Modeling and Control of a Road Wheel Actuation Module in Steer-by-Wire System

Insu Chung, Jungdai Choi and Kanghyun Nam *

School of Mechanical Engineering, Yeungnam University, Gyeongsan 38541, Republic of Korea; aksien@yu.ac.kr (I.C.); 21720533@ynu.ac.kr (J.C.)
* Correspondence: khnam@yu.ac.kr; Tel.: +82-53-811-2455

Abstract: Since the steer-by-wire system removes the mechanical connection and uses electrical signals to drive the system, it has the disadvantage of being less stable in the failure of parts or systems. Therefore, in this paper, we present a methodology for developing a digital model of the road wheel actuator of the steer-by-wire system. First, the detailed dynamics of the road wheel actuator are analyzed and simplified, and the friction model is estimated and compensated to obtain the equilibrium inertia and damping coefficient of the motor and the road wheel actuator. And to verify the accuracy of the digital model developed based on these parameters, the outputs are compared by giving the same inputs under open-loop control. Furthermore, to solve the problem caused by nonlinear disturbance and model uncertainty, a disturbance observer-based position controller is proposed. The validity of the proposed controller and the validity of the digital model development methodology are confirmed by the results of the position control experiment.

Keywords: steer by wire; digital model; simulation in the loop simulation; friction model; disturbance observer

1. Introduction

The automotive environment is undergoing a transition from simple transportation to a dynamic mobility space. These changes boost the advancement of automotive hardware and software technologies, increasing the complexity of the system. This trend increases the space and weight that components occupy in terms of hardware and affects the motion of the vehicle, making it difficult to understand the vehicle system. Researchers are studying technologies like X-by-wire and virtualization as methods to increase the overall performance and efficiency of the vehicle system, overcoming these difficulties.

The X-by-wire is a system that replaces mechanical parts with electrical signals using wires [1,2]. The system attracts attention because it maximizes space utilization and design flexibility to fit different vehicle platforms [3,4], and it has been applied to various vehicle systems such as brakes [5,6] and steering [7]. Unlike traditional steering systems, the steer-by-wire (SbW) system eliminates the mechanical link between the steering wheel and the road wheel and is driven by electrical signals [8]. Figure 1 depicts the fundamental components of the SbW system. The SbW system has two main components. The road wheel actuator (RWA) translates the driver’s steering intent into wheel movement to control the vehicle. The steering feeling actuator (SFA) provides the driver with a reaction force based on the road surface information. SbW offers several advantages. The absence of a shaft connection effectively shields against unwanted vibrations and noise while contributing to weight reduction [9,10]. Additionally, the system allows for an adjustable gear ratio across a wide range based on vehicle speed, eliminating the need for additional hardware [11]. However, SbW systems are dangerous under component and system failure conditions. Electronic power steering can be steered with the driver’s power even if it breaks down, but SbW cannot be steered. To provide stability under fault conditions, a fault...
A digital model is a virtual representation of a physical system that accurately depicts its structure and functions. It is commonly used to analyze and understand complex system behaviors to enhance performance and efficiency [12,13]. This can expedite the development process, allow for quick iterations, and aid in identifying design problems or areas for improvement [14]. Additionally, the digital model can be utilized for simulation purposes, enabling the examination of various situations and design configurations without the necessity of real prototypes [15,16]. Engineers may utilize a computerized representation of a vehicle to test aerodynamic properties or simulate potential crash situations. Similarly, digital models can be used in the development of steering systems to simulate different driving conditions and enhance reaction and safety features [17–19]. These models can utilize data from endurance testing to allow engineers to identify patterns and system failures, thereby improving the dependability of the steering system.

Despite these advantages, digital models often face limitations in accuracy. For example, representing the physical system is difficult due to complex nonlinearities, the computational complexity and time required for detailed simulations, and the limited availability of experimental data for accuracy validation across various scenarios. As a result, digital models are developed as simplified versions of the physical systems, which can lead to reduced accuracy. However, even with these limitations, digital models still offer significant benefits, including cost reduction, faster development cycles, and the ability to test scenarios that would be impractical or impossible with physical prototypes. Because of these advantages, researchers have increasingly focused on digital twins as tools to analyze and predict the motion of complex systems [20]. However, despite these advantages, low accuracy of digital twin models can lead to incorrect diagnostics (such as determining maintenance periods or parts replacement), which can lead to increased maintenance costs or vehicle safety issues.

