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Inverse Fuzzy Model Control: A Data-Driven Learning Framework with Application to Thermal Process Control

Igor Škrjanc ^{1,*}, Daniel Leite ², Fernando Gomide ³  and Simon Tomažič ¹ ¹ Faculty of Electrical Engineering, University of Ljubljana, 1000 Ljubljana, Slovenia; simon.tomazic@fe.uni-lj.si² Department of Computer Science, University of Paderborn, 33098 Paderborn, Germany; daniel.leite@uni-paderborn.de³ School of Electrical and Computer Engineering, University of Campinas, Campinas 1251, Brazil; gomide@unicamp.br

* Correspondence: igor.skrjanc@fe.uni-lj.si

Abstract

Inverse fuzzy model control (IFMC) is an attractive strategy for regulating nonlinear thermal processes, where complex heat-transfer dynamics, disturbances, and actuator constraints challenge conventional control approaches. However, traditional direct and indirect inverse schemes often exhibit limited generalization and sensitivity to noise. This paper presents a data-driven inverse fuzzy control framework for temperature regulation in thermal systems, combining Takagi–Sugeno modeling, functional cancelation feedback, and auxiliary-input optimization. Gaussian antecedents and affine consequents are jointly identified via nonlinear least squares from input–output data generated by the theoretical PHE model. The same theoretical model is used as the numerical plant in all closed-loop simulations, whereas the forward and inverse fuzzy models are constructed only from the generated data and do not use the analytical PHE equations during identification or online inference. The learned inverse model estimates control actions that achieve the desired temperature trajectory while respecting actuator constraints. The method is evaluated in simulation on a plate heat exchanger (PHE) benchmark under tracking and disturbance-rejection scenarios. Results show accurate temperature regulation, smooth actuator behavior, and disturbance rejection in the tested scenarios. Compared with a numerically tuned PI baseline, the proposed approach reduces tracking RMSE by 27.5% and the standard deviation of the primary control current by 9.6% in the reported simulation scenario, indicating improved tracking and smoother primary actuation. In general, the proposed framework provides an interpretable and efficient solution for nonlinear thermal processes, with potential applications in energy systems, heat exchangers, and related thermal engineering technologies.



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1. Introduction

Inverse fuzzy model control (IFMC) schemes are commonly categorized into direct inverse control, where the inverse model directly maps desired outputs to control actions, and indirect inverse control, where a forward model is first identified and subsequently inverted [1,2]. These approaches have been successfully applied in various domains, including robotics, process control, and autonomous systems [3–5].

Despite their potential, inverse fuzzy controllers face well-known challenges related to closed-loop stability, generalization beyond training data, and sensitivity to noise. To address these issues, several enhancements have been proposed, including hybrid neuro-fuzzy schemes [6], evolving fuzzy systems capable of learning from data streams [2,5,7], and constrained learning strategies that incorporate physical priors [8].

More recently, inverse fuzzy control has expanded toward inverse optimal learning control, which seeks to derive optimal control actions without solving the Hamilton–Jacobi–Bellman (HJB) equation, a task known to be computationally demanding. Whereas adaptive dynamic programming and reinforcement learning aim to approximate solutions to the HJB equation directly, inverse optimal control relies on alternative cost formulations to obtain stabilizing control laws. For example, [9] combines indirect adaptive fuzzy control with inverse optimality for uncertain nonlinear systems, [10] proposes a direct adaptive fuzzy inverse optimal scheme, and [11] eliminates auxiliary systems by constructing inverse controllers solely from data and alternative cost functions.

The learning framework proposed in this paper builds on these developments. It employs an offline batch nonlinear least-squares identification procedure in which the nonlinear antecedent parameters and affine consequent parameters of Takagi–Sugeno (TS) fuzzy inverse models are jointly estimated from input–output data using Jacobian-based optimization. In contrast to correntropy-based robust recursive schemes developed for evolving fuzzy systems [12,13], the present approach emphasizes accurate batch modeling that can subsequently be embedded within an inverse control loop. The specific contribution is the simultaneous batch estimation of the multivariate Gaussian antecedents and affine consequents and their integration with inverse feedback and auxiliary-input selection, rather than a new recursive or evolving learning mechanism.

Applications of inverse fuzzy control also extend to renewable energy systems. In [14], a genetic-algorithm-optimized inverse fuzzy controller is applied to an oscillating water column wave energy converter, improving energy conversion efficiency by adapting to nonlinear turbine dynamics and environmental variability. While that study employs univariate Gaussian membership functions, the present work adopts multivariate Gaussian functions with full precision matrix estimation to enhance modeling fidelity. Given the relevance of inverse learning strategies for complex energy and thermal processes, the following section reviews recent advances in control and learning methods applied to thermal systems.

1.1. Literature Review

Thermal processes are characterized by nonlinear dynamics, large thermal inertia, time delays, and sensitivity to disturbances and operating conditions, which makes reliable control particularly challenging in industrial environments. For example, in hybrid heat-integrated distillation systems, variations such as fouling growth and liquid hold-up modify the thermal inertia of the process and can significantly affect the dynamic performance of conventional PID control [15]. Similar issues arise in vapor compression and refrigeration systems, where maintaining appropriate superheat levels avoids compressor damage and ensures high energy efficiency [16]. These difficulties highlight the need for control strategies capable of adapting to varying operating conditions and nonlinear process behavior, motivating advanced and data-driven control approaches for thermal systems.

Among advanced control strategies, model predictive control (MPC) has gained attention due to its ability to handle constraints and optimize system performance. Recent work also targets real-time feasibility by reducing MPC computational effort via robust inactive-constraint removal on laboratory heat exchangers [17]. For example, MPC has been applied to transcritical CO₂ thermal management systems in electric vehicles, where

a data-driven predictive model allows the controller to optimize system operation while maintaining the required cooling or heating capacity [18]. Similar ideas have been explored for compact heat exchangers and sCO₂ power cycles, where predictive models combined with disturbance observers and Gaussian process regression enable improved tracking performance and disturbance rejection [19]. MPC has also been successfully implemented in laboratory heat exchanger setups using self-tunable formulations that reduce computational requirements while improving tracking accuracy and reducing overshoot and settling time [20]. These studies demonstrate the potential of model-based predictive control for managing complex thermal dynamics.

The increasing availability of data has further accelerated the development of data-driven predictive control strategies for thermal systems. In air-cooled data centers, robust data-driven MPC frameworks have been proposed to regulate rack inlet temperatures using input-output measurements, eliminating the need for detailed physical models while maintaining accurate temperature regulation [21]. Similar ideas have been applied to cryogenic cooling systems in fusion devices, where neural network predictors combined with MPC can effectively mitigate oscillatory disturbances in liquid nitrogen cooling loops [22]. Data-driven control has also been validated in real industrial environments, such as waste-to-energy plants, where learning-based control methods demonstrated improved steam temperature regulation and operational stability during a continuous 90-day demonstration, outperforming traditional PID control [23]. In general, the results illustrate the growing maturity of data-driven control approaches for complex thermal infrastructures.

Beyond control design, machine learning techniques have increasingly been used to model the complex dynamics of thermal systems. Neural networks have been used to predict heat exchanger performance under varying operating conditions and to estimate parameters such as heat transfer coefficients or outlet temperatures with high accuracy [24,25]. Similarly, data-driven surrogate models have been developed to support the optimization of plate-fin heat exchangers, where neural network approximations can capture nonlinear thermodynamic relationships while significantly reducing computational costs compared with high-fidelity simulations [26]. This demonstrates the effectiveness of learning-based models in capturing nonlinear thermal behavior and supporting predictive analysis. The wider role of neural modeling is also illustrated by neural methods for constrained nonlinear optimization and catalytic-process prediction [27,28].

Fuzzy and neuro-fuzzy methods offer an alternative modeling paradigm that combines nonlinear approximation capability with interpretability. In thermal energy systems, Takagi–Sugeno and ANFIS-based models have been used to capture nonlinear thermal dynamics and provide predictions of system behavior. For example, ANFIS models have been applied to estimate frost accumulation in finned-tube heat exchangers using environmental and operating parameters [29]. Similar approaches have been used to model solar air heaters with thermal energy storage systems, where TS-ANFIS models accurately predicted outlet air temperature and system efficiency across varying operating conditions [30]. Neuro-fuzzy modeling has also been integrated into digital twin frameworks for photovoltaic facilities, demonstrating high predictive accuracy and low computational cost suitable for real-time monitoring and control [31]. Related hybrid designs combine learned predictors with fuzzy operators to improve temperature-target selection and regulation in thermoelectric cooling under varying environments [32]. Fuzzy-based learning models have shown potential for capturing nonlinear relationships in energy and thermal systems.

In addition to modeling, fuzzy operations have been applied to improve control performance and smooth control actions in nonlinear thermal processes. Adaptive fuzzy PI controllers have been proposed for transcritical CO₂ thermal systems in electric vehicles, where fuzzy inference mechanisms dynamically adjust controller gains to suppress overshoot

and improve settling time under varying operating conditions [33]. Fuzzy multi-model control architectures combined with dynamic sliding mode control have also been developed for thermal and chemical processes with long time delays, providing improved robustness to disturbances and uncertainties [34]. Hybrid learning approaches such as fuzzy neural networks have been explored for battery thermal management systems, allowing improved temperature regulation and reduced energy consumption compared to switching strategies [35]. Hybrid schemes show how fuzzy reasoning can be combined with learning and optimization techniques to enhance control performance in thermal systems.

