



Special Issue Reprint

Railway Dynamics and Ground-Borne Vibrations

Edited by
Aires Colaço, Hassan Liravi and Pedro Alves Costa

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About the Editors

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Editorial

Editorial for the Special Issue of Vibration: Railway Dynamics and Ground-Borne Vibrations

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Rapid urbanisation, the demand for efficient mobility, and the need to mitigate climate change are among the major societal challenges of our time. These challenges have driven the extensive expansion of railway infrastructure in modern cities, raising environmental concerns about the generation and propagation of vibrations and noise, which can adversely affect the comfort, well-being, and quality of life of people living and working in the vicinity of railway lines. In parallel, the increasing demands placed on railway systems, including higher speeds, traffic density, and enhanced operational reliability, have intensified the need for a deeper understanding of railway dynamics and the mechanisms governing the generation, transmission, and mitigation of vibrations.

Addressing these challenges requires multidisciplinary approaches encompassing vehicle-track interaction, wave propagation in soils, structural dynamics, vibration control technologies, and monitoring systems. The Special Issue brings together nine contributions covering a broad spectrum of topics, ranging from fundamental studies of railway dynamic behaviour to practical solutions for vibration and noise prediction, mitigation, and monitoring.

Colaço et al. [Contribution 1] provide a comprehensive review of railway-induced ground-borne vibrations, discussing the mechanisms of vibration generation and propagation, current prediction methodologies, available mitigation measures, and emerging research directions. The authors highlight that harmonised international regulations governing railway-induced vibration remain lacking, and that uncertainty in the prediction procedure remains relatively high. Furthermore, regarding mitigation, although a variety of solutions exist, their effectiveness depends strongly on local conditions and system characteristics. Therefore, mitigation measures should be designed on a site-specific basis.

Valuable insights into the physical mechanisms governing vibration propagation from railway sources through the soil and into adjacent structures, contributing to the validation and improvement of prediction models, can be found in Auersch et al. [Contribution 2]. This work combines field measurements, analytical interpretation, and predictive modelling to investigate the excitation and transmission of train-induced ground and building vibrations.

Tao et al. [Contribution 3] propose a hybrid modelling framework for predicting ground-borne vibrations in metro depots that integrates train dynamics, soil response, pile foundations, and structural interaction. By combining numerical simulations, analytical impedance models, and measurement data, the proposed framework offers a promising approach to addressing the uncertainties inherent in current prediction procedures.

Several contributions focus on vibration and noise mitigation strategies. Sadeghi et al. [Contribution 4] assess the effectiveness of dynamic vibration absorbers for reducing

metro-induced ground vibrations, demonstrating their potential as a practical and efficient mitigation measure. Gu et al. [Contribution 5] investigate how rail track design can be optimised to mitigate vibrations transmitted to buildings directly above a double-deck railway depot, providing practical guidance on track configuration and vibration-isolation solutions. Li et al. [Contribution 6] studied the aerodynamic behaviour of a semi-enclosed sound barrier located in a high-speed railway bridge, contributing to the understanding and design of noise mitigation systems in complex railway geometries. Collectively, these studies emphasise that effective mitigation strategies must be tailored to specific operational and site conditions.

The dynamic behaviour of railway vehicles and infrastructure is also addressed. Dimitrovová [Contribution 7] investigates the potential instability of a moving bogie travelling along a track through semi-analytical analyses, while Galvín et al. [Contribution 8] focus on railway bridges, proposing a time-frequency analysis based on wavelet transforms to separate forced and free vibration components from measured responses. The studies and methodologies provided represent valuable contributions to a deeper understanding of railway infrastructure behaviour.

Finally, Collina et al. [Contribution 9] address the growing importance of monitoring in modern railway systems, specifically investigating the feasibility of detecting contact wire rupture in high-speed railway catenary systems. Their study demonstrates the potential of vibration-based monitoring techniques to improve operational safety and enable the early detection of critical failures in railway infrastructure.

Collectively, the contributions of this Special Issue reflect the breadth and vitality of current research in railway dynamics and ground-borne vibrations. They span multiple spatial scales, from vehicle components and track systems to soil-structure interaction and urban environments, and integrate experimental, analytical, and numerical approaches. Together, they highlight the importance of interdisciplinary research in addressing the challenges posed by modern railway systems and in developing sustainable and resilient transportation infrastructure.

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Article

Ground-Borne Vibration Prediction in a Metro Depot Using Hybrid Train-Soil-Pile-Structure Interactions

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Abstract

Train-induced ground vibrations can propagate into pile foundations, potentially causing undesirable vibration in nearby buildings, laboratories housing vibration-sensitive equipment, and manufacturing facilities for high-precision processes. This paper presents an innovative method for predicting building vibration from free-field ground vibration measurements at locations away from the tracks during train pass-bys. The proposed method accounts for site-specific soil profiles and train-soil-pile-structure interactions and is implemented in four steps. In Step 1, train-induced vibration transmission into the ground is estimated using an axisymmetric finite element model that simulates wave propagation through layered soils from the tracks to free-field ground locations. Step 2 estimates *free* pile head vibration using a three-dimensional finite-element model that captures the ground-borne transmission of track inputs through soil layers to the pile. Step 3 estimates vibration at the junction of the pile head and depot column base using a finite-element model to estimate the pile head impedance and an analytical impedance model for the depot structures supported by the pile. In Step 4, estimates of column-base vibration that transmits into over-track buildings are compared to measured column-base vibration levels obtained during train pass-bys. The method was applied at a metro depot in China, where tracks were in close proximity to columns supporting over-track buildings. Ground and column base vibration levels were measured during multiple train pass-bys. The estimated vibration levels at the base of depot columns closely agreed with the measured vibration levels at the columns during six-car train pass-bys. It demonstrated the potential effectiveness of this hybrid method for assessing vibration transmission into structures atop existing railway tracks. By integrating field measurements, finite element simulations, and analytical impedance models, the proposed hybrid method provides a framework for evaluating the transmission of the train-induced vibration to nearby building structures.

Keywords: train-induced vibration; soil-pile-structure coupling; finite element modeling; building impedance models; ground-borne vibration transmission; over-track buildings

1. Introduction

As megacities face escalating demands for commuter transportation, urban railway systems are being rapidly designed and expanded. Consequently, constructing over-track buildings has become essential in densely populated areas to maximize the utility of the airspace above metro depots. However, train operations induce ground-borne vibrations

that propagate into the depot columns located between the tracks. These vibrations are subsequently transmitted to the platform structures and the overlying buildings, potentially causing discomfort to occupants through perceptible vibrations and audible structure-borne noise [1]. Therefore, there is a critical need for efficient predictive models during the preliminary design phase, before constructing a depot or building structures, to assess potential vibration impacts and guide the implementation of effective isolation and mitigation strategies.

Prior experimental [2] and numerical [3,4] studies have evaluated vibration transmission into buildings. Recent advancements have further refined these simulation techniques. For instance, Chen et al. [5] developed a coupled numerical model of the train-track-depot-building system, incorporating nonlinear mechanical properties and soil layering to predict train-induced vibrations, and validated their approach with field tests. Accurately characterizing the dynamic coupling between ground-borne vibrations and the foundation structures is crucial for estimating vibration transmission. Field measurements investigating soil-foundation coupling were conducted by Colaço et al. [6] and Kuo et al. [7], with a predominant focus on buildings near surface railway lines. Based on such empirical data, the FTA guideline [8] introduced a frequency-independent coupling factor to quantify soil-to-structure transmission. Furthermore, recent comprehensive reviews, such as that by Colaço et al. [9], have assessed analytical, numerical, empirical, and emerging AI-based models for ground-borne vibration prediction, noting that while semi-empirical methods are suitable for preliminary analysis, the complex dynamic soil-foundation coupling requires further development. Much research has been conducted that comprehensively modeled vibration transmission from the ground into structures. Luo et al. [10] investigated vibration transmission and distribution characteristics in double-layer metro depots using field measurements and numerical models. They characterized the propagation of vibrations from the tracks through the bearing columns into over-track structures. López-Mendoza et al. [11] utilized spring-damper elements to characterize the coupling between the soil and buildings.

While the above approaches have evolved to include advanced combinations, such as integrating validated finite element models (FEM) with field-measured databases [12], they still present notable limitations. Existing semi-empirical frameworks (such as the FTA guidelines) often rely on simplified, frequency-independent coupling factors that fail to capture complex, frequency-dependent, dynamic soil-structure interactions. Conversely, fully coupled 3D finite element or boundary element models (modeling the entire train-track-soil-building system) offer high fidelity but are computationally prohibitive. Furthermore, current hybrid approaches often suffer from two critical shortcomings: (1) empirical transfer functions derived from standard ground-impact measurements frequently neglect the crucial impedance of the sub-base under the track bed and rails and (2) the validation of large, fully analytical models from the wheel-rail interaction to the building is rarely demonstrated across all frequencies of interest, particularly the mid-to-high frequencies responsible for structure-borne noise.

To address structural complexities, impedance-based methods have shown promise. Sanayei et al. [13] developed an impedance-based method to predict ground-borne vibrations in a four-story building, accounting for the propagation of axial waves through the columns and the impedance of the intermediate floor slabs. Zou et al. [14,15] accounted for the propagation of axial and bending wave transmission through the columns from the foundation into the upper floors of an over-track building in a metro depot. In both instances, the impedance model predictions were in good agreement with the measured floor vibration levels.

To overcome the limitations of existing hybrid and semi-empirical models, this paper proposes a novel, highly efficient hybrid prediction framework that strategically decouples the source-propagation-receiver system.

(a) The primary contribution of this paper is to estimate train-induced vibration in buildings using easily accessible free-field vibration levels measured at a distance from the tracks during train pass-bys. Specifically, equivalent train inputs under the tracks are back-calculated from free-field ground vibration measurements. These back-calculated inputs are then combined with finite element analysis (FEA) to estimate *free* pile head vibration levels. Finally, an analytical impedance model uses these *free* pile head vibrations to estimate the superstructure's vibration levels. The proposed approach relies solely on safe, accessible, free-field measurements. This allows for accurate predictions without disrupting train operations.

(b) Computational Efficiency and Design Flexibility: By using FEA only up to the pile head and applying analytical impedance models for the superstructure, this method decouples the foundation from the building. This provides a massive quantitative advantage in computational time: engineers can rapidly evaluate several potential building configurations on the same pile foundation in seconds, avoiding the computationally paralyzing task of re-running a fully coupled 3D train-soil-building model for every architectural iteration.

(c) Enhanced Frequency-Dependent Accuracy: Unlike standard semi-empirical transfer functions or FTA guidelines, the back-calculation explicitly captures the impedance of the track sub-base, and the impedance-based building model preserves the frequency-dependent dynamic coupling between the pile foundation and the superstructure. This ensures higher prediction fidelity across the full frequency spectrum of interest.

(d) Ultimately, this framework provides a rigorous, computationally lean, and highly adaptable engineering tool tailored for the dynamic optimization of over-track developments.

2. Hybrid Method for Estimating Train-Induced Vibration Transmission

The present study focuses on vertical vibration induced by train operations, which is the dominant mode of transmission in metro depots. [16–18]. For train-induced vibration, it is generally accepted that vertical vibration dominates the transmission of vibration from the tracks through the ground via Rayleigh wave propagation in the soil, and subsequently into building structures, where vertical vibration transmission to upper floors is important. Building columns, due to their slenderness, inherently have much greater axial impedance than for bending or torsional vibration, which facilitates axial vibration transmission to upper floors [13]. Vibration measurements at free-field ground locations during train pass-bys on existing tracks are utilized to estimate ground vibration inputs that propagate through layered soils into depot foundation structures and over-track buildings. Finite element models estimate vibration transmission through layered soils to free-field ground locations and foundation structures. Finite element models estimate input impedances of depot foundations, which, along with impedance models of transmission into buildings, provide estimates of vibration levels in the buildings. The hybrid method, as shown in the flowchart in Figure 1, involves four steps:

Step 1. Train-induced Inputs into the Ground Beneath the Tracks

Axisymmetric FE models computed transfer functions of vibration transmission through soil layers from locations beneath the tracks to a free-field ground location at a suitable distance from the track and nearby depot or building structures. The transfer functions are the ratios of the measured ground-vibration amplitude to the input ground-vibration amplitude under the track, as a function of frequency. The amplitude of train-induced input vibration at the track is back-calculated by dividing the measured free-field ground amplitude by the transfer function.

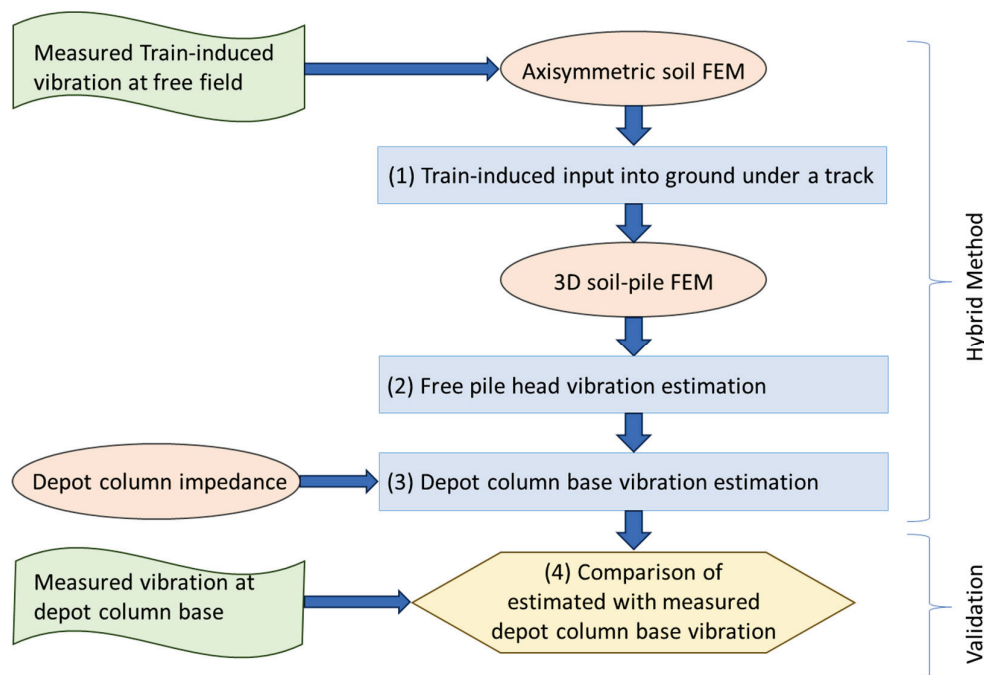


Figure 1. Proposed hybrid method for train-induced vibration prediction.

The axisymmetric FE model, with input at the track and response at the free-field ground location, depends only on the distance between the locations and the frequency. The distance varies for different bogies on the train when positioned along the track where the free-field ground response is greatest. The transfer functions account for soil-layer properties and thicknesses obtained from borehole measurements. The field measurements and soil properties are described in Section 3. The axisymmetric FE model is described in Section 4, along with a discussion of how contributions to the free-field ground vibration from all bogies of a multi-car train are taken into account.

Step 2. Free Pile Head Vibration Estimation

3D FE models are used to compute transfer functions for transmission from track inputs through soil layers to the head of a pile at a depot foundation column. The transfer function is the ratio of vibration at the pile head divided by the input ground vibration amplitude under a bogie at the track. The pile head motion, referred to as the *free* pile head vibration, is computed with *no* depot structure resting on it.

A 3-D model must be symmetric about a vertical plane passing through the track location and the pile. Again, the model depends on the distance from the bogie input at the track input location to the pile. The 3-D model is described in Section 5, alongside a discussion of how contributions to pile head vibration from all bogies on a multi-car train are taken into account.

Estimates of train-induced *free* pile head vibration levels, with no depot columns or depot structures in place, are generated using the transfer functions from the input ground vibration level at the track to a pile head based on the 3D FE model. The model includes the vibration interaction between the pile and soil layers at the depot column location. The *free* pile head vibration is the sum of energy contributions from all bogies. The *free* pile head vibration levels are presented in Section 5.

Step 3. Depot column base vibration estimation

The coupling of the above-ground depot structures to the below-ground pile head is used to estimate pile head vibration during train pass-bys. It requires impedances at the pile head, looking into the ground, and looking up into the depot and building structure.

The impedance of a pile was calculated using an axisymmetric FE model with input at the pile head and no train inputs. It accounts for vibration transmission into the pile and ground, as well as internal losses within both.

An impedance model of the depot and building structures describes the transmission of axial vibration from the pile head into the depot columns and the building. Vibration levels at the pile head, with the depot and building structures in place, were calculated based on the product of *free* pile head vibration levels times a factor containing the ratio of impedances of the depot structures and the pile. Subsequently, the train-induced vibration levels within the above-ground depot structure were derived by multiplying the vibration transmission ratios by the calculated pile head vibration levels, accounting for the depot structure. Pile head vibration predictions are described in Section 6.

Step 4. Comparison of estimated and measured vibrations at depot column base

The validation of the proposed method is presented in Section 6.2, where estimated train-induced vibration levels at the bases of existing ground columns were compared with measured vibration levels obtained during train pass-bys. The validation data was not used in the back-calculation or in other steps in the method that generate vibration estimates at the ground column bases for comparison with the validation data.

3. Train-Induced Vibration Measurements

Figure 2 shows typical high- and low-rise over-track buildings above the Qianhai metro depot in Shenzhen, China, which occupies a total land area of about 489,700 m². There are more than 25 high-rise residential buildings directly supported by pile foundations and over 30 low-rise offices supported above the tracks by depot columns and a transfer floor. The at-grade train tracks in the metro depot lie between depot columns (ground-level columns) that support the depot platform structures and over-track buildings.

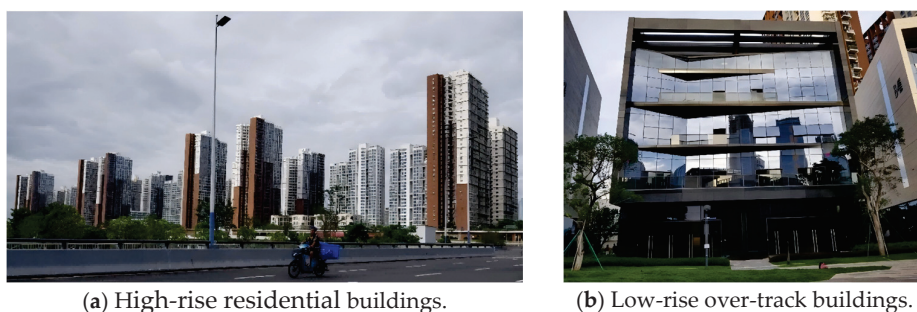


Figure 2. The Qianhai metro depot with over-track buildings in Shenzhen, China.

Train-induced vibration levels were measured at free ground locations for estimating train input vibration levels into the ground at the tracks and at the base of depot columns for validating model predictions. During the experimental study, data were recorded using a wireless data acquisition unit and a JM3873 acquisition system. The wireless unit was equipped with a built-in magnetoelectric accelerometer for vertical and horizontal measurements and supported an external third accelerometer. It featured 1 GB of storage capacity and 1 ms time synchronization accuracy. The sampling frequency was set at 1024 Hz, which, according to the Nyquist theorem, supports a maximum analyzable frequency of 512 Hz. Standard vibration measurement methods, equipment, and signal processing were employed, with further details shown in [19].

For the construction of new train depots, there are often no existing tracks at the proposed construction sites to make train-induced free-field vibration measurements. In this study, the depot tracks and structures were already in place and fully operational. As

discussed in Section 4, train-induced ground vibration measurements at locations between depot columns were utilized to back-calculate train-induced vibration inputs at the tracks. Coulier et al. [20] found that the influence of nearby buildings on free ground vibration measurements could be ignored if the distances between the ground location and the building were larger than the Rayleigh wavelength in the ground. At the highest train-induced vibration frequencies, the Rayleigh wavelength was shorter than the distance to the nearest depot structures, making the measurements representative of free-field ground vibration.

3.1. Plan View of the Throat Area in the Depot

As shown in Figure 3a, a metro depot includes the throat area, a test line, and parking and maintenance areas. Tracks, with switches in the throat area, link the entrance/exit lines to those in the parking and maintenance areas. Train-induced vibration in buildings over the throat area was expected to be higher than in buildings over the parking and maintenance areas [19]. Figure 3b shows the accelerometer measurement locations, including those mounted on customized steel brackets on the ground and attached to platform columns.

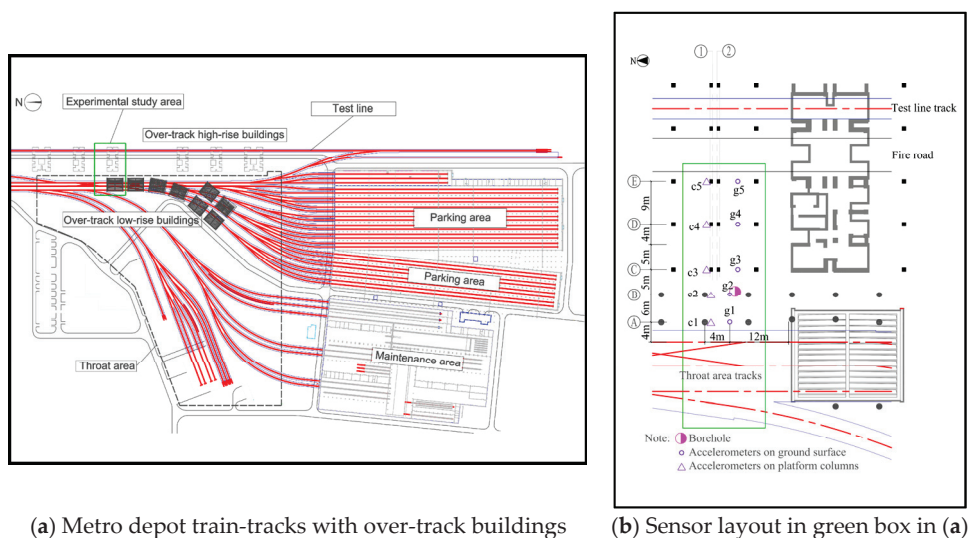


Figure 3. Plan view of the metro depot and measurement layout.

3.2. Soil Profile and Properties

Properties of the soil layers were determined from borehole measurements in between the tracks, free ground, and depot column locations. The geological structure used in the FEM was a layered half-space, based on the data in Table 1. This table presents the soil layer types, approximate thicknesses, Poisson ratios, densities, and their shear and compressional wave velocities.

Table 1. Soil profile and properties.

Soil Type (Layer Number)	Thickness (m)	Cs (m/s)	Cp (m/s)	Poisson's Ratio	Density (kg/m ³)
Back fill (1)	10	186	782	0.47	1800
Sandy clay (2)	6	274	909	0.45	1810
Completely weathered granite (3)	16	424	1556	0.46	1880
Moderately weathered granite (4)	∞	1138	2788	0.40	2400

3.3. Train Layouts

Figure 4 shows a schematic of ground surface inputs from a train with six cars. All cars and bogies were numbered as shown in Figure 4a, for a total of 6 cars with two axles each, 12 bogies, and 24 wheelsets. Wheel-rail contact forces are transmitted through the track tie and ballast structure, resulting in input ground-vibration levels under each bogie, which includes two wheelsets. It was assumed that inputs from individual bogies were statistically independent [21]. For railway vibration predictions, assuming that bogie inputs are statistically independent and exhibit comparable excitation levels is an accepted approach for straight, welded tracks without localized defects, such as unwelded joints or switches. Figure 4b shows the plan view of the train inputs at the point of closest approach with the greatest ground vibration levels during a pass-by. The distance between two wheelsets within the same bogie is 2.5 m. The distance between two bogies within a car is 15.7 m, while the distance between two bogies of two adjacent cars is 6.4 m.

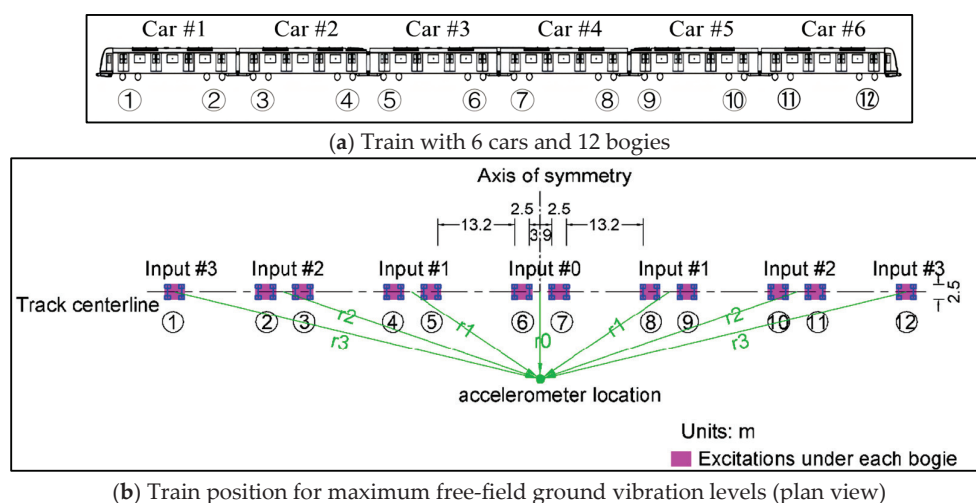


Figure 4. Ground inputs at the bogies of a 6-car train and ground measurement locations.

3.4. Vibration Spectral Analysis

Ground vibration and ambient signals were measured at g1 through g5 during five pass-bys of the train on the nearest track. Figure 5 shows typical train-induced acceleration time histories at ground surfaces g1 and g4. Ground locations g1 through g5 were at 4, 10, 15, 24, and 33 m from the tracks, respectively. If either the ground measurement or the column location is close to the tracks in the near-field of ground vibration, it can potentially result in significant differences between estimates of column vibration and actual measurements. As a result, vibration measurements at g1 and g2 were not used. Only measurement locations g3, g4, and g5 were used to estimate column vibration levels at c3, c4, and c5. The distances from the track to the ground measurement and the column locations were reasonably large, likely beyond the near-field.

Figure 6 show train-induced and ambient vibration spectra along the ground surface from g1 to g5 and along the base of depot columns from c1 to c5, respectively. The measured train-induced vibration levels were significantly greater than ambient levels. Frequency ranges up to 80 Hz are shown in these figures, consistent with the FTA guideline [8].

For the measurements at the ground and column base locations, acceleration levels decreased with increasing distance from the track. Figure 6a shows that measured train-induced vibration levels for multiple pass-bys were closely spaced and repeatable. Ground vibration levels increased up to 40 Hz and then decreased. Figure 6b shows similar trends except that column vibration levels were significantly less than ground levels above 40 Hz, especially for columns c1 and c2, which are closer to the track than other columns.

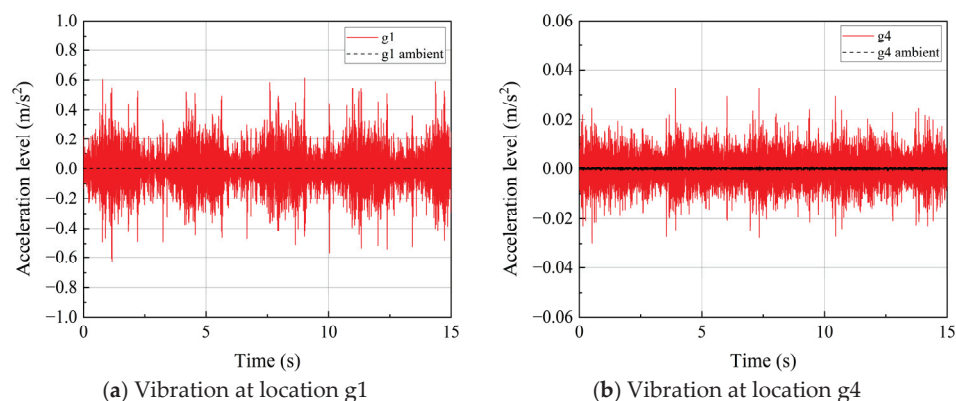


Figure 5. Measured free-field ground vibration during a train pass-by.

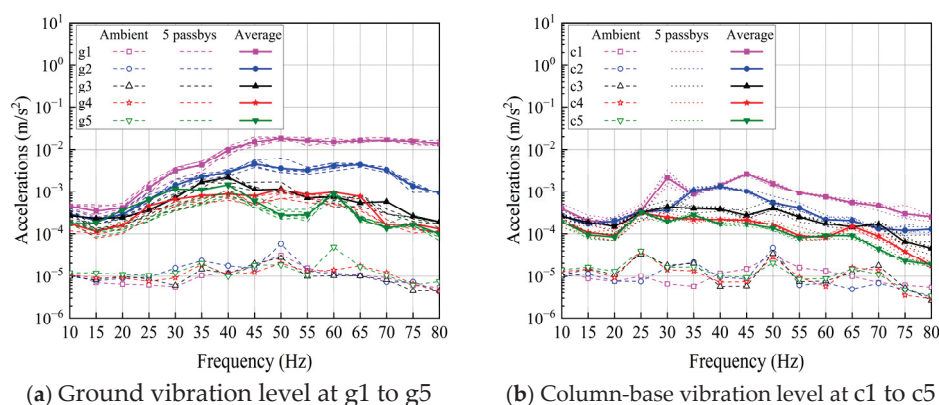


Figure 6. Measured train-induced acceleration levels (RMS).

4. Back-Calculation of Train-Induced Ground Vibration Input at Tracks

The FEM shown in Figure 7 was used to calculate transfer functions from bogie-induced ground input vibration to ground locations at various distances as shown in Figure 4. The axisymmetric model accounts for different properties of the soil layers, as given in Table 1. The bogie, centered at the model's origin, and ground locations at various distances are shown on the ground surface. ABAQUS version 2017 software was used to develop all FE models in this study. A 1% damping ratio [22] was used for saturated soil layers at the depot site, including parts of the top layer, due to its high water table.

To preserve axisymmetric conditions, rollers against horizontal motion were applied at all nodes on the left side of the axisymmetric boundary. The ground input under a bogie was modeled as uniform pressure acting over a circular region of radius 1.41 m at the surface. The actual vibration input into the ground comes from two wheelsets on the track's two rails. At frequencies of interest, differences in the ground vibration induced by four spatially separate wheels in a bogie, in comparison with a uniform input induced by a disc, would be limited to the ground near the bogie. Input to the FE model represents the average total force departed into the ground from the four wheels, whose extent was comparable to, and slightly larger than, that of the bogie. The physical basis for this simplification lies in the load transfer mechanism within the track structure. The bogie vibration is transmitted through the wheels to the rail as discrete point loads. However, the rail, sleepers, and ballast collectively redistribute these point loads into a more continuously distributed pressure acting on the rail subgrade. Therefore, representing the bogie input as a distributed pressure load over an area comparable to the bogie dimensions is a reasonable approximation [18,23] that captures the overall load distribution while enabling an efficient axisymmetric finite element formulation. Infinite elements [24] were used at the outer

radius and on the bottom surface of the model to absorb waves radiating from the input and minimize reflections at the boundaries.

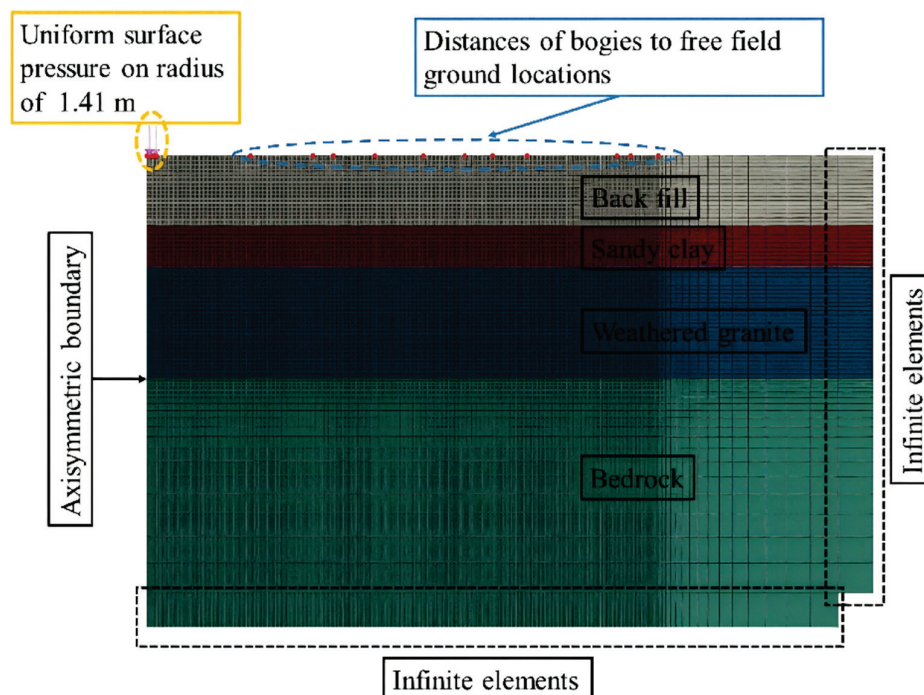


Figure 7. Axisymmetric FEM of vibration transmission from bogies to ground locations.

Table 2 lists the distances between the ground measurement locations and the different bogies on the train when the train is centrally positioned relative to the ground locations. As shown in Figure 4b, when the middle of the train was in line with the ground locations, the distances from bogies symmetrically positioned on either side of the middle of the train to the ground locations were equal. They were also at minimum distances from the ground location to maximize transfer function values consistent with the maximum ground vibration levels during the train pass-bys.

Table 2. Distances from ground measurement locations to each bogie (m).

Ground Measurement Locations	b0	b1	b2	b3
Bogies numbered in Figure 4b	6, 7	4, 5 & 8, 9	2, 3 & 10, 11	1, 12
g3	15	27	46	68
g4	24	33	50	70
g5	33	40	55	74

The FEM is 105 m in the radial direction and extends 68 m deep into the soil layers to bedrock. According to Table 2, the farthest ground location g5 was 33 m away from bogies 6 and 7 (b0) and 74 m away from bogies 1 and 12 (b3). According to Table 1, the shortest shear wavelength at 10 Hz was 18.6 m in the top fill layer. The transfer function for bogies 1 and 12 to ground location g5 was at a distance of 31 m away from the outer radial boundary of the model, which was around 1.7 times the shear wavelength in the top layer at 10 Hz and greater at higher frequencies. The FE model size used was sufficient to avoid adverse effects of reflections at its outer radial boundary [25]. According to Table 1, the shortest wavelength in the top layer at 80 Hz was 2.3 m. To capture the highest-frequency wavelength of interest (80 Hz), fundamental finite-element wave-propagation principles require at least 1/6 to 1/2 of a wavelength per element [25]. Thus, the smallest element size

of 0.2 m in the region of primary interest strictly satisfies these established convergence requirements, ensuring that no significant calculation errors occur in the vibration transfer functions. The model consisted of 31,622 elements.

4.1. Transfer Functions from Bogie Inputs to Ground Locations

In this research, we have used acceleration measurements at g3, g4, and g5. The transfer function to motion of the ground due to input from one bogie is defined in the frequency domain as

$$TF_g(r_{ij}) = \frac{A_{gi}}{A_{bj}} \quad (i = 3, 4, 5; j = 0, 1, 2, 3) \quad (1)$$

The index $i = 3, 4, 5$ identifies the ground location, and $j = 0, 1, 2, 3$ identify the bogie location, as shown in Figure 4b. The numerator is the vibration level at ground location “gi”, and the denominator is the ground vibration input from bogie “bj”. The transfer function is a function of “ r_{ij} ”, i.e., the distance between the ground location “gi” and the bogie location “bj”.

The transfer function as defined in Equation (1) accounts for vibration input from only one bogie to one ground location, while the transfer function in Equation (2) below sums contributions to the ground location from all bogies on the train, assuming they are statistically independent with the same input level, A_{bogie} , and is referred to as the train transfer function ($TF_{gi/train}$) to a ground location.

$$TF_{gi/train} = \frac{A_{gi}}{A_{bogie}} = \sqrt{2[TF_g^2(r_{i0}) + 2TF_g^2(r_{i1}) + 2TF_g^2(r_{i2}) + TF_g^2(r_{i3})]} \quad (2)$$

Figure 8 compares train transfer functions to ground locations g3 to g5 based on Equation (2), using individual bogie transfer functions obtained from the axisymmetric FEM, as shown in Figure 7.

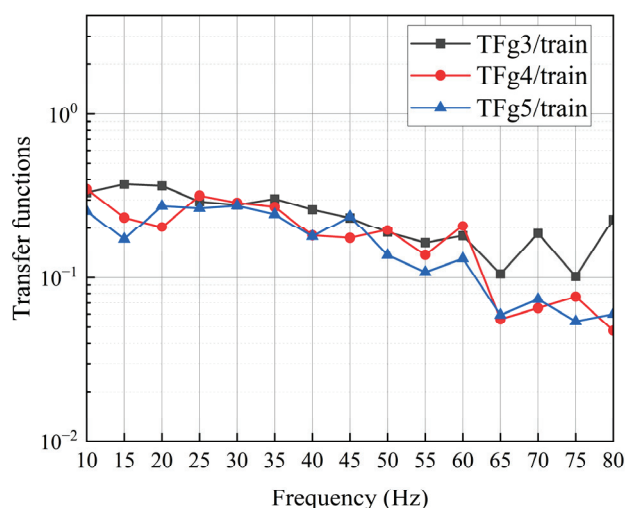


Figure 8. FEM predicted train transfer functions to ground locations from all bogies.

4.2. Back-Calculation of Ground Vibration Input Due to a Bogie

Acceleration spectra on the ground at a bogie A_{bogie} were back-calculated based on each vibration spectrum at ground locations g3, g4, and g5, shown in Figure 6a, using A_{gi} in Equation (2). Train speeds during testing were confined to relatively low speeds of 5–15 km/h. All trains were of the same type with identical marshaling configurations. Individual calculations of bogie vibration inputs based on measured ground vibration levels at locations g3, g4, and g5 were averaged to obtain the bogie’s ground input vibration

level, as shown in Figure 9. Averaging was done to account for possible inhomogeneities in soil properties.

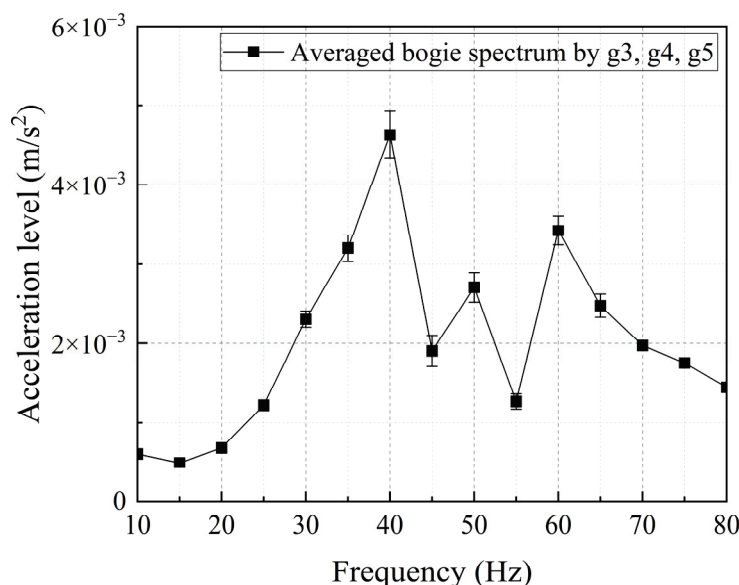


Figure 9. Average of back-calculated ground vibration spectral input at a bogie.

Based on Equation (2), the average back-calculated bogie input vibration level is expressed as

$$A_{bogie} = \frac{1}{3} \sum_{i=3}^5 \frac{A_{gi}}{TF_{gi/train}} \quad (3)$$

Following the first-order Taylor expansion-based error propagation formula, the variance of A_{bogie} is given by

$$\sigma_{A_{bogie}}^2 = \sum_{m=1}^3 \sum_{n=1}^3 \frac{\partial A_{bogie}}{\partial A_{gm}} \frac{\partial A_{bogie}}{\partial A_{gn}} Cov(A_{gm}, A_{gn}) \quad (4)$$

Assuming that the measured surface vibration levels A_{g3} to A_{g5} are independent, the uncertainty in A_{bogie} becomes

$$\sigma_{A_{bogie}} = \frac{1}{3} \sqrt{(\sigma_{A_{g3}}^2 + \sigma_{A_{g4}}^2 + \sigma_{A_{g5}}^2)} \quad (5)$$

Conversely, assuming perfect positive correlation among A_{g3} to A_{g5} yields the upper bound of the uncertainty:

$$\sigma_{A_{bogie}} = \frac{1}{3} (\sigma_{A_{g3}} + \sigma_{A_{g4}} + \sigma_{A_{g5}}) \quad (6)$$

The standard deviations $\sigma_{A_{gi}}$ are derived from the measurements, as illustrated in Figure 6a. This upper-bound case is presented as the vertical error bars in Figure 9. The variability among the measurements at points g3 to g5 is relatively small, resulting in a correspondingly limited variation in the predicted values for A_{bogie} . In practice, when additional uncertainties from the transfer functions are considered, such as those arising from their source using finite element modeling, the overall variability in A_{bogie} is expected to be larger than the estimate that assumes no uncertainty in the transfer functions.

5. Prediction of Free Vibration Levels at a Pile Head

This section presents predictions of train-induced vertical vibration levels at the *free* pile head, with no depot or building structures supported by the pile. An axisymmetric FE

model cannot be used in this section due to the presence of the pile at a distance from a bogie. Transfer functions of pile head vibration divided by the bogie input vibration still depend on the distance from a bogie to the pile.

The pile affects the vertical motion of the ground, where axial vibration of the pile is induced primarily by vertical shear stresses along the interface between the ground and pile as a function of depth, assuming no slippage due to the low soil stiffness compared to the large axial stiffness of the pile. Bending vibration within the pile is also induced by interface stresses involving horizontal shear and normal stresses perpendicular to the pile surface.

The FE model that estimates the transfer functions from bogie to vertical motion at the pile head uses symmetry about a vertical plane that contains both the bogie and the pile head. The symmetry boundary condition was applied by constraining the displacement component normal to the symmetry plane, while the in-plane displacement components were allowed. The use of the symmetry plane reduces the extent of the model because only one side of the plane is modeled. Figure 10 shows the symmetric 3D FE model for pile location c3, with input applied to bogies #6 and 7 (Figure 4b), which are 15 m apart. The same distances in Table 2 apply to both the pile and ground models, as the train was positioned at the minimum-distance location along the track, resulting in maximum vibration level at the pile head during a pass-by.

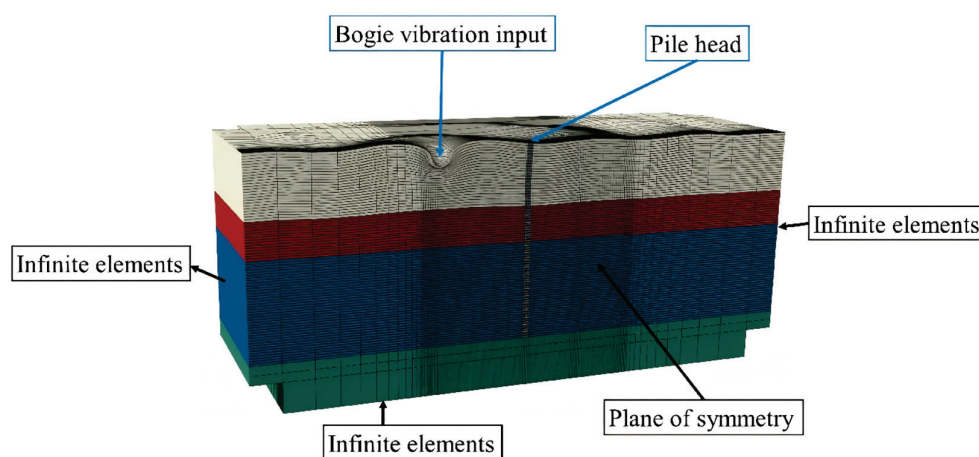


Figure 10. 3D FE model for predicting transfer functions for input at a bogie to vertical motion at a *free* pile head.

The ground input at a bogie, shown in Figure 10, was a uniform pressure on a 1.41 m radius disc. The signal was a sine wave at frequencies of interest from 10 to 80 Hz in 5 Hz increments, as shown in the figures. The plot also shows vertical variation along the pile and in the ground. The surface motion was greatest near the bogie input. The motion decreased significantly with distance from the bogie and with depth into the soil layers. The model was 134 m long along the track, 32 m wide perpendicular to the track, and vertically 40 m long extending into the bedrock. The smallest element size was also 0.2 m to capture ground-borne vibration up to 80 Hz, with a total of 386,854 elements.

A 2% damping ratio was used for the concrete pile. The response of a pile embedded in the ground is much lower than the response level associated with seismic loading. It remained well below the range corresponding to 50% of the concrete capacity. Therefore, the use of a 2% damping ratio for the concrete pile is considered reasonable, in accordance with the value suggested for prestressed concrete structures in Ref. [22].

The bedrock supports the bottom of the concrete pile. The radius and length of the pile are 0.5 m and 32 m, respectively. The mass density of the concrete pile is 2400 kg/m³, Young's modulus is 30 GPa, and Poisson's ratio is 0.2. Soil properties are listed in Table 1.

The transfer function of *free* pile head motion from a ground input at a single bogie is defined in the frequency domain as

$$TF_{ph}(r_{ij}) = \frac{A_{phi}}{A_{bj}} \quad (i = 3, 4, 5; j = 0, 1, 2, 3) \quad (7)$$

Equation (7) is similar to Equation (1). At the same time, the numerator is the averaged acceleration at the *free* pile head location i with subscript "*phi*", and the denominator is the averaged ground acceleration input under bogie "*bj*". Similar to Equation (2), the transfer function of *free* pile head vibration due to the train excitation from all 12 bogies was defined based on an energy summation as

$$TF_{phi/train} = \frac{A_{phi}}{A_{bogie}} = \sqrt{2[TF_{ph}^2(r_{i0}) + 2TF_{ph}^2(r_{i1}) + 2TF_{ph}^2(r_{i2}) + TF_{ph}^2(r_{i3})]} \quad (8)$$

As indicated in Equation (8), the acceleration level at the pile head A_{ph} for a train pass-by can be predicted with $TF_{phi/train}$ and A_{bogie} , where A_{bogie} is the average ground vibration input under a bogie from Section 4. Figure 11 shows plots of transfer functions $TF_{ph3/train}$ to $TF_{ph5/train}$. Subscripts "ph3", "ph4", and "ph5" represent the bogie-to-pile distances of 15 m, 24 m, and 33 m, respectively.

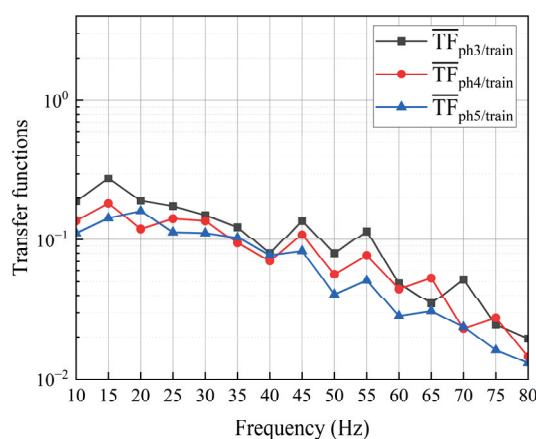


Figure 11. FEM predicted train transfer functions to the pile head.

FEM predicted train transfer functions to the pile head, including contributions from all bogies, decreased more rapidly at higher frequencies than for transfer functions to the ground surface, as shown in Figure 8. The faster attenuation of the pile head vibration transfer function at high frequencies is due to the coupling between the soil and pile. When high-frequency (short-wavelength) stress waves in the soil propagate into an axially stiff (longer wavelength) concrete pile, a severe impedance mismatch occurs at the soil-pile interface. This results in significant scattering and reflection of the incident ground wave, thereby reducing transmission of the ground wave into the pile.

The frequency-dependent transmission behavior is influenced by the types of waves present in the soil. Body waves (P-waves and S-waves) and surface waves (Rayleigh waves) dominate the free-field ground vibration at different depths and distances from the source. When incident waves encounter the pile, the conversion and scattering phenomena depend on the wavelength relative to the pile diameter. At high frequencies, the ground

propagation wavelengths become comparable to or smaller than the pile diameter, where complex wave scattering further reduces the energy transmitted into the pile [26].

The total vibrational energy at the pile head contributed by bogie locations 6 and 7, 4 and 5, and 8 and 9 on both sides accounted for 93.8% of the total vibrational response due to the six-car train, evaluated at the lowest frequency of interest in this study, i.e., 10 Hz. At higher frequencies, the contribution from the middle bogies would be even greater, as the shorter propagation distances further diminish the relative contributions from the end bogies. It indicates that the response from bogie inputs farther along the train could have been omitted with negligible loss of accuracy. This simplification was adopted as a balance strategy between computational cost and prediction accuracy, reducing the computational effort of the finite element model without substantially compromising prediction accuracy.

6. Coupling to Depot Column at Pile Head

The prediction of train-induced vibration levels within over-track buildings above metro depots is based on vibration inputs at the pile heads with the depot structures in place. The free motion of a pile head will potentially be reduced by the impedance of the depot structures supported by the pile. Figure 12 illustrates the prediction of pile head motion with the depot structures in place, V_p , based on the free pile motion, V_{ph} .

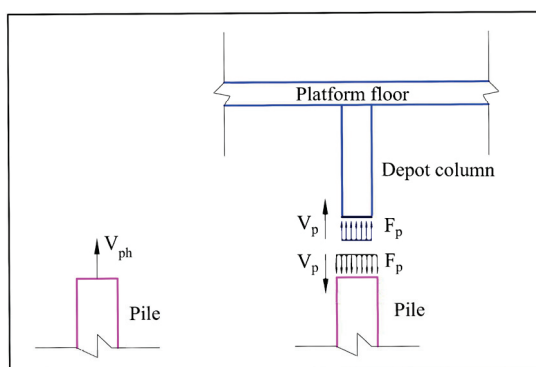


Figure 12. Prediction of pile head velocity without and with depot structure.

The amplitudes of vibration velocity and force at the train inputs, subscript *t*, and the pile head, subscript *p*, are related by the system impedance matrix, $[Z]$:

$$\begin{bmatrix} F_t \\ F_p \end{bmatrix} = \begin{bmatrix} Z_{tt} & Z_{tp} \\ Z_{pt} & Z_{pp} \end{bmatrix} \begin{bmatrix} V_t \\ V_p \end{bmatrix} = [Z] \begin{bmatrix} V_t \\ V_p \end{bmatrix} \quad (9)$$

Equation (9) is used for cases with no depot column to obtain the *free* pile head velocity V_{ph} with $F_p = 0$. V_p is the pile head velocity for the case with a nonzero force F_p at the pile with the depot column in place. The ratio $\frac{V_p}{V_{ph}}$ is equal to the ratio of the corresponding accelerations $\frac{A_p}{A_{ph}}$, as presented in Equation (10):

$$\frac{V_p}{V_{ph}} = \frac{A_p}{A_{ph}} = \frac{1}{1 + \frac{Z_c}{Z_p}} \quad (10)$$

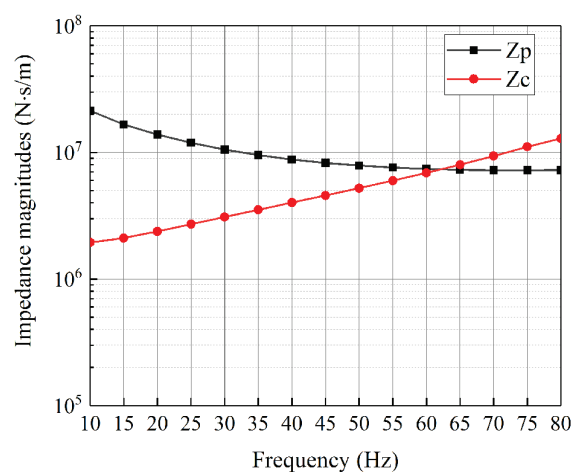
In Equation (10), Z_c is the impedance of the depot structures at the pile head, and Z_p is the finite element evaluation of the impedance of the pile in the soil. A_{ph} is the *free* pile head acceleration level due to train inputs without the depot structures, given in Section 5, and A_p is the acceleration level at the pile head with the depot structures in place. This study focuses on vertical train-induced vibrations, which are dominant in metro depot environments. Accordingly, Equation (6) considers only axial impedance

coupling at the pile-column joint, as vertical vibration energy primarily transmits through axial waves. Bending and torsional impedances are excluded because the excitation is predominantly vertical under low-speed, straight-track conditions, and axial impedance of the columns is significantly larger than bending impedance, rendering any bending or torsional contributions negligible [15,16]. While this simplification is appropriate for the present case, it is acknowledged that bending and torsional effects may become relevant under horizontal excitation or flexible connection scenarios.

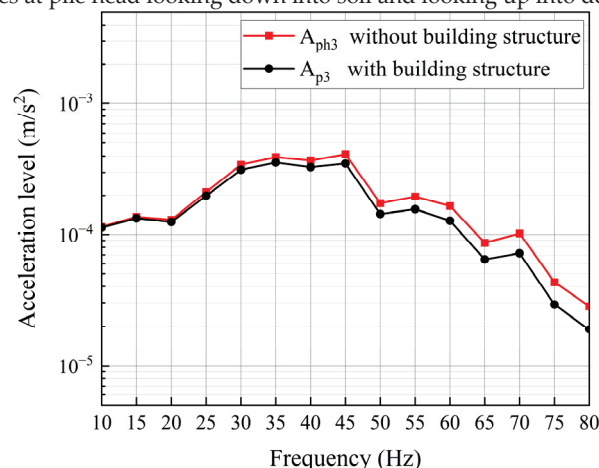
6.1. Predictions of Pile Head Impedance and Train-Induced Depot Column Vibration Levels

An axisymmetric FE model was used to calculate the impedance at the head of a pile buried in soil. The above-ground superstructure considered includes a large-diameter concrete depot column that supports the depot platform floor (and potentially over-track buildings, if present at other locations), as shown in Figure 12. The depot column has a Young's modulus of 32.5 GPa, a density of 2400 kg/m³, and a radius of 0.6 m. The platform floor has a Young's modulus of 31.5 GPa and a density of 2390 kg/m³. The platform floor has a mass-equivalent thickness of 0.788 m and a stiffness-equivalent thickness of 1.381 m [14]. Both the depot column and platform floor have damping loss factors of 4% [27].

Figure 13a compares the FEM calculation of the impedance of the pile head and the impedance model calculation of the impedance of the depot structure, including only a depot column and a platform floor at location c3. There was no over-track building above the platform floor at location c3.



(a) Impedances at pile head looking down into soil and looking up into depot structures



(b) Predicted pile head acceleration level without and with depot structures

Figure 13. Influence of depot structures on pile head acceleration level.

Figure 13a shows that the *free* pile head impedance was significantly larger than the depot structure impedance at lower frequencies up to 50 Hz. At the same time, the impedances were comparable in the range of 50~80 Hz. Equation (10) implies that the building structures did not greatly affect the pile motion below 50 Hz, but possibly affected the *free* pile head motion and transmission into the building in the frequency range above 50 Hz. Figure 13b compares predicted vibration levels of the pile head without and with the depot structures in place at location c3. The depot structures resulted in a negligible reduction in *free* pile head vibration at low frequencies and a limited reduction at higher frequencies, with an approximate 4 dB reduction at 80 Hz.

The impedance of the depot structure was much smaller than the pile impedance below 40~50 Hz, i.e., $Z_C \ll Z_P$ in Figure 13a. In that range, the denominator of Equation (10) was close to 1, and the pile head motion without or with the building structure in place did not differ greatly. When the impedances were comparable (above 40~50 Hz), and the depot column lightly loaded the pile head, the vibration levels were slightly reduced.

6.2. Validation of Predicted Vibration with Measurements at Column Base

The transmission from the pile head to the depot platform slab and building floors was systematically investigated in our previous work [16,28]. Given the complexity of the upstream train-soil-pile-depot column coupling, the present study focuses specifically on this phase. By mathematically linking the proposed method to our previously established depot column-to-superstructure models, the full train-soil-pile-structure vibration transmission pathway is effectively validated. In addition, since the prediction model is formulated in the frequency domain using transfer functions and impedance concepts, validation is appropriately based on frequency-domain characteristics, i.e., the acceleration spectra. Time-domain quantities such as peak values and vibration duration cannot be directly predicted within this framework without additional assumptions, such as phase reconstruction. Predictions of train-induced vibration levels at the bases of depot columns at locations c3, c4, and c5 were compared with measurements, as shown in Figure 14a–c.

The comparison of estimated and measured acceleration levels at the pile head supporting the ground column is shown in Figure 9. Differences between model vibration predictions and field measurements were frequency-dependent and location-dependent, as expected. At higher frequencies, the vibration response is generally more sensitive to local modeling assumptions and measurement noise. Therefore, the observed deviations are likely attributable to a combination of factors, including simplifications in the soil stratification, sensor accuracy and measurement uncertainty, assumed damping ratios in the model, the treatment of soil-pile interaction, and variability in train operating conditions during the measurements. In addition, as shown in Figure 6b, measurements at depot columns c3 to c5 exhibited the maximum signal-to-noise ratio in the frequency range of 30 to 45 Hz, which is also the frequency range with the highest train-induced vibration energy in the far-field, as illustrated in Figure 6a. Correspondingly, prediction accuracy was highest in this frequency range. Since the average field measurements were used to back-calculate train excitation inputs to the subgrade, the discrepancies between predictions and measurements across different depot columns also vary. The maximum absolute errors in the target frequency range at c3, c4, and c5 were 9.3×10^{-5} , 6.8×10^{-5} , and 1.5×10^{-4} , respectively, corresponding to 2.4 dB, 2.3 dB, and 5.3 dB, all occurring at 40 Hz.

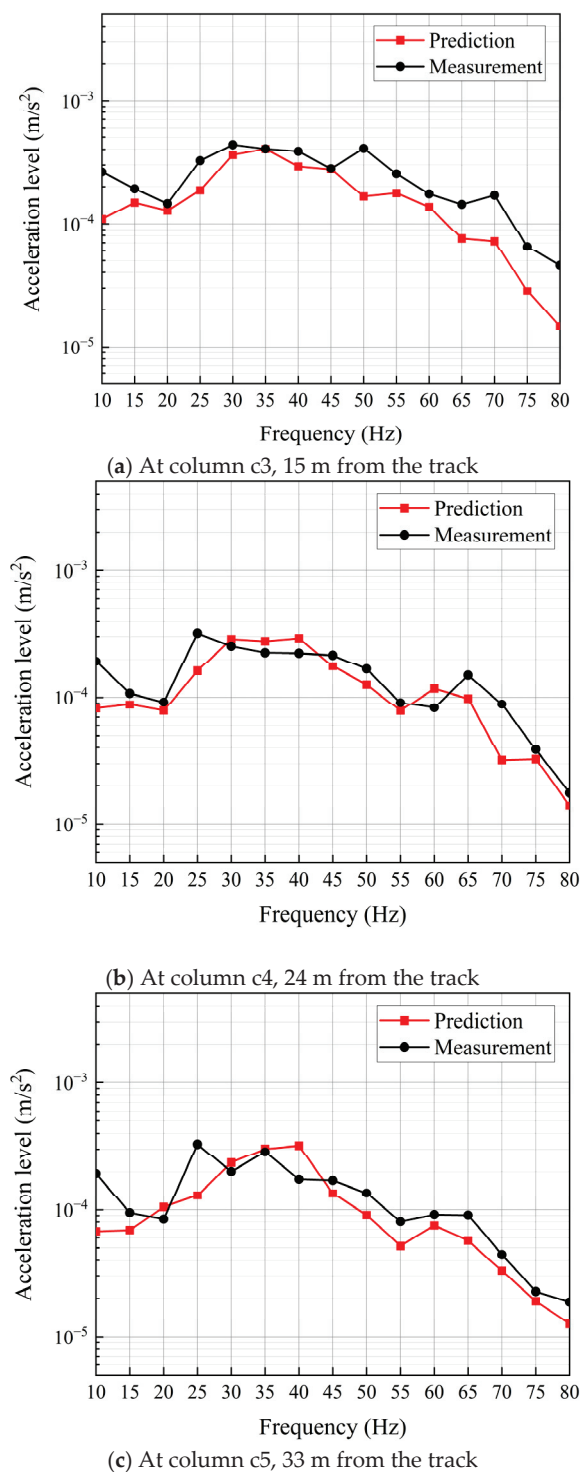


Figure 14. Comparison of predicted with measured pile head acceleration levels.

7. Discussion

The main novelty of the proposed method is predicting train-induced vibration transmission into over-track buildings using free-field measurements, thereby eliminating the need for direct instrumentation at depot columns, a significant practical solution in operational metro depots.

There are other novelties of the proposed method compared with previous studies. Unlike empirical approaches such as the FTA guideline [8], which employs frequency-independent coupling factors, the proposed method accounts for frequency-dependent

soil-pile-structure interactions through detailed finite element and impedance modeling. In contrast to fully analytical models that require extensive parameterization of wheel-rail dynamics and track irregularities [5,12], the proposed method leverages free-field measurements to back-calculate “equivalent” track inputs into the ground, reducing reliance on uncertain input parameters. At the far-field distances evaluated for the field measurements, the input into the ground has a minimal effect on the ratio of FE transfer functions to the free-field and pile locations. It is simply an “equivalent” input for both the simultaneous free-field ground measurement and the ground column measurement above the pile-head. Assuming bogie inputs are equal, they cancel in the ratio of the FE transfer functions, leading to a highly efficient method.

The impedance-based coupling employed in Step 3 builds on prior work by Sanayei et al. [13] and Zou et al. [14,15], which demonstrated the effectiveness of axial impedance models for predicting vibration transmission into a building through columns. The present study extends these concepts to the more complex interactions between train, soil, pile, and depot structures, which have not been systematically addressed in previous hybrid frameworks.

The proposed hybrid method has several key strengths and innovations. First, its modular structure allows individual components (e.g., soil modeling, pile impedance, building impedance) to be refined independently without requiring revalidation of the entire framework. Second, using free-field measurements to back-calculate bogie inputs eliminates the need for trackside instrumentation, which is often not feasible in active depots due to security and operational constraints. Third, the framework’s linear scalability enables applications for different train configurations and speeds. The hybrid method, assuming statistically independent and equal bogie inputs acting on the ground, simplifies the development of the FE models of vibration transmission through the soil.

Unlike earthquake-induced vibrations, train-induced vibrations are classified as “micro-vibrations”, typically well within the linear elastic range. Therefore, using a bonded, non-slip contact model is physically appropriate, mathematically stable, and represents standard practice in environmental vibration predictions. Although this assumption is appropriate for the small-strain vibrations examined, it may lead to an overestimation of transmission in very soft soils or poorly compacted backfill, which was not the case in this study.

The estimated vibration levels at the base of the ground column were in good agreement with measured column vibration levels, particularly in the 30 to 45 Hz frequency range, with important train-induced vibration levels in this case study. The maximum prediction discrepancy in the investigated area and target frequency range was 5.3 dB.

In summary, the hybrid method showed strong potential as a practical tool for vibration screening before the construction of over-track buildings. Future validation under more varied conditions will be beneficial to fully establish the robustness and general applicability of the method.

8. Conclusions

This study developed a hybrid method to predict train-induced vibration levels in over-track buildings at metro depots, based on free-field ground vibration measurements at existing tracks during train pass-bys. This method integrates finite element predictions of vibration transmission through layered soils and pile impedances with analytical impedance models of depot structures. The proposed approach comprises three sequential steps: (a) back-calculation of train-induced ground vibration inputs at the tracks using axisymmetric FE models and free-field measurements, (b) prediction of *free* pile head vibration using 3D FE models that account for soil-pile interaction, and (c) estimation

of pile head vibration with depot structures in place by coupling pile impedances with structural impedances.

The method was applied to the Qianhai metro depot in Shenzhen, China, where free-field ground vibration and depot column base vibration levels were measured during several train pass-bys. The predicted vibration levels at the bases of depot columns showed good agreement with independently measured vibration levels obtained during train pass-bys.

Future work should focus on validating the method across a wide range of metro depots with varying train configurations, operating speeds, axle loads, and soil properties to enhance its general applicability. Systematic parametric sensitivity analyses, including soil shear-wave velocity, pile length and diameter, and structural stiffnesses, are recommended to better understand the train-soil-pile-structure coupling mechanisms and guide the design of vibration mitigation systems.

Author Contributions: Software, formal analysis, and writing—original draft preparation, Z.T.; conceptualization, investigation, and writing—review and editing, J.A.M.; supervision, methodology, finite element modeling guidance, and writing—review and editing, M.S.; validation, investigation, and writing—review and editing, S.B. All authors have read and agreed to the published version of the manuscript.

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Article

Excitation and Transmission of Train-Induced Ground and Building Vibrations—Measurements, Analysis, and Prediction

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Abstract

Measurement results of train-induced vibrations are evaluated for characteristic frequencies, amplitudes and spectra, leading to a prediction which is based on transfer functions of the vehicle–track–soil system, the soil, and the building–soil system. The characteristic frequencies of train-induced vibrations are discussed following the propagation of vibrations from the source to the receiver: out-of-roundness frequencies of the wheels, the sleeper passage frequency, the vehicle–track eigenfrequency, the car-length frequency and multiples, axle-distance frequencies, bridge eigenfrequencies, the building–soil eigenfrequency, and floor eigenfrequencies. Amplitudes and spectra are compared for different train and track types, for different train speeds, and for different soft and stiff soils, where high frequencies are typically found for stiff soil and low frequencies for soft soil. The ground vibration is between the cut-on frequency due to the layering and the cut-off frequency due to the material damping of the soil, but the dominant frequency range also changes with distance from the track. The frequency band of the axle impulses due to the passing static loads obtains a signature from the axle sequence. The high amplitudes between the zeros of the axle-sequence spectrum are measured at the track, the bridge, and also in the ground vibrations, which are even dominant in the far field. A prediction software is presented, which includes all three parts: the excitation by the vehicle–track interaction, the wave transmission through the soil, and the transfer into a building.

Keywords: train-induced vibration; vehicle excitation; track response; bridge resonance; ground vibration; soil–building transfer; floor resonance; axle-sequence spectrum; vehicle–track eigenfrequency; axle impulses

1. Introduction

Trains pass over tracks, and the irregularities of the wheel and the track yield dynamic loads on the track and the soil. Waves are generated and propagate through the soil. The ground vibration excites buildings in the vicinity and can disturb people who live or work inside them. The severity of the problem is increasing as buildings with (even expensive) apartments are built close to railway lines because of limited space in cities and where the vibration would often exceed standards for human comfort. These problems are usually addressed by consultancy firms with engineering experience. Research on railway-induced vibrations has intensified due to high-speed traffic.

The Federal Institute of Material Research and Testing (BAM) has measured vibrations of the vehicle, track, bridge, and soil during several research projects for the German government (for example, [1–4]), the German and Swiss Railways (for example, [5]) and for the European Community [6]. Several measurement campaigns have been performed on a

high-speed line section near Würzburg where the test runs of the ICE were measured to check if special phenomena, for example, a ground vibration boom, could be problematic [1]. A special vibration component has been found for high-speed runs, and the objective of the research project [5] was to look through all BAM measurements to assure the existence of the “axle-distance frequency”. The vibrations of a test train, a track, and soil have been measured simultaneously at a surface, a bridge, and a tunnel section at the same line to find the relation between the vibrations of different railway components and to come to a better physical understanding of the “axle-distance component” and other vibration components [3]. Many building examples with many floors have been measured during consultancy projects. The experimental results and their theoretical understanding have been used to develop a prediction method for the emission, transmission, and immission of train-induced vibrations [4]. The objective was to build a user-friendly prediction software that is simple to handle with only a few input parameters and fast in calculation to give a quick answer.

Several detailed models have been developed to analyse train-induced vibrations and have been compared with measurements at a single site for their verification [7–16], where some useful experimental results about the propagation of train-induced ground vibrations can be found. Larger measurement campaigns at several sites are reported in [17–20]. A special mid-frequency ground vibration component appeared in several measurements of high-speed trains [6,19,21–23], whereas a low-frequency Rayleigh-train effect has been specifically measured at a very soft soil in Sweden [24]. The prediction of train-induced vibrations by simplified models and databases has been addressed in [4,23,25]. The prediction of train-induced building vibration can be found mainly in singular cases [13,26–28] based on a few measurements [29] or for more or less simplified models [30,31]. The deficiencies in the existing literature are that measurement campaigns are restricted to one part of the problem and system, the ground or the building, the soil or the vehicle, and that the prediction is performed by time-consuming detailed calculations or by non-physical methods such as hybrid methods or artificial intelligence [32–35] without understanding the rules.

The present article, which is an extended version of the conference contribution [36], wants to close this gap. It analyses many vibration measurements at different soils and buildings. The structure is as follows: The first part presents the observed frequencies at the vehicle, track, bridge, soil, and building. In the second part, the amplitudes and spectra of the ground vibrations are analysed for the influence of different tracks, trains, and train speeds, and some results for buildings near metro lines are also shown. The third part demonstrates how the prediction of train-induced ground and building vibrations is derived with simplified models and transfer functions. Some formulas and details of the prediction are given in the Appendix B. Numerical analyses behind the prediction are given as references. The originality of the article is the use of many measurements to achieve an understanding and a prediction for all three parts, emission, transmission, and immission of train-induced vibrations, the detection of the scattered axle impulse component yet not known in the railway community, and the concept of a user-friendly and accurate prediction method.

2. Observation of Characteristic Frequencies

2.1. Frequencies of the Vehicle–Track Interaction

The excitation by the vehicle can be found in the axle box accelerations which are shown in Figure 1 as a spectrogram for a 3 km-long line section near Würzburg with a slab track of up to 45 s and a ballast track later on. The horizontal lines are constant frequencies for the constant train speed of 160 km/h. These lines are in a regular distance at $f \approx 15$,

30, 45, 60, 75, and 90 Hz. Below the first out-of-roundness, there are some track alignment irregularities mainly for the ballast track.

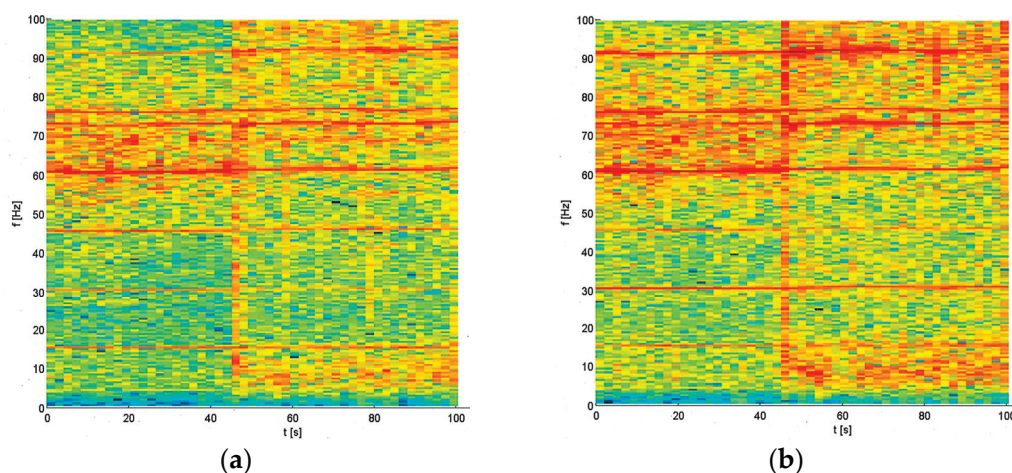


Figure 1. Spectrogram of the axle box accelerations of two consecutive wheelsets (a,b) during a test run with 160 km/h over a slab track (before 45 s) and a ballast track (after 45 s).

A second track component can be clearly found below the fifth out-of-roundness for both track types, the sleeper-distance excitation due to the different track stiffness on and between the sleepers. From the measurements with different train speeds, it is observed that all these excitation frequencies are strictly speed dependent, they increase proportionally with train speed. Another vibration component can be found between 60 and 75 Hz for the slab track and between 75 and 90 Hz for the ballast track. This speed-independent frequency range of amplified amplitudes is due to the resonance of the wheelset on the compliant track, the vehicle–track or so-called P2 resonance. Frequencies above this resonance are reduced, as can be seen for the slab track.

The vibration response of the track is dominated by the impulses of the passing static axle loads which is shown in Figure 2 for a slab and a ballast track. The axle impulses are strongest for the rail, weaker for the sleeper, and much weaker for the track slab. The impulses on the soil are much smoother for the slab track with lower amplitudes and with longer impulse times. For each train, a typical axle-sequence spectrum with characteristic peaks and zeros can be observed in the track measurements (Figure 3). The car length together with the train speed results in a car-length frequency and the multiples give a regular series of peaks in the spectrum, at 3, 6... Hz for the longer ICE3 car and at 4, 8... Hz for the shorter Thalys car. The 3rd, 7th, and 10th peaks are higher due to the bogie distance of the ICE3, and the Thalys has wider bands due to different cars. The distance of the axles in the bogie of 2.5 m (ICE3) or 3 m (Thalys) yields two characteristic zeros and in between a characteristic region of higher amplitudes around 28 and 23 Hz, respectively. These axle-sequence frequencies are speed-dependent and lower for normal passenger trains. The frequency range around the axle distance in a bogie is the important part for the ground vibrations and it is typical for all conventional and articulated trains.

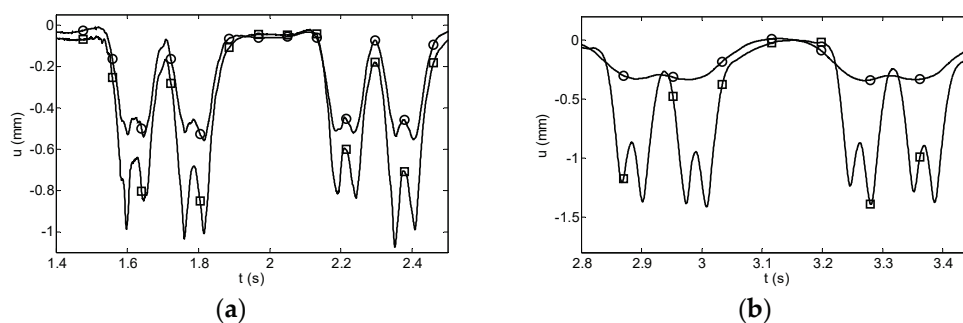


Figure 2. Displacements of (a) a ballast and (b) a slab track during the passage of four bogies, $\square$ rail, and $\circ$ sleeper.

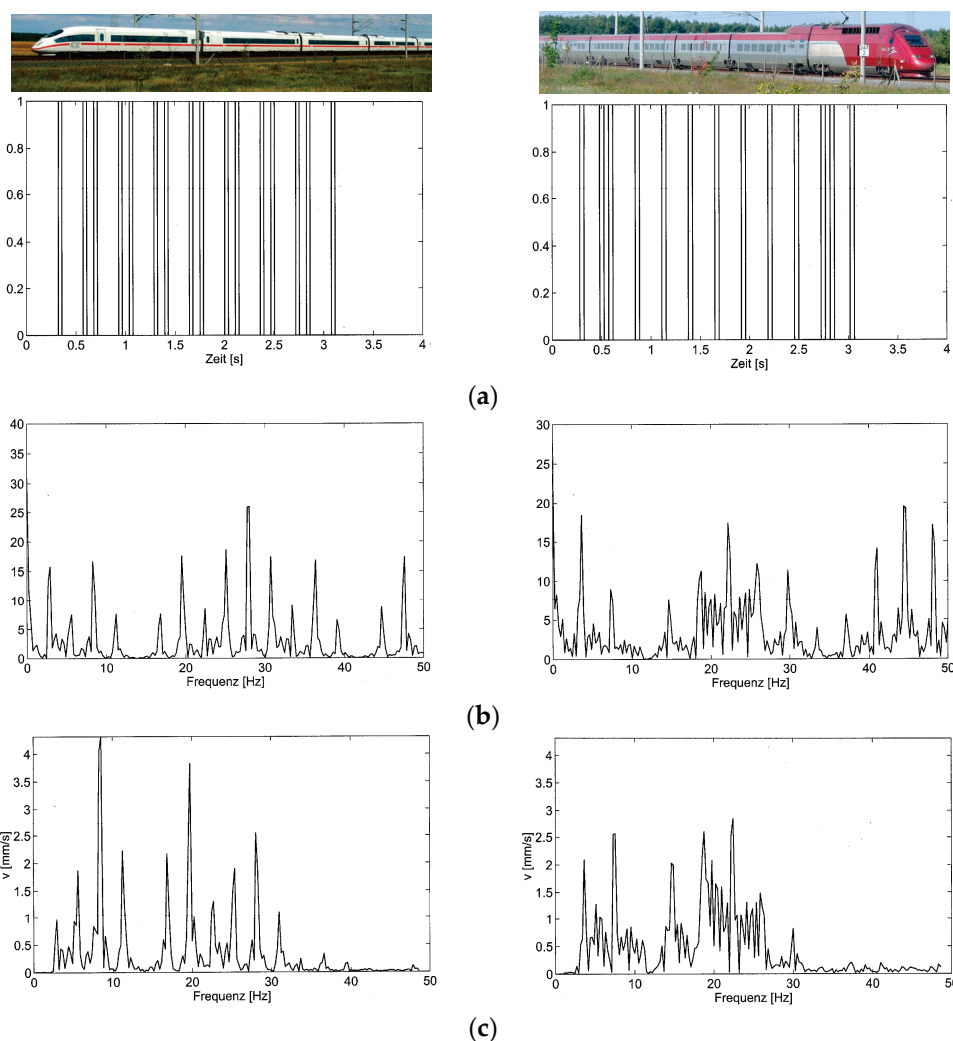


Figure 3. Axle sequence of the ICE3 and the Thalys train in (a) time (b) and frequency, and (c) spectra of the measured sleeper velocities for 250 km/h.

