

Article

Dimensional and Non-Dimensional Implementations for the Differentially Heated Square Cavity Benchmark: Accuracy and Computational Efficiency

Fernando I. Molina-Herrera ¹, Hugo Jiménez-Islas ², María L. López-González ¹, Nora E. Maldonado-Sierra ³, Pedro Yañez-Contreras ⁴, Francisco J. Santander-Bastida ⁵, Juan M. Oliveros-Muñoz ⁶ and Norma L. Flores-Martínez ^{3,*}

¹ Departamento de Ingeniería Agroindustrial, Universidad de Guanajuato, Av. Ing. Javier Barros Sierra 201, esq. Av. Baja California, Ejido Santa María del Refugio, Celaya 38110, Guanajuato, Mexico; fi.molina@ugto.mx (F.I.M.-H.); mdll.lopezgonzalez@ugto.mx (M.L.L.-G.)

² Departamento de Ingeniería Bioquímica y Ambiental, Tecnológico Nacional de México en Celaya, Antonio García Cubas Pte. #600, esq. Av. Tecnológico, Celaya 38010, Guanajuato, Mexico; hugo.jimenez@itcelaya.edu.mx

³ Departamento de Ingeniería en Alimentos, Universidad Politécnica de Guanajuato, Av. Universidad Sur #1001, Cortázar 38496, Guanajuato, Mexico; nmaldonado@upgto.edu.mx

⁴ Departamento de Ingeniería en Calidad y Metrología, Universidad Politécnica de Guanajuato, Av. Universidad Sur #1001, Cortázar 38496, Guanajuato, Mexico; pyanez@upgto.edu.mx

⁵ Departamento de Ingeniería en Manufactura Avanzada, Universidad Politécnica de Guanajuato, Av. Universidad Sur #1001, Cortázar 38496, Guanajuato, Mexico; fsantander@upgto.edu.mx

⁶ Departamento de Ciencias Básicas, Tecnológico Nacional de México en La Laguna, Blvd. Revolución y Av. Instituto Tecnológico de La Laguna s/n, Primero de Cobián Centro, Torreón 27000, Coahuila, Mexico; jmoliverosm@lalaguna.tecnm.mx

* Correspondence: nlflores@upgto.edu.mx

Abstract

This study presents a numerical comparison of dimensional and non-dimensional implementations of the classical benchmark problem of steady natural convection in a two-dimensional differentially heated square cavity over the Rayleigh-number range $10^3 \leq Ra \leq 10^{12}$. The novelty of this work lies in the systematic comparison of both formulations under identical numerical conditions, providing an implementation-oriented assessment of their benchmark accuracy, mesh sensitivity, continuation strategy, and computational efficiency. The governing equations of mass, momentum, and energy conservation were solved under the Boussinesq approximation using primitive variables and the finite-element method. Since both formulations are theoretically equivalent descriptions of the same physical problem, the purpose of this work is not to reassess their physical validity, but to examine their numerical behavior under identical benchmark conditions in terms of mesh sensitivity, continuation strategy, benchmark accuracy, and computational efficiency. In the dimensional implementation, temperature differences of 1, 10, 25, and 50 K were considered, and the corresponding cavity lengths were determined from the Rayleigh-number definition. The main calculations were performed with $\Delta T = 25$ K, while $\Delta T = 50$ K was retained only as an exploratory sensitivity case. Boundary-layer refinement was applied along the vertical walls over the range $10^3 \leq Ra \leq 10^{12}$. The results show that, once the near-wall gradients are properly resolved, both implementations predict essentially identical average Nusselt numbers, with a maximum relative difference of 0.022%. Temperature contours, stream-function distributions, and centerline profiles also exhibited the same structural behavior in both cases. For the reference mesh, the dimensional implementation exhibited a lower computational cost than the non-

Academic Editor: Vincenzo Russo and Kwang-Yong Kim

Received: 6 June 2026

Revised: 10 July 2026

Accepted: 30 July 2026

Published: 5 August 2026

Copyright: © 2026 by the authors. Licensee MDPI, Basel, Switzerland. This article is an open access article distributed under the terms and conditions of the [Creative Commons Attribution \(CC BY\)](https://creativecommons.org/licenses/by/4.0/) license.

dimensional implementation, resulting in an approximately 31% reduction in computation time. These results demonstrate that solving the governing equations directly in dimensional variables provides a numerically efficient and physically interpretable alternative while preserving the benchmark accuracy of the classical non-dimensional formulation.

Keywords: natural convection; differentially heated square cavity; benchmark problem; dimensional implementation; non-dimensional implementation; finite-element method; computational efficiency; Nusselt number

1. Introduction

Natural convection is a fundamental heat-transfer mechanism in fluids, occurring when temperature variations generate density differences that induce fluid motion under the action of gravity. This phenomenon appears in numerous engineering and industrial applications, including cooling systems in electronic equipment, natural ventilation in buildings, thermal energy storage systems, chemical reactors, and grain storage. Because of the strong coupling between velocity and temperature fields, natural convection has been extensively investigated using mathematical models and numerical simulations. In this context, the differentially heated square cavity represents one of the most widely used benchmark problems for studying natural convection in enclosed domains. In this configuration, the vertical walls are maintained at different temperatures, while the horizontal walls are assumed adiabatic, resulting in buoyancy-driven recirculating flow within the cavity. One of the most influential numerical studies of this problem was presented by De Vahl Davis [1], whose work remains a classic reference for validating computational methods for natural convection. In that study, the two-dimensional flow under the Boussinesq approximation was solved using non-dimensional governing equations and the vorticity–stream function formulation [2–5]. The results included temperature contours, streamlines, velocity distributions, and average Nusselt numbers for Rayleigh numbers ranging from 10^3 to 10^6 , establishing benchmark solutions widely used in computational fluid dynamics and heat transfer [6–8].

Subsequently, the analysis of natural convection in cavities expanded toward transient, turbulent, and three-dimensional regimes [9–14]. Hyun and Lee [15] investigated transient natural convection in a square cavity to analyze the temporal evolution of the thermal and velocity fields after the sudden imposition of temperature differences on the vertical walls [16–18]. Their results revealed transient oscillatory behavior associated with internal gravity waves before the system reached steady-state conditions [19,20]. Later, Markatos and Pericleous [3] investigated laminar and turbulent natural convection in enclosed cavities for Rayleigh numbers approaching 10^{10} , showing that increasing buoyancy effects produce more complex convective structures that require turbulence modeling. Similarly, Barakos et al. [4] extended the analysis toward turbulent natural convection in square cavities using κ – ϵ turbulence formulations for Rayleigh numbers up to approximately 10^{11} . Their results demonstrated the importance of turbulence models for predicting heat-transfer behavior at high convective intensities [21–27]. Additional investigations based on κ – ϵ formulations and low-Reynolds-number models were later proposed to improve the numerical representation of near-wall flow behavior and thermal transport mechanisms [23]. However, turbulence models introduce uncertainties into numerical predictions because they rely on empirical closure relationships calibrated for specific flow conditions. Consequently, the prediction of velocity, temperature, and heat-transfer fields may vary depending on the turbulence formulation employed [28–31].

The continuous development of computational resources has enabled more detailed investigations of natural convection using three-dimensional simulations and advanced numerical methods [32–34]. In this context, Janssen and Henkes [35] numerically investigated the transition toward time-periodic behavior in a three-dimensional differentially heated cavity, while Weppe et al. [36] experimentally studied turbulent natural convection in a cubic cavity containing a partially heated internal obstacle. These studies reported the development of thermal boundary layers, thermal stratification, and oscillatory convective structures associated with complex buoyancy-driven flow. Although three-dimensional simulations provide a more detailed description of the flow spatial structure, several studies have shown that two-dimensional models remain a computationally efficient and physically representative approach for investigating natural convection in differentially heated cavities [37,38]. In many cases, two-dimensional simulations adequately reproduce the dominant flow characteristics, including recirculation cells, thermal boundary layers, and global heat-transfer behavior, while significantly reducing computational cost compared with three-dimensional simulations.

Consequently, most numerical studies of natural convection in cavities continue to employ two-dimensional formulations based on non-dimensional governing equations [39–41]. These formulations facilitate comparisons among studies and geometric configurations using characteristic scaling parameters. Nevertheless, although non-dimensionalization has traditionally been considered a convenient mathematical tool for simplifying transport equations, different scaling strategies may lead to distinct numerical behaviors depending on the selected characteristic variables and reference scales [1,3,10,13–16,25–28,34]. At high Rayleigh numbers, the interaction between conductive and convective transport mechanisms produces thin thermal boundary layers and strong velocity gradients near the walls [6,9,14,18]. Under these conditions, the scaling procedure may affect numerical convergence, stability, and predictions of temperature fields, velocity distributions, and Nusselt numbers [6,14,34]. Despite the extensive use of non-dimensional formulations, limited attention has been given to directly solving the governing equations in dimensional form and systematically comparing dimensional and non-dimensional implementations. In many engineering problems, the governing variables, fluid properties, and boundary conditions are naturally defined in dimensional units [42,43]. Therefore, solving the governing equations in dimensional form preserves a closer correspondence to measurable physical quantities such as temperature, velocity, pressure, and heat flux [44,45].

Despite the well-established theoretical equivalence between dimensional and non-dimensional formulations of the differentially heated square-cavity problem, their numerical implementation does not necessarily exhibit the same practical behavior. Limited attention has been paid to how both formulations compare under identical numerical conditions in terms of wall-layer resolution, continuation strategy, convergence behavior, and computational cost, especially at high Rayleigh numbers. Most previous studies have focused primarily on the physical solution itself, usually within a non-dimensional framework, whereas a systematic, implementation-oriented comparison under identical numerical conditions remains insufficiently documented, particularly when benchmark accuracy, mesh resolution, computational efficiency, and solver effort are examined together. This gap is relevant because, even when the same benchmark physics is preserved, differences in scaling may affect the conditioning of the algebraic system, the robustness of the nonlinear solution procedure, and the overall computational efficiency.

Although dimensional or primitive-variable formulations have been used previously in natural-convection studies of square enclosures, the literature has focused predominantly on the physical solution, usually in a non-dimensional framework and over more limited ranges of Rayleigh number. To the best of our knowledge, a systematic, implementation-oriented comparison between dimensional and non-dimensional formulations for the classical

differentially heated square-cavity benchmark, conducted under strictly identical numerical conditions and extended up to $Ra = 10^{12}$, remains insufficiently documented. The novelty of the present work, therefore, does not lie in re-establishing the theoretical equivalence between the two formulations, which is already well known, but in quantifying how they compare in practice with respect to benchmark accuracy, mesh independence, computational efficiency, and accumulated internal solver effort within the same finite-element framework.

Based on these considerations, the present study examines the classical problem of steady natural convection in a differentially heated square cavity by comparing the dimensional and nondimensional implementations of the same two-dimensional Boussinesq model. Since both formulations are theoretically equivalent descriptions of the benchmark problem, the purpose of this work is not to reassess their physical validity, but to evaluate their numerical behavior under identical conditions. Attention is given to benchmark accuracy, wall-layer resolution, continuation strategy, and computational efficiency over the range $10^3 \leq Ra \leq 10^{12}$. In the dimensional implementation, physically plausible temperature differences are specified, and the corresponding cavity lengths are calculated using the Rayleigh-number definition, thereby avoiding unrealistically large wall temperatures while preserving a direct interpretation of the thermal boundary conditions. Within this framework, the study aims to determine whether solving the governing equations in dimensional variables offers practical advantages for the numerical simulation of the adopted benchmark problem.

2. Materials and Methods

2.1. Mathematical Model

The problem considered concerns natural convection in a two-dimensional, differentially heated square cavity; a classical configuration widely used to investigate buoyancy-driven heat transfer mechanisms in fluids. The physical domain consists of a square cavity filled with air with a characteristic length L , whose geometrical configuration is illustrated in Figure 1. To describe the phenomenon, a Cartesian coordinate system is adopted, with the x -axis representing the horizontal direction (left to right) and the y -axis representing the vertical direction (bottom to top). The gravitational field generates buoyancy forces when density variations occur within the fluid [1,13,34,41].

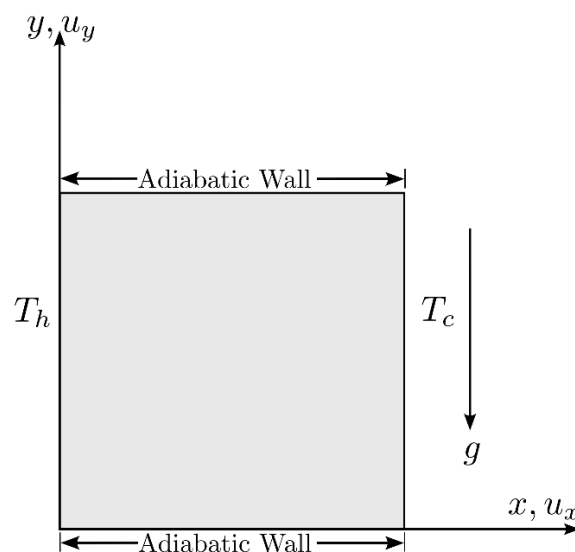


Figure 1. Geometric system.

The convective flow arises from the temperature difference imposed across the vertical walls of the cavity, with the left wall maintained at a higher temperature, T_h , and the right wall at a lower temperature, T_c , where $T_h > T_c$. This temperature difference generates density gradients in the fluid, which, under the Boussinesq approximation, produce buoyancy forces that drive fluid motion within the cavity. As a result, a recirculating flow is established inside the domain, transporting thermal energy from the hot wall toward the cold wall. As the imposed temperature difference between the walls increases, the flow undergoes a progressive transition from a regime dominated by heat conduction to a strongly convective regime, potentially reaching highly unstable behaviors associated with high Rayleigh numbers [13,14,18,19].

2.2. Dimensional Governing Equations

To describe the momentum and heat transfer within the cavity under steady-state conditions, the incompressible Navier–Stokes equations are employed together with the energy equation under the Boussinesq approximation. Under this assumption, the fluid density is constant throughout the domain, except in the buoyancy body-force term, where temperature-induced density variations drive convective motion within the cavity. In this way, the coupling between the thermal field and the velocity field occurs exclusively through the buoyancy term $\rho g \beta (T - T_0)$ while preserving the incompressible formulation of the governing equations [9,14,21,23].

