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Editorial Board Members' Collection Series

Modeling and Simulation of Composite Materials

Edited by
Marcin Kamiński, Haifeng Zhao, Konstantinos Tserpes and Salim Belouettar

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**Editorial Board Members' Collection
Series: Modeling and Simulation of
Composite Materials**

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Guest Editors

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About the Editors

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Marcin Kamiński has been a full professor at the Department of Structural Mechanics, Łódź University of Technology since 2015 and is currently the Head of the Discipline Civil Engineering, Geodesy and Transportation at the Faculty of Civil Engineering, Architecture and Environmental Engineering (since 2019). His main research includes the Stochastic Finite Element Method (and some other discrete stochastic numerical methods), numerical modeling of random composites including homogenization theory, reliability assessment and optimization of civil engineering structures, stochastic aging processes, and also—recently—an application of probabilistic entropy in engineering computations. He has authored more than 200 research papers in various journals and published two monographs. He was the recipient of fellowships from the Foundation for Polish Science (1996 and 1999), the J. Argyris Award from ECCOMAS and Elsevier (2001), the J.T. Oden Scholarship at The University of Texas (Austin, TX, USA, 2004), and also certain Polish national/academic awards including the Bronze Cross of Merit (1998), Silver Cross for Longstanding Service (2017), Medal of Commission of Education (2022), and also very recently The Research Prize of the Ministry of Education and Science (2023). He has been recognized recently (since 2021) as being in the World's Top 2% Scientists by Stanford University, CA, USA. Prof. M. Kamiński mainly teaches courses relevant to Structural Reliability, Optimization and Parametric Design and the Finite Element Method, but also various courses related to Steel Structures, Strength of Materials and Theoretical Mechanics. He co-authored (with prof. B. Rogowski) the academic textbook *Technical Mechanics*. During his career, he has taught many academic courses in Civil Engineering, Environmental Engineering, Architecture and Architecture Engineering. He has served as the promoter for 10 Ph.D. theses as well as for more than 40 M.Sc. and B.Sc. students.

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Preface

Composite materials are now central to modern engineering, with widespread use in aerospace, automotive, civil infrastructure, energy, and biomedical applications. Their superior mechanical performance, design flexibility, and multifunctional potential make them indispensable in high-performance systems. Yet their heterogeneous microstructures, complex constituent interactions, and sensitivity to manufacturing-induced defects present substantial challenges for accurate performance prediction and structural optimization. Experimental methods alone cannot efficiently navigate this vast design space, making computational modeling and simulation essential components of advanced composite development.

This Reprint highlights recent progress in modeling and simulation techniques for a broad range of composite materials. Topics include multiscale constitutive modeling, finite element analysis of damage and failure, machine learning-based property prediction, fatigue and fracture assessment, nondestructive evaluation, and structural optimization. By bringing together analytical, numerical, and data-driven approaches, the volume provides a comprehensive perspective on contemporary strategies for understanding composite behavior across multiple scales and loading conditions. We hope these contributions will foster interdisciplinary collaboration and stimulate further advances toward more reliable, efficient, and sustainable composite systems.

Marcin Kamiński, Haifeng Zhao, Konstantinos Tserpes, and Salim Belouettar

Guest Editors



Editorial

Editorial for the Special Issue on Editorial Board Members' Collection Series: Modeling and Simulation of Composite Materials

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Advanced composite materials, consisting of stiff reinforcements embedded in a compliant matrix, exhibit superior mechanical and functional performance compared with monolithic materials [1,2]. These advantages have led to their widespread adoption in aerospace, automotive, construction, and energy applications [3,4]. However, the same microstructural degrees of freedom that enable property optimization also give rise to a high-dimensional and highly nonlinear design space. Key parameters—including fiber volume fraction, orientation, and multiscale defects—strongly couple across length scales to govern stiffness, thermal response, durability, and failure behavior [5,6]. A systematic experimental characterization of these interactions is costly, time-consuming, and often destructive [7]. Consequently, predictive modeling and high-fidelity simulation have become central to composite mechanics. Recent advances in multiscale constitutive modeling, finite element analysis, and data-driven methods enable the efficient prediction of elastic properties, damage evolution, and residual stresses within hours rather than months [8].

This Special Issue compiles recent research advancements concerning the effective properties and numerical modeling of diverse composite materials. The findings presented herein hold substantial importance for enhancing our understanding of current composite materials, facilitating the optimization of existing formulations and guiding the development of novel multi-component and/or multiphase materials.

Patro et al. proposed a bilinear traction separation law for fiber-reinforced polymer adhesive joints under variable-amplitude loading, which is driven by the fatigue crack growth rate [9]. Their model incorporates a load interaction parameter within a Paris-type cohesive framework. This approach accurately captured damage accumulation trends and significantly improved life prediction accuracy.

Cakiroglu et al. developed highly accurate machine learning models to predict the tensile strength of eco-friendly concrete containing natural fibers and recycled aggregates [10]. This study successfully applies SHAP (Shapley Additive Explanations) analysis to interpret model predictions, identifying key factors such as specimen age and fiber type. Its primary innovation is an accessible online tool that delivers instant strength predictions, addressing a gap in design codes for sustainable composites.

Zitouni et al. reveal that wall survivability under firebrand assault hinges on insulation phase-change and coating emissivity rather than bulk mass [11]. Their transient finite

element model considers primary design parameters for thermal reactions of various wall specimens during firebrand accumulation. The study redirects future codification from prescriptive thickness toward material-specific thermophysical thresholds, offering a rational path to performance-based envelope design.

Tsivouraki et al. present a self-contained numerical twin that couples cohesive-zone fatigue delamination with random vibration response to decode damage diagnosability through virtual testing [12]. Their framework exposes frequency down-shift as a deterministic signature of inter-ply fracture, converting stochastic fatigue scatter into a predictable spectral gradient and enabling the virtual optimization of vibration-based monitoring strategies for thermoplastic composites.

Tomasi et al. welded filament-wound carbon-fiber architecture to an elliptical cross-section, iteratively tuning ply orientation and curvature until the protector met buckling, modal, and regulatory demands at minimum mass [13]. The study shows that composite failure criteria and process constraints can steer the earliest load path definition for an industrial vehicle part. Thus, they achieved the goals of both weight and fuel consumption reduction.

Tarawneh et al. benchmarked five shear design models against a curated set of ultra-high-performance concrete (UHPC) girders, revealing that only the iterative tension localization approach preserves conservatism across reinforcement ratios while alternatives drift toward unsafe predictions as transverse steel increases, thereby establishing a rational basis for updating safety factors in forthcoming UHPC bridge specifications [14].

Jia et al. embedded a ceramic–metal gradient beam within a three-parameter viscoelastic foundation and solved its damped eigenproblem via an n -th-order generalized beam theory coupled to a modified differential quadrature algorithm, demonstrating that foundation damping decouples frequency from decay and enabling frequency-tuned designs without altering amplitude attenuation [15].

Rayhan et al. developed a homogenization scheme with a calibrated deep feed-forward network that mapped constituent stiffness and volume fraction directly to the full elasticity tensor of unidirectional composites, proving that data-driven regression can deliver virtual test results indistinguishable from representative volume element computations and offering a design tool orders of magnitude faster than finite element upscaling [16].

Mofidi et al. recasted the pull-out of post-installed fiber-reinforced polymer anchors as a fracture mechanics eigenvalue problem, deriving a closed-form solution that coupled bilinear bond slip to equilibrium along an evolving debond front [17]. The resulting equation forecasted capacity within a few percentage points of the test data, turning a formerly empirical connection into a design-ready mechanics expression that could be embedded directly in future structural codes.

Sultana et al. interleaved finite element simulations with response surface statistics to reveal that yarn spacing and fabric thickness dominate the stiffness landscape of jute woven composites, whereas shear angle governs only twill architectures, thereby offering a rational route to tailor natural-fiber reinforcements without exhaustive prototyping [18].

Escalante-Tovar et al. developed machine learning models to predict the flexural strength of fiber-reinforced UHPC [19]. They applied SHAP analysis to interpret predictions and identify key influencing factors, as well as build a hybrid neural network that integrated top-performing models, achieving high accuracy validated through independent experiments.

Queirós et al. stacked band-pass filtered shearographic phase maps to suppress speckle noise and magnify nanometer-level strain gradients, revealing barely visible impact, disbond, and hole damage with crisper contours than thermography [20]. By converting

subtle stiffness loss into high-contrast anomaly images, the method enables engineers to size hidden flaws and lighten future composite structures.

Fikry et al. interleaved a polyamide mesh at ply-drop resin pockets, transforming the discontinuity into a crack-deflecting interface that markedly raises interlaminar toughness under shear [21]. Localized inserts redistribute strain and delay delamination, while full embedding sustains load without sacrificing tensile strength, offering a lightweight, manufacturing-friendly route to damage-tolerant composites.

Rahman et al. traced the full-range response of pultruded glass fiber joints with a progressive damage model, revealing how fracture energies steer failure from brittle shear-out to ductile bearing [22]. Their parametric map offers designers a rapid route to optimize end distance and bolt layouts for damage-tolerant composite connections without exhaustive testing.

Chrysochoidis et al. embedded nonlinear contact and frictional slip inside a layer-wise beam model, allowing the delamination faces to open, close, and slide [23]. Guided-wave simulations matched to piezoelectric measurements showed that the resulting time-of-flight stretch and amplitude drop were sensitive to crack length even at minor damage levels. By converting interface kinematics into measurable transient signatures, the study offers a physics-based route to size barely visible delamination in real time.

Yoshida et al. translated ply-curving termination from external to internal drop-offs, revealing that cover plies redistribute through-thickness tension and suppress the curved-fiber cracks which previously limited fatigue benefit [24]. X-ray tomography shows delamination is confined within the curved zone even at high load, extending life tenfold. By converting a stress concentrator into a gradual load transfer region, the study enables lighter and more durable composite tapers for primary aircraft structures.

Felipe-Sesé et al. introduced a novel methodology combining digital image correlation (DIC) with frequency analysis to accurately measure low Poisson's ratios in composites [25]. The key contribution is the successful application of lock-in filtering to DIC data, which provides stable strain distributions and significantly reduces measurement uncertainty. By incorporating frequency-based signal processing into tensile testing, this approach enables a precise characterization of subtle periodic mechanical behaviors at low strain ranges.

Li et al. developed an optimization strategy for CMC (ceramic matrix composite) combustion liners in aero-engines, focusing on maximizing natural frequencies to suppress vibration [26]. Using regression analysis, they identified wall thickness, stiffener thickness, and lamination scheme as key factors. The study demonstrated that a 45° lamination scheme significantly enhanced performance. This study provides a systematic method for optimizing CMC structures, offering practical guidance for vibration-resistant design and maximizing service life in high-temperature applications.

Ribeiro et al. presented a comprehensive finite element scheme to assess the fire behavior of a novel mineral wool composite slab [27]. This unified analytical approach concurrently predicts post-fire load-bearing capacity and associated carbon emissions. Their work harmonizes safety with environmental stewardship and highlights the potential for synergistic optimization in structural fire protection.

Li et al. developed cross-property relations for continuous fiber-reinforced ceramic matrix composites using micromechanics models and finite element analysis [28]. The study reveals that composites with a fixed fiber–interphase volume ratio exhibit nearly linear cross-property relations, while interphase property variations cause these relations to shift downward. This methodology establishes a multiphysics framework that enables the inference of mechanical properties from thermal data or vice versa to address measurement challenges in extreme thermo-mechanical environments.

Despite growing interest in the modeling and analysis of composite materials and the introduction of novel approaches through emerging technologies such as machine learning, significant opportunities remain for further advancement in the modeling and performance evaluation of composite materials. The studies compiled in this Special Issue are expected to facilitate progress in composite material technology, thereby expanding the scope of potential applications.

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Article

A Novel Bilinear Traction-Separation Law for Fatigue Damage Accumulation of Adhesive Joints in Fiber-Reinforced Composite Material Under Step/Variable-Amplitude Loading

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Abstract: Adhesive joints in real-world conditions often experience variable or step loading rather than constant-amplitude fatigue. This study addresses this gap by examining the influence of load sequence and block loading on fatigue damage in adhesive joints of fiber-reinforced polymer (FRP) composites. A novel bilinear traction-separation law based on the Fatigue Crack Growth Rate (FCGR) rule is introduced to predict fatigue failure under step/variable loads, accounting for load history, sequence, and interaction effects. This model was validated using a double-lap joint model under step/variable loading across four experimental scenarios. The proposed model outperformed existing fatigue damage-accumulation models, significantly reducing the Relative Error of Prediction (*REP*). Notably, the proposed model significantly reduced the Relative Error of Prediction (*REP*), achieving reductions from 81.10% to as low as 0.013% in certain cases. The proposed bilinear law exhibited an accelerated damage accumulation rate per cycle for low-to-high loading situations and a decelerated rate for high-to-low loading scenarios, aligning more closely with experimental observations. The proposed model offers practical benefits by improving fatigue life predictions, enabling optimized FRP composite designs, and minimizing overengineering. These advancements are particularly relevant in industries such as aerospace, automotive, and wind energy, where structural durability and safety are paramount. This research represents a significant step forward in the fatigue analysis of composite adhesive joints, paving the way for more reliable engineering solutions.

Keywords: fatigue failure; variable fatigue loading; bilinear traction separation law; fatigue crack growth rate (FCGR); adhesive joints; adhesive material; composite material

1. Introduction

Evaluating the fatigue behavior of adhesives is crucial to ensure the durability and dependability of adhesive joints across various applications. In real-world scenarios, these joints experience complex loading patterns that vary over time, including changes in the load magnitude, frequency, and duration, collectively referred to as variable or step-loading patterns. Such loading conditions represent actual service conditions more accurately than the constant-amplitude loading typically used in laboratory fatigue tests [1–5].

Although constant-amplitude fatigue testing is simpler and more economical, it fails to fully capture the intricacies of real-world loading conditions. This approach has limitations, including the inability to accurately forecast the lifespan and performance of adhesive connections under actual service conditions. This discrepancy occurs because constant-

amplitude tests do not consider the effects of load interactions and sequences, which can significantly affect the fatigue life of the materials [6–9].

Understanding the degradation of materials subjected to cyclic loading depends on the concept of cumulative fatigue damage. Despite its critical importance, this area remains a significant challenge in material science. One of the earliest mathematical approaches to address cumulative fatigue damage was introduced in 1945, using the Palmgren-Miner rule. This model, alternatively referred to as the linear damage rule, can be represented by Equation (1) [10], as follows:

$$D = \sum \left(\frac{n_i}{N_i} \right) \quad (1)$$

where the total damage is denoted by D , while n_i represents the number of fatigue cycles at the i th constant-amplitude loading stage, and N_i signifies the number of fatigue cycles until failure occurs at the i th constant-amplitude loading stage. Xia and Yao assert that a comprehensive model for cumulative fatigue damage must address three aspects: (1) evaluating the magnitude of fatigue damage at a specific stress level, (2) summing up fatigue damage across diverse stress levels, and (3) identifying the critical damage threshold that triggers failure when the material or structure reaches its maximum capacity [11].

Extensive research has been conducted on the fatigue failure of adhesive bonds under variable or block loading conditions [12–18]. The results of these experiments showed considerable differences from the expected outcomes based on Miner's rule for damage accumulation. Multiple factors can contribute to these discrepancies [19]:

1. Failure to consider the history of applied loads.
2. The loading sequence is not considered.
3. Failure to account for interactions between different loads.

For loading sequences that progress from low to high stress ($\sigma_1 < \sigma_2$), Miner's rule generally produces conservative estimates of the fatigue life. However, when applied to sequences transitioning from high to low stress ($\sigma_1 > \sigma_2$), the results tend to be nonconservative. To overcome this shortcoming, various nonlinear damage-accumulation theories have been proposed. Nevertheless, these alternative methods fail to account for all the previously mentioned factors. A typical representation of the High-to-Low and Low-to-High loading cycles and their theoretical damage accumulation is illustrated in Figure 1.

Several factors may explain the observed behavior of the material.

1. Microscopically, intense stress can spark the initiation of small cracks. These microscopic fissures subsequently enlarge during intervals of reduced stress, resulting in more rapid deterioration compared with conditions with consistent stress levels.
2. Applying an increasing number of stress cycles to a material specimen can cause surface degradation during recurring plastic deformation. This surface deterioration creates new potential crack-initiation points that may form under fewer stress cycles.
3. The process of damage accumulation should not be conceptualized as a straightforward, linear progression.

To address this issue, researchers have developed various theories regarding nonlinear damage accumulation. These models, which aim to capture the nonlinear aspects of damage accumulation under step/variable loading conditions, can be classified into seven categories:

1. linear damage rules [10]
2. nonlinear damage curve and two-stage linearization models [16,19–22]
3. life curve modification model [23–28]
4. models based on crack growth concepts [29–36]
5. continuum damage mechanics-based models [37–39,39–42]
6. material degradation models [14,43–46]

7. energy-based models [47–52]

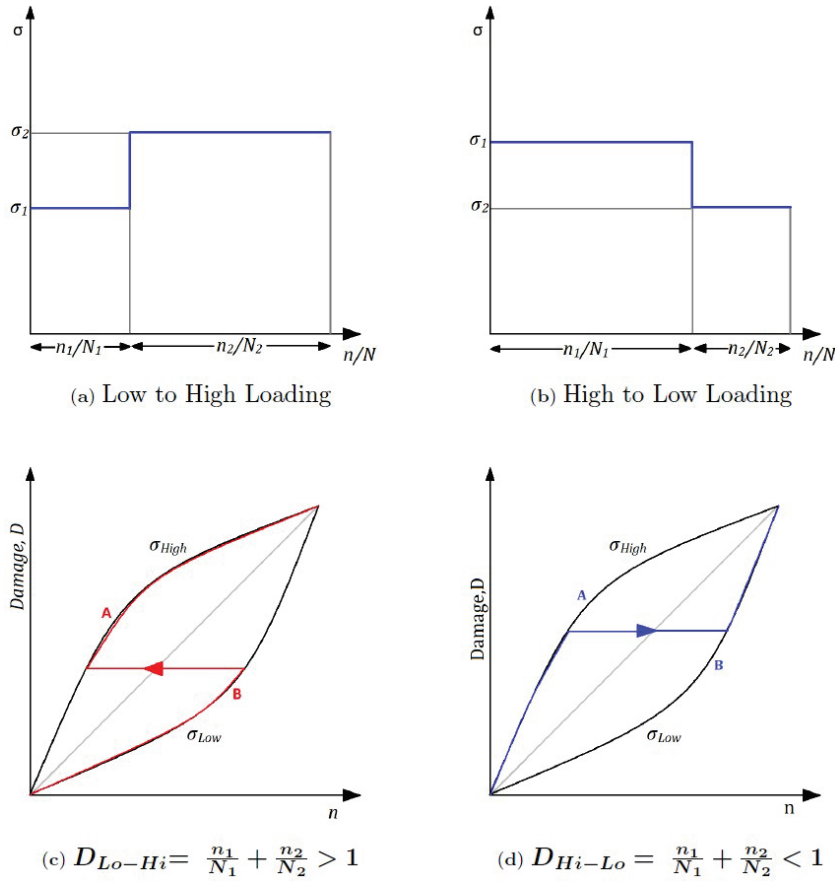


Figure 1. Theoretical High-to-Low and Low-to-High Loading sequences and hypothetical damage-accumulation illustration. The total damage is denoted by D , n_i represents the number of fatigue cycles at the i th constant-amplitude loading stage, and N_i signifies the number of fatigue cycles until failure occurs at the i th constant-amplitude loading stage.

However, these theories overlook one or more of the previously mentioned factors [13–15].

In the next section, the current bilinear traction-separation model with the Fatigue Crack Growth Rate (FCGR) is examined to predict the fatigue failure of adhesive materials. Additionally, It also examines the application of Paris' Law, which is fundamental to FCGR analysis.

The third section presents a novel bilinear traction-separation law for predicting the fatigue failure of adhesive joints in composite joints. To validate the proposed model, a double-lap joint configuration was utilized, and the predictions were compared with experimental data from four distinct test scenarios: two high-to-low loading cases and two low-to-high loading cases. Furthermore, the proposed material model was evaluated against previously established fatigue damage-accumulation models under the step-loading conditions.

2. Literature Review

This section explores current bilinear traction-separation models using the Fatigue Crack Growth Rate (FCGR) to predict the fatigue failure of adhesive materials.

Original Bilinear Traction Separation

Figure 2a shows a typical example of a cohesive zone model. Inspired by the short-crack regime and Paris' Law, Dugdale and Barenblatt introduced separation law to simulate crack initiation and growth in ductile materials [53,54]. The relationship between the total damage parameter $d = d_s + d_f$ and the area enclosed by the cyclic loading curve

in the traction-separation law was examined by Turon to establish the overall damage-accumulation rate d [55]. Harper proposed the concept of the fatigue crack length across an element to define the fatigue damage-accumulation rate d_f .

These studies offer valuable insights into the link between the Fatigue Crack Growth Rate (FCGR) and bilinear cohesive law. Expanding on this foundation, Tabiei and Zhang developed a novel bilinear law that correlates FCGR with the damage parameter [56,57]. Figure 2b illustrates the characteristic bilinear traction-separation law. In this representation, three distinct regions are shown: the blue area indicates damage accumulation during pure mode I separation, the red area represents damage accumulation during pure mode II separation and the green area indicates damage accumulation under mixed-mode separation conditions. Furthermore, in the figure, T represents traction, Δ_n is the separation in the normal direction (mode I), Δ_t is the separation in the tangential direction (mode II), Δ_{mm} is the separation in mixed mode.

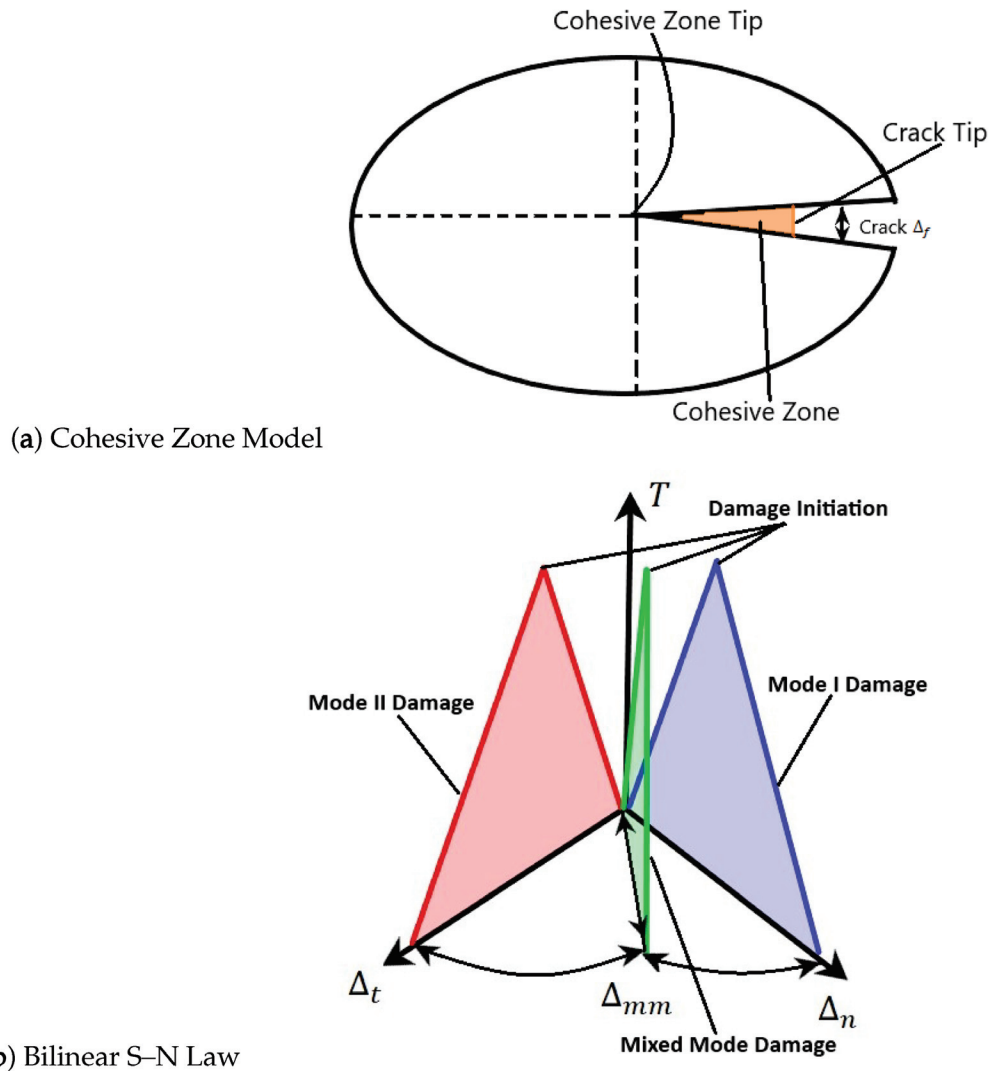


Figure 2. Typical cohesive zone model and its Bilinear Traction-Separation (S–N) Law.

The fatigue damage can be characterized using the traction-separation law as follows:
For Mode I:

$$\frac{\partial d_f}{\partial N} = \frac{b}{A_{cz,I}} \frac{da}{dN} = \frac{32T^2}{9\pi EG_{IC}} \frac{da}{dN} \quad (2)$$

For mode II:

$$\frac{\partial d_f}{\partial N} = \frac{b}{A_{cz,II}} \frac{da}{dN} = \frac{S^2}{EG_{IIC}} \frac{da}{dN} \quad (3)$$

For mixed mode:

$$\frac{\partial d_f}{\partial N} = \frac{b}{A_{cz}} \frac{da}{dN} = \frac{1}{A_{cz}} \frac{da}{dN} \quad (4)$$

where $\frac{\partial d_f}{\partial N}$ is the total fatigue accumulation, $A_{cz,I}$ is the critical area in mode I, $A_{cz,II}$ is the critical area in mode II, A_{cz} is the critical area in mixed mode, $\frac{da}{dN}$ is the total crack propagated and b is a material constant.

Fatigue Crack Growth Rate (FCGR) can be correlated with the Bilinear Traction-Separation Law through the Paris Law regime, which is shown in Figure 3,

$$\frac{da}{dN} = C(\Delta G)^m \quad (5)$$

where C and m are Paris Law parameters, which are experimental obtained for mode I and mode II. In Figure 3, the area shaded in blue represents the total energy released per fatigue load cycle, as described by Paris Law. This ΔG contributes to crack growth within the Fatigue Crack Growth Rate (FCGR) regime.

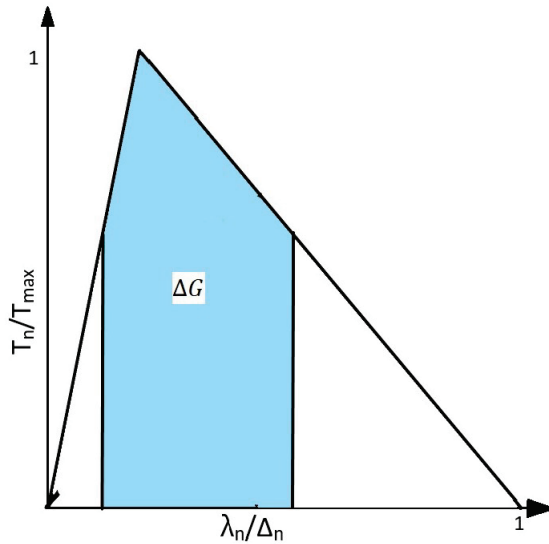


Figure 3. Energy release rate under Paris Law.

According to FCGR, if the load ratio (R) is known, the energy release rate (ΔG) can be calculated by

$$\Delta G = \frac{1}{2E} (T_{max}^2 - T_{min}^2) = \frac{T_{max}^2}{2E} (1 - R^2) \quad (6)$$

where ΔG is the energy release rate, E is the Young's Modulus, T is the traction and R is the loading ratio.

For Normal and Tangential direction, the above equation can be written as,

$$\Delta G_{I,II} = \frac{E \Delta_{n,t}^2}{2} (1 - R^2) (1 - d_s) (1 - d_f) \quad (7)$$

where ΔG_I and ΔG_{II} are the energy release rates in mode I and mode II, respectively. E is the Young's Modulus, R is the loading ratio, d_s is the static damage, d_f is the total fatigue damage, and $\Delta n, t$ is the separation in the normal and tangential direction, respectively.

For mixed mode, the energy release rate can be obtained through a linear interpolation of ΔG_I and ΔG_{II} , which can be expressed as

$$\Delta G_{mm} = \Delta G_I + \beta(\Delta G_{II} - \Delta G_I) \quad (8)$$

Similarly, Paris Law parameters for mixed mode can be found through linear interpolation of mode I and mode II Paris Law parameters,

$$\ln(C) = \ln(C_{II}) + [\ln(C_I) - \ln(C_{II})] \left(\frac{G_{II}}{G_T} \right) \quad (9)$$

$$m = m_I + (m_{II} - m_I) \left(\frac{G_{II}}{G_T} \right) \quad (10)$$

where $G_T = G_I + G_{II}$.

3. New Traction-Separation Law for Variable/Step Loading

The crack growth rate $\left(\frac{da}{dN} \right)$ is a crucial factor in the accumulation of fatigue damage according to the established Bilinear Traction-Separation Law with FCGR. The process of fatigue damage is considered to involve irreversible energy consumption. As cyclic loading occurs, the damage increases monotonically, as

$$\frac{\partial d_f}{\partial N} > 0 \quad (11)$$

Furthermore, under FCGR, fatigue damage can be defined through the Paris Law regime, where

$$\frac{\partial d_f}{\partial N} \propto C(\Delta G)^m \quad (12)$$

Based on Wheeler's work, crack growth is a result of the interaction between the plastic zones at the crack tip, which is created by residual compressive stresses due to overloading [29]. The crack growth under this law can be defined as

$$\frac{da}{dN} = R_i [C(\Delta K)^m] \quad (13)$$

$$R_i = \left(\frac{r_{pi}}{r_{max}} \right)^p \quad (14)$$

where r_{pi} denotes the plastic zone size for the i th loading cycle, whereas r_{max} denotes the distance between the present crack tip and the largest elastic-plastic zone previously created by an overload. The empirical parameter p is influenced by the material characteristics and the loading spectrum. Although this model is useful, it has several limitations. These include failure to account for load interactions, neglect of nonlinear behavior, limited applicability in complex variable-amplitude loading situations, and minimal relevance to small crack growth theories [30,58–62].

To avoid this problem, the authors proposed a new methodology for crack growth. The authors introduced an improvised Paris Law. The authors introduced a new nonlinear load-interaction parameter to the crack propagation rate equation. So, the new proposed crack propagation rate becomes

$$\frac{da}{dN} = C(\Delta G_i)^m * (L_i)^{v_i} \quad (15)$$

where L_i is the load-interaction parameter at the i th level load cycle, and v_i is the exponent parameter considering the loading history. For different loading modes, the load-interaction parameter at i th level load cycle can be defined as

Mode I:

$$L_{I,i} = \frac{T_i}{T_{i-1}} \quad (16)$$

Mode II:

$$L_{II,i} = \frac{S_i}{S_{i-1}} \quad (17)$$

Mixed Mode:

$$L_i = \frac{T_{mi}}{T_{mi-1}} \quad (18)$$

where T and S are Modes I and II loads, respectively. T_{mi} is the mixed-mode traction which can be defined as $T_{mi} = \sqrt{T_i^2 + S_i^2}$.

v_i is a weighted exponent parameter that considers loading history and fatigue life at the respective load. This variable is linked to a decrease in both the elastic modulus and the yield strength during fatigue-induced failure. This is mainly connected to the initiation of cracks and the diminishing load-bearing capacity as these cracks grow under repeated stress cycles. Material degradation can be attributed to a decrease in mobile dislocations and the emergence of barriers to dislocation movements. The declining material toughness is evident in the weakening of strength throughout the fatigue failure process [43]. Similar to Corten-Dolan and Morrow's work, we define the parameter, v_i , as,

$$v_i = \left[\frac{\log N_i}{\log N_{i-1}} * \frac{\sigma_{i-1}}{\sigma_i} \right] \quad (19)$$

where N_{i-1} and N_i are the number of cycles to failure in $(i-1)$ th load (σ_{i-1}) and i th load (σ_i), respectively.

Using the new energy release rate, the crack propagation is redefined as

Mode I:

$$\left(\frac{da}{dN} \right)_I = C \left(\frac{E\Delta_n^2}{2} (1-R^2)(1-d_s)(1-d_f) \right)^m * \left(\frac{T_i}{T_{i-1}} \right)^{\left[\frac{\log N_i}{\log N_{i-1}} * \frac{\sigma_{i-1}}{\sigma_i} \right]} \quad (20)$$

Mode II:

$$\left(\frac{da}{dN} \right)_{II} = C \left(\frac{E\Delta_t^2}{2} (1-R^2)(1-d_s)(1-d_f) \right)^m * \left(\frac{S_i}{S_{i-1}} \right)^{\left[\frac{\log N_i}{\log N_{i-1}} * \frac{\sigma_{i-1}}{\sigma_i} \right]} \quad (21)$$

Mixed Mode

$$\left(\frac{da}{dN} \right)_{mm} = C(\Delta G_I + \beta(\Delta G_{II} - \Delta G_I))^m * \left(\frac{T_{mi}}{T_{mi-1}} \right)^{\left[\frac{\log N_i}{\log N_{i-1}} * \frac{\sigma_{i-1}}{\sigma_i} \right]} \quad (22)$$

For 2 level loading for low-to-high load transition where load 1 (σ_1) < load 2 (σ_2) and number of cycles to failure for load 1 (N_1) > number of cycles to failure for load 2 (N_2), the new crack propagation can be visualized as

$$\left(\frac{da}{dN} \right)_{load I} + \left(\frac{da}{dN} \right)_{load II} * \left(\frac{T_{m2}}{T_{m1}} \right)^{\left[\frac{\log N_2}{\log N_1} * \frac{\sigma_1}{\sigma_2} \right]} > \left(\frac{da}{dN} \right)_{load I} + \left(\frac{da}{dN} \right)_{load II} \quad (23)$$

Because of the slower crack propagation in the above equation, it can be inferred that the number of cycles to fatigue failure is higher than that in the previous model.

Similarly for high-to-low load transition where load 1 (σ_1) > load 2 (σ_2) and number of cycles to failure for load 1 (N_1) < number of cycles to failure for load 2 (N_2), the new crack propagation can be visualized as

$$\left(\frac{da}{dN}\right)_{load I} + \left(\frac{da}{dN}\right)_{load II} * \left(\frac{T_{m2}}{T_{m1}}\right)^{\left[\frac{\log N_2}{\log N_1} * \frac{\sigma_1}{\sigma_2}\right]} < \left(\frac{da}{dN}\right)_{load I} + \left(\frac{da}{dN}\right)_{load II} \quad (24)$$

On the other hand, when transitioning from high to low loads, the crack spreads more rapidly, suggesting that fewer cycles are required for fatigue failure compared to the previous model.

Empirical evidence from various loading conditions revealed that in low-to-high loading sequences, the cumulative damage D exceeded 1, whereas in high-to-low loading sequences, D was less than 1.

Using the new FCGR relation, the fatigue damage-accumulation rate for different loading modes in bilinear traction-separation law can be written as

For Mode I:

$$\frac{\partial d_f}{\partial N} = \frac{32T^2}{9\pi EG_{IC}} C \left(\frac{E\Delta_n^2}{2} (1-R^2)(1-d_s)(1-d_f) \right)^m * \left(\frac{T_i}{T_{i-1}} \right)^{\left[\frac{\log N_i}{\log N_{i-1}} * \frac{\sigma_{i-1}}{\sigma_i} \right]} \quad (25)$$

For mode II:

$$\frac{\partial d_f}{\partial N} = \frac{S^2}{9\pi EG_{IIC}} C \left(\frac{E\Delta_t^2}{2} (1-R^2)(1-d_s)(1-d_f) \right)^m * \left(\frac{S_i}{S_{i-1}} \right)^{\left[\frac{\log N_i}{\log N_{i-1}} * \frac{\sigma_{i-1}}{\sigma_i} \right]} \quad (26)$$

For mixed mode:

$$\frac{\partial d_f}{\partial N} = \frac{1}{A_{cz}} C \left(\frac{E(\Delta_n^2 + \beta(\Delta_t^2 - \Delta_n^2))}{2} (1-R^2)(1-d_s)(1-d_f) \right)^m * \left(\frac{T_{mi}}{T_{mi-1}} \right)^{\left[\frac{\log N_i}{\log N_{i-1}} * \frac{\sigma_{i-1}}{\sigma_i} \right]} \quad (27)$$

Implementation of Fatigue Law into Implicit Time Integration

From a computational standpoint, once the rate of damage accumulation is determined, the overall damage can be computed through

$$d_{f,n+1} = d_{f,n} + \frac{\partial d_f}{\partial N} \quad (28)$$

However, a high cycle fatigue can potentially have up to $\sim 10^8$ cycles during its lifetime. Calculating the total fatigue damage cycle by cycle is computationally very expensive and resource-intensive. As a result, a cyclical jumping technique was utilized. The assumption is made that the rate at which damage accumulates remains constant within a particular cycle range, and subsequently,

$$d_{f,n+m} = d_{f,n} + j \frac{\partial d_f}{\partial N} \quad (29)$$

where j is the number of cycles in which damage is constant.

To minimize computation time, the FEM routine employs an implicit time-integration method. The selected time step was considerably larger than the loading period. When the cyclic load period ΔT remains constant throughout the simulation, the equation remains unchanged.

$$d_{f,i+1} = d_{f,i} + \frac{dt_i}{\Delta T} \frac{\partial d_f}{\partial N} \quad (30)$$

where i denotes the number of time intervals. Rather than employing cyclic loading, it is necessary to provide a load envelope and a load ratio (R). An illustration of the loading curve used to simulate fatigue failure using the cyclic jump method is presented in Figure 4.

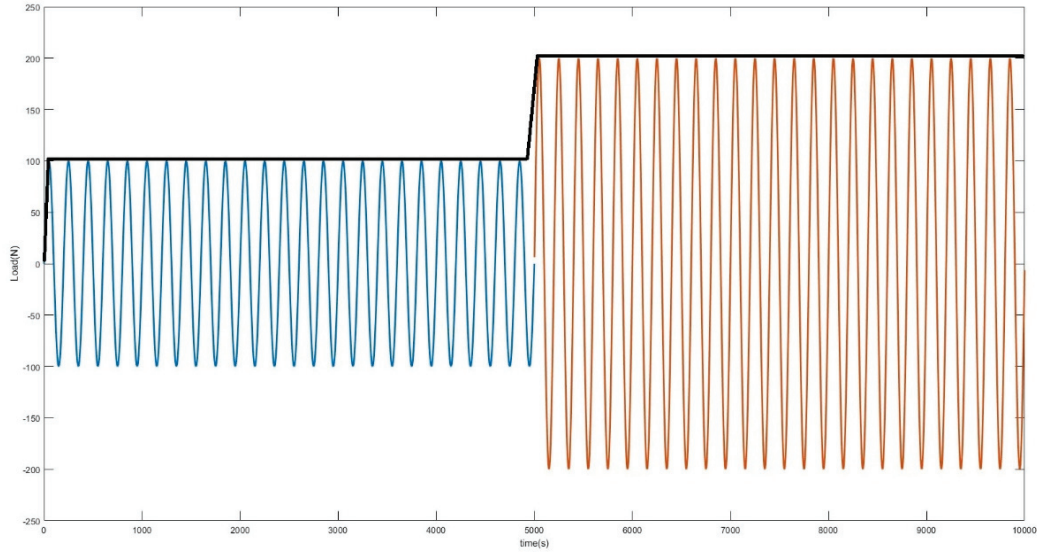


Figure 4. Example of Step Loading used in the simulation.

In implicit time integration, difficulty in obtaining convergence may arise when turning points exist in the material model, particularly with the bilinear cohesive law. To address this problem, the arc-length technique was implemented using LS-DYNA's integrated algorithms. Furthermore, the viscous regularization approach can be applied to enhance convergence [63]. This method introduces a viscous damage variable d_v as a substitute for static damage variable d_s .

$$d_v = d_s - \eta \frac{\dot{\lambda}}{\lambda} \quad (31)$$

where η is viscosity and is set to a value of 10^{-4} , which helps to improve the convergence of the simulation while ensuring that the viscous energy remains negligible. This viscous regularization is an essential component of the UMAT41c material model in LS-DYNA and is used in conjunction with a cohesive model.

An LS-DYNA user material (UMAT) model was created to forecast the fatigue failure in adhesive joints within composite materials subjected to varying loads. This model incorporates the bilinear traction-separation law. The subsequent section elaborates on the specifics of numerical simulation.

4. Fatigue Life Prediction of Adhesive Joint Under Variable Loading

4.1. Experimental Model

A double-lap joint model was used to assess the suitability of the suggested bilinear traction-separation law. Khabbaz et al. conducted experimental research to determine the fatigue lifespan of adhesively bonded FRP structural joints under both steady and fluctuating load conditions [12].

The numerical simulations were performed using the double-lap joint model illustrated in Figure 5. The overall specimen measured 350 mm in length, with GFRP laminate tabs extending 200 mm each. As depicted in Figure 5, a 50 mm long fixed ends is shown in blue, while the load was applied to a 50 mm segment at the load end, represented in

yellow. The adhesive bonds, highlighted in orange in Figure 5, measured 50 mm in length and 2 mm in thickness and effectively joined the GFRP tabs.

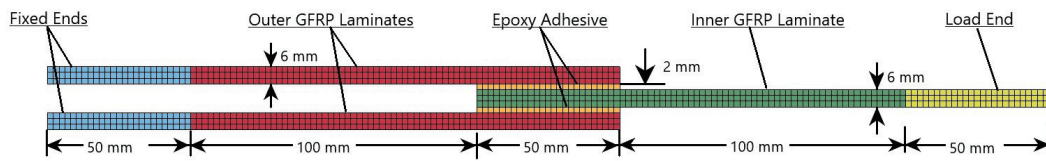


Figure 5. Specification of double-lap joint used in the study.

For the double-lap joint, Fiberline A/S, a Danish company, supplied pultruded GFRP laminates that were used for both inner and outer tabs [12]. These laminates were fabricated using E-glass fibers and isophthalic polyester resin. Table 1 lists the material properties of the GFRP laminates.

Table 1. Mechanical Properties of the Pultruded GFRP Laminate used in the study [12].

Property	Value
Density	2560 kg/m ³
Young's Modulus	25 GPa
Tensile Strength	307.5 MPa
Poisson's Ratio	0.23

The adhesive joints were constructed using SikaDur 330, a two-component epoxy adhesive system produced by Sika AG, a Swiss company [64]. The material properties of SikaDur 330 are presented in Table 2.

Table 2. Mechanical Properties of Sikadur 330 Adhesive used in the study [64].

Property	Value
Flexural Strength	60.6 MPa
Modulus of Elasticity in Flexure	3489 MPa
Tensile Strength	33.8 MPa
Elongation at Break	1.2%
Heat Deflection Temperature	50 °C

Vassilopoulos et al. experimentally determined the Paris Law variables for SikaDur 330 [17,18]. Paris Law parameters for the bilinear traction-separation law used in the model are described in Table 3 below,

Table 3. Paris Law Constants for Sikadur 330 used in the study.

Paris Law Properties	Value
C_I	1
C_{II}	5
m_I	150
m_{II}	8.1

The fatigue experiment was performed under controlled load conditions at a steady frequency of 10 Hz to maintain consistent loading. The load ratio, which represents the

ratio of the maximum to minimum applied cyclic load, was maintained at 0.1. Four samples were examined to assess the fatigue life under constant-amplitude loading. All tests were carried out using an INSTRON 8800 servohydraulic device in a controlled laboratory setting under ambient conditions of $23 \pm 5^\circ\text{C}$ and $50 \pm 10\%$ relative humidity.

At each load level, where $R = 0.1$, a single specimen was equipped on one side using two HBM RDS20 crack gauges. These gauges spanned the entire bonding length and monitored crack initiation and growth throughout the fatigue life. Preliminary findings suggested that cracks formed and spread similarly along both sides of the bond line in each sample. Data collection was performed using LabVIEW and an HBM-Spider8 multichannel electronic measurement system. The total cycle count until failure (N_f) is determined by fitting the power curve to the experimental data. Figure 6 shows a comparison of the experimental and simulation results for various constant loads.

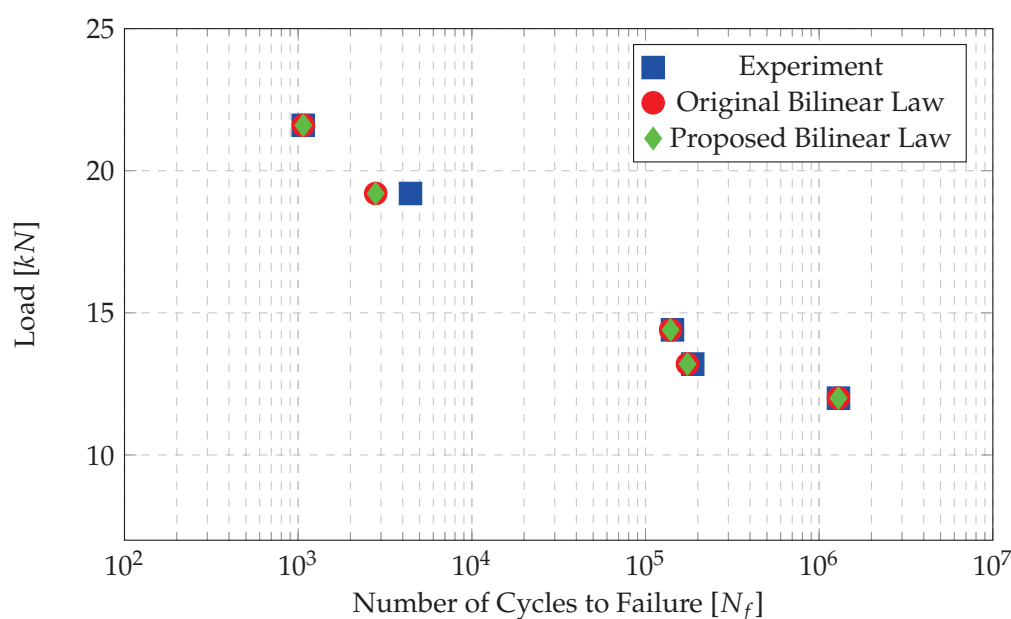


Figure 6. Comparison of Experimental Data and Simulation for constant load fatigue test [12].

Four different two-block loading experiments were considered in this study. The experiments consisted of two low-high-cycle loadings and two high-low-cycle loadings. A constant loading frequency of 10 Hz was used for block loading, and the load ratio was kept constant at 0.1. For each experimental block loading case, three specimens were used to perform fatigue life tests under constant-amplitude loading. Table 4 lists the two-block loading experiments that were used.

Table 4. Experimental data for the four step-loading cases used for validation of the proposed bilinear traction-separation law [12].

	F_{max1} [kN]	n_1	N_1	$D_1 = n_1/N_1$	F_{max2} [kN]	n_2	N_2	$D_2 = n_2/N_2$	D
Low-High Sequence	14.4	48,649	142,978	0.3403	21.6	483	1070	0.4514	0.7917
	12	456,038	1,291,993	0.3530	19.2	2592	4434	0.5846	0.9376
High-Low Sequence	21.6	287	1070	0.2682	14.4	207,559	142,978	1.4517	1.7199
	19.2	1330	4434	0.3000	12.0	4,832,688	1,291,993	3.7405	4.040

Where n_1 represents the number of cycles with the first loading block while n_2 denotes the number of cycles in the second loading block until failure occurs. N_1 indicates the total cycles to failure in the first loading block, and N_2 signifies the same for the second loading block. D_1 refers to the damage inflicted during the first loading block, whereas

D_2 represents the damage caused in the second loading block. Lastly, D symbolizes the cumulative damage as calculated by the Miner Rule, which is the sum of D_1 and D_2 .

To simplify the terminology in this study, we have assigned specific labels to different load transitions. Case 1 denotes the shift from 14.4 kN to 21.6 kN, while Case 2 represents the change from 12 kN to 19.2 kN. Conversely, Case 3 indicates the transition from 21.6 kN to 14.4 kN, and Case 4 signifies the shift from 19.2 kN to 12 kN. These designations will be used throughout the remainder of the study.

4.2. Simulation Results

Simulations were conducted for both the original and proposed bilinear traction-separation models across all four step-loading scenarios. The results for each of these cases, in comparison to their respective experimental data, are displayed in Figure 7.

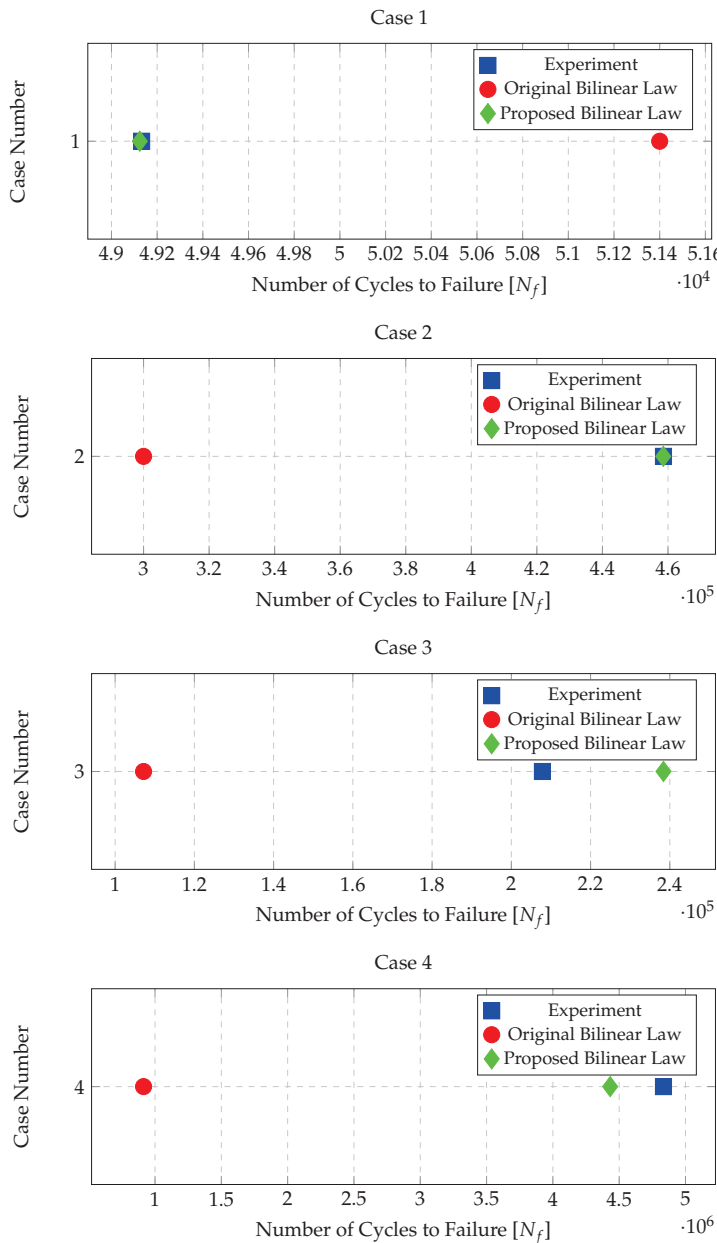


Figure 7. Comparison of fatigue failure of step-loading cases between experimental data, original bilinear traction separation law, and proposed bilinear traction separation law [12]. Low-to-high load transition Cases (**Case 1**) 14400 N to 21600 N and (**Case 2**) 12000 N to 19200 N. High-to-Low Load transition Cases (**Case 3**) 21600 N to 14400 N and (**Case 4**) 19200 N to 12000 N.

The results obtained using the suggested bilinear traction-separation law showed notable improvements over those obtained using the original bilinear traction-separation law, as illustrated previously. To quantify this improvement, the Relative Error of Prediction (*REP*) was used, which was calculated as follows:

$$REP = \frac{N_{exp} - N_{sim}}{N_{exp}} * 100 \quad (32)$$

In Case 1, the proposed bilinear traction-separation law reduced the *REP* from 4.62% to 0.016%. Similarly, in Case 2, the *REP* decreased from 34.59% to 0.013%. For the high-to-low loading scenarios (Cases 3 and 4), the *REP* was significantly reduced from 48.43% and 81.10% to 14.7% and 8.313%, respectively.

The proposed bilinear traction-separation law exhibits different rates of damage accumulation per cycle compared to the original model. In the low-to-high load transition cases (Case 1 and Case 2), damage accumulated at a faster rate per cycle than in the original bilinear traction-separation law, consistent with the experimental results. In contrast, during the high-to-low load transition cases, the rate of damage accumulation per cycle was reduced compared to the original bilinear model, which also matches the experimental observations. The following figure illustrates a comparison of the damage-accumulation rates per cycle for the various step-loading scenarios.

The modified bilinear traction-separation law shows distinct improvements in predicting damage-accumulation rates compared to the original model. For low-to-high loading cycle cases (e.g., Case 1 and Case 2), the damage-accumulation rate per cycle is accelerated, which is consistent with experimental observations. Conversely, for the high-to-low loading cases, the damage-accumulation rate decreased, confirming the trends observed in the experiments. Figure 8 illustrates a comparison of the damage-accumulation rates across the various step-loading cases. Specifically, cycle-to-cycle damage accumulation was evaluated in LS-DYNA for two scenarios: a shift from low to high load (Case 2) and a transition from high to low load (Case 4).

As shown in Figure 8, the proposed bilinear law predicts a faster damage-accumulation rate under low-to-high loading, leading to earlier material failure. In Case 1, the new model anticipated failure after 49,124 cycles compared to 51,400 cycles predicted by the original bilinear law. This result closely matched the experimental failure at 49,132 cycles. However, the proposed bilinear law predicts a slower damage-accumulation rate under high-to-low loading conditions, resulting in delayed failure. For instance, in Case 4, the proposed model predicted failure after 4,432,170 cycles, which is significantly higher than the original model's prediction of 913,610 cycles. This updated prediction aligns with the experimental results, which showed a failure at 4,834,018 cycles.

To validate the efficacy of the proposed model, a comparison was conducted against an existing fatigue damage-accumulation model under varying amplitude fatigue-loading conditions. Eight models were selected based on four key factors, namely widespread use, applicability to adhesive materials, recent advancements, and unique characteristics. Individual UMAT components were created in LS-DYNA for each of the eight fatigue damage-accumulation models to facilitate numerical simulations.

Table 5 presents a comparison of the experimental data, proposed bilinear law, and previously mentioned theoretical models for fatigue damage accumulation under step/variable loading. Additionally, Table 6 illustrates the comparison of the *REP* among the theoretical fatigue damage models.

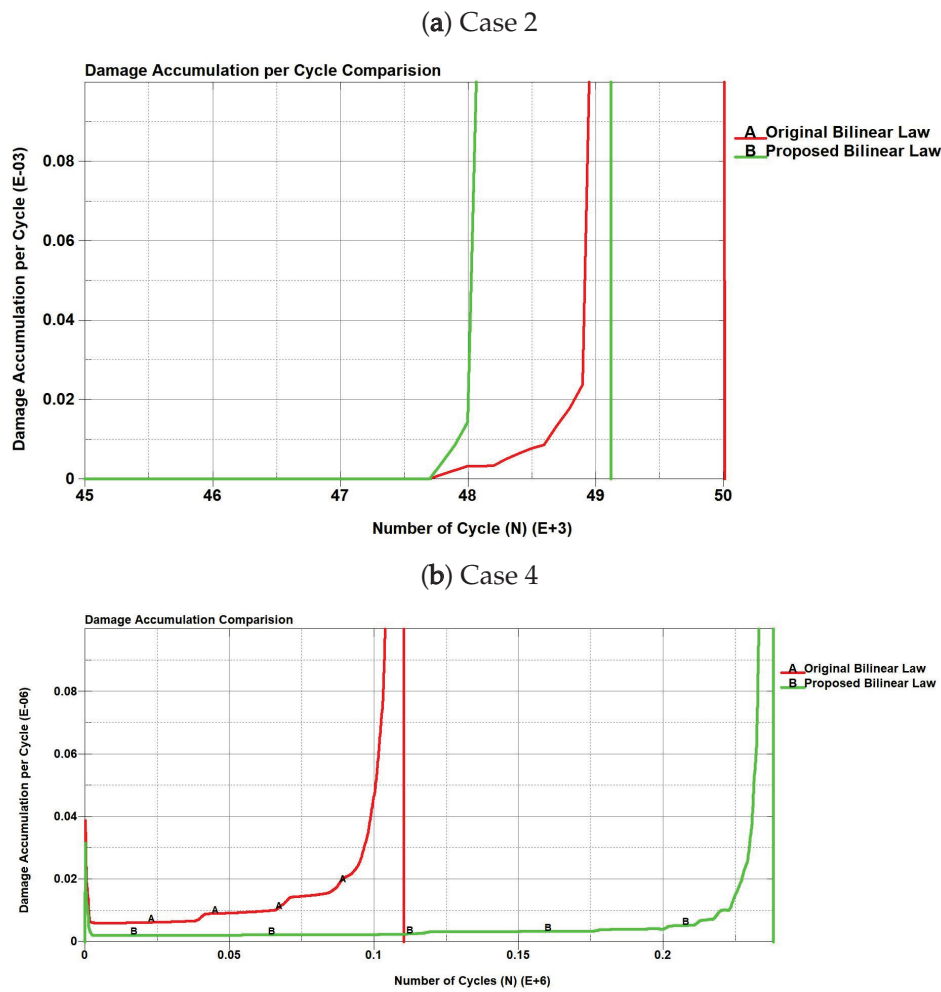


Figure 8. Comparison of damage-accumulation rate per cycle after load transition or load level 2 between the original bilinear traction-separation law and the proposed traction-separation law. Examples (a) Low-to-high loading Case 2 and (b) High-to-Low loading Case 4.

Table 5. Comparison of number of cycles to failure between experimental data, proposed bilinear traction-separation law, original bilinear traction-separation law, and other theoretical models [12].

	Case 1	Case 2	Case 3	Case 4
Experiment	49,132	458,630	207,846	4,834,018
Proposed S–N Law	49,124	458,570	238,400	4,432,170
Original S–N model	51,400	300,000	107,180	913,610
Miner	49,355	458,906	104,915	905,784
Corten-Dolan	48,963	457,158	236,000	2,317,476
Fruedenthal Heller	49,558	459,860	57,277	301,967
Damage Curve Approach	49,516	459,558	127,992	1,109,683
Kwofie	49,064	457,749	178,565	1,517,491
Subramanyan	48,650	456,038	142,978	1,291,993
Dattoma	48,650	456,038	142,978	1,291,993
Xie	49,018	457,465	138,338	1,242,244

Table 6. Comparison of percentage error deviation of number of cycles to failure between experimental data, proposed bilinear traction-separation law, original bilinear traction-separation law, and other theoretical models.

	Case 1	Case 2	Case 3	Case 4
Experiment	0.000	0.000	0.000	0.000
Proposed S–N Law	−0.016	−0.013	14.700	−8.313
Original S–N model	4.616	−34.588	−48.433	−81.100
Miner	0.454	0.060	−49.523	−81.262
Corten-Dolan	−0.344	−0.321	13.546	−52.059
Fruedenthal Heller	0.867	0.268	−72.443	−93.753
Damage Curve Approach	0.782	0.202	−38.420	−77.044
Kwofie	−0.138	−0.192	−14.088	−68.608
Subramanyan	−0.981	−0.565	−31.210	−73.273
Dattoma	−0.981	−0.565	−31.210	−73.273
Xie	−0.232	−0.254	−33.442	−74.302

Tables 5 and 6 demonstrate that the proposed bilinear traction-separation law yielded the most accurate results when evaluated against other fatigue damage-accumulation models. In the low-to-high loading cases (Cases 1 and 2), Kwofie’s fatigue damage-accumulation model produced the best results among the theoretical damage-accumulation models, with an *REP* of approximately 0.15% for both instances. However, the proposed bilinear traction-separation model further decreases the *REP* by 0.014%.

Likewise, Tables 5 and 6 show that for high-to low-loading cases (Cases 3 and 4), the proposed bilinear traction-separation law produced the most accurate results in comparison to other cumulative fatigue damage models. For these cases, Radhakrishnan’s damage-accumulation model exhibited the lowest *REP* among the theoretical cumulative damage-accumulation models, with 13.5% for Case 3 and 52% for Case 4 when compared to the experimental data. Nevertheless, the suggested bilinear traction-separation law further reduces the *REP* to 14.7% and 8.31% for Cases 3 and 4, respectively.

Figure 9 illustrates the comparative analysis of the *REP* between the proposed model and other widely used damage-accumulation models for Cases 1 and 4. This figure highlights the improved accuracy of the proposed bilinear traction-separation law, demonstrating its improved performance for both low- and high and high-to-low load transitions.

Table 7 presents a comparison of simulation durations for the four cases examined in the study, comparing the proposed bilinear traction-separation law with the original bilinear traction-separation law. The overall simulation time experienced a slight increase across all cases owing to the additional internal computations. As evident from the table, the simulation time difference is minimal for the low-to high-load transition cases (Cases 1 and 2). In contrast, for cases with high-to-low load transition (Cases 3 and 4), the simulation time increases markedly, particularly in Case 4. This can be attributed to the significant number of load cycles post-load transition, where the original bilinear traction-separation law predicts earlier failure than experimentally observed.

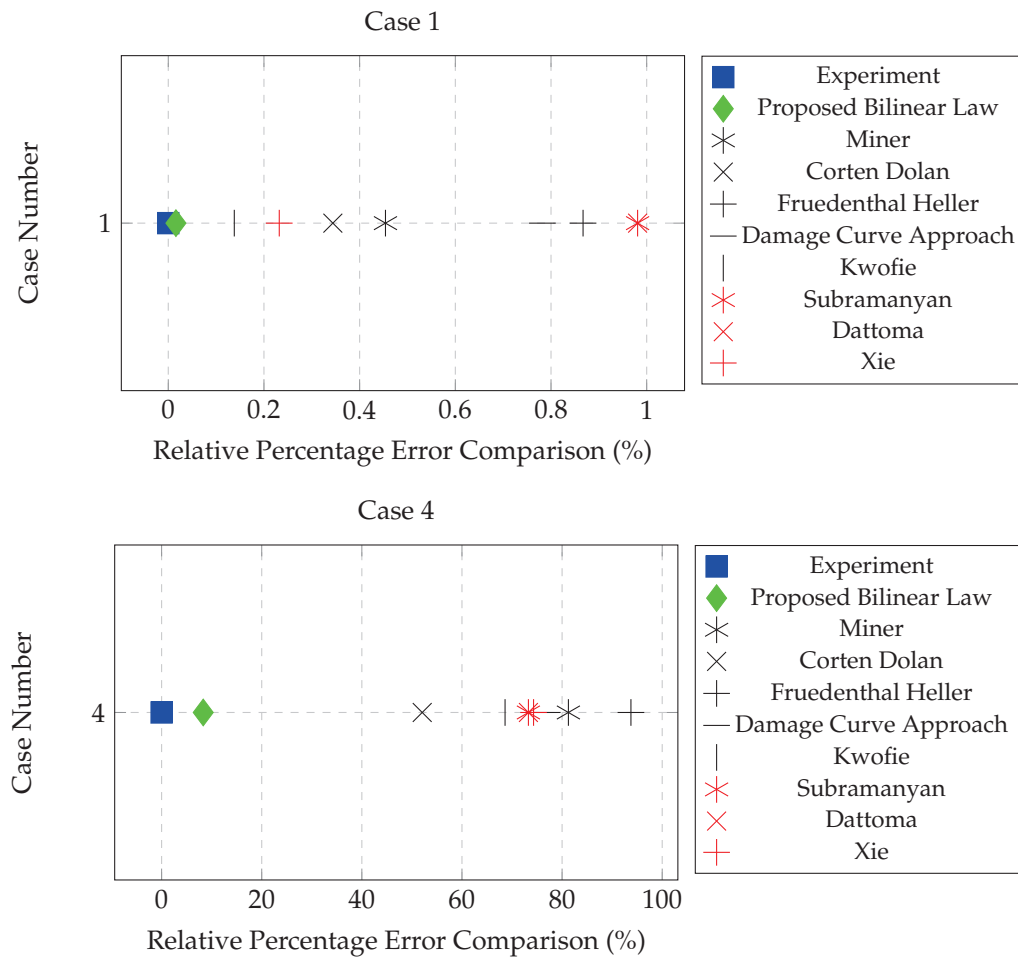


Figure 9. Comparison of percentage error deviation of number of cycles to failure between experimental data, proposed bilinear traction-separation law, original bilinear traction-separation law, and other theoretical models. Low-to-high load transition cases (Case 1) 14400 N to 21600 N and (Case 4) High-to-low load transition cases 19200 N to 12000 N.

Table 7. Comparison of simulation time between the proposed bilinear traction-separation law and the original bilinear traction-separation law.

	Proposed Bilinear Law	Original Bilinear Law
Case 1	20 min 28 s	17 min 56 s
Case 2	1 h 0 min 6 s	1 h 11 min 56 s
Case 3	23 min 40 s	28 min 11 s
Case 4	10 h 1 min 2 s	14 h 38 min 53 s

In low-to-high load transition cases, the accumulation of fatigue damage accelerates the post-load transition process. As a result, the number of cycles before failure after the transition is relatively small, leading to a reduced *REP* compared to high-to-low load transitions. However, the degree of proportional decrease in the *REP* between the highest *REP* and *REP* produced by the proposed bilinear law was similar to that observed in the high-to-low load transitions.

5. Conclusions

This study examined the impact of variable-amplitude loading on the fatigue life of adhesive joints in fiber-reinforced polymer (FRP) composites. A novel bilinear traction-

separation law based on the Fatigue Crack Growth Rate (FCGR) was introduced to enhance fatigue failure predictions by incorporating the load history, sequence, and interaction effects.

The model was validated using a double-lap joint under step/variable loading for four experimental cases: two high-to-low and two low-to-high load transitions. Compared with experimental data and existing models, the proposed method accurately captured damage-accumulation trends, showing accelerated failure in low-to-high scenarios and decelerated failure in high-to-low cases. Among the eight conventional fatigue damage models, the new approach demonstrated superior accuracy, significantly reducing the Relative Error of Prediction (*REP*) across all test Cases from 4.616% to 0.016% (Case 1), 34.88% to 0.013% (Case 2), 48.433% to 14.70% (Case 3), and 81.10% to 8.313% (Case 4). Although the proposed model slightly increased the simulation time in the low-to-high loading cases and high-to-low loading cases, the simulation time increased as the number of fatigue cycles increased. This study combined experimental testing, numerical simulations, and model comparisons, reinforcing its credibility and practical applicability.

The proposed bilinear traction-separation laws improve fatigue life predictions for adhesive joints under variable loads, enabling optimized FRP composite designs that minimize costs and overengineering. These findings enhance the understanding of fatigue behavior in adhesive joints, benefiting industries such as aerospace, automotive, and wind energy by improving structural durability and safety. This study represents a significant advancement in the fatigue analysis of FRP composite adhesive joints, paving the way for more reliable engineering solutions.

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Article

Tensile Strength Predictive Modeling of Natural-Fiber-Reinforced Recycled Aggregate Concrete Using Explainable Gradient Boosting Models

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Abstract: Natural fiber composites have gained significant attention in recent years due to their environmental benefits and unique mechanical properties. These materials combine natural fibers with polymer matrices to create sustainable alternatives to traditional synthetic composites. In addition to natural fiber reinforcement, the usage of recycled aggregates in concrete has been proposed as a remedy to combat the rapidly increasing amount of construction and demolition waste in recent years. However, the accurate prediction of the structural performance metrics, such as tensile strength, remains a challenge for concrete composites reinforced with natural fibers and containing recycled aggregates. This study aims to develop predictive models of natural-fiber-reinforced recycled aggregate concrete based on experimental results collected from the literature. The models have been trained on a dataset consisting of 482 data points. Each data point consists of the amounts of cement, fine and coarse aggregate, water-to-binder ratio, percentages of recycled coarse aggregate and natural fiber, and the fiber length. The output feature of the dataset is the splitting tensile strength of the concrete. Extreme gradient boosting (XGBoost), light gradient boosting machine (LightGBM) and extra trees regressor models were trained to predict the tensile strength of the specimens. For optimum performance, the hyperparameters of these models were optimized using the blended search strategy (BlendSearch) and cost-related frugal optimization (CFO). The tensile strength could be predicted with a coefficient of determination greater than 0.95 by the XGBoost model. To make the predictive models accessible, an online graphical user interface was also made available on the Streamlit platform. A feature importance analysis was carried out using the Shapley additive explanations (SHAP) approach.

Keywords: natural fibers; composites; XGBoost; Shapley additive explanations; recycled aggregate; machine learning

1. Introduction

Concrete is the most widely utilized construction material globally, due to its availability and versatility. However, its inherent weaknesses include low tensile strength, limited ductility, and poor energy absorption. Global urbanization has led to a rapid increase in construction and demolition (C&D) waste, which comprises nearly 30–40% of the total solid waste generated globally [1]. To address these challenges, recycled coarse aggregates (RCA) derived from C&D waste have gained attention as an eco-friendly substitute for natural

aggregates. The use of recycled coarse aggregate (RCA) as a substitute for natural coarse aggregate (NCA) has been extensively investigated in the literature. While RCA offers a sustainable alternative, research indicates that recycled aggregate concrete (RAC) often demonstrates lower performance compared to natural aggregate concrete (NAC) in terms of workability, compressive and tensile strength, unit weight, and modulus of elasticity. This decline is primarily attributed to the presence of attached mortar on the surface of RCA [1–3]. Researchers have experimented with RCA replacement levels ranging from partial (25–50%) to full (100%) substitution [4–6]. It has been observed that replacing NCA with 25–50% RCA has a minimal negative effect on the mechanical properties of concrete, making it a practical and environmentally viable option [2,7,8].

Regarding the tensile properties of concrete, many studies have observed a decrease in tensile strength with the addition of recycled coarse aggregate [1,5,9]. However, fibers can mitigate this strength reduction. For fiber-reinforced recycled aggregate concrete (FRRAC), it was noted that fibers contribute to enhancing tensile and flexural strength. Nonetheless, when RCA content increases for the same fiber dosage, the beneficial influence of the fibers becomes minimal [1,5,9–11].

Another major factor contributing to concrete's insufficient tensile behavior is its low toughness, combined with internal defects. Enhancing the toughness of concrete and minimizing defect size and frequency is essential for improving its overall performance. One proven method for increasing concrete toughness is the incorporation of short fibers (typically 0.5–2% by volume) into the concrete mix during preparation. As a sustainable alternative to synthetic fibers, natural fibers such as bamboo, coconut, straw, and hemp have emerged as a promising solution. On the other hand, there are only a limited number of studies in literature involving the effects of natural fiber addition to concrete. Bittner and Oettel [12] experimentally investigated the flexural behavior of ultra-high performance concrete beams reinforced with bamboo fibers. The addition of bamboo fibers was observed to increase the maximum load up to 37% in these experiments. Joachim and Oettel [13] carried out compressive tests with ultra-high-performance concrete reinforced with bamboo, coir and flax fibers. The addition of natural fibers was observed to reduce the brittle behavior of concrete under compression. Beskopylny et al. [14] evaluated the efficiency of using hemp and flax fibers for concrete reinforcement. The addition of fibers was found to increase the compressive and flexural strength and to decrease the water absorption. Jamshaid et al. [15] investigated the strength, weight loss percentage and surface degradation of jute, sugarcane, coconut, sisal and basalt fibers. It was found that the basalt fibers imparted the greatest compressive strength followed by jute and sisal fibers. Zhao et al. [16] conducted an experimental study to investigate the effects of adding pineapple leaf and ramie fibers on the strength, durability, and permeability of cement-based materials. The addition of fibers was observed to significantly increase capillary absorption and chloride diffusion. Sadouri et al. [17] investigated the effect of juncus fiber addition to cementitious materials on the compressive strength and ultrasonic pulse velocity. The addition of juncus fibers was observed to reduce the ultrasonic pulse velocity and compressive strength. Da Costa Santos and Archbold [18] investigated the effect of surface treatment application on the biodegradability of hemp fibers. The addition of NaOH 10% for 24 h was found to decrease the degradability of hemp fibers in an alkaline solution. Wu et al. [19] carried out experiments with rice straw fiber-reinforced concrete in wall panels. The rice straw fiber was combined with magnesium oxychloride cement to produce a kind of fiber-reinforced concrete with a high content of straw and suitable for light steel keel wall panels. It was found that the mass content of rice straw in concrete used in the wall panel can reach about 12%. Jamshaid et al. [20] carried out experimental investigations related to the effect of jute, sisal, sugarcane, and coconut fibers on the mechanical properties of concrete. Jute and

sisal fibers showed the most significant improvement in tensile and compressive strength (11.6% and 20.2%, respectively) compared to plain concrete when 2% fiber was added to the concrete mix.

In recent years, many studies applied machine learning models to predict the mechanical properties of recycled aggregate and fiber-reinforced concrete. However, to the best of the authors' knowledge, none of these machine learning-based studies investigated the combination of recycled aggregates with natural fibers. Momeni et al. [21] focused on estimating the flexural strength of recycled aggregate concrete beams. They developed a predictive model using an artificial neural network (ANN) optimized by particle swarm optimization and imperialist competition algorithms based on an experimental dataset. Similarly, Dantas et al. [22] built an ANN model to predict the compressive strength of concrete with different curing ages (3, 7, 28, and 91 days) made with construction and demolition waste. A total of 1178 data points were used for modeling the ANN model. Of these, 77.76% were used in the training phase and 22.24% in the testing phase. R^2 scores of 0.928 and 0.971 were achieved on the training and test sets, respectively. Felix et al. [23] introduced an ANN and a nonlinear regression model to estimate the elastic modulus of recycled aggregate concrete. A coefficient of determination of 0.91 could be obtained in predicting the elastic modulus. Dai et al. [24] used multi-layer perceptron and AdaBoost models to predict the compressive strength of high-strength fiber-reinforced concrete using steel fibers. An R^2 score of 0.94 could be achieved by the AdaBoost model. Yuan et al. [25] developed gradient boosting and random forest models to predict the compressive and flexural strength of recycled aggregate concrete using a dataset of 638 mixes. They found that the random forest model most accurately predicted strength, achieving R^2 scores of 0.91 for compressive strength and 0.86 for flexural strength. Abed and Mehryaar [26] used multiple machine learning models including M5p decision trees, ridge and LASSO regression techniques. The models were trained to predict the degradation of concrete's mechanical properties after exposure to high temperatures. The compressive strength, elasticity modulus, flexural strength and splitting tensile strength were predicted with R^2 scores of 0.91, 0.90, 0.94 and 0.959, respectively. Rodsin et al. [27] investigated the use of cotton ropes for confining recycled aggregate concrete by conducting experiments on both full-wrap and strip confinement, and by developing neural network models to predict the material's compressive strength. Their neural network model achieved an R^2 score of 0.931 for fully wrapped specimens and an R^2 score of 0.951 for strip-wrapped specimens. Zhu et al. [28] developed gene expression programming, artificial neural networks, and a bagging ensemble model to predict the splitting tensile strength of recycled aggregate concrete. They found that the bagging ensemble model achieved the highest accuracy with an R^2 score of 0.95.

This study is based on a series of experimental studies recently published by Aayaz [29] which contain the tensile strength of recycled aggregate concrete reinforced by six different types of natural fiber, namely jute, kenaf, bamboo, sisal, coir and ramie (Figure 1). Based on this dataset of experimental results, the current study presents the development of accurate predictive models using state of the art decision tree ensemble machine learning techniques. After presenting the predictive performance of these models, the effects of various input features on the tensile strength were quantified. These input features are the fiber type, superplasticizer amount, natural fiber percentage, cement amount, water-to-binder ratio, fine aggregate amount, coarse aggregate amount and the recycled coarse aggregate percentage in the concrete mixture. Furthermore, an online graphical user interface has been made available which delivers for a certain range of input features the corresponding tensile strength of concrete. The current study is unique in the sense that it combines the effects of recycled coarse aggregate and natural fibers on the mechanical

behavior of concrete. To the best of the authors' knowledge, this is a research area which has not been sufficiently investigated using a data-driven approach. One of the reasons for the lack of sufficient investigation in this area is the biodegradability of natural fibers resulting in pure durability of concrete in the long term. Another reason is that natural fibers exhibit large variability in their mechanical properties which makes it difficult to predict their structural response. The accurate prediction of concrete properties in the presence of recycled coarse aggregate and natural fibers using machine learning techniques can be greatly beneficial for design engineers due to the lack of predictive equations in the design codes. The current study also contributes to the knowledge base related to the application of natural fibers in composites technology. Because of the sustainability benefits of the inclusion of construction and demolition waste and natural fibers in construction, the availability of accurate prediction techniques is highly important.

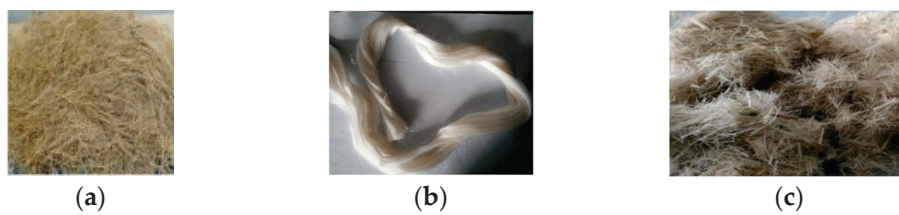


Figure 1. (a) Coir [30], (b) ramie [16], (c) jute [31] fibers.

2. Materials and Methods

In this section, the methods applied in data preprocessing, training the predictive models and interpreting the model outputs are explained. The section starts with a brief theoretical background on the gradient boosting algorithms used in predictive model development. Afterward, the hyperparameter optimization technique is explained followed by a brief background information on the Shapley additive explanations (SHAP) technique used for feature importance analysis. The statistical properties of the dataset are presented and the outlier detection and visualization techniques used for improving predictive model performance are explained.

2.1. Gradient Boosting Algorithms

Gradient boosting models such as XGBoost, LightGBM, and extra trees work as ensembles of decision trees where each subsequent decision tree in the ensemble improves upon the predictions of the previous decision trees. By correcting the predictions of the previous decision trees in the ensemble, the difference between the true and predicted values of the output feature is being minimized [32]. This process can be visualized as in Equation (1) where f_k is a function that determines the output of the decision tree with index k , $\hat{y}$ is the predicted output of the entire decision tree ensemble, N is the total number of decision trees, n is the total number of points in the dataset, and x is a vector of input features.

$$\hat{y} = \sum_{k=1}^N f_k(x), \quad \text{minimize} \left(\sum_{j=1}^n \|y_j - \hat{y}_j\| \right) \quad (1)$$

2.2. Optimization of the Hyperparameters

The performance of a predictive model can significantly fluctuate with respect to the set of hyperparameters. In this study, the blended search strategy (BlendSearch) and cost-related frugal optimization (CFO) algorithm implemented in the FLAML package were used to obtain the optimal hyperparameter configurations for the XGBoost, LightGBM and extra trees models. The BlendSearch and CFO algorithms are used in the exploration and exploitation phases of the optimization, respectively. In this sense, the BlendSearch

and CFO algorithms are combined in the FLAML package for global and local search, respectively. The CFO algorithm uses the randomized direct search technique whereas the BlendSearch algorithm uses global search and enables the optimization process to escape local optimum points. Further details of the CFO and BlendSearch techniques can be found in [33,34].

2.3. Shapley Additive Explanations (SHAP) Methodology

SHAP (Shapley additive explanations) methodology works by assigning importance scores called Shapley values to each input feature. These Shapley values are used as a means for measuring how much each feature contributes to the predictive model's output. The SHAP technique is valuable in that it enables the interpretation of the predictive model's decision-making process. The computation of the Shapley values can be summarized as in Equation (2). In Equation (2), the Shapley value of the variable with index i is denoted with ϕ_i . F denotes the set of all input features, S denotes a subset of the feature set which does not have the feature with index i and x denotes a vector of input variables. The prediction of the machine learning model is represented by the function f . The differences between the model predictions with and without a certain input variable are used as an indicator of the impact of that input variable on the predictive model [35–38].

$$\phi_i = \sum_{S \subseteq F \setminus \{i\}} \frac{|S|!(|F| - |S| - 1)!}{|F|!} [f_{S \cup \{i\}}(x_{S \cup \{i\}}) - f_S(x_S)] \quad (2)$$

2.4. Statistical Analysis of the Dataset

The dataset consisting of 482 data points was collected from the literature [29]. The input features of this dataset are the amounts of cement, fine aggregate, coarse aggregate, superplasticizer, supplementary cementitious materials (SCM), such as fly ash and silica fume, water-to-binder ratio, percentages of recycled coarse aggregate and natural fiber, the type and length of fiber and age of concrete, whereas the output feature is the splitting tensile strength of concrete. The distributions of each one of these features are shown in Figure 2 in histogram format. Each histogram plot contains a kernel density estimator function in addition to the minimum and maximum values of the feature, the standard deviation (σ), the mean value (μ), kurtosis (β) and skewness (γ) values.

The ranges of each input and output feature are also presented in Figure 3, in a parallel coordinates plot. In Figure 3, each data point is represented by a line connecting the corresponding values of the input features with the corresponding output feature. The lines are also colored with respect to the corresponding tensile strength value. As the tensile strength values increase, the line color of the data points changes from tones of purple to tones of yellow. In Figure 3, FT, TS, SP, NF, C, WB, FA, CA and RCA stand for the fiber type, splitting tensile strength, superplasticizer amount, natural fiber percentage, cement amount, water-to-binder ratio, fine aggregate amount, coarse aggregate amount and the recycled coarse aggregate percentage, respectively. The parallel coordinates plot in Figure 3 also presents an alternative visualization for the distribution of the different features. As an example, it can be observed that, data points with fiber types 6 and 5 corresponding to coir and ramie fibers result in higher tensile strength.

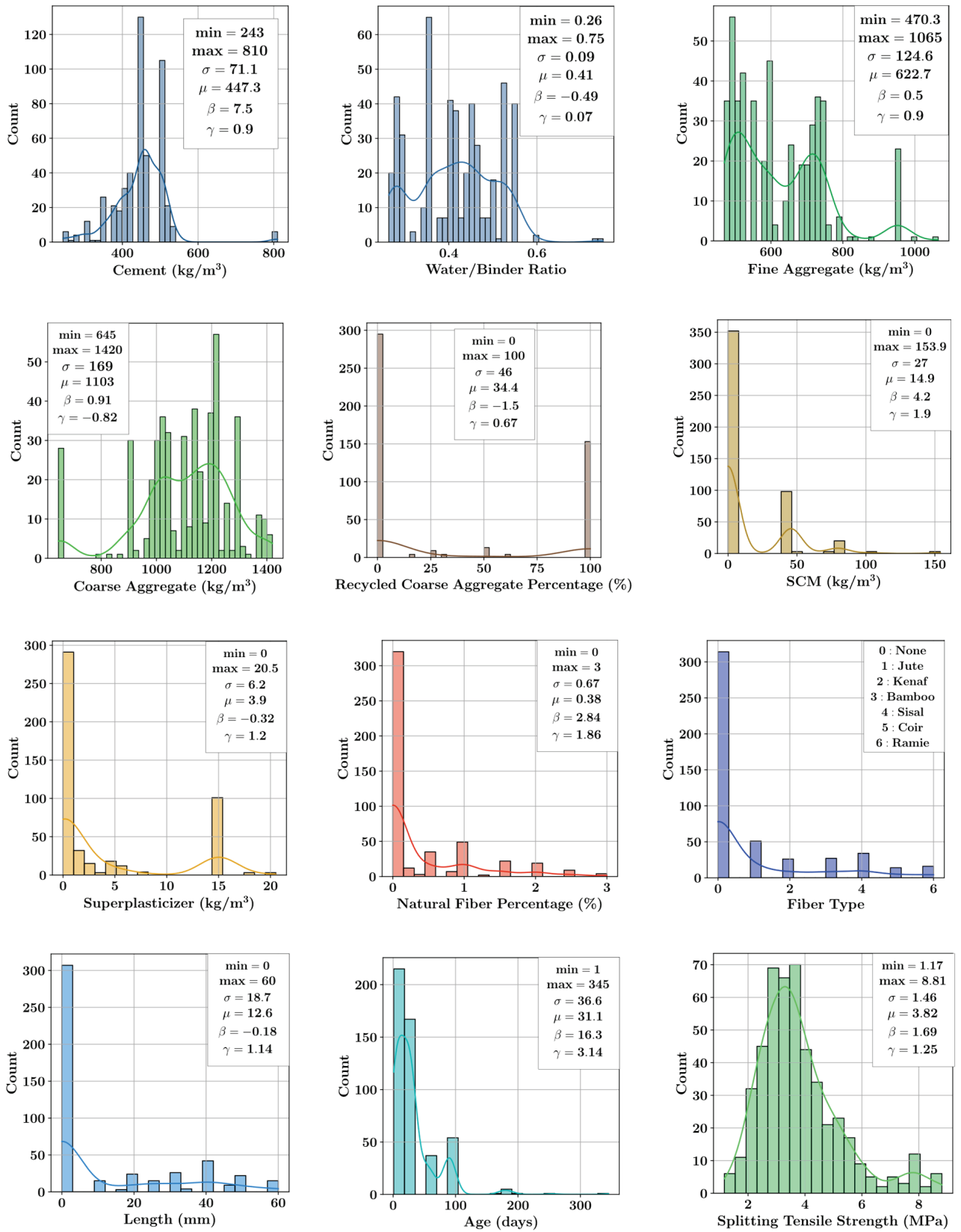


Figure 2. Distribution of the input and output features.

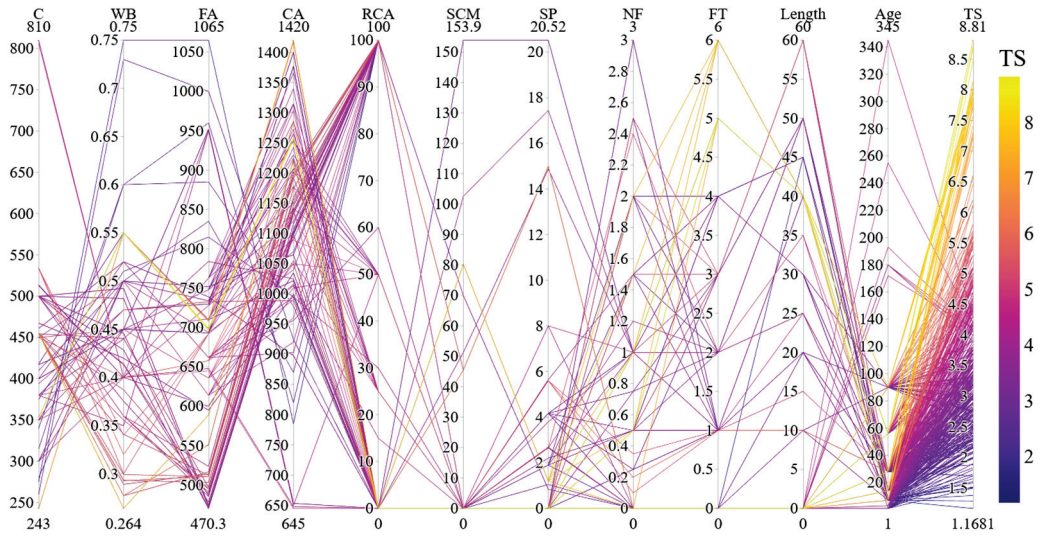


Figure 3. Parallel coordinates' plot of the dataset.

2.5. Outlier Detection Techniques

2.5.1. Principal Component Analysis (PCA)

PCA is a statistical technique used for dimensionality reduction. The method is particularly efficient in the case of datasets with a large number of input features. The PCA algorithm maps the data into a new vector space whose dimensions are called the principal components. Using the PCA technique, the dataset can be mapped into a lower dimensional vector space while avoiding any significant loss of information about the characteristics of the dataset. The principal components are ranked in such a way that the first components capture the largest variance in the dataset. As a result, the PCA technique makes the visualization and interpretation of higher dimensional datasets in a 2 or 3 dimensional vector space possible. However, it should be noted that the omission of the lower ranked principal components in the interpretation of datasets in 2 or 3 dimensions results in a loss of information [39]. In this study, the first 2 principal components have been used for visualization of the data points in the 2-dimensional Euclidean vector space. The positions of data points and their distances from the rest of the points in the dataset in 2-dimensional Euclidean space are used as an indicator of the normalcy or anomaly of a data point. The mapping of a dataset X into its principal components can be summarized as in Equation (3) [40].

$$X' = XW, \quad Cov(X) = \frac{1}{p-1}(X - \mu)^T(X - \mu) \quad (3)$$

In Equation (3), W is a transformation matrix whose columns are the eigenvectors of the covariance matrix of X in the decreasing order of the eigenvalues of $Cov(X)$. The transformed dataset in the vector space spanned by the principal components is denoted by X' . The total number of data points and the matrix of mean values of each input feature are denoted with p and μ , respectively. Further details of the PCA procedure can be found in [40].

2.5.2. Isolation Forest

The isolation forest algorithm is based on separating each data point using binary decision trees with the goal of identifying anomalous data points. The decision trees used for this purpose consist of nodes that include conditions about the values of different features of the data. Each node separates the dataset into two sections. One of these sections includes those data points that fulfill the condition of the node and the other

section includes those data points that do not fulfill the condition. The procedure is repeated with different conditions on the feature values until one of the sections contains only one isolated data point. Since anomalous data points tend to be farther away from the rest of the points in the dataset with respect to distance metrics such as the Euclidean distance, it takes on average a smaller number of node branching to isolate the anomalous points. This concept is further illustrated on a dataset consisting of 2-dimensional data points in Figure 4.

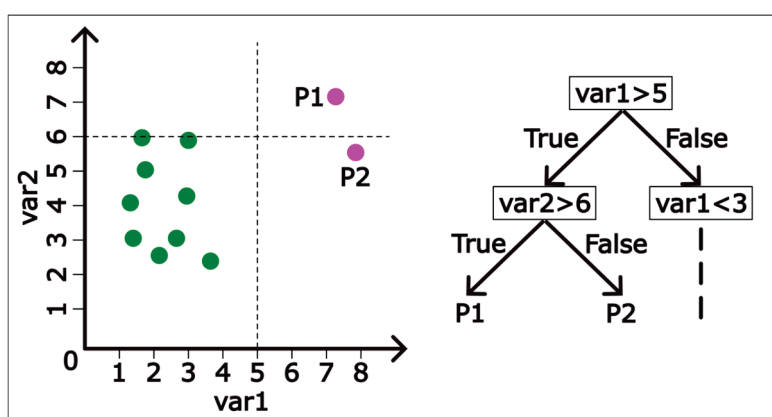


Figure 4. Isolation of data points.

In Figure 4, it can be observed that it took only 2 conditional branches to isolate the data points P1 and P2 which are positioned farther away from the rest of the points in the dataset. In the particular example of Figure 4, the points P1 and P2 could be designated as the outliers of the dataset, whereas the remaining data points are deemed as the inliers. The operation illustrated in Figure 4 could be repeated with different conditions in the nodes of the binary tree. After repeating this operation a certain number of times, the average number of conditional branchings necessary to isolate each point in the dataset is calculated. For the outlier points, this number is expected to be significantly smaller than the inliers which builds the basis of the isolation forest methodology. Further details of the isolation forest algorithm can be found in [41,42]. A summary of the entire machine learning model development and interpretation procedure can be seen in Figure 5.

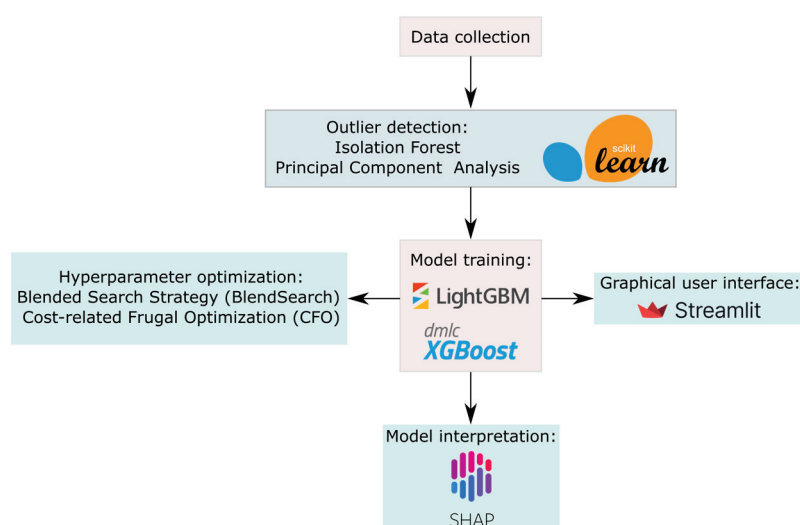


Figure 5. Predictive model development and interpretation.

3. Results

This section presents the performance of the machine learning models in predicting the ultimate tensile strength (f_t). The predictions of XGBoost, LightGBM, and extra trees regressor models were plotted against the experimental measurements. The performances of each predictive model were quantified using state-of-the-art error metrics such as the coefficient of determination and the root mean squared error. The predictive models were ranked according to their performance and the best performing model was analyzed in terms of the feature importance and the impacts of different input features on the model outputs. The Shapley additive explanations (SHAP) technique was used for analyzing the interdependencies of the features and the impacts of the features. The variation in the model output with respect to variations in different input features was visualized using individual conditional expectation (ICE) plots. Furthermore, using genetic programming, a new predictive closed form equation is proposed for the prediction of f_t .

3.1. Outlier Detection Using Isolation Forest and Principal Component Analysis

The performance of a predictive machine learning model can be significantly altered due to outliers in the dataset. Therefore, prior to training the predictive models, the outliers were detected using the isolation forest algorithm and principal component analysis. The isolation forest algorithm was used to determine the indices of the data points that were potentially anomalous. Afterwards, the dataset was mapped into two dimensions using the first two principal components of the dataset in order to facilitate the visualization of the data points. The first two principal components were found to capture 90% of the variance in the dataset. The contribution of each principal component to explain the total variance in the dataset is shown in Figure 6. From Figure 6, it can be observed that the principal components with index 7, 8, 9, 10 and 11 contain near zero information about the variance in the dataset.

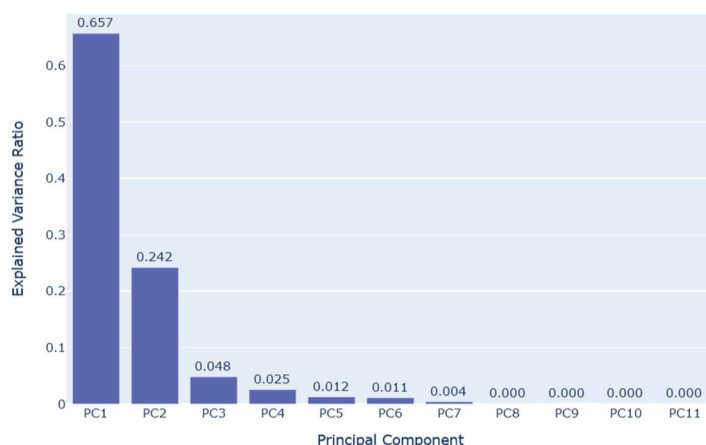


Figure 6. Explained variance ratios.

Figure 7 shows the distribution of the data points with respect to their first and second principal components. In Figure 7, the inlier data points are colored in blue whereas the outlier data points are colored in red. The data points were classified as inliers and outliers based on the results of the isolation forest algorithm. Figure 7 shows the outlier data points for four different levels of the contamination coefficient which determines the number of data points deemed as outliers. The largest number of outliers can be observed in Figure 7a for a contamination of 0.1 which corresponds to 10% of the data points being deemed as outliers.

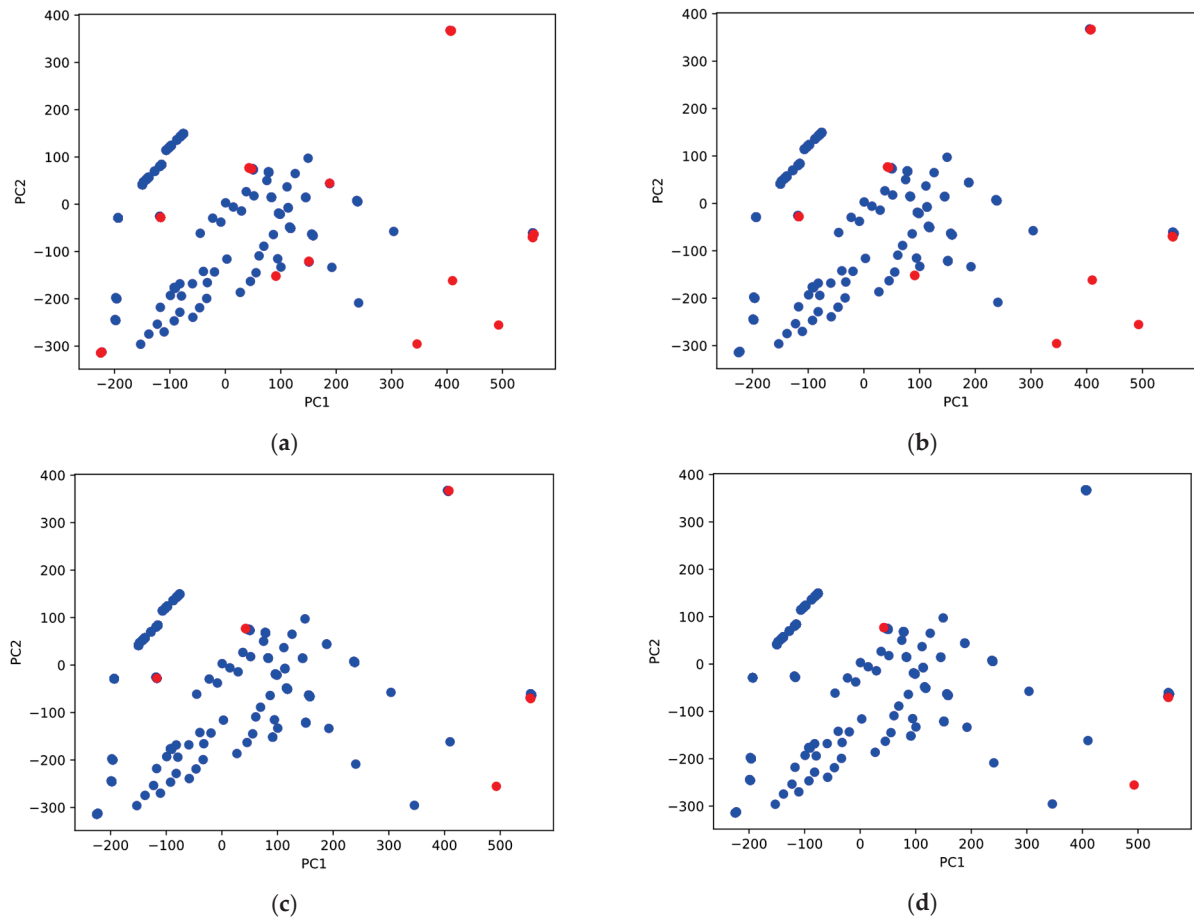


Figure 7. Outliers for (a) contamination = 0.1, (b) contamination = 0.06, (c) contamination = 0.02, (d) contamination = 0.01.

3.2. Predictions of the LightGBM, XGBoost and Extra Trees Regressors

In this section, the predictions of three predictive models are presented. Before training the models, 1% of the data points were removed as outliers. This percentage was selected in order to maximize the model performances. Figure 8 shows the variation in the predictive performances with respect to the amount of removed outliers represented by the contamination variable. In Figure 8, a contamination of 0.01 corresponds to 1% of the data points being removed from the dataset as outliers according to the isolation forest methodology. For each level of contamination in Figure 8, the inlier points of the dataset were randomly split into a training and test set in an 80% to 20% ratio. The model performances were measured on the test set after training the models on the training set. The average model performances resulting from 50 different random splits are presented in Figure 8. For all models in Figure 8, the contamination level of 0.01 delivered the best model performance. Therefore, this level of contamination was adopted for the rest of the analysis in this paper. It should be noted that increasing the contamination level beyond 0.02 is found to adversely affect the predictions.

Figure 8 shows that, on average, the best model performance could be obtained from the extra trees regressor. The fluctuation in the extra trees model performance for 50 different random splits of the inlier dataset can be seen in Figure 9. According to Figure 9, the extra trees model performance fluctuated between 0.948 and 0.987 on the test set with an average value of 0.971. In Figure 9, the maximum, minimum and average values of the R^2 scores are shown with green, red and blue dashed lines, respectively.

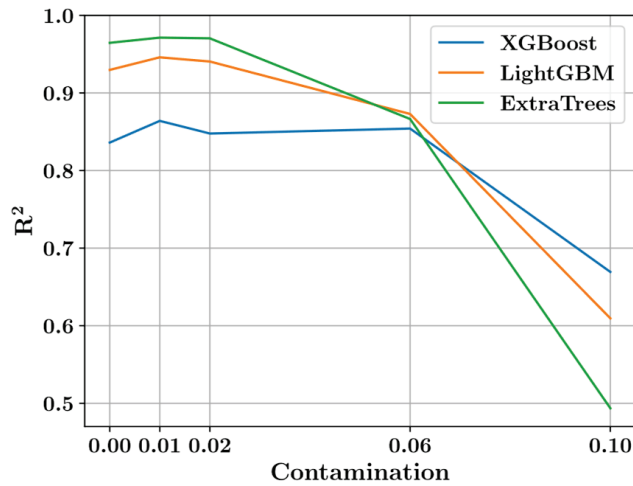


Figure 8. Model performances with respect to contamination.

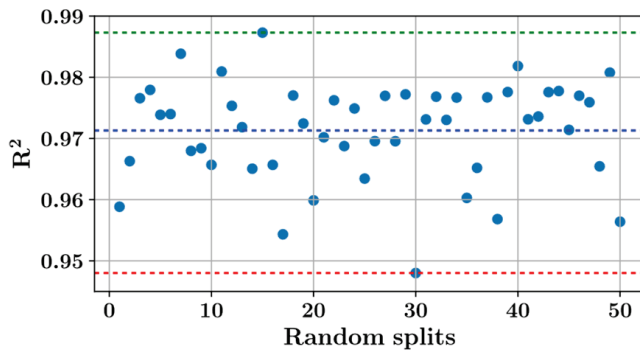


Figure 9. Extra trees model performance fluctuations on the test set.

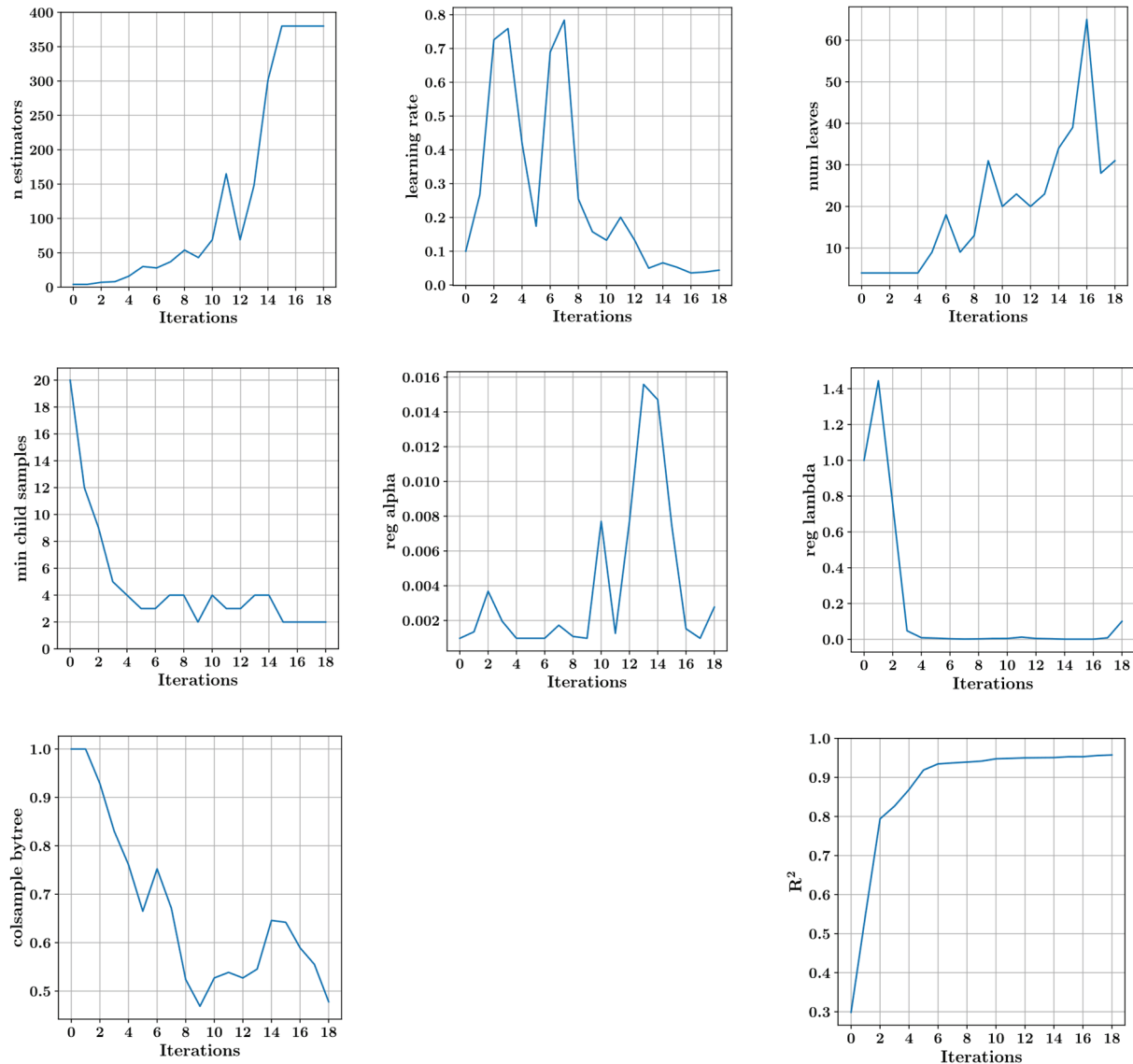
The hyperparameters of the LightGBM, extra trees and XGBoost models were optimized using the blended search strategy (BlendSearch) and cost-related frugal optimization (CFO) algorithm implemented in the FLAML package. The steps of the optimization are presented in Figure 10 for the hyperparameters of the LightGBM model. The optimized values of the hyperparameters for all three predictive models and the performances of the predictive models are listed in Table 1 and Table 2, respectively.

Table 1. Optimal hyperparameters.

Algorithm	Optimal Hyperparameters
LightGBM	n_estimators: 380, learning_rate: 0.0438, num_leaves: 31, max_depth: -1, min_child_samples: 2, subsample: 1.0, colsample_bytree: 0.4779, reg_alpha: 0.0028, reg_lambda: 0.1005, min_split_gain: 0.0, min_child_weight: 0.001
Extra Trees	n_estimators: 13, max_depth: None, min_samples_split: 2, min_samples_leaf: 1, min_weight_fraction_leaf: 0.0, max_leaf_nodes: 191, min_impurity_decrease: 0.0, ccp_alpha: 0.0, max_samples: None
XGBoost	n_estimators: 96, learning_rate: 0.1868, max_depth: 0, subsample: 0.5599, colsample_bytree: 0.6668, reg_alpha: 0.3023, reg_lambda: 0.00097, min_child_weight: 0.218

Table 2. Performances of the optimized models.

Algorithm	R ² Score	
	Training Set	Test Set
LightGBM	0.993	0.946
Extra Trees	0.992	0.971
XGBoost	0.991	0.864

**Figure 10.** Hyperparameter optimization steps.

For each predictive model, the predictions on the training and test set are visualized in Figure 11 for the random split corresponding to the best model performance. The predictions on the training and test sets are plotted in different colors. Furthermore, the $\pm 10\%$ deviation lines from a perfect prediction are plotted as dashed lines. In addition, for each predictive model, a linear curve was fitted which approximates the relationship between the predicted and true tensile strength values. In addition to the R² score, the slope of the fitted curve can be used as an indicator of predictive performance such that slopes closer to 1 indicate more accurate prediction.

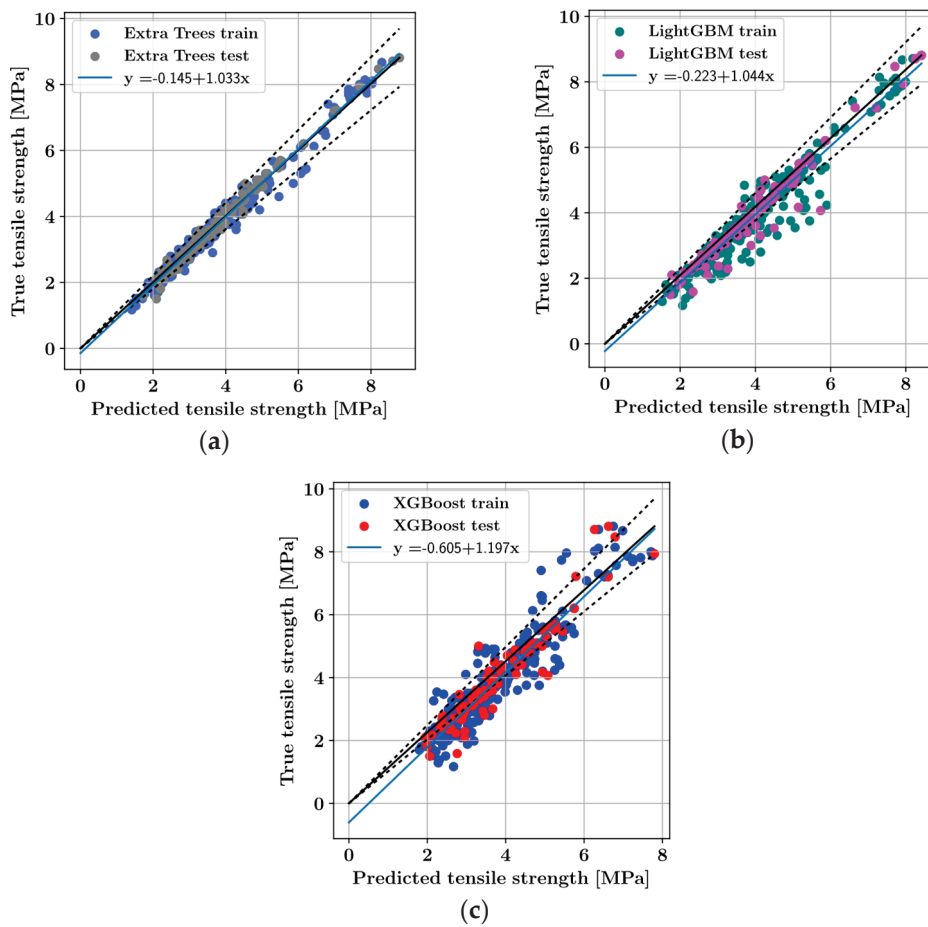


Figure 11. Predicted and true values for (a) extra trees, (b) LightGBM, (c) XGBoost.

3.3. Graphical User Interface

In order to make the developed models available, an online graphical user interface was developed using the Streamlit platform. The online graphical user interface can be accessed through the link <https://splitting-tensile-composite.streamlit.app> (accessed on 6 January 2025). A screenshot of this user interface can be seen in Figure 12. The user interface is responsive to user input such that every time a new input feature value is entered by the user, the output value is automatically updated. The updated tensile strength value is then printed in the lower right portion of the screen in blue color. The input boxes only accept input feature values in certain predefined ranges based on the dataset on which the predictive model has been trained. In this sense, the predictive model is limited by the upper and lower bounds of the feature values included in the training set. The user interface is based on the extra trees regressor which delivered the best performance in terms of the coefficient of determination (R^2 score).

3.4. Feature Importance Analysis Using Shapley Additive Explanations (SHAP)

In this section, the impacts of the input features on the extra trees model predictions were quantified using a feature importance bar plot (Figure 13), a summary plot (Figure 14) and a heatmap plot (Figure 15) generated through the SHAP algorithm. Each one of these plots indicated that the age of the specimen, fiber type and water-to-binder ratio had the greatest impact on the predictive model output. The bar plot in Figure 13 is generated by calculating the SHAP value of each input feature and taking the mean absolute value of these SHAP values over the entire dataset. Figure 13 shows that the amount of cement, the

percentage of the recycled coarse aggregate and the natural fiber percentage had the least impact on the model predictions.

Cement [kg/m^3]	Superplasticizer [kg/m^3]
452.00 — +	0.00 — +
Water/Binder Ratio	Natural Fiber Percentage
0.40 — +	0.00 — +
Fine Aggregate	Fiber Type
600.00 — +	None ▼
Coarse Aggregate	Fiber Length [mm]
1200.00 — +	0.00 — +
Recycled Coarse Aggregate [%]	Age [$days$]
20.00 — +	28.00 — +
Supplementary Cementitious Materials [kg/m^3]	The tensile strength = 3.64 [MPa]
0.00 — +	

Figure 12. Online graphical user interface.

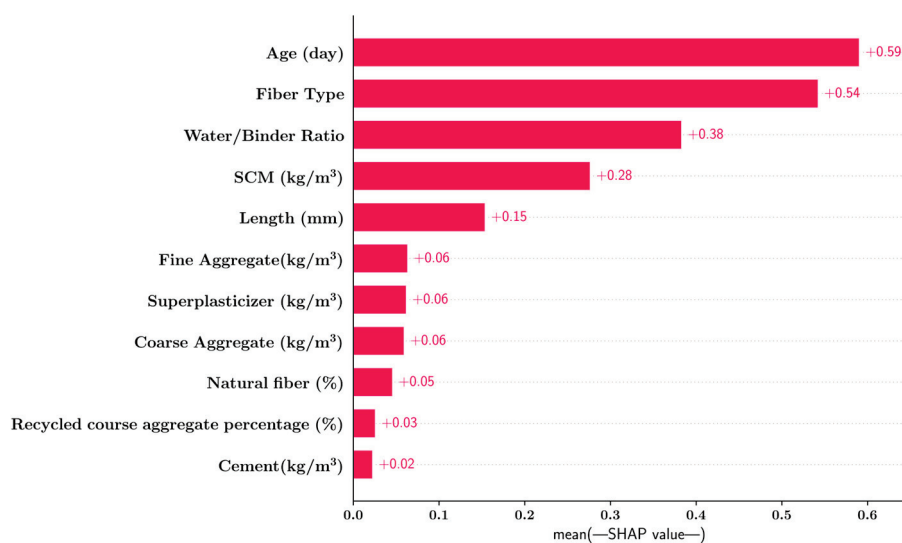


Figure 13. SHAP feature importances.

The SHAP summary plot in Figure 14 contains additional information about the impacts of different input features in a color-coded way. In Figure 14, each data point is represented by a dot which has a certain color ranging between the tones of blue and red. The color is used as an indicator of the feature value in the summary plot such that high values of a feature lead to data points colored in red tones whereas low values lead to data points colored in blue. Furthermore, the position of a dot in the horizontal direction contains information about the impact of an input feature on the model output. For any input feature, as the position of a data point is further away from the zero SHAP value, the impact of that feature on the model output increases. Data points positioned to the right side of the zero SHAP value indicate a positive SHAP value and an increasing effect of

the input feature on the model output. On the other hand, data points positioned to the left-hand side of the zero SHAP value indicate a negative SHAP value and a decreasing effect of that input feature on the model output. Accordingly, it can be observed that the inclusion of the specimen age as a predictive feature has an increasing effect on the model predictions for all data points. On the other hand, there is no clear indication about the effect of the age value on the model prediction since both blue and red colored data points are in equal distance from the zero SHAP point. The inclusion of the water-to-binder ratio is observed to have the opposite effect on the model outputs since all data points are on the left-hand side of the zero SHAP point. The effect of fiber type is such that the coir and ramie type fibers are coded with the integer values of five and six, respectively, as the two highest values of this feature. Since red colored data points at fiber-type level are far from the zero value to the right side, it can be inferred that the usage of coir and ramie as fiber types have an increasing effect on the tensile strength. The fiber types, jute and kenaf are coded with the integer values of one and two, respectively, as the two lowest values of this feature, in addition to the no fiber case coded with 0. Since the blue colored data points at the fiber type level are situated at the left and side of the zero SHAP value, it can be inferred that the usage of jute or kenaf or the omission of fibers has a decreasing effect on the tensile strength.

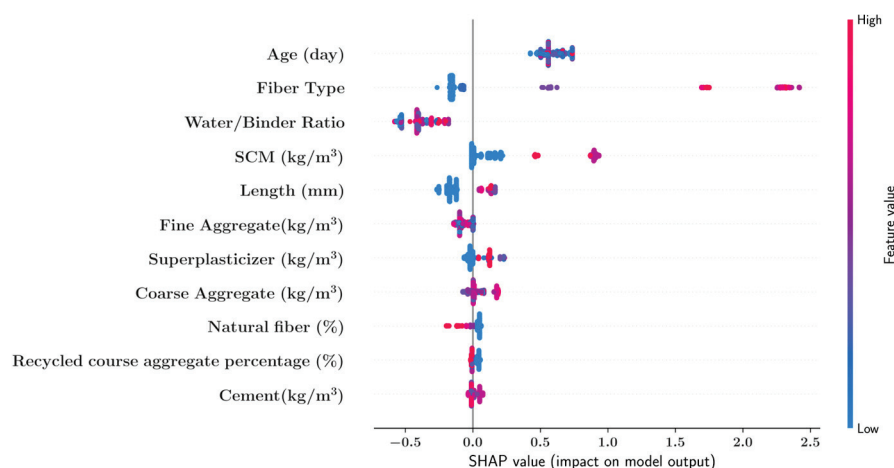


Figure 14. SHAP summary plot.

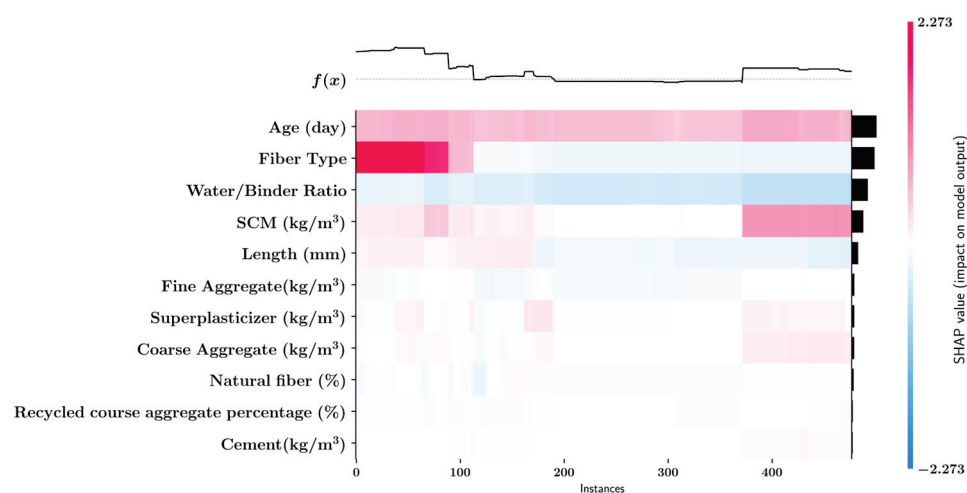


Figure 15. SHAP heatmap plot.

The SHAP heatmap plot in Figure 15 presents an alternative visualization of the feature impacts where each data point is represented with a vertical line colored according to the

SHAP value of a feature. High SHAP values indicating high impact on the model output are color-coded in tones of red whereas low SHAP values are color-coded in blue tones. The heatmap plot also includes the predicted tensile strength values for each data point as the function $f(x)$ at the upper side of the plot. Furthermore, feature importances are included at the right-hand side of the plot as black horizontal bars. It can be observed that features with the lowest impact, such as cement amount and recycled coarse aggregate percentage, are colored in tones of white since these features are associated with low SHAP values.

4. Conclusions

The inclusion of recycled coarse aggregate as a component of concrete in an attempt to replace natural aggregates has the potential to improve the sustainability of the construction industry significantly. However, the replacement of natural aggregates with recycled aggregates has been observed to reduce concrete strength. In order to remedy this effect of aggregate replacement, the addition of fibers into the concrete mix has been proposed as a viable option. However, in the literature, there is only a limited number of studies investigating the effect of natural fibers on the concrete properties. Furthermore, most structural design codes are lacking closed form equations that describe the behavior of concrete with recycled coarse aggregate and natural fiber. This study attempts to address this lack of information about the behavior of concrete using a data-driven approach. To this end, a recently published dataset of 482 experimental results was used to train state-of-the-art decision tree-based machine learning models. The predictions of these models were visualized and their performances quantified using the coefficient of determination (R^2 score) which is a suitable metric for regression tasks since, unlike metrics such as mean absolute error or mean squared error, the R^2 score does not depend on the order of magnitude of the predicted quantity. The main findings and contributions of this study can be summarized as follows:

- (1) The gradient boosting predictive models predicted the splitting tensile strength of natural fiber reinforced recycled coarse aggregate concrete with an R^2 score greater than 0.95. The most accurate predictions were obtained from the extra trees regressor with an R^2 score of 0.971 on the test set.
- (2) The prediction accuracy of the models could be improved by using principal component analysis and isolation forest as outlier detection techniques. By designating 1% of the data points positioned farthest from the rest of the dataset as outliers, the R^2 score of the extra trees model could be enhanced from 0.965 to 0.971 on the test set.
- (3) An online graphical user interface has been made available on the Streamlit platform which can be accessed through the link <https://splitting-tensile-composite.streamlit.app> (accessed on 6 January 2025). It should be noted that the accuracy of the predictions made by the online tool is limited to the range of feature values included in the dataset on which the models were trained. The ranges of the input features are presented in Section 2.4. Further research needs to be carried out in order to expand the range of applicability of the machine learning models.
- (4) The impacts of the input features on the machine learning model predictions were quantified using the SHAP methodology. It was observed that the age of the concrete specimens, the type of fiber used and the water/binder ratio had the highest impact on the predicted tensile strength whereas the amount of cement and the percentages of the recycled coarse aggregate and natural fiber had the least impact.

Although the results of this data-driven approach to the prediction of mechanical properties were accurate, it should be noted that the applicability of the models is limited to the input feature ranges available in the training set. In order to achieve widely applicable data-driven predictive models, it is necessary to significantly expand the dataset to include

wide ranges of input features. Future research in this area should include larger datasets in this area as they become available. As an alternative, the datasets could be enhanced with the results obtained from numerical simulations or by synthetic data generation using generative neural networks. In addition, future research could focus on the compressive strength prediction of natural fiber-reinforced recycled aggregate concrete which is a more important parameter in concrete design compared to the tensile strength.

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Article

Numerical Simulation of the Thermal Effect of Firebrand Deposition on External Composite Walls

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Abstract: This paper presents research concerning simulating the thermal firebrand effect due to its accumulation in exterior construction wall elements by developing a 3D finite element model (FEM) via ABAQUS (2022) software to analyze the exterior walls commonly applied to the exterior of dwellings in southern Europe and South America. A non-linear thermal transient analysis is undertaken, in which the results are directly compared with a previous experimental campaign, in which firebrands are deposited on localized surfaces of construction wall specimens, and the temperature is measured in the several layers of the construction elements. The walls are composite elements, made of different layer combinations of masonry brick and wood, varying the type of thermal insulation in the internal core from cork to classical rigid rockwool and polystyrene foam (XPS). It can be summarized from the results that the FEM effectively simulates the thermal response of brick, normal wood (NW), and cross-laminated timber (CLT) walls when insulated with materials like cork or rockwool coated with mortar against firebrand accumulation. However, the lack of accounting for uncontrolled combustion leads to inconsistent results. Additionally, for walls using XPS as the insulation material, the model requires further refinement to accurately simulate the melting phenomenon and its thermal impact.

Keywords: thermal numerical response; temperature vs. time curve; firebrand accumulation; composite wall elements

1. Introduction

Wildland fires (WLFs) cause significant material losses and human casualties. Despite ongoing efforts by the scientific community to mitigate their risks, the number of devastating WLFs fires has dramatically increased in recent years [1,2]. This surge is primarily attributed to factors such as climate change, inadequate fuel management practices, and the rapid expansion of human development in WLFs areas. Recent WLFs have caused widespread devastation worldwide.

In Portugal, the 2017 fires led to 117 deaths and 204 injuries [3]. In September 2024, Portugal also battled intense WLFs, particularly in the central and northern regions. The combination of unusually high temperatures and severe drought worsened the situation, making it difficult to control the fires as they spread rapidly. At the time of writing, these fires had already consumed over 135,000 hectares of land and resulted in nine fatalities [4]. Canada experienced its worst WLF season in 2023, with over 17.5 million hectares

burned [5]. In the U.S., California and Hawaii were hit by devastating WLFs, including the deadly Maui fire, which claimed at least 97 lives and destroyed 2200 buildings [6]. In South America, Chile faced severe WLFs in February 2023, burning 270,000 hectares and causing 26 deaths, while Bolivia's wildfires severely affected the Amazon and Pantanal wetlands [7]. Recently, wildfires swept through the Los Angeles region, causing unprecedented destruction. The fires claimed at least 24 lives, forced thousands to evacuate, and destroyed over 12,000 buildings, covering an area of more than 160 km², larger than the city of San Francisco. Strong Santa Ana winds intensified the blazes, particularly in the Palisades, Eaton, Kenneth, and Hurst regions. Early estimates suggest these could be the costliest wildfires in U.S. history, with projected economic damages ranging from USD 135 billion to USD 150 billion [8].

Firebrands are a major mechanism in the spread of wildfires and play a crucial role in igniting secondary fires [9]. These embers, carried by the wind, can travel distances ranging from 2 km to 9 km ahead of the original fire source [3]. Firebrands remain incandescent for several hours, further increasing their potential to start new fires far from the initial blaze [10]. This phenomenon presents a significant challenge for wildfire management, as studies suggest that at least half of wildfires are ignited by secondary fires, which are often caused by firebrands [11]. In recent decades, there has been research conducted on the topic of firebrand transportation [12–18]. In this context, studies have focused on utilizing firebrand generators to simulate firebrand ignition in construction elements, aiming to uncover and assess their vulnerabilities to such hazards [19–21]. On the other hand, the scientific community has highlighted another mechanism of firebrand ignition, emphasizing the impact of firebrand accumulation on construction elements. Santamaria et al. [22] conducted an experimental study to evaluate the impact of smoldering embers and wood degradation due to thermal exposure. Their findings revealed that flaming ignition occurred when 60 g of embers had accumulated on the wood. The detailed experiments conducted by Suzuki and Manzello [23] are highly relevant, demonstrating that wind speed has a significant influence on the smoldering combustion intensity of accumulated firebrands.

The investigation of Sayaka et al. and Suzuki and Manzello [24] revealed that accumulated firebrands could cause ignitions on fuel beds with high FMC, and the upper limit of FMC for ignition increased from 6 m/s to 8 m/s. Bicelli et al. [25] conducted a numerical study employing a temperature-dependent finite element method to simulate the thermal effects of firebrand accumulation on flat surfaces, using heat flux transmission via convection and radiation. Their findings demonstrated that moisture content up to 20% significantly impacts temperature distribution, underscoring the potential for thermal optimization in designing wildfire-resistant structures. Zhu et al. [26] conducted experimental and numerical studies to examine the glowing combustion behavior and ignition capabilities of firebrand accumulations. The research focused on two firebrand sizes, four coverage densities, and six airspeeds, revealing that airflow is the primary factor influencing the glowing temperatures of firebrands. Analytical modelling demonstrated that the ignition time is primarily governed by the combustion behavior of the firebrands and the heat transfer to the fuel, with significant influence from firebrand size and airflow, and a lesser impact from coverage density. Arruda et al. [27] conducted a detailed experimental study, followed by numerical validation, to evaluate the impact of firebrand accumulations on construction materials. They found that rigid rock wool, aluminum, and cork could withstand firebrand accumulations for extended periods, exceeding 200 min.

The response of construction elements to firebrand accumulation remains insufficiently understood due to the diverse range of materials used and the numerous influencing factors, such as material properties, geometric configurations, and environmental conditions. Ex-

perimental studies, while critical for understanding firebrand impacts, are often constrained by their high costs and time-intensive processes. This highlights the need for alternative methods, such as numerical simulations. The primary objective of this study is to simulate a previous experimental campaign that investigated the response of exterior dwelling walls to firebrand accumulation using the finite element model (FEM) via ABAQUS (2022) software. This simulation aims to evaluate the effectiveness of finite element software ABAQUS (2022) in accurately modeling the impact of firebrand accumulation on exterior walls. The predicted results from the simulations are compared against the experimental measurements. Material properties were sourced from the existing literature and design codes to serve as input parameters. The analysis incorporates both geometric and material non-linearities, ensuring a comprehensive evaluation of the behavior of the studied wall specimens under firebrand accumulation.

2. Materials and Methods

This section outlines the methodology used to simulate the thermal effects of firebrand accumulation on exterior composite walls. It clearly details the experimental setup, numerical modeling approach, material properties, boundary conditions, and simulation parameters employed in ABAQUS (2022) software.

2.1. Experimental Campaign Summary

The firebrand accumulation tests outlined in this study were carried out in the laboratory of the CERIS-IST research institute in Portugal. The experimental campaign aimed to evaluate the response of exterior dwelling walls to firebrand accumulation. Three types of wall core layers were studied: bricks, designed according to the Exterior Thermal Insulation Composite Systems (ETICS) methodology, cross-laminated timber (CLT), and normal wood (NW), all utilizing the sandwich methodology, as illustrated in Figure 1. Each wall specimen featured a combination of thermal insulation materials, including composite cork, rigid rockwool, polystyrene foam (XPS), and impermeable membranes, paired with a variety of mortars such as Sika, Weber, and the fire-protective Tria mortar, as detailed in Table 1. Full details of the experimental setup, including a description of the chosen materials and layer geometry, are described in [28].

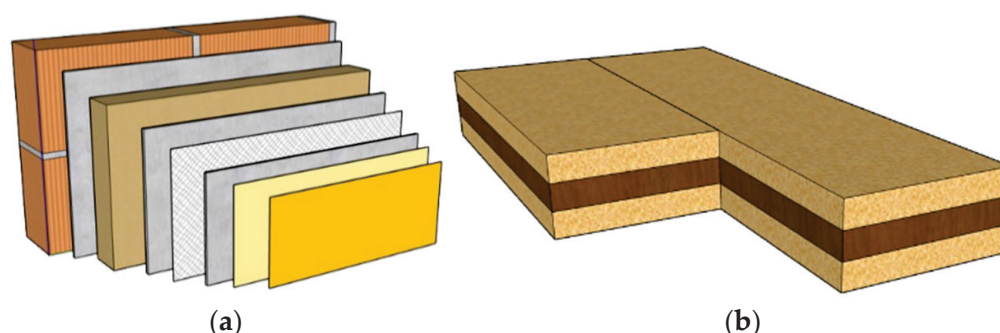


Figure 1. Composition system details of the composite walls. (a) ETICS; (b) sandwich.

Table 1. Specifications of the Tested Brick Wall Specimens.

Wall Designation	Structural Material	Setting Method	Protective Coating	Thermal Insulation Material	Mortar
B	Brick	ETICS	None	None	None
B-W-R	Brick	ETICS	None	Rockwool	Weber
B-S-R	Brick	ETICS	None	Rockwool	Sika
B-W-X	Brick	ETICS	None	XPS	Weber
B-S-X	Brick	ETICS	None	XPS	Sika
B-T-X	Brick	ETICS	None	XPS	Tria
B-W-C	Brick	ETICS	None	Cork	Weber
B-S-C	Brick	ETICS	None	Cork	Sika
NW-N-TP	Normal Wood	Normal	Tria Paint	None	None
NW-S-R-TM	Normal Wood	Sandwich	None	Rockwool	Tria Mortar
NW-S-R	Normal Wood	Normal	None	Rockwool	None
NW-S-TP-R	Normal Wood	Sandwich	Tria Paint	Rockwool	None
NW-S-C	Normal Wood	Sandwich	None	Cork	None
NW-S-C-TM	Normal Wood	Sandwich	None	Cork	Tria Mortar
NW-S-TP-C	Normal Wood	Sandwich	Tria Paint	Cork	None
CLT-N	CLT	Normal	None	None	None
CLT-S-R	CLT	Sandwich	None	Rockwool	None
CLT-N-TM	CLT	Normal	None	None	Tria Mortar
CLT-N-R-TM	CLT	Normal	None	Rockwool	Tria Mortar
CLT-N-TP	CLT	Normal	Tria Paint	None	None
CLT-S-TP-R	CLT	Sandwich	Tria Paint	Rockwool	None
CLT-S-C	CLT	Sandwich	None	Cork	None

Each wall specimen was constructed in an L-shaped configuration ($500 \times 600 \times 400 \text{ mm}^3$), supported by a concrete base slab. Mortar was applied at a fixed thickness of 9 mm, covering both the walls and the connection to the slab to prevent heat flux transmission. The tests followed California Building Code Chapter 7A-4 guidelines [29], exposing each specimen to 1 kg of firebrands at a wind speed of $1.31 \pm 0.13 \text{ m/s}$. Thermocouples were embedded in each wall layer, typically one at the base and two per layer on each side, to monitor and record temperature data at a frequency of 1 Hz; the durations of each test ranged from 100 to 200 min, with variations primarily influenced by safety concerns.

2.2. Numerical Simulation

The experimental campaign described in the previous section was simulated using a 3D FEM in ABAQUS (2022) software [30]. The 3D FEM analyses were performed using several key parameters, as detailed below. Surface convection and radiation according the value provided in Eurocode 4 part 1–2 [31] were incorporated into all simulations. All the thermal analyses were performed in Kelvin, with absolute zero as the reference temperature. Since thermal properties vary with temperature, a non-linear transient thermal analysis was conducted using the incremental Newton–Raphson iterative method. To ensure the accuracy of the numerical solution, the maximum allowable temperature change per increment was limited to 10 K, which may necessitate a reduced time step to achieve a more reliable solution for non-linear thermal behavior [32].

2.2.1. Material Properties

In this section, the thermal properties of all materials used in the numerical models, including density, thermal conductivity, and specific heat, are presented. For brick, NW, and CLT, the thermal properties are based on values at ambient temperature from previous studies, with adjustments for elevated temperatures according to Eurocodes 5 and 6, part

1–2 [33,34]. The variation in thermal properties of the mortars was adopted in accordance with a study on the fire behavior of different mortar types [35]. Regarding rockwool, both density and specific heat remain constant at ambient and elevated temperatures. Additionally, temperature-dependent thermal conductivity values were obtained from the literature [36–38]. The properties of polystyrene foam (XPS) at elevated temperatures were adopted using ambient temperature values of the previous tests [38], and the temperature variations were derived from Duarte et al. [37]. Finally, for cork, the density value at ambient temperature was adopted from previous work [39], and the reduction ratio at elevated temperatures was taken from the literature [40]. The thermal conductivity and specific heat at ambient temperature, along with the variation in reduction ratios at elevated temperatures, were also adopted from prior studies [39]. The thermal properties for all materials at ambient temperature are presented in Table 2, and the variation ratios with the temperature of each thermal property for all materials are illustrated in Figure 2. The values were used as input in ABAQUS (2022) software according to Equation (1).

$$\begin{aligned}\rho(T) &= \rho_{20} \cdot f_{\rho}(T) \\ k_c(T) &= k_{c,20} \cdot f_{k_c}(T) \\ c_p(T) &= c_{p,20} \cdot f_{c_p}(T)\end{aligned}\quad (1)$$

where ρ , k_c , and c_p represent the density, thermal conductivity, and specific heat of the material, respectively. T indicates the given temperature, while 20 refers to the ambient temperature. f denotes the correction ratio for each parameter.

Table 2. Thermal properties of all studied materials at ambient temperature.

Material	Density (kg/m ³)	Thermal Conductivity (W/m·K)	Specific Heat (J/(kg·K))
Brick	1050	0.7	564
NW	510	0.12	1530
CLT	354	0.14	1530
Rockwool	140	0.04	840
XPS	55	0.027	1210
Cork	219.8	0.04	899.5
Weber/Sika	1400	0.45	360
Tria	760	0.122	360

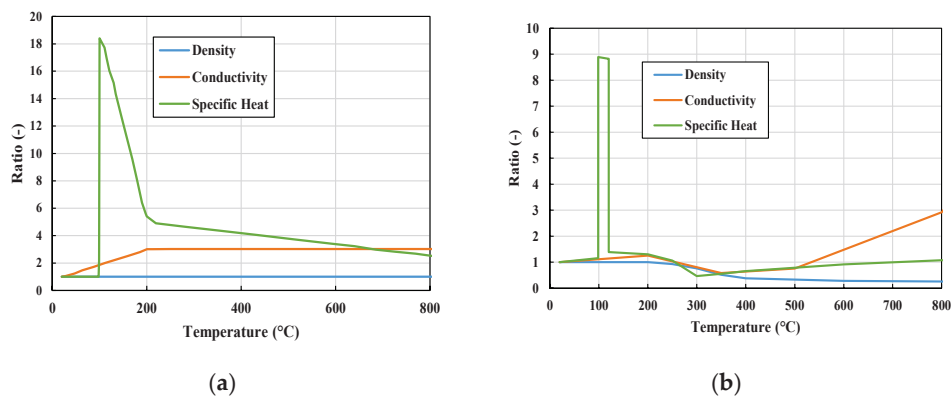


Figure 2. Cont.

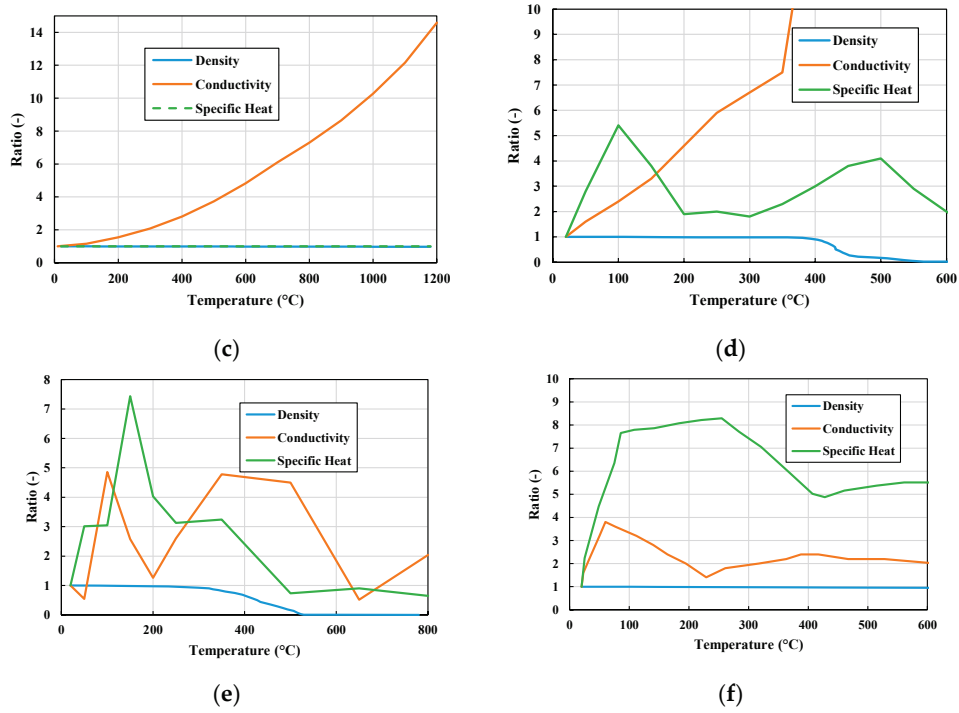


Figure 2. Variation in thermal property ratios with temperature for (a) brick, (b) NW/CLT, (c) rockwool, (d) XPS, (e) cork, and (f) Weber/Sika/Tria Mortar.

2.2.2. Firebrand Input and Boundary Conditions

Based on the studies conducted by Bearinger et al. [41], which demonstrated that firebrand can transfer heat flux on planar surfaces by convection and radiation, convection and radiation boundary conditions were applied to simulate the heat flux transfer between the firebrands and the wall specimen's surface. This approach aligns with standard fire safety design practices outlined in structural codes [42] for structures exposed to nominal or performance-based fire scenarios. The firebrand accumulation on the wall specimens was simulated by applying the average temperatures recorded during the tests for each exposed side. These temperatures were used as inputs for convection and radiation within a triangular area on each exposed side of the modeled wall specimens. Additionally, ambient temperature was applied to all outer surfaces except the fire-exposed surface. The film coefficient was set to $hf = 35 \text{ W}/(\text{m}^2\text{K})$ for the firebrand-exposed surface and $hf = 9 \text{ W}/(\text{m}^2\text{K})$ for the unexposed surface. The Boltzmann constant was $\sigma = 5.67 \times 10^{-8} \text{ W}/\text{m}^2\text{K}^4$, and the emissivity of the fire was $\varepsilon = 1$. Figure 3 provides a detailed illustration of the boundary conditions for the standard wall specimen.

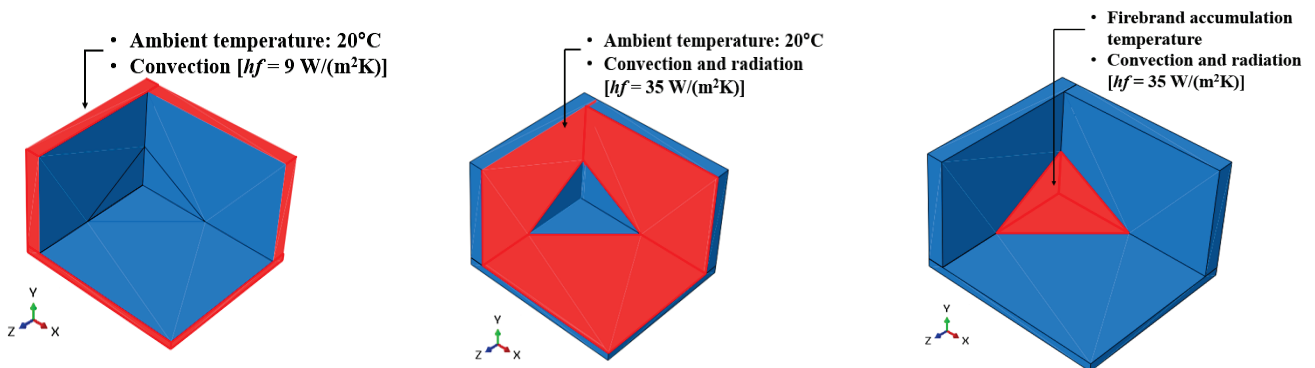


Figure 3. The applied boundary conditions.

The interaction between the materials was simulated using gap radiation, $\varepsilon = 1.0$, a feature in ABAQUS designed to model radiation between surfaces separated by a narrow gap. However, the interaction between the mortar and other materials was treated as full contact. The view factor, which ranges from 0 to 1, represents the degree of radiative heat transfer, where 0 indicates that there is no radiative heat transfer and 1 indicates that there is no gap. For this simulation, a view factor of 0.99 was used, based on the assumption that the gap between surfaces is less than 1 mm [43].

2.2.3. Chosen Mesh

The mesh employed in this FEM is shown in Figure 4. For all materials, the mesh size was set to 30 mm along the longitudinal direction and 10 mm along the transverse direction (specimen depth). For the mortar layer, a finer mesh size of 5 mm in depth was used to optimize convergence and computational efficiency. The element type used was DC3D8 (8-node linear heat transfer brick) with full integration, which is part of the ABAQUS heat transfer family, to accurately capture the temperature field distribution under a significant flux gradient. Since the maximum allowable nodal temperature change per increment was set to 10 in the ABAQUS/Standard implicit method, ABAQUS automatically controlled the time increments to ensure that no node exceeded this value during any analysis increment [30].

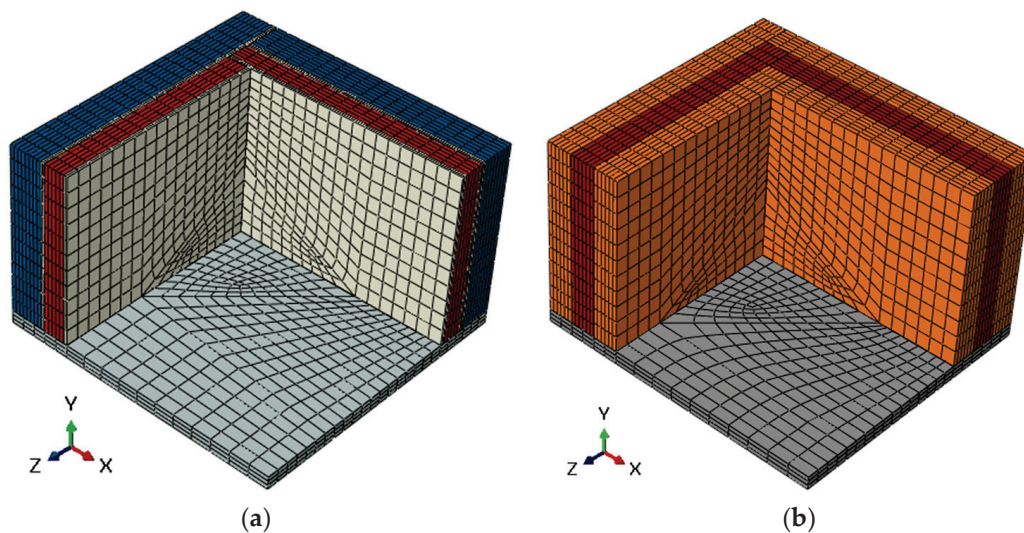


Figure 4. Standard illustration of the wall specimens modelled for (a) bricks walls (ETICS) and (b) wood walls (sandwich).

3. Results

Thermal analysis was conducted on all studied wall specimens to simulate their response to firebrand accumulation. The obtained results were then compared with the measured data from the tests, focusing on the average temperature recorded across each layer of the wall specimens, as illustrated in Figure 5.

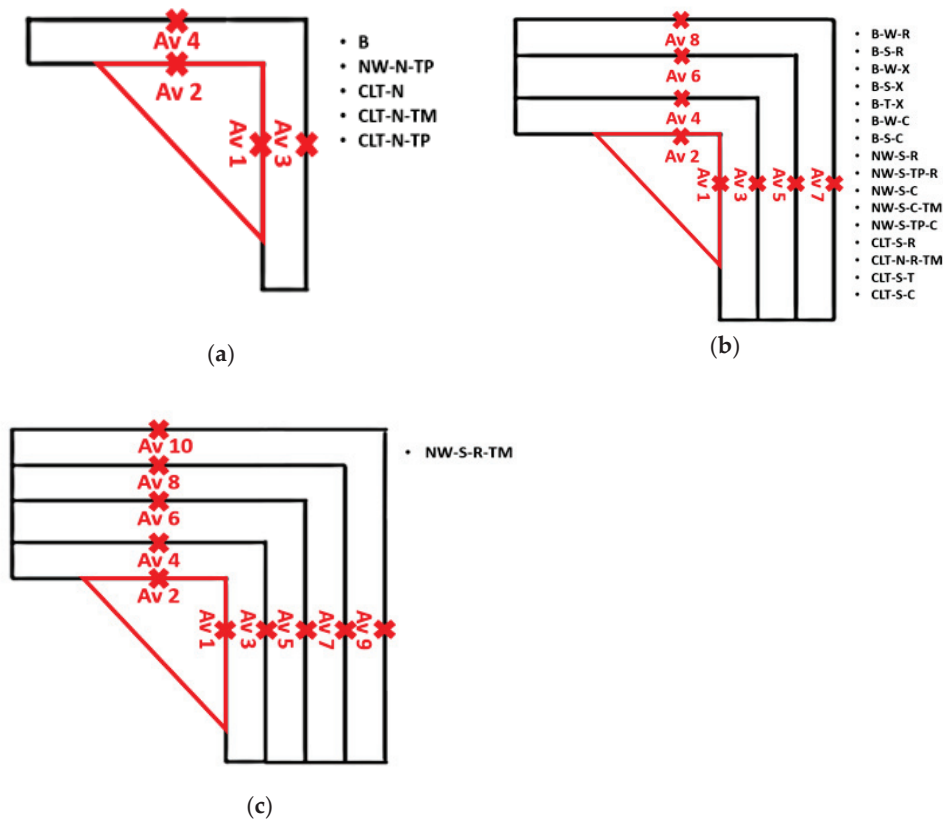


Figure 5. Illustration of the recording locations for average temperatures across each layer of the studied wall specimens: (a) single layer, (b) three layers, (c) four layers.

3.1. Brick Wall Specimens

The comparison between the numerically predicted and experimentally measured test results for the brick wall specimens is shown in Figure 6. It can be noted that the predicted results for the B, B-W-R, B-S-R, B-T-X, B-W-X, and B-S-C specimens demonstrate close agreement with the measured test results. However, for the B-W-X and B-S-X specimens, which incorporate XPS as the thermal insulation material, a more significant inconsistency is observed. This is attributed to the melting of XPS and destruction of the thermocouple, leading to the uncertainty of the test results.

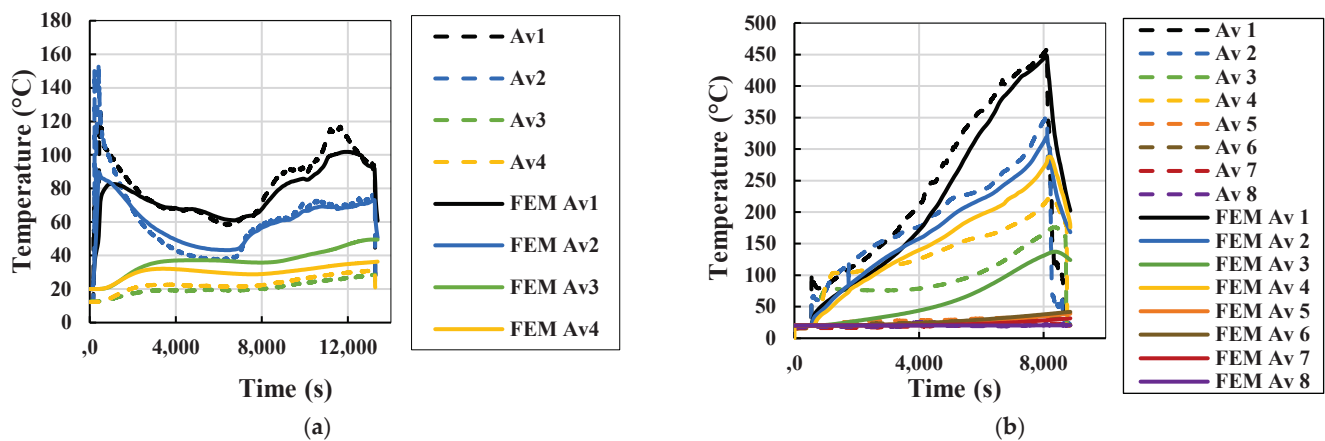


Figure 6. Cont.

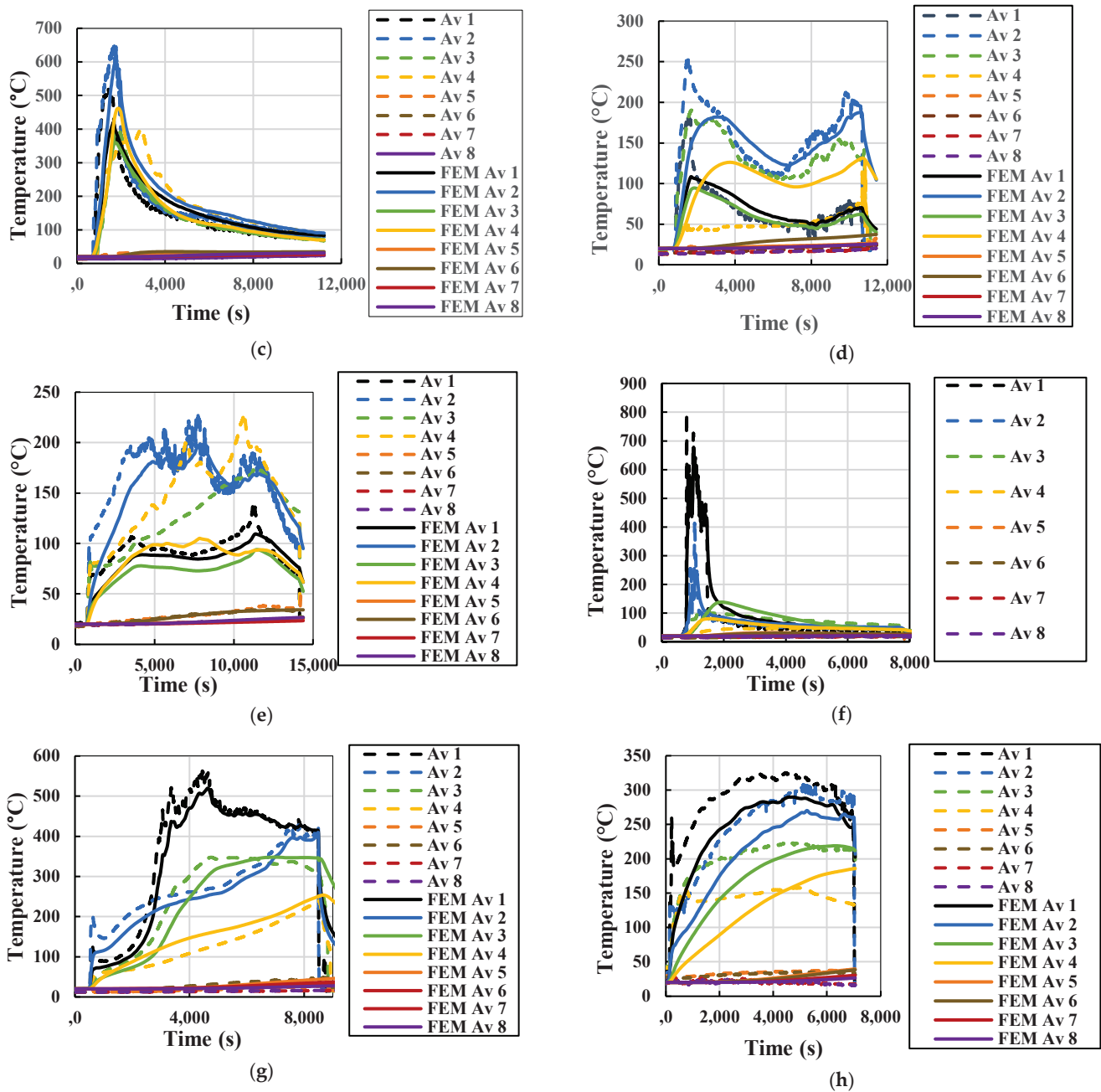


Figure 6. Comparison between predicted results and measured test results for different brick wall specimens: (a) B, (b) B-W-R, (c) B-S-R, (d) B-W-X, (e) B-S-X, (f) B-T-X, (g) B-W-C, and (h) B-S-C.

3.2. NW Wall Specimens

Figure 7 presents a comparison between the numerically predicted and experimentally measured test results for the NW wall specimens. It can be observed from Figure 7 that the predicted results for the NW-S-NW-S-R-TM, NW-S-R, NW-S-C, and NW-S-C-TM specimens closely align with the measured test results. In contrast, for the NW-N-TP, NW-S-TP-R, and NW-S-TP-C specimens, which utilize Tria Paint as a protective coating, a significant inconsistency is observed between the predicted and measured results. This inconsistency arises because the model does not account for Tria Paint. Based on the interpretation of the results, it can be concluded that Tria Paint not only mitigates the combustion of materials but also decreases heat transfer, despite its small thickness. Therefore, its influence cannot be overlooked in finite element simulations.

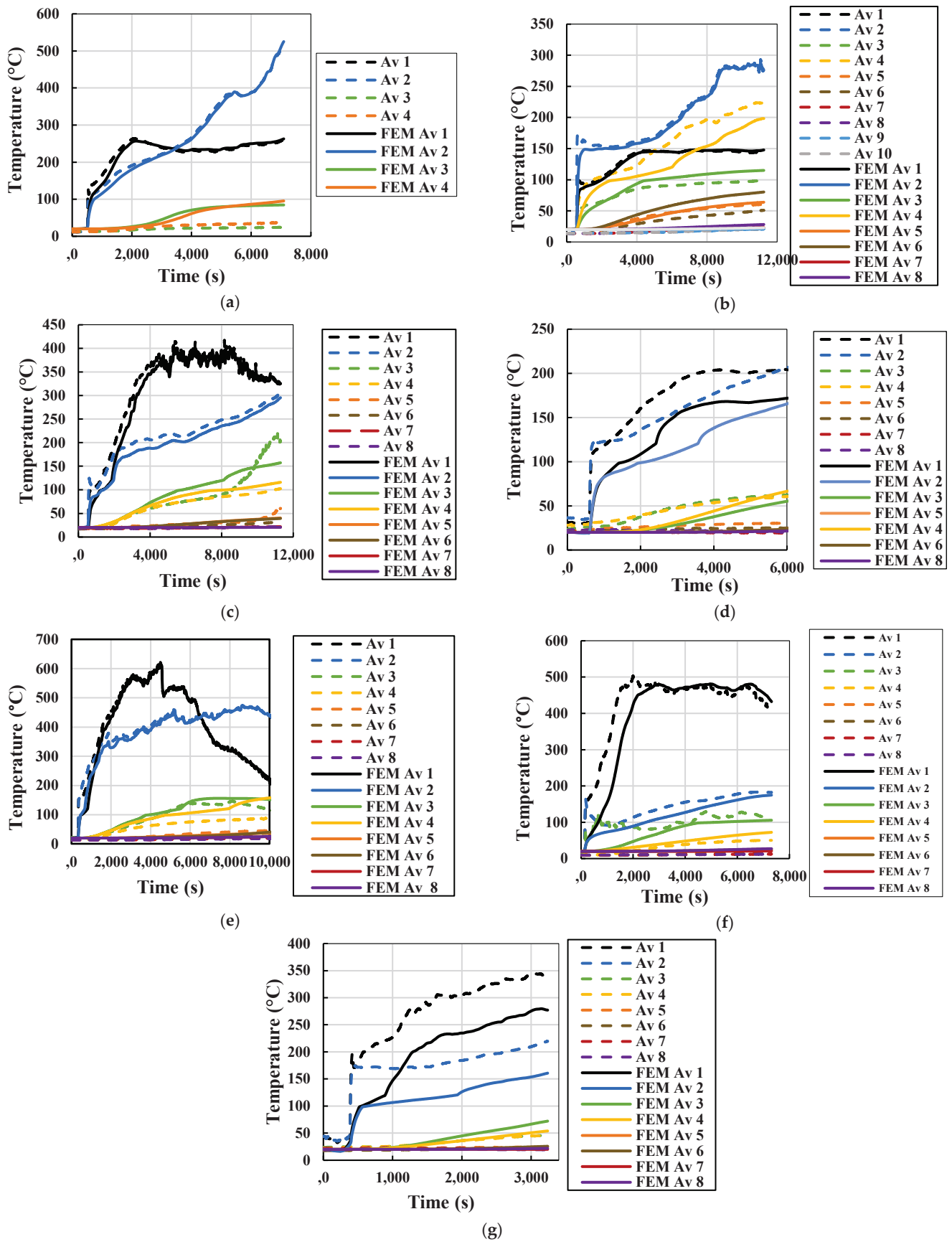


Figure 7. Comparison between predicted results and measured test results for different NW wall specimens: (a) NW-N-TP, (b) NW-S-R-TM, (c) NW-S-R, (d) NW-S-TP-R, (e) NW-S-C, (f) NW-S-C-TM and (g) NW-S-TP-C.

3.3. CLT Wall Specimens

The comparison of the predicted and measured test results for the CLT wall specimens is displayed in Figure 8. From this figure, for the wall specimens containing Tria Paint as a protective coating, a similar observation to that of the NW walls is noted, which is an inconsistency between the predicted and measured test results. Furthermore, the CLT-S-R specimen exhibits a notable discrepancy with the measured test results due to uncontrolled combustion of CLT, which caused a significant temperature rise in the deeper layers during the firebrand accumulation test. This phenomenon was not accounted for in the FEM. In contrast, for the CLT-N-TM, CLT-N-R-TM, and CLT-S-C specimens, the predicted results align well with the measured test results.

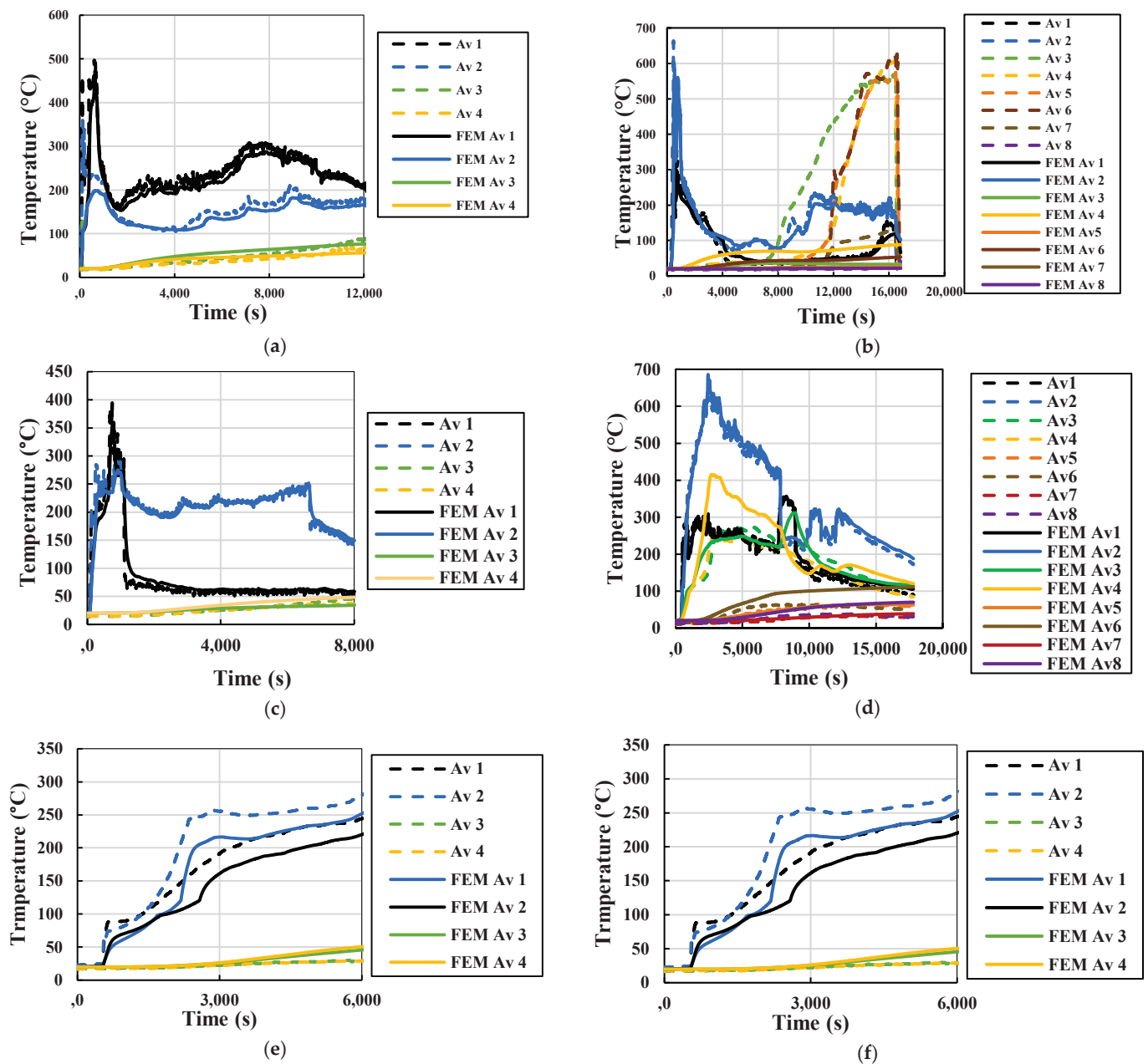
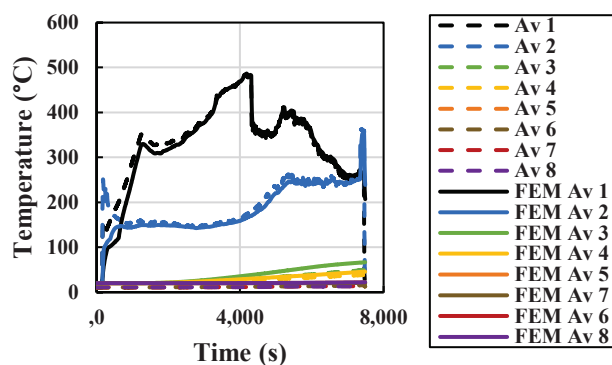


Figure 8. Cont.



(g)

Figure 8. Comparison between predicted results and measured test results for different CLT wall specimens: (a) CLT-N, (b) CLT-S-R, (c) CLT-N-TM, (d) CLT-N-R-TM, (e) CLT-N-TP, (f) CLT-S-TP-R and (g) CLT-S-C.

4. Discussion

The results indicate that the FEM effectively simulates the thermal response of most wall specimens. However, certain discrepancies highlight the limitations of the numerical approach and the need for further refinements.

The inconsistencies observed in XPS-insulated brick walls are due to the degradation and melting of XPS at lower temperatures. This phenomenon aligns with previous studies. Prior research by Wessies et al. [44] and Arruda et al. [17] highlights the severe impact of firebrand accumulation on XPS, which undergoes rapid thermal breakdown in a short duration and at relatively low temperatures. Therefore, future numerical models should incorporate phase-change properties to better reflect experimental observations.

For NW and CLT walls, the impact of Tria Paint as a protective coating was not considered in the numerical model, leading to an underestimation of its fire resistance. Studies by Suzuki et al. [24] demonstrated that protective coatings can significantly reduce heat transfer and delay combustion. Uncontrolled combustion in CLT-S-R specimens is another factor affecting model accuracy. Prior work by Zhu et al. [26] suggests that smoldering combustion in timber structures can alter heat transfer patterns significantly. Implementing combustion kinetics into numerical simulations may improve the model's ability to capture such effects.

4.1. Limitations of the Study

The simulated firebrand accumulation may be more severe than traditional firebrand deposition utilizing an ember generator, but it complies with California Building Code Chapter 7A-4 requirements [29]. Additional wall features like windows, doors, and air vents are not considered, which could alter the overall results.

This work numerically studied a concave exterior wall, which may experience more severe effects than a classical plane wall surface when in contact with firebrand accumulation.

4.2. Future Developments

It is anticipated that new double-brick wall geometries will be numerically tested and showcased to the scientific community. Additionally, tests for firebrand buildup will be conducted on innovative materials for the primary layer, such as cellular concrete and concrete blocks. A comparable numerical fire test operation is also planned for traditional ceramic roofs with various thicknesses of insulation.

5. Conclusions

The purpose of this study was to simulate the response of exterior dwelling walls to firebrand accumulation using the finite element software ABAQUS (2022). Numerical simulation results were compared to data from previous experimental campaigns to evaluate finite element modeling's accuracy and reliability in simulating the thermal reaction of various wall specimens during firebrand accumulation. This study led to the following conclusions:

- The finite element simulations successfully modeled the thermal response of most brick wall specimens, with predicted results showing good agreement with the experimental data. However, discrepancies were observed for wall specimens incorporating XPS insulation, as the numerical model did not account for the melting of XPS during the tests.
- For normal wood wall specimens, the simulations captured the impact of firebrand accumulation with reasonable accuracy. Nonetheless, wall specimens treated with Tria Paint exhibited discrepancies due to the exclusion of the protective coating's effects in the model.
- Similarly, the finite element simulations effectively reproduced the thermal response of most CLT wall specimens. However, the specimen with rockwool and sandwich settings displayed a marked deviation from experimental results, attributed to uncontrolled combustion during firebrand accumulation tests.

