Grey Systems Theory in Business Management

Applying Multiple Criteria Decision-Making Methods

MDPI

Moses Olabhele Esangbedo and Jianwu Xue

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Authors

Moses Olabhele Esangbedo

Jianwu Xue

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Authors
Moses Olabhele Esangbedo
Xuzhou University of Technology
Xuzhou, China

Jianwu Xue
Northwestern Polytechnical University
Xi'an, China

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Dedication

To my wife, Caroline Olufunke Esangbedo; and to my children, David Osemhengbe Esangbedo and Joy Ofure Esangbedo.

Moses Olabhele Esangbedo

To my wife and son; Son, Dad and Mum love you.

Jianwu Xue

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Abbreviations

3PL	Third Party Logistics
AHP	Analytical Hierarchical Process
AMO	Ability, Motivation, and Opportunity
ANP	Analytical Network Process
ARAS-G	Additive-Ratio Assessment method with Grey values
ARAS	Additive-Ratio Assessment
ARWU	Academic Ranking of World Universities
BIM	Building Information Modelling
BWM	Best-Worst MCDM
C&B	Compensation and Benefit
CILOS	Criteria Impact Loss
COPRAS	Complex Proportional Assessment
CSP	Concentrated Solar Panel
CWTS	Centre for Science and Technology Studies
DBP	Doing Business Project
DEA	Data Envelopment Analysis
DEMATEL	Decision-Making Trial and Evaluation Laboratory
DM	Decision Maker
DSS	Decision Support System
EDAS	Evaluation based on Distance from Average Solution
EHFLTS	Extended Hesitant Fuzzy Linguistic Term Set
ELECTRE	ELimination Et Choix Traduisant la REalité—ELimination and Choice Expressing REality
EPC	Engineering Procurement Construction
ERP	Enterprise Resource Planning
EW	Equal Weight
FMEA	Failure Mode Effect Analysis
FSP	Foreign Service Premium
FST	Fuzzy Set Theory
FUCOM	Full Consistent Method
GC	General Contractor
GHRM	Green Human Resource Management
GIS	Geographic Information Systems
GN	Grey Number
GRA-PNR	Grey Relational Analysis with Positive and Negative References
GRA	Grey Relational Analysis
GRFT	Grey Regulatory Focus Theory
GRG	Grey Relational Grade
GST	Grey Systems Theory
GWSM	Grey Weighted Sum Model
HRIS	Human Resource Information System
HRM	Human Resource Management
HR	Human Resource
IDOCRIW	Integrated Determination of Objective Criteria Weights

ILP-GP	Integral Linear Programming with Grey Possibility
ILP	Integral Linear Programming
IoT	Internet of Things
IT	Information Technology
MABAC	Multi-Attributive Border Approximation area Comparison
MAIRCA	Multi Attributive Ideal-Real Comparative Analysis
MCDM	Multiple Criteria Decision-Making
MILP	Mixed Integer Linear Programming
MIS	Management Information System
MODM	Multi Objective Decision-Making
MULTIMOORA	Multi-Objective Optimization by a Ratio Analysis
NRA	Negative Reference Alternative
OWA	Ordered Weighted Averaging
PA	Point Allocation
PRA	Positive Reference Alternative
PROMETHEE	Preference Ranking Organization METHod for Enrichment Evaluation
PRSPWUN	Performance Ranking of Scientific Papers for World Universities
PV	Photovoltaic
QFD	Quality Function Development
QS	Quacquarelli Symonds
RAM	Reliability, Availability, and Maintainability
RFT	Regulatory Focus Theory
RII	Relative Importance Index
ROCS	Rank Order Centroid with Slacks
ROC	Rank Order Centroid
RST	Rough Set Theory
SAW-G	Simple Additive Weighting-Grey
SAW	Simple Additive Weighting
SC	Sub-Contractor
SPC	Solar Panel Contractor
SWARA	Stepwise Weight Analysis Ratio Assessment
THE	Times Higher Education
TOPSIS-G	Technique for Order of Preference by Similarity to Ideal Solution with Grey values
TOPSIS	Technique for Order of Preference by Similarity to Ideal Solution
URAP	University Rankings based on Academic Performance
UR	University Reputation
VIKOR	VIseKriterijumska Optimizacija I Kompromisno Resenje—Multicriteria Optimization and Compromise Solution
WASPAS	Weighted Aggregated Sum Product Assessment
WEEE	Waste from Electrical and Electronic Equipment
WPM	Weighted Product Model
WSM	Weighted Sum Model

About the Authors

Moses Olabhele Esangbedo received a B.Eng. in Electrical/Electronics Engineering from Madonna University, Okija, Nigeria, in 2006. He earned an M.S. in Communication and Information Systems and a Ph.D. in Management Science and Engineering from Northwestern Polytechnical University, Xi'an, China, in 2012 and 2017, respectively. Currently, he serves as an Associate Professor with the School of Management Engineering at Xuzhou University of Technology. From 2018 to 2022, Dr. Esangbedo worked as a postdoctoral researcher at the School of Management of Northwestern Polytechnical University. Before his postdoctoral position, he gained a year of work experience as a human resource manager in Kerui Group—China. During this time, he contributed significantly by writing two notable cases: human resource information system vendor selection and scaling foreign service premium expatriate allowance problems. These cases were published in the Journal of Grey Systems, Expert Systems with Applications, and other works have been published in prestigious journals (See https://orcid.org/0000-0003-2929-0331). He is the author of the monograph entitled "Business Decision-Making Based on Grey System Theory: Methods and Applications," published by China Aviation Publishing and Media Co. LTD. Additionally, Dr. Esangbedo serves as an associate editor of the International Journal of Grey Systems and journal editorial board member for Management Science and Business Decisions. Throughout his academic journey, Dr. Esangbedo has been recognized for his excellence. He was the recipient of full scholarship awards from the Nigerian Scholarship Council from 2008 to 2016 and the Chinese Scholarship Council from 2008 to 2017. He also received an Excellent Student award from the International College of Northwestern Polytechnical University, Creative and Innovative Project grants from Northwestern Polytechnical University, and an Excellent Ph.D. thesis award from Northwestern Polytechnical University.

Jianwu Xue received the M.S. degree in aerospace manufacturing engineering and the Ph.D. degree in management science and engineering from Northwestern Polytechnical University, Xi'an, China, in 1998 and 2011, respectively. Currently, he is leading a research project with the National Social Science Foundation of China and is actively involved in a major National Social Science Foundation project. Dr. Xue holds the position as Professor and the Head of the Management Information Systems department at the School of Management, Northwestern Polytechnical University. Over the course of his career, he has overseen more than 20 scientific research projects, including several at the national level. He has authored or coauthored over 30 academic papers, compiled four specialized textbooks, and has been recognized with three provincial and municipal science and technology achievement awards. Additionally, Dr. Xue holds four software copyrights. His primary research interests encompass information management and data analysis.

Foreword

Decision-making runs through all business activities. The so-called decision-making is to determine the action to be taken according to the actual situation and the predetermined goal. The changing political, economic situation, market conditions and natural environment, as well as the incomplete knowledge of products, services, partners, competitors, etc., lead to uncertainty in business decision-making activities. The grey systems theory, proposed by Professor Julong Deng in 1982, provided a scientific method for solving the decision-making problem under the situation of incomplete information. At present, grey systems theory has been widely used in various fields of social science, natural science and engineering technology. Scholars from more than 100 countries and regions in the world have published hundreds of thousands of research papers on grey systems theory and applications. In business decision-making, grey systems theory has been successfully applied and achieved a lot of results. This book is the research result obtained by an African scholar under the guidance of three professors from Northwestern Polytechnical University. These three professors are Professor Ada Che (Dean of the School of Management), Professor Jianwu Xue (Director of the Information Management Department of the School of Management), and Professor Sijun Bai (Chairman of Xi'an Huading Project Management Consulting Co., Ltd.).

Invited by Professor Naiding Yang, I have attended academic conferences and lectured on grey systems theory at Northwestern Polytechnical University many times. Many scholars including Professor Naiding Yang and Professor Jianwu Xue from Northwestern Polytechnical University also attended academic activities in the field of grey theory and complex equipment development and management at Nanjing University of Aeronautics and Astronautics. After Associate Professor Moses Olabhele Esangbedo learned about the grey systems theory, he developed a strong interest in the uncertainty decision-making method. He has participated in the national first-class courses on grey systems theory through the iCourse International platform, and systematically watched the English version of the online open course "Grey Data Analysis". Dr. Moses uses the grey system method to study the compensation and benefits of multinational companies, and evaluates the business environment of the African continent. He has achieved many valuable results and published academic papers in many high-level English journals. This book is the crystallization of years of his research work.

The main contents of this book include the extended study of the hybrid MCDM method based on grey systems theory, grey rank order centroid, grey stepwise weight analysis ratio evaluation, grey regulatory focus theory, grey complete consistency multi-criteria decision-making and so on. The close combination of theoretical method research and practical application is the characteristic of this book. Combining his work experience as a senior human resources manager in an enterprise, Dr. Moses applied the grey systems theory to the evaluation and development of Human Resource Information System (HRIS), and expanded the foreign service premium of foreign employees of multinational companies. The management innovation and contribution based on grey systems theory earned Moses the company's award.

I was fortunate to read the manuscript of "Grey Systems Theory in Business Management: Applying Multiple Criteria Decision-making Methods" in advance, and was deeply moved by the tireless and unremitting spirit of exploration as a scholar like Moses who came to study in China. I believed that the publication of this book will contribute to the research on business decision analysis in the context of poor information. It is expected that Dr. Moses' work can promote the widespread application of grey systems theory in African countries and make positive contributions to the social and economic development of African countries.

Sifeng Liu
Distinguished Professor of the School of Management
Northwestern Polytechnical University, China
President of International Association of Grey Systems and Uncertainty Analysis

Preface

The book presents a summary of my academic research on Multiple Criteria Decision-Making based on the Grey Systems Theory during my doctoral and postdoctoral research at Northwest Polytechnical University, China.

During my doctoral studies, Professor Che Ada, the Dean of the School of Management at Northwestern Polytechnical University, served as my doctoral supervisor. Under his guidance, I delved into the intricacies of grey systems theory. I was astounded by its wide acceptance and development within the scientific community. My research journey, based on grey systems theory, began during my Ph.D. studies. One notable aspect of my doctoral dissertation, titled "Evaluation Business in Using Improved ROC Weights and WSM," was its focus on assessing Africa's business environment as a research context based on the Rank Order Centriod (ROC) weights and Weighted Sum Model (WSM). Gladly, it received the distinction of being rated as an excellent doctoral dissertation by Northwestern Polytechnical University.

Upon embarking on my postdoctoral research journey, I had the privilege of working with Professor Bai Sijun, who also served as my co-supervisor. Professor Bai, a distinguished professor at the School of Management at Northwestern Polytechnical University and Chairman of Xi'an Huading Project Management Consulting Co., Ltd., collaborated with me on various application cases. These cases ranged from selecting contractors for solar panel installation to utilizing regulatory focus theory for evaluating university reputation. These valuable cases were documented in my postdoctoral research report, titled "Research on Decision Weight Problems and Application Based on Grey System and Regulatory Focus Theories." The research opportunities presented by Professor Bai in such a short timeframe deepened my understanding of these application areas.

As I progressed to the second postdoctoral research, Professor Xue Jianwu, the Director of the Information System Management Department at the School of Management, Northwestern Polytechnical University, became my co-supervisor. His insights brought new dimensions and interpretations to this book, ultimately leading to its publication in the Chinese version. During the period, the research findings were compiled into a comprehensive work report entitled "Improved Grey Relational Analysis With Hybrid Point Allocation Multi Criteria Decision-Making Methods."

As an Associate Professor at Xuzhou University of Technology, I have had the privilege of working in an enriching environment and partially funding my personal goal of making this book freely accessible to the public.

My primary aim is to ensure that these scientific methods are available in English, allowing a broader audience to benefit from them.

This book represents novel approaches to addressing uncertain business problems in practical applications by stitching together the core of my research output over the decade. Grounded in grey systems theory, it introduces a new multi criteria decision weighting and evaluation method. The first chapter of this book introduces the relevant concepts, while the second chapter outlines the evaluation criteria for five major topics, including human resource information systems and contractor selection. Chapters three through six propose four improved hybrid grey weighting methods, including the grey rank order centroid method, grey regulatory focus theory method, grey stepwise weight analysis ratio assessment, and grey point allocation weighting method. Chapters seven through ten introduce four improved hybrid grey evaluation methods, including the grey-weighted sum model, interval grey number relational analysis method, grey regime method, and grey integer linear programming. Chapter eleven delves into other grey methods, and a concluding summary is presented in Chapter twelve. This book serves as a valuable reference for students, researchers, and industry practitioners alike.

Moses Olabhele Esangbedo
Author

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I express my heartfelt gratitude to the Nigerian and Chinese governments for their unwavering support and funding. Both the Nigerian and Chinese governments have played a pivotal role in my academic journey by providing support for my postgraduate and doctoral education research work through the Federal Scholarship Board (FSB) of the Nigerian Federal Ministry of Education, the China Scholarship Council (CSC) as well as my postdoctoral research through the China Postdoctoral Science Fund. These opportunities have created an environment where I could focus on my studies and research.

1. Introduction

1.1. Background on Business Decisions

Business is the development and processing of economic value, which is achieved by sellers providing goods and services to consumers and making a profit [1]. The handling of economic values is a commercial activity for institutions, and profit from the business is the primary motivation. This processing of economic value can be seen as the management of company assets to generate profit by reducing costs and providing dividends for owners or participants involved in the process. A business is a productive organization, which may consist of a single person, with customers to deliver goods or services to [2]. People create and manage business, and they make decisions that determine the growth and survival of the organization. Business involves a company or enterprise in the purchase or sale of manufactured goods and services for profit, which commonly have some monetary value.

The business environment refers to the intricate set of conditions that significantly impact the evolution and sustainability of enterprises. These conditions are characterized by their complexity and dynamic nature. Essentially, the business environment predominantly comprises external forces, factors, and institutions—entities beyond a firm's direct control—which play a pivotal role in shaping its operational landscape. This spectrum of external influences encompasses various facets, such as geographical positioning, governmental interventions, market trends, legal frameworks, technological trajectories, and more. In essence, the business environment encompasses the contextual milieu that exerts both direct and indirect influences on the functions of an enterprise. It is imperative to note that these contextual factors lie outside a company's sphere of influence and encompass a broad range of dimensions, including economic, social, political, legal, demographic, and technological realms.

The decisions or judgments that follow an evaluative process are integral to people's daily activities and fall under the domain of decision-making. Decision-making entails the process by which individuals select courses of action to resolve presented challenges, with the impetus for decision-making arising from the presence of issues requiring resolution. Notably, the essential stages in addressing decision-making quandaries encompass problem identification, the exploration of possible avenues, the assessment of alternatives, and ultimately, the conclusive selection of a resolution. In the realm of Multiple Criteria Decision-Making (MCDM), the fundamental constituents for resolving issues comprise discrete alternatives, each evaluated against predefined criteria, in conjunction with the corresponding weights assigned to these criteria [3]. Over time, a range of classical and hybrid methodologies have evolved within the MCDM framework to grapple with the intricacies inherent in such decision-making challenges.

1.2. Decision-Making Process

In the realm of business, achieving success necessitates a delicate equilibrium between short-term and long-term objectives, particularly in the context of an unpredictable and ever-evolving business environment. For entrepreneurs contemplating investment, where they may have limited prior experience, the decision-making process becomes notably intricate. This complexity arises from the need to assess numerous criteria while navigating varying degrees of uncertainty across different countries. It is crucial to recognize that neglecting uncertainty is a telltale sign of sub-optimal decision-making in this context. In an environment where unpredictability is the norm, acknowledging and effectively managing uncertainties represents the difference between a successful investment venture and a costly misstep. As such, entrepreneurs seeking to enter the market must adopt a strategic approach that not only addresses the multifaceted criteria involved but also embraces the inherent uncertainties to make informed and resilient decisions.

The process of business decision-making typically involves a series of steps, which are not rigid. Prior steps can be revisited throughout the decision process to fine-tune the decision-making process and to ensure the optimal solution is obtained to proceed toward a profitable course of action. The general business decision-making process is as follows:

1. Problem identification: For business opportunities that need to be secured and decision problems to be solved, there must be clarity on the impediments that need to be resolved. This involves ascertaining the capabilities of the company and markets. The pursuit of positive goals can be considered an MCDM problem, for example, a business that wants to reduce waste and another that wants to increase efficiency. Although the term "waste" can have a negative connotation and the term "efficiency" has a positive connotation, both are considered MCDM problems.
2. Information gathering: This involves a quantitative and qualitative approach to searching for and obtaining information as it relates to the decision problem. This can be drawn from internal reports, marketing research, and other sources. The first part at this stage is to investigate if the decision problem has been encountered previously and understand how it was addressed. In academia, it involves conducting a literature review, which may be seen as a qualitative research methodology of searching and gathering information.
3. Evaluation criteria selection: This is the yardstick used to benchmark the performance of the decision alternatives. This can be a direct measure such as temperature in degrees Celsius (°C) for a company that manufactures air conditioners and heaters, or an indirect measurement as a latent construct such as employee satisfaction in a company. To increase the precision of the measure, a hierarchy of decision criteria can be created by dividing the evaluation criteria into sub-criteria. One approach to this subdivision is to use different variables to measure the same criteria, and another approach is to use different variables to contribute to a single criterion. Establishing what is measurable and determining if the performance value of the alternative is obtainable determines the choice of the evaluation criteria.

4. Decision alternative selection: These are the possible solutions to the decision problem. When there are many alternatives, some minimum thresholds can be set for an alternative to be considered for comprehensive evaluation, especially when the evaluation criteria are non-compensatory. This process includes brainstorming and a creative approach being applied by the Decision-Makers.
5. Weighting and evaluation method selection: Different ranking steps can lead to different outcomes. These crucial steps include selecting the appropriate normalization technique, the suitable weighting approach, and the right evaluation method based on the background of the problem and result ones is trying to obtain. For example, the sharpness of a razor blade cannot be compared with that of a chainsaw; both have their strengths and weaknesses. There are situations where the high computational complexity of an MCDM method may not be appropriate for making quick decisions that are not life-threatening.
6. Alternative assessment and ranking: This is where mathematical computation and analysis begin. The conventional sequence is constructing a weighted normalized decision matrix, then aggregating the performance of the alternatives for all of the evaluation criteria and ranking them to obtain the best alternative. Comparison with other evaluation methods should be carried out to ascertain that the results are highly correlated, and sensitivity analysis should be conducted to determine the robustness of the solution.
7. Solution implementation with feedback: While the best alternative may have been selected, the solution should be implemented using a detailed plan. It is possible that the landscape of the business environment can change during implementation, such as in the case of the COVID-19 pandemic. The performance values and weights of criteria must be updated as feedback to respond to swift changes in the overall respective policies. This kind of refinement in the business decision process is continuous.

The business decision-making process is often iterative, as decisions may lead to new challenges or opportunities that require further analysis and action. Effective decision-making is a crucial skill in the business world, as it can determine the success and growth of an organization.

1.3. Factors Affecting Business Environment

In evaluating a business environment, numerous factors exert an influence on the data utilized in the evaluation process. The efficacy of a business environment's performance is intricately interlinked with the very environment it operates within, subsequently dictating the potential success or failure of a business endeavor. The business environment lays down the parameters within which an enterprise operates, outlining the constraints that inevitably shape its trajectory. While certain operational activities might have minimal influence over the broader business environment, it remains incumbent upon businesses to formulate strategies that enable effective operations, particularly when confronted with the uncertainties intrinsic to the business environment. Several pivotal factors that significantly impact the growth and performance of a business encompass the following.

1.3.1. Political System

A nation's political system serves as the cornerstone upon which government policies are formulated, impacting both businesses and citizens in their roles as consumers. Businesses must, therefore, develop a robust strategy in response to the prevailing political system. As Chang et al. [4] demonstrated through their analysis of 14 countries spanning two decades, political parties wield substantial influence over business cycles and the broader business environment. A stable political system not only fosters an environment conducive to foreign investment but also mitigates the costs and risks associated with conducting business. In select instances, certain countries have established free trade zones that offer distinct advantages to businesses. Within the sphere of the political factor are political parties, trade unions, and labour unions, which continue to hold significant sway over the business environment.

1.3.2. Legal System

Laws and regulations constitute binding obligations that businesses are obliged to adhere to, with a focus on avoiding potential penalties associated with legal transgressions. It is imperative for companies to possess a comprehensive understanding of the applicable laws before entering into any legally binding contracts. These laws encompass a broad spectrum, including tax laws, copyright regulations, trademark protections, environmental preservation statutes, corporate social responsibility mandates, consumer protection laws, and more. It is important to acknowledge that the specific legal framework differs across various business environments. The legal system of a business environment encompasses ethical decision-making and corporate social responsibility within its purview.

Breaching the law can not only result in substantial financial losses for a company but also lead to the termination of its operations within a given country [5]. For example, these legal obligations might encompass seemingly straightforward matters like traffic regulations for heavy-duty vehicles. Moreover, in certain business environments, particularly prevalent in many underdeveloped and developing nations, business operations are often informal, and the formal legal system fails to encompass all the regulations pertinent to the business environment. In such scenarios, investors must show respect for both informal and unwritten rules [6].

1.3.3. Social System

The way of life and the belief systems of people in a specific location can exert a profound influence on businesses. This social factor can manifest in various forms, including customs, traditions, belief systems, and literacy levels, among others. In underdeveloped or developing countries, the adoption of the latest and most advanced technologies may not align with the market's needs, and the social infrastructure might not adequately support such technology. However, the widespread use of the internet is contributing to an increase in literacy levels, approaching international standards. Consequently, consumers are increasingly demanding higher-quality products [7]. Social interactions and behaviors are notably observable in the consumption of fashion-related goods. Even in the design of web pages for online marketing, it is essential to consider the cultural context of the users due to its influential role [8].

1.3.4. Government

The governing body that serves as the authoritative entity of a country or state holds significant sway over businesses. It is imperative for businesses to establish themselves in countries with favorable policies, and should these policies undergo changes, businesses must swiftly adapt, as failure to do so may lead to their demise [9]. Within the framework of governance, civil and public services stand as pivotal components that wield a direct impact on the business environment [10]. Furthermore, governments can intervene to safeguard the interests of consumers. For instance, governments may, on one hand, impose operational restrictions on businesses to promote the national interest, even in the absence of any violation of operational laws. Conversely, governments may offer incentives to encourage specific types of businesses, as exemplified by those engaged in sustainable energy practices.

1.3.5. Science and Technology

The relationship between government and its citizens has evolved to align with the contemporary global business landscape. This transformation necessitates the modernization of current business practices. In this context, Information Technology (IT) stands out as the foremost aspect of science and technology that comes to mind. Technological advancements have the capacity to render both products and businesses obsolete in a remarkably short span. For instance, the convenience afforded by mobile phones has significantly reduced the demand for traditional desktop telephone handsets. It is important to note that the realm of technology extends beyond IT. The design of appliances, for instance, must align with environmental standards. In the business environment, there is often an inclination to adopt technological standards from colonial powers, such as British or French standards. IT serves as the foundational platform for developing e-business solutions, which may encompass elements like centralized databases for inventory management [11].

1.3.6. Geographical Location

Geographical location plays a pivotal role in shaping the natural environment, directly impacting various industries. It determines the availability of essential natural resources for mining companies and fertile land for agricultural industries. Proximity to an ocean coast enhances accessibility to sea routes for the import and export of goods through shipping channels. Government policies also play a crucial role in conserving natural resources and preventing pollution in these areas. Moreover, the geographical location of a region offers opportunities for the tourism industry, as well as potential anthropological discoveries that can have implications for businesses. Additionally, understanding the geographical location aids companies in preparing for natural disasters [12].

1.3.7. Demography

The population or population density of a country is intrinsically linked to the size of the market and the availability of a labour force. The demographic composition of a population significantly influences the demand for goods and services. Understanding a country's demography is a vital component of business strategy, creating opportunities for mutually beneficial relationships. For instance, a manufacturing company may leverage a skilled labour force within its production lines. This demographic aspect encompasses factors like age and gender distribution, birth and death rates, and the rate of migration [13]. An illustrative example of this dynamic is how demography dictates the potential market size within a local business environment. Presently, China's population stands at approximately 1.4 billion people, making it the largest market for numerous commodities.

1.3.8. Market

A multitude of factors collectively shape the business environment, encompassing market segmentation, needs, market issues, switching costs, revenue generation, and the dynamics of demand and supply. Additionally, market orientation, which comprises a focus on customers, competitors, and inter-functional coordination within the business and its environment, profoundly influences the business landscape [14,15]. Indeed, the scope of factors influencing the business environment is far-reaching, extending beyond the distribution systems for goods and services. Even internal personnel policies within an enterprise can indirectly impact the business, such as rigid policies that hinder staff from adapting to environmental changes. Consumer preferences for goods and services, particularly in industries like fashion and technology, hold considerable sway over the business environment. Furthermore, the economic infrastructure, encompassing elements like capital availability and global markets, plays a significant role in shaping the business environment.

1.4. Classical MCDM Methods with Applications

1.4.1. Weighting Methods

In addressing MCDM problems, there exists a range of methods for selecting and evaluating alternatives, as documented by Jato-Espino et al. [16], Tzeng and Huang [17], and Zavadskas et al. [18].

One prominent technique for MCDM is the Analytical Hierarchical Process (AHP), a pairwise comparison approach by Thomas Saaty [19]. The AHP is a recognized standard for resolving MCDM challenges. It involves a systematic process where the evaluation is broken down into a distinct hierarchy, considering all conceivable measurable criteria. Subsequently, subjective judgments, typically sourced from experts, are used to compare these criteria. These pairwise comparisons of criteria are then transformed into ratio-scaled weights, which serve as the basis for linear additive evaluation scores of the alternatives and their subsequent ranking [20]. In the AHP, the determination of weights involves constructing a pairwise comparison matrix for the criteria. Afterward, a consistency check is performed, and eigenvalues are calculated. Similarly, AHP extends this process to the pairwise comparison of alternative performances, constructing a matrix, assessing its consistency, and calculating the eigenvalue. Finally, based on these weightings and comparisons, the overall scores and rankings of the alternatives are computed.

As an extension of AHP, the Analytical Network Process (ANP) was introduced by Thomas Saaty to evaluate alternatives that involve interdependent criteria and alternatives [21]. Notably, ANP has found applications in various domains. For instance, Choudhury et al. [22] applied ANP to assess supply chain cells in pharmaceutical companies. Chenglin and Bo [23] utilized ANP for the development of a reverse logistics model for strategic vendor selection, and Sarwar et al. [24] applied the ANP in evaluating the sustainabilty of organizations considering both green and lean methods.

Pairwise comparison is a valuable technique that ensures each criterion receives equal consideration during assessment. Moreover, several MCDM methods have been integrated with Geographic Information Systems (GIS) to tackle spatial MCDM problems effectively. For example, Deniz and Topuz [25] combined the AHP and GIS to conduct a land use suitability analysis. Benti et al. [26] assessed the suitability of wind power plant location in Ethiopia using the AHP. Similarly, the AHP with GIS were combined by Kohno et al. [27] to assess the hazards caused by earthquakes. These instances demonstrate the versatility and applicability of MCDM methods and GIS in various domains, from urban infrastructure and renewable energy to manufacturing processes and spatial analysis.

Structuring a decision problem is indeed a valuable approach to effectively address complex challenges. Several studies have applied various decision analysis methods to structure and tackle different decision problems. For example, Dinçer et al. [28] analyzed the performance of European energy investment policies. They used Quality Function Development (QFD) and the balanced scorecard to assess performance results. Alamoodi et al. [29] conducted a systematic literature review listing the use of various hybrid MCDM methods in medical cases during the COVID-19 pandemic. Gorcun et al. [30] presented a framework that uses blockchain technology for the selection of third-party logistic providers as an MCDM problem that can be employed based a on fuzzy set with the Dombi aggregration method.These studies illustrate the diverse applications of decision analysis methods, such as QFD, DEMATEL, AHP, ANP, and Grey Relational Analysis (GRA), in structuring and resolving complex decision problems across various domains, including energy policy, aviation, expert evaluation, and healthcare.

The issue of dealing with the exponential increase in comparisons among DMs preferences and alternatives performance values for every criteria like AHP and ANP has been addressed through the utilization of the Best-Worst MCDM (BWM) and a novel method called Full Consistent Method (FUCOM). To address this issue, Pamučar et al. [31] developed the FUCOM as a method to approximate criteria weights in MCDM models. The FUCOM is a subjective weighting approach that gauges the deviation from full consistency in comparisons. The number of pairwise comparisons required by the FUCOM is one less than the number of criteria. i.e., it requires only $(n-1)$ pairwise comparisons, making it more efficient than the AHP and BWM. These methods, including BWM and FUCOM, offer solutions to manage the challenges posed by extensive pairwise comparisons in decision-making processes. They provide more efficient and effective ways to determine criteria weights and conduct comparative analyses in various fields, from healthcare and infrastructure to manufacturing evaluations.

1.4.2. Evaluation Methods

There exist numerous MCDM methods designed to address decision problems, and these methods can broadly be categorized as either compensatory or non-compensatory techniques, as outlined by Banihabib et al. [32]. Compensatory MCDM methods allow for the offsetting of a lower-performance value in one criterion by a higher performance value in another. Examples of compensatory techniques include the Weighted Sum Model (WSM), Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS), Evaluation based on Distance from Average Solution (EDAS), and Complex Proportional Assessment (COPRAS). Conversely, non-compensatory MCDM methods lack the ability to balance lower-performance values in one criterion with higher values in another. Notable examples of non-compensatory techniques include the ELimination Et Choix Traduisant la REalité – ELimination and Choice Expressing REality (ELECTRE) method [33], VIKOR (VIseKriterijumska Optimizacija I Kompromisno Resenje; multi-criteria optimization and compromise solution), and the Preference Ranking Organization Method for Enrichment Evaluations (PROMETHEE II) methods [34,35].

The ELECTRE was originally developed in the 1960s by Bernard Roy and has seen several significant iterative improvements to address its limitations, particularly regarding the ranking of alternatives. Various versions of ELECTRE, including ELECTRE I-IV, ELECTRE-IS, and ELECTRE-TRI, have been introduced to enhance its capabilities [36]. For instance, Masri and Khayati [37] used ELECTRE-IV to rank northern African countries according to their attractiveness for foreign direct investment (FDI). Meanwhile, Wei et al. [38] applied ELECTRE-II to develop a ratio for assessing investment portfolios, demonstrating its effectiveness compared to the China Shanghai Shenzhen 300 Stock Index. TOPSIS is another MCDM method introduced by Hwang and Yoon [39]. TOPSIS selects the alternative with the shortest distance to the ideal solution and the farthest distance from the negative ideal solution. Typically, these distances are measured using Euclidean distances [40]. TOPSIS has found applications in various domains, including supplier ranking in sustainable materials for the fashion clothing industry and business model selection for product and service development [41]. These MCDM methods, including ELECTRE and TOPSIS, offer valuable tools for decision-makers in various fields to effectively evaluate and rank alternatives when multiple criteria are involved.

In the realm of MCDM enhanced by Grey Numbers (GNs), Xu and Sasaki [42] extended the TOPSIS by incorporating GNs to measure the coefficient of closeness to the ideal alternative. Liu et al. [43] used the arithmetic mean to aggregate criteria weights provided by a group of decision-makers, selecting the best alternative based on grey possibility degrees, which represent the relative positions of GNs in rankings. Zavadskas et al. [44] integrated GNs into the complex proportional assessment (COPRAS) method for contractor selection. Turskis and Zavadskas [45] applied GNs in the Additive Ratio Assessment (ARAS) method to rank alternatives. Lang et al. [46] introduced a method for ranking interval numbers based on normal distribution and applied it to optimize mining methods. Mousavi et al. Mousavi et al. [47] addressed uncertainty in multi-criteria optimization and proposed a compromise solution using GNs within the VIKOR method for ranking material handling equipment. Oztaysi [48] combined GNs with the AHP to weigh criteria and utilized the Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) for selecting a Content Management System. Sahu et al. [49] developed the Taguchi SAW method to rank cutting speed, feed, and depth of cut in a lathe machine. They subsequently employed the Taguchi TOPSIS method to validate the rankings in their research. Esangbedo and Abifarin [50] applied the Taguchi design of experiments with GRA of a halloysiste nanotube composite in selecting the optimal settings of CNC machine manufacturing.

There are other MCDM methods that can be applied in evaluating the business environment. Superiority and Inferiority Ranking (SIR) is a combination of WSM and PROMETHEE, where WSM is used for the aggregation procedure, and PROMETHEE is employed for ranking systems [51]. Additive Ratio Assessment (ARAS) represents the ratio of the optimal value of the alternatives to the optimal value of the ideal alternative, resulting in utility degrees [52]. Multi-Attribute Utility Theory (MAUT) is a classical method used for evaluating alternatives by aggregating the satisfaction obtainable from each of the criteria [53]. GRA, developed from Grey Systems Theory (GST), employs the relative relational coefficient of evaluated alternatives to the ideal alternative to determine the best alternative [54]. Data Envelopment Analysis (DEA) is a method for evaluating the effective efficiency of a decision-making unit [55]. Decision Making Trial and Evaluation labouratory (DEMATEL) is an evaluation method that can construct and visualize the interrelationships between criteria [56, 57]. Several of these methods may be combined to solve a single problem. For instance, Chang et al. [58] applied Fuzzy AHP, VIKOR, Grey Systems Theory (GST), and TOPSIS for e-book business model selection in Taiwan. Grey Systems Theory (GST) combines the recently developed method based on the removal effects of criteria (MEREC) with multi-attribute ideal–real comparative analysis (MAIRCA) for assessing carbon reduction systems [59]. The choice of the most suitable method depends on the specific problem, the nature of the criteria, and the available data. Researchers and decision-makers often select the method that best aligns with their decision context and objectives.

1.5. Grey Systems Theory

Late Professor Julong Deng's [60] development of Grey Systems Theory (GST) in 1982 was a significant contribution, aiding in how to handle situations where information is limited, partially known, or incomplete. GST provides a framework for dealing with information that falls between completely known (white) and completely unknown (black) parts of a system. This incomplete information could pertain to various aspects of a system, including elements, parameters, structure, behavior, or boundaries. In GST, grey numbers are employed to represent systems with incomplete information. A grey number is essentially an unknown value but with a specified range within which the exact value exists. This approach allows decision-makers and researchers to work with uncertain or imprecise data and make informed judgments despite the lack of complete information. GST has applications in various fields where information is scarce or imprecise, making it a valuable tool for decision-making and analysis in such contexts.

1.5.1. History and Evolution of GST

The theory of the grey system was pioneered by Deng Julong in 1982 and first published in his paper titled "The Control Problems of Grey System" [61]. This paper built upon his previous work titled "Stability of Large Scale System Having Incomplete Parameter via Minimum Information", published in 1979 at a military conference. To consolidate scholarly discussion on the GST, Deng founded the Journal of Grey System as the inaugural Editor-In-Chief, with issues initially distributed exclusively in paper format.

While the late Professor Deng had numerous students during his lifetime, one of the most notable is Professor Sifeng Liu, whom he supervised for his PhD. To this day, Professor Sifeng Liu remains dedicated to the development of GST. Besides several works published in Chinese, among his works are the books titled "Grey System: Theory and Application" (published in 2005) and "Grey Data Analysis" (published in 2010). Professor Liu became the pioneer of the Journal of Grey System: Theory and Application, addressing the limitations of Deng's journal, which was only available in printed format at the time. Currently, Professor Liu serves as the Editor-in-Chief of both journals. Genealogically, the third generation of researchers includes Associate Professor Saad Javed, whose PhD was supervised by Professor Sifeng Liu. He pioneered the International Journal of Grey Systems (IJGS), which disseminates research under the Creative Commons Attribution-NonCommercial (CC BY-NC) license, breaking readership barriers. The journal, in its infancy, welcomes young scholarly research without the rigorous requirement of pushing the boundaries of GST development set in Deng and Liu's journal.

Milestone's literature review of the General System Theory (GST) has been published. In 2013, Mu-Shany Yin [62] conducted a bibliometric analysis on the 15th anniversary of the GST's. She highlighted the global coverage and use of GST in countries such as China, the United States, England, Germany, Japan, Australia, Canada, Austria, Russia, Turkey, the Netherlands, and Iran. While GST is primarily dominated by Chinese and Taiwanese researchers, grey relational analysis (GRA) and grey prediction studies are predominantly conducted by Indian and American researchers, respectively. The main areas of GST application include engineering, computer science, automation control systems, and operations research management science, with over 400 journals featuring GRA studies. Xie and Wang [63] presented a historical review of grey forecasting models, including grey models GM(1, 1) and GM(1, N). Additionally, Ting Kuo [64] reviewed several modified GRA models and concluded that localized GRA models are sufficient in some studies.

Professor Deng and Professor Liu have received numerous awards throughout their careers [65]. Angela Dorothea Merkel, the German Chancellor from 2005 to 2021, acknowledged the GST in her speech at Huazhong University of Science and Technology on September 7, 2019, stating that both Late Professor Deng Julong and Professor Sifeng Liu "have made a profound impact on the world." In the same year, Javanmardi and Liu [66] conducted a review of the application of GST in socio-economic systems. They used keywords such as grey systems, grey relational, grey model, grey prediction, grey control, grey incidence, grey cluster, grey decision, and grey input–output, as well as social, economic, and socioeconomic. Their research explored the application of GST in areas including social networks, healthcare, financial issues, sustainability, tourism, social and cultural sectors, public sectors, urbanization, development, business, economics, demographics, innovation, and entrepreneurship. The application of grey relation in image sparse representation was presented by Li et al. [67]. Subsequently, Javanmardi [68] discussed the application of GST in sustainability, covering sustainable products, sustainability assessment and development, and sectors such as industrial, urban, energy, business, agricultural, tourism, and social sustainability.

The publisher Springer, established in 1842, acknowledges the significance of Grey Systems Theory (GST) and has launched a book series on the subject, edited by Sifeng Liu, Yingjie Yang, and Jeffrey Yi-Lin Forrest. After over 40 years of pioneering work in GST, the book "Grey Systems Analysis: Methods, Models, and Applications" [69], published by these editors, captures four decades of development in the field. Additionally, other notable works in this series include "Emerging Studies and Application of Grey System" [65] and "Advancement of Grey Systems Theory in Economics and Social Sciences" [70]. Recently, the latest addition to the series, "Methodological Aspects of Grey Systems Theory in Management Research" [71], continues to build on this extensive body of knowledge.

1.5.2. Known Application of GST

The GRA method has found extensive applications and extensions across various domains. Researchers have leveraged GRA to evaluate and optimize different aspects of systems and processes. Some examples of how GRA has been applied and extended are covered in the literature. Wang et al. [72] used GRA as a basis for designing a system to capture customer requirements. They assessed a customer's assessment utility, represented as a triangular fuzzy number, using GRA. Li et al. [73] applied GRA to evaluate the work efficiency and medical quality of a hospital in China that operates based on the public–private partnership model. Peng and Shen [74] developed an evolutionary algorithm based on GRA and integrated it into a linear programming solver to solve crew scheduling problems. Li et al. [75] conducted a comparative study on the effectiveness of IoT implementation in different regions of China using GRA. Khuman et al. [76] proposed grey natural language processing using GRA as a method for natural language processing tasks. Wang et al. [72] optimized the cab suspension of self-dumping trucks using GRA as a parameter. Lin and Hu [77] applied GRA to measure the similarity between two patterns, incorporating a tolerance rough set based on an accumulated generating operator. Huang et al. [78] improved the test method for grey relational order, specifically focusing on Grey Relational Grade and probability distribution. Hu et al. [78] developed an aggregation-function-based similarity measure using GRA, which can also be used for prediction tasks. Es et al. [79] introduced GRA-TRI as a multicriteria decision aid classification method, outperforming other methods like ELECTRE-TR-Central. Zhu et al. [80] applied the modified variable-weight clustering method to address issues related to continuation coefficients. These diverse applications and extensions of GRA demonstrate its versatility and effectiveness as a tool for analyzing, evaluating, and optimizing complex systems and processes in various fields.

These newer hybrid MCDM methods that incorporate GRA into their frameworks showcase the versatility of GRA in solving complex decision-making problems across various domains. For instance, Li and Zhu [81] introduced a grey relational decision-making model that utilizes three-parameter interval GNs. They combined AHP and Data Envelopment Analysis (DEA) to determine the weights of criteria used in the three-parameter interval GNs. This approach was applied to analyze aircraft carriers and can be extended to other industries, such as agriculture. Yuan [82] presented a green agricultural structural optimization model based on GRA. This model incorporates an optimization function that significantly improves the evaluation results, making it a valuable tool for optimizing agricultural structures. Kumar et al. [83] used GRA to analyze and optimize the rolling process using carbon tools and steel. This application demonstrates how GRA can enhance the optimization of industrial processes. Suvvari et al. [84] evaluated the performance of 24 life insurance companies in India using various evaluation criteria, including capital adequacy, liquidity, operating efficiency, and profitability ratios. They then employed traditional GRA to rank these insurance companies based on their grey relational grades. Zhang and Yuan [85] applied GRA to provide guidance in establishing a scientific system for college student education. GRA's ability to analyze and relate different factors makes it useful for optimizing educational systems. These applications highlight the adaptability of GRA as a valuable tool for decision-makers and researchers in different fields, providing insights and solutions for complex decision-making problems.

Researchers have been actively advancing GRA and related decision-making methods. Their contributions include a comparative analysis of grey ranking approaches by Darvishi et al. [86], which found that the kernel degree and degree of greyness method can offer more benefits than others. Xi and Wei introduced a systematic approach for determining optimal schemes by selecting the invariant degree of greyness and kernel normalization method. Esangbedo and Wei [87] demonstrated that the choice of normalization method can lead to variations in rankings. They subsequently introduced the concept of grey hybrid normalization and applied this approach to solve a location selection problem. Guo et al. [88] formulated a multi-attribute grey target decision-making method based on the kernel and double degree of greyness, maintaining the properties of three-parameter interval GNs. Wang and Hu [89] improved pattern classification by integrating a genetic algorithm with a multivariate grey prediction model. These developments highlight the ongoing enhancement and adaptability of GRA-based decision-making techniques to tackle diverse challenges across various domains.

GRA has found application in addressing both provincial and national issues, yielding valuable insights into various aspects of development and sustainability. For instance in China, Bao et al. [90] employed GRA to evaluate the industrial structural upgrade in Anhui province, revealing a significant shift towards a post-industrial era over a decade. Xiong and Xiong [91] utilized a driving force, pressure, status, influence, responds (DPSIP) model in conjunction with GRA to analyze ecologically sustainable development and dynamic forecasting in Heifei

Province, China, suggesting sustained growth in the province's development with a focus on sustainability. Hu et al. [92] evaluated air quality across 74 cities in China, using the grey fixed weight clustering analysis model to establish a comprehensive pollution measurement and control strategy. Liu and Cheng [93] conducted an analysis of transportation volume and GDP in China's ports from 2002 to 2017, identifying metal ore as the primary contributor to both transportation volume and GDP in China's port sector. Beyond China, Quartey-Papafio et al. [94] applied grey incidence analysis to assess the impact and control of malaria in sub-Saharan Africa from 2010 to 2017, extending the utility of GRA to address critical health challenges in other regions as well. These diverse applications demonstrate GRA's effectiveness in addressing complex and multidimensional issues at both regional and national levels.

1.5.3. Basic Principle of the Grey Systems Theory

Grey Systems Theory (GST) analyzes systems with partially known information or incomplete information, which is between the category of data that is known information and unknown information consisting of elements, structure, boundary, movement data. A real system has uncertain data as input that amount to grey output that is processed through a whitening operation. This operation aims to reduce uncertainty and enhance clarity, resulting in white output, which represents a clearer and more usable set of information. This process is illustrated in Figure 1.1 and is anchored on the six primary principles of GST enumerated by as Deng Julong's [61] which is as follows.

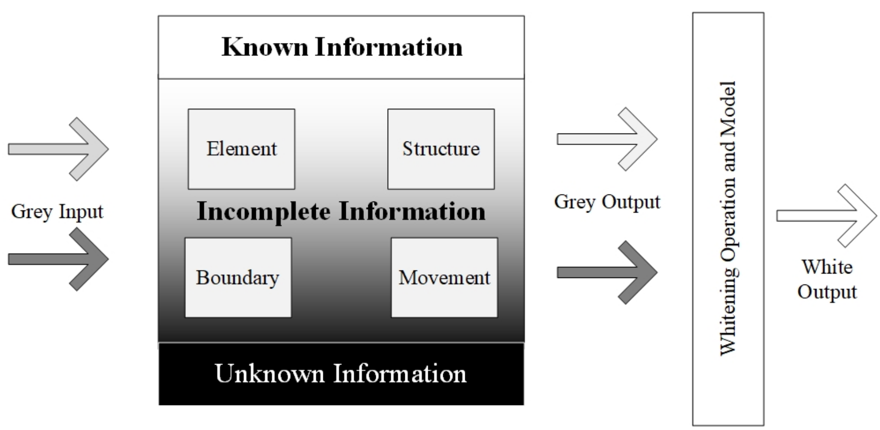


Figure 1.1. Concept of grey system. Source: Figure by authors.

The Principle of Informational Differences

Indeed, information is rooted in the existence of differences. When we perceive disparities between objects or concepts, it signifies the presence of distinct information. In cases where two objects, X and Y, exhibit dissimilarities, it implies that specific information pertaining to X is absent in Y. This principle extends across various domains, encompassing the natural world, scientific inquiries, and business decision-making. For instance, the element of time can introduce differences in information. Consider a scenario where we acquire information, "q", that reshapes our comprehension of a complex business decision. This new piece of information is inherently distinct from our initial understanding of the business problem. In the realm of scientific progress applied to business, the insights generated furnish us with a unique set of information, augmenting our understanding of the intricate business environment. Therefore, the essence of information lies in its capacity to illuminate dissimilarities, facilitating enhanced comprehension and informed decision-making.

The Principle of Non-Uniqueness

The principle that solutions to problems with incomplete or indeterminate information are not unique is a fundamental concept in the application of Grey Systems Theory (GST). It allows for a flexible perspective on problems and enhances problem-solving effectiveness. This flexibility is exemplified through the lens of a grey target. For instance, consider two companies with different approaches to their market goals. Company A aims to provide goods and services to a specific market, while Company B is open to serving multiple markets suitable for their products. Company B has a higher chance of success because there are various combinations of approaches available to achieve its goals. This principle underscores the idea that information can be gleaned through a combination of qualitative and quantitative approaches, providing a versatile toolkit for addressing complex problems characterized by uncertain or incomplete information.

Principle of Minimal Information

A notable characteristic of GST is its capacity to make the most of minimal available information. This approach stands in contrast to probability and statistical methods, which often demand large datasets that conform to strict statistical requirements, such as specified data distributions like normal or uniform. Consider, for instance, the decision-making process for investments. While these may be made in a continent with a wide diversity of countries and regions, GST allows for investment decisions to be made using the available information, even if it is limited. In contrast, probability and statistical approaches necessitate data that adhere to stringent statistical sample sizes and distribution criteria. If these requirements are not met, these methods may not yield acceptable conclusions. GST's distinctive feature lies in its ability to employ little information with a lot of logic, emphasizing that valuable insights and decisions can be extracted even from limited or incomplete information, making it a valuable tool in scenarios where comprehensive data are not readily accessible or feasible to obtain.

Principle of Recognition Base

Information forms the bedrock of human understanding and recognition. This principle underscores the fundamental notion that all forms of recognition rely on information. In the context of business, information serves as the cornerstone for comprehending employees, customers, providing goods and services, and gaining insights into competitive landscapes. Without access to relevant information, businesses would be severely hindered in their ability to function effectively.

Incomplete and uncertain information, while valuable in its own right, can only provide a grey and indeterminate perspective on various phenomena. In essence, when businesses operate with incomplete and non-deterministic information, their understanding and recognition of the situations they encounter are likewise limited, uncertain, and indeterminate—akin to the grey areas in Grey Systems Theory. Thus, the quality and availability of information are pivotal factors in shaping the depth and accuracy of business understanding and decision-making processes.

Principle of New Information Priority

The principle that new information holds greater relevance and utility than old information underscores the dynamic nature of information, where the recency of data carries significant weight. In practical terms, this principle advocates that newer information should replace or be given more importance than older information. Consider the evaluation of two major carbonated drink companies, Pepsi and The Coca-Cola Company. While past performance data are certainly valuable, the principle of new information priority emphasizes that current performance data should be accorded greater weight in decision-making. This recognition of the temporal sensitivity of information aligns with the idea that today's business landscape is evolving rapidly, as evidenced by the changes brought about by the global pandemic. Some companies have faced bankruptcy, altering their positions and prospects in the market. In such uncertain environments, prioritizing newer information by assigning additional weight to it enables more informed and timely business decisions. This principle, "the new replaces the old", encapsulates the essence of prioritizing new information and reflects the importance of staying up-to-date in decision-making processes.

Principle of Absolute Greyness

The principle of information "incompleteness" asserts the inherent and universal nature of incomplete and uncertain information. In the context of managing businesses and understanding the business environment, it is crucial to acknowledge that information is never entirely complete or certain. Managers' comprehension of the business environment is continually shaped through the ongoing process of collecting and augmenting information over time. This recognition is particularly relevant in the contemporary era, often referred to as the age of big data. Even in the face of vast amounts of data, such as those generated by social media and artificial intelligence companies, it is widely accepted that these data remain incomplete. For instance, consider companies profiting from the analysis of human DNA. Despite accumulating extensive databases of genetic information, they can never claim to possess the entirety of human DNA. They continually require more data to refine their decisions and insights. This perpetual cycle of information collection bestows an enduring quality of "greyness" upon business understanding—an acknowledgment that the completeness of information is an unattainable ideal. In essence, the principle of information "incompleteness" underscores that greyness is an inherent and perpetual aspect of business operations and understanding, reflecting the ever-evolving nature of information in this dynamic landscape.

1.5.4. Interval Grey Numbers and Operations

A Grey Number (GN) is an unknown number with information of a known range of the exact number. There are six types of GNs, which are as follows: GNs with only a lower bound, GNs with only an upper bound, interval GNs, continuous and discrete GNs, black and white numbers, essential and non-essential GNs. The interval GNs are used in this book [95,96]. Mathematically, let $\otimes\alpha = [\underline{\alpha}, \overline{\alpha}]$ and $\otimes\beta = \left[\underline{\beta}, \overline{\beta}\right]$, where $\underline{\alpha} < \overline{\alpha}$ and $\underline{\beta} < \overline{\beta}$. Some basic operations of GNs are as follows:

1. Addition:
$$\otimes \alpha + \otimes\beta = \left[\underline{\alpha} + \underline{\beta}, \overline{\alpha} + \overline{\beta}\right], \tag{1.1}$$
2. Subtraction:
$$\otimes \alpha - \otimes\beta = \left[\underline{\alpha} - \overline{\beta}, \overline{\alpha} - \underline{\beta}\right], \tag{1.2}$$
3. Multiplication:
$$\begin{aligned} \otimes\alpha \times \otimes\beta &= [\underline{\alpha}, \overline{\alpha}] \times \left[\underline{\beta}, \overline{\beta}\right] \\ &= \left[\min\left(\underline{\alpha}\underline{\beta}, \underline{\alpha}\overline{\beta}, \overline{\alpha}\underline{\beta}, \overline{\alpha}\overline{\beta}\right), \max\left(\underline{\alpha}\underline{\beta}, \underline{\alpha}\overline{\beta}, \overline{\alpha}\underline{\beta}, \overline{\alpha}\overline{\beta}\right)\right]. \end{aligned} \tag{1.3}$$

   If δ is a white value, i.e., a real number or a crisp value, the product of a crisp value and grey value is the following: $\delta \times \otimes\alpha = [\delta\underline{\alpha}, \delta\overline{\alpha}]$.
4. Division:
$$\otimes \alpha \div \otimes\beta = [\underline{\alpha}, \overline{\alpha}] \times \left[\frac{1}{\underline{\beta}}, \frac{1}{\overline{\beta}}\right]. \tag{1.4}$$

The distance between two arbitrary interval numbers from to is the following [97]:

$$|\otimes\alpha - \otimes\beta| = \max\left(\left|\underline{\alpha} - \underline{\beta}\right|, \left|\overline{\alpha} - \overline{\beta}\right|\right). \tag{1.5}$$

The GRA method using GNs offers a valuable approach for aggregating the weights assigned by a group of experts. These aggregated weights play a pivotal role in the evaluation of the business environment. In the realm of human reasoning, decision-making processes frequently involve factors and data that are not inherently crisp or precisely defined. This holds especially true when measuring preferences, as the judgments made by decision-makers often exhibit a degree of vagueness due to the limited or imprecise information available. In contemporary decision-making, linguistic values have gained significant prominence. They are applied in various systems that necessitate the measurement of decision-makers' preferences expressed in words rather than precise numerical values. In the context of this book, interval GNs serve as a valuable tool for quantifying and measuring the preferences articulated by the decision-makers. By employing interval GNs, this research harnesses a flexible and nuanced approach to capturing the preferences of decision-makers, thereby enhancing the accuracy and applicability of the GRA method in evaluating the complex and multifaceted business environment.

In the realm of system engineering, different types of systems are often used as metaphors to describe the extent of our knowledge about a system. These metaphors help convey the idea of varying degrees of information or transparency within a system. For instance, the black box term describes a system where we have very little or no knowledge about what is inside. This is like a closed container where the internal workings are hidden from view. In this context, it represents a system with extremely limited information. On the other end of the spectrum, a white system is one for which we have perfect or complete information. This is like an open design where every detail is known. Such systems are characterized by full transparency.

Most importantly, a grey system, as the name suggests, falls in between black and white. In a grey system, we have some information about its inner workings, but not all. There are aspects of the system that remain unknown or uncertain. This is akin to having a partially filled book where some chapters are complete, and others are blank or incomplete. The concept of a grey system reflects the reality of many real-world problems. In practice, it is quite common to encounter situations where we have access to certain information but lack a comprehensive understanding of all relevant factors. Grey systems represent these scenarios where some information is known, some is unknown, and there is inherent uncertainty. In essence, it is a way of mathematically modeling and addressing real-life problems that exist within this spectrum of incomplete information.

Grey Systems Theory (GST) employs a mathematical representation through the use of GNs, with interval GNs being a key type utilized in this research. It is important to differentiate between interval numbers and interval GNs. While an interval number encompasses all values within the bounds of a given range, an interval GN represents a single, uncertain value within that interval. In essence, an interval GN is a specific point within a range of certain values. This distinction is crucial when applying Grey Systems Theory (GST), as it enables the handling of uncertainty in MCDM problems under uncertainty. Grey Systems Theory (GST)'s versatility and utilization of interval GNs make it a valuable tool for navigating complex real-world scenarios with incomplete information.

1.6. Outline

This book is dedicated to addressing the central challenge of making informed business decisions within an environment characterized by uncertainty, employing MCDM methodologies. It is structured into four key sections, with the initial two chapters falling within the first part. The second and third parts are grey weighting methods and grey evaluation methods, respectively. The last part includes Chapters 11 and 12, as shown in Figure 1.2.

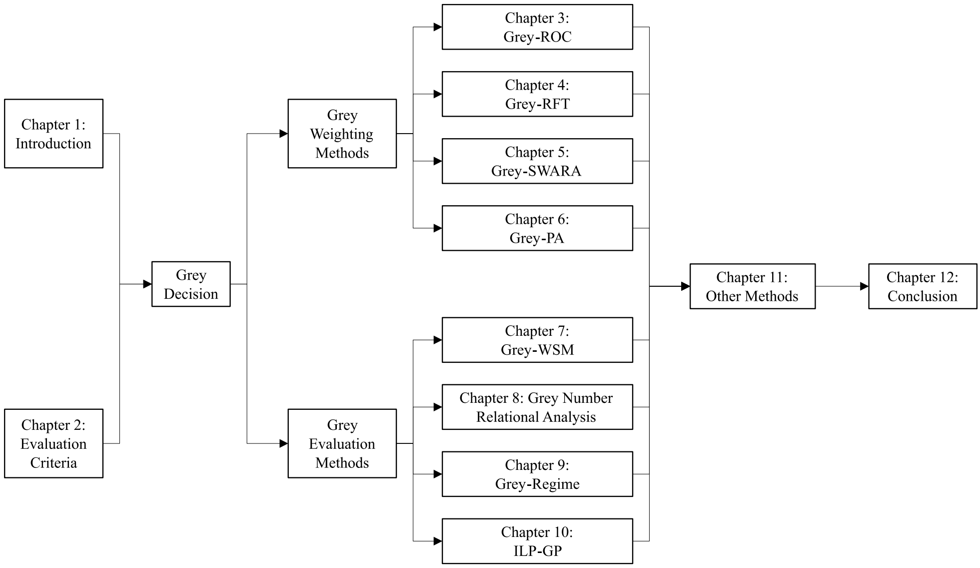


Figure 1.2. Chapter structure. Source: Figure by authors.

In detail, the part first introduces Grey Systems Theory (GST) and explains the evaluation criteria used in the entire book. Chapter 1 serves as an introduction to the book, shedding light on the role of MCDM in practical business applications and emphasizing the fundamental principle of Grey Systems Theory in decision-making under uncertainty. Chapter 2 delves deeper into the subject matter by elucidating the specific criteria employed for the evaluation of the business environment. These criteria encompass a diverse range, including Human Resource Information System (HRIS) indicators, Contractor Selection Indicators, the scaling of foreign service premium in compensation and benefits within the sphere of human resource management, and the critical aspect of university reputation. These criteria form the cornerstone upon which subsequent chapters in this book are built, providing a robust foundation for the exploration of MCDM in various business contexts (Figure 1.3).

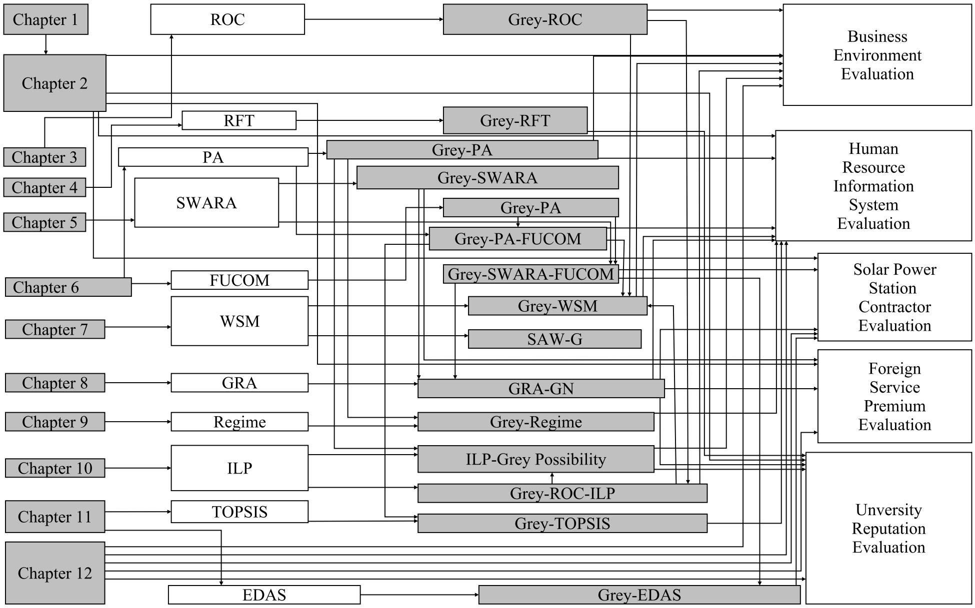


Figure 1.3. Flowchart of grey MCDM methods and applications. Source: Figure by authors.

The second part of this comprehensive book, spanning from Chapter 3 to Chapter 6, focuses on the crucial aspect of weighting methods integrated into MCDM evaluation techniques. This section unfolds as follows: Chapter 3 introduces the Grey ROC method, a framework for allocating weights to the criteria involved in assessing the business environment [98]. It also presents a streamlined evaluation model for business environment appraisal, leveraging the Partial Least Squares (PLS) methodology implemented through specialized PLS software [99]. Chapter 4 delves into the Grey Regulatory Focus Theory (GRFT), an innovative approach considering decision-makers' perspectives from both the promotion and prevention points of view [100]. This theory finds practical application in evaluating university reputation, with an exploration of how this concept extends to assessing the reputations of commercial enterprises. Chapter 5 extends the widely used SWARA method into the realm of GST. This extension takes two forms: firstly, the incorporation of GNs as input variables into the SWARA method, and secondly, the utilization of the FUCOM method to account for uncertainty in weighting methods. Both the grey-SWARA and grey-FUCOM-SWARA methods are applied to practical scenarios, including the scaling of foreign service premium allowances in human resource management and the complex decision of contractor selection for a solar panel installation project [101,102]. Chapter 6 revisits the conventional point allocation method for assigning weights, enriching it with the principles of grey systems theory. The resulting grey point allocation-FUCOM methods represent a hybrid approach that combines straightforward weighting methods with more advanced techniques to capture decision-makers' opinions effectively. The chapter provides a practical illustration of this approach by applying it to the selection of a Human Resource Information System [103].

In the third part of this comprehensive book, spanning from Chapter 7 to Chapter 10, the focus shifts towards the application of grey MCDM evaluation methods. This section unfolds as follows: Chapter 7 introduces two distinct models for evaluating business environments. Firstly, it presents the weighted sum model based on the principle of prioritizing new information. Secondly, the chapter explores the grey weighted sum model, designed to overcome the limitations of the simple additive weighting method with GNs (SAW-G) [98]. These models are then applied to assess the business environment in Africa, integrating the grey-ROC weighting method introduced in Chapter 3. Chapter 8 delves into the Grey Number Relational Analysis Method. This method builds upon classical grey relational analysis but incorporates GNs as input values. It is employed to evaluate not only business environments but also Human Resource Information Systems from multiple vendors, showcasing its versatility. Chapter 9 extends the principles of Grey Systems Theory to the classical regime method. This extension enriches the evaluation of Human Resource Information Systems (HRISs), offering a fresh perspective on assessing complex HRIS implementations. Chapter 10 presents the Grey Integral Linear Programming Approach, incorporating the concept of grey possibility. This method improves both weighting and evaluation procedures and finds application in the evaluation of university reputation and the assessment of business environments [104].

The concluding part of this insightful book, represented by Chapters 11 and 12, serves to consolidate the knowledge and methodologies discussed thus far, leading to a comprehensive understanding and practical application of Grey MCDM methods: Chapter 11 sheds light on additional grey MCDM methods, including grey TOPSIS, grey EDAS, and SAW-G. These methods find practical application in the evaluation of contractors and the selection of Human Resource Information System (HRIS) software. The chapter underscores the versatility of Grey MCDM techniques across various domains. Chapter 12 takes a step back to provide a comprehensive discussion of all the techniques presented throughout the book. It offers a valuable comparison of these methods and conducts sensitivity analyses to showcase their relative strengths and limitations. Furthermore, this chapter addresses the constraints of the presented methods and offers a glimpse into potential avenues for future research and real-world application. In essence, this concluding section ties together the diverse grey MCDM approaches discussed in the book, equipping readers with a robust toolkit for tackling complex decision-making challenges in an uncertain business landscape.

The presentation of the outline and simplified flowchart in subsequent chapters guides the readers through an exploration of this intriguing and practical application of Grey Systems Theory in the realm of implementing decision-making tools.

2. Evaluation Criteria of Applications

This chapter serves as the foundational cornerstone for the subsequent chapters by delineating and defining the evaluation criteria for the five central cases examined throughout the book. These cases span a spectrum of topics, encompassing the evaluation of a nation's business environment, the selection of a Human Resource Information System (HRIS), the choice of a solar panel installation contractor, the scaling of foreign service premium, and the assessment of university reputation. Each case entails its unique criteria and sub-criteria, with the chapter's systematic approach breaking down complex decision problems into manageable components while retaining a holistic perspective. This structure ensures that readers comprehensively grasp the intricacies of each case and the application of grey Multiple Criteria Decision-Making (MCDM) methods to address them, as depicted in Figure 2.1 of the book's outline.

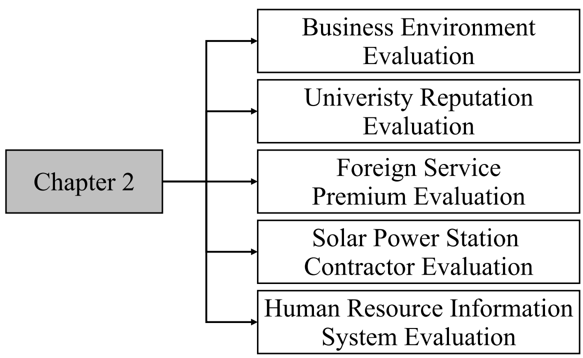


Figure 2.1. Flowchart of Chapter 2. Source: Figure by authors.

2.1. Business Environment Indicators

The World Bank's Doing Business Project (DBP) stands out among various indicators and indexes used to assess business environments due to its global coverage, encompassing all countries. Moreover, what sets it apart even further is the unique characteristic that the data collected for DBP are publicly accessible. The DBP sets the benchmark for evaluating business conditions in a country and serves as a significant influence on the business environment, based on the criteria outlined within DBP. These standards are instrumental in providing valuable insights and comparisons for businesses and policymakers. It is important to note that while the World Bank's Doing Business Project (DBP) may have been discontinued in 2020, the conceptual foundations and principles it introduced remain relevant and applicable in today's business environment. These principles continue to offer valuable insights and guidance for assessing and improving business conditions, even in the absence of the DBP itself.

2.1.1. Starting a Business (A_1)

This indicator is used to measure the processes of starting a business in a country and focuses on several key factors, including the number of procedures, time, costs, and the minimum paid-in capital required to establish a local limited liability company with the assumption of commencing operations with a minimum of ten employees. This indicator provides valuable insights into the ease and efficiency of the business startup process within a country, reflecting the steps and resources required to initiate such a venture. It serves as a useful benchmark for evaluating the business environment and regulatory conditions in different countries and across countries, enabling comparisons and assessments of the startup procedures and associated costs [105].

Procedures ($A_{1\text{-}1}$)

The *Procedures* component of the indicator measures the number of interactions required between the founders of a company and external parties during the process of starting a business in a country. These interactions involve engagement with various entities such as lawyers, auditors, and government agencies. It is important to note that internal interactions within the company are not included in this count. If there are multiple interactions on the same day in the same building but in different sequences, each of these is counted as a separate procedure. The count encompasses all procedures that founders must complete themselves, without the involvement of third parties, unless such third-party involvement is mandated by law. Procedures carried out by professionals on behalf of the company are also included in the count, and electronic procedures are counted separately. If there are multiple procedures conducted through the same website, each is counted individually. Additionally, any shortcut procedures that are considered common practice and available to the public are included in the count. However, if not using such shortcuts would result in a substantial delay, they are not counted. Procedures related to obtaining water, electricity, and complying with environmental requirements are not included in this count.

Time ($A_{1\text{-}2}$)

The *Time* component of the indicator measures the median number of days it takes, as reported by an incorporation lawyer, to complete a procedure necessary for starting a business in a country. This measurement considers the standard completion time with no additional fees or special supervision. The minimum duration for a procedure is counted as one day, while an online procedure is recorded as taking half a day. When multiple procedures are conducted simultaneously, they are counted as different days, except in the case of online procedures. It is important to note that a procedure is considered complete only after the company has received the final incorporation document, such as a registration certificate or tax number. If a process can be expedited by paying a fee, the shortest time it can be completed is used for the country's measurement. This measurement assumes that employees involved in the procedures do not waste time, and it does not account for time spent gathering information.

Cost ($A_{1\text{-}3}$)

The *Cost* component of the indicator represents the percentage of the economy's income per capita required to cover all fees associated with starting a business in a country. These fees encompass charges for professional and legal services, including any costs related to purchasing and legalizing company books if mandated by law. The data sources for these calculations are derived from the national company law, commercial code, and relevant regulations of the country in question. It is important to note that value-added tax (VAT) is excluded from these fee calculations. In cases where government estimates for these charges are unavailable, the indicator uses the median estimate provided by multiple incorporation lawyers.

Paid-in Minimum Capital ($A_{1\text{-}4}$)

The *Paid-in Minimum Capital* component of the indicator represents the minimum amount that a company is obliged to deposit in either a bank or a notary, as stipulated by the commercial code or company law of the country. In cases where certain countries permit partial payment of the required paid-in minimum capital for registration, the amount of partial payment is duly recorded. Any legal restrictions on the organization's activities or decisions associated with meeting the minimum capital requirement are also documented. When the legal minimum capital is specified per share, it is assumed that the company is owned by five investors, and the legal minimum capital is multiplied by five shares for the purpose of the calculation.

2.1.2. Dealing with Construction Permits (A_2)

This indicator measures the difficultly of obtaining permits for constructing a warehouse in a country. It measures the estimated number of procedures, time, and costs that a construction company, fully licensed and insured for construction projects, would need to complete the construction of a warehouse. Additionally, it is assumed that the construction company has a registered architect or engineer as one of its employees to oversee the project. This indicator provides insights into the ease and efficiency of undertaking construction projects within a given country, taking into consideration the regulatory and procedural aspects involved.

Procedures ($A_{2\text{-}1}$)

The *Procedures* in the construction of a warehouse indicator counts the number of interactions a company engaged in constructing a warehouse has with external parties. These external parties may include government agencies, the land registry, utility companies, and architectural firms, as well as public and private inspectors. The count also encompasses procedures related to obtaining water and sewerage services. However, interactions among the company's internal employees involved in the construction project are not included in this count. It aims to quantify the various external steps and interactions involved in the construction process, reflecting the regulatory and administrative complexities associated with construction projects in a given country.

Time ($A_{2\text{-}2}$)

The *Time* in the construction of a warehouse indicator represents the median number of days that local experts estimate as the time required to complete a specific procedure for constructing a warehouse in a given country. This time measurement considers the median duration provided by local experts and is used to understand the typical timeline associated with various procedures. For most procedures, a single day is the minimum duration considered, except for online procedures, which are recorded as half a day. Procedures carried out simultaneously are recorded separately unless they are completed online. The recorded time is the fastest legal procedure, assuming that it adheres to usual practices and regulations, even if it incurs an extra cost (but not bribes). The measurement does not include time spent gathering information, and it assumes that the company follows all procedures promptly without delays. This indicator helps assess the efficiency and timeliness of construction-related procedures in a particular country.

Cost ($A_{2\text{-}3}$)

The *Cost* indicator in the construction of a warehouse metric quantifies the financial burden associated with building a warehouse in a specific country, expressed as a percentage of the total value of the warehouse construction project. This cost factor encompasses all the official fees and expenses required to complete the necessary procedures and fulfill the legal requirements for warehouse construction. It covers a range of expenditures, including those related to land permits, licenses, pre-construction design clearances, construction inspections, utility connections, property registration, and any applicable construction-related taxes. The data sources for calculating this cost typically include the building codes and regulations of the relevant country, as well as insights provided by local experts. By evaluating construction costs in relation to the total project value, this indicator offers valuable insights for businesses and investors assessing the financial feasibility of warehouse construction projects across different regions.

2.1.3. Getting Electricity (A_3)

The *Getting Electricity* indicator pertains to the process of securing a standard electricity supply for a warehouse in a given country. It encompasses several key aspects, including the number of procedures required, the time invested, and the associated costs. These procedures typically involve activities such as arranging inspections and contracts for utility bills. It is important to note that this indicator specifically addresses the establishment of a permanent electrical connection with a single electricity meter. Evaluating these factors provides valuable insights for businesses and investors looking to assess the efficiency and affordability of obtaining electricity for warehouse operations in various locations.

Procedures ($A_{3\text{-}1}$)

The *Procedures* indicator focuses on quantifying the interactions and procedures required to obtain a standard electricity supply for a warehouse. It encompasses the various engagements that involve company employees, electricians, or electrical engineers interacting with external entities such as electricity distribution companies, electrical contractors, and government agencies. These interactions encompass multiple steps, including the submission of necessary documents, the acquisition of installation permits, the completion of notifications for inspection receipt, the procurement of materials for installation, and the signing of supply contracts. Each distinct procedure within these interactions is counted separately, contributing to a comprehensive assessment of the process involved in obtaining electricity supply for the warehouse.

Time ($A_{3\text{-}2}$)

The *Time* component focuses on quantifying the number of days required, either by legal mandate or practical norms, to complete the procedures outlined in obtaining a standard electricity supply for a warehouse. This measurement is based on the premise that there is no prior interaction with the electricity distribution company and that minimal or no follow-up or additional payments are involved in expediting the process. The shortest duration for a procedure is recorded as one day, under the assumption that no time is wasted during the process, and the steps involved are well known, excluding time spent on information gathering from the calculation.

Costs ($A_{3\text{-}3}$)

The *Costs* component of the *Getting Electricity* indicator comprehensively considers all the fees associated with completing the procedures necessary to secure electricity supply for a warehouse. This cost encompasses expenses related to obtaining government clearances, applying for connection and undergoing inspection, and procuring the necessary installation materials and meters. Notably, value-added tax (VAT) and any bribes are excluded from the cost calculation.

2.1.4. Registering Property (A_4)

This indicator assesses the number of *Procedures*, *Time*, and *Costs* associated with a business entity's acquisition of property from another business or the transfer of ownership to the buying business. This transfer of ownership bestows upon the buyer the right to either resell the property or utilize it as collateral for obtaining a loan. The process begins with the seller completing all necessary documentation and concludes when the buyer successfully secures a loan from a financial institution, leveraging the property as collateral. This evaluation assumes that the seller possesses full ownership of the property without any mortgages, that the land has no title disputes, and that no occupants need to be relocated. Additionally, it is presumed that, according to the registering property indicator, the property's selling price is of equivalent value, requiring no renovations after purchase.

Procedures ($A_{4\text{-}1}$)

The *Procedures* component of this indicator quantifies the number of interactions mandated by law or common practice involving the buyer, seller, or their respective agents. These procedures may entail actions such as notarizing the sale agreement, settling associated taxes, and submitting a title application to the city government.

It is presumed that the buyer does not employ external influence to expedite the process, although the engagement of legal counsel or agents representing the parties is permissible.

Time ($A_{4\text{-}2}$)

The *Time* component of this indicator quantifies the number of days it takes for the buyer to finalize the property acquisition procedures without incurring unnecessary delays. The assumptions regarding time include the buyer possessing prior knowledge of the procedural sequence before commencement and the exclusion of time spent on information gathering. The timing concludes upon the buyer's receipt of the registration documents.

Costs ($A_{4\text{-}3}$)

The *Costs* aspect of this indicator is calculated as a percentage of the property's value. It specifically accounts for the official expenses involved in the process, encompassing fees, transfer taxes, duties stamps, and services rendered by professionals such as notaries, public agencies, or lawyers. This cost estimation excludes factors like capital gains or value-added tax (VAT).

2.1.5. Getting Credit (A_5)

This indicator evaluates the legal rights of both borrowers and lenders. It encompasses secured transactions and the sharing of credit information, assessing the Legal Rights Index, Credit Information Index, coverage of public credit registries, and private credit bureaus. This indicator aims to determine the feasibility of lending with collateral and the extent of bankruptcy law coverage. It also gauges the accessibility of credit-related information through credit bureaus and registries. The assumption is that both the lender and borrower are domestic entities with their headquarters located in the largest business city. The data used for this indicator are verified through site visits or teleconference calls, ensuring the accuracy and reliability of the information [106].

Legal Right Index ($A_{5\text{-}1}$)

This indicator assesses the protection of rights for borrowers and lenders, primarily focusing on collateral and bankruptcy laws. The index is derived from surveys conducted among financial lawyers, and the collected data are subsequently verified using publicly available sources. The gathered information is then subjected to in-depth analysis to evaluate the effectiveness and comprehensiveness of these legal provisions, ensuring the safeguarding of both borrowers' and lenders' rights in the context of accessing credit [106].

Credit Information Index ($A_{5\text{-}2}$)

The assessment of credit reporting services involves conducting surveys targeting both banking supervision authorities and the general public. This comprehensive evaluation includes various aspects of credit information, such as the distribution of data related to firms and individuals, the presence of positive and negative credit information, data from utility and retailers companies, the availability of historical credit information, the distribution of data concerning borrowers with amounts below 1% of their income per capita, the legal rights of users to access their data, the accessibility of borrowers' credit information online, and the assistance provided by credit bureaus or registries to users seeking information about lenders' creditworthiness. These factors are systematically analyzed to determine the effectiveness and availability of credit information in a given country [106].

Public Credit Registry Coverage ($A_{5\text{-}3}$)

The coverage of public credit registries is calculated as the percentage of the adult population listed in the registry's database over the past five years. These credit registries typically maintain databases managed by central banks or supervisory authorities, enabling the sharing and supervision of credit information among various financial institutions. The indicator assesses the extent to which a country's adult population is included in this credit database, providing valuable insights into the availability and reach of credit information within the financial system [106].

Private Credit Bureau Coverage ($A_{5\text{-}4}$)

The coverage of private credit bureaus is calculated as the percentage of the adult population listed in the credit bureau's database over the past five years. These credit bureaus are typically private or non-profit organizations that maintain databases containing credit information from both firms and individuals. They also play a crucial role in facilitating the exchange of credit information among various banks and financial institutions. This indicator assesses the extent to which a country's adult population is covered by private credit bureaus, providing insights into the availability and accessibility of credit information within the financial sector [106].

2.1.6. Protecting Investors (A_6)

This indicator assesses the strength of minority shareholders through focusing on the prevention of directors' misuse of corporate assets for personal gain. It measures the transparency of related-party transactions, liability for self-dealing, and shareholders' ability to take legal action against officers and directors for any misconduct. This assessment is based on several sub-indices, including the disclosure index, director liability index, and shareholder suit index. The indicator is designed to provide insights into the legal protections and mechanisms in place to safeguard the interests of minority shareholders within a country's corporate governance framework. It assumes that the Chief Executive Officer (CEO) can act on behalf of investors, even when it is legally mandatory, and that manufacturing companies own their distribution networks [107].

Disclosure Index ($A_{6\text{-}1}$)

This index evaluates and assesses various aspects related to corporate disclosure and transparency. It focuses on the requirements for disclosing related-party transactions and the release of internal, immediate, and periodic transaction information. It also examines whether information about shareholders with 10% or more ownership can be disclosed. Additionally, the index looks into the disclosure of compensation provided to managers, including whether the annual financial statements of the business enterprise undergo external auditing, whether financial statements are distributed with explanatory notes and trend figures, and whether reports containing information on risks and uncertainties are publicly released. These factors collectively contribute to evaluating the transparency and openness of a country's corporate disclosure practices.

Director Liability Index ($A_{6\text{-}2}$)

This index assesses the extent to which minority investors have the legal means and institutions to hold business directors accountable for any detrimental related-party transactions. It measures whether investors can legally pursue charges against a CEO for transactions resulting from negligence or perceived unfairness. Furthermore, it evaluates whether the court has the authority to nullify a transaction if the investor's claim is deemed valid. Additionally, the index considers whether the CEO is obligated to provide compensation for damages if the investor's claim proves successful, including the possibility of fines or legal penalties in such cases. In essence, it gauges the legal protections and avenues available to minority investors to safeguard their interests in corporate transactions.

Shareholder Suit Index ($A_{6\text{-}3}$)

This index evaluates the accessibility of internal corporate documents during legal proceedings and examines the allocation of legal expenses. It assesses whether shareholders with less than a 10% stake possess the legal authority to convene meetings and potentially replace board members during their terms. It also considers the approvals required before new shares can be issued and whether existing shares can be traded before major shareholder meetings. Additionally, the index measures the rights of shareholders in choosing an external auditor. In summary, it examines the legal provisions that empower minority shareholders and influence corporate governance practices.

2.1.7. Paying Taxes (A_7)

This indicator evaluates the annual tax obligations associated with standard business operations, encompassing payments, time expended, and taxes such as corporate income tax, labour tax, social contributions, and other mandatory government levies. It considers both regular and advance tax payments. The assumption is that the company is engaged in typical business activities and does not operate in sectors subject to special taxes, such as liquor or tobacco production. Additionally, the company does not benefit from tax breaks or incentives, as might be available to industries like solar panel manufacturing [107,108].

Payments ($A_{7\text{-}1}$)

This indicator assesses the complexity of tax compliance for a company during its second year of operation, considering factors such as the total number of taxes and contributions paid, the frequency of tax filing, and the number of different agencies involved in the process. The taxes covered include consumption tax, value-added tax (VAT), sales tax, and goods and services tax, among others. Taxes are considered distinct when they are paid under different names or through different agents, and they are counted separately. This assessment includes all taxes paid by the company, including those collected from employees, such as labour taxes and sales tax. It also takes into account online payments and taxes handled by third parties, counting them once regardless of the filing frequency. Joint payments made using the same form are counted as one.

Time ($A_{7\text{-}2}$)

This indicator quantifies the annual hours spent on preparing, filing, and paying income taxes, labour taxes, payroll taxes, value-added tax (VAT), and social contributions. The time spent on tax preparation encompasses tasks related to information gathering and computation. The time for tax filing includes the hours spent on submitting all the necessary tax forms and documents. Lastly, the time for tax payment accounts for the hours required to make payments, whether in person or online.

Profit or Corporate Income Tax ($A_{7\text{-}3}$)

This is the calculated tax liability incurred when a company generates profits, typically based on a predetermined tax rate. Profit, in this context, refers to the income remaining after deducting the costs of goods sold, gross salaries, expenses, provisions, capital gains, and commercial depreciation from the total revenue generated from sales, interest income, and expenses.

Labour Tax and Contributions ($A_{7\text{-}4}$)

This represents the aggregate amount paid by employees as mandatory contributions. For example, it includes pension contributions and insurance premiums. The data encompass charges that organizations are legally obliged to pay, apply to regular business operations, and have an impact on their income statements. These charges extend beyond the scope of taxes for government records and also account for any levies that affect a company's financial statements.

Other Taxes ($A_{7\text{-}5}$)

These miscellaneous taxes can encompass a range of levies, such as vehicle taxes, municipal fees, sanitation taxes, and others.

2.1.8. Trading Across Borders (A_8)

This indicator assesses the ease of exporting and importing goods, considering the number of documents, associated costs, and the time required for international trade. It focuses on a standard 20-foot container used by a domestic company. Exportation includes processes such as packing and loading the container at the warehouse and transporting it to the port of exit, while importation covers activities from the arrival of the vessel at the port of entry to the delivery of the cargo to the warehouse [109]. It is assumed that payment is made using a letter of credit and that the goods being traded are not military or hazardous items. The cargo is a dry cargo weighing 10 tons and valued at USD 20,000, originating from a leading export economy. Data for this indicator are collected from various sources, including local freight companies, customs brokers, port authorities, and shipping lines.

Document for Export ($A_{8\text{-}1}$)/ Document for Import ($A_{8\text{-}4}$)

The documents considered in this indicator encompass a range of paperwork required for both export and import processes. These documents include those related to banking, customs clearance, inspection, port operations, terminal handling, inland transportation, and other documents necessary for exporting or importing goods across international borders. For landlocked countries, this indicator specifically covers documents required at inland borders. Additionally, it includes documents needed to obtain a certificate of origin if the country mandates such certification for trade. Importation-related documents encompass those required from the port of entry where the vessel arrives to the point of cargo delivery at the warehouse. Notably, the annual tax clearance certificate is excluded from this assessment.

Costs to Export ($A_{8\text{-}3}$)/Cost to Import ($A_{8\text{-}6}$)

The costs considered in this indicator include various expenses associated with the export and import processes, such as customs clearance, inspection, port and terminal handling, and as administrative and port-related charges. However, it is important to note that this cost assessment excludes transportation costs related to the actual movement of goods for export and import, focusing instead on the costs associated with document processing, handling, and administrative procedures.

Time for Export ($A_{8\text{-}2}$)/Time for Import ($A_{8\text{-}5}$)

The time measurement for this indicator specifically refers to the number of days it takes for a business to deliver a container to the warehouse across the border. Importantly, this time measurement excludes the time associated with the transportation of goods, focusing solely on the duration of administrative and border-related procedures. Additionally, fast-tracking services that may not be accessible to all companies are not taken into account in this time measurement.

2.1.9. Enforcing Contracts (A_9)

This indicator evaluates the efficiency of the judicial system in resolving a commercial dispute. It considers factors such as the legal framework, the time it takes to resolve a dispute, and the associated costs. The data used for this assessment are gathered from local lawyers and judges. In this context, it is assumed that the court has ruled in favor of the seller in a dispute involving a buyer and a seller. Furthermore, it is assumed that the buyer is required to make a 100% payment based on the contract. This indicator aims to provide insight into the effectiveness and efficiency of commercial dispute resolution within a given jurisdiction [110].

Procedure ($A_{9\text{-}1}$)

This indicator measures the number of procedural steps involved in the resolution of a commercial dispute, starting from the initiation of the dispute before the court's intervention. It encompasses all the steps required by law or common practice, whether undertaken by the parties involved or court officials. This includes procedures conducted by lawyers as well as those internal to the court system. The purpose of this measurement is to assess the complexity and intricacy of the dispute resolution process within a given legal framework.

Time ($A_{9\text{-}2}$)

This indicator evaluates the time required to complete various stages of the dispute resolution process, including filing and service, trial and judgment, and enforcement of the judgment. It encompasses multiple time intervals within these stages, such as the duration between filing and serving the case, the period from serving the case to holding a pre-trial conference, the time for trial activities, the judge's time to issue a written final judgment, and the time limit for appeal. Additionally, it assesses the duration needed for enforcement, which includes obtaining an enforceable copy of the judgment, locating, identifying, seizing, and transporting the losing party's movable assets, advertising, organizing, and conducting an auction, and ultimately achieving full recovery of the claim's value. The waiting period is not excluded from this assessment.

Cost ($A_{9\text{-}3}$)

This indicator assesses the cost associated with resolving a commercial dispute as a percentage of the claim. It considers the average expenses incurred, covering lawyer fees, court costs, and enforcement-related expenses, irrespective of whether the seller or buyer bears these costs. Additionally, it records the average duration of various stages of dispute resolution, including the completion of the service of process, the issuance of judgment, and the moment of payment.

2.1.10. Resolving Insolvency (A_{10})

This indicator evaluates the legal framework for handling a bankruptcy case of a business within the domestic legal system. It considers factors such as the time, costs, and outcome of the recovery rate from insolvency proceeds. Data for this indicator are sourced from insolvency practitioners, laws and regulations, and public information. The scenario assumes that the business in question is a limited liability company with a professional manager and a founder who owns 51% of the shares, along with some personal assets secured by a decade mortgage loan. In this case, the business is experiencing liquidity problems, it will default on a bank loan, and the asset value with the bank equals the outstanding loan. The assumption is that the bank aims to recover its loan quickly and at minimal cost [109].

Time ($A_{10\text{-}1}$)

This indicator measures the number of days it takes for creditors to receive some payment after initiating bankruptcy proceedings. It accounts for any delays caused by debtors, including tactics such as filing dilatory appeals or requesting extensions.

