

Article

Force Control of an Active Suspension Hydraulic Servo System Based on BSO-Optimized ESO-Based SMC

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Abstract

To mitigate the significant impact of system nonlinearities, time-varying parameters, and external load disturbances on the output force of hydraulic servo systems in active hydraulic suspensions for engineering vehicles, this study proposes a beetle swarm optimization (BSO)-optimized extended state observer (ESO)-based sliding mode control (SMC) strategy. A comprehensive mathematical model of the hydraulic servo system is established, and an ESO-based SMC controller is designed, taking into account the coupled effects of chamber pressure dynamics and external loads on the uncertain output force. The stability of the closed-loop system is rigorously analyzed and verified using Lyapunov stability theory. The effectiveness of the proposed control strategy is verified through both numerical simulations and experimental tests. For step inputs of 5000 N and 8000 N, overshoot is significantly reduced compared with the conventional proportional–integral–derivative control and the standard extended state observer-based sliding mode control, while the settling time is shortened by more than 65% in simulations and up to 75% in experiments. Under sinusoidal force excitations at frequencies of 0.5 Hz, 1 Hz, and 2 Hz, the maximum tracking error, mean error, and standard deviation of the tracking error are substantially reduced, with the maximum error reduction exceeding 90%. These results demonstrate that the proposed method achieves high-precision force tracking under external disturbances and pronounced system uncertainties, providing an effective solution for force control of hydraulic servo systems in active suspension applications for engineering vehicles.

Keywords: hydraulic servo system; extended state observer; force tracking control; sliding mode controller; beetle swarm optimization

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1. Introduction

With the increasing demand for the dynamic performance of engineering vehicles, active suspension systems based on hydraulic servo technology have attracted considerable attention due to their high output capability and fast response, offering significant advantages in improving vehicle stability, handling performance, and ride comfort. By actively regulating the suspension force in real time, such systems can effectively cope with complex operating conditions such as road irregularities and load variations, in which accurate force tracking control plays a critical role in ensuring suspension effectiveness and overall vehicle dynamic performance [1]. Compared with the relatively mature

position and velocity control problems in hydraulic servo systems, force control is more strongly influenced by valve-controlled flow nonlinearities, fluid compressibility, friction effects, and time-varying parameters. As a result, the system exhibits high sensitivity to internal and external disturbances, making high-precision and high-robustness force tracking control particularly challenging [2]. Under practical operating conditions, complex road excitations and load variations further intensify system uncertainties and dynamic complexity. Conventional model-based control approaches often struggle to effectively handle strong nonlinearities and unknown disturbances, leading to inherent trade-offs between stability and force tracking performance, which in turn restricts the further improvement of hydraulic servo active suspension systems [3]. Therefore, it is necessary to introduce advanced control strategies with strong disturbance rejection capability. By designing disturbance observers to estimate system uncertainties and external disturbances online and combining them with composite control methods for real-time compensation, the force tracking accuracy and robustness of hydraulic servo active suspension systems can be effectively enhanced [4,5].

In the force control of hydraulic servo systems for vehicle suspensions, load variations, friction nonlinearities, and coupled pressure dynamics introduce pronounced nonlinear behavior, causing the control objective to be directly affected by unmodeled dynamics and external disturbances, which are difficult to eliminate through static modeling. For engineering vehicle suspension systems, the main disturbance sources include random excitations induced by road surface irregularities, load fluctuations resulting from changes in operating conditions, and time-varying system parameters caused by temperature-dependent variations in hydraulic oil viscosity. The combined effects of these factors lead to significant uncertainty in the force control of hydraulic suspension systems and pose substantial challenges to control robustness and disturbance rejection performance. Therefore, introducing observers for online disturbance estimation becomes a key approach to achieving high-precision and stable force tracking. The ESO lumps unknown system dynamics and external disturbances into a total disturbance, enabling real-time estimation and compensation of uncertainties and thereby significantly improving system robustness and tracking performance. The basic idea is to introduce an extended state in which nonlinear terms, parameter uncertainties, and external disturbances that are difficult to model accurately in the original system are lumped into an additional state variable. By employing an observer to reconstruct this extended state online, the controller is able to compensate for disturbances without relying on an accurate system model. Compared with high-gain proportional-integral observer (PIO) and other conventional state observers, ESO exhibits superior estimation accuracy, real-time performance, and robustness under fast-varying disturbances and strong nonlinear conditions [6–9]. Existing studies have demonstrated that ESO can serve as an effective disturbance observation tool, providing a solid technical foundation for force control of complex hydraulic systems. For example, Yao et al. [7] proposed an output-feedback nonlinear control strategy incorporating ESO to achieve high-precision trajectory tracking in hydraulic systems. Wang et al. [8] developed an ESO-based adaptive control method, in which dynamic uncertainties of the hydraulic system and parameter mismatches in the mechanical subsystem are effectively estimated and compensated, resulting in improved tracking performance. In addition, Gu et al. [9] combined ESO with model predictive control (MPC), enhancing system robustness by continuously updating the output prediction model based on disturbance estimation.

Traditional control methods such as proportional-integral-derivative (PID) control are widely used in hydraulic servo systems due to their simple structure and ease of implementation. However, their robustness against system nonlinearities, model uncertainties, and external disturbances is limited, making it difficult to maintain high-precision tracking performance under strongly nonlinear and highly disturbed operating

conditions. Fuzzy control can improve controller adaptability to a certain extent, but its rule design is largely experience-dependent and lacks a systematic framework for handling complex dynamic characteristics. MPC offers advantages in explicitly considering future system behavior; nevertheless, it relies heavily on model accuracy and online optimization capability, and its real-time performance may be insufficient for fast hydraulic servo applications [10–13]. In contrast, SMC, as a representative variable structure control strategy, exhibits inherent robustness to parameter uncertainties and external disturbances. As a result, SMC has been extensively applied to nonlinear control of electro-hydraulic servo systems and has demonstrated superior performance compared with conventional PID and linear control methods in numerous studies [14–17]. For example, Guo et al. [14] proposed an adaptive neural network-based SMC strategy for trajectory tracking of heavy-duty hydraulic manipulators, demonstrating superior performance compared with PID control. Yang et al. [15] developed an SMC scheme incorporating an adaptive high-order neural network for hydraulic rotary drive joints. By enabling online adjustment of control parameters, high-precision position tracking of the hydraulic servo system was achieved, verifying the effectiveness and superiority of SMC in strongly nonlinear hydraulic systems. Nevertheless, the performance of SMC is still highly dependent on the selection of switching gains, and the upper bounds of disturbances are generally difficult to obtain accurately, which may lead to control chattering or performance compromise. Although combining SMC with conventional observers can improve disturbance rejection to some extent, the estimation accuracy and real-time performance of traditional observers remain insufficient under fast-varying disturbances and strong nonlinear conditions. For example, Bakhshande et al. [18] combined a high-gain PIO with SMC to propose a composite control strategy for a nonlinear hydraulic differential cylinder system. This approach, through state estimation and unknown input compensation, effectively improved the system's disturbance rejection and tracking performance, showing superior robustness compared with standard SMC and proportional control methods. However, PIO-based observers typically rely on high observation gains, which can lead to noise amplification and limited estimation accuracy under strong nonlinearity, fast-varying disturbances, and significant parameter uncertainties. Du et al. [19] proposed an extended state observer-based sliding mode controller (ESO-SMC) method and demonstrated that incorporating the extended state observer enhanced the stability and disturbance rejection of a hydraulic drive system for a marine stabilization platform. Therefore, integrating the high-precision disturbance estimation capability of the ESO with SMC not only exploits the inherent robustness of sliding mode control but also compensates for the limitations of conventional observers, making it an effective approach for achieving high-performance control of complex hydraulic systems.

The performance of both the ESO and SMC is highly sensitive to parameter selection, and the optimal parameters vary under different operating conditions. Using fixed parameters may result in compromised performance or reduced robustness. Therefore, the introduction of intelligent optimization algorithms for online adaptive parameter tuning is of significant importance. Commonly used optimization algorithms include particle swarm optimization (PSO), genetic algorithm (GA), and the beetle antennae search (BAS) algorithm. PSO achieves global search through population cooperation but tends to be trapped in local optima and exhibits limited convergence speed in high-dimensional or non-convex problems [20]. GA offers strong global search capability but generally converges slowly, making it less suitable for real-time control [21]. BAS provides fast convergence but has limited global search ability, which may miss the global optimum [22]. To overcome the limitations of individual algorithms in balancing global search and convergence efficiency, the BSO algorithm, which combines BAS and PSO, has been proposed. Zhang et al. [23] introduced BSO by integrating individual beetle antennae search with

swarm cooperation, achieving a good balance between search efficiency and optimization accuracy, demonstrating strong engineering applicability. In a separate study, Zhang et al. [24] systematically compared BSO with PSO, GA, and BAS, showing that BSO achieves a more balanced performance in terms of convergence speed and optimization accuracy.

