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Active Methodologies for the Promotion of Mathematical Learning

Edited by
Francisco D. Fernández-Martín, José-María Romero-Rodríguez,
Gerardo Gómez-García and Magdalena Ramos Navas-Parejo

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Contents

About the Editors

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Special Issue in Education Sciences: Educational Research and Innovation in the First Global Catastrophe of the 21st Century: Committed to Education

Special Issue in Sustainability: Education for Sustainable Development: Commitments and Challenges——Selected Papers from CIEI 2021

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Magdalena Ramos Navas-Parejo

Interests: active methodologies; information and communication technologies (ICT) in education; reading promotion for educational inclusion

Special Issues, Collections and Topics in MDPI journals

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Special Issue in Mathematics: Emerging Technologies in Learning of Mathematics Education

Preface to "Active Methodologies for the Promotion of Mathematical Learning"

For years now, particularly with the rise of information and communication technologies (ICT) in education, new teaching methods and innovations have been emerging in classrooms at all stages of education. This has led teachers to rethink the educational model and teaching tools in order to respond to the way students learn in the 21st century.

Currently, the new methodological currents tend to want to abandon the master class to give way to the type of active and autonomous learning methodologies that are on the rise which provide a better response to the educational demands of today. The suitability of the method will depend on the educational context in which it is applied, so teachers must be familiar with the different types of methodologies and be able to discern the most appropriate one, such that it fits the needs of their classrooms and allows for the fulfilment of the objectives set. The teaching–learning process requires scenarios that promote meaningful learning.

In this sense, research in educational innovation seeks to improve certain aspects and is inclined towards the inclusion of active or emerging methodologies, which achieve greater involvement of students in their learning. These are educational practices in which students play a greater role, learning through experimentation and interaction with their peers. The role of the teacher becomes that of a learning guide, so that the planning part of teaching becomes more relevant than the expository part. To this must be added the need to use ICT in a coherent way.

It can be deduced that active methodologies involve a set of methods and strategies in which the students are the central axis of the didactic process, achieving their active participation, which generates significant and effective learning.

Francisco D. Fernández-Martín, José-María Romero-Rodríguez, Gerardo Gómez-García, and Magdalena Ramos Navas-Parejo
Editors

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Article

E-Learning in the Teaching of Mathematics: An Educational Experience in Adult High School

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Abstract: Currently, the e-learning method, due to the period of confinement that is occurring due to COVID-19, has increased its use and application in the teaching and learning processes. The main objective of this research is to identify the effectiveness of the e-learning method in the teaching of mathematics with adults who are in high school, in contrast to the traditional expository method. The study developed is quantitative, descriptive and correlational. The research design is quasi-experimental, with a control group and an experimental group. The results show that the use of the e-learning method has a positive influence on motivation, autonomy, participation, mathematical concepts, results and grades. It can be concluded that the e-learning method leads to improvement in adult students who are studying the mathematical subject in the educational stage of high school, provided that it is compared with the expository method. Therefore, this method is considered effective for its implementation in adults.

Keywords: emerging methodology; educational innovation; e-learning; educational experimentation; adults; students

1. Introduction

Technological development is a reality today [1]. This fact is reflected in our society [2], specifically in the labour, social and educational fields [3]. This technological advance facilitates, strengthens and speeds up the performance of daily tasks [4].

In the educational field, technological progress is reflected in the development of the so-called information and communication technologies (ICT) [5]. ICTs directly influence the development of teaching and learning processes [6], since they promote innovative pedagogical actions, as well as generate new learning spaces [7]. These pedagogical events enhance the transformation of the classroom as we know it [8], since they allow for the elimination of spatial-temporal barriers [9], as well as access to a large amount of information [10], with different formats [11]. It has also promoted the improvement of students' motivation, autonomy, involvement and attitude towards educational content [12–14].

Among the pedagogical actions based on ICTs is e-learning, which is defined as the pedagogical act that takes place online, thanks to the use of the Internet and technological devices, whether mobile or not, with synchronous or asynchronous connection, and from anywhere [15]. Therefore, the e-learning method becomes a pedagogical tool that facilitates access to learning for the whole of society [16].

The method of not is of recent creation [17], since its beginnings date back to 1993, when it began to be used more assiduously, having a greater impact in the field of education [18]. Prior to that date, distance learning was widely used.

This method of teaching is currently on the rise due to COVID-19 [19]. Its flexibility in terms of location, time, effort and costs [20], makes it the most appropriate option for training and evaluating students [21].

It should be borne in mind that two types of resources are required to develop the e-learning method: digital and technological [22]. Among the digital resources are educational videos, teaching platforms, videoconferences, podcasts, social networks, among many other resources [23]. While technological resources can be the desktop computer, tablet, smartphone, among others [24].

The use of e-learning by the members involved in the teaching and learning process becomes a challenge [17], because an average level of digital competence is required to apply it with guarantees [25]. Therefore, teachers and students need to be trained in the use of the various technological and digital resources [26].

This teaching method has a number of characteristics that make it different from other teaching methods [27]. Some authors see it as an evolution of distance education [28,29]. For others, it is a new teaching modality that differs substantially from face-to-face teaching [30].

Be that as it may, e-learning has a number of characteristics, among which are promoting dialogue and group activities, enhancing students' interpersonal relations [31]; encouraging collaboration among students themselves, achieving joint goals in the elaboration of different tasks [29]; facilitating communication, both synchronous and asynchronous [32]; enabling learning to take place from any location, provided that a technological device is available [33] to encourage the acquisition of digital competence in students [34]; enable adaptation to the individual pace of students [35]; enhance motivation, as the student can develop his or her own learning style [36]; promote the acquisition of learning to learn competence [37]; be adapted to the circumstances of each individual, both personal and occupational [38]; provide access to an unlimited amount of learning resources [39]; facilitate teacher monitoring of student activity [40]; and promote student familiarisation with the use of technological and digital resources [41].

It should also be noted that the e-learning method is a special case of distance learning [42]. There are several reasons for this [43]. In distance learning, email is used to receive the contents of the subject, not having a virtual medium [44]. In addition, a large number of theoretical contents are presented, which are not interactive and whose sequencing is closed [45]. Additionally, contact with the teacher is sporadic, which acts as a mere transmitter of content. In this case, the student is a passive receiver, who usually has a feeling of loneliness [46].

In other words, the teaching-learning process can take place 24 hours per day, every day of the year [47], allowing students to be trained while they are on the move or in a place other than their usual one [48], promoting a change in the teacher-learner mentality, and with it the philosophy of learning, in which the student organizes his or her training process and the teacher guides that action [49], and allowing unlimited access to network resources [50]. Therefore, the use of e-learning totally changes the perspective we had of teaching until now [51].

However, the e-learning method can generate a spatial and temporal gap [52], so it is necessary to personalize the educational experience of the students, trying to keep the learners motivated and committed [53]. Moreover, in developing countries, the use of ICT is not as widespread as in developed countries, leading to a lack of acceptance of technological resources and, therefore, of e-learning, not having the desired effect on educational learning [54,55].

Mathematics in the field of social sciences is considered a necessary instrument to be able to decipher the closest environment and represent various facts, be they social, scientific and technical that occur in today's world [56]. Mathematics facilitate the understanding of various phenomena, be it social reality itself, economic aspects or historical facts, among others [57]. In this case, mathematics becomes an adequate tool to acquire knowledge, reflect on social aspects, and represent facts from the environment [58]. In other words, mathematics tries to convert all these facts into knowledge and information [59]. In addition, the language used in the mathematical field allows the phenomena that occur to be explained in detail and precisely [60].

It should be borne in mind that mathematics is instrumental, and is the basis for acquiring knowledge from other subjects, or in other fields, such as sociology or political science [61].

In addition, mathematics develops the student's intellect, promoting competencies that will allow him to function personally and socially [62]. It also promotes creativity, the development of autonomy, the improvement of self-esteem and entrepreneurship [63].

In the field of mathematics, there are educational actions in which e-learning has been developed as a teaching method [64,65]. One of the ideas is that applied in the MCIEC model (motivation, context, interactivity, evaluation and connectivity), which entails greater student involvement. This model allows the student to increase his or her ability to make an effort to understand mathematical content, thanks to increased interest, motivation and adaptation to the context [64]. The development of the e-learning method presents improvements if it is applied with an appropriate teaching and learning method. An example of this is the development of the e-learning method associated with the GeoGebra resource, which is integrated into the Moodle platform, improving aspects related to assessment, motivation and student interest. It also promotes learning in a more meaningful way and adapts assessment to students' needs [65]. Another similar case is that of the Working Memory Capacity (WMC) method, developed in the e-learning method. This method leads to an improvement in students' abilities to acquire various mathematical concepts. In this case, it improves students' academic performance. This is due to the increase of their involvement and motivation in mathematical contents [66]. Another case is the development of the e-learning method, associated with the Edmodo application, in the field of mathematics. This training process increases participation in learning. This involvement increases the memorization, comprehension, application, analysis, evaluation and creation of mathematical contents. It also increases students' attitude and acceptance of mathematical content [67]. The use of e-learning in the development of mathematics increases the commitment of students themselves, improving performance. It also increases interest, and thus, acquired results. It also improves the acquisition of mathematical content [68]. Another example is pedagogical action, in which e-learning is used with the individualized e-learning environment called UZWEBMAT. This combination promotes individualized attention of students. Moreover, it is adapted to the learning style of the students, improving their comprehension skills. It also increases their responsibility for learning and is reflected in motivation and academic performance [69]. In many cases, student learning, and therefore student outcomes, can be affected by poor connectivity, inflexible scheduling, and inadequate devices [70].

2. Justification and Research Objectives

The use of ICTs today, coupled with the global crisis being experienced by COVID-19, makes e-learning a necessary teaching method. This implies the application of new didactic strategies and pedagogical approaches [71].

This study presents a teaching method based on e-learning for adult students who study high school in the distance mode. In addition, it shows the pedagogical actions developed during the first quarter of the 2019–2020 school year. A contrast is also established with the traditional expository method developed with the students of the night school. All of this was done in the subject of mathematics applied to the social sciences.

The aim of this research is to give continuity to the application of the e-learning method in the teaching of mathematics, with the intention of contrasting the results obtained in other studies with similar characteristics [63–70].

The main objective of this research is to identify the effectiveness of the e-learning method in teaching mathematics to adults who are in high school, in contrast to the traditional expository method. The following specific objectives are established from this objective:

- Determine the degree of motivation;
- Identify the degree of autonomy;
- Analyse the level of collaboration;

- To know the degree of participation;
- To find out the level of problem solving;
- Determine the degree of class time;
- Identify the level of learning of concepts, graphs, scientific data and results;
- To know the capacity of decision in the pedagogical actions; and
- To find out the variation of grades.

3. Method of Investigation

3.1. Research Design and Data Analysis

The study developed is quantitative, descriptive and correlational [72]. The research design is quasi-experimental, with a control group (GC) and an experimental group (Ge), that is, non-equivalent groups. In this case, the research process developed in other previous studies has been followed, where active teaching methods have been applied [73,74]. Unlike the investigations mentioned above, this study tries to know how an active teaching method influences, in this case, the e-learning method in the development of the development of the subject of mathematics. For this, a contrast is established with the exhibition method. The students are divided into two groups: the control group, made up of night school freshmen; and the experimental group, made up of distance school freshmen. In both groups the subject of mathematics applied to social sciences has been developed. In the control group the traditional expository method has been applied. In the experimental group the e-learning teaching method has been developed. The distribution of the students has not been random, because the groups have been formed by the head of studies, according to the registration requested by the students. The criteria for the distribution of the student body is based on the principles of equity and equality. In other words, the management team distributed the groups bearing in mind several criteria, including the length of time the students have been out of official studies and the grades of the last year enrolled. With respect to years of non-study, it established three criteria: (a) more than 10 years not enrolled in official studies; (b) between 10 and 5 years not enrolled in official studies; (c) less than five years not enrolled in official studies. With regard to the qualification, it established four criteria: (a) no subjects passed in the last year enrolled; (b) between 0 and 3 subjects passed; (c) between 4 and 6 subjects passed; (d) all subjects passed. Based on these criteria, it made an even distribution. These criteria are set out in the School Education Project. The information was collected at the end of the first quarter, that is, after the pedagogical intervention, through the application of a post-test (Table 1).

Table 1. Composition of the groups.

Group	*n*	Composition	Pretest	Treatment	Posttest
1- Control	61	Natural	-	X_1	O_1
2- Experimental	71	Natural	-	X_2	O_2

The Statistical Package for the Social Sciences (SPSS) v25 (IBM Corp., Armonk, NY, USA) was used to analyse the data collected. The statistics used are mean (M) and standard deviation, in addition to skewness (S_{kw}) and kurtosis (K_{me}) statistics. Additionally, Student's t-test ($t_{n1+n2-2}$) has been used to compare the means between the established groups. Finally, Cohen's d-test and the biserial correlation (r_{xy}) have been applied, in order to know the effect size and the out-of-association. The significance level applied in the study was $p < 0.05$.

3.2. Participants

The sample applied in this research consists of 132 students. The sampling technique applied is for convenience. This is due to the ease of access to the students. In studies focused on the application of pedagogical methods, the sample size is not a determining factor [75,76].

The students are studying the first year of the adult baccalaureate, specifically the humanities and social sciences, at an adult education centre in Southern Spain. There is a total of 39.39% men and 60.61% women, with an age range between 18 and 33 years old (M = 23.3; SD = 1.89), where 40.15% have work, and 35.61% have family responsibilities.

The research was conducted in the first quarter of the 2019–2020 school year. Previously, permission was requested from both the school management and the students themselves. Both were informed of the objectives of the research. Neither the school nor the students refused to participate.

3.3. Instrument

The instrument used is an ad hoc questionnaire that has had as reference the questionnaires 77 and 78, which consists of 30 items (Appendix A). These are distributed in different dimensions: Socioeducational (five items), oriented to know the socio-educational aspects of the sample; motivation (two items), autonomy (two items); collaboration (two items); participation (two items); resolution (two items); class time (two items), in which the aim is to identify the attitudes, motivations and interests of the student in the application of the teaching method; concepts (two items), scientific data (two items), graphics (two items), results (two items), decision (two items), ratings (three items), which focus on the learning acquired in the subject of mathematics. In addition, teacher-ratings have been taken into account, obtaining the values of the grades established by the teacher. The questionnaire uses a Likert scale, composed of four items (1: None, 2: Few, 3: Enough and 4: Completely).

This questionnaire has been subjected to various statistical tests, for its validation and reliability. At first, the Delphi method was used, with qualitative validity, by eight experts, whose ratings were positive (M = 4.66; SD = 0.16; min = 1; max = 6). Then, the statisticians of Kappa de Fleiss and W de Kendall were used, whose results were adequate (K = 0.89; W = 0.87). Subsequently, it was validated through exploratory factor analysis with varimax rotation, whose data (Bartlett = 2981.09; $p < 0.001$; Kaiser-Meyer-Olkin = 0.89) are adequate. It was finalized using Cronbach's alpha (0.91), McDonald's omega method (0.89), compound reliability (0.85) and mean variance extracted (0.84), showing adequate metrics. Taking into account the statistical tests, the instrument is considered as valid and reliable. The internal consistency of each of the dimensions is: Socio-educational (0.941); motivation (0.884); autonomy (0.861); collaboration (0.952); participation (0.891); resolution (0.948); class time (0.923); concepts (0.891); scientific data (0.901); graphics (0.912); results (0.884); decision (0.896); and ratings (0.911).

3.4. Dimensions and Study Variables

The study focuses research on attitudes and mathematical development. Both aspects have marked the distribution and composition of the dimensions of this study [77,78].

In addition, the dependent and independent variables have been established. The dependent variables are associated with the dimensions indicated for this study. The teaching method developed during this research is established as the independent variable. In order to facilitate the understanding of the results achieved, each of the dimensions is analysed:

- Motivation: Identifies the level of motivation achieved by students in the development of the teaching and learning process;
- Autonomy: Shows the level of autonomy of the student in the development of the tasks posed;
- Collaboration: Shows the ability to work with other colleagues in the development of the task;
- Participation: It identifies the level of involvement and relationship of the student with the contents, with the teacher and with his/her fellow students;
- Resolution: It shows the student's capacity to give an answer to possible problems that may arise in the performance of class activities;
- Class time: It analyses the feeling of time that the student has in the process of teaching and learning;

- Concepts: Identifies the level of acquisition, according to the student, of the contents applied in the pedagogical act;
- Scientific data: Presents the scientific aspects, typical of the mathematics subject, reached by the students;
- Graphics: It gathers the aspects related to the different mathematical graphs developed during the formative period;
- Results: It shows the different actions and mathematical problems developed in the realization of the contents;
- Decisions: Presents the common actions used by the students in order to solve possible activities;
- Ratings: It offers the students' self-evaluation in the teaching and learning process; and
- Teacher-ratings: Presents the qualification given by the teacher to the students in the pedagogical act. In this case, the qualification criteria are taken into account.

3.5. Methodological Procedure

The research process developed began with the validation and reliability of the instrument used. Subsequently, the selection of the sample and the application for permits were made. In this case, the pedagogical proposal was presented to the selected school, which agreed to participate. The centre, itself, requested information on the results achieved in the research.

Then, the pedagogical proposals were developed. On the one hand, the traditional exposition method (Gc), in which the teacher presented the theoretical contents, followed the sequence of the textbook and proposed tasks. On the other hand, there is the e-learning method (Ge), which will be explained in more detail in the next point.

At the end of the first quarter, data was collected using Google Form, which is a Google Drive tool. In other words, the data was collected on the last day of class, in the auditorium of the educational centre, which has a capacity for 300 people. To do this, the students used their own mobile devices. In the cases that they did not have, the centre gave them one to fill out the questionnaire. Indicate that the data collection was carried out at the same time, specifically at 18:10. This data was downloaded in Excel format and transcribed into the format of the selected statistical program. Finally, the various statistical tests were carried out and the results obtained were analysed.

3.6. Pedagogic Procedure

The pedagogical proposal developed with the experimental group is based on the e-learning method. For this purpose, the teacher has made use of the Moodle platform and e-mail. In addition, every week, a schedule was established, consisting of one hour of group attention and two hours of individualized attention. The three hours could be developed in a face-to-face way in the educational centre. It should be noted that these hours were not compulsory. Only those students who considered it necessary came to the centre, and on a voluntary basis. It should be noted that during the study procedure, hardly any students attended the centre to answer questions. The group that developed the expository method, had an hour of tutoring with the teacher of the subject, to solve doubts individually, or attend to the concerns of the students. During this period, the teacher also attended to the student through a virtual platform and by e-mail.

The Moodle platform contained all the content to be dealt with in the subject during the first term, distributed by didactic units. In this case, four didactic units were established for the first quarter. Each one of the didactic units of the Moodle platform was structured in different sections:

- Theory: Formed by theoretical aspects of the subject, presented in pdf format and explanatory videos. The intention was to present all the theoretical aspects of the contents to be worked on, and to reinforce their acquisition through the viewing of videos related to these contents;
- Practice: Composed of activities to show the acquisition of the theoretical contents. These activities were of introduction, development, consolidation, extension and reinforcement. The activities

have been varied, having different types: short answer, long answer, assumptions, problem solving and autocomplete, relate columns and operations, among others. In this case, all the tools available in Moodle have been used;

- To know more. In this section students have been allowed to go deeper into the contents of the subject. This was done through links to web pages on the subject. There were also links to games related to the contents worked on; and
- Forum: This resource has been used in each didactic unit. The intention was to establish a debate, both with the teacher and with other colleagues, on the contents dealt with in the subject. In addition, it has served to resolve doubts and pose small riddles related to the aspects worked on.

The evaluation methods and instruments used have been:

- Written test (50% of the quarterly mark): This test was taken at the end of the quarter. The types of questions were short answer and long answer; and
- Systematic observation (50% of the quarterly mark): Participation in the forum and the development of the activities set out in the Moodle platform were analysed. The instrument used was a heading.

On the other hand, the pedagogical proposal developed by the control group was based on the presentation of theoretical contents by the teacher. In addition, activities have been developed, both from the textbook and from cards given by the teacher. As a method and instrument of evaluation, the following have been applied:

- Written test (50% of the quarterly mark): This test was taken at the end of the quarter. The question type was short answer and long answer; and
- Systematic observation (50% of the quarterly mark): The development and elaboration of the activities proposed by the teacher were analysed. The instrument used was a heading.

4. Results

The data presented in Table 2, after the descriptive statistical analysis, show diversity of response among students who attend both the night school and the distance school. According to the data provided by the asymmetry and kurtosis statisticians, the response distribution is considered normal. This is because the values are between ±1.96, according to [79]. The students in the control group show a mean response that is around 2. Some dimensions are slightly below and others are slightly above. In the control group the dimension with the highest rating is resolution. In contrast, the dimension with the lowest rating is decision. The students in the experimental group show a response tendency that is around 2.5 points. The least valued dimension in the experimental group is decision. The most valued dimension in the experimental group is teacher-ratings. According to the statistic that shows the standard deviation, an even trend of response is observed in the students. This is presented in all the dimensions of the study, both in the control group and in the experimental group. Kurtosis is platykurtic in all study dimensions, both in the control group and in the experimental group.

Table 2. Results obtained for the dimensions of study in GC and EG of high school students.

		Likert Scale *n* (%)				Parameters			
	Dimensions	**None**	**Few**	**Enough**	**Completely**	**M**	**SD**	S_{kw}	K_{me}
Control group	Motivation	24(39.3)	19(31.1)	14(23)	4(6.6)	1.97	0.948	0.552	−0.757
	Autonomy	26(42.6)	17(27.9)	13(21.3)	5(8.2)	1.95	0.990	0.633	−0.760
	Collaboration	20(32.8)	19(31.1)	16(26.2)	6(9.8)	2.13	0.991	0.366	−0.961
	Participation	23(37.7)	22(36.1)	13(21.3)	3(4.9)	1.93	0.892	0.568	−0.569
	Resolution	13(21.3)	16(26.2)	24(39.3)	8(13.1)	2.44	0.975	−0.112	−0.990
	Class time	23(37.7)	19(31.1)	13(21.3)	6(9.8)	2.03	0.999	0.554	−0.804
	Concepts	21(34.4)	19(31.1)	15(24.6)	6(9.8)	2.10	0.995	0.427	−0.921
	Scientific data	26(42.6)	18(29.5)	13(21.3)	4(6.6)	1.92	0.954	0.644	−0.676
	Graphics	24(39.3)	18(29.5)	15(24.6)	4(6.6)	1.98	0.957	0.505	−0.862
	Results	18(29.5)	23(37.7)	13(21.3)	7(11.5)	2.15	0.980	0.462	−0.753
	Decision	28(45.9)	17(27.9)	14(23)	2(3.3)	1.84	0.898	0.620	−0.796
	Ratings [a]	21(34.4)	19(31.1)	16(26.2)	5(8.2)	2.08	0.971	0.396	−0.924
	Teacher ratings [a]	12(19.7)	23(37.7)	17(27.9)	9(14.8)	2.38	0.969	0.190	−0.886
Experimental group	Motivation	6(8.5)	20(28.2)	24(33.8)	21(29.6)	2.85	0.951	−0.296	−0.904
	Autonomy	8(11.3)	11(15.5)	27(38)	25(35.2)	2.97	0.985	−0.680	−0.551
	Collaboration	18(25.4)	24(33.8)	21(29.6)	8(11.3)	2.27	0.970	0.205	−0.994
	Participation	7(9.9)	16(22.5)	25(35.2)	23(32.4)	2.90	0.973	−0.467	−0.782
	Resolution	9(12.7)	19(26.8)	24(33.8)	19(26.8)	2.75	0.996	−0.268	−0.972
	Class time	15(21.1)	32(45.1)	12(16.9)	12(16.9)	2.30	0.991	0.455	−0.768
	Concepts	7(9.9)	16(22.5)	24(33.8)	24(33.8)	2.92	0.982	−0.479	−0.881
	Scientific data	18(25.4)	27(38)	16(22.5)	10(14.1)	2.25	0.996	0.358	−0.878
	Graphics	16(22.5)	29(40.8)	15(21.1)	11(15.5)	2.30	0.991	0.364	−0.847
	Results	5(7)	19(26.8)	21(29.6)	26(36.6)	2.96	0.963	−0.408	−0.956
	Decision	24(33.8)	26(36.6)	13(18.3)	8(11.3)	2.07	0.990	0.582	−0.671
	Ratings [a]	7(9.9)	16(22.5)	23(32.4)	25(35.2)	2.93	0.990	−0.492	−0.838
	Teacher ratings [a]	5(7)	16(22.5)	23(32.4)	27(38)	3.01	0.949	−0.545	−0.736

[a]. Established grade group (None: 1–4.9; Few: 5–5.9; Enough: 6–8.9; Completely: 9–10).

The means presented by the control group and the experimental group show relevant differences. In the control group, there is diversity of means between the study dimensions. The resolution and teacher-ratings dimensions stand out from the total mean. On the other hand, the decision dimension is much lower than the mean. In the experimental group, these differences are more pressing. In this case, the dimensions motivation, autonomy, participation, resolution, concepts, results, ratings and teacher ratings are located above the mean. On the other hand, the collaboration, class time, scientific-data, graphics and decision dimensions are located far below the totalised mean. Furthermore, even ratings are observed, both in the control group and in the experimental group, in the collaboration, class time, scientific-data, graphics and decision dimensions (Figure 1).

To identify the value of independence between the expository-traditional method and the e-learning method, Student's t statistical test has been used. The values present higher averages in favour of the experimental group, although it is not significant in all cases. The dimensions motivation, autonomy, participation, concepts, results, ratings and teacher-ratings show a significant relationship. In all the dimensions where there is a relationship of significance, the force of association is average, if the values of the biserial correlation are taken into account. The size of the effect is low in class time and graphics, and very low in the rest of the dimensions (Table 3).

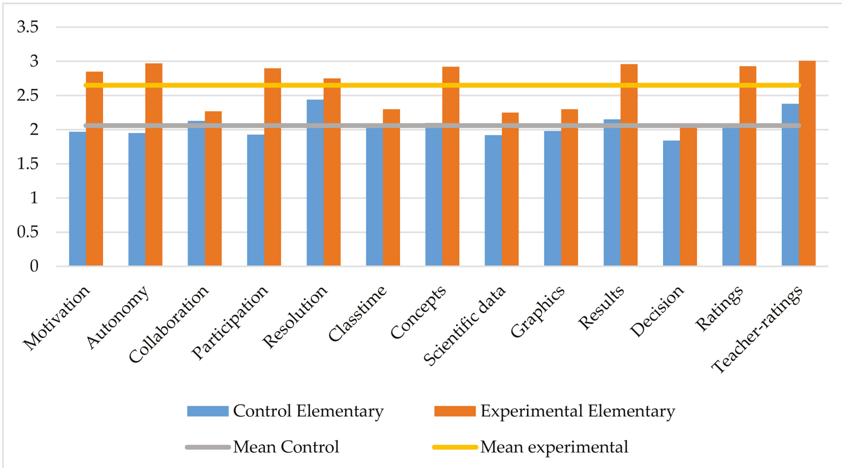

Figure 1. Comparison between control group and experimental group.

Table 3. Study of the value of independence between control group and experimental group.

Dimensions	μ(X1−X2)	$t_{n1+n2-2}$	df	*d*	r_{xy}
Motivation	−0.878 (1.97−2.85)	−5.295 **	130	0.094	0.421
Autonomy	−1.021 (1.95−2.97)	−5.922 **	130	0.062	0.461
Collaboration	−0.136 (2.13−2.27)	−0.798	130	0.036	0.070
Participation	−0.967 (1.93−2.90)	−5.914 **	130	0.048	0.460
Resolution	−0.304 (2.44−2.75)	−1.765	130	0.029	0.153
Class time	−0.263 (2.03−2.30)	−1.514	130	0.115	0.132
Concepts	−0.817 (2.10−2.92)	−4.737 **	130	0.052	0.384
Scientific data	−0.335 (1.92−2.25)	−1.967	130	0.097	0.170
Graphics	−0.312 (1.98−2.30)	−1.833	130	0.105	0.159
Results	−0.810 (2.15−2.96)	−4.780 **	130	0.038	0.387
Decision	−0.234 (1.84−2.07)	−1.415	130	0.081	0.123
Ratings [a]	−0.848 (2.08-2.93)	−4.947 **	130	0.052	0.398
Teacher ratings [a]	−0.637 (2.38−3.01)	−3.809 **	130	−0.008	0.317

**. The correlation is significant at the level 0.01. [a]. Established grade group (None: 1–4.9; Few: 5–5.9; Enough: 6–8.9; Completely: 9–10).

5. Discussion

The rise of information and communication technologies, related to the current situation of confinement caused by COVID-19, makes the e-learning method relevant in recent times, thus promoting innovative educational practices [1–6].

The e-learning teaching method breaks with the classic stereotypes of teaching and learning processes, since it modifies the spaces and time of training, allowing the development of the pedagogical act in any place and at any time. This can be achieved if technological devices and digital resources are available, as well as internet access [22–27].

In the present research, the influence of e-learning in the field of mathematics has been analysed, in contrast to the traditional expository method, in adult students who are studying for high school. As shown in the results obtained, there are significant differences between the values achieved in the control group and the experimental group. These differences have always been in favour of the e-learning method.

In the group where the expository method has been developed, the lowest values have been produced in the decision. In this case, students have difficulties in making decisions by themselves when solving the proposed mathematical problems. On the other hand, the most valued dimension is the resolution, that is, the carrying out of activities in class. This may be due to the fact that the teacher, present in the expository method, can respond to the needs that the students may have during the development of the different practices.

In the group where the e-learning method is developed the dimension with the highest score is teacher-rating. This shows that the students' grades are increasing, being in line with [58]. On the other hand, the less valued dimension, as in the control group, is decision. In other words, neither the expository-traditional method nor the e-learning method allows the student's decision-making to improve when it comes to solving a problem on their own.

Both in the control group and in the experimental group, students have shown a tendency to respond evenly. This shows that the students agree on the teaching methods applied. This does not mean that there are equal values in all the study dimensions. In the control group the means of the dimensions have not been equal to each other. Examples of this are the dimensions resolution and teacher-ratings, which are above the totalised mean. That is to say, for the students who have developed the expository method, in these dimensions, they show a better evaluation. On the other hand, the decision dimension is much lower than the average.

Something similar occurs in the experimental group. The averages thrown between the different dimensions are not equal to each other. In this case, the contrasts are more relevant. For example, the dimensions motivation, autonomy, participation, resolution, concepts, results, ratings and teacher ratings are above the total average. On the other hand, the collaboration, class time, scientific-data, graphics and decision dimensions are much lower than the total average.

If the means of the control group and the experimental group are compared, there are dimensions in which there are no significant differences. This is the case of the collaboration, class time, scientific-data, graphics and decision dimensions, which present evenly distributed means, although always with higher values in the experimental group.

Where there are significant differences, in favour of the e-learning method, are in the dimensions of motivation [36,59], autonomy [35], participation [60], concepts [55], results, ratings and teacher ratings [58]. In other words, the e-learning method favours these aspects in the pedagogical act.

In the dimension where there is a greater contrast, when comparing the expository-traditional method with the e-learning method, it is in autonomy. This may be mainly due to the fact that the e-learning method favours self-regulation of learning [39].

If this study is compared with other studies in which e-learning has been developed, improvements in students can be observed. On the one hand, there is an improvement in motivation, autonomy, participation, concepts, results and grades. All these aspects are reflected in other studies, in which the e-learning method is associated with a clearly defined and structured pedagogical approach. In the studies analysed, student effort, which has an impact on their qualifications, is due to increased motivation and interest. In other words, the pedagogical approach influences whether the student can be more or less motivated. In addition, the fact that the student is more motivated leads to an increase in participation, which will lead to improvements in the acquisition of mathematical concepts. It will also influence the resolution of various activities. All of this is ultimately reflected in the grades, which increase. Therefore, it can be indicated that there is an improvement in students' academic performance. Furthermore, it should be taken into account that the e-learning method will favour the autonomy of the student, adapting to his or her learning style, which implies more individualised attention to the teaching and learning process. What is clear from all this research is that the e-learning method is associated with a clearly defined pedagogical process, as shown in this research [64–70].

6. Conclusions

In general, it can be indicated that the dimensions of motivation, autonomy, participation, concepts, results, self-evaluation and teacher qualification have proved to be significant. That is to say, according to the study group, differences are observed in the evaluations given by the students. It should be borne in mind that these differences may be motivated by the application of the teaching method applied. In one group the expository method has been developed and in the other the e-learning method. The most valued dimensions have been those of the group in which the e-learning method has been developed. This can be due to several reasons. One of them is the applied method, since the e-learning method makes the student the guide of his/her own learning. That is, they have more weight in the teaching and learning process, while the teacher is a guide. This aspect can have a direct influence on motivation, autonomy and participation. This fact, in turn, can lead to a better acquisition of mathematical concepts and results, given that being motivated and having more autonomy in learning, allows the student to increase his or her participation, and in his or her view, to present more interest in the contents being developed. Finally, the improvement in the concepts and results generates an improvement in the qualification of the students, and therefore, an improvement in the self-evaluation of the didactic actions developed. The rest of the dimensions, such as collaboration, resolution, class time, scientific data, graphics and decision, no differences were observed. This may be due to the method itself. In this case, both the e-learning method and the expository method, due to their didactic processes, do not require greater collaboration among students, nor in the feeling of class time. The other dimensions may be due to the fact that neither the expository method nor the e-learning method lead to an increase in the understanding and development of scientific data, the development of graphs or decision-making.

It can be concluded that the e-learning method is an improvement for adult students who are studying mathematics in the educational stage of high school, provided that it is compared with the expository method. In this case, the improvements occur in motivation, autonomy, participation, concepts, results, ratings and teacher-ratings. Therefore, the use of the e-learning method would be effective for its implementation with adults who study mathematics in high school.

The prospective of the research is based on two aspects. On the one hand, the aim is to present the scientific community with new data on the application of innovative teaching methods. In this case, the e-learning method is compared with the traditional expository method for teaching mathematics to adults studying in secondary schools. On the other hand, the aim is to publicise the educational practice developed in this research, so that other teachers, in similar circumstances, can develop it.

The limitations of the study are several. On the one hand, the study sample presents some specific socio-educational characteristics, so one must be cautious when extrapolating the data to other populations. The access to the sample has been for convenience, due to the fact that the educational groups are established by the educational centres themselves. This has prevented the application of other sampling techniques. Finally, the fact of not applying a pretest and posttest study process makes it impossible to be categorical in ensuring that the e-learning method directly influences the dimensions, since there may be other elements that may have been include in the development of the study. Therefore, the results obtained should be treated with caution.

As a future line of research, it is presented to develop this didactic method in other educational stages and in other educational subjects.

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Appendix A

Table A1. The instrument used is an ad hoc questionnaire.

Socio-Educational Dimension					
Variable	Item	Choice			
Gender	Gender	Man Woman			
Age	Age	18–19 years 20–21 years 22–23 years 24 or more years			
Religion	Religion	Christian Muslim Jewish Hindu Atheist Other			
ICT use frequency	How much time do you spend using ICTs every day?	From 0 to 2 h a day From 2 to 4 h a day More than 4 h a day			
Context	What is your socioeconomic level?	Low Level Medium level High level			
Dimensions	Variable	Gradation 1	2	3	4
Motivation	Does the methodology applied affect your motivation with regard to mathematical content? To what extent has the methodology applied improved your motivation with regard to mathematical content?				
Autonomy	How does the methodology applied in the field of mathematics contribute to their autonomy?				
	To what extent has the methodology applied in the field of mathematics contributed to their autonomy?				
Collaboration	How does the methodology developed in the subject of mathematics affect the collaboration of the group? To what extent has the methodology applied in the subject of mathematics contributed to the collaboration of group's collaboration?				
Participation	How has the methodology applied in the field of mathematics contributed to their level of participation? Has the methodology applied to their level of involvement in the subject of mathematics increased?				
Resolution	How does the methodology developed in the field of mathematics affect the resolution of problems that arise during the study? To what extent does the methodology applied in the subject of mathematics contribute to your ability to solve the problems that arise during the study?				
Class-time	How does the methodology developed in the field of mathematics affect the feeling of class time? Do you feel that time passes more quickly in math with the methodology applied?				
Concepts	How does the methodology developed in the subject of mathematics to learning scientific language and mathematical concepts? To what extent has the methodology applied in the field of mathematics contributed to your knowledge of scientific language and mathematical concepts?				
Scientific data	How does methodology applied in the subject of mathematics affect the use of scientific data and processes? To what extent has the methodology applied in the subject of mathematics contributed to the use of data and scientific processes?				

Table A1. *Cont.*

Socio-Educational Dimension		
Graphics	How does the methodology applied in the subject of mathematics affect the ability to analyse and represent graphs?	
	How much has the methodology applied in the subject of mathematics contributed to your ability to analyse and represent graphs?	
Results	How does the methodology applied in the subject of mathematics affect the ability to interpret and reflect the results of the proposed activities?	
	To what extent has the methodology applied in the subject of mathematics contributed to your ability to interpret and reflect the results of the proposed activities?	
Decision	How does the methodology applied in the subject of mathematics to the development of mathematical competence?	
	To what extent has the methodology applied in the subject of mathematics contributed to your ability to make decisions?	
Ratings	What is your average grade in general?	0–4.9 5–5.9 6–8.9 9–10
	What is your general average in the Mathematics subject?	0–4.9 5–5.9 6–8.9 9–10
	What has been the grade you have obtained in the Mathematics subject after the development of the experience?	0–4.9 5–5.9 6–8.9 9–10

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B-Learning in Basic Vocational Training Students for the Development of the Module of Applied Sciences I

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Abstract: Information and communication technologies are a step forward in education, as they have given rise to innovative methodologies, such as blended learning. This type of training can be applied at any stage or educational typology such as basic vocational training. The main objective of this article is to know the degree of effectiveness of this methodology in this stage, specifically in an applied science module. For this purpose, a quasi-experimental design has been applied with a control group and an experimental group with a total of 147 participants. The results show how those students who have worked through b-learning have experienced better results in all the dimensions of the study. In conclusion, the implementation of this methodology in basic vocational training brings benefits, such as motivation and autonomy in the teaching–learning processes of all students.

Keywords: b-learning; ICT; vocational training

1. Introduction

The technological field continues advancing to this day. This fact is reflected in the social field [1], especially in the labour, social and educational fields [2]. This technological and digital boom has made it easier to carry out multiple tasks in the domestic and personal sphere, becoming a tool that facilitates people's daily lives.

This progress is also reflected in the educational field [3]. In this case, the use of technologies in the educational field, as in other fields of study, is called information and communication technologies (ICT) [4]. The use of ICT generates benefits for student training [5], because it facilitates the application of innovative teaching and learning processes, thereby promoting new learning spaces. This leads to new educational proposals [6], transforming educational events as we know them today [7]. This is because ICT promotes the elimination of spatial-temporal barriers [8] and facilitates access to an enormous amount of resources [9] in different kinds of support and media. All of this leads to improvements in student development [10], mainly due to the increase in motivation, autonomy, predisposition and attitude of the students in the treatment of the proposed pedagogical contents [11–13].

Among the different pedagogical methods that the incursion of ICT in the educational field has brought about is b-learning [14], also known as blended learning, combined learning or hybrid learning [15]. When defining it, the authors agree in not considering this teaching method only as a mixture of classroom learning and online learning [16], but rather as a methodological basis that uses the best of the classroom learning teaching process, and the best of e-learning [17]. In order for the b-learning teaching process to be developed with a minimum of guarantees, the characteristics of those involved in the pedagogical act must be analysed [18], identifying the students' learning styles [19], and the changing the role of the teachers and instructors [20], with the student being the organiser of his or her own learning [21], while the teacher must become a guide [22], developing an open methodology [23], attending to the students individually [24], and promoting a more autonomous educational act in the student himself [25], with the intention of facilitating the expansion of the students' knowledge [26]. The manner and proportion of combining classroom and virtual teaching will depend on the needs and characteristics of the environment where the teaching and learning process takes place [27].

Among the main characteristics of b-learning is the fact that the teacher acts as both an online tutor and a traditional teacher [28]; promotes personal links between teacher and student [29] and between the students themselves [30]; encourages the combination of technology, learning development and teacher support [31]; facilitates the application of other teaching methods, complementary to b-learning [32]; allows for the development of synchronous and asynchronous communication [33]; focuses more on the curricular elements that the student should develop than on the environment in which he or she is developing [34]; develops ubiquitous learning [35]; eliminates spatial-temporal barriers [36]; favours the development of the digital competence of teachers and students [37]; promotes digital literacy [38]; adapts to the pace, style and pedagogical development of the learner [39]; facilitates attention to diversity [40]; provides access to a wealth of digital resources [41]; and generates a shift in roles between teachers and learners [42].

The use of b-learning, if not properly applied, can lead mainly to technological dependence for teachers and learners [43] and an increased workload for teachers [44].

In order for the b-learning method to be implemented with a number of guarantees, it first requires a great deal of effort and dedication on the part of the teacher [45], as well as instructional design of the teaching and learning process [46]. The key competence of learning to learn should also be enhanced [47], new roles for teachers and students should be clearly established [48], curricular flexibility should be encouraged [49], different learning styles for students should be addressed [50] and cooperative and collaborative work should be promoted [51].

The b-learning method, in the teaching and learning processes, generates improvements in motivation [52], in academic performance [53], in the relationship between teacher and student [54], in learning autonomy [55] and in collaboration [56].

At the same time, the expository method consists of the presentation of a topic in a structured way with the intention of providing information organised, according to criteria appropriate to the intended purpose. This methodology is fundamentally centred on the verbal presentation by the teacher of the contents of the subject under study. This method is also often referred to as a "master class", to refer to a subject taught by a teacher on special occasions [57]. This method is basically focused on the unidirectional communication of the teacher with the student. The teacher teaches by showing the content to be learned, exposing them, so that the student learns through attentive listening and note taking, and the subsequent completion of tasks [58]. Among the advantages of this method is the saving of time and it also means that the teacher is able to attend to big groups, among other considerations [59]. Among the disadvantages are the low participation of the student, little feedback, difficulty attending individually to the student, not facilitating autonomous learning, a passive position for the teacher, the students receiving such a large quantity of information that they do not have time to assimilate it and exceeding their capacity for attention [60].

In other words, the differences between the b-learning method and the expository method lie mainly in the role of the student in the teaching and learning process. In the b-learning method, the role is active. While in the expository method, the role is passive [28–42,57–60].

In the subject of mathematics, educational experiences based on the b-learning method have been developed [61]. Research shows how the pedagogical process allowed teachers to experience the social and cognitive development of students, through synchronous and asynchronous discussions with their peers and facilitators [62]. In addition, this method improves learning outcomes and attitudes towards learning mathematical content [63].

Ultimately, the application of active teaching methods can generate benefits in the teaching and learning processes. These benefits directly influence the students themselves. An example of this is the b-learning method, which leads to an increase in, among other aspects, motivation, academic performance, the relationship between teachers and between students, learning autonomy and collaboration.

This fact is also reflected in subjects such as mathematics, where students' attitudes improve substantially.

2. Justification and Objectives

The application of information and communication technologies in education has led to the emergence of new methods of teaching. These methods include b-learning [64]. The research presented below is based on analysing the contrast between the b-learning teaching method and the expository method. To this end, this research has focused on students of Basic Vocational Training. All students enrolled at this stage of education are at risk of social exclusion. This is due to their socio-educational characteristics. That is to say, being students with a lack of motivation, with behavioural problems, without study habits and with difficulties in the acquisition of new content.

The pedagogical act has been carried out in the subject of Applied Sciences I, in which they develop pedagogical actions aimed at the acquisition of theoretical and practical competences at a professional, personal and social level. That is, it focuses on science and math.

In the expository method the teacher has had an active role and is always exposing the theoretical contents while the students have a passive role and do not intervene during the class. In b-learning, teachers have developed their pedagogical actions by using a virtual platform. In point 3.6. of the present manuscript, both teaching methods are explained in detail.

It is important to indicate that both pedagogical processes could be observed by the researchers themselves, since they continuously supervised the pedagogical actions, making sure that the established pedagogical processes were adequately developed.

These two methods have been applied in the teaching–learning processes of the educational reality of the participants in the research, so we wanted to measure the influence of both separately, and how this affects the qualifications of the students. This research also tries to present more studies on the use of b-learning in mathematics related subjects [61–63], specifically for students who present adverse socio-educational characteristics, as in the case of students in Basic Vocational Training.

Therefore, the main objective of the present study is to identify the degree of effectiveness of the b-learning method in the module of Applied Sciences I, for Basic Vocational Training students, in comparison with the expository method, and in different areas of socio-pedagogical development. From this general objective, the following specific objectives are developed: (i) to define the level of motivation of the students, both in the control group and in the experimental group; (ii) to specify the level of interaction (teacher–student, student–student, student–content), both in the control group and in the experimental group; (iii) to investigate the level of autonomy of the students, both in the control group and in the experimental group; (iv) to identify the level of collaboration of the students, both in the control group and in the experimental group; (v) to identify the level of deepening of didactic content, both in the control group and in the experimental group; (vi) to discover the level of problem solving in the didactic activities proposed, both in the control group and in the experimental group; (vii) to analyse the perception of the class time developed, both in the control group and in the experimental group; (viii) to specify the influence of the teaching method through the grades, both in the control group and in the experimental group; (ix) to identify the contrast of averages in the different dimensions of study between the group that applies the expository method and the b-learning method.

3. Research Method

3.1. Research Design and Data Analysis

The present research is quantitative, descriptive and correlational [65], applying a quasi-experimental design, by means of control group (Gc) and experimental group (Ge). The study follows the structure and model of previous research [66–69]. The control group has experienced the exposure method. On the other hand, the experimental group has followed the b-learning method. The distribution of students is not random, since the groups were already defined from the beginning of the course. The criteria for the distribution of the students were established in the previous academic year, in a meeting between the different Heads of Studies of the secondary education centres, in the presence of the Education Inspector. At this meeting, the distribution criteria are based on the principle of equity. The information was collected at the end of the educational experience, which took place in January of the 2019/2020 academic year, by means of a post-test (Table 1).

Table 1. Groups' composition.

Group	*n*	Composition	Pretest	Treatment	Posttest
1-Control	25	Natural	-	X_1	O_1
2-Experimental	25	Natural	-	X_2	O_2
3-Control	25	Natural	-	X_1	O_1
4-Experimental	25	Natural	-	X_2	O_2
5-Control	24	Natural	-	X_1	O_1
6-Experimental	23	Natural	-	X_2	O_2

The analysis of the data collected has been carried out using the Statistical Package for the Social Sciences (SPSS) programme, version 25. The statistics used are the mean (*M*), standard deviation (*SD*), skewness (*S*kw) and kurtosis (*K*me). In addition, Student's t-test (tn1 + n2-2) was used to compare the group means. Finally, Cohen's d test and the biserial correlation (rxy) have been applied to identify the effect size and the association force. The significance level applied in the study was $p < 0.05$.

3.2. Participants

The sample used in this study is composed of 147 students. The sampling technique used is a convenience sample, due to the ease of access to the population. In relation to the total number of the sample, the authors [70,71] establish that, in the application of pedagogical methods, the size of the sample is not a determining factor.

The students that make up the two groups that are part of the study are studying the module of Applied Sciences I, of the Basic Vocational Training. The students in this educational stage present specific characteristics, among which the following stand out: having reached fifteen years of age and not exceeding seventeen years of age; having studied the first cycle of Obligatory Secondary Education (ESO), or exceptionally, the second year of ESO; and having been proposed by the teaching team, after acceptance by the parents. These students usually have had a previous negative experience during their stage in ESO, not reaching the necessary competences, with high levels of absenteeism, low academic performance, lack of motivation and no study habits [72].

The Basic Vocational Training student body is made up of 61% men and 39% women, aged between 15 and 17, with an average age of 16.3 years and a standard deviation of 0.432. The students were composed of three professional families or groups of training cycles with common characteristics, on the one hand, the professional family of electricity and electronics (2 groups), the professional family of physical activities and sports (2 groups) and the professional family of personal image (2 groups).

The research was carried out in the first month of the second quarter, in the academic year 2019–2020. It is important to highlight that three teachers participated, so they were trained in b-learning and the Moodle platform, as well as in the expository method. In order to proceed with the research, the corresponding permits were requested and collected, informing all the parties involved of the objectives of the study. There was collaboration at all times between all the people involved in the study.

3.3. Instrument

The instrument used was an ad hoc questionnaire, following the structure of other questionnaires that collected data on active teaching methodologies [66–69]. The qualifications established by the teacher have also been taken into account and it is described in the pedagogical procedure.

The questionnaire is composed of nine dimensions (socio-educational, motivation, interactions, autonomy, collaboration, deepening of content, problem solving, class time and ratings), with 35 items with answer format based on Likert scale (from 1 = None to 4 = Completely).

The instrument was tested for validity and reliability. The Delphi qualitative validity method was applied ($M = 4.46$; $DS = 0.21$; min = 1; max= 5); the Kappa statistic by Fleiss and W by Kendall ($K = 0.88$; $W = 0.86$); the exploratory factorial analysis with varimax rotation (Bartlett = 2. 771.01; $p < 0.001$; Kaiser-Meyer-Olkin = 0.89); Cronbach's alpha (0.89); McDonald's omega method (0.88); the reliability of the compound (0.87); and the mean variance extracted (0.85). Bearing in mind all these values, it is considered a valid and reliable instrument.

3.4. Dimensions and Study Variables

The dimensions used in this study are based on other research [66–69], whose items have been considered as independent variables: socio-educational; motivation; interactions; autonomy; collaboration; deepening of content; problem solving; class time; rating; and teacher ratings. On the other hand, the pedagogical method developed has been considered as a dependent variable. All these variables have been measured through the questionnaire.

3.5. Methods

The methodological procedure applied in the investigation started with the validation and reliability of the questionnaire. Subsequently, the study population and the research sample were selected, requesting at that time all of the corresponding permits, both from the educational centres and from the trainees themselves.

The methodological procedures to be developed were then determined and specified. On the one hand, the pedagogical acts of the expository method (Gc) and the b-learning method (Ge) were established.

Then, data was collected using a form previously developed with the Google Form tool. Finally, they were downloaded in an Excel table, transcribed to the statistical program used, and the statistical tests and analyses were carried out.

3.6. Pedagogical Procedure

In order to develop the b-learning method with the Basic Vocational Training students, each student was assigned a computer at the educational centre, so that they could access a Moodle platform specifically assembled for the development of the module, by means of a username and password. The access to the platform could be done from any place and at any time, as long as they had a device with Internet access. Those students who, due to different circumstances, could not access the platform from their homes, were provided with a corner with computers for their use in the libraries of the educational centres. In this case, the tutor also had access to the platform 24 h a day, being able to enter it outside school hours to correct or solve doubts about activities. The pedagogical development was divided into two clearly defined lines: the virtual sessions and the face-to-face sessions:

For the virtual period, the students had to read the theoretical contents prepared for their study or knowledge, carry out activities to consolidate the acquired contents and ask for the necessary help through the forums, the tutor or a classmate.

For the face-to-face period, the time was dedicated to consolidate the theoretical contents, to carry out cooperative and collaborative activities, to solve the individual difficulties that the students presented before certain types of activities and tasks, as well as to develop activities to attend to the transversal elements, such as reading comprehension, oral and written communication, audio-visual communication, ICT and values education.

The qualification criteria of the Basic Vocational Training students who have developed the b-learning method were:

- A total of 50% of the mark corresponds to the completion of a written test;
- A total of 40% of the mark corresponds to the student's participation in the forums, chat, wiki and other elements related to both synchronous and asynchronous student communication;
- A total of 10% of the mark corresponds to the activities carried out by the students in their notebook.

In the expository method, the teacher has made the theoretical presentation of the contents during the development of the sessions. This theoretical presentation followed what was established in the didactic program as well as what was indicated in the textbook used for the level of the students. On certain occasions, the teacher has used the digital blackboard to show certain contents in an interactive way. In addition, during the development of the classes, the teacher established a series of tasks, both in the development of the session and at home. The time structure of each session was distributed as follows: 10 min to remember the contents worked during the previous session; 10 min to solve the doubts that the students could have or to correct the exercises developed at home; 25 min for the theoretical exposition of the contents; and 10 min for the accomplishment of tasks. The qualification criteria have been:

- A total of 50% of the mark corresponds to the completion of a written test;
- A total of 50% of the mark corresponds to the completion of the student activities in class.

4. Results

The data presented in Table 2, related to the descriptive analysis, shows diversity of scores between the control group and the experimental group. In this case, students in Basic Vocational Training who have developed the educational experience through b-learning present better averages in all the dimensions studied. Although, if we analyse it in detail, we can see that the average of the experimental group is in an intermediate zone, so the scores reached are not very high. These are around 2.5 points. On the other hand, in the control group the averages are relatively low, given that they are in all cases below 2. In the experimental group, the most valued dimension is class time, while the least valued dimension is resolution. In the control group, the dimension with the highest score is the relationship between students. On the other hand, the least valued dimensions are resolution and teacher ratings. If the values of the standard deviation are taken into account, a trend of grouped response is shown, not having dispersion in any of the dimensions. With respect to kurtosis, most of them are platicurtic, although there are also, to a lesser extent, leptokurtic and mesocurtic types. If the values reached in asymmetry and kurtosis are taken into account, it can be established that the distribution of the sample is normal. This is because the values are between ± 1.96, as marked by [73].

Table 2. Results obtained for the dimensions of study in CG and EG of students in Basic Vocational Training.

		Likert Scale *n* (%)				Parameters			
	Dimensions	**None**	**Few**	**Enough**	**Completely**	M	SD	S_{kw}	K_{me}
Control group	Motivation	32(43.2)	27(36.5)	12(16.2)	3(4.1)	1.81	0.855	0.784	−0.154
	Teacher-student	38(51.4)	17(23)	13(17.6)	6(8.1)	1.82	0.998	0.874	−0.477
	Student-content	33(44.6)	28(37.8)	10(13.5)	3(4.1)	1.77	0.837	0.892	0.173
	Student-student	29(39.2)	24(32.4)	17(23)	4(5.4)	1.95	0.920	0.543	−0.731
	Autonomy	31(41.9)	27(36.5)	11(14.9)	5(6.8)	1.86	0.911	0.832	−0.121
	Collaboration	32(43.2)	28(37.8)	11(14.9)	3(4.1)	1.80	0.844	0.826	0.002
	Deepening	35(47.3)	24(32.4)	11(14.9)	4(5.4)	1.78	0.896	0.916	−0.032
	Resolution	36(48.6)	28(37.8)	8(10.8)	2(2.7)	1.68	0.778	0.999	0.553
	Classtime	37(50)	22(29.7)	13(17.6)	2(2.7)	1.73	0.849	0.831	−0.331
	Ratings [a]	37(50)	24(32.4)	10(13.5)	3(4.1)	1.72	0.852	0.997	0.216
	Teacher-ratings [a]	36(48.6)	27(36.5)	10(13.5)	1(1.4)	1.68	0.760	0.820	−0.115
Experimental group	Motivation	10(13.7)	22(30.1)	25(34.2)	16(21.9)	2.64	0.977	−0.142	−0.954
	Teacher-student	16(21.9)	20(27.4)	28(38.4)	9(12.3)	2.44	0.969	−0.070	−0.998
	Student-content	12(16.4)	19(26)	36(49.3)	6(8.2)	2.49	0.868	−0.371	−0.627
	Student-student	11(15.1)	18(24.7)	30(41.1)	14(19.2)	2.64	0.963	−0.278	−0.823
	Autonomy	15(20.5)	18(24.7)	30(41.1)	10(13.7)	2.48	0.973	−0.173	−0.696
	Collaboration	11(15.1)	19(26)	27(37)	16(21.9)	2.66	0.989	−0.232	−0.938
	Deepening	15(20.5)	19(26)	29(39.7)	10(13.7)	2.47	0.973	−0.134	−0.975
	Resolution	18(24.7)	23(31.5)	25(34.2)	7(9.6)	2.29	0.950	0.087	−0.973
	Classtime	5(6.8)	24(32.9)	24(32.9)	20(27.4)	2.81	0.923	−0.149	−0.963
	Ratings [a]	9(12.3)	30(41.1)	20(27.4)	14(19.2)	2.53	0.944	0.153	−0.889
	Teacher-ratings [a]	10(13.7)	28(38.4)	20(27.4)	15(20.5)	2.55	0.972	0.094	−0.977

[a] Established grade group (None: 1–4.9; Few: 5–5.9; Enough: 6–8.9; Completely: 9–10).

The comparison of means shows that the total mean of the experimental group is 2.5, i.e., in the intermediate zone. This indicates that the scores have not been high, but rather average. In contrast, in the control group, the idealised mean is at 1.7, which marks a low response trend. If the totalised means of the control group and the experimental group are compared, a considerable distance is observed, so that the application of one teaching method or another influences the dimensions studied. In the control group, the student–student mean stands out from the idealised mean. The same occurs with class time in the experimental group (Figure 1).

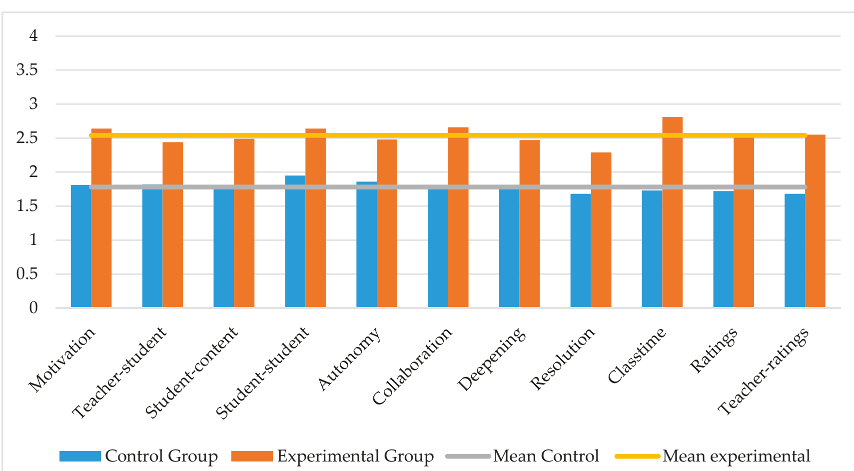

Figure 1. Comparison between control group and experimental group.

By means of the Student's t-test, the relationship of significance, in each of the study dimensions, of the b-learning method in relation to the expository method has been identified. In this case, the statistical values show the relation of significance in all the study dimensions. The greatest difference in means is presented in class time, with up to one point of difference. On the other hand, the dimension with less mean difference is teacher–student. If we take into account the strength of association, we can see that the relationships are medium and low-medium. The dimensions class time, collaboration, ratings, teacher-ratings and motivation have a medium relationship strength. The dimensions with a medium-low relationship are located the rest of the dimensions. According to Cohen's d, the effect size is very low in all dimensions, except in class time, ratings and teacher-ratings, where the effect size is low (Table 3).

Table 3. Study of the value of independence between control group and experimental group.

Dimensions	μ(X1–X2)	$t_{n1+n2\text{-}2}$	*df*	*d*	r_{xy}
Motivation	−0.833(1.81–2.64)	−5.503	145	0.084	0.416 **
Teacher-student	−0.587(1.82–2.41)	−3.614	145	0.123	0.287 **
Student-content	−0.723(1.77–2.49)	−5.141	145	0.059	0.393 **
Student-student	−0.698(1.95–2.64)	−4.949	145	0.057	0.350 **
Autonomy	−0.615(1.86–2.48)	−3.952	145	0.034	0.312 **
Collaboration	−0.860(1.80–2.66)	−5.676	145	0.059	0.426 **
Deepening	−0.682(1.78–2.47)	−4.423	145	0.074	0.345 **
Resolution	−0.612(1.68–2.29)	−4.276	145	0.069	0.335 **
Class time	−1.078(1.73–2.81)	−7.376	145	0.165	0.522 **
Ratings [a]	−0.818(1.72–2.53)	−5.516	145	0.175	0.416 **
Teacher-ratings [a]	−0.872(1.68–2.55)	−6.063	145	0.140	0.450 **

** The correlation is significant in level 0.01. [a] Established grade group (None: 1–4.9; Few: 5–5.9; Enough: 6–8.9; Completely: 9–10).

5. Discussion

The results achieved have been able to show that the b-learning method is an effective teaching and learning process, compared to the expository method. In this case, it is effective with students of Basic Vocational Training, in the module of Applied Sciences I. In other words, it is effective for students who are at risk of social exclusion.

The inclusion of ICT in the educational field is enabling innovative teaching and learning processes to be applied, thus favouring the academic development of students [1–5].

If we analyse each of the groups in detail, we can see that, as in the control group, the results achieved are relatively low in all the study dimensions. This may be due to the characteristics of the students in Basic Vocational Training already indicated by [68], where the students present a poor academic background in previous educational stages. Resolution and teacher ratings are among the least valued dimensions. That is, they present difficulties in the resolution of the various pedagogical actions, and therefore, this is reflected in the qualifications established by the teacher.

An example of this is the b-learning teaching method, considered as a didactic process that mixes the best of the expository method with the best of the e-learning method, allowing, among other aspects, to adapt to the rhythms and learning styles of the students, as well as to provoke a change in the roles of the agents involved in the pedagogical act [14–20].

In the present study, we have analysed how the b-learning method influences the students of Basic Vocational Training, specifically in the module of Applied Sciences I. To this end, a contrast has been established with the expository method. According to the results obtained, it is observed, in general terms, that there are better averages in the experimental group, which has developed the b-learning method, with respect to the control group, which has received a teaching based on the expository method.

Additionally, in the group that has developed the b-learning method, the ratings are higher than those offered by the control group, although it does not present very high ratings. Rather, the scores are average. Even so, there is a contrast between the groups, so it can be considered that the b-learning method favours the academic development of the students. The most valued dimension has been class time. This may be because the method applied may be new to students. On the other hand, the less valued dimension is resolution. In this case, the same happens as in the control group, so the educational base of these students is affected for the development of any educational process they develop, although unlike the control group, the grades are not so affected.

In both groups, the response trend is grouped, so that students maintain the same line of assessment, according to the teaching method applied. That is to say, they agree when giving their opinion about one teaching process or another.

In this case, it can be indicated that the b-learning method, in comparison with the expository method generates an improvement in Basic Vocational Training students in motivation [52]; in the relationship between the teacher and the student [54]; in the relationship between the students and the didactic content [49]; in the relationship between students [51]; in autonomy [55], in collaboration [56]; in the deepening of content [50]; in the resolution of pedagogical acts; in the perception of the class time; in self-evaluation [59]; and in the grades of the module studied [53].

In other words, this study confirms that which has already been established by other authors in relation to the b-learning method and the expository method. With the b-learning method, a positive attitude is produced in the student, since it generates an active process in the formative process. On the other hand, the expository method generates a passive act in the teaching and learning process in the students themselves. The difference of the student's role in these methods provokes significant differences in motivation, in the relationship established between the agents involved in the training process, in the self-management in the collaboration, in the deepening of content, in resolution, in the classtime and in the academic performance.

6. Conclusions

It can be concluded that the b-learning method is effective in the teaching and learning processes of the students of Basic Vocational Training in the module of Applied Sciences I, in comparison with the expository method, having a direct influence on the feeling of the students' own class time. This research shows how important it is to introduce this type of innovative method in the vocational training stage, since it has important advantages for students in many aspects of their learning processes.

The prospective of this study is on two different levels. On the one hand, it tries to provide data to the scientific community on the use of the b-learning method in Basic Vocational Training students. On the other hand, it tries to offer an effective teaching and learning process for teachers working with these types of students.

The limitations of the study are the focus on the selection of the sample, which has been for convenience. In addition, the study population presents specific characteristics, so one must be cautious when extrapolating the results. For future lines of research, it is proposed to analyse this teaching and learning process in the second year of Basic Vocational Training and in other modules.

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mathematics

Article

Analysis of Factors Influencing Students' Access to Mathematics Education in the Form of MOOC

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Abstract: Restricting the movement of students because of COVID-19 requires expanding the offer of online education. Online education should reflect the principles of pedagogical constructivism to ensure the development of students' cognitive and social competencies. The paper describes the preparatory course of mathematics, realized in the form of MOOC. This course was created and implemented based on the principles of pedagogical constructivism. The analysis of the respondents' approach to MOOC revealed a difference between bachelor and master students in the use of MOOC. Bachelors found a strong correlation between their approach to MOOCs and the way they are educated in secondary schools. The results of the research point to the need of more emphasis should be placed on advancing the learner's skills in navigating and analysing information. The questionnaire filled in by the participants also monitored the students' access to learning. The results of the experiment confirmed the connection between the preferred approach to learning and students' activities within the MOOC.

Keywords: constructivism; mathematics learning; MOOC; new teaching techniques; students' access to MOOC

1. Introduction

Recently, many pedagogical experts have questioned traditional teaching methods such as lectures and testing [1] (pp. 167–202), [2] (pp. 3–17). According to Mascolo [2], the basis of pupil-centred education is constructivism. Constructivism is based on the European genetic epistemology of Jean Piaget and American cognitive psychology. Constructivist epistemology includes cognitive constructivism and social constructivism [3] (pp. 241–250). Cognitive constructivism pursues the individual development of knowledge through interaction with the environment, and social constructivism refers to the dialogue of students with each other and the teacher and to the social context in which learning takes place [4] (pp. 61–86). According to Lave and Wenger, an important part of constructivism is social constructivism, which focuses on cultural and social learning conditions, on social interaction in learning [3,5] (pp. 241–250). Pedagogical constructivism is a combination of cognitive and social constructivism and demands that teaching should use authentic problem solving, creative thinking and group work [5]. Medová and Bakusová [6] (pp. 142–150) stressed the role of real-life problems in constructivist mathematics education. Astin [7] also focused on group work in his research.

Recently, we have seen a massive increase in the offer of various online courses, even for university students. There are countries where online courses have become an integral part of teaching, especially at universities. In the USA, for example, more than 30% of university students attend at least one online course [8]. Using state-of-the-art computer technologies, online courses offer students a wide range of engaging and interactive learning environments that have demonstrated support for satisfaction, motivation and persistence among participants [9] (pp. 435–447), [10] (pp. 24–32), [11] (pp. 221–231), [12] (pp. 306–331).

Online courses encourage students to be independent, to develop the skills of personal reflection and abstract conceptualization [13] (pp. 227–243), [14] (pp. 309–328). For more success in using online courses [15] (pp. 1–28) suggests more emphasis should be placed on advancing the learner's skills in navigating and analysing information.

Another, relatively new, but especially effective element in online learning are the so-called massive open online courses or MOOC. The term "massive open online course," or MOOC, was first used to describe a course on learning theory taught by George Siemens and Stephen Downes at the University of Manitoba in 2008. According to Downes, the idea was to "invite the rest of the world to join the 25 students who were taking the course for credit" [16].

MOOC is based on the principle of sharing and freedom. According to Jeffrey [17], this is "self-service learning and crowdsourced teaching".

MOOC courses meet with several positive responses in the professional community. For example, Friedman [18] (pp. 175–186) considers the MOOC a breakthrough in higher education, and Mozoué [19] sees them as an alternative to full-time education and making education accessible to a wider range of society. There are also doubts about their contribution to higher education. Several higher education analysts are sceptical and express their doubts as to whether the MOOC is an adequate alternative to classical higher education or online education, especially in terms of teaching and access to students [8,20] (pp. 7–26), [21] (pp. 87–110). They also point out that the use of MOOC requires participants to be able to work independently and thus have the necessary level of critical literacy and the ability to navigate the course. Therefore, according to Kop, Fournier, & Mak [22] (pp. 74–93), more experienced and independent students are more successful in this environment. It also happens that many participants are struggling with a lack of instructional support at the MOOC and do not complete their courses.

There are currently several empirical studies that evaluate not only the MOOC teaching strategy but also the results of MOOC-related learning. According to Toven-Lindsey [23] (pp. 1–12) and Rhoads and Lozano [21] (pp. 87–110), there are considerable differences in pedagogical approaches, most courses still use elements that are common in traditional classes, including lectures, multi-choice assessments, and discussions about current groups.

Currently, there is already an offer of mass open online courses (MOOC) in Slovakia, but such offer is limited—only in some universities are MOOC offered for selected courses, as a means of supporting the quality of education. This is even though external students make up approximately one quarter of university students. According to [24] (pp. 451–460), due to many online learning opportunities, including MOOC courses, it is necessary to analyse their quality and to improve the effectiveness of education using analytical methods. One of the first Slovak universities involved in the MOOC project since 2013 is the Slovak University of Technology. Slovakia was also involved in the project BizMOOC—Knowledge Alliance to enable a European-wide exploitation of the potential of MOOCs for the world of business Programme: Erasmus+. It was found in the project that one of the major obstacles to using MOOC is the language barrier (see www.bizmooc.eu). The above findings showed the need to create MOOC in the national language in Slovakia. The aim of our research was to create a preparatory MOOC of mathematics that would consider the principles of pedagogical constructivism and to conduct research on the behaviour of students in using this course. In this way, we wanted to find out whether not only the cognitive but also the social component of the student's personality in relation to his/her approach to learning develops within the MOOC. This would determine whether MOOC can be a suitable alternative to full-time education.

2. Materials and Methods

2.1. Objective

For several years, many Slovak universities have introduced the so-called "tutoring mathematics" in the form of various mathematics courses. These courses are intended primarily for students admitted to the 1st year of higher education and are usually organized in full-time form lasting from 3 to 5 days. It was this situation that motivated us to create a pilot preparatory e-learning course in mathematics. In our case, we chose a relatively under-used MOOC model in Slovakia, where students can not only educate themselves but also discuss and present their problem-solving procedures. In addition to the above, we were motivated by idea of Giroux [25], according to which students should not only be educated, but also be active participants in the learning process. Our research team has set a goal to develop a pilot mathematics preparatory course in the form of MOOC and examine its use by students. At the same time, we investigated whether, in addition to the development of cognitive competencies, participants in the course also develop social competencies. In the case of the development of both competencies, the MOOC in the proposed form could be a suitable alternative to the full-time form of education.

As courses of a similar type were not available in Slovakia so far, we developed the course ourselves and offered it to the students and we not only observed the extent to which the students used the course, but we also considered it necessary to find out the students' reactions to the product. We were therefore interested in the extent to which students will use the different parts of the MOOC and what attitude they will take to it.

The MOOC course lasted one month and was made available to students admitted to the first year of undergraduate and graduate study at the technical faculty of a selected university in Slovakia. There were separate MOOC modules for each stage of the study with respect to the achieved education.

During enrolment in the first year (both undergraduate and graduate), students were acquainted with a preparatory course in mathematics in the form of the MOOC containing the "mathematical minimum" needed to master mathematics in the given field of study for which they were admitted. Created MOOC and possibilities of its use were introduced to students by MOOC authors themselves. At the same time, each student received access data to the portal. The access data were anonymous for the research team, only used to monitor the activities of individual students within the MOOC. These data were also used in the final questionnaire. Student activity data served as data for statistical evaluation of MOOC rate and usage. Our MOOC consisted of the following modules: Module 1—algebraic equations and inequalities, Module 2—non-algebraic equations and inequalities, Module 3—functions, Module 4—elemental geometry. Each module was given a week within the MOOC. MOOC was created and launched on the website: https://www.mooc.km.fpv.ukf.sk/, which we were developing for a long time. This training system works both in Slovak and English and the course materials for individual modules were gradually made available. The study materials were divided into two parts. The first part consisted of theoretical bases of the studied problems such as definitions of basic terms (8 pdf files) and assignments of tasks in text form (8 pdf files) and the second part consisted of sample examples in audio-visual form. The video sequences included instructions for solving basic sample examples for individual modules (32 video sequences). Solutions of various problem tasks supporting the construction of new computational strategies for students were the subject of webinars. Every Friday, the webinar was held twice (at 10:00 and 17:00), which was focused on problematic issues related to topics provided to MOOC participants in each week. The webinar was always led by a member of the author team. The aim of the webinar was to support the ability of students to create their own solutions of given tasks with creative use of already acquired theoretical knowledge and skills with solving standard tasks. The principles of pedagogical constructivism were consistently applied to webinars. The heuristic general didactic method was used, in which the teacher acted as a moderator of the participants' discussions. Each registered participant automatically became a member of the MOOC discussion forum without teacher participation. At the same time participants could create different discussion subgroups—these subgroups could be created by the participants. Another possibility through MOOC that we created was the possibility to address the teacher in the form of a question or by requesting to check the correctness of the task. From the questions asked to the teacher, we gradually created the content in the "Frequently Asked Questions" menu. For each of the topics covered, exercises were also available for download with the option to send suggested solutions to the teacher for review.

2.2. Sample

We were interested in the extent to which students will use the individual parts of our MOOC course and what attitude they will take towards it. The research took place in September before the beginning of the winter term of the academic year 2018/2019 at a selected university of the Slovak Republic. Respondents of the research were engineering fields of study students, namely 48 undergraduate students and 35 graduate students. The respondents were between 19 and 26 years of age. Participation in MOOC was voluntary, which was also reflected in students' lower interest from compared commonly used full-time form.

2.3. Information Collection Tools

The data necessary for the evaluation of the research were obtained by monitoring the activity of students involved in the MOOC preparatory course of Mathematics. We monitored the number of views of each video sequence and the number of downloads of study materials. An important source of data was the content and form of discussions among students within the discussion group. The administrator was able to track the overall activity of each MOOC member, so it was possible to determine the priorities of each MOOC member when choosing the options offered within the course. After completing the MOOC course, respondents completed a questionnaire. All students who participated in the MOOC were able to complete the questionnaire, regardless of whether they completed the course or not.

3. Results and Discussion

The basis for evaluating the suitability and usability of MOOC as a preparatory course in mathematics was a questionnaire developed and used by Aharony and Bar-Ilan [26] (pp. 146–152). Just like the authors of the questionnaire, in our case we also observed 5 areas in the questionnaire:

1. Personal data
2. Perception of usefulness questionnaire (PU) (3 questions)
3. Ease of Use Perception Questionnaire (PEOU) (3 questions)
4. Learning Strategies (LS) (14 questions)
5. Cognitive Assessment Questionnaire (CAQ) (9 questions)

The 4th part of the questionnaire—Learning Strategies (LS), which reflects the student's approach to learning and education, was very important for us. According to [26] there is the deep learning versus the surface learning approach; terms that are based on the early work of Marton and Säljö [27]. Deep learners tend to seek for their 'inner self' through the learning process [28,29].

Contrarily, surface learners learn only important and essential facts, applying minimum study efforts [28]. A surface learning approach is associated with students who study only superficial details [30]. They are concerned with the time needed to accomplish the learning task; therefore, they try to choose the quickest way to accomplish their learning assignment, without asking further questions and without fully understanding the text meanings. Surface learners usually memorize facts; thus, meta-cognitive skills are mostly not involved in their learning process [28].

Cognitive appraisals of threat and challenge refer to "dispositions to appraise ongoing relation-ships with the environment consistently in one way or another" [31] (p. 138). Cognitive appraisal addresses the person's evaluation of events for his or her well-being [32].

For the reasons stated above and also in accordance with [26], we divided the fourth area of the questionnaire into two parts (areas), namely: learning strategies: deep learning (LS-D), consisting of questions 1, 3, 6, 8, 12, 13 and surface learning (LS-S), which consisted of questions 2, 4, 5, 7, 10, 11, 14. For the same reasons, we divided the fifth questionnaire into two parts: threat perception (CAQ-T), consisting of questions 1, 2, 3, 4, 6, 7, and challenge perception (CAQ-CH), which consisted of questions 5, 8, 9.

Part of our research was also tracking the activities of students who attended the MOOC course. We divided the activities into two areas: cognitive constructivism (MOOC-CC) and social constructivism (MOOC-SC).

Subsequently, we identified research questions:

Q1: Are there significant differences in student responses in each area of the questionnaire relative to their degree (bachelor or master)?
Q2: Is there a relationship between student responses in each questionnaire area?
Q3: Is there a relationship between students' access to education and their attitudes to using MOOC?
Q4: Is there a relationship between students' access to education and the use of individual areas of activity in our MOOC course?
Q5: Is there a relationship between the perception of new situations in MOOC among students and the use of individual areas of activity in our MOOC course?

To find answers to individual research questions (Q1–Q5) we analysed the results obtained by the questionnaire method as well as by monitoring the respondents' activities. There was an answer to each question on the 5-point Likert scale, where 1 means "absolutely disagree" and 5 means "totally agree". The results obtained in our research by the questionnaire method in both groups of students are illustrated in the following figures (Figures 1–6).

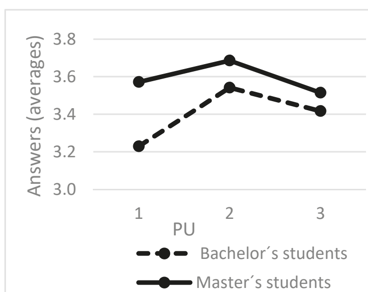

Figure 1. Answers of undergraduate and graduate students in PU (average values).

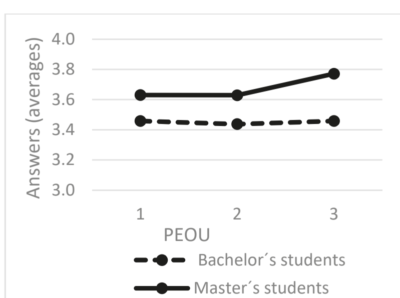

Figure 2. Answers of undergraduate and graduate students in PEOU (average values).

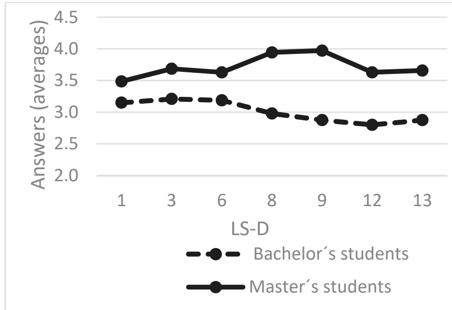

Figure 3. Answers of undergraduate and graduate students in LS-D.

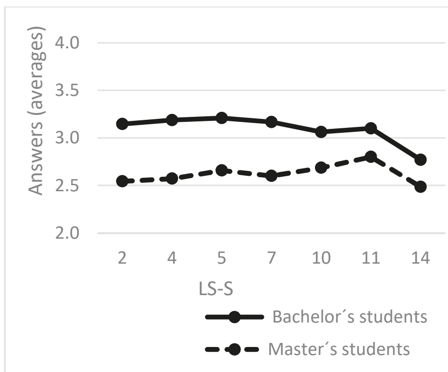

Figure 4. Answers of undergraduate and graduate students in LS-S.

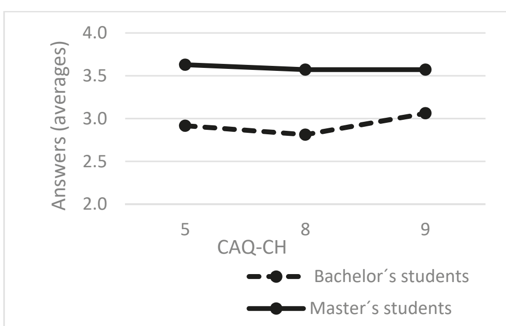

Figure 5. Answers of undergraduate and graduate students in CAQ-CH.

In Figures 1–6 we can see that there are differences between the answers of students of undergraduate and graduate study to questions in individual areas of the questionnaire. We wondered if the differences are statistically significant.

The statistical significance of the differences between the two groups of students in the answers to the questions was verified in each area of the questionnaire (PU, PEOU, LS-D, LS-S, CAQ-CH and CAQ-T) based on calculated values, called total score. As the assumption of a normal distribution of observed traits was not met, we used the non-parametric Wilcoxon two-sample test to verify the Q1 research hypothesis [32].

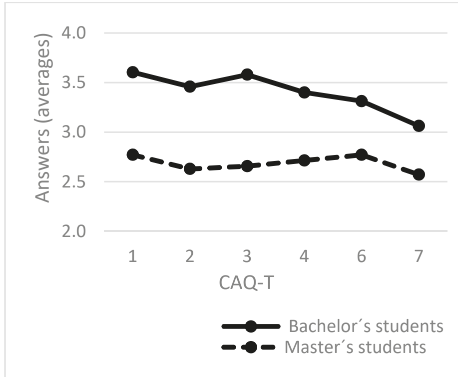

Figure 6. Answers of undergraduate and graduate students in CAQ-T.

In our case, the first selective file consists of undergraduate students and second file consists of graduate students. The results of the selected area of the questionnaire (total score) of both groups of students represent the realization of two mutually independent random samples from continuous distributions. We conducted Wilcoxon's two-sample test in STATISTICA.

The results obtained using Wilcoxon's two-sample test were summarized in the following Table 1.

Table 1. The results of Wilcoxon's two-sample test.

Questionnaire Area	*Z*	*p*
PU	−0.810	0.418
PEOU	−1.600	0.109
LS-D	−3.489	0.000 *
LS-S	1.393	0.164
CAQ-CH	−3.531	0.000 *
CAQ-T	5.368	0.000 *

Note: Values exceeding the critical value are indicated * in the table.

Since the calculated probability value $p < 0.05$, in three cases—in the LS-D, CAQ-CH and CAQ-T areas, the hypothesis H_0 is rejected in all three cases at the significance level $p = 0.01$ and we can say that among the undergraduate and graduate groups is a significant difference in the answers to the questionnaires in the LS-D, CAQ-CH and CAQ-T.

Based on the results obtained in the statistical analysis of PU, PEOU and LS-S questionnaire, the hypothesis H_0 cannot be rejected, i.e., the observed differences are not statistically significant.

Analysis of Student Activity within MOOC

Figures 7 and 8 show the average MOOC visit values for each activity. The activities of undergraduate and graduate students were evaluated separately. This division was based on the results of the questionnaire in the field of LS, where it turned out that undergraduate and graduate students have different approaches to education. While graduate students prefer deep learning, undergraduate students prefer surface learning. Therefore, when answering other research questions, we evaluated the individual parts for undergraduate and graduate students separately.

We divided the activities in the created MOOC course into two parts. The first part was called "Cognitive Constructivism", which included those activities where there was no cooperation with other course participants or with the teacher. They were theory, video, and exercises to practice. The average utilization of the individual activities is shown in Figure 7.

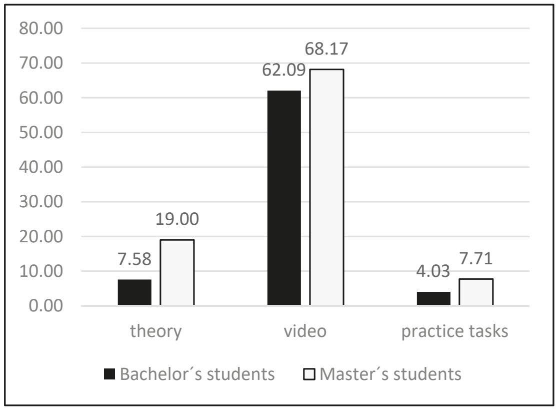

Figure 7. Average visit in options under 'Cognitive constructivism'.

In both groups of students, the most attention was paid to video sequences. In all activities graduate students were more active in all activities. The biggest difference was in the use of the offer of the necessary theoretical knowledge on the individual topics covered within our MOOC.

The second part consisted of activities in which the participants cooperated with each other and possibly with the teacher. We called this part "Social Constructivism" and it included activities: a webinar, a question to the teacher, a discussion forum, frequently asked questions.

Based on the results shown in Figure 8, graduate students were more active in the second part activities, and even more than in the first part. Only in the activity "Question to the teacher" were undergraduate students more active. We interpret this because of the fact that undergraduate students come from a high school (secondary school) environment where the teacher has a dominant position in pupils learning [33].

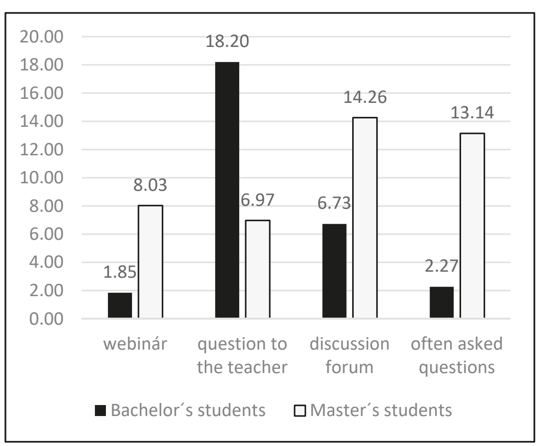

Figure 8. Average visit in options under 'Social constructivism'.

Again, we can see that there are differences between both groups of students (undergraduate and graduate) in both activity areas. We were interested in finding out whether these differences as well as the links between the monitored areas in both groups of students are statistically significant. We used the statistical method—Spearman order correlation coefficient, which expresses the degree of dependence between X and Y.

In our case, we calculated the following Spearman correlation coefficient values for both groups of students (Tables 2 and 3).

Table 2. Spearman correlation coefficient (undergraduate students).

	PEOU	LS-D	LS-S	CAQ-CH	CAQ-T	MOOC-CC	MOOC-SC
PU	0.43	−0.03	0.71 *	0.00	0.21	0.28	0.11
PEOU	1.00	0.00	0.75 *	0.19	0.29	0.38	0.07
LS-D		1.00	−0.05	−0.02	−0.04	0.02	−0.09
LS-S			1.00	0.14	0.35	0.49 *	0.09
CAQ-CH				1.00	0.04	−0.08	−0.27
CAQ-T					1.00	0.40	−0.02
MOOC-CC						1.00	0.05
MOOC-SC							1.00

* $p < 0.05$.

Table 3. Spearman correlation coefficient (graduate students).

	PEOU	LS-D	LS-S	CAQ-CH	CAQ-T	MOOC-CC	MOOC-SC
PU	0.64 *	0.70 *	0.27	0.67 *	−0.43	0.71 *	0.73 *
PEOU	1.00	0.82 *	0.03	0.57 *	−0.30	0.61 *	0.67 *
LS-D		1.00	0.14	0.69 *	−0.41	0.58 *	0.70 *
LS-S			1.00	0.12	0.29	0.15	0.02
CAQ-CH				1.00	−0.48	0.54 *	0.56 *
CAQ-T					1.00	−0.54	−0.35
MOOC-CC						1.00	0.57 *
MOOC-SC							1.00

* $p < 0.05$.

We observe in undergraduate students a high degree of bonding between PU and LS-S ($R = 0.71$) and between PEOU and LS-S ($R = 0.75$). It is the relationship between surface learning and attitudes to the use of MOOC. So, we can state that the more undergraduate students prefer a superficial approach to learning, the more they consider MOOC to be a useful and easy to use tool. Based on the results, a significant degree of binding was also observed between LS-S and MOOC-CC ($R = 0.49$), i.e., between surface learning and cognitive constructivism activities in MOOC. This can also be interpreted as suggesting that undergraduate students with a superficial approach to education tend to use activities that do not interact with other course participants. Further evaluation revealed a significant correlation ($R = 0.5$) when using video in MOOC, which undergraduate students considered the easiest way to obtain information. We can say that undergraduate students approach the use of MOOC rather than a suitable tool to help them master the curriculum with minimal effort. Other connections between observed areas in undergraduate students were not statistically significant.

A significant degree of linkage between several fields of study can be observed in the graduate students. In particular, we observe a high degree of binding between LS-D and PU ($R = 0.7$), PEOU ($R = 0.82$) as well as CAQ-CH ($R = 0.69$). Based on the correlation coefficient values given above, it can be said that the more graduate students prefer a profound approach to learning, the more they perceive the MOOC as a cognitive challenge and consider it a useful and very usable tool for learning. In other words, based on the results we can see that the graduate students prefer deep learning. Equally significant is the degree of binding between LS-D and MOOC-CC (R = 0.58) and also between LS-D and MOOC-SC (R = 0.70), i.e., between deep learning and cognitive and social constructivism activities in our MOOC course,

Unlike undergraduate students, where these links were not confirmed at all. Based on the calculated values of correlation coefficients, we observe significant degrees of binding in the MOOC-CC between the use of tasks and exercises (R = 0.61) and theory (R = 0.53). The use of video had only a slight degree of custody (R = 0.37) regarding given access to education compared to undergraduate students. However, a significant degree of binding has been shown between the use of video and the perception of the usefulness of MOOC (R = 0.57) in graduate students. This means that the more graduate students perceive the usefulness of MOOC, the more they use video sequences in the MOOC course. In the MOOC activities included in social constructivism, there were significant degrees of links between deep learning and webinar (R = 0.63), discussion forum (R = 0.64) and frequently asked questions (R = 0.53). Based on the calculated correlation coefficient values, we got a zero link between LS-S and the activity *question to the teacher*, which was often used by undergraduate students. Based on the above results, we can conclude that in the case of graduate students with a perception of MOOC as a challenge for their education, a positive attitude towards MOOC from the point of view of its usefulness and ease of use also strengthens.

The results of our research correspond to those of [26], which also confirmed the differences between undergraduate and graduate students in LS and CAQ. The behaviour of students within our MOOC was also confirmed by the findings of Kop, Fournier, & Mak [22], that more experienced and independent students are more successful in the MOOC environment. Based on the above, it can be concluded that more frequent use of MOOCs in higher education institutions requires more space should be devoted to activities falling under the principles of social constructivism in secondary schools. In acquiring new knowledge based on the principles of cognitive constructivism, we recommend using the heuristic method in secondary schools, where the teacher acts as a moderator of the pupil's learning. In this way, critical literacy of pupils will be strengthened, which, according to Lewin [34], is one of the important prerequisites for successful mastery of MOOC.

4. Conclusions

Based on the results of the research, it can be stated that students' access to education, as well as the perception of new situations in terms of threat or challenge depends on the level of study. Master students prefer deep learning, so they care more about understanding the subject matter and the value of their knowledge, they are willing to devote more time to study. Bachelor students, on the other hand, prefer surface learning, i.e., the acquisition of the necessary knowledge without a deeper understanding and with the least effort. It was also shown that with the growth of the preference for surface learning among bachelor students and with the growth of the preference for deep learning among master students, there is also a growing perception of the usefulness and applicability of MOOCs. An important element of the research was the students' approach to the activities in our MOOC course, which were divided into cognitive and social constructivism activities. Significant differences between bachelor and master students emerged in this area. Bachelor students showed a positive correlation between their superficial approach to education and the use of MOOC activities included in cognitive constructivism. They used videos the most for their education, which we can consider the easiest way to get information. In the quantitative evaluation of attendance at individual activities of the course, bachelor students more often used the possibility of asking questions to the teacher than masters. On the contrary, master students showed a positive correlation between their perception of MOOC as a challenge and the use of activities of both cognitive and social constructivism in our course. Significant degrees of connection were also shown in the masters between their in-depth approach to education and the use of all activities of our MOOC course with a higher preference of those that were included in social constructivism. Compared to bachelor students, they made much more use of the discussion forum, webinar, theory on the topic as well as tasks and examples.

We believe that differences in behaviour and attitude towards MOOC among undergraduate and graduate students reflect strongly on the way and methodology of education at secondary schools in Slovakia. Undergraduate students attended the MOOC course before they started university (before the semester), so we can attribute their behaviour to high (secondary) school students with very little or no e-learning experience. On the other hand, awareness, and responsibility for the study in terms of its need for the future and self-assertion in life is still low for 19-year-olds. Another aspect may also be the overall atmosphere in society and the speed of time when young people have higher demands on their surroundings, but not towards themselves. On the other hand, graduate students already have a bachelor's degree in higher education and know that they are required to be independent and responsible in their studies. They also have more experience with e-learning and are more open to communicating with their classmates and educators.

Based on our findings, we can state that MOOCs created on the basis of pedagogical constructivism have the potential to be a full-fledged alternative to full-time education. However, future MOOC participants need to be prepared to prefer deep learning.