The dimensional mathematical model is therefore composed of the continuity equation, the momentum conservation equations in the x and y directions, and the energy equation under steady-state conditions, as presented in Equations (1)–(4). In these expressions, the primary variables are the velocity components u_x and u_y with units of m s^{-1} , the pressure p with units of $\text{Pa} = \text{kg m}^{-1} \text{s}^{-2}$, and the temperature T with units of K. The thermophysical properties include the fluid density ρ in kg m^{-3} , dynamic viscosity μ in $\text{kg m}^{-1} \text{s}^{-1}$, thermal conductivity k in $\text{W m}^{-1} \text{K}^{-1}$, specific heat c_p in $\text{J kg}^{-1} \text{K}^{-1}$, thermal expansion coefficient β in K^{-1} , and gravitational acceleration g in m s^{-2} [43,44]. Equations (5a)–(5d) define the boundary conditions of the problem. The no-slip condition is imposed on all cavity walls, such as $u_x = u_y = 0$. The vertical walls are maintained at constant, different temperatures, with the left wall fixed at T_h and the right wall at T_c , thereby generating a horizontal thermal gradient that drives natural convection. The upper and lower horizontal walls are assumed to be adiabatic, which is expressed by the condition $\partial T / \partial y = 0$, indicating the absence of heat flux normal to these surfaces. These conditions define a closed cavity in which air circulation is driven solely by thermal buoyancy, and where the largest temperature and velocity gradients develop within the boundary layers adjacent to the hot and cold walls [9,16,18,28,34].

$$\frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y} = 0 \quad (1)$$

$$\rho \left(u_x \frac{\partial u_x}{\partial x} + u_y \frac{\partial u_x}{\partial y} \right) = -\frac{\partial p}{\partial x} + \mu \left[\frac{\partial^2 u_x}{\partial x^2} + \frac{\partial^2 u_x}{\partial y^2} \right] \quad (2)$$

$$\rho \left(u_x \frac{\partial u_y}{\partial x} + u_y \frac{\partial u_y}{\partial y} \right) = -\frac{\partial p}{\partial y} + \mu \left[\frac{\partial^2 u_y}{\partial x^2} + \frac{\partial^2 u_y}{\partial y^2} \right] - \rho g \beta (T - T_0) \quad (3)$$

$$\rho c_p \left(u_x \frac{\partial T}{\partial x} + u_y \frac{\partial T}{\partial y} \right) = k \left[\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right] \quad (4)$$

$$\text{B.C. 1. } x = 0 \quad u_x = u_y = 0 \quad T = T_h \quad 0 \leq y \leq L \quad (5a)$$

$$\text{B.C.2. } x = L \quad u_x = u_y = 0 \quad T = T_c \quad 0 \leq y \leq L \quad (5b)$$

$$\text{B.C.3. } y = 0 \quad u_x = u_y = 0 \quad \partial T / \partial y = 0 \quad 0 \leq x \leq L \quad (5c)$$

$$\text{B.C.4. } y = L \quad u_x = u_y = 0 \quad \partial T / \partial y = 0 \quad 0 \leq x \leq L \quad (5d)$$

An important aspect of this formulation is that, when working with dimensional variables, the equations involve physical quantities with different units and orders of magnitude. For example, for the air considered in this study, the density is on the order of 1 kg m^{-3} , the dynamic viscosity is on the order of $10^{-5} \text{ kg m}^{-1} \text{ s}^{-1}$, the thermal conductivity is on the order of $10^{-2} \text{ W m}^{-1} \text{ K}^{-1}$, the specific heat is on the order of $10^3 \text{ J kg}^{-1} \text{ K}^{-1}$, and the thermal expansion coefficient is on the order of 10^{-3} K^{-1} . This disparity in scales implies that, within the system of equations, some convective, diffusive, and buoyancy terms may differ by several orders of magnitude, especially at high Rayleigh numbers [13,14,33,34].

Consequently, the numerical solution of the model in its dimensional formulation requires special attention to spatial discretization, mesh refinement, and solution-method stability since the coexistence of variables with very different scales may affect algorithm convergence and the sensitivity of the resulting algebraic system. Nevertheless, maintaining this dimensional formulation preserves a direct relationship between the mathematical model and the problem's physical quantities, facilitating the interpretation of numerical results and their comparison with experimental data or operating conditions. In the present study, the thermophysical properties of air were assumed constant and evaluated at the reference temperature $T_0 = 293.15 \text{ K}$, consistent with the Boussinesq approximation. The air properties used in the simulations were a density of 1.177 kg m^{-3} , thermal conductivity of $26 \times 10^{-3} \text{ W m}^{-1} \text{ K}^{-1}$, specific heat capacity of $1 \times 10^3 \text{ J kg}^{-1} \text{ K}^{-1}$, thermal expansion coefficient of $3.322 \times 10^{-3} \text{ K}^{-1}$, and dynamic viscosity of $1.847 \times 10^{-5} \text{ kg m}^{-1} \text{ s}^{-1}$ [44]. These properties were kept constant throughout all simulations.

It is important to note that, in the dimensional formulation, the Rayleigh number was not directly introduced into the governing equations. Instead, the physical problem was formulated by defining the thermophysical properties of air at a reference temperature T_0 , fixing the cold-wall temperature T_c at T_0 , and selecting physically plausible temperature differences $\Delta T = T_h - T_c$ within ranges consistent with the Boussinesq approximation. In this study, four values of the imposed temperature difference were used: $\Delta T = 1 \text{ K}$, 10 K , 25 K , and 50 K . After setting ΔT , the corresponding hot-wall temperature was determined from:

$$T_h = T_c + \Delta T \quad (6a)$$

$$\text{Ra} = \frac{\rho g \beta \Delta T L^3}{\mu \alpha} \quad (6b)$$

$$L = \left[\frac{\text{Ra} \mu \alpha}{\rho g \beta \Delta T} \right]^{\frac{1}{3}} \quad (6c)$$

where α is the thermal diffusivity. Thus, for each target Rayleigh number, the dimensional problem was defined using a physically plausible temperature difference and a corresponding cavity size, while preserving the thermophysical properties evaluated at the reference temperature T_0 . This procedure avoids unrealistically large wall temperatures in the dimensional formulation and keeps the thermal conditions within a range more consistent with the assumptions of the Boussinesq model. At the same time, it enables direct comparison with the non-dimensional formulation, since both implementations represent the same prescribed Rayleigh number.

For the air properties adopted in this study, the parameter $\beta \Delta T$ takes values of approximately 0.0033, 0.033, 0.083, and 0.166 for $\Delta T = 1, 10, 25$, and 50 K , respectively.

Therefore, the first three values remain clearly within the range typically regarded as well-matched with the small-density-variation requirement associated with the Boussinesq approximation. In this way, the dimensional construction used in the main body of the manuscript is based on thermally conservative conditions, while the extended case $\Delta T = 50$ K is included only to examine the sensitivity of the results to a moderate increase in thermal forcing.

Table 1 summarizes the cavity lengths calculated for the four values of ΔT considered in this work over the range $10^3 \leq Ra \leq 10^{12}$. As expected from Equation (6c), for a fixed Rayleigh number, the required cavity length decreases as the imposed temperature difference increases. Conversely, for a fixed ΔT , the characteristic length increases monotonically with Ra , exhibiting the stronger buoyancy-to-diffusion ratio associated with larger cavities under the same thermal forcing.

Table 1. Characteristic cavity length L (m) required to represent prescribed Rayleigh numbers for selected temperature differences ($\Delta T = 1, 10, 25, 50$ K) in the dimensional formulation under the adopted Boussinesq framework.

Ra	$\Delta T = 1$ K	$\Delta T = 10$ K	$\Delta T = 25$ K	$\Delta T = 50$ K
10^3	0.0218	0.0101	0.0075	0.0059
10^4	0.0470	0.0218	0.0161	0.0127
10^5	0.1012	0.0470	0.0346	0.0275
10^6	0.2180	0.1012	0.0746	0.0592
10^7	0.4696	0.2180	0.1606	0.1275
10^8	1.0118	0.4696	0.3460	0.2747
10^9	2.1799	1.0118	0.7455	0.5917
10^{10}	4.6964	2.1799	1.6062	1.2748
10^{11}	10.1180	4.6964	3.4604	2.7465
10^{12}	21.7990	10.1180	7.4551	5.9172

Among the values considered, $\Delta T = 25$ K was selected as the reference case for the mesh-independence analysis and for the main dimensional results discussed in the following sections. This value provides a suitable balance between physical plausibility under the Boussinesq approximation and moderate cavity sizes across the Rayleigh-number range considered. This representation is especially useful in the present comparative study because it preserves the direct interpretation of the thermal boundary conditions in dimensional units and avoids the excessively high wall temperatures that would arise if the cavity size were fixed and ΔT alone were increased to reach very high Rayleigh numbers. Although the dimensional problem is defined by a variable cavity length L , to preserve numerical stability during the Rayleigh-number sweep, the dimensional problem was implemented on a geometrically normalized computational domain. The physical coordinates x and y were transformed as $X = x/L$ and $Y = y/L$, so that the square cavity was always represented by the unit domain $0 \leq X \leq 1$, $0 \leq Y \leq 1$. Under this transformation, the dependent variables u_x , u_y , p , and T retained their dimensional meaning, while the spatial derivatives were rescaled by the cavity length L . In this way, the physical interpretation of the dimensional formulation was preserved, whereas the computational mesh and geometric topology remained unchanged throughout the continuation procedure. The transformation, therefore, corresponds to a geometric normalization introduced solely for numerical convenience and does not alter the dimensionality of the governing variables or the resulting Nusselt number.

It is also worth noting that alternative dimensional constructions are possible. For example, if the cavity length is fixed and ΔT is calculated for each Rayleigh number, the resulting Nusselt numbers remain essentially unchanged (see Appendix A), provided that

the Boussinesq approximation is preserved. However, this alternative leads to temperature differences that become unrealistically large at high Rayleigh numbers, thereby reducing the physical consistency of the dimensional representation. For this reason, the present study adopts the opposite strategy: physically plausible values of ΔT are fixed, and the corresponding cavity length is calculated from the definition of the Rayleigh number. Additional results for $\Delta T = 1$ K, 10 K, and 50 K are presented in Appendix A to confirm that the predicted Nusselt number is practically insensitive to the specific dimensional construction used, as long as the same Rayleigh number is represented consistently.

2.3. Non-Dimensional Governing Equations

To facilitate the analysis of the convective phenomenon and to compare the results with those reported in the literature, the governing equations can also be expressed in non-dimensional form. The process of non-dimensionalization consists of defining dimensionless variables using characteristic scales of the problem, such as the cavity length, the temperature difference imposed between the walls, and a reference velocity. In this way, the transport equations are rewritten in terms of dimensionless numbers that describe the relationships among the physical mechanisms underlying the phenomenon, thereby allowing the system's behavior to be characterized more generally. As a result of this transformation, characteristic dimensionless numbers arise, particularly the Rayleigh number (Ra) and the Prandtl number (Pr). The Rayleigh number quantifies the balance between buoyancy forces driven by temperature gradients and the dissipative effects of viscosity and thermal diffusion, thereby governing the intensity of convective motion within the cavity [13,14,33,35,38]. The Prandtl number, in contrast, relates momentum diffusion to thermal diffusion, thereby characterizing the fluid behavior involved in natural convection [28–30,39,41].

The introduction of these dimensionless variables reduces the disparity in scales present in the dimensional equations, causing most variables to assume order-unity values, thereby improving numerical stability and facilitating the implementation of discretization methods. However, different non-dimensionalization strategies have been proposed in the literature, depending on the choice of characteristic scales for the problem's variables [38–42]. A recent comparative study analyzed three non-dimensionalization approaches applied to the differentially heated cavity problem, evaluating their influence on numerical convergence, computational time, and predictions of parameters such as streamlines, isotherms, and the Nusselt number. The results showed that although the different formulations yield physically equivalent solutions, the choice of scaling significantly influences the numerical behavior of the system of equations, as each technique produces Jacobian matrices with different conditioning properties that directly affect the stability and convergence time of the solution method. One formulation, referred to as Approach II, exhibited more favorable numerical behavior by keeping the magnitudes of the dimensionless variables within moderate ranges, thereby enabling convergence with fewer iterations, and reduced computational time, even at high Rayleigh numbers [42]. For this reason, this formulation has been identified as one of the most efficient approaches for solving the classical differentially heated cavity problem [6,13,14,41,42].

Laminar Flow Model in Dimensionless Variables

Using the selected non-dimensional formulation, the continuity equation, the momentum conservation equations in the X- and Y-directions, and the energy equation are expressed in Equations (7)–(10). In this representation, the buoyancy contribution appears explicitly in the vertical momentum equation through the dimensionless

temperature θ , while the diffusive terms are scaled by the combined dependence on the Prandtl and Rayleigh numbers. This form provides the basis for numerically solving the natural convection problem in a normalized computational domain.

$$\frac{\partial U_x}{\partial X} + \frac{\partial U_y}{\partial Y} = 0 \quad (7)$$

$$U_x \frac{\partial U_x}{\partial X} + U_y \frac{\partial U_x}{\partial Y} = -\frac{\partial P}{\partial X} + \left(\frac{\text{Pr}}{\text{Ra}}\right)^{1/2} \left[\frac{\partial^2 U_x}{\partial X^2} + \frac{\partial^2 U_x}{\partial Y^2} \right] \quad (8)$$

$$U_x \frac{\partial U_y}{\partial X} + U_y \frac{\partial U_y}{\partial Y} = -\frac{\partial P}{\partial Y} + \left(\frac{\text{Pr}}{\text{Ra}}\right)^{1/2} \left[\frac{\partial^2 U_y}{\partial X^2} + \frac{\partial^2 U_y}{\partial Y^2} \right] - \theta \quad (9)$$

$$U_x \frac{\partial \theta}{\partial X} + U_y \frac{\partial \theta}{\partial Y} = \frac{1}{(\text{Pr Ra})^{1/2}} \left(\frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2} \right) \quad (10)$$

The reference velocity is defined as $u_{ref} = \sqrt{g\beta\Delta TL}$, temperature is written as the reduced variable $\theta = (T - T_c)/(T_h - T_c)$, and spatial coordinates are normalized with the cavity length L , such that $X = x/L$ and $Y = y/L$. Accordingly, the physical domain is transformed into the unit square ($0 \leq X \leq 1$, $0 \leq Y \leq 1$). The relations given in Equation (11) are obtained directly from the definitions of the Rayleigh and Prandtl numbers and, together with the dimensionless variables defined in Equations (8)–(10), allow the formulation to be expressed exclusively in terms of the dimensionless parameters Ra and Pr .

$$(\text{Pr Ra})^{1/2} = \frac{\rho L \text{Pr } u_{ref}}{\mu} \left(\frac{\text{Pr}}{\text{Ra}}\right)^{1/2} = \frac{\mu}{\rho L u_{ref}} \quad (11)$$

The dimensionless model is closed by the boundary conditions given in Equations (12a)–(12d). No-slip conditions are imposed on all boundaries, such as $U_x = U_y = 0$. The vertical walls are differentially heated, with $\theta = 1$ at $X = 0$ and $\theta = 0$ at $X = 1$, while the horizontal walls are adiabatic, $\partial\theta/\partial Y = 0$. Together, these conditions define the dimensionless enclosure problem considered in this study.

$$\text{B.C. 1. } X = 0 \quad U_x = U_y = 0 \quad \theta = 1 \quad 0 \leq Y \leq 1 \quad (12a)$$

$$\text{B.C. 2. } X = 1 \quad U_x = U_y = 0 \quad \theta = 0 \quad 0 \leq Y \leq 1 \quad (12b)$$

$$\text{B.C. 3. } Y = 0 \quad U_x = U_y = 0 \quad \partial\theta/\partial Y = 0 \quad 0 \leq X \leq 1 \quad (12c)$$

$$\text{B.C. 4. } Y = 1 \quad U_x = U_y = 0 \quad \partial\theta/\partial Y = 0 \quad 0 \leq X \leq 1 \quad (12d)$$

2.4. Numerical Implementation

To solve the governing equations describing natural convection in a differentially heated cavity under steady-state conditions, two implementations of the problem were considered: the dimensional and the non-dimensional. The dimensional implementation corresponds to directly solving the conservation equations in their original variables and physical units, such as velocity, pressure, and temperature. The non-dimensional implementation, on the other hand, was obtained through the non-dimensionalization procedure previously reported by Molina-Herrera et al. [42], in which the physical variables are scaled using characteristic quantities of the system. This procedure allows the governing equations to be expressed in terms of dimensionless groups such as the Rayleigh and Prandtl numbers.

The numerical simulations were carried out using COMSOL Multiphysics® version 5.4 on a workstation with an Intel Core i9-14900K processor, 64 GB of RAM, and Windows 11 Professional. The spatial discretization of the governing equations was performed using the finite-element method, a widely used approach for numerically solving systems

of partial differential equations. The nonlinear algebraic system resulting from the discretization was solved using the Newton–Raphson method, which iteratively linearizes the coupled system of equations and updates the dependent variables until numerical convergence is achieved within a prescribed tolerance. In this study, a convergence tolerance of 10^{-5} was used for the system’s residual error. At each iteration, the Jacobian matrix of the system is evaluated, and a successive correction procedure is applied to progressively reduce the residual error. This approach allows stable and accurate solutions to be obtained even for high Rayleigh numbers, where the convective terms introduce strong nonlinearities into the governing equations.

