

Article

Modeling of Light Intensity and Temperature Effects on Algae Growth in Batch and Continuous Bioreactors

Zarook Shareefdeen *  and Salma Mansour

Department of Chemical and Biological Engineering, American University of Sharjah, Sharjah P.O. Box 26666, United Arab Emirates

* Correspondence: zshareefdeen@aus.edu

Abstract

Excessive concentrations of carbon dioxide (CO₂) in the atmosphere lead to adverse environmental effects. Biologically assisted processes that rely on organisms such as microalgae (i.e., *Chlorella vulgaris*) are common in capturing CO₂ from the atmosphere. Microalgae are rich in proteins, vitamins, minerals, and omega-3 fatty acids. Thus, microalgae production serves both health and environmental sectors. Varying light intensity and temperature are shown to influence algae growth. To quantify algae production under different light intensity and temperature conditions, and monitoring or scaling-up of biological reactors, reliable mathematical models are required. In this work, mathematical models that incorporate light intensity and temperature effects on algae growth in batch and continuous bioreactors are developed. Based on the modeling, the growth rate is maximum at T_{opt} = 25 °C, reaching the value of $\mu_{\max} = 0.14 \text{ day}^{-1}$. The growth rate exponentially increases until light intensity (I) reaches around $150 \frac{\mu\text{mol}}{\text{m}^2\text{s}}$, which is approximately the optimal light intensity for *Chlorella vulgaris*. The effect of T on growth rate is found to be more sensitive than light intensity (I) in both batch and continuous reactor systems. When there are too many parameters in models, uncertainties exist and parameter estimation and model predictions become cumbersome. For these reasons analytical solutions to the models are presented in simplified forms and these models are more practical and easier to implement. The novelty of the work is also the presentation of the models in analytical forms. Analytical solutions to the two reactor models (batch and continuous) will help quantify biomass production as a function of time under the varying light intensity and temperature conditions encountered.

Keywords: light intensity; temperature; modeling; algae growth; *Chlorella vulgaris*; carbon dioxide; bioreactors; batch; continuous; analytical solutions



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1. Introduction

Excessive presence of carbon dioxide (CO₂) in the atmosphere leads to adverse environmental effects including climate change, rising sea levels, and extreme weather events. Carbon dioxide (CO₂) can be removed from the atmosphere using multiple techniques including absorption, adsorption, membrane-based separation, cryogenic distillation, electrochemical processes, and biologically assisted processes. Biologically assisted processes that rely on organisms such as microalgae are becoming popular in capturing CO₂ from the atmosphere [1]. Algae thrive in various conditions, both freshwater and highly saline environments [2]. Due to algae's ability to use CO₂ for photosynthesis, carbon capture and bioproduct development, they are vital to ecological systems and they are rich in proteins,

vitamins, minerals, and omega-3 fatty acids [1,3]. Another important sector, namely the energy sector, also benefits from increased microalgal biomass use in the production of biofuels [1,4].

Production of algae can be maximized in bioreactors, in which biomass can be produced in a controlled environment in either batch or continuous mode of operation. Algae bioreactors exhibit several advantages over open cultivation systems [2]. By optimizing parameters like light intensity and temperature, bioreactors can improve algal production. The types of bioreactors used in algae growth include tubular reactors, bubble column reactors, vertical airlift reactors, flat panel reactors and stirred tank reactors. Tubular reactors consist of tubes, arranged horizontally, vertically, or helically [5]. Tubular reactors are prone to algae fouling, which increases maintenance costs and decreases light permeability [2,5]. Horizontal tubular reactors are suitable for outdoor environments and offer a high efficiency, but are energy-intensive compared to vertical tubular reactors [5]. A bubble column reactor consists of a vertical cylindrical vessel with a sparger which provides efficient mixing and CO₂ transfer [2]. Bubble columns typically rely on external lighting. Vertical airlift reactors are modified bubble column reactors, and are composed of the riser and the downcomer [2]. The advantages of a vertical airlift reactor include ample light exposure, which can be provided internally or externally. Flat panel reactors are characterized by their cuboidal shape [5] and they can be aligned horizontally or vertically. The high surface area-to-volume ratio of flat panels allows them to capture more light, thereby increasing the photosynthetic efficiency [5]. Stirred tank reactors consist of a closed tank and an impeller that provides mechanical agitation [2]. The advantages of using a stirred tank bioreactor include uniform mixing and operating conditions [5].

The growth of algae in a bioreactor is subject to the influence of several factors including pH, temperature, nutrient level, and light intensity [6]. The pH level is known to affect CO₂ dissolution, as well as the metabolism of microalgae [6]. It was found that pH levels below 2.5 or above 10.5 inhibit algal growth [6]. As for temperature, Chinni et al. (2019) [7] report optimal growth occurs during daylight hours within the temperature range of 28 to 37 °C, with peak growth at 36 °C. The growth is hindered at temperatures below 10 °C. Microalgae are also in need of essential nutrients, mainly nitrogen and phosphorus, to achieve the maximum growth [8].

Light intensity, which is a measure of energy per area per time, is an important factor that is considered when cultivating algae in a bioreactor [5]. In general, increasing light intensity improves the growth rate [9]. Khoeyi et al. (2012) [10] investigated the effect of light intensity and photoperiod on biomass and fatty acid composition of the microalgae *C. vulgaris*. Results indicated significant differences in biomass and fatty acids at different intensities and photoperiods. Nzayisenga et al. (2020) [9] tested the effects of light intensity on algal growth by measuring lipid production and found that light intensity plays a major role in algae growth; however, constant exposure to light hinders the growth of algae.

The cycle of light, known as the photoperiod, is also a crucial parameter to be considered [5]. Various photoperiods, such as 12/12, 16/8, and 14/10 h (light/dark) cycles, have been examined to optimize light exposure strategies. Additionally, the type of light source utilized also influences algal growth. Atta et al. (2013) [11] studied the effects of various blue light intensities and photoperiods on the growth and lipid content of microalgae *C. vulgaris* using LED (light-emitting diode) in a batch reactor. Cheirsilp and Torpee (2012) [12] studied the effects of light intensity on algal growth in mixotrophic conditions. Although microalgae growth is affected by multiple factors, optimizing light conditions is critical to successfully cultivating microalgae. According to a study by Gonçalves et al. (2014) [13], increasing solar irradiance from 36 to 180 $\mu\text{mol m}^{-2} \text{s}^{-1}$ improved the specific growth rate and increased biomass productivity of *C. vulgaris* and *P. subcapitata*. Here, μmol

refer to photon particles, which relate to light intensity. Moreover, there is evidence that light wavelengths also affect algae growth rates differently based on species. In one study conducted on the microalgae *C. vulgaris*, employing blue light with wavelengths of 400 to 500 nm resulted in a higher growth rate than red, green, or white lights [14]. In another study, Kim et al. (2014) [15] varied light wavelength to enhance growth of *C. vulgaris*. Various light-emitting diodes (LEDs), especially red and blue colors with different light wavelengths, were employed to explore the effects of light source. Blue light illumination led to significantly increased cell size, whereas red light resulted in small-sized cells with active divisions.

In our study we focus on two main factors, namely temperature and light intensity, that significantly affect algae growth in batch and continuous reactors. In order to quantify and enable the scale-up of algae production in large-scale reactors in either batch or continuous mode, reliable mathematical models are needed. Thus, the main objective of this work is to develop mathematical models that incorporate light intensity and temperature effects on algae growth in batch and continuous bioreactors and to obtain analytical solutions of the models that assist in the prediction of biomass concentration as a function of time under various light intensity and temperature conditions. When there are too many parameters in models, uncertainties exist; for this reason, analytical solutions to the models become more practical, are easy to implement and are used in overall process control dynamic studies.