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Measuring Arithmetic Word Problem Complexity through Reading Comprehension and Learning Analytics

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Abstract: Numerous studies have addressed the relationship between performance in mathematics problem-solving and reading comprehension in students of all educational levels. This work presents a new proposal to measure the complexity of arithmetic word problems through the student reading comprehension of the problem statement and the use of learning analytics. The procedure to quantify this reading comprehension comprises two phases: (a) the division of the statement into propositions and (b) the computation of the time dedicated to read each proposition through a technological environment that records the interactions of the students while solving the problem. We validated our approach by selecting a collection of problems containing mathematical concepts related to fractions and their different meanings, such as fractional numbers over a natural number, basic mathematical operations with a natural whole or fractional whole and the fraction as an operator. The main results indicate that a student's reading time is an excellent proxy to determine the complexity of both propositions and the complete statement. Finally, we used this time to build a logistic regression model that predicts the success of students in solving arithmetic word problems.

Keywords: learning; reading comprehension; complexity; problem-solving; arithmetic word problems; fraction operator; technological environment

1. Introduction

Previous work has studied the relationship between performance in mathematics problem-solving and reading comprehension in students of all educational levels [1–3]. Authors such as Pólya [4] and Puig and Cerdán [5] have shown that reading and understanding the statement are key phases of the problem-solving process. The National Council of Teachers of Mathematics (NCTM) [6] determined that, in solving a mathematical problem, many of the necessary skills present in all areas of the educational curriculum are required, such as reading, reflection and understanding. The latest PISA report [7] indeed highlights that a solid reading competence is fundamental for academic achievement in all subjects of the educational system (including mathematics), while being a prerequisite for successful participation in most adult life [8–10].

Our research is framed within the context of arithmetic word problems (from now on AWPs or AWP in singular) and focuses on how to measure the complexity of the statements involved. To this end, we computed the reading comprehension of students through a technological environment and use learning analytics to predict student performance in solving this sort of problems.

1.1. Complexity of Arithmetic Word Problems

AWPs are texts or statements describing real-life situations in which unknown quantities need to be determined from other amounts that are known [5,11,12]. AWPs are some of the first problem-solving activities in the elementary school mathematics curriculum, and as such, they deserve special care and attention.

The complexity of AWPs has been conceptualized through research on the resolution of verbal problems and on the difficulties they present for schoolchildren. Daroczy et al. [13] showed that these difficulties can be caused by either one or a combination of linguistic and numerical complexity. Linguistic complexity refers to the linguistic and morphological aspects of the statement (e.g., how words are combined to form the text). Numerical complexity, in turn, refers to the numerical factors of the statement (e.g., both the quantities and the relationships between them).

According to Castro et al. [14], the complexity of AWPs can be measured following four main approaches:

- The linguistic approach, based on the student's reading ability [15] and the readability of external texts different from the AWP statement [16].
- The structural variables approach, based on the so-called task variables (e.g., syntactic variables or context variables) as defined by Kilpatrick [17] or Goldin and McClintock [18].
- The open sentences approach, based on the situation of the question within the statement [19].
- The semantic approach, based on the semantic structure of the statement, considered as a whole [5] or divided into segments [19], so that an association can be built between keywords and operators in a partial problem-solving process.

To the author's knowledge, none of the previous approaches has yet measured the complexity of AWPs through the students' reading comprehension of the statement itself. This aspect makes our research a novel and original contribution to the state of the art of mathematical problem-solving.

1.2. Measuring the Complexity of AWP Statements through a Technological Environment

To measure the complexity of an AWP, we split its statement into propositions, as follows from the partial semantic approach defined above. Our unit of analysis is thus a proposition, which contains a verb and a quantity associated with the related action. Figure 1 shows an example of an AWP statement [20] divided into three propositions.

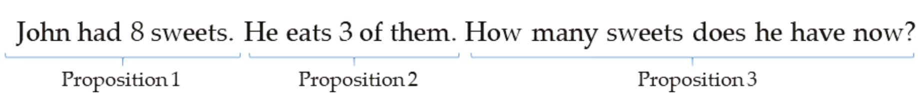

Figure 1. Example of an arithmetic word problem (AWP) statement ([20]) divided into propositions.

Propositions can be classified into levels to facilitate comparability and to determine their complexity. Hunt [21] names these constructions T-units or minimal terminable units of language. T-units each consist of a main clause plus the subordinate clauses it may include, and they can be organized on a number of levels: declarative sentences represent level 0; level 1 adds a subordination to sentences of level 0; level 2 adds a subordination to those of level 1 and so on. The higher the level, the more complex the sentence will be. This way, Proposition 1 in Figure 1 belongs to level 0, and Propositions 2 and 3 are of level 1, since they are respectively subordinate by the terms "of them" and "now".

We measure the complexity of each proposition by obtaining the time per word that students spent while reading the corresponding segment of the AWP statement. The time per word is computed through a technological learning environment able to control which information is displayed at any time and to register the interaction of students with the content. This novel approach is more powerful than to control reading from printed texts.

The use of intelligent tutors or technological learning environments (e.g., Moodle, Edmodo or Bakpax) has increased in recent years across all educational stages [22]. However, these environments have not yet been used to measure reading comprehension and the complexity of AWPs. These tools can usually be accessed through mobile devices and smart screens and allow one to register student–computer, student–teacher or student–content interactions [23–26], thereby giving rise to the so-called learning analytics research field. This field deals with applying data analytics to education and it is defined as the area of investigation in charge of measuring, compiling and analyzing data sets obtained through the use of computer-assisted learning platforms that track and record student digital interactions [27,28].

Technological environments and learning analytics are a cutting-edge approach to detect patterns on student strategies when solving a learning task. They are also helpful in understanding study habits, the use of teaching materials or the time dedicated to the proposed activities [29], sometimes supplemented by information on attendance, participation or motivation [30].

This work focuses on the analysis of the student–computer and student–content interactions obtained through the Read and Learn (R&L) technological environment [24,31]. R&L is a research tool to carry out experiments that analyze the strategies of students when they first have to read a text or problem statement and then answer a series of questions in a digital context.

1.3. Predicting Student Performance When Solving AWPs

Mathematical models have been extensively used to try to predict the probability of correctly solving a learning task. These models are commonly used to build a personalized route that guides students through an adapted teaching–learning process [32].

Logistic and Bayesian knowledge tracing models stand out among the statistical prediction models used for this purpose. The former have been used to predict the probability of success from the students' previous skills and the difficulty of the task [33]. The latter use hidden Markov models to estimate latent parameters and predict student success [32].

Following previous work on the matter [26,34], this work presents a binary logistic regression model to predict student performance from the complexity of an AWP measured by the reading comprehension of its statement.

The remainder of the paper is organized as follows. Section 2 describes the materials and methods used to measure the complexity of AWPs, the features of the R&L technological environment, a validation experiment for a sample population and the tested hypotheses. Section 3 presents the experimental results that determined the feasibility of our approach for assessing the complexity of mathematical problems through reading comprehension. Section 4 shows how to build a logistic model to predict student performance from the complexity computed for an AWP. Finally, discussion and conclusions are drawn in Section 5 in the context of the state-of-the-art literature.

2. Material and Methods

2.1. Procedure for Measuring the Complexity of AWPs

The complexity of an AWP can be derived from the complexity of all the propositions that form its statement. To estimate the complexity of a proposition, we compute the reading time per word for a group of students using the R&L technological environment. The reading time of proposition j in task i (T_{ij} in Equation (1)) thus comprises the time spent by each student (t_{ijs}) in the group (of size n) and the number of words in the proposition (k).

$$T_{ij} = \left\{ \frac{t_{ijs}}{k};\ s \in \{1,\ \ldots,\ n\} \right\} \quad (1)$$

The total complexity of an AWP can in turn be measured by averaging the previous reading times per student for all propositions (Equation (2)), where m represents the number of propositions in the statement.

$$T_i = \left\{ \frac{1}{m} \sum_{j=1}^{m} \frac{t_{ijs}}{k};\ s \in \{1, \ldots, n\} \right\} \tag{2}$$

2.2. Instrument

R&L is a technological environment in which to design research experiments on reading comprehension in text and image-related learning tasks. It is a web tool that can be accessed through mobile devices, computers and smart screens using any browser on any operating system.

Experiments in R&L can include enriched texts with a list of questions and answers. A number of configuration settings are available, such as the possibility of accessing the statement at any time or only under certain conditions, the effect of alternatively hiding and showing parts of texts by clicking on them (Figure 2), the use of open-ended or multiple-choice questions, the number of attempts allowed to complete the task or the definition of feedback to be given after answering the questions.

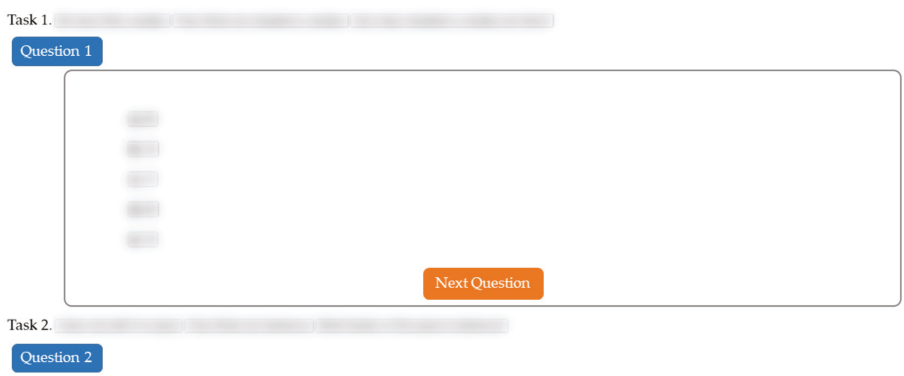

Figure 2. An example of an experimental setting in R&L wherein the texts of both questions and answer options are totally hidden.

R&L records all user interactions with the statements, questions and response options along with timestamps, which allows tracking the access history to the learning content with a level of precision of milliseconds. Any user action is registered, such as displaying a hidden proposition, moving the focus from the statement to the questions and vice versa. This way, we can determine aspects such as: what part of the statement the student is focused on, which point in time a certain proposition is read, how long a student remains in the same proposition, how many times a proposition is consulted and in which order students access the statement, the questions and the answer options.

R&L is able to digest these learning data flows and compute the variables of interest from the previously recorded data (e.g., the time reading a proposition or answering a question). Data can then be exported in CSV so it can be further used in any preferred data analysis software (e.g., R or SPSS). For more details on R&L the interested reader can check out the literature [24] and keep up with our website about data analytics and technological tools in education https://go.uv.es/grimo/datte.

2.3. Experimental Design

To test our proposal we have conducted a descriptive quantitative study involving a group of 70 students, 26 girls and 44 boys, aged between 15 and 16 years old.

At the time of the study, the students belonged to two public secondary schools in Spain selected by a convenience non-probability sampling. One school is located in an upper-middle socioeconomic area of a town of twelve thousand inhabitants. The other one is located in a multicultural suburb with medium-low socioeconomic status in a city of eight hundred thousand inhabitants.

Informed consent was obtained from schools, teachers and students before the start of the experiment. Anonymity of the data was guaranteed by just collecting the year of birth, gender, course and a dummy school code for each student. Any combination of data with a frequency of less than 5 observations was considered subject to statistical secrecy and it was removed to prevent de-anonymization.

The experiment was run individually using the school's computer room. Students were introduced to the R&L technological environment before starting the session. Following fair and ethical practices, participants were made aware that they were involved in a research study. They were clearly informed about the aims of the study and that their performance would not be considered in their grades.

Participants were asked to solve a couple of AWPs presented as two tasks with their corresponding statement and five answer options. The statements were designed taking in to account the mathematical and the grammatical complexity. We built two isomorphic tasks [35] dealing with mathematical introductory concepts related fractional numbers over a natural number, basic mathematical operations with a fractional whole and the fraction as an operator. In addition, we classify the propositions of the statements into levels as defined by Hunt [21], which allows the measured reading comprehension to be compared.

Tasks were written in Spanish since all participants were native Spanish speakers. For the sake of readability, we also show the translation of the statement into English as follows:

- Task 1: We have thirty candies. Two-thirds of them are strawberry flavored. How many strawberry candies do we have? (From the original: *Tenemos treinta caramelos. Si dos tercios son de fresa, ¿cuántos caramelos son de fresa?*) The possible answers are 5, 10, 17, 20 and 45.
- Task 2: I have one-half of a pizza. Two-thirds of it is margherita. What fraction of the pizza is margherita? (From the original: *Tengo media pizza. Si dos tercios son de margarita, ¿qué porción de pizza es de margarita?*) The possible answers are 4/3, 3/5, 1/3, 7/6 and 1/6.

Both tasks have an equal mathematical structure, expressed in terms of the relationships between the variables and quantities involved. This means that they are solved by applying the same rules, procedures, and algorithms. The question is placed at the end of the statement following the pattern $a\ x\ b = ?$ where a and b are known quantities. Note that the semantic relationship between the variables and the unknown quantity, the lack of data in the question and the absence of irrelevant data is equivalent in both statements. The tasks can be classified as two AWPs of multiplicative comparison according to Puig and Cerdán [5]. This sort of problems use a scalar function (I) to link two extensive quantities (E) of the same type of magnitude ($E\ x\ I = E$, the Schwartz relation [36]). For example, the scalar function in task 1 is "two-thirds of," while the two extensive quantities are "thirty candies" and the unknown quantity of "strawberry candies".

The proposed AWPs use the fraction (i.e., two-thirds) as an operator [37] that transforms an initial quantity (i.e., thirty candies or one-half of a pizza) into a final quantity (e.g., strawberry candies or a fraction of the pizza). This transformation is associated with the scalar function and the multiplication operator, as shown in Figure 3. The tasks are consistent [38] since they can be solved by directly translating the key terms in the statement (e.g., are or is) into the operation to be performed, in this case a multiplication.

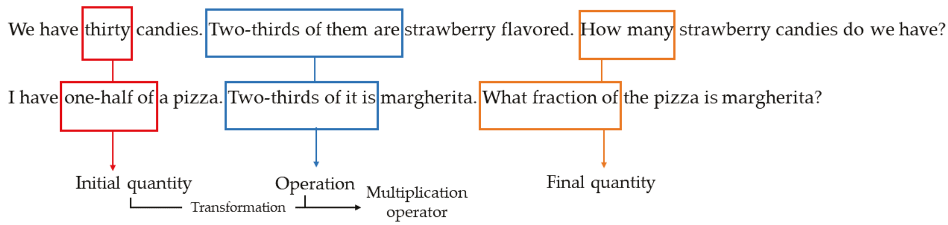

Figure 3. Use of the fraction as an operator in the proposed AWPs.

We can determined the grammatical complexity of the tasks by dividing the statement into propositions and analyzing their syntax. Each statement is composed of three propositions, as shown in Table 1. The first two relate to the informative part of the statement and the third one is the question. We configured the tasks in R&L so that just one proposition could be displayed at a time while the rest of them remained hidden (see the different colored segments in Figure 4).

Table 1. Propositions of tasks 1 and 2.

Prop	Task 1	Prop	Task 2
P11	We have thirty candies *Tenemos treinta caramelos*	P21	I have one-half of a pizza *Tengo media pizza*
P12	Two-thirds of them are strawberry flavored *Si dos tercios son de fresa*	P22	Two-thirds of it is margherita *Si dos tercios son de margarita*
P13	How many strawberry candies do we have? *¿cuántos caramelos son de fresa?*	P23	What fraction of the pizza is margherita? *¿qué porción de pizza es de margarita?*

The length of the informative parts is the same in both statements (i.e., 3 + 6 words for P11 + P12 and P21 + P22 as from the original text in Spanish). The number of words in the question part differs (i.e., 5 to 7 words for P13 and P23 as shown in Table 2) due to the introduction of rational numbers that change the Spanish quantifier "cuántos" by "qué porción de," although it keeps the same length in English.

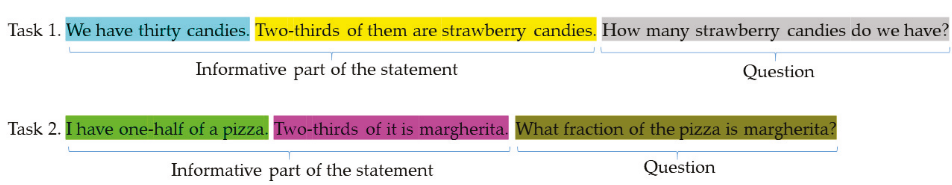

Figure 4. Informative and question parts of the statements.

Table 2. Syntax of the propositions from the original text in Spanish.

Prop	Words	Nouns	Verbs	Numerals	Prepositions	Conjunctions	Level/Type
P11	3	1	1	1	0	0	L0/Declarative
P12	6	1	1	2	1	1	L1/Subordinate
P13	5	2	1	1	1	0	L1/Interrogative
P21	3	1	1	1	0	0	L0/Declarative
P22	6	1	1	2	1	1	L1/Subordinate
P23	7	3	1	1	2	0	L1/Interrogative

The grammatical complexity of each proposition is also represented by the number of nous, verbs, numerals, prepositions and conjunctions in Table 2. The type of sentences can be categorized into levels as defined by Hunt [21]. Propositions P11 and P21 are declarative sentences of level 0. The rest of propositions are level 1 since they include a subordination to the previous sentences by the terms "of them" (P12), "of it" (P22), "candies" (P13) and "of the pizza" (P23) respectively.

2.4. Research Hypotheses

We pose the following hypotheses in line with previous work on the mathematical concepts dealt with by our study:

- **H1: The change from natural to fractional numbers increases the complexity of AWPs.** According to Perera Dzul [39], difficulties begin when students face the study of fractions, without having prior knowledge and enough situations in daily life that present problems related to rational numbers. Gairín and Muñoz [40], in a study on textbooks for the teaching of rational numbers in secondary education in Spain, affirm that rational numbers are overshadowed by the study of procedural aspects, making it difficult to transfer this concept to daily life problems.
- **H2: The use of the fraction as an operator makes statements harder to understand.** Authors like Hart [41] have already shown how challenging a syntagm of the type "two-thirds of them are" can be. Sanz, Figueras and Gómez [42] have also observed that students from 15 to 16 years old find it difficult to tackle this expression when presented literally in simple operative exercises.
- **H3: Operating on a rational whole is more difficult than operating on a natural whole.** Problems arise when the concept of the whole is reformulated. If the whole is not a natural but a fractional number, solving an AWP becomes a more difficult task [43].

Hypotheses 1 and 3 were tested by comparing the average reading times of propositions of the same level. Regarding H1, an increase in complexity from P11 to P21 was due to the mere presence of fractional instead of natural numbers. By comparing the complexity of P12 and P22 we checked the effect of reformulating the whole (H3) from a natural number (i.e., thirty candies) to a fractional one (i.e., one-half of a pizza).

To test H2, we compared the average reading times of level 1 subordinate propositions with that of proposition P21. Propositions P12 and P22 include the syntagms "of them are" and "of them it is" that refer to the use of the fraction as an operator (from now on, we refer only to syntagms "of them are" in order to improve readability). We take proposition P21 as the reference level 0 declarative sentence since it also uses a rational number (i.e., one-half of a pizza), but it does so as a fractional quantity.

3. Analysis and Results

Reading times were rather dispersed in our group of students, as shown by the high standard deviations in Table 3 (values are expressed in seconds per word or s/word). The Kolmogorov–Smirnov test confirmed that the times recorded did not follow a normal distribution (p-value < 0.05) for the propositions (T_{ij}) or the complete statement (T_i). Therefore, we use the median as a good representative of each set of times. We did not use the mean in our analysis, since it is affected by outliers in the obtained asymmetric distributions. For example, see how most of the students read faster than the average reading time (empty circle) in the box-plots shown in Figure 5.

We checked for differences in the reading times due to the socioeconomic context and the gender of students. Differences between school were not statistically significant following the non-parametric Wilcoxon signed-rank test for paired samples (p-values > 0.05). Reading times were also not statistically different between boys and girls (p-values > 0.05). We can then use the data obtained for the whole group to study the complexity of the statements.

Table 3. Reading times (s/word) for each proposition (T_{ij}) and task (T_i).

	T_{11}	T_{12}	T_{13}	T_1	T_{21}	T_{22}	T_{23}	T_2
Mean	5.39	4.72	3.47	4.42	11.55	8.42	7.17	8.46
Median	5.12	2.78	1.73	3.67	6.68	6.01	5.01	7.21
St. Dev.	3.63	5.68	5.96	3.23	12.45	8.22	9.12	5.91
Min	0.37	0.51	0.04	0.88	0.62	0.88	0.27	1.19
Max	14.66	36.42	45.95	19.31	66.38	49.88	70.65	33.47
Q_1	2.36	1.61	0.64	2.50	3.58	3.05	2.03	4.18
Q_3	7.38	4.99	4.83	4.89	15.10	10.45	9.55	10.62

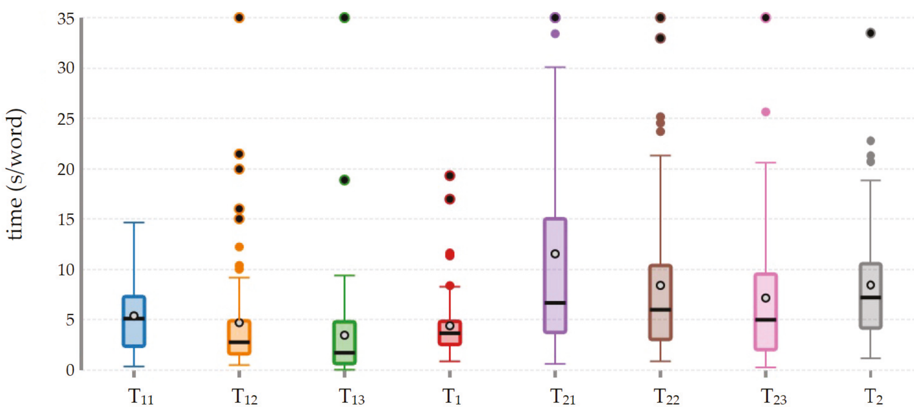

Figure 5. Distribution of reading times for each proposition (T_{ij}) and task (T_i).

By comparing the reading times in Table 3 we can test our hypotheses as follows:

- **H1: The change from natural to fractional numbers increases the complexity of AWPs.** The median reading time of propositions P11 and P21 increases from 5.12 s/word to 6.68 s/word (see also the difference reported in Figure 5). This rise in complexity is due to the change from a natural to a fractional initial quantity. The difference in medians is statistically significant according to the Wilcoxon signed-rank test (p-value $= 0.0001 < 0.05$). The results thus confirm this hypothesis.
- **H2: The use of the fraction as an operator makes statements harder to understand.** The median reading time of propositions that use the fraction as an operator (i.e., 2.78 s/word for P12 and 6.01 s/word for P22) is shorter than that of the proposition using the fraction as a quantity (i.e., 6.68 s/word for P21). The difference in medians is not statistically significant for task 2 according to the Wilcoxon signed-rank test (p-value $= 0.069 > 0.05$). The difference is significant for task 1 (p-value $= 0.004 < 0.05$) mainly due to the ease of operating on a natural whole, as we analyze below in H3. Thus, the syntagm "of them are" does not introduce further complexity to the statements in the AWPs studied.
- **H3: Operating on a rational whole is more difficult than operating on a natural whole.** The median reading time of proposition P22 (i.e., 6.01 s/word) is longer than that of proposition P12 (i.e., 2.78 s/word). Differences are statistically significant according to the Wilcoxon signed-rank test (p-value $= 0.0002 < 0.05$), as is also shown in Figure 5. Those results confirm the hypothesis that it was more complex to operate on a rational whole (e.g., one-half of a pizza) than to operate on a natural whole (e.g., thirty candies).

Student performance was rather good when solving the two proposed tasks. The success rate was 94.3% for task 1 and 62.9% for task 2. The median reading time of all propositions in task 2 was longer than that of task 1 (7.21 s/word and 3.67 s/word respectively) and the distribution was more sparse (e.g., compare T_2 and T_1 in Figure 5). The previous results confirm that solving task 2 was more complicated than solving task 1.

4. Predicting Student Success from the Proposed Complexity Measure

We use a binary logistic regression model to predict the student success when solving an AWP. The model estimates the probability of succeeding (or failing) in completing a task from the complexity of its statement, measured as the reading time per word. The data obtained in our study were used to train a model for each task, as described by Equation (3), where T_{ij} is the time taken by students to read each proposition (j) of the problem (i).

$$P(success = 1) = 1/(1 + e^{-(b_0 + \sum_{j=1}^{m} b_j T_{ij})}) \tag{3}$$

We discarded outliers from our data and kept the results of 58 students to build the model for task 1 and of 57 students for task 2. We trained the models with a random sample of 50 students and validated them with the remaining eight students (task 1) and seven students (task 2). Table 4 shows the relation between the reading time per proposition and the success of students from direct observation of the data. Faster reading times led to better performance in task 1 (indirect relation), whereas slower students were the best performers in task 2 (direct relation). These results are in line with the complexity of the statements analyzed above.

Table 4. Relation between the reading time per proposition and student success.

	T_{11}	T_{12}	T_{13}	T_{21}	T_{22}	T_{23}
Success	inverse	inverse	inverse	direct	direct	direct

The model built for task 1 is shown in Equation (4). It explains between 0.142 (Cox and Snell R^2 value) and 0.424 (Nagelkerke R^2 value) of the dependent variable. It gives an accuracy of 98.3% when calibrating on the train set and it correctly predicts the success of the eight students in the validation set. The sign of the coefficients obtained for each proposition (b_j) reproduces the indirect relation previously found between the reading time and the probability of successfully solving task 1 (see Table 4).

$$P(success = 1) = 1/(1 + e^{-(7.302 - 0.063 \cdot T_{11} - 0.788 \cdot T_{12} - 0.269 \cdot T_{13})}) \tag{4}$$

We analyzed the odds ratio (OR) to understand the magnitude of the effect, that is, how much the probability of success changes as a result of increasing by one second the reading time of a proposition, the rest being constant. An OR greater than one indicates an increase in the probability while an OR less than one implies a decrease. Taking more time to read proposition P12 (i.e., higher values of T_{12}) lowers the probability of success since $OR = 0.455$. Increasing the reading time for propositions P11 and P13 does not affect the student's success that much since OR remains near to one ($OR = 0.939$ and $OR = 0.764$ respectively).

The model built for task 2 (see Equation (5)) is more limited since it explains between 0.056 (Cox and Snell R^2 value) and 0.175 (Nagelkerke R^2 value) of the dependent variable. It gives an accuracy of 65.4% when calibrating on the train set and it correctly predicts the success of four students in the validation set. All coefficients are positive and confirm the direct relation found in Table 4. They are also close to zero, which makes OR rather close to one. For example, increasing the reading time of proposition P22 slightly raises the probability of success ($OR = 1.117$); the time taken to read propositions P21 and P23 does not have any significant effect on student success ($OR = 1.009$ and $OR = 1.059$ respectively).

$$P(success = 1) = 1/(1 + e^{-(-0.896 + 0.009 \cdot T_{21} + 0.111 \cdot T_{22} + 0.057 \cdot T_{23})}) \tag{5}$$

Far from being contradictory, the models represent the different complexities of the two statements. The overall reading time for task 1 was half the overall time for task 2 (e.g., see T_1 and T_2 in Table 3). Students having reading comprehension problems in task 1 thus showed higher probabilities of failure. On the contrary, task 2 appeared as a more complex AWP whose successful resolution could benefit from investing more time in reading its propositions.

5. Discussion and Conclusions

We have presented a novel proposal to measure the complexity of an AWP through the student reading comprehension of its statement. The approach allowed us to predict the students' success from their reading times when solving the task. The students' reading time has demonstrated to be a good proxy to determine the complexity of AWPs and it can become an essential tool for the design of problem statements. By analyzing the statement propositions, one can adjust the level of complexity of the task to focus on certain student profiles.

The paper also introduces the use of the R&L technological environment to compute the complexity of a problem statement, without the need to use traditional paper-and-pencil questionnaires. In addition to that, R&L enables the collection of extensive data on student interactions and opens the way for more data-driven research on the topic.

The results obtained confirm that our procedure for measuring the complexity of AWPs is consistent with previous findings [14]. The two tasks under study can be classified as multiplicative comparison problems according to the semantic approach [5], whose difficulty lies in the introduction of fractional versus natural numbers [39–41].

We identified the complexity of the syntagms "of are" or "of them it is" (or its equivalent "son de" in Spanish), which is related to the multiplication operator and to the concept of "fraction of" or "part of" [37]. These ideas begin to be developed in the school curriculum from the fourth year of primary education. The complexity of this concept, though, increases when it is applied to a fraction. These results may be linked to the design of tasks for current textbooks, where the concept of natural number is introduced through graphic support and considering the whole as a discrete quantity. However, when this concept is introduced over a fraction in the sixth year of primary education, the visual representation is usually removed and the whole becomes a continuum. That results in the mathematical concept being taught through a rote rule, which associates this expression with the multiplication of fractions and leads to possible errors in later courses, as shown by researchers at the Rational Number Project (http://www.cehd.umn.edu/ci/rationalnumberproject/) and the National Assessment of Educational Progress (https://nces.ed.gov/nationsreportcard/). Our work confirmed this issue with a sample group of students of the last year from compulsory secondary education.

The complexity of the statement propositions has been used to build binary logistic regression models that predict the probability of success in solving AWPs. The models confirmed that the propositions that most affect probability are those that involved a more difficult mathematical concept. In our study, these propositions are the ones that deal with the fraction as an operator over both a natural and a rational number.

It is worth noting that our approach also proposes the segmentation of the statement into propositions, whose complexity can be measured and compared following the classification into levels by Hunt [21]. In our study, first level propositions are declarative alphanumeric sentences where the numerical values are either natural numbers or fractions. Second level propositions introduce a subordinate clause through the syntagms "of them are" or "of it is". This fact goes far beyond evaluating the complexity by the success rate [44] and allows comparing the complexity of mathematical concepts within and across AWPs.

This work opens up a line of research on using technological environments and data analytics to determine the complexities of AWPs by measuring the level of understanding of each the statements and dealing with the mathematical concepts that make them more difficult to solve. Next steps include the design of a longitudinal study by students' age that analyzes the evolution of the concepts and the possible blockages that occur. Future work will also help to define an index that allows creating AWPs statements with prefixed complexities by weighting the propositions in the statement according to their level following the classification by Hunt [21].

These sorts of metrics and tools can be implemented by intelligent tutors designed to teach maths through problem-solving. They can help to track personalized teaching–learning paths for each student while using reading comprehension as one of the key drivers for predicting students' skills [26]. Despite the benefits provided by technological environments, the development of digital teaching competence continues to be a challenge for the education system [45,46]. However, the introduction of emerging tools and data analytics is progressively providing teachers and researchers with new experimental scenarios to study, for example, the possible impact of the use of feedback oriented to success when students interact with a given statement [31]. As Alonso et al. pointed out [22], the development of good teaching practices that integrate technology in the classroom can help teachers to start applying digital learning tools effectively and to improve their digital competence.

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mathematics

Article

Learning Mathematics with Emerging Methodologies—The Escape Room as a Case Study

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Abstract: Nowadays, different methodologies are booming in the field of education, and active gamification-based methodologies such as the Escape Room are an example of these methodologies, which is the base of this research. The purpose of this research is to analyze the effectiveness of the use of an Escape Room as an active methodology to learn mathematics. A quantitative research method was performed through an experimental design. Two study groups were set up. With the control group, a traditional training methodology was used, and with the experimental group, an innovative one was used through an Escape room experience. A total of 62 students of the 3rd level of Secondary Education from an educational center in Ceuta (Spain) participated. Results show how the experience developed through the escape room improved achievement, motivation and autonomy in a significant way. It has also reduced learning anxiety significantly. It is concluded that the use of the Escape room in Mathematics improves learning achievement, anxiety, motivation and autonomy, with gender being a variable to be taken into account, especially in motivation and autonomy. Therefore, the escape room has a greater potential than a traditional methodology in Mathematics.

Keywords: active methodology; educational innovation; escape room; gamification; methodological contrast; mathematics; secondary education

1. Introduction

In recent times, traditional education, or more precisely, traditional teaching methods, gave place to a non-central place, and innovation is increasingly taking center stage [1]. Traditional teaching in mathematics is understood as the teacher being the main figure of the educational act, which is a situation more focused on teaching. On the other hand, there is talk of new didactic approaches, when they focus on the student, and on learning as the main action of the educational act. This is causing new innovative teaching modalities to appear that produce a higher incidence in students, such as flipped learning, gamification or problem-based learning [2]. The new didactic environments and approaches allow a greater involvement of the students in the daily life of the classroom and, above all, in the participation of their training process [3]. All this promotes profound changes in education, as well as in people's lives and in their daily actions [4]. Furthermore, this is supported by a total increase in the use of technology [5], as the main support for educational innovation today [6]. For this reason, education approaches the idea of a digital society in which students live daily [7].

This transformation takes place within the framework of constant adaptation of teaching to the digital society and to new ways of life [8]. For all these reasons, fundamental concepts such as that of active methodology derive, as the main source of new forms of teaching, transmission of knowledge and new forms of participation in the educational process of all the agents involved in it. In this new paradigm, the student takes the leading role to become responsible for their training process, always guided by the teacher and always trying to achieve the objectives and strengthen the content [9]. All this tends to have, as its main source, the construction of self-knowledge by the students, and undoubtedly produces new opportunities and means available to students [10]. In this sense, their motivation is increased, causing students a greater interest and a better attitude towards learning [11].

Today's students live in a world surrounded by technologies, as well as with great stimulation caused by this. Therefore, all these actions are aimed at concluding a favorable process for the development of students, with increasingly virtual environments [12]. These innovative learning environments produce a ubiquity in spaces and times dedicated to teaching [13]. Similarly, new methods are generated for a better understanding of the contents [14].

1.1. Gamification as an Innovate Tool

When we talk about gamification, we are referring to one of the active methodologies that, in recent years, has been most developed by teachers and students from all over the world [15]. For this reason, it has earned a place among the most widely used and studied methodologies [16]. As its name suggests, it bases its main focus on the application of the game phenomenon and on converting spaces reserved for formal learning into recreational spaces in which to continue learning [17,18]. Gamification involves the use of tools and elements typical of the game in the classroom, bringing along different benefits as pupils feel more motivated, active in class, and even want to take part in their learning processes in an active way, as within the game structure they feel less pressure in class. [19]. This methodology focuses on facilitating the effort of the students and making their task more enjoyable. For this reason, it focuses on the game as a tool to achieve high levels of learning and achievement of objectives [20]. How could it be otherwise? The game has also undergone a transformation over the years, passing by the classics to the most modern ones, with a great load of virtual reality, an aspect that has been transferred to education [21]. The use of the game, as an educational tool, has had a high success rate, taking into account what has been studied in different investigations, considering it to be an effective and adequate method to put it into practice with students of any age [22,23] and even any educational stage in which their training is developed [12,24].

Gamification allows teachers to develop their own learning program for their students, based on formal knowledge structures, with the certainty of achieving more than acceptable academic indicators in factors as important as the ability to solve problems [25]. In addition, the interactive process that occurs between the agents involved in the educational process [26] and the collaboration between these same agents [27] are benefited. It also increases factors that are influential in the maturational development of the student, such as motivation [28], a positive attitude towards learning [29], interest in knowing their own training [30], the autonomy of the student [31], commitment to the educational act [32], dedication to teaching by teachers and learning by students [33,34], as well as attraction, enjoyment, absence of negative feelings and the satisfaction of facing the task [35,36].

The success of gamification is supported by focusing its method on a system of rewards that make the student increase his attitude and predisposition towards teaching, having a very positive impact on the psychosocial indicators [37] that we have mentioned and which, inescapably, will produce an increase in performance that is obtained from the dedication of the students [38].

In the field of mathematics, recent studies on the use of active and gamified methodologies show very beneficial results for teachers and students who have put it into practice, with all the aforementioned areas and others, such as effective resolution of practices, being developed and promoted [39,40].

1.2. Using the Scape Room as a Gamification Tool

Within gamification, there are many ways to put it into practice. One of them is the Escape Room [41]. This is considered a training modality that is based on the resolution of challenges, tests or problems posed to students by teachers, which give rise to various situations in which students must have adequate knowledge to solve the practice of learning [42]. In an Escape Room, people—students in this case—are locked in a room and they are given several enigma and challenges, which they have to solve to be able to find the way out [43]. Therefore, this method is supported by the game design, with the students having to solve a series of tests, knowing how to self-manage their own knowledge, individually, and collectively to share their knowledge. This causes an increase in the participation of students when solving the aforementioned challenges or problems [44]. The method consists of "locking" the classrooms or spaces ready for practice, where they have to carry out various tasks and activities to solve puzzles and to be able to leave the place in the shortest possible time [45].

Several studies about the application of the Escape Room in the classroom offer us very favorable results in terms of its application in different contexts [46–48]. Thus, its application improves all the indicators previously exposed in relation to gamification. In this line, it produces a high motivation index in students [49], improves their activation and participation in the teaching and learning process [50], produces greater satisfaction for learning and attraction to it [51] and supposes a greater assimilation and reinforcement of the contents [52]. All this, as it cannot be otherwise, leads to a better and greater acquisition of content, positively impacting the student's grades and academic performance [53]. All this, produced by the learning environment, is very favorable for the attitudes of the students, and for their collaborative and personal practice [54].

In the area of mathematics, there are some investigations and training actions that lead to the success of students' learning through the use of the Escape Room. Despite typical problems, a lack of attention to instructions, as occurs in traditional teaching, turns out to be beneficial in its application in the field of mathematics, since it increases competitiveness for learning, motivation and student interest [55].

In addition, the implementation of activities related to the Escape Room in the field of mathematics, promotes the autonomy of students, and facilitates learning, increasing teamwork and the ability of students to resolve conflicts or challenges posed by the teacher [56]. All this favors collaborative work, as well as the autonomy of the students, enabling them to face future learning [57].

1.3. Justification and Objectives

Gamification implies the use of tools, design, and elements of games in classrooms [19]. This teaching methodology eases the students' effort, making it more enjoyable. The implementation of this innovative methodology fosters students' participation and motivation towards their learning processes [11]. Students take the leading role and become more responsible in their learning process, which is guided by teachers [9].

As it has been seen in the previous section, the use of Escape Rooms in education fosters better results in students' motivation [48], activation and participation in their learning processes [49], and satisfaction for learning and attraction to it [50].

This study was carried out at the third level of the Secondary Education stage of the Spanish educational system, due to the lack of motivation students have towards learning and practicing mathematics, as revealed in the scientific literature [58,59].

To check if the lack of motivation was caused by the methodology carried out by the teacher, the main aim of this experimentation is to analyze the effectiveness of the use of educational Escape Rooms in mathematics lessons, as compared to the implementation of a traditional methodology focused on teacher's lectures and presentations without the use of innovative materials and resources. In summary, the following dimensions were measured: learning achievement (as a number obtained in final evaluation of the subject); learning anxiety (example: I have felt nervous during classes); motivation (example: Does the methodology applied affect your motivation with regard to mathematical content?); and autonomy (example: To what extent has the methodology applied in the field of mathematics contributed to their autonomy?).

Bearing in mind this general purpose of this study, the specific aims are:

- To analyze the effects of traditional methodology on mathematics learning;
- To determine if the use of educational Escape Rooms have an effect on mathematics learning;
- To analyze the impact of educational Escape Rooms on academic achievement;
- To analyze the strength or impact of differences between those two methodologies.

In addition to the objectives described, the following research questions arise:

- Does the type of methodology affect learning achievement?
- Does the type of methodology influence learning anxiety?
- Does the type of methodology influence motivation?
- Does the type of methodology influence autonomy?

1.4. Intervention Description

Due to the lack of motivation and active participation of secondary students during mathematics lessons, teachers sought and searched for an innovative methodology that could foster this motivation and participation when learning mathematics. As it has been highlighted in the introduction of this study, this methodology or strategy has been proven to be useful in education, and that is the reason for having chosen it for this experimentation.

In the implementation of the didactic unit carried out, mathematics contents have been worked on and they have been practiced. Within this didactic unit, different challenges and enigmas were designed. Different elements were taken into consideration, following another study about this methodology [57]. These elements were important to design the Escape Room experience, and they were:

- Type of students: It is very important to know what type of students are within the group, to create all the enigmas according to them;
- Time: It will determine the kind of project that can be done;
- Difficulty: The difficulty must be selected or designed regarding the type of students in the group. In this experience, homogeneous groups were organized and some of the enigmas were easy and others a bit more difficult, to maintain attention, but not making them impossible to solve.
- Learning aims: They were to practice mathematics contents, which, according to their teachers, were usually very difficult to pay attention to.
- Theme and space: Recreation in class was important. The recreation of a castle was done.
- Enigmas: They are the core of the Escape Room and they were related to mathematics but from a game point of view.
- Materials and technology: They were chosen according to the ones that can be used in class. Mobile phones were forbidden in school, so they were out of this experience.
- Evaluation: A control list was used on different items to assess the different learning objectives and competences needed.
- Trial: before doing the experience in class, a trail was done, to verify everything was in the right order.

All of these elements were merged into one story to engage the students. The story told by the teacher had a theme centered on action and suspense to encourage the motivation of the students. All this was done with the purpose that the students were immersed in the formative action in an active way.

2. Materials and Methods

2.1. Research Design and Data Analysis

Experimental design through a descriptive and correlational analysis was carried out, which was based on the quantitative perspective following experts within this field [60,61]. The students were classified into two different groups to be analyzed. On one hand, the control group followed traditional teaching methodology. On the other hand, an experimental group followed an Educational Escape Room as a methodology for practicing the mathematics contents. Methodology was defined as an independent variable; other dimensions and the effectiveness of methodologies were selected as dependent variables to be evaluated. Stratified sampling was used to select participants. Stratified sampling is a technique where the researcher divides the entire population into different subgroups. Then, it randomly selects the final subjects from the different strata proportionally. Both groups share a course, work area, content and teachers, so it is established that there is no prior significant difference in both groups.

Statistical Package for the Social Sciences (SPSS) v25 program was used for statistics analysis. For this analysis, the descriptive statistics mean (M) and standard deviation (SD) were used. The measurement of the effect size has been obtained by biserial correlation (r) established by Cohen [62]. In addition, a $p < 0.05$ is established in the study as a statistically significant difference. The value of the effect size of Pearson r correlation varies between −1 (a perfect negative correlation) to +1 (a perfect positive correlation). In this case, after verifying that the data do not follow a normal distribution, non-parametric statistics are used. Specifically, the Mann–Whitney U test is used to compare two groups with no normal distribution [63].

2.2. Participants

The participants were 62 students from secondary education who took part in this experiment. Recently it has been determined by studies of relevant impact that the sample size in these type of investigations does not condition the performance of these experiments [64].

The selection of students was done carrying out an intentional sampling, thanks to the ease of access to the students. They are enrolled in an educational center of the Autonomous City of Ceuta (Spain). One of the workers in this center detected the need of this research after working with these students.

These students were specifically selected from the third level of the Secondary Education stage of the Spanish educational system (n = 62; M_{age} = 15 years; SD = 1.62). The composition of both groups, control and experimental ones, is specified in Table 1.

Table 1. Study groups by sex.

Groups	Boys *n* (%)	Girls *n* (%)	Total *n* (%)
Experimental group	16 (51.61)	15 (48.39)	31 (50)
Control group	13 (41.93)	18 (58.07)	31 (50)
Subtotal	29 (46.77)	33 (53.23)	62 (100)

2.3. Instrument

Data collection was acquired by an ad hoc questionnaire. The design of this tool was done following other validated instruments found within the scientific literature [65,66]. The questionnaire has 32 items in total, which are divided into 20 different dimensions. A Likert scale type is followed with a range of five points (from 1 = Strongly disagree to 5 = Strongly agree).

The instrument was validated first in a qualitative manner and afterwards in a quantitative way. In the first phase, a Delphi method was carried out to do the qualitative validation. Within this procedure, 8 experts in active methodologies in education of different universities were involved. Reviewers rated each item based on its transparency and relevance on a scale of 1 to 6, recommended indicators in the literature [67]. The questionnaire was highly valued by these experts (M = 4.87; SD = 0.21; min = 1; max = 6), and the recommendations given were followed. Kappa de Fleiss and W de Kendall were applied to achieve the indexes of concordance and relevance of observations granted, this showing positive results (K = 0.87; W = 0.89). Afterwards, for the validation, an exploratory and confirmatory factor analysis by the principal components' method with varimax rotation was done quantitatively. The results show an appropriate factorial structure to the initial theoretical approach, and the correlations among factors are positive. The tests determined the dependence between the delimited variables (Bartlett's test of sphericity = 2647.21; $p < 0.001$) and the adequacy of the sample (Kaiser–Meyer–Olkin = 0.86).

More statistical analyses were used to measure the reliability of the questionnaire, such as Cronbach's alpha (α), compound reliability (CR) and average variance extracted (AVE), confirming the internal consistency of all the results of the questionnaire.

2.4. Procedure

The experiment was carried out in several phases. First of all, the ad hoc instrument was designed, and validated. Then, the selection of students who took part in the research was done. Family consent had to be asked to develop this study. In this research, ethical principles of confidentiality were respected. Thus, students were in a random manner divided into two groups with the same number of them, one being established as a control group and the other as an experimental group.

Data collection took place before and after the teaching procedure, with two months' time between them. Teachers taught contents during 10 sessions, through a traditional method with the control group and an innovative method using an Escape Room with the experimental group. The sessions lasted 55 min. The contents taught were related to solving problems using systems of linear equations with two unknowns.

Participants were divided into two groups, which happened to be the group classes where they are enrolled. The control group followed a traditional methodology learning process. In the traditional teaching methodology, the role of the teacher focuses on the presentation of the contents and on the completion of the exercises on the blackboard. All participation in the teaching process has the teacher. The student focuses attention on the actions carried out by the teacher. Therefore, students play a passive role. This hinders and prevents the interaction of educational agents to carry out learning and solve problems in a collaborative way. The students did the problems using systems of equations individually in their own class notebooks. Meanwhile the experimental group followed a learning methodology based on the Escape Room. This fostered the interaction of students with their peers, teamwork, motivation, and participation of students to learn and practice contents.

The Escape room was designed by mathematics teachers with the help of the researchers, who are experts in the field. There were two computers in the room, and a tablet, so the students could surf the net to try to solve the codes that needed its use. There were 5 enigmas hidden in the room, which were based on mathematics problems that needed to be solved, and when one enigma was solved, it led to the other one, because it had a track for the next one. The mathematical problems were problems to practice mathematics in an innovative way. An example of the tasks performed in the Escape Room is as follows: The narrator tells a story to set the students in a haunted house where the doors have been mysteriously closed and a strange noise has been heard. The shadow of a ghost peeks out and tells them that in order to get out of the haunted house they have to solve various problems. The ghost in each test provides students with a card with exercises (Figure 1) that they will have to solve in a satisfactory way to continue advancing in the story and to reach the exit. All the tests, depending on their complexity, have a certain time for the students to solve them. Once the time had elapsed, the students received a clue or puzzle to find the next test.

$$\begin{cases} \dfrac{x+3y}{2} = 5 \\ 4 - \dfrac{2x-y}{2} = 1 \end{cases} \qquad \begin{cases} \dfrac{x+1}{3} + \dfrac{y-1}{2} = 0 \\ \dfrac{x+2y}{3} - \dfrac{x+y+2}{4} = 0 \end{cases}$$

Figure 1. Example of tasks performed.

Teachers were only assessors or guides in the experience, as the students, in groups, had to solve the enigmas with cooperative work. Each group had a color, and the enigmas had the same colors themselves, so each group had to find the tracks of their colors. The enigmas were hidden around the classroom. In order to find them easily, they had to pay attention to the details told in the story. Each time an enigma was solved, the students got a badge, which could be exchanged for a reward afterwards. The rewards could gain extra points in the attitudinal section of the subject in the participation block or new clues to find the enigmas. This was done with the experimental group, meanwhile the control group had the same number of mathematical problems, which was five, displayed on the board in the class, and they had to solve them. These problems were to practice mathematics, so they needed no specific explanation as the processes of those problems were previously explained and they had to practice them. To complete the study, the data were collected and analyzed.

3. Results

All dimensions have had a significant improvement after applying the Escape Room as a pedagogical tool (Table 2). Regarding learning, we find significant values of Learning Achievement (U = 339,000; Z = −2073; p = 0.038) with a large effect size (d = 0.525). Related to Learning Anxiety, we get similar results (U = 339,000; Z = −5754; p = 0.000) with a large effect size (d = 0.528). Motivation and Autonomy show significant differences too (U = 654,000; Z = 2480; p = 0.013 and U = 654,500; Z = 2461; p = 0.014).

Table 2. Mann–Whitney U test for control and experimental groups differences.

	Group	Ranks	U	Z	*p*	*d* *
L Achievement	Control	26.94	339,000	−2073	0.038	0.523
	Experimental	36.06				
L Anxiety	Control	44.61	74,000	−5754	0.000	0.528
	Experimental	18.39				
Motivation	Control	25.90	654,000	2480	0.013	0.653
	Experimental	37.10				
Autonomy	Control	37.11	654,500	2461	0.014	0.655
	Experimental	25.85				

* Cohen's *d* small < 0.20, Medium < 0.50, large > 0.50.

Effects sizes can be considered large d = 0.653 and d = 0.655. In all analyzed dimensions (Achievement: $R_{C\,=}$ 36.06; R_E = 26.94; Motivation: R_c = 25.90; R_E = 26.94 and Autonomy R_c = 37.11; R_E = 25.85), the experimental group obtains higher values than the control group. In anxiety, where these values are different, values are lower in the case of the experimental group (R_c = 44.61; R_E = 18.31). Based on the results, the experience developed has caused significant effects on all the dimensions analyzed.

From another perspective (Table 3), we seek to see if the gender variable could affect some of the changes produced during the experience. The gender variable only influences motivation (U = 52,000; Z = −2620; p = 0.008) and autonomy (U = 33,500; Z = −3365; p = 0.001) in the experimental group. Women are the most motivated and show the highest level of anxiety in general. This effect does not occur in the control group, where there are no differences by gender. Women are the most motivated and show the highest level of anxiety in the control group, although these differences do not become significant.

Table 3. Mann–Whitney U test for control and experimental groups differences by gender.

	Control					
	Group	**Ranks**	**U**	**Z**	***p***	***d* ***
L Achievement	Male	13.81	85,000	−1470	0.175	—
	Female	18.33				
L Anxiety	Male	15.88	118,000	−0.081	0.953	—
	Female	16.13				
Motivation	Male	13.31	77,000	−1754	0.093	—
	Female	18.87				
Autonomy	Male	13.53	80,500	−1567	0.119	—
	Female	18.63				
	Experimental					
	Group	**Ranks**	**U**	**Z**	***p***	***d* ***
L Achievement	Male	16.65	125,500	0.349	0.731	—
	Female	15.53				
L Anxiety	Male	17.81	140,500	0.943	0.352	—
	Female	14.69				
Motivation	Male	11.00	52,000	−2620	0.008	—
	Female	19.61				
Autonomy	Male	9.58	33,500	−0.3365	0.001	—
	Female	20.64				

* Cohen's *d* small < 0.20, Medium < 0.50, large > 0.50.

Finally, it is relevant to know how the different dimensions analyzed are correlated, both for the control group and for the experimental group (Table 4). In the control group, where the Escape Room was not carried out, the different dimensions do not seem to maintain positive or negative correlations ($p > 0.05$). In the experimental group, positive and significant correlations appear. The positive relationship between motivation and achievement stands out ($r = 0.364$, $p < 0.05$), as does autonomy and achievement ($r = 0.404$, $p < 0.05$), and motivation and autonomy ($r = 0.684$, $p < 0.01$).

Table 4. Correlation between dimensions for each group.

	Group		L Achievement	L Anxiety	Motivation	Autonomy
Control	L Achievement	*r*	1	−0.109	0.047	0.295
		p		0.558	0.801	0.107
	L Anxiety	*r*	−0.109	1	−0.162	0.055
		p	0.558		0.383	0.771
	Motivation	*r*	0.047	−0.162	1	0.160
		p	0.801	0.383		0.389
	Autonomy	*r*	0.295	0.055	0.160	1
		p	0.107	0.771	0.389	
Experimental	L Achievement	*r*	1	0.120	0.364 *	0.404 *
		p		0.519	0.044	0.024
	L Anxiety	*r*	0.120	1	0.231	−0.094
		p	0.519		0.212	0.616
	Motivation	*r*	0.364 *	0.231	1	0.684 **
		p	0.044	0.212		0.000
	Autonomy	*r*	0.404 *	−0.094	0.684 **	1
		p	0.024	0.616	0.000	

* Significance with values less than 0.05. ** Significance with values less than 0.01.

4. Discussion

The research carried out has made it possible to achieve the proposed general purpose of analyzing the effectiveness of the use of educational Escape Rooms in mathematics lessons, this compared to the implementation of a traditional methodology in the 3rd year of Secondary Education. The results obtained in this study have confirmed the potential of the literature on teaching innovation and the use of active methodologies in teaching and learning processes [1,2]. In this case, the research has focused on the use of the Escape Room as a methodology to gamify a subject as traditional and rigorous as mathematics.

Despite the acceptance and use of gamification by teachers in general [16], this work acquires its reason for being in the scarcity of studies concerning the use of the Escape Room in the area of Mathematics. Therefore, with the intention of increasing the impact of the literature on the state of the question, this study has been carried out.

This work is based on the analysis of previous research reported on gamification and, specifically, Escape Rooms. It has been conducted in order to analyze the dimensions that offer benefit to the scientific literature on this active methodology. Therefore, this study acquires its potential in presenting some findings on a training approach that has been scarcely studied in the subject of mathematics.

Studies on the implementation of the gamification and Escape Rooms reveal great benefits both in the teaching process [20,22], and in various indicators related to learning [25,27,55]. The latter have to do with an improvement in problem solving [26], in the interaction and collaboration between the people involved [27,28], in activation and attitude [53,57], in motivation [29,52,66] in satisfaction with the environment generated and the task to be carried out [54], and autonomy of students [33,68,69], as well as the results and academic performance achieved [38,64], among the indicators more outstandingly reflected in impact studies.

In particular, this research has been articulated in the analysis of four dimensions, such as learning achievement, learning anxiety, motivation and autonomy. The results show that these dimensions have experienced a significant improvement after the application of the Escape Room as a pedagogical tool in the Mathematics subject. These findings are associated with previous studies on the use of this approach that reveal an improvement in the achievement of students in their formative action [38,56], in the control of learning anxiety [12], in motivation [29,41,52] and in student autonomy [31,56,57].

At a higher level of specificity, the gender variable has only influenced students' motivation and autonomy, as other studies reveal [68,69]. It has been found that women are more motivated and show a higher level of learning anxiety, in analogy with other works [70]. However, this effect produced is not found in the control group, where there are no differences by gender.

At the correlational level, in the control group the correlations are absent. In contrast, in the experimental group, there are correlations between Achievement-Autonomy, Motivation-Autonomy and Motivation-Achievement. This last correlation stands out, which generates a positive effect on students by increasing their motivation for learning and homework due to the achievement attained. These results are in congruence with the specific literature on the analyzed art [12,70]

5. Conclusions

This work shows the analysis of an escape room experience with students. The results obtained allow us to know the students' assessment of the intervention with the escape room and inform about the effect on learning process.

Based on these findings, use of the Escape Room in the Mathematics course has contributed to the improvement in the different dimensions studied, such as learning achievement, learning anxiety, motivation and autonomy (with gender being a variable to take into account), and in a general way, in motivation and autonomy. Therefore, the use of this active gamified methodology is positioned as a didactic approach with greater potential than traditional methodologies.

The potential of Escape Room education has been demonstrated. This resource is effective in both increasing motivation and promoting active learning. As in other studies, there is agreement on the idea that, through this type of game, it is possible to facilitate the learning of a specific topic in a motivating and efficient way.

6. Prospective and Limitations

The prospective of this research focuses on the promotion of the Escape Room and the promotion of gamified practices in the field of mathematics. As revealed in this study, the use of this active methodology benefits several important indicators in the training process. For this reason, the present work encourages the teaching community to use the Escape Room as a methodological alternative in the area of Mathematics. Furthermore, this research contributes to establishing the bases of this active methodology in this specific field, with the purpose of serving as a support for future research by other members of the scientific community.

The limitations of this study are focused on the particularities of the participants, who are involved in a specific context. Furthermore, this study lacks a prior analysis of the groups that have been subjected to experimentation and control, as they were classes already organized by the educational center, which assured its composition was based on heterogeneity. Although the effects are positive, we must take into account the area of work within mathematics. In this case, the positive results were obtained in the work in the areas of algebra, logic and geometry. Therefore, as a future line of research, it is intended to articulate a teaching network to apply the Escape Room in the Mathematics subject with the intention of obtaining a representative sample that allows the results to be generalized to the entire student population. In addition, another of the possible lines of action may focus on the application of this active methodology in different educational stages.

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mathematics

Article

Influence of Musical Learning in the Acquisition of Mathematical Skills in Primary School

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Abstract: In this research we analyze the influence of musical activities in the acquisition of mathematical knowledge and skills of a sample, 50 students from both a public and a private school in A Coruña (Spain), at a cognitive level. Based on a quantitative study with a quasi-experimental design, we evaluated students' knowledge acquisition; we worked with musical activities related to mathematics in the experimental group (EG), and with traditional mathematical activities in the control group (CG). We used a questionnaire that the teachers completed before and after putting the activities into practice, after collecting—writing daily field notes—the mathematical knowledge acquired by the students. The results indicate that there are significant differences between the pretest and the posttest, between CG and EG, but there are no differences between public and concerted schools. In short, it is concluded that music represents an excellent tool in mathematical learning.

Keywords: musical activities; mathematics; learning-teaching; preschool

1. Introduction

The close relationship between music and mathematics has been known since ancient times, Pythagoras (582 BC) being considered as the first to establish connections between the two disciplines when joining them in his school, so that music and arithmetic were studied together until the Middle Ages [1].

However, most of the works developed in this area were specific music treatises formulated by mathematicians (Descartes, Mersenne, Euler, D'Alembert, etc.), or compositions created through mathematics (Bartók and the golden reason, Stochastic music by Xenakis, Halffter's Fibonacciana, etc.), being necessary to wait until the end of the 20th century for it to be envisioned as an emerging study area by Milton Babbitt, David Lewin, and especially John Clough [2].

Recently An, Ma and Capraro [3] or Johnson and Edelson [4] have argued the connection between both disciplines claiming that musical notes, scales and tuning are related to various areas of mathematics, from proportions and integers to geometry and trigonometry. That is, most of the theories are built on the relationships and common structures of both components [5], while studies aimed at teaching and learning of mathematics and music at the same time are scarce [6–9]; even more unusual are those reporting the benefits related to teaching them jointly. This is despite the fact that it has been demonstrated, through neuroimaging, that musicians and mathematicians activate common brain areas [10]. In this line, Winner, Goldstein and Vincent-Lancrin [11] made visible an investigation carried out by Graziano, Peterson and Shaw in which they stated that the improvements in the area of mathematics were greater among students who were also studying piano, especially in subjects related to spatial learning.

Undoubtedly, at a neurological level, music provokes different responses in the areas of the brain that affect both cognitive and emotional levels, since it activates imagination and creativity [12], both very necessary to approach mathematical learning from an affective framework [13], building the foundation from which the processes of cognition act: perception, attention, memory, intelligence, thought and language [3,14].

According to studies, the interdisciplinary approach of teaching music with mathematics positively affects cognitive development and skills related to both disciplines and student's academic results [1,15,16]. Some of the reasons may be that learning is more attractive when the contents are addressed together, which provides emotional security and confidence [17,18]. Benefits are significantly increased if interdisciplinary work begins in the first stage of the education system.

In spite of the above, the contributions, experiences and teaching materials that work between subjects at school in an interdisciplinary way [19] are solely incipient, except for the occasional song to facilitate the processes of memorization of, for example, the multiplication tables. This trend also occurs in Spain, which does not come as a surprise, since an absolute disconnection among the same course subjects is usual [10]. Although the Organic Law 2/2006 of Education of May 3 [20] organizes the curriculum of Infant Education in Spain in three areas of knowledge, indicating that the work must be done in an interdisciplinary manner, it is quite different in real-world scenarios [21].

There are two types of schools depending on their ownership: public and private since 1985, according to the Organic Law Regulating the Right to Education [22]. There is plenty of research that analyzes whether there are differences between both types, the results being torn between those that do find different results [23,24] and those that conclude that there is no significant difference in relation to the ownership of the center [25–27] and claim other inputs to be more relevant.

The edition of PISA [28] goes one step further by adding two parameters: the appreciation of the students (their satisfaction is greater when they study in charter and private centers than in public centers) and the performance in the evaluations (which is lower in public schools).

Calero and Waisgrais [29], Meunier [30] and Salinas and Santín [31] among others justify the differences in academic performance in public centers due to the lower quality of resources and the greater number of students of immigrant origin.

From this referential framework, we have asked ourselves the following research question: How does music and, more specifically, music teaching, influence students' mathematical learning?

The presented work is part of a larger study that analyzes the effects, cognitively, of musical activities in math classes, and how these scaffolds influence student learning depending on certain variables such as the course, sex, ownership of the center, professions and studies of parents [32].

Particularly, the objective of this research is to analyze the influence caused by musical activities, cognitively, in the acquisition of mathematical knowledge in the last course of Pre-school Education, taking into account the ownership of the center.

2. Materials and Method

To analyze the influence on the acquisition of mathematical knowledge, a quantitative methodology has been used, based on a quasi-experimental design with a pretest and a posttest and two reference groups: an Experimental Group (EG), which received the stimulus or treatment; and the Control Group (CG), which only served as a comparison since it did not receive such treatment.

2.1. Context and Participants

The study involved a sample of 50 children between 5 and 6 years old, 25 girls (G) and 25 boys (B), 24 from a Public School (PS) and 26 from a Charter school (CS) of the 3rd year of Early Childhood Education in the province of A Coruña (Spain) and their respective teachers in the school year 2018–2019 (see Table 1). Besides, in this research, centers with similar socio-economic characteristic were selected to avoid possible influences.

Table 1. Distribution of the sample.

	SEX	AGE	CG	EG	Total
PS	G	5	2	3	24
		6	4	3	
	B	5	3	4	
		6	3	2	
CS	G	5	3	3	26
		6	4	3	
	B	5	4	4	
		6	3	2	
	Total		26	24	50

2.2. Procedure

The research carried out refers to a pretest and posttest design. The purpose is to evaluate the mathematical learning before and after the intervention of the teachers to be able to subsequently make a statistical comparison between the students' learning and the possible variations depending on the two groups into which the sample was divided (the CG formed by 26 students and the EG by 24), and the ownership of the center to which they belonged (in the CG, 12 students belong to PS and 14 to CS, and in the EG, 12 to PS and 12 to CS).

In order to obtain the data, the same contents were worked on in both groups, although, to verify if there were differences in the academic results of the students, preselected musical resources were used in certain activities within the classroom—related with mathematics contents—with the experimental group (EG), while with the control group (CG) mathematics content was worked with traditional didactics.

The mathematical contents developed with the strategy of the musical activities worked in the classroom, and with the respective teachers, deal with "Properties of the objects" (PO), "Basic operations with concrete elements" (BO) and "Space-time relations" (STR) and have been elaborated from Decree 330/2009, currently in force in the Autonomous Community of Galicia (Spain).

2.3. Evaluation

The evaluation technique that was used was direct observation; that is to say, the progress of the students during the accomplishment of the programmed activities was contemplated, focusing mainly on the acquired learning, as well as on the rhythm and characteristics of this acquisition. That is why in this work we collected, through field notes, relevant descriptive and reflective information by the teacher-tutors in the day-to-day process. This systematic collection resulted in a class diary of each student that allowed to cover a questionnaire designed by the researchers that would make up the pretest and the posttest.

The questionnaire consisted of 10 items with six response options, ranging from 0 (nothing) to 5 (much), to evaluate some fundamental contents of the mathematics based on LOE [20]. It was then endorsed by a system of inter-rater validation formed by four experts (teachers and professors specialized in mathematics and music). In this way, the most relevant items were selected for their relevance (they should be related to the object of study) and clarity (easily understandable).

The items of the questionnaire are the following:

I_1. To recognize circles, triangles and squares.
I_2. To order objects by size.
I_3. To classify daily use objects by their shape.
I_4. To arrange objects by their height.
I_5. To group items by quantity.

I_6. To create compositions with Cuisenaire rods.
I_7. To associate the numerical name with the number of elements.
I_8. To identify morning, afternoon and evening.
I_9. To use different measuring units.
I_{10}. To recognize before-now-after.

The distribution of the items of the questionnaire regarding the contents worked in the classroom is as follows: PO (I_1, I_2, I_3, I_4), BO (I_5, I_6, I_7), and STR (I_8, I_9, I_{10}).

After the pretest was carried out by the teacher-tutors (in the Spanish educational framework it is the Infant teacher/tutor in charge of teaching music) and the analysis of the data by the researchers, an activity plan was prepared related to the mathematical contents mentioned above (PO, BO and STR), which would be developed three days a week, 30 min per session, for two months with the EG, while in the CG the class sessions related to the same contents were held in a traditional way (cards and textbook). The participating teachers are part of an interdisciplinary research group led by the researchers and had already collaborated in past courses on other innovative experiences with other students. Therefore, the training they had, prior to the start-up of this project, was forged jointly in the whole group, which led to the design of the proposal being developed by all.

At the end of the 8 weeks, and after the achievement of the activities, the teachers—with all the data from the daily observations—covered the posttest test in both groups (Experimental and Control) to determine if the treatment had brought any change in the acquisition of mathematics knowledge. Finally, the tabulation of the data, the statistical analysis and the discussion of the results by the researchers was carried out using the statistical package SPSS v.23.0. Averages and standard deviations were used as descriptive statistics, the Cronbach's alpha is used to find the reliability coefficient, the Wilcoxon T test, appropriate for related variables, and the Student's *t*-test for independent variables. The reliability coefficient obtained by Cronbach's alpha (internal consistency) is satisfactory with values of 0.803 in the pretest and 0.821 in the posttest; for the control group 0.811 and 0.892 for the experimental group, as well as 0.796 for the Public School and 7.01 for the Concerted School.

The development of work with the EG was carried out by formulating a logical sequence of activities structured in sessions using musical instruments, songs, choreographies, working duration, height and intensity, in addition to any situation of interaction with schoolchildren that would respond to their questions.

Each of the sessions consisted of three phases: (1) in the general assembly, an introductory activity was carried out in order to check the previous knowledge, present the contents and stimulate the students. We tried to incite them to action based on what they knew; (2) the development activities were carried out, in which the children demonstrated what they were learning in the previous phases, and (3) finally, the relaxation phase is the highlight in which relaxation tasks were carried out, but without losing connection with the central theme of the program.

Taking Noll [33] into account, all the proposed activities had a playful nature, since at these ages the engine of emotional, intellectual and social developments are games. In addition, they influence knowledge structures and relationships with the environment.

It should be noted that the teacher made an effort to promote a climate of security and trust. At all times, the teacher worried about helping the students when developing the activities, reminding them of the collective rules and guiding those who were blocked by providing them with new patterns of action.

Since music is attractive to students, even more so at early ages, we have proposed a series of musical activities that motivate them to progressively relate them to mathematical concepts. These examples show an example of how the quality of the sound "Duration" has been worked from music and mathematics in an interdisciplinary manner.

2.4. Musical Games

The musical race

Objectives:

- To learn certain musical figures (half note/minim, quarter note/crotchet, eighth note/quaver, rests).
- To understand that each figure has a duration.

Work group: the whole class.

Resources: the song *The musical race.*

Time: 15 min.

Description: We listen to the song The musical race. After the first time, we ask students about figures: did all they run at the same speed? Which one was the fastest? Which one was the slowest? Then, we listen to the song again, this time imitating the figures that are part of the race.

In this activity, through the listening, students have to get to learn, and understand the duration of musical figures (half note, quarter note, eighth note, quarter note rest). Afterwards we check whether that knowledge has been acquired through questions, and finally, through movement, students try to link them to the mathematical concept "Fast-slow", through the experience of a race where they imitate the slower and faster musical figures.

Simon Machine

Objectives:

- To follow a sequence of sounds.
- To obtain a faster or slower sequence of sounds.

Work group: Groups formed of 4 to 5 children.

Resources: Simon machine.

Time: 30 min.

Description: Randomly, the machine illuminates and emits its own sound as it lights. After waiting, the student must repeat the sequence offered by the machine, in the correct order, using theirs visual and audible memory. If the student succeeds, the machine will respond with a longer sequence, and so on. If the student fails, the student must assign his turn to the next classmate. The different levels of difficulty increase the speed of the sequence to be repeated.

In this activity, in addition to trying to involve the student in interdisciplinary work-through the "Fast-slow" sound sequence-it is intended to improve memory, attention and concentration, increasing the difficulty progressively.

Musical domino

Objectives:

- To understand the time that each figure occupies.
- To be able to reproduce the time that figures last by clapping.

Work group: the whole class.

Resources: adapted domino.

Time: 20–25 min.

Description: The domino consists of the following figures: whole, half, quarter, eighth notes and quarter rest, in two colors (red and black). Each student takes 6 tiles that are placed face down. The one that gets the tile with two half notes begins.

The students will have to join the tiles, for example, black red-black red, and they will then clap taking into account the time that each figure occupies; in this case, the student will count to one. When it comes to the silence tile, the student will not clap, but will just wait for the necessary time. The first to finish the tiles wins.

This activity includes the musical figure known as the whole note which implies more difficulty than previous tasks since it is necessary having understood the concept of duration related to quantity.

Let's dance!
Objectives:

- To be able to remember a sequence.
- To internalize ordinal numbers.

Work group: the whole class.
Resources: music player.
Time: 15 min.
Description: There will be a brainstorm of the steps that students want to introduce in the choreography. Once the steps have been proposed, a selection is made and they will be adapted to a song known to children. This was one of the choreographies:
First: We all jump once.
Second: We turn to our right twice.
Third: We move the arms up in circles 3 times.
Fourth: We move the hip to the right and left 4 times.
Fifth: We cover our nose and crouch 5 times.
Repeat as many times as necessary as the children want.
This activity was one of the most attractive for students, since in addition to having fun, they consolidated the musical and mathematical knowledge that the class had been working on, and other activities were added such as laterality, numbering, geometric figures, etc.

3. Analysis and Results

3.1. Averages and Standard Deviations of the Pretest and Posttest by Groups and Ownership of the Center in the Respective Items

The averages and standard deviation show that, with respect to the initial and final tests, there are differences in the academic performance of the Experimental Group, compared to the results obtained by the participants of the Control Group according to the ownership and regarding *I1* (Table 2).

Table 2. Averages and standard deviations of the scores obtained in all Items by group and ownership.

				I1	*I2*	*I3*	*I4*	*I5*	*I6*	*I7*	*I8*	*I9*	*I10*
PS	*Pretest*	CG	M	2.57	1.93	3.25	3.11	1.85	3.66	2.86	2.23	2.85	2.75
			SD	±0.65	±0.79	±0.88	±0.1.23	±0.67	±0.1.36	±0.85	±0.75	±0.85	±0.55
		EG	M	2.50	1.65	2.90	2.84	1.86	2.96	2.75	2.34	2.99	1.87
			SD	±0.51	±0.71	±0.45	±0.87	±0.65	±0.65	±0.65	±0.67	±0.54	±0.80
	Posttest	CG	M	2.74	2.69	3.58	3.52	2.67	3.84	2.90	2.65	2.83	3.02
			SD	±0.68	±0.85	±0.84	±0.143	±0.97	±0.127	±0.85	±0.95	±0.78	±0.41
		EG	M	3.89	3.99	4.34	4.11	4.96	4.30	4.89	4.86	3.12	4.91
			SD	±0.1.52	±0.65	±0.58	±0.76	±0.1.11	±0.85	±0.65	±0.141	±0.76	±0.1.01
CS	*Pretest*	CG	M	2.43	2.16	3.21	3.31	1.96	3.39	3.01	2.31	2.12	2.86
			SD	±0.83	±0.16	±0.12	±0.1.62	±0.83	±0.124	±0.23	±0.87	±0.12	±0.72
		EG	M	2.41	1.23	2.81	2.81	1.92	2.91	2.27	2.11	2.81	1.83
			SD	±0.40	±1.12	±1.28	±0.54	±0.76	±0.43	±1.36	±0.85	±1.21	±0.46
	Posttest	CG	M	2.74	2.51	3.51	3.49	2.74	3.78	3.19	2.83	2.01	3.13
			SD	±0.68	±0.87	±0.87	±1.61	±0.87	±1.00	±0.99	±0.76	±0.89	±0.42
		EG	M	4.73	3.87	4.45	4.29	4.82	4.51	4.83	4.33	2.68	4.82
			SD	±1.19	±0.83	±0.82	±0.1.01	±0.1.36	±0.1.01	±0.76	±0.1.01	±0.87	±0.1.32

Note: M = Mean value; SD = Standard Deviation.

It is stated that the performance in CS has experienced an improvement in the EG (went from 2.41 to 4.43), while in PS (went from 2.50 to 3.89) an improvement is also reflected, although less than in the CS.

The starting scores were higher in the PS. However, in the posttest score the scores obtained by the CS are higher (Table 2).

As it can be seen in Table 2, the scores obtained in the I_2 (to order objects by size) reveal that the EG, starting from very low marks in the *pretest*, has improved more than the CG. If we compare by ownership, we see that in the CG, the students of the PS had lower averages in the *pretest* and higher in the *posttest*. On the other hand, in the EG the scores of the PS sample were and continue to be higher.

The values, in general terms, indicate that the participating sample has a positive attitude towards the use of musical activities in the teaching–learning process of mathematics as shown in the results of Table 2. In this sense, there are differences between CG and EG, in favor of the latter in all cases. The students of the CS stand out again, although all of them experience an increase in their grade (Table 2). While the CG averages are homogeneous in both centers and in both tests, in the EG the differences were greater in the students of the PS at the beginning of the project, but the students of the CS showed superiority in the second test.

Concerning averages and standard deviations of the I_4, we obtain higher values in all cases when the questionnaire is passed a second time. Differences are shown between the Control and Experimental groups—in favor of the first in the pretest and in favor of the second in the posttest—in both centers (Table 2).

In the case of I_5, the EG stands out again with respect to the CG in the posttest result, which analyzes the grouping of elements according to the quantity. Starting from the averages in the initial tests—in which the CS stood out slightly in both groups—the CS CG stood out in the posttest, although in the EG there is a small inclination towards the PS. In addition, if the results are assessed before and after the application of the activities, it is necessary to highlight that the results of the pretest were low in general, while those of the EG posttest are very high, with the PS students reaching scores very close to 5 (Table 2).

In I_6, the average scores obtained by the students according to the interdisciplinary method reflect differences between the CG and the EG, in favor of the latter in all cases. Comparing the results to the type of center being analyzed, the students of the CS stand out again, although all of them experience a increase in the grade (Table 2). While the CG averages are homogeneous in both centers and in both tests, in the EG the differences between the schools were higher in those belonging to the PS at the beginning of the project, but those of the CS showed superiority in the second test. The obtained data are shown in Table 2.

Another aspect included in the instrument used refers to associating the numerical name with the quantity of elements embodied in the responses of item 7 with an average close to the response value 5 (=4.89 and 4.83) equivalent to "Totally agree" in the GE of the PS and the CS.

Based on Table 2, the mean and standard deviations of the scores obtained by group and center typology express that the CG of the CS experienced a slight improvement and no change is perceived in the PS students. Additionally, the EG improved meaningfully and in a special way in PS and CS, since the scores of the initial questionnaire were higher than in the CG than in the EG for both centers. Therefore, it can be inferred that musical activities have been of great benefit to students.

Regarding the question of whether they identify the morning, afternoon or evening, which is shown in Table 2, the analysis show that students quantify it as very positive with values that approach the response 5 (x = 4.86 and 4.33) relative to the option "Totally agree". These results confirm that the perception of the participating students about the intimate relationship between music and mathematics follows the same line of the previous Items regarding ownership and the two groups. While the initial values were homogeneous in PS and CS, in the posttest the EG gets distanced from the CG with noteworthy differences; especially those of the PS are on the verge of the maximum score, although it is true that they started from higher values than those of the CS (Table 2).

Regarding I_9, dissimilar results were found, which contrast with the other values of the study. Such data contrast with the previous values; reflected in Table 2, that in the EC the values in the posttest have decreased in relation to the pretest, both in the CG and in the EG. However, in PS the results increased somewhat in both groups. The reasons given by the teachers of the CS to explain this were convincing, alluding to a very negative family event that had happened to one of the students, which affected the rest of the students.

In reference to I_{10}, which assesses the knowledge of the sample on the fact of recognizing before, now and after, it should be indicated that considering the I_{10} content, the averages and standard deviations of the scores obtained show again that the EG improved in all centers, so we can affirm that the effects of the program of recreational musical activities applied to assess the mastery of the students are noteworthy and positive. In Table 2, we can confirm that the EG started from lower values than the CG in both schools; however, the posttest shows better grades in the EG, mostly in the students of the PS.

3.2. Control Group in Each of the Items

Another aspect analyzed is the comparison of the pretest and posttest between the means obtained by the Experimental Group and the Control Group in each of the items. These results lead us to corroborate that the effects of the program of musical activities focused on mathematics are positive and significative on the performance of the students in all the items that were applied to assess the mastery of basic concepts of mathematics (Figure 1). The initial values of the EG are lower than those of the CG in all items except *I9*, although in *I1*, *I5*, *I8* and *I10* there are few differences between one group and the other. In *I9*, the average mark in the EG pretest is somewhat higher than in the CG. Once the posttest was administered and analyzed, we could see from the obtained results that the EG improved in all the items, especially highlighting the *I1*, *I5*, *I7* over the CG. The *I9* item does not offer high scores in any case for reasons of internal nature, which the teachers kindly explained.

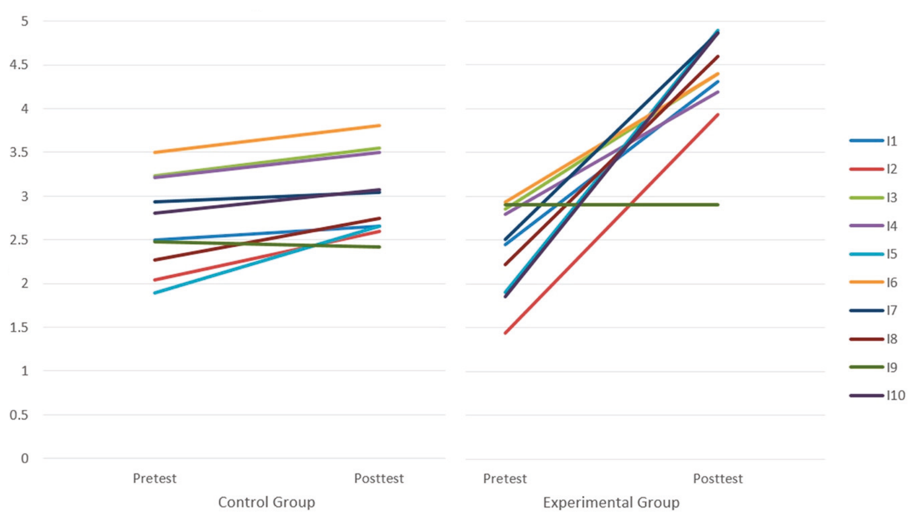

Figure 1. Group interactions by item.

3.3. Comparisons of the Pretest and Posttest between the Means Obtained by the Experimental Group and the Control Group in Each of the Contents

The obtained results, in relation to the worked contents and the groups that make up the study sample, offer the following data: the EG presents worse qualifications in the pretest than the CG in all cases; however, by applying the second questionnaire, the CG increased its performance values on a regular basis and the EG had an important improvement.

The interaction of groups and measurements for performance by content in the pretest—both in the CG and in the EG—show that BO (*I5*, *I6* and *I7*) is the highest, followed of PO (*I1*, *I2*, *I3* and *I4*) and finally STR (*I8*, *I9* and *I10*). Likewise, the posttest also shows that both the EG and the CG remain the highest BO, followed by PO and then STR.

3.4. Comparisons of the Global Means of the EG and CG

It follows that according to the global results, the total averages of the CG (2.67) and the EG (2.36) show higher values in the GC in the pretest. However, the scores of both groups in the posttest differ quite in favor of the EG, with 4.35 for the EG and 2.99 for the CG.

We can therefore affirm that the applied measures had a positive impact, which is in line with other studies that have shown the effectiveness of music to stimulate cognitive functioning [34,35].

3.5. Comparisons of Means for Related Samples

The results obtained in the pretest and posttest were analyzed to check if there are statistically significant differences, through the Wilcoxon test for related samples. Through this analysis, we intend to contrast the null hypothesis of non-existence of significant differences between the means of the subjects before and after applying the proposal of musical activities in the mathematics class.

Observing the results of Table 3, we can verify that the statistic with its associated p-value (0.000) is less than the level of significance (0.05), which allows us to conclude that, between the means obtained in the pretest and the posttest, there are statistically significant differences, favorable to the posttest.

Table 3. Comparisons of means for pretest and posttest.

		Ranges			Contrast Statistics [a]	
		Cases	**Mean Range**	**Sum of Ranges**	**Z**	**Sig. Asint. Bil**
Posttest Pretest	Negative Positive	26 24	2.54 3.67	66.3 88.08	−2.201 [b]	0.000

[a] Wilcoxon signed rank test; [b] Based on negative ranges.

3.6. Comparison of Means for Independent Samples

Next, we proceed to apply the Student's t-test for independent samples, given the parametric nature of the distribution, in order to determine if there is a significant difference between the results of the Public School and the Concerted School, on the one hand, and the Control Group and the Experimental Group on the other.

In both cases, the first assumption of normality is verified, based on the Shapiro-Wilk test, applicable for $n < 30$, since the data have a normal distribution. Additionally, the second assumption, consisting of determining the existence of equality of variances, using Levene's test, resulted in the variances being equal.

When analyzing the data (Table 4), it is observed that there is no significant difference between the mean of the two groups, PS and CS, as p-value = 0.470 > 0.05.

Table 4. Equality of means between the Public School (PS) and the Concerted School (CS).

	Cases	Means	p-Value
PS	24	3.1375	0.470
CS	26	3.0775	

However, there are significant differences between the CG and the EG, since p-value = 0.000 is less than 0.05, which is why differences are shown in favor of the EG (Table 5).

Table 5. Equality of means between the Control Group (GC) and Experimental Group (GE).

	Cases	Means	p-Value
CG	26	2.8525	0.000
EG	24	3.3625	

4. Discussion and Conclusions

The results obtained in this research have allowed us to identify the influence of musical activities (musical instruments, songs, choreographies, working duration, height and intensity, in addition to any situation of interaction with schoolchildren that answered their questions) in the acquisition of mathematical knowledge of students in Early Childhood Education. We can therefore affirm that the objective we set ourselves at the beginning of the study has been fulfilled and corroborate what authors such as Hall [36] are right to point out about the benefits of interdisciplinary work between mathematics and music. Taking into account the results according to the variables group and ownership, we can affirm that the musical teaching has promoted advantages in the mathematical learning of the students.

We can emphasize that, as for the previous mathematical knowledge that the schoolchildren possessed, the sample is homogeneous in both cases with respect to I_1 "To recognize circles, triangles and squares", To I_6 "To create compositions with Cuisenaire rods", to I_8 "To identify morning, afternoon and evening" and I_2 "To order objects by size".

However, differences between groups can be seen in I_{10} "To recognize before-now-after" and in I_4 "To arrange objects by their height". In relation to ownership, there are differences in I_9 "To use different measuring units", I_7 "A To Associate the numerical name with the number of elements" and I_5 "To group items by quantity".

Focusing on the two groups (CG and EG) and the 10 items of the questionnaire, we achieved that the EG showed greater success compared to the first application of the questionnaire, substantially improving compared to the CG, which remained stable or increased its performance minimally. Specifically, the improvement highlights in items 1, 5, 7 and 8, being 8 the one in which a greater difference was evidenced. In items 2, 4, 6, 9 and 10, the CG had better qualifications in the pretest than the EG, however, in the posttest the EG stood out.

Based on the ownership and the items, in general, we appreciate positive effects of the program of musical activities on the performance of the students in the 10 items. In the case of I_9, the average of the posttest performance in PS and CS of the CG decreased, and it increased minimally in the EG, but it was due to a valid and consistent justification, according to the teachers.

With respect to the two groups and contents, the results indicate improvement in the EG with respect to the CG in the three contents: objects properties, basic operations with concrete elements, and space-time relationships.

Likewise, the non-parametric tests reaffirm the aforementioned, giving results that indicate significant differences between the results of the test prior to the implementation of the proposal and the one after it Wilcoxon's T). Regarding the CG and the EG, it is perceived through the Student's t-test that the EG (who received the math class through musical activities) presents significant differences with respect to the CG, in favor of the former. Finally, it should be noted that the Student's t-test, in relation to ownership, shows that there are no significant differences between public and concerted schools. This shows that the differences are not due to the type of school, or the students, but are linked to the fact of working on mathematics through musical activities.

Consequently, it can be concluded that the activities applied had a positive effect, in line with other studies that have shown the effectiveness of rhythmic patterns, intensities, durations, heights, speeds, symmetry, etc., as effective musical mechanisms in the acquisition of mathematical skills [10]. The present investigation echoes these findings and extends them by showing evidence of such benefits in childhood education, age at which studies are not frequent, as the previous authors indicate, in addition to warning that further research is necessary.

It becomes evident after the implementation of this experience the fact that the application of musical activities as a resource in mathematical learning represents an excellent alternative for teachers of Early Childhood Education, who seek to meet the learning needs of children in a fundamental stage for their integral development. In this way, the proposed objective has been achieved, which is an important contribution to the progress of the interdisciplinary work of both subjects.

In short, the basic knowledge of mathematics was achieved through guided and planned musical experiences, which allowed students to be stimulated in a pleasant and conductive environment, since the activities were carried out with high levels of motivation, harmonizing all their dimensions, both physical and emotional. Therefore, we can affirm that for these teachers music represented an excellent alternative, since it had a positive impact on the performance and motivation of the children in these classrooms. Consequently, we confirm that learning should occur in an interdisciplinary and pleasant context. In addition, it must be adapted to the needs that schoolchildren have to explore and get to know their surroundings. Finally, it becomes clear that professionals trained in mathematics didactics and musical didactics are necessary to undertake this challenge [37].

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Article

Mathematical Modeling Projects Oriented towards Social Impact as Generators of Learning Opportunities: A Case Study

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Abstract: This paper presents a case study carried out at an elementary school that led to a characterization of mathematical modeling projects aimed at generating social impact. It shows their potential as generators of mathematical learning opportunities. In the school project, upper-grade students (sixth grade, 11-year-olds) studied the way in which the rest of the students at the institution traveled from their homes to school. Its purpose was to identify risk points from the standpoint of road safety and to develop a set of recommendations so that all the children could walk safely to school. In our study, we identified, on the one hand, the mathematical learning opportunities that emerged during the development of the project and, on the other, the mathematical models created by the students. We discuss the impact of the project on the different groups in the school community (other students, parents, and teachers). We conclude with a characterization of the mathematical modeling projects oriented towards social impact and affirm that they can be generators of mathematical learning opportunities.

Keywords: mathematical modeling; modeling projects; elementary school; learning opportunities

1. Introduction

Mathematical modeling in the classroom encourages students to develop mathematical knowledge through the study of real-life situations, taking advantage of the link between reality and mathematical concepts and procedures (Blum [1]; Doerr and English [2]). Our participation, as researchers in various school projects in which the mathematical analysis of reality is a central part of the work, inspired us to propose the concept of the mathematical modeling project oriented towards social impact (MMPOSI). By way of an initial approach to the concept, we established that the elements defining an MMPOSI are structured around two key ideas: the role of mathematical modeling and its social component. In this article, we exemplify the concept of MMPOSI through a case study developed from a naturalistic perspective at an elementary school. Thus, the case studied allowed us to characterize MMPOSIs and exemplify their potential by showing how they can generate mathematics learning opportunities in elementary school (Cai et al. [3]; Cobb and Whitenack [4]). Given that this research was designed as a case study with an instrumental descriptive character (Merriam [5]; Stake [6–8]), we leave the characterization and discussion of the concept of MMPOSI to the end of the article.

The school where this case study was developed is in the center of Sabadell (Spain), a city with 200,000 inhabitants. The streets around the school are narrow and were laid out prior to motorized vehicles. They are now used by both road traffic and pedestrians. In 2017, an accident occurred in which one of the students was run over by a vehicle while returning home from school. This accident had a huge impact on the school community. New needs emerged both for students and their families, as well as for teachers. Among the various actions taken, the school management team, in coordination with the parents' association, requested the assistance of the first author of this paper to carry out a project in which the pupils would study the difficulties they encountered in their everyday journeys on foot to and from the school. This request provided the basis of the school project "Let's get to school safely", in which upper-grade students (11-year-olds) were given the task of documenting the routes taken by the rest of the students to get to school, then analyzing the findings to identify the pedestrian danger points. The final goal of the project was to generate a set of indications that would ensure that the students could safely walk to the school through the city streets.

The text of this article is organized according to the structure that often appears in reports of instrumental descriptive case studies. In Sections 2 and 3, the conceptual framework of the study is introduced. In the second section, we explain the interpretation of the mathematical modeling that we apply when establishing the definition of MMPOSI and, in the third, the meaning that we give mathematical learning opportunities to justify our methodology. The empirical study is presented in a block consisting of Sections 4 and 5. The fourth section justifies and details the choice of a case study as the research design, and the fifth explains the chronological progress of the project Let's get to school safely. The results, which are essentially descriptive given the nature of the study, are the themes that emerge from the study and are presented in Sections 6 and 7. The sixth section describes the mathematical learning opportunities that emerged during the course of the project and, in the seventh, the models generated by the students. Sections 8 and 9 bring the article to a close. In the eighth section, we discuss the learning opportunities and models identified, and in the ninth, we conclude the article by setting out the basic characteristics of MMPOSIs.

2. Modeling in Mathematics Education

2.1. Background

Mathematical modeling as a research topic in the field of mathematics education began with the work of Pollak [9], who discussed the relationship between the applications of mathematics and the teaching/learning processes. Subsequently, the same Pollak [10] presented a first theoretical framework, which interpreted modeling processes by differentiating between the mathematical domain and the rest of the world. This separation necessarily leads us to the process of mathematizing a phenomenon, moving from reality to the mathematical domain, and to the interpretation within the real context of the models generated in the mathematical domain, as a form of validation.

Following Blum [1], mathematical modeling was established as a research topic of interest in mathematics education, aimed at setting up classroom activities that bring to light the close relationship between mathematics and the world around us. A great deal of research on educational mathematical modeling has been carried out since then, and it has diversified remarkably, in terms of both the goals and the approaches (Abasian, Safi, Bush, and Bostic [11]; Blomhøj [12]; Kaiser and Sriraman [13]). Thus, from the perspective of the mathematization of the environment and mathematical modeling, a clear need has been identified at an international level to link up students' mathematical knowledge with reality (Vorhölter, Kaiser, and Borromeo Ferri [14]).

Recently, Blum [15] reaffirmed that the teaching of applications and modeling has a twofold function: on the one hand, the knowledge of mathematics and its applications is vital to the real world and its advancement, principally with regard to solving real problems and developing complex projects; and on the other hand, the real world and the way it integrates mathematical knowledge are extremely important as a vehicle for giving meaning to the learning of mathematical concepts and, in general, mathematics as a discipline.