It can be summarized from the results of this study that the finite element model can simulate the response of construction brick, normal wood, and CLT walls with a combination of the following insulators, cork or rockwool coated with mortar, against the accumulation of firebrand.

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Data Availability Statement: The raw data supporting the conclusions of this article will be made available by the authors on request.

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Article

A Numerical Tool for Assessing Random Vibration-Based Fatigue Damage Diagnosability in Thermoplastic Coupons

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Abstract: A numerical tool is developed to simulate the random vibration-response-only-based fatigue delamination diagnosability in thermoplastic coupons. That is the ability to both detect damage and identify its current severity, aiming to establish a virtual framework for optimizing diagnosability methods. The numerical tool employs the FE method. It comprises two modules: a fatigue delamination module and a random vibration module. The first module implements a fatigue crack growth model based on the cohesive zone modeling method to predict delamination accumulation, while the second module uses an experimentally verified FE model of the delaminated coupon to predict its random vibration response. Delamination accumulation is evident in the ‘predicted’ FE-based power spectral densities. The model’s capability to diagnose delamination is demonstrated using seven different damage metrics based on simulated random vibration responses, enabling damage detection and severity assessment (increasing trend guides to distinguishing each fatigue state from its counterparts). Comparisons with their experimentally obtained counterparts are also used in the assessment. The procedure clearly suggests that the proposed numerical tool may be reliably used for virtually assessing the efficacy of random vibration-based fatigue damage diagnosability for any given structure and also to aid the user in selecting the method’s parameters for virtual diagnosability optimization.

Keywords: thermoplastic composites; CFRPs; vibration-based monitoring; numerical random vibration model; fatigue damage; damage diagnosis

1. Introduction

Since the launch of the Boeing 787 Dreamliner, in which composites account for nearly 50% of its structural weight, and the Airbus A350 XWB, where composites make up about 52% of its structural weight, Carbon Fiber Reinforced Plastics (CFRP) have become a dominant material in the aeronautical industry [1]. While thermosets are primarily used in aircraft structures, thermoplastics are gaining popularity due to their advantages, particularly in terms of dynamic behavior, recyclability, and weldability.

Fatigue behavior remains a critical design factor for composite aircraft structures due to its stochastic nature, which is further enhanced by the inherent anisotropy of CFRPs. To date, CFRP structures have primarily been designed based on experimental data, as reliable predictive models are still lacking.

Improving safety has become a paramount concern for aircraft manufacturers and operators, particularly in light of recent incidents and accidents that have resulted in the grounding of hundreds of aircraft for extended periods. Structural Health Monitoring

(SHM) offers a valuable solution by providing reliable diagnostic and prognostic tools that can help prevent critical structural failures [2,3]. Additionally, SHM has the potential to enhance the efficiency and effectiveness of maintenance procedures, ultimately contributing to improved operational safety and reduced downtime.

SHM of composite structures has undergone rapid advancements over the past decade [3,4]. Traditionally, offline experimental techniques have been used to detect the initiation of damage, track its progression, and, in some cases, predict the residual fatigue life and strength of structures [5]. Common methods include X-rays [6,7], acoustic emission [8–10], optical and surface observation [11,12], and post-damage analysis through Scanning Electron Microscopy (SEM) [13,14]. In contrast, a promising family of online SHM methods relies on vibration-data-driven methods. These offer two key advantages [15]: Continuous, real-time information on the structure's health state without disrupting normal operation and operating via sensors that are not necessarily in the neighborhood of the damage location. This renders vibration-based methods especially useful for the real-time monitoring of complex composite structures.

Vibration-based SHM has been widely used across various engineering sectors, though predominantly in beam-like structures. Most studies have employed experimental approaches [16–27], with fewer studies utilizing a combined experimental–numerical approach [28–36], an experimental–analytical approach [37], or a purely numerical approach [38–43]. Certain studies [44–46] employed the wavelet transform, while others [47–49] utilized artificial neural networks. The majority of the published works focus on damage detection, with only a small subset [24,27] addressing fatigue damage of composite materials. All experimental–numerical approaches and purely numerical approaches have exploited experimental data to predict damage accumulation or fatigue life of the structures, while only the work of Sartorato et al. [39] employed numerical analyses to support the design of a piezoelectric-based SHM system for metallic materials. Additionally, some important studies [19,50,51] examined the non-linearities, which are inserted in the problem of vibration–fatigue damage.

The goal of this study is the development of an integrated fatigue damage–random vibration numerical tool for the parametric simulation of random vibration-based progressive fatigue damage detectability for thermoplastic coupons. The implementation of the numerical tool requires no experimental data, with the exception of the fatigue model. Following its validation against mechanical fatigue and random vibration tests, the numerical models are used to assess the effects of specific parameters on fatigue damage diagnosability.

The study's innovative aspects may be summarized as follows:

- Development of a numerical methodology of random vibration-response-only-based fatigue damage diagnosability for thermoplastic coupons;
- Development of an efficient simulation process for random vibration experiments;
- Examination of progressive fatigue damage through numerical random vibration responses in the frequency domain;
- Employment of several damage metrics for fatigue damage diagnosability in thermoplastic coupons using numerical random vibration response data;
- Generation of an integrated numerical model/tool, which can be used to support random vibration-response-only-based SHM systems for composite structures.

2. Conceptual Framework

The concept of detecting damage in materials and structures by exploiting random vibration response is based on the premise that damage alters their characteristics (stiffness and damping), thus their dynamical behavior, which subsequently leads to changes in their vibration response characteristics [52]. Yet, as damage characteristics can also be affected by

other factors, such as operational and environmental conditions, care should be exercised to distinguish between actual damage and other external influences [53–58].

Especially, detecting fatigue damage from random vibration responses and modal characteristics, such as frequency, damping ratio, spectral distance, PSD estimates, etc., requires the ability to deeply understand the initial propagation, constant evolution, and the critical damage modes before failure, with the aim of understanding the vibration behavior and the correlation of it with fatigue damage. The proposed fatigue degradation model is based on the loading envelope approach proposed by [59] in which the cyclic loading is simulated by a linear force incremental to the maximum value of applied force. The implementation of the model was undertaken on an element basis and was based on a modified Paris' law. For more details, Section 3 of this present paper, as [59–61], describes the fatigue delamination growth model.

The implementation of the random vibration model also needs a comprehensive understanding of the exact and with which tendency the modal characteristics are influenced by progressive fatigue damage. The random vibration simulation process lies in the modal superposition method to characterize the linear dynamic behavior of the structure [62].

3. The Numerical Tool for Fatigue Damage Diagnosability

Both the Fatigue Delamination Module (FDM) and the Random Vibration Module (RVM) were developed in LS-Dyna. These are two distinct models functioning dependently on each other. The FDM has been described in detail in [61], so only a brief overview of it is presently provided for the sake of presentation completeness.

3.1. The Fatigue Delamination Module (FDM)

The FDM was based on the fatigue crack growth model developed in [60] for simulating interfacial fracture in co-consolidated thermoplastic joints. The model utilizes the cohesive zone modeling method and incorporates a modified Paris' law, expressed as:

$$\frac{da}{dN} = c(G_{\max})^m \quad (1)$$

where $\frac{da}{dN}$ is the fatigue crack growth rate, c and m are the modified Paris' law parameters for each simulated FS, and $G_{\max}$ represents the maximum energy release rate for the affected element at a given FS. The instantaneous energy release rate G_i for each cohesive element is consistently set to $G_{\max}$, reflecting the maximum energy release rate of the actual loading spectrum due to the use of the load envelope technique. The intermediate mixed-mode parameters c and m are derived using the prediction model of Russel and Street [63], which requires only pure mode I and mode II fatigue experimental data as inputs.

Numerical fatigue degradation was implemented using the method of cumulative static and fatigue element damage parameters, described by:

$$d_{\text{tot}} = d_s + d_f \quad (2)$$

where d_{tot} is the total damage variable (ranging from 0 for undamaged to 1 for failed), while d_s and d_f are the static and fatigue contributing damage variables, respectively. The stress state σ for each element in the fatigue-activated zone is then degraded using:

$$\sigma = (1 - d_{\text{tot}})\sigma_{\max} \quad (3)$$

where $\sigma_{\max}$ is the maximum traction of the affected element.

In this study, the modified Paris' law was applied, accounting solely for $G_{\max}$. The fatigue crack growth model was fully validated using the same experimental parameters,

such as stress ratio and maximum load percentage, for the same interfaces as detailed in [60].

Fatigue loading was implemented using the loading envelope technique [59], where the actual sinusoidal spectrum was approximated by a constant force corresponding to the maximum fatigue load value, as illustrated in Figure 1.

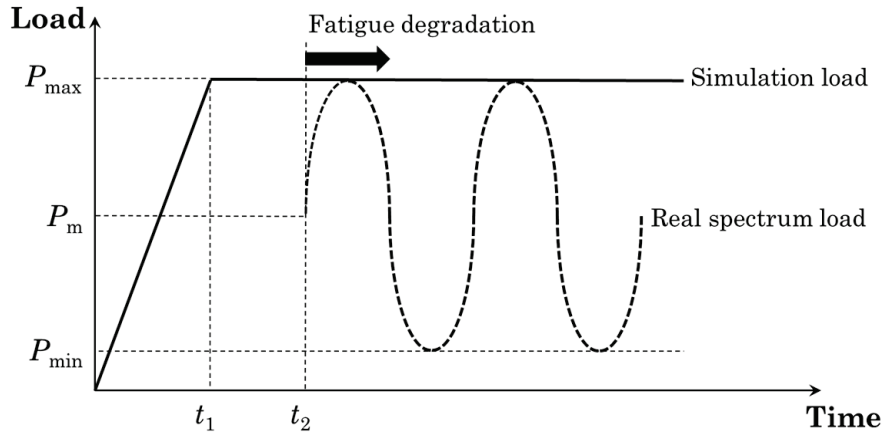


Figure 1. The load envelope for the modeling of fatigue loading.

During each numerical iteration, the rate of fatigue crack growth was calculated using Equation (1), which operates in a singular direction. This resulted in the establishment of an equivalent rate of damage development uniformly across all axes, disregarding the orthotropic nature of thermoplastics and the complexity of three-dimensional interfaces. To address this limitation, a specific length measurement was assigned to each element, guiding the progression of damage by considering input from the cohesive integration points. This measurement is particularly significant in cases where there is considerable variation in element aspect ratios and irregularities within the interfaces. The calculation of the fatigue damage rate, df , requires the characteristic element length, l_e , as described in detail in [29]. l_e was computed for each iteration through an internal loop, which determined the minimum distance from the midpoint of a non-failed element to the midpoint of neighboring elements. Following this, the elements' data, including G_I and G_{II} , were calculated, and a check for fatigue module activation was performed. If either G_I or G_{II} exceeded the threshold value G_{th} , the fatigue module was activated; otherwise, static degradation was applied. The value of G_{th} was based on literature. The stress σ of the cohesive element is governed by the mixed-mode stress, which combines the contributions from both normal and tangential stress components, according to LS-Dyna's MAT_138 formulation [35]. Therefore, cohesive elements subjected to fatigue damage are fully governed by mixed-mode stress (σ) for their response, degradation, and failure. The iterative numerical procedure for the fatigue crack growth model is illustrated in Figure 2.

The pure mode I and mode II modified Paris' laws used in the study are:

$$\left(\frac{d\alpha}{dN}\right)_I = 0.0027 G_{I,max}^{4.23} \quad (4)$$

$$\left(\frac{d\alpha}{dN}\right)_{II} = 0.00040 G_{II,max}^{5.31} \quad (5)$$

where $\left(\frac{d\alpha}{dN}\right)_I$, $\left(\frac{d\alpha}{dN}\right)_{II}$ and $G_{I,max}$, $G_{II,max}$ are the fatigue crack growth rates and the maximum energy release rates per cycle for mode I and mode II, respectively. More details on the derivation of Equations (1) and (2) can be found in [60].

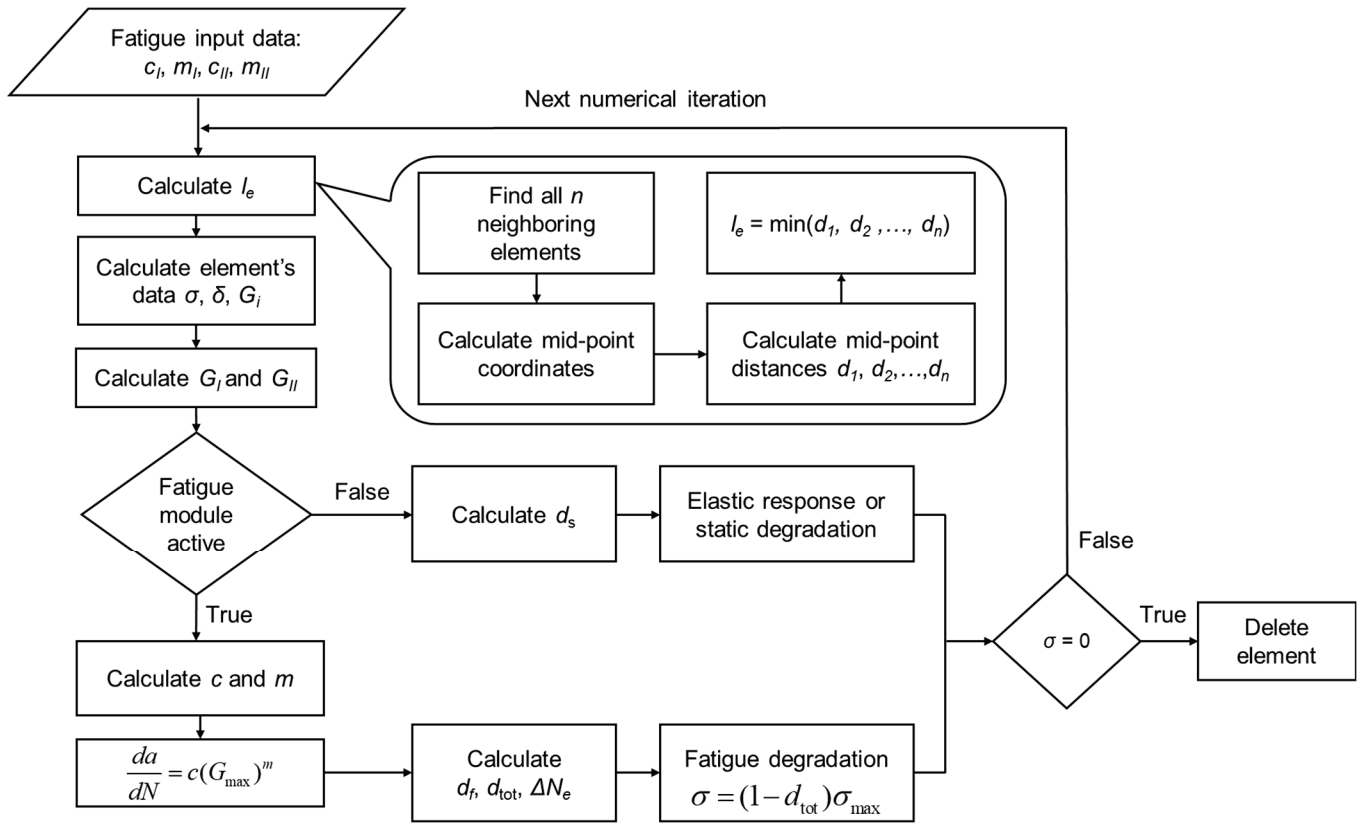


Figure 2. Flowchart of the fatigue crack growth model.

3.2. The Random Vibration Module (RVM)

The RVM was developed using LS-Dyna's graphical interface pre-processor LS-PrePost 4.8. The damping ratio was determined using:

$$\xi = \frac{\Delta f}{2f_n} \quad (6)$$

where f_n is the first natural frequency and Δf is the bandwidth of the half-power points [40]. To prevent hourglass-induced instabilities, a conversion of the excitation amplitude was necessary.

The applied excitation was calculated using [64]:

$$[\text{dB}] = 20 \times \log_{10} \left(\frac{P}{2 \times 10^5} \right) \quad (7)$$

where P is the pressure of white noise in Pascal and 2×10^5 is reference sound pressure in Pascal.

Respectively, the numerical results were converted from [mm/s] to [dB] using [65]:

$$v_{\text{dB}} = 20 \times \log_{10} (v_{\text{mm/s}} \times 10^6) \quad (8)$$

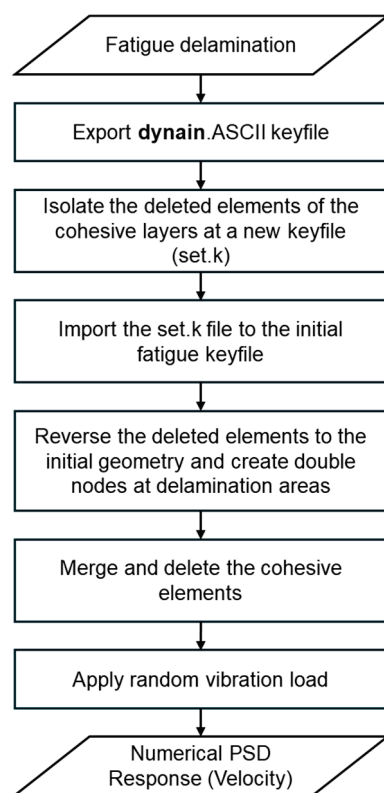
where v_{dB} is the resulting velocity in dB and $v_{\text{mm/s}}$ is the resulting velocity in mm/s.

With the aim of calibrating the model parameters, and checking how basic parameters (such as boundary conditions, measurement points, and mesh density—more details in Section 4.5) influence the coupon's behavior, Frequency Response Functions (FRF) analyses were implemented. The geometry (see below—Section 4.4 and the material model (orthotropic elastic—see below Table 1) were the same as described in the RVM model.

Table 1. Orthotropic elastic properties of the thermoplastic lamina [61].

Property	Value
Density (ton/mm ³)	1.75×10^{-9}
Elastic modulus—a direction (MPa)	95,000
Elastic modulus—b direction (MPa)	8500
Elastic modulus—c direction (MPa)	8500
Shear modulus—ab direction (MPa)	4300
Shear modulus—bc direction (MPa)	3571
Shear modulus—ac direction (MPa)	4300
Poisson's ratio—vba	0.027
Poisson's ratio—vca	0.024
Poisson's ratio—vcb	0.172

The FE modeling procedure for the damaged coupon is outlined in the flowchart of Figure 3. This process combined the RVM with the FDM. For each FS, the FDM was executed to predict the delamination state, which corresponds to deleted cohesive elements representing the interfaces among layers. All cohesive elements were exported to a `dynain.ASCII` file, from which the deleted cohesive elements were isolated into a new keyfile (`set.k`). The `set.k` file was then imported into the initial FE model, and by reversing the deleted elements, duplicated cohesive layers (both healthy and damaged) were created. Finally, the deleted cohesive elements saved in the `set.k` file were removed, and the remaining duplicate nodes were merged. This procedure effectively “inserts” the delaminated (deleted) cohesive elements into the cohesive layers.

**Figure 3.** The RVM development process for the damaged coupon.

An important clarification should be given at this point: the coupons that were submitted to T-T fatigue had transverse and off-axis cracks, except for delamination, which surely contributed to the final failure. The initiation and propagation of fatigue damage, which was validated from C-Scan images, showed that delamination was the main cause

of failure [61]—so, as with the FDM, the RVM also considers only the delamination as damage mode.

4. Numerical Tool Assessment and Experimental Verification

In [27], the concept of using random vibration response to detect fatigue damage in thermoplastic coupons was experimentally demonstrated. The objective of this present work was the numerical development of [27] that comprehensively examines the influence of testing parameters on the method's efficiency and virtually optimizes the process for structural parts. A key difference to be kept in mind is that while in the experimental procedure, all damage modes may be present, the numerical tool presently included only delamination damage. Thus, the assessment was based on the premise that delamination is the predominant damage mode in coupons subjected to tension-tension fatigue—which was validated by C-Scan images [61]. The numerical tool assessment was the application of the above two modules into a thermoplastic coupon, which was experimentally validated at various stages. The FDM was validated by C-Scan images [61], and the RVM was validated by the experimental PSD Welch estimates [27,66]. Any deviation between the numerical and experimental results is justified below.

4.1. Material and Coupons

The coupons employed in the study were fabricated from T700 carbon fiber reinforced low-melt semi-crystalline PAEK (polyaryletherketone) matrix prepregs (Toray Cetex® TC1225) [67–69]. The fiber volume fraction was 66%, and each ply was 0.14 mm thick. The lay-up was quasi-isotropic, with the sequence $[-45^\circ/0^\circ/45^\circ/90^\circ]_2s$. The dimensions of the coupons were 250 mm $\times$ 25 mm $\times$ 2.24 mm. Tabs made of the same material, measuring 50 mm $\times$ 25 mm $\times$ 2.24 mm, were bonded to the coupons. In total, 23 coupons were subjected to mechanical loads and excitation. The properties of the material are provided in Table 1 [61] and the dimensions of the coupon are illustrated in Figure 4.

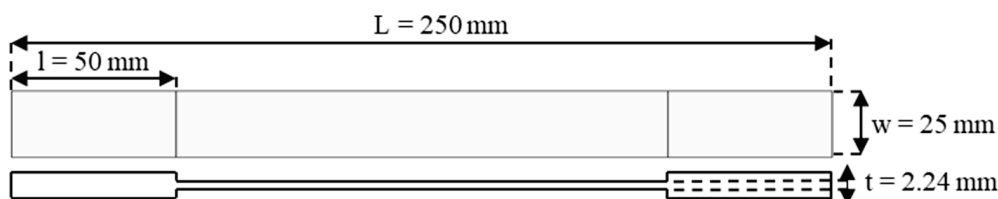


Figure 4. Dimensions of coupons.

4.2. Brief Overview of the Experimental Procedure

The random vibration test setup is illustrated in Figure 5, while Figure 6 provides a sketch of the testing and data acquisition processes. The test setup consisted of a Laser Doppler Vibrometer (LDV) OFV-505 (Polytec GmbH, Karlsruhe, Germany), its OFV-5000 controller (Polytec GmbH, Karlsruhe, Germany), an NI-9230 acquisition card (National Instruments, Austin, TX, USA), and a PC. The applied excitation was a MATLAB (R2024a)-generated band-limited (within the 0–6.4 kHz range) white noise. The coupons were supported via elastic cords to simulate free-free boundary conditions. Vibration velocity measurements were taken using the laser vibrometer, which was focused on a reflective surface positioned at the center of the upper face of each coupon. The random vibration test was implemented every 10,000 fatigue cycles for all coupons.

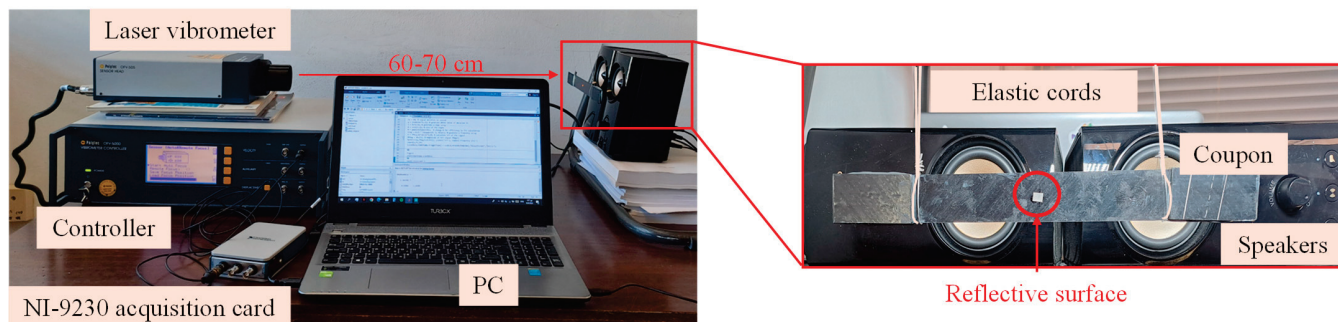


Figure 5. Random vibration test setup (adapted from [27]).

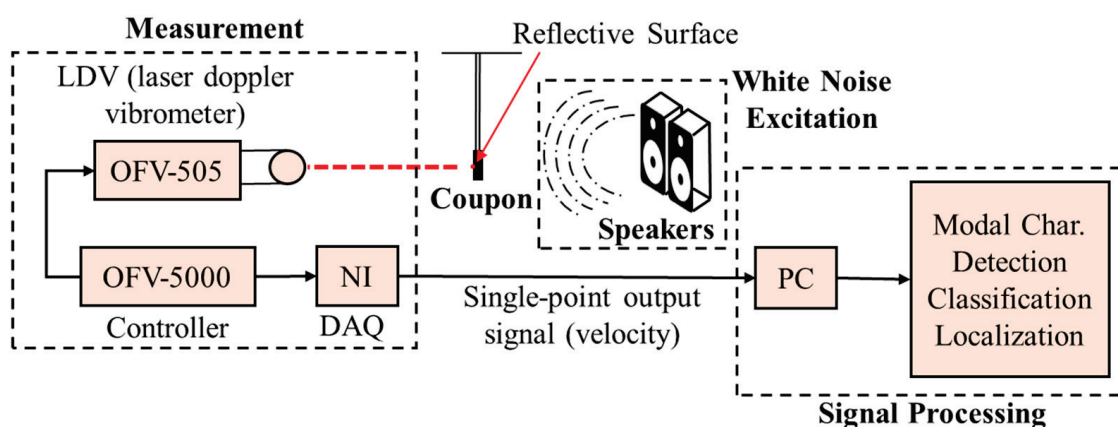


Figure 6. Sketch of the testing and acquisition processes for the random vibration test (adapted from [27]).

Non-parametric signal analysis was performed, based on which Welch-based Power Spectral Density (PSD) estimates were obtained for the Healthy State (HS) and various Fatigue States (FSs). The signal characteristics and the Welch estimation parameters are summarized in Table 2. Progressive fatigue damage detection was conducted on a population of 13 coupons, with one representative sample selected for model comparison and validation. The results across the coupons were grossly consistent, with random deviations attributed to the stochastic nature of fatigue damage accumulation. This variability is discussed by the authors in references [66,70]. The PSD Welch estimates for Coupon #3 under HS and various FSs (70 PSD estimates under the HS and 35 under each FS) are presented in Figure 7. A clear and consistent effect of fatigue damage was observed on the PSD Welch estimates in that certain resonant frequencies (in particular, those in the 3–3.6 kHz range) shifted from higher to lower frequencies as the damage progressed.

Table 2. Signal characteristics and Welch PSD estimation details for random vibration testing [27].

Random Vibration Test with Acoustic Excitation			
Signal Characteristics		Welch Parameters	
Sampling frequency f_s	12,800 Hz	Segment length	25,600 samples
Signal duration	60 s	Overlap	90%
Signal bandwidth	0–6400 Hz	Frequency resolution δf	1 Hz

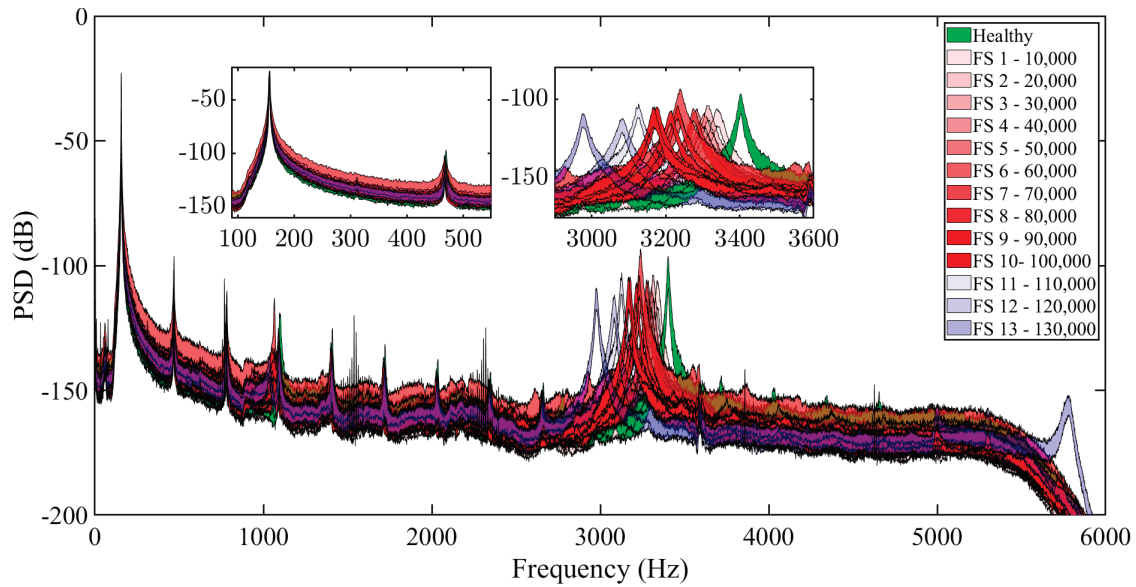


Figure 7. Experimental results: Fatigue effects on random vibration Welch-based PSD estimates for Coupon #3 (Fatigue Stages, FS1–FS13).

4.3. FDM Results

Figure 8 illustrates a typical fatigue delamination propagation pattern predicted by the FDM (Section 3.1) for a thermoplastic coupon. Delamination was initiated at the free edges of the coupon, driven by high through-the-thickness normal and intralaminar shear stresses, and propagated towards the center of the coupon. The propagation pattern was not fully symmetric, which is attributed to the nature of fatigue—repeated loading in specific directions can create progressive damage in certain ply orientations before others, as the material model used here [60,61] is sensitive to element orientation, which can create directional differences in damage evolution. The consecutive Fatigue States (FS) corresponded to the successive fatigue cycles—each FS corresponded to 20,000 cycles. The significance of the fatigue delamination pattern lies in its ability to accurately model delamination growth and capture the characteristics of thermoplastic coupon's fatigue behavior.

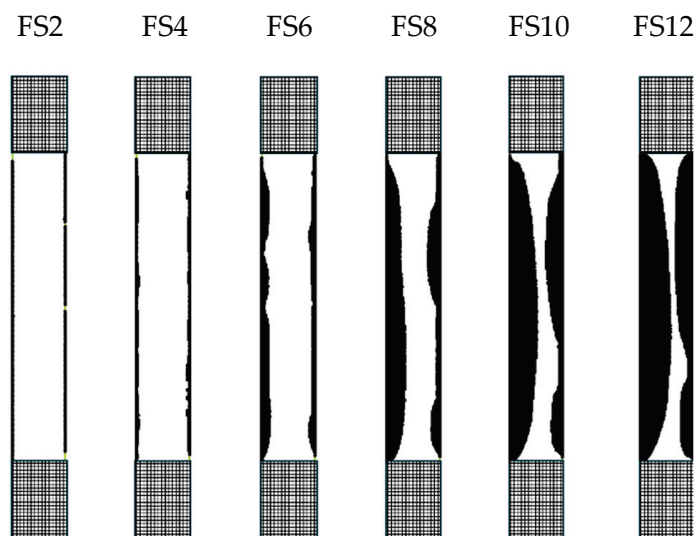


Figure 8. A typical fatigue delamination pattern predicted by the FDM for a thermoplastic coupon (FS: Fatigue States correspond to increasing fatigue cycles).

4.4. RVM Modeling

A typical FE mesh of the coupon is shown in Figure 9. The coupon's damping ratio was calculated as 0.01 from Equation (6). The thermoplastic plies were modeled using solid 8-noded elements with a constant stress formulation (ELFORM 1). The orthotropic elastic material properties used for the elements are listed in Table 1. As boundary conditions, only the nodes that were in contact with the elastic cords were restricted to the y -axis. Velocity was measured at the middle and upper face nodes.

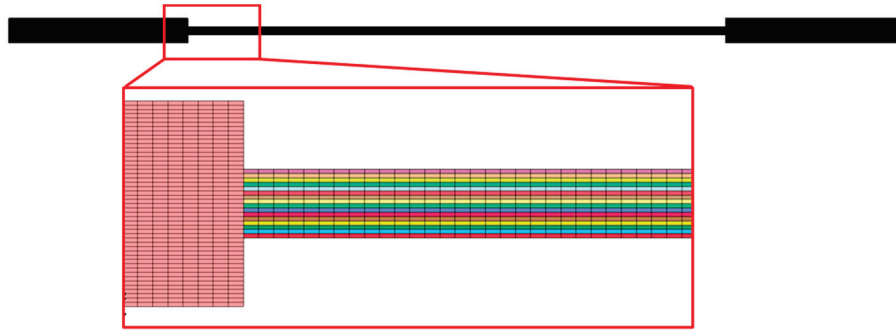


Figure 9. A typical FE mesh of the coupon.

Figure 10 shows the applied white noise excitation in the time and frequency (spectral) domains. Figure 11 shows the applied boundary conditions and the measurement location in the coupon.

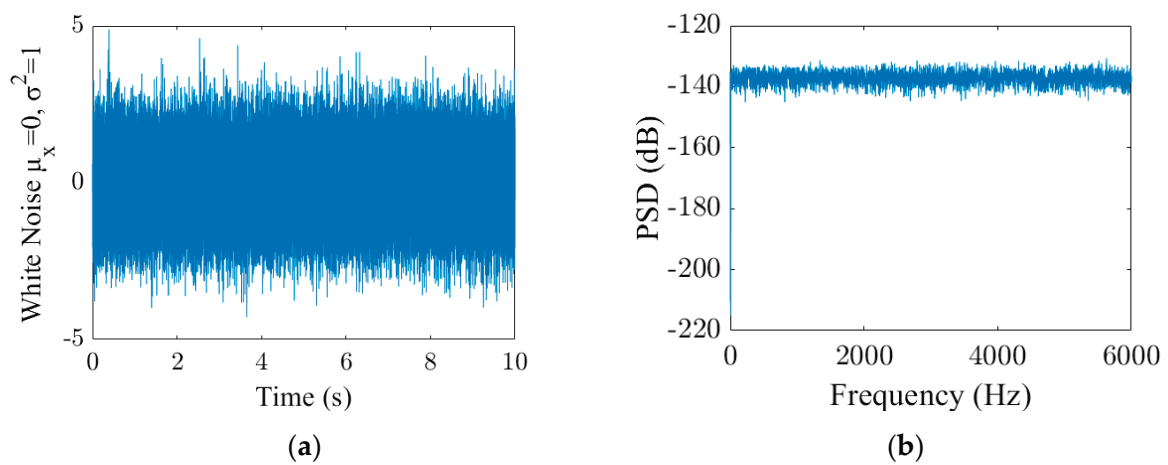


Figure 10. White noise excitation in the time (a) and frequency (spectral) domains (b).

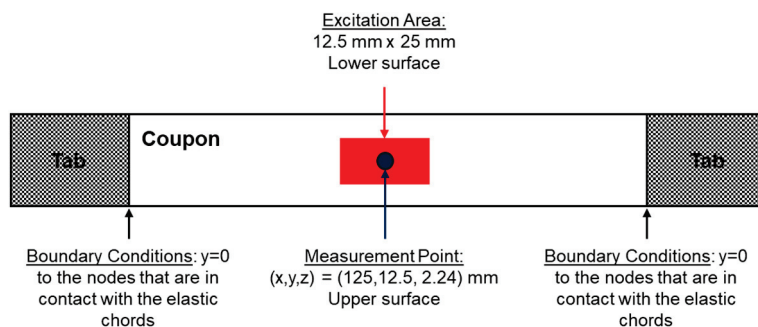


Figure 11. Boundary conditions, excitation region, and measurement point of the FE model.

4.5. RVM Results

4.5.1. Calibration of the RVM

A stepwise validation and calibration process was employed for the RVM. The modeling parameters that were calibrated included the mesh density, the boundary conditions, the measurement point, and the excitation area. To achieve this, in-house experimental results from [27] and laboratory-scale experimental results of a simplified test were utilized.

4.5.2. Parametric Frequency Response Function (FRF) Analysis

As a first step, parametric FRF analysis of the healthy coupon was conducted and the FRF amplitude and phase considering the above parameters were compared. The parameters used are listed in Table 3.

Table 3. Parameter values used in the calibration of the RVM.

Parameter	Values
Measurement location	Center, Upper right corner, Lower left corner
Mesh density	235,000 elements, 432,000 elements, 2,080,000 elements
Boundary conditions	Cord-support, Free, Clamped
Excitation intensity	2×10^{-15} MPa, 79 MPa, 200 MPa

The FRF amplitude and phase for varying mesh densities, measurement locations, boundary conditions, and excitation intensities are compared in Figure 12.

As the mesh density increased, the computed results progressively converged, although the differences between the FE meshes remained minor. Final convergence was achieved with the mesh of 432,000 elements, which was then applied consistently in all subsequent analyses. The comparison between free-free and elastic cord support boundary conditions revealed only minor differences in FRF characteristics, and the elastic cord support was ultimately selected to align with the experimental setup, while fixed boundary conditions affected the dynamic behavior of the coupon. Meanwhile, the location of the measurement point and excitation intensity (expectedly) influenced the magnitude intensity; yet the resonant frequencies (spectral peaks) remained unaffected. The dynamics remained approximately linear under different excitation intensities, as the shape did not change, and the measurement point and the boundary conditions could be replicated in the FEM successfully.

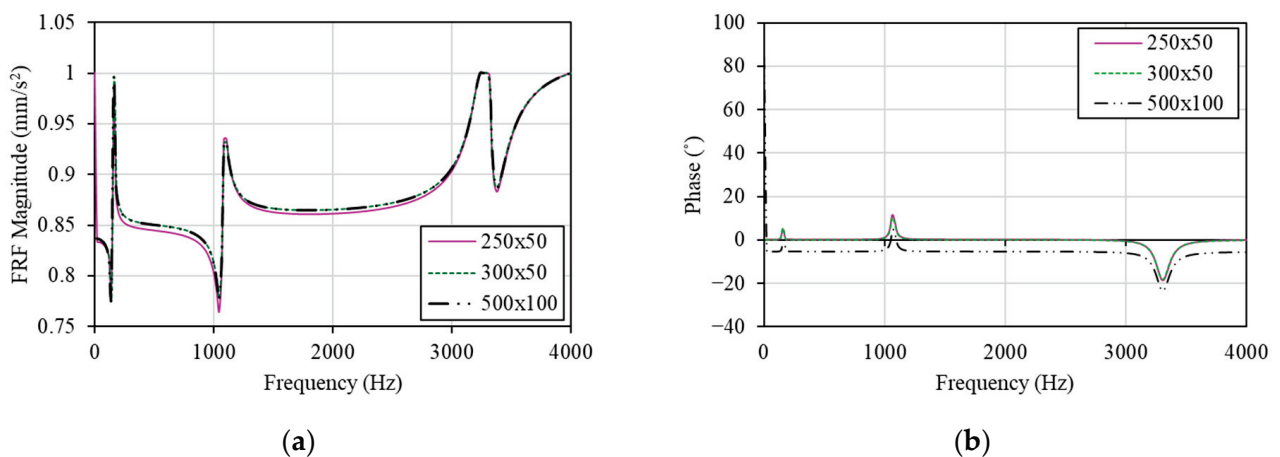


Figure 12. Cont.

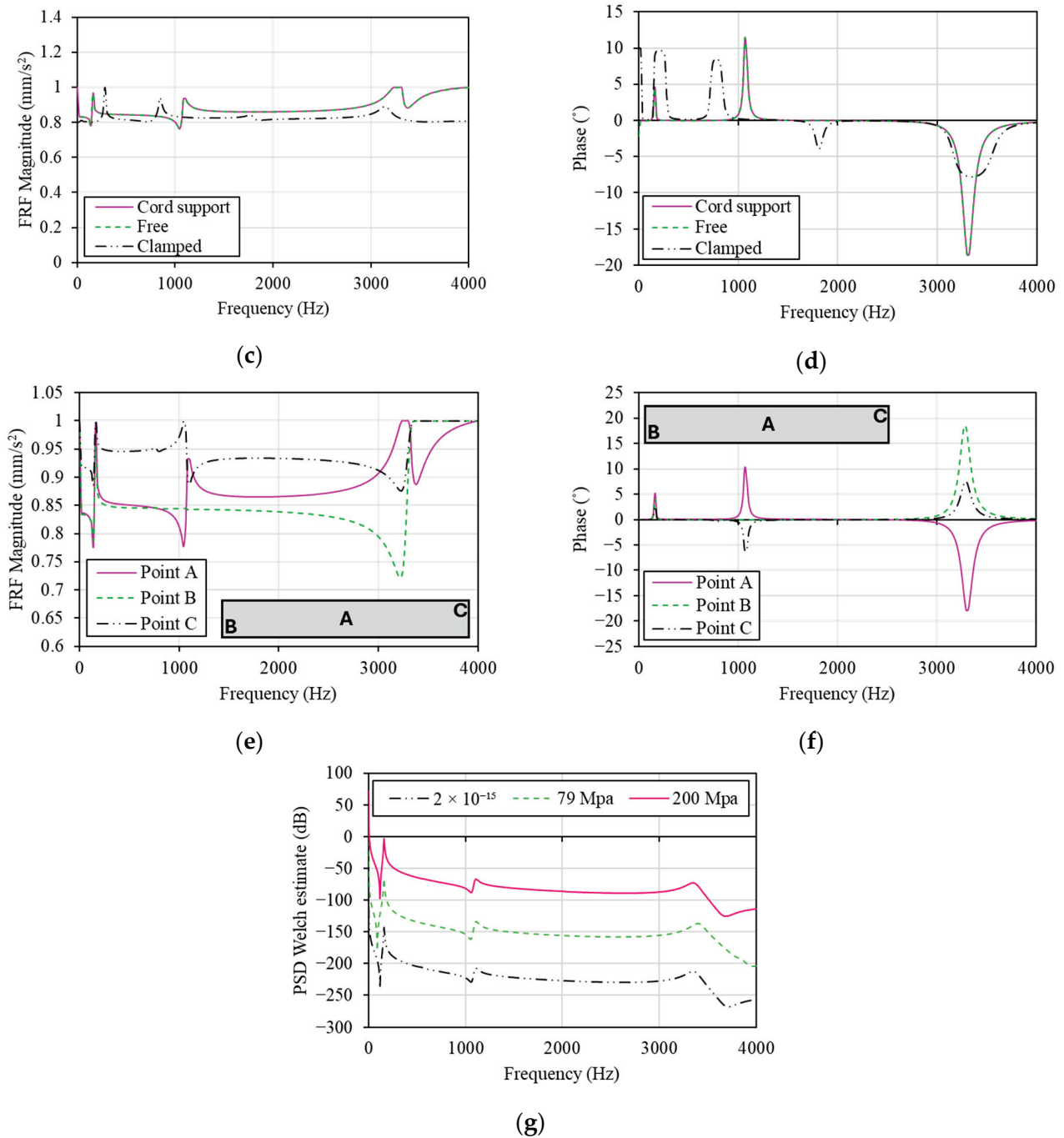


Figure 12. FRF analysis using different mesh densities: (a) Magnitude (mm/s²) and (b) Phase (°), different boundary conditions: (c) Magnitude (mm/s²) and (d) Phase (°), at different measurement points: (e) Magnitude (mm/s²), (f) Phase (°), and (g) RVM analysis.

To ensure consistency with the experimental procedure, the measurement point was positioned at the center of the coupon, corresponding to the setup used in the experiment.

4.5.3. Validation of the RVM (Healthy State)

Given the complexity of the random vibration response that the RVM aims to simulate, validating its accuracy is crucial. To this end, different comparison cases against experimental results were considered, reflecting different vibration scenarios for both healthy and damaged coupons. The validation had two primary objectives: (a) To confirm the model's capacity to accurately represent the random vibration behavior of the thermoplas-

tic coupons and (b) to assess the model's ability to capture the effect of damage on the coupon's vibration behavior.

For the healthy state, two validation cases involving different random vibration-based excitation and support conditions were considered.

In the first case, the random vibration test performed in [27] was considered. Figure 12 compares the numerical-based and experimental PSD estimates. A satisfactory agreement of the intensity of the response amplitude was evident. Furthermore, the model captured three of the resonant frequencies indicated by the peaks in the PSD Welch estimate. The additional peaks in the experimental PSD Welch estimate may be attributed to various factors and uncertainty, including unmodelled phenomena, complicated excitation due to reflections of the white noise acoustic excitation in the room during the experiments, inherent instrumentation effects, and noise in the measurement devices, and so on.

To ensure that the deviation observed in Figure 13 was indeed due to the experimental parameters and conditions of the random vibration tests, and not because the FE model contained misleading information, a second validation case using a simplified random vibration test was considered. This simplified test was conducted on a cantilever coupon. In this case, white noise was generated by an electromechanical exciter (LDS Model V406 with its controller, LDS COMET USB COM-200) and applied to the fixed end (tab) of the coupon. A laser vibrometer was used to measure the velocity at 3 points (the midpoint and 2 points near the right tab) on the upper surface of the coupon. A key difference from the previous experiment is that non-acoustic excitation was used, and the tests were conducted in a significantly larger room. A schematic illustration of the simplified random vibration test and a photograph of the test rig are provided in Figure 14. The excitation signal was recorded and applied with the same intensity as the FE model. The electromechanical exciter stimulated frequencies ranging from 0 to 2 kHz, with direct contact between the exciter and the coupon. Regarding the FE model, the modeling process remained consistent with the previous setup, using the same mesh (432,000 elements), the same thermoplastic coupon, and geometry (Figure 4). The only differences were the location of the excitation point (on the tab), the intensity of the excitation (−100 dB), the boundary conditions (clamped on the left tab), and the measurement points. This simplified setup helped isolate and eliminate factors, such as acoustic reflections, ensuring a more controlled and accurate validation of the FE model's performance.

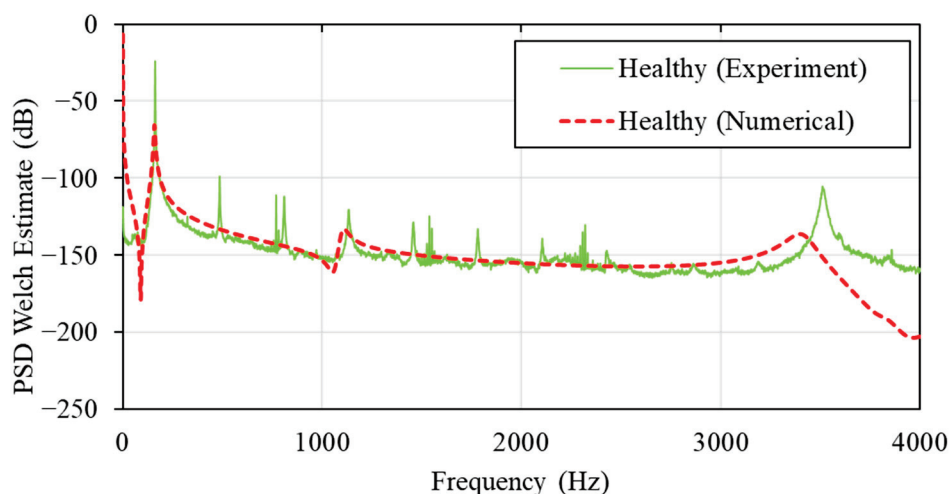


Figure 13. Comparison of PSD Welch estimate of the random vibration test and the FE model.

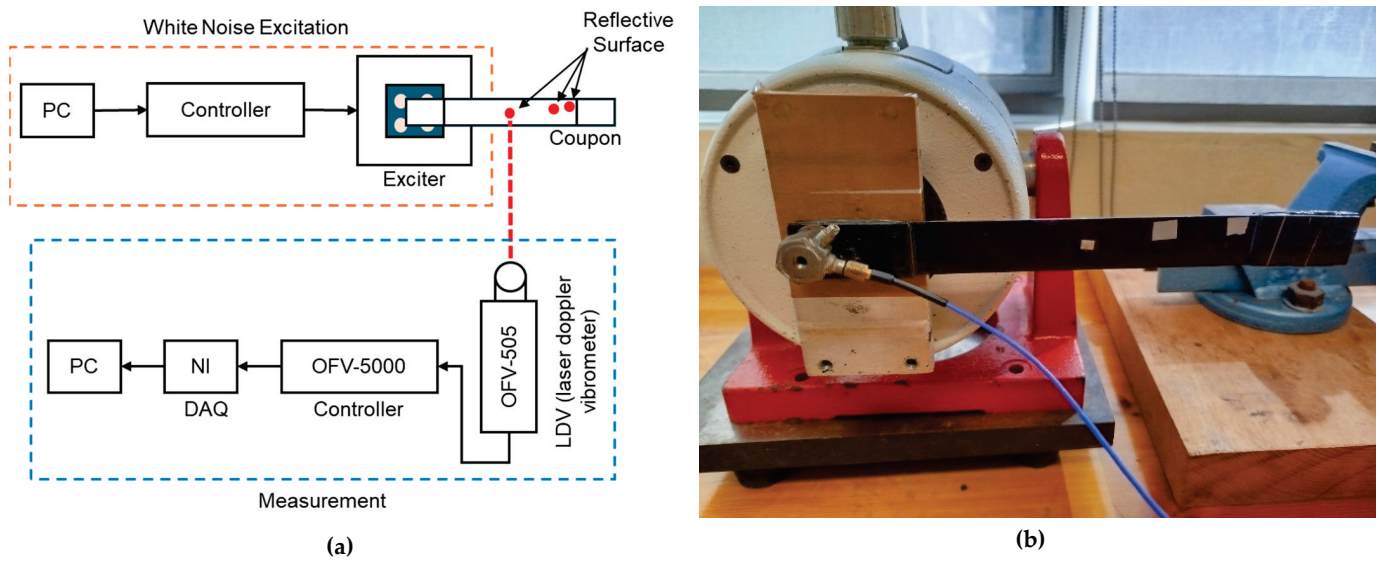


Figure 14. (a) Sketch of the simplified random vibration test, (b) Photo of the test rig of the simplified random vibration test.

The signal characteristics and the Welch PSD estimation details for the simplified random vibration tests are summarized in Table 4. The signal bandwidth was shorter than in the random vibration tests excited by white noise, as the goal was the examination of lower-frequency bandwidth.

Table 4. Signal characteristics and Welch PSD estimation details for simplified random vibration testing.

Simplified Random Vibration Test			
Signal Characteristics		Welch Parameters	
Sampling frequency f_s	2000 Hz	Segment length	4000 samples
Signal duration	10 s	Overlap	90%
Signal bandwidth	0–2000 Hz	Frequency resolution δf	1 Hz

Figure 15 presents the comparison between the FEM-based analytical PSD and its Welch-based estimate obtained from the experimental procedure. Evidently, the resonant frequencies, as well as the overall PSD, were quite accurately captured by the FE model. This validated the FE model's capability to properly represent the coupon dynamics.

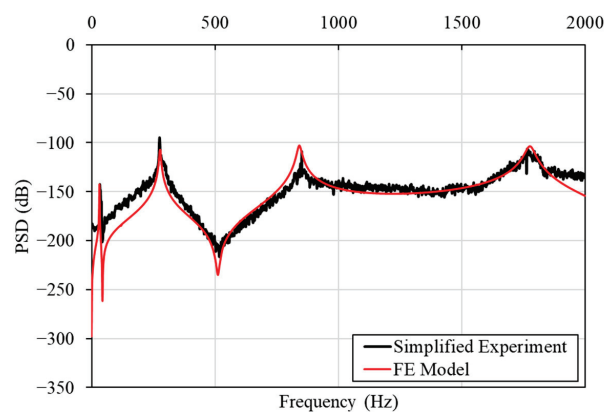


Figure 15. Random Vibration Module validation: FE-based analytical PSD of the random vibration response versus its experimental Welch-based estimated counterpart.

4.6. Numerical Fatigue Damage Identifiability Assessment

As already mentioned, the damage identifiability assessment of the numerical tool was also based on the experimental procedure and the database of our recent work [27] which employed various Fatigue States (FSs) and a more complex acoustic excitation. The complexity of the acoustic excitation, which was not only spatially dependent but also included reflections from the surrounding lab walls, implies that some discrepancies between the experimental and simulated results were expected. Yet, viewed in a positive light, this was intended to provide a sense of the robustness of the assessment results when such discrepancies—that are difficult to model precisely—are present. The assessment results are then presented in Section “Assessment Results with Experimental Validation”.

Assessment Results with Experimental Validation

In Figure 16, the FEM-based analytical PSDs are compared to their experimental Welch-based counterparts for the thermoplastic coupon (Coupon #3) under different Fatigue States (FSs). Even from the early beginning (FS2), the dynamic behavior of the coupon was captured from the FEM-based spectrum, which confirmed the usefulness of the proposed numerical tool. As expected, in the Fatigue State 10 (FS10), the effects of damage on the PSDs were intense.

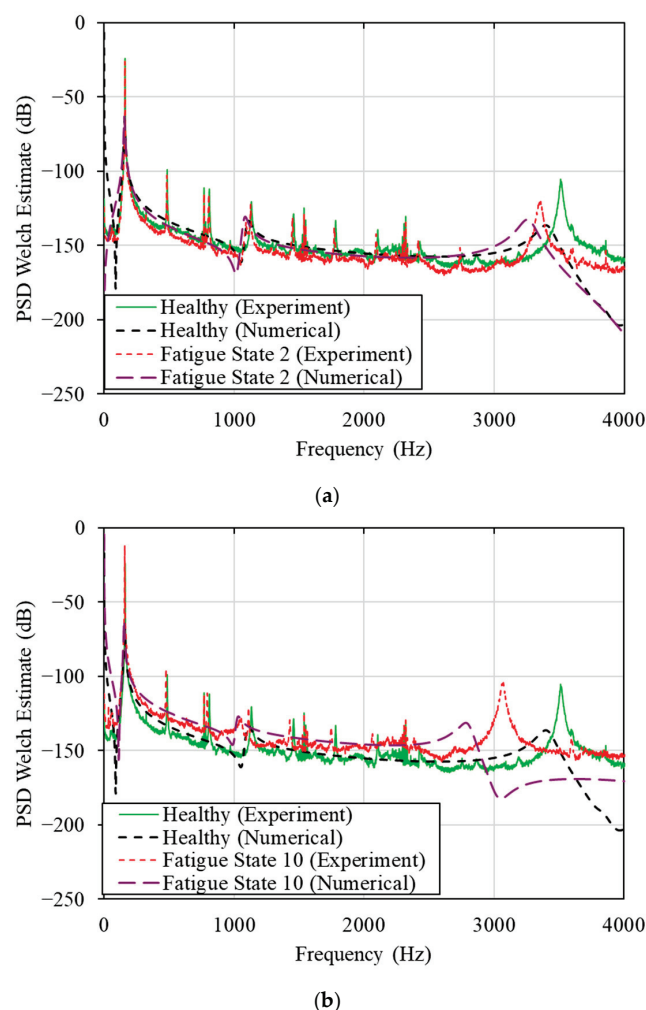


Figure 16. Random Vibration Module validation (a) between Healthy and Fatigue State 2 (FS2), and (b) between Healthy and Fatigue State 10 (FS10): FE-based analytical PSDs of the random vibration response versus their experimental Welch-based estimated counterparts.

As demonstrated in the present section, the RVM's ability to properly model progressive fatigue damage was shown through its effects on the PSD and, in particular, shifts in the resonant frequencies. These are more evident in Figure 17, where the FEM-based spectrums for six FSs are depicted, clearly highlighting the model's ability to model the effects of fatigue damage accumulation on the dynamics.

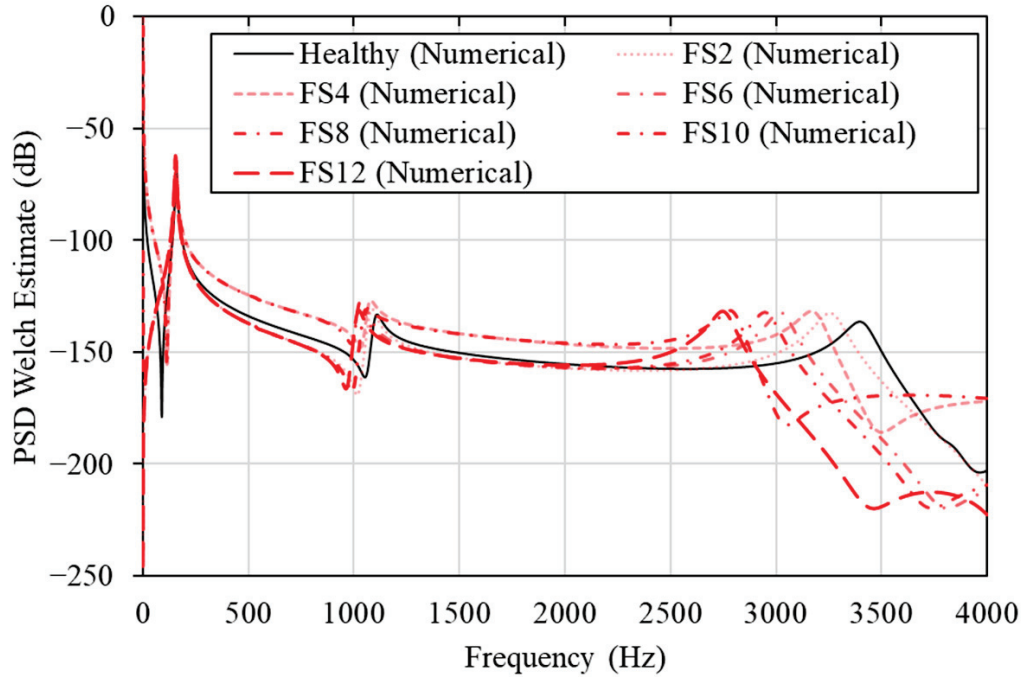


Figure 17. The Random Vibration Module was used: ‘Predicted’ FEM-based PSDs of the random vibration response for the Healthy and various Fatigue States.

The model successfully predicted the decrease in resonant frequencies caused by fatigue damage accumulation. Consistent with the experimental results, the decrease was more pronounced at higher frequencies.

Damage diagnosability is based on dynamics-based Damage Metrics (DMs) that characterize the ‘distance’ of the Healthy State from any Fatigue State (FS) within a proper dynamical feature-based space. Specifically, in this present work, seven Damage Metrics were employed based on the numerical vibration PSD spectrum: Three (Metrics 1 to 3) previously used in [39], and four (Metrics 4 to 7) presently introduced. The DMs were dimensionless, except for DM5. These metrics are defined as follows:

DM 1:

$$\frac{\log(S_{FS_i}(f_i) - S_H(f_i))}{\log(S_H(f_i))} \times 100\% \left[\frac{\text{dB}}{\text{dB}} \right] \quad (9)$$

DM 2:

$$\frac{\sum_{i=1}^n |S_{FS_i}(f_i) - S_H(f_i)|}{\sum_{i=1}^n |S_{FS_i}(f_i)|} \left[\frac{\text{dB}}{\text{dB}} \right] \quad (10)$$

DM 3:

$$\sum_{i=1}^n \frac{1}{f_c} \frac{\sum_{j=1}^i |S_{FS_i}(f_j) - S_H(f_j)|}{\sum_{i=1}^j |S_H(f_j)|} \left[\frac{\text{dB}}{\text{dB}} \right] \quad (11)$$

DM 4:

$$\sum_{i=1}^n (|S_{FS_i}| - |S_H|) [\text{dB}] \quad (12)$$

DM 5:

$$\frac{|S_H - S_{FS_i}|}{S_{FS_i}} \times 100\% \left[\frac{\text{dB}}{\text{dB}} \right] \quad (13)$$

DM 6:

$$\cos(S_H, S_{FS_i}) = \frac{\sum_{i=1}^n S_H \cdot S_{FS_i}}{\|S_H\| \|S_{FS_i}\|} \left[\frac{\text{dB}}{\text{dB}} \right] \quad (14)$$

DM 7:

$$d(S_H, S_{FS_i}) = \frac{\sqrt{(S_H - S_{FS_i})^T C^{-1} (S_H - S_{FS_i})}}{\max \left(\sqrt{(S_H - S_{FS_i})^T C^{-1} (S_H - S_{FS_i})} \right)} \left[\frac{\text{dB}}{\text{dB}} \right] \quad (15)$$

where

S_{FS_i} : FEM-based analytical PSD for the i -th FS;

S_H : FEM-based analytical PSD for the Healthy State;

$|S_{FS_i}|$: absolute of the FEM-based analytical PSD for the i -th FS;

$|S_H|$: absolute value of the FEM-based analytical PSD for the Healthy State,

f_i : a specific frequency;

f_c : the upper (max) frequency within the range of interest;

$\cos(S_H, S_{FS_i})$: cosine of the angle between the two FEM-based analytical PSDs;

$\|S_{FS_i}\|$: l_2 norm of the FEM-based analytical PSD;

d : normalized Mahalanobis distance between two health states;

$S_H - S_{FS_i}$: difference between two FEM-based analytical PSDs;

C : estimated covariance matrix of the two FEM-based analytical PSDs (Healthy and corresponding Fatigue State); and

C^{-1} : inverse of the estimated covariance matrix of the two FEM-based analytical PSDs (Healthy and corresponding Fatigue State).

The values of the Damage Metrics (DM) computed from the FEM-based analytical PSDs for six Fatigue States (FSs) are compared to those of the Healthy State (HS), as well as to their experimentally obtained counterparts, in Figure 18. The DMs for all FSs exhibited considerable differences from the Healthy State, with noticeable deviations starting from FS2, clearly indicating the onset of fatigue damage. DM1, DM2, DM3, and DM5 showed a consistent increase across the FSs, suggesting their suitability not only for detecting damage but also for classifying its severity (extent). In contrast, DM4 displayed random variation, making it unsuitable for damage classification. Numerical-based damage metrics tended to have better behavior, due to a clear mode of damage and the separation of each Fatigue State from one another.

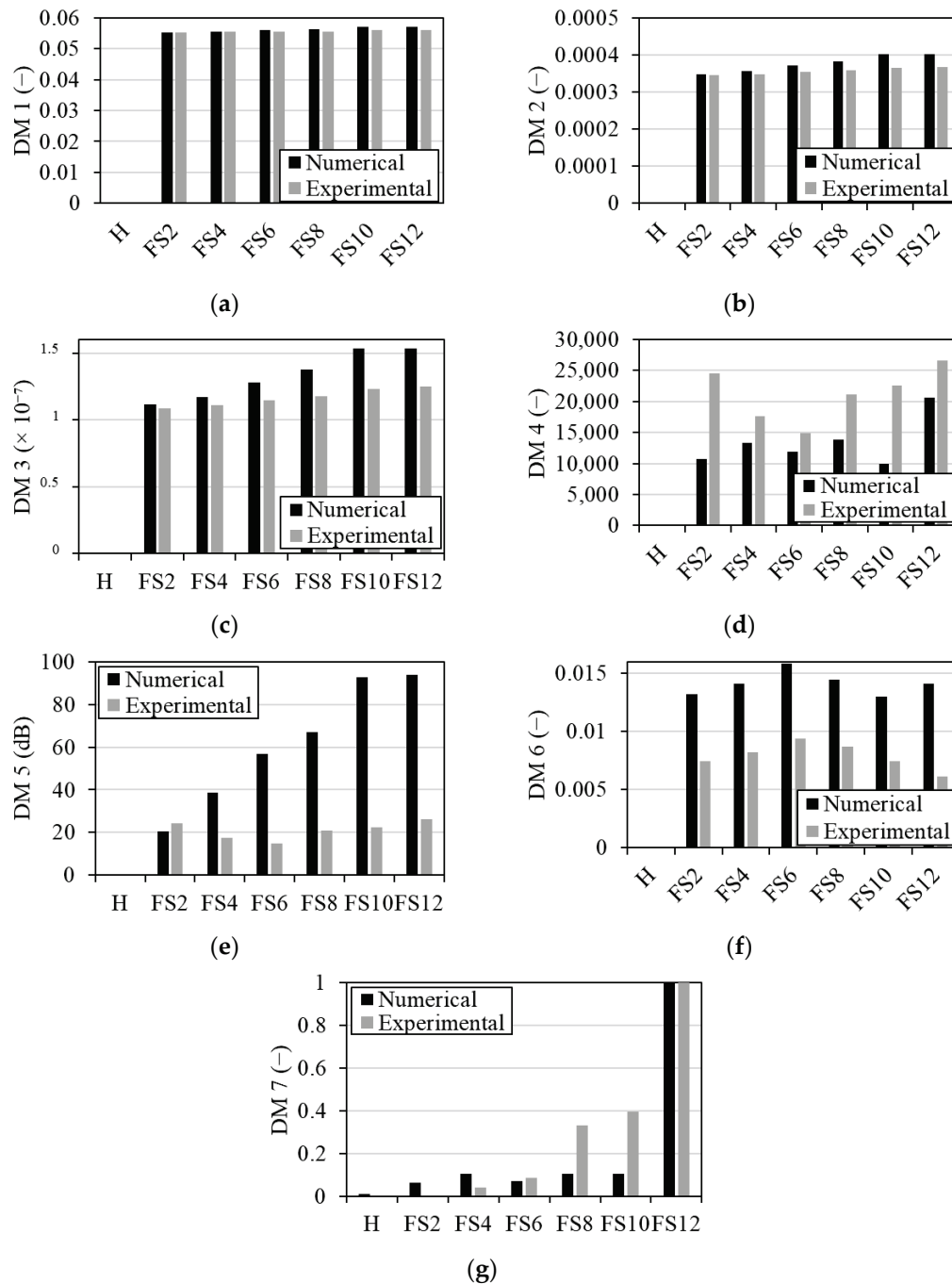


Figure 18. The Random Vibration Module was used: Fatigue damage diagnosability for various Fatigue States via the Damage Metrics (a) DM1, (b) DM2, (c) DM3, (d) DM4, (e) DM5, (f) DM6, and (g) DM7.

5. Concluding Remarks

In this work, we developed an integrated numerical model to simulate random-vibration fatigue damage detectability in thermoplastics. The model comprised two key modules: a fatigue damage simulation module and a random vibration simulation module. This numerical approach digitally replicates a previously established experimental process. Based on the numerical results, the following conclusions can be drawn:

- The Fatigue Delamination Module (FDM) accurately simulates the fatigue delamination initiation and propagation. Furthermore, FDM captures the fatigue behavior of thermoplastic coupons, proving how severe delamination is for the coupon;

- The Random Vibration Model (RVM) accurately simulates the vibration response of both healthy and damaged composite coupons under various excitation conditions. This was validated in two different cases using in-house experimental data. Additionally, the RVM demonstrated similar sensitivity to damage as observed in the experiments, showing a shift in the natural frequencies to the left;
- A parametric study using the FRF model revealed that the Finite Element (FE) mesh density had a minimal effect, while the measurement location, boundary conditions, and excitation intensity significantly influenced the FRF amplitude's intensity;
- All damage metrics, using the FEM-based Power Spectral Density (PSD) spectrums, achieved highly effective damage detectability, reaching 100% performance. Frequency-related damage metrics also showed a monotonic increase, making them suitable for damage classification;
- The proposed numerical tool supports the design and optimization of a random vibration-based Structural Health Monitoring (SHM) system for composite structures through the following steps:
 1. Predicting fatigue damage progression and residual strength as functions of the number of cycles.
 2. Simulating the random vibration response of the healthy structure.
 3. Introducing fatigue damage into the RVM and simulating the random vibration response of the damaged structure.
 4. Maximizing detection efficiency by comparing the FEM-based PSD spectrums of the healthy and damaged states.

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Abbreviations

The following abbreviations are used in this manuscript:

CFRP	Carbon Fiber Reinforced Plastics
DM	Damage Metric
LDV	Laser Doppler Vibrometer
FDM	Fatigue Delamination Module
FE	Finite Element
FEM	Finite Element Model
FRF	Frequency Response Function
FS	Fatigue State
PSD	Power Spectral Density
RVM	Random Vibration Module
SEM	Scanning Electron Microscopy
SHM	Structural Health Monitoring

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Article

Implementation of Composite Materials for an Industrial Vehicle Component: A Design Approach

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Abstract: The aim of this research is to define a design approach for implementing composite materials in a component of an industrial vehicle, having weight reduction as the primary goal. Through the schematisation of the problem and analytical analysis, the definition of a new geometry, a material and production process, and numerical simulations and experimental studies to test the new solution, an optimization process of the chosen geometry is proposed. After the definition of the process, an applicative example is presented, analysing a front underrun protection device in two different solutions: one made of glass-fibre-reinforced polymer and the other of carbon-fibre-reinforced polymer. An economic comparison has also been conducted between the new configurations and the traditional steel version, showing a weight reduction of approximately 55% for the carbon-fibre-reinforced polymer solution and around 18% for the glass-fibre-reinforced polymer solution. These weight reductions are achievable through a reinvestment that can be amortized in less than five years, thanks to fuel consumption savings.

Keywords: composite materials; lightweight; finite element analysis; weight reduction; industrial vehicles components; modal analysis; buckling analysis; optimisation

1. Introduction

The mechanical design process is an essential component of engineering that entails a methodical approach to developing efficient and effective solutions for many applications. As technology advances, the tools currently accessible in the market substantially contribute to the reduction in costs and time spent on research. To ensure their reliability and identify potential issues, it is essential to validate these virtual models through physical testing. Mechanical testing yields critical data that validate simulation accuracy and allow designers to enhance their models, resulting in more resilient and efficient outputs. As a result of the advancements in design and testing methodologies in the last decades, innovative materials have emerged as viable alternatives to traditional ones. Nowadays, the adoption of fibre-reinforced polymer (FRP) composite structures instead of traditional metallic components in structural design offers several advantages, including reduced fuel or oil consumption, emission reductions, and improved performance, due to their lower mass and the low energy costs involved in their production process [1]. In the literature, the study of lightweight machine parts has been applied not only to transport vehicles but also to earth-moving machinery [2,3], hoist equipment [4,5], automotive applications [6,7], and even aviation [8–10].

The optimization process for lightweight components has already been extensively studied in the literature. Saeedi et al. [11] defined general procedures for design-

ing and analyzing fibre-reinforced polymer parts for railroad vehicles, particularly for car bodies, bogies, wheelsets, and other structural components such as rail sleepers. Dragatogiannis et al. [12] analysed a deck reinforcement on a composite yacht for crane installation, proposing a methodology flowchart for the structural analysis. Rahman et al. [13], while studying ship structures made from composite materials, presented a process to evaluate the engineering properties of multidirectional laminates. Gardie et al. [14] investigated the use of composite materials in car wheels, highlighting their advantages compared to aluminium. Ciampaglia et al. [15] developed an innovative method to design a suspension for automotive applications.

Some of the author's previous articles also discuss the optimization process in lightweight design. For example, a new concept for the design of a tank for a firefighting vehicle was defined, considering the use of plastic material and a new geometry to accommodate the boundary conditions imposed by the component [16]. Additionally, lightweight design using composite materials has already been applied to a reconnaissance rover [17] and an industrial vehicle transmission [18], where comparisons of different materials were made to study weight reduction.

These studies propose a design approach aimed at reducing structural weight—the main advantage of the composite materials used—but they lack experimental tests to validate the finite element model or perform an economic evaluation.

This research is presented from this perspective, aiming to suggest a design approach that can be applied to structural components of an industrial vehicle, supported by applicative examples validated through experimental tests. Furthermore, the variety of the outputs can be demonstrated using identical numerical models with differing inputs, such as geometry or material utilized.

The rest of the paper is organized as follows: Section 2 describes the optimization process and the theoretical background of this design approach, in order to show the procedure that the authors want to propose, while Section 3 implements this design approach on an industrial vehicle component—a front underrun protection device. After being described, this component is shown in three different configurations: the traditional steel version, which is the one currently in use, one in GFRP and another in CFRP. Section 4 then discusses the results of the three configurations, with a focus on the economic aspects of the component. Finally, Section 5 provides a brief summary and critique of the results.

2. Materials and Methods

2.1. Theoretical Background of the Design Approach

The design approach, aimed at reducing the weight of a component through the use of composite materials, can be summarized by the flowchart shown in Figure 1. After schematizing the problem, which depends on the boundary conditions, the geometry must be defined. This geometry is correlated with the manufacturing process of the studied component and the material selection. The stresses can then be determined through analytical, numerical, or experimental analysis, taking into account specific conditions. As an example of such specific conditions, in the automotive sector, the component experiences external vibrations, necessitating modal analysis to prevent resonance effects; additionally, buckling analysis is essential to ensure that the component not only endures the applied loads but also remains stable under certain conditions. Once a solution that satisfies all boundary conditions is found, the geometry optimization process begins. This is an iterative process that continues until the new geometry meets the required conditions. These boundary conditions typically depend on the purpose of the component being analysed or the applicable standards.

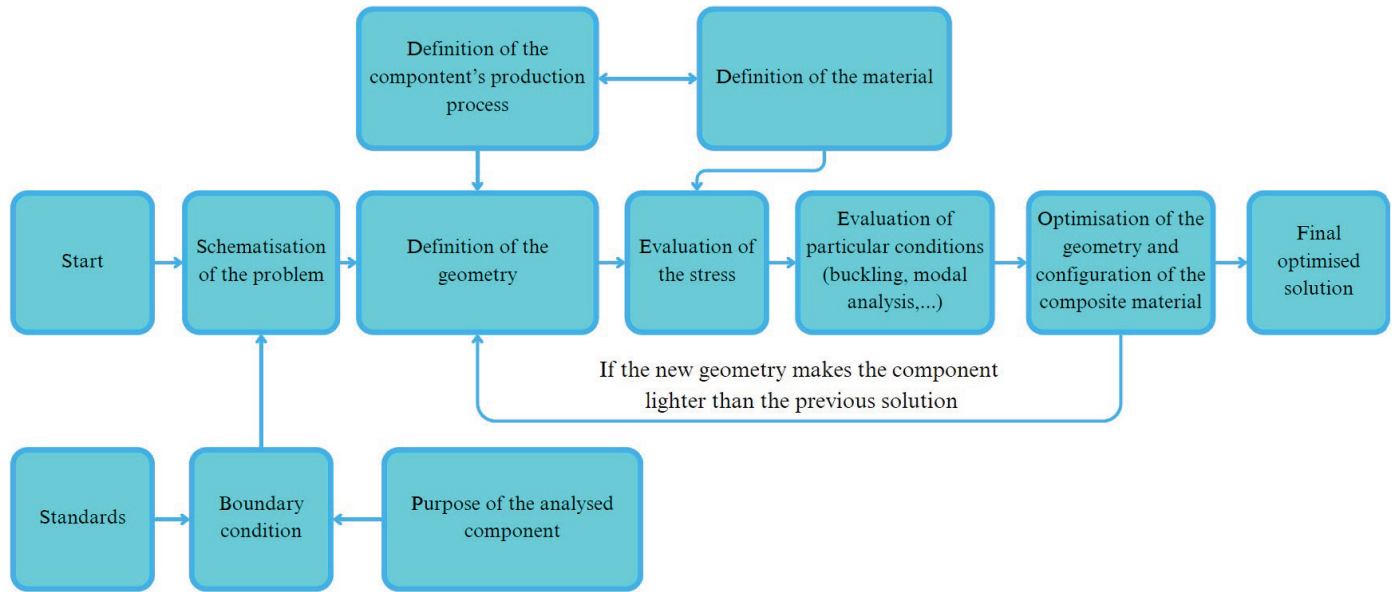


Figure 1. Design approach flowchart.

2.2. Methods and Criteria Adopted

The case study must be analysed with respect to its boundary conditions and different configurations. Through the balance of momentum and angular momentum, the support reactions can be evaluated to determine the force distribution diagrams, which include tensile forces, bending moments, shear forces, and torques. Using these diagrams, the most stressed positions can be identified. After defining the geometry, the stress in the material can be analysed using De Saint Venant's theory, Bredt's formula, and Jourasky's formula [19]. The stress profiles can be described using Equation (1):

$$\sigma_n = \frac{F}{A} \quad ; \quad \sigma_f(y) = \frac{M_f}{J_{nn}} \cdot y \quad ; \quad \tau_{M_t} = \frac{M_t}{2\Omega s} \quad ; \quad \tau_t = \frac{TS^*}{J_{nn}s} \quad (1)$$

where σ_n is the normal stress imposed by force F on the section area A , $\sigma_f(y)$ is the stress due to the bending moment at a distance y from the section's centreline, M_f is the bending moment, J_{nn} is the moment of inertia with respect to the centreline, τ_{M_t} is the stress from the torque, M_t is the torque, Ω is the mean area of the section, s is the profile thickness, τ_t is the shear stress, T represents the shear forces, and S^* is the static moment of the section. This step identifies the worst-case scenario for the component, though a preliminary geometry must be established first based on material selection and production methods.

At this stage of the design process, buckling analysis must also be considered. Buckling refers to lateral deformations in structural elements subjected to compressive forces. In steel structures, where thinness is common, buckling is especially significant. According to elastic stability theory, Equation (2) defines P_{cr} , the critical load at which a perfectly elastic component begins to exhibit non-axial deformations:

$$P_{cr} = \frac{\pi^2 EJ}{L_0^2} \quad (2)$$

where E is the Young's modulus of the material, J is the section's minimum moment of inertia, and L_0 is the length of free vertical displacement, which depends on the beam's constraints. Usually, this value is the product of the beam's length l and a correction factor μ . Figure 2 shows this value for different constraints.

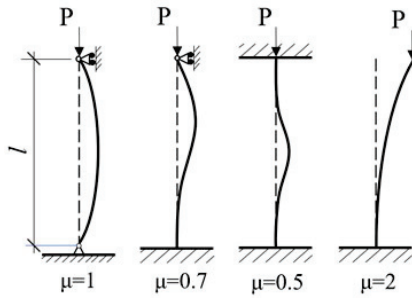


Figure 2. Different configurations and their values of μ .

Equation (2) assumes the load is perfectly applied at the section's centre. However, in reality, this is not always achievable. Additionally, accidental material inhomogeneity, such as pre-deformed fibres, may result in a beam that is not perfectly straight. These are known as second-order effects [20], and to account for them, the criteria in (3) must be verified:

$$\frac{P_a}{P_{cr,m}} + \frac{M_{eq}}{M_u(1-\theta)} \leq 1 \quad \text{where} \quad \theta = \frac{P_a}{P_{cr,m}} \quad (3)$$

where P_a is the axial failure force under a primary bending moment, $P_{cr,m}$ is the buckling load at failure without a primary bending moment, M_{eq} is the equivalent bending moment along with the axial force, and M_u is the ultimate bending moment at failure without an axial force. To define $P_{cr,m}$, Equation (4) must be used:

$$P_{cr,m} = \frac{\chi A f_y}{\gamma_{M1}} \quad \text{where} \quad \chi = \frac{1}{\phi + \sqrt{\phi^2 + \bar{\lambda}^2}} \leq 1 \quad (4)$$

where A is the cross-sectional area, f_y is the material's yield strength, γ_{M1} is the material safety factor, and χ is the reduction coefficient accounting for buckling. The factors ϕ and the dimensionless slenderness $\bar{\lambda}$ are defined in Equation (5):

$$\phi = 0.5 \cdot [1 + \alpha \cdot (\bar{\lambda}^2 - 0.2) + \bar{\lambda}^2] ; \quad \bar{\lambda} = \frac{\lambda}{\lambda_y} = \sqrt{\frac{A f_y}{P_{cr}}} \quad (5)$$

where α represents the effect of imperfections, with values of 0.13, 0.21, 0.34, 0.49, and 0.76 for curves a_0 , a , b , c , and d (European buckling design curves) respectively [20,21]. For the other terms in Equation (3), Equation (6) provides the method for evaluating them:

$$M_u = \frac{f_y W_{pl}}{\gamma_{M1}} ; \quad M_{eq} = C_m \cdot |M_B| \quad \text{where} \quad C_m = 0.6 - 0.4(M_A/M_B) \leq 0.4 \quad (6)$$

where M_A and M_B are the moments at the ends of the beam, and their ratio is negative for single-curvature bending and positive for double-curvature bending. The coefficient W_{pl} represents the plastic resistance modulus of the section, as defined in Equations (7) and (8).

$$W_{pl,elliptical} = 2 \cdot (\pi r s) \cdot \left(\frac{l}{2} + \frac{4r}{3\pi} \right) \quad (7)$$

$$W_{pl,rectangular} = \frac{bh^2}{4} - (b-2t) \left(\frac{h}{2} - t \right)^2 \quad (8)$$

All the variables to evaluate the plastic resistance modulus of the section are the dimensions of the section itself. To understand them, Figure 3 shows the dimensions for a hollow rectangular section and for a hollow elliptical section.

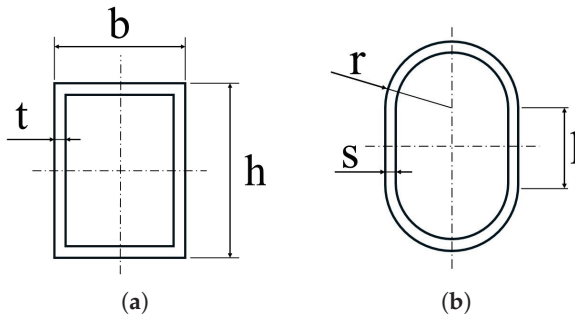


Figure 3. Dimensions of the used sections. (a) Dimensions of the hollow rectangular section. (b) Dimensions of the hollow elliptical section.

Given that the essence of this design methodology involves the utilization of composite materials, plate buckling [22] must be considered. Beginning with classical plate theory and incorporating laminated composite theory, a composite plate is examined, with its dimensions and coordinate system delineated in Figure 4:

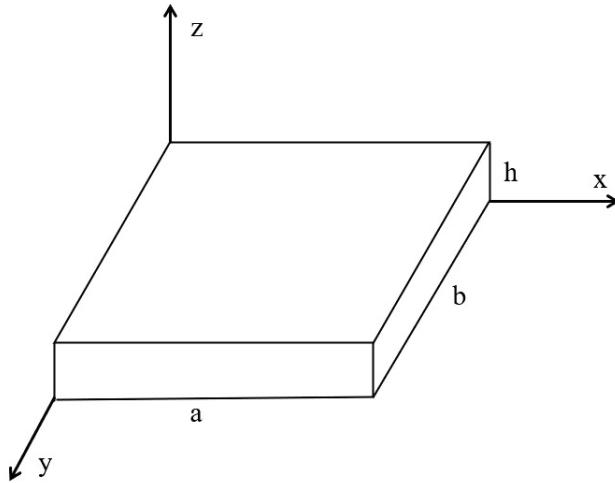


Figure 4. Coordinate systems and dimensions of the composite plate.

The critical load for a slender composite plate subjected to uniaxial compression N_x in the x-direction is articulated as delineated in Equation (9):

$$N_{x,cr} = \frac{2\pi^2}{b^2} \sqrt{D_1 D_2} \left(1 + \frac{D_3}{\sqrt{D_1 D_2}} \right) \quad (9)$$

where $N_{x,cr}$ is the critical buckling load per unit length (N/m) and b is the plate width. D_1 , D_2 and D_3 are dependent on the components of the bending stiffness matrix and are obtained through Equation (10):

$$D_1 = D_{11}; \quad D_2 = D_{22}; \quad D_3 = D_{12} + 2D_{66}; \quad (10)$$

For a composite laminate, each value of the bending stiffness matrix can be determined through Equation (11):

$$D_{ij} = \frac{1}{3} \sum_{k=1}^N (\bar{Q}_{ij})_k (z_k^3 - z_{k-1}^3) \quad (11)$$

where Q_{ij} is the transformed reduced stiffness coefficient for each k-layer (see Equation (21)), z_k is the coordinate of the upper surface of layer k and z_{k-1} is the coordinate of the lower surface of the k-layer.

2.3. Choice of the Materials

Together with defining the geometry, the materials must also be selected. Most structural components in industrial vehicles are made of steel, an isotropic material with higher mechanical properties than other metals. This study aims to implement composite materials instead of metals due to their advantages in weight reduction. The choice of material depends on several factors: the availability of suppliers, the production process, the material's performance in the studied scenario, and its cost compared to other options. A preliminary selection can be done thanks to Ashby's diagrams [23], where the stiffness-density ratio helps to decide on a substitute for the material already implemented in the current solution but choosing in order to lighten the component.

To evaluate the mechanical properties of these materials, it is necessary to know the mechanical properties of both the fibre and the matrix, which are usually provided by suppliers. Assuming the fibres are aligned along the main longitudinal direction, Equations (12) and (13)—derived from the micromechanics theory of orthotropic plates [24]—can be used:

$$\rho = \rho_f V_f + \rho_m V_m; \quad \nu_{12}(= \nu_{13}) = \nu_f V_f + \nu_m V_m; \quad E_1 = E_f V_f + E_m V_m; \quad (12)$$

$$E_2(\approx E_3) = \frac{E_f E_m}{V_m E_f + V_f E_m}; \quad X_T = \sigma_{ft} V_f; \quad Y_C = \sigma_{mc} [1 + (V_f - \sqrt{V_f})(1 - \frac{E_m}{E_f})] \quad (13)$$

$$Y_C = \sigma_{mc} [1 + (V_f - \sqrt{V_f})(1 - \frac{E_m}{E_f})] \quad Y_T = \sigma_{mt} \frac{E_2}{E_m} (1 - V_f^{\frac{1}{3}}); \quad (14)$$

To determine the longitudinal compressive strength X_C , the failure mode must be analysed. Naik and Kumar [25] evaluated the different analytical models to predict the longitudinal compressive strength and compared their predictions to experimental results. They identified that both Xu-Reifsnider [26] and Budiansky [27] models predict the compressive strength quite accurately when the parameter representative of the actual material system is ascertained properly. The first model predicted the compressive strength based on the analysis of microbuckling of a representative volume element using a beam on an elastic foundation model. The effect of matrix slippage and the fibre–matrix bond condition was included by two factors, namely, ξ and η . The final expression in terms of the constituent properties and micro-geometrical parameters is given by Equation (15):

$$X'_C = G_m \left(V_f + \frac{E_m}{E_f} (1 - V_f) \right) \left(2(1 + \nu_m) \sqrt{\frac{\pi \sqrt{\pi} \eta r_f}{3 \frac{E_m}{E_f} (V_f \frac{E_m}{E_f} + 1 - V_f)(1 + V_f \nu_f + \nu_m (1 - V_f))}} + 1 - \xi - \frac{\sin \pi \xi}{2\pi} \right) \quad (15)$$

where ξ is the empirical factor representing matrix slippage, η is the empirical factor representing fibre matrix bond and r_f is the radius of fibre.

The second model unified the Rosen (based on microbuckling of the fibres) and Argon formula (based on the principle that the component of interlaminar shear stress due to the presence of misalignment produces kinking) for an elastic ideally plastic composite. The expression as given by Budiansky is defined by Equation (16):

$$X''_C = \frac{G_m}{(1 - V_f)(1 + \bar{\phi}/\gamma_y)} \quad (16)$$

where $\bar{\phi}$ is the initial fibre misalignment angle in unidirectional composites and γ_y is the matrix yield strain.

The shear strength S and critical load are difficult to determine using formulas due to multiple uncertainty factors and the different behaviour of the matrix and fibre before failure [28,29]. Typically, this value is provided by the composite material supplier or can be evaluated through experimental tests such as single-fibre fragmentation testing, short beam shear, or the Iosipescu shear test [30].

Once the geometry of the component and the percentage of each type of ply are suggested, classical composite theory for two-dimensional laminates [24] can be applied for an initial analysis. By introducing a reference system with the first two axes on the lamina and neglecting the deformation ϵ_3 (which is orthogonal to the lamina plane), the deformations of a specially orthotropic lamina can be evaluated using Equations (17) and (18). The S_{ij} components of the compliance matrix S are calculated from the lamina's elastic constants:

$$\begin{bmatrix} \epsilon_1 \\ \epsilon_2 \\ \gamma_{12} \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & 0 \\ S_{12} & S_{22} & 0 \\ 0 & 0 & S_{66} \end{bmatrix} \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix}; \quad (17)$$

$$S_{11} = \frac{1}{E_1}; \quad S_{22} = \frac{1}{E_2}; \quad S_{12} = -\frac{\nu_{12}}{E_1} = -\frac{\nu_{21}}{E_2}; \quad S_{66} = \frac{1}{G_{12}} \quad (18)$$

To calculate stresses from deformations, the stiffness matrix Q (the inverse of the compliance matrix) must be used. These formulations can be applied to laminas with fibres reinforced along the primary axis. However, if there is a relative angle θ , the rotation matrix T must be used, where $c = \cos \theta$ and $s = \sin \theta$, as described in Equation (19):

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} \begin{bmatrix} \epsilon_1 \\ \epsilon_2 \\ \gamma_{12} \end{bmatrix} \quad T = \begin{bmatrix} c^2 & s^2 & 2sc \\ s^2 & c^2 & -2sc \\ -sc & sc & c^2 - s^2 \end{bmatrix} \quad (19)$$

To adapt the compliance matrix S and the stiffness matrix Q to the new direction, the following Equation (20) must be applied:

$$[\bar{S}] = [T]^T [S] [T] \quad [\bar{Q}] = [T]^T [Q] [T] \quad (20)$$

Furthermore, the expression for calculating stresses in a generic lamina becomes the one described in Equation (21):

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} = \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} \end{bmatrix} \begin{bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{bmatrix} \quad (21)$$

To obtain a first approximation of the deformation values, the extensional stiffness matrix A must first be derived. By setting the total number of laminas N and the composite thickness h , a coordinate system can be defined (as shown in Figure 5), and then A can be evaluated using Equation (22):

$$A_{ij} = \sum_{k=1}^N (\bar{Q}_{ij})_k (z_k - z_{k-1}) \quad (22)$$

where i represents the rows of the matrix, j the columns, and k the number of layers. Additionally, a local reference system has been defined to evaluate stresses. Using the deformation values, obtained analytically by multiplying the inverse of the A -matrix with the stress matrix (calculated using Equation (1) and the laminate thickness), it is possible to

use Equation (23) to calculate the local stresses along each lamina's directions within the entire laminate:

$$\begin{bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{bmatrix} = h [A]^{-1} \begin{bmatrix} \sigma_f \\ 0 \\ \tau_t \end{bmatrix} \rightarrow \begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} = [\bar{Q}] \begin{bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{bmatrix} \rightarrow \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix} = [T] \begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} \quad (23)$$

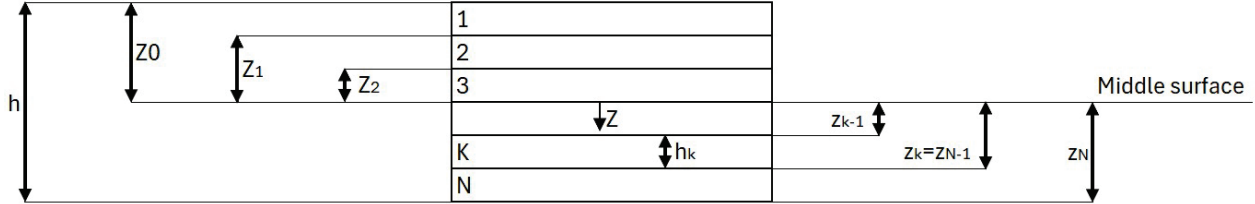


Figure 5. Coordinate system for the position of laminates.

By relating the local reference system to the lamina's system, and considering the maximum bending stress and maximum shear stress (obtained from the component's geometry and boundary conditions), it is possible to apply the Tsai–Hill criterion, described in Equation (24), or the maximum stress criterion, described in Equation (25), based on the material resistance values.

$$FI = \frac{\sigma_1^2}{X_1^2} - \frac{\sigma_1 \sigma_2}{X_1^2} + \frac{\sigma_2^2}{X_2^2} + \frac{\tau_{12}^2}{S^2} ; \quad FOS = \frac{1}{\sqrt{FI}} \quad (24)$$

$$X_C < \sigma_1 < X_T ; \quad Y_C < \sigma_2 < Y_T ; \quad |\tau_{12}| < S \quad (25)$$

where X_1 represents X_T if $\sigma_1 > 0$ or X_C if $\sigma_1 < 0$, while X_2 represents Y_T if $\sigma_2 > 0$ or Y_C if $\sigma_2 < 0$. To determine if the ply will theoretically fail, the factor of safety (FOS) must be less than 1.

Finally, to increase resistance to weathering or sunlight, a protective film described by Komartin et al. [31] or Zhang et al. [32] should be applied to the surface of the component.

2.4. Optimization of the Geometry

After deciding on the material and verifying the boundary conditions, the geometry of the section must be chosen. Since weight reduction is the main goal of the research, it is preferable to select a hollow section rather than a solid one. Furthermore, the shape of the vehicle components also depends on the production process, which is related to the material. When working with composite materials, optimizing the geometry and designing the material itself must be done simultaneously, thanks to the flexibility in choosing different fibre orientations.

The idea is to develop objective functions that need to be minimized or maximized, depending on the goal of the design. These functions could represent the mass of the component or its cost. The variables in the function are the parameters describing the geometry, such as the main dimensions and thickness. The process must adhere to a system of constraints, which are typically imposed by standards or safety factor requirements for the section under analysis.

To illustrate this concept, Figure 6 shows an example: with the weight of the component as the objective function (dependent on its thickness), the goal is to minimize it without exceeding the current configuration's weight and ensuring a safety factor greater than 1. Another constraint is related to the production process, meaning the thickness must be an integer. In this scenario, the optimized solution has a thickness of 6 mm.

This is a simple example; in practice, objective functions, parameters, and constraints can be much more complex.

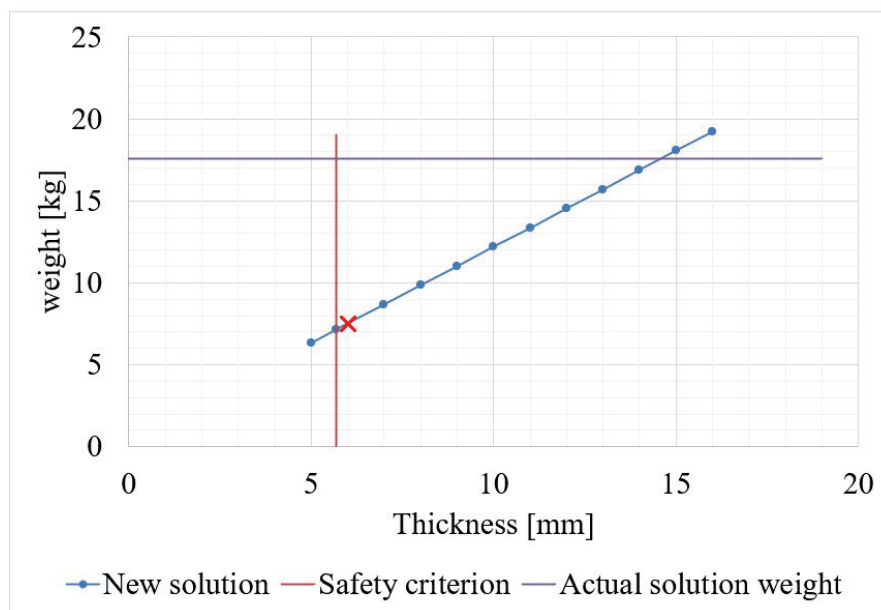


Figure 6. Example optimization chart: here the thickness of a hollow section has been optimized, with the weight reduction as the main goal. Two boundary constraints are shown, one defined by the safety criterion and one defined by the previous solution.

The algorithm adopted and, most of all, the variables in this process depend on the needs of the user. In the literature, an extended review [33] about the implementation of an optimization algorithm for laminated composites is presented: the algorithms suggested are the meta-heuristic ones (such as Genetic Algorithm, Simulated Annealing algorithm, Particle Swarm Optimization, Ant Colony Optimization, Firefly Algorithm), which are more popular due to their efficiency and stability. It was also described that stacking sequence and geometry are considered the key design variables in optimization problems, and in most of the articles, the main goals were the following:

- Minimizing the weight;
- Maximizing the mechanical properties;
- Minimizing the cost of the component.

As an example, in the following scenarios, an objective function that can be taken is the minimization of the weight (which is correlated here to the minimization of the costs), and the Tsai–Hill criterion and maximum displacement can be chosen as constraints. The variables in this research work should be the geometry of the component and the stacking sequence.

3. Implementation of the Design Approach: The Front Underrun Protection Device

3.1. Description of the Component

The front underrun protection device (FUPD) is a component installed on heavy industrial vehicles designed to limit the damage caused to cars in the event of a collision. In the case of a collision between a heavy vehicle and a car, there is a risk that the car may become wedged beneath the heavy vehicle. To prevent this, the underrun protection device was introduced in the position shown in Figure 7. This component was chosen as an applicative example, but the lightweight process of the FUPD is part of a larger project

that aims to reach a weight reduction for each component of an industrial vehicle—some of which have already been completed by the authors [18,34].

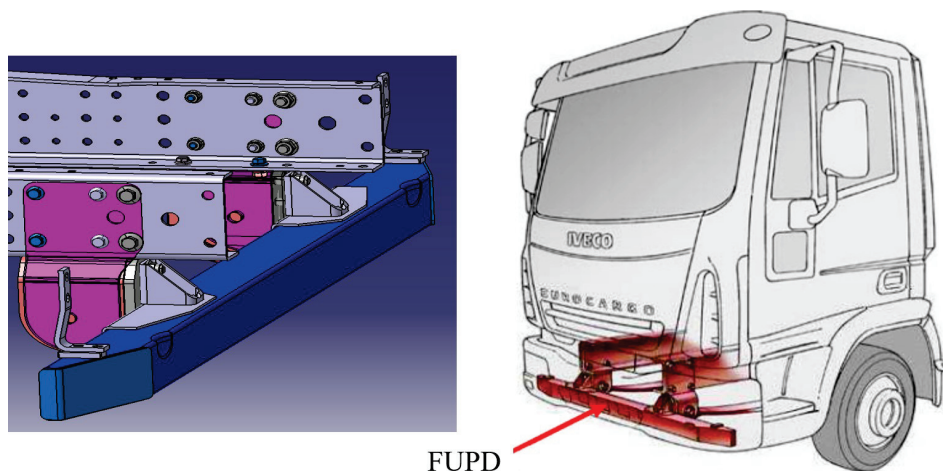


Figure 7. On the left, a 3D view of the studied device; on the right, the location of the FUPD on the industrial vehicle.

The FUPD design needs to meet some requirements to ensure its functionality:

- Be compliant with the United Nations Economic Commission for Europe (UN/ECE) standard n.93 [35];
- Exhibit natural frequencies higher than the characteristic internal and external excitation frequencies of the vehicle;
- Prevent buckling of the structure.

This study focuses on the Eurocargo 120EL category of Iveco® vehicles, which have a maximum weight of 120,000 N, approximately 12,000 kg. UN/ECE standard n.93 [35] outlines the methodology to homologate the analysed component. Three loading conditions are defined: P1, P2, and P3. Forces P1 and P3 must be equal to half the weight of the vehicle at its maximum load, while force P2 must be equal to the entire weight of the vehicle. For a vehicle weighing 12 tons, $P1 = P3 = 60 \text{ kN}$ and $P2 = 120 \text{ kN}$. These forces should be applied longitudinally to the chassis, following the positions illustrated in Figure 8, with an almost static application velocity as defined by the standards.

The loads are applied through a plate positioned between the piston rod and the FUPD surface, with the central point of the plate aligning with the positions described for the loads. The success of the homologation test is determined by ensuring that the maximum displacement of the FUPD is less than 400 mm from the outermost surface of the bumper. Plastic deformation of the component is allowed, but it must not exceed the maximum deflection outlined in the regulations. For composite materials, known for their brittleness, only elastic deformation is acceptable. Any other type of deformation would result in the component being deemed broken and ineligible for homologation.

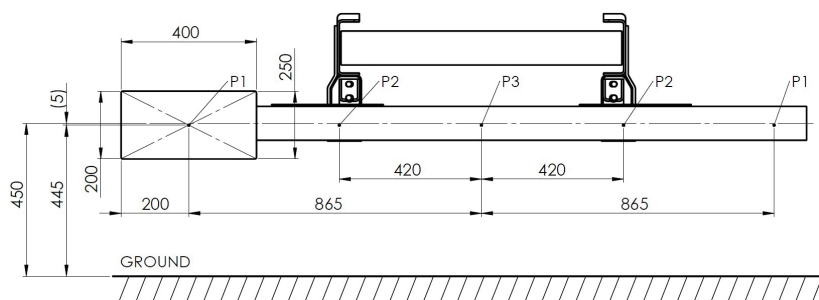


Figure 8. Geometry and location of the three different loads P1, P2 and P3 [mm].

Figure 9 shows where the force is applied. The dimensions are $L_f = 865$ mm, $b = 400$ mm (so $a = 465$ mm), and $F = 60$ kN. Using momentum and angular momentum balance, the support reactions are evaluated through the system of equations in (26).

$$\begin{cases} v_a + v_b - F = 0 \\ v_b \cdot 2b + F \cdot a = 0 \end{cases} \quad (26)$$

Based on these data, we calculate $v_a = 98.875$ kN and $v_b = -34.875$ kN.

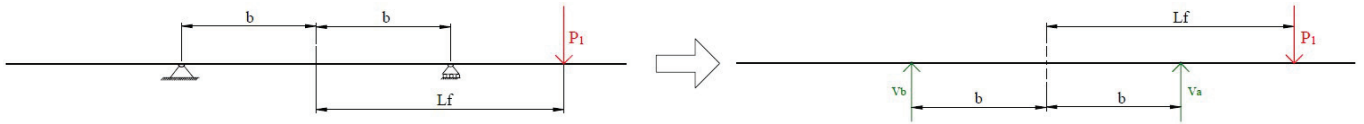


Figure 9. Sketch of the component treated as a beam.

Next, force distribution diagrams are determined. Considering that the device must be no more than 400 mm from the ground and that the beam is 100 mm high, the centreline of the bar is 450 mm from the ground. The load is applied 445 mm from the ground [35], resulting in a torque moment with a 5 mm lever. The torque $M_t = 300$ Nm.

The force distribution diagrams are shown in Figure 10, indicating that the most critical section is where the support reaction v_a is applied. The bending moment is $M_f = 27,900$ Nm, with $T_1 = v_b$ and $T_2 = F$.

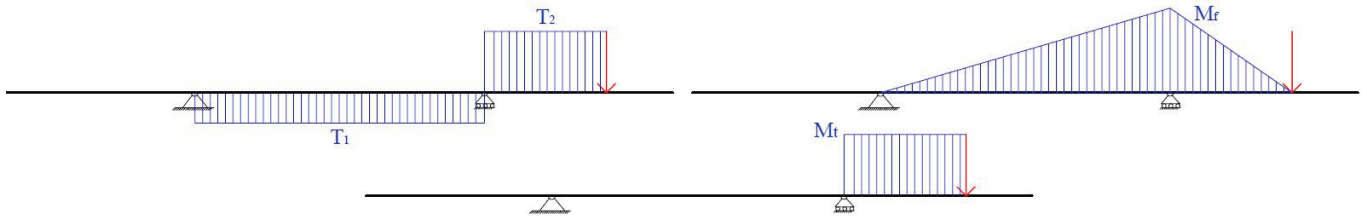


Figure 10. Force distribution diagrams of the analysed component.

The P_1 load [35] was analysed as the most critical of the three loading conditions described in the European Standard due to the highest bending moment on the supports. Specifically, $M_f(P_1) = 27,900$ Nm, $M_f(P_2) = 0$ Nm, and $M_f(P_3) = 24,000$ Nm. P_2 is more critical for the brackets linked to the chassis, where the constraint reaction v_a would be 120 kN instead of 98 kN.

These loads are then used to analyse the structural integrity of the FUPD. In addition to this, a modal analysis is needed to evaluate the natural frequencies and corresponding mode shapes of the structure. Indeed, external excitation and internal power sources, such as engine vibration excitation and road excitation, are the sources of frame resonance. Road vibration frequency of cars is in the range of 0–100 Hz [36] and the vehicle body's first order vibration frequency needs to be higher than 20 Hz, considering the most important road excitations are within this frequency [37]. Equation (27) can be used to determine the excitation frequency of diesel engines, according to pertinent research:

$$f = \frac{2nz}{60\tau} \quad (27)$$

where τ is the number of engine strokes, z is the number of engine cylinders, and n is the engine speed, defined in RPM. A six-cylinder, four-stroke diesel internal combustion engine is used in this design of the truck, and for the calculation, a rated speed of 2500 RPM is chosen in order to be in a critical situation. After these considerations, when moving, the diesel engine vibrates at a frequency of 125 Hz.

Three different configurations of the analysed component were studied: the traditional steel S460Q design, a glass-fibre-reinforced polymer (GFRP) option, and a carbon-fibre-reinforced polymer (CFRP) alternative.

3.2. Traditional Solution

The properties of the traditional steel configuration, also defined as S460Q according to UNI EN 10027-1, are shown in Table 1. This material was adopted both for the FUPD and for the supports, which were welded to each other.

Table 1. Mechanical properties of S460Q.

Ultimate Tensile Strength R_m [MPa]	Yield Strength $R_{p0.2}$ [MPa]	Young's Modulus E [MPa]	Density ρ [kg/m ³]	Poisson's Coefficient ν
620	460	210,000	7860	0.29

Figure 11 shows the geometry of the traditional component design. In this solution, the stress $\sigma_f(y_{max}) = 498$ MPa, assuming $J_{nn} = 2.801 \times 10^6$ mm⁴, which is the moment of inertia related to the section where M_f is maximum. The action of the reinforcement bracket is added to that of the bar, resulting in $\tau_{Mt} = 5.15$ MPa and $\tau_t = 231.74$ MPa.

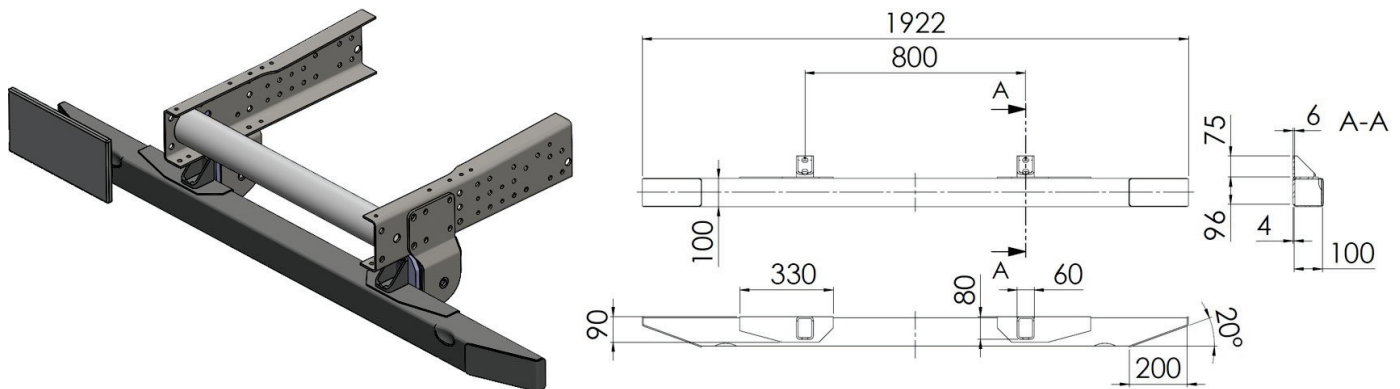


Figure 11. Current solution [mm].

The tensile stress in the critical section is higher than the yield stress of the material but lower than the ultimate tensile strength (620 MPa), making this value acceptable as a preliminary estimate. To study displacements, equations of the elastic line [19,38] were applied. Solving the system reveals a maximum displacement of 16.83 mm, well within the target range, where the maximum allowable displacement is 81 mm, considering the imposed standards and the vehicle's geometry.

3.3. GFRP Innovative Solution

Figure 12 shows the GFRP geometry for the initial study, where the cross-section is a square profile with a 6.5 mm thickness. The reason behind the choice of this material was mainly for the availability of the supplier but also factors such as the mechanical properties of the material and its cost related to the feasibility of high production volumes influenced the decision. Regarding the material of the supports, steel S460Q was adopted in this initial configuration. This configuration was used to compare numerical results from FEM analyses with experimental tests. Concerning the material of the supports, steel S460Q was adopted in this initial configuration. Given that the material and production process are typically chosen together to accommodate high production volumes (estimated at 6000 units per year), pultrusion [39] was selected as the manufacturing process, adopting

the unidirectional fibre as the architecture of the material structure. An isophthalic polyester resin was chosen as the matrix, being a common choice with this production process [40]. The minimal mechanical properties of this material, given directly by the supplier and according to UNI EN 13706-3, are shown in Table 2: presuming the material demonstrates brittle behaviour, the stress–strain response was considered linear-elastic, defined by the chosen modulus of elasticity. Failure was characterised as occurring upon reaching the tensile strength, in accordance with the literature [41,42]. The same logic was applied to the CFRP solution.

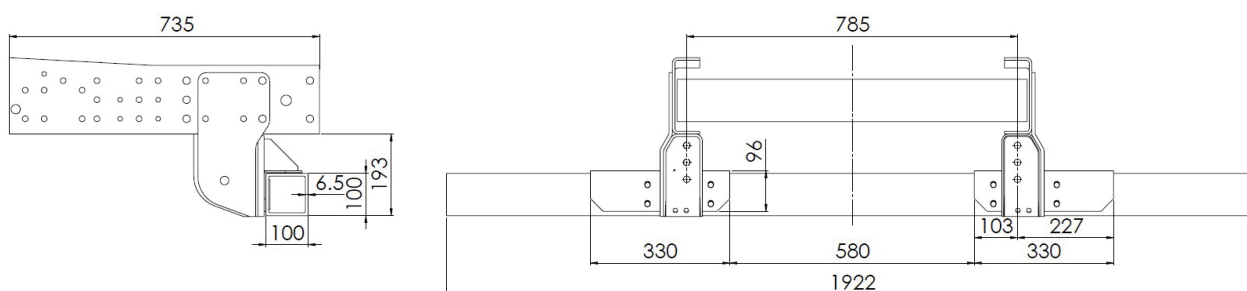


Figure 12. First GFRP assembly developed [mm].

Table 2. Mechanical properties of GFRP.

Mechanical Property	Symbol	Value	UOF
Bending elasticity modulus	E_f	29	GPa
Longitudinal tensile elasticity modulus	E_1	29	GPa
Transversal tensile elasticity modulus	E_2	7	GPa
Shear elasticity modulus	G_{12}	3	GPa
Longitudinal tensile strength	X_T	240	MPa
Transversal tensile strength	Y_T	50	MPa
Longitudinal bending strength	σ_{fX}	240	MPa
Transversal bending strength	σ_{fY}	100	MPa
Density	ρ	1.92	kg/dm ³
Glass fibre percentage	v_f	58	%

Simulations of this configuration were run in Solidworks®, using a solid mesh based on curvature with 265,786 nodes and 135,015 quadratic tetrahedral elements. The final mesh resulted from an h-refinement procedure that concentrated on the FUPD and its supports. It was defined using a chassis with a coarser mesh to conserve computational time and resources. A static analysis was conducted to confirm the numerical model prior to investigating additional properties of the examined component, including resonance frequencies, and subsequently discovered buckling modes. Regarding the connections of the chassis components, bonded interactions were utilised, as the chassis is not the focus of this research paper. Conversely, bolt connections were employed for the connections between the FUPD, brackets, and other components to accurately simulate the actual connections among these parts, except for the plate where the force is applied, where a bonded interaction is chosen to avoid a possible rigid motion of the body. The mechanical properties of the composite material were implemented by the software through the definition of a local reference system and the material's orthotropic behaviour. The local reference system is chosen based on the mechanical properties of the material, so the X-axis was the longitudinal axis along the FUPD, while the Y-axis was the transversal axis on the surface of the beam. The Z-axis defines the dimension of the thickness of the FUPD. Regarding the orthotropic behaviour, the mechanical properties of Table 2 were added in the definition of

the material. This numerical test showed that the most stressed area of the FUPD is the rear surface, specifically the inferior edge, as illustrated in Figure 13.

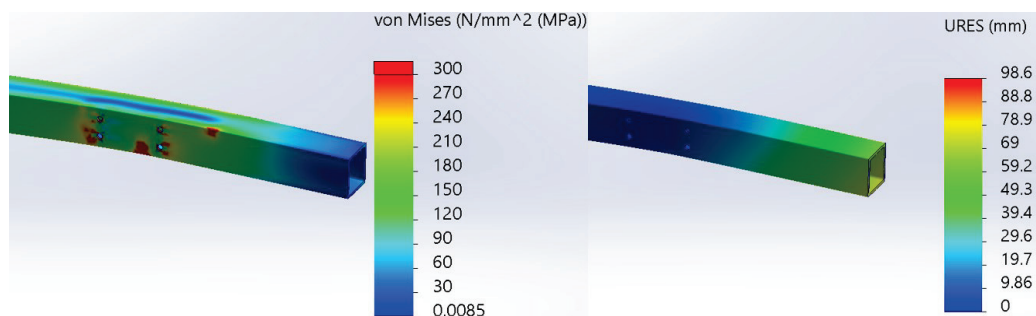


Figure 13. Results of the GFRP first solution at the load application end: on the left is the Von Mises stress, while on the right is the total displacement. It is clear that the component has reached the failure point in some areas.

The most critical stress is the shear stress, which, combined with the longitudinal compressive stress near the side of the support close to the applied load, leads to component failure. The numerical test also identified other highly stressed areas near the bores where the supports are linked to the bar, exceeding the tensile strength of the GFRP material.

To verify the numerical computations, an experimental test was conducted using the 100×100 profile with a 6.5 mm thickness. A system was developed to represent a real vehicle, where the chassis was attached to simulate a fixed support 735 mm behind the front edge, as shown on the left of Figure 14. The FUPD was mounted 400 mm from the ground, matching the conditions of the numerical analysis. On the right of Figure 14, a hydraulic cylinder was used to apply the load, pushing on a plate aligned with the concentrated load. This force is applied directly to the beam and not its supports. The plate was linked to the cylinder via a hinge, allowing it to follow the component's deformation.

As expected, the test showed that the FUPD failed, as represented in the load-displacement diagram in Figure 15, accompanied by images of the failed area.

The graph shows that failure occurred at $F = 28.11$ kN with a displacement of 35.74 mm, the final value on the red line near the theoretical straight line. Afterwards, the load increased due to the steel reinforcement, followed by a collapse of the force curve. However, Figure 15 reveals that the failure was not caused by axial stress from the bending moment but by shear stress on the rear surface of the beam, near the steel reinforcement.

The Tsai–Hill criterion was applied to determine the factor of safety (FOS) for the composite material. Using the FOS from the numerical analysis, the failure load was evaluated and compared to the load reached in the experimental test.

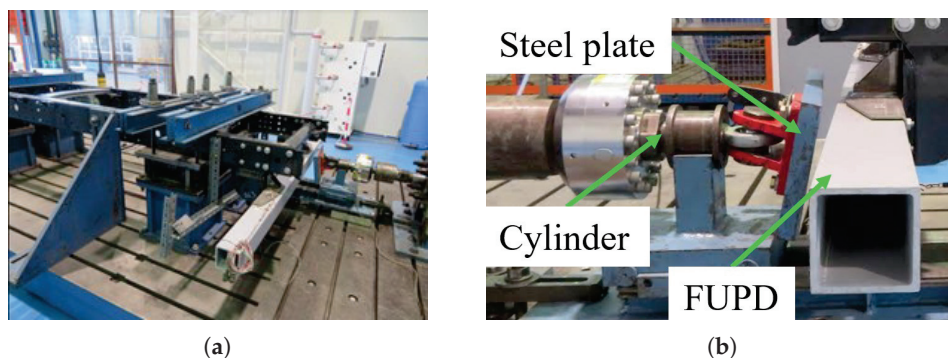


Figure 14. Photos of the experimental test area. (a) Setup for the experimental test on the studied component. (b) Zoom of the cylinder used to apply the loads.

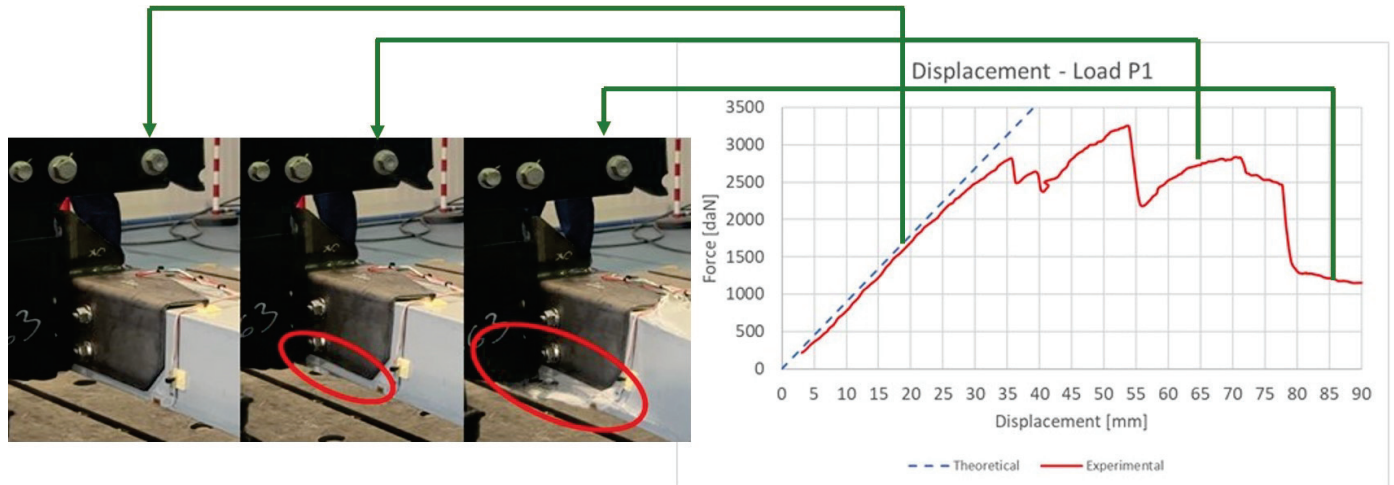


Figure 15. On the (left), a zoom on the failed area of the FUPD (circled in red), and on the (right), the diagram of load versus displacement from the experimental test.

Table 3 shows the minimal difference between the numerical and experimental results, which is less than 5%. The FOS from the numerical computation was taken at the critical edge of the component (Figure 15), while the experimental FOS was obtained by dividing the actual load by the rated load.

Table 3. Comparison between the FEM model and the experimental data.

	FOS	Rated Load [daN]	Actual Load [daN]	Deviation [%]
FEM values	0.446	6000	2676.44	−4.79%
Experimental test values	0.449	6000	2811.16	ref.

Given the strong correlation between the numerical and experimental values, a GFRP model of the FUPD was developed that can withstand the stress imposed by European regulations. The geometry of the new solution is shown in Figure 16.

The new solution has a similar geometry to the previous one, but the FUPD thickness was increased to 9 mm, and the reinforcement bracket was modified as shown in the drawing. The new configuration, particularly the “C” profile replacing the original “L” profile, was chosen to reduce shear stresses on the rear surface of the FUPD, as previously explained.

The reinforcement brackets were made from aluminium 7075-T6 [43], selected for its lighter density and mechanical properties comparable to steel, as shown in Table 4:

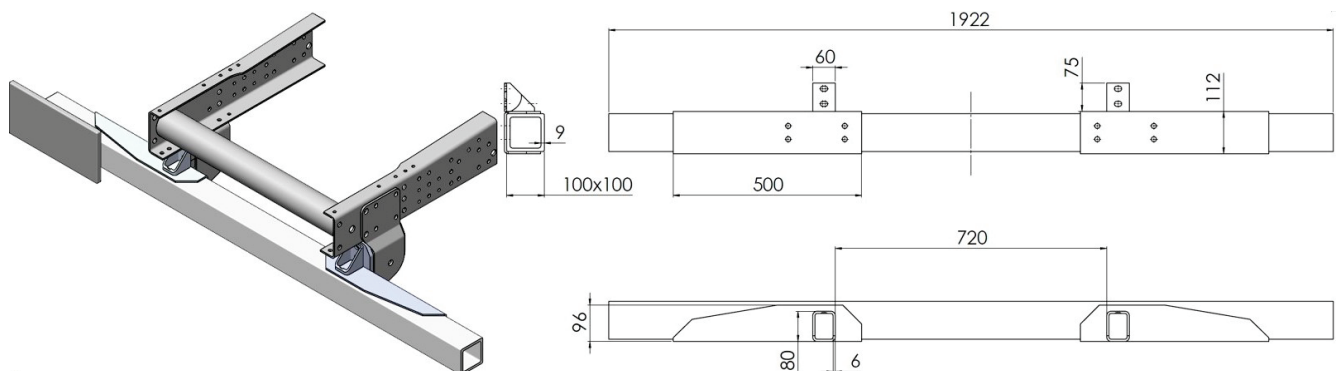


Figure 16. Final GFRP assembly developed [mm].

Table 4. Mechanical properties of Al 7075-T6.

Ultimate Tensile Strength R_m [MPa]	Yield Strength $R_{p0.2}$ [MPa]	Young's Module E [MPa]	Density ρ [kg/dm ³]	Poisson's Coefficient ν
570	505	72,000	2.81	0.33

The results confirm that the FUPD withstands the applied load: the Von Mises stress is lower than the ultimate strength of the GFRP, and the displacement is within the limit imposed by the standard, as shown in Figure 17.

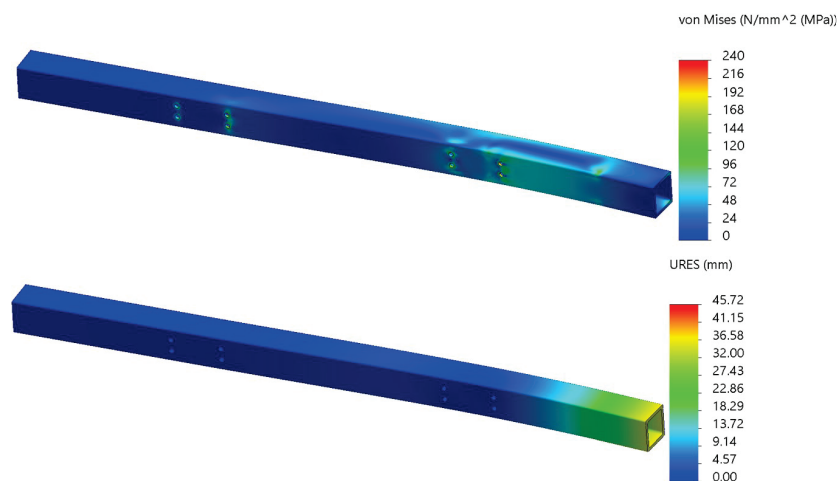


Figure 17. Final GFRP assembly stress: the Von Mises stress is shown at the top, and the total displacement of the FUPD is shown at the bottom.

Due to the higher stress on the brackets exceeding the yield stress of the material, a non-linear analysis was performed. To reduce computational costs, the fitting radii of the brackets were not considered in this analysis, but this simplification did not affect the most stressed areas. The simulation was set as a non-linear, static analysis with iterations based on the Newton–Raphson technique, through displacement control, due to the almost plateau of the aluminium after the yield point. As shown in Figure 18a, the FEM analysis results demonstrate that the tensile stresses on the bracket are lower than the ultimate strength of the material. In Figure 18b, the stress–strain curve of the 7075-T6 aluminium alloy adopted for the non-linear analysis is presented.

As explained previously, modal analyses were performed: Figure 19 shows that the first natural frequency of the structure is far from the highest excitation that the FUPD is subjected to.

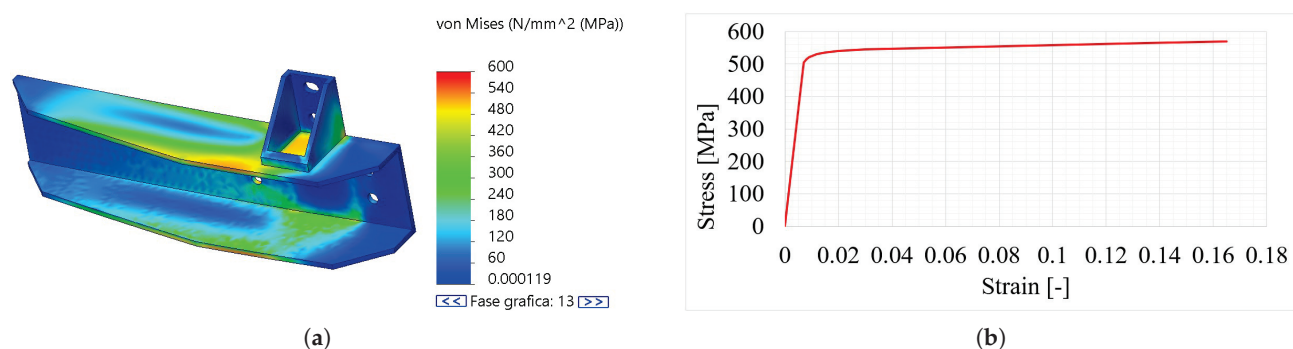


Figure 18. Non-linear FEM analysis on the innovative aluminium bracket. (a) Von Mises stress results from the non-linear analysis of the bracket. (b) Stress–strain curve of 7075-T6 aluminium alloy at quasi-static strain rate.

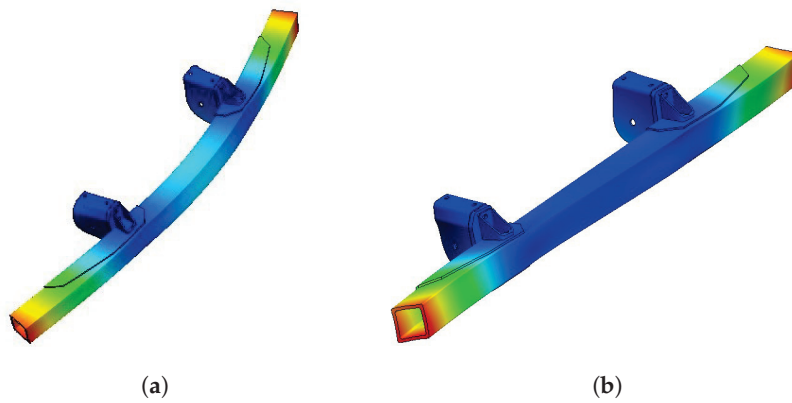


Figure 19. Modal analysis of the GFRP solution. (a) First resonance mode of the GFRP solution: the resonance frequency here is 158 Hz. (b) Second resonance mode of the GFRP solution: the resonance frequency here is 181 Hz.

Global and local buckling were also studied. Figure 20a shows the first global buckling mode of the GFRP solution, with a safety factor of 9.09, while Figure 20b shows the first local buckling mode, with a safety factor of 7.21.

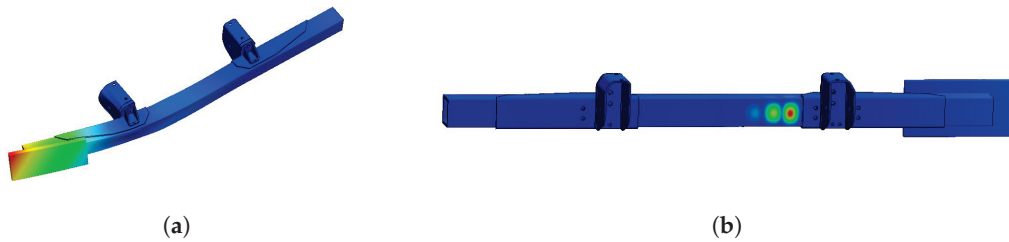


Figure 20. Buckling analysis of the GFRP solution. (a) First global buckling mode of the GFRP solution. (b) First local buckling mode of the GFRP solution.

3.4. CFRP Innovative Solution

The last solution studied was designed using carbon-fibre-reinforced polymer (CFRP). The selection of this material was the result of a compromise between the highest mechanical properties as possible while maintaining an affordable increment on the costs of the component. For this material, the FUPD was manufactured using a process called filament winding [39], adopting the unidirectional fibre as the architecture of the material structure. With this production method, a different geometry was applied, as shown in Figure 21. The new geometry features the innovative elliptical profile where the cross-section is not constant, but the overall length remains the same as the GFRP profile. The decision to alter the section at the points of support was primarily made to increase resistance in those areas.

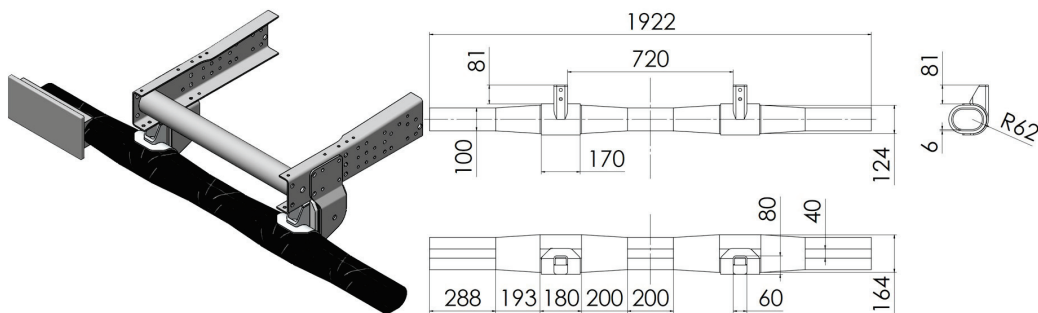


Figure 21. CFRP assembly developed [mm].

Furthermore, the connection between the FUPD and the supports was modified, utilizing a support structure that can adapt to the new geometry and is made of the same

material, as described in Table 4. In this configuration, the supports were designed to position the FUPD 5 mm lower, which avoids the torque moment described in Section 2.1. In the previous solution, this adjustment was not possible due to the similarity of the existing supports, but with the new design of the component, it became feasible. It should be noted that the experimental testing performed on the GFRP solution facilitated the development of a numerical model that closely approximates reality. Upon establishing the model's reliability, it was deployed for various geometries and materials.

The carbon fibre chosen for this study is M50J [44], and the matrix is an epoxy resin (the epoxy resin is commonly used with this production process [40]) SX10 [45], with their mechanical properties detailed in Table 5. For the properties of the ply, see Table 6, evaluated through the equations of Section 2.3.

Table 5. Property of the M50J carbon fibre and SX10 epoxy resin.

Property	Carbon Fibre		Epoxy Resin	
	Symbol	Value	Symbol	Value
Density	ρ_f	1880 kg/m ³	ρ_m	1200 kg/m ³
Young's modulus	E_f	475 GPa	E_m	3.3 GPa
Tensile strength	σ_{ft}	4120 MPa	σ_{mt}	65 MPa
Compressive strength	σ_{fc}	-	σ_{mc}	120 MPa
Poisson ratio	ν_f	0.28	ν_m	0.34

Table 6. Mechanical properties of CFRP.

Mechanical Property	Symbol	Value	U.O.M.
Longitudinal tensile elasticity modulus	E_1	286	GPa
Transversal tensile elasticity modulus	E_2	8.16	GPa
Shear elasticity modulus	G_{12}	3	GPa
Longitudinal tensile strength	X_T	2472	MPa
Transversal tensile strength	Y_T	45	MPa
Longitudinal compression strength	X_C	880	GPa
Transversal compression strength	Y_C	99.2	MPa
Shear strength	S	59	MPa
Density	ρ	1608	[kg/m ³]
Poisson ratio	ν_{12}	0.3	
Carbon fibre percentage	V_f	60	%

After testing various configurations, the analytical studies determined that the optimized solution, with the described geometry, has a thickness of 6 mm and consists of 20 layers (with each ply having a thickness of 0.3 mm). The Layer Stacking Sequence (LSS) includes 35% of the plies oriented at 5°, 35% at −5°, 10% at 45°, 10% at −45°, and 10% at 90° made by filament winding as already done for a hydraulic cylinder [46]. This result was achieved through an optimization process where the main constraint was the Tsai–Hill criterion, and six variables were chosen: the thickness of the section and the percentage of plies in five specific orientations (5°, −5°, 45°, −45°, 90°), selected based on the adopted production process and to maximise strength in the main directions. The function to be minimised was the weight; therefore, the minimum thickness was selected (as it is a variable directly related to the weight). Once a solution was found, an FEM simulation was performed to confirm the LSS and the previously determined thickness. Numerical analysis shows that the studied device can withstand the application of force P1: as shown

at the top of Figure 22, the maximum displacement is around 36 mm, which is below the limit imposed by European regulations.

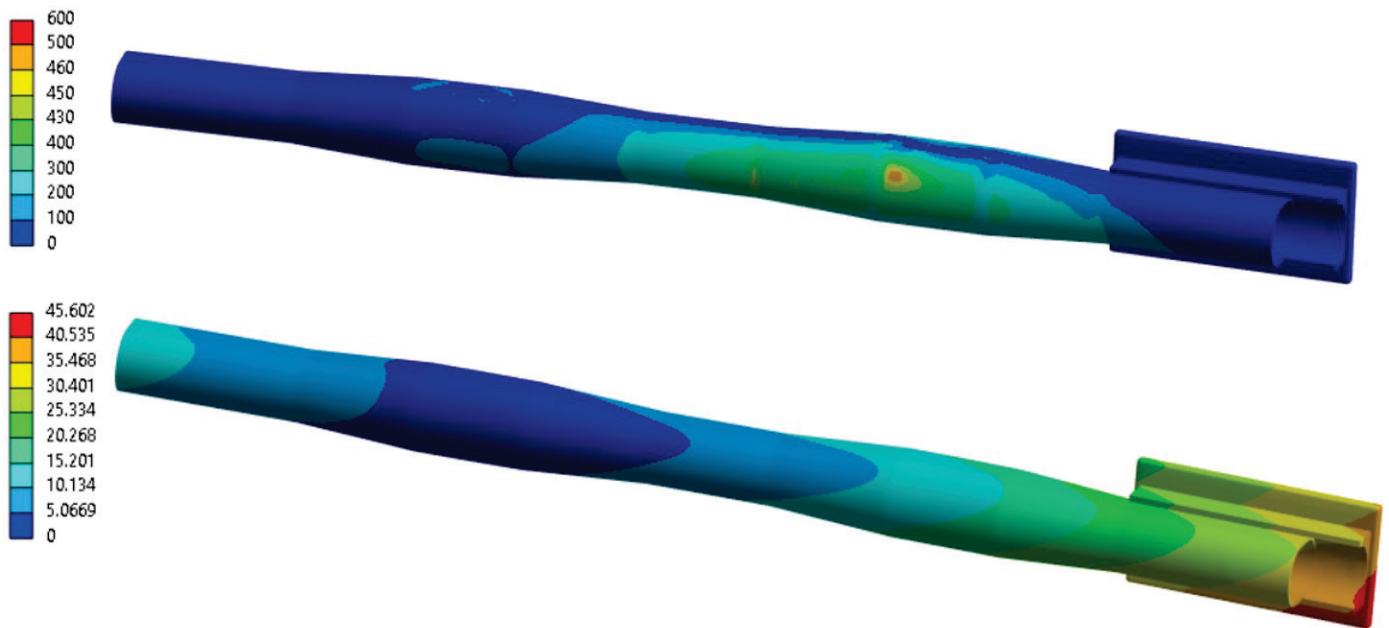


Figure 22. CFRP assembly results: at the (top), the Von Mises stress of the component is presented, while at the (bottom), the total displacement is shown.

The study of this configuration was conducted using Solidworks® for 3D modelling and Ansys® for simulations. A solid mesh was used, based on curvature, with 424,821 nodes and 391,808 total elements. The type of solid elements used varied by component; for the FUPD, linear hexahedral elements were used. The software incorporated the mechanical properties of the composite material by establishing a local reference system and defining the material's orthotropic behaviour. The local reference system is selected according to the material's mechanical properties, designating the X-axis as the longitudinal axis along the FUPD and the Y-axis as the transverse axis on the beam's surface. The Z-axis delineates the dimension of the FUPD's thickness. Each element here represents a segment of the ply, characterised by a finer mesh, which results in augmented computing time, while also accounting for the interactions among each ply. The mechanical properties from Table 6 were incorporated into the specification of the material about its orthotropic behaviour. At the bottom of the same figure, the Von Mises stress of the FUPD is displayed: the most stressed area is at the closest support to the applied load. The stress is less than 410 MPa, meaning the component remains within safe limits due to the material chosen.

A non-linear static analysis for the brackets was also performed under load P2. The results, shown in Figure 23a, indicate that the maximum Von Mises stress in the support is around 550 MPa, which does not exceed the failure threshold.

In this design, the connection between the brackets and the FUPD is made using an epoxy adhesive, as described by Konchakova et al. [47], instead of traditional bolts. This decision was made to avoid excessive stress concentrations around the bolt holes. Figure 23b shows a zoomed-in view of the Von Mises stress graph for the FUPD's linked area, revealing a stress peak near the holes. While the plies can withstand stress along the fibre's longitudinal direction, the FUPD could fail in that area because not all plies share the same orientation. To verify this, it was found that the contact area between the tubular section and the supports is around 10,000 mm², and the P2 load is 120,000 N, as required by European standards. The shear stress in this area is around 12 MPa, well below the

stress limit for Konchakova's adhesive epoxy, as well as the in-plane and interlaminar shear strength of the composite material.

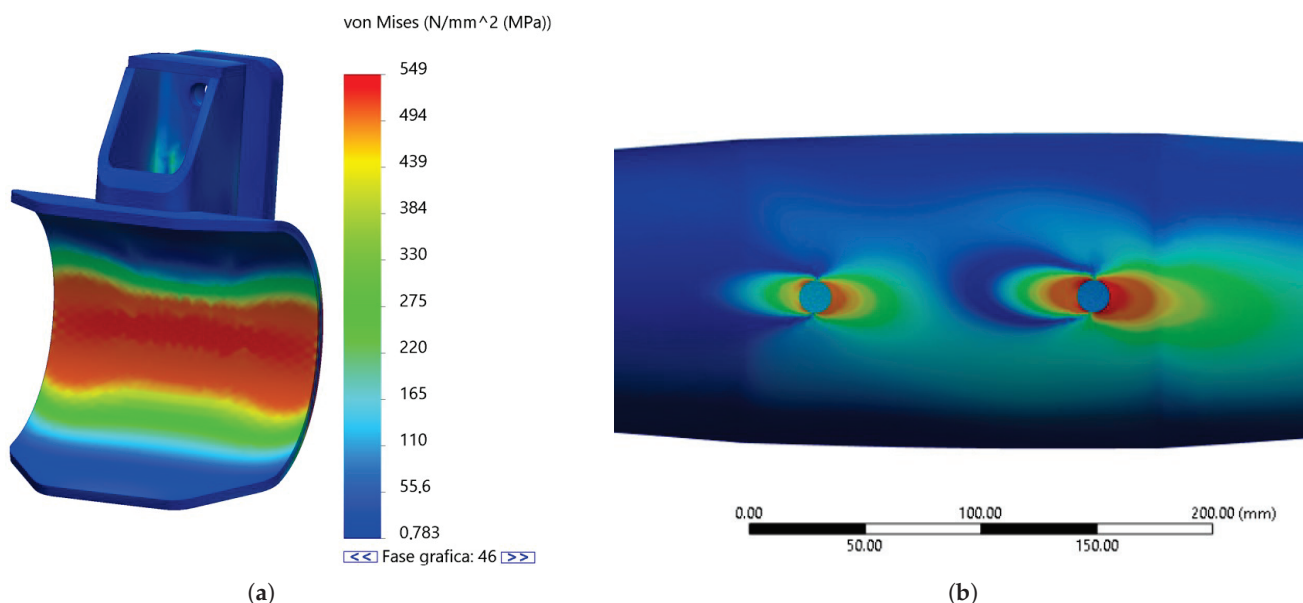


Figure 23. Numerical analysis for the supports of the CFRP solution. (a) Nonlinear analysis results on the bracket. (b) Von Mises stress of the FUPD solution in CFRP with bolt connections to the brackets.

Modal analyses were also conducted: Figure 24 shows the first and second resonance modes of the structure. With the same considerations of Section 3.3, it is clear that the component does not reach the resonance, having different frequencies than the external and internal excitations of the structure.

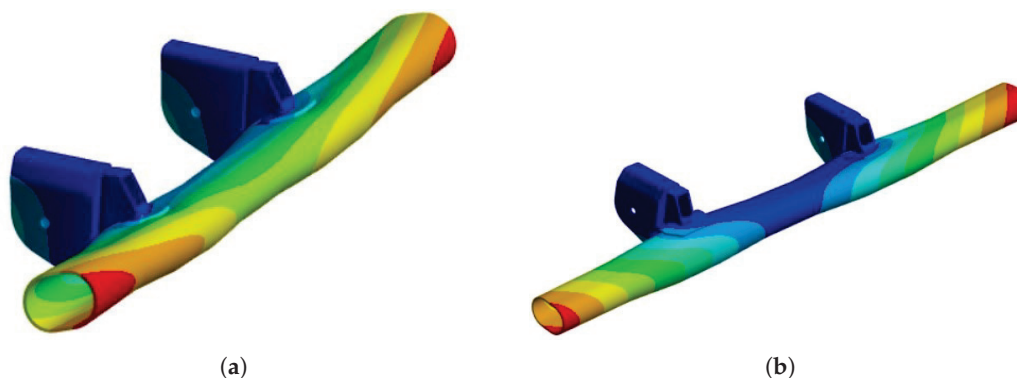


Figure 24. Modal analysis of the CFRP solution. (a) First resonance mode of the CFRP solution: the resonance frequency here is 277 Hz. (b) Second resonance mode of the CFRP solution: the resonance frequency here is 289 Hz.

For the buckling analysis, Figure 25a shows the first global buckling mode of the CFRP solution, with a safety factor of 9.85, while Figure 25b shows the first local buckling mode, with a safety factor of 10.82.

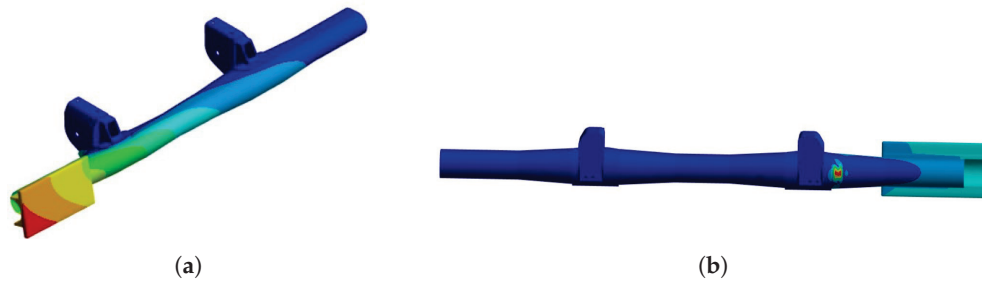


Figure 25. Buckling analysis of the CFRP solution. (a) First global buckling mode of the CFRP solution. (b) First local buckling mode of the CFRP solution.

4. Discussion of the Results

Using data from manufacturer datasheets and the FEM program's 3D models, the mass of the FUPD and the supports for each configuration were determined.

Table 7 presents the weights of the FUPD in the three configurations studied. The percentage of weight reduction (W) and the percentage increase in cost (c) were evaluated as described in Equation (28):

$$W_{Reduction} \% = \frac{W_{newsol} - W_{steel}}{W_{steel}} \cdot 100 ; \quad c_{Increase} \% = \frac{c_{newsol} - c_{steel}}{c_{steel}} \cdot 100 \quad (28)$$

A clear reduction in weight can be seen when comparing the steel solution to the composite material solutions: the glass fibre solution shows a 17.7% reduction in weight compared to the steel solution, and the carbon fibre solution shows a 55.6% reduction. However, the cost comparison reveals that the new FUPD and support designs require a greater initial investment. The cost difference between the innovative solutions and the traditional one is approximately EUR 17.68 for the GFRP solution and EUR 393.95 for the CFRP solution (after subtracting the cost of the traditional FUPD). Nevertheless, considering the economic savings from reduced fuel consumption, the time required to recover the initial investment is less than 5 years for the CFRP solution and less than 1 year for the GFRP solution, as illustrated in Figure 26. The calculations assume a vehicle lifetime of 11 years and an average mileage of 1.49 million kilometers [48], or 135,000 km per year.

Table 7. Costs and weights of the different solutions.

	Steel S460Q		Glass Fibre Reinforced Polymer		Carbon Fibre Reinforced Polymer	
	Weight [kg]	Cost [EUR]	Weight [kg]	Cost [EUR]	Weight [kg]	Cost [EUR]
Brackets	4.38 (S460Q)	14.76	4.59 (Al 7075-T6)	23.3	2.22 (Al 7075-T6)	10.71
FUPD	17.58	59.50	12.36	57.9	7.51	450.6
links	-	-	1.13	10.74	0.02	6.6
Total	21.96	74.26	18.08	91.94	9.75	467.91
Percentage change	-	-	−17.7%	+23.8%	−55.6%	+530.1%

An estimate of annual fuel consumption was made for both the traditional and optimized solutions, with a reduction of approximately 41.21 L/year for the CFRP solution and 13.09 L/year for the GFRP solution, as shown in Table 8. Based on these results, it was possible to estimate how fuel consumption reductions can impact both economic savings and CO₂ emission reductions. This calculation was made using data on diesel engine performance, as diesel is the most common fuel for industrial vehicles. The economic savings were calculated based on the current fuel price of 2 EUR/L (the average price in Italy during early 2024).

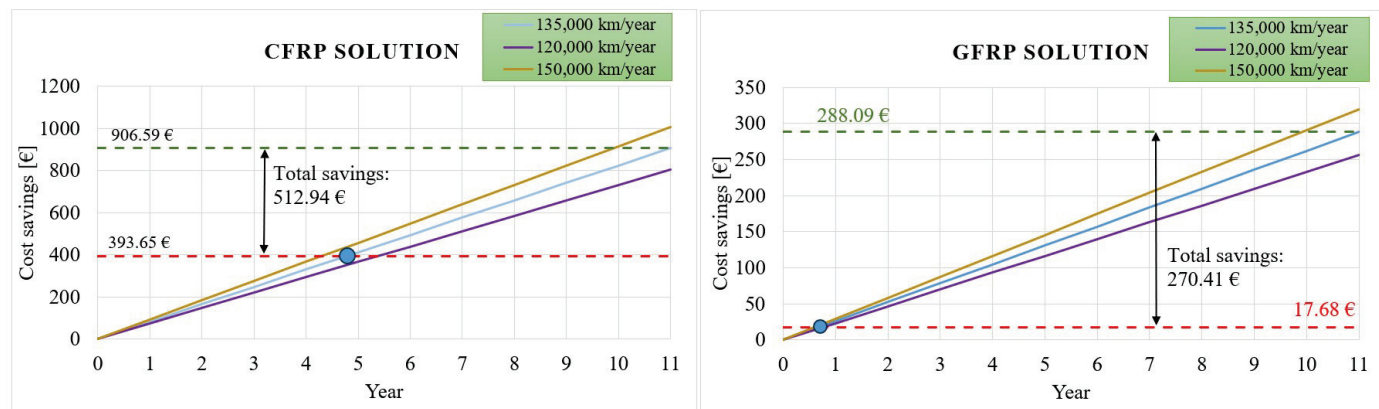


Figure 26. Graphs of the amortization for the two innovative solutions.

Table 8. Evaluation of the economic savings and reduction in CO₂ emissions.

Property	Value CFRP	Value GFRP	U.O.M.
Standard consumption [49]	0.21	0.21	L/km
Annual mileage [48]	135,000	135,000	km/year
Fuel consumption	28,350	28,350	L/year
Fuel consumption variation index for Internal Combustion Engine Vehicles (ICEVs) [50]	0.000025	0.000025	L/(km kg)
Mass decrease with respect to the standard solution	12.21	3.88	kg
Optimized consumption	0.000305	0.000097	L/km
Fuel consumption (optimised semi-trailer)	28,308.79	28,336.91	L/year
Fuel consumption decrease (with respect to the standard solution)	41.21	13.09	L/year
CO ₂ emissions per kilogram of diesel [51]	3.17	3.17	kgCO ₂ /kg _{diesel}
Density of diesel (at 15 °C)	820	820	kg/m ³
CO ₂ emissions per litre of diesel	2.60	2.60	kgCO ₂ /l _{diesel}
Reduction in CO ₂ emissions (with respect to the standard solution)	107.12	34.04	kgCO ₂ /year
Current diesel price	2	2	EUR/L
Economic saving (with respect to the standard solution)	82.42	26.19	EUR/year

5. Conclusions

This research demonstrates a design approach for an industrial vehicle component using composite materials. The analysis began by studying the applied loads, optimizing the geometry to distribute stresses along the component, analysing resonance modes, and considering types of instability. Once the materials were defined, the results were evaluated, and the component's geometry was optimized using specific algorithms. To consolidate the procedure, two case studies—one with GFRP and one with CFRP—were presented, focusing on the economic aspects of these optimized solutions. As shown in earlier sections, this process is not a linear flowchart due to the many variables involved. The ability to modify both the geometry and material configurations allows for alternative solutions, verified for the scenario under study.

The solutions can be compared:

- The GFRP configuration, when compared to the CFRP one, exhibits higher displacement under load due to its lower stiffness;
- However, the glass fibre solution requires a lower economic investment (+23.8% compared to the original configuration) than the CFRP (+530.1%);
- The carbon fibre solution offers instead greater long-term economic savings (equal to 512.94 EUR) by the end of the vehicle's useful life than the glass fibre one (equal to EUR 270.41). This is achieved due to a weight reduction of 55% (compared to the original configuration) for the CFRP solution, while a minor weight reduction of 18% for the GFRP configuration was evaluated.

Another important aspect to consider is the need for experimental tests to compare the numerical results with real-world data. This methodological approach not only validates the reliability of the outcomes but also highlights the commitment to empirical verification.

Regarding the comparison between this approach and alternative ones:

- This method is costlier than alternatives due to the necessity of experimental equipment and prototypes;
- Nevertheless, it does not necessitate expensive optimization tools;
- The selection is dependent upon the requirements of the end user: if optimization is necessary for a singular component, the presented methodology may serve as a viable solution, yielding more dependable outcomes. However, when this method is consistently implemented, the cost of this design strategy is not justified.

The principal benefit of the findings presented in the paper is the flexibility of the approach. The example analysed here is just one structural component of a transport vehicle, but this process can be applied to other components, such as the chassis, rear underrun protection device, or side underrun protection device. Moreover, the methodology can be adapted to different truck models, making it suitable for further exploration and refinement in future studies. These solutions present viable avenues for ongoing innovation within transportation engineering.

Regarding the future developments, a study about the effectiveness of different algorithms for the same component can help to choose the most suitable algorithm given the variable and the constraints considered.

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Abbreviations

The following abbreviations are used in this manuscript:

MDPI	Multidisciplinary Digital Publishing Institute
DOAJ	Directory of open access journals
FUPD	front underrun protection device
GFRP	glass-fibre-reinforced polymer
CFRP	carbon-fibre-reinforced polymer
FEM	finite element method
LD	linear dichroism

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Article

Performance Evaluation of Current Design Models in Predicting Shear Resistance of UHPC Girders

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Abstract: This manuscript delivers a comprehensive evaluation of five different ultra-high-performance concrete (UHPC) shear resistance models: FHWA-HRT-23-077 (2023), ePCI report (2021), French Standard NF-P-18-710 (2016), Canadian Standards A23.3-04 (2004), and Modified Eurocode2/German DAfStb (2023). The models differ in accounting for the steel fiber and shear reinforcement contribution and determining the angle of inclination of the diagonal compression strut. The evaluation was carried out using an experimental database of 198 UHPC specimens and focused on accuracy, conservatism, and ease of use for each considered model. The database included beams with prestressed and steel reinforcement, different shear reinforcement ratios, and a wide range of geometrical and material properties. In order to apply the FHWA method, a utilization tensile stress ($f_{t,loc}$) prediction equation was developed. Generally, the FHWA method showed superior performance to the other models in terms of statistical measures and consistent prediction conservatism across variable ranges. Although the ePCI methods yielded the highest conservatism, it can be said that the ePCI, AFGC, and CSA methods showed similar behavior with different degrees of conservatism. The DAfStb method showed the lowest prediction accuracy and the greatest scatter of data. Except for the FHWA method, all methods showed a reduction in conservatism at a high transverse reinforcement ratio.

Keywords: ultra-high-performance concrete; model comparison; shear strength; prediction models; steel fiber; utilization tensile stress; FHWA

1. Introduction

Ultra-high-performance concrete (UHPC) has been a research subject for over two decades [1]. UHPC is a dense concrete mixture of a high amount of cement, fine aggregate, steel fibers, and reactive powder additives such as pozzolana, fly ash, superplasticizer, and silica fume [2,3]. The water–cement ratio usually ranges between 0.15 and 0.25 [4]. This mixture results in improved properties comprising high post-cracking tensile strength [1], high compressive strength, high ductility, and less wide cracks [3]. The UHPC mixture can be designed to achieve strain-hardening performance under tension [5]. The tensile behavior primarily depends on the fibers' volume fraction, geometry, distribution, and orientation [6,7].

Recently, UHPC has garnered significant attention in bridge construction, which caused it to be called a “Game Changer” in this field due to the ability to design smaller cross-sectional members and longer spans, with less reinforcement and less cost [8], and the ability to design I-girders with very thin webs [5].

Although significant enthusiasm exists for incorporating ultra-high-performance concrete (UHPC) into bridge superstructures, as highlighted in recent research, its application has primarily been confined to smaller projects. This limitation stems from the absence of structural design models validated through extensive experimental testing, particularly regarding shear resistance. Numerous researchers have concentrated on examining the shear behavior of UHPC via experimental methods [9–19]. These studies have investigated various factors, including cross-sectional geometry, prestressing effects, fiber content, shear span-to-depth ratio, the inclusion of shear reinforcement, and reinforcement ratio.

The results of these studies revealed that the primary factors affecting shear failure were the post-cracking tensile strength and strain capacity of UHPC, rather than the compression strut's crushing. Failure typically initiates when a major crack extends through the girder's depth, subsequently leading to fiber pull-out along the principal diagonal crack [15]. An adequate amount of steel fibers within the UHPC matrix contributes to a tensile-strain-hardening behavior after initial cracking, which helps to prevent brittle failure. This property allows for the development of multiple closely spaced cracks even without transverse reinforcement. Furthermore, test results showed that steel fibers can replace or be combined with conventional shear reinforcement [16]. The combined presence of shear reinforcement, prestressing, and steel fibers enhances shear resistance, with their contributions being additive. That study also concluded that the contribution of transverse reinforcement is influenced by the strain capacity of UHPC, where higher strain capacity will enable the transverse reinforcement to exhibit more tensile stress up to yielding [5]. It has also been shown that higher axial stress due to prestressing will increase the shear capacity and reduce the cracking angle.

As shear theories for conventional concrete are not suitable for UHPC [20], several proposed shear models were proposed specifically for UHPC in Europe and North America. The main differences among the proposed models lie in how to account for the tensile strength of UHPC [13].

Previous attempts were made to evaluate the shear capacity and the behavior of UHPC, or composite materials generally, using statistical methods. Kodsý et al. [21] evaluated different UHPC shear prediction models on the basis of standard deviation, average strength ratio (experimental-to-predicted), and plotting ratio versus other variables. On the other hand, machine learning algorithms have also been utilized by Hematibahar et al. [22] and Haruna et al. [23] to generate prediction models for the mechanical properties of UHPC and to evaluate their reliability, respectively. It should be noted that, regardless of the method used in developing a prediction model for shear strength, adopting an extensive and reliable experimental database is very important.

In this study, a statistical evaluation of the shear resistance models proposed for UHPC was conducted. The evaluated models were FHWA-HRT-23-077, 2023 [24], ePCI report, 2022 [25], French Standard NF P 18-710, 2016 [26], Canadian model A23.3-04, 2004 [27], and Modification Eurocode 2/German DAfStb method, 2023 [28]. The statistical evaluation was performed to assess the prediction accuracy, conservatism, and ease of use for each of the design codes in predicting the capacity of 198 specimens compiled from the literature. The experimental database included specimens with and without prestressing reinforcement, and with and without shear reinforcement, and different cross-sectional geometries with a wide range of variables.

2. Shear Resistance According to Various Prediction Models

In the last two decades, worldwide standards have adopted different shear resistance models for UHPC. Despite the fact that most models are based on a modification of the modified compression field theory (MCFT), different assumptions were made in them and

they were calibrated using limited and different experimental databases. These methods include FHWA-HRT-23-077 [24], ePCI report [25], French Standard NF P 18-710 [26], Canadian model A23.3-04 [27], and Modification Eurocode 2/German DAfStb method [28]. This section discusses each of the mentioned methods in detail.

2.1. FHWA-HRT-23-077, 2023

In 2023, the Federal Highway Administration (FHWA) published the FHWA-HRT-23-077 document “*Structural Design with Ultra-High Performance Concrete*” [24]. The shear model adopted in the document is based on the MCFT principles, with a modification to the material constitutive models to apply to UHPC behavior. The model was originally based on a Garybel and El-Helo study [5] in 2022 and was recently adopted also by the AASHTO in 2024 [29]. This method will be referred to as the FHWA method in this study.

The nominal shear resistance of a UHPC member (V_n) according to the FHWA is given in Equation (1), representing the combined contributions of the UHPC tensile strength (V_{UHPC}) as in Equation (2), shear reinforcement (V_s) as in Equation (3), and the vertical component of prestressing force resisting vertical shear (V_p).

$$V_n = V_{UHPC} + V_s + V_p \leq 0.25f'_c b_v d_v + V_p \quad (1)$$

$$V_{UHPC} = \gamma_u f_{t,loc} b_v d_v \cot \theta \quad (2)$$

$$V_s = \frac{A_v f_{v,\alpha} d_v \cot \theta}{s} \quad (3)$$

where b_v is the effective web width (mm), d_v is the effective depth in resisting shear forces (mm), γ_u is a reduction factor that takes into consideration the variability in tensile capacity of UHPC (less than or equal to 1.0), $f_{t,loc}$, $f_{v,\alpha}$ are the localization tensile strength (MPa) and uniaxial stress in the transverse steel reinforcement at nominal shear resistance (MPa), respectively, A_v is the area of transverse reinforcement (mm²), θ is the angle of inclination of diagonal strut (degree), and s is the spacing of shear (mm).

If the beam girder has no shear reinforcement, the angle of the compression strut θ can be determined directly by solving Equation (4) with one unknown. However, if the girder has shear reinforcement, θ and the stress in shear reinforcement $f_{v,\alpha}$ are determined iteratively by solving Equations (4)–(7) (see Figure A1). The $f_{t,loc}$ and $\varepsilon_{t,loc}$ should be evaluated using direct tension testing according to AASHTO T-397 [30]. ε_s , in Equations (4) and (6), is the longitudinal strain at the level of reinforcement and is calculated according to Equation (8). If the value of ε_s calculated from Equation (8) is negative or is positive and less than the elastic tensile strain, $\varepsilon_{t,cr}$, the value should be determined again following Equation (9).

$$\gamma_u \varepsilon_{t,loc} = \frac{\varepsilon_s}{2} (1 + \cot^2 \theta) + \frac{2f_{t,loc}}{E_c} \cot^4 \theta + \frac{2\rho_{v,\alpha} f_{v,\alpha}}{E_c} \cot^2 \theta (1 + \cot^2 \theta) \quad (4)$$

$$\varepsilon_2 = -\frac{2f_{t,loc}}{E_c} \cot^2 \theta - \frac{2\rho_{v,\alpha} f_{v,\alpha}}{E_c} (1 + \cot^2 \theta) \quad (5)$$

$$\varepsilon_v = \gamma_u \varepsilon_{t,loc} - 0.5\varepsilon_s + \varepsilon_2 \quad (6)$$

$$f_{v,\alpha} = \frac{E_c \varepsilon_v}{\sin \alpha} \leq f_y \quad (7)$$

$$\varepsilon_s = \frac{\frac{|M_u|}{d_v} + 0.5N_u + |V_u - V_p| - A_{ps}f_{po} - \gamma_u f_{t,loc} A_{ct}}{E_s A_s + E_p A_{ps}} \quad (8)$$

$$\varepsilon_s = \frac{\frac{|M_u|}{d_v} + 0.5N_u + |V_u - V_p| - A_{ps}f_{po}}{E_s A_s + E_p A_{ps} + E_c A_{ct}} \quad (9)$$

where $|M_u|$ is the absolute value of the factored moment at the critical section (N.mm), N_u is the factored axial force at the design section (N), V_u is the design shear load at the critical section (kN), and A_{ps} , A_s , and A_{ct} are the area of prestressing reinforcement, non-prestressed steel reinforcement, and UHPC in the flexural-tension side (mm^2), respectively. f_{po} is taken as the modulus of elasticity of prestressing steel multiplied by the locked-in difference in strain between the prestressing reinforcement and the surrounding concrete and can be taken as 0.7 of the ultimate tensile strength of the prestressing reinforcement (MPa), and E_s and E_p are the moduli of elasticity of non-prestressed and prestressing steel (MPa), respectively. ε_2 is the diagonal compressive strain in the section, and ε_v is the vertical strain in transverse reinforcement at the design section. It is noteworthy that the elastic modulus of the concrete (E_c) is calculated according to the FHWA report [24].

The FHWA report specifies a minimum UHPC compressive strength of 120.6 MPa, minimum effective cracking strength of 5.2 MPa, and minimum crack utilization strain of 0.0025.

2.2. ePCI-UHPC, 2021

In 2021, a research and development program for utilizing UHPC in bridges by Tadros et al. [25] proposed a shear model that is an adaptation of the current AASHTO provisions. This method will be referred to as the ePCI method in this study. The shear strength of the UHPC beam consists of shear resistance provided by tensile stresses in UHPC (V_{cf}) in Equation (10), shear resistance proved by shear reinforcement (V_s) in Equation (11), and the vertical component of prestressing force opposite to the direction of the applied shear force (V_p). Consequently, the shear capacity of the UHPC beam (V_n) is given in Equation (12). Despite the procedure including the contribution of shear reinforcement with the assumption of yielding stress, the authors discouraged the use of shear reinforcement due to the potential fiber disruption.

$$V_{cf} = \frac{4}{3} f_{rr} b_v d_v \cot\theta \quad (10)$$

$$V_s = \frac{A_v f_y d_v}{s} \cot\theta \quad (11)$$

$$V_n = V_{cf} + V_s + V_p \quad (12)$$

where f_{rr} is the residual tensile strength of UHPC and preferably taken as 5.2 (MPa) for UHPC to meet minimum PCI-UHPC tensile properties requirements. The crack angle is estimated according to Equation (13). The longitudinal strain ε_s is calculated according to Equations (14) (for positive values) and (15) (for negative values) and is bounded by -0.40×10^{-3} in compression and 6.0×10^{-3} in tension, and other parameters are similar to what was described before. The ePCI method specifies a minimum compressive strength of 124 MPa, and a 10 MPa minimum tensile strength.

$$\theta = 29^\circ + 3500\varepsilon_s \quad (13)$$

$$\varepsilon_s = \frac{\left(\frac{M_u}{d_v}\right) + (V_u - V_p) - A_{ps}f_{po}}{(E_s A_s) + (E_p A_{ps})} \geq 0.0 \quad (14)$$

$$\varepsilon_s = \frac{\left(\frac{M_u}{d_v}\right) + (V_u - V_p) - A_{ps}f_{po}}{(E_s A_s) + (E_p A_{ps}) + (E_c A_{ct})} \leq 0.0 \quad (15)$$

2.3. French Standard, NF P 18-710, 2016

The UHPC shear model French Standard model (AFGC) is similar to a simplified version of the MCFT. This method will be referred to as the AFGC method in this study. According to the French Standard model [26], the shear resistance of a UHPC member (V_{Rd}) is the combined resistance of three terms: concrete ($V_{Rd,c}$), steel fibers ($V_{Rd,f}$), and shear reinforcement ($V_{Rd,s}$), as shown in Equations (16)–(21).

$$V_{Rd} = V_{Rd,c} + V_{Rd,f} + V_{Rd,s} \quad (16)$$

$$V_{Rd,c} = \begin{cases} \frac{0.24}{\gamma_c f_{ct} \gamma_E} K \sqrt{f_{ck}} b_v z, & \text{for prestressed} \\ \frac{0.18}{\gamma_c f_{ct} \gamma_E} K \sqrt{f_{ck}} b_v h, & \text{for non-prestressed} \end{cases} \quad (17)$$

$$K = 1 + 3 \frac{\sigma_{cp}}{f_{ck}} \quad (18)$$

$$\sigma_{cp} = \frac{N_{Ed}}{A_c} \leq 0.4 f_{ck} \quad (19)$$

$$V_{Rd,f} = A_{fv} \sigma_{Rd,f} \cot \theta \quad (20)$$

$$V_{Rd,s} = \frac{A_s}{s} z f_y \cot \theta \quad (21)$$

where the comprehensive safety factor $\gamma_{cf} \gamma_E$ shall be 1.5 at the design stage. f_{ck} represents the compressive strength of UHPC (MPa). The term z represents the lever arm of the internal forces and is taken as $z = 0.9 d$ (mm). The prestressing improvement factor k is determined by the external load or prestressing N_{Ed} . $\sigma_{Rd,f}$ is the mean value of the post-cracking strength along the shear crack. A_{fv} is the cross-section of the inclined area on which the fibers act. The inclination of the shear crack θ is fixed as the minimum value of 30° . The minimum compressive and tensile strengths are 150 and 5 MPa, respectively.

2.4. Canadian Standard CSA A23.3-04

The Canadian Highway Bridge Design Code also adopts a modification of the MCFT for the shear design of UHPC. This method will be referred to as the CSA method in this study. According to the Canadian code [27], the shear strength of a member (V_r) in Equation (22) is taken as the combination of the concrete component (V_c) in Equation (23), the shear reinforcement component (V_s) as in Equation (24), and the vertical component of prestressing force (V_p).

$$V_r = V_c + V_s + V_p \quad (22)$$

$$V_c = \varphi_c \lambda \beta \sqrt{f'_c} b_v d_v \quad (23)$$

$$V_s = \frac{A_v \varphi_s f_y d_v}{s} \cot \theta \quad (24)$$

$$B = \frac{0.4}{1 + 1500 \epsilon_x} \left(\frac{1300}{1000 + s_{ze}} \right) \quad (25)$$

$$\theta = (29^\circ + 7000 \epsilon_x) \quad (26)$$

where φ_c is the resistance factor for concrete take = 1.0, λ is a factor to account for low-density concrete = 1.0, β is a factor representing the strain and the size effect and determined according to Equation (25), f'_c is the concrete cylinder strength (MPa), φ_s is the resistance factor for reinforcing steel take = 1.0, f_y is the yield strength of the reinforcement (MPa), s_{ze}

is the influence of aggregate size, and θ is the angle of average principal compression in the beam with respect to the longitudinal axis (degree), determined by Equation (26). To determine θ , the longitudinal strain at mid-depth of the member due to factored loads, ε_s , is required (Equation (27)).

$$\varepsilon_s = \frac{\frac{M_u}{d_v} + V_u - V_p + 0.5N_u - A_{ps}f_{po}}{2(A_sE_s + A_pE_p)} \leq 0.003 \quad (27)$$

If the value of ε_s calculated from Equation (27) is negative, the value shall be recalculated with the denominator replaced by $2(E_sA_s + E_pA_p + E_cA_{ct})$. ε_s shall not be taken as less than -0.2×10^{-3} . It should be noted that f_y shall not exceed 500 MPa except for prestressing reinforcement and the minimum compressive strength is 120 MPa.