2. Modeling

Given that no external limitations are present and all necessary growth factors are available in excess, the rate of growth could be described as proportional to the number of cells at any particular time [16]. As such, a larger number of cells present in a culture will result in higher growth rates. The following differential equation describes the simplest relation between algal cell density and growth rate:

$$\frac{dB}{dt} = \mu B \quad (1)$$

where B = concentration of biomass algae in culture (g/L) at time t (days), and μ = specific growth rate (day^{-1}).

This differential form, however, cannot be applied to most cultures, as it assumes perfect conditions and no cell death, which is not the case. In fact, cultures in experimental settings should resemble the natural environment as far as possible, with a set of rules specifying nutrient quantities and respective ratios to mimic natural conditions. As such, other differential forms are needed to model and represent algal growth in various culture types and conditions. Before delving into the different cultures and their respective models, the growth rate, which is the main factor influencing algae growth, must be optimized and accurately represented.

In this work, the main growth factors, light conditions and temperature, are investigated. Although other significant factors, such as pH and nutrient supply, affect growth, this work focuses solely on light conditions and temperature, meaning that certain assumptions are made with regard to the remaining factors. It is not uncommon to assume the influence of other factors is negligible when there is abundant nutrient supply, oxygen availability, etc. This assumption can be considered as one of the limitations of the models. For simplicity, it will be assumed that pH and nutrient supply are at optimum conditions for maximum growth, to ensure that they do not affect growth rates and thus, any observed effects are linked to both light conditions and temperature only. Thus, the model developed in this study is applicable to these conditions with non-limiting nutrients, stable pH, and sufficient dissolved oxygen.

2.1. Effects of Temperature on Algal Growth

According to Bernard and Remond (2012) [17], the following mathematical equation could be used to represent the temperature-dependent growth term of algae, given that the range of temperatures corresponding to the species satisfies the model equations.

$$\phi(T) = \frac{(T - T_{max})(T - T_{min})^2}{(T_{opt} - T_{min})[(T_{opt} - T_{min})(T - T_{opt}) - (T_{opt} - T_{max})(T_{opt} + T_{min} + 2T)]} \quad (2)$$

Satisfying these two conditions,

$$T_{min} < T < T_{max} \quad \text{and} \quad \frac{(T_{max} + T_{min})}{2} < T_{opt} \quad (3)$$

where T = temperature ($^{\circ}\text{C}$), T_{max} = maximum temperature ($^{\circ}\text{C}$) with observed algal growth, T_{min} = minimum temperature ($^{\circ}\text{C}$) with observed algal growth, T_{opt} = optimal temperature ($^{\circ}\text{C}$) for maximum algal growth and $\phi(T)$ = temperature-specific growth factor [18]. Zalizniak et al. (2025) [18] report an appropriate set of temperatures for *C. vulgaris* is: $T_{max} = 38^{\circ}\text{C}$, $T_{min} = 4^{\circ}\text{C}$ and $T_{opt} = 25^{\circ}\text{C}$. These values satisfy the conditions stated in Equation (3); that is, $\frac{(38+4)}{2} = 21 < 25$.

2.2. Effect of Light Intensity on Specific Growth Rate

Since light intensity affects the algae growth rate, it must be determined accurately and converted into quantifiable values that can help to predict the resultant algal growth rate. The light intensity (I), in units of W/m^2 , can be determined using the following equation:

$$I\left(\frac{\text{W}}{\text{m}^2}\right) = \frac{P}{4 \times \pi \times d^2} \quad (4)$$

where P = power of the light source (W) and d = distance from the light source (m).

It is important to note that when it comes to the growth rate of photosynthetic organisms, the light intensity being used for the actual photosynthesis process is only a fraction of the available light intensity. The common term for the light intensity solely used for photosynthesis is known as Photosynthetically Active Radiation (PAR), and it is measured in units of $\mu\text{mol}/\text{m}^2\cdot\text{s}$. As mentioned earlier, this unit refers to μmol photons (light particles). According to Reis and Ribeiro (2020) [19], the conversion factor that transforms light intensity from a unit of W/m^2 to $\mu\text{mol}/\text{m}^2\cdot\text{s}$ is $1.0 \frac{\text{W}}{\text{m}^2} = 4.25 \frac{\mu\text{mol}}{\text{m}^2\cdot\text{s}}$. This conversion factor assumes that all the energy is being transferred into light energy.

Higher biomass concentrations will result in lower light penetration; therefore, it is also important to account for the effect of biomass concentration on light intensity. Additionally, each microalgae species has distinctive pigments which absorb light differently; therefore, a factor specific to the microalgae species should be added when evaluating the overall light absorbance. As such, the Beer–Lambert law for light absorbance is applicable as proposed by Saccardo et al. (2024) [20]. The Beer–Lambert law is stated as follows:

$$I(z) = I_0 e^{-k_a c z} \quad (5)$$

where k_a is the Beer–Lambert light absorbance constant in units of $\frac{\text{cm}^2}{\text{g}}$, c is the biomass concentration in units of $\frac{\text{g}}{\text{cm}^3}$, and z is the distance from the light source in units of cm . Here, the units are kept for consistency as per literature reference [20]. Noac'h et al. (2025) [21] report that the absorption coefficient (i.e., exponential power, $k_a c z$) can be estimated either by finding the corresponding values for k_a , c , and z or using experimental data. As mentioned above, regarding the growth rate of *C. vulgaris*, employing blue light with wavelengths of 400 to 500 nm resulted in a higher growth rate than red, green, or

white lights [14]; thus, the wavelength of operation is also relevant in modeling. Using the experimental data of Baránková et al. (2020) [22], at optimum conditions (i.e., high light of 8.5 h, and wavelength of 440 nm), the coefficient k_{acz} (unitless) was estimated to be 0.7 or $I_{opt} = I_0 e^{-0.7}$. Noach et al. (2025) [21] proposed a parametrized equation for the light intensity-specific growth rate as follows:

$$\mu(I(z)) = \mu_{max} \cdot \frac{I(z)}{I(z) + \frac{\mu_{max}}{\alpha} \left(\frac{I(z)}{I_{opt}} - 1 \right)^2} \quad (6)$$

According to Muñoz & Bernard (2021) [23], μ_{max} is reported as 0.94 day^{-1} and the optimal light intensity is $142 \mu\text{mol}/(\text{m}^2 \cdot \text{s})$. In Section 3.1, μ_{max} (0.14 day^{-1}) is evaluated in our own lab. This value (0.14 day^{-1}) is consistent with the value reported in reference [14]. However, according to Muñoz & Bernard (2021), μ_{max} is reported as 0.94 day^{-1} for their experimental conditions (i.e., optimal light intensity is $142 \mu\text{mol}/(\text{m}^2 \cdot \text{s})$ [23]. Depending on the light intensity, wavelength, etc., selected, μ_{max} values vary. Equation (6) is applicable for algae growth kinetics since it accounts for hindrance in growth at high light intensities due to photoinhibition. Unlike the Monod growth model, light intensities above the optimal light intensity for the algae will result in lower growth rates rather than a constant positive relationship with increasing light intensities.