Despite significant progress in predictive, data-driven, and fuzzy-based approaches for thermal systems, several challenges remain. Many existing methods rely on complex predictive models or optimization routines that may limit interpretability or computational efficiency. Others focus primarily on forward modeling or predictive control rather than learning inverse control mappings from data. These limitations motivate the development of interpretable inverse modeling frameworks capable of learning control actions directly from input–output data while maintaining robustness and efficiency. In this context, Takagi–Sugeno fuzzy models provide an attractive structure for constructing data-driven inverse control schemes capable of handling nonlinear thermal dynamics while retaining interpretability.

1.2. Main Contributions and Paper Organization

Motivated by these limitations, this work proposes an interpretable data-driven inverse fuzzy control framework based on Takagi–Sugeno models for nonlinear thermal processes. Its main contributions are:

- A data-driven inverse fuzzy control framework based on TS models for nonlinear thermal processes. The method learns the inverse mapping between desired outputs and control actions from data generated by the theoretical PHE model; once generated, the data are used to identify the fuzzy models without using the analytical PHE equations in the identification or online inference steps.
- A unified nonlinear least-squares identification procedure is developed to jointly estimate the antecedent parameters of Gaussian membership functions and the affine consequent parameters of the inverse model. The approach employs multivariate Gaussian membership functions with full precision matrix estimation, allowing improved modeling flexibility and fidelity.
- The learned inverse fuzzy model is integrated into a feedback control loop capable of generating smooth actuator signals while compensating for nonlinear dynamics and disturbances. The framework supports auxiliary-input optimization to handle actuator constraints.
- The proposed method is evaluated in simulation on a nonlinear plate heat exchanger benchmark, demonstrating improved temperature tracking accuracy, disturbance rejection capability, and smoother control signals compared to a tuned PI controller.

This paper therefore focuses on the TS model as the core component of a data-driven inverse control framework. The TS structure provides a mathematically grounded and computationally efficient representation of nonlinear mappings through locally affine models blended by fuzzy membership functions. The remainder of this paper is organized as follows. Section 2 presents the proposed data-driven inverse fuzzy control framework, including TS modeling, nonlinear least-squares learning, and inverse feedback control. Section 3 introduces the plate heat exchanger (PHE) benchmark and its modeling and identification setup. Section 4 describes the control implementation and simulation-based evaluation, including auxiliary-input optimization, trajectory tracking, robustness analysis, and comparison with proportional-integral control. Section 5 concludes this paper and describes directions for future work.

2. Proposed Data-Driven Inverse Fuzzy Control Framework

This section presents the proposed data-driven inverse fuzzy control framework. We first introduce the Takagi–Sugeno fuzzy model used to represent nonlinear input–output mappings then describe the batch nonlinear least-squares procedure employed for joint estimation of antecedent and consequent parameters. Finally, the learned inverse model is embedded within a feedback architecture to realize inverse fuzzy model-based control.

2.1. Takagi–Sugeno Model Structure

The TS fuzzy model represents a nonlinear mapping by blending multiple local affine models through fuzzy inference. Let $\underline{u} \in \mathbb{R}^p$ denote the input vector, $\underline{z} \in \mathbb{R}^s$ the partitioning vector responsible for capturing the process nonlinearities, and $y \in \mathbb{R}$ the scalar output. Each fuzzy rule R_i is defined as

$$R_i : \text{ if } \underline{z} \text{ is } A_i, \text{ then } y_i = \underline{a}_i^\top \underline{u} + b_i, \quad (1)$$

where A_i is a multivariate Gaussian membership function that depends on $\underline{z}$, i.e.,

$$A_i(\underline{z}) = \exp\left[-\frac{1}{2}(\underline{z} - \underline{v}_i)^\top \underline{P}_i(\underline{z} - \underline{v}_i)\right], \quad (2)$$

with $\underline{v}_i$ denoting the center vector and $\underline{P}_i \succ 0$ the precision matrix associated with rule i . Positive definiteness is enforced through the Cholesky parameterization $\underline{P}_i = \underline{L}_i \underline{L}_i^\top$, where $\underline{L}_i$ is lower triangular with positive diagonal entries. The vectors $\underline{a}_i$ and scalars b_i define the local affine consequents.

Let the unnormalized and normalized rule activations be defined as

$$w_i(\underline{z}) = A_i(\underline{z}), \quad \bar{w}_i(\underline{z}) = \frac{w_i(\underline{z})}{\sum_{j=1}^R w_j(\underline{z})}, \quad (3)$$

respectively. The global model output is then given by

$$\hat{y}(\underline{z}, \underline{u}) = \sum_{i=1}^R \bar{w}_i(\underline{z}) (\underline{a}_i^\top \underline{u} + b_i), \quad (4)$$

which ensures smoothness while yielding an interpretable representation of nonlinear functions.

2.2. Batch Offline Nonlinear Least-Squares Learning

Rather than relying on hybrid gradient/recursive least-squares schemes, the proposed method estimates all model parameters jointly through nonlinear least squares (NLS). This includes the nonlinear antecedent parameters $\{\underline{v}_i, \underline{L}_i\}_{i=1}^R$ and the linear consequent parameters $\{\underline{a}_i, b_i\}_{i=1}^R$.

Let $\mathcal{D} = \{(\underline{z}^k, \underline{u}^k, y^k)\}_{k=1}^N$ be the training dataset. The prediction error at sample k is

$$e^k(\underline{\theta}) = y^k - \hat{y}(\underline{z}^k, \underline{u}^k; \underline{\theta}) \quad (5)$$

and the batch cost function is

$$I(\underline{\theta}) = \frac{1}{2} \sum_{k=1}^N (e^k)^2, \quad (6)$$

where $\underline{\theta}$ collects all antecedent and consequent parameters.

Since $\hat{y}(\underline{z}^k, \underline{u}^k; \underline{\theta})$ depends nonlinearly on the antecedent parameters, the gradient of I is evaluated using the Jacobian $\underline{J} = [\frac{\partial e^1}{\partial \underline{\theta}^\top}, \dots, \frac{\partial e^N}{\partial \underline{\theta}^\top}]^\top$, whose rows contain derivatives with

respect to both antecedent and consequent parameters. The parameter update follows a standard NLS scheme such as the Gauss–Newton or Levenberg–Marquardt method,

$$\underline{\theta}^{(\ell+1)} = \underline{\theta}^{(\ell)} - \left(\underline{J}^T \underline{J} + \lambda_{\text{LM}} \underline{I}_{n_\theta} \right)^{-1} \underline{J}^T \underline{e}, \quad (7)$$

where $\underline{e} = [e^1, \dots, e^N]^T$, λ_{LM} is the regularization parameter, and $\underline{I}_{n_\theta}$ is the $n_\theta \times n_\theta$ identity matrix.

Unlike hybrid algorithms that alternate between gradient descent and linear regression, the NLS formulation optimizes the entire parameter set simultaneously, with the stacked parameter vector defined as

$$\underline{\theta} = [\underline{v}_1, \text{vech}(\underline{L}_1), \underline{a}_1, b_1, \dots, \underline{v}_R, \text{vech}(\underline{L}_R), \underline{a}_R, b_R]^T. \quad (8)$$

Positive definiteness of $\underline{P}_i$ is enforced via Cholesky reparameterization, ensuring valid Gaussian membership functions throughout optimization. Here, $\text{vech}(\underline{L}_i)$ contains the $s(s+1)/2$ independent lower-triangular entries, so $n_\theta = R[s + s(s+1)/2 + p + 1]$ and $\underline{J} \in \mathbb{R}^{N \times n_\theta}$.

The entire procedure is performed offline using the full training dataset at each NLS iteration, which provides (i) globally consistent updates without sample-by-sample stochasticity, (ii) improved convergence due to second-order curvature information, and (iii) accurate estimation of both antecedent geometry and local affine consequents. The unified NLS framework offers a consistent batch approach for identifying TS models without alternating separate antecedent and consequent updates while preserving full interpretability of the resulting rule base.

In summary, the TS model combines interpretability, flexibility, and smooth interpolation of local dynamics. Each rule defines an affine model over a localized region, while Gaussian antecedents and normalized activations ensure efficient adaptation and continuity, making TS models well suited for data-driven inverse control.

2.3. Inverse Model-Based Feedback Control

Inverse learning arises naturally in robotics (inverse kinematics), control (inverse dynamics), and machine learning, for example, in imitation learning or behavior cloning, where the objective is to compute inputs that achieve desired outputs. Practical challenges include non-uniqueness of the inverse mapping, overfitting in high-dimensional spaces, and limited interpretability of highly parameterized black-box models. Data-driven TS systems address these issues by combining adaptivity with structural transparency.

Inverse fuzzy model control (IFMC) drives the process using a controller that approximates the inverse plant dynamics. Consider the discrete-time nonlinear process

$$y^{k+1} = f(y^k, u^k), \quad (9)$$

where y^k and u^k denote the output and control input at time k . An inverse model is trained from the inverted dataset

$$D_{\text{inv}} = \{(y^k, y^{k+1}, u^k)\}_{k=1}^N, \quad (10)$$

so as to learn the mapping

$$u^k = f^{-1}(y^k, y^{k+1}), \quad (11)$$

where (y^k, y^{k+1}) forms the input vector and u^k is the output.