Cost ($A_{10\text{-}2}$)

This indicator measures the cost as a percentage of the borrower's estate. It includes all fees related to selling the property, insolvency administration, auctioneers, lawyers' fees, and more. This cost is calculated based on questionnaire responses.

Recovery Rate ($A_{10\text{-}3}$)

This rate is measured as the percentage of a US dollar recovered after foreclosure. It takes into account whether the foreclosure results in the establishment of a new business or if all the assets are sold individually. The rate calculation also includes any value lost due to depreciation of the assets.

2.2. Human Resource Information System Indicators

The evaluation criteria for the HRIS (Human Resource Information Management System) were primarily derived from the existing literature, including the evaluation of ERP (Enterprise Resource Planning) software as outlined by Ayağ and Özdemir [111]. Lessons presented by Carvallo et al. regarding the determination of criteria for software components were also taken into consideration [112]. These lessons emphasized the importance of balanced criteria that are neither overly narrow nor too focused, the recognition of non-technical selection criteria, precise definition of the selection framework, and consideration of the ultimate purpose of organizing software scopes. The evaluation criteria encompass 5 first-level criteria and 27 second-level criteria, which are described as follows.

2.2.1. Human Resource Management Functions (Φ_1)

This primary function represents the core role that an HRIS (Human Resource Information Management System) is designed to fulfill, aligning with the specific needs of the organization. The HR department often serves as a central support division within many organizations. It is important to note that the functions listed below do not encompass all the functions typically provided by the HR department.

Staff Information Management ($\Phi_{1\text{-}1}$)

This function encompasses the creation and management of a comprehensive employee data repository, capturing the entire lifecycle of employee records, starting from initial contact through to reports from their supervisors. It includes storing various types of information such as employee biographies, job applications, job interview records, contracts, academic and medical records, onboarding training materials, email and instant messaging records, job scheduling, and performance evaluations.

Organization Structure and labour Management ($\Phi_{1\text{-}2}$)

This function involves the software's capability to organize employees and their skills within the company's organizational structure, including various types of organograms such as hierarchy, matrix, function, product-based structures, and team-based structures. Additionally, it should provide insights into the organization's workforce strength by reporting areas that may be either under- or overstaffed to the HR manager.

Compensation and Benefits Management ($\Phi_{1\text{-}3}$)

This function involves managing the financial compensation and benefits (C&Bs) that employees receive for their work, which can include direct forms such as wages, salaries, bonuses, and commissions, as well as indirect forms like career development opportunities, recognition, a positive work environment, and favorable working conditions. The software should have the capability to integrate with the finance department for payroll management and administer benefits in compliance with labour laws governing the organization.

Staff Training Management ($\Phi_{1\text{-}4}$)

This function entails maintaining a database of training courses aimed at enhancing employees' skills and knowledge. The software should be capable of recording employee course attendance and engagement, tracking test and grading results, and managing a question bank for assessments. In addition to company-specific courses like enterprise culture, business models, and safety policies, the software should provide access to licensed training courses that can benefit the organization's employees.

Staff Recruiting Management ($\Phi_{1\text{-}5}$)

This function involves the software's capability to facilitate applicant tracking and onboarding processes for candidates looking to join the company. It encompasses managing job analysis, screening, selection of information, referral databases, and professional associations. It also involves the use of Application Programming Interfaces (APIs) for sourcing candidates through recruitment advertising agencies. The software should provide employees with the ability to refer candidates for open positions within the company and offer automatic filtering based on criteria such as years of experience, qualifications, location, and availability. Qualified candidates would receive automatic invitations, and designated staff members would be notified to conduct interviews.

Staff Performance Management ($\Phi_{1\text{-}6}$)

The HRIS should have the capability to track organizational goals through various means, including task tracking, performance appraisal, work hour tracking, reward and punishment tracking, documentation of performance plans, coaching, staff assessment, and performance measurement reporting. This may involve methods like 360-degree evaluation and balanced scorecards. The software should allow for the registration of performance measurement metrics specified in the job descriptions, as agreed upon by the employee and their supervisor. Additionally, the system should maintain records of staff members who have excelled, and these can be used as examples to motivate the team or for promotion and awards. Furthermore, the software should provide insights into employees' weaknesses and how they may contribute to the organization's overall weaknesses. In essence, the HRIS will offer a standardized approach to aligning tasks and objectives with employee performance in line with organizational goals.

2.2.2. Technology (Φ_2)

This criterion assesses the software's capability to stay up-to-date with current information technology trends and leverage them to provide additional advantages to the HRIS. The software should be able to integrate with innovative solutions to optimize the organization's HR operations, potentially enabling digital transformation within the organization. Keeping pace with emerging technologies is crucial in harnessing the latest information technology solutions to gain a competitive edge over rivals.

Big Data Analysis ($\Phi_{2\text{-}1}$)

This criterion evaluates the system's capacity to effectively manage, organize, and analyze unstructured and comprehensive organizational information. The software should be capable of uncovering hidden patterns and identifying high-dimensional correlations within the data, ultimately enhancing decision-making for HR managers. It should also provide timely responses to user queries with high efficiency, surpassing the capabilities of traditional business intelligence platforms.

Artificial Intelligence ($\Phi_{2\text{-}2}$)

AI, here, goes beyond facial recognition systems for entry into and exit from the organization, as well as logging in and out of computer systems; it should be able to analyze employees' facial expression, posture, and give recommendations for appropriate roles. Assisting in summarizing past data, it renders them into a form that can easily be read by people. The HRIS should also create executive reports, such as knowing the estimated number of employees to attain the target profit and sales. More importantly, it should identify causes of and suggest solutions to a problem. The HRIS should improve the company's quality of forecast as the HR manager makes future strategic plans.

Internet of Things ($\Phi_{2\text{-}3}$)

This is the HRIS's ability to provide sensors that can track and log employee activities that give employees their tools with unique identifiers (UIDs) or radio-frequency identification (RFID), thus providing the organization with real-time analytical information. The data generated by IOT would be fed into a big database associated with people and their related processes. This would connect the employees, managers, and HR together to allow seamless tracking of their actives within the organization. Also, this would create a more digitized work environment with improved connection between the employee and managers.

Social Network ($\Phi_{2\text{-}4}$)

This evaluation criterion assesses the HRIS's capability to facilitate informal interactions among employees and with the outside world, enhancing business networking. The system should offer a platform for employees to engage with each other and external networks through APIs for social media platforms like WeChat, Microblogs (Weibo), Facebook, and LinkedIn. The goal of these social connections is to establish a strong link between employees and the organization, enabling HR managers to gain deeper insights into employee engagement, collabouration efforts, and their impact on public perception in HR forums. This also contributes to the company's branding by increasing its visibility in the digital space through external platforms. Additionally, it streamlines HR functions such as recruitment by simplifying job postings, resume acquisition, and providing a comprehensive view of candidates before interviews.

Self-Help Service ($\Phi_{2\text{-}5}$)

This criterion evaluates the HRIS's capability to offer an employee self-service system that minimizes HR manager interaction by automating responses to employee inquiries, such as leave applications and extensions. Employee self-service empowers employees to update their information, take more control over their tasks, and reduce the HR manager's workload. It efficiently records employees' work hours, crucial for accurately computing payments, especially for hourly paid positions. The system should also provide access to company policies and user manuals, decreasing errors by allowing employees to input their personal information. Additionally, the HRIS should handle travel expenses and reimbursements, streamlining the process and improving efficiency.

2.2.3. Software Quality (Φ_3)

This criterion assesses the relative standard of quality that the HRIS provides, considering both functional and non-functional requirements [113]. It evaluates how well the software fulfills its intended functions and addresses the needs of end-users. Quality can be evaluated from two perspectives: software defect management and software quality attributes. Software defects are instances where the software fails to meet user requirements, which can result from design flaws, misunderstood requirements, or coding errors. Quality attributes are considered based on the ISO/IEC 25010:2011 model, which provides a comprehensive framework for evaluating software quality.

Functionality ($\Phi_{3\text{-}1}$)

This criterion evaluates the extent to which the HRIS software aligns with the design requirements of the organization, reflecting its suitability [112,114]. It assesses the software's quality in terms of how well it meets both functional and non-functional requirements. Specifically, it examines whether or not the software fulfills its intended functions and how effectively it does so. This evaluation aims to determine how well the software aligns with the organization's needs and design specifications.

Reliability ($\Phi_{3\text{-}2}$)

This criterion assesses the organization's ability to trust the HRIS software and the information it manages, including its software architecture [111,112,115,116]. It can be measured by the probability of the HRIS operating on the company premises without failure. Reliability also encompasses availability, which is the probability that the system provides services upon request. Availability considers factors such as the number of employees affected and the duration of any system outages. For example, a multinational company should have a HRIS that operates 24/7 to serve staff worldwide. This evaluation also takes into account the HR manager's perception of the software's reliability and any deviations from software specifications as indicators of failure.

Usability ($\Phi_{3\text{-}3}$)

HRIS usability refers to the ease of using the graphical and web-based user interfaces. It aims to limit the need for expert systems for HR managers on the backend while providing an intuitive front end that is user-friendly [111,114,116,117]. Usability should align with the needs and preferences of HR managers and employees, and it should also be in line with the organization's culture and privacy considerations. Striking a balance between the usability and utility of the HRIS is crucial. Employees overwhelmed by functions may have to undergo a relative steep learning curve, so the software should be user-friendly and familiar, encouraging its usability. The time it takes for users to master the software should be reasonable, with features easily discoverable within the HRIS interface.

Efficiency ($\Phi_{3\text{-}4}$)

The HRIS should significantly reduce the time required to complete HR tasks while ensuring they are carried out correctly, ultimately enhancing the overall organizational performance [112,117]. From a technical perspective, the HRIS should utilize computer resources such as memory, Central Processing Unit (CPU), and network bandwidth efficiently to operate effectively. Efficiency, as described by Peter Drucker, is the capacity to do things right, while effectiveness is the capacity to do the right thing. This principle is equally applicable to HRIS. Efficiency can be compromised when there are unnecessary requests for information that should logically be known from previous data stored in the HRIS. For example, the HRIS should be aware of national public holidays and should have records of employee default supervisors, eliminating the need for repeated requests for this information.

Maintainability ($\Phi_{3\text{-}5}$)

Maintainability refers to the ease of maintaining the HRIS, ensuring it remains operational with reliable backup systems that can minimize downtime and prevent unplanned data duplication [111,116]. It also encompasses the ability to extend the lifespan of the HRIS by correcting errors and adapting to changes within the organization over time. After installation, the HRIS provider should be capable of providing service support, addressing and correcting defects recorded in the HRIS logs. Additionally, the provider should be able to diagnose the root cause of any issues and provide solutions to resolve them. This ensures that the HRIS remains functional and effective throughout its lifecycle.

Portability ($\Phi_{3\text{-}6}$)

This refers to the availability of the company's complete HR data to be exported and imported into different software systems without data corruption or loss [111,115,117,118]. This data portability can take various forms, including direct portability, installability, replaceability, and adaptability. Installability involves ensuring that the HRIS is easy to deploy and install. Even if HR managers are not experts in software installation, the process should be transparent and manageable by the IT department. It should not be a black box. Replaceability is the aspect that focuses on avoiding vendor lock-in, allowing the organization to replace the HRIS with alternative solutions if needed. It ensures flexibility in choosing and transitioning to different HRIS platforms. Adaptability involves designing the HRIS with a user-centric approach, ensuring that it can be effectively used on various devices, including mobile screens and computer monitors. It should provide a seamless user experience across different platforms.

2.2.4. Cost (Φ_4)

This refers to the total cost incurred in setting up and maintaining the HRIS software [111,119]. Reducing the total cost of ownership (TCO) is a fundamental concern for organizations. It is essential to note that minimizing costs and maximizing quality and features can be conflicting objectives. To reduce costs, organizations may consider open-source solutions, which can lower initial expenses but may require professional support. Additionally, customization and building additional modules to meet specific feature requirements can increase the overall cost of the HRIS. In summary, managing the TCO of the HRIS involves striking a balance between cost reduction, feature enhancement, and maintaining software quality and performance.

Operation and Maintenance Fees ($\Phi_{4\text{-}1}$)

This refers to the fees incurred for ongoing maintenance and updates to the HRIS software [111,113,114,117]. As organizations evolve, there may be a need to reorganize or update the HRIS to accommodate changes in the organization's structure or processes. For example, if the organization acquires a new company, there may be changes to the organizational hierarchy that require adjustments in the HRIS. Maintenance fees may also cover activities such as regular data backups, especially for HRISs deployed as software-as-a-service (SaaS), or migrating the HRIS to different physical servers to take advantage of new and improved technology. These ongoing maintenance and update fees are essential to ensure that the HRIS remains effective and aligned with the changing needs of the organization.

Licensing Fees ($\Phi_{4\text{-}2}$)

This cost refers to the fees associated with obtaining official permission or licenses to use the HRIS software [111–113,116,120]. These fees can vary depending on the software vendor and the terms of the license.

Some important considerations regarding licensing fees for HRIS software include user limitations and feature restriction. Licensing fees may often be based on the number of users or employees who will use the software. It is important to clarify whether or not there are any restrictions on the number of users covered by the license. Some licensing agreements may restrict access to certain features of the software based on the type of license purchased. Managers should ensure that the selected license provides access to the required features for their organization.

Some vendors offer demo versions of the software that can be used for a limited time or with restricted features. It is necessary to understand the limitations of demo versions and whether or not they can be converted into full licenses. In some cases, vendors may offer free versions of HRIS software for non-commercial or educational use. Managers should make sure they are aware of any usage restrictions associated with such versions.

If the software is proprietary, the should understand the licensing terms, including any restrictions on integration with other platforms or systems. They should consider whether they have the option to obtain full ownership of the HRIS software, which could provide long-term availability. However, they should be aware that there may be additional costs for upgrades or security updates. Clear communication with the software vendor and a thorough understanding of the licensing terms are crucial to avoiding legal issues and ensuring that the HRIS meets the organization's needs.

Consultation Fees ($\Phi_{4\text{-}3}$)

The cost associated with consultations for HRIS software and organizational goal alignment is a crucial aspect of decision-making [111,114,116,117]. These fees can be structured on an hourly or daily basis, or they may be determined on a case-by-case basis depending on the complexity of the solutions provided. Opting for vendor consultation services offers the advantage of tapping into their extensive experience in HRIS deployment. It also serves as a platform for the IT department to gain technical insights into the HRIS. For instance, consultations may be necessary when deploying the HRIS in a server cluster to enhance redundancy or when considering virtualization of the HRIS server. When critical decisions are at stake, engaging HRIS consultants becomes a viable option. However, it is vital to exercise caution when granting consultants administrative remote access to the HRIS to mitigate potential security risks.

Cost of Equipment ($\Phi_{4\text{-}4}$)

In addition to standard computing equipment like computers, servers, laptops, and mobile phones, HRIS implementation often involves specialized hardware such as facial recognition terminals, cameras, proprietary wireless access points for attendance tracking, and smart card reading systems with access control and logging functionalities [111,116–120]. It is advisable to use the hardware recommended by the HRIS vendor, or, alternatively, have the vendor provide this equipment to ensure compatibility and seamless integration. Some vendors may offer discounts when both the HRIS software and hardware, such as servers, are sourced from them. The HRIS also excels in centralized record-keeping. Companies can choose to deploy the HRIS within their premises with remote accessibility through a virtual private network (VPN). Alternatively, the HRIS can be offered as a cloud-based service. Cloud deployment brings the advantage of easy updates and reduced IT infrastructure management efforts for the company. However, both approaches come with their own set of risks. It is important to consider that employee information and sensitive business strategies are company secrets. When deploying critical company data in the cloud, data integrity must be maintained, and data stored in the cloud should ideally be importable to a local test server to mitigate risks associated with remote storage.

Software Training Fees ($\Phi_{4\text{-}5}$)

The cost of training employees, particularly HR and Information Technology (IT) managers, to acquire the necessary skills for fully utilizing all the software features of the HRIS should be factored in [116,117]. Training courses may be available for free during HRIS deployment or included in the learning materials provided within the learning management system. Some vendors might offer advanced training at an additional cost on a per-employee basis. Language considerations are also important when planning training. Online training services provided by the vendor on a subscription basis might be the most cost-effective option for companies with a large number of employees. However, this approach can increase internet traffic usage. Alternatively, hosting training materials locally for a one-time fee could be another option to consider, especially if data usage is a concern.

2.2.5. Vendor Support (Φ_5)

The support provided by the software provider is a critical factor to consider [111,114]. Typically, this support involves technical support services for troubleshooting issues related to the HRIS. This support can come in the form of a guarantee for the HRIS. Commonly, troubleshooting is carried out through remote access, utilizing installation and help functions built into the HRIS. Hotline numbers are often provided so that employees can call for assistance when needed. The quality and effectiveness of vendor support can be assessed through various measures.

Vendor Reputation ($\Phi_{5\text{-}1}$)

The reputation of the software provider, based on their past performance with other clients, is a crucial factor to consider when selecting an HRIS [111,113,119,120]. A reputable HRIS vendor typically showcases its reputation and track record on its website to instill confidence in potential buyers. Information about the HRIS software's history, past implementations, and current clients can be obtained from the vendor. It is essential to choose a reputable vendor with a strong track record in HRIS solutions, as they are more likely to provide reliable and effective software.

Technological Capability ($\Phi_{5\text{-}2}$)

The qualities of the vendor, such as technical skills, managerial competency, and expertise in providing HRIS solutions, play a significant role in the selection process [111,113,115,120]. A vendor's technical capability is crucial, as it reflects their ability to design and deliver HRIS solutions effectively. This capability is often demonstrated through the vendor's past works, which may include design patents and industry-standard modules. A technically capable vendor is more likely to implement HRM processes effectively through the HRIS, using established industry standards rather than reinventing the wheel. In addition to technical skills, the vendor's managerial competency is essential for project management and ensuring the successful deployment of the HRIS. Their know-how and collective expertise, gained through years of experience and their team's skills, contribute to their overall capability in delivering HRIS solutions.

After-Sales Service Commitment ($\Phi_{5\text{-}3}$)

After-sales services are a form of guarantee provided by the vendor to the organization [113,114]. These services encompass the support and assistance that the organization can expect even after purchasing the HRIS. They include various forms of assistance, such as quick training, maintenance, and repair, aimed at ensuring the smooth operation of the HRIS. Offering after-sales services not only benefits the organization but also strengthens the vendor's relationship with the company, fostering customer loyalty. For instance, if a hardware component, like a hard disk or power supply, fails within the server, the vendor should be willing to assist in replacing the spare part at an agreed cost. This proactive approach helps prevent potential issues and ensures compatibility when replacing hardware components. Overall, after-sales services provided by the vendor play a crucial role in maintaining the HRIS's reliability and functionality.

After-Sales Update and Upgrade ($\Phi_{5\text{-}4}$)

Software updates refer to the possibility of enhancing the software by adding new packages, addressing bugs, patching security vulnerabilities, and keeping it in line with recent technological changes [113,114,116]. Updates are focused on improving the performance and reliability of the existing software version. They typically include security patches and bug fixes to ensure the software works better and remains secure. Software upgrades, on the other hand, involve not only updates but also the introduction of new features or a significant change in the software's architecture. Upgrades may bring innovative changes and enhancements to the software. However, they can sometimes be less stable in terms of performance compared to updated versions. The timing and availability of updates and upgrades depend on the vendor's release cycle. It is common for vendors to offer software updates for free, as this helps maintain a positive image by demonstrating their commitment to delivering bug-free and secure software. Upgrades may come at a cost and are usually associated with substantial changes or additions to the software's functionality, making it more modern and compatible with different devices, including mobile devices.

Software Delivery and Service Response Time ($\Phi_{5\text{-}5}$)

This is the period of software implementation, which refers to the time required through process of transitioning from the planning phase through to payment and finally deploying the software for use within an organization [113,115,116]. The implementation process can vary depending on the vendor and the type of software being delivered. Some vendors provide the HRIS as a script that needs to be set up and run on the organization's servers. Others offer the HRIS as a cloud-based solution, where the software instance is hosted and managed on remote servers. Rapid response from the vendor during the implementation phase is a valuable service. Vendors may offer different levels of support, with the top tier providing instantaneous responses when the vendor is contacted. Other tiers may promise responses within specific timeframes. It is essential for organizations to choose a level of support that aligns with their needs and expectations. While software updates are typically provided for free to maintain a positive vendor reputation, the implementation phase may involve additional costs, such as setup fees or customization services, depending on the specific requirements of the organization.

2.3. Contractor Selection Indicators

The primary standard for evaluating the alternatives in an MCDM problem is the criteria. Here, 36 criteria are used to evaluate service providers that will complete a construction job for another party, usually in exchange for payment. These criteria are divided into two levels: six first-level criteria and thirty second-level criteria.

2.3.1. Financial Capabilities (Ψ_1)

Financial capabilities pertain to the competencies and attributes required for efficiently managing the financial aspects of a solar panel installation project [121, 122]. They encompass an understanding of the risks associated with engaging a contractor for the project. The financial capacity of the contractor plays a critical role in influencing both the quality of work and the likelihood of project completion. To assess this capacity, various financial documents and references are scrutinized. These documents can provide insights into the contractor's financial stability, including their historical turnover, credit ratings, and other relevant financial indicators [123]. In essence, this criterion aims to ensure that the selected contractor possesses the financial acumen and stability necessary to execute the project successfully.

Financing and Investment ($\Psi_{1\text{-}1}$)

This criterion pertains to the financial resources allocated to the project to ensure its successful execution. It involves an evaluation of the contractor's funding mechanisms, which may include sources like banks and capital investors. The specific Financing and Investment vehicles, such as debt or equity, are considered in this assessment. Additionally, scrutiny is applied to the legitimacy of shareholders investing in the contractor, ensuring that the funds are obtained through legitimate means [124,125]. In essence, this criterion examines the financial backing and investment strategies of the contractor to assess their capability to fund and deliver the project.

Financial Stability ($\Psi_{1\text{-}2}$)

This criterion assesses the quality of the contractor's funds by examining the stability of their cash flow. It aims to determine whether the contractor's cash flow is consistent and unlikely to experience significant fluctuations or interruptions. A contractor with a stable financial situation is less susceptible to financial crises or unexpected disruptions. This resilience to economic stress ensures that the contractor can absorb the impact of unforeseen events without compromising the terms of the contract. In essence, this criterion evaluates the contractor's financial stability and their capacity to withstand economic shocks during project execution [126,127].

Financial Strength ($\Psi_{1\text{-}3}$)

The financial strength of the contractor is a measure of their ability to withstand financial challenges without facing the risk of bankruptcy. One key indicator for evaluating financial strength is the debt-to-equity ratio, which provides insights into the balance between a contractor's debt and equity. Additionally, the working capital ratio, calculated as current assets divided by liabilities, can be used to assess the contractor's capacity to settle its obligations, including debts, within a year. This ratio takes into account assets such as inventories, accounts receivable, and cash. Evaluating financial strength is crucial to ensuring that the contractor has the financial resilience needed to fulfill their contractual obligations without facing insolvency [121,128].

Financial Status ($\Psi_{1\text{-}4}$)

Evaluating a contractor's financial relationship with financial institutions, such as banks, is essential. This assessment involves a thorough examination of the contractor's financial statements, including the balance sheet, income statement, cash flow statement, and work-in-progress report. The work-in-progress report provides insights into the contractor's ongoing projects, which can indicate their current workload and capacity for taking on additional projects, particularly when subcontracting is subject to strict regulations. Decision-makers should pay close attention to changes in assets, liabilities, profit and loss statements, and the backlog of the contractor's projects that extend over an extended period. Assessing the contractor's financial standing with financial institutions helps determine their credit access and bonding capacity [127,129,130].

Credit Ratio ($\Psi_{1\text{-}5}$)

The contractor's credit ratio, as assessed by banks, reflects their ability to secure loans for a project. This ratio, also known as the credit utilization ratio, indicates how much borrowing capacity the contractor has left, as borrowing is not unlimited. The credit ratio specifically represents the ratio of the contractor's funds guaranteed by credit to the total loans provided by the bank. A higher value in this ratio signifies a safer proposition for banks when engaging in construction projects with the contractor, as it indicates a stronger financial position and borrowing capacity [122,127].

2.3.2. Technical Capability (Ψ_2)

The assessment of the contractor's technical competence involves evaluating their staff's capability to execute the solar panel installation project while adhering to national electrical regulations and standards [124,125]. This assessment takes into account the contractor's technology, resources, and work processes. However, it is important to note that aspects of the contractor's technical capacity that are considered confidential or not provided for evaluation are excluded from this assessment.

Quality Performance ($\Psi_{2\text{-}1}$)

Quality performance evaluation assesses the bidding proposal of the contractor in terms of how well they plan to adhere to national electricity regulatory standards and the precautionary measures implemented to enhance the longevity of the solar panel installation project [125,127]. This evaluation should encompass both project-level and corporate-level aspects. At the project level, there are two dimensions to consider. First, the product dimension evaluates the contractor's facilities in terms of features, esthetics, reliability, durability, conformance to standards, and the serviceability of the facilities. Second, the service dimension assesses the contractor's timeliness, accuracy, responsiveness, accessibility, and overall approach to project completion. On the corporate level, the evaluation extends to the contractor's organizational culture. This involves examining their leadership style, commitment to employee empowerment, and their approach to client interactions and relationships [131].

Training Program ($\Psi_{2\text{-}2}$)

Training program evaluation assesses the extent to which the contractor's contract goes beyond merely installing the solar panel project. It evaluates whether the contractor includes provisions for educating project owners in the skills needed to manage and maintain the installed solar panel system. Additionally, it considers whether the contract incorporates the concept of clean energy and sustainability as part of the training plan [132–134]. The significance of the training program cannot be underestimated. The operation of complex equipment by untrained staff not only poses risks to the company's resources but also endangers the safety of those in the vicinity. Therefore, particular attention should be given to Health, Safety, and Environment (HSE)-related training to ensure the safe and efficient operation of the solar panel system.

Similar Project Performance ($\Psi_{2\text{-}3}$)

The evaluation of the contractor's past performance assesses how well or poorly the contractor has executed similar projects in the past [125,133]. If the contractor's previous performance on similar projects falls significantly below expectations, it may be justifiable to exclude the contractor from participation in the current project under evaluation. However, it is crucial to recognize that past performance is not an absolute indicator of future performance. Therefore, a comprehensive assessment is needed, and appropriate weighting should be assigned to this criterion using a Multiple Criteria Decision-Making (MCDM) method. Technical performance aspects, including product or service quality, cost control, adherence to schedules, management practices, business relationships, subcontracting arrangements, and other factors like reporting, cost data accuracy, tax compliance, and payment history, should all be considered in this evaluation [135].

Qualification of Staff ($\Psi_{2\text{-}4}$)

The Staff Qualification criterion assesses the number of licensed staff available to complete the project [124,125]. Managing a project involves inherent risks, and evaluating the qualifications of the contractor and its key staff can help mitigate avoidable issues that may disrupt the smooth execution of project tasks. This evaluation involves a thorough review of the resumes and verification of the qualifications of the contractor's key personnel. An unqualified contractor may be compelled to subcontract regulated aspects of the project, potentially complicating the project execution process. Ensuring that key staff members possess the necessary qualifications can contribute to more efficient and reliable project execution.

Technical Staff Experience ($\Psi_{2\text{-}5}$)

The technical staff skills criterion evaluates the technical skills of the staff responsible for installing project components [128,136,137]. Technical experience reflects the cumulative time spent working on various projects and the number of projects executed. It is not uncommon for staff members to accumulate knowledge and expertise during the training and execution of previous projects. This knowledge can be transferred and applied to new projects, leading to improved project quality and efficiency. Therefore, assessing the technical skills and experience of the staff is crucial in ensuring the successful execution of the current project.

2.3.3. Management Capability (Ψ_3)

The management capability criterion assesses the contractor's potential to effectively manage the solar panel installation project within the defined scope of the contract [122,124,125,132]. Effective project management is crucial for increasing the project's success rate and reducing execution costs. This criterion evaluates the contractor's ability to organize resources, maintain project focus, and establish appropriate communication channels specific to solar panel installation projects. It goes beyond general management capabilities and focuses on project-specific management skills, which are essential for the successful execution of the project.

Knowledge Management ($\Psi_{3\text{-}1}$)

Knowledge management (KM) is a crucial aspect taken into consideration in sustainable development projects. KM involves creating, acting upon, or dealing with information within the organization based on education and experience [124,138,139]. In sustainable development projects, often led by the government, it becomes essential to effectively capture and organize the lessons learned during project execution. This knowledge can be leveraged to improve future projects, leading to increased innovation and reduced project cycle times through effective knowledge sharing and application. KM plays a pivotal role in enhancing the overall efficiency and effectiveness of sustainable development initiatives.

Current Workload Capacity ($\Psi_{3\text{-}2}$)

The assessment of the contractor's workload capacity is crucial in minimizing subcontracting in a winning bid [124,129]. It is important that the project falls within the contractor's perceived capacity to handle the amount of work required. When a significant portion of the contract is subcontracted, the contractor may lose a considerable level of control and authority over the project. Therefore, aligning the project's scale with the contractor's workload capacity is essential for efficient project management and control.

Managerial Staff Experience ($\Psi_{3\text{-}3}$)

This measures the contractor's ability to run and control the project based on long-term accumulated practical knowledge, and skills are a critical factor [136]. Experienced managers play a vital role in ensuring the seamless operation of daily procedures. They motivate employees, serve as intermediaries between employees and project ownership, and delegate tasks effectively to maximize efficiency. Moreover, these seasoned managers can swiftly identify bottlenecks within the business structure, reduce operating costs, and secure the long-term success of the project. Their accumulated knowledge and skills are invaluable assets in achieving project goals.

Project Management System ($\Psi_{3\text{-}4}$)

This measures a contractor's ability to plan and organize resources for installing solar panels; it is a crucial factor in project success [132,140]. This capability ensures that the project is strategically aligned and provides leadership in the direction needed to complete the project on time. A well-established project management system that evolves based on past successes and failures contributes significantly to the project's quality and reduces associated risks. Efficient resource planning and organization are essential for building a sustainable energy solution through solar panel installation.

Progress Cost Control ($\Psi_{3\text{-}5}$)

The evaluation of the relative cost of the process involved in completing solar panel installation is critical [141,142]. This assessment includes checks and restrictions put in place to reasonably minimize costs throughout the project. An effective progress cost control report generated by the contractor should offer a reasonable budget at various milestones within a specified timeline. In addition to recording project transactions, this report provides a dashboard reflecting the progress of the solar panel installation and highlights any potential problems that may arise during the project. Effective cost control is essential for managing the budget and ensuring the project's financial success.

2.3.4. Health and Safety (Ψ_4)

Ensuring the health and safety of staff during solar panel construction is paramount, and it comes with inherent risks that need to be minimized and compensated for. The primary objective of this criterion is to prevent staff from being injured or falling ill during the construction process. Additionally, it aims to instill a positive health and safety culture within the company, ensuring compliance with legal requirements for employee health and safety protection. This indicator encompasses best practices related to general Health, Safety, and Environment (HSE) during construction to create a safe working environment for all involved.

Safety Planning and Records System ($\Psi_{4\text{-}1}$)

This criterion emphasizes the importance of developing comprehensive safety measures to protect staff members throughout the solar panel installation process and beyond. It involves the creation of detailed plans and procedures that prioritize the safety of workers. Furthermore, it includes the establishment of written records that can be analyzed in the future to identify areas for improvement in both design and execution procedures. These records should be readily accessible both on-site and at the main office to ensure ongoing safety enhancements.

Management Safety Accountability ($\Psi_{4\text{-}2}$)

This criterion involves actively managing and controlling safety measures to ensure the well-being of staff members. It includes contractors having a clear understanding of their responsibilities in terms of assisting and caring for staff if any unforeseen incidents occur during project execution. To achieve safety accountability within management, safety should be established as a fundamental organizational objective. Regular safety inspections, along with a system of rewards and penalties for employees and managers based on their adherence to safety regulations, can help promote a culture of safety within the organization.

Injury, Illness, and Accidents ($\Psi_{4\text{-}3}$)

The criterion "Injury, Illness, and Accidents" assesses the measures contractors have in place to address the well-being of workers who may become unwell or suffer injuries, which can be caused by unforeseen events that lead to harm or damage. These measures typically include insurance coverage and financial compensation provided to affected staff members. Specifically, "injury" refers to the harm experienced by an employee during the project, "illness" encompasses both physical and mental health issues that make employees feel unwell, and "accidents" represent unexpected and unpleasant events occurring during construction that result in injury or damage.

Waste Disposal during Construction ($\Psi_{4\text{-}4}$)

The "Waste Disposal" criterion assesses the contractor's strategy for disposing of materials that are no longer deemed useful for the project. This evaluation focuses not only on contractors who minimize excess material use but also on those who have plans for reusing materials that would otherwise be considered unusable within the same project.

Occupation, Safety, Health, and Management ($\Psi_{4\text{-}5}$)

The "Occupation, Safety, Health, and Management" criterion evaluates the contractor's compliance with statutory requirements related to occupational disease control and work safety acts in the local environment, such as those applicable in China. This evaluation aims to ensure safe and healthful working conditions for workers by setting and enforcing standards, providing training, outreach, education, and assistance. There are two main objectives to be met: first, the contractor should provide construction staff with a hazard-free work environment that is free from recognized hazards that could lead to death or serious physical harm. Second, each employee should comply with occupational safety and health standards and regulations as well as government regulations relevant to their actions and conduct.

2.3.5. Reputation (Ψ_5)

The "Reputation and Professionalism" criterion evaluates the opinion that people have of the contractor based on their past interactions and reputation. Reputable contractors are known for their professional experience and qualified teams, which contribute to the positive image of the project. This criterion emphasizes the importance of a contractor's professionalism and its impact on their brand reputation in the energy market.

Business Development Status ($\Psi_{5\text{-}1}$)

The "Business Development Status of the Contractor" criterion assesses the contractor's growth and advancement over time by considering factors such as firm size, project quality, and industry ranking. This evaluation provides insights into the contractor's capacity and whether their proposal aligns with their business development status. It helps in determining if the contractor is capable of executing the project and whether their proposal is realistic. Proposals that do not align with the contractor's business development status may be deemed unsuitable.

Customer Relationship ($\Psi_{5\text{-}2}$)

The "Customer Relationship" criterion assesses how the contractor interacts with individuals or organizations that utilize their services. This evaluation examines the contractor's approach to customer satisfaction, communication, and overall client management. A positive customer relationship is often indicative of a contractor's commitment to meeting client needs and maintaining a good reputation in the industry [122].

Failure/Success in Project Completion ($\Psi_{5\text{-}3}$)

The "Failure/Success in Project Completion" criterion assesses the contractor's track record by examining their performance in previous projects. This evaluation involves scrutinizing the contractor's history of successfully completed projects as well as any instances of project failures or shortcomings. It serves as an indicator of the contractor's reliability and ability to deliver on their commitments. A contractor with a strong history of project success is typically viewed more favorably than one with a history of project failures [127,143].

Quality Assurance Program ($\Psi_{5\text{-}4}$)

This criterion assesses the contractor's plans and processes for maintaining high standards and excellence in their work. This evaluation considers the quality control measures and procedures the contractor has in place to ensure that the project meets or exceeds established standards. It also includes assessing the contractor's ability to address issues such as troubleshooting in a timely manner to maintain project quality. Also, this criterion evaluates the interactions between the contractor and other stakeholders, including subcontractors and statutory personnel. It assesses how well the contractor collabourates with partners and complies with relevant regulations and requirements related to subcontracting and cooperation. This criterion is essential for ensuring that the contractor can effectively work with others to achieve project objectives and meet legal obligations [124,144].

Cooperation and Subcontractor Relationship ($\Psi_{5\text{-}5}$)

This criterion evaluates the interaction between the contractor and other partners, including subcontractors and statutory personnel. It assesses how well the contractor collabourates with partners and complies with relevant regulations and requirements related to subcontracting and cooperation [129,134,145]. One key indicator of a successful floating solar power installation project is its timely completion. Contractors with a proven track record of completing their tasks on time inspire confidence among their customers. Therefore, it is important to consider a contractor's history of project completion when evaluating proposals. Contractors with a high failure rate of completing their past projects may not be suitable candidates for the project, regardless of the potential benefits of emerging technology. This criterion helps ensure that the selected contractor has a history of successful and timely project delivery.

2.3.6. Clean Power (Ψ_6)

This criterion assesses energy sources used for lighting, heating, and driving machinery that are free from pollutants as by-products. It evaluates the environmental impact and sustainability of the energy sources employed in the project. Choosing clean and renewable energy sources can contribute to reduced pollution and a more sustainable energy solution. This criterion encourages the use of eco-friendly energy sources in the project to minimize its environmental footprint.

Energy Efficiency ($\Psi_{6\text{-}1}$)

This criterion evaluates the efficiency of the solar panels and other equipment used in the project. It assesses the extent to which these components are able to convert sunlight into electricity efficiently. To evaluate the environmental aspects of the project over its life cycle, a Life Cycle Assessment (LCA) technique is employed.

This involves several stages, including defining the goal and scope of the assessment, conducting an inventory analysis to describe material and energy flows, performing an impact assessment, and finally, interpreting the results and conducting sensitivity analysis to gain a comprehensive understanding of the project's energy efficiency and its environmental impact. This criterion encourages the use of energy-efficient components to enhance the sustainability of the project.

Installation Cost and Impact on the Grid ($\Psi_{6\text{-}2}$)

This is criterion assesses the cost of installing a solar panel system and its impact on the electricity grid. It considers whether the contractor's solution can effectively reduce the dependency on the grid or even supply surplus energy to the grid. This criterion evaluates the economic feasibility and grid integration capabilities of the solar panel installation project [133,146–148].

Operation and Maintenance Optimization ($\Psi_{6\text{-}3}$)

This criterion assesses how well the bidder has described their planned activities for efficiently operating and maintaining the installed solar panel system. It considers the bidder's strategy for ensuring that the solar panel system operates at its optimal performance levels, which may involve routine maintenance, monitoring, and necessary repairs. This criterion aims to evaluate the bidder's commitment to long-term system performance and reliability.

Life Cycle Assessment ($\Psi_{6\text{-}4}$)

This criterion assesses the projected lifespan of the solar panel system and how its efficiency may degrade over time. This evaluation considers the bidder's understanding of the long-term performance of the solar panels, including their ability to maintain efficiency and generate electricity over an extended period. It is important to ensure that the solar panel system continues to meet energy production expectations throughout its projected lifespan.

Pollution and Waste Reduction ($\Psi_{6\text{-}5}$)

This criterion evaluates how the contractor plans to maximize energy production and minimize energy loss, such as through the use of energy-efficient technologies. Additionally, it assesses the contractor's approach to waste management, emphasizing the waste management hierarchy. The contractor should prioritize waste reduction at the source, followed by recycling, energy recovery, waste treatment, and finally, waste disposal as a last resort. This criterion focuses on the contractor's commitment to sustainable practices and minimizing environmental impacts in the solar panel installation project.

2.4. Scaling Foreign Service Premium Indicators

Evaluating the locations of overseas branches consists of 5 first-level criteria and 15 second-level criteria, which include 3 second-level criteria for each first-level criterion. The first-level indicators are measured as a formative construct, while the second-level criteria are measured as a reflective construct because they are conceptually correlated. Since the second-level criteria of each criterion are correlated, having more than three second-level criteria yields approximately the same rates in grey numbers. The main reason for using these criteria for evaluation is the availability of data at the evaluated branches and the resources used to obtain these data. The criteria are defined as follows:

2.4.1. Natural Environments (Θ_1)

The natural environment and surroundings are outside the control of the expatriate; however, they must carry out their overseas duties regardless of the nature of the environment, as long as it is not life-threatening. Some experts may be deprived of an excellent natural environment. A clear example of this is an expert working in a coal mine whose life depends on oxygen cylinders, even though oxygen is a freely available gas in the atmosphere.

Clean Cities ($\Theta_{1\text{-}1}$)

Clean cities are measured by the annual mean concentration of fine particulate matter with a diameter of less than 2.5 microns (PM 2.5) in micrograms per cubic meter ($\mu g/m^3$) in a country's urban areas, which is an essential parameter for assessing air quality [149]. High levels of PM 2.5 can pose significant health risks to individuals, as exposure to air pollution can have adverse effects on the respiratory and cardiovascular systems. Some experts advocate for green cities because trees can help improve air quality by absorbing pollutants. However, it is also essential to address issues like improper disposal of public waste, as this can lead to contamination of items people consume and other environmental hazards. Therefore, maintaining clean urban environments is critical for public health and the well-being of the ecosystem.

Environmental Performance Index ($\Theta_{1\text{-}2}$)

This index provides a data-driven summary of the state of sustainability worldwide. It draws on 32 indicators across 11 categories to rank countries based on their environmental health and ecosystem quality. Among the 32 indicators covered are waste management, air quality, drinking water, sanitation, pollution emissions, and more. The ranking includes 180 countries and encompasses quantitative metrics related to pollution control and the management of natural resources. It considers both environmental health and ecosystem vitality, incorporating 24 indicators in these categories [150,151]. Also, this index offers a valuable tool for identifying issues, establishing goals, monitoring trends, assessing results, and recognizing effective policy approaches. Having access to reliable data and conducting evidence-based analysis can aid government officials in fine-tuning their policy priorities, enhancing communication with important stakeholders, and optimizing the impact of environmental investments.

Disaster Risk Index ($\Theta_{1\text{-}3}$)

This index assesses the types of natural disasters that can overwhelm a nation's ability to respond effectively. It utilizes data related to various natural hazard categories and exposure dimensions, including earthquakes, tsunamis, floods, tropical cyclones, and droughts. This index helps in evaluating and preparing for potential disaster risks [152–155].

2.4.2. Conflict State (Θ_2)

This index assesses countries based on their political stability, reflecting the fragility of the government and the risk of its collapse. This index is crucial for evaluating the business environment, as operating in politically unstable regions can be costly due to the need for higher compensation and increased living expenses for expatriates. Additionally, modern conflicts often result in more civilian casualties due to terrorism, even though the number of soldiers killed in conflicts has decreased.

Global Terrorism Index ($\Theta_{2\text{-}1}$)

The Global Terrorism Index analyzes the impact of terrorism in 163 countries, covering nearly 99.7% of the world's population [156]. This index is based on recorded data of terrorist incidents and the death toll, which is maintained by the Institute for Economics and Peace (IEP). It systematically ranks countries based on their level of terrorist activity. For example, in 2020, Afghanistan was ranked first, and the activities of Islamic extremist groups in Nigeria contributed to its high ranking as well. Consequently, expatriates working in countries with high terrorism rankings typically expect to receive higher compensation due to the associated risks.

Failed State Index ($\Theta_{2\text{-}2}$)

This index is an annual assessment of countries facing various pressures that could potentially lead to internal conflicts or instability. It is compiled by the Fund for Peace (FfP) and serves as a valuable forecasting tool for policymakers and analysts when making decisions. It is important to note that the presence of risk does not necessarily mean there is an ongoing war, but rather it reflects the underlying tensions and challenges within a country. These risks can significantly impact the willingness of expatriates to work in such countries. Human interactions and conflicts are complex, and internal ethnic conflicts can arise from a variety of factors, including both internal and external influences.

Global Peace Index ($\Theta_{2\text{-}3}$)

Indeed, assessing a country's stability goes beyond just indicators related to the presence or absence of war. The assessment also takes into account ecological threats and their potential impact on peace and a country's fragility. Additionally, it considers the economic implications of violence, including security expenditures such as the cost of maintaining public safety through police services, law courts, and prisons [157]. Furthermore, the assessment differentiates between positive peace and negative peace. Positive peace encompasses the attitudes, institutions, and structures that foster and sustain peaceful societies. On the other hand, negative peace refers to the absence of violence or the fear of violence. This comprehensive approach allows for a more nuanced understanding of a country's overall stability and peace [156].

Fragile State Index ($\Theta_{2\text{-}2}$)

The Fragile State Index by the Fund For Peace (FFP) organization is a valuable tool for assessing and ranking the stability and fragility of 178 countries. This assessment is based on a combination of quantitative and qualitative data, and is further validated by experts in the field. The index serves the important purpose of promoting security and preventing violence [158]. In addition to traditional indicators of fragility, such as political instability and conflict, the effort to maintain peace in a country is also considered as a criterion. This comprehensive approach provides a more holistic view of a country's stability and helps policymakers and analysts make informed decisions to address and mitigate potential risks.

Global Peace Index ($\Theta_{2\text{-}3}$)

The Global Peace Index (GPI) is a valuable tool for measuring peace worldwide, and it plays a significant role in understanding the relationship between peace and a country's prosperity. This index provides quantitative data that assess various aspects of peace, including domestic and international conflicts, the safety and security of society, and levels of militarization based on funding and access to weapons [156]. By analyzing these factors, the GPI helps promote cultural understanding and can contribute to efforts aimed at increasing peace and stability in the world. It provides a comprehensive assessment of peace-related issues and serves as a valuable resource for policymakers, researchers, and organizations working toward global peace and prosperity.

2.4.3. Economic Performance (Θ_3)

The economic performance indicator assesses the economic condition of countries, which can have a significant impact on expatriates. It primarily evaluates macroeconomic factors such as the country's overall economic growth and development. Additionally, it considers the economy's resilience in responding to unpredictable events, often referred to as economic shocks. This indicator helps gauge the economic stability and potential risks that expatriates might face when working in a particular country.

Consumer Price Index ($\Theta_{3\text{-}1}$)

The Consumer Price Index (CPI) is used to measure changes in the price level of consumer goods and services that are typically purchased by households. It reflects the average change over time in the prices paid by urban consumers for a market basket of these goods and services. In some cases, it may consider the weighted average change in prices. The CPI, as reported by the World Bank, is utilized in this book to assess the cost of living and purchasing power an expatriate should have to live comfortably in a specific country. It helps determine how inflation affects the affordability of consumer goods and services for expatriates. This is computed as follows:

$$\frac{\text{Cost of market basket in a year}}{\text{Cost of market basket in a based year}} \times 100. \tag{2.1}$$

Gross Domestic Product per Capita ($\Theta_{3\text{-}2}$)

This is the market value of the goods and services produced in a year. The economic condition heavily influences business decisions; investing in a growing country can make the funded business enjoy the favorable factors that are creating economic growth. Expatriates are drawn to a country with favourable GDP where they can enjoy the resources in the host country with less of their personal needs been imported from their home country to sustain acceptable living condition will accept relocating with their family because of the benefit, thereby increasing their job retention. GDP per capita ($\Theta_{3\text{-}2}$) is the GDP divided by the midyear population. The GDP is the gross value of all the goods and services produced by a country, with all subsidies excluded [159]. This reflect the economic health of the country, and its production, expenditure, income can be used in determining the GDP, which decision-makers, such as the government, can use in making strategic plans and policy.

Inflation ($\Theta_{3\text{-}3}$)

Inflation is commonly defined as the percentage change in the prices of a basket of goods and services that a typical consumer purchases on an annual basis. The CPI ($\Theta_{3\text{-}1}$) uses a base period of 2010 to depict the fluctuations in the cost for a typical consumer annually buying a basket of goods and services [160]. It reflects the rate at which the general price level of goods and services in an economy is rising, leading to a decrease in the purchasing power of a currency over time. Inflation is an important economic indicator and can impact various aspects of an economy, including the cost of living, interest rates, and investment decisions [161]. An expatriate prefers to be paid at an equivalent rate to that of a stable country, not just the salary decided by the labour market. Since infatuation reduces the purchasing power of money, expatriates would to be paid in a currency with more purchasing power. If an expatriate is paid in the country's local currency, then their income should increase as inflation increases, or else they will prefer working in their home country.

2.4.4. Healthcare (Θ_4)

Healthcare is a critical factor for expatriates when considering job assignments in different countries. Access to quality healthcare services, including hospitals and medical facilities, plays a significant role in their decision-making process. Expatriates often prefer working in countries with reliable and advanced healthcare systems to ensure their well-being and the well-being of their families. Moreover, a strong healthcare infrastructure can also reduce the financial burden on companies in terms of compensation and insurance premiums, especially in high-risk countries where the chances of accidents or health issues may be higher. This consideration underscores the importance of healthcare in expatriate assignments and workforce planning.

Sanitation and Hygiene ($\Theta_{4\text{-}1}$)

Sanitation and hygiene are crucial factors for expatriates when evaluating potential job assignments in different countries. Access to proper sanitation services and good personal hygiene practices are essential for maintaining health and well-being. Expatriates working in countries with inadequate sanitation systems and poor hygiene practices may be at a higher risk of contracting diseases and experiencing health issues [162]. The availability of clean water for personal hygiene, proper waste disposal systems, and overall sanitation infrastructure in a country can significantly impact the health and safety of expatriates. Companies and organizations often take these factors into account when determining compensation packages and insurance coverage for their expatriate employees. Ensuring proper sanitation and hygiene is not only a matter of individual health but also a crucial consideration for workforce planning and risk management in international assignments.

Environmental Pollution ($\Theta_{4\text{-}2}$)

Environmental pollution and its associated mortality rate are significant concerns for expatriates evaluating potential work assignments in different countries. Pollution, whether in the form of air, water, soil, noise, or light pollution, can have detrimental effects on public health. Expatriates often consider the environmental quality and pollution levels in their destination country when making decisions about their assignments [149]. High levels of environmental pollution can contribute to various health problems, including respiratory issues, cardiovascular diseases, and even unintentional poisonings. The COVID-19 pandemic highlighted how disease outbreaks can affect regions differently, and expatriates are likely to be cautious about working in areas with a history of high mortality rates. Companies and organizations sending expatriates abroad often factor in environmental pollution levels when determining compensation, insurance coverage, and the overall safety of an assignment. Maintaining a healthy and safe environment is crucial for the well-being of expatriate employees, and addressing environmental pollution is a key aspect of ensuring their safety and quality of life while working abroad [96].

Drinking Water ($\Theta_{4\text{-}3}$)

Water is essential for the survival of man. Drinking water as reported by the world health organization is the number of individuals in the population that have basic and safe water services within a 30 min walking distance [163]. When there is a scarcity of clean drinking water, then it becomes relatively expensive to obtain it, which directly affects the cost of soft drinks and clean food.

2.4.5. Regulatory Institutions (Θ_5)

These are the arms of a country’s government that have the power to manage the nation. These institutions are responsible for providing transparent outcomes to people in a country efficiently. They are established and managed by the government and play a crucial role in strengthening the authority of the police force and the judicial system. Without regulatory institutions, there would be no law and order in society, and expatriates may experience security challenges. Consequently, some expatriates may be compelled to spend their time in highly secure environments, leading to an increase in their overall expenses.

Public Integrity Index ($\Theta_{5\text{-}1}$)

The Public Integrity Index measures a country's capacity to control corruption and ensure that public resources are spent without corrupt practices. This index includes various factors such as judicial independence, administrative burden, trade openness, budget transparency, citizenship (with electronical records), and freedom of press [164].

Justices System ($\Theta_{5\text{-}2}$)

The World Justice Project-Rule of Law Index provides data for 113 countries that adhere to the rule of law from the perspective of people based on their experiences [165]. A country with a good rule of law is beneficial to expatriates since there is clarity in the regulations' definitions. One can proactively stay out of trouble by not breaking the law. This index captures the government, the police, the courts, the country's openness and accountability, the level of corruption, and the prevalence of common crimes among the public.

Reliability of Police Service ($\Theta_{5\text{-}3}$)

The reliability of a police Service is the extent to which the police force can enforce law and order in a country, based on a World Economic Forum survey [166]. The police force's primary responsibility is to uphold the law and prevent crime in the community by bringing justice to offenders. In practice, expatriates are highly skilled individuals who bring their skills to contribute to a community in return for some benefit. These individuals need to feel safe and be protected. The cost of doing business increases when they have to employ personal bodyguards for these expatriates.

2.5. University Reputation Indicators

Today, universities are currently facing pressure to measure their performance objectives, which have become a crucial part of their strategic framework. When assessing university reputation, various factors come into play, including the institution's image [167] and its standing among universities from different countries [168,169]. The ultimate aim of this assessment is to gauge university reputation, positioned at the top of a hierarchical structure that encompasses 6 primary indicators (C_1 to C_6) and 18 secondary indicators ($C_{1\text{-}1}$ to $C_{6\text{-}3}$). These criteria have largely been derived from prior quantitative and empirical research conducted by scholars like Plewa et al. [170], Verčič et al. [171], Vidaver-Cohen [172], and others. It is crucial to identify metrics that gauge the student experience, including assessments of campus life and meeting expectations. Understanding and acknowledging students' perspectives is paramount in university activities, spanning from enrollment to graduation. These perspectives are often shaped by organizational characteristics.

2.5.1. Social Contribution (C_1)

Social contribution within a university context is a multifaceted response to the needs of its surrounding environment. It encompasses all the actions taken by the university, which can have a significant impact on the community. This contribution involves the institution's ethical vision and sense of responsibility, outlining how it aims to make a positive impact on society as an organization, regardless of whether it is for-profit, not-for-profit, or nonprofit. In essence, social contribution reflects the university's commitment to its broader community. This commitment can manifest in various ways, such as supporting charitable causes, offering open lectures and libraries to the public, and actively engaging as a responsible and ethical member of society. A university that positions itself as a good community citizen can have a profoundly positive influence on society as a whole.

Citizenship ($C_{1\text{-}1}$)

Citizenship, in the context of a university, signifies its responsibility to society and its potential to support charitable initiatives. This can involve actions like making lectures and libraries accessible to the public. When a university actively embraces this role as a responsible and engaged community member, it can have a notably positive influence on society as a whole.

Employment ($C_{1\text{-}2}$)

The employment rate of students from this institutio and the positive feedback from employers on the university are important factors to consider. It reflects how successful students are in securing jobs after graduating from the university and how employers perceive the quality of education provided by the institution. Job seekers who graduate from well-known universities tend to receive more favorable responses from potential employers. Universities play a vital role in contributing skilled and talented individuals to various industries. The reputation of a university significantly influences employers' perceptions of its students. A prestigious university serves as the initial trust-building bridge between students and recruiters. This enhanced reputation increases students' credibility and opens up more job opportunities for them. University reputation indirectly impacts students' job security, their ability to secure higher positions, and the potential for better salaries. It is, therefore, a critical aspect of a student's educational journey.

Alumni ($C_{1\text{-}3}$)

Alumni associations are a result of the strong bond formed through a positive student experience, and they are better positioned to acknowledge the positive impact of university reputation on their degrees. The reputation of a university significantly shapes how employers perceive its students. A prestigious university acts as the initial trust-building factor between students and potential employers, enhancing the students' perceived credibility and their chances of securing a job. Indirectly, university reputation also influences students' job security, leading to opportunities for higher positions and better salaries [170]. Moreover, the perceptions of university alumni regarding the extent of learning and the practicality of the knowledge gained serve as vital indicators of the quality of education and services provided by the university to its former students [173]. As time passes, reputable universities often receive donations from alumni, further contributing to society's well-being and the institution's continued growth and success.

2.5.2. Environments (C_2)

The academic environment encompasses the conditions that have a direct or indirect impact on the growth and development of both students and educators. Students and parents typically expect a safe, clean, and pleasant learning environment as a fundamental requirement for academic institutions [174]. For some students, the educational journey can be quite stressful. Therefore, universities should provide an environment equipped with research tools and facilities to alleviate mental fatigue and stress. These facilities may include sports facilities, the close proximity of student dormitories to classrooms, access to dining options, and school transportation services, such as buses that commute between campuses, among other amenities.

International Learning ($C_{2\text{-}1}$)

A prestigious university should extend its reach beyond its local boundaries, attracting students from diverse countries and cultural backgrounds [175]. This diversity not only enhances the university's reputation but also helps students achieve their personal goals [171]. Offering an electronic learning platform over the internet further underscores the university's international standing. Students who engage in global education are exposed to various cultures, ethnicities, religions, and languages. This exposure enriches their social experiences and broadens their academic horizons. International students can boost their confidence, acquire new languages, and interact with counterparts who speak different languages, fostering a multicultural and multilingual academic environment.

Safety ($C_{2\text{-}2}$)

The safety index assesses the university's capacity to safeguard students from potential dangers [174]. The learning environment plays a crucial role in students' experiences, and it is the university's social and environmental responsibility to ensure safety. Without a secure environment, students cannot fully focus on their studies. When violence or threats are present in the educational setting, it affects all students. School safety is a significant concern for various levels of government, encompassing issues like food safety, violence prevention, and disease control. Student well-being must be prioritized in areas such as fire safety and the services provided by university clinics. Furthermore, in the digital age, digital security is equally essential. Controlling the flow of information on the university network should promote a safe and conducive learning environment for students.

Campus Location ($C_{2\text{-}3}$)

The geographic location of a university campus continues to be a significant factor in students' decisions regarding their higher education. Some students may prioritize attending a college that is reasonably close to their home. The physical location of the university is an essential aspect of the overall student experience and is tied to the institution's social and environmental responsibilities. However, a reputable university should expand its reach beyond local boundaries, attracting students from diverse countries and cultural backgrounds. This not only enhances the university's image but also supports students in pursuing their individual goals. It is important to note that not all universities have fully transitioned to a completely digital format, and physical campuses still play a crucial role in education.

2.5.3. Leadership (C_3)

A prestigious university should possess a well-defined development vision, showcasing strong capabilities and a well-structured organization capable of spearheading innovation and offering tangible solutions to societal challenges. This commitment to excellence is essential for upholding the university's reputation and ensuring its continued growth and impact on society [171,172]. Additionally, having student clubs and organizations within a prestigious university, which mirror what students will encounter in their respective industries, provides an invaluable platform for students to smoothly transition into the professional workforce. These clubs offer students practical experiences and opportunities for networking and skill development, further enhancing the university's role in preparing students for successful careers.

Course Materials ($C_{3\text{-}1}$)

The quality of teaching resources is evaluated through various assessments conducted throughout the academic year [176]. Teaching aids, such as visual materials like pictures, videos, or three-dimensional objects, can significantly enhance students' understanding and mastery of new concepts. Particularly in courses related to science, technology, engineering, and mathematics (STEM), students benefit from the opportunity to engage in hands-on experiments that align with their research interests. It is crucial for the curriculum to be aligned with contemporary technology and teaching methods. This alignment enriches teachers' instructional practices, ensures they are well versed in the latest trends and challenges within their fields, and enables them to develop courses that are both relevant and engaging for students.

Lecturers ($C_{3\text{-}2}$)

Lecturers are the academic professionals responsible for delivering these course materials. Instructors should possess authority in their field, expertise in relevant knowledge and skills, and the ability to apply this knowledge to real-world scenarios. Effective communication skills are equally essential, along with active interaction with students, timely feedback provision, and the encouragement of autonomous learning among students [177]. They play a role in evaluating and enhancing UR by contributing to academic excellence through high-quality teaching, curriculum development, and impactful research.

Administration ($C_{3\text{-}3}$)

Effective administration in a university encompasses all aspects of its services and emphasizes tangible benefits and their repercussions, often serving as a magnet for attracting new students [178]. A well-organized university administration is instrumental in ensuring the delivery of quality education. It necessitates close collabouration between academics and administrators to establish and apply assessment criteria that align with providing students with appropriate learning services.

2.5.4. Funding (C_4)

A prestigious university typically enjoys financial support from various sources, including government funding, contributions from parents, and sponsorships from agencies, which allows them to provide scholarships and subsidize tuition fees [179]. This funding encompasses both core government allocations and external funds, often dedicated to addressing specific research challenges. The financial strength of a renowned university reflects its substantial potential for delivering impactful research outcomes.

Income Level of Parents/Sponsors ($C_{4\text{-}1}$)

This gauges the purchasing power associated with the education offered by the university. It recognizes that most parents are the primary financial contributors to their children's education [180,181]. Parents typically invest significant resources in their child's development and education, and they tend to select a university that aligns with their perception of affordability and quality. Those with higher incomes often aspire to provide their children access to prestigious universities, as long as they can comfortably afford it. In some regions, students may opt to study from home to avoid the additional costs associated with living on campus.

Tuition ($C_{4\text{-}2}$)

This represents the foremost expense incurred for receiving education and associated services, with the expectation of receiving substantial educational value in return [171,182]. The cumulative school fees paid by students collectively contribute to the university's overall income.

It is important to emphasize that tuition payments are a direct and equitable compensation for the perceived value of education and services provided. This comprehensive expense encompasses various aspects such as access to student services, utilization of library resources, engagement in student government activities, and the acquisition and upkeep of school buses. Consequently, the income of the university is substantiated by the tuition payments made by all students during their collegiate pursuits.

Scholarships ($C_{4\text{-}3}$)

Scholarships are a testament to a university's capability to draw in exceptional talent by offering high-quality educational standards [183]. This extends to the provision of diverse scholarships, encompassing not only academic excellence but also the scholarship of teaching and learning [184]. This scholarship is designed to not only foster academic growth but also promote personal and social well-being. They operate within a multifaceted framework, considering organizational, institutional, and external dimensions such as political, economic, social, cultural, and technological aspects. Moreover, research funding plays a pivotal role in expanding a university's capacity to admit more students and attract accomplished faculty members. This, in turn, fuels the generation of research outcomes that contribute directly to national innovation, reinforcing the university's reputation as a distinguished institution.

2.5.5. Research and Development (C_5)

Universities in high-income countries strive to capitalize on publicly funded research to drive knowledge transfer, innovation, entrepreneurship, and economic expansion. The collabouration between industry and universities is instrumental in addressing the intricacies of innovation for a knowledge-based economy, bolstering technological and scientific infrastructure to cater to the demands of a burgeoning economy.

Industry Linkage ($C_{5\text{-}1}$)

Industry and universities collabourate closely to collectively tackle the innovation challenges presented by the knowledge economy. This partnership serves to enhance technological and scientific infrastructure, aligning it with the expanding economic needs. Universities greatly benefit from such collabourations as they can secure research funding and vital financial support. Additionally, it provides university graduates with valuable industry exposure and opportunities to learn from industry researchers through internships and potential employment prospects [185,186].

Key Project ($C_{5\text{-}2}$)

National projects play a crucial role in providing teachers and students with hands-on experience in dealing with real-world practical challenges. Solving these critical issues often necessitates access to empirical data offered by the government and the application of research for the public good. This approach contributes to the development of well-prepared graduates and the enhancement of university labouratories with cutting-edge equipment and facilities. Ultimately, it helps in reducing the overall cost of delivering quality education. Government-led projects extend key plans to universities to work together in finding solutions to these challenges [187,188].

Publications ($C_{5\text{-}3}$)

Universities' research performance can be assessed using various metrics, including the number of publications, cited publications, and collabourations with international and industry partners. These achievements are made available to the academic community. Intellectual contributions that advance the frontiers of science and technology are shared through peer-reviewed academic journals. These groundbreaking research findings are published in international journals, and the volume of publications from a university is often linked to its reputation in the academic world [189].

2.5.6. Students Guidance (C_6)

Students play a pivotal role in shaping university life, although many may have a limited understanding of it when transitioning from high school. They often rely on advice from guidance counselors, individuals who assess and judge a university based on their own experiences and the information accessible about the institution. Furthermore, the quality of academic life within a university can influence recommendations made by students, contributing to the university's reputation.

Recommendations ($C_{6\text{-}1}$)

The perceived service quality at a university significantly impacts students' intentions to recommend it to others and can influence their decision to study at another institution as well [190,191]. Additionally, the quality of academic life plays a role in shaping a university's reputation. Students are typically encouraged to enroll in universities with national and regional accreditation, and international students often seek programs accredited by relevant authorities. The physical location of a university campus also plays a role in students' decision-making processes when choosing where to study.

Parents ($C_{6\text{-}2}$)

Parents and teachers have a role in the evaluation of universities. Parental control can be a strong influence on children's views on UR, and parents use the rankings and reputation of a university as the bases for recommending a university. The number of children is closely related to the number of children parents have and their past life experiences. There is a good chance that parents will obtain and fill out a child's college admissions form. At this point, children may have little influence on their career choices [174,192].

Students ($C_{6\text{-}3}$)

Certainly, students can be influenced by their peers, especially friends, when it comes to their perceptions of a university. The goals and aspirations of friends who are pursuing their own careers can impact a student's choice of college. Additionally, the perceived quality of services provided by a university can affect students' willingness to recommend that institution to others and may also influence their willingness to consider studying at other schools. It s worth noting that many high-achieving graduates may not have a clear understanding of the top criteria for selecting a university, making peer influence and university reputation even more significant factors in their decisions [193,194].

In conclusion, all these criteria are used in subsequent chapters of this book to select the best alternative to the MCDM problems. Specifically, in the next chapter, the evaluation of business environment criteria is conducted, and the evaluation criteria are validated qualitatively.

3. Grey Rank Order Centriod Weights

This chapter primarily introduces the Rank Order Centroid (ROC) weighting method, then integrates it with the Grey Systems Theory (GST) to estimate the weights of criteria in multi-criteria decision-making problems. One of the essential components in decision-making is the selected method to determine the weights of the criteria. Many methods try to determine the true weights of the decision criteria from the Decision Makers (DMs). Since weights are unknown, there is no precise method of obtaining the real weights, but based on logical reasoning, one can propose a better method that can produce better estimates than previously existing methods. Furthermore, in searching for the exact weights, if the DMs are not careful, the method used to estimate the weights may eliminate the actual weights. In situations where estimating weights becomes complex, or when a group of DMs holds differing opinions, ranking becomes essential. When a group of DMs expresses their preferences, the weights used in the evaluation depend entirely on the aggregation method. Weight, from the perspective of decision-makers, denotes the relative importance of criteria. Ranking helps address inconsistencies arising from varying weighting methods because DMs might not reach a consensus on the exact degree of importance of one criterion over another. However, they can readily agree on the ranking order of criteria. The evaluation of a business environment will be incorrect when the weights swing the importance of criteria and the rankings. When considering group decision-making estimating, the weights becomes more complicated. The grey ROC weights are used as surrogate criteria weights and applied in evaluating business environments as depicted in Figure 3.1. In this Chapter, Grey Relational Analysis (GRA) simplifies the process of prioritizing criteria by providing a precise order of their weights.

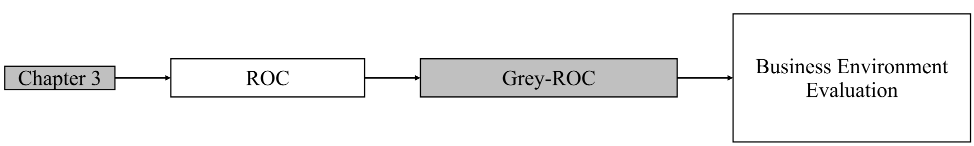


Figure 3.1. Flowchart of Chapter 3. Source: Figure by authors.

3.1. ROC Weights

The ROC weighting method is used to find the central or average weights for a set of criteria. These center weights are determined by exploring a range of possible weight combinations, taking into account any constraints imposed by linear inequalities. The centroid point is calculated as the average coordinate when the criteria are ordered based on their importance, ranging from the most to least important criterion. This method allows for a systematic way to estimate weights that represent the noteworthiness of each criterion in a decision-making process. For a set of criteria with n elements, the rankings of the criteria using their level of importance are given as follows: $w_1 \geq w_2 \geq ...w_n$. This ranking represents the order of importance, where (w_1) is the most important criterion, (w_2) is the second most important, and so on, with (w_n) being the least important criterion. $w_1 \geq w_2 \geq ...w_n$.

The centroid weights for the most important, second most important, and least important criteria in the ROC method are calculated as follows [195],

$$\begin{aligned}
w_1^{ROC} &= \frac{1+\frac{1}{2}+\frac{1}{3}+\ldots+\frac{1}{n}}{n} \\
w_2^{ROC} &= \frac{0+\frac{1}{2}+\frac{1}{3}+\ldots+\frac{1}{n}}{n} \\
&\vdots \\
w_n^{ROC} &= \frac{0+0+0+\ldots+\frac{1}{n}}{n}
\end{aligned}. \tag{3.1}$$

The generalized induction formula for calculating the ROC weights is given as follows:

$$w_i = \frac{1}{n}\sum_{j=i}^{n}\frac{1}{j}, \quad i = 1, 2, \ldots, n, \tag{3.2}$$

where w_i is the ROC weight for the i^{th} criterion, n is the total number of criteria, $\sum_{i=1}^{n} w_i = 1$, and the ROC weights are non-negative, $w_i \geq 0$ [195].

Research has shown that ROC weights can achieve a high level of accuracy, with a 95.7% hit rate when compared to the true weights. This makes ROC weights an excellent choice for estimating unknown weights, especially when compared to equal weights, which tend to perform poorly in such cases. While equal weights may be easier to estimate, it is often unrealistic to assume that all criteria are of equal importance in evaluating decision alternatives [196,197].

3.2. GRA Based on Grey Numbers Combined with ROC Weights

In situations where uncertainty needs to be addressed, such as determining the weights for evaluating a business environment, various mathematical approaches can be considered. Probability and statistics are typically employed when dealing with a large sample of random data, where the goal is to derive weights based on specific data distributions. On the other hand, fuzzy mathematics is used to handle cognitive uncertainty, often expressed through fuzzy membership functions derived from experiential knowledge [198].

However, in the context of evaluating a business environment, Grey Systems Theory (GST) is chosen to determine the weights of the indicators. This choice is made because of several factors:

1. Limited Information: There may be poor or limited information available about the weights of the indicators.
2. Expert Availability: The number of experts available to provide input on the weights may be limited.
3. Data Collection Period: The number of years for which data have been collected by organizations like the World Bank from some developing countries might also be relatively small.

Grey Systems Theory (GST) is a suitable approach in such situations, as it can handle uncertainty and limited data more effectively, allowing for the determination of indicator weights even when traditional methods may not be as applicable.

The process for evaluating criteria weights using Grey Relational Analysis and the Rank Order Centroid (GRA-ROC) weighting method involves several steps. Initially, grey linguistic values, represented by grey numbers, are used to assess criteria weights. A grey weights matrix is constructed based on these values and then standardized. Reference weights for the criteria, as perceived by decision-makers, are determined, and differences between these reference weights and assigned weights are calculated. Grey relational coefficients are calculated to establish relationships between criteria and reference weights, measuring similarity. The Grey Relational Grade (GRG) reflects the overall closeness of criteria weights to the reference weights. Finally, grey relational grades are transformed into ROC weights, representing each criterion's relative importance in the evaluation process. This approach estimates aggregated group weights of criteria, distinguishing it from other applications of GRA that use expert-specified weights for evaluating alternatives. These steps for the computation of the GRA-ROC weighting method are as follows:

Step 1. Construct the criteria hierarchical structure and obtain the weighting data.

Step 2. Construct the grey weights matrix. The grey weights data matrix puts the data in a rectangular array for easy expression.

$$A = \begin{pmatrix} \otimes a_{1,1} & \otimes a_{1,2} & \cdots & \otimes a_{1,n} \\ \otimes a_{2,1} & \otimes a_{2,2} & \cdots & \otimes a_{2,n} \\ \vdots & \vdots & \ddots & \vdots \\ \otimes a_{m,1} & \otimes a_{m,2} & \cdots & \otimes a_{m,n} \end{pmatrix}. \tag{3.3}$$

where $\otimes a_{ij} = \left[\underline{a}_{ij}, \overline{a}_{ij}\right]$ represents the grey number weights provided by the j^{th} decision-maker (DM) for the i^{th} indicator or criterion, $1 \leq i \leq m$ and $1 \leq j \leq n$. Here, m and n denote the numbers of criteria and decision-makers, respectively.

Step 3. Standardize the weight matrix. A standardized weight matrix, S, is calculated, with the standardized element $\otimes S_{ij} = \left[\underline{s}_{ij}, \overline{s}_{ij}\right]$. Interval grey numbers are standardized using a norm based on the minimization of maximum distance [199]:

$$\left[\underline{s}_{ij}, \overline{s}_{ij}\right] = \left[\frac{\underline{a}_{ij}}{\|a_j\|}, \frac{\overline{a}_{ij}}{\|a_j\|}\right], \tag{3.4}$$

where

$$\|a_j\| = \max_{1 \leq i \leq m} \overline{a}_{ij}. \tag{3.5}$$

Step 4. Determine the reference weights for the DMs. The reference weights, also known as the ideal weights, represent the optimal weights indicated by the decision-makers [200]:

$$S_0 = \{\otimes S_{01}, \otimes S_{02}, ..., \otimes S_{0n}\}, \tag{3.6}$$

where

$$\otimes S_{0j} = \left[\max_{1 \le i \le m} \underline{s_{ij}}, \max_{1 \le i \le m} \overline{s_{ij}} \right]. \tag{3.7}$$

Step 5. Calculate the differences between the reference weights and the criteria weights. Calculating the differences between the reference weights and the criteria weights involves measuring the distances between two arbitrary interval numbers, as expressed in Equation (3.8):

$$\Delta_{ij} = \left| \otimes S_{0j} - \otimes S_{ij} \right| = \max \left(\left| \underline{s}_{0j} - \underline{s}_{ij} \right|, \left| \overline{s_{0j}} - \overline{s_{ij}} \right| \right). \tag{3.8}$$

Step 6. Calculate the grey relational grade. The Grey Relational Grade (r_i) is calculated from the grey relational coefficient (γ_{ij}) of the weights using the following formula:

$$r_i = \frac{1}{n} \sum\nolimits_{j=1}^{n} \gamma_{ij}, \tag{3.9}$$

where the grey relational coefficient is [61]

$$\gamma_{ij} = \frac{\min\limits_{1 \le i \le m} \min\limits_{1 \le j \le n} \Delta_{ij} + \zeta \max\limits_{1 \le i \le m} \max\limits_{1 \le j \le n} \Delta_{ij}}{\Delta_{ij} + \zeta \max\limits_{1 \le i \le m} \max\limits_{1 \le j \le n} \Delta_{ij}}. \tag{3.10}$$

Here, the grey distinguishing coefficient ζ is a value between 0 and 1, which reflects the relative emphasis on the minimum score compared to the maximum score. In this study, ζ is set to 0.5 [61].

Step 7. Transform the Grey Relational Grade to ROC weights. To transform the grey relational grades into ROC weights, the criteria are first ranked based on their grey relational grades, and then these rankings are converted into ROC weights using the following formula [196]:

$$W_{ROC} = \frac{1}{n} \sum_{q=1}^{n} \frac{1}{q}, \tag{3.11}$$

where, n represents the total number of criteria, and q signifies the rankings of the criteria.

The process of assigning weights to criteria involves ranking both the first-level and second-level criteria. First, the rankings of the first-level criteria are transformed into ROC weights, which are referred to as the first-level criteria weights (W_F). Subsequently, the second-level criteria within the same first-level criterion group are ranked relative to each other and also transformed into ROC weights, known as the second-level criteria weights (W_S). This hierarchical approach helps establish the importance of criteria at both the first and second levels, facilitating multi-criteria decision-making.

Step 8. Calculate the effective weights of the criteria. To calculate the effective weights of the criteria, one can obtain a visual representation of how each second-level criterion influences the overall goal, which makes up the evaluation of the business environment. The effective weights, denoted as (W_E), are computed by multiplying the first-level criteria weights (W_F) by the second-level criteria weights (W_S) for the second level. The formula for calculating the effective weights is as follows [99]:

$$W_E = W_F \times W_S. \tag{3.12}$$

This calculation will provide the effective weights for each criterion, indicating their influence on the overall evaluation of the MCDM problem.

3.3. Partial Least Squares Algorithm

The Partial Least Squares (PLS) model has a rich history spanning two decades, with its development occurring from the 1960s to the 1980s. This model was initially conceived by Herman Wold and further refined by his research team [201]. Herman's work initially focused on creating a set of methods capable of solving least squares regression problems. During the 1980s, the PLS method found numerous applications in fields such as economics and social sciences. It was later introduced to the field of chemistry, gaining significant acceptance within the scientific community [202].

In contrast to covariance-based Structural Equation Modeling (SEM), PLS modeling does not rely on the assumption of variables following a normal distribution. Moreover, its primary objective is not to minimize the difference between observed sample variance and theoretical model variance. Instead, PLS is geared towards predicting the dependent variables within constructs. Model accuracy is evaluated through data resampling and prediction error analysis. Covariance-based Structural Equation Modeling (SEM) operates under the assumption that the data accurately originate from a predefined theoretical model. Its core objective is to tightly fit the collected data to this theoretical model, which necessitates a strict alignment between the data and the model. Additionally, covariance-based SEM must adhere to various statistical assumptions, including those related to data distribution.

On the other hand, Partial Least Squares (PLS) takes a distinct approach by treating the measured data as a self-contained entity that can be directly interpreted. Instead of forcing the data to conform to a predetermined theoretical model, PLS focuses on understanding the data themselves. This approach serves as an effective dimension reduction technique, providing a concise and meaningful representation of dependent variables that explains the underlying data patterns. PLS is also versatile, capable of analyzing datasets with multiple tables or blocks of data.

Through this analysis, PLS identifies common factors and structures within the dataset, which can then be harnessed to create predictive models.

A very comprehensive explanation of the concepts, methods, and applications of PLS in SEM is given by Lohmöller [201] and Vinzi et al. [202]. The main idea of PLS is that the model is divided into two main parts: the inner model and the outer model. The inner model is the structural model that consists of relationships only between latent variables, while the outer model comprises the measurement variables that explain the latent variables. Mathematically, for a data set, X, with n number of observations (also know as manifest variable) and p number of variables, X can be represented as a matrix $n \times p$. If X is divided into J mutually exclusive blocks, i.e., latent variables, $LV_1, LV_2, \ldots, LV_J$, with block j having $n(j)$ variables, where the block index is represented as $j = 1, 2, 3, \ldots, J$, and the variable index in block j is represented as $k = 1, 2, 3, \ldots, n(j)$, then the linear relationship of the structural model is defined as follows [203]:

$$LV_j = \beta_0 + \sum_{i \to j} \beta_{ji} LV_i + \varepsilon_j. \tag{3.13}$$

From Equation (3.13), it can be observed that the PLS model can be approximated as a recursive algorithm, in which the paths formed by the inner model cannot create loops. The predictor specification of a PLS inner model, expressed as the conditional expectation of LV_j determined by LV_i, is given as follows:

$$E(LV_j | LV_i) = \beta_{0i} + \sum_{i \to j} \beta_{ji} LV_i. \tag{3.14}$$

This can also be viewed as a regression relationship with the following assumption:

$$\mathrm{cov}(LV_j, \varepsilon_j) = 0, \tag{3.15}$$

meaning there is no correlation between latent variables and the residuals, as well as zero covariance between LV_j and ε_j. Based on Equation (3.15), there are no assumptions about the distribution of the data.

The outer model is also referred to as the measurement model, and it establishes relationships solely between the latent variables (LV) and manifested variables. The outer model can consist of formative or reflective blocks.

The path diagram of a latent construct is shown in in Figure 3.2. For a formative block (Figure 3.2a), the observed variables are regarded as the cause of the latent variable. The formative construct can be represented as follows:

$$LV_j = \lambda_{0j} + \lambda_{jk} X_{jk} + \varepsilon_j. \tag{3.16}$$

Here, λ_{jk} represents the loadings, and λ_{0j} is the intercept. The conditional expected value is represented as follows:

$$E(LV_j | X_{jk}) = \lambda_{0j} + \lambda_{jk} X_{jk}.$$

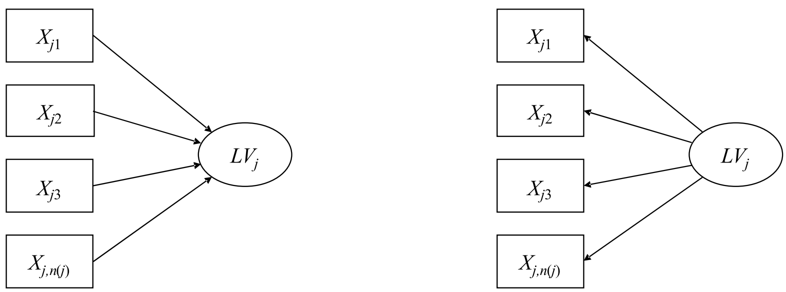
In a reflective block (Figure 3.2b), the latent variable is considered the cause of the observed variables. The reflective construct can be represented as follows:

$$X_{jk} = \lambda_{0jk} + \lambda_{jk} LV_j + \varepsilon_j. \tag{3.17}$$

The conditional expected value is expressed as follows:

$$E(X_{jk}|LV_j) = \lambda_{0jk} + \lambda_{jk} LV_j.$$

The PLS algorithm operates in stages. The first stage involves estimating the scores for the latent variables (LV), followed by the second stage, where the outer weights are estimated, and finally, the third stage, which focuses on estimating the path coefficients.



(**a**) Formative construct

(**b**) Reflective construct

Figure 3.2. Path diagram. Source: Reprinted from [204], used with permission.

3.3.1. First Stage

The first stage begins by estimating the latent variable scores. The LV scores are estimated as the weighted sum of their indicators [203]:

$$LV_j = \sum_k w_{jk} X_{jk}.$$

The iterative process starts with the initial weight values set to 1:

$$\begin{bmatrix} w_{11} & w_{12} & \dots & w_{1m} \\ w_{21} & w_{22} & \cdots & w_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ w_{n1} & w_{n2} & \dots & w_{nm} \end{bmatrix} = \begin{bmatrix} 1 & 1 & \cdots & 1 \\ 1 & 1 & \cdots & \vdots \\ \vdots & \vdots & \ddots & 1 \\ 1 & 1 & \cdots & 1 \end{bmatrix}.$$

Step 1. Outer approximation of the latent variable scores. The external weights are estimated as follows [203]:

$$LV_k \propto X_k \widetilde{w}_k,$$

where $\widetilde{w}_k$ represents the external weights, and α is used to indicate the dependency.

The vector form is as follows:

$$LV_j = \pm f_j \sum_k \widetilde{w}_{jk} X_{jk},$$

where f_j is a scalar used to standardize the values of Y_j, and

$$sign\left[\sum_k sign\{cor(X_{jk}, LV_j)\}\right],$$

is to maintain positive correlation with X_{jk}.

When the standardized LVs are as follows:

$$LV_j = \sum_k \widehat{w}_{jk} X_{jk},$$

where $\widehat{w}_{jk}$ represents the external weights.

Step 2. Estimate the inner weights. With the calculated initial scores Z_j of the latent variable, the weighted aggregate of the LV is given as follows:

$$Z_j = \sum_{i \leftrightarrow j} e_{ij} LV_i,$$

where there is an association between LV_j and LV_i, as indicated by the double arrowhead, and e_{ij} represents the inner weights.

The inner weights can be calculated using centroid, factor, or path schemes.

(a) Centroid Scheme: This scheme only considers the sign direction of the correlations between an LV and its adjacent LVs, given as follows:

$$e_{ji} = \begin{cases} sign\left[cor(LV_j, LV_i)\right] & \text{if } LV_j, LV_i \text{ are adjacent} \\ 0 & \text{otherwise} \end{cases}$$

This scheme is susceptible to errors when a correlation value is close to zero, resulting in continuous sign changes from positive to negative (+1 to −1, and vice versa) during the iterative process.

(b) Factor Scheme: This scheme uses the correlation coefficient, which considers both the sign direction and the strength of the path:

$$e_{ji} = \begin{cases} cor(LV_j, LV_i) & \text{if } LV_j, LV_i \text{ are adjacent} \\ 0 & \text{otherwise} \end{cases}$$

(c) Path Scheme: This scheme takes into account the direction of the relationships between the LVs. However, it is prone to issues with a singular matrix. The advantage of this scheme is that both the direction and the strength of the path are considered.

Step 3. Inner approximation of the latent variable scores:

$$Z_j = \sum_{i \leftrightarrow j} e_{ij} LV_i.$$

Step 4. Estimate the outer weights using the internal estimate for a reflective block:

$$\overline{w}_{jk} = \frac{1}{(LV_j' LV_j)^{-1}} LV_j' X_{jk},$$

and for a formative block:

$$\widetilde{w}_j = \frac{1}{(X_j' X_j)^{-1}} LV_j' X_j.$$

Then, repeat steps 1 to 4 until convergence. Convergence is reached when the following apply:

$$|w_{jk}^{S-1} - w_{jk}^{S}| < 10^{-5},$$

where the steps for iteration are denoted as $S = 1, 2, 3, \ldots$.

3.3.2. Second Stage

The outer weights/loadings and path coefficients are estimated using both single and multiple Ordinary Least Squares (OLS) regressions.

The latent variable scores are calculated as the weighted sum of all manifested variables [203]:

$$LV_j = \sum_k w_{jk} X_{jk}.$$

Here, w_{jk} represents the outer weights.

The weights/loadings and path coefficients can be mathematically represented as:

$$LV_j = \sum_{i \rightarrow j} B_{ji} LV_i,$$

where the path coefficient estimate, B_{ji}, is given by the following:

$$B_{ji} = (LV_i' LV_i)^{-1} LV_i' LV_j.$$

This estimation is performed using OLS in the multiple regression of LV_j on LV_i.

3.3.3. Third Stage

In this stage, the loadings are calculated as correlations between a latent variable and its indicators [203]:

$$\lambda_{jk} = \mathrm{cor}(X_{jk}, LV_j).$$

The goal is to estimate the unobserved variables as a linear combination of the observed variables, taking into account the relationships between the structural and measurement models that best explain both the observed and unobserved variables. The PLS algorithm is employed to calculate the weights of the unobserved variables as a linear combination based on the specifications of the structural and measurement models. These weights and loadings are estimated iteratively until convergence is achieved. The combination of inside and outside approximation elucidates the inner and outer relationships.

3.4. Application of GRA-ROC Method in Evaluating Business Environment

The African business environment is evaluated in this section. Countries in Africa are in significantly different stages of development. Africa's growth potential remains untapped, with over a billion people, accounting for about 16% of the world's population [205]. In many African countries, young people make up nearly three-quarters of the population, making job creation a crucial element on the continent. Companies of all sizes can absorb this population growth and market potential. To achieve a high absorption rate of this population, Africa needs a healthy business environment to grow, thrive, and create jobs. The medium-term outlook for Africa is promising, supported by high commodity prices, growing domestic demand, eased infrastructure constraints, and stronger trade and investment ties. The potential for success is evident when considering emerging economies and improving global economic and regional business environments. However, it is important to note that the prospects for medium-term growth on the continent are still at risk of worsening, especially due to adverse events in the global economy, external shocks, variations in weather conditions, political instability, and civil unrest in some countries [206].

The complexity of the African business environment is rising, partly due to issues such as bribery of foreign public officials. Civil wars and coup threats are prevalent in certain regions, and even democratic leaders often seek to extend their presidential terms. Although democracy faces challenges in many places, Africa has made significant progress in becoming a more democratic continent. Africa stands out as one of the most dynamic regions in the world economy, maintaining sustained growth of 5% or more for over a decade. Some countries are even experiencing growth rates in the double digits, exceeding 10%. Consequently, the African continent has become an attractive destination with a multitude of opportunities across various fields [207].

According to Egan [208], a significant factor that has led to the underdevelopment of most African regions is the legacy of regional colonialism. Many African governments initially pursued import-oriented strategies that failed to evolve into export-driven industrialization. Despite having 16% of the world's population, Africa accounts for only 1% of global trade. The challenges within Africa can be attributed to the legacies left by colonial powers and issues related to governance and political systems, as well as the fragile macroeconomic environment.

