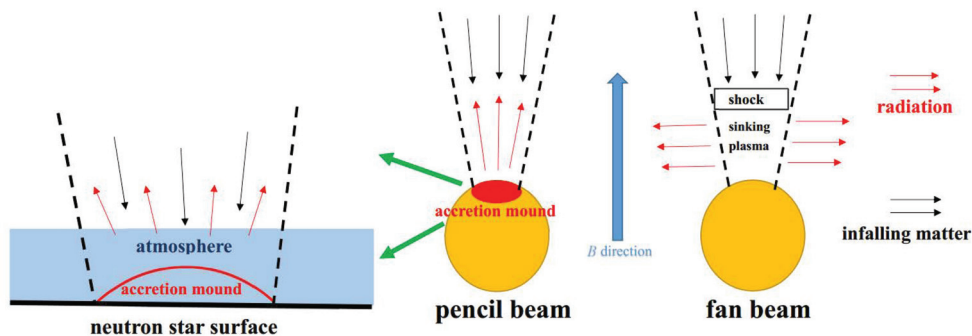


Figure 3. A sketch showing the distinct geometries and radiation patterns for the accretion mound at low accretion rates (middle) and the accretion column at high accretion rates (right). The left panel shows a zoom-in plot of the surface of the neutron star in the case of the accretion mound. The blue arrow indicates the magnetic field direction at the pole. The arrows on the rightmost side are the legend, i.e., black arrows present the direction of the matter movement, and red arrows show the main direction of the radiation.

3. Temporal Properties

As accretion systems, AXRPs exhibit both regular and irregular variability across timescales ranging from seconds to years. Globally, most BeXBs are transients, whereas SGXBs tend to be persistent. In RLO SGXBs, material is steadily transferred from donor stars, forming accretion disks around the NSs, resulting in persistently high luminosi-

ties ($\sim 10^{38}$ erg s $^{-1}$). In contrast, wind-fed SGXBs, which typically have narrower orbits compared to BeXBs, involve NSs embedded in dense stellar winds. Consequently, both disk-fed and most wind-fed SGXBs are persistent sources. However, a fascinating subclass of SGXBs, known as supergiant fast X-ray transients (SFXTs), has been identified, characterized by short, intense flaring emissions (see Section 3.1). Conversely, BeXBs have wide and highly eccentric orbit, and the radii of optical companions are smaller than their Roche lobes. NSs intermittently capture substantial amounts of material from the circumstellar disks of their companions, leading to transient outbursts (Section 3.1). Meanwhile, a small number of BeXBs show persistent and low luminosities ($\sim 10^{34}$ – 10^{35} erg s $^{-1}$), e.g., the prototype of which is X Per/4U 0352+309 [109]. It is worth noting that the classification of specific sources can be dynamic, as outbursts may alter their behavior. For instance, LS V +44 17/RX J0440.9+4431 was initially categorized as a persistent BeXB [110] but subsequently underwent outbursts in 2010–2011 [111] and 2022–2023 [62,112,113]. Despite the differences in long-term behavior, various AXRP subclasses exhibit similar timing and spectral properties. In this section, we summarize their temporal evolution characteristics, including long-term trends, the defining feature of AXRPs—X-ray pulsations, aperiodic variability (e.g., quasi-periodic oscillations, QPOs), and superorbital modulation.

3.1. Long-Term Behavior

The AXRPs in our Galaxy and in the Magellanic Clouds display a broad range of luminosities, from $\sim 10^{32}$ to 10^{39} erg s $^{-1}$. They are inherently variable, showing flares and state transitions. In extreme cases, such as outbursts in BeXBs and SFXTs, their dynamic range (the ratio of peak luminosity during an outburst to quiescence luminosity) can be up to 10^2 – 10^7 .

3.1.1. Outbursts in BeXBs

Due to the occasionally rapid increase of the accretion material, BeXBs leave the quiescence state ($L_X < 10^{35}$ erg s $^{-1}$) and could present two distinct types of outbursting activity, i.e., Type I and Type II outbursts. Type I outbursts are characterized by regular and (quasi-)periodic activities with a moderate intensity ($L_X \sim 10^{36}$ – 10^{37} erg s $^{-1}$). The peak luminosities normally arrive at or close to the binary periastron passages, and outbursts tend to cover only a small fraction of the orbit (~ 0.2 – $0.3 P_{\text{orb}}$). Now, it is widely recognized that Type I outbursts are triggered as enhancement in the mass transferring from a tidally truncated Be disk around periastron passages [46].

Type II outbursting events (also called giant outbursts) are much rarer and occur at any orbital phase. They are known to have much brighter luminosity ($L_X > 10^{37}$ erg s $^{-1}$) and last for several weeks or even for several orbital periods. Recently, some BeXBs, i.e., SMC X-3 [114], RX J0209.6-7427 [26,115] and Swift J0243.6+6124 [116,117], underwent the most violent outbursts with a peak luminosity of $\geq 10^{39}$ erg s $^{-1}$, which is the threshold luminosity for the definition of ULXs. During giant outbursts, both timing and spectral properties display dramatic evolution and may go through a transition around the critical luminosity ($L_{\text{cirt}} \sim 10^{37}$ erg s $^{-1}$, depending on the NS's magnetic field [49,51,52]; Equation (8)). A quick look at the evolution of outbursts can be achieved with the hardness-intensity diagram (HID), in which the source transits from the horizontal branch to the diagonal branch and back. These two branches may reflect two different accretion modes below and beyond the critical luminosity. In the horizontal branch, AXRPs have low intensity and high X-ray variability, and the diagonal branch corresponds to high-luminosity states. The spectral and timing parameters show some significant, although not universal, correlations [118].

The operation of more-sensitive imaging X-ray telescopes (e.g., *Swift*, *XMM-Newton*, and *Chandra*) allows us to monitor giant outbursts in their very low luminosity levels. Some BeXBs were observed to have the luminosity drop sharply (by a factor of more than 100) within a few days at the end of outbursts. Such a phenomenon was usually interpreted as a

transition to the propeller regime, offering another critical tool for estimating the magnetic field of AXRPs [119,120] (see Section 2.2).

3.1.2. SFXTs

Two decades ago, the IBIS/ISGRI instrument onboard *INTEGRAL*, which had a sufficiently wide field of view and high sensitivity, detected recurrent fast X-ray transient events at low Galactic latitudes, effectively excluding the possibility of gamma-ray bursts [121,122]. The subsequent identification of their optical counterparts as supergiants led to the designation of this new subclass of HMXBs as SFXTs [67,123,124]. The hallmark feature of SFXTs is their extreme variability: they spend most of their time in a quiescence state ($L_X \sim 10^{31}–10^{33} \text{ erg s}^{-1}$) [125], and intermittently exhibit sporadic, short-lived, intense X-ray flares with durations of $\sim 10^2–10^4 \text{ s}$ and peak luminosities of $L_X \sim 10^{36}–10^{38} \text{ erg s}^{-1}$ [126]. This variability corresponds to a dramatic increase in X-ray luminosity by 2–6 orders of magnitude within a very short time span, with a low duty cycle in the bright phase ($<5\%$) (see [127] for a review). Additionally, relatively fainter flares ($L_X \sim 10^{33}–10^{35} \text{ erg s}^{-1}$) on longer timescales of days to months have also been recorded [128].

SFXTs share numerous observational similarities with other known AXRPs, particularly their very hard spectral profile, which strongly suggest the presence of NSs in these systems. Indeed, both orbital and spin periods have been determined for approximately half of the known SFXTs through X-ray monitoring data [129–133]. Notably, IGR J11215–5259 displayed recurrent brief outbursts with a periodicity of ~ 165 days. Each outburst lasted several days and comprised numerous X-ray flares of $\sim 10^3–10^4 \text{ s}$ duration. This behavior is best explained by the presence of an elliptical orbit, with X-ray outbursts occurring near periastron, akin to the Type I outbursts observed in BeXBs but on a significantly shorter timescale [130,134].

To date, around two dozen SFXTs have been discovered [27,135]. These systems likely constitute a significant fraction of HMXBs; however, their detection is hindered by several factors, including their significantly lower luminosity compared to classical SGXBs, the rarity and brevity of their outbursts, and heavy absorption [136,137]. It has been proposed that SFXTs represent an evolutionary link between BeXBs and SGXBs, with more than half of SFXTs overlapping with persistent SGXBs, and some overlapping with BeXBs on the $P_{\text{spin}} - P_{\text{orb}}$ diagram [138] (Figure 1).

The intuitive explanation for the low luminosity in quiescence and short bright flares of SFXTs was the peculiar clumpy properties (denser and slower) of the stellar winds and/or orbital characteristics [130,139,140]. Multi-wavelength observations suggest that wind parameters play a decisive role in distinguishing classical SGXBs from SFXTs. For instance, the wind terminal velocity measured in the prototype of SFXTs, IGR 17544–2619 ($\sim 1500 \text{ km s}^{-1}$) was much faster than in the classical SGXBs, Vela X-1 ($\sim 700 \text{ km s}^{-1}$) [141]. However, this interpretation remains uncertain due to a lack of comprehensive studies across a larger sample of sources [142]. On the other hand, it has been argued that the classical Bondi-Hoyle-Littleton accretion with changes in velocity and density could only produce moderately bright flares ($L_X \leq 10^{36} \text{ erg s}^{-1}$) as sources entered a regime dominated by strong Compton cooling [143]. Therefore, alternative mechanisms are required.

Two primary models have been proposed to explain the extreme transient behavior of SFXTs: magnetic gating mechanism and subsonic setting accretion model. In the former scenario, accretion is typically inhibited by a centrifugal barrier, with sporadic dense clumps penetrating the NS magnetosphere to trigger brief flares. Essentially, the magnetic gating mechanism is governed by the spin and magnetic field, i.e., longer spin periods require stronger magnetic fields to sustain the centrifugal barrier. More specially, magnetar-like magnetic fields ($B \sim 10^{14} \text{ G}$) have been suggested for slow rotating pulsars [47]. Interestingly, tentative CRSFs have been reported in several SFXTs, such as IGR J18483–0311 [144], IGR J17544–2619 [145], and SAX J1818.6–1703 [146], implying relatively low magnetized NSs ($10^{11}–10^{12} \text{ G}$). However, these claims remain controversial and have not been consistently confirmed by subsequent observations [147].

The subsonic accretion model was initially proposed to explain the low-luminosity AXRPs, in which a hot, quasi-static shell formed above the magnetosphere due to inefficient radiative cooling at low accretion rates (Section 2.2). The sources switch into a more effective Compton cooling regime, and matter enters magnetospheres due to the Rayleigh–Taylor instability, resulting in a moderate flare with $L_X \leq 10^{36} \text{ erg s}^{-1}$. It was speculated that the plasma lost by the supergiant of SFXTs is non-magnetized for most of the time, while a small fraction of them carried large-scale magnetic fields transported from hot massive stars. The magnetized stellar wind triggers a much more efficient plasma entry rate via the magnetic reconnection, and the entire shell can fall on to the central NS on the free-fall time scale. That is, brighter flares can be produced by the sporadic capture of magnetized density clumps, and the magnetic reconnection triggers shell instability around the magnetosphere. Notably, this mechanism is less dependent on the NS magnetic field strength [58,143].

3.2. X-Ray Pulsations

Material from the companion is captured and channeled by the strong magnetic field onto magnetic poles of the NS surface, forming two X-ray hot spots. When there is a misalignment between the NS's rotational and magnetic axes, the observed X-ray radiation becomes pulsed as the NS spins. Therefore, investigating X-ray pulsations provides a direct access to the matter motions and emerged X-rays in the very close vicinity of pulsars.

3.2.1. Pulse Profile

Ideally, the X-ray pulsations from an isotropic spot would produce a simple, symmetrical, sinusoidal-like pulse profile. However, the actual pulse shapes of AXRPs are often highly structured, vary significantly from one outburst to another, and depend on both luminosity and photon energy. For example, some sources exhibit double peaks at high luminosities and a single peak at low luminosities. This behavior is commonly interpreted as a transition between fan beam and pencil beam emission patterns [51,114]. As discussed in Section 2.3, at low luminosities, radiation predominantly escapes along the magnetic axis, forming a pencil beam. In contrast, under supercritical accretion ($L_X > L_{\text{crit}}$), the accretion column becomes optically thick along the magnetic field lines. Photons then escape from the side of the column in the sinking region, producing a fan beam. At intermediate luminosities, the pulse profile can reflect a hybrid pattern that combines pencil and fan beam components [51]. In reality, however, pulse profiles are far more complex. Factors such as the presence of an accretion disk, occultation and reflection by the NS surface, gravitational light bending, scattering by surrounding matter, or a distorted dipole or multipole magnetic field configuration can significantly alter the pulse shape. These modifications may result in asymmetric, narrow spikes or flare-like features. Usually, X-ray pulsations in the softer band have a more complex shape and a smaller pulsed fraction [148], but non-monotonic correlation is observed between the pulsed fraction and the photon energy emerged in RX J0440.9+4431 [149]. Thanks to *Insight*-HXMT, which has a very large effective area in the hard X-ray band, the pulsations were recently detected beyond 100 keV in RX J0209.6-7427 [26], Swift J0243.6+6124 [150], and 1A 0535+262 (Figure 4) [151].

Since pulse profiles are governed primarily by the accretion process onto the NS surface, they provide valuable insights into the magnetic field geometry and X-ray beam patterns. However, the complex radiative processes, dynamics of the accretion flow, and gravitational light bending make it theoretically challenging to reproduce pulse profiles accurately [152]. Various methods and assumptions have been used to decompose pulse profiles [153–159], with results that can be cross-verified using polarization observations [159–162]. On the other hand, once the beam mode switching can be determined, we will be able to estimate the critical luminosity and thus to measure the strength of the magnetic field. For instance, the 0.25 phase offset observed in the giant outbursts of Swift J0243.6+6124, SMC X-3, and RX J0209.6-7247 was interpreted as the change in photon propagation directions ($\sim 90^\circ$) in the subcritical (pencil beam) and supercritical (fan beam) regimes [163]. However, we note that beam patterns at different energy bands could be

different because the magnetized Comptonization cross-section is highly energy-dependent. This may lead to a large uncertainty in estimating L_{crit} according to the variations in the profile profiles at a narrow energy band [26].

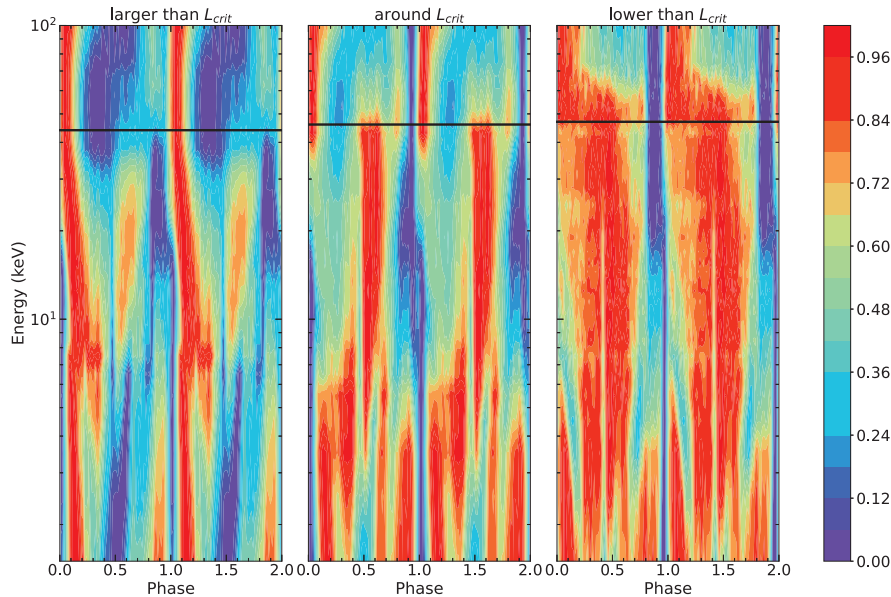


Figure 4. Pulse profiles of 1A 0535+262 plotted as a function of photon energy. From left to right, the three panels show the observations made at the peak luminosity ($L_X > L_{\text{crit}}$), around L_{crit} , and low luminosity ($L_X < L_{\text{crit}}$), respectively. The color bar displays the normalized intensity, and the black lines in the three panels mark the energy of the fundamental CRSFs. This plot was reproduced from Wang et al. [151].

3.2.2. Spin Evolution

Newborn NSs are typically fast rotators, with a spin period $P_{\text{spin}} < 1$ s [164,165], functioning initially as rotation-powered pulsars (ejector state). In a binary, the NS is further spun down as a consequence of the interactions between the pulsar’s magnetosphere and material emitted from the companion star (propeller state). When the spin period of the NS reaches a critical value, i.e., $R_{\text{co}} \approx R_{\text{m}}$, the centrifugal barrier cannot stop the matter from falling onto the NS surface, and the system switches on as an AXRP (Section 2.2). Accretion processes are always accompanied by an energy transformation (from gravitational energy to radiation) and matter/angular momentum transfer. AXRPs experience spin up and spin down due to the torques exerted by accreting material and may achieve an equilibrium spin rate over long times. Historically, *CGRO/BATSE* and *Fermi/GBM* have a full-sky view and have been devoted to continuously monitoring bright AXRPs in the hard X-ray window [38,39]³. Additionally, when sources became active, *RXTE*, *Swift*, *NICER*, *Insight-HXMT*, *AstroSAT*, and other satellites’ pointed observations, which had a high sensitivity and a good time resolution, were triggered to trace the sources from bright to faint. Furthermore, survey campaigns aiming to discover and monitor X-ray outbursts from BeXBs in SMC were performed weekly by both *RXTE* [166] and *Swift* [167]. The gathered data have helped us to depict the decade-long behaviors of AXRPs and reveal stochastic variations and secular spin-up/spin-down evolution on long time scales. Generally, the spin-up rate ($\dot{\nu}$) and the X-ray flux are tightly correlated during the outbursts of transient sources. Specially, the $\dot{\nu}$ increases with the bolometric luminosity, following a relation of $\dot{\nu} \propto L^b$, with an index b of 0.8–1.1 [38,114,163]. For persistent AXRPs, however, the relationship between accretion torque and luminosity can be more complex, including correlations, anti-correlations, or a lack of correlation altogether.

The spin evolution reflects the angular momentum exchange between accreting material and the NS magnetosphere, making it a crucial diagnostic for determining magnetic field strengths in AXRPs [24,88,114,163]. In disk accretion scenarios near the spin equi-

librium (or torque reversal), magnetic fields can be estimated by equating the co-rotation radius with the magnetospheric radius. But, this assumption is not always valid as the secular spin evolution observed in many sources, in particular for the fast spin-up trend recorded in Type II outbursts. More rigorous torque models for both disk [56,87,168] and wind accretion [169], whether close to spin equilibrium or not, have been proposed to investigate magnetic fields of AXRPs (more details in Section 2) [24,88].

It has been well estimated that an NS can reach the longest period of ~ 500 s by assuming a reasonable magnetic field ($B \sim 10^{12}$ – 10^{13} G) and the total spin-down time in the ejector and propeller states being less than the lifetime of the young massive star ($\sim 10^6$ – 10^7 yr) [78,170]. The discovery of several relatively bright, persistent AXRPs with periods in the range of 500 – 10^4 s poses a serious challenge to this framework [10,171–173]. Various scenarios have been proposed to alleviate this contradiction, e.g., the specific quasi-spherical settling accretion model [58], the subsonic propeller model with a normal pulsar [78,174], and the wind-accreting magnetar model [172,175,176].

3.3. Aperiodic Variability

Rapid aperiodic variability, driven by instabilities and fluctuation in the accretion flow, has been extensively studied in LMXBs, particularly in black hole systems. However, it remains less explored in HMXBs. Observationally, QPOs are identified and characterized by fitting power density spectra with a combination of several phenomenological Lorentzian and/or power-law functions. The quality factor Q is defined as the ratio between the characteristic frequency of the component over its width ($Q = \nu / \text{FWHM}$). Broad features are referred to as noise and narrow features with $Q > 2$ are classified as QPOs. For some sources, the timing parameters of broad noise components, such as their characteristic frequency and the fractional amplitude, show significant correlations with spectral and QPOs parameters. These correlations suggest a connection between the inner accretion column and the outer accretion disk [113,118].

Unlike LMXBs, which display diverse types of QPOs over a wide frequency range of ~ 0.1 – 10^3 Hz across various states [6,177,178], QPOs in AXRPs are sporadically detected, predominantly in transient BeXBs and occasionally in persistent sources. These QPOs typically occur at lower frequencies ($\nu_{\text{QPO}} \sim 0.01$ – 1 Hz) [113,179–183]. During the giant outbursts of A0535+262, the center frequency of QPOs was found to be tightly correlated with the X-ray flux and spin-up rates, offering strong evidence of accretion disk [179,184,185]. The two most commonly used models for explaining QPOs in AXRPs are the beat-frequency model (BFM [186]) and the Keplerian frequency model (KFM [187]), in both of which ν_{QPO} is related to the location of the inner edge of the accretion disk. But, in some cases, the QPO frequency has no obvious dependence on the X-ray flux, challenging current theories [182,188]. Interestingly, *Fermi*/GBM recorded the transient QPOs from RX J0440.9+4431, which only appeared at certain spin pulse phases in the supercritical accretion regime. Such spin-phase-dependent QPOs likely originate from the bottom of the accretion column instead of the outer accretion disk [112].

3.4. Superorbital Modulation

Superorbital modulations on time scales of tens and hundreds of days are observed across different types of LMXBs and HMXBs [189–191]. Diverse mechanisms have been proposed to account for the long-term variability, primarily linked to accretion processes, e.g., radiation-induced warping and/or precession disk [192–194], and the scenario of a precessing NS for the unique source Her X-1 [160,195,196]. Alternatively, in BeXBs, superorbital modulation is observed at both X-ray and optical wavelengths, primarily driven by the formation and depletion of the Be star’s circumstellar disk [197]. Notably, the superorbital period (P_{super}) has a nearly linear positive correlation with the orbital period, with different types of systems occupying distinct regions in the $P_{\text{super}} - P_{\text{orb}}$ diagram [191,197] (Figure 5). This relationship underscores the influence of orbital dynam-

ics on long-term variability and is further supported by the correlation between the $H\alpha$ equivalent width and P_{orb} [65,198].

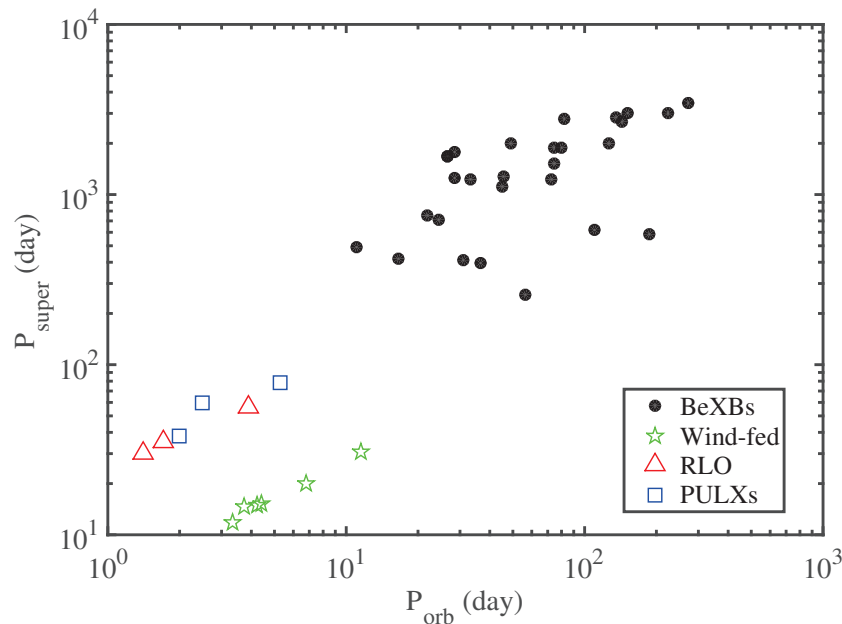


Figure 5. $P_{\text{super}} - P_{\text{orb}}$ diagram for different types of HMXBs and PULXs. The period values of BeXBs (black circles) were adopted from (Rajoelimanana et al. [197], Chen et al. [199] and the references therein), while data on wind accretion SGXBs (green pentagram) and RLO systems (red triangle) were adopted from (Corbet and Krimm [191] and references therein). The data for PULXs (blue square) were adopted from (King et al. [18] and references therein).

4. Spectral Properties

The spectra of AXRPs are characterized by non-thermal radiation, resulting from bulk and thermal Comptonization scatterings of hot plasma surrounding the polar caps with seed photons emitted via blackbody, bremsstrahlung, cyclotron processes [50,53,200]. The bremsstrahlung is generally considered the primary source of seed photons in the accretion column, except for cases at very low accretion rates, where the blackbody becomes more dominant [53]. In addition, it is possible that a substantial fraction of the radiation may be emitted from a more extended region due to the reflection, which is characterized by Compton scattering in the NS atmosphere [201,202]. The spectral shape of AXRPs is highly variable, reflecting changes in the accretion structure and/or modulations caused by absorption and obscuration. This spectral variability can be explored via spectral modeling or a model-independent hardness-intensity diagram [118]. In observations, the spectra of AXRPs generally consist of a continuum component and several narrow features, which are discussed in detail below.

4.1. Continuum

A non-saturated Comptonization results in a power-law-like spectral shape, which cuts off at the plasma temperature. Therefore, some phenomenological models, such as the cut-off power-law (*Cutoffpl*), the high-energy cut-off power-law (*Highecut*), the negative and positive exponential cut-off (*Npex*), and the Fermi Dirac cut-off (*Fdcut*), have been widely used in the literature to describe the continuum photon spectra of AXRPs [203–205]. Their shapes are presented as follows:

$$Cutoffpl(E; K, \Gamma, E_{\text{fold}}) = K E^{-\Gamma} \exp(-E/E_{\text{fold}}) \quad (9)$$

$$Highecut(E; K, \Gamma, E_{\text{cut}}, E_{\text{fold}}) = K E^{-\Gamma} \begin{cases} 1, & \text{if } E \leq E_{\text{cut}} \\ \exp(-\frac{E - E_{\text{cut}}}{E_{\text{fold}}}), & \text{if } E \geq E_{\text{cut}} \end{cases} \quad (10)$$

$$Npex(E; K_1, \Gamma_1, K_2, \Gamma_2, E_{\text{fold}}) = (K_1 E^{-\Gamma_1} + K_2 E^{-\Gamma_2}) \exp(-E/E_{\text{fold}}) \quad (11)$$

$$Fdcut(E; K, \Gamma, E_{\text{cut}}, E_{\text{fold}}) = \frac{K E^{-\Gamma}}{1 + \exp(-\frac{E - E_{\text{cut}}}{E_{\text{fold}}})} \quad (12)$$

where E is the photon energy, and $K, \Gamma, K_1, \Gamma_1, K_2, \Gamma_2, E_{\text{cut}}, E_{\text{fold}}$ are free parameters. Since these models exhibit a similar shape, a given spectrum can often be adequately fitted by multiple models. There are also several physics-motivated models that have been employed to describe the spectra of AXRPs, such as `compTT`⁴, `comptb`⁵, `compmag`⁶, and `bwcycl`⁷. `compTT` is an analytic thermal Comptonization model [206], and `comptb` is more complex, which takes both thermal and bulk Comptonizations into account [207]. `compmag` and `bwcycl` are two models developed specifically for the accretion column of highly magnetized X-ray pulsars assuming different velocity profiles of the infalling material [50,200,208]. They can be used to constrain the physical properties of the accretion column (i.e., the radius, the optical depth, and the electron temperature) and have been successfully applied to many sources, e.g., Her X-1⁸, RX J0440.9+4431, Cen X-3, and 4U 1538-522 [210–214]. Regardless of the model used, the spectral parameters typically show a pronounced dependence on luminosity. For most strong outbursts of AXRPs, the spectral evolution with luminosity reveals two distinct branches (see Figure 6 in [118]), akin to the behavior observed in the hardness-intensity diagram. This can be interpreted in terms of the different accretion regimes discussed in Section 2.3. We caution that luminosity is not the only factor that can impact the underlying physics. For example, the hysteretic effect—characterized by spectral and timing differences between the rising and declining phases of outbursts at similar luminosity states—has been observed in many sources [116,215–218].

The above models have limitations when fitting the spectra obtained from low-luminosity sources because they only consider the photons escaping through the wall of the accretion column, which is dominant when the accretion rate is relatively high. For the faint states of GX 304-1, 1A 0525+262, and X Persei observed at a low luminosity of $\sim 10^{34} \text{ erg s}^{-1}$, their spectra present a double-hump shape, instead of a cutoff power-law shape as mentioned above [219–221] (Figure 6). This spectral evolution at low luminosities is also associated with changes in pulse profiles [222]. Theoretical calculations suggest that the soft thermal hump at low energies originates from the deep layer of the NS atmosphere, and the high-energy hump might be explained as resonant Comptonization in the heated non-isothermal part of the atmosphere and cyclotron photons [107,108]. To the best of our knowledge, the only physical model currently available for data analysis is an updated version of `bwcycl` (BW22), which has been successfully applied in RX J0440.9+4431 [53,113].

In addition to the Compton component, other broad features are observed in AXRPs' spectra. For example, a soft excess commonly appears below 1 keV, which can be modeled with a blackbody component [223]. It may originate from different emission processes, depending on the luminosity, such as the reprocessing of hard X-rays from the NS by the inner region of the accretion disk and emission from the surrounding diffuse gas. Furthermore, in many cases, the spectral fitting is not perfect, and there might be some residuals around 10 keV (known as the “10-keV-feature”; [204]). They can be technically alleviated by including an additional broad emission or absorption line [224]. Their physical origin is still unclear. It is also possible that they just reflect the imperfections of the available continuum models.

The radiation observed from Earth is absorbed by both the interstellar medium (ISM) and the matter surrounding the binary system. Given that the latter might be inhomogeneous and highly clumpy, the absorption appears to be highly variable in many sources. When the equivalent hydrogen column (N_{H}) is high enough, the absorption significantly

influences the soft part of the continuum as $I_{\text{obs}}(E) = I_{\text{source}}(E)e^{-\sigma(E)N_{\text{H}}}$, where $\sigma(E)$ is the abundance-dependent photoelectric cross-section [225]. If only a fraction of the emergent photons is absorbed while the rest are detected directly by observers, a partial covering model (e.g., *pcfabs*⁹) is usually employed. Studying absorption variations at different orbital phases can provide valuable information about the surrounding large-scale accretion structures, such as the accretion wake [74,226–228]. In addition, with high-energy-resolution instruments, such as the gratings on *Chandra* and *XMM-Newton* and the microcalorimeter on *XRISM*, the velocity and structure of fast outflows, i.e., the accretion disk wind, can be resolved through blue-shifted narrow absorptions [229–232]. Furthermore, after the absorption, the re-emission of X-rays (known as “X-ray reprocessing”), e.g., Fe K_{α} fluorescence, also reflects the distribution of the material, being an important diagnostic tool to probe the environment in binary systems [233–236].

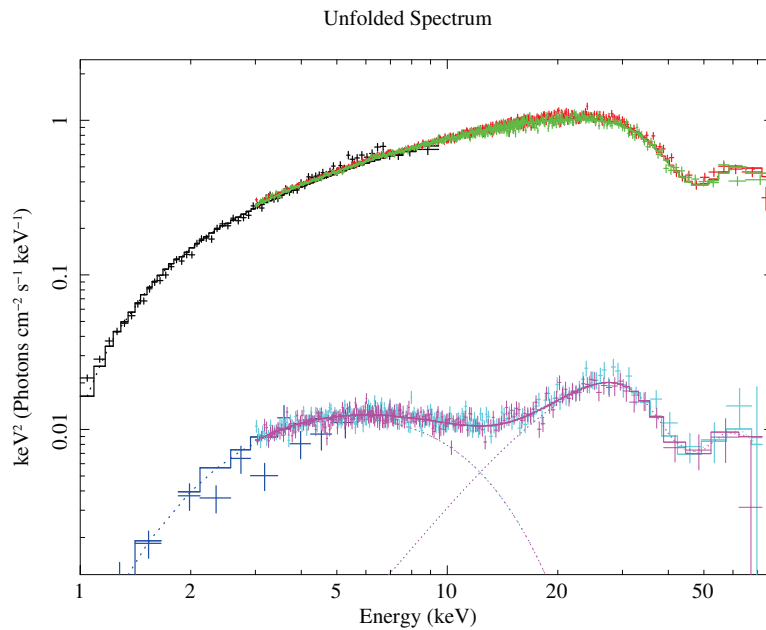


Figure 6. Representative spectra of AXRPs at high- and low-luminosity states. The structure around 45 keV is caused by the cyclotron line. This plot was reproduced using the data from Tsygankov et al. [219].