2.5. Eurocode 2 and German DAfStb Guideline, 2022

A modified procedure to determine the shear capacity of UHPC following the Eurocode 2 and the German DAfStb Guideline was presented in a Metje and Leutbecher study [28]. This method will be referred to as the DAfStb method in this study. Similar to previous concepts, the shear design resistance in this method is determined by combining contributions from three components, the shear resistance of a UHPC member ($V_{Rd,c}$) as in Equations (28)–(31), shear resistance provided by the fibers ($V_{fRd,f}$), and shear resistance provided by the shear reinforcement ($V_{Rd,s}$), as shown in Table 1 and Equations (32) and (33). The minimum compressive and tensile strengths are 150 and 8.5 MPa, respectively.

$$V_{Rd,c} = \left[\left(\frac{0.15}{\gamma_c} \right) k (100\rho_l f_{ck})^{\frac{1}{3}} + 0.12\sigma_{cp} \right] b_v d_v \quad (28)$$

$$k = 1 + \sqrt{200/d} \leq 2.0 \quad (29)$$

$$\rho_l = \frac{A_s}{b_w d_v} \leq 0.06 \quad (30)$$

$$\sigma_{cp} = \frac{N_{Ed}}{A_c} < 0.2f_{cd} \quad (31)$$

$$f_{cftd} = \alpha_{CF} f_{ctk} / \gamma_{CF} \quad (32)$$

$$f_{ctk} = k_F f_{ct0} \quad (33)$$

where f_{ck} and f_{cd} are the characteristics and the design value of the compressive cylinder strength, respectively; γ_C and γ_{CF} are the partial factors for concrete and post-cracking tensile strength, respectively; h is the height of the cross-section; and η_F is a coefficient considering the shape of the shear crack ($\eta_F = 1.0$ for members with I-shaped cross-section and $\eta_F = 0.7$ for members with compact cross-section). α_{CF} is a coefficient taking account of long-term effects on the post-cracking strength, κ_F is a factor considering the orientation of the fibers ($\kappa_F = 1.0$ for I-shape cross-sections and $\kappa_F = 0.5$ for compact cross-sections), and f_{ct0} is the reference value of the post-cracking tensile strength evaluated by testing.

Table 1. Contributions of fibers and shear reinforcement according to the DAfStb method.

Shear Span	$a/d \geq 2$	$0.5 \leq a/d < 2$
Fiber contribution	$V_{Rd,f} = b_v h \eta_f (f_{cftd})$	$V_{Rd,f} + V_{Rd,s} = (\min 0.75a, h) b_v \eta_f f_{cftd} + A_v f_{ywd}$
Stirrup contribution	$V_{Rd,s} = \frac{A_v}{s} z f_{ywd} \cot \theta$ $\cot \theta = 1.2 + 2.4 \frac{\sigma_{cp}}{f_{cd}} \geq 1.0$	

As shown in Table 1, the fiber contribution is considered to correspond to a $\theta = 45^\circ$ ($\cot \theta = 1.0$) when the shear span-to-depth ratio a/d is greater than or equal to 2 and the stirrup contribution with θ calculated according to Table 1. For members with short shear spans (a/d less than 2.0), the fiber contribution is considered effective over the minimum of h or $0.75 a$, and only stirrups within $0.75 a$ are considered.

2.6. Summary of the Prediction Models

The differences and similarities between the mentioned UHPC shear models can be summarized in the following points:

- Generally, the mentioned models agree that the shear resistance is composed of contributions from the tensile resistance of concrete, fibers, shear reinforcement, and prestressing. However, the FHWA, ePCI, and CSA methods have one term for the tensile resistance of concrete and steel fibers combined, while the AFGC and the DAfStb methods have separate terms for each.
- It is worth mentioning that the FHWA method adopts the concept of localization tensile stress for the tensile resistance of concrete, which should be evaluated experimentally. The ePCI method adopts a fixed minimum residual tensile stress for UHPC ($f_{rr} = 5.2$ MPa). In the other codes, the tensile stress is related to the compressive strength of UHPC.
- Regarding the inclination angle of the compression strut θ , the FHWA method adopts an iterative solution to determine it, while the ePCI and the CSA method adopt a similar equation form. In the three methods, the angle depends on the strain of the longitudinal reinforcement, ε_s . The AFGC method adopts a fixed minimum angle of 30 degrees. However, the DAfStb method includes the effect of the shear span-to-depth ratio on the inclination angle θ .
- In the shear reinforcement contributions, all methods assume yielding stress in the shear reinforcement except for the FHWA method, where an iterative solution is required to determine the stress in the shear reinforcement.
- In the FHWA, ePCI, and CSA methods, the effect of the prestressing level on the shear capacity of UHPC is included in determining the strain of the longitudinal reinforcement, ε_s , and as a vertical component resisting vertical shear, V_p . In the AFGC and the DAfStb methods, the prestressing level is included in the concrete contribution equation (σ_{cp}).

3. Database Analysis

To evaluate the effectiveness of the UHPC shear design models discussed earlier, a comprehensive review of the existing literature was carried out to assemble a database of experimental tests of UHPC beams subjected to shear. Only those specimens that failed in shear loading with dominant shear crack were considered. The database was further refined to include beams with a shear span-to-depth ratio (a/d) of 1.5 or higher, while excluding any specimens that incorporated coarse aggregate in their mix design. In addition, only specimens with steel fiber were included as the FHWA and the ePCI methods define UHPC as a mixture that exhibits strain-hardening and they do not have a separate term for steel fibers' contribution. The literature included a total of 198 specimens from 28 references [5,9–13,18,25,27,31–50] (Table 2). It should be noted that only specimens with complete information were included. In addition, the reason for excluding specimens with coarse aggregate is that UHPC usually does not contain coarse aggregate, to reduce the friction in the mix and to achieve a dense, compact, and more flowable mix.

Table 2. UHPC beam database.

<i>Reference</i>	<i># of Tests</i>	<i>Section</i>	<i>f'c (MPa)</i>	<i>V_f l_f/d_f</i>	<i>a/d</i>	<i>h (mm)</i>	<i>b_w (mm)</i>	<i>A_{ps} (mm²)</i>
[11]	9	I-shaped	203–205	130–167	2.5	380	65	900
[31]	8	I-shaped	125–140	75–150	3.2	650	50	840
[32]	6	I-shaped	150–170	82–163	3.3	650	50	1680
[13]	3	I-shaped	103–105	48.8	2	400	70	280
[10]	8	I-shaped	168–189	65–130	2.5–3.4	700	40, 50	1680
[33]	3	I-shaped	193.0	130.0	2.5, 2.8	910	152	2230
[12]	6	I-shaped	125–127	120.00	1.1, 2, 3	300	50	420
[5]	6	I-shaped	137–160	130.0	3.1–3.9	889, 1092	76.2, 101.6	4560
[25]	13	I-shaped	126–155	125, 159, 190	1.2–3.4	863.6	50–101	3640
[9]	5	I-shaped	195–212	97–100	2.5–8.5	380	65	840
[34]	3	Rectangle	108.2	75	1.5–2.5	340	160	140
[35]	15	I, box-shaped	118–155	130, 160, 190	2.5–4	800–1500	50–152	1680
[44]	7	I-shaped	128–145	40, 75, 65, 100	1.60	500	65	372
[37]	2	I-shaped	126.0	130	1.1–3.3	600	70	280
[38]	10	I-shaped	152–189	77, 147	3.0–3.5	600–1000	40	1115
[39]	1	I-shaped	173.2	163	2.50	1067	114	6580
[40]	1	Rectangle	166.9	131.25	3	290	150	0
[41]	13	I-shaped	160–188	75, 150	3.25	350	60	0
[42]	2	Rectangle	100, 200	80	2	350	200	0
[43]	31	T-shaped, Rectangle	115–125	65, 130, 195	1.5–3.0	250	150	0
[44]	8	I-shaped	144–152	65.0, 130.0, 195	4–8.0	380, 460	50	0
[27]	2	I-shaped	147.69	52, 130	1.6–3	220, 400	50	0
[18]	3	Rectangle	140	200	1.23	300	200	0
[46]	8	Rectangle	130–152	172.25	1.5–3.0	350	250	0
[47]	11	Rectangle	127, 131	65, 130, 195, 260	1.5–2.3	250	150	0
[48]	9	I-shaped	160–170	97.5	2.5, 3.5	520, 700	40	0
[49]	2	I-shaped	150	195	2.8	200	20	0
[50]	3	I-shaped	198–211	37.1, 74.2	4	330	30	0
Average	-	-	149.58	130.2	2.8	517.1	91.1	1709.6
Max.	-	-	212	260	8	1422.4	250	6580
Min.	-	-	100	37.1	1.5	200	20	140

In the compiled database, 101 specimens were prestressed, and the remainder were non-prestressed. All shear-tested prestressed specimens in the literature had a straight prestressing profile because UHPC members tend to have thin webs where inclined strands are difficult to achieve. In terms of shear reinforcement, 82 specimens included shear reinforcement, while the remainder did not. Specimens in the database included I-shaped (72%), rectangular (25%), T-shaped (2%), and box (1%) sections. Figure 1 is a graphical illustration of the content of the experimental database.

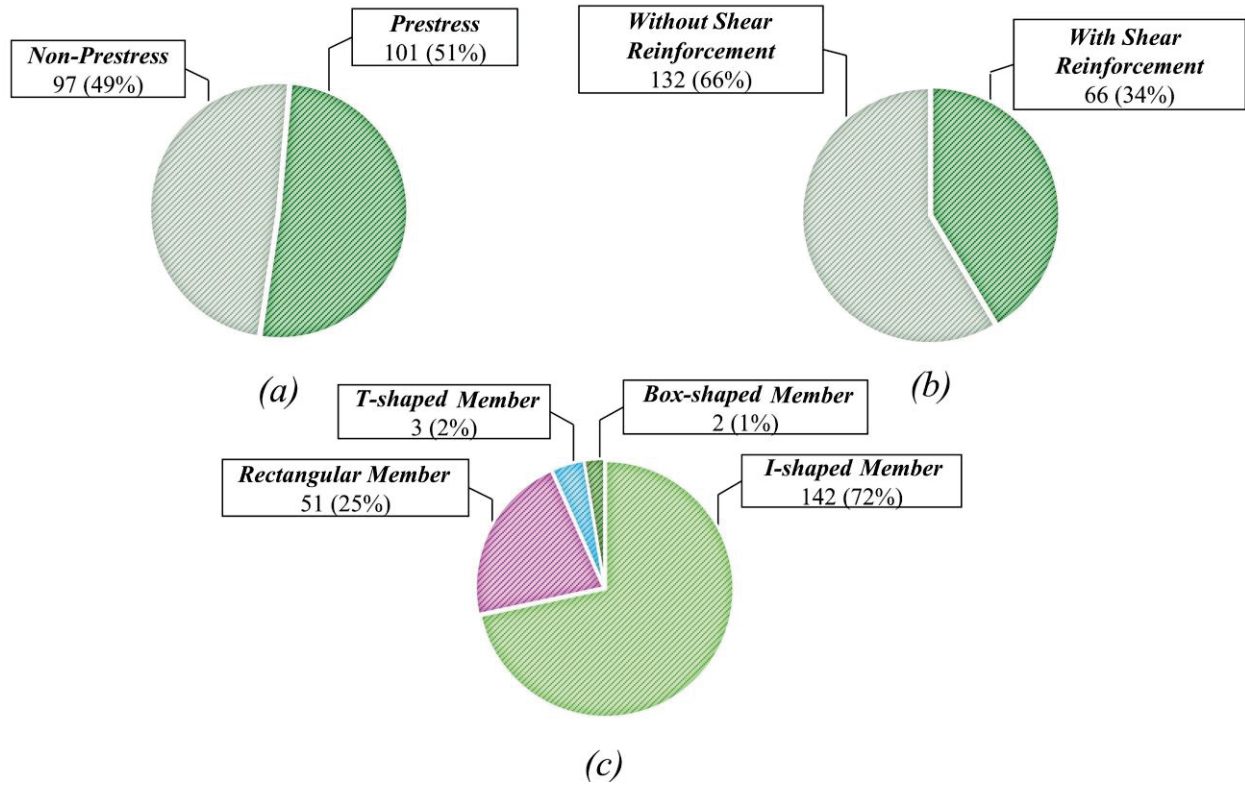


Figure 1. Experimental database according to: (a) reinforcement, (b) shear reinforcement, and (c) cross-section type.

For every specimen included in the database, the collected data encompassed sectional information and reported the compressive strength of the concrete, details of non-prestressed and prestressed reinforcement, volume and aspect ratios of steel fibers, shear reinforcement information (where applicable), shear span-to-depth ratio (a/d), and the experimentally measured shear capacity (V_{exp}). Figure 2 illustrates the distribution of selected variables within the compiled database. The database encompassed a broad spectrum of variables, which is essential for thorough model assessment. It is worth noting that the majority of specimens featured a fiber volume ranging between 1.5% and 2.5%, and none contained coarse aggregate. From Figure 2 it can be seen that 26 specimens had a very thin web with 20 to 40 mm thicknesses. As shown in Figure 2, specimens with shear span-to-depth ratios less than 1.5 were excluded to avoid deep beam behavior. In addition, most of the specimens had a compressive strength greater than 125 MPa. The elastic modulus for all specimens in the database was calculated using the empirical equations proposed in the FHWA report [24] (Equation (34)).

$$E_c = 9100 f_c'^{0.33} \text{ [MPa]} \quad (34)$$

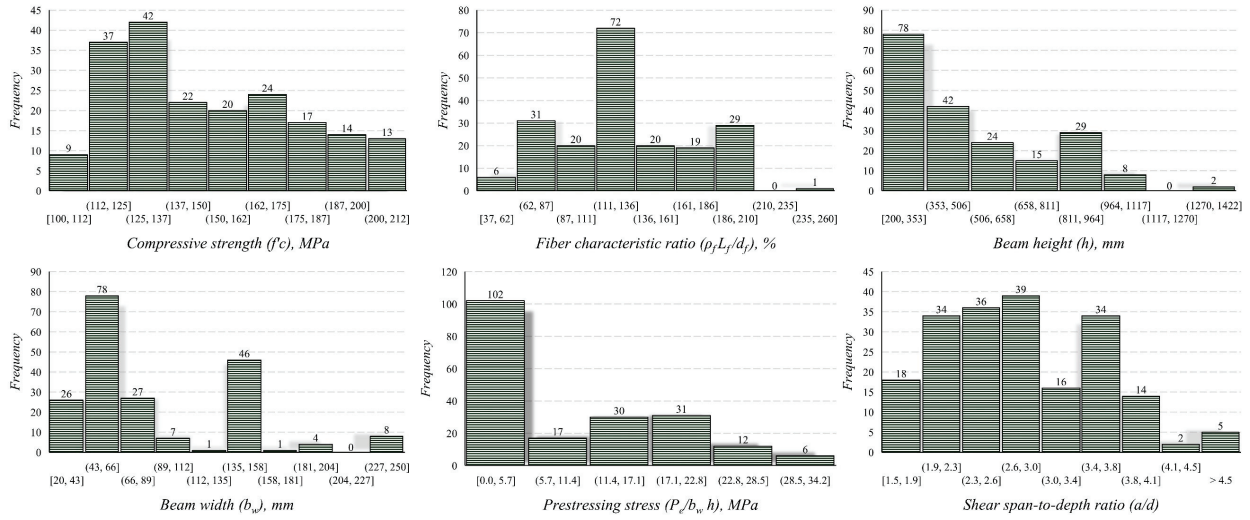


Figure 2. Material and geometric properties of UHPC girders in the compiled database.

4. Localization Tensile Stress and Strain for the FHWA Method

Determining the nominal capacities for beams in the database according to the FHWA requires experimentally obtaining the crack localization strength ($f_{t,loc}$) and strain ($\epsilon_{t,loc}$) through direct tension testing. According to the FHWA, the definition of crack localization strength $f_{t,loc}$ is “the tensile stress value at which the tensile stress continuously decreases with increasing strain” (Figure 3). These values are associated with shear failure that is triggered by a localized and dominant crack developing from existing closely spaced cracks. However, these parameters were not provided in most of the surveyed database.

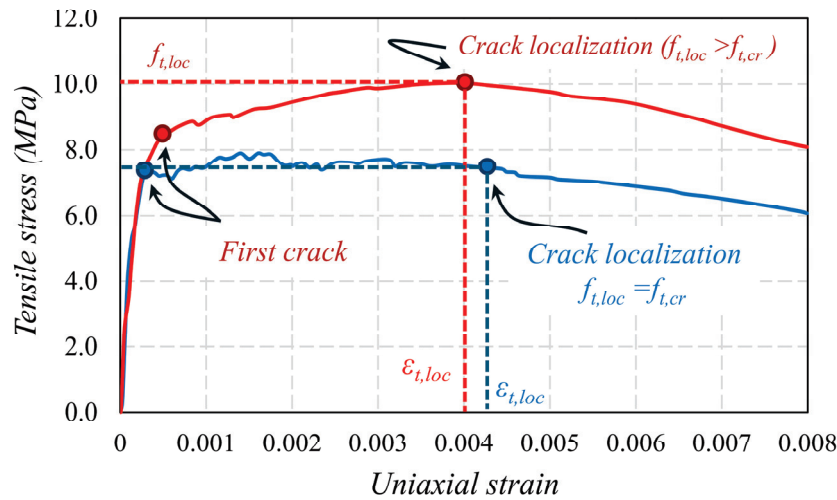


Figure 3. Stress–strain behavior of UHPC under direct tension [5].

Therefore, the authors aimed to develop a prediction equation for the localization strength. Consequently, a secondary database for the direct tension test of UHPC was surveyed. It contained 76 tests from eight different studies [5,51–57] with different variables, including concrete and fiber characteristics. The secondary database is shown in Table 3. Correlation analysis revealed that the localization strength exhibits a predominantly linear relationship with both the compressive strength (f'_c) and the fiber characteristic ratio ($V_f l_f/d_f$). Based on this finding, a linear equation was developed to predict the localization strength, as presented in Equation (35). The developed model demonstrated a coefficient of variation (CoV) of 16.1% and a root mean square error (RMSE) of 1.49 MPa, indicating a

reasonable level of accuracy. It is important to highlight that Equation (35) is valid only within the variable ranges specified in Table 2. Additionally, the term $V_f l_f/d_f$ is expressed in its original form and not converted into a percentage.

$$f_{t,loc} = 0.279 + 0.035f'_c + 3.40V_f l_f/d_f \quad (35)$$

The localization strength for the 198 shear specimens in the database was determined according to Equation (35). Regarding the utilization strain, a fixed value of $\varepsilon_{t,loc} = 0.004$ was assumed for all specimens, as all specimens in Table 3 had $\varepsilon_{t,loc}$ close to 0.004.

Table 3. UHPC direct tension database for the proposed Equation (35).

Reference	No. of Specimens	$V_f l_f/d_f$ (%)	f'_c (MPa)	f_{tloc} (MPa)
[51]	31	86.70–260.0	84.8–170.3	6.4–12.5
[5]	6	130.0	137.0–160.0	8.6–11.5
[52]	8	54.5–121.9	129.0–140.0	5.0–9.4
[53]	12	62.5	171.0–195.0	6.9–11.9
[54]	5	86.7–130.0	93.8–148.0	6.4–12.5
[55]	4	130.0	126.0	11.9
[56]	6	130.0–290.0	150.0	15.1–15.7
[57]	4	130.0	196.5	12.2

5. Performance and Comparison of Prediction Models

To evaluate the performance of the prediction models discussed in the previous sections, the nominal shear capacities of the UHPC girders in the experimental database were determined according to each of them (see Appendix A for FHWA method and ePCI method calculation flowcharts). In all models, all partial safety factors were set to 1.0 for the purpose of comparative analysis.

The experimental versus predicted capacities were plotted for each prediction model in Figure 4. The statistical measures for each model, including the average (*Avg.*), standard deviation (*SD*), *CoV*, and relative root mean square error (*RRMSE*) for the shear strength ratio (V_{exp}/V_{pred}), are summarized in Table 4. In addition, the statistics based on reinforcement type are presented in Table 5.

Generally, all models yielded a safe average shear strength ratio V_{exp}/V_{pred} . The ePCI method resulted in the highest average shear strength ratio of 1.58, while the FHWA method yielded the lowest average of 1.19. The ePCI method resulted in 28 specimens with a shear strength ratio > 2.0 and no specimens falling below 0.75. This is expected due to adopting a conservative constant shear stress of 5.2 MPa. The FHWA method resulted in the lowest standard deviation and coefficient of variation, reflecting the data's lowest scatter (highest prediction accuracy), which also can be seen in Figure 4a. Table 5 also indicates a lower *CoV* for prestressed members compared to non-prestressed members for both models. It should be noted that high conservatism can lead to an uneconomical design.

The DAfStb method showed the highest standard deviation and *CoV*, indicating the lowest prediction accuracy and the greatest scatter of the data. This method also accounted for the largest number of specimens having shear strength ratios above 2.0 and below 0.75. The AFGC and CSA shear methods, on the other hand, demonstrated similar statistical performance. Overall, methods derived from MCFT exhibited better performance in predicting shear strength compared with models that take the effect of steel

fiber reinforcement contribution separately, or models that do not take into account the angle of the compression strut.

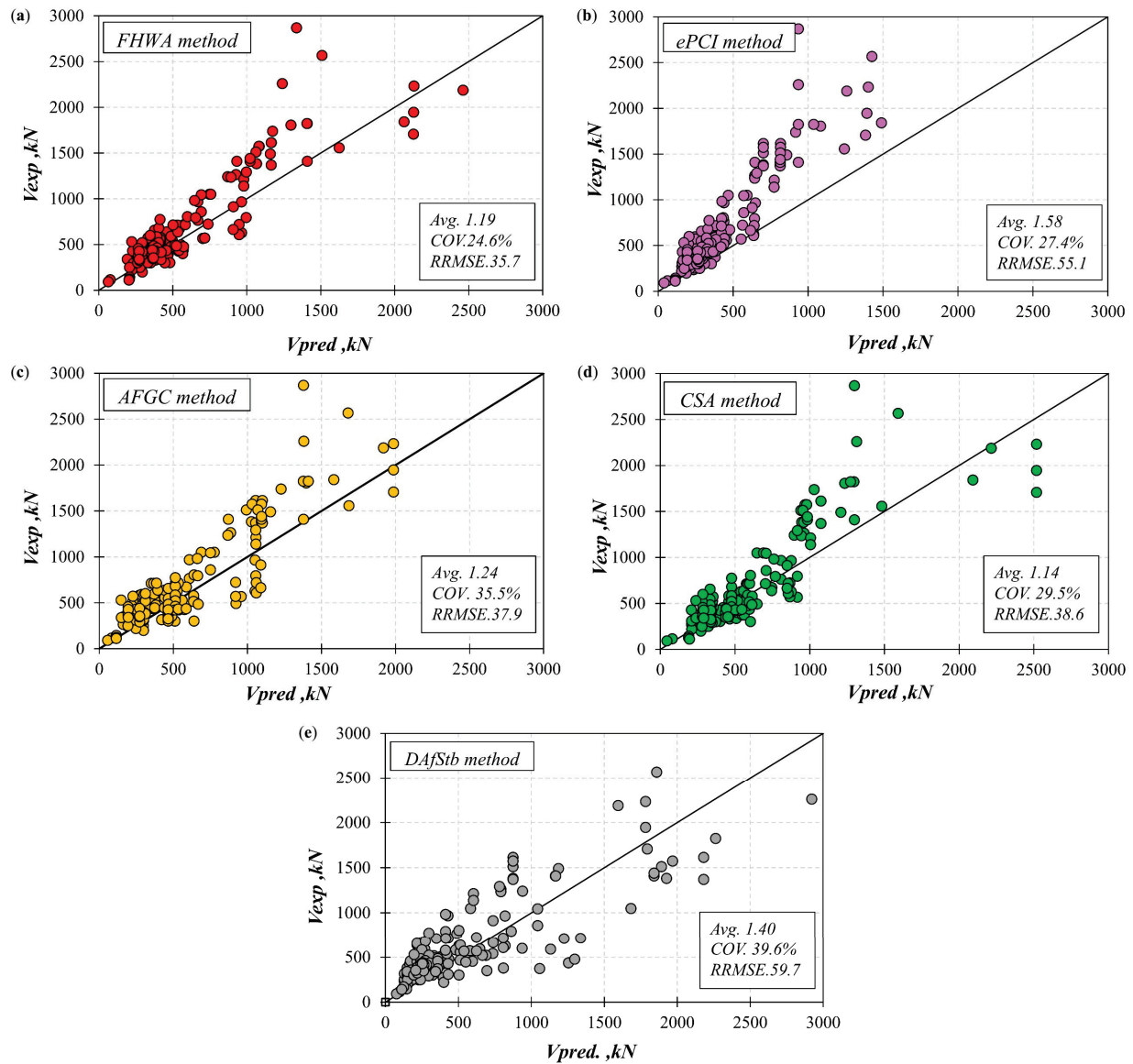


Figure 4. Measured versus predicted shear strength according to prediction models: (a) FHWA, (b) ePCI, (c) French, (d) Canadian, and (e) DAfStb methods.

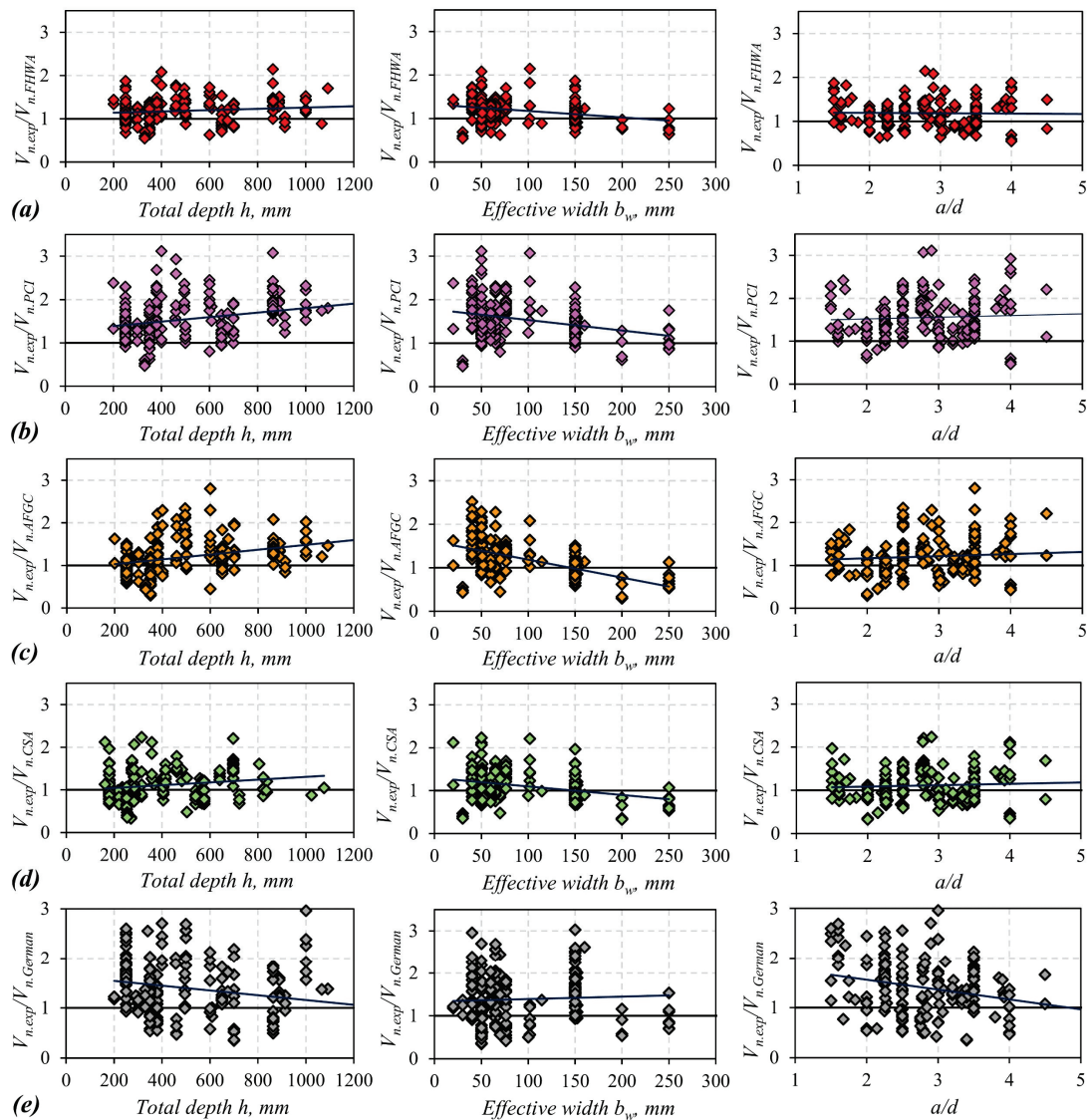
Table 4. Summary of evaluation statistics of prediction models.

Prediction Model	V_{exp}/V_{pred} . Avg.	V_{exp}/V_{pred} . SD.	V_{exp}/V_{pred} . CoV (%)	V_{exp}/V_{pred} . RRMSE (%)	V_{exp}/V_{pred} . >2	V_{exp}/V_{pred} . <0.75
FHWA method [24]	1.19	0.29	24.6	35.7	2	11
ePCI method [25]	1.58	0.43	27.4	55.1	28	0
AFGC method [26]	1.24	0.44	35.5	37.9	13	20
CSA method [27]	1.14	0.34	29.5	38.6	5	15
DAfStb method [28]	1.40	0.55	39.6	59.7	28	23

Table 5. Evaluation statistics for prediction models based on reinforcement.

Method	Non-Prestressed ($n = 97$)						Prestressed ($n = 101$)					
	No A_v ($n = 60$)			With A_v ($n = 37$)			No A_v ($n = 72$)			With A_v ($n = 29$)		
	Avg.	CoV %	<0.75	Avg.	CoV %	<0.75	Avg.	CoV %	<0.75	Avg.	CoV %	<0.75
FHWA [24]	1.17	29.4	5	1.18	22.3	3	1.19	23.9	2	1.27	15.8	1
ePCI [25]	1.64	30.3	0	1.40	19.1	0	1.65	28.4	0	1.54	18.9	0
AFGC [26]	1.19	48.6	14	0.97	21.6	5	1.42	26.8	0	1.25	16.1	1
CSA [27]	1.20	31.7	4	0.94	22.2	3	1.15	28.9	7	1.25	22.1	1
DAfStb [28]	1.40	38.3	6	1.54	28.0	1	1.48	39.2	7	1.01	48.5	9

For a further look at the performance of the prediction models, the shear strength ratios were drawn with respect to geometrical properties (h , b_w , a/d) in Figure 5, and with respect to concrete and reinforcing properties (f'_c , ρ_v , P/A) in Figure 6. The slopes of the trendlines in the figures reflect the conservatism level across the variables' ranges.

**Figure 5.** Shear strength ratios for all models with respect to geometrical properties (h , b_w , a/d): (a) FHWA, (b) ePCI, (c) French, (d) Canadian, and (e) DAfStb models.

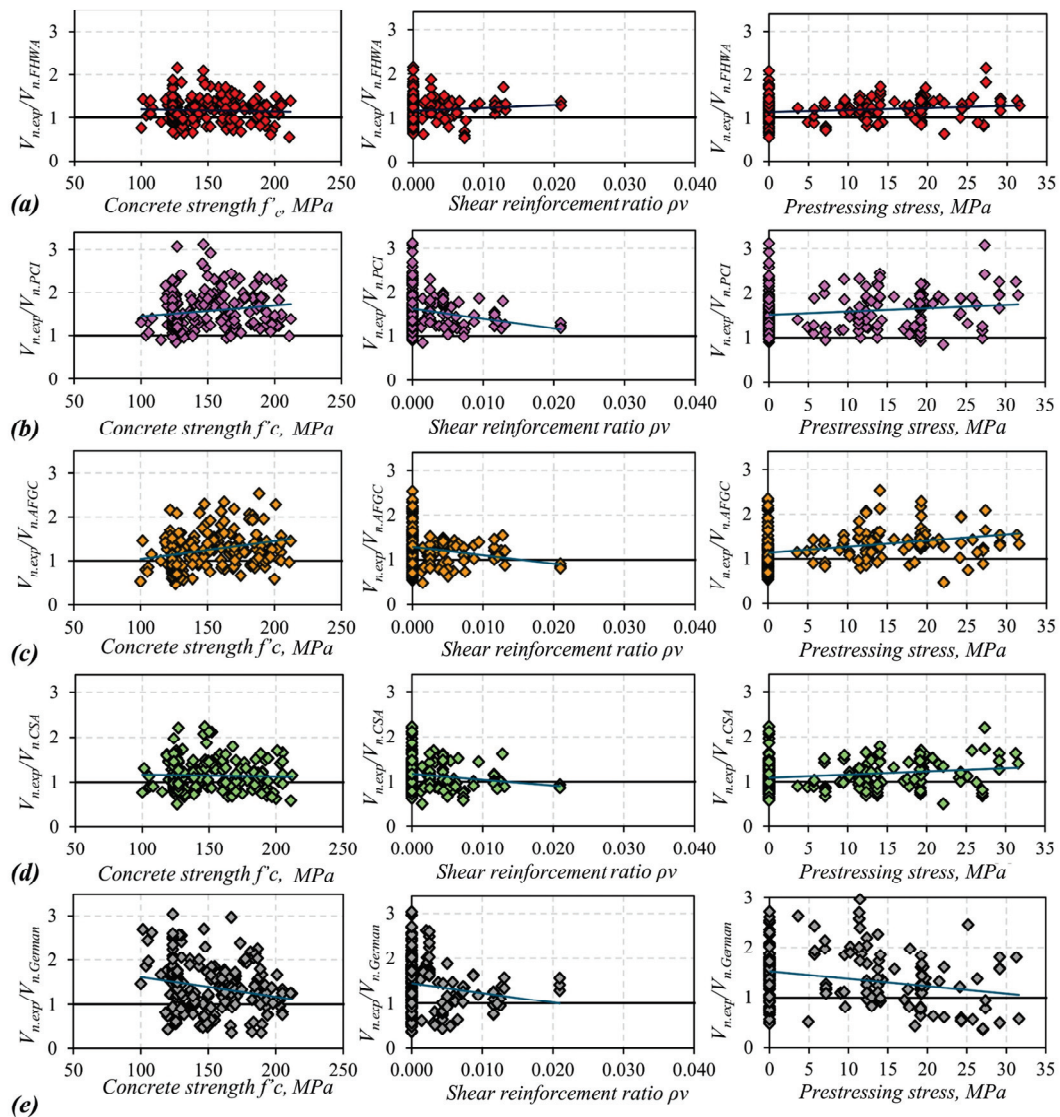


Figure 6. Shear strength ratios for all models with respect to concrete and reinforcing properties (f'_c , ρ_v , P/A): (a) FHWA, (b) ePCI, (c) French, (d) Canadian, and (e) DAfStb models.

With respect to the geometrical properties (Figure 5), it can be seen that the FHWA method yielded relatively flat trendlines, indicating more consistent predictions across variables compared to other models. In addition, the DAfStb method showed a reduction in conservatism at high values of h and a/d . The ePCI, AFGC, and CSA methods showed similar behavior across all variables.

Regarding the concrete and the reinforcing properties (Figure 6), the FHWA method still shows consistent prediction accuracy with relatively flat trendlines compared to other methods. Additionally, the DAfStb method still shows a reduction in conservatism at high values of the variables. Finally, it can be seen that all methods, except for the FHWA method, show a reduction in conservatism at a high transverse reinforcement ratio. This can be attributed to their adopting yield stress for transverse reinforcement compared to the FHWA method which assesses the transverse reinforcement strain and stress.

6. Conclusions

This paper aims to define and evaluate the performance of five UHPC shear design models: the FHWA, ePCI, AFGC, CSA, and DAfStb methods. The evaluation was carried out using a detailed experimental database from 28 different experimental programs

published in the literature. The database included 198 specimens, from which 51% of the specimens were prestressed and 34% of the specimens had shear reinforcement. In addition, all specimens had steel fiber and fine aggregate in their UHPC mix. The following conclusions can be drawn:

1. The FHWA, ePCI, and CSA methods are based on modifications of the MCFT. On the other hand, the AFGC and DAfStb methods were shown to have separate terms for the concrete and steel fiber tensile contributions. The angle of inclination of the diagonal compressive strut (θ) in the FHWA, ePCI, and CSA methods is determined using the strain of the longitudinal reinforcement (ϵ_s), while the AFGC method uses a fixed minimum angle of 30° , and the DAfStb method incorporates the span-to-depth ratio (a/d). In addition, it was shown that all methods assume yielding stress in shear reinforcement except for the FHWA method, which determines the strain of the shear reinforcement.
2. To determine the shear capacity according to the FHWA procedure, the localization strength and strain ($f_{t,loc}$, $\epsilon_{t,loc}$) must be determined experimentally. Conversely, $f_{t,loc}$ and $\epsilon_{t,loc}$ had not been determined for most of the specimens in the database. For that reason, a prediction model $f_{t,loc}$ was developed based on the concrete and steel fiber properties. Regarding the localization strain, a fixed value was assumed ($\epsilon_{t,loc} = 0.004$) for all specimens.
3. The FHWA method showed a superior performance compared to other models in terms of statistical measures and consistent prediction conservatism across variables ranges. The FHWA method resulted in an *Avg.* of 1.19, *SD* of 0.29, and *RRMSE* of 35.7% for the strength ratios ($V_{exp}/V_{pred.}$). Out of 198 specimens, only 14 had a shear strength ratio of either more than 2.0 or below 0.75.
4. The ePCI method showed the highest conservatism with an *Avg.* of 1.58 due to limiting the UHPC residual tensile strength to 5.2 MPa. The ePCI method resulted in 28 specimens with a shear strength ratio > 2.0 and no specimens falling below 0.75.
5. The DAfStb method showed the highest *SD* and *CoV*, indicating the lowest prediction accuracy and the greatest scatter of the data. This method also accounted for the largest number of specimens having shear strength ratios above 2.0 and below 0.75. In addition, this method showed a reduction in conservatism at high geometrical or material properties values, including the shear span-to-depth ratio.
6. It can be said that the ePCI, AFGC, and CSA methods showed similar behavior with different degrees of conservatism; they resulted in *Avg.* shear strength ratios of 1.58, 1.24, and 1.14, respectively.
7. Except for the FHWA method, all methods showed a reduction in conservatism at a high transverse reinforcement ratio. This can be attributed to adopting yield stress for transverse reinforcement compared to the FHWA method which checks its strain.

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Appendix A

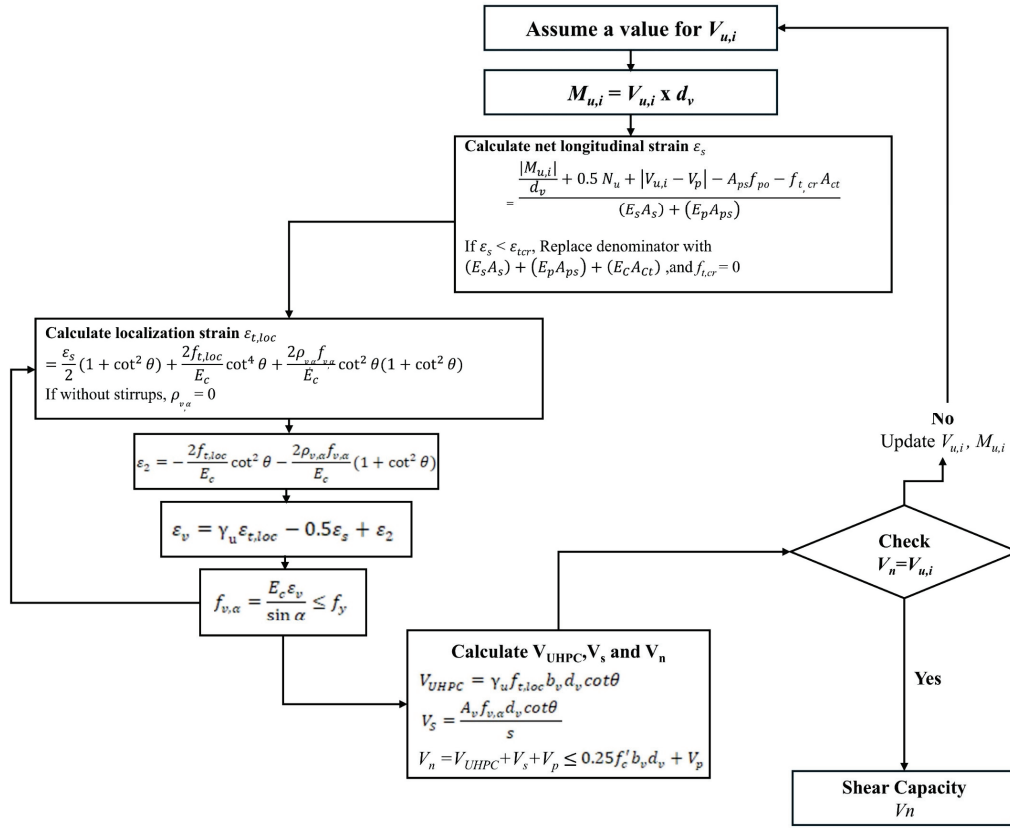


Figure A1. Flowchart for determining shear capacity of UHPC members according to FHWA Shear Provision.

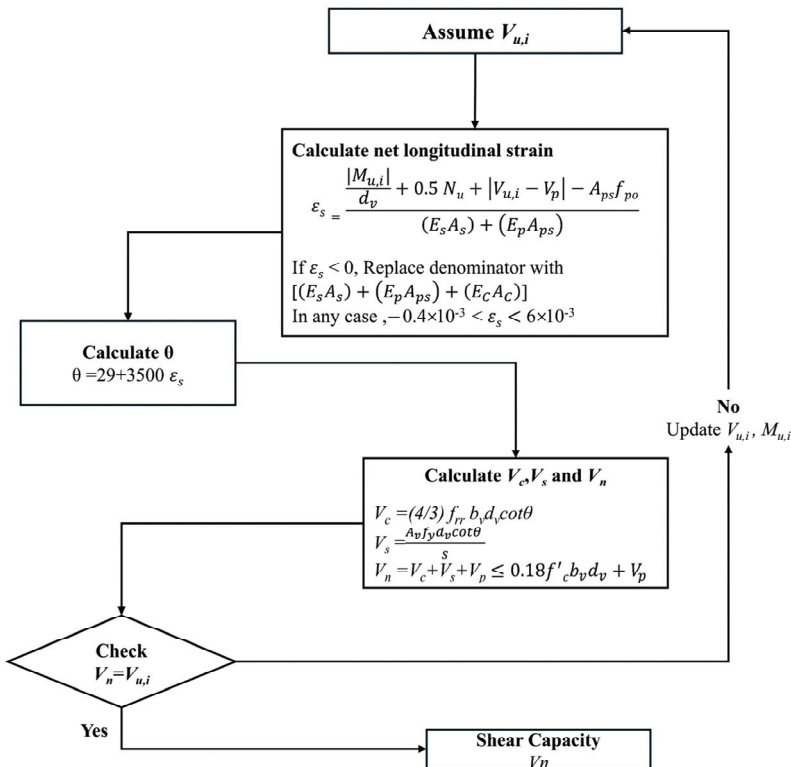


Figure A2. Flowchart for determining shear capacity of UHPC members according to ePCI method.

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Article

Free Vibration Characteristics of Functionally Graded Material (FGM) Beams on Three-Parameter Viscoelastic Foundation

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Abstract: This study numerically investigated free vibration characteristics of functionally graded material (FGM) beams on Winkler–Pasternak three-parameter elastic foundations using the modified generalized differential quadrature (MGDQ) method. To compare the effects of different beam theories on the predicted frequency responses, an n th order generalized beam theory was employed to establish the governing equations of the system's dynamic model within the Hamilton framework. As a pioneering effort, a MATLAB (version 2021a) computational program implementing the MGDQ method was developed to obtain the free vibration responses of foundation-supported FGM beams. Parametric analyses were conducted through numerical simulations to systematically examine the influences of various factors, including beam theories, damping coefficients, foundation stiffness parameters, boundary conditions, gradient indices, and span-to-thickness ratios, on the natural frequencies and damping ratios of FGM beams. The findings provide an essential theoretical foundation for dynamic characteristic analysis and functional design of foundation-supported FGM beam structures.

Keywords: functionally graded material beams; viscoelastic foundation; n -th order generalized beam theory; eigen frequency; critical damping; damping ratio

1. Introduction

Dynamic characteristics analysis of beams on elastic foundations has garnered significant attention owing to their broad range of applications in civil engineering, rail transit, mechanical systems, and biomedical engineering [1–4]. Current elastic foundation models primarily include the Winkler–Pasternak [1], Kelvin [2], Vlasov [3], and Kerr [4] models. Among these, the Winkler–Pasternak two-parameter model is widely adopted by researchers [5,6], as it effectively characterizes both the compressive and shear interactions between the foundation and beam, thus offering a realistic representation of elastic foundation behavior. Notably, Thambiratnam & Zhuge [7] pioneered dynamic analysis of beams under moving loads on elastic foundations; they established a theoretical framework for time-domain response prediction. Kim & Roesset [8] further quantified the influence of frequency-independent damping in foundation models on dynamic responses, highlighting the necessity of viscosity characterization. Furthermore, the damping effects of foundation viscosity are unavoidable in practical engineering. For instance, studies [9–11] employed Euler beam theory (also known as classical beam theory (CBT)) to numerically investigate the vibration characteristics of homogeneous beams on viscoelastic foundations using diverse analytical approaches. Recent advances in viscoelastic modeling emphasize the importance of fractional calculus, as demonstrated by Sorrentino & Fasana [12], who

integrated fractional derivative models into finite element analysis of vibrating systems, capturing frequency-dependent damping effects more accurately than classical integer-order approaches.

Functionally graded material (FGM) beams, featuring spatially continuous gradation of multiphase materials, enable tailored material parameter distributions. This customizable feature has led to innovative applications in cutting-edge fields, such as aerospace thermal protection systems and heterogeneous interfaces in energy equipment. Extending this concept to plate structures, a recent work [13] investigated free vibration of FGM plates on elastic foundations, revealing micromechanical model sensitivity in predicting mode shapes—a finding that underscores the need for refined modeling in FGM beam studies. Precise analysis of the dynamic behavior of FGM beams holds critical theoretical value for overcoming multi-objective optimization challenges under extreme conditions and ensuring operational safety, thus making it a frontier topic in smart material mechanics.

Existing research predominantly adopts forward analysis frameworks, combining analytical and numerical methods to explore the dynamic mechanisms of FGM beams [14–26]. In theoretical modeling, Simsek [14] pioneered a unified Lagrange frequency equation system encompassing seven shear deformation theories. Kahya et al. [15] developed finite element models for free vibration and buckling analysis based on Timoshenko's first-order shear beam theory (FSBT). The integration of advanced viscoelastic models was facilitated by Lesieutre & Lee [16], who developed a finite element method with frequency-dependent constrained layer damping and demonstrated enhanced energy dissipation modeling capabilities for beam structures. For coupled-field analysis, some studies [17–19] systematically investigated vibration responses under thermo-mechanical multiphysical coupling. Pu Yu et al. [20] resolved the dynamic prediction challenges of porous FGM beams under hygro-thermo-mechanical coupling using the n -th order generalized beam theory (GBT) and Navier's analytical method.

Some notable advancements in elastic foundation effects include the following: Ying et al. [21] established a free vibration model for FGM simply supported beams on elastic foundations using state-space methods based on 2D linear elasticity theory, while Pu Yu et al. [22] revealed the regulatory effects of Winkler–Pasternak foundation parameters on vibration modes via 2D differential quadrature methods. Zahedinejad [23] quantified thermal effects on spectral characteristics of foundation beams by integrating Reddy's third-order shear theory (TBT) with differential quadrature techniques. Sun et al. [24] provided critical insights into memory-dependent behaviors through comparative analysis of constant-order versus variable-order fractional models, thus establishing theoretical foundations for selecting viscoelastic constitutive models in foundation-beam interactions. Deng et al. [25] pioneered quantitative stability evaluation of dual-FGM beam systems using the FSBT-based dynamic stiffness matrices. Recently, Pu Yu et al. [26] employed GBT and a modified generalized differential quadrature (MGDQ) method to numerically study the vibration characteristics of FGM beams on elastic foundations under initial axial loads; specifically, they unveiled the dual coupling between buckling and vibrational static-dynamic behaviors.

Moreover, Duc et al. [27] developed a nonlinear dynamic model for piezoelectric FGM hyperbolic shells embedded in elastic foundations under damping-thermal-electrical-mechanical loads. Using stress functions, the Galerkin method, and the fourth-order Runge-Kutta method, they solved the nonlinear vibration problem, emphasizing the effects of material properties and coupled loads on nonlinear dynamic responses. Loghman et al. [28] considered the viscoelastic internal damping of materials and, based on modified couple stress theory and Von Kármán nonlinear relations, applied finite difference and finite element methods to analyze the influence of fractional damping coefficients, scale

parameters, and material gradient indices on the nonlinear free and forced vibration characteristics of FGM microbeams. Despite these advancements, research on the vibration behavior of FGM beams on viscoelastic foundations remains limited.

This work introduces a novel approach by incorporating the foundation's damping effect from an energy dissipation perspective. As an exploratory effort, we developed the system's dynamic model based on n -th order GBT and Hamilton's principle. Through the introduction of boundary condition control parameters, we achieved unified derivation of modified weighting coefficients for beams with three boundary configurations: clamped-clamped (C-C), clamped-supported (C-S), and supported-supported (S-S). This study pioneers the application of the MGDQ method to numerically solve the frequency response of free vibrations in FGM beams resting on Winkler–Pasternak three-parameter elastic foundations. Finally, comprehensive parametric analyses were conducted to investigate the effects of multiple factors, including beam theories, damping coefficients, foundation stiffness parameters, boundary conditions, gradient indices, and span-to-thickness ratios, on the frequency characteristics of FGM beams.

2. Governing Differential Equations

2.1. Governing Equations of Dynamic System

As illustrated in Figure 1, consider an FGM beam resting on a three-parameter viscoelastic foundation, with length L , width B , and height H . The beam comprises a ceramic-rich upper surface and a metal-rich lower surface. A Cartesian coordinate system is defined such that the x -axis aligns with the beam's longitudinal direction; the z -axis corresponds to the thickness direction, and the x - y plane coincides with the geometric midplane of the beam. The beam is subjected to a transversely distributed dynamic load $q(x,t)$. The foundation is characterized by three parameters: the Winkler stiffness coefficient kw , Pasternak shear stiffness coefficient kp , and transverse viscous damping coefficient cd .

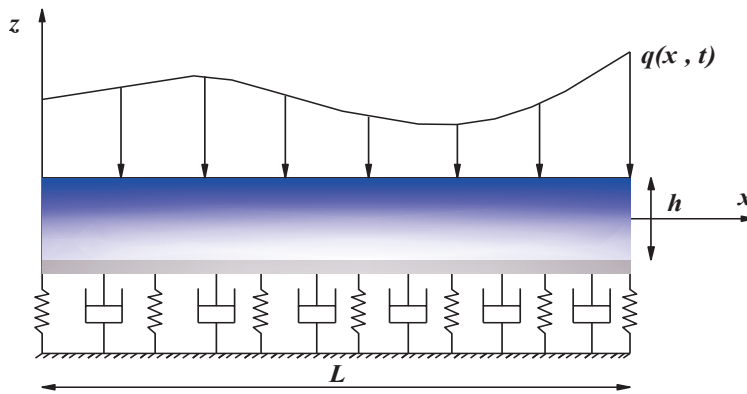


Figure 1. Model of FGM beam on three-parameter viscoelastic foundation.

The material properties (elastic modulus E , Poisson's ratio ν , and density ρ) vary continuously along the thickness direction (z) following the Voigt-type power-law mixture model. These graded properties are expressed as:

$$P(z) = P_m + (P_c - P_m) \left(\frac{1}{2} + \frac{z}{h} \right)^p, \quad (1)$$

where p denotes the material gradient index, and the subscripts m and c represent the metallic and ceramic constituents, respectively.

To compare the predictive differences among various beam theories regarding free vibration responses, this study employs an n -th order GBT framework, where the displacement field at any point within the beam can be expressed as:

$$\begin{cases} u_x(x, z, t) = u(x, t) - z \frac{\partial w}{\partial x} + f(z) \left[\varphi(x, t) + \frac{\partial w}{\partial x} \right] \\ u_y(x, z, t) = 0 \\ u_z(x, z, t) = w(x, t) \end{cases}, \quad (2)$$

$$f(z) = z - \frac{h}{2n} \left(\frac{2z}{h} \right)^n, \quad (3)$$

where $u(x, t)$ denotes the axial displacement at time t ; $\varphi(x, t)$ represents the cross-sectional rotation; and $w(x, t)$ corresponds to the generalized transverse displacements associated with the i -th deformation mode. The function $f(z)$ represents the transverse shear stress shape function corresponding to the n -th order GBT [29]. When shear deformation effects are neglected (i.e., $n = 1$), the GBT reduces to the CBT; when a parabolic distribution of shear strain is assumed through the beam thickness (i.e., $n = 3$), the GBT degenerates into Timoshenko Beam Theory (TBT). For a constant shear strain assumption across the beam thickness (i.e., $n = \infty$).

Based on the geometric equations of small deformation, the expressions for non-zero strain components can be derived as:

$$\begin{cases} \varepsilon_x = \frac{\partial u}{\partial x} - z \frac{\partial^2 w}{\partial x^2} + f(z) \left(\frac{\partial \varphi}{\partial x} + \frac{\partial^2 w}{\partial x^2} \right) \\ \gamma_{xz} = g(z) \left(\frac{\partial w}{\partial x} + \varphi \right) \end{cases}, \quad (4)$$

where $g(z)$ denotes the transverse shear strain shape function.

The constitutive relations are expressed as:

$$\begin{cases} \sigma_x = E(z) \varepsilon_x \\ \tau_{xz} = G(z) \gamma_{xz} \end{cases}, \quad (5)$$

$$G(z) = \frac{E(z)}{2[1 + \mu(z)]}, \quad (6)$$

where $G(z)$ represents the shear modulus varying along the thickness direction.

The variation in the system's kinetic energy (δT), strain energy (δV_ε), and external work and viscoelastic foundation potential (δV) are expressed as:

$$\delta T = \int_0^L \int_A \rho(z) \left(\frac{\partial u_x}{\partial t} \frac{\partial \delta u_x}{\partial t} + \frac{\partial u_z}{\partial t} \frac{\partial \delta u_z}{\partial t} \right) dA dx, \quad (7)$$

$$\delta V_\varepsilon = \int_0^L \int_A (\sigma_x \delta \varepsilon_x + \tau_{xz} \delta \gamma_{xz}) dA dx, \quad (8)$$

$$\delta V = \int_0^L \left[(k_w w - c_d \frac{\partial w}{\partial t} + q) \delta w + k_p \frac{\partial w}{\partial x} \frac{\partial \delta w}{\partial x} \right] dx. \quad (9)$$

Applying Hamilton's principle:

$$\int_{t_1}^{t_2} (\delta T - \delta V_\varepsilon - \delta V) dt = 0. \quad (10)$$

Substituting Equations (2)–(9) into Equation (10), followed by simplification and rearrangement, yields the governing dynamic equation of the system:

$$\begin{aligned} S_1 \frac{d^2 u}{dx^2} + S_3 \frac{d^2 \varphi}{dx^2} - S_7 \frac{d^3 w}{dx^3} - I_0 \ddot{u} - I_2 \ddot{\varphi} - I_6 \frac{d\ddot{w}}{dx} &= 0, \\ S_3 \frac{d^2 u}{dx^2} + S_6 \frac{d^2 \varphi}{dx^2} - S_0 \left(\varphi + \frac{dw}{dx} \right) + S_8 \frac{d^3 w}{dx^3} - \\ I_2 \ddot{u} - I_5 \ddot{\varphi} - I_7 \frac{d\ddot{w}}{dx} &= 0 \\ S_7 \frac{d^3 u}{dx^3} - S_8 \frac{d^3 \varphi}{dx^3} + S_9 \frac{d^4 w}{dx^4} + S_0 \left(\frac{d\varphi}{dx} + \frac{d^2 w}{dx^2} \right) - \\ k_w w + k_p \frac{d^2 w}{dx^2} - c_d \dot{w} - I_0 \ddot{w} + I_6 \frac{d\ddot{u}}{dx} + \\ I_7 \frac{d\ddot{\varphi}}{dx} - I_8 \frac{d^2 \ddot{w}}{dx^2} &= q(x, t) \end{aligned} \quad (11)$$

The elastic coefficients are defined as:

$$\begin{aligned} \{S_i\} &= \int_A E(z) \{1, z, f, z^2, zf, f^2\} dA, (i = 1, 2, \dots, 6), \\ S_0 &= \int_A G(z) g^2 dA, S_7 = S_2 - S_3, S_8 = S_6 - S_5, \\ S_9 &= 2S_5 - S_4 - S_6. \end{aligned} \quad (12)$$

The inertia coefficient is defined as:

$$\begin{aligned} \{I_i\} &= \int_A \rho(z) (1, z, f, z^2, zf, f^2) dA, (i = 0, 1, \dots, 5), \\ I_6 &= I_2 - I_1, I_7 = I_5 - I_4, I_8 = 2I_4 - I_3 - I_5. \end{aligned} \quad (13)$$

Equation (11) governs the forced vibration of a three-parameter viscoelastic foundation-FGM beam under a transverse dynamic load $q(x, t)$. Specific cases include free damped vibration ($q(x, t) = 0$) and free undamped vibration ($q(x, t) = 0$ and $C_d = 0$). Notably, the elastic and inertia coefficients embed tension/compression-shear-bending coupling effects.

Based on Hamilton's principle, the boundary conditions derived from Equation (10) are simplified and categorized into three beam configurations for investigation:

(i) Clamped-Clamped (C-C):

$$u = \varphi = w = \frac{dw}{dx} = 0, \quad x = 0 \text{ and } x = L. \quad (14)$$

(ii) Clamped-Supported (C-S):

$$\begin{aligned} u = \varphi = w = \frac{dw}{dx} &= 0, \quad x = 0. \\ \frac{du}{dx} = \frac{d\varphi}{dx} = w = \frac{d^2 w}{dx^2} &= 0, \quad x = L. \end{aligned} \quad (15)$$

(iii) Supported-Supported (S-S):

$$\frac{du}{dx} = \frac{d\varphi}{dx} = w = \frac{d^2 w}{dx^2} = 0, \quad x = 0 \text{ and } x = L. \quad (16)$$

2.2. Governing Equations of Damped Free Vibration System

For a freely vibrating FGM beam, the displacement components can be assumed as:

$$\begin{Bmatrix} u(x, t) \\ \varphi(x, t) \\ w(x, t) \end{Bmatrix} = \begin{Bmatrix} U(x) \\ \Psi(x) \\ W(x) \end{Bmatrix} \exp^{i\omega t}, \quad (17)$$

where ω is the complex characteristic frequency of the damped free vibration of the FGM beam; $U(x)$, $\Psi(x)$, $W(x)$ are the displacement mode shape functions; and $i = \sqrt{-1}$ is the imaginary unit.

By setting $q(x,t) = 0$ and substituting Equation (17) into Equation (11), the governing differential equation for the free vibration of the FGM beam on a three-parameter viscoelastic foundation is derived as shown below:

$$\begin{aligned} S_1 \frac{d^2 U}{dx^2} + S_3 \frac{d^2 \Psi}{dx^2} - S_7 \frac{d^3 W}{dx^3} + \omega^2 (I_0 U + I_2 \Psi + I_6 \frac{dW}{dx}) &= 0, \\ S_3 \frac{d^2 U}{dx^2} + S_6 \frac{d^2 \Psi}{dx^2} - S_0 (\Psi + \frac{dW}{dx}) + S_8 \frac{d^3 W}{dx^3} + \\ \omega^2 (I_2 U + I_5 \Psi + I_7 \frac{dW}{dx}) &= 0 \\ S_7 \frac{d^3 U}{dx^3} - S_8 \frac{d^3 \Psi}{dx^3} + S_9 \frac{d^4 W}{dx^4} + S_0 (\frac{d\Psi}{dx} + \frac{d^2 W}{dx^2}) - \\ k_w W + k_p \frac{d^2 W}{dx^2} - i c_d \omega W + \\ \omega^2 (I_0 W - I_6 \frac{dU}{dx} - I_7 \frac{d\Psi}{dx} + I_8 \frac{d^2 W}{dx^2}) &= 0 \end{aligned} \quad (18)$$

Clearly, the governing differential equation (Equation (18)), combined with the boundary equations (Equations (14)–(16)), forms a boundary value problem for the damped free vibration of the FGM beam on a three-parameter viscoelastic foundation under three typical boundary conditions.

3. MGDQ Method for Solving for Complex Characteristic Frequencies

3.1. Introduction of Beam Boundary Condition Control Parameters

First, based on the generalized differential quadrature method [29], the k -th order derivatives of the displacement mode shape functions at discrete nodes x_i can be expressed as:

$$\frac{d^k}{dx^k} \begin{Bmatrix} U(x) \\ \Psi(x) \\ W(x) \end{Bmatrix}_{x=x_i} = \sum_{j=1}^N C_{ij}^{(k)} \begin{Bmatrix} U_j(x_j) \\ \Psi_j(x_j) \\ W_j(x_j) \end{Bmatrix}, \quad (19)$$

where $C_{ij}^{(k)}$ is the weighting coefficient for the k -th order derivative of the displacement mode shape function, and N is the total number of discrete nodes.

Second, the weighting coefficients for the 0-th order derivative of any unknown function at $x = 0$ and $x = L$ are defined as:

$$C_{1j}^{(0)} = \begin{cases} 1 & (j = 1) \\ 0 & (\text{else}) \end{cases}; C_{Nj}^{(0)} = \begin{cases} 1 & (j = N) \\ 0 & (\text{else}) \end{cases}. \quad (20)$$

Using these definitions, the boundary equations (Equations (14)–(16)) for C-C, C-S, and S-S beams, respectively, can be uniformly discretized via the GDQ method as:

$$\begin{cases} \sum_{j=1}^N C_{1j}^{(n_0)} U_j = 0 \\ \sum_{j=1}^N C_{1j}^{(n_0)} \Psi_j = 0 \\ W_1 = 0 \\ \sum_{j=1}^N C_{1j}^{(n_0+1)} W_j = 0 \end{cases}, \begin{cases} \sum_{j=1}^N C_{Nj}^{(n_1)} U_j = 0 \\ \sum_{j=1}^N C_{Nj}^{(n_1)} \Psi_j = 0 \\ W_N = 0 \\ \sum_{j=1}^N C_{Nj}^{(n_1+1)} W_j = 0 \end{cases}, \quad (21)$$

where n_0 and n_1 are the boundary condition control parameters. By assigning $n_0 = 0$ or 1, and $n_1 = 0$ or 1, the boundary conditions are defined as follows: $n_0 = 0$ and $n_1 = 0$ for the C-C beam; $n_0 = 0$ and $n_1 = 1$ for the C-S beam; and $n_0 = 1$ and $n_1 = 0$ for the S-S beam.

3.2. Weighting Coefficients in the MGDQ Method

To address the case wherein the total number of discretized equations exceeds the number of unknown displacements (thereby preventing a direct conversion to an eigenvalue problem), this study introduces improved weighting coefficients for the displacement mode shape functions under C-C, C-S, and S-S boundaries. These coefficients are derived [29] as:

$$\begin{cases} P_{ij}^{(k)} = C_{ij}^{(k)} + \frac{C_{i1}^{(k)} \cdot CUK1 + C_{iN}^{(k)} \cdot CUKN}{CUN} \\ Q_{ij}^{(k)} = P_{ij}^{(k)} \\ R_{ij}^{(k)} = C_{ij}^{(k)} + \frac{C_{i2}^{(k)} \cdot CWK1 + C_{i(N-1)}^{(k)} \cdot CWKN}{CWN} \end{cases}, \quad (22)$$

$$\begin{cases} CUK1 = C_{1N}^{(n_0)} C_{Nj}^{(n_1)} - C_{NN}^{(n_1)} C_{1j}^{(n_0)} \\ CUKN = C_{N1}^{(n_1)} C_{1j}^{(n_0)} - C_{11}^{(n_0)} C_{Nj}^{(n_1)} \\ CUN = C_{11}^{(n_0)} C_{NN}^{(n_1)} - C_{1N}^{(n_0)} C_{N1}^{(n_1)} \end{cases}, \quad (23)$$

$$\begin{cases} CWK1 = C_{1(N-1)}^{(n_{0+1})} C_{Nj}^{(n_{1+1})} - C_{N(N-1)}^{(n_{1+1})} C_{1j}^{(n_{0+1})} \\ CWKN = C_{N2}^{(n_{1+1})} C_{1j}^{(n_{0+1})} - C_{12}^{(n_{0+1})} C_{Nj}^{(n_{1+1})} \\ CWN = C_{12}^{(n_{0+1})} C_{N(N-1)}^{(n_{1+1})} - C_{1(N-1)}^{(n_{0+1})} C_{N2}^{(n_{1+1})} \end{cases}, \quad (24)$$

where $P_{ij}^{(k)}$, $Q_{ij}^{(k)}$, and $R_{ij}^{(k)}$ are the improved weighting coefficients for the m -th order derivatives of the displacement mode shape functions $U(x)$, $\Psi(x)$, $W(x)$, respectively, at internal nodes, $x = 0$, and $x = L$, respectively.

3.3. Unified Discretized Equations via the MGDQ Method

Applying the MGDQ method to discretize Equation (18), the unified discretized equations for C-C, S-S, and C-S boundary beams are obtained as follows:

$$S_1 \sum_{j=2}^{N-1} P_{ij}^{(2)} U_j + S_3 \sum_{j=2}^{N-1} Q_{ij}^{(2)} \Psi_j - S_7 \sum_{j=3}^{N-2} R_{ij}^{(3)} W_j + \omega^2 \left(I_0 U_i + I_2 \Psi_i - I_6 \sum_{j=3}^{N-2} R_{ij}^{(1)} W_j \right) = 0, \quad (i = 2, 3, \dots, N-1), \quad (25)$$

$$S_3 \sum_{j=2}^{N-1} P_{ij}^{(2)} U_j + S_6 \sum_{j=2}^{N-1} Q_{ij}^{(2)} \Psi_j + S_8 \sum_{j=3}^{N-2} R_{ij}^{(3)} W_j - S_0 \Psi_i - S_0 \sum_{j=3}^{N-2} R_{ij}^{(1)} W_j + \omega^2 \left(I_2 U_i + I_5 \Psi_i + I_7 \sum_{j=3}^{N-2} R_{ij}^{(1)} W_j \right) = 0, \quad (i = 2, 3, \dots, N-1), \quad (26)$$

$$\begin{aligned} & S_7 \sum_{j=2}^{N-1} P_{ij}^{(3)} U_j - S_8 \sum_{j=2}^{N-1} Q_{ij}^{(3)} \Psi_j + S_9 \sum_{j=3}^{N-2} R_{ij}^{(4)} W_j + \\ & S_0 \left(\sum_{j=2}^{N-1} Q_{ij}^{(1)} \Psi_j + \sum_{j=3}^{N-2} R_{ij}^{(2)} W_j \right) - k_w W_i - i c_d \omega W_i + \\ & k_p \sum_{j=3}^{N-2} R_{ij}^{(2)} W_j + \omega^2 \left(I_0 W_i - I_6 \sum_{j=2}^{N-1} P_{ij}^{(1)} U_j - \right. \\ & \left. I_7 \sum_{j=2}^{N-1} Q_{ij}^{(1)} \Psi_j + I_8 \sum_{j=3}^{N-2} R_{ij}^{(2)} W_j \right) = 0 \end{aligned}, \quad (i = 3, 4, \dots, N-2). \quad (27)$$

3.4. Complex Characteristic Frequency Equation

After discretization, the total number of algebraic equations in the system equals the number of unknown displacement mode shape quantities ($3N - 8$). This system can be expressed as a matrix equation:

$$\{[K] + i\omega[C] + \omega^2[M]\}X = 0, \quad (28)$$

$$X = \{U_2, U_3, \dots, U_{N-1}, \Psi_2, \Psi_3, \dots, \Psi_{N-1}, W_3, W_4, \dots, W_{N-2}\}^T,$$

where K is the sub-stiffness matrix, C is the sub-damping matrix, M is the sub-mass matrix, and X is the nodal displacement vector.

The non-trivial solution condition for Equation (28) yields the complex characteristic frequency equation:

$$\det([K] + i\omega[C] + \omega^2[M]) = 0. \quad (29)$$

Solving Equation (29) provides the frequency response of the damped free vibration of the FGM beam.

For generality, the following non-dimensional parameters are adopted:

$$\lambda = \frac{L}{h}, K_w = \frac{k_w L^4}{E_c I}, K_p = \frac{k_p L^2}{E_c I}, C_d = \frac{c_d L^2}{\sqrt{E_c I \rho_c A}}, \Omega = \omega L^2 \sqrt{\frac{\rho_c A}{E_c I}}, \quad (30)$$

$$\bar{\omega} = -\Omega(\zeta \pm i\sqrt{1 - \zeta^2}), \Omega_R = -\Omega\zeta, \Omega_I = \Omega\sqrt{1 - \zeta^2}, \delta = -\Omega_R,$$

where λ is the span-to-thickness ratio, K_w and K_p are non-dimensional foundation stiffness coefficients, C_d is the non-dimensional viscous external damping coefficient, I is the moment of inertia, A is the cross-sectional area, Ω is the undamped natural frequency, Ω_R is the real part of and reflects the damping-induced decay, Ω_I is the imaginary part of $\bar{\omega}$ and represents the underdamped natural frequency, ζ is the damping ratio, and δ is the decay coefficient.

From the non-dimensional frequency definition in Equation (30), the real part of A is evidently negative, reflecting the influence of the external damping on the decay characteristics of the displacement response. When $0 < \zeta < 1$, the imaginary part of $\bar{\omega}$ corresponds to the underdamped natural frequency, when $\zeta = 1$, the system reaches the critical damping state, and when $\zeta > 1$, the system is in an overdamped state.

4. Analysis and Discussion

In the examples considered in this study, N was taken as 17. An FGM beam composed of metal and ceramic was considered, with the following physical coefficients: $E_m = 70$ GPa; $\rho_m = 2702$ kg/m³; $E_c = 380$ GPa; $\rho_c = 3960$ kg/m³; and $\mu_m = \mu_c = 0.3$ [20].

4.1. Validation of Numerical Results

To address the lack of data on the natural frequencies of FGM beams under viscoelastic damping effects, this study established a multi-dimensional validation framework. First, theoretical self-consistency verification was performed by degenerating the three-parameter foundation model into the classical Euler–Bernoulli beam ($K_w = K_p = C_d = 0$) under a short and thick beam condition with a λ of 5 to establish a baseline model. Subsequently, the numerical convergence was verified. As shown in Table 1, when Reddy's third-order shear deformation theory (TBT, $n = 3$) was adopted and the dimensionless frequency definition was consistent with [14], and the relative errors of the dimensionless fundamental frequencies under C-C and S-S boundary conditions were both less than 0.5%, thereby significantly outperforming the 1.2% error level of the traditional differential quadrature method. This dual verification mechanism not only confirmed the computational stability of the MGDQ algorithm under geometrically constrained conditions but also revealed the

physical consistency of the viscoelastic parameter modeling in this study, thus providing a reliable numerical benchmark for subsequent research on complex damping scenarios.

Table 1. Comparison of dimensionless fundamental frequencies for undamped free vibration in FGM beams ($\lambda = 5$).

Boundary	Method	$p = 0$	$p = 0.2$	$p = 1$	$p = 10$
C-C	Ref. [14] *	10.0705	9.46641	7.95034	6.16515
	This study	10.0756	9.46932	7.95395	6.16797
S-S	Ref. [14] *	5.15274	4.80924	3.99042	3.28160
	This study	5.15274	4.80807	3.99041	3.28160

* The data source is open access and comes from Ref. [14].

4.2. Influence of GBT Order n on Output Response Parameters

Next, the influence of the GBT order n on the free vibration response parameters of the three-parameter viscoelastic foundation FGM beam was explored, considering $p = 1$, $\lambda = 5$, $K_w = 25$, $K_p = 5$, and $C_d = 10$. Figure 2 illustrates the relationship curves of the dimensionless characteristic fundamental frequency (imaginary part Ω_I , real part Ω_R , and damping ratio ζ versus the GBT order n) for a clamped FGM short beam in an underdamped state.

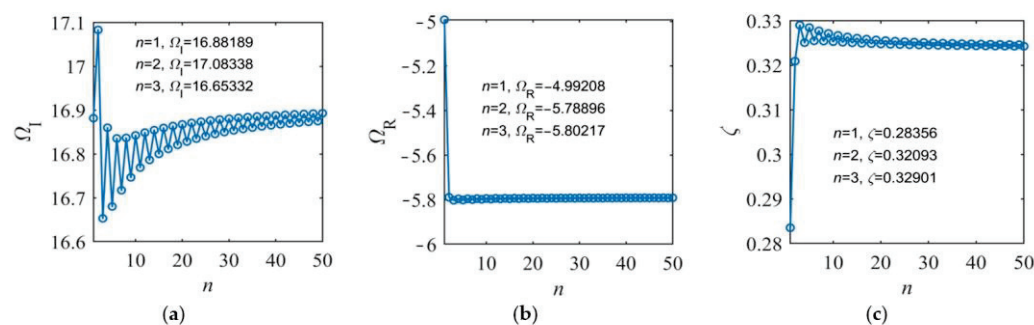


Figure 2. Curves of dimensionless fundamental frequency and damping ratio versus GBT order n for a clamped FGM beam: (a) Ω_I - n ; (b) Ω_R - n ; (c) ζ - n .

From Figure 2a, it can be observed that Ω_I exhibited minor fluctuations and gradually stabilized as the positive integer n increased, eventually approaching the prediction of the FSBT. The maximum Ω_I value occurred at $n = 2$, and the minimum at $n = 3$, with an extreme difference of 0.43006. Additionally, for $\lambda = 5$ (a short and thick beam), the CBT neglected the effects of shear deformation, thereby overestimating the overall beam stiffness. Compared to the FSBT and TBT, the CBT overestimated Ω_I (underestimating the period of underdamped decay vibration in the FGM beam). From Figure 2b, unlike the undamped free vibration, the real part Ω_R of the fundamental frequency in the underdamped state was negative, indicating amplitude decay due to damping. Figure 2b,c show that, except for $n = 1$, the variations in the predicted values of Ω_R and ζ were negligible with changes in n . However, the CBT significantly overestimated Ω_R (underestimating the decay coefficient δ), thereby overestimating the amplitude; furthermore, it underestimated ζ , leading to an overestimation of the fundamental frequency Ω_I . Compared with the TBT, the relative errors in Ω_R and ζ predicted by the CBT reached 13.96% and 13.81%, respectively, which are not negligible.

4.3. Influence of Other Factors on Output Response Parameters

All examples considered herein adopted TBT to explore the influence of parameters, such as beam boundary conditions C_d , p , λ and foundation elastic stiffness on the free vibration response parameters of the FGM beam. Figure 3 describes the relationships

between Ω_L , Ω_R , and ζ , and C_d for FGM beams under the three boundary conditions. With given parameters $p = 1$, $\lambda = 20$, $K_w = 25$, $K_p = 5$, the following observations were made.

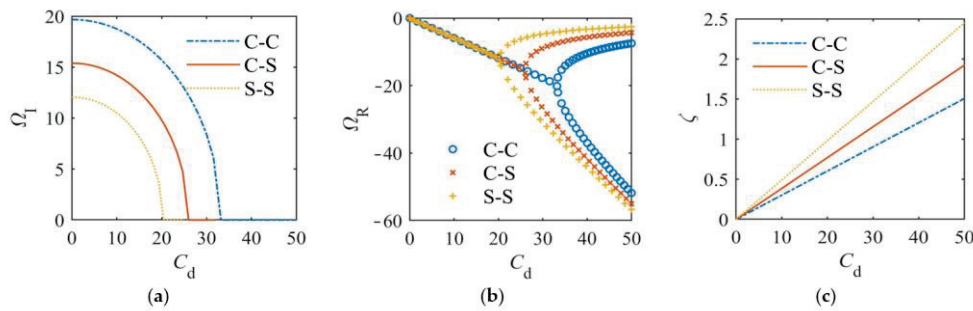


Figure 3. Curves of dimensionless fundamental frequency and damping ratio versus external damping coefficient for FGM beams under different boundary conditions: (a) Ω_I – C_d ; (b) Ω_R – C_d ; (c) ζ – C_d .

From Figure 3a: Ω_I monotonically decreased as C_d increased. When C_d was equal to critical damping C_{cr} , Ω_I reached its minimum value. Increasing C_d further led to an overdamped state, where Ω_I approached zero, thereby indicating purely decaying motion of the beam. Under different boundary conditions, Ω_I was largest for C-C beams, intermediate for C-S beams, and smallest for S-S beams. The values of C_{cr} also varied across the boundary conditions, with C-C beams having the highest C_{cr} , S-S beams the lowest, and C-S beams falling in between.

From Figure 3b: In the underdamped stage, Ω_R was negative for all three boundary conditions and decreased linearly with increasing C_d . This implies that the decay coefficient increased with C_d , reflecting a gradual reduction in the vibrational energy. In other words, the underdamped free vibration of the FGM beam manifested as an amplitude-decaying motion, with densely packed linear segments in the Ω_R – C_d plot.

From Figure 3c: ζ of the three boundary-conditioned FGM beams increased linearly with C_d . For small damping (e.g., $C_d < 5$), boundary conditions had limited influence on ζ . For larger damping (e.g., $C_d > 10$), at the same C_d value, the S-S beams exhibited the highest ζ , C-C beams the lowest, and C-S beams falling in between.

Figure 4 shows the relationship curves of Ω_L , Ω_R , and ζ versus C_d for clamped FGM beams with different gradient index values. The parameters were set as $\lambda = 20$, $K_w = 25$, and $K_p = 5$.

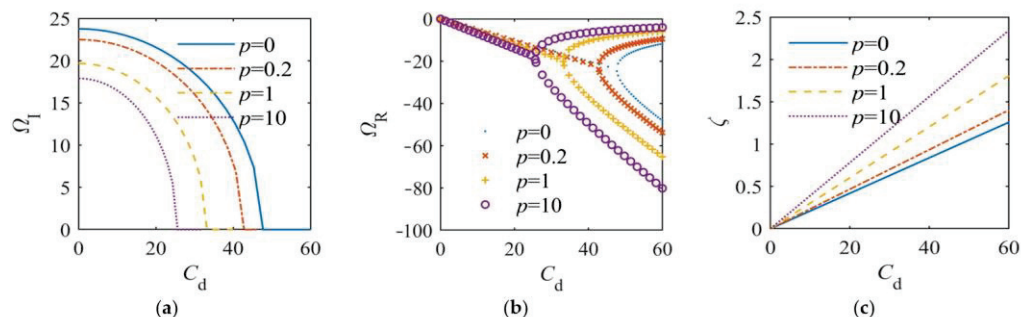


Figure 4. Curves of dimensionless fundamental frequency and damping ratio versus external damping coefficient for clamped FGM beams with different gradient index values: (a) Ω_I – C_d ; (b) Ω_R – C_d ; (c) ζ – C_d .

From Figure 4a: For $p = 0, 0.2, 1$, and 10 , Ω_I decreased monotonically with increasing C_d in the underdamped stage until $\Omega_I = 0$ at their respective critical damping states. At the

same C_d , a larger p resulted in a smaller Ω_I and lower C_{cr} . When the FGM beam entered the overdamped state, $\Omega_I = 0$.

From Figure 4b: At $C_d = 0$, $\Omega_R = 0$. In the underdamped stage ($0 < C_d < C_{cr}$), Ω_R was negative and decreased linearly with C_d . For the same C_d , a larger p led to a smaller Ω_R . In the overdamped stage ($C_d > C_{cr}$), Ω_R assumed two negative real numbers, and the Ω_R – C_d curves opened to the right with significant separation, indicating a pronounced influence of p on displacement decay. Notably, larger p -values resulted in wider curve openings. Furthermore, C_{cr} corresponded to the abscissa of the intersection point between the underdamped linear segment and overdamped curve segment of Ω_R – C_d .

From Figure 4c: ζ increased linearly with C_d . For small damping (e.g., $0 < C_d < 2$), the ζ – C_d linear segments were densely packed, suggesting a limited influence of p on ζ . However, for large damping (e.g., $C_d > 20$), the linear segments diverged significantly, highlighting a strong dependence of ζ on p .

In general, small damping (e.g., $C_d < 5$) had a negligible effect on the natural frequencies and could be ignored in routine calculations. However, the influence of small damping on amplitude decay could not be neglected, as the amplitudes diminished exponentially.

Figure 5 illustrates the relationship curves of Ω_I and Ω_R versus λ for a clamped–simply supported FGM beam in an underdamped state, with parameters $p = 1$, $K_w = 25$, and $K_p = 5$.

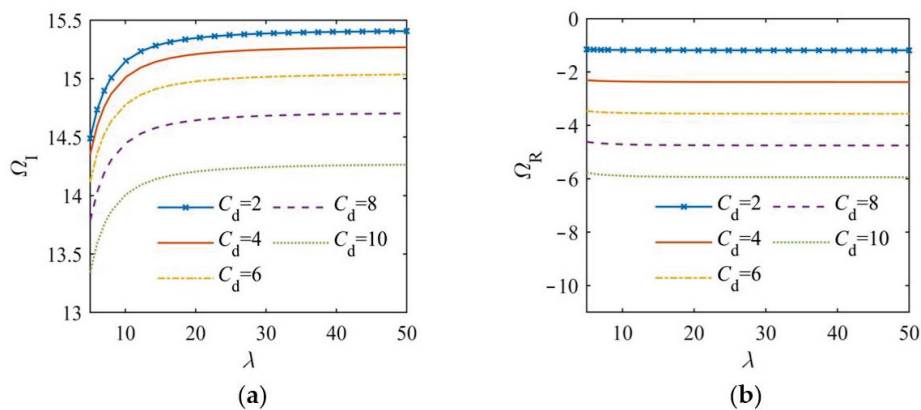


Figure 5. Curves of dimensionless fundamental frequency versus span-to-thickness ratio for a clamped–simply supported FGM beam: (a) Ω_I – λ ; (b) Ω_R – λ .

From Figure 5a: As λ increased, the Ω_I curves initially rose significantly. When $\lambda > 20$, the increase in Ω_I slowed down, indicating that the period of underdamped decay vibration decreased with λ , but became less sensitive for $\lambda > 20$. Additionally, for the same λ , a larger C_d resulted in a smaller Ω_I .

From Figure 5b: The Ω_R – λ curves are nearly flat, suggesting that λ had minimal influence on the amplitude decay. For the same λ , a larger C_d led to a smaller Ω_R (i.e., a larger decay coefficient δ), thereby accelerating the amplitude attenuation.

Figure 6 shows the relationship curves of Ω_I and Ω_R versus p for a clamped FGM beam in an underdamped state, with parameters $\lambda = 20$, $K_w = 25$, and $K_p = 5$.

From Figure 6a: The Ω_I curves monotonically decreased as p increased (i.e., the ceramic content decreased). The most significant reduction occurred in the range $0 \leq p \leq 2$, which could be attributed to the rapid weakening of the FGM beam’s overall stiffness due to the sharp decline in its alumina (Al_2O_3) content.

From Figure 6b: When $C_d = 0$, $\Omega_R = 0$ (a horizontal line), indicating undamped free vibration with a constant amplitude. With damping, Ω_R decreased with p , most notably in the range $0 \leq p \leq 2$. For $p > 2$, the decrease in Ω_R slowed down, reflecting that amplitude attenuation diminished gradually as the ceramic content reduced further.

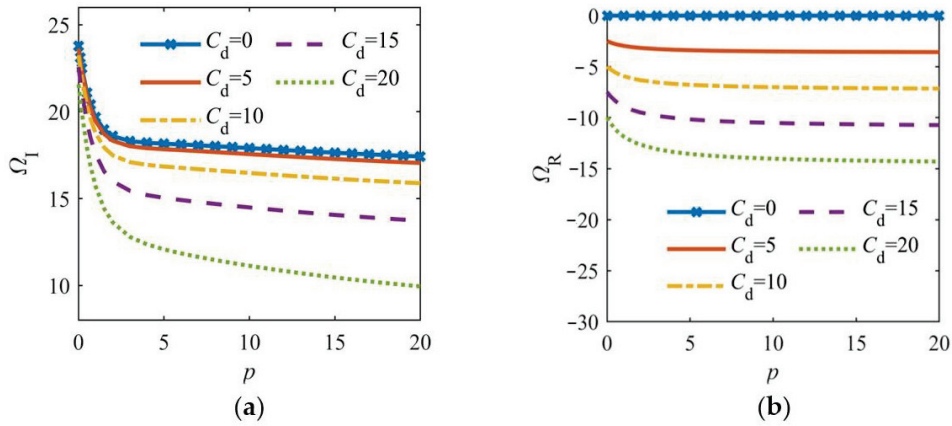


Figure 6. Curves of dimensionless fundamental frequency versus gradient index for a clamped FGM beam: (a) Ω_I - p ; (b) Ω_R - p .

Figure 7 illustrates the relationship surfaces of Ω_I and Ω_R versus the Winkler–Pasternak foundation elastic stiffness coefficients K_w and K_p for the three typical boundary conditions (C-C, C-S, S-S) of the FGM beams in an underdamped state, with parameters $p = 1$, $\lambda = 20$, and $C_d = 10$.

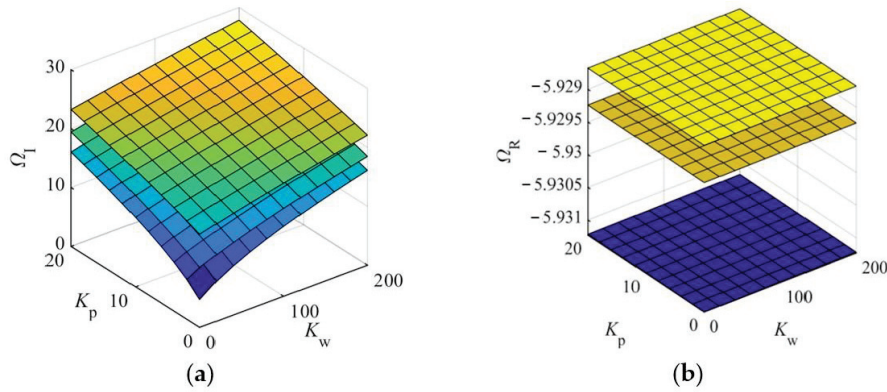


Figure 7. Influence of foundation elastic stiffness coefficients on dimensionless fundamental frequency imaginary and real parts of FGM beams: (a) Ω_I - K_w - K_p ; (b) Ω_R - K_w - K_p .

From Figure 7a: As K_w and K_p increased, Ω_I rose, leading to a gradual reduction in the period of underdamped decay vibration. For a given foundation elastic stiffness, the Ω_I surface values were significantly higher for the C-C boundaries, intermediate for C-S boundaries, and lowest for S-S boundaries. This was attributed to the enhanced equivalent stiffness of the beam structure under strongly constrained boundaries, which would increase the vibrational frequency.

From Figure 7b: The Ω_R surfaces for all three boundary conditions are flat planes, indicating that increasing the foundation elastic stiffness had no effect on amplitude attenuation. Additionally, the Ω_R values were slightly lower for the C-C boundaries, slightly higher for the S-S boundaries, and intermediate for the C-S boundaries. However, the differences in Ω_R across the boundaries were minimal, reaffirming that the boundary conditions had a negligible influence on the amplitude attenuation in the underdamped states.

Figure 8 shows the relationship curves of Ω_I versus C_d for the clamped (C-C) and simply-supported (S-S) FGM beams, with parameters $p = 1$, $\lambda = 20$, $K_w = 100$, and $K_p = 10$.

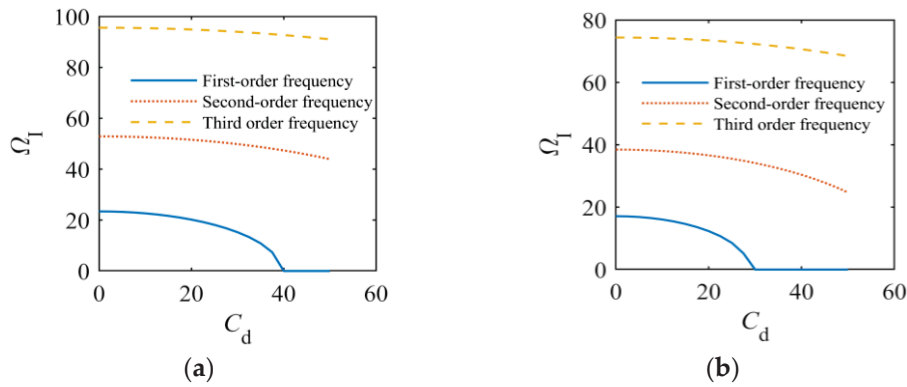


Figure 8. Relationship curves of the first three dimensionless characteristic frequency imaginary parts versus the external damping coefficient: (a) C-C beam; (b) S-S beam.

The first three frequencies of the FGM beams under both these boundary conditions decreased gradually with increasing C_d . The first-order frequency decreased most rapidly, reaching zero at C_{cr} , after which it remained at zero in the overdamped state. However, the second- and third-order frequencies did not reach their respective critical damping states. This indicates that external damping effects more readily attenuated low-order vibrational mode frequencies, while higher-order frequencies were less significantly affected.

5. Conclusions

Based on the n -th order GBT, a systematic dynamic model was established under the Hamiltonian framework. The free vibration characteristics of a three-parameter viscoelastic foundation FGM beam were numerically analyzed using the MGDQ method. The key results are as follows:

(1) Compared to the FSBT and TBT, the CBT overestimated the frequency of underdamped free vibration in short FGM beams, thereby underestimating the period. It significantly underestimated the decay coefficient, leading to an overestimation of the vibration amplitude; moreover, it notably underestimated the damping ratio.

(2) As the external damping coefficient increased, the FGM beam transitioned through underdamped, critical damping, and overdamped states. The characteristic frequency equation exhibited complex roots of different forms. Vibrational behavior was observed only in the underdamped state.

(3) The C-C beams exhibited lower damping ratios and higher critical damping values, while the S-S beams showed higher damping ratios and lower critical damping values; the C-S beams lay between these extremes, thus demonstrating intermediate damping characteristics.

(4) The natural frequencies decreased with increasing external damping coefficient (C_d) and gradient index (p), but increased with span-to-thickness ratio (λ) and foundation elastic stiffness (K_w, K_p). Amplitude decay is significantly influenced by C_d and p , while boundary conditions, λ , and foundation stiffness have minimal impact on attenuation.

(5) Small damping ($C_d < 5$) negligibly affected the natural frequencies and can be ignored in routine calculations. However, its effect on the amplitude decay remained significant due to exponential attenuation.

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Appendix A

To conserve space in the main text, the matrix elements of Equation (28) have been relocated to Appendix A, with the detailed expressions provided below:

$$\mathbf{K} = \begin{bmatrix} \mathbf{K}_u^{11} & \mathbf{K}_\varphi^{12} & \mathbf{K}_w^{13} \\ \mathbf{K}_u^{21} & \mathbf{K}_\varphi^{22} & \mathbf{K}_w^{23} \\ \mathbf{K}_u^{31} & \mathbf{K}_\varphi^{31} & \mathbf{K}_w^{33} \end{bmatrix},$$

where: $\mathbf{K}_u^{11} = S_1 P_{ij}^{(2)} (i, j = 2:N - 1)$; $\mathbf{K}_\varphi^{12} = S_3 Q_{ij}^{(2)} (i, j = 2:N - 1)$; $\mathbf{K}_w^{13} = -S_7 R_{ij}^{(3)} (i = 2:N - 1, j = 3:N - 2)$; $\mathbf{K}_u^{21} = S_3 P_{ij}^{(2)} (i, j = 2:N - 1)$; $\mathbf{K}_\varphi^{22} = S_6 Q_{ij}^{(2)} - S_0 (i, j = 2:N - 1)$; $\mathbf{K}_w^{23} = S_8 R_{ij}^{(3)} - S_0 R_{ij}^{(1)} (i = 2:N - 1, j = 3:N - 2)$; $\mathbf{K}_u^{31} = S_7 P_{ij}^{(3)} (i = 3:N - 2, j = 2:N - 1)$; $\mathbf{K}_\varphi^{32} = S_0 Q_{ij}^{(1)} - S_8 Q_{ij}^{(3)} (i = 3:N - 2, j = 2:N - 1)$; $\mathbf{K}_w^{33} = (S_0 + k_p) R_{ij}^{(2)} + S_9 R_{ij}^{(4)} - k_w (i = 3:N - 2, j = 3:N - 2)$;

$$\mathbf{M} = \begin{bmatrix} \mathbf{M}_u^{11} & \mathbf{M}_\varphi^{12} & \mathbf{M}_w^{13} \\ \mathbf{M}_u^{21} & \mathbf{M}_\varphi^{22} & \mathbf{M}_w^{23} \\ \mathbf{M}_u^{31} & \mathbf{M}_\varphi^{31} & \mathbf{M}_w^{33} \end{bmatrix},$$

where: $\mathbf{M}_u^{11} = I_0 (i, j = 2:N - 1)$; $\mathbf{M}_\varphi^{12} = I_2 (i, j = 2:N - 1)$; $\mathbf{M}_w^{13} = -I_6 R_{ij}^{(1)} (i = 2:N - 1, j = 3:N - 2)$; $\mathbf{M}_u^{21} = I_2 (i, j = 2:N - 1)$; $\mathbf{M}_\varphi^{22} = I_5 (i, j = 2:N - 1)$; $\mathbf{M}_w^{23} = I_7 R_{ij}^{(1)} (i = 2:N - 1, j = 3:N - 2)$; $\mathbf{M}_u^{31} = -I_6 P_{ij}^{(1)} (i = 3:N - 2, j = 2:N - 1)$; $\mathbf{M}_\varphi^{32} = -I_7 Q_{ij}^{(1)} (i = 3:N - 2, j = 2:N - 1)$; $\mathbf{M}_w^{33} = I_0 + I_8 R_{ij}^{(2)} (i = 3:N - 2, j = 3:N - 2)$;

$$\mathbf{C} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \mathbf{C}_w^{33} \end{bmatrix},$$

where: $\mathbf{C}_w^{33} = -c_d (i = 3:N - 2, j = 3:N - 2)$.

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Article

Predicting the Elastic Moduli of Unidirectional Composite Materials Using Deep Feed Forward Neural Network

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Abstract: Elastic moduli are important mechanical properties that describe a material's stiffness and its deformation under elastic loading. In addition to experimental techniques, computational homogenization is commonly used for composite materials to calculate their elastic moduli. This research employs a deep learning algorithm, specifically a Feedforward Neural Network (FNN), to predict the longitudinal and transverse Young's modulus, shear modulus, and Poisson's ratio of various unidirectional (UD) composites. The predictions are based on several features, including the names of the composites, Young's moduli and Poisson's ratios of the fibers and matrices, and the fiber volume fraction. Initially, 20 different UD composites were selected from the existing literature. ANSYS-19 Material Designer was then utilized to calculate the elastic moduli of these materials while varying the fiber volume fraction from 0.2 to 0.7. This process generated a dataset of 1948 samples, with 80% of the data allocated for training the FNN model and the remaining 20% used to evaluate performance metrics of the test data. These metrics include mean squared error (MSE), root mean squared error (RMSE), mean absolute error (MAE), and R^2 score. The results indicate that, with optimized hyperparameters, the FNN model can accurately predict the elastic moduli, demonstrating its effectiveness as a tool for calculating the elastic properties of UD composites.

Keywords: deep learning; FNN; ANSYS Material Designer; elastic modulus; UD composites

1. Introduction

Over the past few decades, composite materials have drawn significant attention in multiple industries, including aerospace, naval, automotive, building and construction, and so on, due to their better mechanical performance. These advantages include mechanical strength, lightweight, corrosion resistance, and the ability to withstand harsh environments compared to conventional or single-constituent materials [1]. In recent times, various types of composite materials have become available. However, based on the fiber reinforcement type, composite materials are classified into unidirectional composites and woven or multidirectional composites. Particularly in unidirectional composites, fibers are aligned in one direction, mainly benefiting them by effectively absorbing loads and offering maximum strength and stiffness [2–4]. Unidirectional composites can be fabricated using various manufacturing methods, including conventional techniques such as injection molding, compression molding, vacuum-assisted resin transfer molding (VARTM), and advanced methods such as 3D printing [2,5]. While 3D printing is more advanced and offers advantageous features like sustainability and environmental friendliness, conventional

methods consistently deliver superior mechanical performance. For example, this research compared conventionally fabricated carbon fiber-reinforced thermoplastic composites with 3D-printed composites and found that conventionally fabricated composites provided significantly better mechanical properties (strength: 1500 MPa, stiffness: 135 GPa) than 3D-printed composites (strength: 986 MPa, stiffness: 64 GPa) [5].

Various theoretical models have previously been proposed and developed to compute the elastic moduli of unidirectional composites, such as the Rule of Mixture (ROM), Modified Rule of Mixture (MROM), Halpin–Tsai model, Chamis model, Mori–Tanaka model (M-T), self-consistent model (S-C), and Bridging model (BM) [6]. ROM is the first theoretical simplified model that enables the calculation of elastic moduli (E_{11} , ν_{12} , E_{22} , G_{12}) by considering an equal amount of experienced strain on both fiber and matrix. Later, the ROM is modified into MROM, incorporating fiber length and orientations into the calculation; the modified theory enables the prediction of the elastic moduli (E_{22} , G_{12}), which was unattainable by ROM [6,7]. The Halpin–Tsai model is another well-known semi-empirical approach proposed for calculating the elastic moduli, specifically the transverse modulus (E_{22}), within a suitable volume fraction range of 0.25 to 0.55 without unnecessary complexity [8]. The Halpin–Tsai model also considers the fiber aspect ratio, inter-fiber spacing, and fiber end gap in the transverse modulus of discontinuous fiber composites [9]. However, compared to other theoretical models and experimental results, it may exhibit negligible deviations in predicting the transverse modulus (E_{22}) [9,10]. Another model, called the self-consistent model, is widely adopted in micromechanics to predict elastic moduli. This model primarily estimates elastic properties by considering inclusions, where a single particle inclusion is embedded in the effective medium [11,12]. Similarly, the Mori–Tanaka model (M-T) is also well-known for predicting elastic moduli [13–15]. The main benefit of this model is that it can consider the multiple filler ratios, fiber length distributions, and orientation of composites in the model [14]. Furthermore, it can also be used to compute the composite's thermal expansion coefficient. For example, this study calculates the thermal expansion coefficient of composites employing the M-T model and found that the M-T model can predict the moduli in an identical manner compared to other theoretical models [13]. However, the M-T model may cause some difficulties, particularly where nonlinear analyses or time dependence are required to consider in composite materials [16]. The Bridging model is another popular micromechanics approach used to predict the transverse modulus (E_{22}), as well as in-plane and out-plane shear modulus (G_{12} , G_{23}) of unidirectional composites [17]. This model is particularly convenient for micromechanical modeling since it can compute the elastic moduli by correlating the average stress states in the constituent fibers and matrix through the bridging matrix [18]. However, in recent times, the Bridging model has also been employed to predict the strength of an interface crack containing unidirectional composites. The investigations found that the Bridging model can predict the transverse moduli even in the presence of an interface crack. In addition, in comparison to experimental outcomes, the Bridging model showed good agreement for predicting off-axis tensile strength [19]. Similarly, the Chamis model has also been effectively used to predict the elastic properties of unidirectional composites [20]. This model enables the calculation of all independent effective properties, including E_{11} , E_{22} , ν_{12} , G_{12} , and G_{23} . Notably, in this model, E_{11} and ν_{12} are calculated in the same formula as ROM, while the other moduli's volume fraction (V_f) is replaced by its square root [6]. The Chamis model has also been further developed into the parallel-series model to compute moduli under more complex conditions, such as fiber undulations [21]. Overall, compared to other theoretical models, the Chamis model may provide better accuracy in its predictions [17].

In computational mechanics, Representative Volume Element (RVE) generation through the finite element method (FEM) is the most straightforward technique for com-

puting the mechanical properties of composite materials, facilitating understanding of the materials' behavior from the micro to macro scale [22]. Previously, significant research and extensive investigations have been conducted on RVE to enhance the accuracy of numerical outcomes compared to theoretical and experimental results [23]. During numerical investigations, factors such as fiber volume fraction, fiber diameter, fiber shape, fiber aspect ratio, and fiber overlapping may significantly influence the moduli of unidirectional composites [22,24,25]. Focusing on fiber volume fractions, substantial research has been conducted. Increasing volume fractions may increase the RVE size, which impacts the elastic moduli and damage initiation strength [26]. In some previous investigations, both real experiments and FEM analyses have suggested that the optimal fiber volume fraction ranges between 0.1 and 0.7 [1,27,28], since excessive volume fractions may lead to losing ductility in final components. In terms of fiber shape in RVEs, a comparison between hexagonal and square RVEs employing ANSYS and theoretical methods has been conducted in this study to calculate the thermal conductivity of a unidirectional composite. The outcomes suggest that while other moduli are unaffected by fiber shape, the hexagonal array aligns more closely with the Hashin model than the ROM and Chawla models [29].

Micro-voids are a type of defect often observed during composite manufacturing or throughout their service life [30]. In RVE generations, micro-voids can significantly influence the elastic modulus. This research considers two types of voids: inter-fiber voids and matrix voids. The investigation found that in RVE composites, void content ranging from 1% to 5% may reduce the elastic moduli, particularly affecting transverse strength [31]. Later on, a similar approach considering different types of micro-voids (circular, elliptical, and arbitrary) in RVEs, along with additional factors such as thermal residual stress, is explored in this research. The investigation found that different voids can influence the properties of the RVE. However, these voids are primarily responsible for creating premature cracks around them in the composites [32]. In addition to the effect of micro-voids, recent research has also emphasized the influence of interface defects and uncertainties on the homogenized elastic and thermal properties of periodic fiber-reinforced composites [33]. The study utilized stochastic finite element methods (SFEMs) and Monte Carlo simulations to analyze how uncertainties propagate during homogenization. Furthermore, Artificial Neural Networks (ANNs) were employed to optimize the fitting of response polynomials that relate input uncertainties—such as defect size and quantity—to the effective material properties. The findings indicated that interface defects significantly impact both the homogenized elastic moduli and thermal conductivity.

In recent times, ML has been extensively used as an important tool to predict the mechanical properties of composites, the effect of manufacturing processes, and to identify damage and structural health monitoring [34]. For instance, three different ML models, namely, Linear, Ensemble, and ANN models, are trained to predict the transverse modulus of UD composite. It was found that the Ensemble model can predict the modulus with an R^2 score of 1 [35]. Another study based on computer-generated microstructures of UD and the corresponding FE analyses suggested that the Machine Learning Algorithm can be an effective tool to predict the elastic moduli with less time and effort. Though the example is presented based on finite element data, it is suggested that the same can be applied based on experiments too [36]. A mean absolute percentage error (MAPE) is used to calculate the error for effective elastic properties of UD composites using the Convolution Neural Network (CNN) model. The fiber volume fraction is varied, and for each fiber volume fraction, elastic properties are predicted, and the error is calculated [37]. Despite the low error percentage (>3%), the problem in the study is that each volume fraction is studied separately, whereas it could be studied together to calculate the elastic properties. Recent research optimized the backpropagation neural network model to accurately predict the

elastic properties of three different UD composites: basalt fiber, carbon fiber, and glass fiber with epoxy. The error percentage is less than 2%; however, the study's main limitation is that the variety of fiber is limited to only three [38].

In summary, there is a lack of literature considering a wide range of fiber and matrix variations to predict the elastic properties of various UD composites combined using ML models. The present study addresses this issue and considers 20 different fiber and matrix combinations of various glass fibers, carbon fibers, natural fibers, etc., to predict the elastic properties of UD using a feedforward deep neural network model. The presentation of the research article is as follows: Section 2 describes the mechanical properties found in the literature, along with numerical setup and validation. Next, in Section 3, the FNN architecture is defined with the data collection strategy for the present study. Finally, Section 4 proposes an optimized FNN model after statistical evaluation.

2. Materials and Methods

2.1. Materials

To predict the elastic moduli of unidirectional composites, various materials have been selected. The fibers include both isotropic and orthotropic types, while the matrices used in this research are isotropic. The selected fibers include polymeric, carbon, glass, and natural fibers. Tables 1 and 2 present the material properties of all the fibers and matrices. Please note that in Table 1, several epoxy matrices with different elastic moduli are shown. Epoxy matrices from different literature sources differ in formulation and curing processes, leading to variation in reported elastic moduli. The exact modulus used for simulation in each composite material is taken directly from its corresponding reference, as indicated.

Table 1. Material properties of unidirectional fibers and matrices (Isotropic).

Sl No.	Composites Combinations	E ₁₁ (GPa)	ν	References
1	Glass-fiber	72	0.2	[39]
	Epoxy-matrix	3.5	0.35	
2	Jute-fiber	20	0.38	[40]
	Epoxy-matrix	5.35	0.25	
3	Sisal-fiber	22	0.32	[41]
	Epoxy-matrix	5.35	0.25	
4	UHMWPE-fiber	25	0.2	[41]
	Polypropylene	1.325	0.43	
5	Hemp-fiber	70	0.4	[42]
	Epoxy-matrix	3.78	0.35	
6	Flax-fiber	27.6	0.45	[42]
	Epoxy-matrix	3.78	0.35	
7	Sisal-fiber	22	0.32	[42]
	Epoxy-matrix	3.78	0.35	
8	Alfa-fiber	19.4	0.34	[42]
	Epoxy-matrix	3.78	0.35	
9	Glass-fiber	73	0.20	[43]
	Polypropylene-matrix	1.308	0.43	
10	Carbon-fiber	23.34	0.25	[32]
	Epoxy-matrix	3.45	0.35	
11	E-glass-fiber	80	0.2	[44]
	Epoxy-matrix	3.35	0.35	
12	T800-carbon-fiber	294	0.3	[45]
	Epoxy-matrix	3.4	0.4	
13	NICALON™-fiber	193	0.22	[46]
	MAS-5	129	0.22	

Table 2. Material properties of unidirectional fibers and matrices (Orthotropic).

Sl No.	Composites Combinations	E_{11} (GPa)	$E_{22} = E_{33}$ (GPa)	$G_{12} = G_{31}$ (GPa)	G_{23} (GPa)	ν_{23}	$\nu_{12} = \nu_{13}$	References
1	IM7-carbon-fiber 8552-epoxy	287 4.08	13.40	23.8	7	0.48	0.29 0.38	[47]
2	AS4-carbon-fiber 3501-epoxy	234 4.2	15	15	7		0.2 0.34	[48]
3	Carbon-fiber Epoxy-matrix	245 3.73	19.8 1.351	29.191	5.922		0.28 0.38	[49]
4	T300-fiber 7901-matrix	230 3.17	15 1.17	15		0.2	0.2 0.35	
5	Aramid-fiber Epoxy-matrix	124.1 3.28	4.1 1.21	2.9		0.35	0.35 0.36	[19]
6	Flax-fiber GE-7118 A-matrix	41.7 3.05	7	3	2	0.75	0.3 0.3	[24]
7	Carbon-fiber Epoxy-matrix	228 3.6	17.2	27.6	5.73	0.5	0.2 0.39	[50]

2.2. Numerical Setup and Validation

The present study employs the ANSYS Material Designer module to calculate the elastic moduli of 20 different combinations of fibers and matrices of a fiber volume fraction ranging from 0.2 to 0.7, providing sufficient data for the Artificial Neural Network (ANN) model analyzed later in this research. One of the module's primary advantages is its ability to automatically generate the Randomized Volume Element (RVE) for UD composites once the user inputs the mechanical properties of the fiber and matrix, along with the fiber diameter and volume fraction. In addition, the geometry type, which is rectangular for the UD composite, must be selected. Following this, the meshing of the element is performed using a block and conformal meshing approach, which is recommended for UD composites [51]. Additionally, a mesh convergence study was conducted to determine the optimal element size for this research, focusing on a glass fiber and epoxy matrix with a fiber volume fraction of 0.5, as detailed in Table 3. The table indicates that variations in the maximum element size do not impact the elastic modulus. However, a slight change is observed in the shear modulus when the element size is decreased from 3 μm to 2 μm . Beyond this point, further reductions in mesh size do not yield any noticeable changes. Therefore, a mesh size of 2 μm has been selected for the numerical calculations in this study. Figure 1a displays a Scanning Electron Micrograph (SEM) of the interface of a unidirectional composite with square-shaped fibers [52]. In Figure 1b, a unidirectional composite RVE is shown. Similarly, in Figure 1c, a meshed unidirectional composite RVE is presented. Since the RVE is designed to represent a repeating structure, periodicity is in composites. Therefore, a periodic boundary condition is applied to avoid any boundary effects. Figure 2a illustrates a schematic representation of the periodic boundary condition applied to the RVE [53], while Figure 2b illustrates the step-by-step setup for calculating the elastic moduli in the ANSYS Material Design Module.

Table 3. Mesh convergence study.

Maximum Mesh Size (μm)	E_{11} (GPa)	$E_{22} = E_{33}$ (GPa)	$G_{12} = G_{31}$ (GPa)	G_{23} (GPa)	ν_{23}	$\nu_{12} = \nu_{13}$
5	37.7	11.4	3.58	2.74	0.31	0.26
4	37.7	11.4	3.58	2.74	0.31	0.26
3	37.7	11.4	3.58	2.74	0.31	0.26
2	37.7	11.3	3.56	2.69	0.32	0.26
1	37.7	11.3	3.56	2.69	0.32	0.26
0.5	37.7	11.3	3.56	2.69	0.32	0.26

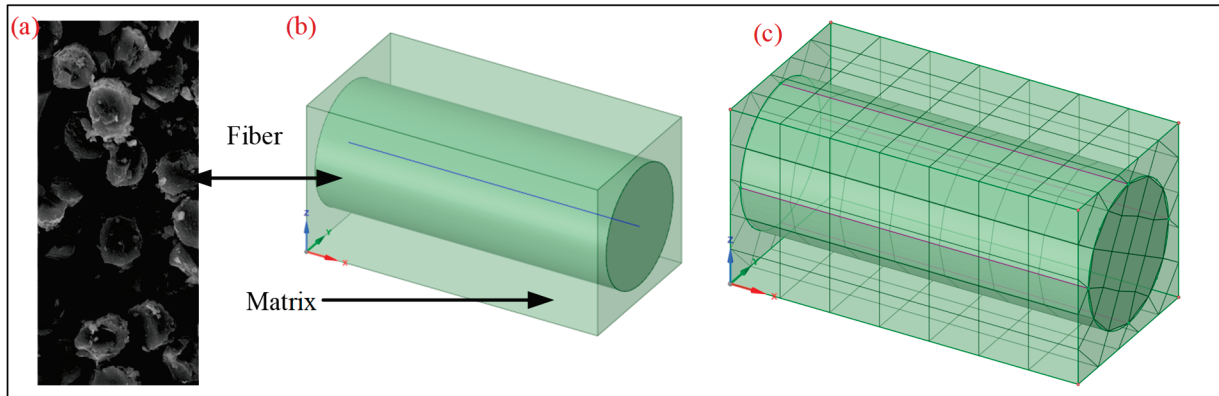


Figure 1. (a) SEM interface of a unidirectional composite, (b) RVE of a unidirectional composite, (c) block and conformal meshed RVE.

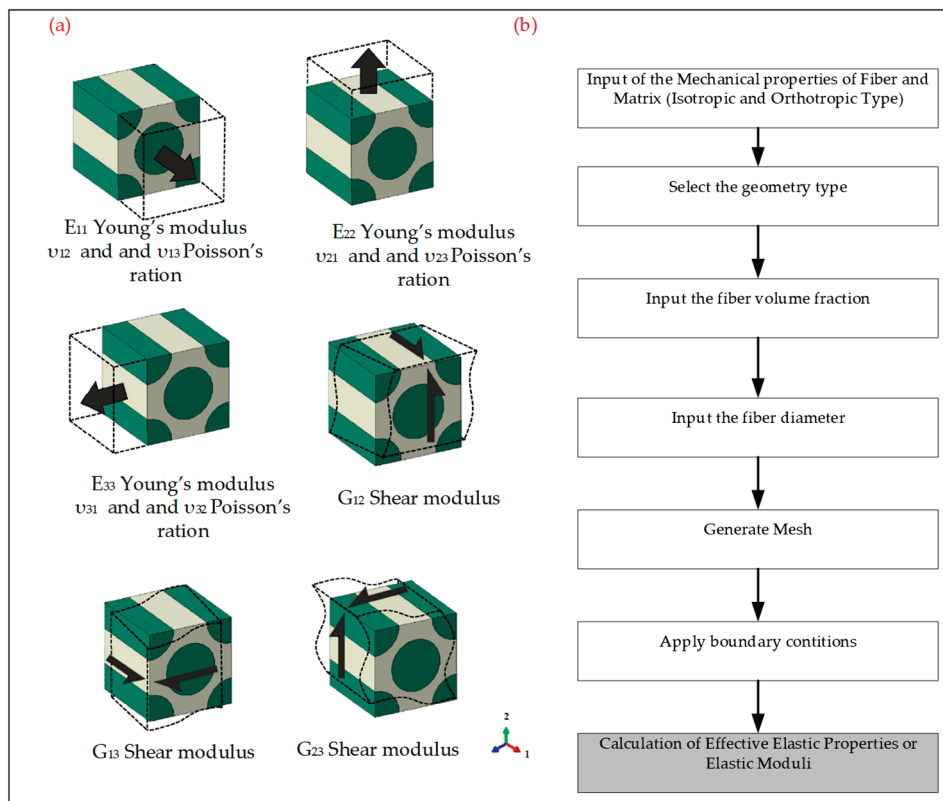


Figure 2. (a) ANSYS Material Designer setup to calculate the elastic moduli, (b) periodic boundary conditions applied to RVE.

To ensure the reliability of our research and simulations, we conducted a numerical, experimental, and theoretical comparison with the findings from the literature [6]. The numerical validation against the experimental results is presented in Table 4. The material properties of the carbon fiber and the epoxy matrix were obtained from the same source. Overall, the ANSYS Design Module demonstrated good agreement with the experimental results, especially for the transverse elastic moduli ($E_{22} = E_{33}$) and Poisson's ratio (ν_{12}), with deviations remaining minimal (between 0% and 1.67%). The longitudinal modulus (E_{11}) showed a low deviation of 1.59% at a fiber volume fraction (V_f) of 0.5 but a slightly higher deviation of 6.57% at $V_f = 0.6$. The shear moduli (G_{12} , G_{31} , G_{23}) were also reasonably predicted, although a higher deviation, reaching up to 13.5%, was noted at higher fiber content. In summary, these results indicate that ANSYS provides reliable predictions for the elastic properties of unidirectional composites, with minor variations considered acceptable.

for numerical simulations. Lastly, for ν_{12} , due to the lack of experimental data, we used the well-known micromechanical model M-T for comparison.

Table 4. Experimental and numerical elastic modulus of carbon–epoxy unidirectional composite.

Elastic Modulus	V_f	Experimental [6]	Ansys	Deviation (%)	Elastic Modulus	V_f	Experimental [6]	Ansys	Deviation (%)
E_{11} (GPa)	0.5	116	114.16	1.59%	E_{11} (GPa)	0.6	148.2	138.45	6.57%
$E_{22} = E_{33}$ (GPa)		9.28	9.27	0.01%	$E_{22} = E_{33}$ (GPa)		10.14	10.31	1.67%
$G_{12} = G_{31}$ (GPa)		4.50	4.94	9.7%	G_{12} (GPa)		6.75	6.34	6.07%
G_{23} (GPa)		3.54	3.82	7.9%	$G_{13} = G_{23}$ (GPa)		4.22	4.79	13.50%
Elastic modulus ν_{12}	V_f	M-T	Ansys	Deviation (%)	Elastic modulus ν_{12}	V_f	M-T	Ansys	Deviation (%)
	0.5	0.32	0.32	0%		0.6	0.30	0.30	0%

3. FNN Architecture and Data Collection Strategy

The Feedforward Neural Network, or in short “FNN”, is a subcategory of the ANN where data are fed as input, processed through one or more hidden layers, and propagated to the output. As the name suggests, data flow only in one direction, and feedback from the output does not reconnect or loop back through the layers to the input. These layers are made up of neurons, which are the building blocks of neural networks, and are connected to each other through synaptic weights. The weighted sum of the input data is processed through an activation function, generating an output that is then sent as input to the connected neurons [54]. To capture the complex behavior and pattern of the datasets, the neural network utilizes a learning algorithm that adjusts the synaptic weights based on the calculated error between the predicted and actual outputs.

The type of error function or cost function depends on the specific task, for instance, cross-entropy for classification or mean squared error (MSE) for regression. The most common learning algorithm for FNNs is backpropagation combined with gradient descent (such as Adam) [55]. During training, the model iterates through the data multiple times, each iteration called an epoch, in which the weights are updated to minimize the cost function. Based on the number of hidden layers, an FNN can be further categorized as a shallow neural network (with one hidden layer) or a deep FNN (with two or more hidden layers) [56], as shown in Figure 3.

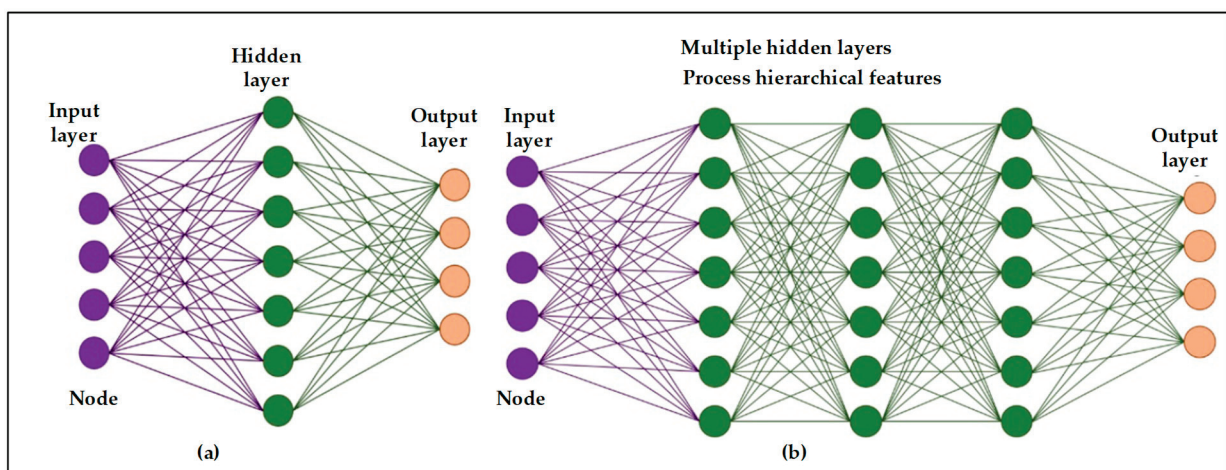


Figure 3. Neural network: (a) shallow neural network with a single hidden layer, (b) deep neural network with multiple hidden layers [56].

To improve the learning capability of neural network models, hyperparameters are configured before the training process begins. These hyperparameters have a significant effect on the learning process, with some being adopted by default within the training model, while others require explicit implementation. A set of hyperparameters used for FNN models is shown in Table 5, along with the ranges studied for the current research.

Table 5. Hyperparameters for FNN.

Hyperparameter	Description	Values Studied for the Current Research
Network Architecture	The network architecture mainly defines the number of hidden layers and neurons assigned to each layer.	66/33/16/8/4/4/1, 33/16/8/4/2/1, 33/16/8/4/1, 16/8/4/4/1, 16/4/1 [Numbers represent the neuron count in each layer, with the sequence representing the input layer, hidden layers, and output layer]
Activation Function	Activation functions in neural networks transform the input sum into a specific range, such as (0,1) for Sigmoid, $(-1,1)$ for Tanh, $(0, \infty)$ for ReLU, and probabilities summing to 1 for Softmax, enabling the network to learn complex patterns.	Softplus, Softmax, ReLU, Sigmoid, Linear [The output layer activation function is Linear for regression tasks]
Cost Function	The cost function measures the difference between predicted and actual outcomes.	For regression-based task, MSE is set as a cost function
Optimizer	An optimizer is an algorithm that updates the weight of the model to minimize the cost function and improve learning. Most popular optimizers, such as Adam, use an adaptive learning rate that adjusts over time, helping the model to train faster with higher accuracy.	Adam, Nadam, AdamW
Learning Rate	The learning rate controls the size of the steps the model takes when adjusting its weights during training to minimize the cost function. If the learning rate is too low, the model will take a longer time to train, which is computationally expensive. On the other hand, if the learning rate is too large, the model will train faster; however, it will lose the ability to recognize patterns and make predictions for complex datasets.	Dynamic value, adjusted by optimizer
Batch Size	The number of batch sizes determines how many subsets the training data are divided into for processing. For instance, if the batch size is set to 20 and the training dataset has 700 samples, the number of batches will be 35. In other words, the dataset will be divided into 35 batches, each containing 20 training samples.	4, 8, 12, 16, 32
Epoch Number	The number of epochs defines how many times the model will process the samples by updating the weights to minimize the cost function. For instance, if the number of batches is 35, then the number of iterations will also be 35, corresponding to one epoch.	100, 200, 300, 400, 500

The present research utilizes a dataset of 1948 samples to predict the effective elastic properties of unidirectional composite materials, namely, elastic moduli E_{11} and $E_{22} = E_3$, Shear moduli $G_{12} = G_{31}$ and G_{23} , and Poisson's ratios ν_{23} and $\nu_{12} = \nu_{13}$. All of them are used as a target variable and excluded from the dataset while training. In contrast, the name of the composites, Young's moduli and Poisson's ratio of fiber and matrix, along with the fiber volume fraction, are utilized as input variables for the training. Since the names of the composites represent categorical data, a one-hot encoding is required to transform them to numerical data [57]. The fiber volume fraction ranges from 0.2 to 0.7, with an average of 98 data points collected within this range for each composite, resulting in a total dataset of 1948 samples. The training data and test data ratio is 80:20.

4. Evaluation of FNN Model

After tuning the hyperparameters, it was found that the relu activation function in the hidden layers and a linear activation in the output layer, combined with the AdamW optimizer, performed best with a hidden layer architecture of 33/16/8/4/1. The optimal batch size was 32, and the number of epochs was 400. Notably, these parameters yielded the highest accuracy in terms of MAE, MSE, RMSE, and R^2 score, which are the core evaluation metrics for predictive analysis. As previously discussed, an adaptive learning rate was maintained, as it proved suitable for the present predictions. The results of the optimized parameters are summarized in Table 6.

Table 6. Optimized parameters.

Name of the Hyperparameters	Ranges
Activation Function	relu
Optimizer	Final hidden layer to output layer: Linear activation AdamW
Number of Batches	32
Hidden Layer Architecture	33/16/8/4/1
Number of Epochs	400

To examine the performance of the FNN model, at first, MAE is utilized, which describes the average magnitude of errors between the predicted and actual values of a dataset. The expression is given as follows [58]:

$$MAE = \frac{1}{n} \sum_{i=1}^n |y_i - \hat{y}_i| \quad (1)$$

where y_i is the actual value, $\hat{y}_i$ is the predicted value, and n is the total number of observations. MAE is a non-negative value, and if the value is 0, then it suggests that no error is found. A value closer to 0 indicates the predicted values are closer to actual values and the prediction is fairly accurate.

For the case of E_{11} prediction, it is found that the value of MAE is 0.09, which suggests that the FNN model can predict the elastic moduli of composite materials with high accuracy, as shown in Table 6. It is important to note that the calculated elastic moduli of different composite materials vary from 3.69 GPa to 190.42 GPa, representing a good variation in magnitude. Despite that, the predictions made by the FNN model are excellent. Similar to E_{11} prediction, $E_{22} = E_{23}$, $G_{12} = G_{31}$, and G_{23} also suggest a very good accuracy with MAE values of 0.0177, 0.0263, and 0.0368, respectively, despite having large magnitude variations, to be exact, 1.7848 GPa to 170.74 GPa for $E_{22} = E_{23}$, 0.55 GPa to 70 GPa for $G_{12} = G_{31}$, and 0.53 GPa to 69.18 GPa for G_{23} . However, in the case of Poisson's ratio, the model outperforms all other predictions and represents an excellent accuracy, with a value

of 0.0002 for $V_{12} = V_{13}$ and 0.0023 for V_{23} . This exceptional observation is mainly due to the fact that the Poisson's ratio variation is relatively small and ranges between 0.2 and 0.45, which helps the FNN model to predict the values with negligible error.

MSE is another statistical parameter that defines the predictive accuracy while penalizing the large errors. The formula to calculate the MSE is as follows [58]:

$$MSE = \frac{1}{n} \sum_{i=1}^n (y_i - \hat{y}_i)^2 \quad (2)$$

Similar to the previous observation, the values of MSE for all cases suggest an excellent accuracy, as shown in Table 7. Notably, for the present observation, MAE values are higher than the MSE values, suggesting that the errors are less than 1 in magnitude for most cases, which leads to this outcome. A similar observation can be found in the literature [58]. Furthermore, RMSE values, which are the root square of MSE, scale the error back to the original unit, which is widely used as an evaluation matrix alongside MAE and MSE [58]. The values of RMSE for test data, as shown in Table 7, suggest that the FNN can predict the elastic moduli with high accuracy. Finally, the coefficient of determination, which is denoted as an R^2 score, can range from 0 to 1, where 0 represents that the model completely fails to capture any variance during testing and 1 suggests that the model can explain the variance with full perfection. The R^2 score is formulated as follows [59]:

$$R^2 = 1 - \frac{\sum (y_i - \hat{y}_i)^2}{\sum (y_i - \bar{y})^2} \quad (3)$$

Table 7. The results of the evaluation metrics.

Effective Elastic Properties	MAE	MSE	RMSE	R^2 Score
E_{11} (GPa)	0.0927	0.0177	0.1332	0.9999
$E_{22} = E_{23}$ (GPa)	0.0177	0.0078	0.0177	0.9999
$G_{12} = G_{31}$ (GPa)	0.0263	0.0016	0.0403	0.9999
G_{23} (GPa)	0.0368	0.0026	0.0514	0.9999
$V_{12} = V_{13}$	0.0002	1.06×10^{-7}	0.0003	0.9999
V_{23}	0.0023	1.01×10^{-5}	0.0031	0.9999

For the present elastic moduli predictions, it is found that the optimized FNN model in the target variables can capture the variance with full perfection, as shown in Table 7.

Next, the results are plotted for training loss and validation loss over a number of epochs during the elastic moduli training and testing, as shown in Figure 4. Training loss represents the learning capability of the model based on the loss function (MSE) with the increment of the epoch numbers, while validation loss calculates the MSE on the unseen data or test data. As shown in the figure, it is apparent that the model starts with high MSE for training and validation loss, and just after running a few epochs (5 to 10), the MSE values significantly drop and start to converge. It also suggests that during the training, the model learns very quickly and improves its performance over time (number of epochs). The same is true for the unseen data, as the model performs well, and the validation loss also starts to decrease sharply. It might seem that 400 epochs are a little more than the required number of epochs for the present training and testing. However, it sometimes takes a high number of epochs to converge fully with the least MSE values for training loss and validation loss.

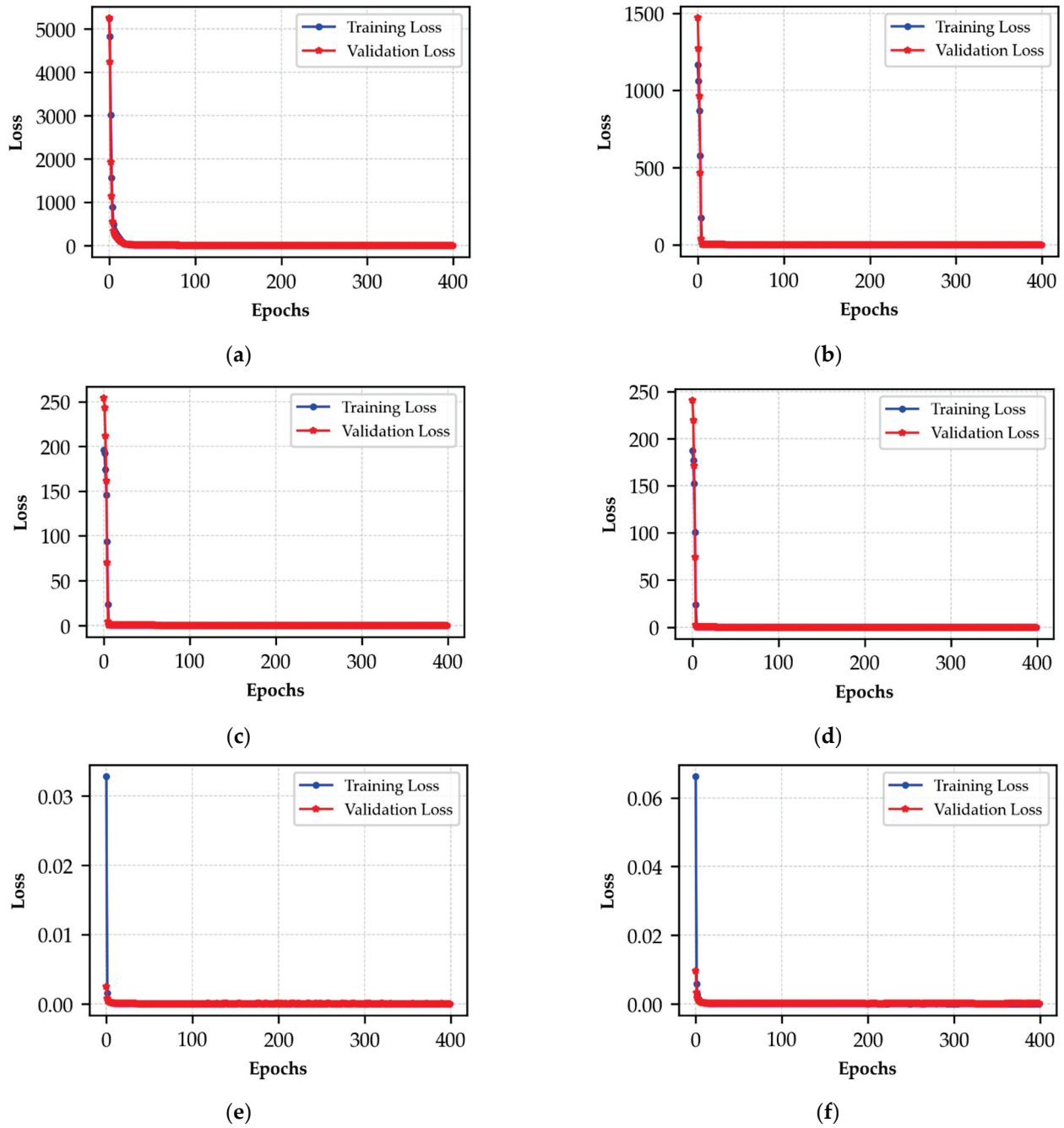


Figure 4. Training loss and validation loss vs. epochs: (a) E_{11} , (b) $E_{22} = E_{23}$, (c) $G_{12} = G_{31}$, (d) G_{23} , (e) $V_{12} = V_{13}$, (f) V_{23} .

Finally, the results are plotted based on the predicted data vs. actual data to visualize the accuracy of the predicted data during training and testing for all elastic properties, as shown in Figure 5. This figure apparently helps to identify whether the optimized FNN model is overfitting, underfitting, or generalizing well to unseen data. In case the model had overfitted, the model would align well with the perfect fit line during training; however, it would fail to follow the trend during testing. In contrast, underfitting would suggest that, both during training and testing, the model's prediction is scattered and away from the perfect line. However, for the present case, both during training and testing, predicted values vs. actual values closely follow the perfect fit for all elastic properties. This suggests that the model performs well on seen and unseen data, and it can accurately

predict Young's moduli [E_{11} , and $E_{22} = E_{23}$], shear moduli [$G_{12} = G_{31}$, and G_{23}], and Poisson's ratios [$V_{12} = V_{13}$ and V_{23}].

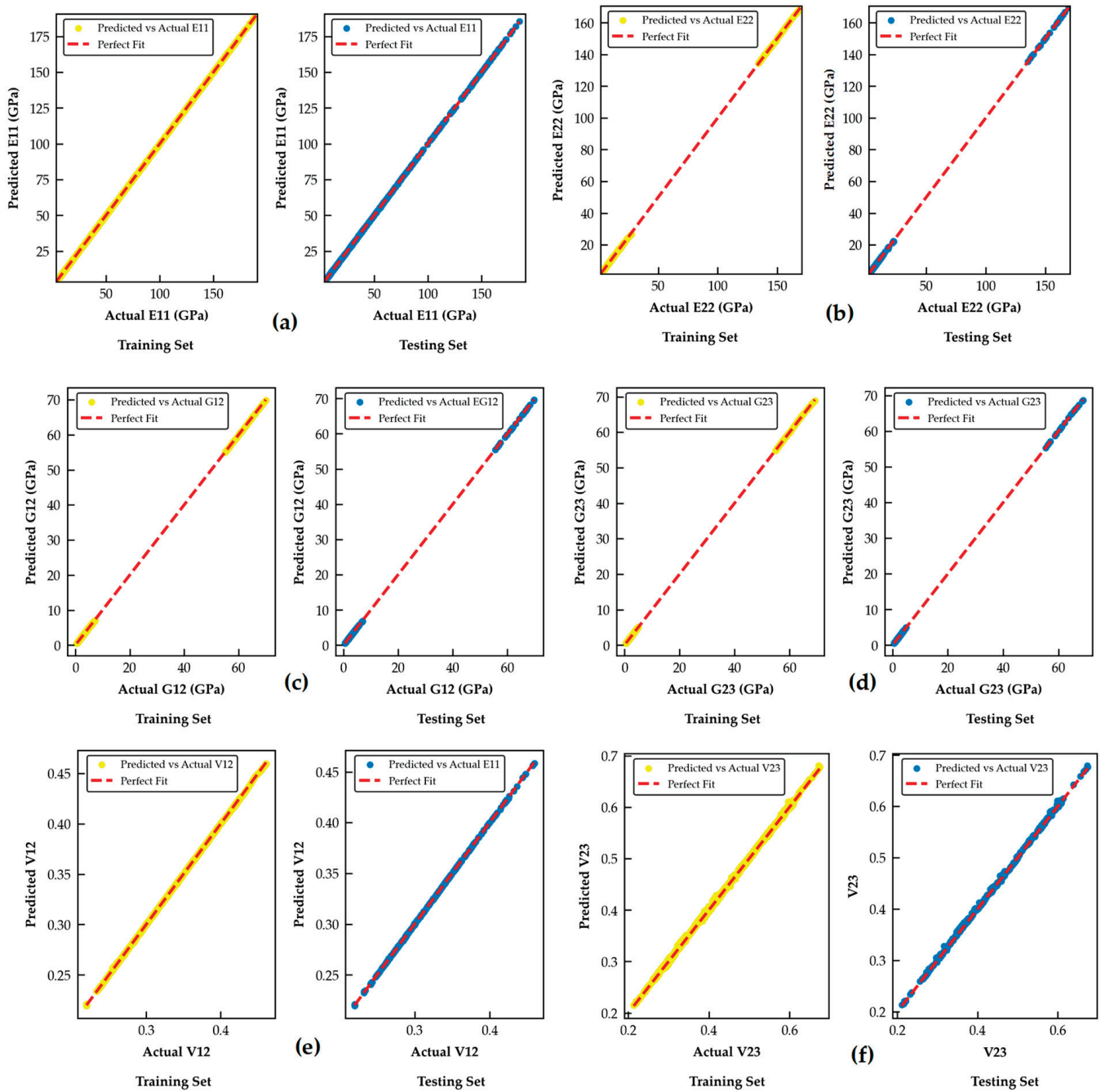


Figure 5. Predicted data vs. actual data during training and testing: (a) E_{11} , (b) $E_{22} = E_{23}$, (c) $G_{12} = G_{31}$, (d) G_{23} , (e) $V_{12} = V_{13}$, (f) V_{23} .

5. Limitations

While the developed Feedforward Neural Network (FNN) model demonstrates excellent predictive performance for the elastic properties of unidirectional (UD) composite materials, several limitations must be acknowledged.

First, the mechanical parameters were determined based on numerical simulations at the microscale using Representative Volume Elements (RVEs) within ANSYS Material Designer. These RVEs represent idealized conditions and do not account for large-scale manufacturing defects, such as voids, resin-rich zones, fiber misalignments, or interface

debonding. Consequently, while the model effectively captures the stiffness properties in a controlled microstructural context, applying these results directly to larger-scale structures may lead to discrepancies due to real-world imperfections.

Second, the model considers only three input parameters: Young's moduli and Poisson's ratios of the fiber and matrix, as well as the fiber volume fraction. It does not include strength properties essential for analyzing failure and damage tolerance. Additionally, the study solely focuses on unidirectional fiber orientations and does not address layup configurations like cross-ply, angle-ply, or woven fiber arrangements. This limitation restricts the model's applicability to more complex laminate designs.

Third, although a diverse range of composite materials—including glass, carbon, natural, and polymeric fibers—was considered, they were grouped into a single dataset. This approach introduces variability that may obscure subtle trends specific to each fiber–matrix composite material. While this grouping is intended to improve the model's generalizability, it may compromise accuracy for particular composites, e.g., unidirectional metal matrix composite. Future iterations of the model could benefit from subgroup classification or hierarchical modeling.

Fourth, while the input dataset includes the bulk properties of the matrix and fiber, it does not consider curing and manufacturing conditions (such as temperature, pressure, and resin infiltration quality), which are known to significantly impact the mechanical behavior of composites. The omission of these factors stems from inconsistent reporting across literature sources, but it limits the model's ability to reflect processing-related variability.

Lastly, the model is trained exclusively on elastic properties. It does not account for time-dependent behaviors (such as viscoelasticity or creep), environmental degradation, or the impact of aging. Incorporating these effects would require more complex data collection and model structures, potentially involving hybrid experimental–numerical approaches and multi-scale modeling.

These limitations outline the model's current applicability boundaries and suggest directions for further research to enhance its physical realism and practical usability in composite design and certification.

6. Conclusions

This study implemented and evaluated an FNN model to predict the elastic properties of twenty different unidirectional composite materials. The optimized FNN model, employing a ReLU activation function in the hidden layers, a linear activation in the output layer, and the AdamW optimizer with an architecture of 33/16/8/4/1, exhibited excellent predictive performance across various evaluation metrics, including MAE, MSE, RMSE, and R^2 . Notably, the model achieved high accuracy in predicting Young's moduli (E_{11} , $E_{22} = E_{23}$), shear moduli ($G_{12} = G_{31}$, G_{23}), and Poisson's ratios ($\nu_{12} = \nu_{13}$, ν_{23}), despite the dataset covering a wide range of magnitudes. The evaluation metrics confirm the model's robustness, as indicated by low MAE, MSE, and RMSE values and an R^2 score of 1 for all predicted properties. Convergence analysis of training and validation loss over multiple epochs further validated the model's learning capability, demonstrating an efficient optimization process. Additionally, the comparison between predicted and actual values indicates that the model generalizes well, showing no signs of overfitting or underfitting.

Overall, the results suggest that the FNN model is an efficient tool for predicting the elastic moduli of UD composite materials. This study demonstrated that if trained with the required dataset, it can generalize to unknown UD composites with minimal input details. Future research will focus on developing a comprehensive database to optimize the model for tailoring UD composite properties to specific requirements.

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Article

Fracture Mechanics-Based Modelling of Post-Installed Adhesive FRP Composite Anchors in Structural Concrete Applications

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Abstract: Adhesively bonded fibre-reinforced polymer (FRP) anchors have emerged as a progressive alternative to traditional steel anchors in concrete structures, owing to their superior corrosion resistance, high tensile strength, and light weight. Despite their increasing use, a robust mechanics-based bond model capable of accurately predicting the load transfer behaviour has not yet been developed. This study presents a fracture mechanics-based analytical bond model for post-installed adhesive FRP anchors embedded in concrete. The model formulation is derived from fundamental equilibrium and compatibility principles, incorporating a bilinear bond–slip law that captures both elastic and softening behaviours. A new expression for the effective bond length is also proposed. Validation of the model against a comprehensive database of direct pull-out tests reported in the literature shows excellent agreement between predicted and experimental pull-out forces ($R^2 = 0.980$; $\text{CoV} = 0.058$). Future research should aim to extend the proposed model to account for confinement effects, long-term durability, the impact of adhesive type, and cyclic loading conditions.

Keywords: adhesive anchors; bond–slip behaviour; fracture mechanics; pull-out strength; FRP composites; post-installed reinforcement; concrete anchors; bond model; ETS FRP

1. Introduction

Over the past decades, fibre-reinforced polymer (FRP) composite anchors have gained increasing attention in structural applications due to their high strength-to-weight ratio, corrosion resistance, and electrical non-conductivity [1–5]. These advantages make FRP anchors particularly suitable for modern construction needs, including: (1) connection of prefabricated FRP or steel elements to existing concrete structures [1]; (2) load transfer across new-to-old concrete joints [2]; and (3) shear strengthening of reinforced concrete (RC) members using embedded through-section (ETS) methods (Figure 1a,b) [3–5].

Although FRP anchors are emerging as an attractive alternative to steel-based systems, much of the existing anchorage design knowledge stems from earlier work on cast-in-place and post-installed mechanical anchors made of steel [6–8]. These systems are now well-established in major building codes and design standards such as ACI 318-25 [9], ACI 355.4-19(21) [10], CSA A23.3:24 [11], Caltrans 2020 [12], and IBC 2024 [13]. In structural practice, advanced composite post-installed adhesive anchors, play a key role in diverse applications such as (1) anchoring base plates on RC pedestals [14]; (2) seismic retrofitting [15]; (3) attachment of façade systems [16]; (4) support of mechanical, electrical, and plumbing components [17]; (5) installation of safety-critical elements like guardrails

and handrails [18]; and (6) integration of new structural elements with existing concrete infrastructure [19].

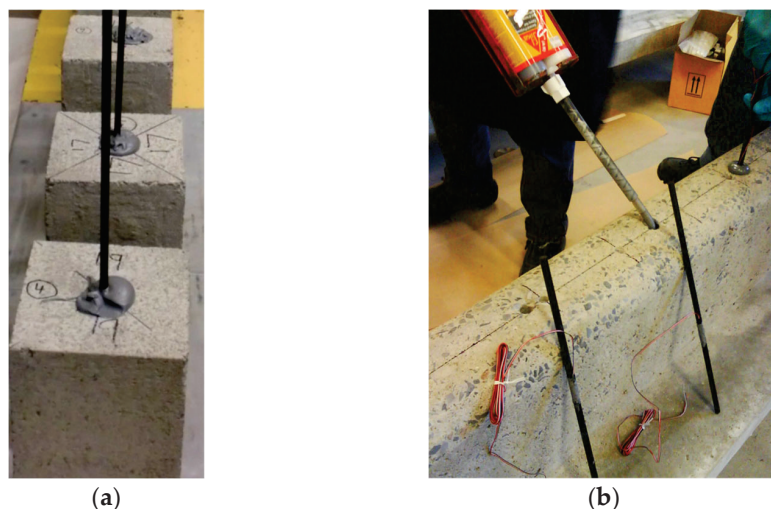


Figure 1. Post-installed anchors in the shape of (a) pull-out test specimen; (b) shear strengthening advanced composite components using the ETS method.

Peier [20] developed one of the earliest advanced bond models for post-installed adhesive steel anchors by benefiting from a nonlinear finite element approach to incorporate concrete fracture and plasticity models. The model simulated the tensile behaviour of three single anchor types: adhesive, expansion, and form-fit anchors. The model's accuracy was verified by comparing it with existing experimental data. To support the formulation of design provisions for adhesive anchors, Eligehausen et al. [21] carried out an extensive programme of numerical simulations and experimental testing focused on the behaviour of adhesively bonded steel anchor systems. This investigation led to the development of the Eligehausen, Popov, and Bertero (BPE) [21] bond-slip relationship for deformed post-installed steel anchors in concrete. In 1997, Cosenza et al. [22] redeveloped the BPE relationship for post-installed adhesive FRP anchors and proposed the Cosenza, Manfredi, and Realfonzo (CMR) [22] bond-slip relationship. However, their investigation did not suggest a relationship for the pull-out force for FRP anchors. Robinson and Said [23] used artificial neural networks (ANNs) to estimate the tensile strength of adhesive anchors in uncracked concrete, considering the influence of geometric and material parameters.

In addition, Ahmed et al. [24] conducted a comprehensive experimental study on the tensile pullout capacity of glass FRP (GFRP) post-installed adhesive anchors in concrete, evaluating the effects of bar diameter, embedment depth, adhesive type, and installation conditions. Their findings demonstrated that adequate performance was achieved with epoxy adhesives in wet conditions and with cement-based adhesives in dry conditions, particularly at embedment depths of 10–15 d_b , where d_b is the diameter of the anchor. Moreover, Valerio et al. [25] investigated the bond performance of adhesive FRP anchors in concrete through a series of direct pull-out tests. Godat et al. [26] conducted thirteen direct-shear test specimens on FRP bars adhesively bonded to concrete blocks. The results of this study revealed that by increasing the concrete strength, the bond strength increased, and the failure mode was changed. Raicic et al. [27] tested RC push-off specimens strengthened with anchored FRP and steel bars across a shear plane to assess the shear capacity and the failure mechanism of the test specimens. Caro et al. [28] tested the bond behaviour of GFRP and carbon FRP (CFRP) bars adhesively anchored into concrete prisms. It was seen that the bond strength and initial stiffness of the bond stress-slip curves decreased with the increase in the embedded lengths. Moreover, Caro et al. [28] proposed an analytical

model for predicting the bond strength of FRP bar anchors and validated the model against experimental results. In addition, Bui et al. [29] conducted experimental and numerical studies on RC beams with steel and GFRP ETS bars using mechanical anchorage at the tension ends. In their study, the effectiveness of their developed bond stress-slip model was demonstrated with the results obtained from finite element modelling of the tested joints. Sogut [30] conducted two-dimensional nonlinear finite element analysis of RC beams strengthened with embedded steel bars in the absence of conventional transverse shear reinforcement. The results indicated that increasing the beam width led to a reduction in the relative shear strength enhancement provided by the ETS bars. Additionally, the study showed that larger beam dimensions corresponded with lower shear stress at failure in both unstrengthened and ETS-strengthened beams. Dutta et al. [31] also employed two-dimensional nonlinear finite element analysis to simulate continuous RC T-beams retrofitted with ETS bars made of either CFRP or steel. Their parametric analysis demonstrated that the predicted shear capacity of both unretrofitted and retrofitted beams improved with higher concrete compressive strength and increased effective depth. Vertical and inclined configurations of ETS bars were both considered in their models.

2. Research Significance

To address the lack of predictive models for FRP anchor systems, this study develops a mechanics-based bond model tailored for post-installed adhesive FRP anchors in concrete. The model is formulated through a rigorous analytical procedure grounded in fracture mechanics and bond-slip theory, capturing both the elastic and softening phases of interfacial behaviour. A central objective of the study is to establish a closed-form equation for predicting the pull-out force of FRP anchors based on material and geometric properties, enabling rational design beyond empirical methods. This study outlines the step-by-step development of an analytical bond model for post-installed adhesive FRP anchor bars, emphasising the methodology used to derive the key parameters within a fracture mechanics-based framework. These parameters are evaluated against the existing experimental results in the literature.

3. Bond Model

In developing the proposed bond model, several assumptions were made as follows: (1) the adherents are considered linear-elastic and homogeneous; (2) bending effects are neglected; (3) normal stresses are uniformly distributed across the FRP composite bar's cross-section; (4) the diameter, width, and thickness of the adherents and adhesive remain constant along the bond line; (5) the thickness of the failure plane provides a uniform concrete cover around the FRP bar and along its pull-out length; (6) both the adhesive and concrete possess sufficient strength to ensure that failure occurs solely by pull-out; and (7) the adhesive serves only to transfer shear stress from the FRP bar to the surrounding concrete.

Considering the anchored FRP/concrete joint shown in Figure 2a,b, the equilibrium equation can be written as follows:

$$\frac{d\sigma_b}{dx} - \frac{\tau L_{PER}}{A_b} = 0 \quad (1)$$

where σ_b and A_b are the normal stress and cross-sectional area of the FRP bar, respectively, and L_{per} , the length of the debonding failure plane is calculated as follows:

$$L_{per} = 2\pi \left(\frac{d_h}{2} + t_f \right) \quad (2)$$

where d_h is the hole diameter and t_f is the thickness of the concrete cover surrounding the FRP bar, the interface at which the shear failure takes place. This relationship can be simplified as $L_{per} = \pi d_f$, where d_f combines the d_h and t_f terms to represent the effective diameter of the failure plane.

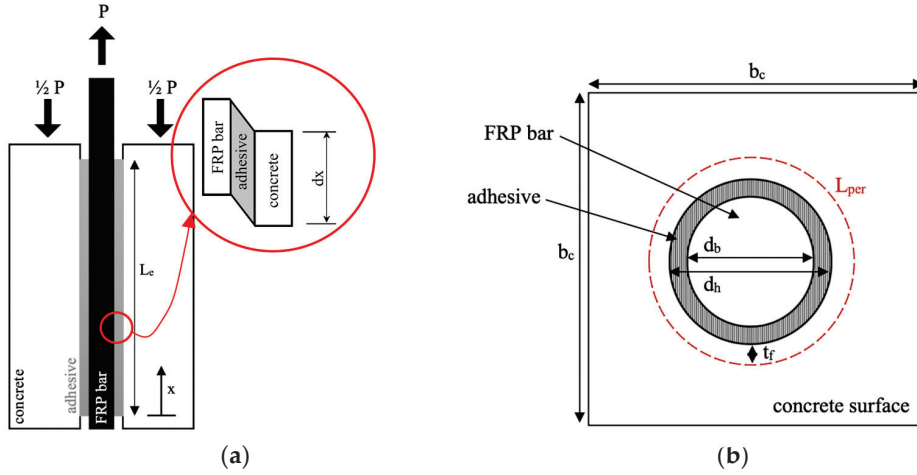


Figure 2. Post-installed adhesive anchor pull-out test specimen; (a) side view; (b) top view.

Since the adherents are assumed to be linear elastic, their elastic extension is illustrated as shown in Figure 3, and the corresponding slip is defined by Equation (3).

$$\delta = u_1 - u_2 \quad (3)$$

where u_1 and u_2 represent the individual displacements of the FRP anchor and the concrete block, respectively, as illustrated in Figure 3.

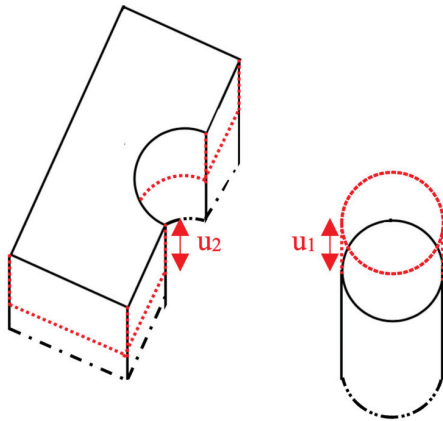


Figure 3. Elastic elongation of adherent segments.

Taking the second derivative of Equation (3) yields:

$$\frac{d^2\delta}{dx^2} = \frac{d^2u_1}{dx^2} - \frac{d^2u_2}{dx^2} \quad (4)$$

Considering Hooke's law for FRP bar ($\sigma_b = E_b \frac{du_1}{dx}$) and for concrete, ($\sigma_c = E_c \frac{du_2}{dx}$), and taking the second derivative of the displacements u_1 and u_2 , leads to Equations (5) and (6), respectively:

$$\frac{d^2u_1}{dx^2} = \frac{1}{E_b} \frac{d\sigma_b}{dx} \quad (5)$$

$$\frac{d^2 u_2}{dx^2} = \frac{1}{E_c} \frac{d\sigma_c}{dx} \quad (6)$$

By substituting Equations (1), (5), and (6) into Equation (4), Equation (7) is obtained:

$$\frac{d^2 \delta}{dx^2} - \tau \left(\frac{L_{per}}{E_b A_b} - \frac{L_{per}}{E_c A_c} \right) = 0 \quad (7)$$

Here, by defining β as shown in Equation (9) and $\tau(x)$ as presented in Equation (10), Equation (7) can be transformed to Equation (8).

$$\frac{d^2 \delta}{dx^2} - f(\delta) \beta = 0 \quad (8)$$

and

$$\beta = L_{per} \left(\frac{1}{E_b A_b} - \frac{1}{E_c A_c} \right) \quad (9)$$

$$\tau(x) = f(\delta) \quad (10)$$

Since $E_c A_c$ is significantly larger than $E_b A_b$, the last term in Equation (9) is negligible and can be omitted. Thus, Equation (9) simplifies to Equation (11):

$$\beta = \frac{L_{per}}{E_b A_b} \quad (11)$$

A double-branched shear-slip curve is adopted for the joint between the FRP anchor and the concrete (Figure 4a,b). Supporting experimental data for the adaptation of such a shear-slip curve is provided in the “Verification of Analytical Bond Model” section of this article. Stage I represents the initial elastic response of the joint, idealized as linear. Stage II captures the softening behaviour: once the joint reaches its τ_{max} at the corresponding slip value (δ_1), interfacial fracture initiates and propagates. This progression is depicted by a gradually descending linear branch, which terminates at δ_2 rather than δ_{max} as all available experimental studies concluded before the complete pull-out of the FRP bar. Therefore, the function $f(\delta)$ in Equation (10) is given by the following equations:

- Stage I, for $0 \leq \delta \leq \delta_1$

$$f(\delta) = \frac{\tau_{max}}{\delta_1} \delta \quad (12)$$

- Stage II, for $\delta_1 < \delta \leq \delta_2$

$$f(\delta) = \frac{\tau_{max}}{(\delta_2 - \delta_1)} (\delta_2 - \delta) \quad (13)$$

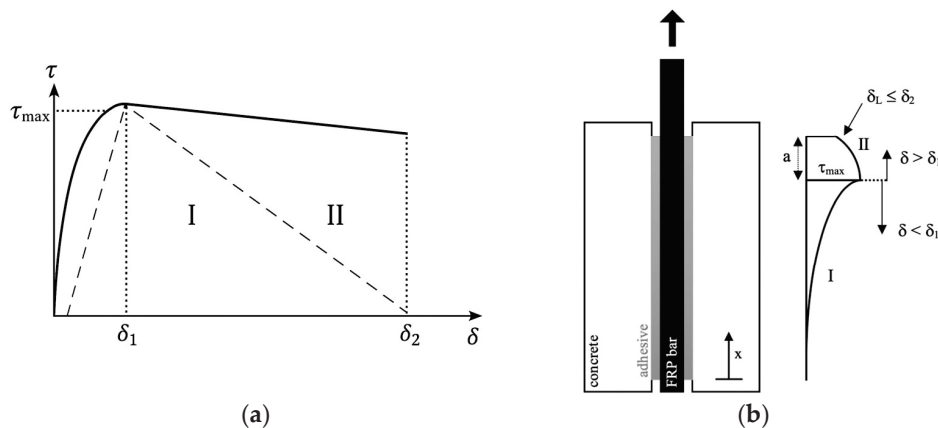


Figure 4. Proposed local bond–slip relationship: (a) curves for adhesively bonded FRP anchor–concrete joints; (b) side view of FRP bar/concrete joint when Stage I and Stage II meet.

3.1. Elastic Stage (Stage I)

By solving Equation (8) for Stage I, Equation (14) is obtained:

$$\frac{d^2\delta}{dx^2} - \frac{\tau_{max}}{\delta_1}\delta\beta = 0 \quad (14)$$

$$\frac{d^2\delta}{dx^2} - \lambda_1^2\delta = 0 \quad (15)$$

where

$$\lambda_1^2 = \beta \frac{\tau_{max}}{\delta_1} \quad (16)$$

Applying the initial boundary condition at $x = 0$ yields:

$$\delta(x) = A \sinh(\lambda_1 x) \quad (17)$$

As shown in Figure 2a, the two curves intersect at $x = L_e - a$ in δ_1 , where ‘ a ’ defines the fracture length. Therefore, $x = L_e - a$ serves as the next boundary condition:

$$\delta_i(L_e - a) = \delta_1 = A \sinh(\lambda_1(L_e - a)) \quad (18)$$

By applying Equation (12), Equation (19) is obtained:

$$\tau_i(x) = \frac{\tau_{max} \sinh(\lambda_1 x)}{\sinh(\lambda_1(L_e - a))} \quad (19)$$

By substituting Equation (19) into Equation (1), Equation (20) is obtained:

$$\sigma_{b,i}(x) = \frac{L_{per} \tau_{max}}{A_b \lambda_1} \frac{\cosh(\lambda_1 x)}{\sinh(\lambda_1(L_e - a))} \quad (20)$$

3.2. Softening Stage (Stage II)

The interfacial shear stress distribution is illustrated in Figure 1a, where Stages I and II meet at τ_{max} . At this point, the bond begins to deteriorate, initiating interfacial fracture propagation within the concrete cover over a width equal to L_{per} , as represented by the descending branch of the bond–slip curve (Stage II). As the fracture progresses, shear stress is gradually transferred to the adjacent fully bonded segment of the FRP anchor, following the Stage I curve and shifting toward the unloaded end. The effects of friction from residual aggregate interlocking and residual friction at the debonded interface are neglected. To establish the initial formulation, the softening branch is substituted into the governing differential equation:

To set up the initial problem, the softening branch must be substituted into the governing differential equation:

$$\frac{d^2\delta}{dx^2} - \frac{\tau_{max}}{(\delta_2 - \delta_1)}(\delta_2 - \delta)\beta = 0 \quad (21)$$

$$\frac{d^2\delta}{dx^2} + \lambda_2^2(\delta - \delta_2) = 0 \quad (22)$$

where

$$\lambda_2^2 = \beta \frac{\tau_{max}}{(\delta_2 - \delta_1)} \quad (23)$$

Applying the initial boundary condition yields:

$$\delta_{ii}(x) = C \cos(\lambda_2 x) + D \sin(\lambda_2 x) + \delta_2 \quad (24)$$

It should be noted that at $x = L_e - a$, where the two curves intersect, the slip and normal stress for both stages satisfy $\delta_i(L_e - a) = \delta_{ii}(L_e - a)$ and $\sigma_i(L_e - a) = \sigma_{ii}(L_e - a)$, with separate boundary conditions applied to solve the problem.

$$\delta_i(L_e - a) = \delta_{ii}(L_e - a) = \delta_1 = C \cos(\lambda_2(L_e - a)) + D \sin(\lambda_2(L_e - a)) + \delta_2 \quad (25)$$

By applying Equation (13), Equation (26) is obtained:

$$\tau_{ii}(x) = \frac{\tau_{max}}{(\delta_2 - \delta_1)} [\delta_2 - (C \cos(\lambda_2(x)) + D \sin(\lambda_2(x)) + \delta_2)], \quad (26)$$

By substituting Equation (26) into Equation (1), Equation (27) is obtained:

$$\sigma_{b,ii}(x) = \frac{\tau_{max} L_{per}}{A_b(\delta_2 - \delta_1)} \left[\frac{-C}{\lambda_2} \sin(\lambda_2(x)) + \frac{D}{\lambda_2} (\cos(\lambda_2(x))) \right] \quad (27)$$

From $\sigma_i(L_e - a) = \sigma_{ii}(L_e - a)$, the second equation is obtained. Solving the simultaneous equations yields the final expression:

$$\sigma_{b,ii}(x) = \frac{L_{per} \tau_{max}}{A_b \lambda_2} \left(\sin(\lambda_2(x - L_e + a)) + \frac{\lambda_2}{\lambda_1} \coth(\lambda_1(L_e - a)) \cos(\lambda_2(x - L_e + a)) \right) \quad (28)$$

Equation (29) can be derived by applying $P = \frac{\sigma_b}{A_b}$, where $x = L_e$ to define the pull-out force along the fully bonded length:

$$P = \frac{L_{per} \tau_{max}}{\lambda_2} \left(\sin(\lambda_2 a) + \frac{\lambda_2}{\lambda_1} \coth(\lambda_1(L_e - a)) \cos(\lambda_2 a) \right) \quad (29)$$

The maximum pull-out force occurs when $\frac{dP}{da} = 0$, and thus:

$$P_{max} = \frac{L_{per} \tau_{max}}{\lambda_2} \left(\sin(\lambda_2 a) + \frac{\lambda_2^2}{\lambda_1^2} \sin(\lambda_2 a) \right) \quad (30)$$

Using a similar approach for large L_e values, and incorporating Φ to account for the properties of the FRP anchor and concrete, P_{max} is derived as shown in Equation (31),

$$P_{max} = \frac{L_{per} \tau_{max}}{\lambda_2} \frac{\delta_2}{(\delta_2 - \delta_1)} \Phi \quad (31)$$

where

$$\Phi = \frac{1}{\sqrt{1 + \frac{E_b A_b}{f'_c A_c}}} \quad (32)$$

and f'_c is the concrete specified compressive strength.

4. Discussions on the Behaviour of the Proposed Bond Model

To evaluate the accuracy of the proposed model, a comparative study was conducted using existing data from direct pull-out tests of post-installed adhesive FRP bar anchors in unconfined concrete blocks.

Predictions of the Proposed Model Versus Experimental Results

To validate the proposed bond model, direct pull-out tests reported by Valerio et al. [25], Godat et al. [26], and Caro et al. [28] were examined. Table 1 summarizes the properties of the FRP anchor pull-out test specimens from the literature. Table 2 presents the predicted P_{max} calculated using Equation (31), which was derived from solving

the governing differential equations. Additionally, Table 2 includes the ratio of P_{exp} to P_{max} . In this study, the experimental maximum bond strengths reported in each referenced study were used to calculate P_{max} and predict the pull-out force for each specimen in the database. However, the authors have generally developed analytical equations to predict τ_{max} and δ_1 as follows:

$$\tau_{max} = A \times E_{frp}^j \times f_c'^f \times (\pi d_b L_e)^k \quad (33)$$

$$\delta_1 = G(\tau_{max})^m \quad (34)$$

where for CFRP anchors, $A = 10^{-7.71}$, $j = 1.81$, $f = 0.49$ and $k = -0.32$; for GFRP anchors, $A = 10^{-47.29}$, $j = 10.58$, $f = 0.38$ and $k = -0.27$; and for combined CFRP and GFRP anchors $A = 0.34$, $j = 0.30$, $f = 0.62$ and $k = -0.24$. Meanwhile, $G = 9.06$ and $m = -0.73$ are curve-fitting parameters calculated through linear regression. Details of the development of equations for the maximum shear stress and the corresponding slip are provided in Mirzabagheri et al. [4].

Table 1. Properties of adhesively bonded FRP anchor pull-out test specimens as reported by Caro et al. [28], Godat et al. [26], and Valerio et al. [25], respectively.

Specimen Name	Bar Type	L_{emb} (mm)	d_b (mm)	d_h (mm)	L_{per} (mm)	f_c' (MPa)	E_{frp} (GPa)	A_b (mm ²)	A_c (mm ²)
C26-15d-CFRP10-1.5d	CFRP	150	10.00	15.0	53.40	26.1	130	78.53	39,922
C25-10d-GFRP12-1.5d	GFRP	120	12.00	18.0	62.82	24.8	40	113.08	9887
C25-10d-CFRP12-1.5d	CFRP	120	12.00	18.0	62.82	24.8	130	113.08	9887
C25-5d-GFRP12-1.5d	GFRP	60	12.00	18.0	62.82	24.8	40	113.08	9887
C25-5d-GFRP12-1.5d	GFRP	60	12.00	18.0	62.82	24.8	40	113.08	9887
C25-5d-CFRP12-1.5d	CFRP	60	12.00	18.0	62.82	24.8	130	113.08	9887
C25-5d-CFRP12-1.5d	CFRP	60	12.00	18.0	62.82	24.8	130	113.08	9887
C46-15d-CFRP10-1.5d	CFRP	150	10.00	15.0	53.40	45.6	130	78.53	7772
C46-10d-GFRP10-1.5d	GFRP	100	10.00	15.0	53.40	45.6	40	78.53	7772
C46-10d-CFRP10-1.5d	CFRP	100	10.00	15.0	53.40	45.6	130	78.53	7772
C46-5d-GFRP10-1.5d	GFRP	50	10.00	15.0	53.40	45.6	40	78.53	7772
C46-5d-CFRP10-1.5d	CFRP	50	10.00	15.0	53.40	45.6	130	78.53	7772
C2-1.50d-9.5S-15d	CFRP	143	9.52	15.0	53.40	42.7	155	71.17	36,100
C2-1.50d-9.5S-5.0d	CFRP	48	9.52	15.0	53.40	42.7	155	71.17	36,100
C2-1.50d-9.5S-10.0d	CFRP	95	9.52	15.0	53.40	42.7	155	71.17	36,100
C2-1.50d-9.5S-20.0d	CFRP	190	9.52	15.0	53.40	42.7	155	71.17	36,100
C60-H500-CFRP7.5-15	CFRP	15	7.50	9.5	36.12	60.0	130	44.17	22,500
C60-H500-CFRP7.5-30	CFRP	30	7.50	9.5	36.12	60.0	130	44.17	22,500
C60-H500-CFRP7.5-45	CFRP	45	7.50	9.5	36.12	60.0	130	44.17	22,500
C60-H500-CFRP7.5-60	CFRP	60	7.50	9.5	36.12	60.0	130	44.17	22,500
C60-H500-CFRP7.5-75	CFRP	75	7.50	9.5	36.12	60.0	130	44.17	22,500

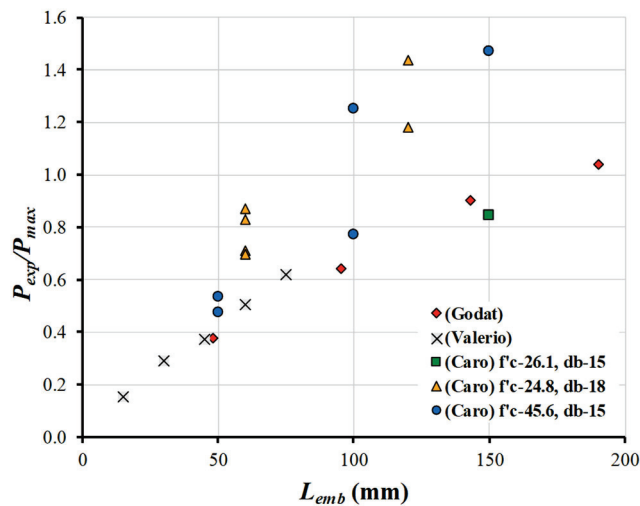
To this end, Equation (33) can be rewritten for combined CFRP and GFRP anchors as follows:

$$\tau_{max} = 0.245 E_{frp}^{0.3} f_c'^{0.62} (d_b L_e)^{-0.24} \quad (35)$$

Figure 5 shows the ratio of P_{exp}/P_{max} versus the embedded length (L_{emb} , in millimetres) for each tested specimen, evaluating the sensitivity of the model predictions to changes in L_{emb} . Data sets from different studies are colour-coded. Except for four specimens from Caro et al. [28], an increase in L_{emb} generally led to improved accuracy across all specimens. The most notable improvement occurred with the specimens from Godat et al. [26], where increasing L_{emb} from 143 mm to 190 mm not only enhanced accuracy but also reduced the gradient of error, indicating convergence toward L_e , in agreement with experimental results.

Table 2. Validating the predicted P_{max} versus the experimental pull-out force.