Michalopoulos and his colleagues [209] have demonstrated a relationship between pre-colonial political centralization and development. Their study provides evidence of a positive correlation between statehood and development in Africa.

Cross-border banking in Africa dates back to the colonial era, with some of these early African banks facilitating trade. Over the years, African banks have expanded their geographical footprint. The role of the financial system in Africa has deepened, thanks to improved cross-border banking services and enhanced monetary policies [210]. For instance, Ecobank operates in 32 countries across Africa, while South Africa's bank has a presence in 16 countries, both boasting the highest number of registered formal businesses [210]. Africa's financial technological innovation has introduced mobile banking that does not require a traditional bank account, agricultural insurance schemes based on rainfall data, or micro-deposit institutions to partner with supermarkets and post offices to deliver financial services [211]. However, inadequate and poor infrastructure in some parts of Africa remains a significant obstacle to the development of competitive industries in the region. It is estimated that Africa loses 1% of per capita economic growth annually due to its infrastructure deficit. Poor infrastructure increases commercial costs and reduces the productivity of African firms by approximately 40%. This challenge is particularly evident in sectors such as energy, water supply, transportation, and communications infrastructure, all of which are crucial for the success of manufacturing businesses [212].

While some African countries have recently experienced positive economic development, they continue to rank poorly in almost every aspect of a business environment that is critical for long-term success. Bah and Fang [212] conducted research on the output productivity of the African business environment and found that the poor business environment is hindering Africa's development. Many firms experience significant profit losses due to government regulations, which misallocate resources and negatively impact productivity. Insufficient infrastructure and unrealistic government regulations also lead to reduced sales, as businesses struggle to produce efficiently. Moreover, a poor business environment creates fertile ground for corruption, crime, and social vices, all of which contribute to the underperformance of some countries in terms of economic growth.

The business environment for industrialization in Africa has undergone significant transformation and can be considered a new environment initiated to promote industrialization in the twentieth century. Multilateral trade agreements, bilateral agreements, and regional trade agreements have reduced the policy space available for promoting industrial development in these countries. Additionally, the global production environment has been transformed to the extent that companies are increasingly facing stiff competition in global export markets due to the reduction in tariff and non-tariff barriers to trade in products [213]. The factors influencing the business environment in Africa include institutional changes that contribute to stability and the recognition of informal institutions. Furthermore, with the recent advancements in information technology, more Africans are becoming involved in online marketing businesses, marking a notable development in this sector.

Africa represents an untapped market with a growing Gross Domestic Product (GDP) per capita and a stabilizing investment environment. The continent boasts a diverse market of over one billion consumers, with increasing demand for various imported products and a growing need for local goods, driven by urbanization and busy lifestyles. Notably, countries such as Algeria, Angola, Ghana, Mauritius, and Nigeria are experiencing exceptionally high projected growth rates, with increasingly affluent consumers who are spending on imported goods. These nations have a wide income range, and their consumption levels are common indicators of higher growth and stability. The substantial level of consumption presents opportunities for foreign investors to explore [214]. However, one of the most significant challenges facing many African countries is the need to improve governance within their institutions and establish a harmonized and attractive business environment. Without the right conditions, lasting development and emergence are difficult to achieve. Africa, as a continent, presents both challenges and opportunities [214].

3.4.1. Valid Criteria for Africa

In the validation of the structure, a covariance-based Structural Equation Model (SEM) is not suitable. Therefore, we choose to employ the Partial Least Squares (PLS) method, which aims to explain the variance in the structure. This choice is made due to the substantial number of variables, totaling 47, for both the first-level and second-level criteria. Using covariance-based SEM in this context can often result in unreliability and issues with convergence [215]. The PLS model comprises formative constructs for the first-level criteria and a reflective construct for assessing the business environment in Africa, making PLS algorithm the appropriate choice [216].

Model validation comprises two distinct parts: the reflective and formative components. In addition to data provided by the World Bank, the African Development Bank (AfDB) operates data platforms across all African countries, further contributing to the comprehensive standards used in evaluating business environments across the continent. The United Nations grouped 54 countries as the member states of the African continent [205]. The countries evaluated in Africa are as follows: Algeria, Angola, Benin, Botswana, Burkina Faso, Burundi, Cameroon, Cape Verde, Central African Republic, Chad, Comoros, Republic of the Congo, Democratic Republic of the Congo, Côte d'Ivoire, Djibouti, Egypt, Equatorial Guinea, Eritrea, Ethiopia, Gabon, Gambia, Ghana, Guinea, Guinea-Bissau, Kenya, Lesotho, Liberia, Libya, Madagascar, Malawi, Mali, Mauritania, Mauritius, Morocco, Mozambique, Namibia, Niger, Nigeria, Rwanda, São Tomé and Príncipe, Senegal, Seychelles, Sierra Leone, South Africa, South Sudan, Sudan, Swaziland, Tanzania, Togo, Tunisia, Uganda, Zambia, and Zimbabwe. In total, 53 countries are evaluated. Somalia is excluded from the evaluation of the business environment in Africa because the World Bank has not provided data on Somalia over the years. These countries are indexed from 1 to 53 in alphabetical order. Algeria has an index number of 1, Angola has an index number of 2, and Zimbabwe has an index number of 53 (see the index column of Table A3 in the Appendix A.1.2). The names of these countries in these study are alphabetically listed and thus refer to the African business environment.

The formative aspects pertain to the performances of all African countries, while the reflective part involves the Distance to Frontier (DTF) values of each country. DTF scores are assigned to every country to measure their relative performance in comparison to the best-performing country at the time of measurement. This score serves to evaluate a country's absolute performance over time and is rated on a scale from 0 to 100. A score of 0 indicates the farthest distance from the best-performing country, while a score of 100 signifies that the country is on par with the best-performing country, with no distance between them. These scores provided to all countries reflect their business environment relative to other nations. For example, a country with a DTF score of 80 points on an indicator is 20% away from being on par with the best-performing country for that specific indicator. Since DTF is a relative score that measures a country's performance concerning an indicator, it is a reflective measurement. In other words, high DTF scores do not cause a country's business environment to be considered good. Instead, a good business environment will result in high DTF scores if the country performs better than others.

Statistical software plays a crucial role in data analysis. The commonly used Statistical Package for the Social Sciences (SPSS), specifically version 23, does not include the implementation of the PLS algorithm suitable for formative analysis. However, there are alternative statistical software options available. These include PLS-Graph, the Partial Least Squares—Path Modelling (PLS-PM) package by the R Foundation for Statistical Computation, and SmartPLS [217]. In this application of PLS, we choose to utilize SmartPLS, specifically version 3.2.3. The standard analysis procedure begins with data screening. In this process, certain criteria are applied to determine which variables will be included in the analysis. One of the first criteria for removal from the analysis is *Getting Electricity* (A_3) due to the presence of missing values in 25% of the cases. These missing values suggest potential bias issues in the recording of *Getting Electricity*-related data in many African countries. Similarly, the *labour Tax and Contribution* ($A_{7\text{-}4}$) and *Other Tax* ($A_{7\text{-}5}$) indicators are excluded from the analysis due to a significant amount of missing data, totaling 87.5%. As a result, the number of first-level indicators is reduced from 10 to 9, and that of second-level indicators is reduced from 37 to 33. Table 3.1 provides details on the parameter settings used in running the PLS algorithm. Following the data screening, the analysis is conducted with a reduced dataset containing 418 cases. The PLS algorithm employs a mean of 0 and a variance of 1, with an initial weight of 1 using the path weighting method. To ensure robust results, a Bias-Corrected and Accelerated (BCa) bootstrap method is used to generate 10,000 samples, with a significance level of 0.05 using a two-tailed test. Bootstrapping is a statistical technique that involves randomly generating samples with replacement from the original data. This allows us to determine the significance of the criteria and the loadings of the second-level indicators on the first-level indicators. It is worth noting that in PLS-SEM, there is no assumption that the data follow a normal distribution, making it suitable for various types of data.

Table 3.1. Parameter settings for data analysis.

Specification	Value
Data Cases	418
Missing Data	None
PLS Algorithm Data	Mean = 0, Variance = 1
Initial Weights	1
Weighting Method	Path
Bootstrap Samples	10,000
Significance Level	0.05
Test Type	Two Tailed
Confidence Interval Method	Bias-Corrected and Accelerated (BCa) Bootstrap

Source: Reprinted from [204], used with permission.

Analysis of the DBP Model for Africa

In the DBP model, the latent variables (LV) represent the overall goal and the first-level indicators, while the measured variables pertain to the second-level indicators, as explained in Figure 3.3 and detailed in Section 2.1.

In formative measurement, the assessment is primarily based on reliability and validity. The reliability of constructs is evaluated using composite reliability, which assesses internal consistency. Unlike Cronbach's alpha, composite reliability does not assume that the indicators themselves are reliable, making it particularly well suited for Partial Least Squares (PLS) analysis. To identify significant criteria, bootstrapping is employed. This technique helps us determine which criteria have a meaningful impact. The weights assigned to each criterion provide insights into their respective contributions to each latent variable (LV). In Figure 3.4, the loadings (λ) and their associated significance levels (p-value in parentheses) for all the criteria can be observed. Latent variables (LVs) are depicted as circles, while observed/measured variables are represented as rectangles.

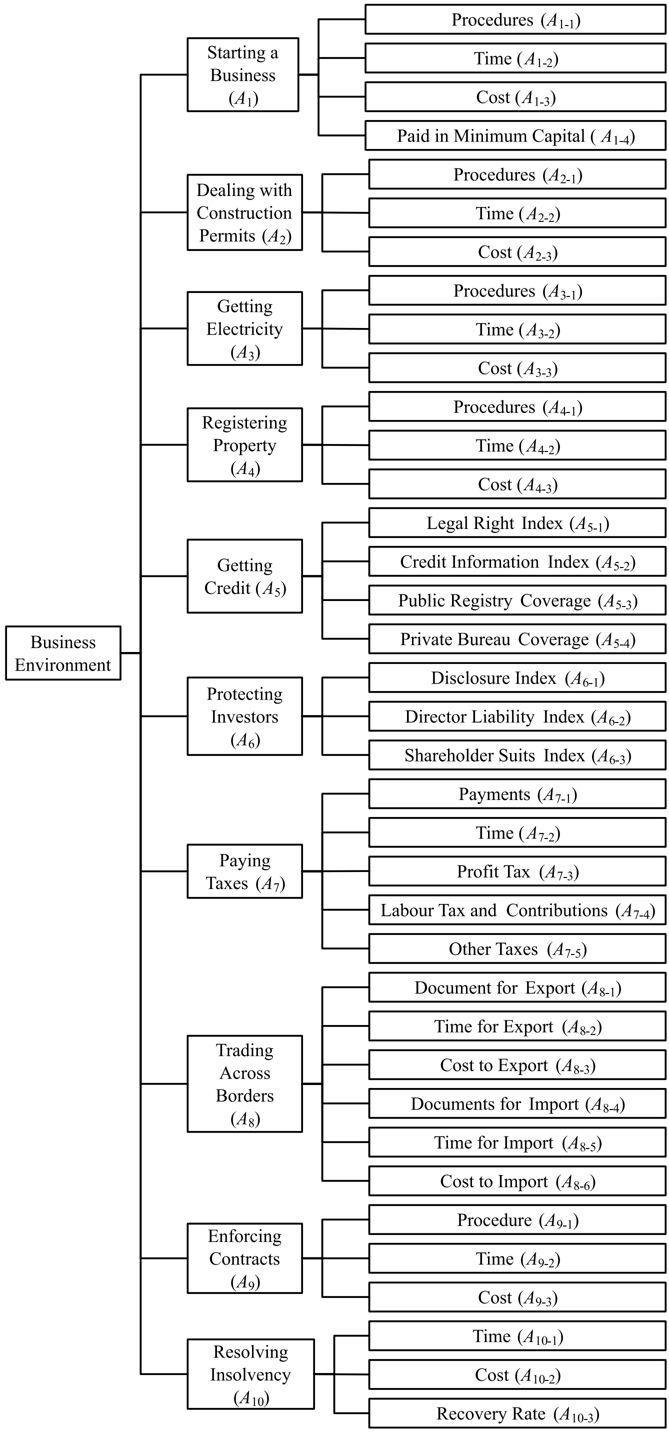


Figure 3.3. DBP hierarchical model. Source: Reprinted from [204], used with permission.

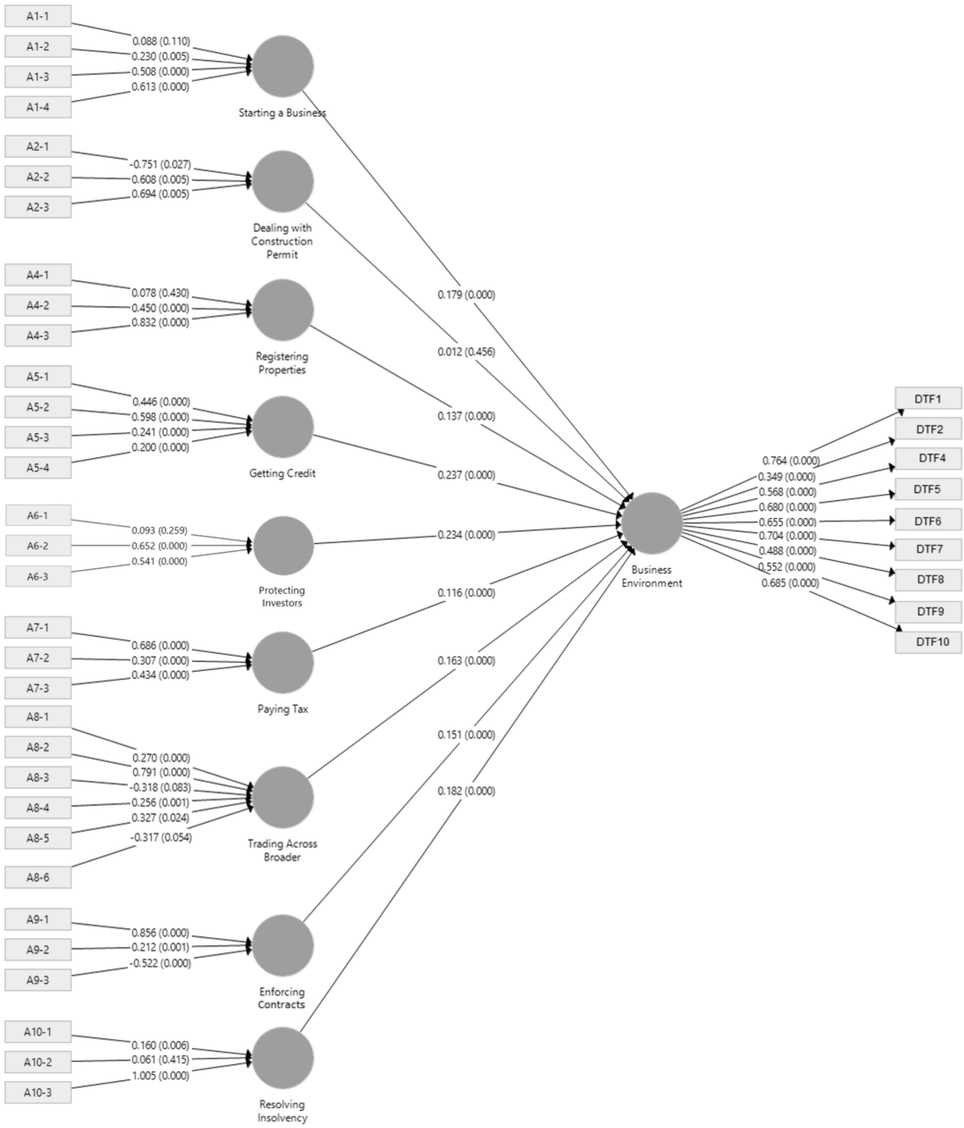


Figure 3.4. DBP model showing the weights, loadings, and p-values. Source: Reprinted from [204], used with permission.

Based on Equation (3.16), we can represent the formative constructs as follows. Starting a business:

$$LV_1 = \lambda_{0,1} + \lambda_{1,1\text{-}1} A_{1\text{-}1} + \lambda_{1,1\text{-}2} A_{1\text{-}2} + \lambda_{1,1\text{-}3} A_{1\text{-}3} + \lambda_{1,1\text{-}3} A_{1\text{-}4} + \varepsilon_1.$$

Dealing with construction permits:

$$LV_2 = \lambda_{0,2} + \lambda_{2,2\text{-}1} A_{2\text{-}1} + \lambda_{2,2\text{-}2} A_{2\text{-}2} + \lambda_{2,2\text{-}3} A_{2\text{-}3} + \varepsilon_2.$$

Registering Properties:

$$LV_4 \;=\; \lambda_{0,4}+\lambda_{4,4\text{-}1}A_{4\text{-}1}+\lambda_{4,4\text{-}2}A_{4\text{-}2}+\lambda_{4,4\text{-}3}A_{4\text{-}3}+\varepsilon_4.$$

Getting Credit :

$$LV_5 \;=\; \lambda_{0,5}+\lambda_{5,5\text{-}1}A_{5\text{-}1}+\lambda_{5,5\text{-}1}A_{5\text{-}2}+\lambda_{5,5\text{-}1}A_{5\text{-}3}+\lambda_{5,5\text{-}1}A_{5\text{-}4}+\varepsilon_5.$$

Protecting Investors:

$$LV_6 \;=\; \lambda_{0,6}+\lambda_{6,6\text{-}1}A_{6\text{-}1}+\lambda_{6,6\text{-}2}A_{6\text{-}2}+\lambda_{6,6\text{-}3}A_{6\text{-}3}+\lambda_{6,6\text{-}4}A_{6\text{-}4}+\varepsilon_6.$$

Paying Tax:

$$LV_7 \;=\; \lambda_{0,7}+\lambda_{7,7\text{-}1}A_{7\text{-}1}+\lambda_{7,7\text{-}1}A_{7\text{-}2}+\lambda_{7,7\text{-}1}A_{7\text{-}3}+\varepsilon_7.$$

Trading Across Borders:

$$\begin{aligned} LV_8 \;&= \lambda_{0,8}+\lambda_{8,8\text{-}1}A_{8\text{-}1}+\lambda_{8,8\text{-}3}A_{8\text{-}2}+\lambda_{8,8\text{-}3}A_{8\text{-}3} \\ &\quad +\lambda_{8,8\text{-}4}A_{8\text{-}4}+\lambda_{8,8\text{-}5}A_{8\text{-}5}+\lambda_{8,8\text{-}6}A_{8\text{-}6}+\varepsilon_8 \end{aligned}$$

Enforcing Contracts:

$$LV_9 = \lambda_{0,9}+\lambda_{9,9\text{-}1}A_{9\text{-}1}+\lambda_{9,9\text{-}2}A_{9\text{-}2}+\lambda_{9,9\text{-}1}A_{9\text{-}3}+\varepsilon_9.$$

Resolving Insolvency:

$$LV_{10} = \lambda_{0,10}+\lambda_{10,10\text{-}1}A_{10\text{-}1}+\lambda_{10,10\text{-}2}A_{10\text{-}2}+\lambda_{10,10\text{-}3}A_{10\text{-}3}+\varepsilon_{10}.$$

Business environments:

$$\begin{aligned} LV_0 =&\lambda_{0,0}+\lambda_{0,1}A_1+\lambda_{0,2}A_2+\lambda_{0,4}A_4+\lambda_{0,1}A_1+\lambda_{0,5}A_5 \\ &+\lambda_{0,6}A_6+\lambda_{0,7}A_7+\lambda_{0,8}A_8+\lambda_{0,9}A_9+\lambda_{0,10}A_{10}+\varepsilon_{10} \end{aligned}.$$

Based on Equation (3.17), the reflective part is represented as follows:

$$DTF_1 = \lambda_{0,1,0}+\lambda_{1,1}LV_0+\varepsilon_{1,1}$$

$$DTF_2 = \lambda_{0,2,0}+\lambda_{2,2}LV_0+\varepsilon_{2,2}$$

$$\vdots$$

$$DTF_{10} = \lambda_{0,10,0}+\lambda_{10,10}LV_0+\varepsilon_{10,10}.$$

The summary of the initial analysis results is presented in Table 3.2, which provides a comprehensive overview of key findings and metrics. Additionally, the quality and goodness-of-fit of the model are assessed and reported in Table 3.3, shedding light on the model's overall performance and reliability.

Table 3.2. Path coefficient of the first-level indicators.

Business Environment	Original Sample (*O*)	Sample Mean (*M*)	Standard Deviation (*STDEV*)	*T*- Statistics ($\|O/STDEV\|$)	*p*-Values
Dealing with Construction Permits	0.012	0.012	0.016	0.764	0.456
Enforcing Contracts	0.151	0.15	0.022	6.833	0
Getting Credit	0.237	0.237	0.021	11.285	0
Paying Tax	0.116	0.118	0.018	6.571	0
Protecting Investors	0.234	0.229	0.024	9.915	0
Registering Properties	0.137	0.138	0.031	4.47	0
Resolving Insolvency	0.182	0.179	0.019	9.743	0
Starting a Business	0.179	0.181	0.016	11.318	0
Trading Across Borders	0.163	0.161	0.026	6.32	0

Source: Reprinted from [204], used with permission.

Table 3.3. Quality of the DBP model for evaluating business in Africa.

Parameters	Original Sample (*O*)	Sample Mean (*M*)	Standard Deviation (*STDEV*)	T- Statistics ($\|O/STDEV\|$)	*p*-Values
R^2	0.943	0.944	0.007	132.102	0
Adjusted R^2	0.941	0.943	0.007	129.078	0
AVE	0.381	0.381	0.016	24.253	0
Cronbach's Alpha	0.789	0.788	0.014	55.182	0
Composite Reliability	0.842	0.841	0.009	90.16	0
SRMR Common Factor Model	0.144	0.148	0.004	39.299	0
SRMR Composite Model	0.126	0.131	0.003	38.63	0

Source: Reprinted from [204], used with permission.

From Table 3.3, it is evident that the *p*-value of the path coefficient from *Dealing with Construction Permit* to *Evaluating Business Environment* is relatively high, measuring 0.455. This value indicates a weak statistical significance for this particular path. To gain insights into the degree of correlation between variables, we use the Variance Inflation Factor (VIF). A high VIF suggests that a variable may be linearly predictable from other variables, potentially indicating overlapping information. Keeping such variables may result in a lack of unique variance in the respective first-level indicator. The following are the VIF values for the inner model variables: Dealing with Construction Permits (A_2) = 1.176; Enforcing Contracts (A_9) = 1.505; Getting Credit (A_5) = 1.689; Paying Tax (A_7) = 1.892; Protecting Investors (A_6) = 1.807; Registering Properties (A_4) = 1.477; Resolving Insolvency (A_{10}) = 1.665; Starting a Business (A_1) = 1.794; Trading Across Borders (A_8) = 1.676; Additionally, we compute the VIF values for the outer model variables as follows: DTF_1 = 1.860, $A_{1\text{-}1} = 1.591$, $A_{1\text{-}2} = 1.672$, $A_{1\text{-}3} = 1.193$, $A_{1\text{-}4} = 1.054$, $DTF_2 = 1.531$, $A_{2\text{-}1} = 1.072$, $A_{2\text{-}2} = 1.042$, $A_{2\text{-}3} = 1.051$, $DTF_4 = 1.732$, $A_{4\text{-}1} = 1.004$, $A_{4\text{-}2} = 1.020$, $A_{4\text{-}3} = 1.017$, $DTF_5 = 1.697$, $A_{5\text{-}1} = 1.056$, $A_{5\text{-}2} = 2.454$, $A_{5\text{-}3} = 1.400$, $A_{5\text{-}4} = 2.325$, $DTF_6 = 1.575$, $A_{6\text{-}1} = 4.466$, $A_{6\text{-}2} = 6.961$, $A_{6\text{-}3} = 4.776$, $DTF_7 = 1.635$, $A_{7\text{-}1} = 1.163$, $A_{7\text{-}2} = 1.135$, $A_{7\text{-}3} = 1.026$, $DTF_8 = 1.449$, $A_{8\text{-}1} = 2.270$, $A_{8\text{-}2} = 6.093$, $A_{8\text{-}3} = 11.314$, $A_{8\text{-}4} = 2.372$,

$A_{8\text{-}5} = 6.220$, $A_{8\text{-}6} = 10.010$, $DTF_9 = 1.507$, $A_{9\text{-}1} = 1.027$, $A_{9\text{-}2} = 1.041$, $A_{9\text{-}3} = 1.024$, $DTF_{10} = 1.503$, $A_{10\text{-}1} = 1.477$, $A_{10\text{-}2} = 1.488$, $A_{10\text{-}3} = 1.026$. Notably, variables $A_{6\text{-}2}$, $A_{8\text{-}2}$, $A_{8\text{-}5}$, and $A_{8\text{-}6}$ all exhibit VIF values exceeding 5, with $A_{8\text{-}6}$ having the highest VIF at 10.010. These variables warrant further investigation. Furthermore, the p-value of *Dealing with Construction Permits* (A_2) is 0.456, prompting additional scrutiny.

Improved Hierarchical Model for Evaluating Africa

The improved evaluation hierarchical model is the results of validating the DBP model (Figure 3.3). The following changes were made after a series of iterative processes: The *Getting Electricity* (A_3) indicator was initially removed due to a high percentage of missing values, suggesting potential data bias. The *Dealing with Construction Permits* (A_2) indicator was subsequently removed after several iterations. It was found that the number of *Procedures* ($A_{2\text{-}1}$), with a weight of -0.751, contributed negligibly to the construct . Additionally, indicators related to the *Time* ($A_{2\text{-}2}$) and *Costs* ($A_{2\text{-}3}$) of Dealing with Construction Permits (A_2) exhibited inconsistencies. The *Dealing with Construction Permits* (A_2) was also removed due to its high p-value of 0.456, suggesting that it did not significantly contribute to the hierarchical structure. *Trading Across Borders* (A_8) was removed based on a comprehensive assessment of its first-level indicators, considering factors such as VIF, loadings, weights, and p-values. The number of *Procedures* ($A_{1\text{-}1}$ and $A_{4\text{-}1}$) for *Starting a Business* (A_1) and *registering property* (A_4) were removed as their contribution was negligible, less than 0.1. *Trading Across Borders* (A_8) was removed due to its negative contribution and inconsistency with its expected role in evaluating the business environment in Africa. The cost of resolving insolvency ($A_{9\text{-}3}$) was dropped due to its negative contribution. The *Cost* ($A_{10\text{-}2}$) for *Resolving Insolvency* (A_{10}) was removed as it had a p-value of 0.415. Throughout these iterations, the following second-level indicators were also dropped: $A_{1\text{-}1}$, $A_{2\text{-}1}$, $A_{2\text{-}3}$, $A_{3\text{-}1}$, $A_{3\text{-}2}$, $A_{3\text{-}3}$, $A_{4\text{-}1}$, $A_{7\text{-}4}$, $A_{7\text{-}5}$, $A_{9\text{-}3}$, $A_{10\text{-}2}$. Additionally, the composite reliability of the original sample, which measured at 0.85, falls within the recommended range of 0.7 to 0.9 [218]. This reliability assessment ensures the robustness of the model.

Assessment of the improved model based on the following statistical measures was employed: Average Variance Extracted (AVE): AVE assesses convergent validity by calculating the average amount of variance explained by the indicators. Notably, the AVE increased from 0.356 to 0.450, indicating a significant increment of 26.4%. This suggests that the latent construct can now explain 26.4% more variance than the DBP model. Cross-loadings: As the construct has only a single reflective latent variable, there are no cross-loadings to consider. Cronbach's alpha: This measure evaluates the extent to which a group of measured variables fits the latent variable. It reflects the average inter-variable correlation between the DTF and the first-level indicators. A Cronbach's alpha above 0.7 is considered satisfactory. R^2 (R-squared): R^2 explains the variance between the latent variables in the structural model. For both models, the R^2 value is approximately 0.94, indicating a satisfactory level of variance explanation. p-values: All p-values in the improved model are below 0.05, indicating statistical significance. Table 3.4 provides details on the Cronbach's alpha, R^2, T-statistics, and p-values for the business environment (LV_0) in the improved model. Additionally, the significance level of all criteria in the improved model is illustrated in Figure 3.5.

Table 3.4. Quality of improved evaluation hierarchical model for Africa.

Parameters	Original Sample (*O*)	Sample Mean (*M*)	Standard Deviation (*STDEV*)	*T*-Statistics ($\lvert O/STDEV \rvert$)	*p*-Values
R^2	0.943	0.945	0.005	176.697	0
Adjusted R^2	0.942	0.944	0.005	173.553	0
AVE	0.45	0.451	0.018	25.107	0
Cronbach's alpha	0.794	0.794	0.015	51.925	0
Composite Reliability	0.85	0.85	0.01	89.101	0
SRMR Common Factor Model	0.129	0.132	0.004	34.614	0
SRMR Composite Model	0.114	0.117	0.004	30.535	0

Source: Reprinted from [204], used with permission.

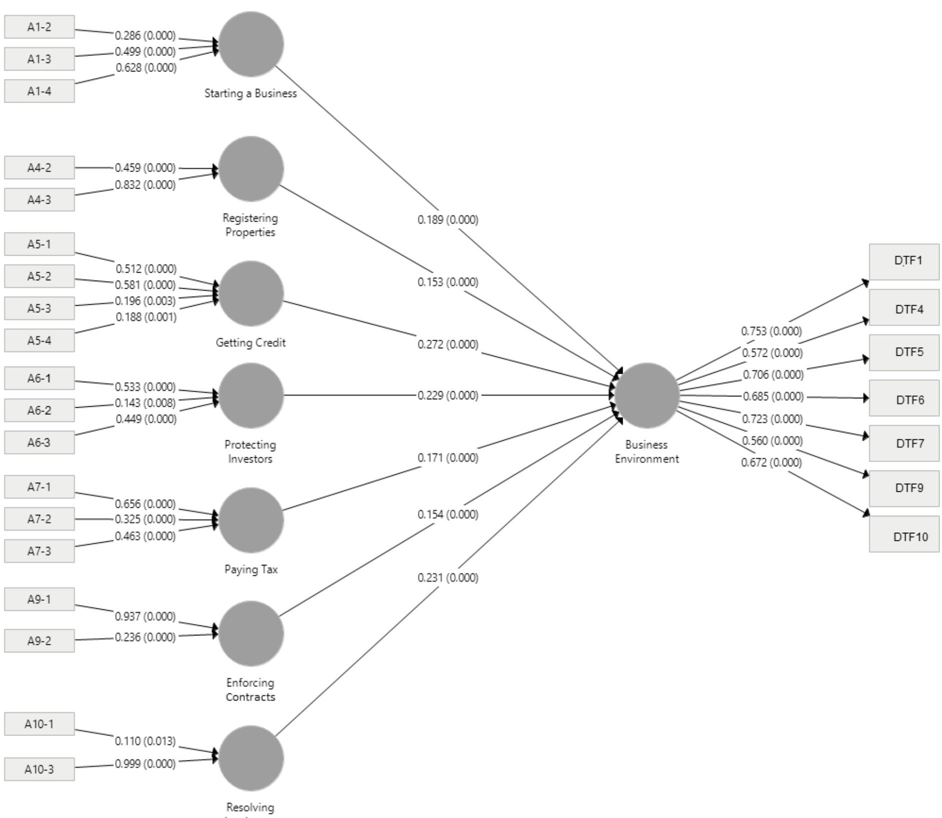


Figure 3.5. Results of improved evaluation hierarchical model. Source: Reprinted from [204], used with permission.

To evaluate multicollinearity among independent variables, we utilize two key measures: Tolerance and the Variance Inflation Factor (VIF). The tolerance measure for an independent variable (i) is calculated as 1 minus the proportion of variance it shares with other independent variables in the analysis. The Variance Inflation Factor (VIF) can be defined as the reciprocal of tolerance [219]. The Standard Root Mean Square Residual ($SRMR$) is the square root of the discrepancy between the sample covariance matrix and the model covariance matrix. While $SRMR$ is not typically required for testing formative constructs, it is reported in Table 3.5 using Table 3.6 as the reference liguistic scale. Now, the VIF is the reciprocal of tolerance and provides insights into the degree of multicollinearity. In our analysis, VIF values within the recommended range of $0.2 < VIF < 5$ indicate an acceptable level of multicollinearity. The VIF values for the inner model, specifically in the context of Evaluating Business Environment, are as follows: Enforcing Contracts, (A_9) = 1.352; Getting Credit, (A_5) = 1.685; Paying Tax, (A_7) = 1.828; Protecting Investors, (A_6) = 1.855; Registering Properties, (A_4) = 1.445; Resolving Insolvency, ($A10$) = 1.446; and Starting a Business (A_1) = 1.731. On the other hand, the VIF for the outer model is as follows: $DTF_1 = 1.678$, $A_{1\text{-}2} = 1.135$, $A_{1\text{-}3} = 1.185$, $A_{1\text{-}4} = 1.054$, $DTF_4 = 1.324$, $A_{4\text{-}2} = 1.017$, $A_{4\text{-}3} = 1.017$, $DTF_5 = 1.520$, $A_{5\text{-}1} = 1.056$, $A_{5\text{-}2} = 2.454$, $A_{5\text{-}3} = 1.400$, $A_{5\text{-}4} = 2.325$, $DTF_6 = 1.571$, $A_{6\text{-}1} = 2.493$, $A_{6\text{-}2} = 1.669$, $A_{6\text{-}3} = 3.479$, $DTF_7 = 1.578$, $A_{7\text{-}1} = 1.163$, $A_{7\text{-}2} = 1.135$, $A_{7\text{-}3} = 1.026$, $DTF_9 = 1.305$, $A_{9\text{-}1} = 1.023$, $A_{9\text{-}2} = 1.023$, $DTF_{10} = 1.430$, $A_{10\text{-}1} = 1.003$, $A_{10\text{-}3} = 1.003$.

Table 3.5. Raw data from decision-makers.

Decision-Makers/Criteria	DM_1	DM_2	DM_3	DM_4	DM_5	DM_6	DM_7
A_1	I	E	I	E	S	M	E
A_4	M	E	E	E	I	M	E
A_5	I	I	I	E	M	I	E
A_6	E	E	E	E	E	E	E
A_7	E	I	E	E	M	I	M
A_9	I	I	E	I	E	E	E
A_{10}	M	M	E	I	M	M	E
$A_{1\text{-}2}$	E	M	I	I	E	I	I
$A_{1\text{-}3}$	I	U	E	I	M	E	M
$A_{1\text{-}4}$	M	E	I	I	M	M	M
$A_{4\text{-}2}$	E	I	I	I	E	E	E
$A_{4\text{-}3}$	I	E	I	I	M	M	I
$A_{5\text{-}1}$	E	E	E	E	E	E	E
$A_{5\text{-}2}$	I	E	E	E	E	I	I
$A_{5\text{-}3}$	M	I	M	E	I	I	I
$A_{5\text{-}4}$	M	S	I	E	I	I	E
$A_{6\text{-}1}$	E	I	I	E	I	E	I
$A_{6\text{-}2}$	E	E	E	I	I	E	E
$A_{6\text{-}3}$	M	I	E	I	E	E	E
$A_{7\text{-}1}$	M	M	I	E	E	M	I
$A_{7\text{-}2}$	M	M	M	I	M	I	M
$A_{7\text{-}3}$	E	I	E	I	I	I	I

Table 3.5. *Cont.*

Decision-Makers/Criteria	DM_1	DM_2	DM_3	DM_4	DM_5	DM_6	DM_7
$A_{9\text{-}1}$	M	I	I	I	I	E	E
$A_{9\text{-}2}$	I	M	I	M	E	I	E
$A_{10\text{-}1}$	M	M	I	I	I	M	I
$A_{10\text{-}3}$	M	I	E	E	E	E	E

Source: Reprinted from [204], used with permission.

Table 3.6. Linguistic variables and grey numbers for weights.

Linguistic Variables	Notation	Grey Numbers
Unimportant	U	[0, 0.2]
Somewhat Important	S	[0.2, 0.4]
Moderately Important	M	[0.4, 0.6]
Important	I	[0.6, 0.8]
Extremely Important	E	[0.8, 1]

Source: Reprinted from [204], used with permission.

In the formative part of our analysis, traditional measures like consistency reliability and convergent validity are not applicable. This is due to the assumption that there are no errors associated with the evaluation criteria. Instead, we focus on verifying whether the second-level criteria genuinely contribute to the formation of the first-level criteria. The level of significance obtained from the bootstrapped samples is presented in Table 3.7, with a threshold of $p < 0.005$ indicating statistical significance. Moreover, the high R^2 value of 0.943 for the model indicates that the path coefficient is significant, further validating our analysis. Rigorous analysis processes ensure the reliability and robustness of the findings in the formative part.

Table 3.7. Bootstrapping output results of improved evaluation hierarchical model.

Business Environment	Original Sample (O)	Sample Mean (M)	Standard Deviation ($STDEV$)	T-Statistics ($\|O/STDEV\|$)	p-Values
Enforcing Contracts	0.154	0.154	0.015	10.102	0
Getting Credit	0.272	0.271	0.021	12.856	0
Paying Tax	0.171	0.171	0.017	10.275	0
Protecting Investors	0.229	0.226	0.023	9.977	0
Registering Properties	0.153	0.153	0.023	6.62	0
Resolving Insolvency	0.231	0.229	0.018	12.601	0
Starting a Business	0.189	0.192	0.017	11.061	0

Source: Reprinted from [204], used with permission.

A frequent error made in designing a hierarchical structure is the inclusion of an extensive list of criteria within an inappropriate sub-criterion. Such overlap can lead to data complexity and render estimations of the best alternative meaningless. After a rigorous analysis and validation process, we successfully streamlined the hierarchical structure of the DBP model. This resulted in a reduction in first-level indicators from 10 to 7, representing a 30% decrease, and a reduction in second-level indicators from 37 to 19, marking a substantial 48.65% reduction. The advantages of this modified model become apparent when considering the cost associated with collecting data from all African countries. This reduction in indicators not only simplifies the data collection process but also enhances the model's efficiency and interpretability. The use of PLS-SEM for validating the model is particularly suitable in our case. PLS-SEM does not assume a typical probability distribution, making it versatile for various data types. Additionally, it does not require large sample sizes for reliable conclusions. Importantly, PLS-SEM does not demand that linear relationships between factors be satisfied before analysis, aligning with the unique characteristics of our model. In the refined model, only the indicators that meet the stringent requirements for a valid formative construct are retained. This ensures the model's accuracy and effectiveness in evaluating the African business environment.

The primary aim of employing PLS is to provide a comprehensive justification of the total variance present in the indicators. This goes beyond merely examining correlations between indicators. PLS allows us to unravel the intricate relationship between the variances found among the first-level criteria and second-level criteria, and the ultimate goal of evaluating the business environment in Africa. This unique property of PLS makes it exceptionally well-suited for our hierarchical structure designed to evaluate the African business environment, which predominantly features formative constructs. PLS enables us to capture and explain the entire spectrum of variances, enhancing the depth and accuracy of our analysis. An optimized hierarchical structure, validated using PLS-SEM, enhances the model's robustness, reduces data collection costs, and focuses on valid formative constructs, making it a powerful tool for assessing the African business environment.

3.4.2. GRA-ROC Weights

The weights for evaluating African businesses were obtained through interactions with prospective Chinese investors. To ensure privacy and anonymity, these DMs were kept anonymous, and their confidentiality is highly upheld. Limited information about these individuals is provided to prevent tracking, identification, or outreach.

DM_1. He possesses over 20 years of experience as a businessman operating in Guinea and Cameroon. His primary business revolves around logging trees in these African countries, which are subsequently shipped to China for processing and utilized in the manufacturing of high-quality furniture. He maintains business relationships with furniture companies that both produce and distribute furniture throughout Africa. Beyond his extensive African business endeavors, he also collabourates with business partners in America, Europe, and various other regions in Asia.

DM_2. He has spent a cumulative period of 15 years in Africa, with business ventures in Nigeria and South Africa. Additionally, he has visited several other African countries, including Ghana and Kenya. His endeavors involve road construction, the development of water networks, as well as electricity transmission and distribution networks. Despite being Chinese by nationality, his family resides in the United States of America. These multicultural experiences have provided him with a unique opportunity to comprehend both Western and Chinese business approaches, which he adeptly applies within the African business environment.

DM_3. She serves as the CEO of a prominent company based in China, which conducts business operations in Nigeria and South Africa. The company specializes in providing smart electricity and water meters, along with electricity energy meter boxes equipped with Electricity Inventory Management Systems (IMSs). Additionally, the company has established a global presence with marketing and customer service networks. Their products are utilized both internationally and within the Chinese domestic electric power and water supply sectors, catering to a diverse range of customers.

DM_4. He is a Chinese professor in the School of Management with an impressive track record. He has numerous publications in top-tier academic journals and is currently supervising a student from Africa who is conducting research on the business environment. During his academic career, he spent a portion of his time outside China and had the opportunity to visit numerous countries around the world.

DM_5. He is a Chinese diplomat stationed in Africa, where he has dedicated more than four years of service in West Africa. In his diplomatic role, he has engaged in numerous interactions with both Chinese and African business individuals. His extensive travels have taken him to over 10 African countries, providing him with valuable insights into the continent. In addition to his diplomatic role, he is the proprietor of a book printing and distribution company based in China.

DM_6. She formerly held the position of a manager at an international freight company based in China, specializing in providing freight services to clients in Africa and the Middle East. Throughout her tenure, she established connections with over 2000 African business individuals, primarily from countries such as Kenya, Ghana, the Democratic Republic of Congo, Nigeria, Burkina Faso, Cote d'Ivoire, and Togo. The scope of her responsibilities encompassed a wide range of services, including bulk cargo shipping, air transport, express delivery, international material procurement, participation in international exhibitions, and various other integrated supply chain services.

DM_7. He transitioned from the role of West African Regional Manager to that of East African Regional Manager at a world-leading telecommunications company. He has dedicated more than six years to working in these African regions, during which he effectively managed the regional offices and engaged closely with individuals across these areas.

The evaluation of criteria weights poses a significant challenge in the process of assessing alternatives. However, the DBP approach employs equal weights for evaluating these countries. To address this limitation, the preferences of DMs are utilized to estimate the weights of the evaluation criteria, leveraging the GRA-ROC and GRA-ILP-ROC (Section 10.2) methodologies.

The fundamental concept behind the assessment of criteria weights, using the combination of GRA with ROC weights, is as follows: grey numbers that represent the weights of the indicators are derived from the linguistic values provided by the DMs. These weights pertain to the improved evaluation hierarchical model outlined in Figure 3.6, and they form the basis for constructing a grey weights data matrix, which is subsequently standardized. By integrating traditional GRA with grey numbers, grey relational grades are calculated and then transformed into ROC weights. Following the steps of the GRA-ROC weights approach, the weights for evaluating the business environment in Africa are estimated.

Step 1. Construct the hierarchical structure for the criteria and gather the raw weight data. This step involves identifying the criteria that need evaluation. The hierarchical structure of the indicators to be evaluated is illustrated in Figure 3.6. The responses from the DMs are provided in Table 3.6 and are subsequently transformed into grey numbers, as presented in Table 3.7.

Step 2. After obtaining the responses for the DMs, a grey preference matrix, A, is constructed using Equation (3.18). The rows of matrix A correspond to the indicators, and the column corresponds to the DMs preferences.

$$A = \begin{pmatrix} \otimes a_{1,1} & \otimes a_{1,2} & \otimes a_{1,3} & \otimes a_{1,4} & \otimes a_{1,5} & \otimes a_{1,6} & \otimes a_{1,7} \\ \otimes a_{4,1} & \otimes a_{4,2} & \otimes a_{4,3} & \otimes a_{4,4} & \otimes a_{4,5} & \otimes a_{4,6} & \otimes a_{4,7} \\ \otimes a_{5,1} & \otimes a_{5,2} & \otimes a_{5,3} & \otimes a_{5,4} & \otimes a_{5,5} & \otimes a_{5,6} & \otimes a_{5,7} \\ \otimes a_{6,1} & \otimes a_{6,2} & \otimes a_{6,3} & \otimes a_{6,4} & \otimes a_{6,5} & \otimes a_{6,6} & \otimes a_{6,7} \\ \otimes a_{7,1} & \otimes a_{7,2} & \otimes a_{7,3} & \otimes a_{7,4} & \otimes a_{7,5} & \otimes a_{7,6} & \otimes a_{7,7} \\ \otimes a_{9,1} & \otimes a_{9,2} & \otimes a_{9,3} & \otimes a_{9,4} & \otimes a_{9,5} & \otimes a_{9,6} & \otimes a_{9,7} \\ \otimes a_{10,1} & \otimes a_{10,2} & \otimes a_{10,3} & \otimes a_{10,4} & \otimes a_{10,5} & \otimes a_{10,6} & \otimes a_{10,7} \end{pmatrix}. \quad (3.18)$$

The DMs preference matrix, A, is as follows:

$$A = \begin{pmatrix} [0.6, 0.8] & [0.8, 1] & \cdots & [0.8, 1] \\ [0.4, 0.6] & [0.8, 1] & \cdots & [0.8, 1] \\ \vdots & \vdots & \ddots & \vdots \\ [0.4, 0.6] & [0.4, 0.6] & \cdots & [0.8, 1] \end{pmatrix}. \quad (3.19)$$

This same approach is used for all the second-level indicators, and are drawn from Table 3.8.

Step 3. Standardize the weighted matrix. This is to make the grey weights matrix a scale of 0–1. Using Equation (3.4), we have $||A_{1\text{-}1}|| = 1$, $||A_{1\text{-}2}|| = 1$, $||A_{1\text{-}3}|| = 1$, $||A_{1\text{-}4}|| = 1$, $||A_{1\text{-}5}|| = 1$, $||A_{1\text{-}6}|| = 1$, $||A_{1\text{-}7}|| = 1$. Then, the standardized weights matrix, S, using Equation (3.20) is as follows:

$$S = \begin{pmatrix} [0.6, 0.8] & [0.8, 1] & \cdots & [0.8, 1] \\ [0.4, 0.6] & [0.8, 1] & \cdots & [0.8, 1] \\ \vdots & \vdots & \ddots & \vdots \\ [0.4, 0.6] & [0.4, 0.6] & \cdots & [0.8, 1] \end{pmatrix}. \quad (3.20)$$

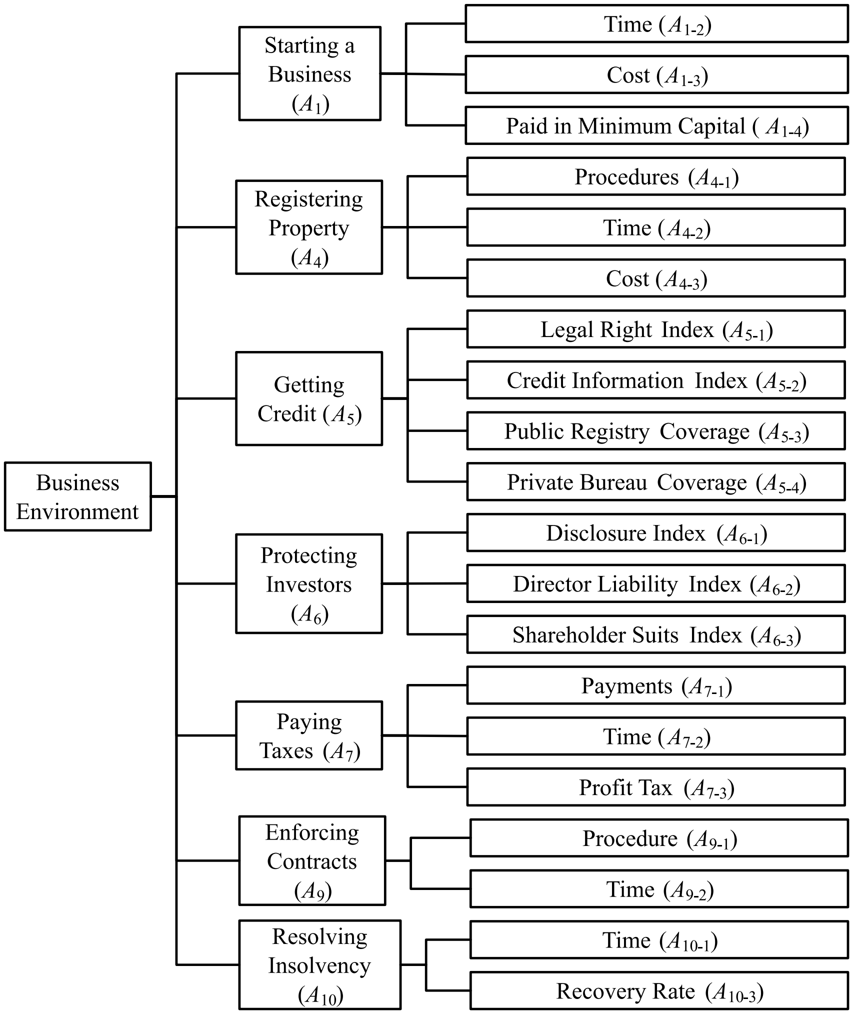


Figure 3.6. Improved evaluation hierarchical model. Source: Reprinted from [204], used with permission.

Table 3.8. Conversion of grey linguistic weights to grey numbers.

Decision-Makers/ Indicators	DM_1	DM_2	DM_3	DM_4	DM_5	DM_6	DM_7
A_1	[0.6, 0.8]	[0.8, 1]	[0.6, 0.8]	[0.8, 1]	[0.2, 0.4]	[0.4, 0.6]	[0.8, 1]
A_4	[0.4, 0.6]	[0.8, 1]	[0.8, 1]	[0.8, 1]	[0.6, 0.8]	[0.4, 0.6]	[0.8, 1]
A_5	[0.6, 0.8]	[0.6, 0.8]	[0.6, 0.8]	[0.8, 1]	[0.4, 0.6]	[0.6, 0.8]	[0.8, 1]
A_6	[0.8, 1]	[0.8, 1]	[0.8, 1]	[0.8, 1]	[0.8, 1]	[0.8, 1]	[0.8, 1]
A_7	[0.8, 1]	[0.6, 0.8]	[0.8, 1]	[0.8, 1]	[0.4, 0.6]	[0.6, 0.8]	[0.4, 0.6]
A_9	[0.6, 0.8]	[0.6, 0.8]	[0.8, 1]	[0.6, 0.8]	[0.8, 1]	[0.8, 1]	[0.8, 1]
A_{10}	[0.4, 0.6]	[0.4, 0.6]	[0.8, 1]	[0.6, 0.8]	[0.4, 0.6]	[0.4, 0.6]	[0.8, 1]

Table 3.8. *Cont.*

Decision-Makers/ Indicators	DM_1	DM_2	DM_3	DM_4	DM_5	DM_6	DM_7
$A_{1\text{-}2}$	[0.8, 1]	[0.4, 0.6]	[0.6, 0.8]	[0.6, 0.8]	[0.8, 1]	[0.6, 0.8]	[0.6, 0.8]
$A_{1\text{-}3}$	[0.6, 0.8]	[0, 0.2]	[0.8, 1]	[0.6, 0.8]	[0.4, 0.6]	[0.8, 1]	[0.4, 0.6]
$A_{1\text{-}4}$	[0.4, 0.6]	[0.8, 1]	[0.6, 0.8]	[0.6, 0.8]	[0.4, 0.6]	[0.4, 0.6]	[0.4, 0.6]
$A_{4\text{-}2}$	[0.8, 1]	[0.6, 0.8]	[0.6, 0.8]	[0.6, 0.8]	[0.8, 1]	[0.8, 1]	[0.8, 1]
$A_{4\text{-}3}$	[0.6, 0.8]	[0.8, 1]	[0.6, 0.8]	[0.6, 0.8]	[0.4, 0.6]	[0.4, 0.6]	[0.6, 0.8]
$A_{5\text{-}1}$	[0.8, 1]	[0.8, 1]	[0.8, 1]	[0.8, 1]	[0.8, 1]	[0.8, 1]	[0.8, 1]
$A_{5\text{-}2}$	[0.6, 0.8]	[0.8, 1]	[0.8, 1]	[0.8, 1]	[0.8, 1]	[0.6, 0.8]	[0.6, 0.8]
$A_{5\text{-}3}$	[0.4, 0.6]	[0.6, 0.8]	[0.4, 0.6]	[0.8, 1]	[0.6, 0.8]	[0.6, 0.8]	[0.6, 0.8]
$A_{5\text{-}4}$	[0.4, 0.6]	[0.2, 0.4]	[0.6, 0.8]	[0.8, 1]	[0.6, 0.8]	[0.6, 0.8]	[0.8, 1]
$A_{6\text{-}1}$	[0.8, 1]	[0.6, 0.8]	[0.6, 0.8]	[0.8, 1]	[0.6, 0.8]	[0.8, 1]	[0.6, 0.8]
$A_{6\text{-}2}$	[0.8, 1]	[0.8, 1]	[0.8, 1]	[0.6, 0.8]	[0.6, 0.8]	[0.8, 1]	[0.8, 1]
$A_{6\text{-}3}$	[0.4, 0.6]	[0.6, 0.8]	[0.8, 1]	[0.6, 0.8]	[0.8, 1]	[0.8, 1]	[0.8, 1]
$A_{7\text{-}1}$	[0.4, 0.6]	[0.4, 0.6]	[0.6, 0.8]	[0.8, 1]	[0.8, 1]	[0.4, 0.6]	[0.6, 0.8]
$A_{7\text{-}2}$	[0.4, 0.6]	[0.4, 0.6]	[0.4, 0.6]	[0.6, 0.8]	[0.4, 0.6]	[0.6, 0.8]	[0.4, 0.6]
$A_{7\text{-}3}$	[0.8, 1]	[0.6, 0.8]	[0.8, 1]	[0.6, 0.8]	[0.6, 0.8]	[0.6, 0.8]	[0.6, 0.8]
$A_{9\text{-}1}$	[0.4, 0.6]	[0.6, 0.8]	[0.6, 0.8]	[0.6, 0.8]	[0.6, 0.8]	[0.8, 1]	[0.8, 1]
$A_{9\text{-}2}$	[0.6, 0.8]	[0.4, 0.6]	[0.6, 0.8]	[0.4, 0.6]	[0.8, 1]	[0.6, 0.8]	[0.8, 1]
$A_{10\text{-}1}$	[0.4, 0.6]	[0.4, 0.6]	[0.6, 0.8]	[0.6, 0.8]	[0.6, 0.8]	[0.4, 0.6]	[0.6, 0.8]
$A_{10\text{-}3}$	[0.4, 0.6]	[0.6, 0.8]	[0.8, 1]	[0.8, 1]	[0.8, 1]	[0.8, 1]	[0.8, 1]

Source: Reprinted from [204], used with permission.

Step 4. Determine the reference weights of the indicators for each expert. Using Equation (3.6), the optimal grey number weights associated with the experts are as follows:

$$s_0 = ([0.8, 1], [0.8, 1], [0.8, 1], [0.8, 1], [0.8, 1], [0.8, 1], [0.8, 1]). \quad (3.21)$$

Step 5. Determine the difference between the reference weights and every weight by the decision-maker. For the first-level indicator, the differences are shown in Table 3.9, and we have $\min\limits_{1 \le i \le m, 1 \le j \le n} \min \Delta_{ij}=0$ and $\max\limits_{1 \le i \le m, 1 \le j \le n} \max \Delta_{ij}=0.6$.

Table 3.9. Difference between the standardized and reference indicators.

Delta	DM_1	DM_2	DM_3	DM_4	DM_5	DM_6	DM_7	$\min\limits_{1 \le j \le n} \Delta_{ij}$	$\min\limits_{1 \le j \le n} \Delta_{ij}$
A_1	0.2	0	0.2	0	0.6	0.4	0	0	0.6
A_4	0.4	0	0	0	0.2	0.4	0	0	0.4
A_5	0.2	0.2	0.2	0	0.4	0.2	0	0	0.4
A_6	0	0	0	0	0	0	0	0	0
A_7	0	0.2	0	0	0.4	0.2	0.4	0	0.4
A_9	0.2	0.2	0	0.2	0	0	0	0	0.2
A_{10}	0.4	0.4	0	0.2	0.4	0.4	0	0	0.4

Source: Reprinted from [204], used with permission.

Step 6. Calculate the grey relational grade. First, the grey relational coefficients are calculated using Equation (3.10) as follows:

$$
\begin{array}{rl}
\gamma_{1j} = & (0.6,\ 1,\ 0.6,\ 1,\ 0.3333,\ 0.4286,\ 1) \\
\gamma_{4j} = & (0.4286,\ 1,\ 1,\ 1,\ 0.6,\ 0.4286,\ 1) \\
\gamma_{5j} = & (0.6,\ 0.6,\ 0.6,\ 1,\ 0.4286,\ 0.6,\ 1) \\
\gamma_{6j} = & (1,\ 1,\ 1,\ 1,\ 1,\ 1,\ 1) \\
\gamma_{7j} = & (1,\ 0.6,\ 1,\ 1,\ 0.4286,\ 0.6,\ 0.4286) \\
\gamma_{9j} = & (0.6,\ 0.6,\ 1,\ 0.6,\ 1,\ 1,\ 1) \\
\gamma_{10j} = & (0.4286,\ 0.4286,\ 1,\ 0.6,\ 0.4286,\ 0.4286,\ 1)
\end{array} . \tag{3.22}
$$

Then, the grey relational grades are calculated using Equation (3.9):

$$
\begin{array}{l}
r_1 = 0.7088, \quad r_4 = 0.7796, \quad r_5 = 0.6898, \quad r_6 = 1, \quad r_7 = 0.7224, \\
r_9 = 0.8256, \quad r_{10} = 0.6163,
\end{array} \tag{3.23}
$$

$$
A_6 > A_9 > A_4 > A_7 > A_1 > A_6 > A_{10}. \tag{3.24}
$$

Step 7. Transform the rankings to weights. Now, we transform the rankings in Equation (3.24) to ROC weights as shown in Table 3.10. Similarly, the grey ROC weights of the second-level indicators are calculated using the same method.

Table 3.10. ROC weights transformation for GRA.

Criteria	q	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**Weights**
A_6	1st	1	0.5	0.3333	0.25	0.2	0.1667	0.1429	0.3704
A_9	2nd		0.5	0.3333	0.25	0.2	0.1667	0.1429	0.2276
A_4	3rd			0.3333	0.25	0.2	0.1667	0.1429	0.1561
A_7	4th				0.25	0.2	0.1667	0.1429	0.1085
A_1	5th					0.2	0.1667	0.1429	0.0728
A_5	6th						0.1667	0.1429	0.0442
A_{10}	7th							0.1429	0.0204

Source: Reprinted from [204], used with permission.

Step 8. Calculate the effective weights. Then, the effective weights of the indicators are computed using Equation (3.12). The results are given in Table 3.10. The local weights are the weights of the second-level criteria in relation to their first-level criteria, and the effective weights are the fraction of contributions by the second-level indicators to the overall weights in relation to the top-level hierarchy as given in Table 3.11. The GRA-ROC weighting method combines the advantages of grey numbers, traditional GRA, and ROC weighting methods. These weights are used for the assessment of the business environment for every country in Africa.

Table 3.11. Effective GRA-ROC weights of indicators.

First-Level Indicator	Weights	Second-Level Criteria	Local Weights	Effective Weights	Index (v)
A_1	0.0728	$A_{1\text{-}2}$	0.1111	0.0081	1
		$A_{1\text{-}3}$	0.2778	0.0202	2
		$A_{1\text{-}4}$	0.6111	0.0445	3
A_4	0.1561	$A_{4\text{-}2}$	0.25	0.039	4
		$A_{4\text{-}3}$	0.75	0.1171	5
A_5	0.0442	$A_{5\text{-}1}$	0.0625	0.0028	6
		$A_{5\text{-}2}$	0.1458	0.0064	7
		$A_{5\text{-}3}$	0.5208	0.023	8
		$A_{5\text{-}4}$	0.2708	0.012	9
A_6	0.3704	$A_{6\text{-}1}$	0.6111	0.2264	10
		$A_{6\text{-}2}$	0.1111	0.0412	11
		$A_{6\text{-}3}$	0.2778	0.1029	12
A_7	0.1085	$A_{7\text{-}1}$	0.2778	0.0301	13
		$A_{7\text{-}2}$	0.6111	0.0663	14
		$A_{7\text{-}3}$	0.1111	0.0121	15
A_9	0.2276	$A_{9\text{-}1}$	0.25	0.0569	16
		$A_{9\text{-}2}$	0.75	0.1707	17
A_{10}	0.0204	$A_{10\text{-}1}$	0.75	0.0153	18
		$A_{10\text{-}3}$	0.25	0.0051	19

Source: Reprinted from [204], used with permission.

4. Grey Systems Theory Integrated with Regulatory Focus Theory

In the realm of business development, assessing a company's reputation holds paramount importance. In the context of Multiple Criteria Decision-Making (MCDM), accurately estimating the weights of evaluation criteria is a crucial step in enhancing the precision of assessment outcomes. However, the conventional practice of assigning standard weights by decision-makers may result in inaccuracies if these weights are not aligned with the decision-makers' pursuit of their goals. This chapter presents a novel approach that combines Grey Systems Theory (GST) and Regulatory Focus Theory (RFT) to estimate criterion weights. The DMs' preferences for weighting are measured from their regulatory orientation in terms of promotion and prevention focus, and these tendencies are expressed as grey numbers. Notably, not all weight measures from decision-makers (DMs) are symmetrical when measured from both a promotional and preventive orientation. To address this, Grey Regulatory Focus Theory (GRFT) weighting method is introduced, which aggregates the preference of the group DMs to estimate the weights of MCDM problems, thereby surmounting the limitation of single-direction measurement. To illustrate the application of the GRFT method, it is applied to assess the reputation of a university, a business entity primarily focused on providing services to its customers, i.e., students. Figure 4.1 visually demonstrates the relationship between GRFT and RFT, highlighting that GRFT is a natural extension of the Regulatory Focus Theory.

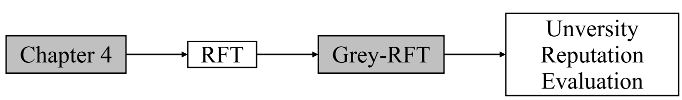


Figure 4.1. Flowchart of Chapter 4. Source: Figure by authors.

4.1. Regulatory Focus Theory

Regulatory Focus Theory (RFT), as articulated by Higgins [220], offers valuable insights into how individuals approach tasks and goals in the context of motivation. It helps us understand how various factors can influence our motivation while pursuing objectives, leading us to adapt our methods and strategies accordingly. RFT posits that motivation is maximized when the approach aligns with an individual's regulatory focus, which can be shaped by their personal goals and characteristics. RFT identifies two primary regulatory orientations that individuals employ to achieve their goals: promotion focus orientation and prevention focus orientation. In essence, promotion focus involves striving for gains and pursuing opportunities, while prevention focus centers on avoiding losses and minimizing risks [221]. These distinct orientations influence how individuals regulate their actions and decisions in pursuit of their objectives. Moreover, RFT plays a crucial role in predicting the effectiveness of persuasive communication. By tailoring messages to align with an individual's regulatory focus, communicators can enhance their persuasive impact.

This means that the success of persuasive efforts may depend on how well the content and style of communication resonate with an individual's personal goals and regulatory orientation [222].

Interestingly, researchers like Forster et al. [223] proposed the concept of the "goal looms larger effect," confirming that people's motivation increases as they approach their goals, a phenomenon closely tied to their regulatory focus. In other words, motivation and task performance tend to peak when the regulatory focus of incentives aligns with an individual's regulatory orientation [224]. For instance, Zhao and Pechmann [225] demonstrated that effective communication hinges on the synergy between the receiver's regulatory focus, the message's regulatory focus, and the framing of the message. This suggests that tailoring communication to match an individual's regulatory focus can enhance its impact. Furthermore, Dijk and Kluger [226] explored the interaction between regulatory focus and task type in influencing motivation and task performance. They hypothesized that task type moderates the relationship between regulatory focus and performance. Their research confirmed that different tasks can be affected by varying regulatory foci, with both regulatory focus and task type contributing to the variability in the feedback effect on motivation and performance. On a different note, Bullens et al. [227] investigated how the reversibility of a decision affects motivation, particularly in terms of promotion and prevention focus. Their findings suggest that reversible decisions tend to strengthen prevention focus more than promotion focus. This implies that individuals faced with reversible decisions may exhibit a stronger inclination toward prevention focus, which can impact their overall choice satisfaction. These studies collectively shed light on the intricate interplay between regulatory focus, motivation, and performance across different contexts, providing valuable insights into how aligning incentives and communication with regulatory focus can enhance goal pursuit and decision-making.

Several studies have applied RFT to decision-making processes and management, aligning with the principles of the upper echelons theory. These investigations shed light on how decision-makers (DMs) consider regulatory focus in their judgments and actions. Kuhn's study [228] explored how DMs make hiring decisions by considering both promotion focus (selecting the good candidates) and prevention focus (rejecting the unsuitable candidates). Ahmadi et al. [229] delved into the motivation of managers when faced with new technological changes. They found that the answers and explanations were contingent upon the regulatory focus and fit of the DMs. Lai et al. [230] empirically investigated the performance of information system development teams. Their research confirmed that transformational leadership tends to foster a promotional focus, whereas transactional leadership encourages a prevention focus. Liao and Long [231] discovered a link between a company's environmental innovation process and the top leader's regulatory focus. Positive influence was observed when leaders exhibited a promotion focus, while a prevention focus had a negative impact. In the study by Song et al. [232], the impact of institution-based trust on community commitment was examined, with regulatory focus orientations serving as moderators. These studies collectively illustrate how RFT is applied in diverse decision-making contexts, offering insights into the role of regulatory focus in managerial judgments and organizational outcomes.

Of particular significance, Higgins and Cornwell [233] conducted an extensive discussion on the frontiers of RFT, exploring the effects of both prevention and promotion focus on decision-making. However, it is noteworthy that their discussion did not encompass the aspect of the MCDM weighting method. In a different vein, Lo-Gerfo et al. [234] conducted neurophysiological testing related to RFT, shedding light on the potential for decision-makers (DMs) to exhibit varying degrees of either promotion or prevention focus. This research addresses a gap in the existing literature by applying RFT to MCDM, specifically in the estimation of DM weights within an uncertain decision-making environment. Notably, this approach considers the DMs' weighting perspectives from both promotion and prevention focus orientations. These studies collectively contribute to our understanding of RFT's applicability in diverse decision-making scenarios, with Lo-Gerfo et al.'s research bridging the gap by introducing RFT concepts into the realm of MCDM, thereby enriching the discourse on DM weight estimation.

4.2. GRFT Weighting Method

The core concept behind the Grey Regulatory Focus Theory (GRFT) weighting method is to capture the preferences of decision-makers (DMs) from both the prevention and promotion focus and represent them as interval grey numbers. Subsequently, these grey weights are standardized so that the sum of the upper bounds of the grey weights equals one. This standardized value is then utilized in the MCDM evaluation process.

The Grey Regulatory Focus Theory (GRFT) weighting method begins by capturing the preferences of decision-makers (DMs) from both the prevention and promotion focus orientations. These preferences are represented as interval grey numbers, denoted as $\otimes a = [\underline{a}, \overline{a}]$ and $\otimes b = \left[\underline{b}, \overline{b}\right]$, where $\underline{a}$ and $\underline{b}$ signify the lower bounds, and $\overline{a}$ and $\overline{b}$ represent the upper bounds. This interval representation accommodates the inherent uncertainty in DMs' preferences. To quantify the dissimilarity between these interval grey numbers, an arbitrary distance is calculated using Equation (1.5). Finally, the grey weights are standardized, ensuring that the sum of the upper bounds of the grey weights equals one. This standardization process establishes a consistent scale for the resulting weights, which are then employed in the MCDM evaluation process. These steps collectively form the foundation of the GRFT weighting method, facilitating the incorporation of both prevention and promotion focus preferences into the decision-making framework while addressing the inherent uncertainty and variability in DMs' inclinations.

To apply the Grey Regulatory Focus Theory (GRFT) weighting method, follow these steps:

Step 1. Obtain the decision-makers' (DMs') ratings for each evaluation criterion. This can be achieved through a questionnaire containing both prevention and promotion focus questions. Direct ratings are allocated as percentage scores (0–100%) to measure DMs' preferences.

Step 2. Transform the DMs' ratings into grey numbers. These interval grey numbers represent the first-level criteria. The grey number ratings for the second-level criteria are measured as both promotion and prevention focus orientations and serve as the lower and upper bounds of the grey numbers. Specifically, the following apply:

- For the first-level criteria, C', the grey ratings consist of α first-level criteria c, and s is the last first-level criterion. This is computed as follows:

$$C' = \otimes c_1, \otimes c_2, \ldots, \otimes c_s. \tag{4.1}$$

  Here, $\otimes c_\alpha = [\underline{c}\alpha, \overline{c}\alpha]$, with $\underline{c}\alpha$ and $\overline{c}\alpha$ as the lower and upper bounds. These lower and upper bounds are given as $\underline{c}_\alpha = \min\limits_{1\le i\le m} (DM_i(C_\alpha))$ and $\overline{c}_\alpha = \max\limits_{1\le i\le m} (DM_i(C_\alpha))$, respectively.

- For the second-level criteria, C'', the grey ratings represent the minimum and maximum ratings by the DMs for each criterion. These are given as follows:

$$C'' = \otimes c_{\alpha\text{-}1}, \otimes c_{\alpha\text{-}2}, \ldots, \otimes c_{\alpha\text{-}t}, \tag{4.2}$$

  where t is the last second-level criteria for each first-level criterion.

  The aggregated grey ratings provided by the group of m DMs for the second-level criteria, $C_{\alpha-\beta}$, are calculated as follows:

$$\otimes c_{\alpha\text{-}\beta} = \left[\underline{c}_{\alpha-\beta}, \overline{c}_{\alpha\text{-}\beta}\right] = \left[\sum_{i=1}^{m} \underline{c}_{\alpha\text{-}\beta\text{-}i}, \sum_{i=1}^{m} \overline{c}_{\alpha\text{-}\beta\text{-}i}\right]. \tag{4.3}$$

  Here, $\underline{c}_{\alpha\text{-}\beta\text{-}i}$ and $\overline{c}_{\alpha\text{-}\beta\text{-}i}$ represent the lower and upper bounds of the DM_i grey ratings, given by $\otimes c_{\alpha\text{-}\beta\text{-}i} = \left[\underline{c}_{\alpha\text{-}\beta\text{-}i}, \overline{c}_{\alpha\text{-}\beta\text{-}i}\right]$.

Step 3. Standardize the grey weights. This involves standardizing the grey weights, ensuring that the sum of the criteria weights equals one unit, or 100%. This standardization process applies to both first-level and second-level criteria weights:

(a) For the first-level criteria. The standardized weight for the first-level criteria, W', is the grey weight for the αth criteria, which is $\otimes w_\alpha = [\underline{w}_\alpha, \overline{w}_\alpha]$, where $\underline{w}_\alpha = \frac{\underline{c}_\alpha}{\sum_{\alpha=1}^{m} \overline{c}_\alpha}$ and $\overline{w}_\alpha = \frac{\overline{c}_\alpha}{\sum_{\alpha=1}^{m} \underline{c}_\alpha}$. Thus, the following apply:

$$W' = \otimes w_1, \otimes w_2, \ldots, \otimes w_s. \tag{4.4}$$

(b) For the second-level criteria. The standardized weight for the second-level criteria, W'', is the grey weight for the β^{th} criteria, which is $\otimes w_{\alpha\text{-}\beta} = \left[\underline{w}_{\alpha\text{-}\beta}, \overline{w}_{\alpha\text{-}\beta}\right]$ where $\underline{w}_{\alpha\text{-}\beta} = \frac{\underline{c}_{\alpha\text{-}\beta}}{\sum_{\beta=1}^{m} \overline{c}_{\alpha\text{-}\beta}}$ and $\overline{w}_{\alpha\text{-}\beta} = \frac{\overline{c}_{\alpha\text{-}\beta}}{\sum_{\beta=1}^{m} \underline{c}_{\alpha\text{-}\beta}}$. Thus, the following apply:

$$W'' = \otimes w_{\alpha\text{-}1}, \otimes w_{\alpha\text{-}2}, \ldots, \otimes w_{\alpha\text{-}t}, \tag{4.5}$$

where k is the last term of the first-level criteria.

Step 4. Compute the overall weights. To compute the overall weights, multiply the local weights of the first-level and second-level criteria.

$$W = W' \times W''. \tag{4.6}$$

This has a grey value for the second-level criteria of $\otimes w_v = \otimes w_\alpha \times \otimes w_{\alpha\text{-}\beta}$, where v is the index of the criteria for the MCDM evaluation method, i.e., the second-level criteria index, $\otimes w_v = [\underline{w}_v, \overline{w}_v]$ and $\sum_{v=1}^{m} \overline{w}_v = 1$:

$$W = \begin{pmatrix} \otimes w_1 & \otimes w_2 & \cdots & \otimes w_n \end{pmatrix}^T. \tag{4.7}$$

The weight transpose is a column matrix that can be used in other MCDM methods such as the Weighted Sum Model (WSM).

4.3. Practical Application of Grey RFT for Evaluating University Reputation

The increasing globalization of universities is often attributed to the diverse perspectives and challenges that the education sector must navigate. Politicians, educational leaders, and commentators alike acknowledge that competition among higher education institutions has grown more intense in recent years [170]. One prominent international university ranking that continually assesses the reputation of universities, both in China and globally, is the Academic Ranking of World Universities (ARWU) developed by Shanghai Jiao Tong University, China.

Reputation can be described as the amalgamation of beliefs, ideas, and impressions that individuals hold about an object, individual, institution, or organization based on past and present experiences. university reputation (University Reputation (UR)), in particular, is an institutional standing that emerges as people construct their perceptions of a university's objectives, ethics, operational approaches, and the treatment accorded to its students. In essence, UR represents a spontaneous, organic character influenced by expectations and interactions that individuals have with the university. As articulated by Cole and Bruch [235], UR is the mental vision, representation, or impression that individuals form based on the information or data they acquire about a university through interactions with its various elements and components. Consequently, universities that are successful in attracting talented individuals or clients are those that maintain and cultivate a positive reputation.

In an increasingly competitive landscape, universities recognize the significance of establishing a strong and consistent institutional image to resonate with their target audiences [236]. The visibility of an institution is crucial, encompassing the presentation of its educational offerings, validation of its scholarly potential, and the grandeur of its facilities, resources, and financial stability. However, many higher education institutions struggle to effectively showcase their wealth and the diversity of their national and international affiliations. With over 2879 colleges and universities in the People's Republic of China in 2017 [237], evaluating the reputation of all these institutions can be a challenging task, especially when comparing lower-tier universities without direct assistance from the Ministry of Education. For many Chinese students, the National College Entrance Examination plays a pivotal role in determining their future, with aspirations to secure a place at the most reputable university that matches their abilities to achieve a sense of psychological satisfaction. Consequently, there is a compelling need to evaluate the reputation of these universities from the perspective of students.

The evaluation criteria for assessing university reputation encompass 6 first-level criteria and 18 second-level criteria, each with four variables. These four variables serve as reflective constructs to gauge university performance in relation to each second-level criterion. In essence, each survey participant answered 72 questions, corresponding to these variables. The selection of these criteria is informed by their capacity to provide a comprehensive assessment of the university's past activities [104,170,171]. Furthermore, these derived criteria capture the university's perceived image by its students. This research specifically focuses on the weighting method based on the regulatory focus theory.

The model for measuring criteria weights follows a hierarchical structure, comprising 6 first-level criteria and 18 second-level criteria. Each first-level criterion is further divided into three second-level criteria. Notably, each of the second-level criteria is assessed from both prevention and promotion focus perspectives, and these assessments are represented as grey interval numbers, as illustrated in Figure 4.2. In total, each decision-maker provided ratings for 42 weighting items, consisting of 6 items related to the first-level criteria and 36 items associated with the second-level criteria (18 items each for both prevention and promotion-focused weightings) in the questionnaire.

The choice to use this method is rooted in the subjective nature of the weights in this study. It is important to note that our intention is neither to endorse nor detract from any of the universities under evaluation.

To assess the reputation of four universities, namely Xi'an AAA University (A_1), Xi'an BBB University (A_2), Xi'an CCC University (A_3), and Xi'an DDD University (A_4), we designed two sets of questionnaires. The first set of questionnaires was administered online through a dedicated website, gathering input on each university's performance from the perspective of students. The second set of questionnaires was distributed in paper and portable document format (.pdf), aimed at capturing the preferences of DMs for weight estimation.

The questionnaire utilized in this study, as outlined by Chen and Esangbedo [104], was administered to a sample of 1200 students, with 300 students randomly selected from each of the four universities under consideration. Data collection commenced on 31 October 2018, and it took an average of 11 days to collect responses from these university students. A total of 51 responses were removed due to being classified as unattended or incomplete, resulting in 1149 valid responses for the evaluation. This yielded a response rate of 95.75% for the questionnaire.

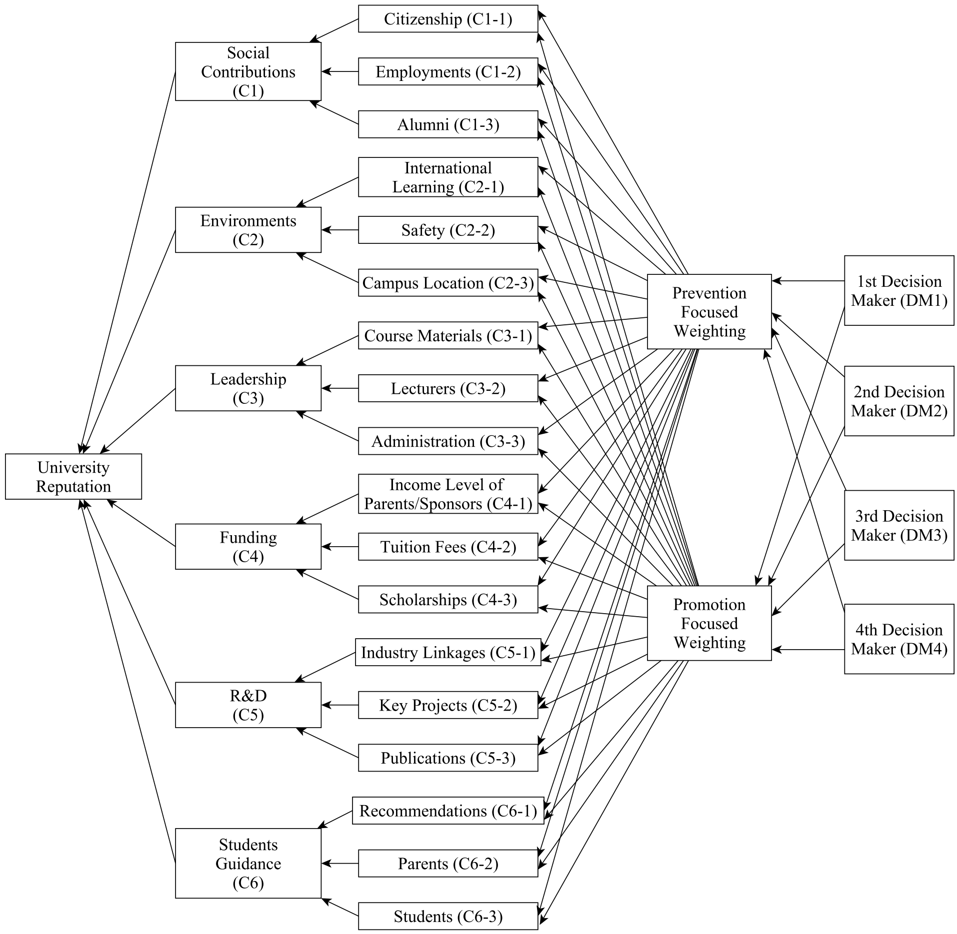


Figure 4.2. GRFT weighting hierarchical model. Source: Reprinted from [100], used with permission.

Likewise, our selection of decision-makers (DMs) was influenced by their years of experience. The four DMs collectively possessed 134 years of cumulative work experience, with some having backgrounds in academia and others serving as top managers in various industries.

For the second-level criteria, Decision Makers (DMs) were asked to provide ratings twice, once from a prevention focus orientation and once from a promotion focus orientation. This rating approach essentially functions as a points allocation method, with points allocated to criteria based on their perceived importance. To ensure consistency, we requested DMs to employ a percentage scale in assigning points to all criteria. Each DM provided ratings (scores) using a 0 to 100 percentage scale, resulting in a total of 42 questions—6 questions for the first-level criteria and 36 questions for the second-level criteria, as shown in Tables 4.1 and 4.2. For a visual overview of the study, refer to the flowchart presented in Figure 4.3. Additionally, we categorized the decision-makers' ratings as either symmetric RFT ratings or asymmetric RFT ratings. Ratings were considered symmetric if the assigned ratings for the prevention and promotion focus questions for a given criterion were equal ($DM_{i,p} = DM_{i,q}$); otherwise, they were classified as asymmetric RFT ratings. Figure 4.4 illustrates that at least half of the DMs' responses were asymmetric. Therefore, it would be inappropriate to measure the DMs' preferences from a single orientation.

Table 4.1. DM ratings for first-level criteria.

First-Level Indicators (C_i)	DM_1	DM_2	DM_3	DM_4
Social Contribution (C_1)	85	90	85	100
Environments (C_2)	90	90	86	95
Leadership (C_3)	85	90	95	100
Funding (C_4)	70	80	96	100
R&D (C_5)	75	80	96	95
Students Guidance (C_6)	70	80	96	100

Source: Reprinted from [100], used with permission.

Table 4.2. DMs' ratings for second-level indicators.

Second-Level Indicators $C_{\alpha\text{-}\beta}$	Second-Level Indicator Index (v)	Prevention Measurements (p)				Promotion Measurements (q)			
		DM_1	DM_2	DM_3	DM_4	DM_1	DM_2	DM_3	DM_4
Citizenship ($C_{1\text{-}1}$)	1	100	90	100	95	100	90	100	98
Employment ($C_{1\text{-}2}$)	2	100	81	85	100	70	50	85	100
Alumni ($C_{1\text{-}3}$)	3	80	90	91	100	100	70	100	100
International Learning ($C_{2\text{-}1}$)	4	90	70	100	90	95	70	100	90
Safety ($C_{2\text{-}2}$)	5	85	90	100	98	85	90	100	98
Campus Location ($C_{2\text{-}3}$)	6	80	60	90	100	85	80	95	90
Course Materials ($C_{3\text{-}1}$)	7	90	90	100	92	70	30	90	85
Lecturers ($C_{3\text{-}2}$)	8	90	70	98	100	90	90	95	100
Administration ($C_{3\text{-}3}$)	9	85	70	90	100	85	90	90	95
Income Level of Parent/Spouse ($C_{4\text{-}1}$)	10	50	70	30	80	50	40	30	90
Tuition Fees ($C_{4\text{-}2}$)	11	50	20	85	80	70	70	88	80
Scholarships ($C_{4\text{-}3}$)	12	70	90	90	100	85	90	95	100
Industrial Links ($C_{5\text{-}1}$)	13	75	40	85	90	70	30	81	90
Key Projects ($C_{5\text{-}2}$)	14	90	80	95	98	90	90	92	100
Publication ($C_{5\text{-}3}$)	15	85	90	95	100	85	80	95	100
Recommendation ($C_{6\text{-}1}$)	16	90	90	100	100	85	50	50	85
Parents ($C_{6\text{-}2}$)	17	80	30	25	80	85	90	50	78
Students ($C_{6\text{-}3}$)	18	100	90	62	100	85	60	82	96

Source: Reprinted from [100], used with permission.

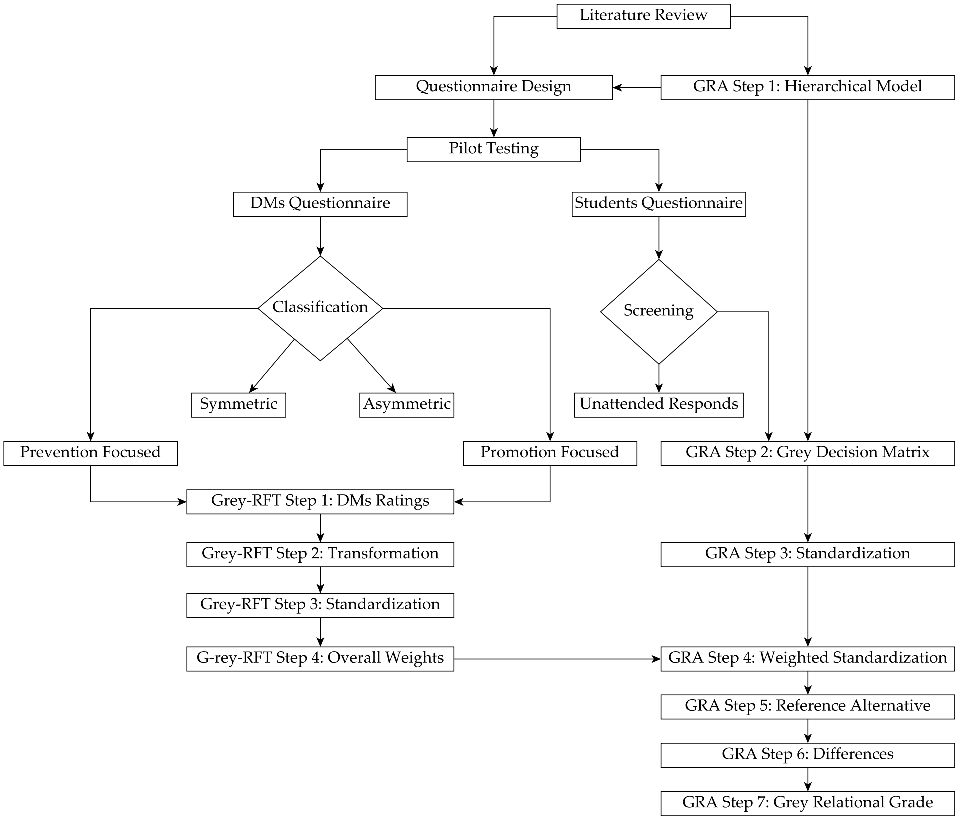


Figure 4.3. Flowchart of the GRFT weights for GRA. Source: Reprinted from [100], used with permission.

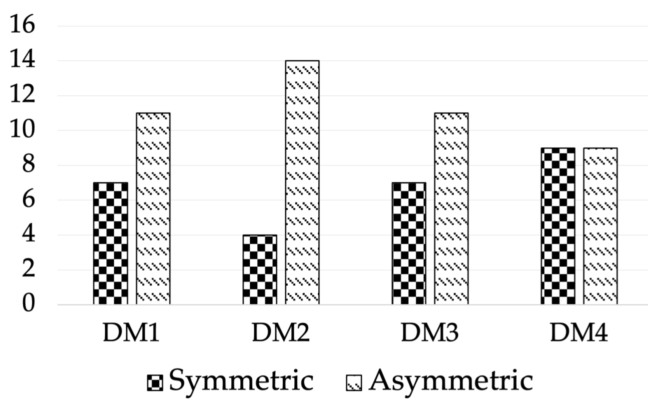


Figure 4.4. Regulatory focus similarity of the DMs' preferences. Source: Reprinted from [100], used with permission.