4.2. Cyclotron Resonant Scattering Features

In many AXRPs, there are broad absorption features seen in the spectra, which are known as cyclotron resonant scattering features (CRSFs) or cyclotron lines. They are caused by the resonant scatterings of photons between discrete Landau levels of electrons' motion perpendicular to the strong magnetic field lines. Since the first discovery in Her X-1 in 1976 through a balloon experiment [54,237], CRSFs have been detected in several tens of sources using broadband observatories, such as *BeppoSAX*, *RXTE*, *INTEGRAL*, *Suzaku*, *NuSTAR*, and *Insight-HXMT* (for a detailed review, see [31]). These features are of significant importance as they provide a model-independent way to directly measure the magnetic fields of AXRPs via the “12-B-12” rule: $E_{\text{cyc}} \approx \frac{n}{1+z} 11.6 \times B_{12}$ keV, where E_{cyc} is the centroid line energy, z is the gravitational redshift, B_{12} is the magnetic field in the line-forming region in units of 10^{12} G, and n is the quantum number. In some sources, multiple cyclotron lines can be observed, e.g., 4U 0115+63 and GX 301-2 [238–240], while their line energies might not always be harmonically spaced [241–243]. Cyclotron lines in AXRPs are broad features due to the Doppler effect for thermal broadening [244] and/or a gradient of the magnetic field in the line formation region. In observations, the CRSF width is typically a few keV and shows a correlation with E_{cyc} [204]. This inevitably results in the slight coupling between the CRSF model and the continuum model.

Theoretical studies of CRSFs have been conducted using both analytical methods and Monte Carlo simulations by different authors assuming different line-forming regions and configurations [202,245–253]. Due to the complexity of physically motivated models, phenomenological models are often employed in observational studies to fit the data. The most commonly used models in the literature are multiplicative models *gabs*¹⁰ and *cyclabs*¹¹, where the former has a Gaussian profile, and the latter is pseudo-Lorentzian. Both models are characterized by the centroid energy, the line width, and the strength (or the optical depth) of the line. Due to the broad nature of CRSFs, the selection of continuum and line models introduces systematic uncertainties in E_{cyc} , typically by a few keV.

It is important to note that the observed E_{cyc} does not directly reflect the intrinsic polar magnetic field but rather measures the local magnetic field where resonant scattering occurs. Consequently, E_{cyc} is expected to vary depending on the line-forming region. In observations, E_{cyc} is typically not constant, showing variations with both the spin phase and luminosity. The spin-phase-dependent line is related to the distortion of the local magnetic field of two poles due to the confinement of the accreted matter [254]. Additionally, in some cases, the CRSF is only detectable at certain spin phases and may not appear in the phase-averaged spectrum, such as the highest CRSF discovered in Swift J0243.6+6124 [255]. The relationship between E_{cyc} and luminosity was initially investigated in Her X-1 [256] and has been reported in other sources (for a summary, see Figure 9 in [31]). Generally, E_{cyc} remains stable or shows a positive correlation with luminosity when the luminosity is below a critical value, which can be explained as variations in the emission region height [257] and/or the Doppler effect [258]. On the other hand, the negative E_{cyc} –luminosity correlation is only firmly identified in bright outbursts of V0332+53 and 1A 0535+262,¹² [216,218,259,261] (as an example, see Figure 7). A lower E_{cyc} at higher luminosities may arise from the radiation emitted by a higher accretion column or from the reflection process occurring away from the magnetic poles, where the local magnetic field is weaker [201]. In addition, in the supercritical regime, both the Doppler effect due to the bulk motion and the gravitational redshift should be considered [262]. We note that the transition between positive and negative E_{cyc} correlations serves as a key indicator for confirming the onset of the accretion column (for details, see Section 2.3).

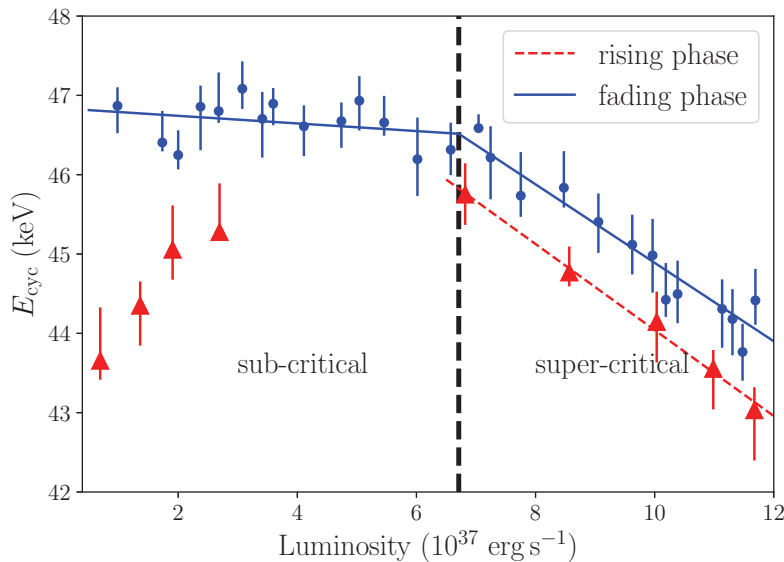


Figure 7. The evolution of E_{cyc} as a function of luminosity (L) observed in 1A 0535+262. Red triangles and blue points present the data from the rising and fading phases of the 2020 giant outburst. Their discrepancy reflects the hysteresis effect. The black dashed line represents the critical luminosity L_{crit} , which roughly separates two accretion regimes. Below $L < L_{\text{crit}}$, E_{cyc} remains stable or exhibits a positive correlation with luminosity, while above $L > L_{\text{crit}}$ a negative correlation when $L > L_{\text{crit}}$. This figure was reproduced based on the data from Kong et al. [218].

The long-term evolution of CRSFs has been reported in Her X-1 and Vela X-1 [222,263–269]. Both sources exhibit an E_{cyc} decay period ($\dot{E}_{\text{cyc}} \sim$ several 0.1 keV per year) and an E_{cyc} -invariant period over a timescale of several decades. The long-term behavior likely reflects a geometric displacement of the line-forming region and/or distortions of the magnetic field around polar caps. Considering the rapid decay rate and the sudden jump of E_{cyc} between 1990 and 1995 in Her X-1, the long-term evolution of E_{cyc} probably follows a cyclic pattern, which should be confirmed in the future [263].

5. Polarimetry

The X-ray emissions of AXRPs were initially presumed to be strongly polarized due to the effects of the plasma and vacuum birefringence as the radiation propagates through a strongly magnetized plasma. These properties made AXRPs ideal candidates for polarization measurements [60,61,270,271]. The polarization degree (PD) and polarization angle (PA) are highly sensitive to the geometry of the emission regions and magnetic fields, offering a unique opportunity to study radiative transport and matter dynamics in the immediate vicinity of pulsars. The launch of the first highly sensitive X-ray polarimeter *IXPE* marked a significant step in this field, allowing researchers to test these theoretical predictions. Over its first two years of operation, *IXPE* observed 10 AXRPs, and impressive achievements have been presented in the latest review [35]. Here, we just repeat some of key findings.

For all observed AXRPs, the measured PD was very low $\sim 5\text{--}20\%$, which is significantly lower than the prediction ($\sim 60\text{--}80\%$) by all existing theoretical models [160,162,272]. Both PD and PA evolve along with the pulse phase. The PD has a rather complex variation and is anti-correlated with flux. On the other hand, the PA usually has a smooth profile and can be described with the rotating vector model (RVM) (but see Doroshenko et al. [62] for situations where an unpulsed polarized component exists). Due to the relatively narrow energy band coverage of *IXPE* ($\sim 2\text{--}8$ keV), a positive relationship between the PD and the photon energy was observed in only a few sources, such as X Persei [273] and Vela X-1 [274]. For most other AXRPs, however, both PD and PA appear largely energy-independent. In addition, a $\sim 90^\circ$ difference in the PA has been found between 2 and 3 keV and above 5 keV in Vela X-1, which may be caused by vacuum resonance [274,275]. As the era of X-ray polarimetry begins, further detailed polarization measurements at varying states, enabled by *IXPE* and the forthcoming *eXTP* observatory [276], promise to deliver transformative insights.

6. Related Binary Systems

Observations across multiple wavelengths have led to the discovery of intriguing types of binary systems beyond AXRPs. Here, we only provide very short discussions on two of the most striking binary systems—gamma-ray binaries and PULXs.

A small yet growing number of HMXBs are found to display bright emissions in nearly all wavelengths (from radio to TeV), being distinguished from the majority of HMXBs. The radiation from these sources with orbital modulation is relatively stable except when the compact objects pass through the periastron. Due to their high-energy output, predominantly above 1 MeV, they are classified as “gamma-ray binaries” [277]. The mechanisms driving particle acceleration and high-energy emissions in these systems remain poorly understood. Key breakthroughs have been achieved with the detection of radio pulsations from PSR B1259-63 (48 ms), PSR J2032+4127 (143 ms), and LS I +61° 303 (269 ms) [25,278,279], and gamma-ray pulsations have also been reported for PSR J2032+4127 [279]. These results clearly confirm the fact that broadband emissions come from shocks formed by the collision of pulsar and stellar winds, ruling out the accretion-powered microquasar scenario. The most essential difference between the AXRPs reviewed in this paper and gamma-ray binaries is that the former is accretion-powered, while the latter is mostly a non-accreting system, although the nature of the other gamma-ray binaries is still under debate.

To date, more than about 1800 ULXs with $L_X \sim 10^{39}–10^{42}$ erg s^{−1} have been discovered in nearby galaxies [280]. The most relevant result was the discovery of the X-ray pulsations from M82 X-2 in 2014, which identified it as the first PULX [281]. Since then, X-ray pulsations have only been detected from about less than 10 PULXs after extensive searching [282–287]. Nevertheless, the orbital periods and superorbital periods were also reported for several sources, e.g., M82 X-2 [281], NGC 5907 ULX-1 [288], and M51 ULX-7 [287,289]. The periodic signal of ~ 64 days detected from NGC 7793 P13 remains contentious, with debates about whether it represents the orbital or superorbital period [240,290,291]. As can be seen in Figures 1 and 5, PULXs and disk-fed SGXBs located at the same regions in both diagrams, indicating the presence of RLO in PULXs. However, care should be taken as the sample of PULXs is very small.

PULXs share a similar spin-up trend with the Galactic AXRPs but with much higher luminosity and spin-up rates [18]. Therefore, many works would like to address these two types of sources together, calling both “PULXs”. But, actually, the canonical PULXs in outer galaxies are sharply distinct from Galactic AXRPs in both spectral and temporal properties: 1. Galactic AXRPs can only occasionally reach the super-Eddington luminosity ($\sim 10^{39}$ erg s^{−1}), while PULXs can stay at the ultraluminous state for a long time with much higher luminosity. 2. In contrast to the complex pulse profiles emerged in AXRPs (e.g., Figure 4), the pulse profiles of all PULXs are nearly sinusoidal. 3. The high signal-to-noise spectra of PULXs display a curvature of around 10 keV and can be phenomenologically described by a two-component (or three-component) model. Apart from the details of diverse models, the radiation of PULXs is usually dominated by soft X-rays [17,292] and tends to become harder when brighter [293]. However, the spectra of AXRPs are significantly harder with a photon index of $\Gamma < 1.0$ and move along the diagonal branch [118]. These observational results suggest that the X-ray radiation in PULXs is not strongly beamed, and the accretion flows have different geometries in PULXs and AXRPs [18, 108,294]. As we discussed for AXRPs, the strength of magnetic fields and the type of optical donors are two key factors in NS accreting systems. These significant differences in observational phenomena are thought to be due to the differences in the strength of their magnetic fields and/or the accretion rate in PULXs and AXRPs. There is still a huge controversy regarding whether PULXs have magnetar-like ($B \geq 10^{13}$ G) or normal magnetic fields as AXRPs ($B \sim 10^{12}$ G) [119,295–298], though potential CRSFs were reported [299–301]. Additionally, the donor stars in PULXs are intriguing but challenging to study due to their distance. For example, the companion star of NGC 7793 P13 was identified as a B9Ia supergiant [302], and a red supergiant donor star was suggested for NGC 300 ULX-1 [303]. Further research is necessary to elucidate these enigmatic systems.

7. Multi-Wavelength Advances

Although the primary radiation of AXRPs is emitted in the X-ray band, a multi-frequency approach is clearly essential to fully understand the physical mechanism driving their behaviors. For instance, in the era of *Gaia*, accurate measurements of the position, distance, and velocity of HMXBs can be obtained. This allows us to estimate their luminosities precisely and enables constraints on natal kicks, birthplaces, and the spatial distribution of NSs [304–306]. On the other hand, optical observations have been performed for investigating the properties of companion stars and directly reflecting the inner-binary environment, such as the truncated and warped accretion disks [307,308].

Recent radio observations strikingly detect the radio counterparts of AXRPs, and the L_X – L_{radio} plane shows a correlation as $L_{\text{radio}} \propto L_X^{0.86 \pm 0.06}$ during giant outbursts [309–314]. This power-law index is similar to those measured in NS and BH LMXBs, but the radio intensity is significantly lower by 2–4 orders of magnitude. Current observations suggest that this radio radiation is likely attributed to jets. In theory, however, accretion systems with high magnetic fields, such as AXRPs, are not expected to launch jets, and the underlying emission mechanism remains unknown.

Theoretically, AXRPs might also emit MeV radiation. This is because, in some cases, the kinetic energy of infalling matter exceeds the binding energy of nucleons within nuclei. This leads to the nuclear spallation of a large portion of the accreted helium and liberates neutrons. Eventually, a redshifted line is expected due to proton–neutron recombinations $p + n \rightarrow D + \gamma$ (2.223 MeV) [315–319]. As of now, several attempts have been made using *COMPTEL*, *RHESSI*, and *INTEGRAL/SPI*, which however only result in upper limits [319–323]. The detection of MeV lines from accreting pulsars remains one of the key scientific projects for future MeV astronomy projects, such as *COSI*, *eASTROGAM*, and *MASS* [324–326].

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Abbreviations

The following abbreviations are used in this manuscript:

AXRPs	Accreting X-ray pulsars
BeXBs	Be X-ray binaries
BHs	Black holes
BFM	Beat-frequency model
CRSFs	Cyclotron resonant scattering features
GPD	Gas-pressure-dominated
IID	Hardness-intensity diagram
HMXBs	High-mass X-ray binaries
IMXBs	Intermediate-mass X-ray binaries
KFM	Keplerian-frequency model
LMC	Large Magellanic Cloud
LMXBs	Low-mass X-ray binaries
NSs	Neutron stars
PA	Polarization angle
PD	Polarization degree
PULXs	Pulsating ultraluminous X-ray sources
QPOs	Quasi-periodic oscillations
RLO	Roche-lobe overflow
RPD	Radiation-pressure-dominated
RVM	Rotating vector model
SFXTs	Supergiant fast X-ray transients
SGXBs	Supergiant X-ray binaries
SMC	Small Magellanic cloud
ULXs	Ultraluminous X-ray sources
XBs	X-ray binaries

Notes

¹ In this paper, we use “Galactic accreting pulsars” to include BeXBs in Magellanic Clouds.

² L_{cirt} is also estimated by Mushtukov et al. [52] considering that L_{cirt} is not associated with the Eddington limit and using an exact Compton scattering cross-section. The result suggests that L_{cirt} is not a monotonic function of B .

- 3 <https://gammaray.nsstc.nasa.gov/gbm/science/pulsars.html>, accessed on 6 December 2024.
- 4 <https://heasarc.gsfc.nasa.gov/xanadu/xspec/manual/XSmodelComptt.html>, accessed on 6 December 2024.
- 5 <https://heasarc.gsfc.nasa.gov/xanadu/xspec/models/comptb.html>, accessed on 6 December 2024.
- 6 <https://heasarc.gsfc.nasa.gov/xanadu/xspec/manual/node155.html>, accessed on 6 December 2024.
- 7 <https://heasarc.gsfc.nasa.gov/xanadu/xspec/manual/XSmodelBwycl.html>, accessed on 6 December 2024.
- 8 We consider Her X-1 in this paper although it is an IMXB [209], because it shares a lot in common with the other AXRPs in HMXBs.
- 9 <https://heasarc.gsfc.nasa.gov/xanadu/xspec/manual/XSmodelPcfabs.html>, accessed on 6 December 2024.
- 10 <https://heasarc.gsfc.nasa.gov/xanadu/xspec/manual/XSmodelGabs.html>, accessed on 6 December 2024.
- 11 <https://heasarc.gsfc.nasa.gov/xanadu/xspec/manual/node247.html>, accessed on 6 December 2024.
- 12 but a possible negative correlation is also seen in the faint states of 1A 0535+262 and GRO J1008-57 [259,260].

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Article

Effects of Thermodynamics on the Concurrent Accretion and Migration of Gas Giants in Protoplanetary Disks

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Abstract: Accretion and migration usually proceed concurrently for giant planet formation in the natal protoplanetary disks. Recent works indicate that the concurrent accretion onto a giant planet imposes significant impact on the planetary migration dynamics in the isothermal regime. In this work, we carry out a series of 2D global hydrodynamical simulations with *Athena++* to explore the effect of thermodynamics on the concurrent accretion and migration processes of the planets in a self-consistent manner. The thermodynamics effect is modeled with a thermal relaxation timescale using a β -cooling prescription. Our results indicate that radiative cooling has a substantial effect on the accretion and migration processes of the planet. As cooling timescales increase, we observe a slight decrease in the planetary accretion rate, and a transition from the outward migrating into inward migration. This transition occurs approximately when the cooling timescale is comparable to the local dynamical timescale ($\beta \sim 1$), which is closely linked to the asymmetric structures from the circumplanetary disk (CPD) region. The asymmetric structures in the CPD region which appear with an efficient cooling provide a strong positive torque driving the planet migrate outward. However, such a positive torque is strongly suppressed, when the CPD structures tend to disappear with a relatively long cooling timescale ($\beta \gtrsim 10$). Our findings may also be relevant to the dynamical evolution of accreting stellar-mass objects embedded in disks around active galactic nuclei.

Keywords: protoplanetary disks; planet-disk interactions; accretion; extrasolar gas giants; black holes

1. Introduction

With several exoplanet populations have been detected so far, the discovery of the close-in super-Earths [1] and hot Jupiters [2] has prompted the theoretical proposal that planets may migrate significantly in the protoplanetary disk (PPD) phase [3,4], although it has been suggested that the giant planet could also migrate due to a more populated asteroid belt [5]. Another compelling evidence for disk-driven migration is the discovery of multiple planets in or near mean motion resonant configurations [6–9]. The migration of giant planets has a substantial dynamic impact on the formation regions and final locations of minor bodies (e.g., [10,11]). For the migration of a low-mass protoplanet, the gravitational interaction with the PPD is usually the dominant factor which leads to the classical type I migration [12]. The torque exerted onto the protoplanet can be divided into two parts, the wave torque [13,14], and the corotation torque or horseshoe drag [15,16], which originate from the asymmetries in different regions of the PPD and usually drives the planet to migrate inward [17–20]. When the protoplanet’s mass is high enough to modify

the density distribution of the PPD, the tidal interaction between the protoplanet and the disk leads to the opening of a gap around the protoplanet's orbit, and the migration is in the realm of type II, which is believed to be locked to the viscous drift velocity of the background disk [13,21].

In the meanwhile, under the widely adopted core accretion model for planet formation, the dynamical runaway gas accretion occurs at a “critical core mass” above which the envelope fails to maintain hydrostatic equilibrium [22–25]. In the phase of the dynamical gas accretion, the gas supplied by the PPD, due to the conservation of angular momentum, could form a disk around the protoplanet, known as the circumplanetary disk (CPD) [26–29]. The accretion onto the protoplanets sensitively depend on the structures of the CPD, which finally determine the asymptotic mass of mature planetary system (e.g., [29–35]). The presence of the deep gap and the strong tidal barrier effect in the type II regime would prevent the gas from reaching the protoplanets' vicinity [13,21], thus impeding their growth (e.g., [35,36]). However, further studies have shown that the gas in the PPD could still pass through the gap through the horseshoe orbit, so that the protoplanet could accrete gas anyway [29,37–41]. Therefore, a new paradigm for the type II migration emerges, which suggests that the type II migration rate is not at the viscous velocity of the disk, but shows a similar scaling law as the type I case after considering the steady gap structure [42].

It has been shown that the accretion process in the CPD is mainly contributed by spiral shock dissipation, which is sensitive to the disk thermodynamics [43]. While most of the studies for the CPD structures and the gas dynamical accretion adopted an isothermal equation of state (EOS), there have been some works including non-isothermal effect and radiation hydrodynamics for the CPD formation (e.g., [28,44,45]). Different treatments of the thermodynamics could also influence the circumbinary disk accretion and dynamics (e.g., [46–48]). Furthermore, the disk thermodynamics could alter the global disk structure [49–52] and thus the planet migration dynamics significantly [53–55]. However, the accretion and migration of planets in these works are typically studied in isolation. In this sense, the coupled evolution between the CPD accretion and planetary accretion are not treated in a self-consistent manner, as also highlighted in Li et al. [56].

Recently, it has been found that the concurrent accretion would impose strong impact on the migration of the planets in PPDs when taking into account of a fully self-consistent treatment for an accreting planet [56,57]. Especially, Li et al. [56] found that the structures of CPD play an important role on the migration of the planets in the isothermal limit. As the thermodynamics of the disk could modify the CPD structures significantly, it is thus still unclear how different thermodynamics affect the concurrent accretion and migration of planet in PPDs, which is the main goal of this work. Since there have been some theoretical arguments suggesting that there could be a population of stellar-mass objects (e.g., stellar-mass black holes, stars) embedded in active galactic nucleus (AGN) disks, our studies should also be relevant to the accretion and migration of stellar-mass objects embedded in AGN disks.

This rest part of the paper is organized as follows. We present the numerical methods in our hydrodynamical simulations in Section 2. The results are shown in Section 3. The conclusions and discussions are presented in Section 4.

2. Numerical Methods

2.1. Hydrodynamical Model

We simulate globally an accreting planet embedded in a thin and non-self-gravitating disk using Athena++ [58]. Most of our simulation setup are similar to Li et al. [56]. Here we only briefly describe the main physics of our models. The simulations are performed in a

cylindrical coordinate system (r, ϕ) , and the origin of the coordinate system is located at the position of the central star with mass M_* . We initialize a constant aspect ratio of the PPD as $h \equiv \frac{H}{r} = 0.05$ over the whole disk, where H is the disk scale height. This leads to an initial temperature profile of $T_{\text{disk}} \propto r^{-1.0}$. The local sound speed is thus $c_s = h v_K$, where v_K is the local Keplerian velocity of the disk. The disk has an axis-symmetric initial profile of $\Sigma(r) = \Sigma_0 (\frac{r}{r_0})^p$ with $p = -0.5$, where r_0 is the typical length scale of the disk, and Σ_0 is the surface density at r_0 . Due to the high uncertainty from the observations, the exponents of the temperature and surface density profiles exhibit considerable variations for different PPDs at different radial locations [59,60]. Therefore, we adopt specific exponents of the profiles to maintain a steady-state accretion disk, remaining within the acceptable range of observational uncertainties.

The planet is fixed at a circular orbit with a semi-major axis of $a_p = r_0$ in a corotating frame with the planet so that the planet's azimuthal angle fixed at $\phi_p = 0$. We consider a planet with 1 Jupiter mass ($m_p = M_J$) which corresponds to a mass ratio between the planet and the solar-type star of $q \equiv \frac{m_p}{M_*} = 0.001$. The gravitational potential of the planet at r_p is in the form,

$$\Phi_p = -\frac{Gm_p}{(|\mathbf{r}_p - \mathbf{r}|^2 + s^2)^{1/2}} + q\Omega_p^2 \mathbf{r}_p \cdot \mathbf{r}, \quad (1)$$

where s indicates the softening length of the potential which is set as $0.1 r_h$ unless otherwise stated, and r_h is the Hill radius of the planet. Other softening lengths have also been tested for the convergence, as shown in the Appendix A.

We adopt a Shakura-Sunyaev viscosity prescription $\nu_g = \alpha c_s H$ with a constant $\alpha = 0.01$ across the whole disk [61]. A relative large viscosity parameter is to ensure the disk reach the quasi-viscous steady state within a feasible computation time. The disk, covering a radial range of $[r_{\text{in}}, r_{\text{out}}] = [0.3, 3.5] r_0$, is resolved with a uniform base grid of 80 and 256 cells in radial and azimuthal directions, respectively. We have implemented four levels of static mesh refinement within a distance of $1 r_h$ from the planet to effectively resolve the CPD region. With such a grid resolution, the Hill radius can be resolved by about 20 cells in each dimension. A test with different grid resolutions have been performed as shown in the Appendix A to confirm the convergence of our simulation results. For all of our simulations, we take the natural units of $G = M_* = r_0 = 1.0$, and the simulation results are scale-free in a non-self-gravitating disk.

2.2. Cooling of the Disk

To explore the effects of thermodynamics, we conduct simulations with locally isothermal and adiabatic EOSs with β -cooling, as adopted in previous works (e.g., [50,52,62]). The EOS for the isothermal case is

$$P = c_s^2 \Sigma. \quad (2)$$

The adiabatic EOS is

$$P = (\gamma - 1)e, \quad (3)$$

where P is the pressure, and e is the specific internal energy of the gas. The sound speed in this case is defined as $c_s = \sqrt{\gamma P / \Sigma}$. In our simulations, we take $\gamma = 1.4$ for Equation (3). In addition, We perform the adiabatic EOS simulations with a simple thermal relaxation

process, such that the gas internal energy is relaxed towards its initial state on a timescale controlled by the parameter β ,

$$\frac{dE(t)}{dt} = -\frac{\Omega}{\beta}(E(t) - E(0)), \quad (4)$$

where Ω is the local orbital frequency of the disk. In the calculation, this is equivalent to the following equation in which E is transformed after each time step as follows,

$$E_{t+dt} = E_t(1 - \frac{\Omega}{\beta}dt) + E_0 \frac{\Omega}{\beta}dt, \quad (5)$$

where E_{t+dt} and E_t are the internal energy of the gas in the grid before and after each time step, E_0 indicates the initial state. So the local cooling timescale t_{cool} of the system is β/Ω . The smaller the value of β , the closer the disk is to the locally isothermal condition. In contrast, a higher value of β brings the disk closer to the adiabatic approximation. Assuming a Minimum Mass Solar Nebula, the cooling timescale is shorter (e.g., $\beta \simeq 10^{-2}$ at 100 au) at the outer disk and much longer at the inner disk (e.g., $\beta \simeq 10^5$ at 1 au) [62]. In our numerical simulations, we examine β values across a broad range from 10^{-2} to 10^2 .

2.3. Planet Accretion and Boundary Conditions

We follow Li et al. [56] to implement the accretion onto the planet and the boundary condition both at the inner and outer edge of the disk. Basically, there are two parameters, which are sink hole radius r_{acc} around the accreting planet, a removal rate f within the sink hole, to describe the dynamical accretion of the planet. The planetary accretion rates $\dot{m}_p$ are recorded on-the-fly during each simulation, which are calculated as (e.g., [29–31,35,56])

$$\dot{m}_p = \int_{\delta r < r_{\text{acc}}} f \Sigma dS, \quad (6)$$

where Σ , dS , δr are the surface density, surface area of each grid, and the distance to the planet, respectively. We adopt an accretion radius as the same of the softening length with $r_{\text{acc}} = 0.1 r_h$ and a removal rate of $f = 5\Omega$, similar to Li et al. [56]. Other accretion prescriptions have also been tested for the convergence as shown in the Appendix A. The accreted mass and angular momentum are not added actively onto the planet, while their effect on the migration dynamics of the planet is analyzed by post-processing the gravitational and accretional torque components as in Li et al. [56].

For the radial boundary, we maintain a steady inflow $\dot{m}_d$ from the outer boundary, and preserve a radially constant inward mass flux at the inner edge as in Li et al. [56]. A wave damping is applied at the outer and inner edges to remove the wave reflection [63]. A periodical boundary is adopted at the azimuthal direction.

3. Simulation Results

We first outline the accretion dynamics of the planet, followed by an examination of the effects of planetary migration under various cooling prescriptions.

3.1. The Accretion of the Planet

The time evolution of the planetary accretion rates $\dot{m}_p$ for isothermal and different β -cooling are shown in Figure 1. All the simulations proceed to 20,000 orbits, which is roughly the viscous timescales at r_{out} (one viscous timescale can be calculated as $(2\pi\alpha h_0^2)^{-1}$ orbits). The accretion rates at 20,000 orbit for different runs are slightly below the mass supply rate $\dot{m}_d = 3\pi\Sigma v$ at the outer boundary, similar to [56]. This suggests that most of the mass supply from the outer disk is accreted onto the planet, with a small fraction of the disk supply accreted onto the central star. The nearly constant accretion rates onto the

planet around 20,000 orbits indicates the viscous quasi-steady state for the CPD accretion. We have further checked the mass flux across the Hill sphere around the planet, which is nearly constant as a function of δr and in good agreement with the accretion rates onto the planet. The accretion rate also matches well with the global disk mass flux jump at the planetary orbit. All of these confirm that the quasi-steady state for the CPD accretion is well established. For different cooling prescriptions, it can be seen that a higher β results in a slightly decrease in the planetary accretion rate $\dot{m}_p$, although strong variabilities appear for high β -cooling runs.

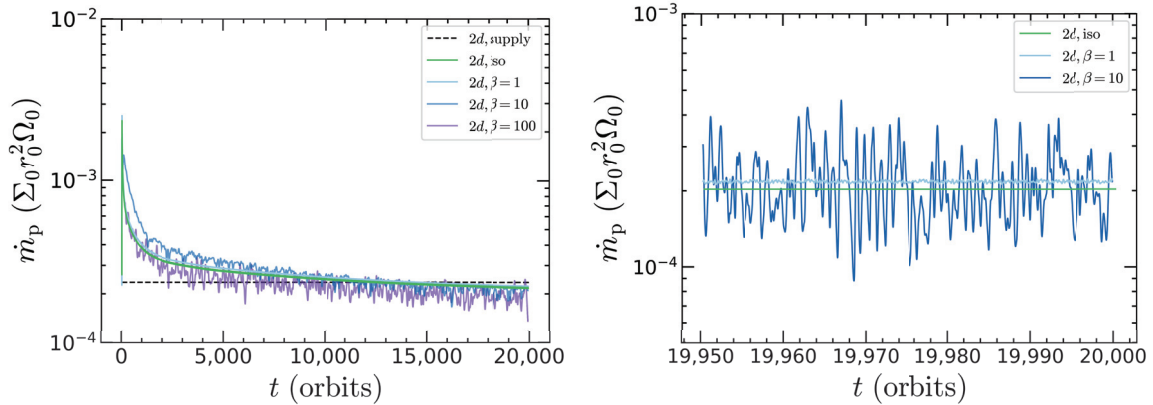


Figure 1. The left panel displays the running time-averaged planetary accretion rates in a locally isothermal disk and disks with varying β -cooling timescales, measured in unit of $\Sigma_0 r_0^2 \Omega_0$. The black dashed line indicates the material supply rate from the outer boundary. The right panel illustrates the short-term fluctuations in accretion rates for several representative simulations, without any time averaging.

The surface density distribution and the velocity field in the frame corotating with the planet are shown in Figure 2. The gas velocity field reveals the presence of a distinct disk structure in close proximity to the planet, known as the CPD, in the isothermal case. The disk structure is particularly prominent under isothermal condition. Additionally, the horseshoe streamline is clearly visible for the isothermal and low- β cases, but it disappears for $\beta \gtrsim 10$. It is evident that the accreted material is brought into the vicinity of the planet from the upper horseshoe orbits and flows outward from the spiral density waves. Notably, the rotational velocity is in the retrograde direction for $\beta \gtrsim 10$, in contrast to the prograde flow observed in the low β cases.