Using desired trajectories (y_d^k, y_d^{k+1}) , the control action becomes

$$u^k = f^{-1}(y_d^k, y_d^{k+1}), \quad (12)$$

which aims to enforce $y^{k+1} \approx y_d^{k+1}$.

Following [36], a singleton inverse TS model is embedded within a feedback structure, shown in Figure 1. The inverse model $u^k = f^{-1}(y^k, y^{k+1})$ compensates (cancels) the process dynamics in the forward path, such that the transfer between the process output and the inverse TS input approaches unity and u_a^k becomes approximately equal to y^k . The closed-loop dynamics are then shaped by

$$\frac{U_a(z)}{E(z)} = \frac{G_r(z)}{1 - G_r(z)}, \quad (13)$$

where $G_r(z)$ specifies the desired closed-loop poles and zeros inside the unit circle. A first-order discrete-time reference model is adopted,

$$G_r(z) = \frac{1 - a_r}{z - a_r}, \quad (14)$$

with tuning parameter a_r and unit steady-state gain for reference tracking. Choosing $a_r = 0$ yields the fastest response, provided actuator saturation is not a concern.

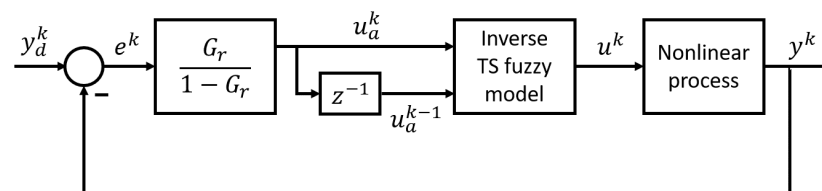


Figure 1. Block diagram of the inverse singleton fuzzy feedback control scheme.

From (13), the equivalent feedback filter is

$$G_f(z) = \frac{U_a(z)}{E(z)} = \frac{G_r(z)}{1 - G_r(z)} = \frac{(1 - a_r)z^{-1}}{1 - z^{-1}}, \quad (15)$$

which in the time domain yields

$$u_a^k = u_a^{k-1} + (1 - a_r)e^{k-1}. \quad (16)$$

This recursion corresponds to a discrete-time integrator that drives the tracking error e^k toward zero. Consequently, y^k follows the desired trajectory y_d^k , with convergence rate governed by a_r . The transfer function $G_r(z)$ is referred to as the reference model.

Stability and performance depend directly on the choice of a_r . Selecting $a_r \in [0, 1)$ ensures stability; smaller values provide faster tracking but increased sensitivity to disturbances, whereas values closer to unity improve robustness at the cost of slower convergence, yielding a practical speed–robustness trade-off. Closed-loop performance also depends on the accuracy of the learned inverse model, motivating the use of adaptive fuzzy structures that provide smooth local interpolation and robustness to modeling errors and process uncertainties.

A bounded-error statement can be given for an imperfect inverse model. For a constant reference and away from sustained actuator saturation, write the realized closed loop as

$$y^{k+1} = a_r y^k + (1 - a_r)y_d + \eta^k, \quad (17)$$

where η^k is the inverse-model mismatch. If $|\eta^k| \leq \bar{\eta}$ and $0 \leq a_r < 1$, then

$$|e^k| \leq a_r^k |e^0| + \frac{1 - a_r^k}{1 - a_r} \bar{\eta}, \quad \limsup_{k \rightarrow \infty} |e^k| \leq \frac{\bar{\eta}}{1 - a_r}. \quad (18)$$

Thus, bounded inverse-model error gives an ultimately bounded tracking error under these stated conditions; this is not a global stability proof in the presence of sustained saturation.

3. Plate Heat Exchanger Simulation Benchmark

This section introduces the plate heat exchanger (PHE) employed as a simulation benchmark for the development and evaluation of the proposed data-driven inverse fuzzy control framework. The theoretical PHE model is used in two roles: it generates every input–output sequence used to identify and validate the forward and inverse fuzzy models, and it serves as the numerical plant in all closed-loop control tests. Consequently, all fuzzy models are constructed exclusively from data generated by the theoretical model, and no measurements from a physical PHE are used in this study. The system represents a nonlinear thermal process with coupled flows, actuator constraints, and measurable disturbances, making it well suited for system identification, forward and inverse modeling, and closed-loop control validation.

3.1. Process Description, Modeling, and Control Formulation

A heat exchanger transfers thermal energy between two fluid streams and appears in numerous applications, including thermal regulation in robotics, HVAC and building climate-control systems, water cooling loops, automotive radiators, and heat management in chemical or pharmaceutical reactors. In this work, only the mathematical PHE model is used: it generates the complete identification and validation dataset and supplies the plant response during controller evaluation. The setup consists of a primary circuit with heated water that transfers heat to a secondary circuit with cold water through metallic plates, as illustrated in Figure 2.

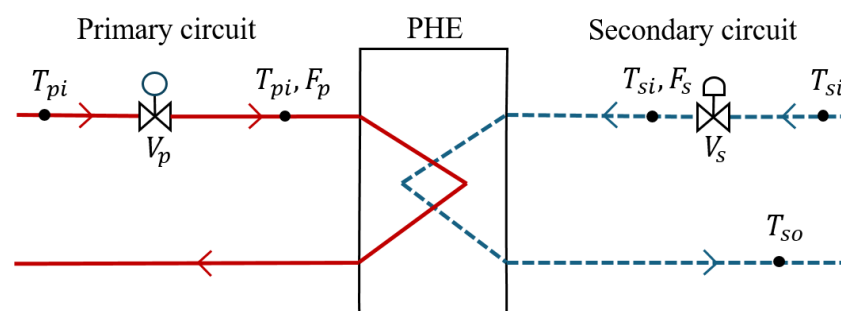


Figure 2. Schematic of the plate heat exchanger represented by the simulation benchmark.

The theoretical model, adapted from [37], describes the PHE as a nonlinear multiple-input single-output system,

$$\tau(t) \frac{dT_{so}(t)}{dt} + T_{so}(t) = \gamma(t)T_{si} + (1 - \gamma(t))T_{pi}(t), \quad (19)$$

$$\gamma(t) = \frac{1 + k_c \left(\frac{1}{F_p(t)} \right)^m}{1 + k_c \left(\left(\frac{1}{F_p(t)} \right)^m + \left(\frac{1}{F_s(t)} \right)^m \right)}, \quad (20)$$

$$F_p(t) = v_1 \left(\frac{1}{\pi} \text{atan}(v_2(V_p(t) - \bar{V}_p)) + \frac{1}{2} \right), \quad (21)$$

$$F_s(t) = v_3 V_s(t). \quad (22)$$

Where, $T_{so}(t)$ is the secondary outlet temperature (system output), $T_{pi}(t)$ the primary inlet temperature, and T_{si} the constant secondary inlet temperature. The primary inlet temperature $T_{pi}(t)$ may exhibit slight oscillations due to upstream on/off heating control. The primary flow $F_p(t) \in [0.1, 1]$ is regulated by a servo valve driven by the current $V_p(t) \in [4, 20]$ mA, while the secondary flow $F_s(t) \in [0.1, 1]$ is adjusted through the pneumatic-valve command $V_s(t) \in [0.1, 1]$, where v_3 is a constant representing the valve characteristics.

The system time constant $\tau(t) = f(T_{so}(t))$ introduces temperature-dependent dynamics, and the parameters k_c , m , v_1 , v_2 , v_3 , and $\bar{V}_p$ (average valve input) characterize flow interactions and valve behavior. The primary inlet temperature is modeled as a sinusoidal disturbance with an amplitude of 2 °C and a period of 360 s around $\bar{T}_{pi} = 90$ °C. Simulated measurement noise $\mathcal{N}(0, 0.1)$ is added to $T_{so}(t)$.

Both flow rates $F_p(t)$ and $F_s(t)$ are normalized to the strictly positive interval $[0.1, 1]$; zero is excluded because the model contains the reciprocal terms F_p^{-m} and F_s^{-m} . The temperature-dependent time constant is given by

$$\tau(T_{so}(t)) = 55 - 0.5 T_{so}(t). \quad (23)$$

The inlet temperature of the secondary circuit is fixed at $T_{si} = 7$ °C. The theoretical benchmark parameters adopted from [37] are $k_c = 0.65$, $m = 1.93$, $v_1 = 1.04$, $v_2 = 0.23$, $v_3 = 1$, and $\bar{V}_p = 10$. The continuous-time model (19)–(22) is discretized using a sampling time $t_s = 2$ s for simulation.

From a control perspective, $V_p(t)$ acts as the primary control input, directly regulating the primary flow rate and thus exerting the strongest influence on the process dynamics (denoted in discrete time as u^k). The signal $V_s(t)$ provides an auxiliary control input that offers additional degrees of freedom to shape the thermal response of the secondary circuit (denoted u_{ac}^k). The primary inlet temperature $T_{pi}(t)$ is treated as an external disturbance (d^k), while the secondary outlet temperature $T_{so}(t)$ is the controlled output (y^k). The control objective is to regulate T_{so} close to its reference while simultaneously maintaining a high secondary flow F_s to ensure efficient heat delivery, which requires coordinated adjustment of the auxiliary input without degrading temperature tracking performance.