To present a methodology for overcoming the limitations described above, this paper proposes a comprehensive methodology for developing, validating, and implementing a digital model of an RWA system. (1) The first is the development and validation of a physical model. A physics-based model of the RWA system was established by integrating analytical dynamics equations and comprehensive friction components. Analytical dynamics equations were analyzed in the frequency domain, and a model-based position control algorithm was designed based on the developed physical model. The designed control algorithm was experimentally validated in a test environment constructed using MATLAB/Simulink 2022b and Speedgoat Baseline devices. (2) The second is the development
and verification of the digital model. A digital model was developed by integrating the developed physical model and control algorithm into the software in the loop simulation (SILs) environment. To verify the accuracy and efficiency of the digital model, the results obtained from the real equipment and the digital model were compared in real time to confirm similarity and inconsistency.

The rest of this paper is organized as follows. In Section 2, we discuss the model for RWA system and friction model, and the digital model development validates the model through experiments. Section 3 presents a position control scheme based on disturbance observer (DOB). Simulation and experimental results are addressed in Section 4 to demonstrate the effectiveness of the proposed control scheme. Section 5 briefly concludes this paper.

2. System Modeling

The RWA translates the driver’s intentions into precise wheel movements. As shown in Figure 2, the structure utilizes a belt drive mechanism, a system that uses a belt to transmit rotational energy, for efficiency and miniaturization. The motor’s rotational energy is transferred through a reduction gear system, which consists of a belt pulley and a ball screw. The belt pulley applies a gear ratio defined as 1:2.28 to power the driven pulley. This amplification of torque allows for reduced motor speed while maintaining the necessary force. The driven pulley rotates the ball screw with its 7 mm threaded pitch. This interaction between the belt pulley and ball screw translates rotational motion into linear displacement, moving the rack gear left and right. The stroke of the rack gear is 160 mm and provides sufficient range for steering movement.

![Figure 2. Configuration of road wheel actuator system.](image)

Detailed specifications for additional components like the torsion bar and rack bush were not incorporated into the physics model. There are several reasons why these components were not incorporated into the physics model, such as the following: these components might affect the RWA motion, such as reducing vibrations; making their exclusion acceptable for initial model development or including them would increase the model’s complexity and computational demands, potentially hindering the initial control design process. However, it is important to note that, in the future, including these elements could help us better understand how the RWA behaves and potentially improve its control. The process of testing the model with experimental data will make the model more accurate and useful for developing the SbW system.

2.1. Dynamic Equation of Road Wheel Actuator System

This section presents the dynamic equations of the RWA system. The RWA system is considered as the composition of the motor, reduction gear, and the load-after load pulley in order to simplify, and the dynamic equations are derived in consideration of the belt drive mechanism, as shown in Figure 3.
Figure 3. Description of belt–pulley system.

Modeling of the RWA system has been studied in some papers [21,22]. According to these papers, the dynamic equation of the RWA system is derived as follows:

\[
\begin{align*}
(J_m + J_{pm}) \ddot{\theta}_{pm}(t) + (J_m + J_{pm}) \dot{\theta}_{pm}(t) + K_b r_{pm} \left\{ r_{pm} \dot{\theta}_{pm}(t) - r_{pl} \dot{\theta}_{pl}(t) \right\} + C_b r_{pm} \left\{ r_{pm} \ddot{\theta}_{pm}(t) - r_{pl} \ddot{\theta}_{pl}(t) \right\} &= \tau_m(t) - \tau_{fm}(\theta, \dot{\theta}) \\
(J_l + J_{bm}) \ddot{\theta}_{pl}(t) + (J_l + J_{bm}) \dot{\theta}_{pl}(t) + K_b r_{pl} \left\{ r_{pl} \dot{\theta}_{pl}(t) - r_{pm} \dot{\theta}_{pm}(t) \right\} + C_b r_{pl} \left\{ r_{pl} \ddot{\theta}_{pl}(t) - r_{pm} \ddot{\theta}_{pm}(t) \right\} &= -\tau_{fb}(\theta, \dot{\theta}) - \tau_{fl}(\theta, \dot{\theta})
\end{align*}
\]