To facilitate a more comprehensive examination of the CPD structures in isothermal and long cooling timescale cases, we plot the azimuthally-averaged rotation curves and density profiles of the CPDs in Figure 3. Based on the rotation curves presented in the left panel, it is evident that the rotation velocity within $\sim 0.2 r_h$ under the isothermal condition closely resembles the Keplerian rotation. This is also true for the cases with $\beta \lesssim 0.1$. This indicates that the CPD is rotationally supported up to $\sim 0.2 r_h$ and exhibits characteristics of a disk structure in these cases, where the centrifugal force effectively counterbalances the gravitational force. In the longer cooling timescale cases, the rotation curve exhibits a significant deviation from the Keplerian curve in all regions within the Hill sphere and the materials even rotate in the retrograde direction for $\beta \gtrsim 10$ as confirmed in Figure 2. Consequently, it is the pressure gradient due to the inefficient cooling in the CPD region that balances the gravity. This is consistent with previous 3D simulations which have shown that the rotationally supported CPD appears mostly under the isothermal EOS, while it transformed into a pressure supported envelope for an adiabatic EOS (e.g., [28]). Actually, based on the temperature (sound speed) profile within the Hill sphere shown in the right panel of Figure 2, the Bondi radius $r_{\text{Bondi}} \equiv m_p / c_s^2$ at the expected CPD size from the planet

(e.g., $\simeq 0.2 r_h$) is smaller than the Hill radius due to the increase of the temperature in the vicinity of the planet, which suggests that the planetary accretion proceeds indeed by the Bondi-like process.

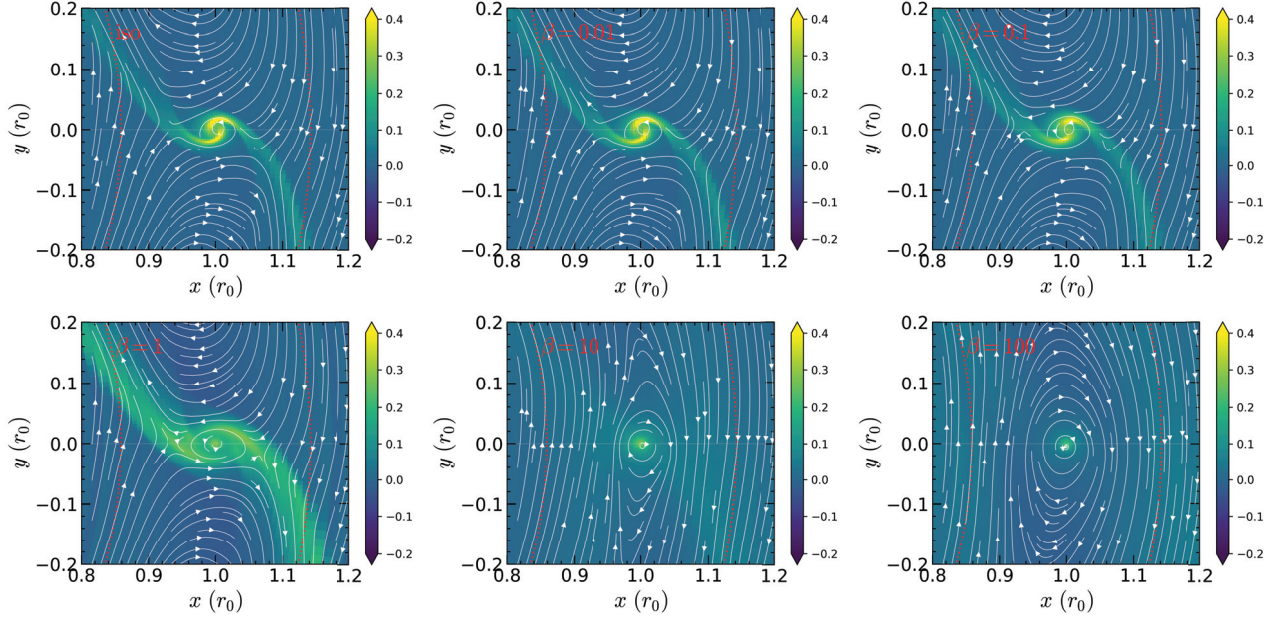


Figure 2. The density distribution $\delta\Sigma$ in the vicinity of the planet with different cooling timescales. $\delta\Sigma = \Sigma - \langle\Sigma\rangle$ where $\langle\Sigma\rangle$ is the azimuthally averaged surface density from the perturbed disk at the same orbital time. The arrows in the plots represent the velocity vectors relative to the planet. All plots are presented at 20,000 orbits. For the cases with $\beta = 10$ and $\beta = 100$, time averages over 100 and 500 orbits, respectively, were performed to achieve steady flow structures. The red dotted lines enclose the classical horseshoe region around the planetary orbit defined as $x_s = a_p(q/h)^{1/2}$ (e.g., [19]).

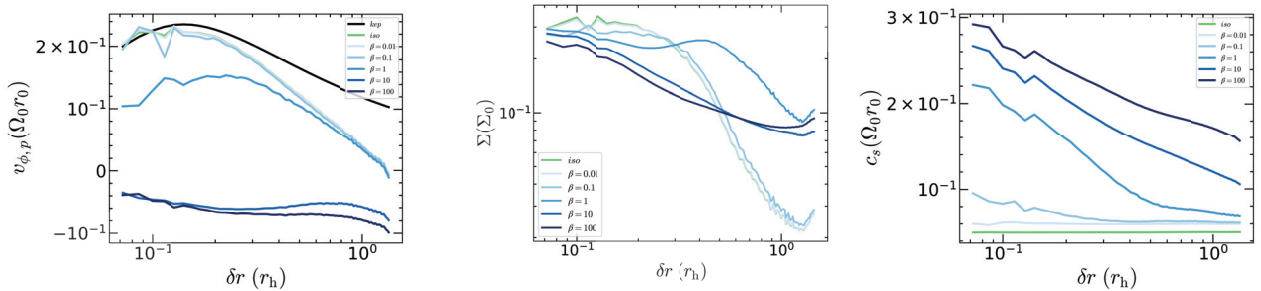


Figure 3. Left panel: the azimuthally-averaged rotation velocity relative to the planet for different models in unit of the local Keplerian velocity orbiting around the star. The black line represents the Keplerian rotation velocity around the planet. Middle panel: the surface density profile around the planet in unit of Σ_0 . Right panel: the azimuthally-averaged sound speed c_s in the vicinity of the planet in code unit.

3.2. Mechanism of the Accretion

To explore the accretion process, we examine the CPD's angular momentum conservation equation in the vicinity of a planet following Zhu et al. [43],

$$\partial_t \langle \Sigma \delta r v'_\phi \rangle_\phi = -\frac{1}{\delta r} \partial_r (\delta r^2 \langle \Sigma v'_r v'_\phi \rangle_\phi) + \langle \delta r \times F \rangle_\phi, \quad (7)$$

where δr is the radial distance relative to the planet and $\langle X \rangle_\phi$ represents $\int_{-\pi}^{\pi} X d\phi$. The quantities with ' indicate that they are measured in the frame relative to the planet. Here we have neglected the viscous stress associated with the explicit alpha viscosity in the CPD

since it is found that the explicit α viscosity is much smaller than the effective viscosity due to shock dissipation. The sum of all external forces, denoted as F , encompasses various components, including stellar gravity, centrifugal forces and Coriolis forces within the reference frame that orbits the star with the planet's orbital period. The planet's gravity does not contribute to the external forces because it is in parallel to the radial vector δr .

By decomposing the velocity in ϕ direction into two components, the averaged velocity $\bar{v}'$ (it is worth noting that we use the azimuthally averaged velocity in the ϕ direction instead of the local Keplerian velocity due to the distinct deviation from Keplerian motion under long cooling timescales) and the disturbance velocity $\delta v'_\phi$, and afterwards multiplying both sides of Equation (7) by δr^2 to ensure angular momentum dimensions, the equation below can be obtained,

$$\delta r^2 \partial_t \langle \Sigma \delta r v'_\phi \rangle_\phi = \dot{M} \delta r \partial_r (\delta r \bar{v}') - \delta r \partial_r (\delta r^2 \langle \Sigma v'_r v'_\phi \rangle_\phi) + \delta r^2 \langle \delta r \times F \rangle_\phi, \quad (8)$$

where $\dot{M}_{\text{CPD}} = -\langle \Sigma \delta r v'_r \rangle_\phi$. It turns out the mass flux in the CPD $\dot{M}_{\text{CPD}}$ well matches with the planetary accretion rate $\dot{m}_p$, which again suggests a steady state for the CPD accretion. The term on the left side represents the time-dependent evolution term of angular momentum, the first term on the right side denotes the angular momentum transfer resulting from accretion, the second term corresponds to the gradient of Reynolds stress or the gradient of angular momentum transfer flow, and the last term represents the external torque.

When the state of accretion equilibrium for the CPD has been established as in our cases, the time-dependent evolution term in Equation (8) is negligible. Consequently, Equation (8) can be expressed as follows,

$$-\dot{M}_{\text{CPD}} \delta r \partial_r (\delta r \bar{v}') \approx -\delta r \partial_r (\delta r^2 \langle \Sigma v'_r v'_\phi \rangle_\phi) + \delta r^2 \langle \delta r \times F \rangle_\phi. \quad (9)$$

By performing integration on both sides of Equation (9), the components of the aforementioned equation is written as,

$$\alpha_{\text{eff}} = \alpha_{\text{rey}} + \alpha_{\text{con}} + \alpha_{\text{T}}, \quad (10)$$

where

$$\alpha_{\text{eff}} = \frac{\dot{M}_{\text{CPD}}}{3\pi \Sigma c_s H}, \quad (11)$$

$$\alpha_{\text{rey}} = \frac{\langle \Sigma v'_r \delta v'_\phi \rangle_\phi}{3\pi \Sigma c_s^2}, \quad (12)$$

$$\alpha_{\text{con}} = \frac{C}{3\pi \Sigma c_s H \delta r \bar{v}'}, \quad (13)$$

$$\alpha_{\text{T}} = -\frac{\int \delta r \langle \delta r \times F \rangle_\phi dr}{3\pi \Sigma c_s H \delta r \bar{v}'}, \quad (14)$$

where α_{eff} represents the effective accretion coefficient of the CPD, while α_{rey} denotes the accretion coefficient resulting from Reynolds stress, α_{T} is the accretion coefficient induced by external force. α_{con} is associated with the accretion coefficient determined by a constant free parameter C associated with the accretion boundary of the CPD.

As shown in Figure 4, it is evident that the total accretion coefficient ($\alpha_{\text{rey}} + \alpha_{\text{T}} + \alpha_{\text{con}}$) by combing different components can be in reasonable agreement with the effective α_{eff} based on measured planetary accretion rate $\dot{m}_p$ for both isothermal and $\beta = 100$ cases. It should be noted that due to the softening lengths and the inner accreting boundary in the vicinity of the planet, the two curves do not align perfectly. Therefore, the consistency of

Equation (10) is another signature of a steady state within the CPD region. Furthermore, we can see that α_{eff} is quite close to α_{rey} in most parts of the disk, and both α_{eff} and α_{rey} is much higher than the explicit α viscosity implemented in the global disk. These suggest that the angular momentum transport and thus the accretion within the CPD could be mainly induced by spiral shocks in the CPD regions instead of the global disk viscosity [43].

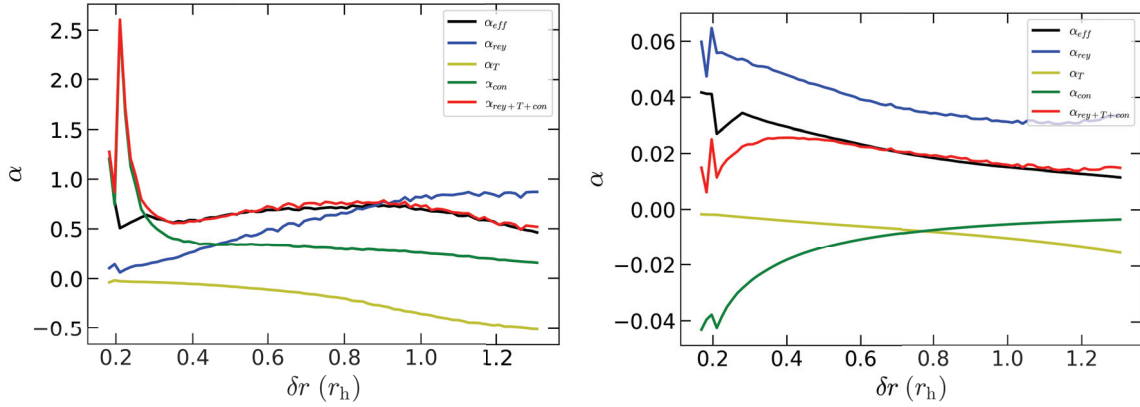


Figure 4. Different accretion coefficient calculated by Equation (10) under isothermal EOS (left panel) and $\beta = 100$ (right panel) cases. The the summation of α_{rey} , α_{con} and α_T is shown as red lines.

The effective accretion coefficients α_{eff} for different β -cooling models are shown in the left panel of Figure 5, illustrating its variation across various thermodynamic factors. It reveals that the effective accretion coefficient is highest under the isothermal condition and progressively decreases as the cooling timescale increases, ultimately reaching its minimum under adiabatic conditions. As the net accretion rates onto the planet vary slightly for different β -cooling, the decreasing of α_{eff} is partly compensated by a higher CPD temperature resulting from the adiabatic heating with a longer cooling timescale, as shown in the right panel of Figure 3. Nevertheless, the mass flux across the CPD and the accretion rate onto the planet under the isothermal limit are still the highest one after considering the lowest disk temperature for allowing the formation of a rotationally supported disk.

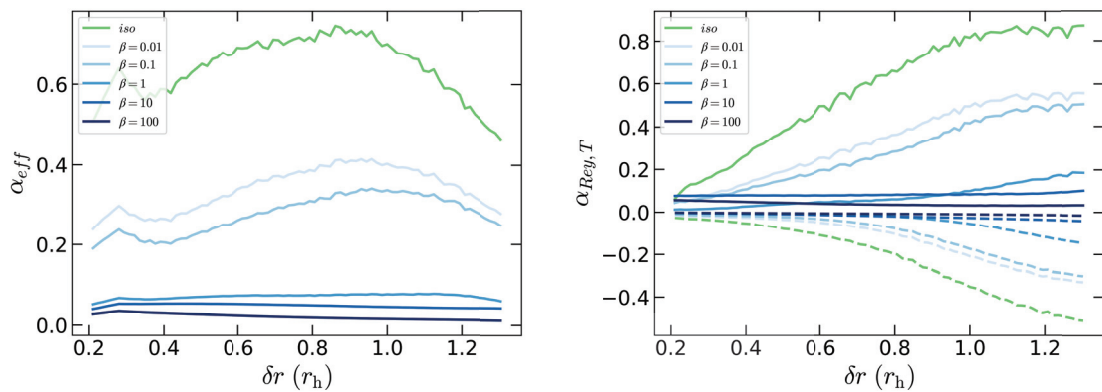


Figure 5. Left panel: the effective accretion coefficients in Equation (11) under different cooling timescales. Right panel: the accretion coefficients resulting from Reynolds stress (α_{rey} ; solid lines) and external forces (α_T ; dashed lines) for different cooling timescales. The same color represents the same cooling timescale.

Furthermore, the Reynolds stress coefficient α_{rey} and the external torque coefficient α_T under different cooling timescales are represented in the right panel of Figure 5. It is evident that the α_T is anti-correlated with the effective accretion coefficient α_{eff} among

different thermodynamical models. The Reynolds stress coefficient α_{rey} , however, exhibits noticeable positive correlation with α_{eff} for different cooling timescales, with the highest values observed under isothermal conditions. It, therefore, again suggests that the CPD accretion for different thermodynamic models is mostly attribute to the Reynolds stress.

In the long cooling timescale case, the gas is unable to efficiently dissipate the compressional heat, leading to an elevated temperature in the vicinity of the planet, as shown in the right panel of Figure 3. The sound speed appeared in Equation (12) is larger compared to the isothermal limit. In addition, as the azimuthal motion relative to the planet is significantly suppressed with a longer cooling timescale, the Reynolds stress in Equation (12) also becomes weaker. As a result, the value of α_{rey} is reduced with the increasing of the cooling timescale, and the shock-driven accretion becomes less efficient.

3.3. The Migration of the Planet

To study the effects of thermodynamics on the migration of the planets, we calculate the torques exerted onto the planet. The torques can be divided into two parts, the first part is from the gravitational force from the PPD, the other one is from the mass and angular momentum transfer in the sink hole area, even though we do not actively add them onto the planet in our simulations [35,64,65]. Considering the small mass ratio between the planet and star ($q = 10^{-3}$), the gravitational torque and accretional torque associated with migration can be calculated as follows [56],

$$\Gamma_{\text{gra}} = \mathbf{r}_p \times \int \Sigma \nabla \Phi_p dS, \quad (15)$$

and

$$\Gamma_{\text{acc}} = \mathbf{r}_p \times \int_{\delta r < r_{\text{acc}}} (\mathbf{v} - \mathbf{v}_p) d\dot{m}_p, \quad (16)$$

where Φ_p is the gravitational potential of the planet, $\mathbf{v}$ is the velocity of the gas within the sink hole radius in the inertial frame. The migration rate for the planet's semi-major axis a_p can be expressed as

$$\frac{\dot{a}_p}{a_p} = \frac{2\ell_p}{\ell_p} = \frac{2(\Gamma_{\text{gra}} + \Gamma_{\text{acc}})}{m_p \ell_p}, \quad (17)$$

where $\ell_p = \sqrt{GM_* a_p}$ is the specific angular momentum of the planet. Here we have neglected the accretion term onto the central star $-\dot{M}_*/M_*$ as this is usually negligible.

The migration rates of the semi-major axis a_p for two typical models are shown in Figure 6. It is evident that both the isothermal case and the one with $\beta = 100$ achieve a quasi-steady state at approximately 20,000 orbits. In the isothermal case, the planet migrates outward, whereas it migrates inward in the case of $\beta = 100$.

We present the results for the migration torques for all different models in Figure 7. The migration torque is usually dominated by the gravitational component except for the cases with $\beta \gtrsim 10$, where the accretional component becomes comparable to the gravitational torque; however, it is still not the primary factor determining the direction of migration. It can be seen that as the cooling timescale increases, the gravitational torque Γ_{gra} and also the total torque Γ_{tot} drop monotonically from positive to negative, with $\Gamma_{\text{tot}} > 0$ for $\beta \lesssim 1$ and $\Gamma_{\text{tot}} < 0$ for $\beta \gtrsim 1$. This suggests that when the cooling timescale exceeds the local orbital timescale (i.e., $\beta \gtrsim 1$), migration transitions to inward. The positive torque for an accreting Jupiter-mass planet in the isothermal case with a high disk viscosity is consistent with the results from Li et al. [56], Laune et al. [57].

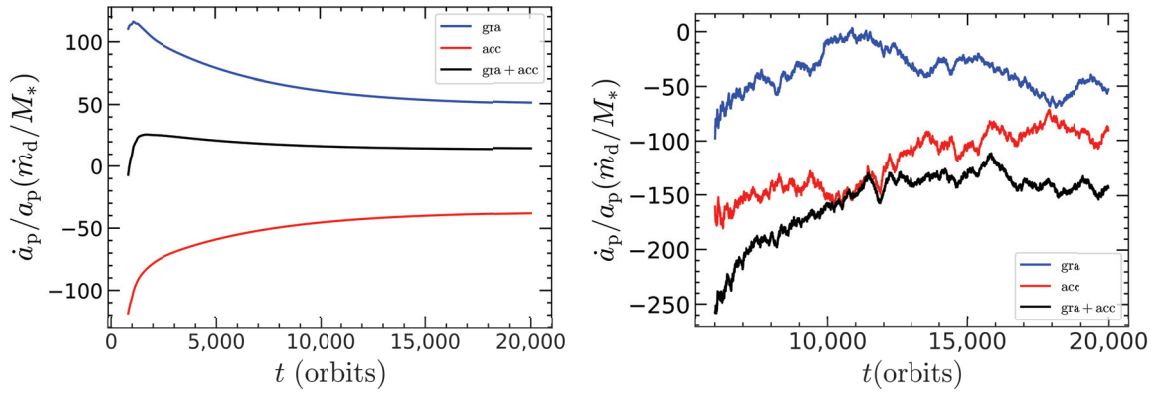


Figure 6. The semi-major axis evolution due to the gravitational (blue lines), accretional (red lines) and total (black lines) torques in isothermal (left panel) and $\beta = 100$ (right panel) cases. For the case of $\beta = 100$, a running-time averaged over 500 orbits is carried out to smooth out the short timescale variability.

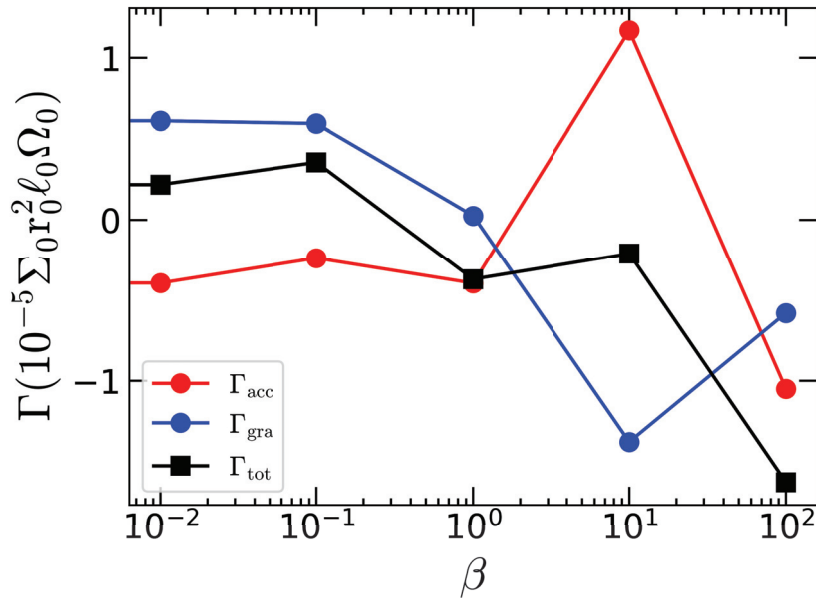


Figure 7. The torque exerted on the planet under different cooling timescales in code unit, where $\ell_0 = r_0^2 \Omega_0$ is the specific angular momentum of the disk at r_0 . The blue and red circle points represent the gravitational torque and accretional torque respectively. The black square points represent the sum of the two torques. The positive (negative) torque means that it drives the planet to migrate outward (inward).

In the type I/II regime for non-accreting planets, the inward migration rate is $\dot{a}_p/a_p \simeq -4q/h_0^2 \Sigma_p r_0^2 \Omega_0 / M_* \simeq -500 \dot{m}_d / M_*$ [18], where the Σ_p is the surface density at the planetary orbit after considering the gap opening effect [42]. We can see that the inward migration rate in the high β -cooling cases is a factor of ~ 3 slower than that of the non-accreting planet. In the adiabatic limit for the non-accreting planet, the migration torque can be positive for our density and temperature profiles due to the non-linear corotation torque [54]. With a long β -cooling timescale, which is close to the adiabatic case, the negative gravitational torque on the accreting planet is due to the suppression of the positive corotation torque associated with accretion.

3.4. How Thermodynamics Affects Migration

The gravitational torque could be the dominant factor that determines the planet migration. To understand what causes the transition of the migration, we explore the influ-

ence of thermodynamics on the gravitational torque. As the regions where the Lindblad torque dominates and the corotation torque dominates overlap with each other, we do not distinguish their contributions in this paper. Following Li et al. [56], we plot the spatial torque distribution in the vicinity of the planet and the results are shown in Figure 8. The CPD structure is clear in the isothermal case in which the leading horseshoe region provides positive torque (red color region) while the trailing region provides negative torque (blue color). In the $\beta = 100$ case, the disk-like structures break down due to thermal pressure and the torque within the Hill sphere is highly symmetrical with respect to the $\phi = 0$ plane. We then calculate the asymmetrical part of the torque defined as $\Gamma_{\text{asy}} = \Gamma(r, \phi) + \Gamma(r, -\phi)$ as shown in the right panels in the figure. It can be seen that in the CPD region for the isothermal case, the positive torque from the $\phi > 0$ hemisphere is stronger than the negative torque from the $\phi < 0$ hemisphere so the net torque is positive, as expected from the asymmetric spiral arms feeding from the global disk into the Hill sphere [56]. While for the high β -cooling case, the net positive torque in the vicinity of the planet almost vanishes, due to that the asymmetric disk-like structure is replaced by a spherical-like envelope within the Hill sphere.

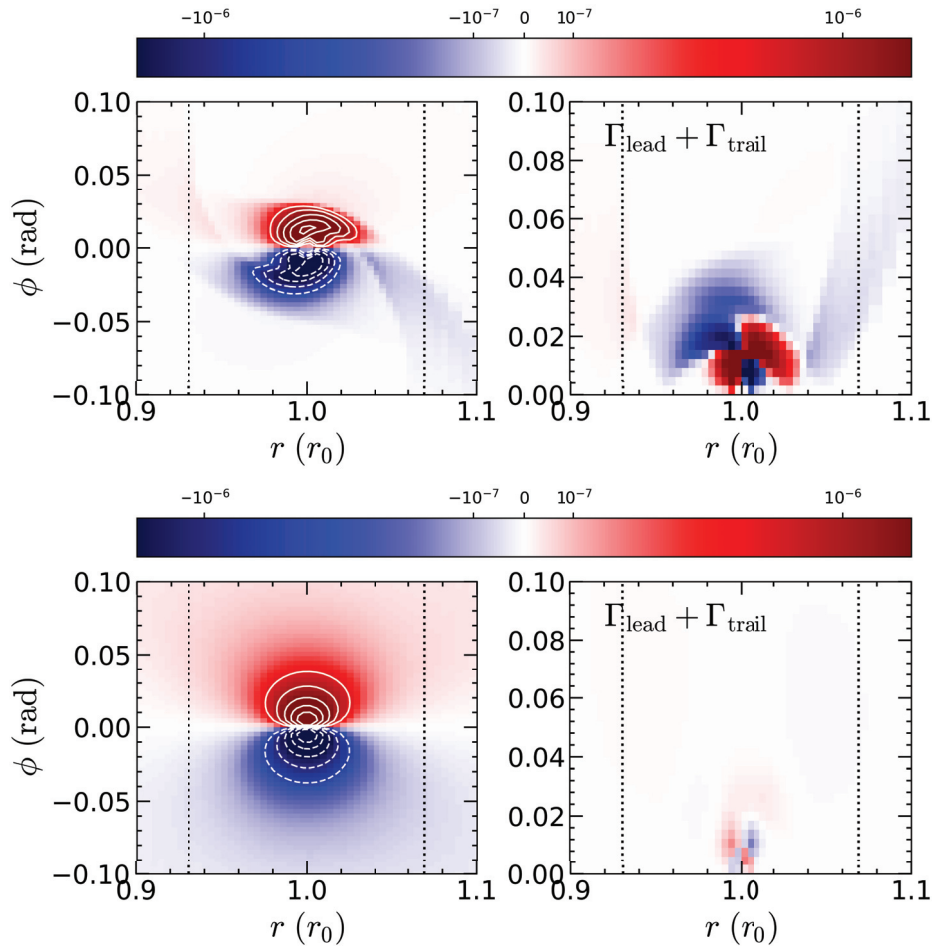


Figure 8. Spatial distribution of gravitational torques in the vicinity of the planet around 20,000 orbits. The upper panel represents the isothermal condition and the lower panel shows the $\beta = 100$ case. A time-averaging over 500 orbits is done for the $\beta = 100$ case. The contour in the left panels represented by solid and dashed lines being the same magnitude of the torques but with different signs. The vertical dotted lines correspond to the region of $1 r_h$ from the planet. The right panels show the net torque by summing the leading (upper) horseshoe and trailing (lower) horseshoe region, i.e., $\Gamma(r, \phi) + \Gamma(r, -\phi)$.

The asymmetry of the gravitational torque comes from the asymmetry of the density wave structure in the vicinity of the planet. We plot the surface density distribution around the planet in Figure 2. The asymmetry in the spiral arms feeding the planet occurs in the isothermal case, but vanishes in the $\beta = 100$ case. In the accretion process of the planet, the materials from the outer regions of the disk approach the planet through outer horseshoe tracks and accumulate at the CPD region. Part of the materials are accreted onto the planet with the left materials moving to the inner horseshoe tracks and will enter the CPD region again after one orbit. Therefore, the asymmetries in density and torque originate from the accretion process of the planet. In contrast, for the $\beta = 100$ case, the CPD structure is destroyed and supported by the thermal pressure gradient, which tends to be nearly isotropic, such that the accretion tends to be Bondi-like.

The cumulative radial torque profiles by azimuthally averaging the 2D spatial gravitational torque distribution $\int_{>r} \int_{\phi} \Gamma(r, \phi) dr d\phi$ as a function of radius for two different models are shown in Figure 9. We can see that the torque outside the Hill sphere ($r < r_p - r_h$, $r > r_p + r_h$), which is dominated by the Lindblad resonance, is negative both in isothermal and long cooling timescales. These differential Lindblad torque usually drives the planet migrate inward. However, as shown in Li et al. [56], the gravitational torque within the planet's Hill sphere is significantly positive in cases close to the isothermal EOS, allowing it to counterbalance the negative differential Lindblad torque. This is found to be associated with the asymmetric structures in the CPD region as discussed above. As the cooling timescale becomes longer (e.g., $\beta = 100$), a rotationally supported CPD disappears, the positive gravitational torque from the CPD region thus becomes too weak to balance the negative differential Lindblad torque, in line with Figure 8. As a result, the planet migrates inward.

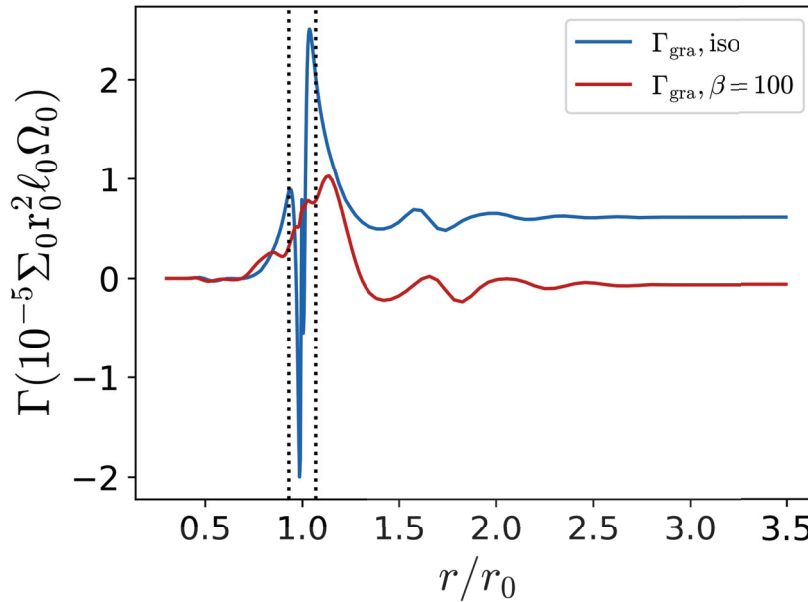


Figure 9. The cumulative radial distribution (integration from r_{in} to r) of the gravitational torque exerted onto the planet, where the blue (red) line represents the isothermal ($\beta = 100$) case. The torque for $\beta = 100$ is time-averaged over 500 orbits around 20,000 orbits. The two vertical dotted lines show the Hill radius. The torque is in code unit with $\ell_0 = r_0^2 \Omega_0$ being the specific angular momentum of the disk at r_0 .

4. Conclusions and Discussion

In this work, we perform a series of 2D global hydrodynamical simulations using Athena++ to study the impact of thermodynamics on gas giants' concurrent accretion

and migration embedded within their natal disks. The disk is evolved over the viscous timescale with a desired mass flux from the outer boundary, allowing the system to reach a quasi-steady state in a self-consistent manner. A parameterized thermal relaxation process with β -cooling is implemented to simulated the critical role of disk thermodynamic on the accretion and migration dynamics of the embedded object. The planetary accretion is modeled as a sink particle, with a high resolution around the planet to resolve the CPD region. Our findings are summarized as follows:

- The planetary accretion rates for varying cooling timescales are primarily constrained by the disk supply rate from the outer disk, exhibiting a slight decrease with increasing cooling timescales, even though different thermodynamical states have a significant impact on the CPD structures.
- The CPD is much smaller in size and less rotationally supported with longer cooling timescales. When the cooling timescale is an order of magnitude longer than the local dynamical timescale (e.g., $\beta \gtrsim 10$), the rotation of the CPD can even shift to a retrograde orbit, which could strongly influence satellite formation within the CPD.
- The planetary accretion is chiefly driven by spiral shock dissipation in the CPD region. We observe that the Reynolds stress coefficient varies significantly with different cooling timescales, following a trend similar to that of the effective accretion coefficients, making it a dominant factor in controlling the accretion process onto the planet.
- We confirm that in an isothermal disk, the planet migrates outward for a high disk viscosity, consistent with recent findings by Li et al. [56], Laune et al. [57]. However, there is a tendency for inward migration as the cooling timescale increases ($\beta \geq 1$), different from the general trend where non-accreting planets migrate outward in the adiabatic limit.
- The transition in migration under the longer cooling timescales is closely linked to the gravitational torque from the co-orbital region of the planet. In longer cooling timescales, the positive gravitational torque from this region is significantly suppressed due to the absence of a rotationally supported CPD. These results highlight the critical role of thermodynamics in the accretion and migration of planets in PPDs.