Subsequently, we computed the grey ratings for the first-level criteria using Equation (4.1), and for the second-level criteria, we employed Equations (4.2) and

(4.3). For instance, let us consider the first criterion of the second-level criteria, Citizenship ($c_{1\text{-}1}$), which had the grey ratings detailed in Table 4.3. Similarly, the last criterion of the second-level criteria, Students ($c_{6\text{-}3}$), is also presented in Table 4.3. The details for other first-level criteria were omitted.

Table 4.3. Grey ratings for second-level indicators.

$\otimes c_{\alpha\text{-}\beta\text{-}i}$	DM_1	DM_2	DM_3	DM_4	$\otimes c_{\alpha\text{-}\beta}$
$\otimes c_{1\text{-}1\text{-}i}$	[100, 100]	[90, 90]	[100, 100]	[95, 98]	[385, 388]
$\otimes c_{1\text{-}2\text{-}i}$	[70, 100]	[50, 81]	[85, 85]	[100, 100]	[305, 366]
$\otimes c_{1\text{-}3\text{-}i}$	[80, 100]	[70, 90]	[91, 100]	[100, 100]	[341, 390]
⋮	⋮	⋮	⋮	⋮	⋮
$\otimes c_{6\text{-}1\text{-}i}$	[85, 90]	[50, 90]	[50, 100]	[85, 100]	[270, 380]
$\otimes c_{6\text{-}2\text{-}i}$	[80, 85]	[30, 90]	[25, 50]	[78, 80]	[213, 305]
$\otimes c_{6\text{-}3\text{-}i}$	[85, 100]	[60, 90]	[62, 82]	[96, 100]	[303, 372]

Source: Reprinted from [100], used with permission.

Then, the first-level criterion weights were standardized using Equation (4.4); see Table 4.4 as well.

Table 4.4. GRFT weights for the evaluation of university reputation.

First-Level Indicators (c_α)	Local Weights (W')	Second-Level Indicators ($c_{\alpha\text{-}\beta}$)	Local Weights (W'')	Effective Weights in % (W)
c_1	[0.1438, 0.1692]	$c_{1\text{-}1}$	[0.3365, 0.3392]	[4.84, 5.74]
		$c_{1\text{-}2}$	[0.2666, 0.3199]	[3.83, 5.41]
		$c_{1\text{-}3}$	[0.2981, 0.3409]	[4.29, 5.77]
c_2	[0.1455, 0.1607]	$c_{2\text{-}1}$	[0.3217, 0.3263]	[4.68, 5.24]
		$c_{2\text{-}2}$	[0.3428, 0.3428]	[4.99, 5.51]
		$c_{2\text{-}3}$	[0.2941, 0.3309]	[4.28, 5.32]
c_3	[0.1438, 0.1692]	$c_{3\text{-}1}$	[0.2466, 0.3336]	[3.55, 5.65]
		$c_{3\text{-}2}$	[0.3184, 0.339]	[4.58, 5.74]
		$c_{3\text{-}3}$	[0.3049, 0.3274]	[4.39, 5.54]
c_4	[0.1184, 0.1692]	$c_{4\text{-}1}$	[0.2179, 0.2614]	[2.58, 4.42]
		$c_{4\text{-}2}$	[0.256, 0.3355]	[3.03, 5.68]
		$c_{4\text{-}3}$	[0.3813, 0.4031]	[4.52, 6.82]
c_5	[0.1269, 0.1624]	$c_{5\text{-}1}$	[0.2618, 0.2802]	[3.32, 4.55]
		$c_{5\text{-}2}$	[0.3478, 0.3623]	[4.41, 5.89]
		$c_{5\text{-}3}$	[0.3478, 0.3575]	[4.41, 5.81]
c_6	[0.1184, 0.1692]	$c_{6\text{-}1}$	[0.2554, 0.3595]	[3.03, 6.08]
		$c_{6\text{-}2}$	[0.2015, 0.2886]	[2.39, 4.88]
		$c_{6\text{-}3}$	[0.2867, 0.3519]	[3.4, 5.95]

Source: Reprinted from [100], used with permission.

$$W' = [0.1438, 0.1692], [0.1455, 0.1607], [0.1438, 0.1692], [0.1184, 0.1692], [0.1269, 0.1624], [0.1184, 0.1692], \quad (4.8)$$

Also, the second-level criteria were standardized using Equation (4.5).

$$W'' = [0.3365,\ 0.3392],\ [0.2666,\ 0.3199],\ [0.2981,\ 0.3409],\ \ldots,\ [0.3478,\ 0.3575]. \quad (4.9)$$

Finally, the effective grey weights for the second-level criteria were calculated using Equation (4.6), incorporating the grey weights as described in Equation (4.7). The results are presented in Table 4.4.

$$W = 100^{-1} \times \left([4.84,\ 5.74],\ [3.83,\ 5.41],\ [4.29,\ 5.77], \ldots,\ [3.4,\ 5.95]\right)^{\mathrm{T}}. \quad (4.10)$$

The weights, expressed in percentages, range from [4.84, 5.74] to [3.4, 5.95], with the sum of the upper bounds totaling 100%. The estimated grey weights of the DMs preferences indicate that the most critical criterion for assessing the reputation of a university is the level of funding (c_4), with "scholarship" ($c_{4\text{-}3}$) having the highest grey weight of [4.52, 6.82]. Conversely, the least important criterion in assessing a university's reputation is "students' guidance" (c_6) from the perspective of parents ($c_{6\text{-}2}$). The level of uncertainty associated with these weights can be observed through the length of the weight bars depicted in Figure 4.5. The shortest bar represents the highest degree of certainty, corresponding to the "safety" ($c_{2\text{-}2}$) condition in university environments (c_2). In contrast, the longest bar represents the highest degree of uncertainty, corresponding to "students' guidance" (c_6) in the context of recommendations ($c_{6\text{-}1}$) that students may receive, which can vary widely among different individuals.

Customer awareness is crucial for any business, and in the case of a university, students are considered its clients. The weights assigned by decision-makers can be elucidated through the lens of psychology. Psychology explores various concepts, including attention, motivation, emotion, brain function, intelligence, personality, relationships, consciousness, and unconsciousness. One unique theory in psychology is the RFT, which elucidates how individuals pursue their goals. It is important to recognize that in real-life decision-making, uncertainty is inherent. Grey Systems Theory (GST) offers a valuable framework for assigning and evaluating weights to account for varying degrees of uncertainty. In Chapter 8, we introduce the grewy number-based GRA evaluation procedure, which further enhances our understanding of these concepts.

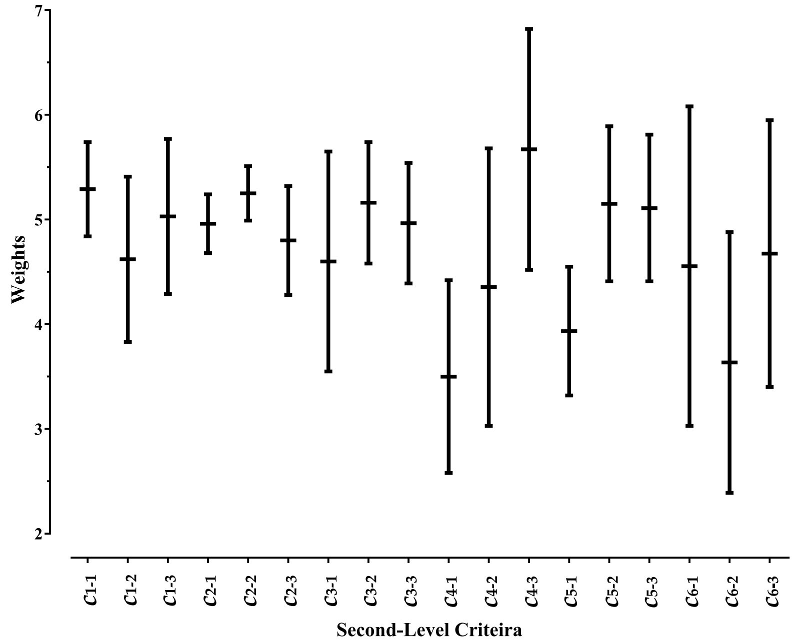


Figure 4.5. Decision-makers' GRFT weights. Source: Reprinted from [100], used with permission.

5. Grey SWARA Weighting Method

In this chapter, an innovative extension of the stepwise weight analysis ratio assessment (SWARA) method is introduced, incorporating the principles of Grey Systems Theory. The context for this method's application lies in the realm of international and multinational corporations, where the challenge of compensating expatriates appropriately in countries with less-than-optimal working conditions is a critical concern. The chapter delves into the intricate issue of determining the foreign service premium ratio, a multi-criteria decision-making problem. Specifically, the use of Grey Systems Theory in the domain of compensation and benefits within the realm of human resource management is explored. The novel approach, the grey SWARA method for uncertain group decision-making, is employed to estimate the weights of various criteria. As this chapter is navigated, insights into how this method enhances decision-making processes related to expatriate compensation are provided, offering valuable perspectives for international companies. The flowchart in Figure 5.1 visually outlines the structure and content.

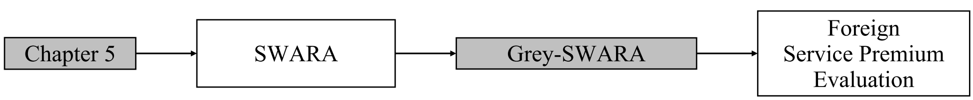


Figure 5.1. Flowchart of Chapter 5. Source: Figure by authors.

5.1. SWARA Method

The SWARA weighting method is initiated by ranking the criteria, followed by pairwise comparisons between the directly higher-ranked criterion and the lower-ranked criterion, which is the directly adjacent criterion. Subsequently, a comparative coefficient is calculated, and the weight is derived and scaled to address MCDM problems. The steps for estimating criteria weights using SWARA are outlined as follows:

Step 1. Rank the criteria. The criteria are ranked based on the preference of the decision-maker (DM), starting with the most important criterion and proceeding to the least important criterion.

Step 2. Determine comparative importance through average values. The comparative importance is ascertained based on the criteria ranked in the second position. Subsequent comparative importance is determined by comparing criterion j to criterion j-1.

Step 3. Determine the comparative coefficient. The comparative coefficient, denoted as coefficient k_j, is calculated using Equation (5.1):

$$k_j = \begin{cases} 1 & j = 1 \\ s_j + 1 & j > 1 \end{cases}, \tag{5.1}$$

where s_j represents the comparative importance of the average value [238].

Step 4. Calculate the weights. The unscaled weight q_j is calculated using Equation (5.2):

$$q_j = \begin{cases} 1 & j = 1 \\ \frac{k_j - 1}{k_j} & j > 1 \end{cases}. \tag{5.2}$$

Step 5. Calculate the scaled weight. Generally, MCDM criteria weights are scaled to one unit, or 100%. The scaled weight, denoted as w_j, is computed using Equation (5.3):

$$w_j = \frac{q_j}{\sum\limits_{k=1}^{n} q_k}. \tag{5.3}$$

Step 6. Compute the effective weight. In group decision-making, the weights of each Decision-Maker (DM) based on the SWARA method are aggregated as a summation and then scaled to obtain effective weights. Equation (5.4) provides a SWARA weights vector for the first-level criteria $W_\alpha^s = w_j(1)$, $w_j(2), w_j(3), \ldots, w_j(v)$ by v DMs for the j^{th} criteria:

$$w_\alpha^s = \frac{w_j(u)}{\sum_{u=1}^{v} w_j(u)}. \tag{5.4}$$

Similarly, for a SWARA weight vector for the second-level criteria $W_{\alpha-\beta}^s = w_j(1), w_j(2), w_j(3), \ldots, w_j(v)$ by v DMs for the jth criteria, Equation (5.5) is applied:

$$w_{\alpha\text{-}\beta}^s = \frac{w_{\alpha\text{-}\beta}(u)}{\sum_{u=1}^{v} w_{\alpha\text{-}\beta}(u)}. \tag{5.5}$$

Hence, the effective SWARA weight is calculated as shown in Equation (5.6):

$$w_v^s = w_\alpha^s \times w_{\alpha\text{-}\beta}^s. \tag{5.6}$$

These steps are employed for estimating criteria weights using the SWARA method for group decision-making, commencing with criterion ranking and subsequent pairwise comparisons of adjacent ranked criteria.

5.2. SWARA Weighting Method with Grey Weights

The SWARA weighting method is extended to group decision-making using grey weight. The steps for estimating criteria weights using the SWARA method in group decision-making are outlined as follows:

Step 1. Rank the criterion based on its level of importance. Criteria are ranked based on their perceived importance, reflecting the preferences of the decision-makers (DMs).

Step 2. Determine the comparative importance of average value. Determine the comparative importance of each criterion concerning the average value. Comparative importance is calculated concerning the ranking of criterion j

relative to criterion j-1. This process begins with the second-ranked criterion.

Step 3. Determine the comparative coefficient.Compute the comparative coefficient, represented as coefficient k_j, using Equation (5.7):

$$k_j = \begin{cases} 1 & j = 1 \\ s_j + 1 & j > 1 \end{cases}, \tag{5.7}$$

where s_j signifies the comparative importance concerning the average value [238].

Step 4. Recalculate the weights. The recalculated weights, represented as q_j, are obtained as follows:

$$q_j = \begin{cases} 1 & j = 1 \\ \frac{k_j\text{-}1}{k_j} & j > 1 \end{cases}.$$

Step 5. Calculate the weights. These weights are scaled to a unit value, ensuring that they are relative to each other. The calculation for each weight w_j is as follows:

$$w_j = \frac{q_j}{\sum_{k=1}^{n} q_k}. \tag{5.8}$$

In the following part, we extend the SWARA method for group decision-making by representing the DM weights using grey interval numbers. Grey inveral numbers capture the uncertainty in weights by computing the weights for each DM and determining the scaled minimum and maximum weights for each criterion across all DMs.

For a weight matrix W of p DMs and n criteria, represented as:

$$W = \begin{bmatrix} w_{11} & w_{12} & \cdots & w_{1n} \\ w_{21} & w_{22} & \cdots & w_{2n} \\ \vdots & \ddots & \cdots & \vdots \\ w_{p1} & w_{p2} & \cdots & w_{pn} \end{bmatrix}, \tag{5.9}$$

the grey weight of the criteria is denoted as:

$$\otimes W^{\Theta} = \begin{pmatrix} \otimes w_1 & \otimes w_2 & \cdots & \otimes w_n \end{pmatrix}, \tag{5.10}$$

where $\otimes w_j = \left[\underline{w_j}, \overline{w_j}\right] = \left[\frac{\min_{1 \le i \le p} w_{ij}}{\sum_{j=1}^{n} \max_{1 \le i \le p} w_{ij}}, \frac{\max_{1 \le i \le p} w_{ij}}{\sum_{j=1}^{n} \max_{1 \le i \le p} w_{ij}}\right]$.

It's important to note that the SWARA method by Dahooie et al. [239] results in crisp weights, i.e., deterministic weights that do not account for uncertainty. In contrast, the implementation of the SWARA method in this paper estimates evaluation weights as interval grey numbers, providing a reasonable margin to capture uncertainty.

5.3. Application of SWARA-GN Weighting Method for Contractor Selection

The primary standard for evaluating alternatives in an MCDM problem is the criteria. In this application, we employ 36 criteria to evaluate contractors. These criteria are categorized into two levels: six first-level criteria and 30 second-level criteria. Figure 5.2 presents the hierarchical diagram used for evaluation, where each first-level criterion comprises five second-level criteria. These criteria have been derived from the existing literature.

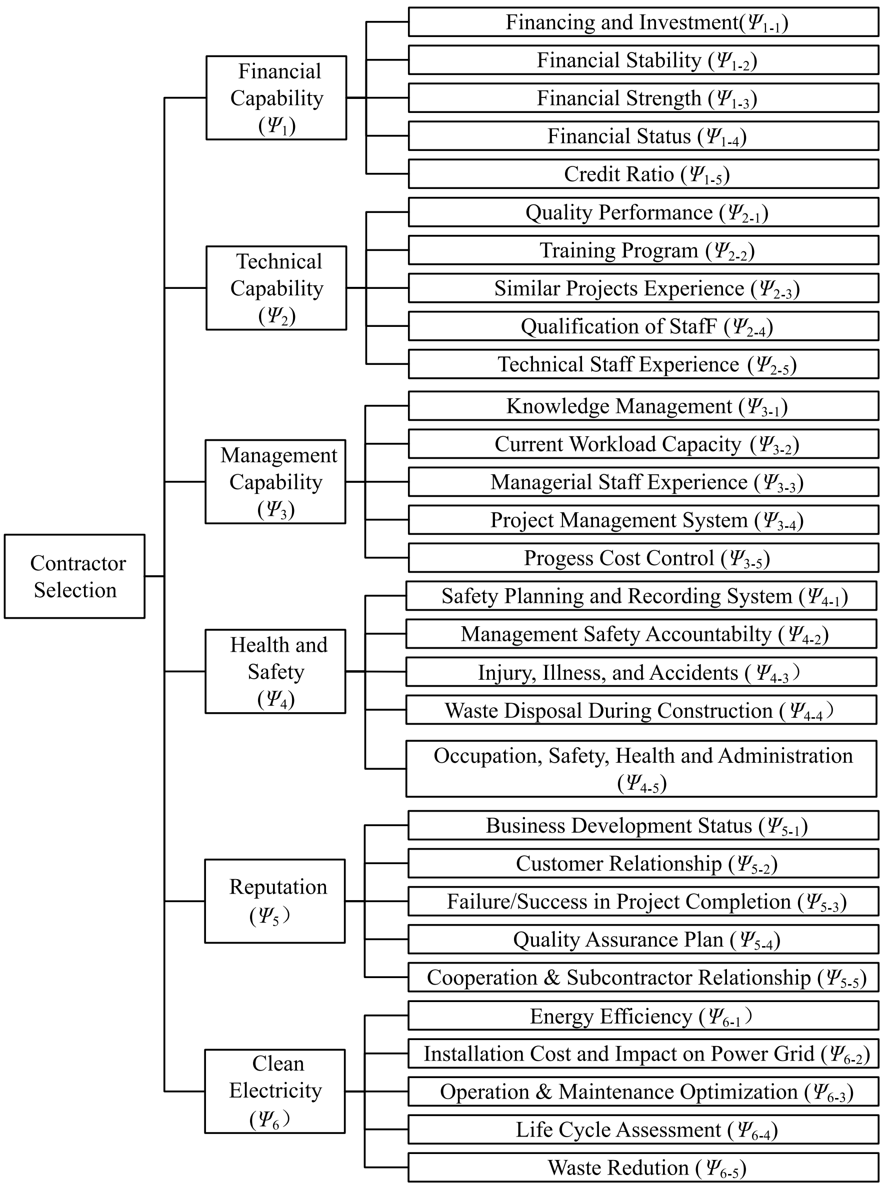


Figure 5.2. Solar panel contractor selection hierarchical model. Source: Reprinted from [102], used with permission.

Two sets of web-based questionnaires were meticulously designed to collect data from the Decision-Makers (DMs). These web-based questionnaires offered the advantage of facilitating criterion ranking by allowing the DMs to easily adjust the importance of criteria by moving them up or down the scale. Prior to distributing the questionnaires to the DMs, multiple revisions and pilot tests were conducted to ensure their clarity and effectiveness. The first set of questionnaires aimed to capture the rankings of criteria, and the obtained ranking data are presented in Table A1 in the Appendix A.1.1.

The second set of questionnaires was specifically designed to elicit the degree to which the criteria directly ranked higher were considered more important than the lower-ranked ones. This assessment was conducted using a nine-point scale given in Table 5.1. The comparative data collected from the DMs are presented in Table A2 in the Appendix A.1.1.

Table 5.1. Nine-point scale for weighting.

Numerical Values	Definitions
1	Important
3	Essentially improtant
5	Weakly important
7	Very strongly important
9	Absolutely important
2, 4, 6, 8	Intermediate values

Source: Adapted from [103], used with permission.

Following the steps outlined in Section 5.2, the weights of Decision-Makers (DMs) were calculated. Table 5.2 displays the weights for the first-level indicators, which were calculated based on the preferences of the first DM. Calculations for the other criteria by additional DMs are not provided here. The effective SWARA weights of the DMs are presented in Table 5.3, demonstrating the collective weights assigned to each DM based on their assessments. Additionally, Table 5.4 showcases the effective SWARA weights assigned to the criteria. These weights represent the overall assessment of criteria importance obtained through the SWARA-GN (grey number) weighting method.

Table 5.2. Estimated weights for DM_1 based on the SWARA weighting method.

Rankings	First-Level Criteria (Ψ_j)	Comparative Importance of Average, s_j	Coefficient, $k_j = s_j + 1$	Re-Calculated Weights, $w_j = {}^{x_{j-1}}/_{k_j}$	Scaled Weights, $q_j = {}^{w_j}/_{\sum_{j=1}^{m} w_j}$
1st	Ψ_5	–	1	1	0.248
2nd	Ψ_4	0.0833	1.0833	0.9231	0.2289
3rd	Ψ_6	0.0556	1.1389	0.8105	0.201
4th	Ψ_2	0.1944	1.3333	0.6079	0.1507
5th	Ψ_3	0.1111	1.4444	0.4208	0.1044
6th	Ψ_4	0.1111	1.5556	0.2705	0.0671

Source: Reprinted from [102], used with permission.

Table 5.3. Computed SWARA weights for each DM.

Criteria	DM_1	DM_2	DM_3	DM_4
Ψ_1	0.0671	0.0679	0.0584	0.0566
Ψ_2	0.1507	0.2461	0.2647	0.1871
Ψ_3	0.1044	0.1037	0.1436	0.1321
Ψ_4	0.2289	0.1959	0.2382	0.0881
Ψ_5	0.248	0.1469	0.0957	0.2443
Ψ_6	0.201	0.2394	0.1994	0.2918
$\Psi_{1\text{-}1}$	0.1146	0.142	0.2088	0.1194
$\Psi_{1\text{-}2}$	0.156	0.1814	0.2437	0.2693
$\Psi_{1\text{-}3}$	0.2037	0.2117	0.1634	0.2034
$\Psi_{1\text{-}4}$	0.249	0.2293	0.264	0.2486
$\Psi_{1\text{-}5}$	0.2766	0.2357	0.1201	0.1592
$\Psi_{2\text{-}1}$	0.1633	0.1694	0.2884	0.1567
$\Psi_{2\text{-}2}$	0.1176	0.1325	0.2032	0.1106
$\Psi_{2\text{-}3}$	0.2042	0.207	0.2596	0.2046
$\Psi_{2\text{-}4}$	0.2439	0.2357	0.0995	0.2779
$\Psi_{2\text{-}5}$	0.271	0.2554	0.1493	0.2501
$\Psi_{3\text{-}1}$	0.1314	0.1304	0.1707	0.2565
$\Psi_{3\text{-}2}$	0.1715	0.1738	0.2884	0.2779
$\Psi_{3\text{-}3}$	0.271	0.2554	0.2663	0.1289
$\Psi_{3\text{-}4}$	0.2501	0.2419	0.2229	0.1718
$\Psi_{3\text{-}5}$	0.2144	0.2124	0.1182	0.2148
$\Psi_{4\text{-}1}$	0.271	0.2554	0.2884	0.2779
$\Psi_{4\text{-}2}$	0.1934	0.1697	0.1108	0.244
$\Psi_{4\text{-}3}$	0.1547	0.13	0.2663	0.0783
$\Psi_{4\text{-}4}$	0.2567	0.2419	0.2178	0.1218
$\Psi_{4\text{-}5}$	0.231	0.2074	0.1601	0.1793
$\Psi_{5\text{-}1}$	0.1339	0.1392	0.2178	0.0834
$\Psi_{5\text{-}2}$	0.171	0.1779	0.2663	0.2274
$\Psi_{5\text{-}3}$	0.209	0.2124	0.1634	0.1228
$\Psi_{5\text{-}4}$	0.271	0.2554	0.1153	0.2779
$\Psi_{5\text{-}5}$	0.2439	0.2419	0.2884	0.1705
$\Psi_{6\text{-}1}$	0.1594	0.1782	0.0909	0.1864
$\Psi_{6\text{-}2}$	0.1992	0.2177	0.2472	0.1342
$\Psi_{6\text{-}3}$	0.2379	0.2419	0.1894	0.0912
$\Psi_{6\text{-}4}$	0.271	0.2554	0.2884	0.2779
$\Psi_{6\text{-}5}$	0.1171	0.1336	0.1364	0.2382

Source: Reprinted from [102], used with permission.

Table 5.4. Effective SWARA weight.

1st Level Criteria	Local Weight	2nd Level Criteria	Index (v)	Local Weights	Effective Weight
		$\Psi_{1\text{-}1}$	1	0.1462	0.0091
		$\Psi_{1\text{-}2}$	2	0.2126	0.0133
Ψ_1	0.0625	$\Psi_{1\text{-}3}$	3	0.1956	0.0122
		$\Psi_{1\text{-}4}$	4	0.2477	0.0155
		$\Psi_{1\text{-}5}$	5	0.1979	0.0124
		$\Psi_{1\text{-}1}$	6	0.1945	0.0413
		$\Psi_{1\text{-}2}$	7	0.141	0.0299
Ψ_2	0.2122	$\Psi_{1\text{-}3}$	8	0.2189	0.0464
		$\Psi_{1\text{-}4}$	9	0.2143	0.0455
		$\Psi_{1\text{-}5}$	10	0.2314	0.0491
		$\Psi_{1\text{-}1}$	11	0.1653	0.02
		$\Psi_{1\text{-}2}$	12	0.2187	0.0264
Ψ_3	0.1209	$\Psi_{1\text{-}3}$	13	0.221	0.0267
		$\Psi_{1\text{-}4}$	14	0.2127	0.0257
		$\Psi_{1\text{-}5}$	15	0.1823	0.022
		$\Psi_{1\text{-}1}$	16	0.2694	0.0506
		$\Psi_{1\text{-}2}$	17	0.177	0.0332
Ψ_4	0.1878	$\Psi_{1\text{-}3}$	18	0.1551	0.0291
		$\Psi_{1\text{-}4}$	19	0.2067	0.0388
		$\Psi_{1\text{-}5}$	20	0.1918	0.036
		$\Psi_{1\text{-}1}$	21	0.144	0.0265
		$\Psi_{1\text{-}2}$	22	0.2112	0.0388
Ψ_5	0.1837	$\Psi_{1\text{-}3}$	23	0.1774	0.0326
		$\Psi_{1\text{-}4}$	24	0.2305	0.0424
		$\Psi_{1\text{-}5}$	25	0.2369	0.0435
		$\Psi_{6\text{-}1}$	26	0.158	0.0368
		$\Psi_{6\text{-}2}$	27	0.2052	0.0478
Ψ_6	0.2329	$\Psi_{6\text{-}3}$	28	0.1954	0.0455
		$\Psi_{6\text{-}4}$	29	0.2808	0.0654
		$\Psi_{6\text{-}5}$	30	0.1607	0.0374

Source: Adapted from [102], used with permission.

5.4. Application of SWARA-GN Weighting Method for Scaling Allowance

This section delves into the application of the SWARA-GN (grey number) weighting method for scaling allowances in the context of human resource (HR) Compensation and Benefits (C&B). C&B encompass all forms of monetary and non-monetary rewards that employees receive for their work. Direct compensation includes regular payments such as wages, salaries, bonuses, and commissions. Indirect compensation covers monetary benefits that are part of the employment agreement, such as leave with pay, insurance, pension plans, training, and employee services. Non-monetary benefits encompass intangible factors like career prospects, opportunities for recognition, and a positive work environment. It's essential to note that what attracts employees to a company may differ from what retains them. Managing and monitoring the work environment, organizational values, competencies, commitment to the mission, motivational aspects, training levels, and career plans are vital challenges in human resource management. Research by

Highhouse et al. [240] highlights the distinction between company attractiveness and company prestige.

Job and pay satisfaction play a pivotal role in employee retention. Studies by Omar and Ogenyi [241] explore pay satisfaction among senior managers in the Nigerian civil service, emphasizing the role of instrumental perception and procedural justice in pay-incentive schemes. Schaubroeck et al. [242] delve into the relationship between pay-for-performance and employee reactions, including pay satisfaction and turnover intention. Emotional stability is a significant factor in salary satisfaction, as evidenced by Shrader and Singer's [243] research on compensation for small-business managers in China and the USA. Effective communication of compensation packages is crucial to justify and validate pay levels and structures.

On the contrary, research has also investigated the impact of pay secrecy on employee task performance. Bamberger and Belogolovsky [244] found that pay secrecy is associated with higher performance, mediated by perceptions of fairness. However, pay secrecy can negatively affect employees sensitive to inequality. Jawahar and Stone [245] also confirmed that informational justice relates to pay-level satisfaction, pay structure and administration, as well as potentially relating to an increase in payment. Studies have shown that informational justice is related to pay-level satisfaction, pay structure and administration, and potentially higher pay. Shen's [246] research presents various models for Chinese expatriate compensation, considering firm-specific factors, host contextual factors, and international human resource management policies and practices.

Worker compensation and assistance programs are essential in organizations, particularly for those involved in physically demanding jobs. Legal protections in the USA ensure that employees, regardless of their condition or nature of employment, receive benefits for work-related injuries or illnesses. Spieler's [247] research highlights the role of worker compensation benefits in preventing employees from falling into poverty due to job-related injuries or illnesses. The impact of C&B on expatriate assignments has been explored, especially in the context of women expatriates in the oil and gas industry. Employees whose work involves a certain degree of physical difficulty are prone to accidents in their jobs. Employees in the USA are protected by the law regardless of their condition or nature of employment, which today is a complete benefit provided to them [248]. Shortland [249] used a triangulated qualitative-research approach to know how women's decision to be an expatriate is affected by Compensation and Benefit (C&B) in the oil and gas industry. Factors like housing quality, salary increments, access to quality education and healthcare, and travel and leave arrangements significantly influence women's decisions to accept overseas assignments.

Some domestic companies opt to transition strategically into multinational and, possibly, global enterprises to expand their market size and increase profitability [250]. During the initial stages of globalization, it becomes essential to adequately prepare foreign staff to align with the company's overall strategy. Sending expatriates to oversee foreign branches can be a cost-effective alternative to relocating all foreign staff to the company's headquarters. This cost reduction arises from the challenges associated with integrating local staff into headquarters when they speak different languages and use technology equipment in a different language, such as

Chinese. Sending expatriates to overseas branches serves the purpose of training foreign staff, operating the company's equipment, and transferring the company's culture, all at a relatively reduced cost [251]. As global expansion leads to an increasing number of branches in various countries, companies and their Human Resources (HR) departments face a significant challenge in defining C&B policies for international mobility processes [252]. Expatriates naturally seek fair compensation, which includes a reasonable allowance for working overseas, especially in remote, challenging, or hazardous locations [253].

Here, the term "Foreign Service Premium (FSP) allowance" refers to the lump sum, among other benefits, provided to expatriates as compensation for working overseas, with the aim of attracting, retaining, and motivating them. Some companies may alternatively label this allowance as a hardship allowance or simply an expatriate allowance. This allowance can be viewed as a form of payment to expatriates to offset the challenges they face when accepting overseas assignments, which can include dealing with different cultures, work environments, and geographical distances from their families. Naturally, expatriates expect to receive varying compensation levels based on whether they accept assignments in underdeveloped countries or developed ones [249]. Here, we use the term "scaling" to refer to the practice of setting different premium allowance levels based on the assignment location. It's important to note that this book does not encompass the entire C&B package, such as the overall salary structure for expatriates.

The process of scaling the Foreign Service Premium (FSP) allowance is treated as a MCDM problem [3]. To effectively evaluate various locations for foreign assignments, numerous factors, which serve as assessment criteria, must be taken into account. The conditions in these locations are characterized by various uncertainties that necessitate consideration, and these uncertainties are quantified using Grey Number (GN). The relative importance of each criterion is determined based on rankings and comparative points provided by a group of decision-makers. These rankings and scores are then combined using the Stepwise Weight Analysis Ratio Assessment (SWARA) method [238]. Traditional Grey Relational Analysis (GRA) is subsequently employed alongside GN to establish ratios for scaling FSP allowances across different locations.

Despite the existing research, there is a notable gap in evaluating the specific locations of overseas assignments and providing a comprehensive scale for compensating and motivating expatriates to accept assignments at various overseas branches. This research aims to address these gaps using the SWARA-GN weighting method.

This research presents a case study focused on scaling FSP allowance within the petroleum equipment manufacturing and service industry in China. The company in question operates globally with 22 branches across 22 different countries. It was noted that some employees were reluctant to work in highly remote branch locations. Consequently, the proposed methodology was applied to address this issue. The branches evaluated, listed alphabetically, are situated in the following countries: Albania, Algeria, Bangladesh, Brazil, Canada, Colombia, France, Indonesia, Italy, Kazakhstan, Malaysia, Mexico, Nigeria, Pakistan, Peru, Poland, Romania, Russia, Ukraine, the United Arab Emirates (UAE), the USA, and Venezuela. Data for this research were collected during the third quarter of 2018.

Four expatriates (DM_1, DM_2, DM_3, and DM_4), who collectively possess over 70 years of work experience, were approached to provide their rankings and comparative points for all the criteria as shown in Figure 5.3 and explained in Section 2.4. The identity of the DM_s has been kept anonymous to ensure their privacy and confidentiality. The rankings assigned by these Decision-Makers are presented in Table 5.5.

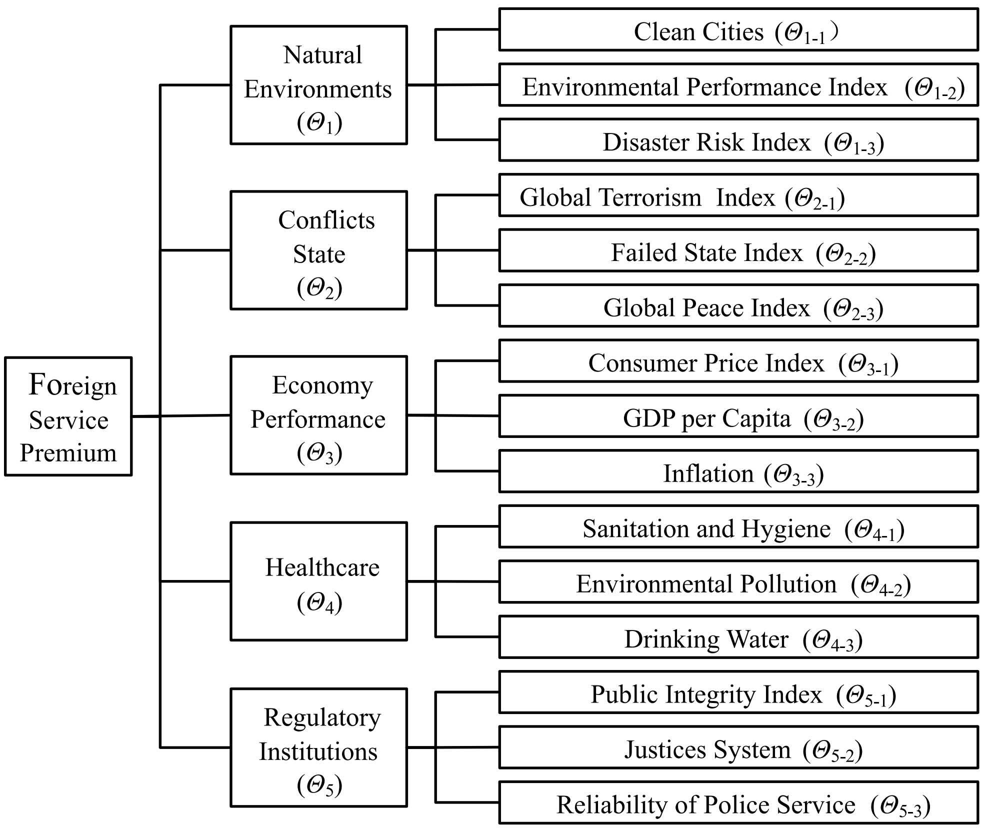


Figure 5.3. Expatriate compensation hierarchical model. Source: Adapted from [101], used with permission.

Table 5.5. Raw rankings of the first-level criteria by the DMs.

Expatriates (DM_i)/First-Level Criteria (Θ_j)	DM_1	DM_2	DM_3	DM_4
Natural Environment (Θ_1)	2nd	2nd	3rd	4th
Conflict State (Θ_2)	1st	1st	1st	1st
Economic Performance (Θ_3)	4th	3rd	2nd	2nd
Health Care (Θ_4)	3rd	5th	4th	5th
Regulatory Institution (Θ_5)	5th	4th	5th	3rd

Source: Reprinted from [101], used with permission.

Following the steps outlined in Section 5.2, the weights of the Decision-Makers (DMs) were calculated. Table 5.6 displays the weights for the first-level indicators, which were computed using the preferences of the first Decision-Makers (DM_1). Calculations for the other criteria by the remaining Decision-Makers are not presented here. The effective SWARA weights for the Decision-Makers are presented in Table 5.3, and the effective SWARA weights are summarized in Table 5.7. Therefore, the grey weight is

$$\otimes X' = \begin{pmatrix} [0.0386,\ 0.1565] & [0.0430,\ 0.1362] & \cdots & [0.0296,\ 0.0815] \\ [0.0858,\ 0.1565] & [0.1148,\ 0.1917] & \cdots & [0.0410,\ 0.0796] \\ \vdots & \vdots & \ddots & \vdots \\ [0,\ 0.0746] & [0.1050,\ 0.2285] & \cdots & [0.1070,\ 0.1413] \end{pmatrix}. \quad (5.11)$$

Table 5.6. Estimated weights for DM_1 based on the SWARA weighting method.

Rankings	First-Level Criteria (Θ_j)	Comparative Importance of Average, s_j	Coefficient, $k_j = s_j + 1$	Re-Calculated Weights, $w_j = {}^{x_{j\text{-}1}}/_{k_j}$	Scaled Weights, $q_j = {}^{w_j}/_{\sum_{j=1}^{m} w_j}$
1st	Θ_2	–	1	1	0.3097
2nd	Θ_1	0.3571	1.3571	0.7368	0.2282
3rd	Θ_4	0.2143	1.2143	0.6068	0.1879
4th	Θ_3	0.2857	1.2857	0.472	0.1462
5th	Θ_5	0.1429	1.1429	0.413	0.1279

Source: Reprinted from [100], used with permission.

Table 5.7. Grey DMs' weights.

Decision-Makers' (DM_i)/First-Level Criteria (Θ_j)	DM_1	DM_2	DM_3	DM_4	$\min_{1\le i\le 4} w_{ij}$	$\max_{1\le i\le 4} w_{ij}$	$\otimes w_j$
Θ_1	0.2282	0.2258	0.2228	0.1969	0.1969	0.2282	[0.1744, 0.2022]
Θ_2	0.3097	0.3126	0.3038	0.2941	0.2941	0.3126	[0.2605, 0.2769]
Θ_3	0.1462	0.1834	0.1885	0.2406	0.1462	0.2406	[0.1295, 0.2131]
Θ_4	0.1879	0.1292	0.1253	0.1476	0.1253	0.1879	[0.111, 0.1665]
Θ_5	0.1279	0.149	0.1595	0.1208	0.1208	0.1595	[0.107, 0.1413]

Source: Reprinted from [100], used with permission.

Practically, MCDM weighting methods ensures a more systematic and objective selection process by incorporating diverse criteria such as cost, quality, reliability, and timeliness. This relates to several theories: decision theory [254], which provides principles and methodologies for making rational choices; utility theory, which focuses on selecting options that maximize the Decision-Maker's utility or satisfaction; system theory [255], which examines complex systems and their interactions; contingency theory [256], which posits that the best course of action depends on specific situational factors; and stakeholder theory [257], which considers the interests of all stakeholders in decision-making processes. This comprehensive approach leads to more informed and balanced decisions, reducing risks associated with contractor performance and enhancing project outcomes.

Theoretically, MCDM weighting enriches decision-making frameworks by integrating quantitative and qualitative factors, fostering a more comprehensive understanding of contractor selection dynamics. It underscores the importance of multi-dimensional analysis in business decisions, promoting a more nuanced and strategic approach to supplier and contractor management. Overall, MCDM weighting aligns contractor selection with broader business goals, improving efficiency and competitiveness. Equity Theory [258], developed by John Stacey Adams, posits that employees seek fairness in compensation relative to their peers. Grey SWARA allows businesses to systematically evaluate and balance multiple factors, such as cost of living, hardship levels, and market conditions, ensuring fair and competitive compensation packages. This can enhance expatriate satisfaction, retention, and performance. Expectancy Theory [259], formulated by Victor Vroom, suggests that employees are motivated when they believe their efforts will lead to desired outcomes. Theoretically, MCDM weighting introduces a structured, quantifiable approach to decision-making in compensation strategy, enriching HR models with greater precision and flexibility. It bridges the gap between qualitative judgments and quantitative analysis, fostering more objective and transparent compensation practices in global business management.

6. Grey Point Allocation FUCOM Weighting Method

In this chapter, the grey Point Allocation (PA) Full Consistent Method (FUCOM) Weighting Method is introduced. This method combines the simple PA approach, commonly used by human resource managers, with the FUCOM, a technique widely endorsed by scholars in Grey Systems Theory (GST). This hybrid approach is employed for evaluating various scenarios, including the selection of solar panel installation contractors and the assessment of standard weights in human resource information systems. Figure 6.1 provides an overview of this hybrid method.

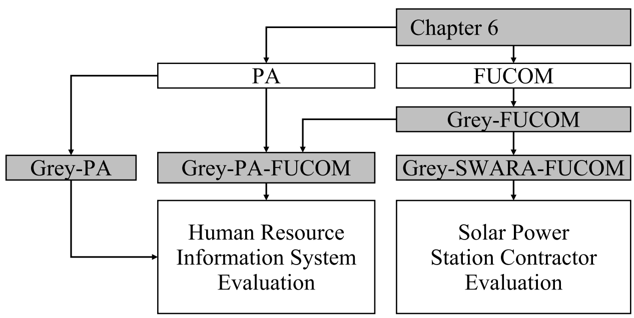


Figure 6.1. Flowchart of Chatper 6. Source: Figure by authors.

6.1. Point Allocation Weighting Method Extended to Group Decision-Making

The PA Weighting Method is introduced, which extends the approach to group decision-making. This method involves directly scoring the evaluation criteria and then scaling the scores to one unit to determine the decision weight. In this research, percentage scores ranging from 0% to 100% are used, and the steps for the PA method are outlined as follows:

Step 1. Obtain the percentage scores for the criteria. The Decision Makers (DMs) provide direct percentage scores, ranging from 0% to 100%, for each criterion.

Step 2. Scale the percentage scores. The percentage scores for each criterion are scaled to a unit value using the following equations.

- For the first-level criteria, Equation (6.1) is employed:

$$x'_p(v) = \frac{x_p(v)}{\sum_{p=1}^{\rho} x_p(v)}. \tag{6.1}$$

- For the second-level criteria, Equation (6.2) is used:

$$x'_{p\text{-}q}(v) = \frac{x_{p\text{-}q}(v)}{\sum_{q=1}^{\sigma} x_{p\text{-}q}(v)}. \tag{6.2}$$

This scaling ensures the following:

$\sum_{p=1}^{\rho} x_p'(v) = 1$ and $\sum_{q=1}^{\sigma} x_{p\text{-}q}'(v) = 1$.

Step 3. Compute the effective scaled weights. The effective scaled weights are calculated as the product of the first- and second-level criteria weights, as described in Equation (6.3):

$$w_j'(v) = x_p'(v) \times x_{p\text{-}q}'(v). \tag{6.3}$$

This calculation ensures the following: $\sum_{j=1}^{n} w_j'(v) = 1$.

6.2. Full-Consistency MCDM Method

Similar to the Stepwise Weight Analysis Ratio Assessment (SWARA) weighting method, this method begins with ranking the criteria and pairwise comparing the directly adjacent criteria, i.e., the upper-ranking criterion to the lower-ranking criterion. Next, the weights are obtained by solving an optimization function that minimizes the deviation from full consistency. The steps for the FUCOM method are as follows:

Step 1. Rank the criteria based on their level of importance. The DMs rank the criteria according to their preferences, from the most important criterion to the least important criterion.

$$C_{j(1)} > C_{j(2)} > C_{j(3)} > \ldots > C_{j(k)} > C_{j(k+1)}. \tag{6.4}$$

Step 2. Determine the comparative priority. This is the lead preference that the upper-ranking criterion $C_{j(k)}$ has over the lower-ranking criterion $C_{j(k+1)}$.

$$\Phi = \left(\varphi_{1/2}, \varphi_{2/3}, \ldots, \varphi_{k/(k+1)}\right), \tag{6.5}$$

where $\varphi_{k/(k+1)}$ is the comparative priority of $C_{j(k)}$ over $C_{j(k+1)}$.

Step 3. Compute the weights. The weights are computed as an optimization function by minimizing the deviation from full consistency (χ), as given below in Optimization Function (6.6):

$$\begin{aligned}
&\min \chi \\
&s.t. \\
&\left|\frac{w_{j(k)}}{w_{j(k+1)}} - \varphi_{k/(k+1)}\right| \le \chi, \forall j \\
&\left|\frac{w_{j(k)}}{w_{j(k+2)}} - \varphi_{k/(k+1)} \otimes \varphi_{(k+1)/(k+2)}\right| \le \chi, \forall j \\
&\sum_{j=1}^{n} w_j = 1, \forall j \\
&w_j \ge 0, \forall j
\end{aligned} \tag{6.6}$$

The first constraint ensures the weight computation. The second constraint ensures the mathematical transitivity of the criteria weights. The third constraint ensures that the criteria weights are fractions that add up to one unit. The last constraint ensures non-negative weights.

Step 4. Compute the effective weight. In group decision-making, the weights of each DM based on the FUCOM method are aggregated as a summation, then scaled to obtain the effective weights using Equation (6.7):

$$w_{\alpha}^{f} = \frac{w_j(u)}{\sum_{u=1}^{v} w_j(u)}. \tag{6.7}$$

6.3. Grey-PA-FUCOM Method

The Grey-PA-FUCOM method represents the weights obtained by both the PA and FUCOM methods in grey numbers. Then, the effective weights for the evaluation criteria are calculated. The steps for computing the Grey-PA-FUCOM weights are as follows.

Step 1. Compute effective PA and FUCOM weights. The raw weights are obtained based on the PA computation in Section 6.1 and the FUCOM weights in Section 6.2.

Step 2. Represent PA and FUCOM weights as grey numbers. The minimum and maximum weights using both the PA and FUCOM methods are scaled to obtain the Grey-PA-FUCOM weights for the first- and second-level criteria, $\otimes w_p^* = \left[\underline{w}_p^*, \overline{w}_p^*\right]$ and $\otimes w_{p\text{-}q}^* = \left[\underline{w}_{p\text{-}q}^*, \overline{w}_{p\text{-}q}^*\right]$, respectively. The lower and upper bounds of the grey numbers for the first-level criteria are computed using Equations (6.8) and (6.9), respectively.

$$\underline{w}_p^* = \frac{\min_{1 \le v \le \vartheta}\left(w_p'(v), w_p''(v)\right)}{\sum_{p=1}^{\rho} \max_{1 \le v \le \vartheta}\left(w_p'(v), w_p''(v)\right)}, \tag{6.8}$$

$$\overline{w}_p^* = \frac{\max_{1 \le v \le \vartheta}\left(w_p'(v), w_p''(v)\right)}{\sum_{p=1}^{\rho} \max_{1 \le v \le \vartheta}\left(w_p'(v), w_p''(v)\right)}. \tag{6.9}$$

Here, w_p' and w_p'' are the PA and FUCOM weights, respectively, for the vth DM. Similarly, for the second-level criteria, the lower and upper bounds of the grey numbers are computed using Equations (6.10) and (6.11), respectively.

$$\underline{w}_{p\text{-}q}^* = \frac{\min_{1 \le v \le \vartheta}\left(w_{p\text{-}q}'(v), w_{p\text{-}q}''(v)\right)}{\sum_{q=1}^{\sigma} \max_{1 \le v \le \vartheta}\left(w_{p\text{-}q}'(v), w_{p\text{-}q}''(v)\right)}, \tag{6.10}$$

$$\overline{w}_{p\text{-}q}^* = \frac{\max_{1 \le v \le \vartheta}\left(w_{p\text{-}q}'(v), w_{p\text{-}q}''(v)\right)}{\sum_{q=1}^{\sigma} \max_{1 \le v \le \vartheta}\left(w_{p\text{-}q}'(v), w_{p\text{-}q}''(v)\right)}. \tag{6.11}$$

Here, $w^{'}_{p\text{-}q}$ and $w^{''}_{p\text{-}q}$ are the PA and FUCOM weights, respectively, for the vth DM.

Step 3. Compute the effective weight. After obtaining the local weights for each of the second-level criteria, they are scaled using Equation (6.12).

$$\otimes w_j = \otimes w^*_p \times \otimes w^*_{p\text{-}q}. \quad (6.12)$$

Here, $\otimes w_j = \left[\underline{w}_j, \overline{w}_j\right]$.

Therefore, for an n number of criteria of the Grey-PA-FUCOM weights, W is given as follows:

$$W = \begin{pmatrix} \otimes w_1 & \otimes w_2 & \cdots & \otimes w_n \end{pmatrix}^T, \quad (6.13)$$

where $\sum_{j=1}^{n} \overline{w}_j = 1$.

6.4. Grey SWARA FUCOM Weighting Method

The grey SWARA -FUCOM method represents different weights obtained using the SWARA and FUCOM methods. The procedure for the grey SWARA -FUCOM weighting method is as follows.

Step 1. Obtain the SWARA and FUCOM weights of the criteria using Equation (5.8) and the Optimization Function (6.6).

Step 2. Represent the SWARA and FUCOM weights as grey numbers. The minimum and maximum weights using both methods are scaled to obtain the grey SWARA -FUCOM weights, $\otimes w_v = [\underline{w}_v, \overline{w}_v]$, respectively. The lower bound and upper bound of the GN are computed using Equation (6.14) and Equation (6.15), respectively:

$$\underline{w}_v = \frac{\min\left(w^f_v, w^s_v\right)}{\sum_{i=1}^{n} \max(w^f_i, w^s_i)}, \quad (6.14)$$

$$\overline{w}_v = \frac{\max\left(w^f_v, w^s_v\right)}{\sum_{i=1}^{n} \max(w^f_i, w^s_i)}, \quad (6.15)$$

where w^s and w^f are the SWARA and FUCOM weights, respectively, v is the index of the criteria for the MCDM evaluation method (i.e., the second-level criteria index), $\otimes w_v = [\underline{w}_v, \overline{w}_v]$, and $\sum_{v=1}^{m} \overline{w}_v = 1$.

Step 3. Form the weight matrix, W, as follows:

$$W = \begin{pmatrix} \otimes w_1 & \otimes w_2 & \cdots & \otimes w_n \end{pmatrix}^{\mathrm{T}}. \quad (6.16)$$

The transpose of the matrix in Equation (6.16) is the effective weight of the decision criteria that can be used in other MCDM evaluation methods, such as GRA and EDAS.

6.5. Application of Grey FUCOM Weighting Method

The evaluation of a solar panel system installation contractor begins with the contracting company (Z Company) conducting background checks on certain contractors and then doing business with those who are satisfied. Four contractors (A_1, A_2, A_3, and A_4) were selected, and the tenders that presented the evaluation criteria in Figure 5.2 were limited to these contractors. In this chapter, the contractor remains anonymous, and no background information about the contractor is provided. The analysis includes two main aspects: The first is the Grey-SWARA -FUCOM weighting method.

6.5.1. Contractor Selection Criteria Weight

This is the floating solar power station contractor selection problem. The criteria weights are estimated, and then the contractors are ranked.

Application of Full Consistency Method Weights

Based on the steps in Section 6.2, the DMs using FUCOM are shown in Table 6.1, and the group's effective FUCOM weights are presented in Table 6.2. For example, the first criteria of DM_1 using the Function (6.6), the objective function is given as follows:

$$
\begin{aligned}
&\min \chi \\
&s.t. \\
&\left|\frac{w_5}{w_4} - \frac{1}{4}\right| \le \chi, \forall j; \ldots ; \left|\frac{w_3}{w_1} - \frac{5}{5}\right| \le \chi, \forall j; \\
&\left|\frac{w_5}{w_6} - \frac{1}{4} \times \frac{4}{3}\right| \le \chi, \forall j; \cdots ; \left|\frac{w_2}{w_1} - \frac{8}{5} \times \frac{5}{5}\right| \le \chi, \forall j \ . \\
&\sum_{j=1}^{6} w_j = 1, \forall j \\
&w_1, w_2, w_3, w_4, w_5, w_6 \ge 0, \forall j
\end{aligned}
$$

Similarly, the objectives for the different criteria of the other DMs can be computed accordingly.

A similar study by Esangbedo and Bai [260] addressed the challenge of subcontractor selection in a decision-making environment characterized by uncertainty. The point allocation weighting method was employed for subjective weighting, and the Rank Order Centroid with Slacks (ROCS) weighting method was used to address limitations associated with the Rank Order Centroid Method. Additionally, they improved grey relational analysis (GRA) by introducing positive and negative reference (PNR) alternatives, deviating from the classical GRA, which used a single reference alternative. This enhancement was implemented to evaluate the selection of the most appropriate subcontractor for the supply of heliostats for photothermal power station construction.

Table 6.1. Computed FUCOM weights for each DM.

Criteria	DM_1	DM_2	DM_3	DM_4	Criteria	DM_1	DM_2	DM_3	DM_4
Ψ_1	0.0879	0.0879	0.0858	0.1581	$\Psi_{3\text{-}3}$	0.48	0.4918	0.1137	0.123
Ψ_2	0.0879	0.4396	0.515	0.0949	$\Psi_{3\text{-}4}$	0.12	0.1639	0.0812	0.1639
Ψ_3	0.1099	0.1099	0.0644	0.0949	$\Psi_{3\text{-}5}$	0.12	0.123	0.0947	0.0984
Ψ_4	0.2198	0.0549	0.103	0.1186	$\Psi_{4\text{-}1}$	0.4724	0.4724	0.5455	0.5788
Ψ_5	0.4396	0.0879	0.103	0.0593	$\Psi_{4\text{-}2}$	0.1575	0.1575	0.1364	0.1364
Ψ_6	0.0549	0.2198	0.1288	0.4743	$\Psi_{4\text{-}3}$	0.1181	0.1181	0.1364	0.1364
$\Psi_{1\text{-}1}$	0.1681	0.0876	0.1282	0.16	$\Psi_{4\text{-}4}$	0.1575	0.1575	0.0909	0.0909
$\Psi_{1\text{-}2}$	0.1261	0.1095	0.1282	0.48	$\Psi_{4\text{-}5}$	0.0945	0.0945	0.0909	0.0909
$\Psi_{1\text{-}3}$	0.1008	0.146	0.1026	0.08	$\Psi_{5\text{-}1}$	0.1515	0.1154	0.0893	0.132
$\Psi_{1\text{-}4}$	0.1008	0.219	0.5128	0.12	$\Psi_{5\text{-}2}$	0.1515	0.1538	0.1339	0.0587
$\Psi_{1\text{-}5}$	0.5042	0.438	0.1282	0.16	$\Psi_{5\text{-}3}$	0.1515	0.1154	0.1071	0.176
$\Psi_{2\text{-}1}$	0.1709	0.1154	0.5683	0.1351	$\Psi_{5\text{-}4}$	0.4545	0.4615	0.1339	0.5279
$\Psi_{2\text{-}2}$	0.0855	0.1538	0.0812	0.1081	$\Psi_{5\text{-}5}$	0.0909	0.1538	0.5357	0.1056
$\Psi_{2\text{-}3}$	0.1282	0.1538	0.1137	0.1081	$\Psi_{6\text{-}1}$	0.1639	0.0968	0.1639	0.1116
$\Psi_{2\text{-}4}$	0.1026	0.1154	0.0947	0.5405	$\Psi_{6\text{-}2}$	0.1639	0.1613	0.1639	0.1116
$\Psi_{2\text{-}5}$	0.5128	0.4615	0.1421	0.1081	$\Psi_{6\text{-}3}$	0.082	0.1613	0.082	0.1394
$\Psi_{3\text{-}1}$	0.16	0.0984	0.1421	0.123	$\Psi_{6\text{-}4}$	0.4918	0.4839	0.4918	0.5578
$\Psi_{3\text{-}2}$	0.12	0.123	0.5683	0.4918	$\Psi_{6\text{-}5}$	0.0984	0.0968	0.0984	0.0797

Source: Table by authors.

Table 6.2. Effective FUCOM weight.

1st Level Criteria	Local Weight	2nd Level Criteria	Index (v)	Local Weights $\left(w^{s}_{\alpha\text{-}\beta}\right)$	Effective Weights in % (w^{s}_{v})
		$\Psi_{1\text{-}1}$	1	0.136	0.0143
		$\Psi_{1\text{-}2}$	2	0.2109	0.0221
Ψ_1	0.1049	$\Psi_{1\text{-}3}$	3	0.1073	0.0113
		$\Psi_{1\text{-}4}$	4	0.2382	0.025
		$\Psi_{1\text{-}5}$	5	0.3076	0.0323
		$\Psi_{1\text{-}1}$	6	0.2474	0.0704
		$\Psi_{1\text{-}2}$	7	0.1072	0.0305
Ψ_2	0.2843	$\Psi_{1\text{-}3}$	8	0.126	0.0358
		$\Psi_{1\text{-}4}$	9	0.2133	0.0607
		$\Psi_{1\text{-}5}$	10	0.3061	0.087
		$\Psi_{1\text{-}1}$	11	0.1308	0.0124
		$\Psi_{1\text{-}2}$	12	0.3258	0.0309
Ψ_3	0.0948	$\Psi_{1\text{-}3}$	13	0.3021	0.0286
		$\Psi_{1\text{-}4}$	14	0.1323	0.0125
		$\Psi_{1\text{-}5}$	15	0.109	0.0103
		$\Psi_{1\text{-}1}$	16	0.513	0.0637
		$\Psi_{1\text{-}2}$	17	0.1457	0.0181
Ψ_4	0.1241	$\Psi_{1\text{-}3}$	18	0.1262	0.0157
		$\Psi_{1\text{-}4}$	19	0.1232	0.0153
		$\Psi_{1\text{-}5}$	20	0.0919	0.0114
		$\Psi_{1\text{-}1}$	21	0.122	0.021
		$\Psi_{1\text{-}2}$	22	0.1245	0.0215
Ψ_5	0.1724	$\Psi_{1\text{-}3}$	23	0.1375	0.0237
		$\Psi_{1\text{-}4}$	24	0.3945	0.068
		$\Psi_{1\text{-}5}$	25	0.2215	0.0382

Table 6.2. *Cont.*

1st Level Criteria	Local Weight	2nd Level Criteria	Index (v)	Local Weights $\left(w^{s}_{\alpha\text{-}\beta}\right)$	Effective Weights in % (w^{s}_{v})
Ψ_6	0.2194	$\Psi_{6\text{-}1}$	26	0.134	0.0294
		$\Psi_{6\text{-}2}$	27	0.1502	0.033
		$\Psi_{6\text{-}3}$	28	0.1162	0.0255
		$\Psi_{6\text{-}4}$	29	0.5063	0.1111
		$\Psi_{6\text{-}5}$	30	0.0933	0.0205

Source: Reprinted from [102], used with permission.

Grey SWARA-FUCOM Weights

Two sets of questionnaires were designed to collect data from the DMs. All questionnaires were web-based, chosen for their ease of use in ranking criteria by moving important factors up the scale. Before administering the questions to the DMs, we underwent several revisions and pilot testing. The first set of questionnaires aimed to rank the criteria, and the data from these rankings are presented in Table A1. The second set of questionnaires was designed to assess the degree to which the directly upper-ranked criteria were more important than the lower-ranked criteria. We used a nine-point scale for this purpose as given in Table 5.1. Tables 5.3 and 6.1 display the comparative data obtained from the DMs.

Following the steps outlined in Section 5.1, the criteria weights by the decision-makers are calculated. Table 5.2 illustrates the weight calculation of the first-level indicators using the preferences of the first decision-maker, with the calculations for other criteria by other decision-makers omitted. Table 5.3 presents the effective SWARA weights for each decision-maker, and Table 5.4 displays the weights used in this study based on Equation (6.16).

$$\begin{aligned}W = (&[0.0075, 0.0118], [0.0109, 0.0182], [0.0093, 0.01], [0.0127, 0.0205], [0.0102, 0.0265],\\ &[0.0339, 0.0579], [0.0246, 0.0251], [0.0294, 0.0381], [0.0374, 0.0499], [0.0403, 0.0715],\\ &[0.0102, 0.0164], [0.0217, 0.0254], [0.0219, 0.0235], [0.0103, 0.0211], [0.0085, 0.0181],\\ &[0.0416, 0.0523], [0.0149, 0.0273], [0.0129, 0.0239], [0.0126, 0.0319], [0.0094, 0.0296],\\ &[0.0173, 0.0218], [0.0177, 0.0319], [0.0195, 0.0268], [0.0348, 0.0559], [0.0314, 0.0357],\\ &[0.0242, 0.0302], [0.0271, 0.0393], [0.021, 0.0374], [0.0537, 0.0913], [0.0168, 0.0307])^{T}.\end{aligned} \quad (6.17)$$

Figure 6.2 displays a plot of the weights. The shortest bar, representing the *Training Program* ($\Psi_{2\text{-}2}$), signifies the closest convergence between the SWARA and FUCOM methods. Conversely, the longest bar, representing the *Life Cycle Assessment* ($\Psi_{5\text{-}4}$), indicates the largest divergence between the SWARA and FUCOM methods. Among the set of criteria, the one with the lowest possible weight is Financing and Investment ($\Psi_{1\text{-}1}$), which has a lower bound of 0.0043. Conversely, the criterion with the highest possible weight is *Life Cycle Assessment* ($\Psi_{5\text{-}4}$), with an upper bound of 0.0913.

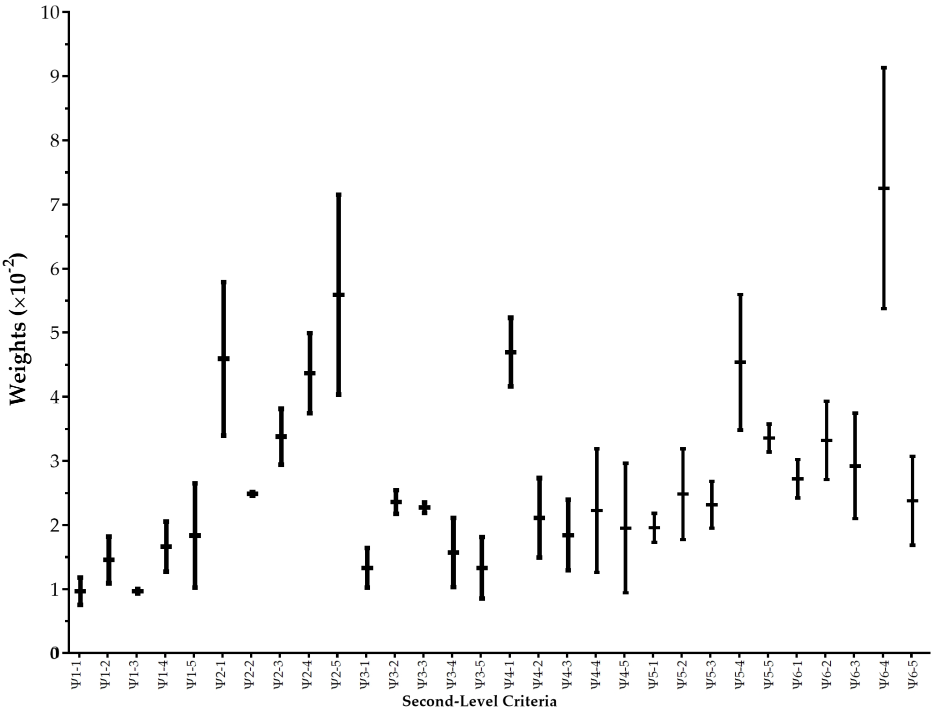


Figure 6.2. Grey point allocation with FUCOM weights. Source: Reprinted from [102], used with permission.

6.5.2. HRIS Criteria Weights

PA Weights

Here, four DMs, including an associate professor of HRM and three HR managers, assigned percentage scores to the criteria for the Human Resource Information Systems (HRISs) evaluation. These percentage scores were then normalized and scaled. The process involved the following steps, as outlined in Section PA Weights:

Step 1. Obtain percentage scores for the criteria. The four DMs provided percentage scores for all the criteria. These scores, which ranged from 0% to 100%, are documented in Table 6.3. The hierarchical model for HRIS evaluation is depicted in Figure 6.3.

Step 2. Scale the percentage ratings. The percentage ratings for each of the local weights were normalized to one unit. The scaling process involved applying Equations (6.1) and (6.2) to obtain the scaled weights for both first-level and second-level criteria. The scaled PA weights for the DMs are presented in Table 6.4.

Step 3. Compute the effective scaled weights. These effective weights are calculated as the product of the first- and second-level criteria weights, as described in Equation (6.3). The resulting effective scaled weights are presented in Table 6.5. Thus, $\sum_{j=1}^{27} w_j'(v) = 1$.

Table 6.3. Raw data of DMs rankings.

Indicators	DM_1	DM_2	DM_3	DM_4
HRM Functions (Φ_1)	100	98	85	90
Technology (Φ_2)	72	100	65	80
Software Quality (Φ_3)	83	100	65	90
Cost (Φ_4)	80	94	75	80
Vendor Support (Φ_5)	79	93	75	90
Staff Information Management ($\Phi_{1\text{-}1}$)	82	100	100	95
Organization Structure and labour Management ($\Phi_{1\text{-}2}$)	57	84	95	100
Compensation and Benefits with Budget Management ($\Phi_{1\text{-}3}$)	81	98	85	75
Staff Training Management ($\Phi_{1\text{-}4}$)	86	85	70	80
Staff Recruiting Management ($\Phi_{1\text{-}5}$)	84	84	75	90
Staff Performance Management ($\Phi_{1\text{-}6}$)	80	100	50	99
Big Data Analysis ($\Phi_{2\text{-}1}$)	90	93	100	80
Artificial Intelligence ($\Phi_{2\text{-}2}$)	100	94	70	80
Internet of Things ($\Phi_{2\text{-}3}$)	91	100	85	95
Social Network ($\Phi_{2\text{-}4}$)	85	94	75	90
Self Help Service ($\Phi_{2\text{-}5}$)	86	93	40	80
Functionality ($\Phi_{3\text{-}1}$)	100	84	90	95
Reliability ($\Phi_{3\text{-}2}$)	84	91	90	90
Usability ($\Phi_{3\text{-}3}$)	86	96	75	80
Efficiency ($\Phi_{3\text{-}4}$)	91	92	60	95
Maintainability ($\Phi_{3\text{-}5}$)	100	96	85	80
Portability ($\Phi_{3\text{-}6}$)	87	93	70	80
Operation and Maintenance Fees ($\Phi_{4\text{-}1}$)	68	94	75	95
Licensing Fees ($\Phi_{4\text{-}2}$)	67	89	65	90
Consultation Fees ($\Phi_{4\text{-}3}$)	74	93	85	80
Cost of Equipment ($\Phi_{4\text{-}4}$)	85	94	75	80
Software Training Fee ($\Phi_{4\text{-}5}$)	79	99	50	80
Vendor Reputation ($\Phi_{5\text{-}1}$)	86	93	95	80
Technological Capability ($\Phi_{5\text{-}2}$)	100	93	75	100
After-sales Service Commitment ($\Phi_{5\text{-}3}$)	100	100	95	100
After-sales Update and Upgrade ($\Phi_{5\text{-}4}$)	92	100	65	100
Software Delivery and Service Response Time ($\Phi_{5\text{-}5}$)	87	100	70	80

Source: Reprinted from [103], used with permission.

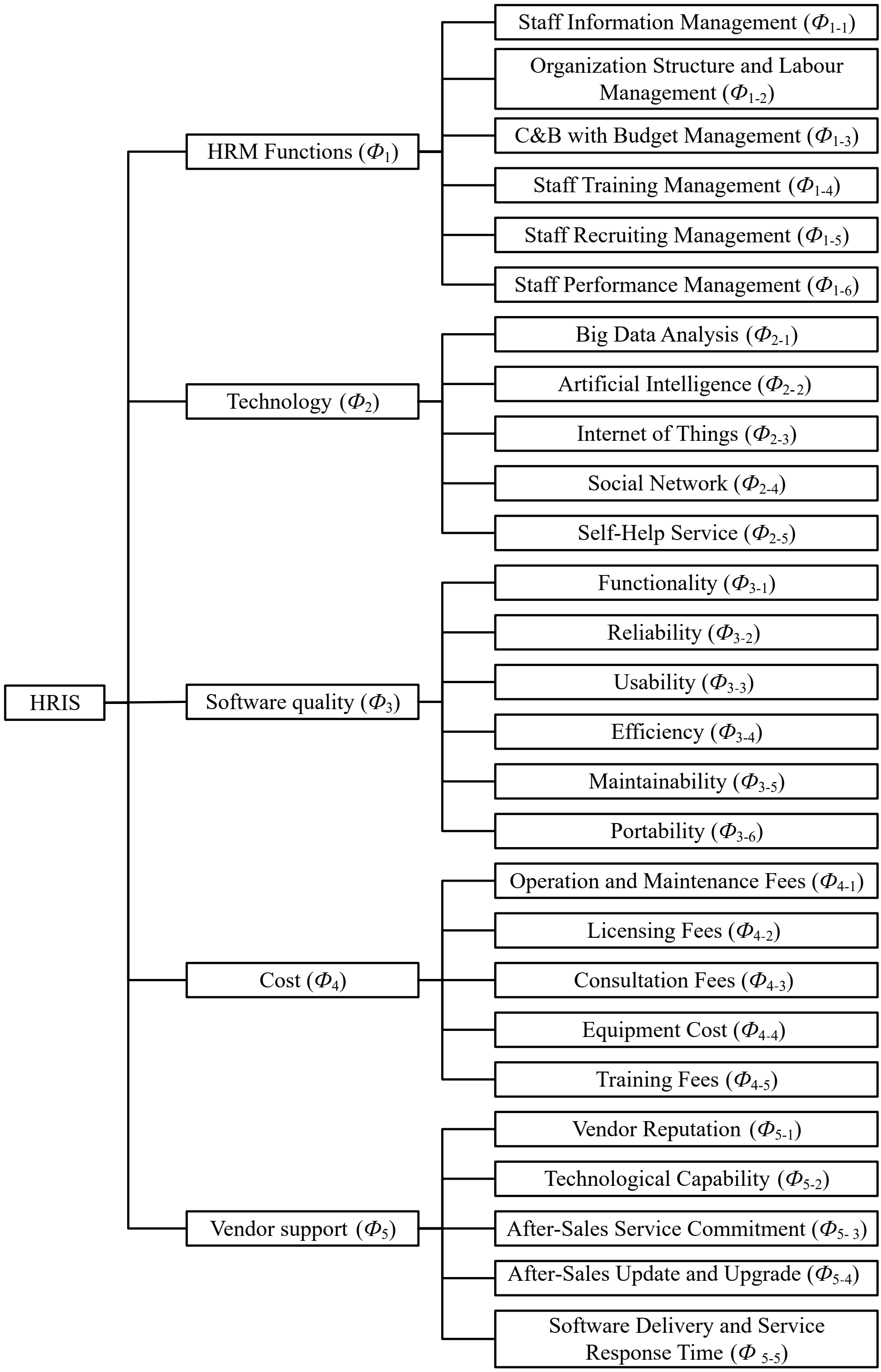


Figure 6.3. Hierarchical diagram for evaluating HRISs. Source: Reprinted from [103], used with permission.

Table 6.4. Scaled point allocation weights.

Criteria/Decision-Makers	DM_1	DM_2	DM_3	DM_4
Φ_1	0.2415	0.2021	0.2329	0.2093
Φ_2	0.1739	0.2062	0.1781	0.186
Φ_3	0.2005	0.2062	0.1781	0.2093
Φ_4	0.1932	0.1938	0.2055	0.186
Φ_5	0.1908	0.1918	0.2055	0.2093
$\Phi_{1\text{-}1}$	0.0421	0.0367	0.049	0.0369
$\Phi_{1\text{-}2}$	0.0293	0.0308	0.0466	0.0388
$\Phi_{1\text{-}3}$	0.0416	0.0359	0.0417	0.0291
$\Phi_{1\text{-}4}$	0.0442	0.0312	0.0343	0.0311
$\Phi_{1\text{-}5}$	0.0432	0.0308	0.0368	0.0349
$\Phi_{1\text{-}6}$	0.0411	0.0367	0.0245	0.0384
$\Phi_{2\text{-}1}$	0.0346	0.0405	0.0481	0.035
$\Phi_{2\text{-}2}$	0.0385	0.0409	0.0337	0.035
$\Phi_{2\text{-}3}$	0.035	0.0435	0.0409	0.0416
$\Phi_{2\text{-}4}$	0.0327	0.0409	0.0361	0.0394
$\Phi_{2\text{-}5}$	0.0331	0.0405	0.0193	0.035
$\Phi_{3\text{-}1}$	0.0366	0.0314	0.0341	0.0382
$\Phi_{3\text{-}2}$	0.0307	0.034	0.0341	0.0362
$\Phi_{3\text{-}3}$	0.0315	0.0359	0.0284	0.0322
$\Phi_{3\text{-}4}$	0.0333	0.0344	0.0227	0.0382
$\Phi_{3\text{-}5}$	0.0366	0.0359	0.0322	0.0322
$\Phi_{3\text{-}6}$	0.0318	0.0347	0.0265	0.0322
$\Phi_{4\text{-}1}$	0.0352	0.0388	0.044	0.0416
$\Phi_{4\text{-}2}$	0.0347	0.0368	0.0382	0.0394
$\Phi_{4\text{-}3}$	0.0383	0.0384	0.0499	0.035
$\Phi_{4\text{-}4}$	0.044	0.0388	0.044	0.035
$\Phi_{4\text{-}5}$	0.0409	0.0409	0.0294	0.035
$\Phi_{5\text{-}1}$	0.0353	0.0367	0.0488	0.0364
$\Phi_{5\text{-}2}$	0.041	0.0367	0.0385	0.0455
$\Phi_{5\text{-}3}$	0.041	0.0395	0.0488	0.0455
$\Phi_{5\text{-}4}$	0.0378	0.0395	0.0334	0.0455
$\Phi_{5\text{-}5}$	0.0357	0.0395	0.036	0.0364

Source: Reprinted from [103], used with permission.

Table 6.5. Computed criteria weights of DMs.

Indicators	Point Allocation Method				FUCOM Method				Grey Weights
	DM_1	DM_2	DM_3	DM_4	DM_1	DM_2	DM_3	DM_4	
Φ_1	0.2415	0.2021	0.2329	0.2093	0.506	0.3846	0.5932	0.3077	[0.1191, 0.3495]
Φ_2	0.1739	0.2062	0.1781	0.186	0.1687	0.0769	0.0847	0.0385	[0.0227, 0.1215]
Φ_3	0.2005	0.2062	0.1781	0.2093	0.1265	0.3846	0.0847	0.3077	[0.0499, 0.2266]
Φ_4	0.1932	0.1938	0.2055	0.186	0.0723	0.0769	0.1186	0.0385	[0.0227, 0.1211]
Φ_5	0.1908	0.1918	0.2055	0.2093	0.1265	0.0769	0.1186	0.3077	[0.0453, 0.1813]
$\Phi_{1\text{-}1}$	0.0421	0.0367	0.049	0.0369	0.0148	0.1374	0.2665	0.0259	[0.0069, 0.1248]
$\Phi_{1\text{-}2}$	0.0293	0.0308	0.0466	0.0388	0.1186	0.0275	0.0666	0.1295	[0.0129, 0.0607]
$\Phi_{1\text{-}3}$	0.0416	0.0359	0.0417	0.0291	0.1186	0.0275	0.0888	0.0647	[0.0129, 0.0556]
$\Phi_{1\text{-}4}$	0.0442	0.0312	0.0343	0.0311	0.1186	0.0275	0.0666	0.0432	[0.0129, 0.0556]
$\Phi_{1\text{-}5}$	0.0432	0.0308	0.0368	0.0349	0.1186	0.0275	0.0381	0.0185	[0.0087, 0.0556]
$\Phi_{1\text{-}6}$	0.0411	0.0367	0.0245	0.0384	0.0169	0.1374	0.0666	0.0259	[0.0079, 0.0644]
$\Phi_{2\text{-}1}$	0.0346	0.0405	0.0481	0.035	0.012	0.0444	0.0569	0.0586	[0.0056, 0.0275]
$\Phi_{2\text{-}2}$	0.0385	0.0409	0.0337	0.035	0.0361	0.037	0.0095	0.0586	[0.0045, 0.0275]
$\Phi_{2\text{-}3}$	0.035	0.0435	0.0409	0.0416	0.0361	0.2219	0.0142	0.1172	[0.0067, 0.1039]

Table 6.5. *Cont.*

Indicators	Point Allocation Method				FUCOM Method				Grey Weights
	DM_1	DM_2	DM_3	DM_4	DM_1	DM_2	DM_3	DM_4	
$\Phi_{2\text{-}4}$	0.0327	0.0409	0.0361	0.0394	0.0361	0.037	0.019	0.0147	[0.0069, 0.0192]
$\Phi_{2\text{-}5}$	0.0331	0.0405	0.0193	0.035	0.006	0.0444	0.019	0.0586	[0.0028, 0.0275]
$\Phi_{3\text{-}1}$	0.0366	0.0314	0.0341	0.0382	0.0202	0.0055	0.0533	0.1295	[0.0026, 0.0607]
$\Phi_{3\text{-}2}$	0.0307	0.034	0.0341	0.0362	0.0101	0.0055	0.0133	0.0259	[0.0026, 0.017]
$\Phi_{3\text{-}3}$	0.0315	0.0359	0.0284	0.0322	0.0101	0.0275	0.0076	0.0185	[0.0036, 0.0168]
$\Phi_{3\text{-}4}$	0.0333	0.0344	0.0227	0.0382	0.0067	0.0055	0.0133	0.0259	[0.0026, 0.0179]
$\Phi_{3\text{-}5}$	0.0366	0.0359	0.0322	0.0322	0.0202	0.0275	0.0178	0.0432	[0.0083, 0.0202]
$\Phi_{3\text{-}6}$	0.0318	0.0347	0.0265	0.0322	0.005	0.0055	0.0133	0.0647	[0.0023, 0.0303]
$\Phi_{4\text{-}1}$	0.0352	0.0388	0.044	0.0416	0.0187	0.0431	0.007	0.0234	[0.0033, 0.0206]
$\Phi_{4\text{-}2}$	0.0347	0.0368	0.0382	0.0394	0.0187	0.0108	0.0098	0.0033	[0.0015, 0.0185]
$\Phi_{4\text{-}3}$	0.0383	0.0384	0.0499	0.035	0.0187	0.0072	0.0488	0.0039	[0.0018, 0.0234]
$\Phi_{4\text{-}4}$	0.044	0.0388	0.044	0.035	0.0562	0.0072	0.007	0.0039	[0.0018, 0.0263]
$\Phi_{4\text{-}5}$	0.0409	0.0409	0.0294	0.035	0.0141	0.0086	0.0122	0.0039	[0.0018, 0.0192]
$\Phi_{5\text{-}1}$	0.0353	0.0367	0.0488	0.0364	0.0402	0.0183	0.0246	0.0013	[0.0006, 0.0229]
$\Phi_{5\text{-}2}$	0.041	0.0367	0.0385	0.0455	0.0402	0.0183	0.0246	0.0119	[0.0056, 0.0213]
$\Phi_{5\text{-}3}$	0.041	0.0395	0.0488	0.0455	0.0402	0.0183	0.0246	0.0119	[0.0056, 0.0229]
$\Phi_{5\text{-}4}$	0.0378	0.0395	0.0334	0.0455	0.008	0.0183	0.0049	0.0119	[0.0023, 0.0213]
$\Phi_{5\text{-}5}$	0.0357	0.0395	0.036	0.0364	0.0402	0.0037	0.0061	0.0013	[0.0006, 0.0188]

Source: Reprinted from [103], used with permission.

FUCOM Weights

A questionnaire template for this research was designed and pilot-tested for the customized questionnaire for each DM. The used steps are those presented in Section 6.2.

Step 1. Rank the criteria. The points assigned to various criteria, as shown in Table 6.3, represent the preferences of the DMs. These preferences are converted into rankings, with 0% indicating the most important criterion and 100% indicating the least important criterion. For DM_1, the rankings for the first-level criteria based on the points given are as follows:

$$C_{1(1)} > C_{1(3)} > C_{1(4)} > C_{1(5)} > C_{1(2)}. \tag{6.18}$$

Similar rankings for other DMs and criteria are not shown here for brevity.

Step 2. Obtain the comparative priority. Comparative priorities are determined based on the responses from the DMs. The scale used for comparisons follows the conventional scale commonly employed in the Analytic Hierarchy Process (AHP) given in Table 5.1. To calculate the comparative priorities accurately, the DMs provided their judgments and preferences. These judgments are utilized to establish the comparative priorities, which are essential for the subsequent weighting process [116]. The comparative priorities are shown in Table 6.6.

Table 6.6. Raw comparison data.

Indicator	Ranking	DM_1	DM_2	DM_3	DM_4
First-Level Indicator	1st	-	-	-	-
	2nd	4	-	5	-
	3rd	7	5	5 *	-
	4th	4	5	7	8
	5th	3	5	7 *	8
HR Management Support System (Φ_1)	1st	-	-	-	-
	2nd	1	- *	4	5
	3rd	8	5	3	5
	4th	1	5	7	7
	5th	7	5	4	3
	6th	1	5 *	4	2
Technology (Φ_2)	1st	-	-	-	-
	2nd	1	6	6	8
	3rd	3	6	4	2
	4th	6	5	3	2 *
	5th	1	5	3	2 *
Software Quality (Φ_3)	1st	-	-	-	-
	2nd	- *	- *	- *	- *
	3rd	3	5	7	1
	4th	4	5	4	6
	5th	2	5	7	6 *
	6th	2	5	4	6 *
Cost (Φ_4)	1st	-	-	-	-
	2nd	4	6	5	7
	3rd	3	6 *	5 *	6
	4th	3	4	7	6 *
	5th	3	5	4	6 *
Vendor Support (Φ_5)	1st	-	-	-	-
	2nd	- *	- *	- *	- *
	3rd	5	- *	7	- *
	4th	1	4	6	9
	5th	1	5	4	9 *

* = Criteria with equal ranking to the preceding criterion. Source: Reprinted from [103], used with permission.

Step 3. Compute the weights. The weights are calculated by solving an Optimization Function aimed at minimizing the deviation from full consistency (χ). The Optimization Function, as outlined in Section 6.2 and described by the Objective Function (6.6), involves a series of constraints and computations.

For instance, consider the computation of the weights for the first criteria of DM_1:

$$
\begin{aligned}
&\min \chi \\
&s.t. \\
&\left|\frac{w_1}{w_3} - \frac{1}{4}\right| \leq \chi, \forall j; \ldots; \left|\frac{w_5}{w_2} - \frac{4}{3}\right| \leq \chi, \forall j; \\
&\left|\frac{w_1}{w_4} - \frac{1}{4} \times \frac{4}{7}\right| \leq \chi, \forall j; \cdots; \left|\frac{w_4}{w_2} - \frac{7}{4} \times \frac{4}{3}\right| \leq \chi, \forall j. \\
&\sum_{j=1}^{5} w_j = 1, \forall j \\
&w_1, w_2, w_3, w_4, w_5 \geq 0, \forall j
\end{aligned}
\tag{6.19}
$$

While the objective functions for the remaining criteria by the other Decision-Makers are not presented here, the results of these computations are summarized in Table 6.5. This table provides an overview of how each criterion's weight was determined based on the full-consistency method for all Decision-Makers involved in the process. Before moving onto computing the effective weights, it is beneficial to visualize the distribution of local weights obtained from both the PA and FUCOM methods. In Figure 6.4, the scatter plot showcases this distribution. The dots and rings represent the local weights calculated using the PA and FUCOM methods, respectively. This visualization provides insights into the variations and commonalities in how different DMs assigned weights to the criteria. It can be observed that the PA swings between two extremes, from the highest to the lowest weights in some criteria. For example, in *HRM functions* (Φ_1), the weight assigned by DM_3 is 0.5932, which is the highest local weight. Conversely, the weight for *Technology* (Φ_2) assigned by DM_4 using the FUCOM method is the lowest, at 0.0385. This emphasizes the level of uncertainty in the weights since it is not guaranteed that FUCOM will always result in the highest or the lowest weights. These weighting extremes are influenced by the FUCOM method.

Step 4. Computing effective weights. Effective FUCOM weights were computed using Equation (5.6) for each DM on the basis of the FUCOM method. Results are given in Table 6.5.

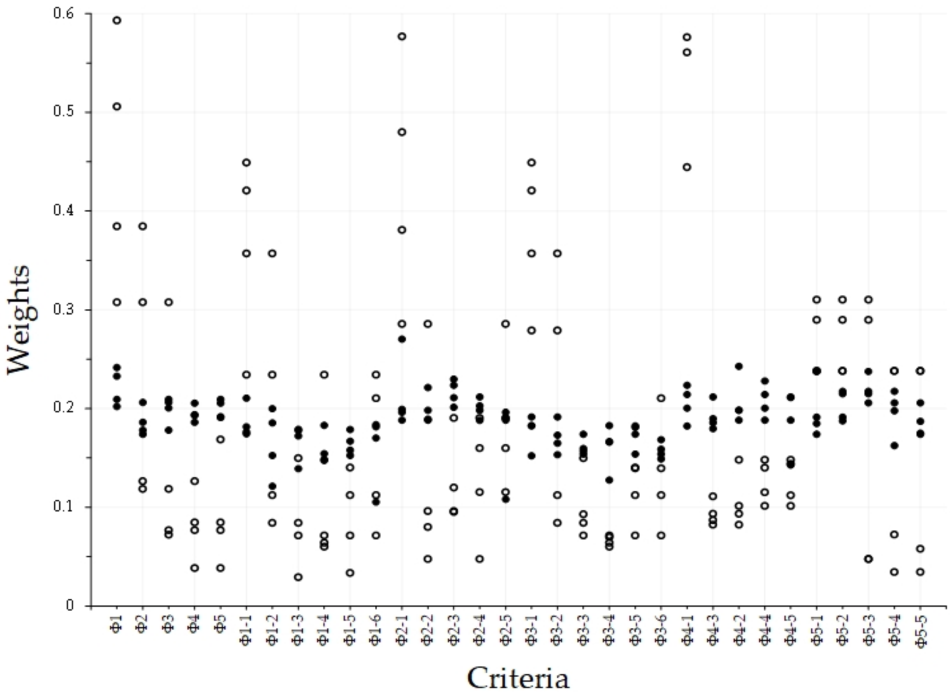


Figure 6.4. Scatter graph of the distribution of DM weights. Source: Reprinted from [103], used with permission.