2.3. Application of Specific Growth Rate Dependent on Light Intensity and Temperature to a Batch Reactor System

Martinez-Ruiz et al. (2023) [24] formulated the following equation, which combines the individual growth rates dependent on temperature (T) and light intensity (I) as follows:

$$\mu(T, I) (\text{day}^{-1}) = \mu(I < P_{max}) \times \phi(T) \quad (7)$$

As shown in Figure 1, a batch system is characterized by no liquid-phase (algae or nutrients) mass transfer into or out of the system, meaning that the original mass of algae in the culture is allowed to grow within the same confined volume over time.

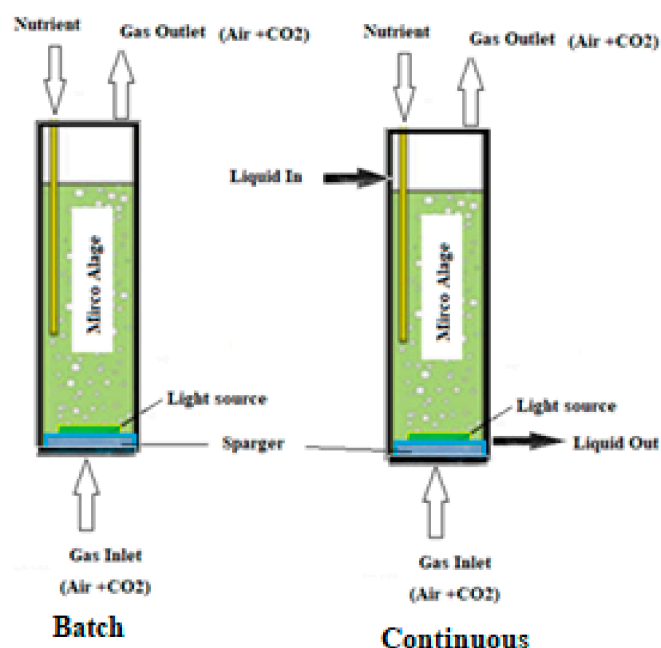


Figure 1. Schematic diagram of batch and continuous reactors.

According to the literature, the algae in batch cultures experience a period of exponential growth, given that the other growth factors are not limiting. The species then experiences a period of stationary growth, where the population growth falls out of an exponential relationship [25]. To accurately model growth kinetics in a batch culture, the overall mass transfer equation for the system is given as follows:

$$\frac{dB}{dt} - \mu(T, I)B + \text{Growth} - \text{Decay} = \text{accumulation} \text{ or } \text{Growth} - \text{Decay} = \text{accumulation} \quad (8)$$

In earlier stages, growth is exponential; however, light saturation eventually decreases with increasing algal mass in the batch culture due to self-shading which heavily affects the growth rate. According to the literature, the following equation represents the growth kinetics in a batch culture where the first term on the right represents the growth and the second term represents the decay or cell death [18]:

$$\frac{dB}{dt} = \mu(T, I)B - \frac{\mu(T, I)B^2}{K} \text{ or } \frac{dB}{dt} = \mu(T, I)B - \lambda B^2 \quad (9)$$

where K = maximum concentration of algae that can be attained in a batch reactor (g/L), and λ = decline coefficient in a batch reactor ($L \cdot g^{-1} \cdot day^{-1}$). In Equation (9), the coefficient $\frac{\mu(T, I)}{K}$ is represented by the symbol λ , which is the decline coefficient. The decline coefficient is limited by a maximum algae mass concentration (K). The above differential equation is solved using the separation of variables methods as follows:

$$\frac{dB}{B\left(\mu(T, I) - \frac{\mu(T, I)}{K}B\right)} = dt \quad (10)$$

$$\frac{1}{B\left(\mu(T, I) - \frac{\mu(T, I)}{K}B\right)} = \frac{A}{B} + \frac{F}{\mu(T, I) - \frac{\mu(T, I)}{K}B} \quad (11)$$

$$1 = A\left(\mu(T, I) - \frac{\mu(T, I)}{K}B\right) + FB = A\mu(T, I) - \frac{\mu(T, I)}{K}AB + FB \quad (12)$$

$$1 = A(\mu(T, I)) \text{ or } A = \frac{1}{\mu(T, I)} \quad (13)$$

$$0 = -\frac{\mu(T, I)}{K}AB + FB \text{ or } 0 = -\frac{\mu(T, I)}{K} \frac{1}{\mu(T, I)}B + FB \quad F = \frac{1}{K} \quad (14)$$

$$\int \frac{dB}{B\left(\mu(T, I) - \frac{\mu(T, I)}{K}B\right)} = \int \frac{1}{\mu(T, I)B} dB + \int \frac{1}{\mu(T, I)K} \left(\frac{1}{1 - \frac{1}{K}B}\right) dB = \frac{1}{\mu(T, I)} \ln B + \frac{1}{\mu(T, I)K} (-K) \ln \left(1 - \frac{1}{K}B\right) \quad (15)$$

$$\int \frac{dB}{B\left(\mu(T, I) - \frac{\mu(T, I)}{K}B\right)} = \frac{1}{\mu(T, I)} \ln \frac{B}{1 - \frac{1}{K}B} \quad (16)$$

$$\int \frac{dB}{B\left(\mu(T, I) - \frac{\mu(T, I)}{K}B\right)} = \int dt \quad \frac{1}{\mu(T, I)} \ln \frac{B}{1 - \frac{1}{K}B} = t + c \quad (17)$$

$$\frac{B}{1 - \frac{1}{K}B} = ce^{\mu(T, I)t} \quad \text{for } t = 0 \quad \frac{B_0}{1 - \frac{1}{K}B_0} = c \quad (18)$$

$$B = \left(1 - \frac{1}{K}B\right) ce^{\mu(T, I)t} \text{ or } B\left(1 + \frac{1}{K}ce^{\mu(T, I)t}\right) = ce^{\mu(T, I)t} \quad (19)$$

This leaves us with the final form, which represents the mass concentration of algae in a batch culture at any given time, shown below where c is a constant:

$$B = \frac{ce^{\mu(T,I)t}}{1 + \frac{1}{K}ce^{\mu(T,I)t}} \quad (20)$$

$$\text{where } c = \frac{B_0}{1 - \frac{1}{K}B_0}.$$

2.4. Application of Specific Growth Rate Dependent on Light Intensity and Temperature to a Continuous Reactor System

As shown in Figure 1, in a continuous reactor system, mass is transferred in and out of the system. A supply of nutrient solution is added to the culture while an equivalent amount is being withdrawn [25]. Generally, removing a specified quantity of the culture ensures that the mass present in the system remains constant and no self-shading occurs. As such, light saturation is always guaranteed to stay at maximum optimal levels, ensuring that light saturation does not hinder growth kinetics. If the concentration of the energy source, rather than the rate of flow, is used to limit the cell density, then the system is called a chemostat. The following equations constitute a model for a chemostat. First, the overall mass transfer equation for the continuous system is stated as

$$In - Out + Growth = accumulation \quad (21)$$

The terms are then substituted with variables obtained from the literature to form the following differential equation [26]:

$$DB_0 - DB + \mu(T, I)B = \frac{dB}{dt} \quad (22)$$

where D = dilution rate (day^{-1}) which is defined as the flow rate over volume. The differential equation is then rearranged and solved as a linear differential equation as shown below:

$$\frac{dB}{dt} + (D - \mu(T, I))B = DB_0 \quad (23)$$

$$\text{let integrating factor} = \int e^{(D - \mu(T, I))dt} = e^{(D - \mu(T, I))t} \quad (24)$$

$$e^{(D - \mu(T, I))t} \frac{dB}{dt} + e^{(D - \mu(T, I))t} (D - \mu(T, I))B = e^{(D - \mu(T, I))t} DB_0 \quad (25)$$

$$\frac{d}{dt} \left(e^{(D - \mu(T, I))t} B \right) = e^{(D - \mu(T, I))t} DB_0 \quad (26)$$

$$\int \left(e^{(D - \mu(T, I))t} B \right) dt = \int e^{(D - \mu(T, I))t} DB_0 dt \quad (27)$$

$$e^{(D - \mu(T, I))t} B = e^{(D - \mu(T, I))t} \frac{D}{D - \mu(T, I)} B_0 + c \quad (28)$$

$$B = \frac{D}{D - \mu(T, I)} B_0 + ce^{(\mu(T, I) - D)t} \quad (29)$$

$$\text{for } t = 0, \quad B_0 = \frac{D}{D - \mu(T, I)} B_0 + c \quad \text{or } c = B_0 - \frac{D}{D - \mu(T, I)} B_0 \quad (30)$$