The physical process in this work could be also relevant to the dynamics of the accreting stellar-mass black holes/compact objects embedded in AGN disks. Their dynamical evolution in AGN disks could provide be an important channel for binary black hole mergers detected by LIGO/Virgo/KAGRA gravitational wave observations (e.g., [46,64,65]). In this case, the mechanical and radiative feedback from the embedded compact objects could be significant to modify the circumsingle disk structures, and thus their dynamical evolution. In addition, the inner solar system are subjected to extreme radiation environments from young stars [66]. Massive asymptotic giant branch stars could also play a significant role in delivering short lived radionuclides to heat rock-forming materials in the disk [67]. All of these would necessitate a more self-consistent approach to cooling and heating with radiative hydrodynamics simulations for the CPD, rather than relying on the β -cooling prescription. Second, a laminar viscous flow with a α -viscosity is assumed in our work. Magnetic turbulence in disks, which could modify the coherent spiral waves lunched by the embedded objects, may also play an important role in the dynamical evolution of embedded objects. Third, our focus has been primarily on a parameter space involving a Jupiter mass planet embedded in a highly viscous disk fixed in a circular orbit. A systematic exploration of different planet masses, disk viscosities, disk scale heights, and planetary eccentricities is still needed, which could be crucial for future population synthesis studies. Lastly, our simulations are limited to 2D cases. While our previous studies have shown that 3D simulations yield similar accretion and migration dynamics as 2D cases with an

isothermal EOS [56], the extent of the 3D effects for our adiabatic equation of state with varying β -cooling remains unclear. All these aspects will be addressed in future work.

Author Contributions: Conceptualization, methodology, supervision, funding acquisition, Y.-P.L.; writing—original draft preparation, H.W.; software, validation, formal analysis, visualization, investigation, resources, writing—review and editing, H.W. and Y.-P.L. All authors have read and agreed to the published version of the manuscript.

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Data Availability Statement: All the data that are used in this article are available from the corresponding author upon reasonable request.

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Conflicts of Interest: The authors declare no conflicts of interest.

Appendix A

Here we test the convergence of the accretion prescription and numerical resolution for a model in the transition region (i.e., $\beta = 1$), mainly focused on their impact on the accretion dynamics. There are two parameters for the planetary accretion, the removal rate f , and the accretion radius (or sink hole radius) r_{acc} .

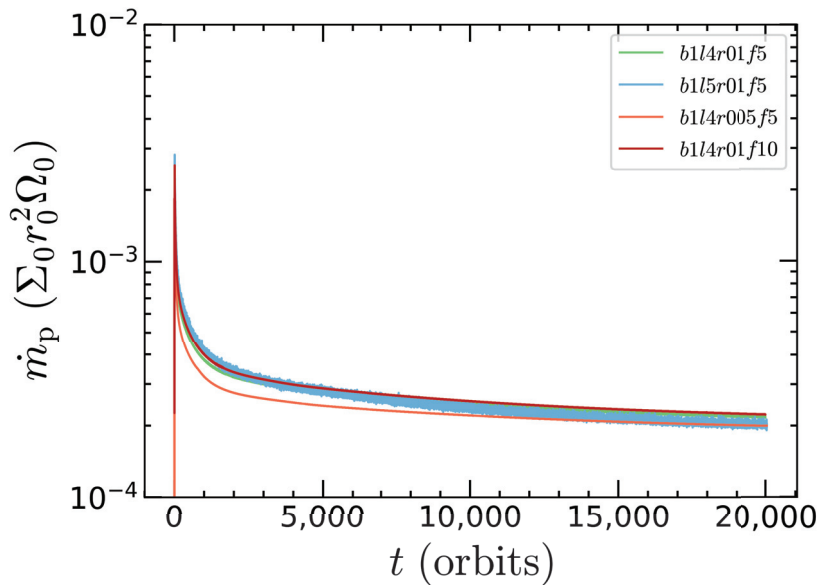


Figure A1. The accretion rates for different parameters. The number following the letter ‘b’ indicates the value of β , ‘l’ refers to the mesh refinement level, ‘r’ denotes the sink hole radius, which is the same as the softening radius, and ‘f’ represents the removal rate of the sink hole region, e.g., ‘b1l4r01f5’ indicates $\beta = 1$, 4 levels of mesh refinement, $r_{\text{acc}} = 0.1 r_h$, and $f = 5$. It is clear that all of these parameters have a negligible effect on planetary accretion rates.

The sink hole radius r_{acc} should be small enough to represent the size of the accreting object accurately. However, using a sink hole radius comparable to that of a Jupiter-mass planet necessitates an extremely high resolution around the planet in our global simulations, which is impractical even with mesh refinement. Therefore, it is advisable to select an appropriate value for r_{acc} that does not significantly affect the dynamical accretion of

the embedded object. By comparing the simulation results with $r_{\text{acc}} = 0.1r_h$, $0.05r_h$, and different removal rates f , we observe no significant differences in the accretion rate, as illustrated in Figure A1. This indicates good convergence with the accretion prescription.

In addition, we have conducted convergence tests at various numerical resolutions in the vicinity of the planet. With 4 and 5 levels of mesh refinement around the planet, we found that the differences in accretion rates among these cases are negligible. Similar tests were performed for different accretion prescriptions and global numerical resolutions (up to $n_r = 1800$, and $n_\phi = 2500$) without any mesh refinement using FARGO3D [68]. All of these indicate satisfactory convergence for our simulation results.

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Article

A Scenario for Origin of Global 4 mHz Oscillations in Solar Corona

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Abstract: We establish a spherically symmetric model of solar atmosphere, which consists of the whole chromosphere and low corona below the 1.25 solar radius. It is a hydrodynamic model with heating in the chromosphere through an artificial energy flux. We performed a series of simulations with our model and found oscillations with a peak frequency of ~ 4 mHz in the power spectrum. We confirmed that this resulted from the p -mode excited in the transition region and amplified in a resonant cavity situated in the height range $\sim 4 \times 10^3$ – 2×10^4 km. This result is consistent with global observations of Alfvénic waves in corona and can naturally explain the observational ubiquity of 4 mHz without the difficulty of the p -mode passing through the acoustic-damping chromosphere. We also confirmed that acoustic shock waves alone cannot heat the corona to the observed temperature, and found mass upflows in the height range $\sim 7 \times 10^3$ – 7×10^4 km in our model, which pumped the dense and cool plasma into the corona and might be the mass supplier for solar prominences.

Keywords: hydrodynamics; methods: numerical; sun: atmosphere; sun: corona

1. Introduction

Since Edlén (1943) [1] showed that the coronium line observed in the Sun is actually a highly ionized iron line emitted at temperatures exceeding one million Kelvins, how to heat and sustain the solar corona millions of Kelvins hotter than the surface, i.e., the coronal heating problem, has become one of the fundamental problems in solar physics. Similarly, there is also a chromospheric heating problem and a solar wind heating problem [2]. Biermann (1946) [3] and Schwarzschild (1948) [4] first proposed that acoustic waves generated by solar granulation could carry energy from the top of the convective zone into the corona. However, this mechanism was first successfully applied to the chromospheric heating problem instead. Ulmschneider [5–7] established a time-dependent hydrodynamic model and discussed the acoustic waves generated in the convection zone and the resulting shock waves as the possible fundamental energy transportation and heating mechanism. Stein and Schwartz [8,9] further considered the effects of ionization using Saha’s equation [10] on the acoustic pulse and periodic wave train. Subsequently, Ulmschneider et al. (1977) [11], Kalkofen and Ulmschneider (1977) [12], and Ulmschneider and Kalkofen (1977) [13] performed systematic research to investigate acoustic waves in the solar atmosphere, and their results supported the short-period acoustic heating theory of

the chromosphere. Anderson and Athay [14,15] improved the empirical loss proposed by Vernazza et al. (1981) [16] and recomputed the chromospheric cooling fluxes, which were found to be consistent with the observational propagating acoustic wave flux obtained by Endler and Deubner (1983) [17,18]. Carlsson and Stein (1992) [19] established the first fully self-consistent one-dimensional radiation–hydrodynamic model to explain the chromospheric bright point phenomenon, and their calculation of acoustic waves reproduced many aspects of the observed Ca II bright point behavior rather well. Fossum and Carlsson (2005) [20] proposed that the acoustic energy flux is too low to balance the radiative losses in the chromosphere based on space observations with the Transition Region and Coronal Explorer (TRACE) [21,22], but their result was later contested by several works [23–25], who attributed the missing flux to the limited spatial resolution of TRACE. Bello González et al. (2010) [26] gave a detailed discussion on the acoustic wave heating debate, and Bello González et al. (2010) [27] found that the energy flux of acoustic waves in the 5.2–10 mHz range, obtained from the wavelet analysis of high-resolution spectropolarimetric data taken with IMA/Sunrise and considered to be lower limits, lies within a factor of two of the energy flux needed to balance the radiative losses from the chromosphere according to the estimates of Anderson and Athay (1989) [15]. Recent observations [28–30] with the Interface Region Imaging Spectrograph (IRIS) found that the acoustic energy flux is sufficient to balance the radiative losses in the quiet chromosphere, though its contribution to the radiative losses in the active-region chromosphere is only 10–30 per cent.

In short, it is widely accepted that acoustic waves, as the fundamental energy transportation and dissipation mechanism, are enough to heat and sustain the solar chromosphere in the quiet regions. Turning to the coronal heating problem, the wave periods observed in the corona exhibit a peak of around three to five minutes [31–35], which is likely connected with photospheric *p*-mode oscillations with the same peak period (Except the *p*-mode, there is another driver of acoustic waves, impulsive trigger, which is mainly responsible the excitation of five minutes oscillation [36] but it is not our study object in this paper). However, the chromosphere is an acoustic cut-off layer, in which waves with periods around, or longer than, three minutes are evanescent and non-propagating. Thus, coronal heating cannot be directly maintained by acoustic waves from the photosphere, and it is widely believed that the magnetic field plays an essential role in the transportation of energy from the photosphere to the corona [37]. At present, almost all possible magnetic field effects, such as Alfvénic waves (e.g., [38–40]), fast/slow magneto-acoustic waves (e.g., [41–43]), and magnetic reconnections (e.g., [44,45]), have been applied to the explanation of coronal heating, which is closely related to various magnetic phenomena observed by ground and space instruments with higher and higher spatial and temporal resolution [46–48].

Although the importance of the magnetic field has become the consensus in solar physics, some observational research has revealed that the contribution of acoustic waves cannot be simply and arbitrarily ignored in corona, for example, the work of Morton et al. (2019) [31]. They measured the Alfvénic wave motions along line-of-sight through the Doppler shift of the Fe XIII emission line above the limb in corona and found that there was always a peak about 4 mHz appearing in the power spectrum of the Doppler velocity time-series taken from the iron line observation. They finally confirmed the ubiquity of 4 mHz in corona with arbitrary power spectral density (PSD) measurements using the data from 2005 to 2015, which cover different phases of the whole 11-year solar cycle. In order to explain these observational results, Morton et al. (2019) [31] suggested magnetohydrodynamical (MHD) wave models where coronal Alfvénic modes are excited at the transition region (TR) through the acoustic-Alfvénic mode conversion [49–51], and the inputting acoustic waves are leaked into the atmosphere from the photosphere by magneto-acoustic portals [52].

However, the point of this suggestion that the inputting acoustic waves are leaked from the photosphere may conflict with the observational ubiquity of 4 mHz, as a chromosphere leaking acoustic waves everywhere is equivalent to being transparent to acoustic waves. Thus, it is necessary to explore the origin of the global 4 mHz oscillations in corona and investigate the possibility of coronal heating via the resulting shock waves. For this goal, we performed global spherically symmetric hydrodynamical (HD) simulations of the solar atmosphere between 1 and 1.25 $R_{\odot}$. For simplicity, we applied an artificial energy flux in the chromosphere, with which the simulated chromosphere could quickly converge to a quasi-steady state that agreed with various observations. The computation time was thus greatly reduced, enabling us to explore the global structure of the solar atmosphere. This paper is arranged as follows: The model is presented in Section 2. The numerical settings, such as the computational grid, initial conditions, and boundary conditions, are presented in Section 3. The numerical results are presented and discussed in Section 4. We summarize and conclude in Section 5.

2. The Model

2.1. Basic Equations

We consider a spherically symmetric HD solar atmosphere. The following are the basic equations adopted in our model, including the mass conservation,

$$\frac{\partial \rho}{\partial t} + \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 \rho v_r) = 0, \quad (1)$$

radial momentum conservation,

$$\frac{\partial(\rho v_r)}{\partial t} + \frac{1}{r^2} \frac{\partial}{\partial r} [r^2 (\rho v_r^2 + p)] = -\frac{GM_{\odot}}{r^2}, \quad (2)$$

and total energy (including internal energy and kinetic energy) conservation equations,

$$\frac{\partial e}{\partial t} + \frac{1}{r^2} \frac{\partial}{\partial r} [r^2 v_r (e + p)] = -\rho v_r \frac{GM_{\odot}}{r^2} + q^- + \frac{\partial F_c}{\partial r}, \quad (3)$$

where t , r , ρ , v_r , p , e , G , $M_{\odot}$, q^- , and F_c are the time, spherical radius, density, radial velocity, pressure, total energy, gravitational constant, solar mass, cooling rate, and artificial chromospheric energy flux, respectively (we ignore the heat conduction in our model, though it is important, especially within the TR¹). We treat the plasma as an ideal gas, so that $e = p/(\gamma - 1) + \rho v_r^2/2$, where $\gamma = 5/3$ is the adiabatic index. The numerical simulations were performed with Athena++ [53] (Specifically, we used its default HD module including the HLLC Riemann solver, time-integrator of second-order accurate van Leer predictor–corrector scheme, and the spatial reconstruction of piecewise linear method applied to primitive variables), and accordingly Equations (1)–(3) are written in numerical conservation forms, and the effects of gravity, radiative cooling, and artificial energy flux are all treated as physical source terms appearing at the right-hand side of the equations.

2.2. Artificial Chromospheric Energy Flux

As mentioned in Section 1, it is widely accepted that the main heating mechanism in the quiet chromosphere is the dissipation of acoustic waves transmitted from the photosphere. Ideally, we should set an acoustic wave generator in the photosphere as the lower boundary condition (e.g., ref. [19] used a sinusoidal pistol as the lower boundary).

However, as explained below, we instead use an artificial chromospheric heating flux in the chromosphere of our simulation (between 0 and 2000 km above the photosphere), which is

$$F_c = k_0 p, \quad (4)$$

where k_0 is a coefficient whose value will be discussed in Section 4, and the photosphere boundary is set to be in hydrostatic equilibrium (see Section 3.2 for more details). The reason we use this artificial flux is as follows. First, our main finding in this paper, as will be discussed in Section 4, is that acoustic shock waves are newly generated at the bottom of the corona, which propagate upward and participate in the coronal heating. Thus, we need to exclude the possibility that these acoustic waves are leaked from the photosphere, and a boundary condition with the acoustic wave input from the photosphere should be avoided. Secondly, we focused our computation on the corona, so that we only had very limited resolution in the chromosphere (~ 20 cells). Had we used the real heating flux via the dissipation of acoustic waves and the corresponding lower boundary condition, this low resolution would make it more difficult to converge to a quasi-steady solution in the chromosphere, where the heating from the dissipation of acoustic waves must be calculated precisely, and make the acoustic waves from the photosphere more likely to leak through the chromosphere. Thus, here, we used an artificial chromospheric heating flux instead. We found that the simulated chromosphere could quickly converge to a quasi-steady state which conformed to various observations (see Section 4.1) with this artificial flux and limited resolution in computation, while avoiding the possible leakage of acoustic waves from the photosphere. Note that this flux was only applied in the chromosphere. Considering that the observed thickness of the solar chromosphere is ~ 2000 km, we only added this energy flux in the height range between 0 and 2000 km above the photosphere ($r = R_\odot$), which was the inner boundary of our calculation. Although the real chromosphere energy flux may not suddenly stop at 2000 km, the pressure there becomes relatively small due to the rapid decrease in density (but the temperature change is not significant in chromosphere), and hence the artificial energy flux, which is proportional to the pressure, also becomes very small and the actual impact of this truncation should not be significant. Meanwhile, this approach of fixing the truncation position at a reliable observed chromospheric thickness, rather than releasing it as a free parameter, simplifies the model and ultimately concentrates any uncertainty in a single parameter k_0 , making it easier to fit the other observations (see Figure 1).

Physically speaking, the energy flux of an acoustic wave is

$$F = \frac{1}{2} \rho v_a^2 c_s \quad (5)$$

where v_a is the velocity amplitude and c_s is the sound speed. The pressure is dominated by gas pressure, so that $p = \rho k T / (\mu m_p)$, where k is the Boltzmann constant, μ is the mean molecular weight, and m_p is the proton mass. At different heights of the chromosphere, the variation in density is much larger than that of the temperature, so that we have approximately $\rho \propto p$, and c_s can also be regarded as a constant. Thus, we have $F \propto p$ when the variation in v_a is neglected. Anderson et al. (1989) [14,15] (also see the review of Narain and Ulmschneider (1996) [46]) found that the solar chromospheric cooling flux is for many scale heights proportional to the gas pressure, which is consistent with the directly observed propagating acoustic wave flux [17,18]. Anderson et al. (1989) [14] attributed this phenomenon to the saturation of the velocity amplitude of acoustic waves near the speed of sound. So, eventually we could approximately obtain $F \propto p$, which is the formula for the artificial chromospheric heating flux we used. The parameter k_0 can be determined from observations (see Section 4).

However, it is worth noting that this artificial energy flux is highly simplified, ignoring many details of chromosphere heating and only applicable to our simplified model. For further studies in the future, it will be necessary to refer to detailed chromosphere heating studies (e.g., [54]).

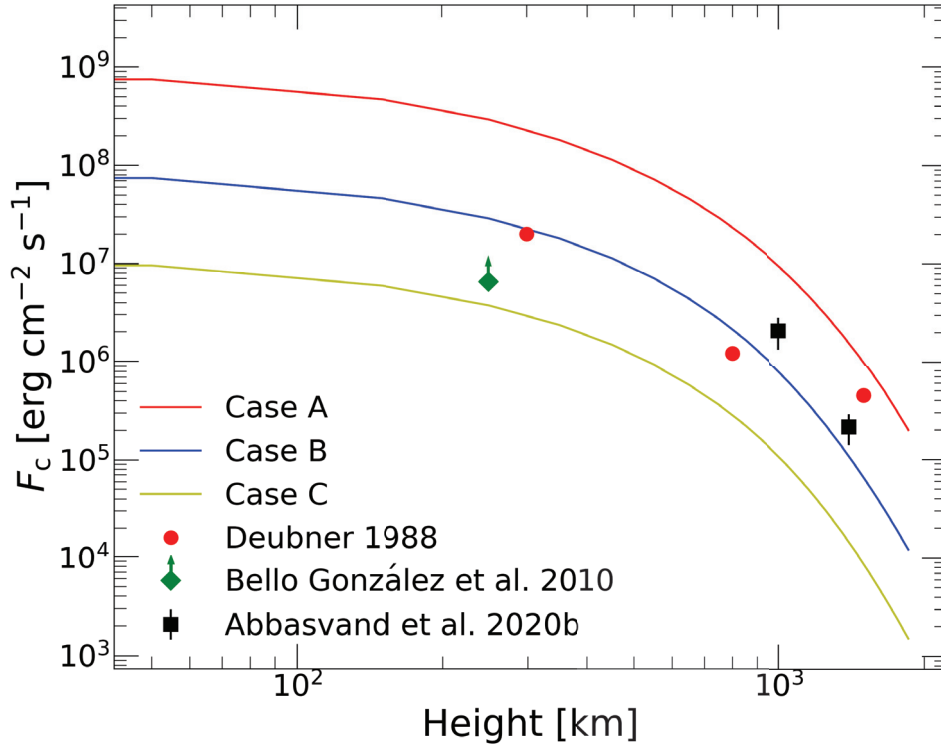


Figure 1. Artificial chromospheric energy fluxes F_c (solid lines) and the observed acoustic energy fluxes (dots, diamonds, and squares). The curves of F_c (only defined in the height range 0–2000 km) were plotted with the mean pressures averaged over the last 100 days of the respective simulations. The dots, diamonds, and squares correspond to the observed acoustic energy fluxes in the solar chromosphere, obtained from [18,27,30], respectively. The arrow in the diamond signifies that the data from [27] are the lower limit, and Deubner (1988) [18] did not provide error information for the three dots.

2.3. Radiative Cooling

We assume that the low-density plasma in the chromosphere and corona is in the collisional ionization equilibrium (CIE) everywhere and ignore radiative transfer. Therefore, the radiative cooling rate can be written as

$$q^- = n_e n_i \Lambda_1, \quad (6)$$

where n_e and n_i are the number densities (in cm^{-3}) of electrons and ions, respectively, and Λ_1 is the normalized cooling rate (in $\text{erg cm}^3 \text{s}^{-1}$). Sutherland and Dopita (1993) [55] calculated Λ_1 under CIE, including significant collisional line radiation, continuum radiation, and recombination heating. They presented a series of models with different metal abundances covering the temperature range of 10^4 – $10^{8.5}$ K. We adopted the model for solar abundances in this paper.

For the cooling of plasma with solar abundances at temperatures below 10^4 K, we adopted the formula of Mashchenko et al. (2008) [56],

$$q^- = n_H^2 \Lambda_2, \quad (7)$$

$$\log(\Lambda_2) = -24.81 + 2.928x - 0.6982x^2, \quad (8)$$

where $x \equiv \log(\log(\log(T)))$, T is temperature (in K), n_H is the density number of hydrogen atoms (in cm^{-3}), and the normalized cooling rate Λ_2 is defined in Equation (8). This formula is valid from 20 to 10^4 K, but we had to stop using it below 5000 K to avoid over-cooling in the simulations. Therefore, the plasma below 5000 K is adiabatic (which is still able to achieve a temperature below 5000 K via adiabatic expansion).

2.4. State of Matter

In the solar atmosphere, the temperature of plasma rises from thousands to millions of Kelvins, and the state of matter can be described by a partially ionized ideal gas. We assumed that the plasma reached local thermal equilibrium (LTE) everywhere, which is compatible with the CIE assumption. As radiative transfer was neglected in our calculation, the radiation pressure was consequently also ignored under LTE, so that the pressure of plasma can be calculated as

$$p = (n_i + n_e)kT, \quad (9)$$

$$n_i = \frac{\mathcal{R}\rho}{\mu_i}, \quad (10)$$

$$n_e = En_i, \quad (11)$$

where k , $\mathcal{R}$, μ_i , E , and T are the Boltzmann constant, gas constant, mean molecular weight, average number of electrons lost per atom, and temperature of the plasma, respectively. We regarded the plasma as a mixture of hydrogen and helium, and ignored the contribution of trace amounts of metal elements, so that E can be calculated as

$$E = Y_H\eta_H + Y_{\text{He}}(\eta_{\text{He}} + 2\eta_{\text{He}^+}), \quad (12)$$

$$Y_H = \frac{4X_H}{4X_H + X_{\text{He}}}, \quad (13)$$

$$Y_{\text{He}} = \frac{X_{\text{He}}}{4X_H + X_{\text{He}}}, \quad (14)$$

where Y_H and Y_{He} are the number fractions of hydrogen and helium; X_H and X_{He} are their mass fractions; and η_H , η_{He} , and η_{He^+} are the ionization fractions of hydrogen and the first- and second-order ionized helium, respectively. Under both LTE and CIE, these ionization fractions satisfy Saha's equations [10]

$$\frac{\eta_H}{1 - \eta_H} \frac{E}{1 + E} = \frac{(2\pi m_e)^{3/2} (kT^{5/2})}{ph^3} e^{-\chi_H/kT}, \quad (15)$$

$$\frac{\eta_{\text{He}}}{1 - \eta_{\text{He}} - \eta_{\text{He}^+}} \frac{E}{1 + E} = 4 \frac{(2\pi m_e)^{3/2} (kT^{5/2})}{ph^3} e^{-\chi_{\text{He}}/kT}, \quad (16)$$

$$\frac{\eta_{\text{He}^+}}{\eta_{\text{He}}} \frac{E}{1 + E} = \frac{(2\pi m_e)^{3/2} (kT^{5/2})}{ph^3} e^{-\chi_{\text{He}^+}/kT}, \quad (17)$$

where m_e , h , χ_H , χ_{He} , and χ_{He^+} are the electron mass, Planck constant, and ionization energies of the corresponding elements, respectively.

In simulations, the density and pressure are known in the cells of the numerical grid. To calculate the cooling rate of each cell with Equations (6) and (7), the key is to evaluate E , which can be achieved by solving Equations (12) and (15)–(17) for unknown quantities E , η_H , η_{He} , and η_{He^+} . In practice, we prepared a 2D table of E for different possible cases of density and pressure through solving these equations with a non-linear system solver. With this table, we could evaluate E with a bilinear interpolation in the computation.

It is worth noting that the real chromosphere is clearly in a non-equilibrium ionization state, in which the ionization/recombination timescale of hydrogen is obviously longer than the HD/MHD timescale [57]. This would make the chromospheric plasma exhibit larger temperature fluctuations than those predicted from LTE when the shock wave sweeps it [58]. However, in our model, the chromosphere is highly simplified, and a lot of real details are ignored, including the acoustic shock from the photosphere. Therefore, we assumed that the impact of LTE on our quasi-hydrostatic chromosphere structure is limited, which can be confirmed by our subsequent numerical results (see Section 4.1).

3. Numerical Settings

3.1. Grid and Initial Conditions

A logarithmically equally spaced grid with 1550 cells was adopted in our simulations from the top of the photosphere $r = R_{\odot}$ to $r = 1.25R_{\odot}$. This grid had a minimum spacing of ~ 100 km at $r = R_{\odot}$, and the chromosphere (thickness of ~ 2000 km) could be resolved with ~ 20 cells. Each simulation run was carried out for long enough to achieve a quasi-steady state. Through repeated attempts, we found that the final result was not sensitive to the initial conditions or the grid resolution. Therefore, we adopted a low-density and low-temperature static homogeneous plasma as the initial condition in our simulations to minimize the computational consumption. Specifically, the plasma in the grid was uniformly initialized to $\rho = 2 \times 10^{-14}$ g cm $^{-3}$, $T = 10^4$ K, and $v_r = 0$ cm s $^{-1}$.

3.2. Boundary Conditions

There are two boundaries in our model. One is the outer boundary at $r = 1.25R_{\odot}$, which is far below the solar wind acceleration region ($r \gtrsim 3R_{\odot}$, see [47]), and the other is the photosphere boundary at $r = R_{\odot}$. The outer boundary was set as an unidirectional outflow boundary through a simple improvement of the classical outflow conditions, in order to prevent fictitious inflow from outside and ensure normal outflow. Specifically, we inserted a function into the integrator loop to monitor the direction of the mass flux at the interface between each boundary cell and the corresponding ghost cell. If fictitious mass inflow from the ghost cell was detected, all of the fluxes, including the mass, momentum, and energy fluxes, were reset to zero before they were used to update the cell-centered conservative variables in the boundary cell. This kind of unidirectional boundary was applied in our previous work [59], where its validity was confirmed.

The photosphere boundary plays an important role in the formation of the solar atmosphere. Since we adopted a second-order-accuracy algorithm in our simulations, there were two ghost cells at the boundary. The centers of these two cells were set inside the photosphere at 50 km and 150 km below $r = R_{\odot}$, respectively. The cells were so thin that the density difference was insignificant, as observed in the photosphere (see the density profile in [16]). Therefore, we assumed that the densities in both ghost cells were the same as that of the photosphere top, which was $\sim 2 \times 10^{-7}$ g cm $^{-3}$ from observations. We also assumed hydrostatic equilibrium in these two ghost cells. Note that the observed photosphere was actually not in hydrostatic equilibrium, and our assumption led to no self-consistent energy flow being generated from this boundary to heat the chromosphere. An artificial chromospheric energy flux (see Section 2.2) was adopted to compensate for this inconsistency, while avoiding unnecessary computational complexity.

4. Numerical Results

4.1. Structure of the Model Atmosphere

In our model, there is only one parameter, the ratio of the artificial chromospheric energy flux to the local pressure, k_0 . Multiple simulation runs with different values of

k_0 were performed, and we found that the larger the value of k_0 , the higher the mean temperature and density of the corona in the quasi-steady state became. This is to be expected, because when k_0 increases, the heating flux in the chromosphere increases and consequently more material with higher temperature is driven into the corona. We also found that there existed a critical lower limit of $k_{0c} = 2.01 \times 10^2 \text{ cm s}^{-1}$, below which the model calculation collapsed due to lack of heating to sustain the chromosphere and subsequently the corona. In order to reveal the effects of k_0 , here, we present the simulation results for three different values of k_0 (Table 1), i.e., Cases A, B, and C, with $k_0 = 1.61 \times 10^4$, 1.61×10^3 , and $2.1 \times 10^2 \text{ cm s}^{-1}$, respectively. Case B was actually the optimal simulation result that we found (see discussion below). Case C corresponded to a value of k_0 slightly greater than k_{0c} , which can be seen as a lower limit. Case A represents the simulation result with a larger value of k_0 , presented as a comparison. Each simulation was carried out for one year (365 days) in physical time, which achieved a quasi-steady state after about 50 days (Our code is serial and does not require much memory from the computer, but due to the high temperature and low density of our research object, coronal plasma, the practical computational time-step is small, so the entire computation of each simulation took about 2 days of CPU time).

Table 1. Important characteristics for three typical cases of simulation. The p -mode frequencies indicate the range of peak frequencies in the velocity PSD measured at different heights. The mass losses are the mean mass outflow rates at the outer boundary of our model. The max temperatures are maximal mean temperatures averaged over the last 100 days of the respective simulations.

Case	k_0 [$\times 10^3 \text{ cm s}^{-1}$]	p -Mode Freq. [mHz]	Mass Loss [$\times 10^{-11} \text{ g cm}^{-2} \text{ s}^{-1}$]	Max Temp. [$\times 10^6 \text{ K}$]
A	16.1	3.39–4.98	6.8	1.1
B	1.61	3.90–4.07	1.4	0.92
C	0.21	3.99–4.38	0.20	0.84

The value of k_0 can be constrained by observations. Several works in literature presented the acoustic energy fluxes obtained from observations, which can be compared directly with those from our simulation results, as presented in Figure 1. Deubner (1988) [18] found the acoustic energy fluxes to be 2.0×10^7 , 1.2×10^6 , and $4.5 \times 10^5 \text{ erg cm}^{-2} \text{ s}^{-1}$ at the heights 300, 800, and 1500 km (also see [46]), respectively, represented by the red dots in Figure 1. Bello et al. (2010) [27] proposed an average acoustic energy flux at 250 km of $6.5 \times 10^6 \text{ erg cm}^{-2} \text{ s}^{-1}$ as the lower limit, represented by the green diamonds. Abbasvand et al. (2020) [30] presented acoustic energy fluxes at the heights of 1000 and 1400 km, which were $2.085 \pm 0.744 \times 10^6$ and $2.14 \pm 0.74 \times 10^5 \text{ erg cm}^{-2} \text{ s}^{-1}$, respectively, in the non-magnetic area, represented by the black squares. The solid lines represent the artificial chromospheric energy fluxes F_c for the three different cases, with the red line for Case A, blue line for Case B, and yellow line for Case C. We can see that the observed fluxes all fall between the profiles of Case A and Case C, while Case B best conformed to the observations.