3.2. Simulated Data and Identification Setup

The following simulated trajectories provide the data used to identify and validate both the forward and inverse fuzzy models. They characterize actuator dynamics, thermal behavior, and operating-range coverage, allowing accurate modeling of process nonlinearities and supporting reliable controller design.

Figure 3 shows the simulated excitation profiles of the current $V_p(t)$ and the pneumatic-valve command $V_s(t)$. The profiles cover the operating region required for modeling primary flow regulation. Observed nonlinearities and possible delays are accounted for during system identification.

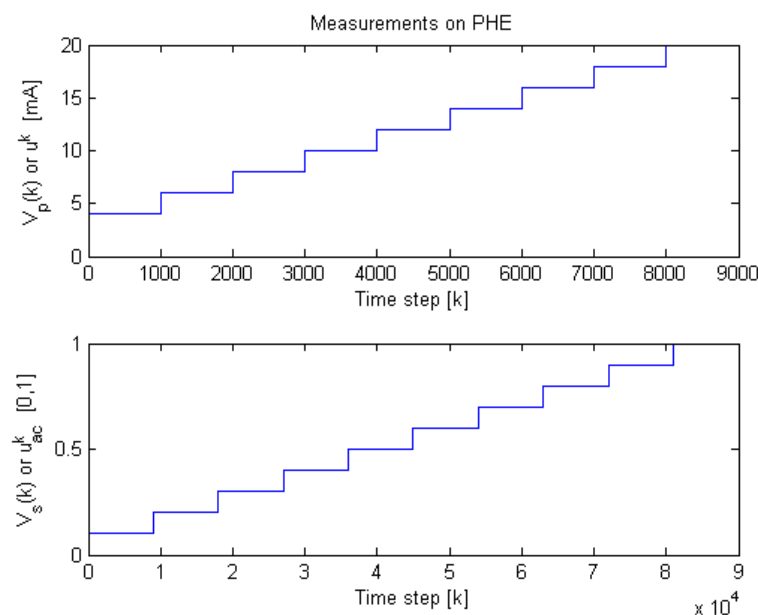


Figure 3. Simulated primary-valve excitation $V_p(t)$ [mA] and secondary-valve command $V_s(t)$ [dimensionless].

Figure 4 presents the simulated thermal response of the process through the secondary outlet temperature $T_{so}(t)$ and the primary inlet temperature $T_{pi}(t)$. Their interaction reflects the heat-exchange dynamics and disturbance propagation, providing the information required for accurate modeling and control design accounting for disturbances.

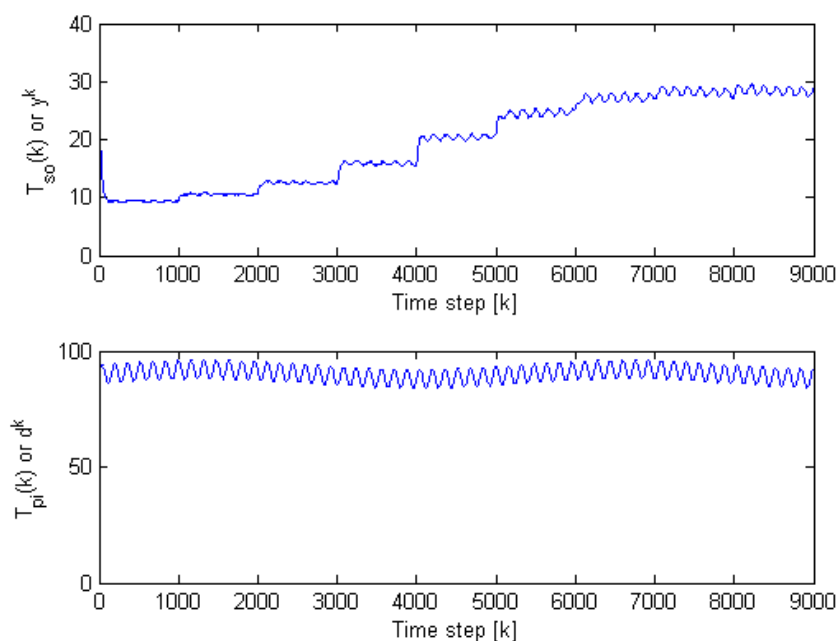


Figure 4. Simulated secondary outlet temperature $T_{so}(t)$ and primary inlet temperature $T_{pi}(t)$ [°C].

Figure 5 illustrates the design of experiments (DOE) adopted to construct the inverse model. The DOE systematically excites the operating space, ensuring sufficient coverage of the nonlinear dynamics and generating a structured dataset for training the fuzzy inverse mapping from desired outputs to control inputs.

Together, these model-generated trajectories define the data basis for the forward and inverse fuzzy models over the admissible operating range. Using the generated data and the TS structure and nonlinear least-squares learning procedure introduced in Sections 2.1 and 2.2,

a forward TS fuzzy model of the process is identified. The resulting model is subsequently evaluated on an independent simulated dataset to assess predictive accuracy and generalization.

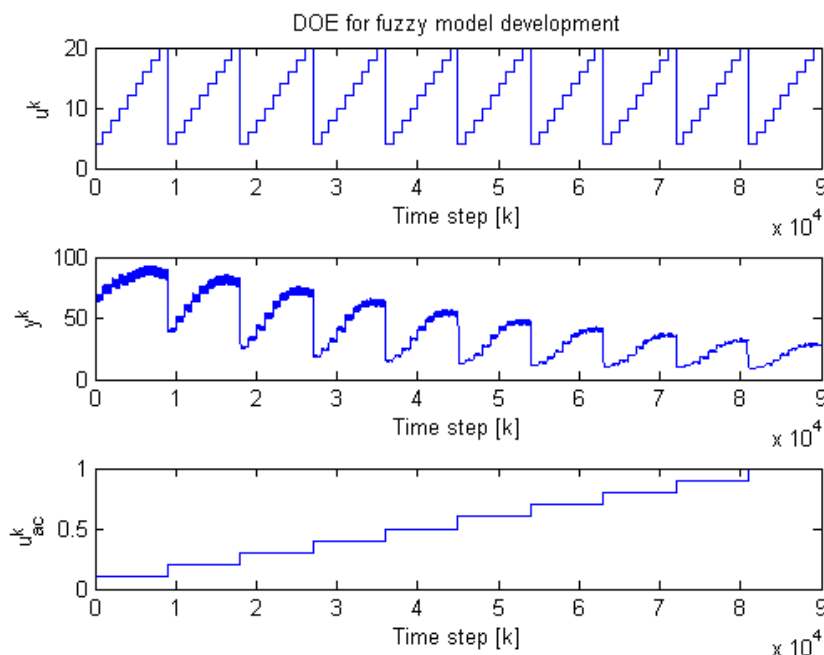


Figure 5. Simulation-based design of experiments (DOE) used for inverse-model identification.

3.3. Forward-Model Validation

The TS fuzzy forward model is validated using an independent simulated dataset not used during training to assess generalization and avoid overfitting. Figure 6 compares the theoretical-benchmark output y^k with the model prediction y_m^k . The close agreement between both signals indicates that the model accurately captures the dominant nonlinear and time-varying dynamics of the process.

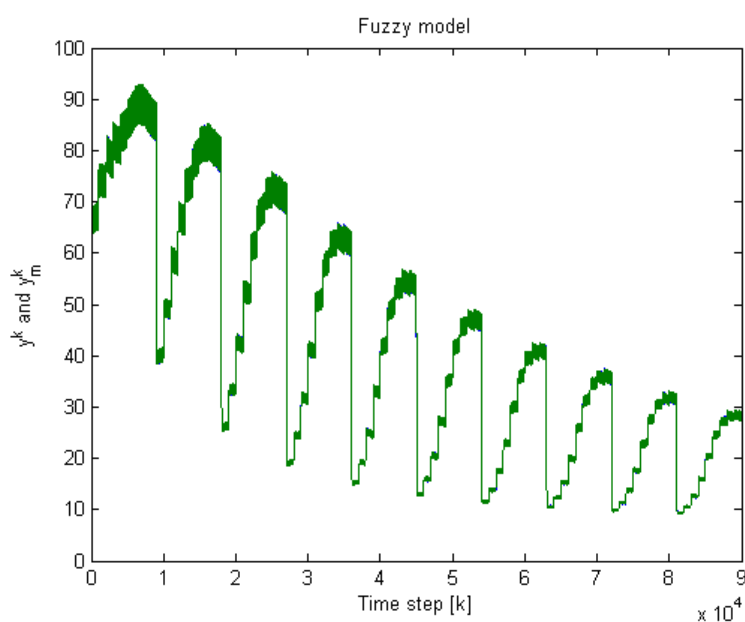


Figure 6. Forward-model validation on simulated data: benchmark output y^k and prediction y_m^k [°C] versus sample index k .

Minor deviations observed during fast transients are attributed to fuzzy approximation and input quantization effects and may be reduced through rule-base expansion or adaptive

refinement. Overall, the results confirm that the model provides sufficiently accurate predictions for integration within the subsequent model-based control framework.