Specimen Name	τ_{max} (MPa)	δ_1 (mm)	δ_2 (mm)	P_{exp} (kN)	β	λ_2	P_{max} (kN)	$\frac{P_{exp}}{P_{max}}$
C26-15d-CFRP10-1.5d	11.9	1.60	5.1	56.20	5.231×10^6	0.0042	66.8	0.8
C25-10d-GFRP12-1.5d	8.0	1.90	5.0	36.30	1.389×10^5	0.0060	30.7	1.2
C25-10d-CFRP12-1.5d	11.0	1.50	5.0	49.60	4.274×10^6	0.0037	34.5	1.4
C25-5d-GFRP12-1.5d	10.1	1.40	5.0	22.80	1.389×10^5	0.0062	32.0	0.7
C25-5d-GFRP12-1.5d	12.0	2.10	5.0	27.10	1.389×10^5	0.0076	38.9	0.7
C25-5d-CFRP12-1.5d	14.0	1.00	5.0	31.60	4.274×10^6	0.0039	36.4	0.9
C25-5d-CFRP12-1.5d	13.3	1.20	5.0	30.10	4.274×10^6	0.0039	36.4	0.8
C46-15d-CFRP10-1.5d	15.9	2.20	5.0	74.80	5.231×10^6	0.0055	51.0	1.5
C46-10d-GFRP10-1.5d	12.9	2.80	4.2	40.40	1.700×10^5	0.0125	52.6	0.8
C46-10d-CFRP10-1.5d	13.8	1.20	2.4	43.50	5.231×10^6	0.0078	34.8	1.2
C46-5d-GFRP10-1.5d	13.8	1.70	2.3	21.60	1.700×10^5	0.0198	45.5	0.5
C46-5d-CFRP10-1.5d	13.4	1.10	5.0	21.10	5.231×10^6	0.0042	39.6	0.5
C2-1.50d-9.5S-15d	22.3	1.05	5.0	91.20	4.841×10^6	0.0052	101.0	0.9
C2-1.50d-9.5S-5.0d	29.9	0.80	5.0	42.80	4.841×10^6	0.0059	113.4	0.4
C2-1.50d-9.5S-10.0d	22.3	0.90	5.0	63.40	4.841×10^6	0.0051	99.1	0.6
C2-1.50d-9.5S-20.0d	18.1	1.65	5.0	102.40	4.841×10^6	0.0051	98.8	1.0
C60-H500-CFRP7.5-15	36.0	0.85	3.9	12.72	6.291×10^6	0.0086	84.2	0.2
C60-H500-CFRP7.5-30	32.0	0.25	4.5	22.62	6.291×10^6	0.0069	77.6	0.3
C60-H500-CFRP7.5-45	28.0	0.70	4.9	29.69	6.291×10^6	0.0065	79.5	0.4
C60-H500-CFRP7.5-60	24.0	0.40	4.3	33.93	6.291×10^6	0.0062	67.0	0.5
C60-H500-CFRP7.5-75	25.0	0.80	4.2	44.18	6.291×10^6	0.0068	71.6	0.6

**Figure 5.** Sensitivity of P_{exp}/P_{max} with respect to changes in L_{emb} of each tested specimen.

As reported by De Lorenzis and Nanni [32], L_e represents the bond length beyond which the bond force no longer increases. A similar trend was observed for Valerio et al. [25], where increasing L_{emb} from 15 mm to 75 mm (approximately 10% to 50% of the effective length) reduced the P_{max} prediction error from 85% to 35%. These findings highlight the fundamental influence of L_{emb} , relative to L_e , on the debonding force.

For the Caro et al. [28] test specimens, a similar trend was observed; however, the model was less effective, as the predictions converged to overly conservative values for both CFRP and GFRP specimens. On the other hand, in the Godat et al. [26] results, the two larger specimens yielded the most accurate predictions, highlighting the critical role of sufficient bond length in the overall performance of P_{max} .

To account for the effect of bond length in the proposed model, the equation developed by Mofidi et al. [33] was adopted to calculate the effective length of ETS FRP in concrete, as shown in Equation (36):

$$L_e = \sqrt{\frac{E_b d_b \delta_1 (1 + \alpha)}{2 \tau_m (1 - \alpha)^2}} \quad (36)$$

where α is a calibration factor taken as -0.6 .

After calculating L_e and comparing it with L_{emb} to assess the adequacy of the embedment length, the modified equation for P_{max} is expressed as follows:

$$P_{max} = k \frac{L_{per} \tau_{max}}{\lambda_2} \frac{\delta_2 L_{emb}}{(\delta_2 - \delta_1) L_e} \Phi \quad \text{If } L_{emb} < L_e, \quad (37)$$

$$P_{max} = k \frac{L_{per} \tau_{max}}{\lambda_2} \frac{\delta_2}{(\delta_2 - \delta_1)} \Phi \quad \text{If } L_{emb} \geq L_e, \quad (38)$$

where k is set to 1.2. The coefficient of determination (R^2) between the predicted results from Equations (36) and (37) and the experimental data is 0.980 (Figure 6), with a coefficient of variation (CoV) of P_{exp}/P_{max} equal to 0.058, which indicates excellent predictive accuracy of the proposed bond model. Table 3 shows the results of the calculations of P_{max} for FRP anchor/concrete joints.

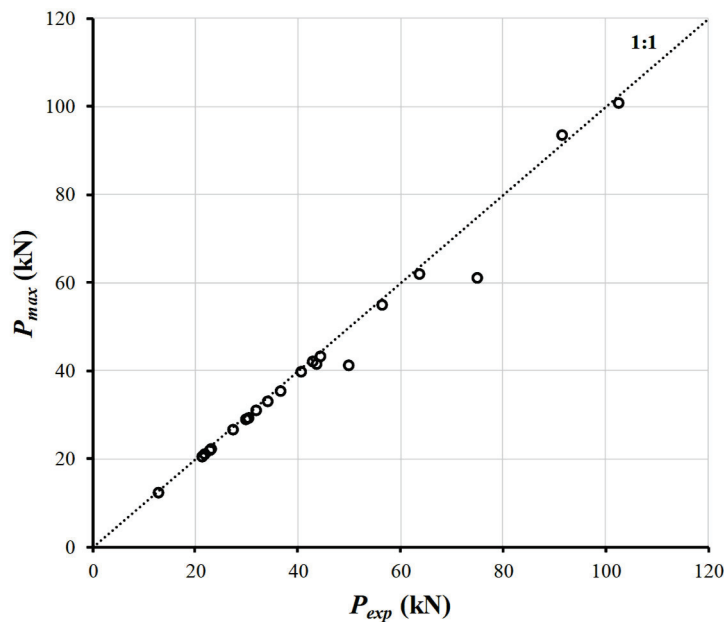


Figure 6. Proposed model predictions of P_{max} versus P_{exp} .

Table 3. Proposed model predictions of P_{max} .

Specimen Name	P_{exp} (kN)	P_{max} (kN)	$\frac{P_{exp}}{P_{max}}$
C26-15d-CFRP10-1.5d	56.2	55.1	1.02
C25-10d-GFRP12-1.5d	36.3	35.5	1.02
C25-10d-CFRP12-1.5d	49.6	41.4	1.20
C25-5d-GFRP12-1.5d	22.8	22.4	1.02
C25-5d-GFRP12-1.5d	27.1	26.6	1.02
C25-5d-CFRP12-1.5d	31.6	31.1	1.02
C25-5d-CFRP12-1.5d	30.1	29.5	1.02
C46-15d-CFRP10-1.5d	74.8	61.1	1.22

Table 3. Cont.

Specimen Name	P_{exp} (kN)	P_{max} (kN)	$\frac{P_{exp}}{P_{max}}$
C46-10d-GFRP10-1.5d	40.4	39.8	1.02
C46-10d-CFRP10-1.5d	43.5	41.8	1.04
C46-5d-GFRP10-1.5d	21.6	21.3	1.01
C46-5d-CFRP10-1.5d	21.1	20.7	1.02
C2-1.50d-9.5S-15d	91.2	93.6	0.97
C2-1.50d-9.5S-5.0d	42.8	42.1	1.02
C2-1.50d-9.5S-10.0d	63.4	62.2	1.02
C2-1.50d-9.5S-20.0d	102.4	101.0	1.01
C60-H500-CFRP7.5-15	12.7	12.5	1.02
C60-H500-CFRP7.5-30	22.6	22.2	1.02
C60-H500-CFRP7.5-45	29.7	29.1	1.02
C60-H500-CFRP7.5-60	33.9	33.3	1.02
C60-H500-CFRP7.5-75	44.2	43.4	1.02

5. Recommendations

Based on the findings of this study, the following recommendations are proposed to enhance the design and application of post-installed adhesive FRP anchors in structural concrete:

- The mechanics-based bond model developed in this study provides a predictive tool for estimating the pull-out capacity of FRP anchors. Practitioners and design engineers are encouraged to incorporate this model into design procedures.
- The model's predictions highlight the critical influence of embedment length on the pull-out force. Designers should consider the calculated effective bond length to ensure adequate anchorage, especially in applications where limited embedment is feasible.
- When selecting adhesive types and concrete strengths, it is crucial to carefully consider adhesive curing conditions and environmental factors, as they significantly impact the overall bond performance and long-term durability of the anchor system.

6. Conclusions

This study developed a novel analytical bond model to predict the pull-out force of adhesively bonded FRP anchors embedded in concrete. The model is based on a bilinear bond–slip relationship that captures both the ascending elastic and descending softening stages of interface behaviour and is expressed as a function of the maximum shear stress and corresponding slip. The proposed model was validated against experimental pull-out tests of CFRP and GFRP anchor–concrete joints, demonstrating high predictive accuracy with an R^2 value of 0.980 for P_{max} with respect to P_{exp} , and a CoV of 0.058 for the P_{exp}/P_{max} ratio. The results showed that the model consistently produced conservative yet accurate estimates of pull-out capacity across a range of bar types, concrete strengths, and embedded lengths.

The key scientific contributions and novelty of this study are as follows:

- This article offers a mechanics-based predictive model specifically tailored to post-installed adhesive FRP anchors, filling a gap not addressed in existing design guidelines such as CSA A23.3:24 or ACI 355-19(21).
- This study proposes a new closed-form expression for predicting the pull-out force of FRP anchors based on material and geometric properties, enabling rational design beyond empirical methods and enhancing the analytical understanding of load transfer mechanisms in FRP–concrete joints.

- The model is not only validated but also formulated in a practical, design-ready format that can support the development of future FRP anchorage standards.

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Abbreviations

The following notations are used in this manuscript:

a	Length of Fracture
A_b	FRP bar's cross-sectional area
A_c	Concrete block cross-sectional area
CoV	Coefficient of variation
d_b	Bar diameter
d_f	Failure plane diameter
d_h	Hole diameter
E_b	FRP bar's Young's modulus
E_c	Concrete's Young's modulus
E_{frp}	FRP rod's Young's modulus
f'_c	Specified compressive strength of concrete
L_e	Effective bond length
L_{emb}	Embedded length
L_{per}	Length of the debonding failure plane
P_{exp}	Experimental pull-out force
P_{max}	Maximum predicted pull-out force
t_f	Thickness of the concrete cover attached to the FRP bar
u_1	FRP bar's displacement
u_2	Concrete block's displacement
α	Factor for calculation of L_e equal to -0.6
β	$L_{per} / (E_b A_b)$
δ	Local slip between FRP and concrete
δ_1	Slip value at τ_f or τ_{max}
δ_2	End of the pull-out test slip value
δ_{max}	Maximum slip
λ_1	$(\beta \tau_{max} / \delta_1)^{0.5}$
λ_2	$(\beta \tau_{max} / (\delta_2 - \delta_1))^{0.5}$
σ_b	FRP bar's normal stress
τ_{max}	Maximum bond stress
Φ	FRP bar and concrete properties coefficient

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Article

Design and Analysis of Natural Fiber-Reinforced Jute Woven Composite RVEs Using Numerical and Statistical Methods

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Abstract: Woven composites and natural fiber-reinforced composites both have widespread applications in various industries due to their appealing load-carrying capacity and performance compared to conventionally manufactured composites, such as polymeric composites. Representative volume element (RVE) generation is one of the most effective and widely adopted methods for estimating mechanical performance in current research. This study aims to explore the effects of three significant factors in woven composite RVEs: yarn spacing (from 0.5 mm to 1.5 mm), fabric thickness (from 0.2 to 0.5 mm), and shear angle (from 3.5 to 15 degrees) through finite element methods and statistical analysis to understand their effectiveness in the elastic moduli's. The validation of this research has been conducted using available literature. The generation of representative volume elements (RVEs) and the calculation of elastic moduli were performed using ANSYS-19, including the material designer feature. The experimental design was carried out using Design-Expert software version 13, which used response surface methodology. The materials selected for this study were jute fiber and epoxy. After obtaining the elastic moduli from the ANSYS material designer, three responses were considered: longitudinal Young's modulus (E_{11}), in-plane shear modulus (G_{12}), and major Poisson's ratio (ν_{12}). ANOVA (Analysis of Variance) and 3D contour graphs were generated to further analyze and correlate the effects of the selected materials on these responses. These investigations revealed that in comparison to twill structure, plain structure in natural fiber-reinforced woven composites could be a good alternative. Additionally, the findings highlighted that yarn spacing and fabric thickness significantly influence the considered moduli in plain-weave NFRC material RVEs. However, in twill-woven composite RVEs, the effects of yarn spacing, fabric thickness, and shear angle were found to be considerable. Moreover, statistical analysis has found the best combinations for both plain and twill structures, while the yarn spacing was 1 mm, the shear angle was 9.25 degrees, and the fabric thickness was 0.35 mm.

Keywords: natural fiber reinforced composite; jute woven composite; representative volume element; response surface methodology

1. Introduction

Natural fiber-reinforced composites (NFRCs) have gained significant attention in various industries, such as automotive, building and construction, and biomedical, due to their advantageous mechanical properties, including availability, high strength-to-weight ratio, sustainability, and recyclability [1]. NFRC materials reduce carbon emissions, support a circular economy, and provide sustainable solutions for a better environment for future generations. Since the beginning of the Industrial Revolution, numerous conventional

and advanced manufacturing techniques have been developed. Among these, three-dimensional printing (3DP), also known as additive manufacturing (AM), has gained significant attention in modern manufacturing industries [2]. However, when it comes to mechanical properties, traditionally fabricated composites or parts typically demonstrate better performance, especially in terms of mechanical strength, than those produced using 3DP [3]. Based on the type of reinforcement material, composite materials are classified as natural fiber-reinforced composites, polymeric composites, and so on. However, based on the fabrication process, composite materials are categorized into various types, including unidirectional composites, particulate composites, woven composites, bidirectional composites, and hybrid composites [4,5]. In woven composites, the structures can be complex in terms of dimensions and weaving patterns. They are typically classified as plain or twill weaves. Plain weaves are structured with a 1×1 thread ratio, while twill weaves follow a 2×2 pattern in both the warp and weft directions [6].

To calculate the mechanical properties of woven composite RVEs without conducting solid experiments, several methods have been developed, including theoretical approaches and finite element methods (FEMs). Compared to solid experiments, finite element methods and theoretical approaches are sufficient to predict the mechanical properties of composite materials [5]. Using the finite element method, the generation of a representative volume element (RVE) is one of the most straightforward techniques for estimating mechanical properties, particularly the elastic moduli, including Young's modulus (longitudinal and transverse), shear modulus (in-plane and out-of-plane), and Poisson's ratio (major and minor) [7]. In both recent and earlier studies, significant research has been conducted on the RVE of woven composites. For example, earlier investigations employed the finite element method (FEM) to generate woven composite RVEs, considering the RVE size and its effects on local and global strain. These studies highlighted the significant role of RVE size in influencing the response of both local and global strains [8]. Apart from RVE size, several factors can significantly influence the elastic moduli of woven composites, including fiber volume fraction, yarn fiber volume fraction, fabric thickness, yarn spacing, and shear angle [6]. Fiber volume fraction refers to the proportion of fibers used to reinforce composite materials. It plays a crucial role in determining the mechanical behavior of these composites, affecting characteristics such as ductility and brittleness after fabrication. A recent study examined the effects of fiber volume fraction on the ballistic performance of woven composite representative volume elements (RVEs). The findings indicated that higher fiber volume fractions generally resulted in better energy absorption and performance outcomes [9]. However, this trend does not apply in all situations. For instance, the present study revealed that excessively high fiber volume fractions can make composites too stiff, leading to premature failure. This suggests that lower fiber volume fractions, ranging from 0.1 to 0.5, may be more favorable for achieving optimal performance [10]. Similarly to fiber volume fraction, yarn fiber volume fraction can significantly affect the elastic moduli of materials. The key distinction between the two is that the yarn fiber volume fraction refers specifically to the proportion of fiber contained within the yarn, whereas the fiber volume fraction encompasses the total fiber amount throughout the entire woven composite [11]. Although the yarn fiber volume fraction typically ranges from 0.45 to 0.65, optimal outcomes are generally achieved at higher fractions [12]. Yarn spacing is also a crucial factor to consider in woven composites, as it can significantly impact the elastic moduli. Yarn spacing refers to the distance between two yarns in the representative volume element (RVE) of woven composites. A recent study comparing different yarn spacings in woven composite RVEs found that the elastic moduli decreased as yarn spacing increased from 1 mm to 3 mm. However, this trend does not hold for every increment. Specifically, while the elastic moduli decreased when yarn spacing increased from 1 mm

to 2.5 mm, there was a noticeable increase in elastic moduli at a spacing of 3 mm [13]. In woven composites, whether they are plain or have more complex structures, the shear angle plays a crucial role in influencing the elastic moduli. The shear angle reflects the angle formed between the two yarns—warp and weft—during the manufacturing process of the woven composites [13,14]. This angle can range from -45 degrees to 45 degrees. However, optimal elastic moduli in woven composites RVE are typically achieved within the lower angle range [11,13,14]. The fabric thickness in the RVE refers to the thickness of the woven composites, which can also have a significant influence on their properties [6]. However, this parameter has not been extensively discussed in recent studies.

Apart from the aforementioned numerical analyses, several recent and earlier studies have developed theoretical methods, which were later compared with numerical and experimental results to predict the elastic properties [15–19]. This earlier study most likely first predicted the elastic modulus of E-glass woven composites by designing a new analytical model, which was later compared with Hashin's concentric cylinder and Halpin–Tsai models. Both models accurately predicted all outcomes without any significant discrepancies [15]. Later, the Mori–Tanaka model and the finite element method were employed, and experimental outcomes were compared in this research. This investigation also revealed that the finite element method, the theoretical method, and experiments can be rigorous tools to predict the outcomes in the RVEs of woven composites [16]. Similarly to the aforementioned studies, some newly developed theoretical models, namely the stochastic fracture model [17], data-driven reduced-order models [18], and the continuum model [19], have been developed in recent years, which have also been validated against the numerical and experimental outcomes in woven composites RVE.

Numerous recent studies have employed various statistical methods to optimize material properties or identify the best combinations of influencing factors. These methods are beneficial because they allow for the design of experiments based on input parameters while simultaneously considering multiple variables and outcomes within a unified framework. Furthermore, they enable the prediction of optimal results by comparing the effects of the selected parameters [5,20–22]. To enhance the outcomes of RVE simulations, a recent study incorporated artificial intelligence-based statistical analysis on random RVEs by employing geometric descriptors and statistical functions [23]. Similarly, an earlier study adopted a statistical approach by considering the density of the RVE as a key variable to improve prediction efficiency [24]. One of the main advantages of using statistical analysis to predict the elastic moduli of an RVE is its ability to generate multiple variables, which can be optimized to enhance predictive accuracy. For instance, previous research explored different RVE sizes and achieved precise estimations of thermal and mechanical properties by applying the integral range approach [25]. However, a key limitation remains in establishing robust correlations among RVE variables, and accurately estimating optimal parameters in the optimization process continues to pose challenges, particularly in the context of woven composites.

It is undoubtedly true that, in recent years, significant research has focused on the development of woven composites by investigating their elastic moduli through numerical and theoretical methods, particularly finite element analysis, often using various types of polymer fibers. Although these studies have provided valuable insights, they have primarily focused on parametric analyses without incorporating a comprehensive statistical approach. This represents a notable gap in the existing body of research. To address this limitation, the present study examines the effects of three key parameters—yarn spacing, shear angle, and fabric thickness—on the mechanical behavior of woven composites. These parameters are analyzed while maintaining other influential factors, such as fiber volume fraction and yarn fiber volume fraction, at constant levels. By employing a statistical

framework, this study aims to offer a more holistic understanding of how these parameters interact and contribute to the overall performance of woven composites (plain and twill structures), which are numerically embedded by jute fiber and epoxy matrix.

2. Finite Element Modeling Setup, Material Properties, and Design of Experiment

All numerical investigations were conducted using ANSYS version 19 FEM software, specifically within the Material Designer tool. First, the material properties of both the fiber and the matrix were defined and added to the material library. These properties were then assigned to the Material Designer to initiate the simulation setup. The fiber volume fraction and yarn fiber volume fraction were kept constant at 0.325 and 0.65, respectively, based on values reported in the literature. Other parameters—yarn spacing, shear angle, and fabric thickness—were varied and defined within the Material Designer interface. The weave structure was also specified at this stage. Following the geometry definition, the repeat count and algorithm were selected. The lenticular algorithm was chosen for this investigation as it produces more accurate and reliable results for woven composite modeling.

Mesh generation in finite element method (FEM) analysis, particularly for representative volume elements (RVEs), is a complex and critical task. Different FEM approaches employ various mesh generation techniques. For instance, in one study, a high-resolution mesh was generated in ABAQUS to achieve more accurate results for the RVE [26]. Considering this complexity, the present research utilized conformal and periodic meshing for all RVEs using the Material Designer tool. Although Material Designer also provides a block meshing option, periodic meshing offers a more refined structure and facilitates property estimation with fewer complications. Moreover, a significant limitation of block meshing is that the Material Designer is unable to properly mesh all the edges of the RVE using this technique. For these reasons, periodic and conformal meshing techniques were adopted in all RVE simulations. The mesh size is set to 0.2 mm, and it was observed that varying the mesh size does not significantly impact the outcomes, especially in the microscale virtual domain. In addition, the cell type is triangular by default. Subsequently, boundary conditions, which are provided by default in Material Designer, were applied. Finally, simulations were run with the constant material output option selected to compute the effective material properties.

Figure 1a illustrates the shear angle to provide insights into the deformation characteristics of woven jute composite RVEs. Figure 1b presents the multiscale approach of the composites, describing the transition from the microscale to the actual plain weave structure. In Figure 1c, yarn spacing is shown in a plain-weave-structure RVE with a spacing of 1.5 mm; here, the repeat count is set to 2 for improved visualization. Figure 1d displays the woven jute composites' twill-weave RVE structure. Figure 1e,f provide a microscale representation of the twill composite captured through scanning electron microscopy. Following this, the difference in fabric thickness in NFRC-woven composites is depicted in Figure 1g,h, showing real structural variations. However, in Figure 1i, a meshed twill RVE was presented, subjected to both conformal and block meshing. Finally, the material properties of the jute fiber and epoxy matrix are listed in Table 1. The material properties are taken as isotropic from the literature [5].

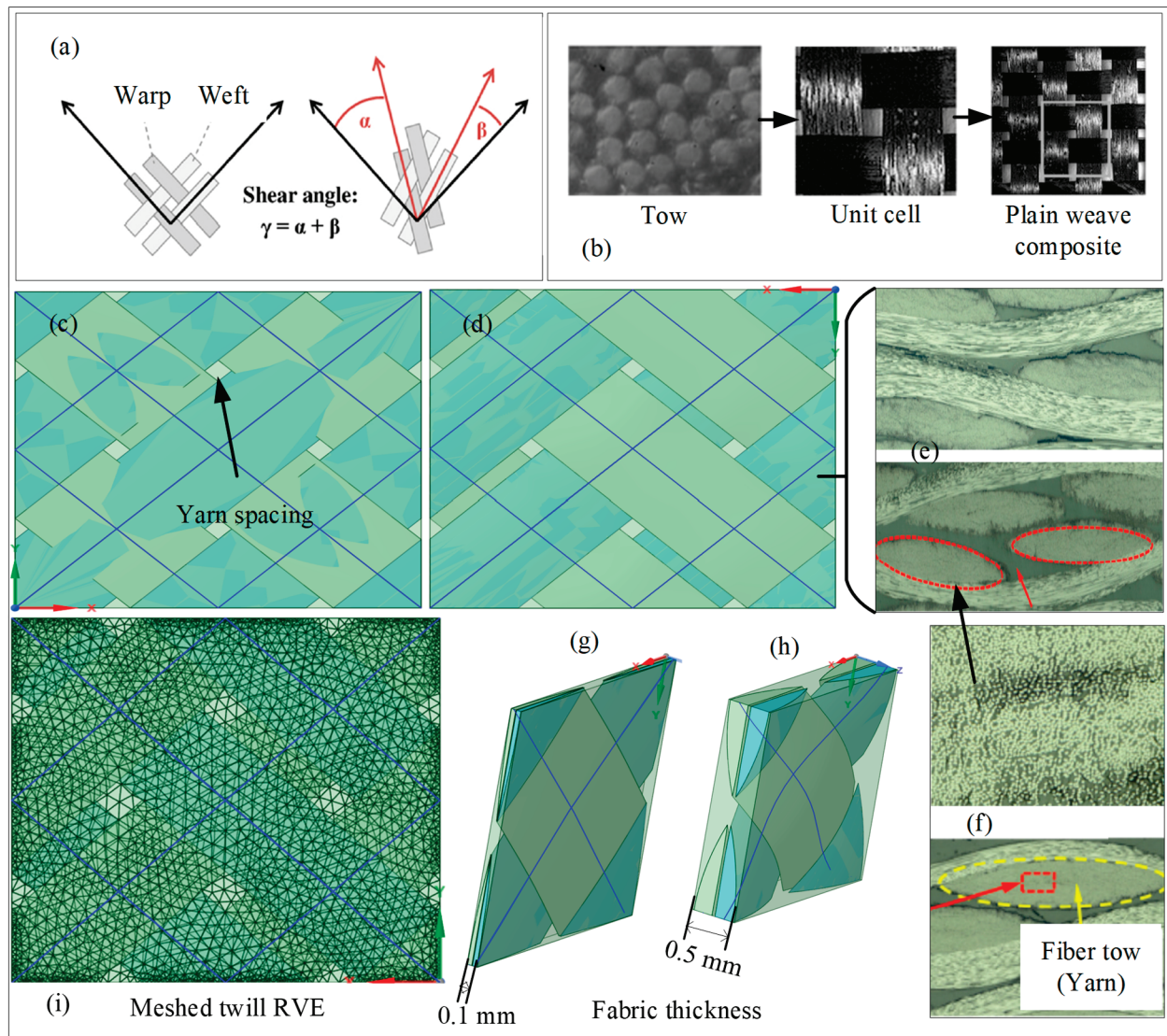


Figure 1. (a) Shear angles in both plain- and twill-woven composite RVEs [13]; (b) multiscale representation of the actual structure of a plain weave-woven composite [15]; (c) realistic depiction of yarn spacing in plain weave-woven composites RVEs of NFRCs; (d) twill-structure RVE of NFRC; (e,f) scanning electron micrographs of twill-composite RVE [27]; (g,h) variations in fabric thickness at 0.1 mm and 0.5 mm; (i) meshed RVE of a twill-composite RVE.

Table 1. Material properties of jute fiber and epoxy matrix.

Moduli's	Jute	Epoxy
Modulus of elasticity	20 GPa	5.35 GPa
Poisson's ratio	0.38	0.25

The design of the experiments (DOEs) is conducted using Design Expert software version 13. After defining the input variables in the software, the response surface methodology has been selected. Later, the chosen design type was a central composite with a quadratic model. The runs mean the number of experiments is determined as 30 by selecting no blocks. The design of the experiment is given in Table 2.

Table 2. The design of the experiment for NFRC materials of plain- and twill-woven composite.

Input Variables	Unit	−1 Level	+1 Level
Shear angle	Degree	3.5	15
Yarn spacing	mm	0.5	1.5
Fabric thickness	mm	0.2	0.5

Although periodic boundary conditions are a default feature in ANSYS Material Designer, they are explicitly presented and discussed in this research [13]. These boundary conditions are primarily applied along the three principal directions or planes of the woven composites: *a*, *b*, and *c*, and discussed in Equations (1)–(7).

The boundary conditions used to predict the longitudinal elastic moduli of woven composites are as follows:

$$u_{11}(a,b,0) = u_{22}(a,b,0) = u_{33}(a,b,0), u_{11}(a,b,x) = \partial_{aa} \quad (1)$$

$$u_{11}(0,b,c) = u_{22}(a,0,c) = u_{33}(a,b,0), u_{22}(a,x,c) = \partial_{bb} \quad (2)$$

Similarly, in the *c* plane, the boundary conditions are as follows:

$$u_{11}(0,b,c) = u_{22}(a,0,c) = u_{33}(a,b,0), u_{33}(a,b,x) = \partial_{cc} \quad (3)$$

For the in-plane and out-of-plane shear moduli (G_{12} , G_{23} , and G_{13}), the periodic boundary conditions are provided below.

$$u_{11}(a,0,c) = u_{22}(a,0,c) = u_{33}(a,0,c) = 0, u_{11}(a,x,c) = \partial_l \quad (4)$$

$$u_{11}(a,b,0) = u_{22}(a,b,0) = u_{33}(a,b,0) = 0, u_{22}(a,b,x) = \partial_t \quad (5)$$

$$u_{11}(a,0,c) = u_{22}(a,0,c) = u_{33}(a,b,0) = 0, u_{22}(x,b,c) = \partial_l \quad (6)$$

However, for both major and minor Poisson's ratios, the following equations are used:

$$\nu = -\partial\epsilon_{\text{trans}}/\partial\epsilon_{\text{axial}} \quad (7)$$

3. Model Validations

The numerical validation in this study was performed by comparing the results obtained from ANSYS Material Designer with those from widely used theoretical models—namely the Mori–Tanaka model and the self-consistent model—as well as available experimental data. Due to the limited number of studies specifically focused on woven composites RVE, the validation was carried out using unidirectional composite material RVE as a reference. This comparison incorporated both numerical and experimental results, with simulations conducted using ANSYS Material Designer [28]. The primary objective of this validation was to demonstrate the capability of ANSYS Material Designer to accurately predict the representative volume element (RVE) behavior of composite materials. To ensure scientific rigor, fiber volume fractions of 0.3, 0.4, 0.5, 0.6, and 0.7 were considered for a carbon–epoxy-based unidirectional composite, and the corresponding elastic properties— E_{11} , G_{12} , and ν_{12} —were evaluated. It is also noteworthy that the entire validation process followed the finite element setup previously described in the Finite Element Setup section.

The resulting elastic moduli were plotted as bar graphs in Figure 2a–c and compared with both theoretical predictions and experimental values. The results revealed a maximum deviation of only 0–5%, which falls well within the acceptable range for numerical simulations [29]. These minimal discrepancies confirm that ANSYS Material Designer is a reliable and efficient tool for predicting the mechanical properties of composite materials RVE. It provides a practical alternative to experimental testing, reducing both complexity and computation time while maintaining high accuracy.

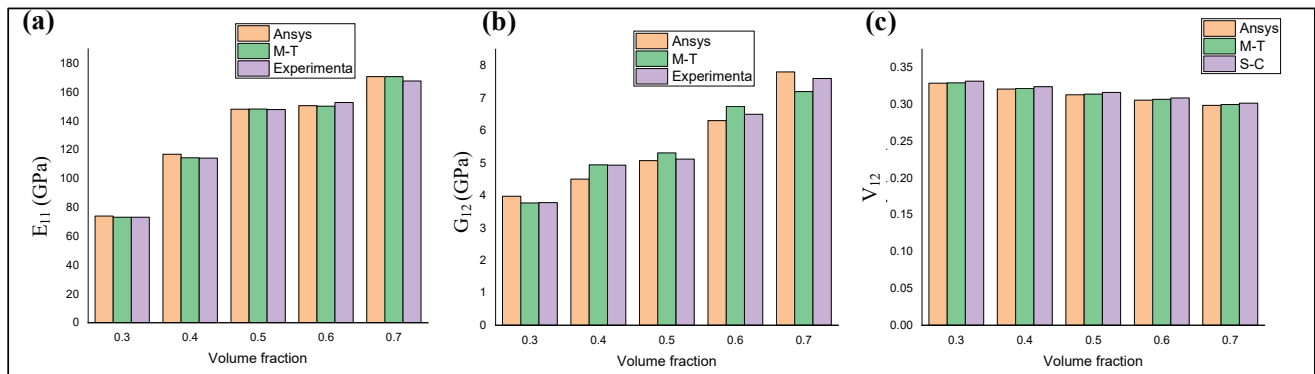


Figure 2. (a) Comparison of longitudinal Young's modulus (E_{11}) obtained from experimental results, ANSYS Material Designer, and the Mori–Tanaka (M–T) model; (b) comparison of in-plane shear modulus (G_{12}) from experimental data, Material Designer, and the M–T model; (c) comparison of major Poisson's ratio (ν_{12}) from Material Designer, the M–T model, and the self-consistent model.

4. Results and Discussions

4.1. Overall Outcomes for Plain and Twill Jute Composite RVEs

After performing simulations based on the design of experiments (DOEs), the results are presented in Table 3. It can be observed that, for all the considered elastic moduli, both the highest and lowest values are obtained from the same DOE configurations for both plain and twill jute composites. This indicates a strong correlation between the selected input variables and the resulting mechanical properties. In Table 3, the highest modulus values are achieved in DOE-1, where the input variables are a shear angle of 9.25 degrees, yarn spacing of 1 mm, and fabric thickness of 0.1 mm. In contrast, the lowest values for both types of woven composites are found in the DOE-10 with a shear angle of 15 degrees, yarn spacing of 0.5 mm, and fabric thickness of 0.5 mm. From these two DOE cases, it can be postulated that a lower shear angle leads to improved mechanical performance. This is likely because a higher shear angle alters the fiber orientation relative to the loading direction, thereby reducing the composite's stiffness [13]. In contrast, at lower shear angles, the fibers remain more aligned with the direction of the applied load, enhancing their load-bearing efficiency. Regarding yarn spacing, moderate values appear to be more effective. For instance, a comparison between DOE-1 and DOE-10 shows that better modulus values are obtained at a yarn spacing of 1 mm rather than 0.5 mm. This is likely due to improved matrix distribution at larger yarn spacing, which helps reduce voids and enhances the composite's stiffness. In terms of fabric thickness, the lowest value considered—0.1 mm—results in the best modulus values. This can be attributed to a more uniform compaction of the composite, which enables better stress distribution across the material and improves overall stiffness. Further explanations and detailed reasoning are provided below.

Table 3. Numerical results and elastic moduli of plain and twill jute composites.

Run	Shear Angle (Degrees)	Yarn Spacing (mm)	Fabric Thickness (mm)	Plain Weave Jute Composite			Twill Weave Jute Composite		
				E ₁₁ (GPa)	G ₁₂ (GPa)	V ₁₂	E ₁₁ (GPa)	G ₁₂ (GPa)	V ₁₂
1	9.25	1	0.1	12	4.61	0.35617	12.1	4.61	0.35709
2	3.5	0.5	0.2	11.5	4.48	0.34187	11.3	4.46	0.33873
3	3.5	1.5	0.2	11.9	4.59	0.35385	11.8	4.58	0.35267
4	9.25	1	0.35	11.7	4.59	0.34591	11.5	4.47	0.34294
5	3.5	0.5	0.5	10.7	4.22	0.32039	10.5	4.26	0.32435
6	9.25	0.75	0.35	11.5	4.44	0.34052	11.3	4.42	0.33798
7	3.5	1.5	0.2	11.9	4.59	0.35385	11.8	4.58	0.35266
8	9.25	1	0.35	11.7	4.49	0.34587	11.5	4.47	0.34294
9	15	1.5	0.2	11.2	4.48	0.35514	12	4.56	0.35394
10	15	0.5	0.5	7.45	3.14	0.28097	10.8	4.23	0.32999
11	3.5	0.5	0.5	10.07	4.22	0.32038	10.5	4.26	0.32434
12	3.5	1.5	0.5	11.6	4.51	0.34511	11.5	4.49	0.34208
13	9.25	1	0.35	11.7	4.49	0.34587	11.6	4.47	0.34296
14	3.5	0.5	0.2	11.5	4.48	0.34187	11.4	4.46	0.33874
15	9.25	2	0.35	11.9	4.57	0.35336	11.9	4.56	0.35176
16	9.25	1	0.35	11.7	4.49	0.34589	11.6	4.48	0.34297
17	15	1.5	0.5	11.8	4.49	0.34734	11.8	4.47	0.34463
18	15	0.5	0.2	11.7	4.45	0.34427	11.6	4.44	0.3417
19	9.25	1	0.35	11.7	4.49	0.3459	11.6	4.47	0.34293
20	9.25	1	0.35	11.7	4.49	0.3459	11.6	4.47	0.34293
21	9.25	1	0.65	11.2	4.35	0.33276	11	4.34	0.33241
22	15	1.5	0.5	11.8	4.49	0.34732	11.8	4.47	0.34463
23	9.25	1	0.35	11.7	4.49	0.34589	11.7	4.47	0.34293
24	15	0.5	0.2	11.7	4.45	0.34427	11.6	4.44	0.34169
25	15	1.5	0.2	11.2	4.58	0.35514	12	4.56	0.35395
26	3.5	1	0.35	11.6	4.5	0.34428	11.4	4.48	0.3412
27	3.5	1.5	0.5	11.6	4.51	0.34512	11.6	4.49	0.34197
28	20.75	1	0.35	11.9	4.47	0.34632	11.7	4.45	0.34406
29	15	0.5	0.5	11.2	4.35	0.3555	10.8	4.23	0.32999
30	9.25	1	0.35	11.7	4.49	0.3459	11.6	4.47	0.34292

4.2. Analysis of Variance (ANOVA) Investigation for Plain Jute Composite

After obtaining the results, this research employed a widely used statistical method, namely Analysis of Variance (ANOVA), for all the considered outcomes, following the methodology outlined in the literature [10]. The aim of this analysis is to determine the significance of the variables for each response. A confidence level of 95% was adopted, meaning that if the p -value is less than 0.05, the corresponding variable is considered statistically significant for that particular response. The longitudinal Young's modulus (E_{11}) of the plain-woven jute composite is presented in the table. As shown in Table 4, the p -values for the variables yarn spacing and fabric thickness in relation to E_{11} are both less than 0.05. This indicates that yarn spacing and fabric thickness are significant factors influencing the E_{11} of the plain-woven jute composite. However, the shear angle is insignificant for E_{11} . This is possibly because, in this composite, the interlacing of yarns at the selected shear angle results in tight crimping and constraint, which inherently minimizes shear deformation. As a result, the shear angle yields an insignificant p -value. Furthermore, the percentage contribution (PC%) of each variable was calculated. For E_{11} , the PC% values are 0.85% for shear angle, 15.67% for yarn spacing, and 12.83% for fabric thickness, respectively. From Tables 4–9, SS represents the Sum of Squares, df is the Degrees of Freedom, MS is the Mean Square, while the F-value and p -value represent Fisher's test statistic and the probability value, respectively.

Table 4. ANOVA analysis in E_{11} for plain-woven jute composite.

Source	E_{11} (GPa)						Conditions
	SS	df	MS	F-Value	<i>p</i> -Value	PC (%)	
Model	6.09	3	2.03	3.6	0.0267		
A-shear Angle	0.176	1	0.176	0.3123	0.5811	0.85%	not significant
B-Yarn Spacing	3.25	1	3.25	5.76	0.0238	15.67%	significant
C-Fabric Thickness	2.66	1	2.66	4.72	0.0392	12.83%	significant
Residual	14.65	26	0.5636				
Lack of Fit	7.42	11	0.6749	1.4	0.2673		
Pure Error	7.23	15	0.482				
Cor Total	20.74	29					

Table 5. ANOVA in G_{12} for plain-woven jute composite.

Source	G_{12} (GPa)						Conditions
	SS	df	MS	F-Value	<i>p</i> -Value	PC (%)	
Model	0.7342	3	0.2447	5.12	0.0064		
A-Shear Angle	0.0639	1	0.0639	1.34	0.2579	3.23%	not significant
B-Yarn Spacing	0.3656	1	0.3656	7.65	0.0103	18.46%	significant
C-Fabric Thickness	0.3036	1	0.3036	6.35	0.0182	15.33%	significant
Residual	1.24	26	0.0478				
Lack of Fit	0.4962	11	0.0451	0.9073	0.5558		
Pure Error	0.7458	15	0.0497				
Cor Total	1.98	29					

Table 6. ANOVA in V_{12} for plain-woven jute composite.

Source	V_{12}						Conditions
	SS	df	MS	F-Value	<i>p</i> -Value	PC (%)	
Model	0.0028	3	0.0009	7.42	0.001		
A-Shear Angle	8.21×10^{-6}	1	8.21×10^{-6}	0.0652	0.8005	0.13%	Not significant
B-Yarn Spacing	0.0015	1	0.0015	12.06	0.0018	24.59%	significant
C-Fabric Thickness	0.0013	1	0.0013	10.12	0.0038	21.31%	significant
Residual	0.0033	26	0.0001				
Lack of Fit	0.0005	11	0	0.2446	0.9884		
Pure Error	0.0028	15	0.0002				
Cor Total	0.0061	29					

Table 7. ANOVA in E_{11} for twill-woven jute composite.

Source	E_{11} (GPa)						Conditions
	SS	df	MS	F-Value	<i>p</i> -Value	PC (%)	
Model	4.82	9	0.5352	200.83	<0.0001		
A-Shear Angle	0.3072	1	0.3072	115.3	<0.0001	6.31%	significant
B-Yarn Spacing	2.34	1	2.34	878.23	<0.0001	48.05%	significant
C-Fabric Thickness	1.67	1	1.67	625.52	<0.0001	34.29%	significant
Residual	0.0533	20	0.0027				
Lack of Fit	0.0145	5	0.0029	1.13	0.3886		
Pure Error	0.0387	15	0.0026				
Cor Total	4.87	29					

Table 8. ANOVA in G_{12} for twill-woven jute composite.

Source	SS	df	MS	G_{12} (GPa)		PC (%)	Conditions
				F-Value	p-Value		
Model	0.285	9	0.0317	472.91	<0.0001		
A-Shear Angle	0.0017	1	0.0017	25.23	<0.0001	0.59%	significant
B-Yarn Spacing	0.143	1	0.143	2135.89	<0.0001	49.95%	significant
C-Fabric Thickness	0.1215	1	0.1215	1814.39	<0.0001	42.44%	significant
Residual	0.0013	20	0.0001				
Lack of Fit	0.0013	5	0.0003	42.91	<0.0001		
Pure Error	0.0001	15	5.83×10^{-6}				
Cor Total	0.2863	29					

Table 9. ANOVA in V_{12} for twill-woven jute composite.

Source	SS	df	MS	V_{12}		PC (%)	Conditions
				F-Value	p-Value		
A-Shear Angle	0	1	0	88.34	<0.0001	0%	significant
B-Yarn Spacing	0.001	1	0.001	1773.28	<0.0001	52.63%	significant
C-Fabric Thickness	0.0008	1	0.0008	1536.98	<0.0001	42.11%	significant
Residual	0	20	5.40×10^{-7}				
Lack of Fit	0	5	2.16×10^{-6}	3901.03	<0.0001		
Pure Error	8.30×10^{-9}	15	5.53×10^{-10}				
Cor Total	0.0019	29					

Table 5 presents the ANOVA analysis and percentage contribution (PC%) for the in-plane shear modulus (G_{12}) of the plain-woven jute composite. The table shows that, similar to the results for E_{11} , yarn spacing and fabric thickness are also significant parameters for G_{12} . However, the shear angle does not significantly influence either response. The PC% values for this response are as follows: shear angle—23%, yarn spacing—18.46%, and fabric thickness—15.33%.

In Table 6, a similar trend is observed for the major Poisson's ratio (V_{12}), consistent with the results for E_{11} and G_{12} . This indicates that yarn spacing and fabric thickness are also crucial parameters for V_{12} . Additionally, the shear angle again shows no significant influence on the response. The PC% values for V_{12} in the plain-woven jute composites are shear angle—0.13%, yarn spacing—24.59%, and fabric thickness—21.31%. From Tables 4–6, it can be concluded that all responses exhibit a similar trend in terms of variable influence.

4.3. Analysis of Variance (ANOVA) Investigation for Twill Jute Composite

Similarly to the plain-woven jute composite, ANOVA was also conducted on the elastic moduli of the twill-woven jute composite, using a 95% confidence level. The results for the longitudinal Young's modulus (E_{11}) are presented in Table 7. As shown in Table 7, the p -values for all considered variables—shear angle, yarn spacing, and fabric thickness—are less than 0.05, indicating that each variable significantly affects E_{11} . In terms of percentage contributions, yarn spacing shows the highest influence at 48.05%, followed by fabric thickness at 34.29% and shear angle at 6.31%. A possible explanation for these results is that the twill weave has a more complex structure compared to the plain weave, which may significantly influence the mechanical properties of the composite.

In Table 8, the ANOVA analysis is presented for the in-plane shear modulus (G_{12}) of the twill-woven jute composite. Similarly to the findings for E_{11} , the p -values for all considered variables—shear angle, yarn spacing, and fabric thickness—are less than 0.05, indicating that each has a statistically significant effect on G_{12} . Furthermore, the percentage

contributions (PC%) for G_{12} demonstrate a consistent trend like E_{11} , shear angle contributes 0.59%, yarn spacing contributes 49.95%, and fabric thickness contributes 42.44%. This highlights that yarn spacing and fabric thickness are the dominant factors influencing the in-plane shear modulus in the twill-woven jute composite.

In Table 9, the ANOVA analysis for the major Poisson's ratio (V_{12}) of the twill-woven jute composite is presented. From the table, it can be observed that the p -values for all variables—shear angle, yarn spacing, and fabric thickness—are less than 0.05, indicating that each variable has a statistically significant effect on V_{12} , like E_{11} and G_{12} . However, in terms of percentage contribution (PC%), the shear angle shows no measurable influence, while yarn spacing provides the highest contribution at 52.63%, followed by fabric thickness at 42.11%.

4.4. Three-Dimensional Contour Graph Analysis for Plain Jute Composite

To further correlate the ANOVA analysis with the considered parameters and responses, the 3D contour graph analysis has been conducted. Figure 3 illustrates the contour graph for E_{11} , considering all the variables. As shown in Figure 3a, yarn spacing exhibits an inclined surface, indicating that an increase in yarn spacing significantly enhances Young's modulus. However, an increase in fabric thickness may lead to an increase in response as shown in Figure 3b. Aside from these two variables, the shear angle does not show any notable interaction point. Overall, the observed outcomes and the percentage of contribution (PC%) follow a similar pattern, further supporting the correlation between the variables and responses in this analysis.

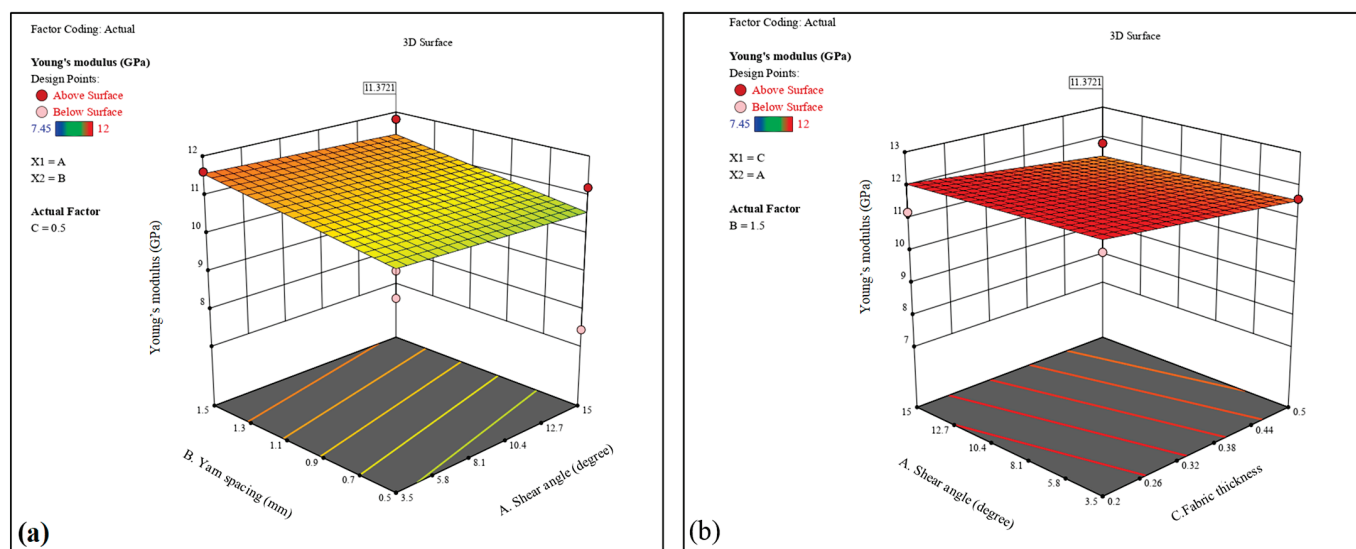


Figure 3. (a) Three-dimensional surface contour plot showing the effect of yarn spacing and shear angle on Young's modulus (E_{11}) for the plain-woven jute composite. (b) Three-dimensional surface contour plot showing the effect of fabric thickness and shear angle on shear modulus (E_{11}) for the plain-woven jute composite.

In Figure 4, the 3D contour graph analysis is presented. A similar pattern to that of E_{11} has been observed for G_{12} . For instance, the graph shows an inclined surface with respect to yarn spacing, indicating that an increase in yarn spacing can lead to an increase in the shear modulus. Fabric thickness also exhibits an inclined surface toward the shear modulus, reflecting its correlation with this response. However, the shear angle has a negligible effect. Overall, the ANOVA results and the 3D contour analysis reveal consistent observations.

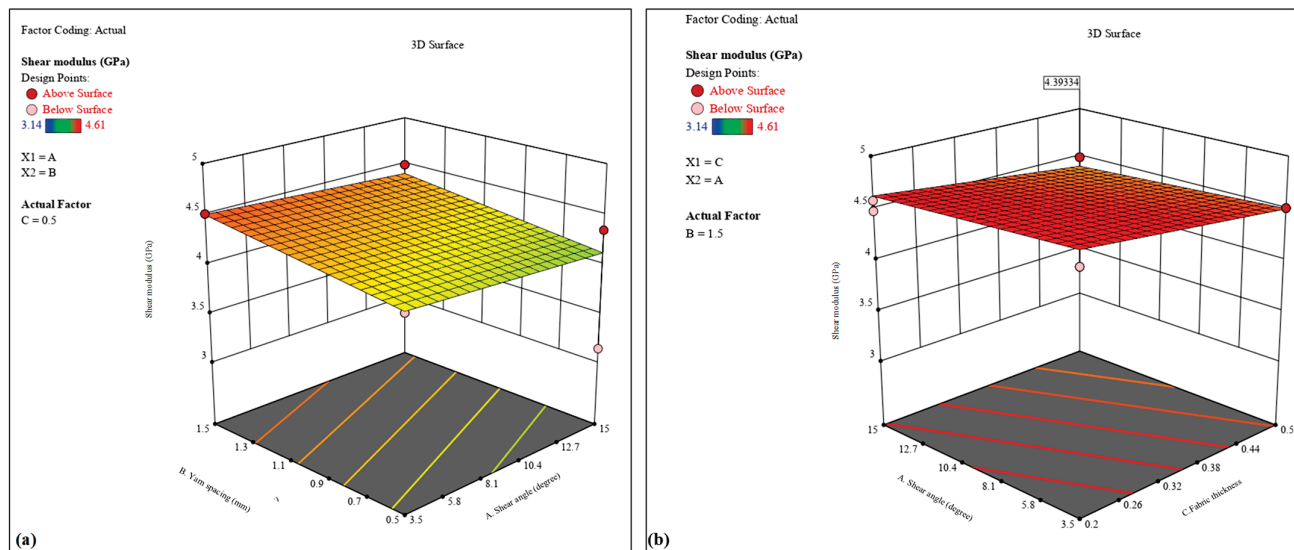


Figure 4. (a) Three-dimensional surface contour plot showing the effect of yarn spacing and shear angle on shear modulus (G_{12}) for the twill-woven jute composite. (b) Three-dimensional surface contour plot showing the effect of fabric thickness and shear angle on shear modulus (G_{12}) for the twill-woven jute composite.

In Figure 5, the 3D contour plot for Poisson's ratio (V_{12}) is presented. The graph appears mostly flat; however, a slight gradient can be observed along the yarn spacing and fabric thickness axes. In contrast, the shear angle does not show any significant effect. Overall, similar to the trends observed in E_{11} and G_{12} , V_{12} also demonstrates a consistent and satisfactory correlation with the input variables.

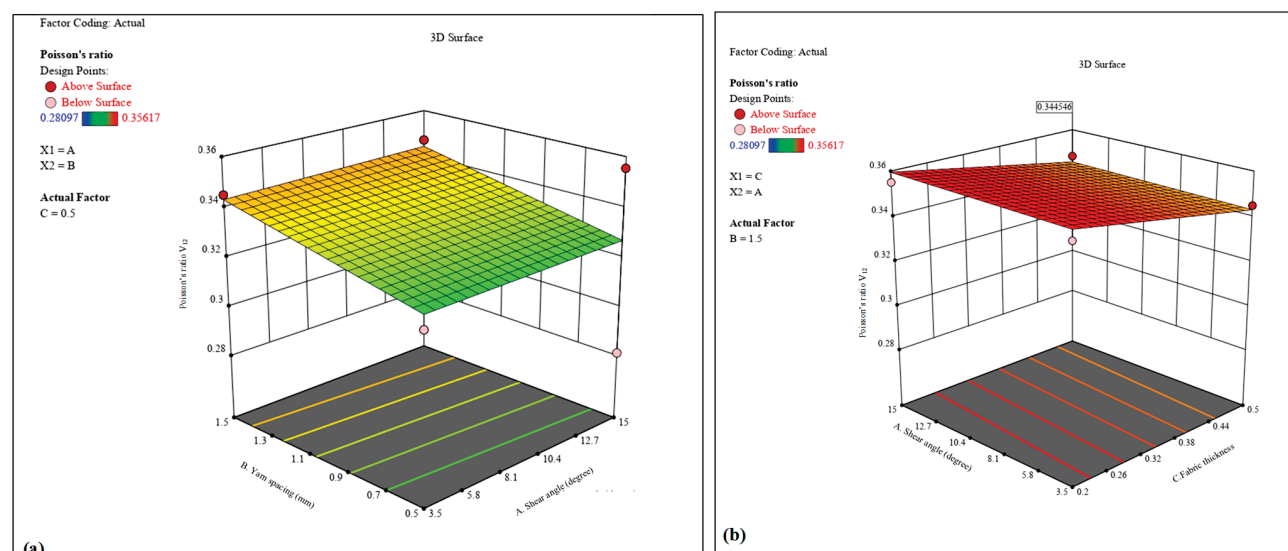


Figure 5. (a) Three-dimensional surface contour plot showing the effect of yarn spacing and shear angle on Poisson's ratio (v_{12}) for the plain-woven jute composite. (b) Three-dimensional surface contour plot showing the effect of fabric thickness and shear angle on Poisson's ratio (v_{12}) for the plain-woven jute composite.

4.5. Three-Dimensional Contour Graph Analysis for Twill Jute Composite

Similarly to the plain jute composite, a 3D contour graph analysis has been conducted for the twill jute composite, as presented in Figure 6. In Figure 6a,b, it can be observed that the highest value of E_{11} is obtained when both the shear angle and yarn spacing are low.

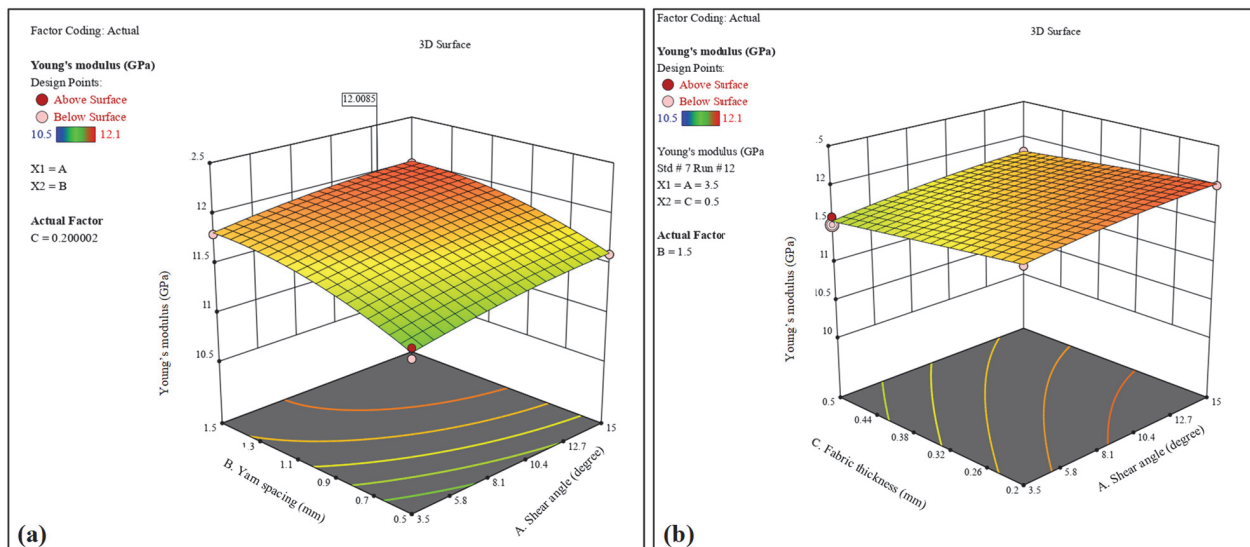


Figure 6. (a) Three-dimensional surface contour plot showing the effect of yarn spacing and shear angle on Young's modulus (E_{11}) for the twill-woven jute composite. (b) Three-dimensional surface contour plot showing the effect of fabric thickness and shear angle on shear modulus (E_{11}) for the twill-woven jute composite.

This can be attributed to the fact that as the shear angle increases, the yarns become more misaligned, resulting in a reduction in the effective elastic modulus. Similarly, greater yarn spacing reduces the number of fibers per unit area, which significantly decreases E_{11} . The graph clearly demonstrates that lower yarn spacing leads to an increase in E_{11} . Furthermore, with respect to fabric thickness, an increase in thickness results in a significant rise in E_{11} . Thicker fabrics contain more fibers, which contributes to the improved or increased value of E_{11} .

In Figure 7a,b, 3D contour graph analyses are presented to illustrate the correlations among all variables. In Figure 7a, it can be observed that the shear modulus (G_{12}) increases as both the shear angle and yarn spacing decrease. However, in Figure 7b, an increase in fabric thickness significantly enhances the shear modulus in the twill-woven jute composite.

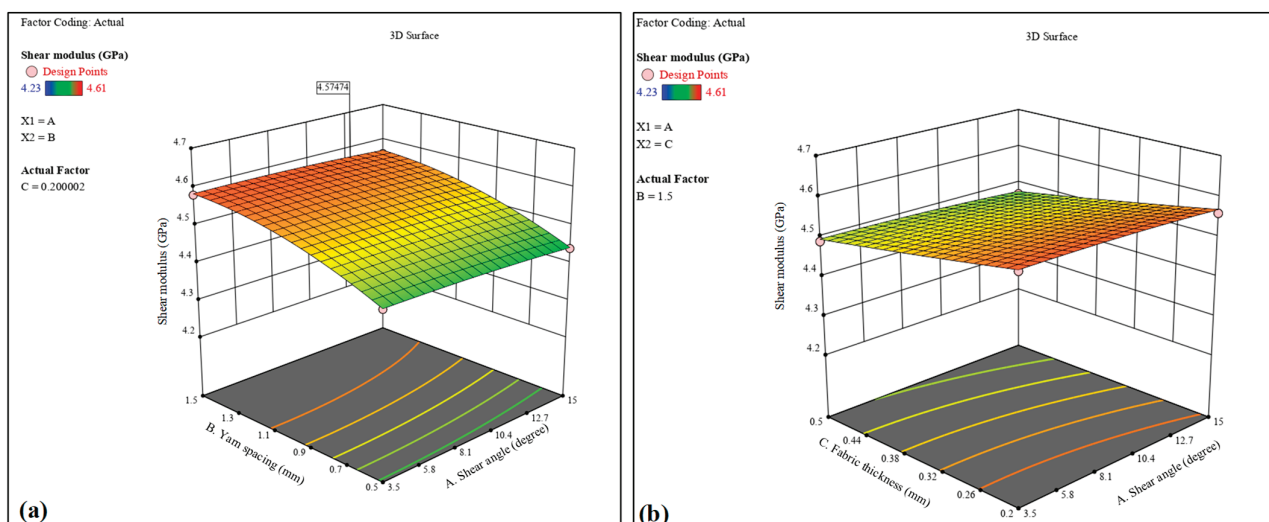


Figure 7. (a) Three-dimensional surface contour plot showing the effect of yarn spacing and shear angle on shear modulus (G_{12}) for the twill-woven jute composite. (b) Three-dimensional surface contour plot showing the effect of fabric thickness and shear angle on shear modulus (G_{12}) for the twill-woven jute composite.

In Figure 8a, the shear angle and yarn spacing exhibit a linear trend with Poisson's ratio (V_{12}) for the twill woven composite. It is evident from the figure that V_{12} increases with increasing shear angle and yarn spacing. However, in Figure 8b, fabric thickness shows only a minor influence, although a slight upward trend in V_{12} is still observed with increasing fabric thickness.

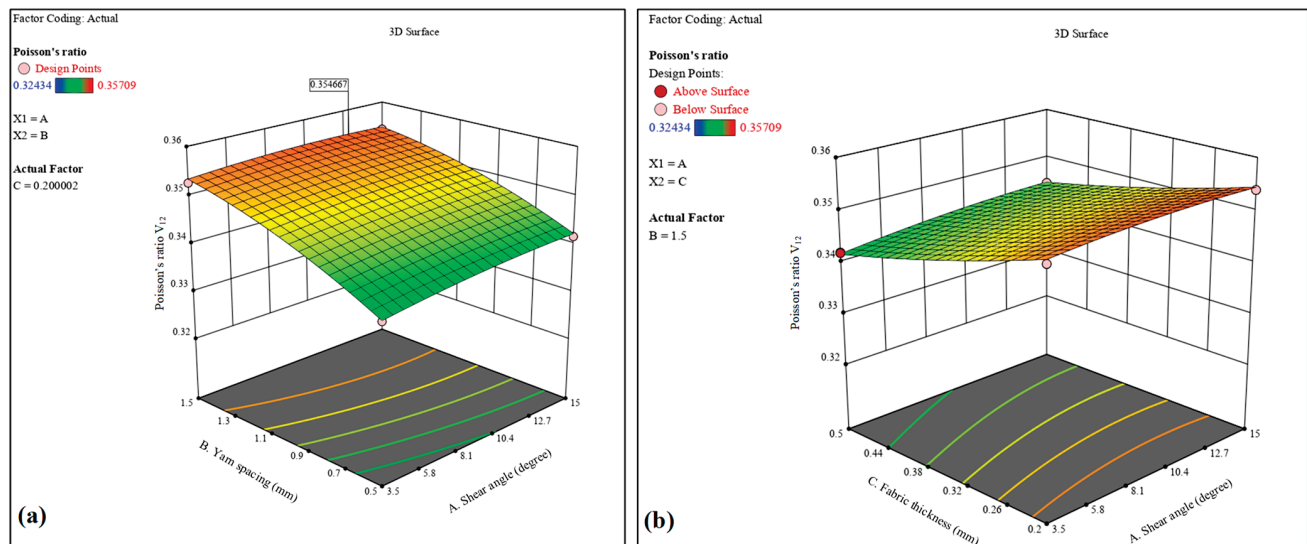


Figure 8. (a) Three-dimensional surface contour plot showing the effect of yarn spacing and shear angle on Poisson's ratio (v_{12}) for the twill-woven jute composite. (b) Three-dimensional surface contour plot showing the effect of fabric thickness and shear angle on Poisson's ratio (v_{12}) for the twill-woven jute composite.

5. Numerical Predictions and Optimizations

After comparing all outcomes using statistical methods such as ANOVA and 3D contour graph analysis, numerical predictions and simulations were conducted to validate the predicted variable responses and compare them with actual simulation results. Table 10 presents the optimized variables and their corresponding responses for the plain-woven jute composite, along with their difference. The differences between predicted and confirmed values range from 0% to 3%, which is within an acceptable margin of error. For this composite, the optimal combination of parameters is achieved at a shear angle of 9.25 degrees, yarn spacing of 1.0 mm, and fabric thickness of 0.35 mm.

Table 10. Numerical predictions and optimized outcomes for the plain-woven jute composite.

Predicted Variables	Values	Responses	Predicted Values	Confirmations	Difference
Shear angle	9.25	Young's modulus (E_{11} GPa)	11.40 GPa	11.10 GPa	2.63%
Yarn spacing	1.0 mm	Shear modulus (G_{12} GPa)	4.43 GPa	4.4 GPa	0.68%
Fabric thickness	0.35 mm	Poisson's ratio (V_{12})	0.34	0.34	0.0%

Similarly, a prediction analysis was also conducted for the twill-woven jute composites. The optimized variables were found to be a shear angle of 9.25 degrees, yarn spacing of 1.0 mm, and fabric thickness of 0.35 mm—identical to those of the plain-woven jute composite. Table 11 presents the optimized parameters, corresponding outcomes, and their differences. For the twill-woven composite, the differences between predicted and confirmed values were negligible, all falling below 1%.

Table 11. Numerical predictions and optimized outcomes for the twill-woven jute composite.

Predicted Variables	Values	Responses	Predicted Values	Confirmations	Difference
Shear angle	9.25	Young's modulus (E_{11} GPa)	11.58 GPa	11.54 GPa	0.35%
Yarn spacing	1.0 mm	Shear modulus (G_{12} GPa)	4.47 GPa	4.47 GPa	0.0%
Fabric thickness	0.35 mm	Poisson's ration (V_{12})	0.34	0.34	0.0%

6. Future Research Scope

This study aims to investigate the elastic moduli of natural fiber-reinforced jute composites using statistical and numerical methods, considering key manufacturing parameters such as shear angle, fabric thickness, and yarn spacing. The authors also recognize several promising directions for future research, particularly relevant to microscale investigations. The potential areas for future research are outlined below:

This research only investigated the plain- and twill-type-woven composite RVE using the ANSYS-19 material designer. Future research could involve solid experimentations and compare them with numerical and theoretical approaches to improve the elastic properties of woven or braided composites embedded in natural fibers.

This investigation analyzed only the interactions between the considered variables and responses. However, the interfacial bonding between fibers and the matrix could be further explored by examining different volume fractions through solid experimentations.

Environmental factors, such as humidity, temperature, and moisture, can significantly affect the mechanical performance of jute composites. Although this study focused on the mechanical properties of jute-woven composites, future research could explore the impact of these environmental factors through real experiments.

The study only considered two regular types of woven composite, namely plain and twill. However, a future comparative analysis can be generated numerically and experimentally with other types of woven structures, namely satin, multiaxial, hybrid, and so on, which would be embedded with natural fibers.

7. Conclusions

Natural fiber-reinforced composite materials have attracted significant attention in both the contemporary composite industry and academic research due to their potential to promote sustainability, renewability, and a reduced carbon footprint, along with offering low fabrication costs. Accordingly, plain- and twill-woven jute composites represent promising alternatives to conventional synthetic or polymeric composites, particularly for real-life applications where environmental sustainability is a primary concern. In this study, a series of numerical simulations were conducted to optimize key processing parameters and achieve the most favorable performance outcomes. These optimized parameters provide valuable insights for future researchers and manufacturers, helping to elucidate the significance and effects of the variables considered and guiding their application during the fabrication process. The main findings of the study are summarized below:

ANSYS Material Designer effectively predicted the elastic moduli of both plain- and twill-woven jute composites RVEs without complications and with minimal computational time. Therefore, this robust tool can be reliably used to determine key parameters for jute woven composites RVEs prior to the manufacturing process.

The plain-woven jute composite yielded superior outcomes across all DOE cases compared to the twill-woven composite. Interestingly, the highest and lowest outcomes for both composites were observed within the same DOE cases, indicating strong correlations

among the considered responses. Despite their structural differences, both composites exhibited their maximum and minimum values in DOE-1 and DOE-10, respectively.

ANOVA for the plain-woven composite, yarn spacing, and fabric thickness was found to significantly influence the elastic moduli, while the shear angle showed minimal impact. Conversely, in the twill-woven composite, all three parameters—shear angle, yarn spacing, and fabric thickness—played critical roles in affecting the elastic properties.

Based on the percentage contribution (PC%) analysis of the variables to the responses, it was observed that, in the plain-woven jute composite, the considered variables contributed approximately 0–24% to the overall responses. In contrast, the twill-woven jute composite exhibited significantly higher contributions, ranging from 0 to 53%.

The 3D contour analysis further supports the ANOVA results by illustrating the relationships between each response and the corresponding variables. Consistent and logical trends were validated through this analysis. Overall, it was observed that yarn spacing and fabric thickness exhibit significant surface interactions with all elastic moduli in both plain- and twill-woven composites. In contrast, the shear angle demonstrated negligible interaction effects across all responses.

The optimized outcomes for both plain- and twill-woven jute composites were obtained at a shear angle of 9.25 degrees, yarn spacing of 1.0 mm, and fabric thickness of 0.35 mm. For the plain-woven composite, the optimized properties were E_{11} —11.10 GPa, G_{12} —4.4 GPa, and V_{12} —0.34. In the case of the twill-woven composite, the optimized results were E_{11} —11.54 GPa, G_{12} —4.47 GPa, and V_{12} —0.34. The differences between the software predictions and numerical confirmations were minimal in both cases, indicating the high accuracy of the prediction method.

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Article

Predicting and Unraveling Flexural Behavior in Fiber-Reinforced UHPC Through Based Machine Learning Models

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Abstract: Predicting the flexural behavior of fiber-reinforced ultra-high-performance concrete (UHPC) remains a significant challenge due to the complex interactions among numerous mix design parameters. This study presents a machine learning-based framework aimed at accurately estimating the modulus of rupture (MOR) of UHPC. A comprehensive dataset comprising 566 distinct mixtures, characterized by 41 compositional and fiber-related variables, was compiled. Seven regression models were trained and evaluated, with Random Forest, Extremely Randomized Trees, and XGBoost yielding coefficients of determination (R^2) exceeding 0.84 on the test set. Feature importance was quantified using Shapley values, while partial dependence plots (PDPs) were employed to visualize both individual parameter effects and key interactions, notably between fiber factor, water-to-binder ratio, maximum aggregate size, and matrix compressive strength. To validate the predictive performance of the machine learning models, an independent experimental campaign was carried out comprising 26 UHPC mixtures designed with varying binder compositions—including supplementary cementitious materials such as fly ash, ground recycled glass, and calcium carbonate—and reinforced with mono-fiber (straight steel, hooked steel, and PVA) and hybrid-fiber systems. The best-performing models were integrated into a hybrid neural network, which achieved a validation accuracy of $R^2 = 0.951$ against this diverse experimental dataset, demonstrating robust generalizability across both material and reinforcement variations. The proposed framework offers a robust predictive tool to support the design of more sustainable UHPC formulations incorporating supplementary cementitious materials without compromising flexural performance.

Keywords: Ultra-High-Performance Concrete (UHPC); flexural strength; machine learning; hybrid fiber reinforcement; SHAP analysis; ensemble models; sustainability

1. Introduction

1.1. Background

UHPC is a complex material that has received a lot of interest within the structural engineering community, particularly due to its enhanced mechanical properties [1,2]. Recent reviews reiterate that, although UHPC routinely attains compressive strengths above 150 MPa and outstanding durability, its broad implementation remains constrained by the high price of specialized ingredients, limited design guidance and the complexity

of mixing/curing protocols [3]. UHPC is commonly utilized in multiple examples of high-performance structures, including high-rise buildings, long-span bridges, and the upgrading of current infrastructure [3,4]. Its exceptional compressive strength, durability, and resistance to extreme environmental conditions make UHPC a suitable material for applications where both performance and long life cycle are critical [5,6]. As UHPC is used in more and more demanding structural conditions, it is thus critical to know and predict how it will behave under different loading scenarios, particularly based on those properties that are key features for UHPC's applications, such as its flexural performance.

One of the major advancements in UHPC has been the use of fibers to improve the mechanical properties of UHPC, such as flexural strength and ductility. The fibers included (such as steel and polymeric, with the proper geometric and mechanical properties) significantly increased the material's flexural strength and ductility, making it appropriate for use in circumstances requiring significant bending resistance to cracking and deformation [7,8]. Fibers, particularly steel fibers, have been shown to improve the flexural performance of UHPC beams, thereby enhancing the material's capacity to withstand repeated stresses [9]. In addition, fibers play a crucial role in delaying crack propagation, which ultimately contributes to enhanced flexural strength and longer-lasting durability [9]. This extraordinary synergy between the fibers and matrix creates a material that is stronger and superior at resisting dynamic loading, compared to conventional concrete.

It is important to note that UHPC's flexural performance is emerging as a critical attribute driving its adoption across a range of applications. In structural systems, its capacity to sustain high bending stresses enables the design of precast bridge deck panels that offer enhanced durability and diminished maintenance requirements [10–12], as well as thin-shell structures capable of spanning significant distances without sacrificing structural stability [13]. Beyond structural use, UHPC's flexural performance is also a key feature for this fiber-reinforced composite deployment in precast façade components, leverages, and urban furniture [14,15]. Moreover, another UHPC application that necessitates a proper UHPC's flexural performance is the road pavement rehabilitation by means of ultra-thin overlays, commonly in the range of just 40 to 50 mm [10,16,17].

However, modeling the flexural behavior of fiber-reinforced UHPC is still a complex problem to solve. The complexity arises from the interactions of the material properties, failure modes, and modeling approach in defining the behavior of UHPC under flexural loading. Furthermore, the interaction between different fiber types and the different UHPC-making materials, such as mineral admixtures, polycarboxylate-based superplasticizers, and others, further complicates predictions, especially as combinations of fibers in varying quantities yield different mechanical properties [18]. These complexities suggest that more robust tools are required to predict the flexural behavior of UHPC more reliably and to make reasonable decisions about the use of UHPC within structures. In this case, ML (Machine Learning) offers a competitive means to predict the flexural behavior of fiber-reinforced UHPC by recognizing more complex patterns and relationships within a dataset. Many traditional analytical models observed in constructions, which theorize flexural behavior through experimental observations, are unable to describe complex interactions between fibers and the matrix, especially with nonlinear, high-dimensional data. Consequently, this study attempts to develop and apply ML models to predict flexural strength more objectively and reliably while leveraging their nonlinear modeling capabilities and insights on the impacts of various material components on flexural behavior.

Given that flexural performance is a defining property of UHPC in both structural and architectural applications, developing reliable models to predict this behavior is essential for effective material utilization and structural design. Due to the increasing use of UHPC in structural applications, accurate prediction of its behavior becomes necessary for the

purpose of optimized mix designs and material selection. In this paper, a complete solution through machine learning models, which have been trained on a database of 566 UHPC mixtures collected from scientific literature, with 41 input variables and an output variable, i.e., the UHPC modulus of rupture (MOR), is provided. First, a series of individual ML approaches are adjusted and evaluated. Then, the better-performing individual models are used to create SHAP (SHapley Additive exPlanations) to visualize the effects of essential mixture parameters on MOR response and provide an understanding of the effects of material property variations on performance. Further, the ensemble learning approaches are employed in this research, by means of better-performing individual models, to increase predictive accuracy when generalized, and a model that is potentially more effective at predicting flexural behavior than the individual ML approaches. Moreover, an experimental campaign is also developed with the intention of serving as an experimental evaluation of the ensembled model as well as the individual ones. By utilizing ML techniques, while learning about material behavior, this work serves as the measurement that pushes toward the future use of UHPC, or further future use of UHPC, in design projects that are demanding, and provides the safety that UHPC can perform under its extreme material properties maximally.

1.2. Bibliographical Overview

On the one hand, a comprehensive multiscale review by Du et al. [19] highlights two complementary research needs: (i) lowering the environmental footprint of UHPC through high-volume supplementary cementitious materials, reduced binder contents, and energy-efficient curing; and (ii) advancing multiscale-physics reinforcement strategies (including fiber hybrid systems) to fine-tune the response of this material.

On the other hand, while considerable advancements have been made in leveraging ML techniques to predict and evaluate the UHPC's mechanical characteristics, most of the existing literature remains centered around compressive strength [20–22]. In contrast, flexural behavior—despite its vital role in structural design and long-term durability—has not been explored as extensively. Moreover, the limited number of studies that do address this aspect often suffer from constraints related to material variation, fiber combinations, dataset size, and the complexity of the employed ML frameworks.

For example, Ergen et al. [23] examined the flexural properties of UHPC reinforced with conventional steel fibers through ML methodologies. However, their investigation is limited by a narrow selection of constituent materials and fiber types, lacking consideration for interactions between multiple fiber categories or variable dosages. Additionally, the relatively modest dataset (just 82 observations) used in their study poses challenges to the robustness and generalizability of their predictive models.

Similarly, Soni and Nateriya [24] performed experimental work on UHPC beams to analyze flexural behavior. While their contribution is noteworthy, it is confined to traditional steel fibers, without addressing hybrid fiber systems or potential synergies arising from multi-fiber configurations. On the other hand, their ML analysis also relies solely on Support Vector Machines (SVM), a method that may not capture complex, nonlinear patterns as effectively as more contemporary ensemble-based models [25].

In another notable effort, Qian et al. [26] utilized Artificial Neural Networks (ANN) to estimate the flexural strength of fiber-reinforced UHPC. Although the incorporation of ML in this context is commendable, the study is restricted by its use of a single fiber type, without considering the hybrid reinforcement system. Additionally, while the sample size of 317 observations is acceptable, a larger dataset could further enhance the model's robustness and generalizability. Furthermore, it lacks an in-depth evaluation of supplementary

cementitious materials (SCMs) such as silica fume (SF), fly ash (FA), and ground granulated blast-furnace slag (GGBFS), all of which are known to significantly affect flexural performance.

Lastly, Das and Abul Kashem [27] adopted hybrid ensemble models, including XGBoost and LightGBM, to predict both compressive and flexural strengths. While their work demonstrates progress in applying advanced ML algorithms, it does not incorporate fiber type diversity or explore the integration of various SCMs. Another important aspect is that the study primarily emphasizes predictive accuracy, without delving into the influence of different mix design parameters on flexural behavior.

To address these gaps, this study significantly expands the scope and depth of current knowledge by integrating a broader range of mineral admixtures and employing advanced machine learning methodologies. A comprehensive dataset comprising 566 UHPC mixtures collected from scientific literature was assembled, with 80% allocated for models training and the remaining 20% reserved for initial models' validation. To further strengthen the evaluation, an independent experimental campaign was conducted, encompassing 26 UHPC series developed with up to three distinct matrix formulations incorporating diverse mineral admixtures, including mixes both with and without fiber reinforcement. This experimental program included three types of fibers and explored hybrid fiber reinforcement systems by combining two different fiber types within a single mix. Models assessed included linear regression, K-Nearest Neighbors, Support Vector Regressor, XGBoost, Random Forest, Extremely Random Trees, and Artificial Neural Networks. A hybrid neural network, constructed by integrating the best-performing individual models ($R^2 > 0.80$ on the testing subset), demonstrated enhanced predictive performance and robustness. Collectively, these advancements not only refine the predictive capabilities for flexural strength in UHPC but also provide critical insights for optimizing mixture design in high-performance, fiber-reinforced concrete applications.

1.3. Scope and Significance of the Research

Recent machine-learning contributions that do report UHPC flexural data remain limited in scope. Ergen and Katlav [23] modeled only 82 mixes with a single steel-fiber morphology; Qian et al. [26] used 317 observations but omitted hybrid fibers and SCM interactions; and Das & Kashem [27] focused mainly on predictive accuracy without external validation. No peer-reviewed study to date has combined (i) a parametric dataset exceeding 500 UHPC mixes, (ii) multiple fiber classes in mono- and hybrid configurations, (iii) a broad suite of mineral admixtures, and (iv) independent laboratory validation.

Accordingly, the present study closes these gaps by assembling 566 mixes with 41 variables, training interpretable ensemble models, and verifying them against 26 new UHPC formulations covering three distinct matrix chemistries and hybrid fiber systems. Therefore, this research proposes a comprehensive predictive framework to assess the UHPC's MOR under flexural loadings by leveraging advanced ML techniques and ensemble learning strategies. Through a detailed examination of structural, matrix-related, and reinforcement parameters, the study identifies the key factors—such as fiber dosage, fiber type, cement content, silica fume proportion, and water-to-binder ratio—that critically influence flexural strength. By elucidating these relationships, the developed models support data-informed material selection and formulation strategies, enabling the design of optimized, high-performance UHPC mixtures for structural elements such as bridge girders, high-strength pavements, and seismic-resistant structures.

Importantly, the study integrates SHAP-based interpretability techniques to provide actionable insights, enhancing the transparency and trustworthiness of ML-driven predictions. This allows engineers and researchers not only to predict performance accurately but also to understand the underlying drivers behind flexural behavior. In doing so, the re-

search promotes a shift toward intelligent, performance-oriented, and sustainable material design in civil engineering practice, ultimately advancing the broader adoption of UHPC in demanding structural applications.

Figure 1 puts forward a flow chart of the study presented herein.

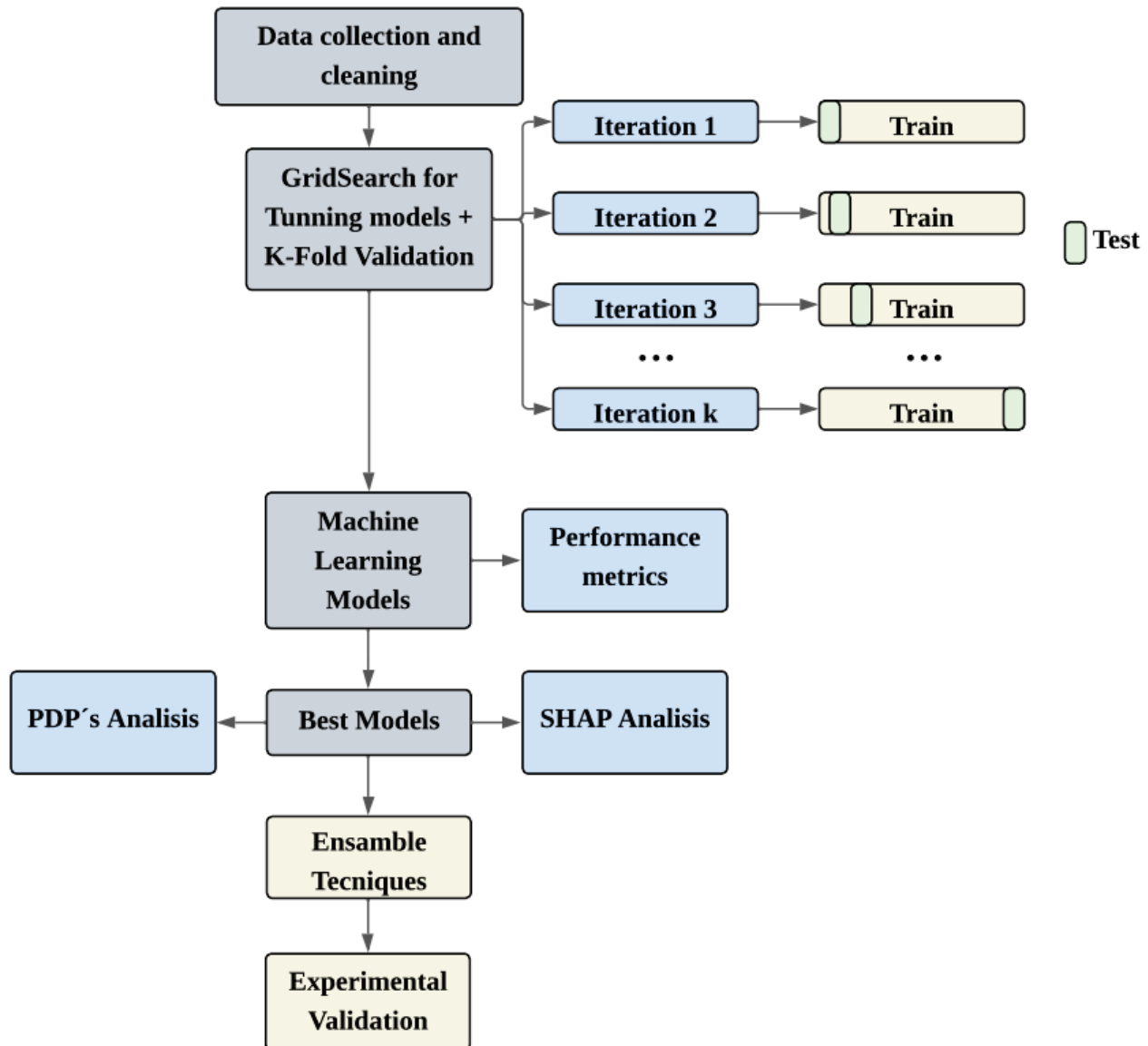


Figure 1. Flow Chart of the study.

2. Materials and Methods

2.1. Data Compilation

2.1.1. Literature-Based Dataset for Models Development

The literature-based dataset employed in this investigation was constructed from a wide-ranging corpus of peer-reviewed articles and proceedings from international conferences centered on UHPC research. In total, 566 experimental data points were extracted from 36 scholarly sources [7,8,28–61], each reporting both UHPC mix compositions and corresponding MOR values, the latter serving as the principal response metric in this study. Studies that did not conform to ASTM C1609/C1609M-19 [62] or equivalent flexural test protocols were not taken into account in the database. This standard specifies a span-to-depth ratio of approximately 4, which ensures the development of a constant-moment region with negligible shear influence.

To capture the complex, multi-scale phenomena governing flexural performance, 41 independent variables (IVs) were identified and harmonized across the dataset. These span from binder composition and admixture content to aggregate features, proportioning parameters, and descriptors of fiber geometry and mechanical properties. All raw values extracted from the 36 source studies were first converted to a common unit system (i.e., constituent contents in $\text{m}^3 \text{m}^{-3}$; fiber mass in kg m^{-3} ; mechanical strengths in MPa). The explanatory variables used in modeling were classified into six main categories:

- Group 1: IV1–IV14 (UHPC paste-making constituents):
- These variables represent the volumetric content (m^3/m^3) of different binder components in the UHPC mix:
 - IV1: Cement.
 - IV2: Silica Fume (SF).
 - IV3: Fly Ash (FA).
 - IV4: Ground Granulated Blast-Furnace Slag (GGBFS).
 - IV5: Ground Recycled Glass.
 - IV6: Rice Husk Ash (RHA).
 - IV7: Fluid Catalytic Cracking Catalyst Residue (FC3R) from petroleum refining.
 - IV8: Metakaolin.
 - IV9: Calcium Carbonate.
 - IV10: Other SCMs not listed in IV2–IV8 (m^3/m^3).
 - IV11: Water content.
 - IV12: Superplasticizer dosage.
 - IV13: Quartz powder.
 - IV14: Other non-reactive additions (excluding IV9 and IV13).
- Group 2: IV15–IV17 (Aggregate):
 - IV15: Fine aggregate (sand) content (m^3/m^3).
 - IV16: Coarse aggregate content (m^3/m^3).
 - IV17: Maximum aggregate size (μm).
- Group 3: IV18–IV19 (Mix Proportioning and Mechanical Properties):
 - IV18: Water-to-binder ratio.
 - IV19: Compressive strength of UHPC (MPa).
- Group 4: IV20–IV21 (Global Fiber Properties):
 - IV20: Total fiber volume fraction (m^3/m^3).
 - IV21: Global fiber factor for the UHPC mix.
- Group 5: IV22–IV31 (Fiber Type 1 Characteristics):
 - IV22: Binary variable indicating if Fiber 1 is straight steel (1 = yes, 0 = no).
 - IV23: Hooked-end steel fiber (1 = yes, 0 = no).
 - IV24: Polymeric fiber (1 = yes, 0 = no).
 - IV25: Tensile strength of Fiber 1 (MPa).
 - IV26: Length of Fiber 1 (mm).
 - IV27: Equivalent diameter of Fiber 1 (mm).
 - IV28: Aspect ratio (length-to-diameter) of Fiber 1.
 - IV29: Mass of Fiber 1 in the mix (kg/m^3).
 - IV30: Volume fraction of Fiber 1 (m^3/m^3).
 - IV31: Fiber factor for Fiber 1.
- Group 6: IV32–IV41 (Fiber Type 2 Characteristics):

These variables replicate the descriptors used for Fiber 1, applied to a second type of fiber (when present):

- IV32: Binary variable indicating if Fiber 2 is straight steel (1 = yes, 0 = no).
- IV33: Hooked-end steel fiber (1 = yes, 0 = no).
- IV34: Polymeric fiber (1 = yes, 0 = no).
- IV35: Tensile strength of Fiber 2 (MPa).
- IV36: Length of Fiber 2 (mm).
- IV37: Equivalent diameter of Fiber 2 (mm).
- IV38: Aspect ratio of Fiber 2.
- IV39: Mass of Fiber 2 in the mix (kg/m^3).
- IV40: Volume fraction of Fiber 2 (m^3/m^3).
- IV41: Fiber factor for Fiber 2.

This diverse and multiscale set of variables enables a detailed representation of UHPC composition and fiber reinforcement strategies, facilitating accurate modeling of its flexural behavior. The inclusion of both cementitious matrix- and fiber-level descriptors ensures that the ML models capture not only the influence of bulk constituents but also the synergistic and hybrid effects of multi-type fiber reinforcement.

Initial statistical insights are provided in Figure 2, which features violin plots for all input variables, allowing visualization of their density distributions. Moreover, Figure 3 presents the distribution of the response variable (MOR).

Figure 3 illustrates the distribution of the MOR values across the compiled dataset. The histogram reveals a near-Gaussian distribution centered around a median value of 22.63, as indicated by the vertical dashed line. This central tendency, reinforced by the kernel density estimate (KDE) curve, highlights the concentration of data points near the median, with relatively few extreme values. Understanding this distribution is critical for training robust machine learning models, as it informs assumptions about data variability, the presence of outliers, and potential skewness, all of which influence algorithm selection and feature engineering.

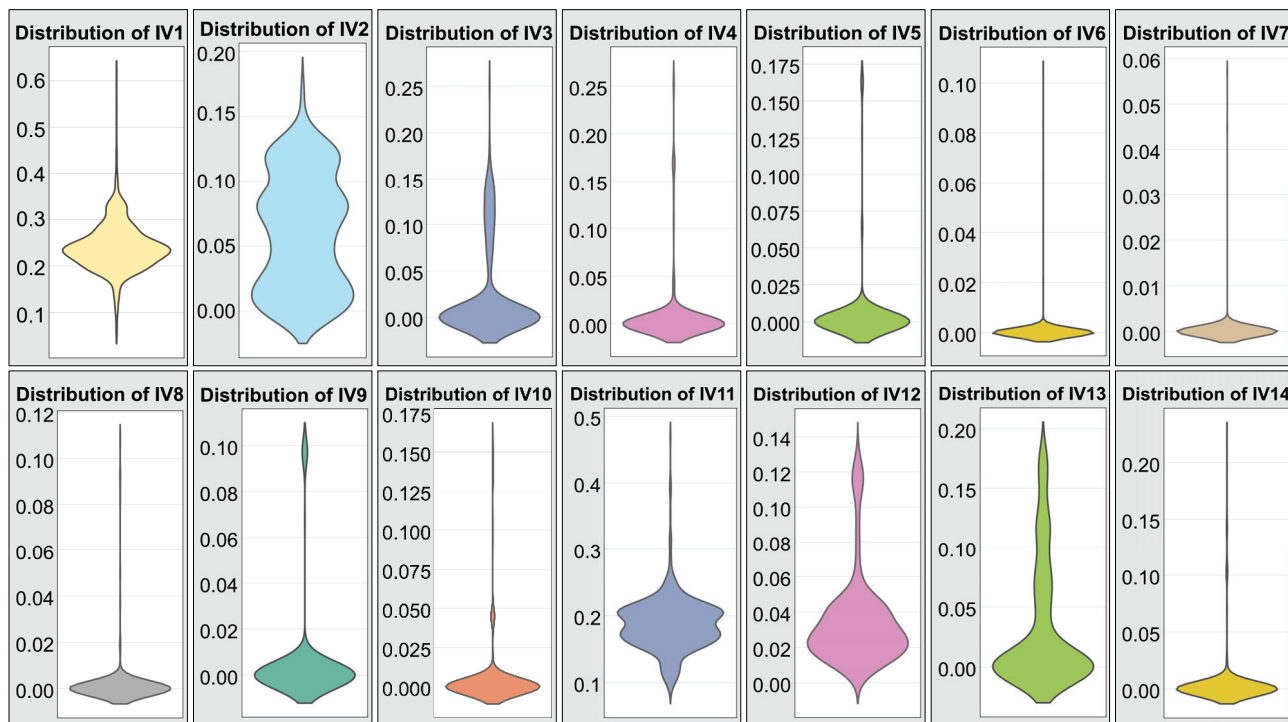


Figure 2. Cont.

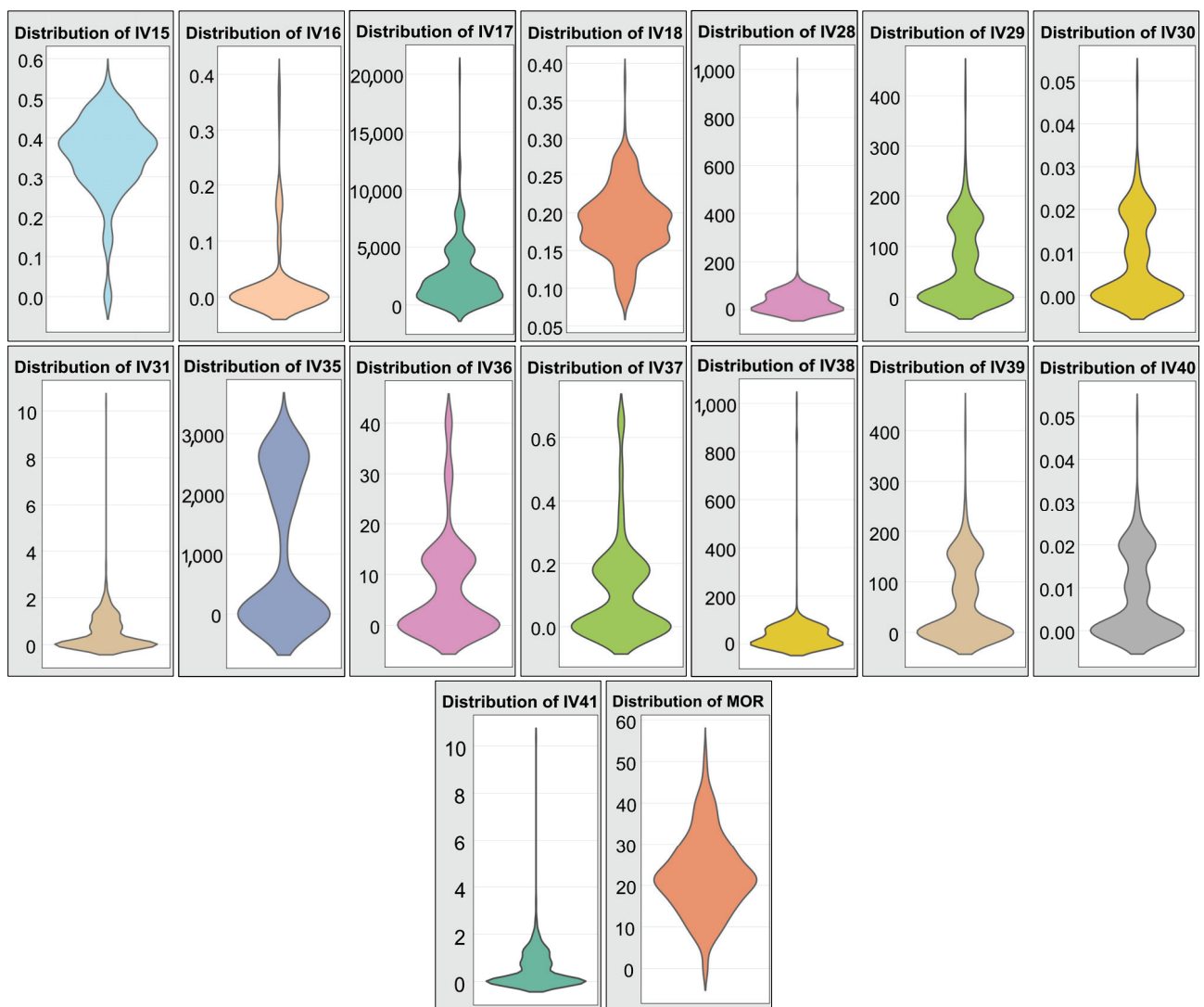


Figure 2. Violin plots distribution of the study's variables.

Before training the models, the data from the database were preprocessed. Specifically, the numeric predictors were centered and scaled ($\mu = 0$, $\sigma = 1$) to eliminate scale-induced bias while preserving the underlying physical relationships.

Figure 4 presents the Pearson correlation matrix, offering a detailed view of linear relationships among the 41 input variables and the MOR. Several features exhibit noteworthy correlations with flexural strength. Notably, cement content (IV1), superplasticizer dosage (IV12), and fiber factor (IV21) show positive associations with MOR, suggesting their reinforcing roles in enhancing flexural capacity. Additionally, compressive strength (IV19) and total fiber volume (IV20) display moderate correlations, aligning with established knowledge of their mechanical contributions.

Conversely, the water-to-binder ratio (IV18) shows a marked negative correlation with MOR, emphasizing its detrimental impact on matrix integrity and flexural response. Similarly, the maximum aggregate size (IV17) exhibits a negative relationship, possibly reflecting poorer fiber dispersion and reduced homogeneity in larger-grained mixes.

It is also important to note that the matrix put forward in Figure 3 serves as a diagnostic foundation for subsequent model development, enabling informed feature selection and deeper interpretability in predicting UHPC's flexural performance through machine learning frameworks.

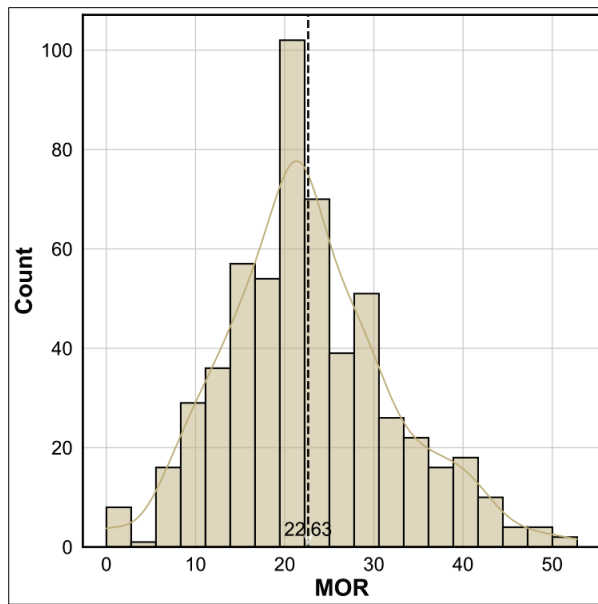


Figure 3. Statistical depiction of distributions of the response variable (MOR).

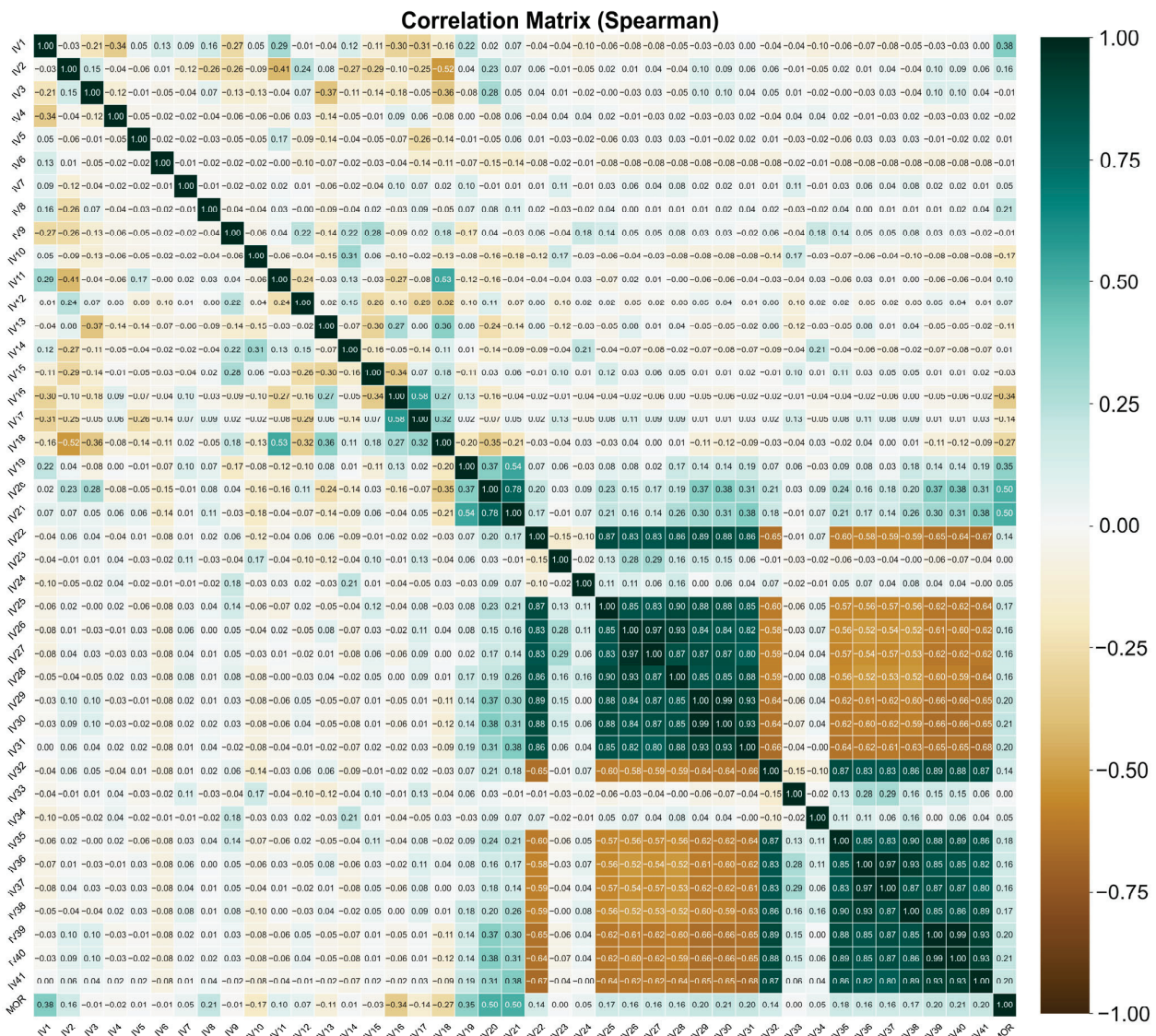


Figure 4. Matrix encoded with Pearson's correlation coefficients.

Finally, it is also relevant to say that for model development, 80% of the compiled dataset was randomly allocated to the training data subset, while the remaining 20% was reserved for testing purposes to evaluate model performance after adjusting with the training data.

2.1.2. Experimental Dataset for External Model Validation

To further validate the predictive performance of the machine learning models, an independent experimental dataset was generated, comprising 26 distinct UHPC mixtures. Flexural tests were conducted on $70 \times 70 \times 350$ mm prismatic specimens (span = 280 mm) under third-point loading conditions, following ASTM C1609/C1609M-19 [62] guidelines. The standard adopts a span-to-depth ratio of 4, which promotes a central region dominated by bending, with minimal shear contribution.

For each mixture, three prismatic specimens were cast and tested under flexural loading using a third-point loading configuration, as presented in Figure 5. For each UHPC mixture, three identical specimens were tested under the same conditions, and the modulus of rupture was reported as the average of these three values, in accordance with ASTM C1609/C1609M-19.

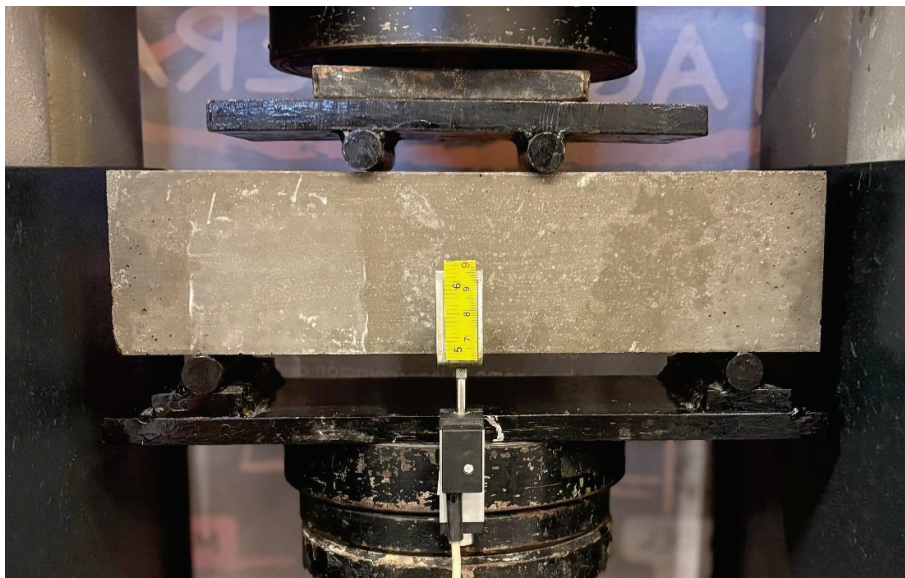


Figure 5. Test setup for flexural loading of UHPC as per ASTM C1609/C1609M-19 [62].

The mixture designs were developed following the modified Andreasen and Andersen particle packing model, aiming to maximize packing density and improve homogeneity. Cement and silica fume were used as the primary cementitious components across all formulations, with selected mixes incorporating additional mineral admixtures—including fly ash, ground recycled glass, and calcium carbonate—to evaluate their effects on flexural behavior.

Three types of fibers were employed as reinforcement strategies: straight high-strength steel fibers, hooked-end steel fibers, and polyvinyl alcohol (PVA) polymeric fibers. Both mono-fiber and hybrid fiber-reinforced systems were investigated, the latter combining two distinct fiber types to explore potential synergistic enhancements in post-cracking performance.

UHPC mixtures were prepared using a 20 L food-type mixer. The mixing process was executed in sequential stages to ensure uniform material dispersion. First, water and superplasticizer were blended at low speed for one minute. All powders—including cement, silica fume, and additional mineral admixtures—were then gradually incorporated and mixed at low speed for another minute. The mixing speed was increased progressively to medium and then high, continuing for approximately 3 to 5 min until a homogeneous

and flowable paste was achieved. Sand was then added and mixed for one minute at low speed, followed by two minutes at high speed. Fibers were incorporated at low speed (speed 1) to avoid clumping or fiber balling. After fiber addition, the speed was raised to medium (speed 3) for two minutes and finally to maximum speed for three minutes. Upon completion of the mixing process, three prismatic specimens of the aforementioned dimensions ($70 \times 70 \times 350$ mm) were cast for each series and tested in accordance with ASTM C1609/C1609M-19 under third-point loading.

Detailed mechanical and geometric properties of the fibers are presented in Table 1. Table 2 summarizes the complete composition and material dosages of the three UHPC matrices used for validation, specifying the contents of cement (IV1), silica fume (IV2), fly ash (IV3), glass powder (IV5), calcium carbonate powder (IV9), water (IV11), superplasticizer (IV12), and sand (IV15), as well as the maximum aggregate size (IV17), water-to-binder ratio (IV18), and compressive strength (IV19). Measurement units are consistent with those outlined in Section 2.1.1. Variables from groups 1 and 2 not explicitly listed in Table 2 were assigned a value of zero across all matrices.

Table 1. Mechanical and geometrical properties of fibers used in the experimental mixtures.

Notation	Material	Type/Shape	Tensile Strength	Length (mm)	Diameter (mm)	l_f/d_f
OL	Steel	Straight	2650 (MPa)	13	0.20	65
H	Steel	Hooked	1800 (MPa)	35	0.50	70
PVA	PVA	Straight	1600 (MPa)	6	0.02	300

Table 2. Composition of UHPC matrices considered (information corresponding to IV's Groups 1, 2 and 3).

Notation	IV1	IV2	IV3	IV5	IV9	IV11	IV12	IV15	IV17	IV18	IV19
A	0.254	0.115	0	0	0.061	0.2	0.036	0.325	600	0.16	157
B	0.254	0.045	0.084	0.061	0.061	0.216	0.021	0.319	600	0.15	153
C	0.231	0.068	0	0.085	0.033	0.226	0.024	0.334	600	0.19	124

Finally, Table 3 presents the UHPC series reinforced with fibers and used for the experimental validation of the machine learning models. The series notation is straightforward: the first character denotes the UHPC matrix type, the second group specifies the primary fiber type and its volume fraction, and, when applicable, the third group identifies the secondary fiber in hybrid systems along with its corresponding dosage. For instance, A0 indicates matrix A without fiber reinforcement (plain UHPC); B-H2 refers to matrix B reinforced with 2% by volume of hooked-end steel fibers; and A-OL1-PVA1 represents matrix A reinforced with a hybrid combination of 1% straight steel fibers (OL) and 1% PVA fibers.

In total, the experimental program comprised 26 series, including three plain UHPC series, eighteen mono-fiber reinforced series, and five hybrid-fiber reinforced series.

In light of the information presented in Tables 2 and 3, Tables 4–6 present information on the IVs of Groups 5, 5, and 6, respectively. The IV values for these groups are zero for dosages A0, B0, and C0, as they do not have fiber reinforcement. Similarly, information on Group 6 is only presented for the hybrid fiber blend series.

Table 3. Fiber-reinforced UHPC series experimentally evaluated.

UHPC's Series Notation	Matrix	Fiber 1 Type	Fiber 1 Volume Fraction	Fiber 2 Type	Fiber 2 Volume Fraction
A0	A	-	-	-	-
A-OL1	A	OL	1%	-	-
A-OL2	A	OL	2%	-	-
A-H1	A	H	1%	-	-
A-H2	A	H	2%	-	-
A-PVA1	A	PVA	1%	-	-
A-PVA2	A	PVA	2%	-	-
A-OL1-H1	A	OL	1%	H	1%
A-OL1-PVA	A	OL	1%	PVA	1%
A-H1-PVA1	A	H	1%	PVA	1%
A-OL1.5-PVA0.5	A	OL	1.5%	PVA	0.5%
A-H1.5-PVA0.5	A	H	1.5%	PVA	0.5%
B0	B	-	-	-	-
B-OL1	B	OL	1%	-	-
B-OL2	B	OL	2%	-	-
B-H1	B	H	1%	-	-
B-H2	B	H	2%	-	-
B-PVA1	B	PVA	1%	-	-
B-PVA2	B	PVA	2%	-	-
C0	C	-	-	-	-
C-OL1	C	OL	1%	-	-
C-OL2	C	OL	2%	-	-
C-H1	C	H	1%	-	-
C-H2	C	H	2%	-	-
C-PVA1	C	PVA	1%	-	-
C.PVA2	C	PVA	2%	-	-

Table 4. Global fiber properties (Group 4: IV20–IV21) for the experimental series.

UHPC's Series Notation	IV20	IV21
A-OL1	0.01	0.65
A-OL2	0.02	1.3
A-H1	0.01	0.7
A-H2	0.02	1.4
A-PVA1	0.01	3
A-PVA2	0.02	6
A-OL1-H1	0.02	1.35
A-OL1-PVA	0.02	3.65
A-H1-PVA1	0.02	3.7
A-OL1.5-PVA0.5	0.02	2.15

Table 4. *Cont.*

UHPC's Series Notation	IV20	IV21
A-H1.5-PVA0.5	0.02	2.55
B-OL1	0.01	0.65
B-OL2	0.02	1.3
B-H1	0.01	0.7
B-H2	0.02	1.4
B-PVA1	0.01	3
B-PVA2	0.02	6
C-OL1	0.01	0.65
C-OL2	0.02	1.3
C-H1	0.01	0.7
C-H2	0.02	1.4
C-PVA1	0.01	3
C.PVA2	0.02	6

Table 5. Fiber Type 1 characteristics (Group 5: IV22–IV31) for the experimental series.

UHPC's Series Notation	IV22	IV23	IV24	IV25	IV26	IV27	IV28	IV29	IV30	IV31
A-OL1	1	0	0	2650	13	0.2	65	78.5	0.01	0.65
A-OL2	1	0	0	2650	13	0.2	65	157	0.02	1.3
A-H1	0	1	0	1800	35	0.5	70	78.5	0.01	0.7
A-H2	0	1	0	1800	35	0.5	70	157	0.02	1.4
A-PVA1	0	0	1	1600	6	0.02	300	11.5	0.01	3
A-PVA2	0	0	1	1600	6	0.02	300	23	0.02	6
A-OL1-H1	1	0	0	2650	13	0.2	65	78.5	0.01	0.65
A-OL1-PVA1	1	0	0	2650	13	0.2	65	78.5	0.01	0.65
A-H1-PVA1	0	1	0	1800	35	0.5	70	78.5	0.01	0.7
A-OL1.5-PVA0.5	1	0	0	2650	13	0.2	65	78.5	0.015	0.975
A-H1.5-PVA0.5	0	1	0	1800	35	0.5	70	78.5	0.015	1.05
B-OL1	1	0	0	2650	13	0.2	65	78.5	0.01	0.65
B-OL2	1	0	0	2650	13	0.2	65	157	0.02	1.3
B-H1	0	1	0	1800	35	0.5	70	78.5	0.01	0.7
B-H2	0	1	0	1800	35	0.5	70	157	0.02	1.4
B-PVA1	0	0	1	1600	6	0.02	300	11.5	0.01	3
B-PVA2	0	0	1	1600	6	0.02	300	23	0.02	6
C-OL1	1	0	0	2650	13	0.2	65	78.5	0.01	0.65
C-OL2	1	0	0	2650	13	0.2	65	157	0.02	1.3
C-H1	0	1	0	1800	35	0.5	70	78.5	0.01	0.7
C-H2	0	1	0	1800	35	0.5	70	157	0.02	1.4
C-PVA1	0	0	1	1600	6	0.02	300	11.5	0.01	3
C.PVA2	0	0	1	1600	6	0.02	300	23	0.02	6

Table 6. Fiber Type 2 characteristics (Group 5: IV32–IV41) for the experimental series.

UHPC's Series Notation	IV32	IV33	IV34	IV35	IV36	IV37	IV38	IV39	IV40	IV41
A-OL1-H1	0	1	0	1800	35	0.5	70	78.5	0.01	0.7
A-OL1-PVA1	0	0	1	1600	6	0.02	300	11.5	0.01	3
A-H1-PVA1	0	0	1	1600	6	0.02	300	11.5	0.01	3
A-OL1.5-PVA0.5	0	0	1	1600	6	0.02	300	11.5	0.005	1.5
A-H1.5-PVA0.5	0	0	1	1600	6	0.02	300	11.5	0.005	1.5

It is important to note that, for prediction purposes, for each experimental series, the value of any IV not described in Tables 2 and 4–6 will be equal to zero.

2.2. Machine Learning

2.2.1. Linear Regression Models

Linear Regression (LR) is among the simplest and most widely adopted statistical models for predicting continuous outcomes in machine learning. It operates under the assumption of a linear relationship between the independent variables (features) and the dependent variable (target). The general form of a linear regression model is expressed in Equation (1):

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \dots + \beta_n x_n + \epsilon \quad (1)$$

where y denotes the predicted output (flexural strength), β_0 is the intercept term, β_1 through β_{41} represent the coefficients corresponding to each input variable x_1 to x_n , and ϵ accounts for the residual error between predicted and observed values.

Model fitting in linear regression can be achieved through analytical methods, such as the normal equation, or iterative optimization techniques like gradient descent. In this work, the coefficients were determined using the normal Equation (2):

$$\hat{\beta} = (X^T X)^{-1} X^T y \quad (2)$$

where β is the vector of estimated regression coefficients, X is the input feature matrix, and y is the vector of observed response values. This closed-form solution enables the direct computation of the optimal parameters, provided that the matrix $(X^T X)$ is invertible.

By establishing a baseline predictive performance, the LR model serves as a reference for evaluating the benefits of more complex machine learning algorithms applied to flexural strength prediction.

2.2.2. K-Nearest Neighbors (KNN) Models

The K-Nearest Neighbors (KNN) algorithm is a non-parametric, instance-based learning technique commonly employed for both classification and regression tasks. Its predictive strength relies on the assumption that similar input instances tend to exhibit similar output behaviors, making it particularly suitable for materials science applications where complex, multi-variable interactions are prevalent [63].

For regression problems, the working principle of KNN can be summarized as follows:

- The algorithm computes the distance between a query point and all points in the training dataset using a metric such as the Euclidean distance;
- Identifies the k closest neighbors;
- Predicts the output as the average of the target values of these neighbors.

Formally, the Euclidean distance between two instances, x_i and x_j , each characterized by n attributes, is calculated as Equation (3):

$$d(x_i, x_j) = \sqrt{\sum_{r=1}^n (ar(x_i) - ar(x_j))^2} \quad (3)$$

where $ar(x)$ denotes the value of the r^{th} attribute of instance x .

For its part, the predicted output for a test point x_{test} is then given by Equation (4):

$$\hat{y}(x_{test}) = \frac{1}{k} \sum_{i=1}^k y_i \quad (4)$$

where y_i represents the target value of the i^{th} nearest neighbor, and k is the number of neighbors considered [64,65]. The optimal value of k is determined through a k -fold cross-validation, selecting the number of neighbors that minimized the prediction error on the validation set [63].

2.2.3. Support Vector Regression Model

Support Vector Regression (SVR) is a supervised learning algorithm derived from Support Vector Machines (SVM), designed to perform regression tasks by identifying a function that approximates the underlying data within a specified margin of tolerance. Rather than minimizing the prediction error for each training point, SVR seeks to fit a model within an ε -insensitive tube, penalizing only deviations exceeding this threshold.

The prediction function for SVR can be expressed as depicted in Equation (5):

$$z(a, w) = (w \cdot \phi(a) + c) \quad (5)$$

where

- $z(a, w)$ is the predicted value for input a .
- w is the weight vector.
- $\phi(a)$ is a feature function that maps the input a to a higher-dimensional space.
- c is a bias term.

2.2.4. Random Forest

Random Forest (RF) is a robust ensemble learning method designed for both regression and classification tasks. Building upon decision trees, RF mitigates overfitting and variance by constructing multiple trees and aggregating their predictions. In regression problems, such as predicting the flexural strength of UHPC, the final output is typically obtained by averaging the individual tree predictions, as illustrated schematically in Figure 6. In this diagram, the yellow circles denote the sequence of internal decision nodes that a specific input vector x_i that traverses in each tree: beginning at the root node, following successive split criteria based on feature values, and ultimately arriving at the terminal leaf node (also highlighted in yellow) whose numeric output contributes to $f_t(x_i)$. All remaining nodes, shown in gray, represent branches not visited by that sample. This ensemble approach significantly enhances model stability and generalization, particularly when dealing with complex and heterogeneous datasets [66].

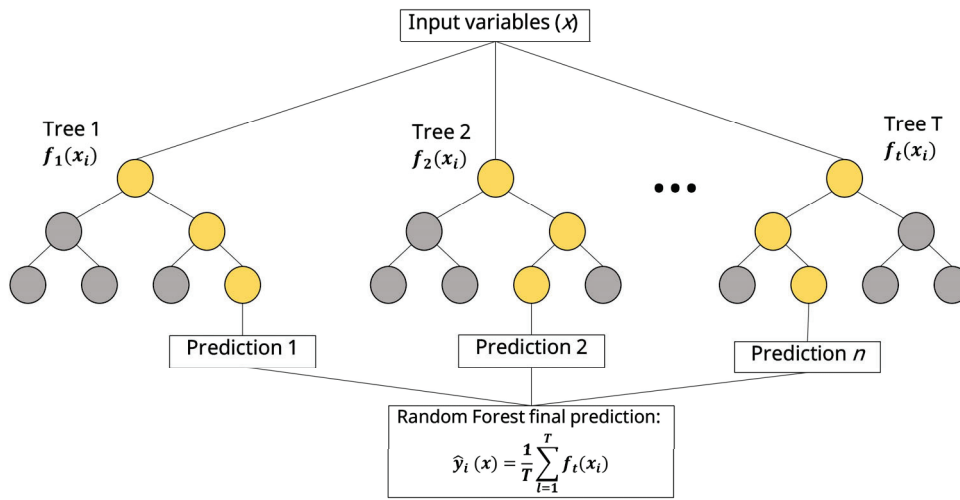


Figure 6. Scheme of an RF model used for regression purposes.

Mathematically, the prediction for the i^{th} instance is given by Equation (6):

$$\hat{y}_i = \frac{1}{T} \sum_{t=1}^T f_t(x_i) \quad (6)$$

where $\hat{y}_i$ is the predicted output for the i^{th} sample, T is the total number of trees in the forest, $f_t(x_i)$ denotes the prediction of the t^{th} tree, and x_i represents the input feature vector for that instance [66].

2.2.5. Extreme Random Trees

The Extremely Randomized Forest (XRF) algorithm is an advanced ensemble learning technique derived from RF approaches, aimed at further reducing overfitting and enhancing predictive performance. Unlike RF, XRF constructs trees by using the entire training dataset and introducing additional randomness through the selection of split thresholds at each node. This approach increases model diversity and stability while maintaining computational efficiency [23,67].

Mathematically, the output of the XRF model can be written as Equation (7) describes:

$$f^t(x) = f^{(t-1)} + f(x) \quad (7)$$

where

- $f^t(x)$ is the final prediction of the model after t -iterations.
- $f^{(t-1)}$ is the prediction from the previous iteration.
- $f(x)$ is the new prediction at the current iteration, generated based on a random subset of the feature [22,65]

2.2.6. Extreme Gradient Boosting

The Extreme Gradient Boosting (XGBoost) is a high-performance ensemble learning algorithm derived from the gradient boosting framework, also based on decision trees, but offering a fundamentally different approach compared to RF. While RF builds multiple independent trees in parallel and aggregates their outputs to reduce variance through bagging, XGBoost constructs trees sequentially, with each new tree focusing on correcting the residual errors of the previous one [68,69].

This sequential boosting process could enable XGBoost to achieve higher predictive accuracy, particularly in complex, high-dimensional datasets. Furthermore, XGBoost

incorporates explicit regularization techniques (L1 and L2 penalties) to control overfitting, a feature not present in standard RF models [65,70]. Optimization in XGBoost is performed through gradient descent on a specified loss function, whereas RF relies purely on averaging without explicit loss minimization [69].

The general formulation for the model prediction is depicted in Equation (8) [67]:

$$\hat{y}_i = \sum_{t=1}^T f_t(x_i) \quad (8)$$

where

- $\hat{y}_i$ is the predicted output for the i th sample.
- $f_t(x_i)$ represents the output of the t th decision tree (weak learner) based on input features x_i .
- T is the total number of trees in the ensemble.

2.2.7. Artificial Neural Networks

Feedforward Artificial Neural Networks (ANNs) are supervised learning models composed of interconnected layers of neurons organized sequentially from input to output, without cycles or feedback loops, as presented in Figure 7. Each neuron computes a weighted sum of its inputs, adds a bias term, and applies a nonlinear activation function, enabling the network to model complex, nonlinear relationships between features and target variables [71].

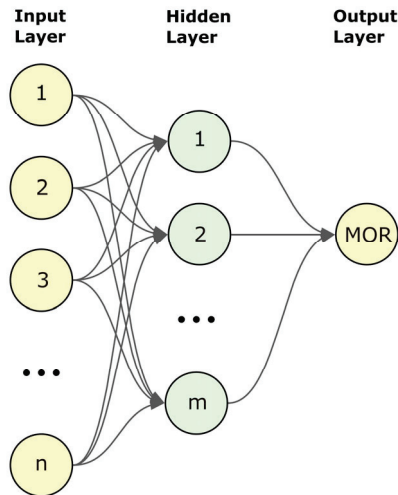


Figure 7. Scheme of a feed-forward neural network.

The formula for the output of a single neuron is presented by Equation (9):

$$y_j = f\left(\sum_{i=1}^n w_{ij}x_i + b_j\right) \quad (9)$$

where

- y_j is the output of neuron j in the current layer.
- w_{ij} represents the weights between neuron i in the previous layer and neuron j in the current layer.
- x_i is the input from neuron i .
- b_j is the bias term for neuron j .
- f is the activation function (such as ReLU, Sigmoid, or Tanh), which introduces nonlinearity into the model.

Training involves adjusting the weights and biases through the backpropagation algorithm combined with optimization techniques like gradient descent, minimizing a defined loss function. For regression tasks, the output layer typically consists of a single neuron producing a continuous value. Therefore, ANNs offer a flexible and powerful framework capable of capturing complex mappings between input variables and outputs, making them particularly effective for predictive modeling tasks in high-dimensional spaces [71,72].

Beyond their conventional role as high-dimensional black-box regressors, ANNs have been increasingly leveraged as surrogate models within probabilistic frameworks, particularly for composite materials characterized by intrinsic uncertainties. For instance, Kushari et al. [73] developed an ANN-based model to simulate first-ply failure in laminated composites under random variations in ply orientation and stacking sequence, effectively capturing variability-driven failure envelopes while substantially reducing computational demands relative to Monte Carlo simulations. Similarly, Gotlib et al. [74] proposed an ANN architecture capable of generalizing the effective thermal and electrical conductivities of disordered composites, even when microstructural geometries are entirely random or non-idealized. More recently, Kamiński [75] applied ANNs within a stochastic homogenization framework to evaluate entropy-based uncertainty metrics—such as Shannon entropy, Kullback–Leibler divergence, and Bhattacharyya distance—for fiber-reinforced composites with imperfect interfacial zones. These works collectively demonstrate that ANNs are not only effective predictive tools but also offer interpretability benefits in uncertainty quantification. Their demonstrated capacity to map the probabilistic behavior of composite systems provides strong support for their adoption in this study as interpretable surrogates for estimating the flexural strength of UHPC mixtures with complex formulations and hybrid reinforcements.

2.3. Model Tunning and Evaluation

2.3.1. Computational Resources and Execution Time

To enhance the transparency and reproducibility of the modeling pipeline, the computational environment used in this study is detailed here. All machine learning models were developed and executed on a personal workstation equipped with an AMD Ryzen 7 7735HS processor, 16 GB of RAM, and an AMD Radeon RX 7700S GPU. This configuration provided sufficient processing power to support both traditional algorithms and more complex ensemble and hybrid neural architectures.

The total average computation time for the complete modeling workflow was approximately 1.8 h. This includes all stages of the process: training of base models, construction of ensemble learners, implementation of hyperparameter tuning via GridSearch, 10-fold cross-validation, generation of one-way and two-way Partial Dependence Plots, and application of SHAP-based global and local interpretability routines.

2.3.2. Model Tunning

Hyperparameter optimization is a critical step in machine learning model development, as it directly influences model architecture, training dynamics, and generalization capacity. In this study, GridSearch was employed to systematically explore combinations of hyperparameters and identify the configurations that yield optimal model performance based on evaluation metrics such as the coefficient of determination (R^2).

GridSearch conducts an exhaustive, structured search across predefined hyperparameter spaces, ensuring that potential sources of overfitting or underfitting associated with arbitrary hyperparameter selection are minimized. Although optimal hyperparameters do

not guarantee perfect predictive accuracy, careful tuning enhances both model reliability and robustness.

Hyperparameter optimization was applied to all machine learning models considered, including ensemble methods and artificial neural networks, providing a foundation for fair comparison and maximizing each model's predictive capabilities.

2.3.3. K-Fold Validation

All machine learning models were trained and evaluated using K-Fold cross-validation with $K = 10$. In this procedure, the dataset is partitioned into ten equal subsets; for each iteration, nine subsets are used for training while the remaining one is used for testing, ensuring that every instance is utilized for both purposes across the validation cycle [62].

This methodology provides a comprehensive assessment of model generalization while minimizing the risk of overfitting. Performance metrics, specifically the coefficient of determination (R^2), were computed for each fold and averaged to obtain a robust estimate of model accuracy [76].

Figure 8 illustrates the distribution of the R^2 scores obtained across the folds for individual models through box-and-whisker plots, offering a visual representation of the variability and stability achieved during the cross-validation process. In these plots, the central line indicates the median R^2 , the box spans the interquartile range (IQR), and the whiskers extend to $\pm 1.5 \times \text{IQR}$; the white circles beyond the whiskers represent outlier fold scores that exceed this range, thereby highlighting instances of extreme performance deviation.

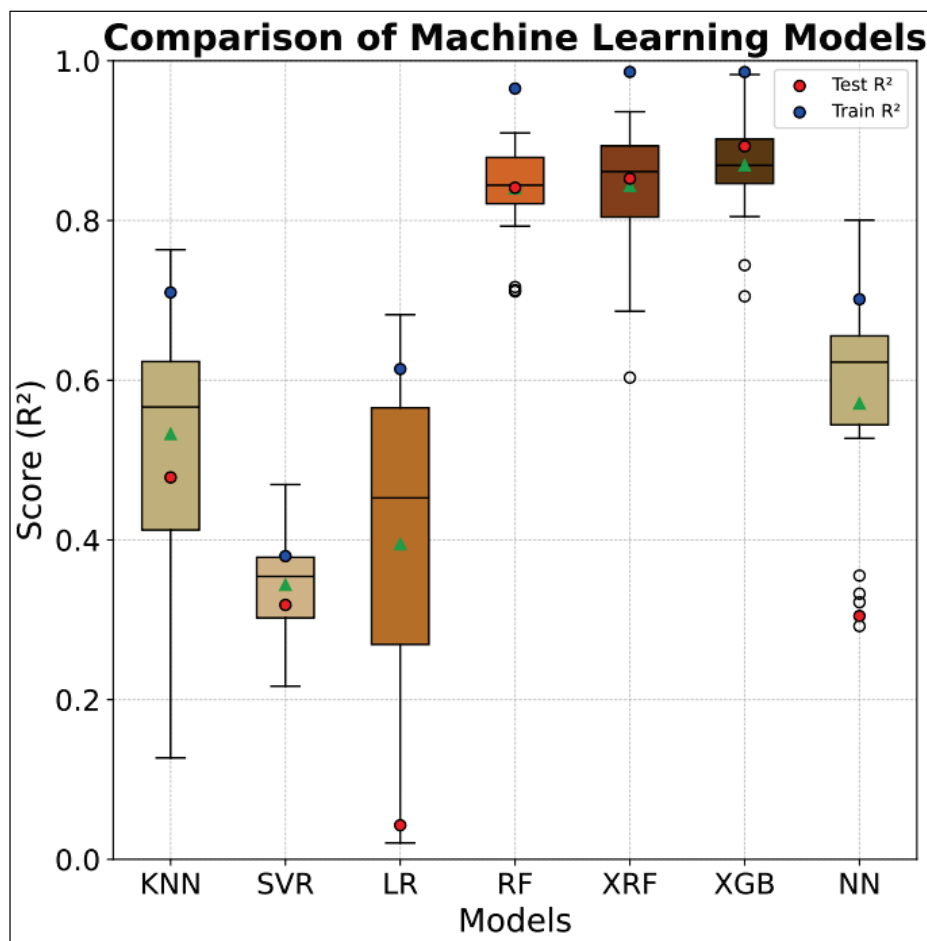


Figure 8. Model performance of the individual ML regression models evaluated.

2.3.4. Prediction Performance Metric

The coefficient of determination (R^2) was used as the primary evaluation metric to assess the predictive performance of the regression models. R^2 quantifies the proportion of variance in the dependent variable that is explained by the independent variables, serving as an indicator of model goodness-of-fit.

The R^2 value ranges from 0 to 1, where higher values denote better explanatory power of the model. A value of 1 indicates that the model perfectly accounts for the observed variance, while a value close to 0 suggests minimal predictive capability.

The computation of R^2 is given by Equation (10):

$$R^2 = 1 - \frac{\sum_{i=1}^n (a_i - \hat{y}_i)^2}{\sum_{i=1}^n \hat{y}_i^2} \quad (10)$$

where a_i represents the observed values and $\hat{y}_i$ the predicted values.

To give a better and stronger evaluation perspective of model performance, additional error metrics were also evaluated: Mean Absolute Error (MAE), Root Mean Squared Error (RMSE), Mean Squared Error (MSE), and Mean Absolute Percentage Error (MAPE). Each one of these measures the prevalence and variance of the prediction errors in its own way.

The MAE represents the average absolute difference between predicted and actual values, providing a direct measure of prediction accuracy is given by Equation (11):

$$\text{MAE} = \frac{1}{n} \sum_{i=1}^n |a_i - \hat{y}_i| \quad (11)$$

The RMSE reflects the square root of the average of the squared differences between observed and predicted values. It penalizes larger errors more severely due to the squaring operation is given by Equation (12):

$$\text{RMSE} = \sqrt{\frac{1}{n} \sum_{i=1}^n (a_i - \hat{y}_i)^2} \quad (12)$$

The MSE is the square of RMSE, often used to capture the overall magnitude of errors, especially when comparing models where large deviations are critical is given by Equation (13):

$$\text{MSE} = \frac{1}{n} \sum_{i=1}^n (a_i - \hat{y}_i)^2 \quad (13)$$

The MAPE expresses the prediction error as a percentage, making it particularly useful when comparing models across different scales or units is given by Equation (14):

$$\text{MAPE} = \frac{100\%}{n} \sum_{i=1}^n \left| \frac{a_i - \hat{y}_i}{a_i} \right| \quad (14)$$

The combined interpretation of the indicators provides a precise assessment of predictive accuracy and reliability to the outlier, robustness to prediction more importantly, the generalizing ability of the predictive model, all of which are salient features in an engineering context, where reliable predictions and the ability to interpret those predictions are of utmost importance.

2.4. SHAP (Shapley Additive ExPlanations)

Shapley Additive Explanations (SHAP) were employed to interpret the contribution of individual input variables to the model predictions. Based on cooperative game theory,

SHAP values provide a consistent and locally accurate attribution of feature importance, enhancing the transparency of complex machine learning models.

Following the framework proposed by Lundberg and Lee [77], the prediction for a given instance can be expressed as given in Equation (15):

$$h(z) = \phi_0 + \sum_{i=1}^N \phi_i z_i \quad (15)$$

where

- $h(z)$ is the prediction made by the model.
- ϕ_0 is the baseline prediction (the average prediction of the model without any features).
- ϕ_i is the Shapley value for feature i , representing its contribution to the prediction.
- z_i is the value of feature i for the given instance.

For its part, the attribution of each feature is determined by calculating the weighted average of its marginal contributions across all possible subsets of features. The mathematical expression for this is given in Equation (16):

$$\phi_i = \sum_{K \subseteq M \setminus i} \frac{|K|!(N - |K| - 1)!}{N!} [g_x(K \sqcup \{i\}) - g_x(K)] \quad (16)$$

where

- ϕ_i is the Shapley value for feature i .
- K is a subset of features without feature i .
- M is the set of all features.
- N is the total number of features.
- $g_x(K)$ Represents the predicted value of the model when the feature set K is used.

SHAP analysis has been widely adopted in scientific and engineering domains due to its solid theoretical foundations, model-agnostic applicability, and ability to reveal nonlinear and interaction effects between features [78]. In this study, SHAP was applied to the best-performing individual learners (RF, XRF, XGB) to uncover the most critical parameters influencing flexural strength in UHPC, both at the global and local levels.

2.5. Partial Dependence Plots

In this study, Partial Dependence Plots (PDPs) served as a supporting interpretability technique to expand the understanding of how identified input variables influenced the predicted flexural performance of UHPC. PDPs, originally introduced by Friedman [79], quantify the marginal effect of one or two input features on the predicted output of a machine learning model while averaging the effect of all other input variables. This model-agnostic visualization allows for an intuitive way to see how important predictors affect the response variable or how the predictors modify the response variable (i.e., in this study, the modulus of rupture (MOR)).

Formally, the partial dependence function of a machine learning model f with respect to a subset of features $S \subseteq \{1, \dots, p\}$ is defined as in Equation (17) [79]:

$$\hat{f}_S(x_S) = E_{x_C}[f(x_S, x_C)] = \int f(x_S, x_C) dP(x_C) \quad (17)$$

where

- x_S denotes the values of the feature(s) of interest.
- x_C represents the complementary features (i.e., the remaining predictors not in S).

- $f(x_s, x_c)$ is the prediction function of the model.
- $P(x_c)$ is the marginal probability distribution of the complementary features.

2.6. Ensemble Learning Techniques

To further enhance predictive performance, this study explores ensemble learning techniques, which combine multiple base models to capitalize on their individual strengths and mitigate their weaknesses. These strategies aim to improve model robustness, stability, and generalization beyond what could be achieved with standalone algorithms.

2.6.1. Bagging

Bootstrap Aggregating (Bagging) is an ensemble learning method designed to enhance the accuracy and robustness of predictive models by reducing variance through the aggregation of multiple learners. It operates by generating diverse versions of a base model, each trained on a random bootstrap sample of the training data.

In Bagging, models are trained independently, and their outputs are combined to produce the final prediction. For regression tasks, the ensemble output is typically the average of the individual model predictions, formulated as in the case of the individual decision tree-based models (see Equations (6) and (8)).

This strategy is particularly effective when applied to strong base models that individually demonstrate high predictive performance, offering a systematic approach to mitigating overfitting and enhancing model stability.

2.6.2. Stacking

Stacking is an ensemble learning technique that combines predictions from multiple base models of different types to exploit their individual strengths and mitigate their weaknesses. In this approach, several base learners are first trained independently on the training dataset. Their respective predictions are then used as input features for a secondary model, known as the meta-model, which generates the final output.

The prediction generated through Stacking is given by Equation (18):

$$\hat{y} = \sum_{i=1}^N w_i f_i(x) + w_0 \quad (18)$$

where

- $\hat{y}$ is the final predicted output.
- $f_i(x)$ represents the prediction from the i^{th} base model.
- w_i are the learned weights of each base model's prediction, learned by the meta-model.
- w_0 is the bias term learned by the meta-model.

By leveraging diverse predictive patterns captured by different models, Stacking often achieves superior generalization performance compared to individual learners.

2.6.3. Boosting

Stacking Boosting is an ensemble learning technique that constructs a strong predictive model by sequentially training a series of weak learners, each focused on correcting the errors of its predecessor. At each iteration, a new model is trained to minimize the residuals of the combined previous models, progressively improving overall predictive accuracy.

The final prediction produced by a Boosting ensemble can be expressed as (Equation (19)):

$$\hat{y}_i = \sum_{t=1}^T \alpha_t f_t(x_i) \quad (19)$$

where

- $\hat{y}_i$ is the final predicted output for the i^{th} sample.
- $f_t(x_i)$ is the prediction from the t^{th} model.
- α_t is the weight assigned to each model's prediction, based on its accuracy.

2.6.4. Hybrid Neural Network

The Hybrid Neural Network (Hybrid NN) is an advanced ensemble approach that integrates the predictions from multiple machine learning models with the original input features to improve predictive accuracy and generalization. This architecture capitalizes on the strengths of individual models by using their outputs as additional features within a neural network framework, thus reducing individual model biases and enhancing overall performance.

Initially, high-performing base models are trained independently, and their predictions are collected alongside the original feature set. The Hybrid NN receives these combined inputs and processes them through multiple hidden layers equipped with nonlinear activation functions. Weight optimization is performed using backpropagation and gradient descent, minimizing a predefined loss function such as Mean Squared Error.

The final prediction generated by Hybrid NN is represented as (Equation (20)):

$$\hat{y}_{\text{hybrid}} = f(\hat{y}_{\text{RF}}, \hat{y}_{\text{XRF}}, \hat{y}_{\text{XGB}}, X_1, X_2, \dots, X_{41}) \quad (20)$$

where

- $\hat{y}_{\text{hybrid}}$ is the final prediction of the Hybrid Neural Network.
- $\hat{y}_{\text{RF}}, \hat{y}_{\text{XRF}}, \hat{y}_{\text{XGB}}$ are the outputs from the base models (RF, XRF, XGB), each with an R^2 value greater than 0.8.
- $X_1, X_2, \dots, X_{41}$ are the original input features, such as fiber content, water-to-binder ratio, cement content, etc.

By combining model outputs with raw features, the Hybrid Neural Network offers a flexible and powerful methodology for capturing complex relationships within predictive modeling tasks.

3. Results

This section may be divided by subheadings. It should provide a concise and precise description of the experimental results, their interpretation, as well as the experimental conclusions that can be drawn.

3.1. Performance of the Individual Machine Learning Models

The predictive performance of the individual machine learning (ML) models developed in this study was assessed using a 10-fold cross-validation scheme applied to the training subset. Figure 8 summarizes the performance metrics through box-and-whisker plots, depicting the distribution of the R^2 scores across the folds for each model, as well as the R^2 values measured on both the training and testing subsets.

The results of the hyperparameter selection for each model by using GridSearch are summarized in Table 7.

Overall, the results demonstrate a wide variability in model performance. Among the models evaluated, RF, XRF, and XGB exhibited superior predictive capabilities, each achieving R^2 scores exceeding 0.80 on the testing subset. Specifically, the R^2 scores recorded for these models on the testing set were 0.8415 for RF, 0.8432 for XRF, and 0.8563 for XGB, indicating excellent generalization performance. In contrast, the other models—KNN, SVR,

Linear LR, and the Feedforward ANN—delivered notably lower R^2 scores, with maximum values of 0.5379, 0.3186, 0.0428, and 0.3714, respectively.

Table 7. Hyperparameter tuning ranges and selected optimal values for each machine learning model using GridSearch.

Model	Selected Hyperparameters
Linear Regression (LR)	• fit_intercept = True • copy_X = True • normalize = False
KNN	• n_neighbors = 7 • weights = 'distance'
RF	• n_estimators = 300 • max_depth = None • bootstrap = True • min_samples_split = 2 • min_samples_leaf = 1 • max_features = 'sqrt'
XRF	• n_estimators = 200 • max_depth = None • bootstrap = False • min_samples_split = 2 • min_samples_leaf = 1 • max_features = 'auto'
XGBoost	• n_estimators = 100 • learning_rate = 0.2 • booster = 'gbtree' • max_depth = 6 • subsample = 1.0 • colsample_bytree = 1.0 • reg_alpha = 0 • reg_lambda = 1
ANN	• Hidden layers: [32,63] • Activation (hidden): ReLU • Activation (output): linear • Optimizer: Adam (lr = 1×10^{-3}) • Loss: MSE; Metric: MAE • Epochs: 100 (early stopping, patience = 10) • Batch size: 32

Given these findings, only RF, XRF, and XGB were selected for further analysis and integration into subsequent ensemble learning strategies. Figure 9 illustrates the regression plots for these best-performing individual models across the testing subset.

The superior performance of RF, XRF, and XGB can be attributed to their inherent capabilities in modeling nonlinear and multivariate relationships, combined with mechanisms for controlling overfitting—such as bootstrap aggregation, random threshold selection, and gradient-boosting optimization—thereby making them particularly well-suited for capturing the complex behavior of fiber-reinforced UHPC under flexural loadings.

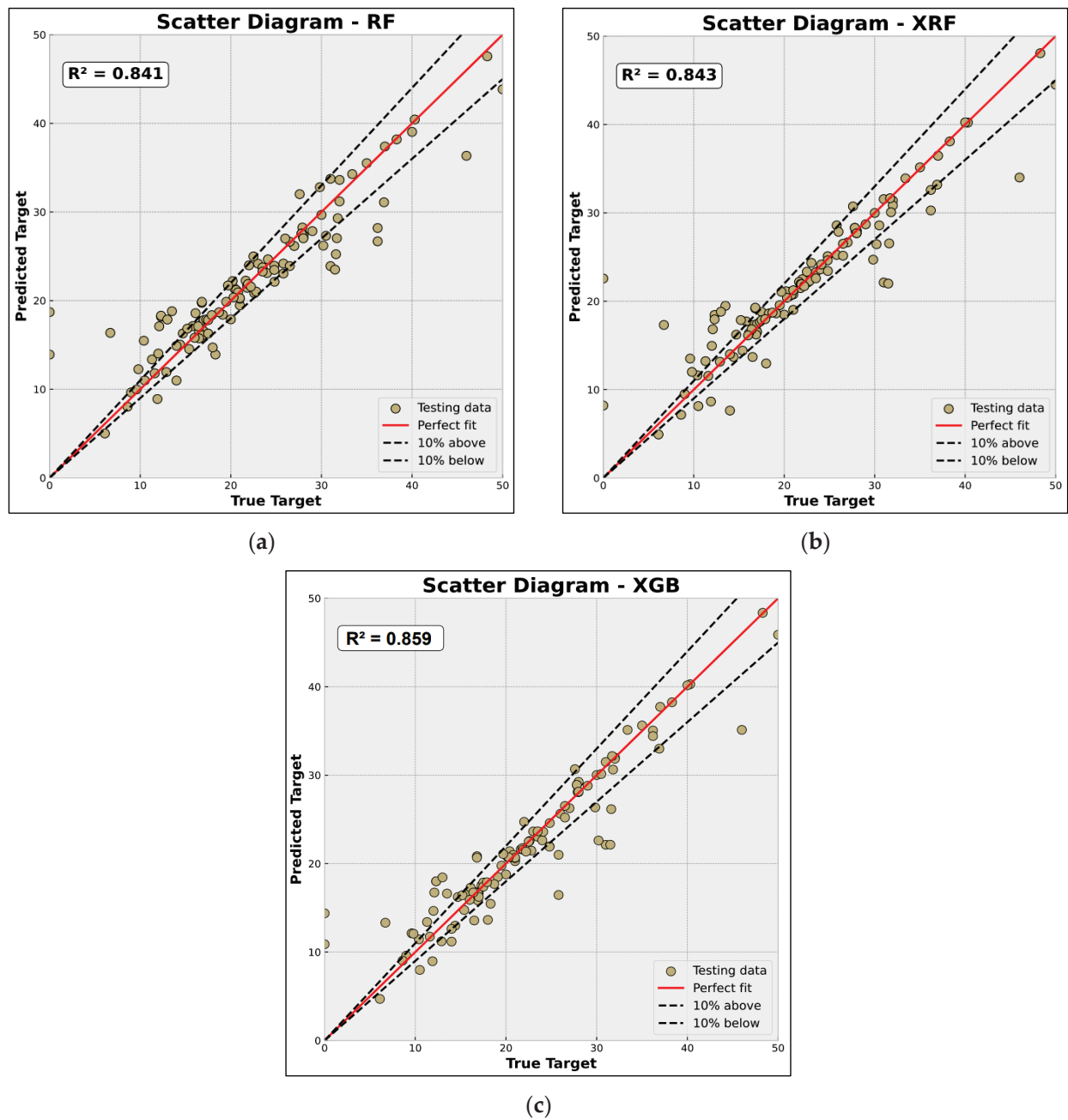


Figure 9. Regression plots of the best-performing individual ML regression models with R^2 over 80% measured on the testing data subset: (a) FR; (b) XRF; and (c) XGB.

3.2. Feature Importance Analysis Using SHAP

3.2.1. Interpretability of Individual Models via Shapley Additive Explanations

To deepen the understanding of the internal predictive mechanisms of the individual machine learning models prior to ensemble integration, Shapley Additive Explanations (SHAP) analysis was performed on the best-performing individual models. This interpretability technique provides a comprehensive assessment of the contribution of each input feature to the model predictions, allowing for a clearer understanding of the critical variables influencing flexural strength predictions.

Therefore, Figures 10–12 present the SHAP summary plots for the RF, (XRF, and XGB models, respectively. In these figures, the top ten most influential variables for each model are highlighted, complemented by brief discussions of additional relevant features.

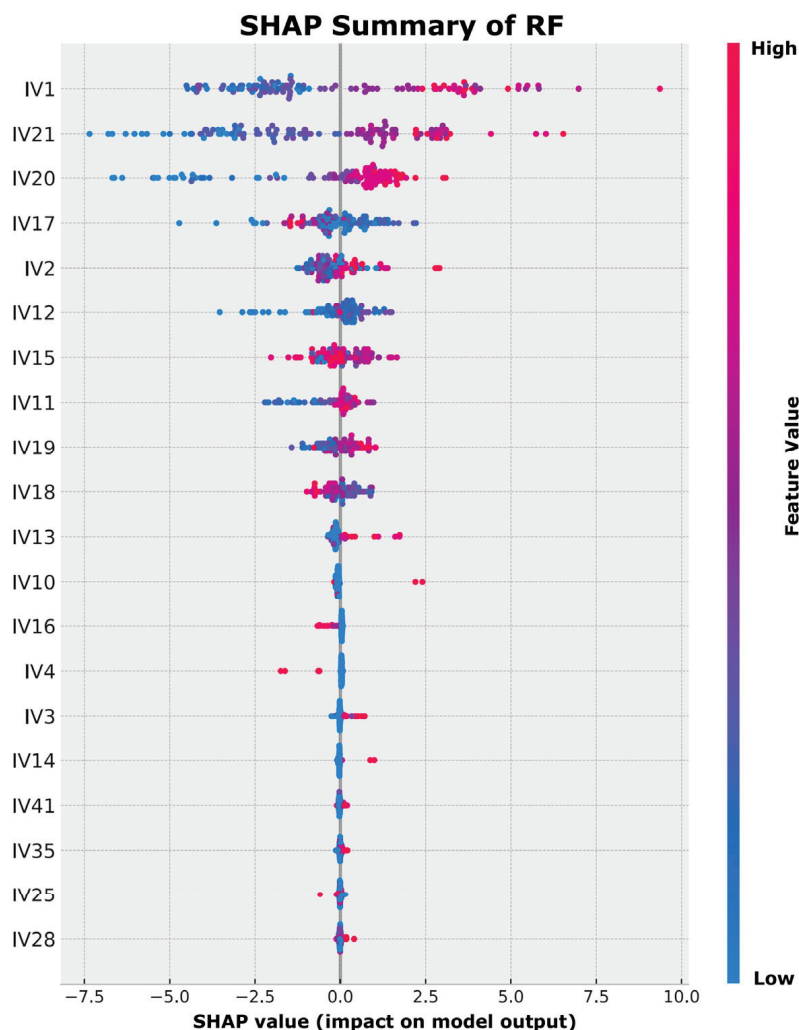


Figure 10. Impact on the RF model's output by SHAP value.

For the RF model (Figure 10), the most influential variable was the cement content (IV1), exerting a positive contribution to the predicted modulus of rupture (MOR). The fiber factor (IV21) and total fiber volume fraction (IV20) followed closely, both enhancing the model's output. Maximum aggregate size (IV17) demonstrated a negative influence, whereas the superplasticizer content (IV12) and silica fume dosage (IV2) showed positive effects. Other influential variables included water content (IV11), compressive strength (IV19), sand content (IV15), and notably, a strong negative impact associated with the water-to-binder ratio (IV18).

In the XRF model (Figure 11), total fiber volume (IV20) emerged as the dominant feature, followed by fiber factor (IV21) and cement content (IV1). Coarse aggregate and maximum aggregate size (IV17) were ranked fourth and fifth, with both negatively impacting flexural strength predictions. Silica fume content (IV2), compressive strength (IV19), water-to-binder ratio (IV18), supplementary cementitious materials (IV10), and superplasticizer dosage (IV12) rounded out the top contributors.

For the XGB model (Figure 12), the fiber factor (IV21) again led the feature ranking, followed by cement content (IV1) and total fiber volume (IV20), confirming the consistency observed across models. Silica fume dosage (IV2) and water content (IV11) were also significant contributors. Superplasticizer dosage (IV12), sand content (IV15), compressive strength (IV19), maximum aggregate size (IV17), and water-to-binder ratio (IV18) were included among the most relevant features, with trends largely aligning with those observed for RF and XRF.

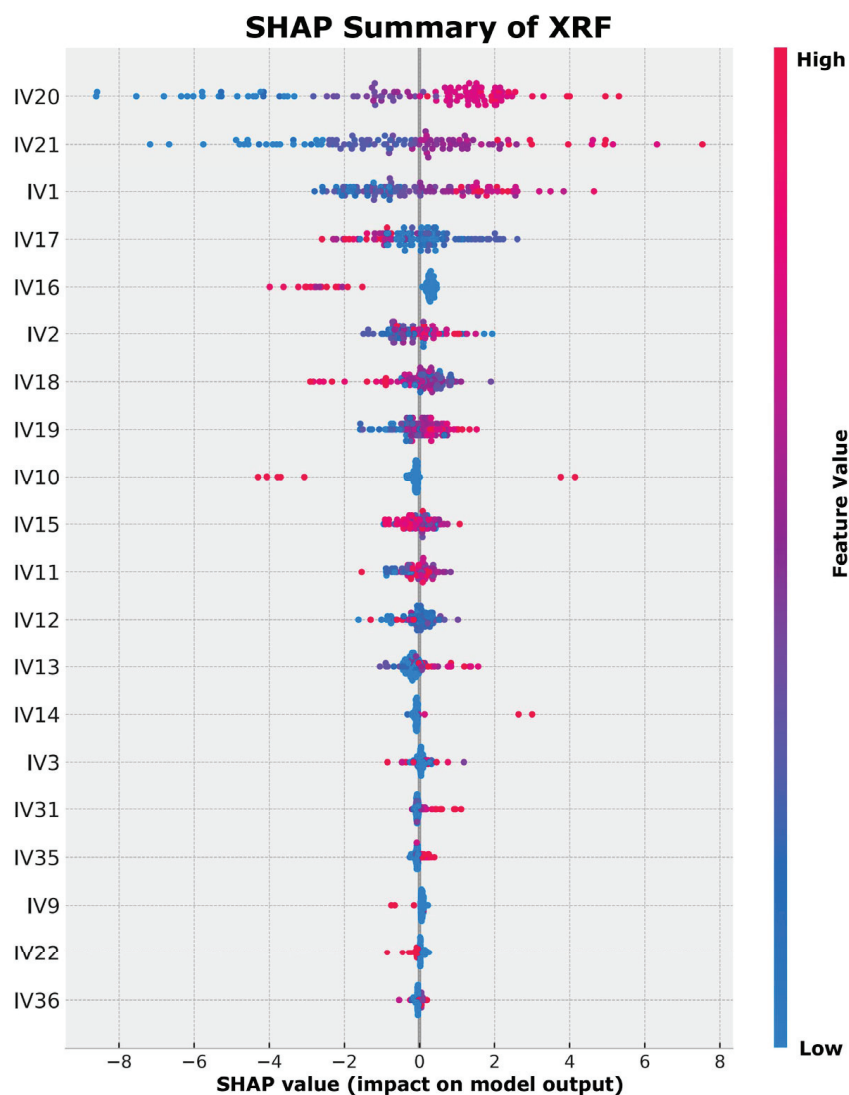


Figure 11. Impact on the XRF model's output by SHAP value.

An important observation across the models is the limited influence of alternative Supplementary Materials, such as fly ash (IV3), ground granulated blast-furnace slag (IV4), and ground calcium carbonate (IV9). Although in some instances minor positive or negative effects were observed (particularly for IV3), their overall impact on flexural strength was marginal. This finding suggests that the incorporation of these alternative materials—while environmentally beneficial for reducing UHPC's carbon footprint—does not significantly compromise flexural performance, aligning with recent studies on sustainable UHPC design.

3.2.2. Scientific Pattern of SHAP-Identified Factors

The results displayed in Figures 10–12 highlight the relevance of several key input variables in determining the flexural performance of UHPC, which can be scientifically supported through previous experimental and theoretical studies.

The consistent prominence of the fiber reinforcement index (IV21) across the models is explained by its definition, which involves the product of the fiber aspect ratio and its volumetric content [80,81]. This index encapsulates both geometric and quantitative aspects of fiber reinforcement, playing a central role in bridging cracks and improving post-cracking behavior [80,82,83]. Its contribution to the fiber-bridging capacity is well

established as a critical factor for energy absorption and flexural strength in fiber-reinforced UHPC [84–87].

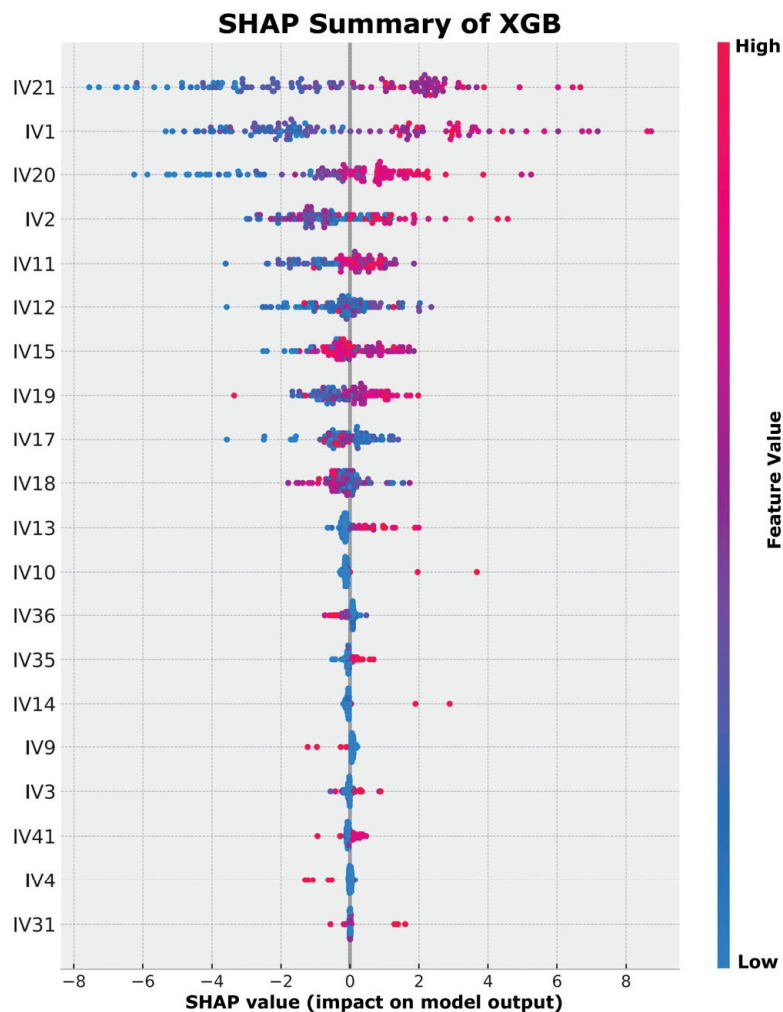


Figure 12. Impact on the XGB model’s output by SHAP value.

Similarly, the cement content (IV1) serves as the primary source of hydration products in the matrix, producing calcium silicate hydrate (C-S-H) and portlandite upon hydration, both of which are essential to mechanical strength development [88–90]. These hydrates interact with pozzolanic materials, such as silica fume, to form additional C-S-H that densifies the matrix and enhances strength.

The positive contribution of silica fume (IV2) aligns with its dual role as a highly reactive pozzolan and micro-filler. Its extremely fine particle size improves particle packing and reduces porosity, while its pozzolanic reaction contributes to additional C-S-H formation, thus strengthening the matrix [88,91]. In addition, silica fume enhances the fiber-matrix bond by improving the interface microstructure, especially when combined with low water content [92–95].

The influence of total water content (IV11) is more nuanced. While a minimum amount of water is necessary to facilitate hydration and support the development of mechanical properties, excessive water leads to increased porosity, reducing strength and interfacial bonding [94,96,97]. In UHPC, the low water-to-binder ratio typically results in dense microstructures and clogged fiber interfaces, thereby enhancing bonding and load transfer [85,98,99]. Moreover, the water dosage must be carefully balanced with silica fume due to its high specific surface area and water demand [88,100].

The negative impact of the water-to-binder ratio (IV8) can be attributed to its strong correlation with matrix porosity. Higher values increase pore connectivity and reduce the compactness of the matrix, which directly impairs mechanical performance [94,96,97].

Superplasticizer content (IV12) contributes positively to flexural strength by reducing the water requirement in the mix, allowing for low-porosity, high-density matrices without compromising workability [88,101]. This effect supports better fiber dispersion and improves the fiber-matrix bond, both of which enhance flexural behavior.

Compressive strength (IV19) also appears as a relevant factor due to its intrinsic relation to the matrix compactness and cracking resistance. Although an overly rigid matrix may limit ductility, an optimal level of compressive strength is necessary to support the multitrack cracking pattern typical of UHPC [102,103]. Studies have shown that fiber-reinforced composites with moderate matrix strength may display superior fiber-matrix behavior due to more effective crack distribution and energy dissipation [92].

The size of the largest aggregate (IV17) typically has a negative effect on flexural strength. Smaller aggregate sizes contribute to a more homogeneous matrix and a stronger interfacial transition zone (ITZ), both of which improve mechanical performance and fiber anchorage [89,94,104]. In contrast, larger aggregates tend to disrupt fiber distribution and reduce the effective interaction between fibers and the matrix, thereby compromising the load-bearing capacity [84,105,106].

Finally, the total fiber volume (IV20) shows a strong positive influence, as expected. Increasing fiber dosage improves both pre- and post-cracking responses, leading to higher performance of the fiber reinforced cementitious composite [83,84,106].

3.3. Partial-Dependence Analysis

3.3.1. One-Way Partial-Dependence Analysis

As per Figures 10–12, Fiber Factor (IV21), Cement Content (IV1), Silica Fume Content (IV2), Maximum Aggregate Size (IV17), Total Fiber Volume (IV20), and Superplasticizer Dosage (IV12)—were noted as the most impactful variables to the prediction of the MOR within the best-performing models.

To better understand how these six variables impact the model output, we conducted and analyzed Partial Dependence Plots (PDPs) for each of the six variables. PDPs also allow for a more visual and interpretable way to illustrate the relationship between single features and the predicted MOR, while still holding the average effect of all other variables constant.

As shown in Figure 13, the flexural strength increases with the cement content, which correlates well with the literature. First, cement serves as the primary source of hydration products in UHPC [101]. Moreover, cement content allows for greater bond properties within the concrete matrix and is able to better distribute loads and increase resistance to bending stresses [107]. However, the graph also shows that the flexural strength decreases with greater than $0.31 \text{ m}^3/\text{m}^3$ cement content, which suggests that the flexural strength decreases due to excessive cement content. There are a few reasons for this pattern. First, when the cement content exceeds a threshold and the water-to-cement ratio is not properly adjusted, the porosity of the cement paste may increase, which consequently reduces the overall strength [108]. Further, higher cement content can alter the micromechanical properties, resulting in lower fracture toughness ratios, which negatively impact the flexural strength [109]. Finally, excess cement can interfere with the proper dispersion and bonding of fibers within the concrete matrix, limiting their effectiveness and contributing to the decrease in strength [110].

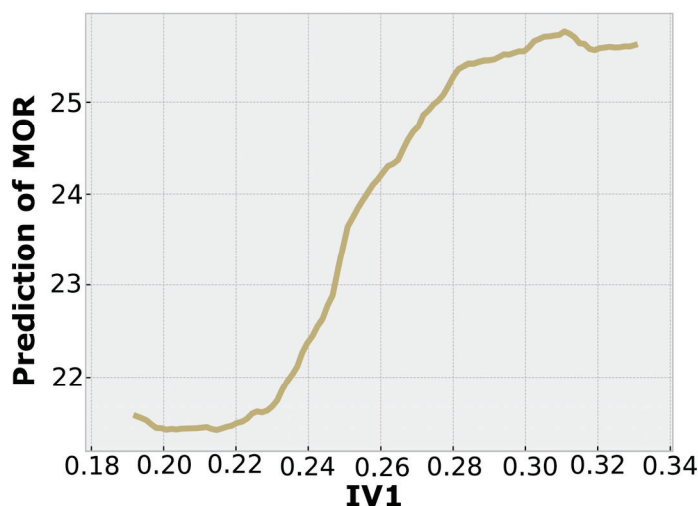


Figure 13. Partial dependence of MOR on cement content (IV1).

For its part, Figure 14 illustrates the influence of silica fume content (IV2) on UHPC flexural performance, showing a clear increase in partial dependence up to an optimal value of approximately $0.093 \text{ m}^3/\text{m}^3$. This trend aligns with prior studies indicating that adequate silica fume additions enhance flexural strength by densifying the matrix, refining pore structure, and improving fiber–matrix interaction [111]. Beyond this point, performance declines as further increases in silica fume content reduce workability and promote agglomeration, which disrupts fiber dispersion and bonding [112]. In addition, the strength loss observed at higher contents can also be attributed to excessive matrix densification, which weakens fiber–matrix adhesion and limits reinforcement effectiveness [113]. Moreover, the resulting poor workability hinders uniform fiber distribution, potentially introducing voids and reducing structural integrity [114]. In addition, high silica fume levels may also promote micro-cracking and increase brittleness, facilitating crack propagation under load [115].

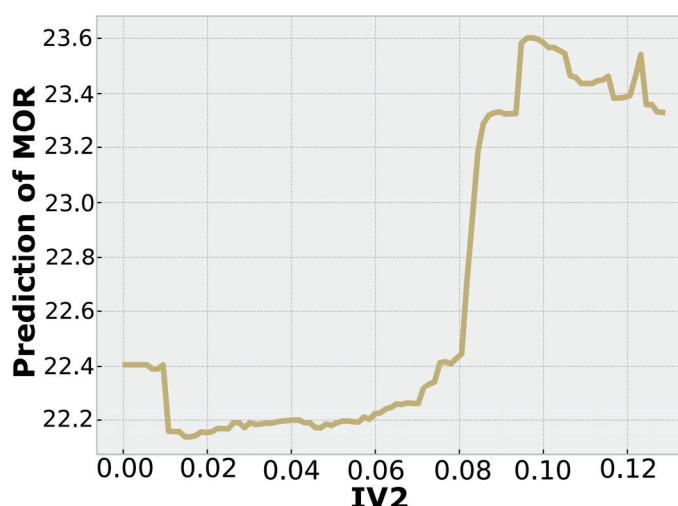


Figure 14. Partial dependence of MOR on silica fume content (IV2).

Figure 15 demonstrates a clear nonlinear relationship between superplasticizer content (IV12) and flexural strength. The partial dependence rises until reaching a peak at approximately $0.04 \text{ m}^3/\text{m}^3$, after which performance declines. When added in proper amounts, superplasticizers improve workability and fiber dispersion, promoting a denser and more cohesive matrix that enhances flexural performance [101,116]. However, exceed-

ing the optimal dosage can lead to mix segregation, non-uniform fiber distribution, and stickier-than-desired mixtures, which negatively impact mechanical performance [101,107].

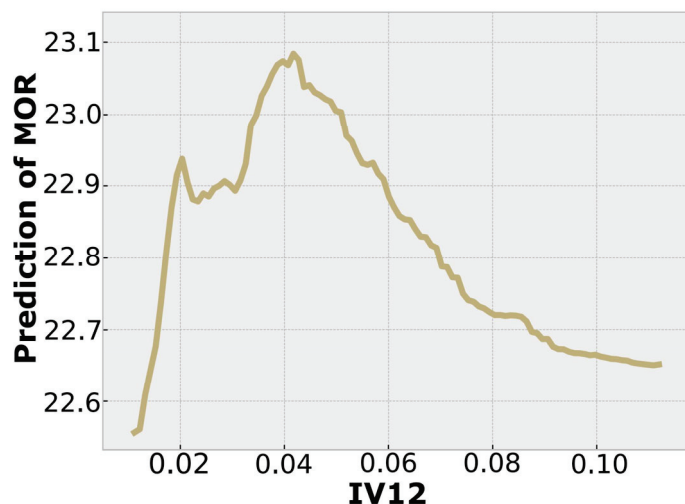


Figure 15. Partial dependence of MOR on superplasticizer content (IV12).