On this basis, considering the characteristics of strong nonlinearity, significant parameter variations under changing operating conditions, and complex external disturbances in engineering vehicle active suspension hydraulic servo systems, this paper establishes a mathematical model of the system and designs a BSO-ESO-SMC-based composite control strategy. For hydraulic servo force control, model uncertainties and load disturbances directly affect output force tracking accuracy, and conventional control methods relying on precise system models struggle to maintain stable performance under complex operating conditions. To address this issue, an ESO is introduced to lump the unknown system dynamics and external disturbances into a generalized disturbance and estimate it online, thereby reducing the dependence on an accurate hydraulic model. This feature is particularly necessary for engineering vehicle suspension systems with pronounced parameter variations. Building upon the disturbance estimation provided by the ESO, SMC is employed to enhance the robustness of the closed-loop system against disturbances and parameter perturbations, ensuring stable operation and rapid force-tracking performance. Despite its robustness advantages, the performance of sliding mode control is highly dependent on appropriate parameter selection, and manual tuning makes it difficult to achieve a satisfactory trade-off between tracking accuracy and chattering suppression. To overcome this limitation, the BSO algorithm is introduced to perform global optimization of the control parameters, thereby improving the overall controller performance under multi-objective constraints. Consequently, the proposed BSO-ESO-SMC framework is capable of effectively addressing uncertainties and disturbances in the hydraulic servo force control from both theoretical and practical perspectives. Finally, the effectiveness of the proposed method is validated through combined simulation and experimental studies, and its performance is compared with several commonly used control strategies.

The remainder of this chapter is organized as follows. Section 2 presents the modeling of the hydraulic servo system. Section 3 details the design of the controller. Section 4 evaluates the system performance through computer simulations. Section 5 validates the system performance via experiments. Finally, Section 6 summarizes the main conclusions.

2. Hydraulic Servo System Model

The force control of the nonlinear hydraulic servo system is modeled, with particular attention to the process by which the servo valve regulates hydraulic oil flow. The flow of hydraulic oil into the cylinder chambers is adjusted by the movement of the valve spool, generating a pressure difference across the piston and producing an external active force. The control input is ultimately represented in terms of the valve spool displacement [25,26]. The hydraulic servo system considered in this study consists of a zero-lap four-edge sliding valve and a symmetric hydraulic cylinder, and its schematic configuration is shown in Figure 1.

In the system schematic shown in Figure 1, p_s and p_r represent the supply and return pressures, respectively; x_v denotes the valve spool displacement; q_1 and q_2 correspond to the flow rates into or out of the rodless and rod chambers of the hydraulic cylinder, respectively; p_1 and p_2 denote the pressures in the rodless and rod chambers; A_1 and A_2 represent the areas of the rodless and rod chambers, respectively; and m is the spring-supported load of the hydraulic cylinder in the active suspension unit. For modeling the valve-controlled asymmetric hydraulic cylinder, separate models are established for piston extension and retraction. Based on the assumptions of incompressible oil, constant supply

pressure, zero return pressure, and negligible pipeline dynamics, the asymmetric flow gain characteristics due to the area difference between the two chambers are analyzed. The valve spool displacement is defined as positive when the piston moves downward.

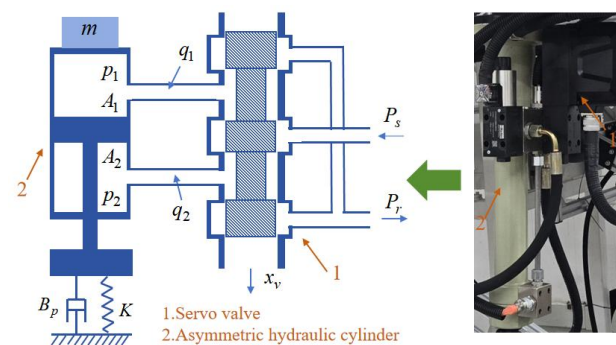


Figure 1. Schematic diagram of the hydraulic servo system in active suspension.

According to Newton's second law, the force balance equation of the actuator output force can be written as:

$$F = p_1 A_1 - p_2 A_2 = m \ddot{x}_p + B_p \dot{x}_p + K x_p + mg + f_L \quad (1)$$

where x_p denotes the displacement of the piston rod relative to the cylinder barrel; B_p is the viscous damping coefficient of the hydraulic fluid; K represents the load stiffness; g is the gravitational acceleration; and f_L represents the mechanical disturbances caused by unmodeled friction forces and external disturbances.

According to the natural frequency of the controlled power mechanism, the transfer function between the spool displacement of the electro-hydraulic servo valve and the input current can be approximated by a second-order oscillatory element, an inertial element, or a proportional element. In this study, the selected electro-hydraulic servo valve has a natural frequency much higher than that of the hydraulic actuator. Therefore, the transfer function between the spool displacement and the input voltage can be approximated as a proportional element, expressed as:

$$x_v = k_{sv} u \quad (2)$$

where k_{sv} denotes the servo valve gain.

When a positive input current $u \geq 0$ is applied to the servo valve, the valve spool moves in the positive direction with $x_v \geq 0$, causing the piston rod to extend. The rodless chamber is connected to the supply line through the servo valve orifice, and the pressure difference across the orifice is $p_s - p_1$. The rod chamber is connected to the return line through the servo valve orifice. Since the return pressure is zero, the pressure difference across the orifice is p_2 . According to the thin-walled orifice flow principle, the flow rate equations of the two chambers of the asymmetric hydraulic cylinder can be expressed as follows:

$$q_1 = C_d \omega x_v \sqrt{\frac{2}{\rho} (p_s - p_1)} \quad (3)$$

$$q_2 = C_d \omega x_v \sqrt{\frac{2}{\rho} p_2} \quad (4)$$

where C_d is the flow coefficient of the servo valve spool orifice; ω is the gradient of the servo valve orifice area; and ρ is the fluid density.

Let $\gamma = C_d \omega \sqrt{\frac{2}{\rho}}$ Equations (3) and (4) can be expressed as follows:

$$q_1 = \gamma x_v \sqrt{(p_s - p_1)} \quad (5)$$

$$q_2 = \gamma x_v \sqrt{p_2} \quad (6)$$

Similarly, when a reverse current is applied to the servo valve input, $u < 0$, the spool moves in the reverse direction, and $x_v < 0$. The rodless chamber is connected to the return line through the servo valve orifice, and the pressure drop across the orifice is p_1 . The rod chamber is connected to the supply line through another servo valve orifice, and the pressure drop across this orifice is $p_s - p_2$. According to the orifice flow equation based on the thin-walled orifice theory, the flow rates into the two chambers of the asymmetric hydraulic cylinder can be expressed as:

$$q_1 = C_d \omega x_v \sqrt{\frac{2}{\rho} p_1} \quad (7)$$

$$q_2 = C_d \omega x_v \sqrt{\frac{2}{\rho} (p_s - p_2)} \quad (8)$$

Substituting γ into Equations (7) and (8), they can be expressed as:

$$q_1 = \gamma x_v \sqrt{p_1} \quad (9)$$

$$q_2 = \gamma x_v \sqrt{(p_s - p_2)} \quad (10)$$

By combining Equations (2), (5), (6), (9) and (10), the flow equation of the servo valve can be expressed as:

$$q_1 = \gamma k_{sv} [s(u) \sqrt{p_s - p_1} + s(-u) \sqrt{p_1}] u \quad (11)$$

$$q_2 = \gamma k_{sv} [s(u) \sqrt{p_2} + s(-u) \sqrt{p_s - p_2}] u \quad (12)$$

where the function $s(u)$ is:

$$s(u) = \begin{cases} 1, & u \geq 0 \\ 0, & u < 0 \end{cases} \quad (13)$$

Considering the effects of actuator leakage and fluid compressibility, the inflow rate and the volume of the hydraulic cylinder chamber can be expressed as:

$$q_1 = A_1 \dot{x}_p + C_{ip} (p_1 - p_2) + \frac{V_1}{\beta_e} \dot{p}_1 \quad (14)$$

$$q_2 = A_2 \dot{x}_p + C_{ip} (p_1 - p_2) - C_{ep} p_2 - \frac{V_2}{\beta_e} \dot{p}_2 \quad (15)$$

where $A_1 \dot{x}_p$ and $A_2 \dot{x}_p$ represent the flow rate changes in the two chambers due to the piston displacement; C_{ip} is the leakage coefficient of the hydraulic cylinder; $C_{ip} (p_1 - p_2)$ is the leakage flow rate generated by the pressure difference between the two chambers across the piston gap; C_{ep} is the external leakage coefficient of the hydraulic cylinder; and $C_{ep} p_2$ is the leakage flow rate generated by the pressure difference

between the rod chamber and the external environment through the gap between the cylinder head and the piston rod. Since the rodless chamber does not experience such leakage, this term does not appear in the flow continuity equation for the rodless chamber. β_e is the fluid elasticity modulus; $\frac{V_1}{\beta_e} \dot{p}_1$ and $\frac{V_2}{\beta_e} \dot{p}_2$ represent the volumes of the rodless and rod chambers, respectively, which can be expressed by the following equations:

$$V_1 = V_{01} + A_1 x_p \quad (16)$$

$$V_2 = V_{02} - A_2 x_p \quad (17)$$

where V_{01} and V_{02} represent the volumes of the rodless and rod chambers, respectively, when the piston is at the initial position.

By selecting the piston rod displacement x_p , velocity $\dot{x}_p$, the pressure in the rodless chamber p_1 , and the pressure in the rod chamber p_2 as the state variables of the system, the state variables can be expressed as $x = [x_1 \ x_2 \ x_3 \ x_4]^T = [x_p \ \dot{x}_p \ p_1 \ p_2]^T$, and the output variables as $y = F$. The state-space equations of the nonlinear model can then be expressed as:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -\frac{K}{m} x_1 - \frac{B_p}{m} x_2 + \frac{A_1}{m} x_3 - \frac{A_2}{m} x_4 - g - d \\ \dot{x}_3 = \frac{\beta_e}{V_1} [-A_1 x_2 - C_{ip} (x_3 - x_4) + g_1(u)u] \\ \dot{x}_4 = \frac{\beta_e}{V_2} [A_2 x_2 + C_{ip} (x_3 - x_4) - C_{ep} x_4 - g_2(u)u] \\ y = A_1 x_3 - A_2 x_4 \end{cases} \quad (18)$$

where $d = \frac{f_L}{m}$ and u are the system inputs, and y is the system output, with $g_1(u) = \gamma k_{sv} [s(u)\sqrt{p_s - p_1} + s(-u)\sqrt{p_1}]$ and $g_2(u) = \gamma k_{sv} [s(u)\sqrt{p_2} + s(-u)\sqrt{p_s - p_2}]$.

3. Design of the Controller

To achieve accurate force tracking control for the active suspension hydraulic servo system, it is necessary to effectively observe internal disturbances and modeling uncertainties and to enhance the tracking accuracy and dynamic performance through advanced control strategies. First, a nonlinear ESO is employed to estimate modeling uncertainties and external disturbances in real time, providing a basis for disturbance compensation. Second, SMC is introduced to suppress the influence of disturbances by exploiting its inherent robustness, thereby ensuring basic tracking performance. On this basis, Lyapunov stability theory is applied to derive sufficient conditions for controller stability, guaranteeing the asymptotic stability of the closed-loop system. Furthermore, a BSO algorithm is utilized to globally optimize key parameters of the ESO and SMC, such as the observer bandwidth and switching gain, enabling adaptive adjustment to variations in system dynamics. This parameter optimization improves tracking accuracy and response speed while maintaining system stability, ensuring effective control performance under complex operating conditions. Accordingly, a control scheme integrating ESO, SMC, and BSO-based parameter tuning is proposed in this study. The detailed structure of the proposed controller is shown in Figure 2.

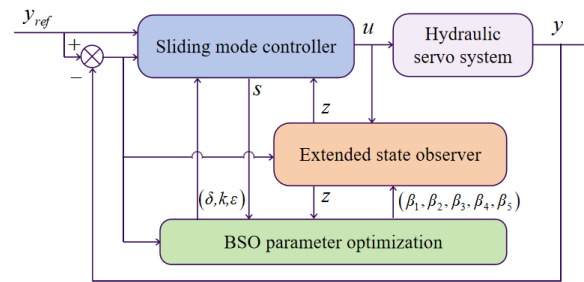


Figure 2. Block diagram of the controller structure.

3.1. Design of the Nonlinear ESO

In the practical operation of electro-hydraulic force servo systems, the output force F is affected by external disturbances, which can degrade the system's dynamic accuracy and control performance [27–29]. To address this issue, an ESO is employed to estimate the external disturbance d , which is defined as $x_5 = d$ and assumed to be a bounded function. By incorporating this external disturbance $\dot{x}_5$ into the state-space equations of the electro-hydraulic force servo system, a state-space model that accounts for the effect of disturbances is obtained.

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -\frac{K}{m}x_1 - \frac{B_p}{m}x_2 + \frac{A_1}{m}x_3 - \frac{A_2}{m}x_4 - g - d \\ \dot{x}_3 = \frac{\beta_e}{V_1}[-A_1x_2 - C_{ip}(x_3 - x_4) + g_1(u)u] \\ \dot{x}_4 = \frac{\beta_e}{V_2}[A_2x_2 + C_{ip}(x_3 - x_4) - C_{ep}x_4 - g_2(u)u] \\ \dot{x}_5 = \dot{d} \\ y = A_1x_3 - A_2x_4 \end{cases} \quad (19)$$

To improve the estimation performance of the ESO, the nonlinear function $\text{fal}(e, \alpha, \delta)$ is introduced in the observer update. For small errors, the function behaves approximately linearly to suppress measurement noise, while for larger errors, the nonlinear feedback is stronger to accelerate observer convergence. Compared with using the error directly, this nonlinear design better balances noise attenuation and fast convergence, enhancing the overall performance and robustness of the ESO.

The function $\text{fal}(e, \alpha, \delta)$ is defined as

$$\text{fal}(e, \alpha, \delta) = \begin{cases} \frac{e}{\delta^{1-\alpha}}, & |e| \leq \delta \\ |e|^\alpha \text{sign}(e), & |e| > \delta \end{cases} \quad (20)$$

where $\alpha \in (0, 1)$ denotes the nonlinearity degree of the error feedback, and $\delta > 0$ specifies the width of the linear region around the origin.

Based on the state-space equations in Equation (19), the state-space mathematical model of the Extended State Observer can be obtained as:

$$\begin{cases} e_y = y_{ref} - y \\ \dot{z}_1 = z_2 + \beta_1 fal(e_y, \alpha_1, \delta) \\ \dot{z}_2 = -\frac{K}{m} z_1 - \frac{B_p}{m} z_2 + \frac{A_1}{m} z_3 - \frac{A_2}{m} z_4 - g - z_5 + \beta_2 fal(e_y, \alpha_2, \delta) \\ \dot{z}_3 = \frac{\beta_e}{V_1} [-A_1 z_2 - C_{ip} (z_3 - z_4) + g_1(u)u] + \beta_3 fal(e_y, \alpha_3, \delta) \\ \dot{z}_4 = \frac{\beta_e}{V_2} [A_2 z_2 + C_{ip} (z_3 - z_4) - C_{ep} z_4 - g_2(u)u] + \beta_4 fal(e_y, \alpha_4, \delta) \\ \dot{z}_5 = \beta_5 fal(e_y, \alpha_5, \delta) \end{cases} \quad (21)$$

where $z = [z_1 \ z_2 \ z_3 \ z_4 \ z_5]$ denotes the estimated value of the state to be observed $x = [x_1 \ x_2 \ x_3 \ x_4 \ x_5]$, and $\beta_1, \beta_2, \beta_3, \beta_4$, and β_5 are the observation gains of the ESO. The parameters of the nonlinear function $fal(\cdot)$ are chosen as $\alpha_1 = \alpha_2 = \alpha_3 = \alpha_4 = \alpha_5 = 0.5$ and $\delta = 0.1$.