The corresponding validation metric is $\text{RMSE}_{\text{fwd}} = \sqrt{N_v^{-1} \sum_{k=1}^{N_v} (y^k - y_m^k)^2}$. A numerical value is not inferred from Figure 6 because the sample-wise validation vectors are not contained in the archived manuscript source.

The discrete-time forward TS model is defined as

$$y_m^{k+1} = f(u^k, d^k, u_{ac}^k, y^k), \quad (24)$$

where the predicted output y_m^{k+1} depends on the control input u^k , disturbance d^k , auxiliary input u_{ac}^k , and the current output y^k .

3.4. Inverse TS Fuzzy-Model Structure

The inverse model predicts the control input from current and past process variables according to

$$u_m^k = f(d^k, u_{ac}^k, y^{k-1}, y^k), \quad (25)$$

where u_m^k is the estimated control action, and the inputs comprise the disturbance d^k , auxiliary control signal u_{ac}^k , and recent outputs y^{k-1} and y^k . This formulation enables inverse control by estimating the input required to achieve the desired system behavior.

The model adopts a TS rule-based structure with R rules of the form

$$R_i : \text{if } \underline{z}^k \text{ is } A_i, \text{ then } u_m^k = \underline{\theta}_i^\top \underline{\phi}^k, \quad (26)$$

where the partitioning vector is $\underline{z}^k = [u_{ac}^k, y^{k-1}]^\top$, the regression vector is $\underline{\phi}^k = [d^k, u_{ac}^k, y^{k-1}, y^k, 1]^\top$, and $\underline{\theta}_i \in \mathbb{R}^5$ contains the consequent parameters.

The antecedent fuzzy sets are defined by multivariate Gaussian membership functions,

$$\mu_{A_i}(\underline{z}^k) = \exp\left(-\frac{1}{2}(\underline{z}^k - \underline{v}_i)^\top \underline{P}_i(\underline{z}^k - \underline{v}_i)\right), \quad (27)$$

where $\underline{v}_i$ and $\underline{P}_i$ denote the center and precision matrix of rule i . Each variable is partitioned into three linguistic regions (low, medium, high), giving the full 3×3 rule base $R = 9$; no pruning is used in the reported model.

The inverse control signal is obtained as the normalized weighted average of the rule consequents,

$$u_m^k = \frac{\sum_{i=1}^R \mu_{A_i}(\underline{z}^k) \underline{\theta}_i^\top \underline{\phi}^k}{\sum_{i=1}^R \mu_{A_i}(\underline{z}^k)}. \quad (28)$$

The model is validated by comparing the predicted input u_m^k with the simulated control signal u^k , as shown in Figure 7, demonstrating an accurate reconstruction of the inverse dynamics throughout the operating range.

For $R = 9$, $s = 2$, and four consequent regressors plus a bias, the inverse model contains 90 scalar parameters, corresponding to approximately 720 bytes in double precision when only the numerical parameter vector is counted. Online inference scales linearly with R ; the dense batch Levenberg–Marquardt step has the usual approximate cost $\mathcal{O}(Nn_\theta^2 + n_\theta^3)$. No rule-selection or pruning stage is used, and processor-specific timing and sensitivity to initialization are left for future study.

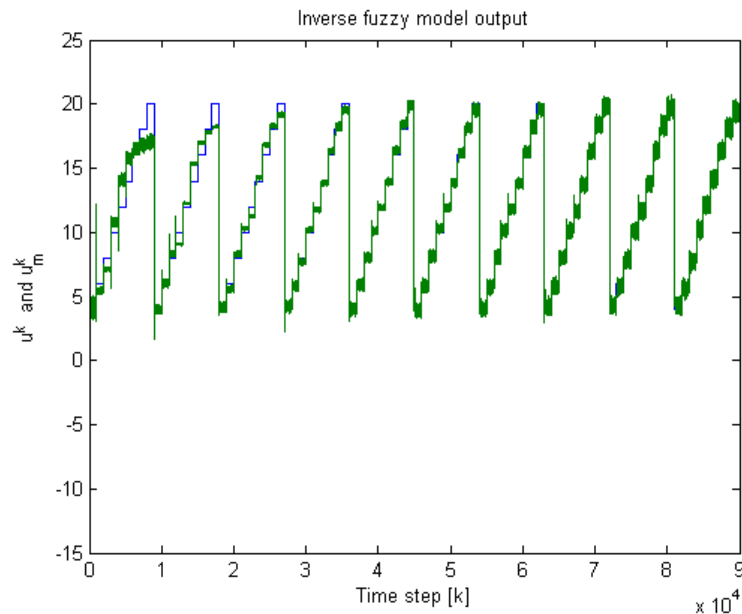


Figure 7. Inverse-model validation on simulated data: primary current u^k (blue) and prediction u_m^k [mA] versus sample index k .

4. Control Implementation and Simulation Results

This section presents the implementation and simulation-based evaluation of the proposed inverse fuzzy control strategy on the PHE benchmark. First, the auxiliary control input is optimized using the learned inverse model to maximize secondary-side flow while respecting actuator constraints. Next, trajectory tracking and disturbance-rejection simulations are conducted to assess closed-loop performance under nominal and perturbed conditions. Finally, the proposed IFMC approach is compared with a conventional PI controller to quantify improvements in tracking accuracy, robustness, and smoothness of actuation.

4.1. Auxiliary Input Optimization via the Inverse Model

In inverse fuzzy model control, the tracking error is first integrated to generate the reference signal for the inverse model,

$$u_a^k = u_a^{k-1} + (1 - a_r) e_{aw}^{k-1}, \quad (29)$$

where a_r is the tuning parameter and $e^k = y_d^k - y^k$ is the tracking error. In the implementation, the error entering this integrator is clipped to $e_{aw}^k = \min\{5, \max\{-5, e^k\}\}$ °C. Thus, the ± 5 °C anti-windup band limits the tracking-error input, not the integrator state itself. This definition applies to the IFMC reference integrator. For the PI baseline, the practical anti-windup/actuator-protection measure is an output-increment limiter. The previous PI command is initialized as $u_{PI}^0 = 9$ mA, corresponding to `u_prev`, and the allowed command change is $\Delta u_{PI} = 2$ mA per sample, corresponding to `delta_u`. This PI safeguard limits the command increment and does not clamp or back-calculate the integral state.

Using the inverse model (25), the primary control input is computed as

$$u^k = f(d^k, u_{ac}^k, u_a^{k-1}, u_a^k), \quad (30)$$

where (u_a^{k-1}, u_a^k) replaces (y^{k-1}, y^k) in the regression vector ϕ^k when the model operates inside the feedback loop.

The auxiliary signal u_{ac}^k provides an additional degree of freedom that can be exploited to increase the secondary-side flow while keeping the primary actuator within its admissible range. Considering the actuator limits

$$u^{\min} \leq u^k \leq u^{\max}, \quad (31)$$

the objective is to select the largest feasible $u_{ac}^k \in \mathcal{U}_{ac}$ that yields a valid control action. This leads to the discrete optimization problem

$$u_{ac,opt}^k = \arg \max_{u_{ac}^k \in \mathcal{U}_{ac}} u_{ac}^k \quad \text{s.t.} \quad u^{\min} + \delta_u \leq f(d^k, u_{ac}^k, u_a^{k-1}, u_a^k) \leq u^{\max} - \delta_u, \quad (32)$$

where δ_u is a safety margin to avoid saturation.

Since u_{ac}^k takes only 19 discrete values in $[0.1, 1]$ (step 0.05), the optimization is efficiently solved by direct enumeration, enabling real-time implementation without numerical solvers. Each candidate can be evaluated directly through (30). With $R = 9$, at most $19 \times 9 = 171$ rule evaluations are required per sample.

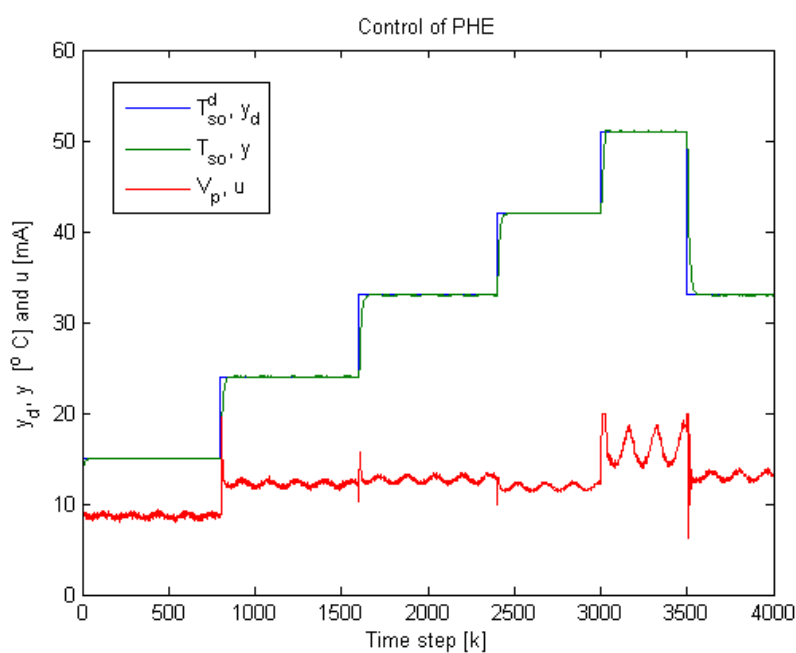
The final control action is therefore

$$u^k = f(d^k, u_{ac,opt}^k, u_a^{k-1}, u_a^k). \quad (33)$$

This strategy maximizes the secondary flow while guaranteeing that the primary actuator remains within operational limits.