The theoretical advances made in the didactic use of modeling are well known. However, their presence has not yet been felt in the majority of classrooms. There are various reasons why the use of mathematical modeling has not increased in elementary and secondary education classrooms. For example, institutional constraints have been identified that make it difficult to fit modeling activities into the normal functioning of educational centers (Barquero, Bosch, and Romo [16]). It has also been observed that teachers' attitudes and their training are key to the regular use of modeling activities (Blum and Leiß [17]; Schmidt [18]). Indeed, many mathematics teachers do not think modeling is an essential component of learning mathematics, and they also question their own mathematical modeling skills. Given that students may come up with many different solutions and it is not easy to identify the focus of the activity in these tasks, teachers regard the implementation of mathematical modeling tasks as rather complex (Ng [19]; Winter and Venkat [20]). These difficulties are possibly more marked among elementary school teachers, since their training is less grounded in mathematics.

2.2. Models and Modeling Activity

Problem solving is a key part of mathematics education (Lester [21]; Schoenfeld [22]). In this paper, we argue that contextualized problem solving activities can be interpreted as mathematical modeling activities, given the type of mathematical constructions that students create to solve them. Our approach is situated in that area of research that explores how students solve mathematical problems, situated in real contexts, when there are no defined heuristics. These problems promote metacognition and help to familiarize students with the methods of applied mathematics (Verschaffel [23]). From this perspective, the fundamentals of modeling are aligned with the principles of project-based learning, while promoting active learning (Krajcik and Blumenfeld [24]) in meaningful contexts that students can relate to their prior knowledge (Blumenfeld et al. [25]), with the focus on the development of mathematical activities and concepts. Thus, we consider a class task to be a modeling activity when students generate or use mathematical models to describe or analyze real phenomena. In this paper, we adopt the definition of a mathematical model proposed by Lesh and Harel [26]:

> "Models are conceptual systems that generally tend to be expressed using a variety of interacting representational media, which may involve written symbols, spoken language, computer-based graphics, paper-based diagrams or graphs, or experience-based metaphors. Their purposes are to construct, describe or explain other system(s).
>
> Models include both: (a) a conceptual system for describing or explaining the relevant mathematical objects, relations, actions, patterns, and regularities that are attributed to the problem-solving situation; and (b) accompanying procedures for generating useful constructions, manipulations, or predictions for achieving clearly recognized goals." (p. 159)

This definition makes it clear that some of the concepts and procedures that make up a model are mathematical. However, the constructed models may also contain non-formal aspects that allow an intuitive description of the reality under study, such as graphic representations or the use of metaphors. Modeling is a process of solving a real problem in which mathematical concepts, methods, and results are involved (Blum and Niss [27]). To this end, the objects, data, and relationships occurring in reality are transferred to the world of mathematics (horizontal mathematization), thereby obtaining a mathematical model. Mathematical methods are then applied to this model to reach a mathematical solution (vertical mathematization), which must be interpreted and validated in the real world where the problem is framed, resulting in a real solution (again, horizontal mathematization).

It is generally agreed in the world of mathematics education that modeling processes are of a cyclical nature (Blum and Leiß [17]; Carreira, Amado, and Lecoq [28]; Doerr and English [2]; Galbraith and Stillman [29]; Greefrath [30]; Kaiser and Stender [31]). During a modeling process, students try to solve a problem by going through different stages in which they move from reality to the mathematical domain, each time re-evaluating the phenomenon under study. The entire process is repeated in different cycles, with the students improving the models and solutions found for the problem they are working on, adapting the models to the requirements of the problem statement (Blum and Borromeo Ferri [32]). Finally, they have to communicate the result of this process. To successfully find their way through these stages, students have to draw on a series of competences that include aspects related to metacognition, motivation, and their own ideas about the nature of mathematics (Maaß [33]).

From the theoretical perspective of models and modeling (M&M) and from a model-eliciting point of view, students are understood to perform multiple cycles of interpretation, descriptions, conjectures, explanations, and justifications that are redefined and reconstructed iteratively as they interact with other students (Doerr and English [2]). Model building involves quantifying, dimensioning, coordinating, categorizing, algebratizing, and systematizing relevant objects, relationships, actions, patterns, and regularities (Mousoulides, Sriraman, and Christou [34]). The M&M perspective also considers data-modeling problems that focus on organizing and representing data, building patterns, and searching for relationships (Lesh, Amit, and Schorr [35]), as well as involving students in statistical reasoning such as decision-making, inference, and prediction.

The problems posed to students in the framework of M&M, contextualized in the real world and with characteristics and demands that make them modeling activities, are called "model-eliciting activities" (MEAs) (Doerr and English [2]). The M&M approach is integrated into problem solving because it considers that an MEA in itself constitutes the process of modeling and obtaining a model. Thus, the problem statement must allow the students to establish adequate criteria that help to decide which solution is the most appropriate among a set of different alternatives. It should also enable the students to judge for themselves whether the answers need improving, refining, or amplifying for a specific purpose. During an MEA, students are asked to work in small groups (Clohessy and Johnson [36]; Zawojewski, Lesh, and English [37]) and are confronted with a problematic situation that is significant and relevant to them, for which they must create, expand, and perfect their own mathematical constructions.

In MEAs, students are encouraged to generate products that go beyond providing brief answers to artificially restricted questions about pre-mathematized situations. It is a question of enabling the creation of models by the students on the basis of their previous knowledge, both mathematical and about the real world. Students' work during a modeling activity should result in productions that are shareable and re-usable in similar situations (Lesh and Lehrer [38]). Depending on the project needs, students may generate models to provide decision-making tools (Mousoulides, Sriraman, and Christou [34]). Students develop these tools from models that fulfill a functional or operational role. This includes drawing up specific action plans to deal with problematic situations and designing the assessment instruments needed to distinguish different scenarios in complex situations where it is necessary to use specific mathematical models.

3. Mathematics Learning Opportunities

There is a wide and continuous spectrum of situations in which learning opportunities have been studied. On the one hand, there are large-scale studies that measure the acquisition of learning. At the opposite extreme, there are micro-studies that address the achievement of specific learning goals, usually in classroom activities. In large-scale studies in general, the goal is to arrive at an interpretation of learning outcomes measured globally through performance tests, either in international comparative studies or accountability studies, or to explain why certain groups (minorities, students with specific needs (Kurz [39]), etc.) do not perform at the same level as the population taken overall. From this perspective, learning opportunities are defined in relation to the the measured contents of the curriculum, the educational level, and the learning conditions. In any case, in this field, the interest is in learning as a product, as a result of certain conditions that include learning opportunities. It is important to note that, under this interpretation, the opportunity is not necessarily thought to imply learning. In fact, Törnroos [40] pointed out that having the opportunity to learn is a necessary prerequisite for learning, but a learning opportunity does not guarantee that students will actually learn. In general, the studies related to learning opportunities try to explain the lack of learning as caused by a lack of opportunities. However, Floden [41] pointed out that other factors influence learning outcomes, including the quality of the teaching and the students' abilities.

At the level of what happens in the classroom, when the term mathematical learning opportunity is discussed, it is initially linked to the analysis of classroom interactions where mathematical knowledge is constructed. Cobb and Whitenack [4] argued that mathematics learning is a process of conceptual self-organization and enculturation. From this perspective, a mathematical learning opportunity is a situation in which students have the opportunity to reorganize their conceptual structures and approaches when solving problems or, in general, when dealing with a new mathematical activity. Therefore, it is a concept closely linked to the content being learned, the learning process, and the characteristics of the learning activity.

Cai et al. [3] stated that any definition of classroom-based learning opportunities must necessarily consider the interactions between three elements: the mathematics tasks, the teaching, and the students. They considered it impossible to separate out the influence of any of these components, since the nature of their interactions will determine whether an activity, or a classroom experience, becomes a learning opportunity for a given group of students in relation to a specific goal. Accordingly, Cai et al. [3] stated that the interactions between the three elements create complexities that can probably only be understood by means of multiple iterations of studies based on successive conclusions. In order to make progress, they suggest that multiple studies, often small-scale ones, must be carried out to move gradually towards more complete and accurate answers. Among the research methods they proposed is the study of the mathematical task set for students.

In recent years, a good number of studies of mathematics education related to mathematical learning opportunities have focused on teaching quality and the resources used, analyzing how the pedagogical and/or curricular characteristics of the teaching facilitate or limit students' opportunities to learn. In this respect, the study carried out by Wijaya, van den Heuvel-Panhuizen, and Doorman [42], who concluded that the lack of tasks in context-based textbooks limits students' learning opportunities, is particularly interesting in our opinion. These authors developed a framework with four perspectives for analyzing the role of context in mathematical activities: the types of context, the purpose of context-based tasks, the information used in tasks, and the type of cognition required by the tasks.

Wijaya, van den Heuvel-Panhuizen, and Doorman [42] concluded that when the procedure to be applied is made more or less explicit, students do not need to determine what the appropriate mathematical procedures might be to solve the task, which means they will not develop their ability to transform a context-based task into specific learning. These authors recommended including more tasks based on real-life contexts in classroom practice, and they set out how these tasks should be introduced. First, they pointed out that they should not only appear immediately after the explanations of concepts or procedures, since then the strategies to be followed seem clear. The quality of the tasks is also important: they should be presented in essential, relevant contexts, which can generate opportunities to mathematize situations or organize them mathematically. In addition, the assignments should, according to these authors, incorporate superfluous information or require a search for new information so that students have the opportunity not only to select what is relevant, but also to identify appropriate procedures. They should be tasks with a high cognitive demand so that students have the opportunity to develop complex reasoning, which requires reflection in relation to real-life contexts.

Research has shown that tasks posing a greater cognitive challenge intensify students' involvement in mathematical ideas (Boaler and Staples [43]; Tarr et al. [44]). Tasks with a high cognitive demand require the connection of procedures to their underlying concepts, or the completion of complex, non-algorithmic tasks; tasks with a low cognitive demand involve the memorization or performance of procedures without connecting them to the underlying concept. The best learning opportunities arise when the task meets two conditions. On the one hand, it should require the use of two or more forms of representation (Lesh, Cramer, Doerr, Post, and Zawojewski [45]) and the translation between them, and on the other, it should oblige students to explain their strategies and thinking (Walkowiak, Pinter, and Berry [46]). Ultimately, mathematical tasks play a central role in the type of interactions that are possible and in the nature of the learning opportunities that emerge (Cai et al. [3]).

Our study is among those that analyze factors related to the quality of teaching, and more specifically, the activities proposed to students. Therefore, we analyze the project as an activity that can generate learning opportunities, based on the study of what the students show they learned during its development. The study of learning opportunities in conjunction with classroom activities has been based traditionally on three basic methods: direct observation in the classroom, teacher's reports, and documentary analysis of different elements, including the students' products (Kurz [39]). Kurz and Elliott [47] suggested that the learning opportunities generated by an activity can be studied according to what students show they are capable of doing when coping with the activity.

4. A Case Study Research Design

Below, we present the case study carried out with the goal of supporting the concept of MMPOSIs as generators of learning opportunities. Whereas this research was a case study (Merriam [5]; Stake [6,7]), the object of study was a bounded system (Stake [8]) where we worked essentially with qualitative data and with the intention of providing a detailed account of the case. Our case was an inclusive one (Cresswell [48]; Merriam [5]; Yin [49]) since we not only focused on an object with a clear entity (a group of people), but we were also interested in studying an activity under development. Case study research can be used to address exploratory, descriptive, and explanatory research questions (Stake [7,50]; Merriam [5]; Yin [49]). Our research was an instrumental descriptive case study since it provided initial insights into an issue (Stake [7]). Then again, our interest went beyond understanding the particular case because we hoped that the project Let's get to school safely would illustrate how MMPOSIs operate as generators of learning opportunities.

All case studies have one thing in common: they focus on a case as a complete unit, just as it exists in its real-life context. When determining the case to be studied, we used purposive sampling, taking advantage of the opportunity to participate in the orientation of the project offered by the center. In this case study, we adopted a naturalistic perspective and took the role of research observers during an activity developed naturally at a school (Hatch [51]). We had no control over the environment or the variables that influenced the students' work, but we were able to obtain a close-up view of how things happened in reality.

In research designed as a case study, the researcher must provide a detailed account of the case. We present a detailed comprehensive description of the development of the project Let's get to school safely in the following section, organizing it around what we call episodes. In each of them, the students worked on a core activity based on the four specific goals of the project. The development of the project was documented with photos, videos of specific moments, and notes from classroom observation. We also collected all the work material produced by the sixth graders, which included maps that showed the routes taken by students to get to school, reports of measurements of danger points in the surroundings of the school, guidelines for safe walking, and presentations created to inform other students at the center.

The research issues that focus this study (Stake [8]) and the findings presented in this article revolve essentially around two ideas: the mathematics learning opportunities promoted by the project, and the mathematical models developed by the students during the project. Consequently, various methods of data collection were used: the primary data comprised the field notes together with the documents produced by the students, while the informal interviews with the teacher and the parents provided the secondary data. We drew on these data to develop what was essentially a content analysis that followed the steps proposed by Miles and Huberman [52], consisting of data reduction, data visualization, and conclusion/verification. Specific codes were used to group the data into the emerging categories that became obvious, and the categories were organized into two themes around the research issues.

In this way, we start with a description of the development of the project Let's get to school safely, and then, we analyze it through different levels of abstraction and provide our interpretation of the development of the project, relating it to our conceptual framework, i.e., the learning opportunities and construction of mathematical models. The discussion of the findings and the themes that emerged from the categories led to the interpretation of the case studied. We conclude by proposing a detailed characterization of MMPOSIs that goes beyond the initial definition.

5. Development of the Project in the Classroom

The project that was the object of our case study was implemented in a school that has one class group per grade and welcomes students from three to eleven years old. As explained in the Introduction, the school is located in the center of a city with a high population density and surrounded by narrow streets shared by cars and pedestrians. Since most students go to and from school on foot, difficulties and potential dangers are generated by the coexistence of cars and pedestrians in a small area. Both the parents' association and the teaching staff had expressed their concern in this respect and proposed various activities to deal with these potential dangers.

Among other activities, the project Let's get to school safely was launched. The older students (sixth grade in elementary education, a group of twenty-six 11-year-olds) would study and analyze the routes to school taken by each of the rest of the students at the school, the goal being to establish appropriate guidelines and recommendations so that the children could get to and from school safely. The project took place over two weeks in six sessions lasting 90 min each. In the first session, the project and its role as a generator of recommendations for the rest of the students at the school were discussed. The teacher and the first author helped the students define the core activities of the project. These were: (i) identify the most common walking routes to the school; (ii) identify potentially dangerous places and situations for students on these routes, (iii) analyze these places to determine safe practices; and (iv) communicate the results of the project to the rest of the school.

Below, we describe how each of the core activities was carried out during the different episodes of project development in order to provide the reader with the details of the case under study.

5.1. Episode I. Identification of Routes

Eight work groups were set up in the sixth grade class. Each of these groups focused on studying one of the classes in the lower grades, which was their target class. Students were encouraged to choose a class where they had connections (siblings, neighbors, friends) so as to make communication easier and encourage greater involvement. As a first step, students asked students in their target class about the routes they use to get to school. This step was particularly challenging for the students who worked with the youngest children. Figure 1 shows a sixth grade student asking a four-year-old girl about the route she follows. Given her age, the younger student still had difficulty describing the route she takes to school. For this reason, the sixth grade student asked questions about well-known buildings, squares, and shops that she might recognize on the route in order to obtain the required information. This was then displayed on a map of the school surroundings.

Figure 1. A sixth grade student asking a four-year-old girl about the route she takes to go to school.

Once all the routes followed by all the students in each target class had been ascertained, the data had to be organized to make decisions. At this point, various interesting findings emerged from the data, such as the distribution of the students' homes or the age at which they begin to learn their route to school in detail. The sixth grade students observed that, below the third grade of primary education (8- to 9-year-olds), there is no guarantee that a child will know the details of the route that he/she usually takes. This prompted other questions that were discarded because they strayed from the main goal of the project. The sixth grade students decided that, for each target class, they would represent the routes followed by the students on a single map in order to structure the subsequent project activities (Figure 2).

Figure 2. A map showing the routes taken by the students in a target class.

5.2. Episode II. Identification of Danger Points

The next phase focused on identifying danger points in the surroundings of the school and examining them to identify the factors that could provoke accidents. To do this, a map of the area around the school was projected onto the classroom screen, and the students discussed the characteristics of the streets and nearby intersections and their own dangerous experiences in these places. When students considered it necessary, they used the Google Maps tool to visualize the streets from an immersive perspective and explore them virtually. The final product of this discussion was a list of danger points requiring further investigation (Figure 3). The danger points were then distributed among the groups for study, bearing in mind in each case that the point had to be part of a route frequented by the target class corresponding to the work group.

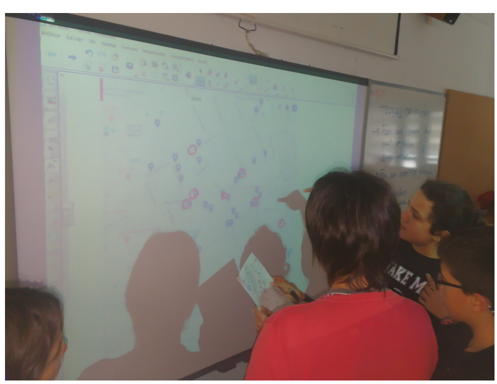


Figure 3. Identifying danger points in the surroundings of the school.

5.3. Episode III. Analysis of Danger Points and Preparation of Recommendations

Episode III consisted of fieldwork. Most of the risk points were street intersections with crosswalks presenting specific characteristics. These characteristics were discussed beforehand in the classroom during Episode II, since upon separating to do the fieldwork in the surroundings of the school, the students were unable to consult the teacher directly. This prior knowledge helped the students to work more efficiently in the field and ensured they obtained the necessary data.

For this activity outside the school, we counted on the assistance of 16 parents from the center who volunteered to make sure the activity went smoothly and looked after the students' safety while they were working in the streets. Each of the groups went to the place to be studied with the intention of drawing up their own map of the area, measuring the relevant features from the standpoint of the passage of vehicles and pedestrians, and indicating the way the latter group moves around. They also took photographs to illustrate their work and recorded videos simulating everyday situations to illustrate the difficulties encountered by pedestrians.

Once they had identified the potential dangers of each one of the points of interest and analyzed the causes, the sixth graders set about identifying safe behaviors that would enable students to move around securely. This search for safe behaviors began in the street and ended later, in the classroom, while the students were preparing to communicate the results of their project.

5.4. Episode IV. Communication of Project Results

During this episode, each group shared its proposals with the other class groups in order to reach a consensus. The information obtained was distributed to help each work group prepare the informative talk that they would give their target class. This collaborative approach made it possible to validate each group's proposals and also optimize the impact of their efforts with the preparation of experts on each risk point. The groups adapted the message content to the age of the target students in order to properly communicate the results of the project. Furthermore, thanks to their personal relationships, they were able to exemplify good practices based on specific cases. The presentations began with an introduction that insisted on the right to be able to move safely around the city. After that, each work group explained the procedure followed during the project, the difficulties encountered at the risk points, and their recommendations for safe passage through them. The talks with the different target classes were organized in parallel on a Friday afternoon, and the parents were also invited to learn about the project and its conclusions.

6. Theme 1: Mathematics Learning Opportunities

In this and the following section, we report the findings of this study, presenting the two emerging themes through a robust description (Merriam [5]) that incorporates the constructed categories and exemplifies them with units of meaning drawn from the data. This enabled us to show that there were learning opportunities and that models were generated.

The first emerging theme comprised the mathematics learning opportunities that appeared during the development of the Let's get to school safely project. Cobb and Whitenack [4] held that mathematical learning is a process of conceptual self-organization and enculturation. From this perspective, a mathematics learning opportunity is a situation in which students have the chance to reorganize their conceptual structures and approaches when solving problems or, in general, when they have to cope with a new mathematical activity. For each episode in the project, we identified mathematics learning opportunities by analyzing the actions taken by the students—understood as mathematical processes—in response to the proposed activity. These are indicated in italicsbelow.

During Episode I—route identification—the sixth grade students prepared the data collection and established a specific way of recording the information they would collect. The students discussed the various options and selected the most relevant ones. Thus, they realized that writing down the home address of each student in the survey was not relevant since it did not determine the route taken. Conversely, they noted that other options, such as recording the information on a printed map, generated useful data that clearly reflected the students' journeys. This discussion represented an opportunity *to reflect on the complexity of the phenomenon under study, the nature of the data to be collected, the data collection procedures that they were familiar with, and how they would need to use these data during the subsequent development of the project*.

When the sixth grade students asked the students in their target classes about their routes, this led to an interaction that contained various types of mathematical content. In all cases, the first step was for the sixth graders *to explain the information on the map and present the way to interpret it*, working on *orientation on a map and obliging the younger students to identify specific landmarks* in the city (parks, buildings, and shops) with points on the map, and *relating the directions of movements on real routes with movements on the map*. The younger students had to visualize their daily route to school and adapt their explanations to the needs of the sixth graders. This process of visualization included *interpreting graphic information on the map and the visual processing of the route they use to go to school*, with these two procedures understood as in Bishop [53] and Gorgorió [54].

Once the sixth grade students had identified all the routes, the mathematical activity focused on understanding the information they had collected. To do this, the students were asked to *organize the maps* and then *classify the identified routes into sets*, where the routes had to have a large section in common. We observed that the students hesitated when grouping routes with slight variations. The *need to reduce the complexity of the situation* was discussed in order to be able to study it with the methods at hand. After creating the sets of routes, each group had to *draw up a map showing the frequency of use* of all the streets around the school (Figure 4). This process of representing information included *visual coding*, since students developed their own graph format. Previously, they had only worked with pie charts and bar graphs in statistics as resources to represent frequency.

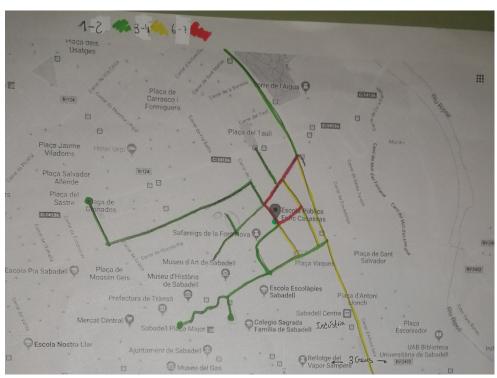


Figure 4. A map showing the intensity of use of the different routes taken to go to school.

In Episode II—danger points—the first mathematical activity consisted of drawing up a proposal for a list of dangerous places for pedestrians in the surroundings of the school. The sixth graders were asked to identify them on their own journeys to and from school and to try to *visualize possible risk situations* for younger students. This obliged the sixth grade students to reinterpret the use of the streets from the perspective of people with more limited mobility and who see the world from a lower height. This activity involved a *visual processing* procedure because it required anticipating movements and lines of sight that would be verified and complemented in the later field study. Before accepting a proposed risk point, students were required to verbally describe a potentially dangerous situation that could occur in that spot. This activity demanded *spatial reasoning*, since students had to describe the movement of different objects (vehicles and people) and their form of interaction. The various validated risk points were situated on a map, thereby repeating the activity of *location on the map*.

In Episode III—analysis of danger points and generation of recommendations—various mathematical activities related to the *use of measurement in context* and the representation of reality on a map were carried out, both as regards urban architecture and the use of the street by vehicles and pedestrians. Students recreated possible risk situations and took various types of measurement that were relevant to understanding how cars could interact negatively with pedestrians. They recorded them on a map of the risk point under study.

The corner in Figure 5 is shown by way of an example. As you can see in the picture, the curb has been lowered to allow cars to turn more easily. The students measured by how much the cars mount the sidewalk.

Figure 5. Fieldwork measuring the width of the sidewalk mounted by cars when turning.

In other cases, students *measured how high a pedestrian needs to be for a driver to see him/her* when waiting at a zebra crosswalk where the adjacent parked cars limit the drivers' field of view. Students also *measured the angle necessary for a pedestrian at that same crosswalk to see a car approaching* and have enough time to stop. In all these cases, the measurements they took were not of objects in the street, but rather measurements of distances that the students determined by *visualizing the interactions between vehicles and pedestrians*, because they offered information relevant to the decision-making in the following part of the project.

Lastly, in Episode IV—communication of the project results—the mathematical activity consisted of the interpretation of the collected data to prepare guidelines and recommendations for the students in the other grades. They used videos, photographs, screenshots of Google Maps, and annotations with measurements to distinguish safe behaviors from risky ones. This implied *using this information in a contextualized manner, interpreting the mathematical model generated in relation to the real world*; they had to consider traffic regulations and also the usual behavior of vehicles and pedestrians.

Figure 6 shows the categories that summarize the mathematics learning opportunities identified during the development of the project.

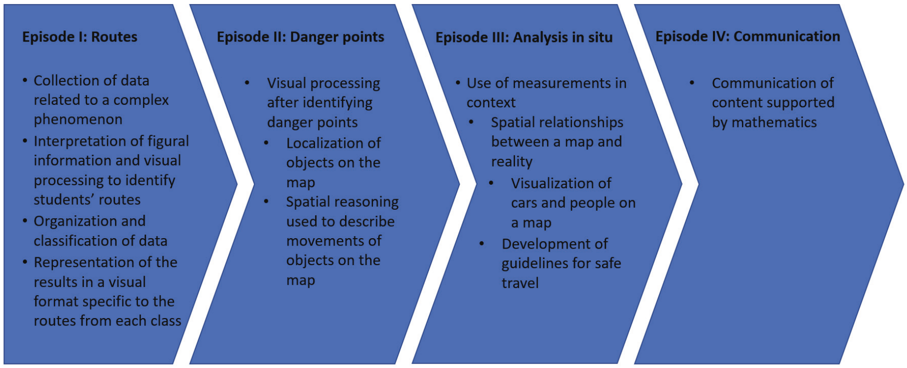


Figure 6. Mathematical learning opportunities identified during the project.

7. Theme 2: Models Generated

The second emerging theme consisted of the mathematical models created by the sixth graders during the development of the Let's get to school safely project. Mathematical models are conceptual systems that describe real phenomena (Lesh and Harel [26]), but from the research standpoint, the identification of the conceptual systems generated by students turns out to be rather complex. However, models are implemented according to specific procedures associated with these concepts. In previous works, we developed a tool for the characterization of mathematical models (Albarracín [55]; Albarracín and Gorgorió [56]; Gallart, Ferrando, García-Raffi, Albarracín, and Gorgorió [57]), which was based on identifying the chains of procedures that students implement and how these procedures are represented. In this study, we observed that sixth grade students created two types of mathematical models to tackle two aspects that were crucial to the development of the project: (i) maps of route use intensity and (ii) safe travel recommendations for other students.

7.1. Maps of Route Use Intensity

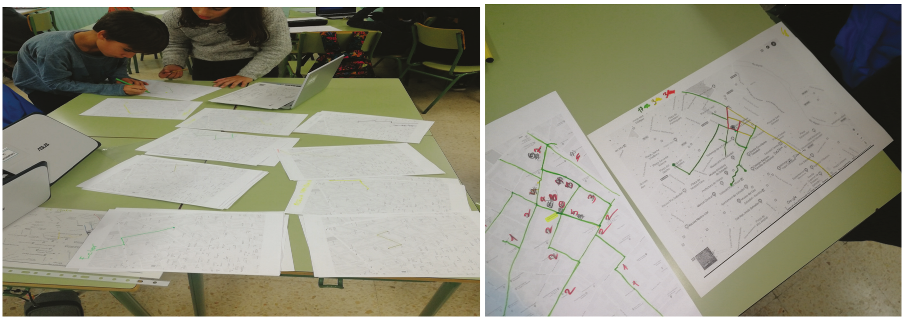
The first mathematical model they created consisted of the *maps of route use intensity* in the streets around the school. These maps were obtained from the data collected from the other students at the school, and they describe the phenomenon of student travel to school. The generation of these maps was crucial to the project because they were what enabled the students to identify the risk points to be studied. Table 1 describes the mathematical procedures that shaped the model.

Table 1. Description of the modeling process used to generate a map of route use intensity in terms of identified procedures.

Procedures of the Maps of Route Use Intensity Model
1. Ask the students about the route they use to go to and from school
2. Collect information so that it can be represented on a map as a clearly defined route
3. Organize all the routes collected from a class into sets that share the area near the school
4. Represent on a map the number of routes generated by the different students who pass through a certain street
5. Change the form of presentation to establish a code that makes the information displayed easy to read

A sample of the work involved in making these maps of route use intensity is shown in Figure 7. On the left, Figure 7a shows two students grouping the maps with the individual routes to make the map of route use intensity. On the right side, Figure 7b shows two different steps in the preparation of the same map. On the left, a count of the number of students passing through each street is shown.

On the right, the finished map is shown, with the numerical information simplified into a color code with the busiest streets marked in red.



(**a**) (**b**)

Figure 7. (**a**) Organizing the collected routes, (**b**) representing them on a single map.

From the perspective of M&M, these maps are clearly the result of modeling work. They display the use of the streets for going to school and are a product that supported subsequent decision-making on how to prioritize the risk points that students were going to study. The sixth grade students used these maps as a starting point to establish a criterion of selection based on determining those risk points most frequented by students in the target classes.

7.2. Recommendations for Safe Travel

The second type of mathematical model generated was the *set of recommendations for safe travel* in the streets. These recommendations emerged from the analysis of the layout of the areas under study and the way cars and pedestrians move through them, and from the predictions made about what they considered safe behavior. Table 2 describes the mathematical procedures that shaped the model.

Table 2. Description of the modeling process used to generate recommendations for safe travel in terms of identified procedures.

Procedures of the Recommendations for Safe Travel Model
1. Imagine and simulate the movement of vehicles and pedestrians on the street
2. Identify possible interactions that put pedestrians at risk
3. Represent the interactions of risk on a map, and take the measurements (distances and angles) that characterize them
4. Propose alternatives for pedestrian movement to avoid risk situations
5. Represent recommendations for safe action on a map, using indicators of movement

Given the diversity of black spots in the area around the school, we show two examples of the recommendations drawn up by the students in their work. In the first example, the students decided that the corner shown in Figure 8 represents a potential risk area for pedestrians heading towards it from the north. Therefore, they indicated this movement with a red arrow in Figure 8. This becomes a danger point if a car mounts the sidewalk at the same time as when a pedestrian coming in the opposite direction turns the corner, since the two paths cross. The students suggested that pedestrians should avoid that corner by using two crosswalks, i.e., following the green arrow. This movement is not really natural as it forces pedestrians to walk a greater distance than the red option, but the field of view of both pedestrians and drivers is better at each crossing.

Figure 8. Scheme with recommendations for use at a corner where cars may mount the sidewalk.

The second example concerns a T-intersection where cars that turn right have to give way to cars coming from their left (Figure 9a). There is a mirror at the intersection so that drivers can see whether it is safe to turn without having to pull halfway out into the other street (Figure 9b). The need to pay attention to the left means that drivers do not pay much attention to the right when turning. As in the previous case, a car could easily mount the sidewalk on the right-hand corner and hit someone walking there. For this reason, the students in their recommendations suggested walking on the left-hand sidewalk (Figure 9c), marking it in green on the map, so that they would always be in the drivers' field of view and be able to use the crosswalks when the vehicles stop.

(**a**) (**b**) (**c**)

Figure 9. (**a**) Aerial view of the corner under study; (**b**) drivers' focus of attention; (**c**) recommendations for use represented by arrows.

The students' set of recommendations was the product of a modeling process that enabled us to distinguish between good and bad practices when moving through the streets. The recommendations were specific to each situation studied, but the procedure for creating them is shareable and reusable in other analogous situations. Beyond this possibility of direct transfer, the generated model could be used for the design of safe behaviors for other users in other situations, such as drivers of cars and other vehicles. Moreover, the sixth grade students communicated the recommendations resulting from their study to the rest of the students, highlighting that their construction was based on mathematical arguments and procedures. In this way, mathematics constituted a validating component of the work carried out.

8. Discussion

The results of this case study showed that the Let's get to school safely project generated a large number of mathematical learning opportunities for the sixth graders. During the project, students had to cope with activities that required the coordinated use of different content and mathematical skills. Statistical data collection, information processing, data coding, visualization, mapping, and measurements, among other processes, were highly relevant to shaping the mathematical models. Each of the work groups was responsible for studying the routes of one of the school classes and for providing age-appropriate recommendations for these students.

During Episodes I and II, the work groups focused on collecting and organizing the data obtained from their target classes, thereby raising opportunities for mathematics learning that had to be consolidated by means of concrete products in order to continue the project. During Episode III, each of the groups was responsible for analyzing a risk point and preparing specific recommendations for safe passage through that point. The sense of responsibility developed during the project was what compelled the various work groups to address the learning opportunities that emerged during the project and transform them into concrete learning that manifested itself in the form of tangible products. In other words, the responsibility of each group in the project was the medium that promoted the construction and reconstruction of mathematical models, which is the main objective of modeling activities that take the M&M approach (Doerr and English [2]; Lesh, Amit, and Schorr [35]; Mousoulides, Sriraman, and Christou [34]).

Some of the ideas and proposals examined to generate the different models that supported the products of the project began with interaction among a small number of students, who were the ones who contributed the ideas in the first place. However, following group discussions involving the entire class, each of the work groups had to consider, interpret, and implement these ideas according to their own needs. Thus, the project encouraged the students to explain their strategies and reasoning (Walkowiak, Pinter, and Berry [46]) relative to the mathematical activities for which the students had decided what information they needed and the methods needed to analyze it (Wijaya, van den Heuvel-Panhuizen, and Doorman [42]). We concur with Törnroos [40] when he stated that the existence of learning opportunities does not imply that learning is consolidated. However, during the project, we saw how the students developed a form of collaboration where the mathematical contents and methods constituted the core of the activity. Therefore, what we observed leads us to think that the students learned to work mathematically as a team. Not only that, they also developed a sense of belonging to the school community.

The project had an impact on other school groups apart from the sixth grade students. The younger students saw how their schoolmates in the last year were interested in a problematic aspect of their daily lives. The sixth grade students asked them about their experiences and later gave them explanations—tailored to their level of comprehension and their practical needs—about factors that they had to take into account to guarantee their own safety when walking the streets. The explanations about the analysis and decision-making process that the sixth graders gave the younger students were an essential aspect of this informative procedure. Thus, mathematics played a clear role as a validator of the project results for the rest of the students. Besides that, some of the children's parents were able to participate actively in one of the activities, and others attended the final informative talks. In all cases, their children received information that made it possible to address safe behavior on the streets from within the family unit, taking the results of the project as a starting point.

From the point of view of the teaching staff, the project brought to light a new way of teaching students to use mathematics in context. Furthermore, the project format, based on the analysis of a social reality in order to generate safe behavior guidelines, provides a guide for teachers. This format could allow them to overcome some of the difficulties identified in the literature, because when students contribute a variety of ideas that are difficult to manage (Ng [19]; Winter and Venkat [20]), the teacher can refer directly to the needs of each episode of the project to decide if the students' proposals point in the right direction and prioritize them appropriately. We understand that teachers must have an active mathematics disposition if they are to adequately guide their students. In other words, they must be open to asking themselves questions and setting themselves problems, have a knowledge of specific cases where mathematics helps to understand real situations, and be able and willing to document themselves or seek specific help when they are not acquainted with the mathematics necessary to interpret or describe a situation. It is also necessary that teachers be trained to work in the classroom using open projects or have the experience needed to do so, even if they are not strictly mathematical projects. Thus, a key aspect of the role played by teachers is to identify the mathematical contents that students generate in order to properly organize and institutionalize them.

9. Conclusions

The case study developed around the project Let's get to school safely permits the characterization of a new type of school project that we labeled mathematical modeling projects oriented towards social impact and informs us of its possibilities, showing us that such projects can be generators of learning opportunities, as explained above. We conclude this article with a description of the basic characteristics of MMPOSIs.

First of all, a modeling project geared towards social impact must tackle the needs of people as a community (for example, road safety, pollution, immigration, etc.) and must be structured with the intention of having a direct impact on the educational community, either on the students themselves, their families, or other close groups of people with similar needs. In this way, the issue tackled in the project arises from the students themselves or the school itself, and the product resulting from the modeling process has a real impact on the community, even beyond the group of students who develop it. Furthermore, it is the fact of developing a sense of responsibility towards the community that leads to the search for solutions, thus favoring the emergence of mathematics learning opportunities and, in particular, stimulating students to take advantage of them. We call this characteristic the principle of social impact.

Then again, for an intervention to be an MMPOSI, it must be structured in such a way that it demands an analysis of the phenomenon from the perspective of mathematical modeling. It should be possible to organize the project into various coordinated activities that act as MEAs (Doerr and English [2]), which encourage the students to generate products based on a mathematical model, which can be communicated to and used by the project target group. Thus, the development of this type of project obliges the students to decide what information is needed to tackle it and what methods are required to analyze it, as well as to explain their strategies, procedures, and reasoning to their peers. We call this second characteristic the principle of mathematical modeling.

The mathematical knowledge gained during the project should support actions that provide a return to the educational community. Mathematical knowledge provides a basis for student decision-making and, at the same time, plays a key role in validating the results of a project when it is presented to other students. We call this third characteristic the principle of mathematical justification. In MMPOSI, mathematical modeling plays a twofold role. On the one hand, it allows students to develop their mathematical competence in complex situations that are familiar and relevant to them, and on the other, mathematics serves as a tool for validating the results. This mathematical validation also offers students a way to defend their products, since it is a procedure that guarantees the suitability of their recommendations for solving the problems under study. In previous studies, we observed that when students produce their own mathematical explanations of social phenomena, they tend to trust their own methods and conclusions to such a point that they generate a framework to support their decisions and freely criticize the results provided by external sources, such as information that appears in the media (Albarracín and Gorgorió [58]).

Producing safe proposals to move around the city demands complex, relevant decision-making, which entails developing a high level of responsibility and social awareness. The process leads students to question the way in which their environment is constructed, and given the doubts raised, they come up with their own ideas about how a city should be organized. Thus, group work, the connection of mathematical knowledge with reality, and the use of different technologies to collect, organize, and interpret data come together so that students can take a reasoned position on relevant social aspects that affect them. This requires the coordination of knowledge and procedures typical of different disciplines, with which MMPOSIs can provide a way of promoting interdisciplinary work without reducing the role of mathematics to a minimum expression. They thereby play an active role as members of the community who begin to make their own decisions and develop their own ideas of what the world they live in should be like.

Finally, we would like to make it clear that because this case study was instrumental, i.e., developed with the intention of illustrating a concept, its findings should not be regarded as all-inclusive or generalizable. Given that we started from naturalistic observation to exemplify and characterize a new theoretical construct, it is obvious that this study needs to be followed up by others to continue exploring MMPOSIs at different educational levels and with different teachers, both in terms of their mathematical knowledge and their approach to mathematics. In particular, we think that it would be interesting to explore how the students' social commitment and sense of belonging to the community impact the emergence of learning opportunities during MMPOSIs and makes these learning opportunities materialize in concrete learning throughout the development of a project of this type. In this research, we saw that the students' survey of the routes led to the study of movements on a map. In this respect, we also think it would be interesting to explore how certain topics could be connected to develop projects with specific mathematical content. For this reason, we think that it would be worth studying teachers' modeling competencies, to see in particular if they can identify a priori these connections between reality and mathematical content.

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mathematics

Article

Enhancing Computational Thinking through Interdisciplinary STEAM Activities Using Tablets

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Abstract: Computational thinking is a highly appreciated skill by mathematicians. It was forecasted that, in the next few years, half of the jobs in science, mathematics, technology and engineering (abbreviated as STEM, including arts as STEAM) will use some kind of computation. It is therefore necessary to enhance the learning of mathematics by collaborative problem-solving activities focused on both learning mathematics and developing computational thinking. The problems in science offer a reasonable context in which to investigate the common overarching concepts (e.g., measuring the length). An interdisciplinary STEAM collaborative problem-solving activity was designed and piloted with 27 lower secondary students aged 13.07 ± 1.21 years. Different levels of willingness to use the technology were observed and the factors influencing it were identified. We found that strong background knowledge implies high demands when controlling the used device. On the other hand, when a nice and user-friendly application was used, students did not need to perceive any control over it. After the intervention, the students' views on the tablet changed and they reported more STEAM-related functions of the device.

Keywords: computational thinking; STEAM education; leisure-time education

1. Introduction

To learn mathematics in the 21st century not only means obtaining mathematical proficiency, but also critical thinking, creativity and technology literacy [1]. Collaborative problem solving is one of the recommended pedagogies to promote the active learning of mathematics. Furthermore, collaborative problem solving led to better performance in standardized tests in mathematics than a traditional transmissive approach [2–5], particularly when the problems were related to the real life of the students [6] and used technology [7]. It was also reported that students educated using collaborative problem solving appreciate their knowledge of mathematics and science even in their future workplace. This influenced their academic performance and career choice [8].

Furthermore, it is predicted that, in the next few years, half of STEAM-related jobs will be in computing [9,10]. Children and young people use smartphones and tablets on a daily basis, but their use is mainly for entertainment, not for learning purposes. On the contrary, mathematicians consider the effective use of technological tools as a "valuable component of the practice of doing mathematics" [11] (p. 9). A similar perception can be given of the scientists [10,12]. This has led to the large-scale development and piloting of materials aimed at fostering computational thinking [13], but not all of them are suitable for problem-based learning. Cápay and Magdin [14] used black boxes as the main concept for tasks developing computational thinking and they provoked very intensive reaction. Burbaite et al. [15] designed an activity where the students could learn about the physical principles of functioning an ultrasonic sensor, connecting knowledge of physics with knowledge from computer science. Students were able to gain conceptual knowledge in physics and design the algorithm at the same time. Another example of an interdisciplinary approach can be found in the work of Lytle et al. [16], aimed at an agent-based simulation with a special focus on student-perceived ownership of developed programs. Students using the use–modify–create approach felt more confident and perceived the code developed in the guided part, with their slight changes, as more familiar compared to the transmissive approach in the control group. Several studies [17,18] have shown that the design-based approach can improve the computational thinking of participating students and enhance the students' awareness of the different tasks that can be performed using the computer and their self-efficacy in using computers.

1.1. Interdisciplinary Teaching

A lot of current scientific problems can be addressed only if experts from several scientific fields collaborate together. New scientific fields (e.g., physical chemistry, biostatistics, and theoretical physics) have even been established. However, the school curriculum is divided into separate subjects. In Slovakia, even science subjects are separated to physics, chemistry and biology in secondary education [19]. Both mathematics and science education aim to enable students to understand the wonder of the world around us. They share strategies for solving problems and for scientific inquiry. These approaches include logical thinking, hypothesizing, observations, analysis and experimentation. Even university students are not used to solving practical problems and, therefore, they are not able to interpret the obtained results [20].

St. Clair and Hough [21] grouped arguments supporting an interdisciplinary approach to education into six groups. An interdisciplinary approach (i) is in agreement with the current body of knowledge about the needs of the secondary students; (ii) offers a substantial learning environment and, therefore, has a positive impact on the learning process as well as on achievement; (iii) provides students with a more holistic approach to problems; (iv) is global in content and better prepares participating students for critical citizenship; (v) improves the skills necessary for problem-solving by demonstrating different views and perspectives; (vi) encourages collaboration among teachers.

1.2. Computational Thinking

Weintrop et al. [9] stress the ability of mathematics and science to develop key skills in computation. Departing from the poor-technological view, we can focus on four main categories: "data practices, modelling and simulation practices, modelling and simulation practices, computational problem-solving practices and systems thinking practices" (p. 127). It is important to realize that computational thinking is more than using technology: it is a way of thinking while solving complex problems [22]. Computational thinking was defined by Aho as "the thought processes involved in formulating problems so their solutions can be represented as computational steps and algorithms" [23] (p. 832).

Similar to proficiency in mathematics [24], computational thinking may be also developed via problem-solving activities [25]. Gretter and Yadav [26] see "collecting, analysing and representing data, decomposing problems, using algorithms and procedures, making simulation" as the key component of computational thinking (p. 511). Bocconi et al. [27], based on studies [28,29], define six components constituting computational thinking: (1) abstraction, (2) algorithmic thinking, (3) automation, (4) decomposition, (5) debugging, and (6) generalization. Abstraction is understood as reducing the details in order to make the artefact more understandable. The essence of abstraction is competence in choosing the proper feature to hide and proper representation, so that the hiding results in easier problems with a suitable solution. Algorithmic thinking is a systematic way of thinking applied to splitting a complex problem into a series of (not necessarily ordered) steps utilising the different tools available in the moment. Automation can be defined as a procedure aimed at saving the labour which the computer is uses to perform a (ordered) set of repetitive tasks instead of the slow and inefficient work of humans. Decomposition is a way of breaking down the artefacts into smaller parts that can be "understood, solved, developed and evaluated separately" [28] (p. 8), which makes complex systems simpler to design. Debugging is a looking-back ability when the outcomes are analysed and evaluated. Generalization, as a part of computer thinking, is connected to "identifying patterns, similarities and connections, and exploiting those features" [28] (p. 8), relating to previous experience with similar problems and adopting developed algorithms to solve the comprehensive class of similar problems. Although the positive effect of the use of technology on students' performance in mathematics and science was confirmed, very few studies investigated the use of these applications for mathematics. The results of Kosko et al. [30] suggest that integration of the applications over a three-week period significantly increased the mathematics achievements of participating students.

The ability to use the technology is not self-developing. The fact that students are able to use tablets or smartphones or any other technology for communication or browsing the internet does not imply an ability to use it for more sophisticated purposes, such as measuring the distance, temperature or size of an angle, calculating repetitive tasks or processing the measured data. It is necessary to provide students with the opportunity to experience this kind of use of the technology. The invention of mobile technologies allowed students to unplug the computers, leave the classroom and move outdoors [31].

The instrumental approach [32] seems to be a reliable framework to understand what is going on during the activities, supporting both mathematical learning and computational thinking. The technology introduced in the classroom can be considered as an artefact. Only when students learn to use it, when they develop the utilization scheme, does the artefact become a tool, an instrument [33]. The development of the utilization scheme can be described as having three levels: (1) usage schemes, (2) instrumental action schemes including gestures and operative invariants, and (3) instrumented collective activity schemes [34]. Usage schemes are directly related to the artefacts themselves. They are developed through manipulations and examinations by the artefact. Instrumented action schemes or instrument-mediated action schemes are higher-order, coherent and meaningful mental schemes, acquired from existing elementary usage schemes when a student manipulates an instrument with the aim of solving the problem. The developed schemes are specific to each activity. When an application is introduced in the classroom, students first have to become familiar with its basic features, developing the usage scheme. Only then they can use it for solving the problem and fostering the instrumental action schemes. Instrumented collective activity schemes or collective instrument-mediated activity schemes are the schemes developed in the context of collective, particularly collaborative, activity. The students are both influenced by artefacts' potentialities and constraints (instrumentation) and influencing the artefact via their preconceptions, knowledge, beliefs and usual ways of work (instrumentalisation) [34]. The two described dual processes are united in the instrumental genesis when the instrument arises as the result of the interactions between the student (subject of the activity) and the artefact [35–38].

The main aim of this study is to demonstrate the potential of interdisciplinary problem-solving activities, including several STEAM disciplines, to develop both the mathematical proficiency and computational thinking of involved students. Various activities were designed to develop computational thinking [13,14], but only a few of them were focused on the students' tendency to use technology to solve problems. In this article, we looked for the answer to the research question formulated as follows: What components of computational thinking may be developed by involving students in interdisciplinary STEAM activities using technology? How is this development manifested?

2. Methodology

The presented research was conducted with a more than 15-year longitudinal study about summer camps for lower-secondary students focused on STEAM, particularly physics and mathematics (mainly as the language of physics). The study based on the design research principles involved the following phases: (i) the preparation and design of instructional materials, (ii) implementation of the materials, and (iii) retrospective analyses, cyclically repeating from 2006 to 2020, described in more detail in the work of Cobb [39,40]. The outputs from each year informed preparation and design in the following year. Development of the design and the overarching topic of each year are summarised in Supplementary Materials (Table S1). The camp leader had an input and primary responsibility for implementing the activity with the group of 3–4 students. In this article, we focus on one particular activity implemented in the year 2019. As the activity was held in the 14th year of the study, the design was informed by each activity implemented in previous years of the camp.

2.1. Participants

The camp in 2019 was attended by 27 students of grade 6–10 (age 13.07 ± 1.21 years), nine of whom were girls. The leaders in the group were a graduate student of Physics Education and a bachelor student of physics. The 27 participating students were divided into three big groups, and each group was further divided into three sub-groups to enable students to try the hands-on activities on their own. For some of the activities, the groups were divided into two smaller subgroups, one with younger students (YS) aged 11–12 and the second with older students (OS) aged 13–14. Written informed consent was collected from the parents of all participating students.

2.2. Preparation Phase: Design of the Activity

During all years, the camp was organized to last from Monday till Friday. Usually, there is an overarching topic for all the activities in the camp. On top of this, the overall design of the 2019 camp was focused on the development of students' skills in the area of project management including the development of related soft skills, e.g., leadership, organization and communication. The main topic of the camp was the construction of a rocket. Students were supposed to design a rocket in an environment simulating a real funding programme. They had to write a project proposal to get funding, prepare the budget and milestones, and defend their project at the ending of the camp-week. One of the required tasks in the final project was to assess their developed rocket based on various indicators, including the maximum height their rocket could achieve.

The design of the activity was led by the categorization of the strategies of interdisciplinary teaching into three groups, as proposed by Nikitina [27]: (1) contextualizing, providing a reasonable context for the interdisciplinary activity (preparation of estimating the height of the rocket), (2) conceptualizing, meaning that the activity is based on an overarching concept that is central for two or more disciplines (i.e., measuring and length) and (3) problem-centring, particularly using real-life or realistic problems in which concepts, processes and ideas of different disciplines have to be used in the solution. The main aim of the activities rooted in this strategy was to create a tangible outcome or product, in our case, a procedure to estimate the height.

In order to prepare students to solve the problem, e.g., to estimate the rocket range height, a series of three smaller activities was designed. The primary designers were two of the authors of this article, who were involved in the longitudinal study since its very beginning and therefore were informed in detail about the outcomes of the previously implemented activities. The camp leaders adjusted the ideas and prepared the worksheets for the students.

2.3. Implementation and Reflection

The designed activity was performed three times during camp-week. After each implementation, the leaders reflected on their experience with the guidance of the designers of the activity. After the first piloting, the activity was slightly changed. The second pilot did not lead to any significant change in the worksheet.

2.4. Data Analysis

The implementation of the activity was audio-recorded and transcribed. The students' actions connected to the use of technology (i.e., tablets) were analysed using the components of computational thinking, as described by Bocconi et al. [27], and the episodes where computational thinking was observable were chosen.

In order to compare the responses of the participating students in the items related to computational thinking in the beginning and at the end of camp-week, a McNemar test of symmetry [41] with Yates correction [42] was performed using the calculations, carried out in the programme STATISTICA 13.3 (StatSoft Inc., TIBCO Software, Palo Alto, CA, USA).

3. Results and Discussion

The activity was designed to teach children the procedure for measuring or estimating the height of the object. The overarching concept involved was measuring the distance. Length was seen as both the physical quantity and also the geometrical characteristics. On the other hand, the main aim of the activity was to estimate the height of the statue, so problem-centring was used as a strategy to create interdisciplinary teaching. The whole activity took approximately 90 min for each group. It was divided into the three phases: motivation and introduction, problem-solving, and concluding. Each of the three strategies took approximately 20 min.

3.1. Strategy 1: Shadow

The first strategy offered to students in order to solve the problem used measuring tape and a pole (140 cm). First, pupils measured the length of the pole when the pole was laid down on the flat surface. After that, the pole was placed to be orthogonal with the floor, and the length of the shadow of the pole was measured. The next step was to measure the shadow of the statue.

The first phase of the strategy was the introduction into similarity. Teacher explained the concept of similarity, and when she found that some of the pupils knew and understood, she left him/her to explain it to the other members of the group. The teacher facilitated the discussion and posed questions to lead the students to come up with the expression $s_h : s_s = p_h : p_s$, where s_h means height of the statue, s_s height of its shadow, p_h means height of the pole and p_s height of the pole's shadow. Students later expressed the height of the statue as $s_h = \frac{p_s}{p_t} \cdot s_s$. The intention was to use the spreadsheet to calculate the height of the statue (s_h). After expressing s_h, the teacher asked the students how to calculate the height of the statue using the measured data.

T	Have you seen this kind of expression? How would you use it?
YS1	This [points finger on $\frac{p_s}{p_t}$] reminds me a fraction, so we might substitute our numbers and we will get something.
T	So, you can start to calculate. Add a column to the table and write down the results.
YS2	Isn't it possible to write the numbers to the table in the tablet, so the tablet can calculate instead of us? (1)
T	Try to find an app which can calculate. What kind of software would you look for?
YS3	I am going to ask some of the older students of the group.
T	Check the installed apps in the tablets. Maybe you can find an appropriate app on your own.

The two students remaining at the stage with the Statue problem started to run the different applications in the tablet. The students had two minutes, so the remaining students did not get a chance to check more than three apps.

OS1	Just find excel or something resembling excel [takes the tablet, starts MS Excel, creates a table] and do not delete it, is will remain for the next group [then he leaves the subgroup of younger members and joins his subgroup]
YS1	I do not know how and based on what that excel calculates (2).
YS2	Neither do I.
YS3	It's kind of a black box (3). It gives the data but we do not know where the formula is hidden.

The students resisted using the spreadsheet prepared by their groupmate without understanding it. The teacher made them insert the data into the table and then showed them the formula written in the last column. Even though the students could see it, they were not familiar with the syntax and they were not able to see the formula they came up with before (see Figure 1). Their previous experience and usual ways of work were not satisfactory to understand the potentialities of the application.

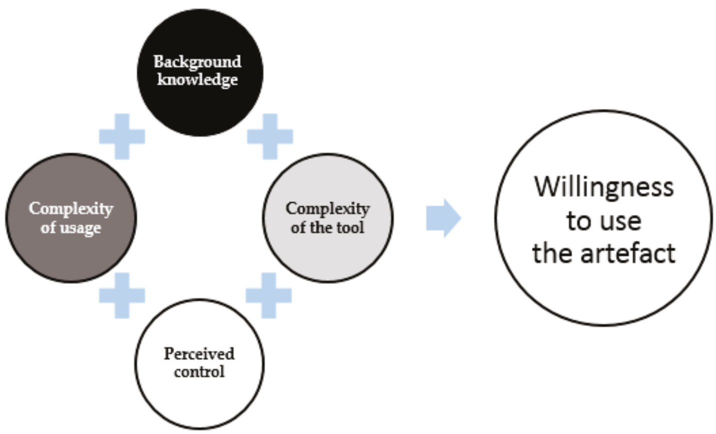

Figure 1. Low student willingness to use the artefact as influenced by the low perceived control.

One of the older students gained this ability and did not expect that the younger members of his group would not understand his product. Therefore, the teacher asked the younger students to create a new file and introduced them to the basics of typing formulae in a spreadsheet. Then, the students were able to see their formula in the prepared spreadsheet and they were willing to use it for calculations (see Figure 2). The knowledge of basic syntax allowed them to use the spreadsheet while fully understanding how it works. As the students were fully aware of the method used to estimate the height of the state, they felt the need to influence the artefact in the process of instrumentalisation. In this case, students understood the formula and how it was developed, they had full background knowledge and resisted using the spreadsheet without controlling the process of computing.

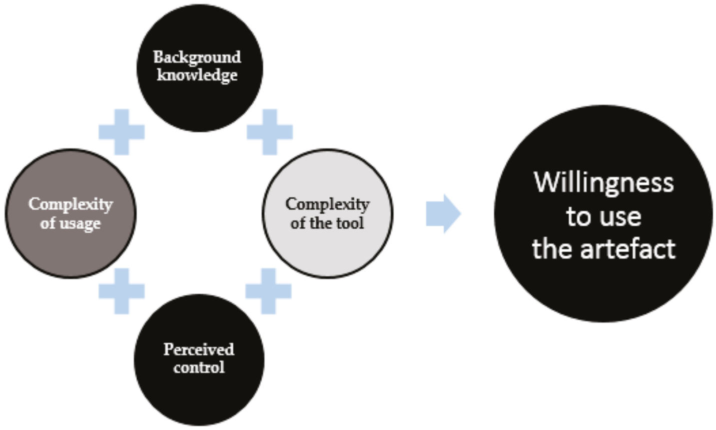

Figure 2. High student willingness to use the artefact as influenced by the high perceived control.

This episode gives us two interesting moments. In contrast with the rest of the group, the young student YS2 did not perceive the tablet as an artefact; she intended to use it as a tool, but did not have the appropriate knowledge. She showed an unexpected level of computational thinking in the area of automation (1), understood as a labour-saving process when the computer executes repetitive tasks instead of humans [29], but she was able to come up with either a concrete application or the kind of software needed. Conversely, the older student OS1 had the utilization scheme at their disposal and was able to develop a needed spreadsheet, so he acted using the tablet as with an instrument.

We consider it a very positive sign that the younger students complained that they did not understand what the spreadsheet developed by their group-mate computed [2], and pushed the teacher to show them how to create it. The process of instrumental genesis was present in both ways: students' knowledge of mathematics led to their own development of the artefact and the unknown calculations provoked the students' perception that they need to understand how the artefact works. This need may indicate some level of evaluation [43], or at least a tendency toward the evaluation of results provided by the software. After a detailed explanation by the teacher, they could understand the process of how the formula was transformed into the spreadsheet. Even though we did not test whether they could create the spreadsheet on their own, they were able to use it, and developed an instrumented action scheme capable of solving the problem.

3.2. Strategy 2: Protractor

The second implemented strategy followed the successful solution by strategy 1, described above. The main objective of the activity was to give the students the opportunity of learning how to estimate the height of the object based on the angle between the horizon and the line connecting the observer's eyes and the object. The intention of this activity was to prepare students to use the application for measuring angles by giving them experience measuring them with a homemade tool.

This strategy did not depend on the weather, as it did not use the length of the shadow. It also needed less information compared to the shadow strategy. Instead, it used other manipulatives: a homemade protractor from cardboard and a rope. With the first of the six groups, we tried to create the protractor, but it took too long, as the students did not have satisfactory skills. As they used corrugated cardboard, the nib of the pen pierced the upper layer of the paper. Therefore, the other groups got the pre-prepared cardboard and had to finish the protractor. The students first fastened the rope to the paper. Then, they had to decide what kind of plummet they would use.

T	I should point out that to ensure the correct measurement the rope should be tight.
YS2	How to ensure that the rope will be tight?
YS3	We can hold it stretched.
OS1	And how would you measure it, if it should be tight all the time? What should we put there?

The teacher provided students with the box of different things, including the 20 g weight. All the groups intended to find an object that was easy to fasten and heavy enough to tighten the rope. One of the three groups wanted to find some object with a spike to point to the exact line. All the groups used the weight, but one of them tried to use the pen first (the nip should serve as the pointer).

After finishing the cardboard protractor, the students could finally measure the angle under which they see the top of the statue. The students were already familiar with the similarity of triangles. All the groups came up with the idea that if they knew the ratio and distance from the statue, they could calculate the height of it. All the groups started to calculate, and they came up with results different than those obtained using strategy 1. The students were confused, but the teacher asked them where they held the protractor. Then, they realized that they should subtract the height of the person doing the measuring. The results were quite different again. In each group, the teacher tried to question the students to lead them to realise that they should subtract the height of the eyes (see Figure 3), not the top of the head of the measuring person. With this correction, all the groups obtained results not very different to the ones estimated by strategy 1.

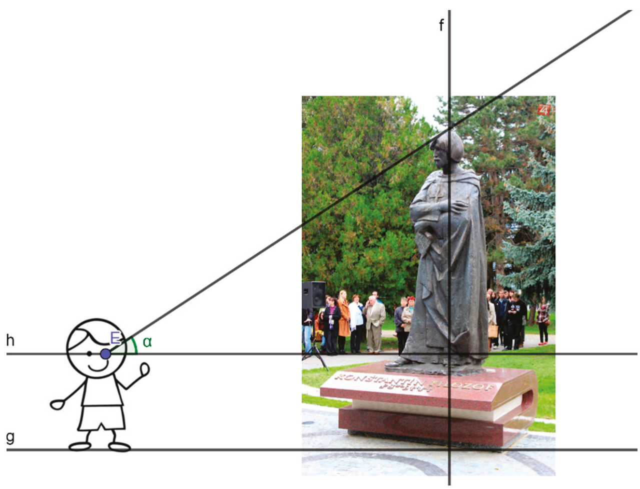

Figure 3. The sketch used to calculate the height of the statue based on the view angle.

In the final discussion after this strategy, the teacher provided the students with the definition of a tangent as the ratio of the length of the opposite side to the length of the adjacent side of the right triangle. The teacher put the formula into the tablet and developed a spreadsheet calculating the height of the statue based on the distance between the statue and observer and the viewing angle. Contrasting with the previous strategy, the children did not ask for an explanation of the formula. This may indicate that they did not understand the idea of the tangent in depth, but accepted the existence of it and believed that the teacher could teach it to them through a worksheet. Their background knowledge was lower and they did not perceive the need to fully control the method (see Figure 4). Furthermore, the worksheet gave the same result as the one they understood. Their knowledge was not deep enough to shape the instrument. The students were able to deal with the spreadsheet, as with the black box providing the same result. We assume that the same instrumented action scheme is needed to use the spreadsheet as in the previous case. However, none of the students tested whether the results would be the same when measuring the height of the known object. This fact may indicate that the students did not have advanced computational thinking in the area of debugging, defined by Csizmadia et al. [28] as the evaluation of developed application by testing, tracing or critical thinking.

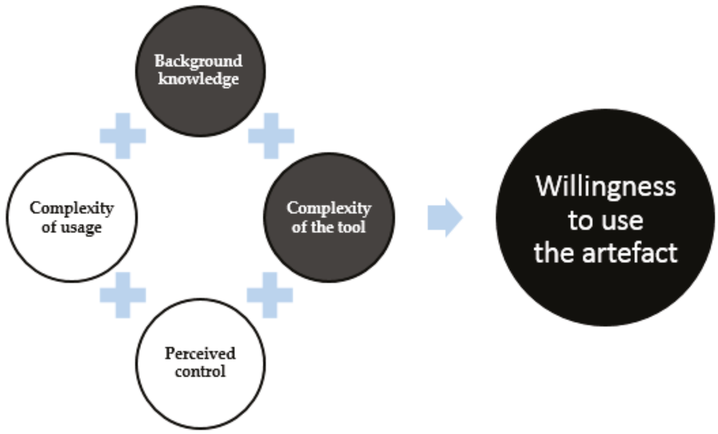

Figure 4. High student willingness to use the artefact as influenced by a similar level of background knowledge and perceived control.

3.3. Strategy 3: Application

The third strategy used to solve the statue problem involved directly measuring the height of the statue with the SmartMeasure application on a tablet. The input for the application is the height of the eyes of the observer. Then, two other measurements are carried out: measuring the distance between the observer and the bottom of the statue and measuring the viewing angle of the observer. The application then gives the height of the statue as the output. We expected that with direct use of the application, the students may be confused by the reason for inputting this particular information. The students were already familiar with the application, as they used it for measuring the length of the corridor. The students were pleased when they found out that they would use the tablets. The teacher instructed them to use the application.

YS1	What should we look for in the app menu?
T	Try to look and guess what we can use.
YS2	There is a picture like the protractor we used, so may be that one (1).

After the confirmation of the teacher, the students launched the application and tried to use it without any previous instruction, but with the guidance of the teacher.

YS3	We need to set here something.
OS2	We need to know the height of the tablet. What height we should use?
YS3	My height.
YS1	Do you have the tablet on your head?
YS3	No, I hold it in my hands. But I think I should put it close to my eyes, same as we measured by paper protractor. (2)
T	So, what height should you set?
YS3	The height of my eyes.

The students very smoothly used the application to measure the distance between the ground and the observer's eyes and use it as the input for the estimation of the height. We assume that the students were able to use the table so naturally because the utilization scheme for the cardboard protractor was very similar to the utilization scheme for the tablets. The relation with the previous experience was signalled by the icon of the software (1). The student YS3 said this explicitly.

The students had the same background knowledge in mathematics as in strategy 2, and the application uses the same principle to estimate the height of the object. On the other hand, their constructed protractor was fully controlled by them but, when using the tablet, they did not know how the technology measures the angle or the source calculating the height. Their background knowledge was even lower than in the previous case (Figure 5). The work was automated by the table, and the tool used was more complex.

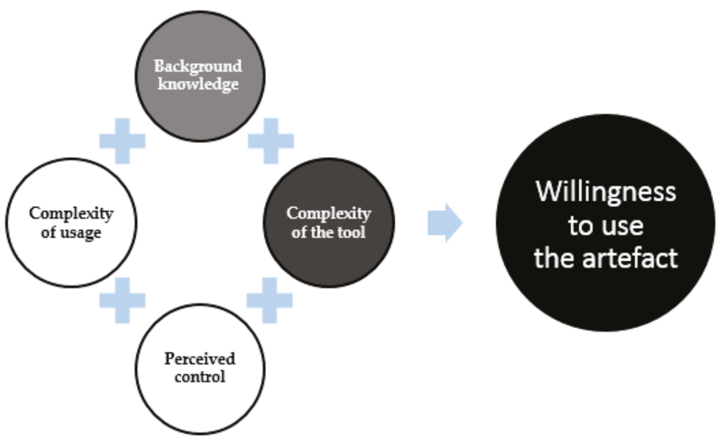

Figure 5. High student willingness to use the artefact as influenced by higher complexity of the tool.

The students demonstrated a certain amount of abstraction when they were able to abstract from the concrete artefact and transfer the utilization scheme from a paper tool to an electronic one. We believe that the previous experience with the cardboard protractor enabled students to adopt the instrumented action scheme with a satisfactory depth for specific transfer. Furthermore, the simplicity of the homemade tool provided students with the opportunity to learn the process behind the estimation, and to concentrate on the mathematical basis used by the application without the disturbance caused by the more sophisticated affordances of the tablet.

The above analysis shows that there are more factors affecting the proper use of technology than the elements of computational thinking. Although students were not able to do this, they wanted to use the tablets for calculations. On the other hand, they refused to use the technology they did not understand. The need to understand the technology was influenced by both (1) the characteristics of subject (students), namely the background knowledge and perceived control over the artefact, and (2) the constraints and potentialities of the tool, its complexity and the complexity of its utilisation schemes. We believe that this is an important factor to be considered when assessing the computational thinking of students, particularly in the area of automation.

3.4. Remarks about the Impacts of the Activity

Besides the identification of the process of instrumental genesis, we would like to show how participation in the activity further influenced students' work and perception of the technologies.

Just after solving the statue problem, the students had to answer the question of which methods they would use to estimate the height of an object. All of them chose using tablets and the SmartMeasure app as the most appropriate approach, so we might assume that the artefact became an instrument. The students' reasoning varied. Students stressed the different potentialities of the application: (1) its simplicity of usage "we did not need to do anything, the tablet did the measure instead of us", "tablet needed just one number", "we can stand at one place", "we did not need any other equipment"; (2) independence of the weather "we can measure using the tablet if there is a shadow or not" or (3) the universal usage "I can install the same app to my smartphone that I always have on disposal". In the feedback for the activity, the students complained that they had to estimate the height of the statue using the shadow or protractor when the tablet offered a very convenient way to measure it. Even though they were not happy with the demanding process, we believe that the procedure gave students the opportunity to learn some new geometry, as well as stimulating their critical thinking in the further use of the technology. They were actors in the situation when they misused the technology by inputting the height of the person instead of the height of his/her eyes. When they inputted the wrong information in the application, the result was wrong.

Their experience with measuring the height of the statue was also observable in other activities performed during the week, mainly during estimating the height of their constructed rockets. When measuring the rocket range height, all the groups chose the tablet. This fact might be considered as partial evidence that the automation and abstraction components of computational thinking were developed. It is worth mentioning that younger students (age 10–11 years) referred to the measuring of the height as "statue", while older students (age 12–13 years) referred to it as "estimating the height by a protractor". The different terminology used may reflect the different levels of cognitive development of the students. While the younger students named the action according to the surface characteristic, the particular object measured in the problem, the older students focused more on the structure of the solution when the angle was used to calculate the height, so, surprisingly for them, the height was estimated by a protractor.

Besides this qualitative evidence, we may support our findings about the changed perception of the tablets from the participating students by the change that occurred in their responses to specific items in the questionnaires administered at the very beginning and at the end of the camp week. The responses are summarized in Table 1. In the first question, students were asked to list different means of measuring distance. In the pre-test, none of the participating students mentioned any electronic device, while in the post-test 17 of the 27 participating students mentioned the tablet or the SmartMeasure application ($chi^2 = 15.059,\ p = 0.0001$). When students listed what tablets can be useful for, they mainly listed calling and messaging in the pre-test, while in the post-test they listed different features useful for solving STEAM problems, e.g., calculator, measuring, GPS or compass $\left(chi^2 = 5.882,\ p = 0.015\right)$. The measurement of different characteristics was listed by 14 of 27 participating students ($chi^2 = 7.118,\ p = 0.008$) in the post-test. The change in students' responses supports the hypothesis that involving students in problem-solving activities when technology (i.e., tablet) is used as a tool to simplify their work can enhance their computational thinking, mainly in the automation component.

Table 1. Students' responses to the chosen items in pre- and post-questionnaire.

Before Intervention	After Intervention		
	Measuring the length		
	using ICT	without ICT	Total
using ICT	0	0	0
without ICT	17	10	27
Total	17	10	27
	Using tablet for		
	no-STEAM	Including STEAM	Total
no-STEAM purposes	4	3	18
including STEAM	14	6	9
Total	17	10	27
	Using tablet for		
	no-measuring	measuring	Total
no measuring	0	0	18
including measuring	14	13	9
Total	14	13	27

The episodes reported in the qualitative analysis indicate that the abstraction can be enhanced too. The process of the instrumental genesis of the students was not studied in great detail, but we assume that using the physical aid before the virtual one enhanced students' understanding of the process of how height is obtained based on the viewing angle. Similar results are reported by Vagova et al. [44], who used 3D-printed manipulatives before the virtual manipulation with cubes in the computer environment. The findings also support Lieban's [45] conjecture that the results obtained using a virtual environment should be confronted by results using physical aids. The implementation of both physical manipulatives and computers enhances the mathematical modelling skills of secondary students [46].

3.5. Limitations of the Study

We are aware of the limitations of the study, mainly the fact that the participants of the study were not regular students, but students choosing a summer camp focused on physics, so their enthusiasm and willingness to solve the problem is much higher than in the common population. Their interest in STEAM subjects allowed us to analyse the processes appearing when introducing technology without the need of demanding classroom management. When generalising the results, one should keep in mind that the sample comprised only 27 students. The arrangement of the activities in the summer camp, where a small group of students work under the supervision of one or two camp leaders, allowed us to see what is going on during the process of instrumentation genesis in detail. On the other hand, neither the teachers' (camp-leaders) work nor the teachers' influence on the process were analysed. Furthermore, the activity in which they were involved provided a plausible context to solve problems in mathematics and/or physics, and we therefore assume that similar processes would happen in usual classrooms.

4. Conclusions

The main aim of the presented study was to demonstrate the effect of incorporating ICT in solving STEM interdisciplinary problems on the computational thinking of the students. The concept of measuring connects mathematics and physics, and the problem-based orientation of the analysed activity supported the interdisciplinary learning of participating students. The mathematical apparatus was used in the procedural way or as a toolbox, but enhanced the mathematical repertoire of models of right-angled triangles by a nontrivial separated model. On the other hand, the students could develop skills in measuring the distance and length by concrete tools, as is usual in physics. The progressive involvement of technology in the problem-solving process allowed students to understand the way the application works and enhanced their computational thinking in a very meaningful way. They were able to progressively develop their utilisation schemes.

The detailed process of instrumental genesis during a problem-solving activity involving mathematics and physics should be studied further. The influence of different orchestrations should be researched in order to maximize the students' gain from the educational activity.

Supplementary Materials: The following are available online at http://www.mdpi.com/2227-7390/8/12/2128/s1, Table S1: Development of the design of the camp in years 2006–2020.

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mathematics

Article

Factors Influencing Mathematics Achievement of University Students of Social Sciences

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Abstract: The paper aims to investigate the main factors influencing the mathematics achievement of social sciences university students in Slovenia. A conceptual model was derived where three categories of variables were taken into account: attitude towards mathematics and math anxiety, engagement in learning activities, and attitude towards involving technology in learning mathematics. Data were collected for seven consecutive academic years and analysed using Structural Equation Modelling (SEM). The results showed a very high coefficient of determination for mathematics achievement (0.801), indicating that variables "Perceived Level of Math Anxiety", "Self-Engagement in Mathematics Course at University", and "Perceived Usefulness of Technology in Learning Mathematics", together, explain 80.1% of the total variance. Based on our findings, we can conclude that teaching in secondary school is a crucial determinant for success in mathematics at university. It is essential to identify the best methods for secondary school math teachers which will help them give future students better entry-level knowledge for universities. These methods will, hopefully, also improve the level of mathematics self-confidence, as well as lower the level of math anxiety, which all considerably affect the performance of students in university mathematics.

Keywords: mathematical education; good practices in mathematics education; mathematics achievement; influencing factors; university; social sciences; structural equation modelling (SEM)

1. Introduction

Mathematical skills have long been recognised as essential not only for academic success but also for efficient functioning in everyday life [1]. By studying mathematics, we train accuracy, consistency, and mental discipline, which are essential skills needed for effective and responsible problem solving and decision making in everyday life. Due to the global awareness of the importance of mathematical knowledge on the one hand, and the concern expressed for many years at various levels of education about underachievement in mathematics [2], the performance of students in mathematics from primary school to higher education is still a topic of concern [3].

After reviewing publicly available databases, we found that the majority of studies on mathematical performance and achievement are focused on either primary or secondary education or both (see, e.g., [4–19]). Studies focusing on higher education (i.e., tertiary or post-secondary education), which is the subject of our research, are less represented (see Section 2.2).

Knowledge of mathematics has often been cited as crucial for several disciplines in higher education, including technical fields, engineering, economics, and finance, as well as agriculture, pharmaceuticals, and health sciences [20–22]. Since mathematical knowledge offers widespread application, social sciences university programs around the world require their students to take at least one mathematics course. Their students gain essential mathematical knowledge and develop the analytical and computational skills they need in their field of specialisation. Unfortunately, mathematics in university courses has often been identified as a significant obstacle for students and as one of the main reasons for dropping out of university [22]. This problem is particularly pronounced in non-scientific university programs, where the failure rate in mathematics can easily exceed 30 percent [23]. Since poor performance in mathematics indirectly affects the overall academic performance of students, there is an urgent need to investigate the factors that have contributed to poor performance in mathematics in higher education.

This study aims to develop a conceptual model to analyse the factors that influence the mathematical performance of university students of social sciences. The background knowledge of secondary mathematics, the attitude towards learning mathematics with technology, the perceived level of math anxiety, and the self-engagement and motivation during the mathematics course were taken into account. In our effort to investigate the relationships between the model components, we applied the Structural Equation Modelling (SEM). The results were then presented and discussed.

The rest of the paper is structured as follows. First, the results of a relevant literature review are outlined. The research model and the proposed hypotheses are developed. Furthermore, the methodology of our empirical study is explained. The results are presented and discussed. Finally, the conclusions are outlined based on research implications, the limitations of the study, and future research recommendations.

2. Review of Related Literature

2.1. Factors that Influence Mathematics Performance

To determine the predictors of mathematics achievement among various groups of individuals, a large body of studies have been conducted over the past several decades. Since education is a complex process with many variables interacting in a way that affects how much learning takes place [24], the authors express the diverse and complex nature of factors associated with mathematics performance. To provide a comprehensive and consistent insight, some authors try to classify the factors into various categories with related properties.

Papanastasiou [24] distinguishes between internal and external factors influencing mathematics performance. Internal factors are those related to the test (exam) material, while external factors refer to the environment which surrounds the individual as well as to his unique persona (e.g., socio-economic level and educational background of the family, the school climate, the language background, and students' attitudes toward mathematics).

Patterson et al. [4] express that factors associated with mathematics achievement range from the dynamics of individual cognitive processes to the social and environmental factors that affect a particular student.

Furthermore, Enu, Agyman, and Nkum [25] ascertained that the successfulness of learning mathematics is contingent on a myriad of factors: students' factors (entry behaviour, motivation, and attitude), socio-economic factors (education of parents and their economic status), and school-based factors (availability and usage of learning materials, school type, and teacher characteristics).

A comprehensive and systematic literature review on influential factors found to be responsible for success or failure in mathematics is provided by Kushwaha [26]. The author divided the factors under three general heads as follows:

- Psychological variables: attitude towards mathematics, intelligence, math anxiety, self-concept, study habits, mathematical aptitude, numerical ability, achievement motivation, cognitive style, self-esteem, interest in mathematics, test anxiety, reading ability, problem-solving ability, mathematical creativity, educational and occupational aspiration, personal adjustment, locus of control, emotional stability, and confidence in math.

- Social variables: socio-economic status, school environment, home environment, parents' education, parental involvement, parents' occupation, parents' income, social status, social relations, type of school, teacher's expectation, and social maturity.
- Biographical and instructional variables: gender, locality, methods of instructions, caste, birth order, teacher effectiveness, and home tutoring.

In many studies on mathematics achievement, the psychological, social, biographical, and instructional variables were studied simultaneously, where the authors have focused on a limited number of factors or themes with the aim of demonstrating their role in the complex process of mathematics education. Kushwaha [26] found out that the most preferential factors of the investigation in the category of psychological variables are intelligence, attitude towards mathematics, self-concept, numerical ability, and math anxiety. Among social variables, the factors which were considered very widely are socio-economic status, parental involvement, and parents' education, while among biographical variables, the most frequently considered factor is gender.

2.2. Investigation of Mathematics Performance at the Tertiary Level of Education

Due to specific characteristics of the target population, we believe that research results relating to different levels of education (primary, secondary, and tertiary) cannot always be directly compared. Therefore, we have limited our attention to studies related to the tertiary level of education, as this is the subject of our research.

Concerns regarding the problem of unsatisfactory mathematics performance have been reported internationally. Most of these studies are related to developing countries, such as Malaysia [2,3,23,27–30], Iran [31], Nigeria [32], Ghana [25], and the Philippines [33,34]. Studies relating to the field of Australia are also quite common [20,21,35–38]. The territory of the USA and Canada seems to be somewhat less represented [39,40], while studies dealing with mathematics achievement in European higher education are very rare. In this regard, we found a few papers from the following countries: the UK [41], Finland [42], Spain [43], Ireland [44], Germany [22], and Sweden [45]. In our opinion, the origin of the study is an important factor that should be considered when comparing the results. Namely, specificities of a particular national education system, as well as cultural differences between different parts of the world, can lead to significant differences in conclusions.

The majority of the studies addressed mathematics performance in relation to certain selected factors. As one of the most influential sources and predictors of underachievement in mathematics at the tertiary level of education, the authors consider the insufficient level of mathematical background from secondary education (see, e.g., [2,20,21,36–40,42,44]).

Furthermore, many authors note that math anxiety also plays an important role in mathematics achievement (see, e.g., [3,23,27,32–34,43,45]).

Among other influencing factors, the following are also exposed: attitudes toward mathematics and/or self-confidence with regard to mathematics (see, e.g., [25,27,40,43]), mathematical self-efficacy and student engagement [41], academic self-beliefs [42], learning motivation (see, e.g., [22,25]), learning strategies and/or availability of teaching resources (see, e.g., [22,25]), importance of mathematics [23], teaching style ([31,33]), parent's profile [33], mathematics class size (see, e.g., [2]), gender (see, e.g., [2,28,32]), and age [32].

In our opinion, the study discipline is also a parameter that should not be neglected when selecting potential influencing factors of mathematical performance. Namely, factors that are relevant for technical, engineering, and other science-oriented studies are not necessarily relevant for students of social sciences and humanities or similar courses of study. In our experience, the first group of students expresses a much higher positive attitude towards mathematics than the second. This is also consistent with the results of Núñez Peña, Suárez-Pellicioni, and Cabré [43], which showed that the students who received good/excellent grades in mathematics were mainly from scientific and technological itineraries, while those who failed had mainly studied the humanistic and social syllabuses.

2.3. Methodology Adopted for Studying the Phenomenon of Mathematics Achievement

A detailed review of the related literature revealed that many researchers used descriptive research methodology. To collect the required data, they used suitable tools (standardised, well-known from the literature, or self-developed). Beside descriptive analyses, collected data have been mostly subjected to independent samples *t*-test (see, e.g., [25]), analysis of variance (see, e.g., [21,31,37,43]), correlation techniques (see, e.g., [22,27,28,33,45]), or to regression (linear, multiple) analyses (see e.g., [2,3,32,34]). Studies which applied factor analysis (see, e.g., [23,30]), principal component analysis (see, e.g., [36]), discriminant analysis (see, e.g., [2,44]), or mixed-effects models [40] are relatively rare.

Undoubtedly, the results of descriptive research would provide a solid basis for selecting the most effective variables and formulating a hypothesis accordingly [26]. However, the influencing factors are often interdependent variables. Therefore, more sophisticated techniques are needed to study the relationships between them. Definitively, one of them is Structural Equation Modelling (SEM), which enables analysis of relationships between latent and observed variables simultaneously [46]. In an in-depth review of the related literature on mathematics performance, we found only three applications of SEM. Two of them, [8,12], refer to secondary education level in a specific geographical area (Turkey, the city of Konya and its surroundings). The only application of SEM for the analysis of mathematical achievement in higher education was found in [42], where SEM was used to examine the relationships between prior knowledge, academic self-beliefs, and previous study success in predicting the achievement of university students participating in an obligatory mathematics course within a mathematics program.

The literature review allows us to conclude the following:

- Studies investigating the factors influencing the mathematical performance of social science students are very rare. With regard to the European education environment, only two studies were found, where both were applied to psychology students [43,45].
- Advanced statistical methods are not used very often. Among all studies that refer to mathematical achievement at the tertiary level of education, we found the only application of SEM in [42]. The results of this study are worthwhile but cannot be directly applied to our case because of the incomparable study discipline (mathematics study program).

These statements provided the fundamental starting points for our research.

3. Research Model Development

It is well established in the literature that mathematical performance is influenced by numerous factors, including psychological, social, biographical, educational, and other factors, which are often not independent of each other and can influence each other. Furthermore, some of the influencing factors are very complex, so it is necessary to divide them into sub-variables and find out how each sub-variable is related to mathematical success [26].

Based on the findings in the literature and the results of our preliminary studies [47], we assume that students' performances in mathematics are influenced by at least the following dimensions: their attitude towards mathematics (including mathematics anxiety), their engagement in learning activities (including background knowledge), and their attitude towards integrating technology into mathematics education.

3.1. Attitude towards Mathematics and Math Anxiety

Much research has been conducted to examine students' attitudes towards mathematics, and most authors agree that it plays a vital role in the process of teaching and learning mathematics [30]. Results show that a positive attitude towards mathematics has a significant impact on effective student engagement and participation and will increase students' success in mathematics (Khoo and Ainley (2005), as cited in [30]). Furthermore, valuing the importance of mathematics was also claimed to have a positive effect on students' mathematics performances [23]. A positive attitude is also related to students' self-confidence, which refers to their belief in their cognitive capacity to learn or perform actions to achieve intended results [42]. We believe that those who have confidence in their ability to perform well also expect success in a particular task.

On the contrary, it has been identified that fear of mathematics or mathematics anxiety, educational issues, and values and expectations towards mathematics can be treated as causes of low mathematics achievements among students [23]. Mathematics anxiety (also math anxiety) can be defined as a person's negative affective reaction to situations involving numbers and mathematical calculations in both academic and daily-life situations [48]. Math anxiety, being considered to have an attitudinal component, is also considered to be one dimension of attitude to mathematics and is considered as one of the severe problems that affects mathematics education (see, e.g., [48–54]).

The majority of the studies that examine the influence of math anxiety on mathematics performance in higher education report a significant relationship between math anxiety, mathematical thinking, and attitudes towards mathematics. Students with a higher level of math anxiety tended to score lower in their mathematical thinking, their attitudes to mathematics, and, consequently, their performance, and vice versa (see, e.g., [3,27,33,34,43]).

Similar to other types of anxiety, math anxiety is a complex set of multidimensional aspects in the form of cognition, affective, somatic, and behavioural reactions [55]. Due to its complexity, there is no unique and transparent measure of math anxiety. Several researchers have argued that a mathematics anxiety instrument should be bi-dimensional and concise, contrary to the unidimensional multiple-item instruments used in the past (Mahmood and Khatoon (2011), as cited in [56]). A systematic and chronological literature review of available instruments is provided by Zakariya [56]. One of the most extensively used mathematics anxiety instruments is the Mathematics Anxiety Rating Scale—MARS [57]—and its revised version, the Revised Mathematics Anxiety Rating Scale—RMARS [58].

3.2. Engagement in Learning Activities

Linnenbrink and Pintrich [59] divide student engagement in the classroom into three distinct components: behavioural engagement, cognitive engagement, and motivational engagement. Behavioural engagement is observable behaviour seen in the classroom that relates to the efforts students are putting into mathematical tasks and students' relations to each other and to the teacher in terms of their willingness to seek help, attendance at the classes, etc. Cognitive engagement recognises that a student appearing to work on a mathematics problem is not necessarily indicative of the student fully engaging mental faculties in trying to complete it. Motivational engagement is the personal interest that the student has in the subject, the utility that the student feels the subject brings, and, finally, the general importance of the subject to longer-term goals or desires. All three components of engagement are likely correlated. That is, if students are cognitively and motivationally engaged, they are likely to be behaviourally engaged. The literature suggests (and is supported by empirical evidence) that all three components of students' engagement are related to outcome measures of learning and achievement (Pajares and Miller (1994), as cited in [41,59]).

However, many authors emphasised the importance of an appropriate mathematical background from secondary school and its influence on success in mathematics at the tertiary level. Studies conducted in various parts of the world documented prior mathematical attainment to be a significant predictor of performance and progress in higher mathematics education [44]. Similarly, a weak mathematical background on entering higher education is reported as one of the fundamental reasons for and predictors of poor student performance [44].

The background knowledge of secondary mathematics is usually measured by achievement in the secondary school leaving qualification. A significant positive correlation has been revealed between students' grades of the secondary school leaving qualification and their performance in mathematics at the university level (see, e.g., [2,37,40]).

Furthermore, some studies report positive and facilitative effects of prior knowledge on learning (Dochy, Segers, and Buehl (1999), as cited in [42]). The authors revealed that students who were able to operate at a higher cognitive level at the beginning of the course, by applying their knowledge and by solving problems, were more likely to perform better than the students whose prior knowledge consisted mainly of facts and a surface-level understanding of the issue. Moreover, inaccurate prior knowledge and misconceptions within a specific domain can make it difficult for students to understand or learn new information.

3.3. Attitudes towards Involving Technology in Learning Mathematics

Computer-based technologies are now commonplace in the classroom, and the integration of these media into mathematics teaching and learning is supported by government policies in most developed countries [60]. The use of technology for learning mathematics is one of the main issues for leading professionals involved in mathematics education at different levels of education (e.g., ERME—European Society for Research in Mathematics Education; NCTM—National Council of Teachers of Mathematics). A review of recent CERME (Congress of the European Society for Research in Mathematics Education) research is presented in [61]. At the same time, Li and Ma [62] provide a systematic literature review and a comprehensive meta-analysis of the existing empirical evidence on the impact of computer technology on mathematics education.

The literature reports many positive effects of integrating technology into mathematics education. It enables educators to create powerful collaborative learning experiences that support problem solving and flexible thinking. Therefore, the use of technology is seen as a useful tool for promoting mathematics learning [62]. Furthermore, Attard and Holmes [63] recently noted that teachers use technological tools to enhance their awareness of students' individual learning needs and to promote student-centred pedagogies, leading to greater student engagement with mathematics. On the other hand, the results of [64] suggest that the use of educational technologies generally has a positive effect on mathematics achievement in comparison to traditional methods, where the most remarkable effect has been experienced with the application of computer-assisted instructions. Barkatsas, Kasimatis, and Gialamas [65] reported positive attitudes among students towards learning mathematics with computers, even if they are not confident in using computers or express negative attitudes towards mathematics. They experienced the benefits of technology in learning mathematics, and they aim to improve mathematics performance via the use of technology.

Additionally, Al-Qahtani and Higgins [66] examined the impact of e-learning, blended learning, and classroom learning on students' achievement to determine the optimal use of technology in teaching. They confirmed a statistically significant difference between the three methods in terms of students' performance, favouring the blended learning method. The use of new and different technologies in studying a subject can increase students' enthusiasm and provide them with additional skills. Similarly, the analyses of Lin, Tseng, and Chiang [67] showed that the blended learning experience benefited the students in the experimental group, as it had a positive effect not only on learning outcomes but also on their attitude towards studying mathematics in a blended environment. Moreover, web-based learning systems and electronic materials allow users to repeat exercises and to learn simultaneously. This learning model helps to overcome time and space constraints in the classroom [60]. However, today's use of ICT coupled with the global crisis being experienced of COVID-19 makes e-learning not only a possible but also a necessary teaching method [68].

3.4. Conceptual Model and Hypotheses

Arising from the discussion in the previous subsections, we present our conceptual model in Figure 1, and summarise the propose d hypotheses as follows:

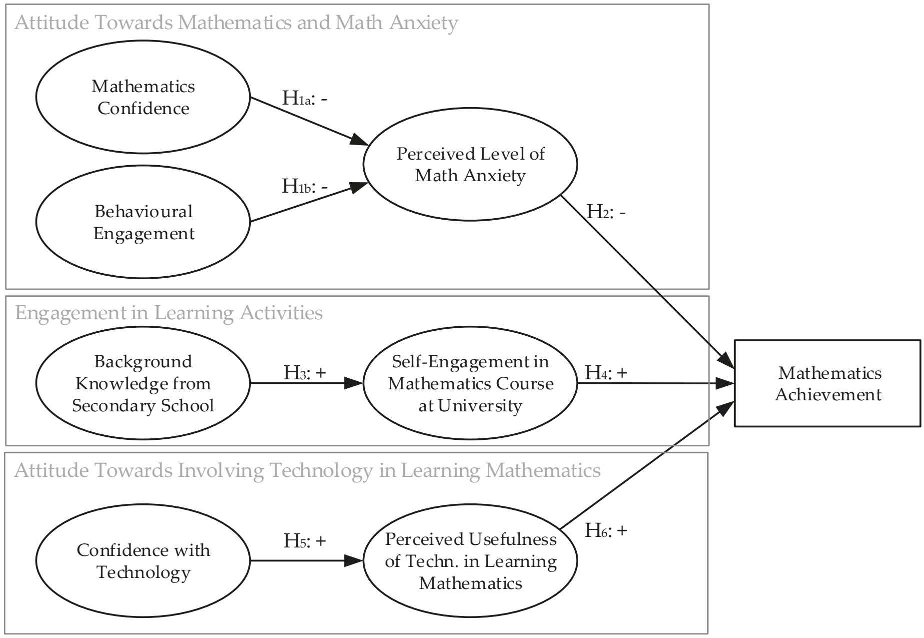

Figure 1. Conceptual model of relationships among the factors influencing social sciences students' mathematics achievement.

H_{1a}: *Mathematics confidence negatively affects perceived level of math anxiety.*

H_{1b}: *Behavioural engagement negatively affects perceived level of math anxiety.*

H_2: *Perceived level of math anxiety negatively affects achievement in mathematics.*

H_3: *Background knowledge from secondary school positively affects self-engagement and motivation to fulfil obligations during a mathematics course at university.*

H_4: *Self-engagement in a mathematics course at university positively affects achievement in mathematics.*

H_5: *Confidence with technology positively affects perceived usefulness of technology in learning mathematics.*

H_6: *Perceived usefulness of technology in learning mathematics positively affects achievement in mathematics.*

4. Materials and Methods

4.1. Measurement Instruments and Data Collection

For our research, a three-part questionnaire was prepared. The first part served to collect students' socio-demographic data (gender, age, year of study, and study course) and data on students' background knowledge in mathematics from secondary school. The second part was designed to measure the level of math anxiety as perceived by students. In the last part, a scale for monitoring students' attitudes towards mathematics, technology, and towards involving technology in learning mathematics was used. In addition, we provide data on students' engagement in the university mathematics course and mathematics achievements, which were added to the database.