From (19) and (21), the ESO error can be obtained. Let $e_i = z_i - x_i$ (where $i=1, 2, 3, 4$) be the error in the state variables. The error of the state observer can be represented as:

$$\begin{cases} \dot{e}_1 = e_2 - \beta_1 fal(e_y, \alpha_1, \delta) \\ \dot{e}_2 = -\frac{K}{m} e_1 - \frac{B_p}{m} e_2 + \frac{A_1}{m} e_3 - \frac{A_2}{m} e_4 - e_5 + \beta_2 fal(e_y, \alpha_2, \delta) \\ \dot{e}_3 = \frac{\beta_e}{V_1} [-A_1 e_2 - C_{ip} (e_3 - e_4)] + \beta_3 fal(e_y, \alpha_3, \delta) \\ \dot{e}_4 = \frac{\beta_e}{V_2} [A_2 e_2 + C_{ip} (e_3 - e_4) - C_{ep} e_4] + \beta_4 fal(e_y, \alpha_4, \delta) \\ \dot{e}_5 = \beta_5 fal(e_y, \alpha_5, \delta) - \dot{d} \end{cases} \quad (22)$$

To analyze the stability of the error system described by Equation (22), we construct a positive definite Lyapunov function:

$$V(e) = \frac{1}{2} (e_1^2 + e_2^2 + e_3^2 + e_4^2 + e_5^2) \quad (23)$$

Taking the derivative of $V(e)$, we obtain:

$$\dot{V}(e) = e_1 \dot{e}_1 + e_2 \dot{e}_2 + e_3 \dot{e}_3 + e_4 \dot{e}_4 + e_5 \dot{e}_5 \quad (24)$$

Substitute the expressions for each $\dot{e}_i$ into the ESO error equation:

$$\begin{aligned} \dot{V}(e) &= e_1 [e_2 - \beta_1 fal(e_y, \alpha_1, \delta)] \\ &+ e_2 \left[\frac{K}{m} e_1 - \frac{B_p}{m} e_2 + \frac{A_1}{m} e_3 - \frac{A_2}{m} e_4 - e_5 + \beta_2 fal(e_y, \alpha_2, \delta) \right] \\ &+ e_3 \left[\frac{\beta_e}{V_1} [-A_1 e_2 - C_{ip} (e_3 - e_4)] + \beta_3 fal(e_y, \alpha_3, \delta) \right] \\ &+ e_4 \left[\frac{\beta_e}{V_2} [A_2 e_2 + C_{ip} (e_3 - e_4) - C_{ep} e_4] + \beta_4 fal(e_y, \alpha_4, \delta) \right] \\ &+ e_5 [\beta_5 fal(e_y, \alpha_5, \delta) - \dot{d}] \end{aligned} \quad (25)$$

Perform term classification analysis on $\dot{V}(e)$: The linear terms are formed by the product of the system's physical parameters and the error, and the negative-definite feedback inherent in the observer structure, combined with Cauchy's inequality and the

positive definiteness of the parameters $(K, m, B_p, C_{ip}, \dots)$, which are all positive real numbers. This allows a more precise bound on the linear term, avoiding the overly conservative positive quadratic estimate: $L(e) \leq -C_1 \|e\|^{\alpha+1} + D\|e\|$, where $C_1 > 0$ is a constant derived from the observer's negative-definite feedback, and D represents the upper bound of the disturbance variation rate. The nonlinear term consists of the product of the nonlinear function $\text{fal}(e_y, \alpha_i, \delta)$ and the observation gain. Since $\text{fal}(e_y, \alpha_i, \delta)$ is an odd and sign-preserving nonlinearity, the nonlinear feedback term naturally has the same sign as the estimation error. The optimization of the observation gains $\beta_1 \sim \beta_5$ mainly affects the magnitude of the nonlinear correction term, thereby influencing the convergence speed and steady-state performance. Thus, there exists $C_2 > 0$ such that: $N(e) \leq -C_2 \|e\|^{\alpha+1}$. Combining the linear and nonlinear terms, the Lyapunov derivative becomes: $\dot{V}(e) \leq -C_1 \|e\|^{\alpha+1} - C_2 \|e\|^{\alpha+1} + D\|e\|$. It is worth emphasizing that the constant C_1 , previously defined as positive, originates from the intrinsic negative-definite feedback structure of the ESO and is preserved throughout the Lyapunov inequality derivation, rather than being introduced by conservative upper bounding. For $\alpha \in (0, 1)$, we have $\alpha + 1 \in (1, 2)$. When $\|e\| \rightarrow \infty$, the negative term $-C_1 \|e\|^{\alpha+1} - C_2 \|e\|^{\alpha+1}$ dominates the linear disturbance term $D\|e\|$, ensuring $\dot{V}(e) < 0$. When the $\|e\|$ is small, the optimized gain properties guarantee: $\dot{V}(e) \leq -\lambda V(e) + M$ where $\lambda > 0$ is the decay rate and $M > 0$ is a bounded constant. According to the Lyapunov final uniform boundedness theorem, integrating the inequality yields: $V(e(t)) \leq V(e(0))e^{-\lambda t} + \frac{M}{\lambda}(1 - e^{-\lambda t})$. As $t \rightarrow \infty$, $V(e(t))$ converges to the bounded set $\left\{e \mid V(e) \leq \frac{M}{\lambda}\right\}$, implying the ESO observation error $e(t)$ is ultimately uniformly bounded. If the total disturbance rate of the system $\dot{d} \rightarrow 0$, then $M \rightarrow 0$, and at this point $V(e(t)) \leq V(e(0))e^{-\lambda t}$, meaning that the error $e(t) \rightarrow 0$, and the ESO achieves asymptotic stability.

3.2. Design of the SMC Controller

SMC is a variable-structure control strategy that dynamically adjusts the controller according to the actual state of the system, driving the system state variables to move along a predetermined sliding surface. This control approach is particularly suitable for valve-controlled hydraulic cylinder systems, as it can effectively handle external disturbances, load variations, and model uncertainties, providing strong robustness, high accuracy, and fast response [30–33]. Therefore, the application of sliding mode control in valve-controlled hydraulic cylinders can ensure system stability and control performance under complex operating conditions. For the electro-hydraulic servo system described above, the sliding surface is designed as:

$$s = e_1^* + \lambda_1 e_2^* + \lambda_2 \int e_1^* dt \quad (26)$$

where $e_1^* = y_{ref} - y$ is the output force error, representing the difference between the desired force and the actual output force; $e_2^* = z_5$ is the estimation of the total system disturbance by the ESO; and λ_1 and λ_2 are design parameters. $\lambda_1 e_2^*$ represents the disturbance compensation term. Using the disturbance estimation value from the ESO, it compensates in advance for uncertainties such as parameter perturbations in the hydraulic system and external load disturbances, in order to enhance the system's disturbance rejection capability and reduce chattering in sliding mode control.

Taking the first derivative of Equation (26) and substituting Equation (18) yields:

$$\begin{aligned}\dot{s} = \dot{y}_{ref} - A_1 \frac{\beta_e}{V_1} [-A_1 x_2 - C_{ip} (x_3 - x_4) + g_1(u)u] \\ + A_2 \frac{\beta_e}{V_2} [A_2 x_2 + C_{ip} (x_3 - x_4) - C_{ep} x_4 - g_2(u)u] + \lambda_1 \beta_5 fal(e_y, \alpha_5, \delta) + \lambda_2 (y_{ref} - y)\end{aligned}\quad (27)$$

For ease of controller design, Equation (27) is rewritten in the form of a nominal term, an input gain, and a disturbance:

$$\begin{aligned}\dot{s} = f(x, \dot{x}, t) + b(x)u + \Delta(t) = \dot{y}_{ref} - A_1 \frac{\beta_e}{V_1} [-A_1 x_2 - C_{ip} (x_3 - x_4)] + A_2 \frac{\beta_e}{V_2} [A_2 x_2 + C_{ip} (x_3 - x_4) - C_{ep} x_4] \\ + \lambda_2 (y_{ref} - y) - \left[A_1 \frac{\beta_e}{V_1} g_1(u) + A_2 \frac{\beta_e}{V_2} g_2(u) \right] u + \lambda_1 \beta_5 fal(e_y, \alpha_5, \delta)\end{aligned}\quad (28)$$

where $f(x, \dot{x}, t)$ is the nominal term composed of known system parameters, states, and reference signals; $b(x)$ is the equivalent gain of the control input; and $\Delta(t)$ represents the total disturbance, including all external disturbances, nonlinearities, and model uncertainties.