4.2. Trajectory Tracking Performance

The trajectory tracking capability of the inverse fuzzy model-based controller under nominal conditions is illustrated in Figure 8. Figure 8a shows the reference temperature y_d^k , the simulated outlet temperature y^k , and the corresponding primary control input u^k , while Figure 8b presents the auxiliary valve position u_{ac}^k in the secondary circuit.



(a)

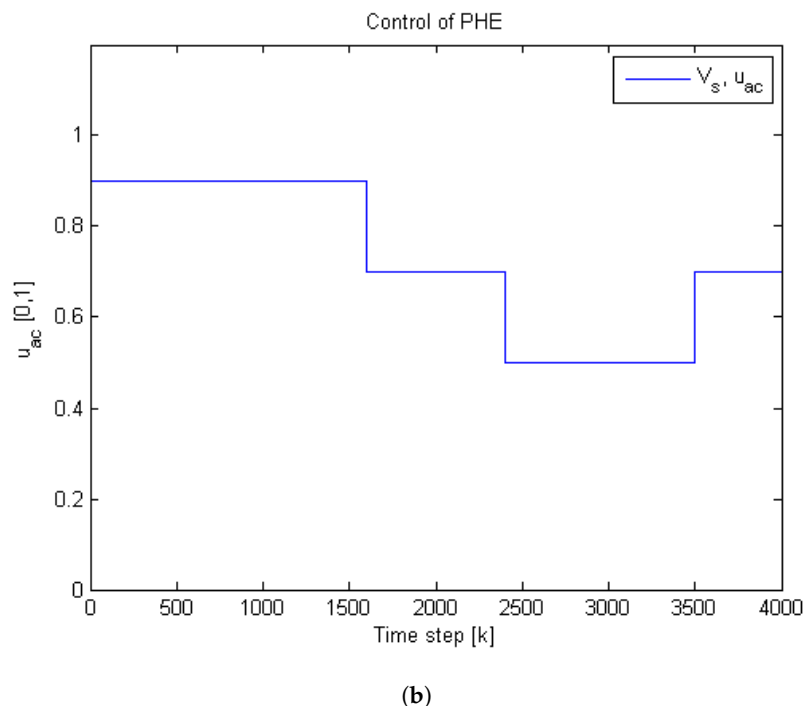


Figure 8. Nominal simulation of trajectory tracking with the inverse fuzzy controller. (a) Reference y_d^k and simulated outlet temperature y^k [$^{\circ}\text{C}$], and primary control current u^k [mA], versus sample index k . (b). Dimensionless auxiliary-valve command u_{ac}^k versus sample index k .

The outlet temperature closely follows the reference with negligible steady-state error and minimal overshoot, while the control input evolves smoothly without saturation. The auxiliary valve signal u_{ac}^k remains well behaved, indicating efficient flow regulation without excessive oscillations. These results confirm that the inverse fuzzy controller effectively handles the nonlinear multivariable dynamics of the PHE and achieves precise thermal regulation.

4.3. Robustness Under Input and Output Disturbances

Robustness is evaluated under simultaneous disturbances acting on both the process input and output. The simulation setup follows the nominal tracking simulation in Section 4.2, with additional perturbations applied to the actuator and measurement channels over a horizon of $N = 5000$ samples.

The hot-side reservoir temperature d^k is modeled as a sum of sinusoidal components to emulate realistic thermal fluctuations. A reference step of $\Delta y_d = 5$ $^{\circ}\text{C}$ is applied at $k = 2500$ (from 30 to 35 $^{\circ}\text{C}$) to assess transient performance. The inverse fuzzy controller combines the learned inverse model with the integrator-like reference filter (29) using $a_r = 0.8$ and the tracking-error clipping $e_{aw}^k \in [-5, 5]$ $^{\circ}\text{C}$ defined above. The primary actuator is constrained to $u^k \in [4, 20]$ mA, with a safety margin $\delta_u = 2$ mA enforced through online selection of the largest admissible auxiliary input.

Input disturbances z_u^k are introduced as piecewise-constant signals $\{+1, -1, +1, 0\}$ mA, while output disturbances z_y^k alternate between $\{-2, +2\}$ $^{\circ}\text{C}$ after the reference step. These combined perturbations constitute a demanding test of disturbance rejection and closed-loop resilience. The corresponding results are shown in Figures 9 and 10.

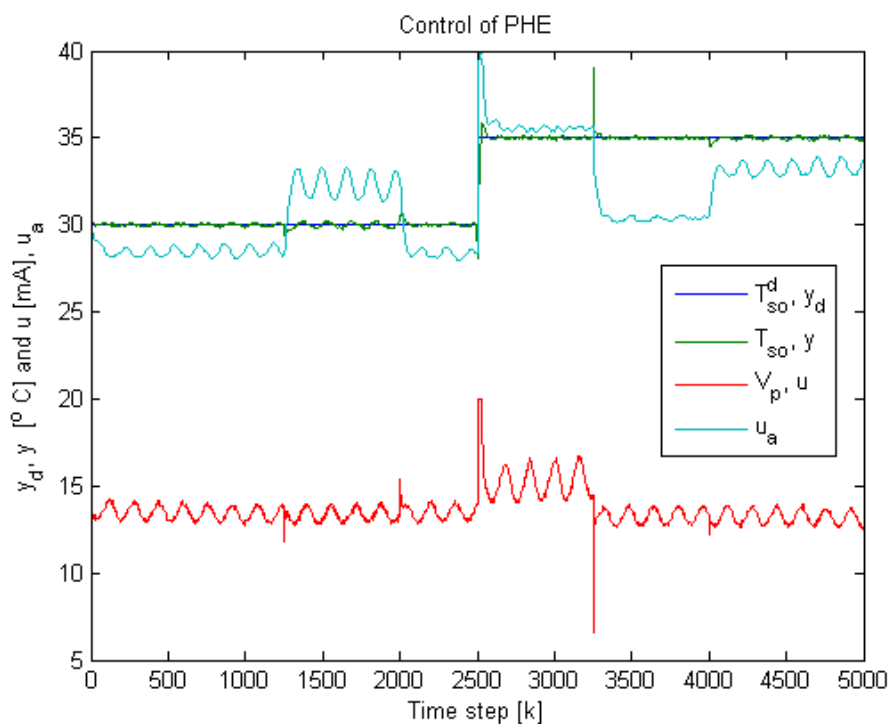
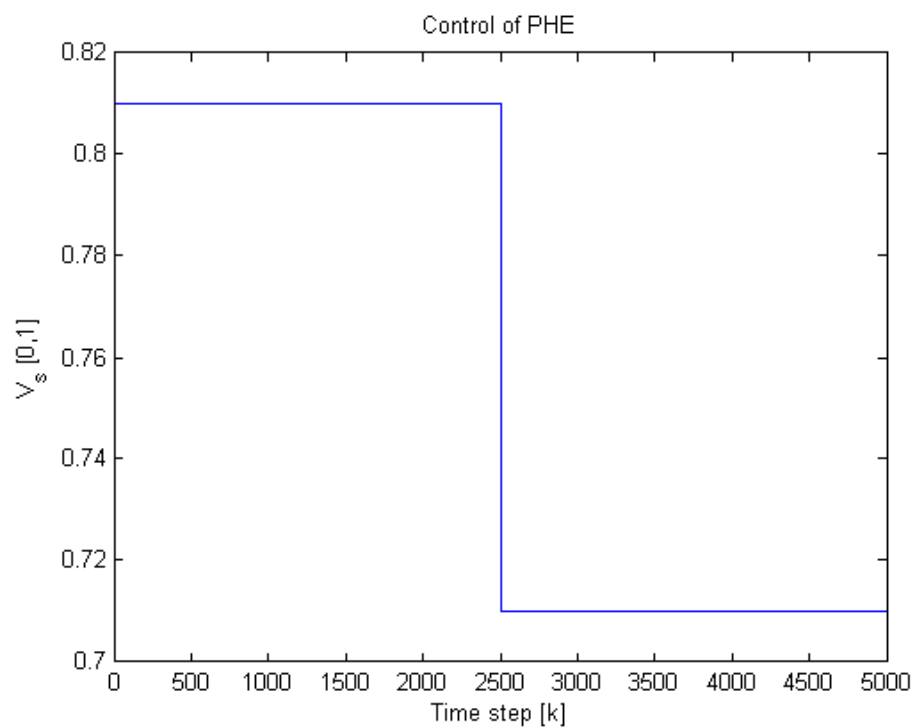


Figure 9. Disturbed simulation: reference y_d^k , process output y^k , model output y_m^k [°C], and primary current u^k [mA] versus sample index k .