4.1.1. Measuring Students' Background Knowledge from Secondary School

Three variables were used to measure students' Background Knowledge from Secondary School (BKSS):

- Grade in mathematics in the final year of secondary school—Grades in Slovenia range between 1 (insufficient) and 5 (excellent), and 2 (sufficient) is the lowest passing grade.
- Grade in mathematics at Matura (i.e., final national school-leaving exam)—There are two types of school-leaving exam in Slovenia: the Matura and the Vocational Matura. The Matura is the Slovene equivalent of the SAT (Scholastic Assessment Test) in the US, enabling candidates with completed general upper secondary education to enrol in all tertiary education programs, i.e., vocational colleges, colleges, and university courses. The Vocational Matura is a national examination for candidates with technical education, enabling them to enrol only in a vocational college. Vocational Matura candidates (among other specified subjects) choose between mathematics and a foreign language, while the Matura requires both. Students' grades can range from 1 to 8, where a grade above 5 can only be achieved by those who choose to take the Matura at a higher level of difficulty.
- Final grade in high school—Overall grade for the last year of secondary school ranging from 1 to 5.

4.1.2. Measuring Students' Level of Math Anxiety

To explore the estimate of the Perceived Level of Math Anxiety (PLMA) among students, we used the reduced version of the Math Anxiety Rating Scale—RMARS [58]—which has been demonstrated to be highly reliable [54]. Baloğlu and Zelhart [69] employed an exploratory factor analysis on a proposed 25-item questionnaire to explore the relationships between items and to identify the underlying factors. Three factors were identified, and five items were omitted from the scale. Therefore, a simplified version of the scale of 20 items was used in our research. This scale evaluates 20 situations which may cause math anxiety, divided into three dimensions:

- Mathematics Test Anxiety (MTA), which includes 10 items reflecting apprehension about taking a test in mathematics or about receiving the results of mathematics tests;
- Numerical Task Anxiety (NTA), which includes 5 items reflecting anxiety about executing numerical operations;
- Mathematics Course Anxiety (MCA), which includes 5 items reflecting anxiety about taking a mathematics course [43,69].

Students were asked to indicate their level of anxiety associated with each item on a 5-point Likert-type scale from 1 ("no anxiety") to 5 ("high anxiety").

4.1.3. Measuring Students' Attitude towards Mathematics, Technology, and Involving Technology in Learning Mathematics

A scale for assessing students' attitudes towards mathematics, technology, and the learning of mathematics with technology was adjusted from [70]. We used four out of the five constructs identified by the factor analysis for the Mathematics and Technology Attitudes Scale—MTAS—in [70]:

- Mathematics Confidence (MC), which includes 4 items referring to students' perception of their ability to attain good results and their assurance that they can handle difficulties in mathematics;
- Confidence with Technology (CT), which includes 4 items reflecting students' extent of confidence when working with computers and other commonly available technology;
- Perceived Usefulness of Technology in Learning Mathematics (PUTLM), which includes 4 items reflecting on the extent to which students consider computers to be relevant in learning mathematics and whether they can contribute to achievements in mathematics;
- Behaviour Engagement (BE), which includes 4 items reflecting students' behaviour during mathematics lectures and their involvement in learning assignments.

Students were asked to indicate their level of agreement with each of 16 statements on a 5-point Likert-type scale from 1 ("I do not agree at all") to 5 ("I agree completely").

4.1.4. Measuring Students' Self-Engagement and Achievement in Mathematics Course at University

The Self-Engagement of students in a Mathematics Course at University (SEMCU) was measured by two variables:

- Additional points for solving mathematical problems—During the course, students were able to voluntarily select problems that were solved individually at home and later presented to the class during tutorials. The texts of mathematical problems were published in advance. At each tutorial, each student could present the solution to one problem. For the correct solution, the student received one point. Each student was able to collect up to 13 additional points in this way;
- eActivities, which include points earned by activities (e-lessons and quizzes) in Moodle—A quarter of the course and both the lectures and tutorials, were organised as e-lessons in the virtual environment Moodle. Moreover, additional quizzes were prepared to test students' progress after each completed chapter. Students were required to solve eActivities in order to take the mid-term exams, but there was no minimum requirement. In our data, we used the average percentage of points (variable labelled eActivities) on a scale from 0 to 100%, obtained from e-lessons as a marker of the degree of self-engagement in learning activities in the mathematics course.

Mathematics Achievement (MA) was measured with the percentage of points achieved in the final exam. There are two ways to obtain a final grade in the university mathematics course. Either the student collects at least 50% of the points in three written mid-term exams or the student passes a final written exam (by achieving at least 50% of the points). Additional points, as described in the previous paragraph, are added to the student's percentage of points obtained in the exam or mid-term exams if they have achieved more than half of the points. In this way, their final grade in the mathematics course can be increased by one grade or, in rare cases, by two grades. The dataset, therefore, contains the value of the variable MA as the percentage of achieved points, regardless of the form of the exam (final or mid-term).

4.2. Data Collection

Data were collected for seven consecutive academic years (from 2013–2014 to 2019–2020) at the beginning of the mathematics course at the Faculty of Organizational Sciences, University of Maribor, Slovenia. All 1st level students were invited to participate in the research. Participation in this research was voluntary. The online questionnaire was distributed to the students via the e-learning environment Moodle. At the end of each academic year, we supplemented the data with the assessment of students' self-engagement and final achievement at the mathematics course. After that, the data were anonymised.

All subjects gave their informed consent for inclusion before they participated in the study. The study was conducted in accordance with the Declaration of Helsinki, and the protocol was approved by the Ethics Committee for Research in Organizational Sciences (514-3/2020/3/902-DJ).

4.3. Statistical Methods

Data were analysed using the two-stage approach to the structural equation modelling (SEM) approach (see, e.g., [71–73]). Analyses were performed using R-package lavaan [74–76] and e1071 [77] for assessing univariate normality.

The standard estimation method in SEM, maximum likelihood, assumes multivariate normality. Tests designed to detect violations of multivariate normality, including Mardia's test, have limited usefulness, since small deviations from normality in large samples could be denoted as significant [73]. Therefore, multivariate normality was assessed by examining univariate frequency distributions, including histograms, skewness, and kurtosis, and values for skewness and kurtosis between −2 and +2 are considered acceptable in order to prove normal univariate distribution [78].

The first step of SEM involves validation of the measurement model. A confirmatory factor analysis (CFA) was used to validate the measurement instrument in order to determine how well the measured items reflect the theoretical latent variables. A construct validity was investigated in order to determine how well a set of measured items actually reflects the corresponding theoretical latent variable. To assess construct validity, convergent validity and discriminant validity were examined. As suggested in [79,80], convergent validity was examined by:

- Estimates of standardised factor loadings, which should exceed 0.5 (or even 0.7).
- Composite reliability (CR) for each latent variable, which should exceed 0.7.
- Average variance extracted (AVE) for each latent variable, which should exceed 0.5.

In the second step of the data analysis, SEM was used to test the structural relationships among the latent variables. The results of SEM are presented with the values of the standardised path coefficient β together with its z-values and denoted the significance level. For each of the endogenous latent variables, a coefficient of determination (R^2) was also calculated, which shows the percentage of the explained variance by the set of variable predictors.

5. Results

5.1. Sample Characteristics

In total, 347 students collaborated in the study. Among them, 45.8% were men, while 54.2% were women. The average age of participants was 21.2 years (with a standard deviation of 1.74 years), ranging from 18 to 32 years.

5.2. Descriptive Statistics

Background Knowledge from Secondary School.

We analysed the respondents' graduation grades from secondary school. The results showed that 9.5% of the respondents completed secondary school with a grade 2 (sufficient), more than half (50.7%) of them achieved a grade 3 (good), 32.9% a grade 4 (very good), and 6.9% a grade 5 (excellent).

In addition, we checked their grades in a mathematics course in the last (fourth) year of secondary school. It turned out that 26.2% of the respondents achieved a grade 2 (sufficient), 41.2% a grade 3 (good), 25.6% a grade 4 (very good), and 6.9% a grade 5 (excellent). The average grade in mathematics in the last year of secondary school was 3.1, with a standard deviation of 0.88.

Finally, we examined the mathematics achievement at the secondary school leaving exam Matura. Only 83.6% of the respondents took mathematics at the Matura or Vocational Matura. Among them, 26.2% received a grade 2 (sufficient), 35.5% a grade 3 (good), 30.7% a grade 4 (very good), and 7.5% a grade 5 or higher (excellent). The average grade in mathematics at the Matura examinations was 3.2, with a standard deviation of 0.92.

The highest absolute values of skewness for three variables describing background knowledge were 0.35 and 0.72 for skewness and kurtosis, respectively, indicating fairly normally distributed variables [78].

5.2.1. Attitude towards Mathematics and Math Anxiety

Descriptive statistics for items related to students' attitude towards mathematics and math anxiety are presented in Table 1. First, eight statements related to students' attitude towards mathematics (represented by constructs MC and BE, which were adjusted from MTAS) are listed. On average, the students agreed most with the statement "I am confident that I can overcome difficulties in math problems" (M = 3.95) and least with the statement "I am confident in my skills at mathematics" (M = 3.19). The lowest anxiety was assessed for RMARS items from the NTA construct, where the lowest average value belongs to the statement "Being given a set of subtraction problems to solve" (M = 1.54). Respondents perceived the highest level of anxiety when "Being given a 'pop' quiz in a math class" (M = 3.80). Values of skewness were in the range from −0.81 to 1.55 and values of kurtosis ranged from −0.91 to 1.70, indicating a normal univariate distributions [78].

Table 1. Descriptive statistics for items related to students' attitude towards mathematics and math anxiety.

	Questionnaire Item	M	SD	Skewness	Kurtosis
Mathematics Confidence (MC)	I have a mathematical mind. (MC1)	3.91	0.817	−0.564	0.477
	I can get good results in mathematics. (MC2)	3.72	0.899	−0.484	0.301
	I know I can handle difficulties in mathematics. (MC3)	3.95	0.812	−0.366	−0.453
	I am confident with mathematics. (MC4)	3.19	1.064	−0.197	−0.304
Behavioural Engagement (BE)	I concentrate hard in mathematics. (BE1)	3.48	0.844	−0.231	0.224
	I try to answer questions the teacher asks. (BE2)	3.48	0.938	−0.341	−0.004
	If I make mistakes, I work until I have corrected them. (BE3)	3.56	0.930	−0.119	−0.553
	If I cannot do a problem, I keep trying different ideas. (BE4)	3.52	0.944	−0.272	−0.254
Mathematics Test Anxiety (MTA)	Studying for a math test. (MTA1)	3.13	1.244	−0.030	−0.910
	Taking the math section of the college entrance exam. (MTA2)	2.70	1.119	0.195	−0.682
	Taking an exam (quiz) in a math course. (MTA3)	2.88	1.130	0.174	−0.637
	Taking an exam (final) in a math course. (MTA4)	3.37	1.144	−0.320	−0.636
	Thinking about an upcoming math test one week before. (MTA5)	2.82	1.237	0.137	−0.906
	Thinking about an upcoming math test one day before. (MTA6)	3.36	1.229	−0.288	−0.848
	Thinking about an upcoming math test one hour before. (MAT7)	3.62	1.270	−0.483	−0.901
	Realising you have to take a certain number of math classes to fulfil requirements. (MTA8)	2.18	1.226	0.770	−0.396
	Receiving your final math grade in the mail. (MTA9)	2.91	1.206	0.059	−0.804
	Being given a "pop" quiz in a math class. (MAT10)	3.80	1.283	−0.805	−0.451
Numerical Task Anxiety (NTA)	Reading a cash register receipt after your purchase. (NTA1)	2.04	1.122	0.834	−0.162
	Being given a set of numerical problems involving addition to solve on paper. (NTA2)	1.56	0.896	1.553	1.696
	Being given a set of subtraction problems to solve. (NTA3)	1.54	0.864	1.521	1.584
	Being given a set of multiplication problems to solve. (NTA4)	1.59	0.874	1.358	1.059
	Being given a set of division problems to solve. (NTA5)	1.71	0.953	1.263	0.988
Mathematics Course Anxiety (MCA)	Buying a math textbook. (MCA1)	1.95	1.206	1.055	0.051
	Watching a teacher work on an algebraic equation on the blackboard. (MCA2)	1.85	1.049	1.079	0.433
	Signing up for a math course. (MCA3)	2.53	1.200	0.413	−0.596
	Listening to another student explain a mathematical formula. (MCA4)	2.07	1.106	0.654	−0.606
	Walking into a math class. (MCA5)	1.82	1.097	1.310	0.976

5.2.2. Attitude towards Involving Technology in Learning Mathematics

Table 2 presents descriptive statistics for eight statements related to students' attitude towards involving technology in learning mathematics (represented by the constructs CT and PUTLM from MTAS). On average, the lowest agreement was expressed with the statement "It is more fun to learn mathematics if we are using a computer" (M = 3.20), while the highest average value belongs to the statement "I can use DVDs, MP3s, and mobile phones well" (M = 4.27). The highest absolute values of skewness and kurtosis were 0.93 and 0.91, respectively, and were considered acceptable in order to demonstrate a normal univariate distributions [78].

Table 2. Descriptive statistics for the items related to students' attitude towards involving technology in learning mathematics.

	Questionnaire Item	M	SD	Skewness	Kurtosis
Confidence with Technology (CT)	I am good at using computers. (CT1)	3.92	0.952	−0.606	−0.173
	I am good at using things like VCRs, DVDs, MP3s, and mobile phones. (CT2)	4.27	0.798	−0.932	0.541
	I can fix a lot of computer problems. (CT3)	3.50	1.126	−0.290	−0.720
	I can master any computer program needed for school. (CT4)	3.60	0.993	−0.207	−0.483
Perceived Usefulness of Technology in Learning Mathematics (PUTLM)	I like using computers for mathematics. (PUTLM1)	3.50	1.141	−0.426	−0.453
	Using computers in mathematics is worth the extra effort. (PUTLM2)	3.29	1.117	−0.212	−0.568
	Mathematics is more interesting when using computers. (PUTLM3)	3.20	1.226	−0.059	−0.910
	Computers help me learn mathematics better. (PUTLM4)	3.25	1.218	−0.187	−0.819

5.2.3. Self-Engagement and Achievement in Mathematics Course at University

More than a quarter of the respondents (28.5%) did not take the opportunity of additional points, while a quarter of the respondents received eight additional points or more. The overall average number of additional points earned was 4.6, with a standard deviation of 4.25. If we consider only those who solved at least one problem given additionally, the average value of additional points is 6.4 (SD = 3.70). The values of skewness and kurtosis were 0.55 and −0.95, respectively. In the left panel of Figure 2, the histogram and boxplot of additional points earned (without zeroes) are presented.

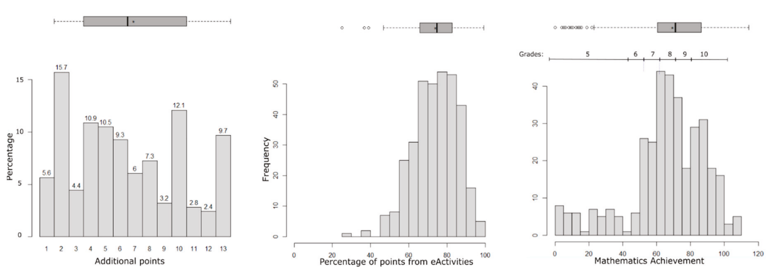

Figure 2. Boxplots (with an average denoted by an asterisk) and histograms for additional points (**left**), points from eActivities (**middle**), and mathematics achievement (**right**).

The respondents collected between 25 and 99.2 points from eActivities (e-lessons and quizzes in Moodle), with an average of 73.9 and a standard deviation of 11.83 points. Half of the respondents earned, on average, between 65.7 and 82.6 points (middle panel in Figure 2). The values of skewness and kurtosis were −0.94 and 0.92, respectively.

Mathematics achievement is measured as the percentage of points in the final examination and can range from 0 to 113 points (with additional points). Five respondents received zero points, while the highest score was 109.7 points. The average score was 66.4, with a standard deviation of 22.86 points. A quarter of the respondents received 58.0 points or less, while the quarter of the most successful respondents received at least 82.7 points (right panel in Figure 2). The values of skewness and kurtosis were 0.55 and −0.95, respectively.

5.3. Construct Validity of the Measurement Model

CFA was used to evaluate the measurement model. Construct validity was examined through evaluation of convergent validity and discriminant validity. First, the standardised factor loadings were examined. Five measured items were sequentially omitted from the model due to factor loadings bellow 0.5: MTA8: λ = 0.287; MCA1: λ = 0.428; BE1: λ = 0.469; BE2: λ = 0.432; and NTA1: λ = 0.494.

The unstandardised and standardised factor loadings of the final measurement model, together with corresponding z-values for each measured item, are presented in Table 3. It can be seen that all standardised factor loadings exceed a threshold of 0.5 for convergent validity, while 78% of values exceed even the threshold of 0.7.

Table 3. Parameter estimates, error terms, and z-values for the measurement model.

Latent Variable	Item	Unst. Factor Loading	Error Term	z-Value	Stand. Factor Loading
Mathematics Confidence (MC)	MC1	1.000	-[a]	-[a]	0.572
	MC2	1.531	0.145	10.589	0.796
	MC3	1.286	0.127	10.129	0.740
	MC4	1.995	0.179	11.158	0.876
Behavioural Engagement (BE)	BE3	1.000	-[a]	-[a]	0.718
	BE4	1.218	0.129	9.412	0.862
Mathematics Test Anxiety (MTA)	MTA1	1.000	-[a]	-[a]	0.735
	MTA2	0.971	0.065	14.830	0.793
	MTA3	0.998	0.066	15.228	0.807
	MTA4	1.026	0.066	15.515	0.820
	MTA5	0.959	0.075	12.848	0.708
	MTA6	1.121	0.073	15.308	0.834
	MTA7	1.060	0.076	14.039	0.763
	MTA9	0.808	0.072	11.217	0.612
	MTA10	0.817	0.077	10.575	0.582
Numerical Task Anxiety (NTA)	NTA2	1.000	-[a]	-[a]	0.926
	NTA3	0.980	0.030	33.070	0.941
	NTA4	0.995	0.030	32.918	0.944
	NTA5	0.986	0.040	24.665	0.858
Mathematics Course Anxiety (MCA)	MCA1	1.000	-[a]	-[a]	0.688
	MCA2	1.225	0.106	11.503	0.737
	MCA3	1.220	0.095	12.878	0.796
	MCA4	1.145	0.093	12.318	0.754
Perceived Level of Mathematics Anxiety (PLMA)	MTA	1.000	-[a]	-[a]	0.809
	NTA	0.464	0.080	5.791	0.414
	MCA	0.798	0.095	8.428	0.817
Background Knowledge from Secondary School (BKSS)	Grade in Mathematics in the Final Year	1.000	-[a]	-[a]	0.809
	Grade in Mathematics at Matura	0.464	0.080	5.791	0.414
	Final Grade in High School	0.798	0.095	8.428	0.817
Self-Engagement in Math. Course at Univ. (SEMCU)	eActivities	1.000	-[a]	-[a]	0.949
	Additional points	0.679	0.082	8.267	0.618
Confidence with Technology (CT)	CT1	1.000	-[a]	-[a]	0.882
	CT2	0.663	0.044	14.903	0.698
	CT3	1.167	0.061	19.179	0.870
	CT4	0.796	0.058	13.679	0.673
Perceived Usefulness of Technology in Learning Mathematics (PUTLM)	PUTLM1	1.000	-[a]	-[a]	0.854
	PUTLM2	0.921	0.050	18.374	0.803
	PUTLM3	1.130	0.051	21.967	0.898
	PUTLM4	1.129	0.050	22.466	0.903

-[a] Indicates a parameter fixed at 1 in the original solution. Fit indices: χ^2 = 1294.9, df = 570, χ^2/df = 2.27, comparative fit index (CFI) = 0.908, root mean square error of approximation (RMSEA) = 0.061, 90% confidence interval for RMSEA = (0.061, 0.065).

The values of CR and AVE for all latent variables of the final measurement model are presented in Table 4. The CR values of each latent variable easily fulfil the criterion CR > 0.7, except for SEMCU being equal to 0.601. AVE values for all nine latent variables are above the desired threshold of 0.5. According to the obtained results, the convergent validity for the set of latent variables and corresponding items in the measurement model can be confirmed. Therefore, all measured items included in the final model are significantly related to the corresponding latent variable.

Table 4. Composite reliability (CR), average variance extracted (AVE), square root of AVE (on the diagonal), and correlations among the latent variables.

			Correlations among Latent Variables								
Construct	**CR**	**AVE**	**MC**	**BE**	**MTA**	**NTA**	**MCA**	**BKSS**	**SEMCU**	**CT**	**PUTLM**
MC	0.850	0.600	0.775 [a]								
BE	0.772	0.632	0.564	0.795 [a]							
MTA	0.915	0.548	−0.589	−0.408	0.740 [a]						
NTA	0.954	0.838	−0.301	−0.209	0.335	0.915 [a]					
MCA	0.833	0.556	−0.594	−0.412	0.661	0.338	0.746 [a]				
BKSS	0.776	0.554	0.400	0.347	−0.299	−0.153	−0.302	0.744 [a]			
SEMCU	0.601	0.500	0.634	0.400	−0.500	−0.256	−0.505	0.321	0.707 [a]		
CT	0.871	0.638	0.236	0.167	−0.171	−0.087	−0.172	0.082	0.260	0.798 [a]	
PUTLM	0.924	0.754	0.253	0.269	−0.080	−0.041	−0.081	−0.004	0.241	0.452	0.868 [a]

[a]—the square root of AVE.

To assess the discriminant validity of the measurement model, the square root of the AVE of each latent variable is compared to the correlations between the latent variables. The correlations among the latent variables are given in the right panel of Table 4, while on the diagonal, the values of the square root of AVE are presented. The values of the square root of AVE for the corresponding latent variables are all greater than the inter-variable correlations. This indicates that the discriminant validity can be determined for all latent variables.

The overall fit of the measurement model was assessed based on a set of commonly used fit indices. Since χ^2 statistics itself is sensitive to the sample size, the ratio of χ^2 to the degrees of freedom (df) was used. An obtained ratio lower than 3 ($\chi^2/df = 2.27$, $\chi^2 = 1294.9$, $df = 570$) indicates an acceptable fit [81]. The value of the comparative fit index (CFI) is above 0.9 (CFI = 0.908) and, hence, according to [80], indicates an adequate model fit. The root mean square error of approximation (RMSEA) of our measurement model is equal to 0.06, and the upper bound of RMSEA 90% confidence interval (0.061, 0.065) is below 0.08, as suggested by [82].

5.4. Evaluation of the Structural Model and Hypotheses Testing

SEM was used to test the predicted relationships (as shown in Figure 1) among the constructs of our model.

First, the goodness of fit of the structural equation model was evaluated. The results show that the model has a good fit according to the following indices: $\chi^2/df = 2.37$ ($\chi^2 = 1458.0$, $df = 614$), CFI = 0.897, and RMSEA = 0.063 with its 90% confidence interval (0.059, 0.067).

Second, the structural paths were evaluated. The results are presented in Table 5 and Figure 3. The values of the standardised path coefficient β and corresponding z-values are listed. Each path coefficient β is interpreted in terms of magnitude and statistical significance. For each endogenous latent variable, the coefficient of determination (R^2) was calculated. The results are shown in Figure 3.

For the second-ordered factor PLMA, the loadings to the three first-ordered factors are written in grey in Figure 3.

Table 5. Summary of hypotheses testing for the structural model.

Hypothesis	Path	Expected Sign	Standardised Path Coefficient	z-Value	Hypothesis Supported?
H_{1a}	MC → PLMA	-	−0.669	−6.6673 **	Yes
H_{1b}	BE → PLMA	-	−0.131	−1.824	No
H_2	PLMA → MA	-	−0.243	−5.307 ***	Yes
H_3	BKSS → SEMCU	+	0.328	3.781 ***	Yes
H_4	SEMCU → MA	+	0.835	9.118 ***	Yes
H_5	CT → PUTLM	+	0.458	7.924 ***	Yes
H_6	PUTLM → MA	+	−0.047	−1.189	No

** $p < 0.01$; *** $p < 0.001$.

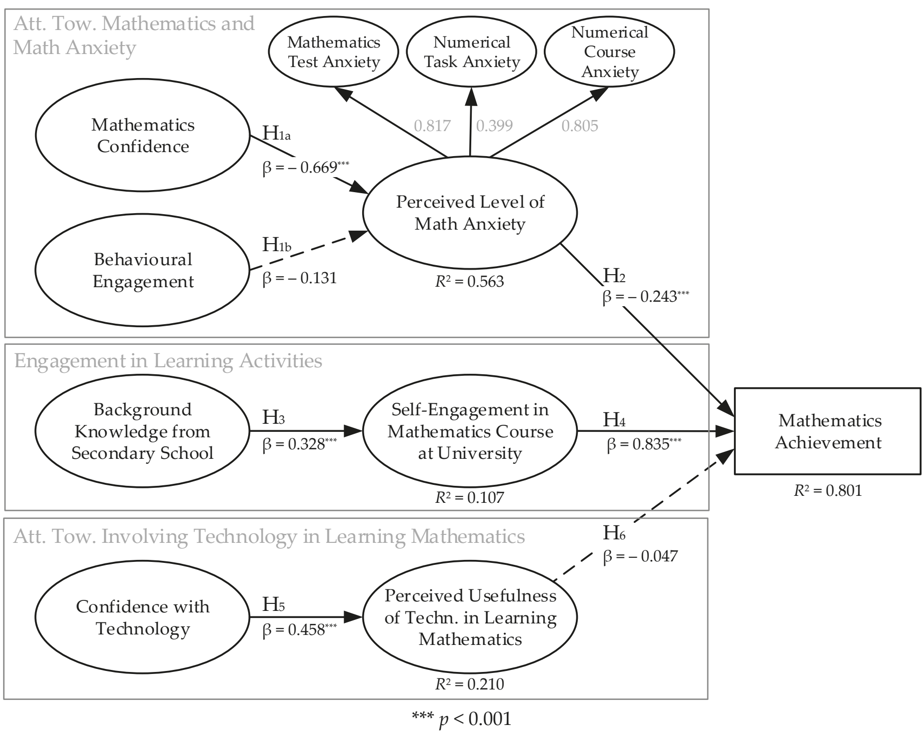

Figure 3. Structural Equation Modelling (SEM) model of relationships among the factors influencing social sciences students' mathematics achievement.

Based on the values of the standardised path coefficients and corresponding z-values, each of the proposed research hypotheses in Section 3.4 is either supported or rejected. A summary of the hypotheses testing is given in Table 5, which shows that 5 out of 7 hypotheses were supported. The predictive capability of the proposed model is satisfactory because all values of R^2 are higher than 0.1 (suggested by Falk and Miller (1992), as cited in [83]), while the coefficient of determination R^2 for Mathematics Achievement is extremely high since it equals 0.801.

The results confirmed that hypothesis H_{1a} could be supported at a 0.1% significance level (H_{1a}: $\beta = -0.669$, $z = -6.667$), while hypothesis H_{1b} could not be supported (H_{1b}: $\beta = -0.131$, $z = -1.824$) at a 5% significance level. The hypotheses from H_2 to H_5 were all supported at a significance level of 0.1% (H_2: $\beta = -0.243$, $z = -5.307$; H_3: $\beta = 0.328$, $z = 3.781$; H_4: $\beta = -0.131$, $z = -1.824$; H_5: $\beta = -0.131$, $z = -1.824$). Furthermore, the hypothesis H_6 could not be supported at a 5% significance level (H_6: $\beta = -0.047$, $z = -1.189$). Finally, we found out that PLMA, SEMCU, and PUTLM, together, explain 80.1% of the total variance in MA.

6. Discussion and Conclusions

In this study, relationships among factors influencing social sciences students' mathematics achievements were examined using Structural Equation Modelling (SEM). The factors considered in the study were divided into three categories:

- Attitude towards mathematics and math anxiety;
- Engagement in learning activities;
- Attitude towards involving technology in learning mathematics.

The first category included three main variables: mathematics confidence, behavioural engagement, and perceived level of math anxiety. Since negative attitudes toward mathematics and the negative influence of math anxiety are often identified in the literature as important predictors of underachievement in mathematics, we assumed that both mathematics confidence and behavioural engagement negatively affect the perceived level of math anxiety, which also negatively affects the mathematics achievement. Our assumptions have only been partially confirmed. The results showed a strong negative influence of mathematics confidence on the perceived level of math anxiety (H_{1a}), while the influence of behavioural engagement does not seem to be significant (H_{1b}). It was also confirmed that the perceived level of math anxiety has a negative effect on mathematics achievement (H_2), meaning that a higher level of math anxiety leads to poorer performance in the mathematics exam. This result is consistent with the findings of many authors (see, e.g., [3,27,34,43]). Of the three dimensions of math anxiety considered in our study, the highest factor loading was determined for mathematics test anxiety and numerical task anxiety.

We can presume that finding ways to enhance mathematics confidence and to reduce math anxiety can significantly improve the students' performance in mathematics, leading to better mathematics exam score. It would, therefore, make sense to focus our further research to this area. The literature suggests several methods and best practices (see, e.g., [54,84]). In our opinion, upgrading the traditional teacher-centred teaching methods with newer, advanced teaching methods (e.g., problem-solving and discovery learning) can strengthen students' self-confidence in mathematics (see, e.g., [85,86]). According to our experience, E-lessons and quizzes in the online classroom are also well received among the students. Such activities can be used as a trigger to achieve more intensive self-engagement of students during the mathematics course and, consequently, lead to their better achievement in mathematics (proved with H_4 in our study). We also think that it is necessary to provide creative learning environments (see, e.g., [87,88]) that will enable the students to experience success in mathematics, support their self-confidence, and develop positive attitudes towards mathematics. Moreover, [45] suggests taking more help from other students, group assignments, study groups, and buddy systems as very beneficial methods for students with high math anxiety. In our opinion, math anxiety can also be reduced by increasing the value of mathematics learning. Lecturers should try to introduce carefully designed activities into the learning process and prepare real-world problems. According to our experiences, such real-world problems are interesting for students and motivate them to understand better the results obtained. This approach is especially important when dealing with students from non-technical or science-oriented disciplines. Namely, some previous research confirmed that math anxiety is more pronounced for students of social sciences and humanities than for students of physical sciences, engineering, and math [49].

Regarding the second category, two variables were taken into account: background knowledge from secondary school and self-engagement in a mathematics course at university. It was confirmed that background knowledge from secondary school positively affects students' engagement in the university mathematics course (H_3). Furthermore, a positive and high-level relationship was found between self-engagement in learning activities at the university and the final achievement in mathematics (H_4). These results support previous studies which indicate that incoming skills measured by grades in high school mathematics are among the most significant predictors of students' success in mathematics and science courses [2,22,37,40,44]. Hence, in order to improve mathematics performance at a higher education level, more attention must be given to the students in secondary school, especially those with weak mathematics results [3].

The third category refers to the students' attitude toward involving technology in learning mathematics. Many authors claimed that educational technologies provide greater opportunities for creating new learning experiences that engage students and generally positively affect mathematics achievement [35,64]. For learning and doing mathematics, technology in the form of mathematics analysis tools can assist students' problem solving, support exploration of mathematical concepts, provide dynamically linked representations of ideas, and can encourage general metacognitive abilities, such as planning and checking [70]. However, a positive association between the perceived usefulness of technology in learning mathematics and mathematics achievement was not confirmed in this study (H_6), although this variable was confirmed to be positively influenced by the confidence with technology (H_5). In our case, the e-learning component accounted only for 25% of the subject's total scope. Hence, we estimate that this percentage was too low to influence the students' achievements in mathematics significantly. Since the value of the H_6 path coefficient is very low, we assume that a more intensive integration of technology into the pedagogical process could lead to different results.

Summary results showed a very high coefficient of determination for mathematics achievement (0.801), indicating that the variables "Perceived Level of Math Anxiety", "Self-Engagement in Mathematics Course at University", and "Perceived Usefulness of Technology in Learning Mathematics", together, explain 80.1% of the total variance of "Mathematics Achievement". These results prove that the variables considered in the model are relevant for our study.

If we summarise our findings, we can conclude that mathematics achievements of university students of social sciences depend on the following factors: math anxiety, mathematics confidence, students' engagement in a mathematics course, and background knowledge from secondary school. This finding, therefore, opens up guidelines for our further research. In our opinion, a great responsibility for improvements lies on university teachers, who must strive to enable students to progress in these segments. However, many studies emphasise the role of secondary school mathematics (see, e.g., [20,37]), which also proved to be important in our study. We agree with the author of [40], who found that teaching in secondary school is a key determinant for the success of university students in mathematics. We believe that secondary school teachers can play an important role in building students' mathematics self-confidence. In addition, their role in preventing and reducing the level of math anxiety among their students is also essential [43]. Therefore, it is very important to identify the best ways in which secondary school math teachers can help students to achieve better incoming skills and, consequently, higher performance at university. One suggestion given in the literature is to train high school teachers to advocate skilfully for the achievement of students by employing practical mathematics learning activities and by developing an appropriate curriculum and educational programs that are focused on how to engage students in solving mathematics problems [40]. Studies in the future may be emphasised in terms of reducing mathematics anxiety, especially emotional factors, from the early stage at primary or secondary school as a possible preventive measure to reduce the level of the severe mathematical anxiety level. Thus, the finding is hoped to provide some useful information to those involved in improving the mathematics performance in higher-level institutions [3].

However, we agree with Awaludin et al. [23], who believe that all mathematics educators, irrespective of the education level, should play a role in raising students' awareness about the importance of mathematics in everyday life, their majors, its applications for other courses in their field of expertise, and also future careers. Consequently, this can contribute to better mathematical literacy in the general population, which has received growing attention in the last few decades [89].

In conclusion, it is clear from both the literature review and the results of our study that the factors that contribute to students' mathematics performance are very complex. Therefore, mathematics performance continues to be an important area of research to support the planning of effective educational programs in mathematics that meet the needs of diverse students and a well-prepared workforce.

Finally, some limitations of our study have to be acknowledged. The first limitation is the population under consideration. The sample cohort was drawn from a single faculty of a single Slovenian university. Consequently, the findings may have limited generalisability to other contexts, nationally and internationally. Replication of the study with a different sample would enable examination of the generalisability of the findings.

The measurement instruments taken from the literature [58,70] were translated from English into Slovenian at the beginning of our research (see Appendix A). It is, therefore, possible that the meaning of a particular questionnaire item was somewhat "blurred" during translation. However, since all respondents answered the same questionnaire, we believe that this fact does not directly affect the results themselves. Nevertheless, a considerable amount of attention is required when we compare our results with the results of research conducted in English (or other) language areas.

Furthermore, the measurement instrument RMARS, which was used to examine math anxiety among the students, is mainly focused on mathematical activities based on numbers and calculations. Therefore, we have not included other areas of mathematics that are not directly related to numbers in our study. In future research, it would be worthwhile to investigate the extent to which such activities generate anxiety to the students.

We are also aware that we have excluded, from our model, some important variables (e.g., achievement motivation and teacher effectiveness) and potential relationships (e.g., background knowledge from secondary school to the perceived level of math anxiety) which may have influenced our results. Further research should address these issues.

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Appendix A

Table A1. Slovenian Translation of the Measurement Instruments.

		Original Statement	Slovenian Translation
Adopted from Mathematics and Technology Attitudes Scale—MTAS [70]		Indicate the extent of your agreement with each statement, on a five-point scale from "strongly disagree" (1) to "strongly agree" (5).	Na 5-stopenjski lestvici od "sploh se ne strinjam" (1) do "popolnoma se strinjam" (5), označite, v kolikšni meri se strinjate s posamezno trditvijo.
	Math. Conf.	I have a mathematical mind.	Znam logično razmišljati.
		I can get good results in mathematics.	Dosežem lahko dober rezultat pri matematiki.
		I know I can handle difficulties in mathematics.	Vem, da lahko premagam težave pri matematiki.
		I am confident with mathematics.	Samozavesten sem glede matematike.
	Behav. Engag.	I concentrate hard in mathematics.	Močno sem osredotočen na matematiko.
		I try to answer questions the teacher asks.	Poskušam odgovoriti na vprašanja, ki jih pri matematiki zastavi učitelj.
		If I make mistakes, I work until I have corrected them.	Če pri matematiki napravim napako, bom delal, dokler je ne odpravim.
		If I cannot do a problem, I keep trying different ideas.	Če pri matematiki ne znam rešiti problema, poskušam z novimi idejami.
	Conf. with Techn.	I am good at using computers.	Dober sem pri uporabi računalnikov.
		I am good at using things like VCRs, DVDs, MP3s and mobile phones.	Dober sem pri uporabi DVD-jev, MP3-jev in mobilnih telefonov.
		I can fix a lot of computer problems.	Odpraviti znam večino težav, povezanih z računalniki.
		I can master any computer program needed for school.	Obvladam vse programe, ki jih potrebujemo za študij.
	Perc. Usef. of Techn. in Learn. Math.	I like using computers for mathematics.	Pri učenju matematike rad uporabljam računalnik.
		Using computers in mathematics is worth the extra effort.	Uporaba računalnika pri učenju matematike je vredna dodatnega truda.
		Mathematics is more interesting when using computers.	Učenje matematike je bolj zanimivo, če uporabljam računalnik.
		Computers help me learn mathematics better.	Računalnik mi pomaga, da se matematiko bolje naučim

Table A1. *Cont.*

		Original Statement	Slovenian Translation
Adopted from Revised Mathematics Anxiety Rating Scale—RMARS [58]		Indicate your level of anxiety in the following situations. There are no right or wrong answers. Do not spend too much time on any one statement but give the answer (on a five-point scale) which seems to describe how you generally feel: "Not at all" (1), "A little" (2), "A fair amount" (3), "Much" (4), "Very much" (5). Ocenite stopnjo nelagodja, ki ga občutite v spodaj navedenih situacijah. Upoštevajte, da ni pravilnih ali napačnih odgovorov. Pri izjavah se ne zadržujte predolgo, ampak na 5-stopenjski lestvici od "nelagodja sploh ne občutim" (1) do "počutim se skrajno nelagodno" (5), preprosto označite odgovor, ki najbolje opisuje vaše počutje v opisani situaciji.	
	Math. Test Anx.	Studying for a math test.	Učim se za izpit iz matematike.
		Taking the math section of the college entrance exam.	Pišem maturo iz matematike.
		Taking an exam (quiz) in a math course.	Pišem kolokvij pri matematiki.
		Taking an exam (final) in a math course.	Opravljam izpit pri matematiki.
		Thinking about an upcoming math test one week before.	Razmišljam o matematičnem izpitu, ki bo čez en teden.
		Thinking about an upcoming math test one day before.	Razmišljam o matematičnem izpitu, ki bo naslednji dan.
		Thinking about an upcoming math test one hour before.	Razmišljam o matematičnem izpitu, ki bo čez eno uro.
		Realising you have to take a certain number of math classes to fulfil requirements.	Ugotovim, da bo za izpolnitev zahtevanih pogojev pri matematiki potrebno prisostvovati določenemu številu matematičnih predavanj.
		Receiving your final math grade in the mail.	Izvem rezultate o končni oceni pri matematiki.
		Being given a "pop" quiz in a math class.	Dobim nenapovedani test pri matematiki.
	Num. Task Anx.	Reading a cash register receipt after your purchase.	Preverjam pravilnost računa po opravljenem nakupu.
		Being given a set of numerical problems involving addition to solve on paper.	V reševanje sem dobil nalogo, kjer se zahteva seštevanje števil.
		Being given a set of subtraction problems to solve.	V reševanje sem dobil nalogo, kjer se zahteva odštevanje števil.
		Being given a set of multiplication problems to solve.	V reševanje sem dobil nalogo, kjer se zahteva množenje števil.
		Being given a set of division problems to solve.	V reševanje sem dobil nalogo, kjer se zahteva deljenje števil.
	Math. Course Anx.	Buying a math textbook.	Kupujem matematični učbenik.
		Watching a teacher work on an algebraic equation on the blackboard.	Gledam profesorja, ki rešuje enačbe na tablo.
		Signing up for a math course.	Prijavljam se na izbirni predmet, ki vsebuje veliko matematičnih vsebin.
		Listening to another student explain a mathematical formula.	Poslušam sošolca, ki razlaga matematično formulo.
		Walking into a math class.	Vstopam v matematično učilnico.

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Review

Impact of the Flipped Classroom Method in the Mathematical Area: A Systematic Review

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Abstract: Currently, the use of technology has become one of the most popular educational trends in Higher Education. One of the most popular methods on the Higher Education stage is the Flipped Classroom, characterised by the use of both face-to-face and virtual teaching through videos and online material, promoting more autonomous, flexible and dynamic teaching for students. In this work, we started to compile the main articles that used Flipped Classroom within the mathematical area in Higher Education, with the aim of analysing their main characteristics, as well as the impact caused on students. To do so, the method of systematic review was used, focusing on those empirical experiences published in Web of Sciences and Scopus. The results indicated that, in most cases, the implementation of Flipped Classroom led to an improvement in students' knowledge and attitudes towards mathematical content and discipline. In addition, aspects such as collaborative work, autonomy, self-regulation towards learning or academic performance were benefited through this method.

Keywords: Flipped Classroom; flipped learning; mathematics; higher education; university

1. Introduction

Nowadays, traditional teaching–learning methods have become obsolete and do not respond to the demands of today's students, who behave passively and are unmotivated and do not encourage critical thinking. Likewise, traditional teaching methods do not adapt to the pace of society, which is advancing at a dizzying rate, experiencing important changes in all areas [1]. The work that teachers had done for decades now also needs to be different in order to provide an adequate response to their students [2]. They need to be kept up to date with methodological innovation, which goes far beyond the master class. These new methods prepare students to successfully face the real world, using their knowledge and enabling them to adapt autonomously to the changing pace of society [3]. Thus, we are looking at a revolution in the way we understand and operate the teaching–learning processes, where different skills are proposed and digital resources are incorporated, which also characterise today's society [4]. Information and Communications Technology (ICT) is essential for this methodological change, as they enable students to work independently and in a personalised way. [5].

These innovative strategies called active methodologies offer students a significant role, making them the principal participants in their own learning [6,7]. The Flipped Classroom method, also known as the hybrid model or blended learning, forms part of one of these types of active methodologies [8]. It is a methodological proposal based on the theory of social learning and constructivism, so that students are the active actors in their learning [9].

If traditionally, during class time the teacher actively presented the theoretical contents and left the practical part for the students to work at home individually, from this new approach, the organisation and management of time is reversed [10], differentiating two parts: in the first, the students work on the theoretical contents individually, asynchronously and autonomously, before the classroom session. In this phase, ICT play a fundamental role, since these theoretical contents reach the students through videos, images, computer graphics or iconic materials [11]. The second part, coincides with the time in class, during which the questions are raised and the practical work is done; developing the competences and solving problems connected to the real world, in which the theoretical contents learned are used, in a collaborative and active way, under conditions of self-regulation and structuring of cognitive scaffolds [12–16].

The role of the teacher using this methodology is to guide learning, adapting teaching approaches to needs and preparing the different learning scenarios [17].

This method includes three lines of learning: (1) Individual learning, which is adapted to the different learning rhythms, since the contents of the first phase can be visualised as often as necessary, and it encourages responsible and autonomous work. (2) Collaborative learning, worked on during the second phase, where in groups we pursue objectives that are agreed upon until the final objective is reached. (3) Problem-based learning, which also takes place in the second part of this method, in which what is learned is put into practice in a contextualised way, enriched by the contributions of the group's colleagues, and it is checked whether the learning has been effective [18].

The advantages that the Flipped Classroom methodology brings to teaching are the following [19]: It respects learning rhythms, since theoretical explanations can be used at any time, promoting oblique learning [5]. Self-evaluation is made possible, providing constructive feedback on their progress and the quality of their work. It develops responsibility in one's own learning [17,20]. It is a methodology which is in accordance with the motivations and interests of the students [21], who tend to prefer virtual environments, to those which they are increasingly used to. It develops autonomy by also increasing their interest and motivation for learning [22], contributing to favour the "learning to learn" competence, so important to acquire in a society that is in continuous change [23]. This autonomy also favours creativity and critical thinking in the student [18]. It develops collaborative teamwork [24] and exposes students to problematic situations that encourage meaningful learning [25], which improves academic performance [26,27].

For this methodology to be effective and for all these educational advantages to be enjoyed, it is essential that teachers have acquired digital competence, which allows them to create audio–visual material and move around content management platforms, as well as having adequate methodological training [28]. Therefore, one of the drawbacks that make the use of this method difficult is the lack of training of the teaching staff in aspects related to innovative methodologies and ICTs, together with the necessary dedication to carry it out and the lack of habit of the students with the invested learning [29,30].

Numerous interventions, particularly in the area of mathematics, at all educational stages, show the benefits of this method for learning in this area [31–35]. Authors such as [36] argue that this form of learning mathematics allows higher levels of Bloom's taxonomy to be worked on in the classroom, such as analysis, which requires more discussion, making the face-to-face class more profitable. To this concept, [37] adds the impact on increasing performance and motivation in this subject, which is often difficult for students to assimilate [38] and together with [39], who also highlight the improvement in the working environment and the attitude of the students.

On the other hand, [40] implemented this method in the differential calculus classroom of higher education, appreciating, from the results obtained, its advantages of motivation and break with the classical routines, as well as the need to develop the methodological foundations of the Flipped Classroom. In this way, the personalization, meaning, idealization and representation of mathematics teaching was not impaired, besides the need to take into account the students' previous knowledge. Ref. [41] reiterates the importance of teacher training, which will lead them to change the traditional methodology, accepting the new active methodological strategies, to which they are not used to.

Ref. [42], after the application of this didactic strategy in the secondary education mathematics classroom, obtained as a result a substantial improvement in the evaluation and attitude of the students, verifying the increase in motivation and skills in the analysis and representation of graphics.

Based on these ideas, the main objective of this work was to locate the main educational experiences that would use the Flipped Classroom method for the promotion of mathematical knowledge within the Higher Education stage. With this purpose in mind, the next research questions were configured:

RQ1. What are the main experiences in which the Flipped Classroom method is being implemented to achieve an acquisition of mathematical knowledge?
RQ2. In which disciplines within mathematics are these experiences framed?
RQ3. What journals have published scientific articles on this field?
RQ4. What has been the impact of the Flipped Classroom method on students?
RQ5. What instruments were used to measure the effectiveness of the Flipped Classroom method?

2. Materials and Methods

Based on the ideas set out above, this work is part of the systematic literature review method, conceived as that which analyses information provided to generate an overview of a certain object of study, specifically information provided in databases or scientific reports [43]. This type of research allows the categorization of the results to date on the topic, as well as measuring the data based on different criteria regarding the relevant issues that need to be clarified [44,45].

For this purpose, the methodological process consists of a sequence of steps that go from the defining of the scope to the classification of the data obtained. For this purpose, the work phases proposed in the PRISMA declaration (Preferred Reporting Items for Systematic reviews and Meta-Analyses) were followed [46].

The examination process carried out was divided into two steps:

- Planning: This protocol consisted of the definition of the research questions, inclusion and exclusion criteria and the development of the descriptors and databases from which the scientific papers would be collected.
- Action: We proceeded to find references in the selected bases, to refine the data using different filters to extract the information to ultimately make the representation of the data. As for the formulation of the inclusion and exclusion criteria, these were configured according to the objectives of the study and the indications contained in the PRISMA declaration.

2.1. Search Strategy

The search of the scientific papers was carried out in the international databases Web of Science (WoS) and Scopus. These two databases were chosen for their potential and international reputation, as well as for the criteria they use to index their articles [47]. In the case of the Web of Sciences, the search was carried out in the Social Sciences Citation Index (SSCI), Science Citation Index Expanded (SCIE) and Arts and Humanities Citation Index (AHCI). The search equation was used, which is composed of the following descriptors: "Flipped Classroom" or "Flipped Learning", and "Mathematics".

The descriptors were applied in the searching engine of both databases in order to filter them further. In order to do so, a list of inclusion and exclusion criteria was set up to limit the study sample (Table 1).

To avoid any bias in the selection of the studies, after other works of systematic review [48], two researchers conducted the systematic review using the identical descriptors and criteria for inclusion and exclusion. The degree of consensus in the inclusion of the article was 95%. The disagreement was addressed by a third researcher who chose to include 100% of the extracted scientific literature.

Table 1. Inclusion and exclusion criteria.

Inclusion Criteria (IC)	Exclusion Criteria (EX)
IC1: Journal articles	**EX1**: Book chapters, books, or other types of non-peer-reviewed publications.
IC2: Articles available in Open Access	**EX2**: Articles not available in Open Access
IC3: Empirical research	**EX3**: Theoretical studies or revisions.
IC4: Articles written in English or Spanish language.	**EX4**: Articles that are not written in English or Spanish language.
IC5: Research that has taken place in the Higher Education stage	**EX5**: Practices that have not been implemented in other education stage.
IC6: Educational experiences in which I used the Flipped Classroom method within the mathematical field	**EX6**: Educational experiences in which I used the Flipped Classroom method in another discipline of knowledge.
	EX7: Duplicate Articles

2.2. Procedure

First, using the Prism Declaration [46,49] as a reference, the procedure was divided into four specific phases. The first, called "Identification", consisted of applying the database search equation, filtering the search for scientific articles (IC1, EX1) in English or Spanish (IC3, EX3), obtaining a total of 10 documents (WoS; Scopus). After that, in the review phase, most of the inclusion criteria (IC2, IC4, IC5) and exclusion criteria (EX2, EX4, EX5) were applied. Finally, duplicate articles were eliminated (EX7) in order to finally obtain a sample of articles to be analysed (n = 10).

In order to shorten this procedure, a flow chart is presented that shows the process described from the initial location of documents to the final scrutiny of the sample of articles that make up the systematic review study (Figure 1).

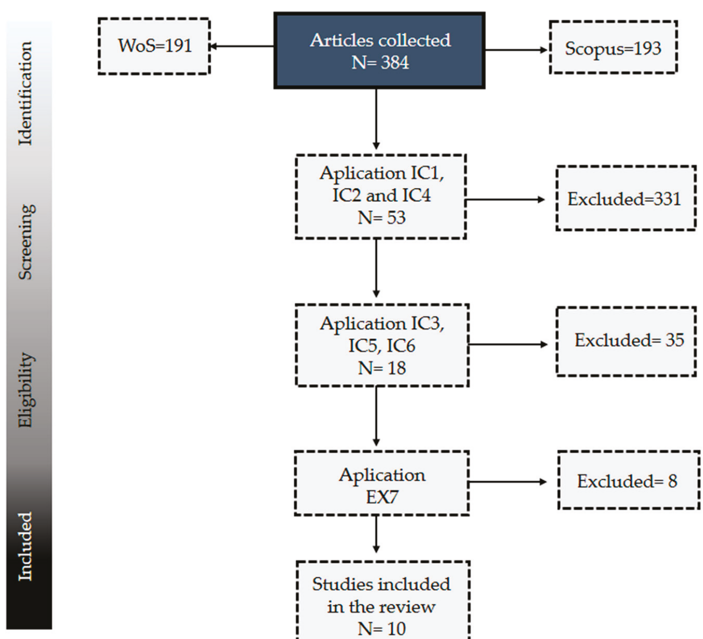

Figure 1. Flowchart of the phases that make up the systematic review.

3. Results

First, the studies were grouped according to the year in which they were published (Figure 2). In this case, it can be seen that the year 2019 was the one with the most contributions, followed by 2016, 2018 and 2017, respectively.

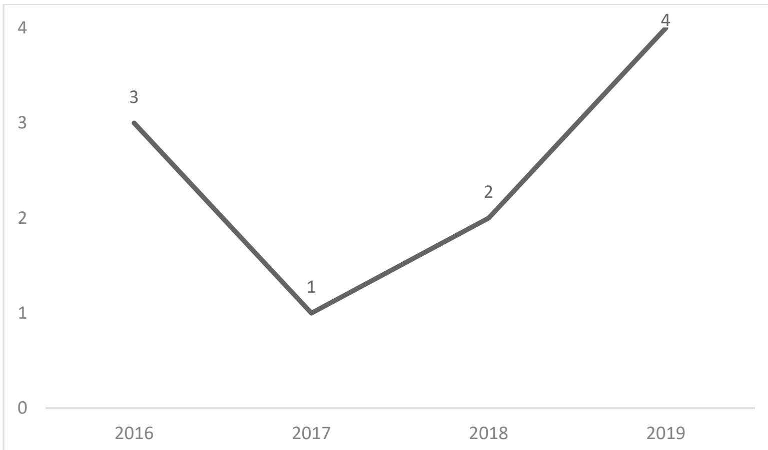

Figure 2. Number of articles per year.

On the other hand, looking at the journals in which these scientific papers have been published (Table 2), it can be seen that the articles have been published in different platforms. In particular, *Educational Technology, & Society* stands out, with a total of two works on this topic. According to the origin of the journals, they correspond to different countries, among which the United Kingdom stands out, with a total of four publications.

Table 2. Journals to which the works belong and country.

Reference	Journal	Country
[50]	*Tecné Episteme y Didaxis: TED*	Colombia
[51]	*Higher Education Pedagogies*	United Kingdom
[52]	*International Journal of Interactive Mobile Technologies*	Germany
[53]	*International Journal of Higher Education*	Canada
[54]	*CBE—Life Sciences Education*	United States
[55]	*Journal of Technology and Science Education*	Spain
[56]	*International Journal of Mathematical Education in Science and Technology*	United Kingdom
[57]	*PRIMUS: Problems, Resources, and Issues in Mathematics Undergraduate Studies*	United Kingdom
[58]	*Research and Practice in Technology Enhanced Learning*	Singapore
[59]	*Teaching Mathematics and Its Applications*	United Kingdom

According to the mathematical contents that have been addressed during the Flipped experiences, it mainly corresponds to the treatment of the derivative and the limit of functions (Table 3). However, there are also works coming from the computer, algebraic and even didactic field.

Table 3. Mathematical content covered in each educational experience.

Reference	Mathematical Content
[50]	The derivative as a ratio of change, incremental coefficient. Optimization of functions
[51]	Limits and derivatives
[52]	Mathematical modelling
[53]	Gaussian elimination method
[54]	Critical thinking in the STEM area
[55]	Didactic programming in the area of Mathematics
[56]	Computer programming
[57]	Language in Mathematics
[58]	Limit of functions, differentiation, applications of derivatives, and theory of integration with applications
[59]	Algebraic calculation

Finally, the objectives of each investigation were analysed in detail, as well as the methodology used and the impact on students after the application of the methodology (Table 4). In general terms, all the studies claim that the application of Flipped Classroom has improved students' attitudes towards the content taught, and in some cases towards the mathematical discipline [51,56]. In addition, parallel aspects of learning benefit, such as self-regulation [57], collaborative learning and the social climate of the classroom [52] and improved academic performance [55]. On the other hand, the methodological plurality found in the methodology employed by the authors is also noteworthy, with qualitative and quantitative works, as well as experimental, quasi-experimental designs, of a comparative nature between academic years.

Table 4. Mathematical content covered in each educational experience.

Reference	Objective	Method	Instrument	Impact of FC Application
[50]	To analyse the effectiveness of the Flipped Classroom method in the assimilation of the derivative in the resolution of application exercises	Qualitative	Written test	Increase in students' motivation, as well as their capacity to argue mathematical problems. However, the freedom of students to use information sources in their "out of class" period, or the traditional character of algorithmic work under this methodology, were not considered as negative aspects.
[51]	What is the influence of Flipped Classroom on the attitude towards mathematics in students depending on their socio-economic level?	Quantitative pre-post test. Existence of control and experimental group	The Attitudes Towards Mathematics Inventory (ATMI)	The groups that experienced Flipped Classroom increased their participation and commitment in class. Students who came from more vulnerable environments rated the pedagogical change more positively.
[52]	Promote through Flipped Classroom a better understanding of mathematical modelling concepts.	Mixed	Self-observation, ad-hoc questionnaire and student diary	The experiment led to an improvement in the social climate of the students, improving collaborative work between them in the classroom. In turn, it allowed students to better control their learning time, as some of them repeated the videos if they did not understand the content. The students' perception of the usefulness of the class was increased.

Table 4. *Cont.*

Reference	Objective	Method	Instrument	Impact of FC Application
[53]	Acquire the Gaussian elimination method through the Flipped Classroom model	Qualitative	Participant observation	Through the previous phase at home, the students acquired the basic notions about the method. Later, in the classroom, through Matlab they were able to deepen their learning. Therefore, the learning was more effective.
[54]	To evaluate the impact of the Flipped Classroom method on the mathematical thinking of life science students, with emphasis on critical thinking	Quantitative	California Critical Thinking Skills Test, & Student Assessment of their Learning Gains (SALG)	Students' critical thinking developed considerably. Especially, when evaluating mathematical information, expressing their arguments and in confidence when expressing their reasoning.
[55]	The purpose of this study has been to analyse student satisfaction after the implementation of the flipped learning model in online learning, as well as to study whether there is an improvement in the performance of these students.	Quasi-experimental	Ad-hoc survey	Student satisfaction improved after the course of the flipped course, and an improvement in student academic performance was noted. Students say they would repeat the experience
[56]	What are the attitudes of students after experimenting with Flipped Classroom?	Quantitative Experimental-Comparative between the application in two different academic years	Student Experience of Course (SEC)	The fact that time in class was used in an active way, which was not contemplated in the traditional methodology, was valued very positively.
[57]	Check the results of the Communications in Mathematics program through following a Flipped Classroom methodology	Mixed with experimental character applying pre-test and post-test.	Motivated Strategies for Learning Questionnaire (MSLQ), and open questions	The students valued the experience positively. There was a slight improvement in student self-regulation towards learning during the course. There was also strong resistance to adapt to the method.
[58]	Checking students' attitudes after the application of Flipped Classroom in learning limits, derivatives and application theory with applications	Mixed	Ad hoc survey	The reaction of the students was positive, who, in generic terms, presented positive attitudes towards the methodology. However, there were students who did not follow the video viewing.
[59]	To evaluate the effectiveness of Flipped Classroom in the development of algebraic calculus of different groups of science students	Quantitative. Quasi-experimental	The Mathematics Attitudes and Perceptions Survey (MAPS), and The Calculus Concept Inventory	Those students with lesser skills in calculus showed significant improvements after the experimentation of the method. However, those students with a higher level did not experience any benefit.

4. Discussion and Conclusions

The implementation of Flipped Classroom is considered as one of the latest and most relevant methodological innovations in recent years. Specifically, in the field of mathematics, the arrival of this teaching methodology has led various Higher Education teachers to incorporate it into their daily work in the classroom [34,35]. The aim of this study was to analyse the main experiences using this methodology when teaching mathematics, and to see what effect it had on students.

Thus, the main findings of the work indicated that in the majority of the investigations analysed, the implementation of Flipped Classroom meant a notable improvement in students' knowledge of the specific content covered, as was the case of algebraic calculation, the derivation and limits of functions, mathematical modelling and mathematical critical thinking, among others. Therefore, this finding adds to the ideas indicated by previous studies that determine the effectiveness of Flipped Classroom in this sense [34,40].

Likewise, the experimentation of the method promoted an improvement in the motivation rates of the students, as well as a more positive evaluation towards the contents and, in general, towards mathematics [51]. Therefore, we continue on a path, as corroborated by the scientific literature, of improvement in the motivation rates after the implementation of active methodologies that integrate technology in their teaching procedures [37,42].

Similarly, the improvement of aspects such as collaborative learning within the classroom activity, as well as autonomous learning, should also be highlighted. With regard to the former, dividing the teaching process into two phases encourages that time in the classroom to be dedicated to active and collaborative learning, in which students take on a greater role, giving rise to a better social climate and better group synchrony [24]. On the other hand, placing part of the activity of the methodology outside the classroom promotes the development of self-regulation and autonomy skills towards learning. However, this assertion cannot be fully corroborated, since in turn, the results of this review determined that although the majority of students fulfilled this purpose, a minority group was found not to follow the Flipped videos and that they presented greater difficulty in incorporating themselves into the new class dynamic [57,58]. In view of this situation, it is necessary to continue working on didactic strategies and the implementation of resources that allow the entire student body to be correctly incorporated into this method.

In short, the implementation of active methodologies such as Flipped Classroom is becoming an emerging practice that is gaining prominence within the mathematical landscape. The arrival of technology in the classroom has changed the concept of teaching, in favour of online education, student autonomy and the practical nature of classroom attendance.

Among the limitations of this work is that it was not possible to analyse those studies that were not in Open Access, which limited the sample of studies to a small number. In this way, there is a part of the scientific literature that cannot be analysed and interpreted. On the other hand, the future lines of research are to continue promoting the use of the Flipped Classroom method within the mathematical branch as one of the main lines of innovation at a didactic level. The proposal of good practices in this sense, will provide teachers of all educational stages in the area of mathematics with ideas to be able to undertake in their classroom and promote an improvement through technology.

In conclusion, the dizzying technological progress experienced by society, and therefore the education system, has led to a profound transformation in the teaching–learning process. Faced with a student body that is very different from that of a few years ago, it is necessary for teachers to explore their interests and motivations in order to plan their teaching. For this reason, the inclusion of technology and its application in teaching methods is seen as a solution that motivates students, increases interest in the subject and the content it covers and, as this research has shown, promotes better knowledge acquisition.

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Using Robotics to Enhance Active Learning in Mathematics: A Multi-Scenario Study

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Abstract: The use of technology, which is linked to active learning strategies, can contribute to better outcomes in Mathematics education. We analyse the conditions that are necessary for achieving an effective learning of Mathematics, aided by a robotic platform. Within this framework, the question raised was "What are the conditions that promote effective active math learning with robotic support?" Interventions at different educational scenarios were carried in order to explore three educational levels: elementary, secondary, and high school. Qualitative and quantitative analyses were performed, comparing the control and treatment groups for all scenarios through examinations, direct observations, and testimonials. The findings point to three key conditions: level, motivation, and teacher training. The obtained results show a very favourable impact on the attention and motivation of the students, and they allow for establishing the conditions that need to be met for an effective relationship between the teacher and the technological tool, so that better learning outcomes in Mathematics are more likely to be obtained.

Keywords: educational robotics; active learning; educational innovation; mathematics learning; case studies

1. Introduction

For a long time, human beings have endeavoured to develop new methods to perform tasks in an easier way, with the aim of doing those activities that benefit him in any area of his life faster and more efficiently. It is then that technology comes into play. Despite the fact that technology has several paths, it can be assured that each and every one of them has a common purpose: to help humanity solve some problem that is inherent in it. Information and communications technologies (ICTs) currently occupy an extremely important place in society and the economy. Their importance has been increasing enormously. The concept of ICT has emerged as a technological convergence of electronics, software, and telecommunications infrastructure. Robotics is one of the expressions of technology whose application has extended to various contexts of life. In the educational field, it becomes a valuable resource to facilitate learning and develop general skills, such as socialisation, creativity, and initiative in students today [1].

Speaking of education, the results in the latest PISA test showed that Mexico's performance is below the OECD average in science (416 points), reading (423 points), and Mathematics (408 points) [2]. According to [3], one of the main causes of school failure in students is a lack of interest and boredom.

This is mainly due to the fact that, in most cases, the current education is not interested in generating innovative activities that favour the participation of their students. For this reason, the use of ICTs has been promoted within the classrooms. The use of new technologies allows for an incentive towards learning in students of different educational levels, and the learning of Mathematics is a very specific area.

One of the most relevant challenges in Mexico is the attitude towards learning Mathematics. Currently, it is a necessity that Mathematics instructors find better ways of teaching; this would allow for students to be more empathic and make sense of this area of knowledge. Thus, in most classes nationwide, teachers do not improve the use of teaching materials, due to a lack of creativity, time, proper training, or planning. Hence, the importance of showing how technology allows for significant improvements in attention and motivation towards Mathematics, which, in turn, allows for an improvement in training programs and teaching practices; thus, achieving a positive impact on student learning. The principles of mathematical modelling include learning on our own; using technological devices and basic productivity tools to investigate and produce learning material in an ethical and efficient manner; and, applying numerical, algebraic, and geometric procedures for the understanding and analysis of real situations. The use of a technological platform would help to fulfil the expected objectives.

Although there are already ways to incorporate technology in the classroom, it is still not very common to use robots as a support tool for lesson delivery. Robotics in the classroom not only allows to study topics of automation and process the control in the area of technology and computer science, but it also serves as an aid in learning different areas of knowledge. The robots arouse interest in students, as they are concrete objects striking. An educational robotic proposal can be implemented under an approach that takes the learning environment, the planning of activities, resources, the time needed for the realisation of each of these, and the methodology to perform them into account. In this framework, active learning with strategies for doing, reviewing, learning and applying can be of support to contribute with the construction of mathematical knowledge. When joining these resources, one could ask oneself: What are the conditions that promote effective active Math learning with robotic support?

1.1. Learning Mathematics in Mexico

Mathematics is a universal language that contributes to the development of logical thinking, the ability to reason, and to face new challenges. Learning Mathematics is a subject that emphasises problem solving. If students think critically, they can solve problems effectively [4]. Mathematical competences, according to the Organisation for Economic Cooperation and Development [5] refer to students' abilities to analyse, reason and communicate effectively when they identify, formulate, and solve mathematical problems in different situations [6]. Mathematical competence "implies the ability and willingness to use mathematical modes of thinking (logical and spatial thinking) and their representation (formulas, models, graphs, and diagrams)" [7] (p. 164). Mathematical competences must be placed as part of the key elements within learning for life.

The Government of Mexico states that the general purposes of learning Mathematics are: (a) to conceive Mathematics as a social construction in which mathematical facts and procedures are formulated and argued; (b) acquire positive and critical attitudes towards Mathematics; and, (c) to develop skills that allow them to pose and solve problems while using mathematical tools, make decisions, and face non-routine situations (Government of Mexico, n.d.). Thus, the profile of the preschool graduate in mathematical thinking is [8]: "he or she counts at least to 20, reasons to solve problems of quantity, builds structures with figures and geometric bodies, and organises information in simple ways" (p. 68). For the primary education profile: "he or she includes concepts and procedures for solving various mathematical problems, and for applying them in other contexts" (p. 74). Finally, for secondary education: "he or she expands his knowledge of mathematical concepts and techniques to pose and solve problems of varying degrees of complexity, as well as to model and analyse situations. He or she values the qualities of mathematical thinking favourable to Mathematics" (p. 80).

In Mexico, not all of the students have the same opportunities to learn. Those who cannot access mathematical content in school are at a lifelong social and economic disadvantage. According to [2], the lack of equal learning opportunities in the Mexican education system leads to the reinforcement of gaps and inequalities in society. Learning in Reading, Mathematics, and Science remains below the international average. According to the PISA report, only 1% of students performed at the highest levels of competence and 35% of students did not achieve a minimum level of competence. The obtained results continue to show a lack of competence in Mathematics [3].

Mexico seeks quality education that is innovative in its pedagogical practices, and that has the necessary means for the integral development and greater well-being of its society. In order to achieve this, socio-educational policies search for new opportunities in order to reduce inequalities between communities, promote training in values, minimise gaps, and promote equity [9]. In this regard, the Secretary of Public Education (SEP) and the National Institute for the Evaluation of Education (INEE), highlighted the importance of reviewing and redesigning existing educational programmes and modalities with the aim of: (a) facilitating access to all citizens, (b) encouraging the development and use of new technologies, and (c) promoting the skills of all students in order to ensure a full life. The integration of technologies and, in the case of educational robotics, helps to develop various skills and promote the construction and acquisition of knowledge in general, and Mathematics in particular. To this end, they designed the National Plan for the Evaluation of Learning (PLANEA), which classifies those evaluated into four levels of mastery: I (insufficient), II (basic), III (satisfactory), and IV (outstanding). This plan aimed to assess the teaching-learning processes and the learning achievements in Language-Communication and Mathematics [10].

Thus, the Secretary of Public Education [8] raises the need to ensure quality of learning in basic education, and educational inclusion and equity for the construction of a more just society. If we focus on the area of Mathematics, we seek to modify the actions with innovative practices, integrating the use of technologies in teaching methodologies for learning them, and promoting the development of mathematical skills.

1.2. Educational Robotics

Educational robotics (ER), which is also known as pedagogical robotics, is a discipline that aims at the conception, creation, and implementation of robotic prototypes and specialised programs for pedagogical purposes [11]. ER is not a new concept, but, rather, it has been growing exponentially in recent years. It has a major impact on learning [12], and it is associated with the STEAM disciplines (Science, Technology, Engineering, Art, and Mathematics) for the development, skills, and understanding of mathematical, physical, engineering, and related concepts [13–15]. Across the various faculties and universities, and in order to reduce the gender gap in STEAM careers, we have to train students in/with robotics related skills in all disciplines. Additionally, in this way, promote learning centred on the student, on his or her interests, and on the demands of society, using innovative methods; and, promoting critical training, in order to develop active and co-participating citizens in today's society [16,17].

It could be defined as "an interdisciplinary discipline, which requires the construction of a technological object with a specific purpose (some authors call it an educational robot, others call it a robotic prototype, others refer to automatisms …); it aims at the pedagogical field; and it develops key competences and skills for the students of the 21st century" [18] (p. 4).

The integration and use of educational robotics in the teaching-learning process in pre-school, primary, and secondary levels can become visible, and be a turning point, as a resource to address the diversity of the classroom, as a means to help the inclusion of all students, as well as keep them active and motivated [15,19–21]. Additionally, it can be utilised as a tool to promote the construction of knowledge and the achievement of results. Therefore, when working with ER, apart from working on science and technology, the aim is to promote other cross-disciplinary skills, such as: creativity, communication, collaboration, critical thinking, teamwork, innovation, the development of solutions to problems, digital skills, and computational thinking [22,23]. In order to do this, teachers must have basic knowledge to be able to teach the contents with robots [24].

Introducing students to the areas of science and technology, in the case of Mathematics, through play and constructionist learning in order to generate new knowledge, is one of the goals of the usability of ER in the classroom. Learning with robots offers students intrinsic motivation, and invites them to investigate, foster their curiosity and imagination, to ask questions, to work in teams, to overcome challenges, to make decisions, and to be responsible of their own process [12,25].

There are many robots on sale, according to shape, size, function, working environment, and autonomy. Depending on the shape, we find: zoomorphic (imitation of a creature, e.g., bee), humanoid (reproduction of the shape of a human and its movements, in this case, the NAO robot), hybrid (combination of the above), and polymorphic (different shapes, adapting its structure according to the task). The size of the robots can be: robots, microrobots, nanorobots, or nanobots (the smallest, nanometric size). Function: domestic, medical, military, entertainment, space, educational, among others. In relation to the work environment: stationary robots (they are fixed and immovable, they are the majority of industrial robots), ground, underwater, air, and microgravity robots. Additionally, in terms of their autonomy: tele-operated (drones) and semi-aquatic [26].

The NAO robot, humanoid robot, was born in 2008 and developed by the company Softbank Robotics, we are currently facing the model evolution v5. The NAO robot measures 58 cm, weighs no more than 5 kilos, speaks, listens, sees, relates to the environment as programmed, and interacts naturally. It is capable of perceiving its environment from multiple sensors. It is composed of two cameras, four microphones, nine tactile sensors, two ultrasonic sensors, eight pressure sensors, an accelerometer, a gyroscope, a voice synthesiser, and two speakers [27]. The robot includes a graphic programming software, Choregraphe, which allows accessible communication with the NAO; this software is a graphic programming interface by means of blocks, which provide specific tasks for the NAO [28].

NAO has two key advantages: (1) its versatility (customising its functions and individualising its uses) and (2) its body language (freedom of movement, adapted to the environment, manageable and friendly, and it is designed for any age). In relation to Education, NAO is designed to be used from the age of five up to university. Its use in the classroom means that the students are more playful, motivated in the learning process, able to interact and communicate, and establish a link between theory and practice. On the part of the teaching staff, they have more engaged students, and more dynamic classes, promoting student interest, and obtaining results of the programming in real time. As far as researchers are concerned, NAO has been used in different universities, as it is considered to be ideal for practical experiments, as it is an intuitive software that offers endless possibilities with multi-language programming [27].

1.3. Active Learning and Robotics in Mathematics

Learning environments for Mathematics require the diversified use of methods, techniques, and strategies that support the acquisition of processes of analysis and construction. One technique that has proven to be effective in developing mathematical skills is active learning. In active learning, the teacher uses a methodology that seeks to promote the participation of the student as a prosumer of knowledge [29]. In this technique, the teacher must plan continuous stimulation activitie, so that the student individually or collectively performs procedures of higher order: analysis, synthesis, interpretation, inference, and evaluation [30]. According to [31], the family and community socio-productive activities in which students participate in on a daily basis constitute areas of experience that demand their incorporation into the didactics of Mathematics focused on problem solving.

It is important that the practical experience that students have had regarding mathematical reasoning be considered within the teaching of Mathematics in a formal school space, with the support of practical research. In the study [32], the impact of the use of an Adaptive Tutoring System (ATS) on the development of three mathematical competences was measured: the use of symbolic language, modelling of mathematical problems, and problem solving through mathematical reasoning. Research [33] suggested increasing the level of student mastery of mathematical concepts, while applying the concept of active learning that involves collaborative metacognitive activities among students during the learning process. Through this strategy, it was possible to improve the understanding and mastery of mathematical concepts.

The current educational practices postulate learning as a construction of knowledge, the attention has special relevance in the storage and integration only of the information that is relevant, becoming an element that the student must actively train. Thus, active, dynamic, participative learning, far from passivity, is done with the attention and concentration focused and directed towards the significant elements of it; only in this way, the acquired knowledge will be permanent and effective. In conclusion, attention and concentration are both basic elements of all learning. Project based learning has a key place in active learning. Project-based learning is an example of active learning and it is a current instructional strategy that is driven by students in an interdisciplinary, collaborative, and technology-based manner [34]. Students who implement project-based learning perform better after using this method in teaching and learning sessions [35].

The phases of active learning are differently defined, depending on the author who writes about them. Nonetheless, the main elements remain common among models. One version that is quite easy to understand is presented in [36,37], and described in detail in Table 1.

Table 1. Stages of active learning.

Stage	What Is It?	Student's Responsibility	Learning about Active Learning
Do	The tasks stimulate the students' activity (games, discussion of cases, dynamics, problem solving . . .)	Students choose and plan their work strategies.	Students are encouraged to observe aspects of their learning while they are involved in the tasks.
Review	Students stop to become aware of what happened in the process, what was important, how they felt.	Students monitor their progress and review their plan.	Students describe what they observed and review their learning (objectives, strategies, feelings, context, etc.).
Learn	The new ideas and perspectives that the activity allowed to generate are made explicit.	Students can identify for themselves what they have learned.	Elements affecting progress are identified and new strategies proposed.
Apply	Future actions are planned in the light of the new findings or knowledge. The possibility of transferring what was learned to other situations is discussed.	Students review their plans taking into account their recent learning.	Students plan how they will continue to observe and experience their learning strategies.

Source: [36] (p. 30).

Furthermore, in [38], it is explained how active learning is not a subject area competence (Table 2), but that there is a wide variety of ways in which it can be used in the life of the classroom: from an individual activity to the development of a group project. In all of them, the same processes and principles are applied: Do, Review, Learn, and Apply.

Table 2. Examples of active learning in a range of subjects.

Subject	Do	Review	Learn	Apply
Maths	Tackle a problem	Review strategies	Compare effectiveness	Prepare for next challenge
English	Create a draft	Try out with a reader	Consider feedback	Redraft and publish
Technology	Construct a product	Test its function	Examine evaluations	Redesign
History	Collect sources	Identify points of view	Synthesise	Make sense of another situation

Source: [38] (p. 76).

The examples that are shown in Table 2 follow a similar process, a process that can be described as a cycle. This cycle models the process of learning from experience with four phases: Do, Review, Learn, and Apply [37,38]. To this cycle, it is also important to add a previous stage in doing, Planning, as shown in Figure 1, below.

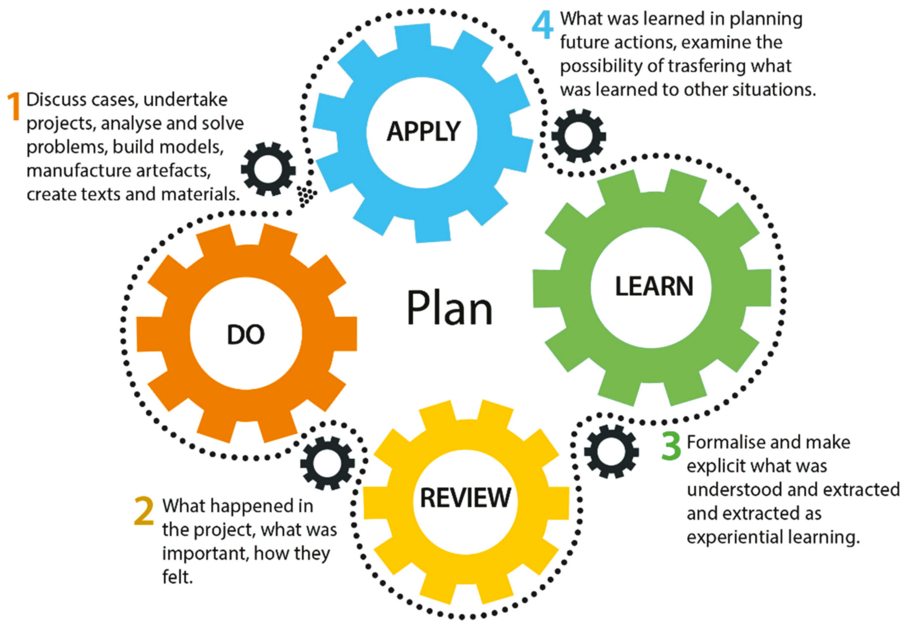

Figure 1. The active learning process (adapted from [36]).

When planning the teaching-learning processes, teachers focus on the experience they want to show and carry out with the students. To this end, the model of the phases presented by [37] can help them to acquire the learning. Active learning has been integrated to support the construction of mathematical knowledge, both at the basic education level while using open educational resources and learning objects [39] and in higher education in virtual and remote laboratory environments [40]. Project-based learning enhances students' academic performance.

When planning the teaching-learning processes, teachers focus on the experience that they want to show and carry out with the students. To this end, the model of the phases that are presented by [36] can help them to acquire the learning.

Math teaching can find great support in the use of technology. Math learning should be done while using instructional media [41], particularly visual learning media, with the support of technology-based ecosystem environments) [42]. Ref. [43] confirmed, through their research, that students who access a multimedia environment provided by their teacher on a blended learning basis can perform significantly better than those that are taught in the classroom. In [44], the design principles that underlie the development and delivery of a blended learning professional development program for high school mathematics teachers were analysed and theoretical frameworks where face-to-face and computer-mediated instruction, teacher identity formation, and structural and basic characteristics of effective teacher professional development are coordinated were identified: the form, duration, and coherence of activities; the nature of teacher participation; focus on content knowledge (mathematics); and, opportunities to engage in active learning. The implementation of visual and technological resources has had good results in the teaching-learning process of Mathematics.

Numerous studies have been conducted on technology applied to the teaching of Mathematics. Ref. [45] developed a research that allowed concluding that cognitive, affective, and metacognitive factors can be modelled and supported by intelligent tutoring systems. The system also helped to improve Math performance on standardised tests, as well as improve student engagement and affective outcomes. In [46], their study found that students who learned using Microsoft Mathematics performed higher on their assessments and a positive effect on student confidence in Mathematics was observed. The work [47] focuses on the description of the principles on which a hypermedia tool (Hipatia) is based and then analyses its impact in three key areas: the learning process of students in Mathematics, their self-management, and affective-motivational variables, such as perceived utility, perceived competence, intrinsic motivation, and anxiety towards Mathematics. The studies focus not only on the cognitive aspect, but they also analyse other elements, such as motivation and confidence.

2. Materials and Methods

In all scenarios, the robot was used as a support tool, which facilitated the teacher's action in the execution of explanations, interactions with students, and review of results. The design and execution of the robot intervention was different for each scenario. It is important to point out that decisions by the administration of each school limited the participation of the protocol in each one of them; therefore, the number of visits, duration of interventions, and application of tests was different, in accordance with the guidelines that each school determined and approved for the execution of the protocol.

2.1. Plan

To develop this project, several aspects were considered, and a selection of topics was carried out for the planning of robot support for the different scenarios. The main aspects to consider are the number of visits, measurement tools, exams, and interviews, as described below.

Visits: in the case of the primary school, two visits were carried out on consecutive days, having almost one and a half hours to carry out the planned routines in mathematical reasoning, through activities to work with distance measurement and fractions. Our project team carried out the visits. For the secondary school case, the robot assisted in four full 50-min. classes, one for each visit. In each session, the teacher used the robot at certain times that were previously defined in the design of the sessions. Finally, in the case of high school, the robot visited the class on a daily basis for a week. Three visits were done at different times of the semester. The first visit was carried out in an introductory way, so that the robot had already been presented with the students for creating a first impression by them. The next two were to support the teacher in the topics established from the beginning. It was the teachers who manipulated the robot when they needed it.

Measurements: in each scenario, a control group and an experimental group were established. Same interventions or classes were carried out with the intervention of the instructor or with the instructor using the robot. The intention was to compare and be able to measure the impact that the robot had among the students. Therefore, tests with numerical results were applied, as well as observation scales in order to measure motivation [48]. Both of the tools were applied in two types of groups. Only in high school were the tests were given before and after starting the robot interventions. In addition, interviews were carried out on the experience carried out with the participating teachers and, in the case of high school, with the students.

Tests: the tests were different for each scenario. At elementary school, a questionnaire was applied, and the students had to solve a mathematical exercise based on the explanation of the topic made by the robot. This exercise was chosen from the Mathematics book that was used by the course teacher, and the number of correct answers, in this questionnaire, was accounted for. For secondary school, an exam was conducted on the topics chosen by the teacher, those where there was support from the robot. Almost all of the exercises were about Analytical Geometry. The design of these exams was done by the teachers in charge of the two groups. In the case of high school, the applied exams were designed and were considered in the planning of the course from the beginning. That is, they were the standard exams that are commonly applied in the subject of Trigonometry, coordinated by the Department of Mathematics of the institution in question.

Interviews: they were carried out in order to find out the opinion of the teachers involved, and the more formal implementation of the structure of the interview, as well as the testimonies of the students and teachers, was established in the high school. As can be appreciated, the protocol became more formal as experience was gained from the previous scenarios. In Section 3.1 of this document, more details of each scenario are given, and a more detailed quantitative analysis of the obtained results is carried out.

The planning for the project is summarised in the following three aspects:

(1) The robot: as mentioned above, a NAO robot was used to support the teacher. The robot delivered the explanation of the topic to be developed, provided that the appropriate environment was put in place. In addition, sometimes it guided the exercises that were established by the teacher. Furthermore, the robot confirmed to the students whether the exercises had correct results. Figure 2 shows the general way in which the robot interacted.

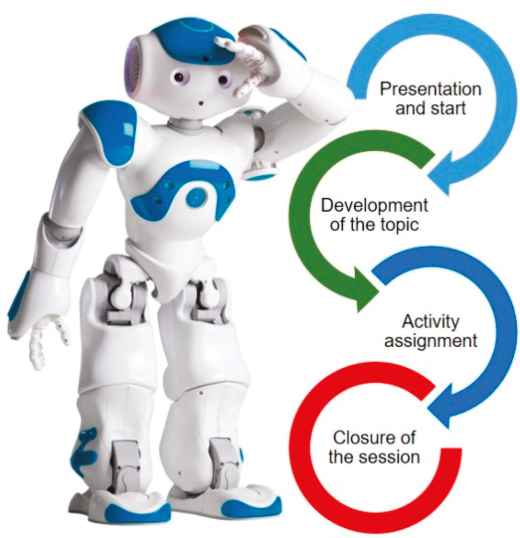

Figure 2. Structure of the sessions.

(2) The scenarios: primary school, two groups of 3rd and 5th grades were visited, with the completion of mathematical reasoning exercises in two consecutive visits. The teacher did not directly use the robot, which carried out demonstrations of previously chosen topics, but was autonomous in carrying out the activities. Before and after, questionnaires were made to the students in order to compare the performance obtained.

For secondary school, four visits were made to the school, with a control group and a treatment group (using the robot), before and after exams were compared, in addition to the observation scales of motivation and attention for the students. The teachers selected and designed Math exercises for their execution.

In high school, performance and motivation were observed in five groups, two experimental and three control groups, during an academic semester with daily visits in full intervention weeks. The subject was trigonometry and the teachers designed and planned the exercises, with greater control over the way the robots intervened. Before and after, the results in the semester were compared as well as the measurement of the students' attention.

(3) The measurements: for quantitative analysis, according to the scenario, teachers applied tests and/or exercises, aimed at measuring the students' learning, and most of them were conventional exams.

In all of the scenarios, a qualitative analysis was carried out, based on previous works [47]. In these analyses, the applied scale was composed of the following indicators: concentration (precision and recall), habituation, withdrawal, distraction (neglect), and interest in the task (motivation and enthusiasm). Based on the definition of care and the indicators that compose it, operational definitions of these indicators were developed to be later translated into observable items of dimensions of care.

The analysis and taking of the scales were carried out by students of Psychology, taking care to observe the guidelines of the reference indicated in terms of human behaviour.

2.2. Methodology

We explored different approaches in three different scenarios in order to shed light on what constitutes a successful application of a robotic platform for enhancing active learning.

The scenarios were divided by educational level. That is, elementary, secondary, and high school settings. Each one of the interventions is described below.

Scenario 1: elementary school. There were two interventions undertaken to two sections of the third and fifth grades, with 28 students each. For these visits, activities were planned without teachers' involvement. They limited their participation to allowing the research group to enter the room and coordinate the activities, manipulate the robotic platform, and perform a brief interview at the end of the session, both for students and teachers. Evaluation was done by applying quizzes to sections where the robot was used, and to others where it was not, so that a results comparison could be made.

Scenario 2: secondary school. For this scenario, visits were performed every Friday for an entire month. This time, two sections were provided use of the robotic platform, and another one was observed as a control with no treatment. Each section had 25 students enrolled. Another variation was that, this time, teachers were involved in planning the activities, including the topic to cover, and the way that the robotic platform was to be used. A group of psychology students was active making observations throughout the process, and capturing both qualitative and quantitative information for later interpretation. Interviews with students were also performed. The emphasis was on assessing the levels of attention and motivation gained by the students translated into measurement scales, as well as performance on the subject matter.

Scenario 3: high school. For this scenario, the actions became more complex. The robotic platform was applied to two treatment groups, and observations were also made to three control groups. The robot was used in every class during an entire week at the beginning of the course, then it was done again in the middle, and at the end of the course. The involvement of teachers in this case was intensive, being trained in the use of the robot, and deciding together the moments and themes where the robot would intervene. Pre- and post-tests were performed for all sections, in order to compare performance results. Additionally, as in the previous scenario, Psychology students undertook behavioural observations and made interviews and testimonials.

For all scenarios, an assessment on how well the process met the requirements of the four cited phases of active learning was made, assigning a rubric-like success categorisation for each. This assessment was made qualitatively, and was based on panel expert discussions, drawing from the experiences that were obtained at each scenario. A comparison was then derived in order to identify the appropriate success factors.

Figure 3 summarises the overall methodological process.

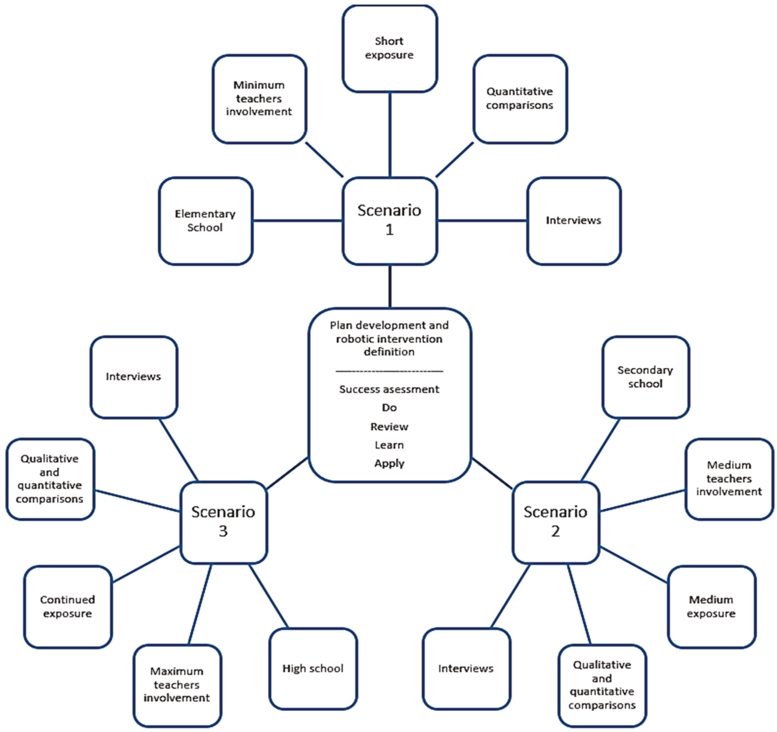

Figure 3. The robotics & active learning exploratory methodological process.

3. Results

3.1. Quantitative Analysis

The use of the analysis instruments that are mentioned in Section 2.1, of the different scenarios grew gradually, starting in primary school, improving aspects of implementation in secondary school, until reaching a more developed scenario, with a longer participation time in high school.

3.1.1. Primary School Tests Performed

Two different sessions were scheduled, taking into account that they were students from third and fifth grades, for a total count of 65. In these sessions, it was evaluated how much attention the students paid to the class, if they retained more information with the help of the robot, and results were compared with a class without robotic help. The sessions were designed in order to address the following topics: propagation of sound, the metric system, and whole number fractions. These topics helped to develop mathematical reasoning and were approved as examples of application by the teachers. All the academic procedures designed by the team followed the following structure: a personal presentation, a presentation of the robot, a brief explanation of the topic, a learning activity, an exam, and a questions session by the students.

For the propagation of sound activity, it was observed how far the sound could be detected, having a “receiver” and a “transmitter”. In the session without the robot, the receivers would be the students and the transmitter would be the teacher. Instead, in the session with the robot, the transmitters would be the students and the receiver the robot. In both cases, the activity would consist of the transmitter speaking at an initial distance and going backwards until the receiver can no longer hear it, once this happens the transmitter will begin to speak through a foam cone, until the same happened, and later on by a paper one. In the case of the metric system topic, the activity consisted of the students having a piece of tape 2-m long having a different colour every 20 cm. An object would be placed at the beginning of this tape, either provided by the teacher or the Nao robot, and it would move forward or backward through each colour. The student should observe the distance from the beginning to the position of the object to attain a better perception of the measurements. Finally, the last activity was to explain the components of a fraction, particularly how to convert a fraction with whole numbers and equivalent fractions. To do this, some examples of fractions, equivalent fractions, and fractions with whole numbers would be seen visually to make the subject clearer. Once the topic was explained, an activity would be carried out reaffirming the knowledge acquired.

In the last session, Psychology students were present to observe the interaction of the teams. The session was applied in two groups, in order to observe differences, and it only occurred in one with the robot. In this session, the topic of fractions would be explained, and examples would be provided for the students to observe. Later, an exercise chosen by the teacher would be carried out, from the book of Mathematical Challenges for fifth grade students. To end the session, a test was applied in order to measure the knowledge acquired.

Some of the results obtained for two of the indicated activities are shown below. Figure 4 shows the percentages of students in a range of scores on the sound propagation test. It can be observed that there was a reduction of 17% in the number of students who obtained a grade that is smaller than seven, thus increasing the proportion of passing grades, which is six or higher.