To reach the highest Rayleigh number considered, a sweeping strategy was employed. This strategy involves solving the problem progressively, starting at a relatively low Rayleigh number. Once a converged solution is obtained for that case, it is used as the initial condition for the next Rayleigh number. This procedure is repeated, increasing the Rayleigh number step by step until the maximum value considered, $Ra = 10^{12}$, is reached. This approach facilitates the numerical convergence of the problem, since each new simulation starts from a solution field that is physically close to the final state [34,42]. It should be emphasized that the highest Rayleigh-number cases considered in this work are intended primarily as benchmark-level numerical stress tests within the adopted steady, two-dimensional Boussinesq framework. Their purpose is to evaluate the robustness of the numerical implementation, mesh resolution requirements, and computational performance under increasingly demanding conditions, rather than to represent specific practical high-Rayleigh-number physical systems.

Accurate resolution of the temperature and velocity gradients associated with natural convection at high Rayleigh numbers requires a mesh specifically designed to capture the near-wall regions where thermal and hydrodynamic boundary layers develop. Figure 2 shows the mesh distribution used for the differentially heated cavity, along with an enlarged view of the refinement near the vertical walls. To adequately resolve the steep gradients generated adjacent to the hot and cold surfaces, boundary-layer (BL) regions with a total thickness of 0.01 m were introduced along both vertical walls [34]. The selection of this thickness was based on numerical tests performed at $Ra = 10^{12}$, which corresponds to the most demanding case analyzed in this study, in which the thermal and velocity boundary layers become extremely thin due to intensified convective transport. Inspection of the isotherms and velocity distributions confirmed that the selected BL thickness was sufficient to encompass the complete thermal and hydrodynamic boundary-layer structure adjacent to the walls. Figure 2b clearly illustrates the local mesh refinement implemented in these regions, where the highest temperature and velocity gradients occur. This refinement is essential for accurately resolving the boundary-layer dynamics and the formation of near-wall vortical structures characteristic of high-Rayleigh-number natural convection [3–8].

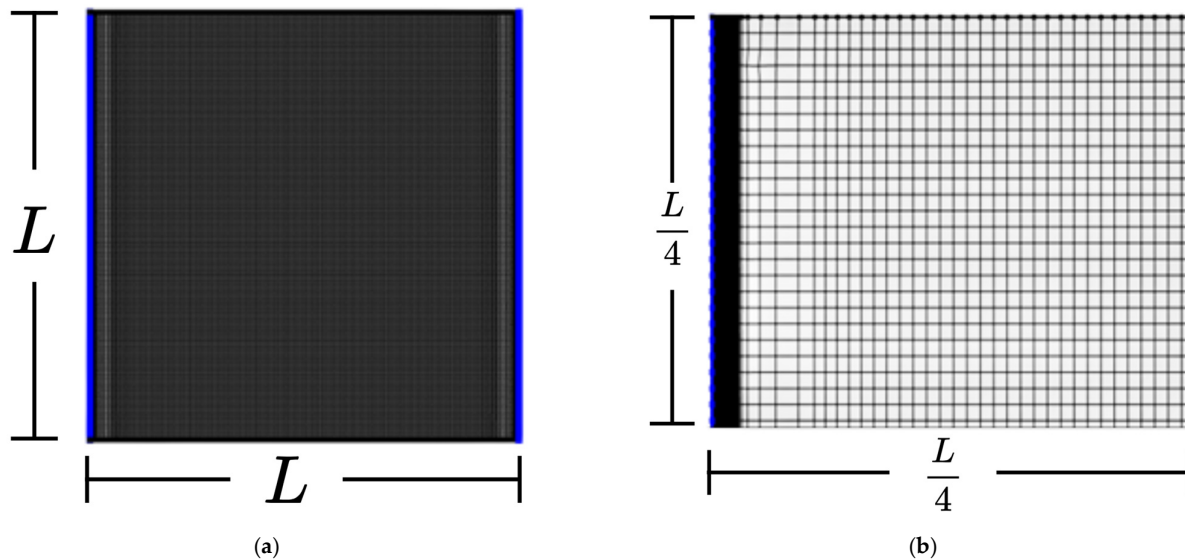


Figure 2. Computational mesh: (a) overall view; (b) boundary-layer refinement.

For the numerical comparison between the two implementations, both cases were solved in COMSOL Multiphysics v5.4 using the stationary solver with the default automatic Newton–Raphson settings. The comparison was performed under identical hardware, mesh, and continuation conditions. The reference computational-efficiency comparison reported in this work corresponds to $BL = 100$, for which the dimensional implementation required 9'40", whereas the non-dimensional implementation required 14'00".

To evaluate the influence of boundary-layer refinement on numerical accuracy, several mesh configurations were tested by progressively increasing the number of boundary-layer elements from $BL = 20$ to $BL = 1000$, as shown in Table 2. Under these conditions, the total number of elements in the computational domain increased from 48,000 to 440,000, while the number of elements specifically located within the boundary-layer regions increased from 880 to 4800. The results indicated that increasing the number of boundary-layer elements significantly improved the resolution of the near-wall thermal and velocity gradients, particularly for the highest Rayleigh numbers, where the boundary layers become extremely thin. However, excessively refined meshes also led to a substantial increase in computational cost without producing significant changes in the global thermal behavior once mesh convergence was achieved. In contrast, the central region of the cavity exhibited smoother variations in temperature and velocity, allowing a more uniform orthogonal discretization without compromising numerical accuracy. Consequently, the remainder of the cavity was discretized using a uniform quadrilateral mesh of " 0.005×0.005 m" to reduce computational cost while maintaining numerical stability. The combined mesh strategy therefore provided an appropriate balance between numerical precision and computational efficiency, ensuring accurate prediction of heat-transfer behavior, thermal boundary layers, and flow structures within the cavity.

Table 2. Grid-independence analysis based on the average Nusselt number for the dimensional implementation at $\Delta T = 25$ K with different boundary-layer (BL) refinement levels.

Rayleigh Number	Coarse BL Meshes		Intermediate		Reference		Extreme
Ra	BL = 20	BL = 40	BL = 60	BL = 80	BL = 100	BL = 120	BL = 1000
Whole-domain elements	48,000	56,000	64,000	72,000	80,000	88,000	440,000
10^3	1.1178	1.1178	1.1178	1.1178	1.1177	1.1177	1.1178
10^4	2.2448	2.2448	2.2448	2.2448	2.2447	2.2446	2.2446
10^5	4.5217	4.5215	4.5215	4.5214	4.5205	4.5203	4.5202
10^6	8.8261	8.8255	8.8252	8.8251	8.8231	8.8220	8.8219
10^7	16.539	16.543	16.545	16.546	16.543	16.546	16.546
10^8	30.271	30.279	30.283	30.286	30.284	30.326	30.328
10^9	54.481	54.475	54.477	54.481	54.475	54.505	54.515
10^{10}	97.639	97.638	97.623	97.617	97.592	97.591	97.590
10^{11}	173.91	174.34	174.33	174.33	174.29	174.18	174.18
10^{12}	NC	NC	309.07	309.06	308.89	308.42	308.29
CPU time	—	—	8'27"	8'41"	9'40"	9'48"	92'17"

An evaluation of the influence of mathematical formulation on the model's computational efficiency was conducted by running simulations with both implementations under identical numerical conditions, including the same computational hardware specifications and mesh configurations used in the domain. The comparison between the two implementations showed that, in general, the dimensional implementation requires less computational time than the non-dimensional implementation for the range of Rayleigh numbers considered. This behavior suggests that the scaling introduced by the non-dimensional implementation may influence the numerical conditioning of the system of equations, which in turn affects the computational efficiency of the solution process.

2.4.1. Nusselt Number

In the analysis of natural convection, one of the most used parameters for characterizing heat transfer is the Nusselt number. This dimensionless number quantifies the intensity of heat transport in a convective system by comparing the total heat transfer from combined conduction and convection to the heat transfer that would occur solely by conduction if the fluid were at rest. In this way, the Nusselt number provides a direct measure of the enhancement in heat transfer caused by fluid motion. In differentially heated cavities, the Nusselt number quantifies the intensity of natural convection driven by temperature gradients between the walls. In general, higher values of this number are associated with thinner thermal boundary layers and larger heat fluxes at the solid surfaces. Since the Nusselt number depends directly on the temperature gradient at the walls, its value is particularly sensitive to the numerical resolution of the near-wall regions. For this reason, the Nusselt number is widely used as a criterion for evaluating the accuracy of numerical simulations and for assessing mesh independence, since variations in spatial discretization may affect thermal gradients and, consequently, the calculated heat transfer values. In this context, the analysis of the Nusselt number allows verification of whether the adopted mesh can accurately reproduce the temperature gradients at the walls and, therefore, whether the obtained numerical solution is sufficiently precise [34,42,45].

In general terms, the Nusselt number is defined as $Nu = hL/k$, where h is the convective heat transfer coefficient. This dimensionless number expresses the ratio of convective to conductive heat transfer. For the present problem, the average Nusselt number is evaluated directly from the temperature gradient normal to the heated wall, as given below. In this expression, L is a characteristic length of the system, and k corresponds to the thermal conductivity of the fluid. This relationship expresses the ratio of convective to purely conductive heat transfer. Equivalently, the Nusselt number can be determined locally from the temperature gradient normal to the solid surface, thereby allowing direct evaluation of the heat flux intensity at the domain walls. In the present study, the Nusselt number for a square cavity with differentially heated vertical walls is calculated using the formulation presented below.

$$\begin{aligned}\overline{Nu}_{dim} &= \int_0^L -\frac{1}{T_h - T_c} \frac{\partial T}{\partial x} \bigg|_{x=0} dy \quad \text{Dimensional} \\ \overline{Nu}_{nd} &= \int_0^1 -\frac{\partial \theta}{\partial X} \bigg|_{X=0} dY \quad \text{Non - Dimensional}\end{aligned}\tag{13}$$

In this framework, Table 2 presents Nusselt number values obtained for the dimensional implementation with $\Delta T = 25$ K, using different mesh configurations with boundary-layer refinement near the heated and cooled walls, for Rayleigh numbers in the range $Ra = 10^3$ to 10^{12} . In addition to the Nusselt number values, Table 2 includes the number of elements in the interior of the domain and in the boundary layers, enabling simultaneous evaluation of solution accuracy and the computational cost for each mesh configuration. The results indicate that the coarser configurations, corresponding to $BL = 20$ and $BL = 40$, are sufficient to reproduce the thermal behavior for low and intermediate Rayleigh numbers; however, they fail to converge for $Ra = 10^{12}$, as evidenced by the absence of results in the table for this case. This behavior indicates that these refinement levels are insufficient to resolve the extremely thin thermal gradients that develop near the differentially heated walls at very high Rayleigh numbers. Consequently, the lack of spatial resolution in the wall-normal direction prevents the attainment of a stable, steady-state numerical solution. In contrast, convergence is achieved from $BL = 60$ onward even for $Ra = 10^{12}$, confirming that a minimum level of refinement in the near-wall layers is required to properly capture the physics of the problem. As the number of boundary layer elements increases from $BL = 60$ to $BL = 120$, the Nusselt number values, particularly for the most demanding cases ($Ra \geq 10^{10}$), show a clear trend toward convergence, with progressively smaller differences between consecutive mesh configurations. This behavior is particularly evident for $Ra = 10^{12}$, where the values obtained with $BL = 80, 100$, and 120 are close to each other.

Indeed, the comparison indicates that the $BL = 100$ configuration reproduces practically the same accuracy as $BL = 120$, while maintaining excellent agreement with the extreme case of $BL = 1000$, which represents a very severe refinement in the vicinity of the walls. These results demonstrate that once the thermal and hydrodynamic boundary layers are properly resolved, further refinement no longer produces significant changes in the global heat-transfer prediction [14,34]. The inclusion of the $BL = 1000$ case allows an independent verification of this asymptotic behavior. Although this extreme configuration considerably increases the number of elements in the near-wall region, the resulting Nusselt numbers are practically identical to those obtained with $BL = 100$ and $BL = 120$, confirming that the solution has already become independent of further mesh refinement. However, this marginal improvement in spatial resolution does not translate into a significant gain in accuracy, while it does lead to a noticeable increase in computational cost. Under these conditions, the $BL = 100$ configuration represents the best compromise between numerical accuracy, convergence stability, and computational

efficiency. This mesh adequately resolves the temperature gradients near the differentially heated walls, reproduces converged Nusselt number values, and avoids the additional computational cost associated with excessively refined meshes. In this sense, the results obtained with the dimensional implementation demonstrate that using original variables and physical units, combined with an appropriate discretization in the near-wall region, constitutes a robust strategy for reliably predicting heat transfer in differentially heated cavities at high Rayleigh numbers [14,34,42].

Based on these considerations, the mesh configuration with $BL = 100$ was selected for all subsequent simulations. The numerical results obtained with this discretization are presented and discussed in the following section.

Following the mesh independence analysis performed for the dimensional implementation, a similar evaluation was conducted using the non-dimensional implementation to assess the influence of mesh refinement under the same physical conditions. For further details, see Molina-Herrera et al. [45]. However, when these results are compared with those obtained using the dimensional implementation, it becomes evident that the convergence of the Nusselt number is equally satisfactory in both alternatives, although the dimensional implementation yields a more favorable numerical and computational performance. Although both implementations yield practically identical converged Nusselt numbers with sufficient refinement, the dimensional implementation is more computationally efficient. The results obtained with the non-dimensional implementation show that increasing the number of boundary layer elements leads to a considerable increase in computational time, particularly for the extreme configuration $BL = 1000$, without yielding a noticeable improvement in the accuracy of the Nusselt number relative to the $BL = 100$ and $BL = 120$ configurations. A similar trend was also observed in the dimensional implementation, although with lower computational times under equivalent numerical conditions.

The reported computational-efficiency difference should be interpreted as an observed, framework-specific result obtained within COMSOL Multiphysics v5.4 using the finite-element method, under the hardware, mesh, solver, and continuation conditions adopted in this study. The present benchmark-oriented analysis was not intended to provide a detailed causal decomposition of the difference in solver performance in terms of nonlinear iteration counts, residual scaling, internal linear algebra behavior, or processor-specific execution effects. Therefore, although the observed computation-time reduction is reproducible within the adopted setup, it should not be interpreted as a universally applicable ranking for other computational fluid dynamics (CFD) platforms, numerical methods, or hardware architectures.

Within the non-dimensional implementation, the $BL = 100$ configuration is also appropriate, since it yields Nusselt numbers that are practically indistinguishable from those obtained with $BL = 120$ and even $BL = 1000$. However, when both implementations are analyzed together, the results confirm that solving the governing equations in dimensional variables yields the most favorable balance among numerical convergence, Nusselt-number accuracy, and computational efficiency. Consequently, the dimensional implementation represents the most suitable alternative for simulating natural convection in the differentially heated cavity over the Rayleigh number range considered in this study [14,34,42].

Comparing the mesh-independence results for the dimensional implementation (Table 3) with those from the corresponding nondimensional simulations shows that both formulations exhibit essentially the same convergence behavior. For both implementations, the coarser mesh configurations ($BL = 20$ and $BL = 40$) failed to converge for the most demanding case ($Ra = 10^{12}$), indicating that these refinement levels are insufficient to resolve the extremely thin thermal and hydrodynamic boundary layers that

develop adjacent to the differentially heated walls. Convergence was first achieved with $BL = 60$, and the average Nusselt number progressively stabilized as the boundary-layer refinement increased to $BL = 100$ and $BL = 120$, demonstrating that a mesh-independent solution had been attained. Although both implementations produced practically identical converged Nusselt numbers, the dimensional formulation consistently required less computational time. For the nondimensional implementation, the computational times were 14'14" ($BL = 60$), 13'51" ($BL = 80$), 14'00" ($BL = 100$), 15'23" ($BL = 120$), and 128'29" ($BL = 1000$) [45], whereas the corresponding dimensional simulations required 8'27", 8'41", 9'40", 9'48", and 92'17", respectively (Table 2). For the reference mesh ($BL = 100$), the dimensional implementation reduced the computational time by approximately 31% while maintaining essentially identical Nusselt-number predictions.