The sliding mode reaching law is designed as:

$$\dot{s} = -ks - \varepsilon \tanh\left(\frac{s}{\delta}\right)\quad (29)$$

where $k > 0$ is the gain of the continuous approaching term, used to adjust the convergence rate of the system toward the sliding surface and to ensure the attractiveness of the sliding surface; $\varepsilon > 0$ is the gain of the switching approaching term, which must satisfy the reaching condition $\dot{s} \leq -\varepsilon|s|$; $\delta > 0$ is a smoothing parameter, which replaces the traditional sign function $\text{sgn}(s)$ with $\tanh\left(\frac{s}{\delta}\right)$: when $|s| \gg \delta$, $\tanh\left(\frac{s}{\delta}\right) \approx \frac{s}{\delta}$, making the switching process continuous and smooth, which mechanically suppresses chattering.

By equating Equation (28) with the reaching law in Equation (29), we have:

$$f(x, \dot{x}, t) + b(x)u + \Delta(t) = -ks - \varepsilon \tanh\left(\frac{s}{\delta}\right)\quad (30)$$

The nominal control law can be obtained as:

$$u = -\frac{1}{b(x)} \left[ks + \varepsilon \tanh\left(\frac{s}{\delta}\right) + f(x, \dot{x}, t) + \Delta(t) \right]\quad (31)$$

where $b(x)$ represents the control gain determined by the physical parameters of the hydraulic servo system. These parameters are strictly positive and bounded in practice, so it is reasonable to assume $b(x) \geq b_{\min} > 0$, which prevents singularity in the control law and avoids excessively large control signals. The terms on the right-hand side are evaluated using the available system states at the current sampling instant.

By further expanding the above equation, the sliding mode controller is obtained as:

$$\begin{aligned}u = -\frac{1}{\left(A_1 \frac{\beta_e}{V_1} g_1(u) + A_2 \frac{\beta_e}{V_2} g_2(u) \right)} \left(-ks - \varepsilon \tanh\left(\frac{s}{\delta}\right) - \dot{y}_{ref} + A_1 \frac{\beta_e}{V_1} [-A_1 x_2 - C_{ip} (x_3 - x_4)] \right. \\ \left. - A_2 \frac{\beta_e}{V_2} [A_2 x_2 + C_{ip} (x_3 - x_4) - C_{ep} x_4] - \lambda_2 (y_{ref} - y) - \lambda_1 \beta_5 fal(e_y, \alpha_5, \delta) \right)\end{aligned}\quad (32)$$

where the term on the right-hand side of the equality denotes the control input at the previous sampling instant.

Using Lyapunov stability theory, we prove that the controller Equation (32) makes the system asymptotically stable. Let the positive definite Lyapunov function constructed by s be:

$$V = \frac{1}{2}s^2 \quad (33)$$

According to the Lyapunov stability condition, it can be seen that by differentiating Equation (33) and substituting Equation (29), we obtain:

$$\dot{V} = s\dot{s} = -ks^2 - \varepsilon \tanh\left(\frac{s}{\delta}\right) \quad (34)$$

From Equation (34), it can be seen that $\dot{V}$ is strictly negative for all $s \neq 0$, and at $s = 0$, $\dot{V} = 0$. Therefore, the designed sliding surface satisfies asymptotic stability. Since the parameters k and ε are both positive constants, the system state will always be attracted to the sliding surface. On the other hand, ESO is introduced in this design to estimate the total system disturbance $\Delta(t)$. ESO tracks the disturbance through a nonlinear error feedback mechanism, and the output disturbance estimate z_3 satisfies boundedness, providing an effective basis for disturbance compensation in sliding mode control. Since the sliding mode control law contains continuous switching terms $\varepsilon \tanh\left(\frac{s}{\delta}\right)$, as long as the switching gain satisfies the upper bound for the disturbance estimation error, the disturbance estimation error will not affect the reachability of the sliding surface, and the system will remain robustly stable. Thus, under the joint action of ESO's disturbance compensation and the sliding mode reaching law, it is ensured that the sliding surface satisfies $s \rightarrow 0$ and $\dot{s} \rightarrow 0$, which leads to the error term $e_1^* \rightarrow 0$, achieving asymptotic tracking of the system output to the desired trajectory.

In summary, by constructing the modified sliding surface and using the Lyapunov function, it can be proven that under the disturbance compensation support of ESO, the Lyapunov derivative of the sliding mode control remains negative definite for all non-zero sliding mode states, thereby ensuring the reachability and asymptotic stability of the sliding surface. Therefore, it can be concluded that the designed ESO-SMC controller ensures robust stability of the closed-loop system in the presence of unknown disturbances and parameter uncertainties and achieves asymptotic tracking of the desired trajectory.

3.3. Controller Parameter Optimization Using the BSO Algorithm

In this study, the BSO algorithm is employed to simultaneously optimize the parameters of the ESO as well as the sliding mode controller parameters k , ε , and δ . The BSO algorithm is a novel swarm intelligence optimization method developed by integrating the BAS algorithm with PSO. BAS achieves efficient local search by mimicking the mechanism of beetles sensing odor gradients through their antennae, whereas PSO realizes global search via population-based cooperation. By combining the respective advantages of BAS and PSO, the BSO algorithm exhibits both rapid local search capability and strong global exploration ability. Moreover, it features a simple structure, fast convergence, and low dependence on algorithmic parameters. As a result, BSO has been widely applied in engineering fields such as controller parameter tuning and system identification.

The BAS algorithm is a class of heuristic search methods inspired by the antennae-based sensing behavior of beetles. In nature, beetles rely on the difference in odor concentration detected by the left and right antennae on their heads to determine the favorable direction of movement. BAS abstracts this biological behavior into a mathematical model and achieves rapid determination of the search direction by analyzing local variations in

the objective function. Owing to its simple structure, low computational cost, and strong directional search capability, BAS has attracted considerable attention in recent years [34].

To enable online optimization of the sliding mode control parameters and the non-linear ESO parameters in the active hydraulic cylinder force control system, an objective function that simultaneously characterizes control performance and disturbance rejection capability must be constructed. Let the particle position vector x represent a set of candidate control parameters; the optimization objective function is then defined as follows:

$$f(x) = \int_0^T (w_1 s^2(t) + w_2 e_1^{*2}(t) + w_3 e_2^{*2}(t)) \quad (35)$$

where $s(t)$ denotes the sliding surface, $e_1^*(t)$ represents the output force error of the hydraulic cylinder, and $e_2^*(t)$ is the disturbance estimated online by the ESO, which can be used to evaluate the disturbance compensation capability. The coefficients w_1 , w_2 , and w_3 are weighting factors introduced to balance the contributions of the three performance indices in the objective function. In this study, their values are empirically selected as 0.5, 0.25, and 0.25, respectively.

To simulate the sensing behavior of beetles, the algorithm first generates a random direction in the search space. Specifically, at iteration k , a direction vector in D is randomly generated and then normalized to represent the current orientation of the beetle:

$$b^k = \frac{\text{rand}(D, 1)}{\|\text{rand}(D, 1)\|} \quad b^k \in \mathbb{R}^n \quad (36)$$

where D denotes the dimension of the optimization variables, and $\text{rand}(D)$ denotes a random vector of dimension D . This vector is used to simulate the orientation of the beetle's head.

The left and right antennae of the beetle are abstracted as two sensing points located at equal distances from the current position x^k :

$$\begin{cases} x_l^k = x^k + d^k b^k \\ x_r^k = x^k - d^k b^k \end{cases} \quad (37)$$

where d^k denotes the antennae distance, which is gradually reduced with the iteration process to improve convergence.

The algorithm evaluates the objective function $f(\cdot)$ at the left and right antenna positions and compares their values to determine the more favorable direction. This differential judgment can be expressed using a sign function as follows:

$$\text{sign}(f(x_l^k) - f(x_r^k)) \quad (38)$$

If the left side is more favorable, the value is positive; if the right side is more favorable, the value is negative; if the difference is small, the value approaches zero.