where \(J_m\) and \(B_m\) are the inertia and damping of the motor, \(J_{pm}\) and \(B_{pm}\) are the inertia and damping of the motor pulley, \(J_{pl}\) and \(B_{pl}\) are the inertia and damping of the load pulley, \(J_l\) and \(B_l\) are the inertia and damping of the part after the belt, \(\dot{\theta}_{pm}\), \(\dot{\theta}_{pl}\), and \(\ddot{\theta}_{pm}\) are angular acceleration, angular velocity, and angular position of the motor pulley, \(\dot{\theta}_{pl}\), \(\ddot{\theta}_{pl}\), and \(\dot{\theta}_{pm}\) are angular acceleration, angular velocity, and angular position of the load pulley, \(r_{pm}\) and \(r_{pl}\) are the radius of the motor pulley and load pulley, \(\tau_m\) is the driving torque of the motor, \(\tau_{fm}\) is the frictional torque affecting the motor pulley, \(\tau_{fb}\) is the frictional torque of ball screw, \(\tau_{fl}\) is the frictional torque affecting the load pulley, \(C_b\) is the viscous friction coefficient of the belt, and \(K_b\) is the modulus of elasticity of the belt.

\[
J_e \ddot{\theta}_{pm}(t) + B_e \dot{\theta}_{pm}(t) = \tau_m(t) - \tau_f(\theta, \dot{\theta})
\]

\[
J_e = J_m + J_{pm} + \frac{r_{pm}^2}{r_{pl}} (J_l + J_{bm})
\]

\[
B_e = B_m + B_{pm} + \frac{r_{pm}^2}{r_{pl}} (B_l + B_{bm})
\]

\[
\tau_f = \tau_{fm} + \tau_{fb} + \tau_{fl}
\]

The sum of Equations (1) and (2) can be expressed as Equation (3). Assuming a higher belt stiffness during the derivation of the equation, it can be considered rigid, and the controller design within the target frequency band can ignore the elastic modulus of the belt. In this case, the inertia and damping coefficients of the motor, motor pulley, and load pulley after the belt are expressed as equilibrium inertia \(J_e\) and damping coefficients \(B_e\), as shown in Equations (4) and (5), reflecting the gear ratio. Also, \(\tau_f\) is the sum of the friction coefficients of the motor pulley, load pulley, and ball screw, which can be grouped into disturbance and model uncertainty, as shown in Equation (6). The transfer function with inertia and damping elements of each component reflecting the gear ratio can be obtained.
by the relationship between the torque input and the angular velocity of the motor pulley. The transfer function is used for the model-based controller and derived as follows:

$$\frac{\dot{\theta}_{pm}}{\tau_m} = \frac{1}{Jes + B_e}$$

(7)

2.2. Friction Model

Equation (3) is a dynamic equation of an RWA system with friction forces that interfere with motion. This friction force is a characteristically nonlinear factor, meaning that its influence on the system is not proportional to the applied torque [23]. Friction can be broadly categorized into three types: static friction, coulomb friction, and viscous friction.

- Static friction: The input force gradually increased from the initial stationary state of the system until the movement began. The force at this point was identified as static friction.
- Coulomb friction: While the system was in motion, there was a resistive force that acted against the object’s movement and dissipated its energy. The velocity steadily diminished until it reached a state of rest, and the force acting at this moment was determined to be coulomb friction.
- Viscous friction: After the movement of the system begins, the coefficient of proportionality between the friction force and the relative velocity is called the discrete friction.