Table 3. Comparison of the Nusselt number obtained using the dimensional and non-dimensional implementations and the corresponding relative percentage error for $BL = 100$.

Ra	$\overline{Nu}_{dim}$ Non-Dimensional	$\overline{Nu}_{nd}$ Dimensional	Error %
10^3	1.1178	1.1177	0.0089
10^4	2.2448	2.2447	0.0045
10^5	4.5212	4.5205	0.0155
10^6	8.8246	8.8231	0.0170
10^7	16.546	16.543	0.0181
10^8	30.289	30.284	0.0165
10^9	54.485	54.475	0.0184
10^{10}	97.613	97.592	0.0215
10^{11}	174.32	174.29	0.0172
10^{12}	308.95	308.89	0.0194

In addition, an inspection of the COMSOL solver statistics over the full continuation procedure, from $Ra = 10^3$ to $Ra = 10^{12}$, showed that, under identical mesh, solver, and hardware conditions, the non-dimensional implementation required more cumulative internal solver effort than the dimensional implementation. For the complete Rayleigh-number sweep, the non-dimensional case accumulated 358 nonlinear iterations, 837 residual evaluations, 358 Jacobian updates, and 999 linear solves, whereas the dimensional case accumulated 312 nonlinear iterations, 629 residual evaluations, 312 Jacobian updates, and 794 linear solves. Since both implementations used the same number of degrees of freedom, these results indicate that the longer computation times observed for the non-dimensional implementation are attributable to greater accumulated solver effort during the continuation process. Although these statistics do not provide a complete causal decomposition of solver behavior, they offer quantitative support for the reported difference in computational cost within the adopted COMSOL framework.

However, when the results are analyzed together, both implementations yield practically identical Nusselt numbers once the thermal boundary layers are properly resolved, confirming that they describe the same physical behavior of the problem. This agreement is further clarified in Table 3, where the Nusselt numbers obtained with both implementations are directly compared on a mesh with $BL = 100$. The results show that the percentage error between the two solutions is extremely small across the entire Rayleigh number range considered, remaining below approximately 0.022%, even in the most demanding case ($Ra = 10^{12}$). This level of agreement confirms that both implementations consistently reproduce heat transfer within the cavity.

The main difference between the two implementations lies in the computational efficiency of the solution process. Although the extreme-refinement $BL = 1000$ produces

Nusselt number values that are practically identical to those obtained with the BL = 100 and BL = 120 configurations, its implementation considerably increases computational cost without providing significant improvements in result accuracy. Consequently, excessively refined meshes do not offer a meaningful advantage once the thermal boundary layers have been adequately resolved. In this context, the joint analysis of the two tables demonstrates that the BL = 100 configuration provides the best balance among numerical accuracy, convergence stability, and computational cost, enabling adequate resolution of the thermal boundary layers even for $Ra = 10^{12}$. Furthermore, the comparison between implementations indicates that although the non-dimensional implementation correctly reproduces the behavior of the Nusselt number, the dimensional implementation is more computationally efficient, achieving the same level of accuracy in less time. These results confirm that solving the problem using dimensional variables, combined with appropriate refinement in the near-wall regions, constitutes a robust strategy for simulating natural convection in differentially heated cavities over a wide range of Rayleigh numbers.

After confirming mesh independence and comparing the dimensional and non-dimensional implementations, an additional validation of the numerical results was performed by comparing them with values reported in the literature. Table 4 presents Nusselt number values reported in the literature, obtained by solving the governing equations of mass, momentum, and energy conservation in nondimensional form. This alternative provides a consistent basis for comparison across a broad range of Rayleigh numbers, since most reference studies present their results in non-dimensional form. In contrast, the current work provides results in both non-dimensional and dimensional forms, enabling direct comparison with the literature while maintaining the physical meaning of the variables. In this context, the agreement in Nusselt number values confirms that the numerical solutions in this work accurately manifest the fundamental physical behavior of the problem.

Table 4. Validation of the average Nusselt number through comparison with numerical studies reported in the literature.

Authors	10^3	10^4	10^5	10^6	10^7	10^8	10^9	10^{10}	10^{11}	10^{12}
De Vahl Davis [1]. Non-dimensional	1.50	3.52	4.51	8.79	-----	-----	-----	-----	-----	-----
Markatos & Pericleous [3]. Non-dimensional	3.54	3.48	4.43	8.7	-----	32.0	-----	156.8	137.5	840.13
Barakos et al. [4]. Non-dimensional	1.11	2.24	4.51	8.81	-----	30.1	54.4	97.6	134.6	-----
Dixit & Babu, [6]. Non-dimensional	1.12	2.28	4.54	8.65	16.79	30.5	57.3	103.6	-----	-----
Goloviznin et al. [14]. Non-dimensional	1.17	2.23	4.51	8.82	-----	30.3	55.6	100.3	174	367
Hernández-López et al. [13]. Non-dimensional	1.11	2.24	4.45	8.86	-----	-----	58.0	137.7	318.53	725.57
Molina-Herrera et al. [42]. Non-dimensional	1.11	2.24	4.51	8.82	16.51	30.2	-----	-----	-----	-----
Molina-Herrera & Jiménez-Islas [34]. Non-dimensional	1.12	2.24	4.52	8.83	16.53	30.23	54.89	100.01	-----	-----
This work Non-Dimensional	1.1118	2.2448	4.5213	8.8346	16.546	30.289	54.485	97.613	174.32	308.95
This work Dimensional	1.1177	2.2447	4.5205	8.8231	16.543	30.284	54.475	97.592	174.29	308.89

In general, the Nusselt number values obtained in the present study show excellent agreement with benchmark results reported in the literature, particularly for Rayleigh numbers in the range $Ra = 10^3$ – 10^6 , where numerous numerical validation studies are available. For example, for $Ra = 10^3$ and $Ra = 10^4$, the values obtained in this work are practically identical to those reported by Barakos et al. [4], Dixit and Babu [6], and Goloviznin et al. [14], confirming that the model accurately reproduces the regime dominated by conduction and weak convection. As the Rayleigh number increases, the convective flow intensifies and the thermal boundary layers thin, requiring higher numerical resolution near the walls. Despite these increasing numerical demands, the values obtained in this study remain in very good agreement with previously reported results at high Rayleigh numbers, showing only minor differences relative to earlier studies using different numerical methods [14]. A particularly relevant aspect of this comparison is that the results from the dimensional and non-dimensional implementations are practically indistinguishable, confirming that both alternatives correctly describe the problem's physical behavior. Nevertheless, the analyses presented in the previous sections showed that the dimensional implementation offers computational advantages, as it allows the governing equations to be solved directly in their original physical variables without additional scaling transformations.

In this sense, dimensional implementation can be interpreted as an efficient strategy for the direct solution of natural convection problems, since it reproduces benchmark results reported in the literature with high accuracy while maintaining strong numerical stability even at high Rayleigh numbers. The excellent agreement observed in Table 4 confirms that the implemented model reliably predicts heat transfer in differentially heated cavities, providing additional validation of the numerical methodology employed in this study. Overall, these results demonstrate that the numerical strategy adopted, based on the solution of the conservation equations using the finite-element method together with an appropriate refinement of the thermal boundary layers, allows Nusselt number predictions that are consistent with the reference values widely reported in the literature, even in the range of high Rayleigh numbers.

2.4.2. Macroscopic Energy Balance

An additional criterion for evaluating the physical and numerical accuracy of the simulations is the analysis of the macroscopic energy balance in the differentially heated square cavity. This balance provides a fundamental verification of the numerical solution's quality, as it allows the global conservation of thermal energy within the computational domain to be directly assessed. The configuration considered consists of a square cavity in which the vertical walls are maintained at constant temperatures, while the upper and lower horizontal walls are assumed to be adiabatic. Under these conditions, the macroscopic energy balance requires that the heat flux entering the domain through the hot wall equals the heat flux leaving the system through the cold wall. Accordingly, the inlet and outlet heat flow rates, Q_{in} and Q_{out} , are evaluated using Equations (14) and (15), respectively, while the corresponding relative error is computed using Equation (16). Consequently, any difference between these two quantities represents a global error associated with the numerical discretization or with the resolution of the thermal field. In addition, the adiabatic condition imposed on the upper and lower walls implies that the heat flux normal to these surfaces must be zero or extremely small. Therefore, the presence of significant heat fluxes at these boundaries would indicate potential limitations in spatial discretization or numerical resolution of the thermal gradients. These verifications confirm that the simulation satisfies global energy conservation within the cavity, providing an additional criterion for assessing the reliability of the calculated Nusselt numbers [34,41,42,45].

$$Q_{In} = \int_0^L \left(-k \frac{\partial T}{\partial x} \Big|_{x=0} \right) dy + \int_0^L \left(-k \frac{\partial T}{\partial y} \Big|_{y=0} \right) dx \quad (14)$$

$$Q_{Out} = \int_0^L \left(-k \frac{\partial T}{\partial x} \Big|_{x=L} \right) dy + d \int_0^L \left(-k \frac{\partial T}{\partial y} \Big|_{y=L} \right) dx \quad (15)$$

$$\% \text{ error} = \left| \frac{Q_{In} - Q_{Out}}{Q_{In}} \right| \times 100 \quad (16)$$

The energy-balance error is evaluated from the conductive heat fluxes integrated along the cavity walls. Under steady-state conditions, the total heat entering the enclosure must equal the total heat leaving it. Therefore, Equations (14) and (15) are formulated as line integrals of the normal temperature gradient along the corresponding boundaries. Any discrepancy between the incoming and outgoing heat rates is attributed to numerical error and is quantified through Equation (16), which provides a direct measure of the global energy conservation achieved by the numerical solution.

Based on verification of the global energy balance, Table 5 presents a comparison of the macroscopic energy balance obtained using three numerical approaches: the orthogonal collocation method, the non-dimensional implementation used in the present study, and the dimensional implementation that solves the governing equations directly in their original physical variables. The results show that for low and intermediate Rayleigh numbers, the energy balance error remains practically negligible across all methods, confirming that global energy conservation is satisfied in this regime. As the Rayleigh number increases, the problem becomes increasingly computationally demanding due to the thinning of the thermal boundary layers and the increase in temperature gradients near the differentially heated walls. Under these conditions, small discrepancies in the energy balance may arise from the problem's increased sensitivity to spatial resolution. Nevertheless, the results obtained using direct numerical simulation based on the finite-element method maintain relatively small errors even for the highest Rayleigh numbers considered [34,42].

Table 5. Comparison of the macroscopic energy balance error obtained using the orthogonal collocation method (OCM) and the dimensional and non-dimensional implementations.

Ra	OCM	Balance Error (%) Non-Dimensional	Balance Error (%) Dimensional
10 ³	6.028 × 10 ^{−6}	0.0	5.00 × 10 ^{−6}
10 ⁴	1.028 × 10 ^{−5}	0.0	0.0
10 ⁵	1.20 × 10 ^{−5}	0.0	0.0
10 ⁶	1.46 × 10 ^{−5}	0.0	0.0
10 ⁷	1.79 × 10 ^{−5}	0.0	0.0
10 ⁸	1.79 × 10 ^{−5}	1.00 × 10 ^{−6}	0.0
10 ⁹	2.20 × 10 ^{−5}	1.00 × 10 ^{−6}	0.0
10 ¹⁰	-----	1.00 × 10 ^{−5}	0.0
10 ¹¹	-----	1.03 × 10 ^{−6}	0.0
10 ¹²	-----	6.88 × 10 ^{−2}	5.12 × 10 ^{−6}

Consistent with the trends previously discussed, the dimensional implementation exhibits negligible errors in the macroscopic energy balance across most of the analyzed range of Rayleigh numbers, confirming that the direct solution of the governing equations in their original physical variables properly preserves global energy conservation within the domain. This behavior supports using a dimensional implementation as a robust strategy for simulating natural convection, since it accurately reproduces energy balances

without introducing significant numerical errors. The non-dimensional implementation also reproduces the energy-balance behavior satisfactorily, although in some cases slightly larger errors are observed at very high Rayleigh numbers. However, these values remain within acceptable limits for energy conservation and do not affect the physical validity of the results. Overall, the analysis of the macroscopic energy balance confirms that the simulations properly conserve thermal energy within the cavity and therefore provide reliable predictions of both the Nusselt number and the convective flow field. This result, together with the mesh independence analysis and comparisons with previously reported results in the literature [1–15,32,34,42], further reinforces the validity of the numerical implementation used in this study.

3. Results

In this section, the temperature and velocity profiles obtained from the numerical solution of the differentially heated square cavity problem are presented primarily in dimensional form. In this representation, temperature is expressed in °C and velocity in m/s, allowing the intensity of the convective phenomenon and the associated heat- and momentum-transfer mechanisms to be interpreted directly in physical units. It should be noted that only the temperature and stream-function contours corresponding to Rayleigh numbers $Ra = 10^6$, 10^8 , 10^{10} , and 10^{12} are presented for comparative purposes between the dimensional and non-dimensional implementations. In contrast, the temperature and velocity profiles are discussed only in terms of the dimensional implementation to avoid repetitive analysis and improve the manuscript's clarity. The non-dimensional temperature and velocity profiles are not discussed separately in this section because previous studies have demonstrated that both dimensional and non-dimensional implementations reproduce equivalent thermal and hydrodynamic behavior for the classical De Vahl Davis [1] problem. Molina-Herrera et al. [45] reported the corresponding nondimensional profiles for a wide range of Rayleigh numbers from 10^3 to 10^{12} , showing nearly identical profile distributions, boundary-layer development, and global heat-transfer characteristics. Therefore, the present analysis focuses on the dimensional profiles, while the non-dimensional implementation is used only as a reference framework for comparison and validation of the predicted behavior. For the reference BL = 100, the dimensional implementation required 9.66 min, whereas the non-dimensional implementation required 14.00 min, corresponding to a reduction of approximately 31% in computation time under identical numerical conditions.

3.1. Temperature and Stream-Function Contours

Figure 3 shows the temperature contours obtained using the dimensional implementation, with temperature expressed directly in °C. In this implementation, the governing equations were solved using dimensional variables, and the corresponding Rayleigh numbers were determined from the imposed temperature differences between the hot and cold vertical walls. This alternative allows the thermal field to be interpreted in terms of measurable physical quantities while maintaining correspondence with the classical benchmark cases $Ra = 10^6$, 10^8 , 10^{10} , and 10^{12} . As Ra increases, the temperature field evolves from a regime with a significant conductive contribution to one dominated by natural convection. At $Ra = 10^6$, the isotherms are slightly deformed, indicating that heat transfer still occurs through the combined action of conduction and buoyancy-driven motion [9,10,12,13]. As Ra increases, the intensification of the convective circulation redistributes heat more effectively inside the cavity, causing the isotherms in the core region to become increasingly horizontal. This behavior indicates the formation of a convective core, in which momentum transfer enhances fluid mixing and reduces temperature gradients away from the walls. In contrast, the largest thermal gradients

become confined near the hot and cold vertical walls, revealing the progressive thinning of the thermal boundary layers. Therefore, the dimensional contours clearly show that increasing Ra strengthens the coupling between momentum and heat transfer, producing stronger near-wall gradients and a more stratified temperature field within the cavity [1,3,4,8,34,45].