Thus, the concentration of algae at any given time in a continuous system is given as follows:

$$B = \frac{D}{D - \mu(T, I)} B_0 + B_0 e^{(\mu(T, I) - D)t} \left(1 - \frac{D}{D - \mu(T, I)} \right) \quad (31)$$

3. Results and Discussion

3.1. Specific Growth Rate at Varying Temperatures and Light Intensities

Temperature-specific growth factor, $\phi(T)$ (no dimension) and $\mu(T, I(z))$ (day^{-1}) as a function of T ($^{\circ}\text{C}$), is plotted in Figures 2 and 3, respectively. $\phi(T)$ is given by Equation (2) and the parameters for *C. vulgaris* are selected from reference [18]. The parameters used for $\phi(T)$ are $T_{\max} = 38$ $^{\circ}\text{C}$, $T_{\min} = 4$ $^{\circ}\text{C}$ and $T_{\text{opt}} = 25$ $^{\circ}\text{C}$. In plotting $\mu(T, I(z))$ (day^{-1}) as a function T ($^{\circ}\text{C}$), the following parameters are used: $\mu_{\max} = 0.14$ day^{-1} , $I(z) = 150$ $\frac{\mu\text{mol}}{\text{m}^2 \text{ s}}$ and $\alpha = 0.039$ day^{-1} [23]. Figure 2 shows clearly the effect of temperature on the temperature-specific growth factor $\phi(T)$, where a maximum of 1 is reached at the optimal temperature $T_{\text{opt}} = 25$ $^{\circ}\text{C}$. The growth rate at different temperatures is represented in Figure 3. The growth rate is maximum at $T_{\text{opt}} = 25$ $^{\circ}\text{C}$, reaching the value of $\mu_{\max} = 0.14$ day^{-1} .

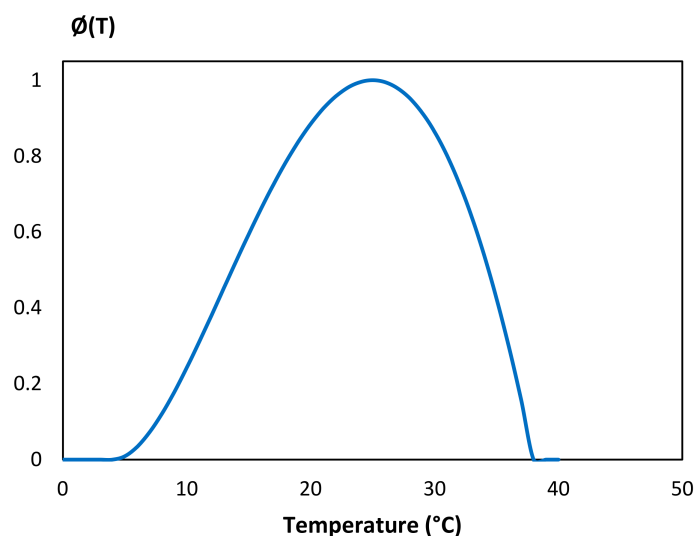


Figure 2. Temperature-specific growth factor $\phi(T)$ as a function of T ($^{\circ}\text{C}$).

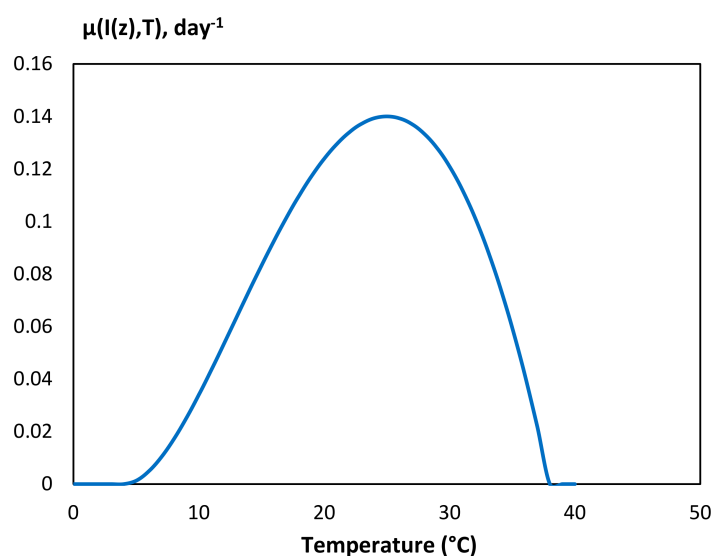


Figure 3. Specific growth rate $\mu(T, I(z))$ (day^{-1}) as a function of T ($^{\circ}\text{C}$).

In Figure 4, specific growth rate is plotted against varying light intensities. For Figure 4, the parameters used were as follows: $T = 25\text{ }^{\circ}\text{C}$, $\mu_{max} = 0.14\text{ day}^{-1}$ and $\alpha = 0.039\text{ day}^{-1}$ from reference data [23]. As seen in Figure 4, the growth rate exponentially increases until $I(z)$ reaches around $150\text{ }\frac{\mu\text{mol}}{\text{m}^2\text{ s}}$, which is approximately the optimal light intensity for *C. vulgaris*. Exceeding $150\text{ }\frac{\mu\text{mol}}{\text{m}^2\text{ s}}$, the specific growth rate starts decreasing slightly, representing photoinhibition. Mathematically, this decrease is because of the factor (i.e., in the denominator; refer to Equation (6)) $\frac{\mu_{max}}{\alpha} \left(\frac{I(z)}{I_{opt}} - 1 \right)^2$ represented in Equation (6) which accounts for photoinhibition at higher light intensities.

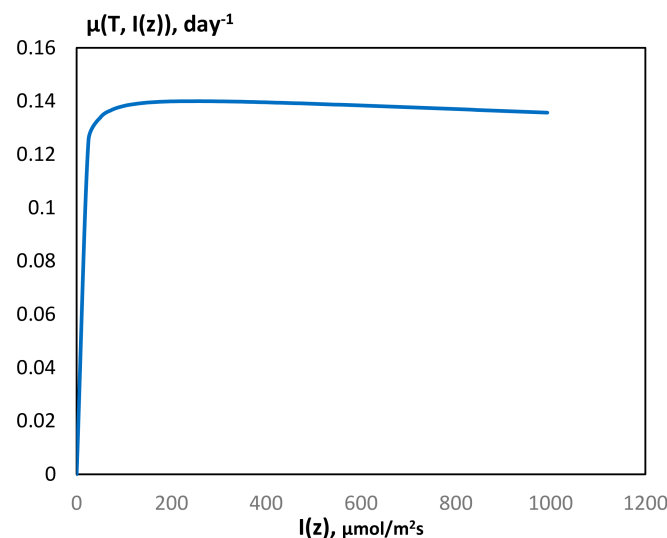


Figure 4. Specific growth rate $\mu(T, I(z))$ as a function of light intensity $I(z)$.