2.2. Bridge Frequencies

Bridges show clear eigenfrequencies in the low frequency range. The 45 m-long concrete box girder bridge has its first bending frequencies at 3.5 and 11 Hz and its first torsional eigenfrequencies at 7 and 13 Hz. When the test train with locomotive, five passenger cars and another locomotive passes over this bridge with different speeds, amplifications and cancellations of the different eigenfrequencies were measured (Figure 4). For the train speed of 100 km/h, the first torsion mode lies in the maximum region of the axle-sequence spectrum, and the second bending mode is in the second minimum. For the train speed of

160 km/h, the first torsion mode is at the first minimum, and the second bending mode lies in the maximum region. In general, the bridge response to the passing static loads is determined by three spectra: the axle-sequence spectrum, the modal force or mode shape spectrum, and the transfer function of the bridge [37]. The axle-sequence and the modal force spectra are speed-dependent, and the peaks of the car-length frequencies can fall into a high- or low-frequency region of the mode shape spectrum, yielding amplification or cancellation. These effects are typical for high-speed trains on short bridges [37], whereas the present example shows the strong influence of the axle distance part of the axle-sequence spectrum.

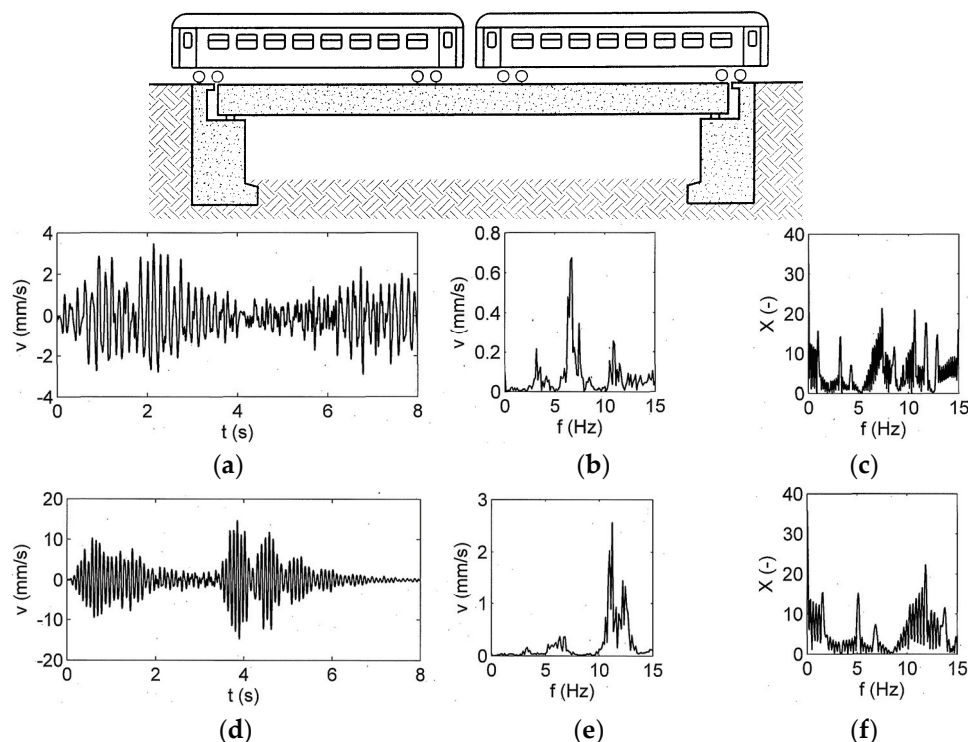


Figure 4. Passage of the test train (lok, 5 cars, lok) over a 45 m-long concrete box girder bridge with (a–c) 100 km/h, (d–f) 160 km/h, (a,d) time histories, (b,e) spectra, and (c,f) axle-sequence spectra.

2.3. Soil Frequencies

The ground vibrations from a surface line are characterised by the cut-on frequency of the layering and the cut-off frequency of the material damping of the soil. The main frequency content of the ground vibration lies between these frequencies. It is at high frequencies for a stiff soil, whereas it is at low frequencies for soft soils. These characteristics can be found in the transfer functions measured by hammer impacts and in the response to the train excitation (Figure 5). The train-induced ground vibration shows two increased frequency regions. At three thirds around 12 Hz, the axle impulses from the passing static loads are dominant, and they show the characteristic minima of the axle sequence at 8 and 20 Hz (Figure 5b for 125 km/h). The sleeper passage component can be found at 64 Hz. Completely different characteristics have been measured for very soft soil (Figure 5c,d). Frequencies below 16 Hz are dominating, and a strong decrease with higher frequencies follows because of the strong effect of the damping. The high amplitudes between the zeros of the axle-sequence spectrum are often measured in the train-induced ground vibrations; see, for example, [19–23].

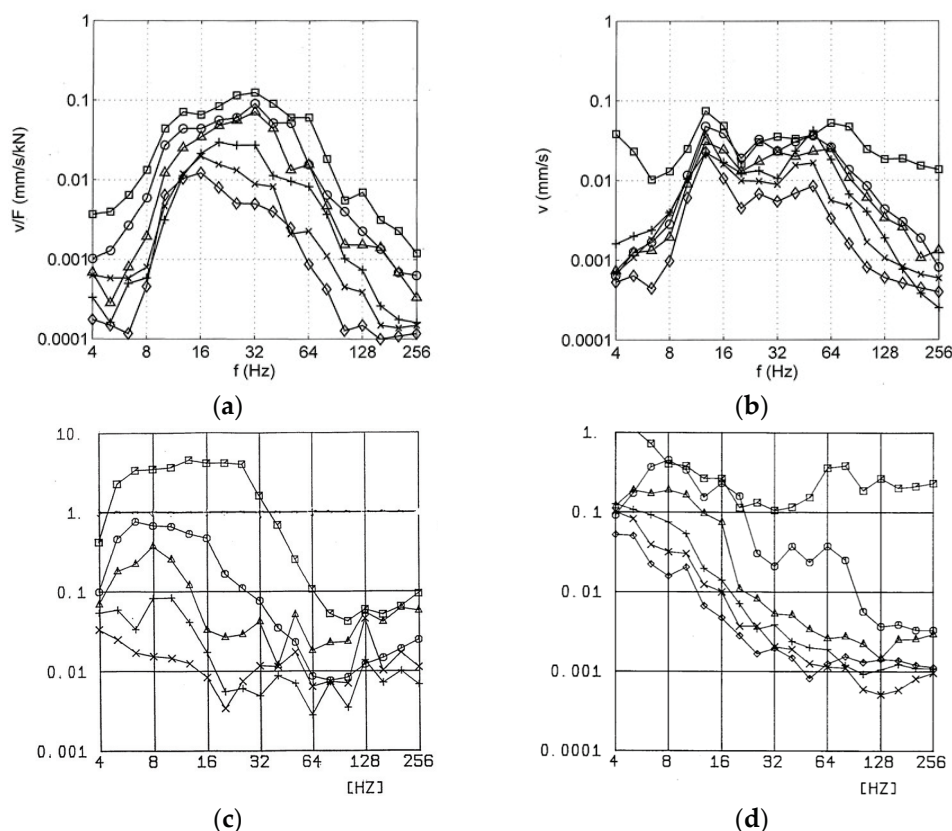


Figure 5. One-third octave spectra of the ground vibrations (a,b) at Wiesenfeld, a stiff soil ($v = 270$ m/s over rock), distance $\square$ 3, $\circ$ 5, $\triangle$ 10, $+$ 20, $\times$ 30, $\diamond$ 50 m, and (c,d) at Prien, a very soft soil ($v = 30$ m/s), distance $\square$ 3, $\circ$ 10, $\triangle$ 20, $+$ 30, $\times$ 50 m ($\times$ 40, $\diamond$ 50 m in (d)) (a,c) hammer impacts, (b,d) train passages.

2.4. Building Frequencies

Buildings near railway lines are excited by the ground vibrations. The transfer functions between building and free field of the soil show a fundamental building–soil eigenfrequency typically below 10 Hz (Figure 6a,b). The disturbance of people is often influenced by the floor eigenfrequencies, which are low for the wooden floors at the first storey (20 Hz in Figure 6c and 12.5 Hz in Figure 6d) and higher for the concrete floors at the ground floor (Figure 6c,d). Higher buildings have a flexible wall or column mode (at 10 Hz in Figure 7a, at 7.5 Hz in Figure 7b,c, at 10 Hz in Figure 7d where the amplitudes increase with the height in the building (Figure 7). In Figure 7b–d, the floor resonance can also be observed at 14, 12.5, and 15–20 Hz.

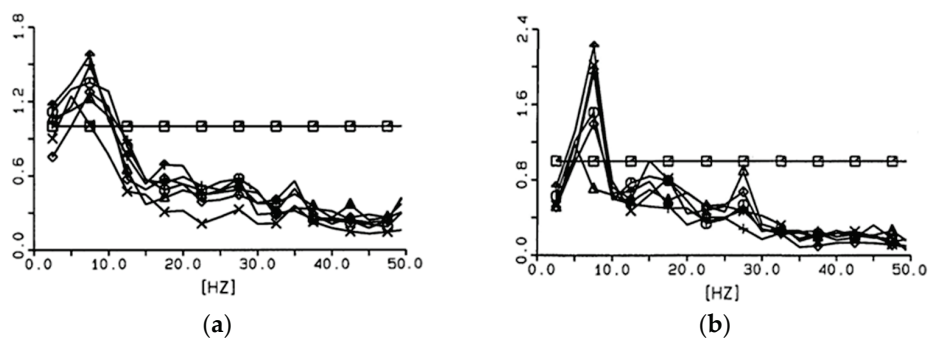


Figure 6. Cont.

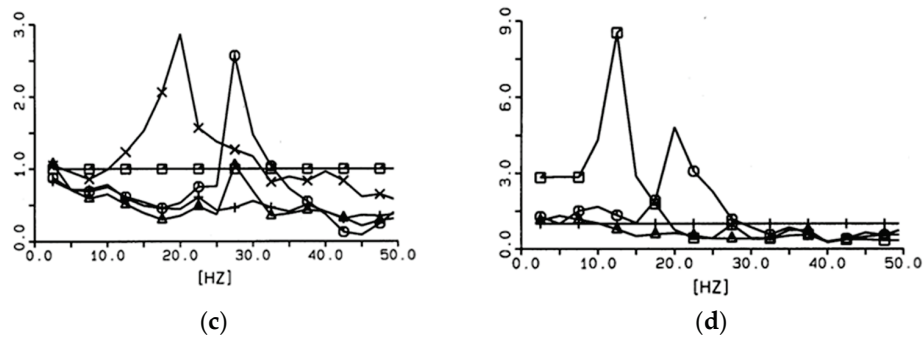


Figure 6. Soil–building transfer function of four small 3-storey residential buildings: (a,b) foundation points, (c,d) wall (+, Δ), wooden ($\times$, $\square$), and concrete ($\circ$) floor points.

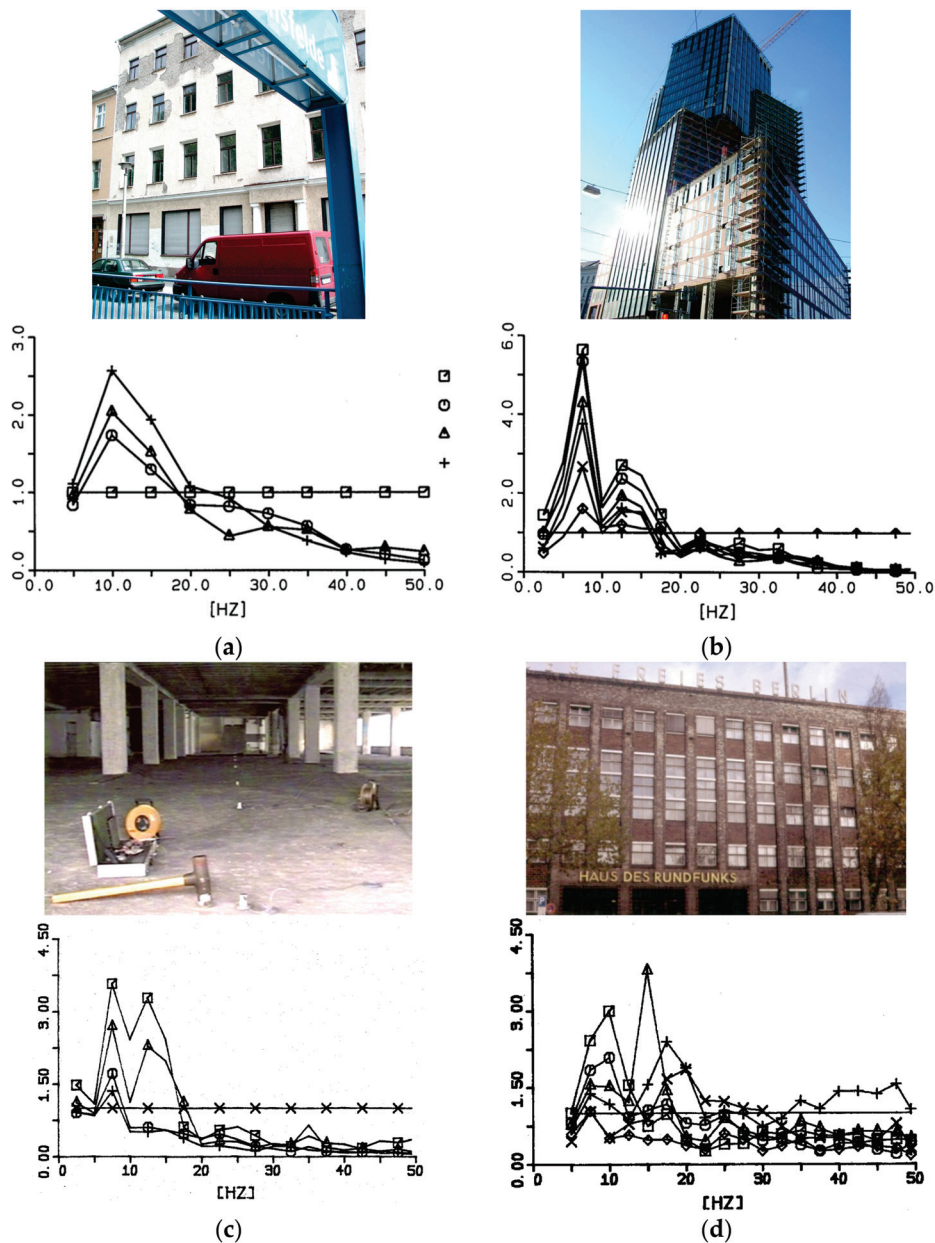


Figure 7. Soil–building transfer function of (a) wall points of $\circ$ 1st, Δ 3rd and + 5th storey in an old 5-storey masonry building, $\square$ soil, (b) floor points of the $\square$ 24th, $\circ$ 21st, Δ 18th, + 15th, $\times$ 12th, $\diamond$ 6th storey in a new 24-storey office tower, (c) first and third floor Δ , $\square$ and column +, $\circ$ of a 3-storey concrete frame office building, $\times$ soil, and (d) $\square$ 5th, $\circ$ 4th, Δ 3rd, + 2nd, $\times$ 1st, $\diamond$ ground floor of the 5-storey steel-frame office building.

3. Analysis of Amplitudes and Spectra

For quantitative results, the measurements are evaluated as broad-band, usually as one-third octave band spectra. In this rather smooth presentation, results for different measurements, different parts, and different distances can be compared. Moreover, the low-frequency part of the axle-sequence spectra up to the second zero are well represented in one-third octave band spectra.

3.1. Vehicle–Track Interaction

Figure 8 shows the axle box accelerations of the simultaneous measurement campaign with different track sections and different train speeds [3]. The axle box accelerations are mainly the result of irregularities of the track and the vehicle. The low-frequency accelerations directly represent the irregularities.

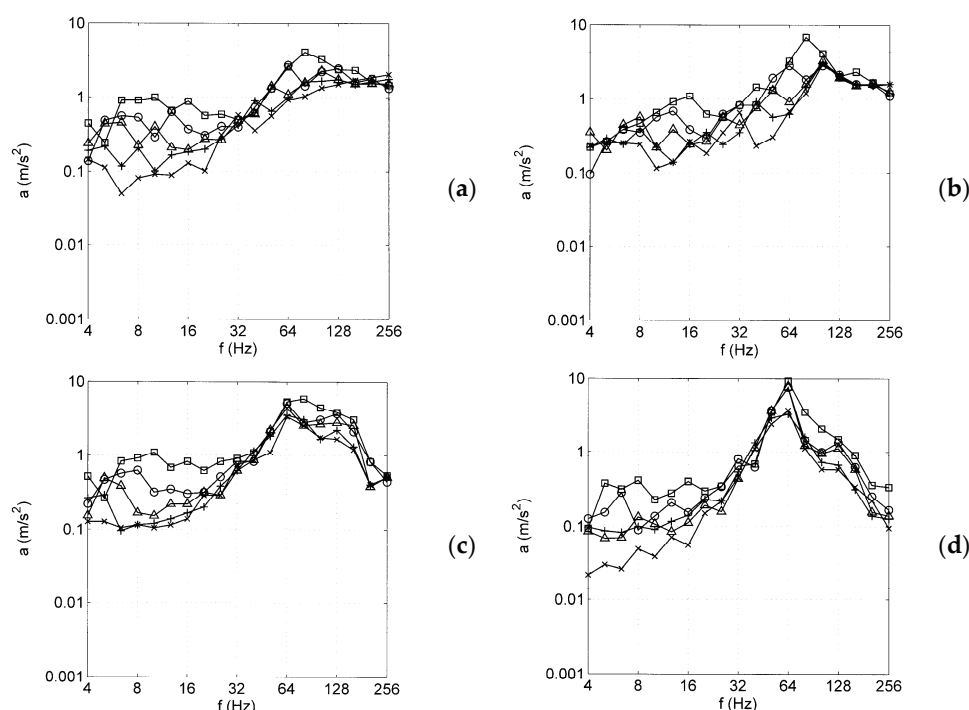


Figure 8. Axle box accelerations for (a–d) different train speeds $\square$ 160, $\circ$ 125, $\triangle$ 100, $+$ 80, $\times$ 63 km/h: (a) wheel 1, surface line; (b) wheel 1, bridge line; (c) wheel 2, surface line; and (d) wheel 2, tunnel line.

Figure 8a shows the results for a ballast track on the soil. The low- and mid-frequency amplitudes increase strongly with train speed. The characteristic frequencies shift with the train speed, e.g., the sleeper passage frequency from 32 to 40, 50, 64 and 80 Hz for speeds from 63 to 160 km/h, or the first out-of-roundness of the wheel with peaks between 8 and 16 Hz. The speed-independent vehicle–track resonance frequency at about 100 Hz is highly damped by the radiation damping of the soil.

The axle box accelerations on a bridge (Figure 8b) show the same features, but the sleeper passage peaks are stronger, and the low-frequency amplitudes (the track irregularities) are lower. Both effects are due to the stiffer support of the ballast track on the bridge.

A slab track (in a tunnel) with soft rail pads (Figure 8d) shows less low-frequency amplitudes (a better track quality), a stronger vehicle–track resonance at 64 Hz independent of the train speed, and a stronger reduction at higher frequencies (less radiation damping) compared to the ballast track (Figure 8a,c).

3.2. Influence of Vehicle, Track, and Speed on the Ground Vibration

Near Dachau, two nearby track sections with a conventional ballasted track and a slab track have been measured for different trains. The soil is rather stiff gravel with shear wave velocities of about 400 m/s. The one-third octave band spectra of the ground vibration at 10 m distance in Figure 9 are small at low frequencies and show an increasing trend up to 64 Hz. The different trains with different speeds show different frequency ranges of higher amplitudes. For frequencies below 10 Hz, the freight train with 70 km/h has the highest amplitudes of all the trains. All the trains show a maximum at the sleeper-distance frequency, which is at 32 Hz for the freight train, at 50 Hz for the urban train with 100 km/h, and at 64 Hz for the intercity train with 130 km/h. The frequency range just below this characteristic frequency has higher amplitudes for the slab track. On the other hand, the ballast track has higher amplitudes between 10 and 25 Hz.

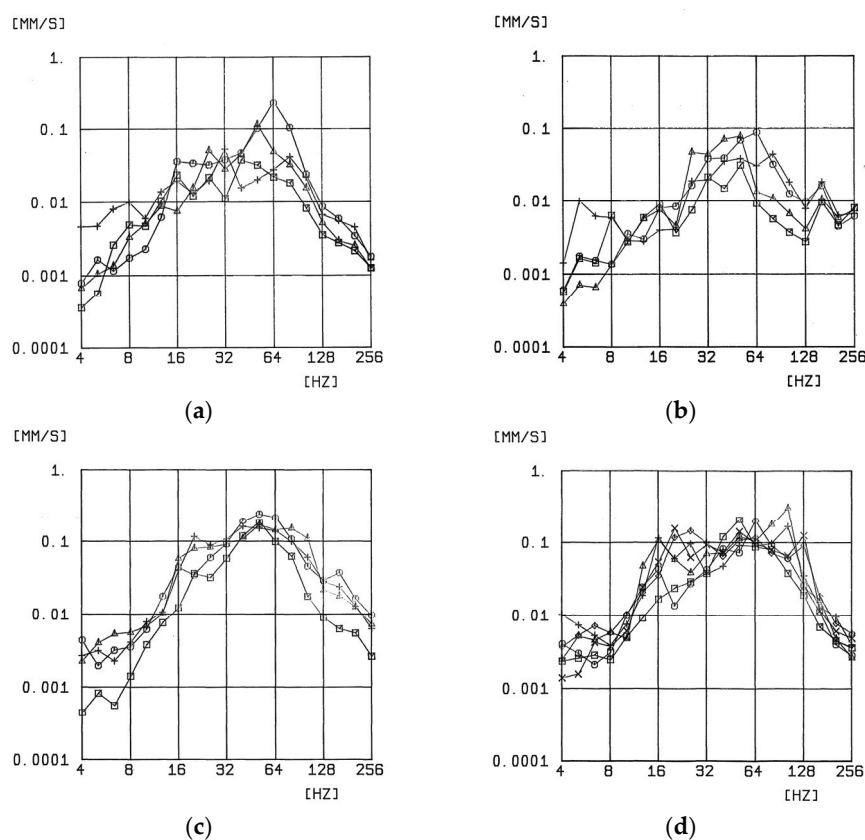


Figure 9. Ground vibration at a distance of 10 m (a,b) at Dachau (soil with $v \approx 400$ m/s), $\square$ passenger train, $\circ$ intercity train (with 130 km/h), $\triangle$ urban train (with 100 km/h), $+$ freight train (with 70 km/h), (a) ballast track, (b) slab track; (c,d) at Wiesenfeld (soil with $v = 270$ m/s) (c) passenger train with $\square$ 100, $\circ$ 150, $\triangle$ 200, $+$ 250 km/h, (d) Intercity Experimental with $\square$ 100, $\circ$ 160, $\triangle$ 200, $+$ 230, $\times$ 250, $\diamond$ 280 km/h.

At the Wiesenfeld site near Würzburg (with a wave velocity of 270 m/s for the top layer), two different trains have been measured for different train speeds, the test train Intercity Experimental in good condition and a conventional passenger train which had some clear out-of-roundness and wheel flats. Therefore, the ground vibration spectra at 10 m distance (Figure 9c) show higher amplitudes between 40 and 80 Hz for the conventional train. The Intercity Experimental, on the contrary, has higher amplitudes at a lower frequency range between 12 and 32 Hz and at a higher frequency range around 100 Hz (Figure 9d). This can be explained by the strong influence of the heavier engine cars of the short five-car ICE. It is concluded that the low- and the high-frequency components are

determined by the static axle load. The low-frequency component are the axle impulses on the track (which are scattered by irregular soil [38]), and the high-frequency component is the sleeper passage component (which is generated by the different deflection on and between the sleepers under the static axle load [39]). The frequencies and amplitudes of the low- and the high-frequency peaks can be studied for the different train speeds of the ICE (Figures 9d and 10a–e). The high-frequency peak is shifted from 50 to 128 Hz for the train speeds of 100 to 280 km/h, and the highest amplitude of 0.4 mm/s (compared to 0.15, 0.3, 0.2, 0.2 mm/s for the other train speeds) is found for 200 km/h where the sleeper passage coincides with the vehicle–track resonance. The low-frequency peaks shift from 16 to 25 Hz for the higher train speeds of 200 to 280 km/h, and the amplitudes are nearly constant at 0.1 to 0.15 mm/s. The proportions of the ground vibration components change at the far field where the high frequencies are strongly reduced due to the material damping of the soil.

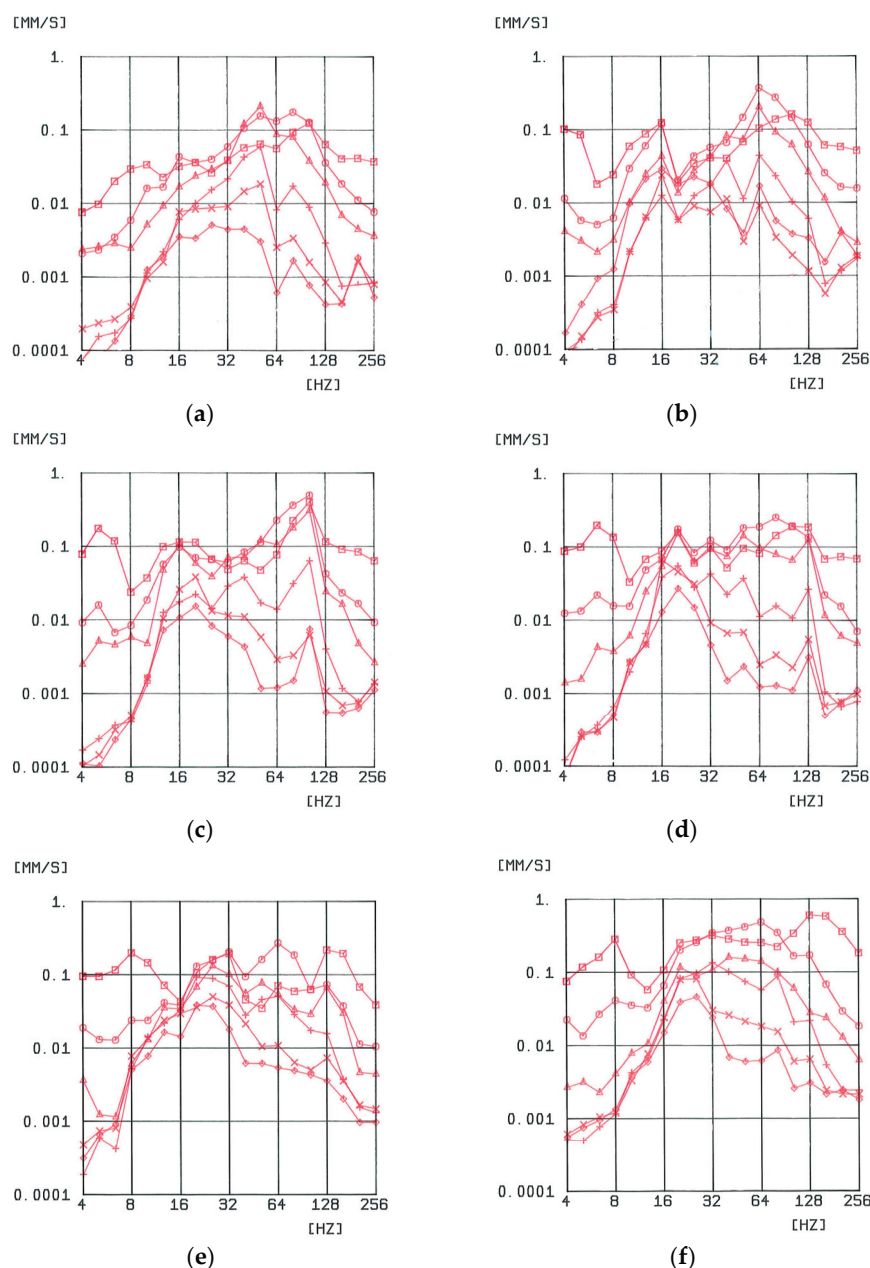


Figure 10. Ground vibrations at Wiesenfeld (soil with $v = 270$ m/s) at distances $\square$ 3, $\circ$ 5, $\triangle$ 10, $+$ 20, $\times$ 30, $\diamond$ 50 m; Intercity Experimental with (a) 100, (b) 160, (c) 200, (d) 250, (e) 300 km/h, (f) passenger train with 250 km/h.

In addition to the ground vibration at 10 m distance in Figure 9, some examples of the whole wave field with distances between 3 and 50 m are shown in Figures 10 and 11. The low-frequency axle impulse component with its frequency shift from 10–16 Hz for 150 km/h (Figure 10b) to 20–32 Hz for 300 km/h (Figure 10e) is the dominant part of the far-field ground vibration. The sleeper passage peak is dominant in the near field and strongly attenuated in the far field by the damping of the soil. At the nearest point, the quasi-static component can be found at 4, 5, 6 and 8 Hz (Figure 10b–e).

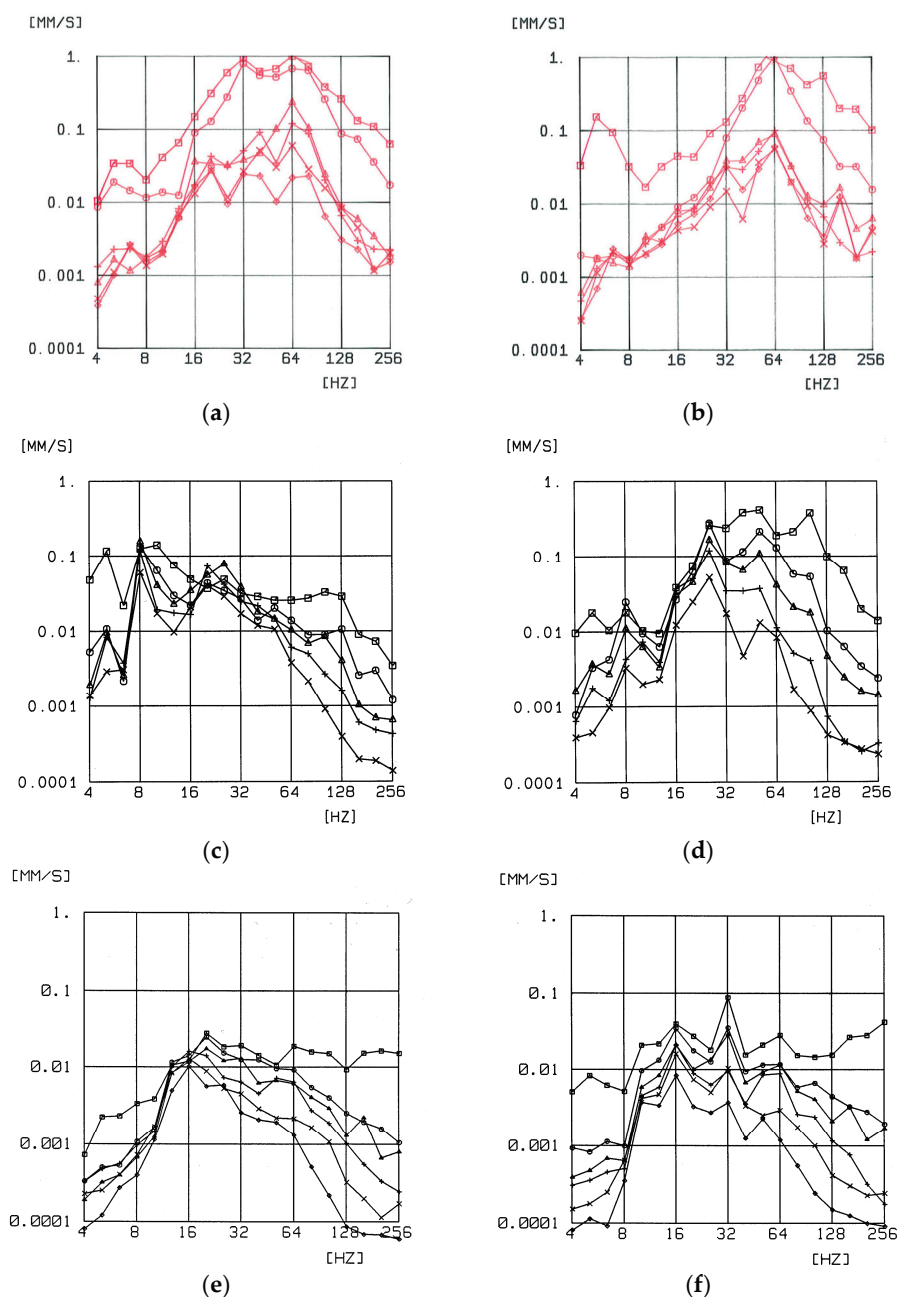


Figure 11. Ground vibrations at different distances, (a,b) at Dachau (soil with $v \approx 400$ m/s) intercity train with 130 km/h, $\square$ 3, $\circ$ 5, $\triangle$ 10, $+$ 20, $\times$ 30, $\diamond$ 50 m, (a) ballast track, (b) slab track, (c,d) at Gardelegen (soil with $v \approx 200$ m/s) intercity express with 250 km/h, $\square$ 4, $\circ$ 8, $\triangle$ 16, $+$ 32, $\times$ 64 m, (c) ballast track, (d) slab track, (e,f) at Wiesenfeld (soil with $v = 270$ m/s), (a) m, (e) passenger car, (f) locomotive with 60 km/h.

The higher amplitudes due to the wheel flats of the conventional passenger train in Figure 10f can be compared with the spectra of the Intercity Experimental with the same

speed (Figure 10d), indicating that the bad vehicle condition can be dominant over the axle impulse component. In Figure 11, the reduction in the low frequencies due to the slab track can be found for the Dachau site (Figure 11a,b) and it is even stronger for the Gardelegen site (Figure 11c,d). Possible reasons include a better track quality or the reduced axle impulses at the slab track. At the Gardelegen site, the high-frequency amplification by the slab track is also strong (Figure 11d), which might be due to the softer rail pads and the less damped P2 resonance. The maximum amplitudes at the slab track are 0.3 mm/s at 50 Hz compared to 0.03 mm/s at the ballast track which means an amplification by a factor of 10. The maximum amplitudes at the ballast track at 8 Hz are 0.1 mm/s, whereas the amplitudes at the slab track are reduced by a factor of 10 to 0.01 mm/s. Finally, Figure 10e,f show the strong influence of the static axle loads when the passage of the locomotive and of the passenger car are analysed separately. The sleeper passage component at 32 Hz (for the train speed of 60 km/h) and the axle impulse component at 10 Hz are clearly stronger for the locomotive. This is in agreement with the theory where these components should be proportional to the static load.

3.3. Building Vibrations at Different Construction Stages

The immission of tunnel-excited ground vibration in buildings has been studied in research project [2] by measurements and by thin-layer finite element models. Some examples are presented in Figure 12 where the depth of the tunnel and the building have been investigated.

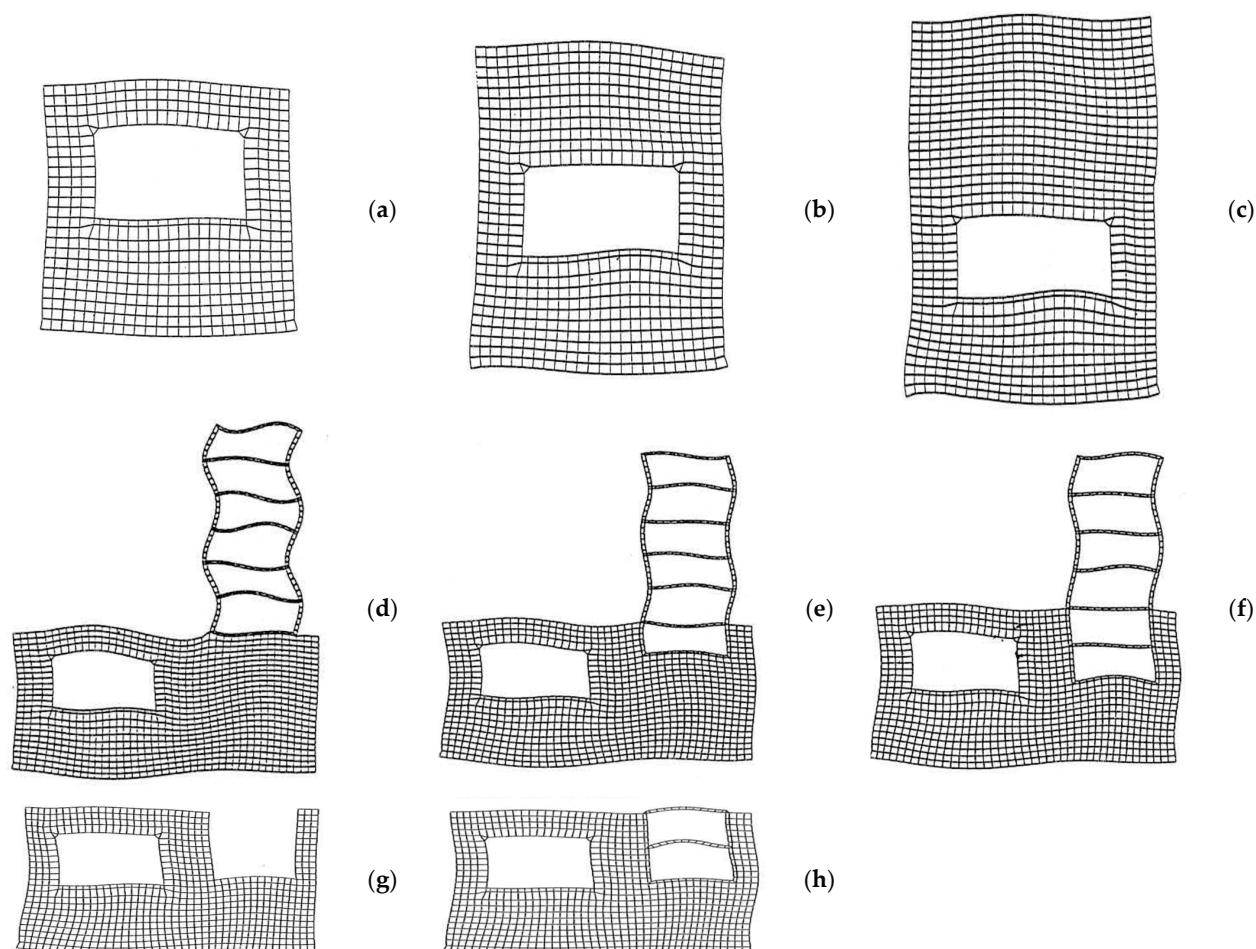


Figure 12. Finite element models for the tunnel–soil–building interaction: variation in (a–c) the depth of tunnel, (d–f) the depth of the building, (g,h) the construction stages, excitation at the right track of the tunnel with 20 Hz.

Different stages of the construction have been analysed by calculation and by measurements (Figure 13). The free-field ground vibration is reduced down to values of 0.1 above 30 Hz by the two basement storeys (probably due to the bending stiffness of the basement slab [40] and the shear stiffness of the basement walls [41]), and the additional storeys of the completed building give no further reduction. On the contrary, an amplification increasing with higher position in the building has been found at around 20 Hz where the fundamental building resonance is estimated. At higher frequencies, the upper building masses are dynamically decoupled from the basement and not effective for reducing ground vibration. In addition, the resonances of the floors are between 15 and 20 Hz, the first bending mode of the wall sections are around 50 Hz, and the second and third bending modes of the floors are between 50 and 100 Hz, the frequency range where secondary noise is usually the main problem of metro vibrations.

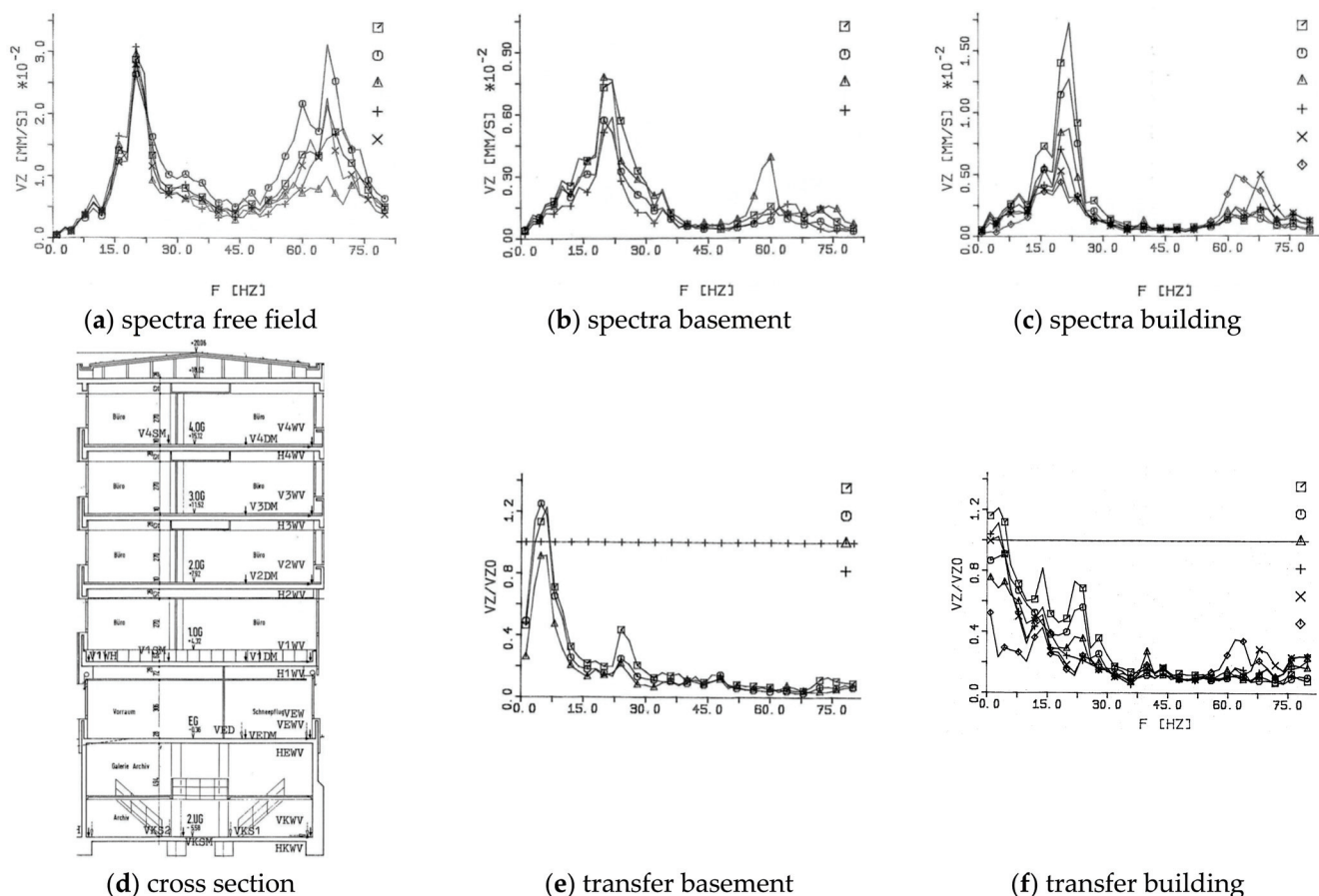


Figure 13. Measurement of the immission from a metro line, (a–c) vertical velocity spectra, (e,f) transfer functions, at different stages of the construction, (a) free field after excavation, (b,e) basement after its construction, (c,f) 7-storey building $\square$ 4th, $\bigcirc$ 3rd, $\triangle$ 2nd, $+$ 1st, $\times$ ground floor, $\diamond$ basement, (d) cross section of the building.

4. Method and Software for the Prediction of Train-Induced Ground and Building Vibrations

The experimental experience and the theoretical analysis have been integrated in a prediction software for train-induced vibrations [4]. Simple models have been developed, which are fast in calculation while presenting detailed calculation results. The prediction is divided into the following modules: “emission” by the vehicle–track–soil interaction, “transmission” through the soil, and “immission” into the building.

The starting point of the emission are the irregularities of the vehicle and the track (Figure 14a). The frequency-dependent transfer function between the irregularities and the dynamic axle loads are calculated by a beam-on-elastic-support model. It is proportional to the unsprung mass of the vehicle (usually the wheelset) up to the vehicle–track frequency, and it is dominated by the dynamic stiffness of the track at higher frequencies. The irregularities decrease with increasing frequency, whereas the transfer function increases with frequency so that the dynamic axle loads are nearly constant (Figure 14b). The increasing train speeds yield strongly increasing force amplitudes at low frequencies and a weaker increase at high frequencies in good agreement with the axle box measurements in Figure 8.

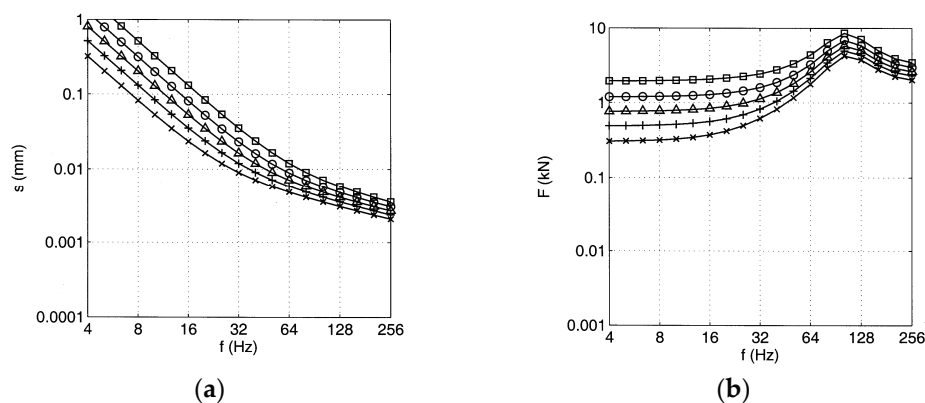


Figure 14. Prediction of the emission: irregularities of the track and the vehicle (a) and resulting dynamic axle loads (b) for different train speeds $\square$ 160, $\circ$ 125, $\triangle$ 100, $+$ 80, $\times$ 63 km/h.

The transmission module starts with the dynamic axle loads from the emission module. The load distribution along and across the track is included approximately and yields the specific attenuation of train-induced ground vibrations with distance. The response of the soil is calculated by a “frequency-dependent half-space model” where the explicit solution of the homogeneous half-space is evaluated for different wave velocities at different frequencies following the dispersion of the layered soil. The transfer functions between the train load and the ground vibrations (Figure 15a,b) represent the measured characteristics of soft and stiff layered soils well (Figure 15c,d).

The immission part calculates the transfer from the free-field ground vibration (at the foundation point under consideration) to the different building parts by a one-dimensional soil–wall–floor model [42], which has been compared with three-dimensional finite element models of wall- and column-type concrete buildings of 2 to 20 storeys and with measurements, for example, the concrete and steel column buildings in Figure 7. Some example results are shown for different soils and for different floors in Figure 16. The soil amplitudes are amplified at the building–soil eigenfrequency and reduced at higher frequencies due to the mass and the dynamic stiffness of the building. At mid-frequencies, the resonance of the floors usually results in the highest amplitudes in the building (Figure 16b,d).

Specific components of railway vibrations can be added as (equivalent) irregularities, for example, the first out-of-roundness of the wheels and the sleeper-distance component, or as equivalent forces (around the axle-distance frequency) for the scattered axle impulse component or empirically as in [23]. These specific components are often found in the measurements (for example, in Figure 15c,d) in comparison with the smooth regular transfer functions (Figure 15a,b).

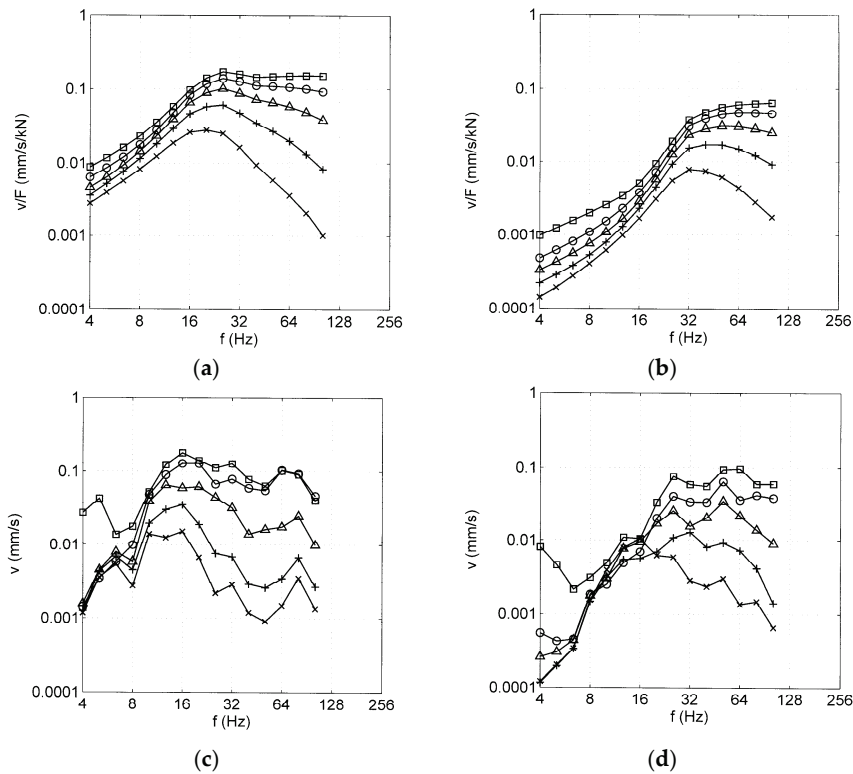


Figure 15. Prediction of the transmission: transfer function between a standard train load and the ground vibrations at $\square$ 4, $\circ$ 8, $\triangle$ 16, $\times$ 32, $\times$ 64 m: (a) soft layered soil $v_{S1} = 150$ m/s, $v_{S2} = 250$ m/s, $h = 3.5$ m), (b) stiff layered soil ($v_{S1} = 325$ m/s, $v_{S2} = 850$ m/s, $h = 5$ m), (c,d) comparison with measurements at Selzach and Hindelbank (Switzerland).

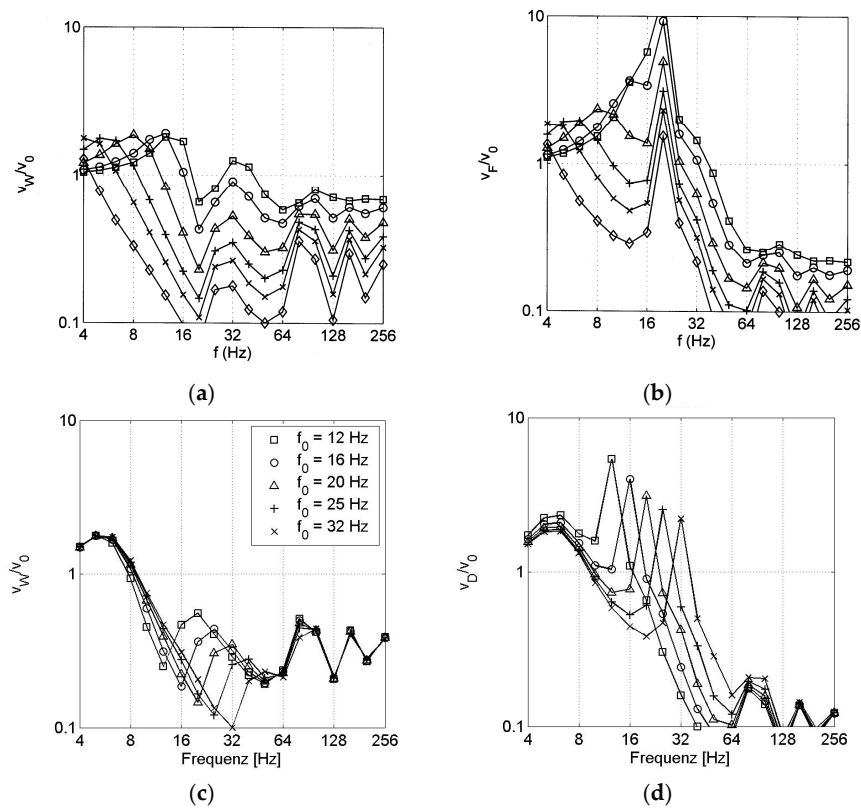


Figure 16. Prediction of building vibrations: Averaged soil-wall (a,c) and soil-floor (b,d) transfer functions of a 6-storey apartment building (a,b) on different soils with $v_s = \diamond 100$, $\times 150$, $+$ 200, $\triangle 300$, $\circ 500$, $\square 700$ m/s, (c,d) with different floor eigenfrequencies $\square$ 12, $\circ$ 16, $\triangle$ 20, $+$ 25, $\times$ 32 Hz.

5. Conclusions

Characteristic frequencies of train-induced vibrations have been shown in the experimental results. The excitation by the train and the track has speed-dependent frequencies, at the wheel out-of-roundness and the sleeper passage component or frequency ranges for the track alignment errors and for the impulses of the passing static axle loads. The excitation is amplified at certain eigenfrequencies such as the vehicle–track eigenfrequency, the bridge eigenfrequencies and the dominant soil frequencies between the cut-on and cut-off frequencies. The building response is amplified at the building–soil, the floor, and the acoustic room resonance. A special role has been found for the axle-sequence spectrum and its car-length frequencies and axle-distance frequencies. The axle-sequence spectrum is clearly found in the track vibration; the excitation of a bridge resonance is strongly influenced by the axle-sequence spectrum, and it can also often be found in the ground vibration indicating the scattered part of the axle impulses. This ground vibration part is dominant at the far field, and it falls typically in the frequency range of floor resonances in buildings, underlining its practical importance.

The influence of the type of the train and the track, the static axle loads, and the speed of the train on the amplitudes of the one-third octave band spectra has been analysed. The influence of different parts of the building has been analysed by measurements and calculations of different construction stages. Specific effects have been found for locomotives, freight trains, slab tracks, tunnel and bridge lines in comparison to passenger trains on a ballast track at a surface line. For the speed dependency, a strong increase for the low-frequency track alignment errors, a limited increase for the sleeper passage component, and constant amplitudes for the axle impulse component have been observed.

The theoretical and experimental experiences have been integrated in a prediction software with simple and fast vehicle, track, soil and building models, which give the transfer functions between irregularities, dynamic axle loads, ground and building vibrations. The excitation by a broad-band irregularity spectrum must be complemented with a higher sleeper passage peak and with the axle impulse component in a certain low-frequency range.

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Conflicts of Interest: The authors declare no conflicts of interest.

Appendix A. Details of the Measurements

Measurement equipment:

The track, soil, bridge, and building vibrations are measured with robust velocity transducers (geophones), a 72-channel measurement system for filtering, amplifying, and digitising, usually 2 kHz sampling and one-third filter frequency. Vehicle measurements with accelerometers BK 4375 on four axle boxes of a bogie. Four train runs with 16, 25, 40, 63, 80, 100, 125, 140, and 160 km/h. Velocity signals from the tracks are transformed to displacements.

Track configurations:

Ballast track at Wiesenfeld with UIC60 rails, stiff rail pads 300 kN/m², concrete sleepers in 0.6 m distance, ballast of 0.35 m height.

Slab track near Wiesenfeld with UIC60 rails, medium soft rail pads 60 kN/m², concrete sleepers in 0.65 m distance, track slab of 0.2 m height.

Track measurement site in Dachau with five different track configurations in a straight line, standard ballast track and ballast track are presented here.

Soil parameters:

Standard values for mass density 2×10^3 kg/m³, Poisson's ratio 0.33, which have little influence, and material damping $D = 2.5\%$ is used in calculation. The shear or Rayleigh wave velocity and the damping are most important and are measured at each site. The damping is important for high frequencies, whereas the layer height is important for low frequencies in case of a layered soil.

Wiesenfeld $v_{S1} = 270$ m/s, $v_{S2} = 1000$ m/s, $D = 5\%$, $h = 10$ m.

Prien $v = 30$ m/s, $D = 5\%$.

Dachau $v = 400$ m/s, $D = 2.5\%$.

Gardelegen $v = 200$ m/s, $D = 1-3\%$

Selzach $v_{S1} = 150$ m/s, $v_{S2} = 250$ m/s, $D = 2\%$, $h = 3.5$ m

Hindelbank $v_{S1} = 325$ m/s, $v_{S2} = 850$ m/s, $D = 2.5\%$, $h = 5$ m.

Appendix B. Details and Formulas of the Prediction Methods

The aim of the methods presented in [4,42] is to provide a user-friendly and accurate prediction of the emission, transmission and immission of train-induced vibrations. Therefore, the input must be simple with only a few parameters, and the calculation must be fast. The challenge was finding simple physical models and rules for the parameters so that the results of detailed models can be presented well. The following models and basic equations have been found.

Emission

The main part of the vehicle for the excitation of ground vibration is the wheelset and the corresponding dynamic stiffness of the vehicle:

$$K_V = -m_W \omega^2$$

The track is modelled as a (multi-)beam-on-support system. For the ballast track with only one beam, the explicit formula

$$K_T = 2\sqrt{2}EI^{1/4}k^{3/4}$$

can be used with bending stiffness EI of the rails and the dynamic stiffness k of the support by rail pads, the sleepers, the ballast, and the soil.

The direct coupling of the vehicle with its dynamic stiffness K_V and the track with K_T yields the transfer function between the irregularities s and the force on the ground F_S :

$$F_S = -\frac{K_V K_T}{K_V + K_T} s$$

Transmission

The core model for the wave propagation through the soil is the homogeneous half-space with its asymptotes

$$\frac{v}{F}(r, f) = \frac{f(1-\nu)}{Gr} \exp(-Dr^*) \begin{cases} 1 & \text{for } r^* \leq r_0^* \\ \sqrt{r/r_0^*} & \text{for } r^* > r_0^* \end{cases} \text{ with } r^* = \frac{2\pi fr}{v_S}$$

for the particle velocities of a soil with the parameters G shear modulus, ν Poisson's ratio, damping D , and shear wave velocity v_s , in the distance r of a force F . Depending on the parameter r^* , we obtain the low-frequency near field or the high-frequency far field. This solution can be used to represent a layered soil. The frequency-dependent wave velocity (the dispersion) of the layered soil is calculated first by approximate formulas, and for each frequency, the soil parameters according to the wave velocity at this frequency are inserted into the formula. This procedure yields reasonable results for a stiffness continuously varying with depth. Some modifications have been found in the case of a clear layering, a resonance function, the near field of a soft layer, and the far field of a stiff layer [42]. The distribution of the dynamic load across the track is included by a reduction factor. Finally, the response to a passing train is obtained by the superposition of several wheel loads.

Immission

The response v_W of the building to the free field v_0 is calculated by a one-dimensional soil–wall–floor model. The explicit solution for the soil–wall model is

$$\frac{v_W}{v_0} = \frac{\cos \omega x/v_L}{\cos \omega H/v_L} \frac{1}{1 + i \frac{Z_W}{Z_S} \tan \omega H/v_L} \quad (\text{A1})$$

with the height H of the building, the position/level in the building, the wave velocity v_L of the wall, Z_W the impedance of the wall and Z_S the impedance of the soil. The frequency-dependent behaviour of the floors can be included in the mass of the wall, and the explicit formula still holds but with frequency-dependent and complex parameters.

Comments

The present prediction holds for a straight line, for different passenger trains with different constant speeds, for surface line on the ground, for different track forms/configurations, for horizontally layered sites with different types of soil, for a single (usually the nearest) building with different number of storeys, with different materials (wood, concrete, masonry, steel), different surface foundations (rigid single footings, strip foundations and plates) on different soils, and floors with different support conditions (point or line support, clamped or hinged). Mitigation measures at the track and at the building are possible but not presented here.

The models have been calibrated against detailed models. Some parameters and some details had to be modified to obtain similar results. When comparing the prediction with the measurements, errors certainly occur. The error levels are different for different frequency ranges and for the different parts of prediction. The emission part can be checked by axle box measurements showing exactly the low-frequency irregularities and (with more uncertainty) the dynamic axle loads at high frequencies. The transmission part typically has stronger high-frequency errors due to the shorter wavelengths and uncertainties about the damping. The immission is the part with the highest uncertainties, as not all of the many parameters of the building are known. To conclude, if the total error of the full chain is considered, the high precision of a single item cannot ensure an overall accuracy.

The prediction models have been compared to all BAM measurements which are available for 50 sites (8 soft sites with $v < 150$ m/s, 30 medium soft sites, 12 stiff sites with $v > 250$ m/s), 40 buildings (20 low-rise buildings with less than four storeys, 18 medium-rise buildings with four to eight storeys, 12 high-rise buildings with more than eight storeys), and 30 tracks (19 ballast tracks, 11 slab tracks). It has been checked that all relevant phenomena are represented sufficiently.

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Article

Vibration Mitigation Through Rail Track Design for Structures Built Directly Above a Double-Deck Railway Depot

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Abstract: This paper reviews, analyses, and suggests practical mitigation techniques at source for reducing vibration-induced annoyance to occupants in building structures that are built on top of significant railway infrastructure. The dynamic characteristics of vibration caused by wheel-rail interaction at metro train depots are different from those on main-lines and conventional studies. Ground-borne vibration in a building directly above a double-deck railway depot was investigated, focusing on vibration attenuation through rail track design, which is more effective and economic compared to treatments at receivers or along propagation paths. A 2.5-Dimensional finite element model was established to simulate vibration transmission using different combinations of track-forms. Source contribution under different train running conditions has been evaluated by computing vibration levels along the main transmission path. Vibration levels at representative positions in the building rooms have been predicted using the numerical model and have been compared against site measurements at the corresponding locations after the completion of the construction of the depot and buildings. It was found that the 2.5D FE model enables a reasonable prediction of ground-borne vibration from the metro depot, and that by appropriate design of the track-form, a good level of vibration attenuation can be achieved in an economical way.

Keywords: metro depot; track-form; dynamic characteristics; frequency range; ground-borne vibration; mitigation measures; train-track interaction; FE model; site measurement; nuisance and discomfort

1. Introduction

1.1. Noise and Vibration from the Metro System

In railway engineering, the two coupled dynamical systems are the train-track system and the pantograph-catenary system. Noise and vibration from the train-track interaction include rolling noise, squeal noise, aerodynamic noise typically from high-speed rail, ground-borne vibration, and structure-borne noise [1]. These different categories of noise and vibration each features its own character frequency range, as shown in Figure 1. Ground-borne vibration features a relatively low frequency range in the railway dynamical system, typically below 100 Hz. On the other hand, structural vibration dominates between 200 to 2000 Hz. In the railway system, it is conventionally defined that low frequencies being below 10 Hz, medium frequencies being in-between 10 and 100 Hz, and high frequency range being above 100 Hz [2].

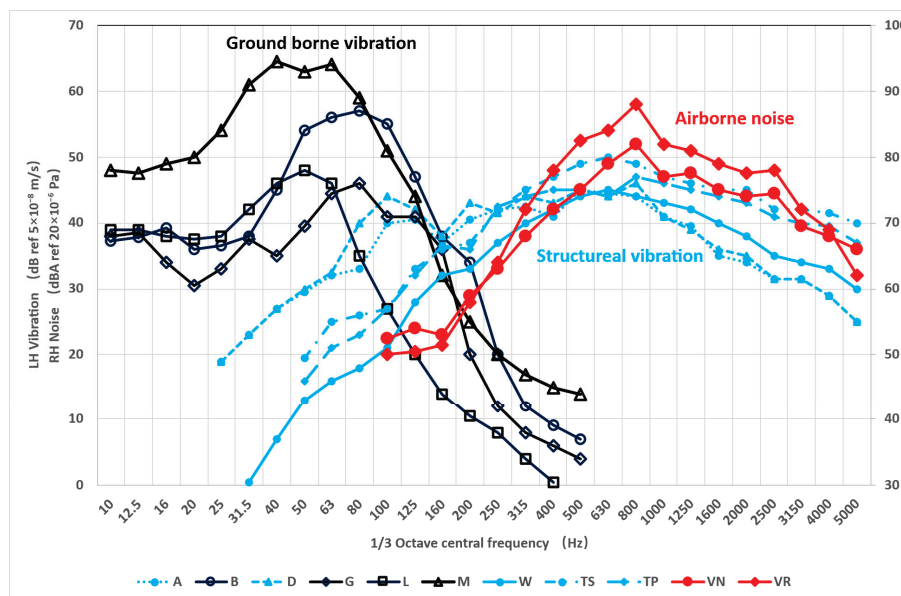


Figure 1. Typical frequency ranges for different types of railway-induced noise and vibration (re-produced from original data in [2]).

Railway-induced ground-borne vibration deals with environmental concerns due to the generation and propagation of vibrations that affect the comfort and life quality of the inhabitants in railway surroundings. Such low frequency ground-borne vibration, typically in the range between 1 Hz and 80 Hz as per standard [3], is mainly due to wheel-rail interaction, especially where rail corrugation and wheel polygonalisation are present.

In a metro system, train-induced ground-borne vibration can be transmitted into nearby buildings through underground soil around a tunnel or the viaduct for elevated lines, causing nuisance and discomfort to inhabitants. A number of characteristic frequencies are excited on any conventional metro system, including bogie-passing, axle-passing, fastener-passing, rail corrugation excitations [2]. P2 resonance, vertical and lateral pinned-pinned frequencies, all above 20 Hz, are characteristic frequencies of long- and short-wavelength rail corrugation, which are responsible for ground-borne vibration due to railway traffic. Bogie-passing, axle-passing, and fastener-passing frequencies are not significant dynamic vibration sources in railways, as according to data [1,4–7], ground-borne vibration response at receivers below 10 Hz are very low, up-to 40 dB below peak magnitude vibration around 50 Hz and 63 Hz 1/3 octave band central frequencies.

In the past decades, research on railway-induced ground-borne vibration and mitigation measures for the control of noise and vibration for buildings along railway tracks have been undertaken [8–16]. Computational simulation has been extensively used to further investigate wheel-rail coupling problems [17–21] and to predict vibrations transmitted from source to receivers [22–25].

In terms of noise and vibration control, in any industry, e.g., industrial noise and vibration, occupational noise and vibration, and building services engineering such as HVAC, it is a common practice and rule of thumb to design and implement mitigation measures at source wherever possible [26–28] (“Law of the People’s Republic of China on the Prevention and Control of Noise Pollution” specifically says “to reduce and control at source”). This is because reducing source intensity is often the most efficient technically and most economical means. Examples include the selection of quieter and low-vibration heavy machinery at factories; the selection of low noise and vibration emission hand-held tools for construction workers, the control of noise and vibration of air conditioning systems in office environments etc. When the control of source emission is insufficient,

further mitigation measures through transmission paths, e.g., noise and vibration insulation techniques, and/or at receivers, e.g., noise reduction earmuffs for construction workers, sound absorption materials inside a room, etc.

1.2. Transit-Oriented-Development (TOD)

A transit-oriented-development (TOD), in a metro system, is a complex of building structures directly connected to a station or a railway depot (Some also define the TOD as all the building property forms within the surrounding 500 m range from a metro station or a train depot.). This particular urban development mode magnifies the amount of residential, commercial, and leisure space within the walking distance of public transport. TOD reduces environmental pollution and energy consumption and provides an effective financing channel for the construction and operation of metro-lines, promoting a sustainable development of the city.

In recent years in China, TOD directly above railway depots of a metro system tends to be popular, wherein the real estate investor and the metro owner work closely from the planning stage of a TOD scheme. A TOD in China typically includes various types of residential houses and buildings, educational facilities such as schools and nurseries, shops, etc.

Up to the end of 2024 in China, 55 cities have metro systems, over 40 of which were with TOD projects. The total real estate value of those TODs exceeds 1 trillion Chinese yuan. One of the key issues related to TOD is the noise and vibration impact from the railway depot, which is significantly different from the main line in the following aspects:

1. The track layout at railway depots is far more complex, with a large number of small-radius curves, rail switches, and rail joints, leading to significant impact load and more frequent and severe vibration excitation.
2. The operation modes at a typical railway depot of the metro system are significantly different from that on main lines. For instance, the majority of activities at the depot are undertaken during night-time hours or very early in the morning with low background noise; the operation hours at the depot also impose more stringent noise and vibration limits.
3. Different train activities at the depot feature a wide range of train speeds, varying from 5 to 10 km/h in the inspection and maintenance warehouse; around 15–20 km/h at the throat zone (amongst the metro-line operators and relevant stakeholders in China, the throat zone is loosely defined as the area where tracks merge and diverge through rail switches between the main line and the railway depot); and up-to 60 km/h on the test track.

To this end, noise and vibration characteristics at source at railway depots are expected to be much more complex than those on the main lines in conventional studies in this field. Yet, noise and vibration impact from railway depots to people in buildings directly above has not yet been studied systematically.

Tao et al. [29] performed a vibration and noise impact assessment with measurements taken at receivers in buildings above a metro depot.

Cao et al. [7] measured ground-borne vibration from a metro depot at receivers. Structural analysis was carried out for key structural elements, including columns and floor slabs at the receiver building, with the conclusion that thicker slabs reduce vibration magnitudes, and therefore a good mitigation effect may be obtained. This falls outside of the scope of this paper, which is on noise and vibration control at source as railway engineers, during the track design stage, to achieve greater technical efficiency at lower costs.

Zou et al. [30] proposed vibration mitigation measures for a metro depot to use trackside wave barriers, which is a concept borrowed from high-speed railways, i.e.,

trenches. However, substantial space is required for the isolation of ground-borne vibration in 1 Hz to 80 Hz frequency range to take effects.

When designing a metro system against environmental noise and vibration impacts, national and industrial standards, and guidance are used. Owing to the cross-disciplinary nature of this type of engineering problem, inconsistencies in design limits exist between the standards in the metro system and in the building industry (Table 1). Lacking communication and coordination at the planning and design stage of the metro depot and property development sometimes leads to inefficient or even insufficient design of noise and vibration mitigation measures, which consequently cause complaints when the depot is under operation. So far, there has not yet been a dedicated standard or code of practice for the design of noise and vibration control for TOD above depots, partly due to the lack of understanding of dynamic characteristics and transmission patterns from the source to receivers.

Table 1. Noise and vibration limit values in different Chinese standards and codes.

Category of Standard	Name of Standard	Limit		
		Residential Buildings	Educational Facilities (Including Schools and Nurseries)	Office and Mixes
Vibration	GB 10070-88 <Standard of environmental vibration in urban area> [31]	Category I: Daytime 70 dB; Night-time 67 dB	Category I: Daytime 70 dB; Night-time 67 dB	Category II: Daytime 75 dB; Night-time 72 dB
Environmental noise	GB 3096-2008 <Environmental quality standard for noise> [32]	Category II: Daytime 60 dB (A); Night-time 50 dB (A)	Category II: Daytime 60 dB (A); Night-time 50 dB (A)	Category II: Daytime 60 dB (A); Night-time 50 dB (A)
Room interior noise	GB 55016-2021 <General code for building environment> [33]	Category ‘Sleeping’: Daytime 40 dB (A); Night-time 30 dB (A); Category ‘Living’: 45 dB (A)	Category ‘Education’: 45 dB (A)	Category ‘Office’: 45 dB (A)
Industrial enterprises noise at boundary	GB 12348-2008 <Emission standard for industrial enterprises noise at boundary> [34]	Category 2: Daytime 60 dBA (A); Night-time 50 dB (A)		
Vibration and structure-borne noise	JGJ/T 170-2009 <Standard for limit and measuring method of building vibration and secondary noise caused by urban rail transit> [35]	Vibration: Category 2: Daytime 70 dB; Night-time 67 dB Structure-borne noise: Category 2: Daytime 41 dB (A); Night-time 38 dB (A)	Vibration: Category 2: Daytime 70 dB; Night-time 67 dB Structure-borne noise: Category 2: Daytime 41 dB (A); Night-time 38 dB (A)	Vibration: Category 2: Daytime 70 dB; Night-time 67 dB Structure-borne noise: Category 2: Daytime 41 dB (A); Night-time 38 dB (A)

This research study aims at gaining a better understanding of the dynamic characteristics at different function areas at metro depots, as well as vibration transmission patterns from source to sensitive receivers at TOD. At this preliminary stage of the project, a numerical model was developed and validated against field data collected at existing railway depots. Vibration source contribution has been examined for trains running on the upper- and lower-deck of a double-deck metro depot. Ground-borne vibration to occupants in buildings above the depot using different track-forms combinations with different vibration isolation levels were quantified. An equivalent parameter method has been used in this study to couple the “train-track-tunnel-ground” model with the building above the TOD, all in 2.5D with reasonable assumptions, enabling quick modelling for assessment. The

work so-far provides a full set of simulation and field measured data, for practical guidance for the design of rail track-form and vibration mitigation measures for TODs in China. The outputs focus on more effective and economical mitigation measures at source where possible, which are expected to shed insights for a design standard or guidance specifically for TOD above metro depots.

2. Research Methodology

2.1. The Double-Deck Railway Depot

This study features a double-deck railway depot shared by several metro lines. The TOD above the railway depot, in this case, consists of high-rise residential buildings and houses; office buildings; a school; a nursery; merchandise shops; and miscellaneous facilities. A graphical illustration of the railway depot and above TOD is shown in Figure 2.

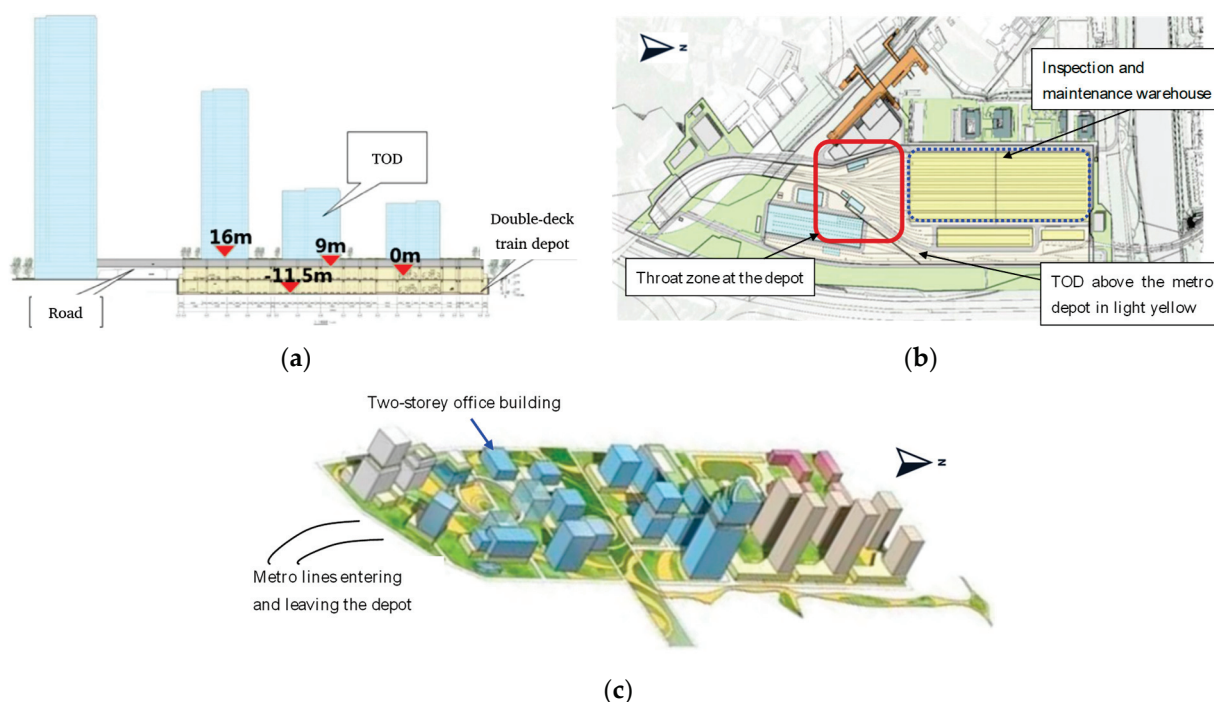


Figure 2. The double-deck railway depot and above TOD: (a) the cross-sectional layout; (b) the plan view of the railway depot; (c) planned buildings above the depot.

Figure 2a shows the front view of the metro-line depot and its above TOD. The upper- and lower-decks of the depot is in light yellow. The ground level of the upper deck is defined as the "0 m plate", the space in grey between the "9 m plate" and the "16 m plate" is a car park for residents and users at the TOD. The "16 m plate" is also called the roof plate of the depot. Figure 2b is the plan view of the depot, with different functional areas. Highlighted in red is the throat zone where tracks between the maintenance and inspection warehouse and the main lines merge and split. As tracks in this area feature a large quantity of rail joints and switches causing impact forces between wheel and rail, it is common practice to avoid tall building structures especially with sensitive noise and vibration receivers above this area at the design stage of TOD. In this case, some low-rise office buildings and merchandise shops are allocated above the throat zone. Inspection and maintenance warehouse is enveloped in dotted blue, at the rear of the depot. Sensitive receivers such as high-rise residential buildings, educational facilities are designed to be put above this area, as shown in Figure 2c.