Figure 16 illustrates the relationship between maximum aggregate size (IV17) and flexural strength, showing an initial increase to a peak, followed by a decline and eventual stabilization. This behavior can be attributed to the disruptive effect of larger aggregates on fiber dispersion and orientation. As aggregate size increases, fibers become less effective in bridging cracks due to impaired alignment and reduced interaction with the matrix, ultimately diminishing their reinforcement contribution [117]. In addition, larger particles can limit the effective embedment length of fibers, compromising their mechanical engagement and reducing the overall structural performance of UHPC [118].

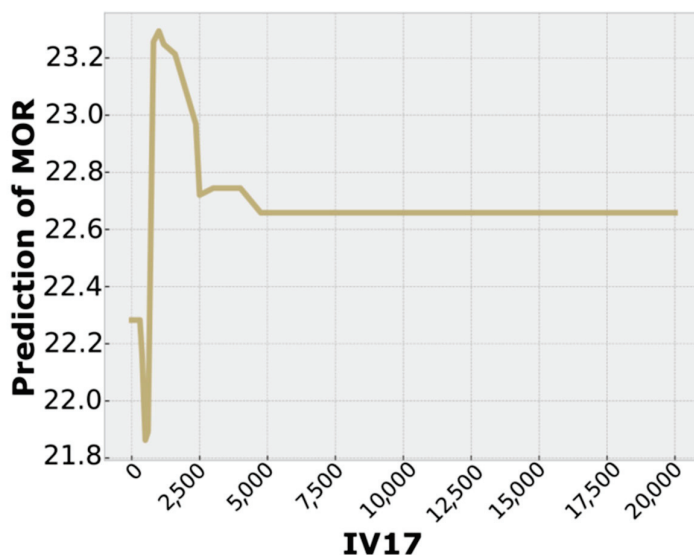


Figure 16. Partial dependence of MOR on maximum aggregate size (IV17).

Similarly, Figure 17 shows that increasing fiber volume leads to higher flexural strength up to a saturation point. Maximum gains are observed between volume fractions of 0.008 and 0.03, consistent with prior research indicating optimal values around 2–3% of the total mix volume [119]. Within this range, fiber content is sufficient to enhance crack-bridging, cohesion, and stress transfer. Beyond this threshold, the contribution of fibers stabilizes as their alignment and dispersion become less efficient, marking the upper limit of their reinforcing effect [120].

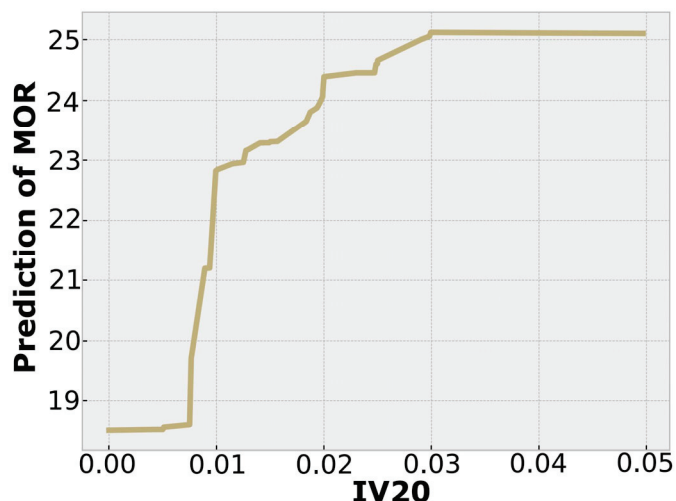


Figure 17. Partial dependence of MOR on fiber volume fraction (IV20).

Finally, as shown in Figure 18, the relationship between fiber factor and flexural strength reveals a sharp increase between values of 1.0 and 1.2, peaking near 2.0, followed by a gradual decline. This trend aligns with previous studies indicating that a higher fiber factor—reflecting fiber volume, length, and aspect ratio—enhances crack-bridging, energy dissipation, and post-crack toughness, thereby increasing flexural strength [121]. However, as discussed in the previous paragraph, beyond the optimal range, performance gains taper off or diminish due to fiber agglomeration, increased air entrapment, and reduced workability and alignment [122]. The observed peak around a fiber factor of 2.0 likely marks the threshold before negative side effects—such as poor fiber dispersion and limited contribution to toughness—begin to outweigh the structural benefits. Once this point is exceeded, the reinforcing efficiency declines, explaining the downward trend in the PDP curve [123].

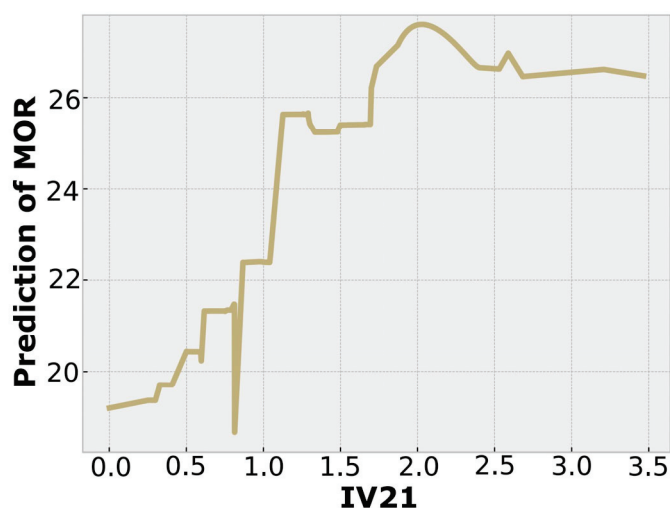


Figure 18. Partial dependence of MOR on composite fiber factor (IV21).

3.3.2. Two-Way Partial-Dependence Analysis

SHAP value analyses (Figures 10–12) have consistently identified the global fiber factor (IV21) as the most influential predictor of MOR across the top-performing individual learners, including the Random Forest, Extremely Randomized Trees, and XGBoost models. Given the central role of IV21 in fiber-reinforced cementitious composites—where it governs

the balance between fiber dosage, aspect ratio, and dispersion efficiency—its interaction with matrix-related parameters warrants detailed investigation.

To this end, three bivariate PDPs were generated, each pairing IV21 with a key matrix property. These plots, presented in Figures 19–21, help elucidate how IV21's influence on MOR is modulated by specific matrix characteristics. The analysis was conducted using the best-performing model for each variable pairing: RF was used for IV21 $\times$ IV17 (maximum aggregate size, μm), XGB for IV21 $\times$ IV18 (water-to-binder ratio), and XRF for IV21 $\times$ IV19 (matrix compressive strength, MPa).

IV21 vs IV17 (10th, 50th, 90th percentiles)

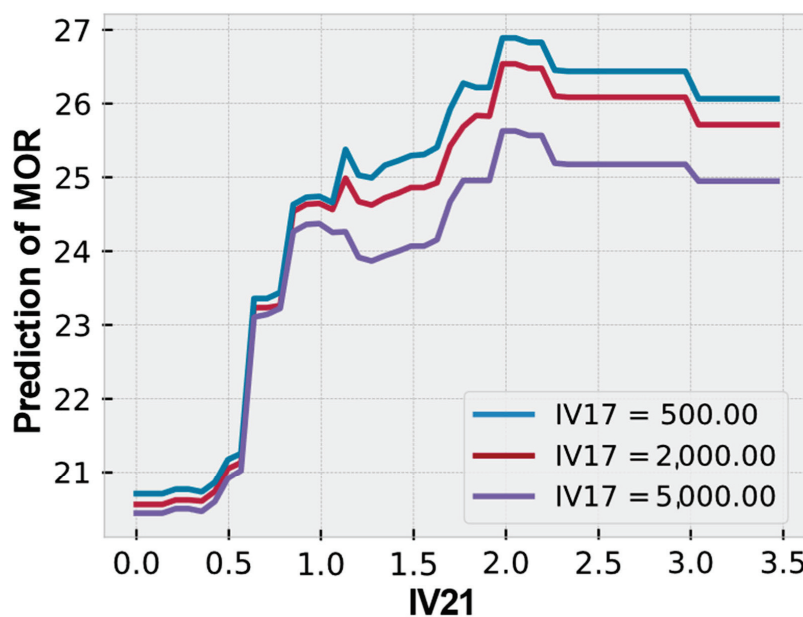


Figure 19. IV21 $\times$ IV17 (maximum aggregate size, μm).

IV21 vs IV18 (10th, 50th, 90th percentiles)

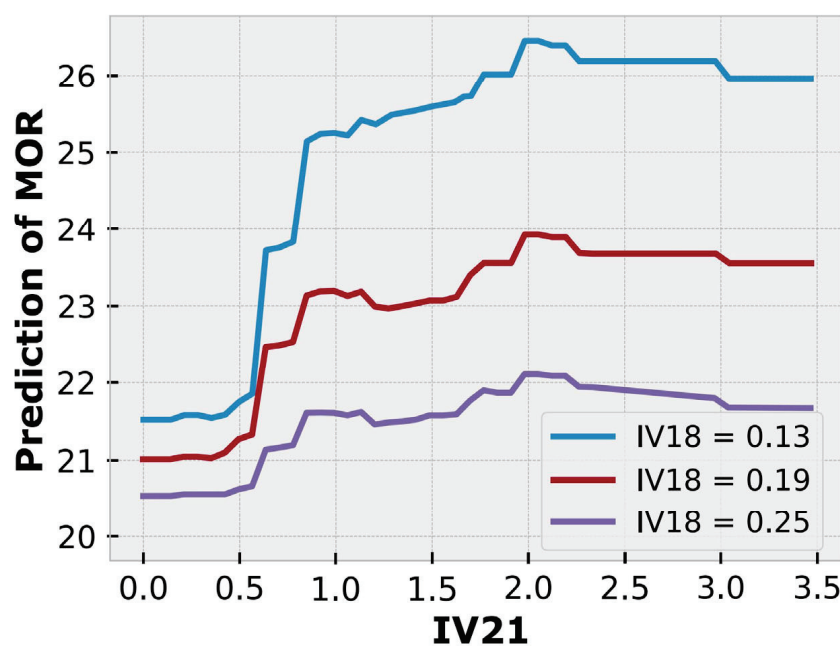


Figure 20. IV21 $\times$ IV18 (water-to-binder ratio).

IV21 vs IV19 (10th, 50th, 90th percentiles)

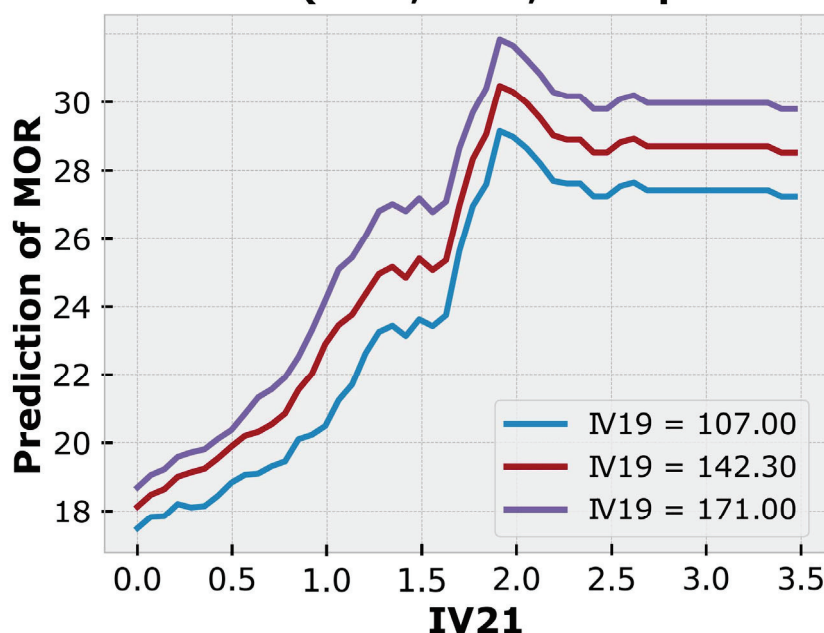


Figure 21. IV21 $\times$ IV19 (matrix compressive strength, MPa).

The first interaction (Figure 19) reveals that smaller aggregate sizes yield higher MOR. This trend is attributed to improved fiber alignment and a higher effective aspect ratio, both of which are disrupted by the presence of larger aggregates. Coarse particles have been shown to weaken the fiber-matrix bond, limiting stress transfer and reducing mechanical efficiency [105]. Furthermore, larger aggregates contribute to heterogeneous fiber dispersion and promote wider, interconnected cracking, thereby diminishing toughness and impairing flexural performance despite the limited impact on compressive strength [117,124].

The second PDP (Figure 20) illustrates how reduced water-to-binder ratios significantly elevate MOR by reinforcing the synergistic effect of higher IV21 values. This enhancement stems from improved fiber-matrix adherence via densified microstructures, reduced pore connectivity, and superior bond-slip behavior at lower w/b ratios, all of which strengthen fiber pull-out resistance and matrix homogeneity [99,125].

The third interaction (Figure 21) highlights a positive monotonic relationship between compressive strength and MOR. As compressive strength rises, so does the matrix's ability to sustain bridging stresses induced by fibers, thereby allowing IV21 to more effectively contribute to flexural strength. This enhancement is attributed to improved crack-bridging capacity and stress transfer efficiency, as stronger matrices better anchor fibers and sustain post-cracking loads [121,126]. Empirical studies confirm that increases in compressive strength—driven by optimized mix designs or advanced fiber types—can yield disproportionately higher gains in flexural performance, in some cases exceeding 100% improvements [123].

Finally, across all three PDPs, a converging trend is evident: the marginal benefit of increasing IV21 begins to diminish as it approaches values between approximately 1.8 and 2.0. This phenomenon is consistent with reports indicating that excessive fiber volumes (i.e., values of fiber factor around 2) may lead to reduced ductility and the onset of brittle failure mechanisms, undermining the deformation capacity of UHPC [127,128]. Additionally, higher fiber contents can disrupt uniform distribution and orientation, inducing anisotropic responses that compromise UHPC's performance [129].

3.4. Ensemble Learning Results

To enhance the predictive performance beyond that achieved by individual machine learning (ML) models, several ensemble learning techniques were employed and evaluated, namely Bagging, Stacking, Boosting, and the Hybrid Neural Network approach. The results are summarized in Figure 22, which presents the R^2 scores obtained for each ensemble method on the testing dataset. Each box-and-whisker plot depicts the distribution of fold-wise R^2 values: the central line marks the median, the box spans the interquartile range, and the whiskers extend to $1.5 \times \text{IQR}$. The colored triangle within each box indicates the mean R^2 across the ten folds, providing a complementary measure of central tendency, while the white circles beyond the whiskers denote outlier fold scores that exceed the $\pm 1.5 \times \text{IQR}$ threshold. As can be appreciated, Bagging achieved an R^2 of 0.848, demonstrating an improvement over the best individual models by reducing variance through the aggregation of multiple predictors. Stacking outperformed Bagging, reaching an R^2 of 0.868, highlighting the advantage of combining heterogeneous base models through a meta-model that optimally integrates their outputs. Boosting, while improving bias correction through sequential training, achieved a slightly lower R^2 of 0.814 compared to Bagging and Stacking.

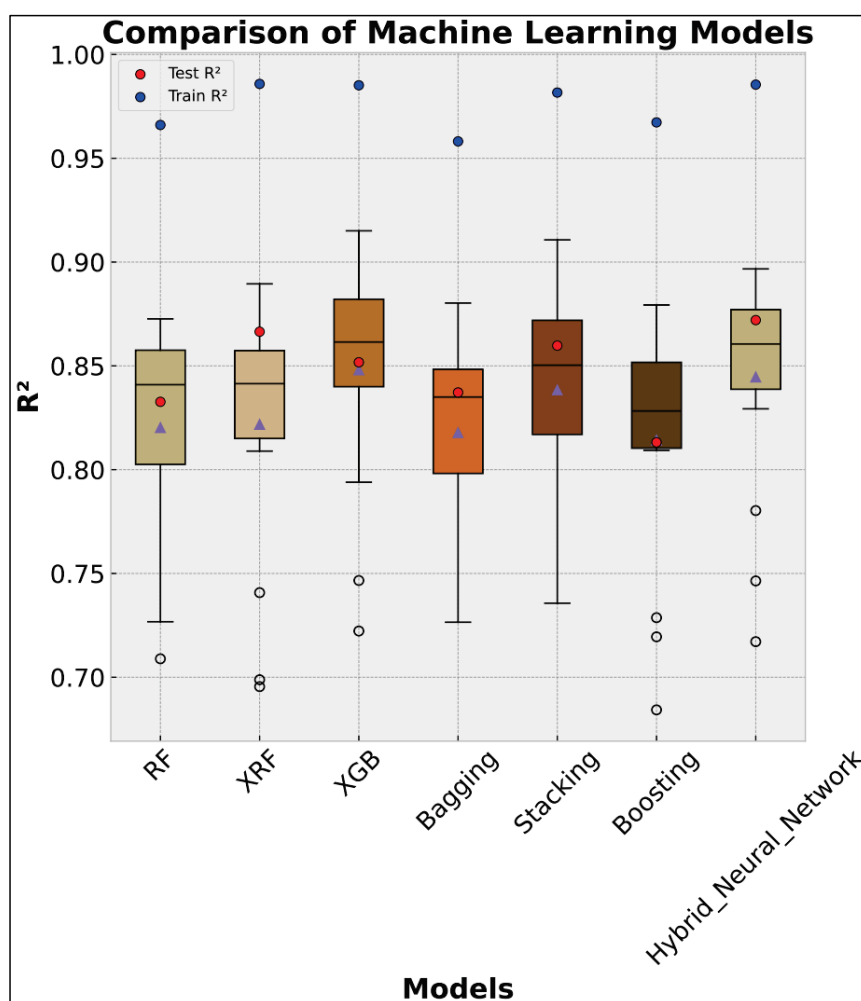


Figure 22. Model performance of the best-performing individual ML regression models along with the ensembled models evaluated.

The best overall performance was achieved by the Hybrid Neural Network, which reached an R^2 value of 0.872 on the testing subset. This model was designed to combine both

the original 41 input variables and the predictions from the three best-performing individual models—Random Forest (RF), Extremely Randomized Forest (XRF), and XGBoost (XGB)—thus incorporating a total of 44 input features. The architecture of the Hybrid Neural Network, shown schematically in Figure 23, includes two hidden layers: the first with 19 neurons and the second with 7 neurons, each utilizing nonlinear activation functions to enhance feature interaction and representation learning.

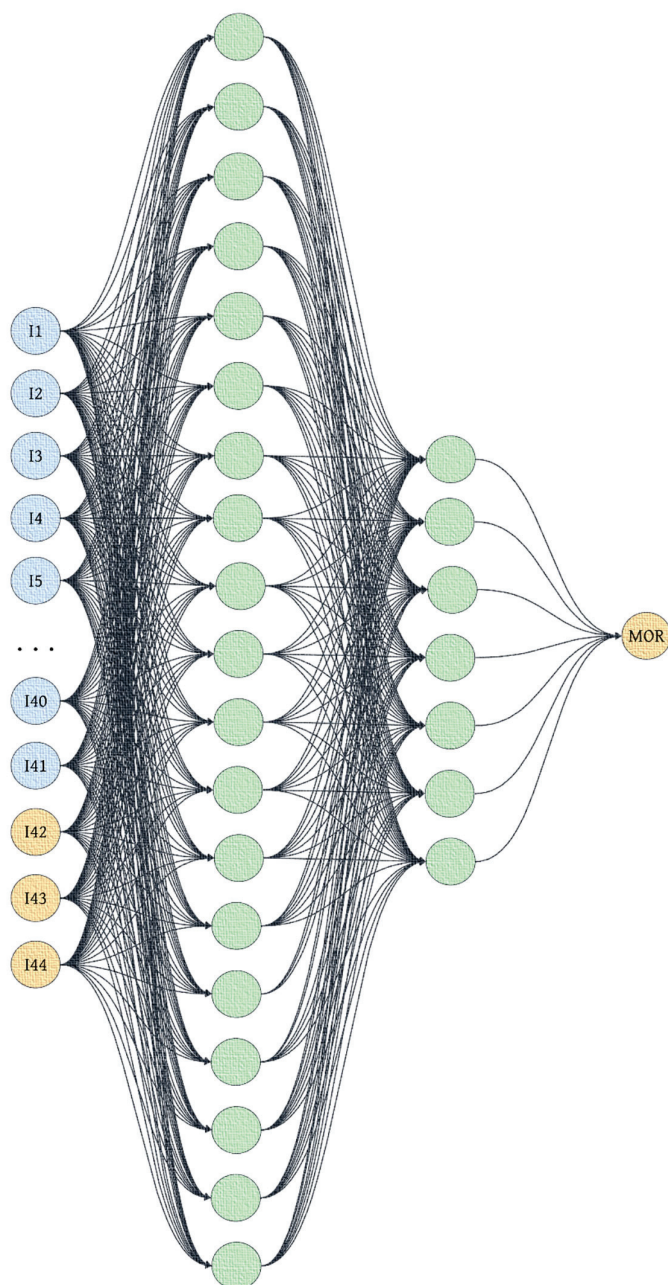


Figure 23. Scheme of the Hybrid Neural Network.

The predictive capacity of the Hybrid Neural Network is illustrated in Figure 24 through regression plots comparing the predicted versus actual values for the testing subset. These results confirm that the Hybrid Neural Network offers superior generalization capabilities, outperforming both individual and traditional ensemble models in accurately capturing the complex relationships governing the flexural behavior of fiber-reinforced UHPC.

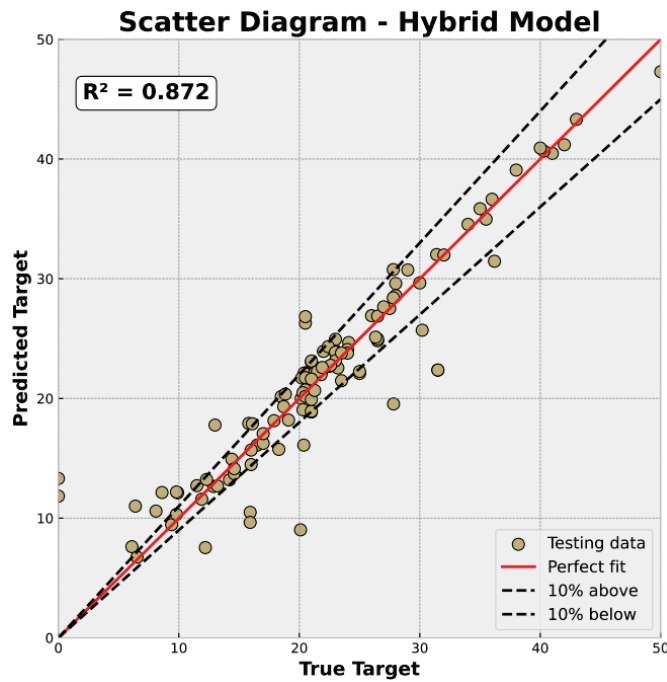


Figure 24. Regression plots of the Hybrid Neural Network on the testing data subset.

In addition, Table 8 presents a comparative analysis between the top-performing individual models and the hybrid model. This evaluation, conducted on the test dataset, includes key performance indicators such as the Mean Absolute Error (MAE), Root Mean Squared Error (RMSE), and the coefficient of determination (R^2)

Table 8. Comparative performance metrics of individual models (RF, XRF, XGB) and the proposed hybrid model, evaluated using R^2 , MAE, and RMSE, measured on the testing data subset.

Model	R^2	MAE	RMSE	MSE	MAPE
RF	0.841	2.3014	3.6531	13.3481	19.4000
XRF	0.843	2.0124	3.5402	12.5290	19.1101
XGB	0.859	1.9468	3.2823	10.7695	16.9485
Hybrid	0.872	1.8597	3.1315	9.8083	16.6920

As shown in Table 8, the hybrid model outperforms the individual models across all three metrics. It achieves the highest coefficient of determination ($R^2 = 0.872$), indicating better predictive accuracy, while also yielding the lowest MAE (1.8597) and RMSE (3.1315), reflecting reduced prediction error on the test set.

3.5. Experimental Validation of the Developed Models

3.5.1. Experimental Findings

To provide an external validation of the developed predictive models, an experimental campaign was conducted involving 26 different UHPC mixtures featuring variations in matrix composition, fiber content, and hybrid reinforcement strategies. The MOR results obtained from these tests are summarized in Figure 25.

The experimental program covered a wide range of flexural strengths, reflecting the influence of key design parameters previously highlighted through SHAP analysis. As identified in the feature importance analysis, variables such as cement content, fiber factor, total fiber volume, water-to-binder ratio, and maximum aggregate size exhibited strong impacts on flexural performance predictions. The experimental results confirmed these trends: higher MOR values were generally associated with increased cement content,

optimized fiber reinforcement (both in type and volume fraction), and lower water-to-binder ratios, while mixtures with larger maximum aggregate sizes tended to exhibit reduced flexural performance. In this sense, Figure 25 reveals that, for the same fiber reinforcement system, UHPC-matrix type A outperforms type B and C. On the other hand, for the same UHPC-matrix type, fiber dosage is a dominant feature.

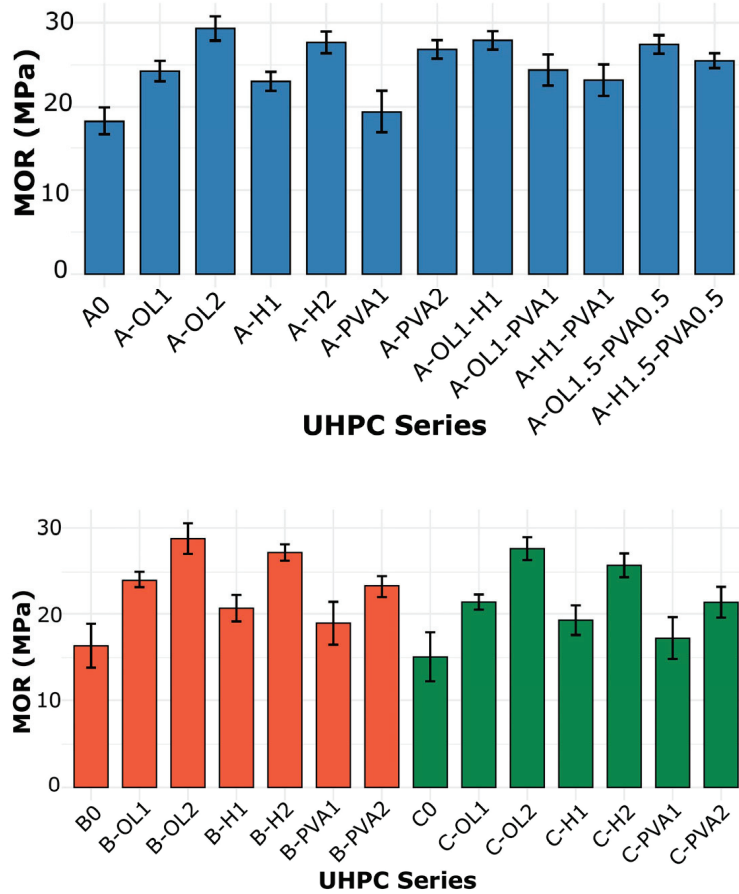


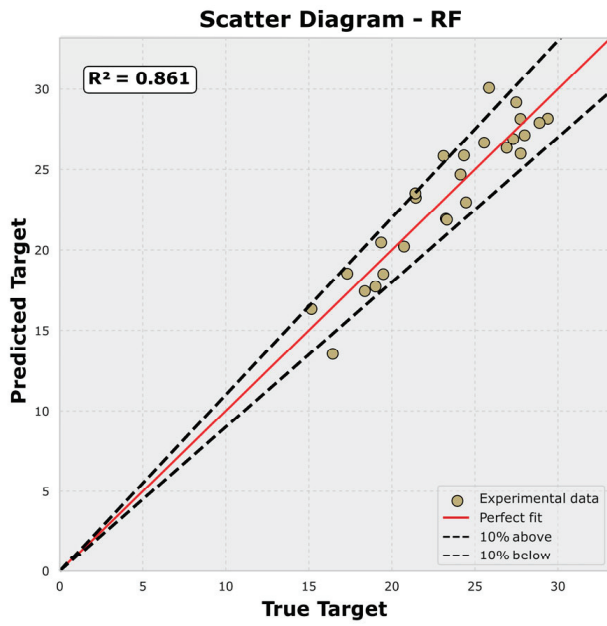
Figure 25. Experimental results of flexural strength (MOR) for the 26 UHPC series evaluated.

Furthermore, the limited influence of supplementary cementitious materials such as fly ash, ground glass powder, and calcium carbonate—previously noted in the SHAP interpretation—was corroborated by the experimental data, where variations in these components produced relatively minor shifts in MOR values.

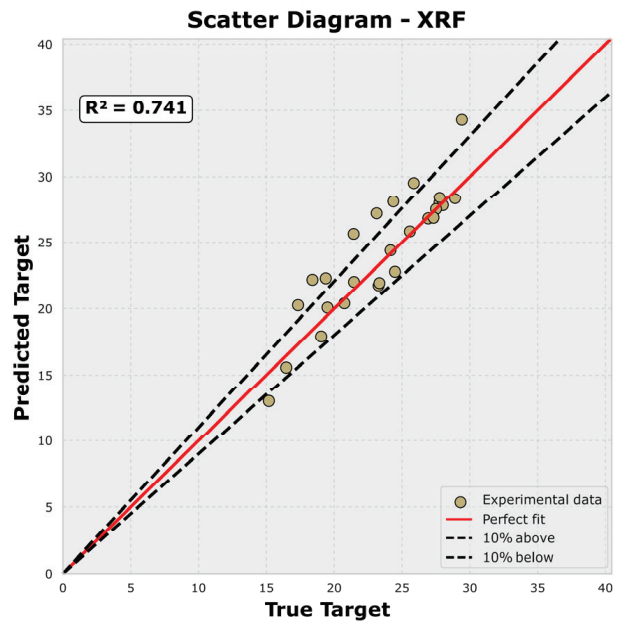
Overall, the experimental findings validate the predictive insights obtained from the machine learning models and the SHAP-based feature analysis, confirming the relevance of the selected variables in governing the flexural behavior of fiber-reinforced UHPC.

3.5.2. Machine Learning Evaluation by Means of the Experimental Findings

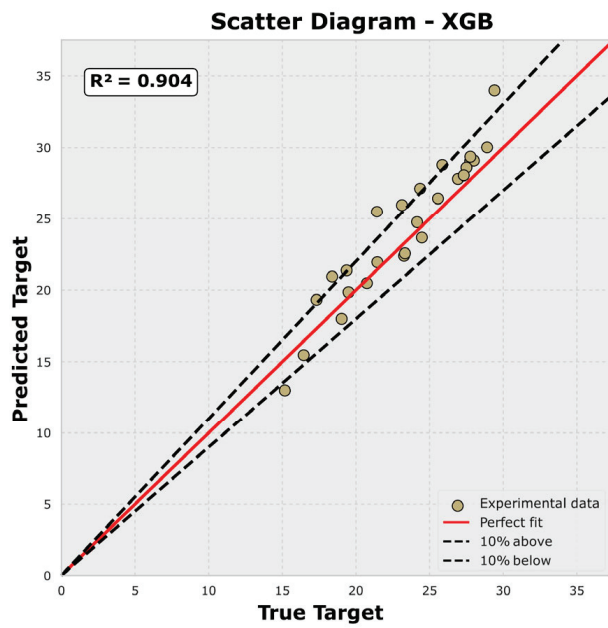
The predictive capabilities of the selected machine learning models—i.e., RF, XRF, XGB, and the Hybrid Neural Network—were further evaluated using the experimental dataset. Figure 26 presents regression plots comparing the predicted versus measured flexural strength values (MOR) for each model, with the dashed line representing perfect agreement between predictions and experimental results. In addition, Table 9 presents additional metrics (i.e., MAE and RMSE) measured on the experimental data for the individual best-performing models and the hybrid one as well.



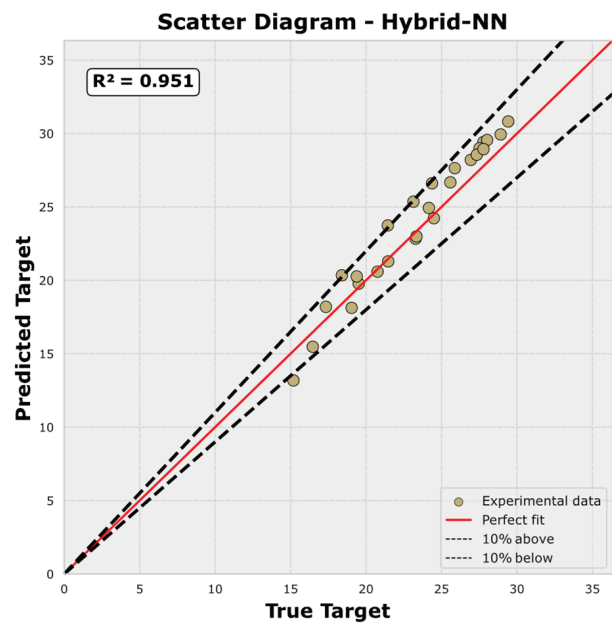
(a) Experimental Data Test—RF



(b) Experimental Data Test—XRF



(c) Experimental Data Test—XGB



(d) Experimental Data Test—HNN

Figure 26. Regression plots of the best-performing individual ML regression models and the Hybrid Neural network on the experimental dataset.

Table 9. Comparative performance metrics of individual models (RF, XRF, XGB) and the proposed hybrid model, evaluated using R^2 , MAE, and RMSE, measured on the experimental data subset.

Model	R^2	MAE	RMSE	MSE	MAPE
RF	0.861	1.6627	2.0372	4.1511	6.2277
XRF	0.741	2.2224	3.7521	14.0733	7.8542
XGB	0.904	1.5267	1.7958	3.2250	5.4667
Hybrid	0.951	1.3892	1.6528	2.7328	3.7331

Among the individual models, XGB exhibited the best performance with an R^2 value of 0.904, closely approximating the experimental measurements. The RF model followed with an R^2 of 0.861, indicating good predictive accuracy but with slightly greater dispersion, particularly at higher MOR values. In contrast, the XRF model showed a markedly lower R^2 of 0.741, reflecting a weaker ability to capture the true flexural behavior across the entire range of data.

The Hybrid Neural Network achieved the highest predictive performance, with an R^2 of 0.951, demonstrating excellent alignment between predicted and actual MOR values. The close clustering of data points around the ideal prediction line in Figure 24 highlights the robustness and enhanced generalization capability of the hybrid approach.

These findings confirm that combining individual model predictions with the original feature set through a hybrid neural network framework significantly improves predictive accuracy, offering a powerful and reliable tool for modeling the complex flexural behavior of fiber-reinforced UHPC materials.

4. Conclusions

This study developed and validated a machine learning-based framework to predict the flexural behavior of fiber-reinforced UHPC, addressing the inherent complexity arising from the interactions among constituent materials, fiber reinforcements, and matrix characteristics. Key findings can be summarized as follows:

- (1) Among the evaluated models, Random Forest (RF), Extremely Randomized Forest (XRF), and XGBoost (XGB) demonstrated superior predictive capabilities, each achieving R^2 scores above 0.80 on the testing subset, significantly outperforming traditional models such as Linear Regression, K-Nearest Neighbors, Support Vector Regression, and conventional Artificial Neural Networks.
- (2) SHAP-based interpretability highlighted cement content, fiber factor, total fiber volume fraction, silica fume dosage, water-to-binder ratio, and maximum aggregate size as the dominant parameters influencing flexural strength. These insights were scientifically consistent with the current understanding of UHPC material behavior.
- (3) The application of ensemble methods further enhanced predictive performance. While Bagging, Stacking, and Boosting showed notable improvements, the Hybrid Neural Network model achieved the highest predictive accuracy with an R^2 of 0.872, outperforming all other approaches on the testing dataset.
- (4) An independent experimental campaign involving 26 novel UHPC mixtures confirmed the robustness of the developed models. The Hybrid Neural Network again provided the best performance, achieving an R^2 of 0.951 against the experimental results, thereby demonstrating exceptional generalization capability and reliability.
- (5) The experimental findings, aligned with SHAP interpretations, confirmed that the incorporation of supplementary cementitious materials such as fly ash, recycled glass powder, and calcium carbonate marginally impacted flexural strength, supporting the potential for sustainable UHPC mix designs without compromising mechanical performance.
- (6) A detailed one-way Partial Dependence Plot (PDP) analysis was conducted on six key variables—cement content, silica fume content, superplasticizer dosage, maximum aggregate size, fiber volume fraction, and fiber factor. Each variable exhibited a characteristic nonlinear effect: cement and silica fume improved strength up to optimal dosages ($\sim 0.27 \text{ m}^3/\text{m}^3$ and $\sim 0.093 \text{ m}^3/\text{m}^3$, respectively), beyond which negative effects such as porosity and agglomeration could emerge. Superplasticizer content showed benefits up to $\sim 0.04 \text{ m}^3/\text{m}^3$ but declined thereafter possibly due to porosity and hydration interference. Coarse aggregate size negatively impacted MOR, which

could be ascribed to poor fiber dispersion and orientation, reducing effectiveness. Fiber volume and fiber factor enhanced flexural strength up to ~3% and ~2, respectively, before plateauing or declining what can be explained by due to poor dispersion and fiber clustering.

- (7) In two-way partial-dependence interaction analyses (PDPs), the interaction of global fiber factor (IV21) with relevant matrix properties—maximum aggregate size, water-to-binder ratio, and matrix compressive strength—was investigated to modulate MOR. These bivariate PDPs showed diminishing marginal returns at a fiber factor of ~1.8–2.0. Other conclusions of the analysis include the following:
 - a. Smaller aggregate sizes enhance MOR due to improved fiber alignment and higher effective aspect ratios.
 - b. Lower water-to-binder ratios amplify IV21's effect by improving fiber-matrix bond strength and matrix density.
 - c. Higher compressive strengths increase MOR by enabling better stress transfer and fiber anchorage.

In conclusion, the integration of advanced machine learning models, ensemble strategies, and interpretable AI techniques offers a powerful, reliable, and transparent framework for predicting and optimizing the flexural behavior of fiber-reinforced UHPC. This study not only advances the predictive modeling of UHPC mechanical properties but also contributes to the design of more sustainable, efficient, and high-performance concrete materials for critical structural applications.

Future research could focus on extending the methodology toward multi-objective optimization frameworks, incorporating durability-related properties, and further exploring the integration of novel, eco-efficient Supplementary Materials in UHPC formulations. In addition, forthcoming work will also explore the use of physics-informed neural network frameworks—particularly the consistent Deep-Energy Method—to simulate fiber pull-out and fracture behavior at the meso-scale.

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Article

Inspection of Damaged Composite Structures with Active Thermography and Digital Shearography

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Abstract: This study comprehensively compares the performance of two non-destructive testing (NDT) techniques—active thermography (AT) and digital shearography (DS)—for identifying various damage types in composite structures. Three distinct composite specimens were inspected: a carbon-fiber-reinforced polymer (CFRP) plate with flat-bottom holes, an aluminum honeycomb core sandwich plate with a circular skin-core disbond, and a CFRP plate with two low-energy impacts damage. The research highlights the significant role of post-processing methods in enhancing damage detectability. For AT, algorithms such as fast Fourier transform (FFT) for temperature phase extraction and principal component thermography (PCT) for identifying significant temperature components were employed, generally making anomalies brighter and easier to locate and size. For DS, a novel band-pass filtering approach applied to phase maps, followed by summing the filtered maps, remarkably improved the visualization and precision of damage-induced anomalies by suppressing background noise. Qualitative image-based comparisons revealed that DS consistently demonstrated superior performance. The sum of DS filtered phase maps provided more detailed and precise information regarding damage location and size compared to both pulsed thermography (PT) and lock-in thermography (LT) temperature phase and amplitude. Notably, DS effectively identified shallow flat-bottom holes and subtle imperfections that AT struggled to clearly resolve, and it provided a more comprehensive representation of the impacts damage location and extent. This enhanced capability of DS is attributed to the novel phase map filtering approach, which significantly improves damage identification compared to the thermogram post-processing methods used for AT.

Keywords: damage identification; impact damage; nondestructive testing; composite structures; active thermography; pulsed thermography; lock-in thermography; digital shearography

1. Introduction

Aerospace, automotive, and renewable energy industries, among others, rely heavily on non-destructive testing (NDT) to ensure structural integrity. Indeed, the pursuit of non-invasive and NDT techniques for early detection of damage and defects in composite structures remains a central focus of contemporary structural health monitoring (SHM). Such damage and defects can severely impact the load-bearing capacity of a structure and ultimately compromise its integrity. NDT techniques are particularly relevant for

inspecting large composite structures such as aircraft wings, fuselage, and other subsurface structural elements.

Both defects and damage negatively impact the ability of a structure to bear loads. Their classification, however, hinges on their source and time. Defects are imperfections or irregularities that arise during various stages of material processing or manufacturing of a structure. Damage, on the other hand, describes the degradation or deterioration that develops during the service life of a structure, which can be induced by applied loads, environmental conditions, or even initiated by an existing structural defect [1]. Composite materials can suffer damage of different sizes and complexities. We can generally group the type of damage into three main levels of severity. The smallest damage encompasses nano-scale damage, which could be very difficult to detect. Simple or primary damage occupies micro-scale to meso-scale levels. These include matrix cracking (tiny cracks in the material's binding resin), fiber splitting (cracks running along individual fibers or bundles), debonding fiber to matrix (separation between fibers and the resin), microbuckling and waviness in fibers (small, localized buckling of fibers under compression), fiber breakage or fracture (actual breaks in the load-carrying fibers), and kink bands (severe, localized bands of buckled and fractured fibers). The final level involves complex, macro scale damage that can be observed with the naked eye. Unfortunately, most of this damage appears beneath the surface, disabling its visual detection. This level includes delamination (separation of layers within the composite, significantly weakening it), stiffness loss (a noticeable drop in the material's overall rigidity, indicating extensive internal damage), and impact damage (often hidden, involving a complex mix of internal delaminations, matrix cracking, and fiber breakage caused by external force) [2]. Delaminations and disbonds are the most frequently identified and structurally significant types of damage. Impact damage is also highly critical, largely due to its often-hidden nature and the severe, complex internal flaws it creates [1]. For the inspection of damage in composite structures, the predominant NDT techniques employed include acoustic emission, ultrasonic testing, infrared thermography, terahertz testing, shearography, digital image correlation, X-ray, and neutron imaging [2].

For this work, infrared active thermography (AT) and digital shearography (DS) were chosen due to their significant advantages. These non-contact, full-field optical techniques ensure safe operation in detecting both surface and subsurface damage and defects. They are both the most prevalent optical techniques actively employed for the in-field aircraft inspection and are often stipulated in composite component inspection manuals. AT is an optical imaging technique that makes use of thermal waves. The basis of thermal imaging lies in detecting the infrared radiation emitted by the inspected surface of the specimen [3]. While often approximated as an ideal blackbody for analytical simplicity, real materials emit radiation proportional to their absolute temperature and their emissivity. Indeed, unlike an ideal blackbody, whose emission depends solely on temperature, the emissivity of real materials varies with factors like material type, surface finish, and wavelength [4]. The sensor unit responsible for receiving and integrating this emitted radiation is typically a bolometer. Modern infrared cameras integrate a rectangular array of elementary bolometers with specialized lenses, commonly made of Germanium or Zinc-selenide. These materials are transparent to infrared wavelengths (while blocking visible light), forming an optical system crucial for infrared image formation [3]. Such systems are designed to record the thermal response with high spatial, temporal, and thermal resolution. An external heat source is commonly used to generate thermal waves that propagate through the thickness of the specimen. If these waves encounter anomalies in the material, they will exhibit a distinct reflection pattern. This pattern results in a temperature variation at the surface that is then captured by an infrared camera. DS is a laser-based optical interferometric method used to measure surface strains resulting from external

stimuli such as thermal, vacuum, or vibrational loads [5]. It uses an optical configuration more resilient to environmental disturbances and vibrations than other interferometric techniques and has become an important diagnostic tool in nondestructive testing [6]. A camera captures the intensity pattern generated by laser speckle interference. When the specimen is subjected to an external load, small deformations are produced on its inspected surface. These deformations are captured by the camera as variations in the intensity patterns. Subsurface damage often leads to the appearance of regions of higher deformation, which indicates the presence of internal flaws like delaminations, cracks, or bond failures.

Hung et al. [7] wrote a comprehensive review of AT and DS techniques, delineating their advantages and limitations. The work directly compares the results yielded by thermal pulse excitation on composite, metallic, and plastic specimens. DS and AT with different excitation sources were used by Georges et al. [8] for the inspection of hybrid composite-metal sandwich structures made of titanium skins and CFRP/GFRP mix core. Both techniques yield good results, and it is suggested that they can be used as separate techniques or under integrated schemes. Wei et al. [9] compared long pulse thermography (LPT) and shearography for defect evaluation. This research focused on detecting disbonding defects in rubber-to-metal and aluminum sandwich plates, as well as flat-bottom hole defects in carbon-fiber reinforced polymer (CFRP) panel. The authors determined that LPT and DS are complementary techniques. They observed that LPT performed best in identifying disbands in rubber-to-metal plates and flat-bottom holes in CFRP panels, whereas DS proved to be more effective for disbond identification in aluminum sandwich plates. Notably, LPT demonstrated a significantly lower error than DS for damage size evaluation. A crucial insight was also the critical role of post-processing methods in improving the defect identification for both techniques. Very recently, Ochana et al. [10] assessed the effectiveness of AT and DS as non-destructive NDT technique for 3D-printed thermoplastic (TP) composites reinforced with continuous carbon fiber. Artificial defects of varying sizes and depths were introduced for benchmark purposes. AT identified defects by observing surface temperature changes. It proved effective for defects up to 1.25 mm deep (and some at 1.5 mm), enhancing their presence through thermal anomalies. However, its limitations included difficulty in detecting deeper or smaller defects and accurately determining their shapes due to thermal diffusion effects. DS successfully identified near-surface defects within the same size range. While effective for these, it required greater expertise for accurate interpretation and, like AT, it struggled with detecting smaller and deeper imperfections.

The present study compares the efficiency of AT and DS, along with their respective post-processing techniques, for identifying various types of damage in three distinct composite specimens: a carbon-fiber-reinforced polymer (CFRP) plate containing multiple flat-bottom holes, an aluminum honeycomb core sandwich plate featuring a circular skin-core disbond, and a CFRP plate presenting two low-energy impacts damage.

2. Materials and Methods

2.1. Inspected Structures

This study aims to compare the effectiveness of two NDT techniques, active thermography (AT) and digital shearography (DS), in identifying different types of damage in composite structures. For this purpose, three plate specimens were inspected. Specimen 1 is a CFRP plate with several flat-bottom holes; specimen 2 is an aluminum honeycomb core sandwich plate with a circular core-skin disbond; and specimen 3 is a CFRP plate with two low-energy impacts damage. These three composite structures were selected for their relevance in the aerospace, automotive, and renewable energy industries. For

instance, sandwich structures can be found in a great variety of aircraft [11], a huge amount of CFRP is replacing metal components in space launchers [12], CFRP structures appear in several parts of cars [13], and the materials commonly used in wind turbine blades are fiber reinforced polymers [14]. Composite structures are particularly susceptible to several types of damage such as delamination, matrix cracking and crushing, fiber breakage, and debonding [15–21]. The three chosen specimens present distinct damage types: the flat-bottom holes in specimen 1 mimic multiple delaminations of various sizes and depths in plates [22,23]; the missing adhesive in the second specimen simulates debonding between the core and the skin [18]; and finally, specimen 3 is subjected to low-energy impacts of 13.5 J and 26.2 J in order to create barely visible impact damage (BVID).

Specimen 1 was made from a single laminated plate consisting of seven layers of prepreg woven carbon fiber/epoxy. The stacking sequence of (45/0/45/0/45/0/45) was chosen to provide nearly isotropic mechanical properties. The fabrication of the plate with flat-bottom holes involved three steps. Initially, the single laminated plate was cut into four equal parts. Subsequently, through-holes of varying dimensions were drilled into three of these parts. Finally, these four parts were glued together using epoxy resin, forming a consolidated plate with the distinct bottom hole configurations. Figure 1a presents a photo of specimen 1 showing the flat-bottom holes on the back surface. The plate has overall dimensions of 206 mm (length) $\times$ 170 mm (width) $\times$ 8 mm (thickness). Its design incorporates three vertical columns, each containing four flat-bottom holes. The flat-bottom holes decrease in diameter with the y coordinate: 25 mm, 20 mm, 16 mm, and 10 mm. The depth of the flat-bottom holes also varies with the x coordinate: 6 mm, 4 mm, and 2 mm. The second specimen consists of an aluminum sandwich plate, comprising a honeycomb core, with 4.33 mm cell dimensions, and two skins, each 0.8 mm in thickness. The overall plate dimensions are 315 mm in length, 220 mm in width, and 12 mm in thickness. An artificial circular debond, 70 mm in diameter, was deliberately introduced during manufacturing by removing the adhesive that bonds the honeycomb core to one of the skins. This flaw simulates a lack of adhesion between the honeycomb core and one of the skins, and is located at coordinates (215 mm, 150 mm). Figure 1b presents a photo of the front surface of specimen 2, highlighting the location and size of the circular damage. Specimen 3 is a CFRP rectangular plate, measuring 276.5 mm in length, 198 mm in width, and 1.825 mm in thickness, with a stacking sequence of (0/90/+45/−45/0/90)₅ and containing two internal damaged areas [24]. Internal damage was generated within the plate by two low-energy impacts. These impacts consisted of dropping a 62.2 mm diameter steel sphere onto the plate. The first impact occurred at coordinates (74 mm, 147 mm) with an energy of 13.5 J and the second at (215 mm, 38 mm) with an energy of 26.2 J. It can be pointed out that according to AIRBUS standards, the impact energy threshold for generating a BVID is 50 J [25]. The plate surface was coated with a thin white powder to aid in identifying the two impact locations. Figure 1c shows a photo of the front surface of specimen 3 after the impacts, where the only visible effect was the absence of white powder in the contact points of the sphere with the plate.

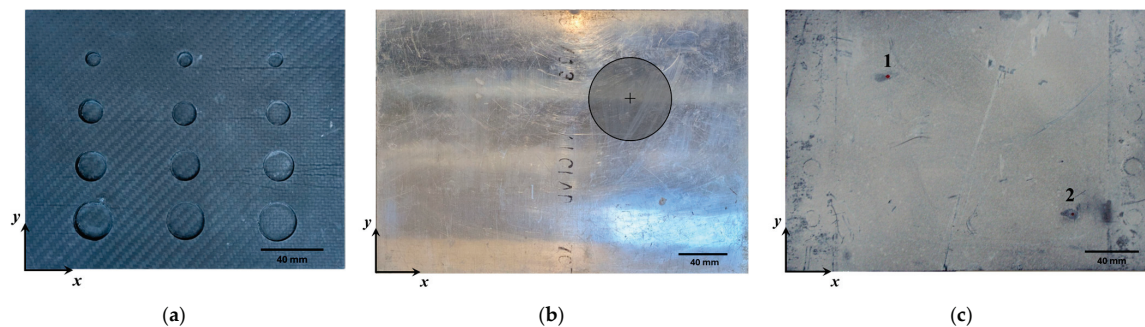


Figure 1. Inspected composite structures: (a) specimen 1 back surface, (b) specimen 2 front surface, (c) specimen 3 front surface.

2.2. NDT Techniques Employed in Inspection

The following two sections describe the NDT techniques used for inspection of the three specimens. The first, AT, uses an external heat source to generate a thermal wave through the thickness; an infrared camera then monitors surface temperature changes over time to reveal hidden damages. The DS technique, on the other hand, employs a laser illumination and an optical interferometer to capture the evolution of surface deformation gradients, often induced by an external heat source, thereby identifying areas of strain concentration indicative of damage.

2.2.1. Active Thermography (AT)

The first NDT technique used to identify damage is AT, which can be subdivided according to the kind of applied thermal excitation [26,27]. In the present work, two types of AT were used in the inspection of specimens: (1) pulsed thermography (PT) and (2) lock-in thermography (LT). In PT, a rectangular pulse thermal stimulation is delivered to the inspected surface, whereas in LT the stimulation is harmonic. Figure 2 shows the experimental setup used in this study. For specimen 1, the inspection was performed on the surface opposite the one containing the flat-bottom holes. For the other specimens, the inspections were on the outer surface of the disbonded skin of specimen 2, and on the impacted surface of specimen 3.

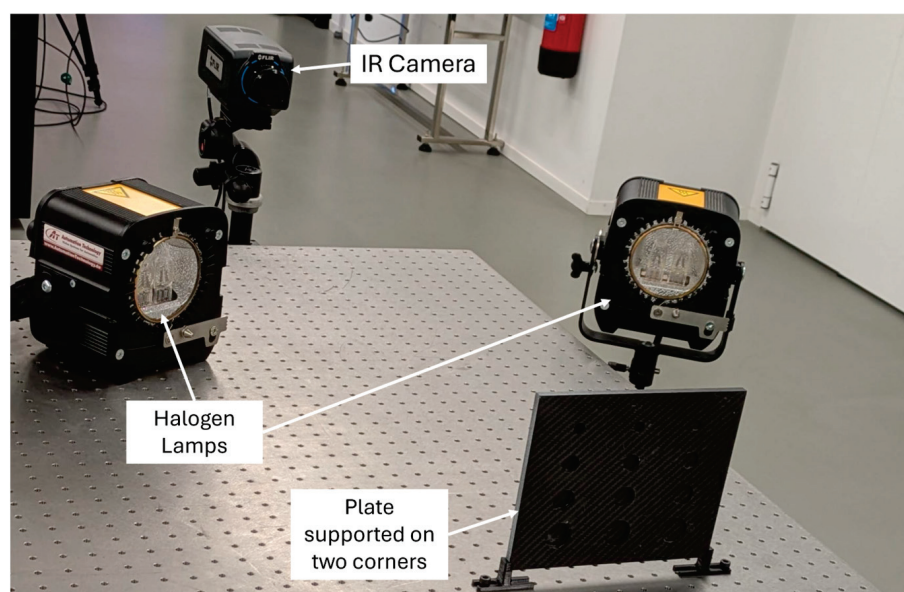


Figure 2. Experimental setup for the acquisition of thermograms.

The thermal images, or thermograms, of the inspected surfaces were recorded using a FLIR® A6751sc infrared camera. This camera has a 640×512 pixel resolution and is sensitive to infrared radiation in the $3\text{--}5\ \mu\text{m}$ spectral band. The thermal excitation was supplied by two 2 kW halogen lamps from Automation Technologies®, powered by an IRX-Box. The National Instruments® NI USB-6251 module provided synchronization between the acquisition of thermograms and the thermal loading. Before the tests, the inspected surfaces were coated with a thin matte black paint to reduce emissivity variations and minimize reflectivity. The specimens were positioned vertically, supported at their lowest bottom corners, and the surface aligned with the camera's optical axis. Two halogen lamps were placed symmetrically around the inspected surfaces at approximately a 45-degree angle and a distance of 50 cm to ensure nearly uniform thermal loading.

Figure 3a shows schematically the sequential acquisition of several thermograms and Figure 3b illustrates typical temperature evolutions at a given pixel located at coordinates (x, y) measured with both AT variants. For measurements with the PT variant, a transient 4 kW excitation was applied to the inspected surfaces, and thermograms were acquired at 7.5 fps just after the thermal excitation. The LT variant involved applying a harmonic wave of two period duration of 2 kW amplitude, being the thermograms acquired at 60 fps during the thermal loading. The thermal loading parameters for achieving the best damage identification results were found by testing various rectangular pulse durations and harmonic frequencies. The optimal parameters for PT pulse duration and LT harmonic excitation were found to be: 12 s and 0.008 Hz for specimen 1; 0.3 s and 0.111 Hz for specimen 2; and 0.5 s and 0.125 Hz for specimen 3, respectively. It should be highlighted that the results are linearly converted into an 8-bit grayscale image for a clear and intuitive visualization of their distribution. This conversion yields a high-contrast representation, thereby facilitating the rapid identification of subtle, damage-induced differences.

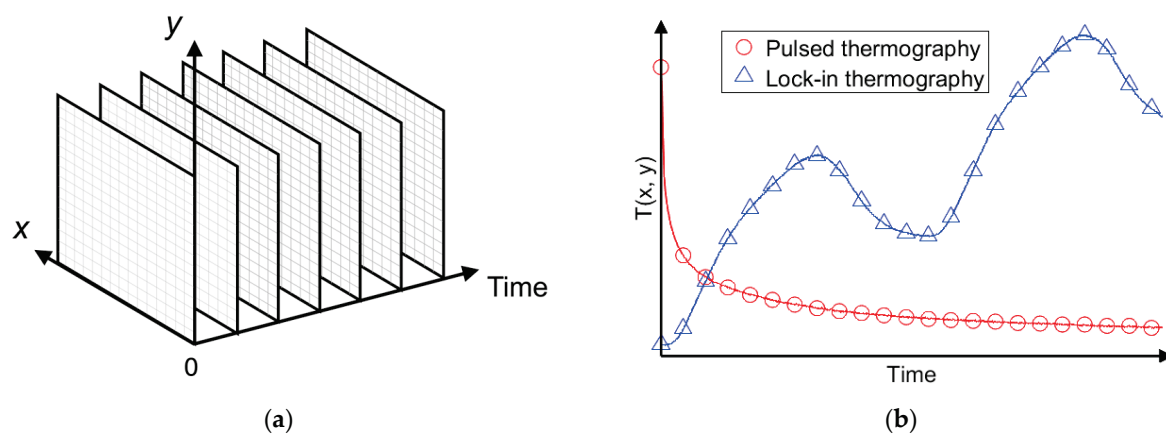


Figure 3. (a) Thermograms sequence; and (b) typical temperature evolutions of a pixel located at coordinates (x, y) .

2.2.2. Digital Shearography (DS)

The second NDT technique presently applied for the identification of damage in the three composite structures is DS with thermal loading. An in-house built DS system, based on a Michelson interferometer and employing temporal phase modulation, was used to monitor the deformation of the specimens during the cooling stage after applying a thermal load. The experimental setup for these measurements is presented in Figure 4. All the devices and the inspected specimens are mounted on top of an optical table with pneumatic suspension to minimize the influence of external vibrations during data acquisition. The specimens were held vertically, secured to the optical table by two supports at their lowest bottom corners. Measurements were carried out on the same surfaces of the specimen, pre-

viously inspected by thermography. The inspected surfaces were covered by a thin layer of white power to create a uniform reflection of the laser light. These surfaces are illuminated uniformly by a 532 nm continuous wave (CW) laser from Coherent (Verdi)[®]. The DS system generates an interference pattern from the reflection wave, and a 4000×3000 pixels digital camera from Basler (acA4112-20um)[®] captures its intensity. Temporal phase modulation of four images with a phase step $\pi/2$ is then used to extract the phase of the interference pattern [28]. This modulation requires the translation of a Michelson interferometer mirror using integrated piezoelectric transducers, actuated by a piezoelectric controller. A 5 mm shearing amount was simultaneously applied in both the x and y directions by slightly tilting the second mirror of the Michelson interferometer. The experimental procedure began with the application of a 10 s rectangular thermal pulse to the inspected surface, using a 500 W halogen lamp. Subsequently, during the 96 s cooling stage of the plate, four images, each corresponding to a phase step of $0, \pi/2, \pi$, and $3\pi/2$, were acquired at 8 s intervals. The acquisition time for each image set was approximately 0.4 s. These images were stored in a computer for later computation of the interference phase. The evaluation of phase maps for each interval, which are directly related to the relative deformation of the inspected surface, involves correlating the interference phases of two consecutive records. In other words, the phase map $\Delta\phi(x, y, t + \Delta t)$, at an instant $t + \Delta t$, is the subtraction between two consecutive interference phases $\Phi(x, y, t + \Delta t)$ and $\Phi(x, y, t)$, where Δt is a predefined time step. In the present study, 13 interference phases were acquired, resulting in 12 phase maps.

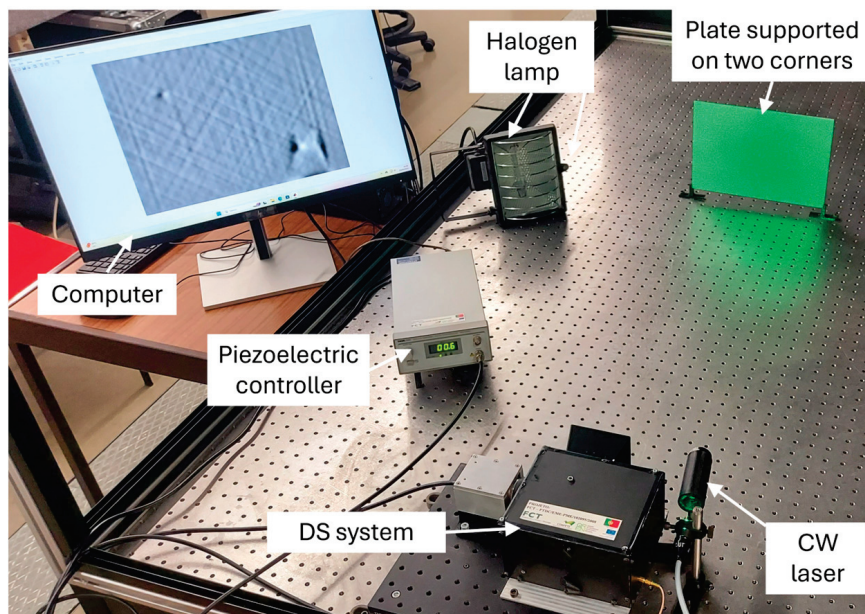


Figure 4. Experimental setup for the acquisition of phase maps.

2.3. Damage Identification Procedure

Identifying structural damage by analyzing the global structural response is challenging because damage typically induces small and localized variations that are difficult to differentiate from the response of the structure in the undamaged state. To enhance damage detectability, it is necessary to post-process the experimental raw data using dedicated tools to extract these damage-induced variations. The following subsections describe the tools and procedures used for damage identification, applied to thermograms from thermography and phase maps from digital shearography.

2.3.1. Post-Processing of Thermograms

The need for thermograms post-processing is illustrated in Figure 5, which shows raw thermograms acquired with PT and LT. It is clear that the damage is only visible in the deeper flat-bottom holes, i.e., the holes on the right column.

In PT, thermographic signal reconstruction (TSR) is frequently applied to improve the quality of thermograms. Typically, TSR involves fitting a 4th or 5th-degree polynomial to the logarithmic temporal decay of the thermal signal at each pixel. This procedure effectively attenuates temporal noise from the IR camera and mitigates artefacts caused by reflection and air convection, resulting in time-filtered thermograms [29]. Basically, TSR is based on approximating the logarithmic time dependence of a pixel in a thermogram by an M th-degree polynomial

$$\ln[T(t)] = \sum_{k=0}^M a_k [\ln(t)]^k \quad (1)$$

and reconstructing the original thermographic signal according to

$$T(t) = \exp\left(\sum_{k=0}^M a_k [\ln(t)]^k\right) \quad (2)$$

where T and t stands for temperature and time, respectively, with a_k being the polynomial coefficients.

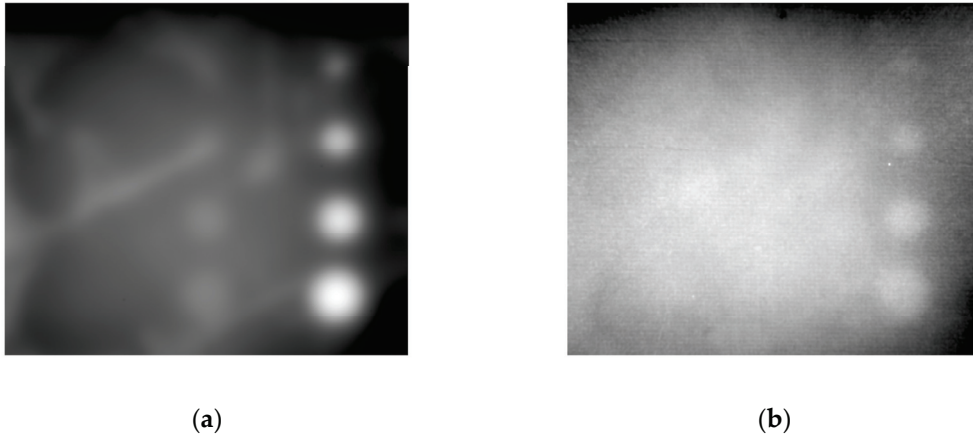


Figure 5. Thermograms of specimen 1 acquired with: (a) PT and (b) LT.

The fast Fourier transform (FFT) is applied to a temporal sequence of thermograms to compute phase in the temperature [30]. In PT, the temporal evaluation of the temperature phase is accomplished by applying a FFT to a sliding set of 15 thermograms. On the other hand, in LT, only a single temperature phase is extracted by applying a FFT to the full set of thermograms. The sequence of thermograms provides a temperature time series, $\{T(x, y, t_0), T(x, y, t_1), T(x, y, t_2), \dots, T(x, y, t_{N_T-1})\}$, for each pixel located at coordinates (x, y) . The length of this series, N_T , corresponds to the number of acquired thermograms. The FFT algorithm efficiently computes the discrete Fourier transform (DFT), which is then applied to the temperature profiles extracted from thermograms. This process transforms these temperature profiles $T(x, y, t)$ into the frequency domain, where each frequency component $F_k(x, y)$ is given by the DFT equation:

$$F_k(x, y) = \frac{1}{N_T} \sum_{n=0}^{N_T-1} T(x, y, t_n) e^{-\frac{i2\pi kn}{N_T}} \quad (3)$$

where i represents the imaginary unit and $F_k(x, y)$ is a complex number that contains both magnitude and phase information for the frequency order k . The magnitude $|F_k(x, y)|$ and phase $\phi_k(x, y)$ are then extracted according to:

$$|F_k(x, y)| = \sqrt{[Re(F_k(x, y))]^2 + [Im(F_k(x, y))]^2}, \quad (4)$$

$$\phi_k(x, y) = \text{atan} \left[\frac{Im(F_k(x, y))}{Re(F_k(x, y))} \right] \quad (5)$$

respectively. The magnitude and phase spectra are obtained by computing each frequency component from order 0 through $N_T/2$. For each pixel with coordinates (x, y) , the specific temperature phase is retrieved from the frequency component with the highest magnitude in its respective spectrum.

Principal component thermography (PCT) [31,32] is a method applied to thermal data to extract the most valuable information, especially when looking for damage. PCT can be interpreted as an eigenvalue and eigenvector decomposition of the thermal data covariance matrix. This decomposition is usually carried out by using the singular value decomposition (SVD) algorithm. SVD is highly favored because it offers superior numerical stability and efficiency, particularly when dealing with large datasets such as the ones found in thermography. The procedure begins by reorganizing the thermogram sequence, initially a 3D matrix. These data are reshaped into a single 2D matrix, often denoted as $\mathbf{D}$. In this matrix, each row represents all pixel values from a single time instant, while each column corresponds to a specific time instant within the sequence. The next step involves centering the temperature for each instant (column) by subtracting the average temperature to all elements in matrix $\mathbf{D}$, creating a centered temperature matrix, $\mathbf{D}_c$. This matrix is then decomposed using SVD, expressed as

$$\mathbf{D}_c = \mathbf{U}\mathbf{\Sigma}\mathbf{V}^T, \quad (6)$$

where $\mathbf{U}$ contains the left singular vectors, interpreted as spatial basis principal components, $\mathbf{\Sigma}$ is a diagonal matrix containing the singular values, which are related to the eigenvalues of the covariance matrix, and $\mathbf{V}$ is the right singular vectors, representing temporal basis principal components scores. The singular values in $\mathbf{\Sigma}$ are arranged in decreasing order, indicating the relative importance or contribution of each principal component. The first component typically represents the global thermal response, whereas the components from the second to around the tenth order are generally used to reveal thermal anomalies induced by the damage. Higher-order components are typically discarded as they are predominantly associated with experimental noise. The thermograms are reconstructed using only the selected temporal basis principal components $\mathbf{V}_s$. The selection of the principal components was based on the visualization of the thermal image of each component. This process effectively retains the dominant thermal patterns while reducing noise and less significant variations. To achieve this, we first compute the principal component scores matrix $\mathbf{PS}$ by projecting the centered temperature matrix $\mathbf{D}_c$ onto the selected principal components $\mathbf{V}_s$:

$$\mathbf{PS} = \mathbf{D}_c \cdot \mathbf{V}_s. \quad (7)$$

Then, these scores are multiplied by the transpose of the selected principal components, transforming the data back into the original temperature space and resulting in the filtered thermograms $\mathbf{D}_f$:

$$\mathbf{D}_f = \mathbf{PS} \cdot \mathbf{V}_s^T. \quad (8)$$

The last step involves reshaping the 2D matrix of the filtered thermograms into a 3D matrix, thus reconstructing the thermogram time sequence. The flowchart in Figure 6a

illustrates the post-processing procedure for PT. Initially, the PT thermograms are time-filtered by TSR to attenuate noise and mitigates artefacts. The TSR-processed thermograms are post-processed using FFT and PCT to extract the phase and amplitude, respectively. On the other hand, we see in Figure 6b that the phase and the amplitude of LT thermograms are extracted directly by FFT and PCT.

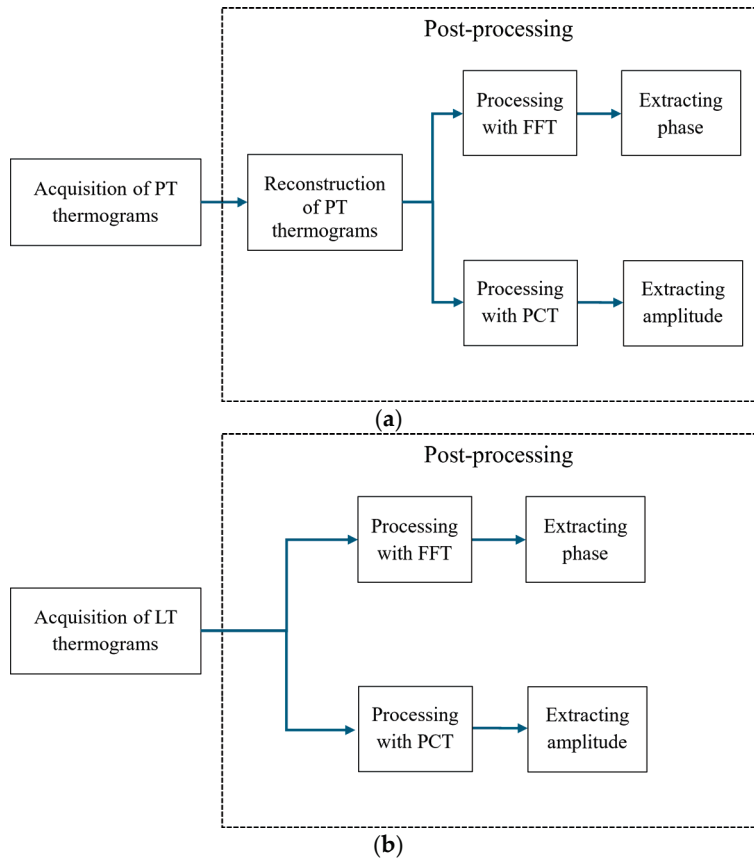


Figure 6. Thermograms post-processing procedure: (a) PT, (b) LT.

2.3.2. Post-Processing of Phase Maps

A DS system produces phase maps, showing how a deformation gradient or slope of a structure changes between two specific moments. The values in these phase maps are wrapped between $[-\pi, \pi]$ (or $[0, 2\pi]$) due to the need of computing the inverse tangent function to extract the interference phase from a series of recorded intensity images [33]. This wrapping is characterized by abrupt phase discontinuities across the map, commonly known as fringes. In addition, the presence of experimental noise, primarily due to speckle decorrelation, contaminates these phase maps. This noise diminishes the clarity of phase values and fringe visibility, thereby making it difficult to interpret the surface deformation gradient. Figure 7 displays a raw phase map of specimen 1. While it shows four distinct vertical fringes and a subtle phase variation along the right fringe, it does not reveal the flat-bottom holes. This demonstrates that post-processing of raw phase maps is essential for extracting damage-induced features and improving damage detectability.

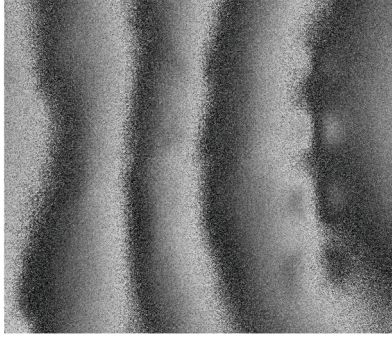


Figure 7. Phase map of specimen 1 acquired 8 s after the cooling stage begins.

The most common post-processing procedure is filtering and unwrapping the phase maps, thus reducing noise and eliminating the discontinuities in the raw phase map [9,15,22–24,33]. However, this improvement may not be substantial if the damage is relatively small. In such cases, another step in the post-processing is to differentiate the continuous filtered and unwrapped phase map [24,33]. However, besides the significant computational effort that this type of post-processing requires, it is difficult to achieve a crucial balance between preserving damage-induced high-frequency signal components and filtering out high-frequency experimental noise. This work employs, for the first time, a simpler, more direct, and very efficient procedure: the application of a band-pass filter to reveal the essential damage-induced phase variations in the maps, which are then summed across different time instants for enhanced visibility.

Typically, low-pass filters are applied to raw phase maps to reduce high-frequency experimental noise while preserving the relevant phase information. However, to effectively handle the cyclic nature of phase and its discontinuities before applying the low-pass filter, the phase maps are often first transformed into their complex exponential representation. This allows the filtering to be performed in the complex domain, which intrinsically deals with the 2π phase wraps and better preserves true phase variations. The raw phase map $\Delta\phi_r(x, y, t)$, evaluated at a generic instant t , is converted to the complex map $Z_r(x, y, t)$ using Euler's formula:

$$Z_r(x, y, t) = e^{i\Delta\phi_r(x, y, t)} = \cos(\Delta\phi_r(x, y, t)) + i\sin(\Delta\phi_r(x, y, t)). \quad (9)$$

A moving average filter, which acts as a low-pass filter, is applied to the complex map $Z_r(x, y, t)$ in the spatial domain using a 2D convolution, with the filtered complex map $Z_f(x, y, t)$ given by:

$$Z_f(x, y, t) = Z_r(x, y, t) * h(M, N), \quad (10)$$

where the symbol $*$ represents the convolution operator, $h(M, N)$ is the window, and M and N denote the vertical and horizontal dimensions of this window, respectively. The filtered phase map, $\Delta\phi_f(x, y, t)$, is retrieved by applying the arctangent function to the imaginary and real components of the filtered complex map $Z_f(x, y, t)$:

$$\Delta\phi_f(x, y, t) = \text{atan} \left[\frac{\text{Im}(Z_f(x, y, t))}{\text{Re}(Z_f(x, y, t))} \right]. \quad (11)$$

The process described by Equations (9)–(11) can be repeated for a specified number of passes to further attenuate the experimental noise. The cutoff frequency of the filter is directly related to both the window size and the number of passes. To generate two filtered phase maps, $\Delta\phi_{lcf}(x, y, t)$ and $\Delta\phi_{hcf}(x, y, t)$, the process described above is applied

twice. The first phase map $\Delta\phi_{lcf}(x, y, t)$ corresponds to a filter with lower cutoff frequency, whereas the second $\Delta\phi_{ucf}(x, y, t)$ is obtained with a filter having a higher cutoff frequency. These two phase maps are then subtracted and corrected, yielding a final filtered phase map:

$$\Delta\phi_{b-p}(x, y, t) = \begin{cases} \left[\Delta\phi_{ucf}(x, y, t) - \Delta\phi_{lcf}(x, y, t) \right] - 2\pi & \text{if } \left[\Delta\phi_{ucf}(x, y, t) - \Delta\phi_{lcf}(x, y, t) \right] > \pi \\ \left[\Delta\phi_{ucf}(x, y, t) - \Delta\phi_{lcf}(x, y, t) \right] + 2\pi & \text{if } \left[\Delta\phi_{ucf}(x, y, t) - \Delta\phi_{lcf}(x, y, t) \right] < -\pi \\ \left[\Delta\phi_{ucf}(x, y, t) - \Delta\phi_{lcf}(x, y, t) \right] & \text{otherwise} \end{cases} \quad (12)$$

The result of subtracting the phase maps is equivalent to applying a band-pass filter, as $\Delta\phi_{b-p}(x, y, t)$ contains only information in the frequency band between the lower and higher cutoff frequencies. Notice that the subtraction and correction yield a continuous map and thus there is no need to unwrap it. Finally, the band-pass filtered phase maps from different time instants are summed to enhance the visibility of phase variations indicative of damage features:

$$\Delta\phi_s(x, y) = \sum_{k=1}^{N_{\Delta\phi}} \phi_{b-p}(x, y, t_k), \quad (13)$$

where $N_{\Delta\phi}$ is the total number of band-pass filtered phase maps that were acquired. A last step consists of converting $\Delta\phi_s(x, y)$ to a grayscale image for a clearer visualization of the phase distribution. The flowchart in Figure 8 depicts the post-processing methodology applied to the acquired phase map data. This procedure comprises three primary operations: defining the band-pass filter cutoff frequencies, sequential processing of the phase maps, and summing the band-pass filtered phase maps. The phase map band-pass filter is implemented by subtracting the phases processed through two low-pass filters with different cutoff frequencies. The selection of these frequencies is empirically driven, based on their observed effect on the first phase maps acquired. Specifically, the higher cutoff frequency is set to attenuate only high-frequency speckle noise, while the lower cutoff frequency is chosen to provide a mild smoothing effect, ensuring that critical information in phase maps is preserved, such as phase discontinuities. Following this, the identical band-pass filter is applied to the phase map sequence. Finally, the resulting band-pass filtered phases are then aggregated through their summation.

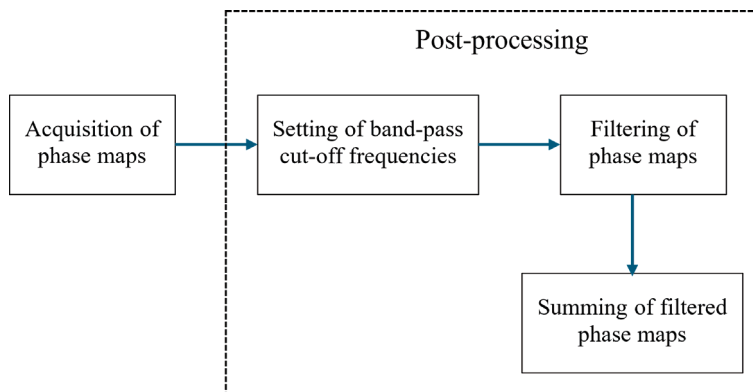


Figure 8. Phase maps post-processing procedure.

3. Results

A comprehensive series of experiments and measured data post-processing was conducted. This included adjusting the thermal excitation and acquisition configurations, as well as selecting the most appropriate post-processing parameters. The most successful outcomes are reported in the following two subsections.

3.1. Active Thermography (AT)

This section presents the best damage identification results, namely temperature phase and temperature amplitude, for the three specimens analyzed. These results were obtained through the post-processing of LT and PT thermograms using the TSR, FFT, and PCT methods. First, TSR is applied to LT thermograms by fitting a 5th-degree polynomial to the logarithmic temporal decay curve for each individual pixel, thus extracting more robust and interpretable thermal signatures. Then, from the FFT, temperature phases were extracted at the frequency of highest magnitude, while from the PCT, temperature amplitudes were obtained by selecting components 4 to 10, which were deemed to contain the most relevant information about the damage. Figures 9–11 display the temperature phase and amplitude images that most clearly reveal the damage. Their selection was based on an analysis of the results over the entire recorded time.

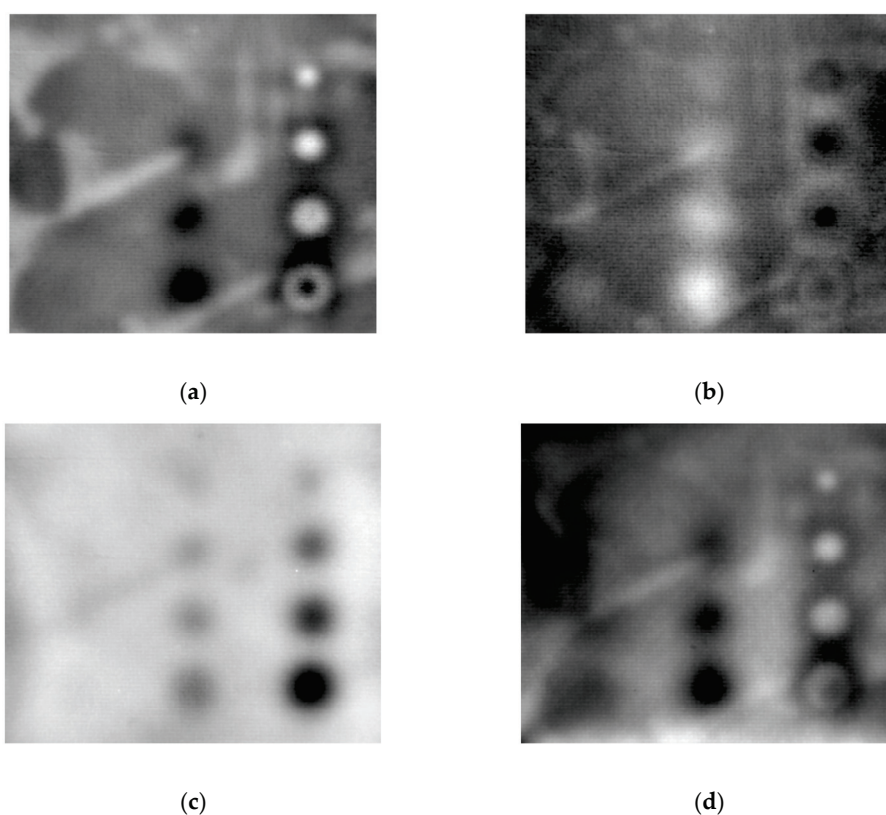


Figure 9. Specimen 1 post-processed thermograms: (a) PT temperature phase, (b) PT temperature amplitude, (c) LT temperature phase, and (d) LT temperature amplitude.

The inspection results for specimen 1, shown in Figure 9, clearly reveal distinct, localized circular anomalies against a relatively uniform background, indicative of subsurface damage. Three main regions can be identified: (1) the right column displays four circles that increase in size and contrast from top to bottom, corresponding to 6 mm deep holes; the middle column contains two to three smaller and fainter spots, indicating 4 mm deep holes; and (3) the left side reveals very diffuse and almost imperceptible variations where the column of 2 mm deep holes is located. The left side of the PT temperature phase image (Figure 9a) exhibits a mid-gray background with noticeable mottled patterns. This pattern is likely attributable to a non-uniform adhesive layer introduced during the plate fabrication process. Specifically, the PT temperature amplitude image reveals highly contrasted circular anomalies, presenting as both bright and dark features, unlike the diffuse appearances observed in other processed thermograms in Figure 9b–d.

Figure 10 shows the inspection results for specimen 2, which was designed to simulate a circular adhesive debonding between the core and one of the skins. Compared to the previous set of results (Figure 9), the anomalies in these images appear more diffuse and less sharply defined, characteristics attributed to the high thermal diffusivity of the aluminum. This observation aligns with the findings of Wey et al. [9], who similarly reported less distinct results when analyzing the thermal response of an aluminum sandwich plate with artificial debondings. The LT temperature phase image (Figure 10c) is notably darker in its central region compared to its lateral edges and exhibits a greater range of brightness variations than the other images. The PT temperature phase image (Figure 10a) reveals a very faint, diffuse, and slightly lighter oval-shaped anomaly in the upper central-right region. A similar oval-shaped anomaly is also present in the LT temperature amplitude image (Figure 10b), though it appears significantly more prominent and less diffuse. On the other hand, this anomaly is not clearly discernible in the highly contrasted LT temperature phase image (Figure 10c). However, the same oval-shaped anomaly appears significantly more prominent, lighter, and less diffuse in the PT temperature amplitude image (Figure 10d). A small anomaly on the image left edge also shows distinct appearances: it is visible as a darker small area in the PT temperature phase image (Figure 10a), a lighter small area in the PT temperature amplitude image (Figure 10b), and a lighter small area, but less distinct due to the inherently high contrast and varied patterns, in the LT temperature phase image (Figure 10c).

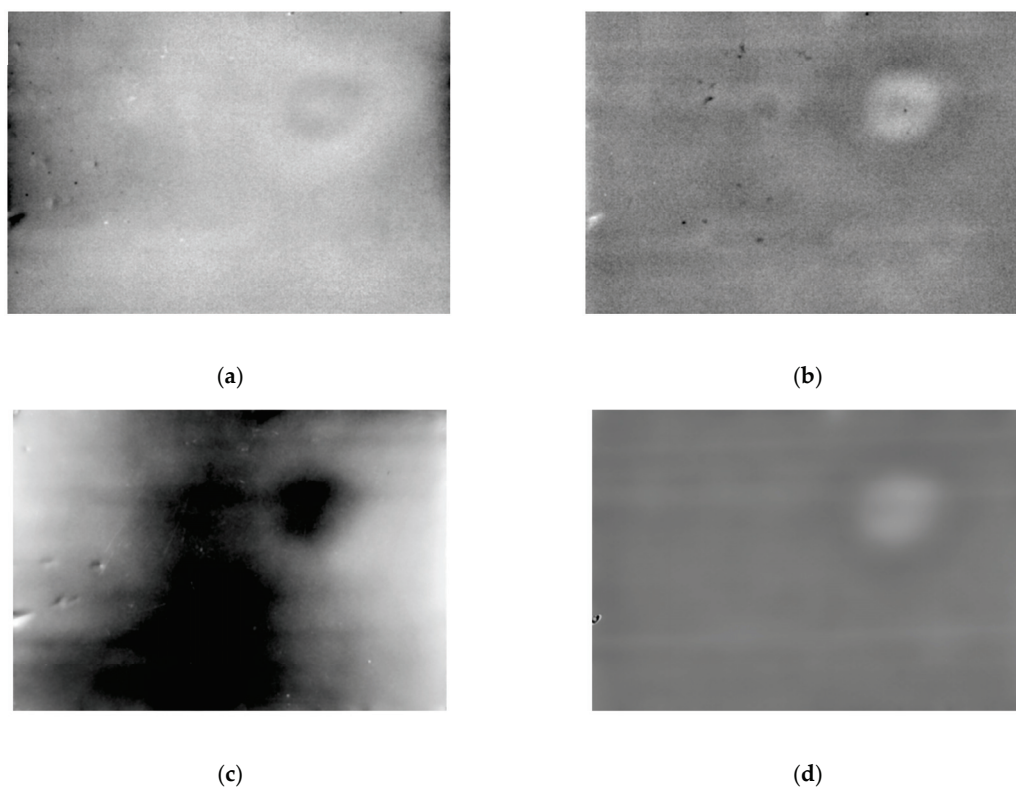


Figure 10. Specimen 2 post-processed thermograms: (a) PT temperature phase, (b) PT temperature amplitude, (c) LT temperature phase, and (d) LT temperature amplitude.

The inspection results for specimen 3 are depicted in Figure 11. The images reveal two distinct anomalies, discernible by their contrast with the surrounding background, which are likely attributable to subsurface damage. Specifically, a large region, containing two contiguous areas with different contrasts and shapes, is consistently observed in the lower right corner, while a small region is visible in the upper-left quadrant of all four

images. The areas of these anomalies extend over the precise locations of the two impacts. Furthermore, the energy of each impact correlates with the observed area and contrast level of its corresponding anomaly. A cross-analysis of the images reveals anomalies with distinct contrast. The PT temperature phase image (Figure 11a) presents prominent brighter anomalies against a darker background. On the other hand, the LT temperature phase image (Figure 11c) generally appears brighter and more diffuse, with its anomalies showing significantly less distinct contrast. Meanwhile, the PT temperature amplitude image (Figure 11b) and the LT temperature amplitude image (Figure 11d) both exhibit sharper and more pronounced features than LT temperature phase image (Figure 11c). A particular challenge is consistently identifying the smaller and circular anomaly located in the upper-left quadrant. Its less pronounced contrast, particularly evident in the brighter and more diffuse LT temperature phase image (Figure 11c), makes its precise identification rather difficult.

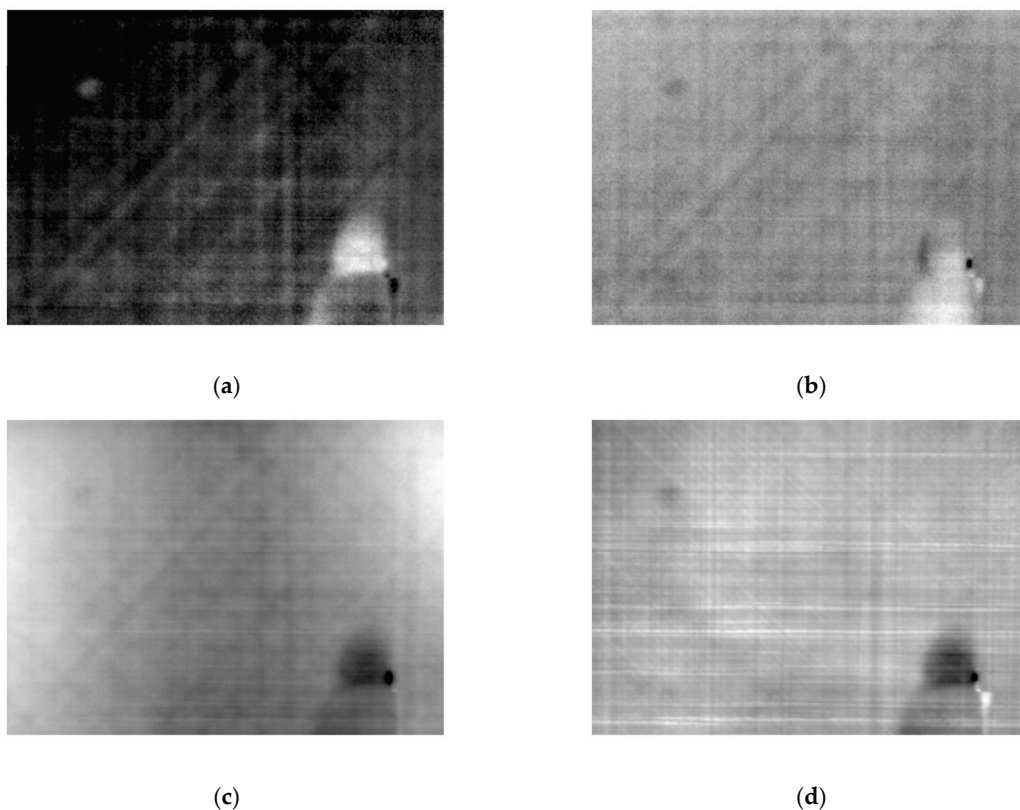


Figure 11. Specimen 3 post-processed thermograms: (a) PT temperature phase, (b) PT temperature amplitude, (c) LT temperature phase, and (d) LT temperature amplitude.

3.2. Digital Shearography (DS)

The DS technique was used to monitor the deformation gradient of the plate during its cooling stage. This involved recording twelve phase maps, each separated by an 8-s interval. The post-processing procedure of these phase maps involved two steps: first, applying a band-pass filter to highlight the damage-induced anomalies, and then summing the resulting filtered maps to enhance the visibility of these anomalies.

The band-pass filtered phase maps, highlighting damage-induced anomalies, were obtained by subtracting the high-frequency signal component (derived from a low-pass filter with a higher cutoff) from the low-frequency signal component (derived from a low-pass filter with a lower cutoff). These low-pass filters were generated individually by applying multiple passes of a moving average with varying window sizes and number of passes. A larger window size leads to more aggressive smoothing and results in a

lower cut-off frequency, which can be further reduced by applying multiple passes. For all measurements, the parameters for the higher cut-off frequency were kept constant, using a window size of 15×15 with 8 passes. However, the parameters for the lower cut-off frequency were adjusted based on observation of phase anomalies in the resulting band-pass filtered phase map for each specimen. Specifically, a 175×175 window with 10 passes was applied for specimen 1, a 95×95 window with 10 passes for specimen 2, and an 85×85 window with 10 passes for specimen 3. Figures 12–14 display two sets of results for specimens 1, 2, and 3, respectively. For each, these figures show the individual filtered phase maps acquired at 8 s, alongside the sum of all twelve filtered phase maps.

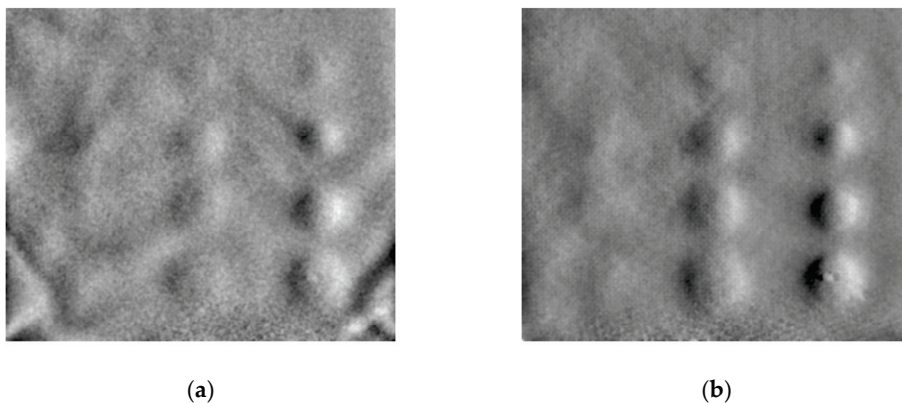


Figure 12. Specimen 1 post-processed phase maps: (a) filtered phase map at 8 s, and (b) sum of 12 filtered phase maps.

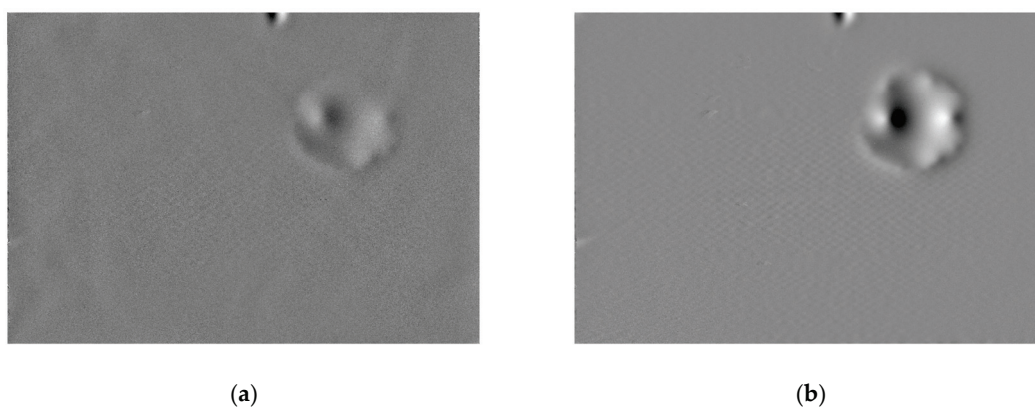


Figure 13. Specimen 2 post-processed phase maps: (a) filtered phase map at 8 s, and (b) sum of 12 filtered phase maps.

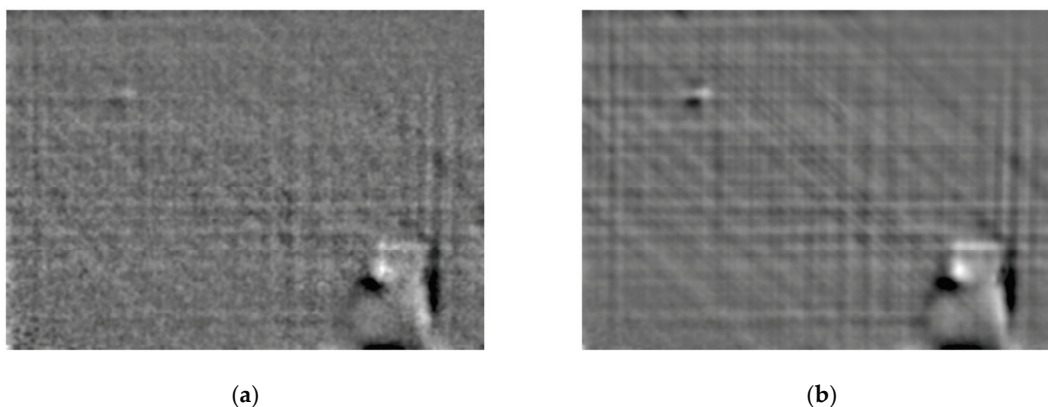


Figure 14. Specimen 3 post-processed phase maps: (a) filtered phase map at 8 s, and (b) sum of 12 filtered phase maps.

The inspection results for specimen 1, displayed in Figure 12, effectively reveal localized circular image anomalies indicative of subsurface damage. These anomalies are distributed along columns and increase in size and visibility from top to bottom. The individual filtered phase map acquired at 8 s (Figure 12a) clearly displays a single column of circular anomalies on the right side, corresponding to flat-bottom holes with a depth of 6 mm. Conversely, the sum of twelve filtered phase maps (Figure 12b) demonstrates an enhancement in their visibility, revealing two columns of circular anomalies across both the right and middle sides of the plate, corresponding to flat-bottom holes with depths of 6 mm and 4 mm, respectively. Indeed, the image of the sum of filtered phase maps (Figure 12b) provides a much clearer and less noisy representation of the underlying image anomalies than the image of individual filter phase maps (Figure 12a). Even with this enhancement, the anomalies from smaller diameter holes, located on the top of the image, remain less distinct. Furthermore, the column of flat-bottom holes with a depth of 2 mm, located on the left side of the plate, remains undetected. A small anomaly is also observed within the circular feature located in the bottom-left corner. This correlates with a small imperfection visible on the surface of the 6 mm deep flat-bottom hole.

Figure 13 displays the inspection results for the aluminum sandwich plate (specimen 2). Consistent with prior observations in Figure 12, the sum of filtered phase map demonstrates a noticeable enhancement in the visibility of image anomalies compared to an individual filtered phase map. In both images, a circular anomaly is evident in the upper central-right region, corresponding to the circular debonding damage introduced between the core and the skin. Additionally, a localized anomaly is observed in the upper center of the image, attributable to a small, visible imperfection in the skin of the inspected surface (see Figure 1b). The anomalies in the filtered phase map at 8 s (Figure 13a) are less prominent, with their circular contours being poorly defined. However, the filtered phase map (Figure 13b) reveals a highly marked and clear anomaly. The small anomaly at the top of the image does not show a distinctive improvement in clarity between the individual and sum of the filtered phase maps.

Figure 14 displays the two filtered phase maps obtained for specimen 3. These reveal two distinctive anomaly regions: a smaller region located in the top-left corner, and a larger region in the lower-right corner, which is defined by its contour and contains a gradient peak. The regions correlate with the coordinates of two impacts and their energy levels, and most likely correspond to the extent of the subsurface damage. Specifically, the lower energy impact (13.5 J) is represented by a smaller region, while the higher energy impact (26.2 J) corresponds to a larger region, with its distinct gradient peak indicating the impact's coordinates. As with specimens 1 and 2, the sum of filtered phase maps significantly improves anomaly visibility. Indeed, the background noise in the single filtered phase map at 8 s (Figure 14a) reveals low visibility of the large region contour in the lower-right corner and obscures the small anomaly region in the top-left corner. However, the sum of filtered phase maps (Figure 14b) effectively suppresses this noise, highlighting the image anomalies and providing more detailed information about their location and size. This enhancement also allows for the observation of subtle background linear patterns in horizontal, vertical, and -45° directions, reflecting the orientation of some unidirectional fibers within the CFRP plies.

4. Discussion

The following discussion focuses on comparing damage identification through qualitative image-based analysis of the results obtained with the different NDT techniques across the three specimens.

The results of AT inspection indicate that the PT temperature phase generally enhances the detection of image anomalies associated with damage in the examined plates. This approach produces images in which anomalies appear brighter relative to the background, facilitating more accurate identification of their location and size, particularly for specimens 1 and 3. In contrast, the results for the aluminum sandwich plate (specimen 2) yield more diffuse images, making it more challenging to distinguish the anomalies. This reduction in image quality is attributed to the high thermal diffusivity of aluminum. For this specimen, the image of PT temperature amplitude provides better identification than the image of PT temperature phase.

The results of DS inspection demonstrate that initial band-pass filtering of a single-phase map successfully extracts and improves the visualization of damage-induced anomalies. Furthermore, the sum of filtered phase maps highlights these image anomalies more effectively by suppressing background noise. This process yields more precise and detailed information regarding the location and size of the anomalies.

The identification of damage using AT and DS inspection techniques relies on fundamentally different physical principles. The AT technique exploits the propagation of induced heat through the structural thickness, where subsurface damage acts as a thermal barrier, leading to distinctive temperatures at the inspected surface. On the other hand, the DS technique measures the deformation gradient of the structure under external loading, revealing subsurface damage as a local increase of this gradient whenever the damage induces a reduction of stiffness. Given their distinct physical principles, the results from the two inspection techniques are expected to yield different levels of identification and sensitivity to damage. Indeed, qualitative image-based comparisons across the three specimens and both techniques often reveal superior performance from the DS inspection technique. Specifically, the image of the sum of filtered phase maps from DS provides more detailed information about damage-induced anomalies than the image of the temperature phase from PT, leading to an improved identification. In specimen 1, the image of the sum of DS filtered phase maps clearly identifies flat-bottom holes at both 6 mm and 4 mm depths through distinct anomalies. In contrast, the image of PT temperature phase only discerns the full column holes at 6 mm depth. Furthermore, a small imperfection within the 6 mm deep flat-bottom hole, located at the image right-bottom corner, is only visible in the image of sum of DS filtered phase maps. For specimen 2, the anomalies in the image of sum of DS filtered phase maps precisely reveal the shape and location of debonding damage, as well as a small anomaly on the top edge of the plate. Conversely, the image of PT temperature phase presents highly diffuse anomalies, making the identification of these damages difficult. In the case of specimen 3, the image resulting from the sum of DS filtered phase maps reveals the region of damage produced by the lower energy impact (13.5 J), the extent of damage produced by the higher energy impact (26.2 J), and the precise internal location of this latter impact. However, the image of PT temperature phase does not provide the same level of information. The anomalies produced by the 13.5 J energy impact are difficult to distinguish from the background. Furthermore, the contours of the 26.2 J energy impact damage are not well defined, and its internal impact location is not revealed.

Table 1 summarizes the damage detection performance of both PT and DS techniques across the three specimens. Overall, the filtered phase maps (DS) demonstrated slightly better damage identification quality than the temperature amplitude and phase (PT). This superior performance is directly attributable to the greater enhancement achieved through phase map post-processing relative to thermogram post-processing. A clear illustration of this can be seen by comparing the raw data for specimen 1 (Figures 5 and 7) with its post-

processed versions in Section 3, where the improvement in damage signature disclosure is notably more substantial with the phase map post-processing algorithm.

Table 1. Comparison of damage identification performance of PT and DS.

	Specimen 1			Specimen 2	Specimen 3	
	Flat-bottom hole depths (mm)			Debonding	Impact of 13.5 J	Impact of 26.2 J
	2	4	6			
PT	✗	✓	✓✓	✓	✓	✓✓
DS	✗	✓✓	✓✓	✓✓	✓✓	✓✓

✗ BALLOT—not identified; ✓—identified; ✓✓—clearly identified.

5. Conclusions

This work compares the performance of two non-destructive testing (NDT) techniques—active thermography (AT) and digital shearography (DS)—along with their respective post-processing methods for identifying different types of damage in composite structures. The implemented post-processing methods significantly improve the detectability of these damages by enhancing the identification of anomalies in the images. Specifically, the fast Fourier transform (FFT) technique enables the extraction of temperature phase, while principal component thermography (PCT) extracts the most significant temperature components. Both approaches generally enhance the detection of anomalies, making them appear brighter and facilitating the identification of their location and size. For DS, applying band-pass filtering to the phase map improves the visualization of phase anomalies induced by damage. Furthermore, summing the filtered phase maps significantly enhances these anomalies by suppressing background noise, providing more precise and detailed information on location and size. The results revealed that the sum of DS filtered phase maps often showed superior performance, providing more detailed information and improved identification compared to pulsed thermography (PT)/lock-in thermography (LT) temperature phase and temperature amplitude. Specifically, for specimen 1, the sum of DS filtered phase maps identified flat-bottom holes at 6 mm and 4 mm depths, whereas PT temperature phase only clearly discerned 6 mm depth holes. Furthermore, a small imperfection was only visible in the sum of DS filtered phase maps. Relatively to specimen 2, the sum of DS filtered phase maps precisely revealed the debonding damage and the small imperfection at the top edge of the specimen, while in PT results these damages were not clearly identified. Both techniques demonstrated different capabilities to detect the two low-energy impacts damage in specimen 3. Indeed, the sum of DS filtered phase maps can clearly reveal both damage locations and sizes, thus presenting a better damage identification than the PT technique. In view of the above conclusions, one can state that the DS outperformed AT in this study, demonstrating a greater ability to identify various damage types across the three composite structures. One of the main reasons for this fact is that the novel phase map filtering approach leads to a significant improvement in damage identification than the post-processing of thermograms. Nevertheless, while the results of this study are promising, it is important to acknowledge that only three composite specimens with specific damage typologies were tested. To further validate the performance and robustness of this new shearography post-processing method, future work should include a broader range of composite specimens, differing in material composition and damage typology, ideally extending this analysis to real aeronautical components.

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editing, J.Q., H.L., L.M. and V.d.S.; supervision, H.L. and V.d.S.; funding acquisition, H.L. and V.d.S. All authors have read and agreed to the published version of the manuscript.

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Conflicts of Interest: The authors declare no conflicts of interest.

Abbreviations

The following abbreviations are used in this manuscript:

AT	Active thermography
BVID	Barely visible impact damage
CFRP	Carbon-fiber-reinforced polymer
CW	Continuous wave
DFT	Discrete Fourier transform
DS	Digital shearography
FFT	Fast Fourier transform
LT	Lock-in thermography
LPT	Long pulse thermography
NDT	Non-destructive testing
PCT	Principal component thermography
PT	Pulsed thermography
SVD	Singular value decomposition
TSR	Thermographic signal reconstruction

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Article

Suppression of Delamination in CFRP Laminates with Ply Discontinuity Using Polyamide Mesh

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Abstract: Carbon fiber-reinforced plastics (CFRPs) offer excellent in-plane mechanical performance, but their relatively low interlaminar fracture toughness makes them vulnerable to delamination, particularly around intralaminar discontinuities such as resin-rich regions or fiber gaps. This study investigates the effectiveness of polyamide (PA) mesh inserts in improving interlaminar toughness and suppressing delamination in CFRP laminates with such features. Two PA mesh configurations were evaluated: a fully embedded continuous layer and a 20 mm cut mesh strip placed between continuous and discontinuous plies near critical regions. Fracture toughness tests showed that PA mesh insertion improved interlaminar toughness approximately 2.4-fold compared to neat CFRP, primarily due to a mechanical interlocking mechanism that disrupts crack propagation and enhances energy dissipation. Uniaxial tensile tests with digital image correlation revealed that while initial matrix cracking occurred at similar stress levels, the stress at which complete delamination occurred was approximately 60% higher in specimens with a 20 mm mesh and up to 92% higher in specimens with fully embedded mesh. The fully embedded mesh provided consistent delamination resistance across the laminate, while the 20 mm insert localized strain redistribution and preserved global mechanical performance. These findings demonstrate that PA mesh is an effective interleaving material for enhancing damage tolerance in CFRP laminates with internal discontinuities.

Keywords: CFRP laminates; interlaminar fracture toughness; polyamide mesh; ply discontinuities; delamination suppression; mechanical interlocking; matrix cracking; strain distribution

1. Introduction

Carbon fiber-reinforced plastics (CFRPs) are exceptionally strong and lightweight, making them ideal for applications such as aircraft and automobiles. Their high strength-to-weight ratio and superior elasticity contribute to reduced fuel consumption and lower carbon dioxide emissions throughout the product lifecycle [1–3]. As CFRPs continue to

be widely adopted across various industries, extensive research has been conducted to understand their damage mechanisms, service life, maintenance, and structural health monitoring [4,5]. A thorough understanding of their complex failure processes is essential for achieving robust and efficient designs in advanced engineering applications.

All CFRP laminates are prone to cumulative microscopic damage, including matrix cracks and delamination, which degrade mechanical properties and limit their use in structures subjected to high strain levels [6–8]. Transverse matrix cracks reduce stiffness and induce residual strains. Delamination, often initiating at the tips of such cracks, serves as a critical secondary damage mode that can propagate rapidly and cause structural failure [9,10]. Suppressing delamination is therefore a key challenge in enhancing the long-term durability and reliability of CFRP laminates, particularly in regions with complex internal architectures such as ply discontinuities [11].

Discontinuities are commonly introduced in industrial composite structures to enable cost-efficient, lightweight designs [12,13]. By selectively cutting prepregs at specific locations, material usage can be optimized, resulting in intentional fiber or ply terminations. The adoption of advanced manufacturing methods such as automated fiber placement has also enabled more flexible and efficient production of complex structures [14,15]. However, it has also introduced new types of defects, including resin-rich gaps and fiber misalignment. These manufacturing-induced flaws, combined with structural discontinuities, further increase local stress concentrations and the likelihood of interlaminar failure [16–18]. In regions where fiber or ply continuity is interrupted, excess resin tends to accumulate during curing, forming resin pockets that fill the spaces left by absent or misaligned fibers. Figure 1 illustrates examples of CFRP laminates with ply discontinuities and resin pockets: Figure 1a shows a resin pocket in a flat laminate, while Figure 1b shows resin pockets formed in tapered laminates. Understanding how ply discontinuities and associated resin pockets influence the damage behavior and mechanical performance of CFRP laminates is therefore essential.

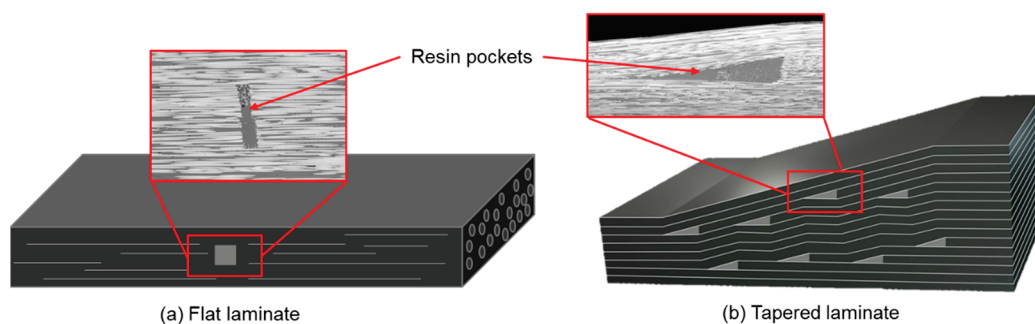


Figure 1. Examples of composite laminates with ply discontinuities and resin pockets: (a) resin pocket formed in a flat laminate due to ply termination; (b) resin pockets observed in tapered laminates resulting from gradual ply drop-offs.

To address these concerns, many studies have examined the effects of fiber or ply discontinuities on damage initiation, propagation, and failure in composite laminates—particularly in regions with ply drop-offs, including those arising from manufacturing defects—focusing on their impact on mechanical properties and damage initiation from discontinuous regions [17–23]. These investigations have shown that factors such as taper angle, step spacing, ply thickness, and resin pocket geometry significantly influence matrix cracking and delamination behavior. In addition to those studies, we have focused on evaluating microscopic damage mechanisms in simple unidirectional CFRP laminates with flat resin pocket configurations [24]. To gain a comprehensive understanding of damage behavior, numerical and analytical approaches have also been used to predict

crack initiation and mechanical damage propagation caused by ply discontinuities [19,25–27].

As ply discontinuities in CFRP laminates trigger interlaminar damage such as delamination, primarily due to localized stress concentrations and resin-rich regions, it is necessary to enhance the interlaminar fracture toughness of laminates at these critical areas to suppress delamination and preserve structural integrity under mechanical loading. Although not specifically targeting critical regions with ply discontinuities, numerous strategies have been developed to enhance the overall properties of composite laminates. One widely studied approach to improving interlaminar fracture toughness involves incorporating interleaves and nanofillers in interlaminar regions to enhance energy dissipation and delay crack propagation. Thermoplastic fiber-based interleaves, such as polyimide, improve toughness through bridging mechanisms that absorb energy via deformation and debonding under load [28]. Similarly, combining nanoscale core-shell rubber particles with microscale short carbon fibers can substantially improve fracture toughness, with Veeramani et al. [29] reporting an increase of up to 127% compared to unmodified laminates. Rubber modified nanofibrous mats have also proven highly effective, with Maccaferri et al. [30] demonstrating a nearly 480% improvement in mode I fracture toughness. Polyetherimide hybrid systems enhance both mode I and mode II toughness [31], highlighting the important role of advanced polymer matrices in improving crack resistance.

Feather-inspired interleaves also show marked toughening capability by mimicking natural structures; Song et al. [32] reported a significant increase in interlaminar fracture toughness using a biomimetic interleaf inspired by feather shafts. Additionally, tailoring the microstructural heterogeneity within the toughening layer has emerged as an effective method, with Ou et al. [33] demonstrating enhanced fracture toughness in unidirectional CFRP laminates through optimized heterogeneity in the interleaf design.

In addition to material modifications, mechanical reinforcement techniques such as stitching and z-pinning offer through-thickness strengthening by inserting threads or pins perpendicular to the laminate layers, thereby restraining delamination and enhancing interlayer bonding [34]. Similarly, three-dimensional fiber architectures, such as woven, knitted, or braided composites, provide multidirectional reinforcement that improves load distribution and suppresses interlayer separation under mechanical or thermal stress [35].

Carbon nanotube interleaves have also gained attention for their multifunctional toughening potential. Functionalized multiwalled carbon nanotubes enhance both crack initiation and propagation resistance, achieving up to a 22% improvement in initiation toughness [36], while also contributing to electrical conductivity, which can support structural health monitoring [37]. However, their performance strongly depends on dispersion quality and matrix interaction, which may be hindered by resin viscosity and processing limitations [38]. Furthermore, interface engineering through precoating techniques has proven effective in improving interlaminar bonding and energy absorption, with Pan et al. [39] reporting that optimized interface treatments can balance mode I and mode II toughness across different laminate configurations.

Polyamide (PA) materials, with high melting points, good resin compatibility, and excellent mechanical properties, have been used in some studies as toughening interleaves in the form of micro/nanoparticles or nanofibers [40–49]. For example, Wang et al. [41] studied the inclusion of PA12 and PA6 particles (5 to 30 μm) in the interlaminar region of composites and identified toughening mechanisms such as crack deflection at the resin/particle interface, particle plastic deformation, and bridging. Zhao et al. [47] utilized low areal density PA66 fiber veils (4 to 10 g/m^2) as interleaving layers in laminated composites, which, under delamination loading, exhibited significant fiber slippage, tearing, bridging, and fracture. These phenomena significantly enhanced the laminates' energy absorption and

interlaminar properties, with the 4 g/m² PA66-interleaved composite showing increases of 13.6% in mode I and 139.8% in mode II fracture toughness, as well as improved shear and tensile strength. Nevertheless, the non-uniform structure and distribution of these PA materials may lead to inconsistent toughening performance. To address this limitation and achieve more reliable enhancement of interlaminar fracture toughness, the present study introduces PA mesh as a novel interleaving material. This approach aims to enhance interlaminar fracture toughness and effectively suppress the initiation of interlaminar damage in CFRP laminates, with particular emphasis on laminates containing intralaminar (ply-level) discontinuities. While many prior studies investigated general toughening approaches applied uniformly across the laminate, the novelty of this study lies in targeting critical regions, specifically ply discontinuities, where delamination is most likely to initiate due to local stress concentrations and resin accumulation.

In this study, the effectiveness of PA mesh in enhancing the interlaminar fracture toughness of CFRP laminates is experimentally evaluated through fracture tests on specimens with inserted mesh layers. Subsequently, PA mesh is strategically applied to critical regions near ply discontinuities, particularly resin-rich zones between continuous and discontinuous plies, to assess its ability to suppress delamination. Mechanical testing and damage observation are conducted to clarify the role of PA mesh in improving damage resistance and structural integrity. Furthermore, digital image correlation (DIC) is used to measure strain distribution during tensile loading, enabling evaluation of the mesh's influence on the overall mechanical performance of the CFRP laminates.

2. Materials and Methods

2.1. Evaluation on the Interlaminar Fracture Toughness

To evaluate the interlaminar fracture toughness of CFRP laminates with inserted PA mesh, double cantilever beam (DCB) tests were conducted for mode I, and end notched flexure (ENF) tests were conducted for mode II.

2.1.1. Mode I Interlaminar Fracture Toughness (DCB Test)

The test specimens were fabricated using T700SC/2592 CFRP prepreg (Toray Industries, Tokyo, Japan) with a fiber volume of 60% and a nominal thickness of 0.14 mm. A unidirectional (UD) [0]₂₀ laminate was prepared by stacking 20 sheets of prepreg, each measuring 200 mm × 200 mm, into two sets of 10 plies. As illustrated in Figure 2, a PA mesh measuring 175 mm × 200 mm was inserted between the two stacked sets. The PA mesh (N-NO175T, NBC Meshtec Inc., Tokyo, Japan) had a fiber diameter of 50 µm, a thickness of 87 µm, and an opening of 95 µm. To introduce an initial crack, a 25 mm × 200 mm polyimide film with a thickness of 25 µm was placed between the upper and lower plies at the mid-plane. The laminate was cured using an autoclave (DANDELION 2010, Nagano, Japan) at 130 °C under a pressure of 0.2 MPa. Finally, the cured laminate was cut into test specimens using a precision cutting machine for composite materials (AC-300CF, Maruto, Tokyo, Japan). A schematic of the DCB test specimen configuration and its dimensions are shown in Figure 3 and Table 1, respectively.

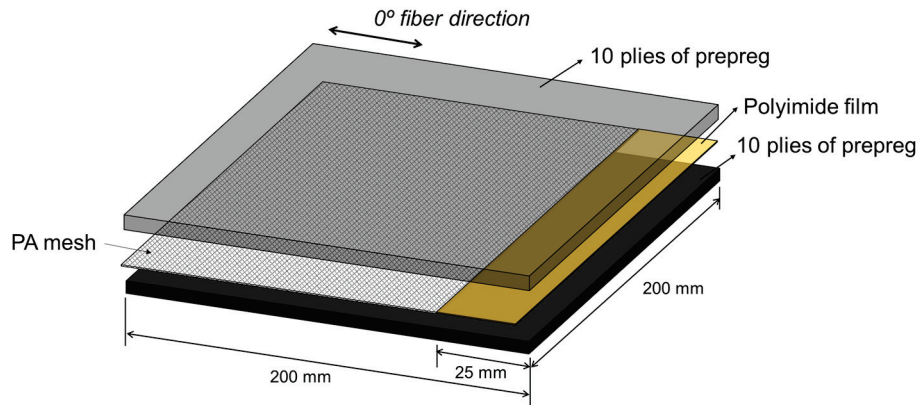


Figure 2. toIllustration of the lay-up structure of a laminate for interlaminar fracture toughness evaluation.

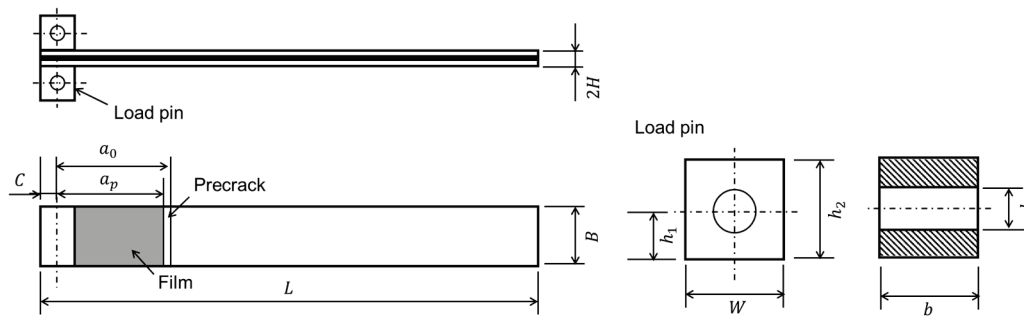


Figure 3. Schematic specimen configuration for DCB tests.

Table 1. Specimen dimensions for DCB tests.

Part	Symbol	Dimension	Size [mm]
DCB specimen	L	Length	90
	$2H$	Thickness	2.3
	B	Width	10
	C	Distance from specimen end load line	6
	a_p	Film crack length	25
	a_0	Pre-crack length	28–32
Load pin	W	Width	12
	b	Length	12
	h_1	Half height	6
	h_2	Height	12
	d	Hole diameter	5

A measuring microscope (VHX-1000, KEYENCE, Osaka, Japan) was used to measure the crack length during the DCB tests. Prior to measurement, the specimen edges were polished to enable crack tip observation for length measurement. Polishing was performed using a polisher (Tegrapol 15, Struers, Tokyo, Japan) with water as the lubricant and waterproof abrasive paper for grinding. The polishing process consisted of 1 min with 500 grit, 1 min 30 s with 1000 grit, 2 min with 2000 grit, and 2 min 30 s with 4000 grit paper. Finally, diamond polishing was performed using a diamond spray (DP Spray P-1 μm , Struers, Tokyo, Japan) for 3 min to enhance the surface finish. Figure 4 shows the polished edge of a specimen.

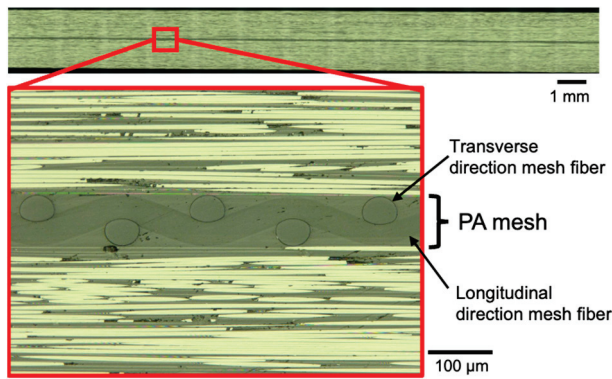


Figure 4. Edge observation of a polished specimen used for interlaminar fracture toughness evaluation.

DCB tests were carried out using a universal testing machine (SC-5H, JT Torsi Co., Kanagawa, Japan). In this study, measurements were taken of the load P , crack length a , and crack opening displacement (COD). Five specimens were tested. The first measurement was taken when the crack had propagated 3 mm from the initial crack, and subsequent measurements were made every 3 mm of crack propagation, with a total of five measurements per specimen. The loading speed was set at 0.5 mm/min up to 6 mm from the initial crack, and increased to 1.0 mm/min for crack propagation beyond 6 mm. Unloading was performed at a rate of 5.0 mm/min after each measurement. Mode I interlaminar fracture toughness G_I was calculated using the following formula:

$$G_I = \frac{3}{2(2H)} \left(\frac{P_C}{B} \right)^2 \frac{(B\lambda)^{\frac{2}{3}}}{\alpha_1}, \quad (1)$$

where P_C is the critical load, λ is the compliance of the COD in the elastic portion of the load–COD curve, and α_1 is the slope of the linear approximation of the cube root of the compliance of the normalized crack length per unit width [50].

2.1.2. Mode II Fracture Toughness Evaluation (ENF Tests)

The materials used for specimen manufacturing in the ENF tests were identical to those used in the DCB tests. In this case, the laminates were cut to the dimensions shown in Figure 5 and detailed in Table 2. The edges of the specimens were then polished using the same technique as described previously.

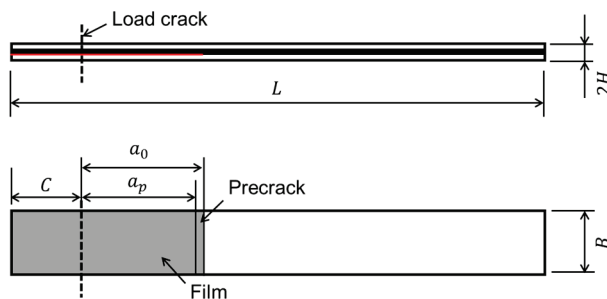
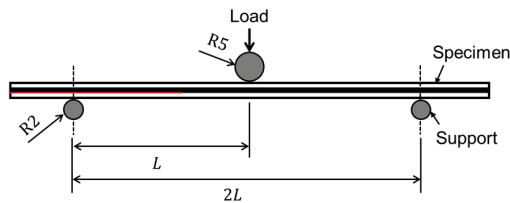


Figure 5. Schematic specimen configuration for ENF tests.

Table 2. Specimen dimensions for ENF tests.

Symbol	Parts	Size [mm]
L	Length	110
$2H$	Thickness	2.3
B	Width	10
C	Distance from specimen end load line	20
a_p	Film crack length	25
a_0	Precrack length	31–35

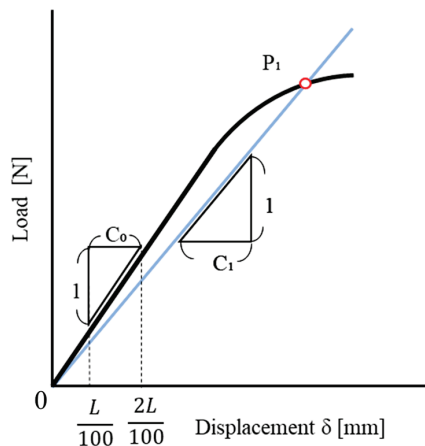
Similar to the DCB tests, ENF tests were carried out using an SC-5H universal testing machine (JT Torsi Co., Kanagawa, Japan). In this study, measurements were taken of the load P and maximum deflection δ . Five specimens were tested. The load was applied at the midpoint of the support span, with a distance of $2L = 70$ mm, as shown in Figure 6. The test speed was set at 0.5 mm/min.

**Figure 6.** Schematic for ENF test setup.

The mode II fracture toughness G_{IIC} was calculated from Equation (2), where a_1 represents the estimated crack length at a specific load, calculated from Equation (3) [50]. Additionally, P_1 represents the specified load, C_0 denotes the compliance of the initial elastic part, and C_1 represents the compliance at a specified load, calculated from the load–COD curve. The maximum load $P_{\max}$ was used as P_1 in this study. Figure 7 illustrates how C_1 and C_0 are determined.

$$G_{IIC} = \frac{9a_0^2 P_1^2 C_1}{2B(2L^3 + 3a_1^3)}, \quad (2)$$

$$a_1 = \left[\frac{C_1}{C_0} a_0^3 + \frac{2}{3} \left(\frac{C_1}{C_0} - 1 \right) L^3 \right]^{\frac{1}{3}} \quad (3)$$

**Figure 7.** Determination of C_0 and C_1 .

For fractographic analysis after the DCB and ENF tests, the specimens were cut into small pieces and coated with gold by sputtering before observation under a scanning electron microscope (SEM) (KEYENCE, VE-7800, Osaka, Japan).

2.2. Application of PA Mesh in CFRP Laminates with Ply Discontinuities

To confirm the suppression of interlaminar delamination by inserting PA mesh, uniaxial tensile tests were conducted on UD CFRP laminates with ply discontinuities.

2.2.1. Laminate Configurations and Manufacturing

The laminate configuration of the specimens used in the uniaxial tensile tests was $[0_2/PA/0_2]_s$. As illustrated in Figure 8, a stack of four UD tape prepreg plies was bisected perpendicularly to the fiber direction. The two halves were aligned with a predefined gap to form a symmetrical laminate structure, with PA meshes and continuous $[0]_2$ UD laminates placed above and below the gap. The gap, or resin pocket length L_0 , was set to approximately 1 mm. Specimens were fabricated with either a 20 mm PA mesh embedded locally or with PA mesh fully embedded throughout the entire structure, as shown in Figure 9. This was performed to investigate whether local embedding of the PA mesh around critical regions such as ply discontinuities is sufficient to suppress delamination, or whether full embedding across the entire laminate is more effective in enhancing interlaminar fracture toughness. The CFRP prepreg, PA mesh materials, and manufacturing process described in Section 2.1 were used. During curing, resin from the adjacent UD plies flowed into the gap, forming a resin pocket that was cured simultaneously with the rest of the laminate. After cooling to room temperature, the laminate was sectioned into specimens, as shown in Figure 10.

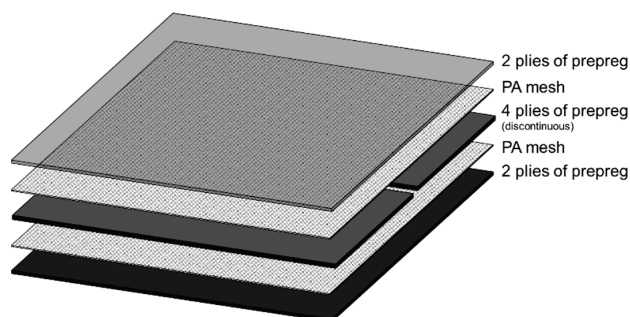


Figure 8. Illustration of laminate with discontinuous plies and embedded PA mesh.

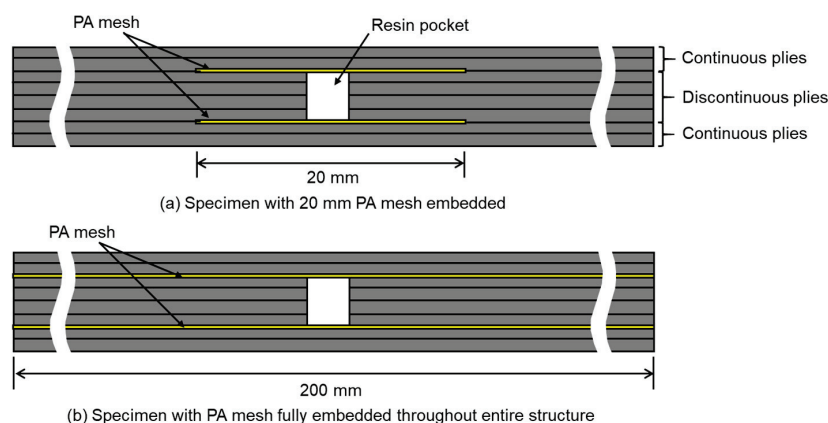


Figure 9. Placement of PA mesh in the specimen: (a) Specimen with 20 mm PA mesh embedded; (b) specimen with PA mesh fully embedded throughout entire structure.

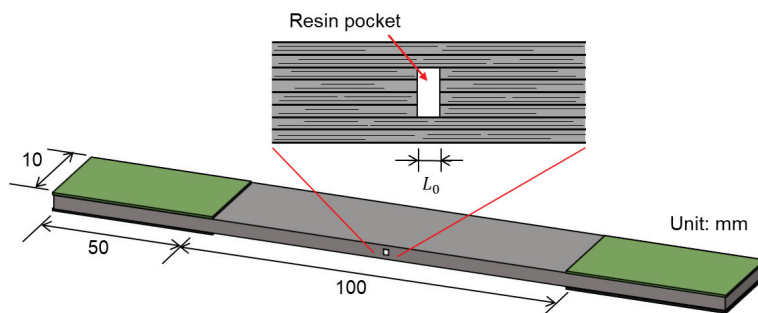


Figure 10. Specimen measurement for uniaxial tensile tests.

2.2.2. Tensile Tests and Damage Observation

Uniaxial tensile loading was applied at a crosshead speed of 0.5 mm/min using a TENSILON RTF-1350 tensile testing machine. To measure strain, two strain gauges, each with a gauge length of 5 mm (KFGS-5-120-C1-11L1M2R, Kyowa, Tokyo, Japan), were used. As shown in Figure 11, one strain gauge was adhered directly above the ply discontinuity area (resin pocket), while the other was affixed 20 mm from the center of the resin pocket to measure strain in the laminate without significant influence from the resin pocket. In situ edge observations were performed using a KEYENCE VHX-1000 optical microscope (lens: MX 7575CS, KEYENCE, Osaka, Japan) to verify the presence of damage.

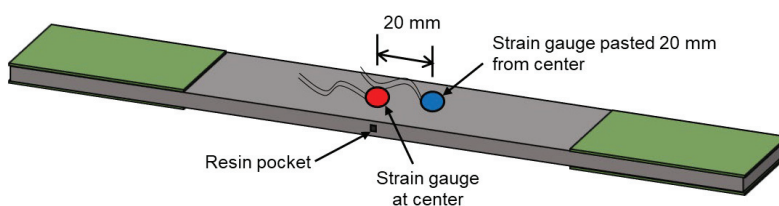


Figure 11. Strain gauge placement on a specimen.

3. Results and Discussion

3.1. Evaluation on the Interlaminar Fracture Toughness

3.1.1. Mode I Interlaminar Fracture Toughness (DCB Tests)

Figure 12 shows the mode I fracture toughness (G_I)–crack length relationship for all specimens. The dotted lines indicate the average values of G_{IC} and G_{IR} , which were 519 J/m² and 336 J/m², respectively. G_{IC} was calculated from the first loading cycle and represents the critical strain energy release rate required to initiate delamination. G_{IR} , derived from subsequent cycles, represents the propagation strain energy release rate needed to sustain crack growth. Figure 13 displays bar charts comparing the G_{IC} and G_{IR} values of CFRP laminates with and without PA mesh inserts [11]. The error bars shown in the figure represent the standard deviations calculated from the test results of five specimens. G_{IC} of the PA mesh-inserted laminate was higher than that of the neat laminate (348 J/m²), indicating improved resistance to crack initiation. However, G_{IR} was lower than that of the neat CFRP (457 J/m²), suggesting reduced resistance to crack propagation.

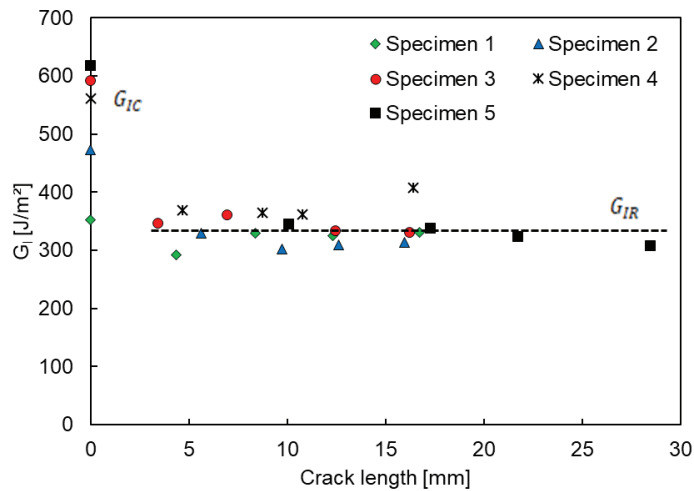


Figure 12. Mode I fracture toughness G_I –crack length relationship for all specimens. Dotted lines show average G_I values.

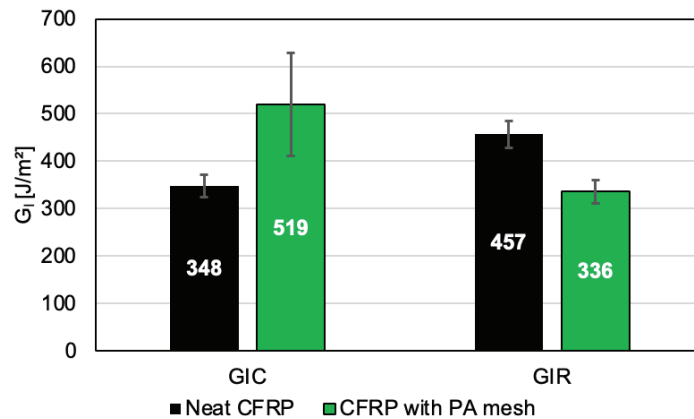


Figure 13. Comparison of mode I interlaminar fracture toughness between neat CFRP laminate and CFRP laminate with PA mesh insert.

Mode I fracture is characterized by crack surface opening perpendicular to the crack plane, often described as peeling. Although the inserted PA mesh increased the energy required for crack initiation, it did not effectively suppress crack propagation under this loading condition. This suggests that the current mesh configuration is insufficient to improve interlaminar toughness in mode I loading. Future work should explore mesh design modifications, such as finer mesh size, adjusted strand thickness, and altered orientation, to optimize both crack initiation resistance and propagation control.

3.1.2. Mode II Interlaminar Fracture Toughness (ENF Tests)

Figure 14 presents the load–deflection curves obtained from ENF tests conducted on five specimens. The peak loads for the specimens are highlighted with red dots. Figure 15 compares the mode II fracture toughness, G_{IIC} , values between CFRP laminates with and without PA mesh inserts. G_{IIC} values are averaged over five specimens and the error bars in the figure represent the corresponding standard deviations. The reference value for the neat CFRP laminate was obtained from [11]. The CFRP laminates incorporating a PA mesh insert exhibited a mode II fracture toughness approximately three times greater than that of the neat laminates.

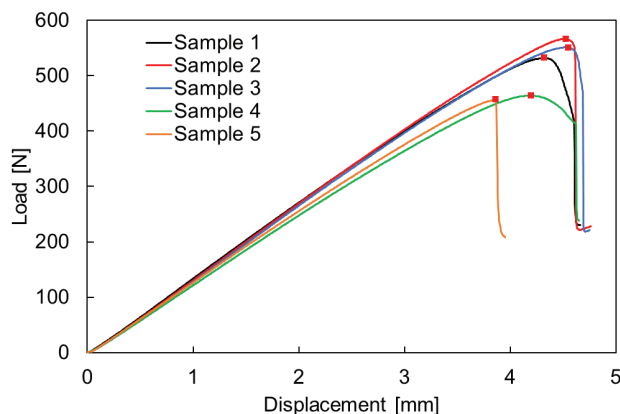


Figure 14. Load–deflection curves from ENF tests. Red dots represent the maximum loads in the tests.

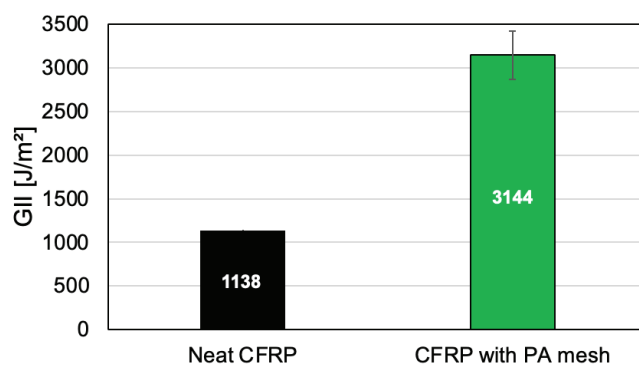


Figure 15. Comparison of mode II interlaminar fracture toughness between neat CFRP laminate and CFRP laminate with PA mesh insert.

To clarify the improvement in fracture toughness, fractography was conducted using SEM. Figures 16 and 17 show the fractography results of a DCB and an ENF test specimen, respectively. Subfigure (a) in the figures presents the top surface, while subfigure (b) shows the bottom surface during the tests. On the top surface, the woven structure of the PA mesh is clearly visible, indicating that the mesh remained intact during fracture. In contrast, the bottom surface displays a clear imprint of the mesh pattern, replicating the geometry of the woven structure despite the absence of mesh on this side. Based on these SEM observations, it can be confirmed that the resin successfully penetrated the PA mesh structure, embedding well between the mesh fibers. Including the edge observation in Figure 4, no voids were observed at the interface, indicating good interfacial bonding quality. The imprint on the fractured surface suggests that the crack path was influenced by the physical presence of the mesh, providing evidence of a mechanical interlocking effect. Based on the SEM images, it can also be seen that the PA mesh fibers in the current study are aligned in the 0° and 90° directions, which reflects the intended fiber orientation during the insertion process.

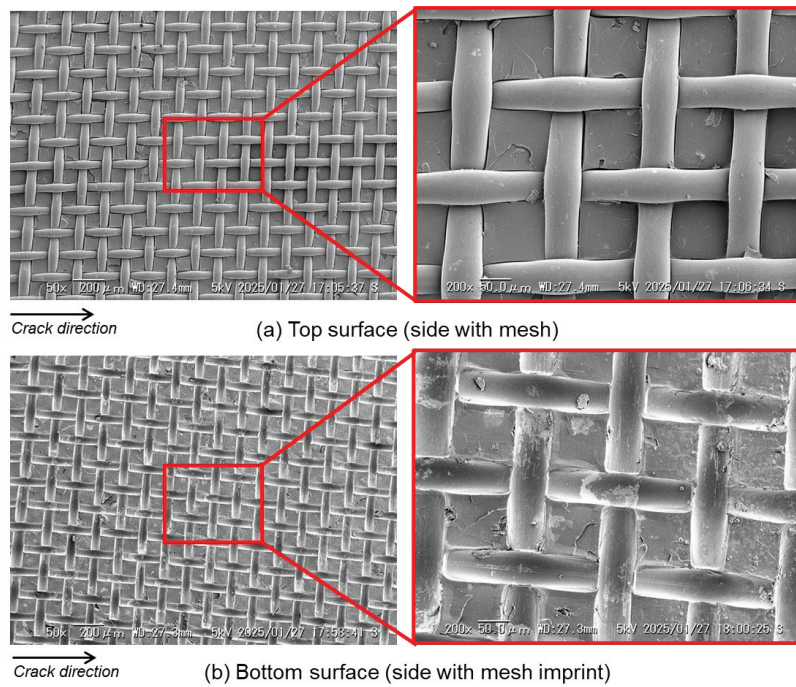


Figure 16. Fractography of DCB test specimen using SEM: (a) top surface (side with mesh); (b) bottom surface (side with mesh imprint).

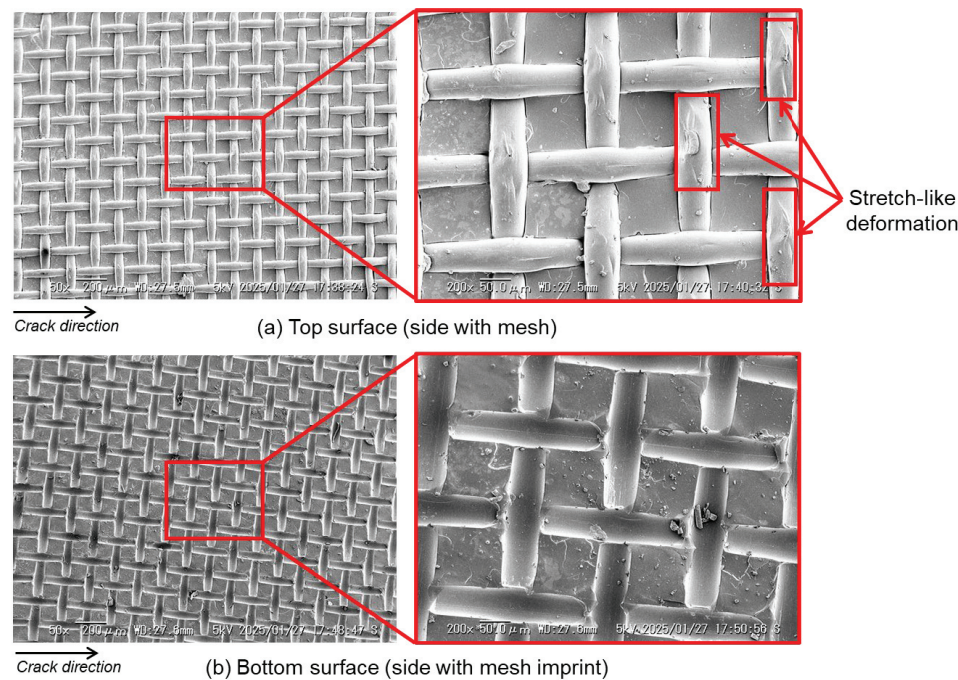


Figure 17. Fractography of ENF test specimen using SEM: (a) top surface (side with mesh); (b) bottom surface (side with mesh imprint).

By comparing the fracture surfaces under mode I (Figure 16a) and mode II (Figure 17a) loadings, it is evident that the PA mesh undergoes stretching deformation under mode II (shear) loading, particularly in the mesh fibers perpendicular to the direction of crack propagation, whereas no such deformation is observed under mode I loading. This indicates that, under shear loading, the PA mesh functions as a mechanical interlocking feature that disrupts the natural crack path. The crack is forced to deviate and navigate around the woven structure, thereby requiring additional energy for propagation. This deviation,

together with interfacial friction contributes to the enhanced mode II fracture toughness. The presence of both the embedded mesh and its mirrored imprint, along with evidence of shear-induced deformation, supports the conclusion that the PA mesh improves resistance to shear-induced delamination through micromechanical toughening mechanisms.

The results clarify the role of PA mesh in enhancing the interlaminar fracture toughness of CFRP laminates under both mode I and mode II loading. While the mesh increased the resistance to crack initiation in mode I, it did not effectively suppress crack propagation due to the peeling nature of the fracture. In contrast, a significant improvement was observed under mode II loading, where the mesh disrupted the crack path through mechanical interlocking, requiring more energy for propagation. In actual structural applications, delamination typically occurs under mixed-mode loading rather than purely mode I or mode II. Considering the combined energy dissipation from both modes, the insertion of PA mesh leads to a clear overall enhancement in interlaminar fracture toughness. This demonstrates the effectiveness of PA mesh in improving the damage tolerance of CFRP laminates under realistic service conditions. However, further optimization of the mesh design and placement could enable better performance across a broader range of loading scenarios.

3.2. Application of PA Mesh in CFRP Laminates with Ply Discontinuities

3.2.1. CFRP Laminate with No PA Mesh Embedded (Neat CFRP)

To validate the effectiveness of PA mesh in suppressing interlaminar delamination, the uniaxial tensile test results of CFRP laminates with embedded PA mesh were compared to those of neat CFRP laminates (without mesh insertion) from previous studies [24,26]. Figure 18 illustrates the representative stress–strain response of a neat specimen, accompanied by microscopic edge observations at selected stress levels during tensile loading.

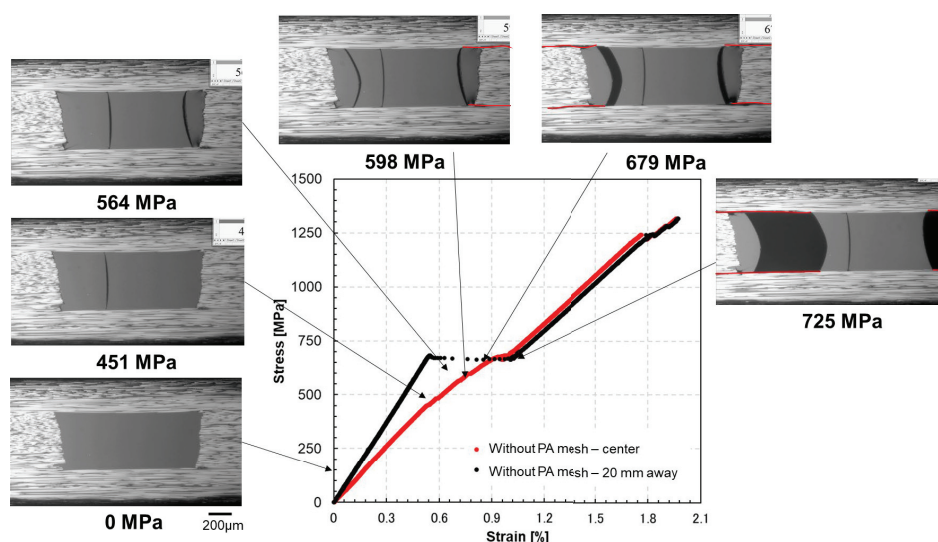


Figure 18. Stress–strain curves and microscopic edge observations (around the resin pocket) captured at selected stress levels during the tensile test of a specimen with no PA mesh embedded [24,26]. Reproduced with permission from M. J. Mohammad Fikry, Composites Science and Technology; published by Elsevier, 2022.

From the nonlinearity (strain jumps) in the stress–strain curve and the corresponding edge observations shown in Figure 18, the onset of the first matrix crack was reported at 451 MPa [24,26]. A second matrix crack within the resin pocket appeared at 564 MPa, from which delamination initiated. The third matrix crack was observed at 679 MPa. In addition to the edge observations, the initiation and propagation of delamination were also indicated

by a reduction in the slope of the stress–strain curve recorded by the centrally placed strain gauge following matrix cracking. Complete delamination between the continuous and discontinuous plies was confirmed to occur at approximately 725 MPa.

3.2.2. CFRP Laminate with 20 mm PA Mesh Embedded

The uniaxial tensile test results of specimens with 20 mm PA mesh embedded were compared to those of neat CFRP specimens. Figure 19 shows the stress–strain curves recorded by strain gauges placed at the center and 20 mm from the center for both types of specimens. Figure 20 presents the stress–strain curves along with edge observation results near the resin pocket at selected stress levels. The edge images were extracted from microscope video recordings captured during the test. The red lines overlaid on the images indicate the delamination propagation paths.

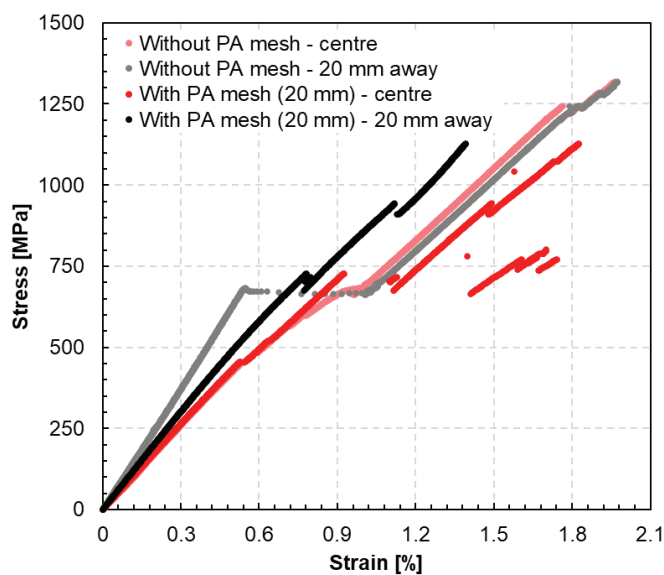


Figure 19. Stress–strain curves from the tensile test of a specimen with a 20 mm PA mesh embedded, in comparison to neat CFRP specimen.

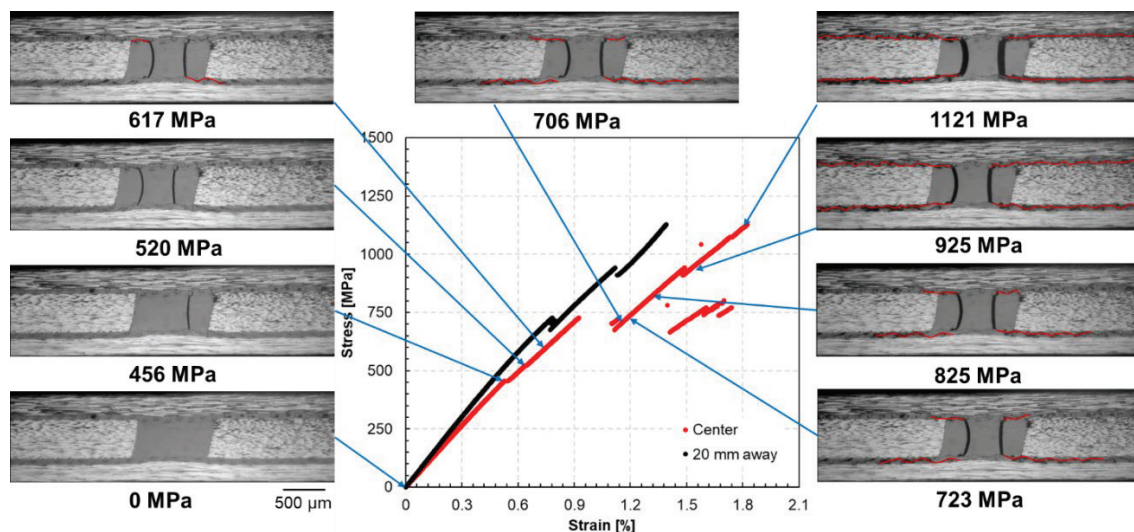


Figure 20. Stress–strain curves and results of edge observations (around the resin pocket) at specific applied stress levels for a specimen with a 20 mm PA mesh embedded. The red lines in the images indicate delamination propagation paths.

As shown in Figure 20, the first matrix crack in the resin pocket occurred at 456 MPa, followed by a second at 520 MPa. Small delamination from the matrix crack tips started to occur at 617 MPa, but the propagation was very slow as the applied stress increased. The decreasing slope of the stress–strain curve corresponds to the progression of delamination, indicating a loss of stiffness as the crack propagates. Above 600 MPa, while the slope of the curve for the neat CFRP specimen tended to decrease, the slope for the specimens with PA mesh inserts remained almost unchanged, showing linear behavior up to high stress levels. In other words, in the neat CFRP specimen, delamination progressed significantly above 600 MPa, whereas for the specimens with PA mesh inserts, it progressed slowly within the mesh layer even up to high stress levels.

A large strain jump was observed at approximately 720 MPa for the strain gauge pasted at the center, indicating a sudden delamination at all four edges of the resin pocket. At this point, the load continued to increase, and delamination progressed slowly up to a very high stress level of 1121 MPa. Complete separation between the continuous and discontinuous plies occurred at this stress level, leading to complete fracture soon after. This stress level is approximately 55% higher than that of the neat CFRP specimen, in which complete delamination occurred at about 725 MPa. This enhancement is attributed to the mechanical interlocking effect provided by the embedded PA mesh, which disrupts crack propagation and increases the energy required for delamination.

3.2.3. CFRP Laminate with PA Mesh Fully Embedded Throughout Entire Structure

The results of the uniaxial tensile test were compared between laminates with PA mesh fully embedded throughout their entire structure and neat CFRP specimens. The stress–strain curves are shown in Figure 21. Figure 22 presents the stress–strain curves and edge observation results (around the resin pocket) at specific applied stress levels.

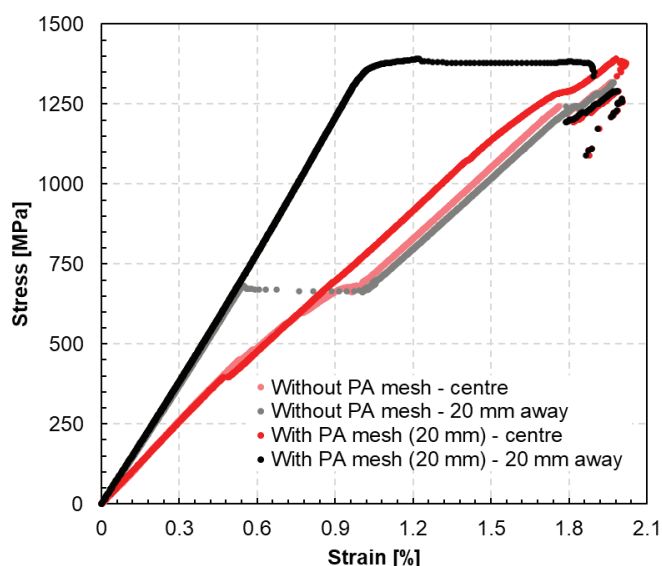


Figure 21. Stress–strain curves from the tensile test of a specimen with PA mesh fully embedded throughout its entire structure, compared to a specimen without an inserted PA mesh interlayer.

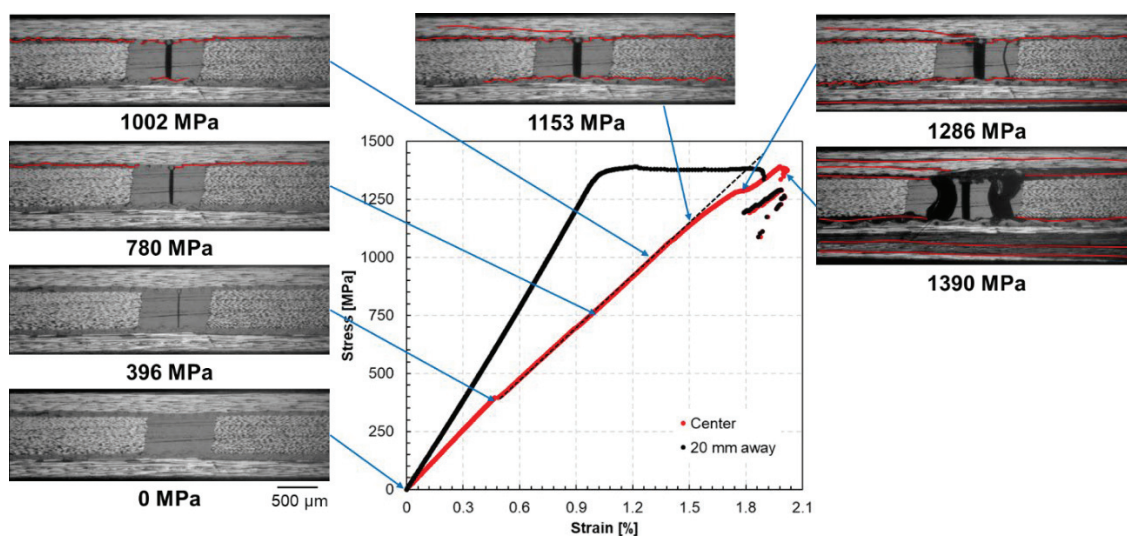


Figure 22. Stress–strain curves and edge observation results at specific applied stress levels for a specimen with PA mesh fully embedded throughout its entire structure.

The first matrix crack in the resin pocket occurred at 396 MPa, similar to the value for the neat CFRP specimen. The second matrix crack occurred at 1286 MPa, significantly higher than in the neat CFRP specimen and the specimen with a 20 mm PA mesh embedded. In those cases, the second crack usually occurred soon after the first, after which delamination started to propagate. However, in the specimen with the PA mesh fully embedded, the second crack occurred after significant delamination propagation. Above 600 MPa, while the slope of the curve for the neat CFRP specimen tended to decrease, the slope for the specimens with PA mesh inserts remained almost unchanged, showing linear behavior up to high stress levels. In other words, for neat CFRP specimens, delamination progressed significantly above 600 MPa, whereas in specimens with PA mesh inserts, it progressed slowly even up to high stress levels.

Delamination was first observed at 780 MPa in the edge observations and progressed slowly as the applied stress increased. At 1153 MPa, delamination also propagated between the continuous plies. This may have occurred because the enhanced interlaminar toughness between the continuous and discontinuous plies, due to the embedded PA mesh, caused the crack to propagate in locations where the fracture toughness was lower. Delamination between the continuous plies continued as the applied stress increased to 1286 MPa and 1390 MPa, as shown in Figure 22. Since delamination progressed very slowly in the specimen, it fractured immediately after complete ply separation at 1390 MPa. This final delamination stress was approximately 92% higher than that of the neat CFRP specimen (725 MPa), indicating a substantial improvement in delamination resistance. In this case, there was no large strain jump, and the stress–strain curves remained linear up to the high stress level of 1390 MPa. As previously noted, this improvement stems from the mechanical interlocking effect of the embedded PA mesh, which disrupts crack propagation and dissipates energy. Nevertheless, the tensile strength and fracture strain remained consistent with those of the neat CFRP laminate, indicating that mesh insertion does not compromise overall structural performance.

4. Effect of Embedded PA Mesh on Delamination and Mechanical Properties

Compared to neat CFRP laminates, those with PA mesh inserts show distinctly different stress–strain responses (Figures 19 and 21). In neat specimens, delamination begins shortly after matrix cracking and progresses rapidly, causing a noticeable reduction in

the slope of the stress–strain curve. In contrast, specimens with PA mesh, whether partially or fully embedded, maintain a stable initial slope, and delamination proceeds much more slowly. This result indicates that PA mesh effectively increases resistance to crack propagation and improves the mechanical performance of the laminate.

Based on the results obtained in this study, the laminates with fully embedded PA mesh suppress delamination more effectively than both the neat CFRP and the laminate with a 20 mm PA mesh insert. In the fully embedded configuration, delamination is significantly delayed and progresses very slowly, even under high applied stress. This enhanced performance is attributed to the mechanical interlocking effect of the PA mesh structure, which absorbs more energy and disrupts crack propagation throughout the entire laminate. Furthermore, the strength of the material, as indicated by the fracture stress and strain, remains comparable to that of the neat CFRP, indicating that the inclusion of PA mesh does not compromise the laminate strength.

Note that the results obtained in this study may not yet represent the optimal performance for delamination suppression. As discussed in Section 3.1.1, the propagation strain energy release rate (G_{IR}) for mode I interlaminar fracture toughness suggests a limited ability to resist crack propagation once initiated. This indicates that the current PA mesh configurations have not been fully optimized for mode I loading conditions. Therefore, further research is needed to systematically investigate the influence of PA mesh dimensions and placement. A factorial design approach should be employed to optimize these parameters and maximize the improvement in fracture toughness, thereby fully exploiting the benefits of the embedded PA mesh structure.

To investigate the effect of embedding PA mesh in CFRP laminates, uniaxial tensile tests were conducted with full-field strain measurements using DIC in the axial, transverse, and shear directions. Figure 23 presents strain distribution maps obtained by DIC at approximately the same applied stress for three types of laminates: neat CFRP, CFRP with 20 mm embedded PA mesh, and CFRP with fully embedded PA mesh. In the axial strain field (Figure 23a), the laminates with 20 mm and fully embedded PA mesh exhibit localized high-strain concentrations originating near the resin pocket, indicating the onset of delamination. In contrast, the neat specimen displays a more pronounced axial strain distribution associated with complete delamination between the continuous and discontinuous plies. Given that complete delamination was observed in other neat specimens around 760 MPa, it is likely that delamination had also already progressed significantly in this case.

In the transverse strain field (Figure 23b), increased negative strain values are evident in the neat specimen, further indicating that the outer plies bear most of the load due to the loss of load transfer across the delaminated region, resulting in more pronounced transverse deformation. The PA mesh–reinforced laminates show different behavior: the fully embedded specimen generally exhibits a higher Poisson’s ratio across the entire area, reflecting a uniform transverse strain distribution, while the specimen with the 20 mm embedded mesh shows a localized increase in Poisson’s ratio around the delaminated region, indicating that transverse strain is more confined to this area.

In the shear strain field (Figure 23c), the laminate with fully embedded PA mesh shows notably high shear strain near the resin pocket, implying that the mesh contributes to interlaminar load transfer. On the other hand, the neat specimen exhibits very low shear strain within the delaminated region itself, while higher shear strain is concentrated in the upper and lower regions. This distribution is consistent with complete delamination, where interlaminar shear deformation can no longer be transmitted across the delaminated interface, causing the surrounding intact regions to accommodate the shear load.

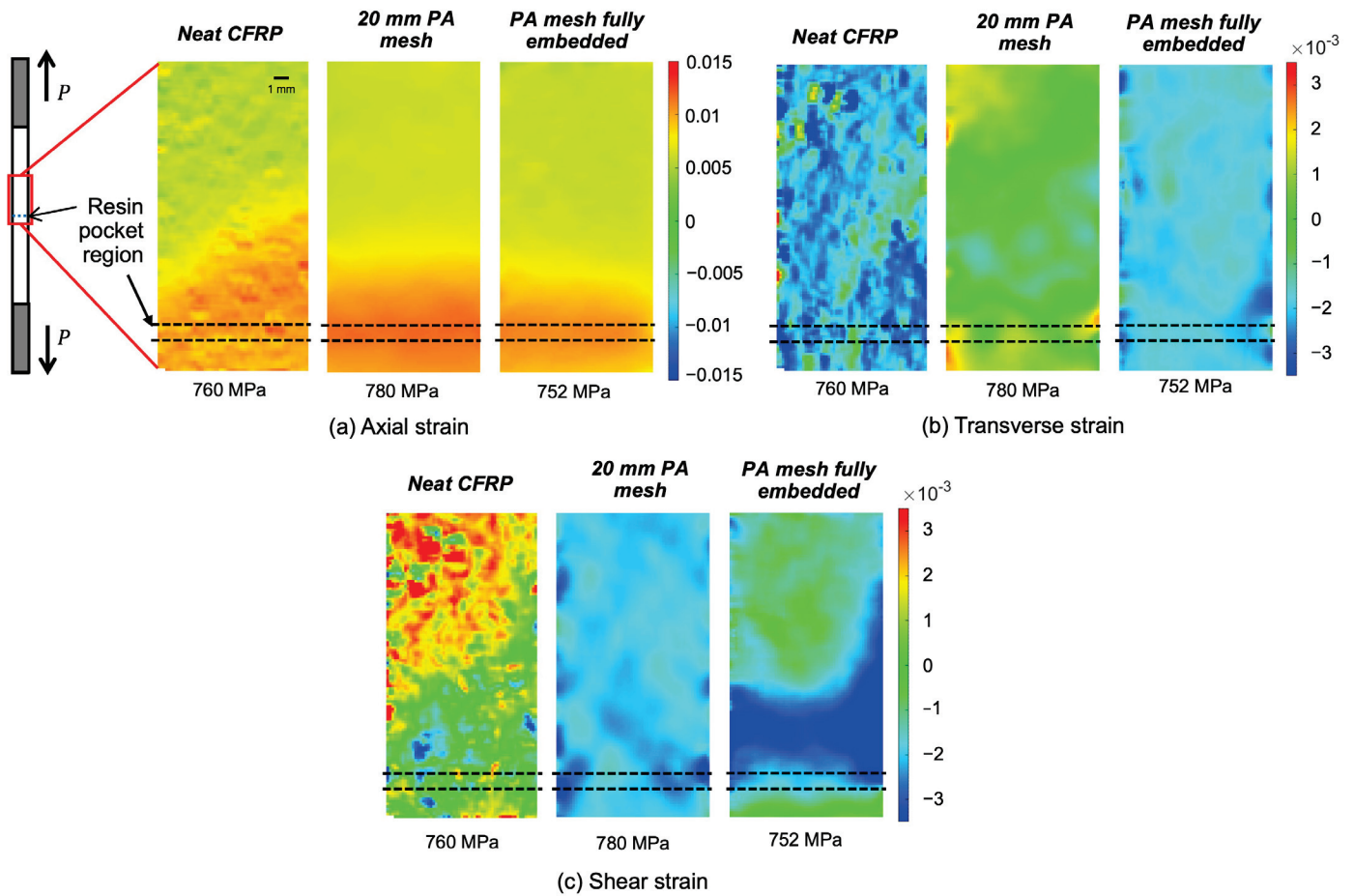


Figure 23. Strain distribution maps obtained by DIC at approximately the same applied stress: (a) axial strain, (b) transverse strain, and (c) shear strain for neat CFRP, CFRP with embedded 20 mm PA mesh, and CFRP with fully embedded PA mesh.

While the fully embedded PA mesh specimens demonstrated superior suppression of delamination, it is important to highlight the advantages of localized insertion, such as the 20 mm mesh used in this study. Localized embedding improves interlaminar performance while preserving the original in-plane mechanical properties. Specifically, compared to the fully embedded mesh, the 20 mm PA mesh increased transverse and shear strain only around the delaminated region. This indicates that strain redistribution was localized and did not affect the global mechanical behavior of the laminate. Such behavior is particularly beneficial for applications where maintaining the original in-plane deformation characteristics is critical.