Based on the above differential information, the update equation of the beetle can be written as:

$$x^{k+1} = x^k - \alpha^k b^k \text{sign}(f(x_l^k) - f(x_r^k)) \quad (39)$$

where α^k denotes the step size of the beetle. In the algorithm, a step size decay strategy is typically adopted to ensure large scale exploration in the early stage and fine-grained search in the later stage: $\alpha^k = \alpha_0 \delta^k$, $0 < \delta < 1$.

PSO is a typical swarm intelligence algorithm in which multiple particles cooperatively iterate in the search space to share and approach the optimal solution. Each particle

is characterized by its position and velocity, and its evolution is jointly influenced by its own historical best position and the global best position of the swarm [35,36].

Particle velocity update equation:

$$v_i^{k+1} = \omega v_i^k + c_1 \text{rand}(D, 1)(p_i - x_i^k) + c_2 \text{rand}(D, 1)(g - x_i^k) \quad (40)$$

where ω is the inertia weight used to balance exploration and exploitation; c_1 and c_2 are learning factors; p_i denotes the particle's own historical best position; and g denotes the current global best position of the swarm.

Particle position update equation:

$$x_i^{k+1} = x_i^k + v_i^{k+1} \quad (41)$$

The essence of the BSO algorithm is to embed the differential foraging mechanism of BAS into the velocity update of PSO, enabling each particle to not only follow the swarm learning pattern of PSO but also utilize the differential direction of BAS to escape local optima. In BSO, each particle retains the standard PSO position and velocity, and additionally generates its own beetle direction vector:

$$b_i^k = \frac{\text{rand}(D, 1)}{\|\text{rand}(D, 1)\|} \quad (42)$$

Construct the left and right sensing points using the beetle's antennae:

$$\begin{cases} x_{i,l}^k = x_i^k + d_i^k b_i^k \\ x_{i,r}^k = x_i^k - d_i^k b_i^k \end{cases} \quad (43)$$

The difference in the objective function values at the left and right sensing points determines the BAS direction:

$$d_i^k = \text{sign}(f(x_{i,l}^k) - f(x_{i,r}^k)) \quad (44)$$

where d_i^k effectively acts as a global perturbation direction, allowing particles to escape local extrema.

By incorporating the differential direction of BAS into the PSO velocity update, the characteristic equation of BSO can be obtained:

$$v_i^{k+1} = \omega v_i^k + c_1 \text{rand}(D, 1)(p_i - x_i^k) + c_2 \text{rand}(D, 1)(g - x_i^k) + \eta \alpha^k d_i^k b_i^k \quad (45)$$

where η is the gain coefficient of the BAS differential direction, used to adjust the influence of BAS on the overall search; d_i^k introduces a global perturbation to enhance the ability to escape local optima; the remaining terms retain the PSO structure, ensuring the algorithm's cooperative learning and convergence capabilities.

Consistent with PSO, the position update of BSO is still governed by the velocity:

$$x_i^{k+1} = x_i^k + v_i^{k+1} \quad (46)$$

To balance the search range in the early stage and the convergence accuracy in the later stage, the antenna length of BAS is still governed by an exponential decay strategy: $d^k = d_0 \delta^k$, $0 < \delta < 1$.

In each iteration, each individual in the particle swarm represents a candidate set of control parameters x_i . To evaluate the quality of this parameter set, it is substituted into the closed-loop system for simulation or online operation, and the corresponding fitness value is computed. For the i particle at iteration $k+1$ with parameter vector x_i^{k+1} , the fitness is calculated as:

$$f_i(k+1) = f(x_i^{k+1}) \quad (47)$$

A lower value of $f_i(k+1)$ indicates that the corresponding parameter set enables the control system to achieve a faster sliding mode convergence, higher force tracking accuracy, and stronger disturbance rejection capability of the ESO. Therefore, the fitness evaluation serves as a critical criterion in the BSO search process.

To exploit particle historical information and swarm cooperation, BSO updates the particle best position (p_{best}) and the global best position (g_{best}) at each iteration, in the same manner as PSO.

If the current fitness value is better than the particle's historical best value, it is updated as follows:

$$p_i = \begin{cases} x_i(k+1), & f_i(k+1) < f(p_i) \\ p_i, & \text{otherwise} \end{cases} \quad (48)$$

The p_{best} represents the best parameter set that each particle has achieved during the search process and serves as a record of its individual experience, which helps prevent the search from deviating from previously identified promising regions.

Among the p_{best} values of all particles, the one with the minimum fitness is selected as the global best:

$$g = \arg \min_i f(p_i) \quad (49)$$

The g_{best} represents the best parameter set currently found by the entire swarm and serves as a key reference for cooperative search. Its continuous updating ensures that the optimization process converges toward the overall optimum.

Based on the mathematical modeling and update mechanisms of the BSO strategy described above, the overall execution of the algorithm can be further summarized as a standard iterative solution procedure. For clarity of understanding and implementation, the optimization process is organized into a flowchart, and the complete operational steps are shown in Figure 3.

It should be noted that BSO is a beetle swarm intelligence optimization algorithm, in which the optimization procedure is jointly completed by multiple beetle individuals. The population size represents the number of beetle agents participating in the search and directly affects both the diversity of the solution space and the computational burden. In this study, the beetle population size is set to 20 in order to maintain sufficient global search capability while avoiding high computational cost. The maximum number of iterations is set to 30 to balance optimization convergence performance and real-time implementation requirements. In addition, the rolling window size is chosen as 10 to achieve a trade-off between parameter update responsiveness and numerical stability. Regarding the optimization parameter ranges, the lower bounds of all parameters are uniformly set to zero to ensure the non-negativity of controller and observer parameters. The upper bounds are determined according to the physical significance of each parameter and the stability constraints of the control system. Specifically, the upper bounds of the ESO-related parameters $\beta_1 \sim \beta_5$ are selected to guarantee that the observer dynamics are sufficiently faster than those of the controlled plant while avoiding noise amplification caused by excessively high gains. The upper bounds of the sliding mode control parameters k , ε , and δ are determined based on the requirements of suppressing control chattering and preventing saturation of the hydraulic actuator. In this study, the upper bounds of the optimized parameters are defined as $\beta_1 = 150$, $\beta_2 = 150$, $\beta_3 = 100$, $\beta_4 = 100$, $\beta_5 = 100$, $k = 200$, $\varepsilon = 50$, and $\delta = 500$. These parameter ranges are predefined prior to the optimization process and remain fixed throughout the entire BSO search procedure.

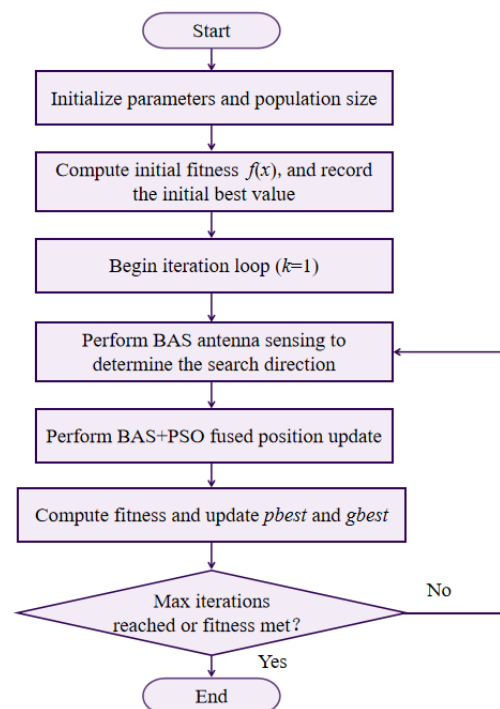


Figure 3. Flowchart of the BSO process.

4. Simulation Results and Analysis

To verify the performance of the proposed controller, a simulation model closely approximating the actual system is established on the MATLAB Simulink platform, and the simulation parameters of the hydraulic servo system are listed in Table 1.