The system requires torque input to overcome and operate these friction elements. This paper mainly focuses on the combined effects of static and coulomb friction. The equation for the friction model is expressed as follows:

If $$|\dot{x}_m| \geq v_{th}$$, then,

$$F_f = \left(F_c + (F_s - F_c) \cdot \exp\left(-c_v |\dot{x}_m|\right)\right) \cdot \text{sgn}(\dot{x}_m)$$  \hspace{1cm} (8)

If $$|\dot{x}_m| < v_{th}$$, then,

$$F_f = \left(F_c + (F_s - F_c) \cdot \exp\left(-c_v |\dot{x}_m|\right)\right) \cdot \text{sgn}(\dot{x}_m)$$  \hspace{1cm} (9)

where $$F_f$$ is the friction force, $$F_c$$ and $$F_s$$ are the coulomb and static friction force, $$c_v$$ is the velocity coefficient, $$v_{th}$$ is the velocity threshold, and $$\dot{x}_m$$ is the speed of the rack bar, which is a linear conversion of motor movement using reduction gear and ball screw.

Equations (8) and (9) in the present study utilize a friction model incorporating a velocity coefficient and a velocity threshold. The velocity coefficient aids in approximating the transition between static and coulomb friction regimes. The velocity threshold defines a linear gradient from zero force to the static friction value. Selecting overly small threshold values can lead to undesirable chattering behaviors near zero velocity. To determine the appropriate parameter values for Equations (8) and (9), a series of experiments were conducted under varied operating conditions. An input force was gradually increased from a static state, and the force at the initiation of movement was recorded. Subsequently, the input force was progressively decreased from a state of motion, and the force at the cessation of movement was measured. Following the convention of coulomb’s friction model, the force determined during the initiation of movement was identified as the maximum static friction, while the force measured at the cessation of movement was considered the dynamic friction coefficient. These experiments thus facilitated the identification of the system’s frictional parameters. Experiments have been conducted to identify the friction model between the motor and the RWA system in order to develop the digital model. The unit conversion constant from voltage to torque is 0.3847 N·m/volt. The parameter values of motor and RWA system in Equations (8) and (9) are as follows:

- Motor positive direction: $$\tau_s = 0.0279$$ Nm, $$\tau_c = 0.0255$$ Nm.
- Motor negative direction: $$\tau_s = -0.0264$$ Nm, $$\tau_c = -0.0255$$ Nm.
- RWA system positive direction: $$F_s = 285.37$$ N, $$F_c = 186.97$$ N.
- RWA system negative direction: $$F_s = -322.76$$ N, $$F_c = -236.17$$ N.
Figure 4 depicts the resulting friction model. Based on this model, a friction compensator was designed as a function of the steering wheel motor’s velocity.

![Friction model of the motor and RWA system](image)

**Figure 4.** (a) Friction model of the motor. (b) Friction model of the RWA system.

2.3. Model Identification

In this section, an experimental result and analysis for model identification of an SbW system are presented to identify the dynamic model. The identification process verifies the compensator’s efficiency using the friction model designed in the previous section. System identification to estimate the model represented by Equation (7) was performed, using a random phase multisine signal with a frequency range of 0.1 Hz to 50 Hz as a torque input, and we measured the angular velocity of the motor at the same time. The equations of a random phase multisine signal are as follows:

$$u(t) = \sum_{k=1}^{\infty} A_r \cos(2\pi f_0 k T_s + \varphi_r)$$  

(10)

where $F$ is the number of frequencies, $A_r$ is amplitude, $f_0$ is the period frequency of the multisine signal, $T_s$ is the sample period of the signal, and $\varphi_r$ is uniformly random distributed in $[0, 2\pi]$.

The frequency response result of the motor and RWA system is shown in Figures 5 and 6. A black solid line represents a response to low input voltage, and a blue dashed line represents a response to high input voltage, respectively. The experiment result for motor model identification tests before applying a friction compensator is shown in Figure 5a, and the performance of the designed compensator as the result of the model identification test after applying the friction compensator is shown in Figure 5b. The nonlinear factor is found in the low-frequency range. When comparing the two results, it can be seen that the test result after applying the friction compensator removed the effect of the friction. The nominal model was designed as a first-order system using Equation (7). In Figure 5b, the parameter values of the motor are obtained as $J_m = 0.0003 \text{ kg} \cdot \text{m}^2$ and $B_m = 0.0008 \text{ N} \cdot \text{m} \cdot \text{s} / \text{rad}$ using a red dashed line, respectively.