This study evaluated only uniaxial loading conditions, whereas actual structures are often subjected to more complex loading scenarios. Therefore, future studies should assess the long-term durability of PA mesh-interleaved laminates, particularly under fatigue loadings, to determine their suitability for realistic service environments. Additionally, the effects of the PA mesh on other mechanical properties such as bending strength, indentation resistance, impact performance, and out-of-plane behavior warrant further investigation. Furthermore, although this study employed a single PA mesh configuration in both the interlaminar fracture toughness evaluation and the application to laminates with ply discontinuities, optimizing key mesh parameters such as fiber diameter, thickness, and mesh opening is essential for further performance enhancement. A systematic investigation involving multiple PA mesh configurations is necessary to determine the optimal design for various laminate architectures and loading conditions.

5. Conclusions

This study examined the effectiveness of PA mesh as an interleaving material to enhance interlaminar performance in CFRP laminates with ply-level discontinuities such as resin pockets. Two configurations were tested: full laminate embedding and a 20 mm local mesh insert between continuous and discontinuous plies. Under mode II loading, the mesh improved interlaminar fracture toughness by approximately 2.4 times due to mechanical interlocking, which altered crack propagation paths and increased energy dissipation. Under mode I loading, the mesh enhanced crack initiation resistance but slightly reduced propagation resistance, likely due to local voids or weak bonding. This indicates the need for better mesh and resin interface design.

Uniaxial tensile tests revealed that while initial matrix cracking occurred at similar stress levels across all specimens, those with PA mesh showed significant suppression of delamination propagation. The 20 mm insert and the full embedding increased the stress at full delamination by approximately 55% and 92%, respectively. Local mesh placement effectively suppressed damage at critical regions without affecting overall tensile strength, while full embedding further improved delamination resistance at the cost of added transverse and shear strain. These findings demonstrate that PA mesh interleaving is a practical and scalable approach for improving delamination resistance in CFRP laminates. Further optimization through mixed-mode testing, fatigue evaluation, and interface engineering is recommended for demanding structural applications.

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Abbreviations

The following abbreviations are used in this manuscript:

CFRP	Carbon fiber reinforced plastic
PA	Polyamide
DCB	Double cantilever beam
ENF	End-notched flexure
DIC	Digital image correlation
SEM	Scanning electron microscopy

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Article

Numerical Study on the Performance and Failure Modes of Bolted Connections in Pultruded-Fibre-Reinforced Polymer (PFRP) Profiles

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Abstract: The use of pultruded-fibre-reinforced polymer (PFRP) composite profiles in structural applications is rapidly increasing, due to their high strength-to-weight ratio, corrosion resistance, and durability. Bolted joints between PFRP play a critical role, as localized high stresses in a material that typically exhibits brittle behaviour—especially in tension and shear—can lead to sudden failure. This study aims to investigate the mechanical performance of such bolted connections (in terms of stiffness, strength, displacement capacities and failure modes), contributing to the development of reliable yet optimized design criteria for structural applications. In particular, numerical analyses of single-bolted connections in PFRP profiles are presented in the paper. To emphasize the general validity of the model and demonstrate its applicability across different configurations, the simulations were validated against experimental results from three separate test campaigns, which varied in both material (three different PFRP composites) and geometry (profile thickness, bolt diameter, and hole–end distance). Finite element models using continuum shell elements in ABAQUS, based on the Hashin failure criteria, successfully captured typical failure modes, including shear-out and pin-bearing. Two analysis approaches—implicit and explicit solvers—were also compared and discussed. Sensitivity analyses were carried out to enhance the model’s accuracy and its computational efficiency. The validated model was then extended to simulate different configurations, investigating the role of the main parameters influencing the connections.

Keywords: composite profiles; bolted connections; numerical simulations; explicit solver; implicit solver

1. Introduction

Fibre-reinforced polymers (FRPs) are composite materials composed of high-strength fibres embedded within a polymer matrix [1–4]. The fibres impart strength and stiffness to the composite, while the matrix binds the fibres together, enabling them to act as a unified material system and protect them from environmental damage. The reinforcing fibres are typically synthetic, such as carbon, glass, aramid, or basalt fibres, although natural cellulose-based fibres (e.g., jute, hemp) are also increasingly utilized for developing sustainable composites. The polymer matrix is generally a thermoset resin, such as epoxy, polyester, or vinyl ester. By varying the type and proportion of fibres and polymer matrix, a wide range of composite materials with tailored mechanical properties can be produced. These composites are manufactured through various processes, including weaving, hand lay-up, resin transfer moulding, vacuum bagging, and pultrusion. Pultrusion is particularly

common for producing continuous composite profiles; in this process, fibres are drawn through a resin bath and subsequently cured as they are pulled through a heated die.

The use of pultruded-fibre-reinforced polymer (PFRP) composite profiles in civil engineering has grown rapidly over the past three decades in civil engineering [1–7]. Initially, their application was limited to the repair, strengthening, and retrofitting of concrete and masonry structures [8–13]. However, in recent years, their adoption has expanded significantly due to their versatile properties, such as their light weight combined with their high strength and stiffness, their resistance to corrosion, their controllable thermal properties, their lower life cycle cost, their enhanced fatigue life and the fact that they are inert towards magnetics [5]. These characteristics make composite profiles highly advantageous for a variety of structural applications. Glass-fibre-reinforced polymer (GFRP) profiles are most commonly used in construction of new infrastructures like pedestrian bridges, communication towers and low-rise buildings.

PFRP profiles can be assembled using various methods, including bolted connections, adhesive joints, or a combination of both. Among these, bolted connections are the most commonly used in civil engineering structures. The preference for bolted connections is primarily due to their ease of construction, faster assembly, and straightforward disassembly for inspection and maintenance purposes, and they are also the most economical and cost-effective [14–16]. However, a critical aspect of using PFRP profiles is the connection itself, as it is often the point where stress concentration occurs. Proper design and analysis of these connections are thus crucial to ensure the structural integrity and performance of the system.

Bolted connections in PFRP profiles can fail in several modes, depending on various factors such as the composition of the material (i.e., fibres amount and orientation), the geometry of the jointed profiles, the position of the bolts and the loading direction. The primary failure modes include pin-bearing, shear-out, cleavage, splitting and net tension, as shown in Figure 1. Extensive research and experimental campaigns have been conducted to understand the conditions under which these failures occur [7,14,15,17–22].

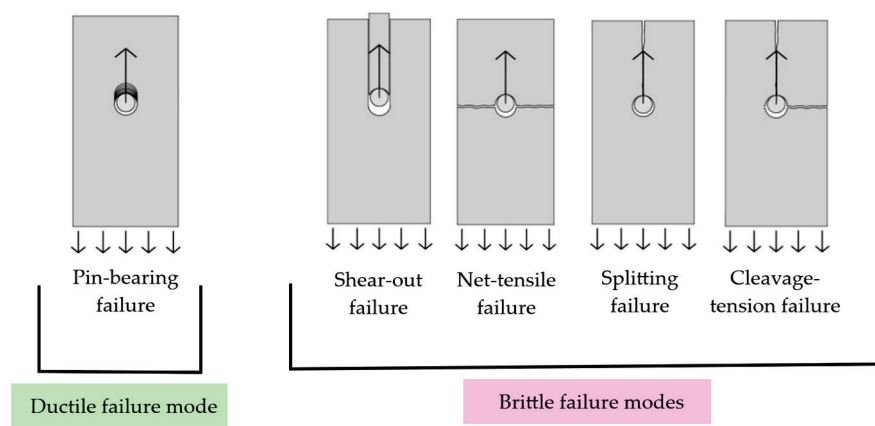


Figure 1. Schematization of typical failure modes of GFRP bolted connections subjected to longitudinal tensile load.

For example, referring to load along the longitudinal pultrusion direction, Rosner and Rizkalla and [14] carried out a comprehensive experimental programme to investigate the effects of geometric parameters, such as end distance (e) and plate width (w), with respect to bolt diameter (d), on the performance of GFRP bolted connections. Single-bolted, double-lap joints were studied: when $e/d = 1\text{--}3$ and $w/d \geq 5$, the dominant failure mode was cleavage failure (i.e., an initial crack parallel to the applied loading direction developed at the end of the plate along the end distance and started growing towards the plate hole, thereby leading to the development of other fractures and cracks around the net

section due to formation of in-plane bending stresses). When $w/d < 5$, net tension failure occurred, regardless e/d ($e/d = 1\text{--}12$); it was characterized by a crack along the net section perpendicular to the applied load. As e/d and w/d both increased to ≥ 5 , the failure shifted towards more favourable bearing failure (i.e., localized crushing of the material in the vicinity of the bolt hole, with an enlargement of the plate hole). They further observed that the connection resistance increased approximately linearly with increasing plate thickness ($t = 9.5$ to 19.1 mm).

Abd-El-Naby et al. [19] carried out an experimental campaign on double-lap joints using two different types of GFRP materials, namely grey (52 specimens) and yellow (45 specimens). The grey material consisted of 73% unidirectional fibres and 27% outer layers of chopped strand mat (CSM), whereas the yellow material contained 59% unidirectional rovings and a higher proportion of CSM (41%). The bolt diameter was fixed at 9.5 mm. For the grey material, e varied from 40 to 140 mm and w between 30 and 70 mm; for the yellow one, $e = 30$ to 70 mm and $w = 20$ to 38 mm. The experimental results revealed three primary failure modes: net tension, shear-out, and bearing. For the yellow material, net tension failure within the CSM layers occurred when the specimens had a small w combined with a large e . Shear-out failure in the central unidirectional fibre layer, characterized by the formation of a hat-shaped wedge, was observed when e was small. When both the e and w were increased, bearing failure occurred in the outer CSM layers, while shear-out failure developed in the central unidirectional fibre layers. For the grey material, w was sufficiently large and only shear-out and bearing failures occurred. When there was a small e , the shear-out failure was abrupt; when there was an intermediate e , it became gradual; for a large e , bearing failure occurred. The critical end distance was defined, for each w , as the distance at which the maximum resistance of the connection was attained, beyond which further increases in the end distance did not enhance strength. The increased thickness of CSM in the yellow material enabled a more uniform distribution of stresses compared to the grey material.

Tajeuana et al. [21] conducted an experimental study on single-bolt GFRP joints, focusing primarily on single-lap configurations to assess the effects of varying end distances and side distances (s), with respect to bolt diameter. In single-lap joints with $e/d = 3$ and $s/d = 2$, cleavage failure was predominant. Occasionally, shear-out and net tension failures also occurred. Increasing both e/d and s/d to 4 definitely shifted the failure mode to shear-out and improved the peak load capacity. A few double-lap joints with $e/d = 3$ and $s/d = 2$ were also tested, showing cleavage failure. Double-lap joints exhibited a more stable load–displacement response and higher load capacity than single-lap joints, due to their symmetrical loading, which minimized out-of-plane bending and other secondary effects.

Martin et al. [17] performed experimental tests on double-lap joints with various end distances ($e/d = 1.9, 3.1, 4.4$, and 8.8) while keeping $w/d = 5$. They observed that for the specimens with the two lower e/d ratios, sudden shear-out failure occurred, with an abrupt loss of resistance after the peak. For the highest e/d ratio, the failure mode was pin-bearing and the load was also almost maintained under a large displacement, when shear-out failure was also observed. For an intermediate e/d ratio, localized pin-bearing failure was initially observed, but it was immediately followed by shear-out failure.

The effect of loading direction has been extensively investigated by several researchers, who have addressed inclined and transverse loading conditions with respect to the main pultrusion direction. For example, Rosner and Rizkalla [14] compared longitudinal (0°), inclined (45°) and transverse (90°) loading conditions, varying the e/d and w/d ratios. Net tension failure occurred for both inclined (45°) and transverse (90°) loading, for configurations with e/d and w/d ratios ranging from 1 up to higher values of 10. When the w/d ratio was further increased (13.4), cleavage failure occurred at $e/d = 1$, and this then

shifted to net tension failure for $e/d = 2-5$. For $e/d = 10.7$, pin-bearing failure occurred. The cleavage failure that typically occurred under longitudinal (0°) loading was suppressed under inclined (45°) and transverse (90°) loading when $w/d \geq 5$ and $e/d < 4$. Cleavage and bearing failure required larger widths ($w/d \geq 13$) to avoid net tension failure. Overall, it was observed that the resistance of the connection under longitudinal loading (0° along the main pultrusion direction) was 18% and 24% higher than that under the inclined (45°) and transverse (90°) loading directions, respectively.

Turvey [18] analyzed single-bolt connections under longitudinal (0°), transverse (90°), and inclined (30° and 45°) loading. Four different width-to-diameter ratios ($w/d = 4, 6, 8$, and 10) were considered. Under the transverse loading condition, the ultimate resistance increased with increasing w/d ratios; however, net tension remained as the failure mode. For the longitudinal loading direction, the failure mode was dependent on the e/d ratio: when $e/d \geq 4$, bearing failure occurred, whereas for $e/d < 4$, shear-out failure was observed. In the case of inclined loading condition (30° and 45°), net tension failure occurred consistently, even for larger e/d ratios of 6 and w/d ratios of 10, highlighting the dominance of transverse tensile stresses in these orientations.

In general, referring to the main loading direction (i.e., the longitudinal one), shear-out failure tends to dominate when the end distance of the connection is small. On the other hand, net tension failure is more likely to occur when the width of the profile is insufficient. If both the end distance and width are inadequate, cleavage failure can result. Conversely, when both the end distance and width are sufficient, bearing failure typically occurs, while in the transverse and inclined loading conditions, net tension failure is more likely to dominate due to the lower transversal tensile strength. Among these failure modes, bearing failure is the most desirable, because it is ductile in nature, allowing for large deformations and providing warnings before final failure, also enabling load distribution in the case of multi-bolt connections. In contrast, shear-out, net tension, cleavage and splitting failures are brittle, leading to sudden and catastrophic failure without any prior indication.

Unlike steel, for which standardized guidelines and well-defined testing procedures are firmly established, FRPs present a significant challenge for universal standardization due to their inherently variable compositions. The mechanical properties of composites are highly sensitive to the type, volume fraction, and orientation of fibres, as well as to the nature of the polymer matrix. As a result, even minor changes in constituent materials or manufacturing processes can lead to substantial variations in structural behaviour. To provide a framework for design, several standards and guidelines have been developed. These include EN 13706 [23], which specifies requirements and test methods for pultruded profiles in structural applications, the EUROCOMP Design Code and Handbook [24], which offers comprehensive design recommendations and classification systems, and CNR-DT 205/2007 [25], an Italian guide that details the design and execution of structures using PFRP elements.

Despite these established documents, the inherent variability of composite materials means that experimental characterization remains indispensable whenever the composition or fibre architecture changes. Testing is essential to accurately determine mechanical and strength properties in the longitudinal, transverse and out-of-plane directions. Moreover, conducting exhaustive experimental campaigns to capture the strength and failure modes of composite connections is essential. But experimental testing is often complex and time-consuming, particularly given the vast array of possible geometric configurations and loading conditions. To address this, researchers have increasingly turned to numerical simulations, leading to the development of a variety of failure criteria to predict damage initiation and evolution in composite structures. FRP-specific material models are required, rather than directly the isotropic models commonly used in civil engineering (e.g., for steel

or concrete), since FRP composites are strongly anisotropic/orthotropic, with properties that vary by direction, and they fail through multiple distinct modes in the longitudinal and transverse directions (fibre tension, fibre compression, matrix tension, and matrix compression). Consequently, accurate prediction of the damage initiation and progression of these failure modes requires anisotropic constitutive laws and mechanism-specific failure criteria combined with progressive-damage formulations. Among these, the Hashin [26], Puck [27], and Larc05 [28] failure criteria are widely recognized and extensively used. These failure criteria assess damage in both fibres and the matrix under different loading conditions. Moreover, in recent years, progressive-damage models have been employed to track damage evolution under loading. ABAQUS [29] includes an in-built Hashin failure criterion; however, its applicability is limited to shell and continuum shell elements. More recently, the Larc05 criterion has also been integrated into ABAQUS versions 2024 and 2025 for shell and continuum shell elements.

Numerical modelling of FRP-bolted joints is carried out mostly in the aircraft industry but also in civil engineering, and it has been widely explored to reduce the need for extensive and often costly experimental campaigns [21,30–47]. Over time, these models have evolved significantly, ranging from simplified two-dimensional (2D) analyses to more detailed three-dimensional (3D) representations, and from purely linear elastic formulations to advanced progressive-damage models capable of capturing nonlinear and failure behaviours.

Among these, Turvey and Wang [31] developed 2D (FE) models by using ANSYS FE software [48] for single-bolt tension joints in pultruded GFRP plates, accounting for factors such as bolt-hole clearance and friction. Their study highlighted the inadequacy of simplified normalized stress-based design approaches, such as those adopted in the EUROCOMP code [24], due to the sensitivity of stress distributions to contact conditions and applied loads. However, their model remained within the elastic regime and did not incorporate damage progression or post-elastic behaviour. To address this limitation, various researchers have developed two- and three-dimensional progressive-damage models, enabling accurate damage assessment in the nonlinear range.

Tajeuna et al. [21] developed a two-dimensional finite element model using ADINA 8.7.3 [49] to simulate single-bolt, single-lap and double-lap GFRP–steel connections to investigate the influence of geometric parameters. The GFRP plate was discretized with nine-node quadrilateral elements, using a refined mesh around the bolt hole to improve accuracy. The material behaviour was modelled as orthotropic plasticity, with input parameters derived from experimentally measured tensile properties, including ultimate strain. The bolt was idealized as a rigid half-cylinder to reduce computational cost, and contact between the bolt and plate was defined using ADINA's surface contact formulation. The model was first validated against experimental results for both joint types, showing good agreement in peak strength and failure modes. A parametric study was then conducted varying both the end distance and side distance in the ranges of $1 \leq e/d \leq 5$ and $1 \leq s/d \leq 5$. While the numerical model successfully captured all observed failure modes, the load–displacement response beyond the peak load was not simulated. This limitation may be attributed to convergence difficulties or the intrinsic restrictions of the analysis setup.

Du et al. [50] developed a three-dimensional progressive-damage model in Abaqus/Explicit to simulate the behaviour of double-lap joints in pultruded GFRP composites. Their approach integrated a nonlinear shear response, the Hashin failure criteria, and a strain-based continuous stiffness degradation law into an explicit vectorized custom-defined user material (VUMAT) subroutine. This allowed the model to capture the progressive damage evolution of the composite under quasi-static loading, which was applied using a gradually increasing displacement. The numerical predictions showed good agreement with the

experimental results, validating the effectiveness of the model. However, a key limitation of this approach is that the progressive-damage formulation is not natively available in ABAQUS. It requires users to compile and integrate the VUMAT subroutine with ABAQUS, which can be technically demanding and presents a significant barrier for many structural engineers without advanced programming or finite element backgrounds.

Recently, Arruda et al. [43] introduced an implicit three-dimensional damage model based on the Hashin failure criterion. This user-defined material (UMAT) model customizes damage initiation and progression by incorporating fracture energies and residual strength parameters in a user subroutine in the implicit solver. To mitigate convergence issues, the model employs an implicit-to-explicit solver. The shear-out and pin-bearing failure were simulated without having any convergence problems.

Liu et al. [45] developed a numerical model for a single-lap joint between a carbon-FRP laminate and a steel plate using countersunk fasteners. In their approach, continuum shell elements were used to capture intralaminar ply failure, while cohesive elements were implemented to simulate interlaminar damage under Mode I (traction) loading. For Mode II (shear) and mixed-mode delamination, a user-defined VUMAT incorporating 3D Hashin failure criteria and the Camanho degradation law [51] was employed. Their simulations, performed using the ABAQUS/Explicit solver [29], showed excellent agreement with the experimental results, and no convergence issues were reported. However, their study was limited to modelling bearing failure as the dominant failure mode.

To the best of the authors' knowledge, the application of ABAQUS/Explicit using in-built failure criteria has not yet been explored in detail for FRP-bolted joints. While several 2D and 3D user-defined material models are available in the literature [43,50] for PFRP composites, they are not directly accessible within ABAQUS. These models often require significant computational effort, specialized expertise, and additional software integration for pre- and post-processing, which limits their practical usability. This paper aims to address this gap by employing the built-in composite damage modelling capabilities of ABAQUS/Explicit to simulate the response of FRP-bolted joints. The main purpose is to evaluate the capability and reliability of the developed model in simulating the performance of bolted joints—including the prediction of the different failure modes and the post-elastic behaviour—and to thoroughly understand the role of the different fundamental mechanical parameters that govern the performance. Different GFRP materials and joint configurations are considered to achieve more thorough and robust validation.

In this manuscript, the general features of the numerical model for single-bolted joints are first presented. This is followed by a description of the three types of composite materials considered for validation, and the corresponding experimental features and results obtained for double-lap connections. The reliability of the numerical model is then proven by comparison with the experimental data for all material types and configurations. Furthermore, comprehensive sensitivity analysis is conducted to optimize computational efficiency while maintaining acceptable accuracy. In addition, parametric analyses are performed to evaluate the influence of the main material's mechanical properties on the performance of the connection.

2. Numerical Model Features

The numerical model of the single-bolted double-lap joint shown in Figure 2a was developed in the latest version of ABAQUS/Explicit version 2025 [29]. The general configuration consists of two steel plates with an inner GFRP pultruded plate. The X-axis was designated as the longitudinal direction of the GFRP plate, aligned with the unidirectional roving, which corresponds to the direction of the major elastic modulus. To reduce computational costs, only one-fourth of the model was developed, with symmetry boundary

conditions applied in both the Y and Z directions. The steel plate was modelled with the full length and thickness but half the width, while the composite plate was modelled with the full length, half the thickness, and half the width. Additionally, one-fourth of the bolt was modelled. The far end of the GFRP composite plate was fully constrained in all directions, and a linearly increasing X displacement was applied to the far end of the steel plate to replicate the experimental loading conditions. A 1 mm gap was maintained between the plates, consistent with the experimental setups.

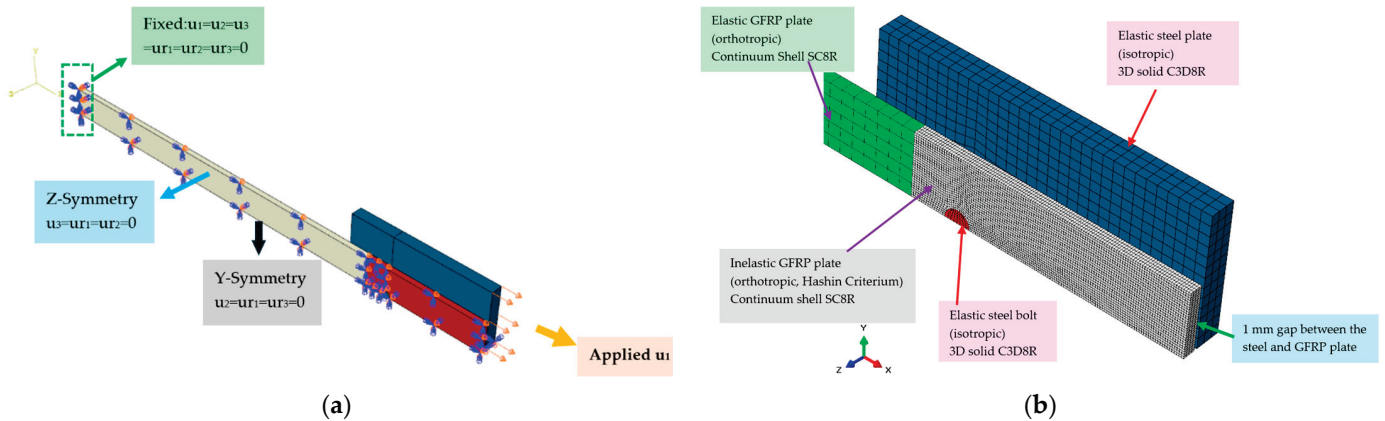


Figure 2. A generalized model description for single-bolted, double-lap joints in ABAQUS: (a) boundary conditions and (b) partial description with meshing.

The continuum shell elements with eight nodes and reduced integration (SC8R) were selected for the composite plate, which was modelled in two separate parts, one being assigned only elastic properties, and the other—surrounding the bolt region, where stress concentrations and failures are expected—being assigned mechanical failure-capable properties (nonlinear behaviour). The elastic and inelastic parts of GFRP plates were tied together. Orthotropic engineering constants were considered for both. Additionally, the built-in Hashin damage failure and progression criteria (applicable only for shell and continuum shell elements) were assigned to the inelastic part. The inelastic plate was assigned tensile, compressive, and shear strength parameters in both the longitudinal and transverse directions for damage initiation. For damage progression, the tensile and compressive fracture energies were specified in the longitudinal and transverse directions. The details of these parameters are explained later in Section 4. On the other hand, continuum three-dimensional solid elements with reduced integration (C3D8R) and with isotropic elastic behaviour were used for both the bolt and steel plate. The combination of SC8R and C3D8R elements ensured both an accurate representation of composite damage mechanisms in the FRP and a reliable simulation of the steel components. The general contact algorithm for the explicit solver was applied for the contact between the bolt and the steel plate and also between the bolt the GFRP plate. Contact interactions between the plate and bolt were defined as hard contact in the normal direction, and a coefficient of friction of 0.2 was applied in the tangential direction, as recommended by various researchers [31,43].

The mesh size for the steel plate was set to $3 \times 3 \times 3 \text{ mm}^3$, while the bolt was meshed at $1 \times 1 \times 1 \text{ mm}^3$. For the elastic region of the composite plate, we used a mesh size of $4 \times 4 \times 4 \text{ mm}^3$, and the region around the bolt was refined to $0.7 \times 0.7 \times 1 \text{ mm}^3$, with 1 mm in the thickness direction. A mesh sensitivity analysis was performed (as detailed in Section 5.3 of the manuscript), and the mesh size was selected as the ideal compromise between accuracy and computational efficiency. The explicit solver with a dynamic procedure was used to perform the simulations. The displacement applied on the steel plate was increased gradually, to avoid dynamic effects, ensuring that the kinetic

energy remained negligible throughout the simulation, with respect to the input energy. Mass scaling with a target increment of 5×10^{-7} was set to reduce the computational time without compromising the accuracy of the analysis (the duration of the simulation on a machine with 8 CPUs was reduced approximately from 24 to 42 h to 2–6 h). However, preliminary sensitivity analyses were performed to compare the capabilities of the explicit solver with those of the implicit one and to properly calibrate the mass scaling and mesh size, as will be discussed later in Section 5.

3. Experimental References

Three different experimental campaigns were selected to carry out the numerical simulations of single-bolted double-lap joints, considering different GFRP materials. This selection was based on the need to cover various composite GFRP materials and various geometric configurations. The mechanical characteristics for the three GFRP materials denoted as Type-A, Type-B and Type-C are shown in Table 1 and are described in the subsequent sub-sections.

Table 1. Mechanical properties of GFRP pultruded profiles.

Property	Type-A	Type-B	Type-C	Standards for Type-A	Standards for Type-B and Type-C
Longitudinal Tensile Strength (MPa)	334	395	360	EN ISO 527-1 [52]	EN ISO 527-1
Longitudinal Compressive Strength (MPa)	316	440	280	ASTM-D695 [53]	EN ISO 14126 [54]
Transverse Tensile Strength (MPa)	29	19	100	--- [43]	EN ISO 527-1
Transverse Compressive Strength (MPa)	51.9	45	35	ASTM-D695	EN ISO 14126
In-plane Shear Strength (MPa)	35	19	50	ASTM-D5379 [55]	EN ISO 14130 [56]
Longitudinal Young's Modulus (MPa)	23,100	32,600	27,000	ASTM-D695	EN ISO 527-4
Transverse Young's Modulus (MPa)	2900	3800	7000	ASTM-D695	EN ISO 527-4
In-plane Shear Modulus (MPa)	3000	3029	5800	ASTM-D5379	EN ISO 14130
Interlaminar shear modulus (MPa)	2500	2500	5800	ASTM-D2344 [57]	--- [58,59]
Interlaminar shear strength (MPa)	33.8	19	30	ASTM-D2344	--- [58,59]
Poisson ratio (ν_{12})	0.28	0.23	0.23	EN ISO 527-1 [52]	EN ISO 527-1 [52]
Poisson ratio (ν_{23})	0.3	0.28	0.28	--- [43]	--- [58,59]

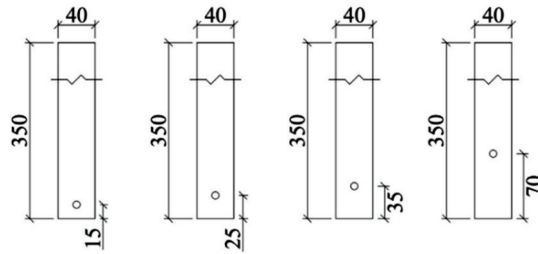
For the mechanical parameters not determined experimentally (---), values from the literature were assumed.

3.1. GFRP Material "Type-A"

Martin et al. [17] carried out a broad experimental testing campaign on single-bolted double-lap joints. The GFRP material, denoted by "Type-A" for simplicity onward in this paper, was thoroughly characterized experimentally, and the detailed properties, along with the test procedures, are reproduced in Table 1. Typical values from the literature [43] were selected when the experimental test was not applicable (transverse tensile strength and Poisson's ratio ν_{23}). The Type-A material was made from E-glass fibres and an isophthalic polyester resin matrix, in which the mass fibre ratio was 55%.

Experimental testing was performed on plates $t = 8$ mm thick and $w = 40$ mm wide, with a bolt diameter $d = 8$ mm. Four different bolt-end distances were investigated, $e = 15, 25, 35$, and 70 mm, as illustrated in Figure 3a. The setup is schematized in Figure 3b: the composite plate was sandwiched between two stainless steel plates, with a 1 mm gap between them. Capacity curves, representing the applied axial load varying with relative displacement between section A-A in the GFRP plate and section B-B in the steel (on a base length of 350 mm), were obtained for all specimens, as shown in Figure 4. The monitored displacement thus included both the local deformation of the connection and some deformation of the plates. The specimens with end distances of 15 mm and 25 mm exhibited shear-out failure, characterized by a sudden drop in the peak load (Figure 4a). In contrast, the specimen with an end distance of 70 mm (Figure 4b) showed a slight reduction

in the peak load, indicating pseudo-ductile behaviour typical of pin-bearing failure, with shear-out failure then occurring at high displacements. The specimen with an end distance of 35 mm (Figure 4b) exhibited the shear-out failure mode, but attained almost the same initial peak as the 70 mm specimen, suggesting transitional behaviour from shear-out to bearing failure.



All series with plate thickness of 8 mm.

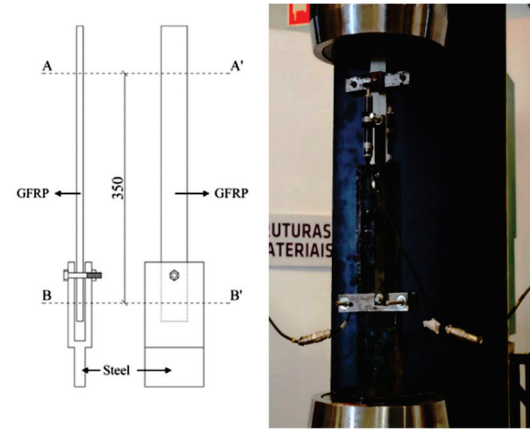
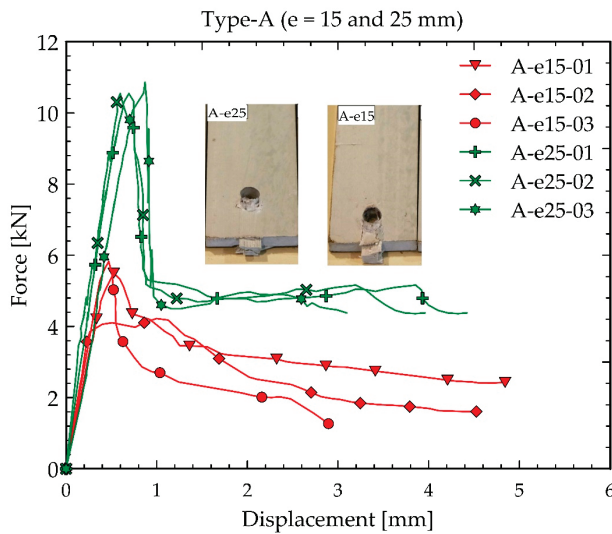
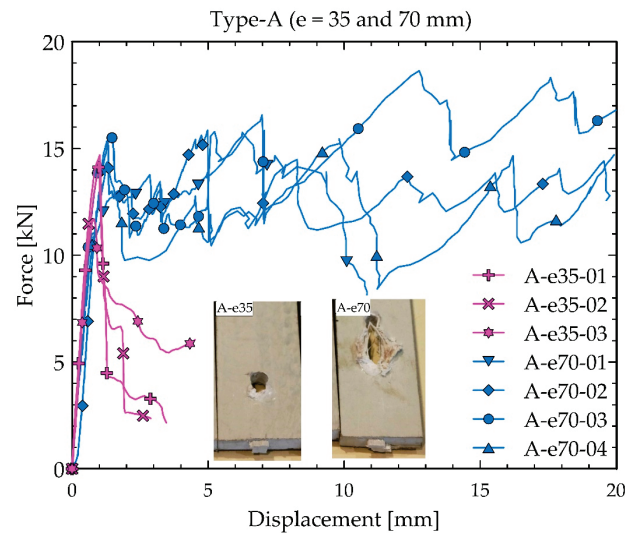


Figure 3. “Type-A” based on tests on bolted connections by Martin et al. [17]: (a) main geometric features of the samples and (b) experimental setup. Reprinted with permission from Martins, D.; Gonilha, J.; Correia, J.R.; Silvestre, Thin-Walled Structures, Elsevier, 2021. © [17].



(a)



(b)

Figure 4. Force vs. displacement graph for “Type-A” tests with end distances of (a) 15 mm and 25 mm; (b) 35 mm and 70 mm.

3.2. GFRP Material Type-B

Gattesco and Boem [22] considered GFRP “Type-B” plates composed of E-Glass fibre and an isophthalic polyester resin, with a fibre weight percentage of 60%. Experimental standard tests were conducted to determine the mechanical properties of the profiles according to EN ISO standards [60], and the results are summarized in Table 1. The performance of the bolted connections was evaluated on plates with dimensions of $10 \times 102 \times 300 \text{ mm}^3$ and $8 \times 78 \times 230 \text{ mm}^3$. A centrally located hole with diameters of 16 mm (M16) or 12 mm (M12) was created in each sample, and a close-fitting metallic pin was inserted in the hole. The bolt diameter-to-plate thickness ratio (d/t) was main-

tained at approximately 1.5 [24]. Additionally, the end distance-to-diameter (e/d) and width-to-diameter (w/d) ratios were kept at about six times the bolt diameter. The samples characteristics and the test setup are illustrated in Figure 5: the specimens were loaded under double shear through a rigid fork mechanism composed of three metallic plates, with 0.5 mm thick steel washers welded on the internal side of the fork brackets, acting as spacers, to prevent the lateral confinement of the GFRP plate. During the tests, the applied axial load and the relative displacement between the GFRP and the steel plates, at the height of the pin axis (i.e., the local deformation of the connection), were recorded.

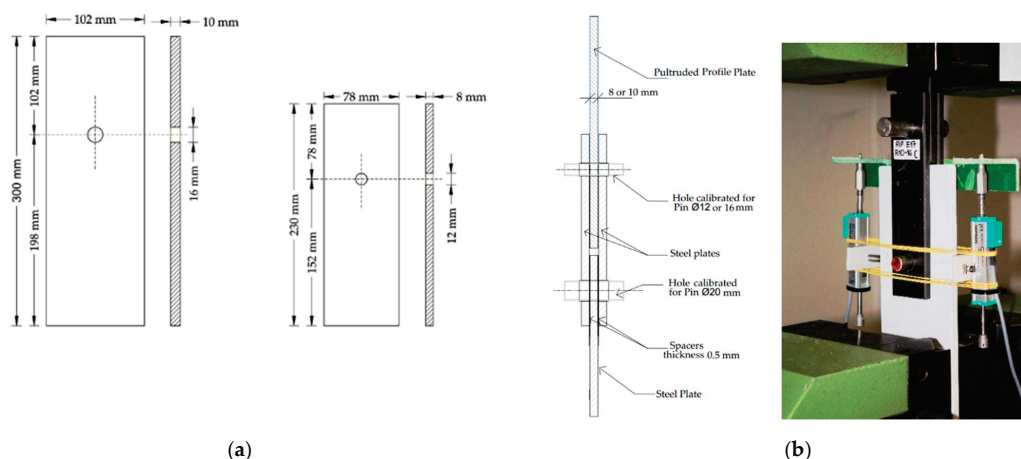


Figure 5. “Type-B” based on tests on bolted connections by Gattesco and Boem [22]: (a) main geometric features of the samples and (b) experimental setup.

The force–displacement curves (Figure 6) show an initial linear elastic response, followed by local plastic deformation in the vicinity of the bolt, due to pin-bearing. A temporary drop in load occurred due to the splintering of the composite material near the bolt, after which the load increased again. Pin-bearing failure emerged, as evidenced from the evident pseudo-ductile post-peak phase. This pin-bearing failure was followed by shear-out failure at a much larger displacement in some samples, as illustrated Figure 6b. The pin-bearing load was found to be influenced by both the bolt diameter and the plate thickness. The configuration with the $d = 16$ mm and $t = 10$ mm resulted in a higher pin-bearing load compared to the configuration with a $d = 12$ mm bolt diameter and $t = 8$ mm. However, the average pin-bearing strength (evaluated as the peak load divided by $d \cdot t$) was lower in the former configuration (149.50 MPa) than in the latter (208.90 MPa).

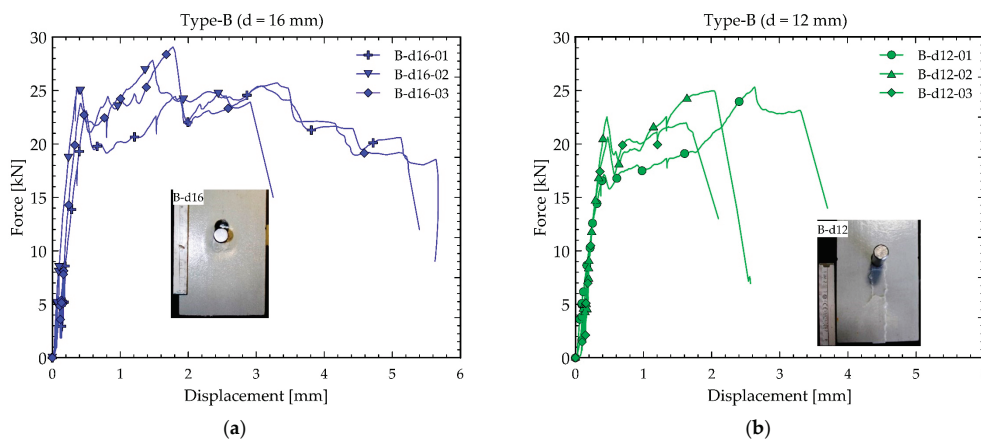


Figure 6. Force–displacement graph for “Type-B” tests, with (a) an M16 bolt with 6d spacing, and (b) an M12 bolt with 6d spacing.

3.3. GFRP Material “Type-C”

The “Type-C” material was composed of E-Glass fibre and an ophthalmic polyester resin, with a fibre weight percentage of 68%; the detailed properties, which were determined from the experimental campaign, are presented in Table 1. Higher transversal and out-of-plane mechanical properties emerged with respect to Type-A and Type-B, as it also incorporates triaxial laminates layers.

Explorative tests were recently performed on single-bolt double-lap joints on GFRP plates with dimensions of $15 \times 70 \times 200 \text{ mm}^3$, as well as with a bolt hole measuring $d = 13 \text{ mm}$ and an end distance of $e = 35 \text{ mm}$, as illustrated in Figure 7a. The test assembly consisted of an inner GFRP plate sandwiched between two outer GFRP plates with the same characteristics. The far end of the inner plate was clamped into the upper jaw of the testing machine (Figure 7b). The far ends of the outer plates were bolted to an inner steel plate measuring $10 \times 50 \times 200 \text{ mm}^3$, clamped into the lower jaw. Both connections had M12 bolts; a small torque of $18 \text{ N}\cdot\text{m}$ was applied, just to ensure proper alignment along the centerline. The applied load was monitored against the relative displacement between the jaws (Figure 8), thus including the deformation of the GFRP and steel plates and possible slip at the clamping heads in the measurement of the local deformation of the two connections.

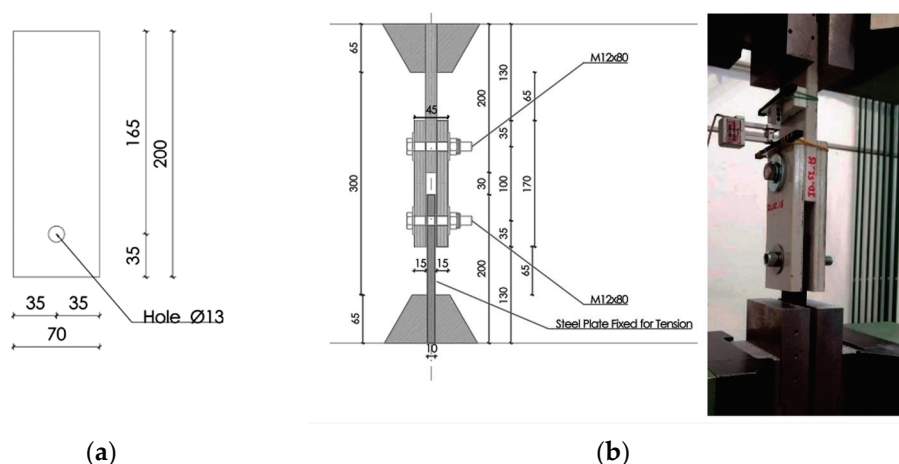


Figure 7. “Type-C” tests on bolted connections: (a) main geometric features of the samples and (b) experimental setup.

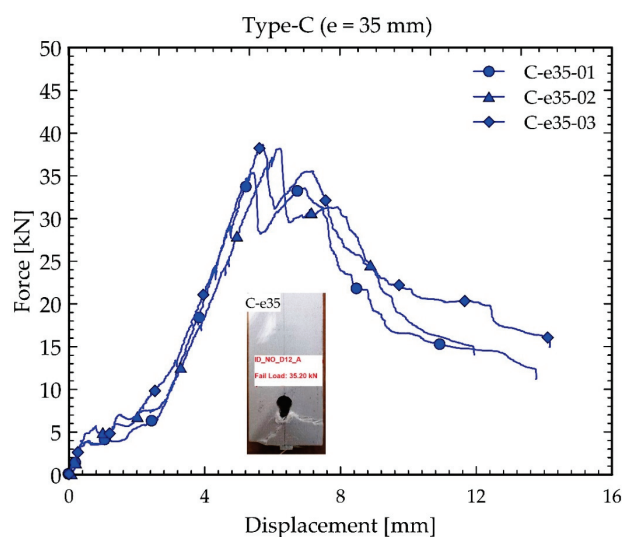


Figure 8. Force–displacement graph for “Type-C” tests, with M12 bolt and end distance of 35 mm.

After an initial elastic phase, the load–displacement curves showed a temporary but relevant stiffness reduction, from approximately 5 to 7 kN, associated with the bolts clearance recovery, once the friction between the plates, due to the small torque, was overcome. Then, the curves resumed their linear trend until the first peak was reached. Aside from this temporary effect, it was verified that the low torque did not affect the performance of the connection. The collapse of the samples was primarily dominated by the shear-out failure at the upper joint, even though with initial pin-bearing damage activation (attainment of the first peak), that immediately progressed toward the complete shear-out failure of the inner GFRP plate (second peak), as depicted in Figure 8.

4. Numerical Results

4.1. Main Parameters of the Numerical Model

The numerical model described in Section 2 was used to simulate the experimental results for the single-bolted joints collected in Section 3. The material densities were defined as 1800 kg/m³ for the GFRP composite and 7850 kg/m³ for the steel components. For the elastic material of the steel plates and bolts, a Young's modulus of 195 GPa and a Poisson's ratio of 0.3 were assigned for Type-A configurations and values of 210 GPa and poisson ratio of 0.3 for the Type-B and Type-C configurations. For the elastic region of the GFRP plate, the orthotropic engineering constant values from Table 1 were defined in three orthogonal directions. A local coordinate system was applied to ensure the accurate representation of the material orientation, aligned with the unidirectional fibre direction.

For the inelastic region of the GFRP plate, material input included the longitudinal tensile and compressive strengths, transverse tensile and compressive strengths, and in-plane and interlaminar shear strengths from Table 1. To simulate damage progression after initiation, tensile and compression fracture energies (FEs) in both the longitudinal and transverse directions were assigned, as presented in Table 2. These values were incorporated by Arruda et al. [43] in their user-defined Hashin-based UMAT model and were obtained experimentally from a similar pultruded GFRP material by Almeida-Fernandes et al. [61]. Since the present work used the same material, the Type-A material, considered by Arruda et al. [43], along with comparable Type-B and Type-C GFRP materials, the same values were assumed as initial input parameters, and subsequently refined through targeted sensitivity-based calibration to improve accuracy (Section 6.1). In general, fracture energies need to be determined for each specific material type through experimental tests, namely compaction tension tests—to characterize intralaminar tensile and compression fracture energies (longitudinal and transverse)—and double-cantilever-beam (DCB) tests to determine interlaminar fracture energies.

Table 2. Fracture energies for longitudinal and transverse directions.

Property	Type-A, B and C
Longitudinal Tensile Fracture Energy (LT-FE) (MPa.mm)	100
Longitudinal Compressive Fracture Energy (LC-FE) (MPa.mm)	600
Transverse Tensile Fracture Energy (TT-FE) (MPa.mm)	5
Transverse Compressive Fracture Energy (TC-FE) (MPa.mm)	20

A small damage stabilization factor value of 1×10^{-4} is also provided to stabilize the damage progression. The explicit solver successfully simulates the analysis of all materials and configurations, as reported and discussed in the following subsections.

4.2. “Type-A” Simulations

For Type-A tests, load–displacement curves were obtained for each considered configuration and compared against the experimental results in Figure 9. Generally, the responses matched closely in the elastic region, and the peak loads were accurately predicted for all end distances.

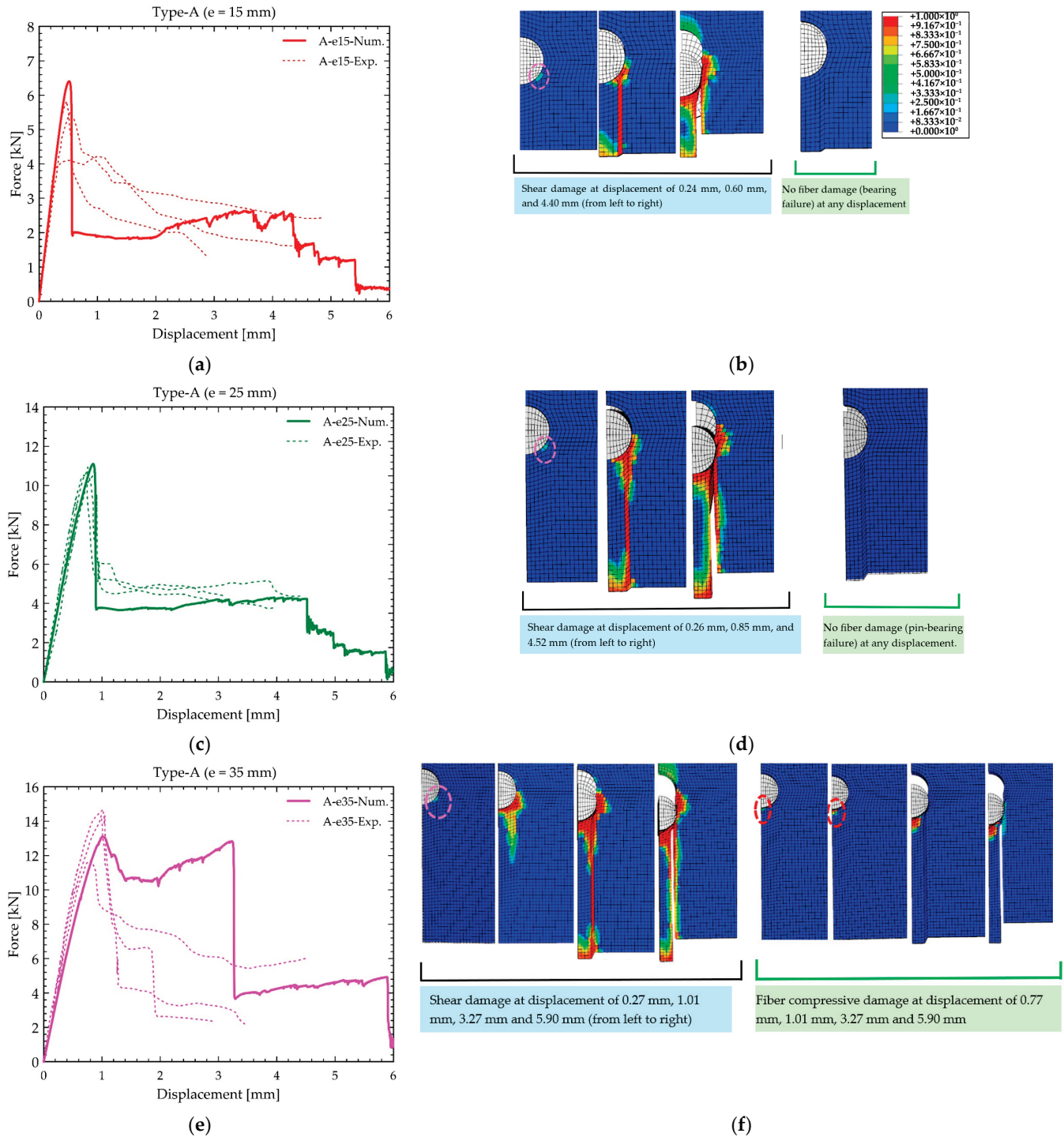


Figure 9. Cont.

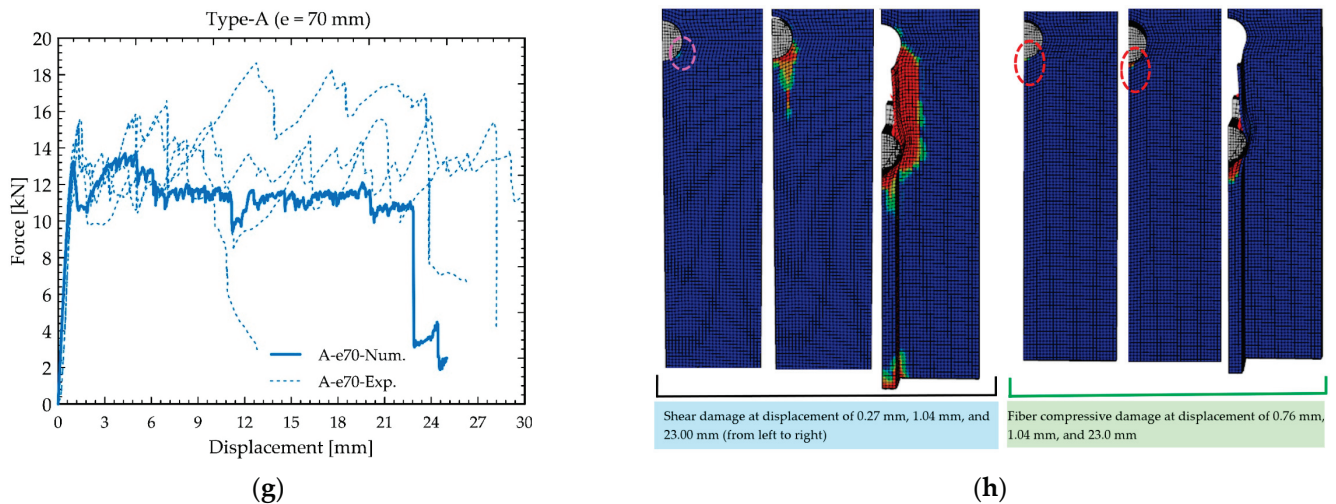


Figure 9. Numerical vs. experimental results for “Type-A” tests: (a,c,e,g) load vs. displacement graphs, and (b,d,f,h) damage at various displacements, for end distances of (a,b) 15 mm, (c,d) 25 mm, (e,f) 35 mm and (g,h) 70 mm.

The sudden load drop, indicative of shear-out failure onset, was well captured for the configurations with $e = 15$ mm (Figure 9a) and $e = 25$ mm (Figure 9c), as evidenced also from the comparison between the numerical and experimental failure patterns (Figure 9b and d, respectively). The shear damage in some elements around the lateral sides of the hole on the compressed edge started at around 3.40 kN at a displacement of 0.24 mm in the model with $e = 15$ mm, as well as at 4.1 kN and a displacement of 0.26 mm in the model with $e = 25$ mm, and then progressed in the longitudinal, loading direction in the elements along the end distance on the shear-out plane (Figure 9b,d). The first sudden reduction in resistance occurred at a displacement of 0.6 mm and a peak load of 6.40 kN (for $e = 15$ mm), or 0.85 mm (for $e = 25$ mm), when all the elements along the end distance of the shear-out planes became damaged and the shear damage variable index became 1. As the shear-out failure occurred, the capacity curve progressed with residual load, until a second reduction occurred at a displacement of 4.40 mm ($e = 15$ mm) or 4.52 mm ($e = 25$ mm), when the separation of the shear-out plane from the main plate occurred, as the elements along the plane attained full damage. Figure 9b,d also show that no fibre compression damage occurred throughout the analysis. Therefore, no bearing failure was activated in the model with $e = 15$ mm and $e = 25$ mm.

On the opposite side, in the case where $e = 70$ mm, the bearing failure was correctly simulated, as can be seen both from the post-peak pseudo-ductile phase of the capacity curves (Figure 9g) and from the concentration of the compressive damage in the fibres around the bolt (Figure 9h), which reflects the experimental damage pattern. The activation of shear damage in elements around the hole started at a displacement of 0.27 mm and at a load around 4.30 kN, as was the case for the model with $e = 25$ mm, and then progressed as illustrated in Figure 9h. Within the elastic region, this shear damage decreased the stiffness of the capacity curve. The longitudinal compression damage in elements around the compressed edge of the plate near the bolt started at around 11.20 kN and became full damage at a load of 13.25 kN (displacement 1.04 mm), where the pin-bearing failure activated. The pin-bearing failure progressed with even more elements becoming damaged under longitudinal compression. In the mean time, the shear damage in elements along the shear-out plane also progressed, almost reaching one-fourth of the end distance at the point of pin-bearing failure (Figure 9h). Moreover, when all elements along the end distance on the shear plane became fully damaged, the final shear-out failure activated at a large displacement (23.00 mm), where the resistance suddenly dropped from 10.58 kN to 3.1 kN.

In the case where $e = 35$ mm (Figure 9e,f), shear damage also started at a displacement of 0.27 mm in elements around the plate hole, which started to decrease the stiffness of the curve within the elastic limit. Fibre compression damage started later at a displacement of 0.77 mm (load is 11.21 kN similar to $e = 70$ mm). At the peak load, pin-bearing failure first became activated in the numerical model at a displacement of 1.01 mm, and was then followed by shear-out failure at a displacement of 3.25 mm. Actually, the post-peak behaviour that resulted was overestimated in the portion before the shear-out failure occurred. However, as can be seen from the capacity curve, the forces corresponding to the activation of the two failure modes were very close. This confirms that the 35 mm end distance is approximately the threshold at which the transition between brittle shear-out and ductile pin-bearing failure occurs in Type-A materials (as also evidenced in Section 3.1). Further sensitivity analysis is required to carefully calibrate the elastic moduli and strength parameters to directly attain shear-out failure without the activation of pin-bearing. However, the purpose here is to show the results obtained using the average mechanical parameters available in the literature, as achieved via the standard characterization tests carried out on the pultruded material. The final shear damage and fibre compression damage at a displacement of 5.90 mm are also shown in Figure 9f, where the separation of some portion from the main plate occurs.

No convergence issues emerged in all the simulated configurations, even when some portion of the main plate became separated from it due to full damage to the elements. Figure 10 contains a plot of the percentage ratio of kinetic energy (KE) to internal energy (IE), as the applied displacement increased, in the four numerical simulations. It emerges that the kinetic energy remained below 1% of the internal energy throughout the analysis, ensuring that the system behaved in a quasi-static manner during the entire simulation, and thus proving the reliability of the stress and strain results of the analyses.

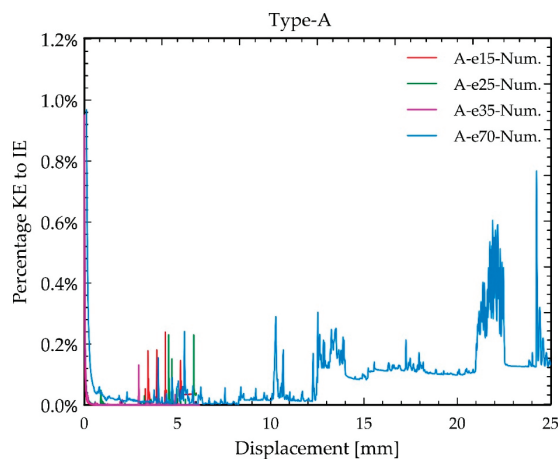


Figure 10. Percentage of kinetic energy (KE) to internal energy (IE) for the “Type-A” models.

4.3. “Type-B” Simulations

The resulting load–displacement curve of “Type-B” configurations (Figure 11a,c) demonstrated quite good agreement with the corresponding experimental data. However, the peak load was somewhat overestimated in the case of the joint with $d = 16$ mm, while in the case of the joint with $d = 12$ mm, the peak load was accurately estimated but the behaviour after pin-bearing was underestimated. The final shear-out failure observed after the large displacement was consistent with the experimental results of the joints. The final failure patterns for both configurations are presented in Figure 11b,d, showing a strong correlation with the experimentally observed damage modes. In Figure 11b, the initiation and evolution of shear and compression damage are shown for the case with $d = 16$ mm. The shear damage around the hole started at around 6 kN, at a displacement

of 0.058 mm, and progressed within the elements along the end distance on the shear-out plane. The shear damage within the elastic limit reduced the stiffness of the capacity curve. Pin-bearing failure occurred at around 0.75 mm (29.50 kN), where the elements around the plate hole on the compressed end reached full compression damage. Then, the shear damage also progressed along the end distance and covered more than half of the end distance. Complete shear-out failure occurred at around 3.20 mm, where all the elements along the end distance on the shear plane became fully damaged under shear. The final shear-out damage and fibre compression damage at a displacement of 5.95 mm are also shown in Figure 11b. Similarly, the initiation and progression of shear and compression damage are also shown for the case with $d = 12$ mm (Figure 11d). In this case, the shear damage started at a displacement of 0.04 mm and load of 3.2 kN, affecting the stiffness of the capacity curve. Pin-bearing failure occurred at a relative displacement of 0.68 mm (18.51 kN); shear-out failure occurred at a displacement of 2.95 mm, where the resistance dropped from 19 kN to 8 kN. The final shear-out damage and fibre compression damage that occurred at a relative displacement of 4.79 mm are also shown in Figure 11d.

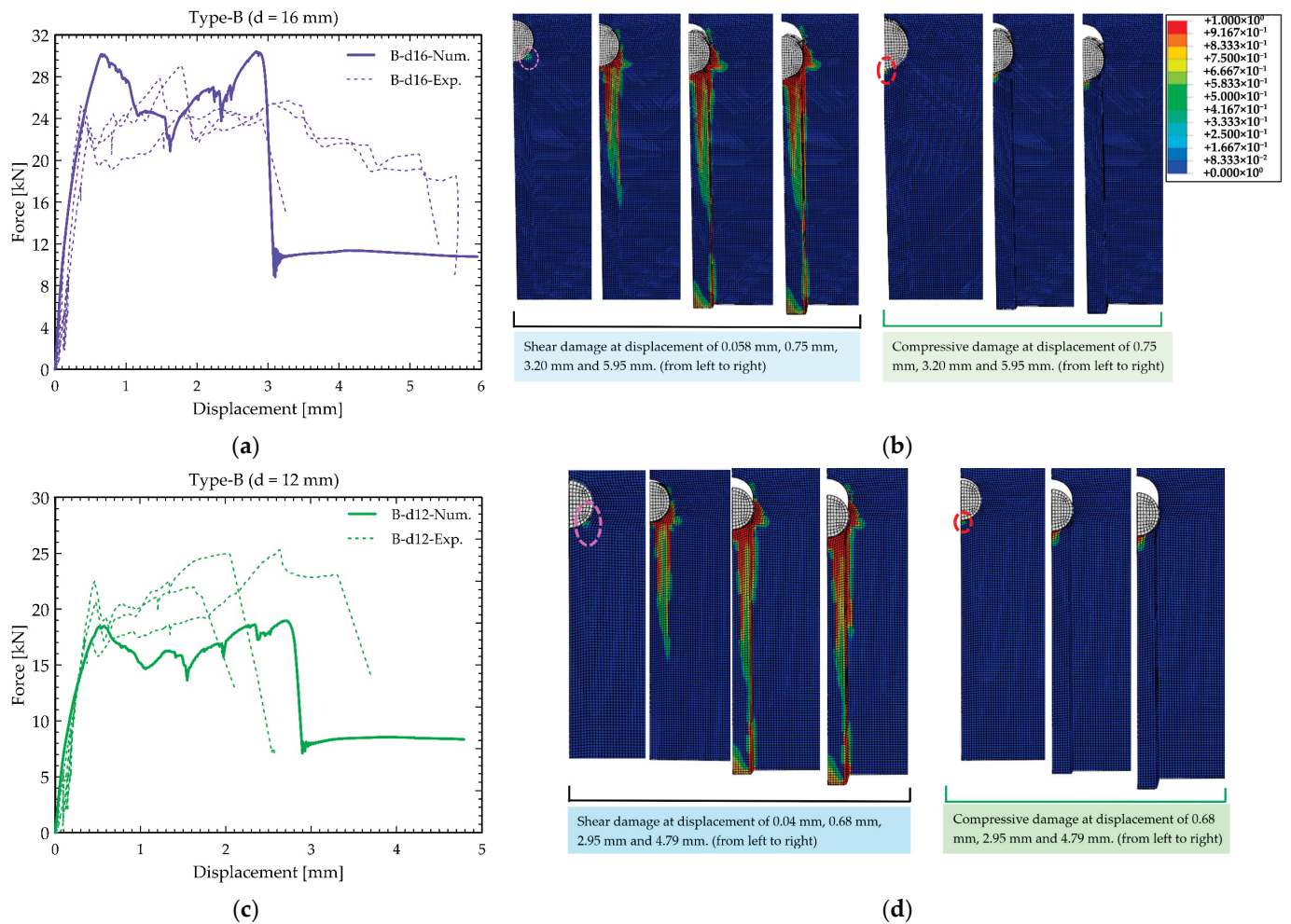


Figure 11. Numerical vs. experimental results for “Type-B” tests: (a,c) load vs. displacement graphs, and (b,d) damage at the end of the test, for (a,b) M16 and (c,d) M12 pins.

No convergence issues were encountered during the simulation, enabling analysis under large-displacement conditions. The percentage of kinetic energy (KE) to internal energy (IE) was also less than 1%, which further confirms the quasi-static behaviour of the model (Figure 12).

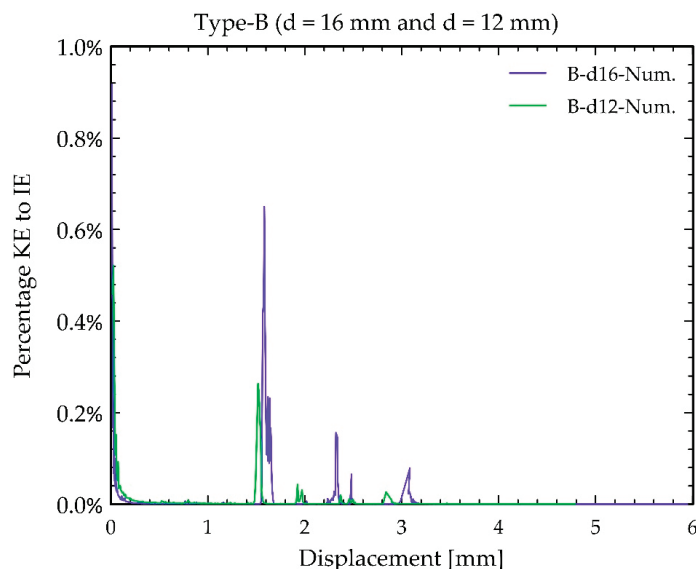


Figure 12. Percentage of kinetic energy (KE) to internal energy (IE) for the “Type-B” models.

4.4. “Type-C” Simulations

The Type-C configuration was modelled and simulated, generally applying the features outlined in Section 2, but the full asymmetric double connection is required to be explicitly modelled. In the model, the inner steel plate was fixed, while gradually increasing displacement was applied to the inner GFRP plate. The two outer GFRP plates, along with the portion of the inner GFRP plate away from the bolt, were assigned purely elastic properties, which are detailed in Table 1, since these regions remained elastic throughout the test. Only the area of the inner GFRP plate surrounding the bolt, where plasticization and failure occurred experimentally, was modelled with nonlinear properties from Tables 1 and 2. To account for the effects of slip at the clamping heads, linear springs were introduced at the far ends of the inner plates. Moreover, to account for the hole clearance, a gap of 1 mm was introduced in the bolt–plate contacts. To simplify the representation of the friction between the plates, due to the small bolt torque, rigid-plastic springs acting in the loading direction were also added to connect adjacent plates.

Following this setup, the model was simulated to obtain the load–displacement response, which was then compared with that in the experimental results, as shown in Figure 13a. The initial trend was captured accurately by the model, and the peak load also showed good agreement with the experimental results. However, the resulting post-peak behaviour was governed by sudden shear-out failure. The damage pattern indicated that bearing failure (compression damage) initiated simultaneously with the onset of shear-out failure, as shown in Figure 13b. Furthermore, shear-out was predominant in the numerical simulation, while in the experiment, it declined linearly. In this case, shear damage started at a relative higher displacement of 2.60 mm (9.1 kN) in contrast to the case for the Type-A and Type-B materials due to the effect of the bolt torque (Figure 13b). Shear-out failure occurred at a displacement of 6.10 mm, where the resistance dropped from 38.42 kN to 16.70 kN. Fibre compression in the elements around the hole on the compressed side of plate started at a displacement of 3.86 mm and at a resistance of 20.70 kN. The elements around the hole were almost completely damaged under fibre compression at a displacement of 5.94 mm (38.33 kN). However, at the same time, at a displacement of 6.10 mm, sudden shear-out failure also occurred. The final shear-out damage and fibre compression damage at a displacement of 7.33 mm are shown in Figure 13b, which shows that some portion of the main plate was separated from it along the end distance.

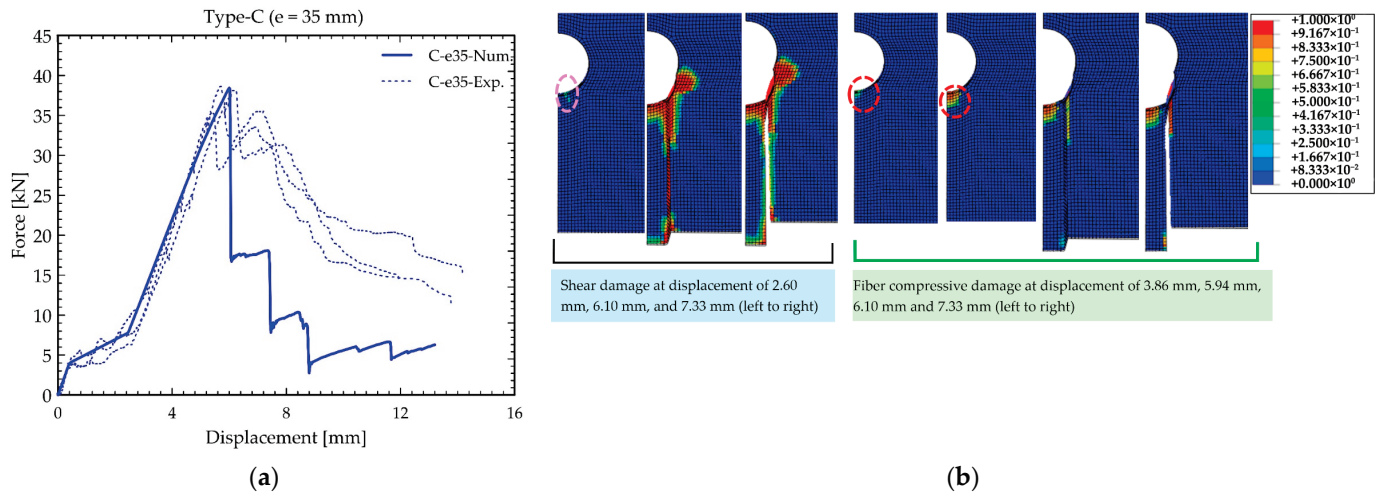


Figure 13. Numerical vs. experimental results for “Type-C” tests: (a) load vs. displacement graphs, and (b) damage at 6.40 mm.

No convergence issues were encountered during the simulation, enabling analysis under large-displacement conditions. The percentage of kinetic energy (IE) to internal energy (IE) was also less than 1%, which further confirms the quasi-static behaviour of the model (Figure 14).

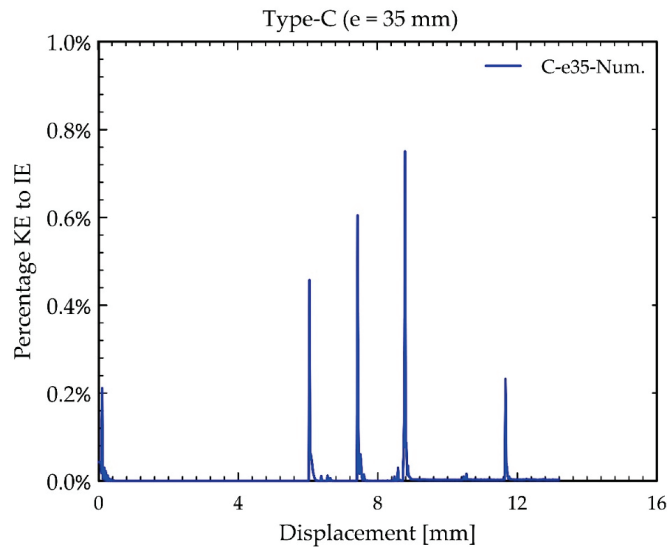


Figure 14. Percentage of kinetic energy (KE) to internal energy (IE) for the “Type-C” model.

5. Sensitivity Analysis for Computational Efficiency

Prior to setting the definitive model features described above, a sensitivity analysis was performed to reduce computational costs while maintaining solution accuracy. Type-A tests were considered as an example. Key parameters such as mesh size and mass scaling (MS) were investigated.

5.1. Implicit Solver

An implicit solver with a general static procedure was used to perform the same simulations with the same parameters as those used for the explicit solver, as this required less computational time. The resulting load–displacement curves were compared with those of the explicit solver, as shown in Figure 15.

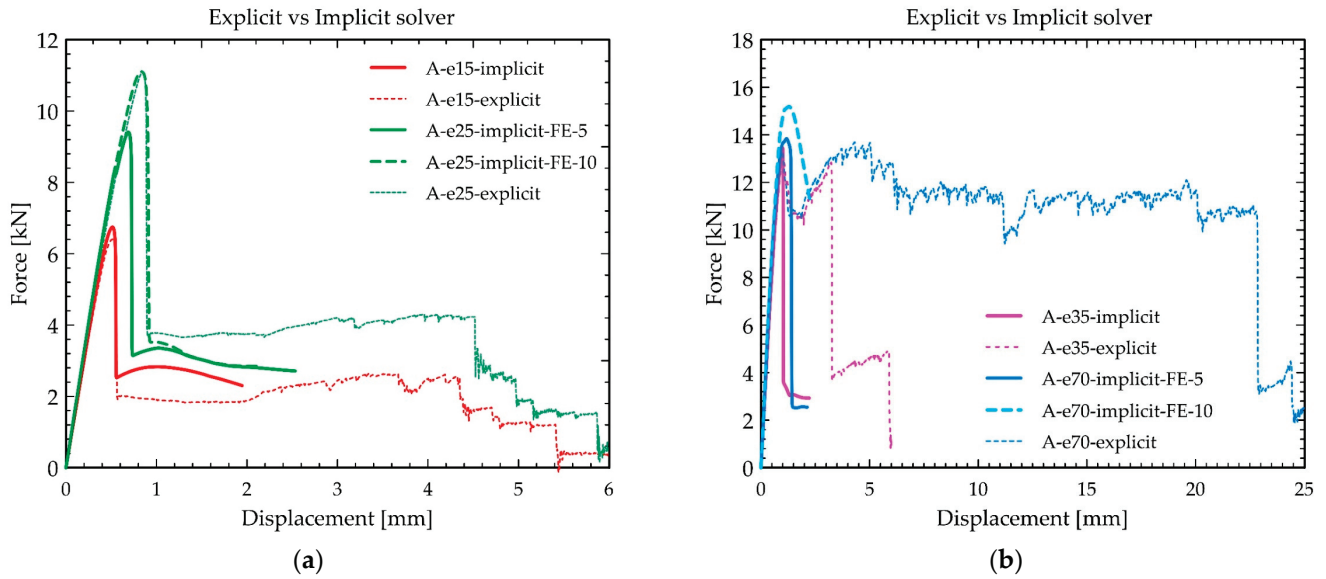


Figure 15. Implicit vs. explicit solver numerical results for “Type-A” tests for end distances of (a) 15 and 25 mm, and (b) 35 and 70 mm.

The results evidence that the implicit solver properly captured the initial loading stage, along with the shear-out failure mode in the configurations with smaller end distances ($e = 15, 25, 35$ mm). However, the peak load in the configuration with $e = 25$ mm was underestimated, and required an increase in the original transverse tensile fracture energy (from 5 to 10 MPa·mm). Moreover, larger differences in the post-peak residual load estimations emerged, and the simulations struggled with convergence issues in the nonlinear range after a certain displacement stage (2–2.5 mm). The differences that arose are attributed to the different solver algorithms used.

In the case of the $e = 70$ mm configuration, the original parameters incorrectly led to shear-out failure; an increase in the original transverse tensile fracture energy (i.e., 10 MPa·mm) would have been required to achieve pin-bearing-dominated failure. However, the main drawbacks occurred in the case analysis performed later, when the implicit solver encountered significant convergence difficulties after damage initiation and peak load attainment, leading to the premature termination of the analysis (at about 2–2.5 mm). The implicit solver in ABAQUS is efficient but, due to convergence issues, was not suitable to simulate the pin-bearing failure of GFRP-bolted connections with the considered material model. Therefore, even though requiring more computational time, ABAQUS/Explicit turned out to be more versatile for simulating the different failure modes of a composite connection.

5.2. Mass Scaling Strategy

Mass scaling (MS) was applied to decrease the analysis duration by introducing artificial mass, which increased the stable time increment and thus reduces the total number of increments required. This technique was implemented with caution, as any mishandling may have affected the reliability and accuracy of the results. According to the ABAQUS documentation, the maximum stable time increment is given by the following:

$$\Delta t_{STABLE} \leq \frac{L_C}{c_d} \quad (1)$$

where Δt_{STABLE} is the stable time increment, L_C is the smallest size in the mesh and c_d is the dilatational wave speed of the material, which can be calculated as follows:

$$c_d = \sqrt{\frac{E}{\rho}} \quad (2)$$

Here, E is the young modulus of the material and ρ is the density of the material. In the case of GFRP, which is an orthotropic material, the stable time increment was determined for both directions, and the smallest stable time increment was selected to remain on the safer and more conservative side. The stable time increments for the longitudinal and transverse direction were 2×10^{-7} and 5.5×10^{-7} , respectively. It is noted that the above expressions are most accurate for materials with a low or zero Poisson's ratio; hence, larger increments were also tested as part of the sensitivity analysis.

For the joint with a 25 mm end distance, the stable time increment without mass scaling was approximately 4.2×10^{-8} , leading to a single simulation runtime of about 24.82 h on an eight-core CPU machine. To address this, mass scaling was applied in separate simulations with target increments of 1×10^{-7} , 2×10^{-7} , 3×10^{-7} , 5×10^{-7} , 1×10^{-6} , and 5×10^{-6} , while keeping all other parameters constant. The total computation time for each case is shown in Table 3. The corresponding structural responses (capacity curves) were plotted to evaluate the impact of mass scaling in Figure 16a. The results show that up to target increment of 1×10^{-6} , they are in high agreement with the non-scaling results. The results for the target increment of 5×10^{-6} showed higher deviations at displacements of 1.2 mm and 4.4 mm, while for the rest of the increments, they matched very well with the results with no mass scaling. The percentages of kinetic energy to internal energy vs. the displacement were also plotted for all mass scaling scenarios (Figure 16b). The percentage ratio was very small up to 0.4% for all target displacements up to 1×10^{-6} ; however, it was very large for a target increment of 5×10^{-6} . Furthermore, the results show that, in the latter case, there was a higher percentage ratio of KE to IE at 1.2 mm and 4.4 mm; the results deviate to a large extent from the results for no mass scaling. The mass scaling sensitivity analysis revealed that the optimum duration for the analysis can be 2.58 h for an end distance of 25 mm, with reliable accuracy.

Table 3. Analysis duration with mass scaling for Type-A model with end distance of 25 mm and 70 mm.

Mass Scaling (MS) (Target Increment)	Computational Time (Hours) for Model ($e = 25$ mm)	Computational Time (Hours) for Model ($e = 70$ mm)
No-MS	24.82	41.65
1×10^{-7}	11.66	17.78
2×10^{-7}	5.65	8.88
3×10^{-7}	3.88	6.11
5×10^{-7}	2.58	3.61
1×10^{-6}	1.11	1.94
5×10^{-6}	0.30	0.81

Similarly, mass scaling sensitivity analysis was carried out for a joint with an end distance of 70 mm (Figure 16c) and target increment times of 1×10^{-7} , 2×10^{-7} , 3×10^{-7} , 5×10^{-7} , 1×10^{-6} and 5×10^{-6} . The analysis without mass scaling required a huge computational time of 41.65 h on a machine with eight-core CPUs. The duration time significantly reduced to 3.61 h, with a target increment of 5×10^{-7} . The dynamic effects started from a target increment of 1×10^{-6} , where the percentage of kinetic energy to internal energy became higher than 1%, as shown in Figure 16d. Also in this case, the

target increment of 5×10^{-7} provided the optimum duration of 3.61 h for the analysis, with reliable accuracy.

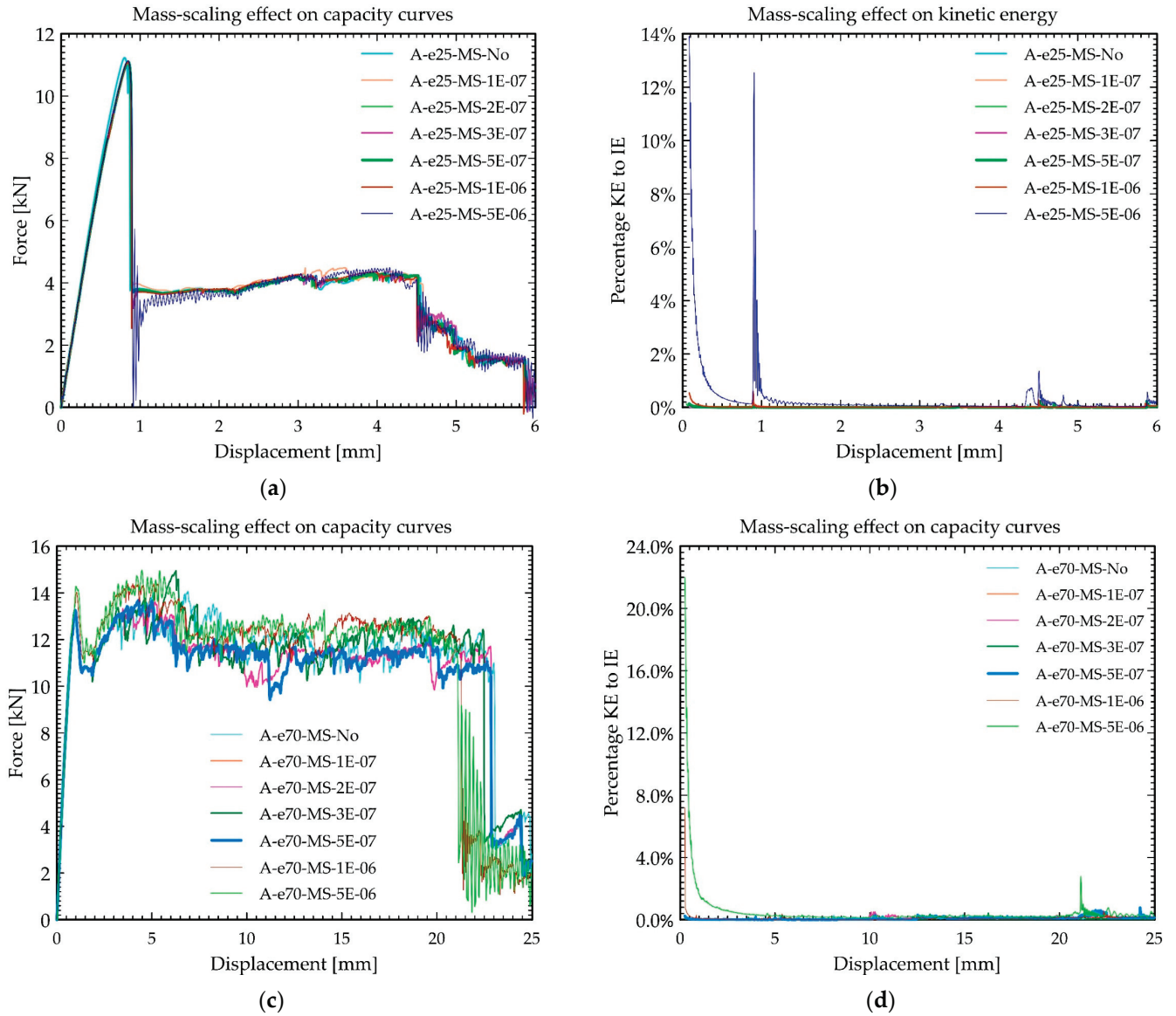


Figure 16. Effect of mass scaling increments: (a,c) load vs. displacement graph and (b,d) percentage of kinetic energy (KE) to internal energy (IE) for “Type-A” models with (a,b) 25 mm and (c,d) 70 mm end distances.

5.3. Mesh Sensitivity Analysis

A mesh sensitivity analysis was performed to evaluate the influence of mesh refinement on the accuracy, reliability, and computational cost of the nonlinear analysis of the GFRP inelastic section only. The two end distances of 25 mm and 70 mm were investigated.

A series of mesh sizes were examined, ranging from very fine to very coarse configurations, while keeping all other parameters constant. The reference mesh was a fine mesh measuring $0.7 \times 0.7 \times 1 \text{ mm}^3$, containing 6040 elements. The finest resolution of $0.5 \times 0.5 \times 1 \text{ mm}^3$, resulting in 11,888 elements, was considered. Subsequently, lower-resolution meshes were tested, including $0.8 \times 0.8 \times 1 \text{ mm}^3$ (4532 elements), $1.0 \times 1.0 \times 1.0 \text{ mm}^3$ (2856 elements) and $1.5 \times 1.5 \times 1.0 \text{ mm}^3$ (1236 elements). The respective mass scaling target increment factors, which were different for each mesh size, were calculated from Equation (1).

The capacity curves for the $e = 25$ mm configuration (Figure 17a) show that for all meshes, the elastic region was the same, followed by very similar peak loads. However, different residual loads were observed in all cases after the first drop. The residual load was the highest with the finest mesh containing 11,888 elements, while it was the smallest for the coarse and very coarse meshes, with 4532, 2656 and 1236 elements. The fine mesh (6080 elements) had an intermediate residual load value. Furthermore, the second reduction was observed only for the finest mesh with 11,888, 6080, and 4532 elements, leading to a second reduction with finest mesh size early on. On the other hand, in the case of the coarse and very coarse mesh sizes, the capacity curves showed an increasing trend instead of a second reduction. This led to the conclusion that the desired behaviour starts with a coarse mesh size with 4532 elements, while a 6080-element mesh size, which shows an intermediate residual load and a definite second reduction, is an optimize mesh size to consider with regards to computation time and accuracy.

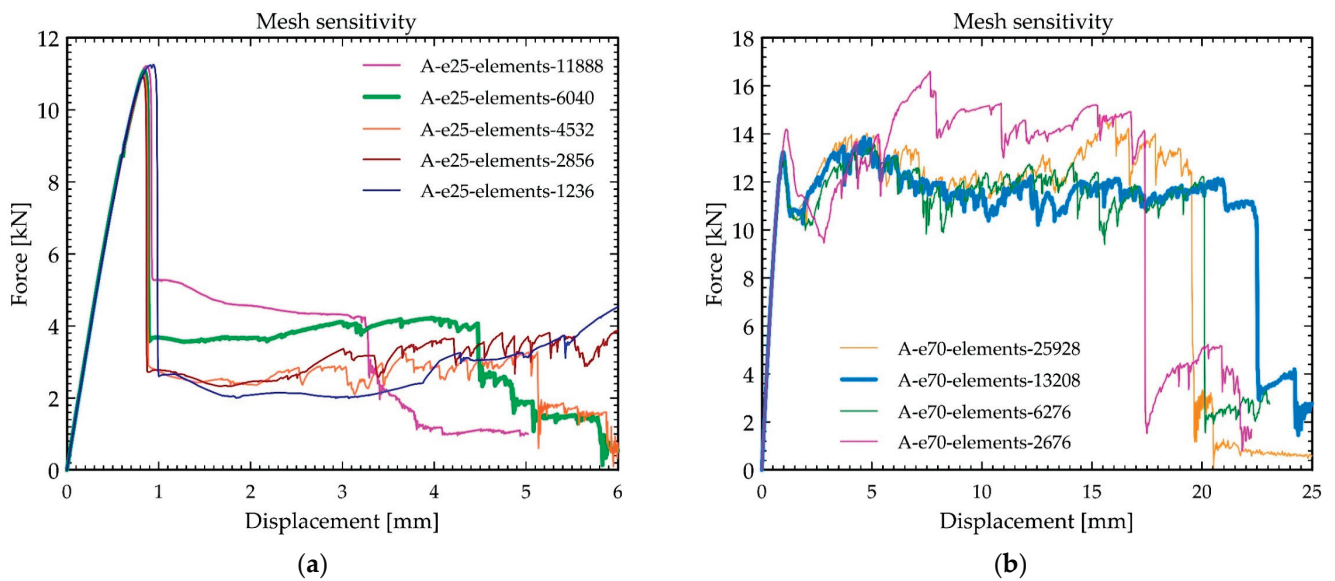


Figure 17. Load vs. displacement graph for model with (a) 25 mm and (b) 70 mm end distances, with varying mesh sizes in inelastic part of GFRP plate.

The capacity curves for the $e = 70$ mm configuration are presented in Figure 17b. The very coarse mesh exhibited a higher peak load, with results that were significantly skewed, showing underestimations after the peak load and overestimations beyond a displacement of 5 mm. Moreover, the very coarse mesh also accelerated the final shear-out failure. The coarser mesh produced a peak load comparable to that of the fine and very fine meshes, though with slightly more scattered results. The fine and very fine meshes displayed nearly identical behaviour, maintaining consistency up to a displacement of 11 mm. The final shear-out failure occurred at slightly different displacements, with the finest mesh size accelerating this shear-out failure compared to the fine mesh. Overall, when comparing the behaviours, the mesh with 13,208 elements provided stable and consistent results closely matching those of the very fine mesh. Consequently, this mesh size was selected for the simulations, to ensure a balance between computational efficiency and accuracy.

6. Mechanical Parametric Study

6.1. Fracture Energy Parameter Sensitivity

To understand the behaviour of the Hashin damage criterion in individual failure modes, a sensitivity analysis was first carried out using a $10 \times 10 \times 10$ mm³ cube model. This preliminary study aimed to evaluate how different failure modes are triggered inde-

pendently, considering that in a bolted joint, all failure modes may be activated even under uniaxial tensile or compressive loading. The cube was modelled using continuum shell elements and subjected first to pure longitudinal tensile loading, and then longitudinal compressive loading followed by transverse tensile and compressive loading. A mesh size of $0.7 \times 0.7 \times 1 \text{ mm}^3$ was used to capture the damage's initiation and progression accurately. The aim of the simulation was to observe the crack/crush initiation point and the post-peak behaviour based on varying values of longitudinal tensile and compression fracture energy (FE), ranging from 50 MPa·mm to 600 MPa·mm. For the transverse direction, the range selected was from 1 MPa·mm (indicating brittle behaviour) to 100 MPa·mm (suggesting ductile response), as the strength and elastic modulus in this direction are significantly smaller than those in the main longitudinal direction.

The obtained stress–strain curves (Figure 18a,b) revealed a typical pattern: an initial linear elastic region, followed by damage initiation at peak load, and then a gradual or steep linear strength reduction depending on the fracture energy. Lower fracture energy led to a sharper post-peak drop, indicating brittle failure, whereas higher fracture energy resulted in a gentler slope, representing more ductile behaviour.

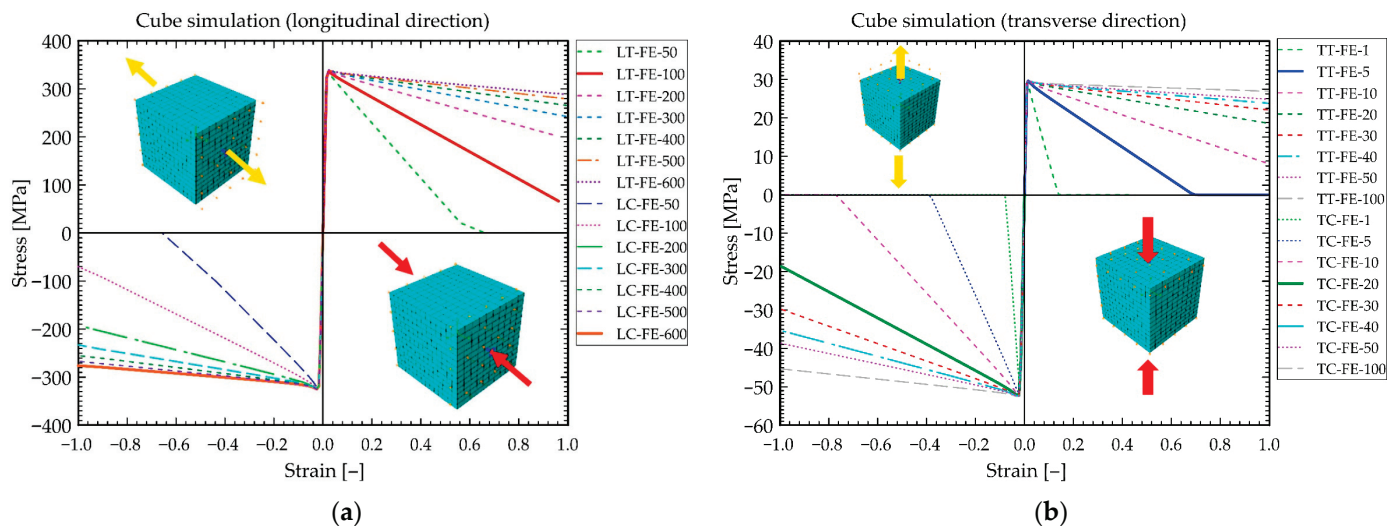


Figure 18. Load–displacement graph for cube model under (a) pure longitudinal tensile and compressive loading; (b) pure transverse tensile and compressive loading.

Experimental evidence and the literature indicate that pultruded GFRP profiles exhibit brittle behaviour under longitudinal tension, with many researchers using a fracture energy value around 100 MPa·mm, which aligns with a steep post-peak decline [43]. It was also observed that under pure longitudinal tensile loading, fracture energies in other directions had negligible influence; the damage's evolution was governed solely by the longitudinal tensile fracture energy. On the other hand, the compressive behaviour of pultruded GFRP is typically less brittle than that under in tension, and thus a relatively higher fracture energy value of 600 MPa·mm was selected for the bolted joint simulations.

The sensitivity analysis for fracture energies was also carried out using the Type-A model with $e = 25 \text{ mm}$ and 70 mm . For the model with $e = 25 \text{ mm}$, the first sensitivity study was performed on the transverse tensile fracture energy, varying it from the original 5 MPa·mm to 1 MPa·mm, 10 MPa·mm, and 20 MPa·mm. The load–displacement graphs (Figure 19a) revealed that, as the transverse tensile fracture energy increased, both the peak load and the residual strength increased. A notable difference was observed between the peaks corresponding to 5 MPa·mm and 1 MPa·mm; with 1 MPa·mm, the second reduction occurred at a much earlier stage (1.21 mm). When increasing the fracture energy beyond 5 MPa·mm to 10 MPa·mm and 20 MPa·mm, the increase in peak load was smaller

compared to the difference observed at 1 MPa·mm. In all cases except 1 MPa·mm, the second reduction occurred at approximately the same displacement of 4.52 mm, as all elements along the end distance on the shear plane became fully damaged. From this analysis, it is concluded that the transverse tensile fracture energy significantly affects the peak load, the residual strength, and the behaviour after the residual load.

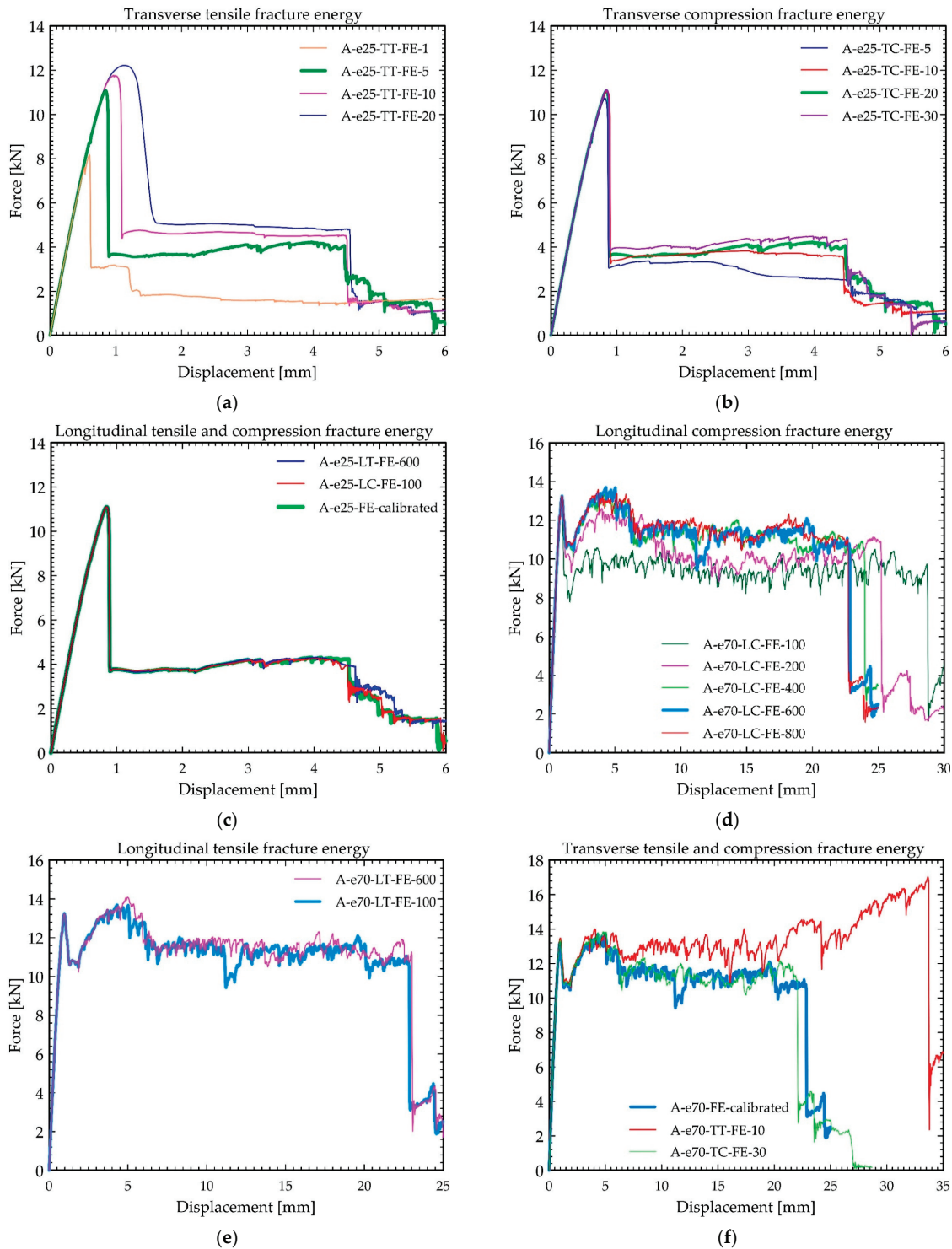


Figure 19. Load–displacement curves from sensitivity analysis of fracture energies (F.E): (a–c) for a 25 mm end distance: (a) transverse tensile F.E, (b) transverse compression F.E, and (c) longitudinal tensile and compression fracture energies; (d–f) for a 70 mm end distance: (d) longitudinal compression F.E, (e) longitudinal tensile F.E, and (f) transverse tensile and compression fracture energies.

Similarly, sensitivity analysis was performed for the transverse compression fracture energy, and the resulting capacity curves were plotted (Figure 19b). The results showed no effect on the peak load; however, the residual load after the peak was slightly influenced. The post-peak behaviour was more noticeably influenced when the fracture energy was raised from 5 MPa·mm to 10 MPa·mm, while it was less prominent when it was further increased from 20 MPa·mm to 30 MPa·mm. The second reductions after the residual behaviour consistently occurred at a displacement of approximately 4.52 mm.

Furthermore, the longitudinal tensile fracture energy was increased from 100 MPa·mm to 600 MPa·mm, and the longitudinal compression fracture energy was decreased from 600 MPa·mm to 100 MPa·mm. As expected, the resulting capacity curves (Figure 19c), compared with the standard fracture energy curve, indicated no significant effect on behaviour. Only a slight increase was observed after the second drop in residual strength in the case of a 600 MPa·mm tensile fracture energy. Overall, it can be concluded that longitudinal tensile and compression fracture energies have minimal impact in joints dominated by shear-out failure.

For the model with a 70 mm end distance, for which the dominant failure mode was bearing failure, the first sensitivity study was conducted by varying the longitudinal compression fracture energy from 100 MPa·mm to 800 MPa·mm. The curves (Figure 19d) show that the first peak load remained unaffected by changes in longitudinal compression fracture energy. However, the first drop after the peak was more pronounced for 100 MPa·mm and 200 MPa·mm than for 400 MPa·mm, 600 MPa·mm, and 800 MPa·mm; in the latter cases, it became stabilized. Additionally, the second peak after the first drop was more prominent and comparable for the 400/600/800 MPa·mm cases, slightly lower for the 200 MPa·mm condition and notably reduced for the 100 MPa·mm condition. The displacement related to the final shear-out failure, which occurred after large deformations, tended to increase as the fracture energy decreased (22.9 mm for 800 MPa·mm, 23.0 mm for 600 MPa·mm, 24.0 mm for 400 MPa·mm, 25.5 mm for 200 MPa·mm, and 29 mm for 100 MPa·mm).

Similarly, the longitudinal tensile fracture energy was increased from 100 MPa·mm to 600 MPa·mm, and the resulting capacity curves were plotted (Figure 19e). This increase had almost no effect on the overall behaviour, although it helped stabilize the small, local load reduction at around 11 mm. This stabilization is attributed to the limited longitudinal tensile damage occurring in a few elements around the hole, which was better controlled by higher tensile fracture energy. Nonetheless, it did not impact the bearing failure mode of the joint. However, it is reasonable to assume that the composite in tension had relatively brittle behaviour, as has already been discussed.

Additionally, the transverse tensile fracture energies, which increased from 5 MPa·mm to 10 MPa·mm, and transverse compression fracture energies, which increased from 20 MPa·mm to 30 MPa·mm, were investigated (Figure 19f). The increase in the transverse tensile fracture energy did not influence the peak load or the subsequent load reduction, which was associated with bearing failure, but led to an increased post-peak load response after reaching about 6 mm, as well as an evident delay in the onset of the final shear-out failure (at 33.6 mm). However, increasing the transverse compressive fracture energy exhibited a negligible impact on both the peak load and the overall load–displacement behaviour just slightly before the final shear-out failure.

In conclusion, the analysis clearly indicates that for the joint with shear-out failure as the dominant mode, the transverse tensile fracture energy played the most significant role. It directly influenced the peak load, the residual strength, and the overall post-residual behaviour of the connection. In contrast, variations in the other fracture energies—namely longitudinal tensile, longitudinal compression, and transverse compression—exhibited

negligible effects on the global response under this configuration. Differently, for the joint where bearing failure governed the response, the behaviour was primarily affected by the longitudinal compression fracture energy. Changes in this parameter notably alter the post-peak behaviour and the progression towards the final shear-out failure. In addition, the transverse tensile fracture energy showed an appreciable influence at large displacements, also affecting the final stage of shear-out failure. On the other hand, the longitudinal tensile and transverse compression fracture energies had a minimal impact on the overall structural behaviour.

6.2. Strength Parameter Sensitivity

The variation in the main strength parameters was investigated for both shear-out and bearing failure-dominated connections, using the Type-A model with $e = 25$ mm and 70 mm, respectively.

At first, the shear strength was varied from the standard 35 MPa to 25 MPa and 45 MPa. Predictably, the results showed that shear strength has a significant impact on the peak load for the model with $e = 25$ mm (Figure 20a). Specifically, the peak load increased with increasing shear strength. The post-peak residual resistance, however, was similar in all cases, as it was primarily influenced by the transverse tensile fracture energy (as previously demonstrated in Section 6.1), which remained the same across the three models with different shear strengths. Notably, for the model with a shear strength of 25 MPa, the second drop occurred earlier (3 mm) than for the 35 MPa and 45 MPa models (4.5 mm). Similarly, for the model with an end distance of $e = 70$ mm, the shear strength had a significant influence on the peak load (Figure 20b). An increase in shear strength not only raises the peak load but also delays the onset of final shear-out failure, resulting in an overall increasing trend for this behaviour in the load–displacement response.

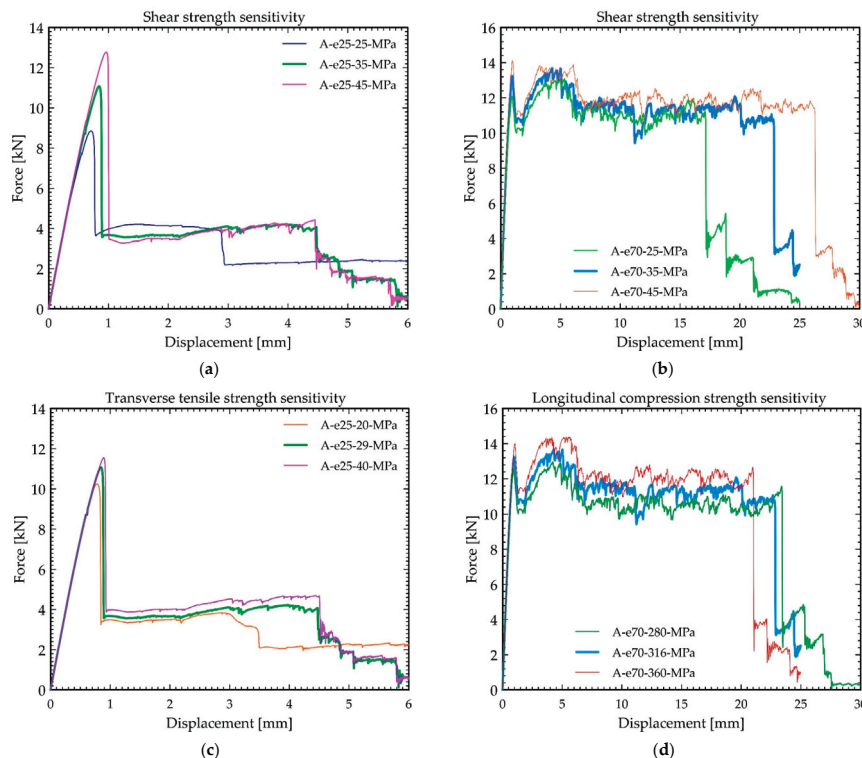


Figure 20. Load–displacement curves from parametric strength values: (a) for shear strength in model with 25 mm end distance, (b) shear strength in model with 70 mm end distance; (c) transverse tensile strength in model with 25 mm end distance; (d) longitudinal compressive strength in model with 70 mm end distance.

The effect of transverse tensile strength was also investigated by varying it from the standard 29 MPa to 20 MPa and 40 MPa for model with $e = 25$ mm. The results (Figure 20c) indicate that the peak load was slightly affected, with a similarly minor impact on the post-peak behaviour, until about 3 mm. However, for the model with the lower transverse tensile strength, the second drop occurred earlier, at a displacement of around 3 mm.

To investigate the influence of the longitudinal compression strength in the model with $e = 70$ mm, the strength value varied from the standard 316 MPa to 280 MPa and 340 MPa. The results (Figure 20d) show that with higher compression strength, the peak load slightly increases, the post-peak behaviour exhibits an upward trend, and the final shear-out failure occurs earlier, consistent with the observations made for variations in longitudinal compression fracture energy.

7. Conclusions

In this study, comprehensive and detailed numerical simulations were carried out to investigate the behaviour of GFRP double-lap bolted joints using ABAQUS/Explicit. Three different GFRP materials with varying geometric configurations for double-lap joints were modelled. The inbuilt Hashin failure criteria were employed for damage initiation and progression, with damage evolution governed by calibrated fracture energies in both the longitudinal and transverse directions. The following key conclusions were drawn:

- Capturing Different Failure Modes in A Simulation:** The explicit solver simulations effectively captured distinct failure modes, including shear-out and pin-bearing failures. The numerical results showed good agreement with experimental observations across most configurations. However, for joints representing a transition between bearing and shear-out dominated failures, further calibration of the elastic moduli, strengths and fracture energies is recommended to improve the predictive accuracy. Furthermore, the shear-out failure in the numerical simulations appeared more abrupt compared to the slightly less steep strength reduction observed in experiments, due to the current inherent limitation of purely brittle intralaminar failure formulations in the ABAQUS material model. Further refinement should include linear or exponential declination. However, shear-out failure is undesirable from the structural design point of view, since this implies a sudden failure of the connection. Conversely, the goal is to design connections governed by a pin-bearing mechanism, which—as clearly proven experimentally—exhibits pseudo-ductile behaviour. This aspect is particularly important in multi-bolt connections, as it enables effective load redistribution. Shear-out failure that appears slightly more brittle than that observed experimentally can be considered, in this sense, a conservative assumption. Additionally, it is important to note that the fracture energy values employed in this study were calibrated and adopted from the existing literature, without direct experimental validation specific to the materials under investigation. Similarly, certain strength parameters were assumed based on typical values reported in the literature, rather than being determined through dedicated experimental characterization. These aspects introduce inherent limitations to the predictive accuracy of the models, particularly when extending the findings to different geometries or loading scenarios.
- Explicit/Implicit Solver Performance:** The explicit solver successfully simulated all models with different end distances and material types, reliably capturing damage initiation and progression without any convergence issues. On the other hand, the implicit solver consistently struggled with post-damage initiation, particularly failing to model bearing failures beyond the onset of damage. The implicit solver also struggled in the case of shear-out failure for larger deformations. This highlights

the explicit solver's outstanding capability for simulating progressive damage in GFRP-bolted joints.

- **Mass Scaling and Mesh Sensitivity:** Mass scaling techniques were successfully applied, reducing the computational time from 24–42 h to approximately 2–6 h without compromising accurateness of the results. Mesh sensitivity analysis was also performed to optimize element counts, ensuring a balance between computational efficiency and numerical precision.
- **Fracture Energy Sensitivity:** Systematic sensitivity analysis was conducted to investigate the role of different fracture energies on the peak load, residual load after the peak load and post-peak failure behaviour. Transverse tensile fracture energy had the most pronounced effect on shear-out failures, directly influencing the peak load, the residual load after the first peak, and the final post-residual load drop. Increasing this parameter also delayed the onset of the final shear-out failure. The other longitudinal tensile and compression fracture energies and transverse compression fracture energies have minimal impact on the overall behaviour of the shear-out failure. Longitudinal compression fracture energy primarily affects the pin-bearing response, while transverse tensile fracture energy has no effect on the peak load and overall behaviour. However, it only delays the final shear-out failure when increased. The other fracture energies have no impact on the behaviour of pin-bearing failure.
- **Parametric Study On Strength Parameters:** Parametric analyses were performed considering the tensile, compression and shear strength. The shear-out-dominated failure was most sensitive to in-plane shear strength and transverse tensile strength: an increase in these parameters elevated the peak load and enhanced the post-peak response. In configurations dominated by pin-bearing failure, both shear strength and longitudinal compressive strength significantly influenced the bearing behaviour. Increasing these strengths raised the peak load and improved the overall load–displacement response, and the shear strength delayed the final shear-out failure.
- **Limitation of Numerical Models:** The developed numerical models are well suited for accurately simulating double-lap bolted joints, where out-of-plane stresses and deformations are minimal. That is why continuum shell elements, which neglect out-of-plane stresses and reduce the computational time, were employed; in this specific case, this simplification does not affect the accuracy of the results. However, for single-lap joints, where significant out-of-plane bending and associated stresses occur, the current modelling approach could become inadequate, and the continuum shell elements may no longer be appropriate. Accurately capturing these three-dimensional stress states would require the use of solid 3D elements in combination with user-defined material subroutines (UMAT or VUMAT) in ABAQUS, since the built-in Hashin failure criteria in ABAQUS apply only to shell and continuum shell elements. This underscores the necessity of using advanced modelling techniques to accurately predict the complex failure mechanisms in single-lap GFRP bolted joints. However, employing UMAT or VUMAT subroutines will substantially increase both the computational cost and complexity of numerical simulations.
- **Future Work:** The present model has so far been established and validated mainly for in-plane loading conditions and specific connection configurations. Its applicability to different geometries and loading scenarios therefore remains to be evaluated. Thus, now that the material models have been validated and the influence of the key parameters has been established through detailed parametric studies, this research will be further extended to investigate multiple bolted connections, as well as the strengthening of single- and multi-bolted joints, e.g., by using externally bonded fibre-reinforced sheets. For this purpose, the modelling assumptions and simplifications of

the model are deemed sufficiently accurate to support reliable simulation. Nevertheless, ongoing study is also directed toward extending the model to transversal and inclined loading conditions. Alongside the numerical simulations, experimental tests are currently being carried out to refine the model on the basis on the specific PFRP material, through comparison with the experimental results of single-bolt tests, and to validate the developed model, through comparison with the experimental results of multi-bolt connections.

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Abbreviations

The following abbreviations are used in this manuscript:

PFRP	Pultruded-Fibre-Reinforced Polymer
CSM	Chopped Stranded Mat
GFRP	Glass-Fibre-Reinforced Polymer
FE	Finite Element
VUMAT	Vectorized User Material
UMAT	User Material
SC8R	Continuum Shell Elements with 8 Nodes and Reduced Integration
C3D8R	Continuum 3D Solid Elements with Reduced Integration
FE	Fracture Energy
LT-FE	Longitudinal Tensile Fracture Energy
LC-FE	Longitudinal Compressive Fracture Energy
TT-FE	Transverse Tensile Fracture Energy
TC-FE	Transverse Compressive Fracture Energy
KE	Kinetic Energy
IE	Internal Energy
MS	Mass Scaling

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Article

Insulation Strategies to Enhance Fire Resistance in Composite Slabs with Reduced Carbon Emissions

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Abstract: Composite slabs have gained popularity in modern high-rise construction due to their superior load-bearing capacity and reduced self-weight. The vulnerability of the unprotected steel deck under fire conditions poses serious challenges, as the rapid reduction in steel strength and stiffness can compromise structural resistance and accelerate fire spread. This study presents a comprehensive numerical simulation to assess the fire behaviour of a novel composite slab and a new proposal for a simplified method. Three insulation techniques are investigated: a steel shield for the thinner part, a steel shield with the cavity filled with mineral wool, and a mineral wool plate applied from below. The simplified method is proposed to evaluate the fire resistance using new empirical coefficients, recalibrated within the framework of the prEN 1994-1-2 to allow for precise temperature predictions in steel components under standard fire. The numerical model, validated against experimental results, shows that the steel shield insulation extends the time to reach critical temperatures by approximately 25%. In contrast, mineral wool insulation proved to be substantially more effective by reducing temperatures in the UPPER 2 region by up to 89% compared to uninsulated slabs, after 60 min of fire exposure. This significant temperature reduction increases the load-bearing capacity during 60 min of fire exposure by 29%, also resulting in a potential reduction of approximately 22% in carbon emissions. The findings underscore and highlight the potential of these insulation systems to enhance the overall safety and resilience of composite slabs under fire, offering valuable insights for structural fire design.

Keywords: composite slabs; steel decking; fire resistance; fire insulation; finite element modelling; simplified method

1. Introduction

Composite slabs with steel decks are highly efficient and lightweight structural solutions compared to conventional reinforced concrete slabs. Given these advantages, this composite slab model has become widely adopted in building projects involving steel structures and high-rise buildings [1]. The composite slab consists of a thin cold-rolled steel deck, typically with thicknesses ranging from 0.6 to 1.2 mm, over which concrete is poured [2]. The concrete is usually reinforced with a light anti-crack mesh and may also include reinforcement bars, typically positioned within the ribs.

The presence of an unprotected steel deck in composite slabs represents a significant disadvantage in terms of fire resistance. When directly exposed to fire, the steel deck's structural stability can be compromised due to the temperature-induced reduction in

strength and stiffness [3]. This issue arises because the steel components are primarily responsible for the load-bearing capacity of the composite slabs, which are crucial for maintaining the structural element stability during a fire. Additionally, these slabs play a vital role in preventing fire from spreading to other compartments within the building.

Fires pose a critical global threat, with over 300,000 annual fatalities attributed to fire-induced burns, predominantly in low- and middle-income countries (LMICs) [4]. Residential fires account for 80% of fire-related deaths in the United States, occurring every 85 s [5,6], while 78% of unintentional fire fatalities in London arise from residential incidents [7]. Human behaviour, particularly unsafe practices, is a primary causative factor [8], exacerbated by evacuation challenges in densely populated residential complexes [6,9]. Fire protection plays a crucial role in building safety, aiming to reduce the likelihood of death, injury, property loss, and environmental damage caused by unwanted fires [10]. Protecting the building's structure and contents is equally vital to prevent the spread of fire and structural collapse, ensuring the structure withstands long enough to evacuate occupants safely [11]. Jiang et al. [12] mentioned that the fire resistance classification of structural elements is typically determined through fire tests, where the element is exposed to a standard fire curve for a specified duration, usually following the fire standard ISO 834 [13]. The fire resistance of an element, measured by its load-bearing capacity (R), insulation (I), and integrity (E), is determined by exposing it to a standard furnace temperature–time curve.

Beyond fire safety, one of the most critical factors is the advancement of low-carbon building practices, as conventional construction releases substantial quantities of carbon into the atmosphere. The construction industry accounts for 40% of global energy consumption and 36% of carbon emissions [14]. Among the industrial sectors, construction is the largest carbon emitter due to the energy-intensive nature of its materials [15]. The production of building materials generates the most carbon emissions (64.5%) over a building's lifecycle. Specifically, the production and use of steel and concrete are the primary contributors to emissions associated with this sector. Notably, steel's recyclability is superior to that of concrete, and the quantity of steel used is significantly lower than that of concrete. Consequently, concrete is the most significant contributor to carbon emissions over a building's lifecycle. To mitigate the carbon footprint of construction, replacing concrete with low-energy materials is imperative.

Recent research highlights the complexities in analysing structural fires in composite slabs. The cooling phase can generate connection failures in frames, and slabs can anticipate collapses [16–18]. Failures in beam–column joints can trigger slab disintegration and fire propagation between floors [19,20], requiring advanced predictive models. Marcilio et al. [1] also noticed that failure may occur during the cooling phase and explored the extension of the simplified EN 1994-1-2 model to natural fire scenarios. However, a broader database is needed, including additional fire curves, different steel slab profiles, mechanical simulations, and experimental tests, to evaluate failure modes [1]. These studies demonstrated how delicate the structures of composite slabs are under fire.

To extend the load-bearing capacity time (R), Fourie (2020) [21] proposed a novel type of insulation for composite slabs with steel decking. The insulation consists of a steel plate, referred to as a steel shield, attached to both ends of the lower regions of the rib, creating an area shielded from direct exposure to fire, as illustrated in Figure 1.

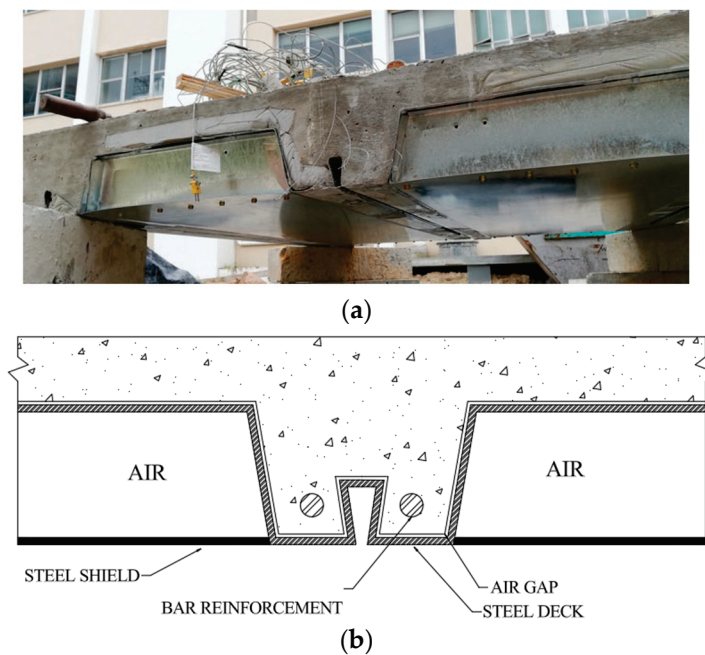


Figure 1. Composite slab model presented by Fourie [21]. (a) Slab before test 2. (b) Schematic representation of the slab proposed by Fourie.

To analyse the thermal behaviour of this new composite slab, Fourie (2020) [21] conducted two experimental tests, where the difference between the tests lies in the presence of the steel shield. Thermocouples were placed on the lower, web, upper, and rebar, with a fire duration of 90 min for test 1 and 80 min for test 2. The results indicate that including steel shields reduces the unexposed surface average temperature by 16% to 41% [21]. This reduction contributes to a 19.35% decrease in concrete usage [21] and a fire resistance increase of 20 min in the insulation criteria.

Based on the studies conducted by Fourie [21], numerical simulations are presented using the ANSYS software (2024 R2) to demonstrate the load-bearing capacity increase provided by active steel shield insulation. In addition to the steel shield, numerical simulations are conducted using mineral wool, allowing a comparison of temperature and mechanical resistance between steel decks insulated with steel shield, mineral wool, and without insulation. The numerical validation and verification were carried out using the experimental and numerical results, respectively, obtained by Fourie [21]. Mineral wool insulation products are widely used in industrial applications due to their excellent thermal and acoustic insulation properties, non-combustibility, cost-effectiveness, and ease of installation [22]. Its favourable performance per unit volume further supports the selection of this insulation type for various engineering applications.

According to Piloto et al. [23], to improve the accuracy of the numerical model used to simulate the debonding effect of the steel deck from the concrete during a fire, an insulating layer (air gap) with a constant thickness should be introduced between the concrete and the steel deck in the numerical models. In the numerical models presented in this study, an air gap of 0.5 mm is assumed to simulate the debonding effects, aiming to provide a more accurate scenario [1].

This new proposed model is not yet included in the next generation of prEN1994-1-2 [24]. There is a lack of design methods for this new steel deck (bottom re-entrant stiffener in the rib) and a lack of design methods when using insulation materials. The present investigation aims to develop a simplified method that ensures new empirical coefficients, “ b_i ” for approximating the average temperature in the steel deck components, and new empirical coefficients, “ c_i ” for approximating the average temperatures in the

rebars. According to Marcílio et al. [1], the equations proposed by EN 1994-1-2 can be used to obtain the average temperatures in steel components as long as the “ b_i ” and “ c_i ” coefficients are appropriately calibrated. Moreover, Jiang et al. [25] emphasise that the simplified formulations proposed by EN1994-1-2 [24] overestimate fire resistance and that the moisture content significantly affects the temperature distribution in composite slabs. This study further examines how the integration of insulation systems can enhance carbon emission reduction in the construction of new buildings.

Thus, this paper presents, as its primary innovation, the development and calibration of new empirical coefficients (“ a ”, “ b ”, and “ c ”) for the simplified method of prEN 1994-1-2, specifically adapted for composite slabs with steel decking using thermal insulation systems. These coefficients, calibrated through finite element analysis and validated by experimental results, enable a more accurate prediction of steel component temperatures and load-bearing capacity under standard fire conditions, addressing a regulatory gap for composite steel decks with special rib profiles and insulation materials. Furthermore, an integrated methodology is proposed that correlates the enhancement in fire resistance with the potential for reducing carbon emissions, offering not only a robust tool for fire safety design but also a contribution to sustainability within composite construction.

2. Numerical Model

This research study is based on the Finite Element Method (FEM) using ANSYS software [26], conducted in four solution steps. The first step involves the development of a validated numerical model using the experimental tests conducted by Fourie [21] and the experimental tests developed by CSTB [27], thereby ensuring the accuracy of the numerical model. The second step involves performing an insulation fire analysis in accordance with prEN1994-1-2 [24], measuring the maximum and average temperatures of the unexposed zones, and determining the fire resistance time, following the ISO 834 standard fire curve [13]. The third step consists of a parametric study, running 21 simulations, including 3 types of slabs and 4 types of insulation, to define the optimal values of “ b_i ” and “ c_i ”, as per prEN1994-1-2 [24], used to determine the temperature in the structural component zones of the composite slabs. The final step involves conducting a simplified load-bearing analysis following the guidelines of prEN1994-1-2 [24], comparing the load-bearing capacity enhancement provided by each type of insulation.

2.1. Geometry and Thermally Affected Regions of the Composite Slab

Figure 2a presents the geometry and temperature-affected regions of the composite steel and concrete slab produced by Voidcon, used by Claasen et al. [28]. This study presents a transient and non-linear thermal analysis of three fire protection types of these composite slabs (steel shield, steel shield with mineral wool above and steel shield with mineral wool plates below).

The composite slab insulation, protected by a steel shield, creates an air cavity in the UPPER 2 and WEB 2 regions, preventing these areas from being directly exposed to fire [21]. The second insulation was also assumed to be made using mineral wool in the cavity region formed by the steel shield. The third proposed insulation involves the addition of a mineral wool board beneath the steel plate, as illustrated in Figure 2b. Within the resulting cavity, radiation and convection phenomena will occur, with the insulation thickness (h_i) being adjustable. For the parametric study, h_i values of 5, 10, 20, and 30 mm were employed.

In the present geometry, as shown in Figure 2a, there are two distinct regions: a general trapezoidal region (T), composed of the lower region, WEB 2, and UPPER 2, and a re-entrant region (R), formed by WEB 1 and UPPER 1. Each region has unique dimensions, which are critical for calculating the average temperature within its respective elements

using the simplified method [24]. This approach relies on accurately determining the view factors, as they directly influence the temperature field.

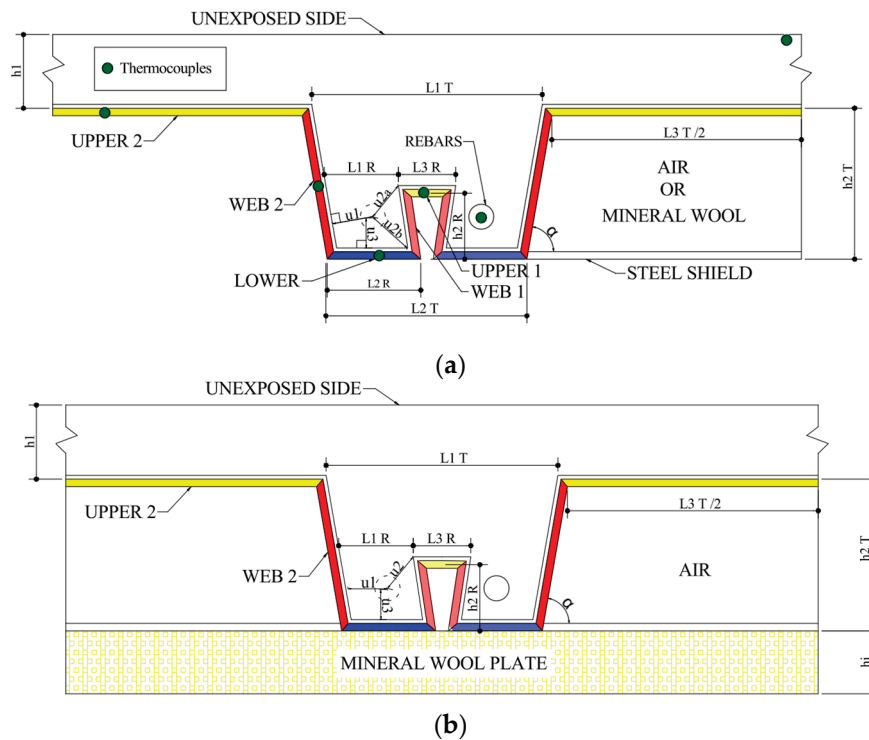


Figure 2. (a) Dimensions and zones of the Voidcon slab. (b) Dimensions of the Voidcon slab with mineral wool insulation plate. Adapted from [21].

Table 1 presents three different types of Voidcon steel decks and the geometry used for this investigation. The concrete cover has been considered constant and equal to 65 mm.

Table 1. Dimensions of the slab produced by Voidcon.

	VP 50	VP115	VP 200
L1 R (mm)	30	50	85
L2 R (mm)	55	75	110
L3 R (mm)	25	25	25
L1 T (mm)	165	200	300
L2 T (mm)	115	150	230
L3 T (mm)	250	400	460
h1 (mm)	65	65	65
h2 T (mm)	50	115	200
h2 R (mm)	50	50	50
u1 (mm)	24	37	47
u2 (mm)	32	32	47
u3 (mm)	25	25	25
α (°)	63.40	77.73	80.07

To calculate the temperature effect in each region, 10 points were collected, and the arithmetic average temperature was determined.

The temperature field is determined by solving the heat conservation equation inside the solid parts (see Equation (1)) through the discretisation of the domain using a finite element mesh as illustrated in Figure 3. The material properties are temperature-dependent, and the solution method is incremental and iterative, using a time increment of 60 s,

reducible to 1 s if required. The convergence criterion was based on heat flow, with a tolerance of 10^{-3} and a minimum reference value of 10^{-6} W.

$$\rho(T)C_p(T)\frac{\partial T}{\partial t} = \nabla \cdot (\lambda(T)\nabla T) \quad (1)$$

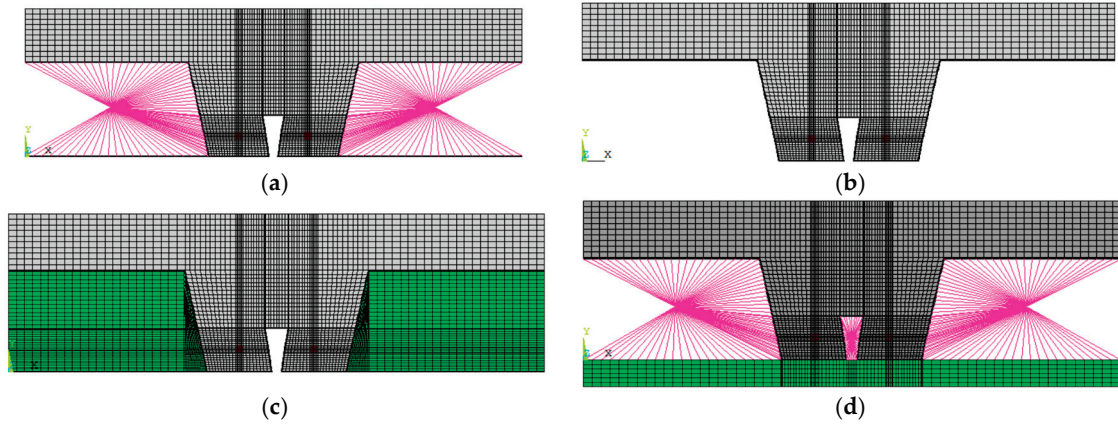


Figure 3. Mesh size for VP 115. (a) Composite slab with steel shield insulation. (b) Composite slab without insulation. (c) Composite slab with mineral wool and steel shield insulation. (d) Composite slab with mineral wool plate and steel shield insulation.

T represents the temperature of each material [K], $\rho(T)$ is the specific mass [kg/m^3], $C_p(T)$ is the specific heat [J/kgK], $\lambda(T)$ is the thermal conductivity [W/mK], t is the time [s], and ∇ is the gradient in 2D space. Equation (1) is based on the heat flow balance for the infinitesimal material volume in each spatial direction.

The boundary conditions are defined in the exposed and unexposed sides of the composite slabs, using Equation (2). The exposed surface is submitted to radiation and convection, while convection is assumed in the unexposed surface with an appropriate convection coefficient to consider the radiation effect. The cavity region is also submitted to convection and radiation, using Equation (2) again, but using average convection coefficients between the exposed and unexposed surfaces. The average view factor $\varnothing$ was determined for each region. The emissivity of the material exposed to radiation ε_m depends on the surface material and the emissivity of the flames ε_f equals 1. The Stefan–Boltzmann coefficient is σ . The bulk temperature is defined by T_∞ .

$$-(\lambda(T)\nabla T) \cdot \vec{n} = \alpha_c(T_\infty - T) + \varnothing\varepsilon_m\varepsilon_f\sigma(T_\infty^4 - T^4) \quad (2)$$

A two-dimensional model is presented, assuming no temperature variation through the span of the composite slab and neglecting the temperature effect near the supports. The temperature field is determined by the weighted residual method, using Galerkin approximation, ending in the solution of a set of algebraic equations. Based on previous 3D, thermal, and mechanical simulations developed by the authors, the 2D finite element analysis is justified by the slab's uniform cross-section, which minimises longitudinal heat flow, concentrating it in the 2D analysis of the cross-section.

2.2. Two-Dimensional Finite Element Mesh

This study used the PLANE55 [26,29] finite element for two-dimensional conduction thermal analysis to evaluate heat conduction in planar regions (cross-section of the composite slab). This finite element has four nodes, one degree of freedom (temperature) per node, and assumes linear interpolating functions and a full Gauss (2×2) integration method.

The element supports both transient and steady-state analyses, including convection and radiation effects.

The finite element LINK31 [26,29] models heat exchange by radiation between surfaces. This two-node finite element incorporates parameters such as radiation area, geometric view form factor, and emissivity, applying the Stefan–Boltzmann Law to calculate radiation heat flow between two nodes. The non-linearity is solved iteratively using the Newton–Raphson method, with emissivity defined as either constant or temperature-dependent. This element has no interpolating function or numerical integration, and uses a direct formula to determine the heat flow [W] by radiation; see Equation (3).

$$q [W] = \sigma \times \varepsilon_R \times \varnothing \times A \times (T_i^4 - T_j^4) \quad (3)$$

The resultant emissivity is defined by $\varepsilon_R = \varepsilon_f \cdot \varepsilon_m$. The area of influence used for each finite element depends on the position of the nodes and should be previously defined [26].

The finite element LINK34 [26,29] simulates uniaxial thermal convection between nodes, featuring a non-linear film coefficient depending on temperature or time. The convective heat transfer rate is determined based on the film coefficient, convection area, and possible empirical coefficients (not used here). This element also has no interpolating function or numerical integration, and uses a direct formula to determine the heat flow by convection; see Equation (4). This equation depends on the convection coefficient used in the cavity region (average value) and on the area of influence defined between the two nodes. This formula is also prepared to use an empirical coefficient E , herein assumed as 1.

$$q [W] = \alpha_c \times A \times E \times (T_i - T_j) \quad (4)$$

Figure 3 presents the finite element mesh adopted for the VP50, VP115, and VP200 configurations, highlighting three fire scenarios. Figure 3a presents a composite slab with steel shield insulation, where LINK31 and LINK34 elements are employed to simulate the heat transfer within the air cavity. Figure 3b presents a composite slab without insulation. Figure 3c presents a composite slab with mineral wool and steel shield insulation, both discretised using PLANE55 elements. Figure 3d presents a composite slab with a mineral wool plate and steel shield insulation, both discretised using PLANE55 elements and LINK31 and LINK34 elements, which are employed to simulate the heat transfer within the air cavity. The mesh distribution and density are optimised to capture the critical temperature gradients in each configuration accurately. The thinner regions are modelled with a minimum of three finite elements, regarding the linear interpolating functions used in PLANE55.

The air gap modelling is produced using thermal conduction using a planar region in ANSYS with PLANE55 to simulate the debonding effect between concrete and steel. The selection of a constant debonding thickness of 0.5 mm follows sensitivity analyses conducted by Balsa et al. [30], which determined this value to provide an optimal balance between model accuracy and computational efficiency for simulating debonding separation behaviour under fire conditions.

2.3. Boundary Conditions

Heat flux by convection and radiation is applied as a primary thermal boundary condition in accordance with EN1991-1-2 [31]. The convection coefficient of 25 W/m²K is assigned to the fire-exposed side, and 9 W/m²K to the unexposed side, accounting for radiative effects. The flame emissivity factor of 1.0 is applied with a Stefan–Boltzmann constant of 5.67×10^{-8} W/m²K⁴, consistent with previous studies [32–34]. The core temperature on the exposed side was defined using the ISO834 standard fire time–temperature

curve, while the non-exposed side was assumed to maintain a constant temperature of 20 °C. Figure 4 illustrates the details of the thermal boundary conditions applied to the steel deck with the steel shield insulation model.

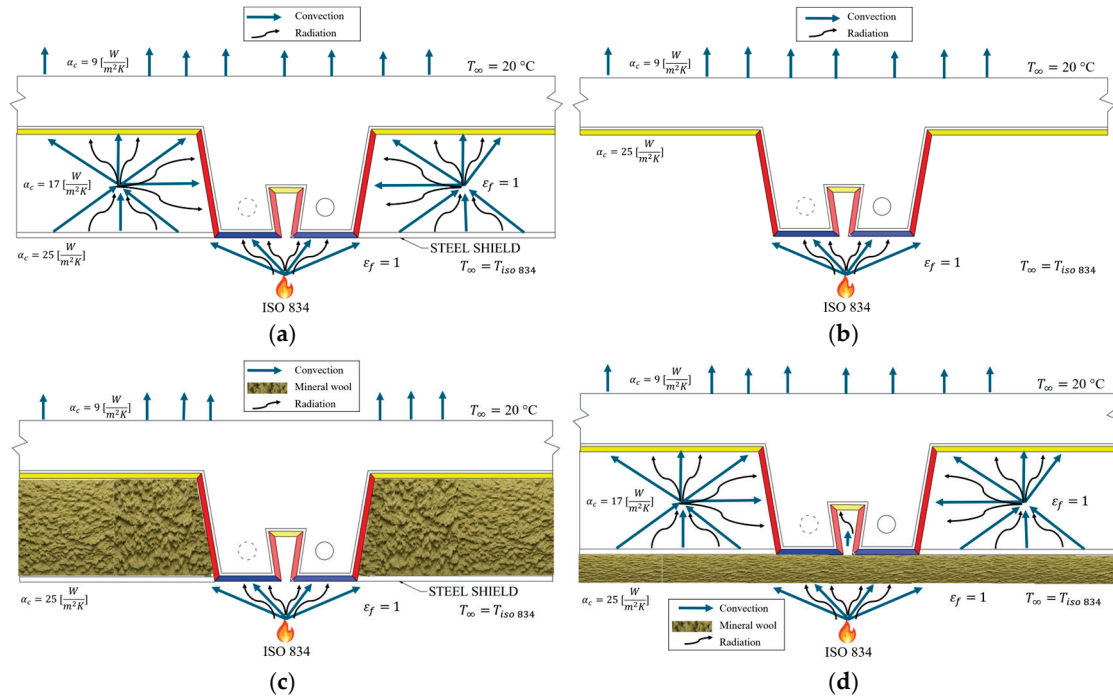


Figure 4. Thermal boundary conditions. (a) Slab with steel plate insulation. (b) Slab without insulation. (c) Slab with mineral wool insulation. (d) Slab with mineral wool plate [13].

The methodological approach adopted in this investigation maintains the bulk temperature on the unexposed side at 20 °C. According to previous experimental temperature measurements [32], this value increases with time, but keeps the temperature very low. Moreover, this assumption provides conservative fire resistance results.

The thermal analysis is conducted as a transient non-linear analysis, configured with full option settings.

3. Simplified Calculation Method

The simplified calculation method used for the (R) criterion presented by prEN1994-1-2 [24] can be applied to simply supported composite slabs when exposed to standard fire ISO-834 [13]. To determine the bending moment capacity of the composite slab (sagging moment), this standard requires the temperature for each component of the steel section (top flange, web, and bottom flange) to be calculated for each required fire rating, according to Equation (5). For the reinforcement components, the temperature θ_s is determined using Equation (6). The temperatures θ_a and θ_s are given in °C. Other required parameters are defined by Equation (7) to Equation (8).

The insulation's fire resistance is determined by Equation (9), and also depends on a series of geometric parameters. The parameter $\varnothing$ is dimensionless and represents the view factor of the steel section component, as given by Equations (10) and (11). The length L_3 is the distance within the ribs and u_3 represents the distance from the centre of the reinforcement to the bottom flange in mm, as shown in Figure 2.

$$\theta_a = b_0 + b_1 \frac{1}{L_3} + b_2 \frac{A}{L_r} + b_3 \varnothing + b_4 \varnothing^2 \quad (5)$$

$$\theta_s = c_0 + c_1 \frac{u_3}{h_2} + c_2 z + c_3 \frac{A}{L_r} + c_4 \alpha + c_5 \frac{1}{L_3} \quad (6)$$

The z factor represents the position of the reinforcement relative to the slab rib using u_{2a} , as shown in Figure 2 and used in Equation (7), given in $\text{mm}^{-0.5}$. α represents the angle between the web component of the steel formwork and the horizontal direction in degrees ($^\circ$), as shown in Figure 2. The ratio A/L_r is the rib geometry factor and represents the relationship between the concrete volume and the area exposed per metre of length of the steel formwork rib, given in mm, and its calculation is performed using Equation (8).

$$\frac{1}{z} = \frac{1}{\sqrt{\frac{1}{u_1}}} + \frac{1}{\sqrt{\frac{1}{u_2}}} + \frac{1}{\sqrt{\frac{1}{u_3}}} \quad (7)$$

$$\frac{A}{L_r} = \frac{h_2 \left(\frac{L_1 + L_2}{2} \right)}{L_2 + 2\sqrt{h_2^2 + \left(\frac{L_1 - L_2}{2} \right)^2}} \quad (8)$$

The coefficients “ a_i ”, “ b_i ”, and “ c_i ” represent the empirical coefficients used by prEN1994-1-2 [24], which depend on the type of concrete (NWC or LWC) and on the standard fire rating. Equations (5) and (6) allow for estimating the average temperatures that affect the strength reduction coefficients in all steel deck components and rebars, consequently enabling an accurate estimation of the load-bearing capacity criteria after a specific time (fire rating). The load-bearing capacity criteria (R) will be based on the reduction coefficients applied to the yield strength of each component.

To determine the insulation fire resistance (I) time in minutes, Equation (9) is used, which employs the empirical coefficients “ a_i ” generally used by prEN1994-1-2 [24].

$$t_i = a_0 + a_1 h_1 + a_2 \varnothing_{UPPER} + a_3 \frac{A}{L_r} + a_4 \frac{1}{L_3} + a_5 \frac{A}{L_r} \frac{1}{L_3} \quad (9)$$

The view factor ($\varnothing$) quantifies the radiation received by an exposed surface. In numerical simulations, specific considerations must be considered and used. For instance, the lower flange is assumed to have a view factor of $\varnothing_{LOWER} = 1$ because it is oriented parallel to and directly exposed to the fire. In contrast, the other components, which are not directly exposed, require their average view factors to be calculated using Equations (10) and (11). This calculation is based on the Hottel crossed-strings method [35]. Table 2 represents the view factor for the thermal zones of all the slabs to be used in the numerical model, corresponding to the respective regions.

$$\varnothing_{UPPER} = \frac{\left(\sqrt{h_2^2 + \left(L_3 + \frac{L_1 - L_2}{2} \right)^2} - \sqrt{h_2^2 + \left(\frac{L_1 - L_2}{2} \right)^2} \right)}{L_3} \quad (10)$$

$$\varnothing_{WEB} = \frac{\left(\sqrt{h_2^2 + \left(\frac{L_1 - L_2}{2} \right)^2} + (L_3 + L_1 - L_2) - \sqrt{h_2^2 + \left(L_3 + \frac{L_1 - L_2}{2} \right)^2} \right)}{2\sqrt{h_2^2 + \left(\frac{L_1 - L_2}{2} \right)^2}} \quad (11)$$

Table 2. View factors used for the composite slabs produced by Voidcon[®] [36].

	VP 50	VP115	VP 200
$\varnothing_{LOWER}$	1	1	1
$\varnothing_{WEB\ 2}$	0.697	0.480	0.480
$\varnothing_{UPPER\ 2}$	0.930	0.780	0.712

Table 2. Cont.

	VP 50	VP115	VP 200
$\varnothing_{WEB\ 1}$	0.058	0.058	0.058
$\varnothing_{UPPER\ 1}$	0.064	0.064	0.064

4. Materials

The thermal conductivity and specific heat from the steel parts are depicted in Figure 5a, and defined according to the standard FprEN1993-1-2 [37]. Additionally, the emissivity of the steel was obtained from Table 3, using the values for the hot-dip-galvanised deck, which is different from carbon steel. For the density of both the steel deck and rebars, a value of 7850 kg/m^3 is assumed at room temperature and remains constant at elevated temperatures. The steel grade used for the steel deck is ISQ 23 Grade 275 galvanised steel, with presents a characteristic strength of 230 MPa; the reinforcement bar steel has a characteristic strength of 450 MPa. The rebars used for all the slabs have a diameter of 10 mm. The thermal conductivity and specific heat of normal-weight concrete are depicted in Figure 5b, and defined according to the standard prEN1994-1-2 [24]. The density of the concrete starts at a temperature of $20\text{ }^\circ\text{C}$ with a value of 2300 kg/m^3 and decreases to 2024 kg/m^3 when reaching a temperature of $1200\text{ }^\circ\text{C}$. The thermal properties of air are also non-linear, as depicted in Figure 5c, as derived from data presented by Çengel [35]. Based on prEN 1995-1-2 [38], the thermal properties of mineral wool manufactured by Rocterm[®] [39], PN 70, with a density of 70 kg/m^3 , presents the thermal properties depicted in Figure 5d.

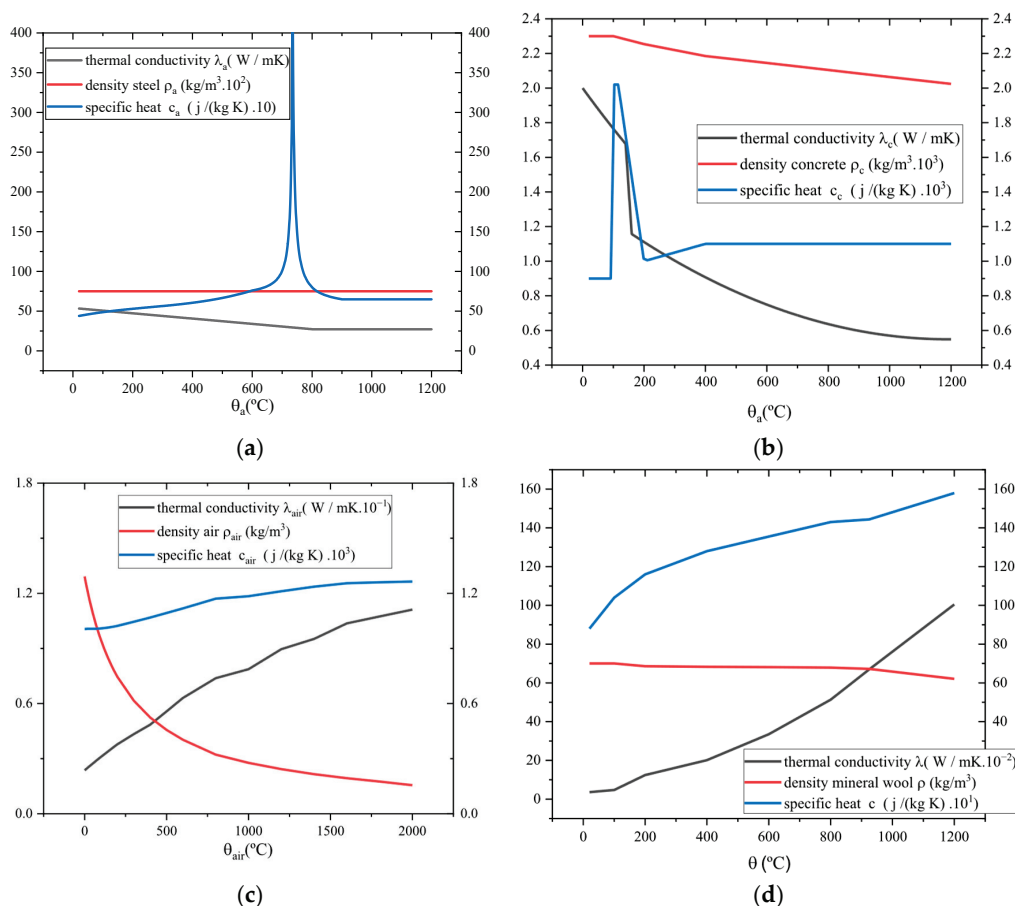


Figure 5. Variation in thermal material properties at elevated temperatures: (a) steel for steel deck and rebars; (b) normal-weight concrete; (c) air; (d) mineral wool.

Table 3. The emissivity of steel.

Type of Steel	$\varepsilon_m (\leq 500\text{ }^{\circ}\text{C})$	$\varepsilon_m (>500\text{ }^{\circ}\text{C})$
Carbon steel	0.7	0.7
HDG steel ^a	0.35	0.7

^a Steel that has been hot-dip-galvanised according to EN ISO 1461(2022) [40] and with steel composition according to Category A or B of ISO 14713-2 (2019) [41].

5. Results

The results of the advanced thermal and simplified structural analyses encompass the behaviour of composite slabs under fire conditions, the performance of the proposed insulation systems, and the load-bearing capacity of the structures over time. The comparison between experimental results, Figure 6, and numerical simulations demonstrates the validation of the models and the calibration of the empirical coefficients “ a_i ”, “ b_i ”, and “ c_i ”, detailing the validation of the numerical model, the analysis of insulation criteria, and the parametric study of mechanical performance during fire exposure.

**Figure 6.** Specimen 2, obtained from Fourie [21], after the test was completed.

5.1. Numerical Validation

Figure 7a compares the temperatures recorded in the unexposed region of the VP115 composite slab with the steel shield, considering the experimental test results obtained both by Fourie [21] and by this investigation using ANSYS. Additionally, Figure 7b compares Fourie’s experimental results with the numerical results from this investigation for the UPPER 2 and WEB 2 zones. It can be noted that, for these regions, the numerical results show good agreement with the experimental results, as illustrated in Figure 7. The results are presented in Figure 7, which displays Fourie’s experimental temperature measurements alongside the numerical prediction. According to Fourie [21], the difference between numerical and experimental results is due to the formation of an air gap during the initial periods of the experiment, a phenomenon also noted by Piloto et al. (2020) [23]. Figure 7c presents the temperature field of the validation test at 60 min for VP 115 with a steel shield.

The air gap size is assumed to be a constant average value, valid throughout the entire interface between steel and concrete. The debonding effect results from the difference in thermal expansion between these two materials. The actual air gap size in the experiments is expected to increase depending on the fire event. The numerical results slightly underestimate the temperature of the unexposed concrete region and overestimate the steel temperature in the UPPER 2 region.

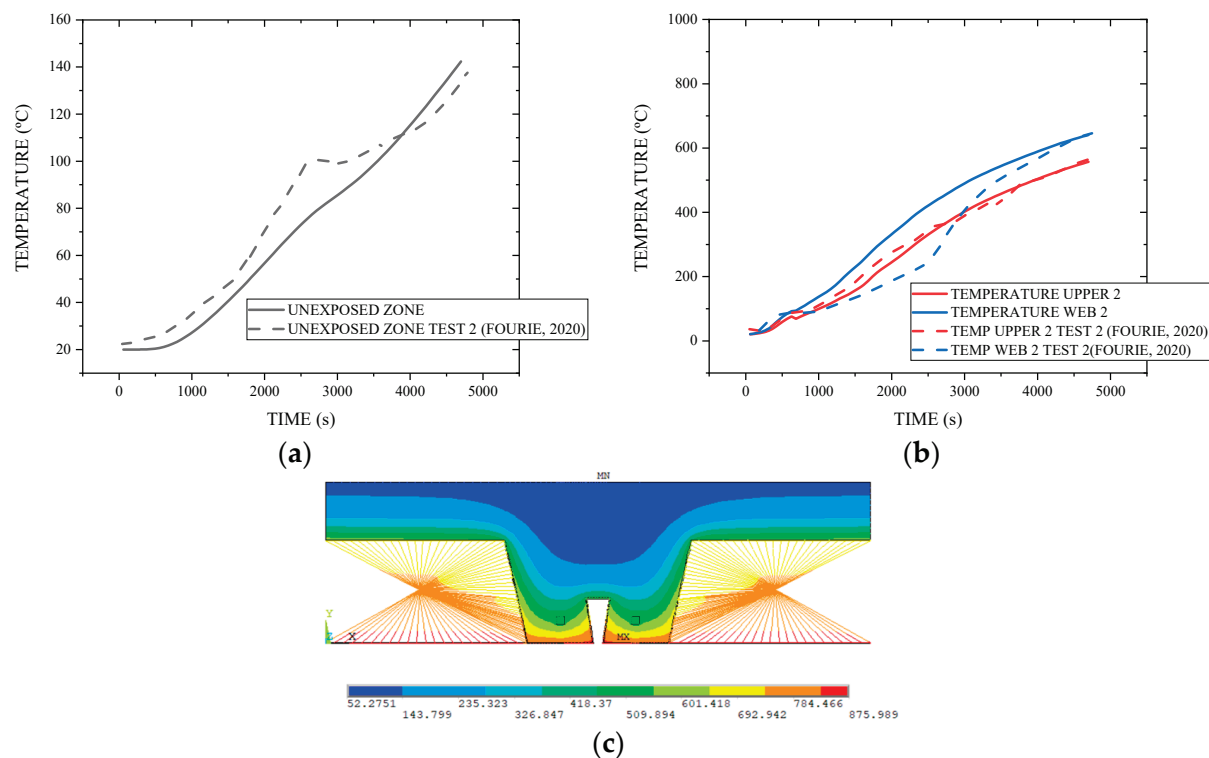


Figure 7. Comparison of the experimental results obtained by Fourie [21] with those obtained using the ANSYS model in this study. (a) Unexposed zone; (b) steel sheet trapezoidal zones; (c) temperature field after 60 min.

The root mean square errors (RMS) are presented in Table 4 for the absolute and relative difference. These values were determined between the numerical model and the experimental results throughout the simulation for the WEB 2, UPPER 2, and unexposed zones. This value reflects the accuracy of the computation model, considering that at any time after the first 10 min of any standard fire test, the temperature recorded by any thermocouple in the furnace should not differ from the corresponding temperature of the standard temperature–time curve by more than 100 °C [42].

Table 4. Root mean square error (°C) between experimental and numerical results of VP 115 with steel shield.

REGION ZONE	RMS (°C)	RMS (%)
WEB 2	82.1	18
UPPER 2	18.1	6
UNEXPOSED	10.4	15

An experimental test report from the structures, safety and fire department at CSTB [27] was used to validate models utilising mineral wool. This report used the COFRADAL 200 steel deck, as shown in Figure 8a. The analysis was conducted using the ANSYS software, with the finite element PLANE55 [26], illustrated in Figure 8b. The boundary conditions applied were identical to those used for the first validation (composite slab VP 115), with convection coefficients of 25 W/m²K for the exposed face and 9 W/m²K for the unexposed face, in accordance with EN 1991-1-2 [31]. A constant view factor of 1 was utilised for radiation, with an emissivity coefficient of 0.7 (assuming carbon steel). The ISO-834 standard fire curve [13] was applied.

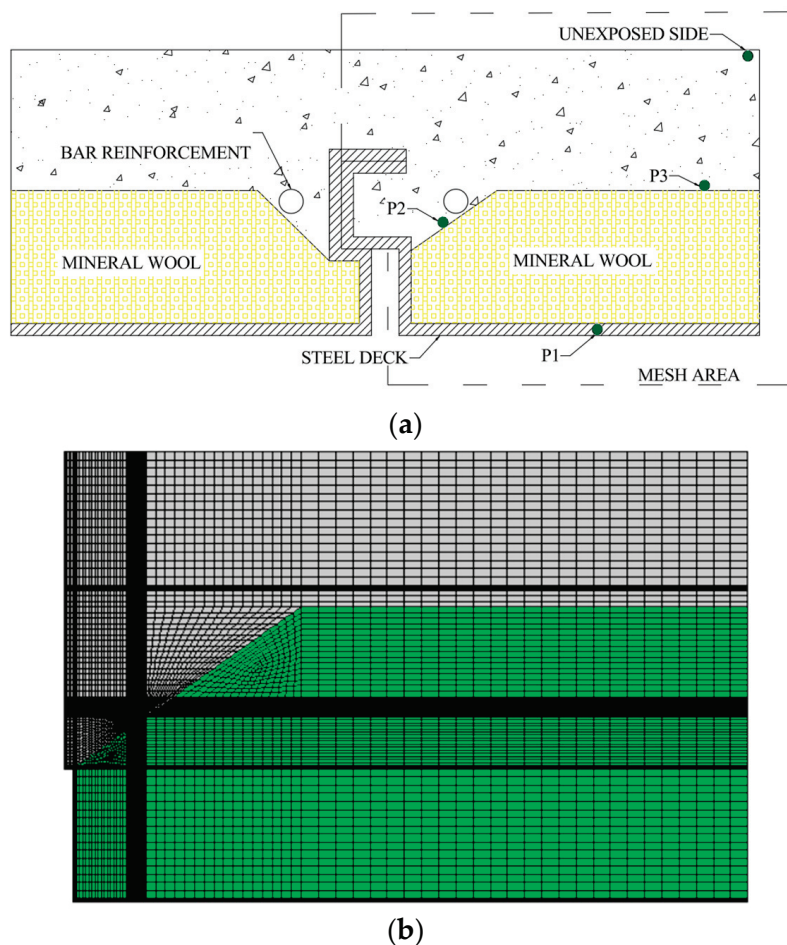


Figure 8. (a) Thermocouples used in the COFRADAL 200 experimental test [27]; (b) mesh size of COFRADAL 200 in ANSYS.

Figure 9a shows the comparison of the temperature evolution at points P1 and P2 between the experiment conducted by CSTB [27] and the numerical simulations performed using ANSYS software. Figure 9b, meanwhile, presents the comparison of the temperature history at point P3 and on the unexposed face. Table 5 presents the RMSE values. The highest value occurs at P1; however, it remains below the 100 °C threshold limit [42]. The other analysed points exhibit low values, demonstrating the reliability of the numerical model and thus enabling a parametric study. Figure 9c shows the temperature field at 60 min from that validation.

Table 5. Root mean square error (°C) between experimental and numerical results of COFRADAL 200.

REGION ZONE	RMS (°C)	RMS (%)
P1	54.5	7
P2	20.3	8
P3	10.3	12
UNEXPOSED SIDE	1.4	3

The discrepancies observed between the numerical and experimental results, namely the overestimations and underestimations, can be attributed to uncertainties in the precise location of the thermocouples, the use of effective thermal properties rather than measured values incorporating degradation, as well as the effect of material moisture evaporation and migration, which are responsible for the higher or lower temperature plateau.

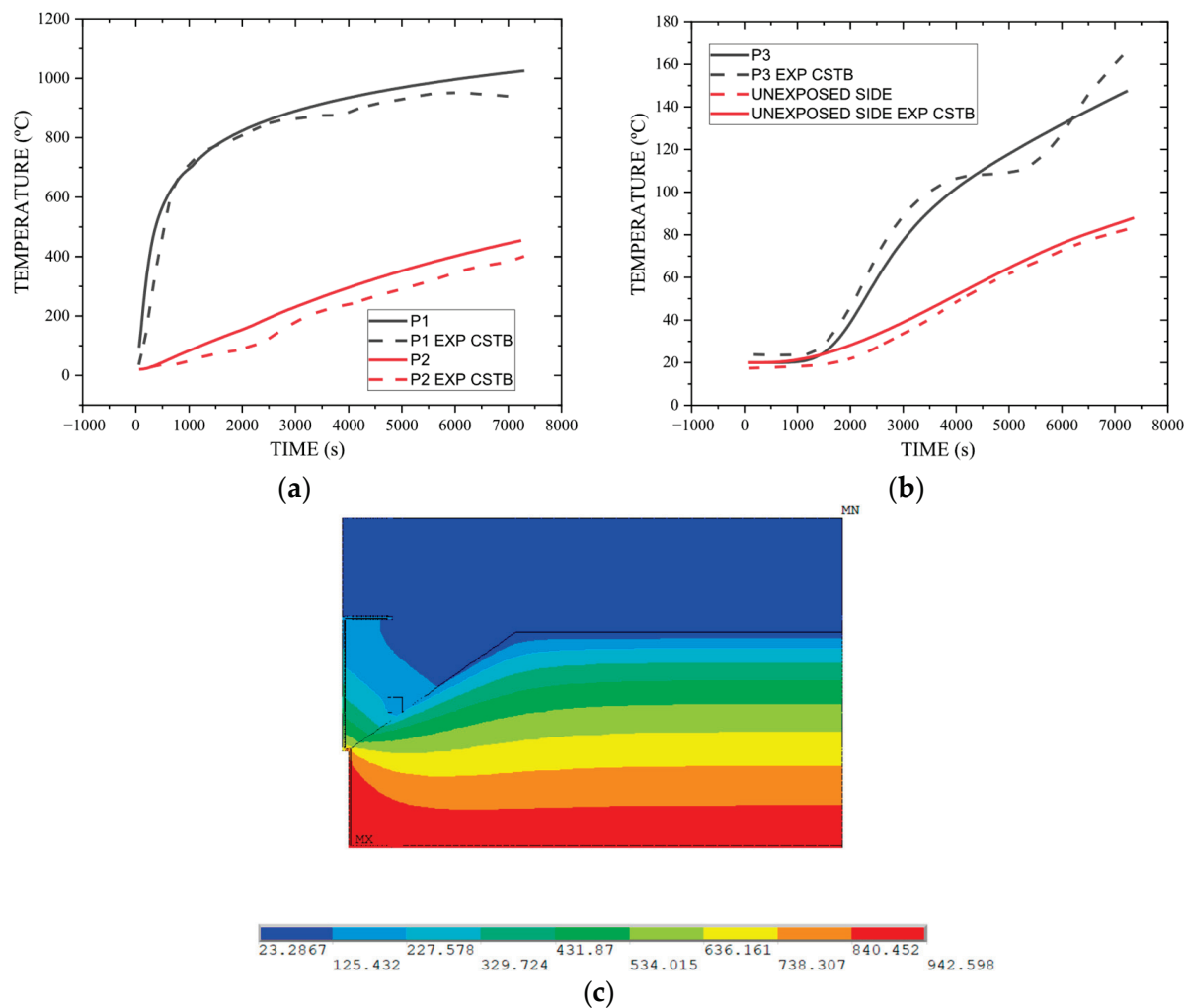


Figure 9. Comparison of the experimental results obtained by CSTB [27] with those obtained using the ANSYS model in this study. (a) P1 and P2; (b) P3 and UNEXPOSED side; (c) temperature field after 60 min.

5.2. Insulation Fire Resistance (I)

Table 6 presents the results for the fire resistance (criterion ‘I’) for the composite slabs and their insulation types. This fire resistance criterion is determined with respect to the average temperature rise, 140 °C above the initial average temperature (T_0), or with respect to the maximum temperature rise, 180 °C above the initial average temperature (T_0), on the unexposed surface of the slab. The lowest value between the two limits determines the fire resistance, according to EN1363-1 (2020) [42]. prEN1994-1-2 [24] also defines the minimum effective thickness as a function of standard fire resistance.

After four hours of simulation using the ISO 834 curve [13], the results showed a time gain for the maximum and average temperatures on the unexposed surface compared to the composite slab without insulation. The steel shield insulation is used in three types of composite slabs, resulting in a time gain of approximately 25% for the maximum temperature criterion in all slabs. The use of mineral wool in the cavity region of the composite slab VP50 with $h_2 = 50$ mm shows a total time gain of 43% compared to the composite slab without insulation. The composite slabs VP115 and VP200 with mineral wool, even after four hours, did not reach the temperature limit for the insulation criterion. The composite slab with mineral wool plate insulation showed significantly improved fire resistance (I), increasing proportionally to the insulation thickness (h_i). On average, if one

is using h_i of 30 mm, the composite slab exhibits a 65% enhancement in fire resistance compared to non-insulated composite slabs.

Table 6. Time limit for the fire resistance (I) criteria.

Type of Slabs	Insulation Type		T ₀ + 180 (min)	T ₀ + 140 (min)
VP 50	without insulation		61	66
	steel shield		78	77
	mineral wool		109	113
	mineral wool plate	hi = 5 mm	96	99
		hi = 10 mm	114	114
		hi = 20 mm	143	143
		hi = 30 mm	180	169
VP 115	without insulation		61	83
	steel shield		80	97
	mineral wool		333 ^a	276 ^a
	mineral wool plate	hi = 5 mm	100	113
		hi = 10 mm	120	133
		hi = 20 mm	155	171
		hi = 30 mm	191	222
VP 200	without insulation		62	102
	steel shield		81	107
	mineral wool		449 ^a	501 ^a
	mineral wool plate	hi = 5 mm	103	143
		hi = 10 mm	124	157
		hi = 20 mm	163	210
		hi = 30 mm	200	255 ^a

^a Exceeded the fire rating of 240 min for criterion I according to standard EN 13501-2 [43].

New empirical coefficients “ a_i ” were determined to calculate the fire resistance insulation time (in minutes) according to the standard prEN1994-1-2 [24]. The simplified method uses Equation (9) to derive these coefficients for insulation fire resistance. The calculation is made using a non-linear evolutionary solver [44], implemented in Excel. Table 7 presents the newly established coefficients “ a_i ”.

Table 7. New coefficients “ a_i ” proposed to determine the insulation fire resistance in composite steel deck under fire curve ISO 834 for a steel shield and mineral wool insulation.

	a_0 (min)	a_1 (min/mm)	a_2 (min)	a_3 (min/mm)	a_4 (mm min)	a_5 (min)
Steel shield model	−28.79	1.72	−3.21	0	−734.50	7.90
Mineral wool model	−1207.42	19.71	−33.10	5,61	−26,611.50	0.008

The proposed equation to determine the insulation time (t_i) of mineral wool plates retains the fundamental structure of Eurocode 4 Part 1-2 [24], employing a linear combination of geometric and thermal parameters to estimate fire resistance. However, the formulation introduces two critical terms explicitly dependent on the insulation thickness (h_i), addressing the limitations in the Eurocode methodology for systems incorporating

mineral wool. The general form of the equation aligns with the EN1994-1-2 framework, and is presented in Equation (12).

$$t_i = a_0 + a_1 h_1 + a_2 \phi_{UPPER\ 2} + a_3 \frac{A}{L_r} + a_4 \frac{1}{L_3} + a_5 \frac{A}{L_r} \frac{1}{L_3} + a_6 \log\left(\frac{hi}{1}\right) + a_7 * hi^{\frac{1}{h^{22}}} \quad (12)$$

The coefficients a_0 to a_5 used in Equation (12) are derived from the steel shield model (Table 6), ensuring continuity between insulation configurations. This methodology ensures that the equation reduces to the steel shield model when the mineral wool insulation thickness hi is zero, maintaining consistency across systems. New coefficients a_6 to a_7 are presented in Table 8, facilitating the calculation of the insulation fire resistance for composite steel decking systems incorporating mineral wool plates.

Table 8. New coefficients “ a_i ” proposed to determine the time of insulation in steel deck under fire curve ISO 834 for mineral wool plate insulation.

	a_6 (min)	a_7 (min)
Mineral wool plate model	112.6	−82.1

5.3. Parametric Study and New Proposal

Figure 10 illustrates the temperature field for the composite slab VP115, under standard fire exposure ISO 834 [13], when using three different insulation types. These results correspond to a fire duration of two hours (7200 s), with the analysis conducted for a concrete thickness of $h_1 = 65$ mm.

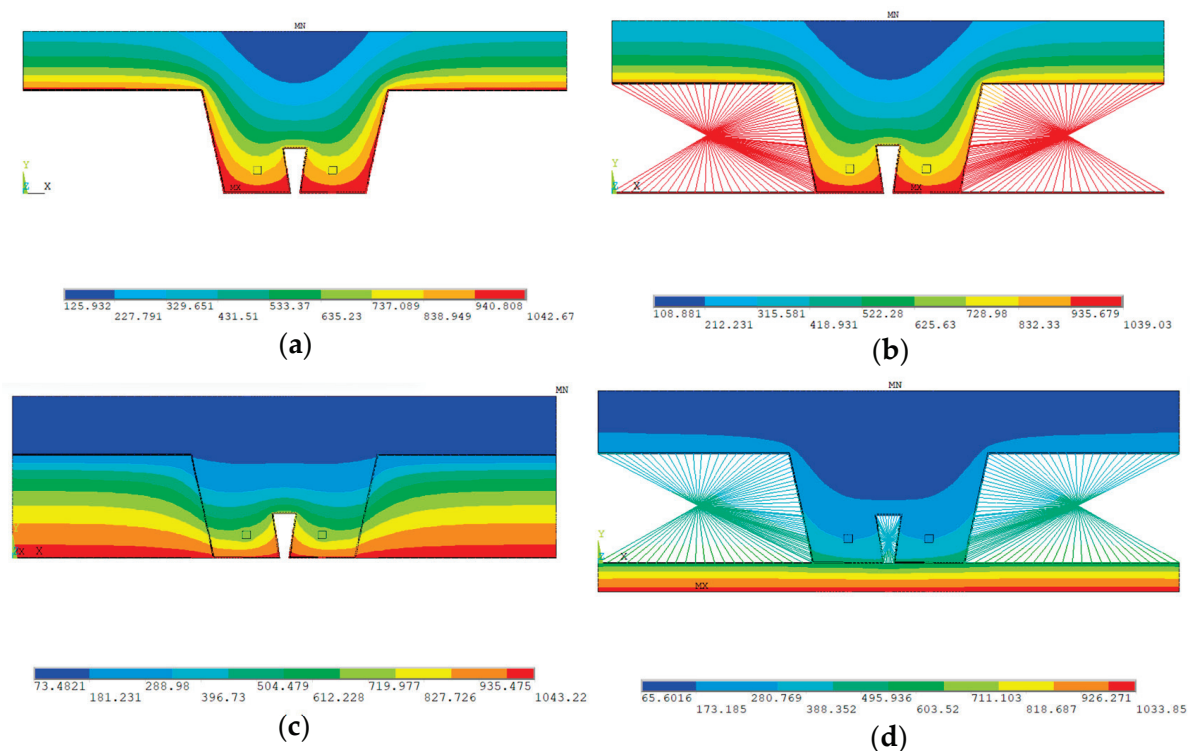


Figure 10. Temperature field over the composite slabs under ISO 834 curves for the same $h_1 = 65$ mm for a time equal to 7200 s: (a) VP115 without insulation; (b) VP115 with steel shield; (c) VP115 with mineral wool; (d) VP115 with mineral wool plate $hi = 30$ mm.