Another constraint was the mass loss rate of the solar atmosphere in our simulation, which was the mass outflow rate at the outer boundary. Cohen (2011) [60] calculated the solar mass loss rate from observations, which was scattered around the value of $2 \times 10^{-14} \text{ M}_\odot \text{ yr}^{-1}$ with a variation factor of 2–5. The center value corresponded to $1.3 \times 10^{-11} \text{ g cm}^{-2} \text{ s}^{-1}$ at the outer boundary ($1.25 R_\odot$) of our simulation. Meanwhile, the mean mass loss rates (see Table 1) were $6.8 \times 10^{-11} \text{ g cm}^{-2} \text{ s}^{-1}$ for Case A, $1.4 \times 10^{-11} \text{ g cm}^{-2} \text{ s}^{-1}$ for Case B, and $0.20 \times 10^{-11} \text{ g cm}^{-2} \text{ s}^{-1}$ for Case C, averaged over the last 100 days of the respective simulations. We can see that Case B also agreed quite well with the observed mass loss rate.

In Figure 2, we compare the structure of the simulated solar atmosphere with the VAL-C model determined from *Skylab* observations of the quiet Sun [16]. The solid lines with different colors represent the mean density and temperature profiles of the simulation results, averaged over the last 100 days of the respective simulations. The dotted lines represent the density and temperature profiles of the VAL-C model. In each simulation, there were some time points when the density and temperature profiles agreed better with those of the VAL-C model. For clarity, here, we only show the profiles on Day 363 of Case B, represented by the dashed lines. While the exact values differed slightly from the VAL-C model, the tendency of the variation and the location of TR agreed well. The tendency of variation in the mean density and temperature profiles (solid lines) also agreed with those of the VAL-C model (dotted lines), though the location of the TR differed slightly. Considering that we have very limited resolution for the chromosphere (~ 20 cells), we think that the agreement is acceptable and the main characteristics of the solar chromosphere were reproduced by our simulations, especially Case B, which also agreed quite well with the other observations, as mentioned above.

It is worth noting that the detailed deviation between our model and the observational case is also obviously shown in Figure 2, though the tendency is the same between them. From the temperature profiles shown in the upper panel, it can be seen that the model temperature at low chromosphere was higher than the real case but it reversed after stopping the artificial energy flux at 2000 km. This reflects the drawback of our model on the estimation of the chromospheric heating–cooling balance. This results from the over simplification of our model chromosphere, which could be removed by further consideration of real chromospheric details in future works.

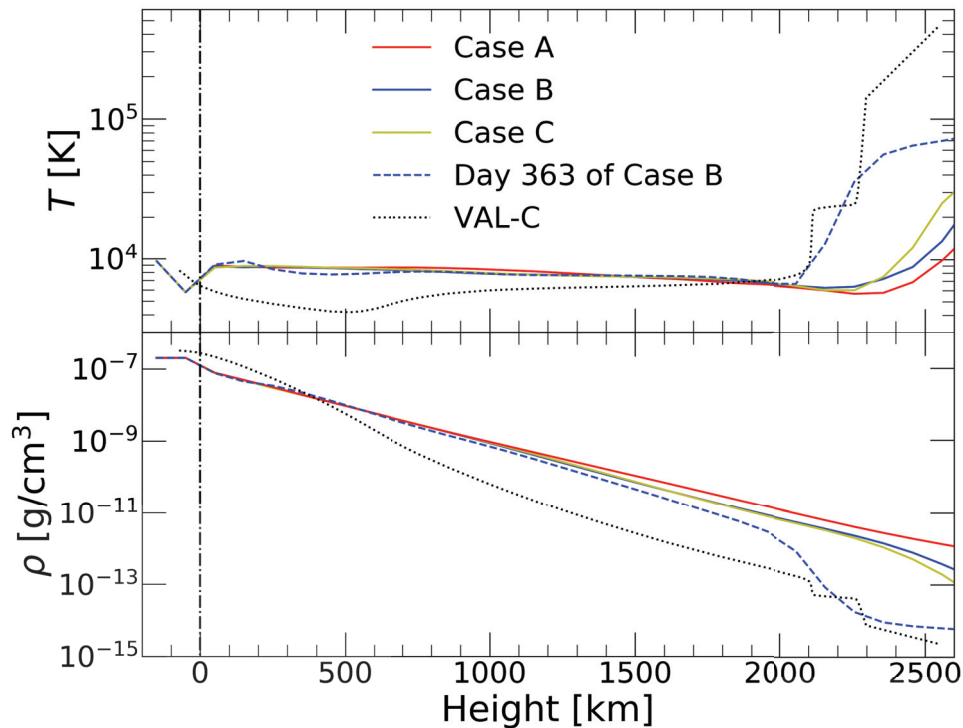


Figure 2. Mean temperature and density profiles for the three cases (solid lines), the temperature and density profiles on Day 363 of the Case B simulation (dashed lines), and the corresponding profiles in the VAL-C model (dotted lines). The vertical dot-dashed line denotes the top of the solar photosphere ($r = R_{\odot}$).

In Figure 3, we present the density, temperature, and radial velocity profiles on Days 354 and 363 of Case B, which exhibit two typical states of the quasi-steady atmosphere. It can be seen that a series of shocks were generated at the bottom of the corona and propagated upward with a speed $\sim 139 \text{ km} \cdot \text{s}^{-1}$, which could be estimated through multiplying the spatial distance between shocks ($\sim 0.05 R_{\odot}$, see the radial velocity profile shown at the bottom panel of Figure 3) by their passing frequency (4 mHz, see Section 4.2). These traveling shocks heated the coronal plasma, since the temperature post shock is always higher than that of the shock front (see the middle panel). While these two states 9 days apart had similar profiles in general, a higher density always resulted in a cooler plasma through a larger radiative cooling under almost the same shock heating (almost the same shock strength, see the bottom panel). The maximum temperatures on these two days both reached 10^6 K .

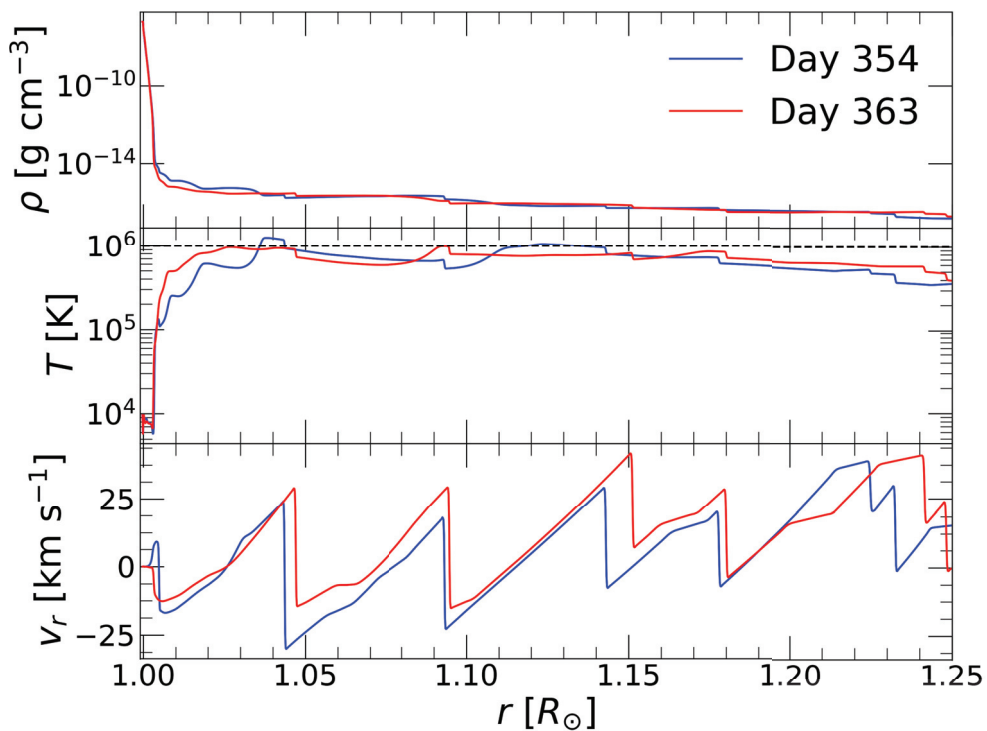


Figure 3. Density, temperature, and radial velocity profiles on Days 354 (blue lines) and 363 (red lines) of the Case B simulation. The horizontal dashed line in the middle panel denotes the temperature of 10^6 K .

In order to show fluctuations in the quasi-steady model atmosphere, we drew the density, temperature, and radial velocity profiles of the last 100 days (from Day 265 to Day 365) in Figure 4. While there was little variance in the model chromosphere ($r < 1.005 R_{\odot}$), obvious fluctuations occurred in the model corona ($r \geq 1.005 R_{\odot}$), with a density ranging from $\sim 10^{-18}$ to $\sim 10^{-15} \text{ g cm}^{-3}$, temperature from $\sim 10^5$ to $\sim 2 \times 10^6 \text{ K}$, and radial velocity from ~ -40 to $\sim 40 \text{ km s}^{-1}$, respectively. The amplitude of the coronal heating shocks in our model was much larger than that required by chromospheric heating, whose typical radial velocity jump is only a few kilometers per second [54], while the density in the corona was much smaller than that in the chromosphere, so it is natural that the model corona could be heated to a much higher temperature. It should be noted that, in observations, the amplitude of acoustic shocks in the corona is not so large (e.g., [61] and references therein). This is because our simulation was hydrodynamic and did not include the effects of magnetic fields. When magnetic fields are added, the energy stored in acoustic waves

can be converted into other forms, such as Alfvénic waves (see Section 4.2), spicules (see Section 4.2), and prominences (see Section 4.3), which explains why such high amplitude acoustic waves are not typically observed.

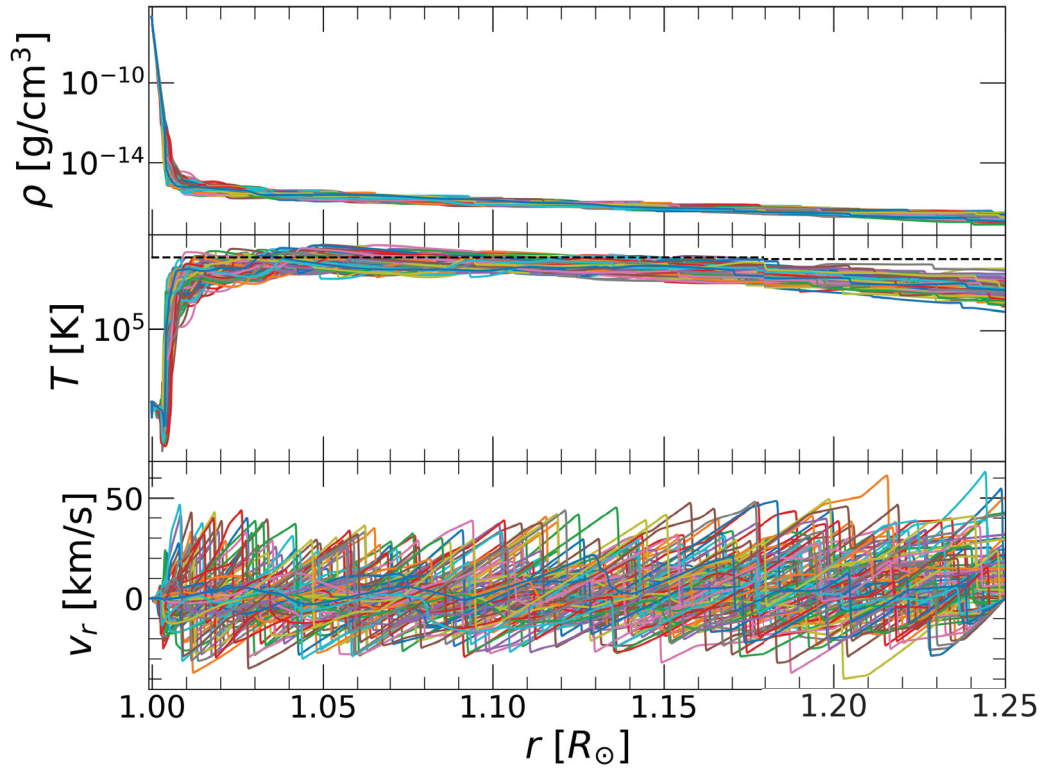


Figure 4. Density, temperature, and radial velocity profiles over the last 100 days (from Day 265 to Day 365) of the Case B simulation.

Observationally, the mean temperature in the quiet solar corona was 1.4–1.8 MK [62]. While Figure 4 indicates that the coronal temperature in the simulation could occasionally reach this range, this does not guarantee that the mean temperature can be maintained in this range. To test this, we calculated the mean temperature and density profiles of the simulations, averaged over the last 100 days for the three cases, as shown in Figure 5. We can see that as k_0 increased, both the mean temperature and the mean density in the simulated corona increased. As mentioned above, this is to be expected, because when k_0 increases, the heating flux in the chromosphere increases and consequently more material with higher temperature is driven into the corona. For the optimal simulation, Case B, the maximum mean temperature was only 0.92 MK (see Table 1), much lower than the observed mean temperature in the quiet corona. Even with a k_0 ten times larger (Case A), the mean temperature could only reach 1.1 MK, still lower than the observed value. We can also clearly see in Figure 5 that the mean temperature dropped monotonically as the height increased past the location corresponding to the maximum value, due to the quick dissipation of acoustic shock waves. Both phenomena indicate that heating via acoustic shock waves alone cannot maintain a solar corona as observed, and other heating mechanisms, e.g., magnetic reconnection and the dissipation of Alfvénic waves, must play an important role in coronal heating.

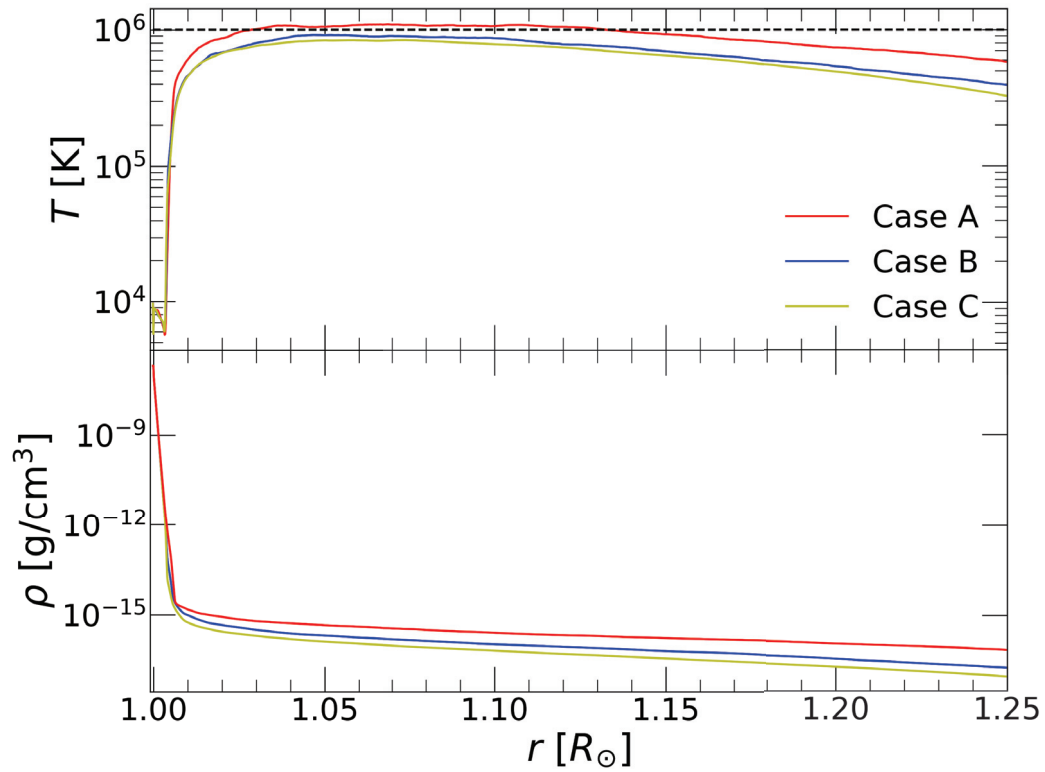


Figure 5. Mean density and temperature profiles, averaged over the last 100 days (from Day 265 to Day 365) for the three cases.

4.2. Origin of Global 4 mHz Oscillation

In our model, the chromosphere is sustained by the artificial chromospheric energy flux, which is also the energy source for maintaining the corona, as our model will collapse when k_0 is less than its critical value, instead of exhibiting a corona without the chromosphere. This also implies that the coronal heating in our model must require the existence of the chromosphere. We can approximately locate the birthplace of the coronal heating shock waves in Figures 3 and 4 between the top of the chromosphere and the bottom of the corona. In this section, we investigate these shock waves in further detail. Below, we focus on a discussion of Case B, while the relevant results for the other two cases are shown in Table 1.

First, we measured the frequency of successive shock waves in our model and compared it with observations. We sampled the instantaneous radial velocities v_r at different heights over the last 100 days of our simulation (Day 265 to Day 365) with a time-step of 10^{-4} day. The data at each height formed a fluctuating velocity profile, as presented in the upper panel of Figure 6, which is the v_r profile at the height of 1.5×10^4 km above the photosphere, and only the data from Day 363 to 363.1 are displayed to show more details. The PSD of the v_r profile was then calculated with fast Fourier transformation at each height, as presented in the lower panel of Figure 6. The peak frequency in the PSD, which can either arise from the local p -mode oscillations or correspond to the periodic fluctuation due to the passing of acoustic waves or successive shock waves, was 4.07 mHz at the height of 1.5×10^4 km (see Figure 6), and lay in the range of 3.9–4.07 mHz at different heights (denoted by the dashed line in Figure 7), consistent with the observed typical wave periods in the corona of around three to five minutes (3.33–5.55 mHz; [31–35]). It should be noted that, as the height increased, while the peak at ~ 4 mHz always existed in the PSD except for places close to the outer boundary at $r = 1.25R_\odot$ (height of $\sim 1.74 \times 10^5$ km),

another peak at ~ 0.3 mHz appeared and gradually became stronger, which we suspect also resulted from boundary effects.

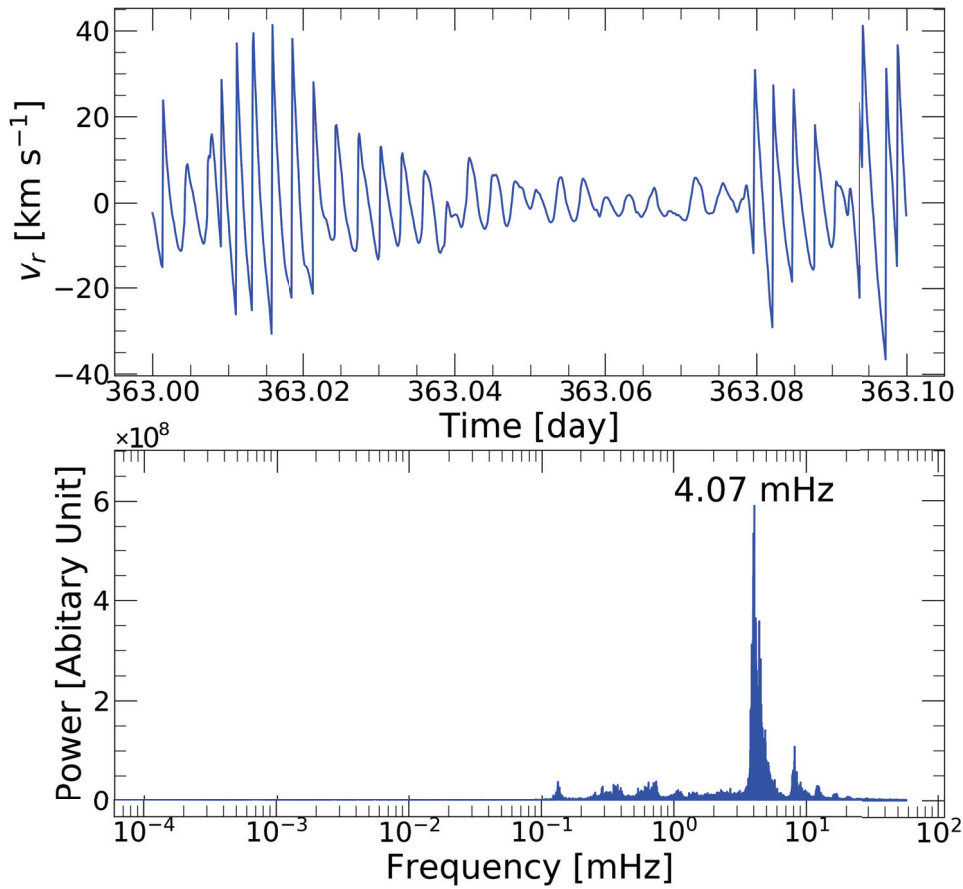


Figure 6. Profile of v_r over time sampled at a fixed height of 1.5×10^4 km above the photosphere (upper panel) and its PSD (lower panel), for Case B. Only data from Day 363 to 363.1 are displayed in the upper panel, to show more details. The PSD in the lower panel was calculated with data from Day 265 to 365, sampled with a time-step of 10^{-4} day. The peak frequency in the lower panel was 4.07 mHz, denoted by a vertical dashed line.

Second, we investigated the propagation of acoustic waves in our model atmosphere. According to the linearized perturbation theory of stellar oscillations (e.g., [63,64]), only p -mode oscillations exist in our spherically symmetric model. The p -mode oscillations excite acoustic waves with the same frequency, which can be investigated through the Brunt-Väisälä critical frequency, above which waves can freely propagate, rather than rapidly damping in the surroundings, which is calculated as [64]

$$N^2 = \left(\frac{GM_\odot}{r^2} \right)^2 \frac{\rho}{p} \left[\nabla_\mu + \frac{4-3\beta}{\beta} (\nabla_{\text{ad}} - \nabla) \right], \quad (18)$$

where N is the critical frequency, β is the ratio of gas to total pressure, ∇_μ represents the inhomogeneity of elements, ∇ is defined as $\nabla \equiv d \ln T / d \ln p$, and ∇_{ad} is the value of ∇ in the adiabatic case. Note that N^2 can be either positive or negative here, where positive values signify that the propagation of acoustic waves with frequencies no less than $\sqrt{|N^2|}$ is permitted in the relevant computing cells, and negative values correspond to places where the waves are evanescent and non-propagating. In our calculation, we ignored the inhomogeneity of elements ($\nabla_\mu = 0$) and radiation pressure ($\beta = 1$), and assumed $\nabla_{\text{ad}} = 0.4$, which is the adiabatic value of a fully ionized ideal gas. Then, we calculated N^2 with the mean values of density, pressure, and temperature from Day 265 to Day 365, as

presented in Figure 7. The dots represent the critical frequencies $\sqrt{|N^2|}$ calculated at the center of the computing cells. Red dots were calculated from positive N^2 , and blue dots from negative N^2 . The dashed line represents the peak frequency in the PSD of the v_r profile at each corresponding height (see Figure 6 as an example). We can see that the region below 3×10^3 km, including the whole chromosphere, was a cut-off layer for acoustic waves, where N^2 were mostly negative and even the places with positive N^2 only allowed the propagation of acoustic waves with frequencies much larger than the peak frequencies sampled from v_r profiles. Thus, it is impossible that the ~ 4 mHz coronal heating shock waves identified in our model came from the photosphere (in fact, our model did not inject any waves from the photosphere boundary). The region above 4×10^4 km also had large damping for the propagation of acoustic waves, with mostly negative N^2 , though the critical frequencies in the limited places with positive N^2 in this region were lower than those in the model chromosphere. On the other hand, the region between $\sim 4 \times 10^3$ and $\sim 2 \times 10^4$ km, which lies at the bottom of the corona, allowed the free propagation of acoustic waves of ~ 4 mHz. We conjecture that this region is a resonant cavity for the acoustic waves excited by p -mode oscillations, which amplifies the oscillations and gives birth to shock waves. Although the acoustic waves cannot propagate stably far outside of this region, shock waves of the same frequency form in this region and successively carry energy to the higher corona, which can be seen in Figure 3.

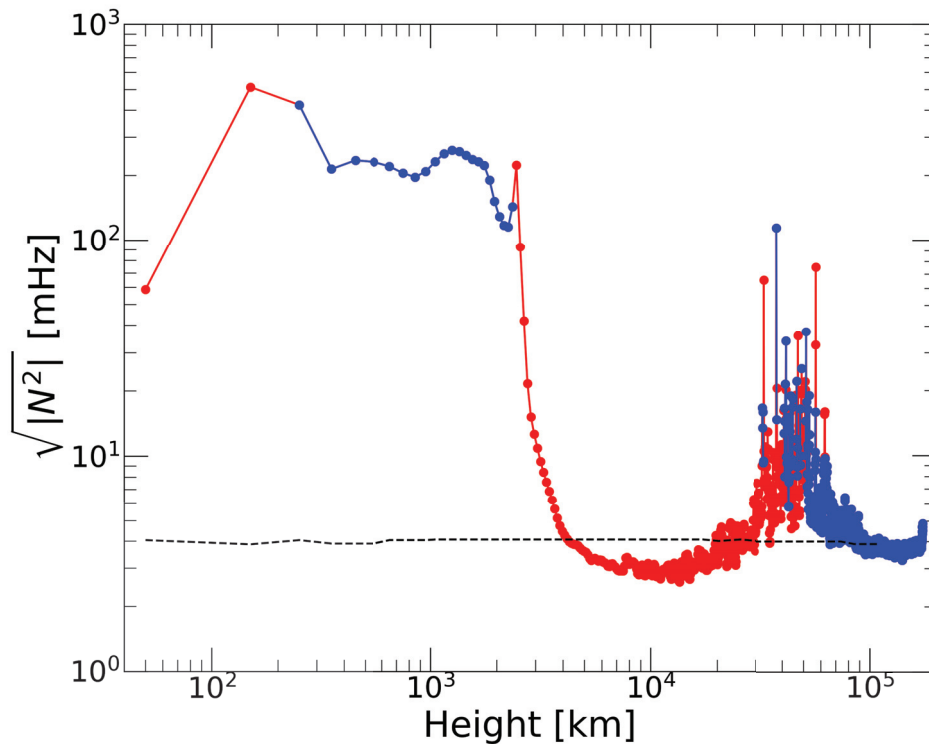


Figure 7. The profile of critical frequencies at different heights for Case B. Red dots were calculated from positive N^2 and blue dots from negative N^2 . Acoustic waves with frequencies higher than the red dots can freely propagate at corresponding heights, while those lower than the red dots are rapidly damped. The heights with blue dots always have large damping. The dashed line represents the peak frequency in the PSD of the v_r profile at each corresponding height.

Alfvénic waves have been ubiquitously observed (both in space and time) in the solar corona [31,35,65,66]. Morton et al. (2019) [31] found an enhancement in power around 4 mHz (mean and standard error of mean are 4.0 ± 0.1 mHz, with a standard deviation of 1 mHz) in the velocity PSD of Alfvénic waves, both from the Coronal Multi-channel

Polarimeter (CoMP) and the Solar Dynamics Observatory (SDO) Atmospheric Imaging Assembly (AIA) data. This enhancement exists in a large majority (>95 per cent) of coronal power spectra (i.e., throughout the entire corona) and is almost invariant in the center values throughout the solar cycle. Traditionally, it was proposed that Alfvénic waves are generated in the solar photosphere (e.g., [67]), though the low-ionization fraction in the photosphere hinders the generation [68] (see [69] for a different view), and the transmission of Alfvénic waves also suffers from a strong reflection at the TR. Several MHD wave models have been proposed to solve this problem (e.g., [49–51,70,71]), where Alfvénic waves can be excited by helioseismic *p*-mode oscillations through double mode conversion and the TR reflection can be greatly reduced under certain conditions. However, these models need to be fine-tuned to provide sufficient Alfvénic waves as observed in the corona, and may have difficulty in explaining the ubiquitous existence of the Alfvénic waves peaked ~ 4 mHz. On the other hand, our hydrodynamic simulation found that acoustic waves were newly excited in a resonant cavity at the bottom of the corona, so that when magnetic fields are taken into consideration, Alfvénic waves could be generated there with processes similar to the excitation of Alfvénic waves in the photosphere or through double mode conversion, while naturally overcoming the traditional problems of the low-ionization fraction in the photosphere and the strong TR reflection. The acoustic waves in our model also peaked at ~ 4 mHz in the PSD, which coincides with the peak in the observed PSD of Alfvénic waves. The energy flux of acoustic waves in our model can be estimated as $\sim 2.84 \times 10^5 \text{ erg} \cdot \text{cm}^{-2} \cdot \text{s}^{-1}$, which is similar to the energy flux of observed Alfvénic waves of $\sim 1.54 \times 10^5 \text{ erg} \cdot \text{cm}^{-2} \cdot \text{s}^{-1}$.² These imply that the acoustic waves generated in our model may be sufficient to become the driving source of the Alfvénic waves observed by Morton et al. (2019) [31].

De Pontieu et al. (2004) [42] proposed that solar chromospheric spicules are driven by the leakage of photospheric *p*-mode oscillations, which can be achieved through the waveguide of magnetic flux tubes. Their model can well explain the observed spicule periodicity, 350 ± 60 s (frequency of 2.86 ± 0.59 mHz). This frequency is similar to that of the shock train in our model (~ 4 mHz). Their simulations showed that the oscillations leaking into the chromosphere can develop non-linearly into upward propagating shocks with a velocity amplitude $\sim 20\text{--}40$ km/s, which is also similar to our model (see the panels showing velocity profiles in Figures 3 and 4). Thus, we think that our model can be regarded as an extension of their model, especially regarding their omission of coronal heating.

4.3. Mass Upflows

Figure 8 displays the radial mass fluxes of Case B at four different heights (solid lines) from Day 265 to Day 365, and the dashed line in each panel denotes the zero-flux level as the reference line of radial outflow (positive) and inflow (negative). While the oscillations of mass fluxes are natural results of shocks waves, we can see that the absolute values of mass fluxes at the peaks are much larger than those at the troughs, indicating that mass upflows are superposed onto the peaks. These mass upflows occurred in our model in the height range $\sim 7 \times 10^3\text{--}7 \times 10^4$ km, which coincided with the observed height of solar prominences ($\sim 10^4\text{--}10^5$ km, [47]). While prominences are believed to be a kind of magnetic field structure formed in filament channels, how their mass is acquired is still an open question. Aschwanden (2005) [47] (also see [72]) proposed three possible scenarios, one of which is injection by chromospheric mass upflows, and there is a lot of observational evidence that mass is continuously entering and exiting the filament magnetic field throughout its lifetime. These features agree quite well with the mass upflows in our model, so we conjecture that they might be the mass supplier of prominences, which

could later evolve in magnetic fields into the observed prominence structure, though these structures could not be obtained in our simulation as we neglected magnetic fields in the corona. These mass upflows also existed for the other two cases, though the exact height range differed slightly.

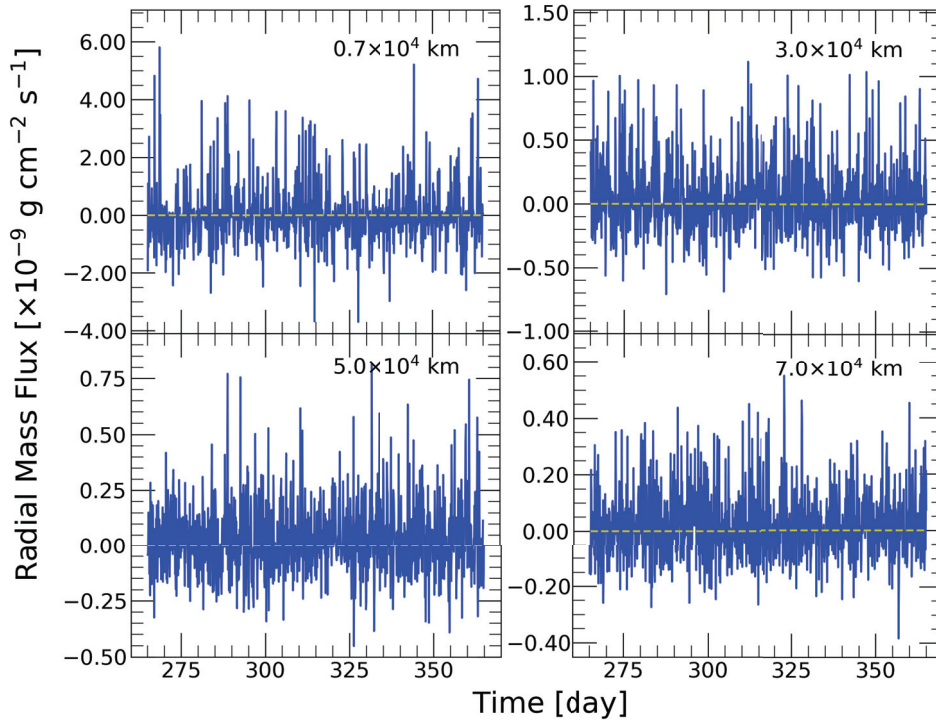


Figure 8. Radial mass fluxes at four different heights (solid lines) and their zero-flux levels (dashed lines) for Case B, recorded from Day 265 to Day 365.