It is a common practice, in railway depot applications, to divide the analysis according to the functionality of vibration sources. The test track, the throat zone, and the inspection and maintenance warehouse are three main function areas where wheel-rail induced vibration is most significant (Fixed plants such as train washing plants are also a major source of noise and vibration, but are relatively easy to treat, due to the steady state nature of the source. Usually fixed plants with quieter motor/source and a vibration isolation platform are sufficient for vibration control, thus is not included in this paper.). The following analysis steps were undertaken:

1. The construction, verification, and validation of the computational model: a 2.5D Finite Element (FE) model was developed using Ansys software 2020 R2 [36]. The vibration level on the “16 m plate”, as per Figure 2a, was compared against field measurement collected from the depot roof plate at a similar depot, to validate the FE model.
2. Using the validated FE model, vibration contribution of trains running on the upper- and lower-decks of the depot was evaluated, to support the design of rail track-forms on both decks.
3. A parametric study was then undertaken wherein the vibration attenuation effect of using different combinations of track-forms at the throat zone with complex track geometry was examined, providing useful information for the design of vibration mitigation measures at source.
4. After the completion of the construction of the depot, numerically computed vibration levels in rooms of the two-storey office building were compared to site measurement at corresponding sensitive receivers.

2.2. The Numerical Model

2.5D models are efficient tools for dynamic analysis of structures that are longitudinally invariant. Such models have been widely used in the studies of ground-borne vibrations caused by railways [25,37–46].

2.2.1. The Rail Track Model

An illustration of the rail track structure-ground model is shown in Figure 3. The material and geometric properties are assumed to be invariant in the x direction, which is the train running direction.

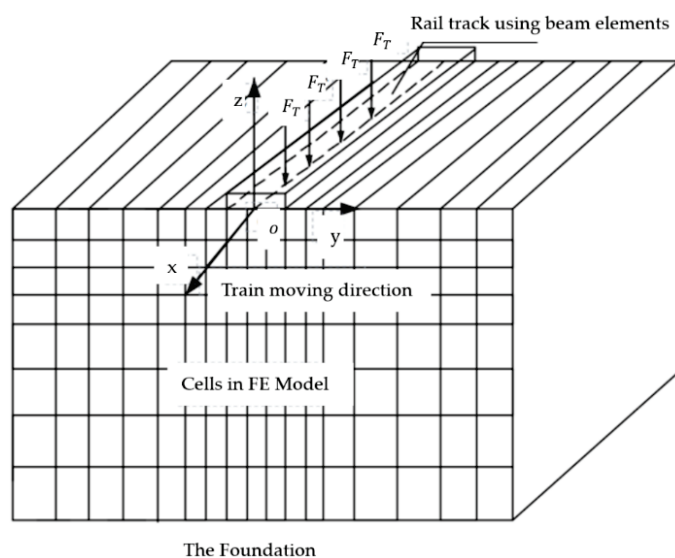


Figure 3. The rail track-ground FE model.

The tracks and sleepers are simplified and modelled with Euler–Bernoulli beams rested on half-space; the Equation of Motion (EoM) can be written as per Equation (1):

$$EI \frac{\partial^4 u_r}{\partial x^4} + m \frac{\partial^2 u_r}{\partial t^2} = F_F(x, t) + F_T(x, t) \quad (1)$$

where E is Young's modulus, I is the second moment of area about y -axis, m is the overall mass of the rail and sleeper per unit length; u_r is the vibration displacement of the rail; F_F is the reaction force from the foundation; and F_T is the train load.

By implementing the Fourier Transform, the above EoM of the beam can then be written as Equation (2) in the frequency domain, and finally represented in the matrix form in Equation (3):

$$(EI \varepsilon_x^4 - m \omega^4) \overline{u_r} = \overline{F_F}(\varepsilon_x, \omega) + \overline{F_T}(\varepsilon_x, \omega) \quad (2)$$

$$\overline{K_r} \overline{U_r} = \overline{F_F} + \overline{F_T} \quad (3)$$

where ε_x and ω are the wave number corresponding to x and circular frequency, respectively, in the frequency domain.

2.2.2. The Foundation Model

The soil foundation is assumed to be isotropic, the EoM in the frequency domain is shown in Equations (4) and (5).

$$\begin{aligned} & \left(C_{11} \frac{\partial^2}{\partial x^2} + C_{66} \frac{\partial^2}{\partial y^2} + C_{44} \frac{\partial^2}{\partial z^2} \right) \overline{u} + (C_{12} + C_{66}) \frac{\partial^2 \overline{v}}{\partial x \partial y} + (C_{13} + C_{44}) \frac{\partial^2 \overline{w}}{\partial x \partial z} = -\rho \omega^2 \frac{\partial^2 \overline{u}}{\partial t^2} \\ & (C_{12} + C_{66}) \frac{\partial^2 \overline{u}}{\partial x \partial y} + \left(C_{66} \frac{\partial^2}{\partial x^2} + C_{11} \frac{\partial^2}{\partial y^2} + C_{44} \frac{\partial^2}{\partial z^2} \right) \overline{v} + (C_{13} + C_{44}) \frac{\partial^2 \overline{w}}{\partial y \partial z} = -\rho \omega^2 \frac{\partial^2 \overline{v}}{\partial t^2} \\ & (C_{13} + C_{44}) \frac{\partial}{\partial z} \left(\frac{\partial \overline{u}}{\partial x} + \frac{\partial \overline{v}}{\partial y} \right) + \left[C_{44} \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) + C_{33} \frac{\partial^2}{\partial z^2} \right] \overline{w} = -\rho \omega^2 \frac{\partial^2 \overline{w}}{\partial t^2} \end{aligned} \quad (4)$$

wherein:

$$\begin{aligned} C_{11} &= E_h \left(1 - \mu_{hv}^2 \frac{E_h}{E_v} \right) (1 + \mu_{hh}) \Delta; \quad C_{12} = E_h \left(\mu_{hh} + \mu_{hv}^2 \frac{E_h}{E_v} \right) (1 + \mu_{hh}) \Delta; \quad C_{13} = \frac{E_h \mu_{hv}}{\Delta}; \\ C_{33} &= E_v \left(\frac{1 + 2\mu_{hv}^2 \frac{E_h}{E_v}}{E_v \Delta} \right); \quad C_{44} = G_v; \quad C_{66} = \frac{E_h}{2(1 + \mu_{hh})}; \quad \Delta = 1 - \mu_{hh} - 2\mu_{hv}^2 \frac{E_h}{E_v} \end{aligned} \quad (5)$$

where $\overline{u}$, $\overline{v}$, $\overline{w}$ are vibration amplitudes of the components of displacement u , v , w along the x , y , z directions, respectively; C_{11} to C_{66} are coefficients of soil reaction of isotropic soil; E_h and E_v are tensile modulus in the horizontal and vertical direction, respectively; G_v denotes shear modulus in the vertical plane; μ_{hh} is the Poisson's ratio in the horizontal plane, and μ_{hv} is the Poisson's ratio of the strain in the vertical plane orthogonal to the normal stress in that plane.

Using the Galerkin method with fully coupled boundary conditions between rail tracks and the soil foundation, and visco-elastic boundary conditions for the soil foundation, a set of differential equation in the frequency–wavenumber domain can be obtained, as per Equation (6):

$$\begin{aligned}
& \int_v \delta \bar{u} (C_{11} \frac{\partial \bar{u}}{\partial x} \frac{\partial \delta \bar{u}}{\partial x} + C_{66} \frac{\partial \bar{u}}{\partial y} \frac{\partial \delta \bar{u}}{\partial y} + C_{44} \frac{\partial \bar{u}}{\partial z} \frac{\partial \delta \bar{u}}{\partial z} + C_{66} \frac{\partial \bar{v}}{\partial x} \frac{\partial \delta \bar{u}}{\partial y} + C_{12} \frac{\partial \bar{v}}{\partial y} \frac{\partial \delta \bar{u}}{\partial x} + C_{13} \frac{\partial \bar{v}}{\partial z} \frac{\partial \delta \bar{u}}{\partial x} + C_{44} \frac{\partial \bar{v}}{\partial x} \frac{\partial \delta \bar{u}}{\partial z} \\
& \quad + \rho \omega^2 \bar{u}) dv + \int_s \delta \bar{u} \bar{F}_{Fx} ds = 0 \\
& \int_v \delta \bar{v} (C_{66} \frac{\partial \bar{u}}{\partial y} \frac{\partial \delta \bar{v}}{\partial x} + C_{12} \frac{\partial \bar{u}}{\partial x} \frac{\partial \delta \bar{v}}{\partial y} + C_{66} \frac{\partial \bar{v}}{\partial x} \frac{\partial \delta \bar{v}}{\partial x} + C_{11} \frac{\partial \bar{v}}{\partial y} \frac{\partial \delta \bar{v}}{\partial y} + C_{44} \frac{\partial \bar{v}}{\partial z} \frac{\partial \delta \bar{v}}{\partial z} + C_{13} \frac{\partial \bar{v}}{\partial z} \frac{\partial \delta \bar{v}}{\partial y} + C_{44} \frac{\partial \bar{v}}{\partial y} \frac{\partial \delta \bar{v}}{\partial z} \\
& \quad + \rho \omega^2 \bar{v}) dv + \int_s \delta \bar{v} \bar{F}_{Fy} ds = 0 \\
& \int_v \delta \bar{w} (C_{13} \frac{\partial \bar{u}}{\partial x} \frac{\partial \delta \bar{w}}{\partial z} + C_{44} \frac{\partial \bar{u}}{\partial z} \frac{\partial \delta \bar{w}}{\partial x} + C_{44} \frac{\partial \bar{v}}{\partial z} \frac{\partial \delta \bar{w}}{\partial y} + C_{13} \frac{\partial \bar{v}}{\partial y} \frac{\partial \delta \bar{w}}{\partial z} + C_{44} \frac{\partial \bar{w}}{\partial x} \frac{\partial \delta \bar{w}}{\partial x} + C_{44} \frac{\partial \bar{w}}{\partial y} \frac{\partial \delta \bar{w}}{\partial y} + C_{33} \frac{\partial \bar{w}}{\partial z} \frac{\partial \delta \bar{w}}{\partial z} \\
& \quad + \rho \omega^2 \bar{w}) dv + \int_s \delta \bar{w} \bar{F}_{Fz} ds = 0
\end{aligned} \tag{6}$$

where $\delta \bar{u}$, $\delta \bar{v}$, and $\delta \bar{w}$ are virtual displacement in the x , y , and z directions, respectively. A discretization of 8 node iso-parametric elements per sleeper bay is used, the EoM in the frequency–wavenumber domain is therefore written in the matrix form per Equation (7):

$$(\bar{K} - \omega \bar{M}) \bar{U} = \bar{F}_F \tag{7}$$

2.2.3. The Building Model

Conventionally, 3D FE models are built for structural analysis for buildings. However, for vibration impact concerning nuisance and comfortableness on habitants in buildings, 2.5D FE models is considered suitable for the following reasons.

- (1) The building in consideration is of simple rectangle shape, without curvature or any complex shape.
- (2) Structural columns within the office building are evenly distributed along the train running direction, which means periodical boundary conditions is applicable.
- (3) The “track-tunnel-ground” excitation source originates from the interaction between the “train-track” system. Since the train has a finite length, only the effect of one train’s length is considered herein. The finite train excitation source is assumed to be an infinite line source. The conditions assumed, here, are that the influence of the train’s length on the effects beyond a certain distance within the frequency range of interest can be considered negligible for a given observation point, e.g., a building. For example, the contribution of far-field excitation beyond 50 m to the near-field response of vibrations above 20 Hz can be considered negligible.
- (4) The frequency range of interests is between 1 Hz and 80 Hz, as per compliance standard; and the actual frequency range of interests from the majority of train-vehicle excitations and responses in the metro system is between 20 Hz and 80 Hz, which means short wavelength excitation at the wheel-rail coupled interface, which contributes the most energy in wheel-rail induce vibration, is dissipated through the transmission paths from the source to the receiver at the TOD.
- (5) The ground-borne vibration decay rate due to railway excitation is some 0.5 dB/m to 1.5 dB/m, i.e., around 10 dB attenuation with the source and receiver 10 m apart, which means that the truncation is irrelevant as long as the computational grid is large enough.

For the rest of the factors that might have an impact on the truncation from 3D to 2.5D, an equivalent parameter method has been used to transform the discontinuous 3D building into 2.5D, as described below.

The principle underlying the 2.5D FEA parameter equivalence is based on the equivalent energy and equivalent modes of the system, ensuring that the responses of the building’s excitations and observation points align with the actual 3D model. The coupling between the “track-tunnel-ground” and the “building” is transformed into 2.5D parameters

by re-interpreting the equivalent “mass-damping-stiffness” parameters in the equation of motion of the 3D building, as per Equation (8).

$$m\ddot{u} + c\dot{u} + ku = F \quad (8)$$

where m , c and k are mass, damping, and stiffness, with $\ddot{u}$, $\dot{u}$, and u being acceleration, velocity, and displacement of the structure; and F is the force.

The main parameters in the building model are:

- (1) Equivalent mass: The equivalent total mass of key structural elements such as beams, columns, slabs, and walls per unit length is equivalent to the actual total mass of the building per unit length. This is achieved by using the equivalent material density.
- (2) Equivalent stiffness: The equivalent stiffness of the 2.5D building per unit length is the same as the actual stiffness of the key structural elements of the building, specifically the equivalent Young’s modulus E and cross-sectional moment of inertia I .
- (3) Equivalent damping: Different physical material damping coefficients are used for various sections within a building structure. The damping parameters in 2.5D FE and 3D FE models are identical in this case.

Equivalent parameter values used in the 2.5D FE building model is in Section 2.2.5.

2.2.4. Excitation Force from Wheel-Rail Interaction

As shown in Figure 4, the train vehicle can be simplified and modelled as seven rigid-bodies, including the main car body, bogies, and wheels, with a total of 21 degrees-of-freedom, two translational, and one rotational for each rigid body, under wheel-rail interaction force in the vertical direction, i.e., z direction in Figure 3.

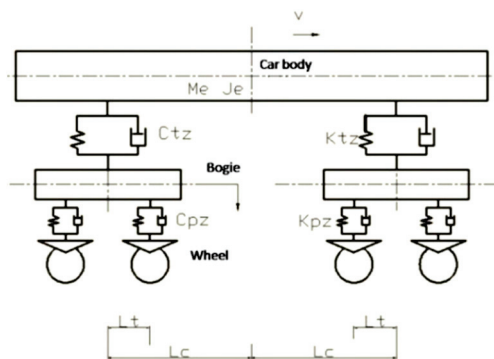


Figure 4. The simplified train vehicle model.

The EoM of the train vehicle can be described by Equation (9):

$$M_T\ddot{U}_T + C_T\dot{U}_T + K_TU_T = F_T(x, t) \quad (9)$$

where M_T , C_T , and K_T are the mass, stiffness, and damping matrices, respectively, while U_T is the configuration parameter related to the track-foundation subsystem. F_T represents the forces acting on the track, including the wheel-rail contact forces. The receptance of the wheel displacement of the train in the frequency domain can be obtained according to Equation (10):

$$A_T(x_{ij}) = \frac{U_{Ti}(\omega)}{F_{Tj}(\omega)} \quad (10)$$

where $A_T(x_{ij})$ is the receptance of wheel i with loading force applied at wheel j ; and A_T is the receptance matrix at the wheels of the train vehicle.

The total wheel-rail force, according to Thompson [1], comprises contributions from the receptance of wheel A_T , the receptance of rail A_r and that of wheel-rail contact forces A_C , and can be expressed by Equation (11).

$$F_T = \frac{R}{A_T + A_r + A_C} \quad (11)$$

2.2.5. Input of the Numerical Model

The double-deck railway depot was constructed with computation points along the main vibration transmission path (Figure 5). In Figure 5b, computation point A is on the ground at 7.5 m from the track centre-line on the “0 m plate”; point B is on the structural column at the “0 m plate”; and point C is on the column at the “9 m plate”, all calculated in the vertical direction.

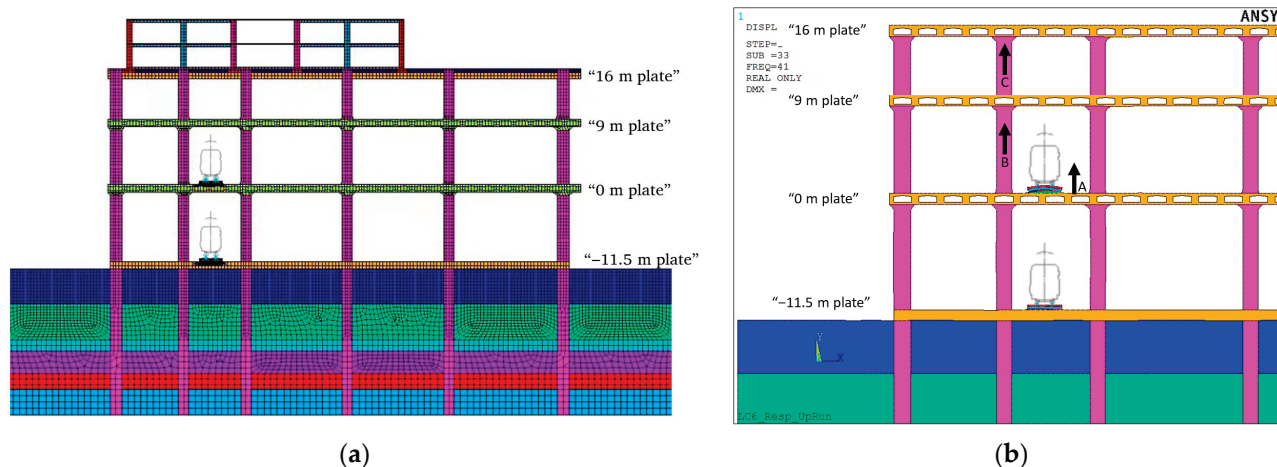


Figure 5. The FE model of the two-storey office building above the throat zone of the depot: (a) the cross-sectional view and (b) computation points along the main vibration transmission path.

Type A train vehicle was used in this study. Properties of the train vehicle are summarised in Table 2. As used in most metro railway depots, the rail is standard CN 50 rail, the mass of which is 50 kg/m. Material properties of key components of the track structure and track-bed are summarised in Table 3. Material properties of key components of the metro plates and key structural elements in the building are summarised in Table 4.

Roughness profile R of rails, for calculating the wheel-rail interaction force, consists of two parts. For wavelengths between 1 m and 300 m, R takes the value from the roughness profile in the FRA manual [47]. For shorter wavelengths, typically from 10 mm to 1 m, which correspond to medium- to high-frequency excitation, site measurements of rail roughness at a similar depot according to [48], using Corrugation Analysis Trolley (CAT) [49], were used. Total roughness profile across all frequency ranges (all wavelengths) is combined and shown in Figure 6.

Table 2. Input parameters of the train vehicle in the FE model.

ID	Name of Parts of the Train Vehicle	Material Property	Unit	Quantity
1	Axle load	Without passengers	kg	11,000
2		Full load	kg	16,000
3	Car body	Mass (without passengers)	kg	23,000
4		Mass (full load)	kg	48,000
5		Length	m	24.4
6	Primary suspension	Vertical Stiffness K_V	N/m	1.229×10^6
7		Lateral Stiffness K_L	N/m	1.542×10^6
8		Vertical Damping C_V	N·s/m	1.229×10^3
9		Lateral Damping C_L	N·s/m	1.542×10^3
10	Secondary suspension	Vertical Stiffness K_V	N/m	0.227×10^6
11		Lateral Stiffness K_L	N/m	0.118×10^6
12		Vertical Damping C_V	N·s/m	0.227×10^3
13		Lateral Damping C_L	N·s/m	0.118×10^3
14	Bogie	Mass (Motor)	kg	4600
15		Mass (Trailer)	kg	2800
16		Distance between bogie centres (2Lc)	m	15.7
17	Wheelset	Unsprung mass (Motor)	kg	1590
18		Unsprung mass (Trailer)	kg	1140

Table 3. Material properties of the track-bed in the 2.5D FE model.

ID	Item	Young's Modulus (GPa)	Poisson's Ratio	Density ($\text{kg}\cdot\text{m}^{-3}$)	Damping Ratio
1	50 kg/m rail	206	0.3	7850	0.005
2	Concrete slab	34.5	0.2	2500	0.05
3	Resilient mat	3×10^{-3}	0.45	2.25×10^{-1}	0.075
4	Standard rail fasteners	7.35×10^{-3}	0.45	1000	0.05
5	Resilient rail fasteners	2.83×10^{-3}	0.45	1000	0.075
6	Ballast	1×10^{-8}	0.35	1800	0.05
7	Steel Spring	3.83×10^{-3}	0.3	1000	0.001
8	Ground	1×10^{-1}	0.25	1500	0.25

Table 4. Material properties of the depot and the building in the 2.5D FE model.

ID	Item	Young's Modulus (GPa)	Poisson's Ratio	Density ($\text{kg}\cdot\text{m}^{-3}$)	Damping Ratio
1	Depot plates (0 m, 9 m, and 16 m plates)	34.5	0.2	2700	0.05
2	Depot structural columns	82.8×10^{-3}	0.2	85	0.05
3	Building walls	33	0.2	2700	0.05
4	Building floor slabs	33	0.2	2700	0.05

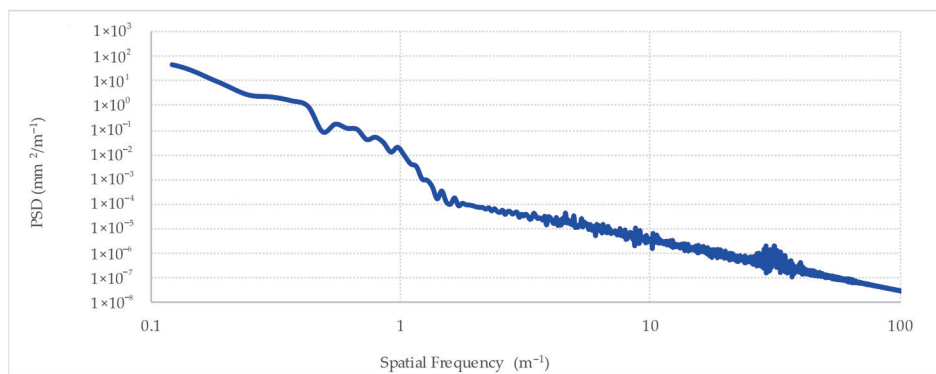


Figure 6. PSD of rail roughness R in the vertical direction.

2.2.6. Boundary Conditions

In the FE model, the foundation depth of the depot track-bed was set to be 20 m (Figure 5a), with three-dimensional visco-elastic artificial boundary conditions.

Dynamic analysis was performed throughout from the underground railway depot to its above TOD building structures.

2.2.7. Validation of the Numerical Model

Numerically predicted total vertical vibration level on the “16 m plate”, directly above the vibration source, was compared against field data collected at the depot roof plate, at 0 m from the track centre-line above a track of the same track-form, at a similar railway depot (Figure 7). Measurement points and device setup are shown in Figures 8 and 9, respectively.



Figure 7. Photos of site measurement at a similar depot: (a) an aerial photo of the TOD and (b) noise and vibration measurement devices on the ground of the TOD.

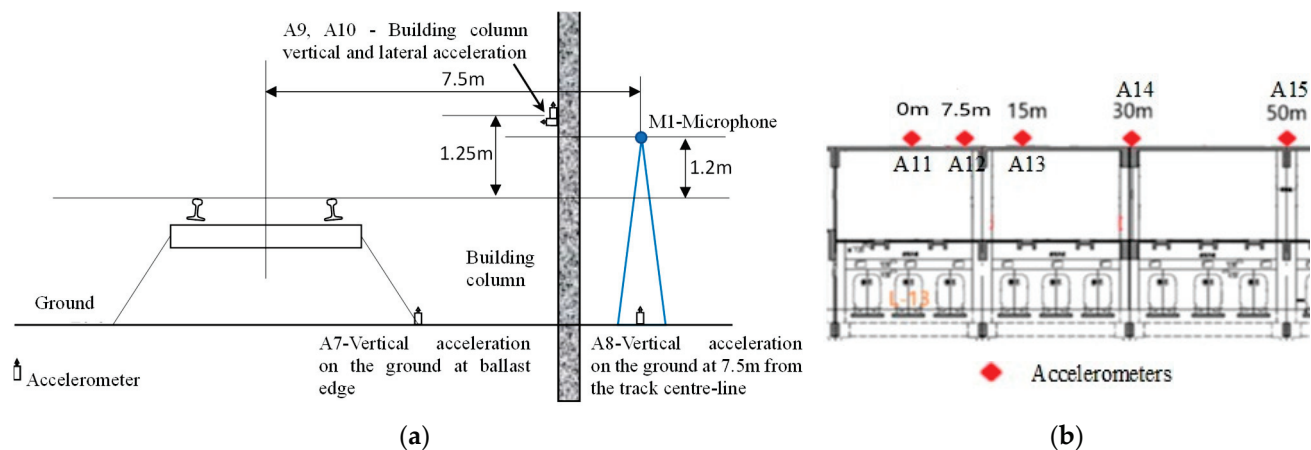


Figure 8. Site measurement points: (a) At vibration source and (b) on the ground of the depot roof plate.

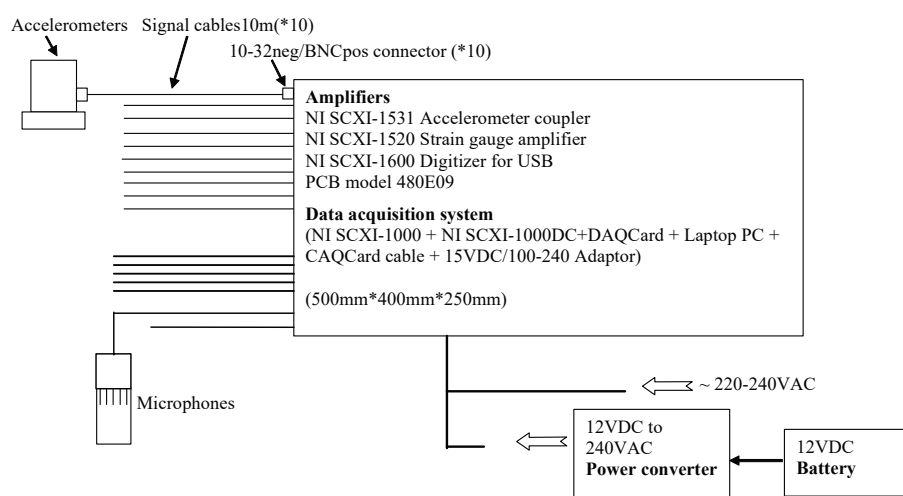


Figure 9. Schematic diagram of the data collection system.

Vibration levels at the throat zone feathering rail switches were measured using PCB type low frequency high sensitivity accelerometers and were recorded with NI data acquisition system. Appropriate sensors were assigned at relevant locations to account for different frequency ranges, as per Table 5. For instance, sensors with a 0.5 g pk measurement range were chosen for low frequency vibration measurement on the ground of building plates (1–80 Hz as per ISO 2631-1:1997 [3]). In other words, the lower the measurement frequency range, the higher the measurement accuracy needs to be achieved.

Table 5. Measurement sensor properties.

Sensor ID	Measurement Location	Measurement Range	Sensitivity	Frequency Range
A1–A4	Rails	± 500 g pk	$(\pm 10\%)$ 10 mV/g	$(\pm 5\%)$ 0.5 to 10,000 Hz
A7–A10	On the ground and at the columns at the source level (Figure 8a)	± 5 g pk	$(\pm 5\%)$ 1000 mV/g	$(\pm 5\%)$ 0.5 to 2000 Hz
A11–A15	On the ground of the depot roof plate (Figure 8b)	± 0.5 g pk	$(\pm 10\%)$ 10,000 mV/g	$(\pm 5\%)$ 0.15 to 1000 Hz

Frequency weighted vibration spectra and total vibration levels from the numerical model and from site measurements are shown in Figure 10. Vibration energy is summed up and presented in 1/3 octave bands, in terms of vibration amplitude in decibels (dB). As per ISO 2631-1:1997, total vibration level L_{dBZ} is obtained by applying frequency weighting W_k along the z-axis in the vertical direction.

Both sets of data show a good agreement, in terms of the total value and the vibration contents in each 1/3 octave band. Therefore the 2.5D FE model is considered successfully validated and is ready to be used for further study.

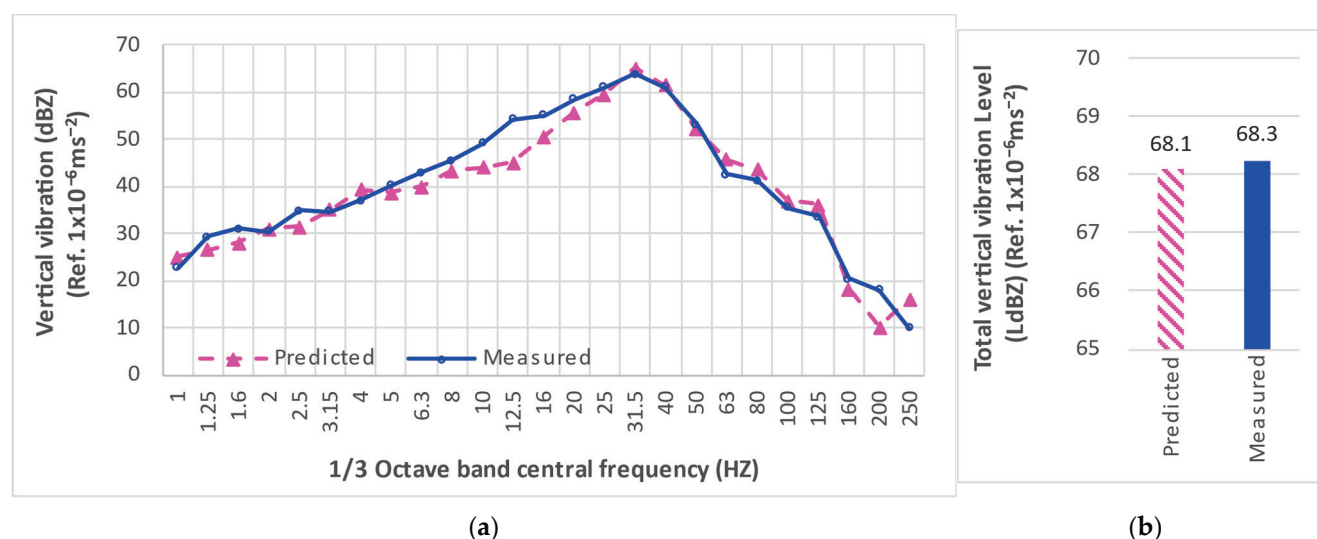


Figure 10. Comparison between predicted vibration level and site measurement: (a) 1/3 octave band spectra; (b) total vibration levels.

3. A Parametric Study of the Vibration Mitigation Effects of Track-Forms

3.1. Track-Forms and Properties

Impact vibration at the throat zone of the depot is mainly caused by switches and rail joints for rails to merge and split. The lateral and torsional impact forces are much more significant compared to those of main-lines, it is more economic and efficient to reduce such vibration at source. Table 6 consists of commonly used track-forms in the metro system in China. Standard concrete slab tracks with non-resilient rail fasteners and concrete sleepers, typically of 10–12 m in length, was used as the reference track-form, i.e., standard track-form without vibration mitigation measures. Ballast provides good vibration isolation, but usually requires regular and sometimes intensive maintenance works, hence higher life-cycle costs. Vibration isolation mat is in general a long-term solution but is almost non-replaceable. Resilient rail fasteners provide some reasonable amount of vibration isolation, hence an economical solution if the vibration attenuation requirement is relatively low. Steel Spring Floating-Slab-Track (SSFST), on the other hand, offers a good level of vibration isolation, but it is expensive and should be applied only where necessary.

Four combinations of track-forms on the upper- and lower-decks were designed as shown in Table 7. In theory, vibration level on the upper-deck contributes the most to vibration levels in buildings at TOD. This has been confirmed by simulating trains running on the upper-deck only, on the lower-deck only, and on both decks. The vibration attenuation capacity of the track-forms in Table 7 is in a gradually increasing order. For the parametric study, a reasonable train speed of 15 km/h was assumed for the throat zone.

Table 6. Track-form list.

Track ID	Track-Form	Formation	Key Parameters
A	Slab track	Standard rail fasteners + concrete sleepers + concrete slab	Reference track-form with no resilient component, under the traffic of an empty type A train vehicle;
B	Ballast track	Standard rail fasteners + concrete sleepers + ballast track-bed	Sleeper dynamic displacement with reference to the ground-level = 0.5 mm; Sleeper equivalent static stiffness = 30 kN/mm;
C	Ballast track with under-mat	Standard rail fasteners + concrete sleepers + ballast track-bed + ballast mat	Track-bed dynamic displacement with reference to the ground-level = 1.0 mm;
D	Slab track with resilient rail fasteners	Resilient rail fasteners + concrete slab	Rail dynamic displacement with reference to the sleeper-level = 1.0 mm; Resilient fastener equivalent static stiffness = 15 kN/mm;
E	Steel Spring Floating-Slab-Track (SSFST)	Standard rail fasteners + Steel Spring Floating-Slab-Track	Slab dynamic displacement with reference to the ground-level = 2.0 mm;

Table 7. Different combinations of track-forms for the upper and lower decks of the depot.

Combo-Track ID	Upper-Deck	Lower-Deck	Track ID
0	Slab track	Slab track	A + A
1	Ballast track with mat	Slab track with resilient rail fasteners	C + D
2	SSFST	Slab track with resilient rail fasteners	E + D
3	SSFST	Ballast track with mat	E + C

3.2. Vibration Contributions from Trains Running on the Upper- and Lower-Deck

Energy contribution of trains running on the upper-deck only; the lower-deck only was also examined.

Total vertical vibration levels at computation points along transmission path (Figure 5b) under the following conditions are summarised in Table 8:

- Test train running on the upper-deck only (TUD);
- Test train running on the lower-deck only (TLD);
- Test trains running on both the upper- and lower-decks simultaneously (TBD).

When trains are running on both the upper- and lower-deck simultaneously, by subtracting vibration response when the train runs on the upper-deck only, from vibration response when trains run on both decks, vibration contribution from the lower-deck (column “TBD-TUD”) as shaded in Table 8 can be obtained. With a baseline vibration level of at least 57.2 dB at point A at the source, vibration contribution from the lower-deck ranges from 0.2 dB to 0.7 dB, considered negligible. The finding confirms that the upper-deck rail tracks require more intensive vibration mitigation measures than the lower-deck rail tracks. Therefore, the rail track design in Table 7 is found to be sensible, with higher vibration mitigation measures such as SSFST at the upper-deck, and mainly resilient rail fasteners for tracks at the lower-deck.

3.3. Vibraton Attenuation at Sensitive Receivers

Vibration levels in the rooms of the two-storey office building in Figure 2c were computed for the comparison of vibration attenuation by different track-forms.

Vibration levels were predicted at computation points closest to the track centre-line, as shown in Figure 11. Calculation points 19034, 19040, 19189, and 19195 are on the floor in the rooms directly above the track centre-line and trains. Total vertical vibration levels at computation points in the building, as well as along the vibration transmission path as per Figure 5b, are summarised in Table 9.

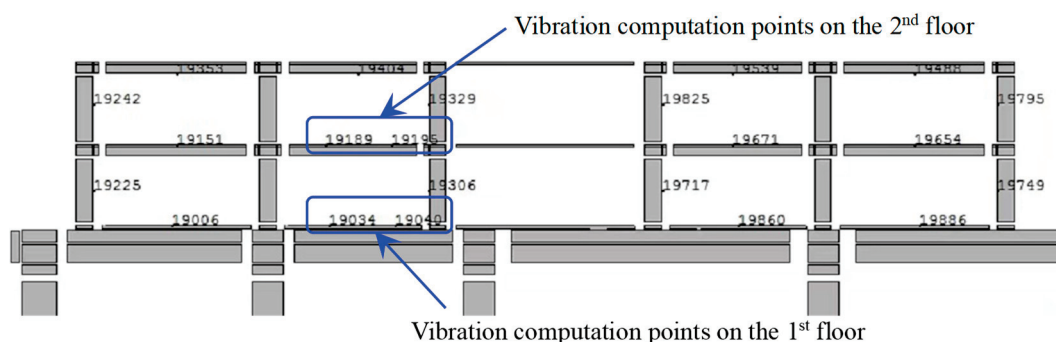


Figure 11. Vibration computation points in the office building.

Total vibration levels on the second floor of the office rooms are slightly higher than the corresponding calculation points underneath on the first floor of the building. This was due to the vibration frequency range being close to the modal frequency of the floor slab of the 2nd floor room and can be alleviated by re-arranging the modal frequency of the floor slab, by changing the thickness of the floor, amongst other means.

From the prediction, the use of ballast track with mat on the upper-deck tracks provides an overall 4.7 dBZ to 8.3 dBZ vibration attenuation effect. The use of SSFST on tracks of the upper-deck provides a vibration attenuation effect between 6.6 dBZ and 8.5 dBZ.

A comparison of the vibration spectrum in 1/3 octave frequency band at location 19034, i.e., in the middle of the room of the first floor office directly above the track centre-line, using the four combinations of track-forms is plotted in Figure 12. Using track-form option 0, i.e., the reference track-form without resilient track components, gives an overall highest vertical vibration in every frequency band. The use of SSFST on tracks on the upper deck provides a relatively good vibration isolation in the lower frequency range.

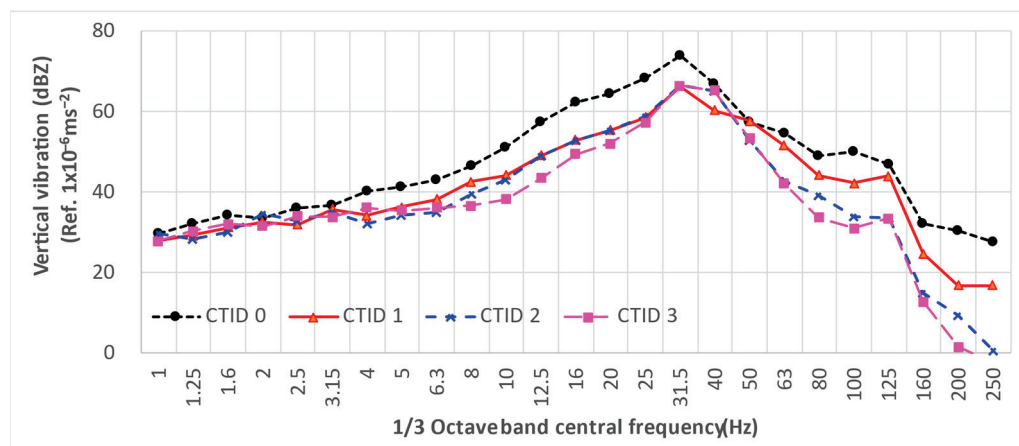


Figure 12. Vibration spectra at computation point 19034 in the office building.

Table 8. Vertical vibration responses with different track-forms and train running conditions.

Combination of Track-Forms		Total Vertical Vibration Level (LdBZ Ref. $1 \times 10^{-6} \text{ ms}^{-2}$)									
Combo-Track ID	Upper-Deck	Lower-Deck	Computation Point A			Computation Point B			Computation Point C		
			TUD	TLD	TBD	TUD	TLD	TBD	TUD	TLD	TBD
0	Slab track	Slab track	71.5	62.7	72.0	0.5	72.3	61.8	72.6	59.7	69.2
1	Ballast track with mat	Slab track with resilient rail fasteners	66.5	57.2	67.0	0.5	68.3	56.4	68.6	54.5	62.9
		Slab track with resilient rail fasteners	65.7	57.2	66.3	0.6	66.3	56.4	66.7	54.5	62.4
3	SSFST	Ballast track with mat	65.7	58.2	66.4	0.7	66.3	54.1	66.5	50.8	62.0

Table 9. Vertical vibration responses from vibration source transmitted to receivers.

Combination of Track-Forms		Total Vertical Vibration Level (LdBZ) (Ref. $1 \times 10^{-6} \text{ ms}^{-2}$)									
CTID	Upper-Deck	Lower-Deck	Through Transmission Path			In the Building			Vibration Attenuation (LdBZ)		
			Ground	Columns	C	1st Floor	2nd Floor	1st Floor	2nd Floor	1st Floor	2nd Floor
0	Slab track	Slab track	72.0	72.6	69.2	76.3	73.7	77.5	77.7	-	-
1	Ballast track with mat	Slab track with vibration isolation fasteners	67.0	68.6	62.9	68.7	66.2	69.2	69.6	-7.6	-8.1
		Slab track with vibration isolation fasteners	66.3	66.7	62.4	69.7	66.5	69.7	70.1	-6.6	-7.6
3	SSFST	Ballast track with mat	66.4	66.5	62	69.4	65.8	69	69.4	-7.9	-8.3

As expected, the use of SSFST features the peak vibration response in the low frequency range between 25 Hz and 50 Hz; and the use of under-ballast mat and rail resilient fasteners feature another peak vibration response at around 125 Hz.

3.4. Validation of the Computation

After the completion of construction of the depot and the TOD, site measurements in the rooms of the office building have been conducted, for comparison to numerical simulation results.

The final design of the track-form is steel spring floating-slab-track on the upper deck, and slab track with resilient rail fasteners on the lower deck. As the track works of the lower-deck is yet to be finished, measurements were undertaken when trains running on the tracks at the upper-deck below the office building. The train was scheduled to run 5 rounds northbound and 5 rounds southbound, that is a total of 10 train passing recordings, at an average train speed of 20 km/h. According to the Chinese national vibration measurement standard GB 10071-1988 <Measurement method of environmental vibration of urban area> [50], the 10 total vibration level readings are averaged to obtain the vibration level. To account for the train running modes and vibration features of southbound and northbound trips, 5 records of each direction was averaged. Site photos are shown in Figure 13.

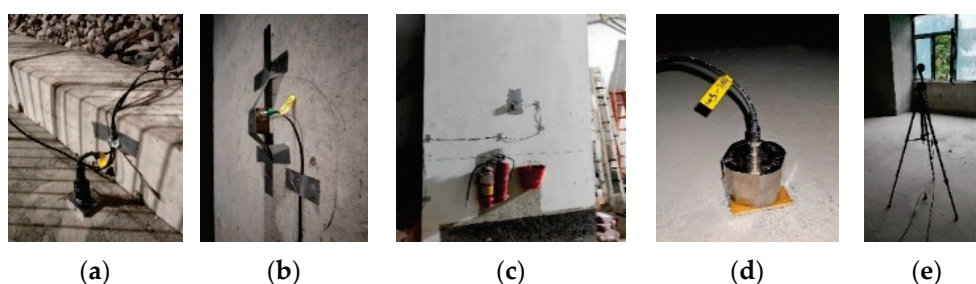


Figure 13. Measurement devices at source and receiver rooms: (a) Accelerometer at 7.5m from the centre-line of the test track; (b) accelerometer on the structural column next to the test track; (c) accelerometer on the 9m plate structural column; (d) accelerometer on the floor slab in 2F room; and (e) microphone and accelerometer in 2F room.

Site measurement results, together with numerical prediction at the same location are summarised in Table 10. It can be seen that in general, southbound trains cause higher vibration levels than northbound trains. Measured data are somehow different from predictions, which can be due to a combination of factors such as uncertainties in train speed in terms of the assumption in the numerical model, and variation in train speed during testing.

Table 10. Total vertical vibration level at different locations.

Location	Numerical Simulation	Total Vertical Vibration Level (LdBZ Ref $1 \times 10^{-6} \text{ ms}^{-2}$)			% Difference
		Southbound	Site Measurement Northbound	Average	
Point A (0m plate 7.5 m ground)	65.7	77.4	70.4	73.9	12%
Point B (0 m plate column)	66.3	60.4	52.9	56.7	15%
Point C (9 m plate column)	61.7	65.3	61.3	63.3	3%
2nd floor office room	69.7	64.5	58.6	61.6	12%

In addition, noise and vibration measurement at the source location, i.e., point A and point B along the cross-section at the test track, was at a certain degree, disrupted by

surrounding environment, concurrent construction work onsite. Vibration levels inside the 2nd floor office room was lower than the prediction level and was well below the 75 dBZ upper limit in relevant standard, which was satisfactory to the property developer.

4. Conclusions

This part of the research work entails a preliminary study employing computational modelling to:

1. Evaluate vibration contribution from trains running simultaneously on the upper- and lower-deck of a double-deck metro-line depot; this output gives an indication for the design of rail track-forms on the upper- and lower-deck.
2. Examine receiver-end ground-borne vibration of using different track-forms representing different vibration mitigation measures at the source.
3. Comparison of numerical simulation results and site measurements.

Key findings and conclusions are summarised below:

1. Outputs from the 2.5D FE model in the study have been successfully validated against site measurements from a similar metro-line depot. Therefore, the model is adequate for the prediction of vibration transmission from the railway depot to the TOD built above the depot.
2. It has been found that when trains run simultaneously on the upper- and lower-deck of the railway depot, vibration impacts from the train-track interaction on the lower-deck contributes no more than 1 dB, thus can be considered negligible.
3. Compared to the reference track-form, using ballast track with vibration isolation mat on the upper-deck may provide up to 8 dB vibration insertion loss, whereas Steel Spring Floating Slab Track (SSFST) provides a similar level of vibration attenuation, i.e., no more than 1 dB difference. Ballast track with isolation mat is therefore a cost-effective option considering its lower capital cost.

Above findings can be used as general guidance in track design against vibration impact for buildings constructed on top of double-deck metro-line depots.

Vibration transmission from train-track source, through structural elements of a double-deck metro depot to closest receiver, i.e., a two-storey office building above the depot, using a successfully validated numerical model has been predicted. Vibration attenuation effects and trends using different track-forms, from non-resilient, to medium- to highly resilient track structures have been compared. Vibration levels measured onsite after the completion of the construction of the depot have been compared with numerically predicted results. Discrepancies were within 15% and were caused mainly by concurrent site construction works and the varying of actual train speeds.

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Abbreviations

The following abbreviations are used in this manuscript:

TOD	Transit oriented development
PSD	Power spectral density
PCB	PCB accelerometers
SSFST	Steel spring floating slab track
TUD	Test train running on the upper-deck only
TLD	Test train running on the lower-deck only
TBD	Test trains running on both the upper- and lower-decks simultaneously

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Article

Wind Load Distribution Characteristics of a Semi-Enclosed Sound Barrier at the Junction of a Single-Track Bridge and Three-Track Bridge of a High-Speed Railway

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Abstract: Due to its effective noise reduction, the semi-enclosed noise barrier is increasingly being applied in the construction of high-speed railways. However, there is still a lack of systematic research on the wind load distribution characteristics under natural crosswind, especially for the complex aerodynamic behavior of the intersection section of multi-line bridges. Therefore, the wind load distribution characteristics on the surface of the sound barrier under crosswind conditions are explored within the engineering context of a semi-enclosed acoustic barrier at the junction of a single-track bridge and a three-track bridge, using a combination of wind tunnel testing and numerical simulation. A rigid-body model with a geometric scale of 1:10 is established for the wind tunnel test. The wind load distribution characteristics of the two acoustic barriers are analyzed from the perspectives of mean wind pressure, pulsating wind pressure, and extreme wind pressure, respectively. FLUENT 2022 software is utilized to model the flow field characteristics of the sound barrier under two working conditions: windward and leeward. The results show that under the action of crosswind, the surface wind load of the sound barrier at the junction of the single/three-line bridge is very prominent, the maximum negative pressure shape coefficient is -4.516 , and its distribution is dominated by negative pressure; that is, the sound barrier mainly bears suction. Compared with the semi-closed sound barrier on the single-track bridge, the extreme wind pressure at the semi-closed sound barrier on the three-track bridge and the junction of the two is more significant, which shows that this kind of area needs special attention in wind-resistant design.

Keywords: railway bridge; semi-enclosed noise barrier; experimental study; numerical simulation; wind load

1. Introduction

With the rapid development of China's high-speed railway and the growing demand for passengers' travel comfort, traffic noise control has become a key link in line design and environmental governance. As a sound insulation structure between the sound source and the receiver, the sound barrier is widely used in the reconstruction of existing lines and new construction projects [1–3]. The traditional vertical sound barrier is limited by height and form, which makes it difficult to meet the noise reduction requirements of higher floors and special noise-sensitive areas. For the semi-closed sound barrier, noise can overflow due to the open side of the structure, and the diffraction area formed by the overflow noise is difficult to interfere with on the closed side [4,5]. Therefore, the semi-closed sound barrier

has dual functions of ensuring the sound comfort inside the vehicle and protecting the sound environment of high-rise buildings on the side of the line, and is especially suitable for noise control of complex bridges and urban railways.

Abbott J P et al. [6] found through experiments that the wind speed amplification effect of the wind fence enclosures and the problem of infrasound noise reduction cannot be ignored. Belloli et al. [7] combined numerical simulation and experimental methods to study the complex fluid–solid coupling phenomenon that occurs when a vehicle passes through the sound barrier. Then, the corresponding theoretical research was gradually improved. Zhang et al. [8] derived the calculation formula for the wind load change on the train body when the train enters or leaves the wind barrier structure and established the dynamic analysis model of the wind–vehicle–bridge system. Based on theoretical research, many scholars have conducted various experimental and numerical simulation studies on the wind load effect of high-speed railway sound barriers. The study by Lü et al. [9] analyzed the influence of EMU speed, the distance between the train and the sound barrier, and the type of EMU on the aerodynamic load characteristics of the noise-reducing barrier through experimental methods. Taking the fully enclosed sound barrier as the research object, researchers explored the influence of wind speed, Reynolds number effect, wind angle of attack, and test section location on the surface wind load of the fully enclosed sound barrier using sectional model wind tunnel tests [10]. They indicated that the Reynolds number effect has a significant influence on the crosswind aerodynamic characteristics and the wind pressure distribution at the top of the fully enclosed sound barrier. The wind angle of attack has a substantial influence on the drag coefficient. Reference [11] conducted a numerical simulation of the air pressure load on an 840 m long fully enclosed sound barrier of a high-speed railway. They demonstrated that the extreme pressure under opposing-traffic conditions is greater than that under single-vehicle conditions, and the variation pattern is more complex. The pressure reaches its maximum when the middle position of the fully enclosed sound barrier is aligned with the vehicle. Yang et al. [12] revealed the pressure relief mechanism of the pressure relief hole on the fully enclosed sound barrier and the buffering mechanism of the buffer structure at the end of the semi-enclosed sound barrier from the perspective of the flow field. Wu et al. [13] also investigated the fatigue life prediction of the semi-enclosed sound barrier of high-speed railways under wind load and proposed a new method for predicting the fatigue life of the sound barrier. Deng et al. [14] used the improvement of the delayed detached-eddy simulation (IDDES) method based on the SST k - ω turbulence model for numerical calculation and the overlapping grid method to realize the relative movement of trains, and studied the influence of the wind barrier of the bridge on the aerodynamic characteristics of high-speed trains when they meet under crosswind. Moreover, to explain the mechanisms and conditions for the occurrence of VIVs in the main girder affected by water-filled barriers, Liang et al. [15] studied the effect of water-filled barriers on the Vortex-Induced Vibration performance of suspension bridges.

Accordingly, most existing research focuses on the wind load characteristics of the fully enclosed sound barrier under the action of train-induced fluctuating wind [3,7,9,16,17]. For the semi-closed sound barrier, although the load caused by the fluctuating wind of the train is relatively small, the effect of natural wind is extremely significant. In addition, compared to the traditional plug-in sound barrier, the external sound barrier is only fixed on the outside of the steel structure, resulting in relatively poor wind resistance performance and low structural reliability under load conditions with high train running speeds and significant aerodynamic wind pressure effects. Therefore, it is necessary to evaluate the wind resistance of the semi-enclosed external sound barrier system.

Furthermore, compared with the above references, this research fills the research gap under specific bridge conditions and adopts a multi-dimensional research method.

Firstly, innovation of research perspective: from “influence caused by a single train [12]” to “wind load under complex bridge conditions”. Secondly, deepening of engineering background: from “single bridge type, and sound barrier on conventional bridges [4]” to “complex intersection area”. Thirdly, integration of research methods: from “single means” to “joint verification of wind tunnel test and CFD”, and “local wind load concentration” is revealed. Finally, sublimation of research objectives: from “performance analysis and focuses on the bridge’s vibration [13]” to “wind-resistant design guidance”. To sum up, this research, through the three-dimensional research framework of “complex bridge condition (single/three-line bridge junction), unique load (natural crosswind), and comprehensive method (test and simulation)”, fills the research gap on the distribution law and mechanism of extreme wind pressure of semi-closed sound barrier under such complex geometric boundaries at the intersection of multi-line bridges. The specific works are as follows:

This paper studies the wind-induced response of the semi-enclosed external sound barrier on a high-speed railway bridge at the junction of a single-track bridge and a three-track bridge. It employs the wind tunnel test method to systematically investigate the wind load distribution characteristics of the sound barrier by analyzing the patterns of average wind pressure, fluctuating wind pressure, and extreme wind pressure on its surface. Then, with the aid of FLUENT software, the SST $k-\omega$ turbulence model is employed to simulate the wind field environment of the semi-enclosed sound barrier on the single-track bridge and the three-track bridge in both windward and leeward directions, and to obtain the flow field characteristics under the two flow conditions. It not only reveals the influence mechanism of different bridge types and incoming flow directions on the wind pressure distribution, but also strongly proves the extreme of local wind load. The research results can provide a reference for wind-resistant design and structural optimization of a semi-closed external sound barrier under complex bridge conditions for high-speed railways.

2. Materials & Methods

2.1. Introduction of Engineering Project

A 1550 m semi-enclosed external sound barrier is installed for noise control near Dongping Station on the Xining-Huangshengguan section of the new Xining-Chengdu Railway Line. Among these, the length of the semi-enclosed sound barrier on the single-track bridge is 350 m, and the height of the bridge is approximately 40 m. The length of the semi-enclosed sound barrier on the three-track bridge is 1200 m, and the bridge height ranges from approximately 10 to 30 m. The design speed of the Xining-Chengdu Railway is 200 km/h; however, due to the line conditions within the mileage range of the semi-enclosed sound barrier, the maximum operating speed is expected to be 120 km/h. The cross-sectional form of the project’s single-track and three-track bridges is shown in Figure 1.

The semi-enclosed sound barrier structure adopts a portal frame configuration with a total frame height of 8.7 m. The vertical sections and the top of the sound barrier are constructed entirely from metal sound barrier panels. The semi-enclosed sound barrier columns for the single-track bridge have a cross-section of HW 250 × 250 × 9 × 14, with a spacing of 2 m along the bridge. The contact network foundation is installed separately, and the sound barrier foundation is connected to the side wall of the bridge by six M27 U-type high-strength bolts. The semi-enclosed sound barrier columns for the three-track bridge have a cross-section of HW 400 × 300 × 10 × 16, with a longitudinal spacing of 2 m. The contact network columns and sound barrier columns on the inside of the bridge sidewalls are integrated to save space, and eight M30 U-type high-strength bolts are utilized to firmly connect the bridge sidewalls to the full-frame sound barrier foundation of the three-track bridge.

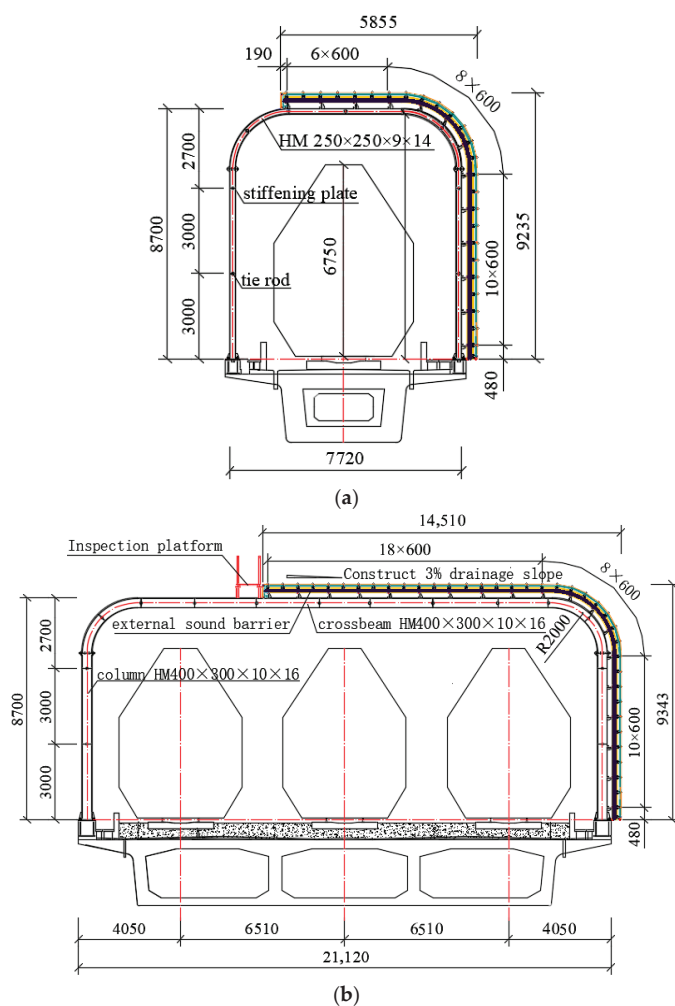


Figure 1. Cross-section diagram of semi-enclosed sound barrier (unit: mm): (a) Single-track bridge; (b) Three-line bridge.

2.2. Wind Tunnel Test Model

The wind tunnel test was conducted in a boundary layer wind tunnel with main technical indices that reached an advanced international level. The dimensions of the test section of the wind tunnel are 36 (L) $\times$ 22.5 (W) $\times$ 4.5 m (H), and the adjustable range of experimental wind speeds is 1.0–16.5 m/s. The wind tunnel is equipped with a simulation device that accurately reproduces the atmospheric boundary layer characteristics of the region where the structure is located through measures such as rough elements and baffle settings. A rotating disk at the base of the wind tunnel allows for wind direction adjustment from 0° to 360°, accommodating the complex requirements of the wind tunnel experiments.

After comprehensive analysis of the wind tunnel test section size and the geometric dimensions of the sound barrier structure to be simulated, the geometric scale of the test model was set at 1:10, which meets the balance between Reynolds number similarity and model detail capture. Based on the provided design drawings, the model was 3D printed using metal pipes, composite materials, plexiglass, and other materials, in accordance with geometric similarity requirements. As the model is a rigid-body pressure measurement model, aerodynamic shape similarity was the primary consideration. The model's blockage ratio (Corrected by the classical correction formula) in the wind tunnel is less than 5%, meeting wind tunnel test standards. The semi-enclosed sound barrier models of the single-track bridge and three-track bridge installed in the wind tunnel are shown in Figure 2. Both

sound barrier models are 4 m in length. The model's height is 30 cm above the ground, equivalent to the actual pier height of 3 m.



Figure 2. Experimental model.

Wind pressure measurement points in the cross-section are arranged on both the inner and outer sides of the sound barrier. The surface of the sound barrier is partitioned to facilitate subsequent analysis and application. Figure 3 illustrates that there are 47 wind load measurement points for wind barriers on the single-track bridge, which are divided into 12 zones and numbered A1 through A12. For wind barriers on the three-track bridge, there are 71 wind load measurement points, which are divided into 18 zones and numbered A1 through A18.

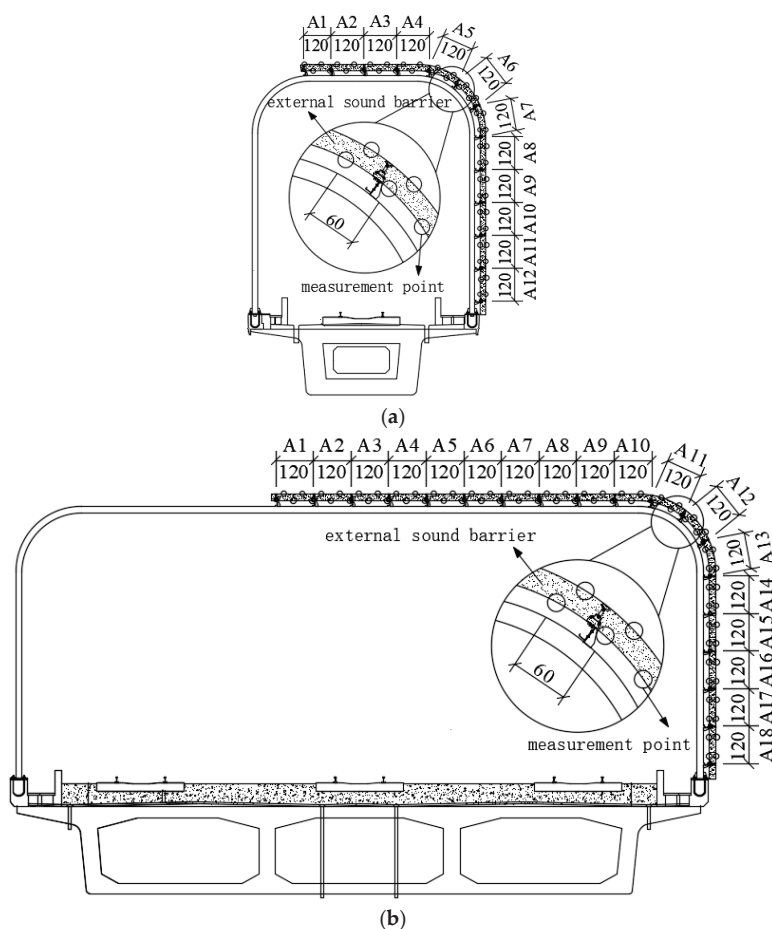


Figure 3. Cross-sectional layout of semi-enclosed acoustic barrier measurement points (unit: mm): (a) Single-track bridge; (b) Three-track bridge.

The vertical arrangement of the measurement points is shown in Figure 4. The measurement points on both the single-track and three-track bridges are arranged in 10 columns along a longitudinal interval of 200 mm. These points are positioned 20 mm on the left and right sides of the junction.

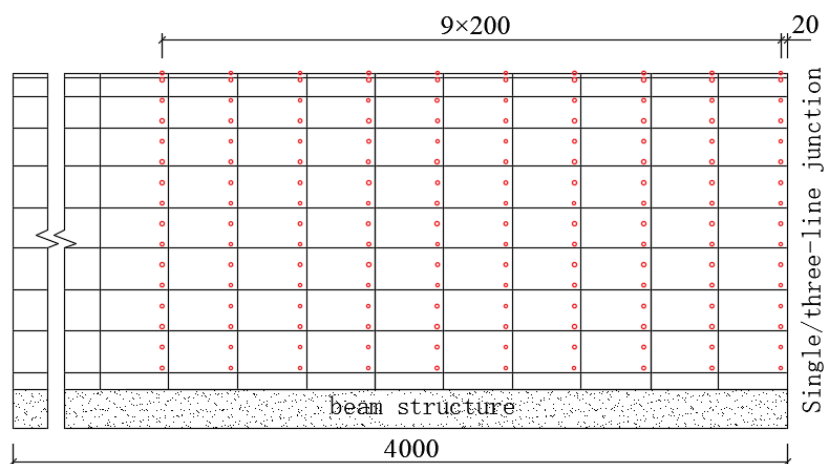


Figure 4. Elevation layout of measuring points (unit: mm).

The model is installed on the turntable of the wind tunnel test section. Each point is sampled for 60 s at 256 Hz, allowing for the simultaneous acquisition of pressure time histories for all measurement points. The setup of the wind direction angle and the model coordinate axis is shown in Figure 5. The incoming flow along the length direction of the sound barrier is defined as $\beta = 0^\circ$, and the wind tunnel test encompasses 36 working conditions, spaced at 10° intervals from 0° to 360° . The test wind speed is 8 m/s, and each wind direction angle is measured twice. The turbulence intensity at the height of the test section model is approximately 15%, and the lateral uniformity of the flow field is about 0.5%.

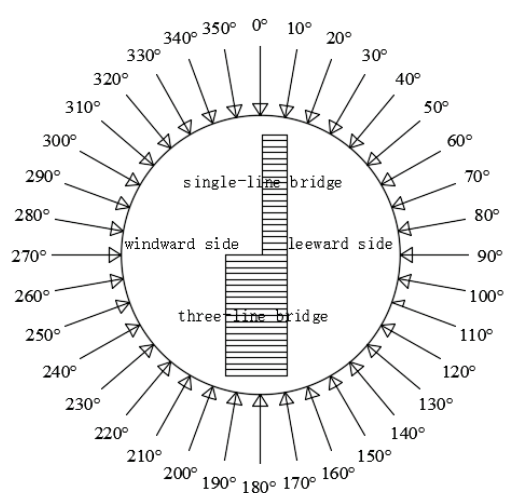


Figure 5. Schematic diagram of test wind angle.

2.3. Wind Pressure Load Test

During the test, the instrument utilized to measure the total pressure and static pressure at the flow field measurement point and the surface of the sound barrier is the

Scanvalve electronic pressure scanning valve from the United States. The wind pressure coefficient of each measurement point is calculated using the following formula:

$$C_p^i = (p_i - p_H) / (0.5\rho V_H^2) \quad (1)$$

where p_i is the pressure at each pressure measurement point of the model; p_H is the reference static pressure at the height; ρ is the air density; V_H is the average wind speed in the undisturbed area in front of the model, which is equivalent to the average wind speed at the maximum height of the sound barrier.

The mean value and root-mean-square of the wind pressure coefficients reflect the average wind pressure and the fluctuating wind pressure characteristics at pressure measurement point i , respectively, and are defined as follows:

$$\bar{C}_p^i = \sum^N C_p^i(t) / N \quad (2)$$

$$\hat{C}_p^i = \sqrt{\sum_{i=1}^N [C_p^i(t) - \bar{C}_p^i]^2 / (N - 1)} \quad (3)$$

where $\bar{C}_p^i, \hat{C}_p^i$ are the average and fluctuating wind pressure coefficients at the measurement point i , respectively; N is the number of sampling points in the pressure measurement test.

Due to the large number of measurement points and the extensive volume of data, the average along the bridge direction of the collected data is used as the final cross-section measurement partition result. The mean wind pressure coefficient and fluctuating wind pressure coefficient for each partition are calculated using the following formula:

$$\bar{C}_p = \sum \bar{C}_p^i A_i / \sum A_i \quad (4)$$

$$\hat{C}_p = \sum \hat{C}_p^i A_i / \sum A_i \quad (5)$$

where A_i is the area of each measuring point; $\sum A_i$ is the sum of the area of each measuring point.

Due to the presence of fluctuating wind pressure, it is necessary to calculate the extreme pressure distribution on the surface of the sound barrier. The extreme pressure ($\tilde{C}_p$) is expressed as the sum of the fluctuating pressure coefficient and the average pressure coefficient, that is,

$$\tilde{C}_p = \bar{C}_p \pm g\hat{C}_p \quad (6)$$

where g is the peak factor. The theory of probability and statistics indicates that when the pressure time series is a smooth Gaussian process, and the peak factor is 3.5, the actual occurrence of the instantaneous pressure does not exceed the extreme value of the pressure determined by Equation (6) at a probability level of 99.9%.

The wind pressure symbol in this study is defined as follows: relative to the surface of the semi-closed sound barrier, pressure that is upward or outward is considered negative, and pressure that is downward or inward is considered positive.

2.4. Computational Fluid Dynamic (CFD) Simulation Calculation of Ambient Wind Loads

This study simulates the wind field environment of the full-frame semi-closed sound barrier on a single-track bridge and a three-track bridge in the leeward direction (wind direction angle 90° in Figure 5) and the windward direction (wind direction angle 270° in Figure 5) to analyze the characteristics of wind pressure distribution on a sound barrier under crosswind using FLUENT software. The modeling process is shown in Figure 6.

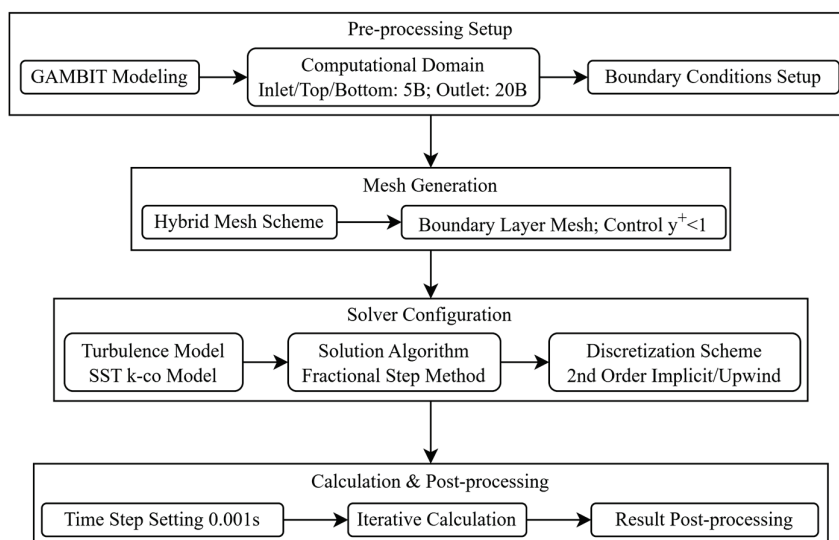


Figure 6. Flowchart of CFD simulation calculation.

The pre-processing software GAMBIT 6.3.26 is utilized to construct the numerical model. Based on the provisions of “D.2 Virtual Wind Tunnel Test Area and Boundary Conditions” in the “Code for Wind-Resistant Design of Highway Bridges (JTG/T 3360-01-2018),” the distance from the entrance and the top and bottom sides to the experimental model is set to $5B$, and the distance from the exit to the model is set to $20B$, where B is the width of the bridge segment model. The inlet of the computational domain is considered as a uniform incoming flow, and the inlet boundary condition is set to velocity-inlet. The pressure difference at the outlet is considered to be zero, and the boundary condition is set to pressure-out. The upper and lower boundaries are set as symmetrical boundaries (Symmetry). The bridge section and its ancillary components are set as non-slip solid wall boundaries (wall).

For the division of the element mesh, grid models with different densities and types were first constructed, and numerical simulations were carried out under the same boundary conditions and calculation settings. The variation in surface wind pressure coefficient with grid density was compared and analyzed. At the same time, the grid density is determined by referring to the grid division standard of similar structural numerical simulation in the relevant literature (such as the y^+ value corresponding to Reynolds number) and combining with engineering experience. The results show that when the hybrid grid scheme is adopted, when the grid density reaches a specific threshold, the calculation result of the wind pressure coefficient on the surface of the sound barrier changes by less than 5%, and the solver has high computational efficiency. At the same time, the calculation results under the grid setting are in good agreement with the wind tunnel test data, indicating that the grid setting adopted can meet the accuracy requirements of this study.

The hybrid mesh scheme considers the accuracy, speed, and flexibility of the solver. Structured meshes and unstructured quadrilateral meshes are utilized to divide the peripheral flow field and the area near the bridge, respectively. The computational Reynolds number is 5×10^4 , and boundary layer mesh control is maintained with y^+ less than 1. The mesh system is shown in Figure 7. The turbulence model selected is the SST k- ω model, which has a wide application range, high precision, good stability, and strong adaptability to flow around bluff body cross-sections. In addition, the SST k- ω model has recognized accuracy in simulating large separation flow and inverse pressure gradient in the blunt body section, which was used in the full-scale three-dimensional computational fluid dynamics simulation of the Queensferry Crossing Bridge, and the aerodynamic behavior

of the bridge midspan and central tower under the action of a real wind field is successfully simulated [18]. The coupled pressure-velocity algorithm used in the solver is the Fractional Step method, which has the same second-order accuracy as the PISO algorithm. The second-order implicit method and the second-order upwind scheme are employed for temporal and spatial discretization, respectively. The time step is set to 0.001 s to meet the Courant number requirement.

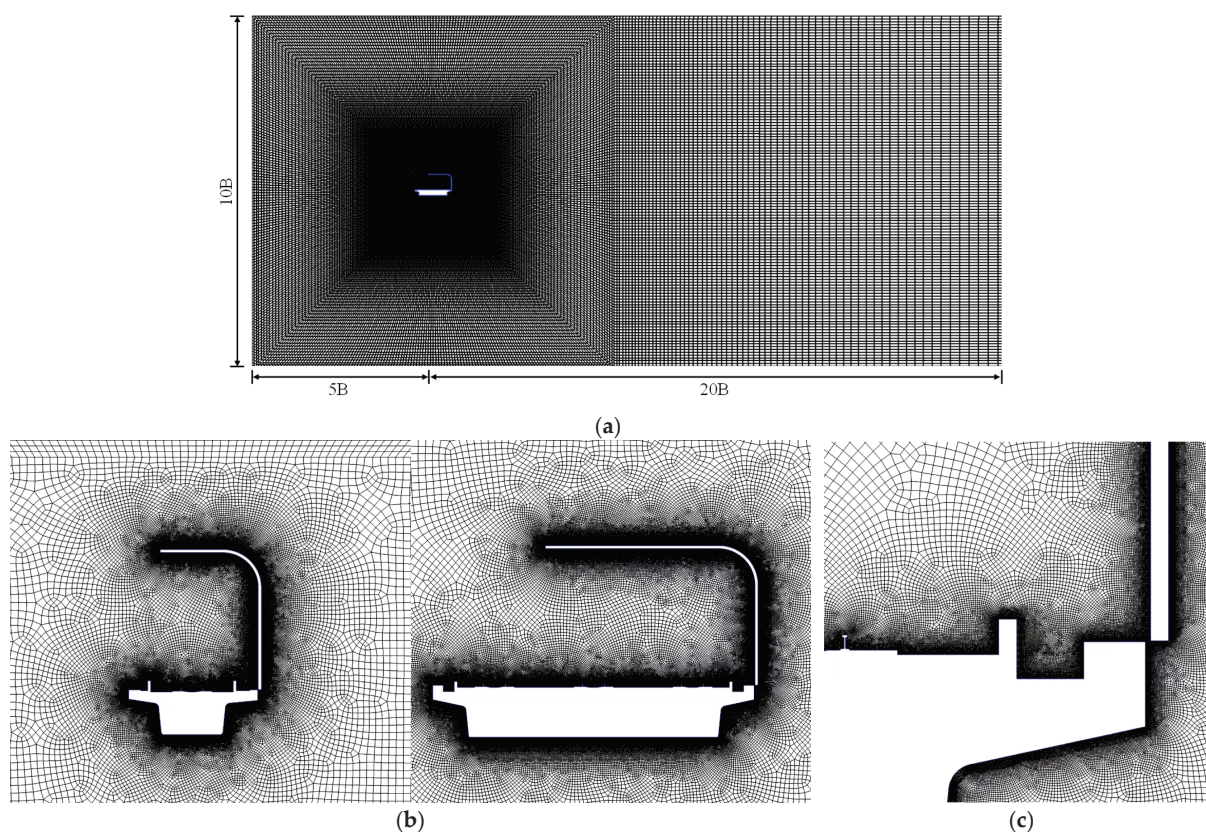


Figure 7. Finite volume model of the semi-enclosed sound barrier for the full frame of a single/three-track bridge: (a) Semi-enclosed sound barrier overall mesh (windward); (b) Single/three-track bridge full-frame semi-closed sound barrier overall mesh (windward); (c) Detailed mesh around section.

3. Test Results

3.1. Mean Wind Pressure Distribution

The characteristics of the mean wind pressure distribution on the surface of the sound barrier are reflected using the mean wind pressure coefficient. The distribution of mean wind pressure coefficients in each partition of the semi-enclosed sound barrier of the single-track bridge is shown in Figure 8.

Figure 8 indicates that with the change in wind direction angle, the area above the sound barrier of the single-track bridge (partitions 1–5) is mainly under suction, and the distribution range of the mean wind pressure coefficient is $[-2.3, 0.1]$. The side area (parts 6–12) primarily exhibits an alternating distribution of pressure and suction, with the distribution range of the mean wind pressure coefficient primarily in the range of $[-2.0, 1.7]$.

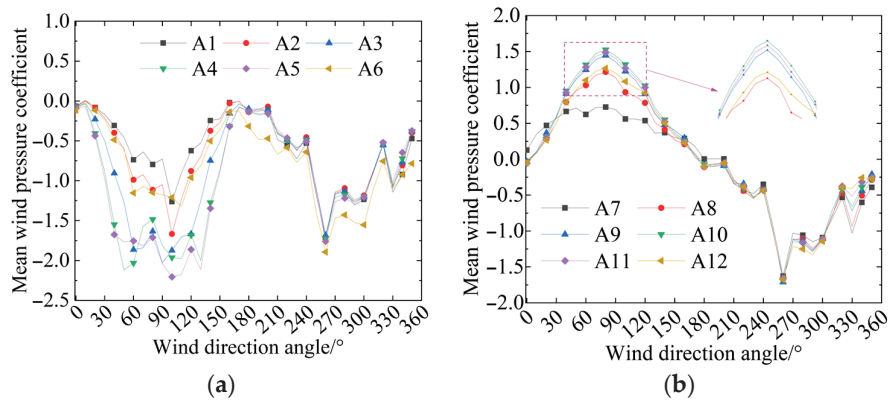


Figure 8. Mean wind pressure coefficient of semi-enclosed sound barrier of single-track bridge: (a) Upper area; (b) Side area.