Grey-PA-FUCOM Method

The Grey-PA-FUCOM method represents the weights obtained using both the PA and FUCOM methods as grey numbers. The procedure for calculating the grey weights for HRIS evaluation is outlined below.

Step 1. Obtain PA and FUCOM weights. The first step involves obtaining the PA and FUCOM weights, which were computed in Section 6.5.2.

Step 2. Represent weights as grey numbers. Next, the PA and FUCOM weights are represented as grey numbers. This is achieved by scaling the minimum and maximum weights obtained from both methods. For the first-level criteria, Equations (6.8) and (6.9) are utilized. Here, $w_p^{'}$ and $w_p^{''}$ represent the PA and FUCOM weights for the decision-makers (DMs). Similarly, for the second-level criteria, Equations (6.10) and (6.11) are applied, where $w_{p\text{-}q}^{'}$ and $w_{p\text{-}q}^{''}$ represent the PA and FUCOM weights for the DMs. The resulting local weights are expressed as grey intervals. The local weights are as follows:

$$w_1^* = \begin{pmatrix} [0.2021, 0.5932] \\ [0.0385, 0.2062] \\ [0.0847, 0.3846] \\ [0.0385, 0.2055] \\ [0.0769, 0.3077] \end{pmatrix}, \ldots, w_{5\text{-}q}^* = \begin{pmatrix} [0.0013, 0.0488] \\ [0.0119, 0.0455] \\ [0.0119, 0.0488] \\ [0.0049, 0.0455] \\ [0.0013, 0.0402] \end{pmatrix}. \quad (6.20)$$

Step 3. Compute effective weight. To compute the effective weights, the scaled weights are multiplied, yielding the effective grey weights according to Equation (6.12). These weights are presented in Equation (6.21), and the summation of upper bounds equals one. The complete column matrix of weights can be found in the last column labeled "*W*" in Table 6.5. Additionally, Figure 6.5 provides a bar graph visualization of the grey weights. The lowest and highest points in each bar of the criteria represent the lower and upper bounds of the intevera weights, respectively, as indicated by interval grey numbers. The middle point represents the value of the white weights using a whiten coefficient of 0.5. The criterion *Staff Information Management* ($\Phi_{1\text{-}1}$) has the highest degree of uncertainty, i.e., a weight of [0.0069, 0.1248] and a white value of 0.0553. On the other hand, *maintainability* ($\Phi_{3\text{-}5}$) has the lowest degree of uncertainty, i.e., a weight of [0.0083, 0.0202] and a white value of 0.0142:

$$W = \begin{pmatrix} [0.0069, 0.1248] \\ [0.0129, 0.0607] \\ [0.0129, 0.0556] \\ \vdots \\ [0.0006, 0.0188] \end{pmatrix}. \tag{6.21}$$

This completes the procedure for calculating the grey-PA-FUCOM weights for HRIS evaluation. The direct percentage scores of the decision-makers (DMs) are clustered around the 3rd and 4th percentiles, as shown in Table 6.3. This skewness may partly be attributed to the fact that the DMs were Chinese and had experienced the Chinese education grading system, where the cut-off mark between failing and passing an examination is 60%. In contrast, some cultures use a 50% cut-off mark, while others may use 40%. To address this subjective perspective, the scores were scaled and then normalized.

Figure 6.4 reveals that the PA and FUCOM methods resulted in different weight distributions. The PA weights were less dispersed, with a standard deviation of 0.0266, compared to the FUCOM weights, which had a standard deviation of 0.1392. Figure 6.5 shows that the weight for *Quality* (Φ_3) based on *Maintainability* (Φ_{3-5}) had the shortest bar, indicating a relative agreement among the DMs, though it is not perfect. On the other hand, *Technology* (Φ_2) based on *IoT* (Φ_{2-3}) had the longest bar, reflecting significant divergence in the DMs' opinions on the advantages of IoT for HRM.

The computational complexity of the proposed methods is further explored here. The PA method for group decision-making, which uses the average point as a weight, has a linear time complexity, $O(mkn)$, where k represents the hierarchy level for m decision-makers (DMs) and n criteria at each level. Similarly, the complexity of the FUCOM method is also $O(mkn)$. Thus, the grey-PA-FUCOM method also has a complexity of $O(mkn)$. This is simpler compared to the AHP method in group decision-making, which has a complexity of $O(mkn^4)$ for m DMs and n decision matrices with k hierarchy levels [261]. However, it is important to note that [261] did not account for uncertainties, such as fuzzy and grey derivatives of the AHP. Therefore, the Grey REGIME method can be considered a viable alternative pairwise comparison MCDM method for group decision-making under uncertainty due to its lower time complexity.

In summary, the analysis highlights the variability in DMs' perspectives influenced by cultural differences in grading systems and underscores the importance of normalization in achieving a balanced evaluation. The significant differences in weight distribution between PA and FUCOM methods emphasize the need for careful consideration of methodological approaches in decision-making processes.

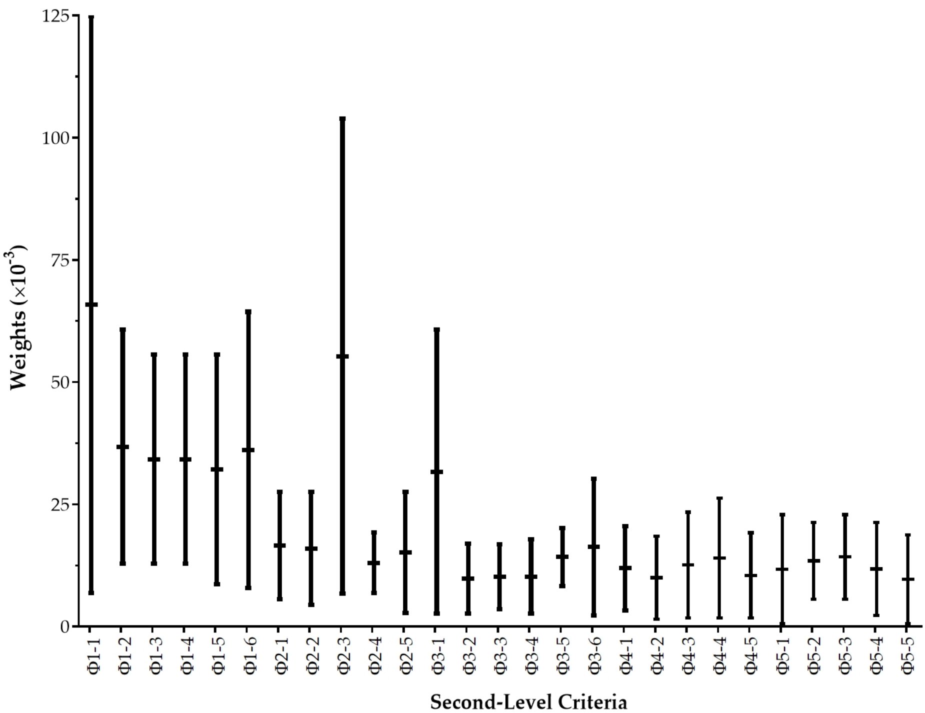


Figure 6.5. Grey-PA-FUCOM weights. Source: Reprinted from [103], used with permission.

7. Grey Weighted Sum Model

In this chapter, the Weighted Sum Model (WSM), also known as the simple additive weighting (SAW) method, is extended with the grey systems theory. This chapter begins with the presentation of the classic WSM, which involves the simple addition of weights. The simple additive weighting grey relations (SAW-G) method is then explored. The Grey Weighted Sum Model (GWSM), designed to assess business environments and Human Resource Information Systems (HRISs), is presented in this chapter. It is worth noting that while the evaluation index weights for HRISs are determined using the Rank Order Centroid (ROC) method in Chapters 3 and 6, the grey point allocation method is applied here to evaluate the weights of evaluation criteria. However, it is essential to acknowledge that the SAW-G method, while informative, is not in this book due to its recognized limitations. Figure 7.1 illustrates the structural framework of Chapter 7.

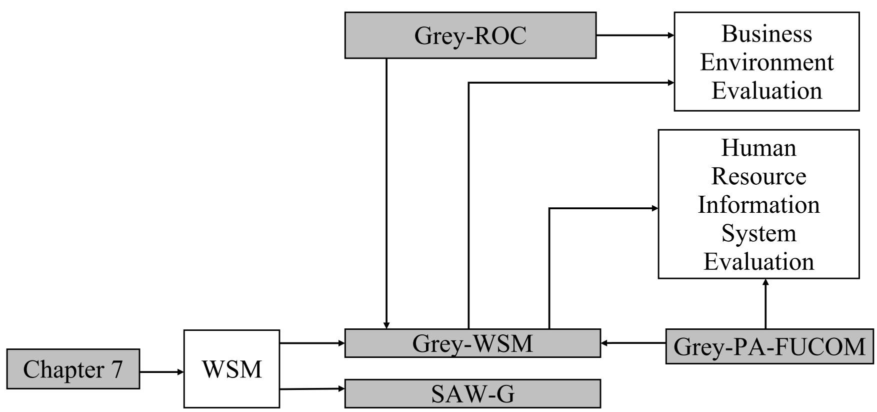


Figure 7.1. Flowchart of Chapter 7. Source: Figure by authors.

7.1. Weighted Sum Model

The weighted sum model (WSM) is a decision-making method that combines criteria scores for alternatives using weighted linear combinations. Essentially, WSM is equivalent to the simple additive weighting (SAW) method and falls into the category of scoring methods [262,263]. It relies on a weighted average approach, representing a Decision-Maker's (DM) preferences through a linear additive function [264]. Research by Triantaphyllou and Mann highlighted [265] WSM as one of the simplest methods for solving Multiple Criteria Decision-Making (MCDM) problems. In WSM, the scores assigned to alternatives are determined by multiplying the weights of the criteria by the normalized values of each alternative's performance across all criteria. For a unidirectional scale used in measuring the criteria of the alternatives, the best alternative can be defined as an alternative with the highest score using Equation (7.1). Mathematically, the score for an alternative (a_i) is calculated as follows:

$$a_i = \sum_{i=1}^{n} x_{ij} w_j. \tag{7.1}$$

Here, x_{ij} represents the normalized score for the ith alternative with respect to the jth criterion, and w_j is the weight assigned to the jth criterion. WSM's simplicity and intuitive nature make it a valuable method for tackling MCDM problems [17]. It is often used for exploring potential solutions and providing comparative assessments. Many MCDM methods, including AHP and PROMETHEE, incorporate a weighted sum aggregation technique to identify the best alternative [263]. Over the years, WSM has been enhanced and applied across various domains. These enhancements include approaches that consider fuzzy numbers, preference ratios, and other modifications to improve its accuracy and utility in real-world decision-making scenarios. WSM's transparency, ease of implementation, and capability to handle well-structured problems with clear objectives, criteria, and alternatives have contributed to its popularity in decision-making processes [266].

The weighted sum model (WSM) has undergone significant enhancements and widespread applications across diverse domains. These advancements include the introduction of a revised weighted method to address dissimilar decision preferences by Goh et al. [267], the integration of fuzzy numbers with WSM by Triantaphyllou and Lin [268], the use of preference ratios for comparing fuzzy numbers by Modarres and Sadi-Nezhad [269], the proposal of a fuzzy simple additive weighting system for facility location decisions by Chou et al. [270], and its application in a Decision Support System for smartphone purchases by Atmojo et al. [271]. Furthermore, the combination of WSM with Data Envelopment Analysis (DEA) to assess power supply technology was introduced by Shakouri et al. [272], and Xu et al. [273] presented a conflict elimination model for determining group expert weights in decision-making, which was applied to WSM for identifying optimal alternatives in group decision contexts. These developments underscore the versatility and adaptability of WSM in addressing a wide range of decision-making challenges and accommodating various data types, including fuzzy and group decision data.

Researchers have frequently integrated fuzzy sets with the weighted sum model (WSM) to tackle Multiple Criteria Decision-Making (MCDM) problems characterized by uncertainty. For instance, Zavadskas et al. [274] combined the weighted product model with the WSM, creating an evaluation approach known as Weighted Aggregated Sum Product Assessment (WASPAS). They subsequently extended WASPAS to accommodate interval-valued intuitionistic fuzzy numbers and grey numbers, acknowledging the inherent vagueness in human judgment and preferences [18,275]. Stanujkic and Zavadskas [276] introduced compensation coefficient values into the WSM, aiding decision-makers in selecting the best-ranking alternatives that align with their preferences. Chen [277] applied the WSM in group decision-making scenarios, incorporating interval type 2 fuzzy sets to handle linguistic and incomplete preference measurements. Wang [278] addressed a limitation of fuzzy multiplication in WSM by utilizing relative preferences to represent criteria weights. Additionally, Zamri and Abdullah [279] combined linear programming with the WSM within the framework of interval fuzzy sets to rank alternatives effectively. These innovative approaches demonstrate the versatility of WSM in handling complex decision problems involving uncertain or imprecise information.

There have been relatively few attempts to integrate grey numbers with the weighted sum model (WSM). Zavadskas et al. [280] introduced the simple additive weighting with grey relations (SAW-G) and TOPSIS grey techniques, which found applications in contractor selection for construction projects. SAW-G was also utilized in the evaluation of rural ICT centers in Iran [281]. However, the SAW-G method falls short in fully capturing the degree of uncertainty in rankings because it does not consider the boundary distance between the lower and upper bounds of grey numbers. For instance, when assigning equal weights to the normalized investment outcomes of two alternatives, the SAW-G technique may yield the same white value for both investments. Nevertheless, this approach fails to account for the differing degrees of uncertainty between the two investments. In reality, one investment may have a higher level of uncertainty than the other, signifying that the outcome of the second investment is more assured. The GWSM seeks to enhance existing grey number and WSM combination methods, enabling them to distinguish the degree of uncertainty by incorporating the boundary information into their assessments.

7.2. Simple Additive Weighting with Grey Relations

The classical simple additive weighting (SAW) method has been extended to Grey Systems Theory (GST) by Zavadskas [280], resulting in the method known as simple additive weighting with grey relations (SAW-G). The primary concept behind SAW-G is to compute the weighted grey decision matrix and aggregate the criteria for the alternatives, after which the weighted values of the alternatives are ranked. The following steps are involved in the SAW-G method:

1. Determine the evaluation criteria, as previously detailed in Chapter 2;
2. Formulate the grey decision matrix, denoted as $\otimes X$;
3. Normalize the grey decision matrix, which involves different processes based on the nature of the criteria.

   - For criteria with benefit preferences (where higher values indicate better performance), normalization is performed using Equation (7.2):

$$\otimes X' = \left[\underline{x}_{ij}, \overline{x}_{ij}\right] = \left[\frac{\underline{x}_{ij}}{\max\limits_{j} \underline{x}_{ij}}, \frac{\overline{x}_{ij}}{\max\limits_{j} \underline{x}_{ij}}\right]. \tag{7.2}$$

   - For criteria with cost preferences (where lower values indicate better performance), normalization is carried out as described in Equation (7.3):

$$\otimes X' = \left[\underline{x}_{ij}, \overline{x}_{ij}\right] = \left[\frac{\min\limits_{j} \overline{x}_{ij}}{\underline{x}_{ij}}, \frac{\min\limits_{j} \overline{x}_{ij}}{\overline{x}_{ij}}\right]. \tag{7.3}$$

4. Obtain the criteria weights, denoted as q_j.
5. Aggregate the weighted and normalized decision matrix using Equation (7.4):

$$\otimes \widehat{X} = \otimes X' \times q_j. \tag{7.4}$$

6. Calculate the optimality criteria, denoted as L_i, using Equation (7.5).

$$L_i = \frac{1}{n} \sum_{j=1}^{m} \frac{\underline{d_{ij}^*} + \overline{d_{ij}^*}}{2}. \tag{7.5}$$

7. Rank the alternatives, with the alternative having the highest value of $\max_i (L_i)$ considered the best.

These steps collectively constitute the SAW-G method, which is utilized to assess and rank alternatives in decision-making scenarios with grey data.

7.3. Grey Weight Sum Model

The conventional weighted sum model (WSM) typically employs real numbers to evaluate alternatives. However, when dealing with decision-making under uncertainty, it becomes essential to provide decision-makers with a comprehensive perspective of the problem, allowing them to make informed and confident choices [282]. To address uncertainty in decision-making, the integration of Grey Systems Theory (GST) with the WSM introduces reasonable slacks to alternative evaluations, facilitating decision-making in dynamic business environments characterized by changing factors such as government policies, natural conditions, tax rates, and exchange rates. GST is specifically designed to analyze systems with incomplete information and systems where the performance ranges are known but uncertain. These uncertain ranges are represented using grey numbers, which are essentially interval numbers encompassing a range of values [198,283].

In the traditional weighted sum model (WSM), both the criteria values and their corresponding weights are considered fixed values. However, in certain Multiple Criteria Decision-Making (MCDM) scenarios, the performance of criteria may fluctuate within a range of values over time. This inherent variability in criteria performance poses a challenge for the WSM, as it is unable to effectively evaluate alternatives with fluctuating or uncertain criteria measurements, which can be represented as interval numbers. This limitation of the traditional WSM is precisely what the developed GWSM aims to address and overcome.

The GWSM represents an enhancement of the traditional WSM by addressing the evaluation of interval values and incorporating uncertainty as an evaluation criterion. The GWSM achieves this by utilizing weighted grey numbers for evaluating criteria and accounting for the degree of uncertainty using the boundary distances of the indicators. The key steps of the GWSM are as follows:

Step 1. Construct the grey decision matrix. The grey decision matrix, X, is where each element, $\otimes x_{ij}$, represents a grey number for the jth criterion of the ith alternative.

$$X = \begin{pmatrix} \otimes x_{11} & \otimes x_{12} & \cdots & \otimes x_{1n} \\ \otimes x_{21} & \otimes x_{22} & \cdots & \otimes x_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \otimes x_{m1} & \otimes x_{m2} & \cdots & \otimes x_{mn} \end{pmatrix}, \tag{7.6}$$

where $\otimes x_{ij}$ represents the grey interval number of the jth criterion of ith alternative. Also, every alternative can be written in vector form:

$$X_i = (\otimes x_{i,1}, \otimes x_{i,2}, \ldots, \otimes x_{i,n}) .$$

Step 2. Normalize the grey decision matrix. The purpose of the normalization step is to ensure the uniform directionality of criteria measurements.

- For benefit preferences (with higher values indicating better performance), normalization is carried out using Equation (7.7):

$$\otimes x^*_{ij} = \left[\frac{\underline{x}_{ij} - \min\limits_{1 \le i \le m} \underline{x}_{ij}}{\max\limits_{1 \le i \le m} \overline{x}_{ij} - \min\limits_{1 \le i \le m} \underline{x}_{ij}}, \frac{\overline{x}_{ij} - \min\limits_{1 \le i \le m} \underline{x}_{ij}}{\max\limits_{1 \le i \le m} \overline{x}_{ij} - \min\limits_{1 \le i \le m} \underline{x}_{ij}} \right]. \tag{7.7}$$

- For cost preferences (with lower values indicating better performance), normalization is carried out using Equation (7.8):

$$\otimes x^*_{ij} = \left[\frac{\max\limits_{1 \le i \le m} \overline{x}_{ij} - \overline{x}_{ij}}{\max\limits_{1 \le i \le m} \overline{x}_{ij} - \min\limits_{1 \le i \le m} \underline{x}_{ij}}, \frac{\max\limits_{1 \le i \le m} \overline{x}_{ij} - \underline{x}_{ij}}{\max\limits_{1 \le i \le m} \overline{x}_{ij} - \min\limits_{1 \le i \le m} \underline{x}_{ij}} \right]. \tag{7.8}$$

This results in a normalized decision matrix X^*:

$$X^* = \begin{pmatrix} \otimes x^*{}_{11} & \otimes x^*{}_{12} & \cdots & \otimes x^*{}_{1n} \\ \otimes x^*{}_{21} & \otimes x^*{}_{22} & \cdots & \otimes x^*{}_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \otimes x^*{}_{m1} & \otimes x^*{}_{m2} & \cdots & \otimes x^*{}_{mn} \end{pmatrix}. \tag{7.9}$$

We define the following:

$$X^*_i = \left(\otimes x^*_{i,1} \otimes x^*_{i,2}, \ldots, \otimes x^*_{i,n}\right). \tag{7.10}$$

Step 3. Determine the weights of the criteria. The criteria weights, denoted as W', can be obtained from experts or through various techniques discussed in earlier Chapters 3 to 6. Also, the proposed GRA-ROC weights or GRA-ILP-ROC weights can be used, as discussed in Chapter 10. These weights are represented as Equation (3.12):

$$W' = \left(w'_1, w'_2, \ldots, w'_n\right)^{\mathrm{T}}. \tag{7.11}$$

1. Aggregate the weighted decision matrix. This involves calculating the sum of the weighted normalized criteria for all alternatives using the basic operation of grey numbers, as shown in Equation (7.18). The basic operation of grey numbers is used; see Equation (1.1) to Equation (1.5).

$$Y = X^{*} \times W', \tag{7.12}$$

$$Y = \begin{pmatrix} \otimes x_{11}^{*} & \otimes x_{12}^{*} & \cdots & \otimes x_{1n}^{*} \\ \otimes x_{21}^{*} & \otimes x_{22}^{*} & \cdots & \otimes x_{21}^{*} \\ \vdots & \vdots & \ddots & \vdots \\ \otimes x_{m1}^{*} & \otimes_{m2}^{*} & \cdots & \otimes x_{mn}^{*} \end{pmatrix} \times \begin{pmatrix} w'_1 \\ w'_2 \\ \vdots \\ w'_n \end{pmatrix}. \tag{7.13}$$

Therefore,

$$Y = \begin{pmatrix} \otimes y_1 \\ \otimes y_2 \\ \vdots \\ \otimes y_m \end{pmatrix}, \tag{7.14}$$

where $\otimes y_i = \left[\underline{y}_i, \overline{y}_i\right]$ is the grey number, which represents the weighted sum of all the criteria for the ith alternative.

2. Obtain the white values of the alternatives. The whitenization process is crucial for transforming grey numbers into white numbers. This is achieved using Equation (7.15), where the whitenization coefficient, λ, lies in the range $[0, 1]$.

$$V_i = \underline{y}_i(1 - \lambda) + \overline{y}_i\lambda. \tag{7.15}$$

3. Determine the boundary distances of the alternatives. The degree of uncertainty, denoted as S_i, for each alternative is computed to quantify uncertainty. Equation (7.16) defines S_i for different values of p, which determines the type of distance metric used.

$$S_i = \sqrt[p]{(\overline{y}_i)^p - \left(\underline{y}_i\right)^p}, \tag{7.16}$$

when $p = 1$, the boundary is the Manhattan distance; when $p = 2$, the boundary is the Euclidean distance; when $p = 3$, the boundary is the Minkowski distance with a 3rd degree. As S_i tends toward zero, theoretically, the stability of the outcome for the future investment tends toward 100%. In other words, there is no uncertainty.

4. Rank the alternatives. In the ranking process, the uncertainty of the white values obtained in Step 5 is considered when calculating rank scores. The rank score (*zi*) is determined as the product of the white value (*Vi*) and the degree of certainty $(1 - S_i)$, as per Equation (7.17):

$$z_i = V_i\,(1 - S_i)\,. \tag{7.17}$$

Additionally, percentage ranking scores (Z_i) are computed to represent the relative rankings of all alternatives in comparison to the best alternative, where the best alternative is assigned a score of 100%. Equation (7.18) outlines the calculation of percentage ranking scores.

$$Z_i = \ [z_i / \ max\,(z_i)] \ \times \ 100. \tag{7.18}$$

It is worth noting that the GWSM can be adapted and combined with various weighting approaches, such as fuzzy weights or pairwise comparison weighting methods, to create hybrid methods tailored to specific decision-making scenarios [284–287]. These hybrid approaches can provide more flexibility and robustness in solving Multiple Criteria Decision-Making (MCDM) problems.

7.4. Application of GWSM

The African region serves as our sample; the data for these countries are obtained from the DBP database, and are then transformed into grey numbers. This transformation is achieved by using the lowest and highest measurement values of these countries for each criterion between the years 2008 and 2015 as the lower and upper bounds of the grey numbers. Additionally, the decision criteria are depicted in Figure 3.6. In total, there are 19 second-level criteria indexed from 1 to 19. The GWSM method is employed to evaluate the business environment of African countries. The criteria weights help in recognizing the relative importance of these criteria, and they directly influence the rankings of alternatives. The grey numbers associated with these criteria and the corresponding weights are aggregated to obtain the weighted grey number for each country. The steps are as follows.

Step 1. Construct the grey decision matrix. Transform the values of the second-level criteria obtained from the DPB/World Bank database from 2008 to 2015 into grey numbers:

$$\otimes x_{ij} = \left[\underline{x}_{ij}, \overline{x}_{ij}\right], \tag{7.19}$$

where $\underline{x}_{ij} = \min\left(v_{ij}^{2008}, v_{ij}^{2009}, \ldots, v_{ij}^{2015}\right)$ and $\overline{x}_{ij} = \max\left(v_{ij}^{2008}, v_{ij}^{2009}, \ldots, v_{ij}^{2015}\right)$.

Here, v_{ij} represents the value for the second-level indicators, j, for country i provided by the DBP for the years 2008 to 2015, where $1 \leq i \leq 53$ and $1 \leq j \leq 19$.

$$X = \begin{pmatrix} [22.4, 24] & [10.813, 13.2] & \cdots & [41.741, 41.7] \\ [66.83, 123.5] & [123.5, 343.7] & \cdots & [0, 13.8] \\ \vdots & \vdots & \ddots & \vdots \\ [86, 93] & [114.6, 676.1] & \cdots & [0, 13.8] \end{pmatrix}. \tag{7.20}$$

The first element of vector X_1 is $\otimes x_{1,1\text{-}2}$, corresponding to the time ($A_{1\text{-}2}$) it takes to start a business (A_1) in Algeria, with a lower bound of 22 days and an upper bound of 24 days. All elements of matrix X have similar corresponding lower and upper bounds for the second-level indicator of every country. The complete grey data are provided in Table A3 in Appendix A.1.2.

Step 2. Normalize the grey decision matrix. A normalized grey decision matrix is constructed using Equation (7.9).

$$X^* = \begin{pmatrix} [0.9216, 0.9294] & [0.9891, 0.9911] & \cdots & [0.6187, 0.6187] \\ [0.6902, 0.7568] & [0.7091, 0.8956] & \cdots & [0, 0] \\ \vdots & \vdots & \ddots & \vdots \\ [0.6510, 0.6784] & [0.4275, 0.9031] & \cdots & [0, 0.2048] \end{pmatrix}. \tag{7.21}$$

Based on Equation (7.10), a vector form of matrix X^* is obtained. For example, X_1^* = ([0.9234, 0.9312], [0.9891, 0.9911], [0.9555, 0.9763], [0.8, 0.8514], [0.7, 0.716], [0.1818, 0.2727], [0, 0.5], [0.0278, 0.0278], [0, 0], [0.5, 0.5], [0.1111, 0.1111], [0.4, 0.4], [0.6557, 0.6557], [0.5674, 0.6416], [0.8879, 0.8879], [0.7297, 0.7297], [1, 1], [0.2463, 0.2463], [0.2097, 0.2097]). Other data are omitted here as well.

Step 3. Determine the weights of the criteria. The weights based on the GRA-ILP-ROC weights presented in Section 10.4.1 for all the second-level indicator weights are used.
W' = (0.0445, 0.0202, 0.0081, 0.0390, 0.1171, 0.0028, 0.0064, 0.0230, 0.0120, 0.2264, 0.1029, 0.0412, 0.0632, 0.0253, 0.1391, 0.0271, 0.0814, 0.0051, 0.015)$^{\mathrm{T}}$.

Step 4. Aggregate the weighted grey decision matrix. The aggregated weights are calculated using Equation (7.16): $Y = (\otimes y_1, \otimes y_2, \otimes y_3, \otimes y_4, \otimes y_5, \otimes y_6, \otimes y_7, \otimes y_8, \otimes y_9, \otimes y_{10}, \otimes y_{11}, \otimes y_{12}, \otimes y_{13}, \otimes y_{14}, \otimes y_{15}, \otimes y_{16}, \otimes y_{17}, \otimes y_{18}, \otimes y_{19}, \otimes y_{20}, \otimes y_{21}, \otimes y_{22}, \otimes y_{23}, \otimes y_{24}, \otimes y_{25}, \otimes y_{26}, \otimes y_{27}, \otimes y_{28}, \otimes y_{29}, \otimes y_{30}, \otimes y_{31}, \otimes y_{32}, \otimes y_{33}, \otimes y_{34}, \otimes y_{35}, \otimes y_{36}, \otimes y_{37}, \otimes y_{38}, \otimes y_{39}, \otimes y_{40}, \otimes y_{41}, \otimes y_{42}, \otimes y_{43}, \otimes y_{44}, \otimes y_{45}, \otimes y_{46}, \otimes y_{47}, \otimes y_{48}, \otimes y_{49}, \otimes y_{50}, \otimes y_{51}, \otimes y_{52}, \otimes y_{53})^{\mathrm{T}}$

$Y =$ ([0.60 4, 0.6575], [0.497, 0.6201], [0.5132, 0.6131], [0.7201, 0.8668], [0.557, 0.6474], [0.4283, 0.7829], [0.5208, 0.618], [0.5008, 0.5907], [0.3947, 0.5983], [0.4495, 0.5602], [0.4248, 0.5749], [0.2738, 0.6462], [0.4342, 0.6039], [0.5369, 0.6541], [0.5063, 0.5728], [0.5721, 0.7534], [0.5122, 0.6159], [0.5348, 0.6014], [0.5427, 0.5915], [0.5456, 0.6689], [0.3696, 0.5524], [0.748, 0.8013], [0.4756, 0.5835], [0.4739, 0.6201], [0.5476, 0.6114], [0.5114, 0.6294], [0.5091, 0.5604], [0, 0.4585], [0.6652, 0.7156], [0.6274, 0.706], [0.5074, 0.6552], [0.5421, 0.6079], [0.752, 0.8351], [0.5943, 0.7196], [0.6369, 0.7036], [0.6474, 0.7161], [0.5625, 0.6489], [0.516, 0.6141], [0.5737, 0.8774], [0.4398, 0.5202], [0.4522, 0.591], [0.6584, 0.7162], [0.4357, 0.7249], [0.8361, 0.9033], [0, 0.5044], [0.4776, 0.5207], [0.4585, 0.5983], [0.5732, 0.6134], [0.4947, 0.6202], [0.5238, 0.7524], [0.6156, 0.6635], [0.6309, 0.7053], [0.55, 0.6953])$^{\mathrm{T}}$.

Step 5. Obtain the white values of the alternatives. From Equation (7.15), with the center whitenization coefficient of $\lambda = 0.5$, we have the following: $V_1 = 0.6984$, $V_2 = 0.6183$, $V_3 = 0.6234$, $V_4 = 0.8783$, $V_5 = 0.6667$, $V_6 = 0.6704$, $V_7 = 0.6304$, $V_8 = 0.6042$, $V_9 = 0.5496$, $V_{10} = 0.5589$, $V_{11} = 0.5533$, $V_{12} = 0.5092$, $V_{13} = 0.5746$, $V_{14} = 0.6592$, $V_{15} = 0.5973$, $V_{16} = 0.7336$, $V_{17} = 0.6244$, $V_{18} = 0.6289$, $V_{19} = 0.6278$, $V_{20} = 0.6722$,

$V_{21} = 0.5103$, $V_{22} = 0.8575$, $V_{23} = 0.5862$, $V_{24} = 0.6055$, $V_{25} = 0.6415$, $V_{26} = 0.6314$, $V_{27} = 0.592$, $V_{28} = 0.2538$, $V_{29} = 0.7643$, $V_{30} = 0.738$, $V_{31} = 0.6435$, $V_{32} = 0.6365$, $V_{33} = 0.8785$, $V_{34} = 0.7272$, $V_{35} = 0.7419$, $V_{36} = 0.7547$, $V_{37} = 0.6705$, $V_{38} = 0.6255$, $V_{39} = 0.8032$, $V_{40} = 0.5314$, $V_{41} = 0.5774$, $V_{42} = 0.7609$, $V_{43} = 0.6423$, $V_{44} = 0.9628$, $V_{45} = 0.2792$, $V_{46} = 0.5525$, $V_{47} = 0.585$, $V_{48} = 0.6568$, $V_{49} = 0.6171$, $V_{50} = 0.7064$, $V_{51} = 0.708$, $V_{52} = 0.7396$, $V_{53} = 0.6893$.

Step 6. Determine the boundary distances of the alternatives. Using the Manhattan distance for measuring the distance defined in Equation (7.16), we have the following: $S_1 = 0.0588$, $S_2 = 0.1362$, $S_3 = 0.1105$, $S_4 = 0.1624$, $S_5 = 0.1001$, $S_6 = 0.3926$, $S_7 = 0.1076$, $S_8 = 0.0995$, $S_9 = 0.2254$, $S_{10} = 0.1225$, $S_{11} = 0.1661$, $S_{12} = 0.4123$, $S_{13} = 0.1879$, $S_{14} = 0.1297$, $S_{15} = 0.0736$, $S_{16} = 0.2007$, $S_{17} = 0.1148$, $S_{18} = 0.0737$, $S_{19} = 0.054$, $S_{20} = 0.1364$, $S_{21} = 0.2023$, $S_{22} = 0.0589$, $S_{23} = 0.1195$, $S_{24} = 0.1619$, $S_{25} = 0.0706$, $S_{26} = 0.1305$, $S_{27} = 0.0568$, $S_{28} = 0.5076$, $S_{29} = 0.0558$, $S_{30} = 0.087$, $S_{31} = 0.1637$, $S_{32} = 0.0728$, $S_{33} = 0.092$, $S_{34} = 0.1387$, $S_{35} = 0.0739$, $S_{36} = 0.076$, $S_{37} = 0.0957$, $S_{38} = 0.1086$, $S_{39} = 0.3362$, $S_{40} = 0.089$, $S_{41} = 0.1536$, $S_{42} = 0.064$, $S_{43} = 0.3201$, $S_{44} = 0.0745$, $S_{45} = 0.5583$, $S_{46} = 0.0477$, $S_{47} = 0.1548$, $S_{48} = 0.0444$, $S_{49} = 0.1389$, $S_{50} = 0.2531$, $S_{51} = 0.053$, $S_{52} = 0.0824$, $S_{53} = 0.1608$.

Step 7. Rank the alternatives. The rank scores, z_i, are calculated using Equation (7.17): $z_1 = 0.6574$, $z_2 = 0.5341$, $z_3 = 0.5545$, $z_4 = 0.7357$, $z_5 = 0.5999$, $z_6 = 0.4072$, $z_7 = 0.5625$, $z_8 = 0.5441$, $z_9 = 0.4257$,$z_{10} = 0.4904$, $z_{11} = 0.4614$, $z_{12} = 0.2993$, $z_{13} = 0.4666$, $z_{14} = 0.5737$, $z_{15} = 0.5534$, $z_{16} = 0.5864$, $z_{17} = 0.5528$, $z_{18} = 0.5826$, $z_{19} = 0.5939$, $z_{20} = 0.5805$, $z_{21} = 0.407$, $z_{22} = 0.807$, $z_{23} = 0.5161$, $z_{24} = 0.5075$, $z_{25} = 0.5962$, $z_{26} = 0.549$, $z_{27} = 0.5584$, $z_{28} = 0.125$, $z_{29} = 0.7217$, $z_{30} = 0.6738$, $z_{31} = 0.5382$, $z_{32} = 0.5902$, $z_{33} = 0.7976$, $z_{34} = 0.6263$, $z_{35} = 0.6871$, $z_{36} = 0.6974$, $z_{37} = 0.6063$, $z_{38} = 0.5575$, $z_{39} = 0.5331$, $z_{40} = 0.4841$, $z_{41} = 0.4887$, $z_{42} = 0.7122$, $z_{43} = 0.4367$, $z_{44} = 0.891$, $z_{45} = 0.1233$, $z_{46} = 0.5262$, $z_{47} = 0.4944$, $z_{48} = 0.6276$, $z_{49} = 0.5313$, $z_{50} = 0.5276$, $z_{51} = 0.6705$, $z_{52} = 0.6787$, $z_{53} = 0.5784$.

The percentage rank scores, Z_i, are calculated using Equation (7.18): $Z_1 = 73.78\%$, $Z_2 = 59.94\%$, $Z_3 = 62.23\%$, $Z_4 = 82.57\%$, $Z_5 = 67.33\%$, $Z_6 = 45.7\%$, $Z_7 = 63.13\%$, $Z_8 = 61.06\%$, $Z_9 = 47.78\%$, $Z_{10} = 55.04\%$, $Z_{11} = 51.78\%$, $Z_{12} = 33.59\%$, $Z_{13} = 52.37\%$, $Z_{14} = 64.38\%$, $Z_{15} = 62.1\%$, $Z_{16} = 65.81\%$, $Z_{17} = 62.03\%$, $Z_{18} = 65.38\%$, $Z_{19} = 66.65\%$, $Z_{20} = 65.15\%$, $Z_{21} = 45.68\%$, $Z_{22} = 90.57\%$, $Z_{23} = 57.93\%$, $Z_{24} = 56.95\%$, $Z_{25} = 66.91\%$, $Z_{26} = 61.61\%$, $Z_{27} = 62.66\%$, $Z_{28} = 14.03\%$, $Z_{29} = 80.99\%$, $Z_{30} = 75.62\%$, $Z_{31} = 60.4\%$, $Z_{32} = 66.23\%$, $Z_{33} = 89.52\%$, $Z_{34} = 70.29\%$, $Z_{35} = 77.12\%$, $Z_{36} = 78.26\%$, $Z_{37} = 68.05\%$, $Z_{38} = 62.57\%$, $Z_{39} = 59.83\%$, $Z_{40} = 54.33\%$, $Z_{41} = 54.84\%$, $Z_{42} = 79.93\%$, $Z_{43} = 49.01\%$, $Z_{44} = 100\%$, $Z_{45} = 13.84\%$, $Z_{46} = 59.06\%$, $Z_{47} = 55.49\%$, $Z_{48} = 70.44\%$, $Z_{49} = 59.63\%$, $Z_{50} = 59.21\%$, $Z_{51} = 75.25\%$, $Z_{52} = 76.17\%$, $Z_{53} = 64.92\%$.

The ranking of these countries from the first position is as follows: South Africa > Ghana > Mauritius > Botswana > Madagascar > Seychelles > Namibia > Mozambique > Zambia > Malawi > Uganda > Algeria > Tanzania > Morocco > Niger > Burkina Faso > Kenya > Ethiopia > Mauritania > Egypt > Eritrea > Gabon > Zimbabwe > Côte d'Ivoire > Cabo Verde > Liberia > Nigeria > Benin > Djibouti > Equatorial Guinea > Lesotho > Cameroon > Mali > Angola > Rwanda > Togo > Tunisia > Sudan > Guinea > Guinea-Bissau > Swaziland > Chad > Senegal > São Tomé and Príncipe > Congo, Rep. > Comoros > Sierra Leone > Central African Republic > Burundi > Gambia > Congo, Dem. Rep. > Libya > South Sudan.

Although Tanzania is ranked the 13th position, Tanzania has the highest degree of certainty with a value of 0.0444. Rwanda is ranked the 53rd country and has the highest degree of uncertainty with a value of 0.5583, as reflected by the boundary distance.

The WSM has many derivatives. While the WSM may be passively mentioned even if an MCDM process has a step that requires the aggregation of the weighted normalized performance values of the alternatives, the WSM, also known as the simple additive weighting (SAW) method, assumes that there are no uncertainties. In this chapter, the limitations of the WSM based on this assumption are addressed using grey systems theory. The weights obtained from Chapters 3 and 6 are combined with the Grey-WSM. The grey relational analysis (GRA) method under uncertainty is introduced in the next Chapter.

8. Grey Relational Analysis with Grey Numbers

In this chapter, traditional Grey Relational Analysis (GRA) is introduced, and an extension known as Grey Number Relational Analysis (GNRA) is presented. The GNRA is a method that utilizes grey numbers to enhance the process of GRA. The application of GNRA as a hybrid approach, combined with the Grey SWARA (Step-Wise Weight Assessment Ratio Analysis) and grey SWARA-FUCOM (full-consistency method) weighting methods, is explored. These techniques will be used to evaluate various scenarios, including the Human Resource Information System (HRIS), contractor selection for solar panel installation, the scaling of foreign premium allowances, and assessments of university reputation. The aim is to provide a comprehensive understanding of GNRA and its practical applications in decision-making processes, as illustrated in Figure 8.1.

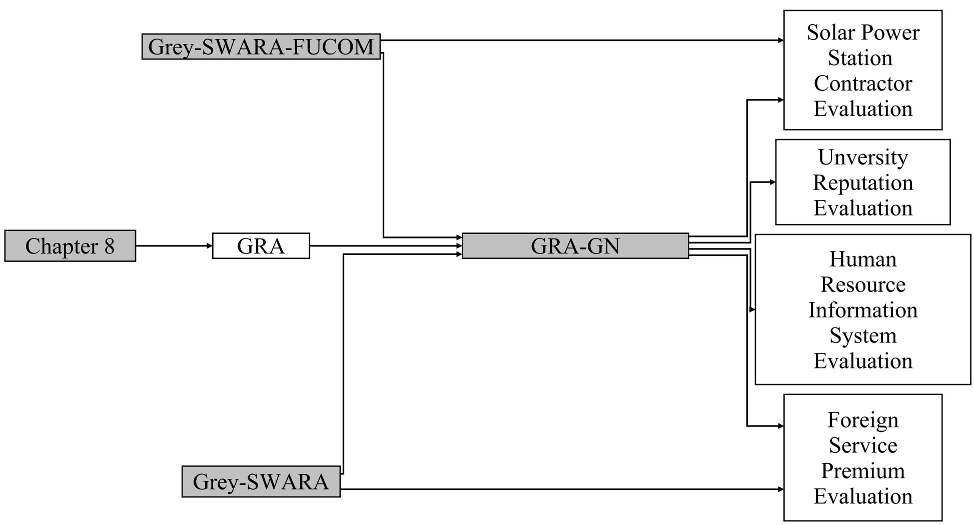


Figure 8.1. Flowchart of Chapter 8. Source: Figure by authors.

8.1. Grey Relational Analysis

Grey Systems Theory (GST), founded by Professor Julong Deng in 1982, is a field of study that deals with scenarios involving small sample sizes and limited information. This information can be described as partially known or incomplete. GST operates in the space between complete and unknown information, where incomplete information pertains to elements, parameters, structures, behaviors, or boundaries of a system. To handle systems with incomplete information, GST employs grey numbers. A grey number represents an unknown value within a known range.

The GST has different sections that include GRA, grey decision, grey programming, grey prediction, and grey control. The traditional GRA involves constructing a decision matrix, normalizing it, assigning criteria weights, comparing it to a reference alternative, computing grey relational grades, and ultimately selecting the best alternative. This method is invaluable for decision-making when dealing with incomplete information, offering a systematic approach to handle such complexities. The following are the steps involved in traditional GRA.

Step 1. Determine the evaluation sample and gather raw data to create a data matrix. This matrix represents the performance of all alternatives based on the evaluation criteria.

$$Y = \begin{pmatrix} y_1(1) & y_1(2) & \cdots & y_1(v) \\ y_2(1) & y_2(2) & \cdots & y_2(v) \\ \vdots & \vdots & \ddots & \vdots \\ y_u(1) & y_u(2) & \cdots & y_u(v) \end{pmatrix}, \tag{8.1}$$

where $y_i(k)$ is the precise data of the kth the criteria for the ith alternative, $1 \leq k \leq v$ and $1 \leq i \leq u$. Also, u and v are the number of alternatives and criteria, respectively.

Step 2. Normalization of the data. Normalization makes the data uniform, usually within a range of 0 to 1. Two different normalization formulas are used based if the criteria are beneficial or cost i.e. whether larger values, considered better or smaller values, are preferred.

- For beneficial criteria, i.e., when larger values are better values, we use Equation (8.2):

$$y_i^*(k) = \frac{x_i(k) - \min\limits_{1\leq k\leq v} x_i(k)}{\max\limits_{1\leq k\leq v} x_i(k) - \min\limits_{1\leq k\leq v} x_i(k)}. \tag{8.2}$$

- For cost criteria, i.e., when the smaller values are better values, we use Equation (8.3):

$$y_i^*(k) = \frac{\max\limits_{1\leq k\leq v} x_i(k) - x_i(k)}{\max\limits_{1\leq k\leq v} x_i(k) - \min\limits_{1\leq k\leq v} x_i(k)}. \tag{8.3}$$

Thus, the normalized data matrix brings the different criteria onto a common scale, ensuring fair comparisons:

$$Y^* = \begin{pmatrix} y_1^*(1) & y_1^*(2) & \cdots & y_1^*(v) \\ y_2^*(1) & y_2^*(2) & \cdots & y_2^*(v) \\ \vdots & \vdots & \ddots & \vdots \\ y_u^*(1) & y_u^*(2) & \cdots & y_u^*(v) \end{pmatrix}. \tag{8.4}$$

Then, the comparative series for matrix Y^* can be written as follows:

$$\begin{aligned} Y_1^* &= \{y_1^*(1), y_1^*(2), ..., y_1^*(v)\} \\ Y_2^* &= \{y_2^*(1), y_2^*(2), ..., y_2^*(v)\} \\ &\vdots \\ Y_u^* &= \{y_u^*(1), y_u^*(2), ..., y_u^*(v)\} \end{aligned} . \tag{8.5}$$

Step 3. Obtain the weighted normalized data. The weights (W) are assigned to criteria, reflecting their relative importance in the decision-making process, and are used to multiply the normalized data series (Y_u^*). That is,

$$W_h(k) = W(k) \times y_h^*(k). \tag{8.6}$$

The weight series can be written as follows:

$$\begin{aligned} W_1 &= \{w_1(1), w_1(2), ..., w_1(v)\} \\ W_2 &= \{w_2(1), w_2(2), ..., w_2(v)\} \\ &\vdots \\ W_u &= \{w_u(1), w_u(2), ..., w_u(v)\} \end{aligned} . \tag{8.7}$$

Step 4. Determine the reference data. For the weighted data, $W_1, W_2, \ldots, W_u$, the reference normalized weight data are as follows:

$$W_0 = \{w_0(1), w_0(2), ..., w_0(v)\}, \tag{8.8}$$

where $w_0(k) = \max_{1 \le i \le u} w_i(k)$.

Step 5. Determine the series differences. The normalized decision matrix is compared with a reference alternative, usually representing the desired outcome. The difference between the reference alternative and all alternatives are calculated to obtain the minimum and maximum difference:

$$\Delta_i(k) = w_0(k) - w_i(k). \tag{8.9}$$

Step 6. Calculate the Grey Relational Grade (r). The Grey Relational Grade indicates its closeness to the reference alternative. This is obtained using the grey relational coefficient (γ) for each alternative:

$$r_i = \frac{1}{v} \sum_{k=1}^{v} \gamma\left(w_0(k), w_i(k)\right), \tag{8.10}$$

where (γ) considers both the minimum and maximum differences and is defined as,

$$\gamma\left(w_0(k), w_i(k)\right) = \frac{\min\limits_{1\leq i\leq u}\min\limits_{1\leq k\leq v}\Delta_i(k) + \zeta\max\limits_{1\leq i\leq u}\max\limits_{1\leq k\leq v}\Delta_i(k)}{\Delta_i(k) + \zeta\max\limits_{1\leq i\leq u}\max\limits_{1\leq k\leq v}\Delta_i(k)}, \tag{8.11}$$

and ζ is the grey distinguishing coefficient.

8.2. Grey Number Relational Analysis

Grey Number Relational Analysis (GNRA) is a subset of the Grey Systems Theory (GST), and it deals with uncertainties represented as grey numbers (GNs). In classical GRA, a weighted and normalized decision matrix is compared to a reference alternative, and grey relational grades are used to rank alternatives [101]. It is worth noting that while interval numbers and interval grey numbers share a similar concept, they have distinct differences. Interval numbers encompass all possible numbers within a given range, whereas a grey interval number represents a single number within a range. The GRA ranking method employing interval GNs is a modified version of the traditional GRA method. Table 8.1 provides an overview of the three types of GRAs.

Table 8.1. Types of GRA with grey numbers.

GRA Types		Performance Value	
		White Decision Matrix	**Grey Decision Matrix**
Weights	White numbers	Classical GRA	Type-I GRA
	Grey number	TYPE-II GRA	Type-III GRA

Source: Table by authors.

8.2.1. Grey Weights with White Performance Values

Type II GRA is where the alternatives are ranked based on the grey relational grade. The steps for using GRA with grey numbers (GNs) in type II GRA are as follows.

Step 1. Construct a decision matrix. Begin by creating a decision matrix from raw data, considering the criteria and performance of the alternatives. The decision matrix is represented as follows:

$$X = \begin{pmatrix} x_1(1) & x_1(2) & \cdots & x_1(n) \\ x_2(1) & x_2(2) & \cdots & x_2(n) \\ \vdots & \vdots & \ddots & \vdots \\ x_m(1) & x_m(2) & \cdots & x_m(n) \end{pmatrix}, \tag{8.12}$$

where $x_i(k)$ represents the precise data of the kth criterion for the ith alternative, with $1 \leq k \leq n$ and $1 \leq i \leq m$.

Step 2. Normalize the decision matrix: Normalize the decision matrix to ensure that the preference is unidirectional and evenly distributed within the range of 0 to 1. For benefit preferences (where larger values are better), use Equation (8.13), and for cost preferences (where smaller values are better), use Equation (8.14).

- Benefit preference:

$$x_i^*(k) = \frac{x_i(k) - \min\limits_{1\leq k\leq n} x_i(k)}{\max\limits_{1\leq k\leq n} x_i(k) - \min\limits_{1\leq k\leq n} x_i(k)}. \tag{8.13}$$

- Cost preference:

$$x_i^*(k) = \frac{\max\limits_{1\leq k\leq n} x_i(k) - x_i(k)}{\max\limits_{1\leq k\leq n} x_i(k) - \min\limits_{1\leq k\leq n} x_i(k)}. \tag{8.14}$$

This results in the normalized data matrix, X^*:

$$X^* = \begin{pmatrix} x_1^*(1) & x_1^*(2) & \cdots & x_1^*(n) \\ x_2^*(1) & x_2^*(2) & \cdots & x_2^*(n) \\ \vdots & \vdots & \ddots & \vdots \\ x_m^*(1) & x_m^*(2) & \cdots & x_m^*(n) \end{pmatrix}. \tag{8.15}$$

Step 3. Construct the grey decision matrix. From the normalized data matrix, construct the grey decision matrix, $\otimes X$, using Equation (8.16):

$$\otimes X = \begin{pmatrix} \otimes x_{1,1} & \otimes x_{1,2} & \cdots & \otimes x_{1,n} \\ \otimes x_{2,1} & \otimes x_{2,2} & \cdots & \otimes x_{2,n} \\ \vdots & \vdots & \ddots & \vdots \\ \otimes x_{m,1} & \otimes x_{m,2} & \cdots & \otimes x_{m,n} \end{pmatrix}. \tag{8.16}$$

Each entry, $\otimes x_{ij}$, in the matrix represents a grey interval number, where $[\underline{x}ij, \overline{x}ij] = [\min_{1\leq k\leq h} C_{j\text{-}k}, \max_{1\leq k\leq h} C_{j\text{-}k}]$. Here, C_j is the jth first-level criterion, and $C_{j\text{-}h}$ represents the last term for the ith alternative.

Step 4. Calculate the weighted normalized grey decision matrix. Determine the weight for each criterion using a Multiple Criteria Decision-Making (MCDM) weighting method. Multiply the normalized decision matrix (X^*) by the transposed weights matrix (W^{T}) to obtain the weighted normalized grey decision matrix ($\otimes X'$) using the following formula:

$$\otimes X' = \otimes X^* \times \otimes W^{\mathrm{T}}. \tag{8.17}$$

$$\otimes W = (\otimes w_1, \otimes w_2, ..., \otimes w_n),$$

$$\otimes X' = \begin{pmatrix} \otimes x'_{1,1} & \otimes x'_{2,1} & \cdots & \otimes x'_{1,n} \\ \otimes x'_{2,1} & \otimes x'_{2,2} & \cdots & \otimes x'_{2,n} \\ \vdots & \vdots & \ddots & \vdots \\ \otimes x'_{m,1} & \otimes x'_{m,2} & \cdots & \otimes x'_{m,n} \end{pmatrix}.$$

That is, $\otimes x'_{k,h} = \otimes x^*_{k,h} \times \otimes w_h$. In vector form, the series can be written as follows:

$$\begin{aligned} \otimes X'_1 &= \left\{ \otimes x'_{1,1}, \otimes x'_{1,2}, ..., \otimes x'_{1,n} \right\} \\ \otimes X'_2 &= \left\{ \otimes x'_{2,1}, \otimes x'_{2,2}, ..., \otimes x'_{2,n} \right\} \\ &\vdots \\ \otimes X'_m &= \left\{ \otimes x'_{m,1}, \otimes x'_{m,2}, ..., \otimes x'_{m,n} \right\} \end{aligned} .$$

Step 5. Determine the reference alternative. Calculate the reference alternative as follows:

$$\otimes X'_0 = \{\otimes x'_{01}, \otimes x'_{02}, ..., \otimes x'_{0,n}\}, \tag{8.18}$$

where $\otimes x'0j = [\max 1 \leq i \leq m\underline{x}ij, \max 1 \leq i \leq m\overline{x}_{ij}]$.

Step 6. Determine the series differences: Calculate the differences between the reference alternative and the other alternatives to obtain the difference matrix using the following:

$$\begin{aligned} \Delta_{ij} &= \left| \otimes x_{0j} - \otimes x_{ij} \right| \\ &= \max \left(\left| \underline{x}_{0j} - \underline{x}_{ij} \right|, \left| \overline{x}_{0j} - \overline{x}_{ij} \right| \right). \end{aligned} \tag{8.19}$$

Step 7. Calculate the Grey Relational Grade (GRG): Compute the GRG (r_i) based on the grey relational coefficient (γ_{ij}) for each alternative using the following:

$$r_i = \frac{1}{n} \sum_{j=1}^{n} \gamma_{ij}, \tag{8.20}$$

where the grey distinguishing coefficient is ζ, and the grey relational coefficient is as follows:

$$\gamma_{ij} = \frac{\min\limits_{1 \leq i \leq m} \min\limits_{1 \leq j \leq n} \Delta_{ij} + \zeta \max\limits_{1 \leq i \leq m} \max\limits_{1 \leq j \leq n} \Delta_{ij}}{\Delta_{ij} + \zeta \max\limits_{1 \leq i \leq m} \max\limits_{1 \leq j \leq n} \Delta_{ij}}. \tag{8.21}$$

8.2.2. Grey Weight with Grey Performance Value

The steps for using the GRA with GNs are as follows.

Step 1. Construct a decision matrix. Create a decision matrix from raw data based on the criteria and the performance of alternatives. The decision matrix is represented as follows:

$$X = \begin{pmatrix} x_1(1) & x_1(2) & \cdots & x_1(n) \\ x_2(1) & x_2(2) & \cdots & x_2(n) \\ \vdots & \vdots & \ddots & \vdots \\ x_m(1) & x_m(2) & \cdots & x_m(n) \end{pmatrix}, \tag{8.22}$$

where $x_{i(k)}$ represents the precise data of the kth criterion for the ith alternative, with $1 \leq k \leq n$ and $1 \leq i \leq m$, and u and n are the numbers of alternatives and criteria, respectively.

Step 2. Normalize the decision matrix. Normalize the decision matrix to ensure that the preference is unidirectional and evenly distributed within the range of 0 to 1.

- For benefit preferences (where larger values are better), use Equation (8.23):

$$x_i^*(k) = \frac{x_i(k) - \min\limits_{1\leq k\leq n} x_i(k)}{\max\limits_{1\leq k\leq n} x_i(k) - \min\limits_{1\leq k\leq n} x_i(k)}. \tag{8.23}$$

- For cost preferences (where smaller values are better), use Equation (8.24):

$$x_i^*(k) = \frac{\max\limits_{1\leq k\leq n} x_i(k) - x_i(k)}{\max\limits_{1\leq k\leq n} x_i(k) - \min\limits_{1\leq k\leq n} x_i(k)}. \tag{8.24}$$

This results in the normalized data matrix, X^*:

$$X^* = \begin{pmatrix} x_1^*(1) & x_1^*(2) & \cdots & x_1^*(n) \\ x_2^*(1) & x_2^*(2) & \cdots & x_2^*(n) \\ \vdots & \vdots & \ddots & \vdots \\ x_m^*(1) & x_m^*(2) & \cdots & x_m^*(n) \end{pmatrix}. \tag{8.25}$$

Step 3. Construct the grey decision matrix. From the normalized data matrix, construct the grey decision matrix, $\otimes X$, using Equation (8.26):

$$\otimes X = \begin{pmatrix} \otimes x_{1,1} & \otimes x_{1,2} & \cdots & \otimes x_{1,n} \\ \otimes x_{2,1} & \otimes x_{2,2} & \cdots & \otimes x_{2,n} \\ \vdots & \vdots & \ddots & \vdots \\ \otimes x_{m,1} & \otimes x_{m,2} & \cdots & \otimes x_{m,n} \end{pmatrix}. \tag{8.26}$$

Each entry, $\otimes x_{ij}$ in the matrix represents a grey interval number, where $[\underline{x}_{ij}, \overline{x}_{ij}] = [\min_{1\leq k\leq h} C_{j\text{-}k}, \max_{1\leq k\leq h} C_{j\text{-}k}]$. Here, C_j is the jth first-level criterion, and $C_{j\text{-}h}$ represents the last term for the ith alternative.

Step 4. Calculate the weighted normalized grey decision matrix. Determine the weights for each criterion using a MCDM weighting method. Then, multiply the normalized decision matrix (X^*) by the transposed weights matrix (W^{T}) to obtain the weighted normalized grey decision matrix ($\otimes X'$) using Equation (8.27):

$$\otimes X' = \otimes X^* \times \otimes W^{\mathrm{T}}, \tag{8.27}$$

$$\otimes W = (\otimes w_1, \otimes w_2, ..., \otimes w_n)\,, \tag{8.28}$$

$$\otimes X' = \begin{pmatrix} \otimes x'_{1,1} & \otimes x'_{2,1} & \cdots & \otimes x'_{1,n} \\ \otimes x'_{2,1} & \otimes x'_{2,2} & \cdots & \otimes x'_{2,n} \\ \vdots & \vdots & \ddots & \vdots \\ \otimes x'_{m,1} & \otimes x'_{m,2} & \cdots & \otimes x'_{m,n} \end{pmatrix}. \tag{8.29}$$

That is, $\otimes x'_{k,h} = \otimes x^*_{k,h} \times \otimes w_h$. In vector form, the series can be written as follows:

$$\begin{array}{ll} \otimes X'_1 & = \{\otimes x'_{1,1}, \otimes x'_{1,2}, ..., \otimes x'_{1,n}\} \\ \otimes X'_2 & = \{\otimes x'_{2,1}, \otimes x'_{2,2}, ..., \otimes x'_{2,n}\} \\ & \vdots \\ \otimes X'_m & = \{\otimes x'_{m,1}, \otimes x'_{m,2}, ..., \otimes x'_{m,n}\} \end{array}.$$

Step 5. Determine the reference alternative: Calculate the reference alternative as shown in Equation (8.30):

$$\otimes X'_0 = \{\otimes x'_{01}, \otimes x'_{02}, ..., \otimes x'_{0,n}\}, \tag{8.30}$$

where $\otimes x'0j = [\max 1 \leq i \leq m\underline{x}ij, \max 1 \leq i \leq m\overline{x}_{ij}]$.

Step 6. Determine the series differences. Calculate the differences between the reference alternative and other alternatives to obtain the difference matrix using Equation (8.31):

$$\Delta_{ij} = \left|\otimes x_{0j} - \otimes x_{ij}\right| = \max\left(\left|\underline{x}_{0j} - \underline{x}_{ij}\right|, \left|\overline{x}_{0j} - \overline{x}_{ij}\right|\right). \tag{8.31}$$

Step 7. Calculate the Grey Relational Grades (GRGs). Compute the GRGs (r_i) based on the grey relational coefficient (γ_{ij}) for each alternative using Equation (8.32):

$$r_i = \frac{1}{n}\sum_{j=1}^{n} \gamma_{ij}. \tag{8.32}$$

The grey distinguishing coefficient is ζ, and the grey relational coefficient is calculated using Equation (8.33):

$$\gamma_{ij} = \frac{\min\limits_{1\leq i\leq m}\min\limits_{1\leq j\leq n}\Delta_{ij} + \zeta\max\limits_{1\leq i\leq m}\max\limits_{1\leq j\leq n}\Delta_{ij}}{\Delta_{ij} + \zeta\max\limits_{1\leq i\leq m}\max\limits_{1\leq j\leq n}\Delta_{ij}}. \tag{8.33}$$

The grey distinguishing coefficient, $\zeta \in [0,1]$, is the degree to which the minimum score to the maximum score is stressed. The grey distinguishing coefficient is usually 0.5: $\zeta = 0.5$ [61].

8.3. Application of GRA with Grey Numbers

Grey Systems Theory (GST) has found applications in various fields of business management, including project management, planning, stock market analysis, and portfolio selection [54,62,288]. Zhang [199] provided an example of using GRA with grey interval numbers for selecting an enterprise planning system, where expert-provided weights played a crucial role. Wu et al. [289] utilized GRA to determine the weights of business performance indicators for wealth management banks, leveraging the core features of GRA. Shuai and Wu [290] employed grey entropy to evaluate hotel performance and its influence on e-marketing strategies. Zhang and Jia [291] applied Grey Systems Theory (GST) to measure business synergy by analyzing the coordinating degree between resource sub-systems and performance sub-systems, identifying opportunities for enhancing coordination among different business systems. Bai and Sarkis [292] combined Grey Systems Theory (GST) with the Decision-Making Trial and Evaluation labouratory (DEMATEL) to assess critical success factors in business processes, incorporating grey numbers for linguistic measurements. The work of Yin [62] emphasized the increasing role of Grey Systems Theory (GST) in decision-making, highlighting the versatility of grey numbers in quantifying both qualitative and quantitative features in complex systems with limited information.

8.3.1. Contractor Selection

A contractor selection problem is encountered in the installation of a Floating Solar Panel Energy System. The grey SWARA-FUCOM method, as introduced in Section 8.2.1, is applied for Grey Number Relational Analysis (GNRA). Following the outlined steps, the hierarchical model is presented in Figure 5.2, and the decision matrix, D, is constructed from the data in Table 8.2, in accordance with Equation (8.12).

$$D = \begin{pmatrix} d_{1,1} & d_{1,2} & d_{1,3} & \cdots & d_{1,30} \\ d_{2,1} & d_{2,2} & d_{2,3} & \cdots & d_{2,30} \\ d_{3,1} & d_{3,2} & d_{3,3} & \cdots & d_{3,30} \\ d_{4,1} & d_{4,2} & d_{4,3} & \cdots & d_{4,30} \end{pmatrix} = \begin{pmatrix} 86.25 & 82.25 & 82.5 & \cdots & 88.33 \\ 70 & 56.25 & 57.5 & \cdots & 61.25 \\ 78.5 & 77.5 & 73.75 & \cdots & 81.25 \\ 84.25 & 80 & 80 & \cdots & 78.75 \end{pmatrix}. \quad (8.34)$$

Subsequently, the normalized decision matrix is obtained through the utilization of Equation (8.15):

$$D' = \begin{pmatrix} d'_{1,1} & d'_{1,2} & d'_{1,3} & \cdots & d'_{1,30} \\ d'_{2,1} & d'_{2,2} & d'_{2,3} & \cdots & d'_{2,30} \\ d'_{3,1} & d'_{3,1} & d'_{3,3} & \cdots & d'_{3,30} \\ d'_{4,1} & d'_{4,2} & d'_{4,3} & \cdots & d'_{4,30} \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 & \cdots & 1 \\ 0 & 0 & 0 & \cdots & 0 \\ 0.5231 & 0.8173 & 0.65 & \ddots & 0.7386 \\ 0.8769 & 0.9135 & 0.9 & \cdots & 0.6462 \end{pmatrix}. \quad (8.35)$$

Table 8.2. Ratings (%) of the contractors based on proposals.

Criteria	Index	A_1	A_2	A_3	A_4
$\Psi_{1\text{-}1}$	1	86.25	70	78.5	84.25
$\Psi_{1\text{-}2}$	2	82.25	56.25	77.5	80
$\Psi_{1\text{-}3}$	3	82.5	57.5	73.75	80
$\Psi_{1\text{-}4}$	4	82.5	61.25	75	80
$\Psi_{1\text{-}5}$	5	86.25	71.25	72.5	75
$\Psi_{2\text{-}1}$	6	82.5	61.25	71.25	88.75
$\Psi_{2\text{-}2}$	7	86.25	65	78.75	87.5
$\Psi_{2\text{-}3}$	8	80	58.75	75	81.25
$\Psi_{2\text{-}4}$	9	86.25	60	77.5	85
$\Psi_{2\text{-}5}$	10	87	58.75	78.75	83.75
$\Psi_{3\text{-}1}$	11	84.5	62.5	80	80
$\Psi_{3\text{-}2}$	12	86.25	67.5	77.5	77.5
$\Psi_{3\text{-}3}$	13	85.5	67.5	77.5	79.5
$\Psi_{3\text{-}4}$	14	82.5	62.5	77.5	75
$\Psi_{3\text{-}5}$	15	82.5	68.75	73.75	75
$\Psi_{4\text{-}1}$	16	83.75	67.5	73.75	78.75
$\Psi_{4\text{-}2}$	17	80	68.75	70	75
$\Psi_{4\text{-}3}$	18	81.25	60	78.75	78.75
$\Psi_{4\text{-}4}$	19	82.5	65	77.5	75
$\Psi_{4\text{-}5}$	20	82.5	65	76.25	78.75
$\Psi_{5\text{-}1}$	21	82.5	68.75	83.75	82.5
$\Psi_{5\text{-}2}$	22	79.33	70	82.5	85
$\Psi_{5\text{-}3}$	23	85	67.5	82.5	77.5
$\Psi_{5\text{-}4}$	24	88.33	65	75	75
$\Psi_{5\text{-}5}$	25	76.67	73.75	80	77.5
$\Psi_{6\text{-}1}$	26	81.67	66.25	76.25	80
$\Psi_{6\text{-}2}$	27	88.33	67.5	77.5	80.75
$\Psi_{6\text{-}3}$	28	80	62.5	81.25	78.75
$\Psi_{6\text{-}4}$	29	83.33	60	78.75	81.25
$\Psi_{6\text{-}5}$	30	88.33	61.25	81.25	78.75

Source: Reprinted from [102], used with permission.

The computation of the weighted standardized decision matrix is performed using the grey SWARA–FUCOM weight, as specified in Equation (6.17). The grey weight is as follows:

$$W = \begin{pmatrix} [0.0075,\ 0.0118] & [0.0109,\ 0.0182] & [0.0093,\ 0.01] & \cdots & [0.0168,\ 0.0307] \end{pmatrix}^{\mathrm{T}}.$$

$$\otimes D^* = \begin{pmatrix} \otimes d^*_{1,1} & \otimes d^*_{1,2} & \otimes d^*_{1,3} & \cdots & \otimes d^*_{1,30} \\ \otimes d^*_{2,1} & \otimes d^*_{2,2} & \otimes d^*_{2,3} & \cdots & \otimes d^*_{2,30} \\ \otimes d^*_{3,1} & \otimes d^*_{3,2} & \otimes d^*_{3,3} & \cdots & \otimes d^*_{3,30} \\ \otimes d^*_{4,1} & \otimes d^*_{4,2} & \otimes d^*_{4,3} & \cdots & \otimes d^*_{4,30} \end{pmatrix}$$
$$= \begin{pmatrix} [0.0075,\ 0.0118] & [0.0109,\ 0.182] & [0.0093,\ 0.0100] & \cdots & [0.0168,\ 0.0307] \\ 0 & 0 & 0 & \cdots & 0 \\ [0.0039,\ 0.0062] & [0.0089,\ 0.0149] & [0.0060,\ 0.0065] & \cdots & [0.0124,\ 0.0227] \\ [0.0066,\ 0.0103] & [0.0100,\ 0.0166] & [0.0084,\ 0.0090] & \cdots & [0.0109,\ 0.0198] \end{pmatrix}, \tag{8.36}$$

where $\otimes d_{ij}^* = d_{ij}' \times \otimes w_{ij}$.

Next, the weighted grey reference alternative is determined:

$$D_0^* = \left\{ \otimes d_{0,1}^*, \otimes d_{0,2}^*, \ldots, \otimes d_{0,30}^* \right\}$$
$$= \left\{ \begin{array}{ccccc} [0.0075,\ 0.0118] & [0.0109,\ 0.0182] & [0.0093,\ 0.01] & \cdots & [0.0168,\ 0.0307] \end{array} \right\}, \quad (8.37)$$

where $\otimes d_{0j}^* = \left[\max\limits_{1 \le i \le 4} \underline{d_{ij}^*}, \max\limits_{1 \le i \le 4} \overline{d_{ij}^*} \right]$. Then, weighted alternative differences are calculated:

$$\Delta = \begin{pmatrix} \delta_{1,1} & \delta_{1,2} & \delta_{1,3} & \cdots & \delta_{1,30} \\ \delta_{2,1} & \delta_{2,2} & \delta_{2,3} & \cdots & \delta_{2,30} \\ \delta_{3,1} & \delta_{3,2} & \delta_{3,3} & \cdots & \delta_{3,30} \\ \delta_{4,1} & \delta_{4,2} & \delta_{4,3} & \cdots & \delta_{4,30} \end{pmatrix}$$
$$= \begin{pmatrix} 0 & 0 & 0 & \cdots & 0 \\ 0.0118 & 0.0182 & 0.0100 & \cdots & 0.0307 \\ 0.0056 & 0.0033 & 0.0035 & \cdots & 0.0080 \\ 0.0015 & 0.0016 & 0.0010 & \cdots & 0.0109 \end{pmatrix}. \quad (8.38)$$

Finally, the grey relational coefficient is calculated to determine the Grey Relational Grade, which is utilized for ranking. The grey relational coefficient is as follows:

$$\gamma_{ij} = \frac{\min\limits_{1 \le i \le 4} \min\limits_{1 \le j \le 30} \delta_{ij} + \zeta \max\limits_{1 \le i \le 4} \max\limits_{1 \le j \le 30} \delta_{ij}}{\delta_{ij} + \zeta \max\limits_{1 \le i \le 4} \max\limits_{1 \le j \le 30} \delta_{ij}}, \quad (8.39)$$

where the grey distinguishing coefficient, $\zeta = 0.5$, is calculated. Thus, the Grey Relational Grade is $r_i = \frac{1}{30} \sum_{j=1}^{30} \gamma_{ij}$:

$$r_1 = 0.9703, r_2 = 0.6288,\ r_3 = 0.8256,\ r_4 = 0.8816. \quad (8.40)$$

$r_1 > \mathrm{r}_4 > r_3 > r_2$, the $A_1 \succ A_4 \succ A_3 \succ A_2$, i.e., contractors A_1, A_4, A_3, A_2 are ranked as the first, second, third and fourth positions, respectively.

8.3.2. Scaling Foreign Service Premium

This section presents a case study focusing on scaling the foreign service premium (FSP) allowance within the petroleum equipment manufacturing and service industry based in China. The company operates globally with 22 branches established in 22 different countries. It was observed that certain staff members were reluctant to work in very remote branches. Consequently, GRA was applied to address this issue. The branches under evaluation, listed alphabetically, are located in the following countries: Albania, Algeria, Bangladesh, Brazil, Canada, Colombia, France, Indonesia, Italy, Kazakhstan, Malaysia, Mexico, Nigeria, Pakistan, Peru, Poland, Romania, Russia, Ukraine, United Arab Emirates (UAE), USA, and Venezuela. The data used were collected during the third quarter of 2018.

The performance assessments for all the alternatives, with regard to every second-level criterion, were obtained. The performance scores for these countries are provided in Table 8.3. Notably, in this evaluation, three values are missing: the Justice System ($\Theta_{5\text{-}2}$) in Algeria, the Public Integrity Index ($\Theta_{5\text{-}1}$) in the UAE, and the GDP per Capita ($\Theta_{3\text{-}2}$) of Venezuela. These missing values were disregarded, as second-level criteria are conceptually correlated with their respective first-level criteria. It is important to emphasize that the absence of these values is not expected to significantly impact the results, and their exclusion is unlikely to skew the outcome. As a result, the evaluation was conducted using 99.09% of the available data, involving 327 out of a total of 330 values. The evaluation process, based on the steps outlined in Section 8.2.2, is detailed as follows.

The decision matrix, denoted as X, is formulated using the data from Table 8.3 in accordance with Equation (8.12):

$$X = \begin{pmatrix} x_1(1) & x_1(2) & \cdots & x_1(15) \\ x_2(1) & x_2(2) & \cdots & x_2(15) \\ \vdots & \vdots & \ddots & \vdots \\ x_{22}(1) & x_{22}(2) & \cdots & x_{22}(15) \end{pmatrix} = \begin{pmatrix} 18.20 & 65.46 & \cdots & 5.20 \\ 34.50 & 57.18 & \cdots & 4.70 \\ \vdots & \vdots & \ddots & \vdots \\ 16.80 & 63.89 & \cdots & 1.80 \end{pmatrix}.$$

Then, the normalized decision matrix is obtained using Equation (8.25):

$$X^{*} = \begin{pmatrix} 0.2216 & 0.3400 & 0.7742 & \cdots & 0.2766 \\ 0.5356 & 0.4922 & 0.7742 & \cdots & 0.3830 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0.1946 & 0.3688 & 0.0000 & \cdots & 1.0000 \end{pmatrix}.$$

The grey data were computed utilizing Equation (8.16) and are presented in Table 8.4. Subsequently, the grey decision matrix, denoted as $\otimes X$, was constructed using the data from Table 8.4:

$$\otimes X = \begin{pmatrix} \otimes x_{1,1} & \otimes x_{1,2} & \cdots & \otimes x_{1,5} \\ \otimes x_{2,1} & \otimes x_{2,2} & \cdots & \otimes x_{2,5} \\ \vdots & \vdots & \ddots & \vdots \\ \otimes x_{22,1} & \otimes x_{22,2} & \cdots & \otimes x_{22,5} \end{pmatrix}$$

$$= \begin{pmatrix} [0.2216,\ 0.7742] & [0.1651,\ 0.492] & \cdots & [0.2766,\ 0.5768] \\ [0.4922,\ 0.7742] & [0.4407,\ 0.6923] & \cdots & [0.383,\ 0.5631] \\ \vdots & \vdots & \ddots & \vdots \\ [0,\ 0.3688] & [0.4032,\ 0.8252] & \cdots & [1,\ 1] \end{pmatrix}.$$

Table 8.3. Performance of the alternatives for second-level indicators.

Countries (i)/ Second-Level Criteria ($\Theta_{j\text{-}k}$)	**Index (n)**	**Albania (1)**	**Algeria (2)**	**Bangladesh (3)**	**...**	**Venezuela (22)**
Clean Cities ($\Theta_{1\text{-}1}$)	1	18.2000	34.5000	58.6000	...	16.8000
Environmental Performance Index ($\Theta_{1\text{-}2}$)	2	65.4600	57.1800	29.5600	...	63.8900
Disaster Risk Index ($\Theta_{1\text{-}3}$)	3	9.5000	9.5000	1.6900	...	36.2800
Global Terrorism Index ($\Theta_{2\text{-}1}$)	4	1.4870	3.9700	6.1810	...	3.6320
Failed States Index ($\Theta_{2\text{-}2}$)	5	60.0793	75.7851	90.3128	...	86.2069
Global Peace Index ($\Theta_{2\text{-}3}$)	6	1.8490	2.1820	2.0840	...	2.6420
Consumer Price Index ($\Theta_{3\text{-}1}$)	7	115.0843	142.3842	161.1360	...	2740.2740
GDP per Capita ($\Theta_{3\text{-}2}$)	8	4537.8625	4123.3899	1516.5134	...	-
Inflation ($\Theta_{3\text{-}3}$)	9	1.2828	6.3977	5.5135	...	254.9485
Sanitation and Hygiene ($\Theta_{4\text{-}1}$)	10	98.0000	87.0000	47.0000	...	95.0000
Mortality From Environmental Pollution ($\Theta_{4\text{-}2}$)	11	104.7000	40.3000	103.4000	...	28.9000
Drinking Water ($\Theta_{4\text{-}4}$)	12	91.0000	93.0000	97.0000	...	97.0000
Public Integrity Index ($\Theta_{5\text{-}1}$)	13	6.4800	4.9400	5.1700	...	1.9300
Justices System ($\Theta_{5\text{-}2}$)	14	0.5078	-	0.4091	...	0.2863
Reliability of Police Service ($\Theta_{5\text{-}3}$)	15	5.2000	4.7000	3.3000	...	1.8000

Source: Reprinted from [101], used with permission.

Table 8.4. Grey decision data.

Criteria (Θ_j)/Countries (i)	Θ_1	Θ_2	Θ_3	Θ_4	Θ_5
Albania (1)	[0.2216, 0.7742]	[0.1651, 0.492]	[0.0031, 0.9479]	[0.0299, 0.6254]	[0.2766, 0.5768]
Algeria (2)	[0.4922, 0.7742]	[0.4407, 0.6923]	[0.0135, 0.9551]	[0.1779, 0.2121]	[0.383, 0.5631]
Bangladesh (3)	[1, 1]	[0.3982, 0.8776]	[0.0206, 1]	[0.0909, 0.791]	[0.5298, 0.7654]
Brazil (4)	[0.0983, 0.9003]	[0.1745, 0.6021]	[0.0185, 0.8568]	[0.0909, 0.209]	[0.434, 0.617]
Canada (5)	[0, 0.7334]	[0, 0.3283]	[0.0019, 0.2499]	[0, 0.0303]	[0, 0.0426]
Colombia (6)	[0.2023, 0.8641]	[0.621, 0.7589]	[0.0095, 0.9175]	[0.0909, 0.2388]	[0.3309, 0.6596]
France (7)	[0, 0.985]	[0.1367, 0.662]	[0, 0.3629]	[0, 0.073]	[0.0131, 0.1702]
Indonesia (8)	[0.1869, 0.8569]	[0.269, 0.6478]	[0.0134, 0.9598]	[0.303, 0.4776]	[0.3875, 0.5594]
Italy (9)	[0.1285, 0.9824]	[0.2204, 0.3053]	[0.0007, 0.4754]	[0, 0.2363]	[0.135, 0.4255]
Kazakhstan (10)	[0.1503, 0.9164]	[0.3275, 0.5348]	[0.0237, 0.8738]	[0.0299, 0.2926]	[0.4267, 0.5651]
Malaysia (11)	[0.2042, 0.8182]	[0.1381, 0.5368]	[0.0048, 0.8547]	[0, 0.1425]	[0.2128, 0.5241]
Mexico (12)	[0.2736, 0.5999]	[0.3654, 0.6773]	[0.0089, 0.8727]	[0.0606, 0.1642]	[0.3512, 0.8723]
Nigeria (13)	[0.5367, 0.763]	[0.8395, 1]	[0.0408, 0.9922]	[1, 1]	[0.6386, 0.7447]
Pakistan (14)	[0.854, 0.9725]	[0.9324, 0.9547]	[0.019, 0.9995]	[0.3333, 0.6831]	[0.4935, 0.7985]
Peru (15)	[0.405, 0.9483]	[0.2824, 0.6195]	[0.007, 0.9129]	[0.303, 0.3433]	[0.2917, 0.8298]
Poland (16)	[0.277, 0.3648]	[0.0426, 0.2545]	[0.0011, 0.7881]	[0.0299, 0.4281]	[0.1829, 0.5106]
Romania (17)	[0.1676, 0.9104]	[0, 0.3553]	[0, 0.8397]	[0, 0.7554]	[0.1684, 0.4468]
Russia (18)	[0.1541, 0.8777]	[0.5915, 1]	[0.0233, 0.841]	[0.1212, 0.4948]	[0.4238, 0.6535]
Ukraine (19)	[0.2447, 0.8604]	[0.6512, 0.9737]	[0.0488, 0.9806]	[0.0597, 0.8464]	[0.3803, 0.5969]
UAE (20)	[0.0173, 0.8627]	[0.2066, 0.6026]	[0, 0.0109]	[0, 0.0653]	[0, 0.1523]
USA (21)	[0.4606, 0.8627]	[0.0234, 0.272]	[0.0024, 0.3246]	[0, 0.0083]	[0, 0.3051]
Venezuela (22)	[0, 0.3688]	[0.4032, 0.8252]	[1, 1]	[0.0746, 0.0987]	[1, 1]

Source: Table by authors.

The weighted grey decision matrix was computed using Equation (8.17). The weights (W) were determined through the application of the Stepwise Weight Analysis Ratio Assessment (SWARA) weighting method for group decision-making, as specified in Equation (5.11). The resulting weighted grey matrix is as follows,

$$\otimes X' = \begin{pmatrix} [0.0386,\ 0.1565] & [0.0430,\ 0.1362] & \cdots & [0.0296,\ 0.0815] \\ [0.0858,\ 0.1565] & [0.1148,\ 0.1917] & \cdots & [0.0410,\ 0.0796] \\ \vdots & \vdots & \ddots & \vdots \\ [0,\ 0.0746] & [0.1050,\ 0.2285] & \cdots & [0.1070,\ 0.1413] \end{pmatrix},$$

and the reference country based on Equation (8.18) is as follows:

$$\otimes X'_0 = ([0.1744, 0.2022], [0.2429, 0.2769], [0.1295, 0.2131], [0.1110, 0.1665], [0.1070, 0.1413]).$$

The series differences based on Equation (8.19) are presented in Table 8.5.

Table 8.5. Differences between reference country and evaluated countries.

Criteria (Θ_j)/Differences (Δ_{ij})	Θ_1	Θ_2	Θ_3	Θ_4	Θ_5	$\min_{1\le j\le 5} \Delta_{ij}$	$\max_{1\le j\le 5} \Delta_{ij}$
Δ_{1j}	0.1636	0.2339	0.2127	0.1632	0.1117	0.1117	0.2339
Δ_{2j}	0.1164	0.1621	0.2114	0.1468	0.1003	0.1003	0.2114
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
Δ_{22j}	0.2022	0.1719	0.0836	0.1582	0.0343	0.0343	0.2022
$\min_{1\le i\le 22} \min_{1\le j\le 5} \Delta_{ij}$	–	–	–	–	–	0.0278	–
$\max_{1\le i\le 22} \max_{1\le j\le 5} \Delta_{ij}$	–	–	–	–	–	–	0.2769

Source: Reprinted from [101], used with permission.

The Grey Relational Grade (GRG) using the relational coefficient $\zeta = 0.5$ is as follows:

$$r_i = (r_1, r_2, r_3, ..., r_{22}), \tag{8.41}$$

= 0.5372, 0.5920, 0.6638, 0.5432, 0.5002, 0.5838, 0.5082, 0.5614, 0.5263, 0.5542, 0.5298, 0.5610, 0.7284, 0.7321, 0.5716, 0.5286, 0.5175, 0.5845, 0.5938, 0.5129, 0.5315, 0.6590.

As the Grey Relational Grade increases, the location becomes less favorable, and consequently, the compensation required is higher. The location rankings of the branches, ranging from the most favorable (first position) to the least favorable (22nd position), are as follows: Canada, France, USA, Romania, Italy, Poland, Malaysia, UAE, Albania, Brazil, Kazakhstan, Mexico, Indonesia, Peru, Colombia, Russia, Algeria, Ukraine, Venezuela, Bangladesh, Nigeria, and Pakistan. These rankings, along with the proposed compensation scale for expatriates, are illustrated in Figure 8.2. Additionally, a heatmap depicting the premium allowance is presented in Figure 8.3.

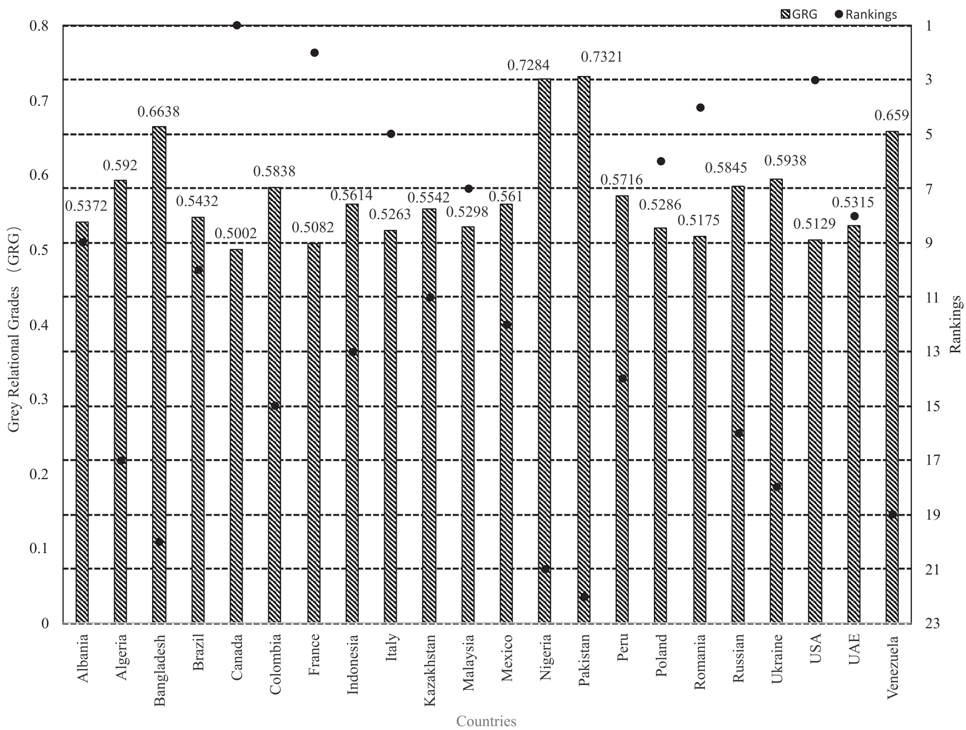


Figure 8.2. Scaling and rankings of overseas branches. Source: Figure by authors using Microsoft Excel.