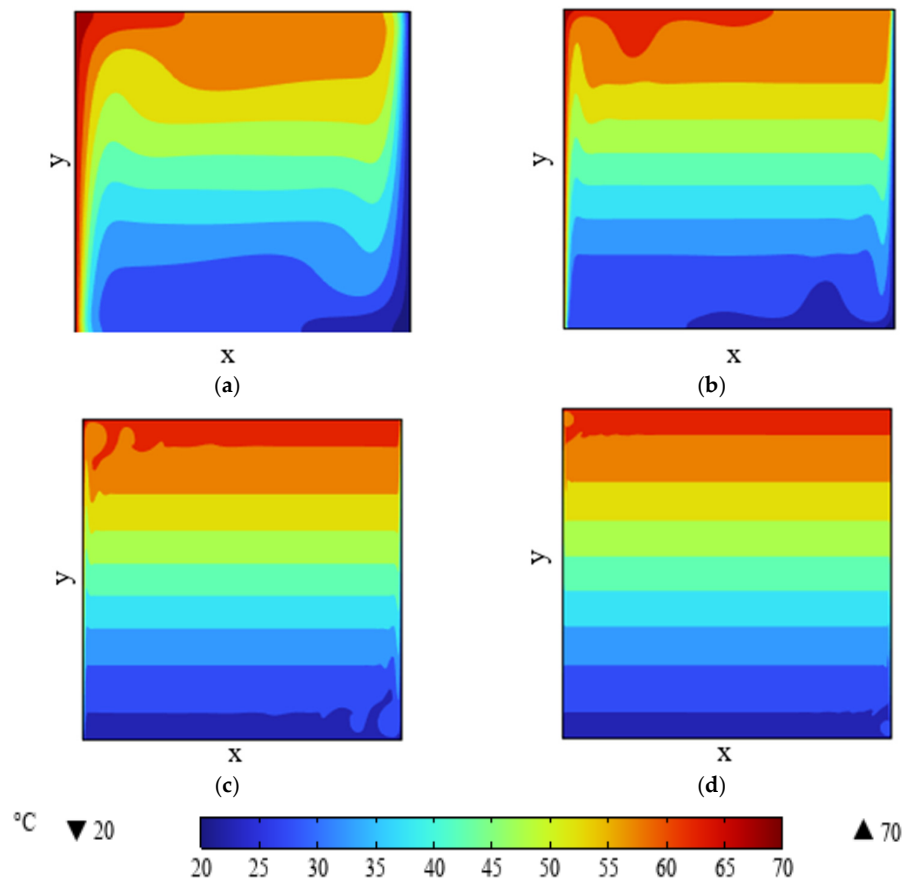


Figure 3. Temperature contour maps predicted by the dimensional implementation ($^{\circ}\text{C}$) for increasing Rayleigh numbers: (a) $Ra = 10^6$, (b) $Ra = 10^8$, (c) $Ra = 10^{10}$, and (d) $Ra = 10^{12}$.

After analyzing the temperature contours obtained from the dimensional implementation, Figure 4 presents the corresponding temperature contours calculated using the non-dimensional implementation, where the thermal field is represented by the dimensionless temperature variable θ , ranging from $\theta = 1$ at the hot wall to $\theta = 0$ at the cold wall. These contours are used here as a reference framework to verify the consistency of the dimensional results [3,5,45]. The comparison between Figures 4 and 5 shows that both implementations reproduce the same thermal evolution: slight isotherm curvature at lower Ra , progressive horizontal alignment in the cavity core, and confinement of the strongest temperature gradients near the differentially heated walls as Ra increases [45]. This agreement confirms that the dimensional implementation preserves the physical behavior obtained with the non-dimensional implementation, while offering a more direct interpretation of temperature differences and thermal gradients in physical units. Thus, the dimensional results provide the main basis for the present discussion, whereas the non-dimensional contours support the consistency of the predicted heat-transfer mechanisms with the classical behavior reported for differentially heated cavities [1,2,7,9,12,34,42].

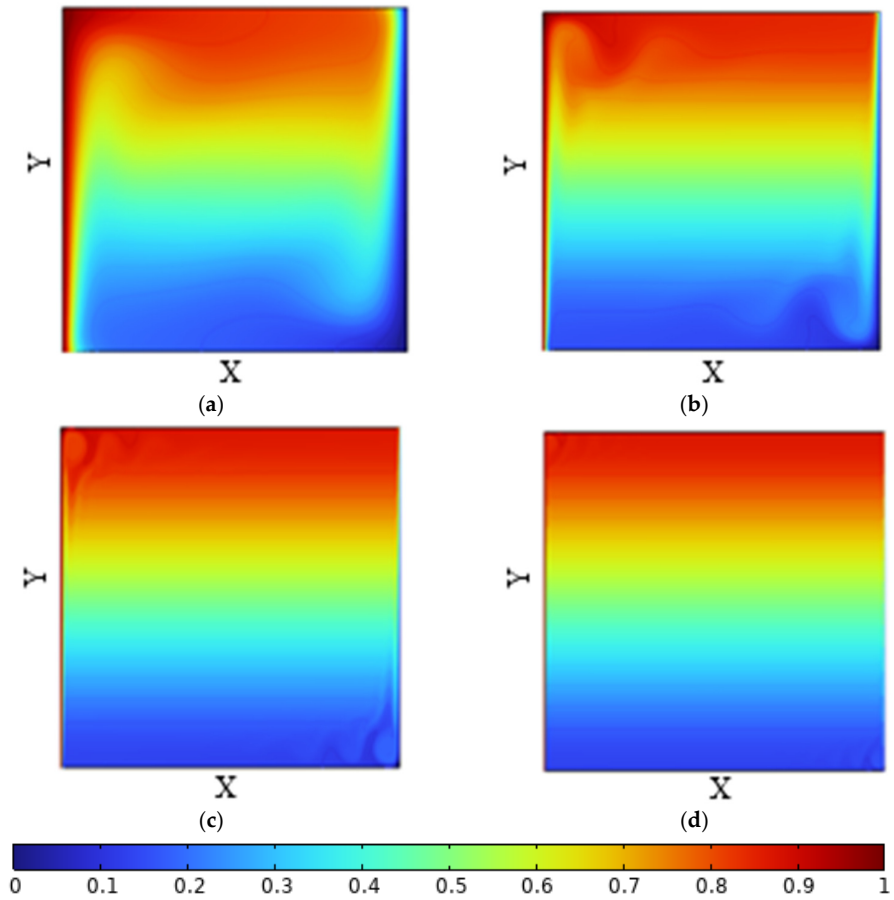
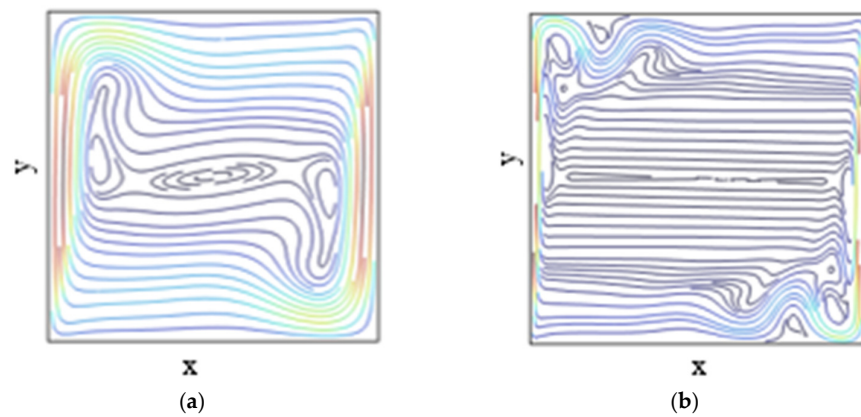


Figure 4. Temperature contours calculated using the non-dimensional implementation for increasing Rayleigh numbers: (a) $Ra = 10^6$, (b) $Ra = 10^8$, (c) $Ra = 10^{10}$, and (d) $Ra = 10^{12}$.



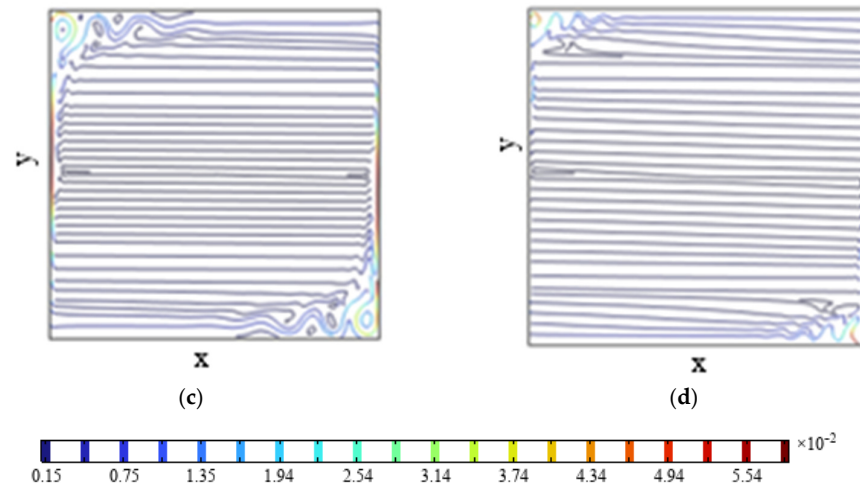


Figure 5. Stream-function contours calculated using the dimensional implementation (m^2/s) for different Rayleigh numbers: (a) $Ra = 10^6$, (b) $Ra = 10^8$, (c) $Ra = 10^{10}$, and (d) $Ra = 10^{12}$.

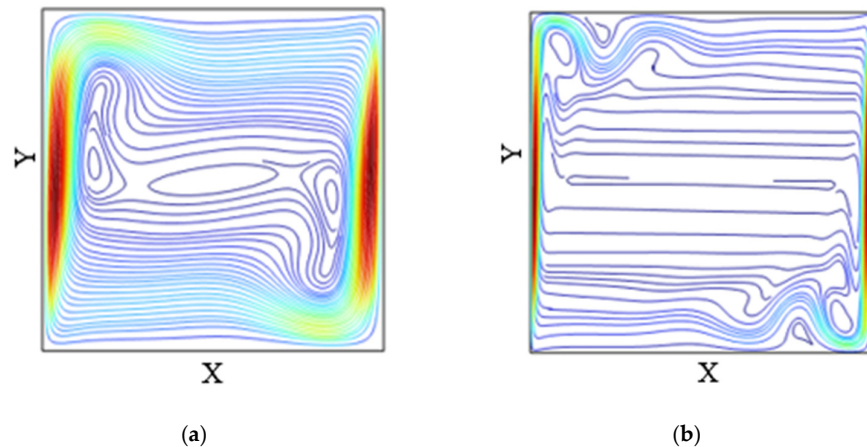
To examine the flow structure within the cavity in greater detail, this section presents the stream-function contours, also referred to as streamlines, corresponding to the solution of the differentially heated square cavity problem. Unlike temperature contours, which reveal the thermal distribution within the domain, stream-function contours provide a clearer representation of fluid circulation, enabling more accurate visualization of the intensity and evolution of convective patterns as the Rayleigh number increases. These representations facilitate the identification of recirculation cells and changes in the intensity of fluid motion associated with increased buoyancy effects [3,6,9,10,13,14].

Continuing with this analysis, the stream-function contours obtained from both the dimensional and non-dimensional implementations of the problem are presented. In the case of the dimensional solution, the stream function is expressed in physical units of m^2/s , allowing the flow intensity within the cavity to be interpreted directly. In contrast, the non-dimensional implementation describes the flow structure in normalized form, facilitating comparison with the classical results reported in the literature for the De Vahl Davis problem. Consistent with the analysis of temperature contours, the stream-function contours are presented for the same Rayleigh numbers considered earlier, namely $Ra = 10^6$, 10^8 , 10^{10} , and 10^{12} . The results obtained using the dimensional implementation are discussed first, followed by the corresponding non-dimensional contours, to compare the flow structure predicted by both representations.

Figure 5 presents the stream-function contours obtained using the dimensional implementation, with the stream function expressed in physical units of m^2/s . These contours complement the temperature-field analysis because they describe the transfer of momentum inside the cavity and allow the evolution of buoyancy-driven circulation to be directly visualized. At $Ra = 10^6$, the flow is characterized by a primary recirculation cell occupying most of the cavity. Under these conditions, the hot fluid rises along the heated wall, moves across the upper region, descends near the cold wall, and returns through the lower part of the cavity, completing the convective cycle. This pattern indicates that natural convection is already relevant, although the flow remains relatively organized and the hydrodynamic boundary layers are still moderately developed [31,34,45]. As the Rayleigh number increases to $Ra = 10^8$, the streamlines become more concentrated near the vertical walls, indicating stronger velocity gradients and thinner hydrodynamic boundary layers. This behavior results from the intensification of buoyancy forces, which accelerate the fluid adjacent to the heated and cooled walls. Secondary recirculation

structures also begin to appear near the upper-left and lower-right corners, showing that the interaction between the primary circulation and the wall boundary layers becomes more complex. For $Ra = 10^{10}$, the flow field becomes more strongly dominated by convective momentum transport. The central region exhibits nearly horizontal streamlines, while most of the velocity variation is confined to the vertical walls and corners. This indicates that the flow core behaves as a convective circulation region, whereas the walls control the main momentum and heat-transfer resistance. At $Ra = 10^{12}$, this behavior becomes more pronounced in the core, where the flow is almost parallel, and the strongest flow variations are restricted to very thin near-wall regions. The corner vortices become more defined, reflecting the increased interaction between the thermal and hydrodynamic boundary layers at very high Rayleigh numbers [1,3,4,10,18,27].

To complement the dimensional analysis, Figure 6 presents the corresponding stream-function contours calculated using the non-dimensional implementation. These contours are used as a reference framework to verify the consistency of the flow patterns predicted by the dimensional implementation. The comparison between Figures 5 and 6 shows that both implementations reproduce the same evolution of the flow structure: a dominant primary recirculation cell at $Ra = 10^6$, progressive concentration of streamlines near the vertical walls as Ra increases, development of secondary corner vortices, and nearly parallel streamlines in the core region for $Ra = 10^{10}$ and $Ra = 10^{12}$ [36,45]. This agreement confirms that the dimensional implementation preserves the momentum-transfer behavior predicted by the non-dimensional implementation, while allowing the intensity of fluid motion to be interpreted directly in physical units. Therefore, the dimensional stream-function results provide the main basis for the present discussion, whereas the non-dimensional contours support the consistency of the predicted flow mechanisms with the classical behavior reported for natural convection in differentially heated cavities [1,2,7–10,21,27].



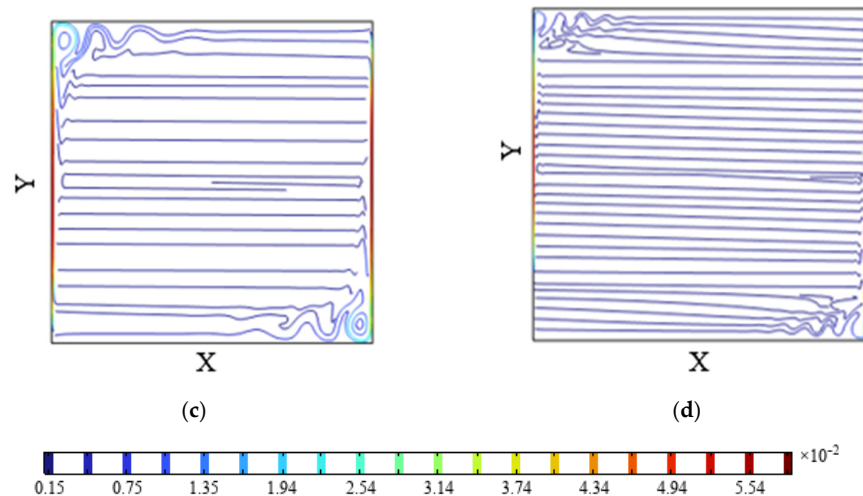


Figure 6. Stream-function contours calculated using the non-dimensional implementation for different Rayleigh numbers: (a) $Ra = 10^6$, (b) $Ra = 10^8$, (c) $Ra = 10^{10}$, and (d) $Ra = 10^{12}$.

3.2. Temperature and Velocity Profiles

In the following section, the temperature and velocity profiles obtained from the numerical solution of the differentially heated square cavity problem are presented primarily in dimensional form. In this representation, temperature is expressed in $^{\circ}\text{C}$ and velocity in m/s , allowing the intensity of the convective phenomenon and the associated heat- and momentum-transfer mechanisms to be interpreted directly in physical units. The non-dimensional temperature and velocity profiles are not discussed separately in this section because previous studies have demonstrated that both dimensional and non-dimensional implementations reproduce equivalent thermal and hydrodynamic behavior for the classical De Vahl Davis [1] problem. Molina-Herrera et al. [45] reported the corresponding nondimensional profiles for a wide range of Rayleigh numbers from 10^3 to 10^{12} , showing nearly identical profile distributions, boundary-layer development, and global heat-transfer characteristics. Therefore, to avoid repetitive discussion and improve the clarity of the manuscript, the present analysis focuses on the dimensional profiles, while the non-dimensional implementation is used only as a reference framework for comparison and validation of the predicted behavior [45].