When the wind direction angle is in the range of $[170, 350]$, the mean wind pressure coefficients of each partition in the upper region are relatively consistent. When the wind direction angle is in the range of $[0, 170]$, the distribution of the mean wind pressure coefficient is more irregular, and the most unfavorable wind pressure coefficients vary across different parts. The variation in the mean wind pressure coefficient of each partition in the side area is generally similar under different wind direction angles. Since partitions A6 and A7 are located at the corner, flow separation occurs at these positions, resulting in a complex flow field distribution and a noticeable difference in wind pressure coefficients compared to other locations.

Figure 9 shows the distribution of the mean wind pressure coefficients for each partition of the semi-enclosed sound barrier on the three-track bridge. It demonstrates that the trend of the mean wind pressure coefficient of the three-track bridge is similar to that of the single-track bridge. With the change in wind direction angle, the upper region (partitions 1~12) is mainly subject to suction, and the mean wind pressure coefficient distribution range is $[-2.4, 1.1]$. The wind direction angle at which maximum suction occurs appears near 260° , and this variation is caused by the upstream and downstream positions of the single-track bridge and the three-track bridge.

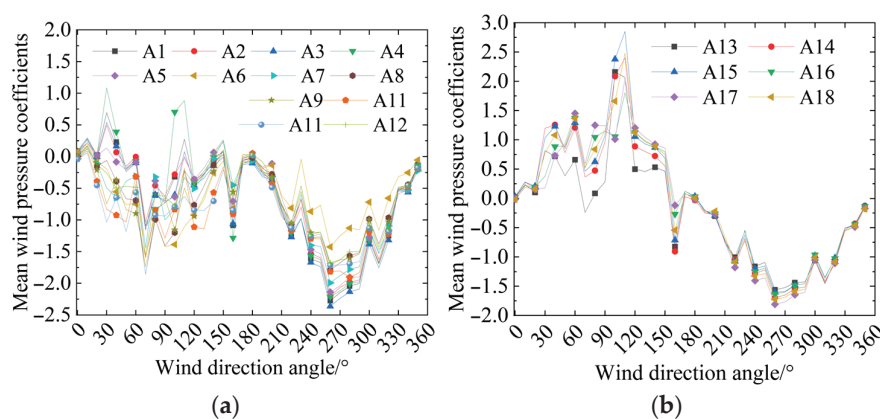


Figure 9. Mean wind pressure coefficient of semi-enclosed sound barrier of three-track bridge: (a) Upper area; (b) Side area.

The maximum average negative pressure of the two sound barriers in the windward and leeward wind field environments appears near the corner between the horizontal and vertical sections, namely, the single-track bridge A5 and A6 measurement point partitions and the three-track bridge A11 and A12 measurement point partitions. The pronounced negative pressure peaks observed at critical corner regions can be attributed to a well-

defined flow separation mechanism. As the crosswind impinges on the windward edge of the semi-enclosed barrier, the flow separates, forming a shear layer that detaches from the surface. This shear layer subsequently rolls up into a transiently anchored vortex core, which remains near the corner. The low-pressure region associated with this vortex core is directly responsible for the high suction peaks recorded. Conversely, the adjacent side areas experience a more attached flow, resulting in a relatively moderate and uniform pressure distribution. In general, for both the single-track bridge and the three-track bridge sound barriers, the mean wind pressure distribution exhibits a similar trend with the wind direction angle. The wind direction angles corresponding to the maximum mean wind pressure in the side area differ, but are all near the 90° wind direction angle. This is because, when the wind direction angle is 90° , the incoming flow is directly obstructed by the sound barrier, and the static wind force acting on the sound barrier is greater. When there is a deviation of more than 90° from the 90° wind direction angle, a three-dimensional effect occurs. Specifically, after the incoming flow is rolled up on the leeward side of the single-track bridge, it interferes with the stable flow field behind the semi-closed sound barrier of the three-track bridge, generating vortices that increase the static wind suction behind the sound barrier. Therefore, the maximum mean wind pressure coefficient in the side area of the three-track bridge sound barrier appears near 110° . However, the box body of the three-track bridge is significantly wider than that of the single-track bridge. The flow field after separation on the leeward side of the three-track bridge has little effect on the wake of the single-track bridge, resulting in the maximum mean wind pressure coefficient in the side area of the single-track bridge sound barrier still appearing near 90° .

3.2. Fluctuating Wind Pressure Distribution

The fluctuating wind pressure coefficient can be utilized to reflect the adverse effects of fluctuating wind pressure on the sound barrier structure. Figures 10 and 11 show the distribution of the fluctuating wind pressure coefficient on the surface of the semi-closed sound barriers of the single-track bridge and the three-track bridge under different wind direction angles.

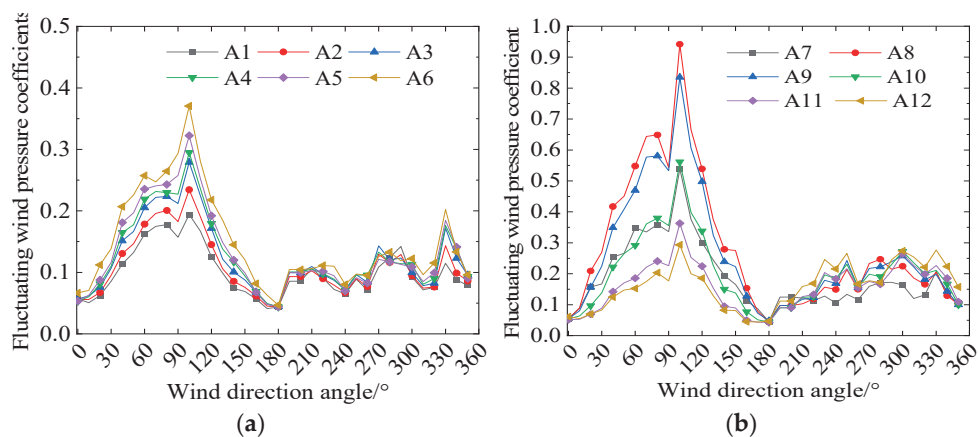


Figure 10. Fluctuating wind pressure coefficient of semi-enclosed sound barrier of single-track bridge: (a) Upper area; (b) Side area.

Figure 10 exhibits that the distribution range of the fluctuating wind pressure coefficient in the upper area of the semi-closed sound barrier of the single-track bridge is $[0.05, 0.37]$, and in the side area is $[0.05, 0.96]$. Between the wind direction angles of 50° and 120° , the fluctuating wind pressure coefficient in each partition of the single-track bridge is relatively high. The fluctuating wind pressure coefficients of partitions A5–A6 and A8–A9 are greater than those of the other partitions. Under the action of fluctuating

wind, wind-induced vibration is more likely to occur at these locations, potentially resulting in the loosening and shedding of bolts on the sound barrier.

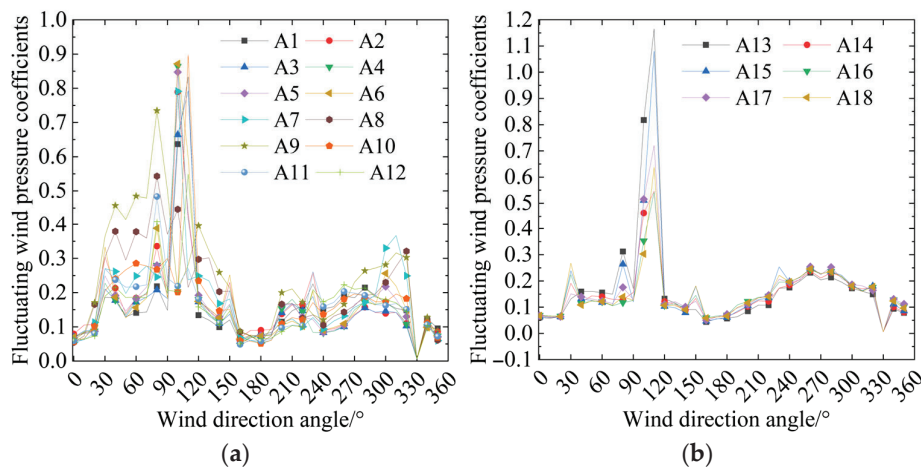


Figure 11. Fluctuating wind pressure coefficient of the semi-enclosed sound barrier of the three-track bridge: (a) Upper area; (b) Side area.

Figure 11 reveals that the distribution range of the fluctuating wind pressure coefficient in the upper area of the semi-closed sound barrier of the three-track bridge is $[0.05, 0.9]$, and in the side area is $[0.05, 1.15]$. The distribution pattern of the fluctuating wind pressure coefficient on the surface of the semi-closed sound barrier of the three-track bridge is similar to that of the single-track bridge, but the values are higher. Therefore, the influence of fluctuating wind on the semi-closed sound barrier of the three-track bridge is more pronounced, and wind-induced vibration is more likely to occur.

Comparing Figures 10 and 11 reveals that the fluctuating wind pressure distribution of the single-track bridge and the three-track bridge sound barrier changes abruptly near the 100° wind direction angle. This occurs because when the wind direction angle is 90° , the incoming flow is orthogonal to the bridge, and the three-dimensional effect of the wake generated around both the single-track bridge and the three-track bridge is weaker. Moreover, at this angle, the wake of the single-line bridge interacts strongly with the incoming flow of the three-line bridge, forming a complex vortex merging and interference effect, which leads to a sharp change in the wind pressure on the surface of the downstream sound barriers. When there is a 10° deviation in the wind direction, the box of the single-track bridge is narrower than that of the three-track bridge. The wake formed after the flow is rolled up on the leeward side of the single-track bridge floor interferes with the stability of the flow state downstream of the semi-closed area of the three-track bridge sound barrier, resulting in a complex turbulent flow and increasing the fluctuating wind pressure acting on the semi-closed sound barriers of both bridge types. As the wind direction angle increases further, the interference effect diminishes, and the fluctuating wind pressure is reduced.

3.3. Extreme Wind Pressure Distribution

The extreme wind pressure refers to the wind pressure generated by the maximum wind speed that the structure may encounter during the design life, which is the benchmark load for the wind-resistant design of the structure. Extreme wind pressure is a critical parameter for the wind-resistant design of the external sound barrier structure. Figure 12 shows the distribution of the extreme wind pressure coefficients for each partition of the semi-closed sound barrier of the single-track bridge and the three-track bridge at wind direction angles of $0-360^\circ$.

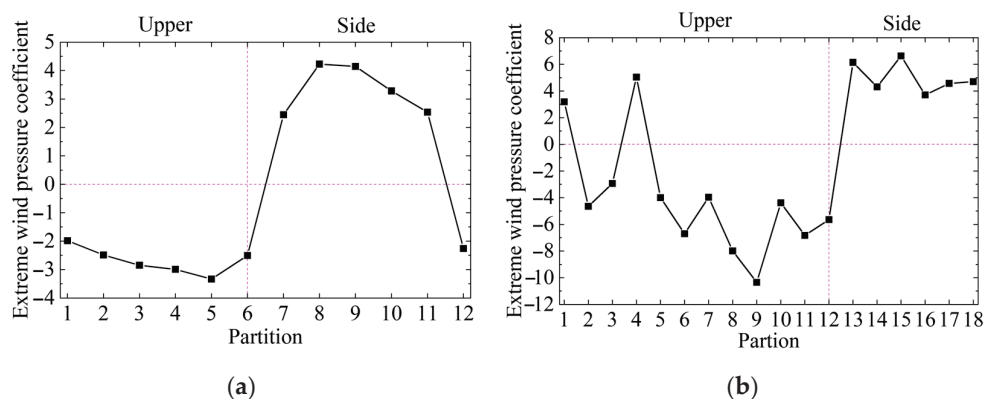


Figure 12. Extreme wind pressure coefficients of two types of semi-enclosed sound barriers: (a) Single-track bridge; (b) Three-track bridge.

The extreme wind pressure in the upper area of both sound barriers is negative, while in the side area it is positive. A comparison reveals that the extreme wind pressure coefficients for each partition of the semi-closed sound barrier of the three-track bridge are significantly higher than those of the single-track bridge sound barrier. Based on the previous analysis, under specific wind direction deviation angles, both the mean wind pressure and fluctuating wind pressure distribution of the three-track bridge sound barrier are increased compared to those of the single-track bridge. This is due to the three-dimensional effect formed by the interference of the single-track bridge wake with the three-track bridge, which significantly increases the extreme wind pressure coefficient.

4. Discussion

The time-averaged streamline diagram represents the mean flow field after time averaging, and its characteristics help explain the distribution of mean pressure on the surface of the structure. The simulation results of the time-averaged streamline and the mean wind pressure distribution for the single-track bridge and the three-track bridge sound barrier in the windward and leeward wind fields are shown in Figures 13 and 14, respectively. Comparing the experimental results indicates that the mean wind pressure calculated by CFD is consistent with the distribution trend of the experimental values. However, in general, the CFD results are slightly higher than those from the wind tunnel tests because the CFD analysis is two-dimensional.

When the windward flow occurs, the flow separates at the windward angle of the sound barrier, and two separation bubbles are formed in the upper parts of the horizontal and vertical sections, respectively. The degree of turbulence increases, and the pressure on the suction surface of the sound barrier rises, while the pressure on the pressure surface inside the sound barrier remains essentially unchanged, resulting in a reduction in the pressure difference on the surface of the sound barrier. In the leeward inflow, the flow through the arc section of the sound barrier remains attached without separation, maintaining a laminar flow state. The flow on the upper suction surface accelerates, the pressure decreases along the flow direction, a pressure gradient forms, and the pressure difference on the surface of the sound barrier increases. Therefore, the maximum average negative pressure of the two sound barriers in the leeward inflow wind field environment is greater than that in the windward inflow.

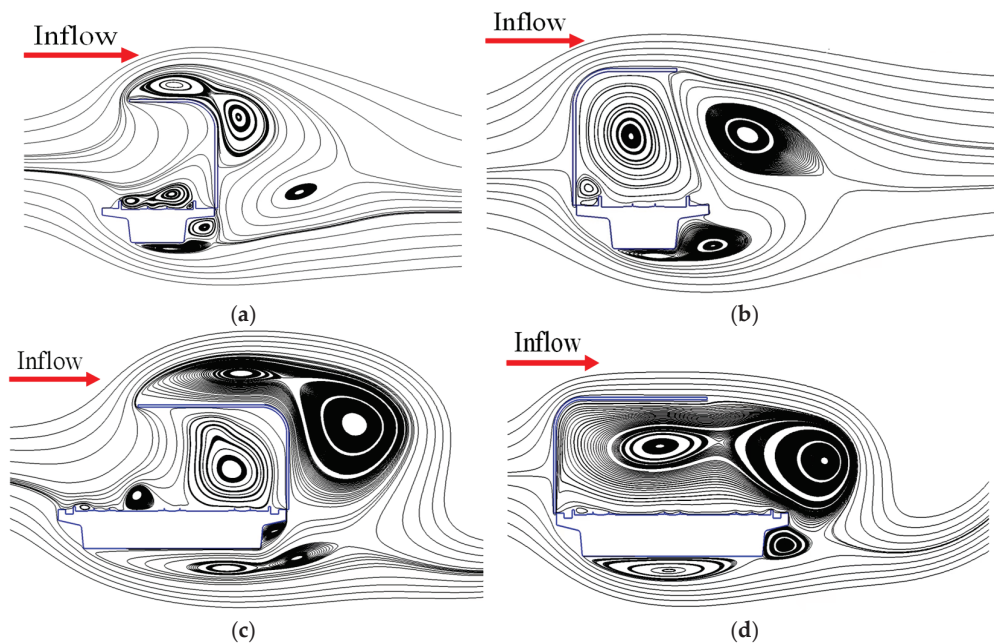


Figure 13. Average flow field of single/three-track bridge: (a) Windward flow; (b) Leeward flow; (c) Windward flow; (d) Leeward flow.

In addition, in the windward flow, although the separation bubble size and reattachment length of the three-track bridge sound barrier increase compared to the single-track bridge sound barrier, the vortex core is located farther from the surface of the sound barrier, so the maximum mean negative pressure of the two sound barriers does not differ significantly. In the leeward flow, the vortex center in the inner recirculation zone of the single-track bridge sound barrier is clearly closer to the sound barrier, resulting in a greater maximum mean negative pressure for the single-track bridge sound barrier compared to the three-track bridge sound barrier.

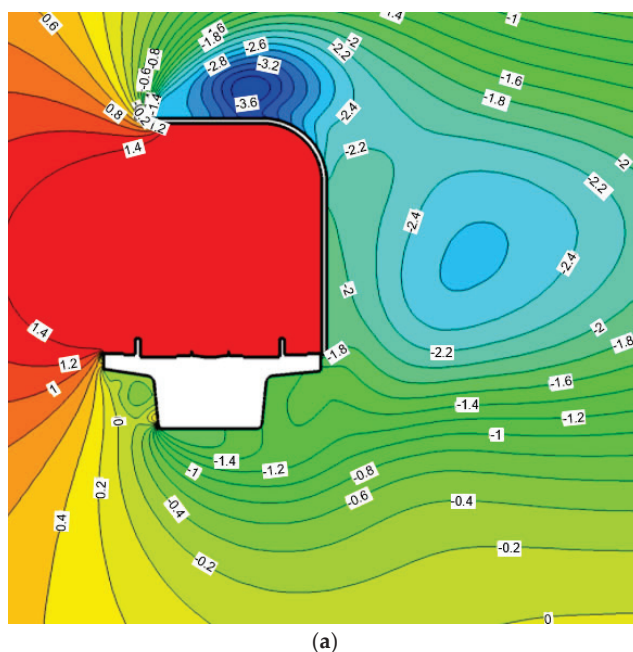


Figure 14. Cont.

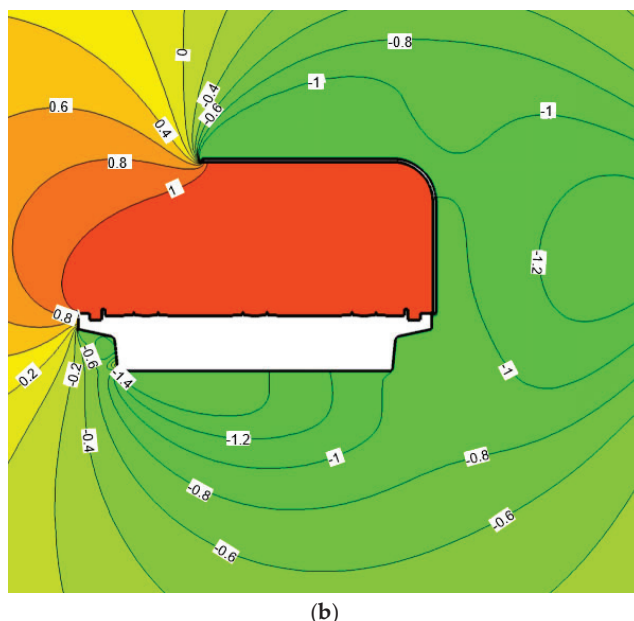


Figure 14. Mean pressure distributions of single/three-track bridge: (a) Single-track bridge windward flow; (b) Three-track bridge windward flow.

Accordingly, the wind tunnel test and numerical simulation results obtained in this study can provide an important basis for the wind-resistant design of high-speed railway semi-enclosed external sound barriers. The results show that the extreme wind pressure on the three-track bridge sound barrier is significantly higher than that on the single-track bridge, especially in the corner area, where the structural strength under wind suction should receive greater attention. Areas with considerable fluctuating wind pressure are prone to fatigue damage, and it is recommended to optimize the connection structure to suppress vibration. In addition, the three-dimensional flow field effect at the junction of single/three-track bridges aggravates the non-uniformity of wind load distribution, and the design should incorporate local reinforcement to improve structural performance.

5. Conclusions

This study examines the distribution characteristics of mean wind pressure, fluctuating wind pressure, and extreme wind pressure acting on the surface of the semi-enclosed sound barriers of single-track and three-track bridges through synchronous pressure measurement wind tunnel tests. It analyzes the flow field characteristics under windward and leeward flow conditions through numerical simulation. The main conclusions are as follows:

(1) The mean wind pressure in the upper area of the semi-enclosed sound barrier is mainly negative, indicating a dominant influence of wind suction, with the maximum suction occurring near a wind direction angle of 90° . The side exhibits an alternating distribution of pressure and suction as the wind direction angle increases.

(2) The fluctuating wind pressure coefficients of each partition of the semi-enclosed sound barrier of the single-track bridge and the three-track bridge are larger between 50° and 120° . When the wind direction angle exceeds 160° , the fluctuating wind pressure coefficient of each partition of the single/three-track bridge remains very low.

(3) The calculation results of the wind pressure coefficient of the sound barrier in different zones do not show significant differences, mainly because its geometry is relatively simple and there is no significant aerodynamic shape change, resulting in a stable and similar flow field in most areas. As the key characteristic parameter of the flow field, the consistency of the pressure coefficient indicates that the overall flow structure under different partition schemes remains similar, which indicates that the dominant flow mechanism

in the studied area does not change substantially due to the change in partition modes. The applicability of this conclusion may be limited to the current research conditions and needs to be verified in a wider range of parameters.

(4) The study reveals that the maximum extreme wind pressure coefficient for both single- and three-track bridge sound barriers occurs on the upper surface near the corner—specifically at partitions A5 and A6 for the single-track bridge and A11 and A12 for the three-track bridge—which is consistent with numerical simulations. These areas' extreme wind pressure is the key load to control the fatigue design of structures and check the strength of connecting bolts. The three-track bridge exhibits a significantly higher extreme wind pressure coefficient than the single-track bridge, whereas values in side regions are relatively similar for both bridge types. Therefore, the structural wind resistance design requirements for the upper surface of the three-track bridge near the corner should be enhanced to address this localized wind load concentration, such as through local load coefficient adjustments, structural stiffness reinforcement, and optimized wind-resistant detailing, to ensure structural safety and durability under extreme wind conditions. Furthermore, the boundary configuration of the bridges introduces a three-dimensional effect, resulting in complex cross-sectional wind pressure distributions along the semi-enclosed sound barrier, with a pronounced influence on the three-track bridge. Accurate determination of extreme wind pressure loads thus requires wind tunnel testing. The scale models, measurement point layouts, and testing methodologies developed in this study offer a reference for the design of comparable structures.

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Conflicts of Interest: Botao Li was employed by the company China Railway First Survey and Design Institute Group Co., Ltd. The remaining authors declare that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

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Article

Time–Frequency Analysis of Railway Bridges Forced and Free Vibrations Identified by Wavelet Transform

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Abstract: In this work, a time–frequency analysis of two railway bridges included in the InBridge4EU project database is presented. The study focuses on the identification of modal parameters from free responses after train passages and their comparison with estimations obtained from ambient vibration data. The wavelet transform is introduced as a valuable tool for detecting both free and forced bridge responses due to different train passages, as well as for conducting time–frequency analysis. This approach is particularly relevant for the identification of structural damping, given its dependence on vibration amplitude, as it enables the estimation of realistic values representative of bridge behavior under operational conditions. Additionally, the paper examines the complementary use of free vibrations for identifying natural frequencies and comparing them with results from ambient vibration tests. Wavelet analysis further reveals the predominant frequencies in the structural response before, during, and after train crossings, thereby capturing the influence of the moving vehicle on bridge dynamics.

Keywords: wavelet; modal parameters; railway bridges; dynamic loads; resonance

1. Introduction

The InBridge4EU project (<https://inbridge4eu.eu/>) started on 1 September 2023. The project is funded by Europe’s Rail Joint Undertaking under Horizon Europe research and innovation program. The overall objective of InBridge4EU is to develop a dynamic interface between railway bridges and rolling stock by proposing new methods that are compatible with existing regulations—namely, INF TSI [1], LOC&PAS TSI [2], EN 15528 [3], EN 1990-Annex A2 [4], and EN 1991-2 [5]—while also addressing the analysis of existing infrastructures, whose role is critical for the sustainability of the European rail system. The project is articulated in seven work packages, including the following objective: identification of critical bridge parameters for the assessment of the economic impact of the new dynamic train categories (DTCs).

During the first year of the project, an extensive and representative set of European railway bridges was selected, and relevant data were retrieved and stored in a database. Starting from the beginning of the second year, time-step calculation (TSC) transient dynamic analyses were performed over the complete database under real train models/multiple unit (MU) classes and under the new DTCs. Based on the analysis of the database, realistic worst-case combinations of critical parameters will be identified for use in parametric

studies. From the detection and evaluation of the bridges that do not meet the economic technical acceptance criteria, an estimation of the resources required for implementing each DTC on a particular line will be presented. Relevant results will also be presented regarding the suitability of the classical beam model, along with various model updates, for analyzing specific bridge configurations.

So far, ten railway lines from five EU countries have been selected, covering different maximum design speeds. Relevant information from about 50 bridges per line has been retrieved with the intensive participation of the corresponding infrastructure managers, including technical drawings and experimental data. In parallel, a cloud database has been designed and made accessible to all the partners in the consortium. The database will accommodate the bridge data and the results of posterior analyses.

To understand the bridge's behavior, it is crucial to identify the modal parameters. Natural frequencies, mode shapes, and modal dampings of large scale structures are usually estimated from operational modal analysis (OMA) using the ambient vibration response. A comprehensive review of the modal identification algorithms utilized within the OMA framework for these structures can be found in References [6,7]. These methods assume linearity and time-invariant conditions. In the case of high-speed railway bridges, modal identification is performed using two types of recorded signals: (i) the long duration pure ambient vibration between train passages and (ii) the free vibration response after their passage of short duration. Due to the high stiffness of railway bridges, the amplitude of the natural ambient response is low, and certain mode shapes may not be excited. Additionally, this scenario of low vibration amplitude is not representative of the operational conditions of a train crossing a bridge, particularly in terms of amplitude of vibration. This can lead to uncertainties in the identification of modal parameters. Liu et al. [8] provide a comparison between the identified natural frequencies and damping ratios in a composite multi-span railway bridge, using different vibration recordings comprising pure ambient vibration measurements, free vibrations after train passages, and forced vibrations during the circulation of trains. The results showed a decrease in the identified first bending frequency when the forced vibration recording was used. Leqia et al. [9] performed a vibration measurement campaign on the Jalon viaduct, a six-span box girder railway bridge for high-speed trains at Calatayud, Spain. From the pure ambient measurements, 11 modes were identified. However, when the free vibration response after the train passages was combined with a small interval of ambient vibrations, six more modes were extracted. Rebelo et al. deal with the dependency of the modal parameters with the amplitude of vibration for certain bridges [10]. In the cited work, the identification was performed using two types of recorded signals (pure ambient and free vibration) and different identification techniques (enhanced frequency domain decomposition, EFDD, and free decay analysis). The identification of modal damping becomes particularly challenging as various energy dissipation mechanisms take place during train passages. A number of works are devoted to the study of efficient techniques for a reliable estimation of this modal parameter from experimental measurements [11–13].

Vibration signals generated by circulating trains constitute complex dynamic data. These signals are inherently non-stationary, characterized by rapidly changing spectral content as the moving load traverses the span. However, the application of modal identification techniques that account for the time-varying nature of railway bridge vibrations under operating conditions is still limited. The use of the wavelet transform for modal identification was proposed and validated in the scientific literature by several researchers (e.g., [14,15]). This mathematical tool enables the capture of the temporal evolution of frequency contributions to the structural response during train passages, i.e., when the train is on the bridge and after it leaves. More recent works have also applied this technique

to detect the non-stationary variations in the bridge vibrations [16,17]. From the view of the authors, there is still a lack of time–frequency characteristics analysis on bridge response.

For this reason, this work presents a time–frequency analysis of the experimental responses of two bridges included in the database developed within the framework of the InBridge4EU research project. Both structures had previously been investigated by the authors in [18,19], where their modal parameters were identified by ambient vibration. The present study extends those analyses by incorporating the free responses recorded after train passages. A comparative evaluation is carried out to identify modal contributions that may arise under one excitation condition but not under the other. For this purpose, the wavelet transform is employed to discriminate between free and forced responses of the bridges induced by different train crossings. The proposed methodology enables the estimation of actual structural modal damping values, which exhibit a strong dependence on vibration amplitude, thereby allowing a more comprehensive assessment of bridge behavior under service loads. In addition, the wavelet analysis provides insight into the predominant frequencies governing the structural response before, during, and after the passage of trains, while explicitly accounting for the influence of vehicle characteristics.

2. Description of the Bridges: Experimental Set-Up and Ambient Vibration Analysis

In this section, the structures under study are briefly presented. More information about them can be found in [18,19]. In all the cases, the railway track is a classical ballasted track. The rails have a UIC60 cross section and are continuously welded. The rails are supported on rail pads and fixed with clips on monoblock concrete sleepers with a spacing of $d_s = 0.60$ m.

As per the acquisition equipment, a portable acquisition system LAN-XI of Brüel and Kjaer was used. The acquisition system fed the accelerometers, and it also performed the analog/digital conversion (A/D). The A/D was carried out at a high sampling frequency that avoided aliasing effects using a low-pass filter with a constant cut-off frequency. The sampling frequency was $f_s = 4096$ Hz. The acquisition equipment was connected to a laptop for data storage. Endevco model 86 piezoelectric accelerometers were used with a nominal sensitivity of 1000 mV/g and a lower frequency limit of approximately 0.1 Hz. The structural response recordings were decimated by a factor of 16 to enable efficient analysis of the data within the frequency range of interest. Structural responses were filtered applying two third-order Chebyshev filters with high-pass and low-pass frequencies of 1 Hz and 30 Hz, respectively, in order to study the bridges within the frequency range of interest defined in [20], Appendix A2.4.4.2 (criteria for traffic safety).

The modal properties of the bridges were identified from ambient vibration measurements using state-space models [6]. Ambient responses were recorded for 3600 and 4184 s per channel during the tests conducted on the Jabalón and Arroyo Bracea I bridges, respectively, while no train passages occurred. Particular attention was devoted to modal phase collinearity, complexity, damping ratios, and MAC values [21], which were used as criteria for distinguishing real natural frequencies from spurious peaks. Since excitation forces were not measured, only unscaled mode shapes could be estimated, and an additional procedure is required to determine the scaling factor [22].

During the recording time, RENFE Class 100 (S100), 102 (Talga 350 or S102), 112 (S112), 103 (ICE 3 or S103), 104 (S104), and 130 (Talga 250 or S130) trains in single and duplex compositions crossed the bridges with circulating velocities in the range [100, 290] km/h. Table 1 shows the characteristic distance of each train. The characteristic distance of a train depends on its typology: for regular trains (S102/S112/S130), it corresponds to the

distance between axles; for articulated trains (S100), it is defined as the distance between bogie centers; and for conventional trains (S103/S104), it is taken as the car length.

Table 1. Train characteristic distance.

Train	d [m]
S100	18.7
S102/S112/S130	13.14
S103	24.8
S104	25.9

2.1. Jabalón HSL Bridge in Madrid–Sevilla HS Line

Jabalón HSL Bridge (Figure 1), is a railway bridge composed of three identical S-S bays of 24.9 m equal spans. The structure crosses Jabalón River with a 134° skew angle. Each deck consists of a cast-in-situ concrete slab with dimensions $11.6\text{ m} \times 0.3\text{ m}$ (wide $\times$ thickness). The slab is supported by five prestressed concrete I-girders with a height of 2.05 m separated 2.625 m. The girders lean on the supports through laminated rubber bearings. The substructure consists of two outer reinforced concrete abutments and two inner wall piers.



Figure 1. HSL bridge over Jabalón River ($38^\circ 53' 51.3''$ N $3^\circ 57' 53.0''$ W).

Figure 2 shows the distribution of sensors, all of which were installed on the first span. Unfortunately, it was only possible to record the response of the first span due to the pier's height and the river flow.

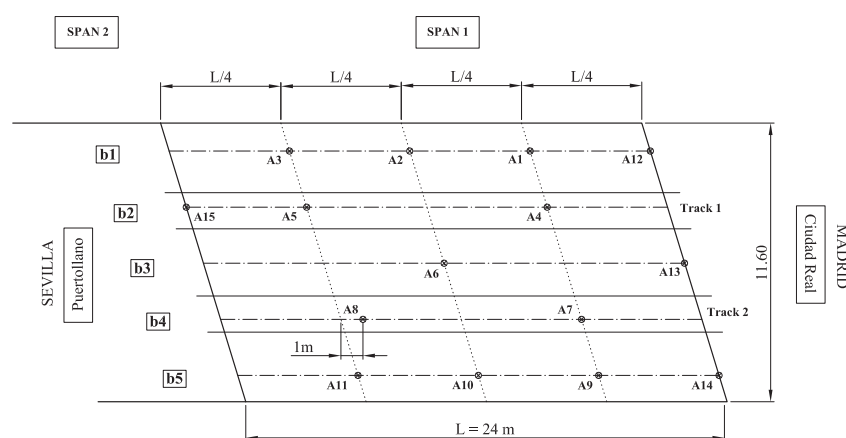


Figure 2. Location of the sensors on Jabalón HSL Bridge. Dimensions in meters.

Figure 3 presents the identified modal parameters of the structure by ambient loads. The first, second, and third modes in frequency order correspond to the first longitudinal,

first torsion, and first transverse bending modes, typical in S-S decks with an orthotropic plate-type behavior. Modal damping has been estimated for the first six modes (modes under 30 Hz). The identified damping ratio for the fundamental mode (3.2%) is higher than the value prescribed by standards (1%) [20].

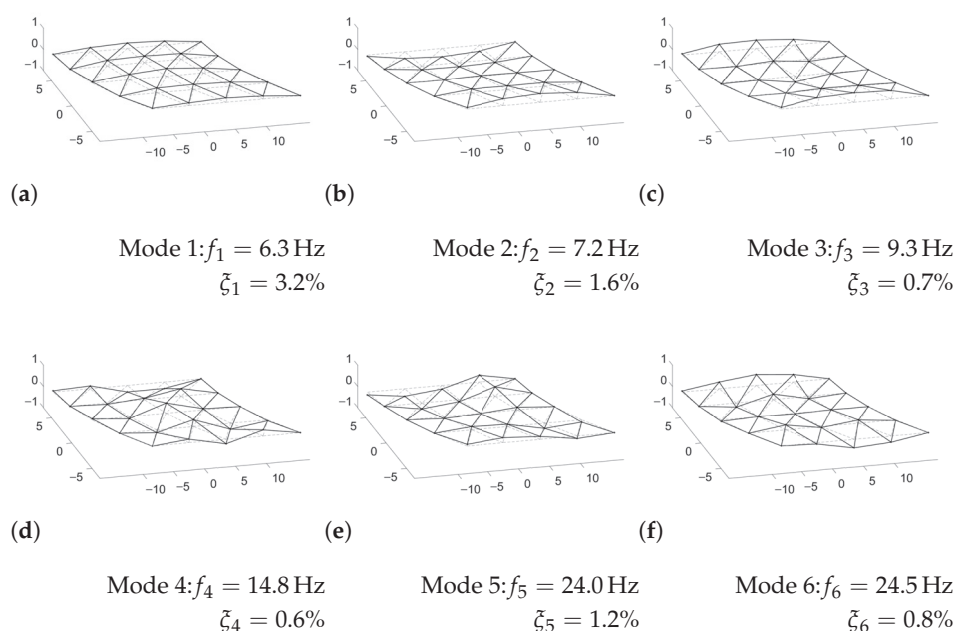


Figure 3. Identified natural frequencies, modal damping ratios, and mode shapes of Jabalón HSL Bridge from ambient vibration.

2.2. Arroyo Bracea I Bridge in Madrid–Sevilla HS Line

The structure under study, designated as Arroyo Bracea I bridge (Figure 4), is a railway bridge composed of two identical simply supported (S-S) bays of 15.25 m equal spans. The structure crosses the Bracea stream with a 45° skew angle. Each deck consists of a 25 cm thick, 11.6 m width concrete slab resting over five 1.05 m height prestressed concrete I girders. The girders lean on the supports through laminated rubber bearings. The slab carries two ballasted tracks with an equal eccentricity of 2.15 m from its longitudinal axis, two sidewalks, and handrails. As per the substructure, the bridge deck is supported on reinforced concrete abutments in its outermost sections, and the inner sections of both bays lean on a pile foundation.



Figure 4. Arroyo Bracea I bridge (39°28′19.18″ N 3°49′48.39″ W).

During the experimental campaign, the vertical acceleration response of the bridge deck was recorded at 11 measurement points distributed along the lower horizontal face of

the flange of the prestressed concrete girders (points 1-11 in Figure 5) and at the pile foundation upper horizontal surface, close to the central girder support (point 12 in Figure 5).

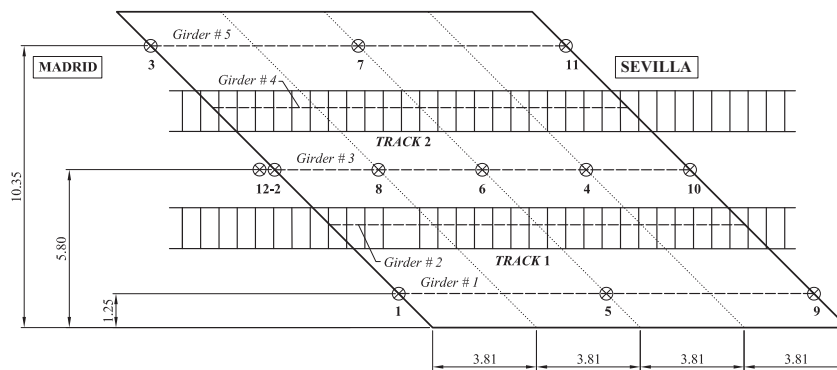


Figure 5. Sensor locations for Arroyo Bracea I bridge (Dimensions in meters).

First, the modal parameters of the bridge were identified from ambient vibration. The deck presents five modes in the frequency range from 1 Hz to 30 Hz, where the lowest ones in frequency order correspond to the first longitudinal bending, first torsion, and first transverse bending mode shapes. In Figure 6, the identified natural frequencies and the damping ratios are included for the first six modes. Mode six was not taken into account in [18] because of its proximity to 30 Hz. The identified damping for the fundamental mode reaches 2.0%, a value higher than that prescribed by standards for design purposes for this length and bridge typology (1.3%) [20].

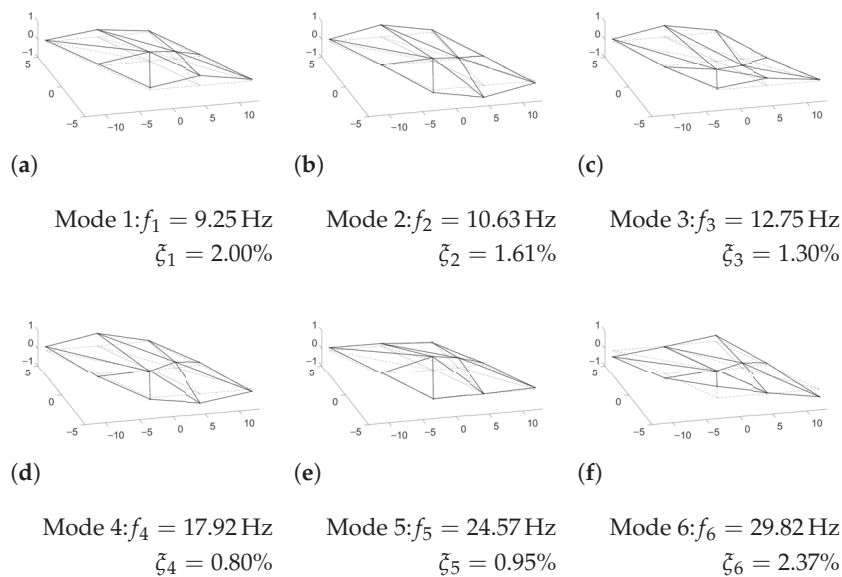


Figure 6. Identified natural frequencies, modal damping ratios, and mode shapes of Arroyo Bracea I bridge from ambient vibration.

3. Free and Forced Vibration Analysis Through Wavelet Analysis

The behavior of railway bridges while the train crosses (forced vibration) differs from right after the train leaves the structure (free vibration). In the latter case, the response of the bridge is influenced by the modal frequencies, which are determined by the stiffness, mass, damping, and support conditions. In the former case, in addition, the characteristic frequencies of the train excitation, which are a function of the passing speed and axle distances, contribute to the response of the structure. In order to determine the modal parameters of the structure, the determination of the free vibration response is decisive.

From this response, using different methodologies, the natural frequencies and modal dampings of the structure can be determined. In order to identify the modal damping, it is also necessary to analyze the maximum amplitude of the free vibration response and the duration of the recording, as this is a parameter that presents considerable uncertainty in its estimation. To identify forced and free vibrations, the use of the wavelet transform is proposed in this work. The wavelet transform has been previously used by the authors to detect damage [23–27] and non-stationary behavior of railway bridges during vehicle crossing events [17]. Other authors [16] have also applied the wavelet transform to analyze the vibrations of railway bridges.

Basic concepts about the wavelet transform can be found in Reference [28]. In addition, the drawbacks that arise when applying the wavelet transform to data analysis, as well as some proposed solutions, have been discussed by some of the authors in [23–26]. What follows is a brief presentation of some of them.

3.1. Continuous Wavelet Transform

The Continuous Wavelet Transform (CWT) of a function $f(t)$ can be defined as

$$CWT_f(u, s) = \frac{1}{\sqrt{s}} \int_{-\infty}^{+\infty} f(t) \Psi^* \left(\frac{t-u}{s} \right) dt \quad (1)$$

where Ψ^* indicates the complex conjugate of the wavelet function. $CWT_f(u, s)$ indicates the content of the scaled wavelet shape in the original function $f(t)$ at a specific instant. This can be understood in the sense of time evolution of frequency content. Parameter u is known as the translation parameter. Parameter s is defined as the scale of the wavelet transform. It defines the shrinking or stretching of the wavelet function. The resulting wavelet transformed function will depend on scale and translation parameters s and u , respectively. The inverse of s can be interpreted as a pseudo frequency, since it modulates the frequency content of the wavelet function. A value of the corresponding pseudo frequency for each scale can be obtained from the following expression:

$$F_a = \frac{F_c}{a} \quad (2)$$

where F_a is the pseudo frequency for scale a , and F_c is the center frequency of the wavelet. The center frequency is a convenient and simple characterization of the dominant frequency of the wavelet.

The wavelet transform gives time information for the frequency content of the signal. On the other hand, the wavelet analysis can provide higher time resolution for higher frequencies and lower time resolution for lower frequencies, since the convolution parameter (u/s) can be adapted according to the frequency.

In this paper, the analytic Morse wavelet [29] with the symmetry parameter $\gamma = 3$ and the time-bandwidth product equal to 120 has been used. This wavelet allows us to analyze signals with wider variance in time. The wavelet coefficients are normalized to the maximum value in each scale.

The main advantage of applying the wavelet transform to railway bridge analysis lies in its ability to capture the temporal evolution of frequency contributions to the structural response during train passages, i.e., when the train is on the bridge and after it leaves. This aspect is particularly relevant because the modal properties of railway bridges depend on multiple uncertain factors, such as soil–structure interaction, vehicle–track interaction, and vehicle–bridge interaction. Some of these factors give rise to non-linearities that affect the identification of modal properties and cannot be detected under ambient or free vibration conditions. They may only become apparent during train crossings.

The transformation of the time history response of the bridge using the wavelet function makes it possible to detect changes that occur in the structure. Since the entry and exit of the train severely affects the behavior of the bridge, which was previously only subjected to ambient loads, the wavelet transform allows us to identify the entrance and exit instants of the convoy. Figure 7a shows the time history of the acceleration at point 10 induced by Renfe S130 train circulating along track 1 of Jabalón bridge at $v = 236$ km/h. The time history of the maximum coefficient of the wavelet transform to the power of 4 is presented in Figure 7b. The time span during which the train is on the bridge is defined as the time window in which the maximum wavelet coefficient of the acceleration exceeds 25% of its standard deviation as is indicated in Figure 7 by vertical dashed lines.

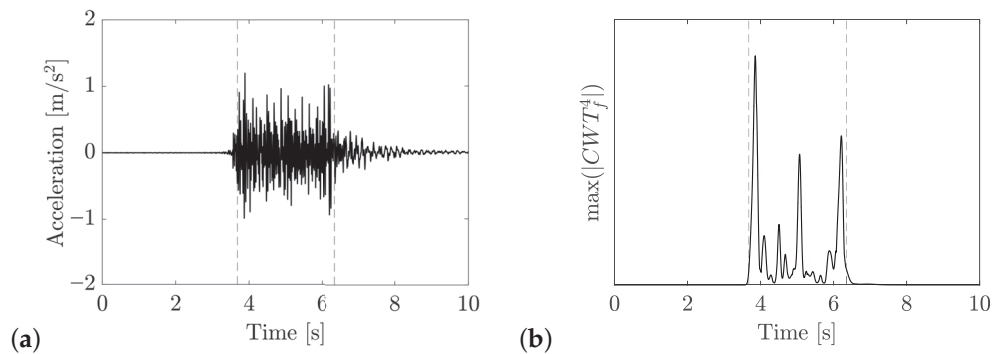


Figure 7. (a) Time history and (b) maximum wavelet coefficient of the acceleration at point 10 induced by Renfe S130 train circulating on track 1 of Jabalón bridge at $v = 236$ km/h.

3.2. Time and Frequency Response

In the following, the response of Jabalón and Arroyo Bracea I bridges due to two different train passages is presented. Figure 8 shows the time history responses due to Renfe S102 Duplex and Renfe S103 trains. The black trace corresponds to the recording period before the train arrival. The green line represents the time span of forced vibration. Finally, the blue line shows the free vibration, once the train has left the structure. In Figure 9, the wavelet scalograms and frequency responses induced by the two passages in both bridges are shown. As in Figure 8, the green line is the frequency response associated with forced vibration, and the blue line is obtained from the free vibration identified by the wavelet transform analysis previously presented.

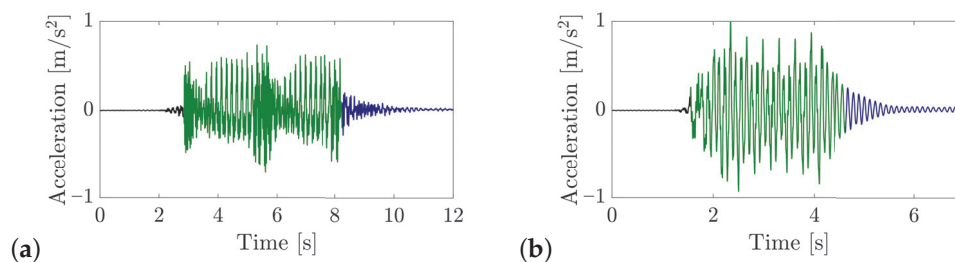


Figure 8. Acceleration time history induced by (a) Renfe S102 Duplex train circulating on track 2 of Jabalón bridge at $v = 265$ km/h at point 4 and (b) Renfe S103 train circulating on track 1 of Arroyo Bracea I bridge at $v = 279$ km/h at point 6. The forced vibration is shown by the green trace.

The response shown in Figures 8a and 9a from a particular train passage (Renfe S102) over Jabalón bridge does not induce resonance. In this case, the excitation frequency associated with the axle distance ($d = 13.14$ m) is 5.6 Hz and Jabalón bridge's fundamental frequency is 6.3 Hz. Both peaks are visible in the scalogram and in the frequency response. Before the arrival of the train, the bridge is subjected to ambient vibration, and the fundamental frequency of the bridge at 6.3 Hz dominates its response. As the vehicle crosses the

bridge, it is clear that the dominant frequency shifts to a lower value, i.e., the frequency associated with the passage of the axles (5.6 Hz), with its harmonic also appearing at 11.2 Hz. Once the train leaves the bridge, the contribution of the fundamental frequency is clearly appreciated again, the contribution of a natural frequency close to 19 Hz being also relevant. This frequency content was not identified in [19] and will be discussed hereafter.

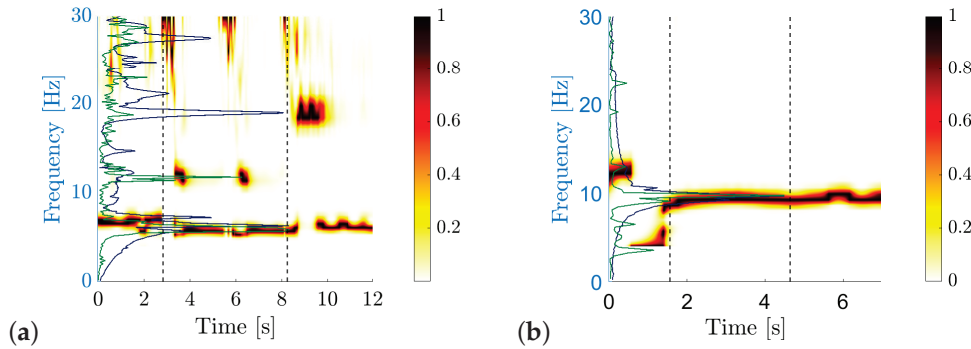


Figure 9. Wavelet scalogram and frequency response induced by (a) Renfe S102 Duplex train circulating on track 2 of Jabalón bridge at $v = 265$ km/h at point 4 and (b) Renfe S103 train circulating on track 1 of Arroyo Bracea I bridge at $v = 279$ km/h at point 6. The forced and free responses are shown by the green and blue traces, respectively.

On the other hand, Figures 8b and 9b show a resonant response of the Arroyo Bracea I bridge due to Renfe S103 passage traveling along track 1 at $v = 279$ km/h. The third theoretical resonant (or critical) speed of the fundamental mode is given by $v_{1,3}^{res} = d \times f_1 / 3 = 24.8 \text{ m} \times 9.25 \text{ Hz} \times 3.6 / 3 = 275.3 \text{ km/h}$, where 1 denotes the mode number, 3 the resonance order, and d the characteristic vehicle distance (Table 1). The calculated resonant speed closely matches the estimated train speed of $v = 279$ km/h. In this case, frequency contents at 9.25 Hz predominate both the forced and the free vibrations. Moreover, the frequency response of the forced vibration exhibits peaks at the lower and higher harmonics: $279 \text{ km/h} / 3.6 / 24.8 \text{ m} \times i \text{ Hz}$ where $i = 1, 2, \dots$

Free and forced vibration responses may be automatically detected from the presented methodology based on the wavelet transform. Figure 10 shows the amplitude of the peaks at the frequency response of the forced vibration for all train passages and sensors in both bridges. The frequency is normalized, accounting for the passing frequency of the train. It can be observed that the amplitude of the response is similar in both bridges. However, the response at accelerometers 4 and 8 prevails in the case of the Jabalón bridge in the complete frequency range, while accelerometers 6 and 7 contribute to the response mainly for Arroyo Bracea I bridge, at the frequencies of the second and third harmonics of the excitation.

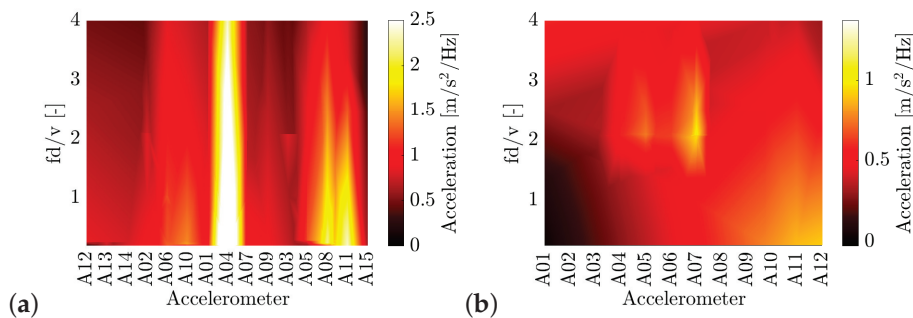


Figure 10. Frequency response main peak amplitudes of the forced vibration for all train passages and sensors: (a) Jabalón and (b) Arroyo Bracea I bridges.

Figure 11 shows, for the sensors that contribute most to the structural response, the position of the peaks that appear in the forced vibration response in the frequency domain, differentiating the type of train. In the case of Jabalón (Figure 11a), it can be seen that peaks appear mostly in the axle-passing frequency and also in its second and third harmonics. The S102 and S112 trains (very similar and with a short characteristic distance) lead to peaks at all three frequencies mentioned. The longer characteristic distance of the S104 train causes amplifications or peaks mainly at the second harmonic of the axle-passing frequency. In the case of Arroyo Bracea I (Figure 11b), with a higher fundamental frequency, the S100 train induces higher responses at the pass frequency, the S102 and S130 types at the second harmonic, while the S103 and S104 types cause peaks at the third harmonic.

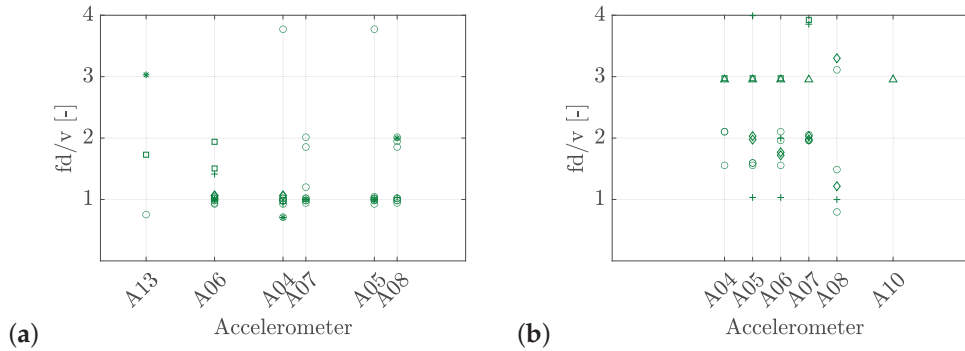


Figure 11. Amplitude of the peaks at the frequency response of the forced vibration for all train passages (+: S100, o: S102, △: S103, □: S104, *: S112, ◇: S130) and sensors: (a) Jabalón and (b) Arroyo Bracea I bridges.

Following, Figures 12 and 13 show the maximum acceleration recorded at the decks of the bridges for resonant and non-resonant passages, respectively. Consistently with Figure 10, the resonant response is localized in the region where the excited mode has its maximum amplitude. In the case of Arroyo Bracea I bridge, the resonance associated with the fundamental mode at 9.25 Hz and the third order is again considered. For Jabalón bridge, this train passage corresponds to a speed close to the resonant speed for mode 1 and resonance order 1, $13.1 \text{ m} \times 6.3 \text{ Hz} \times 3.6/1 = 297 \text{ km/h}$. However, in the case of a non-resonant response, there is no predominant participation of a single mode, but different modes contribute with lower frequency response amplitudes. This is also reflected in the space distribution of the maximum.

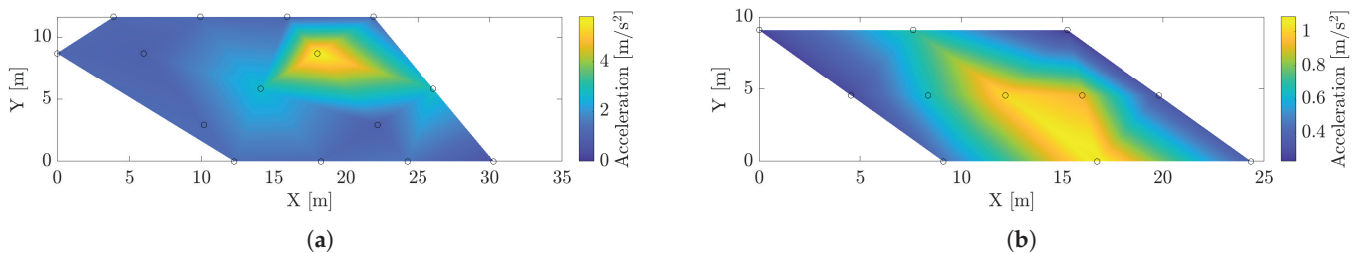


Figure 12. Maximum acceleration recorded in a resonant scenario: (a) Jabalón (S102 at $v = 290 \text{ km/h}$) and (b) Arroyo Bracea I (S103 at $v = 279 \text{ km/h}$) bridges.

In the next section, this procedure is used for the estimation and validation of the modal parameters of the bridges.

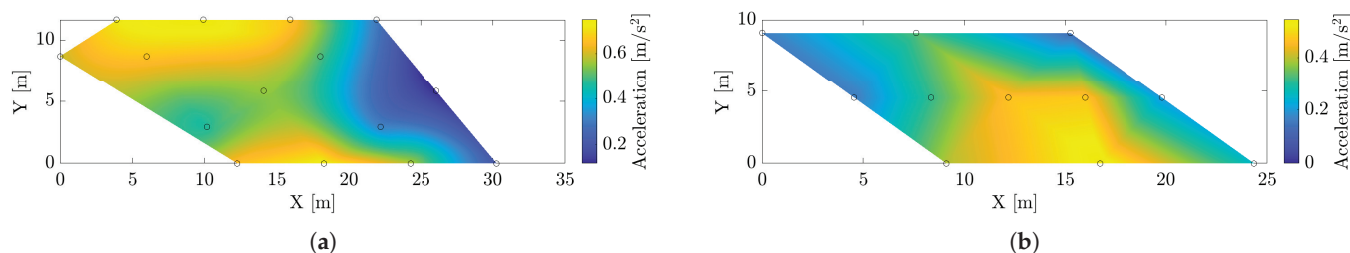


Figure 13. Maximum acceleration recorded in a non-resonant scenario: (a) Jabalón (S102 at $v = 240$ km/h) and (b) Arroyo Bracea I (S104 at $v = 252$ km/h) bridges.

4. Analysis of Modal Parameter Estimation from Combined Ambient and Free Vibrations

In this section, from the records of the bridge response in free vibration, the natural frequencies of the structure and their corresponding modal dampings are estimated. In relation to the natural frequencies, a validation procedure is proposed, taking into account that normally, an analysis of the bridge subjected to ambient loads is carried out in addition to the analysis of the bridge response due to the train passages. Regarding the identification of modal damping, two techniques have been applied: half-power bandwidth method [30] and random decrement technique [31].

The natural frequencies from free vibrations are identified by the peak picking methodology [32] based on the fact that when the frequency response function reaches a peak at a certain frequency, it can be associated to the force or to a resonance frequency of the structure. This procedure produces a good estimate of natural frequencies for weakly damped structures.

The half-power bandwidth method [30] provides a straightforward approach for damping estimation. This technique evaluates the peak width of the frequency response around the natural frequencies, attributing the response to the corresponding vibration mode. The method was originally formulated based on the frequency response of a single-degree-of-freedom system, yet it is widely employed for damping estimation in complex structural systems [33].

An interesting overview of the random decrement technique was recently published by Brincker and Amador [34]. It is a time-domain method for extracting modal parameters from random responses. In this analysis, segments of the signal that exceed a trigger level—set at 0.5 times the standard deviation—are averaged. This averaging removes the random excitation, leaving a deterministic signal that represents the structure's free vibration, from which damping ratios can be estimated.

Figure 14 presents the peaks identified in the frequency response of the free vibration for all the sensors and train passages. The horizontal lines stand for the natural frequencies estimated by operational modal analysis using ambient excitation. It is clearly visible that some natural frequencies were not identified in the case of Jabalón bridge (Figure 14a). These modes were obtained from the stabilization diagram analyzed in [19] but discarded due to their high complexity. However, the structural response indicates that these seem to be real mode shapes.

The operational modal analysis is extended in this case, taking into account both the response to ambient loads and the free responses after train passages. Figure 15 shows the singular value decomposition of the spectral density matrices obtained considering both responses (red lines) and only ambient loads (blue lines). It should be noted that the acceleration scales for the two compared spectra are different. The peaks appear clearer than when only the ambient load response is taken into account, as the structural

modes contribute more significantly to the overall dynamic response. Figure 16 shows the additional mode shapes identified in the Jabalón bridge using both ambient and free vibration after train passages. Some of them are visible in Figure 9. In the case of Arroyo Bracea I bridge (Figure 14b), the mode shapes were identified with reasonable accuracy.

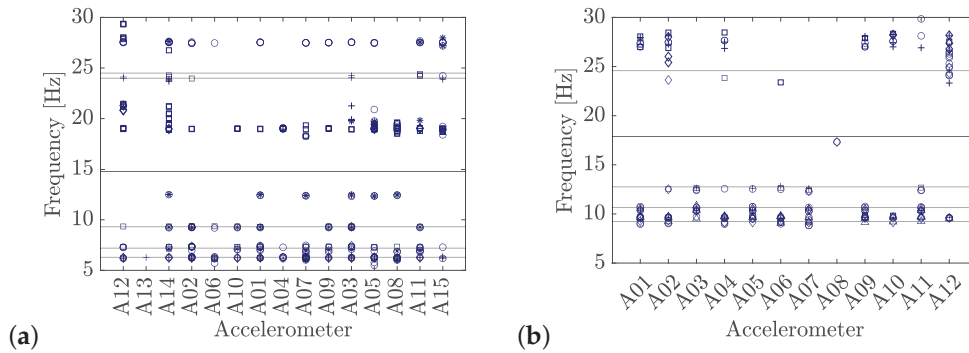


Figure 14. Peaks at the frequency response of the free vibration for all train passages (+: S100, o: S102, △: S103, □: S104, *: S112, ◇: S130) and sensors: (a) Jabalón and (b) Arroyo Bracea I bridges.

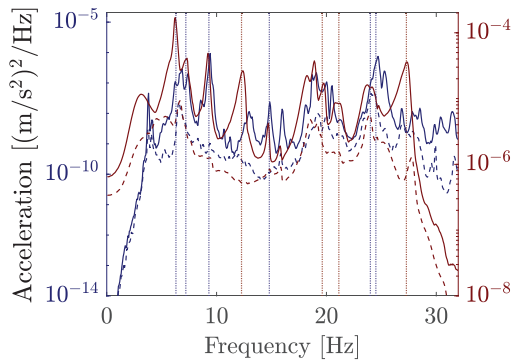


Figure 15. (Solid lines) first and (dashed lines) second singular values of the power spectral density matrices obtained considering (red) ambient and free responses after train passages and (blue) only ambient loads. The estimated natural frequencies in both cases are indicated using vertical lines.

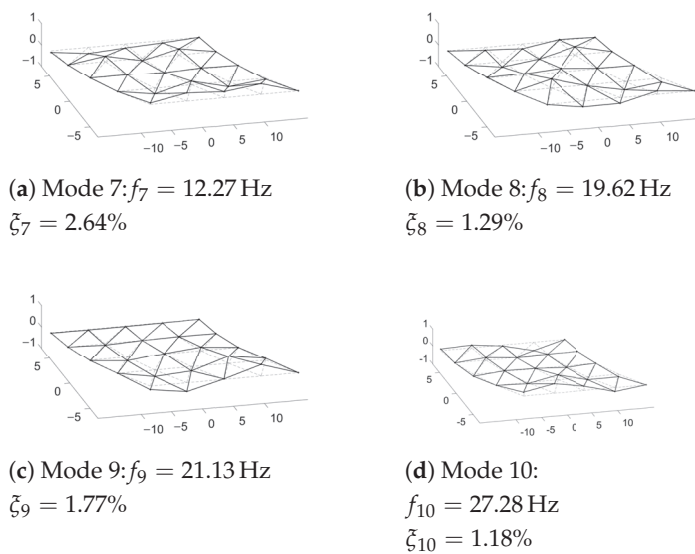


Figure 16. Further mode shapes identified in the Jabalón bridge using both ambient and free vibration after train passages.

Figure 17 shows the influence of the maximum amplitude of the free vibration on the estimation of the natural frequency. This effect was previously discussed in [16], which showed a decrease in the natural frequencies with increasing amplitude. For the cases considered in the present work (high-speed railway bridges that are stiffer than those presented in [16]), the natural frequencies do not depend on the amplitude of the response within the frequency range of interest. The mass of the Jabalón bridge can be estimated as $29,000 \text{ kg/m} \times 24.9 \text{ m} = 722,100 \text{ kg}$ and, in the case of the Arroyo Bracea I bridge, $26,256 \text{ kg/m} \times 15.25 \text{ m} = 400,404 \text{ kg}$. The mass of a high-speed train is at most 2500 kg/m . Therefore, the mass of the train entails less than 10% of that of the bridge deck, and the effect of the vehicle on the natural frequencies during the train passage is expected to be lower than 5%, and here, it has no significance.

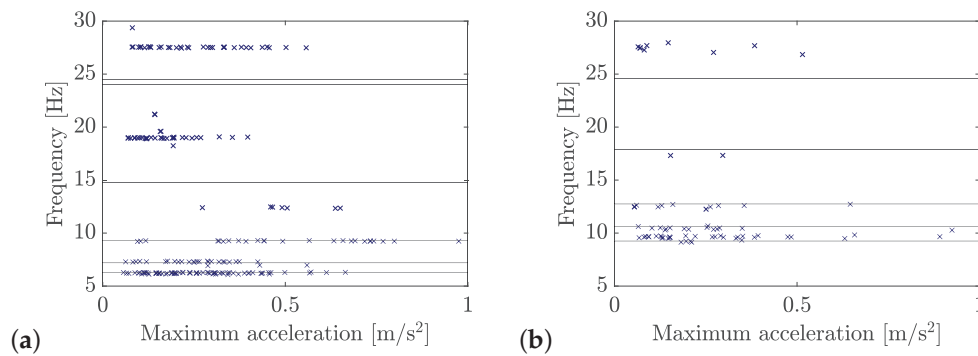


Figure 17. Peaks at the frequency response considering the maximum amplitude of the free response: (a) Jabalón and (b) Arroyo Bracea I bridges.

Finally, Figure 18 presents the estimations of the modal damping ratios by the half-power bandwidth method and the random decrement technique using all experimental campaign recordings. The pentagrams indicate the mean value. The results present high dispersion due to uncertainty of the damping ratio, which is affected by several variables. However, the mean values for Jabalón bridge (Figure 18a) are similar to those obtained in [19] using a lower number of measurements from the free vibration after the train passages. In general, the mean values of the modal damping ratios obtained for the Arroyo Bracea I bridge (Figure 18b) are higher than those reported in [18]. This can be explained by the fact that in this reference, only values from ambient vibration were obtained.

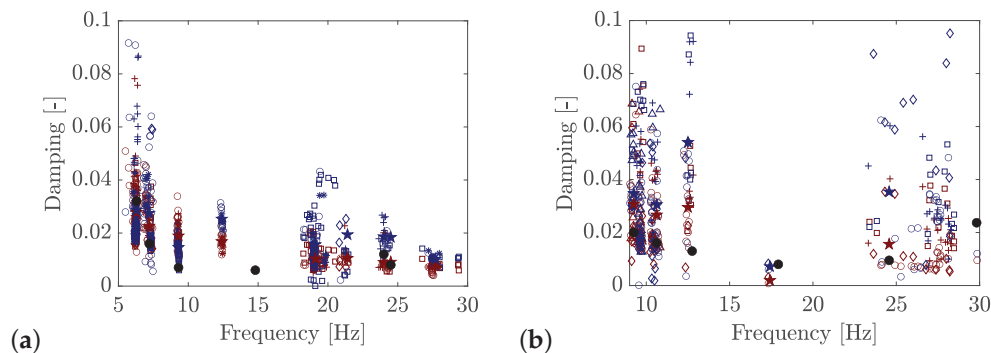


Figure 18. Modal damping estimations by (blue) the half-power bandwidth method and (red) the random decrement technique from all train passages (+: S100, ○: S102, △: S103, □: S104, *: S112, ◇: S130) and mean values: (a) Jabalón and (b) Arroyo Bracea I bridges. The black circles correspond to the estimations based on ambient vibration.

The analysis of the response of the bridges due to rail traffic has clarified uncertainties that were encountered during the post-processing of the operational modal analysis.

5. Conclusions

This work includes the time-frequency analysis of two bridges included in the In-Bridge4EU project database. The dynamic response of a bridge under the circulation of a railway convoy is complex and is affected by several factors. The most evident and certain ones are the bridge properties, the geometric configuration of train axles, and the circulation speed. Additionally, there are other factors whose determination is much more uncertain and that significantly affect the response of the bridge, such as structural damping and various interaction mechanisms, which are not trivial and are currently under investigation, the most relevant being vehicle–structure, track–structure, and soil–structure interaction effects. Some of these are only appreciable when the train is crossing the bridge because they are triggered by a certain amplitude response. This research has analyzed the identification of modal parameters from free responses after train passages and from ambient responses. The wavelet transform has been employed to distinguish between free and forced regimes. Moreover, the wavelet transform enables the characterization of the time history of the frequency response. The methodology has been useful to clarify some results obtained during the ambient modal analysis and has been especially significant for the estimation of structural modal damping, highlighting its dependence on vibration amplitude as well as on the analysis methodology. The wavelet transform is a powerful tool to be considered for studying other important effects on the structural behavior, such as the influence of the vehicle itself.

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Article

Effectiveness of Dynamic Vibration Absorber on Ground-Borne Vibration Induced by Metro

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Abstract: The application of dynamic vibration absorbers (DVAs) is a countermeasure to suppress vibrations induced by railway traffic. A key advantage of the DVA application is that it does not require any changes to the path of vibration propagation or the receiver of vibration. A review of the literature reveals the necessity of deriving the optimum properties of DVA to mitigate railway vibrations. To this end, the optimum DVA properties were investigated through the development of a two-dimensional finite element model of the track-tunnel-soil system. The model was validated using the results of a field test. A parametric study was made to obtain the optimum properties of DVA for different soils surrounding the tunnel. The results of the model analysis indicate that the DVA has better vibration reduction for metro tunnels built in soft soils as compared to those surrounded by medium and stiff soils. Also, the results disclose that the DVA reduces vibration radiated on the ground surface when the DVA natural frequency is tuned to a low frequency. Using the results of the parametric study, graphs are suggested to select the optimum properties of the DVA as a function of the soil around the tunnel.

Keywords: ground-borne vibrations; metro; railway; finite element analysis; DVA

1. Introduction

In recent decades, underground railway networks (subway and metro transportation systems) have been extended significantly. Also, the development of metro lines has solved heavy traffic and reduced greenhouse gas emission challenges [1]. However, ground-borne vibrations induced by underground railways are unavoidable challenges for urban rail transit (as shown in Figure 1a). In fact, the vibrations induced by the underground railway are propagated within the soil surrounding the tunnel and transmitted to buildings in the vicinity of the railway. The vibrations cause discomfort and annoyance to people as shown in Figure 1b [2]. Vibrations with low frequency content, specifically those of less than 20 (Hz), have adverse influences on buildings and are, importantly, undesirable to people living adjacent to the railway [3,4]. Residents of nearby buildings perceive the vibrations due to the oscillation of building elements in the frequency range of 1 to 80 (Hz) [5,6]. Also, the oscillation leads to structure-borne noise in the frequency range of 16 to 250 (Hz) [5,6].

There are several countermeasures to dissipate ground-borne vibration propagated from underground railway tunnels [7–14]. That is, the application of resilient layers such as rail pads, under sleeper pads, ballast mat, and under-slab elastomeric layer is a method to suppress vibrations in the source. The use of unfilled or in-filled trenches or a wave barrier is an approach to mitigate vibrations within the path. Finally, the application of a base isolator or resilient layer under foundation is a solution to control vibration in

the receiver of vibrations. The application of a dynamic vibration absorber (DVA) is an influential countermeasure to limit/suppress vibrations of structures subjected to dynamic loads [15–19]. The DVA plays as a secondary oscillating system that is connected to a main system called the host structure [20,21]. Typically, DVAs are used to moderate the response of the host system in a specific frequency range [22,23].

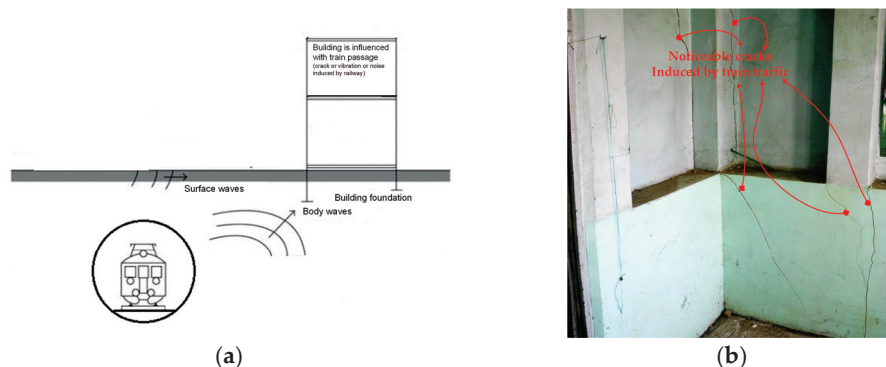


Figure 1. Challenges made by urban rail transit. (a) Schematic image of wave propagation due to metro-induced vibration, and (b) apparent cracks on wall plaster of residential building in the vicinity of metro tunnel (adapted from [2]).

The survey of the literature reveals that there are some studies on the application of DVAs in the vibration attenuation of railway vehicles and tracks. That is, to control vehicle vibration, DVA was used near the unsprung masses in the rolling stock [19]. Moreover, DVAs with negative stiffness were installed on rail in a ballasted track to attenuate the vibration propagation and noise [17]. Also, the application of DVAs in a floating slab track for limiting track vibration was studied [18,24,25]. Moreover, Noori et al. [16,26] studied the application of DVA in limiting underground railway-induced vibration. That is, DVAs were used in a double-deck circular railway tunnel. The Pipe-in-Pipe (PiP) model, which is a full-space model, was employed to study DVA vibration suppression. The PiP model cannot consider vibration transmitted to the ground surface. Also, the soil surrounding the tunnel was only considered as a homogeneous medium, and multilayered soil was not considered [26,27]. The survey of the literature elucidates that there is insufficient research on the study of optimum DVA for the mitigation of ground surface vibrations induced by the railway tunnel. In fact, there is a need to tune DVA properties as a function of soil condition (i.e., vibration path) around the metro tunnel. To fill this research gap, the optimum properties of DVA for different soils around the metro tunnel were obtained. To this end, parametric studies were conducted using the development of a two-dimensional (2D) finite element (FE) model of a railway tunnel surrounded by soil. The model was validated through measurement results derived from a field test. Design graphs were obtained to acquire the mass, stiffness, and damping values for the optimum DVA as a function of the soil properties. The solution suggested in this research is a practical method to attenuate train-induced vibrations, especially the installation of DVAs in the metro tunnel which does not make any change in the track and tunnel structure, and train traffic is not interrupted.

2. Numerical Modeling

A 2D FE model of an underground railway cross-section, including the railway slab track, tunnel, and soil around the tunnel (a track-tunnel-soil system) was developed using Abaqus FE software (Abaqus 2018, Dassault Systèmes, Vélizy-Villacoublay, France). The model is shown in Figure 2. The results from the model were verified with measurements from a field test. After confirming the model's accuracy, parametric studies were performed

on the DVA properties to find the best features of the DVA. The following describes the details of the model development.

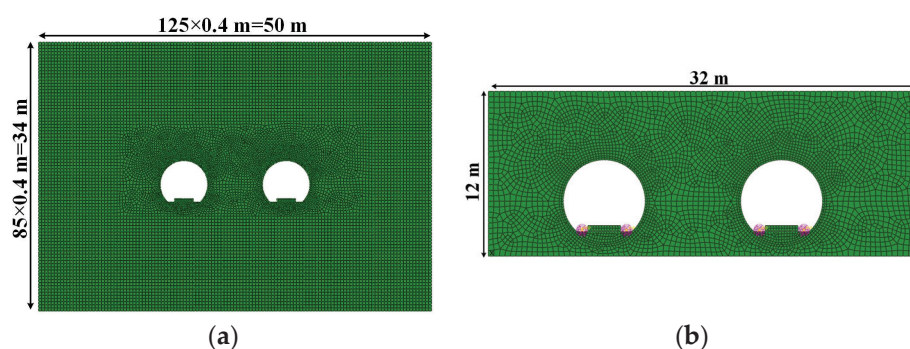


Figure 2. A 2D FE model of track-tunnel-soil system for wave propagation: (a) General view of the FE model; (b) position of DVA (in pink) near the slab track.

2.1. Finite Element Model

A cross-section of a metro twin tunnel and surrounding soils was simulated. Both tunnels have circular shapes. Plane strain elements were used to model the slab track, tunnel lining, and soil. These elements have four nodes with two translational degrees of freedom for each node. The model size was chosen to allow free wave propagation and ensure results were not influenced by boundary conditions or the size of the model. Sensitivity analysis was performed on the vertical and horizontal dimensions. The model's dimensions are shown in Figure 2: a width of 50 m, a depth of 34 m, a tunnel inner-radius of 3 m, a tunnel-lining thickness of 0.3 m, and a center-to-center tunnel distance of 13 m. The tunnel centers are 18 m below the ground surface. The slab track's thickness and width are 0.4 m and 2.4 m, respectively. To simulate the interaction between the tunnels and soil, the tunnel nodes were tied to the soil nodes at the contact edge.

Since only small strains occur due to railway vibrations, the material behavior of the concrete slab track, tunnel lining, and soil was assumed to be linear elastic [27,28]. To prevent wave reflection, energy-absorbing boundary conditions were set up at the boundaries [29,30]. Normal and tangential spring-dashpot elements were used at all nodes on the boundaries to absorb waves. That is, the boundary conditions ensure that compression and shear incident waves are not reflected into the interior domain and are efficiently absorbed by dashpot elements. The vibration propagation problem in the soil around the tunnel was solved using an implicit time integration method (Newmark method) in Abaqus. Implicit methods are unconditionally stable [31], but the time step must be smaller than the inverse of the highest frequency of interest. For ground-borne vibrations, frequencies from 1 to 80–100 (Hz) are influential [5,32–34], so the time step was selected to be less than 0.01 s. Residents of buildings near the railway perceive vibrations in this frequency band [5,32,34,35]. To excite the track, the impulse load was applied on the rail seat on the slab, creating a load with constant amplitude within 1 to 100 (Hz).