5. Summary and Conclusions

We performed global spherically symmetric HD simulations of the solar atmosphere, to explore the origin of global 4 mHz oscillations in corona and investigate the possibility of corona heating via the resulting shock waves of these oscillations. The range of the simulation was set between 1 and $1.25 R_{\odot}$, so that the chromosphere and corona could evolve as a whole. The heating in the chromosphere was simplified by an artificial energy flux proportional to local pressure, $F_c = k_0 p$ (applied in the height range 0–2000 km above the photosphere; see Section 2.2 for its physical interpretation), which simulated the heating flux of acoustic waves in the chromosphere without adding an acoustic wave generator in the photosphere boundary. We used this artificial flux for two reasons. First, with F_c we could ensure that the acoustic waves in the simulated corona were newly generated, instead of being leaked from the photosphere boundary. Secondly, the simulated chromosphere could quickly converge to a quasi-steady state with this form of F_c . While this assumption is very simple, the artificial energy fluxes in the simulation actually agreed quite well with the observed acoustic energy fluxes (Figure 1) for the optimal value of $k_0 = 1.61 \times 10^3 \text{ cm s}^{-1}$ (Case B) that we found. The mass loss rate at the outer boundary also agreed with observed value, and we obtained a chromospheric structure which roughly agreed with the VAL-C model determined from *Skylab* observations of the quiet Sun (see Figure 2). Therefore, we think that our simple assumption reflects part of the physical nature of the solar chromosphere, and that the simulation results are valid. However, it should be noted that we ignored the heat conduction in our model, which is important, especially within the TR. According to our computational practice, the effect of ignoring heat conduction did not alter the generation of 4 mHz oscillations appearing in the early stages of our simulations,

but we cannot remark about its long-term effects through comparing runs with and without the heat conduction.

We found that shock waves are produced from acoustic waves excited by p -mode oscillations at the bottom of the corona and propagate upwards, heating the coronal plasma along the way, occasionally up to >1 MK. However, the maximum mean temperature was only 0.92 MK for the optimal simulation, Case B. Even with a k_0 ten times larger (Case A; the larger k_0 , the larger the heating flux in the chromosphere), the mean temperature could only reach 1.1 MK. As a comparison, the observed mean temperature in the quiet solar corona is 1.4–1.8 MK [62]. The mean temperature also dropped monotonically as the height increased past the location corresponding to the maximum value, due to the quick dissipation of acoustic shock waves. Both phenomena indicate that the heating via acoustic shock waves alone cannot maintain a solar corona as observed, and other heating mechanisms such as magnetic reconnection and the dissipation of Alfvénic waves must play an important role in coronal heating. However, it should be noted that this kind of shock wave heating may still have been exaggerated, due to the one-dimensional nature of our simulations.

For Case B, the frequencies of the shock waves were found to be in the range of 3.9–4.07 mHz (see Table 1 for the results of the other two cases), in agreement with typical wave periods observed in the corona, which peak at around three to five minutes [31–35]. We also investigated the Brunt-Väisälä cut-off frequency for acoustic waves at different heights. While the regions below the height of 3×10^3 km and above the height of 4×10^4 km were largely opaque for acoustic waves of ~ 4 mHz, the region between $\sim 4 \times 10^3$ and $\sim 2 \times 10^4$ km allowed their free propagation. Thus, this region could act as a resonant cavity for p -mode oscillations of ~ 4 mHz and give birth to shock waves of the same frequency, which could then carry energy into the higher corona with a speed ~ 139 km $\cdot$ s $^{-1}$. As the source of acoustic waves involved in the coronal heating is moved from the photosphere to the top of the chromosphere, our model naturally overcomes the problem of chromospheric damping and blocking of acoustic waves. The amplitude of the coronal heating shocks in our model is in the order of tens of kilometers per second in the v_r profile, much larger than that required by chromospheric heating (a few kilometers per second, see [54]), while the density in the corona is much smaller than that in the chromosphere, so it is natural that the model corona can be heated to a much higher temperature. Note that our simulation was purely hydrodynamic and did not include the effects of magnetic fields. When magnetic fields are added, as in the real solar atmosphere, the energy stored in acoustic waves can be converted into other forms, such as Alfvénic waves, spicules, and prominences, which explains why such high amplitude acoustic waves are not typically observed.

Our model also helps to explain the ubiquitously (both in space and time) observed Alfvénic waves in the solar corona. Morton et al. (2019) [31] found an enhancement in power around 4 mHz in the PSD of Alfvénic waves throughout the entire corona, which was almost invariant in the center values throughout the solar cycle. Traditionally, it was proposed that Alfvénic waves are generated in the solar photosphere, though the low-ionization fraction in the photosphere hinders the generation, and the transmission of Alfvénic waves suffers from a strong TR reflection. Several MHD wave models have been proposed to solve this problem (e.g., [49–51,70,71]), where the Alfvénic waves can be excited by helioseismic p -mode oscillations through double mode conversion and the TR reflection can be greatly reduced under specific conditions. However, these models need to be fine-tuned to provide sufficient Alfvénic waves as observed in the corona, and may have difficulty in explaining the ubiquitous existence of the Alfvénic waves peaking at ~ 4 mHz. On the other hand, our hydrodynamic simulation found that acoustic waves are newly

excited in a resonant cavity at the bottom of the corona, so that when magnetic fields are taken into consideration, Alfvénic waves can be generated there with processes similar to the excitation of Alfvénic waves in the photosphere or through double mode conversion, while naturally overcoming the traditional problems of the low-ionization fraction in the photosphere and the strong TR reflection. The acoustic waves in our optimal simulation (Case B) also peaked at ~ 4 mHz in the PSD, which coincides with the peak in the observed PSD of Alfvénic waves.

We also found that mass upflows existed in the height range $\sim 7 \times 10^3$ – 7×10^4 km for Case B, which might be the mass supplier of solar prominences. These mass upflows also existed for the other two cases, though the exact height range differed slightly.

Finally, it is necessary to note the theoretical rationality of our results in this paper. We think that our scenario of general 4 mHz oscillations in solar corona is physically reasonable, but its accompanying acoustic shock heating may still be exaggerated due to the one-dimensional nature of our model. Shock waves can only propagate radially outward in our model, which allows them to be enhanced through superposition during collisions in the same direction, resulting in greater model coronal heating. On the contrary, real shock waves can travel in any directions in the real solar atmosphere, which makes the superposition enhancement less efficient [73]. However, we still think that this kind of heating may have significant effects in certain special one-dimensional situations, such as in magnetic flux tubes, but not in global solar corona.

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Notes

- ¹ There are two reasons for this omission. One is that the resolution of our model is too low to accurately reveal the TR. Another is that the tiny time-step due to the diffusion calculation for the heat conduction is unfavorable for long-term physical time integration. According to our computational practice, the effect of ignoring heat conduction does not alter the generation of 4 mHz oscillations, which appear in the early stages of our simulations, but the code of the heat conduction is too slow to reveal its long-term effects through sufficient long time-integration.
- ² We used Equation (5) to calculate the estimation of the acoustic energy flux. The quantities required in the equation were obtained from the profiles of Day 354 (blue lines in Figure 3) at $r = 0.004R_{\odot}$ (~ 2800 km, the birthplace of acoustic waves). They are the density $\rho \simeq 10^{-14}$ g, temperature $T \simeq 10^5$ K, and velocity amplitude $v_a \simeq 25$ km $\cdot$ s $^{-1}$, respectively. The energy flux of Alfvénic waves was obtained from Supplementary Table 2 of [31].

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Article

Reconciling the Waiting Time Peaks Variations of Repeating FRBs with an Eccentric Neutron Star–White Dwarf Binary

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Abstract: Fast radio bursts (FRBs) are luminous radio transients with millisecond duration. For some active repeaters, such as FRBs 20121102A and 20201124A, more than a thousand bursts have been detected by the Five-hundred-meter Aperture Spherical radio Telescope (FAST). The waiting time (WT) distributions of both repeaters, defined as the time intervals between adjacent (detected) bursts, exhibit a bimodal structure well-fitted by two log-normal functions. Notably, the time scales of the long-duration WT peaks for both repeaters show a decreasing trend over time. These similar burst features suggest that there may be a common physical mechanism for FRBs 20121102A and 20201124A. In this paper, we revisit the neutron star (NS)–white dwarf (WD) binary model with an eccentric orbit to account for the observed changes in the long-duration WT peaks. According to our model, the shortening of the WT peaks corresponds to the orbital period decay of the NS–WD binary. We consider two mass transfer modes, namely, stable and unstable mass transfer, to examine how the orbital period evolves. Our findings reveal distinct evolutionary pathways for the two repeaters: for FRB 20121102A, the NS–WD binary likely undergoes a combination of common envelope (CE) ejection and Roche lobe overflow, whereas for FRB 20201124A the system may experience multiple CE ejections. These findings warrant further validation through follow-up observations.

Keywords: fast radio bursts; transients; white dwarfs; compact binaries

1. Introduction

Fast radio bursts (hereafter, FRBs) are luminous transients with millisecond duration (see reviews, e.g., [1–3]). Because the dispersion measures of FRBs exceed the contributions of our Galaxy, and certain FRB host galaxies have been identified at redshifts in the range $z \simeq 0.03 - 1$ (e.g., [4,5]), FRBs are thought to be extragalactic origins (except FRB 20200428 detected from the Galactic magnetar SGR J1935+2154; [6,7]). Based on the quantity of detected bursts, FRBs can be classified as repeating FRBs and apparently one-off (hereafter, simply “one-off”) FRBs. Recently, nearly 500 unique FRB sources have been detected by the Canadian Hydrogen Intensity Mapping Experiment (CHIME) (i.e., the CHIME/FRB Catalog 1¹). This sample is excellent for exploring the statistical properties of FRBs. Some statistical differences between one-off FRBs and repeating ones have been found based on this sample, including the differences in the distributions of the peak fluxes [8], the spectral indices [9], and the luminosities [10,11]. Additionally, the time–frequency drift (“sad-trombone” effect) commonly seen in repeating FRBs (e.g., [12]) is mostly absent in one-offs. All of these differences strongly suggest that one-off and repeating FRBs likely have distinct emission mechanisms and progenitors (e.g., [13,14]).

Periodic activities are an interesting property of repeaters that can provide important clues for revealing their physical mechanisms. Analysis of the burst arrival times

of FRB 20180916B shows a ~ 5 -day activity window² that recurs every 16.35 days [16]. FRB 20121102A, the first detected repeater coincident with a compact persistent radio source [17,18], was found to exhibit a candidate periodicity of ~ 160 days [19,20]. Some models have been proposed to explain these long activity periods, including orbital motion (e.g., [21–26]), rotational period (e.g., [27]), and precession-based mechanisms (e.g., [28–31]).

The Five-hundred-meter Aperture Spherical radio Telescope (FAST) has conducted continuous monitoring of several active repeating FRBs, including FRBs 20121102A and 20201124A (e.g., [32–34]). Over the course of 59.5 h of observation time between 29 August 2019 and 29 October 2019, Li et al. [32] detected 1652 independent burst events from FRB 20121102A. These bursts have a bimodal energy distribution that can be well-fitted by combining a log-normal function with a Cauchy function [32]. Moreover, the distribution of the waiting time, defined as the time interval between two adjacent (detected) bursts, demonstrates a bimodal structure. The two fitted peak waiting times are ~ 3.4 ms and ~ 70 s (the fitted peak of the waiting time distribution of the high-energy component ($E > 10^{38}$ erg) is ~ 220 s; [32]). For FRB 20201124A³, 1863 separate burst have been detected by FAST during a total observation time of 82 h from 1 April 2021 to 29 May 2021 [33]. The waiting time distribution of FRB 20201124A similarly reveals a bimodal structure, which can be well-fitted by two log-normal functions peaking at 39 ms and 106.7 s, respectively [33]. Despite the detection of thousands of bursts, no significant periods have been found in the millisecond to the second range for either FRBs 20121102A or 20201124A (e.g., [32,33,36]).

For FRB 20121102A, a total of 478 independent burst events were detected by the 305-m Arecibo telescope⁴ over 59 h of observation between December 2015 and October 2016 [37]. The waiting time distribution of these bursts similarly exhibits a bimodal structure, which can be fitted by two log-normal functions peaking at ~ 24 ms and ~ 95 s, respectively [37]. For FRB 20201124A, following the detection of a new burst by CHIME/FRB on 21 September 2021⁵, Zhang et al. [34] used FAST to monitor this source, initiating observations on 25 September 2021. In all, 881⁶ independent burst events from FRB 20201124A were detected in the first four days (i.e., from 25 September 2021 to 28 September 2021; [34]) of the 17-day monitoring campaign. The waiting time distribution of these bursts can also be fitted by two log-normal functions peaking at 51.22 ms and 10.05 s, respectively [34]. The time scales of the long-duration (second-level) peaks of the waiting time distributions of these two active repeaters both decrease with time, which would be a sign that both FRB sources are becoming increasingly active.

These observed similarities in burst characteristics suggest that a common physical mechanism likely exists for both FRBs 20121102A and 20201124A. In this paper, we explain the common features observed in two repeaters using the NS-WD binary model with an eccentric orbit [22]. We consider that the time intervals between adjacent bursts correspond to the orbital period of the NS-WD binary, as the intermittent type of Roche lobe overflow can commonly occur in the semi-detached systems. Consequently, the decreases in the long-duration wait time peaks are linked to the decreases in the orbital periods of the NS-WD binary. To investigate the evolution of the NS-WD binary, we consider two distinct forms of mass transfer, namely, stable and unstable mass transfer.

The remainder of this article is organized as follows: the NS-WD binary model with an eccentric orbit is illustrated in Section 2; the evolution of the NS-WD binary with stable mass transfer is studied in Section 3; the evolution of the binary with a massive WD based on a form of unstable mass transfer is shown in Section 4; finally, conclusions and discussion are presented in Section 5.

2. NS-WD Binary Model

Gu et al. [22] proposed a compact binary model with an eccentric orbit to explain the periodic activity observed in repeating FRBs. The system contains a magnetic WD and an NS with strong dipolar magnetic fields. At the periastron, the mass transfer occurs from the WD to the NS through the inner Lagrange point L_1 when the WD fills its Roche lobe. At other positions along the eccentric orbit, no mass transfer is supplied, as the Roche lobe is not filled. When the accreted magnetized materials approach the NS surface, magnetic reconnection is triggered, releasing strong electromagnetic radiation via curvature radiation as the electrons move along the NS magnetic field lines. The WD may be kicked away after a Roche lobe overflow process if $q < 2/3$ [40], where q is the mass ratio defined as $q = M_{\text{WD}}/M_{\text{NS}}$, with M_{NS} and M_{WD} respectively denoting the masses of the NS and WD. The WD can replenish its Roche lobe through gravitational radiation; therefore, the next transfer process can recur. The intermittent nature of Roche lobe overflow appears to be a common phenomenon in the semi-detached NS-WD system, as the NS mass is normally larger than $1.4 M_{\odot}$ and the WD mass distribution peaks at $0.6 M_{\odot}$. According to this model, the duty cycle of the burst activity is related to the orbital period of the binary. An extremely high eccentricity ($e > 0.95$) of orbit is required to explain the 16.35-day periodicity of FRB 20180916B [22].

The wait time distributions of the active repeaters FRBs 20121102A and 20201124A both exhibit a bimodal structure, i.e., a millisecond peak and a tens-of-second peak. A possible explanation for the millisecond peak is that due to the high time resolution of the radio telescopes, e.g., the $98.304\text{-}\mu\text{s}$ sampling resolution of FAST [32], the individual burst events with long-duration pulse widths and multi-component pulse profiles (e.g., FRB 20191221A including 3-second duration and nine components; [41]) can be considered to be multiple separate bursts. Following the spirit of Gu et al. [22], these bursts with multiple pulse components can be produced in the mass transfer process. Specifically, the viscous processes during mass transfer are necessary to help the accreted materials lose angular momentum and eventually fall onto the surface of the NS. During this process, some massive accreted materials may be fragmented into a number of smaller portions. When the fragmented materials originating from the same initial bulk of material approach the NS at different times, multiple bursts with short intervals may be triggered, which can be regarded as a single burst event containing multiple sub-bursts.

For the tens-of-second peaks, an intriguing phenomenon is that the long-duration peaks decay with time. In our hypothesis, these peak waiting times may correspond to the orbital periods P_{orb} of the NS-WD binary. As a result, the variations in the orbital periods of the NS-WD binary induced by the mass transfer are hypothesized to induce variations in the long-duration wait time peaks of repeating FRBs. The reasons for this are as follows. For the semi-detached NS-WD system, the intermittent type of Roche lobe overflow may be a common phenomenon (e.g., [22,40,42]). The time interval between adjacent mass transfer processes T_{mt} is proportional to the mass of the transferred material from the WD ΔM_{WD} [26]. For $M_{\text{WD}} \sim 0.6 M_{\odot}$, it can be evaluated that $T_{\text{mt}} \lesssim P_{\text{orb}}$ (see Equation (9) of [26]). Moreover, the supply of the accreted materials to the NS is expected to exhibit a uniform temporal lag. Therefore, the time interval between two adjacent bursts should be equivalent to P_{orb} .

On the other hand, due to the remaining angular momentum, some of the fragmented materials cannot not fall to the surface of the NS while surrounding it; that is, even though no mass transfer occurs in the binary system, some materials surrounding the NS arrive on its surface episodically, where they stochastically produce radio bursts with wait times ranging from a few seconds to thousands of seconds.

3. Stable Mass Transfer

3.1. Orbital Period

The dynamic equation of the NS-WD binary system is provided by

$$\frac{G(M_{\text{NS}} + M_{\text{WD}})}{a^3} = \frac{4\pi^2}{P_{\text{orb}}^2}, \quad (1)$$

where G is the gravitational constant and a is the semimajor axis of the eccentric orbit. The Roche lobe radius for the WD R_{L2} at the periastron can take the following form [43]:

$$\frac{R_{\text{L2}}}{a(1-e)} = 0.462 \left(\frac{M_{\text{WD}}}{M_{\text{NS}} + M_{\text{WD}}} \right)^{1/3} \quad (2)$$

where e is the eccentricity of the orbit. The WD radius R_{WD} can be expressed as [44]

$$R_{\text{WD}} = 0.0114 R_{\odot} \left[\left(\frac{M_{\text{WD}}}{M_{\text{Ch}}} \right)^{-2/3} - \left(\frac{M_{\text{WD}}}{M_{\text{Ch}}} \right)^{2/3} \right]^{1/2} \times \left[1 + \frac{7}{2} \left(\frac{M_{\text{WD}}}{M_{\text{p}}} \right)^{-2/3} + \left(\frac{M_{\text{WD}}}{M_{\text{p}}} \right)^{-1} \right]^{-2/3}, \quad (3)$$

where $M_{\text{Ch}} = 1.44 M_{\odot}$ is the Chandrasekhar mass limit and $M_{\text{p}} = 0.00057 M_{\odot}$ [45].

At the periastron, it is assumed that the WD fills its Roche lobe, i.e., $R_{\text{WD}} = R_{\text{L2}}$. The orbital period P_{orb} can be derived by Equations (1)–(3) after M_{NS} , M_{WD} , and e are given. The relationships between the orbital period P_{orb} and the eccentricity e of the orbit for the distinct WD masses are shown in Figure 1. The black lines represent different WD masses, i.e., $0.1 M_{\odot}$ (solid line), $0.6 M_{\odot}$ (dashed line), and $1.2 M_{\odot}$ (dotted line). The blue and red lines correspond to the peak waiting times of FRBs 20121102A and 20201124A, respectively, at the distinct activity epochs. It can be seen from Figure 1 that the orbital period P_{orb} of the NS-WD binary shortens with decreasing eccentricity e for a constant WD mass.

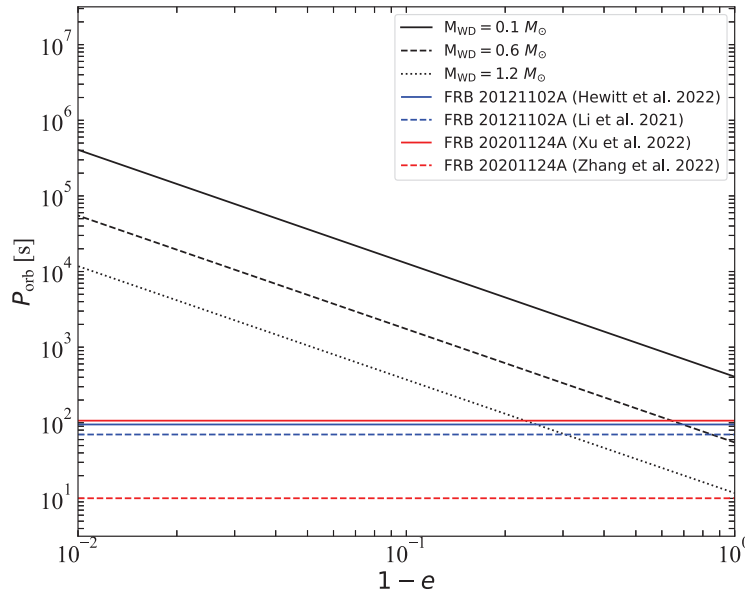


Figure 1. Relationship between the orbital period P_{orb} and eccentricity e for the different WD masses M_{WD} (where $M_{\text{NS}} = 1.4 M_{\odot}$) when the WD fills its Roche lobe at periastron. The black lines represent difference WD masses, i.e., $0.1 M_{\odot}$ (solid line), $0.6 M_{\odot}$ (dashed line), and $1.2 M_{\odot}$ (dotted line). The blue lines correspond to the peak waiting times of FRB 20121102A reported by Hewitt et al. [37] (solid line) and Li et al. [32] (dashed line), respectively. The red lines represent the peak waiting times of FRB 20201124A reported by Xu et al. [33] (solid line) and Zhang et al. [34] (dashed line), respectively.

Observations of the active repeating FRBs 20121102A and 20201124A reveal that the long-duration wait time peaks both shorten over time. In the NS-WD binary model, the WD mass remains quasi-constant for short-term (e.g., a few years) stable mass transfer processes, whereas the eccentricity e of the NS-WD binary system can gradually decrease, as the dynamical mass transfer and gravitational radiation may cause orbital circularization. Thus, the orbital period P_{orb} can decrease gradually. As shown in Figure 1, the orbital period P_{orb} varies from hundreds to tens of seconds as the eccentricity decreases when $M_{\text{WD}} \gtrsim 0.6 M_{\odot}$, which is comparable to the variations in the long-duration peaks of the wait time distributions of FRBs 20121102A and 20201124A.

3.2. Orbital Evolution of NS-WD Binary

The orbital angular momentum of the NS-WD binary is provided by [46]

$$J_{\text{orb}} = M_{\text{NS}} M_{\text{WD}} \left[\frac{Ga(1-e^2)}{M_{\text{tot}}} \right]^{1/2}, \quad (4)$$

where $M_{\text{tot}} = M_{\text{NS}} + M_{\text{WD}}$ is the total mass of the binary. The orbital angular momentum evolution of a compact binary is dominated by the changes due to the gravitational wave radiation ($\dot{J}_{\text{GW}}$), mass transfer ($\dot{J}_{\text{mt}}$), and coupling between the spin of the (primary) accretor and the orbit ($\dot{J}_{\text{so}}$) (e.g., [45]). The changes in the orbital angular momentum can be expressed as follows:

$$\dot{J}_{\text{orb}} = \dot{J}_{\text{GW}} + \dot{J}_{\text{mt}} + \dot{J}_{\text{so}}. \quad (5)$$

Because the magnitude of the orbital angular velocity of eccentric binaries is a periodic function of time, it is unlikely for the stellar rotation rate to be synchronized with the orbital motion during all orbital phases [47]. Therefore, for NS-WD binaries with an eccentric orbit, we ignore the effect of the coupling between the spin of the NS and the orbit on the evolution of the orbital angular momentum.

We can deduce the time derivative of the orbital angular momentum $\dot{J}_{\text{orb}}$ based on Equation (4), i.e.,

$$\frac{\dot{J}_{\text{orb}}}{J_{\text{orb}}} = \frac{1}{2} \frac{\dot{a}}{a} + \frac{\dot{M}_{\text{NS}}}{M_{\text{NS}}} + \frac{\dot{M}_{\text{WD}}}{M_{\text{WD}}} - \frac{1}{2} \frac{\dot{M}_{\text{tot}}}{M_{\text{tot}}} - \frac{e\dot{e}}{1-e^2}, \quad (6)$$

where $\dot{M}_{\text{tot}} = \dot{M}_{\text{NS}} + \dot{M}_{\text{WD}}$ and $\dot{M}_{\text{WD}} < 0$. The change of the orbital angular momentum caused by the gravitational wave radiation is provided by [46]

$$\frac{\dot{J}_{\text{GW}}}{J_{\text{orb}}} = -\frac{32}{5} \frac{G^3}{c^5} \frac{M_{\text{NS}} M_{\text{WD}} M_{\text{tot}}}{a^4} \frac{(1 + \frac{7}{8}e^2)}{(1-e^2)^{5/2}}. \quad (7)$$

When the NS-WD binary is a detached system, the gravitational wave radiation can shrink the orbital separation by dissipating the angular momentum of the system. With orbital decay due to gravitational wave radiation, the radius of the Roche lobe gradually decreases. Eventually, mass transfer occurs from the WD to the NS through the inner Lagrange point L_1 when the WD fills its Roche lobe at the periastron. The angular momentum transferred from the WD to the neighborhood of the accreting NS along with the transferred matter can induce a change in the orbital angular momentum, which is provided by [44,48]

$$\dot{J}_{\text{mt}} = -\sqrt{GM_{\text{NS}}R_{\text{h}}}\dot{M}_{\text{NS}}, \quad (8)$$

where R_h is an equivalent radius of the accreted matter orbiting the accreting NS. This radius R_h can be written as [44]

$$\frac{R_h}{a} = r_h = 0.0883 - 0.04858 \log q + 0.011489 \log^2 q + 0.020475 \log^3 q, \quad (9)$$

where $10^{-3} < q < 1$. In the case of a fully developed accretion disk around the accreting NS, the angular momentum caused by the mass flow is canceled by a reverse flow [44,48,49], i.e., $R_h = 0$.

On the other hand, as one of the most active repeating FRBs to date, the isotropic peak luminosities of FRB 20201124A are very high (e.g., from $5 \times 10^{37} \text{ erg s}^{-1}$ to $3 \times 10^{40} \text{ erg s}^{-1}$; [33]), indicating that the emission mechanism of this FRB source should be efficient. Based on the bursts reported by Xu et al. [33] and Zhang et al. [34], the average accretion rates of two active episodes can be estimated to be $\sim 1.4 \times 10^{-4} M_\odot \text{ yr}^{-1}$ and $\sim 5.2 \times 10^{-5} M_\odot \text{ yr}^{-1}$, respectively, assuming that $L = 0.1 \dot{M}_{\text{NS}} c^2$. Thus, the outflows should be taken into account in the NS-WD binary with super-Eddington accretion rates ($\dot{M}_{\text{Edd}} \sim 10^{-8} M_\odot \text{ yr}^{-1}$). We use the accretion efficiency ϵ to describe the fraction of the matter lost by the WD that is accreted by the NS, i.e.,

$$\dot{M}_{\text{NS}} = -\epsilon \dot{M}_{\text{WD}} \quad (10)$$

and

$$\dot{M}_{\text{tot}} = (1 - \epsilon) \dot{M}_{\text{WD}}, \quad (11)$$

where $0 \leq \epsilon \leq 1$. If $\epsilon = 1$, then all matter transferred from the WD is accreted by the NS, that is, the total mass of the system is conserved (i.e., $\dot{M}_{\text{NS}} = -\dot{M}_{\text{WD}}$ and $\dot{M}_{\text{tot}} = 0$). The only way to lose orbital angular momentum in this case is through gravitational wave radiation. If $\epsilon < 1$, then the binary gradually loses mass. In addition to the gravitational wave radiation, the system can lose angular momentum along with the ejected matter:

$$\frac{\dot{J}_{\text{loss}}}{J_{\text{orb}}} = \gamma \frac{\dot{M}_{\text{tot}}}{M_{\text{tot}}}. \quad (12)$$

In the particular case where the ejected matter carries the specific orbital angular momentum of the accretor, $\gamma = q$ (e.g., [48]).

Combining Equations (6)–(12), the change of the semimajor axis of the eccentric NS-WD binary during the mass transfer process can be written as follows:

$$\frac{\dot{a}}{2a} = \frac{\dot{J}_{\text{GW}}}{J_{\text{orb}}} - \frac{\dot{M}_{\text{WD}}}{M_{\text{WD}}} \left[(1 - \epsilon q) - \frac{(1 - \epsilon)q}{1 + q} \left(\gamma + \frac{1}{2} \right) - \epsilon \sqrt{\frac{(1 + q)r_h}{1 - e^2}} \right] + \frac{e\dot{e}}{1 - e^2}. \quad (13)$$

It is seen that the orbital separation can decay due to the gravitational wave radiation, mass loss, and orbital circularization.

At the periastron, the mass transfer from the WD to the NS occurs through the inner Lagrange point L_1 when the WD fills its Roche lobe. The radii of WD and its Roche lobe can change due to the mass loss of the WD. To ensure the stability of mass transfer, the change in the Roche lobe radius of the WD R_{L2} should exceed the change in its radius R_{WD} , i.e.,

$$\dot{R}_{L2} \gtrsim \dot{R}_{\text{WD}}. \quad (14)$$

By dividing both sides of Equation (14) by $R_{L2} = R_{\text{WD}}$ and $\dot{M}_{\text{WD}}/M_{\text{WD}} < 0$, a well known criterion for the stability of mass transfer can be obtained (e.g., [44,50]):

$$\zeta_{L2} \lesssim \zeta_{\text{WD}} \quad (15)$$

where $\zeta_{L2} = \partial \ln R_{L2} / \partial \ln M_{WD}$ is the response of the Roche lobe radius of the WD to the mass loss and $\zeta_{WD} = \partial \ln R_{WD} / \partial \ln M_{WD}$ is the response of the WD radius to the mass loss. If the WD does not obey this criterion to start transferring matter via Roche lobe overflow, then the increase in WD radius after mass loss is larger than the effective expansion of its Roche lobe. In this case, the Roche lobe overflow tends to be unstable.

Based on Equation (3), the logarithmic differentiation of WD radius ζ_{WD} can be written as

$$\zeta_{WD} = \frac{\partial \ln R_{WD}}{\partial \ln M_{WD}} = -\frac{1}{3} \left[\frac{1 + \left(\frac{M_{WD}}{M_{Ch}} \right)^{4/3}}{1 - \left(\frac{M_{WD}}{M_{Ch}} \right)^{4/3}} \right] + \frac{2}{3} \left[\frac{1 + \frac{7}{3} \left(\frac{M_{WD}}{M_p} \right)^{1/3}}{1 + \frac{7}{2} \left(\frac{M_{WD}}{M_p} \right)^{1/3} + \left(\frac{M_{WD}}{M_p} \right)} \right]. \quad (16)$$

The factor ζ_{L2} can be derived by logarithmic differentiation of Equation (2). We can deduce the critical mass ratio at which the unstable mass transfer occurs based on $\zeta_{L2} \lesssim \zeta_{WD}$. Figure 2 shows the logarithmic change in the radii of the WD and its Roche lobe due to mass transfer. In the NS-WD binary, we set the WD mass in the range of $0.01 - 1 M_\odot$ and $M_{NS} = 1.4 M_\odot$. The black and colored lines describe the logarithmic change in the WD radius and Roche lobe radius, respectively, while the solid and dashed lines represent the NS-WD binary with circular and eccentric orbits (e.g., $e = 0.5$), respectively. The blue lines describe the logarithmic change in the Roche lobe radius when the total mass of the system is conserved, i.e., $\epsilon = 1$. The red and green lines describe the logarithmic changes in the Roche lobe radius when the total mass of the system gradually decreases, i.e., $\epsilon = 0.5$ (red) and $\epsilon = 0.1$ (green). Unstable mass transfer occurs when the black line describing the response of the WD radius falls below the colored lines describing the changes in the Roche lobe radius. As shown in Figure 2, mass transfer is dynamically unstable if $M_{WD} > 0.9 M_\odot$, corresponding to $q \gtrsim 2/3$ (e.g., [40,48]). Moreover, the critical WD mass M_c in a NS-WD binary with an eccentric orbit is larger than that in a binary with a circular orbit. We can find that the critical WD mass decreases as the accretion efficiency ϵ decreases, i.e., $\epsilon = 0.5$, $M_c \sim 0.85 M_\odot$, and $\epsilon = 0.1$, $M_c \sim 0.8 M_\odot$.