Following the introduction of the temperature and velocity profiles described above, Figure 7 presents the temperature profiles evaluated along the horizontal centerline of the cavity ($y = L/2$) using the dimensional implementation, where temperature is expressed directly in $^{\circ}\text{C}$. These profiles provide a clearer description of the thermal stratification and heat-transfer mechanisms developing inside the cavity as the Rayleigh number increases from $Ra = 10^3$ to 10^{12} . For the lowest Rayleigh numbers, particularly in the range $Ra \approx 10^3$ – 10^4 , the temperature distribution remains nearly linear between the hot and cold walls. This behavior is characteristic of a conduction-dominated regime, in which thermal energy is transferred primarily through molecular diffusion, producing a relatively uniform thermal gradient across the domain [1,3,7,8,14]. Under these conditions, fluid motion remains weak, and the influence of buoyancy-driven convection on the thermal field is still limited [21,30,34,42]. As the Rayleigh number increases to the range $Ra \approx 10^5$ – 10^7 , the temperature profiles progressively deviate from the linear conductive distribution. In this regime, the transfer of momentum generated by buoyancy forces intensifies the fluid circulation within the cavity, modifying the thermal field through the upward motion of hot fluid near the heated wall and the downward motion of cold fluid near the cooled wall. Consequently, the central region of the cavity approaches a nearly uniform temperature, while stronger thermal gradients

develop adjacent to the vertical walls. This behavior reflects the progressive transition from conduction-dominated heat transfer toward a regime increasingly controlled by natural convection.

For higher Rayleigh numbers, particularly in the range $Ra \approx 10^8$ – 10^{12} , the temperature profiles exhibit a nearly isothermal core region with temperatures close to the mean thermal value inside the cavity. Under these conditions, convective heat transport dominates the thermal behavior, while the largest temperature variations become confined to very thin regions near the hot and cold walls. These regions correspond to the thermal boundary layers, where the strongest thermal gradients and heat-transfer rates are concentrated [1,13,14,17,34,42]. As Ra increases, the temperature profiles in the central region tend to converge toward a similar distribution, indicating that the thermal structure within the convective core becomes progressively less sensitive to increases in buoyancy forces. Therefore, the increase in Rayleigh number mainly affects the thickness of the thermal boundary layers rather than the temperature level within the cavity core. The use of dimensional variables additionally allows the thermal gradients and temperature levels reached inside the cavity to be interpreted directly in physical units, facilitating comparison with practical heat-transfer systems and engineering applications [14,17,18,25,33].

To complement the dimensional analysis, the corresponding non-dimensional temperature profiles were also evaluated and compared with the dimensional solution. The non-dimensional results reproduce the same thermal evolution observed in Figure 7, including the nearly linear conductive profiles at low Rayleigh numbers, the progressive distortion of the temperature field as natural convection intensifies, and the development of thin thermal boundary layers for $Ra \geq 10^8$ [12,15,22,27,30]. In particular, the non-dimensional profiles exhibit temperatures approaching $\theta \approx 0.5$ in the cavity core, confirming the formation of a thermally mixed convective region surrounded by thin near-wall thermal layers. These trends are fully consistent with the corresponding non-dimensional results reported by Molina-Herrera et al. [45] for Rayleigh numbers ranging from 10^3 to 10^{12} , in which both the dimensional and non-dimensional implementations produced nearly identical thermal distributions and boundary-layer behavior. Consequently, the dimensional profiles presented here preserve the same physical heat-transfer mechanisms predicted by the non-dimensional implementation while providing a more direct interpretation of the thermal field in measurable physical quantities [1,2,4,5,34,42].

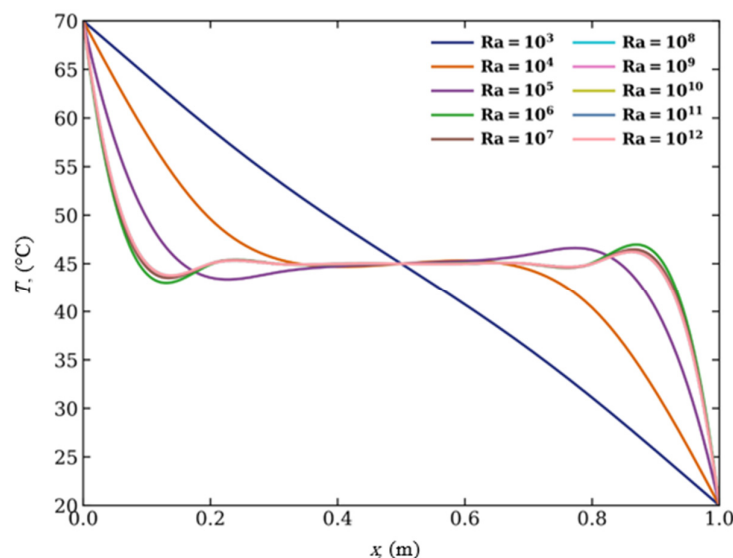


Figure 7. Temperature profiles along the cavity centerline computed using the dimensional implementation.

Continuing the analysis of the dimensional velocity profiles discussed above, Figure 8 presents the vertical velocity component (u_y) profiles, evaluated along the horizontal centerline of the cavity using the dimensional implementation, where velocity is expressed directly in m/s. These profiles provide important information on the transfer of momentum and the redistribution of convective flow within the cavity as the Rayleigh number increases from $Ra = 10^3$ to 10^{12} [2,4,6,13,25,28,30,33,40]. An important feature observed in the results is that the largest velocity magnitudes along the cavity centerline occur for intermediate Rayleigh numbers, particularly in the range $Ra \approx 10^4$ – 10^6 , whereas for higher Rayleigh numbers the magnitude of u_y along this line decreases considerably. Although this behavior may initially appear contradictory, its physical interpretation becomes clear when the flow structure and the evolution of the hydrodynamic boundary layers are analyzed.

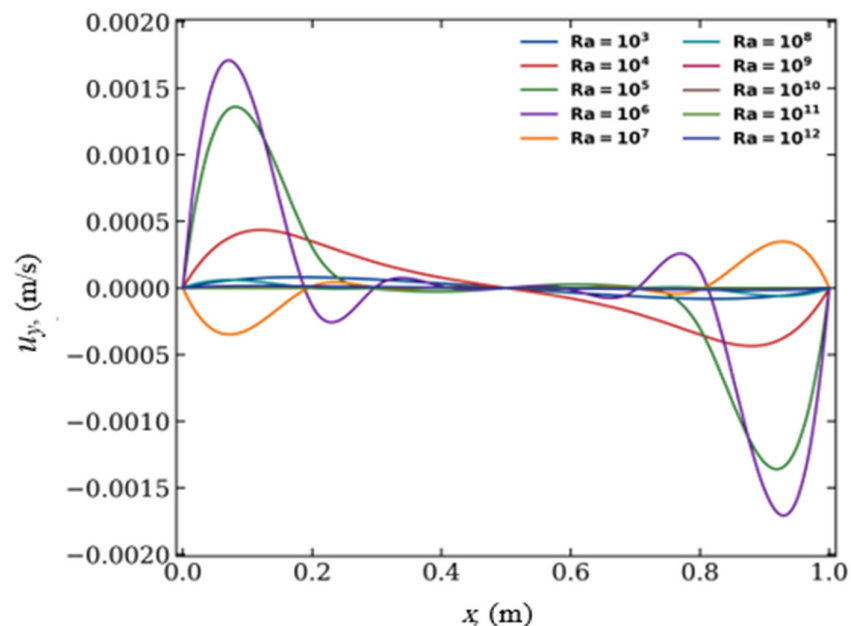


Figure 8. Vertical velocity profiles along the horizontal centerline of the cavity, computed using the dimensional implementation.

At low and intermediate Rayleigh numbers, the primary recirculation cell occupies a large fraction of the cavity interior. Under these conditions, the buoyancy forces generated by the temperature difference between the vertical walls produce upward motion of the hot fluid near the heated wall and downward motion of the colder fluid near the cooled wall. Since the recirculating flow remains distributed across a relatively large portion of the domain, the horizontal centerline intersects regions where the vertical velocity component still exhibits appreciable magnitudes. Consequently, the profiles for $Ra \approx 10^4$ – 10^6 display the highest values of u_y , indicating that the convective circulation extends deep into the cavity interior [1,3,4,9,10,13,14]. As the Rayleigh number increases further, particularly for $Ra \geq 10^7$, the momentum transport becomes progressively concentrated within increasingly thin hydrodynamic boundary layers adjacent to the hot and cold walls. Under these conditions, the core region of the cavity tends to behave as a region of weaker or nearly uniform recirculation, while the strongest velocity gradients become localized near the vertical walls and corner regions. Therefore, although the overall convection inside the cavity intensifies, the horizontal centerline no longer intersects the regions where the maximum vertical velocities occur. As a result, the local values of u_y measured along the centerline decrease as Ra

continues to increase. This behavior clearly illustrates the progressive confinement of convective motion to thin near-wall regions as buoyancy forces increase [8,15,17,18,27,34,45].

To complement the dimensional analysis, the corresponding non-dimensional vertical velocity profiles were also evaluated and compared with the dimensional solution. The non-dimensional results reproduce the hydrodynamic evolution observed in Figure 8, including the larger centerline velocity magnitudes at intermediate Rayleigh numbers and the subsequent reduction in local velocity values as the flow becomes increasingly confined within thin boundary layers at higher Ra . In the non-dimensional implementation, this behavior is further influenced by the scaling imposed by the characteristic velocity used in the normalization. Nevertheless, the overall redistribution of flow and the progressive confinement of convective circulation near the walls remain physically equivalent across both implementations. These trends are fully consistent with the non-dimensional results reported by Molina-Herrera et al. [45] for Rayleigh numbers ranging from 10^3 to 10^{12} , in which both dimensional and non-dimensional implementations yielded nearly identical hydrodynamic behavior, boundary-layer development, and recirculation patterns. Consequently, the dimensional profiles presented here preserve the same physical momentum-transfer mechanisms predicted by the non-dimensional implementation while allowing the flow intensity to be interpreted directly in physical units [31,35,36,41,45].

Continuing with the analysis of the velocity profiles, Figure 9 presents the horizontal velocity component u_x evaluated along the vertical centerline of the cavity using the dimensional implementation, where velocity is expressed directly in m/s. This profile complements the analysis of the vertical velocity component by describing how the flow redistributes momentum horizontally to complete convective recirculation within the cavity. Unlike the vertical momentum equation, which contains the buoyancy term $\rho g \beta (T - T_0)$, the horizontal momentum equation does not include buoyancy explicitly. Consequently, the horizontal velocity component arises from the dynamic adjustment of the flow required to satisfy mass conservation and maintain the closed circulation pattern within the cavity.

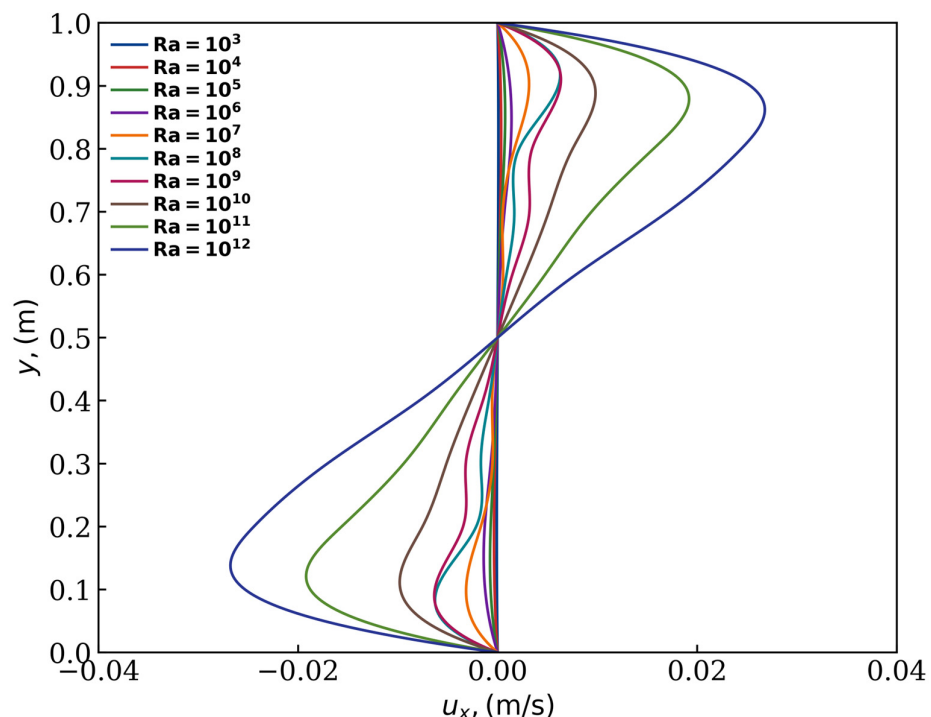


Figure 9. Horizontal velocity profiles along the cavity centerline computed using the dimensional implementation.

The profiles shown in Figure 9 exhibit a change in sign approximately at the cavity mid-height, indicating that the horizontal fluid motion reverses direction between the lower and upper regions of the domain. This behavior reflects the structure of the primary recirculation cell, where the fluid moves horizontally in opposite directions above and below the cavity centerline, thereby closing the convective circulation loop. For relatively low Rayleigh numbers, the convective motion remains distributed across a large portion of the cavity interior. Under these conditions, the vertical centerline intersects regions where the horizontal velocity retains appreciable magnitudes because the primary recirculation cell occupies nearly the entire domain [1,3,4,9,10,13,14]. As the Rayleigh number increases, the flow progressively reorganizes, becoming concentrated within progressively thinner hydrodynamic boundary layers adjacent to the hot and cold walls. Consequently, although global convective intensity increases, the horizontal velocity measured along the vertical centerline tends to decrease. This behavior occurs because the dominant flow structures shift toward near-wall regions, while the central portion of the cavity approaches a region of weaker or more uniform recirculation. Therefore, the local value of u_x along the centerline does not directly represent the overall increase in convective intensity but rather reflects the spatial redistribution of momentum transport inside the cavity as buoyancy effects intensify [8,15,17,18,27,34,45].

To complement the dimensional analysis, it should be noted that the corresponding non-dimensional horizontal velocity profiles are not presented separately in this section because they were previously reported by Molina-Herrera et al. [45] for the same range of Rayleigh numbers analyzed in the present study using the non-dimensional implementation. That study demonstrated that both dimensional and non-dimensional implementations reproduce nearly identical velocity distributions, boundary-layer evolution, and recirculation structures over a wide range of Rayleigh numbers. In particular, the non-dimensional implementation reproduced the same hydrodynamic behavior observed in the dimensional solution, including the reversal of the horizontal velocity direction near the cavity mid-height and the progressive reduction in the centerline velocity magnitude as the Rayleigh number increases. These results confirm that both implementations preserve the same momentum-transfer mechanisms and flow-redistribution patterns associated with natural convection within the cavity [14–16,23,30]. Therefore, the dimensional profiles presented here accurately reproduce the same physical hydrodynamic behavior predicted by the non-dimensional implementation while allowing the velocity field to be interpreted directly in measurable physical units.

4. Discussion

From a comparative perspective, the non-dimensional temperature and stream-function contours reproduce the same thermal and hydrodynamic structures observed in the dimensional implementation, confirming the consistency between both representations. In both implementations, increasing the Rayleigh number intensifies the convective circulation within the cavity, promotes the thinning of the thermal and hydrodynamic boundary layers, and leads to the development of secondary vortical structures near the cavity corners due to interactions between the primary recirculation cell and the near-wall regions [1,2,6,34,42]. As convection becomes stronger, a large portion of the momentum and heat transfer progressively concentrates near the hot and cold vertical walls, while the cavity core tends to behave as a region with relatively uniform thermal and hydrodynamic conditions.