3.2. Biomass Growth in Batch Operation Systems

Using Equation (20), biomass concentration (B) in a batch reactor is calculated against time (days) and the results are shown in Figure 5. The following parameters are used in producing Figure 5: $T = 25\text{ }^{\circ}\text{C}$; $I(z) = 150\text{ }\frac{\mu\text{mol}}{\text{m}^2\text{ s}}$; initial biomass concentration, $B_0 = 0.414\text{ g/L}$; $\mu_{max} = 0.14\text{ day}^{-1}$; $\alpha = 0.039\text{ day}^{-1}$ and $K = 4.25\text{ g/L}$. The figure shows that after nearly 50 days, B (g/L) reaches the maximum concentration of algae (K) with a value of 4.25 g/L in the batch reactor.

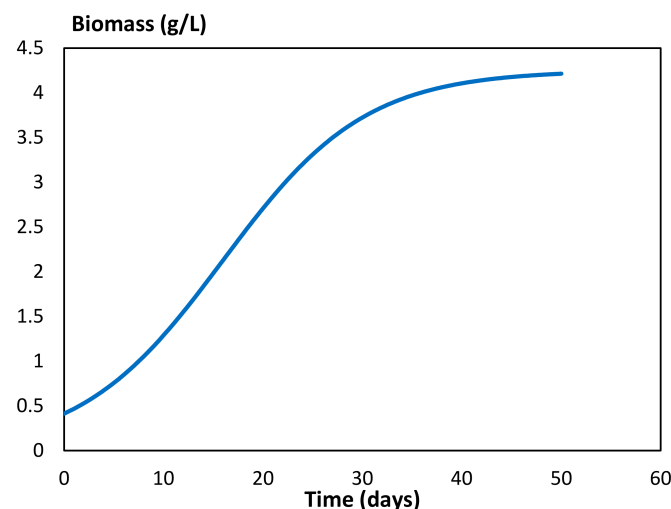


Figure 5. Biomass (g/L) vs. time (days) in batch operation. (Conditions: $T = 25\text{ }^{\circ}\text{C}$, $I(z) = 150\text{ }\frac{\mu\text{mol}}{\text{m}^2\text{ s}}$, $K = 4.25\text{ g/L}$, $B_0 = 0.414\text{ g/L}$).

In Figure 6, experimental growth data for a microalgae species from the literature [27] are compared with the model predictions. The parameters used for the model predictions are $\mu(T, I(z)) = 0.1 \text{ day}^{-1}$, $K = 2.25 \text{ g/L}$, and $B_0 = 0.20 \text{ g/L}$. The figure shows a reasonable agreement between the model predictions and experimental data. Although the microalgae species used in the published data [27] is different from *C. vulgaris*, the figure shows that with suitable model parameters, bioreactor performance can be easily predicted.

To further investigate the effect of temperature and light intensity on biomass growth, Figures 7 and 8 were produced. For Figure 7, the light intensity = $150 \frac{\mu\text{mol}}{\text{m}^2 \text{ s}}$, $K = 4.25 \text{ g/L}$, $B_0 = 0.414 \text{ g/L}$, $\mu_{\text{max}} = 0.14 \text{ day}^{-1}$, and $\alpha = 0.039 \text{ day}^{-1}$. The biomass growth over time (days) was plotted at three different temperatures: $T = 15^\circ\text{C}$, 25°C , and 35°C . As shown in Figure 7, the highest biomass growth was seen at $T = 25^\circ\text{C}$, which is expected as it is the optimal temperature for *C. vulgaris*. Additionally, increasing the temperature above optimal levels resulted in much lower biomass growth than when operating at below optimal temperatures. The increased stress at higher temperatures amplifies photoinhibition and therefore results in lower biomass growth over time. Comparing the graphs, the time it took for the biomass to reach half the final value (i.e., 4.25 g/L) for $T = 15^\circ\text{C}$, 25°C , and 35°C was 27, 16, and 38 days respectively.

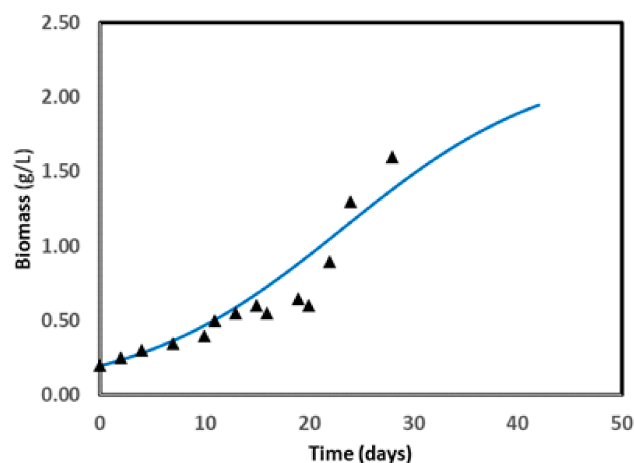


Figure 6. Biomass (g/L) vs. time (days) in batch operation: model predictions (solid line) against experimental data [27]. (Conditions: $\mu(T, I(z)) = 0.1 \text{ day}^{-1}$, $K = 2.25 \text{ g/L}$, $B_0 = 0.20 \text{ g/L}$).

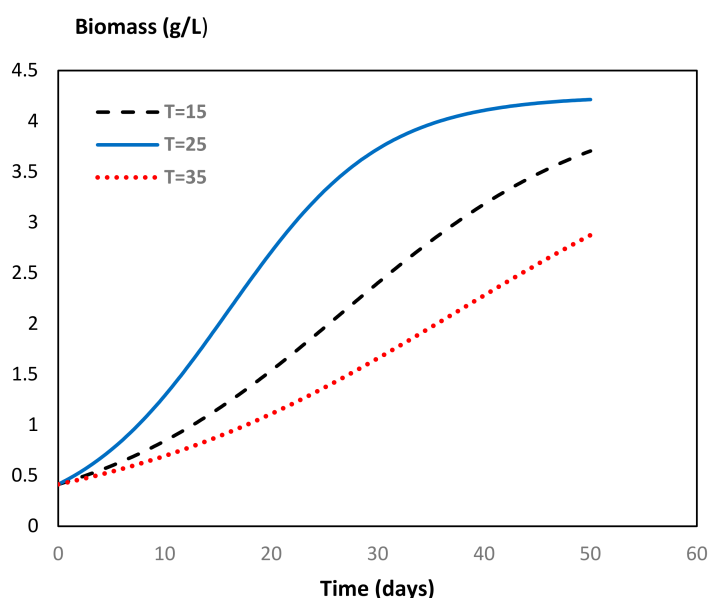


Figure 7. Biomass (g/L) vs. time (days) in batch operation at $T = 15^\circ\text{C}$, $T = 25^\circ\text{C}$, and $T = 35^\circ\text{C}$. (Conditions: $I(z) = 150 \frac{\mu\text{mol}}{\text{m}^2 \text{ s}}$, $K = 4.25 \text{ g/L}$, $B_0 = 0.414 \text{ g/L}$).