Furthermore, Figure 11 presents the temperature history, from 0 to 7200 s, at the UPPER 2 region, for three different geometries of composite slabs, each with a distinct insulation system, all considering the same concrete thickness of $h_1 = 65$ mm under the ISO834 standard for fire exposure.

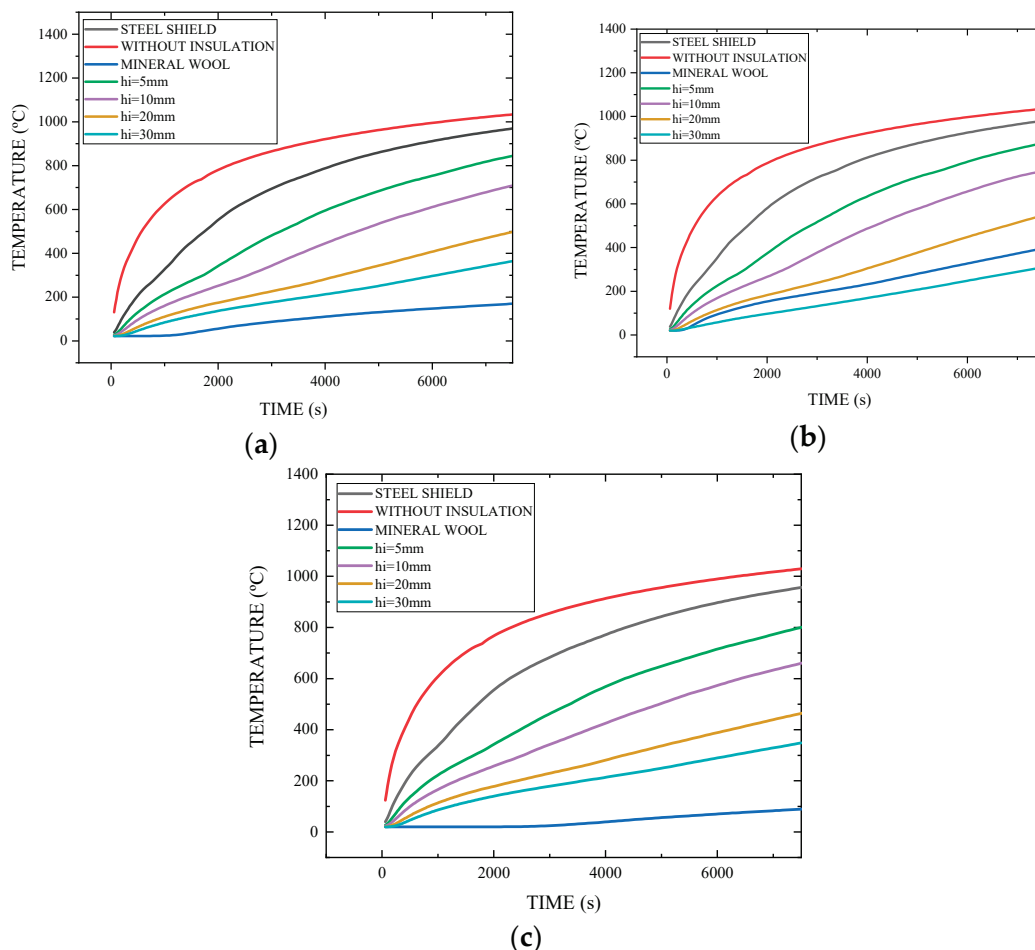


Figure 11. Comparison of temperature history in the UPPER 2 zones for the composite slabs under ISO834 fire standard for the same $h_1 = 65$ mm for a time from 0 to 7200 s: (a) VP115; (b) VP50; (c) VP200.

The numerical results obtained from ANSYS software are averaged for each component of the composite slabs. The average temperatures are determined for the lower flange (LOWER zone), the web (WEB zone), and the upper flange (UPPER zone). Coefficients “ b_i ” and “ c_i ” are introduced to determine the average temperatures in the steel deck components insulated with steel shield and mineral wool, as well as in the reinforcing bars, considering the composite slabs under these fire conditions. The proposed coefficients are obtained using Excel, which minimises the sum of the absolute differences between the numerical temperatures (obtained from the steel components in the parametric study) and the temperatures calculated by the model corresponding to Equation (5) or Equation (6). This minimisation is also performed using the non-linear evolutionary solver [44], available in the Excel SOLVER tool. The tool iteratively adjusted the values of “ b_i ” and “ c_i ” with the objective of achieving the lowest possible absolute error between the simplified method (with the new coefficients) and the numerical results.

Table 9 compares the effect of the steel shield and mineral wool insulation for the VP115 composite slab. The mineral wool reduces UPPER 2 temperatures from 906.3 °C (without insulation) to 99.3 °C (3600 s), while the steel shield achieves 751.0 °C. Similar

efficiency is observed in WEB 2 (e.g., 888.8 °C vs. 391.3 °C at 3600 s). The lower region shows negligible differences (<1%) between methods. The mineral wool has superior thermal resistance in critical zones, highlighting its suitability for structural fire protection. The VP50 and VP200 composite slab geometries perform similarly to VP115.

Table 9. Comparison in temperature between the model without insulation and the model with a steel shield for VP115.

Time (min)	Region	Without Insulation (°C)	Steel Shield (°C)	Difference % (a)	Mineral Wool (°C)	Difference % (b)
60	LOWER	923.2	920.6	0.27	917.8	0.65
	WEB 2	888.8	778.9	12.36	391.3	44.00
	UPPER 2	906.3	751.0	17.13	99.3	89.07
90	LOWER	994.4	990.6	0.40	987.6	0.70
	WEB 2	966.9	897.5	7.14	491.2	49.14
	UPPER 2	979.3	881.1	10.01	135.9	86.21
120	LOWER	1039.7	1038.1	0.09	1035.1	0.384
	WEB 2	1018.7	969.7	4.81	567.2	44.20
	UPPER 2	1025.9	958.1	6.60	163.2	84.07

(a) Difference between slabs without insulation and with steel shield. (b) Difference between slabs without insulation and with mineral wool.

Table 10 summarises the temperature differentials between the non-insulated configuration and the mineral wool plate-insulated composite slab.

Table 10. Comparison in temperature between the model without insulation and the model insulated with mineral wool plate $h_i = 30$ mm for VP115.

Time (min)	Region	Without Insulation (°C)	Mineral Wool Plate $h_i = 30$ mm (°C)	Difference %
60	LOWER	923.2	332.9	64.0
	WEB 2	888.8	254.8	71.0
	UPPER 2	906.3	195.8	78.0
90	LOWER	994.4	447.9	55.0
	WEB 2	966.9	332.9	66.0
	UPPER 2	979.3	269.5	72.0
120	LOWER	1039.7	538.0	48.0
	WEB 2	1018.7	411.0	60.0
	UPPER 2	1025.9	348.1	66.0

Table 11 presents the proposed “ b_i ” coefficients used to determine the temperature in steel deck components when dealing with the composite slab with steel shield, maintaining their use in Equation (5). It is noted that in the lower region, the “ b_2 ” parameters are small, since the rib is not protected, making the geometric parameters less relevant to the fire resistance.

Table 12 presents the proposed “ b_i ” coefficients for determining the temperature in steel deck components when dealing with the composite slab with the cavity protected by mineral wool, maintaining their use in Equation (5). The coefficient “ b_1 ” is still big in comparison to the other coefficients, due to the insulation effect in the L3T regions.

Table 11. New coefficients “ b_i ” proposed to determine the temperature in steel deck under ISO 834 fire curve for a steel shield insulation.

TIME (min)	STEEL DECK REGION	b_0 (°C)	b_1 (°C.mm)	b_2 (°C.mm)	b_3 (°C)	b_4 (°C)
60	UPPER 2	655.63	−734.815	−0.32	193.9096	−52.90
	WEB 2	898.73	−18,071.9	−1.57	276.0602	−210.84
	LOWER	953.79	−868.995	−0.13	17.31293	−40.132
90	UPPER 2	770.62	−17,598.3	−1.14	511.3276	−186.22
	WEB 2	892.69	−12,026.7	−1.09	491.0991	−404.97
	LOWER	1047.78	−473.694	−0.09	7.664627	−57.90
120	UPPER 2	897.64	−16,998.5	−1.20	443.8899	−178.63
	WEB 2	973.99	−5684.49	−0.61	228.0133	−188.25
	LOWER	980.55	−170.244	−0.05	75.57389	−14.705

Table 12. New coefficients “ b_i ” proposed to determine the temperature in steel deck under ISO 834 fire curve for a mineral wool insulation.

TIME (min)	STEEL DECK REGION	b_0 (°C)	b_1 (°C.mm)	b_2 (°C.mm)	b_3 (°C)	b_4 (°C)
60	UPPER 2	0	−80,115.3	−5.54224	1182.233	−0.00016
	WEB 2	546.542	−5772.11	−4.82394	291.80	−0.00262
	LOWER	953.796	−868.995	−0.13103	17.31	−40.1328
90	UPPER 2	0	−131,140	−8.22638	1844.938	−0.00041
	WEB 2	474.912	−0.3311	−4.53379	718.85	−378.888
	LOWER	1047.79	−473.694	−0.09809	7.66	−57.907
120	UPPER 2	0	−189,471	−11.2556	2582.478	0
	WEB 2	802.465	−13,254.1	−5.70221	325.1772	−0.00084
	LOWER	980.556	−170.244	−0.05367	75.57389	−14.7057

For the regions UPPER 1 and WEB 1, which are areas without any insulation, the calculations can be performed using the coefficients presented in the new approach proposed by Carlos Balsa et al. [45].

Tables 13 and 14 present new coefficients to determine the average temperatures of the rebars for the case of composite slabs with steel shield and cavity insulation using mineral wool, respectively, maintaining their use in Equation (6).

Table 13. New coefficients “ c_i ” are proposed to determine the temperature in steel deck under ISO 834 fire curve for a steel shield insulation.

TIME (min)	c_0 (°C)	c_1 (°C)	c_2 (°C.mm ^{0.5})	c_3 (°C.mm)	c_4 (°C/°)	c_5 (°C.mm)
60	947.1	−201.6	−268.1	−3.25	4.17	−4585.2
90	1027.5	−198.8	−176.1	−3.98	3.63	−14,491.4
120	1153.7	−271.6	−158.3	−3.92	2.94	−12,363.0

Table 14. New coefficients “ c_i ” proposed to determine the temperature in steel deck under ISO 834 fire curve for a mineral wool insulation.

TIME (min)	c_0 (°C)	c_1 (°C)	c_2 (°C.mm ^{0.5})	c_3 (°C.mm)	c_4 (°C/°)	c_5 (°C.mm)
60	930.078	−292.355	−117.716	−3.74446	0.936562	−7273.17
90	963.447	−143.803	−177.807	−1.52391	1.434412	−10.1888
120	1007.2	−253.709	−198.434	−3.50322	4.105537	−8.06392

For the case of composite slabs with mineral wool plates (70 kg/m^3), the calculation of the temperature at the steel deck components includes two additional factors, which depend on the thickness of the insulating material. The governing Equation (13) for this insulation approach employs the coefficient “ b_i ” from Table 11, corresponding to steel shield insulation, and includes the parameters “ b_5 ” and “ b_6 ”; see Table 15.

$$\theta_a = b_0 + b_1 \frac{1}{L_3} + b_2 \frac{A}{L_r} + b_3 \varnothing + b_4 \varnothing^2 + b_5 * hi^{\frac{1}{h^2}} + b_6 \log\left(\frac{hi}{1}\right) \quad (13)$$

Table 15. New coefficients “ b_i ” proposed to determine the temperature in steel deck under ISO 834 fire curve for a mineral wool plate insulation.

TIME (min)	STEEL DECK REGION	b_5 (°C)	b_6 (°C)
60	Upper 1–2	81.8	−437.8
	Web 1–2	205.1	−518.687
	Lower	270.8	−580.242
90	Upper 1–2	261.2041	−598.867
	Web 1–2	295.8592	−605.429
	Lower	336.9763	−596.251
120	Upper 1–2	337.2489	−646.264
	Web 1–2	357.831	−641.31
	Lower	353.4428	−572.978

Equation (14) provides the temperature for the steel rebar component when dealing with composite slabs using mineral wool plates, where the coefficients “ c_0 ” to “ c_5 ” are obtained from Table 13. The new coefficient “ c_6 ” is presented in Table 16.

$$\theta_s = c_0 + c_1 \frac{u_3}{h_2} + c_2 z + c_3 \frac{A}{L_r} + c_4 \alpha + c_5 \frac{1}{L_3} + c_6 \log\left(\frac{hi}{1}\right) \quad (14)$$

Table 16. New coefficient “ c_6 ” proposed to determine the temperature in rebars under ISO 834 fire curve for a mineral wool plate insulation.

TIME (min)	c_6 (°C)
60	930.078
90	963.447
120	1007.2

5.4. Load-Bearing Capacity Time (R)

To determine the load-bearing capacity of the slab ($M_{Rd,fi,t}$) under fire, the method presented in prEN1994-1-2 [24] involves using the temperatures defined by Equations (5) and (6) to determine the temperature of the steel deck elements and the rebar components, respectively. With the defined temperatures, the method requires the determination of the reduction factors for the yield stress (k_y) according to prEN 1994-1-2. The method neglects the effect of temperature on concrete. The first step is to define the neutral axis using Equation (15), where “ A_i ” represents the effective area of the steel stress zone, adjusted by the factor (k_y); “ A_j ” is the effective area of concrete resisting to compression; and “ α ” is the coefficient weighting the contribution of the tension zone, taken as 0.85. Once the neutral axis is established as a function of temperature, the second step is to determine

the load-bearing capacity of the structure using Equation (16), using the distance from the plastic neutral axis to the centroid of the elemental area (z_i and z_j).

$$\sum_{i=1}^m A_i k_{y,i} \left(\frac{f_{y,i}}{\gamma_{M,y}} \right) + \alpha_{slab} \sum_{j=1}^n A_j k_{c,j} \left(\frac{f_{c,j}}{\gamma_{M,c}} \right) = 0 \quad (15)$$

$$M_{Rd,fi,t} = \sum_{i=1}^m A_i z_i k_{y,i} \left(\frac{f_{y,i}}{\gamma_{M,y}} \right) + \alpha_{slab} \sum_{j=1}^n A_j z_j k_{c,j} \left(\frac{f_{c,j}}{\gamma_{M,c}} \right) \quad (16)$$

The graphs presented in Figure 12 demonstrate that the implementation of insulation systems, whether through steel shielding or the application of mineral wool, delays the heating of critical elements, particularly with mineral wool, which maintains higher load-bearing capacity ($M_{Rd,fi,t}$) for extended periods compared to the composite slab without insulation. This approach, compatible with prEN1994-1-2 [24], effectively integrates thermal and mechanical effects, offering a robust tool for designing and assessing structural fire resistance and addressing the gap regarding the lack of design methods for composite slabs with special ribs and insulation methods.

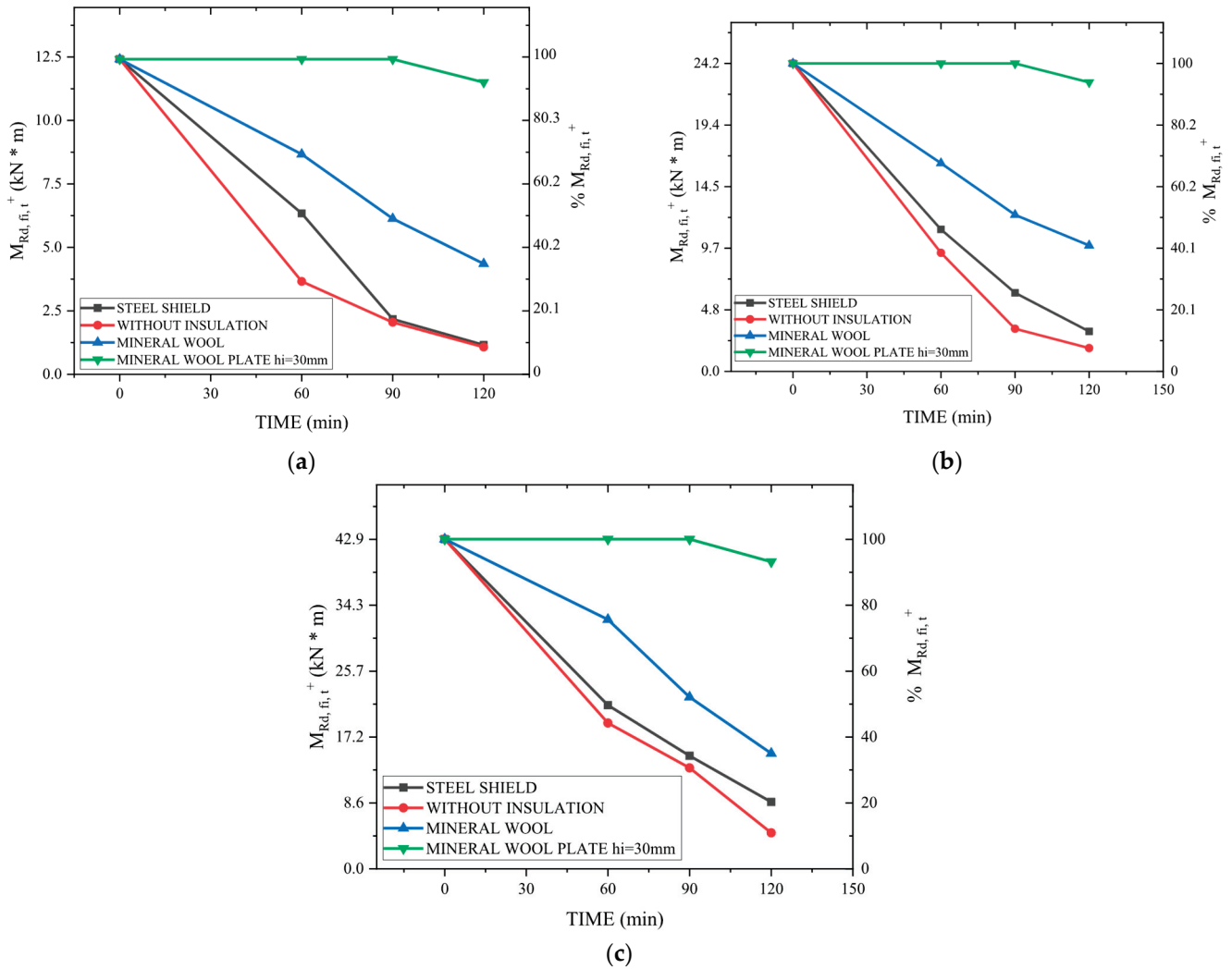


Figure 12. Evolution of the loss of load-bearing capacity of the slab ($M_{Rd,fi,t}$) over time under the ISO 834 fire curve for three conditions: without insulation, with a steel shield, and with mineral wool insulation. (a) VP50; (b) VP115; (c) VP200.

The mechanical analysis shows the fire resistance effect of different insulation strategies to leverage composite slabs' gain in mechanical strength under fire exposure, optimising and reducing concrete usage for sustainability purposes.

Conventional concrete emits approximately 430 kg/m³ of CO₂ [46]. A reduction in concrete volume may be achieved by replacing a composite slab made with VP 200 (without insulation) with a VP 115 slab with 30 mm thick mineral wool board insulation ($h_i = 30$ mm) to ensure 60 min fire resistance.

Table 17 presents a comparison of CO₂ emissions and the mechanical strength enhancement for slabs with 30 mm mineral wool board insulation and composite slabs without insulation. The carbon footprint per square metre of slab is calculated by summing the CO₂ emissions from the concrete and mineral wool. The mineral wool insulation ($h_i = 30$ mm) has a CO₂ emission of 3.09 kg/m² [47].

Table 17. Carbon emissions in comparison to the load-bearing of the steel deck slabs.

STEEL DECK PLATE	Model Without Insulation		Mineral Wool Plate $h_i = 30$ mm	
	Emissions of CO ₂ (Kg/m ²)	Load-Bearing After 60 min (KN * m)	Emissions of CO ₂ (Kg/m ²)	Load-Bearing After 60 min (KN * m)
VP 50	34.76	3.65	37.85	12.41
VP 115	42.40	9.34	45.49	24.27
VP 200	58.21	21.31	61.30	42.93

In the case of replacing VP 200 composite slabs with VP 115 composite slabs, including insulation, a reduction of approximately 20% in CO₂ emissions is achieved. This represents a significant figure, particularly in light of current demands for CO₂ reduction, given that cement is not only the most widely used material globally but also the largest contributor to CO₂ emissions within the construction sector.

6. Conclusions

This investigation presents an integrated and robust approach for assessing the thermal and mechanical behaviour of composite slabs with steel decks subjected to fire conditions, applying validated finite element models using ANSYS. One of the main objectives was to enhance the prediction of temperatures in critical components, such as the steel deck parts and reinforcement, through the calibration of new empirical coefficients (" a_i ", " b_i ", and " c_i ") used in the simplified methods proposed by prEN1994-1-2, including the effect of a new stiffened steel deck and the use of insulation materials.

Specifically, the numerical modelling was divided into five stages: (i) Model validation was performed using experimental results from Fourie [21] and from Duponchel (CSTB) [27], which yielded a mean difference (RMS) of only 10.47 °C in the unexposed zone, despite larger deviations in regions such as WEB 2 (RMS $\approx$ 82.1 °C). (ii) Thermal analyses were implemented using PLANE55, LINK31, and LINK34 elements to accurately capture the temperature field in all the composite slabs using different insulation methods. (iii) A parametric study was conducted that compared the effectiveness of two insulation systems—the steel shield and mineral wool—revealing that, while the steel shield achieved a time gain of approximately 25% for the maximum temperature criterion. The mineral wool demonstrated a more pronounced effect by reducing temperatures in the upper zone (UPPER 2) by up to 89% compared to the uninsulated condition. (iv) The empirical coefficients were calibrated using a non-linear solver in Excel, which minimised the absolute difference between the simplified values and numerical results. (v) Strategies

were proposed for mitigating CO₂ emissions by utilising thermal insulation materials within composite slabs. Some strategies can reduce CO₂ emissions by more than 15 kg/m², enabling a spectrum of potential design configurations. This conclusion, however, is conditional on the specific geometries and materials of the parametric study, which revealed that while mineral wool above the steel shield can exceed standard requirements (EN13501-2), a 30 mm thickness may be insufficient for the highest fire resistance class (I240).

The results not only validate the effectiveness of the numerical model but also demonstrate that the introduction of insulation systems significantly enhances thermal integrity and, consequently, the load-bearing capacity of the slabs during fire events. The proposed methodology allows for a more accurate prediction of the fire resistance time, thereby enabling safer design of composite slabs in complex high-rise buildings. The choice of a constant air gap of 0.5 mm, addressing the debonding effect between the concrete and the steel, proved to be adequate, as already demonstrated by Piloto et al. [23].

Among the three insulation strategies evaluated, the mineral wool plate applied below the steel deck proved to be the most effective in enhancing both thermal insulation (I) and load-bearing capacity (R). It reduced temperatures in critical zones by up to 89% and increased fire resistance time proportionally with thickness, outperforming both the steel shield and cavity-filled mineral wool. The steel shield, while beneficial, offered the least improvement, with an estimated 25% gain in fire resistance time. These findings are consistently supported across all slab geometries (VP50, VP115, VP200) and insulation configurations tested, confirming the superior performance of mineral wool plates for fire-safe and sustainable composite slab design.

Finally, the development of new empirical coefficients and the validation of the simplified method represent a significant contribution to the field of structural fire safety. The proposed model coherently integrates thermal and mechanical effects, providing a robust tool for improving design and analysis methods for structures exposed to fire. Future studies may extend this approach by incorporating sensitivity analyses and additional experimental investigations to refine the adopted assumptions further, ultimately contributing to developing more precise and reliable design standards and guidelines.

7. Limitations and Future Works

This study identifies several limitations requiring further investigation. The practical applicability of the proposed insulation systems—encompassing constructability, durability, cost-effectiveness, adhesive performance at elevated temperatures, and seismic resilience—requires real-world verification. Model assumptions, such as a constant air gap for debonding and the neglect of thermal contact resistance at interfaces, alongside validation discrepancies (e.g., RMS up to 82.1 °C), indicate a need for enhanced thermal modelling, including dynamic gap evolution and three-dimensional effects (e.g., edge radiation, load redistribution). Critical structural aspects overlooked include shear failure mechanisms, particularly at rib roots, and the exclusion of cooling-phase effects where insulation may exacerbate thermal gradients, residual stresses, and delayed failures. Analyses were also limited to the ISO 834 standard fire, neglecting natural fire scenarios involving compartment interactions, moisture effects, and cooling-phase behaviour.

Future research should focus on developing 3D thermo-mechanical models incorporating dynamic air gaps and shear failure criteria (e.g., EN 1992-1-2); quantifying cooling-phase impacts on structural recovery; validating findings through full-scale testing across varied geometries and insulation types; and testing insulation robustness under cyclic thermal exposure, including bonded mineral wool plates and alternative materials (e.g., gypsum boards) or hybrid solutions. Investigating natural fires with compartment interactions and moisture content, alongside establishing design guidelines for fire-induced displace-

ments of steel shields, is essential to enhance the safety and sustainability of fire-resistant composite slabs.

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Article

Time Response of Delaminated Active Sensory Composite Beams Assuming Non-Linear Interfacial Effects

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Abstract: A layerwise laminate FE model capable of predicting the dynamic response of delaminated composite beams with piezoelectric actuators and sensors encompassing local non-linear contact and sliding at the delamination interfaces was formulated. The kinematic assumptions of the layerwise model enabled the representation of opening and sliding of delamination interfaces as generalized strains, thereby allowing the introduction of interfacial contact and sliding effects through constitutive relations at the interface. This realistic FE model, assisted by representative experiments, was used to study the time response of delaminated active sensory composite beams with predefined delamination extents. The time response was measured and simulated for narrowband actuation signals at two distinct frequency levels using a surface-bonded piezoceramic actuator, while signal acquisition was performed with a piezopolymer sensor. Four different composite specimens, each containing a different delamination size, were used for this study. Experimental results were directly compared with model predictions to evaluate the performance of the proposed analytical approach. Damage signatures were identified in both the signal amplitude and the time of flight, and the sensitivity to delamination size was examined. Finally, the distributions of axial and interlaminar stresses at various time snapshots of the transient analysis are presented, along with contour plots across the structure’s thickness, which illustrate the delamination location and wave propagation patterns.

Keywords: composite beam; layerwise mechanics; delamination; nonlinear; interfacial contact; interfacial slip; wave propagation; piezoelectrics

1. Introduction

The effect of existing delamination on the time response of realistic active sensory composite beams was thoroughly investigated by the authors, both analytically and experimentally. The work comprises two parts: (I) development of an analytical model and (II) experimental validation with delamination detection. An analytical model was developed using layerwise mechanics to simulate composite beams that include through-width delamination cracks and interactions with piezoelectric devices. The novelty of this model lies in the natural contact conditions at the delamination interfaces, which prevent penetration in a realistic manner. Additionally, an interlaminar shear stress field was applied during contact, controlling the free axial movement. This paper presents results that explore the effect of local non-linear interactions on the global structural response and validates the analytical model predictions against representative experimental measurements.

Various works investigated the effect of delamination on the structural response using experimental measurements or analytical simulations [1–3]. The majority of these studies investigated the effect of delamination on the frequency parameters [4,5] and presented changes in the natural frequencies of composite beams with delamination [6–10], or even the vibration mode shape [11,12]. The effect of delamination was additionally studied both analytically and experimentally on another critical dynamic parameter, like the modal damping, by Saravanos and Hopkins [13]. The introduction of piezoelectric devices operating either as actuators or sensors led to the development of the more widespread SHM setup for damage detection in composite structures, known as the “pitch and catch configuration”. The pioneers in this direction were Keiler and Chang [14], who presented an experimental study to identify the damage effect on the frequency response functions of delaminated composite beams. Kessler et al. [15] presented an analytical and experimental study revealing the effect of various damage types on the frequency response functions and mode shapes of damaged composite beams. Also, Chrysochoidis and Saravanos [16] in an experimental study investigated the sensitivity of the modal characteristics to the size of delamination as well as the efficiency of different testing setups.

Over the last three decades, many studies have been published on modeling delamination in composite structures. Early works focused on the effects of a single delamination on the natural frequencies of composite beams [4,17,18] and plates [19] using experimental results and simple analytical beam and plate models of the so-called “four-region” approach. Barbero and Reddy [20] reported a layerwise laminate theory to describe plates with multiple delamination cracks between the layers. A generalized composite beam model for predicting the effect of multiple delaminations on the modal damping and modal frequencies of composite laminates was reported by Saravanos and Hopkins [13], which treated the opening and sliding at the delamination interfaces as additional degrees of freedom. A higher-order theory to model matrix cracking and delamination in laminated composite structures was presented [21,22]. Ghoshal et al. [23] modeled delamination using a Fermi–Dirac distribution of the axial displacements at the crack interface to obtain smoother transitions in the displacement and strain fields, and Chrysochoidis et al. [24] used a coupled electromechanical model to simulate delaminated beams and the damage local modes, with piezoelectric actuators and sensors, neglecting contact at interfaces. A layered solid spectral element was used to simulate the wave propagation of delaminated composite structures with piezoelectric wafers [25], while Shen and Cesnic [26] proposed a hybrid local interaction approach that considers local delamination dynamics to simulate the response of delaminated structures. A high-fidelity computational tool was published [27] that utilized spectral finite elements for the rapid simulation of wave propagation characteristics in damaged composite plates. With the modelling focus on the debonding appearing between the composite laminate and the piezoelectric patch [28,29], an efficient layerwise electromechanical FE model was proposed using the recently developed facet shell element based on coupled third-order zig-zag theory.

Focusing on the simulation of interactions occurring locally at the delamination interface, limited research has been conducted. The first study [30] concentrated on the modeling at the zone of delamination, assuming negligible opening and a shear stress applied between the sliding interfaces, while Ghohal et al. [23] and Chattopadhyay et al. [31] presented a model simulating the contact between the delamination faces of a composite, as a function of the relative position of the two crack faces. In another high-order layerwise approach, a penalty constraint was introduced at the interface, preventing penetration [32]. However, none of the available studies realistically accounted for the relative local movements (contact and slip).

The investigation into the effect of delamination on the global transient dynamic response and wave propagation, conducted through both computational and experimental studies, has resulted in various publications. Ostachowitz et al. [33] developed a four-region type analytical model to simulate the transient response of delaminated composite beams. Chrysochoidis and Saravanos [34,35] presented the transient response of smart composite beams experimentally and analytically, revealing the effect of damage for various delamination sizes. Rekatsinas et al. [27] developed a cubic spline layerwise time domain spectral element to study the effect of delamination in guided wave mode conversion. Special attention should be paid to capturing experimentally the nonlinear interactions that occur at the delamination zone [36–39].

On the same path as the research above, Ramadas et al. also moved [40] by studying the effect of the primary anti-symmetric guided wave mode in the presence of delamination in composite strips, a study which was later expanded to plates by the same authors [41]. As a consequence of the presence of a delamination, many researchers were inspired to develop quantitative approaches in terms of Time of Flight (ToF) [42–44]. By measuring the change in ToF, the through-thickness position of a delamination crack could be revealed. Another approach to delamination detection was proposed by Shoja et al. [45], which utilizes low-frequency guided waves along with the Pearson correlation coefficient.

The current state of the art reveals a variety of methods and approaches to studying and understanding the presence and characteristics of delaminations; on the other hand, it also suggests that there is room for more efficient tools and practices to emerge. Thus, the current paper addresses interfacial contact and sliding at delamination faces directly through the layerwise theory without the introduction of external terms. The formulation leverages new terms introduced in interlaminar generalized strains, which are directly related to crack opening and sliding, and are derived from the kinematic assumptions. The developed, realistic, and fully electromechanically coupled FE model is capable of describing actuation via a piezoceramic actuator, wave propagation across the length of the beam, interaction with the delamination interfaces, as well as the reception of the propagated wave at the piezoceramic sensor. Simulations are correlated with representative experiments for various delamination sizes, validating the model performance. Additionally, delamination signatures in predicted time responses, such as amplitude and time of flight, are detected through both analytical and experimental results. At the same time, the continuous propagation of axial and shear waves is presented across delaminated active sensory beams, with contour plots provided to map the exact position of the damage.

2. Theoretical Formulation

An analytical model was developed for analyzing the transient response of active sensory delaminated composite beams, which assumed contact between the delamination faces and control of the relative horizontal slip [46].

2.1. Equations of Equilibrium

The variational statement of the equations of equilibrium for the piezoelectric structure is as follows:

$$\int_A -\delta\mathcal{H}dA + \int_A \delta\mathbf{u}(-\rho\ddot{\mathbf{u}})dA + \oint_{\Gamma_\tau} \delta\bar{\mathbf{u}}^T\bar{\boldsymbol{\tau}}d\Gamma + \oint_{\Gamma_D} \delta\bar{\varphi}\bar{D}d\Gamma = 0 \quad (1)$$

where A is the x - z surface of the beam; $\mathcal{H}$ is in general the electric enthalpy of the material; $\bar{\boldsymbol{\tau}}$ and $\bar{D}$ are, respectively, the surface tractions and charge, acting on the boundary surface Γ_τ, Γ_D ; $\mathbf{u} = \begin{Bmatrix} u \\ w \end{Bmatrix}$ is the displacement vector.

The electric enthalpy includes the components of elastic strain energy and electric energy. Thus, the electric enthalpy in the m-th layer is expressed in variational form as follows:

$$\int_A -\delta \mathcal{H} dA + \int_A \delta \underline{u}(-\rho \ddot{\underline{u}}) dA + \oint_{\Gamma_\tau} \delta \underline{u}^T \bar{\tau} d\Gamma + \oint_{\Gamma_D} \delta \bar{\varphi} \bar{D} d\Gamma = 0 \quad (2)$$

where z_m and z_{m+1} indicate the bottom and top surface location of the layer, respectively.

2.2. Governing Materials Equations

Each ply or layer is assumed to exhibit linear piezoelectric behavior. In the current beam case, in-plane and interlaminar shear strains are considered in the elastic field. The piezoelectric materials are assumed equivalent to monoclinic class 2 crystals with the poling direction coincident with the z-axis. The constitutive equations have the following form:

$$\begin{cases} \sigma_1 \\ \sigma_5 \end{cases} = \begin{bmatrix} C_{11}^E & 0 \\ 0 & C_{55}^E \end{bmatrix} \cdot \begin{cases} S_1 \\ S_5 \end{cases} - \begin{bmatrix} 0 & e_{31} \\ e_{15} & 0 \end{bmatrix} \cdot \begin{cases} E_1 \\ E_3 \end{cases} \\ \begin{cases} D_1 \\ D_3 \end{cases} = \begin{bmatrix} 0 & e_{15} \\ e_{31} & 0 \end{bmatrix} \cdot \begin{cases} S_1 \\ S_5 \end{cases} + \begin{bmatrix} \epsilon_{11}^S & 0 \\ 0 & \epsilon_{33}^S \end{bmatrix} \cdot \begin{cases} E_1 \\ E_3 \end{cases} \quad (3)$$

σ_1 , σ_5 and S_1 , and S_5 are the axial and shear mechanical stresses and strains respectively; E_1 and E_3 are the electric field vectors; D_1 and D_3 are the electric displacement vectors; C_{11} , C_{33} , and C_{55} are the elastic stiffness coefficients; e_{15} and e_{31} are the piezo-electric coefficients; and ϵ_{11} , ϵ_{33} are the electric permittivity coefficients of the material. Superscripts E and S indicate constant electric field and strain conditions, respectively. The above system of equations encompasses the behavior of a beam's piezocomposite layer. The electric field vector is the negative gradient of the electric potential φ :

$$E_k = -\frac{\partial \phi}{\partial x_k} \quad k = 1, 3 \quad (4)$$

2.3. Kinematic Assumptions

Kinematic hypotheses of a coupled layerwise theory for piezoelectric laminates [47] are adopted as the basis for the present model, admitting piecewise linear (zig-zag) fields through the thickness. A typical laminate is assumed to be subdivided into N discrete layers, where each discrete layer may be defined to contain a single ply, a sub-laminate, or a sub-ply, depending on the desired detail of approximation. Linear fields are assumed in each discrete layer for both the in-plane and electric potential fields throughout the laminate thickness. In addition, multiple delamination cracks are also considered at the interfaces of adjacent plies. In such a case, additional terms are included in the displacement field to represent the interlaminar discontinuities in the displacement fields, due to delaminations. The resultant zig-zag field maintains continuity across the discrete layer boundaries, yet allows for different in-plane displacement slopes within each discrete layer, and admits sliding and opening across a delamination crack (see Figure 1). Considering a grading of N discrete layers and taking into account N_d delaminations, the assumed displacements and electric field take the following form:

$$u(x, z) = \sum_{i=1}^N \left(u^i(x) \psi^i(z) \right) + \sum_{k=1}^{N_d} \left(\tilde{u}^k(x) \psi^k(z_k) H(z - z_k) \right) \quad (5)$$

$$w = w^o(x) + \sum_{k=1}^{N_d} \tilde{w}^k H(z - z_k) \quad (6)$$

$$\varphi(x, z) = \sum_{i=1}^N \left(\Phi^i(x) \psi^i(z) \right) \quad (7)$$

where superscripts $i = 1, \dots, N$ indicate the $N - 1$ discrete layers, $k = 1, \dots, N_d$ the number of the delaminations through the thickness, and “o” the mid-plane. In (5), the term u^i is the in-plane displacement at the interface of each discrete layer, effectively describing the extension and rotation of the layer. Also, $\tilde{u}^k$ and $\tilde{w}^k$ are new degrees of freedom describing the sliding and opening across the faces of the k^{th} delamination. $H(z)$ is the Heaviside’s step function and z_k is the distance of the k^{th} delamination from the mid-plane. Finally, ψ^i and ψ^k are the linear interpolation functions through the laminate thickness for the i^{th} layer and the k^{th} delamination, given by the following:

$$\psi^i(z) = \begin{cases} \frac{z - z^{i-1}}{z^i - z^{i-1}}, & z \leq z^i \\ \frac{z^{i+1} - z}{z^{i+1} - z^i}, & z > z^i \end{cases} \quad (8)$$

$$\psi^k(z) = \frac{z^{k+1} - z}{z^{k+1} - z^k} \quad (9)$$

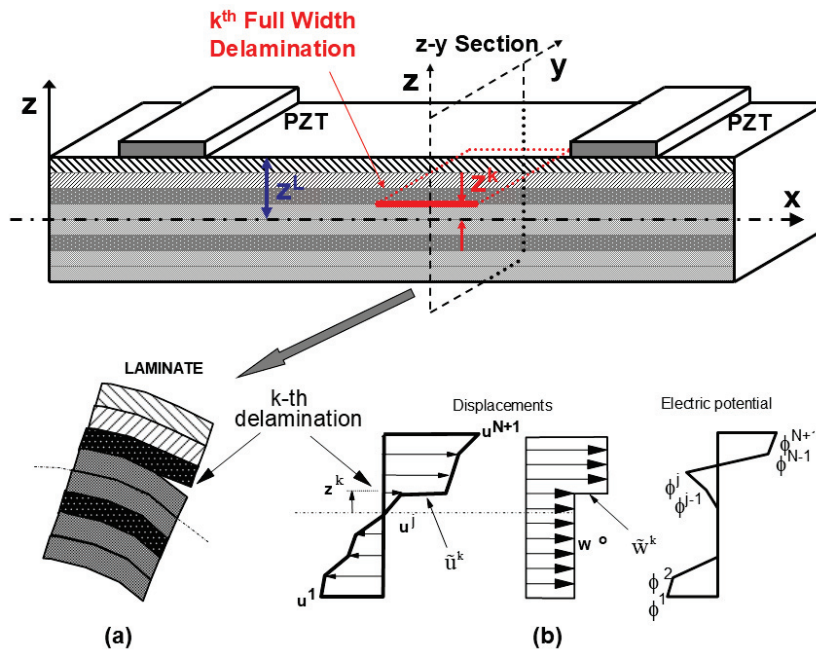


Figure 1. Illustration of the assumed displacement and electrical fields through the thickness of a delaminated composite laminate. (a) Smart laminate, (b) Discrete-layer laminate theory assumptions.

2.4. Strain–Displacement Relations and Electrical Field

In the context of kinematic assumptions (5) and (6), the axial and shear strains S_1 and S_5 of a laminate consisting of N layers and N_d delamination cracks take the following form:

$$S_1 = \sum_{i=1}^N S_1^i \psi^i + \sum_{k=1}^{N_d} \tilde{S}_1^k \psi^k H(z - z_k) \quad (10)$$

$$S_3 = \sum_{k=1}^{N_d} \tilde{w}^k \delta(z - z_k) \quad (11)$$

$$S_5 = w_{,x}^o + \sum_{i=1}^N u^i \psi_{,z}^i + \sum_{k=1}^{N_d} \left[\left(\tilde{w}_{,x}^k + \tilde{u}^k \psi_{,z}^k \right) H(z - z_k) + \tilde{u}^k \psi^k \delta(z - z_k) \right] \quad (12)$$

where $\delta(z)$ is the Dirac function. In the axial strain Equation (10), $S_1^i = u_{,x}^i$ represents generalized axial strains of a healthy laminate, whereas $\tilde{S}_1^k = \tilde{u}_{,x}^k$ are new strain components expressing the effect of the k^{th} delamination on the axial strain above the crack. We further focus attention on new generalized terms appearing in interlaminar strains. In the normal

strain Equation (11), $\tilde{w}^k \delta(z - z_k)$ is a new component expressing the relative transverse movement of the delamination crack faces. We define $\tilde{S}_3^k = \tilde{w}^k$ and take advantage of this term in order to naturally describe the contact phenomena between the delamination interfaces as explained below. In the shear strain Equation (12), the generalized strain term is as follows:

$$\tilde{S}_5^k = \tilde{w}_{,x}^k + \tilde{u}^k \psi_{,z_k}^k \quad (13)$$

This represents a constant shear strain added to all layers above the crack by the presence of the k^{th} delamination, and the last generalized strain term is as follows:

$$\hat{S}_5^k = \tilde{u}^k \quad (14)$$

This is multiplied by the Dirac function, which represents the relative sliding between delamination faces, and has meaning only at the delamination interface.

Also, combining Equations (4) and (7), the transverse electric field in each discrete layer is derived as follows:

$$E_3^i = - \sum_{i=1}^N (\Phi^i \psi_{,z}^i) \quad (15)$$

This is a constant approximation of the transverse electric field in each discrete layer. The previous strain and electrical field equations were included in the variational form of equilibrium equations, and the equivalent stiffness and mass matrices were derived.

2.5. Delamination Interface Interactions

Up to this point, the formulation described above does not account for interfacial effects and interactions between the crack faces, which allow for penetration between them. To remedy this weakness, and take advantage of strain terms $\tilde{S}_3^k$ and $\tilde{S}_5^k$ in Equations (11) and (12), offering a realistic model for delaminated beams, the generalized stress $\hat{\sigma}_3^k$ and $\hat{\sigma}_5^k$ are considered in the formulation for the k^{th} delamination crack, which is further related to the generalized strains $\tilde{S}_3^k$ and $\tilde{S}_5^k$, respectively, through the following constitutive relationships:

$$\hat{\sigma}_3^k = \begin{cases} \hat{A}_{33}^k \tilde{S}_3^k, & \tilde{S}_3^k < 0 \\ 0, & \tilde{S}_3^k > 0 \end{cases} \quad \hat{\sigma}_5^k = \begin{cases} \hat{A}_{55}^k \tilde{S}_5^k, & \hat{\sigma}_3^k \neq 0 \\ 0, & \hat{\sigma}_3^k = 0 \end{cases} \quad (16)$$

where $\hat{A}_{33}^k$ and $\hat{A}_{55}^k$ are coefficients to be determined, and their values are defined in the next section. These generalized constitutive relations effectively provide nonlinear contact and sliding stiffness, thereby controlling penetration and free axial slip when the two delamination faces are in contact. Due to the presence of the Dirac function, the stresses $\hat{\sigma}_3^k$ and $\hat{\sigma}_5^k$ only have meaning at the delamination interfaces and represent the contact stresses at the interface. The above system of generalized stresses works as follows: when the faces of the k^{th} delamination is open, $\tilde{S}_3^k$ is positive, and both $\hat{\sigma}_3^k$ and $\hat{\sigma}_5^k$ vanish; hence, no stresses are applied at the interface. On the other hand, when the crack closes ($\tilde{S}_3^k \leq 0$), $\hat{\sigma}_3^k$ becomes nonzero and resists penetration; at the same time, the shear stress $\hat{\sigma}_5^k$ acts along the axial direction of the crack faces to resist slipping. The described system of constitutive Equations (16) above is nonlinear and introduces nonlinearity into the formulation.

2.6. Laminate Electric Enthalpy and Laminate Matrices

Considering the assumptions of this work and combining Equations (4), (10)–(12), and (14), the electric enthalpy of the laminate in each discrete layer i takes the following form:

$$\delta \mathcal{H}^i = \sum_{\lambda=1}^{N_p^i} \int_{z_\lambda}^{z_{\lambda+1}} \left(\delta S_1^\lambda C_{ij}^\lambda S_j^\lambda - \delta E_3^\lambda e_{31}^\lambda S_1^\lambda - \delta S_1^\lambda e_{31}^\lambda E_3^\lambda - \delta E_3^\lambda \epsilon_{33}^\lambda E_3^\lambda \right) dz \quad (17)$$

where $i, j = 1, 5$ and λ denotes the λ^{th} ply in the i^{th} discrete layer; N_p^i denotes the plies of the i^{th} discrete layer. Also, assuming that, in most cases, continuous electrodes exist on the surface of a piezoceramic layer or wafer, which imposes a uniform axial voltage distribution, the axial component of the electrical field in Equation (4) was ignored. Interfacial contact and friction between the delamination faces are introduced via the terms and constitutive Equations (16) described previously. Combining Equations (10)–(12), (14) and (17), integrating through the thickness of each discrete layer and collecting the common terms, the variation of laminate enthalpy is related to the resultant laminate stiffness, piezoelectric and permittivity matrices:

$$\begin{aligned} \delta \mathcal{H}_L = & \delta w_{,x} A_{55} w_{,x} + \sum_{m=1}^N (\delta w_{,x} B_{55}^m u^n + \delta u^m B_{55}^m w_{,x}) + \sum_{m=1}^N \sum_{n=1}^N (\delta u_{,x}^m D_{11}^{mn} u_{,x}^n + \delta u^m D_{55}^{mn} u^n) + \\ & \sum_{k=1}^{N_d} \left(\delta w_{,x} \bar{A}_{55}^k \tilde{w}_{,x}^k + \delta w_{,x} \bar{B}_{55}^k \tilde{u}^k + \delta \tilde{w}_{,x}^k \bar{A}_{55}^k w_{,x} + \delta \tilde{u}^k \bar{B}_{55}^k w_{,x} \right) + \\ & \sum_{m=1}^N \sum_{k=1}^{N_d} \left(\delta u_{,x}^m \bar{D}_{11}^{mk} \tilde{u}_{,x}^k + \delta u^m \bar{F}_{55}^{mk} \tilde{w}_{,x}^k + \delta u^m \bar{D}_{55}^{mk} \tilde{u}^k + \delta \tilde{u}_{,x}^k \bar{D}_{11}^{mk} u_{,x}^m + \delta \tilde{w}_{,x}^k \bar{F}_{55}^{mk} u^m + \delta \tilde{u}^k \bar{D}_{55}^{mk} u^m \right) + \\ & \sum_{k_1=1}^{N_d} \sum_{k_2=1}^{N_d} \left(\delta \tilde{u}_{,x}^{k_1} \tilde{D}_{11}^{k_1 k_2} \tilde{u}_{,x}^{k_2} + \delta \tilde{w}_{,x}^{k_1} \tilde{A}_{55}^{k_1 k_2} \tilde{w}_{,x}^{k_2} + \delta \tilde{w}_{,x}^{k_1} \tilde{B}_{55}^{k_1 k_2} \tilde{u}^{k_2} + \delta \tilde{u}^{k_1} \tilde{B}_{55}^{k_1 k_2} \tilde{w}_{,x}^{k_2} + \delta \tilde{u}^{k_1} \tilde{D}_{55}^{k_1 k_2} \tilde{u}^{k_2} \right) + \\ & \sum_{k_1=1}^{N_d} \sum_{k_2=1}^{N_d} \left(\delta \tilde{u}^{k_1} \hat{D}_{55}^{k_1 k_2} \tilde{u}^{k_2} + \delta \tilde{w}^{k_1} \hat{A}_{33}^{k_1 k_2} \tilde{w}^{k_2} \right) + \\ & \sum_{m=1}^N \sum_{n=1}^N (\delta \Phi^m P_{31}^{mn} u_{,x}^n + \delta \Phi^m G_{33}^{mn} \Phi^n) + \sum_{m=1}^N \sum_{k=1}^{N_d} \left(\delta \Phi^m \bar{P}_{31}^{mk} \tilde{u}_{,x}^k \right) \end{aligned} \quad (18)$$

In the previous equation, D_{11}^{mn} is the in-plane generalized laminate stiffness, and A_{55}^{mn} , B_{55}^{mn} , and D_{55}^{mn} are the interlaminar generalized laminate stiffness matrices of the undamaged section. In the delaminated region, additional stiffness terms exist: $\bar{D}_{11}^{mk}$ is the generalized in plane laminate stiffness, and $\bar{A}_{55}^k$, $\bar{B}_{55}^k$, $\bar{F}_{55}^{mk}$, and $\bar{D}_{55}^{mk}$ are the generalized shear stiffness matrices. Each one of the overbar terms is a new stiffness term in the damage section, representing coupling between pristine and delamination degrees of freedom. Additionally, $\tilde{D}_{11}^{k_1 k_2}$, $\tilde{A}_{55}^{k_1 k_2}$, $\tilde{B}_{55}^{k_1 k_2}$, and $\tilde{D}_{55}^{k_1 k_2}$ are new laminate stiffness matrices introduced by the delamination degrees of freedom. Similarly, P_{31}^{mn} are the generalized piezoelectric coefficients of the pristine laminate, and $\bar{P}_{31}^{mk}$ represents additional generalized effective piezoelectric coefficients of the laminate due to delamination. The above overbar terms are new and describe the effect of displacement disconnect on the effective direct and converse piezoelectric effect of the respective piezoelectric layer, and G_{33}^{mn} are the generalized permittivity laminate matrices. All the above matrices are provided in the Appendix A section of this paper. Special attention is required for the matrices $\hat{A}_{33}^{k_1 k_2}$, and $\hat{D}_{55}^{k_1 k_2}$, which are the new generalized stiffness terms due to interfacial contact and slip between the delamination surfaces as described in the previous section, and are given from the following:

$$\hat{A}_{33}^{k_1 k_2} = \sum_{l=1}^{N_p} \int_{z_l}^{z_{l+1}} C_{33}^l \delta(z - z_{k_1}) \delta(z - z_{k_2}) dz \quad (19)$$

$$\hat{D}_{55}^{k_1 k_2} = \sum_{l=1}^{N_p} \int_{z_l}^{z_{l+1}} a_5^l \delta(z - z_{k_1}) \delta(z - z_{k_2}) dz \quad (20)$$

In the equations above C_{33}^l and a_5^l are coefficients that need to be determined. The first coefficient, without loss of generality, is assumed equal to the normal elasticity modulus E_{33} .

The second coefficient depends on the quality of the two interfaces that are in contact and is activated only when $\tilde{S}_3^k$ is negative. The authors have performed various computational studies, concluding that the value of the term a_5^1 should vary in the range 1×10^8 to 1×10^{10} , as for lower values the response remains insensitive to the presence of this term, and on the other hand, if the value of this term is over the range eliminates the axial movement.

2.7. Finite Element Formulation

A beam finite element was formulated based on the previous laminate mechanics, implementing C_1 continuous Hermitian polynomials $H^i(x)$ for the local approximation of the transverse displacements and C_0 shape functions $N^i(x)$ for the remaining axial and electric DOFs. In this manner, the local approximations of the generalized state variables in the element take the following form:

$$\begin{aligned} u^i(x, t) &= \sum_{m=1}^L u^{im}(t) N^m(x) \\ \tilde{u}^k(x, t) &= \sum_{m=1}^L \tilde{u}^{km}(t) N^m(x) \\ w^o(x, t) &= \sum_{m=1}^L \left(w^{om}(t) H_1^m(x) + \frac{L_e}{2} w_{,x}^{om}(t) H_2^m(x) \right) \\ \tilde{w}^k(x, t) &= \sum_{m=1}^L \left(\tilde{w}^{km}(t) H_1^m(x) + \frac{L_e}{2} \tilde{w}_{,x}^{km}(t) H_2^m(x) \right) \\ \Phi^k(x) &= \sum_{i=1}^L \Phi^{ki} N^i(x) \end{aligned} \quad (21)$$

where $i = 1, \dots, N+1$ and $k = 1, \dots, N_d$ indicate discrete layer and delamination; $m = 1, \dots, L$ is the node number; L_e denotes the element length. A 2-node ($L = 2$) beam element was further developed and encoded in a prototype research code employing linear interpolation functions N^i and cubic Hermitian polynomials H_1^i, H_2^i .

Substituting Equations (18) and (21) into the governing equations of equilibrium (1), then collecting the common coefficients, the coupled piezoelectric system was expressed in the standard discrete matrix form:

$$\begin{aligned} &\begin{bmatrix} [M_{uu}] & 0 \\ 0 & 0 \end{bmatrix} \begin{Bmatrix} \{\ddot{u}(t)\} \\ \{\ddot{\phi}^S(t)\} \end{Bmatrix} + \begin{bmatrix} [C_{uu}] & 0 \\ 0 & 0 \end{bmatrix} \begin{Bmatrix} \{\dot{u}(t)\} \\ \{\dot{\phi}^S(t)\} \end{Bmatrix} + \begin{bmatrix} [K_{uu}]^{(t)} & \begin{bmatrix} K_{u\phi}^{FF} \\ K_{\phi\phi}^{FF} \end{bmatrix} \\ \begin{bmatrix} K_{\phi u}^{FF} \\ K_{\phi\phi}^{FF} \end{bmatrix} & \end{bmatrix} \begin{Bmatrix} \{\bar{u}(t)\} \\ \{\phi^F(t)\} \end{Bmatrix} = \\ &= \begin{Bmatrix} \{P(t)\} - [K_{u\phi}^{FA}] \{\phi^A(t)\} \\ \{Q^F\} - [K_{\phi\phi}^{FA}] \{\phi^A(t)\} \end{Bmatrix} \end{aligned} \quad (22)$$

Submatrices $[K_{uu}]$, $[K_{u\phi}]$, and $[K_{\phi\phi}]$ indicate the elastic, piezoelectric, and permittivity matrices of the structure; $[M_{uu}]$ indicates the mass matrix. It is worth noting that the previous system matrices implicitly incorporate the delamination parameters into the system response. A proportional damping matrix $[C_{uu}]$ is also assumed to be calculated through mass and stiffness matrices for a specific frequency range. The piezoelectric and capacitance matrices have been partitioned into active and sensory parts. Superscripts F and A indicate, respectively, sensory (free) and active (applied) electric potential components; $\{P\}$ is the applied mechanical forces vector, and $\{Q^F\}$ is the applied electric charge at the sensors.

Also, the above system is nonlinear, since the stiffness matrix $[K_{uu}]$ depends on the local delamination displacements. For the solution of this non-linear problem, a modified Newmark implicit time integration method is used, combined with Newton–Raphson iterations [48], to calculate the model response when the delamination faces contact criteria

are activated. The convergence of the Newton–Raphson solution scheme was satisfied in each time step.

3. Materials, Experimental Methods, and Simulation Parameters

Experiments were conducted in order to investigate the time response of delaminated composite beams. Composite beam specimens with a single delamination crack (Figure 2a) were tested using piezoceramic devices, as shown in Figure 2b. The beams had a $[0/90/45/-45]_S$ laminate configuration, with T300/934 graphite epoxy plies of fiber volume ratios in the range of 0.57–0.63, a nominal thickness of 0.127 mm, a length of 280 mm, and a width of 25 mm. Four different sizes of delamination were artificially created at the mid-plane of the composite using Teflon tape during the pre-preg manufacturing procedure, covering the full width across the beam. The Teflon tape was later removed to avoid compromising the validity of the experimental measurements. Four specimens were tested: one undamaged and three with small, medium, and large delamination cracks covering 0%, 10.9%, 21.8%, and 43.6% of the total beam length, respectively.

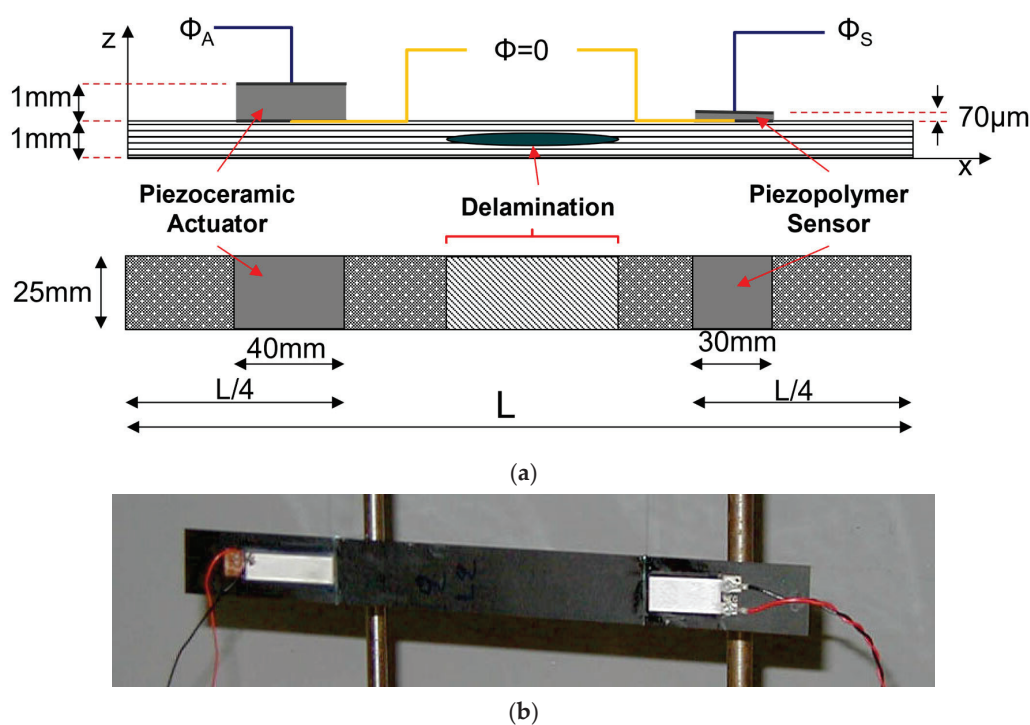


Figure 2. Active sensory delaminated composite beam. (a) Schematic illustration of the delamination position. (b) Photo of the tested specimens.

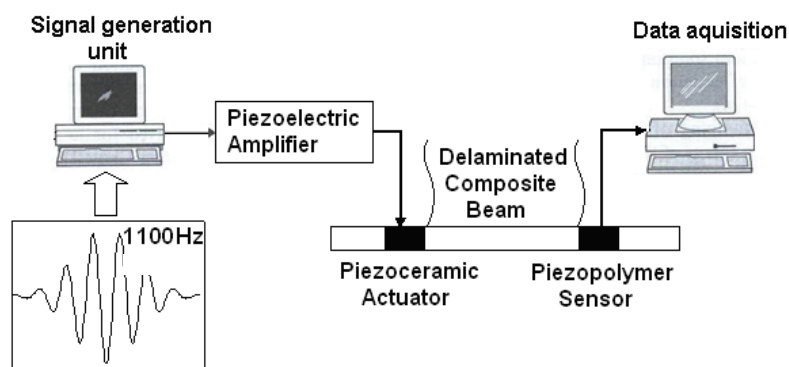
These specimens were tested under a free-free supporting configuration in order to eliminate friction due to supports. Specimens were excited via piezoceramic devices having dimensions $40 \times 10 \times 1$ mm. The piezoceramics were the PZ27 models of the Ferroperm-Piezo brand. Additionally, to complete the “pitch and catch” testing configuration, the authors used piezopolymer sensors attached to the specimen’s surface to measure the specimen’s vibration. Both the actuator and the sensor were bonded near the modal points of the 1st mode shape, as illustrated in Figure 2a. The piezopolymer dimensions were 30 mm $\times$ 12 mm $\times$ 70 μm , while the mechanical properties of the piezocomposite materials, as well as the mechanical and electrical properties of the piezoelectric devices, are incorporated in Table 1.

Table 1. Properties of composite and piezoelectric materials.

Property	T300/934	Pz27 (Ferroperm)	PVDF (LDT1_028K)
E ₁₁ (GPa)	127	58.82	3
E ₃₃ (GPa)	7.9	43.1	6
ν_{13}	0.275	0.371	0.3
G ₁₃ (GPa)	3.4	22.98	1
d ₃₁ (m/V)	0	-170×10^{-12}	-23×10^{-12}
d ₃₃ (m/V)	0	425×10^{-12}	33×10^{-12}
d ₁₅ (m/V)	0	506×10^{-12}	
ϵ_{31} (farad/m)	0	15.94×10^{-9}	106×10^{-12}
ϵ_{11} (farad/m)	0	15.94×10^{-9}	106×10^{-12}
ρ (kg/m ³)	1578	7700	1780

3.1. Testing Apparatus

In order to obtain the free supporting configuration, the specimens were supported with strings attached approximately at the modal points of the first bending mode. A virtual instrument was developed based on Labview® to digitally generate tone-burst excitation signals having various central frequencies. The signal was converted to analog using a 16-bit DAQ card. The actuation signal was amplified through an appropriate piezodrive amplifier, and it was applied at the terminals of a piezoceramic actuator epoxied on the surface of the composite specimen. The response of the beam was acquired at the other free end of the specimen, measuring the voltage at the terminals of the PVDF piezopolymer film sensor. The measured voltage at the piezoelectric sensors was digitized through a high-speed DAQ board. The described testing configuration is illustrated in Figure 3.

**Figure 3.** Experimental setup for the measurement of the time response.

3.2. Simulation Parameters

To simulate the response of the active-sensory composite beam with the delamination debonding, the in-house model used the following discretization:

- (1) Through the thickness, following the linear layerwise assumptions and the requirement for accurate displacement and stress distributions, 30 discrete layers were used; 24 layers (3 for each ply) evenly distributed for the composite laminate, 2 layers on top of the laminate (6.3 μm thick each) for the bonding adhesive of the piezelectrics and 4 layers for the piezoceramic actuator or 1 layer for the piezopolymer sensor.
- (2) Across the length, for the FEM mesh, we used 30 beam elements distributed across the length. The length of each beam was selected appropriately to simulate the exact position of the actuator–sensor pair, as well as to provide a denser mesh in the delamination zone.

The time response was performed with a time step of $\Delta t = 0.02$ ms, and the actuation signal was applied from the beginning of the analysis.

4. Results and Discussion

This section evaluates the performance of the analytical model against the experimental measurements. The numerical predictions of both linear and non-linear systems are compared, and the derived outcome reveals the necessity of the interfacial contact proposed by the authors.

4.1. Analytical Model Efficiency

In this section, the transient response of smart delaminated composite beams with piezoelectric devices was measured. Four different sizes of delamination were tested, covering 0%, 10.9%, 21.8%, and 43.6% of the total specimen length. The derived actuation voltage was a narrowband 7-cycle tone burst signal with a central frequency of around 1 kHz. Based on the previous work of the authors performed on similar specimens [24], most of the dominant modal frequencies of these beams appear in the from range 80 to 500 Hz, so the excitation frequency level of 1 kHz was selected in order to avoid the excitation of dominant modal frequencies. The time signal, as well as its frequency content, is presented in Figure 4a and Figure 4b, respectively. Additionally, the transient response of the specimens mentioned above was simulated via the realistic analytical model.

Two different predictions were assumed. In the first case, the transient response of composite beams allowing crack faces to penetrate was simulated. In contrast, in the second scenario, a non-linear system was assumed through the introduction of interfacial contact and a shear control of the axial movement, simulating a type of friction. The values used for the applied shear force, as presented in Equation (20), were for the small delamination $a_{55} = 1 \times 10^{10}$, and for the other two damage sizes $a_{55} = 1 \times 10^9$. Additionally, proportional damping matrices were calculated for 1KHz.

Figure 5a–d presents both experimental measurements and simulation results for each of the four specimens. Initially, for the pristine specimen (Figure 5a), the two time signals are nearly identical, especially during the first six or seven cycles. After this initial period, the simulation appears to be more damped compared to the experiment. This is due to the damping, which was not analytically formulated but was incorporated as proportional damping through the stiffness and mass matrices. Continuing with the rest of the plots, there is an excellent agreement between the experiment and the model. The introduction of non-linear interactions at the local delamination area enhances the correlation, particularly for beams with small or medium-sized damage. The analytical model accurately captures key characteristics of the time response, such as oscillation frequency, the time of flight, and amplitude, especially for the first six or seven cycles of the transient analysis. The disagreement is evident in the free vibration amplitude due to the damping modeling, as discussed previously. These results demonstrate that the developed realistic model can efficiently simulate the transient response of delaminated active sensory composite beams.

Additional simulations and measurements were obtained for the same set of composite specimens by increasing the central frequency of the applied actuation signal to 5 kHz, as demonstrated in Figure 4c,d. This frequency was selected in order to excite the local oscillations at the delamination interface, as it was previously found [24] that the opening modes of the delamination exist in the range from 4 to 6 kHz, varying with the delamination length. The respective results are illustrated in Figure 6a–d. Each plot represents a different delamination length. Simulations in this case were only for the non-linear model using the same shear force parameters as previously. In this case, for the intact strip and the one with small delamination sizes, the simulations are satisfactorily correlated with the measurements. However, this good agreement disappears after the initial period, presenting differences in both the oscillation amplitude and frequency. For the case of large delamination size (43.6%), differences in comparison appear from the beginning, as each signal requires a different time to travel from the actuator to the sensor. Generally, it was observed for both excitation central frequencies selected for

simulations and testing that significant disagreements were captured in the time responses for large delamination sizes. These disagreements were probably owed to the way friction was simulated at the delamination interface. The shear force applied while the interfaces were in contact always had the same magnitude in all cases. Probably, simulating the magnitude of this force as a function of the normal force at the damage interface, assuming a pre-sliding and sliding region, provides more accurate predictions. Nevertheless, the results suggest that improvements in the sliding model may be required, and this could be a topic for future work.

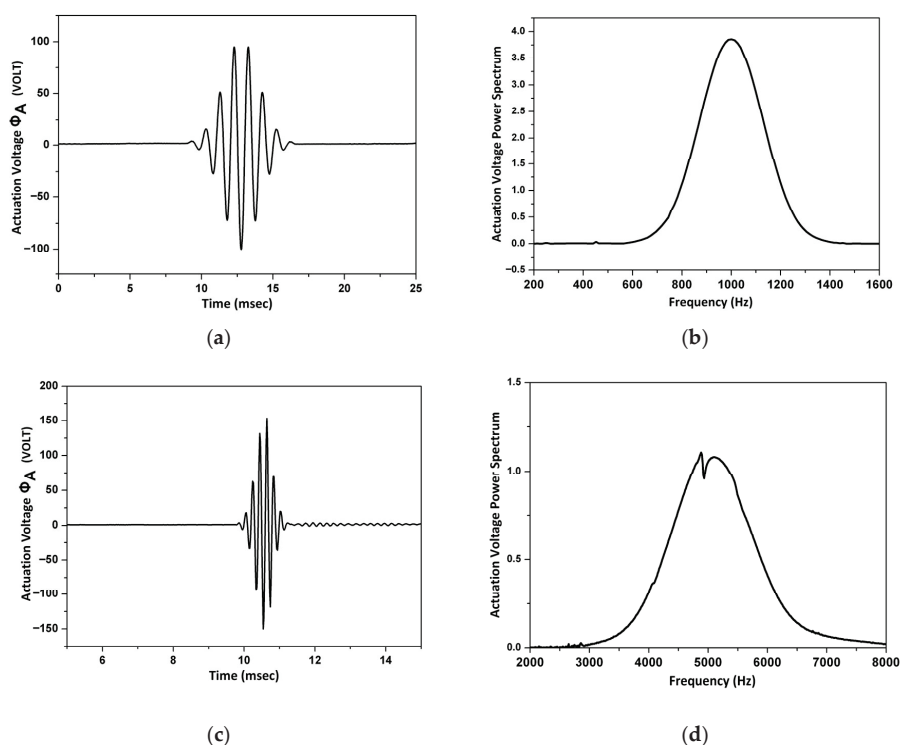


Figure 4. Tone burst actuation signals (a) 1 KHz time waveform, (b) 1 KHz power spectrum, (c) 5 KHz time waveform, (d) 5 KHz power spectrum.

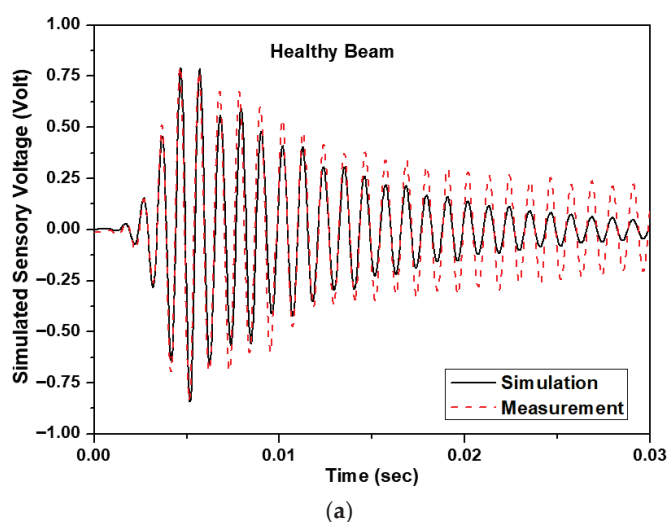


Figure 5. Cont.

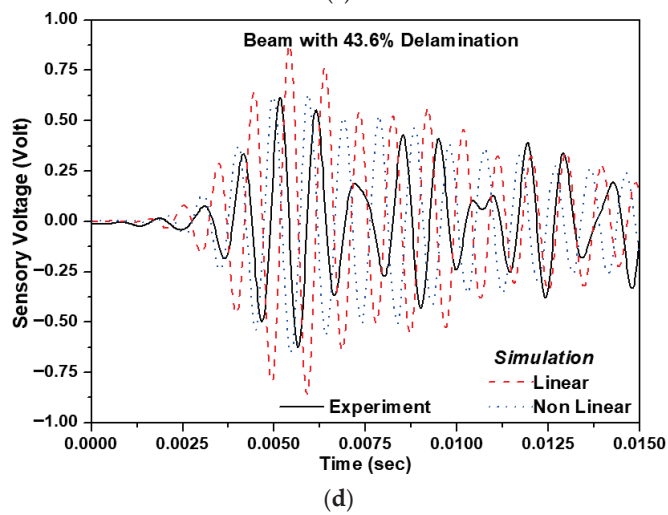
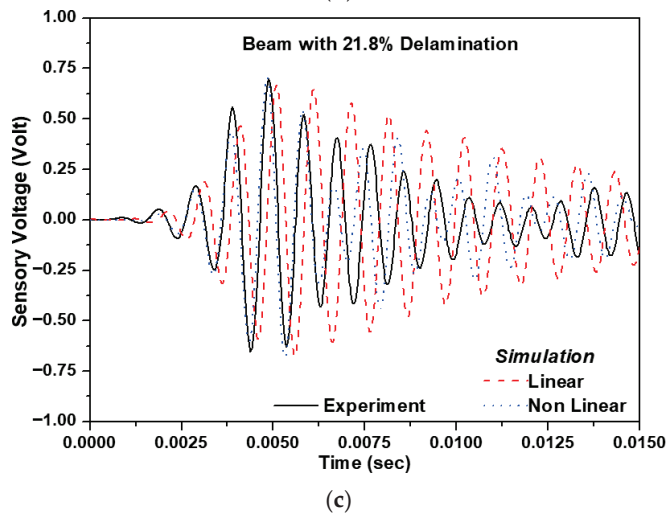
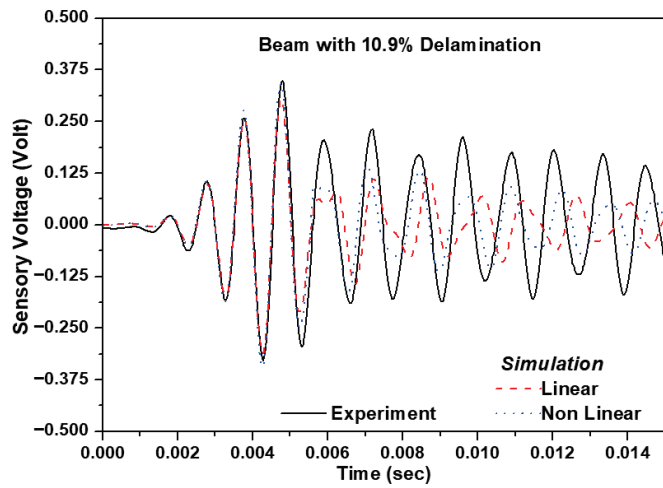


Figure 5. Performance of analytical model to capture experimental predictions under 1 KHz actuation for a beam (a) healthy or with (b) 10.9%, (c) 21.8%, and (d) 43.6% delamination crack, respectively.

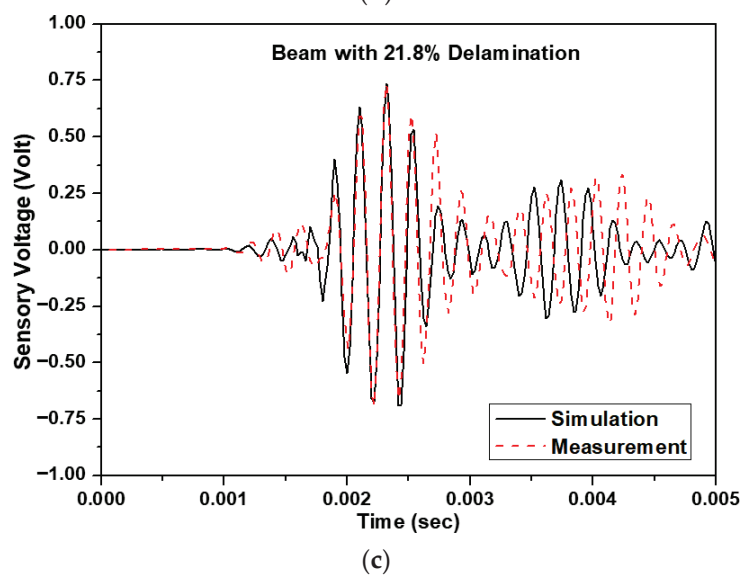
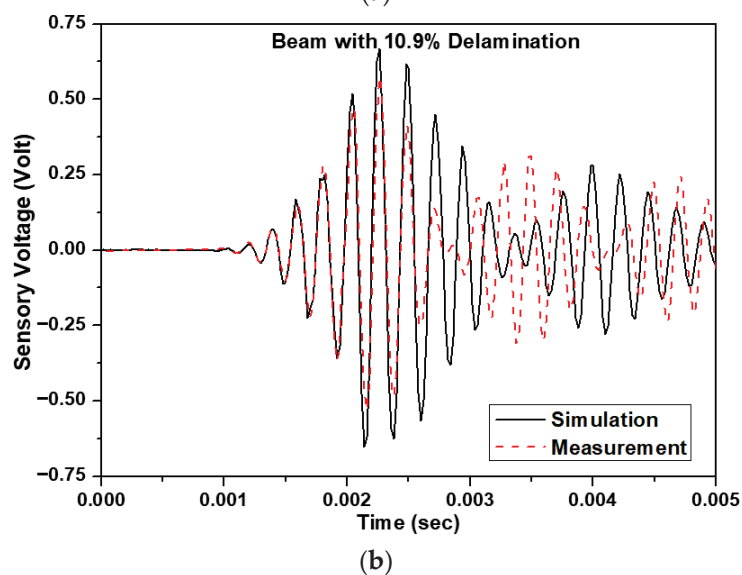
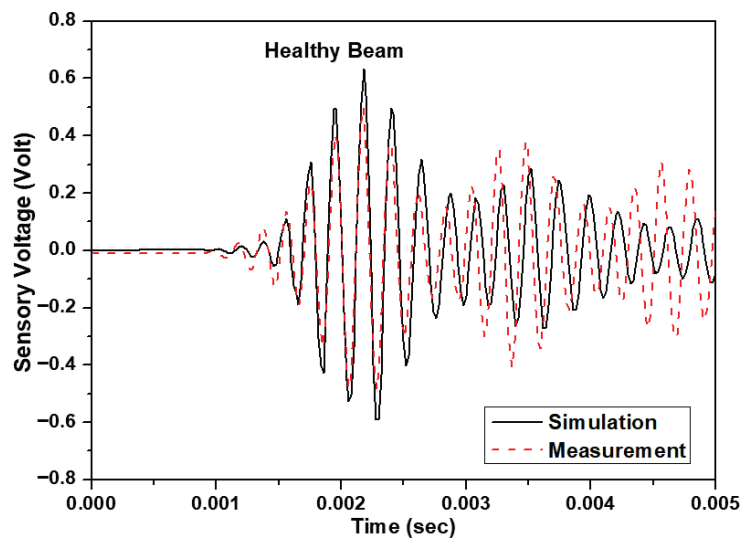


Figure 6. Cont.

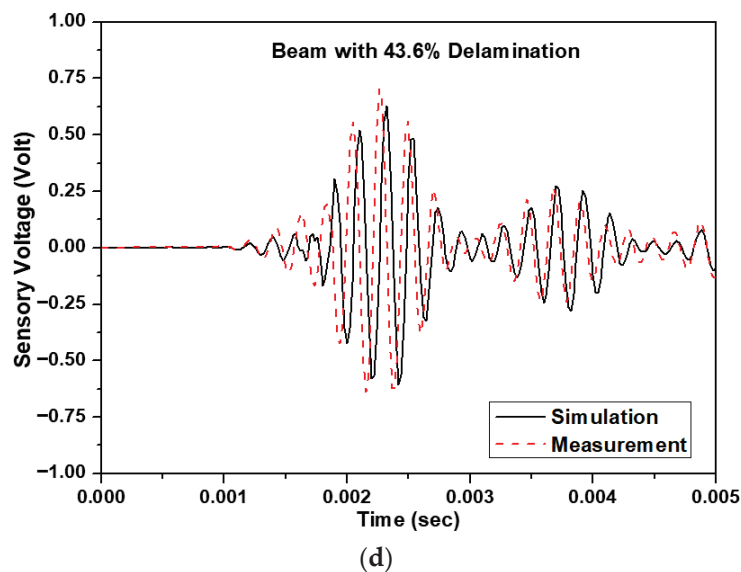


Figure 6. Performance of analytical model to capture experimental predictions under 5KHz actuation for a beam (a) healthy or with (b) 10.9%, (c) 21.8%, and (d) 43.6% delamination crack, respectively.

4.2. Delamination Detection

Following a different perspective on the same results, the authors attempted to elucidate the effect of delamination on the transient response. Delamination indices were investigated on various parameters of the time response, such as (i) the signal amplitude, (ii) the time of flight, i.e., the time required for the signal to travel from the actuator to the sensor, and (iii) the frequency of the voltage signal oscillation. From the two actuation signals, time responses were obtained and used only for the 1KHz actuation frequency.

Initially, for the specimen entailing a small delamination crack length, Figure 7a,b presents analytical predictions and experimental measurements, respectively. Each one of these figures illustrates two different curves representing the healthy and the delaminated beam. It is evident from the results that the introduction of damage causes a drastic reduction in the signal amplitude. Additionally, slight differences are observed in the signal frequency and the time of flight. Both the simulations and the measurements lead to the same conclusions. Similar predictions and simulations for the specimens with the medium and large damage sizes are depicted in Figure 8a,b and Figure 9a,b. Focusing on the investigation of the delamination effects on the time of flight, Figure 10a,b presents the time response for each of the four specimens for the initial 2.5 ms via analytical simulations or experimental measurements, respectively. To assist the observation, we consider the time of flight to be the period between the beginning of excitation and the reception of the first cycle at the sensor. It is evident that the ToF increases as the size of the delamination increases with $t_{PR} < t_S < t_M < t_L$. This conclusion is also validated by the experimental measurements shown in Figure 10b.

For medium and large damage sizes, discrepancies appear in the amplitude, period, and time of flight, which become more pronounced in the case of large delamination sizes. Correlations of time responses for strips with various damage sizes versus the healthy specimen results provide the following conclusions: (i) as the size of delamination increases changes at time of flight and period are also increased and (ii) signal amplitude cannot consist of an indicator of the damage size, as this parameter changes does not appear to be relative to the delamination length. The amplitude of the measured and simulated sensory voltage appears to be more sensitive to another system characteristic, such as the sensor position. However, in any case, the time response appears to be capable of revealing even small delamination sizes using the described pitch and catch configuration. This is very

important, assuming the weak effect of small delamination size on the modal characteristics as presented previously [34].

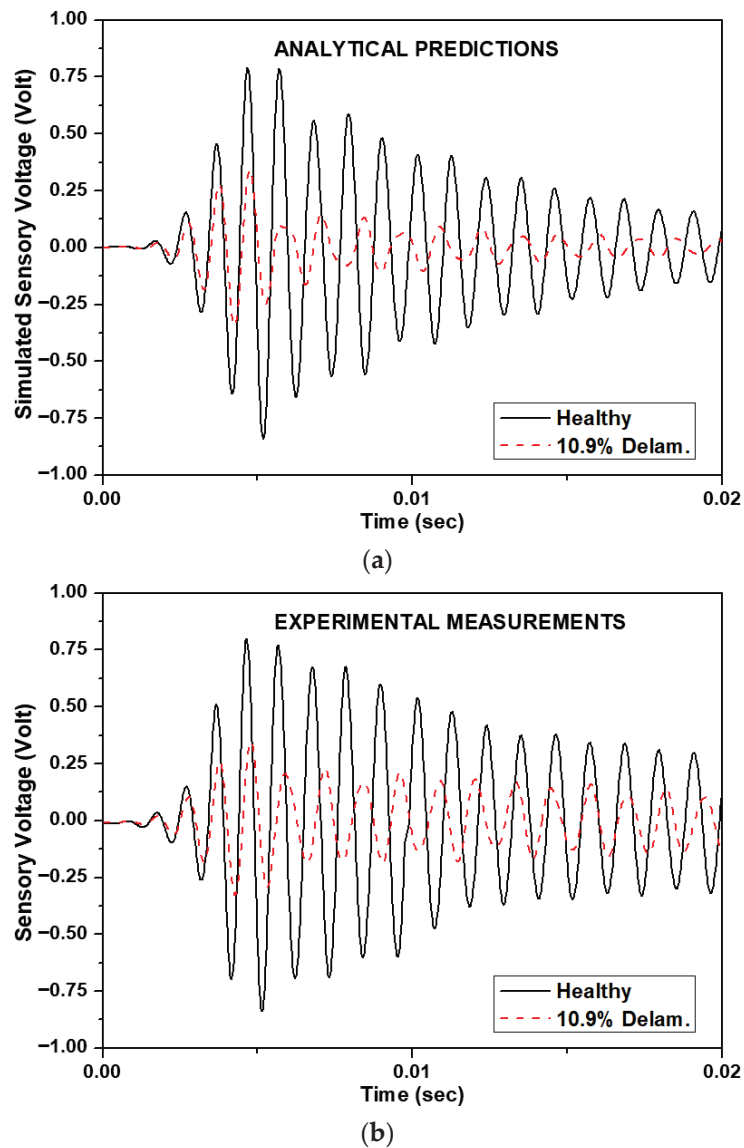


Figure 7. Effect of a delamination covering 10.9% of the total beam length on the (a) predicted and (b) experimentally measured transient response, respectively.

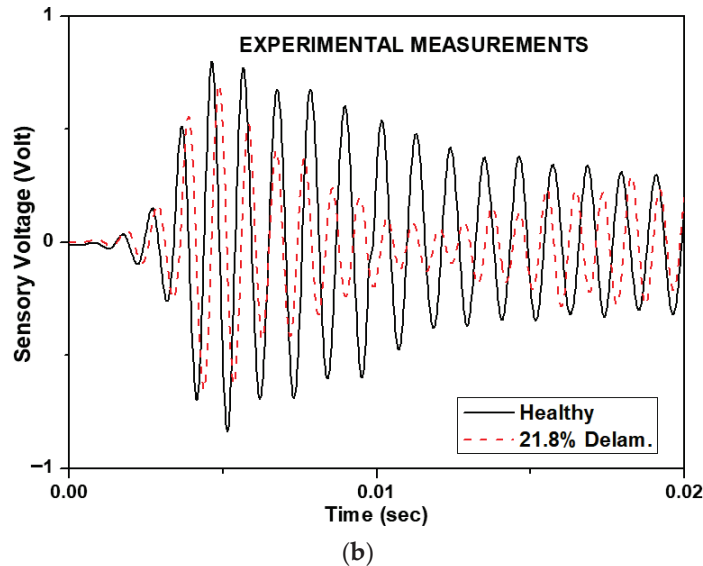
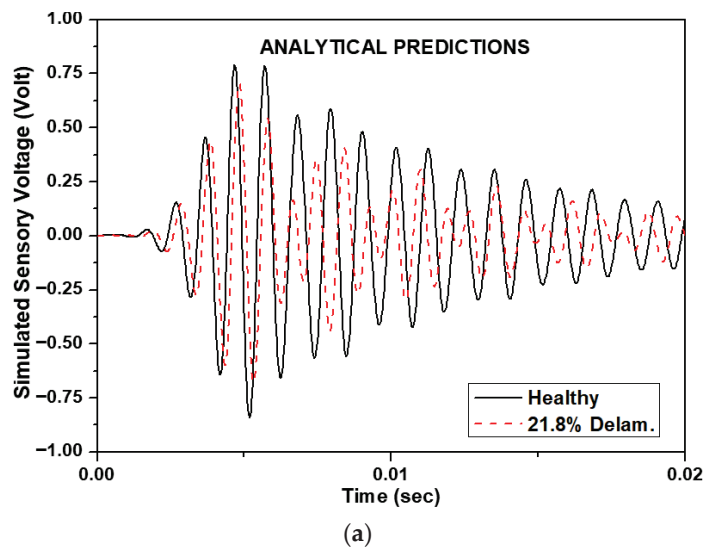


Figure 8. Effect of a delamination crack covering 21.8% of the total beam length on the (a) predicted and (b) experimentally measured transient response, respectively.

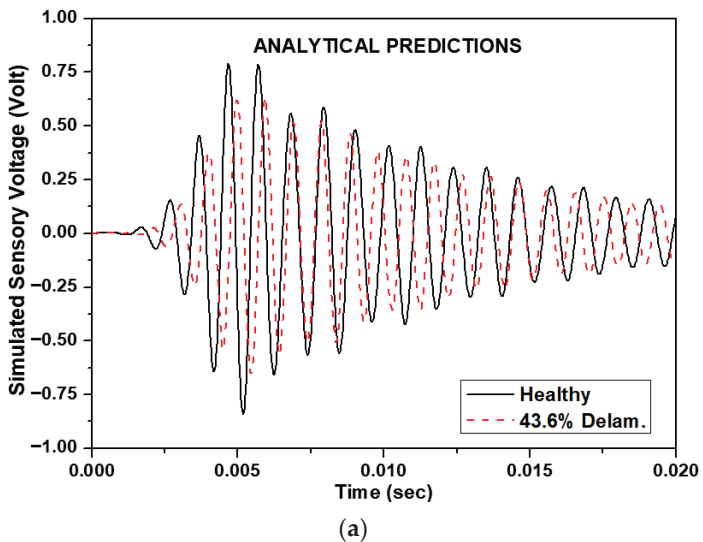


Figure 9. Cont.

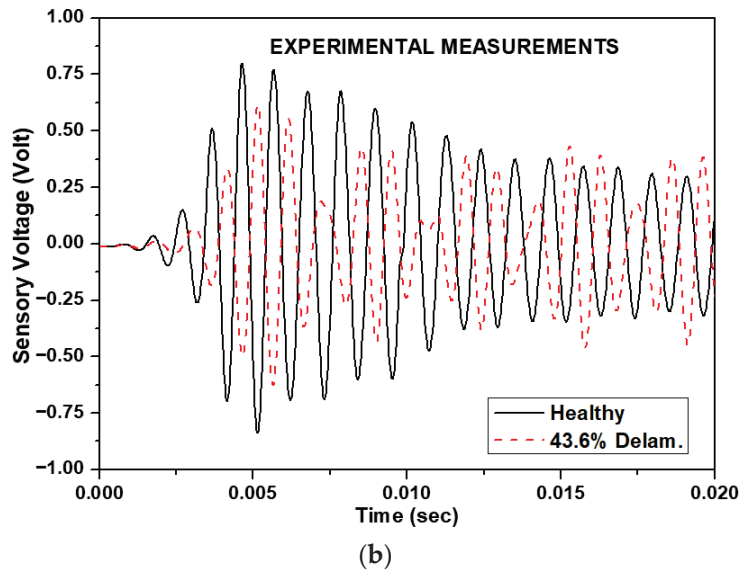


Figure 9. Effect of a delamination crack covering 43.6% of the total beam length on the (a) predicted and (b) experimentally measured transient response, respectively.

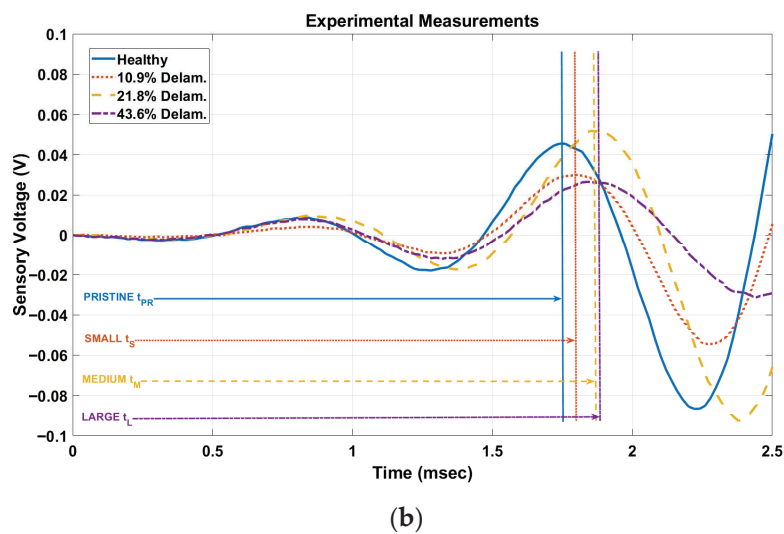
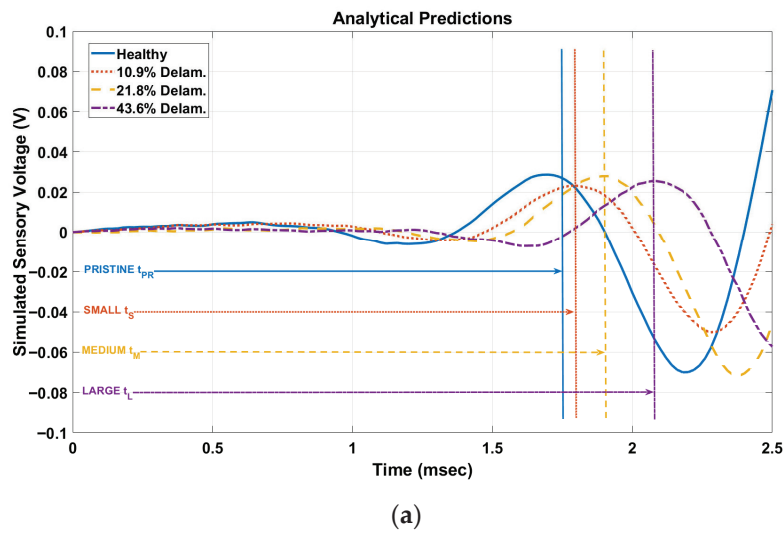


Figure 10. Comparison of time of flight. (a) Simulated results and (b) experimental measurement.

4.3. Axial and Shear Wave Propagation

This section continues the investigation of delamination characteristics in the time response of delaminated strips. Taking into consideration the scope of the present section, it is assumed that only the axial variable field was considered from the analytical model; hence, a study on the propagation of axial and shear waves at a low frequency range is presented. The healthy and the beam entailing small delamination sizes were modeled as described previously, with the difference that the thickness of the piezoelectric actuators was simulated to be 0.1 mm. The through-the-thickness propagation of axial waves stresses through a section located at the midspan of the beam and at the middle of the delamination, as well (see Figure 11), is presented in Figure 12a–j. Simulations were performed for 1 kHz central frequency tone burst, as previously described. The axial stresses are simulated at the top and bottom faces of each discrete layer. The light horizontal lines illustrate the different ply orientation (eight plies) as each ply was simulated with three individual layers, and a vertical line was added to demonstrate the zero stress plane. Each one of the figures presents a different snapshot of the wave propagation. In any one of the distributions through the thickness, a discontinuity appears at the middle, indicating the position of the crack through the thickness. Also, as the transient response continues, the amplitude of predicted axial stresses increases. Additionally, wave propagation presents a signal initially driving towards the right, from the actuator to the sensor. After reaching the right boundary of the specimen, it reflects and drives towards the other end, the left direction, and this vibration continues till the burnout of the vibration. Discontinuity at the middle indicates that two separate axial waves are vibrating separately on the two sides of the delamination.

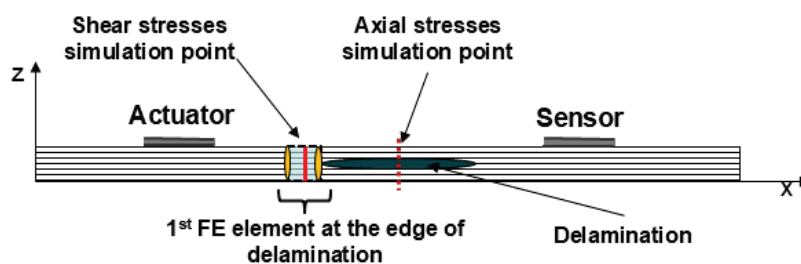


Figure 11. The simulation x-z plane side view of a composite active sensory beam.

Similarly, Figure 13a–j illustrates the propagation of a shear wave through a section located at the integration point of a 2-node finite element at the left edge of delamination with both nodes without delamination DOFs (see Figure 11). Shear stresses are simulated at the middle of each layer. Similar to the axial stresses, the light horizontal lines illustrate the different ply orientation (eight plies) as each ply was simulated with three individual layers, and a vertical line was added to demonstrate the zero stress plane. Variation in the shear stresses indicates the position of damage, resulting in a different distribution compared to the healthy beam. Initially, due to the excitation period and the actuator position on the top beam's surface, the signal is not symmetric for both the healthy and delaminated beams. Similarly to axial stresses, the magnitude of shear stresses increases during the period of transient analysis, and the signal propagates from left to right and vice versa. The most important conclusion of the shear wave propagation is that shear stresses are sensitive to revealing the existence of delamination using stress distribution, even if the measuring position is away from the damage. Additionally, it is crucial to investigate the magnitude of interlaminar stresses that appear at the crack edges during free vibration and determine the critical stress values for crack propagation.

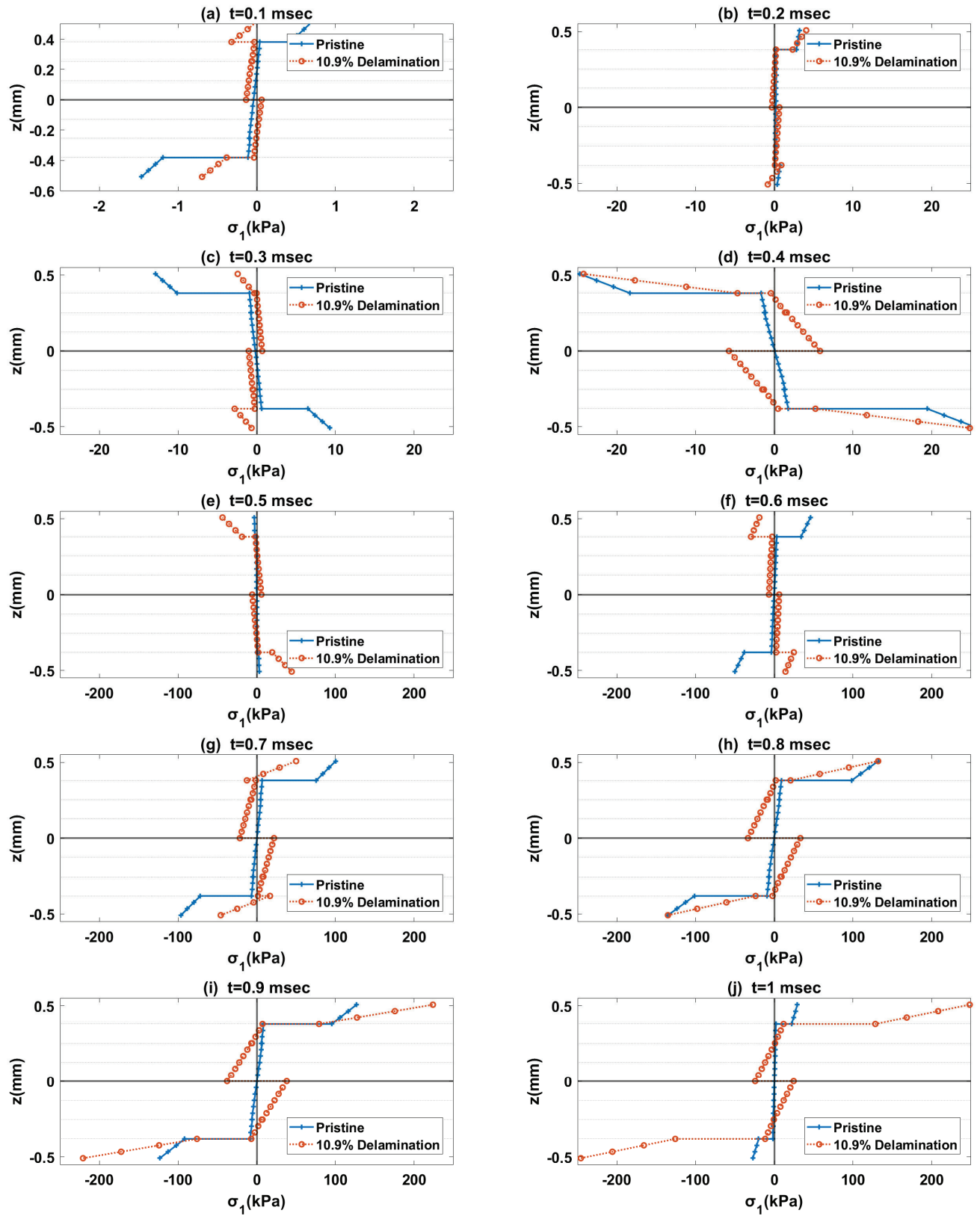


Figure 12. Snapshots of the propagation of an axial wave through a section at the middle of the beam's length.

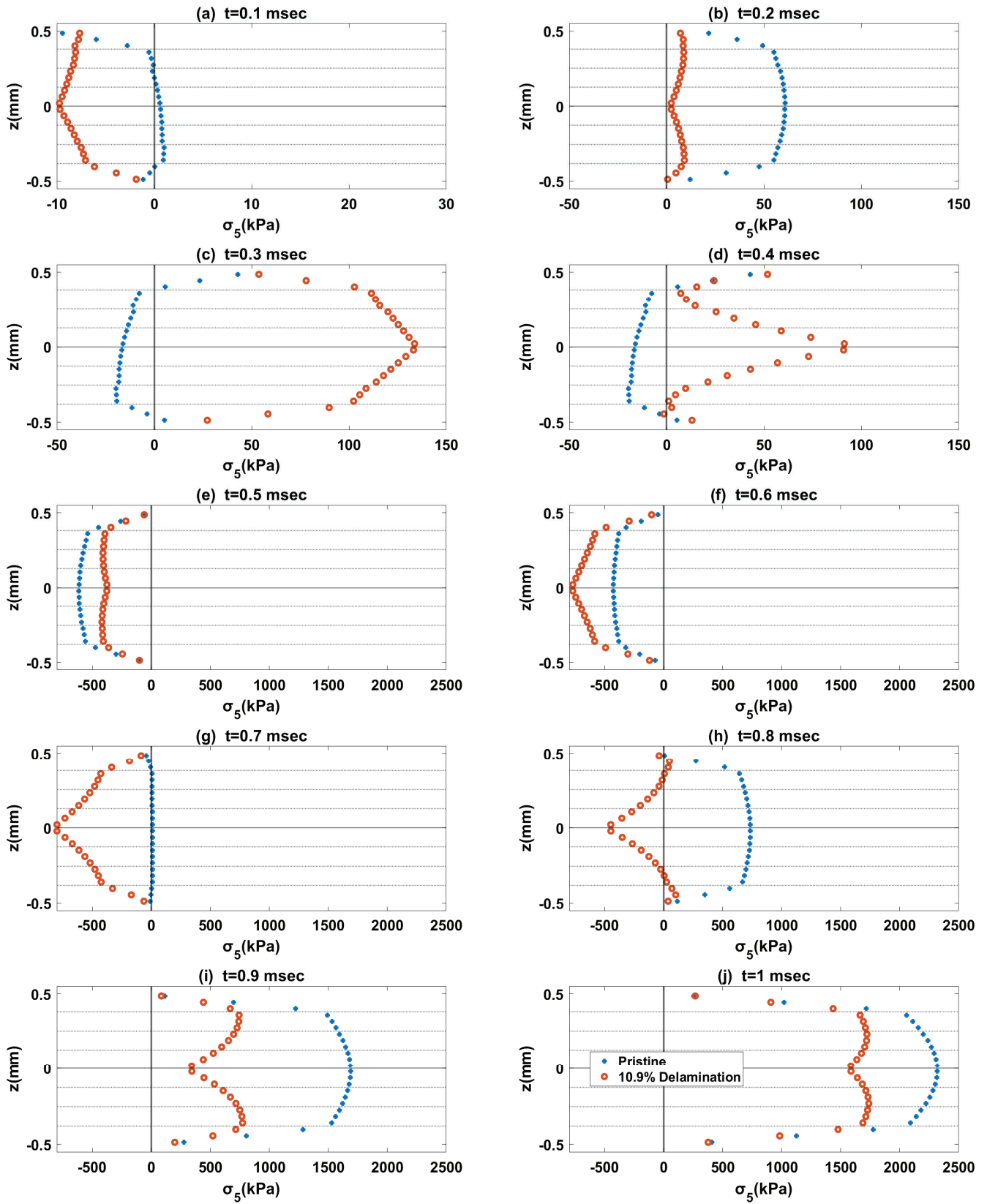


Figure 13. Snapshots of the interlaminar wave propagation through a section at the middle of a healthy element located close to the left edge of the crack.

4.4. Contour Plots of Axial and Shear Stresses

To complete this study of delamination signatures on the low-frequency time response of delaminated composite beams, contour plots were also created. For the same excitation

as in the previous case, and two delamination sizes, the small (10.9%) and the medium (21.8%) axial and shear stresses were predicted in any position through the beam's thickness. Figure 14a–f presents contour plots of the simulated axial stresses. Each plot illustrates the magnified x-z view of the composite beam as presented in Figure 11. Piezoceramic devices, including actuators and sensors, are similar to the previous case, and their positions are also indicated in each graph. In Figure 14, plots (a), (c), and (e) represent, for $t = 0.5$ ms of the transient analysis, the healthy beam and the beams with small and medium delamination sizes, respectively. Similarly, plots (b), (d), and (f) of Figure 14 are the axial stress contours for $t = 1.6$ ms. From the first three plots, representing the $t = 0.5$ ms snapshot since the beginning of the transient analysis, the exact position of delamination is mapped when the signal arrives at the crack. The extent of delamination crack, as well as its position through the thickness, is extensively indicated. Similar conclusions are drawn from the second set of plots, which present a snapshot of the system during free structural vibration after the relaxation of the transient phenomena.

As described in the previous case for axial stresses, Figure 15a–f presents contour plots of the interlaminar stresses for any point through the beam thickness at two different time moments of the transient analysis. Similarly, the x-z view is presented. The first snapshot, at $t = 1.8$ ms, shows the initial arrival of the shear wave at the delamination region during its propagation, providing a slight indication of the exact extent of the damage. At the second snapshot, at $t = 8.6$ ms, the contour shows a stationary shear wave propagating across the beam. When a delamination crack exists through the thickness, it appears as a disturbance in the distribution of shear stresses. The exact crack position and extent of damage are presented. Also, damage affects the magnitude of shear stresses in a region around the crack, indicating the appropriate positions to measure the shear stresses and reveal damage. Finally, comparing the contours of the axial and shear waves, it is clear that the axial wave signal propagates faster across the beam. It is a logical conclusion as a longitudinal wave propagates faster than a transverse wave.

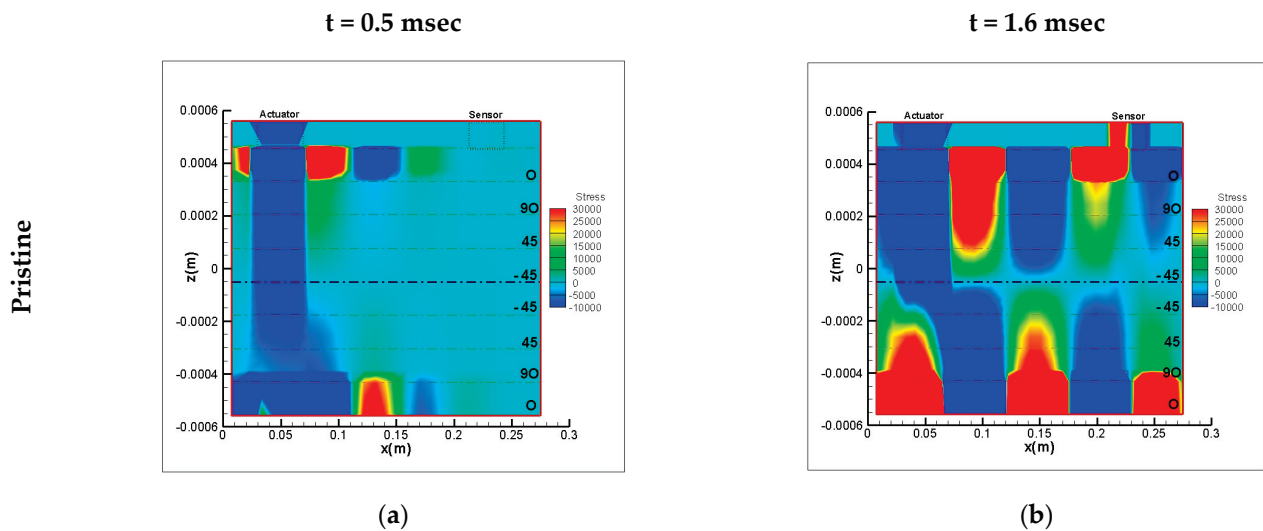
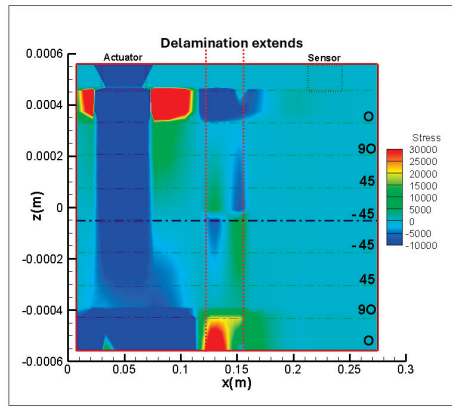
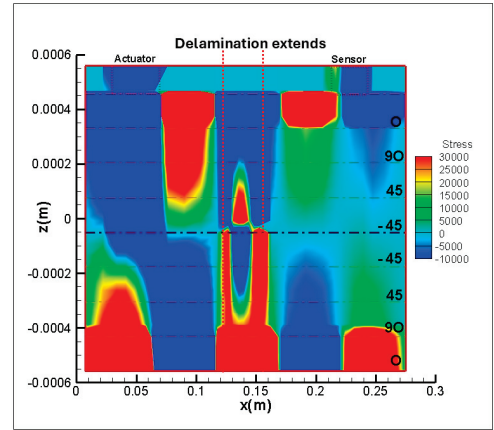


Figure 14. Cont.

10.9% Delamination length

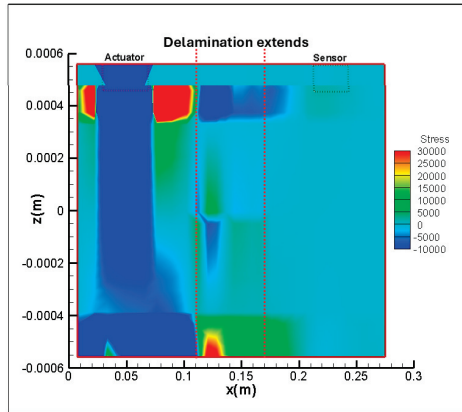


(c)

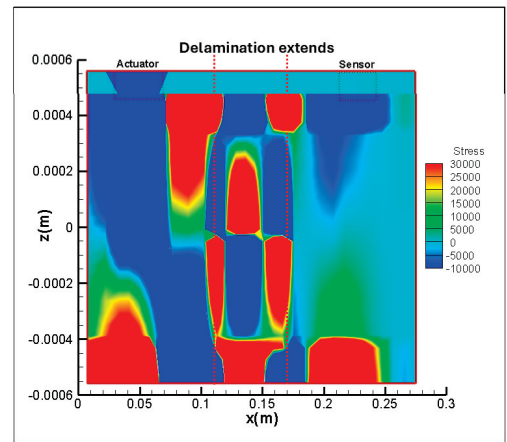


(d)

21.8% Delamination Length



(e)



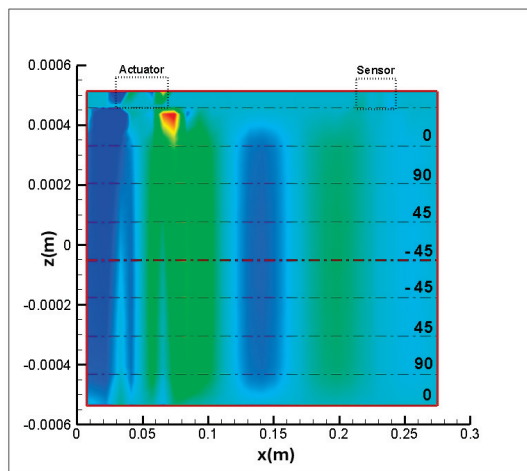
(f)

Figure 14. Contour plots of the axial wave propagation; LEFT COLUMN snapshots at $t = 0.5$ ms; RIGHT COLUMN snapshots at $t = 1.6$ ms. Extents of delamination are illustrated with vertical dotted lines, and dotted boxes demonstrate the actuator-sensor exact position.

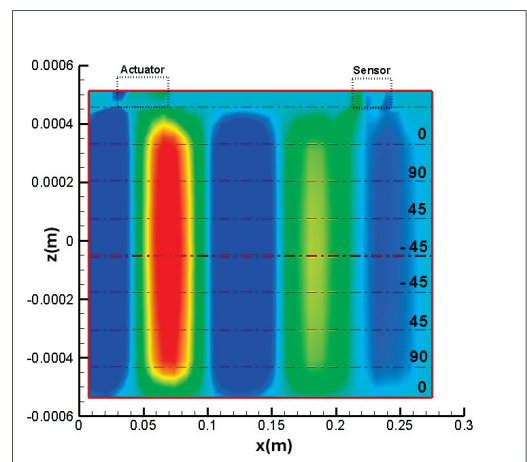
$t = 1.8$ msec

$t = 8.6$ msec

Pristine



(a)



(b)

Figure 15. Cont.

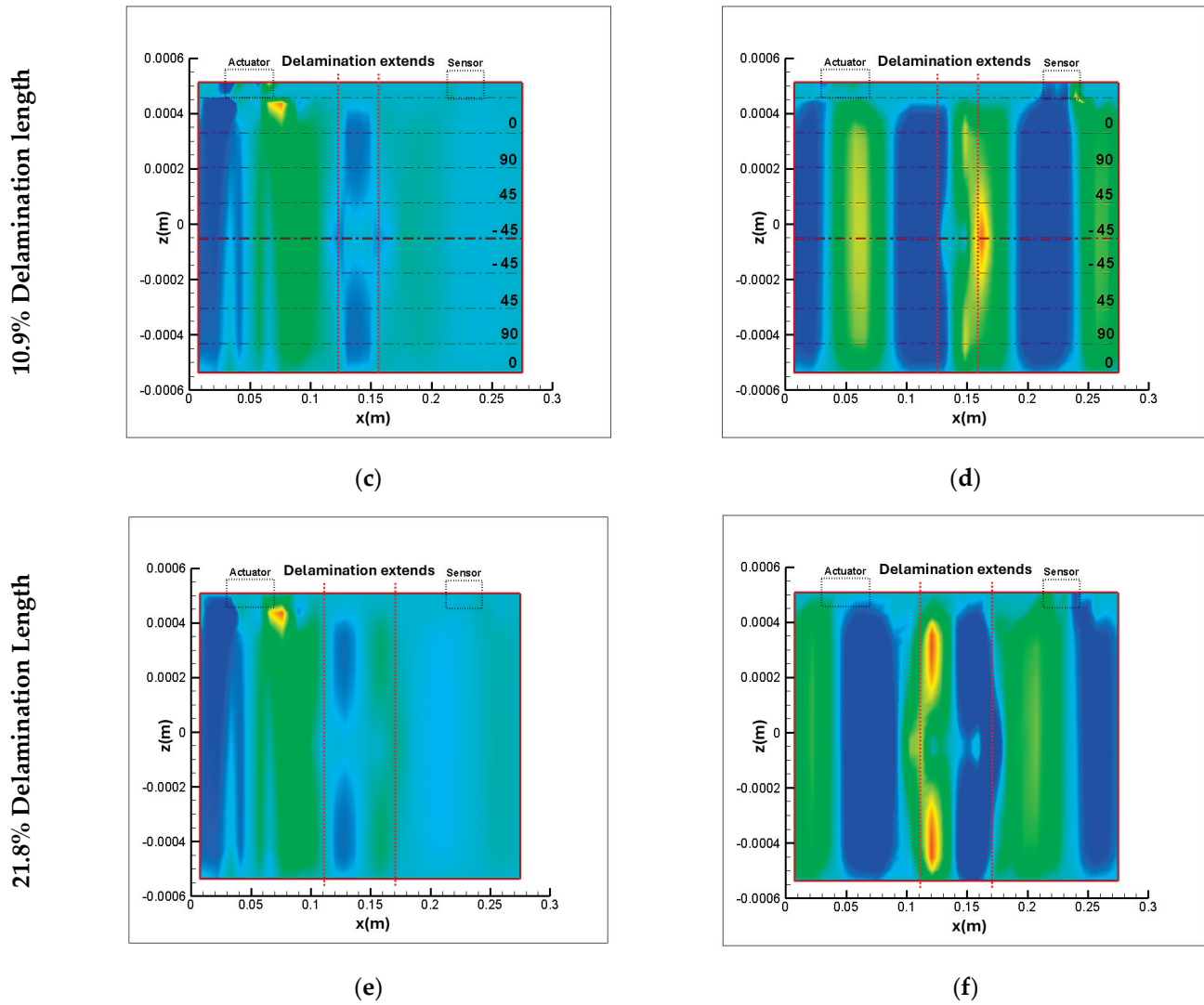


Figure 15. Contour plots of the shear wave propagation; LEFT COLUMN snapshots at $t = 1.8$ ms; RIGHT COLUMN snapshots at $t = 8.6$ ms. Extents of delamination are illustrated with vertical dotted lines, and dotted boxes demonstrate the actuator-sensor exact position.

5. Summary and Conclusions

This paper presents an analytical and experimental study of the time response of delaminated active-sensory composite beams. A previously developed analytical model capable of simulating delaminated composite beams with piezoelectric devices was extended to naturally include contact and sliding at the delamination interfaces. The resultant electromechanical system was non-linear due to the interface contact, and the stiffness matrix was updated in each time step of the transient analysis. The time response of this non-linear system was predicted using a modified Newmark time integration method, combined with Newton–Raphson iterations in each time step, until the least squares error was minimized. Representative experiments were further performed, and the time response of composite beams was measured using an experimental setup that provided actuation via a piezoceramic actuator and vibration acquisition using a piezopolymer sensor, both of which were attached to the beam surface. Four different specimens were tested: one healthy and three delaminated, each with a different damage length. Tests and simulations were conducted under free-supporting conditions to eliminate disturbances in the response from the boundaries.

An excellent agreement was observed between predictions and measurements, especially for the 1 kHz actuation. The non-linear analytical model, which assumes contact and control of axial movement at the delamination region, improved correlation with measurements. A good performance of the simulation in capturing the experiment was also presented for higher frequencies (5 kHz). However, for large delamination sizes, differences appeared between simulations and measurements. This was probably due to the way friction was modeled at the damage interface and to the assumed constant normal displacement field. Additionally, due to the model's weakness in capturing the oscillation amplitude during the free vibration period, damping modeling would probably improve correlations.

Signatures of damage were detected in the time of flight, oscillation frequency, and amplitude of the sensory signal. Changes in these characteristics presented sensitivity even for small delamination sizes. Also, captured modifications on the time of flight and period were relative to the delamination length. These conclusions were duplicated from the experimental measurements.

Continuing the axial and shear wave propagation was predicted through sections along the damaged and healthy composite beams. Axial and shear waves exhibited sensitivity to the existence of cracks, and especially the shear stress distributions through the thickness were sensitive to revealing damage, even when the simulation position was away from the damage. Finally, contour plots of the axial and shear stresses were illustrated at various snapshots. These figures demonstrate the propagation of an axial or shear wave from the actuator towards the sensor, as well as the wave interactions with the delamination crack. These plots demonstrate interactions throughout the thickness and can provide valuable information about the optimal positions of actuators and sensors, facilitating the construction of an efficient health monitoring system.

Author Contributions: Conceptualization, N.A.C. and D.A.S.; methodology, N.A.C., C.S.R. and D.A.S.; software, N.A.C.; validation, N.A.C. and C.S.R.; investigation, N.A.C.; writing—original draft, N.A.C.; writing—review and editing, N.A.C., C.S.R. and D.A.S.; visualization, N.A.C. and C.S.R.; supervision, D.A.S. All authors have read and agreed to the published version of the manuscript.

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Appendix A

Generalized laminate stiffness matrices of the undamaged section:

$$D_{11}^{mn} = \sum_{l=1}^{N_p} \int_{z_l}^{z_{l+1}} C_{11}^l \psi^m(z) \psi^n(z) dz$$

$$A_{55} = \sum_{l=1}^{N_p} \int_{z_l}^{z_{l+1}} C_{55}^l dz$$

$$B_{55}^m = \sum_{l=1}^{N_p} \int_{z_l}^{z_{l+1}} C_{55}^l \psi_{,z}^m(z) dz$$

$$D_{55}^{mn} = \sum_{l=1}^{N_p} \int_{z_l}^{z_{l+1}} C_{55}^l \psi_{,z}^m(z) \psi_{,z}^n(z) dz$$

Generalized stiffness matrices at the delamination region:

$$\begin{aligned}\bar{D}_{11}^{mk} &= \sum_{l=1}^{N_p} \int_{z_l}^{z_{l+1}} C_{11}^l \psi^m(z) \psi^k(z_k) H(z - z_k) dz \\ \bar{A}_{55}^k &= \sum_{l=1}^{N_p} \int_{z_l}^{z_{l+1}} C_{55}^l H(z - z_k) dz \\ \bar{B}_{55}^k &= \sum_{l=1}^{N_p} \int_{z_l}^{z_{l+1}} C_{55}^l \psi_{,z}^k(z) H(z - z_k) dz \\ \bar{F}_{55}^{mk} &= \sum_{l=1}^{N_p} \int_{z_l}^{z_{l+1}} C_{55}^l \psi_{,z}^m(z) H(z - z_k) dz \\ \bar{D}_{55}^{mk} &= \sum_{l=1}^{N_p} \int_{z_l}^{z_{l+1}} C_{55}^l \psi_{,z}^m(z) \psi_{,z}^k(z) H(z - z_k) dz\end{aligned}$$

Coupling stiffness matrices between healthy and delaminated regions:

$$\begin{aligned}\hat{D}_{11}^{k_1 k_2} &= \sum_{l=1}^{N_p} \int_{z_l}^{z_{l+1}} C_{11}^l \psi^k(z) \psi^k(z) H(z - z_k) dz \\ \hat{A}_{55}^{k_1 k_2} &= \sum_{l=1}^{N_p} \int_{z_l}^{z_{l+1}} C_{55}^l H(z - z_{k_1}) H(z - z_{k_2}) dz \\ \hat{B}_{55}^{k_1 k_2} &= \sum_{l=1}^{N_p} \int_{z_l}^{z_{l+1}} C_{55}^l \psi_{,z}^{k_1}(z) H(z - z_{k_1}) H(z - z_{k_2}) dz \\ \hat{D}_{55}^{k_1 k_2} &= \sum_{l=1}^{N_p} \int_{z_l}^{z_{l+1}} C_{55}^l \psi_{,z}^{k_1}(z) \psi_{,z}^{k_2}(z) H(z - z_{k_1}) H(z - z_{k_2}) dz\end{aligned}$$

Generalized piezoelectric matrices:

$$\begin{aligned}P_{31}^{mn} &= \sum_{l=1}^{N_p} \int_{z_l}^{z_{l+1}} e_{31}^l \psi_{,z}^m(z) \psi^n(z) dz \\ \bar{P}_{31}^{mk} &= \sum_{l=1}^{N_p} \int_{z_l}^{z_{l+1}} e_{31}^l \psi_{,z}^m(z) \psi^k(z_k) H(z - z_k) dz\end{aligned}$$

Generalized permittivity laminate matrices:

$$G_{33}^{mn} = \sum_{l=1}^{N_p} \int_{z_l}^{z_{l+1}} \epsilon_{33}^l \psi_{,z}^m(z) \psi_{,z}^n(z) dz$$

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Article

Fatigue Damage Suppression by Ply Curving Termination in Covered Composite Ply Drop-Off

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Abstract: Ply Curving Termination (PCT) is an effective method to suppress stress concentration at composite ply drop-offs by locally curving the reinforcing fibers to reduce the stiffness. A previous study by the authors confirmed that PCT can suppress fatigue delamination failure in composite ply drop-off. However, the specimens used were external ply drop-offs without cover plies and did not reflect practical structural configurations. Following the basic study, this current study evaluated the fatigue damage suppression characteristic of PCT in practically relevant internal ply drop-offs with cover plies. Finite element analysis, fatigue testing, and detailed observation of the failure process using X-ray CT showed that PCT is effective in suppressing fatigue failure of internal ply drop-offs. In particular, delamination propagation from matrix cracks along the curving fibers, a weak point of PCT, is suppressed in the external ply drop-off. Finite element analysis indicated the importance of stress transfer from the cover ply to the ply drop-off, confirming that the fatigue damage suppression effect of PCT is enhanced in practical composite ply drop-off configurations.

Keywords: composite laminate; ply drop-off; delamination; fatigue; damage suppression

1. Introduction

In a lightweight composite aerostructure, the laminate thickness is changed according to the applied load, and tapered laminates are widely used [1–5]. There are two types of tapered laminates with different ply drop-off positions, as shown in Figure 1: external ply drops, in which the ply is dropped-off externally, and internal ply drops, in which the ply is dropped-off internally. Load in the base laminate is transferred to the dropped ply by out-of-plane shear stresses at the interfaces of the dropped ply. It is well known that the ply drop-off causes stress concentration due to the shape discontinuity, and delamination can occur from the edge of the dropped ply [6]. From the perspectives of lightweighting and manufacturing, it is desirable to use a steep taper. However, increasing the number of plies dropped at a single location or reducing the spacing between adjacent ply drop-offs has been found to reduce the strength of the tapered laminate [7,8]. Therefore, a gentle taper is currently used, which is a barrier to further weight reduction in composite structures.

By introducing a mechanism to suppress delamination from the ply drop-offs, a steeper taper can be used. The laminate thickness can be designed with a high degree of freedom, leading to lighter composite structures. Therefore, many studies have been conducted on ideas to suppress delamination [9–14]. One approach is to optimize the order in which plies are dropped off. Fish et al. confirmed the effectiveness of arranging ply drop-offs in a stepped configuration and of overlapping the edges of dropped plies [15]. Another approach is to modify the materials used in the laminate or to introduce additional

materials [16,17]. Thin-ply prepreg enhanced the static tensile strength [11,18], and addition of nanoclay to the matrix resin improved the fatigue life [19]. The other interesting approach is to alter the characteristics and shape of the dropped ply edges. Researchers at the University of Bristol chamfered the edges of dropped ply to reduce the geometry discontinuity [20], and have demonstrated improvements in various mechanical properties, including tension, compression, fatigue, and impact resistance [21–24].

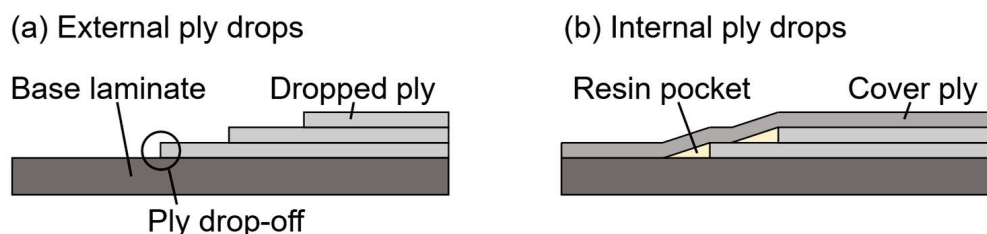


Figure 1. Side view of typical tapered laminates: (a) external ply drop-off; (b) internal ply drop-off.

The authors' research group has also proposed one approach that leverages the anisotropy of composite materials to alter the characteristics of ply edge, called Ply Curving Termination (PCT) [25,26]. PCT locally changes the fiber direction at the edge of the dropped ply (Figure 2). The longitudinal stiffness of the dropped ply edge is significantly decreased, suppressing the stress concentration at the ply drop-off. PCT is simple, requiring no additional materials, and can be combined with other approaches as described above.

In order to improve the technical maturity of PCT, evaluations under practically relevant fatigue conditions are essential [27–29]. In our previous work [30], the fatigue properties of PCT were evaluated using the simplest form, the external ply drop-off specimen (Figure 1a). The results clearly showed that PCT delays the onset of initial damage and also significantly increases the cycles to final failure. However, for practical structures, internal ply drop-off with a cover ply on top of the ply drop-off is commonly used (Figure 1b). Therefore, it is necessary to confirm that PCT is effective in improving fatigue properties of external ply drop-offs. It is important to note that in our previous test using the external ply drop-off specimens [30], the fatigue failure modes transitioned and fatigue properties changed significantly with loading level. Specifically, delamination propagated from the ply drop-off under low loading conditions, whereas under high loading conditions, delamination propagated from the inside of the cracks that formed along the curving fibers (Figure 2). This result implies that detailed observation of the fracture process is necessary to demonstrate the effectiveness of PCT at internal ply drop-offs.

This study first calculates the stress differences between the two forms of ply drop-off (i.e., external and internal) by finite element analysis to examine the effectiveness of PCT at internal ply drop-off. Next, fatigue specimens are fabricated and the process from the onset of initial damage to final failure is evaluated using X-ray CT at high and low loading levels.

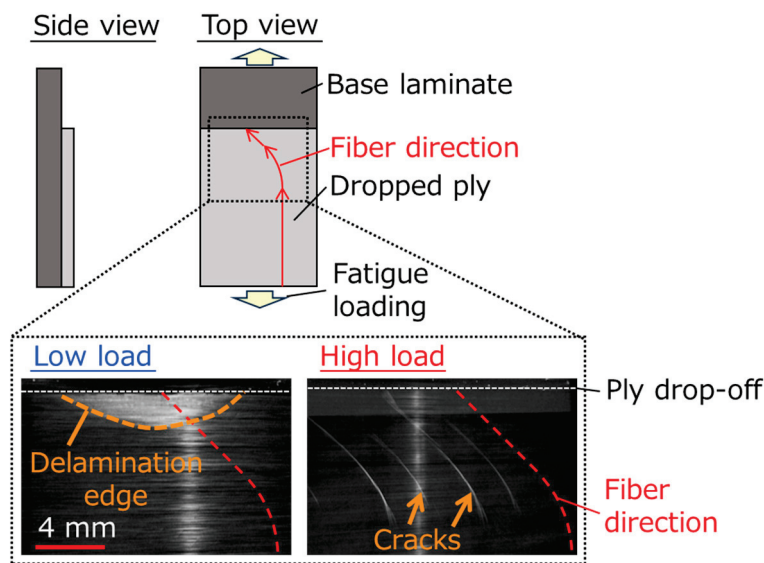


Figure 2. Schematic of PCT and transition of fatigue damage mode in external ply drop-off [30].

2. Finite Element Analysis

2.1. Finite Element Model

The out-of-plane stress at the interfaces of the two kinds of tapered laminates, external and internal ply drop-offs (Figure 1), under tensile loading was calculated. The specimen geometry was the same as that of the tensile fatigue experiment described later. The stacking sequence of the External model (Figure 3a) was $[0_6]$ for the base laminate and $[0_3]$ for the dropped ply. In the Internal model (Figure 3b), the base laminate, the dropped ply and the covered ply all had a stacking sequence of $[0_3]$. So, both the External model and the Internal model had 9 plies in the thick part, and 3 plies were dropped-off at one location, resulting in 6 plies in the thin part. The thickness of one ply was 0.125 mm. The taper angle of the Internal model was 2.5° . Two types of models were created for both the External and Internal models: Normal model with a 0° fiber direction at the end of the dropped ply and PCT45 model with 45° PCT angle. The PCT geometry parameters including the PCT angle are shown in Figure 4. In this study, the PCT radius and modified zone length were set to 8 mm and 8 mm, respectively. In a region where the fiber direction of the dropped ply changes (fiber curving zone), the model was divided into small parts at 0.1 mm intervals in the x-direction, and the corresponding fiber direction was set for each part to model PCT. The material was carbon fiber/epoxy (T700SC/2592, Toray Industries, Inc., Tokyo, Japan), and its material properties are given in Table 1. The triangular column space between the base laminate and the cover ply at the end of the ply drop-off was filled with 90° plies. The edge of the thick section was fixed by the x-symmetry boundary condition, and uniform displacement in the x-direction was applied to the edge of the thin section so that the average tensile stress in the thin section was 1000 MPa (0.8% tensile strain in the thin section). Abaqus 2023 (Dassault Systems Inc., Vélizy-Villacoublay, France) was used to perform static geometric nonlinear analysis. The models were meshed with hexahedral first-order elements (C3D8) except for the edges of the triangular column space between the base laminate and the cover ply in the Internal model, where 6-node linear triangular prism elements (C3D6) were used. One ply was divided by five solid elements in the thickness direction. The mesh was refined until the stress distribution converged. The total number of elements was 215,700 for the External model and 243,440 for the Internal model.

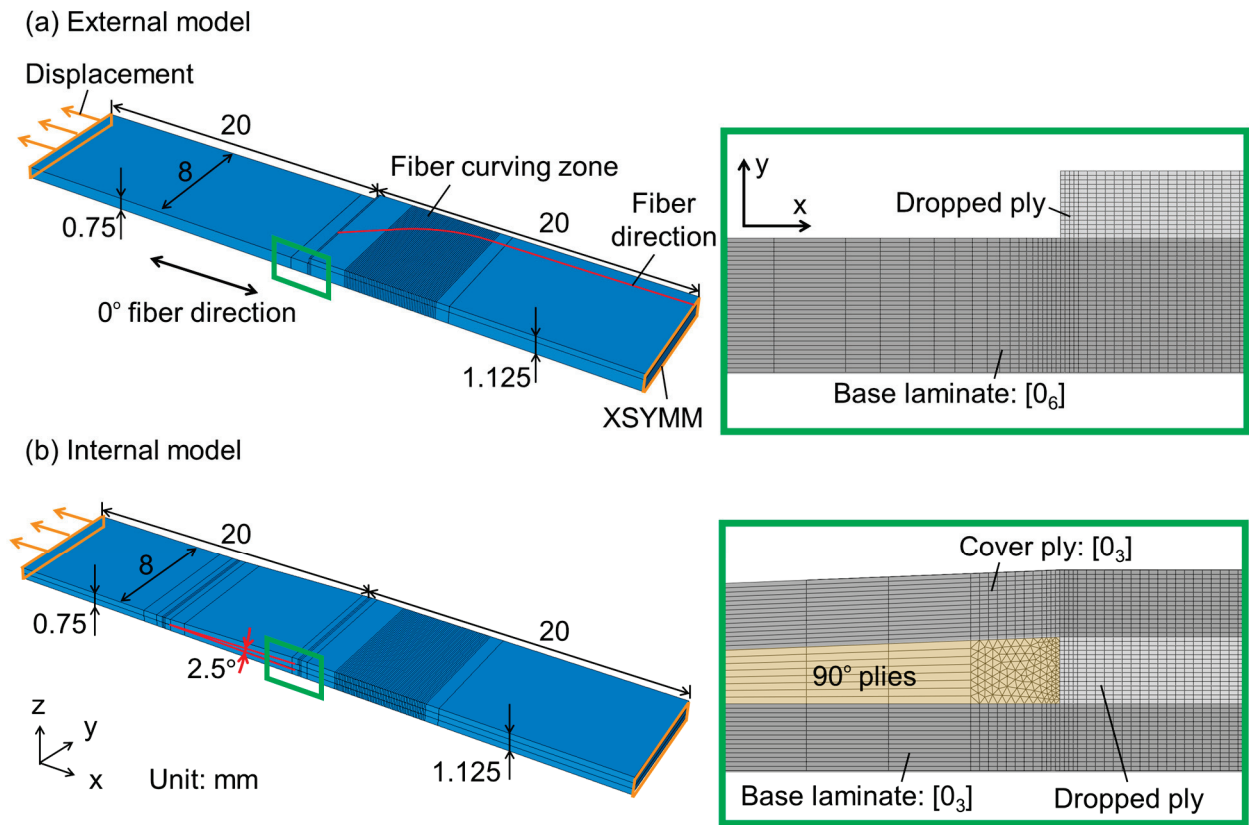


Figure 3. Finite element model. (a) External model. (b) Internal model.

Top view

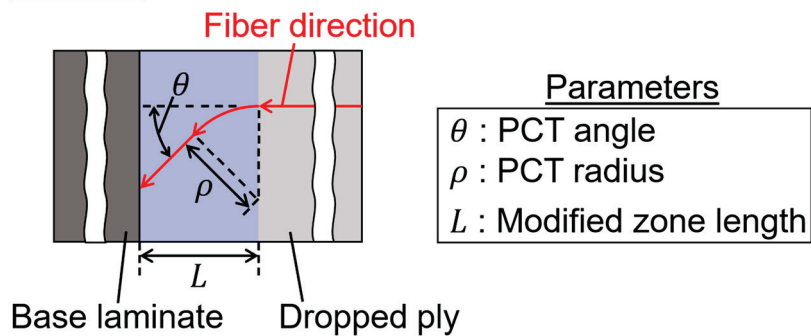


Figure 4. PCT geometry parameters.

Table 1. Material property of T700SC/2592 [31].

Elastic Moduli [GPa]				Poisson's Ratio	
E_{11}	$E_{22} = E_{33}$	$G_{12} = G_{13}$	G_{23}	$\nu_{12} = \nu_{13}$	ν_{23}
135	8.5	4.8	2.7	0.34	0.49

2.2. Results

Figure 5 shows the calculated results of σ_{zz} and τ_{zx} along the interface at the specimen center. Note that σ_{zz} is the peel stress and τ_{zx} is the interface shear stress that contributes to the tensile load transfer to the dropped ply.

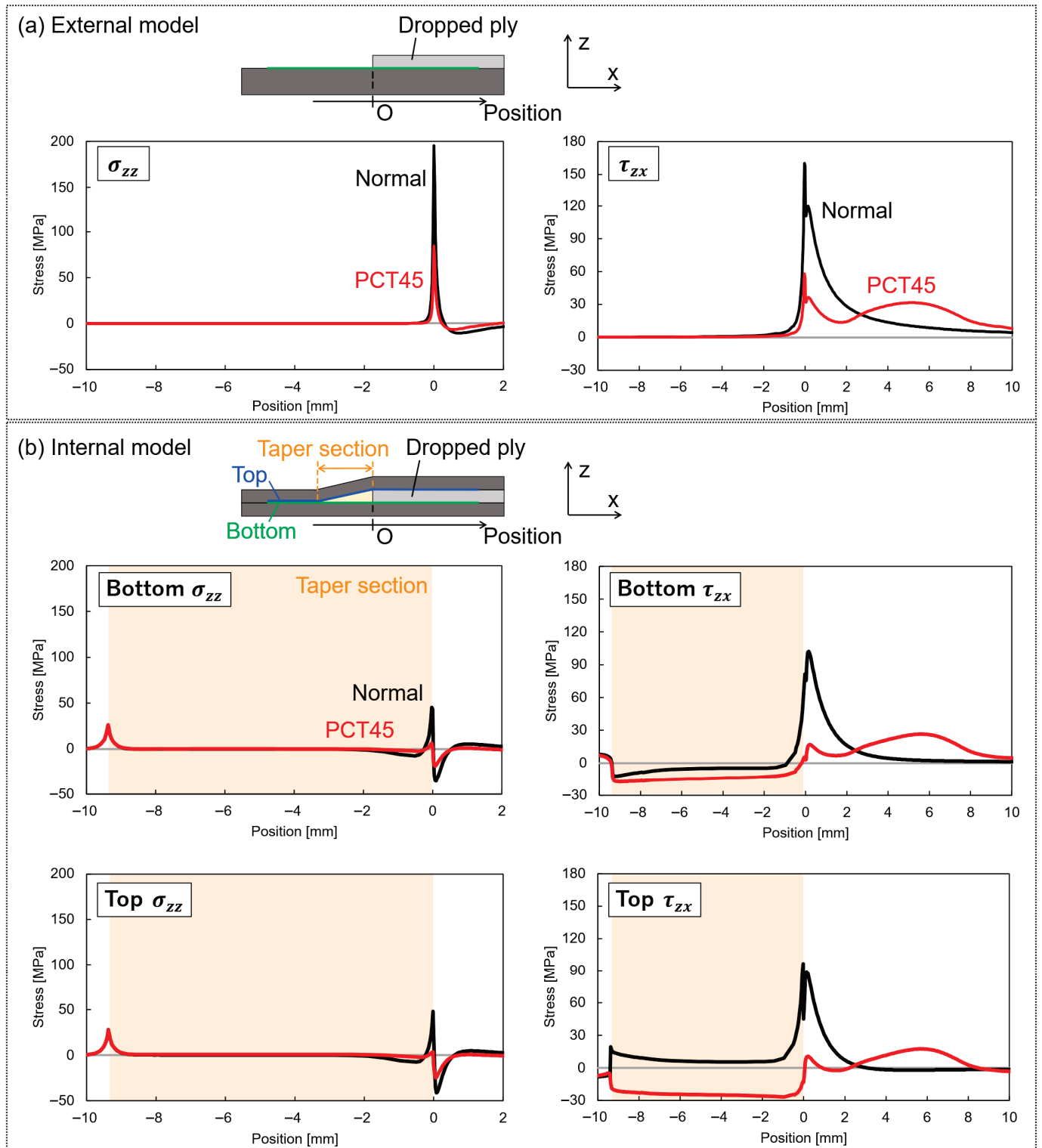


Figure 5. Interface stress calculated using finite element analysis.

In the Normal of External model (Figure 5a), both σ_{zz} and τ_{zx} had sharp peaks near the ply drop-off (i.e., Position = 0 mm). In contrast, PCT45 of External model had much lower σ_{zz} and τ_{zx} peaks near the ply drop-off, which is the effect of PCT reducing the longitudinal stiffness at the edge of the dropped ply [26,30]. τ_{zx} of PCT45 had another peak between 3 mm and 8 mm as the fiber angle of the dropped ply gradually changed from 45° to 0°. This was because the gradual increase in the longitudinal tensile stiffness of the dropped

ply dispersed the position where the tensile load was transferred from the base laminate to the dropped ply. Because PCT reduces both the out-of-plane stresses σ_{zz} and τ_{zx} , PCT is able to suppress delamination of the external ply drops [26,30].

Similarly to the External case, in Normal of Internal model (Figure 5b), both σ_{zz} and τ_{zx} had sharp peaks near the ply drop-off (i.e., Position = 0 mm). However, compared to External, the peak of σ_{zz} was much lower and the peak of τ_{zx} was slightly lower. This indicates that internal ply drops are more resistant to delamination than external ply drops, and that Mode-II delamination propagation is dominant for the internal ply drop-off. Furthermore, the stress distributions were approximately the same at both the top and bottom interfaces of the dropped ply, indicating that tensile load was transferred from both the base laminate and the cover ply to the dropped ply. In PCT45 of Internal model, as in the External case, both σ_{zz} and τ_{zx} peaks near the ply drop-off were significantly reduced, suggesting that PCT is effective in suppressing delamination in internal ply drops as well.

Based on the results of the finite element analysis described above, it is expected that PCT improves the fatigue properties of internal ply drop-offs. However, as observed in the tensile fatigue tests of external ply drop offs (Figure 2, [30]), the introduction of PCT may change the fatigue failure mode of internal ply drops. Therefore, tensile fatigue testing is conducted in the next section to clarify the fracture behavior of internal ply drops with PCT and compare the effectiveness of PCT in internal ply drop-off and external ply drop-off.

3. Tensile Fatigue Test

3.1. Materials and Methods

3.1.1. Specimen Preparation

The specimens were fabricated using carbon fiber/epoxy prepreg (T700SC/2592, Toray Industries, Inc.). The specimen geometry is shown in Figure 6. Two types of specimens were prepared: External specimen with external ply drop-off (Figure 6a) and Internal specimen with internal ply drop-off (Figure 6b). The stacking sequence of External specimens was $[0_6]$ for the base laminate and $[0_3]$ for the dropped ply. Preliminary tests confirmed that damage did not occur in PCT specimens when the dropped ply was one ply, so the dropped ply was set to three plies to make damage more likely to occur. In the Internal specimen, the base laminate, the dropped ply and the covered ply all had a stacking sequence of $[0_3]$. Three plies were dropped simultaneously at one location as in the case of External specimen. So, the External and Internal specimens had comparable load carrying capacity, with the thick section being $[0_9]$ and the thin section being $[0_6]$. Two types of specimens were prepared for each of the External and Internal specimens: Normal specimen with a 0° fiber direction at the end of the dropped ply and PCT45 specimen with 45° PCT angle, 8 mm PCT radius and 8 mm modified zone length (Figure 4). Note that PCT can be introduced into the specimen by shearing the edge of the pre-cured dropped ply prepreg in the in-plane direction with a custom-made sliding mechanism [26,30].

In a preliminary experiment, out-of-plane fiber wrinkles were observed when the dropped ply and base laminate were co-cured. The mechanical properties of the laminate change due to the fiber wrinkles [32], and the magnitude of wrinkles is different whether PCT is introduced or not. To eliminate the effect of fiber wrinkles and evaluate the sole effect of PCT, the base laminate was first cured and then the dropped plies were co-bonded. Specifically, peel plies (Peel Ply PA90, Diatex SAS, Saint Genis Laval, France) were laminated on the surface of stacked base laminate prepreg and the assembly was vacuum-bagged on an aluminum plate. The base laminate was cured in an autoclave (Ashida MFG Co., Ltd., Nara, Japan) under a pressure of 0.3 MPa. The temperature was increased to 130°C at a rate of $2^\circ\text{C}/\text{min}$ and maintained at 130°C for 2 h. After curing, the peel plies were removed. The method of co-bonding the dropped ply onto the cured base laminate differed

between External and Internal specimens. In the case of External, dropped ply prepreg was laminated on the peeled surface without adhesives. The assembly was then cured using the same curing cycle used for the base laminate. In contrast, in the case of Internal, the cured base laminate and the dropped ply prepreg and the cover ply prepreg were laminated on a mold with a 2.5° taper as shown in Figure 7. It is well known that the taper angle significantly changes the mechanical properties of tapered laminates [6], so the mold was used to accurately control the taper angle. Furthermore, as shown in Figure 7, the 90° prepreg sheets were stacked in a staircase-like manner in the triangle space between the cover ply and the base laminate to prevent the taper from collapsing. The assembly was then cured using the same curing cycle used for the base laminate. Figure 8 shows cross-sectional micrographs of the tapered sections in the Internal specimens after curing. The triangle space between the cover ply and the base laminate was filled with 90° layers and no voids were observed. The taper angle was approximately 2.5° .

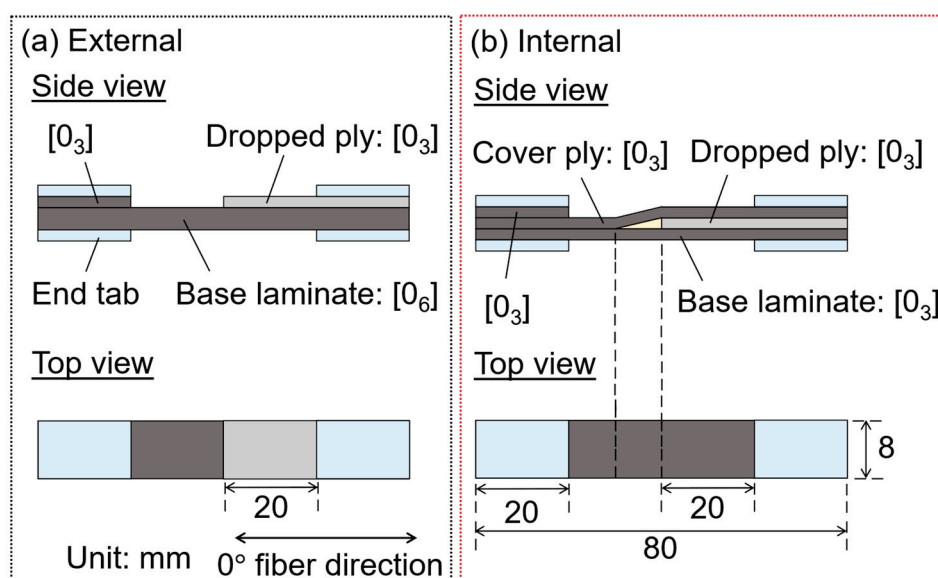


Figure 6. The geometry of specimens used for the tensile fatigue test. (a) External. (b) Internal.

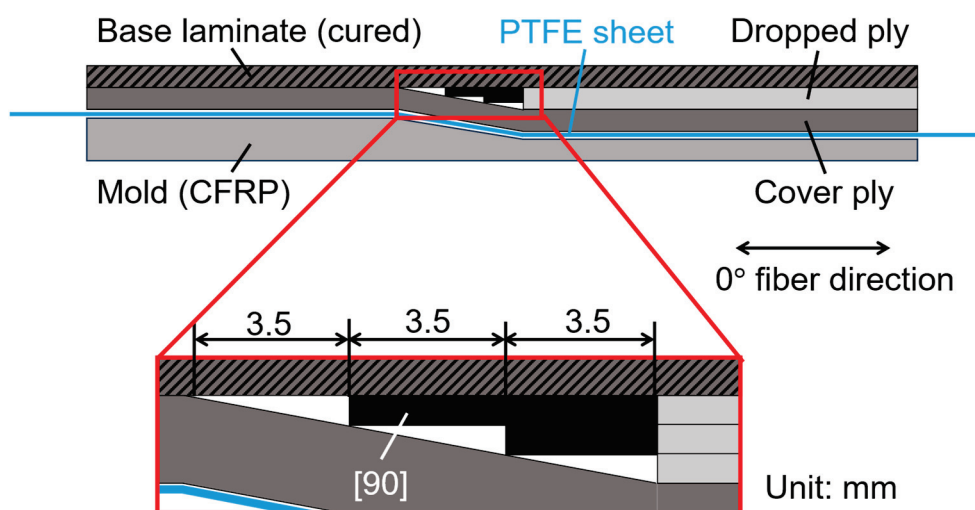


Figure 7. Specimen and mold setup used for co-bonding process to prepare Internal specimen.

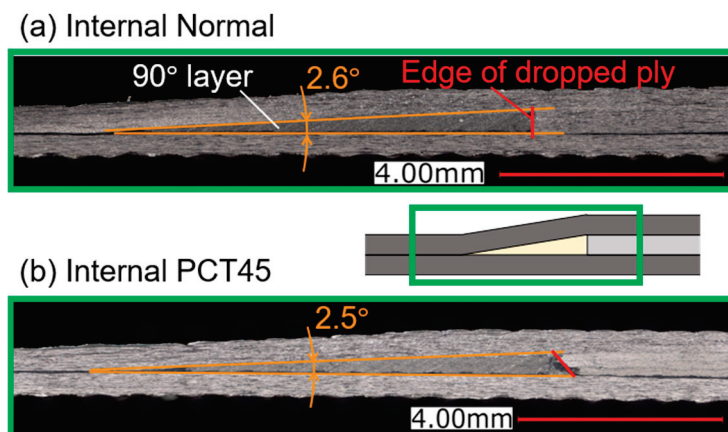


Figure 8. Cross-sectional micrographs of tapered sections in Internal specimens. (a) Internal Normal. (b) Internal PCT45.

3.1.2. Test Method

Tensile fatigue tests were conducted using a hydraulic servo fatigue testing machine (EHF-EB5, Shimadzu Corp., Kyoto, Japan). The stress ratio R was set to 0.1, and the loading frequency was 8.0 Hz. Under these conditions, there was no increase in the specimen temperature. Two loading conditions were used where the maximum tensile stress in the thin section of the specimen was 680 MPa and 1190 MPa. The tensile strain in the thin section was 0.5% at 680 MPa and 0.9% at 1190 MPa. These two conditions were determined based on the finding of the previous research by the authors [30], where the initial damage mode changed from delamination from the ply drop-off to matrix cracking along the curving fibers at a tensile strain of 0.7% (Figure 2).

The fatigue failure process was evaluated by a combination of visual observation and X-ray CT imaging. For delamination propagation assessment, one side of the specimen was coated with thin white paint, and scale marks were drawn at 2 mm intervals. During the test, the damage initiation and delamination propagation on the painted surface were observed with a magnifying glass. Damage initiation from areas other than ply drop-offs is difficult to observe visually during test, and delamination propagation is not always uniform in the width direction. Therefore, the specimen was removed from the fatigue testing machine after a certain fatigue cycle, and X-ray CT imaging (Skyscan1272, Bruker Corp., Billerica, MA, United States) was performed. Before X-ray CT imaging, a contrast agent was applied to the specimen to enhance the visibility of damage.

3.2. Results and Discussions

3.2.1. Low Loading Condition (Maximum Stress 680 MPa, Maximum Strain 0.5%)

At this load level, the matrix crack at the ply drop-off was the initial damage (Figure 9) and the starting point of delamination for both the External and Internal specimens, regardless of whether PCT was introduced or not. Table 2 shows the number of cycles at which the initial damage occurred. The initial damage was delayed for PCT45 compared to Normal for both Internal and External. This was because the introduction of PCT suppressed the stress concentration at the edge of the dropped ply (Figure 5).

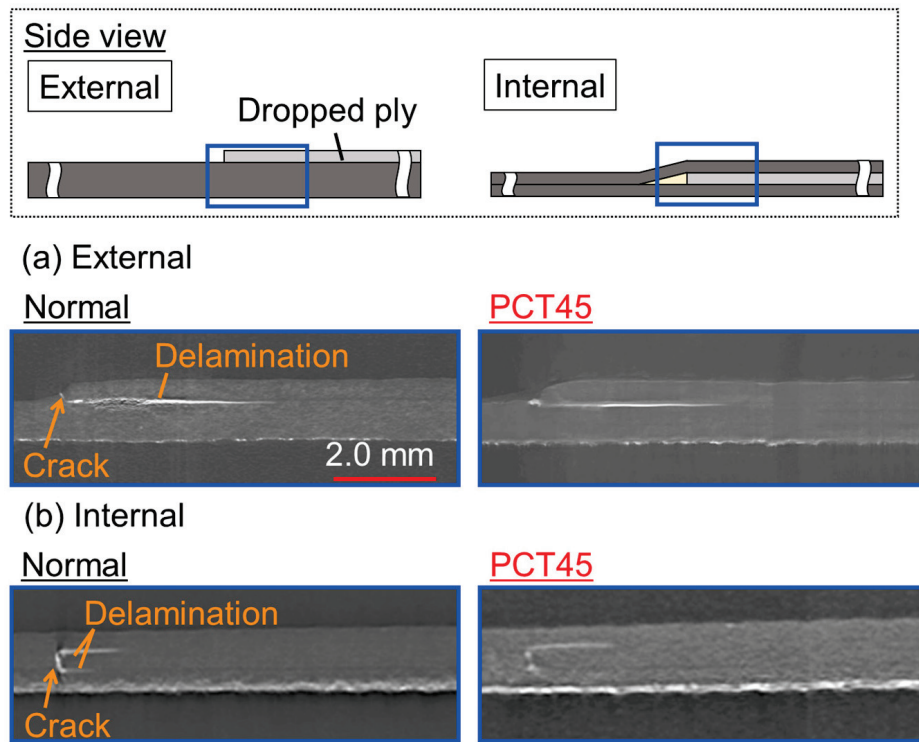


Figure 9. X-ray CT images of initial damage under low loading condition. (a) External. (b) Internal.

Table 2. The number of cycles at which initial damage (matrix crack at ply drop-off) occurred.

	External Normal	External PCT45	Internal Normal	Internal PCT45
Initial damage cycles	200	2400	100	5000

Figure 10 shows the delamination length defined as the distance between the ply drop-off and the point where delamination propagated the most (delamination edge, upper right of Figure 10). Internal specimen has two lines because delamination propagated at both the top and bottom interfaces of the dropped ply. All lines in Figure 10 started with a delamination length of 0 mm because the initial damage was cracking at the ply drop-off. Figure 10 indicates that, overall, the delamination propagation was significantly slower for Internal than for External. In the case of Internal, although delamination propagation was slightly slower at the lower interface, the behavior of delamination propagation at the upper and lower interfaces was similar. The PCT45 had a significantly slower delamination propagation rate for both External and Internal, which was as expected from the results of the finite element analysis in Section 2. Especially, the delamination suppression effect of PCT in Internal was remarkable, and delamination was limited to the area where PCT was introduced (i.e., less than 8 mm) even when PCT45 was loaded with 10 times the number of cycles that delamination reached the edge of the specimen in Normal. This suppression of delamination growth is due to the stress concentration relief effect by PCT. The above results successfully confirmed that PCT is highly effective for internal ply drop-offs under relatively low loading conditions, where delamination starts from the ply drop-off.

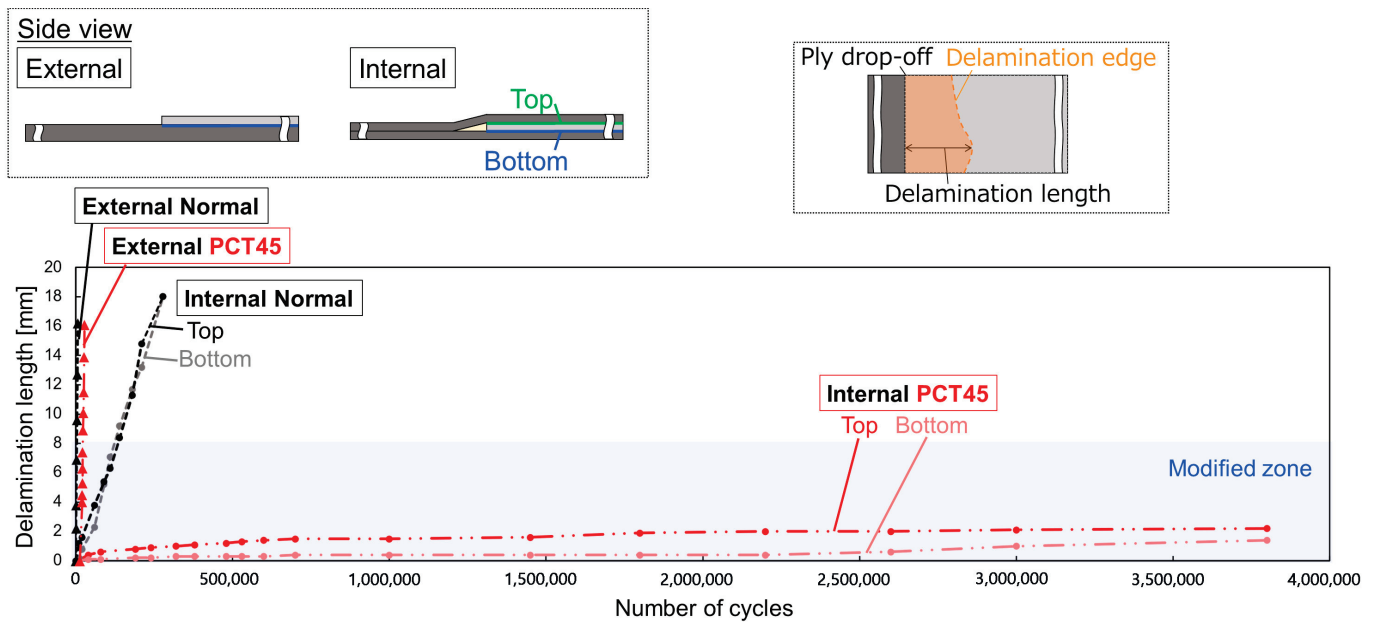


Figure 10. Delamination length under low loading condition.

3.2.2. High Loading Condition (Maximum Stress 1190 MPa, Maximum Strain 0.9%)

Figure 11 shows the delamination length. In Normal specimens of both External and Internal (Black lines), the initial damage was matrix cracking at the ply drop-off, as in the low loading case (Figure 9). So, the delamination length of Normal in Figure 11 started from 0 mm and increased in proportion to the increase in the number of cycles. In contrast, the failure behavior of PCT45 was different from that under the low loading condition. In the case of External, as in the authors' previous study [30], cracks occurred along the curved fibers and delamination propagated from the inside of the cracks (Figure 12a). This is the reason why the delamination length of External PCT45 started at 6 mm in Figure 11. Since the delamination occurred from the end of the PCT modified zone, the delamination edge quickly reached the rigid area, where the fiber direction of the dropped ply was 0° and the delamination propagation was fast, limiting the delamination suppression effect of PCT [30]. In the case of Internal PCT45, however, the initial damage was not cracking along the curved fibers. As in the low loading case, ply drop-off cracking was the first to appear, from which delamination developed (Figure 12b). Interestingly, at 10,000 cycles, as in the External case, a crack developed along the curved fibers, but delamination did not propagate from the inside of the crack. Delamination starting from the matrix crack at the ply drop-off propagated slowly and then accelerated as the delamination edge approached the end of the PCT modified zone and the longitudinal stiffness of the dropped ply at the delamination edge increased. Finally splitting of the base laminate occurred and the specimen failed. As a result, in the Internal case, PCT45 failed at 10 times the cycles of Normal, as shown in Figure 11.

So, the high loading resulted in cracks along the curved fibers for both External PCT 45 and Internal PCT 45, but the delamination did not propagate from the inside of the cracks in Internal PCT45 (Figure 12). To investigate the reason for this, additional finite element analysis was performed using the PCT models in Figure 3. Note that, to simulate the high-loading test conditions, the x-directional displacement was increased so that the stress at the thin section was 1190 MPa. In addition, a seam crack was introduced at the edge of the dropped ply in the Internal model to simulate the ply drop-off cracking (Figure 9). Figure 13 shows the distribution of the transverse stress σ_2 and the out-of-plane shear stress τ_{23} along the thickness direction at the position where the fibers finish curving.

At this position, delamination propagation inside the crack was most pronounced in the External case at the early stage (Figure 12a). Note that the stress deformation state after crack initiation is equivalent to the superposition of (a) the state before crack initiation and (b) the state where the stress at the crack initiation position is reversed and applied to the crack surface (Figure 14). In PCT, as shown in Figure 5, as the fiber angle of the dropped ply decreases from 45° and the stiffness in the tensile direction increases, tensile load is transferred from the base laminate to the dropped ply through interfacial shear stress. Due to this shear lag effect, in the case of the External PCT, σ_{22} was higher closer to the top surface (Figure 13a) and τ_{23} was large (Figure 13b). These stresses act to warp the inside of the crack (Figure 14a), causing delamination to propagate [26]. In contrast, in Internal PCT, σ_{22} was maximum at the center of the thickness direction (Figure 13a). This was because the tensile load was transferred from both the top and bottom interfaces of the dropped ply. As a result, the effect of the shear lag was smaller than in the External case, where the tensile load was transferred only from the bottom interface. This also led to the smaller shear stress τ_{23} (Figure 13b). Therefore, the stress to open the crack was smaller (Figure 14b), and the delamination did not propagate from the inside of the crack.

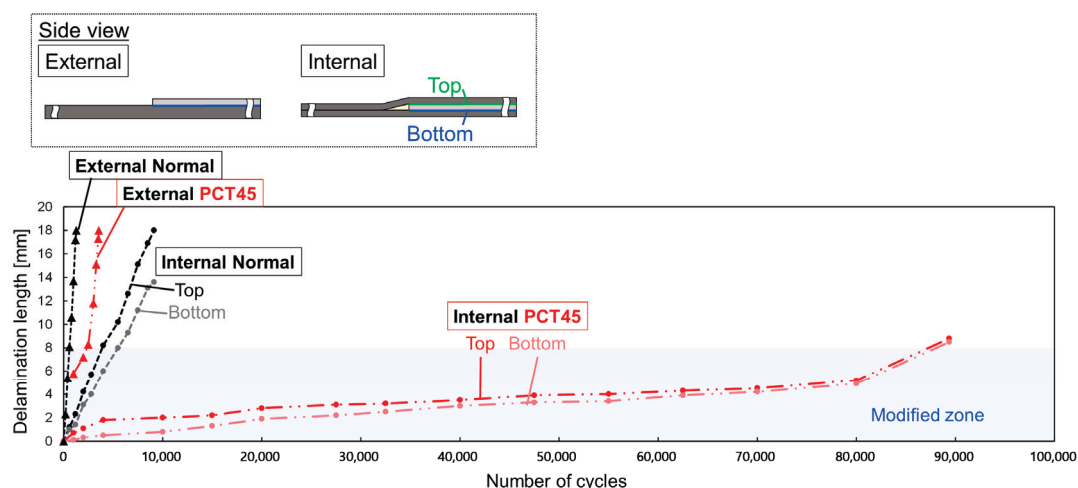


Figure 11. Delamination length under high loading condition.

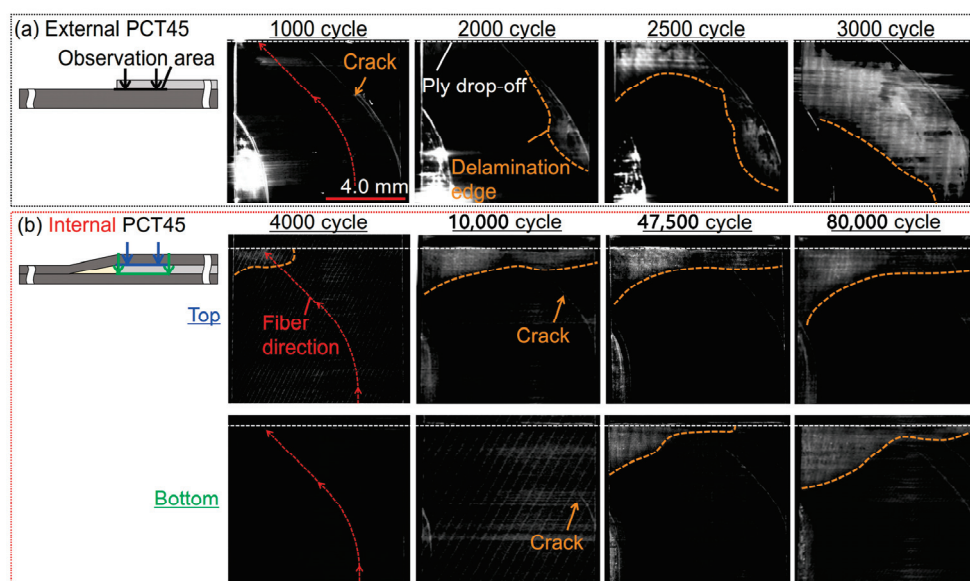


Figure 12. X-ray CT images of damage observed in PCT45 under high loading condition. (a) External PCT45. (b) Internal PCT45.

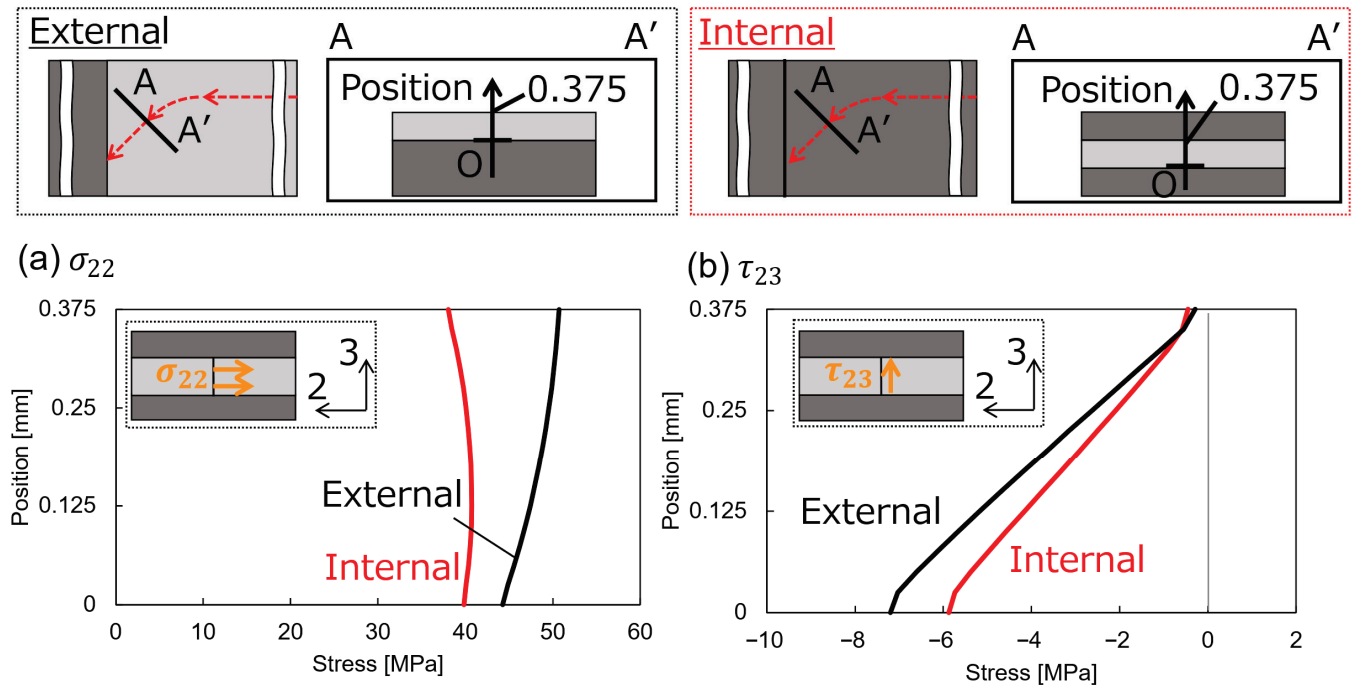


Figure 13. Stress distribution along thickness direction at position where fibers finish curving. (a) σ_{22} . (b) τ_{23} . By reversing this stress and applying it to the crack face, the stress deformation state after crack initiation can be explained (Figure 14).