The temperature contours show that, for low Rayleigh numbers, heat transfer is dominated primarily by conduction, producing smoother temperature gradients across the cavity. However, as the Rayleigh number increases, buoyancy forces intensify the fluid circulation and progressively enhance convective heat transport. Under these conditions,

the isotherms become increasingly horizontal within the cavity core, while the strongest thermal gradients become confined to thin regions adjacent to the vertical walls, corresponding to the thermal boundary layers. Simultaneously, the stream-function contours reveal that the fluid motion becomes progressively concentrated near the walls and corners, where the largest velocity gradients and secondary vortical structures develop [1,3,4,9,10,13,14]. These flow structures are associated with interactions between the primary circulation cell and increasingly thin hydrodynamic boundary layers. Although the non-dimensional implementation facilitates direct comparison with the classical benchmark studies reported in the literature, the dimensional representation provides the additional advantage of expressing the temperature and velocity fields in measurable physical units, such as °C for temperature and m/s for velocity. This characteristic allows the intensity of the thermal gradients, the magnitude of the velocity field, and the proximity to critical thermal conditions to be interpreted more directly in engineering and thermodynamic terms. In addition, the dimensional implementation avoids excessive scaling between variables during the numerical solution process, facilitating direct interpretation of the governing equations and reducing additional transformations associated with the normalization procedure [1,2,14,17,22,32,42]. Consequently, the present study places particular emphasis on the dimensional implementation, while the non-dimensional representation is used primarily as a reference framework to validate the numerical consistency and physical behavior of the predicted solution.

Therefore, the present results confirm that the dimensional implementation accurately reproduces the characteristic heat- and momentum-transfer mechanisms associated with the classical differentially heated square cavity problem. Similar conclusions were reported by Molina-Herrera et al. [45], where both dimensional and non-dimensional implementations were shown to produce equivalent thermal distributions, boundary-layer development, and flow structures over a wide range of Rayleigh numbers.

It should be noted that the general structure of the profiles obtained using the non-dimensional implementation adequately reproduces the behavior observed in the dimensional solution. However, while the non-dimensional representation facilitates comparison with previous studies and allows the phenomenon to be analyzed in terms of normalized variables, the dimensional implementation provides a more direct physical interpretation because the results are expressed in measurable quantities, such as temperature in °C and velocity in m/s. For this reason, the present study places particular emphasis on the dimensional profiles, whereas the non-dimensional implementation is used mainly as a reference framework to validate the consistency of the numerically predicted thermal and hydrodynamic behavior. Similar conclusions were reported by Molina-Herrera et al. [45], where both dimensional and non-dimensional implementations reproduced equivalent thermal distributions, velocity profiles, and boundary-layer development over a wide range of Rayleigh numbers [1,4,6,13,14,23,34,45].

The temperature profiles show that increasing the Rayleigh number progressively transforms the heat-transfer mechanism from a conduction-dominated regime toward a convection-dominated regime. For low Rayleigh numbers, the temperature distribution remains nearly linear across the cavity, indicating that thermal energy is transferred mainly through molecular diffusion. As buoyancy forces increase, convective circulation intensifies and redistributes heat more effectively within the cavity. Under these conditions, the cavity core approaches a nearly uniform temperature, while the strongest thermal gradients become confined to thin regions adjacent to the hot and cold walls. These regions correspond to the thermal boundary layers, where the local heat-transfer

rates become significantly larger. As a result, the increase in Rayleigh number mainly affects the thickness and intensity of the thermal boundary layers, whereas the central region tends to remain thermally stratified with relatively small temperature variations. The velocity profiles reveal a similar redistribution process associated with the evolution of the convective circulation. In the case of the vertical velocity component u_y , the largest centerline magnitudes occur for intermediate Rayleigh numbers because the primary recirculation cell still occupies a large portion of the cavity interior. Under these conditions, the centerline intersects regions where the upward and downward fluid motion remains significant. However, as the Rayleigh number increases further, the flow progressively concentrates within progressively thinner hydrodynamic boundary layers near the vertical walls and in corner regions. As a result, the centerline no longer intersects the regions of highest velocity, and the local value of u_y decreases despite the overall strengthening of convection inside the cavity. Therefore, the velocity profiles should be interpreted as a local measure of flow redistribution rather than as a direct indicator of the global convective intensity. A similar behavior is observed for the horizontal velocity component u_x . The profiles exhibit an antisymmetric distribution about the cavity midpoint, reflecting the structure of the primary recirculation cell: fluid moves horizontally in opposite directions above and below the cavity centerline to close the convective circulation loop. For low Rayleigh numbers, the convective motion remains distributed over a large region of the cavity, allowing appreciable horizontal velocities to persist near the center of the domain. In contrast, for high Rayleigh numbers, the flow becomes increasingly confined near the walls because of the progressive reduction in viscous diffusion relative to convective transport. Consequently, the strongest velocity gradients become concentrated inside thin hydrodynamic boundary layers, while the cavity core behaves as a transition region with weaker local velocities. This explains why the horizontal velocity measured along the centerline decreases even though the overall convection within the cavity becomes stronger. From a numerical perspective, the non-dimensional implementation introduces additional scaling through the characteristic velocity and the dimensionless groups governing the problem. In particular, the viscous diffusion terms are weighted by coefficients that scale as $(\text{Pr}/\text{Ra})^{1/2}$, whose magnitude decreases as the Rayleigh number increases [6,7,34,39,45]. As the Rayleigh number increases, convective effects progressively dominate the flow dynamics, and the regions of strongest momentum transfer shift toward the near-wall boundary layers. Although the non-dimensionalization process modifies the relative magnitude of the profiles, it does not alter the underlying physical behavior. Instead, both implementations preserve the same thermal and hydrodynamic trends, including the confinement of heat and momentum transfer near the walls and the redistribution of the recirculation structure as buoyancy effects intensify. Therefore, the similarity between the dimensional and non-dimensional profiles confirms that both implementations reproduce the same physical mechanisms governing natural convection in the differentially heated cavity. Nevertheless, the dimensional implementation provides the additional advantage of interpreting the thermal and velocity fields directly in physical units, facilitating comparison with engineering applications, experimental conditions, and thermodynamic analyses while preserving consistency with the classical benchmark behavior reported in the literature [1,7,8,10,24,28,34,41,42].

5. Conclusions

This work presented a numerical comparison of dimensional and non-dimensional implementations of the classical benchmark problem of steady natural convection in a two-dimensional differentially heated square cavity under the Boussinesq approximation. Since both descriptions are theoretically equivalent at the continuous level, the purpose

of the study was not to question their physical validity, but to examine their numerical behavior under identical benchmark conditions.

The results showed that both implementations reproduce the same heat-transfer and flow behavior once the near-wall gradients are properly resolved. In particular, the average Nusselt numbers obtained with both implementations were practically identical over the range $10^3 \leq Ra \leq 10^{12}$, with relative differences remaining below approximately 0.022% for the reference $BL = 100$. Likewise, the temperature contours, stream-function distributions, and centerline profiles exhibited the same structural trends in both cases, confirming that both implementations consistently describe the adopted benchmark problem.

A second relevant outcome is that the dimensional implementation was successfully constructed using physically plausible temperature differences and corresponding cavity lengths derived from the Rayleigh-number definition. This strategy avoided unrealistically large wall temperatures and provided a more physically interpretable dimensional representation of the benchmark. In the revised analysis, $\Delta T = 25$ K was selected as the reference case because it offers a suitable balance between consistency with the Boussinesq approximation and moderate cavity sizes over the Rayleigh-number range considered, while $\Delta T = 50$ K was retained only as an exploratory sensitivity case.

The grid-independence analysis showed that adequate refinement of the thermal and hydrodynamic boundary layers is essential for both implementations, especially at high Rayleigh numbers. For the present benchmark, $BL = 100$ offered the best compromise among numerical accuracy, convergence stability, and computational cost. Under identical numerical conditions, the dimensional implementation had a lower computational cost than the non-dimensional implementation, yielding approximately a 31% reduction in computation time. This result indicates that although both implementations produce the same benchmark solution, their practical numerical performance differs. However, this computational performance advantage was observed within the adopted COMSOL finite-element framework and may not generalize to other CFD platforms, discretization approaches, or numerical solution strategies.

The results of this study demonstrate that the dimensional implementation constitutes a numerically efficient and physically interpretable alternative for solving the adopted square-cavity benchmark. Within the scope of the present investigation, both formulations reproduced the same benchmark physics with equivalent accuracy while exhibiting different computational behavior. The conclusions are limited to the adopted steady two-dimensional Boussinesq model, a single benchmark geometry, and the specific numerical implementation employed in COMSOL. In addition, the simulations performed at Rayleigh numbers up to 10^{12} should be regarded primarily as benchmark-level numerical stress tests within the adopted framework rather than as direct representations of practical physical systems. These findings provide useful guidance for the numerical treatment of natural-convection benchmark problems and establish a basis for future studies involving more complex geometries, transient effects, compressibility effects, three-dimensional configurations, and alternative numerical frameworks.

Author Contributions: Conceptualization, F.I.M.-H., and H.J.-I.; methodology, J.M.O.-M., and M.L.L.-G.; software, H.J.-I., N.L.F.-M., and J.M.O.-M.; validation, F.I.M.-H., N.E.M.-S., and H.J.-I.; formal analysis, H.J.-I. and M.L.L.-G.; investigation, F.I.M.-H.; resources, P.Y.-C., F.J.S.-B. and M.L.L.-G.; writing—original draft preparation, F.I.M.-H., P.Y.-C. and N.E.M.-S.; writing—review and editing, H.J.-I., F.I.M.-H. and P.Y.-C.; supervision, F.J.S.-B., N.L.F.-M. and H.J.-I.; project administration, N.L.F.-M. All authors have read and agreed to the published version of the manuscript.

Funding: The APC was funded by Universidad Politécnica de Guanajuato, Mexico.

Data Availability Statement: The original contributions presented in this study are included in the article. Further inquiries can be directed to the corresponding author.

Acknowledgments: The authors acknowledge SECIHTI's support via the Postdoctoral Fellowships for the Training and Consolidation of Researchers in Mexico and TecNM for research support.

Conflicts of Interest: The authors declare no conflicts of interest.

Nomenclature

c_p	specific heat, [J·kg ⁻¹ ·K ⁻¹]
g	gravitational acceleration, [m·s ⁻²]
h	convective heat transfer coefficient, [W·m ⁻² ·K ⁻¹]
k	thermal conductivity, [W·m ⁻¹ ·K ⁻¹]
P	dimensionless pressure
p	pressure, [N·m ⁻²]
Pr	Prandtl number, dimensionless
L	length and height of the cavity, [m]
$\overline{Nu}_{dim}$	dimensional average Nusselt number, $\int_0^L (1/T_h - T_c)(\partial T/\partial x) _{x=0} dy$
$\overline{Nu}_{nd}$	non-dimensional average Nusselt number, $\int_0^L -(\partial \theta/\partial X) _{X=0} dY$
ΔT	temperature difference, [K]
Ra	Rayleigh number, dimensionless
T	temperature, [K]
T_h, T_c	hot and cold wall temperatures, [K]
u_{ref}	buoyancy reference velocity, [m·s ⁻¹]
$U_x, U_y, \overline{U}_x, \overline{U}_y$	horizontal and vertical velocity, dimensionless
$u_x, u_y, \overline{u}_x, \overline{u}_y$	dimensional horizontal and vertical velocity, [m·s ⁻¹]
x, y	dimensional coordinates, [m]
X, Y	dimensionless coordinates
Greek letters	
α	thermal diffusivity, [m ² ·s ⁻¹]
β	thermal expansion coefficient, [K ⁻¹]
ΔT	temperature difference between the hot and cold walls, [K]
ρ	density, [kg·m ⁻³]
μ	dynamic viscosity, [kg·m ⁻¹ ·s ⁻¹] or [Pa·s]
θ	dimensionless temperature

Abbreviations

The following abbreviations are used in this manuscript:

BL	Boundary Layer
CFD	Computational Fluid Dynamics
NC	Convergence not reached

Appendix A

Appendix A presents complementary dimensional results used to assess the formulation's sensitivity to the construction of the prescribed Rayleigh number from dimensional variables. In all cases reported in Tables A1–A4, the same optimized mesh was used, consisting of a uniform core discretization of “0.005 × 0.005 m” combined with BL = 100 boundary-layer elements near the heated and cooled walls. Tables A1–A4 show the cavity length required to represent each Rayleigh number for physically plausible temperature differences of $\Delta T = 1$ K, 10 K, 25 K, and 50 K. The results confirm that, although the dimensional construction changes the cavity size, the predicted Nusselt numbers remain essentially unchanged for a given Rayleigh number. This behavior supports the conclusion that the benchmark heat-transfer response is governed primarily

by the Rayleigh number itself, provided that the dimensional representation remains consistent with the Boussinesq approximation.

Table A1. Characteristic cavity length L at $\Delta T = 1$ K.

Rayleigh Number	Cavity Length (m)	Nusselt at the Hot Wall	Nusselt at the Cold Wall
10^3	0.0218	1.1178	1.1178
10^4	0.0470	2.2448	2.2448
10^5	0.1012	4.5214	4.5214
10^6	0.2180	8.8249	8.8249
10^7	0.4696	16.5470	16.5470
10^8	1.0118	30.2900	30.2900
10^9	2.1799	54.4880	54.4880
10^{10}	4.6964	97.6130	97.6130
10^{11}	10.1180	174.3200	174.3200
10^{12}	21.7990	308.9500	308.9700

Table A2. Characteristic cavity length L at $\Delta T = 10$ K.

Rayleigh Number	Cavity Length (m)	Nusselt at the Hot Wall	Nusselt at the Cold Wall
10^3	0.0101	1.1178	1.1178
10^4	0.0218	2.2448	2.2448
10^5	0.0470	4.5214	4.5214
10^6	0.1012	8.8249	8.8249
10^7	0.2180	16.5470	16.5470
10^8	0.4696	30.2900	30.2900
10^9	1.0118	54.4880	54.4880
10^{10}	2.1799	97.6130	97.6130
10^{11}	4.6964	174.3100	174.3100
10^{12}	10.1180	308.9400	308.9500

Table A3. Characteristic cavity length L at $\Delta T = 25$ K.

Rayleigh Number	Cavity Length (m)	Nusselt at the Hot Wall	Nusselt at the Cold Wall
10^3	0.0075	1.1178	1.1178
10^4	0.0161	2.2448	2.2448
10^5	0.0346	4.5214	4.5214
10^6	0.0746	8.8249	8.8249
10^7	0.1606	16.5470	16.5470
10^8	0.3460	30.2900	30.2900
10^9	0.7455	54.4880	54.4880
10^{10}	1.6062	97.6130	97.6130
10^{11}	3.4604	174.3200	174.3200
10^{12}	7.4551	308.9500	308.9600

Table A4. Characteristic cavity length L at $\Delta T = 50$ K.

Rayleigh Number	Cavity Length (m)	Nusselt at the Hot Wall	Nusselt at the Cold Wall
10^3	0.0059	1.1178	1.1178
10^4	0.0127	2.2448	2.2448
10^5	0.0275	4.5214	4.5214
10^6	0.0592	8.8249	8.8249
10^7	0.1275	16.5470	16.5470
10^8	0.2747	30.2900	30.2900
10^9	0.5917	54.4880	54.4880
10^{10}	1.2748	97.6130	97.6130
10^{11}	2.7465	174.3200	174.3200
10^{12}	5.9172	308.9600	308.9600

Tables A5–A7 present an alternative dimensional construction in which the cavity length is fixed at $L = 1.0, 0.5$, and 10 m, respectively, and the temperature difference ΔT is computed for each Rayleigh number. These calculations also reproduce the expected Nusselt values with very good accuracy, confirming the invariance of the benchmark solution with respect to the specific dimensional path used to represent the same Rayleigh number. However, for fixed cavity lengths of 1.0 and 0.5 m, the required temperature differences become extremely large at high Rayleigh numbers and fall outside the range of physical plausibility associated with the Boussinesq approximation. For this reason, these cases are included only as complementary numerical evidence. In contrast, the strategy adopted in the main body of the manuscript, namely prescribing physically plausible values of ΔT and calculating the corresponding cavity length, provides a more physically consistent dimensional representation of the benchmark problem.