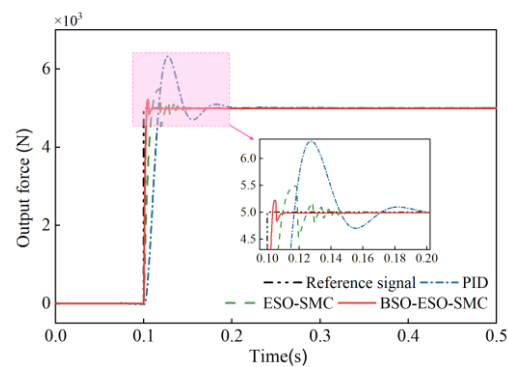
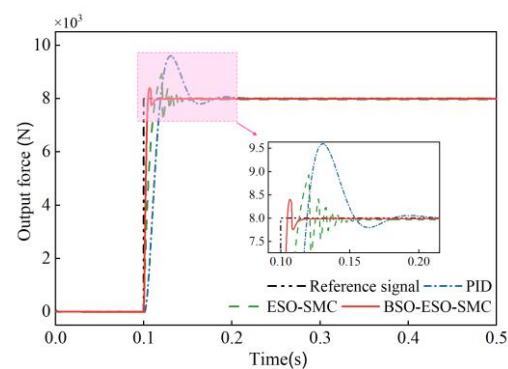
In the simulation study, the lumped disturbance $d(t)$ is not explicitly introduced as an independent disturbance input. According to its definition $d = f_L / m$, where f_L includes unmodeled friction and external disturbances, the effect of $d(t)$ is implicitly reflected through modeling uncertainties (e.g., unmodeled friction characteristics) and external load variations in the hydraulic servo system. Therefore, the lumped disturbance acting on the system arises naturally from these factors and is estimated online by the ESO during the simulation process. The performance of the designed BSO-ESO-SMC controller is evaluated under both step response and sinusoidal response conditions. For comparison, the baseline ESO-SMC and PID controllers are also included. The PID controller, as a classical control method with a simple structure, serves as a reference and its parameters are kept unchanged. The ESO-SMC controller, due to its more complex nonlinear structure and sensitivity to parameter settings, is the focus of BSO-based optimization. The main purpose of this comparison is to demonstrate the performance improvement achieved by applying BSO-ESO-SMC, while the PID controller provides a standard baseline for evaluation. This setup allows a clear assessment of the effectiveness of the proposed optimization strategy under typical operating conditions. The PID controller parameters are set to $k_p = 5.1$, $k_i = 0.57$, and $k_d = 0$. The parameters of the ESO-SMC controller are set to $\beta_1 = 55$, $\beta_2 = 20$, $\beta_3 = 18$, $\beta_4 = 18$, $\beta_5 = 22$, $k = 30$, $\varepsilon = 5$, and $\delta = 50$. For the BSO-ESO-SMC controller, the parameters are chosen as $d^k = 0.5$, $\alpha_0 = 0.3$, $\delta^k = 0.8$, $\omega = 0.6$, and $c_1 = c_2 = 1.5$; the BSO algorithm parameters are set according to the configurations specified in the algorithm design section to ensure consistency and reproducibility.

Table 1. Simulation parameters of the hydraulic servo system.

Parameter (Unit)	Value
m (kg)	200
A_1 (m ²)	8.04×10^{-4}
A_2 (m ²)	3.13×10^{-4}
V_{01} (m ³)	1.21×10^{-4}
V_{02} (m ³)	4.7×10^{-5}
Bp (N·s/m)	3400
K (N/m)	1.5×10^4
βe (Pa)	7×10^8
C_{ip} (m ³ ·s ⁻¹ ·Pa ⁻¹)	3.3×10^{-13}
C_{ep} (m ³ ·s ⁻¹ ·Pa ⁻¹)	2.03×10^{-13}
C_d	0.7
ω (m)	1.12×10^{-2}
k_{sv} (m/A)	0.4
ρ (kg/m ³)	860
P_s (Pa)	15×10^6

4.1. Simulation Results of Step Response

During the simulation, the initial output force of the system is set to 0 N. At $t = 0.1$ s, step excitation signals with amplitudes of 5000 N and 8000 N are applied, respectively. The total simulation duration is 0.5 s, and the corresponding response results are shown in Figures 4 and 5.


Figure 4. Simulated tracking under 5000 N step input.

Figure 5. Simulated tracking under 8000 N step input.

As shown in Figures 4 and 5, the tracking performances of the three control strategies exhibit significant differences under step reference signals of 5000 N and 8000 N. Under the 5000 N step condition, the PID controller produces a peak output force of

approximately 6450 N, corresponding to an overshoot of 29% and a settling time of 0.097 s. The ESO SMC controller achieves a peak value of about 5500 N, with an overshoot of 10% and a settling time of 0.044 s. In contrast, the BSO ESO SMC controller yields a peak value of only 5250 N, with an overshoot of 5% and a settling time of 0.015 s. These results indicate that the BSO ESO SMC controller exhibits superior performance in terms of overshoot suppression and response speed. Under the 8000 N step condition, the peak output force of the PID controller reaches 9650 N, with an overshoot of 20.63% and a settling time of 0.128 s. The ESO SMC controller attains a peak value of approximately 8910 N, corresponding to an overshoot of 11.38% and a settling time of 0.058 s. The BSO ESO SMC controller further reduces the peak value to about 8420 N, with an overshoot of 5.25% and a settling time of 0.02 s. The simulation results further demonstrate that, even under higher amplitude reference commands, the BSO-ESO-SMC controller consistently maintains the smallest overshoot and the fastest settling behavior, highlighting its favorable command adaptability and robustness.

In summary, the BSO-ESO-SMC controller exhibits the best overall dynamic performance under different step reference signals, and its control structure demonstrates a clear advantage in coordinating response speed and steady state accuracy.

4.2. Simulation Results of Sinusoidal Response

Sinusoidal signals are commonly employed to evaluate the dynamic performance of advanced controllers. In this study, three representative frequencies, namely 0.5 Hz, 1 Hz, and 2 Hz, are selected, with the amplitude uniformly set to 5000 N, to conduct force tracking simulation experiments. The tracking curves and corresponding error curves for the 0.5 Hz sinusoidal signal are shown in Figure 6 and Figure 7, respectively. The corresponding results for the 1 Hz signal are presented in Figures 8 and 9, while the tracking performance and error analysis for the 2 Hz signal are illustrated in Figures 10 and 11.

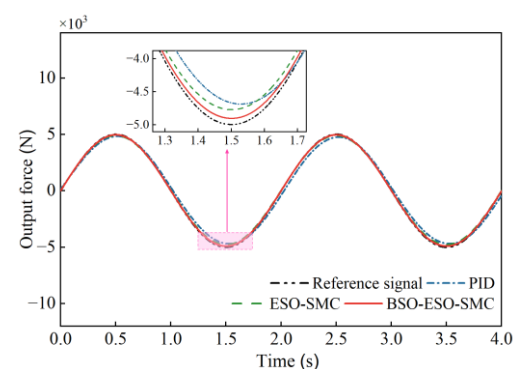


Figure 6. Force tracking performance at the 0.5 Hz sinusoidal signal.

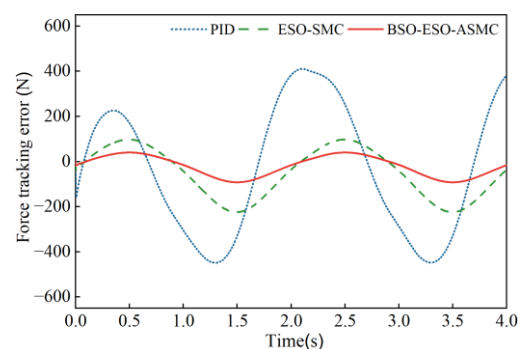


Figure 7. Tracking error under the 0.5 Hz sinusoidal signal.

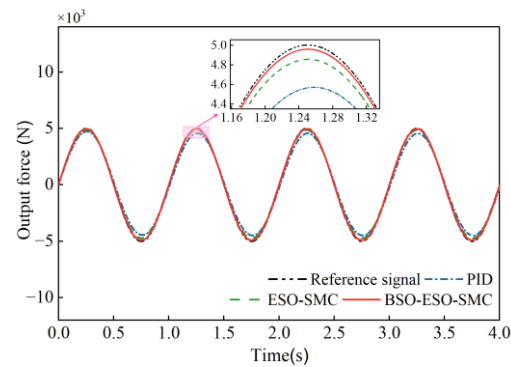


Figure 8. Force tracking performance at 1 Hz sinusoidal signal.