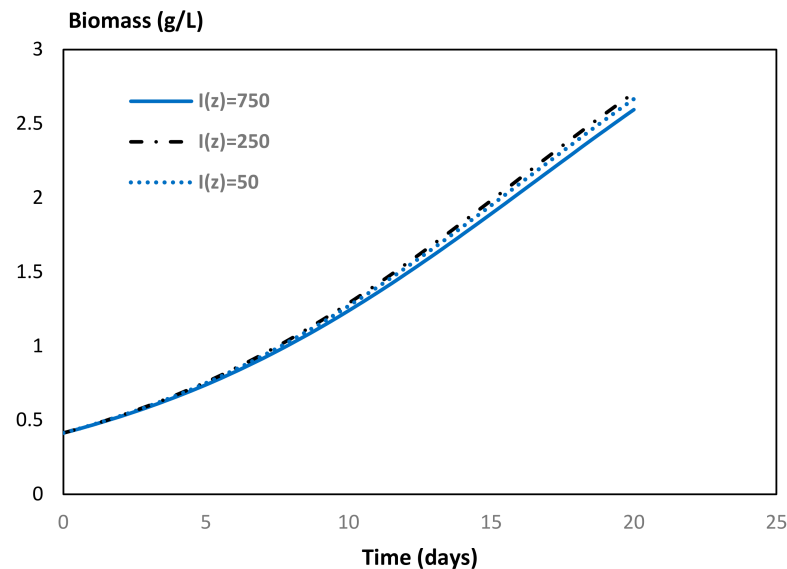


Figure 8. Biomass (g/L) vs. time (days) in batch operation at $I(z) = 50, 250,$ and $750 \frac{\mu\text{mol}}{\text{m}^2 \text{s}}$. (Conditions: $T = 25^\circ\text{C}$, $K = 4.25 \text{ g/L}$, $B_0 = 0.414 \text{ g/L}$).

In Figure 8, the effects of different light intensities on biomass growth are presented. To generate this graph, the parameters used were $T = 25^\circ\text{C}$, $K = 4.25 \text{ g/L}$, $B_0 = 0.414 \text{ g/L}$, $\mu_{\text{max}} = 0.14 \text{ day}^{-1}$, and $\alpha = 0.039 \text{ day}^{-1}$, and three different light intensities, $I(z) = 50, 150$ and $750 \frac{\mu\text{mol}}{\text{m}^2 \text{s}}$, were used. The difference in the plotted graphs for each respective light intensity is far less than the difference observed for biomass growth at different temperatures. As such, it can be deduced that changes in temperature have higher effects on the biomass growth than changes in light intensity. The lowest biomass growth was observed at $I(z) = 750 \frac{\mu\text{mol}}{\text{m}^2 \text{s}}$, followed by the growth at $I(z) = 50 \frac{\mu\text{mol}}{\text{m}^2 \text{s}}$, with the highest growth observed at the optimal light intensity $I(z) = 150 \frac{\mu\text{mol}}{\text{m}^2 \text{s}}$. These changes occur because at $I(z) = 50 \frac{\mu\text{mol}}{\text{m}^2 \text{s}}$, light saturation is not yet reached as it is below the optimal light intensity for *C. vulgaris*, meaning that the light intensity is a limiting growth factor. Biomass growth was the lowest at $750 \frac{\mu\text{mol}}{\text{m}^2 \text{s}}$. This decrease in growth is due to the fact that light saturation has already been reached at $I(z) = 150 \frac{\mu\text{mol}}{\text{m}^2 \text{s}}$, meaning that any light intensity exceeding the optimal limit will result in photoinhibition. It is also important to note that the effects of photoinhibition reduce the overall biomass growth compared to operation below optimal light intensities. For a more quantifiable comparison, the time needed to reach half of the final biomass concentration for $I(z) = 50, 150,$ and $750 \frac{\mu\text{mol}}{\text{m}^2 \text{s}}$ was recorded at 16.6 days, 16.0 days, and 16.2 days respectively.

Further analysis was carried out to determine the effects of different K values on the overall biomass growth over time. As previously explained, K is the maximum allowable biomass concentration in the reactor. The main factors that determine the value of K include the initial limiting nutrient availability, temperature, and self-shading of the culture. In batch operation, increases in biomass result in depletion of nutrients. Considering that one of the main assumptions of this model is the abundance of nutrients, K is therefore limited by the self-shading properties of the culture. Since biomass accumulates over time, less light penetrates the culture due to self-shading, therefore limiting the maximum allowable biomass quantity. Realistically, higher K values will result in higher biomass growth since more algae content is present throughout the growing phase. Figure 9 demonstrates the effects of different K values on overall biomass growth. In order to generate Figure 9, two K values, $K = 4.25 \text{ g/L}$ and $K = 2.25 \text{ g/L}$, were used, with the other parameters set as follows: $T = 25^\circ\text{C}$, $B_0 = 0.414 \text{ g/L}$, $\mu_{\text{max}} = 0.14 \text{ day}^{-1}$, $\alpha = 0.039 \text{ day}^{-1}$, and $I(z) = 150 \frac{\mu\text{mol}}{\text{m}^2 \text{s}}$. As

seen in Figure 9, after 20 days of operation, the total biomass concentration recorded for $K = 4.25$ g/L and $K = 2.25$ g/L was 2.7 g/L and 1.8 g/L respectively.

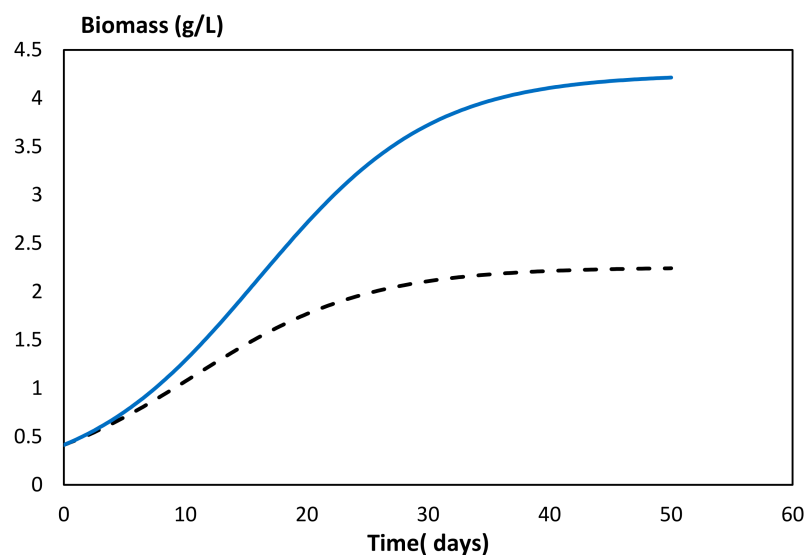


Figure 9. Biomass (g/L) vs. time (days) in batch operation at $K = 4.25$ g/L (solid line) and $K = 2.25$ g/L (dashed line). (Conditions: $T = 25$ °C, $I(z) = 150 \frac{\mu\text{mol}}{\text{m}^2 \text{ s}}$, $B_0 = 0.414$ g/L).

3.3. Biomass Growth in Continuous Systems

In continuous operations, the system has inflow and outflow of biomass which reduces self-shading significantly compared to batch operations. As such, the biomass is allowed to increase over time without a limiting carrying capacity. Figure 10 demonstrates the overall biomass growth according to Equation (31) in continuous operation at $T = 25$ °C, $I(z) = 150 \frac{\mu\text{mol}}{\text{m}^2 \text{ s}}$, $B_0 = 0.414$ g/L, $\mu_{\text{max}} = 0.14 \text{ day}^{-1}$, $\alpha = 0.039 \text{ day}^{-1}$, and $D = 0.05 \text{ day}^{-1}$.