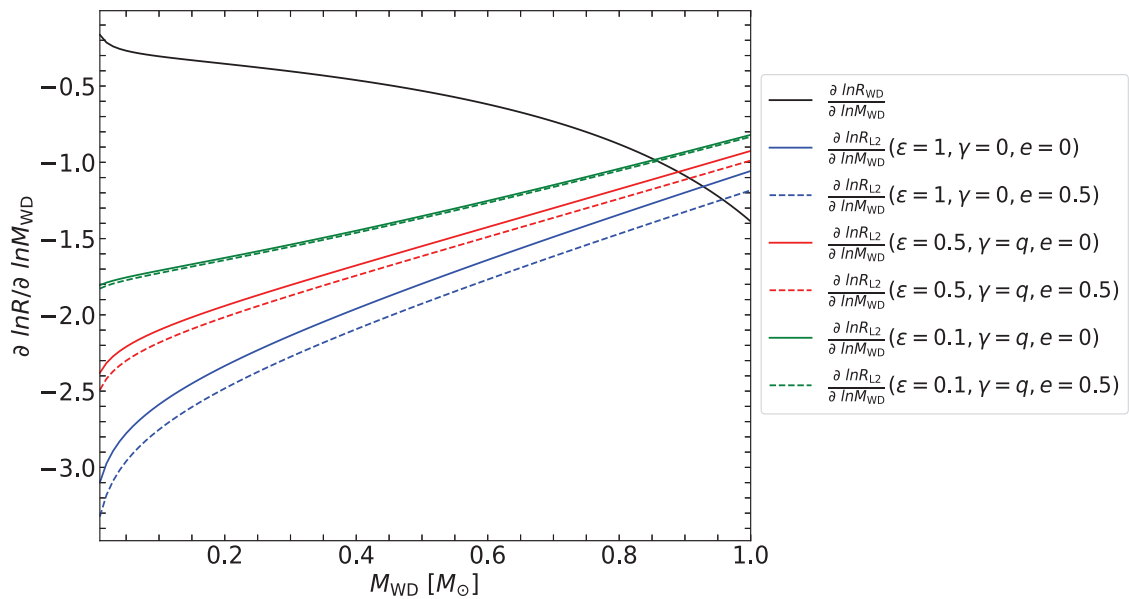


Figure 2. Logarithmic changes of distinct radii due to mass transfer for different WD masses. The black line describes the logarithmic change in the WD radius via mass loss when $M_{NS} = 1.4 M_\odot$. The solid and dashed lines represent the NS-WD binary with circular and eccentric orbits (e.g., $e = 0.5$), respectively. The blue lines describe the logarithmic changes in the radius of the Roche lobe when the total mass of the system is conserved. The red and green lines describe the logarithmic changes in

the Roche lobe radius when the total mass of the system gradually decreases, i.e., $\epsilon = 0.5$ (red) and $\epsilon = 0.1$ (green). Unstable mass transfer occurs when the black line describing the response of the WD radius falls below the colored lines describing the changes in the Roche lobe radius.

Based on the criterion $\zeta_{L2} \lesssim \zeta_{WD}$, the change of the eccentricity e during the stable mass transfer process can be written as

$$\frac{\dot{e}}{1+e} \lesssim \frac{2\dot{J}_{GW}}{J_{orb}} - \frac{2\dot{M}_{WD}}{M_{WD}} \left[\left(\frac{5}{6} - \epsilon q \right) - \frac{(1-\epsilon)q}{1+q} \left(\gamma + \frac{1}{3} \right) - \epsilon \sqrt{\frac{(1+q)r_h}{1-e^2}} - \frac{\partial \ln R_{WD}}{\partial \ln M_{WD}} \right]. \quad (17)$$

Therefore, we can combine Equations (13) and (17) to evaluate the orbital evolution of the NS-WD binary during the stable mass transfer process.

3.3. Mass Transfer Rate

For the NS-WD binary with an eccentric orbit, mass transfer from the WD to the NS occurs when the WD fills its Roche lobe at the periastron, i.e., $R_{WD} = R_{L2}$. We can define the overfill factor to indicate how much the donor overfills its Roche lobe by, i.e.,

$$\Delta = R_{WD} - R_{L2}. \quad (18)$$

Mass transfer occurs when $\Delta > 0$. The adiabatic approximation of the mass transfer rate of the donor can be written as [45]

$$\dot{M}_{WD} = -f(M_{NS}, M_{WD}, a, R_{WD}) \Delta^3, \quad (19)$$

which increases monotonically with Δ . As the system evolves, the mass transfer rate changes, leading to variations in the overfill factor. The change of the overfill factor is provided by [45]

$$\frac{d\Delta}{dt} = R_{WD} \left[(\zeta_{WD} - \zeta_{rL}) \frac{\dot{M}_{WD}}{M_{WD}} - \frac{\dot{a}}{a} \right], \quad (20)$$

where $\zeta_{rL} = \partial \ln(R_{L2}/a) / \partial \ln M_{WD}$. A good approximation for the mass transfer rate can be obtained by setting the right-hand side of Equation (20) to 0 [45,51]. Therefore, we obtain

$$\frac{\dot{M}_{WD}}{M_{WD}} = \frac{\dot{J}_{GW}/J_{orb}}{\frac{(\zeta_{WD} - \zeta_{rL})}{2} + (1 - \epsilon q) - \frac{(1-\epsilon)q}{1+q} \left(\gamma + \frac{1}{2} \right) - \epsilon \sqrt{\frac{(1+q)r_h}{1-e^2}}}. \quad (21)$$

In the case of mass transfer in eccentric orbits, the evolution of the mass transfer rate has a Gaussian-like (or delta function) behavior, with the maximum mass transfer rate occurring at periastron [47,52,53]. Thus, we assume that Roche lobe outflow occurs only at periastron. The duration of the Roche lobe overflow can be evaluated as follows (e.g., [54]):

$$t_{RLOF} \simeq R_p / v_p = \left(\frac{R_p^3}{2GM_{NS}} \right)^{1/2}, \quad (22)$$

where $R_p = a(1-e)$ is the distance between the binary components when the WD approaches periastron and $v_p = (2GM_{NS}/R_p)^{1/2}$ is the WD's orbital speed at R_p . Combining Equations (21) and (22), we can estimate the transferred mass from the WD during periastron passage.

3.4. Results

We numerically integrate Equations (13), (17) and (21), starting from the onset of mass transfer. The short-term evolution (about one hundred years) of the NS-WD binary with an eccentric orbit is shown in Figure 3. We assume that the original system consists of a $1.4 M_{\odot}$ NS and a $0.6 M_{\odot}$ WD with an eccentricity $e = 0.3$. In this case, the initial orbital period of the binary is ~ 94 s, comparable to the long-duration peaks of the wait time distributions of FRBs 20121102A [37] and 20201124A [33].

In Figure 3, the different colored lines represent the evolution of the physical parameters under various accretion efficiencies ϵ and γ , i.e., $\epsilon = 1, \gamma = 0$ (blue), $\epsilon = 0.5, \gamma = q$ (red), and $\epsilon = 0.1, \gamma = q$ (green). For $\epsilon = 1$ and $\gamma = 0$, the total mass of the system is conserved and all matter transferred from the WD is accreted by the NS. In this case, the only mechanism for losing orbital angular momentum is gravitational wave radiation. For the other two cases, the ejected matter provides an additional way to lose orbital angular momentum, resulting in a more significant reduction in both the orbital period and the eccentricity of the NS-WD binary. It can be seen from Figure 3 that the physical parameters of the NS-WD binary, including the orbital eccentricity e , semimajor axis a , and orbital period P_{orb} (panels a, b, and d of Figure 3, respectively), can be significantly reduced during the mass transfer process; moreover, the variations of these parameters in the two cases of $\epsilon = 0.5$ and $\epsilon = 0.1$ are greater than those in the case of $\epsilon = 1$. On the other hand, the WD mass loss rate $\dot{M}_{\text{WD}}$ for the binary system with mass ejection is greater than that for the mass-conserving system (see panel c of Figure 3).

Although the orbital period of the NS-WD binary can be significantly reduced by the short-term Roche lobe overflow, the magnitude of the orbital period shrinkage is still smaller than that of the reductions in the long-duration waiting-time peaks of FRBs 20121102A and 20201124A (see panel d of Figure 3).

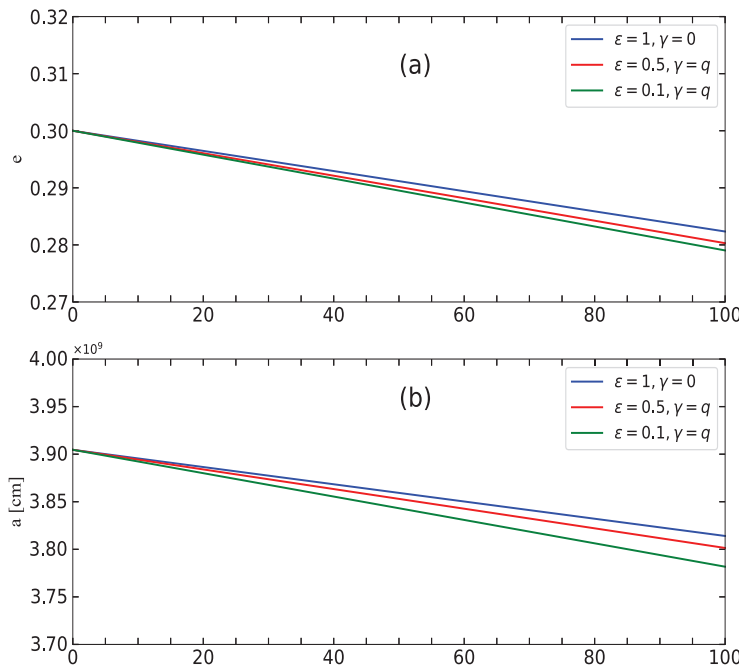


Figure 3. Cont.

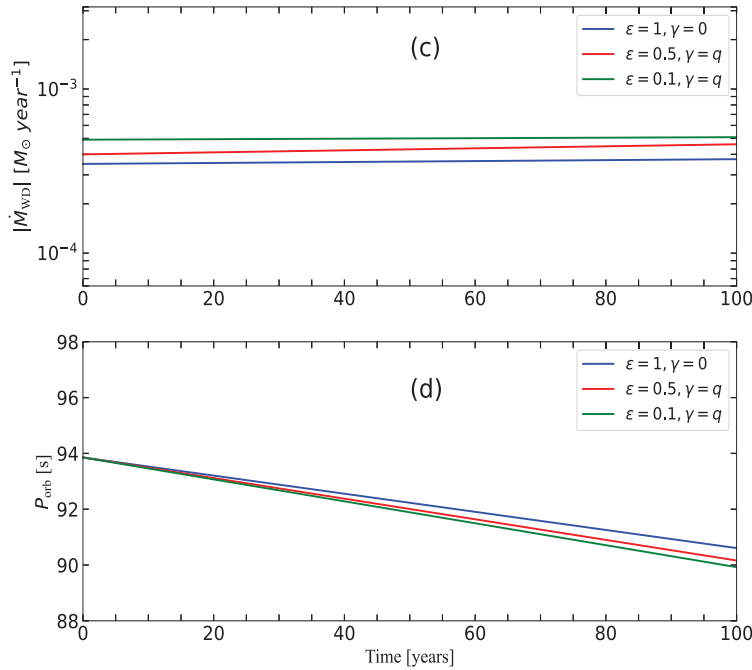


Figure 3. Evolution of the physical parameters of the NS-WD binary, including contributions from gravitational wave radiation, mass transfer, and mass ejection. The system with an eccentricity of $e = 0.3$ consists of a $1.4 M_{\odot}$ NS and a $0.6 M_{\odot}$ WD. The panels show (a) the orbital eccentricity e , (b) the semimajor axis a , (c) the WD mass loss rate $\dot{M}_{WD}$, and (d) the orbital period P_{orb} . The different colored lines represent the various accretion efficiencies ϵ and γ , i.e., $\epsilon = 1, \gamma = 0$ (blue), $\epsilon = 0.5, \gamma = q$ (red), and $\epsilon = 0.1, \gamma = q$ (green).

4. Unstable Mass Transfer

As shown in Figure 1, in the case of $M_{WD} = 1.2 M_{\odot}$ (dotted line), the range of the orbital periods of the NS-WD binaries with different eccentricities is consistent with the range of variations in the long-duration wait time peaks of both repeaters. The presence of a $1.2 M_{\odot}$ WD suggests that the binary system undergoes an unstable mass transfer process (see Figure 2), i.e., $\zeta_{L2} > \zeta_{WD}$, leading to evolution on dynamical time-scales. In this case, the WD would expand dramatically over about tens of orbital periods [55], inducing a sharp increase in the mass transfer rate (e.g., $\dot{M}_{WD} \gtrsim 100 M_{\odot} \text{ yr}^{-1}$; [55]). The NS is unable to accrete all the transferred materials, which subsequently pile up on the surface of the NS and begin to expand, eventually overfilling its Roche lobe. This results in the binary system becoming engulfed by a common envelope (CE). Due to the friction drag within the CE, the orbital energy of the embedded binary is reduced and deposited in the CE. If the total deposited energy can overcome the binding energy of the envelope, then orbital decay occurs, producing a binary with a very short orbital period following CE ejection. In contrast, if the CE fails to eject, then the inner binary coalesces into a single fast-rotating star (e.g., [56,57]).

The standard CE equation (known as the α -mechanism) based on energy conservation can be written as [58,59]

$$\alpha_{CE} \left(\frac{GM_2 M_{1,f}}{2a_f} - \frac{GM_{1,i} M_2}{2a_i} \right) = \frac{GM_{1,i} (M_{1,i} - M_{1,f})}{\lambda R_1}, \quad (23)$$

where $M_{1,i}$ and M_2 are the initial masses of the donor and the accretor, respectively, $M_{1,f}$ is the final mass of the donor that lost its envelope, a_i and a_f are the initial and final binary separations, respectively, and R_1 is the radius of the donor at the onset of CE. The left-hand term of Equation (23) is the energy ΔE_{orb} released from orbital shrinkage, the

right-hand term of Equation (23) is the binding energy, and λ is a parameter describing the structure of the envelope. The CE ejection efficiency α_{CE} is defined as the fraction of reduced orbital energy that is used in ejecting the CE. This α -mechanism can be used to explain the formation and evolution of some close interacting binaries, including cataclysmic variables, low-mass X-ray binaries [60], and double-degenerate WD binaries [59]. However, some uncertainties exist in the α -mechanism; for example, the exact values of α_{CE} and λ are hardly determined, and the exact boundary between the stellar envelope and its core is hard to constrain (e.g., [57]).

If the angular momentum of the orbit is so large that the common envelope can easily be made to co-rotate with the orbit, then there are no drag forces anymore [61]. In this case, it is assumed that the entire envelope is lost completely from the binary system [61,62]. The mass loss process reduces the angular momentum of the system in a linear way, which can be written as follows:

$$\frac{J_i - J_f}{J_i} = \gamma_{\text{CE}} \frac{\Delta M}{M_{\text{tot}}} \quad (24)$$

where $\Delta J = J_i - J_f$ is the decrease in the angular momentum due to CE ejection and ΔM is the envelope mass lost from the binary. This γ -mechanism is based on the angular momentum conservation, which was used in [61] to reconstruct the evolutionary histories of three double-He WDs with γ_{CE} in the range of 1.4 to 1.7. The change in orbital period is provided by [61,62]

$$\frac{P_f}{P_i} = \left(\frac{M_{1,f} M_{2,f}}{M_{1,i} M_{2,i}} \right)^{-3} \left(\frac{M_{1,f} + M_{2,f}}{M_{1,i} + M_{2,i}} \right) \left(1 - \gamma_{\text{CE}} \frac{\Delta M}{M_{1,i} + M_{2,i}} \right)^3, \quad (25)$$

where P_i and P_f are the orbital periods of the binary system before and after CE ejection, respectively, and $M_{2,i}$ and $M_{2,f}$ are the initial and final masses of the accretor, respectively.

Based on Equation (25), we can obtain the relationship between the ratio of the final to the initial orbital period P_f/P_i and the mass loss ΔM_{WD} due to CE ejection (Figure 4). The component masses of the NS-WD binary at the onset of CE are $M_{\text{NS}} = 1.4 M_{\odot}$ and $M_{\text{WD}} = 1.2 M_{\odot}$, respectively. As shown in Figure 4, different colored lines represent different γ_{CE} , i.e., $\gamma_{\text{CE}} = 1$ (black), 2 (blue), 3 (green), and 4 (orange). A larger value of γ_{CE} indicates that a greater fraction of the angular momentum of the initial binary is carried away by the ejected material [61]. The red and purple dashed lines indicate the ratios of the peak waiting times in two different activity phases for FRBs 20121102A and 20201124A, respectively.

4.1. FRB 20121102A

To make the change in the orbital period of the NS-WD binary comparable to the variation in the wait time peaks of FRB 20121102A, the mass of the ejected material ΔM_{WD} should range from $\sim 0.5 M_{\odot}$ to $\sim 0.7 M_{\odot}$ when γ_{CE} varies between 3 and 4 (see the red dashed line in Figure 4). After CE ejection, the remaining WD mass ($M_{\text{WD}} \sim 0.5\text{--}0.7 M_{\odot}$) falls below the critical mass ($M_c \sim 0.9 M_{\odot}$) and the mass transfer process of the NS-WD binary becomes stable. Subsequently, the eccentricity e of both the orbit and the orbital period decrease as the orbital angular momentum decreases.

The possible evolutionary history of FRB 20121102A is as follows. A massive WD ($M_{\text{WD}} = 1.2 M_{\odot}$) may be tidally captured by the NS (e.g., [63]), as large eccentricity e of the orbit is required to interpret the initial orbital period of the system. After the formation of a detached NS-WD binary, the orbital separation can be reduced by the gravitational wave radiation, resulting in a steady decrease in the Roche lobe radius of the WD. In this stage, mass loss from the WD may occur through a tidally enhanced wind (e.g., [64]). The interaction between the wind and the magnetosphere of the NS could produce coherent

radio emissions [65]. When the WD fills its Roche lobe, dynamic unstable mass transfer occurs. A fraction of the transferred material approaches the NS surface, producing strong electromagnetic radiation by curvature radiation [22], whereas the majority of the transferred material cannot be accreted by the NS, leading to the formation of a CE. The angular momentum and the orbital period of the NS-WD binary both decrease due to the CE ejection. Therefore, the time interval between two adjacent bursts decreases as the orbital period shortens. After the CE ejection, the residual mass of the WD falls below the critical mass M_c . Stable mass transfer can occur when the WD fills its Roche lobe at the periastron. Radio emissions can be produced when the accreted materials approach the surface of the NS. This burst phase could correspond to the burst storm observed by Arecibo [37] and FAST [32]. The orbital angular momentum of the NS-WD binary system is further reduced due to gravitational wave radiation and mass loss. When the eccentric orbit of the NS-WD binary system turns into a circular orbit, the mass transfer process always proceeds if the WD fills its Roche lobe. The FRB source may enter an extremely active phase, leading to a shorter time interval between two adjacent bursts.

In summary, the NS-WD binary model with an eccentric orbit can reconstruct the change in the burst activity of FRB 20121102A through the combined effects of CE ejection and Roche lobe overflow.

4.2. FRB 20201124A

For FRB 20201124A, as shown in Figure 4, the mass of the ejected material exceeds $1 M_\odot$ when $\gamma_{CE} = 3\text{--}4$, which can account for the changes in the waiting time peaks. After CE ejection, the binary system containing an NS and a naked O-Ne WD [55] is in a circular orbit. Because the envelope of the WD is completely ejected, the mass transfer rate of the remaining component ($M_{WD} < 0.2 M_\odot$, $\dot{M}_{WD} \sim 10^{-7} M_\odot \text{ yr}^{-1}$; [55]) is lower than that of the previous activity episodes, leading to a sharp decrease in the event rate of FRB 20201124A [66].

Nelemans et al. [61] considered two CE episodes to reconstruct the evolutionary histories of three double He WDs with γ_{CE} ranging from 1.4 to 1.7. In our model, the NS-WD binary system may also undergo two or more CE episodes with γ_{CE} ranging from 3 to 4, which would account for the variations in the wait time peaks of FRB 20201124A. After a CE ejection, the orbital separation decreases as the orbital angular momentum is reduced by the ejected material. Then, the Roche lobe radius of the WD would be reduced due to the gravitational wave radiation, causing the WD to fill its Roche lobe and triggering the next unstable mass transfer. In this scenario, the orbital period of the NS-WD binary becomes comparable to the waiting time peak after two or more CE episodes.

The orbital angular momentum of the evolved NS-WD binary is further reduced by gravitational wave radiation. Finally, the NS-WD binary merges. If the final remnant is a magnetar, then the NS-WD merged channel could produce one-off FRBs (e.g., FRB 20180924B; [67]).

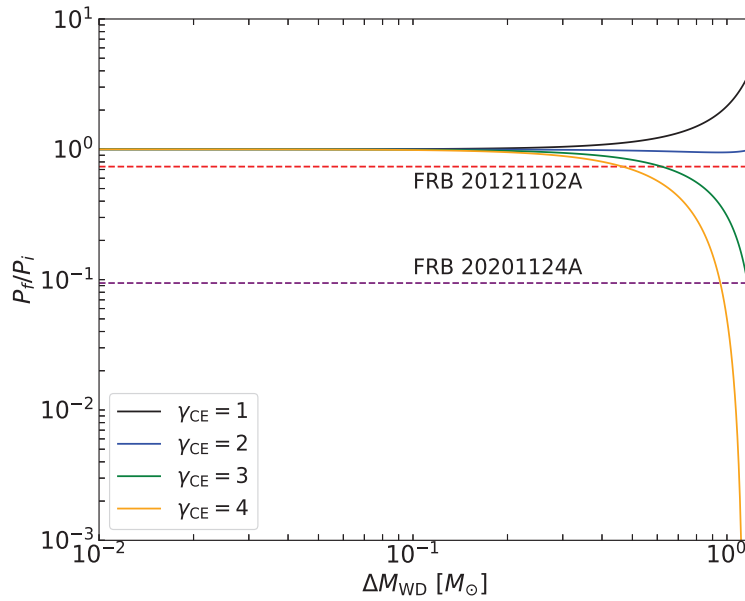


Figure 4. Relationship between the orbital period ratio, defined as P_t/P_i , and the envelope mass lost from the binary ΔM . Different colored lines represent different γ_{CE} , i.e., $\gamma_{CE} = 1$ (black), 2 (blue), 3 (green), and 4 (orange). The initial physical parameters of the NS-WD binary system are $M_{NS} = 1.4 M_{\odot}$ and $M_{WD} = 1.2 M_{\odot}$. The red and purple dashed lines indicate the ratios of the wait time peaks in two different activity epochs for FRBs 20121102A and 20201124A, respectively.

5. Conclusions and Discussion

In this paper, we have revisited the NS-WD binary model with an eccentric orbit to explain the observed variations in the long-duration wait time peaks of FRBs 20121102A and 20201124A. In our model, the decreases in the wait time peaks correspond to the decays in the orbital periods of the binary system. We consider two modes of mass transfer, namely, stable and unstable, to study the evolution of the orbital period of the binary system. Our main results are summarized as follows:

1. In the case of stable mass transfer (e.g., $M_{WD} = 0.6 M_{\odot}$), although the short-term Roche lobe overflow can significantly shorten the orbital period of the NS-WD binary, the magnitude of the orbital period shrinkage cannot fully account for the observed reductions in the long-duration wait time peaks of FRBs 20121102A and 20201124A (see Section 3).
2. The CE ejections provide an additional and more efficient mechanism for angular momentum loss in the system with a massive WD (e.g., $M_{WD} = 1.2 M_{\odot}$). By applying the γ -mechanism to the CE ejections, the variations in the orbital period of the NS-WD binary, which are comparable to the changes in the wait time peaks of two repeaters, can be reconstructed with γ_{CE} ranging between 3 and 4. Furthermore, our analysis also suggests distinct evolutionary pathways for the two sources; for FRB 20121102A, the binary likely undergoes a combination of CE ejection and Roche lobe overflow, while for FRB 20201124A the system may experience multiple CE ejections (see Figure 4, Section 4).
3. For FRB 20121102A, the remnant WD mass falls below the critical mass M_c after the CE phase. Stable mass transfer occurs when the WD fills its Roche lobe at the periastron. For FRB 20201124A, the final binary system consisting of an NS and a stripped O-Ne WD should be in a circular orbit. Following the complete ejection of the WD envelope, the remaining mass transfer rate decreases significantly, leading to a sharp decrease in the event rate of FRB 20201124A (see Sections 4.1 and 4.2).

The observations of the two burst episodes for FRBs 20121102A and 20201124A are discontinuous [32–34,37], suggesting that the variations in the fitted wait time peaks across different activity epochs are also discontinuous. Thus, uncertainties remain in determining the evolutionary stage of the NS-WD binary based on the wait time peaks, such as estimating the WD mass before and after mass transfer.

Our evaluations indicate that the critical WD mass distinguishing stable from unstable mass transfer is $M_c \sim 0.9 M_\odot$. Furthermore, ejecting a sufficient amount of transferred material from the system could reduce the stability of mass transfer. Many studies have explored the stability of mass transfer in WD-NS systems. For example, Verbunt & Rappaport [44] showed that if the transferred angular momentum is efficiently returned to the binary orbit by an accretion disk (i.e., $R_h = 0$), then the critical WD mass is $\sim 0.6 M_\odot$. In contrast, the critical WD mass without an accretion disk drops to around $0.3 M_\odot$. Taking into account the isotropic re-emission mass transfer, van Haaften et al. [48] suggested a WD mass limit of $\sim 0.8 M_\odot$. Based on hydrodynamic simulations, given that the angular momentum of the binary is lost by the disk wind, Bobrick et al. [55] reported a significantly lower critical WD mass of $\sim 0.2 M_\odot$.

In addition to the variations in the long-duration wait time peaks, FRB 20201124A exhibits dramatic variations in the Faraday rotation measure on a day timescale, similar to FRB 20121102A (e.g., [68]). These findings suggest the presence of a complex, dynamically evolving, and magnetized environment within the host galaxy [33]. Wang et al. [69] proposed that FRB 20201124A may be produced by a binary system containing a magnetar and a Be star surrounded by a decretion disk. The orbital period is assumed to be 80 days, with orbital eccentricity $e = 0.75$. The interactions between radio bursts, the decretion disk, and various magnetic field components can naturally explain the variable rotation measures. In our model, the violent and unstable accretion process combined with the interactions between the radio bursts, the materials of the common envelope, and the magnetic field could explain the varying rotation measures [70]. After CE ejection, a non-evolving rotation measure can also be expected due to the vanishing of the CE [38].

Moreover, a massive WD-NS merger could produce some gamma ray bursts (GRBs), e.g., GRB 211211A [71,72]. A millisecond magnetar is a likely outcome of such a merger if the magnetic field of the post-merger product is amplified through a mechanism such as an α - Ω dynamo. Then, at least a subset of FRB 180924-like FRBs could be produced by the magnetar within the framework of flaring magnetars [67]. On the other hand, due to their short orbital periods, NS-WD binaries are important sources of gravitational waves. The relationship between the gravitational wave frequency and the orbital period is given by $f = 2/P_{\text{orb}}$. Thus, the gravitational wave frequency of the closest NS-WD binary lies in the range of 10^{-2} – 10^{-1} Hz, which will potentially be observable by the Laser Interferometer Space Antenna (LISA) (10^{-4} – 10^{-1} Hz; [73]) in the future.

Similar to FRBs 20121102A and 20201124A, the wait time distributions of the active repeaters FRBs 20220912A and 20240114A also exhibit bimodal structures [74,75]. Double-peaked profiles of the wait time distributions seem to be a universal feature of active repeating FRBs. Xiao et al. [76] suggested that this asymmetric shape can be explained by the propagation effect in the magnetosphere of magnetars. Luo et al. [77] considered a rotation-modulated starquake model in magnetars to construct the wait time distribution. In future work, we aim to provide a detailed explanation of this bimodal structure based on the NS-WD binary system.

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Abbreviations

The following abbreviations are used in this manuscript:

CE	Common Envelope
CHIME	Canadian Hydrogen Intensity Mapping Experiment
FAST	Five-hundred-meter Aperture Spherical radio Telescope
FRB	Fast Radio Burst
GRB	Gamma Ray Burst
LISA	Laser Interferometer Space Antenna
NS	Neutron Star
WD	White Dwarf
WT	Waiting Time

Notes

- ¹ <https://www.chime-frb.ca/catalog> (accessed on 20 April 2025) [8].
- ² The length of the activity window depends on observing frequency. For example, the activity phase of the bursts detected by LOFAR in the 110–188 MHz is ~25%, whereas the activity window of bursts detected by CHIME/FRB (400–800 MHz) is ~30% of the activity cycle [15].
- ³ This repeater was first discovered by CHIME/FRB [35].
- ⁴ The detection threshold for Arecibo is different from that of FAST, i.e., 0.015 Jy ms for FAST [32] and 0.057 Jy ms for Arecibo [37].
- ⁵ <https://www.chime-frb.ca/repeaters/FRB20201124A> (accessed on 20 April 2025).
- ⁶ The number of bursts from FRB 20201124A detected by FAST is distinct among the four papers in this series, i.e., [34,36,38,39] as distinct criteria have been adopted by the different groups for different scientific purposes.

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Article

Cosmological Simulations with Massive Neutrinos: Efficiency and Accuracy

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Abstract: Constraining neutrino mass through cosmological observations relies on precise simulations to calibrate their effects on large scale structure, while these simulations must overcome computational challenges like dealing with large velocity dispersions and small intrinsic neutrino perturbations. We present an efficient N -body implementation with semi-linear neutrino mass response which gives accurate power spectra and halo statistics. We explore the necessity of correcting the expansion history caused by massive neutrinos and the transition between relativistic and non-relativistic components. The above method of including neutrino masses is built into the memory-, scalability-, and precision-optimized parallel N -body simulation code CUBE 2.0. Through a suite of neutrino simulations, we precisely quantify the neutrino mass effects on the nonlinear matter power spectra and halo statistics.

Keywords: cosmology; neutrinos; dark matter; large scale structure of Universe; methods; numerical

1. Introduction

The Λ CDM model is the current standard model of cosmology. Increasingly large and precise observational results have largely confirmed its accuracy, such as cosmic microwave background (CMB), large scale structure and gravitational lensing. Future observation projects lead us into an era of higher precision in order to detect potential deviations from a pure cold dark matter (CDM) model, such as nonzero neutrino masses, primordial non-Gaussianities, and variations of gravity. Among these, the neutrino mass is proven to be nonzero by particle physics and neutrino mass effects on cosmology are guaranteed to exist. In laboratory experiments, the neutrino mass-squared differences are determined by measuring the atmospheric and solar neutrino oscillations [1], indicating that the total mass of neutrinos is nonzero, with a lower bound of $M_\nu \equiv \Sigma m_{\nu,i} \gtrsim 0.06$ eV, although they cannot determine the absolute neutrino mass or the mass hierarchy. The tritium β -decay experiment, like KATRIN, gives an upper limit of 0.8 eV at a 90% confidence level [2]. On the other hand, cosmological observations provide an upper limit on the total neutrino mass $M_\nu \lesssim 0.09$ eV [3]. The absence of lower bound constraints on neutrino mass in cosmology implies that the absolute mass of neutrinos remains unmeasured. Detecting the cosmological effects of neutrino mass and precisely constraining its value are key objectives of next-generation galaxy surveys.

From the theoretical side, neutrinos affect cosmic evolution through two primary mechanisms. In the early Universe, relativistic neutrinos have high velocity dispersion, behaving as radiation. They modulate the expansion history by changing the matter–radiation equality [4], altering the acoustic and damping angular scales of CMB [5]. In the late Universe, neutrinos cool down and become non-relativistic, while their thermal velocities are still higher than those of CDM; this free-streaming mechanism suppresses their gravitational clustering and thus damps the small-scale matter power spectrum and the high multipoles of the CMB anisotropy, and modulates the halo mass function (HMF) and halo bias [6,7]. Apart from traditional methods, various novel methods have been proposed to constrain the lower bound of neutrino mass, for example, a dipole asymmetry in the cross-power spectrum due to the velocity field between neutrinos and CDM [8], wakes formed as neutrinos passing through dark matter halos [9], misalignments in angular momentum between neutrino and CDM halos due to differing tidal fields [10], and differential neutrino condensation between neutrino-rich and neutrino-poor halos [11]. Neutrino perturbations couple to nonlinear CDM clustering, and thus, the above effects are also nonlinear, so we must use simulations to accurately calibrate these effects. Additionally, a combination of variations of cosmological parameters may also mimic neutrino mass effects, so a large number of simulations spanning a high-dimensional parameter space will also be needed. To achieve this and meet the scientific requirements for next-generation surveys, such as CSST [12], we need a method to simulate the effects of massive neutrinos that balances accuracy and efficiency.