2.2. Field Test for Validation of Model Results

To verify the accuracy of the model, its results were compared with field test measurements. The field test was conducted in the Isfahan metro network. That is, the measurement was made in Line 1 between Daneshgah and Azadi stations, as shown in Figure 3. In this railway section, there is a twin tunnel, each with a single railway slab track. The track is tangent and has no longitudinal slope. It consists of S49 rails attached every 0.6 (m) to a reinforced concrete slab. At the test location, the ground surface is 18 (m) above the center of the tunnel. The tunnel has an inside diameter of 6.0 (m), with a lining thickness of

0.3 (m). The elastic moduli of the tunnel lining, slab track, and soil are 32.5 (GPa), 35 (GPa), and 150 (MPa), respectively. The densities of tunnel lining, slab track, and soil are 2500, 2500, and 2000 (kg/m^3), respectively [36]. These parameters were used as the input data for the model.

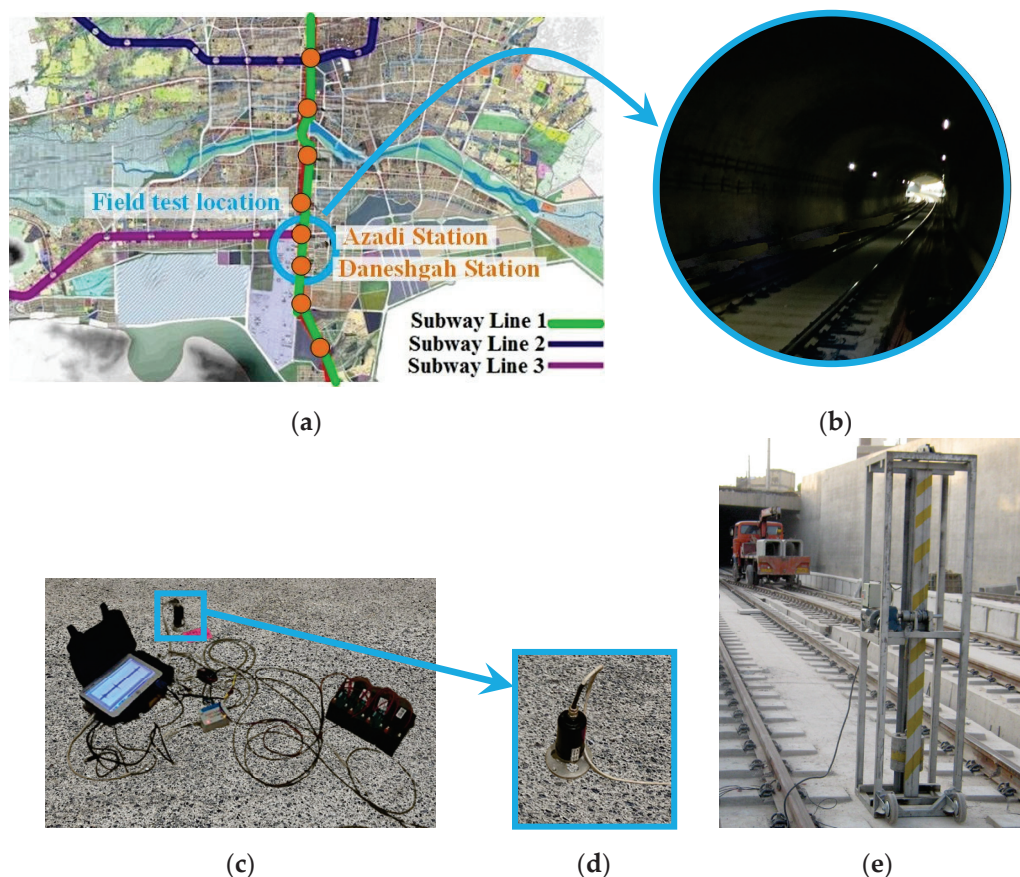


Figure 3. In situ test setup: (a) Map of Isfahan metro network; (b) general view of metro line in the field test; (c) equipment for vibration measurement (Data acquisition system and uniaxial velocimeter); (d) uniaxial velocimeter used for vibration measurement on ground surface; and (e) impact hammer device (before installation in metro tunnel).

For vibration measurement, uniaxial velocimeters (that have the commercial title of MTSS-1001 geophone, (R-sensors, Moscow region, Russia) were placed on the ground surface at several points, as shown in Figure 3c. These sensors measure maximum speeds of ± 30 (mm/s) within a frequency range of 1–300 (Hz) and can sense low-amplitude vibrations. The Geo-Arm-D24 data acquisition system, with a sampling rate of 200 (Hz), recorded the ground vibrations (Figure 3a).

In the field test, ground vibrations due to an impulse from an impact hammer were measured. The hammer, a falling weight device, generated an impulse with a maximum load of 32 (kN), producing measurable vibrations. The impact load was recorded by a load cell installed on the hammer. As shown in Figure 4, the impact load was almost constant across 1 to 100 (Hz). The time history of the impact load was imported into the model as an excitation.

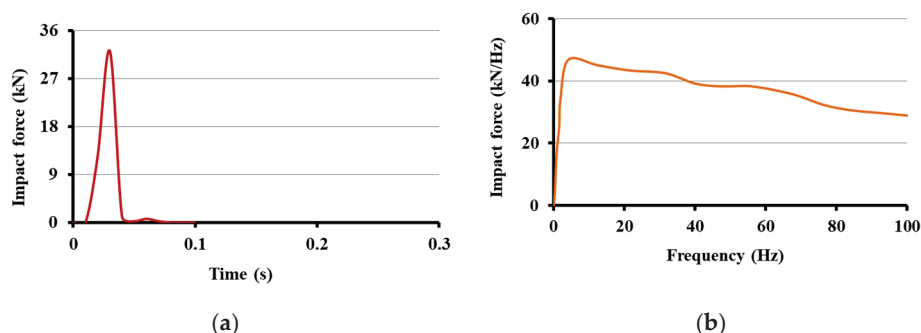


Figure 4. The impulse load derived from the in situ test: (a) the impact load in the time domain; and (b) the impact load in the frequency domain.

To allow vibration waves to propagate within the soil in the frequency range of 1 to 100 (Hz), the soil element size around the tunnel was selected to be small enough. In fact, the maximum soil element size was chosen based on the minimum wavelength of vibration, which is related to the shear wave velocity of the soil and the highest frequency of interest. That is, the maximum soil element size was chosen to be less than one-fourth to one-eighth of $\lambda_{\min}$ of soil [28]. To find the best mesh size, a sensitivity analysis was performed. The soil mesh size was varied considering $\lambda_{\min}$ and $f_{\max}$ criteria, and the velocity response on the ground surface was derived from the analysis. In fact, the soil mesh size varied from 20 (cm) to 50 (cm). By investigating the results obtained from models with various meshes, it was derived that a mesh size of 40 (cm) gives good convergence. To verify the model results, the vertical velocity of the ground surface at 5 (m) from the tunnel's centerline was measured due to the impact load. The comparison was made between model results and the measurements at the receiver point, as shown in Figure 5.

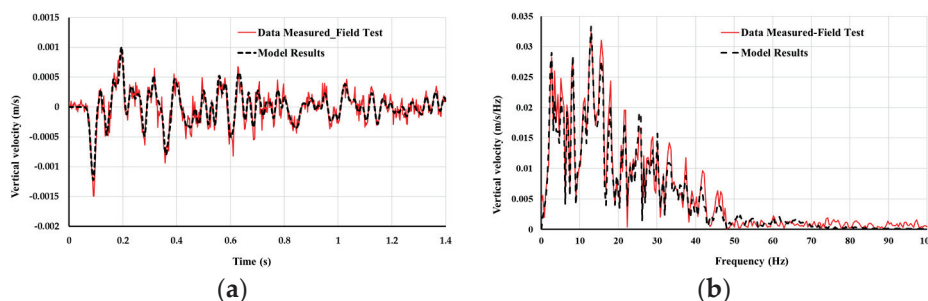


Figure 5. Results obtained from the field test and those derived from the model: (a) Vibration response in the time domain; and (b) vibration response in the frequency domain.

It should be noted that for the dynamic analysis, a time step of 0.0025 s was used. This time step corresponds to a sampling frequency of 400 (Hz). So, the model results can possess a frequency content of up to 100 (Hz). As presented in Figure 5, there is a good agreement between the model and field test results, with differences of up to 6%, which is considered acceptable for this study. The discrepancies between the results of field measurement and numerical analysis arose from uncertainties in the material properties. Some of the differences also appeared because of simplifications made in the simulation of boundary conditions.

3. Parametric Analyses

In this section, the effect of DVAs on reducing the vibration energy propagated upwards from the metro tunnel was evaluated. To this end, the kinetic energy of multiple points on the ground surface located at different distances from the axis of the metro

tunnel was obtained. Kinetic energy (representative of vibration energy) is derived using Equation (1) as follows [37]:

$$E_K = \int_V \frac{1}{2} \rho \vec{V} \cdot \vec{V} dV \quad (1)$$

In this equation, V and $\vec{V}$, respectively, are the volume and velocity of the element. Parametric analyses of the model were used to derive the optimum characteristics (including mass, stiffness, and damping ratio) of a DVA that can efficiently moderate kinetic energy. To this end, kinetic energy on the ground surface was examined while the properties of DVA were changed. Generally, DVA is simulated as a single degree of freedom (SDOF) [23]. In this study, DVA was simulated as a SDOF that was installed on the track foundation adjacent to the slab track, as illustrated in Figure 2b. Properties of the slab and tunnel were selected to be the same as those in the field test. Excitation and boundary conditions in the model that had DVA were the same as those in the validated model. The properties of soil play a significant role in the ground-borne vibration induced by the metro. Various soil profiles around the tunnel were taken into account. That is, stiff, medium, and soft soil surrounding the metro tunnel were considered, as presented in Table 1. The soil properties were adapted from the codes of practice [38] to consider a wide range of soils in practice. In parametric studies, the characteristics of DVA (i.e., mass, stiffness, and damping value) were varied for each soil type. The optimum DVA is the one that attenuates metro-induced vibration (i.e., kinetic energy) the most.

Table 1. Properties of soil neighboring the tunnel.

Soil Type	Modulus of Elasticity [MPa]	Density [kg/m ³]	Poisson's Ratio	Shear Wave Velocity [m/s]
Soft	150	2000	0.3	170
Medium	300	2000	0.3	240
Stiff	450	2000	0.3	294

In accordance with design methods available in the literature [21,23], to achieve efficient performance of TMD or DVA, the natural frequency of DVA should be selected in harmony with the natural frequencies of the dominant modes of host (primary) structure. The natural frequency (f_{nd}) of DVA is derived based on SDOF behavior. That is, $f_{nd} = \frac{1}{2\pi} \sqrt{\frac{k_d}{m_d}}$, in which k_d and m_d stand for stiffness and mass of DVA. In this study, eigenvalue analysis of the slab track-tunnel system was performed to derive the natural frequencies of the host system. In Figure 6, the natural frequencies and mode shapes of the slab track-tunnel system are shown.

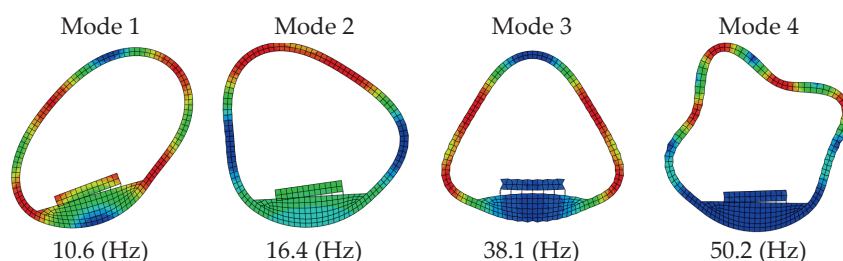


Figure 6. Natural frequencies and mode shapes of the slab track-tunnel system.

In accordance with the natural frequencies of the primary system shown in Figure 6, the natural frequency of DVA was changed from 1 (Hz) to 50 (Hz). In practice, the ratio of DVA mass to the mass of the host structure was selected in the range of 1% to 10% [21,23].

To begin parametric studies, light-mass DVAs (i.e., mass ratios of 0.01 and 0.05) were considered. The vibration of a point on the ground surface near the tunnel built in soft soil was investigated. The time increment in the implicit time integration method was set to 0.0025 (s), which is the same as the time increment considered in the validated model.

The natural frequency of DVA was tuned to be the same as the natural frequency of the track-tunnel system in the first mode of vibration (i.e., 10.6 (Hz)). In Figure 7a,b, the time history curves of kinetic energy for the point on the ground surface at 4 (m) from the tunnel axis are illustrated. DVAs with damping ratios of 0.01 and 0.05 were separately analyzed. Considering Figure 7a,b, kinetic energy is not influenced by DVA, and there is no significant change in the energy when light-mass DVA was installed near the track. Also, similar results (i.e., no significant changes) were derived when kinetic energy results were obtained for points located at the other distances from the tunnel axis.

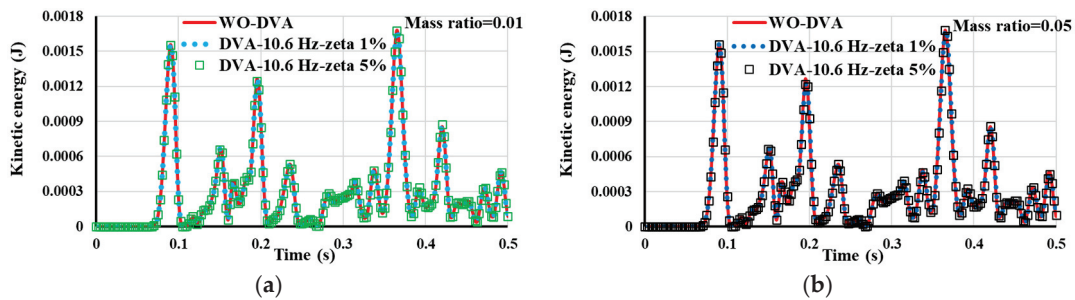


Figure 7. Time history of kinetic energy for the point at 4 (m) from the tunnel surrounded with soft soil: (a) Mass ratio of DVA equals 0.01; and (b) Mass ratio of DVA equals 0.05.

To further study the impact of light-mass DVA on ground-borne vibration, DVAs with different dynamic properties were simulated. That is, DVAs were tuned to have a natural frequency the same as the natural frequency of the slab-tunnel system for the second, third, and fourth modes of vibration. Damping ratios of 0.01 and 0.05 were considered. The results of analyses elucidate that DVAs with low mass do not have influential effects on vibration deterioration and cannot efficiently suppress ground-borne vibration. As shown in Figure 7a,b, an increase in the damping ratio of DVAs did not result in improvement of DVA effectiveness.

Because light-mass DVA has a negligible effect on vibration mitigation, the mass ratio of DVA gradually increased from 0.05 to 0.1 while the damping ratio of DVA was considered to be 0.01. The DVA natural frequency was also adjusted to closely match the slab-tunnel system's natural frequencies and operate over a broad frequency band (i.e., from 1 (Hz) to 50 (Hz)). The results of the parametric studies are presented in Figure 8a–f.

To better show vibration suppression provided with DVA, the insertion loss index was derived. According to the literature [39], the insertion loss is a common index that represents changes in the mean vibrational energy. The insertion loss (*IL*) was derived using Equation (2):

$$IL = 10 \log_{10} \left(\frac{\bar{E}_{DVA}}{\bar{E}_{wo-DVA}} \right) \quad (2)$$

In Equation (2), $\bar{E}_{DVA}$ and $\bar{E}_{wo-DVA}$ are the root mean square (RMS) of the kinetic energy for a certain point when the track is equipped with DVA and when there is no DVA, respectively.

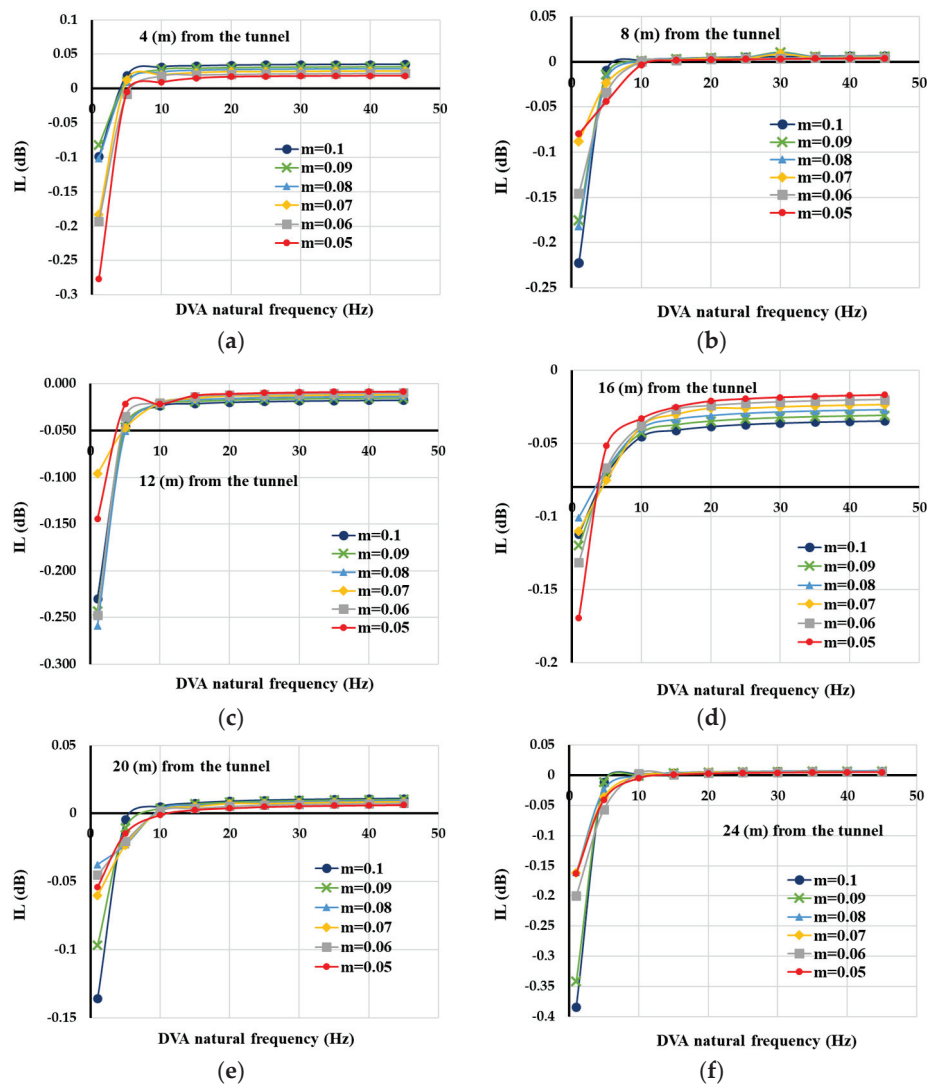


Figure 8. Insertion loss against the natural frequency of DVA when the tunnel is surrounded with soft soil: (a) Receiver point at 4 (m) from tunnel axis; (b) receiver point at 8 (m) from tunnel axis; (c) receiver point at 12 (m) from tunnel axis; (d) receiver point at 16 (m) from tunnel axis; (e) receiver point at 20 (m) from tunnel axis; and (f) receiver point at 24 (m) from tunnel axis.

As presented in Figure 8a–f, insertion loss was significantly influenced by changes in the natural frequency of DVA and distance from the source of vibration. Generally, the most efficient condition was achieved when the DVA natural frequency was set to 1 (Hz), regardless of the distance of the receiver point from the tunnel and the DVA mass ratio. As the natural frequency of DVA increases more than 1 (Hz), DVA performance significantly deteriorates. In fact, except for the results obtained for a point at 16 (m) from the tunnel, when the DVA natural frequency is higher than 5 (Hz), the insertion loss was approximately unchanged. In other words, vibration mitigation was not influenced by DVA natural frequency and mass ratio. For the receiver point at 16 (m) from the tunnel, vibration suppression was slightly augmented with an increase in DVA natural frequency and mass ratio. To further investigate the influence of DVA properties on insertion loss, DVA mass ratios were gradually increased from 0.1 to 0.15, and parametric investigations were made on DVA characteristics. Results of parametric studies are illustrated in Figure 9a–f.

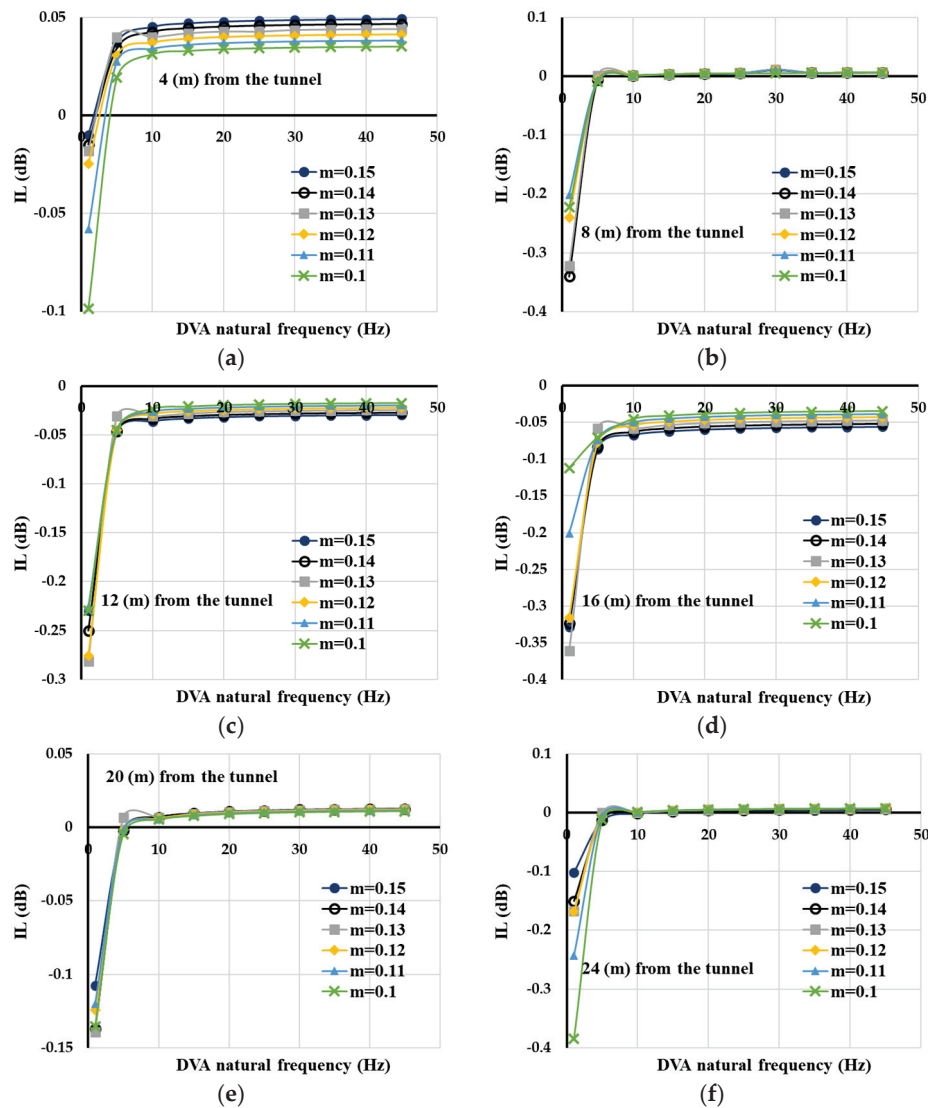


Figure 9. Insertion loss against natural frequency of DVA when the tunnel surrounded with soft soil: (a) Receiver point at 4 (m) from tunnel axis; (b) receiver point at 8 (m) from tunnel axis; (c) receiver point at 12 (m) from tunnel axis; (d) receiver point at 16 (m) from tunnel axis; (e) receiver point at 20 (m) from tunnel axis; (f) receiver point at 24 (m) from tunnel axis.

As per the results elaborated in Figure 9, the most influential DVA was derived when the natural frequency was tuned to 1 (Hz) for all distances from the tunnel and for all DVA mass ratios. As the natural frequency is increased, DVA efficacy importantly falls. Results shown in Figure 9a reveal that when the receiver point was close to the tunnel, DVA that has a lower mass ratio has slightly better vibration dissipation for all natural frequencies. However, with an increase in the distance of the receiver point from the tunnel, similar to the results shown in Figure 8, DVA effectiveness is nearly constant. That is, as natural frequency increases more than 5 (Hz) while mass ratio increases, the insertion loss does not change. In fact, with an increase in DVA natural frequency, insertion loss becomes insensitive to DVA mass. Because the best result for insertion loss was obtained about -0.38 (dB) (which is lower than expected), the next round of parametric investigations was conducted to reach superior vibration reduction. In this round, the DVA mass ratio was gradually increased from 0.1 to 0.35, and the results derived are illustrated in Figure 10a–f.

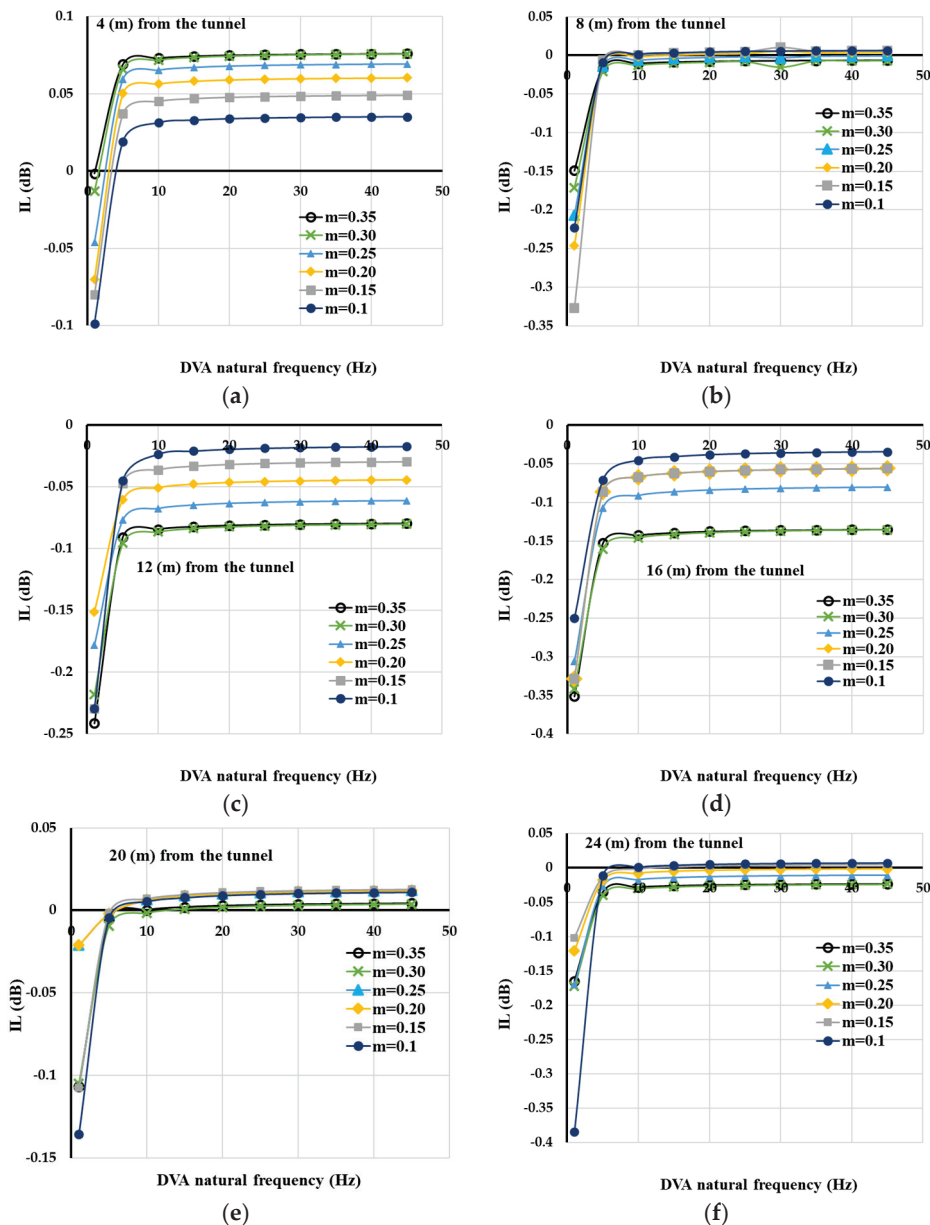


Figure 10. Insertion loss against natural frequency of DVA when the tunnel is surrounded with soft soil: (a) Receiver point at 4 (m) from tunnel axis; (b) receiver point at 8 (m) from tunnel axis; (c) receiver point at 12 (m) from tunnel axis; (d) receiver point at 16 (m) from tunnel axis; (e) receiver point at 20 (m) from tunnel axis; (f) receiver point at 24 (m) from tunnel axis.

The trend of results illustrated in Figure 10a–f is similar to that depicted in Figures 8 and 9. That is, the most effective DVA was achieved for DVA with the natural frequency of 1 (Hz). Also, when the natural frequency exceeds 5 (Hz), DVA effectiveness importantly drops and remains unchanged. For points at 12 (m) and 16 (m) from the tunnel, it was obtained that the most efficient DVA is the one that has the greatest mass ratio. Nevertheless, for the point at 4 (m) from the tunnel, the best performance of DVA was obtained when the mass ratio was reduced. This issue implies that DVA effectiveness is an important function of distance from the vibration source and mass ratio.

A survey of the obtained results unveils that the oscillation of DVA with high natural frequency cannot effectively dissipate vibration energy radiated on the ground surface. Also, the results disclose that the most effective DVA does not have a natural frequency close to the natural frequencies of the slab-tunnel system. This implies that assuming a

complicated system such as a slab tunnel equipped with DVA is a two-degrees-of-freedom system is somewhat misleading. In fact, ground-borne vibration is highly dependent on soil characteristics and Rayleigh wave radiation. As a result, tuning DVA natural frequency to the natural frequencies of the slab-tunnel system cannot lead to the most vibration mitigation.

Hereafter, vibration propagated from metro tunnels surrounded with medium and stiff soil influenced by DVA was investigated. As depicted in Figure 11a,b, vibration amplitude was approximately constant and was not nearly varied when the track was equipped with DVA. Results of extensive parametric studies conducted on DVA natural frequency indicate that DVA effectiveness is insignificant as the tunnel is enclosed with medium and stiff soil. For instance, the vibration of a point on the ground surface at 4 (m) from the tunnel was not significantly changed by a DVA with a mass ratio of 0.1 and a natural frequency of 1 (Hz), as shown in Figure 11a,b. This discloses that the application of DVA in a tunnel surrounded with medium and stiff soil cannot efficiently lead to vibration dissipation. However, when the wave propagation medium is soft soil (i.e., the soil's shear wave speed is less than 170 (m/s)), it can benefit from DVA to limited vibration.

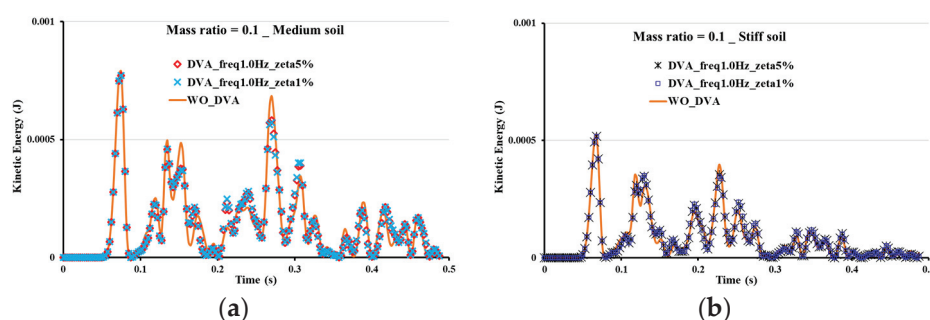


Figure 11. Time history of kinetic energy for the point located at 4 (m) from the tunnel surrounded with: (a) Medium soil and (b) stiff soil.

4. Discussion

Outlining the results obtained from parametric studies indicates that the most vibration attenuation was gained when the DVA natural frequency is 1 (Hz) while the tunnel is enclosed with soft soil. In Figure 12, the optimum mass ratios of DVA for points at different distances from the tunnel are illustrated. As presented in Figure 12, due to the complex nature of wave propagation within the soil, the optimum mass ratio varied significantly with the horizontal distance of the receiver point from the source of vibration. Although Rayleigh wavelength was constant throughout the soil, the variation in insertion loss for different receiver points is not derived as a function of Rayleigh wavelength.

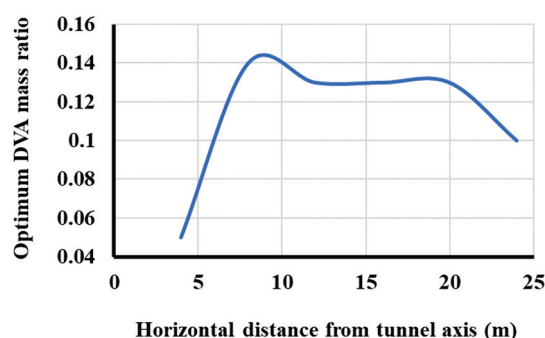


Figure 12. Optimum values for DVA mass ratio.

In the present study, the highest level of vibration dissipation was achieved with a DVA that has a mass ratio of 0.1 and a natural frequency of 1 (Hz) for a point at 24 (m) from the tunnel (as presented in Figure 10f). In this condition, vibration decreased by 0.38 (dB). To evaluate the efficacy of DVA suggested in this study, the results derived here were compared with those available in the literature. Zhou et al. [24] implemented DVA under a slab track; their numerical studies indicate that DVA reduced slab acceleration by 40% in a narrow frequency range of 55 to 65 (Hz). However, the efficiency of the DVA suggested in [24] became negligible outside of this narrow frequency band. Zhu et al. [3] attached DVAs to a floating slab track. Their numerical analyses elucidate that slab accelerations were decreased by up to 20% in the frequency band of 10 to 20 (Hz); the slab acceleration was not influenced by DVA out of the frequency band [3]. Noori et al. [26] modeled a series of DVAs in a double-deck tunnel and derived that DVAs reduced vibration energy by 6.6 (dB) in a narrow frequency band in the vicinity of 31.5 (Hz). Kouroussis et al. [19] numerically assessed whether DVAs are more effective in the vehicle or on the track to mitigate vibration. Kouroussis et al. [19] obtained a reduction of 2 (dB) for the motorized wheel at the frequency of 20 (Hz) [19]; they derived that the response of the vehicle was negligibly reduced due to DVA being installed on the rail or sleeper [19]. A scrutiny of studies on the application of DVA in suppressing railway vibration reveals that DVA efficiency is limited to a narrow frequency band.

5. Conclusions

The main goal of this research is to evaluate the efficiency of using DVAs in underground railway tunnels to suppress vibrations induced by train passage. To this end, numerical modeling was performed in Abaqus FE software. The model includes a slab track, tunnel, and soil. The model was 2D and consisted of four-node plane strain elements. Artificial boundary conditions of damper-spring were used for model boundaries. The model results were validated by the use of the results derived from field measurements conducted in the Isfahan metro network. To investigate DVA performance, an impact load, same as the loading condition in the field test, was simulated as an excitation. Also, DVA was simulated near the slab track. Parametric studies were conducted on the natural frequency, mass ratio, and damping ratio of DVA in order to derive optimum properties of DVA that effectively mitigate ground-borne vibrations. In addition, three different soil properties (i.e., soft, medium, and stiff soils) were considered in parametric investigations. The natural frequency of DVA was tuned in harmony with the natural frequencies of the tunnel-track system that were derived from eigenvalue analysis. Also, natural frequencies of DVA were selected to be in a wide frequency band (i.e., from 1 (Hz) to 50 (Hz)). Results indicate that when a tunnel is surrounded with soft soil, DVAs with a mass ratio of 0.01 have negligible effects on vibration reduction. However, as the mass ratio of DVA is increased to 0.1, the vibration is decreased by 0.38 (dB). The results obtained reveal that augmentation of the damping ratio of DVA from 0.01 to 0.05 has no important effect on DVA efficiency. For medium and stiff soils, ground-borne vibration was not influenced by DVA. For further analysis of the DVA effect on vibration reduction, the DVA mass ratio was increased gradually to 0.35. The derived results disclose that the most effective DVA has a natural frequency of 1 (Hz). Also, the optimum DVA mass ratio ranges from 0.05 to 0.14 and depends on the receiver point's distance from the tunnel. Results indicate that increasing the DVA mass ratio beyond 0.14 does not result in significant vibration suppression. Due to the practical limitations in underground tunnels, especially regarding tunnel clearance, DVAs with moderate mass ratios are recommended for metro tunnels. According to the results obtained, DVA with a natural frequency of 1 (Hz) and a damping ratio of 0.01 can moderately limit ground-borne vibration induced by metro tunnels built

in soft soils. The results reveal that installing DVA adjacent to the slab track in the railway tunnel does not result in a significant reduction in vibration on the ground surface. Finally, it can be concluded that although DVA can control ground-borne vibration, the level of vibration suppression by DVA is limited in practice.

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Abbreviations

The following abbreviations are used in this manuscript:

DVA	Dynamic vibration absorber
TMD	Tuned mass damper
FE	Finite element
RMS	Root mean square
SDOF	Single degree of freedom
2D	Two-dimensional
PiP	Pipe-in-Pipe

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Article

Feasibility Analysis of Monitoring Contact Wire Rupture in High-Speed Catenary Systems

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Abstract: The rupture of the contact wire (CW) of a railway overhead contact line (OCL or catenary) is expected to be a rare event. However, when it occurs, and a pantograph transits under the already broken section of the CW, this can have catastrophic consequences for the pantograph which in turn can cause a further extension of the damaged portion on the OCL with a consequent disruption in the service and cause there to be a long time before the operating condition can be restored. Therefore, the prevention of such events through effective catenary monitoring is gaining significant attention. The purpose of this work is to investigate the feasibility of a monitoring system that can be installed at each end of an OCL section which is able to detect the occurrence of a broken CW event, sending an alert to the management traffic system, so as to stop the train traffic before the damaged catenary is reached by other trains. A nonlinear dynamic analysis is employed to model the OCL's response following a simulated CW rupture and identify a set of variables that can be measured at the line's extremities related to the occurrence of breakage in the CW. Several locations of the rupture of a CW section along the line are simulated to investigate the influence on the time pattern of the measured variables and consequently on the extraction of a signature. Finally, a proposed measurement setup is presented, combining accelerometers and displacement transducers, instead of the direct measurement of the axial load of the OCL conductors.

Keywords: rupture of OCL contact wire; OCL monitoring; nonlinear dynamics

1. Introduction

An overhead contact line (OCL) is a key component of the railway infrastructure's energy subsystem, supplying electrical power to the motors of electric trains. The main components of an OCL include a contact wire (CW), which is suspended to the messenger wire (MW) by a series of droppers spaced along the line. The MW is supported by suspensions attached to structures such as poles or portals, typically at intervals of 50 m to 60 m. Each suspension consists of a frame, insulators, and a steady arm, with the latter imposing the lateral position (stagger) of the CW. Figure 1 reports a schematic of an OCL with its main components.

The purpose of the OCL is to supply electrical power to train motors through a sliding contact between the contact wire (CW) and the pantograph collector strip.

Maintaining the correct geometry and structural integrity of the OCL is essential for safe and efficient railway operation. Any failure or misalignment of one or more OCL components can damage the pantograph, and in the worst case, cause additional damage to the OCL itself. Such damage can lead to service interruptions, disrupting traffic flow.

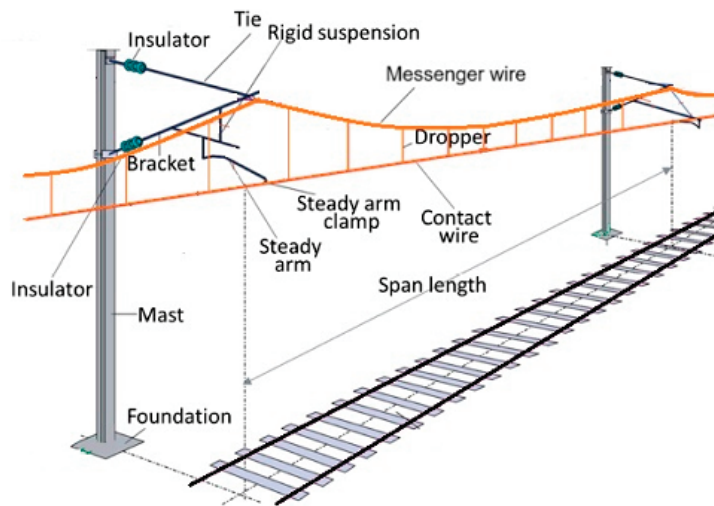


Figure 1. Schematic of an overhead contact line and its main components.

Different types of failures and damages can affect the OCL, each with different impacts on railway operation. These can range from the accelerated wear of individual components to the severe damage or breakage of critical OCL elements.

Failures that may occur in the OCL can be categorized into two main types: (i) failures arising from initial damage to the pantograph, and (ii) failures caused by improper geometrical configuration of the OCL. In these cases, the damage may be limited to the pantograph or extended to the structural integrity of the OCL itself. Other types of failure may occur independently of pantograph transit, often as a result of accumulated damage, leading to a break in the contact wire (CW). Such damage may also be triggered by external factors, such as ice buildup, wind, uncompensated temperature variation and associated overstress, or local overheating due to high current intensity. The failure of the CW can cause severe damage on a pantograph passing beneath the broken CW section, potentially resulting in a catastrophic event requiring the suspension of traffic. Therefore, it is crucial to prevent the transit of the pantograph under a damaged section of the OCL with a broken CW. In such extreme cases, it is important to note that a CW break may not be detectable by an electrical voltage drop from the power station since electrical continuity is maintained unless the messenger wire (MW) is also damaged.

More common failures of the OCL involve the droppers [1–4] or elements of the suspension of the span [5]. On the other hand, although the rupture of the contact wire (CW) is a rare phenomenon, with a probability on the order of 10^{-8} (see [5]), it must still be considered in a proper risk analysis due to its potential consequences. Localized wear and fatigue failure are considered the most common causes of CW rupture. Wear and fatigue are concurrent damage mechanisms [6]; when wear rate is low, as in the case of a high-speed (HS) line, fatigue failure is more likely to develop from microcracks beneath the CW surface.

Possible causes of crack initiation and subsequent fatigue failure in the CW include the continuous occurrence of arcing, combined with bending of the CW [7] during pantograph transit and the subsequent decay that can be lengthy due to the low structural damping of the OCL. Other causes of fatigue are related to the presence of a clamp (either temporary for maintenance or permanent). In such cases, CW failure may occur unexpectedly and remain undetected during standard inspections.

Some research devoted to improving mechanical behavior and performances to prevent failures of wires and overhead line components can be found in the literature [8–10]; but, while these actions are able to reduce the probability of failure related to the fatigue

failure of single OCL components, they are not able to avoid failure related to accidents caused by external factors such as contact between the OCL and damaged pantographs or failures caused by improper geometrical configuration of the OCL.

Examples of the consequences of CW breakage can be found in investigations conducted by safety surveillance authorities or transport boards, usually linked to the National Ministry of Transport.

The first example comes from a regional network close to Rome, Italy ([11], reference in Italian), reported by DIGIFEMA, the Italian office branch of the Ministry of Transport, which is responsible for investigating rail and maritime transportation accidents. In this case, the CW broke due to the transit of a damaged pantograph. Subsequent pantograph transit further damaged the CW, which remained electrically connected to the system and at line voltage. The main following secondary effects were electric arcs and the start of fires. In the meantime, passengers spontaneously evacuated the train, with the power line still in operation due to bilateral connections to the power system and a defective switch. The electrical voltage was only switched off after the on-board train manager called for assistance and safely evacuated the passengers.

A second example, reported by DAIB in the UK [12], involved a train driver who noticed the damaged CW ahead and stopped the train before passing under the broken section. After notifying the signaller staff pertinent to the area, the driver left the train to refer to the exact location of the problem. However, the low-hanging CW was still electrically live, and the driver suffered serious injuries after coming into contact with the rails. The subsequent investigation revealed that the wire had broken due to fatigue, likely initiated at a high-stress area near a clamp supporting the overhead wire.

A third example of accident involving the breakage of the CW of an OCL concerns a freight train in the Netherlands on 14 September 2022 [13]. During the train transit the CW broke, causing a fire in the freight cars being transported. The CW breakage occurred at multiple points and the pantograph subsequently drew 800 m of OCL, further extending the damage.

A subsequent investigation by ProRail [13] revealed that the breakage began at temporarily repaired sections where the contact wire was below the permitted thickness, and supplementary parallel short lengths of CW were set aside with additional clamps. The combination of electrical arc erosion and the presence of clamps eventually led to the CW breaking during pantograph transit.

The fire was triggered by a short circuit caused by a switch operation, which occurred without the operators being aware of the ongoing issue.

The above-mentioned examples underscore the severe consequences of CW breakage and highlight the importance of developing monitoring systems for the OCL that can alert traffic control rooms and prevent trains from passing under a broken CW section.

In the literature, some monitoring systems are proposed to monitor continuously the OCL to detect CW cracks analyzing the changing law of eddy current signal characteristics [14] or, in general, to minimize potential risks that the infrastructure faces by using suitably developed fiber Bragg grating (FBG) sensors-based solutions [15].

The present paper examines the feasibility of a monitoring setup, using standard sensors as accelerometers and displacement transducers, designed to detect a rupture event in the CW and consequently transmit an alert to the traffic management unit. The goal is to prevent train transit under the broken section, minimizing potential harm to passengers and operators and reducing the damage to the OCL, which could delay the restoration of normal operation.

The proposed monitoring system is conceived to be installed at each end of the OCL regulation to detect the dynamic effect caused by a CW break, which disrupts its

mechanical continuity. Consequently, the mechanical tension (axial load) of the portion of the CW is subjected to a sudden change, with a pulse-like pattern, that travels along the OCL and reaches the extremity of the regulation, where measurements are intended to be taken. This paper investigates the dynamic behavior of the OCL after the occurrence of a failure in the CW, simulating the after-breakage transient by means of nonlinear dynamic analysis to define the pattern of the acceleration, displacement, and axial force at a line's extremities. Several locations of CW rupture are examined to analyze the resulting time history of variables that can be measured at the regulation's end, comparing them under different prospects in terms of feasibility, CW rupture detection capability, and challenges in processing the measured variables.

The paper is organized as follows:

- Considerations on the monitoring setup potentially applicable to the OCL;
- Description of the OCL model and of the procedure for simulating the CW breakage;
- Results from simulations in the examined scenarios;
- Final remarks.

The aim of the paper is to present the concept of a monitoring system capable of detecting the occurrence of a rupture in the CW to raise an alert to traffic management and halt train traffic in the affected section, preventing further damage and ensuring passenger safety.

2. Some Consideration on the Monitoring Setup in Relationship to OCL Structure

The arrangement of a measurement and monitoring system on a single structure, such as a bridge or a tunnel, is a common practice today. However, implementing such systems on an OCL is more challenging because of several reasons. The main reason is that the active components of the OCL structure are at a high voltage, with typical nominal voltages of 1.5 kV or 3 kV DC, or 15 kV or 25 kV AC. This characteristic poses issues concerning (i) the insulation of the transducers to be used for monitoring, (ii) how to power them, and (iii) how to transmit the signals to the data acquisition and processing system.

Another critical challenge is the large territorial extent of the infrastructure. For this reason, the use of inspection trains or vehicles is the most efficient way to monitor large areas. This approach is particularly effective for monitoring those parameters that show continuous development, such as the wear of the CW or its geometrical configuration. However, since the interval between inspections depends on the availability of inspection trains, the network extension, and the line category, periodic inspections are not suitable to detect sudden failures. Under this respect, continuous monitoring would be more effective. The challenge, then, is to set a balance between infrastructure coverage and the ability to detect and report alerts quickly to control sites in order to halt the traffic before the section involved in the CW breakage is traversed by a subsequent train.

A monitoring system needs to consider the particular structure of the OCL, which is typically divided into sections (called "regulations") with lengths ranging from 1000 m to 1400 m, consisting of mechanically tensioned uninterrupted conductors, namely the CW and the messenger wire (MW). The length of the CW and the MW corresponds to the length of the entire regulation, so that there are no junctions on the conductors inside a regulation. The CW and the MW are unwrapped from a cable reel, brought along the line by special vehicles during the construction stage. In the final configuration, axial load is applied to the conductors by a series of masses whose gravity force is transferred and amplified through a device consisting of pulleys and a chain (Figure 2). The loading device is connected to the end side of the conductors through an electrical insulator. This tensioning device allows for thermal expansion and contraction associated with temperature changes without

affecting the overall axial load of the conductors. When considering on-site monitoring of the OCL to detect CW ruptures, the first issue that arises is the high cost and impracticality of monitoring every span, regardless of the type of transducer used. This is due not only to the number of transducers required but also to the need to power a distributed network of sensors and transmit their signals. Additionally, maintaining such a monitoring network would add further complexity. A more practical solution would be to concentrate measurements in a single section, selected based on the structure of the OCL.

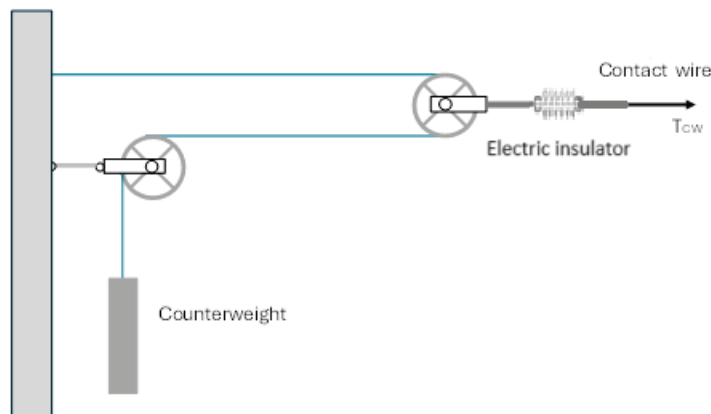


Figure 2. Sketch of a generic OCL regulation with 1:2 ratio force reduction.

Given the structure of the OCL, a suitable solution for transducer placement would be at the end section of the regulation, considering that the effect of CW breakage can be potentially measured also at large distance, enabling the monitoring of half of the regulation (from the fixed point to the load regulation device at each end of the section) with a single measurement system.

The consequences of the breakage of the CW are of two types: a dynamic response due to an impulse-like effect and the change in statical configuration. That considered, the proposed candidate variables worth being measured at the end of the regulation are as follows:

- The dynamic variation in mechanical tension on both the CW and MW;
- The longitudinal displacement of the CW and MW;
- The vertical acceleration of the CW.

However, several challenges must be considered when installing transducers on an OCL. The most significant issues are outlined below:

- Application of transducers on a structure at high voltage. Transducers could be placed before the insulators at the ground potential or after the insulator on the CW. In the latter case, issues related to power supply and signal transmission for standard sensors must be carefully addressed and solved.
- Powering of transducers. Continuous power is required for the transducers to ensure that the alert function always remains active: a combination of batteries and solar panels can be used
- Metrological parameters of the transducers. These include frequency range, long-term bias, and sensitivity to temperature.
- Local processing of the acquired signal. This includes computational effort, management of data acquisition, and signal processing.
- Signal transmission and alerting. Efficient transmission of the signal or index and the consequent alert must be ensured.

- Installation, durability, maintenance, and diagnostics of the transducers. Considerations related to the installation process, long-term durability, ongoing maintenance, and the ability to perform diagnostics on the transducers must be considered.

Considering the above-mentioned issues, some recommendations on the transducers suitable to be used are given below.

- For the measurement of the axial load in the CW and in the MW an optical fiber-based strain gage is the preferable option to measure strain, avoiding the issues related to electrical insulation, with the main drawback being the cost of the signal conditioning device, depending on the type of fiber optics used, since the dynamic behavior needs to be properly measured. FBG requires f.i., a rather costly conditioner, that in turn needs to be powered on-site. As will be shown later in the report, a combination of displacement and acceleration measurement is proposed in spite of direct axial load measurement.
- For the measurement of the longitudinal displacement of a CW and MW section at an OCL end, a long-range optical system can be used without requiring a high resolution due to the expected displacement. The target could be the insulator or the counterweight (see Figure 2), and the measuring device could be located before the insulator in the zone at ground potential so that no insulation issue is present.
- As for the acceleration measurement on the CW, its location is clearly in a live voltage area. An available and convenient option is given by a wireless accelerometer which is able to transmit the signal avoiding the use of electrical cables. Use of such devices has been already demonstrated in real operating conditions.

3. Model of the OCL

The OCL considered in this paper is the Italian HS line operating a 25 kV AC system. The CW and the MW are tensioned at regulated tensions of 20 kN and 16 kN, respectively. The main parameters of the CW and the MW are summarized in Table 1.

Table 1. Main parameters of the Italian high-speed OCL conductors.

Conductor	Cross-Section [mm ²]	Linear Mass [kg/m]	Mechanical Tension [kN]
Contact wire (CW)	150	1.34	20
Messenger wire (MW)	120	1.08	16

A finite element model was created using Abaqus[®] 2022 version, where beam elements are adopted to represent the conductors and insulators while connectors were used to model the droppers to simulate their slackening during the dynamic transient following the breakage of the CW. Only one half of the regulation OCL has been modeled, and the symmetrical boundary conditions have been considered. The model, shown in Figure 3, consists of 420 nodes and 412 B31 beam linear elements to model the CW, MW, and insulators according to the detailed cross-sections reported in Table 2. To model the droppers, 83 nonlinear axial connectors, 1 for each dropper, have been used as shown in the detail of Figure 3. These axial nonlinear connectors are defined by the initial undeformed distance between the two connected nodes; the first node is on the MW and the second node is on the CW. If the distance decreases, the connector exerts no reaction force, corresponding to the dropper slackening, while an increasing distance results in a reaction force proportional to the dropper's axial stiffness. This model allows for a simple yet effective representation of the dropper's behavior under tension and relaxation. The current model does not account for the stagger, or the zig-zag arrangement, of the OCL conductors.

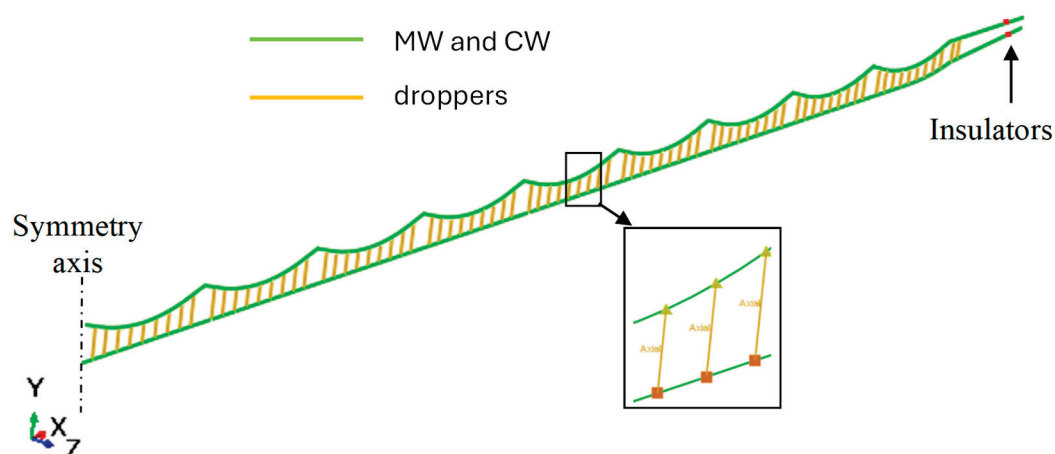


Figure 3. General view of the scheme of the half regulation of the OCL. The fixed point section (left side) realizes a longitudinal restrain for the conductors. Mechanical tensions are applied by means of the constant forces at the regulation's ends.

Table 2. Details of the FE model equivalent cross-section for the conductors and the insulators.

	Cross-Section Profile	Equivalent Diameter [mm]	Inertia Moment [mm ⁴]	Young Modulus [MPa]	Poisson Coefficient	Shear Modulus [MPa]
Contact wire (CW)	Generalized	13.8	$J_{11} = 2865$ $J_{polar} = 5730$	118,850	0.3	45,430
Messenger wire (MW)	Circular	12.4	-	118,850	0.3	-
Insulators (I)	Circular	40	-	24,200	0.3	-

Regarding the damping of the OCL structure, Rayleigh damping is assumed with parameters $\alpha = 0.1$ and $\beta = 0.001$, i.e., the damping matrix $[C]$ can be defined as follows:

$$[C] = \alpha[M] + \beta[K] \quad (1)$$

where $[M]$ and $[K]$ are, respectively, the mass and stiffness matrices of the OCL numerical model. The assumed values of the damping parameters are in the high range of the usual damping ratio, corresponding to a value of 2%, while other reported damping values are around 0.5%.

To model half regulation of the OCL symmetry boundary conditions have been applied at the symmetry axis (Figure 3). To model the suspension configuration that permits the longitudinal motion of the MW and accounts for the effect of the stagger arm, which only restricts lateral movement, the connection points with the supporting structure are represented as stiffness elements. These elements are applied in the vertical and lateral directions for the MW, and in the lateral direction only for the CW. The corresponding boundary conditions are shown in Figure 4a.

Additionally, the applied mechanical tension is modeled as a constant force at the extremities of the CW and MW. The kinematic boundary condition at these extremities is simplified to a longitudinal sliding constraint, as illustrated in Figure 4b.

The analysis performed is an implicit static analysis, which accounts for both the gravitational load and the axial load in the CW and MW. The resulting static displacement and stress field serve as the initial conditions for the subsequent explicit dynamics analysis, which simulates the breakage of the CW.

The simulation of the CW breakage clearly requires a full nonlinear analysis in the time domain, where the rupture of the CW is modeled using a beam connector, with an assigned failure criterion.

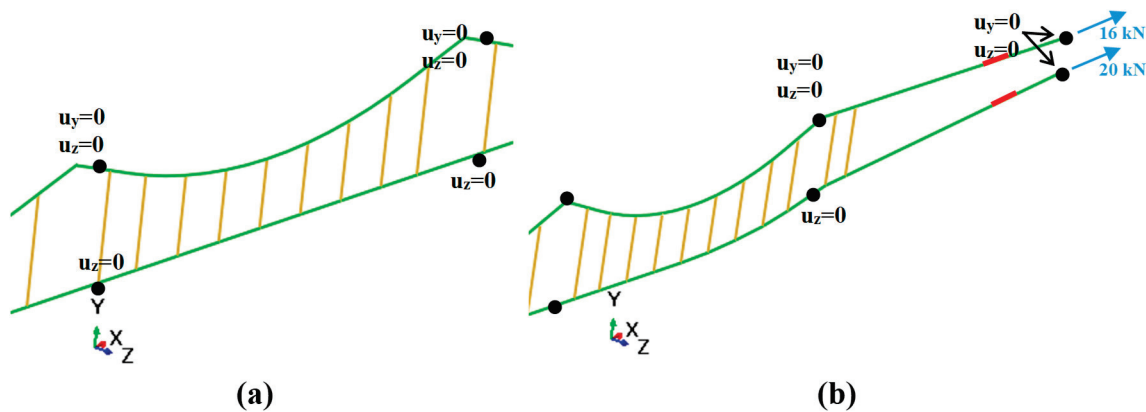


Figure 4. Applied boundary conditions. (a) Standard span boundaries, (b) end span boundaries.

The analysis procedure employs the explicit central-difference integration method for solving the equations of motion of the system, combined with diagonal (“lumped”) element mass matrices. The explicit central-difference operator ensures that the dynamic equilibrium equations are satisfied at the beginning of each increment at time t . The accelerations calculated at time t are then used to update the velocity solution to time $t + \Delta t/2$, and the displacement solution to time $t + \Delta t$.

While the cause of the CW breakage is not addressed in this paper, the focus is on the dynamic transient response following the CW rupture. The aim is to provide practical guidance on the type of measurement setup that should be installed on the OCL to detect when the CW breakage occurs.

The connector element introduced to simulate the CW failure is set to fail at a load level slightly above the static configuration value. To trigger the rupture, an extra tensile load of 100 N is applied across the connector on the CW, causing it to open and so causing CW rupture in the selected section. During the breaking process, as further approximation, no other failures, such as those in the droppers or in any of the elements of the suspension, are considered.

Three rupture locations were simulated (Figure 5): (i) in the first span (Scenario A), (ii) in the last span (Scenario B), and (iii) in the middle (Scenario C). These scenarios were designed to investigate the capability to reveal the rupture and provide insights into the potential failure location. Within the span where the break occurs, the rupture is positioned at the dropper immediately before the suspension. This location is chosen as the most likely point of failure due to the presence of the dropper clamp, along with the higher local deflection of the CW relative to the MW, resulting in a higher bending moment on the CW.

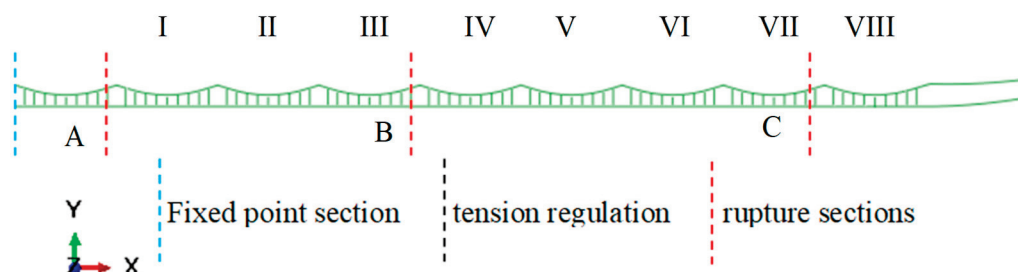


Figure 5. Location of the section of the CW ruptures along the model.

A separate explicit dynamics analysis has been performed for each breakage position considered. For each analysis, the initial conditions are taken from the final increment of the static analysis, as previously mentioned. The failure is assumed to occur at time 0 of

the explicit dynamics analysis. The transient response, including stress, displacement, and acceleration time histories of the modeled system, is then simulated for 10 s using stable time increments on the order of 10^{-6} s.

4. Results of the Simulated Scenarios

The results of the simulation of the three scenarios, in terms of the time histories of the observed variables, i.e., axial tension in the conductors, longitudinal displacement and vertical acceleration of the CW close to the end of the regulation, are presented below. Additionally, some considerations on the processing of these variables are provided, based on the time-domain patterns of the signals obtained from the simulation.

4.1. General Motion Response

The rupture of the CW causes an abrupt disruption of the equilibrium condition. The transient occurring between the initial static position and the final one is the primary focus of this numerical investigation. Vertical motion is the most significant one resulting from CW breakage. The motions of the CW and MW are coupled through the droppers, and for this transient it is crucial to account for the slackening of the droppers. In this analysis, droppers are modeled as elements that are active only in tension and provide no stiffness in compression once the value of the static preload is exceeded during the dynamic response.

To provide an overview of the transient dynamics, the vertical motion of the CW at midspan of each span is plotted as a function of time for the three scenarios considered, as shown in Figure 6. The vertical displacement of the CW at midspan is analyzed from span I to span VIII, where span I is the first span after the rupture and span VIII is the one closest to the end of the line. The reported variables are not intended for alert purposes as it is impractical to measure the vertical displacement of the CW at all midspans. However, it can be observed that moving away from the rupture section, located before span I, the maximum displacement amplitude decreases by 30% initially, then more gradually toward the end of the line. A comparison of the three time-domain diagrams reveals the following common features:

- The maximum amplitude occurs in the span involved in the CW rupture and, for scenarios B and C, also in the adjacent spans, as shown in Figure 7a, reporting the difference between maximum and minimum vertical CW displacement along the span, assumed as p–p amplitude. The latter is about 1.8 m, corresponding to a roughly 3% of the span length.
- The vertical motion induced by the CW rupture is transmitted to both sides of the regulation, with lower amplitude as the distance from the rupture section increases, as clearly shown in Figure 7a.
- The waveform is asymmetrical, due to the slackening of the dropper, which links the motion of the CW with the MW in the downward direction. In the upward direction, the unrestrained relative motion allows the CW to vibrate with a higher amplitude. Another reason for a non-symmetrical waveform is that the final vertical position of the conductors differs from their initial position. This aspect is reflected in the mean value of the vertical displacement of the CW, which is always positive, and is larger close to the span of the rupture section, as shown in Figure 7b where the mean value of the oscillation is reported for all the eight spans.

Considering now the overall OCL configuration, it is evident that the broken CW significantly disrupts the normal static configuration. Under normal operating conditions, the droppers are vertically aligned. However, following the CW rupture, the droppers remain still vertical in the region between the rupture section and the fixed point. In the portion between the rupture section and the end of the regulation, the droppers incline,

transmitting the horizontal component of the axial load to the MW, which remains anchored to the central fixed point. This results in a complex final configuration, where the axial tension, measured at regulation extremity, is still very close to the nominal value, as determined by the tensioning pulley system.

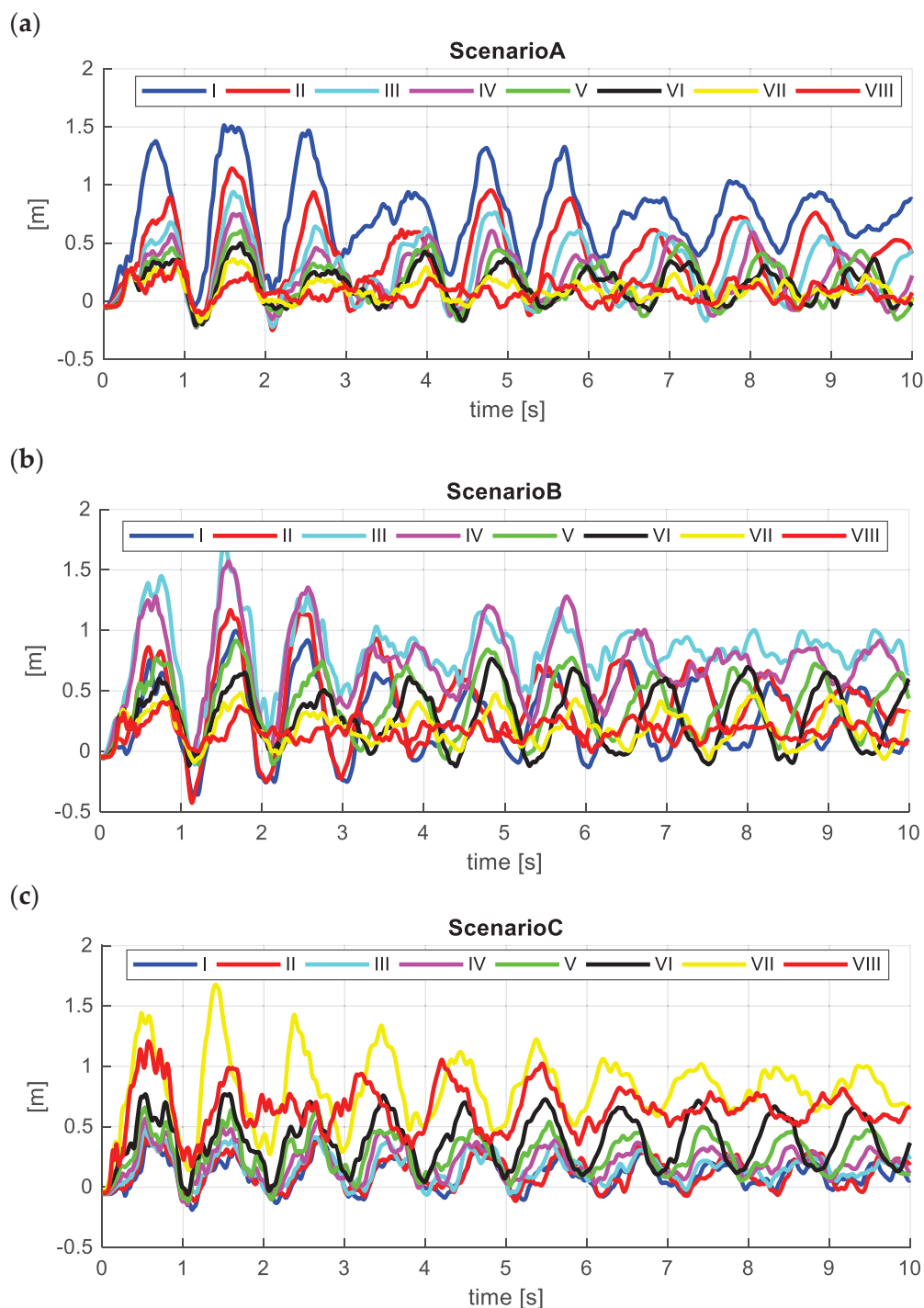


Figure 6. Vertical displacement of the CW at each midspan of the considered span in the numerical model (numbered from I to VIII), following the CW breakage: (a) Scenario A, (b) Scenario B, and (c) Scenario C.

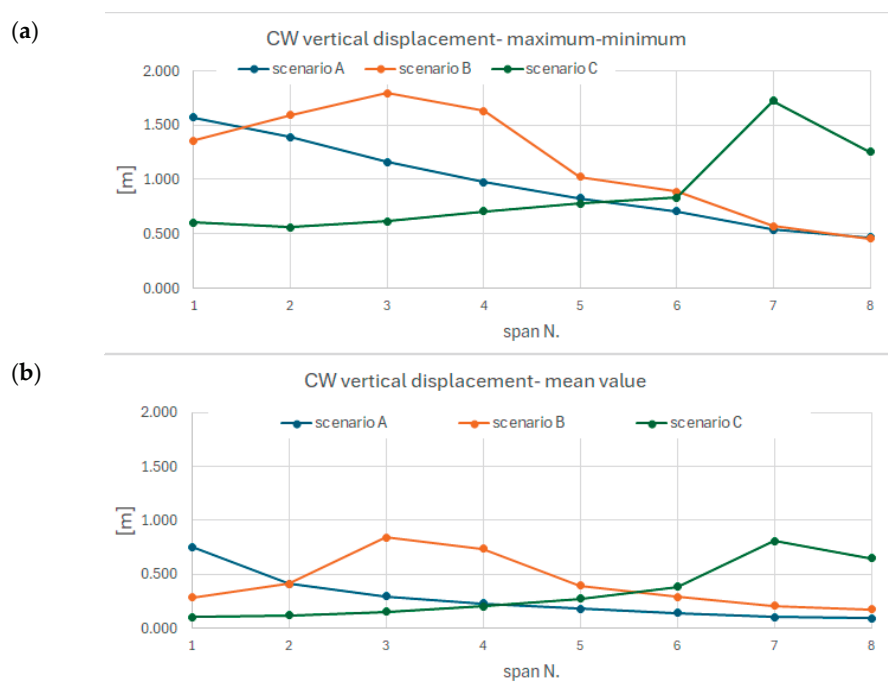


Figure 7. Vertical displacement of the CW. (a) Maximum p-p amplitude, (b) overall mean value along the time history.

The first key detail focuses on the CW breakage, as shown in Figure 8, where only the deformed configuration is shown. The vertical component of the displacement is amplified for highlighting it, along with the vertical dimension. The axial load on the CW between the rupture section and the central fixed point becomes zero as no axial load is applied. Meanwhile, the axial load applied by the loading devices at the regulation is progressively transferred from the CW to the MW through the droppers. These droppers, instead of remaining vertical, take on an inclined configuration, as illustrated in Figure 9. Similarly, in the area close to the tensioning setup, the droppers appear inclined (see Figure 10) due to the progressive transfer of the axial load from the CW to the MW.



Figure 8. OCL position towards the end of the transient for scenario B. The two boxes refer to Figures 9 and 10.

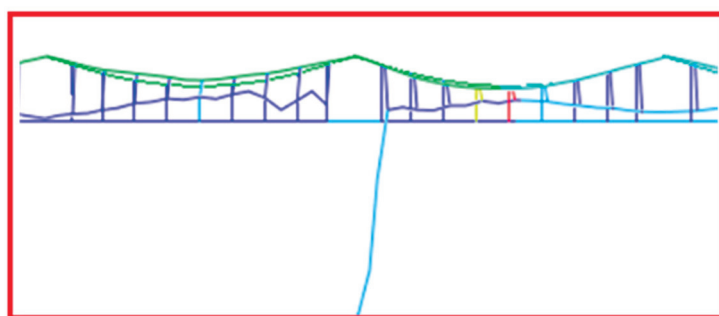


Figure 9. Detail of the OCL position in the broken CW section, for scenario B with the broken CW hanging. Vertical scale is increased for a better view.

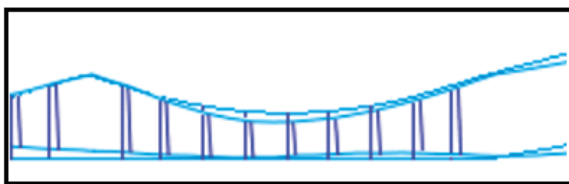


Figure 10. Detail of the OCL position in the end section, for scenario B, showing the longitudinal shift between the CW and MW. Vertical scale is increased for a better view.

Therefore, the measurement of the applied axial load cannot rely upon a final difference in the tension of the CW; instead, a dynamic measurement is required. On the other hand, a difference in the longitudinal position of the conductors could serve as a potential indicator of a CW failure occurring along the line.

4.2. Longitudinal Motion of the Conductors

The longitudinal motion of the MW and CW following the CW rupture event is reported in Figure 11.

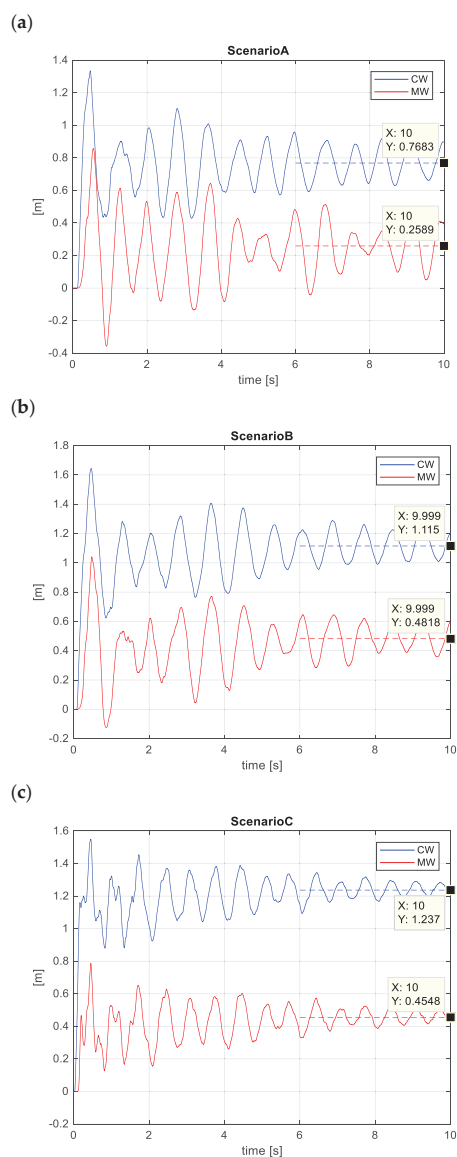


Figure 11. Axial displacement of the CW and the MW following the CW breakage: (a) Scenario A, (b) Scenario B, and (c) Scenario C.

The final positions of the conductor extremities differ, with the CW exhibiting a larger displacement than the MW. As previously noted, the difference between the final longitudinal positions of the CW and MW is reflected in the inclined orientation of the droppers.

4.3. Axial Load in the Conductors

The dynamic axial force in the MW and CW are shown in Figure 12 as a function of time for the three scenarios. The breakage occurs immediately after the integration begins, resulting in a negative pulse in the axial load of the CW after a time interval corresponding to the distance between the measurement section and the rupture section, divided by the propagation velocity of the longitudinal wave c in the CW. The propagation velocity is expressed as follows:

$$c = \sqrt{\frac{E}{\rho}} \quad (2)$$

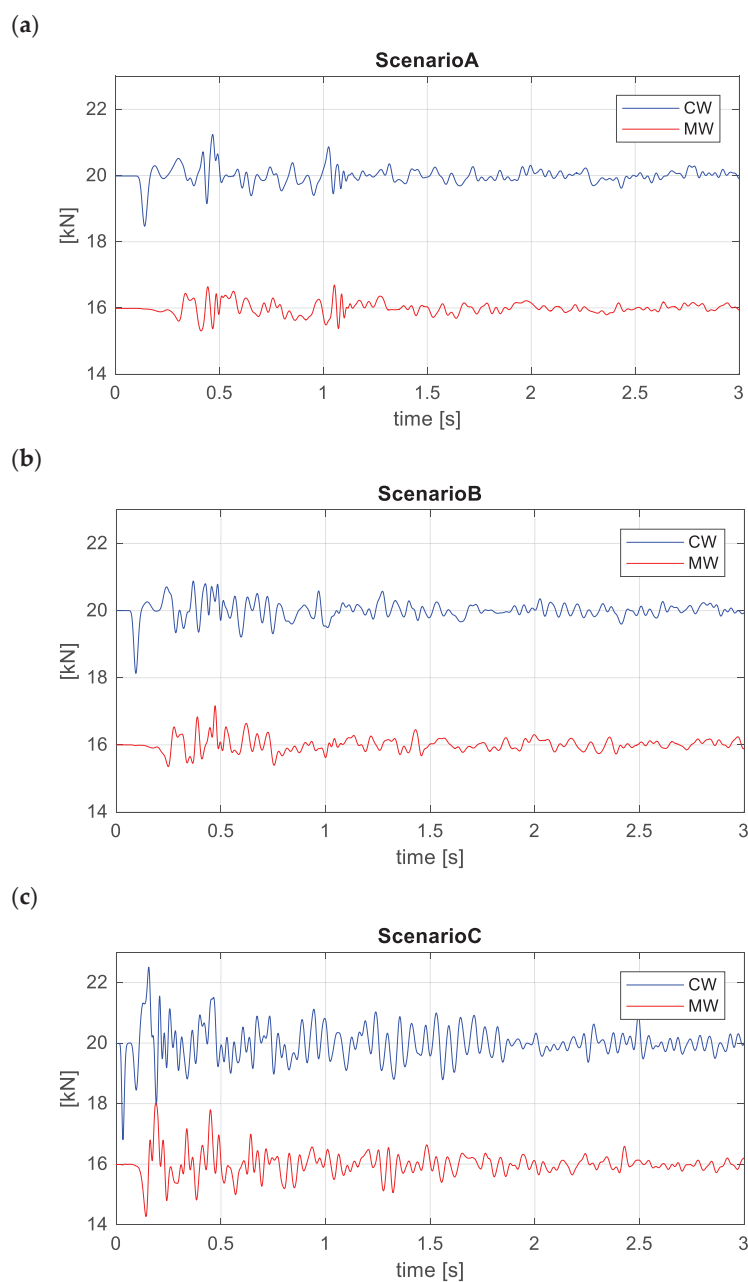


Figure 12. Axial load of the CW and the MW following the CW breakage: (a) Scenario A, (b) Scenario B, and (c) Scenario C.