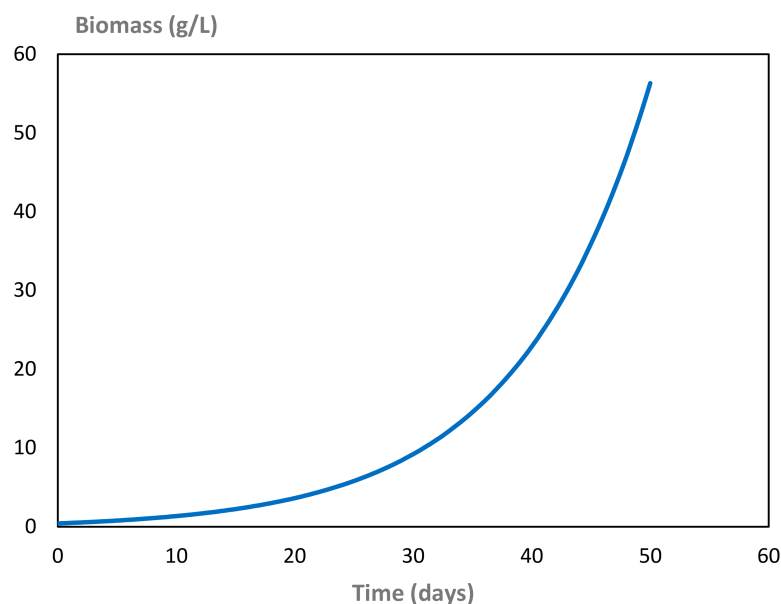


Figure 10. Biomass (g/L) vs. time (days) in continuous operation at $T = 25$ °C. (Conditions: $I(z) = 150 \frac{\mu\text{mol}}{\text{m}^2 \text{ s}}$, $B_0 = 0.414$ g/L, $D = 0.05 \text{ day}^{-1}$).

Figure 11 demonstrates the effects of temperature variation on biomass growth in continuous operation. To generate this graph, three temperatures, $T = 15$ °C, 25 °C, and 35 °C, were used, while other parameters were set as follows: $B_0 = 0.414$ g/L, $D = 0.05 \text{ day}^{-1}$, $I(z) = 150 \frac{\mu\text{mol}}{\text{m}^2 \text{ s}}$, $\mu_{\text{max}} = 0.14 \text{ day}^{-1}$, and $\alpha = 0.039 \text{ day}^{-1}$. The same trend that was applied

in the batch system was also used for continuous operation. Operating at temperatures below optimal levels makes the light intensity a limiting growth factor, therefore reducing the overall biomass growth. As for temperatures above optimal levels, high stress increases photoinhibition and reduces the overall biomass growth. Interestingly, lower temperature and light intensity affect biomass growth in continuous operations more drastically as compared to batch operations. For better comparison, the time required to reach a biomass concentration of 2.125 g/L was recorded for operation at $T = 15\text{ }^{\circ}\text{C}$, $25\text{ }^{\circ}\text{C}$, and $35\text{ }^{\circ}\text{C}$ as 29.5 days, 14.5 days, and 53.8 days respectively. From these values, it is evident that at optimal conditions, the overall biomass growth in continuous systems is greater than that in a batch system.

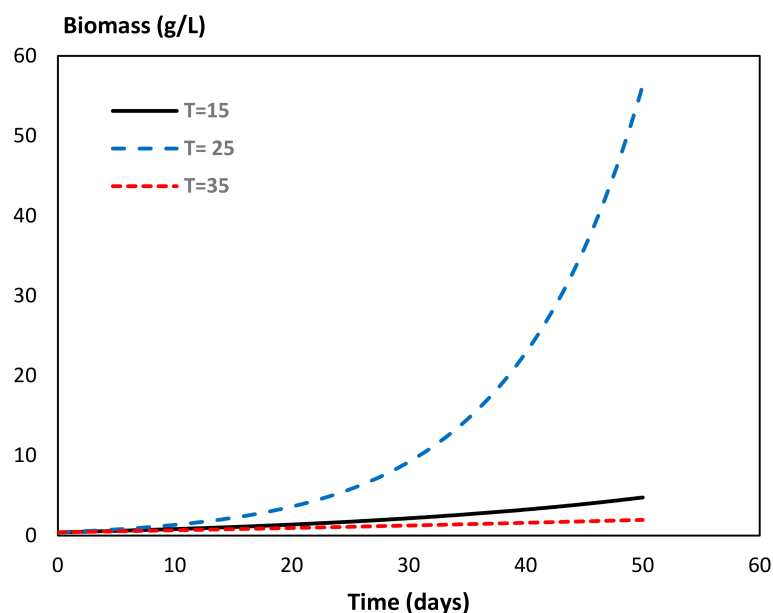


Figure 11. Biomass (g/L) vs. time (days) in continuous operation at $T = 15\text{ }^{\circ}\text{C}$, $T = 25\text{ }^{\circ}\text{C}$, and $T = 35\text{ }^{\circ}\text{C}$. (Conditions: $B_0 = 0.414\text{ g/L}$, $D = 0.05\text{ day}^{-1}$, $I(z) = 150\text{ }\frac{\mu\text{mol}}{\text{m}^2\text{ s}}$).

The effects of varying light intensities on biomass growth in continuous operation were also tested and demonstrated in Figure 12. For Figure 12, the temperature was set at $25\text{ }^{\circ}\text{C}$, $B_0 = 0.414\text{ g/L}$, $D = 0.05\text{ day}^{-1}$, $\mu_{max} = 0.14\text{ day}^{-1}$, $\alpha = 0.039\text{ day}^{-1}$, and three different light intensities were used: $I(z) = 50, 150$, and $750\text{ }\frac{\mu\text{mol}}{\text{m}^2\text{ s}}$. The overall change in biomass concentration due to varying light intensity is higher than that in batch operation, highlighting how changes in temperature- and light-specific growth rate affect the biomass growth in continuous operation to a higher degree. Comparing the time required to reach 2.1 g/L biomass concentration, the times recorded for $I(z) = 50, 150$, and $750\text{ }\frac{\mu\text{mol}}{\text{m}^2\text{ s}}$ were 15.2, 14.5 and 14.7 days respectively. Interestingly, the effects of photoinhibition at higher than optimal light intensities seem to affect the overall biomass in continuous operation less than when operating at lower than optimal light intensities. This deduction is because in continuous operation, the overall biomass in the culture is regulated by dilution rate D , limiting excessive accumulation while also mitigating high light attenuations.

The effect of different dilution rates (D) on the overall biomass growth is demonstrated in Figure 13. Generally, the dilution rate is what regulates biomass accumulation in a continuous system. Referring to Equation (31), the $(D - \mu_{max})$ term determines the overall biomass growth. For continuous operation, there exists a critical value for D , which is equal to μ_{max} . At values lower than the critical value, the biomass accumulates in the system since the rate of biomass production is higher than the dilution rate. Consequently, higher than critical D values will result in washout, where the overall biomass produced will de-

crease over time since dilution rate is higher than the production rate. Additionally, lower D values will result in higher biomass accumulation over time. For Figure 13, biomass concentration (g/L) is plotted against time (days) at different dilution rates of $D = 0.001$, 0.05 , and 0.12 day^{-1} with the other parameters set as follows: $T = 25 \text{ }^{\circ}\text{C}$, $I(z) = 150 \frac{\mu\text{mol}}{\text{m}^2 \text{ s}}$, $B_0 = 0.414 \text{ g/L}$, $\mu_{max} = 0.14 \text{ day}^{-1}$, and $\alpha = 0.039 \text{ day}^{-1}$. As seen in Figure 13, the highest biomass accumulation corresponds to $D = 0.001 \text{ day}^{-1}$, with the lowest biomass accumulation corresponding to $D = 0.12 \text{ day}^{-1}$. This trend is expected since 0.12 day^{-1} is much closer to the critical value $D = 0.14 \text{ day}^{-1}$. The biomass accumulation is less since higher D values move the system closer to washout. However, it is important to note that very low D values will result in less biomass harvesting since self-shading will be more prominent as accumulation will be significantly higher. Comparing the time required to reach 2.1 g/L , the results for $D = 0.001$, 0.05 , and 0.12 day^{-1} were 11.8, 14.5, and 23.4 days respectively.