Table A5. Characteristic ΔT (K) at cavity length $L = 1.0$ m.

Rayleigh Number	$\Delta T = [T_h - T_c]$ (K)	Nusselt at the Hot Wall	Nusselt at the Cold Wall
10^3	1.0359×10^{-5}	1.1178	1.1178
10^4	1.0359×10^{-4}	2.2448	2.2448
10^5	1.0359×10^{-3}	4.5214	4.5214
10^6	1.0359×10^{-2}	8.8249	8.8249
10^7	1.0359×10^{-1}	16.5467	16.5467
10^8	1.0359×10^0	30.2902	30.2902
10^9	1.0359×10^1	54.4870	54.4870
10^{10}	1.0359×10^2	97.6152	97.6152
10^{11}	1.0359×10^3	174.3166	174.3166
10^{12}	1.0359×10^4	308.9494	308.9485

Table A6. Characteristic ΔT (K) at cavity length $L = 0.5$ m.

Rayleigh Number	$\Delta T = [T_h - T_c]$ (K)	Nusselt at the Hot Wall	Nusselt at the Cold Wall
10^3	4.1435×10^{-5}	1.1178	1.1178
10^4	4.1435×10^{-4}	2.2448	2.2448
10^5	4.1435×10^{-3}	4.5214	4.5214
10^6	4.1435×10^{-2}	8.8249	8.8249
10^7	4.1435×10^{-1}	16.5467	16.5467
10^8	4.1435×10^0	30.2902	30.2902
10^9	4.1435×10^1	54.4870	54.4870
10^{10}	4.1435×10^2	97.6152	97.6152
10^{11}	4.1435×10^3	174.3166	174.3166
10^{12}	4.1435×10^4	308.9511	308.9457

Table A7. Characteristic ΔT (K) at cavity length $L = 10.0$ m.

Rayleigh Number	$\Delta T = [T_h - T_c]$ (K)	Nusselt at the Hot Wall	Nusselt at the Cold Wall
10^3	1.0359×10^{-7}	1.1170	1.1166
10^4	1.0359×10^{-6}	2.2468	2.2445
10^5	1.0359×10^{-5}	4.5212	4.5213
10^6	1.0359×10^{-4}	8.8249	8.8249
10^7	1.0359×10^{-3}	16.5467	16.5467
10^8	1.0359×10^{-2}	30.2902	30.2902
10^9	1.0359×10^{-1}	54.4870	54.4870
10^{10}	1.0359×10^0	97.6152	97.6152
10^{11}	1.0359×10^1	174.3166	174.3166
10^{12}	1.0359×10^2	308.9500	308.9475

References

- De Vahl Davis, G. Natural convection of air in a square cavity: A benchmark numerical solution. *Int. J. Numer. Methods Fluids* **1983**, *3*, 249–264. <https://doi.org/10.1002/fld.1650030305>.
- De Vahl Davis, G. Laminar natural convection in an enclosed rectangular cavity. *Int. J. Heat Mass Transf.* **1968**, *11*, 1675–1693. [https://doi.org/10.1016/0017-9310\(68\)90047-1](https://doi.org/10.1016/0017-9310(68)90047-1).
- Markatos, N.; Pericleous, K. Laminar and turbulent natural convection in an enclosed cavity. *Int. J. Heat Mass Transf.* **1984**, *27*, 755–772. [https://doi.org/10.1016/0017-9310\(84\)90145-5](https://doi.org/10.1016/0017-9310(84)90145-5).
- Barakos, G.; Mitsoulis, E.; Assimacopoulos, D. Natural convection flow in a square cavity revisited: Laminar and turbulent models with wall functions. *Int. J. Numer. Methods Fluids* **1994**, *18*, 695–719. <https://doi.org/10.1002/fld.1650180705>.
- Bilgen, E.; Oztop, H. Natural convection heat transfer in partially open inclined square cavities. *Int. J. Heat Mass Transf.* **2005**, *48*, 1470–1479. <https://doi.org/10.1016/j.ijheatmasstransfer.2004.10.020>.
- Dixit, H.; Babu, V. Simulation of high Rayleigh number natural convection in a square cavity using the lattice Boltzmann method. *Int. J. Heat Mass Transf.* **2006**, *49*, 727–739.
- Dorfman, A.; Renner, Z. Conjugate problems in convective heat transfer: Review. *Math. Probl. Eng.* **2009**, *2009*, 927350. <https://doi.org/10.1155/2009/927350>.
- Erdogdu, F.; Uyar, R.; Palazoglu, T.K. Experimental comparison of natural convection and conduction heat transfer. *J. Food Process Eng.* **2010**, *33*, 85–100. <https://doi.org/10.1111/j.1745-4530.2008.00309.x>.
- Ji, Y. CFD modelling of natural convection in air cavities. *CFD Lett.* **2014**, *6*, 15–31.
- Choi, S.; Kim, S. Turbulence modeling of natural convection in enclosures: A review. *J. Mech. Sci. Technol.* **2012**, *26*, 283–297. <https://doi.org/10.1007/s12206-011-1037-0>.
- Pickett, A. *Finite Element Theory and Practical Analysis with Open Source Codes: Including Basic CFD Analysis*, 3rd ed.; Institute of Aircraft Design: Stuttgart, Germany, 2023.
- Fox, R.W.; McDonald, A.T.; Mitchell, J.W. *Fox and McDonald's Introduction to Fluid Mechanics*, 10th ed.; John Wiley & Sons: Hoboken, NJ, USA, 2020.
- Hernández-López, I.; Xamán, J.; Álvarez, G.; Chávez, Y.; Arce, J. Analysis of laminar and turbulent natural, mixed and forced convection in cavities by heatlines. *Arch. Mech.* **2016**, *68*, 27–53. <https://doi.org/10.24423/aom.1905>.
- Goloviznin, V.M.; Korotkin, I.A.; Finogenov, S.A. Parameter-free numerical method for modeling thermal convection in square cavities in a wide range of Rayleigh numbers. *J. Appl. Mech. Tech. Phys.* **2016**, *57*, 1159–1171. <https://doi.org/10.1134/S0021894416070063>.
- Hyun, J.M.; Lee, J.W. Numerical solutions for transient natural convection in a square cavity with different sidewall temperatures. *Int. J. Heat Fluid Flow* **1989**, *10*, 146–151. [https://doi.org/10.1016/0142-727X\(89\)90009-X](https://doi.org/10.1016/0142-727X(89)90009-X).
- Lam, C.K.G.; Bremhorst, K. A modified form of the k- ϵ model for predicting wall turbulence. *J. Fluids Eng.* **1981**, *103*, 456–460. <https://doi.org/10.1115/1.3240815>.
- Xin, S.; Le Quére, P. Numerical simulations of 2D turbulent natural convection in differentially heated cavities of aspect ratios 1 and 4. In *Direct and Large-Eddy Simulation I. Fluid Mechanics and Its Applications*; Springer: Dordrecht, The Netherlands, 1994; pp. 423–434. https://doi.org/10.1007/978-94-011-1000-6_36.
- Igci, A.A.; Arici, M.E. A comparative study of four low-Reynolds-number k- ϵ turbulence models for periodically fully developed duct flow and heat transfer. *Numer. Heat Transf. B* **2016**, *69*, 234–248. <https://doi.org/10.1080/10407790.2015.1097141>.
- Villa Ortiz, A.; Koloszar, L. RANS thermal modelling of a natural convection boundary layer at low Prandtl number. *Comput. Fluids* **2023**, *254*, 105809. <https://doi.org/10.1016/j.compfluid.2023.105809>.
- Khoubani, A.; Mohanan, A.V.; Augier, P.; Flór, J.-B. Vertical convection regimes in a two-dimensional rectangular cavity: Prandtl and aspect ratio dependence. *J. Fluid Mech.* **2024**, *981*, A10. <https://doi.org/10.1017/jfm.2023.1056>.
- Hosseini, S.; Di Felice, R. CFD simulation of high gas flow rate in large-scale rotating packed beds. *ChemEngineering* **2025**, *9*, 126. <https://doi.org/10.3390/chemengineering9060126>.
- Wilcox, D.C. *Turbulence Modeling for CFD*; DCW Industries: La Cañada, CA, USA, 1993.
- Nie, X.; Li, L. A comparison of low Reynolds number k- ϵ models. In *Proceedings of the 2015 4th International Conference on Computer, Mechatronics, Control and Electronic Engineering*; Advances in Engineering Research; Atlantis Press: Dordrecht, The Netherlands, 2015; pp. 1334–1339. <https://doi.org/10.2991/iccmcee-15.2015.250>.
- Sheremet, M.A.; Pop, I.; Mahian, O. Natural convection in an inclined cavity with time-periodic temperature boundary conditions using nanofluids: Application in solar collectors. *Int. J. Heat Mass Transf.* **2018**, *116*, 751–761. <https://doi.org/10.1016/j.ijheatmasstransfer.2017.09.070>.

25. El Hattab, M.; Lafdaili, Z. Turbulent natural convection heat transfer in a square cavity with nanofluids in presence of inclined magnetic field. *Therm. Sci.* **2022**, *26*, 3201–3213. <https://doi.org/10.2298/TSCI210825326E>.
26. Oran, E.S. Numerical simulation of flames: Current status and future perspectives. *Pure Appl. Chem.* **1990**, *62*, 877–887. <https://doi.org/10.1351/pac199062050877>.
27. Pérez-Segarra, C.; Oliva, A.; Costa, M.; Escanes, F. Numerical experiments in turbulent natural and mixed convection in internal flows. *Int. J. Numer. Methods Heat Fluid Flow* **1995**, *5*, 13–33. <https://doi.org/10.1108/EUM0000000004114>.
28. Basak, T.; Roy, S.; Balakrishnan, A.R. Effects of thermal boundary conditions on natural convection flows within a square cavity. *Int. J. Heat Mass Transf.* **2006**, *49*, 4525–4535.
29. Trias, F.X.; Soria, M.; Oliva, A.; Pérez-Segarra, C.D. Direct numerical simulations of two- and three-dimensional turbulent natural convection flows in a differentially heated cavity of aspect ratio 4. *J. Fluid Mech.* **2007**, *586*, 259–293. <https://doi.org/10.1017/S0022112007006908>.
30. Henkes, R.A.W.M.; Hoogendoorn, C.J. Laminar natural convection boundary-layer flows along a heated vertical plate in a stratified environment. *Int. J. Heat Mass Transf.* **1989**, *32*, 147–155. [https://doi.org/10.1016/0017-9310\(89\)90098-7](https://doi.org/10.1016/0017-9310(89)90098-7).
31. Tian, Y.S.; Karayiannis, T.G. Low turbulence natural convection in an air-filled square cavity. Part II: The turbulence quantities. *Int. J. Heat Mass Transf.* **2000**, *43*, 867–884. [https://doi.org/10.1016/S0017-9310\(99\)00200-8](https://doi.org/10.1016/S0017-9310(99)00200-8).
32. Ampofo, F.; Karayiannis, T.G. Experimental benchmark data for turbulent natural convection in an air-filled square cavity. *Int. J. Heat Mass Transf.* **2003**, *46*, 3551–3572. [https://doi.org/10.1016/S0017-9310\(03\)00147-9](https://doi.org/10.1016/S0017-9310(03)00147-9).
33. Cintolesi, C.; Petronio, A.; Armenio, V. Large eddy simulation of turbulent buoyant flow in a confined cavity with conjugate heat transfer. *Phys. Fluids* **2015**, *27*, 095107. <https://doi.org/10.1063/1.4930815>.
34. Molina-Herrera, F.I.; Jiménez-Islas, H. Direct numerical simulation of the differentially heated cavity and comparison with the κ - ϵ model for high Rayleigh numbers. *Modelling* **2025**, *6*, 66. <https://doi.org/10.3390/modelling6030066>.
35. Janssen, R.J.A.; Henkes, R.A.W.M. Transition to time-periodicity of a natural-convection flow in a 3D differentially heated cavity. *Int. J. Heat Mass Transf.* **1993**, *36*, 2927–2940. [https://doi.org/10.1016/0017-9310\(93\)90111-i](https://doi.org/10.1016/0017-9310(93)90111-i).
36. Weppe, A.; Moreau, F.; Saury, D. Experimental investigation of a turbulent natural convection flow in a cubic cavity with an inner obstacle partially heated. *Int. J. Heat Mass Transf.* **2022**, *194*, 123052. <https://doi.org/10.1016/j.ijheatmasstransfer.2022.123052>.
37. Roux, B.; Grondin, J.C.; Bontoux, P.; Gilly, B. On a high-order accurate method for the numerical study of natural convection in a vertical square cavity. *Numer. Heat Transf. Part B Fundam.* **1978**, *1*, 331–349. <https://doi.org/10.1080/10407797809412169>.
38. Ghoben, Z.K.; Hussein, A.K. Natural convection inside a 3D triangular cross-section cavity filled with nanofluid and included cylinder with different arrangements. *Diagnostyka* **2022**, *23*, 2022205. <https://doi.org/10.29354/diag/149734>.
39. Wang, Q.; Xia, S.-N.; Yan, R.; Sun, D.-J.; Wan, Z.-H. Non-Oberbeck-Boussinesq effects due to large temperature differences in a differentially heated square cavity filled with air. *Int. J. Heat Mass Transf.* **2019**, *128*, 479–491. <https://doi.org/10.1016/j.ijheatmasstransfer.2018.06.079>.
40. Hu, N.; Fan, L.-W.; Zhu, Z.-Q. Can the numerical simulations of melting in a differentially heated rectangular cavity be rationally reduced to 2D? A comparative study between 2D and 3D simulation results. *Int. J. Heat Mass Transf.* **2021**, *166*, 120751. <https://doi.org/10.1016/j.ijheatmasstransfer.2020.120751>.
41. Sandoval-Hernández, M.A.; Jiménez-Islas, H.; Molina-Herrera, F.I. Aplicación de ChatGPT en la enseñanza de fenómenos de transporte en ingeniería química. *Rev. Investig. Innov. Ing. Tecnol.* **2025**, *13*, 25–44.
42. Molina-Herrera, F.I.; Quemada-Villagómez, L.I.; Navarrete-Bolaños, J.L.; Jiménez-Islas, H. Comparative analysis of nondimensionalization approaches for solving the 2-D differentially heated cavity problem. *Int. J. Numer. Methods Fluids* **2024**, *96*, 1276–1303. <https://doi.org/10.1002/fld.5285>.
43. Aris, R. *Vectors, Tensors and the Basic Equations of Fluid Mechanics*; Dover Publications: Garden City, NY, USA, 1990.
44. Bird, R.B.; Stewart, W.E.; Lightfoot, E.N. *Transport Phenomena*, 2nd ed.; John Wiley & Sons: New York, NY, USA, 2006.
45. Molina-Herrera, F.I.; Montes-Rosales, H.; Maldonado-Sierra, N.E.; Sandoval-Hernández, M.A.; Martínez-González, G.M.; López-González, M.L.; Jiménez-Islas, H. Direct numerical simulation of the two-dimensional differentially heated square cavity at high Rayleigh numbers ($Ra \leq 10^{12}$). *Rev. Mex. Ing. Quím.* **2026**, *25*, Fen26778.

Disclaimer/Publisher's Note: The statements, opinions and data contained in all publications are solely those of the individual author(s) and contributor(s) and not of MDPI and/or the editor(s). MDPI and/or the editor(s) disclaim responsibility for any injury to people or property resulting from any ideas, methods, instructions or products referred to in the content.