Figure 4. Test result: sound propagation.

Figure 5 shows the results of the three parts of the session: initial prior knowledge test (three questions), final test for comparison with the initial test (three questions), and final question asked during the session.

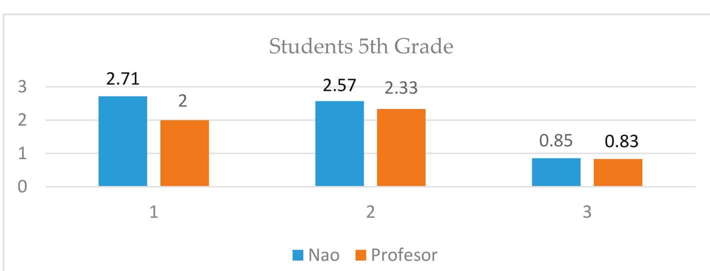

Figure 5. Test result: fractions with whole numbers.

For the results of attention to the class with the robot, an observation scale made up of 34 questions was used, which are grouped and measured the effect in the following dimensions: (1) concentration (precision),

(2) concentration (recall), (3) habituation, (4) de-habit, (5) distraction, (6) interest in the task (enthusiasm), and (7) interest in the task (motivation) [28].

In Figure 6, the results obtained on the observation scale are shown.

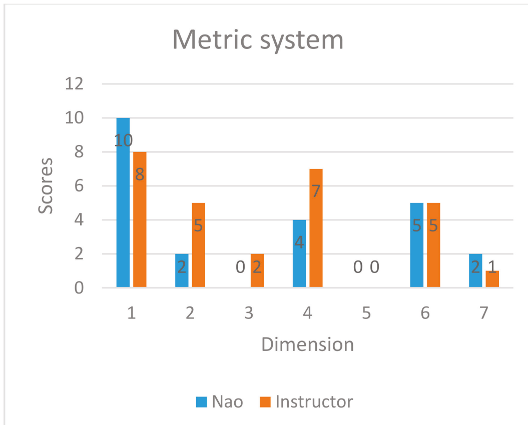

Figure 6. Results of the observation of the activity of the metric system.

When observing the set of all the graphs, it was determined that the favourable points for the NAO robot are: greater concentration, less habituation and dishabituation, and greater interest in the task (motivation). It has to be noted that lesser habituation and dishabituation are desirable, since they indicate that the students will not become bored with the robot with time, and they will not create a dependency on the robot either. There are no apparent differences for distraction and enthusiasm. This can be the result of the influence of observers in the classroom, so it should be more deeply looked into in future studies.

3.1.2. Secondary School Tests Performed

In collaboration with the psychology group, the directors of the institution and the teachers, it was decided that the robot would interact with the students during four sessions of the Mathematics class, on Friday at 7:35 p.m. The group that did not interact with the robot was observed during their Math class on Friday at 2:30 p.m. Before each session, both of the teachers explained the objective of the class and the robot would be prepared for its interaction (analytical geometry). Among the forms of interaction were mainly the dictation of exercises, the response to those same exercises, answers to questions that may have arisen during the class, and even participating as a student during the class. Finally, during the sessions, the psychologists were in charge of observing the students and filling in an observation scale.

Through the tests carried out before and after the sessions, the results of the tests that were applied to the students were averaged, and that both of the observed groups obtained during the investigation.

The observation scales that were performed by the Psychology students turned out to be a more influential tool for hypothesis verification. Thanks to these questionnaires, significant differences could be found in the behaviour of both groups, as shown in Figure 7. This graph shows how behaviours were present and how they varied through the different sessions. It is indicated in the "yes" part of each session, and the frequency in which the behaviour was observed. Similarly, the "no" section indicates the absence of each behaviour at certain point in time for each session. Thus, it is important to note the increase in the "no" part for habituation (from 7 to 11) and distraction (from 4 to 5), from session 1 to session 4, with this being favourable since they are unwanted behaviours.

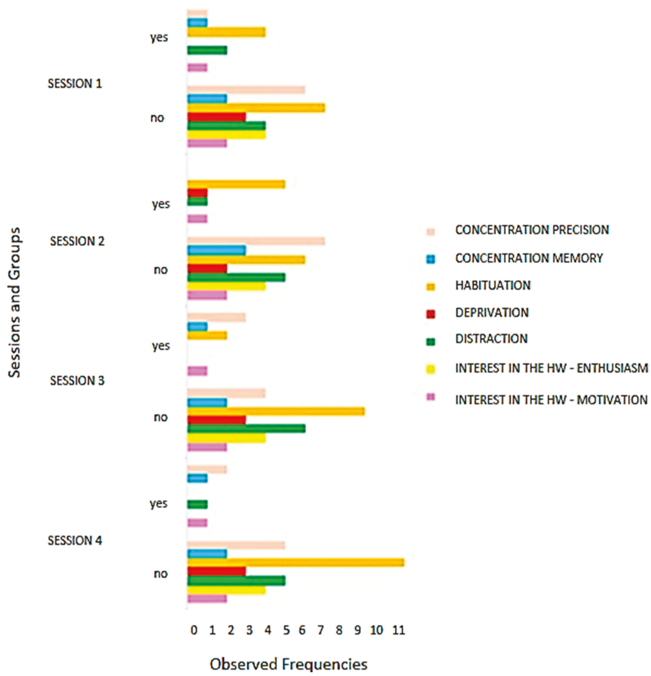

Figure 7. Comparison of the observed behaviour dimensions.

3.1.3. High School Tests Performed

The robot was a support tool for the teacher in the teaching of a hybrid teacher-robot class where two topics of the subject were addressed. During the first visit, the topic would be the statistical analysis of graphs. The disciplinary competencies to be developed are graphical representation of statistical data and the generic competence was collaborative work. For the second visit, the collaborative activity would be dealing with the right triangle solution in the measurement of inaccessible distances within the campus facilities. The disciplinary competencies to be developed were right triangle solution and generic collaborative work.

At this educational level, the teachers were co-designers of the entire research strategy, even controlling the robot to a much greater extent than in the other scenarios. For this, experimental tasks were carried out, divided by periods of six months each. The first was the design and planning of the sessions to begin preparing the programming of the robots, following a structured script by the teachers in charge of the groups. This task focused on the detection of errors in the programming of the robot and dynamics of class teaching and their respective evaluation. The second was the teaching of class by a NAO robot and a Math teacher, following a script that was similarly structured by the high school teaching team. It was expected that the explanations and topics given by the robotic platform would follow the same thematic guide that was used in the high school session plan.

Together, the behaviour of the students was analysed, through the application of a behaviour observation protocol more appropriate for this scenario. The project was carried out in three control and two experimental groups, with the intention of observing more aspects in a population of around 140 students in total.

Figure 8 shows the results of the exams applied as pre-test and post-test at the end of the semester. In all cases, it is clear that the bar for the experimental groups is higher than the one for the control groups. This means that students scored higher with the use of the NAO. This is more evident in the case of the second teacher, where the grades vary in a range of three to four points between exams.

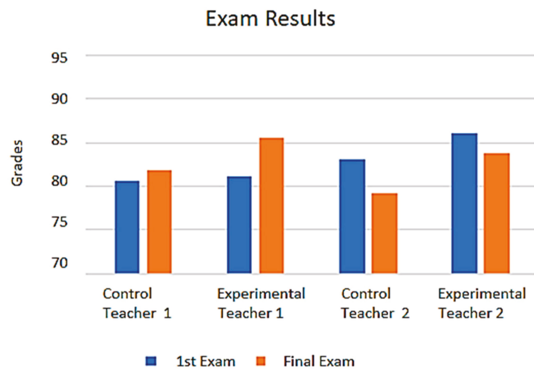

Figure 8. Ratings obtained by the groups.

Figure 9 shows the relative percentage of occurrence of the different behaviours that were organised by the dimensions of the observation scale during a class session (concentration, habituation, withdrawal, distraction, and motivation). The percentage is used since the groups are not equivalent in number of students and in the total frequency of the behaviours, then it is weighted on a percentage, so that all are comparable.

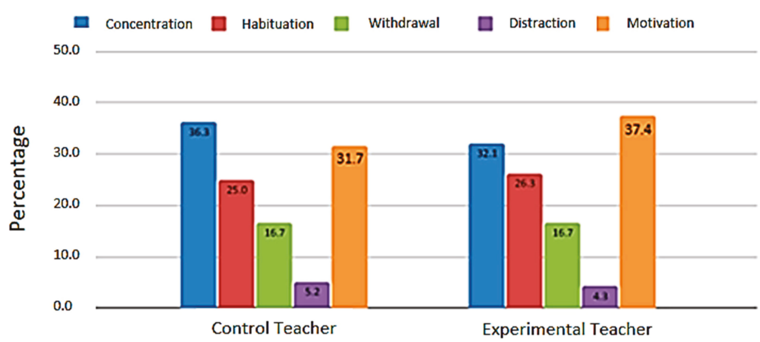

Figure 9. Behavioural observations by dimensions and group.

It is observed that the most concurrent dimensions are concentration and motivation among all groups. However, there is also a certain tendency for increase in these dimensions from the control group to the experimental group in the respective groups of each teacher.

3.2. Qualitative Analysis

Based on the observations made, it was clear that attention and motivation were boosted for the students at all levels. Nonetheless, the differences in the planned activities played an important role in the level of success achieved for each one of the four phases of active learning amongst scenarios. Now, we will discuss all three scenarios for each phase at a time.

Phase 1: Do

At this stage, it is expected that some stimulating activities will trigger motivation and interest by the students. It is expected that students themselves will decide on their learning strategy, and they will reflect on the results along the way, while they are still performing the task.

For the elementary level case, the activities posed and the presence of the robot did, in fact, stimulate the interest of the students. Nonetheless, they had almost no saying in the learning strategy. Their autonomy was limited to trying to solve the problems in the best way possible, once the instructions were given and understood. At the end of the class, they were to reflect a little on how they felt during the activity, and an example was analysed for application in the real world. Hence, a medium level of success was obtained in strategy decisions and in reflecting on one's own learning process.

The secondary level case was equally motivating in terms of the presence of the robot. For this scenario, the teachers solved problems, which then needed to be addressed by the students with the aid of the robot. During the process, partial results were analysed with the teacher, and corrections were made accordingly. This represented an increase in learning awareness by the students. However, it can still be considered to be a medium level of success.

Finally, when considering the high level case, challenges and exercises were presented for the students to solve, which were designed by the teachers, and they used the robot as an aid for their solution. Most exercises were trigonometric problems with examples. In this occasion, students were responsible for establishing the route to solve the problem, using the robot as well as the information that were provided in the class. The robot served as a tool for verifying results. Students received feedback during the task, not only about results, but about the theme's objective, thus restating the results. This process was quite satisfactory for the independence of the students in deciding their own learning strategy, and medium success was achieved for reflecting on their knowledge while doing the task.

Phase 2: Review.

In this phase, it is expected that students will take breaks during the task to become aware of what has just happened, what was important, and how they felt. Subsequently, they monitor their own progress and review their plan. Finally, they document their observations and learning outcomes.

For the elementary school scenario, students commented on the routine that was shown to them and explained, in their own words, what the session consisted of, and how well they believed they understood the lesson. In the second session, they only mentioned how the robotic platform had helped. On the first visit, the students discussed the experience, and commented about what was about to happen in the second visit, but had no say in deciding what that would be, or relating it to their learning process. No documentation was made by the students. The general success score was medium for stopping to evaluate progress, and for monitoring it. Furthermore, it is low for documenting their knowledge acquisition process.

For the second scenario, there were time stops to review and share solutions to the rest of the class, discuss their efficacy, and reflect on the problems and results. They discussed their progress and their experience with the robot. These results were within the desired range. However, documentation focused more on their experience with the robot than in assessing their own learning process.

For the last scenario, the high school students discussed questions that are triggered by the robot or the teacher. The class planning included review and adjustment times, and the students received new exercises that were applied to alternative contexts in order to improve their comprehension of the themes. Constant documentation was included in the process, making this stage considered to be successful.

Phase 3. Learn

For this phase, it is expected that the new ideas that are generated by the activity be made explicit. Students are responsible for identifying on their own their learning outcomes. Additionally, they identify barriers for their progress and propose new strategies.

In the elementary case, the objectives of the exercise were made explicit by students with the authorisation of their teacher. Additionally, there was a conversation space provided for them to verbalise what the experiment consisted of, but suggestions were only given as commentaries by the students. These elements drive to give a score of medium, in terms of the first two elements, but deficient in the proposition of new strategies.

The secondary school scenario, on the other hand, counted with a planned strategy to develop via the robot and the teacher together to make the learning outcomes explicit. Even though it was positive overall, it was not as effective as expected, since it resulted in being somehow confusing for the students. Some exercises were assigned to externalise what was being learned, and students were interviewed at the end. The comments gathered had no influence in the planning of the next sessions. This phase was considered to be positive in making learning explicit, but medium for identifying barriers and deciding strategies.

For the high school level, a space was given to analyse the learning process with the students, and together define strategies for the next sessions. The identification of barriers fell, in turn, a little short.

Phase 4. Apply

This stage is certainly the most complex, and it requires the greatest maturity of the students in the use of active learning. Therefore, it is the most difficult to achieve. First, future actions are planned based on new discoveries and learnings, and the possibility of transferring the knowledge gained to other situations is examined. Students are in charge of reviewing their plans, building on their recent learning experiences, and they move forward to plan future observations and experimentation of their learning strategies.

For the elementary school scenario, this was not covered, since only some documentation of the experience was made by the research team and considered for application in other similar contexts. Nonetheless, this activity did not include teachers or students, making it deficient.

The results improved marginally for the secondary level students, where the experience lived allowed for preparing and improving the process for future situations. However, no significant participation by the students was observed for experimenting learning strategies, beyond some reflections on the effectiveness of the robotic platform as an aid for their learning process.

Finally, the outcomes are far better for the high school scenario, where time was given to reflect on lessons learned to assess the applicability of the process to other contexts and define pertinent improvements. This was explicit also as an activity requested to the students where they had to describe how to apply the knowledge obtained to other domains. The weak point, considered to be a medium success, is that related to the relatively low impact of students' recommendations to redefine the course strategies in future scenarios.

Table 3 presents the summary of the success scores assigned in each case. The first score is the predominant one, and the one after the hyphen indicates there is a small component in a higher or lower category.

Table 3. Success scores.

Scenario	Do	Review	Learn	Apply
Elementary school	Medium-high	Medium-low	Medium-low	Low-medium
Secondary school	Medium-high	High-medium	Medium-high	Low-medium
High school	High-medium	High	Low-medium	High-medium

In summary, Table 4 shows the most relevant aspects regarding the tests carried out in the three scenarios, not only qualitative and quantitative, but also a comment that we highlight about each experience obtained.

Table 4. Summary of observations and test results.

Scenarios	**Comparison**		
	Quantitative results	Qualitative results	General comments
Elementary school	Even though the results obtained in this scenario cannot be considered conclusive because the study is exploratory and no statistical analyses are performed, it is reasonable to derive from the observations that students have an improvement in grades when the robotic platform is present in the session. This we can observe with the group average, of the test that was applied to all groups, since it was always higher in the groups with robotic interaction.	By observing the results obtained, it was determined that the favourable points for the robot were: T-Concentration (memory) -Less distraction -Interest in the task (enthusiasm) -Interest in the task (motivation)	The motivation was clearly increased; perhaps a not so adequate trait is that the control over the group was not so favourable. Teachers somehow also became an audience for the robotic platform.
Secondary school	It can be seen that there was an improvement between the pre-test and post-test results in both groups. This result was expected because, during the application of the tests, the students did not have enough knowledge to solve the exam. Nonetheless, there were many confounding factors, since groups varied in scheduled hours of instruction and assigned teachers, so results should be viewed with caution.	The results showed that the majority considered the presence of the robot made the class more interesting. Some students assured that the robot helped to understand the issues, and most denied feeling uncomfortable with the presence of the robot.	The teacher's comments stand out, as he had not used this type of robotic tool before. He was more interested in knowing how the robot works, and held a favourable position on the introduction of new technologies in education, so he stated that the group was more attentive and it was easier to control
High school	The results are only descriptive, that is, analysing the distributions of grades in the groups (minimum and maximum score, and the concentration of the majority of the students between certain scores), the results were better grades in the experimental groups, and in this case, the participation of the teachers was fundamental.	When comparing the general results of the traditional groups with the experimental one, the robotic platform is useful for the improvement of interactions between students. Furthermore, when comparing the test results and the opinions of the students, it is preferable to use the platform in fewer and more specific sessions	We observed that students are protagonists of their own learning and the technological platform allows a constructive dialogue between student and teacher, promoting reflection on the contents reviewed in each session scheduled for a visit of the robot. Similarly, in the design of each of the classes, the teachers included activities where the student is the axis of their own learning

4. Discussion

These results lead to think that there are many factors at play that need to be considered. The robotic solution will never result in significant learning improvement unless accompanied by the right strategy. However, it is a great tool for attracting interest and motivating students to participate, regardless of level. The active learning process, on the other hand, needs to be carefully planned according to level, since the cognitive capabilities and styles of the students vary greatly depending on it. Invariably, the robotic platform and the active learning strategy have a great potential to generate synergy and be more effective when well harmonised.

Another issue that is worth mentioning is the level of involvement of the teachers. The more prepared and comfortable to use the robot, the better they can plan and adapt their strategy, based on the feedback and outcomes that were provided by the students. This allows for the flexibility needed to customise learning strategies to each student and makes them responsible for their own learning. In the long run, they may develop the right level of maturity to be real active learners. It is also important to note that digital skills acquisition has greatly increased its relevance, not only for students, but for teachers. Today's digital agendas introduce two essential axes to work: on one hand, the trend towards a digital education, and on the other, the acquisition of digital competences and skills for digital transformation [49]. Studies, such as [50,51], highlight the shortage of digital competences in initial training and the lack of knowledge and skills on educational technologies for teaching practice in pre-school, primary, and secondary classrooms.

The time of exposure also seems to be an important issue. Those who were exposed longer to the robot, and to well planned activities, obtained more significant results. One or two sessions might even be counterproductive, as they raise expectations that will no longer be met, causing discouragement of the students.

Finally, capturing behavioural data and observations may be greatly enriched when combining techniques, even including the students themselves as self-documenters. Quantitative and qualitative techniques and assessments will provide a better panorama of the situation. The documents obtained always need be shared and discussed, to gain collective intelligence, and provide more robust changes.

5. Conclusions

In previous years, different scenarios have been developed, in which a humanoid robotic platform has been used in order to increase motivation and interest in students towards Mathematics. The presented case studies show that the proper use of a robotic platform, together with an appropriate teacher participation, can mean giving high-quality hybrid classes, enhancing the student's attention to the topics that are exposed by changing the stimulus, and obtaining effective learning. The results presented above show a numerical improvement in the scales that are used to assess the presence of specific behaviours, and performance in all scenarios. Even though these results should be looked at with caution, they provide a good perspective of the potential usefulness of Robotics in Mathematics teaching. The ultimate goal is to make learning more meaningful, which should translate into better grades and better abilities for students overall.

The study highlights the motivation of teachers to learn more about the use of robots. Beyond the motivation that can be considered on the part of the students, it was the teachers themselves who expressed an attitude of learning in order to integrate them into the educational experiences. The robot is a mediating tool, but the teacher is the one who has the capacity of inventiveness to integrate it in the classes. This integration helps to promote hybrid systems in learning environments, with the teacher-robot binomial (this was observed, to a greater extent, in the upper-middle level).

This study was guided by the question: what are the conditions that promote effective active Math learning with robotic support? The findings point to three key conditions: level, motivation, and teacher training: (a) Encouraging active learning with emerging technologies (in this case, with robotics), involves considering the educational level where the learning environments are targeted, with the profile of students (levels of construction of learning) as a central focus. (b) The motivation, according to the profile of the students of the formative experience, where robotics helps personalised learning. The introduction of robotics supports presenting the contents in a different way in order to change how they are taught in an ordinary situation. (c) The formation of teachers as a relevant aspect, where the pedagogical foundation goes beyond "programming the robot", the teachers are the ones who "should be programmed" for the didactic use that starts from the planning, the articulation of strategies, the strategic arrangement of the moments in which the use of the robots is integrated, and the evaluation of the effects.

Thus, robotics is one of many technologies that can support the processes for increasing mathematical learning, where strategies cannot be left aside, regardless of the technology that is being integrated. Critical thinking skills, digital skills, and teamwork skills are reinforced with the introduction of these technologies, linked to the stages of doing, reviewing, learning, and applying that are encouraged through active learning. Future studies could be geared towards deepening the exploration of the effects of the use of robotics on the stages of active learning, and their contribution to concrete mathematical knowledge acquisition. Similarly, it is worth analysing the teaching processes in order to demonstrate the results of theoretical and practical knowledge when applying emerging technologies, such as robots in their classrooms. It is required, in itself, to expand the studies of management, psycho-pedagogical, and socio-cultural issues, in the application of technologies in innovative learning environments.

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mathematics

Article

The Possibilities of Gamifying the Mathematical Curriculum in the Early Childhood Education Stage

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Abstract: The addition of gamification to the classroom as a methodological tool means that the teacher's opinion about this has become an inflection point that can affect its use or not in the classroom. In this sense, the main objective of the present article is to explore the opinion of future Early Childhood education teachers on the use of this resource for the development of the mathematics curriculum at this education stage and to obtain an explanatory model that explains it. The design of the study utilized a descriptive perspective and a cross-sectional quantitative focus through a quantitative exploratory study. For data collection, an ad hoc questionnaire was utilized, which was administered to a sample of 232 teachers-in-training. The main result obtained was that the future early childhood education professionals considered that gamification could be a resource for the learning of specific mathematics contents, and this was shaped around a model of two elements. On one hand, the development of mathematical thinking, and on the other, the establishment of relationships between mathematical concepts. Lastly, we can conclude that elements such as age or gender do not determine the perception of the use of gamification in the early childhood education classroom.

Keywords: gamification; videogame; early childhood education; mathematics

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1. Introduction

The diverse social, cultural and technological changes currently experienced by society, as indicated by [1], are shaping students and having an effect on their learning processes from different points of view. In this sense, games, in the broadest sense, have evolved as a methodological resource at the same time that society has moved forward. As of today, their use in a digital format has become very important, as the students in the classroom, from diverse educational stages, have been cataloged as technological youth, adolescents, and children. This is why the education community has begun to think of gamification as a valid strategy for the development of teaching-learning processes.

From the start, it must be indicated that current literature recognizes two ways of adding gamification to the classrooms, one supported by traditional games [such as Monopoly, Risk, cards, Parcheesi, the game of the goose, dungeon master, etc.], and the other supported by digital ones (Minecraft, Mario Bros, Fortnite, Pokemon Go, etc.) [2,3], with the objective of this article focusing on the latter trend.

The use of digital gamification or videogames, per se, to the learning process of the students, begins with the search for a transformation in education that promotes the growth of immersive learning of the students, and a demand for the improvement of the teachers' methodology. In addition, other researchers consider that the use of videogames will allow the students to be initiated in curricular competencies, which is the digital competence in this case [4–6], so that their learning process improves from another perspective.

At first, the digital gamification of the classrooms at any educational level implies accepting the use of digital games or videogames, a resource that has been demonized, hated, and loved by the education community and society at large [7–10]. It also implies that one must consider that the strategies utilized in gamification, such as the use of awards or badges, could be yet another element that motivates the students in their learning processes [3]. Thus, we must think beyond the thought that the final intent is to gamify the classroom. The use of gamification in the classroom, from a digital point of view, implies the use of videogames or strategies such as *Escape Room* or educational *Breakouts* to a level where the main objective is to introduce the curricular contents in a manner that is attractive to the student [1,3]. Thus, the contents that are considered to be difficult, either due to their nature, the manner of teaching them, or the manner of learning them, could be presented in a more motivational way, with their virtues shown to the students [11]. This will get the student's attention and promote internal strategies of assimilation and comprehension of this content.

In this sense, various successful experiments on the use of videogames in the classrooms exist, as a tool for propitiating effective learning, and at the same time, for favoring the cohesion and integration of the content to the social reality of the student. Authors such as [12,13] have pointed out that the gamified classroom promotes improvements in learning, metacognition, evaluation, and the process of conceptual support of the students. Along this line, [14] conducted a study with teachers from the area of special education and concluded that when they implemented gamification strategies into their classes, an improvement was achieved in the reasoning processes of the students. As a result, we could say that gamification, per se, tries to promote motivation for the content and the creativity of the individuals. On the other hand, it is interesting to note how the studies that have focused their attention on gender have underlined the importance of their use to foment the mathematical talents of girls [15].

Focusing our attention on the area of mathematics, we find the work by [4], who underlines how a game created more than two decades ago, The Lemmings, helps with the initiation of the basic contents of mathematics, just as the data presented by [11], who, after using various digital games, verified that significant learning of algebra content was achieved. On their part, [16] conducted a pretest-posttest study utilizing QR codes placed on cards, which allowed the students to perform different mathematics activities. The main result was that an improvement was observed in the acquisition of the rational number concept, improving the connections between its different representations such as fractions, decimals, and percentages. Along this line, [17], utilizing experimental and control groups, concluded that significant improvements had been achieved on the comprehension of basic concepts of mathematical logic of the students with whom digital games had been used as a digital resource, as compared to those who had followed a traditional methodology of learning. These results [18] show that the use of game-based interactive materials in the mathematics classroom promotes the improvement of the comprehension of mathematical concepts of the students. Therefore, the use of digital resources [19], defined as manipulative materials that allow us to visualize mathematical concepts more easily and in a more attractive manner, signifies a helping tool in the process of abstraction of mathematical concepts when coming into existence as virtual models of mathematical concepts [20].

Focusing our interest on the early childhood stage, it should first be indicated that this stage is characterized by being a point in time in which immersion into the curricular contents begins, which will be further developed in higher education stages. It is in this initial stage when the teachers begin to observe the first differences in the act of teaching and learning, meaning that different levels of learning and understanding of the contents taught to begin to appear [12,21,22]. As indicated by [23], learning is conducted due to curiosity, exploration, and immersion into the content; experimentation occurs, and initiation into research begins in a playful manner, as games are the main elements in learning processes [24]. For this reason, we can consider that gamification in the early childhood classroom will provide a new learning scenario where fiction comes closer to the educational reality of the student, thereby promoting a creative learning process, which is vital in the first years of socialization of individuals. As for the area of mathematics, we are in agreement with [24] in that the process of logic-mathematics acquisition is conducted through a reflective process that is never forgotten so that the use of different types of resources could promote this reflection in a more effective manner.

Diverse research studies [4,25,26] have pointed out that the use of gamification for teaching the curricular content in the area of mathematics in the early childhood stage promotes experiencing the content, which results in a positive view of the students towards this subject matter. We are in agreement with [27,28] that the use of digital games in the area of mathematics implies that the student learns, in a playful manner, concepts such as probability while they play, so that learning is produced in a manner that is more motivating and personal to the student, helping with the overcoming of obstacles during this learning process.

The objectives of the present research study are:

- To determine the perceptions of the teachers-in-training about the question of if the use of videogames allows for the development of the curriculum in the early childhood stage in the area of mathematics;
- To explore the existence of different dimensions about the use of videogames, to make progress feasible in the curriculum of the early childhood stage, in the area of mathematics;
- To learn about the behavior of these factors, considering their relationships and the existence of an explanatory model for them.

The following hypothesis has been posited with the objectives described above:

Starting hypothesis: There are significant differences in mathematical thinking, depending on the establishment of relationships between concepts, without it being influenced by the age and gender of the study subjects.

2. Materials and Methods

The research study utilized a descriptive design and a cross-sectional quantitative focus, given that a survey was utilized as the data collection instrument.

2.1. Sample

The sample was selected by utilizing a nonprobabilistic, convenience sampling method [29], given that sample was accessed through the classroom where the virtual teaching was done.

The sample was composed of 232 students enrolled in the early childhood education degree at the University of Cordoba (Spain). If the sampling error calculation for finite populations is lower than 3%, as in our case, it is understood that the sample is significant, considering what is mentioned in [30]. In the sample, we found that 88.8% were women, 10.8% men, and 0.4% identified themselves as transgender. From the start, it can be indicated that there was a bias towards the female gender. Nevertheless, it should be pointed out that as indicated by [31,32], this university degree tends to have a higher female component than other university studies, just as with engineering degrees, where the presence of men tends to be higher [32]. As for the academic year, 50.4% were in their first year, and 49.6% in their second year. The mean age of the participants was 19.69 years old (SD = 2.408), which was distributed, as shown in Table 1.

Focusing our attention on the devices they possessed, and which could be used to play, we found that 24.1% had a laptop and a smartphone, while only 0.4% had a smartphone + videogame console or a laptop + smartphone, or a desktop + tablet + smartphone. It is interesting to note that 11.2% had access to all the devices shown (laptop, desktop, smartphone, videogame console, and tablet) (see Table 2).

Table 1. Distribution of the sample according to age.

		Gender				
		Age [1]	Men	Women	Transgender	N.
Academic year	First year	17	0	14	0	14
		18	7	38	1	46
		19	5	16	0	21
		20	3	15	0	18
		21	1	6	0	7
		22	2	4	0	6
		23	0	2	0	2
		24	0	2	0	2
		38	0	1	0	1
		Total	18	98	1	117
	Second year	18	0	16	0	16
		19	1	41	0	42
		20	1	15	0	16
		21	2	14	0	16
		22	1	11	0	12
		23	0	4	0	4
		24	1	3	0	4
		25	1	0	0	1
		26	0	1	0	1
		27	0	2	0	2
		34	0	1	0	1
		Total	7	108	0	115

[1] N total = 232.

Table 2. Digital devices available for playing.

Digital Devices Available for Playing	f. [1]	%
Laptop computer	18	7.8
Smartphone	10	4.3
Laptop computer + tablet + smartphone	34	14.7
Laptop computer +tablet + smartphone + videogame console	27	11.6
All the devices	26	11.2
Laptop computer + smartphone + videogame console	29	12.5
Laptop computer + smartphone	56	24.1
Laptop computer + desktop computer + smartphone + videogame console	4	1.7
Laptop computer + desktop computer +tablet + smartphone	9	3.9
Laptop computer +tablet+ smartphone	2	0.9
Laptop computer + desktop computer + smartphone	6	2.6
tablet + smartphone	4	1.7
Smartphone + videogame console	1	0.4
Laptop computer + smartphone	1	0.4
Laptop computer + tablet	4	1.7
Desktop computer + tablet + smartphone	1	0.4

[1] N = 232; f. = frequency.

As for their characterization as videogame players, in our sample, 44% occasionally played, 24.1% sometimes, and 4.3 and 1.3% often or very much, respectively. It is notable that 26.3% indicated that they had never played. As a function of these results, and when asked about the number of hours they played, we found that 88.4% and 90.5% played between 1 and 3 h throughout the week and throughout the weekend, respectively. It is interesting that 0.9% played less than 10 h during the week, and 1.3% during the weekend (see Figure 1).

Lastly, when asked about the type of videogames they tended to use, 72.8% utilized the one from the videogame platforms, 23.3% fighting games, and 3.9% strategy games.

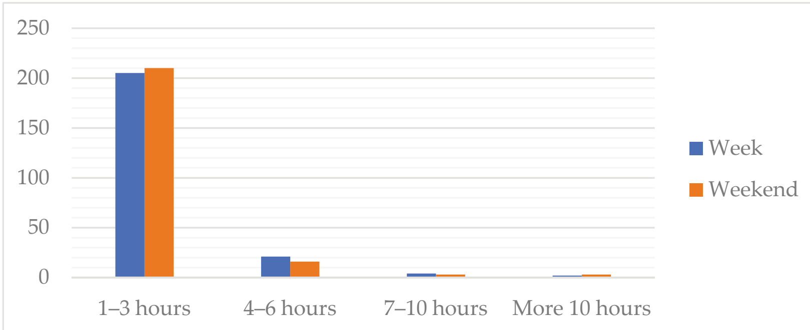

Figure 1. Distribution of the hours of play during the week and the weekend.

2.2. Instrument

The instrument utilized for the collection of data was the survey created ad hoc under the parameters found in [33], which sets the minimum education levels in Early Childhood Education in Spain, summarized in [34], which regulates the curriculum of Early Childhood Education in Andalusia. This instrument was composed of 2 dimensions comprised of 25 items, written in an affirmative, closed and polythematic character. This was an anonymous questionnaire, which was administered online without the in situ assistance of the researcher. A Likert-type response scale was utilized, composed of five response options (where 1 indicated complete disagreement and 5 complete agreement). This type of response scale with this number of options will allow for coming close to the assumption of continuity. In addition, the instrument contained a set of independent variables which allowed us to describe the participating sample: gender, age, academic year, digital devices available for playing (laptop computer, desktop computer, Tablet, Smartphone, and Videogame console), hours spent playing during the week and the weekend, and type of games they like to play.

To determine the validity of the instrument created, an exploratory factor analysis (EFA) was performed through the use of polychoric matrices, along with an "optimal implementation of parallel analysis" [35] and "non-weighted least squares" with a "weighted Oblimin rotation" [36] to determine the number of factors, utilizing the statistical package SPSS 23 and the Factory Analysis (10.10.03) software. This analysis allowed for the verification of the viability of the construct through the correlation matrix 0.000; Bartlett's sphericity test, with a significance of 0.000; and KMO = 0.931, as well as the root mean square of residuals (RMSR) = 0.038, with extracted factors that explained 63.27% of the variance, and whose rotated factors had loads higher than 0.3, providing us with the following two-factor structure (see Table 3).

Therefore, through the EFA, a structure of the instrument was obtained, shaped by two factors, which are:

- Development of mathematical thinking: this dimension encompasses a set of items that refer to the videogame as a facilitator for the development of mathematical thinking of the students through the acquisition of notions such as quantity, order, magnitudes or measurements, among others; as well as the representation and interpretation of reality through mathematical elements;
- Establishment of relationships between concepts: this dimension spans a series of items that allude to the ability of videogames to facilitate the creation of relationships between the qualitative and quantitative mathematical concepts, as well as to collect and represent data and information.

In addition, a confirmatory factorial analysis (CFA) was performed, which allowed us to compare the fit indices of the model obtained in the EFA, taking into account the following tests: $\chi 2$ test/degrees of freedom, comparative fit index (CFI), incremental fit index (IFI), normed fit index (NFI), the Tucker-Lewis index (TLI), the root mean square error of approximation (RMSEA), and the expected cross-validation index (ECVI). In the first analysis, the results found indicated that items 2, 3, 6, and 14 from the first dimension should be eliminated, as well as items 18 and 25 from the second dimension, as the modification indices indicated the existence of covariances between the errors associated with the items that belonged to different factors. Once the model was reformulated, with a total of 19 items, the following results were obtained: $x^2 = 171.97$; df = 131; $p = 0.009$; $X^2/df = 1.31$; CFI = 0.975; IFI = 0.976; NFI = 0.905; TLI = 0.967; RMSEA = 0.053; and ECVI = 2.63. Taking into account the values found, adequate results were observed in that model, as a value lower than 0.060 was found for RMSEA, with values higher than 0.90 in CFI, IFI, NFI and NNFI [37,38].

Table 3. Matrix of rotated factors.

Variables	Factor 1	Factor 2
V 1	0.632	
V 2	0.615	
V 3	0.631	
V 4	0.729	
V 5	0.630	
V 6	0.419	
V 7	0.384	
V 8	0.709	
V 9	0.772	
V 10	0.761	
V 11	0.600	
V 12	0.635	
V 13	0.396	
V 14	0.303	
V 15		0.568
V 16		0.326
V 17		0.569
V 18		0.543
V 19		0.443
V 20		0.390
V 21		0.387
V 22		0.545
V 23		0.893
V 24		0.828
V 25		0.499

On the other hand, and to verify if the reliability of the instrument was appropriate, Cronbach's alpha was calculated to study the internal consistency, with a value of $\alpha = 0.962$ obtained for the instrument as a whole and values of $\alpha = 0.962$ for factor 1, and $\alpha = 0.932$ for factor 2, indicating high-reliability values [39].

Lastly, alluding to the data collection procedure, it should be noted that 15 min were provided for completing the questionnaire, with the study researchers present through a videoconference call to resolve any possible doubts that could arise during this process.

2.3. Analysis Performed

Once the collection of data were complete, the following statistical analyses were performed to provide an answer to the objectives set in this study:

- A descriptive study of the two dimensions addressed in the questionnaire through the measurements of central tendency (mean) and dispersion (standard deviation).
- A correlational study between the two dimensions of the instrument through bivariate correlations.
- ANOVA, ANCOVA and a regression study to establish the explanatory model of dimension 1 as a function of the values from dimension 2 through linear regressions.

3. Results

3.1. Descriptive Study

In the first place, the results (mean, standard deviation, kurtosis, and asymmetry) of the descriptive study are shown, which was conducted with the final version of the instrument, composed of 19 items (see Appendix A, Table A1).

As observed in Appendix A (Table A1), only the items that referred to reader comprehension (items 2 and 3) were evaluated low by the teachers-in-training, with the rest accepted indifferently, in disagreement with the findings from [12], highlighting those that referred to the help provided by this resource to the early childhood students, for the establishment of relationships between different matters and sizes (items 16 and 17).

As for the dimensions established by the CFA, it was verified that the descriptive values showed high values for the mean, as shown in Table 4, with the results being:

Table 4. Descriptive study of the dimensions.

Factors	M.	SD	Asymmetry		Kurtosis	
Development of mathematical thinking (factor 1)	3.87	0.741	−0.747	0.160	1.147	0.318
Establishment of relationships between concepts (factor 2)	3.95	0.719	−0.769	0.160	1.494	0.318

3.2. Inferential Study

After a Student's *t*-test was performed according to gender, and with the objective of verifying if hypothesis 1 is accepted or not, the results showed that no significant differences were found. Therefore, it can be inferred that the gender variable did not determine the use of the videogames as a curricular tool in the early childhood education stage.

With respect to the starting hypothesis, an analysis of variance (ANOVA) and an analysis of covariance (ANCOVA) were performed, which did not show differences when using either age or gender as the discriminatory variable. Thus, the hypothesis set forth must be rejected, and it must therefore be pointed out that age was not a determining variable between mathematical thinking and the use of video games as a resource in the early childhood classroom.

3.3. Correlational Study

The correlational study performed referred to Pearson's correlation test for bivariate relations between the factors. As observed, the relation between the dimensions is positive. As one increases, so do the other by a high [39] and a significant amount (r = 0.710 and $p < 0.001$).

3.4. Regression Analysis

In the search for an explanatory model that possesses the best parsimony possible, the multivariate analysis of linear regressions shows that factor 1 (development of mathematical thinking) can be explained by factor 2 (establishment of relationships between concepts) and gender and age. This analysis obtained the following parameters: $F(1, 230) = 337.63$ and $p < 0.001$; with a corrected coefficient of determination $R^2 = 0.593$, and a Durbin–Watson value = 1.9, which indicates the interdependence of the residues [40]. The only variable that intervenes in the Factor 2 equation has the following statistical values: t = 18.37 and $p < 0.001$.

Analyzing what is shown in Table 5, the equation explains the development of mathematical thinking, factor 1 = 0.59 + 0.81 factor 2, with a root mean square error of 0.22. Likewise, to generalize the explanatory model, the residues were studied, observing the non-multicollinearity with the VIF value = 1.000, and the independence of the residues. On the other hand, their linearity and the homoscedasticity of the residues, observed in figures, comply with these assumptions [41] just as their values of normality through the Kolmogorov–Smirnov test (Z = 0.078 and $p = 0.200$).

Table 5. Linear regression of the development of mathematical thinking [a] in early childhood education with the use of video games.

Variables	B	E.S.	Beta	t	Sig.	Zero Order	Partial R	Semi-Partial R	Tolerance	VIF
Constant [b]	0.587	0.180		3.257	0.001 *					
Factor 2 [b]	0.813	0.044	0.771	18.375	0.000 *	0.771	0.771	0.771	1.000	1.000

Note. [a]. Dependent variable: development of mathematical thinking. [b]. Predictors: (constant) factor 2 (establishment of relationships between concepts). * Level of significance, $p = 0.05$.

Given the complexity of the relation between the dependent variable (F1) and the independent variable (F2), an analysis of covariance (ANCOVA) was performed. Initially, an ANOVA was performed through a univariate linear model with a full factorial design between the main effect F2 and the interaction between F1 and F2, whose results are shown in Table 6.

Table 6. Tests of the inter-subject effects between F1 and F2.

Origin	Sum of Squares Type II	Gl	Quadratic Mean	F	Sig. *	Partial Eta-Squared
Corrected model	77.419 [a]	4	19.355	87.346	0.000	0.606
Intersection	3437.280	1	3437.280	15,512.005	0.000	0.986
F2	77.419	4	19.355	87.346	0.000	0.606
Error	50.301	227	0.222			
Total	3565.000	232				
Total corrected	127.720	231				

Note. [a]. R-squared 0.606 (R-squared corrected = 0.599). Dependent variable F1 *. Level of significance $p = 0.05$.

Utilizing the development of mathematical thinking (F1) as the dependent variable and the establishment of relationships between concepts (F2) as the independent one, it can be observed that the main effect $F(4227) = 87.346$ and $p < 0.005$, as well as the interaction, are significant.

Afterward, the univariate linear model was repeated, with the ANCOVA performed with the covariables gender and age, utilizing the development of mathematical thinking (F1) as the dependent variable and the establishment of relationships between concepts (F2) as the independent one. The results obtained are shown in Table 7.

Table 7. Tests of the inter-subject effects between F1 and F2 with covariables gender and age.

Origin	Sum of Squares Type II	Gl	Quadratic Mean	F	Sig.	Partial Eta-Squared
Corrected model	78.871 [a]	13	6.067	27.075	0.000	0.618
Intersection	0.432	1	0.432	1.929	0.166	0.009
F2	0.124	1	0.124	0.555	0.457	0.003
Gender	0.025	1	0.025	0.111	0.739	0.001
Age	0.104	1	0.104	0.463	0.497	0.002
F2 * Gender	0.154	1	0.154	0.688	0.408	0.003
F2 * Age	0.132	1	0.132	0.588	0.444	0.003
F2 * Gender * Age	0.186	2	0.093	0.416	0.660	0.004
Error	48.849	218	0.224			
Total	3565.000	232				
Total corrected	127.720	231				

Note. [a]. R-squared 0.618 (R-squared corrected = 0.595). Dependent variable F1 *. Level of significance $p = 0.05$.

In light of the data comparisons shown in Tables 6 and 7, it can be concluded that the effect between the dependent variable F1 and the independent variable F2 is altered by the covariables, given that significance does not exist in the model. Thus, the results indicate that a study must be made with the covariables separately to verify if any of the covariables has an influence on the dependent variable, with the results shown in Tables 8 and 9.

Table 8. Tests of the inter-subject effects between F1 and F2 with covariable gender.

Origin	Sum of Squares Type II	Gl	Quadratic Mean	F	Sig.	Partial Eta-Squared
Corrected model	77.906 [a]	7	11.129	50.046	0.000	0.610
Intersection	111.282	1	111.282	500.406	0.000	0.691
F2	0.374	2	0.187	0.840	0.433	0.007
Gender	0.343	1	0.343	1.540	0.216	0.007
F2 * Gender	0.144	2	0.072	0.324	0.724	0.003
Error	49.814	224	0.222			
Total	3565.000	232				
Total corrected	127.720	231				

Note. [a]. R-squared 0.610 (R-squared corrected = 0.598). Dependent variable F1 *. Level of significance $p = 0.05$.

Table 9. Tests of the inter-subject effects between F1 and F2 with covariable age.

Origin	Sum of Squares Type II	Gl	Quadratic Mean	F	Sig.	Partial Eta-Squared
Corrected model	78.211 [a]	8	9.776	44.035	0.000	0.612
Intersection	45.692	1	45.692	205.806	0.000	0.480
F2	0.771	3	0.257	1.157	0.327	0.015
Age	0.377	1	0.377	1.700	0.194	0.008
F2 * Age	0.414	3	0.138	0.622	0.602	0.008
Error	49.509	223	0.222			
Total	3565.000	232				
Total corrected	127.720	231				

Note. [a]. R-squared 0.612 (R-squared corrected = 0.598). Dependent variable F1 *. Level of significance $p = 0.05$.

Tables 8 and 9 show that none of the covariables has an influence on the dependent variable of the univariate linear model. In summary, the results show that factor 1 (development of mathematical thinking) can be explained by factor 2 (establishment of relationships between concepts), without gender or age having an influence.

On the other hand, the participating sample was divided as a function of the student's gender, for a more itemized study of the development of mathematical thinking in the use of the video games, obtaining a variation in this variable as a percentage which is explained with factor 2.

In the case of the men, the general model only explained 17.5% ($R^2 = 0.175$), with the parameters being $F(1,23) = 6.092$ and $p = 0.021$; factor 2, which is the only variable that intervenes in the equation, obtained the following statistics: $t = 2.46$ and $p = 0.021$. After analyzing Table 10, the equation that explains the development of mathematical thinking for the men is factor 1 = 1.63 + 0.60 factor 2, with a root mean square error of 0.27.

Table 10. Linear regression of the development of mathematical thinking [a,c] in early childhood education for men.

Variables	B	E.S.	Beta	t	Sig. *	Zero Order	Partial R	Semi-Partial R	Tolerance	VIF
Constant [b]	1.629	0.998		1.632	0.116					
Factor 2 [b]	0.595	0.241	0.458	2.468	0.021	0.458	0.458	0.458	1.000	1.000

Note. [a]. Dependent variable: development of mathematical thinking. [b]. Predictors: (constant) factor 2 (establishment of relationships between concepts). [c]. Selection of cases for which the gender = men. * Level of significance, $p = 0.05$.

Likewise, to try to generalize this explanatory model as a function of the men, the residues were studied, observing the non-multicollinearity with a VIF value = 1.000 and the independence of the residues in the Durbin–Watson values = 1.953. In addition, their linearity and the homoscedasticity of the residues observed in the graphics complied with these assumptions [41], just as their values of normality through the Kolmogorov–Smirnov test (Z = 0.076 and $p = 0.200$).

In the meantime, for women, factor 2 explains 62% ($R^2 = 0.620$) of the general model of the development of mathematical thinking, where the parameters are F(1204) = 335.923 and $p < 0.001$; factor 2, which is the only variable that intervenes in the equation, obtained the following statistics: t = 18.328 and $p < 0.001$.

Table 11 shows the values that make up the equation that explains the development of mathematical thinking for women, which is factor1 = 0.55 + 0.82 factor 2.

Table 11. Linear regression of the development of mathematical thinking [a,c] in early childhood education with the use of video games for women.

Variables	B	E.S.	Beta	t	Sig. *	Zero Order	Partial R	Semi-Partial R	Tolerance	VIF
Constant [b]	0.548	0.182		3.017	0.003					
Factor 2 [b]	0.818	0.045	0.789	18.328	0.000	0.789	0.789	0.789	1.000	1.000

Note. [a]. Dependent variable: development of mathematical thinking. [b]. Predictors: (constant) factor 2 (establishment of relationships between concepts). [c]. Selection of cases for which the gender = women. *. Level of significance, $p = 0.05$.

Likewise, to try to generalize this explanatory model as a function of the women, the residues were studied, observing the non-multicollinearity with a VIF value = 1.000 and the independence of the residues in the Durbin–Watson values = 2.066. In addition, their linearity, and the homoscedasticity of the residues observed in the graphics complied with these assumptions [41], as do their values of normality through the Kolmogorov–Smirnov test (Z = 0.078 and $p = 0.200$).

4. Discussion

The initiation of learning of mathematical contents in the early ages is an important matter for the education community, given that the establishment of prior knowledge in this area will allow the teacher to detect and determine future hurdles in the acquisition of concepts that will become more complex as the students make progress in the curricular content [4,11].

On the other hand, the addition of the methodologies defined as active, based on the inclusion of diverse digital resources, will provide the teachers with a set of tools that will allow them to bring the social reality experienced by the students closer to the educational reality where they are immersed in during a considerable period of time in their lives [42,43]. However, the inclusion of a digital resource in the classroom methodology will be determined by the perception of the teachers [44]. Thus, knowing their opinion is of the utmost importance. In this sense, to discover their predisposition towards the addition of videogames to the curricular development of the subject of mathematics in the childhood education stage, the profile of the education professional must be understood. Our results showed that the participants in this study had a low level as videogame players, given that they spent a small amount of time playing videogames, as opposed to works by [45,46], where the teachers-in-training spent a greater amount of time playing videogames and digital games.

Focusing our attention on the first objective of this work (Determine the perceptions of university students about the question of if the use of videogames makes possible the development of the curriculum in the early childhood stage in the area of mathematics), it can be verified that the teachers-in-training considered that the videogames would help students aged from 3 to 6 years old to understand and represent some logical and mathematical notions and relationships [47], which can be linked to their everyday lives [48].

On the other hand, they also considered that it would allow students aged from 3 to 6 years old to develop mathematical skill and knowledge [27] and reading comprehension and language, as opposed to the findings by [42], but in line with those found by [49]. In addition, in agreement with [50], the early childhood teachers-in-training believed that videogames would help children to acquire thinking schemes that will bring them closer to the basic notions of order, quantity, number series and functions [51], as well as problem-solving [52].

As for the second objective (To explore the existence of different dimensions about the use of videogames, to make progress feasible in the curriculum of the early childhood stage in the area of mathematics), the existence of two dimensions was corroborated, which brought together, on one hand, the items related to the skills linked with the comprehension of concepts where other curricular elements intervened, such as language, writing, and what was named "development of mathematical thinking" [52]. Moreover, a second dimension is linked to the "establishment of relationships between concepts" [27]. As for the grouping of the items into two dimensions, we verified that a line was followed as set by [6], given that this author also discusses experimentation and discovery on one hand and the relationships between concepts on the other.

With respect to the third objective of the work (To learn about the behavior of these factors, considering their relationships and the existence of an explanatory model for them), it was verified that the two-factor general explanatory model obtained pointed out that the women were closer to it than the men.

Lastly, focusing our attention to the starting hypothesis (There are significant differences in mathematical thinking depending on the establishment of relationships between concepts, without it being influenced by the age and gender of the study subjects), it is accepted and is also in line with the results obtained by [43].

In conclusion, gamification can increase both the cognitive load and the levels of performance, and generally, the students have positive beliefs with respect to the gamification strategies. This is a methodological strategy that allows creating work habits, fomenting participation and autonomy in problem-solving, promoting continuous learning, developing self-confidence and the ability to self-evaluate, promoting mathematical abilities and skills, and it could even motivate the Student's to perform activities that seemed boring to them before [11,12,25].

5. Limits

The field of Social Sciences has limitations in the development of research studies, which is related to the size of the sample. In relation to this, in the present study, a specific sample of teachers-in-training was utilized, which initially allowed us to validate the measurement instrument created, and whose results, although not able to be generalized, can serve as the basis of future research studies which utilize random samples of the population as the starting point. On the other hand, the data are unbalanced and have no element of randomization, so the results of the inferential study are very tenuous.

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Appendix A

Table A1. General descriptive study.

	M.	SD	Asymmetry		Kurtosis	
Videogames will help with understanding and representing some logical and mathematical notions and relationships that refer to situations of everyday life, coming closer to problem-solving strategies.	3.57	0.937	−0.791	0.160	0.179	0.318
Videogames will help with the use of spoken language in manners that are more adequate to the different communication situations to understand and be understood by others.	2.97	1.077	−0.116	0.160	−0.900	0.318
Videogames will help with coming closer to reading and writing in situations of everyday life	2.87	1.121	0.053	0.160	−1.018	0.318
Videogames will help with the development of mathematical and logical skills and knowledge	3.71	0.868	−0.995	0.160	1.278	0.318
Videogames will help with everyday situations, compare, put into groups, order, select, especially elements in their surroundings.	3.72	0.890	−0.833	0.160	0.615	0.318
Videogames will help with the acquisition of specific thinking schemes that bring closer the basic mathematical notions of comparisons between object collections	3.72	0.859	−0.884	0.160	0.911	0.318
Videogames will help with the acquisition of specific thinking schemes that bring closer the basic mathematical notions of order	3.69	0.852	−0.840	0.160	0.834	0.318
Videogames will help with the acquisition of specific thinking schemes that bring closer the basic mathematical notions of quantity	3.69	0.878	−1.044	0.160	1.160	0.318
Videogames will help with the acquisition of specific thinking schemes that bring closer the basic mathematical notions of number series	3.73	0.815	−1.069	0.160	1.285	0.318
Videogames will help with the acquisition of specific thinking schemes that bring closer the basic mathematical notions of functions	3.51	0.892	−0.524	0.160	−0.039	0.318
Videogames will help with the acquisition of specific thinking schemes that bring closer the basic mathematical notions of measurement	3.65	0.884	−0.807	0.160	0.466	0.318
Videogames will help with learning how to use conventional or unconventional mathematical codes as tools for expressing and understanding qualitative and quantitative relationships that can be established between objects and elements	3.48	0.944	−0.572	0.160	0.005	0.318
Videogames will help with using conventional or unconventional mathematical codes as tools for expressing and understanding qualitative and quantitative relationships that can be established between objects and elements	3.48	0.912	−0.589	0.160	0.089	0.318
Videogames will help with establishing the relationships between time and the surrounding with quantifiable objects	3.56	0.905	−0.761	0.160	0.232	0.318
Videogames will help with establishing the relationships between the different types of magnitudes	3.51	0.892	−0.574	0.160	0.142	0.318
Videogames will help with establishing the relationships between the different types of matters	3.50	0.926	−0.648	0.160	0.023	0.318
Videogames will help with establishing the relationships between the different types of sizes	3.90	0.752	−1.127	0.160	2.256	0.318
Videogames will help with learning how to collect data and information	3.95	0.818	−0.870	0.160	0.897	0.318
Videogames will help with learning how to represent data and information	3.68	0.894	−0.598	0.160	−0.039	0.318

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Improving the Teaching of Hypothesis Testing Using a Divide-and-Conquer Strategy and Content Exposure Control in a Gamified Environment

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Abstract: Hypothesis testing has been pointed out as one of the statistical topics in which students present more misconceptions. In this article, an approach based on the divide-and-conquer methodology is proposed to facilitate its learning. The proposed strategy is designed to sequentially explain and evaluate the different concepts involved in hypothesis testing, ensuring that a new concept is not presented until the previous one has been fully assimilated. The proposed approach, which contains several gamification elements (i.e., points or a leader-board), is implemented into an application via a modern game engine. The usefulness of the proposed approach was assessed in an experiment in which 89 first-year students enrolled in the Statistics course within the Industrial Engineering degree participated. Based on the results of a test aimed at evaluating the acquired knowledge, it was observed that students who used the developed application based on the proposed approach obtained statistically significant higher scores than those that attended a traditional class (p-value < 0.001), regardless of whether they used the learning tool before or after the traditional class. In addition, the responses provided by the students who participated in the study to a test of satisfaction showed their high satisfaction with the application and their interest in the promotion of these tools. However, despite the good results, they also considered that these learning tools should be considered as a complement to the master class rather than a replacement.

Keywords: education; learning environments; educational games; engineering students

1. Introduction

Statistics is a core subject in almost all university degrees. As an example, Garfield and Ahlgren pointed out that the University of Minnesota offered 160 statistics courses, taught by 13 different departments [1]. Learning statistics is important as it provides the capacity of conducting logical reasoning and critical thinking, enhancing interpretation and evaluation skills and facilitating dealing with highly abstract concepts [2].

However, many students find learning statistics difficult and unpleasant as it can be appreciated in several studies that reported high levels of statistical anxiety [3–6] or "statisticophobia" [7–9]. Dykeman observed that, compared to students enrolled in general education courses, students in statistics courses had lower levels of self-efficacy and higher levels of anxiety [10]. Ferrandino indicates that the teaching of statistics can be difficult for many reasons that have persisted over time, across disciplines, and across borders [11]. Ben-Zvi and Garfield pointed out different causes as, for example, that some statistical ideas are complex and/or counter-intuitive, that students lack of the required mathematical knowledge, or that they struggle in dealing with the context of the problem [12]. These difficulties can also be noticed in the large number of research articles devoted to explain students' misconceptions related to several statistical concepts [13–16]. These misconceptions appear in almost all the statistical topics, including descriptive statistics [17,18], probability [1,19], or statistical inference [14]. Therefore, developing tools that facilitate the acquisition of statistical concepts is essential. This need increases with the proliferation of online courses on statistics [20–22]. As an example, Reagan points out that several colleges are offering online statistics courses to enable students to complete their academic curriculum [23]. Mills and Raju also highlight the need to learn about how to effectively implement statistical courses [24].

To facilitate the understanding of the different statistical concepts and to correct the misconceptions, several methodologies have been proposed. Two of the most popular approaches are active learning [25] and the flipped classroom [26]. Active learning was initially defined as "anything that involves students in doing things and thinking about the things they are doing" [27]. A more recent and meaningful definition is the one proposed by Felder and Brent [28]. They define active learning as "anything course-related that all students in a class session are called upon to do other than simply watching, listening and taking notes". The use of active learning techniques in statistical courses has raised students' confidence in understanding statistics [29], increased knowledge retention [30], and improved students' performance [31].

In the flipped classroom methodology, knowledge acquisition takes place out of the classroom so that the class time can be devoted to more interactive activities. These can be designed, for instance, to polish the knowledge or to increase students' motivation. The flipped classroom has shown several benefits when it has been used to teach statistics. Wilson's [26] implementation of a flipped classroom in an undergraduate statistics course had a positive impact in students' attitude and performance. Winquist and Carlson found that students who took an introductory statistics course using the flipped classroom approach outperformed the ones that took the same course in a traditional lecture-based approach [32]. They observed that the implementation of a flipped classroom in an undergraduate statistics course had a positive impact in students' attitude and performance. McGee, Stokes, and Nadolsky adapted the flipped classroom methodology to identify students' misconceptions so they can correct them during the master class [33]. The results in Reference [34], when studying mathematics achievement and cognitive engagement, indicate that students in a flipped class significantly outperformed those in a traditional class and those enrolled in the online independent study class. Moreover, flipped learning with gamification promoted students' cognitive engagement better than the other two approaches. Faghihi et al. conclude that students who used the gamified system scored above the median, and their performance was greater than with the alternative method [35], whereas the results reported by Jagušt, Boticki, and Sin suggest that gamification improved student math performance and engagement [36]. Sailer et al. affirm that gamification is not effective per se, but that different game design elements can trigger different motivational outcomes [37]. In particular, their results show that badges, leaderboards, and performance graphs positively affect competence need satisfaction, as well as perceived task meaningfulness.

This article focuses on hypothesis testing which, according to Brewer [38], is probably the most misunderstood and confused of all statistical topics. This assertion can also be appreciated in the review conducted by Castro-Sotos et al. about the different students' misconceptions of statistical inference [14]. Difficulties in distinguishing sample and population [39], confusion between the null and the alternative hypothesis [40], misunderstandings about the significance level [41], and misinterpretation of the p-value [42] are only some of the different misconceptions related to hypothesis testing.

Apart from the above-described general purpose methodologies that have been used to facilitate the learning of any statistical topic, there are some research works devoted to teach hypothesis testing. One of the first works is the one conducted by Loosen, who built a physical device to explain hypothesis testing [43]. He found that his students were enthusiastic about the apparatus, but he did not report any quantitative measure. Schneiter [44] built two applets for teaching hypothesis testing. However, just like Loosen, she did not report about their performance. A more recent work is the one conducted by Wang, Vaughn, and Liu [45]. These researchers showed that the use of interactive animations improves the understanding of hypothesis testing but not students' confidence. Other works have proposed the use of specific examples, such as a two-headed coin [46] or a fake deck that only contains red cards (instead of half deck being red cards and the other half being black cards), to demonstrate hypothesis testing [47].

In this article, a new approach is proposed to facilitate the understanding of hypothesis testing, based mainly on the divide-and-conquer strategy, on the content exposure control, and on the use of a gaming environment that increases student motivation.

2. Material and Methods

The proposed approach is supported by six pillars. These aim at providing a solution to the factors that we assume are responsible for the difficulties in learning hypothesis testing: individual learning differences, misconceptions, knowledge gaps, and lack of attitude and motivation. The six pillars are:

- *Divide and conquer.* Our approach requires an analysis of the subject to be taught, in our case hypothesis testing, to extract which are the main concepts and which are the dependencies between them. In this way, it is possible to plan the acquisition of knowledge in a progressive way, so that students can build the necessary scaffolds to master the subject [48].
- *Flipped classroom methodology.* Not all students assimilate concepts at the same speed. This causes that, in the master class, many students tend to tune out. The fact that, in the proposed approach, the statistical concepts are explained with short videos [49] allows students to watch them as many times as necessary, following their own pace, which facilitates self-directed learning [50]. These videos cover the key concepts of hypothesis testing, as suggested by Pfannkuch and Wild [51].
- *Content exposure control.* In almost all learning platforms, the student can freely move from one concept (video/chapter) to another. In hypothesis testing, this approach might cause major complications because the different concepts are presented sequentially and each is based on the previous ones. If the student has an erroneous concept, its negative effects will extend to the following concepts. For this reason, in the proposed approach, each new concept will not be presented until the student demonstrates sufficient knowledge of the current concept.
- *Formative assessment and feedback.* Our approach evaluates the acquired knowledge as the student learns and provides useful formative feedback to assist the learning process. Whenever students make a mistake, they are informed and appropriate feedback is provided to narrow the student's possible knowledge gap, favoring the learning based on scaffolding [48].
- *Active learning.* The proposed approach adopts a problem-based learning scheme [52]. After each video, the student receives a corresponding set of questions that mimics real situations. These questions are useful for both formative and summative assessment [53]. On the one hand, the feedback can be used by students to improve their learning. On the other hand, these questions also help to assess students' knowledge.
- *Gamification.* It has been shown that the use of game elements increases students' engagement [54]. In a recent review paper, Boyle et al. showed the potential benefits of including games, animations, and simulations in the teaching of statistics [2]. For those reasons, several game elements are included to increase students' motivation and attitude (e.g., scores, leader-board, or a ranking).

The following section describes how the proposed approach, based on the previous pillars, is implemented in an application. After that, in Section 2.2, the design of an experiment, aiming at evaluating the performance of the proposed approach, is described. The obtained results are presented in Section 3. Finally, the article concludes in Section 4 with a discussion of the obtained results.

2.1. The Hypothesis Testing Learning Tool

The learning tool has been developed using the game engine Unity3D [55] and the Next-Gen User Interface (NGUI) library [56]. Following, its main components are described, beginning with an explanation of the main screen and its elements: buttons that lead to video lectures and problems, and the game elements. After that, the video lectures, problems, and content exposure control mechanisms are explained in more detail.

2.1.1. The Main Screen

Figure 1 shows the main screen of the application, in which its different elements are shown. In particular, it contains the following elements:

Figure 1. Main screen of the application.

- **Buttons**. The application has two types of buttons: circular and square. Each circular button leads to a video in which a key concept of hypothesis testing is explained. The concept explained in each video is indicated on the label placed under each button. The courseware includes five concepts that are: type of contrast (mean, variance, proportion), sample vs. population, the null hypothesis, the alternative hypothesis, and the *p*-value and its interpretation to obtain conclusions. The square buttons present the student with a series of questions related to the concept explained in the previous circular button. As it will be detailed, these questions are intended to ensure

that the student does not misunderstand any concept. Until the student demonstrates sufficient knowledge of these questions, the following videos and problems are blocked.
- **Stars**. Next to each circular button, there are two stars that can be illuminated or not. If they are not lit, it means that the student does not have enough knowledge about the current concept yet. In this case, the following buttons are deactivated so that the student cannot access to them. When the two stars are illuminated, the student has successfully completed all the problems at this level. If only one of the two stars is illuminated, it means that the student has demonstrated a sufficient level of the current concept, even if he did not correctly completed all the questions asked. When the student gets one or two stars, the following concept is unlocked.
- **Ship's boy and flag bearer**. The icon of a ship's boy indicates to students their progress in the course. In this way, it is easy to appreciate the concepts that are already mastered and those that still need to be completed. In addition, there is an icon of a flag bearer pirate that indicates the position of the most advanced student in the session. This second icon allows students to know their position with respect to the leader student of his session.
- **Score and leader-board**. Students receive points as they complete the various problems according to a scoring system that will be explained later. In the lower part of the screen, the achieved points can be seen, whereas, in the upper right corner, there is a leader-board that shows the classification of the top 10 students of the session based on their scores.

2.1.2. Videolectures and Questions

As mentioned, the buttons lead to video lectures and assessment questions. Next, we describe their functioning.

Videolectures

Each video explains one of the five above discussed concepts and has an approximate average duration of 6 min. Students can watch each video as many times as desired so that if they are not able to solve the problems related to a concept, they can watch the associated video again. The only restriction is that students can only watch the videos that they have unlocked.

Problems and Questions

As mentioned, our approach is based on a problem-based learning strategy [52]. For the construction of the problems, the difficulty level has been designed to stay within the student's Zone of Proximal Development (ZPD) [57], keeping it understandable and reachable, but challenging enough.

Each of the five key concepts is evaluated with six questions. These questions are obtained from six practical problems that the student must solve. One of these problems is shown in Figure 2. It can be observed that, in this problem, the student must identify the type of contrast, recognize the data sample, formulate the null hypothesis, enunciate the alternative hypothesis, and, based on the *p*-value, determine if it is possible to reject the null hypothesis or not. It is important to note that students do not initially receive all the questions related to a problem at once. Instead, they only receive the question, for each problem, corresponding to the current concept, remaining within their ZPD. Questions related to more advanced concepts appear invisible until the student has reached that level. That is, if a student has currently watched the video that explains the concept of the data sample, he/she will only receive the second question for each of the six problems. Below, these questions are described in more detail.

1. **Type of contrast**. The first question evaluates whether the student is able to recognize each one of the three types of contrasts that are explained in the first video (mean, variance and proportion). For this, the student selects the type of contrast that he considers associated with the problem statement from a drop-down window.
2. **Data sample**. The second question evaluates whether the student is able to differentiate between sample and population, both with regards to the concepts and to their notation. To do this,

the student must identify the problem values belonging to the sample and enter them in the correct text boxes, that is, in the boxes labeled with the corresponding notation to the sample statistics and the sample size. In addition to these text boxes, there are other text boxes related to different population parameters in which the student must not include any value.

3. **Null hypothesis $\mathbf{H_0}$**. The objective of the questions associated with this third concept is to determine if the student is able to recognize the null hypothesis and to write it correctly, using the appropriate notation of the population and not using the one in the sample. For this, the student must recognize the data of the problem that corresponds to the null hypothesis and place it in the corresponding box.
4. **Alternative hypothesis $\mathbf{H_1}$**. The fourth question evaluates if the student is able to identify the alternative hypothesis H_1.
5. ***p*-value and conclusion**. Finally, students must determine according to the answers given to the previous questions, and to the *p*-value that is provided, if the null hypothesis can be rejected or not at a significance level α of 0.05. To do this, they must choose the option that they consider correct between the two possibilities presented to them.

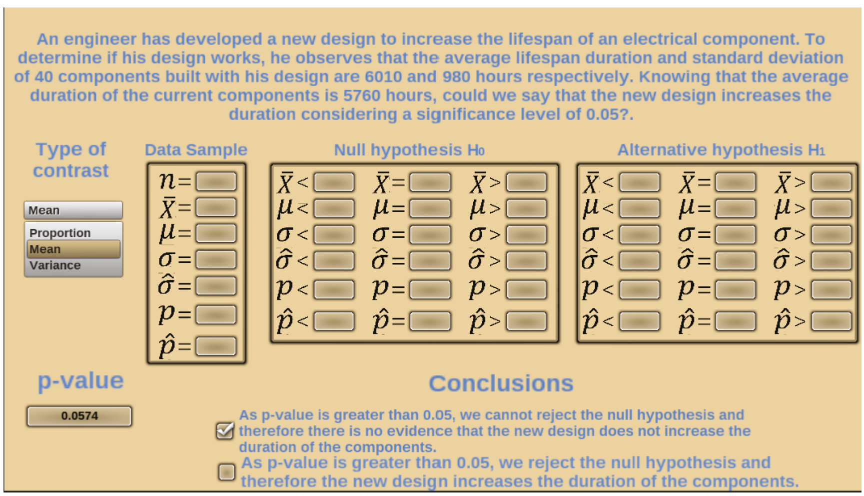

Figure 2. One of the problems (translated to English) included in the courseware.

As previously mentioned, these questions evaluate students' knowledge about each concept and are responsible for determining if a student can advance to the next concept or not. When a student correctly answers the six questions, he/she receives two stars, and, when he answers four or five questions correctly out of the six, the student receives one star. In these two cases, the following concept is unlocked, and the student can move forward. When the student answers less than four questions correctly, he/she does not receive any star and must redo these questions. Following the scaffolding approach, each time the student answers a question erroneously, he/she receives formative feedback in the way of a message that helps him/her to identify his/her error, facilitating his progress. Therefore, it is important to emphasize that the questions do not only assess students' knowledge; the fact of receiving feedback and having to repeat the questions until achieving a minimum knowledge allows students to better understand the concept and to correct their misconceptions.

In addition, the student receives 10 points per question answered correctly on the first attempt. As there are 6 questions, it is possible to obtain a maximum of 60 points per level. The questions answered correctly on the second and subsequent attempts provide 5 points. The student must repeat the level if he/she was not able to correctly answer up to four questions. Students that correctly answered four or five questions can repeat a level, until having correctly answered the six questions and obtaining the two stars.

2.2. Evaluation of the Approach Application

An experiment was conducted to evaluate the benefits of applying the proposed approach with respect to a traditional master class. Following, the details of the experiment are presented.

2.2.1. Sample

A total of 89 students, who were studying the subject of Statistics belonging to the degree of Industrial Engineering, participated in the study. Students were informed that their participation was voluntary and was under no circumstances considered in their academic evaluation. In order to make the data collected anonymous, a random number was given to each student to access the application and sign the tests with it. None of the researchers involved in data collection and analysis were teaching the students who participated in the experiment. After the study, all students were granted access to the application and all the material.

2.2.2. Assessment of Learning Methods

In order to assess the students' knowledge acquisition, two comparable problems were prepared. These two problems were intended to determine whether the student had been able to identify: (i) the type of contrast; (ii) the data pertaining to the sample together with the appropriate notation; (iii) the null hypothesis; (iv) the alternative hypothesis; and, finally, (v) the interpretation of the p-value in relation to the problem in question. Each of the problems was scored either with 1 (totally correct), 0.5 (partially correct), or 0 (incorrect). These two problems were:

Question (Q1): The average expenditure per customer in a store was 89 euros before the recession. Currently, taking a sample of 70 shopping carts, an average of 86 euros with a standard deviation of 9 euros is obtained. According to these data, and assuming a significance level of 0.05, could we affirm that you can see the effect of the current recession? Provide your answer by identifying the type of contrast, the sample data, the null and alternative hypothesis, assuming a p-value of 0.003.

Question (Q2): The ideal weight for 1.80 m tall men is 75 kg. Given a sample of 45 men that are 1.80 m tall in Spain, the average weight turns out to be 77 kg with a standard deviation of 8.5. According to these data, can we say that the Spanish are too fat? Provide your answer by identifying the type of contrast, the sample data, the null and alternative hypothesis, assuming a p-value of 0.06 and a significance level of 0.05.

2.2.3. Experimental Set-Up and Study Groups

The 89 students who participated in the experiment were divided in 3 groups according to their interest in using the application and their time availability. Students who decided to use the application indicated their time availability to participate before and/or after the master class. With this information, we tried to balance the number of participants in each of the groups as much as possible. The first subgroup, which will be referred as G1 from hereafter, was composed of 10 students. The students belonging to this group used the developed application some days before the class during a session of one hour. These students did not have any prior knowledge about hypothesis testing when they participated in this activity. Days later, they attended to the master class in which hypothesis testing was explained. The second group, which will be referred as G2, was the control group. It was made up of 60 students who attended only to the master class. Finally, the group G3 contained the remaining 19 students who enrolled for attending to the master class and days later had access to the application for one hour. There were two students of the G1 who used the application, but they did not attend later to the master class. There were also 4 students from group G3 who attended to the master class, but they did not appear later to use the application.

Next, we describe the order in which each group answered the assessment questions. Group G1 received the Q1 question immediately after their session (prior to the lecture) and the Q2 question after the lecture. Groups G2 and G3 received the Q1 question after the lecture class (none of them had performed used the application). In this way, it was possible, on the one hand, to use Q1 to compare the application of the proposed approach and the master class. On the other hand, it also allowed comparing the performance of groups G2 and G3, which is important to ensure that there are no differences between students who had signed up for using the application from those who did not, as both groups had simply attended the master class by the time they answered the question Q1. Lastly, students belonging to G3 answered question Q2 after using the application. In this way, it was possible to analyze if it is preferable that students to conduct the session with the application before or after the lecture. The design of the experiment is shown in Figure 3. From now on, we will refer to the different sets of scores through the pair formed by the student group and the question presented.

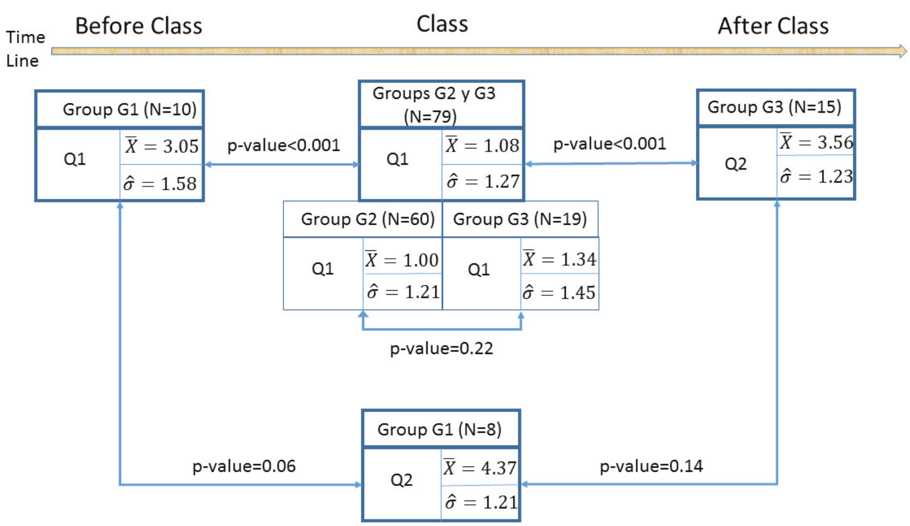

Figure 3. Experimental design including the 3 study groups (G1, G2, and G3), the two assessment questions (Q1 and Q2), and the results of the statistical analysis.

Additionally, the students who participated in the extra session filled out a questionnaire to evaluate the acceptance of the application. This questionnaire contained eight five-point Likert items and two open-ended questions.

3. Results

In this section, we present the results obtained. Figure 3 shows the mean and the standard deviation of the scores obtained for each one of the participant groups in the different phases of the study. Before examining these data and the significance of the differences, it is important to present a result obtained when analyzing the distribution of the scores. This analysis revealed that the distribution of the scores obtained by the students who used the application followed a Gaussian distribution, whereas, when students had only attended to the lecture class, it followed an exponential distribution. Concisely, the goodness-of-fit test to normality Kolmogorov–Smirnov yielded p-values of 0.99, 0.38, and 0.51 for the answers obtained in the pairs-question group G1-Q1, G1-Q2, and G3-Q2, respectively. Similarly, a Chi-square goodness-of-fit test showed that the distribution scores in G2-Q1 and G3-Q1 followed an exponential distribution (p-values 0.14 and 0.18, respectively).

After analyzing the distribution of scores, several hypothesis tests were performed to understand the results obtained. First, we compared the students' scores in the scenario (G1, Q1) with those obtained in (G2 and G3, Q1). A t-test showed that the differences were significant (p-value < 0.001). This result indicates that the proposed approach seems to be more effective to teach hypothesis test concepts than the lecture class.

Similarly, a t-test also showed that the differences between (G2 and G3, Q1) and (G3, Q2) were also significant (p-value < 0.001). This result also indicates that, similarly to the previous result, the application of the proposed approach after the master class significantly improves the knowledge acquired by students during the lecture.

Another interesting comparison was the one between the scores obtained in (G1, Q1) and (G1, Q2). The corresponding t-test provided a p-value close to 0.05, which seems to indicate that the lecture class reinforces the knowledge obtained initially with our approach.

In the comparison of the scores (G1, Q2) and (G3, Q2), the t-test indicated that the results were not significant (p-value 0.14). However, although this difference is not significant, performing the application prior to the class seems to provide better results, which is consistent with the flipped classroom methodology approach.

Finally, the scores obtained in (G2, Q1) and (G3, Q1) were compared to determine if the participants who used the application after the master class and those who participated only in the master class were similar. Since the scores in these groups followed an exponential distribution, the non-parametric Mann–Whitney test was used to analyze the difference. This test showed a p-value of 0.22, indicating that there were no significant differences between these two groups.

Regarding the subjective acceptance questionnaire about the application of the proposed approach, Table 1 shows the obtained results. This table shows the eight items that were presented to the students together with the mean and the standard deviation of their responses. These numbers indicate that the students considered that the application helps to improve assimilation of concepts and that it should be promoted. This can be seen in the values obtained for items 1 and 2, in which the means were 4.30 and 4.80, respectively. The students also considered that the experience was positive (item 8). However, despite these numbers, the students consider this application as a complement to the course but not as a replacement to the teacher (items 5, 6, and 7). On the other hand, despite that explanatory messages were clarifying (item 3), this questionnaire also allowed discovering elements that need to be improved, such as its comfort of use (item 4). In particular, the participants complained about the fact that our video player did not have the option of reproducing only certain parts of the video.

Table 1. Results from questionnaire about the use of the application.

SATISFACTION QUESTIONNAIRE
Date:
User Id:
Please, answer the following questions related to the application with a number from 1 to 5, using the following codes:
1-strongly disagree, 2-disagree, 3-neither agree nor disagree, 4-agree, 5-strongly agree

Item	**Mean Score (Std)**
1. I think that this type of application helps to improve the knowledge assimilation of the course	4.30 (0.61)
2. This kind of applications should be promoted	4.80 (0.40)
3. The explanatory messages have been sufficiently clarifying and concise	4.26 (0.72)
4. The application is comfortable to use	3.69 (0.73)
5. This application should not be mandatory	3.24 (1.23)
6. This type of programs is good complementary material for the course	4.53 (0.50)
7. This type of material should replace the tutor / professor	2.15 (0.73)
8. I would rate the experience positively	4.30 (0.47)

In addition to the eight items above, students answered two open-ended questions. The first question asked the student how he/she had felt when his/her name moved up on the leader-board. This question was answered by 14 students, 13 of whom (92.8%) provided answers related to increased motivation, while the remaining student replied that he did not paid attention to the leader-board.

The second question asked students if they had felt the need to overtake other participants on the leader-board. Of the 16 students who answered this question, 13 (81.2%) answered affirmatively and justified their response on the basis of competitiveness. The remaining three students answered that their goal was either to learn or their personal improvement.

The students' subjective satisfaction, together with the objective values shown above assessing students' knowledge, show the benefits of applying the proposed approach.

4. Discussion and Conclusions

In this article, a new approach has been proposed for the teaching of hypothesis testing. It has taken into account different elements that have been proposed in the literature to facilitate statistical learning, such as the inclusion of video lectures that allow students to learn at their own pace and a problem-based learning approach to encourage active learning. Additionally, the proposed approach includes a mechanism for content exposure control. In this way, students cannot access to any new concept until they demonstrate having understood the previously presented concepts. This mechanism provides several benefits. First, it avoids that, if the student has understood a concept in a wrong way, this error spreads to the following concepts. Second, when students make an error, they receive immediate formative feedback about it and have the opportunity to correct errors as they occur, providing a good basis to understand the next concept. In addition, it increases the students' confidence making them capable of solving the different questions. Finally, the application that incorporates the proposed approach contains elements of gamification to increase student's motivation.

In the experiment, carried out to identify the advantages and disadvantages of the proposed approach with respect to a traditional class, it was observed that students assimilated the concepts much better when they benefited from the proposed approach. This was true regardless of whether our approach had been done before or after the traditional class, although results seem to indicate that the advantage is greater if it is performed earlier, which is consistent with the flipped-class learning approach.

The students who used the application based on our approach were able to complete almost perfectly the six questions that were formulated with respect to the five concepts involved in the resolution of a hypothesis test. Conversely, in the traditional class, they correctly answered 1.34 concepts on average, in the best of cases. A possible explanation of this remarkable difference can be attributed to the fact that, in a master class, the different concepts are explained sequentially. If students are not able to fully understand one of them, they will have serious difficulties in understanding the subsequent ones, limiting their participation to taking notes that they will use later to try to understand the subject. Regarding the gamification elements, more than 50% of the students expressed that these elements enhanced their competitiveness and made learning more entertaining.

If we compare our proposal with others in the literature, we observe that our students also experimented difficulties understanding the statistical concepts when following the master class, corroborating the conclusions of the review made by Castro-Sotos et al. [14]. In addition, as suggested in Boyle et al. [2] and Sailer et al. [37], we included game design elements, such as stars as game badges, a leader board, a scoring system to indicate performance, and even a flag carrier to show the progress of the most advanced student in the session. Regarding performance, though most studies on learning statistics propose improvements, few carry out a formal evaluation. Wang, Vaughn, and Li [45] did evaluate performance after using different animation interactivity techniques, and their results indicated that the increase of animation interactivity could enhance student achievement improvement on understanding and lower-level applying but not on student remembering and higher-level applying. In terms of confidence improvements, there were no significant differences between their four groups. In turn, our results, depicted in Figure 3, reflect high student satisfaction and a mean significant difference of about 2 points out of 5 (p-value < 0.001). As discussed above, our work stands out for dividing complexity into simpler concepts that students can only access in a controlled, progressive manner, and for including elements of gamification, demonstrating a beneficial effect on understanding complex issues and obtaining positive results in terms of student satisfaction.

Some limitations of this research should be noted. First, the generalization of the conclusions of this study to other research contexts, subjects, and group sizes should be studied, due to our limited sample size. Despite this fact, our results showed significant improvement in performance. Another current limitation is that most learning platforms do not include facilities to implement our proposal to control content exposure, which means that it must be specifically implemented for each case. In this study, we created a small learning engine. We hope that, as more studies demonstrate the usefulness of this approach, tools, such as Moodle or EdX, could consider implementing such functionality.

These results show the need to rethink traditional lectures to include the benefits derived from methodologies that incorporate current technologies. The acquisition and understanding of difficult concepts by dividing them into steps or simpler concepts allows students to build the learning. The content exposure strategy not only controls the progress of students, but it is also a very interesting tool for the teacher to see what concepts are hard to master, as well as to evaluate whether it is worth dividing them even further or generating additional material. In addition, it offers the advantage of providing an overview of the overall progress of the group. Our work shows that, just by using the application that implements the methodology, the students' results are reasonably good. This could offer the possibility of redesigning the master class in a different way, to be more oriented to clarify concepts or to increase the cognitive level of learning.

We plan to apply the proposed approach for learning different subjects. In addition, not only gamification elements could be included, but also video games themselves. For example, we are currently developing a video game to teach quality control. In this video game, the student is responsible for controlling the filling of a bottling company. To do this, using an avatar, the student must collect various samples, measure them, and create the control charts. And later, he/she will have to detect whether or not the process is under control.

This work is only a first step, but its extensions are immediate. For example, since the application has been well accepted by the students and its usefulness has been demonstrated even without the intervention of the teacher, it could be offered as a self-learning tool aimed at pre-balancing basic statistical knowledge among students before starting other courses. It would also be interesting to dynamically create specific itineraries for each student, tailored according to the particular results obtained. Future versions can include the adaptive selection of exercises that are based on the detected errors, fostering deliberative practice [58], since this would enable students to pay more attention to their weaker areas. Furthermore, with the growth of online courses, especially with COVID-19, providing learner-centered tools that ensure good understanding becomes even more relevant. We want to finish this article with a phrase that we heard a student say to another when the proposed activity ended: "Now, finally, I understood it all".

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Abbreviations

The following abbreviations are used in this manuscript:

ZPD	Zone of Proximal Development

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mathematics

Article

Application in Augmented Reality for Learning Mathematical Functions: A Study for the Development of Spatial Intelligence in Secondary Education Students

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Abstract: Spatial intelligence is an essential skill for understanding and solving real-world problems. These visuospatial skills are fundamental in the learning of different Science, Technology, Engineering and Mathematics (STEM) subjects, such as Technical Drawing, Physics, Robotics, etc., in order to build mental models of objects or graphic representations from algebraic expressions, two-dimensional designs, or oral descriptions. It must be taken into account that spatial intelligence is not an innate skill but a dynamic skill, which can be enhanced by interacting with real and/or virtual objects. This ability can be enhanced by applying new technologies such as augmented reality, capable of illustrating mathematical procedures through images and graphics, which help students considerably to visualize, understand, and master concepts related to mathematical functions. The aim of this study is to find out whether the integration of the Geogebra AR (Augmented Reality) within a contextualized methodological environment affects the academic performance and spatial skills of fourth year compulsory secondary education mathematics students.

Keywords: augmented reality; spatial intelligence; STEM; mathematics; Geogebra AR; secondary education

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1. Introduction

The term function in mathematics is defined as any relationship between two or more variables that can be represented graphically. Function learning provides students in Compulsory Secondary Education (ESO) with their first contact with the identification, visualization, and interpretation of the relationship between two independent variables and is therefore a key point of transition within mathematical development figure [1]. The cognitive transition of graphically representing a constant, linear, affine, quadratic, exponential, absolute value, inverse proportionality, and logarithmic function from its algebraic expression is included in the curriculum of this educational stage and tends to be a challenge for most students.

This study is based on research integrating ICT in the classroom, where we can detect their benefits and drawbacks, design resources to help implement these technological tools, collect and analyze data, and reflect on the results. These action research elements provide a backdrop for teachers to recreate a digital and proactive environment in the classroom within a contextualized methodology that favors the teaching-learning processes of mathematics, with the aim of making students the protagonists in the construction of their knowledge.

Several studies claim that the inclusion of ICT in the teaching and learning of mathematics helps students to visualize how changes in one variable affect others immediately, thus improving their experience and interaction with learning compared to solving formulas so as to obtain the answer [2–6]. It is common for students to associate the representation of functions with a collection of isolated points rather than a single entity, making it difficult to visualize and interpret graphically [7–9]. As a consequence, students often do not visualize and interpret correctly the representations of graphic functions as a solution in itself, therefore they do not manage to conceive the transition process from algebraic language to visual language and vice versa. Therefore, we pose the following questions: How could ICT based on augmented reality facilitate the process of representation, visualization, and analysis of algebraic functions? Is this cognitive-visual process linked to students' spatial intelligence?

1.1. Spatial Intelligence

According to Bishop's theory, an individual acquires the capacity for spatial visualization through three distinct stages of development [10]. In the first stage, children learn topological spatial visualization, where they can understand the relationship between different objects in space, i.e., the location of an object within a group of objects, the isolation of the object, etc. In the second stage of development, they acquire projective representation, where they can conceive how an object will look from different perspectives. Finally, the final stage of the development of spatial visualization is based on combining spatial projection skills with distance measurement.

On the other hand, spatial intelligence corresponds to one of the eight intelligences of the model proposed by Gardner [11] in the theory of Multiple Intelligences (MI). This type of intelligence implies having the capacity to perceive the visual world with accuracy, to mentally recreate objects or models, even in the absence of physical stimuli, and to carry out transformations or modifications of them.

In the study of the so-called knowledge areas of Science, Technology, Engineering, and Mathematics, better known by its popular homonym in English as STEM, this type of intelligence is fundamental for students to develop the ability to transfer numerical data and two-dimensional projections to three-dimensional objects with ease [12,13]. Within the contents of the subjects of Secondary Education, this skill has numerous applications, such as the conception and construction of spatial models, the analysis of geometric objects, the interpretation of diagrams, and the identification of functions among others.

The term spatial intelligence covers five fundamental skills: Spatial visualization, mental rotation, spatial perception, spatial relationship, and spatial orientation [14].

Spatial visualization [15] denotes the ability to perceive and mentally recreate two- and three-dimensional objects or models. Several authors [16,17] use the term spatial visualization to indicate the processes and abilities of individuals to perform tasks that require seeing or mentally imagining spatial geometric objects, as well as relating these objects and performing geometric operations or transformations with them.

Shepard and Metzler [18] define mental rotation as the cognitive ability to rotate ideal representations of dimensional and/or three-dimensional objects or models, and can be described as the movement of representations through the brain to help conceive each of its views or perspectives regarding a turn.

According to Gibson [19], spatial perception is defined as the ability to visually perceive and understand external spatial information, such as characteristics, properties, measurements, shapes, the position, and movement of an object in relation to an individual.

On the other hand, the spatial relationship determines how an object is located in space in relation to another reference object and this skill is the basis of cognitive development for walking and trapping objects in space [20].

Finally, we can refer to spatial orientation as a fundamental ability to move and locate oneself in space [21,22], being necessary for such common activities as writing straight, reading, differentiating between right and left, and, in general, locating objects and orienting them in space.

These five skills are malleable and can therefore be reinforced through the use of multi-sensory tools or applications that stimulate and improve these abilities [23]. However, the traditional method for teaching visual and spatial skills to students is based on analyzing and interpreting two-dimensional images, orthogonal views, and graphics on a blackboard or paper. This method has obvious limitations, as it hinders the conceptualization and assimilation of contents due to the lack of interaction between students and the representations [24].

This study relates the development of spatial skills to the representation of two and three-dimensional functions in mathematics, and demonstrates that augmented reality technology contributes to the improvement of spatial skills and the understanding of highly visual content. This might be due to the observation and experimentation of the models from different angles and relative positions, respecting the individual learning pace of each student. Some studies [25,26] state that visual and spatial abilities can be improved by emerging technologies such as augmented reality. The integration of this technology in the classroom favors a constructivist approach to learning by allowing teachers to introduce tangible and proactive experiences in the classroom where students interact and manipulate with the learning object. As educators, we must show a positive attitude towards the integration of ICT in education, as it effectively changes the way students learn [27], however, a lot of work still needs to be done in order to achieve a systematic development of augmented reality for educational purposes.

1.2. Augmented Reality as a Methologocial Resource in Teaching-Learning Processes

Augmented Reality, AR henceforth, offers multiple benefits that support the teaching-learning process. The applications of AR allow the human-machine interaction to be more natural by enabling the preservation of the user's environment, providing a real frame of reference which the user can rely on to perform certain actions. This process can be achieved through the superimposition of virtual objects in a real environment. Students can experience the ability to combine their real environment with a virtual one designed, in this case, by themselves.

This technology allows any real environment to be enriched with digital information through the use of a camera and software that in recent years has focused its development on mobile devices which, due to their portability, contribute to off-site learning, where any scenario can be transformed for training purposes [26,28,29].

The reports of New Media Consortium [30–35] that identify and describe the trends, challenges, and technological advances in education, estimate that AR technology will be established in secondary and higher education classrooms in the short term as an information access tool that will generate new applications of technology in the learning process.

This indicator, together with the omnipresence of mobile devices, which have become the main tool for accessing information in different formats and in an immediate form, can be used as access portals to Open Educational Resources (OER) that adapt the pace of learning to the needs of each user; it combines an AR-mobile device binomial that equates access to learning opportunities and facilitates the provision of mobile, interactive, individualized, and adapted learning services [26].

The integration of AR technology into the field of education has enabled an evolution of the educational model. Initially, this technology was used only as a tool for immediate access to digital information, involving students in the theories of behaviorism and objectivism. Recently the applications of this technology are undergoing some changes, with students moving from being recipients to providers of knowledge and the teacher taking on the role of guide and tutor with the objective that students generate knowledge using this technology in an interactive way, where the main theories of this new model are: Cognitivism, constructivism, and constructionism [36].