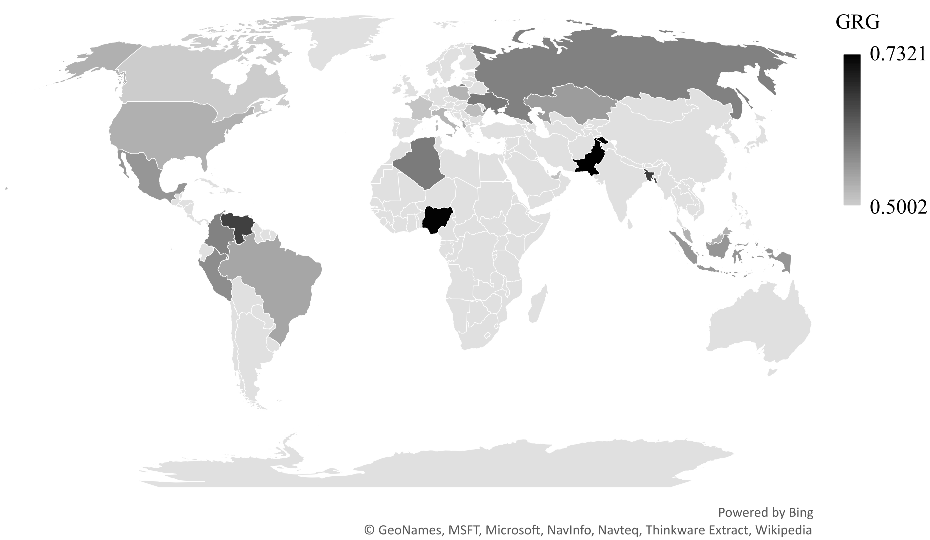


Figure 8.3. Heat map of premium service allowance. Source: Reprinted from [204], used with permission.

It is interesting to note that all the decision-makers (DMs) ranked *Conflict States* (Θ_2) as the most important criterion, with an allocated grey weight of [0.2605, 0.2769].

On the other hand, *Regulatory Institutions* (Θ_5) was deemed the least important criterion, with a grey weight of [0.107, 0.1413]. Canada secured the first position in the rankings, followed by France in second place and the USA in third place.

From the rankings, it is evident that more allowance should be allocated to expatriates who accept assignments in Nigeria and Pakistan, which are ranked 21st and 22nd, respectively. As the Grey Relational Grade (GRG) increases, the locations become less favorable, and consequently, higher compensation is warranted. The ratio of the foreign service premium (FSP) allowance for compensating expatriates is determined by the GRG. For example, if an expatriate accepts an assignment in Albania and is paid CNY 53,720 (Yuan—RMB), then the expatriate should be paid CNY 65,900 if they accept an assignment in Venezuela. Similarly, using the same ratio, an expatriate accepting an assignment in Canada should receive CNY 50,020, while CNY 73,210 should be allocated if the assignment is in Pakistan.

8.3.3. University Reputation Ranking

The evaluation of university reputation as an Multiple Criteria Decision-Making (MCDM) problem involves measuring the performance of each university for every criterion and assessing the preferences of the decision-makers (DMs) for the entire set of criteria. The Grey Regulatory Focus Theory (GRFT) weighting method combined with the GRA is applied, and these results are reported anonymously. It is important to note that these weights are subjective, and we do not endorse or discredit any of the universities under evaluation.

To gather data, two sets of questionnaires were designed to assess the reputation of four universities: Xi'an AAA University (U_1), Xi'an BBB University (U_2), Xi'an CCC University (U_3), and Xi'an DDD University (U_4). The first set of questionnaires was administered online through a dedicated website to collect the perspectives of students regarding the performance of each university. The second set of questionnaires, in PDF format, was designed to obtain the preferences of the DMs for weight estimation. The questionnaire used in this study was adapted from the research by Chen and Esangbedo [104]. It was distributed to 300 students from each of the four universities, with a total of 1200 students. These students were randomly selected, and data collection started on the 31st of October 2018. On average, it took approximately 11 days to collect data from these students across the universities.

Figure 8.4 is the hierarchical model explained in Section 2.5. Figure 8.5 illustrates how the second-level indicator citizenship ($C_{1\text{-}1}$) is measured as a reflective construct, while other second-level indicators are omitted. A zoomed-out view of the formative construct presented in Figure 8.4 is shown in Figure 8.4. Out of the collected data, 51 samples were removed due to unattended responses, resulting in 1,149 usable samples for evaluation. This corresponds to a response rate of 95.75% for the questionnaire. Furthermore, the selection of DMs in this study was biased and primarily based on their years of experience. The four DMs collectively possessed 134 years of work experience, including academic and top managerial roles in the industry.

The raw survey data obtained were transformed into Grey Number (GN) by determining the minimum and maximum values of the average value for each criterion in each university. This transformation was employed because the measurement items for the second-level criterion are reflective constructs. In other words, the measurement items are correlated, and grey numbers were used to capture the range within which the crisp values could fall. The transformation of responses into grey numbers was carried out using Equation (8.42), and the resulting grey decision table is presented in Table 8.6. The grey element in Equation (8.42) is used to construct the grey decision matrix as represented by Equation (8.43):

$$\otimes d_{ij} = \left[\min_{1 \le \delta \le \eta} \overline{C}_{\alpha\text{-}\beta\text{-}\delta}, \max_{1 \le \delta \le \eta} \overline{C}_{\alpha\text{-}\beta\text{-}\delta} \right]. \tag{8.42}$$

Here, α and β represent the first and second-level references of the criteria C, while $\overline{C}$ is the mean of C with the last term η representing the measured variable δ. This equation was used to transform the data into grey numbers for analysis.

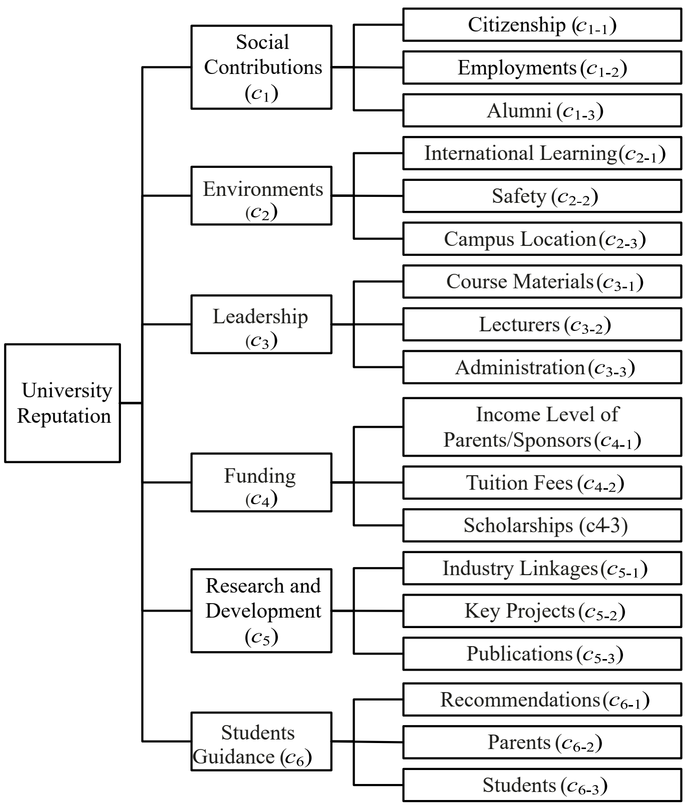


Figure 8.4. Hierarchical diagram for evaluating university reputation. Source: Reprinted from [104], used with permission.

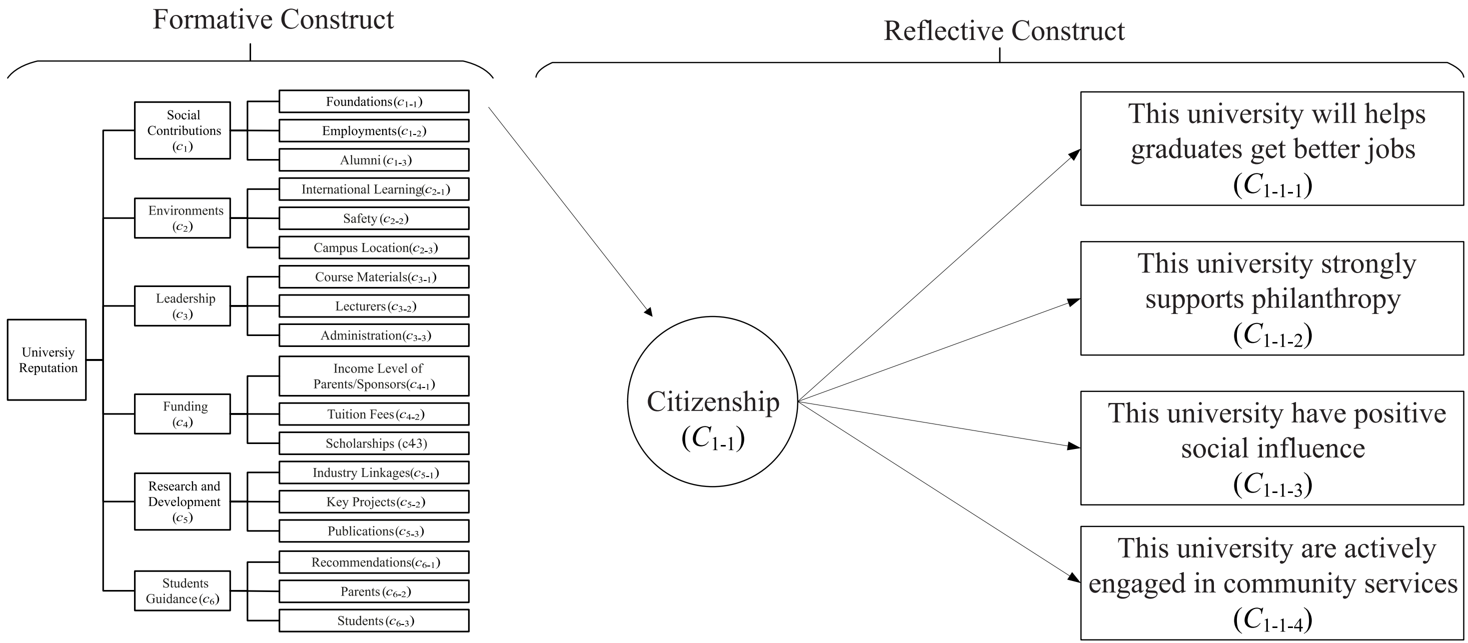


Figure 8.5. Measurement construct. Source: Reprinted from [104], used with permission.

Table 8.6. Grey performances for the universities.

Criteria Index (v)/ Universities	A_1	A_2	A_3	A_4
1	[4.0548, 4.5871]	[1.7762, 2.1434]	[1.669, 2.1725]	[1.8451, 2.3204]
2	[3.2, 4.2677]	[2.0559, 2.7028]	[2.0986, 2.9437]	[2.2746, 2.9507]
3	[4.2839, 4.471]	[1.8497, 2.042]	[1.8275, 2.1127]	[2.0317, 2.3627]
4	[3.9677, 4.7548]	[1.7762, 3.1608]	[1.9472, 3.3521]	[1.5634, 3.5599]
5	[4.1323, 4.4419]	[1.8077, 2.3007]	[1.9014, 2.3415]	[1.9014, 2.1585]
6	[3.4419, 4.2645]	[2.4056, 3.0175]	[2.3592, 2.7852]	[2.3662, 3.1408]
7	[3.8097, 4.0839]	[2.2238, 2.521]	[2.1408, 2.5845]	[2.3204, 2.6092]
8	[3.9839, 4.4161]	[2.1189, 2.3636]	[2.1338, 2.4401]	[2.2077, 2.6092]
9	[3.5355, 4.2065]	[2.2448, 2.6119]	[2.1338, 2.4401]	[2.243, 2.5845]
10	[3.8161, 4.3419]	[1.9685, 2.7657]	[2.1056, 2.7782]	[1.8697, 2.919]
11	[2.4903, 4.2968]	[1.7343, 3.0699]	[1.8345, 3.1831]	[1.7852, 3.2218]
12	[4.2226, 4.529]	[1.7308, 2.2343]	[1.75, 2.1796]	[1.7324, 2.1338]
13	[3.5774, 4.1097]	[2.1434, 2.2762]	[2.2042, 2.4085]	[2.2958, 2.5282]
14	[4.0419, 4.5097]	[2.0699, 2.465]	[2.1338, 2.4859]	[2.2254, 2.507]
15	[4.1903, 4.4452]	[2.0385, 2.2692]	[2.2535, 2.4366]	[2.331, 2.5387]
16	[3.0484, 4.1677]	[2.3531, 3.0559]	[2.4401, 3.2077]	[2.5739, 3.3944]
17	[2.1645, 3.9903]	[2.3497, 3.3741]	[2.5176, 3.6761]	[2.5739, 3.5352]
18	[3.771, 4.0806]	[2.2797, 2.6538]	[2.2782, 2.7746]	[2.4542, 2.9824]

Source: Reprinted from [104], used with permission.

$$D = \begin{pmatrix} \otimes d_{1,1} & \otimes d_{1,2} & \cdots & \otimes d_{1,18} \\ \otimes d_{2,1} & \otimes d_{2,2} & \cdots & \otimes d_{2,18} \\ \otimes d_{3,1} & \otimes d_{3,2} & \cdots & \otimes d_{3,18} \\ \otimes d_{4,1} & \otimes d_{4,2} & \cdots & \otimes d_{4,18} \end{pmatrix}$$

$$= \begin{pmatrix} [4.0548,\ 4.5871] & [3.2,\ 4.2677] & \cdots & [3.771,\ 4.0806] \\ [1.7762,\ 2.1434] & [2.0559,\ 2.7028] & \cdots & [2.2797,\ 2.6538] \\ [1.669,\ 2.1725] & [2.0986,\ 2.9437] & \cdots & [2.2782,\ 2.7746] \\ [1.8451,\ 2.3204] & [2.2746,\ 2.9507] & \cdots & [2.4542,\ 2.9824] \end{pmatrix}. \quad (8.43)$$

A standardized grey matrix, $\otimes D'$, is calculated, with the standardized element $\otimes d'_{ij} = \left[\underline{d'}_{ij}, \overline{d'}_{ij}\right]$.

$$D' = \begin{pmatrix} \otimes d'_{1,1} & \otimes d'_{1,2} & \cdots & \otimes d'_{1,n} \\ \otimes d'_{2,1} & \otimes d'_{2,2} & \cdots & \otimes d'_{2,n} \\ \vdots & \vdots & \ddots & \vdots \\ \otimes d'_{m,1} & \otimes d'_{m,2} & \cdots & \otimes d'_{m,n} \end{pmatrix}. \quad (8.44)$$

In GRA, interval numbers are standardized through a norm based on a minimization of the maximum distance [199]:

$$\left[\underline{d'}_{ij}, \overline{d'}_{ij}\right] = \left[\frac{\underline{d}_{ij}}{\|d_j\|}, \frac{\overline{d}_{ij}}{\|d_j\|}\right], \quad (8.45)$$

where

$$\|d_j\| = \max_{1 \le i \le m} \overline{d}_{ij}. \tag{8.46}$$

Thus, the standardized grey decision matrix based on Equation (8.44) is as follows:

$$D' = \begin{pmatrix} [0.8528,\ 0.9647] & [0.6730, 0.8976] & \cdots & [0.7931, 0.8582] \\ [0.3736, 0.4508] & [0.4324, 0.5684] & \cdots & [0.4795, 0.5581] \\ [0.3510, 0.4569] & [0.4414, 0.6191] & \cdots & [0.4791, 0.5835] \\ [0.3880, 0.4880] & [0.4784, 0.6206] & \cdots & [0.5162, 0.6272] \end{pmatrix}.$$

Next, the weighted standardized decision matrix is computed using Equation (8.47), for which the weights in percentages used are the GRFT weights presented in Equation (4.10):

$$D^* = \begin{pmatrix} \otimes d^*_{1,1} & \otimes d^*_{1,2} & \cdots & \otimes d^*_{1,n} \\ \otimes d^*_{2,1} & \otimes d^*_{2,2} & \cdots & \otimes d^*_{2,n} \\ \vdots & \vdots & \ddots & \vdots \\ \otimes d^*_{m,1} & \otimes d^*_{m,2} & \cdots & \otimes d^*_{m,n} \end{pmatrix}, \tag{8.47}$$

where

$$\otimes\, d^*_{ij} = \otimes d'_{ij} \times \otimes w_{ij}.$$

In vector form, the series can be written as follows:

$$\begin{matrix} D^*_1 = \left\{ \otimes d^*_{1,1}, \otimes d^*_{1,2}, \ldots, \otimes d^*_{1,n} \right\} \\ D^*_2 = \left\{ \otimes d^*_{2,1}, \otimes d^*_{2,2}, \ldots, \otimes d^*_{2,n} \right\} \\ \vdots \\ D^*_m = \left\{ \otimes d^*_{m,1}, \otimes d^*_{m,2}, \ldots, \otimes d^*_{m,n} \right\} \end{matrix},$$

$$D^* = \begin{pmatrix} [4.1275, 5.5376] & [2.5776,\ 4.8558] & \cdots & [2.6965,\ 5.1063] \\ [1.8080,\ 2.5875] & [1.6560, 3.0752] & \cdots & [3.6301,\ 3.3209] \\ [1.6989,\ 2.6226] & [1.6904, 3.3493] & \cdots & [1.6291,\ 3.4720] \\ [1.8782, 2.8012] & [1.8322, 3.3573] & \cdots & [1.7549,\ 3.7321] \end{pmatrix}.$$

Based on Equation (8.48), the weighted grey reference alternative is as follows:

$$D^*_0 = \left\{ \otimes d^*_{01}, \otimes d^*_{02}, \ldots, \otimes d^*_{0n} \right\}, \tag{8.48}$$

where

$$\otimes\, d^*_{0j} = \left[\max_{1 \le i \le m} \underline{d^*_{ij}}, \max_{1 \le i \le m} \overline{d^*_{ij}} \right],$$

$$D^*_0 = \left([4.1275,\ 5.5376], [2.5776,\ 4.8558], \ldots, [2.6965,\ 5.1063] \right).$$

Calculate the differences between the weighted reference alternative and weighted standardized decision matrix. This is measured as the distances between two arbitrary interval numbers, as given in Equation (1.5):

$$\delta_{ij} = \left|\otimes d_{0j}^{*} - \otimes d_{ij}^{*}\right| = \max\left(\left|\underline{d}_{0j}^{*} - \underline{d}_{ij}^{*}\right|, \left|\overline{d_{0j}^{*}} - \overline{d_{ij}^{*}}\right|\right).$$

The difference between the reference university and the evaluated university are computed using Equation (8.31):

$$\Delta = \begin{pmatrix} \delta_{1,1} & \delta_{1,2} & \cdots & \delta_{1,n} \\ \delta_{2,1} & \delta_{2,2} & \cdots & \delta_{2,n} \\ \vdots & \vdots & \ddots & \vdots \\ \delta_{m,1} & \delta_{m,2} & \cdots & \delta_{m,n} \end{pmatrix} = \begin{pmatrix} 0 & 0 & \cdots & 0 \\ 2.9500 & 1.7805 & \cdots & 0.6324 \\ 2.9149 & 1.5064 & \cdots & 0.3225 \\ 2.7364 & 1.4985 & \cdots & 0.4671 \end{pmatrix}.$$

The Grey Relational Grade is calculated using Equation (3.9):

$$\begin{aligned} r_i &= r_1, r_2, r_3, r_4 \\ r_i &= 0.9941, 0.4713, 0.4857, 0.4965 \end{aligned}\ ,$$

i.e., $r_1 > r_4 > r_3 > r_2$. This implies that Xi'an AAA University is ranked in the first position, Xi'an DDD University is ranked in the second position, Xi'an CCC University is ranked in the third position, and Xi'an BBB University is ranked in the forth position, i.e., $U_1 \succ U_4 \succ U_3 \succ U_2$. Xi'an AAA University is ranked the most reputable university from the perspective of students and the subjective weight of the Decision Makers (DMs). The most reputable university is consistent with Academic Ranking of World Universities (ARWU) [293], Centre for Science and Technology Studies (CWTS) Leiden Ranking [294], Performance Ranking of Scientific Papers for World Universities (PRSPWUN) [295], Quacquarelli Symonds (QS) [296], Times Higher Education (THE) [297], and University Rankings based on Academic Performance (URAP) [298].

Classical GRA has found application in various MCDM problems and was designed to evaluate crisp values, as shown in Figure 8.6a. GRA has been extended to incorporate the use of grey numbers, as depicted in Figure 8.6b. However, there is a problem with using a single reference alternative with grey numbers. For example, consider two alternatives, $\otimes A$ and $\otimes B$, evaluated with a reference alternative, represented as a green point in Figure 8.6b. From Figure 8.6b, we might assume that alternative $\otimes A$ is better than alternative $\otimes B$ based on a single point of reference. Surprisingly, when a second reference point is introduced, shown as a red point in Figure 8.6c, it becomes evident that alternative $\otimes B$ is better than $\otimes A$ since it is further from the negative reference alternative. To address this limitation, Esangbedo et al. [260] introduced bi-reference GRA, also known as GRA with positive and negative references.[1]

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[1] This extension is mentioned for the sake of completeness.

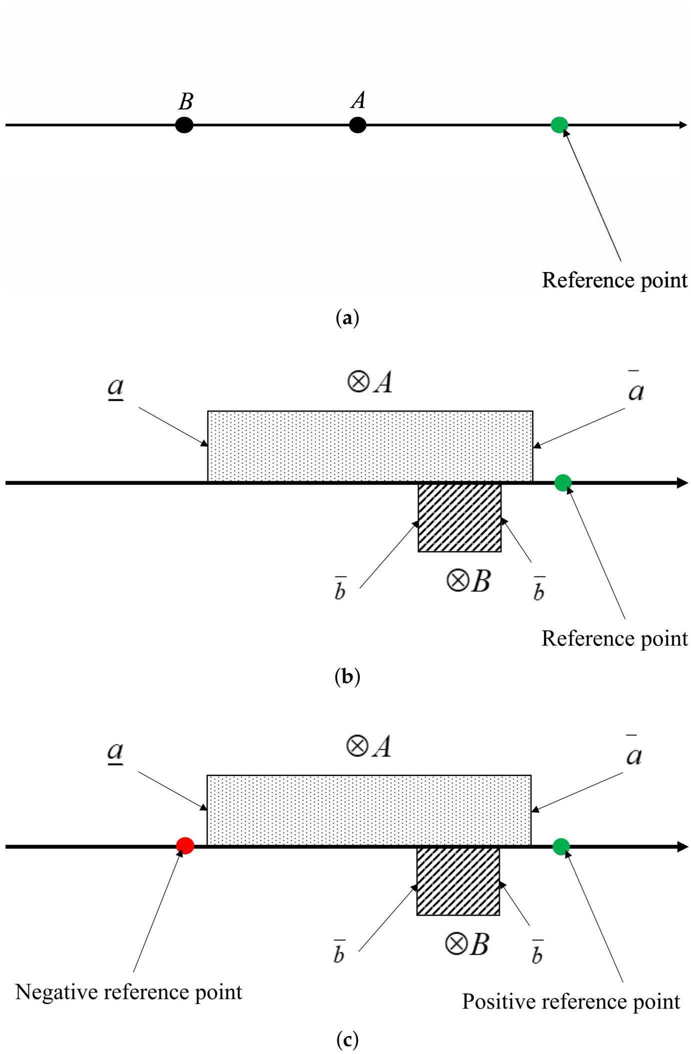

Figure 8.6. GRA with positive and negative reference in comparison with GRA with internval grey numbers. (**a**) Classical GRA; (**b**) GRA with GNs; (**c**) improved GRA. Source: Reprinted from [260], used with permission.

9. The Grey REGIME Method

In this chapter, the Grey REGIME (grey system-based regime) method is introduced, which extends the traditional regime approach by incorporating the principles of Grey Systems Theory (GST) to address uncertainties in the evaluation of Human Resource Information System (HRIS). The focus is on HRIS offerings from five different vendors, and the grey-point allocation-full-consistency (grey-PA-FUCOM) weighting method introduced in Chapter 6 is employed for the weighting process. Both the grey-PA-FUCOM and the hybrid Grey REGIME Multiple Criteria Decision-Making (MCDM) methods are designed to handle ordinal data as inputs. The objective is to rank the five HRIS vendors based on their performance, and Figure 9.1 provides a visual representation of how this section aligns with the overall structure of this book.

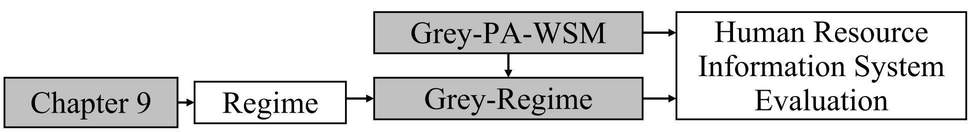

Figure 9.1. Flowchart of Chapter 9. Source: Figure by authors.

9.1. The Regime Method

The regime method, initially developed by Hinloopen and et al. in 1983 [299], offers an MCDM (multiple criteria decision-making) approach for evaluating alternatives when dealing with ordinal data and qualitatively establishes relative impacts. This method involves several key steps, starting with the construction of a regime matrix, which entails a pairwise comparison of alternatives from the impact matrix. The impact matrix, in turn, represents the measurement of the effect of each alternative on the evaluation criteria using ordinal values. Ultimately, the regime method helps determine the best alternative based on these pairwise comparisons. An application of the regime method can be found in the work of Alinezhad and Khalili [300]. The following are the steps involved in the regime method.

Step 1. Construct the decision matrix, denoted as X. The decision matrix is expressed as Equation (9.1):

$$X = \begin{pmatrix} r_{1,1} & r_{1,2} & \cdots & r_{1,n} \\ r_{2,1} & r_{2,2} & \cdots & r_{2,n} \\ \vdots & \vdots & \ddots & \vdots \\ r_{m,1} & r_{m,2} & \cdots & r_{m,n} \end{pmatrix}, \tag{9.1}$$

where i ranges from 1 to m, representing the element decision matrix for m alternatives, while j ranges from 1 to n, representing n criteria.

Step 2. Compute the superiority index, $\tilde{E}_{fl}$. These are the criteria where alternative A_f is better than alternative A_l. Since the performance values of the alternatives are uncertain and are represented using numbers, there are situations where alternative A_f may be better or worse than alternative A_l.

Step 3. Determine the superiority identifier, $\widehat{E}_{fl}$. This corresponds to the weights for the superiority index, as given in Equation (9.2):

$$\widehat{E}_{fl} = \sum_{\substack{j=1 \\ j\in\tilde{E}_{fl}}}^{n} w_j. \tag{9.2}$$

Step 4. Construct the impact matrix. This is achieved by determining the ordinal values of the alternatives. Since the performance values are represented as numbers, the ordinal values are determined using the possibilities, as given in Equation (9.10).

Step 5. Construct the regime matrix. The regime matrix is created using the elements defined in Equation (9.3):

$$E_{fl,j} = \begin{cases} -1 & \text{if} \quad r_{f,j} < r_{l,j} \\ 0 & \text{if} \quad r_{f,j} = r_{l,j}. \\ +1 & \text{if} \quad r_{f,j} > r_{l,j} \end{cases} \tag{9.3}$$

where $j = 1 \ldots n$.

Step 6. Determine the guide index, E'_{fl}. This represents the weighted regime matrix, and its elements are computed using Equation (9.4):

$$E'_{fl} = \sum_{j=1}^{n} w_j E_{fl,j}; j = 1...n. \tag{9.4}$$

Step 7. Sort and rank. In this step, the alternatives are arranged in sequential order from the highest possibility to the lowest. The alternative with the highest superiority possibility is considered the best.

9.2. Grey REGIME Methods

The Grey REGIME evaluation methods employed share certain initial commonalities, including the collection of raw data from the decision-makers (DMs) and the creation of a grey decision table. This process begins with the use of a scoresheet or questionnaire to gather raw data from the DMs. Subsequently, a grey decision table is constructed, as illustrated in Table 9.1, where performance values are represented as grey numbers obtained using Equation (9.5):

$$\otimes d_{i,j} = \left[\underline{d}_{i,j}, \overline{d}_{i,j}\right] = \left[\min_{1\le v\le\vartheta} d_{ij}(v), \max_{1\le v\le\vartheta} d_{ij}(v)\right]. \tag{9.5}$$

Here, $\underline{d}_{i,j}$ and $\overline{d}_{i,j}$ correspond to the upper and lower bounds of the elements in the decision matrix for m alternatives and n criteria.

Table 9.1. Grey decision table.

Criteria/Alternatives	A_1	...	A_i	...	A_m
c_1	$\otimes d_{1,1}$	$\cdots$	$\otimes d_{i1}$	$\cdots$	$\otimes d_{m1}$
$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\ddots$	$\vdots$
c_j	$\otimes d_{1j}$	$\cdots$	$\otimes d_{ij}$	$\cdots$	$\otimes d_{mj}$
$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\ddots$	$\vdots$
c_n	$\otimes d_{1n}$	$\cdots$	$\otimes d_{i,n}$	$\cdots$	$\otimes d_{mn}$

Source: Reprinted from [103], used with permission.

In a stepwise fashion, the Grey REGIME method extends the regime method by incorporating grey systems theory to address uncertainties in both the measurement of performance values for alternatives and pairwise comparisons. The following are the steps of the Grey REGIME method.

Step 1. Construct the grey decision matrix, D. The grey decision matrix is directly obtained from Table 9.1 and is represented as Equation (9.6):

$$D = \begin{pmatrix} \otimes d_{1,1} & \otimes d_{1,2} & \cdots & \otimes d_{1,n} \\ \otimes d_{2,1} & \otimes d_{2,2} & \cdots & \otimes d_{2,n} \\ \vdots & \vdots & \ddots & \vdots \\ \otimes d_{m,1} & \otimes d_{m,2} & \cdots & \otimes d_{m,n} \end{pmatrix}, \tag{9.6}$$

where $\otimes d_{i,j} = \left[\underline{d}_{i,j}, \overline{d}_{i,j}\right] = \left[\min_{1\le v\le \vartheta} d_{ij}(v), \max_{1\le v\le \vartheta} d_{ij}(v)\right]$. $\underline{d}_{i,j}$ and $\overline{d}_{i,j}$ correspond to the upper and lower bounds of the decision matrix elements for m alternatives and n criteria.

Step 2. Normalize the grey decision matrix. The purpose of the normalization of D is to obtain the normalized grey decision matrix, $\widehat{D}$. Normalize the benefit and cost preference scores using Equations (9.7) and (9.8), respectively:

$$\otimes d^*_{ij} = \left[\frac{\underline{d}_{ij} - \min_{1\le i\le m} \underline{d}_{ij}}{\max_{1\le i\le m} \overline{d}_{ij} - \min_{1\le i\le m} \underline{d}_{ij}}, \frac{\overline{d}_{ij} - \min_{1\le i\le m} \underline{d}_{ij}}{\max_{1\le i\le m} \overline{d}_{ij} - \min_{1\le i\le m} \underline{d}_{ij}}\right], \tag{9.7}$$

$$\otimes d^*_{ij} = \left[\frac{\max_{1\le i\le m} \overline{d}_{ij} - \overline{d}_{ij}}{\max_{1\le i\le m} \overline{d}_{ij} - \min_{1\le i\le m} \underline{d}_{ij}}, \frac{\max_{1\le i\le m} \overline{d}_{ij} - \underline{d}_{ij}}{\max_{1\le i\le m} \overline{d}_{ij} - \min_{1\le i\le m} \underline{d}_{ij}}\right]. \tag{9.8}$$

The normalized decision matrix is given in Equation (9.9):

$$\widehat{D} = \begin{pmatrix} \otimes \widehat{d}_{1,1} & \otimes \widehat{d}_{1,2} & \cdots & \otimes \widehat{d}_{1,n} \\ \otimes \widehat{d}_{2,1} & \otimes \widehat{d}_{2,2} & \cdots & \otimes \widehat{d}_{2,n} \\ \vdots & \vdots & \ddots & \vdots \\ \otimes \widehat{d}_{m,1} & \otimes \widehat{d}_{m,2} & \cdots & \otimes \widehat{d}_{m,n} \end{pmatrix}. \tag{9.9}$$

Step 3. Compute the superiority index, $\tilde{E}_{fl}$. This represents the criteria where alternative A_f is better than alternative A_l. Since the performance values of the alternatives are uncertain and are represented using grey numbers, there are various possibilities where alternative A_f may be better or worse than alternative A_l.

Now, we denote the possibilities of $f < l$, $f > l$, and $f = l$ as $p(f < l)$, $p(f > l)$, and $p(f = l)$. Then, for grey numbers, the possibility of $\otimes\alpha$ being superior to $\otimes\beta$, i.e., $\otimes\alpha > \otimes\beta$, is defined as $\otimes\alpha > \otimes\beta$, which is defined as $p(\otimes\alpha \succ \otimes\beta)$ in Equation (9.10):

$$p(\otimes\alpha \succ \otimes\beta) = \begin{cases} 1 & \underline{\alpha} \geq \overline{\alpha} \geq \underline{\beta} \geq \overline{\beta} \\ \frac{\underline{\beta}-\underline{\alpha}}{\overline{\alpha}-\underline{\alpha}+1} + \frac{\overline{\alpha}-\underline{\beta}+1}{\overline{\alpha}-\underline{\alpha}+1}\left(0.5\frac{\overline{\alpha}-\underline{\beta}+1}{\overline{\beta}-\underline{\beta}+1} + \frac{\overline{\beta}-\overline{\alpha}}{\overline{\beta}-\underline{\beta}+1}\right) & \underline{\alpha} \geq \underline{\beta} \geq \overline{\alpha} \geq \overline{\beta} \\ \frac{\underline{\beta}-\underline{\alpha}}{\overline{\beta}-\underline{\alpha}+1} + 0.5\frac{\overline{\beta}-\underline{\alpha}+1}{\overline{\beta}-\underline{\alpha}+1} & \underline{\alpha} \geq \underline{\beta} \geq \overline{\beta} \geq \overline{\alpha} \\ 0 & \underline{\beta} \geq \overline{\beta} \geq \underline{\alpha} \geq \overline{\alpha} \\ 0.5\frac{\overline{\beta}-\underline{\alpha}+1}{\overline{\alpha}-\underline{\alpha}+1}\frac{\overline{\beta}-\underline{\alpha}+1}{\overline{\beta}-\underline{\beta}+1} & \underline{\beta} \geq \underline{a} \geq \overline{a} \geq \overline{\beta} \\ 0.5\frac{\overline{\beta}-\underline{\alpha}+1}{\overline{\beta}-\underline{\beta}+1}\frac{\overline{\beta}-\overline{\alpha}}{\overline{\beta}-\underline{\beta}+1} & \underline{\beta} \geq \underline{a} \geq \overline{a} \geq \overline{\beta} \\ 0.5 & \underline{a} = \overline{a} = \underline{\beta} = \overline{\beta} \end{cases}. \tag{9.10}$$

Step 4. Determine the superiority identifier, $\otimes\widehat{E}_{fl}$. This represents the grey weights for the superiority index and is calculated as shown in Equation (9.11):

$$\otimes\widehat{E}_{fl} = \sum_{\substack{j=1 \\ j\in\tilde{E}_{fl}}}^{n} \otimes w_j. \tag{9.11}$$

Step 5. Construct the impact matrix. This is obtained by determining the ordinal values of the alternatives. Since the performance values are represented using grey numbers, the ordinal values are determined using the grey possibilities given in Equation (9.10).

Step 6. Construct the regime matrix. The regime matrix is built using the elements in Equation (9.12):

$$E_{fl,j} = \begin{cases} -1 & \text{if} \quad r_{f,j} < r_{l,j} \quad \text{iff} \quad p(\otimes\widehat{d}_{fl} \prec \otimes\widehat{d}_{fj}) < 0.5 \\ 0 & \text{if} \quad r_{f,j} = r_{l,j} \quad \text{iff} \quad p(\otimes\widehat{d}_{fl} = \otimes\widehat{d}_{fl}) = 0.5 \\ +1 & \text{if} \quad r_{f,j} > r_{l,j} \quad \text{iff} \quad p(\otimes\widehat{d}_{fl} \succ \otimes\widehat{d}_{fl}) > 0.5 \end{cases}, \tag{9.12}$$

where $j = 1 \ldots n$.

Step 7. Determine the grey guide index, $\otimes E'_{fl}$. This is the weighted regime matrix, and its elements are computed using Equation (9.13):

$$\otimes E'_{fl} = \sum_{j=1}^{n} \otimes w_j E_{fl,j}; j = 1...n. \tag{9.13}$$

Step 8. Sort and rank the alternatives. In this step, the alternatives are arranged sequentially from the highest possibility to the lowest. The alternative with the largest superior possibility is considered the best.

9.3. Application of Grey REGIME Method for HRIS Evaluation

An oil and gas company in China, which employs over 7000 individuals, sought to enhance its global HRM processes by automating them. To maintain confidentiality, specific details about the company are withheld. Criteria for evaluation were chosen based on the existing literature, collective work experiences, and the company's specific requirements. Weights were assigned to these criteria using the PA method, as outlined in Section 6.1. Subsequently, the global HR director and select HR managers conducted background checks on various HRIS providers. Five HRIS vendors were invited to present their solutions, including quick software demonstrations. The DMs had the opportunity to interact with the software, ask questions, and evaluate the vendors based on predefined criteria. The scores assigned by the DMs were then used to rank the vendors. Customized questionnaires were developed for each DM to collect comparative data, which were utilized in the FUCOM method. Figure 9.2 illustrates the flowchart outlining the evaluation process.

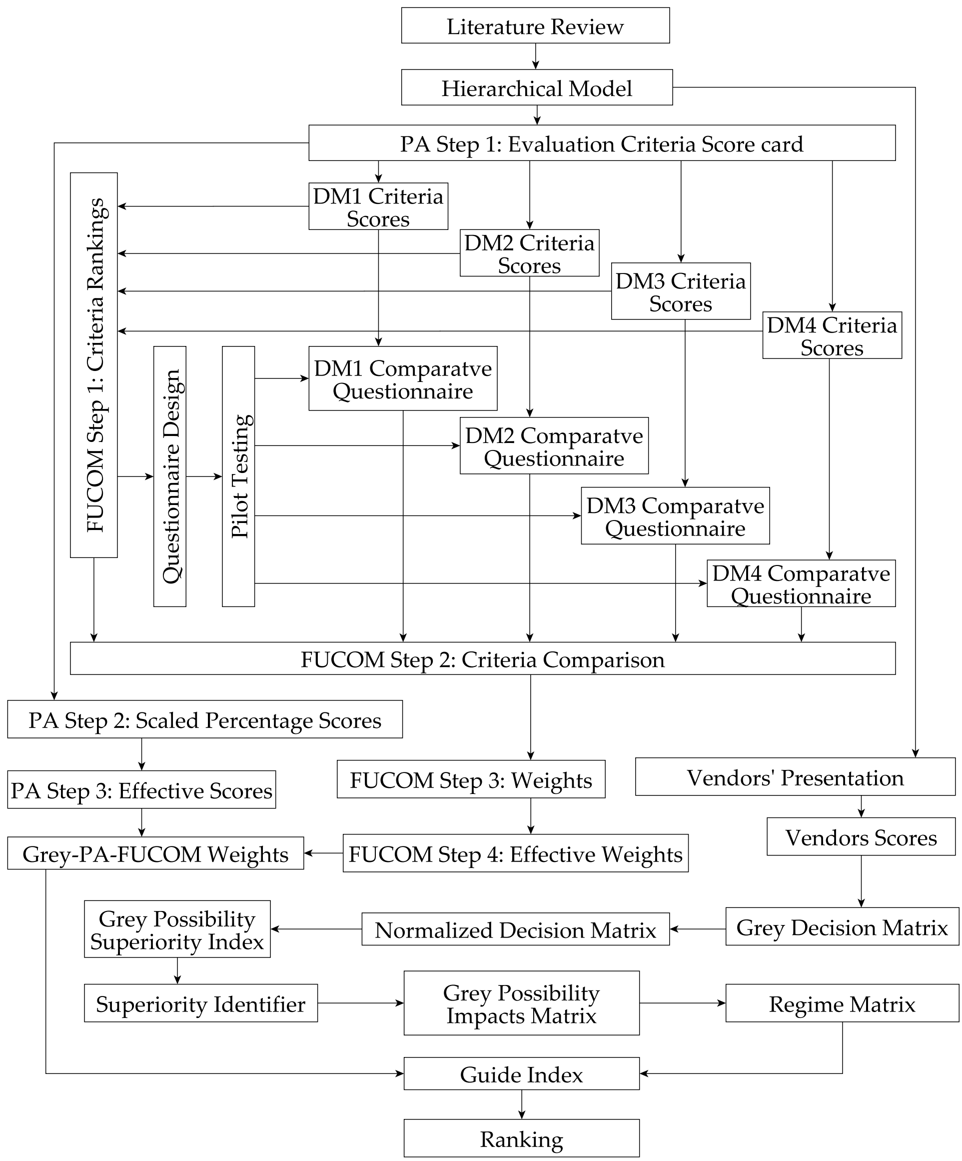

Figure 9.2. Flowchart of HRIS evaluation. Source: Reprinted from [103], used with permission.

Once the criteria weights were determined, the evaluation of the five HRIS solutions provided by the vendors was carried out using the Grey REGIME methods, as detailed in Section 9.2.

To begin, the raw data for the alternatives were obtained from the decision-makers (DMs) using scoresheets. The grey performance values for the alternatives were then calculated based on Equation (9.5) and are presented in Table 9.2. The evaluation of the HRIS solutions provided by the five vendors followed the steps outlined in Section 9.2.

Step 1. Formulate the grey decision matrix. For example, the ratings of the four DMs for the first vendor (A_1) regarding the staff information management criterion ($C_{1\text{-}1}$) were 59, 65, 75, and 60, respectively. These ratings were converted into a grey preference $\otimes d_{1,1} = [59, 75]$. Similarly, the ratings given by the four DMs for the fifth vendor (A_2) for the software delivery & service response time criterion ($C_{5\text{-}5}$) were 70, 80, 85, and 70, respectively, resulting in a grey preference of $\otimes d_{5,27} = [70, 85]$. The other grey preferences are provided in Table 9.2, and they were used to construct the grey decision matrix based on Equation (9.6). The grey decision matrix is presented in Equation (9.14).

$$D = \begin{pmatrix} [59,75] & [60,70] & \dots & [70,85] \\ [60,80] & [65,90] & \dots & [70,80] \\ [80,90] & [60,85] & \dots & [70,90] \\ [70,95] & [65,90] & \dots & [80,90] \\ [60,85] & [70,80] & \dots & [70,85] \end{pmatrix}. \quad (9.14)$$

Here, $\otimes dij = \left[\underline{d}ij, \overline{d}ij\right] = \left[\min_{1 \le v \le 4}, dij(v), \max_{1 \le v \le 4}, d_{ij}(v)\right].$

Step 2. Normalize the grey decision matrix. The normalized grey decision matrix was computed using Equation (9.9):

$$\hat{D} = \begin{pmatrix} [0.1429, 0.6556] & [0.2857, 0.6667] & \dots & [0.5714, 0.8182] \\ [0.4545, 0.6667] & [0.7222, 0.9091] & \dots & [0.5714, 0.8571] \\ [0.7273, 0.8889] & [0.2857, 0.8333] & \dots & [0.5714, 0.9091] \\ [0.5714, 1.0000] & [0.5714, 0.8571] & \dots & [0.5714, 0.9091] \\ [0.2857, 0.7143] & [0.4286, 0.8333] & \dots & [0.5714, 0.8182] \end{pmatrix}, \quad (9.15)$$

where $= \left[\min_{1 \le v \le 4} \widehat{d}_{ij}(v), \max_{1 \le v \le 4} \widehat{d}_{ij}(v)\right].$

Step 3. Compute the superiority index, $\tilde{E}_{fl}$. This index is based on the grey superior possibility defined in Equation (9.10):

$$\begin{aligned}
\tilde{E}_{12} &= C_{1\text{-}4}, C_{1\text{-}6}, C_{2\text{-}5}, C_{3\text{-}5}, C_{3\text{-}6}, C_{4\text{-}5}, C_{5\text{-}2} \\
\tilde{E}_{13} &= C_{3\text{-}4}, C_{5\text{-}2} \\
\tilde{E}_{14} &= C_{3\text{-}5}, C_{3\text{-}6}, C_{5\text{-}2}, C_{5\text{-}4} \\
\tilde{E}_{15} &= C_{3\text{-}3}, C_{3\text{-}4}, C_{3\text{-}5}, C_{3\text{-}6}, C_{4\text{-}3}, C_{5\text{-}2} \\
&\vdots \\
\tilde{E}_{54} &= C_{1\text{-}3}, C_{1\text{-}4}, C_{1\text{-}5}, C_{1\text{-}6}, C_{2\text{-}1}, C_{2\text{-}4}, C_{3\text{-}2}, C_{3\text{-}5}, C_{4\text{-}1}, C_{4\text{-}2}, C_{4\text{-}5}, C_{5\text{-}1}, C_{5\text{-}4}.
\end{aligned}$$

Table 9.2. Grey decision table for HRIS evaluation.

Criteria/ Alternatives	Index (j)	A_1	A_2	A_3	A_4	A_5	W
$\Phi_{1\text{-}1}$	1	[59, 75]	[60, 80]	[80, 90]	[70, 95]	[60, 85]	[0.0069, 0.1248]
$\Phi_{1\text{-}2}$	2	[60, 70]	[65, 90]	[60, 85]	[65, 90]	[70, 80]	[0.0129, 0.0607]
$\Phi_{1\text{-}3}$	3	[58, 80]	[70, 85]	[70, 95]	[60, 86]	[60, 90]	[0.0129, 0.0556]
$\Phi_{1\text{-}4}$	4	[60, 70]	[60, 60]	[70, 90]	[70, 85]	[65, 90]	[0.0129, 0.0556]
$\Phi_{1\text{-}5}$	5	[60, 75]	[60, 80]	[65, 90]	[60, 75]	[60, 90]	[0.0087, 0.0556]
$\Phi_{1\text{-}6}$	6	[60, 70]	[58, 70]	[67, 90]	[68, 80]	[60, 85]	[0.0079, 0.0644]
$\Phi_{2\text{-}1}$	7	[0, 60]	[0, 60]	[60, 70]	[58, 70]	[80, 80]	[0.0056, 0.0275]
$\Phi_{2\text{-}2}$	8	[0, 60]	[0, 60]	[80, 80]	[70, 90]	[0, 60]	[0.0045, 0.0275]
$\Phi_{2\text{-}3}$	9	[0, 60]	[0, 60]	[0, 60]	[60, 70]	[0, 60]	[0.0067, 0.1039]
$\Phi_{2\text{-}4}$	10	[60, 80]	[60, 80]	[60, 75]	[60, 75]	[60, 80]	[0.0069, 0.0192]
$\Phi_{2\text{-}5}$	11	[55, 85]	[60, 80]	[70, 90]	[60, 80]	[60, 80]	[0.0028, 0.0275]
$\Phi_{3\text{-}1}$	12	[60, 80]	[60, 90]	[75, 80]	[70, 85]	[60, 85]	[0.0026, 0.0607]
$\Phi_{3\text{-}2}$	13	[60, 85]	[60, 90]	[70, 85]	[60, 85]	[70, 90]	[0.0026, 0.017]
$\Phi_{3\text{-}3}$	14	[60, 80]	[70, 85]	[60, 90]	[60, 90]	[65, 70]	[0.0036, 0.0168]
$\Phi_{3\text{-}4}$	15	[60, 85]	[70, 85]	[60, 85]	[60, 90]	[65, 80]	[0.0026, 0.0179]
$\Phi_{3\text{-}5}$	16	[70, 90]	[60, 85]	[75, 85]	[60, 80]	[60, 85]	[0.0083, 0.0202]
$\Phi_{3\text{-}6}$	17	[70, 95]	[0, 70]	[75, 90]	[70, 90]	[60, 80]	[0.0023, 0.0303]
$\Phi_{4\text{-}1}$	18	[60, 80]	[60, 80]	[70, 90]	[60, 85]	[70, 80]	[0.0033, 0.0206]
$\Phi_{4\text{-}2}$	19	[50, 90]	[50, 90]	[65, 85]	[60, 85]	[60, 80]	[0.0015, 0.0185]
$\Phi_{4\text{-}3}$	20	[60, 90]	[60, 90]	[65, 85]	[60, 90]	[60, 80]	[0.0018, 0.0234]
$\Phi_{4\text{-}5}$	21	[60, 85]	[50, 90]	[70, 85]	[65, 80]	[65, 80]	[0.0018, 0.0263]
$\Phi_{4\text{-}5}$	22	[70, 90]	[70, 90]	[65, 95]	[70, 95]	[70, 90]	[0.0018, 0.0192]
$\Phi_{5\text{-}1}$	23	[70, 85]	[70, 85]	[80, 90]	[75, 85]	[85, 90]	[0.0006, 0.0229]
$\Phi_{5\text{-}2}$	24	[70, 90]	[0, 70]	[65, 80]	[70, 85]	[0, 60]	[0.0056, 0.0213]
$\Phi_{5\text{-}3}$	25	[60, 80]	[60, 80]	[70, 90]	[70, 90]	[70, 85]	[0.0056, 0.0229]
$\Phi_{5\text{-}4}$	26	[70, 85]	[70, 90]	[70, 85]	[60, 80]	[60, 90]	[0.0023, 0.0213]
$\Phi_{5\text{-}5}$	27	[70, 85]	[70, 80]	[70, 90]	[80, 90]	[70, 85]	[0.0006, 0.0188]

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Step 4. Determine the superiority identifier, $\overset{\frown}{E}_{fl}$. This identifier is calculated based on the computed superiority index, $\tilde{E}_{fl}$.

$$
\begin{aligned}
&\overset{\frown}{E}_{12} = [0.0805, 0.2369]\ \overset{\frown}{E}_{13} = [0.0383, 0.0405]\ \overset{\frown}{E}_{14} = [0.0431, 0.1123] \\
&\overset{\frown}{E}_{15} = [0.0531, 0.1333]\ \overset{\frown}{E}_{21} = [0.0962, 0.4304]\ \overset{\frown}{E}_{23} = [0.0386, 0.1359] \\
&\overset{\frown}{E}_{24} = [0.0654, 0.2236]\ \overset{\frown}{E}_{25} = [0.0768, 0.2777]\ \overset{\frown}{E}_{31} = [0.1231, 0.7237] \\
&\overset{\frown}{E}_{32} = [0.1073, 0.718]\ \overset{\frown}{E}_{34} = [0.1054, 0.5881]\ \overset{\frown}{E}_{35} = [0.107, 0.7121] \\
&\overset{\frown}{E}_{41} = [0.1102, 0.7383]\ \overset{\frown}{E}_{42} = [0.0877, 0.6652]\ \overset{\frown}{E}_{43} = [0.0552, 0.2418] \\
&\overset{\frown}{E}_{45} = [0.0844, 0.6311]\ \overset{\frown}{E}_{51} = [0.1117, 0.5883]\ \overset{\frown}{E}_{52} = [0.0913, 0.4867] \\
&\overset{\frown}{E}_{53} = [0.0397, 0.1473]\ \overset{\frown}{E}_{54} = [0.0687, 0.2599].
\end{aligned}
$$

Step 5. Construct the impact matrix, I:

$$
I = \begin{array}{c} \\ \\ \\ \\ \\ \end{array}
\begin{array}{cccccc} c_1 & c_2 & c_3 & c_4 & \dots & c_{27} \end{array}
$$
$$
I = \begin{pmatrix} 5 & 5 & 5 & 4 & \dots & 3 \\ 4 & 1 & 2 & 5 & \dots & 5 \\ 1 & 4 & 1 & 1 & \dots & 2 \\ 2 & 1 & 4 & 2 & \dots & 1 \\ 3 & 3 & 3 & 3 & \dots & 3 \end{pmatrix} \begin{matrix} A_1 \\ A_2 \\ A_3 \\ A_4 \\ A_5 \end{matrix}. \tag{9.16}
$$

Step 6. Construct the regime matrix (R), as shown in Equation (9.3):

$$\begin{aligned} E_{1\text{-}2} &= -1, -1, -1, 1, \cdots - 1, \\ E_{1\text{-}3} &= -1, -1, -1, -1, \cdots - 1, \\ E_{1\text{-}4} &= -1, -1, -1, -1, \cdots - 1, \\ E_{1\text{-}5} &= -1, -1, -1, -1, \ldots 1, \\ &\vdots \\ E_{5\text{-}4} &= -1, -1, 1, 1, \cdots - 1 \end{aligned}$$

which is

$$R = \begin{array}{c} \begin{matrix} c_1 & c_2 & c_3 & c_4 & \ldots & c_{27} \end{matrix} \\ \begin{pmatrix} -1 & -1 & -1 & 1 & \ldots & -1 \\ -1 & -1 & -1 & -1 & \ldots & -1 \\ -1 & -1 & -1 & -1 & \ldots & -1 \\ -1 & -1 & -1 & -1 & \ldots & 1 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ -1 & -1 & 1 & 1 & \ldots & -1 \end{pmatrix} \end{array} \begin{matrix} E_{1\text{-}2} \\ E_{1\text{-}3} \\ E_{1\text{-}4} \\ E_{1\text{-}5} \\ \vdots \\ E_{5\text{-}4} \end{matrix} . \quad (9.17)$$

Step 7. Calculate the grey guiding index:
$\otimes E'_{1\text{-}2} = [0.0254, 0.1396]$, $\otimes E'_{1\text{-}3} = [-0.6332, -0.0874]$,
$\otimes E'_{1\text{-}4} = [-0.4762, -0.0494]$, $\otimes E'_{1\text{-}5} = [-0.1762, -0.0294]$,
$\otimes E'_{2\text{-}1} = [0.0512, 0.4716]$, $\otimes E'_{2\text{-}3} = [-0.5208, -0.0656]$,
$\otimes E'_{2\text{-}4} = [-0.33, -0.0048]$, $\otimes E'_{2\text{-}5} = [-0.0134, 0.0014]$,
$\otimes E'_{3\text{-}1} = [0.1106, 0.9194]$, $\otimes E'_{3\text{-}2} = [0.079, 0.7286]$,
$\otimes E'_{3\text{-}4} = [0.0752, 0.5168]$, $\otimes E'_{3\text{-}5} = [0.0784, 0.7058]$,
$\otimes E'_{4\text{-}1} = [0.0756, 0.6682]$, $\otimes E'_{4\text{-}2} = [0.0398, 0.5532]$,
$\otimes E'_{4\text{-}3} = [-0.344, -0.034]$, $\otimes E'_{4\text{-}5} = [0.0332, 0.4806]$,
$\otimes E'_{5\text{-}1} = [0.0704, 0.5534]$, $\otimes E'_{5\text{-}2} = [0.044, 0.408]$,
$\otimes E'_{5\text{-}3} = [-0.498, -0.065]$, $\otimes E'_{5\text{-}4} = [-0.373, -0.024]$.

Step 8. Sort and rank the alternatives. In this step, the HRISs are ranked based on their cumulative grey guiding indexes. The cumulative grey guiding index represents the overall evaluation of each HRIS, considering all the criteria and their respective weights. The alternatives are sorted and ranked in descending order based on their cumulative grey guiding index. The HRIS with the highest cumulative grey guiding index is considered the best choice. $E_{1\text{-}i} = [-0.1408, -1.146]$, $E_{2\text{-}i} = [-0.0178, -0.3926]$, $E_{3\text{-}i} = [0.3432, 2.8706]$, $E_{4\text{-}i} = [0.1146, 1.358]$, and $E_{5\text{-}i} = [0.0254, 0.0904]$. A plot of the cumulative grey guiding index for the five HRISs is given in Figure 9.3, and the third alternative (A_3) is the best, i.e., $A_3 > A_4 > A_5 > A_2 > A_1$. Although the fifth vendor (A_5) exhibited the least uncertainty, indicated by the shortest bar in Figure 9.3, the lower bound of the best vendor (A_3) exceeded A_5's upper bound.

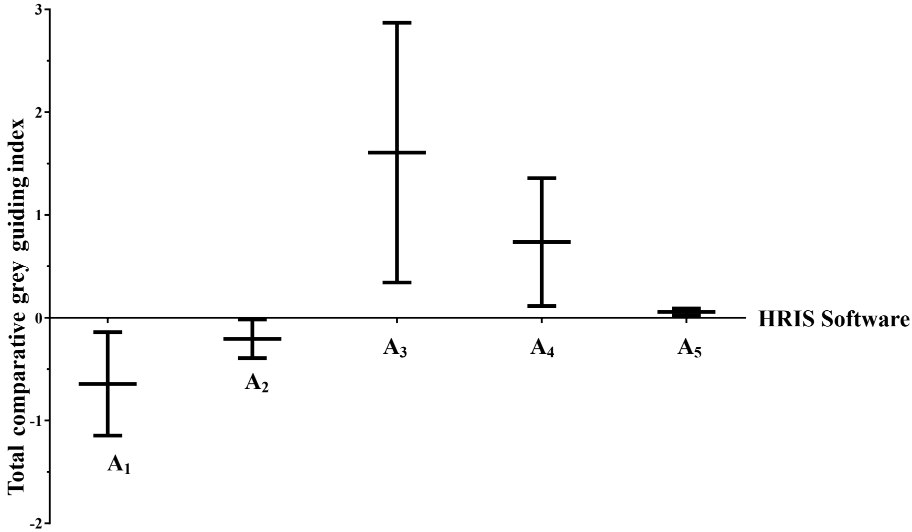


Figure 9.3. Cumulative grey guiding index of alternatives. Source: Reprinted from [103], used with permission.

9.4. HRIS Confirmatory Ranking-Based GRA with Grey Numbers

The second evaluation involved the Grey Relational Analysis (GRA) steps presented in Section 8.2.2 and was applied in evaluating the five HRIS as follows.

Step 1. Construct the grey decision matrix. The grey decision matrix was formulated using Equation (9.6), which was derived from Table 9.2.

$$D = \begin{pmatrix} [0.1429, 0.6556] & [0.2857, 0.6667] & \dots & [0.5714, 0.8182] \\ [0.4545, 0.6667] & [0.7222, 0.9091] & \dots & [0.5714, 0.8571] \\ [0.7273, 0.8889] & [0.2857, 0.8333] & \dots & [0.5714, 0.9091] \\ [0.5714, 1] & [0.5714, 0.8571] & \dots & [0.5714, 0.9091] \\ [0.2857, 0.7143] & [0.4286, 0.8333] & \dots & [0.5714, 0.8182] \end{pmatrix}. \quad (9.18)$$

Step 2. Normalize the grey decision matrix. The grey normalized decision was computed using Equation (9.9), i.e.,

$$\widehat{D} = \begin{pmatrix} [0, 0.5981] & [0, 0.6111] & \dots & [0, 0.7308] \\ [0.3636, 0.6111] & [0.7002, 1] & \dots & [0, 0.8462] \\ [0.6818, 0.8704] & [0, 0.8785] & \dots & [0, 1] \\ [0.5, 1] & [0.4583, 0.9167] & \dots & [0, 1] \\ [0.1667, 0.6667] & [0.2292, 0.8785] & \dots & [0, 0.7308] \end{pmatrix}. \quad (9.19)$$

Step 3. Compute the weighted decision matrix. For weights in Equation (6.21), using Equation (11.7), the weighted–normalized decision matrix was computed as follows:

$$D^{*} = \begin{pmatrix} [0, 0.0746] & [0, 0.0371] & \ldots & [0.0022, 0.0505] \\ [0.0025, 0.0763] & [0.009, 0.0607] & \ldots & [0, 0.0432] \\ [0.0047, 0.1086] & [0, 0.0533] & \ldots & [0.0109, 0.0556] \\ [0.0035, 0.1248] & [0.0059, 0.0556] & \ldots & [0.0086, 0.0541] \\ [0.0012, 0.0832] & [0.003, 0.0533] & \ldots & [0.0109, 0.0556] \end{pmatrix}. \quad (9.20)$$

In vector form, the series are as follows:

$$D_1^{*} = \{[0, 0.0746], [0, 0.0371], \ldots, [0.0022, 0.0505]\}$$
$$D_2^{*} = \{[0.0025, 0.0763], [0.009, 0.0607], \ldots, [0, 0.0432]\}$$
$$D_3^{*} = \{[0.0047, 0.1086], [0, 0.0533], \ldots, [0.0109, 0.0556]\}$$
$$D_4^{*} = \{[0.0035, 0.1248], [0.0059, 0.0556], \ldots, [0.0086, 0.0541]\}$$
$$D_5^{*} = \{[0.0012, 0.0832], [0.003, 0.0533], \ldots, [0.0109, 0.0556]\}$$

Step 4. Determine the reference alternative. This is the optimal or ideal HRIS for all criteria, and it is obtained using Equation (8.18):

$$D_0^{*} = \{[0.0047, 0.1248], [0.009, 0.0607], \ldots, [0.0109, 0.0556]\}, \quad (9.21)$$

where $\otimes d_{0j}^{*} = \left[\max_{1 \leq i \leq 5} \underline{d_{ij}^{*}}, \max_{1 \leq i \leq 5} \overline{d_{ij}^{*}} \right]$.

Step 5. Determine the distance from the reference alternatives to the other alternative. This distance is computed using Equation (8.19):

$$\Delta = \begin{pmatrix} 0.0502 & 0.0236 & \ldots & 0.0051 \\ 0.0485 & 0 & \ldots & 0.0029 \\ 0.0162 & 0.009 & \ldots & 0 \\ 0.0013 & 0.0051 & \ldots & 0 \\ 0.0416 & 0.0074 & \ldots & 0.0051 \end{pmatrix}, \quad (9.22)$$

where $\delta_{ij} = \left| \otimes d_{0j}^{*} - \otimes d_{ij}^{*} \right| = \max \left(\left| \underline{d_{0j}^{*}} - \underline{d_{ij}^{*}} \right|, \left| \overline{d_{0j}^{*}} - \overline{d_{ij}^{*}} \right| \right)$.

Step 6. Calculate the grey relational grade. The overall performance of the HRIS is determined using Equation (8.20):

$$r_i = \frac{1}{27} \sum_{j=1}^{27} \gamma_{ij}, \quad (9.23)$$

$$r_i = \begin{pmatrix} 0.8385 \\ 0.8566 \\ 0.9239 \\ 0.9421 \\ 0.8786 \end{pmatrix} \approx \begin{pmatrix} 5\text{th} \\ 4\text{th} \\ 2\text{nd} \\ 1\text{st} \\ 3\text{rd} \end{pmatrix}. \quad (9.24)$$

Step 7. Rank the alternatives. This step sorts the HRISs from best to worst, where A_2 is the best HRIS provided by the second vendor.

One sign of a poor decision is the failure to consider uncertainty, which includes uncertainties in group decision-making, computational methods, and performance values. For example, implementing an HRIS system can create new issues within the organization, necessitating change management to mitigate the risks associated with the new technology. This lack of consideration is a significant shortcoming of the classical regime method. Additionally, the classical regime method is not suited for group decision-making. Both internal organization experts and theoretical experts, such as HR professors, can contribute to a group decision-making approach.

In summary, addressing uncertainties in decision-making processes is vital for achieving effective and reliable outcomes. The Grey REGIME method provides a robust solution by integrating these uncertainties and offering lower computational complexity compared to that of popular methods like the AHP. As organizations face increasingly complex decision-making scenarios, adopting methods that account for uncertainty will improve decision quality and adaptability. Since there are various methods for validating the best alternative, subsequent chapters introduce more grey Multiple Criteria Decision-Making (MCDM) evaluation methods, and the ranking of A_2 as the best contractor is further confirmed in Chapter 11.

10. Grey Integer Linear Programming

In this chapter, the Integral Linear Programming with Grey Possibility (ILP-GP) method is introduced. The assumption of a fixed value (e.g., 0.5) for the grey distinguishing coefficient (ζ) in Grey Relational Analysis (GRA) is not required by the ILP-GP method. Grey point allocation weights are utilized in conjunction with the ILP-GP for the evaluation of the business environment. An illustration is provided of the application of the ILP-GP method with a numerical example. Specifically, the reputations of five universities are evaluated based on data collected from 1565 students in Shaanxi Province, China. Importantly, the alignment of our findings with the rankings in in the existing literature demonstrates the effectiveness of the ILP-GP approach. Furthermore, the grey-Rank Order Centroid (ROC)-Integral Linear Programming (ILP) weighting method is introduced. This method complements the Grey Weighted Sum Model (GWSM) and the ILP-GP method in the evaluation of the business environment. This combined approach is depicted in Figure 10.1.

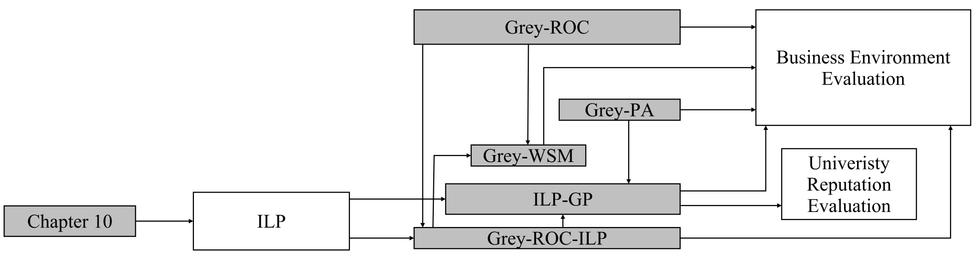


Figure 10.1. Flowchart of Chapter 10. Source: Figure by authors.

Grey Systems Theory (GST), introduced by Deng [301], provided a grey distinguishing coefficient within the interval of 0 and 1, denoted as $\zeta \in (0,1)$. However, for over two decades, an explicitly accepted method for calculating the grey distinguishing coefficient has not emerged. The difficulty in determining the grey distinguishing coefficient has become evident over the years. Consequently, researchers have generally resorted to a simplification, using a default value of 0.5 as the distinguishing coefficient. Çaydaş and Hasçalık [302] mentioned the common use of a grey distinguishing coefficient of 0.5, as did Yang [303]. In their work on evaluating the friction stir welding process of aluminum alloy, Ghetiya et al. [304] also applied GRA and noted that a grey distinguishing coefficient of 0.5 is often employed when all evaluation criteria have equal preferences. While numerous references have utilized 0.5 as the grey distinguishing coefficient, no concrete evidence supports its use, and it is important to acknowledge that the rankings of alternatives are dependent on the chosen grey distinguishing coefficient [304–309]. The ILP approach, on the other hand, is employed for evaluating grey numbers, as it does not necessitate the use of a grey distinguishing coefficient.

First and foremost, when comparing grey numbers, both numbers can be considered absolutely equal if their lower and upper bounds are equal, as illustrated in Figure 10.1. The value of 0.5 represents the last condition for the superior possibility of two grey numbers. When comparing grey numbers, both numbers can be considered absolutely equal if their lower and upper bounds satisfy the conditions $\underline{x} = \underline{y}$ and $\overline{x} = \overline{y}$. Thus, we can consider equal two grey numbers that are symmetric, satisfying Equation (10.1):

$$\left|\underline{x} - \underline{y}\right| = |\overline{x} - \overline{y}|\,. \tag{10.1}$$

This implies that two grey numbers that are absolutely equal satisfy $\left|\underline{x} - \underline{y}\right| - |\overline{x} - \overline{y}| = 0$. Equal grey numbers can be represented on a line graph, as shown in Figure 10.2, where the shaded region is denoted as $\otimes A$ and the striped region is denoted as $\otimes B$.

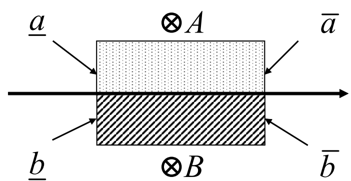
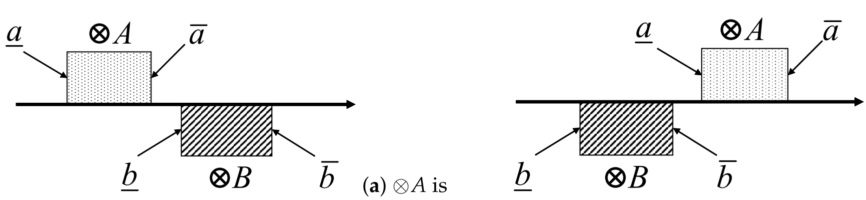


Figure 10.2. Absolute equalities. Source: Reprinted from [204], used with permission.

Absolute inequalities between two grey numbers occur in a disjointed case. $\otimes A$ is less than $\otimes B$ when $\overline{a}$ is less than $\underline{b}$. Similarly, $\otimes A$ is greater than $\otimes B$ when $\overline{a}$ is greater than $\underline{b}$. In other words, $\otimes A < \otimes B$ if and only if $\overline{a} < \underline{b}$, and $\otimes A > \otimes B$ if and only if $\overline{a} > \underline{b}$. Figure 10.3 illustrates the absolute inequalities between two grey numbers, $\otimes A$ and $\otimes B$.



(**a**) $\otimes A$ is less than $\otimes B$. (**b**) $\otimes A$is greater than $\otimes B$

Figure 10.3. Absolute inequalities. Source: Reprinted from [204], used with permission.

There are various cases of inequalities between two grey numbers, as illustrated in Figure 10.4. These cases raise questions about possibility. When expressing these equalities and inequalities as possibilities, we denote the grey possibility that $\otimes A$ is inferior to $\otimes B$ as $\otimes A \prec \otimes B$, and the grey possibility that $\otimes A$ is superior to $\otimes B$ is denoted as $\otimes A \succ \otimes B$. The possibility degree lies within the interval of 0 and 1. Here, $p_{\otimes A \prec \otimes B}$ represents the degree of possibility that $\otimes A$ is inferior to $\otimes B$, and

$p_{\otimes A \succ \otimes B}$ represents the degree of possibility that $\otimes A$ is superior to $\otimes B$. Figure 10.4 displays the seven conditions of possibilities between two grey numbers. It is worth noting that Li et al. [200] proposed a different type of grey possibility degree with four conditions for evaluating alternatives. They aggregated the weights of the criteria using the arithmetic mean. In contrast, seven conditions are presented in Equation (10.2), and under these conditions, the possibility degree that $\otimes A$ may be inferior to $\otimes B$ is as follows:

$$p_{\otimes A \prec \otimes B} = \begin{cases} 1 & \underline{a} \le \overline{a} \le \underline{b} \le \overline{b} \\ \frac{\underline{b}-\underline{a}}{\overline{a}-\underline{a}+1} + \frac{\overline{a}-\underline{b}+1}{\overline{a}-\underline{a}+1}\left(0.5\frac{\overline{a}-\underline{b}+1}{\overline{b}-\underline{b}+1} + \frac{\overline{b}-\overline{a}}{\overline{b}-\underline{b}+1}\right) & \underline{a} \le \underline{b} \le \overline{a} \le \overline{b} \\ \frac{\underline{b}-\underline{a}}{\overline{b}-\underline{a}+1} + 0.5\frac{\overline{b}-\underline{a}+1}{\overline{b}-\underline{a}+1} & \underline{a} \le \underline{b} \le \overline{b} \le \overline{a} \\ 0 & \underline{b} \le \overline{b} \le \underline{a} \le \overline{a} \\ 0.5\frac{\overline{b}-\underline{a}+1}{\overline{a}-\underline{a}+1}\frac{\overline{b}-\underline{a}+1}{\overline{b}-\underline{b}+1} & \underline{b} \le \underline{a} \le \overline{a} \le \overline{b} \\ 0.5\frac{\overline{b}-\underline{a}+1}{\overline{b}-\underline{b}+1}\frac{\overline{b}-\overline{a}}{\overline{b}-\underline{b}+1} & \underline{b} \le \underline{a} \le \overline{a} \le \overline{b} \\ 0.5 & \underline{a} = \overline{a} = \underline{b} = \overline{b} \text{ or} \\ & \left|\underline{a}-\underline{b}\right| = \left|\overline{a}-\overline{b}\right| \end{cases} \tag{10.2}$$

We say $\otimes A$ is inferior to $\otimes B$ when $p_{\otimes A \prec \otimes B} > 0.5$, and $\otimes A$ is superior to $\otimes B$ when $p_{\otimes A \prec \otimes B} < 0.5$. The sum of the inferior and superior possibilities is equal to 1, i.e.,

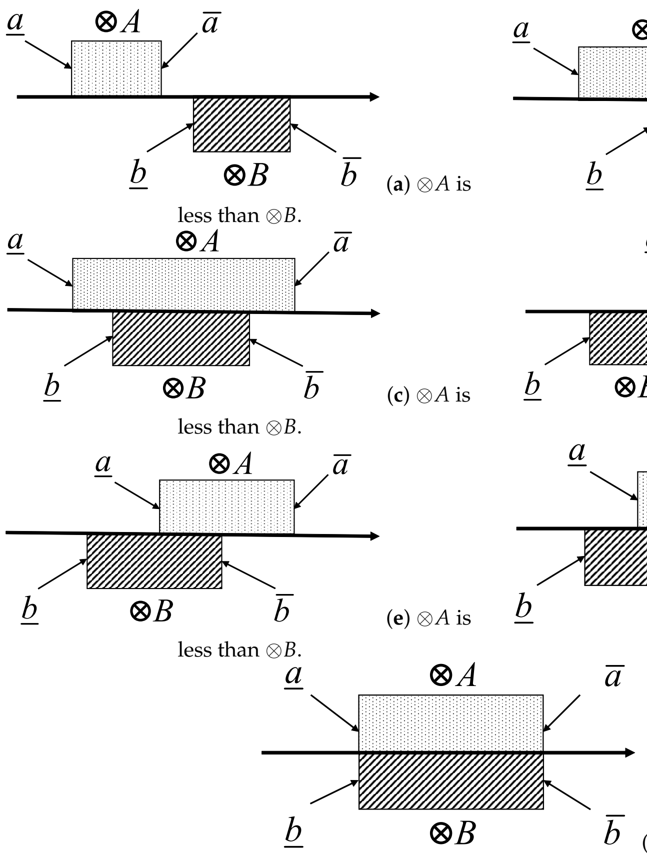
$$p_{\otimes A \prec \otimes B} + p_{\otimes A \succ \otimes B} = 1. \tag{10.3}$$

For a grey decision matrix, X, represented as

$$X = \begin{pmatrix} \otimes x_{1,1} & \otimes x_{1,2} & \cdots & \otimes x_{1,n} \\ \otimes x_{2,1} & \otimes x_{2,2} & \cdots & \otimes x_{2,n} \\ \vdots & \vdots & \ddots & \vdots \\ \otimes x_{m,1} & \otimes x_{m,2} & \cdots & \otimes x_{m,n} \end{pmatrix}, \tag{10.4}$$

the inferior and superior possibility degrees of matrix X are denoted as P^- and P^+, respectively. Additionally, the sum of the inferior and superior possibility degree matrices, when combined with an identity matrix, I, results in a unit matrix:

$$\begin{pmatrix} \otimes p^+_{11} & \otimes p^+_{12} & \cdots & \otimes p^+_{1n} \\ \otimes p^+_{21} & \otimes p^+_{22} & \cdots & \otimes p^+_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \otimes p^+_{m1} & \otimes p^+_{m2} & \cdots & \otimes p^+_{mn} \end{pmatrix} + \begin{pmatrix} \otimes p^-_{11} & \otimes p^-_{12} & \cdots & \otimes p^-_{1n} \\ \otimes p^-_{21} & \otimes p^-_{22} & \cdots & \otimes p^-_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \otimes p^-_{m1} & \otimes p^-_{m2} & \cdots & \otimes p^-_{mn} \end{pmatrix}$$
$$+ \begin{pmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \end{pmatrix} = \begin{pmatrix} 1 & 1 & \cdots & 1 \\ 1 & 1 & \cdots & 1 \\ \vdots & \vdots & \ddots & \vdots \\ 1 & 1 & \cdots & 1 \end{pmatrix}. \tag{10.5}$$



(a) $\otimes A$ is less than $\otimes B$. (b) $\otimes A$ is less than $\otimes B$. (c) $\otimes A$ is less than $\otimes B$. (d) $\otimes A$ is less than $\otimes B$. (e) $\otimes A$ is less than $\otimes B$. (f) $\otimes A$ is less than $\otimes B$. (g) $\otimes A$is greater than $\otimes B$

Figure 10.4. Grey possibilities. Source: Reprinted from [204], used with permission.

This equation can also be expressed as follows:

$$P^+ + P^- + I = 1. \tag{10.6}$$

The main idea of ILP-GP is to solve a MCDM problem using an ILP approach that incorporates grey possibility degrees. Consider a decision matrix,

$$\Phi = \begin{pmatrix} \otimes\phi_{1,1} & \otimes\phi_{1,2} & \cdots & \otimes\phi_{1,n} \\ \otimes\phi_{2,1} & \otimes\phi_{2,2} & \cdots & \otimes\phi_{2,n} \\ \vdots & \vdots & \ddots & \vdots \\ \otimes\phi_{m,1} & \otimes\phi_{m,2} & \cdots & \otimes\phi_{m,n} \end{pmatrix}, \tag{10.7}$$

where $\otimes\phi_{ij} = \left[\underline{\phi}_{ij}, \overline{\phi}_{ij}\right]$ represents the grey number of the j^{th} criterion of an alternative. The i^{th} alternative is represented as $\Phi_i = (\otimes\phi_{i,1}, \otimes\phi_{i,2}, \ldots, \otimes\phi_{i,n})$.

Next, the decision matrix, Φ, is normalized based on preference types. For benefit preferences, where higher values are better, the following apply:

$$\otimes\phi_{ij}^{*} = \left[\frac{\underline{\phi}_{ij} - \min\limits_{1\le i\le m}\underline{\phi}_{ij}}{\max\limits_{1\le i\le m}\overline{\phi}_{ij} - \min\limits_{1\le i\le m}\underline{\phi}_{ij}}, \frac{\overline{\phi}_{ij} - \min\limits_{1\le i\le m}\underline{\phi}_{ij}}{\max\limits_{1\le i\le m}\overline{\phi}_{ij} - \min\limits_{1\le i\le m}\underline{\phi}_{ij}}\right]. \tag{10.8}$$

For cost preferences, where lower values are better, the following apply:

$$\otimes\phi_{ij}^{*} = \left[\frac{\max\limits_{1\le i\le m}\overline{\phi}_{ij} - \overline{\phi}_{ij}}{\max\limits_{1\le i\le m}\overline{\phi}_{ij} - \min\limits_{1\le i\le m}\underline{\phi}_{ij}}, \frac{\max\limits_{1\le i\le m}\overline{\phi}_{ij} - \underline{\phi}_{ij}}{\max\limits_{1\le i\le m}\overline{\phi}_{ij} - \min\limits_{1\le i\le m}\underline{\phi}_{ij}}\right]. \tag{10.9}$$

This normalized decision matrix is denoted as follows:

$$\Phi^{*} = \begin{pmatrix} \otimes\phi^{*}_{11} & \otimes\phi^{*}_{12} & \cdots & \otimes\phi^{*}_{1n} \\ \otimes\phi^{*}_{21} & \otimes\phi^{*}_{22} & \cdots & \otimes\phi^{*}_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \otimes\phi^{*}_{m1} & \otimes\phi^{*}_{m2} & \cdots & \otimes\phi^{*}_{mn} \end{pmatrix}. \tag{10.10}$$

Now, we calculate the inferior possibilities of the normalized decision matrix, Φ^{*}, for criterion k and a pair of alternatives i and j, denoted as Q_{ijk}. For two alternatives, $\otimes\alpha = [\underline{\alpha}, \overline{\alpha}]$ and $\otimes\beta = \left[\underline{\beta}, \overline{\beta}\right]$, the possibility that alternative $\otimes\alpha$ is inferior to $\otimes\beta$ is given by Equation (10.2).

The ILP-GP method formulates an optimization problem using logical constraints represented as 0 and 1 variables. Selecting the best alternative is the primary objective of this maximization ILP model. The ILP problem includes integer constraints $\mu_{ij} = 0$ or 1, which are binary constraints. The decision variables for pairwise comparisons of alternatives are defined as $\mu_{ij} = 1$ if alternative i is preferred to j, and $\mu_{ij} = 0$, and otherwise, where $i = 1, 2, 3, \ldots, n$. This implies that in a comparison between two alternatives, one must be preferred over the other, signifying that two alternatives cannot be equally important. In other words, two alternatives cannot coexist as the best alternative, and this is represented as the constraint $\mu_{ij} + \mu_{ji} = 1$, indicating that one alternative must be the best.

Furthermore, pairwise comparisons are conducted for all pairs of alternatives. To determine the rankings of the alternatives in a sequence, comparisons involving three alternatives are required. For example, consider alternatives i, j, and l. After pairwise comparisons of alternatives i and j, as well as j and l, if alternative i is preferred over alternative j and alternative j is preferred over alternative l, then alternative i must be preferred over alternative l, and thus, alternative i is the best alternative. These constraints can be represented as $\mu_{ij} + \mu_{jl} = 2$ and $\mu_{il} = 1$. If these two constraints are satisfied, then the ranking sequence for the alternatives is $i > j > l$. By combining these two equations, an additional constraint, $\mu_{ij} + \mu_{jl} \le \mu_{il} + 1$, is added.

To determine the ranking sequence for all the alternatives, a combination of n criteria constraints must be added, which is $C_n^3 = \frac{n!}{3!(n-3)!}$. The best alternative is the one with the highest performance across all criteria. This is expressed as a maximization objective function, as we aim to find the best alternative. The objective function for identifying the best alternative is defined as the highest value for the summation of the weights of the criteria, the possibilities that the alternatives are superior, and the binary variables indicating the preference of each alternative over its corresponding pair. The LP model can be defined as follows:

$$\max_{i,j} \quad \sum_{k=1}^{n} w_k Q_{jik} \tag{10.11a}$$

$$\text{s.t.} \qquad \mu_{ij} + \mu_{ji} = 1 \qquad \text{for} \quad i,j \in A; i \neq j, \tag{10.11b}$$

$$\mu_{ij} + \mu_{jl} - \mu_{il} \leq 1 \qquad \text{for} \quad i,j,l \in A; i \neq j \neq l, \tag{10.11c}$$

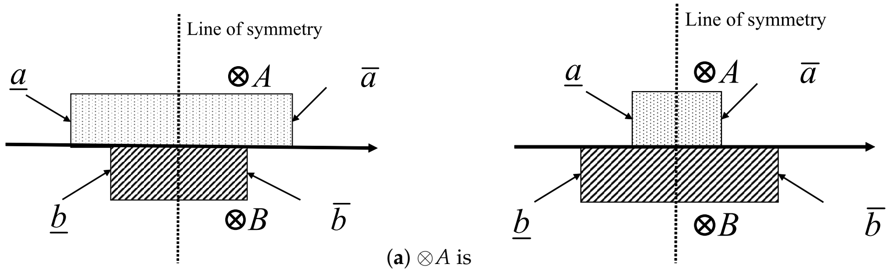
$$\mu_{ij} \in \{0,1\} \quad \text{for} \quad i,j \in A; i \neq j. \tag{10.11d}$$

In this model, the following apply: n represents the number of criteria, A is the set of alternatives, μ_{ij} equals 1 if one prefers A_i over A_j, and 0 otherwise, and w_k represents the weight assigned to criterion k. The objective function in Equation (10.11a) aims to maximize the linear aggregation of the criteria weights (w_k) and the possibilities that the alternatives are inferior to each other, denoted as Q_{jik}. This objective function enforces that a higher value of Q_{jik} (indicating a greater possibility of alternative j being less important than alternative i) corresponds to a greater possibility that $\mu_{ij} = 1$, signifying that alternative i is more important than alternative j. The summation of the possibilities for all alternatives that are inferior is zero because if an alternative is less preferred, then $\mu_{ij} = 0$. When comparing two alternatives, either one of the alternatives must be preferred, as indicated by Constraint (10.11b). Constraint (10.11c) represents the sequencing constraint. Constraint (10.11d) indicates that μ_{ij} are binary variables. This approach to solving MCDM problems, based on grey numbers and the possibilities arising from pairwise comparisons, is implemented in C++. The source code for solving this type of problem is provided in Appendix A.2. The method outlined in this section is known as the ILP-GP approach for rankings.

10.2. GRA-ILP-ROC Weighting Method

The GRA-ROC weights described in Section 3.2 require the use of the grey distinguishing coefficient, and the rankings of the criteria depend on this coefficient. However, the grey distinguishing coefficient is unknown, and a generalized value of 0.5 is typically used. Our major improvement to the GRA-ROC weights is achieved by combining them with the ILP approach, eliminating the need for the grey distinguishing coefficient.

The basic idea behind the GRA-ILP-ROC approach is that the performances of alternatives under uncertain decision-making environments are represented by grey interval numbers. Then, the rankings of the alternatives are based on pairwise comparisons of the alternatives as grey interval numbers. An approach for comparing two grey numbers is introduced, which measures the possibilities of two grey numbers being inferior or superior. For two interval grey numbers to be equal, the lower bounds of both grey numbers must be equal, and the upper bounds of both grey numbers must also be equal. The equality of two grey numbers, $\otimes A$ and $\otimes B$, is defined as follows: $\otimes A = \otimes B$, if and only if $\underline{a} = \underline{b}$ and $\overline{a} = \overline{b}$, as shown in Figure 10.4. However, two unequal grey number can have equal superior (inferior) possiblity when $\underline{a} - \underline{b} = \overline{a} - \overline{b}$ as dipicted in Figure 10.5.



(**a**) $\otimes A$ is less than $\otimes B$ (**b**) $\otimes A$ is greater than $\otimes B$

Figure 10.5. Unequal grey numbers with equal possibilities. Source: Reprinted from [204], used with permission.

The core idea for evaluating the weights of criteria using the GRA-ILP-ROC approach is as follows: Grey linguistic values represented by grey numbers are used to evaluate the weights of the criteria. A grey weights data matrix is constructed and standardized. The weights provided by the group decision-makers (DMs) are pairwise compared, and the grey inferior possibilities are calculated based on the pair of DMs' preferences. An ILP objective function is formulated to optimize the weights in a way where if one criterion is more preferred, the others are less preferred. The criteria are then sorted and converted into weights. The use of ILP in GRA-ILP-ROC improves Step 7. in the GRA-ROC method described in Section 8.2.2. Given a standardized decision matrix based on Equation (3.6),

$$\Phi = \begin{pmatrix} \otimes\phi_{1,1} & \otimes\phi_{1,2} & \cdots & \otimes\phi_{1,n} \\ \otimes\phi_{2,1} & \otimes\phi_{2,2} & \cdots & \otimes\phi_{2,n} \\ \vdots & \vdots & \ddots & \vdots \\ \otimes\phi_{m,1} & \otimes\phi_{m,2} & \cdots & \otimes\phi_{m,n} \end{pmatrix}, \tag{10.12}$$

the inferior possibilities of a pair of two criteria i and j for DM k are denoted as p_{ijk}, and they are calculated using Equation (10.2).

The main concept behind the ILP is to formulate the problem using logical constraints represented as binary variables (0 and 1). An ILP with integer constraints, $y_{ij} = 0$ or 1, is a specific type of ILP approach known as binary linear programming. In the case where DMs need to select the most important criterion from n criteria, the decision variables for the pairwise comparison of criteria are defined as $y_{ij} = 1$ if criterion i is more important than criterion j, and $y_{ji} = 0$ otherwise, where $i = 1, 2, 3, \ldots, n$. This implies that in comparing two criteria, one criterion is more important than the other, and this condition is logically true. In other words, two criteria cannot be equally important, and this is represented by the constraint $y_{ij} + y_{ji} = 1$, which ensures that only one criterion can be the most important.