For the copper of the CW, the assumed values are a Young's modulus $E = 1.18 \times 10^{11} \text{ N/m}^2$ and a material density $\rho = 8890 \text{ kg/m}^3$. The resulting propagation velocity is 3643 m/s. By measuring the time intervals between the start of the simulation and the occurrence of the first negative peak in the CW's axial load, these intervals can be related to the distance between the rupture section and measurement section through the axial wave propagation velocity c . The slope of the linear relationship equals the theoretical propagation velocity, confirming that the adopted mesh is suitable for the analysis (Figure 13).

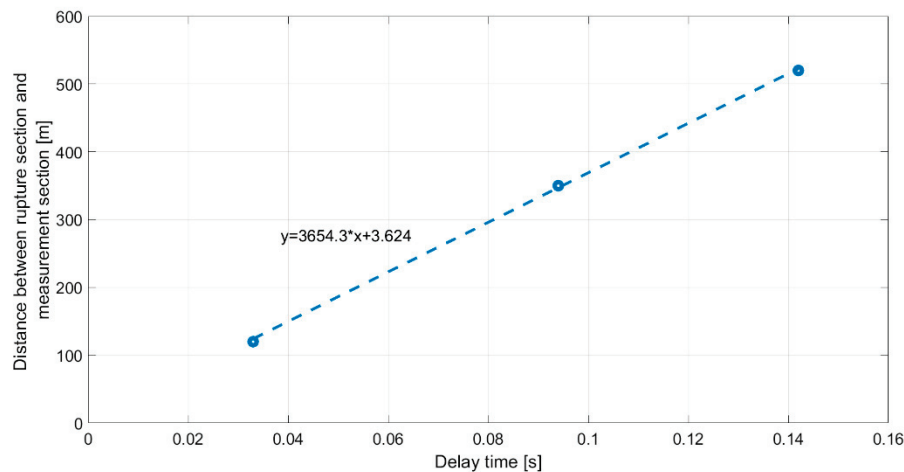


Figure 13. Relationship between the time of the occurrence of the axial load peak in the CW and the distance between the location of the rupture section and the measurement section. 'o' marThe value of the slope reported within the diagram corresponds to an estimated propagation speed of the axial wave along the CW equal to 3654 m/s.

Different from the CW, no pulse is observed in the axial load of the MW. Both the MW and CW axial loads exhibit a fundamental frequency approximatively double that of the vertical motion frequencies of the MW and CW. This is due to the increase in axial load as the MW and CW experience vertical motion, reaching their maximum displacements upward and downward. Additionally, higher-frequency components become more pronounced as the broken CW section moves closer to the half-regulation extremity.

The visual observation of the time history of the axial load clearly reveals the presence of an impulse deriving from the CW breakage. From the point of view of real time monitoring, although the downward impulse is clearly visible it may not be clearly distinguished from the subsequent dynamic variation, making the automatic detection more difficult by simple algorithms like peak value or r.m.s. It is believed that more sophisticated time-domain analysis would be required, making the on-site processing unit more costly.

4.4. Vertical Acceleration of the CW

The final parameter relevant to the actual measurement setup is the vertical acceleration of the CW close to the end of the regulation. The acceleration pattern is oscillatory, so it is more robust to analyze a processed version of this signal. In Figure 14, the moving root mean square (r.m.s.) is superposed to the time history (red line in Figure 14), considering windows of 0.10 s, and shifted with a step equal to the length of the window. The moving r.m.s. trend helps to highlight the mean power of the signal during the transient vibration development. The purpose of processing the raw signal with a moving r.m.s over a short time window is to decrease the sensitivity to the peak value, applying a filtering effect, while keeping the peak value of the processed signal clearly distinguishable with respect to the following decay. The selected time window length is selected as about twice the time interval necessary to reach the peak value of the raw acceleration. The length of the time

window should be a balance between the possibility of clearly highlighting the acceleration peak and an excessive filtering effect that makes the automatic recognition of the peak more difficult.

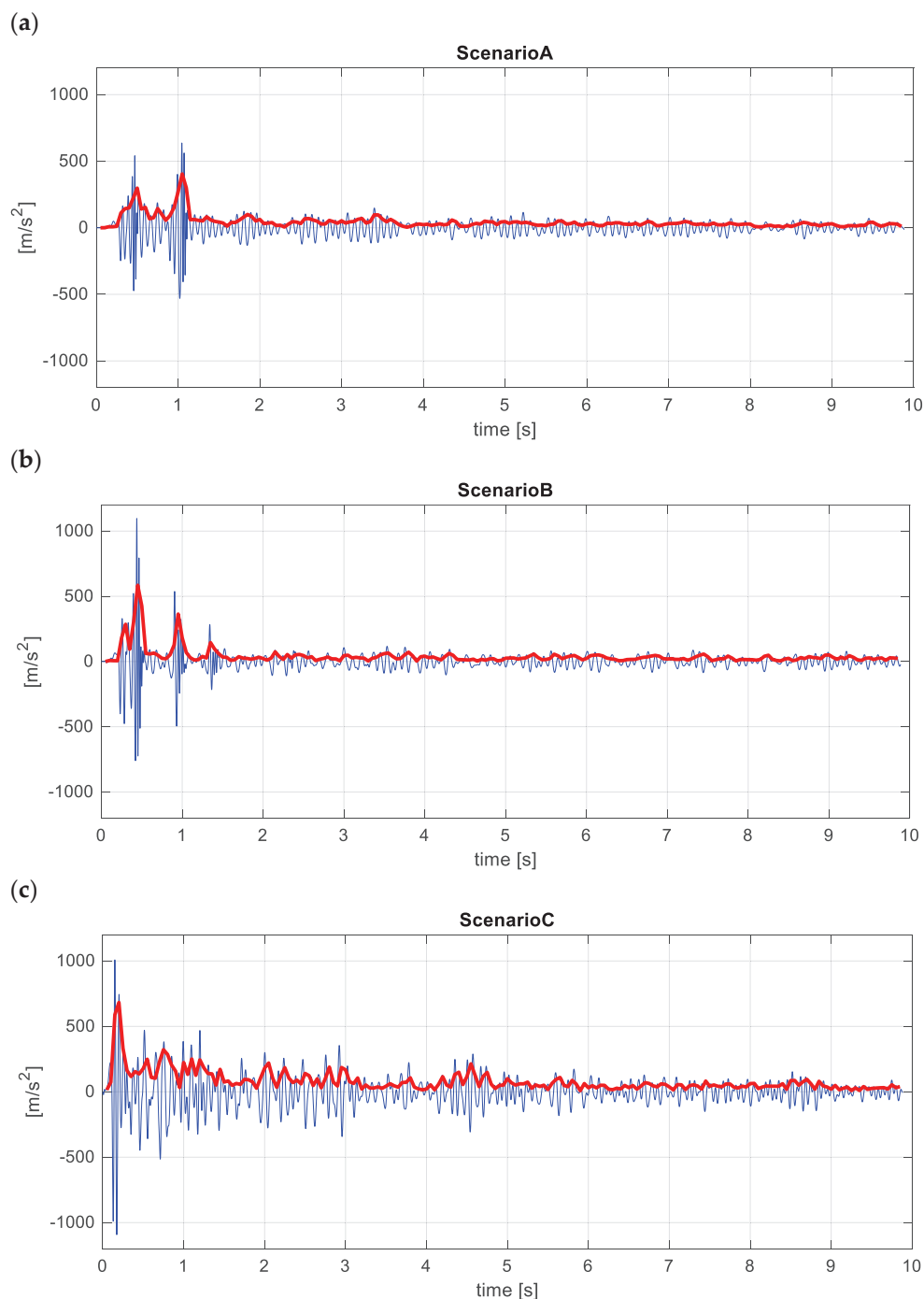


Figure 14. Time history of the vertical acceleration in the CW close to the end section. Red line indicates the moving rms processing of the acceleration with a 0.15 s time window: (a) Scenario A, (b) Scenario B, and (c) Scenario C.

5. Summary and Conclusions

This paper presents a feasibility analysis of a monitoring setup capable of sending alerts regarding a CW rupture event on an OCL. The aim is to halt traffic and prevent the transit of a pantograph beneath the broken CW section. The potential to detect a CW breakage at any position along an OCL regulation has been analyzed, simulating different

scenarios where the transient dynamics following the rupture of the CW in a high-speed OCL have been carried out by means of numerical simulation in Abaqus®.

Different output variables that can be measured at the ends of the regulation, where the tensioning devices apply axial load to the conductors (the CW and the MW), have been analyzed. These variables were compared with respect to their ability to reliably detect the rupture and the methods for processing raw data to derive a meaningful and robust index.

The outcomes of the simulated transient following the CW rupture are as follows:

- The axial load in the CW reveals the presence of a negative force pulse (resulting from the rupture) between 8% and 16% of the static value, followed by a dynamic pattern. While the identification of this pulse could reveal the rupture, it provides a weak alert signal due to the relatively small percentage of deviation from the mean value. Additionally, the subsequent transient may feature peaks higher than the initial pulse.
- The extremities of both conductors show significant longitudinal motion during the transient dynamics. The predominant frequency corresponds to the first vertical frequency of the single span, which is linked to the longitudinal motion through the second-order geometrical effect associated with the shape of the catenary. A detection process should focus on changes in the longitudinal position of conductor ends within a time interval of 10–20 s, excluding gradual thermal expansion of a similar magnitude.
- The vertical acceleration of the CW close to the tensioning devices connection is dominated by frequencies higher than the first flexural one. A moving r.m.s. applied to the raw acceleration signal can provide a more robust indication, in terms of vibratory energy, rather than directly analyzing acceleration amplitude. The overall increase in signal power appears to be a more reliable indication than raw acceleration levels alone.

The longitudinal displacement provides a clear indication on the CW rupture but requires long-range transducers to accurately track the longitudinal motion of the MW and CW. Considering that towards the end of the transient the oscillation is set around different mean values, it is conceivable to monitor at defined intervals the mean value of the conductor positions. This can also provide insights into the location of the rupture along the line as it is related to the final static configuration of the half regulation involved. Combining the acceleration analysis and displacement evaluation offers a more robust indication of the CW rupture's occurrence, giving at the same time an indication of the location of the broken CW section.

The model developed in this work captures the main features of the phenomenon: (i) large displacement nonlinear analysis, (ii) a consideration of dropper slackening, and (iii) the dynamic effect of the CW breakage on the variables (displacement, force, and acceleration) that can be measured at the end of the regulation. The results provide a basis for designing a measurement setup at the ends of each regulation. This setup would combine vertical acceleration measurement of the CW and MW, which, when properly processed, can trigger the alert. It would also include the measurement of large displacement mean values, which, when showing a clear gradient with respect to the previous measurement results, would enhance the robustness to the alert.

As for the possible implementation of the monitoring setup proposed here, two orders of issues are to be considered. The first one concerns the required transducers and their installation on the OCL.

Transducers suitable for detecting acceleration are to be installed on the CW under high voltage, but this has been already demonstrated as feasible [16,17] when adopting wireless transducers. The issue of power is tackled by combining a long-term battery supply with a photovoltaic panel, as experienced in ultra-high voltage electrical lines [18]. The main challenge is the dimensioning of the battery and the photovoltaic panel so as to guarantee

its continuous working. As far as the longitudinal displacement of both conductors is concerned, the possibility of measuring the displacement of the counterweight with a commercial long range and low-cost laser transducer is an option, avoiding problems related to electrical insulation. It is also to be considered that accuracy is not an issue for this measurement. This is relatively easy in cases where each conductor has its own loading system, like in the Italian HS line. In cases of a loading device that connects both the CW and MW, a different analysis has to be performed, although keeping the concept of a sudden variation in the position of the counterweight.

The second category of issues is related to the transmission chain sending the alert to the traffic management unit. This heavily depends on the type of signaling considered. A general option for HS lines would be to communicate the alert to the radio block center (RBC) that directly manages the traffic in its area of competence, and at the same time to the control room that overviews the traffic along a specific route. A second issue consists of the time required to really set off the alarm and halt the traffic. To make some considerations regarding the response time to give the alert, a minimum theoretical time headway of 5 min can be considered, corresponding to a traffic flow of 12 trains/hour. Currently the real traffic flow in the peak hour is 9 trains/hour for the Italian HS service.

Such an interval is compatible with an automatic data flow transmission, but a bottleneck in the decision chain may come if confirmation by a traffic manager operator from the control room is needed. Although these operational issues are very important, they are outside the scope of this paper.

The work presented in the paper was the first step and further improvement can be applied to the model and the definition of the measurement setup.

To further refine the analysis and define the transducer requirements, improvements could be made in modeling the suspension system, as well as incorporating the dynamics of the supporting poles. Additionally, the representation of the tensioning device in more detail could be enhanced by considering its kinematic aspects and the inertia of the counterweights. Finally, it is worth remarking that no experimental evidence was available to the authors to validate the dynamic model in the considered scenario. The design of a scale dynamic model is ongoing. Among the different available scaling approaches, the most promising one is that which considers the equality of acceleration between the scaled model and full-scale system. The reason is that it makes it easier to keep the same ratio between gravity loads and applied axial loads, which, together with inertia forces, are the actions relevant for dynamic behavior. It is worth remarking that, following the same approaches for aeroelastic models tested in wind tunnels, the mass can be adjusted with additional masses on the conductor.

Despite the simplifications used, the core principles of the monitoring system based on acceleration and displacement measurements, despite direct axial load measurement, remain valid, considering that the main nonlinear dynamic aspects have been adequately considered.

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Abbreviations

The following abbreviations are used in this manuscript:

OCL	Overhead contact line
CW	Contact wire
MW	Messenger wire
DC	Direct current
AC	Alternate current

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Article

Instability of a Moving Bogie: Analysis of Vibrations and Possibility of Instability in Subcritical Velocity Range

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Abstract: This paper analyzes vibrations induced by a moving bogie passing through a single-layer railway track model. The emphasis is placed on the possibility of unstable behavior in the subcritical velocity range. All results are presented in dimensionless form to encompass a wide range of possible scenarios. The results are obtained semi-analytically, however, the only numerical step involves solving the roots of polynomial expressions. No numerical integration is used, allowing for the straightforward solution of completely undamped scenarios, as damping is not required for numerical stability. The vibration shapes are presented in the time domain in closed form. It is concluded that increased foundation damping worsens the situation. However, in general, the risk of instability in the subcritical velocity range for a moving bogie is lower than that of two moving masses, particularly for higher mass moments of inertia of the bogie bar and primary suspension damping. The study also examines how the results change when a Timoshenko-Rayleigh beam is considered instead of an Euler-Bernoulli beam. Although some cases may appear academic, it is demonstrated that instability in the supercritical velocity range cannot be assumed to be guaranteed.

Keywords: contour integration; integral transforms; critical velocity; instability; transient vibrations; analytical solution

1. Introduction

One effective way to combat climate change and reduce the carbon footprint is to prioritize rail transport over road transport. This naturally necessitates an increase in railway network capacity by enhancing travel velocity, the number of trains, and axle loads. These increased demands emphasize the importance of ensuring passenger safety and comfort. The upper limit for travel velocity is typically derived from the critical velocity, which, in this context, corresponds to the critical velocity of a moving force. This velocity equals the lowest wave propagation velocity in the supporting structure, resulting from the dynamic interaction of all involved components, and can be referred to as the critical velocity for resonance.

Another commonly used indicator for ensuring safe train passage is the receptance curve, which, however, only indicates the natural frequencies of the supporting structure. The frequencies that cause unstable behavior cannot be identified using receptance curves, as they depend on the mass and velocity of the moving object. Therefore, it is necessary to study the transient vibrations caused by the moving vehicle and the associated so-called induced frequencies, which can determine whether the motion will be stable or unstable. Nevertheless, the problem of instability has often been overlooked, as it was shown that a single moving mass or oscillator can exhibit unstable behavior only in the supercritical

velocity range. Since this region is a priori excluded from track design, instability has not been considered a critical issue.

However, the dynamic interaction of two moving proximate masses can lead to a different scenario, one that surprisingly worsens with increased foundation damping. Such cases have already been identified in the author's previous works.

The study of moving load problems remains a highly active topic in transportation engineering, particularly in rail and road transport. Various simplifications have been introduced to improve computational efficiency and present results in a more compact form suitable for optimization and sensitivity analyses. These simplifications apply to the supporting structure, the moving vehicle, or both, depending on the problem being investigated. In addition to analyzing the influence of several parameters on dynamic performance, key topics of interest include the critical velocity for resonance, the critical frequency for resonance, instability, and the dynamic interaction of proximate inertial objects.

Typical simplifications at the level of the supporting structure have led to layered models consisting of beams, point masses, and spring-damper elements. These models are widely used due to their simplicity and computational efficiency. Such simplified models, along with simplified vehicle models, are used in both practical and theoretical studies. The applicability of the three-layer model in railway applications is analyzed in [1], while an overview of the features of one-, two-, and three-layer models is provided in [2]. Notably, these simplified models have proven adequate for modeling several railway dynamics issues, as demonstrated by other researchers, including in classical works [3,4]. Other works dealing with vehicle-track dynamic interaction modeling include [5–13], further demonstrating the usefulness of simplified models. A classification of layered models is introduced in [4], with validation against experimental results provided in [14]. Studies aimed at defining the properties of such models consider the ballast pyramid model, as referenced in [15,16], and the Saller assumption from 1932, which can be consulted in [17]. The pyramid model is further detailed in [18] under the name “stress cone model”, where it is used to define vertical ballast stiffness and identify dynamically activated ballast mass. Further improvements involve the superposition of cones between adjacent sleepers, i.e., in the longitudinal direction [14].

Numerous research studies have been conducted over the past decades; thus, this review primarily focuses on the most recent publications.

In [19], a critical discussion on developments in the field of the dynamic response of rigid road pavements to moving vehicles is presented. Rigid pavements consist of an elastic concrete plate or jointed plates resting on a supporting foundation. Similar models are also used in railway engineering, albeit with different parameters. The supporting medium can be modeled as a system of spring-damper elements or as a homogeneous or layered half-plane/space with infinite or finite depth. The paper summarizes works in which the material behavior of the subgrade layers is considered isotropic, anisotropic, viscoelastic, poroelastic, or inelastic. In [20], the same authors provide a more detailed review of both rigid and flexible pavements. These works simulate vehicles using either constant concentrated forces or distributed forces over a finite line/area, omitting their inertial effects and, consequently, the possibility of instability. In [21], a plane strain model is used to analyze instability issues in the moving mass problem.

In [22], although the study focuses on road pavements, the guiding structure model corresponds to a one-layer model commonly used in railway applications. The model is traversed by a two-mass oscillator with a contact spring-damper element, a configuration frequently encountered in railway applications. Harmonic roughness of the roadway is considered, but neither instability nor critical velocity is analyzed. Since only a single contact point is involved, the dynamic interaction of proximate objects cannot be observed.

The study primarily analyzes the effects of foundation nonlinearity using the Adomian decomposition method. Moving coordinates are implemented, and results are limited to the steady-state response, as only a single Fourier transform is applied. This approach effectively removes the inertial effects of the moving object. The study finds that there is a critical value for nonlinear stiffness, which increases with vehicle velocity and foundation damping.

Regarding railway applications, the nonlinearity of the supporting structure is analyzed in [23]. A two-layer model is implemented, and a simplified vehicle model is used. A novel nonlinear stiffness model is developed to comprehensively characterize ballast properties, incorporating displacement-dependent stiffness, frequency-dependent stiffness, hysteresis, and time/space-varying features. The model is validated using experimental measurements by processing ground-penetrating radar signals with the adaptive optimal kernel time–frequency representation method.

Moving force problems are simpler than moving mass problems, particularly for finite-length supporting structures, where analytical solutions exist and can be formalized using modal expansion. However, a relatively recent study [24] still analyzes this problem for a double-beam structure, focusing on the effects of various boundary conditions. Although the motion of a mass on finite Euler-Bernoulli beams without an elastic foundation and damping is a classical problem addressed by numerous researchers, new insights and results continue to emerge, as shown in [25]. Furthermore, in [26], a new equivalent lumped parameter model is proposed to describe the vibrations of simply supported beams without an elastic foundation under moving force action. This classical problem has a fully analytical solution. In [27], vibrations of a similar structure subjected to a moving mass are solved using the Green's function method to analyze the effects of boundary conditions, mass velocity, and its ratio with respect to the beam mass.

More practical studies typically address railway line degradation, such as transition zones, rail corrugation, maintenance optimization, and potential failures. For example, in [28], the effect of temperature on concrete cracking in ballastless lines is analyzed, while in [29] the failure of railway vehicle suspension dampers is examined using cross-correlation analysis of bogie accelerations.

Regarding transition zones, it is well known that a sudden change in the vertical stiffness of railway tracks increases dynamic loads and leads to numerous defects in ballasted tracks. The main geometric parameters of the approach slab for short-span bridges were optimized in [30]. A finite element model was developed and subjected to a series of moving forces. The model was validated using field measurements obtained through a laser/camera-based measuring technique. The geometric sensitivity analysis results indicate that rail and ballast displacements can be significantly reduced by employing a geometrically optimized approach slab along the transition zone of a railway bridge.

In [31], a recently proposed revolutionary design aimed at mitigating the negative effects of transition zones is evaluated under extreme conditions. This transition structure, called SHIELD (Safe Hull Inspired Energy Limiting Design), was designed to reverse propagating waves by inverting the shape of the embankment zone, thereby directing wave energy toward the foundation instead of upwards, where it could damage the railroad. The study found that this novel design effectively reduces dynamic amplifications and ensures a smooth distribution of deformation energy across subcritical, critical, and supercritical velocity regimes in both travel directions. This results in a more uniform traffic-induced degradation along the longitudinal direction.

Transition zones are directly linked to the floating sleeper phenomenon, for which a novel modeling approach is presented in [32]. This study provides an accurate mathematical model with a minimal number of degrees of freedom, supported by extensive field

measurements of a railway track. Although the observed phenomenon is highly complex, a simplified one-degree-of-freedom mathematical model has been found sufficiently accurate for its characterization.

Regarding rail corrugation, a new hypothesis for the formation process of short-pitch rail corrugation is proposed in [33]. A dynamic finite element (FE) wheel-track model is used to verify the hypothesis by replicating the initiation and consistent growth of corrugation. The FE model consists of discrete elements, similar to other works employing semi-analytical or analytical solutions. The supporting structure is represented as a two-layer model extended to three dimensions, with rail pads and the foundation (corresponding solely to the ballast layer) modeled using spring-damper elements acting in three directions. The wheelset and rails are detailed using 3D elements, while the remaining vehicle mass is lumped and placed above the primary suspension. The study finds that longitudinal compression modes are responsible for corrugation initiation, and the proposed mechanism successfully explains field observations. The FE model is validated using a downscaled V-Track test rig in [34].

Transition zones are also related to transition radiation. In this context, Refs. [35,36] are worth mentioning. Additionally, Ref. [37] provides an insightful analysis using a one-layer model, with adequate justification regarding how results obtained from simplified models should be interpreted. It is well known that the one-layer model significantly overestimates the critical velocity of a railway track. In light of this, the results should be understood qualitatively, as the ratio of train velocity to critical velocity is more relevant than the absolute values of these quantities.

In terms of maintenance, ballast tamping remains the most common method for restoring ballast bed geometry. In [38], a novel test device is introduced that enables real-time evaluation of the mechanical properties of ballast beds, which is essential for optimizing tamping efforts in railway maintenance. The evaluation is based on measuring the impact force on the tamper, and the results are validated numerically using a coupled model where the ballast is represented using the discrete element method.

In [39], a closed-form solution is derived using the residue theorem and Green's function for the dynamic response of an infinite beam resting on a viscoelastic foundation subjected to a harmonic moving load. However, closed-form results have already been presented in [40]. In [39], these results are used to determine the most appropriate span length of an analogous finite beam to best approximate the deflection of an infinite beam for practical applications. As demonstrated in [41], this can be problematic for a constant moving force, and a solution is proposed in that work. However, for a moving inertial load, no solution is provided, despite previous studies by the author of this paper, where validation was always performed on finite beams. The only precaution taken was to initiate the load slightly farther from the support, as explained in [42]. When considering initial conditions, this distance must be extended to ensure that the entire relevant portion of the initial shape [43] is contained within the beam at time zero.

Similar models are discussed in [44], but only steady-state vibrations are analyzed, thereby eliminating the possibility of instability. However, the model is more detailed, incorporating rail roughness and contact springs. Rail corrugation effects are also considered in [45].

Additionally, other works analyze the limitations of classical beam theories [46–48], large deformations [49], non-uniform motion of an inertial object [50], and railway tracks beyond classical ballasted ones [51].

The stability of a moving inertial object on an elastic structure in railway applications has been investigated using various representations of the vehicle (moving object) and the track (supporting or guiding structure). Simplified vehicle models include a moving

mass [52,53], a two-mass oscillator [40,54], and a three-mass oscillator [55]. These models have only one contact point, meaning dynamic interaction cannot be observed, unlike models considering two moving masses [42,56] or a sequence of two-mass oscillators [57–60]. A more complex vehicle model can also be employed for stability analysis [61]. The guiding structure is modeled using an Euler-Bernoulli infinite beam on a continuous foundation with one elastic layer [53,55–57], two layers [42,62], or three layers [2]. Alternatively, the Timoshenko beam can be used instead of the Euler-Bernoulli beam [56,63]. To account for the influence of sleeper bays, an infinite beam model with discrete supports should be considered for railway tracks [64].

The instability of a moving bogie has been analyzed in previous studies, but none have provided complete vibration shapes, closed-form time-domain analytical solutions, alternatives to the D-decomposition method, validation on finite beams, etc. A pioneering study on railway vehicle stability [65] focused on lateral stability, a topic not covered in this paper, which only examines vertical vibrations. However, the model presented here is similar to the one in that study. The model in [65] incorporates rail irregularities but restricts calculations to forced steady-state vibrations induced by harmonic irregularities. Since transient vibrations are not evaluated, instability caused by anomalous Doppler waves in the beam cannot be identified. In [66], the influence of track gauge and rail irregularities on hunting motion instability is analyzed, which is beyond the scope of this paper's plane model.

The instability of a moving bogie passing through a single-layer model is examined in [67] from multiple perspectives, including a comparison with a moving oscillator and an analysis of critical primary suspension stiffness. In [68], the authors investigate how a more complex foundation structure affects critical stiffness values. However, only isolated values are provided without detailed comparison to [67]. The critical velocity for resonance differs significantly from that of the one-layer model, requiring a different approach to comparison. The same reference input data for the moving bogie as in [67] are used. Due to the large distance between the wheels, the dynamic interaction between them is suppressed, and instability is only detectable for realistic values in the supercritical velocity range. In [69], more complex moving objects are introduced, but results are limited to a few discrete values of critical primary suspension stiffness. The effects of foundation damping and instability in the subcritical velocity range are not addressed in these studies.

The present paper analyzes a moving bogie to clarify the possibility of instability in the subcritical velocity range. The results are presented as analytically as possible, building upon previous developments in [40,53,56] while avoiding the Green's function method used by several other researchers due to its limitations. The primary drawback of this method is its dependence on numerical integration, which necessitates the introduction of damping for numerical stability. When damping is introduced, true resonance cannot be precisely identified or analytically determined. A comparison of both methods is available in [70]. Recent research has shown that the instability of two proximate moving masses on a layered track model can occur in the subcritical velocity range [42,56], contradicting the conventional belief that unstable velocities always exceed the lowest critical velocity for resonance. While instability is mathematically impossible for a single moving object with one contact point, it can occur for multiple moving objects due to dynamic interaction. Additionally, increased foundation damping unexpectedly worsens the situation. This paper's novel contribution is analyzing how this behavior changes when the moving masses are combined into a bogie, a more realistic representation. The study aims to identify scenarios in which bogie instability occurs in the subcritical velocity range and determine the parameters contributing to this unexpected phenomenon, as well as those that push instability into the expected supercritical velocity range.

The paper is organized as follows. Section 2 describes the model along with the governing equations and their solutions, utilizing the new method from [53] to present the results in closed-form formulas. Dimensionless parameters are introduced to cover a broad range of possible scenarios, allowing the results to be applied without the need for recalculations. Section 3, which presents the results, is divided into several parts. It first defines the induced frequencies and then validates the results through analysis on finite beams. The effects of transitioning from the Euler-Bernoulli beam to the Timoshenko-Rayleigh beam are then examined, followed by a presentation of selected cases along with their corresponding induced frequencies. Next, the conditions leading to instability in the subcritical velocity range for two moving proximate masses are summarized and compared with those for a two-axle bogie. The influence of suspension damping, beam theory, and other input parameters is subsequently analyzed. The results section concludes with a comparison to other published works, particularly [67]. Additional cases are provided in Appendix A due to the length of the paper. Finally, the paper concludes with a discussion of the results, followed by the conclusions.

2. Description of the Model and Governing Equations

The model considered in the present paper is depicted in Figure 1. In part (a), two proximate masses pass through the one-layer model, while in part (b), a single bogie moves across the model. The model consists of a beam characterized by its bending stiffness EI and mass per unit length m , following the Euler-Bernoulli theory. The foundation is described by its vertical k_f and shear stiffness k_s , as well as its viscous damping coefficient c_f . Both the moving masses and the moving bogie travel with a constant velocity v .

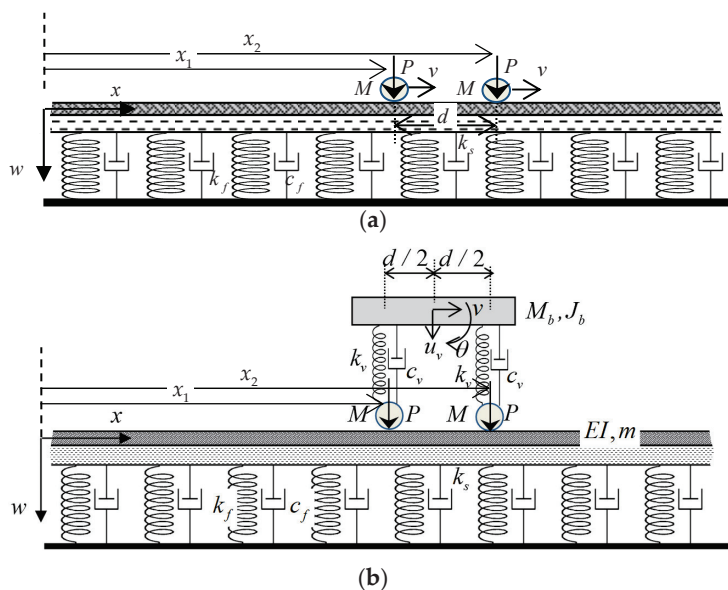


Figure 1. One-layer model: (a) traversed by two masses; (b) traversed by a single bogie.

The bogie comprises a rigid bar of finite length with mass M_b and mass moment of inertia J_b , along with spring-damper elements that characterize the primary suspension. These elements are represented by stiffness and damping coefficients, k_v and c_v , respectively. Additionally, the bogie includes two point masses M representing the wheels, positioned symmetrically at a distance $d/2$ from the bogie bar's mass center. The model in Figure 1a uses only the moving masses, however, when necessary, their values are adjusted to maintain the same total mass as the bogie for comparison purposes.

For simplicity and comparison between models, it is assumed that the moving masses have equal values and are subjected to identical constant forces. The primary suspension

of the bogie is modeled with spring-damper elements, each supporting a point mass that represents a wheel in permanent contact with the beam. These supports are considered identical and symmetrically positioned relative to the bar's mass center.

The problem of two moving masses has been previously solved in [56]. The problem of the moving bogie is governed in fixed coordinates by the following equations:

$$EIw_{,xxxx}(x, t) + mw_{,tt}(x, t) + c_f w_{,t}(x, t) + k_f w(x, t) = p(x, t) \quad (1)$$

$$p(x, t) = \left(P - Mw_{01,tt}(t) - k_v \left(w_{01}(t) - u_v(t) + \theta(t) \frac{d}{2} \right) - c_v \left(w_{01,t}(t) - u_{v,t}(t) + \theta_{,t}(t) \frac{d}{2} \right) \right) \delta(x - x_1) + \left(P - Mw_{02,tt}(t) - k_v \left(w_{02}(t) - u_v(t) - \theta(t) \frac{d}{2} \right) - c_v \left(w_{02,t}(t) - u_{v,t}(t) - \theta_{,t}(t) \frac{d}{2} \right) \right) \delta(x - x_2) \quad (2)$$

$$M_b u_{v,tt}(t) - k_v (w_{01}(t) + w_{02}(t) - 2u_v(t)) - c_v (w_{01,t}(t) + w_{02,t}(t) - 2u_{v,t}(t)) = 0 \quad (3)$$

$$J_b \theta_{,tt}(t) + k_v (w_{01}(t) - w_{02}(t) + \theta(t) d) \frac{d}{2} + c_v (w_{01,t}(t) - w_{02,t}(t) + \theta_{,t}(t) d) \frac{d}{2} = 0 \quad (4)$$

Equation (1) represents the beam equilibrium, Equation (2) describes the loading term, Equation (3) is the vertical equilibrium of the bogie bar and Equation (4) is the moment equilibrium of the bogie bar around its mass center. The unknown variables in this system include the beam's vertical displacement w , the bogie bar's mass center vertical displacement u_v , and its rotation θ , with positive orientations as indicated in Figure 1. Derivatives are represented using subscript notation preceded by a comma. The Dirac delta function is denoted by δ , while the variables representing the positions of the rear and front masses in the fixed coordinate system are denoted as x_1 and x_2 , respectively, where $x_2 = x_1 + d$. The initial conditions are assumed to be homogeneous, and the boundary conditions state that both deflection and slope vanish at the beam's endpoints when $x \rightarrow \pm\infty$.

Furthermore, it is assumed that the point masses remain in permanent contact with the beam. The displacements of the contact points are denoted as w_{01} (for the rear mass) and w_{02} (for the front mass). To eliminate these additional unknowns, the correct expansion of time derivatives must be used:

$$w_{01,t}(t) = vw_{,x}(x, t) + w_{,t}(x, t), \quad w_{01,tt}(t) = v^2 w_{,xx}(x, t) + 2vw_{,xt}(x, t) + w_{,tt}(x, t) \quad (5)$$

where $x = x_1 = vt$

$$w_{02,t}(t) = vw_{,x}(x, t) + w_{,t}(x, t), \quad w_{02,tt}(t) = v^2 w_{,xx}(x, t) + 2vw_{,xt}(x, t) + w_{,tt}(x, t) \quad (6)$$

with $x = x_2 = vt + d$

Substituting these expressions into Equations (2)–(4) yields:

$$p(x, t) = \left(P - M(w_{,tt} + 2vw_{,xt} + v^2 w_{,xx}) - k_v \left(w - u_v + \theta \frac{d}{2} \right) - c_v \left(vw_{,x} + w_{,t} - u_{v,t} + \theta_{,t} \frac{d}{2} \right) \right) \delta(x - vt) + \left(P - M(w_{,tt} + 2vw_{,xt} + v^2 w_{,xx}) - k_v \left(w - u_v - \theta \frac{d}{2} \right) - c_v \left(vw_{,x} + w_{,t} - u_{v,t} - \theta_{,t} \frac{d}{2} \right) \right) \delta(x - vt - d) \quad (7)$$

$$M_b u_{v,tt}(t) - k_v (w(vt, t) + w(vt + d, t) - 2u_v) - c_v (vw_{,x}(vt, t) + w_{,t}(vt, t) + vw_{,x}(vt + d, t) + w_{,t}(vt + d, t) - 2u_{v,t}) = 0 \quad (8)$$

$$J_b \theta_{,tt}(t) + k_v (w(vt, t) - w(vt + d, t) + \theta d) \frac{d}{2} + c_v (vw_{,x}(vt, t) + w_{,t}(vt, t) - vw_{,x}(vt + d, t) - w_{,t}(vt + d, t) + \theta_{,t} d) \frac{d}{2} = 0 \quad (9)$$

To simplify the problem, it is convenient to switch to moving coordinates, setting $r = x - vt$, fixing the zero position on the rear mass and properly expanding the time derivatives as follows:

$$w_{,t}(x, t) = -vw_{,r}(r, t) + w_{,t}(r, t), \quad w_{,tt}(x, t) = v^2 w_{,rr}(r, t) - 2vw_{,rt}(r, t) + w_{,tt}(r, t) \quad (10)$$

The system of equations then takes its final form:

$$EIw_{,rrrr} + m(w_{,tt} - 2vw_{,rt} + v^2w_{,rr}) + c_f(w_{,t} - vw_{,r}) + k_fw = p(r, t) \quad (11)$$

$$p(r, t) = \left(P - Mw_{,tt} - k_v \left(w - u_v + \theta \frac{d}{2} \right) - c_v \left(w_{,t} - u_{v,t} + \theta_{,t} \frac{d}{2} \right) \right) \delta(r) \\ + \left(P - Mw_{,tt} - k_v \left(w - u_v - \theta \frac{d}{2} \right) - c_v \left(w_{,t} - u_{v,t} - \theta_{,t} \frac{d}{2} \right) \right) \delta(r - d) \quad (12)$$

$$M_b u_{v,tt}(t) - k_v(w(vt, t) + w(vt + d, t) - 2u_v) - c_v(w_{,t}(vt, t) + w_{,t}(vt + d, t) - 2u_{v,t}) = 0 \quad (13)$$

$$2J_b \theta_{,tt} + k_v(w(vt, t) - w(vt + d, t) + \theta d) + c_v(w_{,t}(vt, t) - w_{,t}(vt + d, t) + \theta_{,t} d) = 0 \quad (14)$$

To non-dimensionalize the system, the following parameters are introduced:

$$\chi = \sqrt[4]{\frac{k_f}{4EI}}, \quad v_{ref} = \sqrt[4]{\frac{4k_f EI}{m^2}} = \frac{1}{\chi} \sqrt{\frac{k_f}{m}}, \quad w_{st} = \frac{P\chi}{2k_f}, \quad \eta_P = \frac{P}{P} = 1 \quad (15)$$

$$\xi = \chi r, \quad \tilde{d} = d\chi, \quad \tau = \chi v_{ref} t = \sqrt{\frac{k_f}{m}} t, \quad \tilde{w} = \frac{w}{w_{st}}, \quad \tilde{u} = \frac{u_v}{w_{st}}, \quad \tilde{\theta} = \frac{\theta}{w_{st}\chi}, \quad \alpha = \frac{v}{v_{ref}} \quad (16)$$

$$\eta_M = \frac{M\chi}{m}, \quad \mu_v = \frac{M_b\chi}{m}, \quad \tilde{J}_b = \frac{\chi^3}{m} J_b, \quad \kappa_v = \frac{k_v\chi}{k_f}, \quad \eta_v = \frac{c_v\chi}{2\sqrt{mk_f}}, \quad \eta_f = \frac{c_f}{2\sqrt{mk_f}} \quad (17)$$

This results in a system of three equations:

$$\tilde{w}_{,\xi\xi\xi\xi} + 4\alpha^2\tilde{w}_{,\xi\xi} + 4\tilde{w}_{,\tau\tau} - 8\alpha\tilde{w}_{,\xi\tau} + 8\eta_b(\tilde{w}_{,\tau} - \alpha\tilde{w}_{,\xi}) + 4\tilde{w} = \\ 4\left(2\eta_P - \eta_M\tilde{w}_{,\tau\tau} - \kappa_v\left(\tilde{w} - \tilde{u} + \frac{1}{2}\tilde{\theta}\tilde{d}\right) - 2\eta_v\left(\tilde{w}_{,\tau} - \tilde{u}_{,\tau} + \frac{1}{2}\tilde{\theta}_{,\tau}\tilde{d}\right)\right)\delta(\xi) \\ + 4\left(2\eta_P - \eta_M\tilde{w}_{,\tau\tau} - \kappa_v\left(\tilde{w} - \tilde{u} - \frac{1}{2}\tilde{\theta}\tilde{d}\right) - 2\eta_v\left(\tilde{w}_{,\tau} - \tilde{u}_{,\tau} - \frac{1}{2}\tilde{\theta}_{,\tau}\tilde{d}\right)\right)\delta(\xi - \tilde{d}) \quad (18)$$

$$\mu_v\tilde{u}_{,\tau\tau}(\tau) - \kappa_v\left(\tilde{w}(\alpha\tau, \tau) + \tilde{w}(\alpha\tau + \tilde{d}, \tau) - 2\tilde{u}\right) - 2\eta_v\left(\tilde{w}_{,\tau}(\alpha\tau, \tau) + \tilde{w}_{,\tau}(\alpha\tau + \tilde{d}, \tau) - 2\tilde{u}_{,\tau}\right) = 0 \quad (19)$$

$$2\tilde{J}_b\tilde{\theta}_{,tt} + \kappa_v\left(\tilde{w}(\alpha\tau, \tau) - \tilde{w}(\alpha\tau + \tilde{d}, \tau) + \tilde{\theta}\tilde{d}\right)\tilde{d} + 2\eta_v\left(\tilde{w}_{,\tau}(\alpha\tau, \tau) - \tilde{w}_{,\tau}(\alpha\tau + \tilde{d}, \tau) + \tilde{\theta}_{,\tau}\tilde{d}\right)\tilde{d} = 0 \quad (20)$$

The solution is obtained using integral transforms. First, the Laplace transform is applied:

$$\tilde{F}(\xi, \bar{q}) = \int_0^\infty f(\xi, \tau) e^{-\bar{q}\tau} d\tau \text{ with } \bar{q} = iq \quad (21)$$

Then, the Fourier transform

$$F(p, \bar{q}) = \int_{-\infty}^\infty \tilde{F}(\xi, \bar{q}) e^{-ip\xi} d\xi \quad (22)$$

is applied to the beam equilibrium equation. This transformation leads to:

$$W(p, q) = \frac{1}{D(p, q)} \left(\frac{8\eta_P}{iq} + 4(\eta_M q^2 - (\kappa_v + 2i\eta_v q)) \tilde{W}(0, q) + 2(\kappa_v + 2i\eta_v q) (2\tilde{U} - \tilde{\Theta}\tilde{d}) \right. \\ \left. + \left(\frac{8\eta_P}{iq} + 4(\eta_M q^2 - (\kappa_v + 2i\eta_v q)) \tilde{W}(\tilde{d}, q) + 2(\kappa_v + 2i\eta_v q) (2\tilde{U} + \tilde{\Theta}\tilde{d}) \right) e^{-ip\tilde{d}} \right) \quad (23)$$

where:

$$D(p, q) = p^4 - 4p^2\alpha^2 - 4q^2 + 8\alpha pq + 8i\eta_f(q - \alpha p) + 4 \quad (24)$$

The other two governing equations in this transformed space are:

$$\left(\mu_v q^2 - 2(\kappa_v + 2\eta_v i q)\right) \tilde{U} + (\kappa_v + 2\eta_v i q) \left(\tilde{W}(0, q) + \tilde{W}(\tilde{d}, q)\right) = 0 \quad (25)$$

$$\left(-2\tilde{J}_b q^2 + (\kappa_v + 2\eta_v i q)\tilde{d}^2\right) \tilde{\Theta} + (\kappa_v + 2\eta_v i q) \left(\tilde{W}(0, q) - \tilde{W}(\tilde{d}, q)\right) \tilde{d} = 0 \quad (26)$$

Applying the inverse Fourier transform

$$\tilde{F}(\xi, i q) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(p, i q) e^{i p \xi} dp \quad (27)$$

to Equation (23) leads to:

$$\begin{aligned} \tilde{W}(\xi, q) = & \frac{1}{2\pi} \left(\frac{8\eta_P}{i q} + 4(\eta_M q^2 - (\kappa_v + 2i q \eta_v)) \right) \tilde{W}(0, q) + 2(\kappa_v + 2i q \eta_v) \left(2\tilde{U} - \tilde{\Theta} \tilde{d} \right) K(\xi, q) \\ & + \frac{1}{2\pi} \left(\frac{8\eta_P}{i q} + 4(\eta_M q^2 - (\kappa_v + 2i q \eta_v)) \right) \tilde{W}(\tilde{d}, q) + 2(\kappa_v + 2i q \eta_v) \left(2\tilde{U} + \tilde{\Theta} \tilde{d} \right) K(\xi - \tilde{d}, q) \end{aligned} \quad (28)$$

where:

$$K(\xi, q) = \int_{-\infty}^{\infty} \frac{e^{i p \xi}}{D(p, q)} dp \quad (29)$$

The final step involves the inverse Laplace transform, performed analytically using contour integration. To do so, the unknowns $\tilde{W}(0, q)$, $\tilde{W}(\tilde{d}, q)$, $\tilde{U}$ and $\tilde{\Theta}$ must first be determined by solving the system obtained by substituting $\xi = 0$ and $\xi = \tilde{d}$ into Equation (28) and adding Equations (25) and (26):

$$\begin{bmatrix} A_{11} & A_{12} & A_{13} & A_{14} \\ A_{21} & A_{22} & A_{23} & A_{24} \\ A_{31} & A_{32} & A_{33} & A_{34} \\ A_{41} & A_{42} & A_{43} & A_{44} \end{bmatrix} \cdot \begin{bmatrix} \tilde{W}(0, q) \\ \tilde{W}(\tilde{d}, q) \\ \tilde{U} \\ \tilde{\Theta} \end{bmatrix} = \begin{bmatrix} \frac{4\eta_P}{i q} K(0, q) + \frac{4\eta_P}{i q} K(-\tilde{d}, q) \\ \frac{4\eta_P}{i q} K(\tilde{d}, q) + \frac{4\eta_P}{i q} K(0, q) \\ 0 \\ 0 \end{bmatrix} \quad (30)$$

where:

$$\begin{aligned} A_{11} &= \pi - 2(\eta_M q^2 - \kappa_v - 2\eta_v i q) K(0, q) \\ A_{12} &= -2(\eta_M q^2 - \kappa_v - 2\eta_v i q) K(-\tilde{d}, q) \\ A_{13} &= -2(\kappa_v + 2\eta_v i q) K(0, q) - 2(\kappa_v + 2\eta_v i q) K(-\tilde{d}, q) \\ A_{14} &= (\kappa_v + 2\eta_v i q) K(0, q) \tilde{d} - (\kappa_v + 2\eta_v i q) K(-\tilde{d}, q) \tilde{d} \end{aligned} \quad (31)$$

$$\begin{aligned} A_{21} &= -2(\eta_M q^2 - \kappa_v - 2\eta_v i q) K(\tilde{d}, q) \\ A_{22} &= \pi - 2(\eta_M q^2 - \kappa_v - 2\eta_v i q) K(0, q) \\ A_{23} &= -2(\kappa_v + 2\eta_v i q) K(\tilde{d}, q) - 2(\kappa_v + 2\eta_v i q) K(0, q) \\ A_{24} &= (\kappa_v + 2\eta_v i q) K(\tilde{d}, q) \tilde{d} - (\kappa_v + 2\eta_v i q) K(0, q) \tilde{d} \end{aligned} \quad (32)$$

$$\begin{aligned} A_{31} &= \kappa_v + 2\eta_v i q \\ A_{32} &= \kappa_v + 2\eta_v i q \\ A_{33} &= \mu_v q^2 - 2\kappa_v - 4\eta_v i q \\ A_{34} &= 0 \end{aligned} \quad (33)$$

$$\begin{aligned} A_{41} &= \kappa_v \tilde{d} + 2\eta_v i q \tilde{d} \\ A_{42} &= -\kappa_v \tilde{d} - 2\eta_v i q \tilde{d} \\ A_{43} &= 0 \\ A_{44} &= -2\tilde{J}_b q^2 + \kappa_v \tilde{d}^2 + 2\eta_v i q \tilde{d}^2 \end{aligned} \quad (34)$$

Solving system (30) allows for substitution back into Equation (28) for the inverse Laplace transform. It is convenient to switch coordinates, as shown below in Equation (35). Using contour integration, the final result can be expressed as a sum of residues:

$$\tilde{w}(\xi, \tau) = \frac{1}{2\pi i} \lim_{T \rightarrow \infty} \int_{a-iT}^{a+iT} \tilde{W}(\xi, \bar{q}) e^{\bar{q}\tau} d\bar{q} = \frac{1}{2\pi} \int_{-ia-\infty}^{-ia+\infty} \tilde{W}(\xi, q) e^{iq\tau} dq = \sum_j \text{res} \left(i\tilde{W}(\xi, q) e^{iq\tau}, q_{M_j} \right) + I_{bc} \quad (35)$$

where a is a small real number introduced to ensure that all poles and discontinuities lie on the right-hand side of the integration line in the complex q -plane, that is, after switching, above the integration line. The terms q_{M_j} represent the induced frequencies. It has been demonstrated in [40] that there are step discontinuities in the K -function, which must be removed by branch cuts. These discontinuities occur along the line where the imaginary part of q is equal to η_f . This yields to an additional term in Equation (35), I_{bc} , which must be computed through numerical integration along the branch cuts. In most cases, I_{bc} can be neglected, particularly when matching initial conditions exactly at the earliest times. Another situation where I_{bc} becomes relevant is when some of the expected induced frequencies are missing.

To obtain the closed-form solution for the rear mass deflection (after neglecting I_{bc}), it is necessary to evaluate and sum the residues, as indicated in Equation (36):

$$\tilde{w}(0, \tau) = \sum_j \text{res} \left(\frac{4B_2 \left[2B_5 \left(\tilde{d}^2 B_4^2 + B_1 B_3 \right) + \pi B_3 (K_- + K_0) \right] e^{iq\tau}}{q E_{chr}}, q_{M_j} \right) \quad (36)$$

where:

$$\begin{aligned} B_1 &= \eta_M q^2 - \kappa_v - 2iq\eta_v; \quad B_2 = \mu_v q^2 - 2\kappa_v - 4iq\eta_v; \quad B_3 = -2\tilde{J}_b q^2 + \kappa_v \tilde{d}^2 + 2iq\eta_v \tilde{d}^2; \\ B_4 &= \kappa_v + 2iq\eta_v; \quad K_0 = K(0, q); \quad K_+ = K(\tilde{d}, q); \quad K_- = K(-\tilde{d}, q); \\ B_5 &= K_+ K_- - K_0^2; \quad B_6 = K_+ + K_- + 2K_0; \quad B_7 = K_+ + K_- - 2K_0; \end{aligned} \quad (37)$$

and:

$$E_{chr} = 4B_5 \left(\tilde{d}^2 B_4^2 + B_1 B_3 \right) \left(2B_4^2 - B_1 B_2 \right) + \pi \tilde{d}^2 B_4^2 B_2 B_7 + 2\pi B_4^2 B_3 B_6 + \pi B_2 B_3 (\pi - 4K_0 B_1) \quad (38)$$

The denominator in Equation (36) corresponds to the simplified determinant of system (30). Its roots indicate the poles of the system, which determine both the steady-state and transient components of the solution. There is always one obvious pole at $q_M = 0$, which identifies the steady-state part $\tilde{w}_{sd}(0)$ of the full solution:

$$\tilde{w}_{sd}(0, \tau) = \tilde{w}_{sd}(0) = \left. \frac{4B_2 \left[2B_5 \left(\tilde{d}^2 B_4^2 + B_1 B_3 \right) + \pi B_3 (K_- + K_0) \right]}{E_{chr}} \right|_{q=0} \quad (39)$$

with $q = 0$ substituted.

Furthermore, if q_M is a root of $E_{chr} = 0$, then the complex conjugate of its opposite value $(-q_M)^*$ is also a root. These pairs form a harmonic vibration referred to as the unsteady harmonic part of the full solution, $\tilde{w}_{usd}(0, \tau)$. The term “unsteady” is used to indicate that this component belongs to the transient solution. I_{bc} is then called the truly transient part to distinguish it from the unsteady harmonic part. Therefore, the transient solution is the sum of $\tilde{w}_{usd}(0, \tau)$ and I_{bc} . $\tilde{w}_{usd}(0, \tau)$ reads as:

$$\tilde{w}_{usd}(0, \tau) = \sum_j \left. \frac{4B_2 \left[2B_5 \left(\tilde{d}^2 B_4^2 + B_1 B_3 \right) + \pi B_3 (K_- + K_0) \right] e^{iq\tau}}{q E_{chr, q}} \right|_{q=q_{M_j}} \quad (40)$$

To evaluate Equation (40), the derivative of E_{chr} with respect to q must be considered. This naturally involves derivatives of K_0 , K_+ and K_- . Each of these values (without derivatives) is also determined using contour integration. When taking the derivative, all poles are doubled. However, an analytical formula exists for evaluating residues at double poles, making the full evaluation entirely analytical.

Similar formula as for $\tilde{w}_{usd}(0, \tau)$ can be deduced for the front mass deflection:

$$\tilde{w}_{usd}(\tilde{d}, \tau) = \sum_j \text{res} \left(\frac{4B_2 \left[2B_5 \left(\tilde{d}^2 B_4^2 + B_1 B_3 \right) + \pi B_3 (K_+ + K_0) \right] e^{iq\tau}}{q E_{chr}}, q_{M_j} \right) \quad (41)$$

for the mass center of the bogie bar vertical displacement:

$$\tilde{u}_{usd}(0, \tau) = \sum_j \text{res} \left(\frac{-4B_4 \left[4B_5 \left(\tilde{d}^2 B_4^2 + B_1 B_3 \right) + \pi B_3 B_6 \right] e^{iq\tau}}{q E_{chr}}, q_{M_j} \right) \quad (42)$$

and for the rotation:

$$\tilde{\theta}_{usd}(0, \tau) = \sum_j \text{res} \left(\frac{4\pi \tilde{d} B_2 B_4 (K_+ - K_-) e^{iq\tau}}{q E_{chr}}, q_{M_j} \right) \quad (43)$$

The way q is introduced implies that the solution becomes unstable as soon as the imaginary part of q_M becomes negative. I_{bc} can never destabilize the solution because the branch cuts are located in the region of the complex q -plane where the imaginary part is non-negative.

After defining all displacements at the active points, it is possible to substitute back into Equation (28) and determine the full deflection shapes of the beam at any time using a closed-form formula:

$$\tilde{w}_{sd}(\xi, \tau) = \tilde{w}_{sd}(\xi) = \frac{8\eta_P E_{chr} + E_1}{2\pi E_{chr}} \bigg|_{q=0} K(\xi, 0) + \frac{8\eta_P E_{chr} + E_2}{2\pi E_{chr}} \bigg|_{q=0} K(\xi - \tilde{d}, 0) \quad (44)$$

where:

$$E_1 = 4B_1 B_2 \left(8B_5 \left(\tilde{d}^2 B_4^2 + B_1 B_3 \right) + 4\pi B_3 (K_- + K_0) \right) - 2B_3 \left[8B_3 \left(4B_5 \left(\tilde{d}^2 B_4^2 + B_1 B_3 \right) + \pi B_3 B_6 \right) - 4\pi \tilde{d}^2 B_2 B_4 (K_- - K_+) \right] \quad (45)$$

$$E_2 = 4B_1 B_2 \left(8B_5 \left(\tilde{d}^2 B_4^2 + B_1 B_3 \right) + 4\pi B_3 (K_+ + K_0) \right) - 2B_3 \left[8B_3 \left(4B_5 \left(\tilde{d}^2 B_4^2 + B_1 B_3 \right) + \pi B_3 B_6 \right) + 4\pi \tilde{d}^2 B_2 B_4 (K_- - K_+) \right] \quad (46)$$

and:

$$\tilde{w}_{usd}(\xi, \tau) = \sum_j \left[\frac{8\eta_P E_{chr} + E_1}{2\pi q E_{chr,q}} \bigg|_{q=q_{M_j}} K(\xi, q_{M_j}) + \frac{8\eta_P E_{chr} + E_2}{2\pi q E_{chr,q}} \bigg|_{q=q_{M_j}} K(\xi - \tilde{d}, q_{M_j}) \right] e^{iq_{M_j} \tau} \quad (47)$$

3. Results

Several cases are selected to demonstrate when instability can occur in the subcritical velocity range. However, this is not the sole objective. The study also illustrates how the obtained results relate to the case of two moving masses, how they are influenced by input parameters, and how they are affected by considering a Timoshenko-Rayleigh beam instead of an Euler-Bernoulli beam. Several additional cases are presented to highlight the ease of using the closed-form formulas. The results are validated through analysis of long finite beams, where the solution is obtained by modal expansion. Finally, the results are compared with previously published works

3.1. Induced Frequencies

To apply the closed-form formulas derived in Section 2, it is necessary to determine the complex roots of the characteristic equation:

$$E_{chr} = 0 \quad (48)$$

These roots are referred to as induced frequencies. As established in the author's previous works, these frequencies can be efficiently determined using iterative procedures described therein. Generally, there are four pairs of roots, and as explained earlier, instability arises if at least one of these pairs is unstable.

3.2. Validation on Finite Beams

Similar to previous studies, validation is performed on long finite beams using modal expansion. Given that the number of modes must be substantial, 500 modes were implemented in all tested cases to ensure accuracy while optimizing computational efficiency. It was confirmed that 500 modes were sufficient for all tested scenarios.

To solve the problem, the governing equations are retained in their original parameters and converted to dimensionless form only for comparison purposes. As in prior studies, without loss of generality, the beam is considered simply supported, as its vibration modes can then be expressed as sine functions, with modal frequencies and numbers determined analytically. In accordance with previous works, the loading is applied slightly off the left support to minimize its influence.

The beam equilibrium equations (Equations (1) and (7)) are multiplied by a mode shape and integrated over the beam length. In Equations (8) and (9), only the modal expansion of the beam deflection is introduced, after which all equations are assembled into a matrix form:

$$\mathbf{M}(t) \cdot \mathbf{q}_{,tt}(t) + \mathbf{C}(t) \cdot \mathbf{q}_{,t}(t) + \mathbf{K}(t) \cdot \mathbf{q}(t) = \mathbf{p}(t) \quad (49)$$

The key step involves introducing additional variables:

$$Q_{1,tt} = P - \sum_k M w_k(vt) q_{k,tt} - \sum_k (2M v w_{k,x}(vt) + c_v w_k(vt)) q_{k,t} - \sum_k (M v^2 w_{k,xx}(vt) + c_v v w_{k,x}(vt) + k_v w_k(vt)) q_k \quad (50)$$

$$Q_{2,tt} = P - \sum_k M w_k(vt + d) q_{k,tt} - \sum_k (2M v w_{k,x}(vt + d) + c_v w_k(vt + d)) q_{k,t} - \sum_k (M v^2 w_{k,xx}(vt + d) + c_v v w_{k,x}(vt + d) + k_v w_k(vt + d)) q_k \quad (51)$$

resulting in the governing matrices for mass $\mathbf{M}(t)$, damping $\mathbf{C}(t)$, and stiffness $\mathbf{K}(t)$ with the vector of unknowns $\mathbf{q}(t) = [q_1, \dots, q_n, u_v, \theta, Q_1, Q_2]^T$ and of the loading term $\mathbf{p}(t) = [0, \dots, 0, 0, 0, P, P]^T$.

$$\mathbf{M}(t) = \begin{bmatrix} \delta_{jk} & 0 & 0 & -w_j(vt) & -w_j(vt + d) \\ 0 & M_b & 0 & 0 & 0 \\ 0 & 0 & J_b & 0 & 0 \\ M w_k(vt) & 0 & 0 & 1 & 0 \\ M w_k(vt + d) & 0 & 0 & 0 & 1 \end{bmatrix} \quad (52)$$

$$\mathbf{C}(t) = \begin{bmatrix} \delta_{jk} \frac{c_b}{m} & -c_v w_j(vt) - c_v w_j(vt+d) & \frac{1}{2} d c_v w_j(vt) - \frac{1}{2} d c_v w_j(vt+d) & 0 & 0 \\ -c_v w_k(vt) - c_v w_k(vt+d) & 2c_v & 0 & 0 & 0 \\ \frac{d}{2} (c_v w_k(vt) - c_v w_k(vt+d)) & 0 & \frac{1}{2} c_v d^2 & 0 & 0 \\ 2Mv w_{k,x}(vt) + c_v w_k(vt) & 0 & 0 & 0 & 0 \\ 2Mv w_{k,x}(vt+d) + c_v w_k(vt+d) & 0 & 0 & 0 & 0 \end{bmatrix} \quad (53)$$

$$\mathbf{K}(t) = \begin{bmatrix} \delta_{jk} \omega_j^2 & -k_v w_j(vt) - k_v w_j(vt+d) & \frac{d}{2} k_v w_j(vt) - \frac{d}{2} k_v w_j(vt+d) & 0 & 0 \\ -k_v (w_k(vt) + w_k(vt+d)) & 2k_v & 0 & 0 & 0 \\ -c_v v (w_{k,x}(vt) + w_{k,x}(vt+d)) & 0 & \frac{1}{2} k_v d^2 & 0 & 0 \\ \frac{d}{2} k_v (w_k(vt) - w_k(vt+d)) & 0 & 0 & 0 & 0 \\ + \frac{d}{2} c_v v (w_{k,x}(vt) - w_{k,x}(vt+d)) & 0 & 0 & 0 & 0 \\ Mv^2 w_{k,xx}(vt) + k_v w_k(vt) + c_v v w_{k,x}(vt) & 0 & 0 & 0 & 0 \\ + Mv^2 w_{k,xx}(vt+d) + k_v w_k(vt+d) & 0 & 0 & 0 & 0 \\ + c_v v w_{k,x}(vt+d) & 0 & 0 & 0 & 0 \end{bmatrix} \quad (54)$$

3.3. Timoshenko-Rayleigh Beam

When a Timoshenko beam is considered, Equation (1) is replaced by a set of equations:

$$-G\bar{A}w_{,xx} + G\bar{A}\varphi_{,x} + mw_{,tt} + c_f w_{,t} + k_f w = p(x, t) \quad (55)$$

$$-G\bar{A}w_{,x} + G\bar{A}\varphi + J\varphi_{,tt} - EI\varphi_{,xx} = 0 \quad (56)$$

where G is the shear modulus, $\bar{A}$ is the reduced cross-sectional area, φ is the cross-sectional rotation angle and J is the mass moment of inertia of the cross-section, which can be rewritten as mr^2 with r being the radius of gyration.

By transitioning to moving coordinates, the equations transform into:

$$-G\bar{A}w_{,rr} + G\bar{A}\varphi_{,r} + m(w_{,tt} - 2vw_{,rt} + v^2w_{,rr}) + c_f(w_{,t} - vw_{,r}) + k_f w = p(r, t) \quad (57)$$

$$-G\bar{A}w_{,r} + G\bar{A}\varphi + mr^2(\varphi_{,tt} - 2v\varphi_{,rt} + v^2\varphi_{,rr}) - EI\varphi_{,rr} = 0 \quad (58)$$

The introduction of additional dimensionless parameters:

$$\tilde{\varphi} = \frac{\varphi}{w_{st}\chi}, \quad \tilde{S} = \frac{G\bar{A}}{2\sqrt{k_f EI}}, \quad \tilde{r} = r\chi \quad (59)$$

further simplifies the problem, leading to:

$$-4\tilde{S}\tilde{w}_{,\xi\xi} + 4\tilde{S}\tilde{\varphi}_{,\xi} + 4\alpha^2\tilde{w}_{,\xi\xi} + 4\tilde{w}_{,\tau\tau} - 8\alpha\tilde{w}_{,\xi\tau} + 8\eta_f(\tilde{w}_{,\tau} - \alpha\tilde{w}_{,\xi}) + 4\tilde{w} = \tilde{p}(\xi, \tau) \quad (60)$$

$$-4\tilde{S}\tilde{w}_{,\xi} + 4\tilde{S}\tilde{\varphi} + 4\tilde{r}^2(\alpha^2\tilde{\varphi}_{,\xi\xi} - 2\alpha\tilde{\varphi}_{,\xi\tau} + \tilde{\varphi}_{,\tau\tau}) - \tilde{\varphi}_{,\xi\xi} = 0 \quad (61)$$

Applying integral transforms allows the solution of $\tilde{\varphi}$:

$$\tilde{\Phi} = \frac{4ip\tilde{S}}{4\tilde{S} - 4\tilde{r}^2(\alpha p - q)^2 + p^2} \tilde{W} \quad (62)$$

which is then substituted into the first equation. The subsequent steps follow the same derivation process as before.

3.4. Tested Cases

Multiple cases were analyzed to meet the objectives of this study. To define an admissible range of parameters, a broad spectrum of real-world values was considered. For the supporting structure, the parameters EI and m fall within a relatively narrow interval, approximately $EI \in \langle 4.7; 6.4 \rangle \text{ MNm}^2$ and $m \in \langle 54; 60 \rangle \text{ kg/m}$, respectively. The parameter k_s is sometimes omitted in similar models, allowing for a zero value, while variations are primarily assigned to $k_f \in \langle 0.22; 1000 \rangle \text{ MN/m}^2$.

Regarding the moving bogie, combining all possible parameter values would be impractical, as the bogie bar's mass should relate to the wheel masses and, along with the wheel spacing, determine its mass moment of inertia. To account for this, two additional coefficients, cf_1 and cf_2 , are introduced to link these values:

$$\mu_v = cf_1 \eta_M, \tilde{J}_b = cf_2 \eta_M \tilde{d}^2 \quad (63)$$

Using Equations (16) and (17), this also implies that:

$$M_b = cf_1 M, J_b = cf_2 M d^2 \quad (64)$$

The primary suspension stiffness spans a wide range of values, and two representative values were selected: 0.02 for a soft suspension and 0.2 for a stiffer suspension. The damping parameters η_f and η_v , as well as the velocity ratio α , vary based on the effects being analyzed. Table 1 presents the input data for all tested cases, while Table 2 summarizes the induced frequencies used in the residue evaluation.

Table 1. Input data of cases analyzed in time series.

Parameter	η_f	η_M	$\tilde{d}$	cf_1	cf_2	κ_v	η_v	α
Case 1	0.1	20	1	1.7	0.01	0.02	0	0.8
Case 2	0	20	2	2	0.1	0.2	0	0.5
Case 3	0	20	5	2	0.1	0.2	0	0.5
Case 4	0	20	5	1.7	0.1	0.2	0	0.5
Case 5	0.05	40	1.5	1.7	0.01	0.2	0	0.7
Case 6	0.05	40	1.5	1.7	0.01	0.2	0.05	0.7
Case 7	0.05	30	2	1.7	0.01	0.2	0.05	0.4
Case 8	0.05	30	2	1.7	0.01	0.2	0	0.4
Case 9	0	30	2	1.7	0.01	0.2	0.05	0.4
Case 10	0	30	2	1.7	0.01	0.2	0	0.4
Case 11	0.05	25	2	1.7	0.01	0.2	0.05	0.4
Case 12	0.15	50	1.5	1.7	0.1	0.2	0	0.85
Case 13	0.15	15	1	1.7	0.1	0.2	0	0.95

The selection of cases shown in the time series, as well as the parameters listed in Table 1, was made as follows. Case 1 was selected as the only instance where the moving bogie performs worse than moving masses. Cases 2 and 3 were chosen to demonstrate that the strong dynamic interaction discussed in [56] for two moving masses is also present in the case of a moving bogie. Case 4 was selected for comparison with Case 3 to examine the effect of cf_1 . Cases 2–4 are undamped, therefore, all amplitudes remain unchanged, the induced frequencies are real (Table 2), and these cases also serve to illustrate how

undamped cases can be handled without numerical difficulties. Cases 5 and 6 were chosen such that Case 5 is unstable for subcritical velocity, but by adding damping to the primary suspension while keeping the other parameters unchanged, stability is achieved, leading to Case 6. Cases 7–10 demonstrate the effect of damping at different levels. A low damping level of 5% was chosen, and all four combinations were tested by applying damping to the foundation and/or primary suspension. Case 11 complements Case 7, highlighting the effect of changing the moving masses (wheels) value. The final two cases, Case 12 and Case 13, were selected to demonstrate instability in the subcritical velocity range, however, unlike Cases 1 or 5, instability arises here even for a higher mass moment of inertia of the bogie bar.

Table 2. Induced frequencies of analyzed cases.

Parameter	$q_{M_1}^r$	$q_{M_1}^i$	$q_{M_2}^r$	$q_{M_2}^i$	$q_{M_3}^r$	$q_{M_3}^i$	$q_{M_4}^r$	$q_{M_4}^i$
Case 1	± 0.307227	0.021025	± 0.222538	-0.000130	± 0.184817	0.028535	± 0.033910	0.000014
Case 2	± 0.319059	0	± 0.290698	0	± 0.200435	0	± 0.094072	0
Case 3	± 0.310632	0	± 0.294334	0	± 0.200910	0	± 0.094035	0
Case 4	± 0.310639	0	± 0.294718	0	± 0.200910	0	± 0.101860	0
Case 5	± 0.505305	0.000454	± 0.206528	-0.000472	± 0.174711	0.007546	± 0.070460	0.000121
Case 6	± 0.501188	0.063327	± 0.206521	0.000448	± 0.174683	0.008301	± 0.070451	0.001137
Case 7	± 0.577601	0.084896	± 0.255723	0.003197	± 0.236572	0.004611	± 0.083388	0.001565
Case 8	± 0.583823	0.000110	± 0.255859	0.001562	± 0.236478	0.004163	± 0.083397	0.000047
Case 9	± 0.577617	0.084792	± 0.255809	0.001646	± 0.236376	0.000436	± 0.083390	0.001518
Case 10	± 0.583817	0	± 0.255864	0	± 0.236346	0	± 0.083394	0
Case 11	± 0.631225	0.101825	± 0.279230	0.003868	± 0.257027	0.005435	± 0.091336	0.001877
Case 12	± 0.186008	-0.000482	± 0.148407	0.016963	± 0.122981	0.013168	± 0.062675	0.000636
Case 13	± 0.354144	0.025958	± 0.249576	-0.000631	± 0.167778	0.148504	± 0.113114	0.004749

The general trend of the frequency lines is illustrated in Figure 2, using the parameters of Case 1 but extending over the entire velocity range. Since the values are significantly different, they are plotted separately. In general, four pairs of induced frequencies can be observed. Their evolution is interrupted when their imaginary part reaches η_f , as demonstrated in [40].

Unlike more complex models, the critical velocity for resonance in this study is well-defined by an analytical formula, referring to the critical velocity of a single constant force moving at a constant velocity over the one-layer model:

$$\alpha_{cr}^{E-B} = \alpha_{ref} \sqrt{1 + \eta_s} \text{ with } \alpha_{ref} = 1 \quad (65)$$

For the Timoshenko-Rayleigh beam, the critical velocity is presented, for instance, in [71] or [67]. Expressed in the dimensionless parameters used here, it becomes more explicit how this value relates to that of the Euler-Bernoulli beam:

$$\alpha_{cr}^{T-R} = \alpha_{ref} \frac{\sqrt{\tilde{r}^2 - \tilde{S} - 8\tilde{r}^2\tilde{S}^2 + 4\tilde{S}^{3/2}\sqrt{4\tilde{r}^4\tilde{S} - \tilde{r}^2 + \tilde{S}}}}{2(\tilde{S} - \tilde{r}^2)} \text{ with } \alpha_{ref} = 1 \quad (66)$$

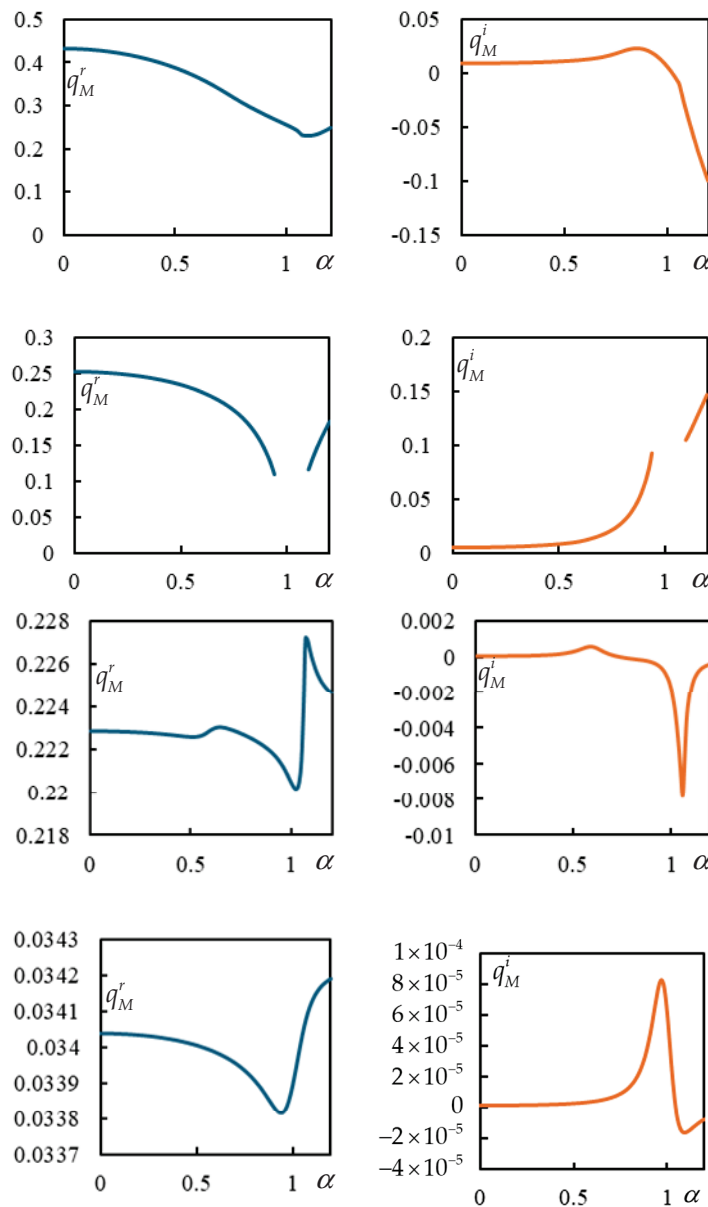


Figure 2. Real (blue) and imaginary (orange) parts of the induced frequencies in Case 1. Only one root of each pair is plotted; the other root has the same imaginary part but an opposite real part.

3.5. Comparison with Moving Masses

Unlike a single moving inertial object with a single contact point on the supporting structure, which cannot experience instability in the subcritical velocity range, multiple moving objects can interact dynamically, leading to subcritical instability. Instability at subcritical velocity is common but, in many cases, requires unrealistically high mass values, as shown in [56].

To summarize the negative effects of foundation damping, Figure 3 illustrates the instability lines for different levels of damping. To illustrate the analyzed features and provide insight into general tendencies, the vertical axis is extended up to $\eta_M = 200$, which is already an academic value. The parameter α extends up to 1.2, where $\alpha = 1$ corresponds to the critical velocity for resonance. For simplicity and to maintain consistency with the moving bogie case, two equal moving masses subjected to the same constant forces are considered. In dimensionless parameters, when the effect of shear stiffness is neglected, the only variables are the distance between the masses and foundation damping.