The fact that the educational scene is one in which the acquisition of digital competences is particularly relevant must be noted [37], although the vast majority of technological tools and resources do not promote the same learning opportunities for all. The Sustainable Development Goal 4 aims to ensure inclusive, equitable, and quality education and promote continuous learning opportunities for all. Mobile devices are driving a revolution in education, allowing learners to access learning resources anywhere, anytime. Therefore, the role of mobile learning is relevant, as it has the ability to help break down economic barriers, differences between rural and urban areas, as well as functional limitations. The omnipresence of mobile devices is changing the way people interact with information and their environment. In addition, the continuous improvement of the hardware of these devices and their reduction in cost, positions them as the first tool for accessing the most widespread information worldwide [26]. Consequently, in order to conduct this study, mobile devices were chosen as the learning platform, since all students had one or had access to them, thus guaranteeing access to training for all students.

Thanks to new technologies, we enter for the first time a place where we interact with real objects and at the same time with virtual ones, which allow us to remember previous learning and restructure our thinking, thus giving meaning to what we perceive from the surrounding world. As Vigotsky [38] stated, people develop ways of interpreting and strategies to relate to physical and cognitive space in such a way that this type of interaction can be established with tools and systems that provide various types of stimulation, thus it is certain that the use of AR will lead to substantial changes in the way knowledge is accessed, interpreted, and communicated, which must be considered in the field of education [39].

AR as an integrated technology in teaching acquires a dimension that emphasizes sensory transformation, so if it is integrated into the teaching-learning processes it could promote meaningful and contextualized learning acquired through multiple sensory experiences [40].

This technology can be used in education to represent 3D models of objects that, because of their size, cost, danger, distance and tangibility, are not within the real reach of students. Moreover, working in contexts with AR, there is a direct interaction with the environment or the object of study, making learning more meaningful.

With the representation of objects in 3D through AR technology we have the freedom of spatial exploration, so students can really perceive and understand space as it is. In addition to spatial perception, students can view models in space and modify parameters that alter their geometry. In this way, the spatial visualization is exercised and they can rotate or flip these representations to visualize each of their perspectives or views, thus promoting spatial rotation. At the same time, and while the user observes the parameters that correlate various objects recreated in space and places the designs in the plane, the skills of spatial relationship and orientation are also developed. With all this, we stimulate, work, and enhance all the fundamental fields of spatial skills established by Maier in 1994 [14] through a multi-sensorial tool, such as AR and mobile devices.

1.3. Geogebra AR as a Tool to Support the Learning of Mathematical Functions

In accordance with the constructivist theory, it is believed that technology can help students in teaching-learning processes. One of the first technological tools for learning functions is graphical calculators, which emerged as an instrument to enable students to solve systems of equations, represent graphs, and perform other tasks with variables [41]. Despite their benefits, these calculators have limitations when solving and representing certain expressions due to their small output interface. In addition, they must be implemented cautiously, as many students have difficulties when using symbols, which can be counterproductive and slow down the resolution of operations [42].

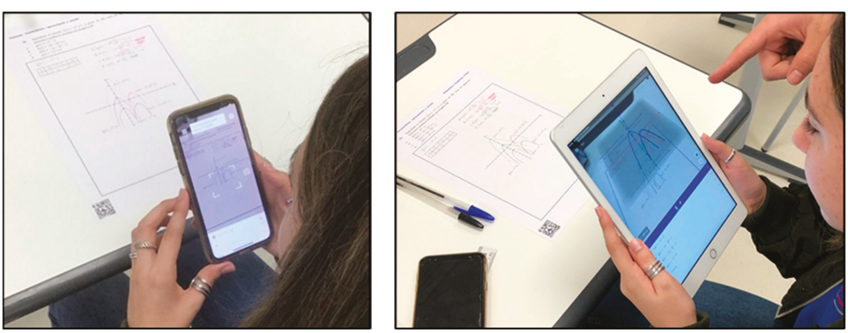
The most recent graphical interfaces offer direct manipulation mechanisms for the representation of mathematical functions, allowing users to interact intuitively and directly in the visualization they are editing, providing immediacy and simplicity when obtaining results, and helping their interpretation and learning. The term direct manipulation describes a style of interaction that stands out for the following characteristics: Continuous representation of objects and actions of interest; change from complex command syntax to manipulation of objects and actions; fast, incremental, and reversible actions that have an immediate effect on the selected object [43]. Therefore, direct manipulation is, by far, the most common type of interaction in mobile applications, and it is found to a greater extent in AR interfaces, since it provides us with an immediate handling of virtual objects in our real environment.

Numerous research studies claim that didactics through AR applications positively influence students' attitude and motivation towards learning [44–53], providing an active teaching environment where the capacity for enquiry and research is encouraged, while promoting the development of autonomous student work in their learning [26,54]. Likewise, several studies state that the correct integration of AR applications in the classroom improves students' learning results [55–59].

Despite the numerous research studies cited on AR resource didactics, few are concerned with the possible impact of AR technology on spatial intelligence [12,60] and, thus, there is an interest in conducting research so as to determine if there is a real contribution of AR to the acquisition of spatial skills.

In order to explore the development of spatial intelligence in relation to mathematical learning, our classroom experience revolves around the open source application, Geogebra AR, for mobile devices which helps students learn analysis, geometry, algebra, and calculus. This mathematical application is specifically designed for educational purposes. It allows the dynamic drawing of geometric constructions of all kinds, as well as the graphic representation, algebraic treatment, and calculation of functions in a simple and effective way, which permits us to use it as a support tool for the study, promoting mathematical self-learning. There is a large volume of research that has shown that Geogebra, in its version for personal computers, has been effective for the teaching-learning of mathematics [61–65], improving the understanding of abstract concepts and enabling their correlation through a meaningful and effective learning experience.

In its AR version, it allows us to generate 3D objects and mathematical functions, which we can place on an imaginary plane in our real environment (Figure 1a) and then experiment with them in a tangible way, being able to visualize and rotate them with total freedom, which helps to improve the understanding of the function itself through manipulative learning. The user interface of the Geogebra AR application is direct and intuitive. At the bottom of the screen, it includes a section where we can introduce the algebraic expressions of our naturally defined functions, as they appear in the textbooks or as they are written by the teacher on the blackboard, through a virtual keyboard incorporated in the mobile device, generating immediately the graphic representations of the introduced functions (Figure 1b).



(**a**) (**b**)

Figure 1. Geogebra AR (Augmented Reality) interface: (**a**) Surface detection and (**b**) introduction and representation of functions.

Through the application menu, located in the upper left corner, we can search and open existing resources, save and share our work, as well as make changes to the program settings (hide or show axes, change the coordinate grid, distances between axes, hide or show descriptions or labels, etc.).

The application design promotes the learning and analysis of mathematical functions, not only generating them in AR, but also emphasizing the cognitive-visual process that occurs when an object is built in space. In particular, introducing the algebraic expression of defined functions, representing them in space and interacting with them in AR, is a major cognitive step in the transition from algebraic expression, through 2D linear designs, to the 3D object representation that covers the five fundamental skills of Maier's spatial intelligence [14].

2. Materials and Methods

2.1. Researh Design

The research approach adapted for this study is based on a quasi-experimental design. Two pre-test/post-test models were applied to each of the two ordinary class groups, formed by students who do not have any type of special educational need, that participated in the study: One to assess the level of spatial ability and the other to determine the level of learning of mathematical functions. The experimental group underwent a contextualized methodology that integrated the binomial RA-mobile devices for the use of the Geogebra AR application in the study of mathematical functions, while in the control group, a traditional teaching-learning methodology was used. At the end of the experience, the experimental group completed a questionnaire in order to obtain the students' perceptions after using Geogebra AR.

2.2. Researh Objectives

The research question posed is whether there is a significant difference between students who use the application of Geogebra AR in a contextualized methodological environment and those who use traditional teaching-learning methods with regard to their spatial intelligence and the level of learning acquired. In order to assess the scope of these research objectives, the following hypotheses are established:

- **H0** (null hypothesis): *There is no statistically significant difference in the performance and spatial intelligence scores of students exposed to the Geogebra AR application and those not exposed to it*;
- **H1** (alternative hypothesis): *There is a statistically significant difference in the performance and spatial intelligence scores of students exposed to the Geogebra AR application and those not exposed to it*.

2.3. Sample

The total number of participants was 48 students, who were taking the subject Academic Mathematics in their 4th year of ESO, taught by one of the teachers who conducted this study. Out of the total number of participants, the 47.92% (f = 23) belonged to the experimental group and 52.08% (f = 25) belonged to the control group, presenting no significant curricular adaptations. The sample used in the research is non-probabilistic and, as a consequence, the results cannot be generalized with statistical precision [66].

2.4. Data Collection Instrument

The study uses three different instruments to collect information: A pre-test/post-test model to evaluate spatial intelligence, a second pre-test/post-test model, which is a written test to detect previous knowledge, and another one to evaluate the learning standards of the functions block within the curriculum of the subject Academic Mathematics in the 4th year of ESO. Finally, the students were given a questionnaire to detect the motivation levels of the experimental group.

There are several standardized tests to measure a person's ability in the first two stages of spatial development. For our study the Purdue Spatial Visualization Test: Rotations (PSVT:R) has been used because of its design to evaluate a person's ability in the second stage of spatial development [67]. Figure 2 presents a random question extracted from the PSVT:R test. This 12-item test has been used as an evaluation instrument at the beginning and end of the experience in the experimental and control group, with the aim of identifying the level of visualization and spatial rotation that the students started from, and to evaluate the impact on the spatial intelligence of the students through the experience in the classroom with the Geogebra AR application, as an aid for the analysis and study of mathematical functions.

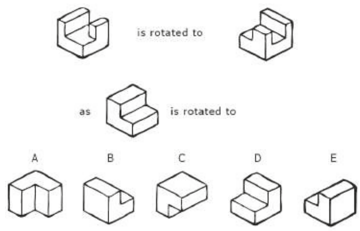


Figure 2. Sample Purdue Spatial Visualization Test: Rotations (PSVT:R) test question (correct answer D).

Likewise, and in the perspective of evaluating the learning of mathematical functions within the block of contents of functions in the curriculum of Academic Mathematics in the 4th year of ESO in Spain established by the Royal Decree 1105/2014 [68], an individual written test of detection of an initial assessment of knowledge and another final assessment test made up of 8 items that includes the evaluable learning standards were used as data collection instruments, having been both instruments designed by the authors of the study.

After the final test, the experimental group carried out a 10-item Likert scale questionnaire with 6 answer options so as to identify the feasibility, motivation, and students' perception of the experience, thus evaluating the AR enriched learning environment. The questionnaire focused mainly on determining the following aspects:

1. The use of AR technology in the teaching-learning process;
2. The contribution of AR tools for a better visualization of the contents;
3. Impact of AR technology on the degree of motivation;
4. The difficulty of using the Geogebra AR application.

Finally, the reliability of the evaluation instruments designed by the authors of the research (written test and Likert questionnaire) is established by means of Cronbach's internal consistency coefficient α [69], considered by several researchers to be one of the most appropriate statistical methods to obtain quality values [51,70,71]. Table 1 shows that the internal consistency reliability indexes are adjusted to a high level for each one of the scales that constitute the evaluation instruments elaborated.

Table 1. Internal consistency reliability coefficient for designed tests.

Dimension	Cronbach's α
Curriculum evaluable learning standards	0.893
AR as a teaching-learning tool	0.762
AR as a spatial visualization tool	0.838
Motivation and stimulation of learning through AR	0.921
Difficulty using the app	0.874

Once the data from the PSVT:R test and the individual written test were collected, they were analyzed using descriptive and inferential statistics. The descriptive statistics are composed of the mean obtained from the pre-test and post-test results, the standard deviation, the range, etc. On the other hand, for inferential statistics, a student *t*-test with a 5% confidence level is used along with a bilateral test to test the study hypothesis.

2.5. Learning Experience

In May 2019, the classroom experience was carried out with 4th year ESO Academic Mathematics students, distributed in 12 class sessions within a three-week period. The objective of this trial was to determine the scope and limitations of integrating the mobile device in the classroom with the Geogebra AR application (Figure 3), as a support for the analysis and study of mathematical functions, in addition to checking its impact on the spatial intelligence of the students.

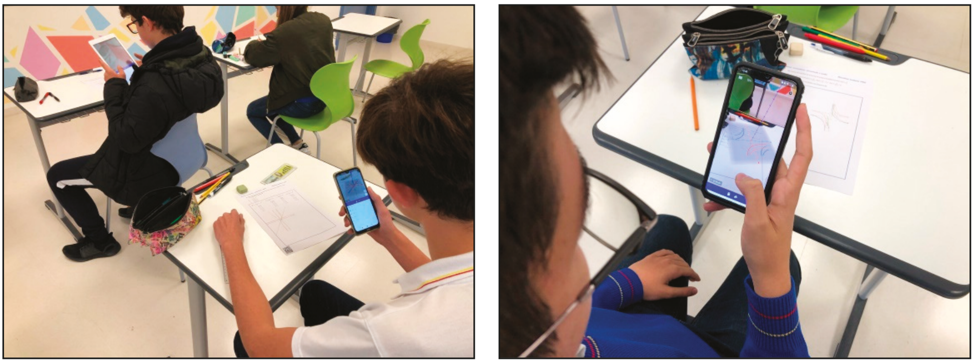


Figure 3. Students working in the classroom during the development of the experience.

The learning standards that are evaluated within the block of content of functions of the curriculum of the subject of Academic Mathematics in the 4th of ESO in Spain, established by Royal Decree 1105/2014, explicitly indicates that students must explain and graphically represent the relationship model between two magnitudes for cases of linear, quadratic, inverse proportionality, exponential, and logarithmic relationship, using technological means, if necessary. This makes it flexible enough to allow the introduction of other teaching methods such as approaches based on new technologies, in our case Geogebra AR, which facilitates the exploration, representation, and analysis of functions among other things. Therefore, by integrating Geogebra AR as a support to the teaching-learning of functions, students can explore and develop cognitive schemes that allow them not only to draw graphs of functions, but also to enhance proactive self-learning by achieving a progression in the development of analysis, application, reflection, and interpretation of knowledge.

2.6. Generated Material

To carry out the experience in the classroom, worksheets were generated, integrating the mobile device as a platform for access to classroom learning through the application Geogebra AR in order to solve the proposed activities. In relation to the above, it should be noted that the teachers do not necessarily have to follow the textbook, but they can create their own work material, in this case cards linked to objects in AR. In order to do this, teachers must have enough knowledge. In this sense, some authors design their own activity cards or OER work materials in what they call "production of augmented materials" which is generally systematic and sequential, adapting to the learning rhythm and needs of each user [12].

The collection of contents generated deals with aspects such as the representation, study, and analysis of functions such as: Constant, affine, linear, quadratic, absolute value, inverse proportionality, exponential, logarithmic, and trigonometric. These materials were used in paper format (Figure 4), so that the students could solve the activities in written form while superimposing in the work card the graphic representations in AR generated by Geogebra AR. A QR code was located at the bottom of each worksheet, giving access to downloading the application.

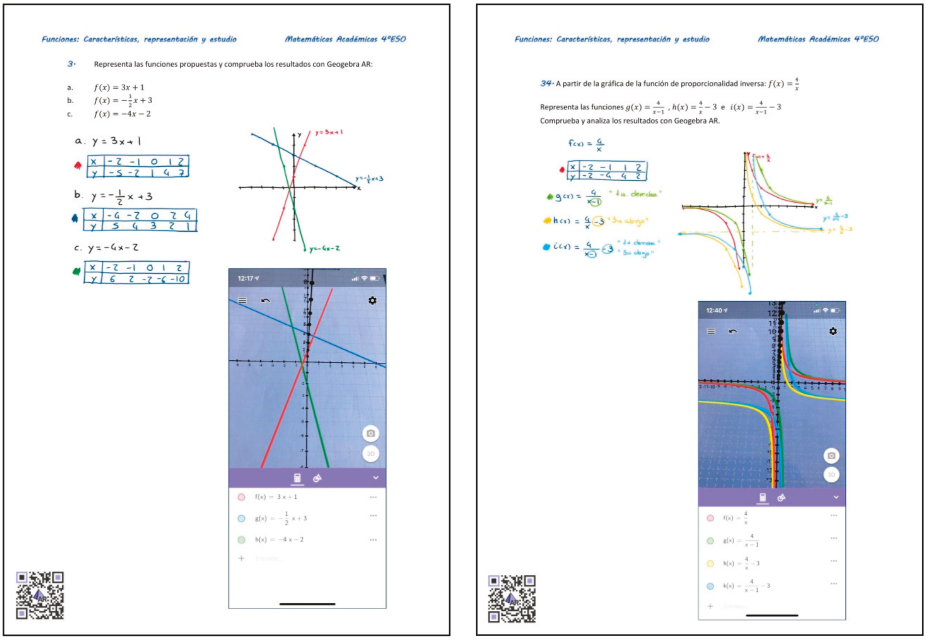


Figure 4. Worksheets with RA content, with QR code for access to the Geogebra AR application.

In this way, students interact directly with the object of study with total freedom of spatial exploration, rotating or flipping the representations to visualize the function in total detail and from any perspective. It is important to emphasize that the activities that are part of the collection of exercises are not far from a traditional teaching methodological framework of mathematical functions, which gives a great advantage when integrating technological AR tools as Geogebra AR.

Although students had never used interactive mathematical software in AR as a teaching tool before, Geogebra AR's smooth learning curve allowed us to design a classroom experience with a discovery-based learning format. Therefore, instead of dedicating teaching sessions to explain the operation, tools, or elements of the program interface, a routine was established in the classroom based on brief instructions and directed activities through proactive and tangible learning that made students gradually master the software according to the demands of each activity, their needs, and inquiry. As in the development of any other training unit, students were assigned tasks to perform outside school hours. The use of the binomial RA-mobile devices allows students to access information regardless of where they are, thus combining classroom work with online work, which results in an educational model closer to the needs of new generations known as b-learning [72]. This has a greater significance nowadays due to the change of paradigm that the educational system is facing in times of Covid-19, and due to the leading and essential role that technologies have taken, we are facing a scenario in which we must help strengthen self-learning and autonomy in students, as well as motivate them to help capture their interest and enhance their desire to investigate [73].

3. Results

During the execution of the experience it was observed in the experimental group that, firstly, the students quickly learned to generate graphic functions through the application as an alternative to the traditional system of representation. Secondly, students learned to visualize and analyze graphical solutions as an alternative to algebraic solutions. Thirdly, students moved from conceiving a graph as a collection of isolated points, to thinking of a graph as an entity, which caused them to begin doing comprehensive studies and analysis of function behavior. Fourthly, students understood the conceptualization of a function and understood the relationship of variables over them. Fifthly, it was detected that the students experimented freely and autonomously with the Geogebra AR application and contributed to the rest of the group with their perception of the operations carried out. It should be noted that these interpretations were typical of students from higher education levels.

Finally, one of the findings observed in the experimental group is that students related the different solutions between the systems of equations through their graphic representations. This shows us that students are able to visualize and identify a point or a line of intersection in a graph as a solution to a system of equations (Figure 5).

Figure 5. Students in the classroom working the graphical intersection through Geogebra AR.

3.1. Analysis of the Variation in the Teaching-Learning of Mathematical Functions

The descriptive statistical results of the initial knowledge assessment test within the function content block for both the experimental and the control group are shown in Table 2.

Table 2. Descriptive statistics of the results obtained in the initial evaluation test.

Initial Eval. Test	N	Maximum	Minimum	Median	Mean	Std. Deviation	Std. Error Mean
Experimental Control	23 25	8.7 8.6	2.3 2.5	5.8 6.1	5.7478 6.0921	1.52729 1.58559	0.31846 0.31712

The experimental group with 23 participants obtained a mean score in the initial evaluation test of 5.7478, while the control group obtained a mean score of 6.0921. A *t*-test for independent samples was carried out to determine if there was a significant difference between the mean score of the two groups in the initial assessment test with a level of reliability of 5%. These results are shown in Table 3.

According to the results of Table 3, the Levene test has a value of 0.818, which is higher than 0.05, therefore assuming that the group variations are equal. The value of the test for bilaterality for the experimental and control group is 0.448 for both cases, which implies that the difference in measurements is not statistically significant at a probability of 0.05. The results show that there is no statistically significant difference ($p > 0.05$) between the mean value of the two groups based on the results of the initial evaluation test. This statistically indicates that students in both groups had similar performance levels at the beginning of the research. Therefore, any difference in performance observed later can be attributed to the use of the Geogebra AR application.

Table 3. T-test of results obtained in the initial evaluation test.

Initial Eval. Test	Levene's Test for Quality of Variance		*t*-Test for Equality of Means						
	F	Sig.	*t*	df	Sig. (2-Tailed)	Mean Difference	Std. Error Difference	95% Confidence Interval of the Difference	
								Lower	Upper
Equal variance assumed	0.054	0.818	−0.765	46	0.448	−0.34417	0.45014	−1.2503	0.5619
Equal variance not assumed			−0.766	45.895	0.448	−0.34417	0.44942	−1.2489	0.5605

Table 4 compares the descriptive statistics of both groups according to the results obtained by the students in the final assessment test that collects the evaluable learning standards. The experimental group obtained a mean score in the final test of 7.3391, a standard deviation of 1.61125, and a mean error of 0.33597. On the other hand, the mean score of the control group was 6.0841, the standard deviation was 1.52334, and the mean error was 0.30467. The mean score obtained in the final evaluation test by the students of the experimental group is significantly higher than that of the control group.

Table 4. Descriptive statistics of the results obtained in the final assessment test.

Final Eval. Test	N	Maximum	Minimum	Median	Mean	Std. Deviation	Std. Error Mean
Experimental	23	9.6	4.6	7.1	7.3391	1.61125	0.33597
Control	25	8.7	3.2	6.1	6.0841	1.52334	0.30467

The results obtained from the t-test for independent samples are shown in Table 5. The statistic of the Levene test is 0.034, which is less than 0.05 and therefore, it is not assumed that the group variations are equal with respect to the results obtained in the final knowledge evaluation test. The bilateral value is less than 0.05, which implies that the difference in means is statistically significant at a level of 0.05. These results indicate that the students of the experimental group achieved higher scores than the students of the control group. Therefore, according to the results of the t-test, we can reject the null hypothesis (there is no statistically significant difference in the performance scores of the students exposed to the Geogebra AR application and those who are not exposed to it) in favor of the alternative hypothesis (there is a statistically significant difference in the performance scores of the students exposed to the Geogebra AR application and those who are not exposed to it).

After analyzing the existence of a relationship between the groups, it is worth asking the intensity of their relationship, for which we use the mean of the effect size in ANOVA. The results of this test are shown in Table 6, where it can be observed that 20.4% of the variation in the teaching-learning of student functions can be attributed to the use of the Geogebra AR application.

Table 5. T-test of the results obtained in the final evaluation test.

Final Eval. Test	Levene's Test for Quality of Variance		*t*-Test for Equality of Means						
	F	Sig.	*t*	df	Sig. (2-Tailed)	Mean Difference	Std. Error Difference	95% Confidence Interval of the Difference	
								Lower	Upper
Equal variance assumed	0.412	0.034	2.774	46	0.008	1.25513	0.45246	0.34438	2.16588
Equal variance not assumed			2.767	45.102	0.008	1.25513	0.45354	0.34171	2.16855

Table 6. Measures of association between groups.

Teaching-Learning Process of Mathematical Functions	Eta	Eta Square
Experimental or Control Score	0.452	0.204

3.2. Analysis of the Variation of Visualization and Spatial Rotation Skills

In the same way, an analysis using descriptive statistics and the *t*-test of the pre-test and post-test PSVT:R was carried out in order to find out if there were any significant differences. By doing so, the impact of the Geogebra AR application is evaluated with the aim of improving the capacity of visualization and spatial rotation of the students.

The descriptive statistical analysis of the pre-test based on the PSVT:R model for the experimental and control groups is shown in Table 7. The participants in the experimental and control groups obtained a mean score of 4.9643 and 5.3332, respectively.

Table 7. PSVT:R pre-test descriptive statistical results.

Pre-Test PSVT:R	N	Maximum	Minimum	Median	Mean	Std. Deviation	Std. Error Mean
Experimental	23	7.5	2.5	5	4.9643	1.40999	0.29400
Control	25	8.33	2.5	5.83	5.3332	1.73455	0.34691

A t-test for independent samples was conducted so as to determine if there was any significant difference between the mean score of the two groups of the pre-test based on the PSVT:R model with a 5% confidence level, the results are shown in Table 8. The Levene test had a value of 0.137 which, being higher than 0.05, means that the group variations are equal. The result of the bilaterality test was 0.425 for equal variances and 0.422 for different variances, so the difference in the means is not statistically significant with a probability of 0.05. Along with the results of the t-test carried out with the scores of the initial evaluation test, it was detected that the groups had a similar level of spatial intelligence at the beginning of the investigation. In this case, any difference detected later in terms of the improvement of the students' visualization and spatial rotation skills can be attributed to the integration of the Geogebra AR application in the classroom methodology.

Table 8. T-test results obtained in the PSVT:R pre-test.

Pre-Test PSVT:R	Levene's Test for Quality of Variance		*t*-Test for Equality of Means						
								95% Confidence Interval of the Difference	
	F	Sig.	*t*	df	Sig. (2-Tailed)	Mean Difference	Std. Error Difference	Lower	Upper
Equal variance assumed	2.291	0.137	−0.804	46	0.425	−0.36885	0.45871	−1.29218	0.55448
Equal variance not assumed			−8.11	45.341	0.422	−0.36885	0.45474	−1.28455	0.54684

Table 9 shows the results of the descriptive statistical analysis according to the scores obtained in the PSVT:R post-test for the two groups. The experimental group obtained a mean score in the post-test of 7.0652, a standard deviation of 1.60574, and a mean error of 0.33482. On the other hand, the mean of the control group was 5.6664, the standard deviation was 1.73463, and the mean error was 0.34693. It should be noted that the mean score obtained by the experimental group in the PSVT:R post-test was significantly higher than that of the control group.

Table 9. PSVT:R post-test descriptive statistical results.

Post-Test PSVT:R	N	Maximum	Minimum	Median	Mean	Std. Deviation	Std. Error Mean
Experimental	23	9.17	4.17	7.5	7.0652	1.60574	0.33482
Control	25	8.33	2.5	5.83	5.6664	1.73463	0.34693

The results obtained from the *t*-test for independent samples in relation to the scores obtained from the two groups in the PSVT:R post-test are shown in Table 10. The value of the Lenvene test is 0.029, which is lower than 0.05, so it is detected that the group variations are not equal. The bilateral test has a value of less than 0.05, implying that the difference in means is statistically significant at a probability of 0.05. For the results obtained in the *t*-test in relation to the scores obtained in the final written test, the students of the experimental group reached higher scores in the PSVT:R test than the students of the control group, therefore, according to the results of the *t*-test, the null hypothesis (there is no statistically significant difference in the level of spatial intelligence of the students exposed to the Geogebra AR application and those not exposed to it) was rejected in favor of the alternative hypothesis (there is a statistically significant difference in the level of spatial intelligence of the students exposed to the Geogebra AR application and those not exposed to it) with respect to the improvement of the students' visualization and spatial rotation skills.

Table 10. T-test results obtained in the PSVT:R post-test.

Post-Test PSVT:R	Levene's Test for Quality of Variance		*t*-Test for Equality of Means					95% Confidence Interval of the Difference	
	F	Sig.	*t*	df	Sig. (2-Tailed)	Mean Difference	Std. Error Difference	Lower	Upper
Equal variance assumed	0.101	0.029	2.892	46	0.006	1.39882	0.48373	0.42513	2.37251
Equal variance not assumed			2.901	45.997	0.006	1.39882	0.48214	0.42831	2.36932

In addition, Table 11 shows the impact of the Geogebra AR application in the obtained scores, which shows that 21.3% of the improvement of the visualization and spatial rotation skills can be attributed to the integration of the Geogebra AR application in the classroom methodology.

Table 11. Measures of association between groups for PSVT:R.

PSVT:R	Eta	Eta Square
Experimental or Control Score	0.462	0.213

3.3. Descriptive Analysis of the Evaluation Questionnaire

Finally, Table 12 presents the results obtained in relation to the data obtained from the Likert scale questionnaire in order to determine the motivation, feasibility, and perception of students in relation to the experience with AR technology. A total of 52.17% of students agreed to use AR resources for content learning and 65.21% believe that AR tools have helped improve their visualization and spatial rotation skills. It is noteworthy that virtually all students report having worked with great motivation and interest, and the vast majority of them confirm the ease of use of the application Geogebra AR.

Table 12. Descriptive analysis of the results of the application of the questionnaire.

Items	In Disagreement (%)		Indifferent (%)		In Agreement (%)	
	1	2	3	4	5	6
AR as a teaching-learning tool		4.36	13.04	30.43	34.78	17.39
AR as a spatial visualization tool		4.36	8.69	21.74	34.78	30.43
Motivation in learning through AR			4.36	8.69	52.17	34.78
Easy to use			8.69	21.74	39.14	30.43

4. Discussion

The results of the final evaluation test and the post-test demonstrate that there is a statistically significant difference in the level of achievement reached by students in the experimental group compared to those in the control group. From the findings of the study, it is evident that students who were exposed to a learning methodology with Geogebra AR (the experimental group) obtained better results both in the level of learning achieved in the formative unit functions and in their visualization and spatial rotation skills, compared to those students who were not exposed to learning supported by AR tools (the control group). Therefore, this finding suggests that the use of the Geogebra AR application as a support in the process of teaching and learning mathematical functions improved the academic performance and spatial intelligence of the students. This finding is related to the findings of Kaufmann and Schmalstieg [22] and del Cerro and Morales [12] about the effectiveness of AR tools in the teaching-learning processes in STEM knowledge areas and, especially, in all subjects where spatial intelligence is fundamental for the development of learning. In addition, the results of this study coincide with those obtained by Hohanwarter [74], who through the use of graphic software improved student performance in the study of functions. Likewise, the findings of this study also coincide with previous research where the software Geogebra was used in its version for personal computers with the objective of improving learning results in the subject of mathematics [61,75,76].

The students in the experimental group were exposed to a not yet fully established educational technology, which most likely captured their attention and interest during the lessons in which it was incorporated into classroom methodology. The interactive and dynamic nature of Geogebra AR allowed students in the experimental group to represent, visualize, rotate, analyze, and compare graphs of mathematical functions with ease. This allowed students to better understand the concept of function, identifying a greater number of characteristics of the function in relation to its form than the control group. In addition, the integration of this technology managed to enhance the proactive learning of students, as well as awakening their inquiry and need to know more. The students of the experimental group had the possibility to verify and evaluate the correction and accuracy of the results of their exercises in an autonomous way through Geogebra AR. Similarly, the experimental group was able to draw and analyze several graphs at the same time without having to perform algebraic calculations, tables of values, or draw by hand each one of them through the application, Geogebra AR. This, in general, made them complete the proposed activities in class in a shorter time than the control group, a factor that may have contributed to the deepening of contents and the higher score in the final written test than the one obtained by the control group.

For all these reasons, it is recommended that teachers integrate tools such as Geogebra AR as support in the resolution of activities for teaching mathematical functions, since it has proven to be effective in improving learning by reducing the effort of students in the tedious task of drawing functions manually, allowing them to focus on other more relevant elements, such as exploring and analyzing them.

Before the study, we discovered that not many ESO students could manipulate and use mathematical software effectively due to their lack of training, but this was not the case with Geogebra AR, in which most students excelled in its intuitive and simple operation, obtaining great results.

The integration of tangible tools such as Geogebra AR in a classroom changes the role of teachers, relocating them as a permanent guide that gives students more freedom and autonomy, as well as encouraging critical and creative thinking, instead of just being a transmitter of knowledge.

As authors, we can assert that the process of integrating Geogebra AR into classroom methodology has been simple and satisfactory. However it must be taken into account that educators must be well trained in the use and integration of ICT, such as mobile devices [77] and AR, in the teaching-learning processes. In this sense, if they are applied in an adequate way and always within a contextualized methodology, it can be very useful in not only facilitating teaching-learning processes, but also making them more interactive, motivating, and interesting [26].

5. Conclusions

Our study explicitly sought to transform the teaching-learning processes of mathematics, with the purpose of promoting mathematical skills linked to spatial intelligence, instead of focusing only on learning specific mathematical content. The integration of Geogebra AR through a contextualized methodology in the teaching-learning process of mathematical functions meant a significant difference in the levels of academic achievement and spatial intelligence of the students exposed to it [12,26]. The results also showed that the students had a positive perspective on the use of the application which managed to capture their attention and increase their motivation from the beginning.

AR technology has come to transform the concept of what, until now, was not possible to implement in the subject of mathematics, allowing efficient and effective learning experiences in the classroom, which must be accompanied by appropriate resources and methods to deepen and stimulate the skills of students [29]. The study evaluates the academic and cognitive achievement of students through the scores obtained in each of the tests and addresses other factors, such as motivation, which have influenced students to obtain this performance. Therefore, we can say that the value of any technology integrated in the classroom depends largely on the level of student engagement.

Lastly, Geogebra AR has proven to be an effective tool in teaching mathematical functions and improving students' spatial intelligence. Therefore, we recommend that teachers integrate this software in the development of learning activities, which can also be adapted for the development of other concepts, with other curricula at different teaching levels. Therefore, its relevance in the field of mathematics covers a wide range of possible uses.

This study was developed around the subject Academic Mathematics of the 4th year of ESO in order to investigate the effect of integrating Geogebra AR in the teaching-learning of functions. Given the scope and potential of the models learned in an interconnected and ubiquitous environment not yet established, the conclusions drawn from this work should be taken with prudence [78]. Therefore, the generalization of the results of this study to other content and levels of mathematical education should be made with caution.

Our findings can be used as a starting point for future research. For example, studies can be carried out to analyze the impact of the Geogebra AR application through mobile devices as part of the learning of mathematics in different situations and contexts inside and outside the classroom (b-learning). This includes integrating our study to the current educational context to effectively stimulate self-learning, improve levels of attention, and motivate students through the paradigm shift caused by the Covid-19 pandemic.

Finally, we recommend that future studies perform qualitative meta-analyses to assess educators' perceptions towards the use and integration of emerging ICTs, such as AR, in the teaching of STEM areas.

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Classroom Methodologies for Teaching and Learning Ordinary Differential Equations: A Systemic Literature Review and Bibliometric Analysis

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Abstract: In this paper, we develop a review of the research focused on the teaching and learning of ordinary differential equations with the following three purposes: to get an overview of the existing literature of the topic, to contribute to the integration of the actual knowledge, and to define some possible challenges and perspectives for the further research in the topic. The methodology we followed is a combination of a systematic literature review and a bibliometric analysis. The contributions of the paper are given by the following: shed light on the latest research in this area, present a characterization of the actual research lines regarding the teaching and learning of ordinary differential equations, present some topics to be addressed in the next years and define a starting point for researchers who are interested in developing research in this field.

Keywords: teaching differential equations; teaching mathematics; mathematical modeling; solving problem

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1. Introduction

The teaching and learning of ordinary differential equations has experienced a dramatic change in the last two decades [1–19]. The motivation for innovation in the traditional teaching obey different reasons, at least three of those are given below. First, from the second half of the 20th century until now, the ordinary differential equations have been recognized as useful tools for teaching and learning mathematical models arising in different areas of science like physics, biomathematics, engineering, and chemistry [20–26]. Second, in our current era, the development of information technologies has strongly influenced and modified the traditional ways of inquiring in science. In particular, the information technologies have increased the innovation and application of numerical methods which are essential to solve a wide class of differential equations and are also useful to understand some qualitative properties [10,11,19,27–30]. Third, as a consequence of the above, in the last years, more attention has been given to the transformation of teaching and learning mathematical concepts by incorporating didactic methodologies to encourage students to be actively engaged in their learning process [21,31–34]. Thus, in a brief sense, the changes in the teaching of differential equations has been mainly influenced by the incorporation of active learning didactic methodologies and technology enriched learning environments.

Traditionally the curricula in many careers, like engineering, physics, mathematics, or statistics, begin with three courses of calculus (differential, integral, and several variables) and they are followed by an ordinary differential equations course. From the last decade of the twentieth century, several efforts to change the calculus curriculum have been proposed and conducted by numerous authors worldwide [15,35,36]. Specifically, in the teaching of differential equations, the changes consider new contents, new pedagogical methods, and the incorporation of the exploration of dynamical systems concepts with graphical (or qualitative) and numerical approximations by using technological resources. Nowadays, the study of curricular modifications that must be undertaken in order to adequately overcome the diverse deficiencies and difficulties in the teaching and learning of differential equations is a very active topic of research with different subjects and perspectives.

Despite the interest in the curricular innovation for the teaching and learning of ordinary differential equations, we have found that the research lines for the next years are still diffusely stated. Some advances for integration of the findings from the research can be seen in the articles [21,37]. In [21], the authors developed an extensive bibliographic survey of 16 works published between 2000 and 2011. Meanwhile in [37], the authors focused on the factors that influence the problem solving abilities for undergraduate students in differential equations. However, to the best of our knowledge, there is not a literature review with an open period of time and the specific didactic methodologies are missing in those works. In other words, a literature review to find, critically evaluate, and synthesize the relevant research topics related to the teaching and learning of ordinary differential equations remains open.

Consequently, in the light of the increasing development of research related to teaching methods and, in order to lead emerging trends and challenges of teaching and learning ordinary differential equations, it is evident the lack of a systematic study of the existing literature. To shed light on this gap, in this paper we present an analysis of the literature related to teaching and learning differential equations, based on a systematic review and a bibliometric investigation. We propose a systematic arrangement of the main existing literature. Thus, we follow the methodology of five steps introduced in [38]: (i) framing questions for a review, (ii) identifying relevant work, (iii) assessing the quality of studies, (iv) summarizing the evidence, and (v) interpreting the findings. For step (i), we considered three questions. In the case of (ii), we retain 120 articles that come from the following databases: Web of Science, Scopus, Qualis, Zbemath, and Scielo. In step (iii), we provide some statistical properties of the retained literature. In steps (iv) and (v), we expose explicitly the subjects of ordinary differential equations covered by the research, the teaching methodologies used in the classroom, and also we present the answers to the questions of step (i). Then, in step (v), we summarize our findings.

We survey a set of 120 articles from 1970 to 2020, where initially two standard classifications for teaching differential equations were identified. The first considers the traditional and contemporary teaching methodologies pointed, for example, by [1]. The second classification includes the separation in analytic, graphical, and numerical approaches advised by some authors [39,40]. However, these classifications are currently imprecise, since the actual state-of-the-art in the research is extensive and there are works which are out of those classes, for instance the teaching under the mathematical modeling as a cyclic process can gather the analytic, graphical, and numerical approaches as a particular phase of the cycle.

The main improvements for the research field which are established in this article are described below. From our analysis of the set of 120 articles we mainly obtained the following contributions:

1. We propose a classification for the research in teaching and learning ordinary differential equations according to the didactic methodologies: traditional teaching and learning methodology, graphical and numerical approaches to the teaching, active learning methods, mathematical modeling-based methodology, information and communication technology-based methodologies, and project-based learning.
2. We introduce five groups given a categorization of the mathematical topics addressed in the papers: basic concepts of ordinary differential equation, biomathematical models, scalar-based models, systems-based on physic models, and other concepts.
3. We found that results about effectiveness of innovation were reported only in a few articles.

Moreover, the review of the literature shows an increasing trend since the first research around 1990. The best ranked journal regarding to the h-Index in the area of Mathematical

Education is "The journal for research in mathematics education" and the most prolific author is Chris Rasmussen with 13 articles in the collected list, and the article with the largest number of cites in Google scholar is [15] with a total of 196 citations. Some conclusions are established by bridging the different influential perspectives of the main works. We also highlight some possible challenges and perspectives for further research of the topic.

The paper is organized as follows. In Section 2, we describe the methodological approach used in this research. In Section 3, we formulate the questions that guide the review. In Section 4, we describe how the relevant work was identified. In Section 5, we develop a bibliometric analysis of the literature. In Sections 6 and 7, we summarize the review and present a discussion. Finally, in Section 8, we draw some conclusions with short comments about some possible challenges and perspectives for further research.

2. Research Methodology

In order to define the methodology supporting this research, we recall that there are at least three approaches related with the literature review: the bibliometric analysis, the systematic literature review, and narrative review [41,42]. The goal of a bibliometric analysis is to develop a quantitative research by applying statistical methods in order to evaluate several characteristics of specific bibliographic information like journals, research institutions, geographic location, and other characteristics [43]. The narrative literature review is developed to provide an overview of a large spectrum for some specific topic chosen by the author and is based on available literature on their particular interest, is descriptive, and written in a friendly readable format [44]. Meanwhile, the systematic literature review has two principal goals: to develop an extensive literature search with a very detailed process; and to give a critical evaluation of the selected literature. Moreover, the researchers who develop literature review recognize that the systematic reviews contain an explicit a priori strategy which is detailed and comprehensive, reducing the appraising when identify the relevant studies.

For the present study, our methodology is a combination of a systematic review and a bibliometric analysis. More precisely, firstly we develop a systematic review of the literature following the five steps introduced in [38]:

Step 1. Framing questions for a review.
Step 2. Identifying relevant work.
Step 3. Assessing the quality of studies.
Step 4. Summarizing the evidence.
Step 5. Interpreting the findings.

Particularly, in Step 2 we generate a list of references that was explored using a bibliometric analysis with particular well defined quantitative indicators in Step 3.

3. Framing Questions for a Review (Step 1)

We follow the discussion given by Benitti [45] to establish the following three research questions:

Question 1: What are the studies developed for teaching and learning of ordinary differential equations with a reported classroom experiences? What types of didactic methodologies have been used in those studies?
Question 2: What topics of ordinary differential equations have been explored in the previous studies?
Question 3: What are the results for the effectiveness of traditional and new didactic methodologies to teach and learning ordinary differential equations, as reported in previous studies?

4. Identifying Relevant Work (Step 2)

To answer our research questions, we drew on multiple resources to identify the topics of differential equations and the teaching methodology that were most mentioned in the papers. We proceed in several steps as is specified below (see Figure 1 for a summary):

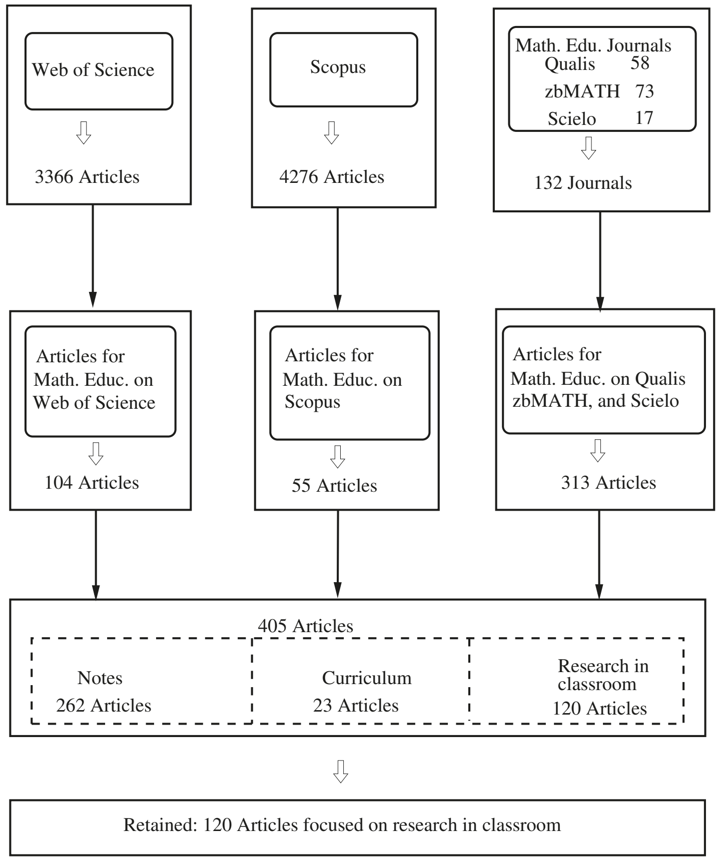

Figure 1. Schematic presentation of the process used for identify the relevant work. For specifications, consult the items (a) to (e) in Section 4.

(a) We have selected the following databases:
- Web of Science (https://clarivate.com/products/web-of-science accessed on 8 August 2020),
- Scopus (https://www.elsevier.com/solutions/scopus accessed on 8 August 2020),
- Qualis (http://qualis.capes.gov.br accessed on 8 August 2020)
- Zbmath (https://zbmath.org accessed on 8 August 2020), and
- Scielo (https://scielo.org accessed on 8 August 2020).

(b) In the case of Web of Science and Scopus, we have derived two major keywords to answer *Questions 1–3* and we have replaced them in the search engine of databases by some synonyms and some alternative terms, as specified below:

Ordinary differential equations. Differential equation; solution to differential equations; graphical interpretation; graphical solution; qualitative solutions; numerical solutions; analytic solutions; first order equations; higher order equations; Laplace transform; power series method; variable separable equation; reducible to variable separable equation; homogeneous equation; reducible to homogeneous equation; exact equation; reducible to exact equation; Bernoulli equation; linear equation; Ricatti equation; phase plane; isoclines; slope fields;

equilibrium; stability of solutions; initial value problems; boundary value problems; scalar equations; systems of equations; linear; nonlinear.

Didactic methodologies. Teaching methodologies; students' understanding and difficulties; interpretation of solutions; registers of representations; mathematical modeling; mathematical models; problem-based learning; problem solving; error analysis; mathematics teaching practices; real world situation; computational resources; mathematical application; classroom discourse; didactic of differential equations; critical discourse analysis.

More specifically the strings are given in Appendix A. First we searched the list of selected words in all fields of the search engine of databases, i.e., in titles, article keywords, abstracts, author, topic, and full paper text.

The search on Web of Science was restricted to all journals indexed to "Science Citation Index Expanded (SCI-Expanded)", "Social Sciences Citation Index (SSCI)", "Arts & Humanities Citation Index (A&HCI)", and "Emerging Sources Citation Index (ESCI)". We get a total of 342,179 publications. Then, we refined the results using the "Document Types" option by "article" and the option "Web of Science Categories" by "Social Sciences Mathematical Methods or Education Educational Research or Education Scientific Disciplines" generating a list of 3366 articles.

In Scopus, when restricting the search to Document Type "article", a total of 23,967 publications were found. Then, we refined the option "subject area" by selecting "psychology or "social sciences", getting a list of 4276 articles.

(c) In the case of Qualis, zbMATH, and Scielo. we selected the journals associated to Mathematics Education as specified below. In the database Qualis, we find that a total of 1434 journals are associated to quadrennium 2013–2016 and are classified as A1, A2, B1, B2, B3, B4, B5, and C in the evaluation area Teaching (ensino). Then, we selected a list of 58 journals associated with Mathematics Education, see Table 1. For zbMATH database, we used the list of journals suggested by Godino [46], where the author present a list of journals from zbMATH classified in two sections labeled as "Serie A" and "Serie B" journals. Moreover, in each category there are three groups or types of journals called A, B, and C, the total of journals of each serie and the corresponding types are summarized in Table 1. Now, from Scielo database we have selected a total of 17 journals associated with the scope in Mathematics Education. Thus, combining the three list of journals and deleting the duplicated ones, we get a list of 132 journals, see Table A1 in Appendix B.

(d) We examined the titles, abstract, and full paper text in the list of papers from Web of Science and Scopus generated in step (b). Then, we retained the paper if it was related to the teaching and learning of ordinary differential equations. After a careful examination, we have identified 104 and 55 articles from Web of Science and Scopus, respectively. Moreover, in the case of the selected journals of step (c), we have applied two types of searches: (i) we consulted the index of each volume of the journal from the years specified on the column labeled as "Years Consulted" in Table A1 and (ii) we have searched for key words in the search engines of each journal. As a result, a total of 313 articles were considered to be analyzed.

(e) Combining the three list of articles and deleting the duplicated ones, we get a list of 405 articles. Then, in order to focus our analysis on classroom methodologies, we classified the 405 articles in three types: *notes, curriculum, and research in classroom*. We consider that an article is a *note* or a *classroom note*, when there is a proposal for teaching some concepts related to differential equations, but there is not a specific didactic methodology or at least, it was never implemented in the classroom. In the class *curriculum*, we consider all works where the aim of the paper was the curriculum innovation proposal and there is not an specific application in the classroom. Meanwhile, we assume that a paper is of the type *research in classroom*, when there is a proposal to teach some topic of ordinary differential equations, there is an explicit didactic methodology, and also includes the implementation in the classroom with a well detailed report of the experience. Thus, by a revision of all 405 papers, we deduce that there were a total of 262, 23 and 120 articles belonging to types classroom notes, curriculum, and research in classroom, respectively. In Figure 2, we present a classification by year and by decade from 1970 to 2020. An isolated case, which is not presented in Figure 2, is the classroom note [47] published in 1913.

Table 1. Number of journals for Mathematics Education on Qualis and zbMATH databases, see also Appendix B.

Classification Qualis	**A1**	**A2**	**B1**	**B2**	**B3**	**B4**	**B5**	**C**	**Total**
J. of Math./Prob.	120	145	260	229	170	135	179	196	1434
J. Math. Educ.	15	15	9	1	8	4	3	3	58
Classification for zbMATH [46]	**A**	**B**	**C**	**Total**					
Serie A: Didact. of Math.	3	14	16	33					
Serie B: Related Areas	27	4	9	40					

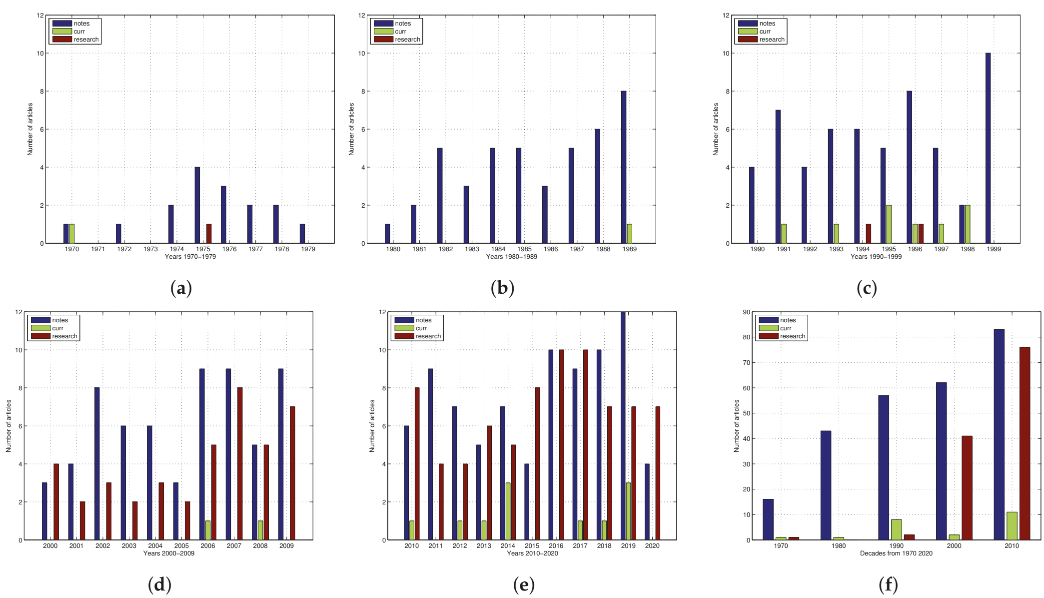

Figure 2. Number of articles from 1970 to 2020. Here the abbreviation notes, curr, and research are used for the types *notes*, *curriculum* and *research in classroom*, respectively. (**a**) Articles from 1970 to 1979. (**b**) Articles from 1980 to 1989. (**c**) Articles from 1990 to 1999. (**d**) Articles from 2000 to 2009. (**e**) Articles from 2010 to 2020. (**f**) Articles by decades from 1970 to 2020.

On the other hand, we also have identified and counted the geographic location declared by the authors in the corresponding affiliation of each article, see Figure 3. We registered the affiliations of each coauthor and then we counted all coincidences of a given region location. The regions with the highest number of records are United States of America (USA), United Kingdom (UK), and Australia with 110, 86, and 29 records, respectively. The ranking is followed by Brazil, Denmark, Germany, India, Israel, Mexico, Spain, and Turkey, which have between 6 and 29 records, see Figure 3a for percentages. Moreover, the following 50 regions have at most 6 records (less than 2%):

Argentina, Azerbaijan, Bahrain, Brunei, Canada, Chile, China, Colombia, Costa Rica, Cuba, Czechia, Ethiopia, France, Ghana, Grece, Holland, Hungary, Iceland, Iran, Iraq, Italy, Kenya, Lebanon, Libya, Lithuania, Malaysia, Netherlands, New Zeland, Nigeria, Norway, Perú, Poland, Portugal, Romania, Russia, Saudia Arabia, Serbia, Singapore, Slovakia, Slovenia, South Africa, South Korea, Spalj, Sweden, Switzerland, Taiwan, Ukraine, United Arab Emirates, and Uruguay.

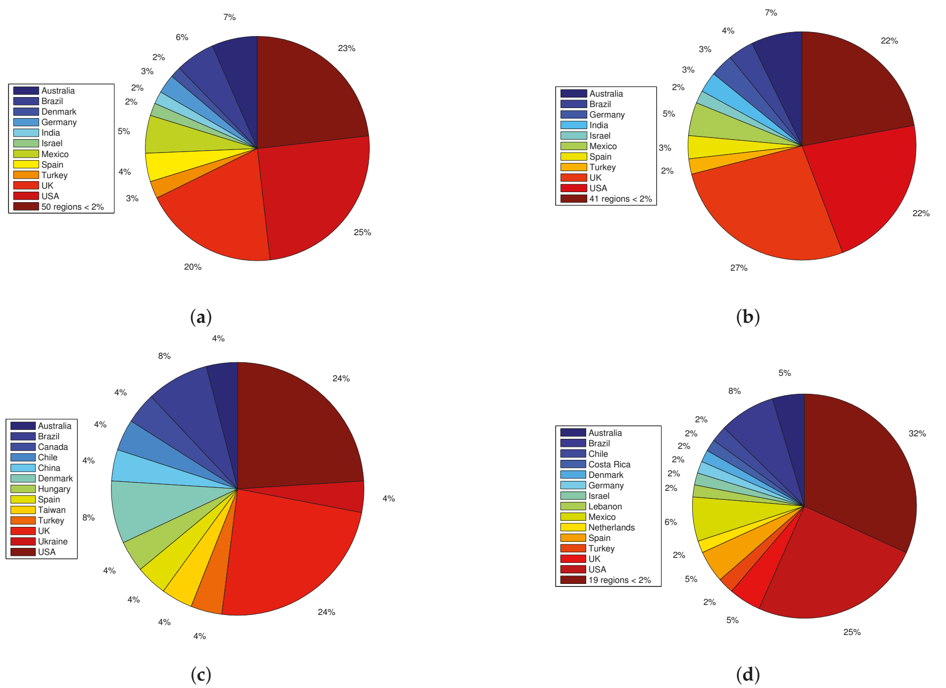

Figure 3. Percentages of the number of papers according to the geographic location declared by the authors in the corresponding affiliation of each article. We remark that the percentages are rounded off by its integer part, then apparently in (**a**) and (**d**) the total percentages are more than 100%. (**a**) Regions for authors with all types, (**b**) regions for authors with articles of *notes* type, (**c**) regions for authors with articles of *curriculum* type, (**d**) regions for authors with articles of *research in classroom* type.

In the case of *notes*, we find that UK (74 records), USA (61 records), and Australia (20 records) are the regions with the highest number of records. Brazil, Germany, India, Israel, Mexico, Spain, and Turkey, appear with more than 4 and less than 20 records. Moreover, we get that the following 41 regions have less than 4 records each, see Figure 3b:

Argentina, Azerbaijan, Bahrain, Brunei, Canada, China, Colombia Cuba, Denmark, Ethiopia, France, Ghana, Grece, Holland, Hungary, Iceland Iran, Italy, Kenya, Libya, Lithuania, Malaysia, Netherlands, New Zeland, Nigeria, Norway, Peru, Poland, Portugal, Russia, Saudia Arabia, Serbia, Singapore, Slovakia, South Africa, South Korea Spalj, Switzerland, United Arab Emirates, and Uruguay.

In the ranking for regions with publications related to *curriculum*, the first two places are for UK, USA, Brazil, and Denmark with a total of 6, 6, 2, and 2 records, respectively. Moreover, each of the following 9 regions: Australia, Canada, Chile, China, Hungary, Spain Taiwan, Turkey, and Ukraine, have associated 1 record, Figure 3c. Now, corresponding to articles of type *research in classroom*, the regions with highest number of registered affiliations are USA, Brazil, and Mexico with 43, 14, and 11 records, respectively. The ranking of *research in classroom* type regions is followed by Australia, Chile, Costa Rica, Denmark, Germany, Israel, Lebanon, Netherlands, Spain, Turkey, and UK with the percentages given in Figure 3d. Moreover, the following 19 regions appear with less than 2 records: Argentina, Colombia, New Zeland, South Korea, Sweden, Ukraine, Canada, Cuba, Czechia, France, Iran, Iraq, Malaysia, Norway, Romania, Singapore, Slovakia, Slovenia, and Taiwan.

Hereinafter, unless stated otherwise, **the retained list** or **the retrieved list** refer to the 120 papers which will be analyzed and are explicitly given by the following references: [1–23,27–34,39,48–135]. The other 285 articles (*notes* and *curriculum*) will be presented and analyzed in a forthcoming work by the authors.

5. Assessing the Quality of Studies (Step 3)

In this section, in order to assess the quality of the 120 articles of *research in classroom* type retrieved and selected in Section 4, we develop a bibliometric study by considering several characteristics to capture the impact of articles, authors, and journals. Amongst the literature characteristics and indicators, which are frequently used in bibliometric analysis, we consider the number of citations, the ranking of authors, the ranking of journals, and the geographic location [43]. Thus, we identify the following characteristics in the analyzed documents:

(i) *Total number of publications by geographic location.* In Section 4, we present initial information regarding the location of origin, which is declared in the affiliation of authors. The top three regions are USA, Brazil, and México with percentages of coincidences of 25%, 8%, and 5%, respectively; see Figure 3d.

(ii) *Total number of publications by year.* In Figure 2, we present six histograms considering all the articles (*notes*, *curriculum*, and *research in classroom*) decade by decade from 1970 to 2020. In Figure 2f, we can see an increase in the number of publications along the years. The first publications are from 1970s, one related to *research in classroom* and one related to *curriculum*, see Figure 2a. Regarding *research in classroom*, the first article retained is from 1975 and is unique in the 1970s. The 2010s have the largest number of publications with a record count of 76 papers, where 2016 and 2017 were the years with more publications with 10 records each year, see Figure 2f,e, respectively. The graphs also show how the research related to *curriculum* has evolved slowly.

(iii) *The most prolific journals.* There are 46 journals associated to the retained list of papers. Table 2 shows the 13 journals which are in the top four positions according to the number of published articles. The first three journals show a similarity in the declared scope, all of them are focused on teaching and learning mathematics. These journals publish research regarding learning and teaching mathematics for different scholar levels and particularly for undergraduate mathematics. These coincidences in the journals' aims are probably the reason which they have the most publications in the area of differential equations, which is traditionally a topic of undergraduate mathematics. Additionally, we found 33 journals with less than 3 publications each one, which are distributed as follows: 9 and 24 journals with 2 and 1 articles, respectively.

(iv) *Ranking of journals by the H index.* In Table 3 we show the top 11 retained journals according to the H index of SCImago Journal & Country Rank (https://www.scimagojr.com/ accessed on 1 September 2020), where the indicator SJR 2019 is also included, quartil, and subject area of those journals.

(v) *The most prolific authors.* Table 4 shows the most prolific authors in the retained list of *research in classroom* articles. The top author in the field is Chris Rasmussen with 12 articles (9.52%). Moreover, we observe that there are four authors from the USA which is naturally related with the higher impact of the research developed in the field by institutions from the USA, see Figure 3d.

(vi) *The impact of articles.* In Table 5, we show the top 10 articles, where the ranking is established by the number of citations reported in google scholar in September 2020. We observe that the research line introduced by Rasmussen and collaborators in the

2000s decade is one of the most prolific, since 8 of the top 10 articles are authored or coauthored by Chris Rasmussen.

Some additional bibliometric characteristics of the *research in classroom* articles, are the following: 92 articles are written in English, 18 in Spanish, and 10 in Portuguese, from which 3 articles [29,61,66] are applied for teaching and learning ordinary differential equations in high school students and the rest of articles (117) for undergraduate students.

Table 2. The top four journals according to the number of published articles.

Rank	Journal	Record Count	% of 120
1°	Teaching mathematics and its applications	16	13.33%
	International journal of mathematical education in science and technology	16	13.33%
2°	The journal of mathematical behavior	13	10.83%
3°	ZDM–Mathematics Education	6	5.00%
4°	Computer applications in engineering education	3	2.50%
	Educação matemática pesquisa	3	2.50%
	Educación matemática	3	2.50%
	Enseñanza de las ciencias: revista de investigación y experiencias didácticas	3	2.50%
	European journal of engineering education	3	2.50%
	International electronic journal of mathematics education	3	2.50%
	International journal of science and mathematics education	3	2.50%
	PRIMUS: problems, resources, and issues in mathematics undergraduate studies	3	2.50%
	Revista latinoamericana de investigación en matemática educativa	3	2.50%

Table 3. The top 11 journals according the H index reported by Scimago Journal & Country Rank, where particularly there are 7 journals in the subject area of Education. The quartil and subject area of Physical review special topics is not assigned yet.

Rank	Journal	H Index	SJR 2019	Quartil	Subject Area and Category
1°	American journal of physics	88	0.51	Q2	Physics and astronomy (miscellaneous)
2°	Journal of chemical education	77	0.47	Q2	Physics and astronomy (miscellaneous)
3°	Journal for research in mathematics education	74	2.92	Q1	Education
4°	Educational studies in mathematics	60	1.57	Q1	Education
5°	Journal of science education and technology	56	1.17	Q1	Education
6°	Advances in physiology education	55	0.52	Q2	Education
7°	International journal of engineering education	47	0.45	Q1	Engineering (miscellaneous)
8°	European journal of engineering education,	41	0.7	Q1	Engineering (miscellaneous)
	Physical review special topics-physics education research	41	–	–	–
9°	Journal of professional issues in engineering education and practice	37	0.45	Q2	Civil and structural engineering
10°	ZDM–Mathematics Education	36	1.08	Q1	Education
11°	International journal of science and mathematics education	35	0.9	Q1	Education
	Journal of mathematics teacher education	35	1.96	Q1	Education

Table 4. Authors with the highest number of articles in the retained list.

Author	Institution	Number of Articles
Chris Rasmussen	San Diego State University, USA	13
Matías Camacho-Machín	University of La Laguna, Spain	4
Samer Habre	Lebanese American University, Lebanon	4
Debasree Raychaudhuri	California State University, USA	4
Lourdes Maria Werle de Almeida	State University of Londrina, Brazil	3
Carolina Guerrero-Ortiz	Potificia Universidad Católica de Valparaiso, Chile	3
Karen Allen Keene	North Carolina State University, USA	3
Karen King	Michigan State University, USA	3
Oh Nam Kwon	Seoul National University, South Korea	3
José Arturo Molina-Mora	Universidad de Costa Rica, Costa Rica	3

Table 5. Top 10 articles followed by the number of citations in Google scholar.

Article Title and Reference	Number of Cites
New directions in differential equations: a framework for interpreting students' understandings and difficulties [15].	196
Advancing mathematical activity: a practice-oriented view of advanced mathematical thinking [15].	187
An inquiry-oriented approach to undergraduate mathematics [109].	186
Classroom mathematical practices in differential equations [125].	184
Teaching mathematical modeling through project work [53].	174
Knowledge needed by a teacher to provide analytic scaffolding during undergraduate mathematics classroom discussions [124].	171
Social and sociomathematical norms in an advanced undergraduate mathematics course [133].	160
Students' retention of mathematical knowledge and skills in differential equations [89].	136
Locating starting points in differential equations: a realistic mathematics education approach [16].	135
Classroom mathematical practices and gesturing [106].	106

6. Summarizing the Evidence (Step 4)

To approach the answer to the questions presented in Section 3, we gathered and selected the relevant information from the retained list of publications (see last paragraph of Section 4). In Table 6, a synthesis with focus on didactic methodology and topics taught or evaluated is showed. More details related to the didactic methodologies (traditional methodology, mathematical modeling, etc.) will be presented in Section 7. The articles with empty topic are those where the topic covered was not specified. Moreover, related to the question of the reported effectiveness of the new didactic methodologies in comparison with the traditional methodology, we found that few articles address explicitly this topic. From the list in Table 6, the following articles: [33,34,55,57,64,66,76,79,84,89,92,123,126,135] provide an explicit treatment of effectiveness.

Table 6. Summary of didactic methodologies and the topics of ordinary differential equations declared on the list of retained list of papers (see last paragraph of Section 4). Here Ref. is used for abbreviation of the reference number in the list of references.

Ref.	Didactic Methodology	Topics Taught or Evaluated
[1]	Traditional methodology	Scalar: first order, second order, orthogonal curves, existence and uniqueness theorem
[2]	Traditional methodology	Scalar: first order, second order, orthogonal curves, existence, and uniqueness theorem
[3]	Geometric and qualitative solutions, Active learning, Information and communication technology	Scalar: first order, applications to exponential decay problems
[4]	Geometric and qualitative solutions, Mathematical modeling, Information and communication technology	Scalar: Malthus model, logistic generalized
[5]	Mathematical modeling	Scalar: second order and applications to electronic circuits
[6]	Active learning	Scalar and systems: Laplace Transform
[7]	Active learning	
[8]	Mathematical modeling	Scalar: first order, applications to mixing problems, freefall problems
[9]	Mathematical modeling	Scalar: first order, applications to mixing problems, second order
[10]	Information and communication technology	Systems: Lotka–Volterra model
[11]	Information and communication technology	Systems: plane phase, linear system, qualitative behavior
[12]	Information and communication technology	Scalar: first order, slope fields, asymptotic behavior
[13]	Mathematical modeling, Information and communication technology	Scalar: first order, logistic generalized
[14]	Geometric and qualitative solutions, Active learning	Scalar: rate of change
[15]	Active learning	Scalar and systems: several topics
[16]	Active learning	Scalar and systems: several topics
[17]	Mathematical modeling	Scalar: first order, applications to electronic circuits
[18]	Information and communication technology	Scalar: first order, applications to electronic circuits
[19]	Information and communication technology	Scalar: first order, second order, graphical solution, Laplace transform
[20]	Mathematical modeling	Scalar: Malthus model, Verhulst model, equilibrium analysis.
[21]	Others	
[22]	Geometric and qualitative solutions Mathematical modeling	Systems: applications for asthma
[23]	Active learning	Scalar: first order
[27]	Information and communication technology	Systems: applications for Chemical reactions
[28]	Information and communication technology	Scalar: first order, second order, graphical solution, Laplace transform
[29]	Information and communication technology	Scalar: first order, freefall problems
[30]	Project-based learning	
[31]	Projects-based learning	Scalar: first order, applications to tumor growth, Gompertz model, graphical solutions, bifurcation
[32]	Information and communication technology	Scalar and systems: several topics
[33]	Information and communication technology	Scalar and systems: first order, Laplace transform, application to chemical reaction and control
[34]	Information and communication technology	Scalar and systems: several topics
[39]	Traditional methodology, Geometric and qualitative solutions	Scalar: first order, applications to exponential decay problems
[48]	Mathematical modeling	Scalar: Newton's law of cooling and laws for velocity, acceleration and volume

Table 6. *Cont.*

Ref.	Didactic Methodology	Topics Taught or Evaluated
[49]	Information and communication technology	Scalar: Verhulst model, generalized logistic.
[50]	Active learning	Scalar: first order, linear, Bernoulli
[51]	Active learning	Scalar: first order, applications to exponential decay problems
[52]	Projects-based learning	
[53]	Projects-based learning	Scalar and systems: populations model, linear system
[54]	Mathematical modeling	Scalar: first order, applications to mixing problems
[55]	Active learning	
[56]	Geometric and qualitative solutions	Scalar: first order
[57]	Traditional methodology	Scalar: linear higher order
[58]	Mathematical modeling	Systems: equilibrium
[59]	Traditional methodology	Scalar: first order
[60]	Information and communication technology	Scalar: applications to electronic circuits
[61]	Others	Scalar: First order
[62]	Others	
[63]	Mathematical modeling	Scalar: Malthus model
[64]	Traditional methodology, Geometric and qualitative solutions	Scalar: first order, second order, graphical solution, slopy fields
[65]	Mathematical modeling	
[66]	Active learning	Scalar: first order, second order, several applications (biomedical, scientific, and social-economic contexts)
[67]	Active learning	Scalar: First order, Verhulst equation
[68]	Mathematical modeling	Scalar and systems: linear, exponential, logistic, and ecology applications
[69]	Geometric and qualitative solutions	Scalar: first order
[70]	Geometric and qualitative solutions	Scalar: first order
[71]	Geometric and qualitative solutions, Active learning	Scalar and systems: first order, autonomous differential equations, slope fields, Lotka–Volterra models
[72]	Geometric and qualitative solutions	Scalar and systems: graphical solutions
[73]	Active learning, Information and communication technology	Scalar: first order, applications to mixing problems
[74]	Information and communication technology	Systems: first order, validation with real data
[75]	Information and communication technology	Scalar: first order
[76]	Traditional methodology	Scalar: first order, applications to kinetics
[77]	Others	Scalar: Laplace transform
[78]	Mathematical modeling	Systems: first order
[79]	Active learning	Scalar: first order, second order, slope fields, several applications
[80]	Traditional methodology	Scalar: first order
[81]	Active learning	
[82]	Traditional methodology	Scalar: first order, higher order
[83]	Others	
[84]	Geometric and qualitative solutions, Active learning	Scalar: first order, autonomous differential equations, slope fields
[85]	Active learning	Scalar: first order, Newton's law of cooling
[86]	Active learning	Systems: first order, linear, slope fields, Lotka–Volterra models
[87]	Active learning	Scalar: first order, autonomous differential equations, slope fields
[88]	Mathematical modeling	Systems: Lotka–Volterra model, phase plane, equilibrium solutions, phase trajectories

Table 6. *Cont.*

Ref.	Didactic Methodology	Topics Taught or Evaluated
[89]	Active learning	Scalar and systems: first order, second order, slope field, linear system
[90]	Projects-based learning	
[91]	Mathematical modeling	Scalar: first, order, Malthus model, Newton's law of cooling
[92]	Information and communication technology	Systems: second order, applications to vibration of a two-mass two-spring problems
[93]	Active learning	Scalar: first order, zombies models
[94]	Mathematical modeling, Information and communication technology	Scalar and systems: first order, freefall problems, generalized Lotka–Volterra
[95]	Active learning, Information and communication technology	Scalar: second order
[96]	Active learning	Scalar: first order, second order
[97]	Geometric and qualitative solutions	Scalar and systems: first order, Lotka–Volterra model, Euler methods
[98]	Mathematical modeling	Scalar: first, order, Malthus model, AIDS models
[99]	Information and communication technology	Scalar and systems: amplitude and phase in second order equations, linear phase portraits in linear system, Fourier coefficients, vibrations applications
[100]	Mathematical modeling, Information and communication technology	Laplace Transform, Newton's law of cooling
[101]	Information and communication technology, Projects-based learning	Systems: Lotka–Volterra model, pancreatitis model
[102]	Information and communication technology	Scalar: first order, applications to energy balance, chemical process and control fundamentals
[103]	Information and communication technology	Systems: applications to epidemics
[104]	Others	Scalar: first order
[105]	Others	Scalar: firs order
[106]	Active learning	
[107]	Active learning	Scalar and systems: Verhulst equation, bifurcation
[108]	Active learning	
[109]	Active learning	Scalar and systems: first order, slope fields, second order with spring-mass applications, linear systems, straight-line solutions, Lotka–Volterra models
[110]	Geometric and qualitative solutions, Active learning	
[111]	Active learning	Scalar: existence and uniqueness theorem of first order
[112]	Active learning	Scalar: concept of solution of first order equation
[113]	Active learning	Scalar: first order
[114]	Active learning	Scalar and systems: several topics
[115]	Mathematical modeling, Information and communication technology	Systems: applications to electronic circuits
[116]	Mathematical modeling	Scalar: second order, applications of Newton's second law
[117]	Others	Scalar and systems: second order, applications of Newton's second law
[119]	Mathematical modeling	Scalar: first order
[118]	Mathematical modeling	Scalar: first order
[120]	Information and communication technology	Scalar: first order, Laplace transform
[121]	Active learning	Scalar: first order, applications to electronic circuits
[122]	Information and communication technology	Scalar: first order, applications to molar and energy balances

Table 6. *Cont.*

Ref.	Didactic Methodology	Topics Taught or Evaluated
[123]	Traditional methodology, Information and communication technology	Scalar: definition of differential equations, graphical solution, applications.
[124]	Active learning	Scalar: first order, Malthus model
[125]	Active learning	Scalar: first order
[126]	Traditional methodology	Scalar: first order
[127]	Mathematical modeling	Scalar: first order
[128]	Mathematical modeling	Scalar: first order, Bernoulli's equation
[129]	Traditional methodology	Several topics
[130]	Mathematical modeling	Scalar: first order, population mathematical model
[131]	Traditional methodology	Scalar: first order, freefall problems
[132]	Active learning	Scalar: first order
[133]	Active learning	Scalar: first order
[134]	Mathematical modeling	Scalar and systems: first order, applications to exponential decay problems, Lotka–Volterra model
[135]	Projects-based learning	Several topics of noise and vibrations concepts

7. Interpreting the Findings (Step 5)

After gathering, filtering, synthesizing, and analyzing the main contributions of each paper of the retained list, in this section we address the answers to the framing questions introduced in Section 3.

7.1. Question 1: What Are the Studies Developed for Teaching and Learning of Ordinary Differential Equations with a Reported Classroom Experiences? What Types of Didactic Methodologies Have Been Used in Those Studies?

To answer this question, we recall Section 4 where we identified 405 articles which were classified in *notes* (262), *curriculum* (23), and *research in classroom* (120), see Figure 1. In the case of *notes* and *curriculum* types of articles, there are no reported empirical applications of classroom experiences. Thus, there are 120 articles with classroom experiences, which are explicitly specified at the end of Section 4 and in the first column of Table 6. Now, regarding the didactic methodologies, we have identified seven groups:

- the traditional teaching and learning methodology,
- graphical or qualitative and numerical approach of teaching,
- active learning methods,
- The mathematical modeling-based methodology,
- information and communication technology-based methodologies,
- project-based learning, and
- other methodologies,

Each classification is discussed below. There are many works that can be included in more than one classification, so we decided to include the paper in a group according to the aim declared by the authors.

7.1.1. The Traditional Teaching and Learning Methodology

The traditional teaching is focused in solving ordinary differential equations by applying algebraic or analytic methods, where solving means that we can find an explicit or implicit expression for the unknown function [69]. Those methods are characterized by being algorithmic, procedural, symbolic, and particularly related with a specific type of differential equation. For instance, the traditional teaching of first-order ordinary differential equations can be summarized in two steps: (i) the educator introduces the general form of the equation by writing the following two equivalent forms

$$\frac{dy}{dx} = f(x,y) \quad \text{or} \quad M(x,y)dx + N(x,y)dy = 0,$$

where f, M and N are given functions from $D \subset \mathbb{R}^2$ to $\mathbb{R}$, followed by the introduction of the classification as separable, homogeneous, exact, linear, Bernoulli and others, depending on the functions f, M, N, see Table 7; and (ii) the educator teaches the students their own algorithmic solution technique for each class of equation, where the algebraic manipulation and the integration of functions are essential techniques common to all classes. Two similar steps of teaching are also applied to higher-order ordinary differential equations and for first-order systems of differential equations. Thus, according to [123], the traditional approach to teaching differential equations consists of the use of a wide variety of algebraic or analytic methods for solving different type of problems.

Table 7. Typical classification of first-order ordinary differential equations.

Class	Properties of Functions f, M or N
Separable	$f(x,y) = h(x)g(y)$ with h and g real functions
Homogeneous	$f(\lambda x, \lambda y) = f(x,y)$ for all $\lambda \in \mathbb{R}$ and $(x,y) \in D$
Exact	$\partial_y M = \partial_x N$
Linear	$f(x,y) = p(x)y + q(x)$ with p and q real functions
Bernoulli	$f(x,y) = p(x)y + q(x)y^n$ with p, q real functions and $n \in \mathbb{R} - \{0,1\}$

The articles [1,2,39,57,59,64,76,80,82,123,126,129,131] address aspects related to the traditional approach to teaching ordinary differential equations. Such as, development of algebraic abilities, student's difficulties of learning, uses of different mathematical representations, among others. The articles [1,2] are in the boundary between traditional and new didactic methodologies of teaching and learning differential equations, since the author discusses the relationship between procedural and conceptual learning. In [57], the authors propose a didactic material to develop skills for solving non-homogeneous higher-order ordinary differential equations by the use of indeterminate coefficient and constant variation methods. In a broad sense, the didactic material proposed by the authors consist of a list of algebraic exercises to select the appropriate method and apply the corresponding algorithmic technique. In [59,82], the author's aim was to measure the undergraduate student's mathematical knowledge through several tests. Although, the authors do not give information about the pedagogical methodology used to teach ordinary differential equations, we observed that the questions in their tests evaluate the processes of finding solutions rather than evaluating the concepts. In the article [64], the authors discuss the prevalence of traditional teaching based on analytic methods and the slow incorporation of geometric methods, they argue that the incorporation of new teaching techniques require a new learning communication skills. A similar approach to [64] is presented in [80,129], where the authors establish a study to identify the difficulties of students to develop a conceptual understanding and to use symbolic representations, meanwhile, learning differential equations based on a procedural teaching. For their part, the authors of [39,123] introduced a widely documented discussion about the characteristics of traditional methods and describe the main disadvantages. In the papers [76,131], a new method to get an analytic solution of first order differential equations is proposed. In [126], the author investigates a mnemonic acronym designed for the pedagogy of first-order ordinary differential equations. The aim in this paper is to develop a critical analysis, and propose a pedagogical model with the potential to move mnemonics from being viewed as a particular tricks where learners repeat some information which they do not understand altogether; towards a deeper, more conscious experience where learners are fostered to think beyond the mnemonic.

On the other hand, several authors have developed a broad research and discussion related to the constrains of traditional learning of differential equations. Here we mention some of the main concerns reported in the literature: the students prefer to learn algebraic methods of solution because it gives them an exact answer, however, these methods present difficulties to converting symbolic information into graphical information and vice versa [72]; student learning with the traditional method is limited because it is focused on applying and mastering algebraic procedures [2]; the main difficulties of students are related with the unsuitable choice of the solution method or an incorrect integration [3]; and the students learning in traditional methodology present some difficulties to contextualize the concepts of ordinary differential equations because they are not able to interpret correctly the terminologies out of the algebraic meaning [2,119]. Consequently, the students develop misunderstandings and learning difficulties related to differential equations [15]. It is widely documented that traditional methods for teaching and learning of ordinary differential equations are not suitable for conceptual learning, and therefore other methodologies are required [1,16,69]. Aspects like the learning in different classroom environments, the design of instructional sequences of activities, and the prompting to rethink theoretical issues as graphical representations, mathematical modeling, and even social interactions, need a further theoretical and empirical investigation [15].

Even though the traditional method of teaching and learning ordinary differential equations has several disadvantages, specifically it is passive to develop concept learning, should not be discarded entirely, since the learning of differential equation concepts needs capability in calculus concepts and skills [136]. Moreover, any change in the teaching methodologies (lecture notes, worksheets, and demonstration materials) should be implemented carefully, considering that although the students may have knowledge on concepts and skills to work with functions, differentiation, integration, and graphical representation of the derivative function, they may be unable to utilize these resources in a differential equations course [3,96].

7.1.2. Qualitative and Numerical Approach to Teaching Differential Equations

As noted in various sources, the traditional teaching of ordinary differential equations has been focused in the teaching of analytic methods, however is also know that those methods are restricted to solve only few types of of equations. In the last decades, we have witnessed the incorporation of graphical and numeric solutions methods to the teaching of differential equations. The practice of these qualitative methods is becoming more frequent in the classroom due to its potential to approach solutions of several types of ordinary differential equations [39,40]. However, in practice, there are some drawbacks. For instance, the order and the non-linearity of the equation which does not permit the universal application of those methods. In our list, 14 articles are focused on exploring the teaching of graphical solution, qualitative behavior and numerical solution of ordinary differential equations [3,4,14,22,39,56,64,69–72,84,97,109]. In the articles related to the teaching of qualitative analysis of ordinary differential equations, the focus is mainly in the learning of several concepts like graphical solution, direction fields, stability, and increasing or decreasing behavior of the solution, interpretation of situations based on the behavior of solutions. Meanwhile the articles on numerical solution are focused to introduce the concept of numerical solution and the construction of the numerical solution by application of the standard schemes like Euler and Runge–Kutta.

There are some works related to qualitative approaches that deserve special mention [137–140]. These works were pioneers in the exploration of new teaching and learning methods for the teaching and leaning differential equations, but they do not appear with our search criteria. The works [137–139] are out of the selected databases where we looked (see Section 4, item (a)) and the work [140] belongs to *notes* type of articles.

In recent years, the list of papers about the teaching of graphical and numerical solution of ordinary differential equations has been increased by the incorporation of technology. Those articles will be presented below on the Section 7.1.5.

7.1.3. Active Learning Methods

In the literature, there is not a unique definition of active learning, although this term is frequently used to refer the classroom practices that engage students in learning activities, such as reading, writing, discussion, or problem solving, that promote higher-order thinking [141]. The active learning methods are student-centered teaching methodologies which provide the students the opportunity to participate in mathematical investigation or problem-solving groups, where they construct and share knowledge in communities while maintaining an appropriate feedback on their work from experts and peers. Several research studies conducted in the last years have evidenced that active learning environments developed for students present better performance and retention than traditional and passive teaching.

In the last decades, a great number of instructional strategies have been proposed to foster the "active learning" approach. For instance, the inquiry-based learning, problem-based learning, the collaborative learning, the flipped classroom, problem solving and modeling activities, thinking-based learning, competencies-based learning, etc. Particularly, in the case of the teaching ordinary differential equations, we found 36 works [3,6,7,15,16,23,30,50,51,55,64,66,67,71,73,79,81,84–87,89,96,106–114,121,124,125,132], which are organized as follows:

(a) **Inquiry-based learning.** The "inquiry-based learning" is one kind of active learning methodology with several implementations in math classroom and its particular form of implementation is the "inquiry-based instruction" [71]. The methodology of inquiry-oriented instruction consists of four main steps: the generation of ways for reasoning of students, the analysis of student contributions, the development of a shared understanding, and the connection of finding in the development of research tasks to standard mathematical language and notation. Thus, the inquiry-oriented instruction generates classroom environments where the students practice an authentic research mathematical activity meanwhile they discover mathematical concepts, answering to purposefully designed tasks.

The inquiry-based instruction for ordinary differential equations is researched in the following articles [15,16,71,79,81,87,89,106–110,124,125]. In [71], the author reports the findings about the students' work with concepts related to slope fields, horizontal and vertical translation of solutions, systems modeling species interaction, and graphical solution of scalar autonomous differential equations. The author concludes that several advantages are generated by the inquiry oriented environment. Particularly he pointed out the following results: the students showed a notable cognitive gain in understanding and thinking; through the intervention of the instructor guiding the discussion the students reinvented knowledge; and they expressed their satisfaction with the inquiry instruction environment. In [79], the authors focus on the teaching of slope direction fields and the conception of solutions. Through a quantitative analysis, they showed that the students were able to successfully identify direction fields when the ordinary differential equation was given in analytical form, matching the appropriate direction field and the solution curve. They also found that students improved their understanding of the concept of solution for an ordinary differential equation as a result of the inquiry oriented intervention. The authors claim that the training had a long-lasting impact. In [81], discourse analysis is used to study the students mathematical narratives when learning the basic concepts of ordinary differential equations in a inquiry-oriented classroom environment, particularly the student's positions and beliefs related to learning mathematics. The articles [15,16,87,89,106–110,125] are part of the line of research introduced by Chris Rasmussen and collaborators. These papers are mainly focused on studying the retention of mathematical knowledge, students reasoning with mathematical ideas, and conceptual understanding, in the context of learning differential equations. From these studies, the inquiry-oriented methodology stands out for its potential to facilitate the development of mathematical reasoning ability and fostering meaningful learning. With a different perspective, in the article [124], the authors discuss the knowledge and capacity of the instructor to manage whole-class discussions concluding that the teacher's knowledge is a valuable component to be considered in the curricular reforms or in the classroom reforms under the inquiry-oriented perspective.

(b) **Problem-based learning.** The problem-based learning is an innovation of the pedagogical teaching and learning process which is learning student centered, promoting significant learning, and developing important skills and abilities which will be useful in the student's professional careers. The principle of problem-based learning is the use of problems as a starting point for the acquisition and integration of new knowledge [142]. The methodology is developed through students work in small groups where they participate in a cooperative learning experience with the aim to solve a problem proposed by the instructor, meanwhile they get a self-learning process. The self-learning process takes several steps like: read and analyze the problem, a focus group, make a list with the known and unknown facts about the problem, make a list of tasks to do, give a formal definition of the problem, get new information, and give a solution to the problem. From our list, 3 articles [55,73,93] are focused on the teaching and learning of ordinary differential equations under the problem-based learning methodology.

(c) **Other active learning methodologies.** Here we included other works related with research on active learning [3,6,7,23,30,50,51,64,66,67,84–86,96,111–114,121,132,133]. In [3,23,30,51,121], the authors apply the problem solving methodology. In [50], the authors develop a methodology based on the analysis of errors. In [6], the authors use the actions-processes-objects-schemas (APOS) theory. In [7], a competences-based methodology is used. In [64], a knowledge-guided based on discursive strategies is implemented. In [66], a guided small-group tasks perspective is applied. In [67], a methodology based on inquiry approach to learning in the context of community of practice theory is used. In [84], the authors compare the students performance when using three different methods for visualizing differential equations and their solutions, they also introduce a new method of visualization called Dynamic Method. In [85], a problem-centered methodology is used. In [86], the author presents a characterization of dynamic reasoning to improve student understanding in time related areas of mathematics. In [96], a discovery-based approached is applied for constructing the solutions of first and second-order linear ordinary differential equations and in [132] a learning methodology supported in embodied cognition and conceptual metaphors are discussed. Now, in the articles [111–114], innovative active learning methodologies are introduced in order to teach advanced topics of ordinary differential equations. For instance, in [111], the called framework of layers concepts–conditions–connectives–conclusions is presented, which was used to teach the interpretation and usage of existence and uniqueness theorems for ordinary differential equations.

The works related with the active methodologies of mathematical modeling, flipped classroom, and projects-based learning will be commented on in Sections 7.1.4–7.1.6, respectively.

7.1.4. The Mathematical Modeling Based Methodology

The mathematical modeling has a long history and a wide spectrum of applications in modern science. However, modeling is not defined in a unified single sense and, in the context of mathematics education, it has been conceptualized in a variety of ways, for instance as a process, a skill, and as a theory for student learning [8]. Over the last decades, research in mathematical modeling has increased highlighting several approaches to the teaching of mathematics and developing of students' modeling abilities. Mathematical modeling has become part of the educational standards in many institutions worldwide, being included in the curriculum of different scholar levels and careers from pedagogy, science, technology, and engineering. The researchers in mathematical modeling have emphasized different pedagogical goals as developing of modeling competencies through centered subject activities, orquestation of teaching and learning processes, developing of critical understanding of different situations, and students' motivation [143,144].

In the context of Mathematics Education, mathematical modeling has also been considered as a didactic methodology where we can find many approaches. Here we mention at least two of these: (i) research works motivated in curricular reasons and use some contextualized examples arising from validated mathematical models and, (ii) the papers that propose implementing mathematical modeling to involve the students in the treatment of real-world or life problems enhancing their career formation abilities [145]. Notice that in the case of (i) and (ii) the modeling can act as a vehicle for teaching mathematics or as content to be learned. This is, in the case (i), the modeling is a mean for attainment curricular contents and, in (ii), the modeling seeks first to nurture and enhance the ability of students to solve authentic real-world or life-like problems. In the case of (ii), the mathematical modeling process has been described as a cyclic process involving phases which are well discussed in [8,9,143,144,146]. A wide and documented discussion of meanings, approaches, priorities, challenges, and research perspectives associated with the mathematical modeling is presented in [145].

In the conceptualization of mathematical modeling cycle, there are several phases involving the process and sub-process of learning [146]. An example of the representation of the modeling process is presented in Figure 4 which was introduced by [147] and cited in [9]. The mathematical modeling is used to transit between two systems called the real world and the mathematical theories or representations. The process of mathematical modeling typically starts when the modeler has a question in the real world, which is referred as real-world situation on the diagram. Then, the modeler observes the situation mathematically by exploring the characteristics of the system which can be described by mathematical quantities and determine the relation between those quantities. After that, in the process known as mathematization or abstraction, the modeler considers some "conditions and assumptions" and replaces the real world by a mathematical entity (mathematical model) in terms of mathematical properties and parameters. The mathematical model is analyzed by applying the specific mathematical theory, deducing some mathematical conclusions which are transferred back to the real-world situation by examining if the conclusions of the mathematical model have a coherent answer to the original question. If the answer is ambiguous or has clear limitations, the modeler can repeat the cycle by considering new and more insightful observations and then improving the mathematical model.

Specifically, in the retained list, the articles [4,5,8,9,12,13,17,18,20,22,48,49,54,56,58,63,65,68,74,78,88,91,94,98,100,101,115,116,118,119,127,128,130,134] are related to some approaches to the mathematical modeling for the teaching of ordinary differential equations. These works were developed between the years 2004 and 2019, with the exception of [78,130]. The inclusion of [78] in the list of mathematical model papers for teaching ordinary differential equations obey to the fact that the author introduced an example of a real-life problem which is analyzed by the application of ordinary differential equations. Meanwhile, in [130], the author addressed the teacher training and recommended to include tests questions to enhance students to experience higher thought levels. Particularly, he exemplified and analyzed a question related with mathematical models for describing population dynamics with ordinary differential equations. The rest of articles (i.e., the works from 2004 to 2019) have diverse and disperse approaches for mathematical modeling. However, we can distinguish some similar characteristics which allow the definition of the following four groups:

(a) **Development of skills for mathematical modeling.** We find some articles where the aim was to study the development of mathematical modeling abilities in order to solve real problem models by employing mathematical theory knowledge related to ordinary differential equations [8,17,20,54,63,65,68,88,91,100]. The papers [20,63] are focused on the teaching and learning of mathematical models, particularly in the construction and application of mathematical models through mathematical activities.

In [20], the authors present two activities, one of them is based on mathematical models already known in the literature of ordinary differential equations and, the other one is based on the treatment of quantitative information for a new situation, concluding that different approaches to mathematical modeling lead to different actions of the students. In [8], the author introduces the methodological tool "Modeling Transition Diagrams" for capturing and representing the individual modeling process which uses this tool to examine the mathematical thinking while the students participate in modeling activities. The authors of article [65] are interested in the experience of implementing a mathematical modeling course, they report that the students adopt different approaches to learn mathematical models and conclude that after the experience, the students appreciate mathematical models, and suggest the usage of mathematical modeling to engage students into higher level learning approaches. The authors of [68,88] report the results of an innovative approach for teaching mathematical modeling with emphases in topics of environment, ecology, and epidemiology. Particularly, in [88] the students were involved in the solution of real-life problems adjusted to their region, by using the mathematical modeling tools were encouraged to pay attention to environmental issues like survival and sustainability. The paper [91] is focused on how to use ordinary differential equations as a pedagogical strategy to introduce students to the concepts of mathematical modeling. The author of [100] presents an application of mathematical modeling as a contextualized activity in several topics of an integral calculus with a small introduction to some topics of ordinary differential equations. In [17], the author studies the transposition of the mathematical modeling process used by the experts into the learning and teaching of mathematical modeling for undergraduate students.