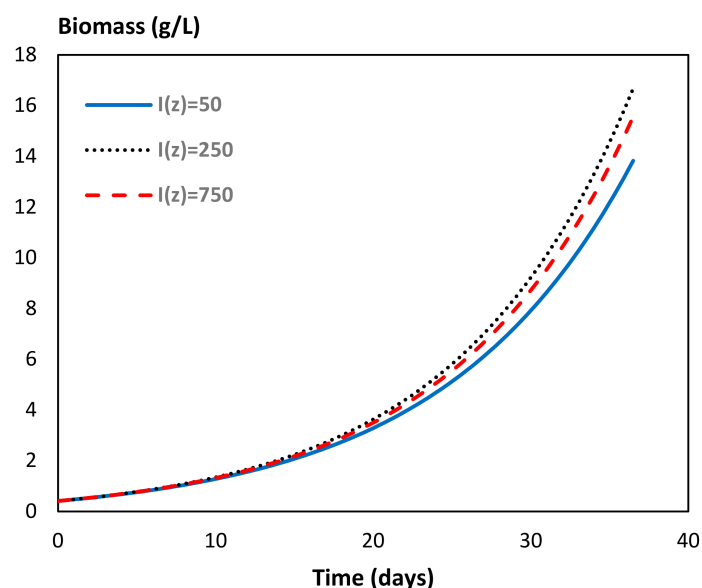


Figure 12. Biomass (g/L) vs. time (days) in continuous operation for $I(z) = 50, 250$, and $750 \frac{\mu\text{mol}}{\text{m}^2 \text{ s}}$. (Conditions: $T = 25 \text{ }^{\circ}\text{C}$, $D = 0.05 \text{ day}^{-1}$, and $B_0 = 0.414 \text{ g/L}$).

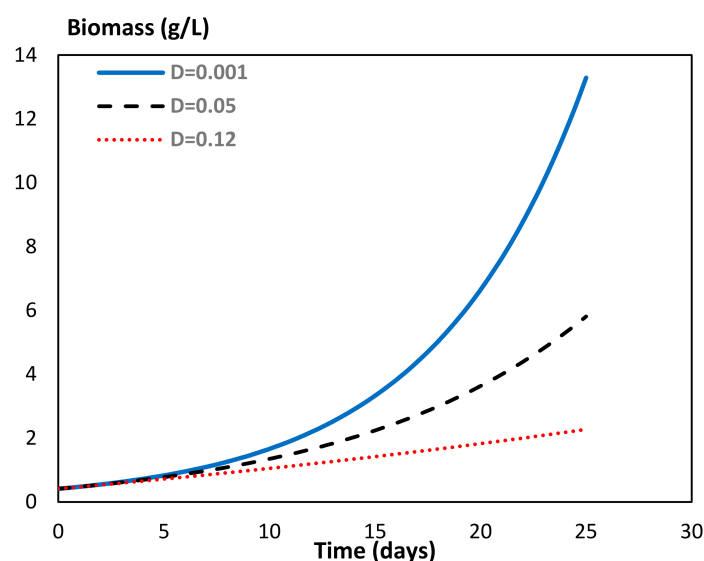


Figure 13. Biomass (g/L) vs. time (days) in continuous operation for $D = 0.001, 0.05$, and 0.12 day^{-1} . (Conditions: $T = 25 \text{ }^{\circ}\text{C}$, $I(z) = 150 \frac{\mu\text{mol}}{\text{m}^2 \text{ s}}$, and $B_0 = 0.414 \text{ g/L}$).

4. Conclusions

In this work, mathematical models that incorporate light intensity and temperature effects on algae growth for batch and continuous bioreactors are developed. Analytical solutions to the models are derived and these models help to quantify biomass production as a function of time under various light intensity and temperature conditions. We acknowledge the limitation of the models, which are based on the assumption of excess nutrients, oxygen, etc. Regardless, these models can be used easily to test data against experiments by engineers/scientists who deal with algae growth that depends on light intensity and temperature. Model comparison against experimental studies under different conditions will further enhance the validity and accuracy of the models. The novelty of the work is also related to presentation of the models in analytical forms which are very practical to implement. The effect of temperature on the temperature-specific growth factor $\phi(T)$ and specific growth rate are evaluated. Specific growth rate is evaluated against varying light intensities. When the optimum light intensity is exceeded, specific growth rate decreases due to photoinhibition. Batch and continuous reactor simulations show that increasing the temperature above optimal levels results in much lower biomass growth than when operating at below optimal temperatures. The increased stress at higher temperatures amplifies photoinhibition and therefore results in lower biomass growth over time. The results also show that changes in temperature have greater effects on the biomass growth than changes in light intensity. In continuous operations, the system has inflow and outflow of biomass which reduces self-shading significantly compared to batch operations. The overall biomass in continuous systems is regulated by the dilution rate which limits excessive accumulation and a low dilution rate will result in less biomass harvesting due to self-shading. With the proper selection of kinetic and model parameters, the models presented in this work will help advance the knowledge on the effects of temperature and light intensity on algae growth in bioreactor systems for various types of algal species.

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Nomenclature

Variable	Definition	Unit
B ₀	Initial mass of algae in culture	g/L
B	Mass of algae in culture	g/L
d	Distance from light source	m
D	Dilution rate of continuous reactor system	day ^{−1}
I	Light intensity	$\frac{W}{m^2}$
I(z)	Effective light intensity absorbed by microalgae	$\frac{\mu mol}{m^2 s}$
I _{opt}	Optimal light intensity for <i>C. vulgaris</i> determined experimentally	$\frac{\mu mol}{m^2 s}$
K	Maximum carrying capacity of algae that can be attained in batch operation	g/L
KI	Correlation factor	$\frac{\mu mol}{m^2 s}$
P	Power of light source	W
t	Time	days
T	Ambient temperature of experimental setup	°C
T _{max}	Maximum temperature with observed algal growth	°C

T_{min}	Minimum temperature with observed algal growth	$^{\circ}\text{C}$
T_{opt}	Optimal temperature for maximum algal growth	$^{\circ}\text{C}$
z	Distance from light source (for Beer–Lambert equation)	cm
α	Constant in parametrized equation for light intensity-specific growth rate	day^{−1}
$\mu(I(z))$	Specific growth rate dependent on light conditions	day^{−1}
$\mu(max)$	Maximum growth rate at optimal conditions in experimental models	day^{−1}
$\mu(T,I(z))$	Specific growth rate dependent on temperature and light intensity	day^{−1}
λ	Decline coefficient in batch reactor	g^{−1}day^{−1}
$\phi(T)$	Temperature-specific growth factor	dimensionless

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