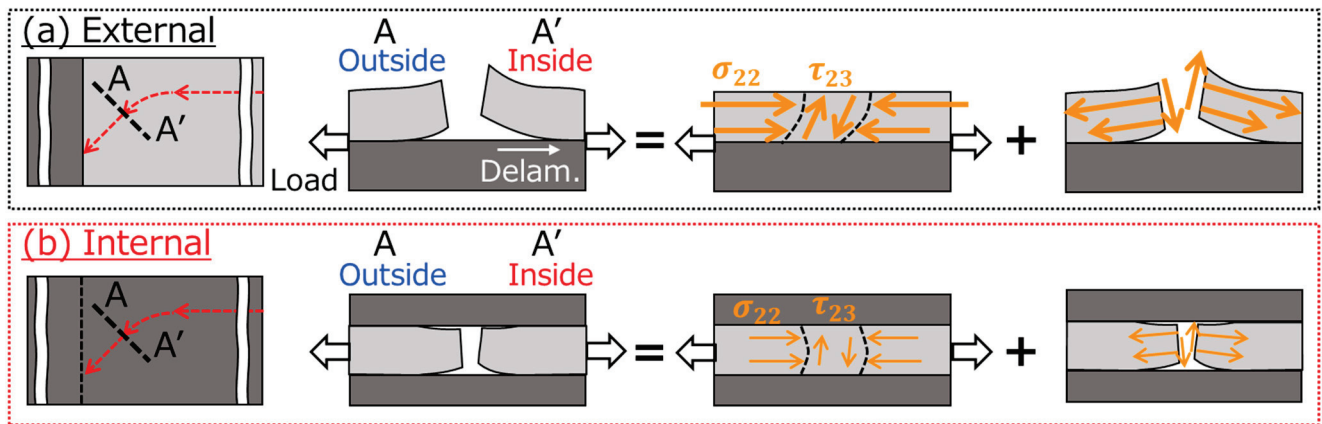


Figure 14. The principle of superposition used to explain stress deformation state after crack initiation. The stress deformation state after crack initiation (left) is equivalent to the superposition of the state before crack initiation (middle) and the state where the stress at the crack initiation position (Figure 13) is reversed and applied to the crack surface (right). In the case of (a) External (upper), σ_{22} , which has high value near the upper surface (Figure 13a), and τ_{23} , which has large value overall (Figure 13b), warp the inside of the crack (A' side), causing delamination to propagate [26]. In contrast, in the case of (b) Internal (lower), additional stress transfer from the cover ply to the dropped ply balances σ_{22} in the thickness direction (Figure 13a), and reduces τ_{23} (Figure 13b), preventing delamination from propagating.

These results clearly demonstrate the advantages of internal ply drop-off in applying PCT. In the case of external ply drop-off, the failure occurs early due to the propagation of delamination from inside of the cracks along curving fibers, whereas in the case of internal ply drop-off, the delamination from cracks along curving fibers is suppressed and the failure occurs after slow propagation of delamination from the ply drop-off. This allows the delamination suppression effect of PCT to be fully utilized.

4. Conclusions

Following the authors' basic study using external ply drop-off without cover plies [30], this study evaluated the fatigue damage suppression characteristic of PCT in internal ply drop-off with cover plies to confirm the effectiveness of PCT in practical composite laminate configurations.

First, finite element analysis was performed to confirm that PCT in internal ply drop-off effectively suppresses out-of-plane stresses at the interface that cause delamination. In subsequent fatigue tests, PCT showed significantly enhanced fatigue damage suppression characteristic at the internal ply drop-off than at the external ply drop-off. In particular, under a high loading condition, the delamination propagation from the inside of the matrix crack along the curved fibers, which significantly reduces the effectiveness of PCT in external ply drop-off [30], was suppressed. Finite element analysis confirmed that the additional stress transfer from the cover ply to the dropped ply suppressed the curved matrix crack opening, which in turn delayed the delamination propagation.

These results demonstrated that the fatigue damage suppression effect of PCT is enhanced in practical composite ply drop-off configurations with cover plies.

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Article

Low Poisson's Ratio Measurement on Composites Based on DIC and Frequency Analysis on Tensile Tests

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Abstract: Accurate determination of elastic properties, especially Poisson's ratio, is crucial for the design and modeling of composite materials. Traditional methods often struggle with low strain measurements and non-uniform strain distributions inherent in these anisotropic materials. This research work introduces a novel methodology that integrates Digital Image Correlation (DIC) with frequency analysis techniques to improve the precision of Poisson's ratio determination during tensile tests, particularly at low strain ranges. The focus is on the evaluation of two distinct frequency-based approaches: Phase-Based Motion Magnification (PBMM) and Lock-in filtering. DIC + PBMM, while promising for motion amplification, encountered specific challenges in this application, particularly at very low strain amplitudes, leading to increased variability and computational demands. In contrast, the DIC + Lock-in filtering method proved highly effective. It provided stable, filtered strain distributions, significantly reducing measurement uncertainty compared to traditional DIC and other conventional methods like strain gauges and Video Extensometers. This study demonstrates the robust potential of Lock-in filtering for characterizing subtle periodic mechanical behaviors leading to a reduction of approximately 70% in the standard deviation of the measurement. This work lays a strong foundation for more precise and reliable material characterization, crucial for advancing composite design and engineering applications.

Keywords: mechanical characterization; digital image correlation; composites; frequency analysis; lock-in filtering; Poisson's ratio

1. Introduction

Composites, defined as the combination of two or more distinct macroscopic materials with an interface between them, allow for designing materials with tailored properties, optimizing aspects such as stiffness, strength, weight, and corrosion resistance in a way that is not possible with individual materials [1]. This is why the scientific interest in composite materials is unquestionable nowadays. In fact, different industries have echoed this potential and applied composites for addressing specific performance requirements, demanding more applied research about it, for instance, in the aerospace, military, or medical industries [2–4]. These are just a few examples; the versatility of composites makes them indispensable in demanding applications across diverse industrial landscapes.

Moreover, most woven composite materials exhibit complex mechanical behavior, and the development of predictive models or mechanical simulations for them represents a significant scientific challenge for the practical design of efficient parts [1]. Specifically, within mechanical characterization, the tensile test is a fundamental method [5]. Among

other properties, Young's modulus, tensile strength, and Poisson's ratio are particularly important parameters to define a material's behavior under complex stress states and a key input for finite element analysis and material modeling [5,6]. However, the accurate determination of Poisson's ratio in composites, which offers the fundamental metric by which to compare the performance of any material when strained elastically, can present several complications, such as low, nonlinear, or negative values [6–9]. Traditional methods using strain gauges or extensometers require the physical attachment of these sensors to the material, which can influence the mechanical response [1]. Additionally, strain gauges provide average strains over a set gauge length and lack the spatial resolution needed to capture non-uniform strain distributions in composites, which can be significant due to the size of the yarns [10].

To overcome some of these difficulties, advanced optical techniques such as Digital Image Correlation (DIC) allow the measurement of full-field strain without contact [11], which has led to numerous previous investigations related to mechanical characterization of composites [10,12,13]. Among its principal advantages, DIC enables the acquisition of detailed information about the strain distribution on the material's surface, revealing stress concentrations and complex deformation patterns. It is based on tracking a random pattern, typically a speckle pattern, applied to the material's surface. In DIC experimentation, digital images are captured before and during deformation, and correlation algorithms find the correspondence between small areas (subsets) in the images. By comparing the position of these subsets, displacement and strain are calculated across the surface. Therefore, the uncertainty and accuracy of DIC measurements are heavily influenced by image resolution. Higher resolution (more pixels per unit area) allows for more precise tracking of subset movement. With higher resolution, interpolation errors are reduced, and smaller displacements can be detected. Nowadays, an accuracy of 0.01 pixels or better can be achieved using commercial software that deals with distortion correction and higher-order spline interpolation functions. However, the subset size is a key consideration: smaller subsets increase spatial resolution but amplify noise, while larger subsets reduce noise but may miss local strain variations. Therefore, proper selection of image resolution and subset size is crucial for minimizing uncertainty and maximizing accuracy in DIC measurements [10,11,14]. These issues, along with speckle pattern quality and out-of-plane motion, are parameters to optimize for accurately measuring small strains, such as transverse strain in tensile tests [15,16]. Therefore, DIC has emerged as a powerful tool for full-field, non-contact measurement of displacement and strain, offering significant advantages over traditional methods for composites [5,10,17] due to, among others, its ability to capture high resolution and simultaneous longitudinal and transverse strains, enabling the direct calculation of Poisson's ratio [5,10,12]. However, some composite materials exhibit a low Poisson's ratio, leading to a reduced transversal strain during the tensile test, so DIC could struggle with the resolution and accuracy required for such measurements, or it could even be interesting to accurately measure Poisson's ratio evolution along different strain ranges [10,15]. Previous investigations indicated that the combination of dynamic mechanical methods with DIC would allow a deeper understanding of material behavior and the simultaneous determination of multiple elastic properties [17].

In this work, to improve the accuracy of low Poisson's ratio values, a methodology is proposed to merge frequency data analysis with tensile tests. This proposal consists of the performance of a sine of low strain amplitude during the elastic region in the tensile tests. Then, the data obtained from the DIC analysis of that sine section would be suitable to be analyzed using frequency filtering or enhancement. Specifically, Lock-in filtering [18] and Phase-Based Motion Magnification (PBMM) [19] are here evaluated since they have previously obtained good results in integration with DIC.

Lock-in is a simple, efficient frequency analysis method usually employed for Thermoelastic Stress Analysis (TSA) under cyclic loading [20]. It is a common tool for filtering the thermoelastic signal and extracting information at the frequency of interest, as the thermoelastic signal is often very subtle and contaminated by noise. Its popularity stems from the simplicity of Lock-in amplifiers. A wide variety of uses of Lock-in and TSA have been found in the literature for the study of fracture and fatigue phenomena, as well as for the evaluation of stress fields in the modal shapes of individual resonances [20,21]. Lock-in requires a reference signal to filter noise out at that frequency. The result is a data field where the magnitude represents the amplitude of the signal and the phase shift is between the filtered signal and the reference signal. The same idea could be applied to obtain displacement maps. In fact, Fruehmann et al. [18] explored the application of a Lock-in amplifier to process DIC data obtained from components under cyclic loading. Their aim was to obtain precise surface deformation maps in fatigue tests without the need for high-speed cameras. The technique involves applying the Lock-in amplifier to the deformation data derived from images captured at low frame rates. The results demonstrated that this method allows for obtaining precise deformation maps even with loading frequencies higher than the camera's frame rate. The application of the Lock-in algorithm achieves an improvement in deformation resolution compared to a standard static test. However, recent investigations have shown that it is not necessary to use a Lock-in amplifier and an off-line filtering process could be applied to the displacement maps [20].

Moreover, Phase-Based Motion Magnification (PBMM) is a Eulerian methodology for processing periodic motion in videos. Its main goal is to reveal subtle and imperceptible movements that occur in image sequences by creating a magnified version of these movements [19]. It is a tool with great potential in the analysis of monotonic periodic events and helps in interpreting the deformations occurring in machine parts. In fact, Molina-Viedma et al. [14] explored this combination to analyze mode shapes at high frequencies with DIC. The authors pointed out that while DIC provides quantitative motion information with greater sensitivity than the naked eye, its accuracy decreases at high frequencies due to low displacements. Therefore, they investigated the combination of DIC and PBMM to provide numerical information in magnified videos and perform mode shape characterization by DIC at unprecedented frequencies, by increasing the amplitude of displacements. It was demonstrated that PBMM contributes to improving the DIC visualization of cantilever beam mode shapes. Particularly, higher-order mode shapes could be visualized, expanding the limits of DIC to frequencies up to 6710 Hz in that work. Furthermore, the authors also applied that methodology in 3D-DIC [22] and in the integration of DIC with Fringe Projection [23].

Specifically, this work explores the potential of DIC in combination with PBMM and Lock-in for the obtaining of a low Poisson's ratio, which is common in composites. The proposal consists of the inclusion of at least one stage during a traditional tensile test where the specimen is also subjected to a sinusoidal excitation controlled by the engineering strain or displacement. The amplitude and strain levels should be reduced compared to the material's failure strain to avoid generating internal damage on specimens. Images obtained during the sinusoidal excitation are processed with DIC, DIC + PBMM, and Lock-in for filtering strain values during the sine stage, thereby yielding a reliable Poisson's ratio. This work demonstrates a promising method for accurate material characterization.

2. Fundamentals

2.1. Lock-In Filtering

As previously mentioned, Lock-in amplification is a signal processing technique used to extract a signal with a known frequency from a noisy environment [18,20]. It is

particularly effective when the signal of interest is small and embedded in much larger noise at other frequencies.

Lock-in processing requires a reference signal at the exact frequency of the signal to be measured. This reference signal can be derived from the source generating the signal (e.g., the load signal in a mechanical testing machine) or be a synthetically generated signal at the frequency of interest, which leads to off-line Lock-in [20].

Traditionally, Lock-in amplification was performed using dedicated electronic instruments called Lock-in amplifiers [24], which contain the necessary multipliers, filters, and phase-shifting circuitry. With advancements in computing, Lock-in amplification can also be implemented in software through off-line processing of recorded data based on Fourier series. Thus, the signal can be decomposed as sines at the discrete frequencies $f_n = n \frac{f_s}{N}$, according to the sampling frequency, f_s , and the number of samples, N , with $n = 0, \dots, N/2$ when N is even, which is the most common in practice. Using the subscript Y to indicate the frequency of interest, the signal can be decomposed as follows [20]:

$$y(j) = A_Y \sin(2\pi f_Y t_j + \phi_Y) + \sum_{n=0}^{N/2} (1 - \delta_{nY}) A_n \sin(2\pi f_n t_j + \phi_n) .$$

$$t_j = \frac{j}{f_s} \quad j = 0, 1, \dots, N-1.$$

$$\delta_{nY} = \begin{cases} 1 & n = Y \\ 0 & n \neq Y \end{cases} \quad (1)$$

where A and ϕ represent the amplitude and phase of each term. Now, consider a reference monotonic signal, x , and its quadrature, x' :

$$x(j) = 2 \sin\left(2\pi f_X \frac{j}{f_s} + \phi_X\right).$$

$$x'(j) = 2 \cos\left(2\pi f_X \frac{j}{f_s} + \phi_X\right). \quad (2)$$

DC terms only depend on the amplitude of the signal, A_Y , and the phase shift, $\Delta\phi$, at the frequency of interest [20]. In addition, this term can be obtained from both modulated signals as the mean value for the N values of the whole sequence, so that

$$R = \frac{1}{N} \sum_{j=0}^{N-1} y(j)x(j) = A_Y \cos \Delta\phi.$$

$$I = \frac{1}{N} \sum_{j=0}^{N-1} y(j)x'(j) = A_Y \sin \Delta\phi. \quad (3)$$

from which the amplitude and phase shift can be obtained as follows:

$$A_Y = \sqrt{R^2 + I^2}.$$

$$\Delta\phi = \text{atan}\frac{I}{R}. \quad (4)$$

When this process is performed at every pixel signal, an image of amplitude $A_Y(p_{col}, p_{row})$ and phase shift $\Delta\phi(p_{col}, p_{row})$ is obtained for the frequency of interest. Using complex algebra, a complex image is obtained, with R and I as the real and imaginary parts, respectively.

2.2. Phase Motion Magnification

Phase-Based Motion Magnification (PBMM) is a computational technique designed to reveal and amplify subtle motions in videos that are often imperceptible to the naked eye [19]. Unlike traditional methods that rely on intensity variations or optical flow, PBMM operates by analyzing and manipulating the phase information within a video sequence [25]. This approach has proven more robust to noise and capable of handling larger magnification factors compared to intensity-based methods.

The first crucial step in PBMM is to decompose the input video frames into a multi-scale and multi-orientation representation using complex steerable pyramid filters [19]. This is an overcomplete linear transformation that breaks down each image into a set of coefficients corresponding to basic functions localized in position, spatial scale, and orientation. These complex coefficients can be expressed in terms of their amplitude and phase [26].

The phase of these complex coefficients holds critical information about the local spatial structure and, importantly, about motion. The fundamental principle behind PBMM is that small motions in the image sequence correspond to subtle variations in the local phase over time. Once the local phase is extracted for each spatial location, scale, and orientation across the video sequence, temporal signal processing is applied to these phase signals. Typically, a temporal bandpass filter is used to isolate phase variations that occur within a specific frequency band of interest. This allows for the amplification of motions occurring at particular frequencies, such as the vibration frequency of a structure. To avoid phase-wrapping issues, the phases are often unwrapped temporally before filtering [26].

After isolating the phase variations in the desired frequency band, these phase differences are multiplied by a magnification factor (α). According to the Fourier shift theorem, shifts in the spatial domain correspond to linear changes in phase in the frequency domain. By amplifying the temporal variations in the phase, PBMM effectively synthesizes a new video in which these subtle motions are exaggerated approximately in the magnitude $(1 + \alpha)$ [23]. The relationship between phase shift and translation is approximately linear for small motions.

Finally, the modified phase information (with amplified variations) is used along with the original or processed amplitudes of the complex steerable pyramid coefficients to reconstruct the frames of a new video sequence. This reconstruction is essentially the inverse transformation of the complex steerable pyramid decomposition.

3. Methodology

3.1. Materials Studied

In this work, off-line Lock-in and PBMM were evaluated for the analysis of Poisson's ratio in cases where strains measured are low or the actual ratio is low. Specifically, it has been evaluated in two different materials, the GF-PA6 and CF-PC.

The GF-PA6 is a woven glass fiber (twill 2/2) reinforced polyamid-6 composed of BASF SE with a layer thickness of 0.25 mm. The fiber weight rate was 50%/50%. According to initial tensile tests, the ultimate strength of the material was 830 MPa and the strain for failure was 21×10^{-3} strains. The CF-PC is a unidirectional carbon fiber reinforced polycarbonate composed by Covestro AG Leverkusen, Germany (CF GP 1000T) with a fiber volume fraction of 44% and a layer thickness of 0.17 mm. According to initial tensile tests, the ultimate strength of the material was 22 MPa and the strain for failure was 4.4×10^{-3} strains. Specimen shape (25 mm width and 2 mm thickness) and hills colocation were considered according to standard DIN EN ISO 527-4 [27]. The geometrical specifications of the specimens and the orientation of fibers are illustrated in Figure 1.

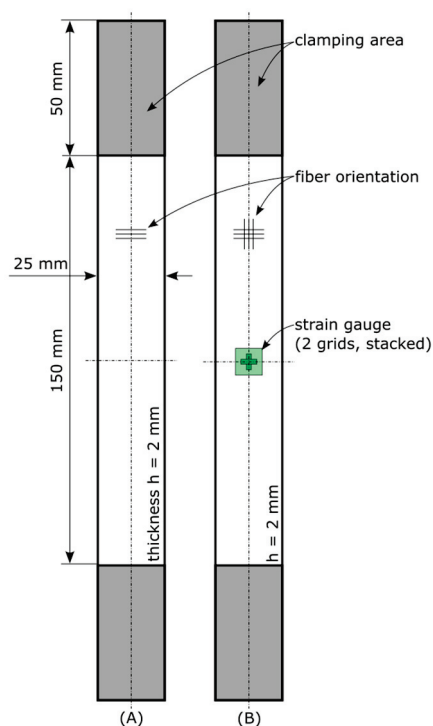


Figure 1. General layout of the specimens and fiber orientation relative to the direction of stress in the tensile tests for (A) CF-PC (B) GF-PA6.

3.2. Experimental Procedure

Since the focus was placed on improving the Poisson's ratio measurement using periodic methodologies such as PBMM and Lock-in, cyclic sinusoidal tests at a low frequency were performed with low strain amplitudes, ensuring that they were well below the respective strain at failure values to avoid generating collateral defects. For the GF-PA6 material two tests at the ranges of 0.05–0.1% and 0.35–0.4% of engineering strains measured by the test machine were commanded. These were designated Test A and Test B. In the case of CF-PC, the other two tests at ranges 0.05–0.1% and 0.2–0.3% of engineering strains were also performed called Test C and Test D, respectively. For all cases, the frequency of the commanded sinusoidal excitation was 0.2 Hz, and a total of 15 load cycles were performed. Tensile tests were executed in a ZwickRoell Z250 universal test machine with a maximum load capacity of 250 kN. The set up employed is shown in Figure 2.

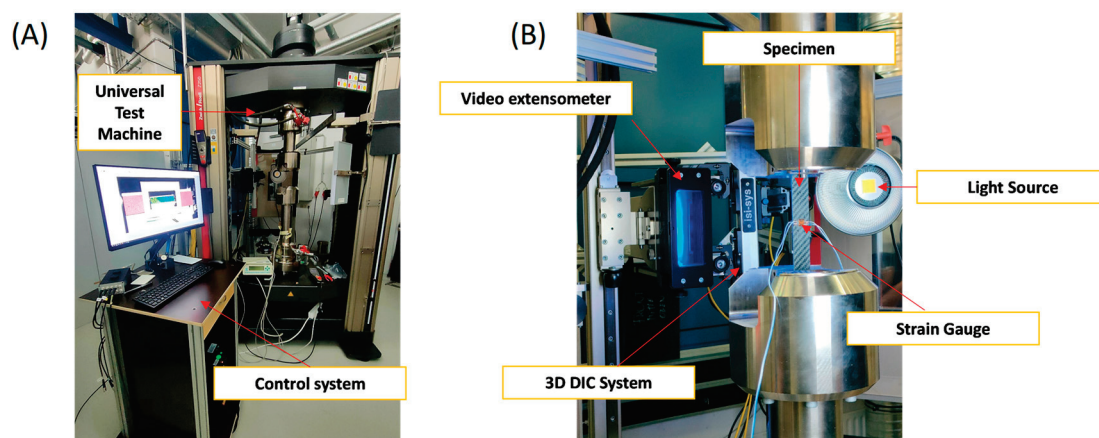


Figure 2. (A) General view of the set up (B) Detail of the specimen and measuring systems.

The primary experimental system for strain measurement during the tests was an ISI-SYS 3D Digital Image Correlation set up, employing VIC 3D 8 software and two Basler acA4096-40 um cameras of 9 Megapixel resolution with Schneider Xenoplan 2.8/50 lenses recording at 10 fps. The resolution of each camera was 30.6 px/mm. The calibration of the system was performed by employing a 7 mm space target. The parameters of the DIC processing were a 29 px subset size and 7 px spacing according to recommendations in the bibliography [13], giving a grid of 221×74 data points.

For the GF-PA6 specimens, the 3D-DIC, PBMM, and Lock-in results were compared with those obtained from two other experimental systems. One of them was a 2-grid stacked strain gauge (1-XY91-6/120 from Hottinger Brüel & Kjaer GmbH, Nærum, Denmark) covering an area of 8×8 mm as illustrated in Figure 1. This size value is important since it is in the order of magnitude of the composite weft. The test machine recorded the ε_{xx} and ε_{yy} data measured by strain gauge and data were synchronized with the load and displacement. An additional experimental system employed was a Video Extensometer, which used two cameras for longitudinal strain (with a resolution of 29.8 px/mm) and one camera for transverse strain (with a resolution of 23.7 px/mm) (VideoXtens biax 2-150 HP by ZwickRoell GmbH, Ulm, Germany). As in the case of the strain gauge, the ε_{xx} and ε_{yy} data measured were synchronized with data obtained by the test machine.

3.3. Methodology for PBMM and Lock-In Analysis

To evaluate the performance of PBMM, 200 pairs of images (set of images of Camera 0 and Camera 1 framed with a red dashed line box in Figure 3) corresponding to four cycles were magnified using PBMM with a magnification factor of $\alpha = 2$ and for CF-PC material also of $\alpha = 1$. For the PBMM procedure (framed with a purple dashed line box in Figure 3), the PBMM algorithm was adapted for use with image sequences from the code made available by its authors [19]. The sets of original images captured by each camera were magnified independently, which required approximately 40 min on a machine equipped with an Intel i7 9700K CPU and 32 GB of RAM. After magnification, the same images representing the unloaded specimen were considered as the initial reference, similar to the non-magnified image sets. Finally, the magnified sets of images were processed using VIC 3D 8 with the same parameters as the non-magnified image sets. To evaluate the off-line Lock-in filtering (framed with a green dashed line box in Figure 3), an in-house code written in Matlab was developed following the methodology proposed by Molina-Viedma et al. [20]. The inputs for the algorithm were the strain maps obtained from 3D-DIC (procedure framed with a black dashed line box in Figure 3), corresponding to the same four cycles as used for PBMM. The results were one ε_{xx} and one ε_{yy} filtered maps representing the respective amplitudes; the computation time required was negligible. Figure 3 shows the schematic representation of the procedure explained above to obtain results from DIC, DIC + PBMM, and DIC + Lock-in.

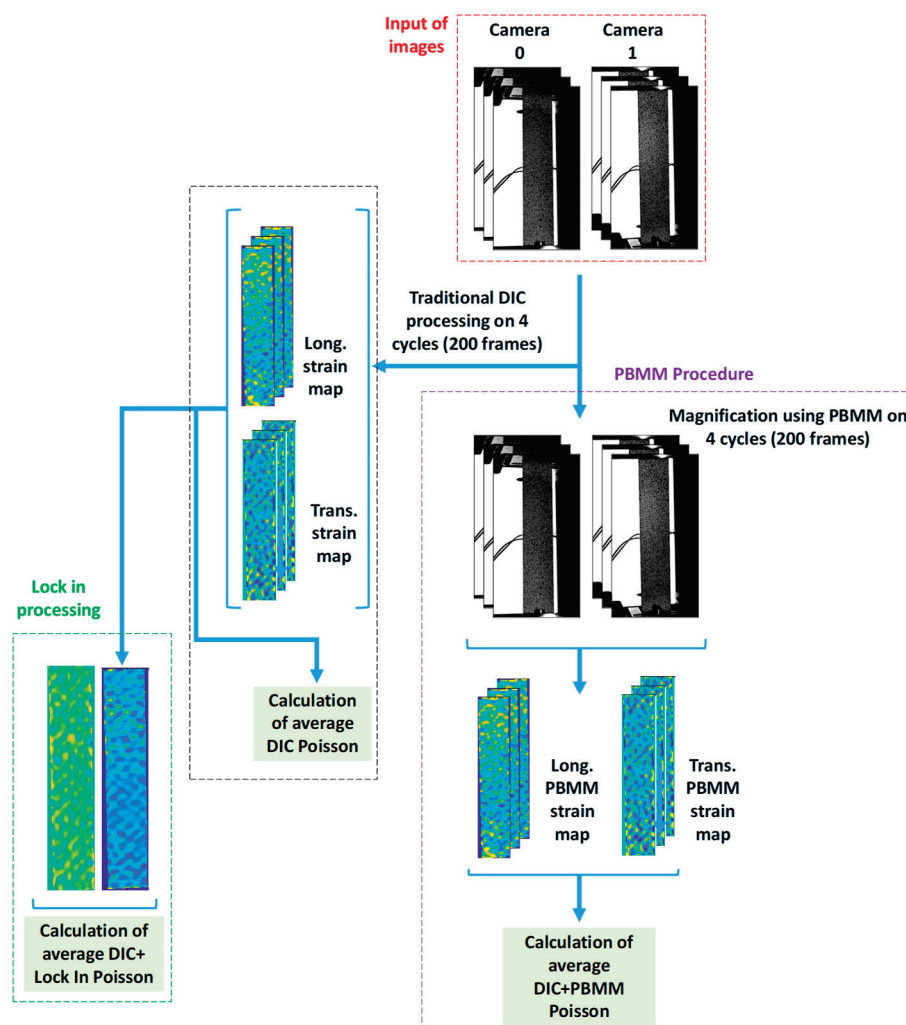


Figure 3. Pipeline of the processing performed to obtain results from DIC, DIC + PBMM and DIC + Lock-in.

4. Results

4.1. Results for GF-PA6 Material

For illustration, Figure 4 shows the stress–strain results during the test performed on GF-PA6 material, specifically Test A and Test B. As observed, the red lines illustrate the engineering strain measured by the testing machine, which corresponds to the commanded data discussed previously. Nevertheless, as expected, the real strain measured by DIC, strain gauge, and Video Extensometer is significantly lower. Focusing on the details of each graph, the strain measured by Video Extensometer is higher than that measured by DIC, which is also slightly higher than the one measured by strain gauge. It is important to note that the tests were conducted at very low strains, well below the onset of damage or plasticity, as intended.

Focusing on the analysis of the Poisson coefficient, Figure 5 shows the Poisson’s ratio measured from the ϵ_{xx} and ϵ_{yy} values measured with the stain gauge, Video Extensometer, DIC on its traditional procedure, DIC + Lock-in, and DIC + PBMM. The results from the full-field techniques, i.e., DIC and DIC + PBMM, were calculated by averaging each data field corresponding to each couple of images captured. DIC + Lock-in Poisson’s ratio results are calculated by relating the ϵ_{xx} - and ϵ_{yy} -filtered data maps as illustrated on Figure 3.

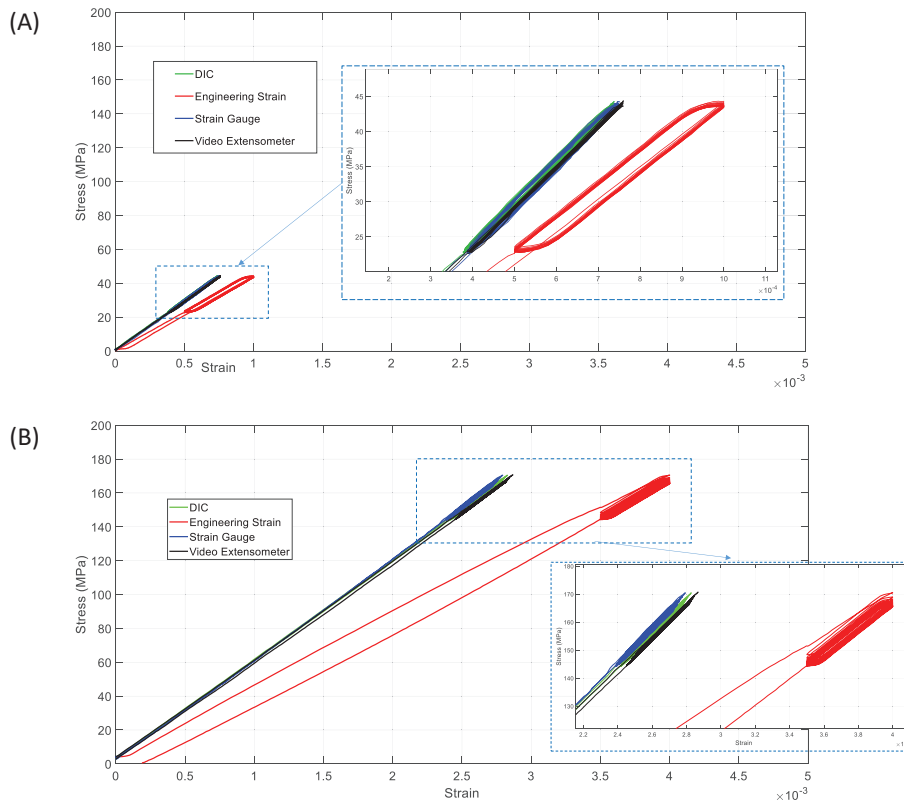


Figure 4. Stress–strain results from Test A (A) and B (B) on GF-PA6 material.

Figure 5 shows the results for the frames corresponding to the images where PBMM was applied. Additionally, Table 1 presents the average of the measurements calculated over the same range as presented in Figure 5. Comparing the results on the graph, it is clear that the results of Test A were noisier than those obtained in Test B. This is mainly due to the strain level, which is lower in the former. In general, it is also observed that strain gauge obtains Poisson's ratios which are slightly lower than results obtained by the DIC techniques, but it is also observed that the results obtained from Video Extensometer are notably overestimated with respect to any of the other systems. In fact, the values from Video Extensometer are 346% and 196% times those obtained from strain gauge. This could probably be due to the lower resolution of the system, which is more notorious at lower strains. Nevertheless, the strain gauge size is in the order of magnitude of the composite yarns, so its strain values could be affected by the exact measuring point. Another issue observed is the peaks on the PBMM results for Test A that are not present in Test B. This behavior is answered with the strain level achieved, which was lower in the former. For PBMM results, since displacements were magnified, the lower displacement values tend to zero value, so the relation $\nu = -\varepsilon_{trans}/\varepsilon_{long}$ tends to infinity. To enhance graph clarity, the Poisson's ratio values higher than 0.1 were not displayed and removed from further calculations. This aspect is not observed in Test B, mainly due to the fact that the average strain during the test was high enough to ensure strains remained sufficiently far from zero values even after magnification. Therefore, tests involving very low strains should be considered with caution when using PBMM, as they represent a critical point-of-care for this method.

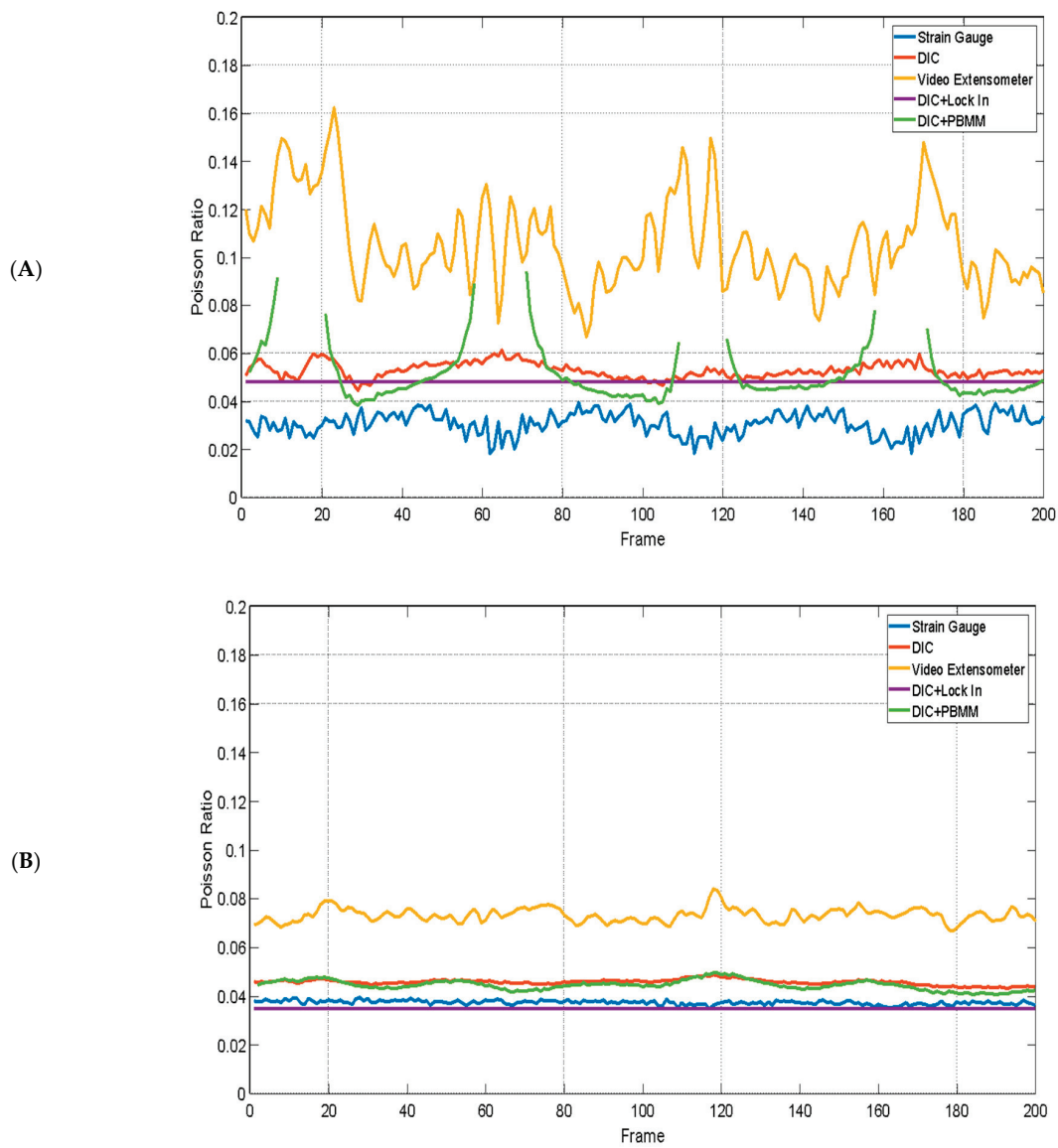


Figure 5. Poisson's ratio measured along the 200 frames corresponding to four cycles for Test A (A) and Test (B).

Table 1. Table resume of Poisson's ratio obtained in test A and test B.

	Strain Gauge	DIC		DIC + PBMM		DIC + Lock-In	
	Average	Average	STD	Average	STD	Average	STD
Test A	0.0307	0.0528	0.0668	0.1423	0.415	0.0479	0.0225
Test B	0.0373	0.0456	0.0391	0.0444	0.0302	0.0346	0.0210
Average	0.0333	0.0493	0.0530	0.0471	0.216	0.0433	0.01275

Regarding Table 1, Video Extensometer results are not considered since the results could be considered as not suitable for this specific application. The uncertainties of the original measurement systems are not considered either since, in this work, the improvement from original results employing the periodical analysis of PBMM and Lock-in procedures is evaluated. From the two tests, A and B, the average is also calculated together with the standard deviation, considering the two tests for each measurement. As observed, the value of the techniques based on DIC is normally higher than that obtained from strain gauge;

however, for the case of DIC + Lock-in in Test B, the Poisson's ratio is significantly closer (7% of difference with respect to strain gauge). In fact, the standard deviation obtained from DIC + Lock-in was reduced to 37% of that which was obtained from DIC in this case. For the case of DIC + PBMM, for Test A, results were not improved with respect to traditional DIC as commented; however, for Test B, both the mean and standard deviation values were reduced, making them closer to those from DIC + Lock-in but not as close to strain gauge.

4.2. Results for CF-PC Material

As in the previous section, firstly it is presented in Figure 6 the stress–strain results during the tests performed on CF-PC material, which are Test C and Test D. In these tests, only the DIC system was employed as the measuring technique for real strains. The data from images or from strain maps is employed following the pipeline presented in Figure 3 to process PBMM or Lock-in, respectively. As in the previous test, the real strain measured by DIC is significantly lower than the engineering strain measured by the test machine, as predicted by theory.

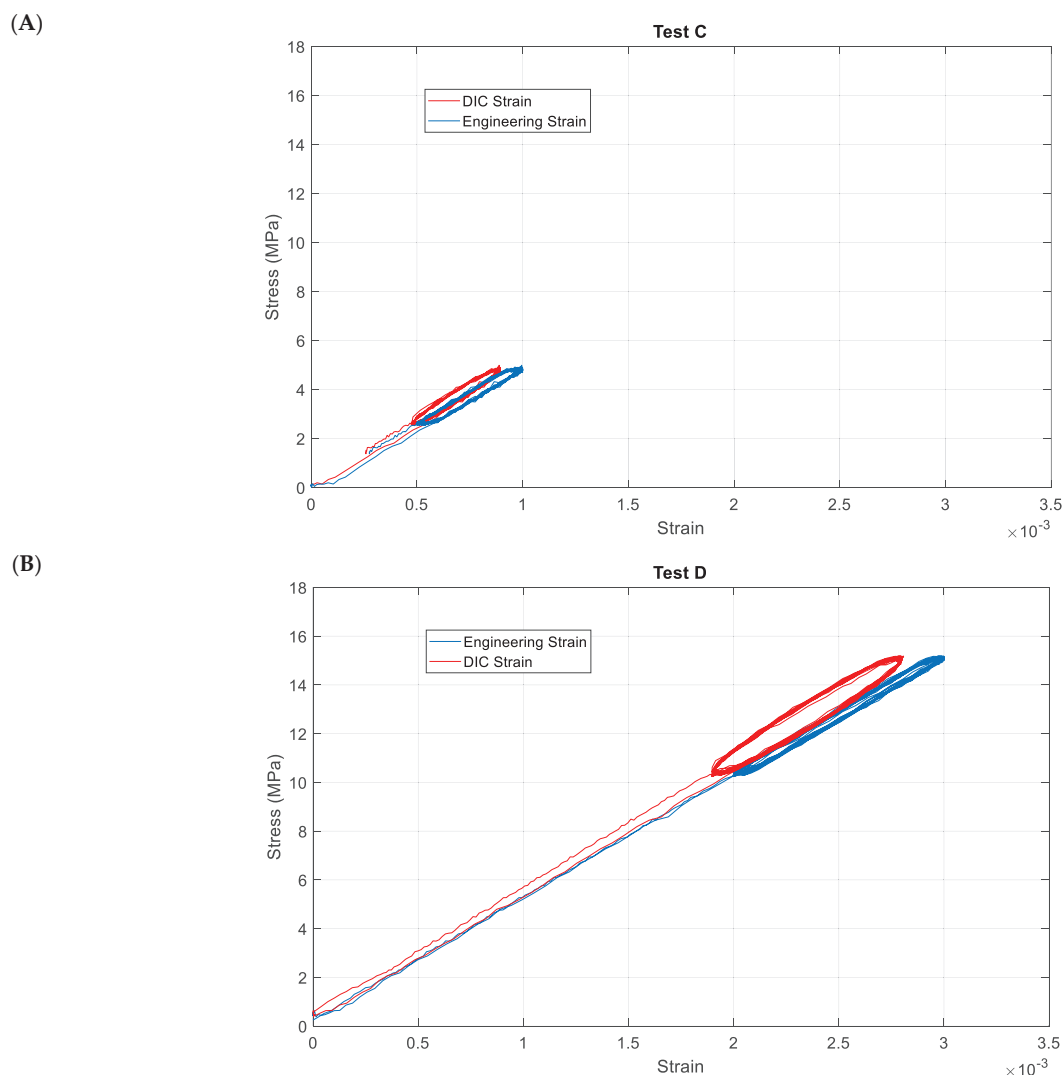


Figure 6. Stress–strain results from Test C (A) and D (B) on CF-PC material.

Moreover, Figure 7 shows the Poisson's ratio measured along the 200 frames corresponding to four cycles of loading. It is notorious that the Poisson's ratio for this material

is much lower than that from previous tests. Results for the two tests, Test C and Test D, were obtained by direct DIC technique, DIC + Lock-in, and DIC + PBMM. In this case, two magnification factors have been explored, $\alpha = 1$ and $\alpha = 2$, in order to evaluate if a lower magnification factor could elude previous drawbacks observed when low strains were achieved after magnification. As observed, the graph highlights the heterogeneity in the Poisson's ratio calculated with DIC + PBMM as occurred in Test A, when lower strains mean that, after magnification, minimum values of $\varepsilon_{\text{transveral}}$ tend to 0, leading to incorrect high ν values. Higher peaks were obtained when the magnification factor was $\alpha = 2$. Nevertheless, for magnification values of $\alpha = 1$, even when reduced, there were also visible some oscillations that could affect the mean value. It is appreciable that direct DIC values are more stable along the frames analyzed, and, logically, DIC + Lock-in present a single filtered value for the complete analyzed period.

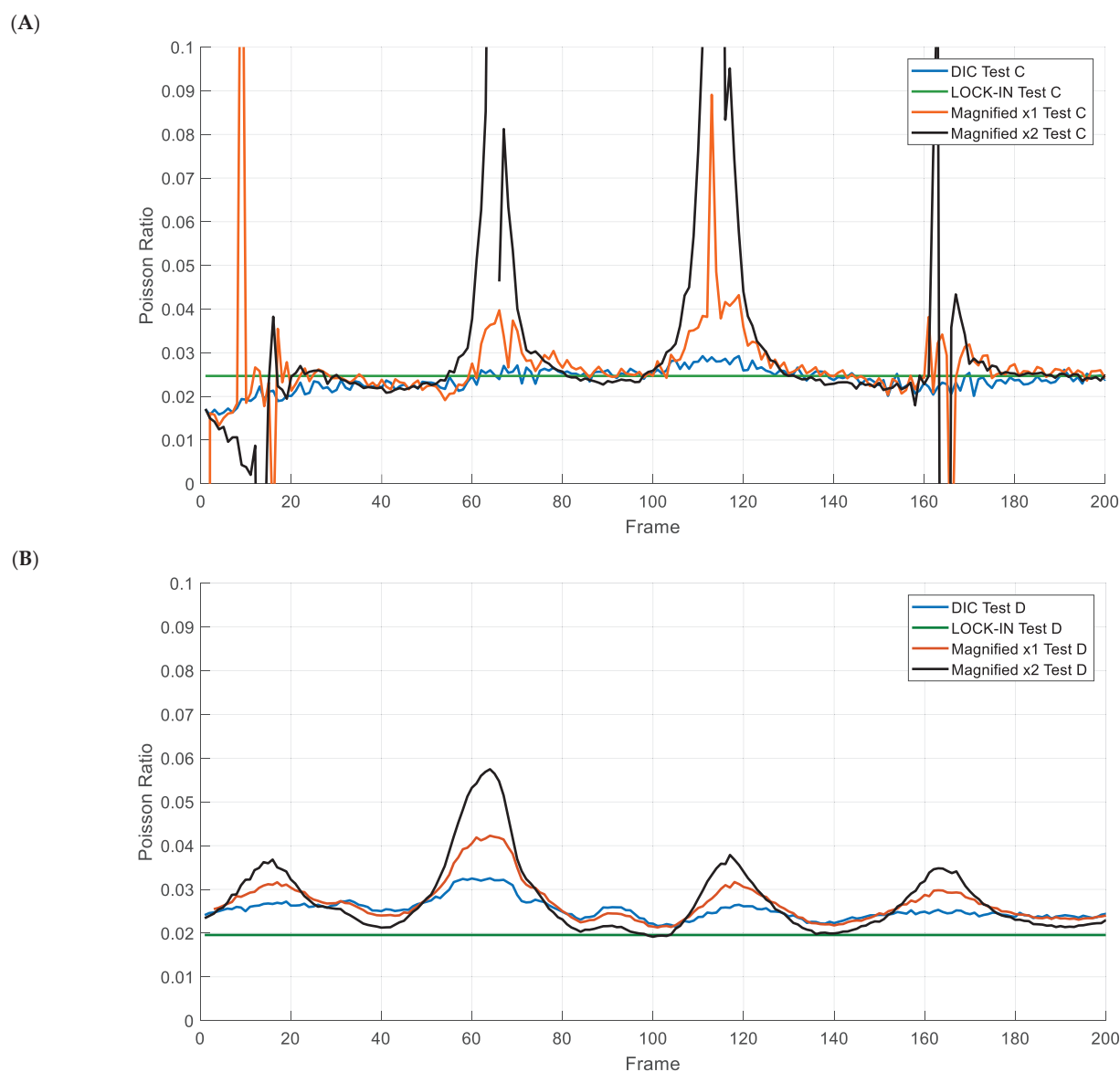


Figure 7. Poisson Ratios Measured along the 200 frames corresponding to 4 cycles for Test C (A) and D (B) on CF-PC material.

In Table 2, the main statistics from the results are summarized. As observed, results for DIC + PBMM were generally affected by a much higher standard deviation due to the

oscillations generated. The Poisson's ratio measured with DIC + Lock-in in Test C was close to that obtained with DIC. However, in Test D, a reduction to 77.1% of the DIC value is observed. Regarding the standard deviation, it was clearly reduced to 27.7% of the original value from DIC.

Table 2. Table resume of Poisson's ratio obtained in Test C and Test D for CF-PC material.

	DIC		DIC + PBMM $\times 2$		DIC + PBMM $\times 1$		DIC + Lock-In	
	Average	STD	Average	STD	Average	STD	Average	STD
Test C	0.0237	0.0231	0.0286	0.6806	0.0173	0.4483	0.0247	0.0036
Test D	0.0254	0.0097	0.0277	0.0235	0.0190	0.1012	0.0196	0.0055
Average	0.0246	0.0164	0.0282	0.352	0.0182	0.275	0.0217	0.00455

5. Conclusions

This work successfully demonstrated a novel approach to accurately determine elastic properties, specifically the Poisson's ratio, at low strain ranges in composite materials. By integrating frequency analysis into traditional tensile testing, it aims to overcome common challenges associated with measuring subtle transverse strains.

The investigation specifically evaluated the potential of two frequency-based methodologies, Phase-Based Motion Magnification (PBMM) and Lock-in filtering, when combined with Digital Image Correlation (DIC). The key findings are summarized as follows:

- It has uniquely employed frequency analysis techniques in conjunction with tensile testing for the precise determination of elastic properties, particularly at the challenging low strain ranges relevant to composite materials. This represents a significant methodological contribution to material characterization.
- While DIC + PBMM shows promise for magnifying subtle motions, its application for low Poisson's ratio measurements presented notable challenges. The primary limitation arose when the low-amplitude strains, especially transverse strains, approached zero after magnification, leading to significant fluctuations and impossibly high (tending to infinity) Poisson's ratio values. Furthermore, the computational time required for PBMM processing was considerable, making it less practical for routine analysis in this specific application.
- In contrast, the integration of DIC with Lock-in filtering emerged as a powerful and highly effective tool for accurately determining Poisson's ratio. This methodology demonstrated no significant drawbacks and incurred negligible computational cost. It consistently provided a filtered and stable strain distribution, leading to more reliable Poisson's ratio measurements, particularly at low strain amplitudes. In fact, the standard deviation of the measurements was reduced by up to 27.7% of the measurement of the original DIC measurements. The fidelity of Lock-in filtering, already well-established in other fields like Thermoelastic Stress Analysis (TSA) for detecting subtle periodic behaviors, is strongly supported by its successful application and performance in this study. This method offers a robust solution for capturing the elusive characteristics of low Poisson's ratios in composites.

This work represents a first successful step towards enhancing the precision of Poisson's ratio measurements in challenging scenarios. Future research could explore the applicability of this method across a wider range of composite types, loading conditions, and frequencies, as well as investigate the long-term stability and automation potential of the technique. This study lays a strong foundation for more precise material modeling and design in industries reliant on composite performance.

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Article

Natural Frequency Optimization of Stiffener Structure for Ceramic Matrix Composites Combustion Liner in Aero-Engines

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Abstract: To mitigate vibration in thin-walled composite combustion liners of aero-engines, this study proposes an optimization strategy for stiffener design to maximize natural frequencies and suppress resonance. The approach enhances structural dynamics by installing transverse and longitudinal stiffeners along the tubular wall, with their dimensions and orientations systematically optimized. Design variables were chosen: combustion liner wall thickness, stiffener thickness, transverse stiffener width/angle, longitudinal stiffener width, and composite lamination layup scheme. The orthogonal experiments were completed and followed by range analysis and variance analysis. The results demonstrated that wall thickness had the most significant impact on the natural frequency, and the 45° lamination scheme showed a superior performance compared to other configurations. Finally, a predictive equation was developed using a multiple linear regression model. The optimized stiffener configuration markedly enhances natural frequencies, mitigating vibration-induced instability. This methodological framework provides a systematic basis for designing optimized stiffener layouts in composite combustion liners for aero-engines.

Keywords: aero-engine; combustion liner; ceramic matrix composite; stiffener design; natural frequency; structure optimization; ply layup

1. Introduction

Owing to their superior thermomechanical properties, ceramic matrix composites (CMCs) have been the focus of extensive research for high-temperature aero-engine components worldwide since the 1980s [1–3]. These components include combustion chamber, combustion liners, turbine outer rings, turbine guide vanes, turbine rotor blades, turbine shrouds, control/sealing vanes, mixers, and shrouds, as well as afterburner flame stabilizers. As alternatives to high-temperature alloys in critical aero-engine components, CMCs offer low density, high strength, and superior resistance to heat, wear, corrosion, and oxidation, thereby extending service life and improving emission efficiency [4,5].

As a critical component of combustion chambers and one of the engine's most heat-intensive parts, the combustion liner has seen structural complexity increase with advancing aviation engine technology. Typically featuring annular thin-walled construction with membrane holes in the tube walls, this component endures simultaneous exposure to high-temperature gases, high-speed cooling airflow, and mechanical forces. These combined

stresses make it prone to issues like cracking failures and deformation imbalance, making combustion liner failures an increasingly prominent technical concern [6].

Current research on combustion liners by scholars worldwide primarily focuses on simulating and designing gas film orifice flow fields, optimizing wall cooling systems, and predicting/fostering fatigue durability [7]. The General Electric (GE) Corporation developed the SiC/SiC combustion chamber combustion liner, which underwent 5000 h full lifecycle testing at 1200 °C operating conditions and 500 h of high-temperature exposure [8]. Results demonstrated that this design could withstand wall temperatures up to 1316 °C while reducing structural weight by 30%, significantly enhancing overall engine chamber performance. Van Roode et al. [9] investigated the impact of manufacturing processes and Environmental Barrier Coatings (EBCs) on SiC/SiC combustion liners. Tests showed that melt infiltration (MI) SiC/SiC combustion liners without EBCs outperformed chemical vapor infiltration (CVI) SiC/SiC counterparts in durability, with EBCs extending service life by 2–3 times. Li Gong's team [10] created a CMC floating wall combustion liner using floating tile technology. The CMC design features simplified structures for easier machining and gaps around tiles to minimize thermal stress. Simulation results indicated that the chamber could withstand 1400 K temperatures while requiring 10% less cooling gas than high-temperature alloy combustion chambers. Edward et al. [11] addressed the issue of excessive or uneven radial deformation in thin-walled combustion liner structures by designing reinforcing ribs to enhance both circumferential and radial stiffness. This approach ensures stable combustion and uniform outlet flow field distribution without significant weight gain. Andrei et al. [12] developed a device for evaluating cooling efficiency of air film holes in combustion liners. The setup involves positioning the workpiece under high-temperature airflow with parallel upper airflow and perpendicular lower cooling airflow, using optical pressure-sensitive material brightness on the workpiece surface to assess cooling effectiveness. Xie Jie et al. [13] conducted numerical studies on convective heat transfer characteristics within air film holes. Their results demonstrated that the surface heat transfer coefficient increases with rising airflow Reynolds number, while increasing the hole inclination angle significantly enhances convective heat transfer coefficients across the entire hole perimeter.

In the field of stiffener optimization design for thin-walled structures, relevant optimization theories have become relatively mature and are gradually being applied in aerospace and other manufacturing industries [14–16]. Conventional stiffener configurations include evenly distributed linear stiffeners, orthogonal rectangular grid stiffeners, and triangular grid stiffeners. With the development of parametric methods, Mulani et al. [17] utilized spline curves to optimize stiffener designs for T-beam and I-beam structures, employing B-spline curves as geometric descriptors. Zhang et al. [18] achieved optimized stiffener layout design for three-dimensional curved thin-walled shell structures using geometric background mesh methods. Aage et al. [19] published a *Nature* paper on wing box stiffener topology optimization, employing nearly 1.1 billion solid element design variables to optimize stiffener topology for the Boeing 777 wing. Afonso et al. [20] adopted shell unit thickness as a design variable to optimize stiffener layouts for plate-shell structures. Inspired by the branching growth patterns of plant canopies, Ding et al. [21] proposed an adaptive biomimetic stiffener growth design method, which not only optimizes structural layout but also generates new stiffeners, thereby expanding the design space. To enhance structural dynamic performance against vibrational resonance, Liu et al. [22] proposed a multiscale topology optimization framework for maximizing the natural frequency of multi-morphology lattice structures, addressing the problems of numerical convergence, microstructural connectivity, etc. Similarly, Wang et al. [23] proposed a data-driven topology optimization approach to enable the multiscale cellular designs with multiple choices

of microstructure classes with a newly proposed latent-variable Gaussian process. Also, Shah et al. [24] considered natural frequency in multi-material topology optimization and solved complex three-dimensional (3D) problems.

However, most works considering natural frequency optimization are realized on theoretical models, and thin-walled engineering structures such as combustion liners have rarely been considered. The challenges include the local heat/fluid boundary conditions and composite material manufacturing constraints.

This study conducted simulation and optimization of vibration buckling issues in thin-walled structures of ceramic matrix composite combustion liners, proposing a ribbed wall design scheme. The optimization objective is set to be maximization of the first order natural frequency, thus enhancing the dynamic performance against resonance. The design focuses on three key parameters: tube wall thickness, longitudinal rib thickness and width, and circumferential rib thickness and width, as well as lamination direction. With the fundamental frequency of structural vibrations as the design objective, orthogonal experiments were designed and optimized through analysis to evaluate the impact of each parameter.

The remainder of this paper is organized as follow: Section 2 describes in detail the design of the stiffener and simulation setups in this paper. In Section 3, a series of orthogonal experiments is designed and performed with the results illustrated. In Section 4, the analysis of range, variance and multiple linear regression is conducted. The influence of stiffener parameters is analyzed. The conclusions of this paper are drawn in Section 5.

2. Materials and Methods

Figure 1 shows a simplified model of a combustion liner, formed by two straight segments and a transition arc. Stiffener design patterns along the tube wall are illustrated in Figure 2, featuring longitudinal and transverse stiffeners. The longitudinal stiffeners span the entire axial direction, consisting of four groups evenly distributed at 90° intervals. Transverse stiffeners are exclusively arranged in the lower smooth section, forming two symmetrically positioned groups along the circumferential direction with adjustable angles relative to the horizontal plane.

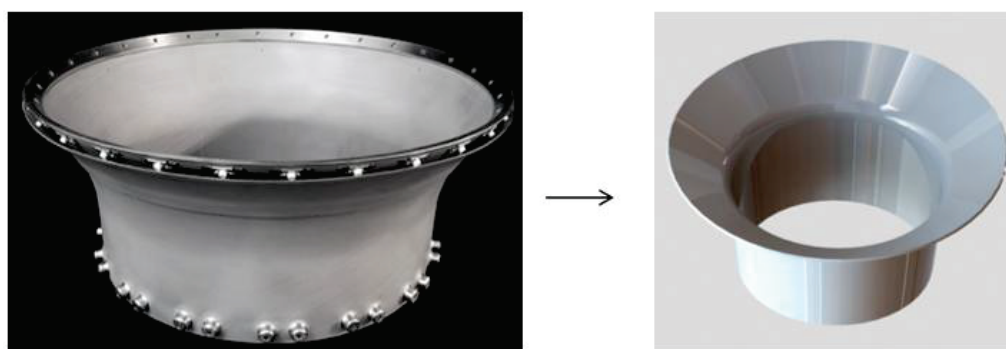


Figure 1. Schematic diagram of simplified combustion liner model: Real structure in the left and simplified model in the right.

The material of the combustion liner structure is set as ceramic matrix composite. The material properties of unidirectional ply are taken according to the data in literature [25], measured from the material Prepreg HiPerComp™ (General Electric Company, Boston, MA, USA) of 8-ply laminates, with a balanced [0-90-90-0]_s stacking of uniaxial plies, and nominally 22–25% by volume of Hi-Nicalon™ (Nippon Carbon Company, Tokyo, Japan) fibers. Here, the material data is used for informative purposes only, and not for engineering specifications. The fiber main direction is taken as 1, the vertical fiber direction

in the lamination plane is taken as 2, and the normal direction in the lamination plane is taken as 3 (Figure 3). The specific parameters are shown in Table 1.

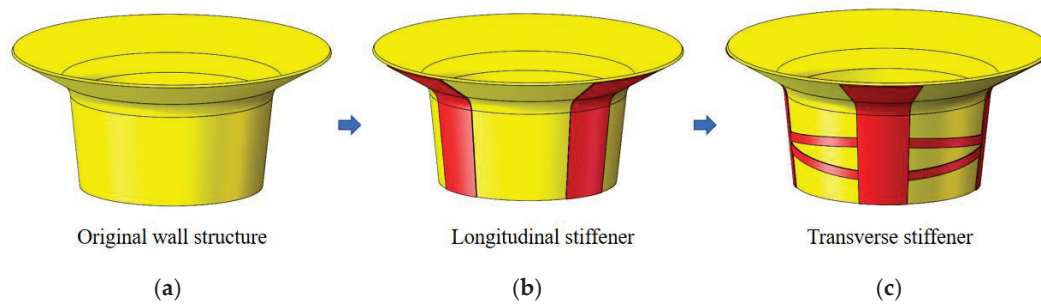


Figure 2. Stiffener design of combustion liner, with red color denoting stiffener areas.

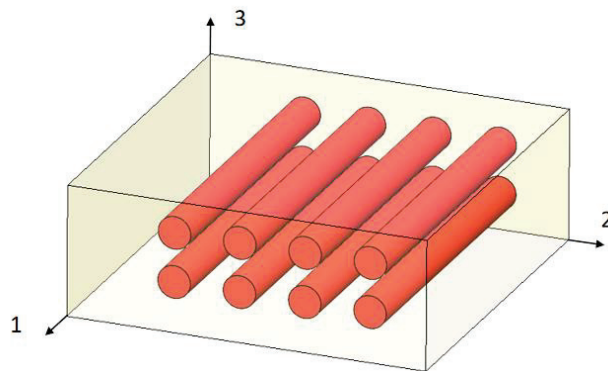


Figure 3. Material coordinate system of CMC unidirectional ply with red columns denoting fibers.

Table 1. Parameters of ceramic matrix composites.

Material Properties	Value	Temperature/°C	Value	Temperature/°C
Density	2.80 g/cm ³	25	2.76 g/cm ³	1200
Tensile modulus E_{11}	285 GPa	25	243 GPa	1200
Tensile modulus E_{22}	285 GPa	25	243 GPa	1200
Tensile modulus E_{33}	201 GPa	25	171 GPa	1200
Shear modulus G_{12}	14.5 GPa	25	12.3 GPa	1200
Shear modulus G_{13}	12.8 GPa	25	10.9 GPa	1200
Shear modulus G_{23}	12.8 GPa	25	10.9 GPa	1200
Poisson's ratio ν_{12}	0.12	25	0.12	1200
Poisson's ratio ν_{13}	0.24	25	0.24	1200
Poisson's ratio ν_{23}	0.24	25	0.24	1200

The design parameters selected include combustion liner basic wall thickness d_0 , stiffener thickness d_1 , transverse stiffener width b_1 , transverse stiffener angle α , longitudinal stiffener angle β , and fiber layer scheme M . All stiffener cross-sections are defined as rectangular. All stiffener structures are employed on the outer surface of the combustion liner, thus ensuring the heat/fluid boundary conditions of inner surface. A finite element model was established to conduct natural frequency simulation analysis, investigating how different stiffener configurations affect the fundamental frequency of the combustion liner. The schematic diagram of design parameters is shown in Figure 4. The computational analysis in this work is fulfilled by commercial software ABAQUS (version 6.14), and the optimization procedure is implemented by the development of Python (version 2.7) scripts embedded in ABAQUS.

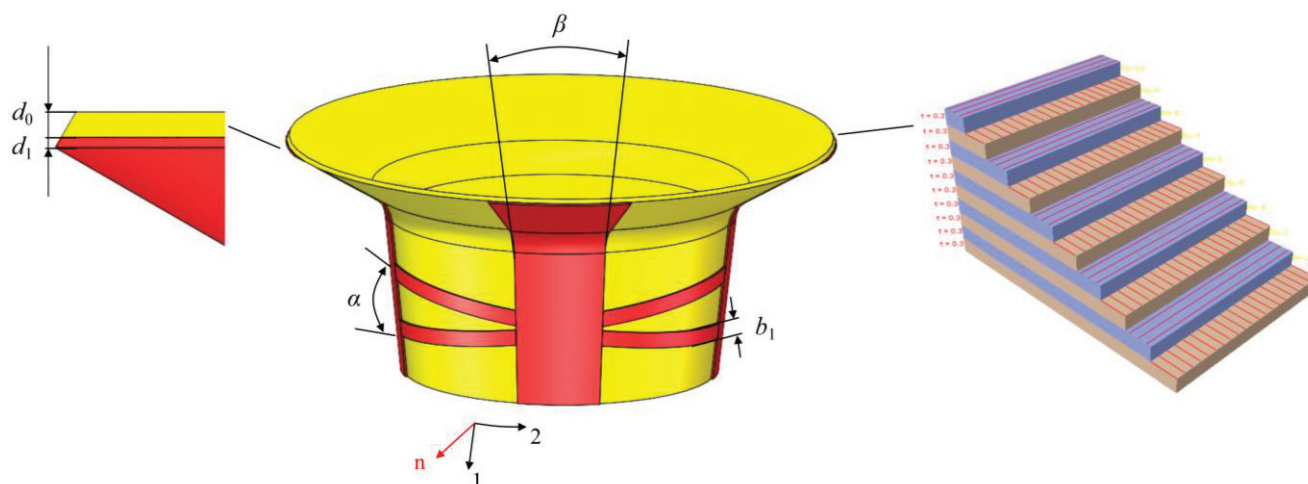


Figure 4. Design parameters of combustion liner. The red color denotes stiffener areas, and the schema of layer scheme of $0^\circ/90^\circ$ with layer thickness t is illustrated on the right (vertical and horizontal lines denoting the fibers in each layer).

3. Results

Orthogonal Experiment Design

Firstly, we conducted a qualitative analysis of how design parameters affect the fundamental frequency. The CMC single-layer thickness was set to be a uniform value of 0.3 mm, and the unreinforced $0/90^\circ$ layered structure was chosen as the original baseline design (Case 1). The modal displacement cloud diagrams and corresponding 1st order natural frequency for the unreinforced structure at room temperature are shown in Figure 5. The modal cloud was rendered at double thickness to clearly demonstrate thickness effects and stiffener variations. Subsequently, three randomized stiffener designs were implemented as illustrated in Figure 6. It is evident that all designs realize a certain degree of improvement in fundamental frequency compared to the original case (control group).

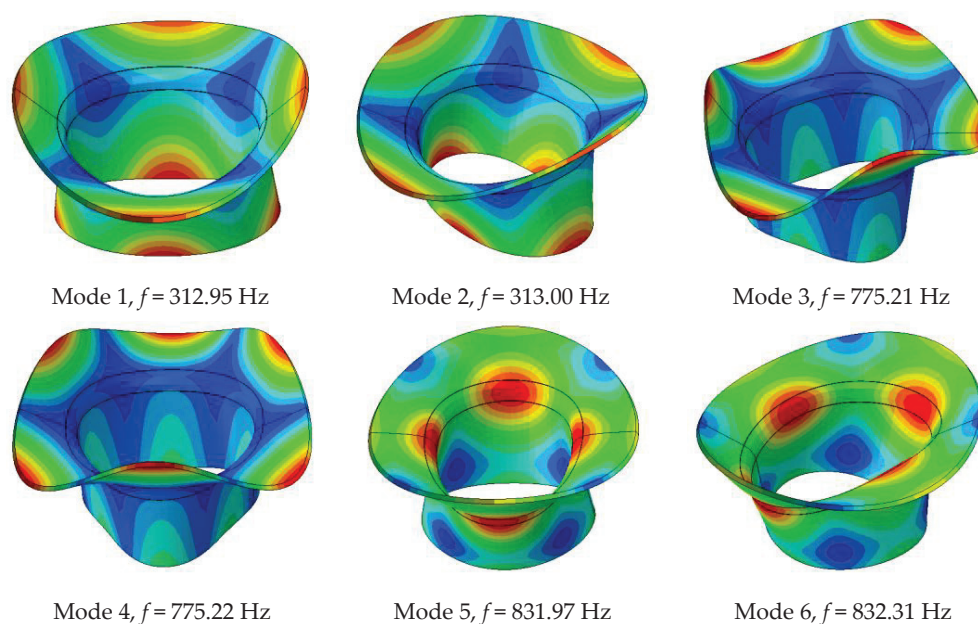


Figure 5. Modal displacement plot of the original structure without stiffener of the combustion liner, with red color denoting the maximum modal displacement areas.

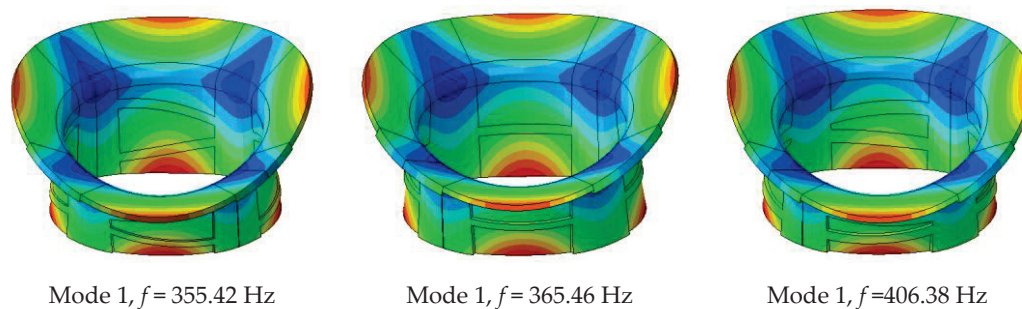


Figure 6. Modal displacement plot of three random stiffener designs, with red color denoting the maximum modal displacement areas.

Consequently, the influence of each stiffener parameter was analyzed as a single-factor variable, as shown in Figure 7. Compared with the unstiffened structure, the effects of design variables on the fundamental frequency were essentially linear, with wall thickness and lamination scheme exerting the greatest influence on the 1st-order natural frequency.

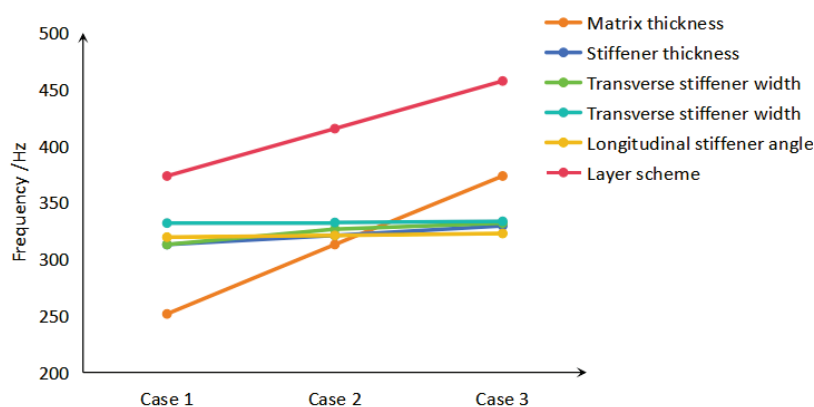


Figure 7. Influence of design parameters on the fundamental frequency.

Thereafter, quantitative analysis was conducted on the orthogonal experimental designs for each parameter, with selected parameter levels as shown in Table 2. The study involved six design parameters, each containing three levels (L18 (3.6) orthogonal experiment design), as detailed in Table 3. Simulation experiments were performed based on the orthogonal table, with experimental data recorded under 25 °C and 1200 °C conditions presented in Table 4 and Figure 8. The results demonstrate that combustion liner fundamental frequency under high-temperature conditions consistently exhibits lower values compared to those obtained at room temperature.

Table 2. Design parameter level selection.

Design Parameter	Wall Thickness d_0 (mm)	Stiffener Thickness d_1 (mm)	Longitudinal Stiffener Angle β (°)	Transverse Stiffener Width b_1 (mm)	Transverse Stiffener Angle α (°)	Layer Scheme M (°)
Level 1	2.4	0.6	25	5	0	0/90
Level 2	3	1.2	30	10	15	0/90/45/−45
Level 3	3.6	1.8	35	15	30	45/−45

Table 3. Orthogonal test table.

Number	Factor 1	Factor 2	Factor 3	Factor 4	Factor 5	Factor 6
1	1	1	1	1	1	1
2	1	1	2	2	3	3

Table 3. Cont.

Number	Factor 1	Factor 2	Factor 3	Factor 4	Factor 5	Factor 6
3	1	2	1	3	3	2
4	1	2	3	1	2	3
5	1	3	2	3	2	1
6	1	3	3	2	1	2
7	2	1	1	3	2	3
8	2	1	3	1	3	2
9	2	2	2	2	2	2
10	2	2	3	3	1	1
11	2	3	1	2	3	1
12	2	3	2	1	1	3
13	3	1	2	3	1	2
14	3	1	3	2	2	1
15	3	2	1	2	1	3
16	3	2	2	1	3	1
17	3	3	1	1	2	2
18	3	3	3	3	3	3

Table 4. Results of orthogonal test.

No.	Wall Thickness d_0 (mm)	Stiffener Thickness d_1 (mm)	Longitudinal Stiffener Angle β (°)	Transverse Stiffener Width b_1 (mm)	Transverse Stiffener Angle α (°)	Layer Scheme M (°)	25 °C Fundamental Frequency f (Hz)	1200 °C Fundamental Frequency f (Hz)
1	2.4	0.6	25	5	0	0/90	264.18	245.62
2	2.4	0.6	30	10	30	45/−45	322.03	299.40
3	2.4	1.2	25	15	30	90/45	307.13	285.55
4	2.4	1.2	35	5	15	45/−45	321.64	299.04
5	2.4	1.8	30	15	15	0/90	347.46	323.05
6	2.4	1.8	35	10	0	90/45	353.32	328.50
7	3	0.6	25	15	15	45/−45	401.09	372.91
8	3	0.6	35	5	30	90/45	350.86	326.21
9	3	1.2	30	10	15	90/45	385.21	358.15
10	3	1.2	35	15	0	0/90	367.85	342.00
11	3	1.8	25	10	30	0/90	379.90	353.21
12	3	1.8	30	5	0	45/−45	394.63	366.91
13	3.6	0.6	30	15	0	90/45	437.71	406.97
14	3.6	0.6	35	10	15	0/90	394.04	366.32
15	3.6	1.2	25	10	0	45/−45	476.79	443.33
16	3.6	1.2	30	5	30	0/90	402.16	373.83
17	3.6	1.8	25	5	15	90/45	456.88	424.84
18	3.6	1.8	35	15	30	45/−45	499.78	464.67

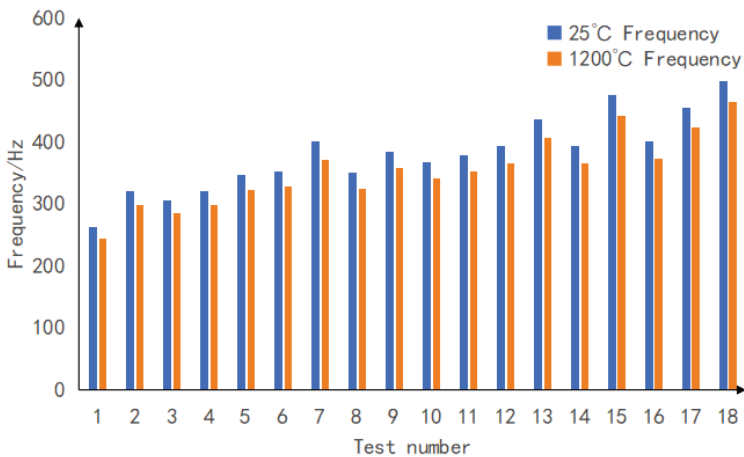


Figure 8. Comparison of fundamental frequency at different temperatures.

4. Discussion

4.1. Range Analysis

A range analysis was conducted based on the orthogonal experimental results. The indicator K_{avg} measures the influence of six experimental factors on the combustion liner fundamental frequency at different levels, while the range R reflects the impact of factor-level variations on experimental outcomes. A larger range value indicates more significant effects of the corresponding factor, whereas smaller ranges suggest relatively minor impacts. As shown in Tables 5 and 6, the wall thickness has the most substantial effect on the combustion liner fundamental frequency, followed by stiffener thickness, layering configuration, and transverse bar width, with longitudinal bar angle and transverse bar inclination showing minimal influence. Notably, the fundamental frequency at 1200 °C was lower than that at 25 °C due to decreased material elastic modulus under high temperatures. This degradation effect caused by high temperature would also result in negative effect on the material's long-term durability and fatigue life. However, the trend of parameter effects on fundamental frequency remained consistent across different temperature conditions.

Table 5. Range analysis results (25 °C).

Project	Wall Thickness d_0 (mm)	Stiffener Thickness d_1 (mm)	Longitudinal Stiffener Angle β (°)	Transverse Stiffener Width b_1 (mm)	Transverse Stiffener Angle α (°)	Layer Scheme M (°)
Horizontal numbers	3	3	3	3	3	3
Number of tests per level	6	6	6	6	6	6
Kavg	319.29	361.65	381.00	365.06	382.41	359.27
Level 1	379.92	376.80	381.53	385.22	384.39	381.85
Level 2	444.56	405.33	381.25	393.50	376.98	402.66
Level 3	3	3	3	3	2	3
Best level	125.27	43.68	0.54	28.45	7.41	43.40
R						

Table 6. Range analysis results (1200 °C).

Project	Wall Thickness d_0 (mm)	Stiffener Thickness d_1 (mm)	Longitudinal Stiffener Angle β (°)	Transverse Stiffener Width b_1 (mm)	Transverse Stiffener Angle α (°)	Layer Scheme M (°)
Horizontal numbers	3	3	3	3	3	3
Number of tests per level	6	6	6	6	6	6
Kavg	296.86	336.24	354.24	339.41	355.55	334.00
Level 1	353.23	350.32	354.72	358.15	357.38	355.04
Level 2	413.33	376.86	354.45	365.86	350.48	374.38
Level 3	3	3	3	3	2	3
Best level	116.47	40.62	0.48	26.45	6.91	40.37
R						

After analyzing the influence of design parameters, the aforementioned results can be interpreted as follows: The base thickness determines the overall stiffness of the structure and plays a crucial role in determining the fundamental frequency of thin-walled structures. Increasing the basic wall thickness significantly enhances the fundamental frequency of the combustion liner, but this also leads to the gain of mass. Therefore, it is essential to optimize thickness design while maintaining performance. Stiffener thickness similarly has a substantial impact on the combustion liner's fundamental frequency. By increasing stiffener thickness, local stiffness of the combustion liner can be enhanced, thereby improving its fundamental frequency. The selection of lamination schemes also significantly affects the combustion liner's fundamental frequency. Since the first-order mode is dominated by lateral oblique vibrations, a 45° lamination significantly increases oblique stiffness and raises the corresponding vibration frequency. The transverse stiffener width strongly affects the combustion liner's natural frequency, with increased width enhancing tube stiffness. In

contrast, longitudinal and transverse stiffener angles have a smaller impact but should still be considered to ensure optimal overall performance.

4.2. Analysis of Variance

Although range analysis can evaluate the sensitivity of different design parameters on the objectives, it has limitations in determining the depth and significance of such impacts. This study employs analysis of variance (ANOVA) on orthogonal experiment results, with the core principle being to decompose experimental data variance into two components: differences caused by varying factors and random errors. The selection of ANOVA is justified for this analysis as the investigation involves natural frequency optimization across multiple kinds of combustion liner parameters, i.e., thickness, angle, and material layup. ANOVA is the appropriate tool because it can efficiently determine the significant differences between these parameters. The total factor variance (Sum Square, SS) represents the variance between individual factor levels' means and the overall mean, while the mean square error (Mean Square, MS) is calculated by dividing the total variance by degrees of freedom, reflecting the degree of variation among observed values across different factor levels. Significance is ultimately determined through F-values and *p*-Values derived from MS calculations, where *p* < 0.05 indicates significant effects. As shown in Tables 7 and 8, the significant trends in parameters at 25 °C and 1200 °C align with the findings from factor analysis in Section Orthogonal Experiment Design. The wall thickness demonstrates particularly significant effects on fundamental frequency, followed by stiffener thickness and layer design schemes, while other factors show no significant impact. These ANOVA results correspond to the conclusions drawn from factor influence analysis in Section Orthogonal Experiment Design.

Table 7. Analysis of variance results (25 °C).

Project	Wall Thickness d_0 (mm)	Stiffener Thickness d_1 (mm)	Longitudinal Stiffener Angle β (°)	Transverse Stiffener Width b_1 (mm)	Transverse Stiffener Angle α (°)	Layer Scheme M (°)
Factor degrees of freedom	2	2	2	2	2	2
Level 1	3839.73	384.44	0.07	262.46	1.33	483.73
Quadratic sum	1.78	19.91	0.08	15.65	9.78	0.35
Level 2	4007.03	579.34	0.00	149.93	18.34	458.01
Level 3	47,091.27	5902.16	0.87	2568.21	176.72	5652.54
Sum square	23,545.63	2951.08	0.44	1284.11	88.36	2826.27
Mean square	100.51	12.60	0.00	5.48	0.38	12.06
F Value	0.000	0.011	0.987	0.052	0.787	0.012
<i>p</i> Value	Highly significant	Significant	Not significant	Not significant	Not significant	Significant
Significance						

Table 8. Analysis of variance results (1200 °C).

Project	Wall Thickness d_0 (mm)	Stiffener Thickness d_1 (mm)	Longitudinal Stiffener Angle β (°)	Transverse Stiffener Width b_1 (mm)	Transverse Stiffener Angle α (°)	Layer Scheme M (°)
Factor degrees of freedom	2	2	2	2	2	2
Level 1	3319.25	332.47	0.05	226.92	1.17	418.92
Quadratic sum	1.54	17.27	0.06	13.53	8.48	0.32
Level 2	3463.87	501.27	0.00	129.63	15.96	396.20
Level 3	40,707.95	5106.03	0.68	2220.48	153.63	4892.56
Sum square	20,353.98	2553.01	0.34	1110.24	76.82	2446.28
Mean square	100.40	12.59	0.00	5.48	0.38	12.07
F Value	0.000	0.011	0.987	0.052	0.787	0.012
<i>p</i> Value	Highly significant	Significant	Not significant	Not significant	Not significant	Significant
Significance						

4.3. Multiple Linear Regression

Furthermore, a multiple linear regression model was established based on the orthogonal experimental results for regression analysis. The model assumes that the response (fundamental frequency f) exhibits linear relationships with the following factors: wall thickness d_0 , stiffener thickness d_1 , longitudinal stiffener angle β , transverse stiffener width b_1 , transverse stiffener angle α , and layer scheme M .

$$f = B + w_1 \cdot d_0 + w_2 \cdot d_1 + w_3 \cdot \beta + w_4 \cdot b_1 + w_5 \cdot \alpha + w_6 \cdot M \quad (1)$$

To ensure dimensional consistency, all factors underwent normalization mapping, with their values scaled to levels 1, 2, and 3. The coefficients and analysis results of the multiple linear regression are presented in Table 9. The physical meaning of the coefficients $w_1, \dots, w_6$ represents the influence level of each design variable to natural frequency, in which the sign represents positive or negative influence, and the absolute value represents the sensitivity. The model demonstrates an R^2 value of 0.974, indicating that the wall thickness, stiffener thickness, longitudinal stiffener width-angle, transverse stiffener width, transverse stiffener angle, and pavement design scheme collectively account for 97.4% of the variation in the fundamental frequency. The F-test results confirm the model's validity ($p = 0.000 < 0.05$), demonstrating that at least one of the six factors influences the fundamental frequency. Multicollinearity tests revealed all Variance Inflation Factors (VIF) below 5, confirming no multicollinearity issues. Additionally, D-W values near 2 suggest no autocorrelation, indicating no inherent correlations between sample data points, thus validating the model's overall robustness.

Table 9. Results of multiple linear regression.

Project	25 °C		1200 °C	
	Coefficient Values	VIF	Coefficient Values	VIF
Constant B	145.659	-	135.423	-
Wall thickness coefficient w_1	62.633	1.000	58.233	1.000
Stiffener thickness coefficient w_2	21.838	1.000	20.313	1.000
Longitudinal stiffener angle coefficient w_3	0.127	1.000	0.107	1.000
Transverse stiffener width coefficient w_4	14.222	1.000	13.225	1.000
Transverse stiffener angle coefficient w_5	-2.718	1.000	-2.538	1.000
Layer scheme coefficient w_6	21.698	1.000	20.186	1.000
R^2	0.974	-	0.974	-
F Value	70.005	-	69.913	-
p Value	0.000	-	0.000	-
D-W	1.937	-	1.938	-

Finally, based on the positive/negative coefficients of the linear regression model, we optimized stiffener parameters to achieve maximum fundamental frequency response. The design selected maximum values for wall thickness, stiffener thickness, longitudinal stiffener angle, transverse stiffener width, and pavement layout scheme while minimizing the transverse stiffener angle. Finite element modal analysis was conducted. As the 1200 °C and 25 °C modal cloud images showed remarkable similarity, only the 25 °C results are presented here. Figure 9 illustrates the optimized modal displacement results of the combustion liner stiffener structure. Compared to unreinforced structures, this design achieves approximately 60% improvement in fundamental frequency while swapping the sequence of modes 3, 4 with modes 5, 6.

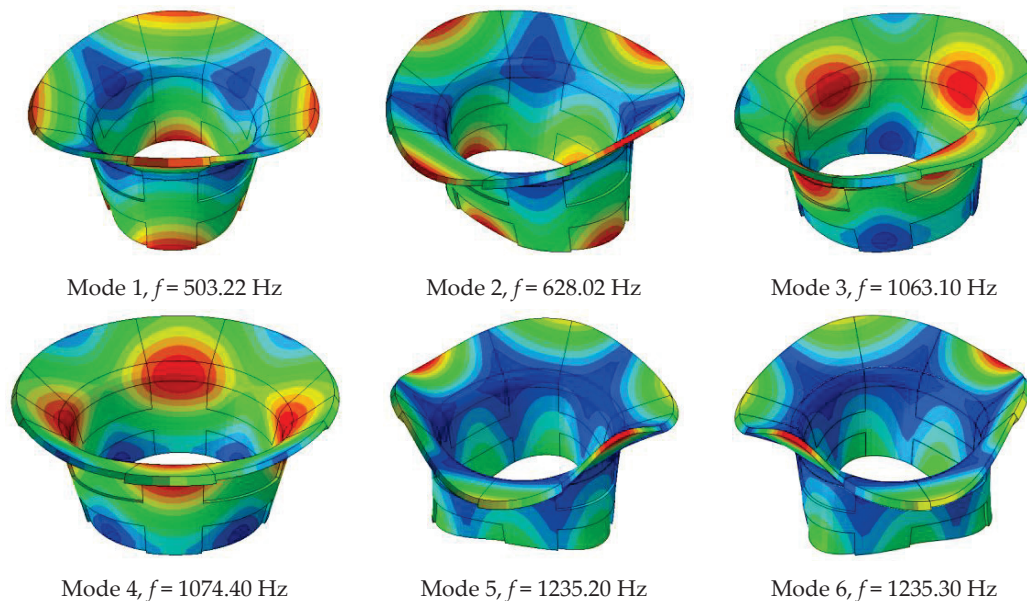


Figure 9. Plot of modal displacement of optimal stiffened combustion liner structure, with red color denoting the maximum modal displacement areas.

5. Conclusions

This study optimized reinforced configurations of CMC combustion liner structures to enhance natural frequencies. Using an orthogonal experimental design with six variables—wall thickness, stiffener thickness, transverse stiffener width and angle, longitudinal stiffener width, and lamination scheme—the analysis showed that wall thickness had the greatest influence, followed by stiffener thickness, lamination scheme, and transverse stiffener width. Notably, the 45° lamination scheme showed superior performance compared to other configurations in improving structural frequency. Multivariate linear regression analysis effectively characterized the frequency response through these six variables, explaining 97.4% of variation while demonstrating no multicollinearity or self-correlation issues, validating the model's effectiveness. Under 1200 °C conditions, although the stiffener design achieved identical results as at 25 °C, reduced material stiffness at elevated temperatures led to decreased fundamental frequency. The regression model ultimately identified optimized stiffener parameters that significantly improved frequency performance while altering low-order modal order characteristics.

This study establishes a theoretical basis for optimizing composite combustion liner structures in aero-engines and provides practical guidance for similar designs. By tailoring stiffener configurations and lamination schemes, natural frequencies can be effectively increased, enhancing vibration resistance and extending service life. Future work will explore diverse stiffener geometries and layering strategies, with the goal of implementing and validating these designs in operational aero-engine systems.

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Abbreviations

The following abbreviations are used in this manuscript:

CMC	Ceramic matrix composites
GE	General Electric
EBC	Environmental Barrier Coating
MI	Melt infiltration
CVI	Chemical vapor infiltration
ANOVA	Analysis of variance
SS	Sum square
MS	Mean square

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Article

Cross-Property Relations for Continuous Fiber-Reinforced Ceramic Matrix Composites

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Abstract

Continuous fiber-reinforced ceramic matrix composites (CFRCMCs) are extensively applied in high-temperature fields such as aerospace, energy, and transportation due to their superior mechanical and thermal properties. The explicit and numerical cross-property relations of CFRCMCs are established by integrating a generalized self-consistent scheme, effective-medium conduction models, and finite element analysis. The results reveal that CFRCMCs with a fixed fiber–interphase volume ratio exhibit nearly linear cross-property relations. While the random distribution and diameter of fibers have minimal effects on these relations, a decrease in interphase properties causes the cross-property curves to shift downward. The cross-property relation surfaces for CFRCMCs with varying fiber–interphase volume ratios are validated through finite element analysis. In summary, cross-property relations in CFRCMCs facilitate the prediction of challenging-to-measure physical properties from more readily accessible parameters, substantially simplifying material characterization. This methodology not only optimizes experimental workflows but also delivers a rigorous theoretical framework for multiphysics applications, particularly in extreme thermo-mechanical environments where coupled properties are critical to performance.

Keywords: cross-property relations; continuous fiber-reinforced; ceramic matrix composites; elastic modulus; thermal conductivity; interphase

1. Introduction

Ceramic matrix composites have garnered considerable attention and application across various fields, including chemistry and mechanical engineering, due to their remarkable mechanical, thermal, and electromagnetic properties and chemical inertness [1,2]. Traditional ceramics encompass a wide array of products, including porcelain, pottery, and glass. More generally, advanced ceramics are defined as materials derived from inorganic non-metallic substances through high-temperature sintering processes. This classification includes a diverse range of materials primarily composed of oxides, such as magnesium oxide, aluminum oxide, and silicon dioxide [3]. Ceramic matrix composites are a rapidly evolving category of advanced materials. The integration of reinforcement phases, such as fiber whiskers, particulates, and platelets, into these composites leads to enhanced toughness, reduced weight, and improved fatigue resistance. These advancements enable

ceramic matrix composites to meet the stringent property standards required in various technological domains [4].

Continuous fiber-reinforced ceramic matrix composites (CFRCMCs) represent a notable category of composite materials that employ continuous fibers as the reinforcement phase to improve their material properties. Prominent examples of these composites include Cf/SiC, SiCf/SiC, and Al₂O₃f/Al₂O₃, among others [5]. For most CFRCMCs, the fibers and the matrix do not have an ideal connection. Interfaces are artificially formed or generated through chemical reactions [6]. The growing demand for CFRCMCs in high-temperature industries is becoming increasingly significant [7]. This trend is particularly observable in the automotive and railway sectors, where CFRCMCs are used in brake pads; the aerospace sector, where they are applied in engine components, thermal protection systems, and structural elements of satellites; the microelectronics field, where they function as cooling components for various devices; and the energy sector, where they play a crucial role in the fabrication of heat exchangers [8].

In the Non-Metallic Turbine Engine Project conducted by NASA, a CFRCMC material reinforced with 20% SiC fibers was employed in the fabrication of a high-pressure turbine (HPT) nozzle and a cooled doublet vane [9]. General Electric has actively participated in the research and application of CFRCMC materials. For instance, the HPT shrouds of the LEAP engine are constructed from CFRCMC materials, which have collectively achieved over 50 million flight hours to date. CFRCMC components are utilized in five distinct parts of the GE9X engine, comprising one inner combustor liner, one outer combustor liner, the shrouds and nozzles of the HPT Stage 1, and the nozzles of HPT Stage 2 [10]. Zhang et al. have engineered scalable ceramic-polymer composites that utilize three-dimensional interconnected piezoelectric microfoams, which possess the ability to concurrently harvest thermal and mechanical energy. This dual-energy harvesting functionality facilitates their utilization in a diverse range of sensing and energy-generating applications [11].

Hypersonic aerospace vehicles, which operate at velocities surpassing Mach 5, predominantly utilize ultra-high temperature ceramic matrix composites (UHTCMCs) as key materials. These ceramics generally incorporate C/SiC fibers as the reinforcement phase, and ZrB₂, ZrC, and other ultrahigh temperature ceramics comprises the matrix materials [12]. Their operational temperatures can exceed 1800 °C, and the melting points of the oxides generated can reach up to 3000 °C [13]. Finally, the German Aerospace Center has successfully developed a UHTCMC material, which incorporates carbon fibers and ZrB₂, utilizing the reactive melt infiltration method. This material maintains a flexural strength of 190 MPa even at temperatures as high as 900 °C [14].

Clearly, the predominant areas in which CFRCMCs are applied center on thermo-mechanical coupling at elevated temperatures, a focus that is determined by their superior mechanical and thermal properties. However, the measurement of material properties at high temperatures is often challenging and costly. Consequently, the development of cross-property relations may increase measurement difficulties [15]. Bristow introduced the concept of cross-property correlations, focusing on the relations between elastic modulus and electrical conductivity in solids with low-density, randomly oriented microcracks [16]. Berryman and Milton expanded upon this work by establishing the relations between geometric parameters and material properties in two-phase composite materials [17]. Sevostianov and Kachanov further extended the cross-property relations to anisotropic materials [18].

The properties of composite materials can be described as functions of the same microstructural parameters [19]. By establishing distinct functions for both mechanical and thermal properties and subsequently eliminating microstructural parameters, cross-property relations can be established in the material [20]. This approach allows other

properties of the material to be rapidly determined based on one property that is easier to measure [21]. The investigation of cross-property relations has gained considerable attention and been extensively explored in the literature. Sevostianov et al. examined the influence of the tortuosity parameter on effective electrical conductivity and the overall mass transfer coefficient in two-phase materials, subsequently establishing cross-property relations by eliminating this parameter [22].

There are also some studies that focus on the cross-property relations of ceramic matrix composites. Uhlířová et al. examined the relations between elastic properties and conductivity in alumina–zirconia ceramics with varying grain size ratios [23]. Pabst et al. investigated the cross-property relations of alumina ceramics and employed thermal conductivity to predict the shear and bulk moduli of the material [24]. However, current research mainly focuses on two-phase materials composed of fibers and matrices, without considering the significant influence of the interphase on macroscopic property prediction and the establishment of cross-property relations. Moreover, relatively limited attention has been paid to continuous fibers.

In this research, we review current micromechanics property prediction models for CFRCMCs with interfaces. Corresponding representative volume elements (RVEs) are then constructed using voxel elements to verify the models, establishing both explicit and numerical cross-property relations of CFRCMCs. Consequently, one physical property (e.g., mechanical or magnetic) can be predicted from another, more easily measurable quantity (e.g., thermal or electrical). This approach significantly mitigates the challenges associated with measuring material properties in CMCs. Furthermore, by establishing explicit connections between different physical properties, cross-property relations streamline the complexities of multi-objective optimization for these materials. Consequently, they offer substantial promises for advancing multiphysics applications, particularly in demanding thermo-mechanical environments, among others.

This paper is organized as follows: In Section 2, the micromechanics prediction models for CFRCMCs with interfaces are first reviewed. Then, corresponding representative volume elements (RVEs) are constructed using voxel elements to verify the models. Section 3 establishes the cross-property relations of CFRCMCs, including CFRCMCs with a fixed fiber–interphase volume ratio and volume ratios. Finally, the work of this article is summarized, and significant results are presented for guiding engineering measurement and design.

2. Methodology

2.1. The Theoretical Approach Based on Micromechanics Models (MM)

CFRCMCs primarily consist of reinforcing fibers and a ceramic matrix; however, ideal interfacial bonding between these two constituents is rarely achieved. Instead, an interphase region invariably exists, which may originate from chemical interactions between the constituents or be intentionally introduced in the form of a coating to enhance material properties. Consequently, it is essential to conceptualize as three-phase composite systems comprising a fiber phase, matrix phase, and interphase (like a coating). To achieve this, a variety of micromechanics effective medium models can be employed to predict the properties of these three-phase composites. These models have been extensively developed and discussed in previous studies [25,26].

2.1.1. Voigt–Reuss (VR) Bound

Voigt bounds were first proposed by Voigt in 1889. They assume that, under longitudinal loading conditions, the strain in the fiber and the matrix of a composite material is equal to the strain in the effective medium, thereby providing an upper limit for the

properties of composite materials. The Voigt upper bounds of the elastic modulus E_{upper} and thermal conductivity k_{upper} for three-phase composites can be derived using Equation (1) [27]:

$$\begin{aligned} E_{upper} &= V_f E_f + V_i E_i + (1 - V_f - V_i) E_m \\ k_{upper} &= V_f k_f + V_i k_i + (1 - V_f - V_i) k_m \end{aligned} \quad (1)$$

where V_f , V_i and V_m represent the Young's modulus of the fibers, the interphase and the matrix. E_f , E_i and E_m represent the Young's modulus of the fibers, the interphase and the matrix. k_f , k_i and k_m represent the thermal conductivities of the fibers, the interphase, and the matrix.

Reuss bounds, proposed by Reuss in 1929, assume that in a composite material subjected to transverse loading, the stress in the fibers and the matrix is equal to the stress in the effective medium, thereby providing a lower limit for the properties of composite materials. The Reuss lower bounds of the elastic modulus E_{lower} and thermal conductivity k_{lower} for three-phase composites are calculated using Equation (2).

$$\begin{aligned} E_{lower} &= \frac{E_m E_f E_i}{V_f E_m E_i + V_i E_m E_f + (1 - V_f - V_i) E_f E_i} \\ k_{lower} &= \frac{k_m k_f k_i}{V_f k_m k_i + V_i k_m k_f + (1 - V_f - V_i) k_f k_i} \end{aligned} \quad (2)$$

Voigt and Reuss (VR) bounds correspond to the series and parallel connections of the constituent phases, respectively, and are often referred to as the Rule of Mixtures and inverse Rule of Mixtures. Similar bounding expressions can be applied to other mechanical parameters, including the bulk modulus and shear modulus [28]. Since the VR bounds are based on the two extreme idealizations of uniform strain and uniform stress, they generally provide broad estimates of the effective properties [29].

2.1.2. Effective Elastic Properties

The VR bounds identify the limits for the properties of materials. However, in practical scenarios, the notably inferior property of the interphase compared to the other two phases leads to a broad range of bounds, which primarily serve as benchmarks for computational validation [30]. In the context of CFRCMCs, geometric relations among the constituent phases are generally well-defined, which inhibits the realization of these extreme conditions. By examining an RVE with predetermined phase configurations, it is possible to derive analytical expressions for effective material properties. Common approaches for this analysis include the Mori–Tanaka model, generalized self-consistent scheme (GSCS), and bridging model [31]. In this study, GSCS was utilized to establish cross-property relations.

Hashin provided a complete set of calculation formulas for GSCS, wherein the interphase was not considered as a separate phase [32]. Instead, it influenced the stress transfer within the material through interface parameters, thereby simplifying the problem into a two-phase system with non-ideal interfaces. This methodology offers a wider range of applicability, particularly in scenarios where the interphase is challenging to identify and measure. When the properties of the interphase are established, the corresponding property parameters can be efficiently derived from these properties. The subsequent calculation of elastic properties is mainly based on Hashin's results, and the specific calculation formula is as follows:

For a transversely isotropic thin elastic interphase with a thickness of t , its three interface parameters η , μ_A and μ_T are expressed as Equation (3). The A subscripts correspond to the axial fiber direction, which is designated as Direction-1, and the T subscripts correspond

to the normal direction of the fiber, which is designated as Directions-2 and Directions-3. A schematic diagram showing these different directions is provided in Figure 1.

$$\begin{aligned}\eta &= \frac{k_i + G_i}{t} \\ \mu_A &= \frac{2G_i}{t} \\ \mu_T &= \frac{G_i}{t}\end{aligned}\quad (3)$$

After the interface parameters are considered, the effective shear modulus of the fiber with non-ideal interfaces can be expressed as follows:

$$G_e = \frac{G_f \mu_A a}{\mu_A a + 2G_f} \quad (4)$$

where a is the radius of fibers. Hashin considered that in some composites, the fibers can be considered as an axis-symmetric structure whose mechanical properties are isotropic in the plane perpendicular to the fiber axis. For transversely isotropic fibers and an isotropic matrix, the effective Young's modulus E_A^* , Poisson's ratio ν_A^* , shear modulus G_A^* and bulk modulus K_A^* can be calculated in the axial direction of the material as follows:

$$\begin{aligned}E_A^* &= E_m V_m + E_{Af} V_f + \frac{4(\nu_{Af} - \nu_m)^2 V_m V_f}{\Theta} \cong E_m V_m + E_f V_f, \quad \Theta = \frac{V_m}{K_e} + \frac{V_f}{K_m} + \frac{1}{G_m} \\ \nu_A^* &= \nu_m V_m + \nu_{Af} V_f + \frac{(\nu_{Af} - \nu_m)(\frac{1}{K_m} - \frac{1}{K_e}) V_m V_f}{\Theta} \\ G_A^* &= G_m + \frac{V_f}{\Delta}, \quad \Delta = \frac{1}{G_e - G_m} + \frac{V_m}{2G_m} \\ K^* &= K_m + \frac{V_f}{\Gamma}, \quad \Gamma = \frac{1}{K_e - K_m} + \frac{V_m}{K_m + G_m}\end{aligned}\quad (5)$$

Considering the continuity of displacement and stress of the material at the phase boundaries, the shear modulus of the material satisfies the following system of eight linear equations:

$$\begin{aligned}-C_0 - D_0 + A_m + B_m + C_m + D_m &= \beta, \\ -\frac{\xi_0}{1+\xi_0} C_0 + D_0 + \frac{2+\xi_m}{1-\xi_m} A_m + B_m + \frac{\xi_m}{1+\xi_m} C_m - D_m &= \beta, \\ \frac{2g^*}{1+\xi_0} C_0 + 3g^* D_0 + B_m - \frac{2}{1+\xi_m} C_m - 3D_m &= \beta g^*, \\ -\frac{g^*}{1+\xi_0} C_0 - 3g^* D_0 + \frac{3}{1-\xi_m} A_m + B_m + \frac{1}{1+\xi_m} C_m + 3D_m &= \beta g^*, \\ B_m - \frac{2}{(1+\xi_m)c} C_m - \frac{3}{c^2} D_m - gB_f &= 0, \\ \frac{3c}{1-\xi_m} A_m + B_m + \frac{1}{(1+\xi_m)c} C_m + \frac{3}{c^2} D_m - \frac{3gc}{1-\xi_f} A_f - gB_f &= 0, \\ cA_m + B_m + \frac{1}{c} C_m + \frac{1}{c^2} D_m - cA_f - (p+1)B_f &= 0, \\ \frac{(2+\xi_m)c}{1-\xi_m} A_m + B_m + \frac{\xi_m}{(1+\xi_m)c} C_m - \frac{1}{c^2} D_m \\ - \frac{(3q+\xi_f+2)}{1-\xi_f} A_f - (q+1)B_f &= 0.\end{aligned}\quad (6)$$

where

$$g^* = \frac{G_T^*}{G_m}, \quad g = \frac{G_f}{G_m}, \quad c = V_f + V_i, \quad p = \frac{2G_f}{\eta a}, \quad q = \frac{2G_f}{\mu_T a}, \quad \xi = \frac{G}{k}, \quad \beta = \varepsilon_0^{23} \quad (7)$$

Here, g^* represents the normalized shear modulus. Under the condition that all fibers possess uniform diameters, GSCS posits that the average strain is consistent across all fibers. As a result, the integration of this premise leads to the derivation of the expression for g^* :

$$g^* = 1 + \left[\frac{3(g+q-1)}{2(1-\xi_f)} A_f + \left(g + \frac{q+p}{2} - 1 \right) B_f \right] \frac{c}{\beta} \quad (8)$$

The substitution of Equation (8) into Equation (6) indicates that a solution for Equation (6) exists solely under the condition that $C_0 = 0$. Therefore, the numerator determinant of Equation (6) must vanish. Thus:

$$\begin{vmatrix} 1 & -1 & 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & \frac{2+\xi_m}{1-\xi_m} & 1 & \frac{\xi_m}{1+\xi_m} & -1 & 0 & 0 \\ g^* & 3g^* & 0 & 1 & -\frac{2}{1+\xi_m} & -3 & 0 & 0 \\ g^* & -3g^* & \frac{3}{1-\xi_m} & 1 & \frac{1}{1+\xi_m} & 3 & 0 & 0 \\ 0 & 0 & 0 & 1 & -\frac{2}{(1+\xi_m)c} & -\frac{3}{c^2} & 0 & -g \\ 0 & 0 & \frac{3c}{1-\xi_m} & 1 & \frac{1}{(1+\xi_m)c} & \frac{3}{c^2} & -\frac{3gc}{1-\xi_f} & -g \\ 0 & 0 & c & 1 & \frac{1}{c} & \frac{1}{c^2} & -c & -(p+1) \\ 0 & 0 & \frac{(2+\xi_m)c}{1-\xi_m} & 1 & \frac{\xi_m}{(1+\xi_m)c} & -\frac{1}{c^2} & -\frac{(3q+2+\xi_f)c}{1-\xi_f} & -(q+1) \end{vmatrix} = 0 \quad (9)$$

The resolution of Equation (9) results in the values of g^* and G_T^* . Subsequently, the formulas for the computation of E_T^* and v_T^* are outlined as follows:

$$\begin{aligned} E_T^* &= 4 / \left(\frac{1}{G_T^*} + \frac{1}{K^*} + \frac{4v_A^{*2}}{E_A^*} \right) \\ v_T^* &= v_A^* \frac{E_A^* - v_A^* E_T^*}{(1-v_A^*) E_A^*} \end{aligned} \quad (10)$$

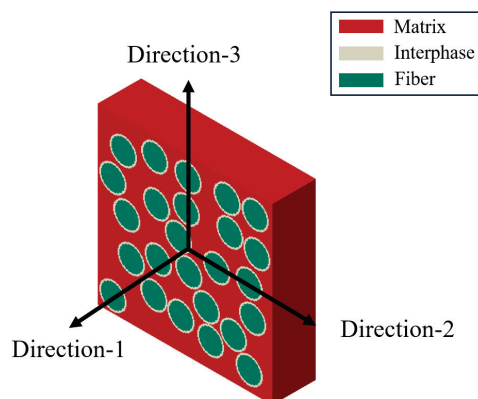


Figure 1. Directions of RVEs.

2.1.3. Effective Thermal Conductivity

Currently, a variety of models have been proposed to predict the effective thermal conductivity of CFRCMCs [33]. Notably, the Hasselman–Johnson model and Markworth model explicitly incorporate the interphase. These methodologies primarily consider the interphase in relation to its geometric characteristics and corresponding thermal resistance, a conceptual framework that parallels the modeling of mechanical properties.

Hasselman and Johnson are pioneers in quantifying the impact of inclusion size on effective thermal conductivity [34]. The material is treated as a two-phase system comprising fibers and matrix, with the interphase exerting its influence through the fiber dimension a and boundary conductivity h_c . When the Hasselman–Johnson model is

utilized in the context of CFRCMCs, the in-plane effective thermal conductivity is expressed as follows:

$$k_T^* = k_m \frac{N - MV_f}{M + MV_f}, \quad M = 1 - \frac{k_f}{k_m} + \frac{k_f}{ah_c}, \quad N = 1 + \frac{k_f}{k_m} + \frac{k_f}{ah_c} \quad (11)$$

Markworth employed effective-medium theory and treated the material as a three-phase system [35]. The entire model is composed of a coaxial cylindrical structure made of three materials. The outermost layer is the matrix phase, the innermost layer is the fiber phase, and the middle layer is the interphase. The expression for the in-plane effective thermal conductivity derived from the Markworth model is as follows:

$$k_T^* = k_m \frac{f}{g} \quad (12)$$

$$f = 2k_i(k_i + k_f)\left[1 + \frac{r_f^2}{r_m^2}\left(1 + \frac{w}{r_f}\right)^2\right] + [k_i - k_m + \frac{r_f^2}{r_m^2}\left(1 + \frac{w}{r_f}\right)^2(k_i + k_m)][(1 + \frac{w}{r_f})^{-2}(k_f - k_i) - (k_f + k_i)]$$

$$g = 2k_i(k_i + k_f)\left[1 + \frac{r_f^2}{r_m^2}\left(1 + \frac{w}{r_f}\right)^2\right] + [k_i - k_m - \frac{r_f^2}{r_m^2}\left(1 + \frac{w}{r_f}\right)^2(k_i + k_m)][(1 + \frac{w}{r_f})^{-2}(k_f - k_i) - (k_f + k_i)]$$

where w is the thickness of the interphase, r_f and r_m represent the radius of the fiber and matrix. When w is significantly smaller than r_f , $\frac{r_f^2}{r_m^2}$ represents the fiber volume fraction.

The Hashin model is expressed through mathematical analogy. Although originally formulated for two-phase materials [36], it can be extended to predict the effective thermal conductivity of three-phase materials by systematically considering pairwise phase combinations as follows:

$$k_c = k_f \frac{k_f V_{cf} + k_i(2 - V_{cf})}{k_f(2 - V_{cf}) + k_i V_{cf}}, \quad V_{cf} = \frac{V_f}{V_f + V_i} \quad (13)$$

$$k_T^* = k_m \frac{k_m V_m + k_c(2 - V_m)}{k_m(2 - V_m) + k_c V_m}$$

It should be noted that all three previously discussed heat conduction models provide information on in-plane effective thermal conductivity. Consequently, the fiber-direction value, which aligns with the microstructural configuration described by the Voigt upper bound, can be directly assessed using the classical Voigt formula.

2.2. Computational Micromechanics Model Using Voxel Elements (CMM-VE)

2.2.1. Generation of RVE

To thoroughly validate the theoretical framework, it is essential to create an RVE that can undergo finite element analysis (FEA) to ascertain its effective mechanical and thermal properties. The RVE is constructed to accurately represent the microstructural characteristics of CFRCMCs and comprises three distinct phases in accordance with the theoretical formulation: the fiber, interphase, and matrix.

Determining an appropriate RVE size is essential to ensure the reliability of FEA. An RVE that is insufficiently small is unable to adequately represent the stochastic characteristics of fiber distribution, while an excessively large RVE imposes prohibitive computational demands. Heinrich et al. conducted a systematic investigation into the impact of RVE size on the predicted effective properties, concluding that a statistically significant RVE for unidirectionally random fiber distributions should encompass no fewer than twenty-five fibers [37]. Consequently, most of the RVEs utilized in this study contain twenty-five fibers. The validity of this selection is further supported in Section 3.1.2, where the predicted responses of multiple RVEs are analyzed; the slight variation observed among the results indicates that the chosen RVE dimensions are representative. This study examines the stochastic phase distribution, specifically concentrating on the random arrangement of

fibers within the matrix. It is important to note that the interfacial layer on the surface of the fibers in actual CFRCMCs exhibits a non-uniform nature. However, the thickness of this interfacial layer is significantly less than that of the fiber itself. As a result, both the fiber and the interphase are modeled as coaxial cylinders within the RVE.

To efficiently produce RVEs that meet the specified morphological criteria and facilitate subsequent meshing processes, a voxel-based discretization approach is applied. Initially, the distributions of fibers and interphases are stochastically generated on a two-dimensional plane, which is subsequently partitioned into a uniform grid composed of square elements. Each element acquires the material properties associated with the phase located at its centroid.

The main limitation of using voxel grids lies in that the boundaries of the grids are either horizontal or vertical, making it difficult to accurately simulate the boundaries of curves. While this discretized representation results in a geometric discrepancy compared to the actual cylindrical fiber morphology, the associated error diminishes significantly when the fiber radius surpasses ten element lengths ($R \geq 10$ elements). Finally, the two-dimensional mesh is extended along the fiber axis to create a comprehensive three-dimensional voxel grid. To optimize the allocation of computational resources, a high in-plane resolution is utilized to precisely represent the cross-sectional geometry of the fibers and interphases, and a relatively coarser discretization is implemented in the fiber direction. The procedure for generating the RVE is presented in Figure 2.

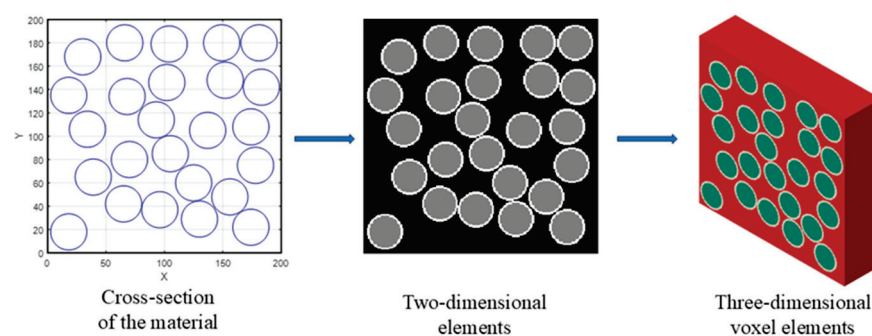


Figure 2. Model generation of RVEs.

2.2.2. Estimation of Elastic Property

Following the construction of the RVE, the mechanical model is validated by assessing the mechanical properties and volume fractions of every constituent phase, as summarized in Table 1. These values were obtained from reference [38]. Although the properties of the fibers found in the current literature are presented in an isotropic material, the computational model established in Section 2.1.2 for transversely isotropic fibers are still holds true.

Table 1. The mechanical properties and volume fractions of phases.

	Hi-Nic Type S Fiber	BN Interphase	SiC Matrix
Young's modulus (GPa)	385	10	327
Poisson's ratio	0.17	0.23	0.22
Volume fractions	28%	13%	59%

In the cited reference, the fiber exhibits a diameter of 12 μm , which is enveloped by a coating that is 1.26 μm thick. Accordingly, a mesh size of 0.6 $\mu\text{m} \times 0.6 \mu\text{m} \times 0.6 \mu\text{m}$ is chosen; each fiber consists of 20 elements arranged throughout its diameter, which is encased in a two-element-thick surface coating. The RVE dimensions are determined from

the fiber volume fraction, yielding an RVE of $168 \times 168 \times 50$ elements with an actual physical size of $100.8 \mu\text{m} \times 100.8 \mu\text{m} \times 30 \mu\text{m}$.

Subsequently, periodic boundary conditions (PBCs) are applied to the RVE to mitigate edge effects and facilitate the determination of its effective macroscopic mechanical properties. The homogenized mechanical properties are ascertained by independently applying unit mechanical loads along various directions to the RVE. The FEA process is conducted within ABAQUS utilizing the easyPBC plugin [39].

Taking the computational result of E_{22} as an example, the corresponding field of stress and strain is illustrated in Figure 3. Here, the maximum tensile stress within the fibers occurs along the loading direction, while the maximum strain occurs transversely to the loading direction. When fibers are sufficiently close in the direction parallel to the applied load, their individual stress-concentration zones overlap and increase in intensity; a concomitant increase is also experienced in strain. Therefore, optimizing the arrangement of the fibers, such as implementing a hexagonal packing configuration, has the potential to improve the effective stiffness of the composite.

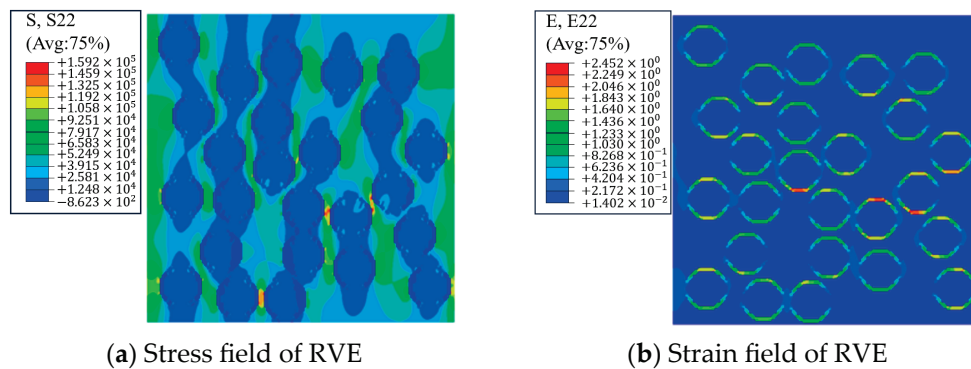


Figure 3. FEA of E_{22} for the RVE of mechanical properties.

The effective properties of the RVE are subsequently estimated utilizing the theoretical model outlined in Section 2.1.2, and these predictions are compared with the results obtained from the FEA. A summary of these comparisons is presented in Table 2. The overall error observed is relatively minor, with all values maintained within a 10% threshold. The most significant errors are noted in G_{12} and G_{13} , which exhibit errors of 7.2% and 7.8%. These discrepancies may be attributed to the varying material properties present between the voxel grids in the oblique direction. In general, the results obtained from FEA and the theoretical solutions corroborate each other, indicating their suitability for the subsequent establishment of cross-property relations.

Table 2. Comparison between mechanical theoretical results and FEA.

	E_{11} (GPa)	E_{22} (GPa)	E_{33} (GPa)	ν_{12}	ν_{13}	ν_{23}	G_{12} (GPa)	G_{13} (GPa)	G_{23} (GPa)
FEA	305.65	160.23	160.14	0.213	0.213	0.240	73.87	73.36	59.00
GSCS results	303.13	160.18	160.18	0.217	0.217	0.246	79.60	79.60	59.01
Relative error	0.9%	0.03%	0.03%	1.8%	1.8%	1.8%	7.2%	7.8%	0.002%

2.2.3. Estimation of Thermal Conductivity

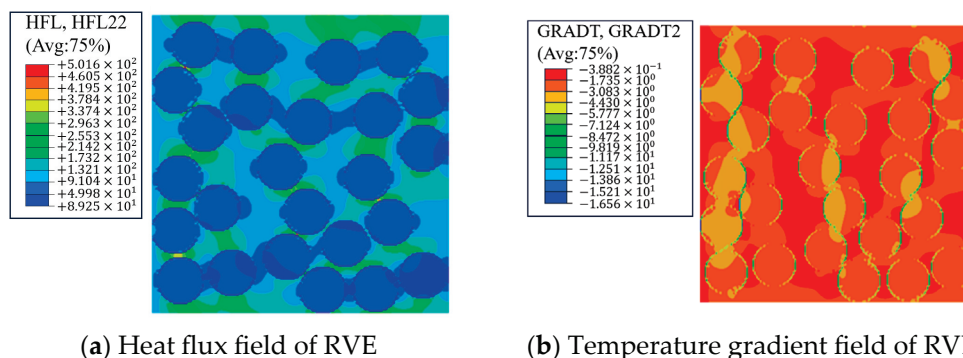
The RVE of thermal properties was constructed using the same methodology and subsequently validated. The thermal property and volume fractions of the phases were derived from reference [40], as illustrated in Table 3.

Table 3. The thermal properties and volume fractions of phases.

	Hi-Nic Type S Fiber	PyC/SiC Interphase	CVI SiC Matrix
Thermal conductivity ($\text{WM}^{-1}\text{K}^{-1}$)	22	6	80
Volume fractions	46%	5%	49%

The fibers utilized in the material have a diameter of $10\text{ }\mu\text{m}$. The surface coating of these fibers comprises five layers of pyrolytic carbon (PyC) and four layers of SiC, with a total thickness of $0.28\text{ }\mu\text{m}$; the entire coating is considered as a single entity. Given that the dimensions of the coating are significantly smaller than those of the fiber, the size of a single voxel unit is set to $0.28\text{ }\mu\text{m} \times 0.28\text{ }\mu\text{m} \times 0.28\text{ }\mu\text{m}$. Consequently, the fiber diameter is represented by 36 voxel units. Based on the fiber volume fraction, the size of the RVE is determined to be $225 \times 225 \times 70$ voxel elements, which corresponds to an actual physical size of $63\text{ }\mu\text{m} \times 63\text{ }\mu\text{m} \times 19.6\text{ }\mu\text{m}$.

Analogous to the calculation of effective mechanical properties, PBCs are imposed on the RVE. In detail, a prescribed temperature is fixed at one end of the RVE and a steady surface heat flux is applied to the opposite face, enabling the calculation of the effective thermal conductivity. Taking the calculation of k_{22} as an example, the resulting heat flux field and temperature gradient are illustrated in Figure 4.

**Figure 4.** FEA of k_{22} for the RVE of the thermal property.

Fiber-direction effective thermal conductivities are calculated using the Voigt bound, and in-plane effective thermal conductivities of the material are subsequently calculated using the Hasselman–Johnson, Markworth, and Hashin models. The results of these calculations are presented in Table 4.

Table 4. Comparison between thermal theoretical results and FEA.

	k_{11} ($\text{WM}^{-1}\text{K}^{-1}$)	k_{22} ($\text{WM}^{-1}\text{K}^{-1}$)	k_{33} ($\text{WM}^{-1}\text{K}^{-1}$)
FEA	47.56	41.10	41.30
Voigt bound	50.42	-	-
Hasselman–Johnson Model	-	30.09	30.09
Markworth Model	-	31.55	31.55
Hashin Model	-	41.96	41.96

The numerical results indicate that the effective conductivity in the fiber direction is slightly lower than the Voigt bound, indicating that the latter can be utilized for predictions. For the in-plane properties, both the Hasselman–Johnson and Markworth models demonstrated significant variances, which could be attributed to the high inclusion volume fraction (46% fiber + 5% interphase = 51%), which is in agreement with findings from the literature [33]. The reason for this is that their main limitations are what they fundamentally

rely on to solve individual inclusions in an infinitely large matrix. This can be extended to the case of a finite volume fraction through various reasonable but not strictly rigorous assumptions. However, when the volume fraction is relatively high, the deviations brought about by these assumptions become increasingly significant [41].

By contrast, the Hashin model exhibits a significantly smaller deviation, and therefore, it is selected for the subsequent determination of effective thermal conductivity to establish cross-property relations.

3. Results

3.1. Cross-Property Relations of CFRCMCs with a Fixed Fiber–Interphase Volume Ratio

For a subset of CFRCMCs in which the interphase thickness on each fiber is meticulously designed, the ratio of the fiber radius to interphase thickness is effectively constant. As a result, the volumetric ratio between the fiber and interphase is also constant. In these materials, the fiber volume fraction is sufficient to ascertain the volumetric fractions of all three constituent phases.

3.1.1. Establishment of Cross-Property Relations

In the current theoretical frameworks employed for predicting the effective properties of multiphase materials, the apparent behavior is assumed to be governed solely by the intrinsic properties and volume fractions of the constituent phases, with the intrinsic properties fixed at the outset of material design. For CFRCMCs characterized by a prescribed fiber–interphase volume ratio, once the fiber volume fraction is specified, the effective properties can be unambiguously determined, thereby enabling the construction of the material’s cross-property relations.

Owing to the limited availability of open-literature data reporting the constituent-phase properties of CFRCMCs, the data properties and volume fractions used to establish the cross-property relations are drawn from the reference of CMCs and subsequently simplified, with a fiber diameter of 9.4 μm , as shown in Table 5 [42].

Table 5. The properties of phases.

	Sylramic Fiber	CVI-BN Interphase	MI-SiC Matrix
Young’s modulus (GPa)	380	62	345
Thermal conductivity ($\text{WM}^{-1}\text{K}^{-1}$)	43	6.9	70
Volume fractions	34%	13%	53%

Initially, the effective properties of the materials are predicted using theoretical models. Accounting for the fiber-to-interphase volume ratio, the fiber volume fraction is prescribed over the interval [0, 0.69] with a uniform increment of 0.01. The corresponding effective properties are presented in Figure 5.

As illustrated in Figure 5, aside from the significant deviation observed in Young’s modulus predictions at a very low fiber volume fraction, most of the calculated values fell within the Voigt–Reuss bounds and can, therefore, be considered reliable. By eliminating the variable of relative density, cross-property relations of mechanical properties and thermal properties for CFRCMCs can be established, as shown in Figure 6.

As illustrated in Figure 6, the cross-property relations derived for the selected material parameters are essentially rectilinear. The slope of the axial cross-property relation is smaller than its in-plane counterpart, and the slope characterizing the cross-property relation between Young’s modulus and thermal conductivity is marginally lower than that governing the relation between shear modulus and thermal conductivity.

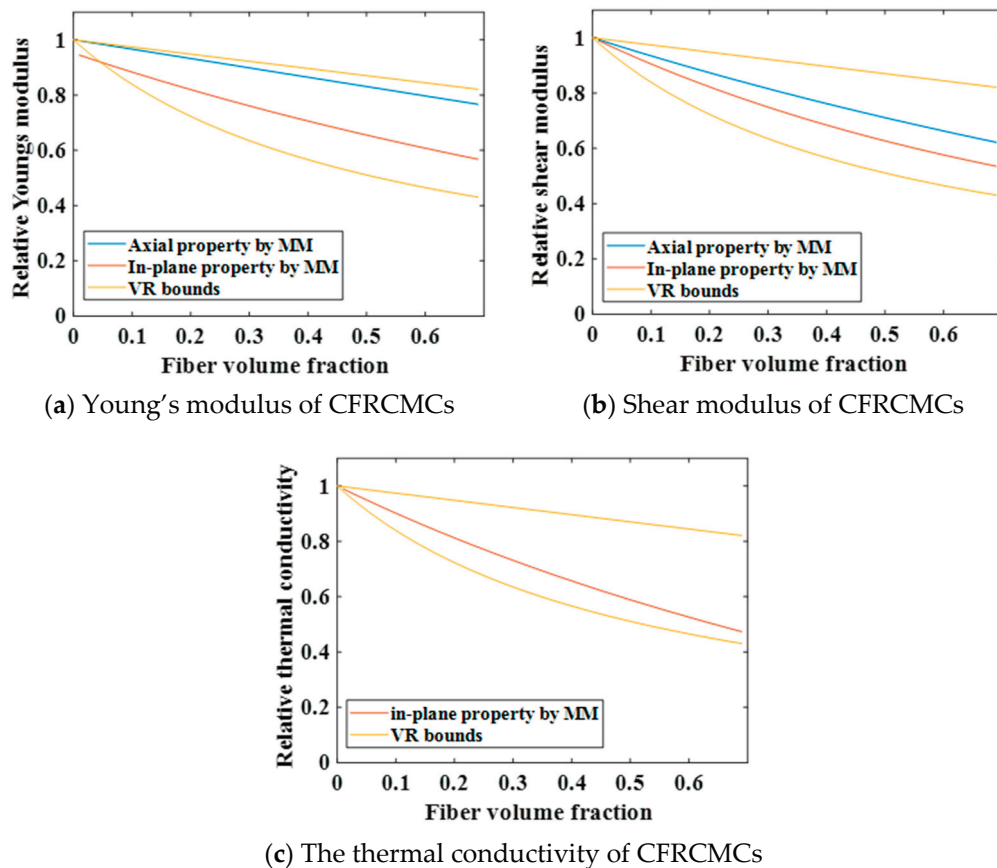


Figure 5. The effective properties of CFRCMCs with a fixed fiber–interphase volume ratio.

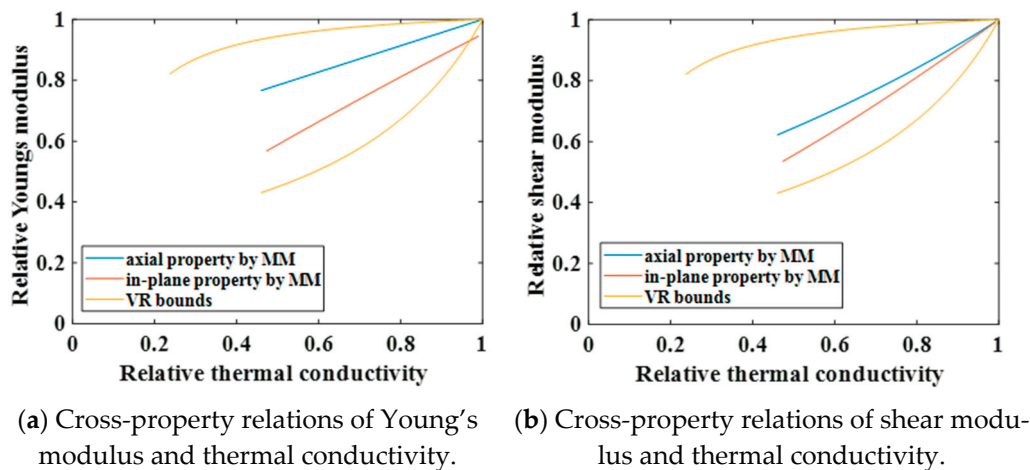


Figure 6. Cross-property relations of CFRCMCs with a fixed fiber–interphase volume ratio.

3.1.2. Random Distribution

For the present material, considering the fiber diameter, the dimensions of a single voxel unit are established at $0.47\ \mu\text{m}$. Consequently, the fiber diameter corresponds to 20 voxel units, while the interface phase thickness is 2 voxel units. The RVE is thus determined to have a size of $155 \times 155 \times 50$ voxels, which translates to a physical size of $72.85\ \mu\text{m} \times 72.85\ \mu\text{m} \times 23.5\ \mu\text{m}$. To substantiate the selected RVE size, it is essential to clarify the impact of random fiber distribution on effective material properties. Accordingly, five RVEs are generated at the phase volume fractions corresponding to the pristine material, varying solely in the spatial configuration of the fibers. Figure 7 illustrates

the phase distributions within each RVE, with the fiber, interphase, and matrix phases represented by green, white, and red colors, respectively.

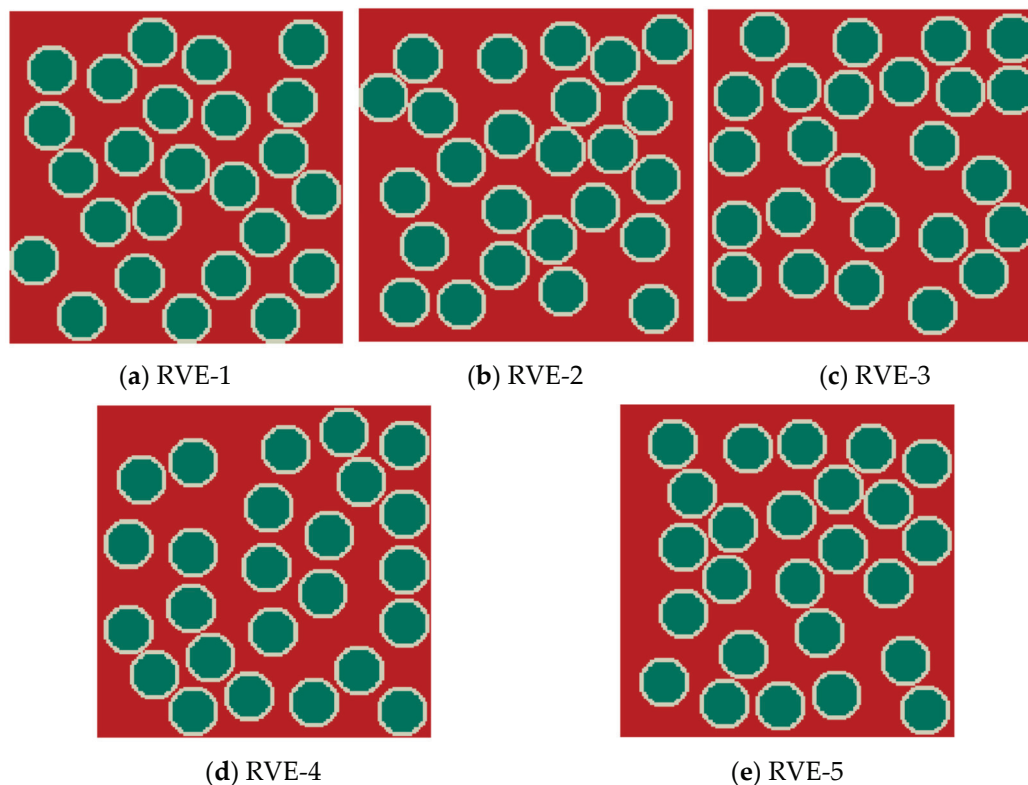


Figure 7. The phase distributions of different RVE models.

Finite element analysis was performed on the generated RVEs, and the computed properties were subsequently compared with the established cross-property relations. The results are presented in Figure 8. It should be noted that the axial shear modulus and the in-plane thermal conductivity did not completely correspond. The axial cross-property relations established are those between G_{12} , G_{13} , and k_{11} , and the in-plane cross-property relations are those between G_{23} and k_{22} , k_{23} .

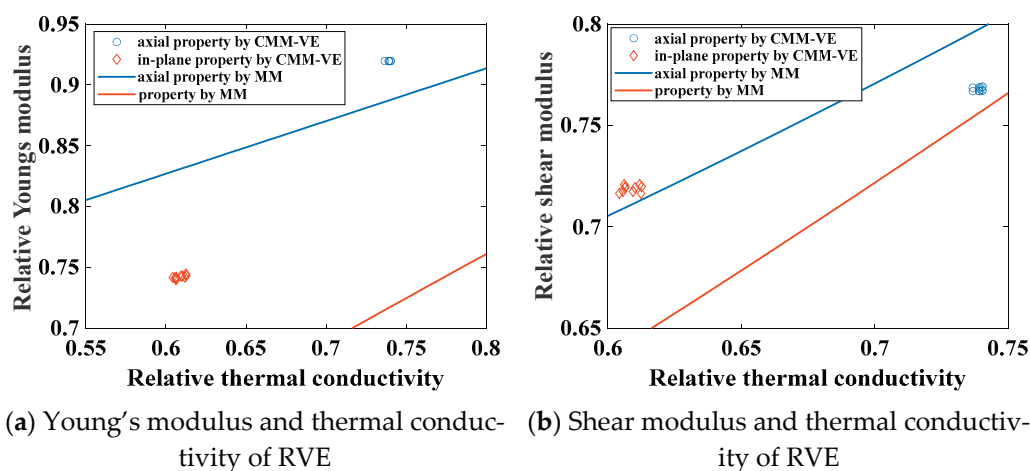


Figure 8. Cross-property relations of RVE.

Based on the effective properties obtained from homogenization, the distribution of RVE properties is highly concentrated. The largest discrepancy is observed for in-plane thermal conductivity, the maximum value ($42.887 \text{ WM}^{-1}\text{K}^{-1}$) occurs for k_{22} of RVE-4,

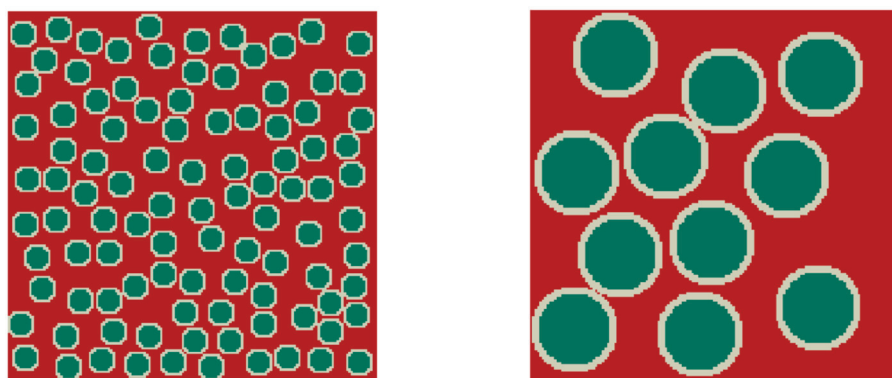
and the minimum value ($42.3 \text{ WM}^{-1}\text{K}^{-1}$) occurs for k_{33} of RVE-5. The relative difference between these two extremes is limited at 1.39%. Consequently, the stochastic spatial distribution of fibers has a negligible influence on effective properties at the current RVE size, confirming their appropriateness.

The data obtained on effective properties can also be employed to validate the applicability of cross-property relations. Assuming that the thermal properties are precisely determined through the experiments, the mechanical properties can be predicted directly via cross-property relations and compared with the FEA results. The largest relative deviation occurred in direction 3 of RVE-1: the shear modulus G_{23} inferred from k_{33} is 94.052 GPa, which is 11.5% lower than the FEA value of 106.276 GPa. It can be considered that the material conforms to cross-property relations.

In the present analysis, the theoretical model exhibits certain inaccuracies when predicting the shear modulus of the material. Although these deviations remain within an acceptable range for composite material property measure, they have given rise to an intriguing observation: the in-plane shear modulus calculated using CMM-VE is significantly higher than that obtained from MM, whereas the axial shear modulus is correspondingly lower. As a result, in the cross-property relations of shear modulus, the axial results from CMM-VE closely resemble the in-plane results from MM, and vice versa. This phenomenon is primarily attributed to the overly idealized treatment of the interphase in MM when predicting effective properties in cases where both the fiber phase and the interphase have relatively high volume fractions. Additionally, the inherent properties of all constituent phases contribute collectively to this behavior, further influencing the observed discrepancy.

3.1.3. Fiber Diameter

The fiber diameter constitutes a critical parameter in CFRCMCs. To investigate the effect of fiber diameter on the effective properties, it is varied by $\pm 50\%$ while maintaining a constant RVE size and fiber–interphase volume ratio. Specifically, the fiber diameter is adjusted to $4.7 \mu\text{m}$ and $14.1 \mu\text{m}$, corresponding to 5 and 15 voxels, respectively. Corresponding RVEs are constructed, as shown in Figure 9. In response to variations in fiber diameter, the quantity of fibers in the RVEs is adjusted proportionally.



(a) RVE with fiber diameter of $4.7 \mu\text{m}$ (b) RVE with fiber diameter of $14.1 \mu\text{m}$

Figure 9. The RVEs of different fiber diameters.

FEA is performed on the newly developed RVEs, with the effective properties of RVE-1 serving as a proxy for the original fiber diameter. The effective property results obtained are shown in Figure 10. From this, it can be observed that at smaller fiber diameters, the results align more closely with the theoretical model, which is attributable to a more uniform

distribution of stress and heat. Nevertheless, the fiber diameter does not compromise the validity of cross-property relations.

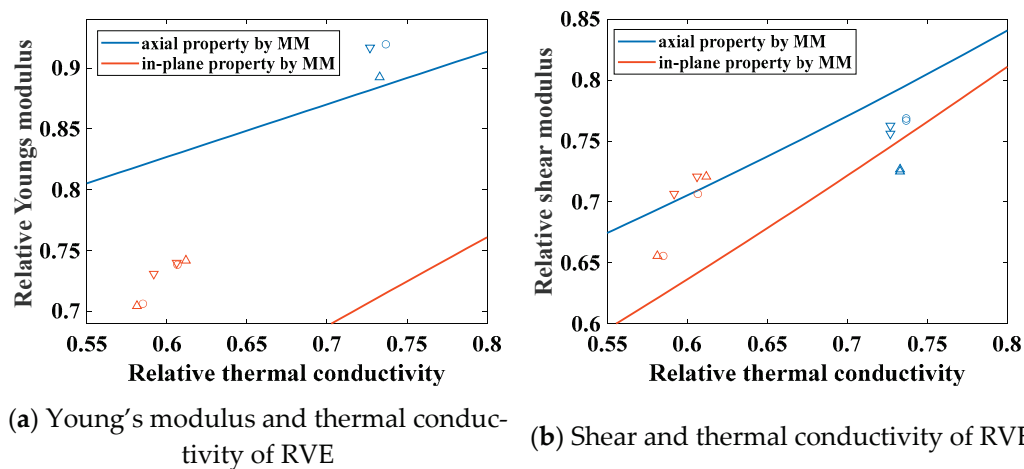


Figure 10. The properties of RVEs with different fiber diameters. Here, 'Δ', 'O', and '▽' denote the result of CMM-VE with the fiber diameters of 4.7 μm, 9.4 μm, and 14.1 μm; blue symbols denote the properties of the axial direction; and orange symbols denote the properties of the in-plane direction.

3.1.4. Interphase

In practical engineering applications, the properties of the fiber and matrix are readily measurable, whereas those of the interphase remain elusive. This difficulty stems from the inherent challenge of precisely defining the interphase domain and the impossibility of isolating it for independent characterization. Consequently, the interphase parameters obtained are unavoidably biased, necessitating a rigorous assessment of their influence on cross-property relations.

Taking RVE1 as the RVE, calculations were performed with the interphase property sets listed in Table 6 to elucidate the influence of interphase properties on cross-property relations. The results obtained from FEA and cross-property relations are both presented in Figure 11. Specifically, property set-1 and set-2 correspond to the interphase properties measured at 600 °C and 1200 °C. Therefore, this result also has some reference value for cross-property relations under high-temperature conditions.

Table 6. Property sets of the interphase.

	RVE1	Property Set-1	Property Set-2
Young's modulus (GPa)	62	41	18
Thermal conductivity ($\text{WM}^{-1}\text{K}^{-1}$)	6.9	5.2	3.5

As illustrated in Figure 11, variations in the properties of the interphase exert a relatively minor influence on the axial cross-property relations between Young's modulus and thermal conductivity. Especially for the axial cross-property relations, it is approximately coincident in Figure 11e. However, these variations significantly affect all other cross-property relations. A reduction in the interphase properties results in a downward shift in the cross-property relation curves, accompanied by an expansion in the range of these curves. This phenomenon occurs because, for a constant interphase volume fraction, a decrease in the interphase property leads to a decrease in the effective material properties. However, its effect remains comparatively limited.

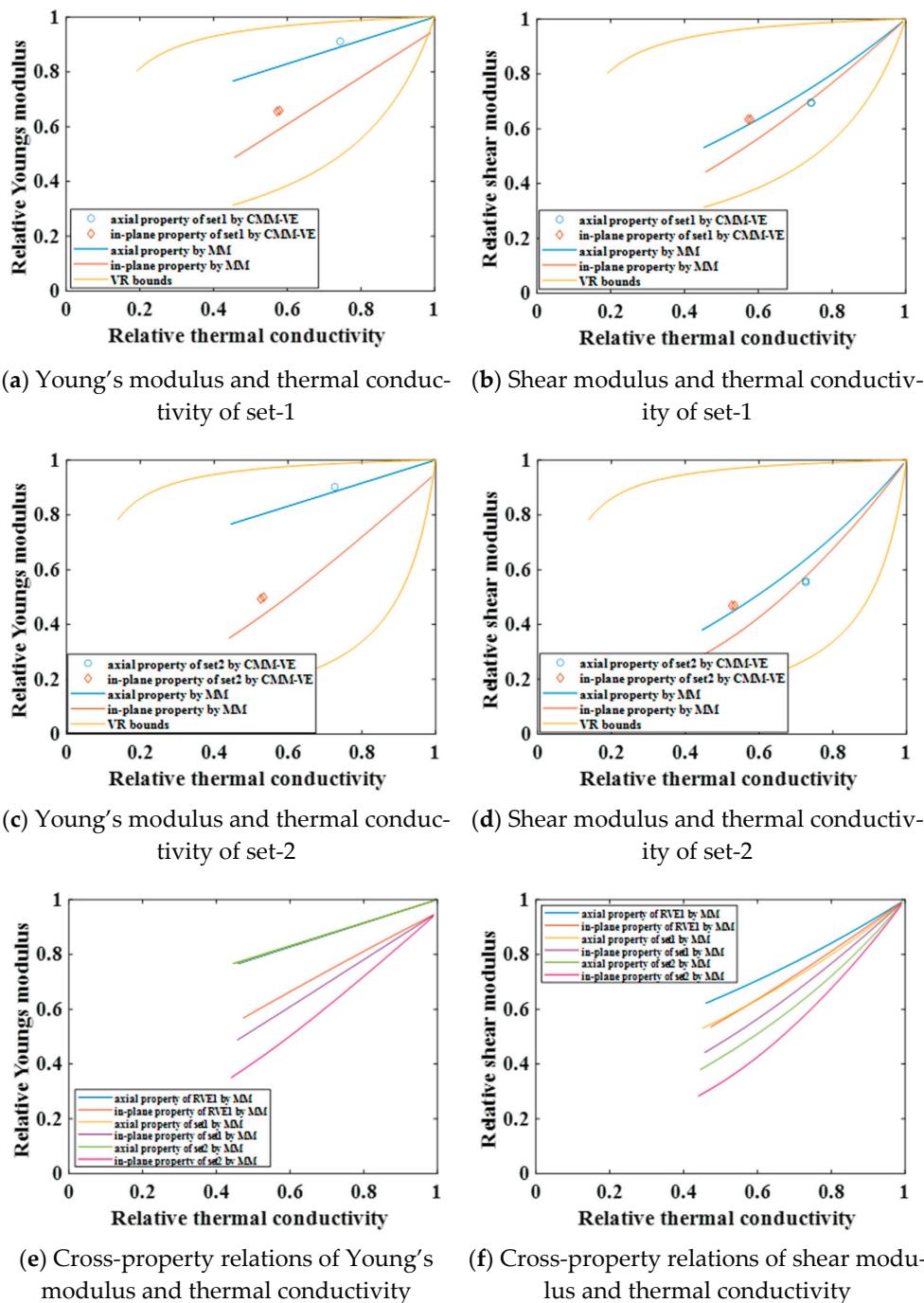


Figure 11. The influence of interphase properties on cross-property relations.

3.2. Cross-Property Relations of CFRCMCs with Varying Fiber–Interphase Volume Ratios

3.2.1. Establishment of Cross-Property Relations

The preceding discussion focused on the materials of CFRCMCs with a fixed fiber–interphase volume ratio. However, in practical engineering applications, the proportions of each constituent phase exhibit considerable variability in various CFRCMCs. The volume fractions of the fibers and interface phases span a certain range. Consequently, it is essential to develop cross-property relations for CFRCMCs with varying fiber–interphase volume ratios.

The material properties employed are consistent with Section 3.1. For CFRCMCs with varying fiber–interphase volume ratios, effective property calculations necessitate that two of the three constituent phase volume fractions be prescribed. Considering the potential

range of component volume fractions, an analysis of existing CFRCMC materials was conducted. The results indicated that a fiber volume fraction within the interval $[0, 0.7]$ and an interface phase volume fraction within the interval $[0, 0.3]$ adequately encompass the majority of currently available CFRCMCs. Analogously, the effective properties are computed at increments of 0.01 over the prescribed ranges of fiber and interphase contents, with the results presented in Figure 12.

As illustrated in Figure 12, the effective properties predominantly remain within the VR bounds, indicating the validity of the calculations. Nevertheless, there are instances during the computational process where some properties surpass the VR bounds. This is primarily attributed to inappropriate volume fraction configurations, such as an exceptionally low fiber content coupled with a relatively high proportion of interphase. It is also worth noting that for CFRCMCs with varying fiber-interphase volume ratios, the thickness of the interphase will affect the surface/volume ratio of the equivalent fibers, and thus may have an impact on their effective properties and corresponding cross-property relations.

Additionally, the present model exhibits certain discrepancies in the estimation of the in-plane Young's modulus. The effective properties of each composite are jointly influenced by the fiber volume fraction and the interphase volume fraction. As a cross-property relation eliminates only one independent variable, the fiber volume fraction is adopted as the Z-axis, thereby permitting the three-dimensional surface representation of the cross-property relations in the property space. Based on these calculated effective properties, the corresponding cross-property relations are subsequently established, as illustrated in Figure 13.

As illustrated in Figure 13, apart from localized regions where the in-plane cross-property relations between Young's modulus and thermal conductivity marginally exceed the VR bounds, the entire surface remains within the theoretical limits, providing a preliminary validation of the results. Contour lines are further superimposed on the surface. For a composite with a prescribed fiber volume fraction, intersecting the cross-property surface with a horizontal plane at the corresponding height yields the desired curve of cross-property relations for that material. The figure further reveals that, with the selected material parameters, every cross-property relation except for the axial relation between Young's modulus and thermal conductivity exhibits a marked decrease in the slope of its curve as the fiber volume fraction diminishes. This progressive flattening induces the surface to curl toward a configuration that is increasingly parallel to the thermal conductivity axis.

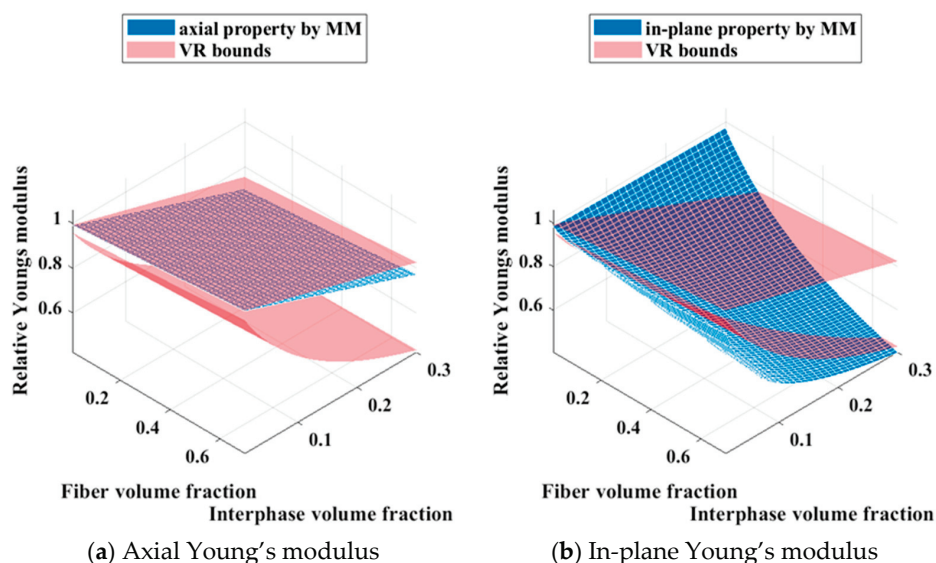


Figure 12. *Cont.*

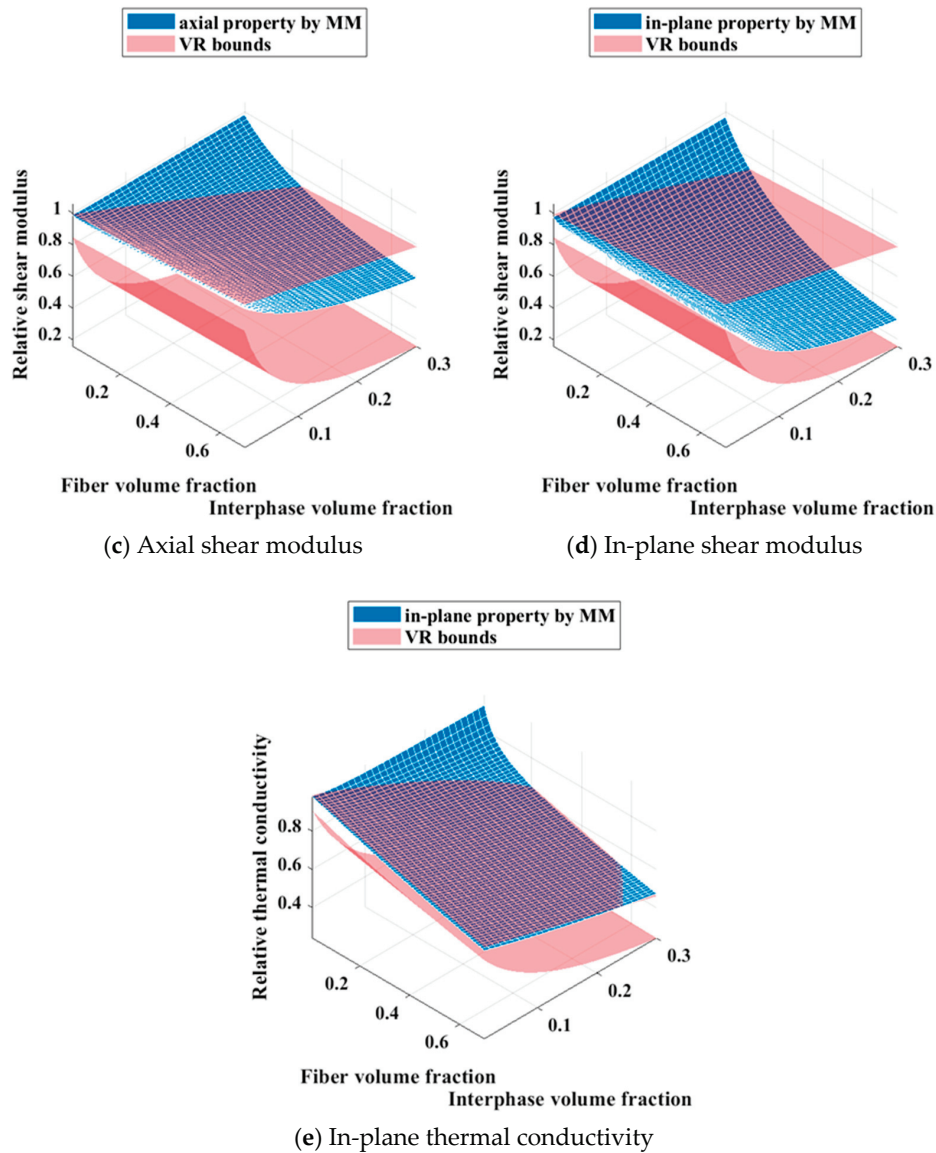


Figure 12. The effective properties of CFRCMCs with varying fiber–interphase volume ratios.

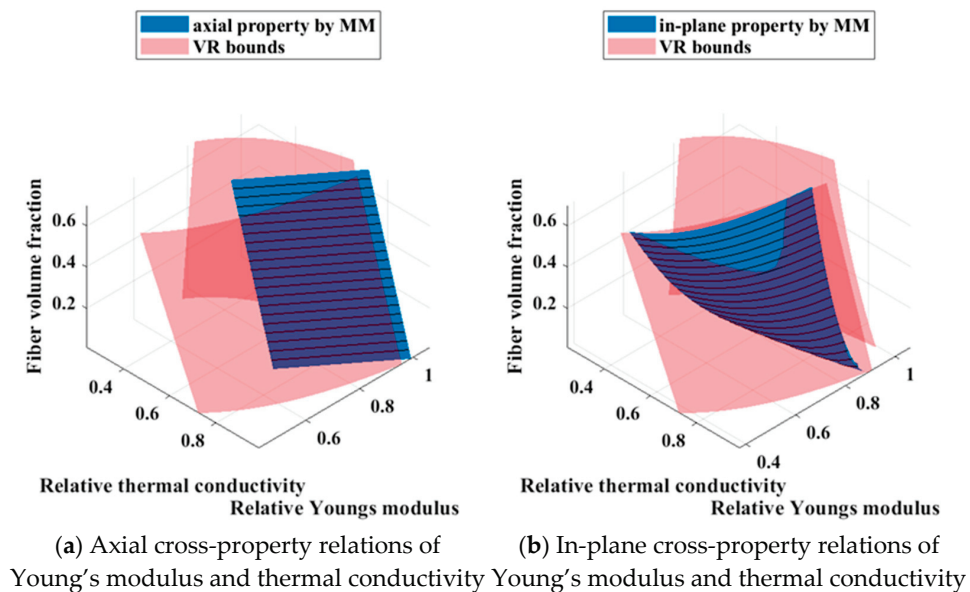


Figure 13. Cont.

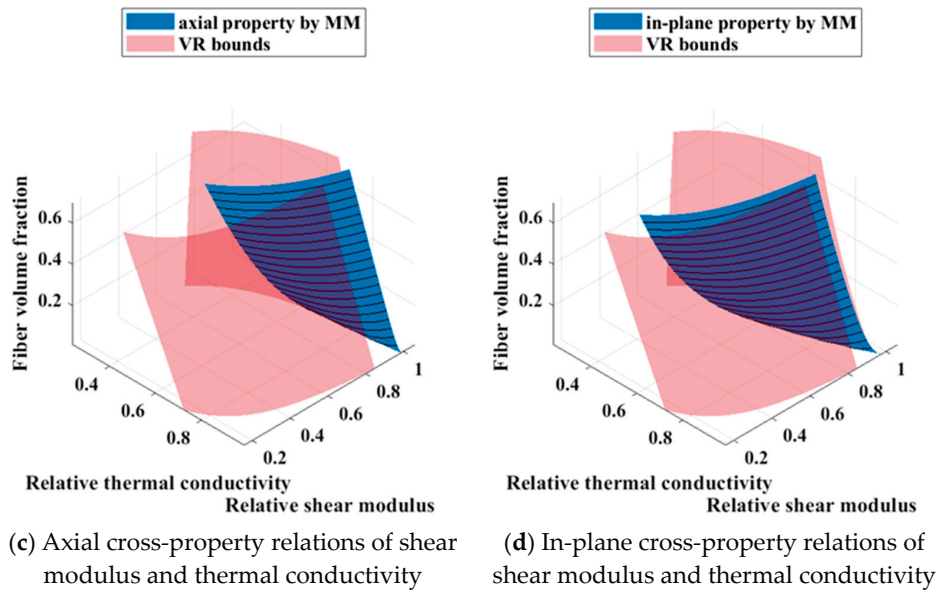


Figure 13. The cross-property relations of CFRCMCs with varying fiber–interphase volume ratios.

3.2.2. CMM for CFRCMCs with Varying Fiber–Interphase Volume Ratios

To verify the validity of the cross-property relations established for CFRCMCs with varying fiber–interphase volume ratios, the RVE sizes were held constant and the volume fractions of the fiber and interphase phases were systematically varied. Nine distinct RVEs corresponding to different combinations of fiber and interphase contents were utilized. Their respective volume fractions are listed in Table 7, and the material distributions are illustrated in Figure 14.

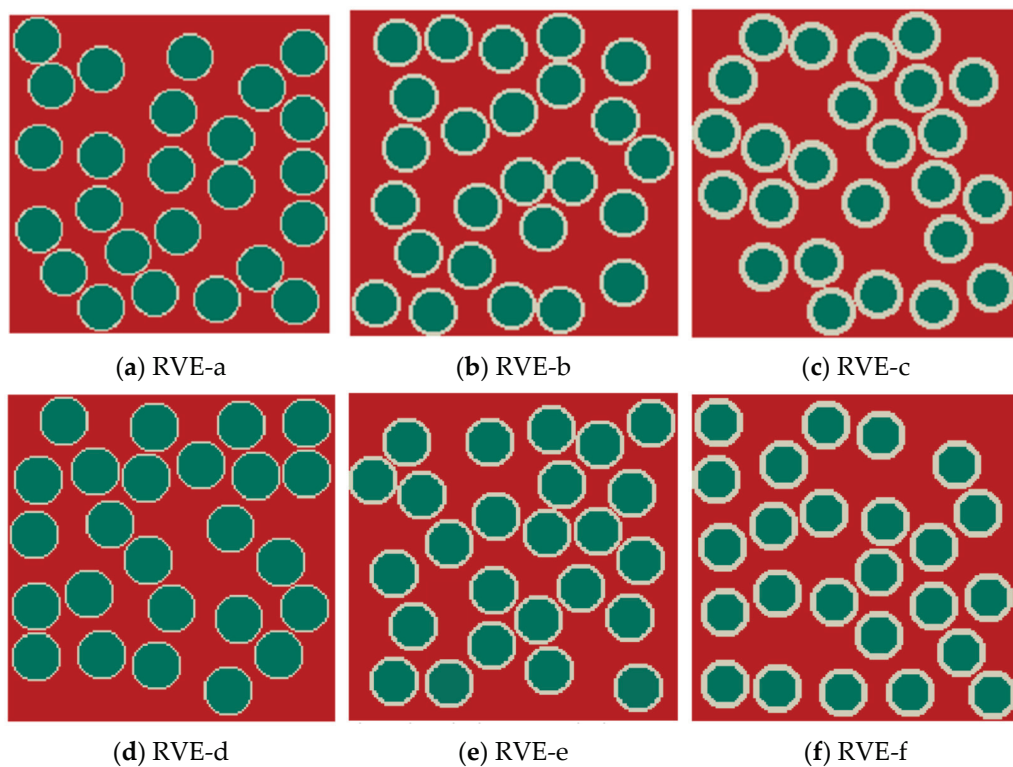


Figure 14. Cont.

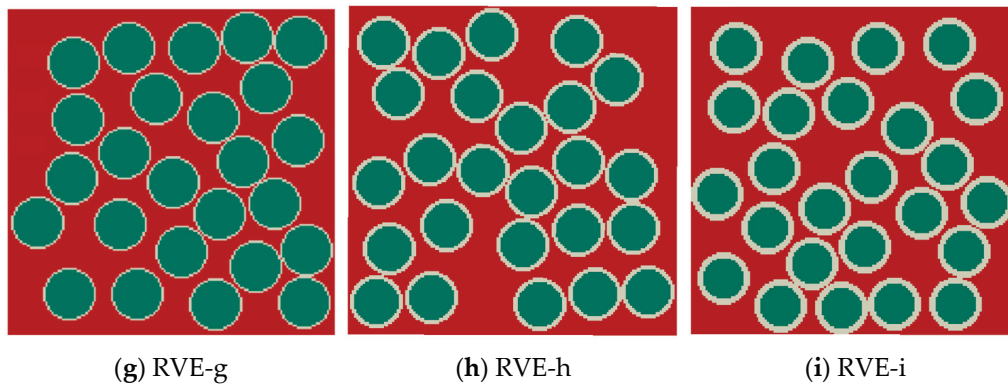


Figure 14. The phase distributions of different RVE cases.

Table 7. Volume fractions of the constituent phases for the different RVE cases.

	Fiber Diameter with Interphase (Voxel)	Fiber Volume Fraction	Interphase Thickness (Voxel)	Interphase Volume Fraction
RVE-a	23	36.04%	1	7.19%
RVE-b	23	29.50%	2	13.73%
RVE-c	23	23.62%	3	19.61%
RVE-d	24	39.56%	1	7.52%
RVE-e	24	32.69%	2	14.38%
RVE-f	24	26.48%	3	20.60%
RVE-g	25	43.23%	1	7.85%
RVE-h	25	36.04%	2	15.04%
RVE-i	25	29.50%	3	21.58%

Homogenization analyses were subsequently performed on each generated RVE to determine the corresponding effective properties, and the results are presented in Figure 15. For the FEA results, the errors associated with in-plane cross-property relations exceed those of their axial counterparts, and Young's modulus exhibits a marginally higher deviation than the shear modulus. These discrepancies intensify as the fiber volume fraction diminishes, with the in-plane Young's modulus showing the most pronounced sensitivity. From the perspective of data evolution, the variation trend exhibited by the FEA results closely parallels that of the cross-property relation surface as the fiber volume fraction changes, thereby confirming the validity of the established cross-property relations of CFRCMCs with varying fiber–interphase volume ratios.

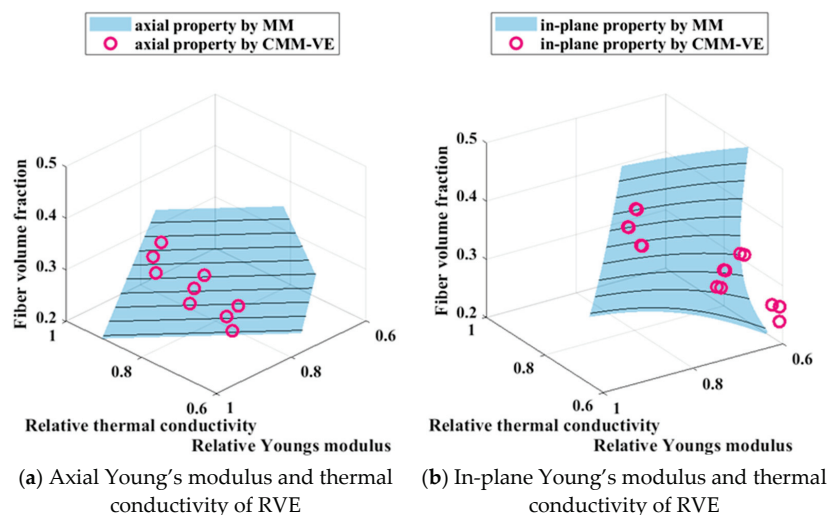


Figure 15. Cont.

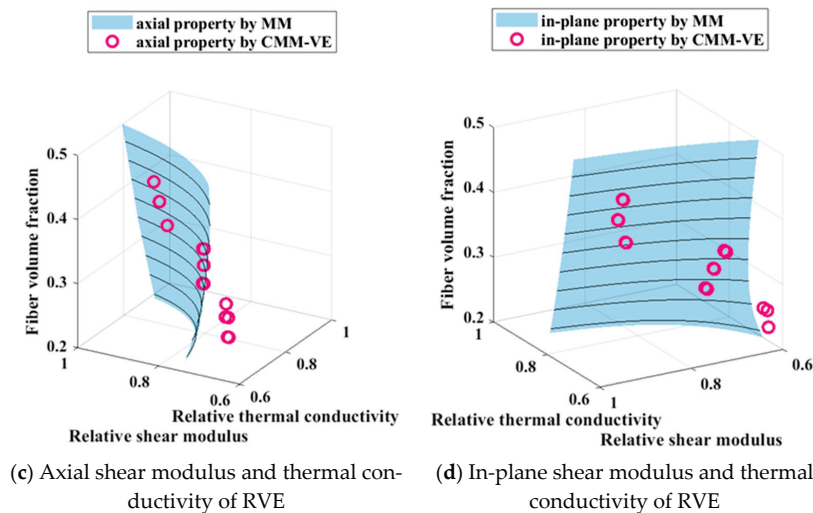


Figure 15. Sensitivity analysis of RVEs with varying fiber–interphase volume ratios.

4. Conclusions

In this work, a systematic framework is established to derive and validate cross-property relations for CFRCMCs that simultaneously couple elastic and thermal responses. By integrating GSCS, effective-medium conduction models, and statistically representative voxel-based finite-element analysis, the following major findings are obtained:

1. For CFRCMCs, self-consistent micromechanics and effective-medium conduction models like the Hashin model can predict the mechanical and thermal properties of materials. However, for cases where the volume fraction of fibers or interphases is too high, the prediction errors are relatively large.
2. For CFRCMCs with a fixed fiber–interphase volume ratio, cross-property relations are close to being linear. The slope of the axial cross-property relation is smaller than that of its in-plane counterpart, and the slope characterizing the cross-property relation between Young's modulus and thermal conductivity is marginally lower than that governing the relation between the shear modulus and thermal conductivity.
3. The influence of the random distribution of fibers, fiber diameter, and interphase properties on cross-property relations was analyzed separately. The influence of the former two parameters is relatively small, while a decrease in the interphase properties causes the curve of cross-property relations to shift downward.
4. For CFRCMCs with varying fiber–interphase volume ratios, cross-property relation surfaces were established, and FEA verified their correspondence. The errors associated with in-plane cross-property relations exceed those of their axial counterparts, while Young's modulus exhibits a marginally higher deviation than the shear modulus.

Overall, the findings indicate that CFRCMCs are applicable to cross-property relations. Consequently, one physical property (e.g., mechanical or magnetic) can be predicted from another, more easily measurable quantity (e.g., thermal or electrical). This approach significantly mitigates the challenges associated with measuring material properties in CMCs. Furthermore, by establishing explicit connections between different physical properties, cross-property relations streamline the complexities of multi-objective optimization for these materials. Consequently, they could contribute to advancing multiphysics applications, particularly in demanding thermo-mechanical environments.

To validate this relation, ceramic matrix composites with different fiber volume fractions should be fabricated, and their properties should be characterized individually to evaluate their consistency with the currently established cross-property relations. In addi-

tion, more accurate models will be developed to account for the influence of parameters such as interphase thickness on these relations.

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