(b) **Modeling as pedagogical strategy to teach concepts of ordinary differential equations.** In these papers, the authors are focused on several topics of ordinary differential equations which are taught by using mathematical modeling. In a broad sense, the authors deduce several advantages in the teaching and learning process and also present some conclusions that promise a continuous development of mathematical modeling as a pedagogical methodology for the following years. Among the advantages pointed out by the authors, we highlight that mathematical modeling is a pedagogical methodology that promotes meaningful learning and, it is a significant and concrete alternative to the questioned traditional teaching. In this group of papers, we have include the following articles [9,54,94,127,134]. In [54] is presented a research about how mathematical modeling as teaching and learning methodology can provide meaningful learning for the students. In [9], the author develops a comparative study of two instructional approaches used in the teaching of ordinary differential equations for engineering students. In one classroom, decontextualized techniques are emphasized, while in the other one, the teaching is based on modeling principles. She concludes that mathematical modeling practice as an instructional approach is a technique that can be used to circumvent several cognitive obstacles identified in the learning of differential equations. The authors of [94] develop a preliminary study of the application of mathematical modeling as a pedagogical tool for teaching several concepts of applied mathematics, particularly the geometric solutions of scalar and systems of ordinary differential equations. In [127], the author is interested in the students' understanding when learn ordinary differential equations under the mathematical modeling perspective. She develops an analysis using the APOS theory and mainly concludes that the modeling stimulates discussion, reflection, and the construction of new processes, objects, and schemes. Based on the didactic engineering perspective, the authors of [134] present the results of experimenting mathematical modeling process as didactic methodology for teaching ordinary differential equations.

(c) **Language games, representations, and relations of mathematics with other sciences.** There are some papers paying attention to some aspects like the different language games developed by the students involved in modeling activities [48], the usage of registers of representation for making relationships between the context and elements in ordinary differential equations [13], and the role of mathematical modeling to establish a relation between mathematics and other sciences [4,5,98].

(d) **Modeling activities using ordinary differential equations to teach other concepts.** Other articles are focused on the study of mathematical models based on ordinary differential equations for teaching concepts of other areas of mathematics or even other disciplines. More precisely, in [22] a study where the students were involved in the learning of concepts like drug administration by using simulations of the mathematical was developed. This experience was supported on modeling drug administration regimes for asthma through systems of coupled differential equations. In [115], the authors are focused in the teaching of concepts from cardiovascular physiology by using an analogous mathematical model to electronic circuits. In [116], some concepts of mechanics are introduced to the students through modeling fighter pilot ejection. In [118,119], the authors study how students understand units and rate of change when working with ordinary differential equations. In [30], some concepts of physical dynamic systems like the stability using mathematical models based on ordinary differential equation systems are studied; and in [128] the authors study some concepts of fluid dynamics using models based on the Bernoulli equation.

The articles [12,18,74,101] will be commented on Section 7.1.5; and [54,134] are presented on Section 7.1.3 and [49,56] on Section 7.1.2.

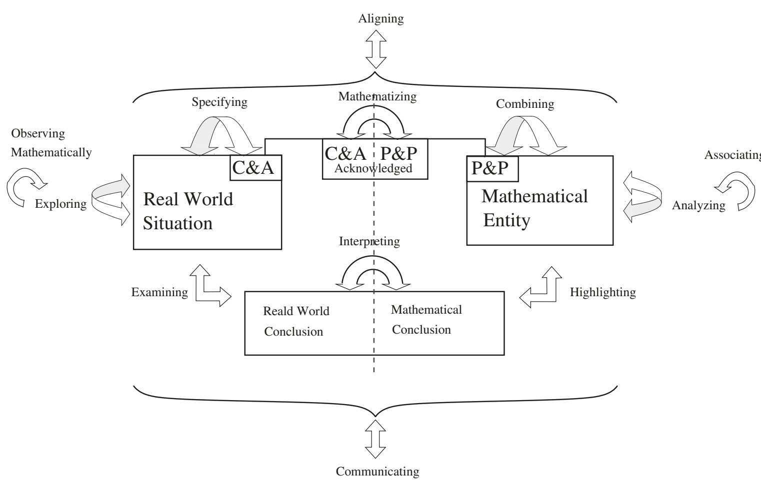

Figure 4. A diagram for the mathematical modeling cycle introduced in [147] and cited in [9]. The notations C&A and C&A are used for "conditions and assumptions" and "properties and parameters", respectively.

7.1.5. Information and Communication Technology-Based Methodologies

The increase of technology has challenged researchers worldwide to explore the roles technology plays and how transforms the teaching and learning of mathematics [148]. Particularly, in the case of ordinary differential equations, the information and communication technology has also become one of the essential hallmarks of contemporary educational landscape and several studies have been developed in the last years [32]. The studies of advantages, effectiveness, and other properties of technology are dynamic and have been constantly improved in recent years. For instance, an advantage of a simulation software as a learning platform is that students can solve more problems and develop abilities to achieve higher-level learning in less time than before when using traditional platforms [27].

The pedagogical methodologies based on the information and communication technology are diverse, including some learning activities like the following ones: the implementation of algorithms by writing computer codes, the analysis of some statements problems to be translated into a computer program, use of an specific software to solve problems or to learn some concepts, split a complex problem in a more small problems which integration permits the solution, conjecture some properties, and simulate the solutions in order to support the development of the proofs. Now, in the case of ordinary differential equations, it is well-known the existence of at least three approaches to solve an equation: the analytic, the qualitative, and the numeric solutions. With support on the information and communication technology, it is possible to implement pedagogic methodologies that address these approaches to the solution of ordinary differential equations. More precisely, from the retained list of papers, the articles related with information and communication technology are: [3,4,10–13,18,19,27–29,32–34,49,60,73–75,92,94,95,99–103,115,120,122,123], which can be arranged in three groups:

(a) **Computer algebra system.** The concept of computer algebra system is widely used to refer a type of software package that is used in learning some concepts by the manipulation of some appropriate mathematical formulae, and it is used in those cases where the algebraic, graphic, or algorithmic manipulations are tedious tasks with a low level of learning [149]. There are several papers focused in the usage of technological tools to find the analytic, numeric, or graphical solution of differential equations or even to analyze the qualitative behavior. Specifically, the articles [3,4,10–13,19,28,29,32,49,73,74,94,95,99–101,103,120,123] are related to the computer algebra system approach. In [28], the use of the software "Scientific Notebook" is studied to obtain the analytic and graphical solution of ordinary differential equations. The authors of [49] are focused on researching the teaching of differential equations through mathematical modeling in a computer enriched environment. In [29], it is reported a study where the students were encouraged to develop simulations of freefall problem by using a spreadsheet based on mathematical models. The authors study if the activities contribute to the mathematical, physical, and technological knowledge of students. The paper [3] discusses the cognitive process developed by students when participating in a teaching module for ordinary differential equations, which is based on problem solving and the usage of the VoyageTM200 calculator. The authors of [4,11] are interested in analyzing the different representations developed by students when learned ordinary differential equations using a computer algebra system as mediator. Indeed, in [4] some results about the application of spreadsheets and the HPGSolver software for visualizing and interpreting the properties of a given phenomenon arising in population dynamics are reported, and [11] contributes to study the connections between symbolic and graphical representations. The authors of [10,94] use the software Modellus to teach some properties of a Lotka–Volterra type system by using numerical simulations. In the research developed in [12,13], it is reported how the students were able to use several digital tools such as Excel, Derive, Wolfram-Alpha, Geogebra, to explore ordinary differential equations and their solutions. Particularly in [12], the students used an Applet to visualize and interpret the behavior of solutions of ordinary differential equations, some students' difficulties were found in this work; and in [13] the students were encouraged to use different digital tools as mentioned before and a computer package "GeomED" particularly designed to visualize and analyze the direction fields. In the research reported in [73] the software called STELLA was used to simulate the physical cascade system. In [74], the authors are focused on teaching mathematical models building for some given physical situations and in the numerical validation using technology. In [95], the authors use Maple to assist students in understanding the construction of analytic solution into the classroom. The authors of [99] present the experience of a project for teaching mathematics at the Massachusetts Institute of Technology and particularly present the result of a developed software called "mathlets" which was used for teaching concepts of dynamical systems. The author of [32,100,101] presents an experience of teaching several topics of calculus and ordinary differential equations using an integrated learning environment enriched with projects, mathematical modeling, and information and communication technology. In the article [103], some innovative ways to use free network computing laboratory called NCLab to the teaching of differential equations and applications are presented. In [120], the authors research how Maple helps the students in algebraic skills and construction of graphs, meanwhile the students learn some concepts related with the Laplace transform. The authors of [123] investigate the usage of Web-based simulations to learn ordinary differential equations. In [19], the authors studied the development of several mathematical thinking processes when the students learn ordinary differential equations using the software Maxima.

(b) **Simulation-based learning for teaching applications of ordinary differential equations.** There are some articles where the simulation-based learning or computer-assisted learning methodologies are used to teach the applications of ordinary differential equations to several areas like physics, biology, chemistry, or related areas. In those papers, the emphasis of teaching is given on concepts which are not included in a traditional course of differential equations. The numerical simulations are typically used to develop the understanding in the students by providing a visual animation and also for develop the intuition with respect to the change of some parameters, for instance, the initial conditions or the coefficients in an specific ordinary differential equations. The papers of this type are [18,27,60,75,92,102,115,122]. In [27], the authors review the traditional engineering textbooks and propose the computer simulations to teach the systems of ordinary differential equations arising in polymer molecular reaction dynamics. The authors of [60] are focused on the teaching several concepts of electric circuits theory by using some concepts of mathematical modeling, the Laplace transform, numerical simulations with MATLAB, and experiments. In [75], the aim was teaching some concepts of hydrostatic and atmospheric theories by using some mathematical models based on ordinary and partial differential equations and their simulation using spreadsheets. The authors of [92] are focused on helping to understand the applications of eigenvalue problems and develop a software using Visual BASIC for a simulation of solutions for the ordinary differential equations system modeling the problem of the two-mass two-spring physical system. The software simulates the vibration of the physical system, allowing the introduction by the user of some parameters such as the body masses and spring constants, solves the mathematical model, and shows on the screen the numerical and graphical results. In [102], it is reported the application of spreadsheet simulations to teach some topics of differential equations arising in a course of chemistry for undergraduate students. In [115], the authors propose the computer-based simulations to teach physiological processes like capacitance and resistance, and also suggest the introduction of those kind of teaching in undergraduate cardiovascular physiology courses. The authors of [18] study the simulation of electric circuits by using the construction of a physical laboratory model and a graphical calculator. In [122], the authors use Phyton to develop a software called REAJA, which is used for teaching some concepts in the undergraduate course of Chemical Processes.

(c) **Flipped classroom.** The pedagogical methodology called "flipped classroom" or "inverted classroom" has been widely used in the last decades to replace traditional lectures given in the classroom by an active learning. The main feature of this methodology is that the responsibility for learning the rest is on the learners, through the design of meaningful activities students have opportunities to control their own processes of leaning before the class. In principle, the activities may or may not be technology-based. However, the advances of information and communication technologies in the last years have increased individual instruction computer-based. The traditional lectures given in the classroom are temporally displaced by videos or similar resources which are previously available for students in a server, then the activities inside the classroom are developed on interactive groups of learning.

Particularly, in [33,34] the authors apply the flipped classroom to study the teaching of topics related to ordinary differential equations. In [33], the authors study the effectiveness of flipped classroom to develop skills related to the application of MATLAB/Simulink in the solution of ordinary differential equation mathematical models arising in a chemical course. Meanwhile, in [34], the authors combine the flipped classroom methodology with the cycle of mathematical model in order to study the introductory concepts of ordinary differential equations. In both works, supported on strong evidence, the authors conclude that the flipped classroom improves the active learning achievement of students.

Additionally, we observe that there are some papers in which digital tools are used without reporting particular results about the use of technology on their studies.

7.1.6. Project-Based Learning

According to the philosophy, concepts and examples of research projects in calculus are provided in [150], we can describe a research project as a multistep take-home assignment which is developed individually or in groups with a concerted effort in long period of time, for instance one or two weeks. The statements of the projects are carefully designed and include some parts expecting to get stuck even in the best students, such that the learners seek for help from their instructors, from whom receive hints, additional exercises, and supplementary readings. Moreover, the projects can be designed for different learning goals. Some projects consider real world problems in order to help the students to discover the applications of mathematics and their utility to study the affine sciences like physics, biology, chemistry, or engineering. One of the key goals when working with projects is to guide the learners to construct formal proofs by exploration of particular examples. For major details on project-based learning in calculus, we refer to [150].

Concerning the application of project-based learning in differential equations, we refer to the following articles from our retained list: [30,31,52,53,90,101,135]. The authors of [31] use mathematical projects arising in biology in the context of modeling tumor growth by differential equations. In [52,53], the authors combine the ideas of mathematical modeling and project-based learning methodologies to design projects to teach some concepts of ordinary differential equations. The authors argue that the project itself contributes to the development of students' competency for project work in science even in the introductory university courses. The authors of [90] are focused into researching the perceptions of the students when writing projects in the context of a differential equations course and conclude that the methodology is appropriate to develop some skills beyond the usual academic content of concepts and procedures. The students participating in the project recognized that they improved their capacity of scientific communication with each other when analyzing and solving real-life problems. An increase in their critical thinking was also observed. In [101], similar to [52,53], is also integrated modeling and project-based methodologies in the context of classroom environment based on the information and communication technology. The authors of [30] give a preliminary report of a series of projects applied in a course of ordinary differential equations. In [135], the author uses the methodology of projects to teach some concepts such as noise, vibration, and harshness, which are part of an undergraduate course in the mechanical engineering program. Particularly, the author studies the mathematical knowledge of students related to differential equations and linear algebra and evaluates the effectiveness of the methodology.

7.1.7. Other Methodologies

In the list of retained articles, we have that the works [21,61,62,77,83,104,105,117] are out of the groups presented before, although their topic of research is related to the teaching of ordinary differential equations and applications. However the didactic methodologies used are not explicitly presented or their goals are not precisely the teaching and learning ordinary differential equations in classroom experiences, for instance [21] is a review or [117] presents the results of a pilot research project.

7.2. Question 2: What Topics of Ordinary Differential Equations Have Been Explored in the Previous Studies?

From our retained list of 120 chosen articles, we can distinguish five groups for the topics covered in the teaching of differential equations:

(a) **Basic concepts of ordinary differential equation.** We refer to as basic concepts the definition of ordinary differential equation and their solutions. For instance, in [72], the author analyzed the answer of students to the question "What comes to your mind when you are asked to solve an ODE?" in two instants of a course, at the beginning and after the intervention. He found that firstly all students think about concepts related to the analytic solution and in the second two-thirds of students consider a change of their answers including some concepts related with the qualitative approach. A similar study was conducted in [69], where the answers of students to the following exam question were analyzed:

> *In your own words, define a differential equation. Explain what constitutes a solution to a differential equation. How can you represent geometrically a differential equation? Can the geometric representation of the differential equation help in sketching approximate solutions? In your opinion, how would you solve a differential equation?"* [69] (p. 654)

In the same study, the results of a semi-structured interview to the students who were asked six questions related with the definition of ordinary differential equation, the solution concept, the concept of geometric solution, and feeling of learning differential equations were also presented. In relation to the student construction of the concept solution a framework of four facets (context-entity-process-object) is introduced to analyze that type of constructions developed, see also [114]. The teaching of the concept of equilibrium solution in the case of scalar equations was investigated in [87]. More recently in [79], the authors research on the students conceptions about the solution of ordinary differential equations. Moreover, there are some works focused in the basic concepts related with graphical and numerical solution of an ordinary differential equation. In the case of graphical solution, researchers explore new ways for the students to interpret and give meaning to the information represented by a slope field. The initial value problem or Cauchy problem, autonomous differential equations, and the asymptotic behavior of solutions are also widely studied [12,71,84]. Regarding the numerical solution, the students have been introduced to learn the concepts of stability of the solution with respect to the initial condition and the coefficients of the equation by empirical examples [29].

Other concepts related with analytic solutions of first order (exact equations, linear, Bernoulli, etc.) and higher order (homogeneous, no homogeneous, coefficients variation, etc.) are treated in [9,19,57,64,79,82,89,95].

(b) **Biomathematical models.** There are several works that introduce some models arising in biomathematics which are based on differential equations. It is possible to find different types of population growth models, for example models from epidemics transmission. In those papers, the authors also pay attention to the introduction of qualitative analysis of solutions.

In the case of scalar models we have the articles [4,12,13,20,31,49,63,91,98,124], where the authors introduce the Malthus or Gompertz models and the Verhulst type models. Firstly, related with Malthus or Gompertz models, in [31] is presented research where the students are introduced in the study of population models according to:

$$\frac{dN}{dt} = rN, \quad (1)$$

$$N(0) = N_0, \quad (2)$$

contextualized to the case of $N(t)$ representing the density of carcinogenic cells of a tumor at the time t, with N_0 the measured initial density and r is a positive constant.

A similar topic of ordinary differential equations is also developed by [63,91,98,124]; particularly in [98] the authors study a model for disinfection and modify the assumption on r by considering that r is a negative constant. Now, concerning with Verhulst type models, in [20] the authors use the mathematical modeling to teach the population models of the form

$$\frac{dN}{dt} = rN\left(1 - \frac{N}{K}\right) - p(N), \tag{3}$$

$$N(0) = N_0, \tag{4}$$

where $N(t)$ is the number of individuals at time t living in a given bounded region; r and K are positive constants used for the increasing rate and the caring capacity, respectively; $p(N)$ is the predation function; and N_0 is the initial population. The attention in [20] is reduced to predation function satisfying the properties $p(N) \to 0$ when $N \to 0$ and $p(N) \to \beta$ when $N \to \infty$, with β a positive number, for instance considering $p(N) = BN^2/(\alpha^2 + N^2)$ with α a positive constant. We notice that when $p(N) = 0$ the model (3)–(4) is reduced to the Verhulst or logistic equation, which is also treated by [49]. A similar model is taught by [4,12,13] where $p(N) = 3/2$ and $p(N) = 2$, respectively.

On the other hand, in the case of systems of differential equations, we have the Lotka–Volterra model in competence of species and epidemiology, which are treated by [10,71,86,88,94,97,101,109,134]. In [10], the authors use mathematical modeling for describing the transmission of Malaria to the humans by the female mosquitoes of the genus Anopheles, given by the following system

$$\frac{dX}{dt} = \frac{ap}{N}Y(N - X) - gX, \tag{5}$$

$$\frac{dY}{dt} = \frac{ac}{N}X(M - Y) - \nu Y, \tag{6}$$

$$X(0) = X_0, \tag{7}$$

$$Y(0) = Y_0, \tag{8}$$

where $X(t)$ is the number of infected humans in time t; $Y(t)$ is the number of (female) mosquitoes infected at time t; N is the total population of humans; M is the total population of mosquitoes; and a, c, p, g and ν are positive constants. The system (5)–(8) is a particular example of the wide class of the models well known as Lotka–Volterra like systems and is used to model competence of species, which are also treated by [71,86,88,94,97,101,109,134].

Other common topics covered by the articles in teaching biomathematical modeling are related to some advances in model design and mathematical analysis. In the case of mathematical modeling, the core of teaching is focused on the simplification of some biological phenomenon using mathematical concepts recognized by the group of students involved in the experience. Related with the mathematical analysis, the works draw attention to understanding the meaning of the equations in the biology context and to the characteristics of the behavior of the solutions. For instance, in [10] the students belong to a course in an undergraduate program in Biology. The students had a previous knowledge about the disease of malaria caused by a parasite of the genus Plasmodium from a female mosquitoes of the genus Anopheles and they also mastered some concepts of calculus. The research reports, that firstly the aim of the modeling design was to increase the relations that the students could build between calculus concepts and Biology elements. In addition, the most important simplifications associated to Biology were stated as follows: the period of incubation is discarded; the human natality and mortality are ignored; the progressive acquisition of immunity in humans is ignored; and infected mosquitoes will prevail infected until death. Then, precisely stating the variables and parameters and, considering the behavior of populations interactions students formulated the model given by (5)–(8). The main two dependent variables at time t are the infected humans and the infected (female) mosquitoes populations given by $X(t)$ and $Y(t)$, respectively. Two parameters to be considered are total population of humans and mosquitoes given by N and M, respectively. To deduce the equation (5), describing the change over time of population for infected humans by interaction with mosquitoes, it is assumed that and infected mosquito bites a health human with a certain probably and the sick persons are recovered. The factors $N - X$ and ap/N represent the health human and the number of bites given by a mosquito per unit of time a/N with a probability of health humans to be infected equal to p, respectively. Meanwhile, the recovered of infected humans is described by the term gX with g a parameter for the recovery rate. Similar arguments are used to deduce the Equation (6), mainly the term $(ac/N)X(M - Y)$ is the change of infected mosquitoes when a non-infected mosquito bites into an infected human in a unit of time a/N with a probability to be infected equal to c, and the term νY is the infected mosquitoes that die at mortality rate ν. Second, concerning the mathematical analysis of (5)–(8), the authors observe that the system is non-linear and prevents the students from achieving analytical solutions and allows them access to the solutions using the software Modellus. The students worked with Modellus were guided by a set of activities that strengthen the concepts of calculus like functions, tangent line, derivative, and maxima and minima.

(c) **Scalar-based models**. We have some work using mathematical models based on scalar differential equations to teach some concepts of differential equations.
For mathematical models based on first order scalar equations, we have four groups of articles. Firstly, we have the increasing (or decreasing) mathematical models based on an ordinary differential equation of the form

$$\frac{d\alpha}{dt} = k\alpha, \quad \alpha(0) = \alpha_0, \tag{9}$$

where k is a positive (or negative) constant, t is the time, and α is the measurement of some physical quantity such that the initial time is α_0. In [51], the authors propose five activities in the context of problem solving and guided discovery methodologies, where particularly the four labeled activities are contextualized to radioactive decay modeled by (9) with α the quantity of radium in a body which is decreasing in time. The radioactive decay in the context of mathematical modeling is also considered by the authors of [39] where α is the number of radioactive atoms. A close problem is the model for uranium decay $p'(t) = -0.0003p(t) + 0.3$ explored in [3], which is described as a variation of (9), with $p(t)$ the amount of mercury in a given reservoir at any instant of time t. Related with the increasing behavior we have the works Malthus or Gompertz type described in the Biomathematical models, see the works for (1)–(2). Moreover, in [76] the authors use a difference equation of the form

$$\frac{[A]_{t_2} - [A]_{t_1}}{t_2 - t_1} = -k\Big([A]_{t_2} - [A]_{t_1}\Big)^m, \quad k > 0,\ m > 0,$$

arising in kinetic reactions and introduce the teach of convergence of discrete models to continuous models of the form (10) or to teach the relation of difference and differential equations. A second group of works are [3,8,17,29,39,48,51,85,100,116,117,131,132], where the authors use mathematical models based on first order differential equations. Here we distinguish four types of mathematical models. Firstly, we have the well known "freefall mathematical model", which is given by a differential equation of the type

$$m\frac{dv}{dt} = mg - bv^2, \quad v(0) = v_0 \tag{10}$$

with m denoting the mass of a body, g is the acceleration due to gravity, b is a constant associated to air resistance, v_0 is the initial velocity of the body, t is the time, and the unknown v is the velocity of the body. In [29], the author uses numerical methods to simulate the solution of (10) in the case of vacuum ($b = 0$) and with air resistance ($b > 0$). Ref. [8] is focused on the research of mathematical thinking process when the students analyze and solve a freefall problem, and in [131] the authors are focused on the analytic solution of (10) by the variable separation method. Third, the model for describing "Newton's law of cooling" given by a differential equation of the form

$$MC\frac{d\theta}{dt} = -h(\theta - \theta_a), \quad \theta(0) = \theta_0, \tag{11}$$

where h is a positive constant called the convective cooling coefficient, θ_a represents the environment temperature of cooling medium, M is the mass of the body, C is the specific heat, and $\theta(t)$ is the unknown temperature of the body in a time t with known initial condition θ_0. The model of type (11) is treated in [39,85,91,100]. The fourth type of mathematical model is based on "Kirchoff and Ohm laws" given by

$$\frac{dU_c}{dt} + \frac{1}{RC}U_c = 0, \quad U_c(0) = E,$$

with RC as the constant for the resistance of the capacitor, the unknown U_c is the voltage in the capacitor, and E is the voltage of the capacitor at $t = 0$; this equation is studied in [17,18].

On the other hand, a second group of scalar models of second order are presented in [5,99], where the authors use mathematical models arising in electric circuits and vibration problems, respectively. Indeed, in [5] the authors consider the model

$$I''(t) + 2\lambda I'(t) + \omega^2 I(t) = 0, \quad I(0) = 2, \quad I'(0) = 0,$$

where the I is the current intensity crossing the circuit and in [99] the authors use an interactive software for explore the equation

$$x''(t) + bx'(t) + kx(t) = k\cos(\omega t), \quad x(0) = x_0, \quad x'(0) = x_1,$$

where b, k, ω, x_0 and x_1 are constants and x is the displacement of the mass from equilibrium in a spring-mass system. In the case of [5], the authors study physical concepts such as the inductance and resistance and in [99] the authors study some concepts of Mechanical Vibration Theory like amplitude and phase.

(d) **Systems based on mechanical theory**. The works [92,117] consider second order systems arising in Mechanical Vibration Theory. To be more precise, in [92] the authors consider a system modeling a two-mass two-spring vibration system of the following type

$$\frac{d^2}{dt^2}\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} -(k_1+k_2)/m_1 & k_2/m_1 \\ k_2/m_2 & -k_2/m_2 \end{pmatrix}\begin{pmatrix} y_1 \\ y_2 \end{pmatrix}$$

where m_1, m_2 are the masses of two bodies connected by two springs with constants k_1 and k_2 and fixed at the top and y_1 and y_2 are the displacement from the equilibrium of the bodies. Moreover, in [92] several concepts like amplitude, modes of vibration, period, and frequency are taught.

(e) **Other concepts**. There are some works focused on the teaching and learning of other topics of differential equations like the Theorems of existence and uniqueness [1,2,111,112], Laplace transform [6,19,120], and bifurcation concept [31,135].

7.3. Question 3: What Are the Results for the Effectiveness of Traditional and New Didactic Methodologies to Teach and Learning Ordinary Differential Equations, as Reported in Previous Studies?

The effectiveness of a new methodology is usually an implicit motivation. However, in a practical research, the aim of a specific paper is usually defined explicitly in terms of other topics which are considered relevant to study in order to improve the teaching and learning process. Then, given that the effectiveness is implicitly transversal to all articles proposing innovative didactic methodologies for ordinary differential equations, here the works where effectiveness was explicitly mentioned were included [33,34,55,57,64,66,76,79,84,89,92,123,126,135].

Concerning the evaluation of the effectiveness, we distinguish four groups of articles: (i) works where only the effectiveness of the new didactic methodology was evaluated [33,55,66,76,79,123,135]; (ii) works where only the effectiveness of the traditional didactic methodology was evaluated [57,126]; (iii) works comparing the traditional and the new didactic methodologies without introducing a measurement of each didactic methodology alone [89,92]; and (iv) works where the authors introduce a quantification of the effectiveness for each didactic methodology and also a comparison [34,64,84].

8. Conclusions

The followed research methodology allowed us to identify and analyze the papers addressing the teaching and learning of ordinary differential equations. We retrieved and reviewed 120 papers from 1970 to 2020 which are associated with Web of Science, Scopus, Qualis, ZbMath, and Scielo. We recognized the didactic methodologies pointed out in each paper. When doing this, the most explored concepts and topics associated to ordinary differential equations and the effectiveness of didactic methodologies reported by the authors were identified. We noticed an increase in research where the attention has been given to the design of new didactic methodologies which have also been strengthened by the development of digital tools. The research related to teaching and learning differential equations has transitioned from exploring elements associated to the teaching in traditional classrooms to the introduction of a qualitative and numerical approach, active learning methods, modeling, and use of technology, emphasizing the importance of student participation in their own learning. As a result of the nature of differential equations for describing several phenomena, it also stands out in research modeling and interdisciplinarity. It should be noted that the characterization presented is not unique and many papers could be organized in one or more category.

The most relevant features achieved of the present article are the identification of works that address the subject of teaching and learning of ordinary differential equations, the recognition of the most explored mathematical content, and the synopsis of teaching methodologies that have used to teach the topic over the years. However, through our review analysis, we have found that there are also some issues that have received little attention. For example, little evidence is found regarding the retention, in terms of learning and skills development, that students achieve after being involved in learning with a particular methodology, which requires considering the validation and improvement of the implemented methodologies. Another element to consider is the update of the university curriculum considering the research results that involve the new teaching methods and use of information and communication technologies (for instance, those indicated in Section 7.1.5) or the relevance of the processes involved in the transition from the learning of calculus to the learning of ordinary differential equations. In relation to the teachers who are normally in charge of teaching ordinary differential equations, the research does not give importance to the fact that in many cases they are engineers or mathematicians, without or a little knowledge of didactic. Then, it is necessary to pay attention to the desired knowledge (didactic, pedagogical and mathematical) that these teachers need to teach the subject, which will allow them to become aware of the learning difficulties that students may face. Teachers of ordinary differential equations still need to be encouraged to experiment and enrich their classes with different teaching methodologies to support the students developing knowledge to respond the challenges that the academic or work field demands of them. Therefore, more research is currently needed in the classroom, in relation to the use of technology, development of simulations, resources for online teaching, and interdisciplinary projects.

The research on the teaching of differential equations is an active area with an increasing number of articles in the last decade. However, there is still much to do toward addressing the challenges in teaching and learning differential equations. We set out three issues that need more detailed exploration. Firstly, we found that some advanced topics of ordinary differential equations are incipient developed in the research. For instance the teaching of the existence an uniqueness Theorems for scalar equations of first order are treated only in [1,2,111,112] and an introduction to bifurcation concept is presented only in [31,135]. However, in the reviewed references, there is not a treatment of other relevant concepts, techniques, and classic results associated to the study of qualitative behavior of solutions, and some properties of the solutions deduced from the qualitative behavior. To name a few concepts, the teaching of linear and non-linear equations is implicitly treated by some articles. The teaching of concepts as autonomous and not autonomous systems and the concepts around stability in non-linear systems are still open topics to research. The teaching of advanced techniques and results to study non-linear systems like Lyapunov functions, topological degree methods, and the Hartman–Grobmann theorem, are still open. We did not find research regarding the teaching of analysis of equilibrium points for nonlinear systems, the periodicity of solutions, and the asymptotic behavior of solutions. Thus, briefly, there is still open the didactic transposition of several topics of ordinary differential equations theory. Second, in the teaching of modeling from physical and biological problems, the topic of existence of positive solutions is uncovered yet. For instance, in [10] the authors do not consider as part of the set of activities the basic aspect of the biological phenomenon: the existence of positive solutions of the system (5)–(8). Thirdly, regarding the systematic literature review, our short-term goal is to analyze the remaining 285 articles (*notes* and *curriculum*) which were found in the search of references given in Section 4. Since in our actual analysis some representative works were excluded, we plan to extend our search to other indexations including books, book chapters, and theses.

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Appendix A. String Search Used in Web of Science and Scopus

The string search used in Web of Science is the following

ALL FIELDS: ("differential equation*" or "solution* to differential equation*" or "graphical interpretation" or "graphical solution*" or "qualitative solution*" or "numerical solution*" or "analytic solution*" or "first order equation*" or "higher order equation*") OR ALL FIELDS: ("Laplace transform" or "power series method" or "variable separable equation*" or "reducible to variable separable equation*" or "homogeneous equation*" or "reducible to homogeneous equation*" or "exact equation*" or "reducible to exact equation*" or "Bernoulli equation*") OR ALL FIELDS: ("linear equation*" or "Ricatti equation*" or "phase plane" or "isocline*" or "slope field*" or "equilibrium" or "stability of solution*" or "initial value problem*" or "boundary value problem*" or "scalar equation*" or "systems of equations" or " linear" or "non-linear") AND ALL FIELDS: ("teaching methodologies" or "students' understanding and difficulties" or "interpretation of solutions" or "registers of representations" or "mathematical modeling" or "mathematical models" or "problem-based learning" or "problem solving") OR ALL FIELDS: ("error analysis" or "mathematics teaching practices" or "real world situation" or "computational resources" or "mathematical application" or "classroom discourse" or "didactic of differential equations" or "critical discourse analysis").

Meanwhile the string search for Scopus is given by

(TITLE-ABS-KEY ("differential equation" OR "solution* to differential equation*" OR "graphical interpretation" OR "graphical solution*" OR "qualitative solution*" OR "numerical solution*" OR "analytic solution*" OR "first order equation*") OR ALL ("higher order equation*" OR "Laplace transform" OR "power series method" OR "variable separable equation*" OR "reducible to variable separable equation*" OR "homogeneous equation*" OR "reducible to homogeneous equation*" OR "exact equation*") OR TITLE-ABS-KEY ("Bernoulli equation*" OR "linear equation*" OR "Ricatti equation*" OR "phase plane" OR "isocline*" OR "slope field*" OR "equilibrium" OR "stability of solution*" OR "initial value problem*") OR TITLE-ABS-KEY ("boundary value problem*" OR "scalar equation*" OR "systems of equations" OR " linear" OR "nonlinear") AND TITLE-ABS-KEY ("teaching methodologies" OR "students' understanding and difficulties" OR "interpretation of solutions") OR TITLE-ABS-KEY ("registers of representations" OR "mathematical modeling" OR "mathematical models" OR "problem based learning" OR "problem solving" OR "error analysis" OR "mathematics teaching"))

Appendix B. List of Journals from Qualis, zbMATH, Scielo, WOS, and Scopus Datbases

Table A1. List of journals from Qualis, zbMATH, and Scielo database. The notation A1, A2, B1, B2, B3, B4, B5, and C are the classification of Qualis. The notation AA, AB, and AC (or BA, BB, and BC) are used for journals considered in the Serie A (or Serie B) and types A, B, and C (or A, B and C) in the classification given by [46]. The "Journal code" is a abbreviated reference code of the corresponding journal which is introduced by citation convenience.

	Journal Title	ISSN	Qualis Class	zbMATH Class	Scielo	Years Consulted
1	Academia journal of educational research	2315-7704	B3			2013-2020
2	Acta scientiae	2178-7727	A2			1999–2019
3	Actualidades investigativas en educación	1409-4703			SC	2011–2020
4	American mathematical monthly	0002-9890		BA		1894–2020
5	Applied measurement in education	0895-7347		BA		1988–2020
6	Australian journal of education	0004-9441		BC		1957–2019
7	BOLEMA : Boletim de educação matemática	1980-4415	A1	AA	SC	1985–2019
8	Boletim cearense de educação e história da matemática	2357-8661	B3			2014–2019
9	Boletim online de educação matemática	2357-724X	B1			2013–2019
10	Boletín das ciencias	0214-7807	B3			1988–2019
11	British educational research journal	6469-3118		BA		1975–2019
12	British journal of educational psychology	2044-8279		BA		1931–2020

Table A1. *Cont.*

	Journal Title	ISSN	Qualis Class	zbMATH Class	Scielo	Years Consulted
13	British journal of educational technology	1467-8535		BA		1970–2019
14	Child development	1467-8624		BA		1990–2020
15	Ciência & educação	1980-850X	A1		SC	1988–2019
16	Ciencia, docencia y tecnología	1851-1716			SC	2000–2019
17	Cognition	0010-0277		BB		1972–2020
18	Cognition and instruction	0737-0008		BA		1984–2020
19	Comparative education	0305-0068		BA		1964–2020
20	comparative education review	0010-4086		BA		1957–2020
21	Cpu-e. revista de investigación educativa	1870-5308			SC	2005–2020
22	Cuadernos de investigación educativa	1510-2432			SC	1997–2019
23	Economics of education review	0272-7757		BA		1981–2020
24	Educação e matemática: revista da associação de professores de matemática	0871-7222	B1			1987–2019
25	Educação matemática em foco	1981-6979	B3			2017–2019
26	Educação matemática em revista	2317-904X	A2			1983–2019
27	Educaçao matematica pesquisa	1516-5388	A2	AB		1999–2019
28	Educación	1019-9403			SC	1992–2020
29	Educación matemática	1665-5826		AC	SC	1989–2019
30	Educación y educadores	0123-1294			SC	1997–2019
31	Educar em revista	1984-0411	A1			1977–2019
32	Educational measurement: issues and practice	1742-3992		BB		1982–2020
33	Educational research	0013-1881		BA		1958–2020
34	Educational studies in mathematics	0013-1954	A1	AB		1968–2020
35	Educational technology research and development	1556-6501		BB		1953–2019
36	Educational technology: the magazine for managers of change in education	0013-1962		BC		1960–2017
37	Elementary school journal	0013-5984		BA		1914–2019
38	Em teia-revista de educação matemática e tecnológica iberoamericana	2177-9309	B1			2010–2019
39	Enseignement mathematique, l'	0013-8584		BC		2009–2019
40	Enseñanza de las ciencias	0212-4521	A1	BA		1983–2019
41	Ensino da matemática em debate	2358-4122	B4			2010–2019
42	Epsilon	2340-714X		AC		1984–2019
43	Estudios-centro de estudios avanzados. universidad nacional de córdoba	1852-1568			SC	1993–2019
44	Focus on learning problems in mathematics and science teaching	0272-8893		BC		1988–1991
45	For the learning of mathematics	0228-0671	A1	AB		1980–2017
46	Formação docente	2176-4360	B1			2009–2019
47	Hiroshima journal of mathematics education	0919-1720		AB		1993–2020
48	IEEE revista iberoamericana de tecnologias del aprendizaje	2255-5706	B3			2006–2012
49	Insegnamento della matematica e delle scienze integrate, l'	1123-7570		BC		1970–2020
50	Integración y conocimiento	2347-0658	C			2012–2020
51	Interciencia	0378-1844	A1			2009–2020
52	International electronic journal of mathematics education	2468-4945	C			2006–2020
53	International journal of engineering education	0949-149X	A1			1991–2020

Table A1. *Cont.*

	Journal Title	ISSN	Qualis Class	zbMATH Class	Scielo	Years Consulted
54	International journal of engineering research and applications	2248-9622	C			2011–2020
55	International journal of mathematical education in science and technology	0020-739X	A1	BB		1970–2020
56	International journal of science and mathematical education	1571-0068	A1			1970–2019
57	International statistical review	1751-5823		BA		1990–2020
58	Jornal internacional de estudos em educação matemática	2176-5634	A2			2009–2020
59	Journal for research in mathematics education	0021-8251		AA		1970–2020
60	Journal für mathematik-didaktik	0173-5322		AC		1980–2020
61	Journal of computers in mathematics and science teaching	0731-9258		BC		1981–2020
62	Journal of educational psychology	0022-0663		BA		2002–2020
63	Journal of educational research	0022-0671		BA		1920–2020
64	Journal of mathematics teacher education	1386-4416		AB		1998–2020
65	Journal of recreational mathematics	0022-412X		BC		1968–2014
66	Journal of research in science teaching	1098-2736		BA		1960–2020
67	Journal of statistics education	1069-1898		AC		1993–2015
68	Journal of the learning sciences	1050-8409		BA		1991–2020
69	Journal of urban mathematics education	2151-2612	B1			2008–2019
70	Learning and instruction	0959-4752		BA		1991–2020
71	Matemática e estatística em foco	2318-0552	B5			2013–2019
72	Matematica e la sua didactica, la	1120-9968		AC		2016–2020
73	Mathematical journal of interdisciplinary sciences	2278-9561	B5			2012–2020
74	Mathematics education research journal	0021-8251		AB		1989–2020
75	Mathematics in school	0305-7259		AC		1971–2014
76	Mathematics teacher	0025-5769		AC		1990–2020
77	Mathematics teaching	0025-5785		AC		1871–2020
78	Mathematics teaching in the middle school	1072-0839		AC		1994–2019
79	Mathematical thinking and learning	1098-6065		AB		1999–2020
80	Mediterranean journal for research in mathematics education	1450-1104		AB		2002–2020
81	Numeros	0212-3096		AC		1981–2020
82	Paradígma	1011-2251	A2		SC	1997–2019
83	Perspectivas da educação matemática	2359-2842	B1			2008–2019
84	Petit X	0759-9188		AC		1986–2007
85	Phi delta kappan	0031-7217		BA		2000–2020
86	Plot: mathematiques et enseignement	0397-7471		AC		1987–2017
87	PNA: revista de investigación en didáctica de la matemática	1887-3987	A2			2006–2020
88	Professor de matemática online	2319-023X	B4			2013–2019
89	Psychology in the schools	1520-6807		BA		1964–2020
90	Quadrante	2183-2838		AB		1992–2020
91	Recherches en didactique des mathematiques	0246-9367		AB		2000–2019
92	Redimat- revista de investigación en didáctica de las matemáticas	2014-3621	A2			2012–2020
93	REEC. revista electrónica de enseñanza de las ciencias	1579-1513	A2			2002–2019
94	Remat: revista eletrônica da matemática	2447-2689	B3			2015–2020

Table A1. *Cont.*

	Journal Title	ISSN	Qualis Class	zbMATH Class	Scielo	Years Consulted
95	Rematec. revista de matemática, ensino e cultura (ufrn)	1980-3141	B2			2006–2019
96	Rencimat	2179-426X	A2			2010–2019
98	Revemat : revista eletrônica de educação matemática	1981-1322	A2			2006–2020
99	Revista de ciência & tecnologia (unig)	1519-8022	B5			1995–2019
100	Revista de ciências da educação	2317-6091	B1			2012–2020
101	Revista de educação, ciências e matemática	2238-2380	A2			2011–2019
102	Revista de produção discente em educação matemática	2238-8044	B3			2012–2019
103	Revista digital de investigación en docencia universitaria	2223-2516			SC	2005–2019
104	Revista docência do ensino superior	2237-5864	B1			2011–2020
105	Revista electrónica de investigación educativa	1607-4041	A1			1999–2020
106	Revista electronica de investigacion en educacion en ciencias	1850-6666	A2		SC	2006–2019
107	Revista eureka sobre enseñanza y divulgación de las ciencias	1697-011X	A1			2004–2020
108	Revista iberoamericana de educación superior	2007-2872			SC	2010–2020
109	Revista internacional de aprendizaje en ciencia, matemáticas y tecnología	2386-8791	B3			2014–2019
110	Revista latinoamericana de investigación en matemática educativa	2007-6819	A2	AA	SC	1997–2020
111	Revista mexicana de investigación educativa	1405-6666			SC	1996–2020
112	School effectiveness and school improvement	0924-3453		BA		1990–2020
113	School psychology quarterly	2578-4218		BA		1986–2020
114	School science and mathematics	1949-8594		BC		1901–2020
115	Science education	1098-237X		BA		2001–2020
116	Science journal of education	2329-0897	B4			2013–2020
117	Sociology of education	0038-0407		BA		2004–2020
118	Statistics education research journal	1570-1824		AB		2002–2020
119	Suma	1130-488X		AC		1988–2019
120	Teaching and teacher education	0742-051X		BA		1985–2020
121	Teaching children mathematics	1073-5836		AC		1954–2019
122	Teaching mathematics and its applications	0268-3679	A1			1982–2020
123	Thai journal of mathematics	1686-0209	B4			2003–2020
124	The college mathematics journal	0746-8342		BC		1984–2020
125	The electronic journal of mathematics & technology	1933-2823	B1			2007–2020
126	The journal of mathematical behavior	0732-3123	A1	AB		1994–2020
127	Uniciencia	1011-0275			SC	1984–2020
128	Unión revista iberoamericana de educación matemática	1815-0640		AC		2005–2019
129	Uno. revista de didactica de las matematicas	1133-9853		AC		1994–2019
130	Young children	0044-0728		BA		1964–2001
131	Zentralblatt fur didactic der mathematik	1863-9690	A1	AB		1997–2021
132	Zetetiké	2176-1744		AB		1993–2020

Table A2. List of journals associated to WOS and Scopus databases which appear when we search articles related with the teaching and learning of ordinary differential equations by applying the strings given in Appendix A and are not included in the list of Table A1.

Journal Title	ISSN	Journal Title	ISSN
Advances in physiology education	1043-4046	International journal of research in undergraduate mathematics education	2198-9745
American journal of physics	0002-9505	Journal of chemical education	0021-9584
Biochemistry and molecular biology education	1470-8175	Journal of professional issues in engineering education and practice	1052-3928
CBE-Life sciences education	1931-7913	Journal of science education and technology	1059-0145
Computer applications in engineering education	1061-3773	Mathematics teaching-research journal online	2573-4377
Computers & education	0360-1315	Physical review-physics education research	2469-9896
Education for chemical engineers	1749-7728	PRIMUS: problems, resources, and issues in mathematics undergraduate studies	1051-1970
Eurasia journal of mathematics science and technology education	1305-8215	Research in mathematics education	1479-4802
European journal of engineering education	0304-3797	Research in science & technological education	0263-5143
European journal of physics	0143-0807	Resonance-journal of science education	0971-8044
Global journal of engineering education	1328-3154	Revista brasileira de ensino de fisica	1806-1117
IEEE transactions on education	0018-9359	Revista cientifica	0124-2253
Information technologies and learning tools	2076-8184	Revista conrado	1990-8644
Interdisciplinary science reviews	0308-0188	Revista electronica de humanidades educacion y comunicacion social	1856-9331
International journal for technology in mathematics education	1744-2710	Revista publicando	1390-9304
international journal of education and information technologies	2074-1316	Teaching of mathematics	1451-4966
International journal of electrical engineering education	0020-7209	The American mathematical monthly	0002-9890
International journal of engineering pedagogy	2192-4880	The physics teacher	0031-921X
International journal of mechanical engineering education	0306-4190	The Turkish online journal of educational technology	2146-7242
International journal of psychosocial rehabilitation	1475-7192		

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Article

Formative Assessment of Pre-Service Teachers' Knowledge on Mathematical Modeling

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Abstract: This document reports how formative assessment strategies promote the knowledge of modeling of pre-service mathematics teachers. This knowledge is understood from content and vehicle points of view. Formative assessment strategies were designed and experimented with 14 participants in a mathematical modeling course offered to pre-service teachers in a Colombian university. Thematic analysis was conducted on lesson plans built by pre-service teachers. In those plans, they evinced knowledge of class management, mathematics teaching, problem solving, and modeling teaching. Finally, the collective construction of assessment rubrics is highlighted. Its contributions and limitations as a formative assessment tool are reported. The role played by the advisors' feedback and support to pre-service teachers is also presented.

Keywords: formative assessment; mathematical modeling; teacher education; teachers' knowledge

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1. Introduction

Research on mathematics teachers' knowledge has produced models regarding characteristics, dimensions, components, and facets of teachers' teaching knowledge have emerged. Pino-Fan, Assis, and Castro [1] explored some dimensions and theoretical-methodological tools suggested by the didactic-mathematical knowledge (DMK) model for the analysis, characterization, and promotion of teacher's knowledge, intended to efficiently develop their teaching practices. Carrillo-Yañez and his team [2] presented the mathematics teacher specialized knowledge model (MTSK); the authors proposed a framework that considers mathematical-knowledge specialization as a model-inherent property which extends to all subdomains. Such models are ways to investigate, understand, analyze, and evaluate teachers' mathematics knowledge. Some models transcend a descriptive dimension and offer tools for intervention in training programs that promote teacher knowledge development. In those cases, continuous evaluation of teachers' knowledge becomes a tool to study and promote the evolution of such models.

In a complementary perspective, assessment of teachers' knowledge is associated with the knowledge they have developed to accredit, certify, or get promoted in their profession. To this end, research methods have been developed to measure teachers' knowledge and produce valid and useful results for policy formulation [3]. Mesa and Leckrone [3] offer an overview of six types of processes, methods, and components to be assessed regarding mathematics teachers' knowledge.

In another perspective, training programs are concerned not only with determining teachers' knowledge, but also have the objective of promoting it. In this regard, assessment of teachers' knowledge can be considered both summative and formative. Accordingly, a course to promote teachers' mathematical modeling knowledge was developed and related formative-assessment strategies were implemented. To analyze the contribution of these strategies, a study was developed to answer the question: how can pre-service teachers' knowledge on mathematical modeling be assessed in a formative way?

2. Theoretical Background

2.1. Teacher's Knowledge on Mathematical Modeling in Mathematics Education

International research on modeling in mathematics education has revealed the opportunities that modeling offers for learning and development of students' competencies, supporting of institutional needs, and fostering of teacher training (ICTMA collection). Blum [4] pointed out that the integration of modeling in school implies open and demanding environments which require complex teaching abilities and, consequently, ways of evaluation capable of facing those requirements. Certainly, mathematical and extra-mathematical knowledge is also required, as well as some familiarity with the selected modeling tasks. Research has also highlighted that teachers require experiences to transcend the use of routine and stereotyped tasks, so they can promote in their students' critical views and help them to solve real-life problems, to use mathematics in society [5–7], and connect mathematics and other STEM areas [8]. In their research, Romo-Vázquez, Barquero, and Bosch [5] point out that teachers require to transcend the rigidity of the curriculum, strict time schedules, lack of adapted assessment devices, problems in the use of ICT, multidisciplinary challenges, among other aspects.

Cetinkaya, Kertil, Erbas, Korkmaz, Alacaci, and Cakiroglu's literature review [6] reported that teachers have limited professional knowledge about the nature of mathematical modeling and about how to use it in mathematics teaching and learning. These authors suggested to pay greater attention to modeling-related learning opportunities for pre-service and in-service teachers through training programs. In their research, theses authors grouped a significant part of the modeling research into the following topics: (i) knowledge of the cognitive demands of certain modeling activities in order to select tasks and appropriate curricular materials for promoting specific concepts in students; (ii) knowledge about how to manage tasks and organize speech during modeling activities; (iii) knowledge on how to promote adaptive activities, make strategic interventions and foster independence as a form of scaffolding and promotion of the principle of minimal teacher assistance; (iv) knowledge of productive modeling ways (contrasted with less productive ones) to help students differentiate between more and less useful ideas, as well as to make connections between them; (v) recognition of unexpected solving approaches to modeling and development of strategies to deal with crises in the modeling process; (vi) mathematical and extra-mathematical knowledge and abilities to use information and communication technologies (ICT) effectively during the modeling processes.

Teacher's knowledge on mathematical modeling must also include at least two intersecting dimensions, namely: conceptions about the nature of modeling and students' training purposes [7]. In this study, the nature of modeling involves a conception of the object and the tool [9]; regarding training purposes, it is assumed that future teachers should not only learn mathematics, they should also learn to use modeling in their professional practice; that is, teachers should promote mathematical thinking as well as modeling skills and competencies. In Figure 1, this perspective of the teacher's modeling knowledge is represented.

In this framework, the intersection between the conception of the tool and the purpose of mathematics training implies the design of learning environments that allow future teachers, through modeling, to conceptualize, to solve problems, and to generalize mathematical concepts. The intersection between the conception of modeling as a teaching tool and as a professional tool suggests the need to promote the development of knowledge in which the (future) teacher uses mathematical modeling in the design of tasks, classes, and environments for mathematics learning, considering all the facts that this implies (students learning, curriculum, context, among others). The intersection between the object and mathematics teaching perspectives implies the design of environments in which (future) teachers can "learn to do modeling"; this also implies the development of a sensitivity to identify and delimit problems, to select relevant variables, techniques, procedures and ways to build models, to solve problems using mathematics, to validate the results, etc. Finally, in the intersection between modeling as an object and as a professional practice, knowledge about the nature of modeling for teaching can be considered, including the type of tasks, type of environments according to contextual and institutional needs, among others.

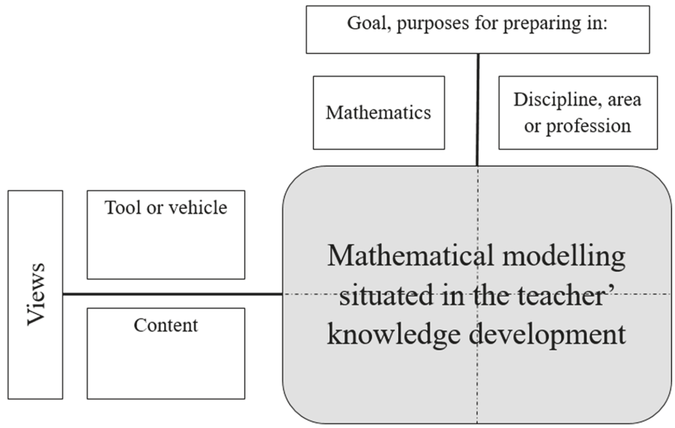

Figure 1. Representation of a perspective of the teacher's modeling knowledge.

It is expected that for each of the abilities that the literature suggests for mathematics teachers, these perspectives and purposes can be identified, so each one can fit in some of the intersections shown in Figure 1. Figure blocks do not represent disjoint compartments in the teacher's knowledge, but analytical categories for the design of learning environments for those professionals. Due to the nature of the question that motivated this study, the intersection between tool and object perspectives will be used to train future teachers.

2.2. *Formative Assessment for the Teacher's Knowledge*

For Black and Wiliam [10,11] formative assessment or assessment for learning demands from teachers and students an active interpretation and use of evidence about their performance to make decisions during the processes. This is a practice that seeks a constant improvement of teaching and learning, tracking students' development in order to make decisions and reformulate tasks according to the observed results [10,11].

In this study, teacher educators and pre-service mathematics teachers were considered key actors in the process of formative assessment. According to Black and Wiliam [11], formative assessment involves several stages, namely: the establishment of training goals or purposes, information gathering about students' thinking and knowledge, and a plan proposal (methods, strategies, environments). Pre-service teachers were allowed to participate in the stage planning, that is, they participated in the delimitation of the evaluation criteria and the procedures and strategies to achieve compliance with this purpose. Black and Wiliam [11] argued that five principles can be recognized in the design of environments for formative assessment, namely: (i) clarify and share learning intentions and criteria for success; (ii) design effective classroom discussions and other learning tasks that provide evidence of student understanding; (iii) provide feedback to helps students progress; (iv) promote students interaction to improve learning; (v) mobilize students to empower themselves in their learning.

Formative assessment, as a means of supporting the development of teachers' knowledge, considers the strategies, media, environments, and roles of teachers as learners and of teacher educators as teachers. In a synthesis of contributions from a special issue on formative assessment and professional learning of teachers (Teachers and Teaching, Vol 19, No 2), Tigelaar and Beijaard [12] found that in the context of professional learning, teachers can be considered as learners, given that the evidence of learning that is being collected during formative assessment processes provides them with an idea of how are they performing, where do they need to move, and what can they do to get there. Regarding strategies, the authors highlight the presence of heuristic diagrams, self-evaluations combined with co-evaluation, formative feedback, negotiated evaluation, among others.

3. Methodology

3.1. Context and Participants

This study was carried out during the second semester of 2019, in a mathematical modeling course for pre-service teachers. The course was part of a Bachelor program offered by a school of Education at a public university in Medellín, Colombia. In Colombia, mathematics teachers are prepared in Bachelor programs offered either schools of Education or Mathematics Sciences or both (more details about the Colombian Mathematics teacher preparation system see Guacaneme-Suárez et al. [13]).

Throughout the course, the pre-service teachers had to develop modeling tasks [14] and analyze their own experience based on theoretical and empirical constructs studied during the process. They also participated in workshops and discussed with modeling researchers and with in-service teachers who had modeling experience. During the course, students had to develop a modeling project [14] and design a lesson plan.

The course was distributed in 16 sessions of 4 h each. In the first session, the objectives of the course, the methodology, and the evaluation products were presented. The meaning of mathematical modeling and their experience in previous courses were also discussed. In sessions 3, 8, 15, and 16 oral presentations about their progress in the projects and lesson plans were developed. Both the teachers and the pre-service teachers could comment, suggest and argue about the progress of their classmates. Based on the approach of Black and Wiliam [11], the course followed the phases and roles for teacher educators and pre-service teachers. The main aspects of formative assessment during the course can be found in Table 1.

Table 1. Aspects of formative assessment adapted to this research.

	Training Purposes	Where Is the Student Now	How to Get There
Teacher Educator	1. Clarify learning intentions for success. The teacher educators specify at the beginning of the course the objectives, methodologies, and tasks to be carried out. A "class by class" is created where the objectives and purposes of each session are specified.	2. Design effective classroom discussions and other learning tasks that provide evidence of student understanding The teacher educator designed training environments for pre-service teachers. The task involved assessment and selection of relevant tasks, actions, interactions, class, and extra-class strategies, and class management.	3. Provide feedback that makes students move forward. The teacher educator offered continuous advice in order to promote reflection and problematization/continuous questioning; teachers also offered feedback. All this was done both in class and in extra-class spaces.
Peers	Understand the intentions and participate in the construction of assessment criteria and learning expectations. (Collaborative rubric)	4. Students actions as training resources for others. Students participated in the development of tasks and reflections on what they know, why they know it, and why what they know is useful for their future professional practice.	4. Students actions as training resources for others. Students actively participated in joint sessions; they commented, criticized, and made suggestions to other classmates' actions. They also participated in the construction of criteria for the rubrics of products of the course.
Pre-Service Teachers	Understand the intentions and participate in the construction of assessment criteria and learning expectations; design paths and strategies to meet these commitments collaboratively (Development of projects, reports, and lesson plans)	5. Empower students as responsible for their own learning They participated in readings, discussions, and workshops on "what should be known" and why it is important to know it.	5. Empower students as responsible for their own learning They got continuous advice for the development of the proposed professional tasks (projects, design of class plans).

Fourteen pre-service teachers (11 female and 3 male) participated in the course and were informed of the ethical protocols, signing an informed consent. The names used in this article are pseudonyms. The mathematics education program was a five-year BSc program, the students (pre-service teachers) were selected according with their scores from university entrance examination. 7 of the participants in their fourth year, and 7 were in the fifth year of the program. The pre-service teachers' ages ranged from 19 to 23 years. All participant had completed mathematics courses (e.g., geometry, arithmetic, mathematical analyses), mathematics education courses (e.g., Didactics of algebra, geometry, statistics), and a part of pedagogical courses (e.g., curriculum, educational politics, culture and education). Only six participants reported that they were coursing *practicum*. None of them reported work experiences as teacher.

3.2. Data

The pre-service teachers committed themselves to the development of the modeling tasks, the projects, and the lesson plans. The collective construction of the rubrics was made around the seventh-class session, after studying theoretical aspects of mathematical modeling and developing related tasks. Each session of the course was videotaped, therefore, for the lesson plans developed by the students, videos of the discussion sessions and of the evaluation rubric agreements were recorded.

Each workgroup participated in at least one advisory space with the teachers. A video that records the interaction between the trainers and the pre-service teachers was recorded. There were four work teams in the course. Each one developed a class plan that was reported in a written document and video-recorded while presented to classmates and teachers.

3.3. Data Analysis

To analyze the data (videos and documents), a category system with its respective coding was developed in an iterative process of going back and forth between predefined concepts (see the second section of this article) and data. The three researchers reached a common understanding on the codes and categories, later, the second author of this article organized and coded the data. He performed the first analysis of each lesson plan separately. The three researchers were regularly meeting to discuss and negotiate agreements and disagreements about the evidence, and data interpretations in light of the theory.

With the data from each lesson plan, a thematic analysis was carried out [15,16], the information was organized by themes, and points of convergence and divergence were sought. This allowed the emergence of other categories of analysis in light of the theoretical aspects described above. Then, the entire team of researchers conducted a cross-sectional analysis of the four lesson plans. The final system of topics and categories is detailed in Table 2. In the results section, the meaning of the categories is illustrated in greater detail with fragments of conversations extracted from the videos and the lesson-plans documents.

Table 2. Category and code system.

Themes	Categories	Codes
Knowledge in the design of lesson plans.	Knowledge on modeling as a vehicle Knowledge on modeling as a content or object	Use. mathematical concepts Sub-processes. Abilities (others) Simplification. Experimentation. Delimitation of problems. Abstraction. Context. Mathematization. Communication (others).
Formative assessment of pre-service teacher's knowledge on mathematical modeling.	Contributions from rubrics construction. Contributions from advise sections	Orientations. Share goal. Limitations. Feedback Questioning. Reflection. Limitations.

4. Results

The results of this study are presented in two sections: the first one presents the results of the analysis of each lesson plan; in the second one, an analysis of the formative assessment of the knowledge of pre-service teachers is made from a joint interpretation of the four lesson plans.

4.1. Analysis of the Four Lesson Plans

4.1.1. Lesson Plan 1: Clash Royale. Mathematical Modeling Experience in the Classroom

This team designed a class based on the use of the Clash Royale video game. The objective of the class was "To record and interpret numerical data from the environment offered by the Clash Royale video game" (Document 1—Class Plan). In their report, the students argued their design on the need for learners to build and compare representations, and to solve arithmetic problems that involve calculation and estimation strategies [17].

Pre-service teachers argued that the need to know a game and build winning strategies enables students to face a challenge. The class design was structured in three stages, each one one-hour long. The first stage was based on the recognition of the video game, its components, rules, players, etc. The second stage involved the delimitation, collection, and organization of data; according to the pre-service teachers "the students will have to extract different numerical data from the game environment: elixir production, cost (in elixir), attack speed, resistance and damage produced by the characters of the cards. The data obtained will be recorded in tables . . . " (Document 1—Class Plan). The third stage was organized through questions about the strategy to play the game efficiently.

This team proposed an evaluation of the class with scores according to the following game criteria: exploration and systematization of numerical data (10 points), analysis of situations (10 points), development and implementation of strategies (20 points), and communication of proposals by the students, during the dialogue spaces in each stage (10 points).

An analysis of this lesson plan allows to infer students' understanding of mathematical modeling as the solution of problems using mathematics; in the context of the video game, mathematical modeling was represented by the construction of a strategy to improve performance. Despite this, aspects such as mathematical work and validation of results were absent. During the modeling process, pre-service teachers took into account elements such as data collection and its organization, identification of variables to reach the solution, reasoning, and communication. In a broad understanding of mathematical modeling, these processes are part of modeling learning. Additionally, considering Colombian curricular guidelines, this team proposed to promote in students the creation of representations to solve problems. These aspects are key in modeling processes as a tool to achieve some curricular goals.

The lesson plan included considerations about assessment related to professional knowledge. For the team, the assessment was present in the three stages of the class. It was based on criteria to assess what students can do; however, it was not in line with the proposed objective or with the stated standards of the class. In this case, knowledge on the assessment during the modeling process is a key aspect in the professional training of pre-service teachers and is related to the intersection between this component and the modeling-as-an-object perspective presented in Figure 1.

4.1.2. Lesson Plan 2: Impacts on a Person's Life Expectancy Caused by Tobacco Use

The team designed a class to promote reflection on the consequences of tobacco use and the understanding of linear functions. In this case, pre-service teachers relied on Colombian curricular guidelines [17]. From this document, they extracted the notion of "learning evidence" that guided the assessment proposal.

The lesson plan was structured in four stages. In the first one, students became familiar with the context, identified a smoker, and interviewed her/him to obtain data on their age, habits, and motivations for smoking. In the second stage, students were invited to deepen in the context understanding; To do this, teachers proposed to observe a video and to answer three questions about the consequences of tobacco use, life expectancy, and its decrease due to tobacco. In the third stage, the students used the rates of change and percentages included in the video (years of life per amount of tobacco use) and, based on the data obtained in the interview, they concluded on the life expectancy of the interviewed person. In the fourth stage, students constructed tables of values and other representations of the obtained data set. After constructing Cartesian graphs, students were asked to "Show your model below, and explain how you got there" (Document 2—Lesson Plan).

An analysis of this lesson plan shows the intention of pre-service teachers to design a modeling task to promote reflections on health care. This purpose is within the scope of the socio-critical perspective of modeling that was studied during the course. In the class plan, there is also an interest in delimiting stages and tasks that students perform, which are gradually designed for the development of the activity. There is an interest in using change ratios to interpret data tendencies and construct linear functions; all of this describes a perspective of mathematical modeling as a tool to understand a situation, to mathematize it through linear functions, and to reflect on the impact of tobacco consumption.

On the other hand, the ordering of data, its organization in tables, and the identification of trends in generated graphs was encouraged. These elements are important for the learning of modeling as an object. Aspects such as experimentation, delimitation of a context, validation, and communication of the results were not observed in this lesson plan. Nor was it observed the creation of a space for reflection on the learning process by students or the promotion of actions or campaigns for health care, aspects that could strengthen the socio-critical scope of the modeling process.

4.1.3. Lesson Plan 3: Get Oriented and Take Tours inside the University of Antioquia

This team proposed a class to study spatial location, including direction, distance, position in space, and representation of space. These themes were based on Colombian curricular guidelines [17].

The class plan was designed based on a fictitious situation in which school children would visit the university facilities, the place where pre-service teachers carry out their studies. The tasks were organized in four stages. The first stage consisted on tracing a path through a 6×6 squared mesh; only horizontal and vertical displacements were allowed. The second stage involved a tour of several places of the University. In the third stage, in the classroom, students must mark on a map the most significant places during the tour. Finally, in the fourth stage, a plenary session was proposed in which they describe what they learned about the more meaningful, faster, and shorter routes. This team considered that evaluation should be used at every stage. They consider, as pre-service teachers, to be attentive to what children do and say, so that they could make timely recommendations. They would pay attention to the way they communicate, during the fourth stage, their actions, and recommendations to other classmates.

In the analysis of this lesson plan, knowledge on modeling was identified as a vehicle to promote spatial location skills in students. Although it was a possible scenario for mathematical work, the activity was not conceived to build mathematical models as representations, but to use notions of laterality and their mental representation. Students supported their choice in the course bibliography. In the class plan, modeling in primary school was described differently as conceived in higher grades; modeling was understood as "a mathematization of reality", according to Parra-Zapata and Villa-Ochoa [18]. Stages were planned so the children gradually gained experience, represented their knowledge on maps, and communicated them to their peers. Regarding modeling as an object, opportunities to explore, position one-self, and move inside the environment are worth noting.

Unlike the first two teams, in this lesson plan, no evaluation rubrics were identified, but there was a continuous effort to be attentive to students' actions and reflections to offer feedback; this evinces comprehension of formative assessment as a permanent activity throughout the modeling process.

4.1.4. Lesson Plan 4: Mobile Operators in Colombia

Unlike the previous ones, this lesson plan focused on solving a problem through an authentic context, supporting students to understand the phenomenon of mobile phone consumption in the country. The design was supported by the course bibliography. The pre-service teachers determined the topics that would include the process, namely: directly proportional magnitudes, conversion of measurement units, collection, and interpretation of data; however, they reported that such topics should emerge as part of the solution, but they were not the main objective of the designed task. Like the other teams, design criteria were justified in the Colombian curricular guidelines [17]. Unlike the other teams, in this lesson plan, the pre-service teachers provided information about what they considered a classroom environment should be: they described the way they conceived the active role of the students, the role of the teachers as helpers, and how to promote collaborative work and good use of resources by the students.

The lesson plan included five class sessions. In the first session, they created a fictional character (Carlos) who needed a mobile phone and wanted to purchase a plan. To help him, the team proposed to the students to inquire about operators, plans, costs, and other relevant facts. They would also assess Carlos' needs and determine how each plan could or could not satisfy his needs. In the second session, students were invited to fill out a table containing information about Gigabytes, prices, duration, among others. Based on the table, students should generate proposals to solve Carlos' needs. The third session focused on Carlos' need to use the internet to upload photos. Students should offer responses according to the number of files to upload and the number of messages received and sent. The fourth session was called "decision making", students were invited to determine Carlos' internet consumption and, based on that, offer him recommendations to make a decision.

In the analysis of this lesson plan, the pre-service teachers created a fictitious character as a way of delimiting the activity so that it became semi-open, that is, it had intentionality and facilitated the knowledge of the phenomenon, the identification of variables, and some simplification according to the initial intention. It is worth noting the effort of pre-service teachers to create not only a working guide for students but also to consider criteria to consolidate a participatory learning environment. That way they, as teachers, could regulate their actions while following and supporting students' performance. This course of action is related to what Cetinkaya et al. [6] call spaces that promote adaptive interventions.

In this lesson plan, opportunities offered by "experimentation" with the phenomenon are highlighted. Pre-service teachers propose to students to identify variables, obtain and organize data, and make inferences about them. The construction of models was guided by the identification of patterns in the data and inductive reasoning. Nevertheless, little emphasis was put on promoting communication of the results to the fictitious character and offering mathematical generalization of the generated algebraic model. All these elements are related to the perspective of modeling as an object.

In these four lesson plans, pre-service teachers show their knowledges on mathematical modeling. These knowledges include several understandings about modeling (process, problem solving) and purposes (introduce a content or developing critical and other skills) [19,20]. It also notes several of types and uses of contexts for the development of modeling (e.g., realistic, authentic [14,21]). The inclusion of tasks and phases was a common aspect in the lesson plans; assessment strategies were also included in all plans. The following section reports how the formative assessment strategies of the course promote knowledge on mathematical modeling.

4.2. Analysis of Formative Assessment of Pre-Service Teacher's Knowledge on Mathematical Modeling

The lesson plans provided information about the knowledge that future teachers developed about teaching (through) modeling, that is, modeling as a teaching vehicle and modeling as a teaching content or object [9]. However, in the context of a teacher-training course, it is not only interesting to identify the generated knowledge, but also how it was promoted; in other words, it is important to consider a formative assessment.

As showed in the previous section, the four lesson plans were guided by a similar framework. This framework included title, class objective, alignment with Colombian curricular guidelines, class development, assessment, and bibliographic references. Additionally, lessons included student's work guides and a justification of the design based on the course's theoretical references. This structure of the four lesson plans included a guide for the student. The similarity in the structure of the lesson plans is due to the agreements reached for the construction of the rubric.

As reported in the methodological section, the pre-service teachers participated in the construction of the rubric, where the components of the lesson plans and evaluation criteria were established. As an example, Amelia pointed out that "A class must have a clear objective, which is expected to be achieved in one or more sessions. In every class that we have had, they presented an objective, the development of the class and the methodology, and, well, the evaluation" (Video, 4 July 2019, negotiation of the guide). Also, Carlos pointed out that "In the tasks that we have read, we see that the authors always state their purpose and establish the tools to measure the achievements of the modeling tasks" (Video, 4 July 2019, negotiation of the guide).

An analysis of the video of the rubric-construction session allowed us to infer the main guidelines on which students relied to consolidate the rubric and the structure of the lesson plans. These results are presented in Table 3.

Table 3. Lesson-plans elements and supports.

	Lesson-Plan Elements	Theoretical Support
I	State a theoretical approach to the way of modeling assumed in the lesson plan.	Villa-Ochoa, Castrillón-Yepes y Sánchez-Cardona [14]
II	In the Colombian context, relate or support your lesson plan according to theoretical and methodological guidelines defined by the Ministry of National Education.	Ministry of Education [17]
III	Define the materials, resources and times necessary to achieve the lesson plan objective.	Bassanezi [22], Biembengut y Hein [23]
IV	Describe how the evaluation process is carried out in the lesson plan.	Aydogan Yenmez et al. [24], Diefes-Dux et al. [25]

Rubrics are instruments designed to help assessors, teachers, and students to judge the quality and progress in student's performance [26]. These instruments are used for both summative and formative assessment. The participation of pre-service teachers in the design of the rubric was intended to promote formative assessment about their modeling knowledge. This participation produced the structural components of the lesson plans (components to be evaluated) and detailed criteria for evaluating them (descriptions of student's performance). The consolidated rubric is presented in Appendix A.

The participation of pre-service teachers in the construction of the lesson-plan structures and its corresponding rubric offered them opportunities to anticipate what would be the evidence of their learning about the use of modeling in teaching; in the words of Black and Wiliam [10,11], this participation contributed to the principle of "clarifying learning intentions". As shown in the previous section, in the lesson plans, certain knowledge became evident: knowledge about the management of the class (lesson plan 4); knowledge on the use of modeling to teach mathematical content (lesson plan 1, 2, 3) and knowledge on problem solving (lesson plan 4). To a lesser extent, knowledge about the teaching of modeling was evidenced, including subjects such as: knowledge of the context (lesson plans, 1, 2, 3, and 4); exploration of conditions and variables (lesson plans, 1, 2, 3, and 4); construction of a model (lesson plans 1 and 4) and use of mathematical information to understand the implications of a situation (lesson plan 2). Despite this, processes such as reasoning and communication, which are fundamental in modeling, were not noticeable in all the designed plans. Table 4 summarizes the knowledge evidenced in the lesson plans designed by pre-service teachers.

Table 4. Knowledge in the lesson plans.

Lesson Plan	Class Management	Teaching of Mathematical Content	Problem Solving	Modeling Teaching			
				Context	Variables	Model Construction	Information Use
1		☑		☑	☑	☑	
2		☑		☑	☑		☑
3		☑		☑	☑		
4	☑		☑	☑	☑	☑	

In these results, participation in the construction of the rubric played a normative role. In this study, it was observed that the rubric offers guidance on what will be evaluated and how it will be evaluated; also, it seems to promote the appearance of other modeling knowledge not directly declared in the rubrics, but which can be valuable for pre-service teachers. This result recalls the criticism that Panadero and Jonsson [26] have called standardization and reduction of the curriculum. According to the authors, it is questionable the way rubrics standardize assessments by providing simple lists of criteria for complex skills and by creating a tendency on students and teachers to guide their actions exclusively towards those criteria.

Another characteristic of the pre-service teacher's formative assessment was the continuous feedback achieved. During the course, in all class activities (workshops, homework, readings, discussions), there were reflections on: What was learned? Why was it important? And how could this be integrated into their future profession? Additionally, spaces for continuous advice were created in class and extra-class times. During the class, oral presentations were made about progress in the lesson plans; both teachers and pre-service teachers could comment and criticize each team. In extra-class spaces, pre-service teachers dialogued with teachers about their progress. Teachers permanently invited pre-service teachers to reflect on: why to do what is proposed? What does the literature say about it? etc. This allowed a reflection on the nature of modeling in mathematics school teaching. As an example, Josefina, a member of the lesson plan 2, indicated:

> *Josefina:* We want to propose our class for third grade children, we liked the document we read about geometry and modeling in primary school, so we would like to do something similar with the children.
>
> *Teacher Educator*: But, how is modeling conceived there (in the document)? What is the most relevant thing the authors talked about? What is different from other ways of modeling?
>
> *Josefina:* Well, what most caught our attention is that the authors showed that modeling allows students to establish a relationship with space, in such a way that geometric notions become a means of decision.
>
> *Teacher Educator:* And what does that mean? How did the authors propose it? Is it a matter of getting the students to move in space or is there something else that requires planning?

In response to these questions, in their lesson plan document, the team described in greater detail the arguments they extracted from that bibliographic reference to design the four stages of the plan and the transition between the real displacement and the map location activity. A similar situation happened while giving advice to the team of class 3.

> *Alexander:* Teacher, we don't know how to integrate the assessment part into our lesson plan, we don't want the assessment to focus only on mathematical concepts; we don't want the assessment to scare students either.
>
> *Teacher Educator*: Alexander, but according to what we have experienced in the course, how do you think your processes have been assessed? What tools and forms of assessment have we used or studied? Ideally, everything we have developed in the course contributes to the construction of your lesson plans.
>
> *Alexander:* Teacher, you have accompanied us with questions that guide us or questions that make us realize the errors or weaknesses we have.
>
> *Teacher Educator*: Accordingly, how should assessment processes be included in your lesson plans?
>
> *Alexander:* Teacher, then it would be like not even telling the students that they are being assessed, but teachers should be very attentive and assess what the students are doing and try to redirect what may not lead them in the right direction. But in that scenario, don't we have to apply an exam or a rubric or a final assessment?
>
> *Teacher Educator*: The idea is that you make the decision about how you will carry out the assessment process and, in general, how you will build your lesson plan. But what is clear is that you do not have to use the rubric as an evaluation instrument, you can use other resources or instruments. What is necessary is that you indicate how the evaluation process would be developed in your lesson plan.

The third team's lesson plan showed that the elements discussed in advise sessions offered clarity to the students (pre-service teachers). In particular, this work team integrated, during the four stages of the lesson plan, feedback processes, and support to the students and made possible an assessment that facilitated orientation and success of the students.

Feedback can be considered a key strategy within formative assessment [10,11]. In the case of the present study, the feedback was conceived as a continuous dialogue and questioning about what pre-service teachers were proposing, thereby offering them opportunities to reflect on their proposals and helping them to improve their arguments and actions. Pres-service teacher's arguments were based on the reviewed literature and also on the projection of other variables present in the institutional context. According to Romo-Vázquez et al. [5], teacher training should not only be based on the design of tasks and its implementation in class, but also on knowledge of the curriculum and other institutional considerations. Despite these reflections, no important evidence of the presence of such knowledge was included in the lesson plans. This can be justified by the fact that pre-service teachers had not yet had contact with school environments and, therefore, were unaware of the diversity of institutional conditions that may be present in daily school life.

5. Conclusions

In the first part of this article, conceptions about the notion of teachers' knowledge assessment were presented. Those conceptions are aligned with the notion of measurement and certification of teachers' knowledge and abilities. It also debated the need for this notion to transcend into a formative assessment of teacher's knowledge in the context of training courses and professional programs.

This article offers evidence that, in the context of a course, the notion of formative assessment of pre-service teachers' knowledge requires a conceptual delimitation of the knowledge that is expected to be achieved and the strategies to achieve it. The courses, by their nature, are delimited in space and time; therefore, their purposes, methodologies, and scope are also conditioned. In the case of this study, a conceptualization of two broad categories of modeling knowledge in teaching was offered: modeling as a tool and modeling as a learning object. In this framework, this study offers evidence of the knowledge showed by pre-service teachers in their lesson plans and on the contributions and limitations of rubrics and feedback in the strengthening of this knowledge. In this regard, this study highlights two important results.

The first result that stands out is the local character of the knowledge that is achieved in a course for pre-service teachers about teaching of (and through) modeling. The literature has shown the complexity involved in integrating modeling into everyday school life and the high demands that it implies for teachers. Faced with this panorama, the scope of a course is only part of that knowledge; the teaching practice will be conditioned by the opportunities and limitations that pre-service teachers have about school practice. It will also depend on the environment and strategies implemented during the course. In this sense, the second important result derived from this study is related to the opportunities and limitations offered by continuous advice and participation in the construction of rubrics. As argued in this study, some research supports the use of rubrics for student learning, academic performance, and self-regulation; however, rubric design requires care. In this study, participation in the rubrics contributed to the development of pre-service teacher's knowledge about "teaching by and through modeling" and conditioned the appearance of other important knowledge in this category. Regarding advise sessions, its contributions to continuous feedback were important, but it also became clear that these contributions may be conditioned by the possible existence of other knowledge, for instance, the institutional context. These results can be used by mathematics teacher educators as an insight to the opportunities and limitations of the formative assessment for developing preservice teacher knowledge on mathematical modeling. Some formative assessment strategies would need to be reworked to afford a generation of other knowledges on mathematical modeling among pre-service teachers.

One limitation of the study is that pre-service teacher knowledge was analyzed through lesson plans. Other studies could analyze pre-service teacher knowledge in professional authentic situations (for instances, practicum) that might provide more differentiated descriptions of their prospective professional work; but as our interest was in the knowledge on modeling as both object and content we found lesson plans more appropriate. The variety of knowledge found in the participants informs about contributions of rubric and feedback, but we cannot generalize all our findings to other formative strategies uses or mathematics teacher education programs. In this sense, this study suggests the need for new research that accounts for the contributions of other strategies to the development of pre-service teachers' knowledge. New studies on the design of rubrics are suggested, to address the participation of pre-service teachers and the formative/normative tension described in this article.

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Informed Consent Statement: Informed consent was obtained from all subjects involved in the study.

Data Availability Statement: Not applicable.

Conflicts of Interest: The authors declare no conflict of interest.

Appendix A

Table A1. Rubric built collaboratively with pre-service teachers. Rubric for classroom assessment of mathematical modeling experiences.

Assessed Aspect	Naive	Novice	Apprentice	Expert	Recomendations
COMMUNICATION MOMENT (15%)—Presentation the Lesson Plans to Group					
Purposes, objectives or goals of the classroom experience	Describe without detail the purposes, objectives or goals of the classroom experience.	Describe the purposes, objectives or goals of the classroom experience.	Presents the relationship between class experiences and objectives, purposes or gools of the class.	Explains to the group how class experiences are articulated with the objectives, purposes or goals of the class.	
Theoretical approach to the way of modeling	Describes without detail the conception of mathematical modeling that is assumed in the classroom experience.	Describes clearly the conception of mathematical modeling that is assumed in the classroom experience.	Presents the conception of mathematical modeling that is assumed in the classroom experience.	Explains the conception of mathematical modeling that is assumed in the classroom experience and recognizes its scope and limitations.	
Relationship between the classroom experience and the guiding documents	The planning of the modeling experience does not state the relationship with the guiding documents.	Describes the expected school grade for the modeling experience.	Articulates the planning of the described modeling experience with the guiding documents and the school grade to which the activity is intended.	Explains the articulation between the planning of the modeling experience and the guiding documents and states the school grade to which the activity is intended.	
Resources and strategies to be implemented during the modeling experience	The planning of the modeling experience describes the necessary resources for its development.	The planning of the modeling experience presents the resources and some strategies to develop.	The planning of the modeling experience defines the resources and some strategies to develop.	The planning of the modeling experience defines the resources and specifies the strategies that will be implemented during the development of the experience.	
Evaluative and feedback process during the modeling experience.	Highlight some elements of the modeling experience that will be evaluated.	Highlights which elements will be taken into account in the evaluation process of the modeling experience.	Describes how the evaluation and feedback process will be carried out in the modeling experience.	Presents an evaluation instrument and a description of how the feedback process will be carried out in the modeling experience.	
Questions	Answer intuitively questions without theoretical or scientific support.	Answer questions without theoretical or scientific support.	Answer questions appropriately on the topic.	Answer questions with deep analyses, synthesis capacity, knowledge of the topic, among others.	

Table A1. *Cont.*

Assessed Aspect	Naive	Novice	Apprentice	Expert	Recomendations
SISTEMATIZATION MOMENT (5%)—Instrument for Student					
Instrument for the student	Creates an instrument for the student where the sequence of activities is presented. This instrument is not coherent with the presented activities.	Creates an instrument for the student where the sequence of class activities is presented	Elaborates an instrument for the student where the sequence of class activities is presented. The instrument is articulated with the activities of the modeling experience.	Elaborates an instrument for the student where the sequence of class activities is presented. The instrument is articulated with the activities of the modeling experience and with the presented modeling conception.	

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mathematics

Review

Influence of Game-Based Learning in Mathematics Education on Students' Affective Domain: A Systematic Review

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Abstract: In modern education nowadays, the use of game-based learning as a teaching and learning method is popular in all school subjects, including mathematics. There are numerous studies dealing with the influences of this teaching method on the students' achievements. Modern teaching theories consider an important effect of education on the development of students' affective domain, connected with the subject and its teaching. In this work, the author studies journal articles that the use game-based learning in mathematics to assess its effects on the students, with the aim to analyze its impact on students' affective domain. To achieve this, a systematic review with the use of a PRISMA statement is applied. The data sources are 57 journal articles from the area of interest listed in the Web of Sciences and Scopus. The results indicate that 54% of the articles consider the affective domain in the measurement of the effects of game-based learning in mathematics education. These articles report mostly (84%) the positive influences of game-based learning on students' motivation, engagement, attitudes, enjoyment, state of flow, etc. The rest of the articles show mixed results, with the authors' conclusions possibly affected by flaws in the research instruments, selection of study groups, and game design, therefore, stressing the importance of these elements in future research on this topic.

Keywords: game-based learning; affective domain; mathematics education; systematic review

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1. Introduction

Based on the fast progress of sciences and technologies, continuous innovations of the content, methods, and goals of school mathematics education are needed. One of the promising methods to achieve the active participation of students in learning activities and their higher motivation is game-based learning [1,2].

Game-based learning encourages active learning and engagement by providing students with possibilities to place problem-solving within the context of play [3].

The idea of games as an educational tool is not a new one, it was originally devised by Hellenic philosophers, Plato and Aristotle. In more recent history, game-based education has been part of the educational theories of important figures in this scientific area, such as J. A. Comenius (1592–1670), J. H. Pestalozzi (1746–1827), F. W. Fröbel (1782–1852), H. Spencer (1820–1903), K. Groos (1861–1946), M. Montessori (1870–1952), J. Piaget (1896–1980), L. S. Vygotsky (1896–1934), and J. Dewey (1859–1952) [4].

The impetus for the vast integration of game-based learning nowadays is driven by other factors, including the inclusion of digital technologies in education. Digital games support learning by giving students an opportunity to develop knowledge and cognitive skills, to learn by problem-solving, and to experience situational learning [5,6].

With this huge increase in game-based learning applications come natural questions about their effects on students. Many studies and reviews of existing research in this area have been conducted, mostly focused on the effect of games on students' performance compared with that of traditional classroom instruction [7].

Randel et al. [8], in their review, compared the effect on student's performance of games with that of traditional teaching in 68 studies up to 1991. The results were mixed;

the beneficial effects of games were mostly found when specific content was targeted. The review study of Hays [9], including 105 instructional gaming articles, also reported that the use of games in specific areas can provide effective learning, but the general conclusion was that there is no evidence that games are a preferred instructional method in all situations.

In the area of mathematics, educational games were identified as suitable to promote mathematic achievements in various domains, e.g., problem-solving and algebra skills [10], strategic and reasoning abilities [11], geometry skills [12], arithmetic [13,14], and critical thinking [15]. These studies are mostly focused on game-based learning's influences on mathematical achievements in the form of knowledge. But other important parts of mathematic education are affective factors such as students' motivation, beliefs, and attitudes towards mathematics and its teaching, as these factors can have a big impact on students' mathematical skills and their future mathematic learning [16–18]. Therefore, the question arises of how and to what extent game-based learning in mathematics education influences students' affectional dimension.

This question has already been discussed in some studies and reviews. The research of Garris et al. [19] found out that games could improve the engagement of students, advocating that games could influence outcomes including attitudes. These games were used in the setting of school education, but also in adult training. The study of Vogel et al. [20] reported positive effects of games vs. traditional teaching methods for both cognitive gains and attitude. However, the authors of the study considered the reliability to be low. The study of Ke and Grabowski [21] dealt with the effect of game-based learning on fifth-grade students' mathematics performance and attitudes. The results indicated that the integration of games positively influenced attitudes towards mathematics, mostly in cooperative structured groups. Based on a review of previous research studies, Vandercruysse [22] suggests that educational games positively affect students' attitudes, though just three journal articles support this suggestion.

The above-mentioned studies imply possible influences of game-based learning on students' affective factors connected with mathematics and its teaching process. However, these indications are fragmented and do not give a general overview of the influences of game-based learning on students' mathematics. Therefore, there is a need for a systematic review of the journal papers dealing with game-based learning in mathematics, focusing on the present state of research on the influences of game-based learning upon student's affective domain.

Based on this need, in the current paper, the following questions are investigated:

Q1: To what extent do the research studies dealing with the effects of game-based learning in the field of mathematics education address the influence of this teaching method on students' affective domain, connected with mathematics and its teaching?

Q2: Which journals have published scientific articles in this field?

Q3: What have been the influences of game-based learning on students' affective domain?

Q4: What research instruments were used to measure the influences of game-based learning on the affective factors?

To present answers to the research questions, the paper has the following structure. The next chapter details the materials and methods used for this systematic review. The third chapter states the result of the systematic review concerning the research questions. The last chapter discusses the results, comparing them with the outcomes of other studies, summarizes the limitations of the study, and proposes ideas for future research on this topic.

2. Materials and Methods

To answer the research questions, a systematic review was selected as the most appropriate research method, as it was developed for identifying and synthesizing research evidence by taking a systematic approach, following transparent and rigorous processes [23]. As the protocol for this systematic review, the author used the Preferred Reporting Items for Systematic Reviews and Meta-Analysis (PRISMA) framework [24]. The reason behind the choice of PRISMA over other existing protocols was its comprehensiveness, its use in several disciplines worldwide beyond the medical field for which it was originally developed, and its capability to increase consistency across reviews. The PRISMA checklist used in this paper reflects the fact that it is a systematic review, not a meta-analysis. Therefore, checklist items 12–16 and 19–23 were not included [24] (p. 3). This is in accordance with the recommendations on PRISMA applications for systematic review studies [25].

The eligibility criteria for the papers included in this systematic review are specified in Table 1.

Table 1. The criteria for eligibility.

Criterion	Inclusion	Exclusion
Document type	Published journal article (both empirical and review)	Book, book chapter, conference presentation, or other type of non-peer-reviewed or unpublished publication
Language	English	Articles not written in English
Accessibility	Open-access version of the article	Source with no open-access version available

According to the criteria, only published versions of journal articles are selected. This means that books, chapters in books, conference proceedings, etc., are excluded. This particular criterion is based on the higher scientific validity of peer-reviewed published journal articles when compared with other types of reports. For the journal articles, both empirical data articles and review articles are included, to cover the biggest possible range of data. The second selection criterion is the language; journal articles are accepted that are written in English. This is to avoid any confusion due to problems with translation and misunderstanding. The third inclusion criterion is that the open-access version of the article is used, to enable the extraction of all relevant data during the data collection process. The timeframe of selected articles is not limited, again, to achieve the biggest collection of relevant sources.

As information sources for this systematic review study, the author selected two major databases, namely Scopus and Web of Science. This selection was based on the broad range of the themes and journals covered in these databases, and their high scientific recognition. The items in these databases are systematically structured and search algorithms are provided that offer the possibility of covering all three eligibility criteria.

The search of the selected databases was implemented on 16 March 2021. The search terms were 'game-based learning' or 'games' combined with a Boolean operator AND with the terms 'mathematics' or 'math'. The exact search strings and limits used are listed in Table 2.

Table 2. The search strings and limits used for the electronic search process.

Database	Search String and Limits
Scopus	TITLE ((“game” OR “game-based”) AND (“mathematics” OR “math”)) AND (LIMIT-TO (PUBSTAGE, “final”)) AND (LIMIT-TO (OA, “all”)) AND (LIMIT-TO (DOCTYPE, “ar”)) AND (LIMIT-TO (LANGUAGE, “English”)) AND (LIMIT-TO (SRCTYPE, “j”))
Web of Science	TI = ((“game” OR “game-based”) AND (“mathematics” OR “math”))) *AND* **LANGUAGE:** (English) *AND* **DOCUMENT TYPES:** (Article) **Refined by: Open Access:** (OPEN ACCESS) **Timespan:** All years. **Indexes:** SCI-EXPANDED, SSCI, A&HCI, CPCI-S, CPCI-SSH, BKCI-S, BKCI-SSH, ESCI, CCR-EXPANDED, IC.

3. Results

The initial search highlighted 68 document results in the Scopus database and 37 results in the Web of Science database. The search results including the abstracts were exported for further screening in MS Excel. The next stage was the identification of duplicates, which led the author to find 20 duplicate records; those were excluded. The remaining records' abstracts were carefully studied to judge their relevance. A total of 27 articles were excluded as they did not focus on the effects of game-based learning in mathematics education. The remaining 58 papers were retrieved in the full text form. They were screened to further inspect their relevance. During this process, one paper was excluded since it was only partially written in English. This process left 57 articles; those are included in the qualitative synthesis described in the next part of the paper. A graphical representation of the flow of citations reviewed during the systematic review process is presented in Figure 1.