Experiments were conducted to confirm the inertia and damping coefficients of the system according to the belt tension of the RWA. The measurement experiment of the friction model was conducted on the belt tension, which is normal, and the parameter values were checked as follows.

In the previously performed motor frequency response results, a black solid line represents a response to low input voltage, and a blue dashed line represents a response to high input voltage, respectively. Figure 6a shows the result of the model identification test after applying the friction compensator when the belt tension of the RWA system is normal. Figure 6b shows the result of the model identification test after applying the friction compensator when the belt tension is low. The parameter values according to the high tension are obtained as $J_e = 0.0006 \text{ kg} \cdot \text{m}^2$ and $B_e = 0.0025 \text{ N} \cdot \text{m} \cdot \text{s} / \text{rad}$, respectively.
and the parameter values according to the low tension are obtained as $J_e = 0.0006 \text{ kg} \cdot \text{m}^2$ and $B_e = 0.0017 \text{ N} \cdot \text{m} \cdot \text{s} / \text{rad}$, respectively. These two frequency response results are used as parameters to determine the range of changes in the system model with belt tension.

![Figure 5](image1.png)

*Figure 5.* Frequency response result of motor (a) without a friction compensator and (b) with a friction compensator.

![Figure 6](image2.png)

*Figure 6.* Frequency response result of RWA system with a friction compensator: (a) high-tension case; (b) low-tension case.

2.4. Digital Model Development

In this section, SILs is established by modeling and simulation of the RWA system using MATLAB/Simulink. The rotational and translational domains are represented in light green and dark green in this framework, respectively.
The connection between components allows force to be transferred in both directions. As shown in Figure 7, the developed model applied motor inertia and damper, gear ratio of the belt–pulley system, pitch of the ball screw, mass of the rack bar, and friction model of the RWA system.

The verification of the developed digital model is divided into two parts: motor and RWA system. The verification process is performed by comparing the results of the frequency response of the digital model with the actual equipment. In Figure 8, the black line is the simulation result of the digital model and the red line is the nominal model obtained from the physical model. The comparison results of the two models in the frequency domain appear similar, and can be used as a basis for the similarity of the digital models.

Figure 7. Simscape model of RWA system.

Figure 8. Frequency response result of digital model with a friction compensator: (a) motor; (b) RWA system.
After the verification process, the accuracy was evaluated through a comparative test with the actual system under open-loop control. After applying the same sine wave input signal at 1 Hz, as shown in Figure 9, in both the experimental results of the physical system and the digital model, their responses were analyzed in terms of position and speed. These comparative analysis results in the time domain show that the outputs of the two models show a high concordance rate.

![Figure 9](image_url)

**Figure 9.** Verification of the developed Simscape model using sine input.

3. Control System Design

Figure 10 depicts the overall controller block diagram employed in this system. The transfer function of the second-order system with the input torque–motor position relationship was used as the plant. To enhance responsiveness and tracking performance, the controller architecture was designed as a feedforward, feedback, and DOB. The feedforward controller is designed to improve control response time. The equation for the PD controller is as follows:

\[
C(s) = K_p + \frac{K_d}{s}
\]

The feedback controller is designed as a PD controller using the pole–zero cancellation technique. The position controller block diagram is as shown in Figure 10.

![Figure 10](image_url)

**Figure 10.** Position controller block diagram.
The feedback controller is designed as a PD controller using the pole–zero cancellation method, which places the pole and zero of the RWA system on the controller to offset the system dynamics and improve the response with minimal phase delay. The plant used the transfer functions of voltage input and position output, which are integral forms of Equation (7). The equation for the PD controller is as follows:

$$C_{FB}(s) = \frac{\omega_{fb}}{s + \omega_{fb}} \times \left( \frac{\omega_i}{s} \right) \left( I_s s^2 + B_s s \right)$$ (11)

The feedforward controller is designed to improve control response time. The equation for the controller is as follows:

$$C_{FF}(s) = \frac{\omega^2_{ff}}{s^2 + 2\zeta\omega_{ff}s + \omega^2_{ff}} \left( I_s s^2 + B_s s \right)$$ (12)

The disturbance observer is composed of an inverse nominal model and a second-order low-pass filter called a Q filter. The Q-filter is designed as follows:

$$Q(s) = \frac{\omega^2_q}{s^2 + 2\zeta\omega_q s + \omega^2_q}$$ (13)

4. Experiment Result and Discussion

In this section, the configuration of the RWA system test equipment is explained and presents the results of experiments carried out using the method proposed in this paper.