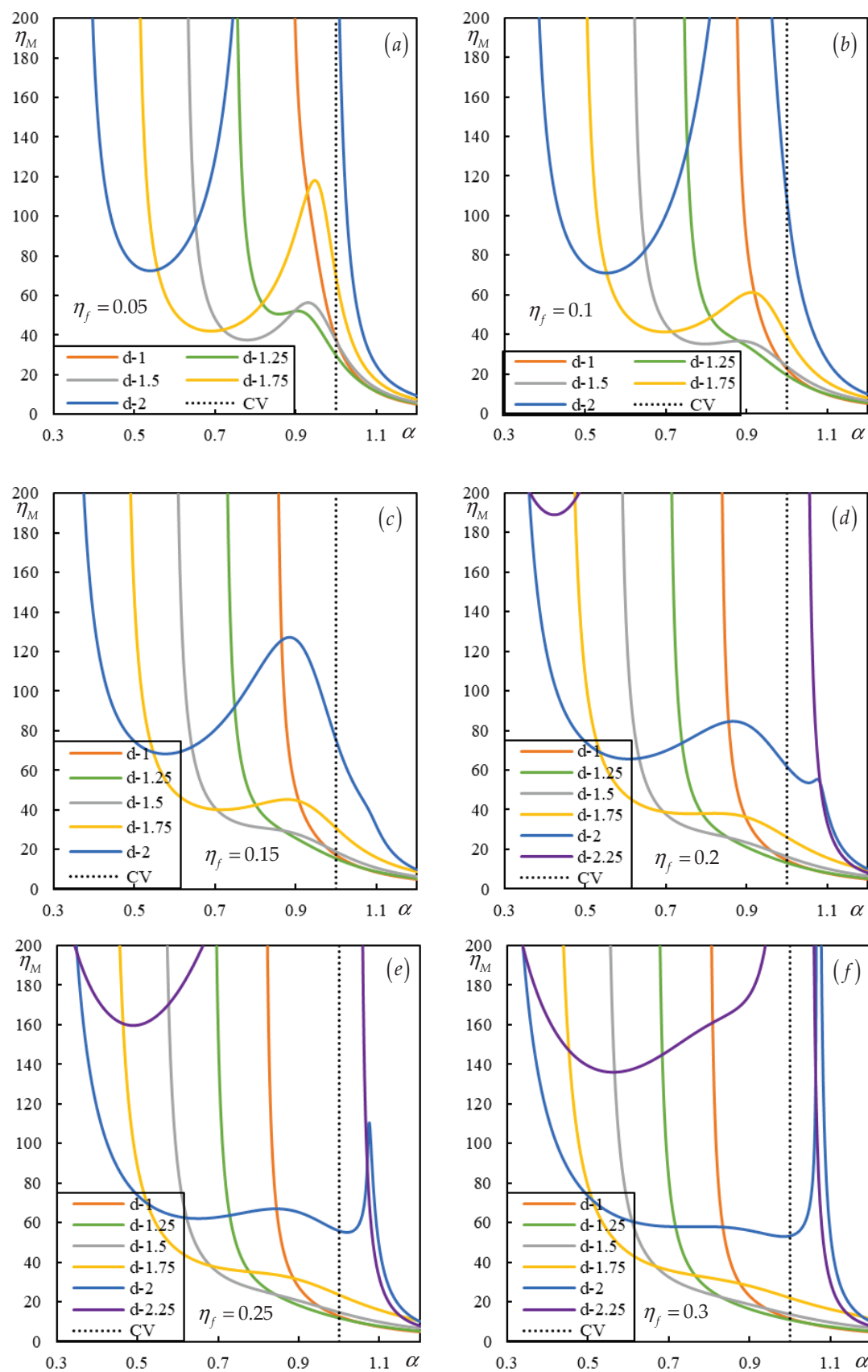


Figure 3. Instability lines for two moving proximate masses of equal value. Different colors represent various distances $\tilde{d} = 1 : 0.25 : 2.25$, as indicated in the legend. The black dotted line marks the critical velocity. Parts (a–f) correspond to $\eta_f = 0.05 : 0.05 : 0.3$.

The instability lines were traced by identifying real-valued induced frequencies, which in damped cases necessarily alter the number of unstable frequencies by one. This method clearly delineates stable and unstable velocity intervals for specific moving mass ratios and is both more effective and simpler than the traditional D-decomposition method used by other researchers (e.g., [67–69]).

Figure 3 demonstrates that increasing damping shifts all instability branches to lower moving masses while also making them straighter. Consequently, lower masses occupy larger velocity intervals, proving that, contrary to expectations, foundation damping worsens instability. However, in closed intervals, the degree of instability is generally lower than in semi-closed intervals.

In Figure 3, only relevant distances between masses are included, specifically those that still contain part of the branch within the defined scale in the subcritical velocity range. A broader analysis of distances between masses reveals that the starting point of the first instability branch shifts leftward as the distance increases, while at the same time, the moving masses increase.

When considering the moving bogie, additional parameters come into play. To make the comparison more realistic, the total mass should remain the same. Figure 4 presents the case where $cf_1 = 1.7$, requiring the instability lines from Figure 3 to be recalculated accordingly, to be comparable. In addition to $cf_1 = 1.7$, the moving bogie is characterized by $cf_2 = 0.01$ and $\kappa_v = 0.02$, chosen to minimize its influence as much as possible.

It can be concluded that even for weak primary suspension stiffness and a low mass moment of inertia of the bogie bar, all cases with a moving bogie perform better—meaning the instability lines are positioned above those for moving masses—except for the lowest tested distance $\tilde{d} = 1$. The instability lines in Figure 4 are represented only up to the critical velocity. The relevant instability lines are those that contain a segment within the chosen scale in the subcritical velocity range. The following rule applies: if a particular instability line ($\tilde{d}$) is relevant for moving masses but not for a moving bogie, then $\tilde{d} + 0.25$ is not plotted for moving masses, even if it would otherwise be considered relevant.

To analyze in detail the only case where the moving bogie performs worse than moving masses, Case 1 from Table 1 is selected. This case is unstable. However, as shown in Table 2, the unstable frequency has a relatively low imaginary part (in absolute value). This implies that the instability becomes significant only at later times, while initially, this case behaves similarly to a highly damped system. Figure 5 presents time series obtained using closed-form formulas, as well as results from a finite beam model for validation. The high-time behavior shown in Figure 6 is calculated exclusively using closed-form formulas, as performing these calculations on a finite beam would be computationally prohibitive due to the excessive number of vibration modes. Similarly, numerical integration would be impractical due to limitations in numerical precision.

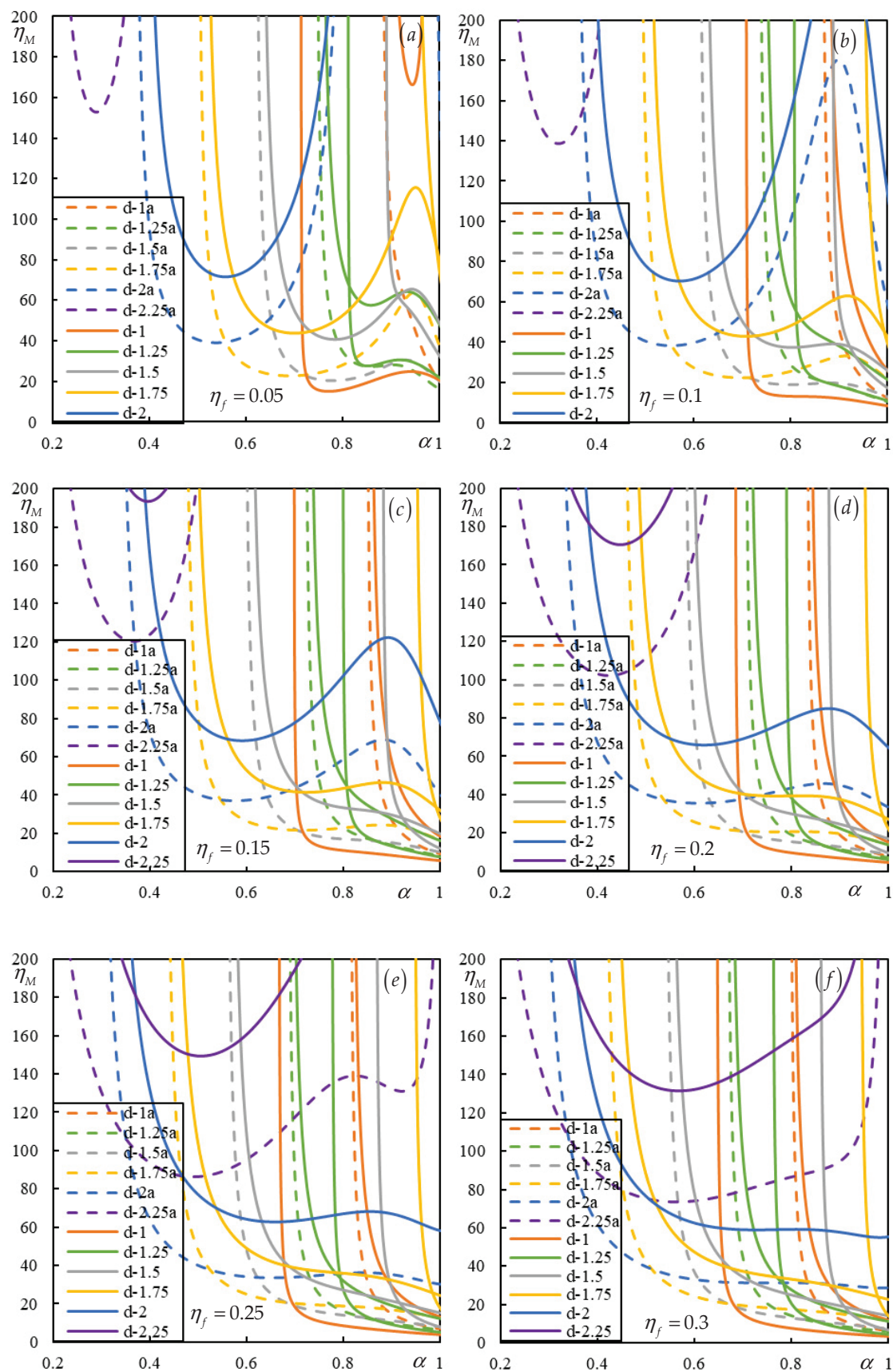


Figure 4. Instability lines for a moving bogie (solid lines) compared to two moving masses (dashed lines). Different colors represent various distances $\tilde{d} = 1 : 0.25 : 2.25$, as indicated in the legend. Curves labeled 'a' have been adapted to represent the same total mass. Parts (a–f) correspond to $\eta_f = 0.05 : 0.05 : 0.3$.

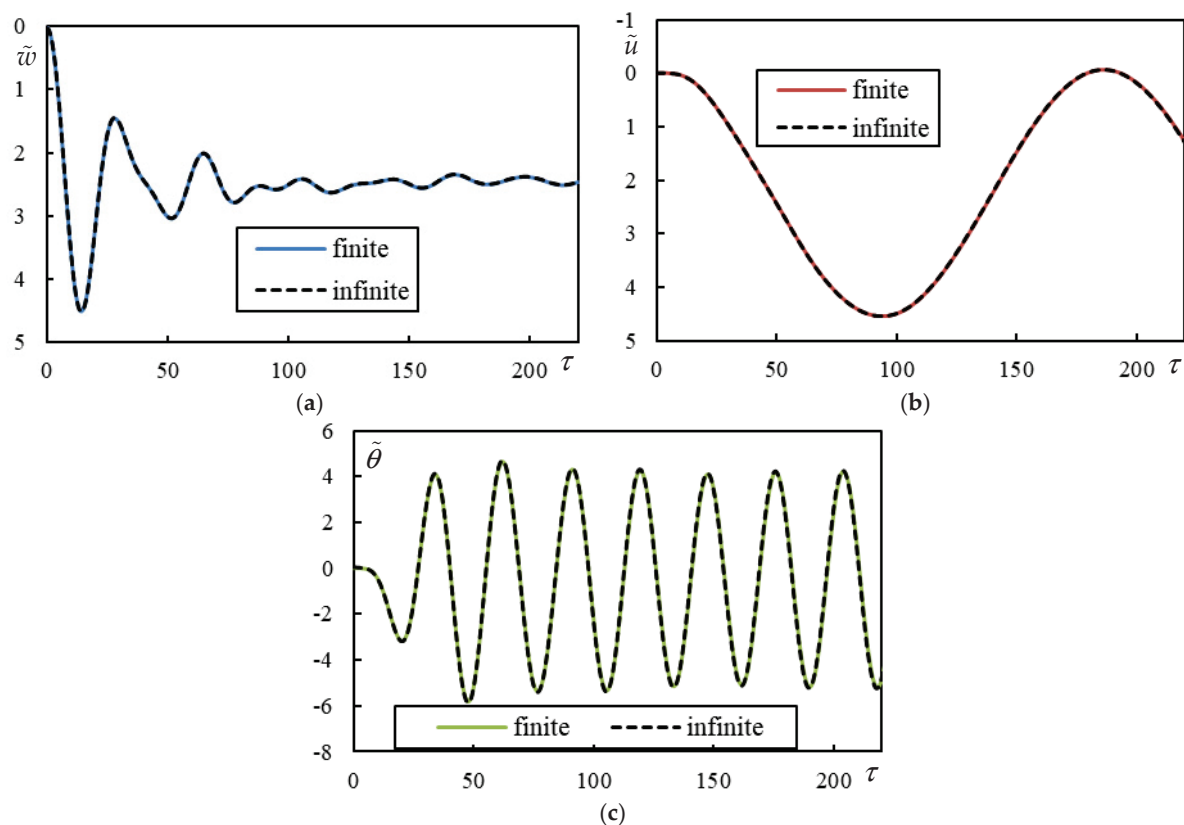


Figure 5. Time series in Case 1. Solid lines represent a finite beam, while dashed lines correspond to an infinite beam calculated using closed-form formulas. (a) Beam deflection at the rear mass; (b) bogie bar mass center displacement; (c) bogie bar mass center rotation.

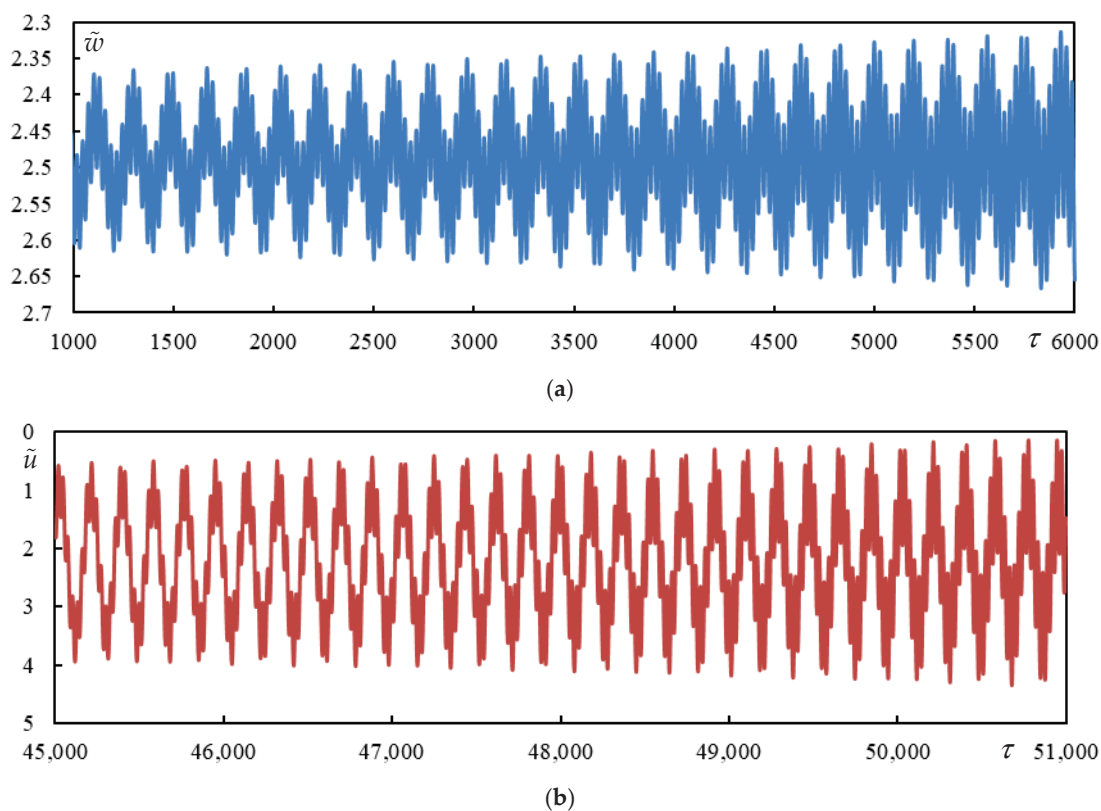


Figure 6. Time series in Case 1. (a) Beam deflection at the rear mass; (b) bogie bar mass center displacement.

3.6. Effect of Suspension Damping

Suspension damping has a significantly positive impact on bogie stability, an effect further enhanced by a higher mass moment of inertia of the bogie bar. To demonstrate this, $cf_1 = 1.7$, and the other coefficient varies between $cf_2 = 0.01$ and $cf_2 = 0.1$. A weak foundation damping of $\eta_f = 0.05$ and a stiffer primary suspension stiffness of $\kappa_v = 0.2$ are implemented.

Figure 7 clearly shows that, with a higher mass moment of inertia of the bogie bar, nearly all instability lines for the cases with primary suspension damping are removed from the subcritical velocity range.

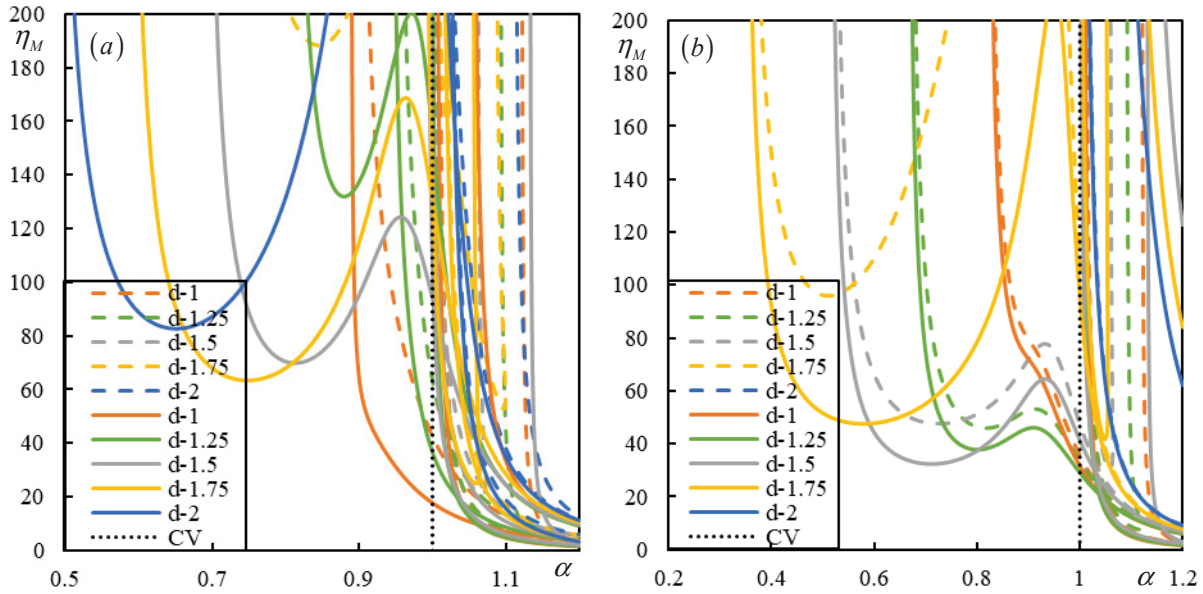


Figure 7. Instability lines for a moving bogie. Different colors represent various distances $\tilde{d} = 1 : 0.25 : 2$, as indicated in the legend. Solid lines correspond to no damping in the suspension, while dashed lines correspond to a suspension damping value of $\eta_v = 0.05$. (a) $cf_2 = 0.1$; (b) $cf_2 = 0.01$.

3.7. Impact of Beam Theory on the Results

To assess the influence of beam theory on the results, two cases are analyzed in Figure 8. Since the additional parameters in the Timoshenko-Rayleigh theory must be chosen consistently, as is clear from Equation (59), the values for a softer foundation $\tilde{S} = 100$ and $\tilde{r} = 0.03$, and for a stronger foundation $\tilde{S} = 10$ and $\tilde{r} = 0.09$, were selected. The other parameters are: $cf_1 = 1.7$ and $cf_2 = 0.1$. Weak foundation damping of $\eta_f = 0.05$ and a stiffer primary suspension stiffness of $\kappa_v = 0.2$ with no damping are used.

Only the subcritical velocity range is shown. However, since the difference in critical velocity in the Timoshenko-Rayleigh case using Equation (66) is very small—0.993 and 0.980, respectively—the graphs are plotted up to the value of 1, as before.

The results indicate that for a softer foundation, there is virtually no difference between the Euler-Bernoulli and Timoshenko-Rayleigh theories. Some differences are noticeable for a stronger foundation, but they do not appear to be fundamental.

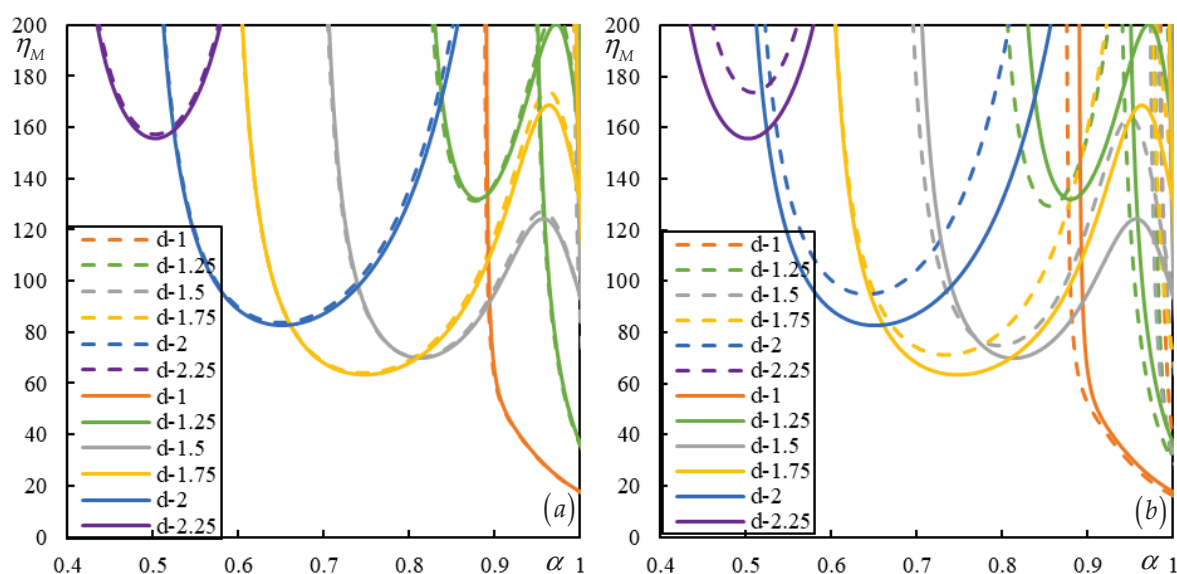


Figure 8. Instability lines for a moving bogie comparing Euler-Bernoulli (solid lines) and Timoshenko-Rayleigh (dashed lines) theories. Different colors represent various distances $\tilde{d} = 1 : 0.25 : 2.25$, as indicated in the legend. (a) $\tilde{S} = 100, \tilde{r} = 0.03$; (b) $\tilde{S} = 10, \tilde{r} = 0.09$.

3.8. Time Series of Other Cases from Table 1

This subsection has three objectives: to analyze the effect under consideration, to demonstrate the ease of using closed-form formulas, and to verify that the contribution of integration along the branch cuts is negligible by comparing results with finite beam analysis.

Further time series analyses are conducted for Cases 2–6 from Table 1, comparing finite and infinite beam models. These cases explore dynamic interactions, stability effects, and induced frequency variations under different conditions. Cases 2 and 3 are selected to show that the dynamic interaction discussed in previous works for two moving masses is maintained for a moving bogie. A slightly higher parameter $cf_1 = 2$ is introduced, but the changes are not significant. Two distances are compared ($\tilde{d} = 2$ in Figure 9 and $\tilde{d} = 5$ in Figure 10). Other parameters are included in Table 1.

Figure 9 shows a strong dynamic interaction. In Figure 10, this effect is less pronounced, however, similar to the two moving masses case, dynamic interaction persists up to relatively high distances. Only when K_+ and K_- start to have negligible values relative to K_0 , then the interaction becomes negligible. This proves that the function envelopes for $\tilde{\theta}$ still show a very strong interaction in Figure 10 because K_0 is not included in Equation (43).

Case 4 is selected for comparison with Case 3 to assess the effect of $cf_1 = 1.7$ with respect to $cf_1 = 2$. In addition to the time series (Figure 11), differences in induced frequencies are also examined in Table 2.

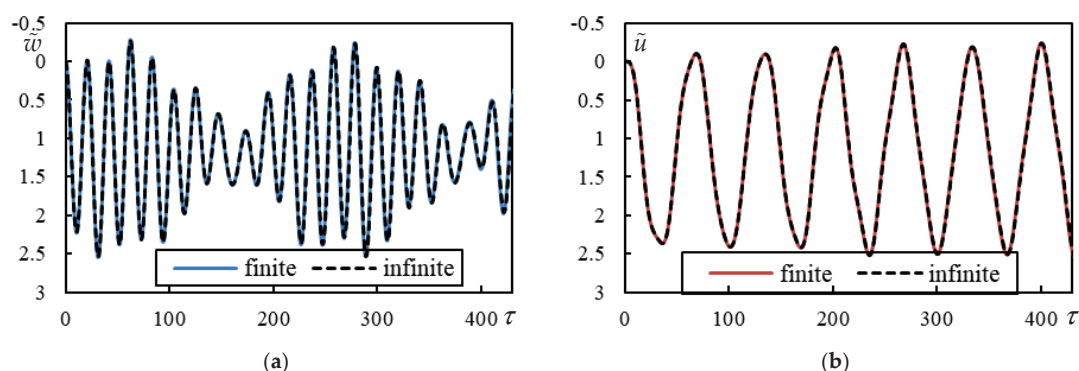


Figure 9. Cont.

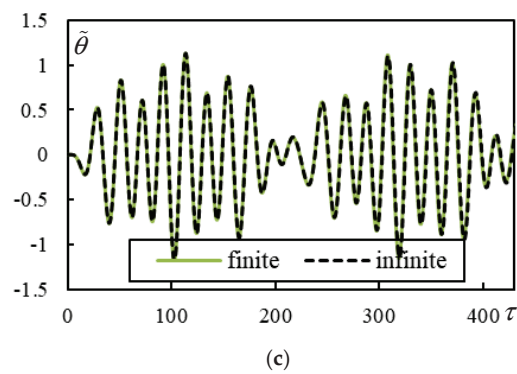


Figure 9. Time series in Case 2. Solid lines represent a finite beam, while dashed lines correspond to an infinite beam calculated using closed-form formulas. (a) Beam deflection at the rear mass; (b) bogie bar mass center displacement; (c) bogie bar mass center rotation.

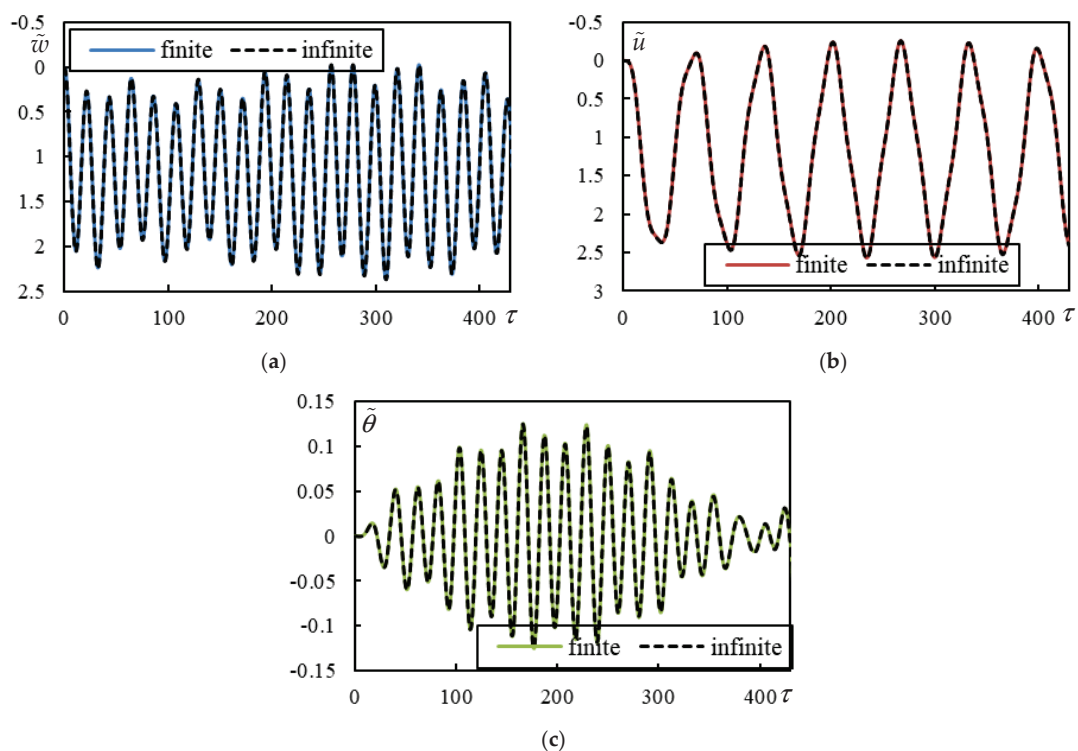


Figure 10. Time series in Case 3. Solid lines represent a finite beam, while dashed lines correspond to an infinite beam calculated using closed-form formulas. (a) Beam deflection at the rear mass; (b) bogie bar mass center displacement; (c) bogie bar mass center rotation.

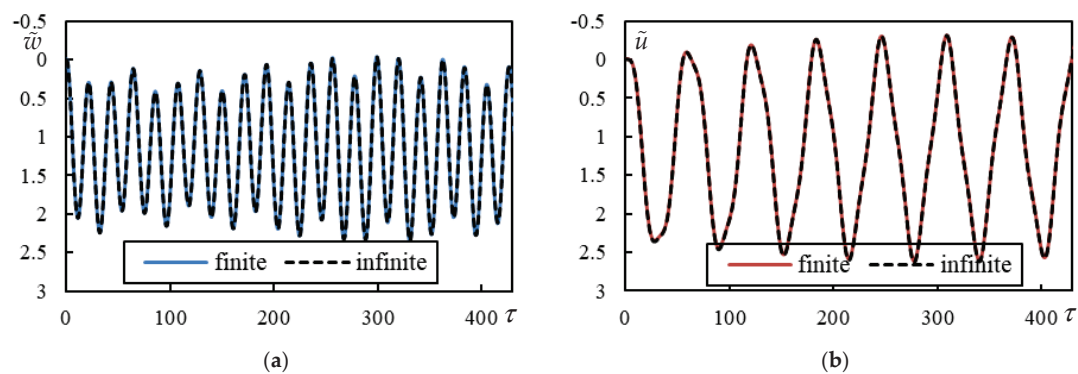


Figure 11. Cont.

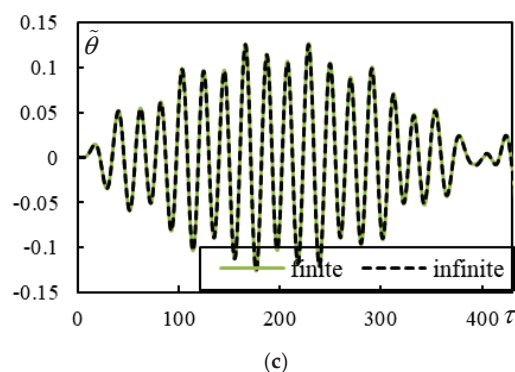


Figure 11. Time series in Case 4. Solid lines represent a finite beam, while dashed lines correspond to an infinite beam calculated using closed-form formulas. (a) Beam deflection at the rear mass; (b) bogie bar mass center displacement; (c) bogie bar mass center rotation.

Cases 2–4 are undamped, therefore, all amplitudes remain unchanged, and the induced frequencies are real (Table 2). These cases also serve to demonstrate that undamped cases can be handled without numerical problems.

Cases 5 and 6 are chosen such that Case 5 is unstable for subcritical velocity, but by adding damping to the primary suspension while keeping other parameters unchanged, it becomes stable (Case 6). Time series of Case 5 are shown in Figure 12. During the initial stages, there is no indication of instability as the rate of instability is very low (Table 2). The unstable frequency surpasses the stable ones only at higher times (Figure 13).

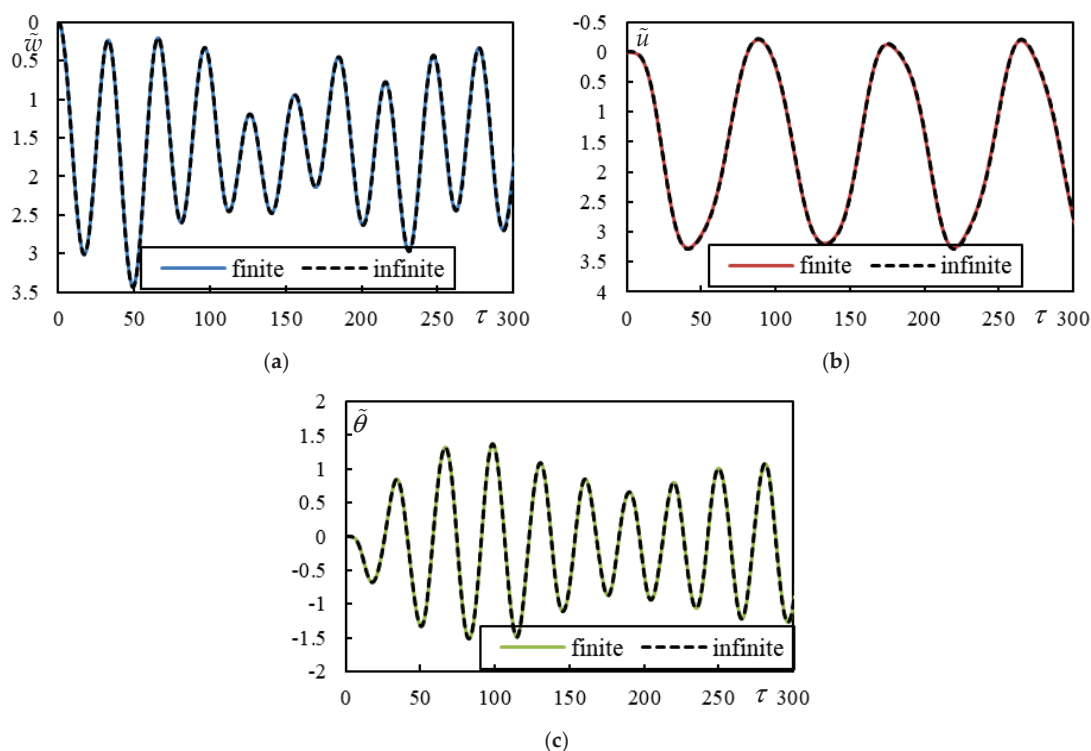


Figure 12. Time series in Case 5. Solid lines represent a finite beam, while dashed lines correspond to an infinite beam calculated using closed-form formulas. (a) Beam deflection at the rear mass; (b) bogie bar mass center displacement; (c) bogie bar mass center rotation.

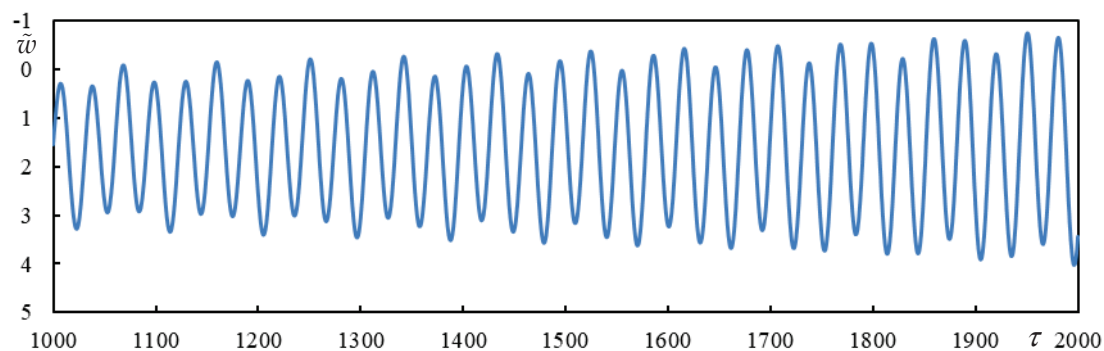


Figure 13. Beam deflection at the rear mass in Case 5.

Case 6 is already stable. Times series are shown in Figure 14 for initial stages and in Figure 15 for higher times.

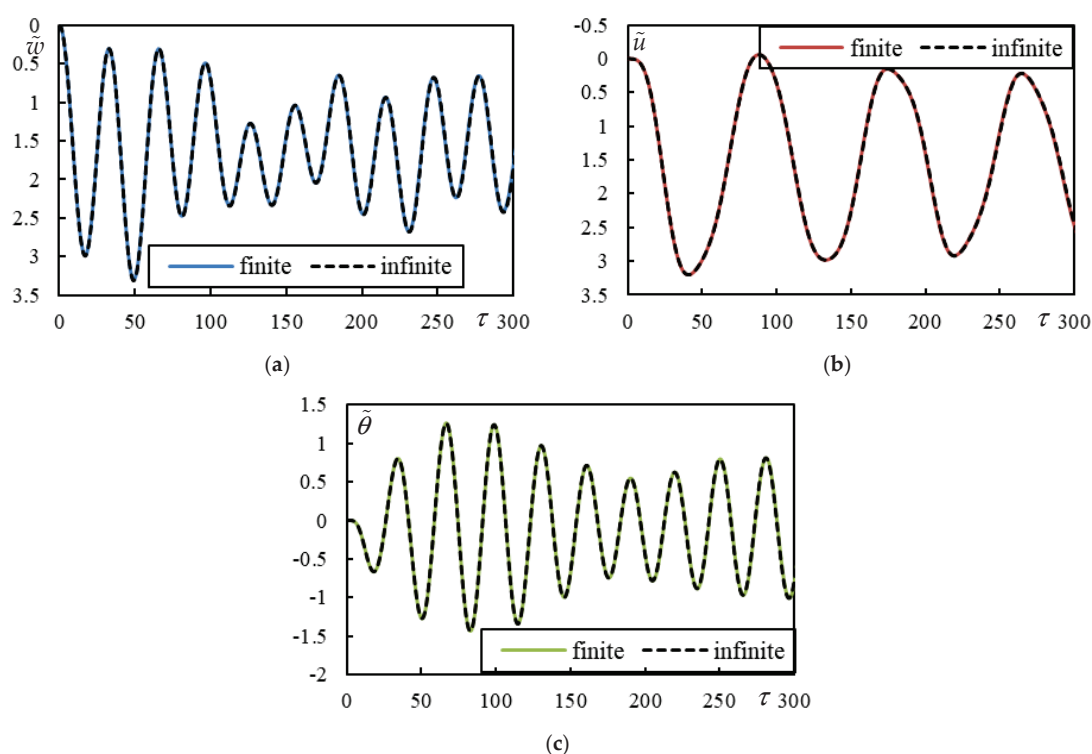


Figure 14. Time series in Case 6. Solid lines represent a finite beam, while dashed lines correspond to an infinite beam calculated using closed-form formulas. (a) Beam deflection at the rear mass; (b) bogie bar mass center displacement; (c) bogie bar mass center rotation.

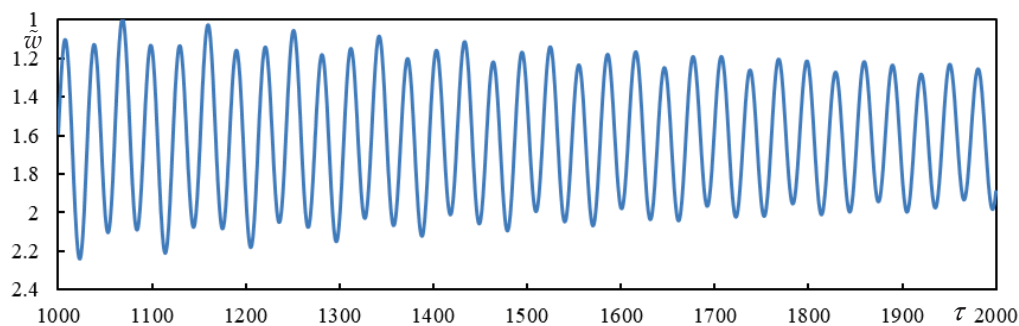


Figure 15. Beam deflection at the rear mass in Case 6.

The other cases from Table 1 are summarized in Appendix A.

3.9. Comparison with Other Published Works

While [68] presents only isolated cases, Ref. [67] offers an evolution over a range of velocities, providing better insight into the problem. However, the reference case in [67] assumes a large distance between wheels (15 m), making it more of a simplified vehicle with two axles than an actual bogie. The evolution of the critical stiffness of the primary suspension is chosen for comparison. The results are shown in Figure 16 using the input data from [67].

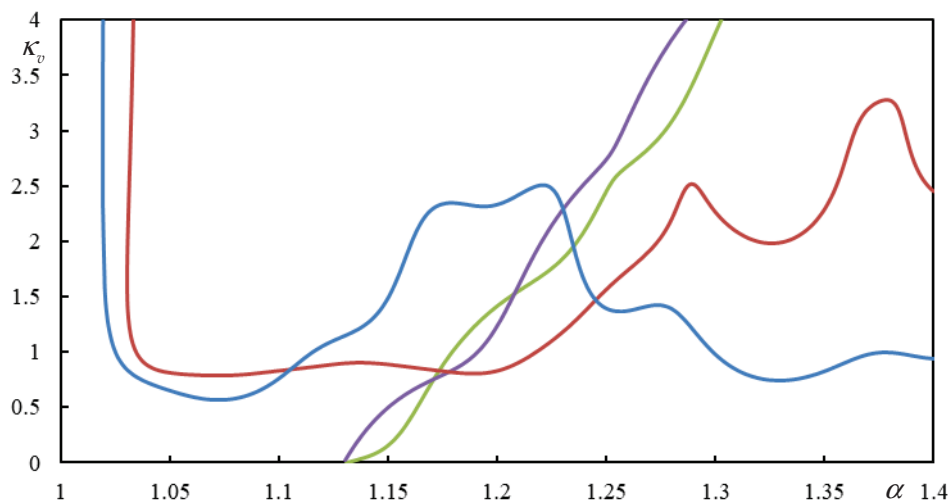


Figure 16. Critical stiffness of the primary suspension using data from [67] (different colors indicate different roots of the characteristic equation).

It can be concluded that there is very good agreement after recalculating to dimensionless data as in [67]. Minor differences can be attributed to improvements in symbolic software, which now allow for higher accuracy and faster evaluation compared to when [67] was written.

In [67–69], the distance between wheels is so large that dynamic interaction is not significant, and as a result, instability in the subcritical velocity range does not occur.

4. Discussion

This paper presents a detailed analysis of the instability of a bogie passing through a one-layer model, focusing on the possibility that this may occur in the subcritical velocity range. All presented results were obtained (semi)analytically, quickly, and with very high accuracy.

The results of two moving proximate masses are used as a reference case, and it is shown that in general, the more realistic the moving object, the lower the risk of instability at such low velocities. This was confirmed even under the conditions of neglected suspension damping, reduced suspension stiffness, and small mass moment of inertia of the bogie bar. On the other hand, it was confirmed that the negative effect of foundation damping found in the two moving mass cases is preserved for the moving bogie. This effect is in contrast to the effect of foundation damping in the case of a single moving inertial object, where increased damping always shifts the onset of instability to higher velocities.

It has been demonstrated in selected examples that instability lines related to moving bogies have more branches than those related to moving masses, which complicates the analysis.

It has been shown in selected examples that suspension damping improves the unstable behavior in all cases, but it is much more efficient with a higher mass moment of inertia of the bogie bar.

As each of the presented examples covers a large range of possible scenarios due to its unidimensional nature, these conclusions might be considered general. Nevertheless, further studies are required as it would be exhaustive to explore the full admissible domain of all involved parameters.

Regarding the distances between the masses, it is not necessarily true that the lowest values indicate worse stability. Larger distances shift the starting point of the first instability branch to lower velocities, but at the same time, the smallest moving mass ratio of such a branch increases. Therefore, it is necessary to balance these two tendencies.

It has been proven that there are cases with realistic parameters where instability occurs in the subcritical velocity range. This article serves as a warning that instability at supercritical velocities cannot be taken for granted a priori. Nevertheless, it is necessary to bear in mind that the model, especially for the supporting structure, is very simple, and further research and validation are required.

Unlike all other works published by other researchers (e.g., [67–69]), where instability is determined by the D-decomposition method, here the so-called lines of instability are traced in the plane of velocity-moving mass ratios by identifying the real-valued induced frequencies. In a damped case, these frequencies necessarily alter the number of unstable induced frequencies by one, so that stable and unstable velocity intervals are clearly visible for any specific moving mass ratio. This approach is much more powerful and simpler than the D-decomposition method.

The presented analysis can be easily extended to other types of supporting structures since all expressions can be used for more complicated foundation models by simply replacing Equation (24) with an equation reflecting the new foundation model. There is only one complication: in other foundation models, the analytical formulas have limited use since discontinuities in the K -function prevent resolving all necessary induced frequencies for all velocities. This is only relevant for the time series—the instability lines are not affected. As an alternative for the time series, either numerical integration or finite models can be used.

5. Conclusions

In this paper, the possibility of instability in the subcritical range of velocities was discussed in detail. The results presented are easy to use, as the dimensionless parameters cover a wide range of possible scenarios, and the instability lines provide an overall view of the onset or aggravation of unstable behavior or the recovery of stable behavior. Such comprehensive information is very useful, as readers do not have to repeat the calculations and can use their dimensionless parameters for the case they are analyzing. However, it would be exhaustive to explore the full admissible domain of all involved parameters. Therefore, it would be incorrect to state that all possible cases are covered by the graphs presented in this paper.

The validation of the results on finite beams is clearly described and easily applicable to other researchers. The author of this paper believes that this is simpler than numerical integration associated with the inverse Laplace transform, which is numerically sensitive.

The unstable induced frequencies compared to the stable ones provide an initial estimate of the degree of instability. Indeed, it can be seen in Table 2 that the unstable frequencies of the selected cases have a relatively low imaginary part (in absolute value). As has been shown, the effect of foundation damping worsens instability. However, during the initial stages, it has a strong impact on the stable frequencies, making the vibrations initially resemble heavily damped cases. The unstable frequency only surpasses the stable ones at higher times. This is important for assessing the severity of instability and can be used in vibration mitigation measures. Since the analysis allows for the determination of

vibration amplitude, together with the degree of instability, it identifies the time required for the unstable harmonics to overcome the stable ones. For this reason, it is important to have analytical formulas that are not affected by numerical approximations during numerical integration so that the time response can be extended as needed.

The main limitation of the results presented in this paper is the simplicity of the supporting structure, modeled as a one-layer system. It is well known that the one-layer model significantly overestimates the critical velocity of the railway track. In light of this, the results should be understood in a qualitative manner, as the ratio of the train's velocity to the critical velocity is important, rather than the absolute values of these quantities. In order to make more general conclusions, it is necessary to extend the model to at least a three-layer system and include nonlinear behavior. The method is easily extendable to other layered models, and the nonlinearity is planned to be incorporated using Adomian decomposition. This will be the subject of further research.

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Appendix A

The aim of this section is to summarize the time series of additional cases listed in Table 1 that are not discussed in the main text. Cases 7–10 demonstrate the effect of damping at different levels. A low damping level of 5% is chosen, and all four combinations are tested by applying damping to the foundation and/or primary suspension. Case 7 has damping at both levels, and its results are presented in Figure A1.

Figure A2 compares all four cases together.

From Figure A2, it can be concluded that, as expected, the foundation damping has a larger influence than the primary suspension damping, and all the curves are approximately in phase. This is also evident from Table 2, where the real parts of the frequencies are very similar, and the noticeable differences are only in the imaginary parts. The dynamic interaction is also very pronounced.

Case 11 complements Case 7 and shows the difference caused by changing the moving masses representing the wheels, from those in Case 7 ($\eta_M = 30$) to those in Case 11 ($\eta_M = 25$). This is illustrated in Figure A3.

It can be observed that the movements are no longer in phase because the frequencies are different: lower for higher moving masses and vice versa.

The last two cases, Case 12 and Case 13, are chosen to demonstrate unstable cases in the subcritical velocity range. However, unlike Case 1 or 5, these cases involve a higher mass moment of inertia of the bogie bar, that is, for $cf_2 = 0.1$. In Case 12, the moving mass is rather academic (Figures A4 and A5), while in Case 13, it is more realistic (Figures A6 and A7). Nevertheless, both cases exhibit a relatively low degree of instability, as can be concluded from Table 2. Therefore, higher times must be shown to confirm the instability, as illustrated in Figures A5 and A7.

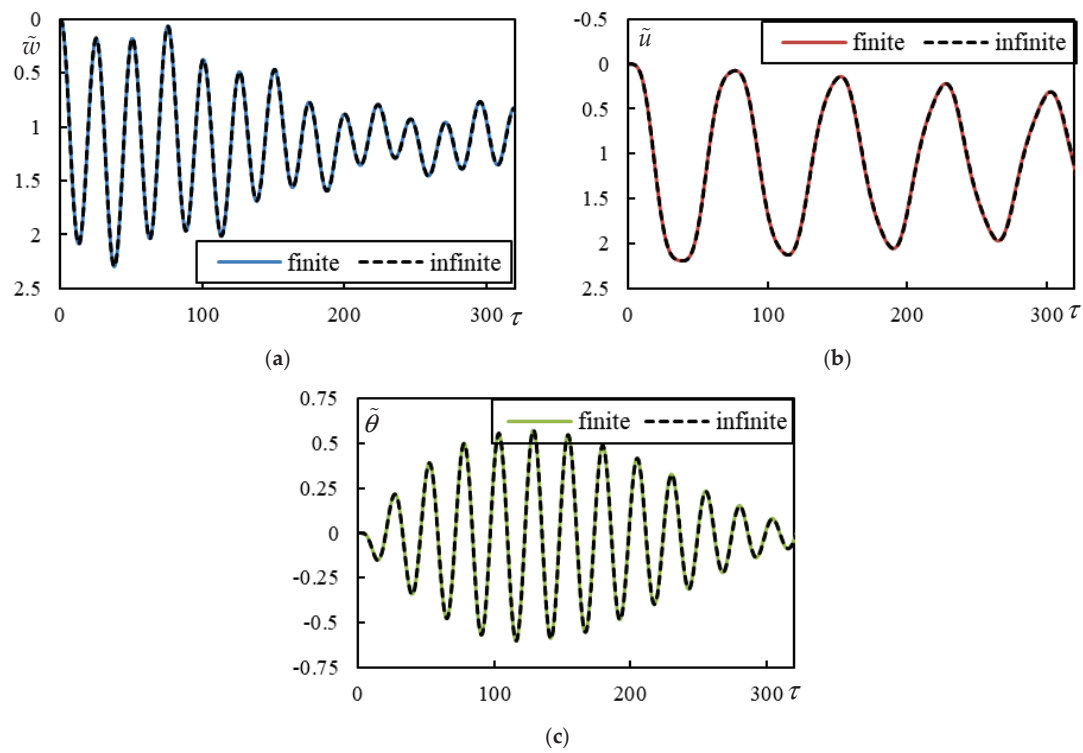


Figure A1. Time series in Case 7. Solid lines represent a finite beam, while dashed lines correspond to an infinite beam calculated using closed-form formulas. (a) Beam deflection at the rear mass; (b) bogie bar mass center displacement; (c) bogie bar mass center rotation.

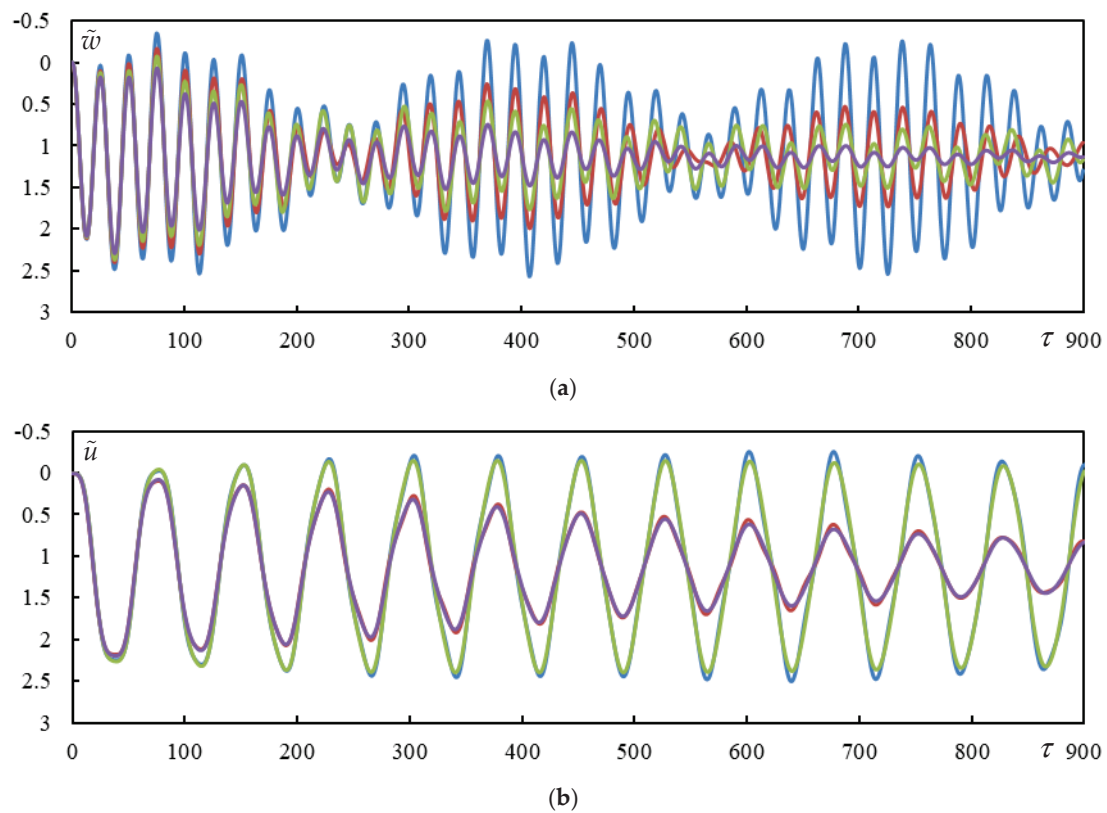


Figure A2. Cont.

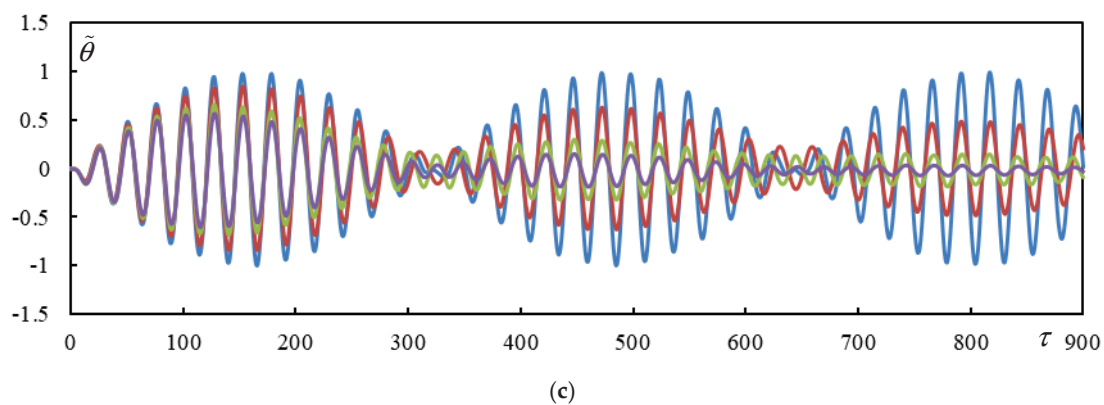


Figure A2. Time series in Cases 7–10. Case 7: violet; Case 8: green; Case 9: red; Case 10: blue. (a) Beam deflection at the rear mass; (b) bogie bar mass center displacement; (c) bogie bar mass center rotation.

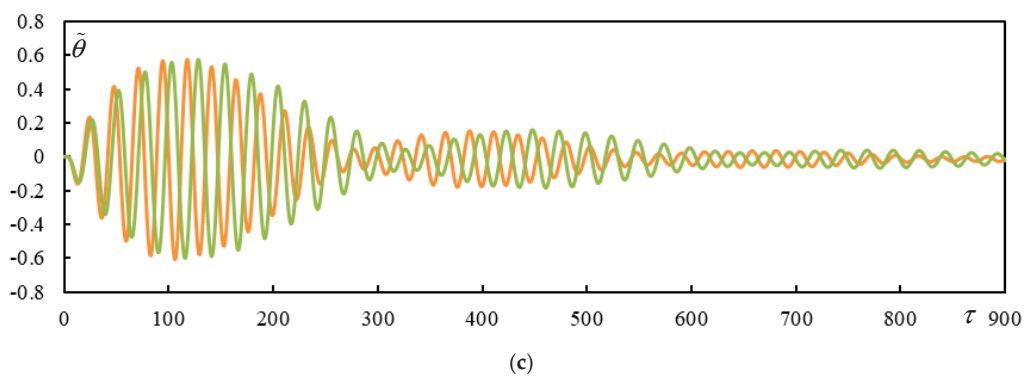
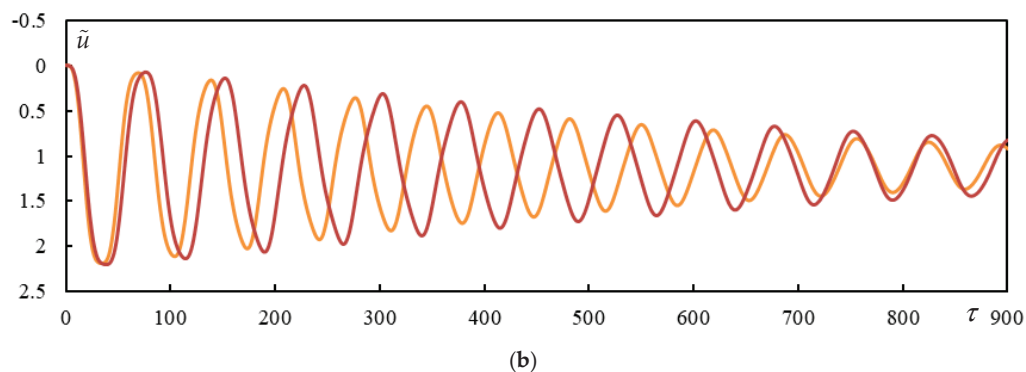
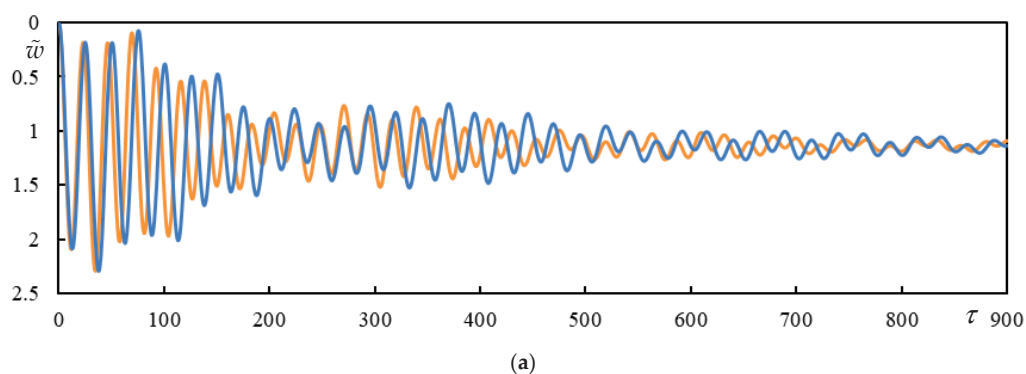


Figure A3. Time series in Cases 7 and 11. Case 11: orange line in all parts of the figure. (a) Beam deflection at the rear mass (Case 7: blue line); (b) bogie bar mass center displacement (Case 7: red line); (c) bogie bar mass center rotation (Case 7: green line).

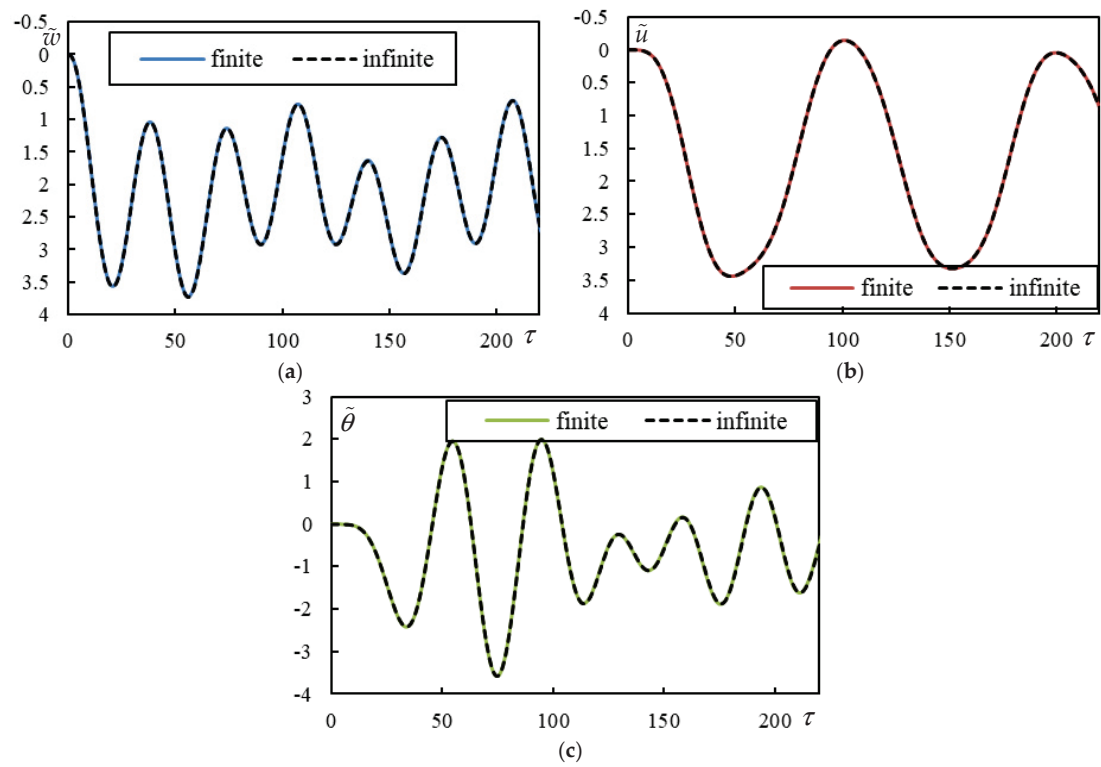


Figure A4. Time series in Case 12. (a) Beam deflection at the rear mass; (b) bogie bar mass center displacement; (c) bogie bar mass center rotation.

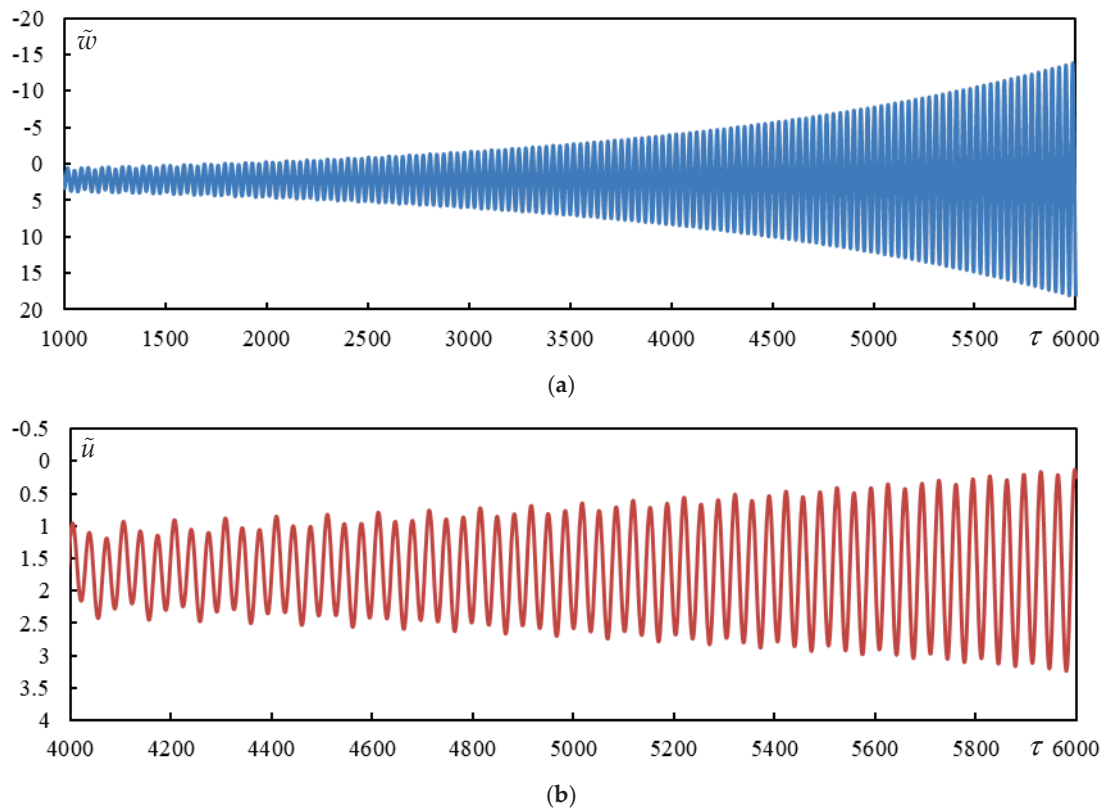


Figure A5. Time series in Case 12. (a) Beam deflection at the rear mass; (b) bogie bar mass center displacement.

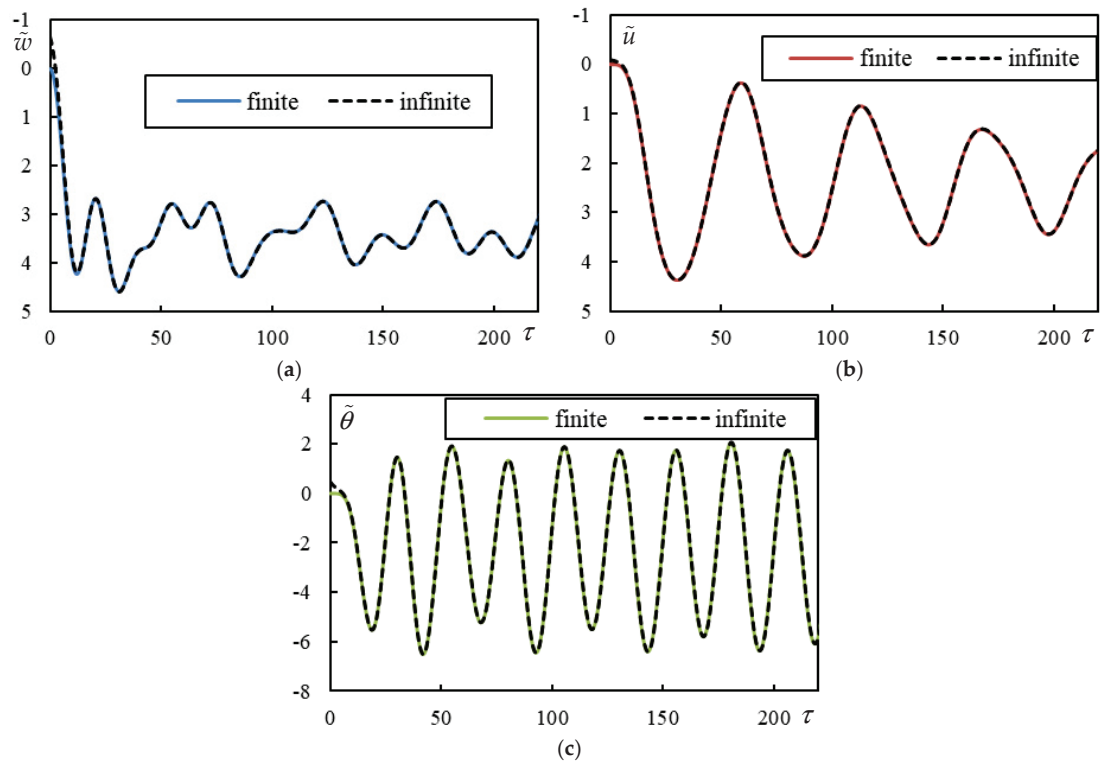


Figure A6. Time series in Case 13. (a) Beam deflection at the rear mass; (b) bogie bar mass center displacement; (c) bogie bar mass center rotation.

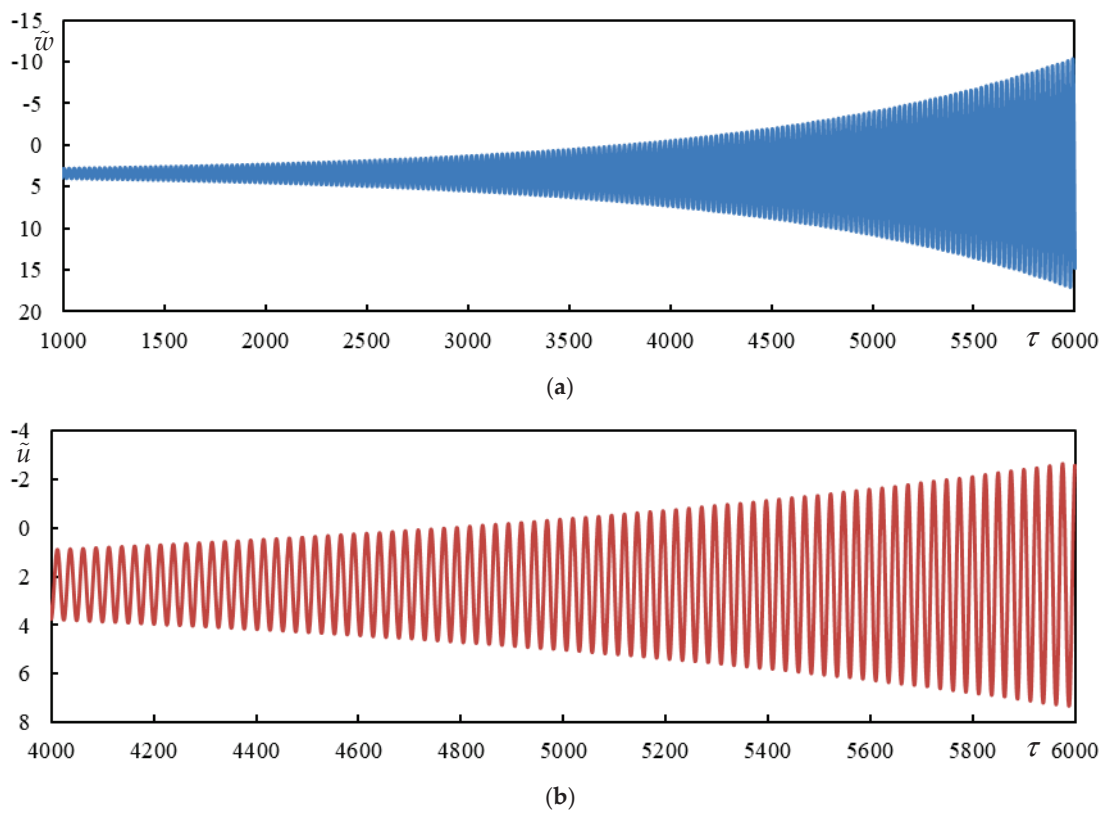


Figure A7. Time series in Case 13. (a) Beam deflection at the rear mass; (b) bogie bar mass center displacement.

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Ground-Borne Vibrations Induced by Railway Traffic: Impact, Prediction, Mitigation and Future Perspectives

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Abstract: Ground-borne vibrations caused by railway traffic represent a significant environmental concern, particularly in densely populated or vibration-sensitive urban areas. These phenomena can lead to discomfort and annoyance among residents, interfere with the operation of sensitive equipment, and even threaten the integrity of heritage sites or structurally vulnerable buildings and infrastructures. Building on these concerns, this paper presents a comprehensive review of the current state of knowledge on the subject. It begins by examining the impacts of ground-borne vibrations on both people and structures, followed by an overview of the regulatory frameworks implemented in different countries to manage these effects, with a focus on four examples from Europe and North America. The review then systematically explores the key factors associated with the generation and propagation of ground-borne noise and vibrations. Furthermore, prediction methodologies are categorised into four groups—analytical and semi-analytical, numerical, empirical and AI-based models—and critically assessed. Finally, the paper reviews mitigation strategies applied at the source, along the propagation path, and at the receiver, assessing their effectiveness in reducing the identified impacts.

Keywords: ground-borne vibration; railway; regulatory framework; mitigation measures; prediction models

1. Introduction

1.1. General Overview: Problem Description

The numerous economic, social, and environmental benefits of urban rail transport make it a widely recognised and highly efficient mode of transportation, particularly in densely populated urban areas. However, the exploration of rail infrastructures is associated with the ground-borne noise and vibrations generated by passing rolling stock, which propagate through the ground and reach surrounding buildings. This transmitted energy can be perceived inside the built environment as structural vibrations and/or re-radiated noise. These phenomena may lead to discomfort among occupants, interfere with the operation of vibration-sensitive equipment, and, under extreme conditions, cause architectural damage.

From a general point of view, the problem of ground-borne vibration generated by railway traffic can be summarised as follows: (i) the movement of the train along the track generates a source of vibration; (ii) this vibration propagates through the surrounding

ground; and (iii) the resulting vibration field reaches the surrounding buildings, inducing structural vibrations and re-radiated noise, along with all the associated drawbacks previously mentioned. A schematic representation of the problem is illustrated in Figure 1.

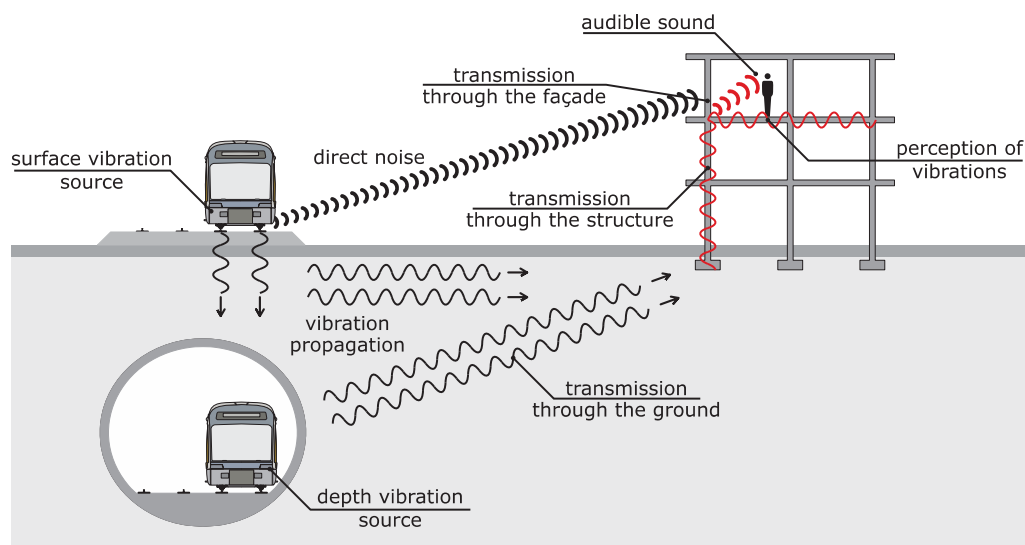


Figure 1. Generation and propagation of ground-borne vibrations.

1.2. Impact on Residents and Structures

Light rail traffic can induce vibration velocity levels higher than 76 dB in the areas surrounding the track [1], as can be seen in Figure 2. This value is higher than the human perception threshold, which is approximately 70 dB (ref. 10^{-8} m/s) [1]. However, when dealing with freight traffic, vibration levels can increase up to 93 dB (ref. 10^{-8} m/s), a high range value when compared to human perception [1].

In terms of long-term effects, vibrations generated by railway traffic generally do not pose a threat to the structural integrity of buildings. They mainly constitute an environmental disturbance, with resident discomfort and the malfunction of sensitive equipment being the primary concerns. Damage to buildings would require extremely high vibration levels, which occur only under unusual operating conditions, as indicated by the typical ground-borne vibration levels in Figure 2.

Although residents often worry about potential structural damage, such fears are generally unfounded, as vibration levels capable of causing damage are one to two orders of magnitude higher than those associated with human perception [2,3]. Therefore, structural damage from railway-induced vibrations is not expected. Nevertheless, historic and heritage structures require special consideration due to their age, materials, and construction techniques, which can make them more sensitive to long-term cumulative vibration effects [4].

As depicted in Figure 2, vibrations caused by railway traffic are usually well below the threshold for structural damage in buildings, making them primarily a source of discomfort rather than a structural concern. However, persistent vibrations may cause material fatigue, potentially leading to the development of cracks and long-term structural weaknesses [5]. Several issues are associated with structural vibrations, including discomfort and disturbance in daily activities, which typically reduce the life quality and work performance. In extreme cases, prolonged exposure can even lead to serious health problems [6,7].