Moreover, to determine the order of importance among the criteria, the sequence of listing the criteria must be determined, and at least three criteria need to be compared. Consider three criteria: i, j, and l. After a pairwise comparison of these three criteria, if criterion i is more important than criterion j and criterion j is more important than criterion l, then criterion i must be more important than criterion

l, making criterion i the most important. This comparison can be represented as $y_{ij} + y_{jl} = 2$ and $y_{il} = 1$. If these two equations are true, then the sequence for ranking these criteria is $i > j > l$. Additionally, by combining these two equations, the constraint $y_{ij} + y_{jl} \leq y_{il} + 1$ is obtained. This constraint ensures that if $y_{ij} = 1$ and $y_{jl} = 1$, then $y_{il} = 1$ must hold. To obtain the sequence for all the criteria, at least three combinations of n constraints are added, which is represented as $C_n^3 = \frac{n!}{3!(n-3)!}$.

The most important criterion is the one that is estimated to be the most preferred by all the DMs. This is expressed as a maximization objective function, where the objective is to achieve the highest value for the summation of the possibilities that the criteria are more superior, along with the binary variable indicating whether an alternative is more or less important than its corresponding pair. The ILP model can be defined as follows:

Objective function:

$$\max_{i,j} \quad \sum_{k=1}^{n} w_k Q_{jik} \tag{10.13a}$$

$$\text{s.t.} \quad y_{ij} + y_{ji} = 1 \quad \text{for} \quad i,j \in C; i \neq j, \tag{10.13b}$$

$$y_{ij} + y_{jl} - y_{il} \leq 1 \quad \text{for} \quad i,j,l \in C; i \neq j \neq l, \tag{10.13c}$$

$$y_{ij} \in \{0,1\} \quad \text{for} \quad i,j \in c; i \neq j. \tag{10.13d}$$

Here, n represents the number of decision-makers (DMs), C is the set of criteria, and y_{ij} takes a value of 1 if one prefers C_i over C_j, and 0 otherwise. The objective function (10.13a) serves as a maximization function, computed as the summation of a linear combination of possibilities where the criteria are considered inferior to each other (p_{jik}). This objective function asserts that when p_{jik} is greater (indicating a higher possibility of criteria j being less important than criteria i), there is a higher likelihood of y_{ij} equaling 1, signifying that criteria i is more important than criteria j. Consequently, when comparing two alternatives, one of them must be preferred, as stipulated by Constraint 10.13b. Constraint 10.13c represents the sequencing constraint, and Constraint 10.13d specifies that y_{ij} are binary variables.

10.3. ILP with Grey Possibilities for Rankings

The primary concept behind ILP-GP is to address Multiple Criteria Decision-Making (MCDM) problems using an ILP approach that incorporates grey possibility degrees. Given a decision matrix, Φ, shown in Equation (10.7), which has been normalized according to Equation (10.10).

Now, the inferior possibilities of the normalized decision matrix, Φ^*, for criterion k and a pair of alternatives, i and j, are denoted as Q_{ijk}. Let $\otimes\alpha = [\underline{\alpha}, \overline{\alpha}]$ and $\otimes\beta =$

$[\underline{\beta}, \overline{\beta}]$ represent two alternatives, respectively. The possibility that alternative $\otimes\alpha$ is inferior to $\otimes\beta$ is given as follows:

$$Q_{\otimes\alpha\prec\otimes\beta} = \begin{cases} 1 & \underline{\alpha} \le \overline{\alpha} \le \underline{\beta} \le \overline{\beta} \\ \frac{\underline{\beta}-\underline{\alpha}}{\overline{\alpha}-\underline{\alpha}+1} + \frac{\overline{\alpha}-\underline{\beta}+1}{\overline{\alpha}-\underline{\alpha}+1}\left(0.5\frac{\overline{\alpha}-\underline{\beta}+1}{\overline{\beta}-\underline{\beta}+1} + \frac{\overline{\beta}-\overline{\alpha}}{\overline{\beta}-\underline{\beta}+1}\right) & \underline{\alpha} \le \underline{\beta} \le \overline{\alpha} \le \overline{\beta} \\ \frac{\underline{\beta}-\underline{\alpha}}{\overline{\beta}-\underline{\alpha}+1} + 0.5\frac{\overline{\beta}-\underline{\alpha}+1}{\overline{\beta}-\underline{\alpha}+1} & \underline{\alpha} \le \underline{\beta} \le \overline{\beta} \le \overline{\alpha} \\ 0 & \underline{\beta} \le \overline{\beta} \le \underline{\alpha} \le \overline{\alpha} \\ 0.5\frac{\overline{\beta}-\underline{\alpha}+1}{\overline{\alpha}-\underline{\alpha}+1}\frac{\overline{\beta}-\underline{\alpha}+1}{\overline{\beta}-\underline{\beta}+1} & \underline{\beta} \le \underline{\alpha} \le \overline{a} \le \overline{\beta} \\ 0.5\frac{\overline{\beta}-\underline{\alpha}+1}{\overline{\beta}-\underline{\beta}+1}\frac{\overline{\beta}-\overline{\alpha}}{\overline{\beta}-\underline{\beta}+1} & \underline{\beta} \le \underline{\alpha} \le \overline{\alpha} \le \overline{\beta} \\ 0.5 & \underline{\alpha} = \overline{\alpha} = \underline{\beta} = \overline{\beta} \text{ or} \\ & \left|\underline{\alpha} - \underline{\beta}\right| = \left|\overline{\alpha} - \overline{\beta}\right| \end{cases} \quad (10.14)$$

The ILP-GP method formulates an optimization problem using logical constraints represented as 0 and 1 variables. It bears similarities to the ILP model formulated in Section 10.2. Selecting the best alternative is the primary objective function in this maximization ILP. The ILP problem involves integer constraints, μ_{ij}, which can take value 0 or 1, making them binary constraints. The decision variables for pairwise comparisons of alternatives are defined as μ_{ij} = 1 if alternative i is considered more important than alternative j and μ_{ij} = 0 otherwise, where $i = 1, 2, 3, \ldots, n$. This implies that when comparing two alternatives, one must be superior to the other, indicating that two alternatives cannot be equally important. In other words, two alternatives cannot simultaneously be the best, and this is represented as the constraint $\mu_{ij} + \mu_{ji} = 1$, signifying that one alternative must be superior.

Furthermore, only two alternatives can be compared at a time, resulting in a pairwise comparison of all alternatives. Additionally, to establish the rankings of alternatives in a sequence, comparisons among three alternatives are necessary. For example, consider alternatives i, j, and l. After a pairwise comparison of these alternatives, if alternative i is deemed more important than alternative j and alternative j is considered more important than alternative l, it follows that alternative i must be superior to alternative l, and hence, alternative i is the best choice. These constraints can be expressed as $\mu_{ij} + \mu_{jl} = 2$ and $\mu_{il} = 1$. When these two constraints are satisfied, the ranking sequence for the criteria becomes $i > j > l$. Combining these two equations, an additional constraint, $\mu_{ij} + \mu_{jl} \le \mu_{il} + 1$, is introduced. To obtain the sequence for ranking all alternatives, three combination n criteria constraints must be added, that is, $C_n^3 = \frac{n!}{3!(n-3)!}$.

Now, the best alternative is the one that exhibits the highest performance across all criteria. To achieve this, a maximization objective function is employed, as we seek the best possible alternative. The objective function for identifying the best alternative is expressed as the highest value obtained by summing the weights of the criteria, the possibilities that alternatives are superior, and the binary variables indicating a preference for one alternative over its corresponding pair. The linear programming model is defined as follows.

Objective function:

$$\max_{i,j} \quad \sum_{k=1}^{n} w_k Q_{jik} \mu_{ij} \tag{10.15a}$$

$$\text{s.t.} \qquad \mu_{ij} + \mu_{ji} = 1 \qquad \text{for} \quad i, j \in A; i \neq j, \tag{10.15b}$$

$$\mu_{ij} + \mu_{jl} - \mu_{il} \leq 1 \qquad \text{for} \quad i, j, l \in A; i \neq j \neq l, \tag{10.15c}$$

$$\mu_{ij} \in \{0, 1\} \quad \text{for} \quad i, j \in A; i \neq j. \tag{10.15d}$$

Here, n represents the number of criteria, A is the set of alternatives, and μ_{ij} takes a value of 1 if one prefers A_i over A_j and 0 otherwise. w_k denotes the weight assigned to criteria k. The objective function (10.15a) aims to maximize the linear aggregation of the criteria weights (w_k) and the possibilities of one alternative being inferior to another, denoted as Q_{jik}. This objective function implies that the greater the value of Q_{jik} (i.e., the higher the possibility of alternative j being less important than alternative i), the more likely μ_{ij} equals 1, indicating that alternative i is more important than alternative j. The summation of possibilities for all alternatives that are inferior is zero because if an alternative is less preferred, then μ_{ij} is set to 0. When comparing two alternatives, one of them must be preferred, as indicated by Constraint (10.15b). Constraint (10.15c) enforces sequencing, and Constraint (10.15d) indicates that μ_{ij} represents binary variables.

This approach for solving MCDM problems based on grey numbers and possibilities through pairwise comparisons is implemented in C++. The source code for solving such problems is provided in Appendix A.2. The method described in this section is the ILP-GP approach for rankings.

The ILP approach, combined with grey possibilities, GRA, and ROC weights, represents an intersection of uncertainty assessment, weighting, and optimization methods, as illustrated in Figure 10.4. Both the GRA-ROC and GRA-ILP-ROC weights share the same initial three steps, as well as the same final steps, which are presented in a flow diagram depicted in Figure 10.6.

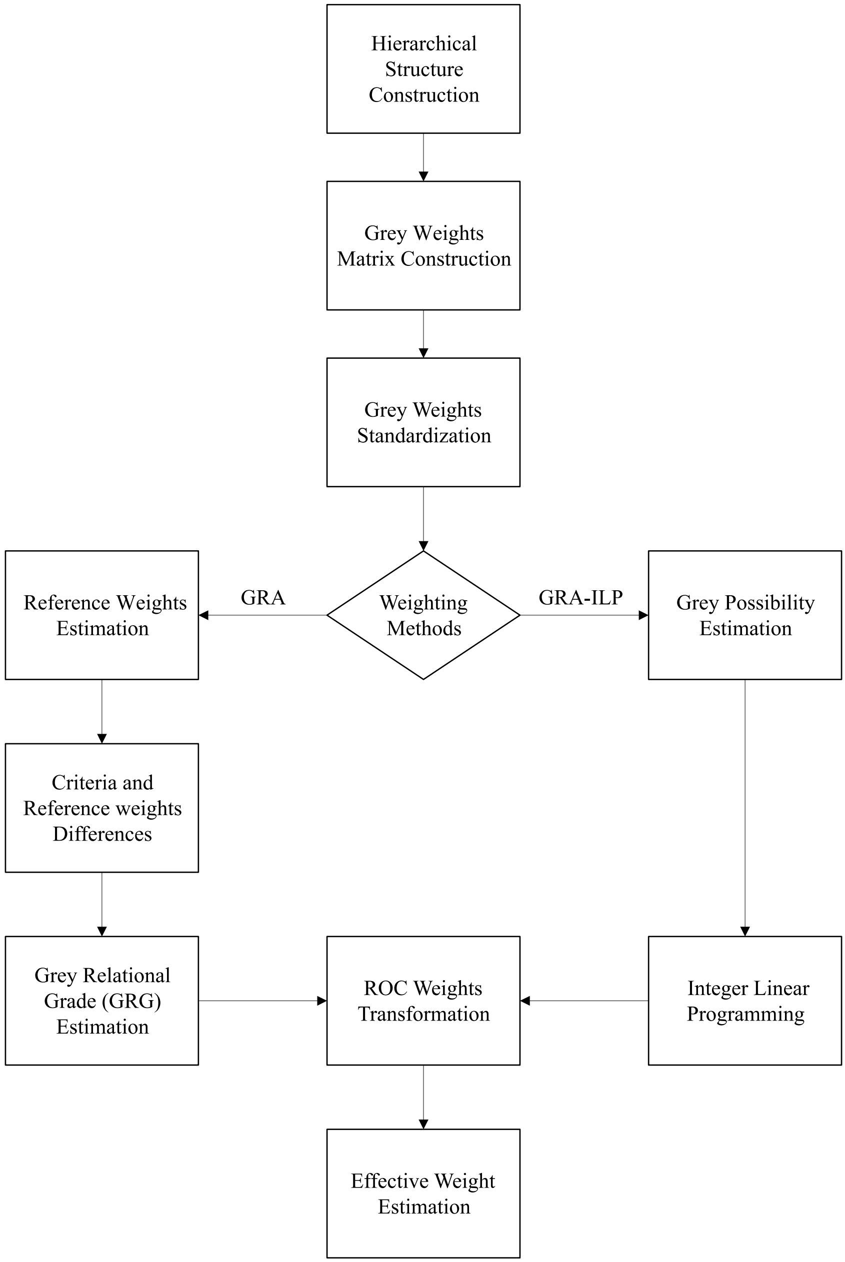


Figure 10.6. Flow chart of the improved ROC weights. Source: Reprinted from [204], used with permission.

Either the GRA-ROC or GRA-ILP-ROC weights can be employed in an MCDM problem. The primary distinction between the GRA-ROC and GRA-ILP-ROC weighting methods is that the GRA-ILP-ROC method does not necessitate the use of a grey distinguishing coefficient. The GRA-ILP approach can be applied more broadly to GRA problems involving uncertainties represented as grey numbers.

The GRA-ROC weights, on the other hand, represent an intersection between Grey Systems Theory (GST) and the ROC method, as depicted in Figure 10.7.

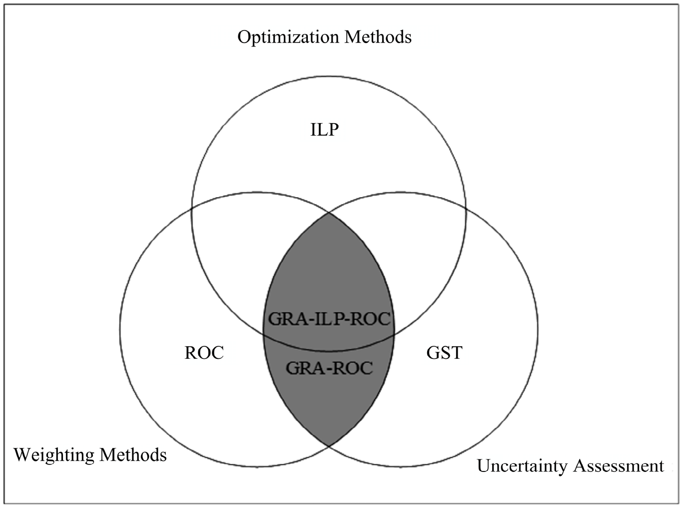


Figure 10.7. Intersection of uncertainty, weighting, and optimization approaches. Source: Reprinted from [204], used with permission.

10.4. Application of Grey Possibility

10.4.1. GRA-ILP-ROC for Evaluating Business Environments

In the GRA-ILP-ROC method for evaluating the business environment, grey numbers are employed to represent the weights of indicators derived from linguistic values provided by the decision-makers (DMs). These weights are those obtained based on the improved hierarchical model shown in Figure 8.4 . The process involves constructing a grey weights data matrix, which is subsequently standardized. The inferior possibilities of the criteria are estimated and used in the ILP objective function to determine the rankings of the criteria. These rankings are then transformed into ROC weights. The following are the steps for estimating the weights for evaluating the business environment in Africa.

Step 1. Construct an objective hierarchical structure and collect the raw weight data.

Step 2. Create the grey weights matrix. This matrix arranges the data in a rectangular format for ease of expression. The grey weights matrix is constructed based on Equation (3.3), and it can be represented as follows:

$$A = \begin{pmatrix} [0.6, 0.8] & [0.8, 1] & \cdots & [0.8, 1] \\ [0.4, 0.6] & [0.8, 1] & \cdots & [0.8, 1] \\ \vdots & \vdots & \ddots & \vdots \\ [0.4, 0.6] & [0.4, 0.6] & \cdots & [0.8, 1] \end{pmatrix}. \tag{10.16}$$

This same approach is applied to all the second-level indicators; however, they are omitted here.

Step 3. Standardize the weights matrix. Calculate a standardized weights matrix, S, using Equation (3.4), which can be expressed as follows:

$$S = \begin{pmatrix} [0.6, 0.8] & [0.8, 1] & \cdots & [0.8, 1] \\ [0.4, 0.6] & [0.8, 1] & \cdots & [0.8, 1] \\ \vdots & \vdots & \ddots & \vdots \\ [0.4, 0.6] & [0.4, 0.6] & \cdots & [0.8, 1] \end{pmatrix}. \quad (10.17)$$

Step 4. Calculate the grey possibilities and solve the ILP objective function. The inferior possibilities of the weight matrix, A, for criterion k, and a pair of alternatives, i and j, are denoted as P_{ijk}. These possibilities are calculated using Equation (10.2). For the first-level indicators, the LP model is defined as follows: Objective function:

$$\max \sum_{k=1}^{7} p_{jik} y_{ij},\ for\ i, j \in C, 1 \leq i \leq 7, 1 \leq j \leq 7, \quad (10.18)$$

subject to the following:

$$y_{ij} + y_{ji} = 1 \quad for \quad i, j, \in C; i \neq j, \quad (10.19)$$

$$y_{ij} + y_{jl} - y_{il} \leq 1 \quad for \quad i, j, l \in C; i \neq j \neq l, \quad (10.20)$$

$$y_{ij} \in \{0, 1\} \quad for \quad i, j, \in C; i \neq j. \quad (10.21)$$

The LP model above indicates that there are seven DMs, and seven first-level indicators. Solving using CPLEX, the generated LP is as follows: Objective function.

Max

$$\begin{aligned} &4y_{1,2} + 3y_{1,3} + 4y_{1,4} + 3y_{1,5} + 5.5y_{1,6} + 1.5y_{1,7} + 4.5y_{2,1} + 2.5y_{2,3} + 5.5y_{2,4} \\ &+ 1.5y_{2,5} + 3y_{2,6} + 4y_{2,7} + 3y_{3,1} + 4y_{3,2} + 5y_{3,4} + 2y_{3,5} + 1.5y_{3,6} \\ &+ 5.5y_{3,7} + 4y_{4,1} + 3y_{4,2} + 2y_{4,3} + 5y_{4,5} + 6y_{4,6} + y_{4,7} + 4y_{5,1} + 3y_{5,2} \\ &+ 4.5y_{5,3} + 2.5y_{5,4} + 2y_{5,6} + 5y_{5,7} + 1.5y_{6,1} + 5.5y_{6,2} + 2y_{6,3} + 5y_{6,4} \\ &+ \mathrm{y}_{6,5} + 6y_{6,7} + 4y_{7,1} + 3y_{7,2} + 2y_{7,3} + 5y_{7,4} + 1.5y_{7,5} + 5.5y_{7,6} \end{aligned} \quad (10.22)$$

Subject to
Constraint 1: $y_{1,2} + y_{1,3} = 1$
Constraint 2: $y_{1,4} + y_{1,5} = 1$
Constraint 3: $y_{1,6} + y_{1,7} = 1$
$\vdots$
Constraint 251: $-y_{6,4} + y_{6,7} - y_{7,6} \geq -1$
Constraint 252: $-y_{7,2} + y_{7,4} - y_{7,6} \geq -1$
$y_{ij} = \{0, 1\}, \quad for \quad 1 \leq i \leq 7, 1 \leq j \leq 7, i \neq j.$

For the first-level indicators based on the preferences of the seven DMs, CPLEX generated an LP function with 252 constraints and 42 binary variables. After solving and sorting, the ranking is as follows:

$$A_6 > A_9 > A_4 > A_7 > A_5 > A_1 > A_{10}. \quad (10.23)$$

Step 5. Transform the rankings of the criteria to ROC weights. Based on the obtained rankings, the transformed ROC weights can be represented in Table 10.1. Similarly, the GRA-ILP-ROC weights of the second-level indicators are calculated using the same method.

Step 6. Calculate the effective weights of the criteria. The effective weights of the indicators are computed, taking into account both local weights (weights of the second-level criteria in relation to their first-level criteria) and effective weights (the fraction of contributions by the second-level indicators to the overall weights in relation to the top-level hierarchy) using Equation (3.12). The results are given in Table 10.1. These GRA-ILP-ROC weights are then used for assessing the business environment for countries in Africa.

Table 10.1. ROC weight transformation for GRA-ILP.

Criteria	q	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**Weights**
A_6	1st	1	0.5	0.3333	0.25	0.2	0.1667	0.1429	0.3704
A_9	2nd		0.5	0.3333	0.25	0.2	0.1667	0.1429	0.2276
A_4	3rd			0.3333	0.25	0.2	0.1667	0.1429	0.1561
A_7	4th				0.25	0.2	0.1667	0.1429	0.1085
A_5	5th					0.2	0.1667	0.1429	0.0728
A_1	6th						0.1667	0.1429	0.0442
A_{10}	7th							0.1429	0.0204

Source: Reprinted from [204], used with permission.

10.4.2. GRA-ROC and GRA-ILP-ROC Weight Comparison

A comparison is made between the weights obtained using different methods for assessing the business environment of African countries. Three sets of weights are under comparison, GRA-ROC weights, GRA-ILP-ROC weights, and equal weights, as utilized by the Doing Business Project (DBP). The effective GRA-ROC, GRA-ILP-ROC and equal weights of the second-level indicators for evaluating the business environment in Africa are given in Table 3.11, Table 10.2 and Table 10.3, respectively. Additionally, for comparison, effective GRA-ROC, GRA-ILP-ROC, and equal weights are plotted in Figure 10.8.

Table 10.2. Effective GRA-ILP-ROC weights of indicators.

First-Level Indicator	Weights	Second-Level Criteria	Local Weights	Effective Weights	Index (v)
A_1	0.0728	$A_{1\text{-}2}$	0.1111	0.0445	1
		$A_{1\text{-}3}$	0.2778	0.0202	2
		$A_{1\text{-}4}$	0.6111	0.0081	3
A_4	0.1561	$A_{4\text{-}2}$	0.25	0.039	4
		$A_{4\text{-}3}$	0.75	0.1171	5
A_5	0.0442	$A_{5\text{-}1}$	0.0625	0.0028	6
		$A_{5\text{-}2}$	0.1458	0.0064	7
		$A_{5\text{-}3}$	0.5208	0.023	8
		$A_{5\text{-}4}$	0.2708	0.012	9
A_6	0.3704	$A_{6\text{-}1}$	0.6111	0.2264	10
		$A_{6\text{-}2}$	0.1111	0.1029	11
		$A_{6\text{-}3}$	0.2778	0.0412	12
A_7	0.2276	$A_{7\text{-}1}$	0.2778	0.0632	13
		$A_{7\text{-}2}$	0.6111	0.0253	14
		$A_{7\text{-}3}$	0.1111	0.1391	15
A_9	0.1085	$A_{9\text{-}1}$	0.25	0.0271	16
		$A_{9\text{-}2}$	0.75	0.0814	17
A_{10}	0.0204	$A_{10\text{-}1}$	0.75	0.0051	18
		$A_{10\text{-}3}$	0.25	0.0153	19

Source: Reprinted from [204], used with permission.

Table 10.3. Effective equal weights of indicators.

First-Level Indicator	Weights	Second-Level Criteria	Local Weights	Effective Weights	Index (v)
A_1	0.1429	$A_{1\text{-}2}$	0.33	0.0476	1
		$A_{1\text{-}3}$	0.33	0.0476	2
		$A_{1\text{-}4}$	0.33	0.0476	3
A_4	0.1429	$A_{4\text{-}2}$	0.5	0.0714	4
		$A_{4\text{-}3}$	0.5	0.0714	5
A_5	0.1429	$A_{5\text{-}1}$	0.25	0.0357	6
		$A_{5\text{-}2}$	0.25	0.0357	7
		$A_{5\text{-}3}$	0.25	0.0357	8
		$A_{5\text{-}4}$	0.25	0.0357	9
A_6	0.1429	$A_{6\text{-}1}$	0.33	0.0476	10
		$A_{6\text{-}2}$	0.33	0.0476	11
		$A_{6\text{-}3}$	0.33	0.0476	12
A_7	0.1429	$A_{7\text{-}1}$	0.33	0.0476	13
		$A_{7\text{-}2}$	0.33	0.0476	14
		$A_{7\text{-}3}$	0.33	0.0476	15
A_9	0.1429	$A_{9\text{-}1}$	0.5	0.0714	16
		$A_{9\text{-}2}$	0.5	0.0714	17
A_{10}	0.1429	$A_{10\text{-}1}$	0.5	0.0714	18
		$A_{10\text{-}3}$	0.5	0.0714	19

Source: Reprinted from [204], used with permission.

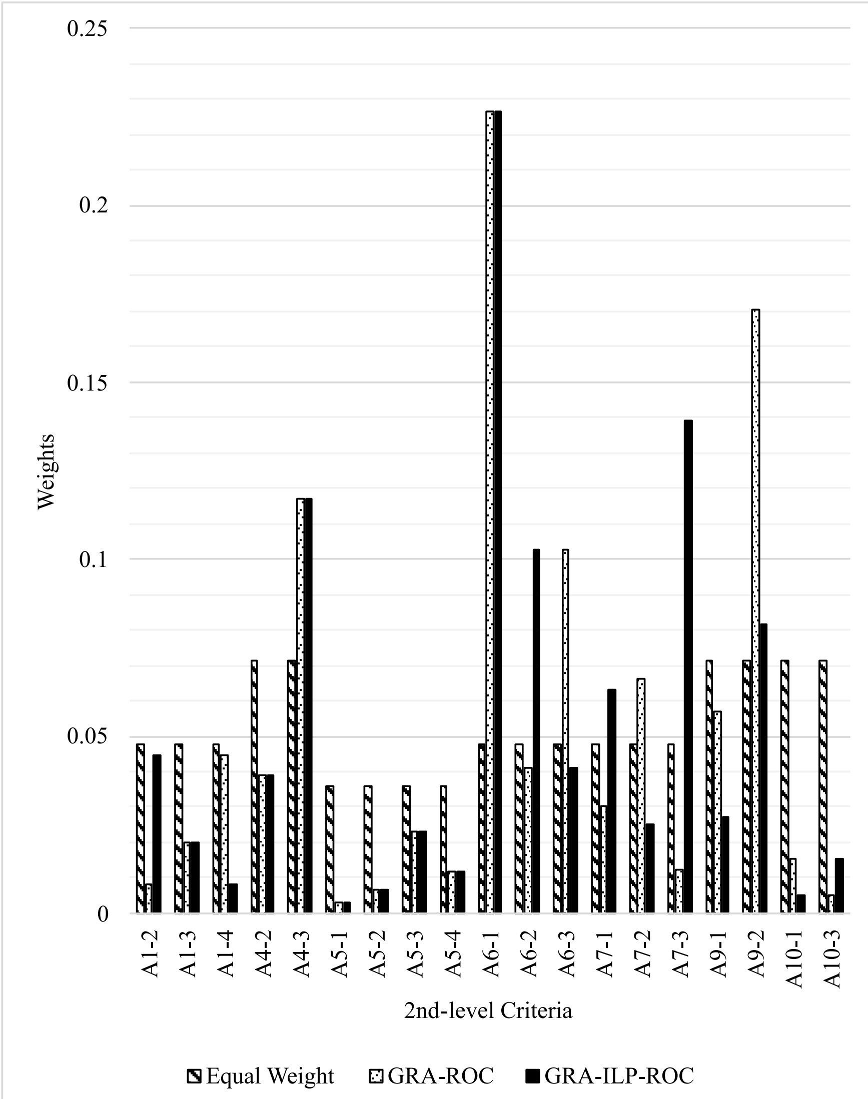


Figure 10.8. Criteria weight comparison. Source: Reprinted from [204], used with permission.

Equal weights are assigned to all criteria, resulting in a uniform weight distribution across the indicators. The equal weighting method implies that the *Time* ($A_{4\text{-}2}$) and *Cost* ($A_{4\text{-}3}$) associated with dealing with construction permits, the number of *Procedures* ($A_{9\text{-}1}$) and *Time* ($A_{9\text{-}2}$) for *Enforcing Contracts* (A_9), as well as the *Time* ($A_{10\text{-}1}$) and Recovery *Rate* ($A_{10\text{-}3}$) for *Resolving Insolvency* (A_{10}), all hold the same level of importance for decision-makers, with weights of 0.0714 assigned to each. However, a notable disparity arises when comparing equal weights to the newly developed methods in terms of the *Director Liability Index* ($A_{6\text{-}1}$) within the Protecting Investors (A_6) indicator. The GRA-ROC and GRA-ILP-ROC methods attribute a weight value of 0.2264 to $A_{6\text{-}1}$, whereas the equal weighting method assigns a much lower weight of 0.0476.

From a general perspective, as shown in Figure 10.8, it is evident that all three weighting methods differ. For instance, when considering the number of *Procedures* ($A_{9\text{-}1}$) within the *Enforcing Contracts* (A_9) indicator, GRA-ROC assigns a weight of 0.0567, GRA-ILP-ROC assigns 0.0271, and the equal weighting method assigns 0.0714. However, the most striking similarity among these methods is observed within the Starting a Business (A_1) indicator. In this case, GRA-ROC assigns a weight of 0.0445 to the *Paid-in Minimum Capital* ($A_{1\text{-}4}$) component, while the equal weighting method assigns a weight of 0.0476. Similarly, the GRA-ILP-ROC method assigns a weight of 0.0445 to the *Time* ($A_{1\text{-}2}$) required for *Starting a Business* (A_1), aligning closely with the equal weighting method's weight of 0.0476.

The GRA-ROC and GRA-ILP-ROC weighting methods exhibit significant similarities. These similarities are not only apparent in the steps used for weight calculation but also in the resulting weights assigned to most indicators. For example, when examining the local weights of first-level indicators in both the GRA-ROC and GRA-ILP-ROC methods, we find identical values, as depicted in Tables 3.10 and 10.1. Notably, both methods assign the highest importance to the *Director Liability Index* ($A_{6\text{-}1}$) within the *Protecting Investors* (A_6) indicator, while ranking the *Legal Right Index* ($A_{5\text{-}1}$) within the Getting Credit category as the least important to decision-makers. Moreover, several other first-level indicators, such as the Credit Information Index ($A_{5\text{-}2}$) and *Private Bureau Coverage* ($A_{5\text{-}4}$) in *Getting Credit* (A_5), the *Cost* ($A_{1\text{-}3}$) of *Starting a Business* (A_1), and *Public-Registry-Coverage* ($A_{5\text{-}3}$), share identical weight assignments in both the GRA-ROC and GRA-ILP-ROC methods. Specifically, the *Time* ($A_{4\text{-}2}$) and *Cost* ($A_{4\text{-}3}$) associated with Registering a Property receive the same weights in both methods, as illustrated in Figure 10.8.

While the GRA-ROC and GRA-ILP-ROC weighting methods exhibit overall similarity in their weight assignments, a notable divergence arises in the case of the *Paying Profit Tax* ($A_{7\text{-}3}$) indicator. Here, the GRA-ILP-ROC method assigns a substantial weight value of 0.1391 to this indicator, whereas the GRA-ROC method allocates a significantly lower value of 0.0121 to *Paying Profit Tax* ($A_{7\text{-}3}$).

10.4.3. ILP-GP with Applications

ILP-GP finds primary application in evaluating both the African business environment and university reputation. It is important to note that the university reputation survey presented in Chapter 4 exclusively pertained to Xi'an City. In contrast, the ILP-GP application introduced in this section offers a broader scope compared to that of Section 4.3, encompassing a survey conducted in Shaanxi Province.

Business Environment Evaluation

Based on the procedure outlined in Section 10.2, countries in Africa are ranked according to their business environment. The grey decision matrix is represented in Equation (7.18). For simplicity, vector notation is employed as follows:

$$\Phi_i = \left(\otimes\phi_{i1}, \otimes\phi_{i2}, ..., \otimes\phi_{i,19}\right). \tag{10.24}$$

For example, Φ_1 = ([22, 24], [10.8, 13.2], [24.1, 45.2], [55, 74], [7.1, 7.5], [2, 3], [0, 0], [0.2, 2.4], [0, 0], [5, 5], [6, 6], [4, 4], [27, 27], [451, 451], [6.6, 6.6], [630, 630], [45, 47], [2.5, 2.5], [41.7, 41.7]).

Subsequently, the matrix, X, is normalized using Equation (10.10), resulting in the normalized matrix presented in Equation (7.21).

Objective function:

$$\max_{i,j} \quad \sum_{k=1}^{19} w_k Q_{jik} \mu_{ij}, \tag{10.25a}$$

$$\text{s.t.} \quad \text{Constraints(10.13b)} - \text{(10.13d)}. \tag{10.25b}$$

The objective function involves the assessment of 19 indicators for 53 countries. Upon solving with CPLEX using the C++ code provided in Appendix A.2, the resulting LP function contains 2756 binary variables and 14,312 constraints. The objective function can be represented as follows.

Objective function:

$$\max_{i,j} \quad \sum_{k=1}^{19} w_k Q_{jik} \mu_{ij} \tag{10.26a}$$

$$\text{s.t.} \quad \mu_{1,2} + \mu_{1,3} = 1, \tag{10.26b}$$

$$\mu_{1,4} + \mu_{1,5} = 1, \tag{10.26c}$$

$$\mu_{1,6} + \mu_{1,7} = 1, \tag{10.26d}$$

$$\vdots, \tag{10.26e}$$

$$-\mu_{53,48} + \mu_{53,50} - \mu_{53,52} = -1, \tag{10.26f}$$

$$\mu_{ij} = \{0, 1\}, \quad \text{for} \quad 1 \le i \le 53, 1 \le j \le 53, i \ne j. \tag{10.26g}$$

The rankings for evaluating the African business environment are as follows: South Africa > Botswana > Mauritius > Rwanda > Ghana > Namibia > Mozambique > Zambia > Seychelles > Madagascar > Egypt > Morocco > Uganda > Cabo Verde > Tunisia > Zimbabwe > Gabon > Mali > Niger > Burkina Faso > Sierra Leone > Nigeria > Malawi > Ethiopia > Burundi > Kenya > Tanzania > Lesotho > Swaziland > Sudan > Equatorial Guinea > Guinea-Bissau > Togo > Cameroon > Senegal > Mauritania > Côte d'Ivoire > Algeria > Liberia > Eritrea > Djibouti > Benin > Comoros > Congo, Rep. > Angola > São Toméand Príncipe > Guinea > Gambia > Congo, Dem. Rep. > Chad > Central African Republic > South Sudan > Libya. In this ranking, South Africa is considered the best alternative, while Botswana, Mauritius, and Libya hold the 2nd, 3rd, and 53rd positions, respectively.

University Reputation Evaluation

In the evaluation of university reputation using the ILP-GP method, five universities (represented as alternatives) are assessed: AAA University (A_1), BBB University (A_2), CCC University (A_3), DDD University (A_4), and EEE University (A_5). The universities' names are kept anonymous to maintain objectivity.

The evaluation process begins with the development of a questionnaire, which incorporates measurement variables from previous research [170,294,310]. The questionnaire is pilot-tested and revised iteratively. A total of 1592 out of 1600 distributed questionnaires are recovered, with 13 questionnaires having unanswered questions and 14 questionnaires containing unattended responses. Therefore, data from 1,565 respondents (97.8% of the distributed questionnaires) are used for analysis.

Second-level criteria are measured as reflective constructs, meaning that these indicators are influenced by the questionnaire responses. For instance, the second-level indicator *Citizenship* ($C_{1\text{-}1}$) is measured as a reflective construct, as shown in Figure 8.5. Other second-level indicators are measured similarly.

Reflective variables are highly correlated, meaning that excluding some of the measured variables will not significantly affect the results. To represent these variables as grey numbers, the average of the measured variables for each alternative is calculated. The minimum and maximum of these averages are used to define the lower and upper bounds of the grey number. This process is applied to all second-level indicators, and the resulting grey data are computed using Equation (8.42).

The following is an example of the transformation of sample data to grey numbers for the *Social Contribution* of the Citizenship indicator ($C_{1\text{-}1\text{-}1}$) for each university (Table 10.4).

Table 10.4. Transformation of sample data into grey numbers.

Measured Variable/ Universities	$C_{1\text{-}1\text{-}1}$	$C_{1\text{-}1\text{-}2}$	$C_{1\text{-}1\text{-}3}$	$C_{1\text{-}1\text{-}4}$	$\otimes\phi_{i1}$
A_1	4.3612	3.9910	4.1104	3.9761	[3.9761, 4.3612]
A_2	4.1378	3.8910	4.4391	4.2436	[3.8910, 4.4391]
A_3	4.2000	4.0164	4.4787	4.4	[4.0164, 4.4787]
A_4	4.1176	3.7451	4.317	4.0163	[3.7451, 4.3170]
A_5	4.2500	3.9333	4.4500	4.2733	[3.9333, 4.45]

Source: Reprinted from [104], used with permission.

Similar transformations are performed for other second-level indicators. The computed grey data for all second-level indicators are shown in Table 10.5.

Table 10.5. Grey data for evaluation of university reputation.

Φ_{ij}	Φ_{1j}	Φ_{2j}	Φ_{3j}	Φ_{4j}	Φ_{5j}
Φ_{i1}	[3.9761, 4.3612]	[3.891, 4.4391]	[4.0164, 4.4787]	[3.7451, 4.317]	[3.9333, 4.45]
Φ_{i2}	[3.8583, 4.1164]	[3.3141, 4.1058]	[3.3443, 3.9607]	[3.4052, 3.9281]	[3.6133, 4.2]
Φ_{i3}	[3.9015, 4.2478]	[4.141, 4.266]	[4.1246, 4.282]	[4.0163, 4.2876]	[4.29, 4.4867]
Φ_{i4}	[3.6507, 4.0716]	[3.4263, 4.4263]	[3.3919, 4.4721]	[3.0523, 4.4379]	[4.0233, 4.5667]
Φ_{i5}	[3.9612, 4.1403]	[4.0353, 4.234]	[4.2787, 4.3967]	[4.0817, 4.1667]	[4.1533, 4.3]
Φ_{i6}	[3.7224, 3.9612]	[2.9006, 3.8397]	[3.4984, 3.9082]	[3.268, 4.0654]	[3.43, 4.1333]
Φ_{i7}	[3.7612, 4.0119]	[3.5149, 3.9068]	[3.6689, 3.8787]	[3.3954, 3.6765]	[3.78, 4.0767]
Φ_{i8}	[3.8537, 4.1194]	[4.0128, 4.1571]	[3.9443, 4.0656]	[3.6405, 4.1176]	[4.02, 4.3733]
Φ_{i9}	[3.7851, 4.4119]	[3.6154, 4.1699]	[3.6623, 4.0754]	[3.1961, 3.8725]	[3.5833, 4.2267]
Φ_{i10}	[3.8776, 4.2448]	[2.8942, 3.8237]	[3.1672, 4]	[2.6667, 3.9902]	[3.88, 4.3133]
Φ_{i11}	[3.7522, 4.0478]	[2.7244, 4.1474]	[3.0393, 4.0787]	[3.0425, 4.134]	[3.0333, 4.22]
Φ_{i12}	[3.9552, 4.2269]	[4.0128, 4.3343]	[4.1481, 4.2984]	[3.9739, 4.2843]	[4.2233, 4.3567]
Φ_{i13}	[3.8448, 4.0687]	[3.5385, 3.8846]	[3.7443, 3.8492]	[3.6471, 3.7353]	[3.4833, 4.0633]
Φ_{i14}	[3.8537, 4.3134]	[3.5545, 4.2286]	[3.8098, 4.1738]	[3.6993, 4.4641]	[3.9467, 4.48]
Φ_{i15}	[3.9045, 4.1821]	[3.9167, 4.2083]	[3.6951, 4.0465]	[3.8235, 4.1111]	[4.1733, 4.3167]
Φ_{i16}	[3.7701, 3.8866]	[2.9551, 3.7051]	[3.0623, 3.6098]	[2.8987, 3.781]	[3.2867, 3.9933]
Φ_{i17}	[3.4836, 3.9403]	[2.4103, 3.5417]	[2.6131, 3.5148]	[2.4183, 3.7157]	[2.5133, 3.9433]
Φ_{i18}	[3.7881, 4.0746]	[3.5032, 4.2244]	[3.3508, 4.066]	[3.3072, 4.1176]	[3.77, 4.2367]

Source: Reprinted from [104], used with permission.

The implementation of the ILP-GP approach for solving MCDM problems based on grey numbers and pairwise comparisons of grey numbers is carried out in C++. IBM ILOG CPLEX Optimization Studio Version 12.6.2.0 is used for solving the ILP-GP problem. The following steps outline the process.

Step 1. Construct the grey decision matrix. The values of the second-level criteria are obtained from Table 10.5, where each element, $\otimes\phi i, j$, of the decision matrix for second-level indicators corresponds to a university (A_i) and a specific second-level indicator ($C_{1\text{-}1\text{-}j}$) obtained from the surveys. For example, the first element of the grey decision matrix corresponds to the grey value of *Social Contributions* ($C_{1\text{-}1\text{-}j}$) in the *Citizenship* (C_1) of AAA University (A_1), with a lower bound of 3.9761 units and an upper bound of 4.3612 units. All elements of the matrix Φ have similar corresponding lower and upper bounds for the second-level indicators of each university.

Step 2. Normalize the grey decision matrix. The grey decision matrix, Φ, is normalized using Equation (10.27). Then, based on Equation (10.28), a vector representation of the normalized matrix is obtained:

$$\Phi^* = \begin{pmatrix} \otimes\phi^*_{11} & \otimes\phi^*_{12} & \cdots & \otimes\phi^*_{1n} \\ \otimes\phi^*_{21} & \otimes\phi^*_{22} & \cdots & \otimes\phi^*_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \otimes\phi^*_{m1} & \otimes\phi^*_{m2} & \cdots & \otimes\phi^*_{mn} \end{pmatrix}, \tag{10.27}$$

where the following apply:

a. To normalize the benefits preferences, where a higher value is considered better, the following procedure is applied:

$$\otimes\phi_{ij}^{*} = \left[\frac{\underline{\phi}_{ij} - \min\limits_{1\le i\le m} \underline{\phi}_{ij}}{\max\limits_{1\le i\le m} \overline{\phi}_{ij} - \min\limits_{1\le i\le m} \underline{\phi}_{ij}} \frac{\overline{\phi}_{ij} - \min\limits_{1\le i\le m} \underline{\phi}_{ij}}{\max\limits_{1\le i\le m} \overline{\phi}_{ij} - \min\limits_{1\le i\le m} \underline{\phi}_{ij}} \right].$$

b. To normalize the cost preferences, where a smaller value is considered better, the following procedure is applied:

$$\otimes\phi_{ij}^{*} = \left[\frac{\max\limits_{1\le i\le m} \overline{\phi}_{ij} - \overline{\phi}_{ij}}{\max\limits_{1\le i\le m} \overline{\phi}_{ij} - \min\limits_{1\le i\le m} \underline{\phi}_{ij}} \frac{\max\limits_{1\le i\le m} \overline{\phi}_{ij} - \underline{\phi}_{ij}}{\max\limits_{1\le i\le m} \overline{\phi}_{ij} - \min\limits_{1\le i\le m} \underline{\phi}_{ij}} \right].$$

In vector form, we define the following:

$$\Phi_i^* = (\otimes\phi_{i,1}^* \ \otimes\phi_{i,2}^*, ..., \ \otimes\phi_{i,n}^*). \tag{10.28}$$

Using Equation (10.29), we obtain a vector form of matrix Φ^* as follows:

$$\Phi^* = \begin{pmatrix} [0.6017, 0.7808] & [0.5747, 0.6739] & \cdots & [0.5373, 0.6627] \\ [0.6763, 0.8606] & [0.5115, 0.7473] & \cdots & [0.5609, 0.7853] \\ \vdots & \vdots & \ddots & \vdots \\ [0.6822, 0.8522] & [0.5647, 0.7753] & \cdots & [0.6308, 0.7761] \end{pmatrix}. \tag{10.29}$$

Based on Equation (10.29), a vector form of matrix Φ^* is obtained. As an example, Φ_1^* is represented as follows: Φ_1^* = ([0.6017, 0.7808], [0.5747, 0.6739], [0.5988, 0.7134], [0.4923, 0.6622], [0.5973, 0.6644], [0.5313, 0.6241], [0.5306, 0.6296], [0.5684, 0.6672], [0.5318, 0.7893], [0.5687, 0.7177], [0.5328, 0.6368], [0.5831, 0.7182], [0.5604, 0.6602], [0.56, 0.7475], [0.5659, 0.6903], [0.5373, 0.5876], [0.44, 0.6067], [0.5373, 0.6627]). The data for other elements are omitted here for brevity.

Step 3. Determine the criteria weights. The weights used in the evaluation are the average weights assigned by the experts, as presented in Section 4.2. These weights correspond to all the second-level indicators. The weight vector, W', is as follows:

W' = (3.2011, 6.9605, 3.7122, 5.1321, 11.4677, 8.0917, 6.7702, 6.4145, 6.3351, 5.5306, 4.3476, 3.5338, 7.2917, 8.5165, 6.6081, 2.2673, 1.0728, 2.7467)^T.

Step 4. Calculate grey possibilities and solve the LP problem.

Based on the procedure in Section 10.1, the evaluation of the five universities is conducted using the second-level indicators and the assigned weights, W'. The objective function for this evaluation is formulated as follows. Objective function:

$$\max_{i,j} \sum_{k=1}^{18} w_k Q_{jik} \mu_{ij} \quad \text{for} \quad i,j \in A, \quad 1 \le i \le 5, \quad 1 \le j \le 5 \tag{10.30a}$$

$$\text{s.t.} \quad \mu_{ij} + \mu_{ji} = 1 \quad \text{for} \quad i,j \in A; i \neq j, \tag{10.30b}$$

$$\mu_{ij} + \mu_{jl} - 1 \le \mu_{il} \quad \text{for} \quad i,j,l \in A; i \neq j \neq l, \tag{10.30c}$$

$$\mu_{ij} \in \{0,1\} \quad \text{for} \quad i,j \in A; i \neq j. \tag{10.30d}$$

The objective function considers 18 second-level indicators and evaluates 5 alternatives. After solving this linear programming (LP) problem using CPLEX, the generated LP function involves 20 binary variables and 80 constraints.

$$\max \quad 0.6668\mu_{1,2} + 0.3332\mu_{1,3} + 0.6469\mu_{1,4} + 0.3531\mu_{1,5} \tag{10.31a}$$

$$+ 0.5737\mu_{2,1} + 0.4261\mu_{2,3} + 0.7457\mu_{2,4} + 0.2543\mu_{2,5} \tag{10.31b}$$

$$+ 0.5612\mu_{3,1} + 0.4388\mu_{3,2} + 0.4846\mu_{3,4} + 0.5154\mu_{3,5} \tag{10.31c}$$

$$+ 0.5415\mu_{4,1} + 0.4585\mu_{4,2} + 0.3904\mu_{4,3} + 0.6096\mu_{4,5} \tag{10.31d}$$

$$+ 0.5588\mu_{5,1} + 0.4411\mu_{5,2} + 0.6662\mu_{5,3} + 0.3338\mu_{5,4} \tag{10.31e}$$

$$\text{s.t.} \quad \mu_{1,2} + \mu_{1,3} = 1, \tag{10.31f}$$

$$\mu_{1,4} + \mu_{1,5} = 1, \tag{10.31g}$$

$$\mu_{1,6} + \mu_{1,7} = 1, \tag{10.31h}$$

$$\vdots,$$

$$\mu_{1,6} + \mu_{1,7} = 1, \tag{10.31i}$$

$$\mu_{ij} = \{0,1\}, \quad \text{for} \quad 1 \le i \le 5, 1 \le j \le 5, i \neq j. \tag{10.31j}$$

Step 5. Sort the results to obtain the rankings. Rank the universities.

Finally, the results are sorted to obtain the rankings for the five universities. In the evaluation, the universities are ranked as follows: $A_5 > A_3 > A_2 > A_4 > A_1$. This implies that EEE University is ranked as the top university, while CCC University, BBB University, DDD University, and AAA University occupy the 2nd, 3rd, 4th, and 5th positions, respectively. The ranking of EEE University as the best university is consistent with the rankings of Academic Ranking of World Universities (ARWU) [293], Centre for Science and Technology Studies (CWTS) Leiden Ranking [294], Performance Ranking of Scientific Papers for World Universities (PRSPWUN) [295], Quacquarelli Symonds (QS) [296], Times Higher Education (THE) [297] and University Rankings based on Academic Performance (URAP) [298].

In today's globalized and highly competitive higher education landscape, universities must continually strive to attract and retain students, faculty, and financial support. University rankings serve as valuable reference points for assessing institutional reputation. It is crucial for universities to meet the expectations of students by providing high-quality education, fostering civic engagement, and building a strong reputation. Additionally, a positive reputation can enhance staff engagement, performance, and commitment, contributing to an institution's overall success [310,311].

10.4.4. ILP-GP and GWSM Evaluation Comparison

In the comparison of the GWSM and ILP-GP methods based on GRA-ILP-ROC weights, several observations emerge as illustrated in Figure 10.9. According to the GWSM method, the top-four-ranked countries are South Africa, Ghana, Mauritius, and Botswana, in that order. In contrast, the ILP-GP approach ranks the top three countries as South Africa, Botswana, and Mauritius, with Rwanda occupying the 4th position. One consistent observation is that South Africa maintains its top ranking in both evaluation methods, suggesting a robust and high-performance business environment in the country. Notably, Rwanda's ranking differs between the two methods. GWSM places Rwanda in the fourth position, while ILP-GP ranks it fifth. This variance could be due to differences in the weighting and evaluation procedures between the two methods. Overall, both methods provide valuable insights into the evaluation of the business environment, with some variations in rankings, especially in the top positions.

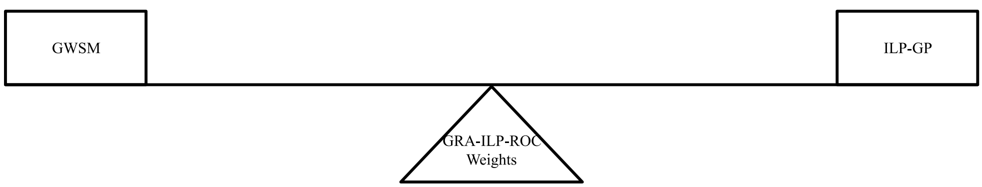


Figure 10.9. Evaluation method comparison based on GRA-ILP-ROC weights. Source: Figure by authors.

11. Other Grey Decision-Making Methods

This chapter introduces additional grey decision-making methods to validate the results obtained in solving Multiple Criteria Decision-Making (MCDM) problems. To ensure the robustness of MCDM outcomes, it is advisable to employ multiple methods for selecting the best alternative. In this chapter, the Evaluation based on Distance from Average Solution (EDAS) and the Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS) methods extended with Grey Systems Theory (GST) are presented. Specifically, the Grey EDAS and Grey TOPSIS methods are utilized to validate the rankings for evaluating Human Resource Information System and contractor selection in a solar panel installation project, as depicted in Figure 11.1. Moreover, the Grey Stepwise Weight Analysis Ratio Assessment (SWARA)-Full Consistent Method (FUCOM) method introduced in Chapter 6 is employed as a hybrid approach with EDAS. Additionally, the Grey Point Allocation Weighting method discussed in Chapter 6 is integrated with Grey TOPSIS. These methodologies offer a comprehensive evaluation and validation framework for addressing various MCDM scenarios.

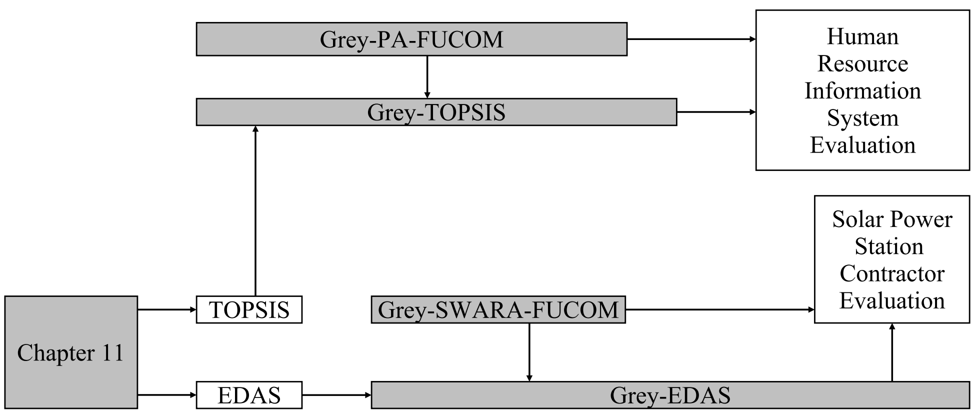


Figure 11.1. Flowchart of Chapter 11. Source: Figure by authors.

11.1. EDAS Using Grey Weights

The EDAS method has found wide-ranging applications across various domains, showcasing its versatility and effectiveness in addressing complex decision-making problems. Researchers have adopted the EDAS method to evaluate and rank alternatives in different contexts. For instance, Trinkuniene et al. [312] utilized EDAS to assess the quality assurance of different contractors and compared the results with other methods, including Weighted Sum Model (WSM), TOPSIS, and Complex Proportional Assessment (COPRAS). They incorporated both objective and subjective weights, incorporating methods such as entropy, Integrated Determination of Objective Criteria Weights (IDOCRIW), and fuzzy Analytical Hierarchical Process (AHP) for weight determination. Liang et al. [313] presented a hybrid approach combining EDAS and ELimination Et Choix Traduisant la REalité – ELimination and Choice Expressing REality (ELECTRE) for evaluating cleaner production performance in gold mines. Barauskas et al. [314] applied EDAS to rank parking lots, while Chen et al. [315] used EDAS in conjunction with a normalized Weighted Aggregated Sum Product Assessment (WASPAS) method to select tea houses, with weights obtained from experts' surveys. Ouenniche et al. [316] applied the EDAS method for risk analysis in various domains, including finance, investment, internet security, fraud detection, and medical diagnosis, considering both parametric and non-parametric classification.

In supply chain management, Stević et al. employed EDAS to evaluate suppliers, with criteria weights determined using the Decision-Making Trial and Evaluation Laboratory (DEMATEL) method. Sremac et al. [317] used EDAS as a confirmatory method for evaluating Third Party Logistics (3PL) providers, alongside rough WASPAS and SWARA methods. Ecer [318] selected 3PL providers using an integrated model of fuzzy AHP and EDAS.

Various operators have been extended to the EDAS method, enhancing its applicability to decision-making problems with uncertainty. For instance, Li et al. [319] combined power-weighted averaging and geometric operators with EDAS for group decision-making. Feng et al. [320] integrated EHFLTS with EDAS using the OWA operator. Ghorabaee et al. [321,322] extended EDAS to handle fuzzy MCDM problems, interval type 2 fuzzy sets, and interval-valued neutrosophic fuzzy numbers. Ilieva et al. [323] analyzed classified and fuzzy EDAS modifications, proposing simplified calculations for trapezoid fuzzy numbers. Karasan et al. [324] extended EDAS to interval-valued neutrosophic fuzzy numbers.

EDAS has also been applied to inventory classification problems, including stochastic scenarios [322,325]. Kutlu Gündoğdu et al. [326] extended EDAS to hesitant fuzzy sets for organ transplant selection in a hospital. Panchal et al. [327] utilized EDAS for fuzzy lambda-tau in the reliability, availability, and maintainability (RAM) parameters of systems using FMEA. While the EDAS method has been widely employed in various applications, only a limited number of studies have explored its integration with Grey Systems Theory (GST), including works by Peng et al. [328] and Stanujkic et al. [329]. These integrations extend the method's capabilities to handle more complex decision-making scenarios.

In contrast to Grey Relational Analysis (GRA), which compares alternatives with the optimal alternative, the EDAS method evaluates alternatives by comparing them with the average performance of all alternatives. The steps for applying this method are outlined below.

Step 1. Construct the hierarchical criteria model, as illustrated in Figure 5.2.

Step 2. Create a decision matrix, as defined in Equation (8.12).

Step 3. Determine the average alternative performance. The average performance value of each alternative is calculated as the arithmetic mean of its scores across all criteria.

$$\overline{D} = \begin{pmatrix} \overline{d}_{i1} & \overline{d}_{i2} & \cdots & \overline{d}_{im} \end{pmatrix}, \tag{11.1}$$

where $\overline{d}_{ij} = \frac{1}{n} \sum\limits_{i=1}^{n} d_{ij}$.

Step 4. Compute the distances from the average alternative performance. Both positive and negative distances are computed, taking into account whether the criteria are beneficial or cost-related.

a. The positive distance from the average alternative performance is as follows:

$$D^{+} = \begin{pmatrix} d^{+}_{1,1} & d^{+}_{1,2} & \cdots & d^{+}_{1,n} \\ d^{+}_{2,1} & d^{+}_{2,2} & \cdots & d^{+}_{2,n} \\ \vdots & \vdots & \ddots & \vdots \\ d^{+}_{m,1} & d^{+}_{m,2} & \cdots & d^{+}_{m,n} \end{pmatrix}, \tag{11.2}$$

where $d^{+}_{ij} = \frac{\max(0,(d_{ij}-\bar{d}_j))}{\bar{d}_j}$ and $d^{+}_{ij} = \frac{\max(0,(\bar{d}_j-d_{ij}))}{\bar{d}_j}$ are beneficial and cost criteria, respectively.

b. The negative distance from the average alternative performance is calculated as follows:

$$D^{-} = \begin{pmatrix} d^{-}_{1,1} & d^{-}_{1,2} & \cdots & d^{-}_{1,m} \\ d^{-}_{2,1} & d^{-}_{2,2} & \cdots & d^{-}_{2,m} \\ \vdots & \vdots & \ddots & \vdots \\ d^{-}_{n,1} & d^{-}_{n,2} & \cdots & d^{-}_{n,m} \end{pmatrix}, \tag{11.3}$$

where $d^{-}_{ij} = \frac{\max(0,(\bar{d}_j-d_{ij}))}{\bar{d}_j}$ and $d^{-}_{ij} = \frac{\max(0,(d_{ij}-\bar{d}_j))}{\bar{d}_j}$ are beneficial and cost criteria, respectively.

Step 5. Calculate the weighted sum of the distances. This is achieved through matrix multiplication, separately for positive and negative distances.

a. The weighted positive distance is as follows:

$$\otimes D^{*+} = \otimes W \times D^{+},$$

$$\otimes D^{+*} = \begin{pmatrix} \otimes w_1 \\ \otimes w_2 \\ \vdots \\ \otimes w_m \end{pmatrix}^{T} \times \begin{pmatrix} d^{+}_{1,1} & d^{+}_{1,2} & \cdots & d^{+}_{1,m} \\ d^{+}_{2,1} & d^{+}_{2,2} & \cdots & d^{+}_{2,m} \\ \vdots & \vdots & \ddots & \vdots \\ d^{+}_{n,1} & d^{+}_{n,2} & \cdots & d^{+}_{n,m} \end{pmatrix}.$$

b. The weighted negative distance is as follows:

$$\otimes D^{*-} = \otimes W \times D^{-},$$

$$\otimes D^{-*} = \begin{pmatrix} \otimes w_1 \\ \otimes w_2 \\ \vdots \\ \otimes w_m \end{pmatrix}^{T} \times \begin{pmatrix} d^{-}_{1,1} & d^{-}_{1,2} & \cdots & d^{-}_{1,m} \\ d^{-}_{2,1} & d^{-}_{2,2} & \cdots & d^{-}_{2,m} \\ \vdots & \vdots & \ddots & \vdots \\ d^{-}_{n,1} & d^{-}_{n,2} & \cdots & d^{-}_{n,m} \end{pmatrix}.$$

Step 6. Normalize the values of the weighted sum. The normalized positive and negative weighted sums are obtained.

a. The positive weighted sum is as follows:

$$\otimes D'^{+} = \left(\otimes d_1'^{+} \quad \otimes d_2'^{+} \quad \cdots \quad \otimes d_n'^{+}\right)^T, \tag{11.4}$$

where
$$d_1'^{+} = \frac{\otimes D^{*+}}{\max\left(\overline{d}_i^{*+}\right)}.$$

b. The negative weighted sum is as follows:

$$\otimes D'^{-} = \left(\otimes d_1'^{-} \quad \otimes d_2'^{-} \quad \cdots \quad \otimes d_n'^{-}\right)^T, \tag{11.5}$$

where
$$d_1'^{-} = 1 - \frac{\otimes D^{*-}}{\max\left(\overline{d}_i^{*-}\right)}.$$

Step 7. Calculate the appraisal scores for each alternative.

$$\otimes A_i = \left(\otimes a_1 \quad \otimes a_2 \quad \cdots \quad \otimes a_n\right)^T,$$

where $\otimes a_i = \frac{1}{2}\left(\otimes d_1'^{+} + \otimes d_1'^{-}\right)$.

Step 8. Rank the alternatives based on their appraisal scores. To enhance the interpretability of the results, the appraisal scores are whitened using a parameterized equation (11.6), with the best alternative being selected based on the highest scores:

$$s_i = \underline{a}_i(1-\lambda) + \overline{a}_i\lambda, \tag{11.6}$$

where the whitenization coefficient $\lambda \in [0,1]$.

The steps outlined above represent the EDAS Multiple Criteria Decision-Making (MCDM) evaluation method using grey weights. While this approach may entail increased computational complexity, it offers the advantage of accounting for uncertainty within a group decision-making context. Furthermore, the Grey SWARA-FUCOM method can be seamlessly integrated with other MCDM evaluation methods for enhanced decision support.

11.2. EDAS for Solar Panel Contractor Selection

This analysis comprises two main components: first, the application of weights using the Grey SWARA-FUCOM method, and second, evaluation based on the GRA and EDAS methods with grey numbers (GNs). A flowchart of this analysis is presented in Figure 11.2.

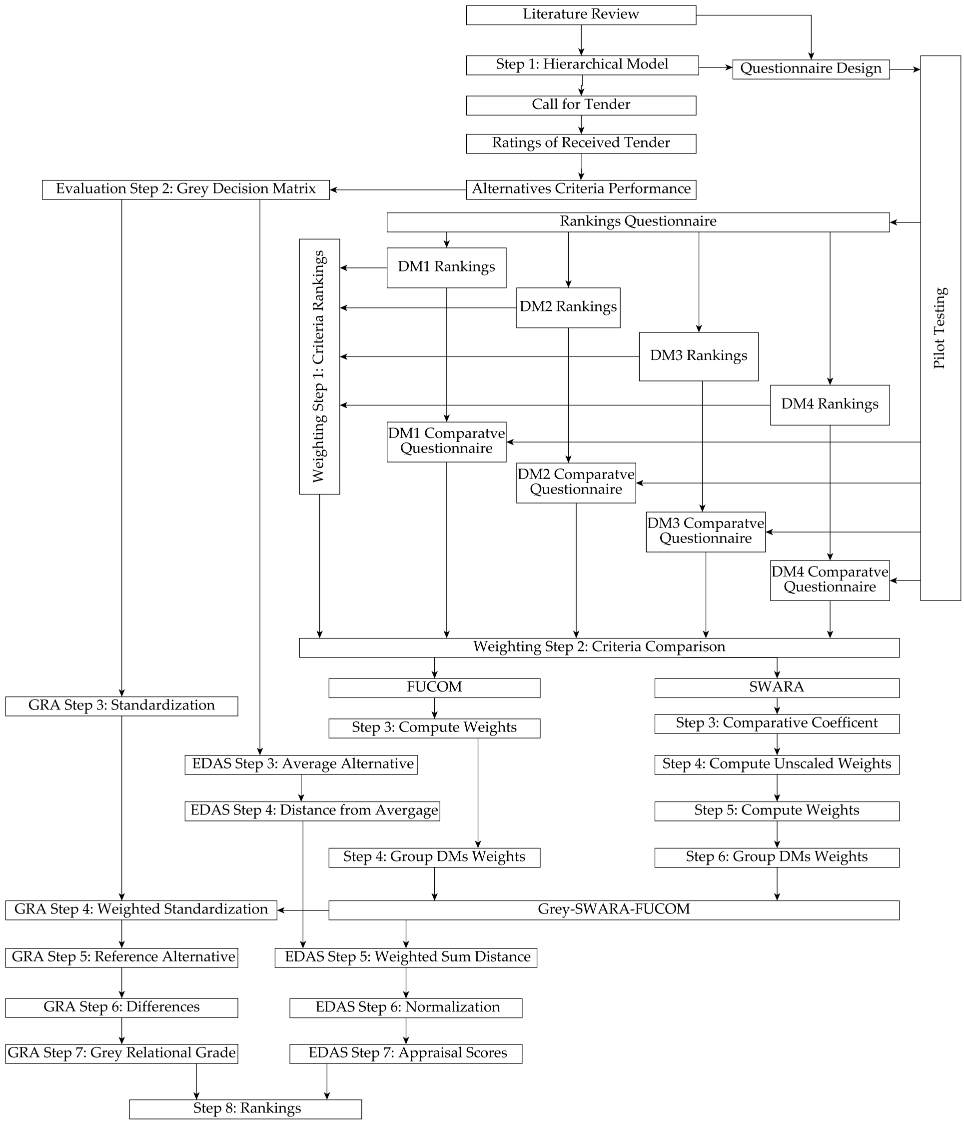


Figure 11.2. Flowchart of the Grey-SWARA-FUCOM with GRA and EDAS. Source: Reprinted from [102], used with permission.

The flowchart beings with a comprehensive literature review, a pivotal part of this research, which informed the design of both the hierarchical model in Figure 5.2 and the questionnaires. Following this, a call for tender was formulated based on the hierarchical model as the basis for contractor requirements. Subsequently, the contract awarding committee assigned scores to the submitted bids. Customized questionnaires for ranking and pairwise comparisons were meticulously designed and pilot-tested before their utilization. The initial round of questionnaires involved ranking the evaluation criteria. Subsequently, the second round of questionnaires comprised customized comparative assessments tailored to individual Decision

Makers (DMs) rankings. These assessments were used to estimate group DM weights employing the Grey SWARA-FUCOM method outlined in Section 6.4. Finally, the computed weights in Section 6.5.1 given in Equation (6.17) were applied in the evaluation and ranking of contractors using the EDAS methods.

The hierarchical diagram is depicted in Figure 5.2, and the decision matrix is provided in Equation (8.34). Subsequently, the average performance value of contractors is computed, representing the arithmetic mean of all criteria scores:

$$\overline{D} = \begin{pmatrix} \overline{d}_{i,1} & \overline{d}_{i,2} & \overline{d}_{i,3} & \cdots & \overline{d}_{i,30} \end{pmatrix} = \begin{pmatrix} 79.7500 & 74.0000 & 73.4375 & \cdots & 77.3950 \end{pmatrix}.$$

Computation of distances from the average. The distances from the average are calculated in the following manner:

a. Positive distance from the average:

$$D^{+} = \begin{pmatrix} d^{+}_{1,1} & d^{+}_{1,2} & d^{+}_{1,3} & \cdots & d^{+}_{1,30} \\ d^{+}_{2,1} & d^{+}_{2,2} & d^{+}_{2,3} & \cdots & d^{+}_{2,30} \\ d^{+}_{3,1} & d^{+}_{3,2} & d^{+}_{3,3} & \cdots & d^{+}_{3,30} \\ d^{+}_{4,1} & d^{+}_{4,2} & d^{+}_{4,3} & \cdots & d^{+}_{4,30} \end{pmatrix} = \begin{pmatrix} 0.0815 & 0.1115 & 0.1234 & \cdots & 0.0579 \\ 0 & 0 & 0 & \cdots & 0 \\ 0 & 0.0473 & 0.0043 & \cdots & 0.0744 \\ 0.0564 & 0.0811 & 0.0894 & \cdots & 0.0413 \end{pmatrix}.$$

b. Negative distance from the average:

$$D^{-} = \begin{pmatrix} d^{-}_{1,1} & d^{-}_{1,2} & d^{-}_{1,3} & \cdots & d^{-}_{1,30} \\ d^{-}_{2,1} & d^{-}_{2,2} & d^{-}_{2,3} & \cdots & d^{-}_{2,30} \\ d^{-}_{3,1} & d^{-}_{3,2} & d^{-}_{3,3} & \cdots & d^{-}_{3,30} \\ d^{-}_{4,1} & d^{-}_{4,2} & d^{-}_{4,3} & \cdots & d^{-}_{4,30} \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & \cdots & 0 \\ 0.1222 & 0.2399 & 0 & \cdots & 0.2086 \\ 0.0157 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 \end{pmatrix}.$$

Next, the weighted sums of the distances are determined as follows:

a. Weighted sum of positive distance:

$$\otimes D^{*+} = \otimes W \times D^{+}$$

$$\otimes D^{+*} = \begin{pmatrix} \otimes w_1 \\ \otimes w_2 \\ \vdots \\ \otimes w_{30} \end{pmatrix}^{T} \times \begin{pmatrix} d^{+}_{1,1} & d^{+}_{1,2} & d^{+}_{1,3} & \cdots & d^{+}_{1,30} \\ d^{+}_{2,1} & d^{+}_{2,2} & d^{+}_{2,3} & \cdots & d^{+}_{2,30} \\ d^{+}_{3,1} & d^{+}_{3,2} & d^{+}_{3,3} & \cdots & d^{+}_{3,30} \\ d^{+}_{4,1} & d^{+}_{4,2} & d^{+}_{4,3} & \cdots & d^{+}_{4,30} \end{pmatrix}$$

$$= \begin{pmatrix} [0.0617, 0.0975] & [0, 0] & [0.0128, 0.0210] & [0.0349, 0.0533] \end{pmatrix}^{T}.$$

b. Weighted sum of positive distance:

$$\otimes D^{*-} = \otimes W \times D^{-},$$

$$\otimes D^{-*} = \begin{pmatrix} \otimes w_1 \\ \otimes w_2 \\ \vdots \\ \otimes w_{30} \end{pmatrix}^{T} \times \begin{pmatrix} d^{-}_{1,1} & d^{-}_{1,2} & d^{-}_{1,3} & \cdots & d^{-}_{1,30} \\ d^{-}_{2,1} & d^{-}_{2,2} & d^{-}_{2,3} & \cdots & d^{-}_{2,30} \\ d^{-}_{3,1} & d^{-}_{3,2} & d^{-}_{3,3} & \cdots & d^{-}_{3,30} \\ d^{-}_{4,1} & d^{-}_{4,2} & d^{-}_{4,3} & \cdots & d^{-}_{4,30} \end{pmatrix}$$

$$= \begin{pmatrix} [0.0001, 0.0001] & [0.1029, 0.1609] & [0.0057, 0.0095] & [0.0007, 0.0013] \end{pmatrix}^{T}.$$

Following that, the normalized values for both positive and negative distances are computed using Equation (11.4) and Equation (11.5), respectively:

a.

$$\otimes D'^{+} = \left(\otimes d_1'^{+} \quad \otimes d_2'^{+} \quad \otimes d_3'^{+} \quad \otimes d_4'^{+}\right)^T$$

$$= \left([0.6329, 1] \quad [0, 0] \quad [0.1317, 0.2158] \quad [0.3579, 0.5470]\right)^T,$$

b.

$$\otimes D'^{-} = \left(\otimes d_1'^{-} \quad \otimes d_2'^{-} \quad \cdots \quad \otimes d_n'^{-}\right)^T$$

$$= \left([0.9991, 0.9992] \quad [0, 0.3606] \quad [0.9412, 0.9647] \quad [0.9922, 0.9956]\right)^T.$$

Finally, the appraisal scores are calculated as follows:

$$\otimes S_i = \left(\otimes s_1 \quad \otimes s_2 \quad \oplus s_3 \quad \otimes s_4\right)^T$$

$$= \left([0.8160, 0.9996] \quad [0, 0.1803] \quad [0.5364, 0.5902] \quad [0.6751, 0.7713]\right)^T.$$

A whitenization coefficient of 0.5, denoted as $\lambda = 0.5$, is used to compute the final scores for each contractor: $s_1 = 0.9078, s_2 = 0.0901, s_3 = 0.5633, s_4 = 0.7232$. $s_1 > s_4 > s_3 > s_2$; $r_1 > \mathrm{r}_4 > r_3 > r_2$, the $A_1 \succ A_4 \succ A_3 \succ A_2$, Now, let's compare the scores to determine the rankings. Higher scores indicate better rankings: $s_1 > s_4 > s_3 > s_2$. In terms of rankings, the following apply: $r_1 > r_4 > r_3 > r_2$. Therefore, based on the results obtained using both the GRA and EDAS methods, and considering the grey SWARA-FUCOM weights, it can be concluded that contractor A_1 is the best choice to be awarded the project. This ranking is consistent between both methods, reinforcing the selection of contractor A_1 as the top choice.

Based on the subjective opinions and ratings of the DMs, the most crucial criterion for the installation of a floating solar panel energy system is *Life Cycle Assessment* ($\Psi_{6\text{-}4}$). This criterion involves a holistic evaluation of the system's components, from the environmental impact of raw material extraction to its end-of-life considerations. The assessment specifies the impacts and effects of the contractors' solutions on climate change, human health, ecosystem quality, and non-renewable resources. This enables DMs to mitigate the negative impact of new products, identify areas for improvement in existing components, and avoid modifications that could lead to significant issues later in the solar panel's lifespan. Additionally, it allows for the comparison of the environmental footprint of similar solutions. The second most important criterion, according to the DMs, is *Technical Staff Experience* ($\Psi_{2\text{-}5}$). This criterion highlights the significance of having a contractor with a skilled and experienced technical team. Following closely in importance is the criterion *Contractor's Quality Performance* ($\Psi_{2\text{-}1}$). This criterion emphasizes the importance of the contractor's track record in delivering high-quality work.

Conversely, the least important criterion, as determined by the DMs using the grey SWARA-FUCOM method, is *Financing and Investment* ($\Psi_{1\text{-}1}$). This may be attributed to the fact that the DMs are Chinese, and in general, the Chinese economy is robust with access to ample funds for capital investment projects. China's economic growth has allowed the nation to make substantial financial decisions, such as establishing the Asia Infrastructure Investment Bank (AIIB), which has the potential to rival the International Monetary Fund (IMF). Consequently, large-scale projects in China often encounter fewer funding obstacles compared to projects in less developed countries. The second least important criterion, according to the DMs' perceptions, is *Progress Cost Control* ($\Psi_{3\text{-}5}$). Interestingly, the company involved in this project prioritizes making a positive environmental impact over immediate cost savings, which aligns with the DMs' preferences.

11.3. TOPSIS-Grey

In the realm of MCDM, various studies have integrated Grey Systems Theory (GST) into different decision-making methods. For example, researchers such as Hsu et al. [330] proposed a hybrid approach that combined Decision Making Trial and Evaluation Laboratory (DEMATEL), Analytical Network Process (ANP), andmodified GRA for outsourcing airline provider services in Taiwan. They aimed to incorporate real-world realities into their decision-making process by modifying the traditional GRA method. Zakeri and Keramati [331] applied both fuzzy and grey TOPSIS in the selection of electrical wire manufacturers. Nguyen et al. [332] presented a hybrid MCDM method that combined fuzzy ANP and COPRAS with GRA for selecting machine tools. They compared the evaluation results using TOPSIS-Grey and SAW-Grey hybrid methods, as well as the GRA method. Kirubakaran and Ilangkumaran [333] combined fuzzy AHP, GRA, and TOPSIS techniques to choose the best maintenance scheme for pumps used in paper manufacturing. Fuzzy AHP was utilized to determine the criteria weights, while the GRA-TOPSIS method was employed to evaluate the maintenance schemes. Yazdani et al. [334] integrated Quality Function Deployment (QFD) with GRA to demonstrate core supply chain criteria in an uncertain environment, particularly in an agricultural-production system project.

TOPSIS-Grey is a method used to evaluate alternatives in an uncertain environment. It achieves this by comparing the distances of each alternative from both the positive and negative ideal solutions. In essence, the best alternative is the one with the shortest grey distance to the positive ideal solution and the longest distance from the negative ideal solution. The steps involved in TOPSIS-Grey are as follows:

Step 1. Construct the grey decision matrix. The grey decision matrix is directly obtained from the data table and represented in Equation (9.6).

Step 2. Normalize the grey decision matrix. The normalization process ensures that all criteria, whether they are benefit or cost criteria, are transformed into a common scale for further analysis. Decision matrix D is normalized to obtain a normalized grey decision matrix, $\widehat{D}$. Each element, $\otimes d^*_{i,j}$, in the normalized grey decision matrix represents the normalized value for the benefit criterion at the intersection of alternative i and criterion j. Benefit and cost preference scores are normalized as shown in Equation (9.7) and (9.8), respectively.

Step 3. Calculate the weighted decision matrix. The criteria weights are represented as a column vector, W, where each element $\otimes w_i$ corresponds to the weight of criterion i,

$$W = \begin{pmatrix} \otimes w_1 & \otimes w_2 & \cdots & \otimes w_n \end{pmatrix}^T.$$

The weighted decision matrix is Equation (11.7):

$$D^* = \begin{pmatrix} \otimes d^*_{1,1} & \otimes d^*_{1,2} & \cdots & \otimes d^*_{1,n} \\ \otimes d^*_{2,1} & \otimes d^*_{2,2} & \cdots & \otimes d^*_{2,n} \\ \vdots & \vdots & \ddots & \vdots \\ \otimes d^*_{m,1} & \otimes d^*_{m,2} & \cdots & \otimes d^*_{m,n} \end{pmatrix}. \tag{11.7}$$

Here $\otimes d^*_{ij} = d'_{ij} \times \otimes w_{ij}$. The series can be written in vector form as in Equation (9.4).

Step 4. Compute the ideal positive and negative ideal solution. These positive and negative ideal solutions provide reference points for evaluating the alternatives in the decision-making process.

(a) The positive ideal solution (D^+) is calculated using Equation (11.8):

$$D^+ = \{\otimes d^+_1, \otimes d^+_2, ..., \otimes d^+_n\}, \tag{11.8}$$

where $\otimes d^+_j = \left[\max_{1 \le i \le m} \underline{d^*_{ij}}, \max_{1 \le i \le m} \overline{d^*_{ij}}\right]$.

(b) The negative ideal solution (D^-) is calculated using Equation (11.9):

$$D^- = \{\otimes d^-_1, \otimes d^-_2, ..., \otimes d^-_n\}, \tag{11.9}$$

where $\otimes d^-_j = \left[\min_{1 \le i \le m} \underline{d^*_{ij}}, \min_{1 \le i \le m} \overline{d^*_{ij}}\right]$.

Step 5. Compute the separation from the ideal. The separation from the ideal for each alternative is calculated from both the positive and negative directions. Here, the arbitrary distances are used as given in Equation (1.5):

(a) For the positive ideal point (D^+_i),

$$D^+_i = \left(\sum_{i=1}^{n} \left(\otimes d^*_{ij} - \otimes d^+_j \right)^{\mu} \right)^{\frac{1}{\mu}}. \tag{11.10}$$

(b) For the negative ideal point (D^-_i),

$$D^-_i = \left(\sum_{i=1}^{n} \left(\otimes d^*_{ij} - \otimes d^-_j \right)^{\mu} \right)^{\frac{1}{\mu}}, \tag{11.11}$$

where the parameter μ determines the type of distance measurement used. In most cases, Euclidean distance ($\mu = 2$) is applied.

Step 6. Calculate similarities to the positive ideal solution. The similarities to the positive ideal solution are computed using Equation (11.12) as follows:

$$T_i = \frac{D_i^-}{D_i^- + D_i^+}. \quad (11.12)$$

11.4. TOPSIS-G for HRIS

The TOPSIS method was developed by Hwang [39] and was extended to GST Zavadskas et al. [280]. The steps for the Technique for Order of Preference by Similarity to Ideal Solution with Grey values (TOPSIS-G) are as follows.

Step 1. Construct the grey decision matrix using Equation (9.14).

Step 2. Normalize the grey decision matrix:

$$\widetilde{D} = \begin{pmatrix} [0.6211, 0.7895] & [0.6667, 0.7778] & [0.6105, 0.8421] & \ldots [0.7778, 0.9444] \\ [0.6316, 0.8421] & [0.7222, 1] & [0.7368, 0.8947] & \ldots [0.7778, 0.8889] \\ [0.8421, 0.9474] & [0.6667, 0.9444] & [0.7368, 1] & \ldots [0.7778, 1] \\ [0.7368, 1] & [0.7222, 1] & [0.6316, 0.9053] & \ldots [0.8889, 1] \\ [0.6316, 0.8947] & [0.7778, 0.8889] & [0.6316, 0.9474] & \ldots [0.7778, 0.9444] \end{pmatrix}, \quad (11.13)$$

where $\otimes d_{ij}^* = \dfrac{\otimes d_{ij}^*}{\max\limits_{1 \le i \le 5} \overline{d}_{ij}}$.

Step 3. Calculate the Weighted Normalized Grey Decision Matrix: $\otimes d_{ij}^* = d'_{ij} \times \otimes w_{ij}$.
Step 4. Compute the positive and negative ideal solutions.

(a) The positive ideal solution is as follows:

$$\begin{aligned} D^+ &= \{\otimes d_1^+, \otimes d_2^+, ..., \otimes d_{27}^+\} \\ &= \{[0.0058, 0.1248], [0.01, 0.0607], [0.0095, 0.0556], \ldots [0.0005, 0.0188]\}, \end{aligned} \quad (11.14)$$

where $\otimes d_j^+ = \left[\max\limits_{1 \le i \le 5} \underline{d_{ij}^*}, \max\limits_{1 \le i \le 5} \overline{d_{ij}^*}\right]$.

(b) The negative ideal solution is as follows:

$$\begin{aligned} D^- &= \{\otimes d_1^-, \otimes d_2^-, ..., \otimes d_{27}^-\} \\ &= [0.0043, 0.0985], [0.0086, 0.0472], [0.0079, 0.0468], \ldots [0.0005, 0.0167]\}, \end{aligned} \quad (11.15)$$

where $\otimes d_j^* = \left[\min\limits_{1 \le i \le 5} \underline{d_{ij}^*}, \min\limits_{1 \le i \le 5} \overline{\hat{d}_{ij}^*}\right]$.

Step 5. Compute the separation from the ideal solutions. Both positive and negative distances are obtained.

(a) The positive ideal points are as follows:

$$\begin{aligned} D^{+} &= \begin{pmatrix} D_1^{+} & D_2^{+} & D_3^{+} & D_4^{+} & D_5^{+} \end{pmatrix}^T \\ &= \begin{pmatrix} 0.2862 & 0.2912 & 0.2998 & 0.3026 & 0.2965 \end{pmatrix}^T, \end{aligned} \tag{11.16}$$

where $D_i^{+} = \left(\frac{1}{2} \sum_{i=1}^{n} \left(\otimes d_{ij}^{*} - \otimes d_j^{+} \right)^{\mu} \right)^{\frac{1}{\mu}}$ and μ represent the type of distance. The Euclidean distance ($\mu = 2$) is usually used.

(b) The negative ideal points are as follows:

$$\begin{aligned} D^{-} &= \begin{pmatrix} D_1^{-} & D_2^{-} & D_3^{-} & D_4^{-} & D_5^{-} \end{pmatrix}^T \\ &= \begin{pmatrix} 0.2588 & 0.2642 & 0.2749 & 0.2785 & 0.2707 \end{pmatrix}^T, \end{aligned} \tag{11.17}$$

where $D_i^{-} = \left(\frac{1}{2} \sum_{i=1}^{n} \left(\otimes d_{ij}^{*} - \otimes d_j^{-} \right)^{\mu} \right)^{\frac{1}{\mu}}$.

Step 6. Calculate the similarities to the positive ideal solution. Calculate the similarity of the Human Resource Information System (HRIS) software to the positive ideal software using Equation (11.18):

$$\begin{aligned} T &= \begin{pmatrix} 0.4749 & 0.4757 & 0.4783 & 0.4792 & 0.4772 \end{pmatrix} \\ &\approx \begin{pmatrix} 5\text{th} & 4\text{th} & 2\text{nd} & 1\text{st} & 3\text{rd} \end{pmatrix}^T, \end{aligned} \tag{11.18}$$

where $T_i = \frac{D_i^{-}}{D_i^{-} + D_i^{+}}$.

Based on the ranks assigned by the evaluation methods, Spearman's correlation was computed. The rankings from the Grey REGIME method, GRA, and GWSM were found to be perfectly correlated. However, the rankings from the Grey REGIME method and TOPSIS-G had a Spearman's correlation of 0.9. Therefore, when considering the results from the proposed grey regime, GRA, GWSM, and TOPSIS-G methods, the order of preference for the HRIS vendors is as follows:

$$A_3 > A_4 > A_5 > A_2 > A_1.$$

According to the preferences of the decision-makers, the HRIS provided by the third vendor, A_3, is the most preferred, while the one provided by the first vendor, A_1, is the least preferred.

The results indicate that researchers and practitioners have differing preferences when it comes to assigning points to evaluation criteria and selecting MCDM methods. While researchers may prioritize accuracy and complexity in their methods, Human Resource (HR) managers within organizations may prefer a more straightforward approach for estimating the weights of evaluation criteria. It is important to note that the advancements made by researchers in the field of

MCDM can provide valuable insights and tools for HR practitioners. However, many individuals, including HR managers, may lack the necessary skills to fully understand and apply these newly developed MCDM methods found in the academic literature. In practice, advanced MCDM methods, such as those involving complex computational tools like IBM CPLEX and MATLAB, are rarely used by HR managers for their day-to-day decision-making. Instead, organizations may rely on experts from other departments who have the expertise to handle these complex computations. As a result, there is often a gap between the advanced methods developed in academia and their practical application in HR management. Therefore, it would be beneficial for academia to take an additional step by providing user-friendly interfaces, in the form of computer programs or software, that allow HR practitioners to input their data into the model and receive intuitive and easily interpretable outputs. This would bridge the gap between academic research and practical use, making advanced MCDM methods more accessible and applicable to HR professionals.

Furthermore, given that the successful operation and survival of an enterprise are closely tied to Human Resource Management (HRM), the selection of an HRIS to support the company's operations should be approached with utmost seriousness, utilizing improved evaluation methods developed in the academic literature. It is important to recognize that an HRIS is a tool, and its deployment should adhere to the industry's best practices to prevent implementation failures. Additionally, it is crucial to remember that HRIS encompasses both HRM and Management Information Systems (MISs), meaning that the HR and IT departments must collabourate closely. From a managerial perspective, it is essential to ensure the continuity of HRM practices alongside the enforcement of IT practices. This can be achieved by directly assigning a proficient IT specialist to the HR department, who would take full responsibility for maintaining the system. For instance, the IT department can oversee the implementation of a fault-tolerant HRIS server, ensuring redundancy in the system. Simultaneously, the HR department should independently create and regularly test backups of its departmental data. This is particularly critical because a company's strategies heavily rely on HR data, and the loss of employee records can lead to a preventable disaster. To reiterate, fault tolerance as Redundant Array of Independent Disk (RAID) should not be mistaken for a backup solution; the HR department should fully own the responsibility of backing up data and securely storing those backups, while the IT department can focus on ensuring uninterrupted system operation. This collabourative effort ensures the reliability and effectiveness of the HRIS while safeguarding crucial HR-related information.