Efficiently simulating neutrinos is challenging due to their high thermal velocities [13,14]. Particle-based approaches partition neutrino phase space into N -body particles; however, they suffer from prohibitive shot noise on small scales due to the neutrino thermal velocities that contaminate small-scale signals. Increasing the number of neutrino particles to suppress this noise is computationally expensive [15]. Alternative methods have been developed to simulate neutrino effects. The grid-based method computes only CDM particles and incorporates neutrino effects by adding an independently evolved linear neutrino density field in Fourier space, which is a fully linear approach [16]. The semi-linear method improves the linear approach by taking into account the nonlinear evolution history of matter density perturbations to estimate neutrino density perturbations [17]. It is more accurate than the fully linear method and is still efficient [7,18]. A recent extension, the SuperEasy method [19], further simplifies the linear response implementation by introducing a closed-form rational function to approximate neutrino clustering, achieving sub-percent accuracy with minimal computational cost. The hybrid method treats neutrinos as linear fields in the early Universe, computes their evolution on grids, and converts some of the slower neutrinos into particles at low redshift during simulation, thereby capturing their nonlinear clustering properties more effectively [7,20]. The fluid method models different neutrino species as fluids rather than particles, solving the Boltzmann equations directly on a grid [13,21,22]. While this method can accurately reproduce nonlinear clustering, its numerical complexity often results in instability and artificial numerical artifacts in simulations. The Newtonian motion gauge approach [23–25] provides a simple method to include massive neutrino effects in N -body simulation. It incorporates the additional potential contribution of neutrinos via modification of the dark matter initial conditions and by employing a dynamically evolving coordinate system [6], avoiding direct neutrino simulations. It can also capture the full impact of linear neutrino perturbations on the nonlinear evolution of CDM particles in N -body simulation.

Given the fact that no neutrino effects have been discovered observationally from large scale structure, we tend to focus on lower-order effects. The semi-linear method captures most of the neutrino-CDM nonlinear coupling, although it neglects out-of-phase

correlations. Despite this, it can accurately model key nonlinear neutrino effects, such as HMF modulation and neutrino torque. We have incorporated a revised version of the semi-linear neutrino method into CUBE 2.0, a parallel, high-precision, and memory-efficient N -body algorithm. Our implementation enables efficient high-resolution large scale structure simulations with varying neutrino masses and hierarchies. In Section 2, we briefly introduce the code and relevant algorithm. In Section 3, we analyze the accuracy of the simulation. Discussions are presented in Section 4.

2. Methods

2.1. CDM Evolution

We use CUBE 2.0 as the basic structure to evolve the overall large scale structure, and the neutrino module is an add-on functional. CUBE 2.0 is an efficient, parallel cosmological N -body simulation code based on the Particle-Mesh-Particle-Particle (P³M) algorithm. The preceding version, CUBE [26], optimized the memory usage by the fixed point structure (FPS) storage, reducing the memory footprint as low as 6 bytes per particle. CUBE 2.0 implements an adaptive innermost grid (PM3) to complement the original two-layer PM (PM1/PM2) framework to substantially alleviate the particle-particle (PP) computation. The resolution of the adaptive grid matches the local density, ensuring that the total computation time of both PP and PM3 is minimized. Furthermore, by employing the optimized Green's function [27] for gravitational calculations, the code achieves maximum relative errors below 5% (with mean errors below 2%) in pairwise gravitational interactions between particles.

The advantages of our cosmological simulations lie primarily in their highly memory-efficient storage of particle information. On average, our method requires only 20 bytes per particle—a significant reduction compared to the widely used Gadget2, which consumes approximately 80 bytes per particle [28]. This represents a several-fold optimization in memory usage for dark matter particle storage [29].

In terms of computational efficiency, CUBE 2.0 has been validated for large scale simulations (over 16,000 cores in 512 nodes) [Yu et al. in prep.], demonstrating the robustness of its three-layer parallel structure. It achieves strong and weak scalability exceeding 90%, while maintaining its advantage in the memory economy.

2.2. Semi-Linear Neutrino Modeling

The neutrino module is built on the above CDM evolution. We consider the following components of the universe—the cosmological constant (Ω_Λ), radiation (Ω_γ), CDM (Ω_c), baryonic matter (Ω_b) and neutrinos (Ω_ν). Since we do not distinguish CDM and baryons in this work, we can safely absorb baryons into CDM, and they are collectively referred to as “cold matter” ($\Omega_{cb} = \Omega_c + \Omega_b$). Neutrinos can be divided into relativistic (Ω_ν^r) and non-relativistic (Ω_ν^{nr}) components, with cosmic time dependency. Therefore, the total matter component is ($\Omega_M = \Omega_{cb} + \Omega_\nu^{nr}$).

To better understand the two components of neutrinos, we describe neutrinos as a fluid with an effective pressure p_ν . The equation of state is given by $p_\nu = \omega_\nu \rho_\nu$, where $\omega_\nu(a)$ is a function that decreases with a . Relativistic neutrinos can thus be described as

$$\Omega_\nu^r(a) = 3w_\nu(a)\Omega_\nu(a). \quad (1)$$

The evolution of neutrinos satisfies $\rho_\nu(a) \propto a^{-3[1+w_\nu(a)]}$, or equivalently,

$$3(1 + w_\nu) = -\frac{d \ln \rho_\nu}{d \ln a} = 4 - y \frac{d \ln \mathcal{F}(y)}{dy}, \quad (2)$$

where $y = aM_\nu / (\Gamma_\nu N_\nu k_B T_{\gamma,0})$, and $T_{\nu,0}, \Gamma_\nu \equiv T_{\nu,0}/T_{\gamma,0}$ is the neutrino temperature and the neutrino-to-photon temperature ratio today. N_ν is the number of massive neutrinos, k_B is the Boltzmann constant, and

$$\mathcal{F}(y) \equiv \int_0^\infty \frac{x^2 \sqrt{x^2 + y^2}}{1 + e^x} dx. \quad (3)$$

We also define the fraction of non-relativistic neutrinos

$$f^{\text{nr}} \equiv \frac{\Omega_\nu^{\text{nr}}}{\Omega_\nu^{\text{r}}} = (1 - 3\omega_\nu) = \frac{y}{\mathcal{F}(y)} \frac{d\mathcal{F}(y)}{dy}. \quad (4)$$

The semi-linear response method models neutrino effects by correcting the CDM density (no neutrinos) field into the total matter density field $\delta_M(\mathbf{k}, \tau) = F_\nu(k) \delta_{\text{cb}}(\mathbf{k}, \tau)$, where the correction function $F(k)$ is defined as

$$F_\nu(k) \equiv \sqrt{P_M/P_{\text{cb}}}. \quad (5)$$

This enables CDM particles to evolve under the gravitational potential of the total matter field. Incorporating the non-relativistic fraction f^{nr} from Equation (4), the correction function resolves as

$$F_\nu(k, a) = \frac{(1 - f_\nu) P_{\text{cb}}^{1/2}(k, a) + f_\nu f^{\text{nr}}(a) P_\nu^{1/2}(k, a)}{P_{\text{cb}}^{1/2}(k, a)}. \quad (6)$$

Here $P_{\text{cb}}(k)$ is the CDM power spectrum calculated in simulation (see Section 2.3). $P_\nu(k)$ is the neutrino power spectrum using equation form Equation (63) of [17] derived from the integral of the total matter power spectrum evolution.

To test the accuracy of the finite integral, we estimate the neutrino power spectrum using the semi-linear method, derived from the theoretical nonlinear matter power spectra generated by CAMB, and test the interpolation strategy (labeled as “interp.”). In Figure 1, we present the residuals of the square root of the power spectrum (rather than the power spectrum itself, as they are more closely related to errors in the correction function), which agree with the predictions of the nonlinear theory on scales $k < 1$, while significant deviations emerge on small scales. However, these small-scale discrepancies are negligible, as demonstrated by the redshift $z = 0$ correction function; for $k > 1$, the correction function drops to $1 - f_\nu$, indicating that neutrinos behave nearly homogeneously on this scale. This ensures that, despite slight deviations in the power spectra estimates, the overall correction function aligns perfectly with theoretical predictions, with the relative error controlled below 10^{-4} . Since the wavenumber at the particle-particle (PP) scale is evidently much greater than 1, we assume that neutrinos are completely homogeneous at this scale. Accordingly, the effective mass contributing to the gravitational interaction between particles should be scaled by a factor of $(1 - f_\nu)$, leading to a correction factor of $(1 - f_\nu)^2$ for the PP interaction force.

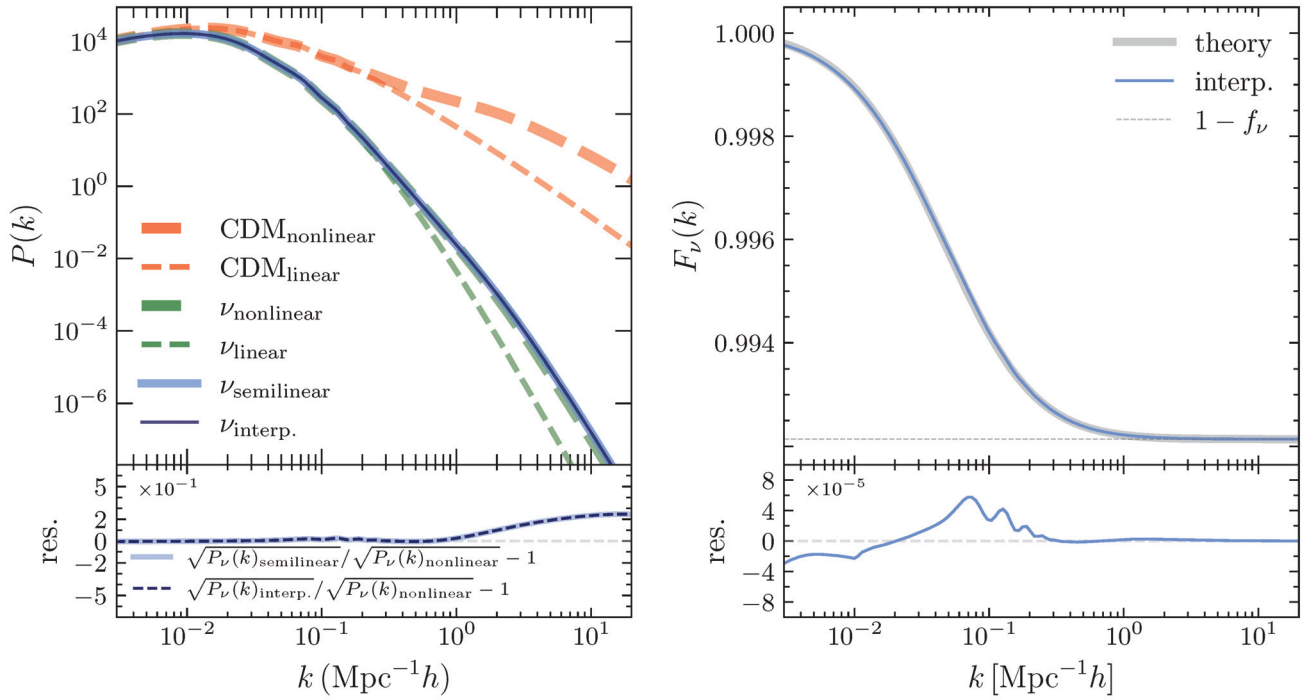


Figure 1. (Left panel): Theoretically predicted CDM (orange) and neutrino (green) power spectra are represented by dashed lines, while the power spectrum predicted by the semi-linear method is shown with solid lines. The thin solid line represents the predicted power spectrum from interpolation. The lower panel displays the residuals between the two predictions. **(Right panel):** Massive neutrino correction function from theoretical predictions (gray) and semi-linear simulations (blue) at $z = 0$, and residual plots showing difference in lower panel.

The expansion history of the universe, which couples with the effects of neutrinos, influences the clustering of matter, making it necessary to use an accurate expansion history. CUBE solves the expansion history during simulation steps (on-the-fly computation) and introduces subtle discrepancies in the expansion history under varying simulation resolutions and precisions. Although these variations diminish with reduced time-step size and remain acceptable in pure CDM simulations, they become problematic for massive neutrino simulations that require higher accuracy and stability. To address this, we implement a precomputed expansion history obtained by solving the Friedmann equation:

$$H(a) = H_0 \left[\Omega_\Lambda + \Omega_{\text{cb}} a^{-3} + \Omega_\gamma a^{-4} + \frac{15}{\pi^4} \Gamma_\nu^4 N_\nu \Omega_{\gamma,0} \mathcal{F}(y) a^{-4} \right]^{1/2}, \quad (7)$$

where the final term accounts for the neutrino component [30].

For convenience, we set the present superconformal time $s(a) \equiv \int_0^{t(a)} a^{-2} dt$ to zero, and we integrate backward in time to obtain the expansion history at higher redshifts:

$$a(s) = \int_0^s a^3 H(a) (-ds). \quad (8)$$

We show the differences in the expansion history and Hubble expansion rate between the on-the-fly algorithm and the new algorithm in Figure 2. The error between the two algorithms reaches a maximum of around 10%, which can lead to significant differences in large scale structure evolution. On the other hand, the variations in the expansion history for different neutrino masses are on the order of $\lesssim 1\%$, meaning that the integration error must be controlled below 10^{-4} . Therefore, choosing an integration step size of 5×10^{-4} is sufficient to meet this requirement.

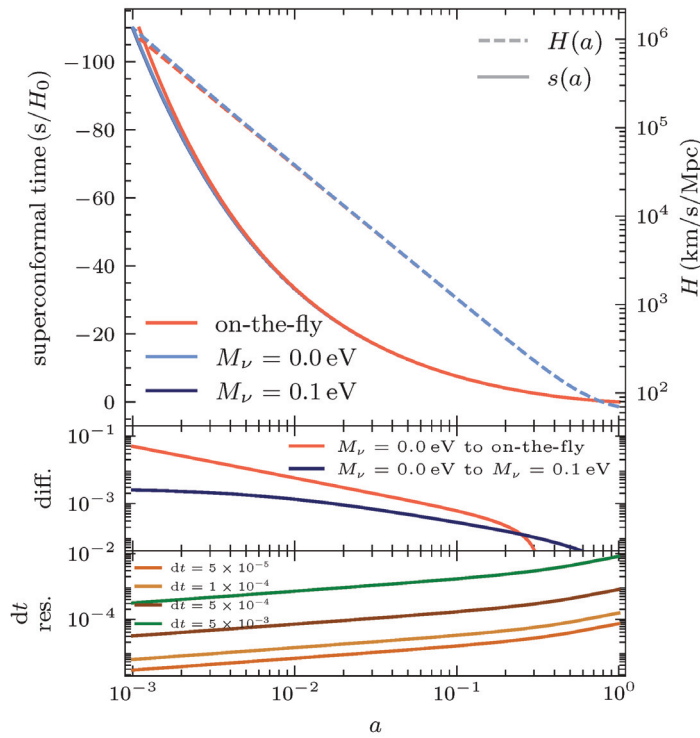


Figure 2. Expansion history (solid line) and Hubble expansion rate (dashed line), as functions of scale factor a , computed using the on-the-fly method (red). Different neutrino masses (blue) cause a 1% variation in the expansion history. The bottom panel shows the impact of the integration time-step size on accuracy.

2.3. Power Spectrum Estimation

The semi-linear method requires integrating the matter power spectrum to obtain the neutrino power spectrum. To account for effects such as cosmic variance, we need to compute the matter power spectrum from simulations rather than using a theoretical one. However, calculating the global power spectrum at the same resolution as the simulation is time- and memory-consuming, and can be comparable to executing a simulation step at low redshift. Therefore, computing the global power spectrum at each step is not practical. Reducing the resolution of the global power spectrum would improve computational efficiency, but would also result in the loss of much nonlinear information. To address this, we calculate the global power spectrum at specific redshifts z_p and use interpolation and extrapolation to estimate the power spectrum at other redshifts. Our analysis compares various z_p selection schemes and interpolation algorithms, revealing that quadratic interpolation maintains the relative error of the matter power spectra within 0.2% while achieving neutrino power spectrum accuracy at the 10^{-4} level relative to non-interpolated methods. This approach yields correction functions with residual errors below 10^{-5} , satisfying the precision requirements for most cosmological studies.

To control the memory consumption in power spectrum calculations during simulations, we develop a power spectrum combining approach by extending Welch's method [31] to three dimensions and integrating it with buffer designs from Particle-Mesh (PM) algorithms. This involved the creation of novel 3D window functions that maintain uniform weighting across all data points with minimized variance. Our approach accelerates the process by a factor of 10 compared to full-resolution global calculations while maintaining equivalent accuracy (Figure 3), crucially avoiding additional memory overhead since density field computations are intrinsic to PM algorithms. Our combining power spectrum implementation proceeds as follows.

1. Compute the low-resolution global power spectrum $P_{\text{PM1}}(k)$ at the PM1 grid points.
2. Calculate tiered spectra $P_{\text{PMx}}(k)$ at standard and high resolutions (PM2/PM3 grids) through all subvolumes:
 - (a) Apply the window function to the density field within the subvolume.
 - (b) Calculate $P_{\text{raw},i}(k)$, then perform a noise correction [32] to obtain $P_{\text{denoised},i}(k)$.
 - (c) Average all $P_{\text{denoised},i}(k)$, then apply the alias correction [32] to obtain $P_{\text{PMx}}(k)$.
 - (d) Automatically apply a cut-off on the power spectrum to account for grid effects.
3. Combine the three levels of $P_{\text{PMx}}(k)$ based on the sampling count and obtain $P_{\text{combine}}(k)$.

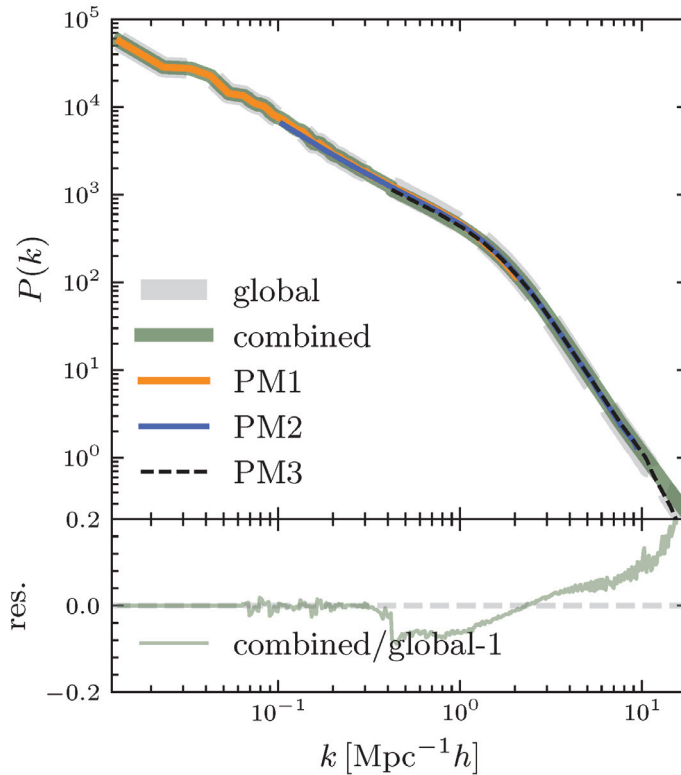


Figure 3. Multi-scale power spectrum combination (thick green curve), compared with the power calculated from the full-resolution algorithm (thick gray curve). Thinner curves show the power spectra calculated by different PM grids. The bottom panel shows the residual.

Due to the fact that semi-linear corrections require the application of correction functions to the grid, the k -modes of the power spectrum cannot perfectly match all k -modes of the density field, especially when smoothing the k -modes via binning. We choose to retain all actual k -modes below $k_{\text{grid}} < 6$ to prevent interpolation errors in sparsely sampled low-frequency regions, implementing interpolation only at higher frequencies where spectral sampling density permits reliable estimation.

3. Simulations

The above neutrino module is built into the parallel N -body code CUBE 2.0. By using it, we run a series of simulations with different resolutions, box sizes, and neutrino masses (see Table 1 for specific parameters). In tuning neutrino masses, we set the density parameters of total matter and baryons to $\Omega_{\text{M}} = 0.279$ and $\Omega_{\text{b}} = 0.0436$, respectively. In the simulation with massless neutrinos, $\Omega_{\text{M}} = \Omega_{\text{b}} + \Omega_{\text{c}}$. In the simulation with massive neutrinos, $\Omega_{\text{M}} = \Omega_{\text{b}} + \Omega_{\text{c}} + \Omega_{\nu}$, where a portion of the dark matter is replaced by neutrinos. The

dimensionless Hubble constant h is set to 0.7. This section presents the data results from these simulations, starting with an analysis of the matter clustering and then examining the halo statistics.

Table 1. Neutrino simulation configurations.

Simulation	Box [$h^{-1}\text{Mpc}$]	N_{CDM}	Σm_ν [eV]
600,512	600	512^3	0.05
			0.10
			0.15
300,512	300	512^3	0.10
1200,512	1200	512^3	0.10
600,1024	300	1024^3	0.10

3.1. Matter Clustering

In Figure 4, we visualize the results of simulations by CDM and neutrino overdensities. The top left panel shows the CDM overdensity from a slice of $1.2 h^{-1}\text{Mpc}$ thickness, taken from the massless neutrino simulation, showing the obvious cosmic web structure. Although there would be trace differences if it were taken from non-zero-neutrino-mass simulations, they are indistinguishable by eye, so we plot only one of them. The rest three panels show neutrino overdensities from the simulation with sum of neutrino masses set to 0.05 eV, 0.10 eV, and 0.15 eV, taken from the same slice from the simulation volume. It can be seen that a neutrino follows the structure of CDM on large scales, while their small scale structure is suppressed by their velocity dispersions. This effect is more prominent for lighter neutrino masses. Similar to CDM, total matter overdensity remains visually indistinguishable across simulations as well; however, quantitative analysis can reveal up to 10% variations in clustering strength, confirming that neutrinos modulate both their own, but also total clustering dynamics.

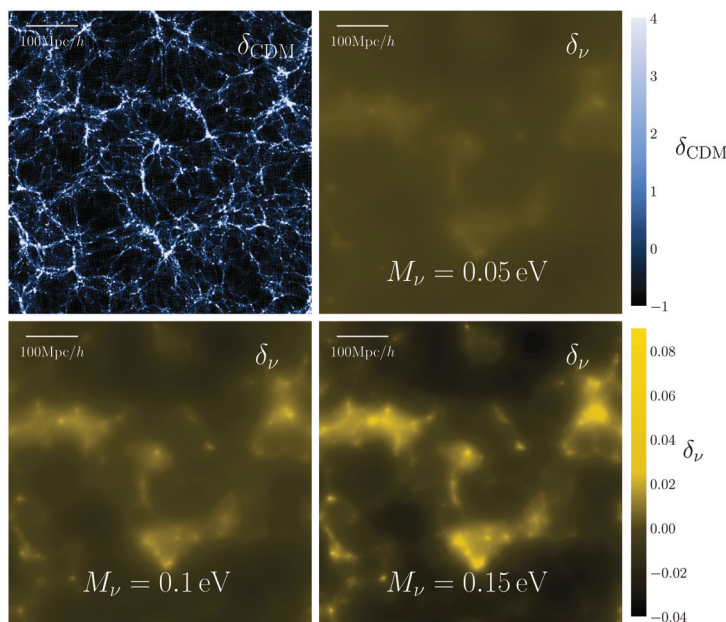


Figure 4. CDM (top left panel) and neutrino overdensity projections with a thickness of $1.2 h^{-1}\text{Mpc}$. The CDM overdensity is obtained from the simulation whose neutrino mass set to zero, while it is indistinguishable from the ones with neutrino mass. The rest of the panels show neutrino overdensities from respective simulations.

The power spectrum effectively quantifies the impact of neutrinos on the clustering of total matter. In Figure 5, we show the total matter power spectrum at redshift $z = 0$ for all simulations and the power spectrum suppression for different neutrino masses. The results from simulations show that the suppression amplitudes are consistent with Halofit predictions [33], robust across resolutions and box sizes. Although from Figure 4, the overdensities of more massive neutrinos are stronger, their greater fractional contribution to the total matter leads to a deeper total matter power suppression.

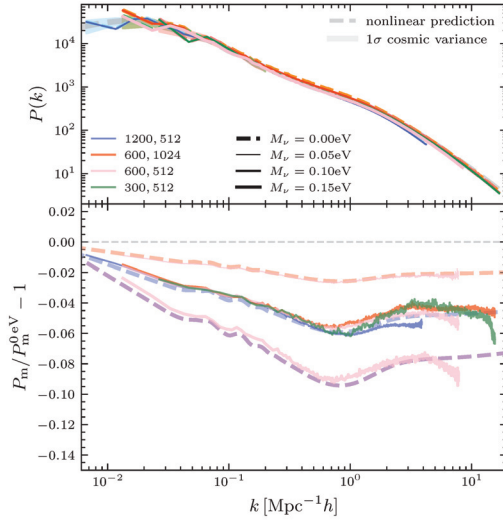


Figure 5. (Top Panel): Simulated power spectra at redshift zero, color-coded by simulation configuration and line thickness corresponding to neutrino mass. Dashed lines denote nonlinear theoretical predictions, while shaded regions represent cosmological variance. **(Bottom Panel):** Residuals from neutrino-induced suppression of the power spectrum.

3.2. Halo Statistics

The neutrino mass effects on halo statistics are shown in Figure 6. It compares two cosmological scenarios: one with massless neutrinos and the other with massive neutrinos of total mass $M_\nu = 0.1\text{eV}$; furthermore, the evolution of the HMF from $z = 2$ to $z = 0$ is shown.

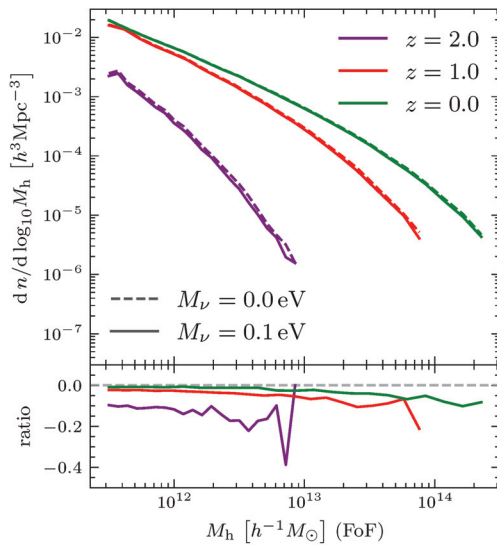


Figure 6. HMF form neutrinos mass $M_\nu = 0.1\text{eV}$ (solid line) and $M_\nu = 0.0\text{eV}$ (dashed line) in $z = 2$ (purple), 1 (red) and 0 (green). The ratios between different neutrino mass HMF ($M_\nu = 0.1$)/HMF($M_\nu = 0.0$) are shown in the bottom panel.

The results reveal a suppression of the HMF in the presence of massive neutrinos compared to the massless neutrino case. This suppression is more pronounced for higher halo masses, as massive halos typically originate from high-density peaks in the initial matter distribution. The presence of massive neutrinos reduces the amplitude of these high-density peaks, thereby diminishing the formation of massive halos. Furthermore, the reduction is more substantial at higher redshifts, reflecting the cumulative impact of neutrinos on structure formation over cosmic time.

The suppression of the HMF due to massive neutrinos has important implications for observational probes such as galaxy clustering and weak gravitational lensing, as these observables are sensitive to the underlying halo population. Current and upcoming large scale structure surveys with higher precision, such as DESI and CSST, may provide constraints on neutrino masses by measuring the HMF at different redshifts.

To ensure the consistency with observational studies that often select galaxy samples based on number density, we construct four halo samples with number densities of $3 \times 10^{-3} h^{-3} \text{Mpc}^3$, $1 \times 10^{-3} h^{-3} \text{Mpc}^3$, $6 \times 10^{-4} h^{-3} \text{Mpc}^3$, and $3 \times 10^{-4} h^{-3} \text{Mpc}^3$ at redshift $z = 0$. For each sample, we calculated the halo two-point correlation function, as shown in Figure 7.

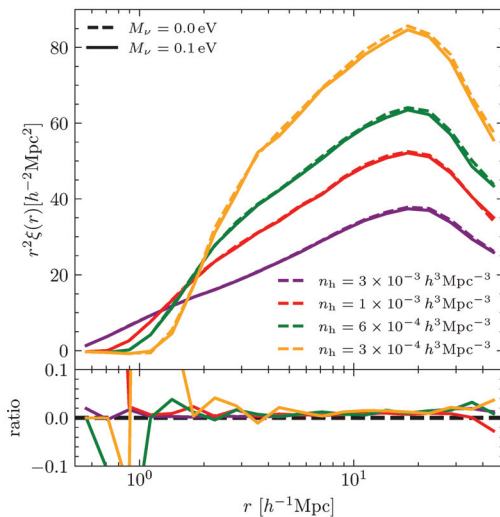


Figure 7. Two-point correlation function of halos in different number density from the simulation with the sum of neutrino mass $M_\nu = 0.1 \text{ eV}$ (solid line) and $M_\nu = 0.0 \text{ eV}$ (dashed line). The bottom panel shows the ratio between different neutrino masses.

Interestingly, we find that the two-point correlation function in the massive neutrino cosmology is slightly higher (by a few percent) than the massless neutrino cosmology. This result appears counterintuitive, as neutrinos, due to their free-streaming behavior, are expected to suppress matter clustering and delay structure growth, as evidenced by the reduction in the matter power spectrum discussed in Section 3.1. The observed increase in the halo two-point correlation function can be attributed to the enhanced halo bias in the presence of massive neutrinos. Specifically, massive neutrinos increase the clustering strength of halos relative to the underlying matter distribution, which compensates for the suppression of the matter power spectrum and leads to a net increase in the halo two-point correlation function [6].

4. Conclusions

Building on the semi-linear method, we extend the algorithm to incorporate the influence of massive neutrinos on the expansion history of the universe and the evolu-

tion of their non-relativistic components, implementing these effects within a hierarchical multi-level PM grid structure, CUBE 2.0. We further assume that neutrinos are completely homogeneous on small scales and introduced corresponding corrections to the PP gravitational calculations.

To enhance the efficiency of matter power spectrum estimation, we develop a segmented power spectrum estimation algorithm. By taking advantage of the intrinsic grid hierarchy of the multi-level PM scheme, this method introduces negligible additional memory or computational costs. Correction functions at each simulation step are obtained through interpolation/extrapolation of the power spectrum.

By a series of cosmological simulations featuring a range of neutrino masses, resolutions, and box sizes, the incorporation of massive neutrinos incurs minimal additional computational costs. The resulting matter power spectra and suppression features not only affirm the accuracy of our simulations but also highlight the robustness of the numerical framework.

The evolution of the HMF with redshift shows that neutrinos suppress halo formation, and this suppression gradually weakens at lower redshifts. This suggests that statistical analysis at different redshifts may offer a way to constrain the neutrino mass. Additionally, the two-point correlation function of halos indicates enhanced halo bias in massive neutrino cosmologies.

A key advantage of our cosmological simulation is its highly efficient memory usage for particle storage. While conventional codes like Gadget2 require approximately 80 bytes per particle [28], our implementation needs only 20 bytes per particle. Additionally, our neutrino simulation employs a power spectrum combining technique, eliminating the need to compute the global power spectrum and ensuring that neutrino-related calculations introduce negligible memory overhead. Thanks to these memory optimizations, our framework enables subsequent high-resolution simulations with a large number of particles.

Leveraging the computational efficiency of our approach, future work will aim to carry out large-volume simulations to explore neutrino-induced parameter degeneracies and their interplay with other cosmological observables.

Author Contributions: Neutrino simulation code, power spectrum estimation, simulation testing and validation, power spectrum analysis B.-H.C.; prerequisite verification, halo statistical analysis J.-J.Z.; theoretical and methodological guidance H.-R.Y., Y.L., J.-H.H., Y.J.; CUBE 2.0 code development H.-R.Y. All authors have read and agreed to the published version of the manuscript.

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