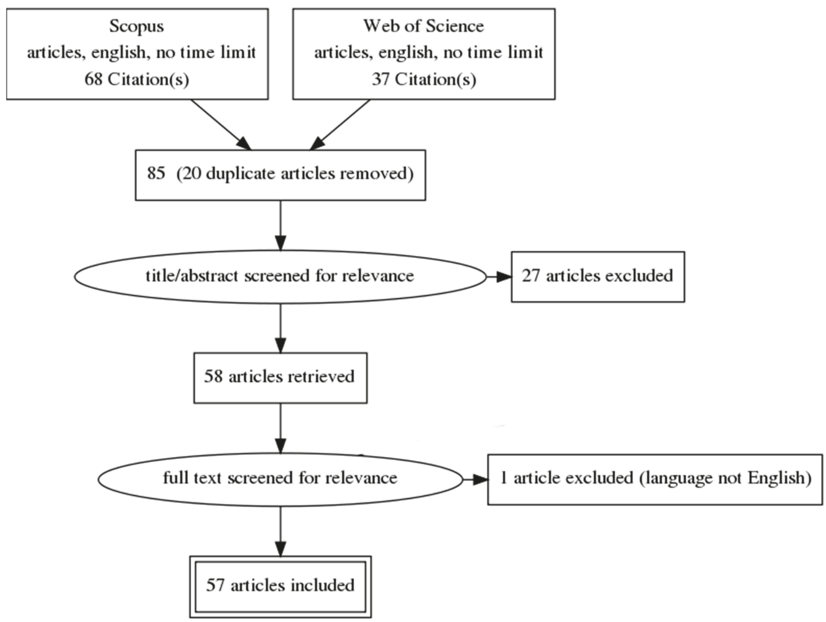

Figure 1. Systematic review flow diagram. Generated by PRISMA Flow Diagram Generator. Available online: http://prisma.thetacollaborative.ca/ (accessed on 20 March 2021).

The 57 included articles were thoroughly studied to identify the results connected with the research questions of this paper. For this analysis, the software ATLAS.ti 9 was used, mainly because it enabled the analyst to solve a range of methodological challenges, such as working with large datasets and supporting deeper levels of analysis than are possible by hand [26]. During analysis, the author found that 26 articles [27–52] did not discuss the direct effects of game-based learning on the affective domain. So, to answer research questions Q2–Q4, the 31 remaining studies are investigated [6,53–82]. Table 3 summarizes the main characteristics of these game-based learning studies, with detailed information concerning the research questions.

Table 3. Study characteristics.

Ref.	Journal	Influence on MRAF [1] [Positive/Negative/Mixed]	Measured Aspects of Affective Domain	Instruments to Measure MRAF [1]
[53]	Aust. J. Teach. Educ.	positive	motivation, engagement, state of flow, self-efficacy	questionnaire, video data
[54]	Aust. J. Teach. Educ.	positive	motivation, attitudes	interview
[55]	Br J Educ Technol	positive	motivation, attitudes	questionnaire
[56]	Br. J. Educ. Technol.	positive	motivation, engagement	observations
[57]	Education Tech. Research Dev.	mixed	motivation	questionnaire
[58]	EJEL	positive	attitudes	questionnaire
[59]	Eurasia J. Math. Sci. Technol. Educ.	mixed	attitudes, anxiety	questionnaire
[60]	Eurasia J. Math. Sci. Technol. Educ.	positive	motivation	unspecified
[61]	IEEE Access	positive	motivation, engagement, attitudes	questionnaire
[62]	iJOE	positive	motivation	literature review
[63]	IJSG	positive	motivation	video data
[64]	Informatics Educ.	positive	motivation, engagement, enjoyment	questionnaire, interview
[65]	Int. J. Artif. Intell. Educ.	positive	motivation	unspecified
[66]	Int. J. Multimedia Ubiquitous Eng.	positive	motivation	literature review
[67]	Int. J. Technol. Enhanc. Learn.	positive	state of flow	questionnaire
[68]	J Educ Psychol	positive	motivation, engagement, attitudes	questionnaire
[69]	J Educ Techno Soc	positive	self-efficacy, state of flow	questionnaire
[70]	J Math Didakt	positive	motivation, engagement	video data
[71]	J. Comput. Educ.	positive	motivation, self-efficacy, attention	questionnaire
[72]	J. Educ. E-Learn. Res.	mixed	attitudes	questionnaire
[73]	J. Educ. Techno. Soc.	positive	motivation, enjoyment, engagement	questionnaire, active participation
[74]	J. Inf. Organ. Sci.	positive	motivation, attitudes	literature review
[75]	J. Math. Educ.	positive	attitudes	questionnaire
[6]	Math. Ed. Res. J.	mixed	attitudes	questionnaire, interview, game data
[76]	MJLI	mixed	motivation	video data
[77]	MJLI	positive	motivation	unspecified
[78]	Rev. Colomb. Comput.	positive	motivation	questionnaire
[79]	Simul. Gaming	positive	motivation	active participation
[80]	Technol. Knowl. Learn.	positive	enjoyment	game data
[81]	Technologies	positive	engagement, enjoyment	questionnaire
[82]	Univers. J. Educ. Res.	positive	motivation	questionnaire

[1] Mathematics-related affective factors.

3.1. Extent of the Studies Considering the Affective Domain

As mentioned previously, of the 57 studies included in this systematic review, 31 (54%) addressed the affective domain in game-based learning in mathematics education, while 26 (46%) did not study the influences of game-based learning on the affective domain. The papers dealing with the affective domain were mostly from the last decade, and their number has slowly increasing trend, as can be seen in Figure 2.

3.2. Journals Publishing Studies Considering the Affective Domain

The 31 identified studies with research directly targeting the affective domain were published in scientific journals. Four journals (15%) contained two studies, while the remaining 23 journals (85%) included a single study. The scientific orientation of these journals was mostly Education and Educational Research, followed by Computer Science Interdisciplinary Applications, Psychology and Educational Sciences, and other similar scientific disciplines.

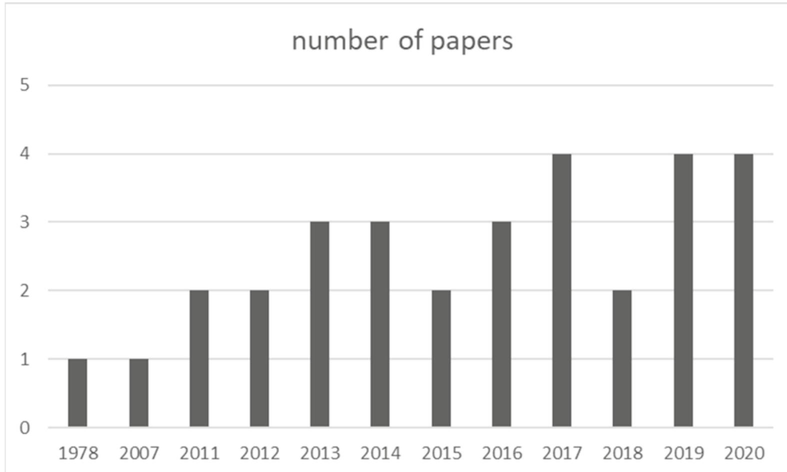

Figure 2. Publishing years of the source studies concerning the affective domain.

3.3. Influences of Game-Based Learning on the Affective Domain

The majority (26, i.e., 84%) of the journal articles dealing with the affective domain reported positive influences of game-based learning on students. These positive results related to the students' motivation (20 studies), engagement (eight studies), attitudes (seven studies), enjoyment (four studies), state of flow (three studies), and attention (one study). As for the number per study, one study reported four of the above-mentioned affective domain elements with positive results due to game-based learning, five studies reported three elements, seven studies considered two elements, and the remaining 13 studies focused on just one element of the affective domain.

The remaining five (16%) journal articles stated mixed results of game-based learning in mathematics education on students' affective domain. These results related to attitudes (three studies), motivation (two studies), and anxiety (one study). Of these studies with mixed results, one study addressed two elements of the affective domain and four studies just one element.

3.4. Instruments Used to Measure the Influences of Game-Based Learning on the Affective Domain

The most widely used research tool to study the influences of game-based learning on students' affective domain in mathematics education was a questionnaire, used in 18 studies (58%). The next most often used instrument was the analysis of video data (four studies), followed by an interview with the game participants (three studies), analysis of the data and metadata from the game (two studies), analysis of students' participation during game-based activities (two studies), and observation of students learning processes (one study). Three studies included literature reviews and three studies did not specify their research instruments.

Considering the number of instruments per study, one study used three of the above-mentioned instruments, three studies use two instruments, and the remaining studies used just one instrument or did not specify the instruments used.

4. Discussion

This systematic review concerns the influences of game-based learning on the affective domain, as studied in 54% of the journal articles in the area of game-based learning in mathematics education. These articles are mostly from the last decade and there is a slowly increasing trend of their number per year. The articles are published in various scientific journals with a broad scientific scope. This underlines that researchers in this field understand the importance of the affective domain for effective teaching, as many include this important dimension in their studies. There is a trend of increasing research focused on the affective domain [16–18].

Considering the research instruments, the author found that the studies included mainly questionnaires (58%), interviews (12%), and analysis of video data (10%), or a combination of these instruments, which are standard in the assessment of the affective domain. This finding follows a general trend in research on this topic [83].

One very promising result of this review is the fact that the majority (84%) of the studied journal articles report positive effects of game-based learning on students' affective domain. These results mostly include increases in motivation and engagement, and improvements in students' attitudes related to mathematical content and its teaching. Although some of the articles report mixed results, none report a negative impact. The positive influences of game-based learning on the affective domain are in accordance with previous research in this area [19–22].

Those with mixed results note that playing the selected game did not have a discernible effect on students' motivation to learn math [57]. The authors conclude that this is because they incorporated into the game features that they believed would be entertaining, but that proved not to be the case for students. Alternatively, the questionnaire was not appropriately framed to allow the researchers to detect any effects on motivation. Another study [59] reports that in terms of learning anxiety, significant differences between students in the high-score and low-score groups may be a result of family factors, as most of the students in the high-score group were very familiar with computer operations, compared with students in the low-score group who had limited experience using computers. The other study with mixed results [72] shows that the use of mathematical games in math courses does not change students' attitudes towards mathematics courses in terms of the content that they are learning. However, in contrast, it was observed that students were much more active and had fun when learning; in addition, informal interviews with students showed that students had positive feelings and thoughts about their mathematics lessons. The authors conclude that this conflict in the results occurs since primary school students who are still in the concrete process period may not be able to fully internalize the scale items in the questionnaire used in the research. The article [76] states, as a reason for the mixed results, different play preferences and motivation, based on the content of the game and to what extent it motivates students and matches their preferences.

The limitations of this study concern the focus on journal articles, thus omitting other sources as books, book chapters, and conference papers. Moreover, this study focuses only on articles written in English. The author was also not able to include articles that were not open-access, which again limits the sample. However, the included databases Scopus and Web of Science are highly representative, and English is used in most of the scientific journals, therefore, these limitations should not influence the overall comprehensive nature of the study.

In conclusion, this systematic review indicates mostly positive influences of game-based learning on students' affective domain (84% of studies). Studies with mixed results, according to their authors, are mostly influenced by inappropriate research instruments, mistakes with the game design, or special conditions within the study groups. Based on this, the author can conclude that if those limiting factors are not present, then it is likely that positive influences of game-based learning will be recorded on students' affective domain. Therefore, proper research instruments, selection of study groups, and game design are recommended for future research on the topic of the influences of game-based learning.

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Article

EXPLORIA, a New Way to Teach Maths at University Level as Part of Everything

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Abstract: The main objective of this article has been to evaluate the effect that the implementation of the EXPLORIA project has had on the Engineering Degree in Industrial Design and Product Development. The EXPLORIA project aims to develop an integrated competence map of the learning process, where the subjects are no longer considered as isolated contents, by elaborating an integrated learning process where the competences and learning outcomes of the subjects are considered as a whole, global and comprehensive learning. The EXPLORIA project connects the competencies of the different STEAM subjects that make up the degree, designing a learning process as a logical, sequential and incremental itinerary. Through concepts on which the foundations of design are based—shape, volume, colour, space and structure—the competencies of the different subjects are defined in incremental learning levels: understanding, applying, experimenting and developing, all taken from Bloom's taxonomy. Mathematics is linked to the rest of learning through active learning methodologies that make learning useful. This new methodology changes the student's affective domain towards mathematics in which positive emotions are transformed into positive attitudes that will improve the learning result and therefore, the students' academic results. To validate it, at the end of the paper, the academic results compared with previous years are shown, as well as an ad hoc survey of the students' assessment of the new teaching methodology.

Keywords: EXPLORIA; STEAM; active methodologies; university level; afective domain

1. Introduction

Mathematics is described by the National Council of Teachers of Mathematics (NCTM) as "Maths for Life" ([1], p. 4). This means that mathematics is essential for life as it helps in decision-making, planning, mathematical thinking and problem solving, which are necessary in different professional areas and daily life [2,3]. In [4] they add that mathematics is related to other sciences, not only numerical such as engineering or statistics, but also to arts, drawing, commerce, medicine, and so forth.

1.1. Affective Domain

The affective domain is defined as a set of feelings, moods and states of mind, understood as something other than pure cognition, and among which three specific elements stand out: attitudes, beliefs and emotions [5,6]. In [5,6] it is explained that these factors interact in a cyclical way, in the way we perceive mathematics, as we can see in Figure 1.

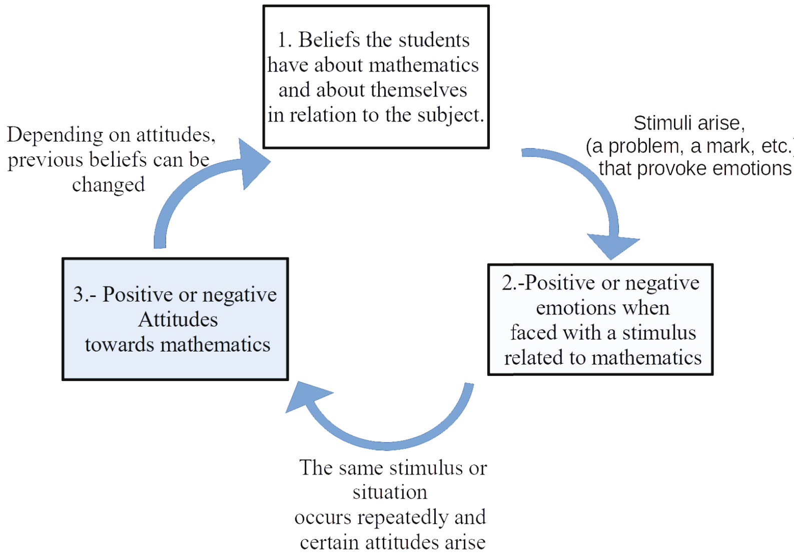

Figure 1. Attitudes and beliefs in mathematics.

Beliefs can be understood as a knowledge or feeling of certainty acquired and determined by past situations, which gives meaning to its own world, and which generates typical reactions without being fully aware of it [7].

Emotions are affective and automatic responses that arise from a significant event for the individual, and which result from complex learning, social influence and interpretation itself [8].

Regarding attitude, there is no unified definition in the literature, however, most authors agree in defining attitude as a disposition or predisposition towards mathematics, as for example in [8].

Attitudes are considered as one of the variables that most explains performance in mathematics [9–13]. In [10] it is estimated that attitudes constitute 30% of the explanatory factors of performance, concluding that students who display a more positive attitude towards mathematics will obtain higher mathematical performance.

1.2. Rejection towards Mathematics

In [12], an in-depth study is carried out on the rejection and negative attitudes towards mathematics. In this study, the number of participating students was 3187, belonging to all education cycles, primary, secondary, high school and the first year of university. The study was carried out in Spain in 10 different autonomous communities. The first item analysed was the taste for Mathematics at the different educational levels. The results show a high taste for mathematics in the initial levels at 87%; however, the taste for mathematics decreases as students go up in level, with 57% when they reach the first year of university. In [12], other subjects were also analysed but this decrease in the negative perception of Mathematics was not detected. In [12] the students' self-perception of mathematics skills is also analysed when answering the question, do I consider myself good at mathematics? In this case, the decrease is higher, going from 80% in primary education to 24% at university level. Finally, in [12] the level of the teacher's responsibility is analysed. In this case, the responsibility is about 15% at primary school level, reaching its maximum in the first year of university with about 60%.

The results obtained in [12] were later corroborated in [14], which showed that 67% of the students dislike mathematics and stated that they did not fully understand it. On the contrary, only 38 % showed an interest and liking for this discipline.

Recently, in [2] a study has been carried out on attitudes towards mathematics in university students. In the study, 1293 students (830 women and 453 men) of different degrees, Agri-food Engineering, Biology, Food Science and Technology, Pre-school and Primary Education, IT and Tourism were analysed. As a result, the average percentage in attitude obtained was 54% which shows that, in general, men have a more positive attitude towards mathematics, agreeing with other existing studies in this regard, such as in [15,16]. In [2] it was also found that students taking engineering degrees showed a better attitude towards mathematics than the rest, agreeing with other studies such as [17]. These degrees tend to have a greater number of men than women.

1.3. Trends in Learning: STEM, STEAM, STREAM

STEM (Science, Technology, Engineering and Mathematics) is a curriculum based on the idea of educating students in four specific disciplines: science, technology, engineering and mathematics, in an interdisciplinary and applied approach. Rather than teaching the four disciplines as separate and discrete subjects, STEM integrates them into a cohesive learning paradigm based on real-world applications.

The implementation of STEM learning generated an in-depth debate on how the four disciplines should be integrated. Two different approaches were established: the traditionalist approach, in which the four disciplines are developed independently, and the integrative approach, in which the four disciplines are developed together [18]. Of the two approaches, the integrative approach is currently the most widely accepted where the four disciplines constitute a single teaching-learning practice [19]. Still, there are researchers who believe that a fair interaction is the right thing to do [20] and others place one discipline above the other [21]. In [19] they observed that, although the disciplines were treated jointly, there was no true connection among them and [22] considered that educational institutions did not agree on how to establish and connect the four disciplines. To solve this problem, in [18] it is proposed to include Art as a new discipline in the STEM context, which was renamed STEAM. In STEAM learning, Art, in addition to promoting interdisciplinarity, facilitates communication and understanding of reality and provides creative strategies and solutions [23]. The concept of Art proposed by [18], is a very broad concept that encompasses, in addition to the so-called fine arts, other fields such as language and social sciences. The combination of scientific and artistic disciplines, apparently opposed, provides "the variety and diversity necessary for innovative product design" [24] and they complement each other because "science provides a methodological tool in art and art provides a creative model in the development of science " [25]. The European Parliament [26] considers the inclusion of art essential as it leads to the acquisition of key competences. They consider that art in STEAM is primarily concerned with creativity and creativity includes divergent thinking [27] which leads to multiple solutions to a single problem.

STREAM incorporates another component to STEM and STEAM by integrating R (Reflective learning) into the equation [28].

The study developed in [29] revealed that the Greek elementary school curriculum in science and mathematics was lacking a connection to the real-life problems that the students have encounter outside of school. This disconnection with the real-life make STEAM projects of little value if they are not connected to the real problems and do not promote critically thinking citizens. Lessons that address issues of equity, gender, cultural diversity and in general, SDG(Suatainable Development Goals) are the key of R in the STREAM projects introduced in [28].

Whichever form of STEM education we are speaking of, STEAM, STREAM or other, it is a definite "plus" with respect to traditional education. The key principle is "integration": subjects and real society problems are not taught separately but form part of an integrated curriculum [30].

1.4. Active Methodologies

STEAM projects in general promote the use of so-called active methodologies, encouraging the active participation of the student, who becomes the protagonist of the teaching-learning process and develops his/her own knowledge. Active methodologies place students at the centre of this process and make them protagonists of the discovery, rather than passive recipients of information [31].There are different teaching strategies for creating an active learning environment and engaging students in it. The most common ones are project-based learning, problem-based learning, collaborative learning, and so forth [31].

Active methodologies, such as challenge-based learning, project-based learning, problem-based learning, collaborative learning or flipped classroom, are revealed as effective tools to generate meaningful learning and train critical and creative people who will be prepared to face current and future challenges and will be able to work in a team, communicate, discuss, evaluate.

One of these active methodologies is challenge-based learning, which, based on an initial and global question or challenge, sets out the objective of guiding the students' learning to focus them on an achievable and upcoming challenge, which allows them to get personally involved in the search for effective and plausible solutions. Learning is based on a complete process of research, ideation, documentation and communication, also enhancing personal skills such as teamwork, consensus, negotiation and leadership, as key elements of emotional intelligence. Challenge-based learning allows the process to be approached in a creative and innovative way, so that the process allows the detection of other challenges or problems to be solved. It therefore implies a broader vision than project-based learning.

Project-based learning starts from an initial question or challenge and raises the objective of generating a final product, generating learning through the tasks that are carried out to develop it. If any of these tasks, in addition to being part of the project, pose a new challenge or problem to solve, we will need to overcome these with techniques from another methodology, the problem-based learning. Both methodologies, project-based learning and problem-based learning, use the large methodological umbrella of cooperative learning and therefore for their implementation we need a new organizational structure of the classroom, a different way of managing times and evaluation systems as well as changing the role of teachers and their training.

These methodologies allow the development of practical knowledge, critical thinking, through formal analysis, creative thinking, through empirical analysis and complete active learning.

This type of learning emerges from university education and is, in turn, an active methodology that focuses on the student and generates a good dose of meaningful learning. Both methodologies, project-based learning and challenge-based learning, use the great methodological umbrella of cooperative learning and they need a new organizational structure of the classroom for its implementation, a different way of managing time and evaluation systems and changing the role of teachers and their training.

1.5. Previous Works. STEAM Projects in Educational Systems

The main limitation for the use of STEAM projects in compulsory education is their absence in the national curricula. For that reason, these type of projects are usually part of extracurricular activities and are not integrated into the normal functioning of the classroom. In our previous works [32–34], the curricula of 4th, 5th and 6th grade of Primary Education in Spain, in particular in the Valencian Community were analyzed to determine the areas of opportunity for STEAM learning projects. From the 11 detected opportunity areas, the opportunity area "Sustainability" was selected in [34], for the development of the STEAM project called "Sustainable City". The board in Figure 2 reproduces the block of a city that must be organized so that when the robot travels around its perimeter street, the different elements that make it a sustainable city are activated.

The central core of the platform is composed of nine tiles, six of which are robotic tiles that pose six sustainability and robotics challenges: (1) wind power, (2) sustainable roof, (3) photo-voltaic field, (4) mobility control, (5) selective collection, and (6) urban lighting. Students must work the tiles alternating cooperative and individual work. Subsequently, they begin the assembly and programming of the complete board.

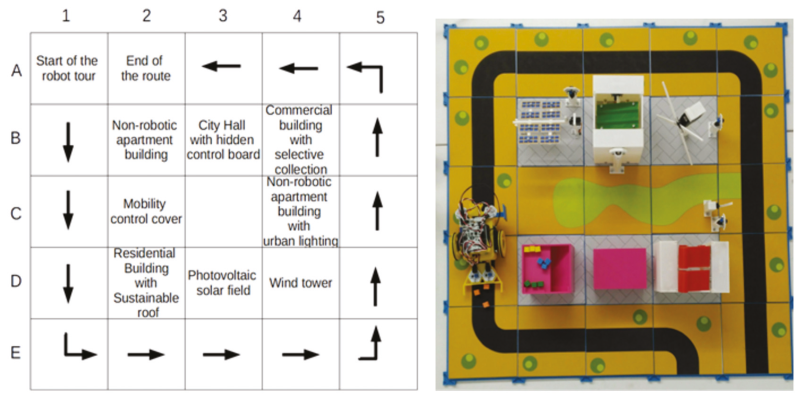

Figure 2. "Sustainable City" STEAM project developed in [33].

The "Sustainable City" STEAM project was tested in [34] in a real classroom. The participants were 30 students (aged 10-11) from 5th year of Primary Education and the project consisted of 14 sessions in which different active methodologies such as project-based learning, collaborative learning and the flipped classroom were carried out. The project included a comprehensive and complete evaluation system with eight questionnaires covering three flipped classroom sessions, two group and two individual self-evaluations, an explanation to the base team, the presentation of the final product and a final test. The average rating of the questionnaires was satisfactory, obtaining an average of 7.23. Throughout all the sessions, a very high degree of motivation and interest has been observed in all the students who have felt highly identified with the project, have discussed among their peers,have solved problems in a collaborative way and have shared objectives. Maths are involved in the project as a part of everything, as a part of STEAM project, but with the stimulus of the "sustainable city" project, seeing the applicability of maths in the real world.

2. EXPLORIA Project. A New Way of Conceiving the University

The EXPLORIA project was born from the need to update university learning methodologies to the new trends, such us active methodologies and STEAM project based learning, among others.

In this sense, the CEU Universities (CEU San Pablo, CEU Cardenal Herrera and CEU Abat Oliva), are developing different pilot projects in degrees such as Advertising, Political Science, Business Administration, Journalism, and so forth, rethinking the processes of university student learning. A group of teachers was formed for each degree to rethink how to do it and which of all new trends in learning methodologies are better for each one. Among the pilot degrees, there is the degree of Engineering in Industrial Design and Product Development, a degree that integrates subjects that coincide with the STEAM classification. That is why STEAM learning process was selected as a way to improve the learning process in this degree.

Project EXPLORIA in the Degree of Engineering in Industrial Design and Product Development

In our previous works [32–34], it was shown that it is possible to transform national curricula into STEAM projects and to improve the learning process, where maths is learned through a project that show its value in real life, in a "sustainable city". In this context, STEAM projects applied to the university environment can be the way to generate positive emotions in the students that change their perception of mathematics and improve their academic performance. There are no STEAM experiences in the literature, based on the authors' knowledge, integrated into the curriculum at university level to improve the understanding and perception of mathematics.

The EXPLORIA pilot project in the Industrial Design and Product Development degree aims to develop STEAM learning process an integrated competence map based on National curricula, in which the subjects are no longer considered as isolated contents, by elaborating an integrated learning process where the competences and learning outcomes of the subjects are considered as a whole, global and comprehensive learning.

In this way, active learning allows students to make the necessary connections to address the resolution of various challenges and problems that require the integration of knowledge from various disciplines. Active learning also enhances motivation, the need to discover, and the autonomy of learning, placing the students at the centre of their development. It transforms the passive attitude, from receiving knowledge and instructions, to an active attitude, in which searching, inquiring, creativity and innovation are present throughout the process.

The pilot project makes use of integrated learning, of temporal sequences focused on different learning objectives linked to Bloom's taxonomy: understanding, applying, experimenting and developing. In this way, through active methodologies, the student addresses all levels of learning, learning by doing. Students develop critical and creative thinking, through formal and empirical analysis, they develop creativity and innovation, and the capacity for global and multidisciplinary analysis, essential in the current context.

The teacher assumes the role of a learning guide, a teacher who accompanies students in their personal and professional development process. The teacher abandons the role of instructor, encouraging students to discover, the motivation to learn and the awareness of the need to learn from each challenge, stage or new situation that may arise. In this way the student is prepared to respond to complex problems, in changing, unstable and equally complex contexts.

3. Research Objectives

As shown in the previous section, the perception and predisposition of students towards mathematics is low when entering university, mainly motivated by the beliefs that the students have about mathematics, which come from previous training cycles. The stimuli that the students receive and their emotions can worsen their results even more, generating negative attitudes that increase the failure of students in this subject, see Figure 1.

The objective of our research is to develop an EXPLORIA pilot project in the Industrial Design and Product Development degree using STEAM learning based on the competences of the Spanish law. The EXPLORIA project connects the competencies of the different STEAM subjects, designing a learning process as a logical, sequential and incremental itinerary. Through concepts on which the foundations of design are based: shape, volume, colour, space and structure, the competencies of the different subjects are defined in incremental learning levels: understanding, applying, experimenting and developing, all taken from Bloom's taxonomy.Each of the learning periods of the fundamentals of design ends with a Milestone based on the Challenge-Based Learning methodology, where students actively and autonomously, and working in teams, integrate the skills acquired, using the learning to propose their solutions.

The goal of the present paper is to analyze the effect of the EXPLORIA pilot project has in Maths in the Industrial Design and Product Development degree. Mathematics is linked to the rest of learning through active learning methodologies that make learning useful, generating positive stimulation and emotions, which lead to positive attitudes of the students and improve their academic performance in all subjects, but especially in mathematics.

4. Materials and Methods

4.1. Research Design and Data Analysis

An experimental design was carried out, following the experts in this field [35]. A qualitative, quantitative and mixed analysis was also carried out, following the experts in this field [36].

The students were classified into two different groups in order to be assessed. On the one hand, the control groups followed the traditional methodology. On the other hand, an experimental group followed the EXPLORIA pilot learning as a methodology. The methodology was defined as an independent variable. Both groups share course, content and teachers, so it is established that there is no prior significant difference between the control and experimental groups.

R was selected as the data analysis language. Descriptive statistics on graphics, mean and standard deviation were used for this analysis. The effect size measure was obtained using Kruskal-Wallis, where a $p < 0.05$ is established as a statistically significant difference. Cronbach's alpha test is also used to see if multiple-question Likert scale surveys are reliable. In that test, the reliability is achieved if $p > 0.7$.

4.2. Participants

The participants in the study were the students of the degree in product design engineering from the courses 2018–2019, 2019–2020, 2020–2021, where the course 2020–2021 is the experimental group in which STEAM learning was applied and the other two courses are the control courses that followed the traditional methodology. The number of students in each sample group was 23, 27 and 31 respectively.

4.3. Scope of Application

STEAM learning has been planned and applied to the first four-month period of academic year 2020–21 in which the following subjects are included, see Table 1.

Table 1. First year subjects.

Subject	ECTS
Maths	6
Physics	6
Art	6
Basic design	6
Art History	6

Where the syllabus of the mathematics subject is as follows, see Table 2.

Table 2. Syllabus of the mathematics course.

Item	Content
1	Vectors
2	Matrices and applications
3	Isometries. Compositions of isometries. Ratio
4	Functions
5	Differential calculus
6	Introduction to integral calculus
7	Basic and advanced trigonometry
8	Linear Algebra

4.4. Instrument

Data collection was obtained through an ad hoc questionnaire. The design of this tool was carried out following other validated methods found in the scientific literature, such as [37]. There are 9 items in the questionnaire. A type of scale is followed depending on the question, some of the questions had the option of YES, NO, others allowed to enter comments in an open format and the rest followed a Likert-type format with a range of five points (from 1 = Strongly disagree to 5 = Strongly agree).

The final grades obtained by the students in each of the courses were also used. In the case of the control courses, 2018/2019 and 2019/2020, they were carried out with a final standard exam while in the experimental group, course 2020/2021, an evaluation by projects and acquisition of skills was carried out. The ways of evaluating, although different in each of the groups, seek to measure the level of acquisition of competences by students.

5. Design and Implementation of EXPLORIA Pilot Project

The EXPLORIA project was born from the need to update university methodologies to new trends and market needs. In this sense, the University CEU has started different pilot projects in degrees such as marketing, law, political science in order to rethink the way of teaching at university. Among the pilot qualifications, there is the degree in design engineering in which the formative character of the subjects coincides with the STEAM classification.

The EXPLORIA project connects the competencies of the different STEAM subjects, see Table 3, where the standard subjects disappear, designing a learning process as a logical, sequential and incremental itinerary. In this learning process, teachers do not have a fixed weekly schedule but rather their schedule is based on the designed itinerary.

Table 3. First-year subjects of design engineering degree.

Cuatrimestral Term 1	STEAM Classification	Cuatrimestral Term 2	STEAM Classification
Physics	S,T,M	Physics Extension	S,T,M
Maths	M	Maths Extension	M
Art History	A	Anthropology	S
Basic design	A,S,T	Design Extension	A,S,T
Shape representation	A	Descriptive geometry	A,S,T

The EXPLORIA project has been designed based on the specification and synthesis of the specific and general competencies of each subject included in the study plan of the Degree in Industrial Design and Product Development, it was estimated that, aiming at obtaining a significant and integrated learning result, it was appropriate to group these skills according to a learning process based on Bloom's Taxonomy relating to the verbs understand, apply, experiment and develop.

On the other hand, and according to the learning objective established by the degree for the student who completes the 1st year of the Degree in Industrial Design and Product Development, we decided to include five concepts that will articulate the itinerary of this course, making them coincide with the basic fundamentals of design: shape, volume, colour, space and structure. In order to adjust to the academic calendar that divides the course into two semesters, we divided the learning itinerary of design fundamentals into two modules. These in turn are divided into three acts as shown:

MODULE I

- Act I: Shape
- Act II: Volume
- Act III: Colour

MODULE II

- Act IV: Space
- Act V: Structure
- Act VI: Project

In addition, to strengthen the objective of each of the fundamentals worked on and obtain a global vision of the related competences, we decided to introduce a milestone at the end of each Act. This milestone is a challenge-based methodology in which students, actively and autonomously, and based on a general topic raised by teachers, respond to their own concerns through a challenge. This challenge is formalized and sustained through the application in a project of the skills and learning acquired by the student during the weeks that have made up each act. In this activity, the role of the teacher is to accompany and guide the student according to the needs required by each phase of the project, being flexible when intervening and adapting to the requirements of the teams depending on their specialization. Since one of the pillars that sustains the EXPLORIA program is the creation and consolidation of the learning community, it is therefore appropriate to develop the milestone within a team. It is in this way that transversal competences such as decision-making, communication, critical thinking, and so forth, are integrated. In addition, the group is changed for each Act, which allows the students to vary their role depending on the idiosyncrasy of the team and obtain different experiences. The project developed based on the challenge is exposed by each team to the community (other teams and teachers) and evaluated on the one hand by the teaching staff, who will determine the cohesion of the acquired competencies and the learning results established for the Act through a rubric designed for this activity. The other teams, using the Post Motorola tool, will qualitatively evaluate what items worked or not, what can be improved and what we have learned, determining a quantitative score based on the responses. Finally, the team itself, and based on an attitudinal and aptitude rubric, carries out a self-evaluation and co-evaluation. The weighting of all these results will be the final grade of each student.

5.1. Mathematics in EXPLORIA

Mathematics, as a basic subject in a first year of Engineering, is part of this project, which has required analysing the role of the subject and how to connect it with the students' learning. Mathematics is a core element not only in the necessary knowledge for the learning of other subjects, but also necessary for thought processes that allow solutions to problems of various kinds to be achieved. In the specific case of the EXPLORIA project, mathematics is essential for understanding physical principles, understanding concepts such as proportion, harmony, present in nature, the objects that surround us and art. The mathematical calculation has a "utility" that can be perceived by students when integrating it and needing it to tackle other type of learning.

Mathematics sessions have a general structure that covers 1 h of theoretical concepts (lectures) and 1 h of practice (seminars). The contents taught are distributed according to the theme of the act in which the subjects take part in a systematic way that determines which curricular concepts should be emphasized. The Milestone makes it possible to evaluate the acquired mathematics competencies, applied to real problems in relation to other competences developed in the other subjects.

5.1.1. Description of Sessions and Timing

We detail the sessions and subjects involved in Table 4 in which you can see a summary of the sessions:

Session 1

1. In the theoretical session of mathematics the concepts of proportionality are explained. Golden ratio.
2. In physics, a practical exercise to measure the proportions of the body is carried out.
3. The authors who use the golden number in art are studied and analysed.
4. In representation of shapes the concept of proportion is used to establish the proportionality of the shape.

Session 2

1. In mathematics we develop the concepts of vector and matrix as set of coordinates of an object. The isometries.

2. In Basic Design, the matrices associated with the shape are applied and also to modular structures.

Session 3

1. In the maths session we study how to build tessellations using isometries. The concept of homothecy is explained.
2. In the basic design subject, tessellations are developed. Exercises using homothecy: gradation and similarity are also performed.

Session 4

1. In the theoretical session of mathematics, the concept of function is developed, seen from two approaches: approximation and the matrix that represents a function.
2. The concept of composition of functions is studied making reference to the composition of isometries.
3. In physics the concept of function is used to approximate the CLO.
4. In basic design, the concept of function and composition of functions is used to develop tessellations with a higher level of complexity than in session 3.

Session 5: MILESTONE I. Sport: Students develop a design project related to sport in groups where they must apply the knowledge acquired in sessions 1,2,3,4. This activity lasts one week and concludes with a defence of the project before a panel of teachers. During the presentation, the students must explain how they have applied the knowledge acquired in the project.

Session 6

1. In mathematics, there is a session in which derivatives are explained and optimization problems are carried out.
2. The concepts are applied to basic design with problems related to space. In physics with problems related to electricity.

Session 7

1. In the theoretical part of mathematics, the concepts of integral are explained as a problem of approximation of areas.
2. In basic design the integral is applied to calculate an approximation to the volume of an object using the serial planes (cross-sectional area).

Session 8

1. In mathematics, Pappus Guldin's theorem is explained.
2. In another basic design session, students analyse shapes of revolution.
3. In the Physics session they calculate the volume and surface area of the shapes of revolution seen in basic design.

Session 9: MILESTONE II. Light: Students develop a design project related to light in groups in which they must apply the knowledge acquired in sessions 5,6,7,8. This activity lasts one week and concludes with a defence of the project before a panel of teachers. During the presentation, the students must explain how they have applied the knowledge acquired in the project.

Session 10

1. Mathematics: The concepts related to Venn diagrams are explained. Logical operations. RGB diagrams are studied.
2. With the logical operations they study the primary, secondary additive and subtractive colours used with Photosop in basic design.
3. Representing shapes are used to make additive and subtractive mixtures.

Session 11

1. Mathematics: Basic trigonometry is explained. HSV/HSL models in which saturation and hue are explained as polar coordinates.
2. In basic design they use different colour gradations and modify them using saturation and hue and making the changes indicated in the maths part.

Session 12

1. Mathematics: they study the colour wheel and establish its polar coordinates. Converting to Cartesian coordinates. The law of sines and cosines is used to establish the distance on the colour wheel.
2. The distance on the colour wheel is used in basic design to find visual harmonies.

Session 13

1. Maths session: vector spaces are explained. RGB is used as a generator space to explain the different combinations of a given value and generate the colour range in RGB. The concept of base and the change of base are studied, in particular, the opponent colour space.
2. In physics the concept of linear combination seen in mathematics is used to measure colours of objects by using an app. This session looks at how to describe them according to their RGB ratios.
3. In representation of shapes, the opponent space is related to 'discomfort' and negative emotions.

Session 14: MILESTONE III. Well-being: Students develop a design project related to well-being in groups where they must apply the knowledge acquired in sessions 10,11,12,13. This activity lasts one week and concludes with a defence of the project before a panel of teachers. During the presentation, the students must explain how they have applied the knowledge acquired in the project.

Table 4. Sessions, timing and subjects involved.

Week	Content	Subjects
	Act I.—Shape	
S1	Proportion, Vectors	Physics, Art, Basic Design
S2	Flat isometries. Matrices connected to isometries. Inverse matrix	Basic Design
S3	Tessellations (DB) Homothecy (gradation)	Basic design
S4	Composition of isometries. Functions.	Basic Design, Physics
S5	Milestone I.—port	
	Act II.—Volume	
S6	Differential Calculus	Physics, Basic Design
S7	Integral Calculus: Approximation of areas and volumes	Physics, Basic Design
S8	Shapes of revolution. Calculation of areas and volumes. Pappus-Gouldinus theorems.	Physics, Basic Design
S9	Milestone II.—Light	
	Act III.—Colour	
S10	Venn Diagrams. Primary, secondary colours Additive and subtractive colours. RGB System	Basic Design, Physics Shape Representation
S11	Flat trigonometry. HSV/HSL Models Polar coordinates. Change from RGB to HSV/HSL	Physics, Basic Design
S12	Laws of sines and cosines. Polar distance. Chromatic harmonies. Colour Wheel	Basic Design, Physics Shape Representation
S13	Vector spaces, Generator system: RGB space. Combination of colours Bases of a vector space. RGB space. Change of coordinates. Opponent space.	Basic Design, Physics Shape Representation
S14	Milestone III—Well-being	

5.1.2. Assessment Methodology

The assessment methodology is one of the most relevant factors introduced in the EXPLORIA project because the traditional exam is replaced by the activities developed in each session as well as the marks obtained in each milestone.

Each session has a theoretical part and a practical part where the student must apply the maths in an exercise related with other subjects (Physics, Basic Design, etc.). The exercises must be done in class. The most important part is that the exercises for each student must be original and different between them. Therefore, the students needs to learn maths to solve their own exercise.

Milestone projects are developed in groups and the mark obtained by the group is the same for each other. However, the teachers check in class the implication of each student in the activity. At the end of the semester, 14 exercises and 3 Milestone projects are used to evaluate the students. The final mark is obtained by 75% exercise sessions and 25% Milestone projects.

5.1.3. Some Milestone Project Examples

Appendix A shows some project examples presented by the Students in Milestone II and III. The projects are:

1. VITALIA: This project is for the Milestone III. Well-being. The students developed a bottle of water that it is able to measure the quantity of water that you drink in a day and alerts you when you should drink because to be hydrated is important for the well-being, see Figure A1.
2. Youmood: This project is for the Milestone III. Well-being. The students develop a product with a bottle of water where each box or bottle has a color and each color is related with art and emotions, see Figure A2.

In these projects, mathematical concepts are applied in different parts of the project. For instance Pappus-Guldin is used to compute the volume and the surface area of the prototype. Both are important for the prototype design. The first one to know the material required if you want to develop the prototype with Polispan. The second one is necessary if you want to develop your prototype with a 3D printer. Students compute both and choose the way to develop their prototype. Color are selected using cosine law and doing linear combinations in order to represent it. In Figures A1 and A2.

6. Results

6.1. Student Perception Survey

To carry out an evaluation of the new teaching methodology, a Microsoft Forms form has been made, in this form the student indicated whether the previous year they had taken the mathematics subject or not. They were also asked what their perception of mathematics was before starting the course and whether that perception had improved after or not. Finally, there were more specific questions about this experience, such as the degree of satisfaction with the activity carried out, perception of learning and appreciations about the educational model. Finally, there is an open question for the student to comment on the experience. The questions asked in the form are shown below in Table 5.

Table 5. Questions asked in the questionnaire for the students.

ID	Question
1	In the previous year, did you study mathematics?
2	What was your perception of mathematics before the course?
3	Have you improved your perception of mathematics after the course?
4	It has been more motivating to solve math problems with the EXPLORIA methodology than with the traditional method.

Table 5. *Cont.*

ID	Question
5	I feel more involved in this type of learning.
6	My perception of learning has improved.
7	Would you propose to keep on using the new learning model next year?
8	In the connection of mathematics with the rest of the subjects, what connection has helped you the most to understand it?
9	What do you think of the application of this new teaching experience?

Table 6 shows the results of question 1, YES/NO answers. Table 7 shows the answers to the question about prior perception of mathematics. Table 8 shows the answers to the Liker-type questions. Finally, Table 9 shows us which of the links with the rest of the subjects have been more useful in understanding mathematics. Cronbach's alpha tests of the multiple-question was performed to assess internal reliability of the questionnaire about perception of mathematics (p = 0.7791).

Table 6. Answers to YES/NO questions of the questionnaire.

Question	Yes	No
1	18	1

Table 7. Student questionnaire responses related to the previous perception of mathematics.

Question	Very bad	Bad	Neutral	Good	Very good
2	1	3	7	6	2

Table 8. Student questionnaire responses, Liker-type questions.

Question	Strongly Disagree	Disagree	Neutral	Agree	Strongly Agree
3	0	0	3	9	7
4	0	1	0	11	7
5	0	0	1	10	8
6	0	1	0	11	7
7	0	0	1	5	13

Table 9. Answers to the question about which link was the best.

Question	Physics	Basic Design	Art	Shape Representation
8	11	7	1	0

If we focus on the answers given by the students of the 2020–2021 course, regarding the evaluation of the EXPLORIA educational model, the survey was answered by 60% of the enrolled students.

The data show that practically all the students have studied mathematics in the previous year. Regarding the perception of mathematics before starting university, 60% show a neutral or negative perception of mathematics, but 85% acknowledge that their perception has improved thanks to the EXPLORIA project.

In the question about if this way of working was more fun than the traditional way, 95% of the students who took the questionnaire completely agreed or agreed with it. In the same way, practically all the students believed that their perception of learning had improved significantly with this new methodology.

Finally, practically all the students expressed the opinion that they would like it to continue in later courses.

Finally, 57% indicate that the connection that has helped them the most to understand mathematics has been with physics, while 36% indicate that the most useful connection has been with basic design. This result is indicative that the applicability of mathematics is key to understanding.

These are some of the answers given to the open question:

- "By coordinating and intertwining subjects, it is easier to relate concepts among them"
- "I think it improves learning, due to its method based mainly on experimentation."
- "In my opinion this new method due to being more practical is more focused on the world of work and what we will do in the future."
- "It is a new method that allows you to see the application of all the theory given in subjects such as mathematics and physics"
- "It is another way of acquiring knowledge in a more practical way in which it becomes more enjoyable and dynamic every day than doing it in an exam"
- "This new methodology in my opinion is much better, since in all subjects there are things that have to do with each other, this means that you can see how the knowledge that is given can be applied in different fields"
- "I think this new methodology makes us more involved in the learning process"
- "It is very effective, because you really learn the meaning and use of mathematics in the professional field"

As a final result, the vast majority of the students have improved the student's belief about mathematics, specifically, 86% of the students, corroborating what is shown in Figure 1 and in the research by [5,6], a positive stimulus or emotion generates positive attitudes that allow the student's beliefs about mathematics to be changed.

6.2. Comparison of Academic Results in the Last Three Years

Table 10 shows the academic results obtained in the last 3 years.

Table 10. Results of Ordinary Exam Call.

	Course 2020–2021	Course 2019–2020	Course 2018–2019
Students attending	93.6%	85.2%	91.3%
Pass	94%	70%	74%
Fail	0%	30%	26%
Average pass mark	7.85	6.55	6.49

In order to determine whether there are significant differences in the average marks obtained, we have used the Kruskal-Wallis test giving a p-value of 0.02168, which allows us to conclude that there are significant differences among the average final grades in the different courses and therefore the methodology introduced in EXPLORIA for learning mathematics has had a positive impact. Figure 3 shows the distribution of grades in the different courses.

As we can see, the distribution pattern of grades, as well as the percentages in the control groups, years 2018–2019 and 2019–2020, are similar, 17% vs. 15% of fail grades, 30% vs. 26% of pass grades 31% vs. 37% of B grades. However, the pattern changes significantly in the experimental group, in the academic year 2020–2021, we can see that the percentage of A grades has increased significantly, reaching 29%, and there is a 3% of grades with honours and finally, the percentage of fail grades has disappeared, reaching 0%.

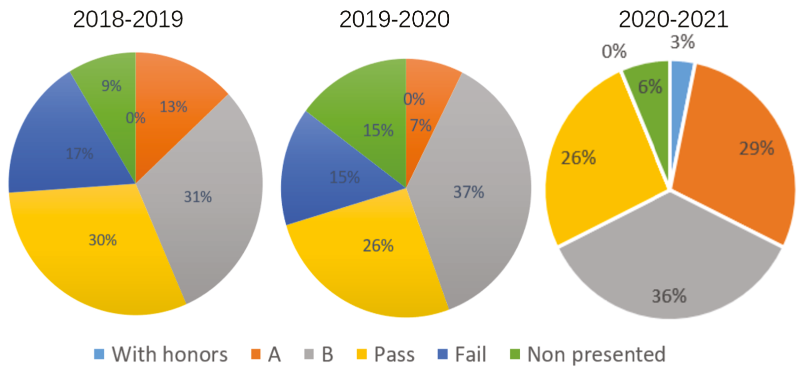

Figure 3. Distribution of grades.

7. Discussion

The EXPLORIA project implemented in the degree of Engineering in Industrial Design and Product Development produce a great impact in the learning process. The vast majority of the students have improved the student's belief about mathematics as well as to understand why it is necessary to learn maths. The present study scores in control and experimental groups and, although it is true that the assessment methods are different because in the control group there are no exams, in the experimental group, the teacher has 14 activities developed in class where the most important part when the teachers design these exercises is that the exercises for each student must be original and different between them. Therefore, the students needs to learn maths to solve their own exercise and then the teacher could evaluate if the student understand the concepts. this is as if an examination exercise of the classical methodology were done in each session of the new methodology. Moreover, the exercise levels are higher than in a traditional exam because the teacher is focused in one concept.

The main limitation for the use of STEAM projects in compulsory education is their absence in the national curricula. For that reason, these type of projects are usually part of extracurricular activities and are not integrated into the normal functioning of the classroom. The first STEAM project developed taking into account national curricula was developed in our previous works [32–34] and in the same way, developed for university level in the present study. Therefore, it is not possible to compare with similar experiences in other universities.

One of the important limitation of the present methodology is that the teachers' schedule is not fixed and it is determined by the learning sequence. If some students fail one part, it is not easy to recover a single part because all the learning process is intertwined between subjects. This could generate organizational problems for the university.

The construction of the learning process could also be a problem for the university because requires a deep effort for the teachers. If an effective learning process is to be achieved, the teachers of the different subjects must act as a single teacher and must know what the other teachers want to achieve from their students.

8. Conclusions and Further Developments

This article shows the design and evaluation of the EXPLORIA project, based on STEAM learning in the degree of product design engineering. The development of an integrated competence map of the learning process, where the subjects are no longer considered as isolated contents, by elaborating an integrated learning process in which the competences and learning outcomes of the subjects are considered as a whole, a complete and global learning, this has allowed a change in students' perception of mathematics, increasing their motivation and their commitment to discovering. The results obtained by the students have improved substantially, both due to the almost absolute decrease in the drop-out rate of the subject, as well as the very low rate of students who failed, as well as the average grades that have increased substantially. In addition to the results, the most significant aspect of the implementation of this project is the change in students' perception of mathematics, which has resulted in a change in attitude and, therefore, in an improvement in academic results. Generating positive emotions through active methodologies in which the students can see the application of mathematics, improves their taste for mathematics and their understanding.

In future works, we intend to implement the EXPLORIA methodology in other degrees related to STEAM learning, such as Architecture. In addition, we are going to transform our STEAM projects into STREAM projects, including Critical reflective learning, mainly focused in the SDG (Sustainable Development Goals).

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Appendix A

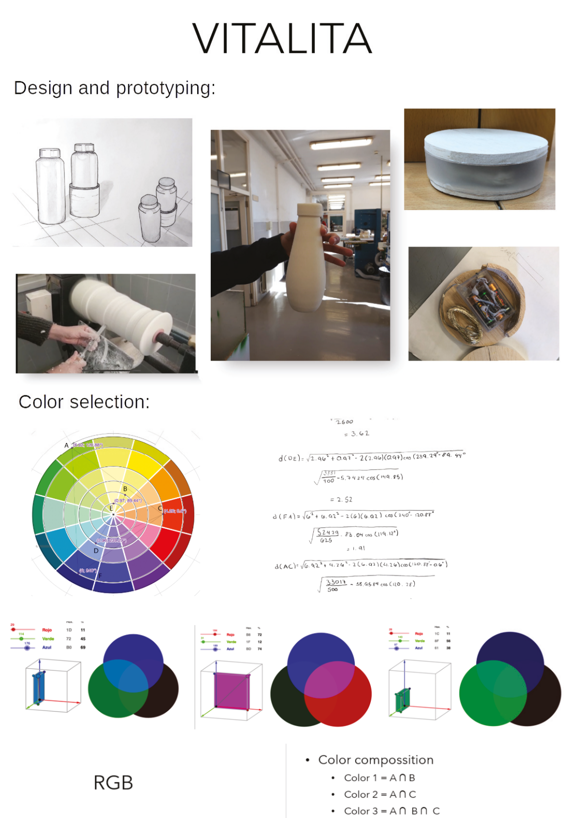

Figure A1. VITALIA. A bottle of water that it is able to measure the quantity of water that you drink in a day and alerts you when you should drink.

Figure A2. YOUMOOD. A product with a bottle of water where each box or bottle has a color related with art and emotions.

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mathematics

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Article

Improving the Teaching of Real Valued Functions Using Serious Games. Binary Who Is Who?

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Abstract: The study presented makes an original, new and exhaustive analysis of the adaptation of a classical board game which has been named Binary Who is Who? This proposal shows a very useful tool for the consolidation of mathematical concepts related to the study of real-valued functions that are treated in the different levels of the teaching of mathematics (first and second year of superior secondary studies and the first years of some university degrees). The use of games as a means for learning is the authors' proposal. The aim is to offer teachers the chance of using the games as a method of teaching mathematical concepts, as well as a motivating instrument for them. This game has been created to be played face-to-face in the classroom and it has also been programmed to create a video game which allows the students to play virtually.

Keywords: gamification; mathematical teaching methodologies; educative innovation; learning through video games; real-valued functions

1. Introduction

The objective of this proposal is to use the educational value of certain games for the presentation and consolidation of the knowledge of some concepts of the subjects of mathematics that students must know, and proficiency related to logical–mathematical reasoning. We want to offer students the opportunity to play and beat challenges as a way of working with the concepts of daily lessons in the classroom, as well as a way of training their reasoning abilities.

There have been studies that confirm that people are naturally playful and, thus, we are open to all proposals that are related to games and competition. The Dutch historian, Johan Huizanga, showed in his book Homoludens [1] that making tends to include games in culture and society. Thanks to works like this, game-based learning and gamification are being incorporated in sectors like education, business, and digital commerce, among others [2–10].

The basic ingredient of gaming consists of the challenge and what it can represent for the individual.

If the users consider that the game that is presented to them puts their abilities to the test, they will show interest in it and see how far they can get [11].

Focusing on the use of games for the teaching and learning of mathematics, it should be noted that a good game, a game that has well-defined rules and has an approach rich in logical content, needs to include, on one hand, certain mathematical concepts, and on the other hand, a type of analysis whose characteristics are very similar to those that are needed to solve typical problems of this science. Mathematics is, to a large extent, a game and the game can, in many cases, be analyzed by means of mathematical instruments. In games, we look for fun, the possibility to quickly perform actions, and competition.

This can be used to establish interest of the students in mathematical concepts, and secure the learned concepts. In this work, we take advantage of stimuli and the motivation that the spirit of games can infuse in students [12] and they are used to introduce and reinforce certain concepts that form part of the curriculum of this subject in different levels of education. What is intended, as stated by Marín [13], is not to "ludify" education, but "to promote learning processes based on the use of games for the development of effective teaching-learning processes, which facilitate cohesion, integration, motivation for content, and enhance the creativity of individuals".

Some questions that are posed about proposals of this type are: Can games really be used in the teaching of mathematics? How? What games? What objectives can be achieved through the games? [14,15]. In the present article, these questions are answered through an exhaustive analysis of a concrete proposal made by the authors.

We support game-based learning in mathematics [16–23]. Iriondo-Otxotorena [24] already did in an experiment in which an introduction to algebra was proposed through the solving of puzzles and enigmas. Our experience allows us to affirm that it is necessary to incorporate new methodological tools that are attractive for students. It is very common for students to find the subject of mathematics difficult to understand, boring, and impractical, so they get discouraged, stop paying attention to the teachers' explanations, and neglect their studies.

The incorporation of games into the mathematics classroom can be done one of two ways: through traditional games and through digital ones [25–27]. This article presents the possibility of combining these two modalities, by creating a game that is based on a traditional one to be played in the classroom, but which also has been adapted digitally to be played online. This combination has made it possible to work using both perspectives, taking advantage of the benefits provided by both.

Statement of the Problem

The study of real functions of real variables is taught in high school programs and the first courses of scientific and technical degrees. The proposal that has been developed adapts the foundations of a classic game such as "Who is Who?" with the objective of using its educational value, mainly for the consolidation of knowledge related to the study of real functions of real variables. We want to offer students the opportunity to play and overcome challenges, for which their mathematical knowledge on this subject will be of great help, while providing teachers a tool that they can adapt to their needs and to help them improve performance in their classes. The adaptation of the game has been proposed so it can be played not only in the classroom with other students but also as a video game that allows students to play individually as many times as they want.

In most games, participants play to win, and to overcome the challenges that the game presents. To win the game that is presented, it is necessary to resort to skills that are related to mathematics, which students should know or are learning in their classes. In addition, they will have to observe the possibilities, deduce, generalize results, plan future options, etc., all elements necessary for their academic training.

Thus, with this proposal, we want to offer teachers and students the opportunity to use and create games with the purpose of working within the daily lessons in the classroom, as well as carrying out a training method for their reasoning abilities. This way, the presented proposal aims to answer the following questions:

- Do the creation and use of games in the classroom improve the mathematical skills of the students?
- Can mathematical contents of the curriculum be reinforced with the use of games?

We believe in the need for another form of education, in which the emphasis is placed on the essential skills of people, promoting creativity, personal initiative, and self-learning, and where educational innovations and initiatives are integrated, in which games have an important role to carry out. Guzmán [28] pointed out that the factor with the highest influence is the teaching method. The work to come is framed in this line: games as an adequate instrument to learn. In this case, we are focused on students of different ages.

2. Materials and Methods

2.1. Theoretical Framework

Undoubtedly, one of the teaching fields where more work remains to be done is that of mathematics. While in other scientific disciplines, there has been great advances in their teaching in recent years, mathematics classes are moving in the same direction but much more slowly. Still, they often remain unrelated to daily reality, becoming a cluster of mechanical and disconnected exercises. Therefore, we consider it important to develop new tools or adapt some existing ones that are useful and easy to implement in the classroom, and always related to the curriculum.

According to Brull and Finlayson [29], game-based learning allows students to participate and learn, enjoying the freedom to experiment and fail in an enjoyable environment. Learners have the opportunity to interact with experiences that keep them motivated. There is evidence that students involved in gamified environments improve their learning, and increase their motivation and engagement [30].

Werbach and Hunter's proposal [31] suggests that in order to create a successful game, three fundamental elements must be introduced: game components, mechanics, and dynamics. However, before choosing these elements, six steps must be followed: defining objectives, defining the desired behaviors, describing the type of players at whom the game is aimed, choosing the activities to be carried out, including an element of fun in the activities, and developing tools (Figure 1). Thanks to its structure, this methodology is flexible and can be adapted to almost any context, particularly mathematics. These steps, as explained below, have been followed in this work for the development of the proposal.

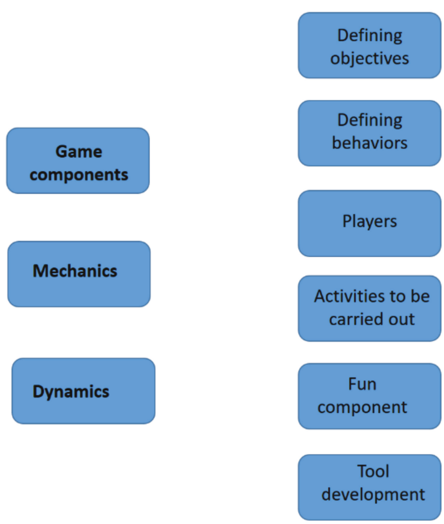

Figure 1. Steps to follow for the start-up process.

We have made several innovative educational proposals through games that allow us to understand and apply different concepts and mathematical theories.

The methodology applied in the project presented in this paper has been put into practice in different stages:

- Search for board games existing in the market that can be suitable from a mathematical point of view and adapted for teaching and learning of various topics. The choice of games which are familiar to students means that the game mechanics are already familiar to them.

- Creation of the new adaptations and proposals.
- Proposal of collaboration, to a group of students, in the physical creation of the game and rules.
- Approach and implementation in the classroom. The students play with the proposals and they are guided in their resolution through the mathematical approach.
- Development of a video game based on the proposal.

The educational courses and levels at which the project is directed include:

- Students of Mathematics II–second year Spanish science baccalaureate.
- Mathematical subjects related to the study of real functions of real variables from the first courses of different grades [32,33].

Concepts tackled:

- Numerical systems: decimal and binary.
- Didactic unit: real functions of real variables: domain, asymptotes, growth, decrease, maximum and minimum, among others.

These steps are outlined in Figure 2. A further development of them, specifically for the game developed, can be found in Section 2.3.

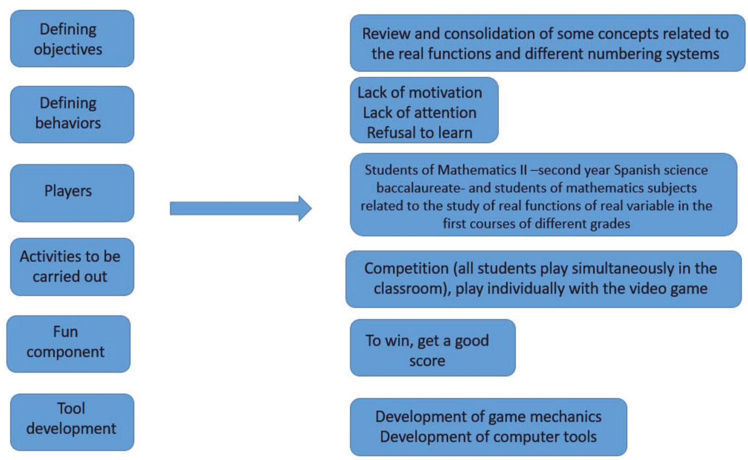

Figure 2. Steps to follow for the implementation of the game.

2.2. Procedure: How Do You Work with Students?

A session of “Games in the Classroom” will be presented to the students. For this, a group of students must be selected who will be involved in the realization of the game and in the organization of the game. The rest of the students will be those who will attend the game day as participants.

We will work with the group in charge of carrying out the proposal in various guided sessions.

When the day of the games session arrives, it will be adjusted to the agenda developed in class, and a competition will be proposed to the students in the classroom.

After the competition, the most appropriate mathematical strategy for the game played is explained. Mathematical contents that propose these strategies are remembered and students are allowed to analyze and think about them.

In this paper, we will focus on one of the games presented to the students: Binary Who is Who? We develop its design below.

2.3. Methodological Design

The design of the game is based on the classic game Who is Who? whose rules are the following:

Each player has a series of cards with different characters. One of the participants chooses a card with a character and places it without the other player seeing it. The other player performs the same operation.

The objective of the game is to guess which character the other player has chosen.

In each turn, questions are asked about the features of the character the player wants to discover.

The other player answers these questions with a yes or no: if the player answers yes to a question, the cards of the characters that do not have that feature are removed; if the player answers no, the tiles of the characters that do have that feature are removed.

2.3.1. Adaptation of the Game

We wanted to adapt the idea of this game to the framework of the study of real functions of real variables, also including some other concepts such as numerical systems, specifically the binary system, which is of great use in computing (the same could be done with other topics).

For this, instead of characters, the idea is to work with defined functions both analytically and graphically. The first step is to choose the characteristics (traits in the classic game) about which the player will ask in order to discover the function chosen by the opponent. In our proposal, the selected ones are: sign of the function, range, monotony, continuity, derivability and existence of a vertical asymptote. Therefore, the six questions to be asked about the function, from which the desired function should be discovered, would be:

- Is it a non-negative function?
- Is it bounded?
- Is it monotone?
- Is it continuous?
- Is it derivable?
- Does it have a vertical asymptote?

Each of these questions will be answered with a yes (1) or a no (0). With this, each of the functions that are part of the game will have a definition in binary code that will consist of a vector of six digits, zeros or ones, depending on the answers to each of the questions to be asked for that function (Figure 3).

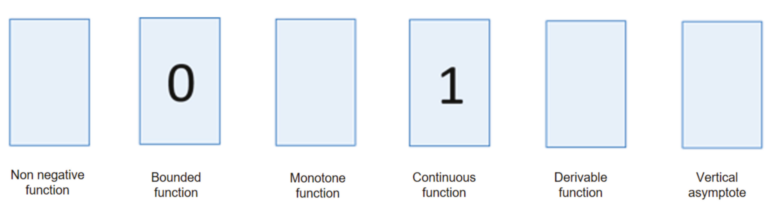

Figure 3. Binary vector that is updated each moment with the known data.

The number of possibilities that exist in the game will be the number of variations obtained with repetition of two elements {0, 1} taken from six in six: $2^6 = 64$ (Table 1).

Table 1. Possible game responses.

(0,0,0,0,0,0)	(0,1,0,0,0,0)	(0,0,0,1,0,0)	(0,1,0,1,0,0)	(0,0,0,0,0,1)	(0,1,0,0,0,1)
(1,0,0,0,0,0)	(1,1,0,0,0,0)	(1,0,0,1,0,0)	(1,1,0,1,0,0)	(1,0,0,0,0,1)	(1,1,0,0,0,1)
(0,0,1,0,0,0)	(0,1,1,0,0,0)	(0,0,1,1,0,0)	(0,1,1,1,0,0)	(0,0,1,0,0,1)	(0,1,1,0,0,1)
(1,0,1,0,0,0)	(1,1,1,0,0,0)	(1,0,1,1,0,0)	(1,1,1,1,0,0)	(1,0,1,0,0,1)	(1,1,1,0,0,0)
(0,0,0,0,1,0)	(0,1,0,0,1,0)	(0,0,0,1,1,0)	(0,1,0,1,1,0)	(0,0,0,0,1,1)	(0,1,0,0,1,1)
(1,0,0,0,1,0)	(1,1,0,0,1,0)	(1,0,0,1,1,0)	(1,1,0,1,1,0)	(1,0,0,0,1,1)	(1,1,0,0,1,1)
(0,0,1,0,1,0)	(0,1,1,0,1,0)	(0,0,1,1,1,0)	(0,1,1,1,1,0)	(0,0,1,0,1,1)	(0,1,1,0,1,1)
(1,0,1,0,1,0)	(1,1,1,0,1,0)	(1,0,1,1,1,0)	(1,1,1,1,1,0)	(1,0,1,0,1,1)	(1,1,1,0,1,1)
(0,0,0,1,0,1)	(0,1,0,1,0,1)	(1,0,0,1,0,1)	(1,1,0,1,0,1)	(0,0,1,1,0,1)	(0,1,1,1,0,1)
(1,0,1,1,0,1)	(1,1,1,1,0,1)	(0,0,0,1,1,1)	(0,1,0,1,1,1)	(1,0,0,1,1,1)	(1,1,0,1,1,1)
(0,0,1,1,1,1)	(0,1,1,1,1,1)	(1,0,1,1,1,1)	(1,1,1,1,1,1)		

They seem like too many functions to generate. We will see in the next section that, thinking a bit, this number is largely reduced.

2.3.2. Creation of the Game and Its Collaborations

The first work to be done will be with the group of students who have agreed to collaborate in the realization of the game. They will be asked:

1. Carry out the necessary study to limit the number of functions that the game will offer. They must think what affirmative or negative answers to certain questions necessarily imply a certain response to other questions. For example, a non-continuous function implies that it will not be derivable either. With this, vectors of Table 1 with elements (-,-,-,0,1,-) must be removed from the list. Carrying out this work supposes an exhaustive review of all the theories of functions that are studied in the classroom from a new perspective. Students are also familiarized with the concept of binary numbers (in base 2) and, in general, with the different bases for expressing numbers.

A manageable number of possibilities, say 28, is selected from Table 1 (see Table 2). This is a suitable number to play.

Table 2. Possible feasible game responses.

(0,0,0,0,0,0)	(0,1,0,0,0,0)	(0,0,0,1,0,0)	(0,1,0,1,0,0)	(0,0,0,0,0,1)	(1,0,0,0,0,0)
(1,1,0,0,0,0)	(1,0,0,1,0,0)	(1,1,0,1,0,0)	(1,0,0,0,0,1)	(0,0,1,0,0,0)	(0,1,1,0,0,0)
(0,0,1,1,0,0)	(0,1,1,1,0,0)	(0,0,1,0,0,1)	(1,0,1,0,0,0)	(1,1,1,0,0,0)	(1,0,1,1,0,0)
(1,1,1,1,0,0)	(1,0,1,0,0,1)	(0,0,0,1,1,0)	(0,1,0,1,1,0)	(1,0,0,1,1,0)	(1,1,0,1,1,0)
(0,0,1,1,1,0)	(0,1,1,1,1,0)	(1,0,1,1,1,0)	(1,1,1,1,1,0)		

2. Define the 28 functions that will represent each of the 28 resulting vectors and which will be the game cards. This definition will sometimes be carried out through a formula (analytical definition of the function), and other times through a graph.

Once the functions are obtained, the chips of the game will be produced physically or virtually (if the game is going to be made with the support of a computer). Each of these cards will have an assigned definition of the function and the number in the decimal system that corresponds to the binary number of the vector that gave rise to the function. For example, the function associated with the vector (0,1,0,0,0,0) will be written with the number $16 = 0 \times 2^5 + 1 \times 2^4 + 0 \times 2^3 + 0 \times 2^2 + 0 \times 2^1 + 0 \times 2^0$ (Figure 4).

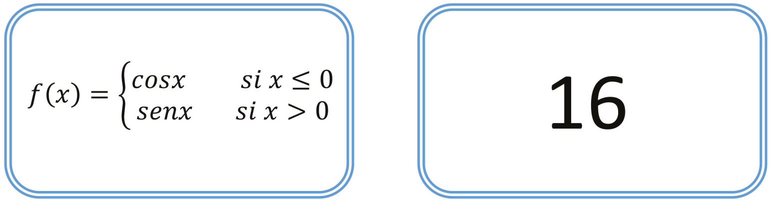

Figure 4. Image of an analytical game sheet.

This corresponds to a function that is not positive, that is bounded and that is not monotonous, nor continuous, nor derivable and it does not have vertical asymptotes.

The function associated with the vector (1,0,0,0,0,0) has the written number $32 = 1 \times 2^5 + 0 \times 2^4 + 0 \times 2^3 + 0 \times 2^2 + 0 \times 2^1 + 0 \times 2^0$ (Figure 5).

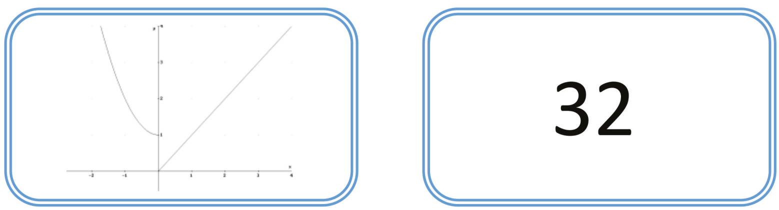

Figure 5. Image of a graphic card of the game.

This corresponds to a function that is non-negative and not bounded and is not monotonous, nor continuous, nor derivable and that has no vertical asymptotes.

2.3.3. Development of the Competition

There are several ways to introduce this game in the classroom. We have chosen one that allows all students to play simultaneously, so that no student is unable to participate. Each participant will have a template with the 28 cards corresponding to the functions of the game and a box of six positions to complete. In addition, the cards can be made in a digital format and will be shown to the participants through a projector. On this screen, the cards are projected on the side of the functions in an arrangement of rows and columns, and students are told that they are numbered from left to right and top to bottom. (Figure 6).

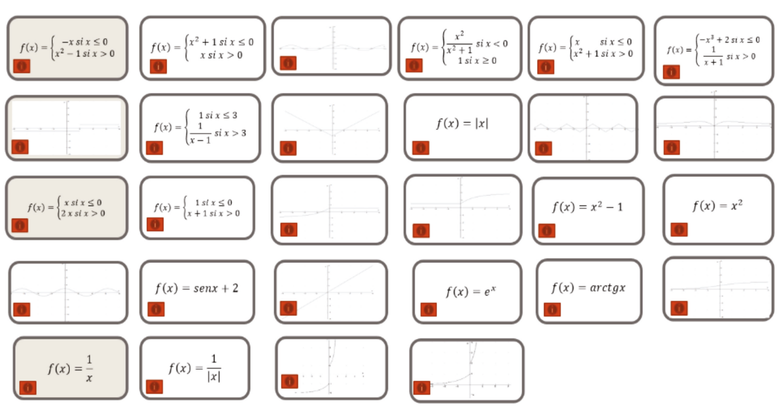

Figure 6. Projection in digital form of the functions selected for the game.

Steps to follow:

- The students can see all the functions which make up the game. The teacher will select one of them (its associated number in the decimal system will be written on a piece of paper that will be saved without any participant seeing it).
- An orderly question time begins for the students. Each student will cross out the functions that do not match the indicated characteristic and, in addition, will be completing the vector of six positions with the answers that are obtained and applying their knowledge (some answers may implicitly give answers to other unasked questions).
- The moment that one of the students thinks that he/she knows the chosen function, he/she will select it on the screen and will give the number in the decimal system that corresponds to it. This means that the participant must change the vector of zeros and ones, corresponding to the answers to the questions asked, to the number in the corresponding decimal system.
- First, the tab of the chosen function will be flipped.
- If the numbers do not coincide, the student is eliminated and the game continues.
- If the numbers match, the teacher will see if it is the number saved at the beginning. If it is, there will be a winner of the game. Otherwise the student is eliminated and the game continues.

The search for a good game strategy makes the participants reflect broadly on the concepts related to the real functions of real variables:

A good knowledge of the theory allows the participant to ask the right questions that lead to a quick resolution. In the same way, one can make beneficial use of the questions asked by colleagues and the answers received. For example, if a student asked about the existence of vertical asymptotes, receiving an affirmative answer, it is supposed, for someone familiar with the subject, that the answers to continuity and derivability of the function will be negative. One can complete the sequence of the binary number without asking questions.

2.4. Video Game

It is an assumed fact that the current generation of students feels a great attraction to computer games. This can be used to motivate the teaching of mathematics [34,35]. In this way, we have adapted the proposed game to the virtual environment by programming a videogame that recreates the "Who's That Function" challenge. This allows the students to become acquainted with the concepts related to the study of functions in a proactive way and to improve their performance through the obtained marks in the online game.

The instructions are displayed in Spanish and the game has free access by the following link: https://flyingflamingo.itch.io/whos-that-function, accessed on 11 March 2021 (Figure 7).

Figure 7. Screenshots of the online game.

Technical Description:

- Programming language:

The game has been programmed by using Godot Engine, whose scripting language is GDScript (similar to the Python language). To execute it in the navigator, the system allows for creating an executable file translated to HTML5.

- Operative environment:

Any standard desktop navigator can serve as an operative environment. Therefore, the online game can be played on platforms such as GNU/Linux, Windows or MacOS. It has been tested in Firefox 82 and Chromium 85.

- Flow chart (see Figure 8):

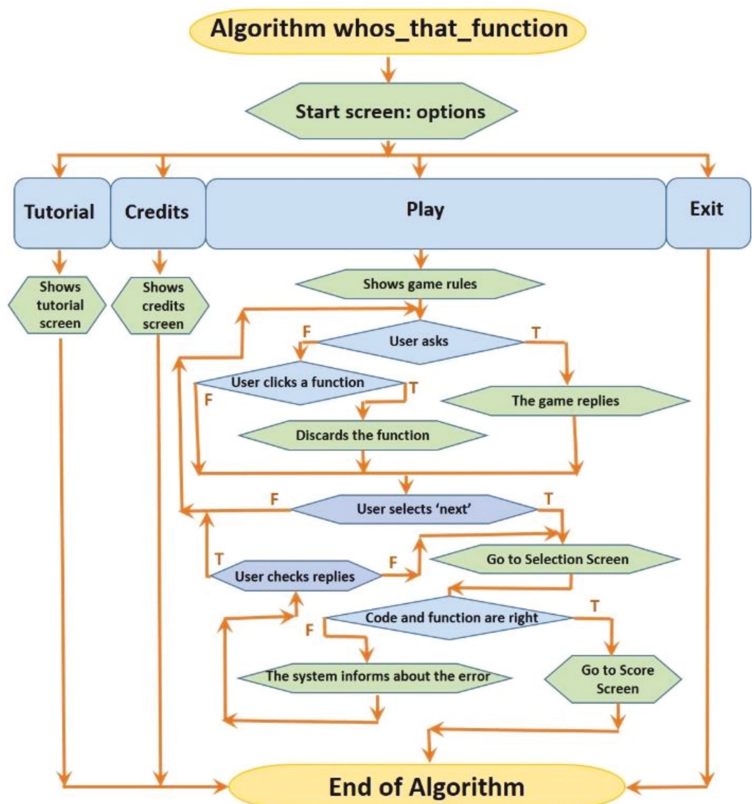

Figure 8. Flow chart for the "Who's That Function" online game.

2.5. Educational Content

The game to be implemented has been designed to be able to assess the students' knowledge once the analysis unit has been completed. The main contents of this didactic unit are:

Limits of functions
Calculation of asymptotes of a function
Continuity
Derivability
Growth and decay

Concavity and points of inflection
Domain and range
Monotonicity
Intervals of constant sign, regions
Graphical representation

As explained above, the objective of the game is for the students to find out the function that has been previously selected either by the teacher if the game is played in the classroom, or by the computer, by answering six key questions.

It is essential that when given an analytical representation of a function, a student can produce its graphical representation, and also know how to specify its characteristics. This first objective is achieved by a student matching the function cards with their analytical and graphical representation. A student who is unable to match both representations will not receive a good result and will not score a mark.

The decision of which questions should be asked responds to the need to assess the maximum amount of key knowledge of the didactic unit. It is essential for the student to evaluate whether a function is continuous or has discontinuities. As such, one of the questions that may be asked is: Is the function continuous? After the game gives the answer to this question, the player must be able to eliminate the incorrect functions by applying their knowledge of continuity. Similarly, for the concept of the derivability of a function, there is the direct question: Is it derivable?

The calculation of the limits of a function and the concept of asymptotes are addressed by the question: Does it have a vertical asymptote? To correctly respond, the student must be able to manage these concepts both graphically and analytically.

The range of a function is reviewed with the question: Is it bounded? Growth and decay and monotonicity are analyzed with the question: Is it monotone?

To put the student's knowledge about regions of constant sign for a function into practice, they are asked the question: Is it a non-negative function?

Additionally, the game works with the theoretical implications of function properties. For example, a derivable function is continuous, a function with a vertical asymptote is not continuous in R, a discontinuous function is not derivable, a function with a vertical asymptote will not be bounded, etc. The student must put this knowledge into practice. When playing in the classroom, using this knowledge will save time and allow the student to respond more quickly to the challenge. In the case of the video game, the format is designed to give the highest score to the player who gets the correct answer with the fewest questions asked.

The concepts of domain, concavity and points of inflection are not directly included in the questions asked in the game, but are addressed in a tangential way. For example, a common misconception among students is to think that, at a point where the function has a vertical asymptote, the function is necessarily undefined, or that a point of discontinuity does not necessarily belong to the domain of the function. Additionally, some students make the mistake of thinking that an inflection point marks a change in the growth of the function (e.g., from increasing to decreasing). The graphic cards in the game, as well as the teacher's guidance as the game is played, can be helpful in correcting these misunderstood concepts.

Additional content that is worked on in the game is the representation of a number in different bases (positional notation). This content is part of the number theory unit, which is useful for preparing students for technical degree programs such as computer engineering or mathematical engineering. Even though it is not present in the game cards, it is evaluated at the end of the game, since the students have to convert a number in base 2 to base 10 to obtain the final result.

Table 3 shows the most relevant information on the key mathematical concepts included in the game, their relationship with the game design and the score given to the players.

Table 3. Concepts tackled in the game.

Key Mathematical Concept	Game Design Element	Final Score
Being able to analyze the continuity of a function.	In the game, the student answers the question: Is it continuous?	In the video game, getting the function correct scores points. The number of points decreases as the number of attempts increases.
Being able to analyze the derivability of a function.	In the game, the student answers the question: Is it derivable?	
Being able to calculate limits of functions and asymptotes. Recognizing these concepts through the visualization of the graph of a function.	In the game, the student answers the question: Is there a vertical asymptote?	
Being able to analyze the range of a function.	In the game, the student answers the question: Is it bounded?	
Being able to recognize the growth and decay of a function and its monotonicity.	In the game, the student answers the question: Is it monotonous?	
Being able to analyze regions of constant sign for a function.	In the game, the student answers the question: Is it a non-negative function?	
Being able to study a function through its analytical or graphical representation.	The function cards are offered in one of two ways, mixing both in the set.	
Being able to recognize implications of different characteristics of the function.	The game allows the student to get the function without answering all the questions.	Extra points will be given to the player who gets the correct function without having needed all the answers.
Being able to change the base of a number representation.	Each function is assigned a number in base 10, which must be guessed from a number in base 2 obtained from the answers of the game.	In the video game, if the numbering is correct, the player gets points. The number of points decreases as the number of attempts increases.

3. Results

The implementation of this proposal has been carried out during the course 2018/2019, on:

- four groups of eighty students each in their first year majoring in civil and territorial engineering at the Polytechnic University of Madrid (UPM) taking Calculus I.
- eight workshops with a capacity for twenty students of the 2nd year of high school in various educational centers of the Community of Madrid (Spain).

After the experience ended, in order to carry out an assessment of the opinions of the students and the results, a survey was carried out for all the participants (to both those who contributed to the creation of the game and those who played it). The survey and its analysis are attached in the Appendix A. It is worth mentioning the good evaluation of the students who were involved in the design of the game who, in a major way, valued very positively the learning that their accomplishment gave them.

Regarding the students who played the game, around 90% of them agreed or strongly agreed that the game helped them to understand the unit and be motivated by mathematics:

92% say it has helped them to review the concepts.
84% consider it appropriate or very appropriate for their level.
85% would use more of these types of games in the course.
86% would find it appropriate to use these types of games in other subjects.
87% have increased their interest in mathematics.
90% would recommend the game.

We reached the objectives that we set ourselves and can be summarized as:

- Awaken the interest in learning mathematics.
- To enable students to apply the acquired concepts.
- Enhance skills based on mathematical reasoning: strategy, planning, decision making, etc.

- Apply the dynamics and principles of games to improve the motivation, interest and involvement of students in subjects with mathematical content.
- Create a series of playful proposals (games) that promote mathematical knowledge and the playful approach to this science.

The achievement of the aforementioned objectives entails the facilitation of the students' learning process, which can be done through an enjoyable, playful, flexible, dynamic and interactive mechanism, which is expected to attract students and promote their involvement in the subject.

4. Discussion and Conclusions

We are sure that the actions proposed in this project will contribute to the approach of students towards understanding essential basic subjects in their studies. The application of games is highly motivating and is a good reinforcement if applied to mathematical subjects. In addition, it allows students to integrate and relate to each other since it promotes team actions.

The groups of students to whom we have presented the activity have shown great receptivity and an excellent understanding of the content presented, which has improved their perception of mathematical topics.

These good results are supported by the surveys presented to the participants (see the Appendix A). We emphasize that the students who collaborated in the realization of the game showed greater satisfaction with the experience and what it gave them.

As future actions, we want to involve students in the design or invention of new proposals (new games) where they must use the contents they have learned or are learning in the subjects related to mathematics. In this way, we want to stimulate creativity and mathematical knowledge.

5. Patents

The software shown in Section 2.4 has been registered in the Intellectual Property Registry. File number: 09-RTPI-08550.8/2020.

Author Contributions: Conceptualization, S.L., M.L., S.M. and J.R.; methodology, S.L., M.L., S.M. and J.R.; software, S.L. and M.L.; formal analysis, S.L., M.L., S.M. and J.R.; investigation, S.L., M.L., S.M. and J.R.; resources, S.L., M.L., S.M. and J.R.; data curation, S.L., M.L. and S.M.; writing—original draft preparation, S.L., M.L., S.M. and J.R.; writing—review and editing, S.L., M.L., S.M. and J.R. All authors have read and agreed to the published version of the manuscript.

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Appendix A

Survey conducted with students who participated in one way or another in the proposal.

Survey on the activity "Binary Who is Who"

1. Rate from 1 to 5 your level of interest towards the Mathematics subject, where 1 is nothing and 5 is a lot.
Mark only one box.
1 2 3 4 5

2. Do you think that this game has helped you to better understand the functions? Rate from 1 to 5, where 1 is not useful and 5 very useful.
Mark only one box.
1 2 3 4 5

3. Do you think this game has helped you to review the contents of the subject? Rate from 1 to 5, where 1 is not useful and 5 very useful.
Mark only one box.
1 2 3 4 5

4. Do you think the difficulty of the game is appropriate to your level? Rate from 1 to 5, where 1 is not adequate and 5 is very adequate.
Mark only one box.
1 2 3 4 5

5. Would you like to be able to have games of this style for other subjects of Mathematics? Rate from 1 to 5, where 1 is I would not like it and 5 I would like it very much.
Mark only one box.
1 2 3 4 5

6. Would you like to be able to have games of this style for other subjects? Rate from 1 to 5, where 1 is I would not like it and 5 I would like it very much.
Mark only one box.
1 2 3 4 5

7. Do you think this game has contributed to increasing your interest in Mathematics? Rate from 1 to 5, where 1 is nothing and 5 a lot.
Mark only one box.
1 2 3 4 5

8. Would you recommend this game for students of your same level? Rate from 1 to 5 where 1 is not recommended and 5 highly recommended.
Mark only one box.
1 2 3 4 5

Attached is the analysis of the surveys made to the students of the 1st year of Civil and Territorial Engineering degree of the UPM (2018-2019 academic year):

(a) Students who participated in the making of the game.

Number of students who participated: 6.

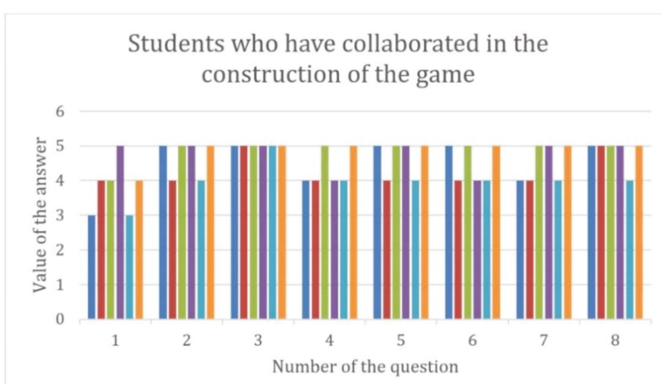

Figure A1. Results of the survey of the students who collaborated in the making of the game.

(b) Students who have played "Binary Who is Who".

Number of students surveyed: 100.

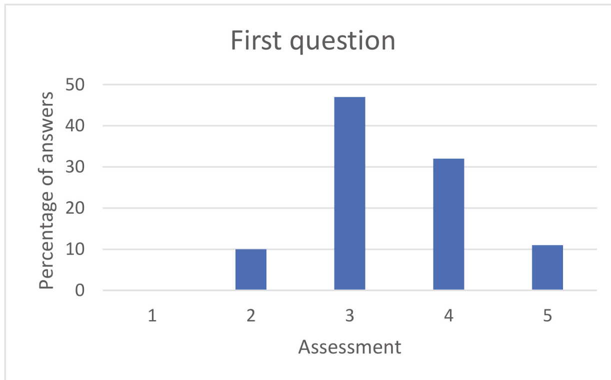

Figure A2. Results of question 1.

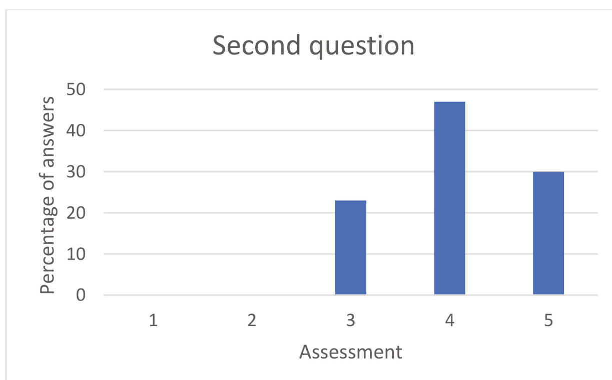

Figure A3. Results of question 2.

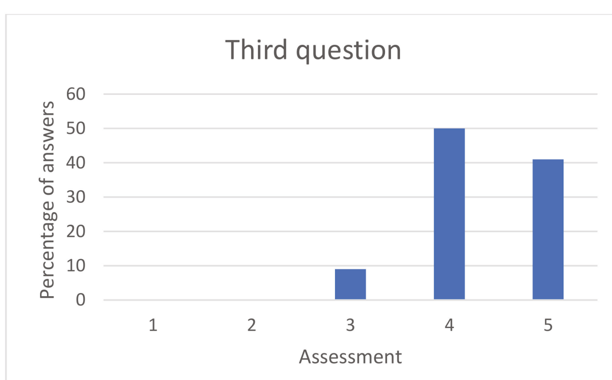

Figure A4. Results of question 3.

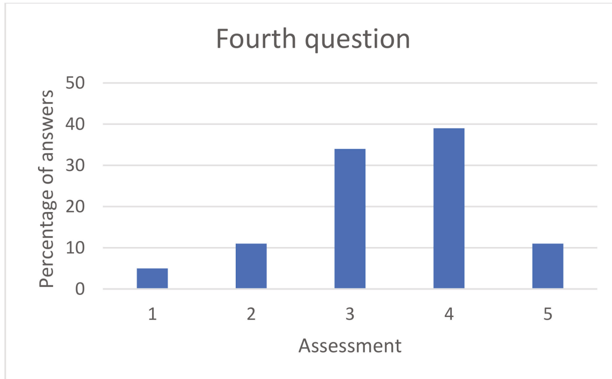

Figure A5. Results of question 4.

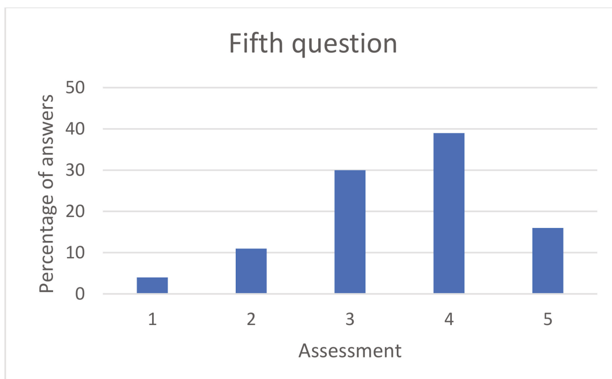

Figure A6. Results of question 5.

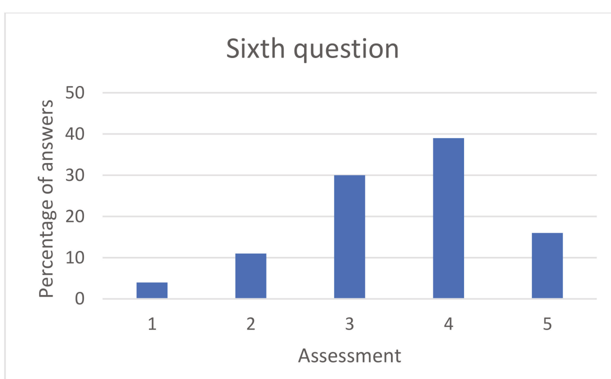

Figure A7. Results of question 6.

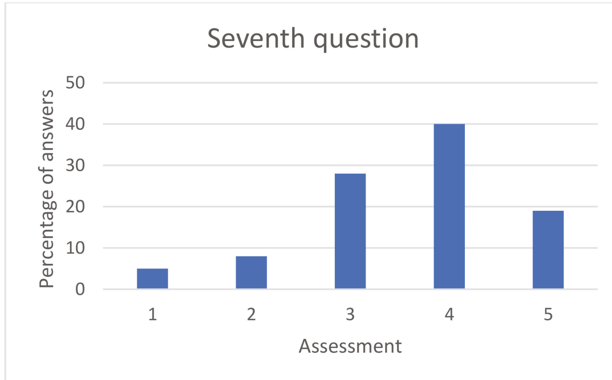

Figure A8. Results of question 7.

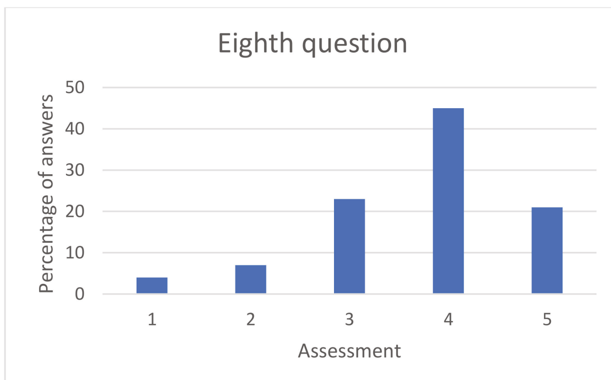

Figure A9. Results of question 8.

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