(a)

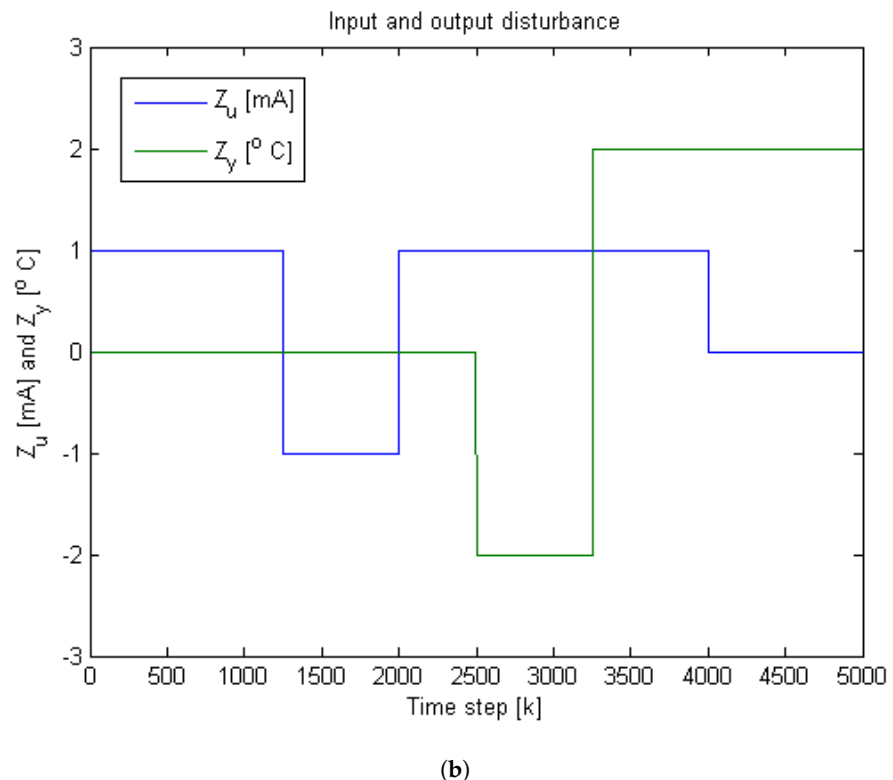


Figure 10. Auxiliary control action and disturbance profiles during the perturbed simulation. (a) Dimensionless pneumatic-valve command u_{ac}^k versus sample index k . (b) Input disturbance z_u^k [mA] and output disturbance z_y^k [°C] versus sample index k .

As seen in Figure 9, the output y^k closely tracks the reference y_d^k despite significant perturbations, while the model prediction y_m^k remains consistent with the process dynamics. The control input adapts smoothly and stays within actuator limits, indicating effective disturbance compensation without oscillations. Figure 10a shows well-damped auxiliary valve motion, whereas Figure 10b illustrates the disturbance profiles. Temperature regulation is maintained with only minor transient deviations.

In general, the results demonstrate strong disturbance rejection, smooth actuation, and reliable closed-loop performance, supporting the disturbance-rejection capability of the proposed inverse fuzzy control strategy for the tested PHE simulation scenario.

4.4. Comparison with PI Control

This subsection compares the proposed inverse fuzzy model-based control (IFMC) strategy with a conventional proportional-integral (PI) controller under identical operating conditions, including reference trajectory tracking and simultaneous input and output disturbances. The PI baseline was tuned numerically on the same theoretical PHE model by minimizing the finite-horizon quadratic tracking-error criterion

$$J_{PI} = \sum_{k=1}^N (y_d^k - y^k)^2.$$

The same sampling time, reference and disturbance sequences, primary-input saturation, and prescribed auxiliary-flow profile were used for both controllers. The recovered gains and implementation constants were

$$\begin{aligned}
 K_p &= 1.7531, & K_i &= 0.124, \\
 u_{\text{PI}}^0 &= 9 \text{ mA}, & \Delta u_{\text{PI}} &= 2 \text{ mA/sample}, \\
 u^k &\in [4, 20] \text{ mA}.
 \end{aligned}$$

The implemented discrete-time PI structure was

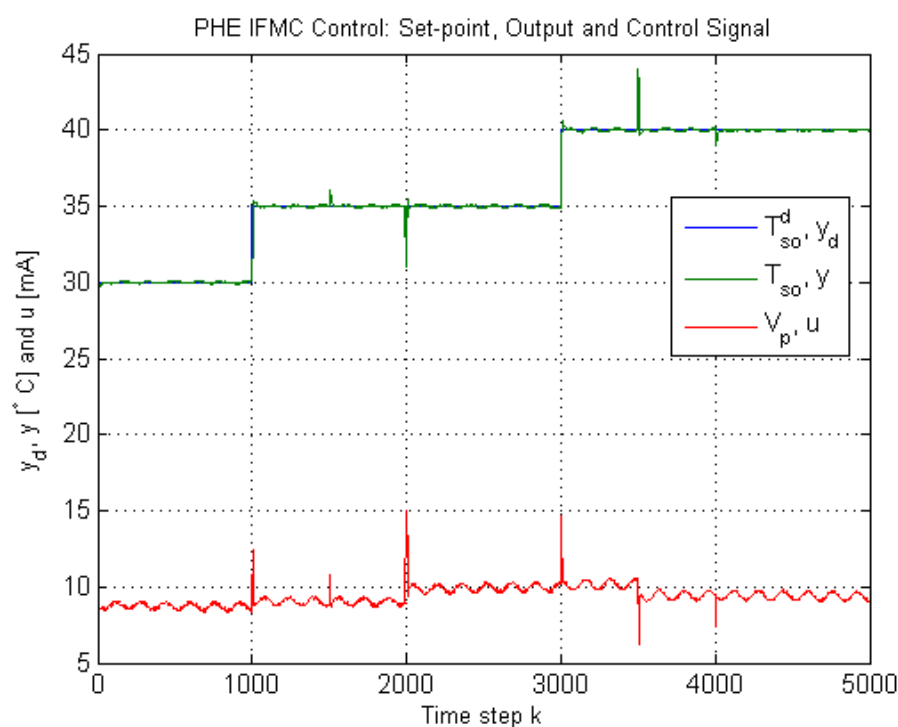
$$\begin{aligned}
 e_{\text{PI}}^k &= y_d^k - y^k, \\
 I_e^k &= I_e^{k-1} + e_{\text{PI}}^k, \\
 \tilde{u}_{\text{PI}}^k &= K_p e_{\text{PI}}^k + K_i I_e^k.
 \end{aligned}$$

The output-increment limiter was implemented as

$$u_{\text{PI}}^k = \begin{cases} u_{\text{PI}}^{k-1} + \text{sgn}(\tilde{u}_{\text{PI}}^k - u_{\text{PI}}^{k-1}) \Delta u_{\text{PI}}, & |\tilde{u}_{\text{PI}}^k - u_{\text{PI}}^{k-1}| > \Delta u_{\text{PI}}, \\ \tilde{u}_{\text{PI}}^k, & |\tilde{u}_{\text{PI}}^k - u_{\text{PI}}^{k-1}| \leq \Delta u_{\text{PI}}. \end{cases}$$

This output-increment limiter is the practical PI anti-windup/actuator-protection mechanism used in the simulation. It acts after the integral update and does not clamp or back-calculate the integral state. The PI controller uses the common prescribed auxiliary-flow profile and does not perform the IFMC auxiliary-input optimization. The archived optimizer settings and search bounds were not retained; therefore, the controller is described as numerically tuned rather than globally optimized.

Figure 11 contrasts the closed-loop responses of both controllers in the PHE process. The IFMC achieves tighter reference tracking and more effective rejection of the disturbances z_u^k and z_y^k , whereas the PI controller exhibits larger tracking deviations and more pronounced oscillatory behavior, particularly during transients.



(a)