4.1. Experimental Setup

Elmo Gold Solo Twitter (G-SOLTWI25/100EE) is used as the motor driver, and the RWA system is part of the control system hardware interface, as depicted in Figure 11. The Speedgoat control board was used as the DAQ device, and real-time control with a sampling rate of 1 ms was carried out. The proposed control logic was set up in MATLAB/Simulink and Simscape Block.

Figure 11. Experimental setup.
4.2. Experimental Test

In order to evaluate the similarity between the physical model and the digital model, position control was performed using a designed controller. The position control experiment was performed using a trapezoidal command, and the signal was input as a steering angle with a maximum magnitude of 90 degrees and a slope of 300 degrees/s. In addition, a first-order low pass filter with 10 Hz was applied to the sharp point. A comparison of the two models’ responses to the position command, position errors, and the magnitude of the input voltage required for control is presented in Figure 12. It is confirmed that there is a similarity between the two models as the difference between the position error and the input voltage is small.

Figure 12. Verification of the development simulation model using trapezoidal command.

5. Conclusions

In this paper, a methodology to develop a digital model of the RWA system through an experiment was presented. A simplified model considering the dynamics of the RWA system was proposed, and a friction model between the motor and the RWA system was presented and a nominal model was obtained by using it. Using these, a digital model was developed and it was verified that the accuracy was high in the frequency and time domains compared with the output results of the actual equipment. Nevertheless, in order to reduce the influence due to the existing nonlinear disturbance and model uncertainty, a DOB-based position controller was proposed. Finally, the validity of the digital model was verified by comparing the position control results and the magnitude of the input voltage of the two models. In future works, we develop an integrated model to analyze the effectiveness of additional components that can affect model accuracy through simulations or experiments with actual equipment and utilize them in diagnosing failures in the system. Fault diagnosis based on the model and data will increase the stability in the driving environment of the vehicle by enabling the early detection of potential issues through real-time comparison of the digital model with actual sensor data. This approach involves identifying deviations from expected behavior, analyzing discrepancies with algorithms, and predicting fault progression to suggest corrective actions. Effective fault diagnosis enhances vehicle stability and safety by allowing timely interventions and preventive maintenance.
Author Contributions: Conceptualization, I.C. and K.N.; methodology, I.C.; software, I.C.; validation, I.C. and J.C.; formal analysis, I.C. and J.C.; investigation, J.C.; resources, I.C.; data curation, I.C.; writing—original draft preparation, I.C.; writing—review and editing, K.N.; visualization, I.C.; supervision, K.N.; project administration, I.C.; funding acquisition, K.N. All authors have read and agreed to the published version of the manuscript.

Funding: This research was supported by “Regional Innovation Strategy (RIS)” through the National Research Foundation of Korea (NRF) funded by the Ministry Education (MOE) (No. 2022RIS-006). This work was supported by the 2022 Yeungnam University Research Grant (No. 222A380089).

Institutional Review Board Statement: Not applicable.

Informed Consent Statement: Not applicable.

Data Availability Statement: Data are contained within the article.

Conflicts of Interest: The authors declare no conflicts of interest.

References

20. Chaiprabha, K.; Chancharoen, R. A Deep Trajectory Controller for a Mechanical Linear Stage Using Digital Twin Concept. Actuators 2023, 12, 91. [CrossRef]


Disclaimer/Publisher’s Note: The statements, opinions and data contained in all publications are solely those of the individual author(s) and contributor(s) and not of MDPI and/or the editor(s). MDPI and/or the editor(s) disclaim responsibility for any injury to people or property resulting from any ideas, methods, instructions or products referred to in the content.