HRISs offer the advantage of centralized and accurate record-keeping. There are two primary deployment options to consider: First are the on-premise HRISs. In this scenario, an HRIS is installed and hosted on the company's premises, allowing for remote accessibility through a Virtual Private Network (VPN). Second are cloud-based HRISs. Alternatively, an HRIS can be deployed as a cloud-based service. This approach offers the benefit of easy updates and minimal effort in managing the IT infrastructure that supports the software. However, it is crucial to carefully evaluate the risks associated with both approaches. It is important to keep in mind that employee information and business strategies are sensitive and confidential company assets. When deploying critical data on the cloud, this means that core aspects of the company's business are managed on another company's servers. In this context, maintaining data integrity and security is of utmost importance. Moreover, it is advisable to ensure that data stored in the cloud can also be imported and restored locally on a test server if needed. This ensures that the company retains control and access to its data, even in the event of unforeseen circumstances or the need to transition away from cloud-based services.

Certainly, aligning an HRIS with organizational objectives is crucial for enhancing operational efficiency, and this entails substantial implications for managerial practices. Beyond software selection, effective change management is imperative throughout the implementation journey. This involves vigilant monitoring of software performance during deployment, ensuring it continuously aligns with organizational needs. Transitioning from an old system to the new HRIS can be smoother when both systems operate concurrently, minimizing disruptions. Employee training, ideally integrated into the onboarding process, is vital to encouraging the use of self-service HR functions, empowering staff to manage HR tasks efficiently. Moreover, training should extend to those who train others within the organization, as their expertise in software deployment and HR processes is invaluable. Ultimately, this holistic approach ensures that the HRIS not only meets organizational objectives but also becomes a valuable and profitable investment for the company.

Nonetheless, while an HRIS offers numerous benefits and enhances efficiency, it can also present challenges. Specifically, if a company's growth does not justify the current size of the HR department, an HRIS can lead to downsizing as fewer staff may be needed to handle HR tasks. In this context, it is essential for management to have a strategic plan in place to facilitate a seamless downsizing process within the HR department when necessary. This plan could involve transferring affected staff to other departments and halting the recruitment of new HR personnel whose responsibilities have been automated. Ultimately, employees who are resistant to change, clinging to old practices and using the HRIS only partially, may face the possibility of job displacement within the company.

Finally, despite the accelerated decision-making facilitated by an HRIS, there may still be instances where the company's internal data alone are insufficient to make robust probabilistic and statistical decisions. In such scenarios, methodologies from Grey Systems Theory (GST) designed for decision-making under uncertainty can be applied, as Grey Systems Theory (GST) specializes in addressing decision problems with limited information. The simplicity and quick assessment capability of the Point Allocation (PA) method suggest that it will likely remain in use in the foreseeable future. However, modern MCDM weighting methods, like FUCOM, offer computational efficiency advantages over traditional techniques such as AHP, which have been in use for decades. Therefore, the managerial implication is that while top-level management may use PA for rapid decision-making, it is crucial for the HR department to incorporate uncertainty considerations into their decision-making processes. In this context, the Grey REGIME method is one of the grey hybrid MCDM methods that facilitates pairwise comparisons of decision alternatives, offering a valuable tool for addressing uncertainty in HRIS-related decisions.

12. Conclusions

In this concluding chapter, a general overview of Grey Systems Theory (GST) is presented. Furthermore, conclusions related to the five cases presented in the book are illustrated in Figure 12.1. To enhance the clarity of the conclusions, some sensitivity analysis is incorporated.

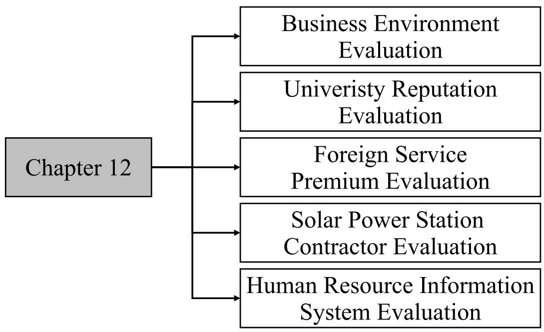


Figure 12.1. Flowchart of Chapter 12. Source: Figure by authors.

The foreign service premium allowance and the ranking of overseas branches of a company were determined using the hybrid method, Stepwise Weight Analysis Ratio Assessment (SWARA) method, and Grey Relational Analysis (GRA) with Grey Numbers (GNs). These results were obtained from a case study of an international company, which was found to be satisfying for both top management and the staff union. A numerical example was presented for the evaluation of the reputation of five universities using data obtained from 1149 students in Shaanxi Province, China. The most reputable university based on the sample data, the weighting and evaluation methods was consistent with other ranking indexes. Furthermore, the grey Regulatory Focus Theory (RFT) weighting method was applied to rank four universities in Xi'an city of China based on 1149 students' responses. Also, the most reputable university was consistent with other established ranking indexes. The ranking of contractors was found to be the same using both methods, confirming the results presented in this book. The use of the grey SWARA-Full Consistent Method (FUCOM) weighting method, combined with the GRA and Evaluation based on Distance from Average Solution (EDAS) methods, increased the confidence of Decision Makers (DMs) in determining the installation of the solar panel energy system as the top-ranked contractor. Finally, to validate the results of this study, grey relational analysis with grey numbers, the grey weighted sum model, and a technique for order performance based on the similarity to the ideal solution with grey values were employed.

12.1. Sensitivity Analysis

Sensitivity analysis is conducted to demonstrate the robustness and the extent of uncertainty encompassed by the results of the GWSM and ILP-GP. Within this analysis, ranges of values for the input parameters and coefficients are determined, such that they will not impact the rank value. Furthermore, a deeper understanding of the results is gained through sensitivity analyses involving periodic variations, whitening, and distance measurement.

12.1.1. Period Sensitivity on Rankings

Grey interval numbers were employed to represent the values of indicators from the year 2008 to 2015, spanning a period of 8 years. However, the effects of representing shorter periods, such as 6 years (2010–2015) and 3 years (2012–2015), with grey numbers are examined.

Firstly, a period sensitivity analysis on rankings using GWSM is conducted. In Figure 12.2, the period sensitivity on rankings is depicted. For the period from 2008 to 2015, South Africa, Mauritius, and Ghana held the first, second, and third positions, respectively. However, during the period from 2010 to 2015 and 2012 to 2015, Mauritius, Botswana, and Ghana occupied the second, third, and fifth positions, respectively. Burundi demonstrated the highest sensitivity to period changes, as it was ranked 49th for the years 2008–2015 but progressed to the 10th position for the years 2012–2015. The rankings of the Democratic Republic of Congo, Libya, and South Sudan remained unchanged for all periods, in the 51st, 52nd, and 53rd position, respectively.

Secondly, we conducted period sensitivity analysis on rankings using the ILP-GP approach. South Africa was consistently ranked as the top country for all the periods considered (2008–2015, 2010–2015, and 2012–2015). For shorter evaluation periods (2010–2015 and 2012–2015), Mauritius and Botswana held the second and third positions, as shown in Figure 12.3. However, over longer evaluation periods, Ghana and Mauritius are ranked in the second and third positions, respectively. Mauritania exhibits the most significant changes in rankings, as it is ranked 36th for the period from 2008 to 2015 but 6th for the period from 2010 to 2015. Libya, Cameroon, and Chad maintain the 53rd position for all the periods under consideration.

Both the GWSM and ILP-GP methods are sensitive to time, with the GWSM being the most sensitive to time within the context of the African business environment. Long-term investment decisions should be grounded in results obtained from longer periods, while short-term investment decisions should be based on results derived from shorter periods.

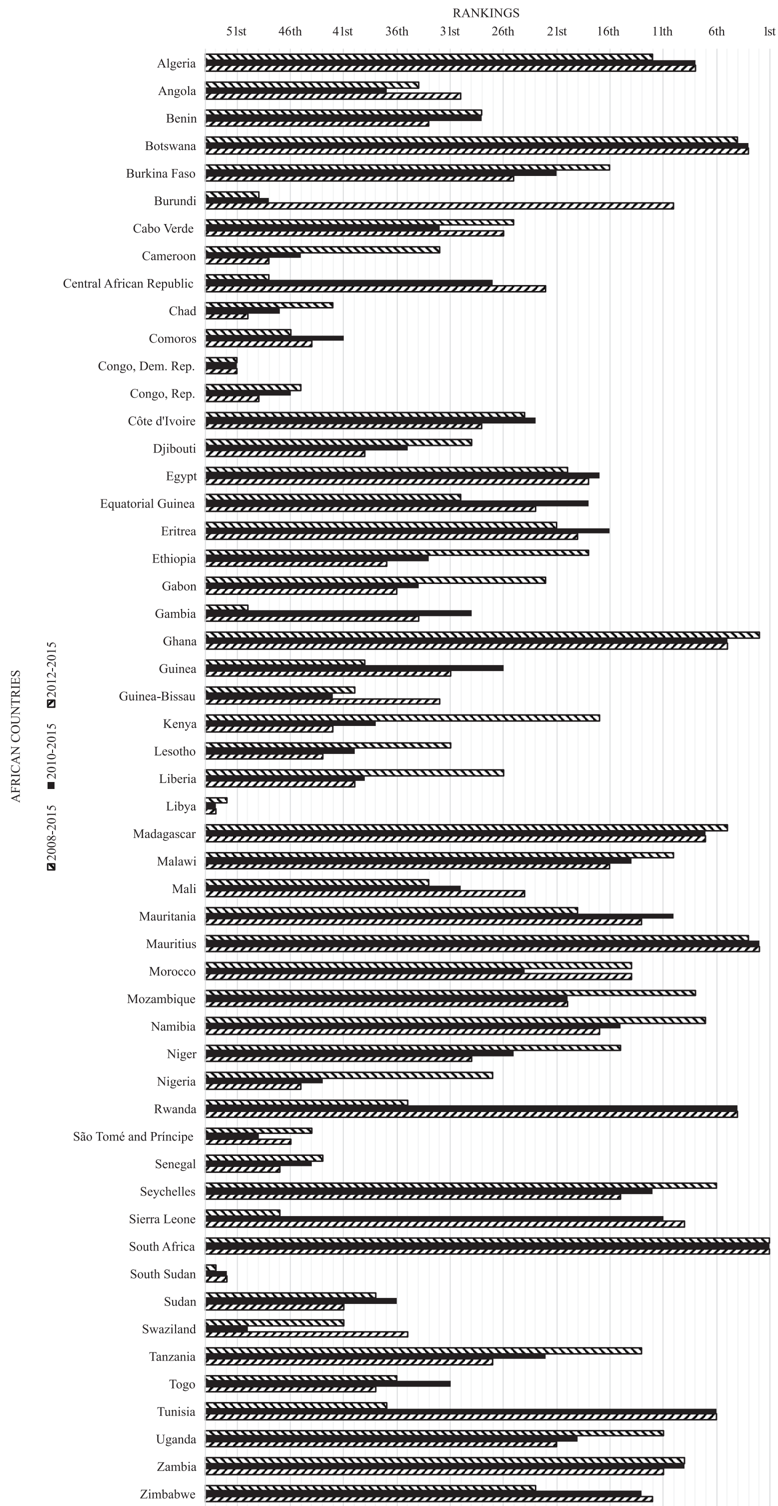


Figure 12.2. Period sensitivity based on GWSM. Source: Reprinted from [204], used with permission.

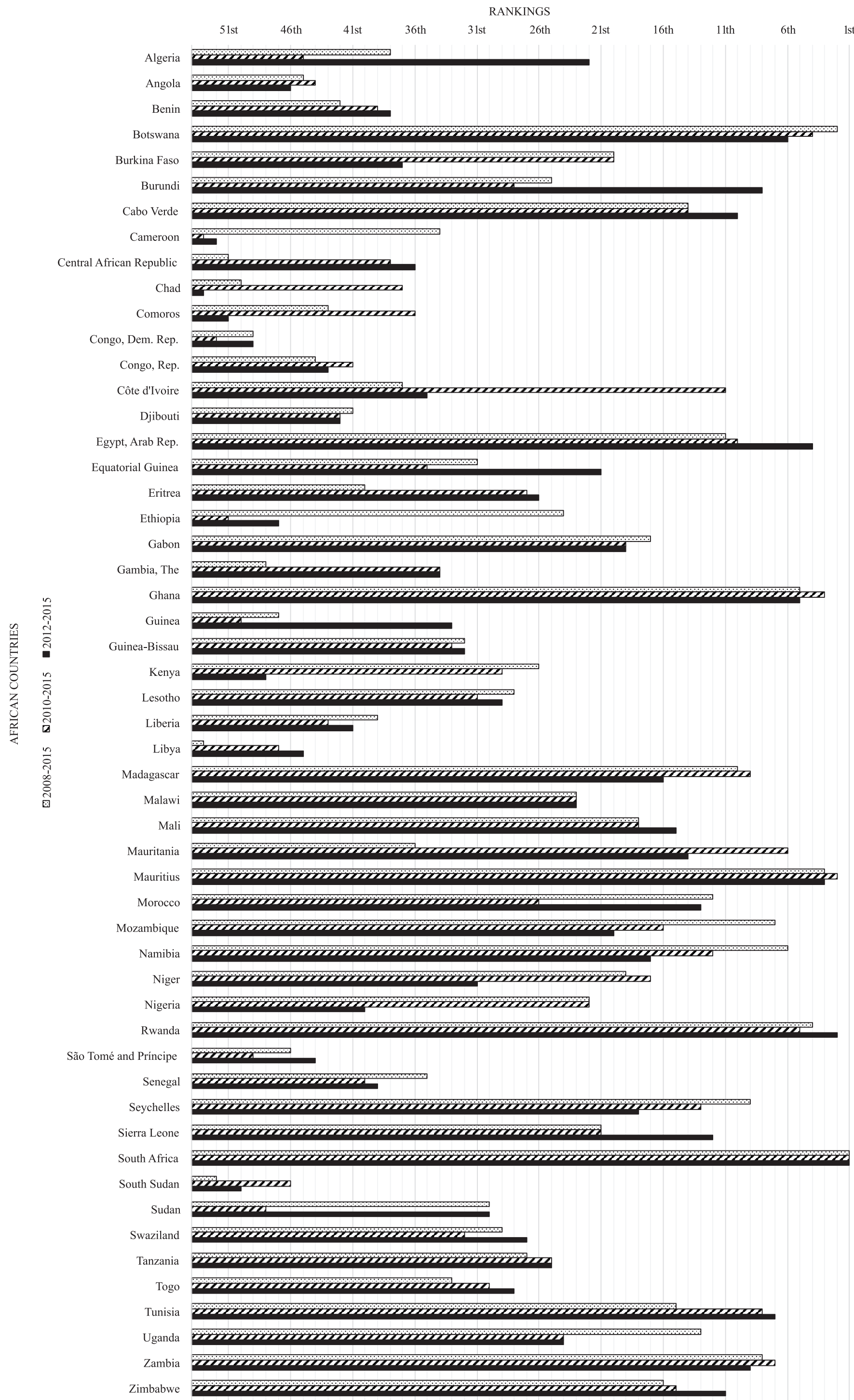


Figure 12.3. Period sensitivity based on ILP-GP. Source: Reprinted from [204], used with permission.

12.1.2. Whitenization Sensitivity on Rankings for GWSM

The whitenization (also known as whitening) sensitivity analysis is exclusively applied to the GWSM method. For a GN $\otimes A = [\underline{a}, \overline{a}]$, the whitenization value is defined as $\otimes\tilde{A} = \underline{a}\lambda + (1-\lambda)\overline{a}, \lambda \in [0,1]$. When the whitenization coefficient is set to half, i.e., $\lambda = \frac{1}{2}$, it is referred to as equal-weight mean whitenization [95]. Since the whitenization value depends on the whitenization coefficient (λ), the coefficient is systematically increased in increments of 0.2, while keeping other input parameters constant. The results of the whitenization sensitivity analysis for various λ values, specifically 0, 0.2, 0.4, 0.6, 0.8, and 1, are provided in Table 12.1. This table illustrates the impact of (λ) on the rankings. Notably, the ranking of South Africa remains unaffected by changes in the whitenization coefficient values. As the whitenization coefficient increases, the rankings of Algeria, Botswana, Burundi, Cabo Verde, Egypt, Libya, Mauritius, and Zambia increase. In contrast, the rankings of Angola, Chad, Congo Dem. Rep., Djibouti, Ethiopia, Ghana, Kenya, Liberia, Madagascar, Mozambique, Namibia, São Tomé and Príncipe, Seychelles, Sudan, Tanzania, and Uganda decrease as the whitenization coefficient increases. Ethiopia demonstrates the most significant change in rankings, progressing from the 42nd position when $\lambda = 0$ to the 14th position when $\lambda = 1$.

12.1.3. Distance Sensitivity on Rankings for GWSM

The measurement of distance between the upper and lower bounds of a GN directly influences the level of uncertainty. The effects of distance measurement, considering the Manhattan, Euclidian, and Minkowski distance measurement methods on rankings using various distance measurements (p) based on Equation (12.1), are presented in Table 12.2.

$$S_i = \sqrt[p]{(\overline{y}_i)^p - \left(\underline{y}_i\right)^p} \underset{p\to\infty}{=} \overline{y}_i. \tag{12.1}$$

For the Manhattan and Euclidian distances, South Africa maintains the first position, but it drops to the sixth position when the Minkowski distance ($p = 3$) is employed. Likewise, Ghana remains in the second position for both Manhattan and Euclidian distances, but it advances to the first position with the Minkowski distance. As p increases, Mauritius regresses from the 3rd position to the 4th position and then to the 15th position using the Manhattan, Euclidean, and Minkowski distances, respectively. Madagascar, which is in the fifth position using the Manhattan distance, advances to the third position using the Euclidian distance and then becomes the second position with the Minkowski distance measurement. Botswana experiences the most significant change in rankings, dropping from the 4th position to the 39th position as p increases. The Democratic Republic of Congo, Libya, and South Sudan show a slight improvement, moving from the 51st, 52nd, and 53rd positions to the 49th, 50th, and 51st positions as p increases.

Table 12.1. Whitenization sensitivity on rankings.

Countries	Index (i)	0	0.2	0.4	0.6	0.8	1
Algeria	1	7	12	12	12	12	12
Angola	2	38	36	36	34	34	34
Benin	3	28	29	27	29	29	29
Botswana	4	4	4	4	4	5	6
Burkina Faso	5	22	17	16	16	18	19
Burundi	6	46	48	49	50	50	50
Cabo Verde	7	33	26	25	26	26	25
Cameroon	8	47	35	33	32	31	32
Central African Republic	9	20	49	48	48	47	47
Chad	10	50	43	43	42	42	40
Comoros	11	34	46	46	46	46	45
Congo, Dem. Rep.	12	53	51	51	51	51	51
Congo, Rep.	13	45	44	45	45	45	46
Côte d'Ivoire	14	14	23	24	24	23	24
Djibouti	15	36	34	31	28	27	23
Egypt	16	16	15	18	21	24	28
Equatorial Guinea	17	12	30	29	30	30	31
Eritrea	18	13	24	23	20	20	20
Ethiopia	19	42	22	19	18	16	14
Gabon	20	26	20	21	22	21	22
Gambia	21	30	50	50	49	49	49
Ghana	22	3	2	2	2	2	2
Guinea	23	19	39	39	39	37	37
Guinea-Bissau	24	37	38	40	40	39	39
Kenya	25	40	19	17	17	17	16
Lesotho	26	31	31	30	31	32	33
Liberia	27	43	33	28	25	22	21
Libya	28	48	52	52	52	52	52
Madagascar	29	6	5	5	5	4	4
Malawi	30	11	10	10	10	11	11
Mali	31	24	32	34	33	35	36
Mauritania	32	8	21	20	19	19	18
Mauritius	33	2	3	3	3	3	3
Morocco	34	18	13	13	14	14	15
Mozambique	35	15	8	8	8	8	8
Namibia	36	10	7	7	7	7	7
Niger	37	23	16	15	15	15	17
Nigeria	38	35	27	26	27	28	27
Rwanda	39	39	25	32	37	40	44
São Tomé and Príncipe	40	52	45	44	43	41	38
Senegal	41	41	42	42	44	44	43
Seychelles	42	9	6	6	6	6	5
Sierra Leone	43	32	47	47	47	48	48
South Africa	44	1	1	1	1	1	1
South Sudan	45	49	53	53	53	53	52
Sudan	46	44	40	38	35	33	30
Swaziland	47	51	41	41	41	43	42
Tanzania	48	27	14	14	13	13	13
Togo	49	25	37	37	36	36	35
Tunisia	50	29	28	35	38	38	41
Uganda	51	17	11	11	11	10	9
Zambia	52	5	9	9	9	9	10
Zimbabwe	53	21	18	22	23	25	26

Source: Reprinted from [204], used with permission.

Table 12.2 makes it evident that distance measurement influences the assessment of decision alternatives, especially as p tends toward infinity (∞). In this scenario, the boundary distances also tend to the upper bound of the weighted aggregated sum of criteria, as demonstrated in Equation (12.1). The methods developed are distinctive and were applied to evaluate business environments. The GWSM and ILP-GP models introduce reasonable slacks, represented as GNs. Finally, the main distinction between the GWSM and the ILP-GP approach lies in the fact that the GWSM assigns scores to all alternatives, and the most preferred alternative is the one with the highest score, while the ILP-GP approach involves pairwise comparisons of all alternatives, and a sequencing constraint is employed to determine the most preferred alternative.

Table 12.2. Distance sensitivity on rankings.

Countries	Index (i)	1	2	3	Countries	Index (i)	1	2	3
Algeria	1	12	8	7	Libya	28	52	52	50
Angola	2	34	35	34	Madagascar	29	5	3	2
Benin	3	28	27	25	Malawi	30	10	12	17
Botswana	4	4	22	43	Mali	31	33	38	39
Burkina Faso	5	16	20	21	Mauritania	32	19	15	16
Burundi	6	49	50	52	Mauritius	33	3	4	15
Cape Verde	7	25	24	23	Morocco	34	14	23	32
Cameroon	8	32	26	22	Mozambique	35	8	10	9
Central African Republic	9	48	47	46	Namibia	36	7	7	10
Chad	10	42	37	33	Niger	37	15	18	20
Comoros	11	46	43	41	Nigeria	38	27	25	24
Congo, Dem. Rep.	12	51	51	49	Rwanda	39	35	49	53
Congo, Rep.	13	45	44	42	São Tomé & Príncipe	40	44	32	26
Côte d'Ivoire	14	24	29	28	Senegal	41	43	42	36
Djibouti	15	29	21	19	Seychelles	42	6	5	5
Egypt	16	20	41	45	Sierra Leone	43	47	48	48
Equatorial Guinea	17	30	28	27	South Africa	44	1	1	6
Eritrea	18	21	17	18	South Sudan	45	53	53	51
Ethiopia	19	18	13	8	Sudan	46	38	19	13
Gabon	20	22	30	31	Swaziland	47	41	40	37
Gambia	21	50	46	44	Tanzania	48	13	9	3
Ghana	22	2	2	1	Togo	49	36	36	35
Guinea	23	39	33	29	Tunisia	50	37	45	47
Guinea-Bissau	24	40	39	40	Uganda	51	11	6	4
Kenya	25	17	14	14	Zambia	52	9	11	12
Lesotho	26	31	31	30	Zimbabwe	53	23	34	38
Liberia	27	26	16	11					

Source: Reprinted from [204], used with permission.

Although both evaluation methods, GRA and EDAS, lead to the same rankings, EDAS employs a wider range of the evaluation scale, ranging from 0.0901 to 0.9078 for Appraisal Scores (ASs). In contrast, GRA utilizes a narrower range of the evaluation scale, spanning from 0.6288 to 0.9703 for the Grey Relational Grade (GRG), as illustrated in Figure 12.4. Additionally, sensitivity analyses were carried out for the grey distinguishing coefficient (ζ) in the case of GRA and the whitenization coefficient (λ) for EDAS, as shown in Figure 12.5 and Figure 12.6, respectively. It is worth noting that, in this study, changes in ζ and λ did not significantly impact the rankings. This reinforces the findings that the first contractor is indeed the best choice.

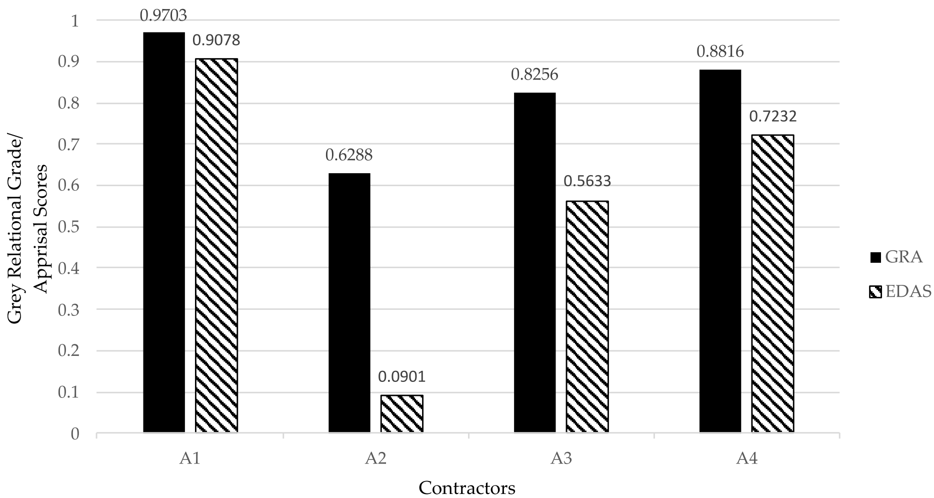


Figure 12.4. Ratings via GRA and EDAS based on grey SWARA-FUCOM weights. Source: Reprinted from [102], used with permission.

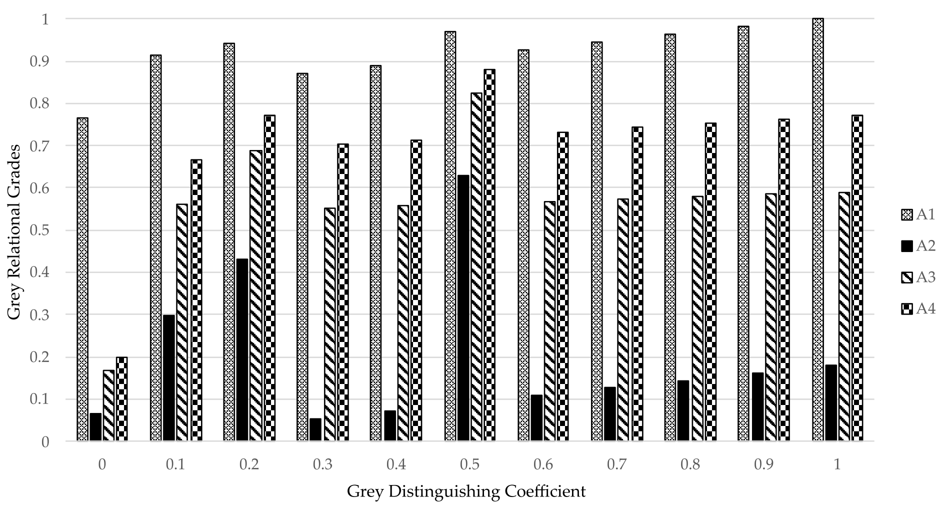


Figure 12.5. Grey distinguishing coefficient sensitivity analysis of GRA. Source: Reprinted from [102], used with permission.

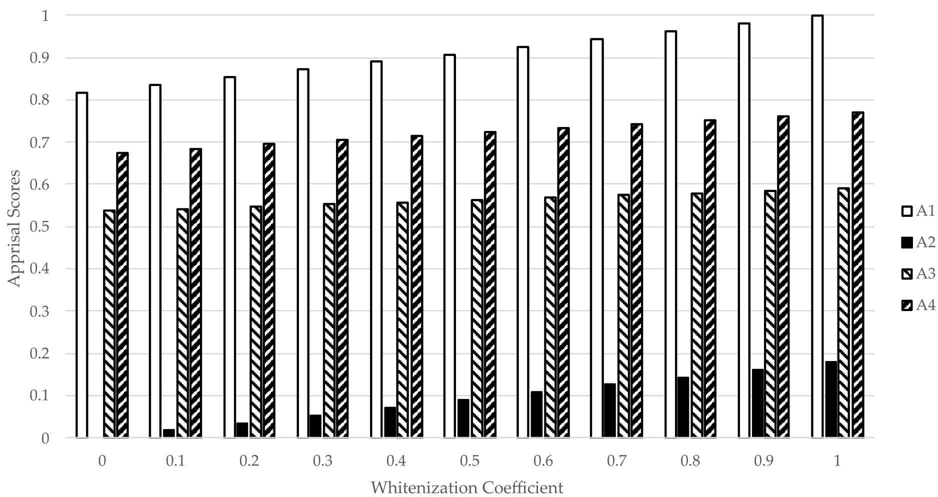


Figure 12.6. Whitenization coefficient sensitivity analysis for EDAS. Source: Reprinted from [102], used with permission.

Creating a favorable business environment globally is an essential endeavor in the contemporary world. Whether they are large multinational corporations or small- to medium-sized enterprises, businesses around the world show a strong interest in entering various markets. However, they often face uncertainties and doubts regarding market entry strategies, location choices, and contacts. Governments can incentivize business startups to enhance the quality of economic development. The business climate worldwide directly influences education, which, in turn, fosters economic growth, development, and well-being.

Enhancements in education can invigorate sectors like Information and Communications Technology (ICT), encompassing industrial policies, e-governance systems, and platforms for greater international business engagement. Ensuring a robust regulatory and administrative framework and secure business transactions is vital for the private sector globally. Developing innovative infrastructure is a crucial step toward achieving better economic performance on a global scale. Promoting industrial development is key to harnessing the benefits of changing global dynamics, including reduced transportation costs and advancements in ICT [335]. Considering the substantial youth population worldwide, it is essential to develop growth models that can generate employment opportunities for the youth in all regions.

Investment growth, not limited to any specific region, is a global phenomenon. Many countries worldwide welcome investors and appreciate their contributions to economic development. This appreciation extends to various sectors, including construction and information technology services, where low-cost technologies and skilled workers willing to tackle challenging conditions are often offered. Rather than managing isolated projects, investors worldwide are increasingly inclined to provide comprehensive solutions, covering areas like transportation, energy production, resource extraction, and refining. Access to long-term capital is a common attribute among many investors globally. Leaders in different parts of the world appreciate efficient models of economic cooperation and development assistance, marked by low administrative costs, mutual respect, adherence to commitments, and a lack of conditionalities. These factors explain the generally warm reception extended to investors around the world.

12.2. Discussion

Universities operate within a highly competitive and globalized environment. The presence of international students seeking education abroad has made university rankings a quick reference for evaluating a university's reputation. These institutions must compete to attract different stakeholders, including students, faculty, and financial supporters. For students, attending university is not only about academic education but also about the overall university experience, which may include civic engagement. The reputation of a university plays a pivotal role in attracting and retaining the best students, teachers, and staff [168]. A positive institutional reputation fosters strong bonds, enhances performance, and leads to higher commitment, involvement, and cohesion among staff [311].

Many people have a general understanding of what reputation is and believe it should be safeguarded. Universities appearing in such rankings use their presence as a marketing tool, while others may question the value of these rankings. The challenge of obtaining direct data for every university limits the number of institutions that can be ranked. The evaluation of university reputation involves numerous criteria, and introducing more variables and uncertainties complicates the process.

To explore students' perceptions of university reputation and the preferences of decision-makers, psychological insights can be valuable. Psychology delves into concepts like attention, motivation, emotion, brain functioning, intelligence, personality, relationships, consciousness, and unconsciousness. An interesting psychological theory is "Regulatory Focus Theory," which explains how individuals pursue their goals. In real-life decision-making, one constant factor is uncertainty. GST accommodates various levels of uncertainty by providing reasonable flexibility in weighting and evaluating variables. In this book, uncertainty is represented using interval grey numbers.

In addition, the significant findings of this work suggest that expatriates accepting assignments in developed countries should receive a lower foreign service premium allowance compared to those taking assignments in underdeveloped regions with harsh and risky work environments. While the Foreign Service Premium (FSP) allowance may not be the sole factor explaining the high turnover rate in the Nigeria and Pakistan branches, the FSP allowance ratio indicates that a fairer approach would be to increase the FSP allowance for expatriates in these branches substantially. Although initially, paying expatriates less may appear cost-effective, in the long term, the loss incurred due to expatriates leaving the company can be significant [336]. The method presented in book enables decision-makers to justify adjusting the FSP allowance for expatriates, instead of responding to individual requests for pay raises. Furthermore, this approach brings the company closer to meeting the expectations of its expatriate employees [337].

For companies to foster innovation, they must attract and retain the right talent while providing appropriate compensation. Human Resource Information System (HRIS) can extend their support beyond standard Human Resource Management (HRM) processes like leave applications and time records. They can also facilitate more expert-level tasks such as succession planning, intern talent pool management, and talent reviews.

In conclusion, sustainable energy solutions should prioritize the responsible use of raw materials, ensuring that future generations have access to these resources. However, it is important to note that not all sustainable energy solutions are equal. There is room for improvement in the decision-making process to maximize benefits while minimizing associated costs. This introduces the challenge of selecting both the best solution and the most suitable contractor for project execution. It is well established that different weighting and evaluation methods can yield different criteria weights, affecting the ranking of outcomes. Such uncertainties can lead to the selection of suboptimal alternatives. Interestingly, the GRA and EDAS methods resulted in identical rankings.

12.3. Limitation and Future Works

The allocation of weights to DMs in Multiple Criteria Decision-Making (MCDM) scenarios emerged as a critical focus. A significant contribution lies in the estimation of DM weights based on their orientations according to RFT within group decision-making contexts. Additionally, the development of Grey Regulatory Focus Theory (GRFT) for weighting represents an innovation. This novel method can complement other MCDM evaluation techniques within the existing literature. Notably, its application to assess university reputation exhibited consistency with other established ranking indices.

The book introduced a hybrid MCDM method that combines SWARA and GRA with GN. This innovative approach is particularly suited for group decision-making in complex and uncertain settings. A noteworthy practical outcome was the proposition to scale FSP allowances for expatriates in overseas branches. Beyond the transparency this brings to global compensation, it fosters employee willingness to undertake assignments in challenging and less-favored environments, addressing the problem of high turnover.

Another significant contribution involves bridging the gap between traditional weight-assessment methods commonly used by Human Resources (HR) managers in practice and advanced methods proposed by academics. The introduction of a new hybrid method, grey-PA-FUCOM, facilitated the evaluation of HRIS offered by diverse vendors. This method offers the potential to standardize and enhance HR decision-making processes.

Furthermore, hybrid methods were unveiled, such as the grey-SWARA-FUCOM designed for the evaluation of contractors for the installation of floating solar panel energy systems. This method, integrated with traditional GRA and EDAS methods, demonstrated consistent rankings between the two approaches. The primary contributions of this book lie in the development of grey weighting methods, which can be seamlessly incorporated into a wide array of Multiple Criteria Decision-Making (MCDM) evaluation methods found in the existing literature.

Despite these significant strides, certain limitations must be acknowledged. The potential inaccuracy in measuring university performance using abstract instruments, common in psychological research, poses a generic challenge. Furthermore, relying on student perspectives for ranking universities worldwide may not be entirely representative, as these perspectives are often shaped by their local environments. Additionally, the work primarily applied the traditional GRA method, introducing only a single new variable, the GRFT weighting method. Future research could explore the integration of GRFT into advanced MCDM evaluation methods like TOPSIS and the latest version of ELECTRE.

On the horizon of future research, one avenue to explore is the provision of different scaling factors for male and female staff, recognizing that the determinants for accepting foreign assignments can differ by gender. Another dimension to investigate is the influence of long service allowances on staff retirement decisions, potentially addressing this as a mathematical programming model with scaling FSP as a set partitioning formulation, incorporating budget constraints.

Additionally, future studies can delve into the benefits of the Internet of Things (IoT) in Human Resource Management (HRM) and approaches for optimizing HRM practices through IoT. Establishing a repository of software packages for MCDM methods that can be seamlessly integrated into word-processing suites like Microsoft Excel would make advanced methods more accessible to HR managers.

Research could be extended to encompass the weighting methods in Chapter 3 to Chapter 6 to various MCDM contexts and address diverse MCDM challenges. Longitudinal studies may evaluate the impact of shadowing a significant portion of a water body on parameters such as pH and oxygen levels, and its implications for aquatic habitats. Further exploration of the economic and environmental consequences of the health of entire water bodies and their support for local communities offers fertile ground for future research.

Finally, grey systems theory can be extended to various other frameworks such as the Pareto principle, cost–benefit analysis, network analysis, large group decision-making [338], Kahneman's System-2 thinking [339] and other operations research methodologies. This extension enhances its applicability, allowing for more robust and comprehensive decision-making processes across diverse fields. By integrating GST with these established frameworks, practitioners can better navigate uncertainty and incomplete information, ultimately leading to more effective and informed outcomes in both strategic planning and operational execution in business management.

Appendix A

Appendix A.1. Data

Appendix A.1.1. Weight Data for Contractor Selection of Floating Solar Panel Energy System Installation

Table A1. Raw data of the DMs' rankings.

Criteria	DM_1	DM_2	DM_3	DM_4	Criteria	DM_1	DM_2	DM_3	DM_4
Ψ_1	6th	6th	6th	6th	$\Psi_{3\text{-}3}$	2nd	1st	3rd	2nd
Ψ_2	4th	1st	1st	3rd	$\Psi_{3\text{-}4}$	1st	2nd	2nd	3rd
Ψ_3	5th	5th	4th	4th	$\Psi_{3\text{-}5}$	3rd	3rd	5th	4th
Ψ_4	2nd	3rd	2nd	5th	$\Psi_{4\text{-}1}$	3rd	1st	4th	1st
Ψ_5	1st	4th	5th	2nd	$\Psi_{4\text{-}2}$	2nd	4th	3rd	2nd
Ψ_6	3rd	2nd	3rd	1st	$\Psi_{4\text{-}3}$	4th	5th	5th	5th
$\Psi_{1\text{-}1}$	5th	5th	3rd	3rd	$\Psi_{4\text{-}4}$	5th	2nd	2nd	4th
$\Psi_{1\text{-}2}$	1st	4th	5th	4th	$\Psi_{4\text{-}5}$	1st	3rd	1st	3rd
$\Psi_{1\text{-}3}$	4th	3rd	2nd	1st	$\Psi_{5\text{-}1}$	2nd	5th	1st	1st
$\Psi_{1\text{-}4}$	2nd	2nd	1st	2nd	$\Psi_{5\text{-}2}$	3rd	4th	4th	3rd
$\Psi_{1\text{-}5}$	3rd	1st	4th	5th	$\Psi_{5\text{-}3}$	1st	3rd	5th	2nd
$\Psi_{2\text{-}1}$	2nd	4th	5th	1st	$\Psi_{5\text{-}4}$	4th	1st	2nd	5th
$\Psi_{2\text{-}2}$	5th	5th	3rd	4th	$\Psi_{5\text{-}5}$	5th	2nd	3rd	4th
$\Psi_{2\text{-}3}$	1st	3rd	4th	2nd	$\Psi_{6\text{-}1}$	1st	4th	1st	1st
$\Psi_{2\text{-}4}$	3rd	2nd	2nd	5th	$\Psi_{6\text{-}2}$	4th	3rd	5th	3rd
$\Psi_{2\text{-}5}$	4th	1st	1st	3rd	$\Psi_{6\text{-}3}$	3rd	2nd	2nd	2nd
$\Psi_{3\text{-}1}$	4th	5th	1st	1st	$\Psi_{6\text{-}4}$	2nd	1st	3rd	4th
$\Psi_{3\text{-}2}$	5th	4th	4th	5th	$\Psi_{6\text{-}5}$	5th	5th	4th	5th

Source: Reprinted from [102], used with permission.

Table A2. Raw data of the comparison.

Criteria	Rankings	DM_1	DM_2	DM_3	DM_4
First–level	1st	–	–	–	–
	2nd	4	2	5	8
	3rd	3	8	4	5
	4th	8	5	8	5
	5th	5	4	5	4
	6th	5	5	6	3
Second-level of Ψ_1	1st	–	–	–	–
	2nd	5	2	4	4
	3rd	5	3	4	6
	4th	4	4	5	3
	5th	3	5	4	3
Second-level of Ψ_2	1st	–	–	–	–
	2nd	5	4	5	5
	3rd	4	3	7	5
	4th	3	4	4	4
	5th	6	3	6	5
Second-level of Ψ_3	1st	–	–	–	–
	2nd	4	3	4	4
	3rd	4	4	5	5
	4th	4	4	5	3
	5th	3	5	6	4
Second-level of Ψ_4	1st	–	–	–	–
	2nd	3	3	4	6
	3rd	3	5	6	9
	4th	4	3	6	5
	5th	3	4	4	4
Second-level of Ψ_5	1st	–	–	–	–
	2nd	5	3	4	9
	3rd	3	4	6	5
	4th	3	3	5	3
	5th	3	4	4	4
Second-level of Ψ_6	1st	–	–	–	–
	2nd	6	3	7	7
	3rd	3	3	6	5
	4th	3	5	4	5
	5th	5	5	5	4

Source: Reprinted from [102], used with permission.

Appendix A.1.2. Business Environment Grey Data

Table A3. Grey data for the performances of African countries.

Countries	**Index**	A_{12}	A_{13}	A_{14}	A_{42}	A_{43}
Algeria	1	[22, 24]	[10.8, 13.2]	[24.1, 45.2]	[55, 74]	[7.1, 7.5]
Angola	2	[66, 83]	[123.5, 343.7]	[20, 50.5]	[190, 335]	[3, 11.6]
Benin	3	[12, 34]	[55.8, 198.1]	[6.3, 354.2]	[120, 120]	[11.7, 11.9]
Botswana	4	[59, 105]	[1, 9.3]	[0, 0]	[10, 15]	[5, 5.1]
Burkina Faso	5	[13, 18]	[44.5, 82.1]	[306.2, 458.8]	[67, 182]	[12.3, 15.1]
Burundi	6	[5, 13]	[13.4, 241.2]	[0, 0]	[23, 104]	[3.2, 7.4]
Cape Verde	7	[10, 52]	[13.5, 40.1]	[0, 53.4]	[22, 83]	[3.7, 7.8]
Cameroon	8	[15, 38]	[34.3, 151.5]	[156.4, 191.8]	[86, 93]	[19, 19.3]
Central African Republic	9	[22, 24]	[162, 244.9]	[411.4, 607.3]	[75, 75]	[11, 18.6]
Chad	10	[53, 62]	[165.6, 273.3]	[201.7, 398.4]	[44, 44]	[15.2, 18.8]
Comoros	11	[15, 30]	[114, 192.3]	[226.7, 280.8]	[30, 30]	[10.4, 16.6]
Congo, Dem. Rep.	12	[16, 132]	[30, 935.4]	[0, 909.1]	[44, 51]	[9.5, 15]
Congo, Rep.	13	[38, 161]	[52.1, 150.1]	[78.5, 206.3]	[55, 55]	[10.3, 22.1]
Cǒte d'Ivoire	14	[7, 42]	[20, 135.8]	[3.4, 219.8]	[30, 51]	[9.6, 14]
Djibouti	15	[14, 44]	[175.2, 251.6]	[0, 530.8]	[39, 39]	[12.8, 13.2]
Egypt	16	[8, 10]	[9.2, 28.9]	[0, 12.9]	[63, 194]	[0.7, 1]
Equatorial Guinea	17	[133, 155]	[98.2, 101.4]	[11.7, 23.2]	[23, 23]	[12.5, 12.5]
Eritrea	18	[83, 84]	[41.5, 125.8]	[182.1, 488]	[78, 78]	[9.1, 9.2]
Ethiopia	19	[15, 18]	[29.8, 267.5]	[164.4, 960]	[41, 43]	[2.1, 3.4]
Gabon	20	[50, 57]	[12.5, 25.6]	[19.3, 38.2]	[38, 103]	[10.5, 17.5]
Gambia	21	[26, 32]	[131.2, 279]	[0, 0]	[66, 66]	[7.6, 7.7]
Ghana	22	[11, 14]	[19.2, 38.9]	[2.8, 20.9]	[46, 46]	[0.7, 1.3]
Guinea	23	[8, 40]	[82.6, 147.7]	[313.8, 519.1]	[44, 59]	[8.6, 14.4]
Guinea-Bissau	24	[9, 259]	[42.2, 465.7]	[338, 1015]	[51, 210]	[10.5, 11.6]
Kenya	25	[30, 44]	[38.2, 46.8]	[0, 0]	[72, 72]	[4.2, 4.3]
Lesotho	26	[29, 39]	[9.4, 37.9]	[0, 14.5]	[43, 101]	[7.9, 8.7]
Liberia	27	[4.5, 68]	[17.4, 489.6]	[0, 0]	[44, 50]	[12.9, 13.3]
Libya	28	[35, 35]	[19.1, 31.7]	[31, 74.1]	[0, 0]	[0, 0]
Madagascar	29	[7, 9]	[6.2, 22.7]	[0, 333.4]	[74, 134]	[9, 11.6]
Malawi	30	[36, 40]	[90.9, 188.7]	[0, 0]	[49, 88]	[1.9, 3.6]
Mali	31	[8, 25]	[76.7, 115.2]	[295.2, 434.6]	[29, 29]	[11.4, 20.5]
Mauritania	32	[9, 50]	[19.8, 83.9]	[314.4, 503.1]	[49, 49]	[4.7, 5.2]
Mauritius	33	[6, 7]	[2.1, 5.3]	[0, 0]	[14, 210]	[10.6, 10.8]
Morocco	34	[11, 12]	[9.2, 20.6]	[0, 15]	[40, 75]	[4.9, 5.9]
Mozambique	35	[13, 29]	[17.1, 25.2]	[0, 12.2]	[39, 42]	[6.9, 12.5]
Namibia	36	[66, 99]	[13.1, 22.3]	[0, 0]	[32, 52]	[9.8, 13.8]
Niger	37	[15, 23]	[76.7, 174.8]	[492, 735.6]	[35, 35]	[9, 11.1]
Nigeria	38	[25, 40]	[31.1, 87.7]	[0, 0]	[45, 79]	[11.8, 21.2]
Rwanda	39	[6.5, 16]	[4.7, 171.5]	[0, 0]	[32, 370]	[0.1, 9.4]
São Tomé and Príncipe	40	[4, 144]	[17.5, 94.5]	[0, 385.7]	[62, 64]	[8.9, 12.6]
Senegal	41	[6, 58]	[63.1, 107]	[19, 255]	[71, 145]	[15.2, 20.6]
Seychelles	42	[38, 39]	[10.7, 29.7]	[0, 0]	[33, 33]	[7, 7]
Sierra Leone	43	[12, 26]	[44.1, 1180.7]	[0, 0]	[67, 236]	[10.9, 14.9]
South Africa	44	[19, 46]	[0.27, 7.1]	[0, 0]	[23, 23]	[5.6, 8.9]
South Sudan	45	[14, 14]	[192.3, 372.1]	[0, 0]	[50, 50]	[15.4, 16.2]
Sudan	46	[35, 37]	[25.1, 57.9]	[0, 0]	[9, 9]	[2.8, 3.2]
Swaziland	47	[30, 60]	[23.3, 38.7]	[0.4, 0.6]	[21, 44]	[7.1, 7.1]
Tanzania	48	[26, 30]	[23.8, 52.8]	[0, 0]	[67, 73]	[4.4, 4.5]
Togo	49	[10, 84]	[94.9, 251.3]	[37.5, 559.9]	[295, 295]	[9.3, 13.9]
Tunisia	50	[11, 11]	[4.1, 8.3]	[0, 25.3]	[39, 39]	[6.1, 6.1]
Uganda	51	[24, 33]	[64.4, 100.7]	[0, 0]	[43, 74]	[2.6, 2.7]
Zambia	52	[6.5, 33]	[26.9, 32.7]	[0, 2.2]	[42, 73]	[6.6, 13.6]
Zimbabwe	53	[86, 93]	[114.6, 676.1]	[0, 0]	[30, 36]	[7.6, 25]

Table A3. *Cont.*

Index	A_{51}	A_{52}	A_{53}	A_{54}	A_{61}	A_{62}	A_{63}
1	[2, 3]	[0, 0]	[0.2, 2.4]	[0, 0]	[5, 5]	[6, 6]	[4, 4]
2	[1, 3]	[0, 0]	[1.8, 2.7]	[0, 0]	[4, 4]	[6, 6]	[6, 6]
3	[3, 6]	[1, 1]	[7.8, 10.9]	[0, 0]	[6, 7]	[1, 1]	[3, 5]
4	[5, 6]	[4, 6]	[0, 0]	[51.7, 60.7]	[7, 8]	[2, 8]	[3, 3]
5	[3, 6]	[0, 0]	[1.7, 2.1]	[0, 0]	[6, 7]	[1, 1]	[4, 6]
6	[2, 3]	[0, 0]	[0.2, 3.9]	[0, 0]	[4, 8]	[1, 7]	[4, 5]
7	[2, 3]	[2, 6]	[16.7, 23]	[0, 0]	[1, 1]	[5, 5]	[6, 6]
8	[3, 6]	[0, 1]	[1, 9.1]	[0, 0]	[6, 7]	[1, 1]	[6, 7]
9	[3, 6]	[0, 0]	[1.2, 3.1]	[0, 0]	[6, 7]	[1, 1]	[4, 5]
10	[3, 6]	[0, 0]	[0.2, 2.1]	[0, 0]	[6, 7]	[1, 1]	[3, 4]
11	[3, 6]	[0, 0]	[0, 0]	[0, 0]	[6, 7]	[1, 1]	[5, 6]
12	[3, 6]	[0, 0]	[0, 0.2]	[0, 0]	[3, 7]	[1, 3]	[3, 4]
13	[3, 6]	[0, 2]	[2.4, 9.4]	[0, 0]	[6, 7]	[1, 1]	[3, 4]
14	[3, 6]	[0, 0]	[2.6, 3.2]	[0, 0]	[6, 7]	[1, 1]	[3, 4]
15	[1, 1]	[0, 0]	[0.2, 0.3]	[0, 0]	[4, 4]	[3, 3]	[0, 0]
16	[2, 3]	[0, 8]	[1.7, 5.8]	[0, 21.8]	[4, 8]	[3, 3]	[3, 3]
17	[3, 6]	[0, 2]	[1.9, 5.1]	[0, 0]	[6, 7]	[1, 1]	[4, 6]
18	[0, 2]	[0, 0]	[0, 0]	[0, 0]	[4, 4]	[5, 5]	[5, 5]
19	[3, 4]	[0, 0]	[0.1, 0.2]	[0, 0]	[3, 3]	[0, 0]	[3, 4]
20	[3, 6]	[0, 2]	[2.4, 53.8]	[0, 0]	[6, 7]	[1, 1]	[3, 4]
21	[4, 5]	[0, 0]	[0, 0]	[0, 0]	[2, 2]	[1, 5]	[5, 5]
22	[7, 8]	[0, 6]	[0, 0]	[0, 14.1]	[7, 7]	[5, 5]	[7, 8]
23	[3, 6]	[0, 0]	[0, 0]	[0, 0]	[6, 7]	[1, 1]	[1, 2]
24	[3, 6]	[0, 0]	[0.9, 1.1]	[0, 0]	[6, 7]	[1, 1]	[5, 6]
25	[7, 10]	[0, 0]	[0, 0]	[1.5, 4.9]	[3, 3]	[2, 2]	[9, 10]
26	[5, 6]	[0, 0]	[0, 0]	[0, 0]	[2, 3]	[1, 4]	[8, 9]
27	[4, 7]	[0, 0]	[0, 1.7]	[0, 0]	[4, 4]	[1, 1]	[6, 6]
28	[0, 1]	[0, 0]	[0.5, 0.5]	[0, 0]	[1, 1]	[1, 1]	[3, 4]
29	[1, 2]	[0, 0]	[0, 0.2]	[0, 0]	[6, 6]	[6, 6]	[5, 5]
30	[5, 7]	[0, 0]	[0, 0]	[0, 0]	[4, 4]	[7, 7]	[5, 6]
31	[3, 6]	[0, 0]	[2.5, 4.1]	[0, 0]	[6, 7]	[1, 1]	[3, 4]
32	[2, 3]	[0, 0]	[0.1, 4.6]	[0, 0]	[6, 6]	[1, 1]	[4, 4]
33	[6, 6]	[2, 7]	[10.2, 71.9]	[0, 0]	[6, 6]	[8, 8]	[8, 9]
34	[2, 3]	[0, 6]	[0, 2.4]	[0, 21.1]	[5, 6]	[2, 2]	[1, 6]
35	[1, 3]	[0, 5]	[0.9, 5.7]	[0, 0]	[5, 5]	[4, 4]	[9, 10]
36	[5, 7]	[4, 6]	[0, 0]	[57.7, 66.2]	[5, 5]	[5, 5]	[6, 7]
37	[3, 6]	[0, 0]	[0.7, 1]	[0, 0]	[6, 7]	[1, 1]	[3, 4]
38	[6, 9]	[0, 6]	[0, 0.1]	[0, 5.8]	[4, 4]	[7, 7]	[5, 7]
39	[2, 11]	[0, 7]	[0.2, 2.4]	[0, 15.7]	[2, 7]	[5, 9]	[1, 3]
40	[0, 2]	[0, 0]	[0, 0]	[0, 0]	[3, 3]	[1, 1]	[6, 6]
41	[3, 6]	[0, 0]	[1, 4.6]	[0, 0]	[6, 7]	[1, 1]	[2, 6]
42	[2, 4]	[0, 0]	[0, 0]	[0, 0]	[4, 4]	[8, 8]	[5, 5]
43	[5, 7]	[0, 0]	[0, 0.9]	[0, 0]	[3, 6]	[6, 8]	[6, 8]
44	[5, 7]	[6, 8]	[0, 0]	[52, 64.8]	[8, 8]	[8, 8]	[8, 8]
45	[2, 3]	[0, 0]	[0, 0]	[0, 0]	[2, 2]	[1, 1]	[4, 5]
46	[3, 4]	[0, 0]	[0, 0]	[0, 1.3]	[0, 0]	[6, 6]	[4, 5]
47	[4, 6]	[5, 7]	[0, 0]	[35.7, 47.8]	[0, 2]	[1, 5]	[5, 6]
48	[5, 7]	[0, 0]	[0, 0]	[0, 0.6]	[2, 2]	[6, 6]	[8, 8]
49	[3, 6]	[0, 0]	[2.5, 3.1]	[0, 0]	[6, 7]	[1, 1]	[4, 6]
50	[2, 3]	[2, 5]	[13.7, 30.2]	[0, 0]	[0, 5]	[4, 7]	[5, 6]
51	[6, 7]	[0, 0]	[0, 0]	[0, 4.9]	[3, 3]	[5, 5]	[6, 7]
52	[7, 9]	[0, 7]	[0, 0]	[0, 12]	[4, 4]	[6, 6]	[7, 7]
53	[5, 7]	[0, 3]	[0, 0]	[1.5, 5.8]	[8, 8]	[2, 2]	[4, 4]

Table A3. *Cont.*

Index	A_{71}	A_{72}	A_{73}	A_{91}	A_{93}	A_{101}	A_{103}
1	[27, 27]	[451, 451]	[6.6, 6.6]	[630, 630]	[45, 47]	[2.5, 2.5]	[41.7, 41.7]
2	[30, 30]	[272, 282]	[25.3, 25.3]	[1011, 1296]	[46, 46]	[0, 0]	[0, 0]
3	[55, 55]	[270, 270]	[15.9, 15.9]	[750, 825]	[41, 42]	[4, 4]	[16.7, 22.6]
4	[34, 34]	[140, 152]	[21.7, 21.7]	[625, 987]	[28, 29]	[1.7, 1.7]	[57.1, 62.7]
5	[45, 46]	[270, 270]	[16.2, 16.2]	[446, 446]	[37, 37]	[4, 4]	[15.3, 19.3]
6	[25, 32]	[140, 274]	[34.7, 39.4]	[558, 832]	[44, 44]	[5, 5]	[6.8, 8.4]
7	[30, 44]	[186, 186]	[18.2, 18.2]	[425, 425]	[37, 37]	[0, 0]	[0, 0]
8	[44, 45]	[630, 654]	[30, 30]	[800, 800]	[42, 43]	[2.8, 3.2]	[13.3, 15.4]
9	[54, 56]	[483, 504]	[0, 0]	[660, 660]	[43, 43]	[4.8, 4.8]	[0, 0]
10	[54, 54]	[732, 732]	[31.3, 31.3]	[743, 743]	[41, 41]	[4, 4]	[0, 0]
11	[33, 33]	[100, 100]	[32.1, 32.1]	[506, 506]	[43, 43]	[0, 0]	[0, 0]
12	[40, 50]	[308, 348]	[27.5, 33.4]	[610, 670]	[43, 43]	[0, 0]	[0, 0]
13	[49, 61]	[602, 606]	[17.9, 18.5]	[560, 560]	[44, 44]	[3.3, 3.3]	[17.8, 17.9]
14	[63, 67]	[270, 270]	[8.8, 8.8]	[525, 770]	[32, 33]	[2.2, 2.2]	[31.3, 37.6]
15	[23, 35]	[66, 90]	[17.7, 17.7]	[1225, 1225]	[40, 40]	[2.3, 2.3]	[37, 38.5]
16	[29, 36]	[392, 711]	[13.6, 16.7]	[1010, 1010]	[42, 42]	[2.5, 4.2]	[16.6, 27.4]
17	[46, 46]	[492, 492]	[0, 0]	[475, 475]	[40, 40]	[0, 0]	[0, 0]
18	[30, 30]	[216, 216]	[9.2, 9.2]	[405, 490]	[39, 39]	[0, 0]	[0, 0]
19	[30, 30]	[198, 306]	[26.2, 26.5]	[530, 690]	[38, 39]	[1.8, 1.8]	[36.1, 41.5]
20	[26, 26]	[488, 488]	[15.8, 18.4]	[1070, 1070]	[38, 38]	[5, 5]	[15.2, 15.2]
21	[50, 50]	[376, 376]	[6.1, 6.1]	[407, 434]	[33, 33]	[2, 2]	[27.8, 28.2]
22	[32, 33]	[224, 304]	[18.6, 18.6]	[487, 710]	[36, 38]	[1.9, 1.9]	[23.7, 26.9]
23	[57, 57]	[416, 440]	[0, 0]	[276, 276]	[49, 49]	[3.8, 3.8]	[17.1, 22]
24	[46, 46]	[208, 208]	[15.1, 15.1]	[1715, 1715]	[40, 41]	[0, 0]	[0, 0]
25	[30, 41]	[201.5, 432]	[30.8, 30.8]	[465, 465]	[40, 44]	[4.5, 4.5]	[24.7, 31.6]
26	[20, 32]	[324, 324.3]	[10.8, 10.8]	[615, 875]	[40, 41]	[2.6, 2.6]	[26, 29]
27	[33, 33]	[150.5, 158]	[16.5, 21.2]	[1280, 1280]	[40, 41]	[3, 3]	[7.8, 8.6]
28	[19, 19]	[889, 889]	[20.8, 20.8]	[690, 690]	[43, 43]	[0, 0]	[0, 0]
29	[23, 26]	[183, 238]	[13.3, 14]	[871, 871]	[38, 38]	[2, 2]	[11.7, 17.9]
30	[23, 35]	[157, 370]	[20.4, 21.3]	[432, 432]	[42, 42]	[2.6, 2.6]	[12.1, 18.5]
31	[35, 59]	[270, 270]	[10.1, 10.1]	[620, 710]	[36, 39]	[3.6, 3.6]	[20.9, 25]
32	[37, 49]	[696, 734]	[0, 0]	[370, 400]	[46, 46]	[0, 0]	[0, 0]
33	[8, 8]	[152, 161]	[11.2, 11.2]	[519, 750]	[34, 37]	[1.7, 1.7]	[55, 67.4]
34	[6, 28]	[232, 358]	[25.3, 25.4]	[510, 510]	[40, 40]	[1.8, 3.5]	[23.6, 35.3]
35	[37, 37]	[230, 230]	[31.3, 31.3]	[760, 1010]	[30, 31]	[5, 5]	[13.9, 17.7]
36	[26, 37]	[314, 339]	[17.5, 18]	[460, 500]	[33, 33]	[2.5, 2.5]	[31.1, 35.2]
37	[41, 42]	[270, 270]	[21.3, 22]	[545, 545]	[39, 39]	[5, 5]	[14, 21.9]
38	[35, 47]	[747, 1120]	[21.6, 21.6]	[447, 720]	[40, 41]	[2, 2]	[26.8, 28.2]
39	[17, 25]	[107, 168]	[26.3, 26.3]	[230, 310]	[23, 24]	[2.5, 3]	[3.1, 19.5]
40	[42, 45]	[424, 424]	[20.2, 20.2]	[1065, 1185]	[43, 43]	[6.2, 6.2]	[4.1, 7.5]
41	[58, 59]	[620, 696]	[16.2, 16.2]	[740, 780]	[43, 44]	[3, 3]	[23, 25.5]
42	[22, 28]	[76, 88]	[20.9, 25.6]	[720, 915]	[36, 37]	[2, 2]	[37.7, 39.7]
43	[29, 33]	[353, 399]	[18.8, 18.8]	[515, 515]	[39, 40]	[2.3, 2.6]	[8.4, 10.5]
44	[7, 11]	[200, 350]	[21.4, 21.7]	[600, 600]	[29, 30]	[2, 2]	[32.2, 35.7]
45	[32, 36]	[218, 218]	[7.1, 7.1]	[228, 228]	[48, 48]	[0, 0]	[0, 0]
46	[42, 42]	[180, 180]	[11.5, 11.5]	[810, 810]	[53, 53]	[2, 2]	[31.5, 33.2]
47	[33, 33]	[104, 110]	[28.6, 28.6]	[956, 972]	[40, 40]	[2, 2]	[34.9, 38.7]
48	[48, 49]	[172, 185]	[20.7, 20.7]	[515, 515]	[38, 38]	[3, 3]	[20.5, 22]
49	[50, 50]	[270, 270]	[9.5, 10]	[588, 588]	[40, 41]	[3, 3]	[26.3, 30.6]
50	[8, 22]	[144, 268]	[15.4, 15.4]	[565, 565]	[39, 39]	[1.3, 1.3]	[51.5, 52.3]
51	[31, 32]	[161, 237]	[25.2, 25.2]	[490, 535]	[38, 38]	[2.2, 2.2]	[36, 41.1]
52	[37, 38]	[132, 183]	[1.3, 1.3]	[471, 611]	[35, 35]	[2.4, 2.7]	[27.2, 39.3]
53	[49, 51]	[242, 270]	[19.2, 19.3]	[410, 410]	[38, 38]	[3.3, 3.3]	[0, 13.8]

Source: Reprinted from [204], used with permission.

Appendix A.1.3. Foreign Service Premium Data

Table A4. Data for evaluating the allowances of the experts in each country.

Countries (i)/ Second-Level Criteria ($C_{j\text{-}k}$)	**Index (n)**	**Albania 1**	**Algeria 2**	**Bangladesh 3**	**Brazil 4**	**Canada 5**	**Colombia 6**
Clean Cities ($C_{1\text{-}1}$)	1	18.2	34.5	58.6	11.8	6.7	17.2
Environmental Performance Index ($C_{1\text{-}2}$)	2	65.46	57.18	29.56	60.7	72.18	65.22
Disaster Risk Index ($C_{1\text{-}3}$)	3	9.5	9.5	1.69	5.14	10.91	6.39
Global Terrorism Index ($C_{2\text{-}1}$)	4	1.487	3.97	6.181	1.572	2.958	5.595
Failed States Index ($C_{2\text{-}2}$)	5	60.0793	75.7851	90.3128	68.7131	21.5056	76.6279
Global Peace Index ($C_{2\text{-}3}$)	6	1.849	2.182	2.084	2.16	1.372	2.729
Consumer Price Index ($C_{3\text{-}1}$)	7	115.0843	142.3842	161.136	155.6688	111.9848	131.8776
GDP per Capita ($C_{3\text{-}2}$)	8	4537.8625	4123.3899	1516.5134	9821.4077	45032.12	6301.5899
Inflation ($C_{3\text{-}3}$)	9	1.2828	6.3977	5.5135	8.7395	1.4288	7.5175
Sanitation and Hygiene ($C_{4\text{-}1}$)	10	98	87	47	86	99	84
Mortality From Environmental Pollution ($C_{4\text{-}2}$)	11	104.7	40.3	103.4	31	14.7	34.4
Drinking Water ($C_{4\text{-}3}$)	12	91	93	97	97	99	97
Public Integrity Index ($C_{5\text{-}1}$)	13	6.48	4.94	5.17	5.83	8.74	6.54
Justices System ($C_{5\text{-}2}$)	14	0.5078	-	0.4091	0.5368	0.8097	0.5035
Reliability of Police Service ($C_{5\text{-}3}$)	15	5.2	4.7	3.3	3.6	6.3	3.4

Table A4. *Cont.*

Index (n)	**France 7**	**Indonesia 8**	**Italy 9**	**Kazakhstan 10**	**Malaysia 11**	**Mexico 12**	**Nigeria 13**	**Pakistan 14**	**Peru 15**	**Poland 16**
1	12.4	16.4	15.7	14.5	17.3	20.9	46.3	56.2	29	21.5
2	83.95	46.92	76.96	54.56	59.22	59.69	54.76	37.5	61.92	64.11
3	2.21	6.64	2.3	4.58	7.98	15.53	11.24	2.64	3.48	26.7
4	5.964	4.55	2.75	2.95	3.334	3.292	9.009	8.4	2.544	0.384
5	32.227	72.2977	43.8	63.4385	63.5948	71.5181	99.9082	96.3381	70.073	41.4612
6	1.909	1.853	1.766	1.974	1.619	2.583	2.873	3.079	1.986	1.727
7	106.8645	142.1824	108.7149	169.2833	119.6051	130.1978	214.2321	156.9123	125.4172	109.6377
8	38476.659	3846.8643	31952.976	8837.4573	9944.9043	8902.8308	1968.5577	1547.8534	6571.9286	13811.664
9	0.1833	3.5258	−0.1233	14.5102	2.1277	2.8217	15.6969	3.7526	3.5957	−0.6097
10	99	68	99	98	100	89	33	58	77	98
11	25.2	80.8	48.7	56.8	35.2	33	158.6	113	58.3	76.3
12	100	90	100	91	96	98	67	89	90	98
13	8.73	6.15	7.89	5.88	6.67	6.4	4.42	5.42	6.81	7.56
14	0.7368	0.5169	0.6483	0.5139	0.5354	0.4547	0.4368	0.3918	0.5221	0.6707
15	5.7	4.3	4.5	4.1	5.5	2.4	3	3.3	2.6	4.1

Table A4. *Cont.*

Index (n)	Romania 17	Russia 18	Ukraine 19	UAE 20	USA 21	Venezuela 22
1	15.4	14.7	19.4	7.6	37.2	16.8
2	64.78	63.79	52.87	71.19	58.9	63.89
3	4.79	5.92	6.52	6.44	6.44	36.28
4	0	5.329	6.557	5.429	0.211	3.632
5	49.3614	77.1655	72.5616	37.7	42.8292	86.2069
6	1.596	3.16	3.113	2.3	1.82	2.642
7	113.9464	168.1717	235.2992	112.4116	113.3035	2740.274
8	10,813.717	10,743.097	2639.8243	59,531.662	40,698.849	
9	−1.5384	7.0498	13.895	1.2616	2.2	254.9485
10	82	89	96	100	100	95
11	123.4	85.9	136.5	24.1	15.9	28.9
12	100	96	98	99	100	97
13	7.66	5.9	6.2	8.82		1.93
14	0.6544	0.4677	0.4973	0.73	0.65	0.2863
15	4.4	3.8	3.7	5.9	6.5	1.8

Source: Adapted from [101], used with permission.

```
/* Please see Equation (5-38).
ILP-GP problem solved using IBM ILOG CPLEX Optimization Studio
    ↪ Version: 12.6.2.0 Build id: 0
data53.txt is the possibilities pairwise comparison for all
    ↪ alternative and criteria */

#include <iostream>
#include <time.h>
#include <ilcplex/ilocplex.h>

ILOSTLBEGIN

const IloInt nbC = 19;
const IloInt nbA = 53;

char* dataDir = "~/data53.txt";
char* resuDir = "~/result53.txt";
char* outDir = "~/output53.txt";

ifstream dataFile(dataDir, ifstream::in);
ofstream resuFile(resuDir, ofstream::out);
ofstream outFile(outDir, ofstream::out);

int ini_Fun(double p[nbA][nbA][nbC]);
IloNum Max_Cplex(double p[nbA][nbA][nbC]);

int main()
{
        double p[nbA][nbA][nbC];
        if (!outFile){
                cerr << "error: unable to open input file:" << endl;
                return(-1);
        }
        clock_t start, end;
        IloNum timeUsed;
        start = clock();
        ini_Fun(p);
        Max_Cplex(p);
        end = clock();
        timeUsed = (double)(end - start) / CLOCKS_PER_SEC;
        outFile << "Computation time is: " << timeUsed << "s" << endl;
        return (1);
```

```
}

int ini_Fun(double p[nbA][nbA][nbC])
{
        resuFile << "p[i][j][k]" << endl;
        for (IloInt k = 0; k < nbC; ++k){
                for (IloInt i = 0; i < nbA; ++i){
                        for (IloInt j = 0; j < nbA; ++j){
                                dataFile >> p[i][j][k];
                                resuFile << "p[" << i << "][" << j <<
                                    ↪ "][" << k << "]= " << p[i][j][k]
                                    ↪ << ", ";
                                resuFile << endl;
                        }
                }
        }
        return(1);
}

IloNum Max_Cplex(double p[nbA][nbA][nbC])
{
        IloEnv env;
        IloNum MaxP = 0;

        try{
                IloModel model(env);

                /* ------!------!------here define the variables
                    ↪ ------!------!------ */
                IloArray<IloBoolVarArray> x(env, nbA);
                for (IloInt i = 0; i < nbA; ++i){
                        x[i] = IloBoolVarArray(env, nbA);
                }

                /* ------!------!------here add the constraints
                    ↪ ------!------!------ */
                for (IloInt i = 0; i < nbA; ++i){
                        for (IloInt j = 0; j < nbA; ++j){
                                if (i == j)continue;
                                model.add(x[i][j] + x[j][i] == 1);
                        }
                }

                for (IloInt i = 0; i < nbA; ++i){
```

```
        for (IloInt j = 0; j < nbA; ++j){
                if (i == j)continue;
                for (IloInt l = 0; l < nbA; ++l)
                if (j == l)continue;
                else if (i == l)continue;
                else model.add(x[i][l] >= x[i][j] + x[j][
                    ↪ l] - 1);
        }
}

/* ------!------!------here add the object
    ↪ ------!------!------ */
IloExpr sumF(env);
IloExpr sumF1(env);
IloExpr sumF2(env);
IloExpr sumF3(env);
IloExpr sumF4(env);
IloExpr sumF5(env);
IloExpr sumF6(env);
IloExpr sumF7(env);
IloExpr sumF8(env);
IloExpr sumF9(env);
IloExpr sumF10(env);
IloExpr sumF11(env);
IloExpr sumF12(env);
IloExpr sumF13(env);
IloExpr sumF14(env);
IloExpr sumF15(env);
IloExpr sumF16(env);
IloExpr sumF17(env);
IloExpr sumF18(env);
IloExpr sumF19(env);

for (IloInt i = 0; i < nbA; ++i){
        for (IloInt j = 0; j < nbA; ++j){
                if (i == j)continue;

                sumF1 += x[i][j] * p[j][i][0];
                sumF2 += x[i][j] * p[j][i][1];
                sumF3 += x[i][j] * p[j][i][2];
                sumF4 += x[i][j] * p[j][i][3];
                sumF5 += x[i][j] * p[j][i][4];
                sumF6 += x[i][j] * p[j][i][5];
                sumF7 += x[i][j] * p[j][i][6];
```

```
                sumF8 += x[i][j] * p[j][i][7];
                sumF9 += x[i][j] * p[j][i][8];
                sumF10 += x[i][j] * p[j][i][9];
                sumF11 += x[i][j] * p[j][i][10];
                sumF12 += x[i][j] * p[j][i][11];
                sumF13 += x[i][j] * p[j][i][12];
                sumF14 += x[i][j] * p[j][i][13];
                sumF15 += x[i][j] * p[j][i][14];
                sumF16 += x[i][j] * p[j][i][15];
                sumF17 += x[i][j] * p[j][i][16];
                sumF18 += x[i][j] * p[j][i][17];
                sumF19 += x[i][j] * p[j][i][18];
        }
}

sumF = 0.0081*sumF1 + 0.0202*sumF2 + 0.0445*sumF3 +
    ↪ 0.039*sumF4 + 0.1171*sumF5 + 0.0028*sumF6 +
    ↪ 0.064*sumF7 + 0.023*sumF8 + 0.2264*sumF9 +
    ↪ 0.2264*sumF10 + 0.0412*sumF11 + 0.1029*sumF12 +
    ↪ 0.0301*sumF13 + 0.0663*sumF14 + 0.0121*sumF15 +
    ↪ 0.0569*sumF16 + 0.1707*sumF17 + 0.0153*sumF18 +
    ↪ 0.0051*sumF19;
sumF1.clear();
sumF2.clear();
sumF3.clear();
sumF4.clear();
sumF5.clear();
sumF6.clear();
sumF7.clear();
sumF8.clear();
sumF9.clear();
sumF10.clear();
sumF11.clear();
sumF12.clear();
sumF13.clear();
sumF14.clear();
sumF15.clear();
sumF16.clear();
sumF17.clear();
sumF18.clear();
sumF19.clear();

IloObjective obj = IloMaximize(env, sumF);
model.add(obj);
```

```
IloCplex cplex(env);
cplex.extract(model);

//Export the model into a file
cplex.exportModel("~/realTime17.lp");

//Solve the model and output the results
cplex.solve();

if (cplex.getStatus() == IloAlgorithm::Optimal){
       MaxP = cplex.getObjValue();

       //to the screen
       env.out() << "\nSolution status = " << cplex.
           ↪ getStatus() << "\nMaxP = " << cplex.
           ↪ getObjValue() << endl;
       env.out() << "\nThe computation time = " <<
           ↪ cplex.getCplexTime() << endl;
       env.out() << "\nThe Number of Nodes = " << cplex.
           ↪ getNnodes() << endl;
       env.out() << "\nNumber of Binaries = " << cplex.
           ↪ getNbinVars() << endl;
       env.out() << "\nNumber of Constraints = " <<
           ↪ cplex.getNrows() << endl;
       env.out() << "\nNumber of Variables = " << cplex.
           ↪ getNcols() << endl;

       //to a extra file
       outFile << "\nSolution status1 = " << cplex.
           ↪ getStatus() << "\nMaxP = " << cplex.
           ↪ getObjValue() << endl;
       outFile << "\nThe computation time1 = " << cplex.
           ↪ getCplexTime() << endl;
       resuFile << "\nThe Number of Nodes1 = " << cplex.
           ↪ getNnodes() << endl;
       resuFile << "\nNumber of Binaries1 = " << cplex.
           ↪ getNbinVars() << endl;
       resuFile << "\nNumber of Constraints1 = " <<
           ↪ cplex.getNrows() << endl;
       resuFile << "\nNumber of Variables1 = " << cplex.
           ↪ getNcols() << endl;

       for (IloInt i = 0; i < nbA; ++i){
              for (IloInt j = 0; j < nbA; ++j){
```

```
                                        if (i == j)continue;
                                        resuFile << "x[" << i << "][" << 
                                            ↪ j << "]=" << cplex.
                                            ↪ getValue(x[i][j]) << endl;
                                }
                        }
                }
                else{
                        if ((cplex.getStatus() == IloAlgorithm::
                            ↪ Infeasible) || (cplex.getStatus() ==
                            ↪ IloAlgorithm::InfeasibleOrUnbounded))
                        cerr << "The model is infeasible, Please check
                            ↪ it carefully!" << endl;
                        else{
                                cerr << endl << "**** error happend
                                    ↪ *****" << endl;
                                throw(-1);
                        }
                }
        }

        catch (IloException& e){
                cerr << "Concert exception caught: " << e << endl;
        }

        catch (...){
                cerr << "Unknown exception caught" << endl;
        }
        env.end();
        return (MaxP);
}
```

Glossary

Alternatives Options or choices available within a decision-making process, typically evaluated using various methods to determine the best choice among them.

Black Number A system with incomplete information, analogous to a closed container where internal workings are hidden from view.

Business The development and processing of economic value through providing goods and services to consumers, aimed at making a profit by the sellers.

Business Environment The business environment refers to the intricate set of conditions that significantly impact the evolution and sustainability of enterprises. It is comprised predominantly of external forces, factors, and institutions beyond a firm's direct control, including geographical positioning, governmental interventions, market trends, legal frameworks, and technological trajectories, which play a pivotal role in shaping its operational landscape.

Business Management The administration of an organization's resources, workforce, and strategies to achieve its goals efficiently and effectively, often involving multi-criteria decision-making methods to address complex and dynamic business environments.

Criteria The standards or principles by which something is judged or decided, often used in the context of decision-making processes involving multiple factors or attributes.

Decision maker Individuals who select courses of action to resolve challenges based on evaluative processes and predefined criteria.

Distance to Frontier Distance to Frontier (DTF) scores are assigned to every country to measure their relative performance in comparison to the best-performing country at the time of measurement. This score evaluates a country's absolute performance over time and is rated on a scale from 0 to 100. A score of 0 indicates the farthest distance from the best-performing country, while a score of 100 signifies that the country is on par with the best-performing country, with no distance between them.

Experts Individuals recognized for their extensive knowledge and skills in a specific field, often consulted for their expert opinions and judgments in decision-making processes.

Floating Solar Panel A solar panel system designed to float on water bodies, typically used for generating renewable energy while minimizing land usage and reducing evaporation of water resources.

Foreign Service Premium The lump sum allowance provided to expatriates as compensation for working overseas, aimed at attracting, retaining, and motivating them, often referred to as hardship or expatriate allowance.

Formative Construct A type of measurement model where observed variables are viewed as causing the construct rather than being caused by it.

GRA-ILP An approach that combines Grey Relational Analysis (GRA) with Integer Linear Programming (ILP) to evaluate performance under uncertain decision-making environments using grey interval numbers and pairwise comparisons to determine rankings and weights without the need for a grey distinguishing coefficient.

GRA-ILP-ROC An approach combining Grey Relational Analysis (GRA) with Integer Linear Programming (ILP) and Rank Order Centroid (ROC) methods to handle uncertainty and optimize weights in multi-criteria decision-making problems.

GRA-ROC A method combining Grey Relational Analysis with Rank Order Centroid for determining criteria weights in decision-making processes involving uncertainty.

Grey Number An unknown number with information of a known range of the exact number, used in Grey Systems Theory to represent systems with incomplete information.

Grey Possibility When expressing equalities and inequalities as possibilities, the grey possibility that one grey number is inferior to another and the grey possibility that one grey number is superior to another. The possibility degree lies within the interval of 0 and 1, with the degree of possibility representing the likelihood that one grey number is inferior or superior to another.

Grey Rank Order Centroid An extension of the Rank Order Centroid (ROC) weighting method using the Grey Systems Theory to estimate the weights of criteria in multi-criteria decision-making problems to address uncertainties and inconsistencies in group decision-making.

Grey Regime A method that extends the traditional regime approach by incorporating principles of the Grey Systems Theory to address uncertainties.

Grey Regulatory Focus Theory A method that integrates the Grey Systems Theory and Regulatory Focus Theory (RFT) to estimate criterion weights by capturing decision makers' preferences from both prevention and promotion focus orientations, representing these as interval grey numbers and standardizing them for use in MCDM weighting.

Grey Relational Analysis An MCDM method based on Grey Systems Theory that involves constructing a decision matrix, normalizing it, assigning criteria weights, comparing it to a reference alternative, computing grey relational grades, and ultimately selecting the best alternative for decision-making with incomplete information.

Grey Relational Coefficient A measure used in grey relational analysis to indicate the closeness of an alternative to the reference alternative, considering both minimum and maximum differences with a formula involving a grey distinguishing coefficient.

Grey Relational Grade A measure used in Grey Relational Analysis to indicate the closeness of an alternative to the reference alternative, calculated using the grey relational coefficient.

Grey SWARA A method for estimating evaluation weights in MCDM, incorporating the Grey Systems Theory to account for uncertainty by computing weights as interval grey numbers.

Grey System A system that contains both known and unknown information to model and analyze systems with incomplete or uncertain information.

Grey Systems Theory A mathematical framework used for dealing with problems that lack information and have uncertain characteristics.

Grey Weighted Sum Model An extension of the classic Weighted Sum Model (WSM) using the Grey Systems Theory to handle decision-making under uncertainty, incorporating interval grey numbers to represent uncertain criteria values and weights.

Human Resource Information System An integrated system designed to provide information used in HR decision-making, incorporating functions such as staff information management, organization structure management, compensation management, training management, and performance management.

Integral Linear Programming A mathematical optimization technique in which the objective function and the constraints are linear, and all the variables are required to be integers.

Interval Grey Number A grey number representing an uncertain value within a known range between lower and upper bounds.

Interval Number A numerical value defined within a known range. It represents all possible values within the bounds of a given range.

Management The process of dealing with or controlling things or people, often within an organizational context to achieve goals efficiently and effectively.

Multi-criteria Decision-making A process for making decisions in the presence of multiple, often conflicting criteria, aimed at finding the best possible compromise solution.

Normalization The process of scaling data to a uniform range, usually between 0 and 1, using different formulas based on whether higher or lower values are preferred, to ensure fair comparisons across criteria.

Partial Least Squares A statistical method primarily used for predictive modeling and dimension reduction, focusing on maximizing the covariance between response variables and predictors.

Performance Value A metric used to compare the performance of different alternatives in decision-making processes, often normalized and weighted to provide a comprehensive evaluation.

Point Allocation A method for determining the relative importance of criteria by assigning a certain number of points, typically on a percentage scale, to each criterion based on their perceived importance by decision makers.

Rank Order Centroid A method used in multi-criteria decision analysis to approximate the weights of criteria based on their rank order.

Reflective Construct A type of measurement model where the latent variable is considered the cause of the observed variables.

Regime A method for evaluating alternatives when dealing with ordinal data and qualitatively established relative impacts, involving steps like constructing a regime matrix and determining superiority indexes to rank alternatives.

Regulatory Focus Theory A theory articulated by Higgins that explores how individuals approach tasks and goals in the context of motivation, positing that motivation is maximized when the approach aligns with an individual's regulatory focus, which can be shaped by their personal goals and characteristics, identifying two primary orientations: promotion focus (striving for gains and opportunities) and prevention focus (avoiding losses and minimizing risks).

Standardization The process of making the (grey) decision matrix a scale from 0–1 using a standardized method to ensure consistency in evaluation and analysis.

SWARA Step-wise Weight Assessment Ratio Analysis, a Multi-Criteria Decision-Making method used to determine the importance of criteria through expert judgments in a step-wise manner.

University Reputation The amalgamation of beliefs, ideas, and impressions individuals hold about a university based on past and present experiences, encompassing the institution's image, performance objectives, and its standing among global universities.

Weight The assigned value indicating the relative importance or significance of a criterion or indicator in decision-making processes, as determined through methods reflecting their comparative influence on overall evaluation outcomes.

Weighted Sum Model A mathematical approach that combines multiple variables, each multiplied by a respective weight, to form a single composite score.

White Number A white number is a real or crisp value with complete information. Output of transforming grey numbers using a specific whitenization process, representing complete information about the parameter under consideration.

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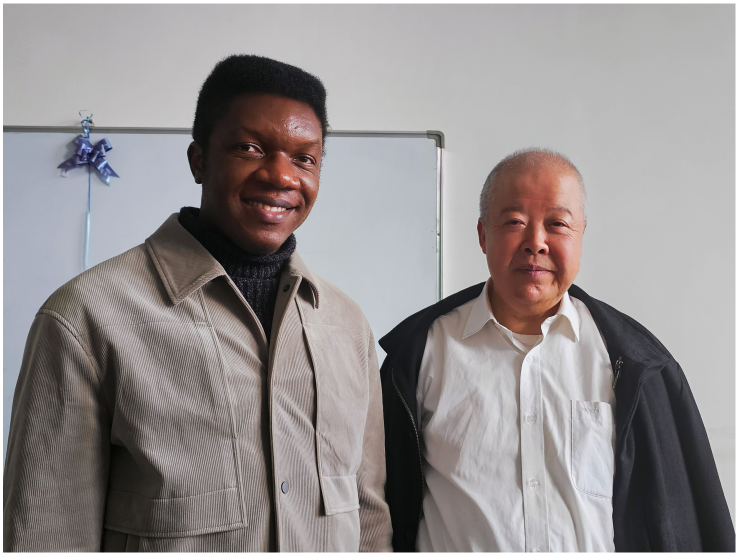
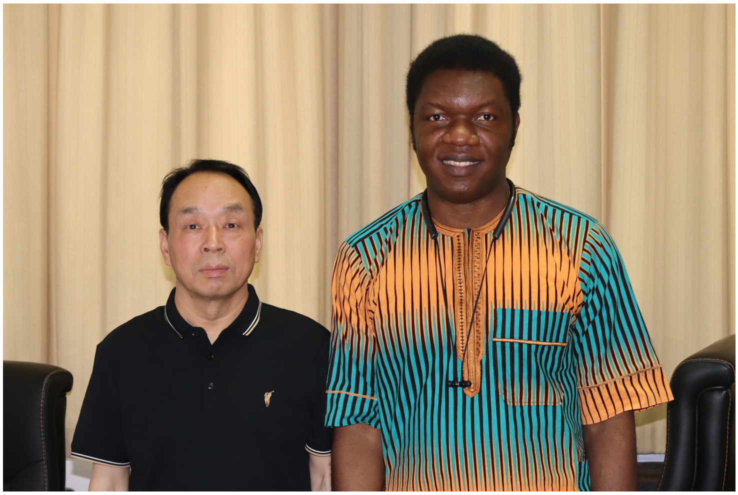
Photo Gallery



Moses O. Esangbedo and Professor Sifeng Liu at Northwestern Polytechnical University on the 14th of November, 2023.

Moses O. Esangbedo and Professor Ada Che at Northwestern Polytechnical University on the 30th of April, 2017.



Professor Jianwu Xue and Moses O. Esangbedo at Northwestern Polytechnical University on the 23th of May, 2022.

Professor Sijun Bai and Moses O. Esangbedo at Northwestern Polytechnical University on the 8th of July, 2022.

MDPI AG
Grosspeteranlage 5
4052 Basel
Switzerland
Tel.: +41 61 683 77 34

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