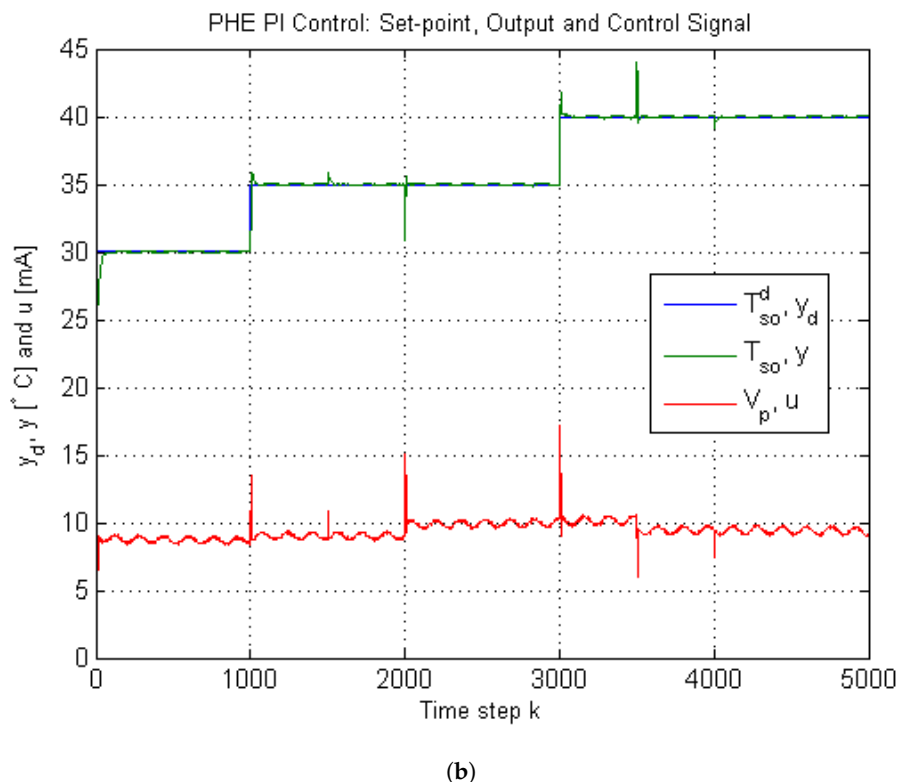
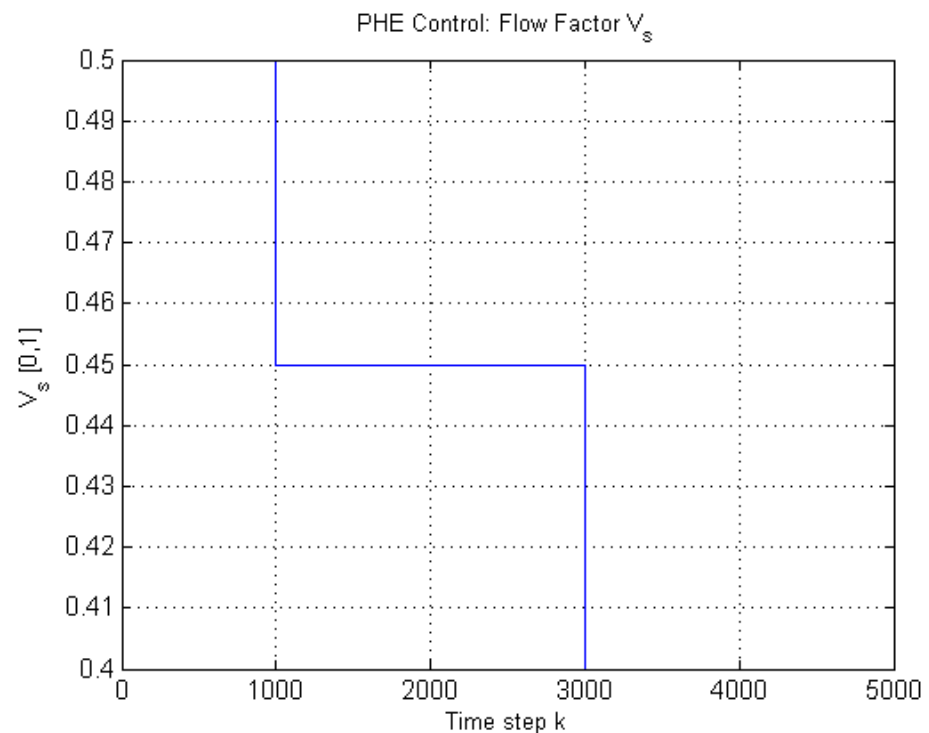


Figure 11. Closed-loop temperature tracking under disturbances z_u^k and z_y^k : comparison between the proposed IFMC and the tuned PI baseline. (a) Response of the proposed IFMC. (b) Tuned PI baseline response.

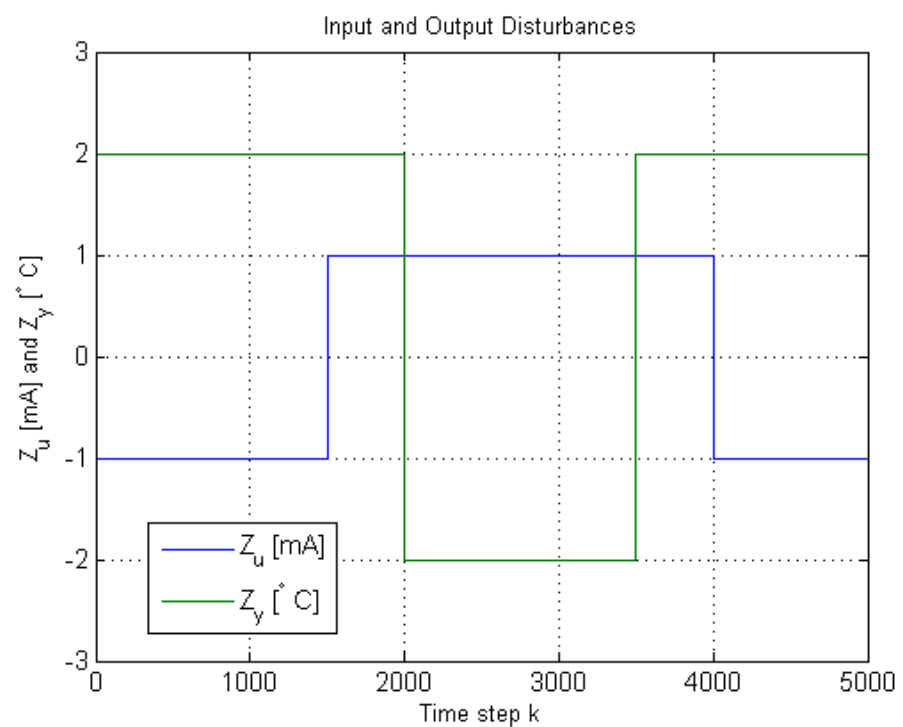
The superior tracking accuracy of IFMC is quantitatively confirmed by the root mean square error (RMSE), which decreases from 0.2867 °C for the PI baseline to 0.2078 °C for IFMC, corresponding to an improvement of approximately 27.5% in the reported simulation scenario. This improvement stems from the ability of IFMC to better handle nonlinear process dynamics and external disturbances through fuzzy partitioning, smooth rule activation, and nonlinear data-driven identification.

Figure 12 shows the auxiliary valve motion together with the applied disturbance profiles. Although both controllers operate around the same average secondary flow ($\bar{F}_s = 0.44$), the IFMC produces noticeably smoother actuation. The reported variability values refer to the primary control current $u^k = V_p^k$, not to the dimensionless auxiliary command. The mean primary current is 9.3777 mA for PI and 9.3832 mA for IFMC, while its standard deviation decreases from 0.6357 mA to 0.5745 mA, respectively, corresponding to a reduction of approximately 9.6%. This supports a statement of smoother primary actuation in the tested scenario but not a claim of reduced mechanical stress.

For the single reported simulation scenario, IFMC gives lower tracking RMSE and lower primary-input variability than the tuned PI baseline under the same reference, disturbance, and actuator conditions. The percentages are specific to the tested 5 °C reference step and disturbance amplitudes. Multi-run statistics, sensitivity to noise, rule number and initialization, and comparison with a reproducibly implemented modern nonlinear controller are left for future work. For thermal systems with substantially longer delays or larger inertia, delayed regressors or multi-step inverse targets would also be required.



(a)



(b)

Figure 12. Auxiliary-flow schedule and disturbance profiles for the IFMC and PI simulations. (a) Dimensionless auxiliary-valve command u_{ac}^k (secondary flow): IFMC and PI versus sample index k . (b) Applied disturbances z_u^k [mA] and z_y^k [°C] versus sample index k .

5. Conclusions

This work presented a data-driven inverse fuzzy control strategy for a nonlinear plate heat exchanger (PHE) process. The controller relies on a Takagi–Sugeno fuzzy inverse model with adaptive Gaussian membership functions, whose antecedent and consequent

parameters are identified via nonlinear least squares. The resulting framework allows for direct estimation of the control action required to achieve the desired temperature trajectory from fuzzy models identified entirely on data generated by the theoretical PHE model. The same theoretical model is used as the numerical plant, while its analytical equations are not used in fuzzy-model identification or online inference. Simulation studies demonstrated accurate temperature regulation with negligible steady-state error, smooth actuator behavior, and stable valve dynamics under both nominal and disturbed operating conditions. Compared with a tuned PI controller, the proposed approach achieved improved tracking accuracy and disturbance rejection, reducing the RMSE from 0.2867 to 0.2078 (a 27.5% improvement) and decreasing the standard deviation of the primary control current by approximately 9.6%, indicating smoother actuation in the tested simulation scenario.

These results highlight several advantages of the IFMC method: (i) effective compensation of nonlinear heat-transfer dynamics and variable coupling without using analytical plant equations in fuzzy-model identification and online inference; (ii) consistent identification of the data structure through a partition-based architecture and unified parameter estimation; and (iii) smooth actuation that respects operational constraints. Together, these properties make the proposed inverse fuzzy controller a robust and interpretable solution for nonlinear thermal process control.

Future work will focus on real-time experimental deployment in plate heat exchanger systems, incremental online adaptation of the fuzzy model, and the incorporation of evolving rule structures to further enhance autonomy and long-term adaptability in thermal energy systems operating under drift, disturbances, and varying operating conditions. Sensitivity studies over reference-step and disturbance amplitudes, noise and initialization, together with comparison against a modern nonlinear baseline, will also be considered.

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Abbreviations

The following abbreviations are used in this manuscript:

IFMC	Inverse fuzzy model control
NLS	Nonlinear least squares
PHE	Plate heat exchanger
PI	Proportional-integral
RMSE	Root mean square error
TS	Takagi–Sugeno

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