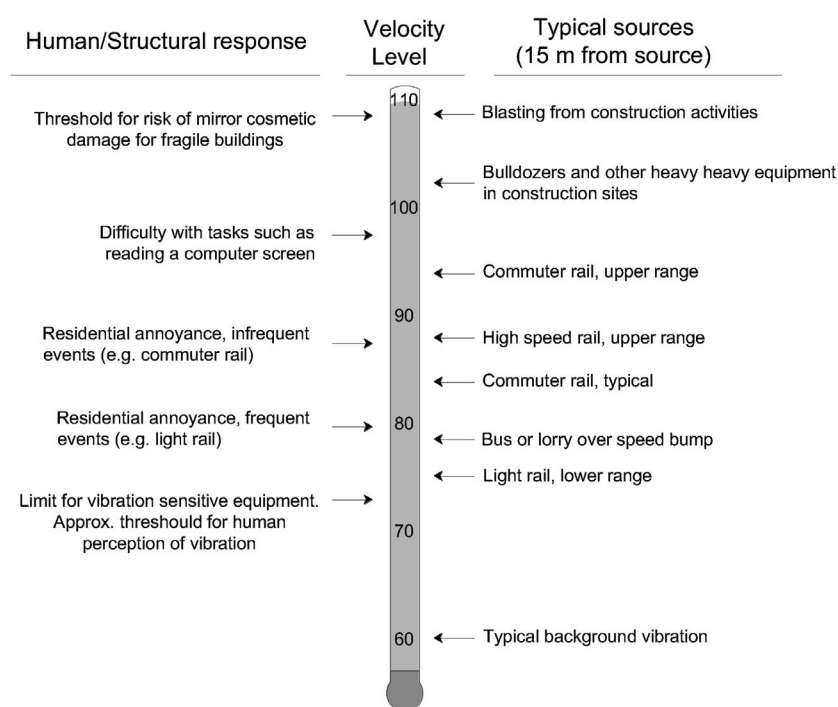


Figure 2. Typical ground-borne vibration levels (adapted from International Union of Railways (UIC) [1]).

The human response to vibrations is influenced by various factors. Some of these are related to the characteristics of the vibration itself, which can be objectively measured, such as amplitude, duration and frequency content. On the other hand, subjective factors (from economic, personal or social nature) lead to a variable perception of vibrations from individual to individual [8]. To overcome this particularity, a statistical approach based on exposed–response curves’ cause–effect can be an effective solution [9,10].

The study of exposed–response relationships involves the consideration and analysis of different levels of discomfort revealed by the building’s users when subjected to prescribed levels of structural vibration, quantifying the percentage of individuals who perceive the vibrations. Recent studies, which have contributed to establishing vibration velocity limit values in international manuals, such as Federal Transit Administration (FTA) [11] and Federal Railroad Administration (FRA) [12], indicated that floors with vibration levels higher than 83 dB (dB ref. 1×10^{-8} m/s) are intolerable from a comfort perspective, particularly for frequent events. Table 1 summarises the human response to different levels of vibrations and ground-borne noise induced by railway traffic. It is important to note that noise values only represent the noise levels directly generated by the vibration of the walls and slabs that delimit dwellings and do not account for other sources of noise such as direct noise induced by traffic.

The information in Table 1 indicates that the impact of vibration and ground-borne noise is frequency dependent, as human response varies accordingly. For example, vibration-induced noise from structural components may cause annoyance even when the vibration itself is imperceptible, whereas low-frequency vibrations can be disturbing even if the associated ground-borne noise is minimal. The values presented in Table 1 refer to residential buildings. Naturally, buildings with different functions have distinct tolerance levels for vibrations. For a frequency range of up to 200 Hz, railway-induced vibrations can interfere with the normal operation of sensitive equipment inside buildings near the infrastructure. These vibrations may damage equipment or hinder their proper functioning. Possible effects include reduced precision and reading speed, or even an inability to take

readings, thus rendering the equipment inoperable. Therefore, when it comes to certain more sensitive equipment, vibrations can make their use impractical [13].

Table 1. Vibration and ground-borne noise levels and associated human responses [11,12].

Vibration Velocity Level (Running RMS) (ref. 10^{-8} m/s)	Ground-Borne Noise Level		Human Response
	Low Freq. * (dBA)	Mid Freq. ** (dBA)	
73.1 VdB	25	40	Approximate threshold of vibration perception for many humans. Low-frequency sound usually inaudible, mid-frequency sound excessive for quiet sleeping areas.
83.1 VdB	35	50	Approximate dividing line between barely perceptible and distinctly perceptible. Many people find train vibration at this level unacceptable. Low-frequency noise acceptable for sleeping areas, mid-frequency noise annoying in most quiet occupied areas.
93.1 VdB	45	60	Vibration acceptable only if there are an infrequent number of events per day. Low-frequency noise unacceptable for sleeping areas, mid-frequency noise unacceptable, even for infrequent events with institutional land uses such as schools and churches.

* Approximate noise level when vibration spectrum peak is near 30 Hz. ** Approximate noise level when vibration spectrum peak is near 60 Hz.

For precision instruments, such as those used in research laboratories, microelectronics, nanotechnology, medical, and biopharmaceutical industries, including electron microscopes, even vibration levels well below the threshold of human perception can pose a serious concern. In Figure 3, the images captured by an electron microscope with a magnification of 150,000 are shown, under normal conditions and when disturbance occurs in its operation due to vibration [13].

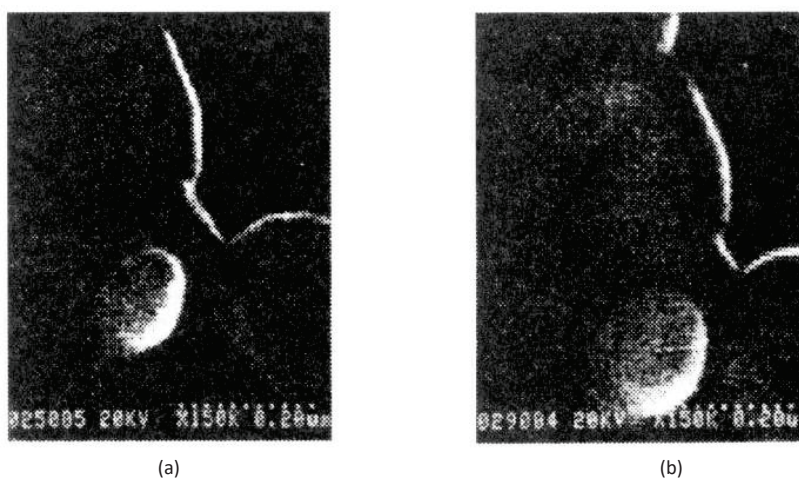


Figure 3. Effect of vibrations on sensitive instruments: (a) normal image; (b) image with disturbance [13].

2. Regulatory Framework

2.1. Generalities

The phenomena associated with continued exposure to ground-borne vibrations induced by railway traffic are addressed within a regulatory framework, which is closely linked to the frequency range in which these vibrations occur. Specifically, building vibrations are perceived by the human body as mechanical vibrations within the frequency range of 1 Hz to 80 Hz, whereas in the range of 16 Hz to 250 Hz, such vibrations may be perceived as sound (ground-borne noise), induced by the vibration of walls, floors, and ceilings [7]. The following sections provide a general overview of the main regulations adopted in different countries. Nevertheless, consultation of the RIVAS project deliverables [14] is recommended in order to access a more exhaustive compilation and description of standards,

regulations, and guidelines. An initial systematisation of the key features of the different regulations is presented later. It is important to highlight that there is a wide variety of descriptors (e.g., maximum running RMS, vibration dose value) based on different physical quantities (such as acceleration or vibration velocity). These differences stem from the historical development of standards, the dominant application areas in each country, and the frequency ranges considered relevant.

2.2. U.S. Regulations

In the United States, FTA manual [11] serves as a comprehensive guideline for transport infrastructure projects. It provides recommendations and analysis procedures for addressing ground-borne noise and vibrations induced by rail traffic that follow the actual state of the art.

The FTA manual classifies the analysis into three levels: a screening procedure, general vibration assessment, and detailed vibration analysis. The screening procedure serves as the initial step to delineate the study area for any subsequent vibration impact evaluation. The general vibration assessment and detailed vibration analysis are used to determine the extent and severity of the impact. In most cases, the general vibration assessment is sufficient. However, if the project is located near vibration-sensitive land uses, and the general assessment suggests that the impact may be significant, it is advisable to conduct a more comprehensive, detailed analysis to accurately characterise the potential effects.

Depending on the level of analysis, different vibration criteria are established. For the general vibration assessment, the FTA manual outlines distinct thresholds for ground-borne noise and vibrations, taking into account factors such as land use and the event frequency (e.g., train passages). These limit values are detailed in Table 2.

Table 2. Impact criteria for general vibration assessment of Indoor Ground-Borne Vibration (GBV) and Ground-Borne Noise (GBN) [11].

Land Use Category	GBV Impact Levels (VdB) ($dB_{ref} = 10^{-8}$ m/s)			GBN Impact Levels (dBA) ($dB_{ref} = 20 \times 10^{-6}$ Pa)		
	Frequent Events ¹	Occasional Events ²	Infrequent Events ³	Frequent Events ¹	Occasional Events ²	Infrequent Events ³
Category I: Buildings where vibration would interfere with interior operations.	73.1 VdB	73.1 VdB	73.1 VdB	N/A	N/A	N/A
Category II: Residences and buildings where people normally sleep.	80.1 VdB	83.1 VdB	88.1 VdB	35 dBA	38 dBA	43 dBA
Category III: Institutional land uses with primarily daytime use.	83.1 VdB	86.1 VdB	91.1 VdB	40 dBA	43 dBA	48 dBA

¹ Frequent Events: more than 70 vibration events of the same kind per day. ² Occasional Events: between 30 and 70 vibration events of the same kind per day. ³ Infrequent Events: fewer than 30 vibration events of the same kind per day.

Ground-borne vibration limits are expressed in terms of Running RMS (Root Mean Square) of the vibration velocity (in dB, ref. 10^{-8} m/s). A one-second averaging period is commonly used to calculate the mean squared amplitude of the vibration signal. Ground-borne noise limits are expressed as A-weighting sound pressure in dB units.

With regard to ground-borne noise, the equivalent pressure level spectra should be assessed against the threshold values defined in Table 2.

2.3. Portugal

In Portugal, there is currently no specific regulatory framework addressing ground-borne noise and vibrations induced by railway traffic. However, the National Laboratory of Civil Engineering (LNEC) provides a set of guidelines aimed at ensuring hu-

man comfort in residential and intellectual environments during the exploration of a railway infrastructure [15].

LNEC distinguishes between underground and at-grade railway projects, proposing different threshold limits and evaluation criteria for ground-borne noise and vibrations. The first set of criteria is summarised in Table 3.

Table 3. Ground-borne noise and vibrations criteria proposed by LNEC [15].

Track Type	Ground-Borne Vibration		Ground-Borne Noise	
	Day	Night	Indirect Criteria	Direct Criteria
Underground	0.28 mm/s	0.28 mm/s	0.025 mm/s	27 dB(A)
At-grade	0.28 mm/s	0.11 mm/s	0.05 mm/s	36 dB(A)

Thresholds for ground-borne vibrations are defined in terms of the average root mean square (RMS) vibration velocity, expressed in millimetres per second (mm/s). These values apply to the vertical or horizontal component of the velocity, whichever is more significant.

- Indirectly, by limiting the RMS vibration velocity levels integrated over 1/3-octave frequency bands with centre frequencies between 16 Hz and 250 Hz, measured at the building's structural slabs.
- Directly, through acoustic measurements, by calculating the equivalent continuous A-weighted sound pressure level (dB(A)) for the same 1/3-octave bands.

The second set of criteria applies to frequency-domain analyses and is valid for both underground and at-grade railway lines. LNEC recommends the use of the base curve from ISO 2631-2: 1989 [8], with multiplicative factors to account for intermittent vibrations in residential buildings. The allowable limits depend on the time of day and the centre frequency of each 1/3-octave band, as detailed below:

- Night period (20:00–08:00):
 - 0.14 mm/s for centre frequencies between 8 Hz and 80 Hz
 - 0.40 mm/s at 2 Hz
 - 0.80 mm/s at 1 Hz
- Daytime period (08:00–20:00):
 - 0.20 mm/s for centre frequencies between 8 Hz and 80 Hz
 - 0.56 mm/s at 2 Hz
 - 1.12 mm/s at 1 Hz

To assess compliance with the various criteria presented, the average value of the 10 most unfavourable vehicle passages, in both directions, shall be determined. For each individual passage, the measurement time must correspond to at least a 10 dB decrease from the maximum level [15].

2.4. Germany

At present, Germany does not have specific regulations for the assessment of railway-induced ground-borne vibrations. Nevertheless, standard DIN 4150-2 [16] is commonly employed to evaluate the effects of vibrations on people within buildings and on structures, respectively.

DIN 4150-2 provides guidelines for assessing vibrations in the frequency range of 1 Hz to 80 Hz that affect occupants of buildings. This standard recommends the use of the running RMS value of the frequency-weighted vibration signal, defined by the following expression [16]:

$$KB_F(t) = \sqrt{\frac{1}{\tau} \int_0^t KB^2(\xi) e^{-\frac{t-\xi}{\tau}} d\xi}, \quad (1)$$

where the symbol $KB_F(t)$ denotes the weighted vibration velocity, calculated using a time constant $\tau = 0.125$ s. It should be noted that the weighted velocity signal $KB(t)$ is derived by frequency (f) weighting (filtering) as:

$$H_{KB}(f) = \frac{1}{\sqrt{1 + \left(\frac{5.6}{f}\right)^2}} \quad (2)$$

Although no unit is explicitly stated in the standards, the associated unit is clearly m/s (more commonly expressed as mm/s). The level of comfort can then be assessed by comparing the maximum value $KB_{F_{\max}}$ with three guideline limits denoted by A_u , A_o , and A_r , which are used both for an overall evaluation and for short-term frequency vibrations, as shown in Table 4. It should be noted that A_r , A_u , and A_o correspond to the lower reference value, upper reference value, and the reference value for rarely occurring impacts, respectively.

Table 4. Reference values “A” for the assessment of vibration emissions in dwellings and similarly used rooms—adapted from [16].

Line	Location of Effect	Day			Night		
		A_u	A_o	A_r	A_u	A_o	A_r
1	Locations of impact, in whose surroundings only commercial facilities and, if applicable, exceptionally residences for owners and managers of the businesses as well as for supervisory and on-call personnel are accommodated.	0.4	6	0.2	0.3	0.6	0.15
2	Locations of impact, in whose surroundings predominantly commercial facilities are accommodated.	0.3	6	0.15	0.2	0.4	0.1
3	Locations of impact, in whose surroundings neither predominantly commercial facilities nor predominantly residences are accommodated.	0.2	5	0.1	0.15	0.3	0.07
4	Locations of impact, in whose surroundings predominantly or exclusively residences are accommodated.	0.15	3	0.07	0.1	0.2	0.05
5	Particularly vulnerable locations of impact, e.g., in hospitals, health clinics, as far as they are located in specially designated special areas.	0.1	3	0.05	0.1	0.15	0.05

2.5. UK

The BS 6472 [17] defines the term vibration dose value (VDV) as a metric that provides a consistent assessment of continuous, intermittent, occasional, and impulsive vibrations, and correlates well with subjective human response. The VDV is given by Equation (3):

$$VDV_{b/d, \text{ day/night}} = \left(\int_0^T a^4(t) dt \right)^{0.25}, \quad (3)$$

where $a(t)$ is the frequency-weighted acceleration, and T is the total period during the day or night over which vibration may occur.

According to BS 6472 [17], the VDV associated with a perception of “uncomfortable” falls within the range of 0.8 to 1.6 $\text{m}\cdot\text{s}^{-1.75}$. The assessment is based on the probability that the experienced vibration dose will provoke adverse comment from those affected, as summarised in Table 5. Below the threshold of 0.2 $\text{m}\cdot\text{s}^{-1.75}$, annoyance is not expected, whereas above 1.6 $\text{m}\cdot\text{s}^{-1.75}$, it is considered very likely that residents will be disturbed by the perceived vibrations.

Table 5. Vibration dose value ranges which might result in various probabilities of adverse comment within residential buildings [17].

Place and Time	Low Probability of Adverse Comment $\text{m}\cdot\text{s}^{-1.75}$	Adverse Comment Possible $\text{m}\cdot\text{s}^{-1.75}$	Adverse Comment Probable $\text{m}\cdot\text{s}^{-1.75}$
Residential buildings 16 h day	0.2 to 0.4	0.4 to 0.8	0.8 to 1.6
Residential buildings 8 h night	0.1 to 0.2	0.2 to 0.4	0.4 to 0.8

2.6. Summary

This section presents a summarised comparison of the standards provided by different countries. As shown in Table 6, key features including frequency range, measured quantity, indicator, and measurement are evaluated across the considered national standards.

Table 6. Standards and guidelines (adapted from RIVAS D1.4 [14]).

Key Feature	USA FTA (2018)	Portugal LNEC (2024)	Germany DIN 4150-2 (1999)	UK BS 6472-1 (2008)
Frequency range	1–80 Hz	1–80 Hz	1–80 Hz	0.5–80 Hz
Measured quantity	Velocity	Velocity	Velocity	Acceleration
Indicator	General assessment: maximum running RMS	Average root mean square (RMS) vibration velocity	Maximum weight vibration velocity	Vibration dose value
Measurement	Near the centre of a floor span where the vibration amplitude is the highest	Vertical or horizontal component of the velocity (whichever is more significant)	Three directions with horizontal x- and y-axes parallel to the walls as much as possible	Highest expected level. Central part of the floor (one or two measurements)

3. Generation and Propagation of Ground-Borne Noise and Vibrations

3.1. Sources of Railway-Induced Ground-Borne Vibration

Ground-borne vibrations are generated through the dynamic interaction between the train and the track. This dynamic action encompasses multiple components, each linked to specific excitation mechanisms, which can be classified into three types: quasi-static, dynamic and parametric excitations [18].

In general terms, quasi-static excitation results from the movement of loads corresponding to the self-weight of the train, distributed over its axles. This type of excitation exhibits a constant magnitude over time, and the dynamic character of the system's response arises from the temporal variation in stress and strain fields at a fixed point in the domain as the vehicle moves. Quasi-static excitation typically occurs at low frequencies (below 20 Hz) and is mainly governed by axle loads and track stiffness.

In contrast to quasi-static excitation, dynamic excitation is caused by the inertial forces generated within the vehicle, as illustrated in Figure 4. This type of excitation can stem from various sources, the most common being geometric or stiffness irregularities in the track, as well as imperfections in the train wheels. Dynamic excitation spans a wide frequency range, from low to high frequencies, with the dominant frequencies depending on the specific source, train speed, train type, among others. For example, it has been suggested that wheel unevenness plays a more significant role than track irregularities in the case of freight traffic [18].

Among the mechanisms of dynamic excitation, parametric excitation is often referenced. It originates from the discrete nature of rail support (except in cases where embedded rail systems are used). This condition gives rise to a dynamic interaction force that can be

distinguished from the other types due to its dependence on a geometric parameter of the track. As an illustrative example, the sleeper-passing frequency can be simply estimated as the ratio $\frac{v}{d}$, where d is the sleeper spacing and v the train speed [18]. Nevertheless, only residual significance is attributed to the latter component in the presence of irregularities in the track and the vehicle [19], which aligns with the classification of excitation mechanisms described by Auersch [20].

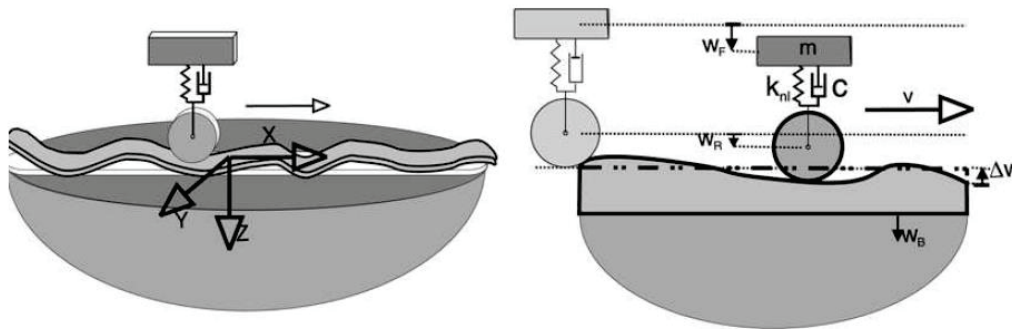


Figure 4. Dynamic excitation mechanisms caused by track irregularities. (v —train speed; Δw —unevenness of the beam surface; K and C stiffness and damping of the SDOF system; W_F , W_R and W_B —displacements).

From the previously outlined concepts, it is evident that the condition of both the train and the track significantly influences the generation of ground-borne vibrations. These conditions affect the characteristics and intensity of the dynamic interactions at the wheel-rail interface. In this context, Table 7 provides a systematic review of the literature, aimed at identifying the main factors related to rolling stock and track components that contribute to vibration generation.

Table 7. Source-related factors influencing vibration levels.

Component	Factor	Influence	References
Rolling stock	Speed	Higher train speeds result in increased vibration levels. Peak ground vibration acceleration in the time domain tends to increase approximately linearly with speed. However, the influence of speed decreases with distance from the track.	Xia et al. [21], Zhai et al. [22], Alexandrou et al. [23], Volberg [24], Yokoyama et al. [25], Mirza et al. [26], Zhai and Cai [27]
	Suspension and vehicle masses	The primary suspension system has the greatest influence on vibration levels. An increase in the unsprung masses leads to a significant rise in vibration levels. At distances close to the track, quasi-static excitations are prevalent and therefore the unsprung mass has little effect. In addition, as distance from the track increases, the frequency content exhibits much larger changes for different unsprung masses. As offset increases, these frequency changes occur over an increasingly broad band of frequencies. This is because the unsprung mass is highly influential in the generation of dynamic excitation, which becomes increasingly dominant with track offset.	Costa et al. [28], Mirza et al. [26], Kouroussis et al. [29], Colaço et al. [30], Zhai and Cai [27]
	Axle and bogie spacing	Vibratory response is amplified at specific frequencies associated with the spacing between axles and bogies.	Kouroussis et al. [29], Mirza et al. [26], Milne et al. [31]
	Wheel conditions	Irregular or out-of-round wheels significantly influence vibration levels by increasing dynamic wheel–rail interaction forces. These effects are particularly pronounced in the medium- to high-frequency range.	Kouroussis et al. [32], Alexandrou et al. [23], Nielsen et al. [33], Mosleh et al. [34]

Table 7. *Cont.*

Component	Factor	Influence	References
Track	Rail irregularities	Rail surface irregularities generate dynamic interaction forces between the wheel and rail. Maintaining good rail conditions helps reduce induced vibration levels.	Xu et al. [35], Sadeghi et al. [36], Grassie [37], Xing et al. [38], Clark et al. [39], Colaço et al. [40], Kouroussis et al. [41]
	Track type	Track type is a key factor influencing vibration levels. Stiffer track systems typically emit higher levels of vibration. The implementation of resilient elements, such as fasteners, mats, and floating slabs, can significantly reduce vibration. These elements function by introducing a new resonance frequency into the system, thereby improving performance at higher frequencies.	Ntotsios et al. [42], Thompson and Jones [43], Costa et al. [28]
	Track location	Railway lines can be constructed in three primary configurations: underground, at ground level, or elevated. Each configuration leads to distinct patterns of vibration generation and transmission:	Lopes [44], Eitzenberger [45], Chen et al. [46], Olivier et al. [47]
	Tunnel track: Vibrations are generated and transmitted directly into the surrounding soil mass.		
	At-grade track: Both airborne noise and ground-borne vibrations are emitted.		
	Elevated track: Airborne noise is produced, and vibrations can be transmitted through the supporting structure, such as a viaduct, before reaching the ground.		

From a modelling perspective, it should be noted that train dynamic loads are still commonly represented using simplified approaches, such as the moving point load [48], lumped parameter model [49], or the vertical train–track coupled dynamic model [50]. However, in recent years, increasing attention has been devoted to the development of time-domain train–track–tunnel–soil interaction models, which aim to comprehensively capture the coupled dynamic behaviour of the train–track and tunnel–soil systems. Such models significantly improve the prediction accuracy of train-induced vibrations within the tunnel–soil system, whether formulated analytically [51], semi-analytically [52–54], or numerically [55–57]. Such advancements in train–track coupled dynamics have facilitated the adoption of more realistic representations of train loading. Nevertheless, considerable potential remains for improving both computational efficiency and the accuracy of train load estimation, particularly in cases involving curved railway alignments.

From the reviewed documents, and particularly regarding the modelling of rolling stock, it is important to emphasise some conclusions reported in Colaço et al. [30]. In many cases, a complete characterisation of the mechanical parameters of a vehicle is not feasible. This limitation arises from several factors, including the lack of data provided by railway operators, the restricted availability of information from manufacturers, and the inherent difficulties associated with experimental testing. The study concluded that the adoption of simplified numerical models, in which only the unsprung masses are considered, is suitable for the prediction of ground-borne vibrations.

From another perspective, track irregularities play a decisive role in vibration levels. Different approaches for defining the power spectral density (PSD) of track irregularities can be found in the literature, as reported by Kouroussis et al. [41]. After analysing some of these approaches, Colaço et al. [40] concluded that the selection of an appropriate PSD is a critical step. Indeed, prospective studies of ground vibrations carried out without specific information on rail irregularities may lead to markedly different results, depending on the PSD function adopted and the assumed track condition. It is therefore essential to remain aware of this limitation in order to support reliable engineering judgments.

3.2. Propagation Through the Soil and Structure

Vibrations generated at the source are transmitted into the surrounding ground, giving rise to the propagation of elastic waves through the soil. In the case of vibrations induced by railway traffic, the strain levels involved are generally low enough for the soil to be modelled as a linear viscoelastic medium (see Table 8). This assumption simplifies the analysis and enables the use of linear wave propagation theories to model vibration transmission.

Table 8. Validity range of constitutive models as a function of the shear strain (adapted from [58]).

Shear Strain	10^{-6}	10^{-5}	10^{-4}	10^{-3}	10^{-2}	10^{-1}
Phenomenon	Wave propagation, vibrations			Cracking, differential settlements	Slippage, compaction, liquefaction	
Mechanical characteristics	Elastic			Elastoplastic	Failure	
Properties	Shear modulus, Poisson's ratio, material damping				Friction angle, cohesion	
Simulation models	Linear (visco) elastic model			Equivalent linear viscoelastic model	Non-linear cyclic model	

The vibrations travel through the propagation medium until they reach a receiver, such as a building. The propagation path is influenced by a variety of factors, with geotechnical properties playing a particularly significant role. The most influential geotechnical parameters include the mechanical properties of the soil, especially stiffness and damping ratio, as well as stratification, depth to bedrock, or the position of the groundwater table. A systematic literature review summarising the key factors influencing ground-borne vibration propagation is presented in Table 9.

Table 9. Factors in the propagation path influencing vibration levels.

Component	Factors	Influence	References
Ground	Mechanical properties	Vibration levels generally increase as soil stiffness decreases. Nevertheless, at larger distances, vibration levels in both softer and stiffer soils tend to converge and become comparable.	Kouroussis et al. [59], Gupta et al. [60], Lopes et al. [61]
	Depth to bedrock	Vibration levels tend to attenuate more rapidly with distance from the track when propagating through soil than through rock. This effect is more pronounced in the presence of a strong stiffness contrast between soil layers.	Kouroussis et al. [59]
	Stratification	Soil stratification can have a substantial impact on vibration levels, as the dynamic properties of individual layers may differ significantly. These variations can result in the amplification of vibrations at certain frequencies, depending on the layering profile.	Kouroussis et al. [59], Thompson et al. [62]
	Water table level	The water table can have a substantial impact on vibration transmission. Vibration levels in saturated soils are lower than in unsaturated soils due to the incompressibility of the saturated medium.	Schevenels et al. [63], Li et al. [64], Bayindir [65], Gupta et al. [60]

The mechanical properties of the track play a crucial role in determining vibration levels. An extensive investigation of this subject was carried out by Kouroussis et al. [59]. As a first approach, and assuming a homogeneous ground, the study shows that vibration levels decrease as the shear modulus and damping increase. When a more realistic case of a layered ground is considered, the authors examined several specific configurations:

- A soil layer overlying a stiffer substratum;
- A layered ground where rigidity increases with depth;
- A layered ground where rigidity decreases with depth.

From these representative scenarios, a number of general conclusions were drawn, which are summarised in Table 10.

Table 10. General conclusions about the trends of ground vibration for different geotechnical scenarios [59].

Scenario	Vibration Level
Shear modulus sensitivity	<i>Homogeneous ground</i> The vibration level diminishes with increasing stiffness.
Soil layer over bedrock	<i>Layered ground</i> An important decrease when the top layer height is relatively small.
Rigidity increasing with depth	A relative decrease with the number of layers.
Rigidity decreasing with depth	A relative increase with the number of layers.

3.3. Dynamic Response of the Receiver

The wide variety of building typologies, coupled with the dynamic interaction between soil and structure, makes the analysis of ground-borne noise and vibrations within buildings a highly challenging task. This analysis involves modelling two distinct media, soil and structure, with distinct mechanical properties and behaviours. When subjected to an incident wave field, the interaction between these two media gives rise to two main effects: kinematic interaction and inertial interaction, as schematically represented in Figure 5.

Kinematic interaction refers to the disturbance of the free-field ground motion caused by the presence of structural elements with stiffness different from that of the surrounding soil. This leads to wave scattering, as rigid foundations are generally unable to follow the ground deformations induced by seismic waves.

In contrast, inertial interaction arises when the structure, subjected to vibrations at its base, generates inertial forces that in turn induce additional deformations in the surrounding soil. This mutual movement between soil and structure characterises the inertial component of the interaction [66].

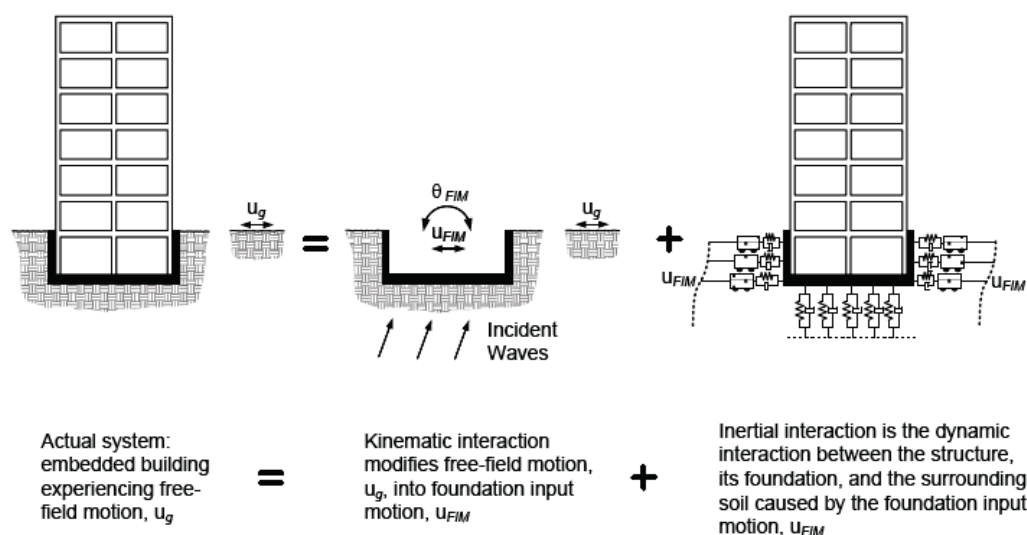


Figure 5. Free-field motion and its relationship to kinematic interaction and inertial interaction [67]. (u_g —free-field motion; u_{FIM} —foundation input motion; θ_{FIM} —foundation rotation).

Generic adjustment factors are often applied to free-field vibration levels to account for soil-structure coupling effects, the attenuation/amplification of vibrations within the building and the conversion of vibration levels into noise levels [11]. However, for more accurate assessments, numerical simulations are typically used to predict vibration responses at specific locations within the structure. Table 11 provides general considerations regarding the key factors that influence the dynamic behaviour of buildings.

Table 11. Receiver characteristics influencing vibration levels.

Component	Factors	Influence	References
Building	Foundation	The greater the mass of the building applied to the foundations, the stronger the dynamic coupling between the soil and the foundations, which leads to a reduction in vibration levels within the building.	Hussein et al. [68], Coulier et al. [66], FTA [11]
	Construction characteristics	The largest displacements in structural elements typically occur at their resonant frequencies. Vibrations associated with the lowest natural modes of slabs often lead to maximum displacements at mid-span. The natural frequencies of slabs depend on their geometry, stiffness, and support conditions.	Auersch [69], Kuo et al. [70], López-Mendoza et al. [71]
	Number of building floors	Vibration levels generally decrease with height, decreasing progressively from the base to the upper floors. However, in buildings with low mass, this attenuation can be minimal or even negligible. In certain cases, vibration levels may actually increase on higher floors due to structural amplification effects.	Quagliata et al. [11], Xia et al. [21], Hanson et al. [12], Kurzweil [72], Auersch [69]
	Acoustic response	Ground-borne noise is typically classified as low-frequency noise, generally occurring below 250 Hz.	Colaço et al. [73,74], Ghangale et al. [75], Nagy et al. [76], Fiala et al. [77,78]

As mentioned in Table 11, the receiver response is strongly influenced by building properties. A key factor in this context is the amplification effect of floors. Studies by Kuo et al. [70] and López-Mendoza et al. [71] provide clear evidence of the significant role of floor amplification in shaping the vibration levels perceived by inhabitants. The dynamic coupling interaction between the soil and the structure must also be considered. Both authors suggested the adoption of a global factor that accounts for slab amplification effects and soil–structure coupling. Since both effects are frequency-dependent, the computation of a transfer function for buildings over the relevant frequency range is essential, highlighting the importance of building properties in the assessment of vibration perception.

4. Prediction Methods

Although experimental vibration tests provide a practical and highly accurate means of assessing vibration levels, there are circumstances in which such methods are impractical. For instance, limited site accessibility, high background noise in urban settings, and complex configurations of tunnels and buildings can pose significant challenges to their implementation [79].

As an alternative, railway-induced noise and vibration can be predicted using a range of prediction approaches. These approaches are generally classified into five main categories: analytical and semi-analytical, empirical models, numerical models, artificial intelligence (AI)-based approaches, and hybrid models. In the following sections, the current state of the art in each of these categories is comprehensively reviewed.

4.1. Analytical Models

To estimate ground vibration levels generated by underground trains, Metrikine et al. [80] developed a two-dimensional (2D) analytical model, treating the tunnel as an infinitely long Euler–Bernoulli beam embedded in a viscoelastic soil layer over a rigid base. While this assumption is reasonable for long tunnels, deviations may occur in cases involving short or curved tunnels due to boundary and geometric effects. Comparative study with 3D models [81] suggests that although 2D models provide results that qualitatively agree with those of 3D models at most frequencies, such simplifications typically lead to significant discrepancies in certain frequencies. Later, this model is refined by introducing a half-space ground beneath the beam and employing a wavelet-based approach to perform the inverse Fourier transform, thereby converting the responses from the wavenumber domain to the spatial domain [82]. Given that 2D analytical embedded beam models cannot fully capture

the complex dynamics of real tunnel structures, there is a clear need for the development of a three-dimensional (3D) underground railway model. Such a model should incorporate the dynamic interactions between the train, the track, the tunnel, and the surrounding soil, while retaining the computational and conceptual advantages of an analytical framework.

Several years later, He et al. [83] developed a general analytical solution for a tunnel embedded within a multi-layered half-space. In their formulation, the tunnel is idealised as an infinitely long, elastic, hollow cylinder, while the surrounding soil is represented as a stratified half-space composed of multiple layers.

4.2. Semi-Analytical Models

One of the most prominent semi-analytical models employing a two-and-a-half-dimensional (2.5D) approach is the Pipe-in-Pipe (PiP) model [84,85]. This model offers a computationally efficient means of analysing underground railway tunnels with circular cross-sections embedded within an unbounded medium. In the PiP model, the surrounding soil is represented as a 3D, homogeneous, isotropic elastic solid, idealised as a thick-walled cylinder. The cylinder has an inner diameter matching that of the tunnel and an outer diameter extending to infinity, as illustrated in Figure 6a.

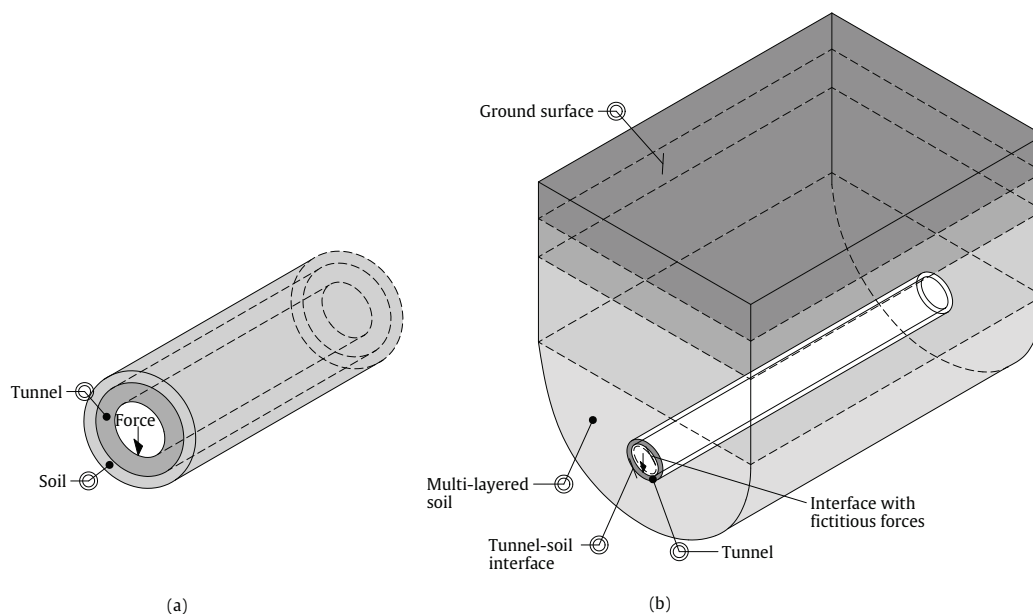


Figure 6. Schematic representation of the PiP model (a) and fictitious force method (b).

An extension of the PiP model to a layered half-space was later introduced by Hussein et al. [86], employing a fictitious force method. This approach provides accurate results when the tunnel is located sufficiently far from the free surface or adjacent layer interfaces. A schematic illustration of this model is presented in Figure 6b. Additionally, it demonstrates a substantial improvement in computational efficiency compared with the coupled FE–BE model; in particular, this method has been observed to be nearly 20 times faster than the coupled FE–BE approach.

Most recently, He et al. proposed a novel 2D semi-analytical method to compute ground vibrations generated by a circular tunnel in both a homogeneous half-space [87] and a layered half-space [88] with an irregular surface. The method enables the simulation of topographies with arbitrary shapes by discretising only the complex, irregular portion of the ground surface. This characteristic enhances the computational efficiency of the approach, making it a robust tool for investigating the propagation of train-induced vibrations across complex ground. Nevertheless, these semi-analytical approaches are limited to very simple tunnel geometries, making them unsuitable for modelling complex tunnel-soil interactions

or irregular configurations. Therefore, numerical alternatives, whether mesh-based or mesh-less, are typically required to handle cases involving complex geometries. Comparisons of semi-analytical models against measurement data demonstrate that these approaches achieve very accurate performance in predicting railway-induced vibrations [89].

4.3. Numerical Models

4.3.1. Mesh-Based Approaches

Various mesh-based numerical modelling strategies have been proposed to simulate train-induced vibrations originating from underground tunnels, either in 2D or 3D [90–94]. A schematic representation of the 2D and 3D finite element method (FEM) meshes used for modelling underground railway tunnels is shown in Figure 7 [92]. In this framework, a modular approach was developed to estimate these vibrations, in which a 3D FE model of the tunnel is first used to derive equivalent Timoshenko beam parameters. To properly simulate the unbounded domain and mitigate spurious wave reflections in the FEM model, viscous boundaries are employed. These parameters are then employed in a 2D side-view beam model of the track, coupled with a mass–spring representation of the train, to calculate the under-sleeper forces [95]. To improve the computational efficiency of full 3D FE simulations, a vehicle–track–tunnel–soil model was later introduced to predict vibrations and stresses induced by wheel–rail roughness in the tunnel–soil system [54]. Another approach utilises a 3D coupled finite element–boundary element model, where the tunnel is modelled using the FEM and the surrounding soil with the boundary element method (BEM), enabling efficient and accurate prediction of ground-borne vibrations from underground railways [81,96]. Both at-grade and underground railway tunnel systems are often assumed to be longitudinally invariant. This characteristic enables the use of 2.5D modelling approaches to assess soil–structure interaction (SSI). A schematic illustration of the 2.5D SSI approach is shown in Figure 8. In recent years, 2.5D modelling has gained increasing attention due to its advantages over fully 3D models, including reduced computational cost and complexity while retaining sufficient accuracy for many practical engineering applications.

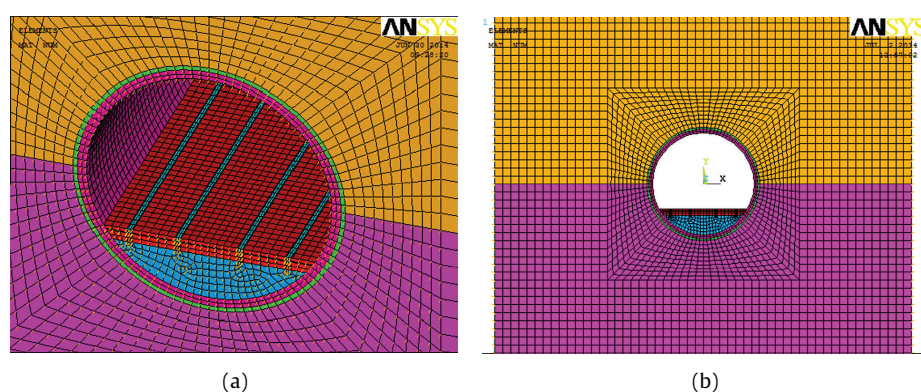


Figure 7. Schematic representation of underground tunnel using 3D FEM (a) and 2D FEM (b) as appearing in Ref. [92] (Figures 6 and 9 in Ref. [92]).

The applicability of 2.5D modelling strategies to SSI problems was first investigated by Hwang et al. [97], who presented the initial work on 2.5D FEM, along with an application of the proposed methodology to soil–structure systems. Later, Yang and Hung [98] developed a 2.5D approach based on finite and infinite elements to model longitudinally invariant unbounded systems subjected to moving loads. In their approach, the infinite elements are used to represent the unbounded domain. Furthermore, a 2.5D FEM approach was introduced by Gavrić to compute the dispersion curves associated with longitudinally invariant structures with thin-walled [99] and solid [100] cross-sections.

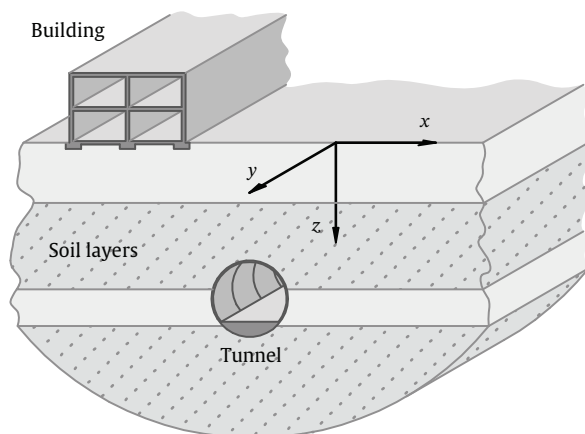


Figure 8. Schematic illustration of underground tunnel–soil interaction in a 2.5D domain.

A well-established method for addressing SSI problems is the coupled FEM-BEM approach, where the FEM models the structure, and the BEM is used to account for the soil medium. Sheng et al. [101,101] proposed a FEM-BEM approach within the 2.5D domain. François et al. [102] also introduced a 2.5D FEM-BEM approach that employs the Green's functions of a layered half-space as fundamental solutions, rather than full-space ones, significantly reducing the number of boundaries to be meshed. The application of this method to predict railway-induced vibrations is explored in [103].

An alternative method for obtaining the Green's functions required in 2.5D BEM in elastodynamics is the thin layer method [104]. Another approach to address SSI problems was proposed by Lopes et al. [105], who studied the vibrations induced in buildings by underground railway traffic using the 2.5D FEM coupled with the Perfectly Matched Layer (PML) technique. In this method, the FEM is used to model the structure, while the PML accounts for the unbounded soil domain. PML is a layer of finite thickness placed beyond the boundary, designed to absorb outgoing waves and prevent reflections at the model boundaries.

In 2.5D models, the structure and surrounding soil are assumed to be invariant in the longitudinal direction. However, due to the physical periodicity of the railway tracks, this assumption does not hold for all frequencies. Specifically, for typical railway tunnel structures, the periodicity of the system does not influence the transfer functions at frequencies below 80 Hz [106]. Alternatively, periodic modelling can be employed to simulate the response of railway systems at higher frequencies. In this regard, Gupta et al. [107] compared the results obtained from a coupled FEM-BEM periodic model of the track-tunnel-soil system with those from the PiP model, highlighting the benefits and drawbacks of both models. Furthermore, Gupta and Degrande [106] evaluated the efficiency of continuous and discontinuous floating slab tracks using a periodic model of the track-tunnel-soil system. When mesh-based numerical methods are compared with experimental measurements, discrepancies of around 3 dB have been reported when using 2D FEM [108], 3D FEM [93], and approximately 3 dB when employing 2.5D FE–BE [109] for the response obtained at the free field.

To overcome the limitations of mesh-based approaches and enhance the computational efficiency of integration-based numerical strategies, meshless methods have been proposed to model wave propagation.

4.3.2. Meshless Approaches

Meshless methods, as an alternative to mesh-based approaches, have gained increasing attention from the research community over the past few decades. In meshless methods,

there is no inherent dependence on a specific mesh topology, which leads to simpler formulations and computational implementation procedures. In recent years, numerous studies have explored novel meshless approaches. The Method of Fundamental Solutions (MFS), Singular Boundary Method (SBM) and Generalised Finite Difference Method (GFDM) are perhaps the most widely used methods in this category. In the following, a general conceptual description of these two methods is provided, along with a review of previous studies in the literature. A summary of alternative meshless methods is also presented thereafter.

The MFS is a meshless method that uses the fundamental solution of the governing equation of interest as the interpolation basis function (Figure 9b). It is particularly effective for solving wave propagation problems in unbounded or partially unbounded domains. This method offers two main advantages over the BEM in terms of numerical efficiency: firstly, it does not require discretising the boundary or performing integration over it, and secondly, the system of equations to solve is typically much smaller than the one required in BEM. The MFS relies on a distribution of collocation points, which evaluate the boundary conditions at discrete locations, and a set of source points (or virtual forces in elastodynamic problems), where the strengths of sources are calculated to satisfy the boundary conditions at the collocation points. In the MFS, the collocation points are placed on the boundary, while the source points are positioned outside the domain.

To extend the capabilities of the MFS, methodologies combining it with mesh-based approaches to address SSI problems have been developed. Godinho et al. [110] presented a 2D FEM-MFS modelling approach for such problems. An extension of this method to the 2.5D domain was proposed by Amado-Mendes et al. [55], where a methodology is introduced that models the structure using 2.5D FEM and the surrounding soil with 2.5D MFS. Godinho et al. [111] also presented a fully meshless method to solve SSI problems in the frequency domain, where the MFS is used to model the soil and the meshless local Petrov–Galerkin method is employed to model the structure. More recently, Liravi et al. [112] proposed a 2.5D FEM-BEM-MFS approach, in which the BEM is used to obtain the soil stiffness matrix for SSI, while the MFS is employed to determine the radiated field in the soil induced by the system. This work also introduces a control methodology to reduce the errors associated with MFS predictions. However, both the FEM-MFS and FEM-BEM-MFS methods face challenges when dealing with complex boundary shapes, due to the difficulties in selecting an appropriate distribution of virtual sources for these geometries.

The SBM is a novel and emerging meshless boundary collocation method for solving boundary value problems that, unlike the MFS, locates the virtual source points on the physical boundary. The SBM inherits key advantages from both the BEM and the MFS. On the one hand, it avoids the computationally expensive integration procedure required in BEM-based methods. On the other hand, the SBM addresses the limitations of the MFS related to the positioning of the virtual source points by placing them on the physical boundary, overlapping the collocation points. While this technique eliminates the challenges associated with the distribution of source points, it introduces singularities in the fundamental solutions on the boundary due to the overlap between collocation points and virtual forces. To overcome these singularities, a regularisation technique is necessary. In the SBM, this is achieved by introducing the concept of the origin intensity factor (OIF) [113].

The SBM has been extensively explored for the prediction of railway-induced noise and vibration in previous studies. The FEM–SBM methodology represents the first attempt to employ SBM in this context, where the FEM is used to model the structural components, and SBM is applied to model wave propagation in the surrounding medium in both 2.5D [114] and 3D [115] formulations. To enhance the accuracy of SBM for highly complex geometries, it was subsequently combined with the MFS, resulting in a novel approach called 2.5D SBM–MFS,

as shown in Figure 9c. In this method, the SBM is employed to handle the complex portions of the geometry, while the MFS is applied to the smoother regions. This hybrid method has proven to be a promising alternative to conventional SBM techniques for simulating both elastic [116,117] and acoustic wave propagation [118,119]. A schematic illustration of the MFS, SBM, and combined SBM–MFS approaches is provided in Figure 9.

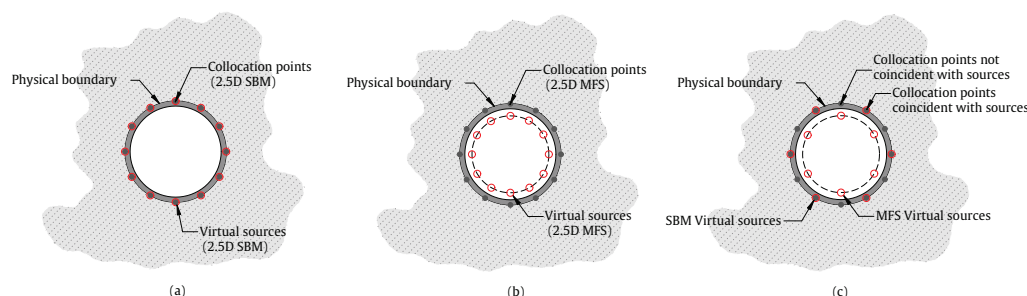


Figure 9. Schematic representation of the SBM (a), MFS (b) and hybrid SBM-MFS (c) approaches.

The GFDM corresponds to an advancement of the classical finite difference method, founded on Taylor series expansions and the moving least squares technique. This meshless approach eliminates the need for numerical integration and is free from the constraints of a predefined mesh. The core idea of the GFDM lies in approximating the derivatives of unknown variables as a linear combination of their values at neighbouring nodes within a local stencil, often referred to as a star. Furthermore, the coefficient matrix arising from the GFDM is typically sparse, which contributes to substantial computational efficiency when compared to the dense matrices commonly generated by boundary-type meshless methods [120]. In evaluating the performance of meshless approaches against experimental measurements, it has been observed that the 2.5D FEM–MFS method is able to predict the vibration velocity amplitude levels at the free field with good accuracy [121].

4.4. Empirical Models

Despite substantial recent progress in the development of numerical models for railway-induced ground vibrations, their practical application remains predominantly within the realm of academic research. In contrast, empirical methods continue to prevail in engineering practice because of their simplicity and low associated engineering costs [122]. When significant uncertainties exist in model parameters, numerical approaches may struggle to yield reliable results. In such cases, empirical methods can offer a practical alternative for estimating the propagation of vibrations and noise from underground railways. Examples of empirical methods include those developed by the FRA and the FTA of the U.S. Department of Transportation [11,12], the Swiss Federal Railways (SBB) [123], the method proposed by Madshus et al. [124] based on measurements conducted in Norway and Sweden, and the approach by Hood et al. [125], which was developed within the framework of the Channel Tunnel Rail Link project in the UK. The procedures established by the FRA and FTA align with the three assessment levels defined in the ISO 14837-1 standard [126]. The Detailed Vibration Assessment, in particular, is based on prediction techniques developed by Bovey [127] and Nelson and Saurenman [128], offering a more comprehensive methodology for predicting ground-borne vibrations and re-radiated noise within buildings. The SBB method differentiates between two models: VIBRA-1 and VIBRA-2. The latter provides a more detailed analysis by incorporating, for example, frequency-dependent attenuation models. Similarly, the empirical approaches in [124,125] follow a comparable structure to that of the SBB method, while also addressing the issue of prediction uncertainty. Nevertheless, it should be noted that empirical models are generally not highly accurate. For example, considerable discrepan-

cies have been reported between measurements and vibration predictions obtained using the FTA model, owing to its limitation of employing a single base curve for the assessment of vibrations in both at-grade railways and underground systems [36].

4.5. AI-Based Models

Although numerical analysis methods can predict vibrations with high accuracy, their drawbacks include complex models, long computation times, and limited practicality. In recent years, there has been growing interest in the application of data-driven and machine learning (ML) approaches for modelling and predicting train-induced ground-borne vibrations. These methods offer promising alternatives to conventional numerical models, particularly in scenarios where field data are available but physical models may be computationally expensive or uncertain due to subsurface variability.

Traditional ML algorithms, such as support vector machine (SVM), artificial neural network (ANN), and random forest (RF), among others [129,130], have been extensively explored for the prediction of ground-borne vibrations. In early studies within this framework, SVMs combined with wavelet transforms were employed to predict train-induced vibrations in multi-storey buildings [131]. Subsequently, ANN models were developed to estimate ground vibration amplitudes caused by urban railways [132] and high-speed trains [133]. More recently, an RF model optimised using Bayesian techniques was proposed to analyse vibration data from elevated high-speed railways [134].

Beyond traditional ML, deep learning techniques have been employed to analyse and predict ground vibrations induced by railway systems. A deep learning-based model has been utilised to identify ground-borne vibrations generated by metro trains, incorporating an uncertainty quantification process to enhance the reliability of the predictions [135]. Furthermore, it is considered a promising approach to train deep neural network (DNN) models using both simulated and measured vibration data via transfer learning (TL) strategies. This enables the model to leverage the strengths of both datasets, thereby improving the accuracy and robustness of vibration predictions [136]. However, it should be noted that, according to previous studies [137], the train–test split ratio has a significant influence on the classification performance of the three pre-trained networks. In most cases, ratios of 80–20% and 90–10% yield better results. Moreover, track-based measurements can be used to infer ground response, where deep learning techniques based on rail acceleration signals are applied to estimate track vibration characteristics [138].

In recent years, physics-informed neural networks (PINNs) have transformed the way that partial differential equations are solved by embedding the underlying physical laws directly into the training process of neural networks. This approach enables the model to learn solutions that are consistent with the physics of the problem, even in the absence of extensive training data. PINNs have demonstrated strong robustness to noise in the data, achieving reasonable identification accuracy even with noise corruption levels of up to 10% [139]. Moreover, PINNs offer the advantage of strong generalisation in data-scarce regimes [140]. In the context of elastodynamics, PINNs have proven particularly effective for modelling elastic wave propagation, offering mesh-free formulations that bypass the complexities of conventional numerical discretisation. By working in the frequency domain, this method captures wave behaviour with high accuracy while significantly reducing computational overhead compared to traditional PINNs [141]. Moreover, advancements in PINN frameworks have introduced capabilities to simulate wave fields in complex media without relying on labelled data, addressing one of the major challenges in data-scarce environments [142].

The uncertainty quantification of ML-based models has also gained attention. Liang et al. [143] integrated Bayesian neural networks with deep learning to assess un-

certainties in vibration predictions. Wu et al. [144] used probabilistic methods to analyse metro-induced vibrations, offering insights into reliability analysis under varying operational conditions. It is worth noting that the uncertainty of prediction models can be further reduced by incorporating field data for the training and validation of ML models [145,146].

Surrogate models have been employed to support vibration prediction, particularly in the contexts of optimisation and uncertainty analysis. Two recent examples involve the integration of a Bayesian optimisation framework with surrogate modelling for underground railway tunnel design [147] and acoustic barriers [148], illustrating the synergy between data-driven prediction and design optimisation. In this approach, RF regression is used as the surrogate model, and the overall framework is illustrated in Figure 10 [147]. Assessing the performance of the RF model in predicting railway-induced vibrations, in comparison with measurement data, demonstrates its capability to provide accurate and instantaneous predictions [134]. Another application is the development of efficient scoping models to aid planners during the early design stages, providing rapid vibration assessments across varying soil conditions near high-speed rail lines [149,150].

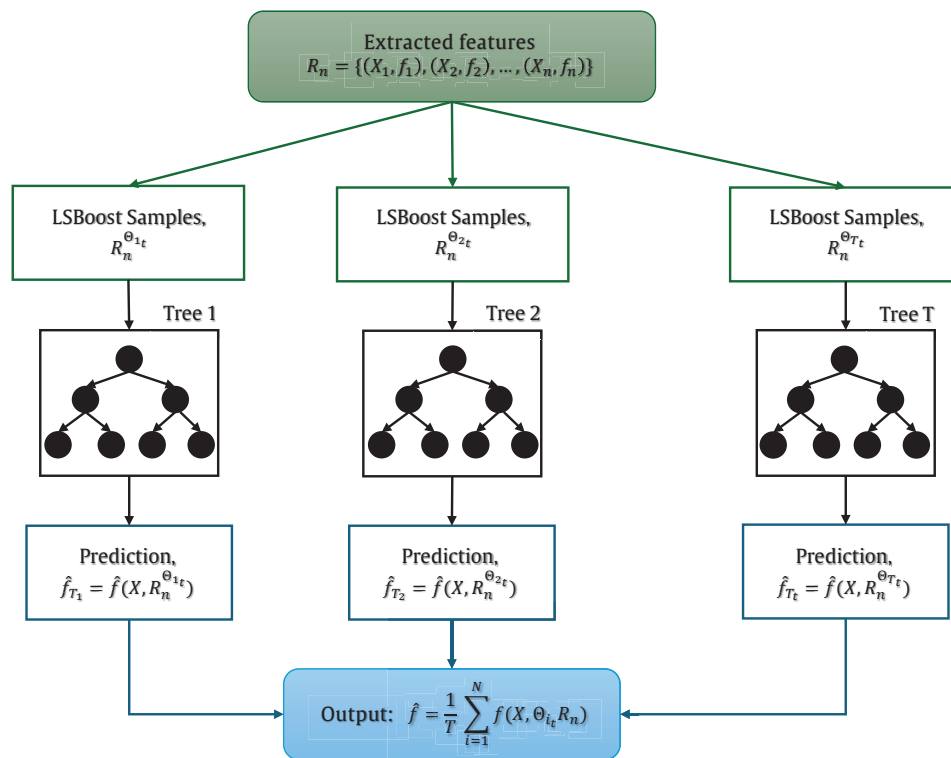


Figure 10. Illustration of Random forest regression construction as appearing in [147] (Figure 4 in [147]). Here, Θ denotes the random variables governing the probabilistic mechanism, T is the finite number of trees, and $(R_n^{\Theta_{1t}}, \dots, R_n^{\Theta_{Tt}})$ represent the bootstrap samples generated by the least-squares algorithm.

The literature reflects a significant transition from conventional physics-based simulations to hybrid and data-driven approaches. While physics-informed and deep learning models offer flexibility and predictive potentiality, challenges remain in model interpretability, generalisation to unseen scenarios, and integration with field data.

4.6. Hybrid Models

The uncertainty in railway-induced ground-borne vibration predictions in buildings, primarily due to limited knowledge of local subsoil conditions, has been highlighted in numerous studies. This challenge can be addressed by integrating field data with numerical models, thereby leveraging the strengths of both approaches [122,151–154]. A variety of

hybrid modelling techniques have been developed to predict ground-borne vibrations in buildings resulting from railway activity, particularly in the presence of SSI. One such approach integrates pre-calculated results from detailed numerical simulations, an experimental database compiled from multiple field measurements, and specific analytical models [155]. Another method represents buildings using finite axial rod elements, with floor impedance derived from infinite thin plate theory, while the incident wave field is characterised by either known column base forces or measured vibrations at key interface locations [156]. In addition, a virtual moving source can be utilised to simulate building vibrations through back-analysis procedures, significantly reducing the influence of uncertain parameters on prediction accuracy. This technique combines the versatility of numerical simulations with the realism of field measurements [157]. Another notable hybrid framework integrates recorded data with numerical predictions, following the source–path–receiver decoupling strategy as defined by the FTA. This method effectively separates the vibration generation, propagation, and reception mechanisms for improved analysis and interpretation [70]. When such hybrid models are compared with measurement data, they are generally found to be accurate, with discrepancies between the numerical model and experimental results typically reported to be below 4 dB [158,159].

To improve computational efficiency, a modal superposition-based model has been introduced to estimate railway-induced ground-borne vibration levels in buildings, particularly under scenarios where the incident wave field is known, either through numerical simulations or ground surface measurements. This approach incorporates SSI effects by introducing spring-damper elements at the building foundation [160]. An extension of this method includes a more rapid model employing a 3D time-domain FEM for structural modelling, alongside enhanced spring-damper representations to better capture SSI effects [71]. Additionally, the use of meshless methods as the numerical component of hybrid models offers a promising alternative that not only reduces uncertainties in the predictions but also improves computational efficiency [121,122,158,159], as illustrated in Figure 11. For the meshless approach, the continuity of displacement and the equilibrium of forces at the interface are employed to couple the FEM and MFS approaches. The soil–foundation numerical model is experimentally validated by comparing the numerical transfer functions with the corresponding experimental measurements [159].

While previous hybrid models typically combine semi-analytical or numerical simulations with field measurements, the computational cost of these approaches often remains high. However, real-time prediction capabilities are essential in certain scenarios, particularly when environmental vibrations may impact sensitive structures. To address this, recent developments have focused on hybrid strategies that enable real-time prediction while simultaneously reducing the uncertainty associated with vibration modelling by using a deep learning technique [161] or multi-point transfer ratio of vibration response [162].

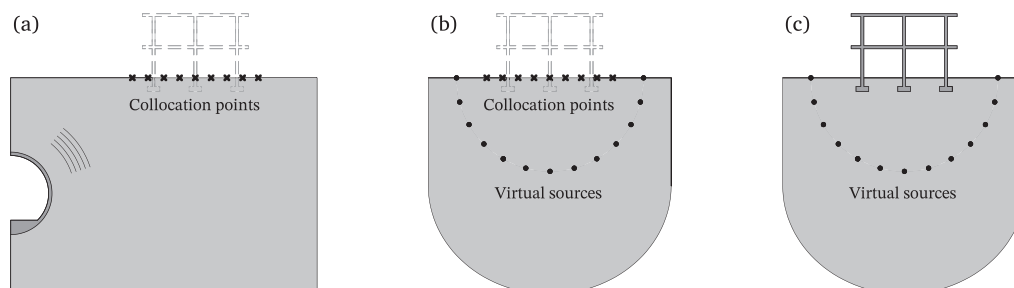


Figure 11. Schematic representation of tunnel-soil (a), soil (b) and building-soil (c) models considered for the hybrid method application as appearing in Ref. [122] (Figure 4 in Ref. [122]).

4.7. Overview of Prediction Methods

In this section, the advantages and disadvantages of the previously reviewed prediction methods are summarised from a general perspective in Table 12.

Table 12. Comparison of modelling approaches for railway-induced vibrations.

Method	Advantages	Disadvantages	References
Empirical models	Simple and fast to apply; low computational cost; useful for scoping studies and regulatory assessments.	Limited accuracy for complex geometries and heterogeneous soils; strongly dependent on available measurement data; poor extrapolation beyond calibrated cases.	[43,163]
Semi-analytical/analytical models	Provide physical insight into wave propagation and soil–structure interaction; lower computational cost than full numerical models; suitable for parametric studies.	Require simplifying assumptions (e.g., homogeneous soil, linearity, infinite/regular geometry); accuracy decreases in complex or strongly heterogeneous conditions.	[41,164]
Numerical models	High fidelity; capable of handling complex geometries, layered soils, and non-linear behaviour; can model both frequency- and time-domain responses.	Computationally expensive; require detailed material and soil data; sensitive to boundary conditions and meshing strategies.	[165–167]
AI/Data-driven models	Once trained, allow very fast predictions; can approximate highly non-linear behaviour; enable rapid scenario exploration and optimisation; useful as surrogates for numerical simulations.	Require large and representative training datasets; generalisation outside training domain may be poor; reduced interpretability compared to physics-based models.	[77,168,169]

5. Mitigation Measures

5.1. Overview

Based on a solid understanding of the phenomena and the modelling approaches previously discussed, it becomes possible to identify effective solutions for their mitigation, particularly with regard to structural vibrations and the associated noise. In problems of this nature, mitigation strategies can be categorised according to the location at which they are implemented. Accordingly, GBN and GBV can be controlled at three distinct levels: at the source, along the transmission path or at the receiver.

Despite considerable progress in the study of mitigation measures, a holistic approach of the entire system, accounting for both GBV and GBN, remains essential. Only through such a holistic perspective, the effectiveness of mitigation strategies can be properly assessed. It is important to note that, in some cases, certain solutions may reduce the problem within specific frequency ranges while significantly amplifying them in others. Moreover, combining multiple mitigation measures across different levels presents an additional challenge that can be addressed through an integrated approach.

5.2. Mitigation Measures at Source

For the purpose of the present section, the vibration source domain comprises both the vehicle and the track. The interaction between these components constitutes a highly complex dynamic system. A comprehensive analysis of this system is essential to gain a deeper understanding of the mechanisms responsible for vibration generation at the source. Such insight is crucial for the design and development of more effective and targeted mitigation strategies. Table 13 summarises a set of vibration mitigation measures applied at the source level.

Table 13. Mitigation measures of ground-bone vibrations applied at the source.

Location	Component	Measures	References
Vehicle	Wheelsets	Good maintenance practices.	Nielsen et al. [170,171], Satis et al. [172], Ju et al. [173], Suarez et al. [174], Kouroussis et al. [168,175], Wilson et al. [176], Colaço et al. [30], RIVAS [33]
	Unsprung masses	Improvement of suspension systems and reduction in unsprung masses.	
	Operating speed	Decrease in vehicle speed.	
	Resilient wheels	Replacement of conventional wheels with resilient wheels.	
	Dynamic Vibration Absorbers (DVA)	Design and installation of DVAs on the vehicle bogie.	
Track	Rail	Proper maintenance or replacement of the rail. Use of embedded rail systems. Installation of rail dampers.	Marshall et al. [177], Lakušić et al. [178], Bentn et al. [179], Hanson et al. [12], Satis et al. [172], Bongini et al. [180], Nelson et al. [181], Grootenhuis et al. [182], Hemsworth et al. [183], Wang et al. [184], Costa et al. [185], Dahlberg et al. [186], Ferro et al. [187], Mateus et al. [188], Shamayleh et al. [189], Auersch [190], Wang et al. [191], He et al. [192]
	Rail fastening system	Use of fasteners with resilient components to prevent direct contact between rail and sleeper.	
	Sleepers and ballast	Use of composite or wooden sleepers, installation of resilient elements beneath sleepers, installation of ballast mats.	
	Track type	Increase in ballast layer thickness, installation of floating slab tracks, increase in tunnel depth.	
	Dynamic Vibration Absorbers (DVA)	Design and placement of DVAs at specific rail defect locations.	

When addressing mitigation measures at the source, a common solution, particularly in urban environments, is the use of a resilient mat, either beneath the slab in slab track systems or beneath the ballast in ballasted tracks. As indicated by Costa et al. [185], the efficiency of these elements depends strongly on their stiffness and exhibits a clear frequency-dependent behaviour. Moreover, the inclusion of mats influences not only the vibrations transmitted to the ground but also the dynamic behaviour of the track itself and, consequently, the train–track interaction loads. Since most studies on the efficiency and design of such measures rely on receptance and mobility concepts, without accounting for changes in the overall compliance of the train–track system, the reported findings are of substantial practical relevance.

5.3. Mitigation Measures at Propagation Path

In situations where it is not feasible to implement mitigation measures directly at the source to improve its condition and reduce vibration generation, alternative strategies can be applied along the propagation path. The primary objective of these measures is to change the transmission characteristics of elastic waves as they travel through the ground.

Interventions along the propagation path are typically implemented as close as possible to the vibration source and are, therefore, most commonly applicable to at-grade railway infrastructures. These measures work as barriers to wave propagation, placed between the source and the receiver, with the aim of reducing or redirecting the transmitted elastic waves. Table 14 presents some relevant mitigation strategies that can be implemented along the propagation path.

Table 14. Vibration mitigation measures applied at the propagation path.

Component	Measures	References
Embankment and platform	Increasing the height of the embankment or reinforcing the foundation soil of the platform aims to create a wave-guiding effect, thereby conditioning wave propagation through the medium.	Lakušić et al. [178], Olivier et al. [47], Connolly et al. [193], Lyratzakis et al. [194,195]
Barrier	The presence of obstacles along the propagation path provides scattering and/or damping effects on incident waves. This effect can be achieved by introducing materials or systems specifically designed for this purpose into the ground, leveraging their mass, stiffness or geometric configuration. Examples of such mitigation measures include trenches and barriers, large mass elements, soil stiffness enhancement techniques, and, more recently, metamaterials.	Çelebi et al. [196], Thompson et al. [197], Yang et al. [198], Jayawardana et al. [199], Guo et al. [200], Ahmad and Al-Hussaini [201], E Richart [202], Beskos et al. [203], Barbosa et al. [204], Hassoun et al. [205], François et al. [206], Coulier et al. [207–209], Kattis et al. [210], Takemiya et al. [211], Krylov et al. [212], Dijkmans et al. [213,214], Adam et al. [215], Ma et al. [216], Barbosa et al. [204], Jones [217], Kouroussis et al. [218], Castanheira-Pinto et al. [219], Albino et al. [220]
Soil	Enhancing the mechanical properties of the soil has a direct impact on vibration propagation. Increasing soil stiffness can significantly reduce vibrations in the corresponding shadow zone. The effectiveness of this measure depends on the stiffness contrast between the natural soil and the improved zone, as well as the thickness of the treated layer.	Takemiya [211], Coulier et al. [207], Thompson et al. [62], Tang et al. [221], Barbosa et al. [204]

As evidenced in Table 14, a wide range of solutions exist for mitigating vibrations along the propagation path. A common approach is the construction of a trench parallel to the railway line, aimed at protecting the structures located behind it. A current topic of investigation concerns the effectiveness of filling the trench, as well as the influence of its depth and position relative to the track [197,204]. Although open trenches generally provide the best results across different scenarios, their practical applicability is limited. Conversely, the stratification and stiffness of the soil in a given context have a significant influence, which restricts the generalisation of conclusions regarding trench depth and track distance. Therefore, an appropriate design should be prioritised for each specific case.

5.4. Mitigation Measures at Receiver

The receiver is defined in this study as the part of the system where vibrations generated by railway traffic are perceived. Traditionally, this issue was addressed primarily in buildings with high sensitivity to vibrations, such as hospitals, research facilities or concert halls. In such cases, mitigation measures were integrated into the construction or rehabilitation process to ensure compliance with service performance requirements.

However, in recent years there has been growing awareness of the adverse effects of prolonged and repeated exposure to vibration, alongside with regulatory requirements from licensing authorities for buildings located near railway lines. Simultaneously, the increasing availability of a wide range of technological solutions has led to the broader adoption of vibration mitigation measures in a more diverse set of buildings. These interventions not only ensure high service standards but also help to preserve or even to enhance property value, even when located in close proximity to vibration sources such as railways. Table 15 summarises a range of mitigation measures applicable at the receiver level to reduce ground-borne vibrations and noise.

Table 15. Vibration mitigation measures at the receiver.

Location	Component	Measures	References
Building	Foundations	Thickening of foundation slabs; local improvement of the foundation soil; vibration isolation of the superstructure using springs or elastomeric supports.	Fiala et al. [78,222,223], Lopez et al. [224], Talbot et al. [225], Ulgen et al. [226], Yang et al. [227], Persson et al. [228], Statis et al. [172], Wilson et al. [229], Soares et al. [159], Reynolds et al. [230]
	Slabs	Reduction in the flexibility of the building slabs; active mass damping of floors.	
	Room	Vibration isolation of a room or part of the structure using a box-in-box system.	

When mitigation measures focus on protecting the building itself, only a limited number of reliable solutions are available. Among them, base isolation can be highlighted as an effective approach [159], although it requires appropriate design and specialised expertise. From a numerical perspective, Talbot et al. [225] provide important guidelines, stressing that: (i) the building and its foundation must be considered as an integrated system; and (ii) the effectiveness of isolation is highly dependent on the choice of isolation frequency, which can significantly influence performance depending on the characteristics of the excitation.

5.5. Summary

Mitigation measures can be classified into three main categories: at the source, along the transmission path, and at the receiver. For source-based strategies, the most common solutions are applied at the track level (with the possible exception of controlling operating speed, since the definition of specific vehicle characteristics is rarely feasible). These measures are generally effective in reducing vibration levels and can be implemented along the entire track or in selected sections. However, their application in operating lines is often challenging, as they typically require significant interventions in the infrastructure. According to the literature, vibration reductions from source-based strategies can range widely from 2 to 40 dB, depending on the particular technique employed [231].

Measures applied along the transmission path (e.g., trenches, barriers, etc.) can attenuate vibrations effectively within specific frequency ranges, but their performance is highly site-dependent and often limited by construction feasibility. Such mitigation measures can result in vibration reductions in the range of 3–14 dB [231].

Mitigation measures at the receiver (e.g., building isolation) directly address vibration perception. However, they are usually costly and intrusive, which restricts their use to particularly sensitive buildings. It should be noted that improvements at the receiver location can achieve vibration reductions of between 2 and 26 dB, largely depending on the technique employed [231].

Overall, each of these approaches presents both advantages and limitations, highlighting the need for integrated and optimised mitigation strategies tailored to the characteristics of each site.

6. Perspectives and Future Research

In light of recent studies in the field of railway-induced noise and vibrations, Table 16 presents potential topics that warrant further investigation. It should be noted that the future perspective of the field is derived from the limitations of the most recently published literature and the ways in which these shortcomings can be addressed.

Table 16. Emerging research trends in railway-induced noise and vibration.

Category	Research Direction	Details
Human health and perception	Combined effects of noise and vibration Epidemiological realism	Investigate cardiovascular, mental health effects, annoyance, and sleep disturbance under joint exposure to vibration and noise. Explore realistic noise-vibration exposure scenarios.
Material and structural modelling	Advanced material modelling Realistic construction detail integration Rod-sprung-mass model refinement	Incorporate non-linear behaviour and fatigue in ballast, sub-ballast, and building materials under cyclic loading. Include connection details, material heterogeneity, and SSI in simulations. Improve high-frequency accuracy and include spatial complexity, high-order floor modes, coupling effects, and non-structural elements.
Vibration mitigation and control	Vibration control systems Tuned Rail Dampers (TRDs) Meta-wedge and novel barriers Composite vibration isolation walls	Develop optimised building designs with passive/active mitigation (e.g., tuned mass dampers, active mass dampers, base isolators, viscoelastic layers). Overcome TRD limitations by developing long-term performance data and design guidelines. Refine idealised metamaterial-based countermeasures for practical and realistic implementation. Test alternative materials, ratios, and layering, especially for non-uniform soil conditions.
Modelling and simulation	Non-linear SSI Numerical uncertainty quantification Combination of traditional prediction models with ML Digital Twin integration	Extend existing linear models to non-linear dynamics for extreme events (e.g., earthquakes, explosions). Incorporate track curvature, speed, bogie distances, and observation geometry in stochastic frameworks; support with field data. Integration of the classical prediction models with AI to leverage the benefits provided by each one. Use Digital Twin technologies for visualisation and modelling of noise and vibration propagation.
Experimental methods and monitoring	Measurement point design Mode identification techniques Full structural characterisation	Develop practical guidelines for optimal sensor placement in buildings; investigate impact on monitoring outcomes. Validate predictive modes through ambient vibration tests; extend beyond vertical global modes. Broaden vertical modal identification to capture entire building response.
Geotechnical and site-specific effects	Heterogeneous ground conditions Structural-foundation-soil systems	Address limitations of current models based on stratified, uniform soil layers by including lateral geological variability. Investigate effects of structure size, foundation type, and soil conditions to inform load modelling modifications.

7. Conclusions

Ground-borne noise and vibrations caused by railway traffic continue to pose significant environmental and societal challenges, particularly in urban areas and locations with vibration-sensitive equipment. This review has highlighted the main aspects to be considered in a railway project when addressing these phenomena. The paper has been structured as a step-by-step guide, resembling a manual for practitioners and researchers in the field. The key topics and insights can be summarised as follows:

- **Regulatory frameworks:** Although many countries have established regulations, significant disparities remain in assessment indicators, threshold values, and applicability, limiting the prospects for a harmonised international approach.
- **Generation, propagation, and reception:** The fundamental mechanisms of how vibrations and noise are generated, transmitted through the ground, and perceived at the receiver have been discussed. The general tendencies identified for each influencing factor are highly valuable as a first step in the analysis of specific cases.
- **Prediction methodologies:** Prediction models, although increasingly sophisticated, still face limitations in accounting for complex site-specific variables. As a result, the uncertainty of predictions remains relatively high.
- **Mitigation measures:** A variety of solutions exist, but their effectiveness depends strongly on local conditions and system characteristics. While some general guidelines can be identified, site-specific designs should be prioritised.

Several key areas of development are expected to drive future progress in the management of railway-induced ground-borne noise and vibrations. These include advances in the lifecycle assessment of noise and vibration emissions from both railway infrastructure and rolling stock; the development of new methodologies for long-term monitoring that

enable the incorporation of real data into prediction models; and the creation of more effective and adaptable mitigation measures, along with their integration into predictive tools. Achieving these goals will require stronger collaboration between engineers, researchers, infrastructure managers, and policymakers.

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