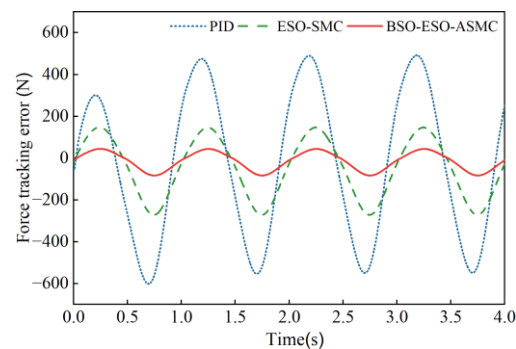


Figure 9. Tracking error under the 1 Hz sinusoidal signal.

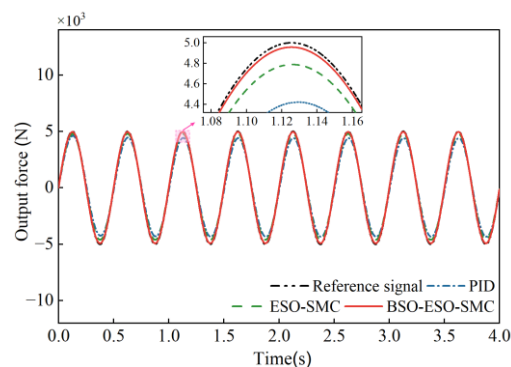


Figure 10. Force tracking performance at 2 Hz sinusoidal signal.

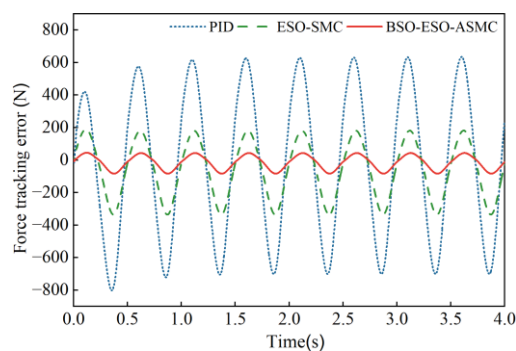


Figure 11. Tracking error under the 2 Hz sinusoidal signal.

As shown in Figure 6, the force tracking performances of three controllers, namely PID, ESO-SMC, and BSO-ESO-SMC, are compared under a 0.5 Hz sinusoidal signal. The results demonstrate that the BSO ESO SMC controller achieves the best tracking

performance, accurately following the target sinusoidal loading curve within the shortest time. Compared with the other controllers, it exhibits higher tracking accuracy and better stability, enabling a more effective response to the 0.5 Hz signal variation. Similar observations can be drawn from the tracking results under 1 Hz and 2 Hz sinusoidal signals shown in Figure 8 and Figure 10, respectively, where the BSO-ESO-SMC controller consistently maintains superior force tracking performance. As illustrated in Figures 7, 9 and 11, the force tracking errors of the controllers under different sinusoidal excitations are further compared. The results show that the BSO-ESO-SMC controller has the smallest error fluctuation range and a significantly lower overall error level than the other controllers, which further confirms its advantages in terms of tracking accuracy and stability. Combined with the tracking performance shown in Figures 6, 8 and 10, it can be concluded that the BSO-ESO-SMC controller consistently achieves low tracking error and reduced oscillations under different frequency inputs, demonstrating excellent tracking performance and robustness.

To analyze the control performance of the controllers and ensure a fair comparison, a quantitative evaluation is performed based on the force tracking error data. The following three performance indices are adopted:

1. Maximum tracking error Me ;

$$Me = \max(|e(i)|) \quad i = 1, 2, \dots, N \quad (50)$$

where $e(i)$ denotes the tracking error at the i th sampling point, and N represents the total number of sampling points.

2. Mean absolute tracking error μ ;

$$\mu = \frac{1}{N} \sum_{i=1}^N |e(i)| \quad (51)$$

3. Standard deviation of absolute tracking error σ ;

$$\sigma = \sqrt{\frac{1}{N} \sum_{i=1}^N (|e(i)| - \mu_e)^2} \quad (52)$$

To further verify the tracking performance of the controllers under sinusoidal signals with different frequencies, Table 2 presents the values of Me , μ , and σ for each controller under input frequencies of 0.5 Hz, 1 Hz, and 2 Hz. The values in parentheses for the ESO-SMC and BSO-ESO-SMC controllers indicate the percentage reduction in error relative to the PID controller.

Table 2. Tracking performance indices under sinusoidal signals with different frequencies.

Frequency	Controller	Me (N)	μ (N)	σ (N)
0.5 Hz	PID	449.12	253.39	132.67
	ESO-SMC	224.39 (50.03%↓)	100.97 (60.15%↓)	67.55 (49.08%↓)
	BSO-ESO-SMC	92.36 (79.44%↓)	41.24 (83.72%↓)	27.83 (79.02%↓)
1 Hz	PID	601.93	311.84	185.61
	ESO-SMC	271.16 (54.95%↓)	116.07 (62.68%↓)	90.13 (51.35%↓)
	BSO-ESO-SMC	85.63 (85.77%↓)	38.56 (87.63%↓)	24.64 (86.72%↓)
2 Hz	PID	801.65	407.17	210.80
	ESO-SMC	335.39 (58.17%↓)	142.34 (65.18%↓)	97.34 (53.81%↓)
	BSO-ESO-SMC	83.67 (89.57%↓)	38.34 (90.58%↓)	24.55 (88.35%↓)

As can be observed from Table 2, the BSO-ESO-SMC controller exhibits the best force tracking performance at frequencies of 0.5 Hz, 1 Hz, and 2 Hz. In terms of the maximum error, mean value, and standard deviation, the BSO-ESO-SMC controller consistently outperforms the other controllers, demonstrating superior high-frequency response capability and stability.

The simulation results demonstrate that the proposed BSO-ESO-SMC controller exhibits superior force tracking performance under sinusoidal excitations with different frequencies. It outperforms the comparative controllers in terms of maximum tracking error, mean absolute error, and error standard deviation, achieving lower error levels and enhanced stability. These results verify the effectiveness of the proposed control strategy and provide a theoretical basis for subsequent experimental investigations.

5. Experimental Validation and Analysis

5.1. Experimental Platform for Suspension Hydraulic Servo System Performance Evaluation

The performance test platform of the suspension hydraulic servo system is shown in Figure 12.

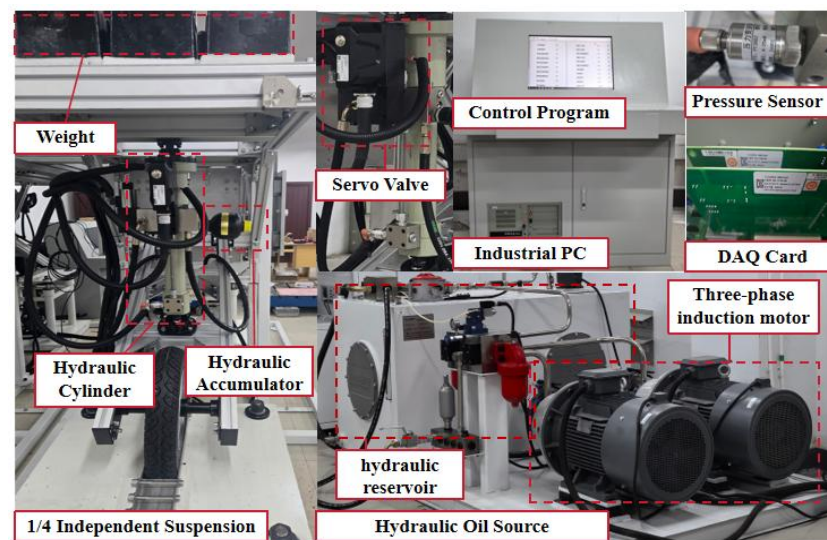


Figure 12. Performance test platform of the suspension hydraulic servo system.

It consists of a hydraulic power supply system, a servo valve, a hydraulic cylinder, a data acquisition system, and a control system. During the experiments, data acquisition and control signal output are implemented using LabVIEW software on an industrial computer, and the controller algorithms are programmed and deployed within the LabVIEW environment. The servo valve employed in this study is a Parker D1FPE50FA9NS00 (Parker Hannifin Corporation, Cleveland, OH, USA) model. Pressure sensors installed in the upper and lower chambers of the hydraulic cylinder are PT3002 (Shanghai Ainstein Sensor Technology Co., Ltd., Shanghai, China) sensors, with a measurement range of 0–30 MPa, an output current of 4–20 mA, and a measurement accuracy of $\pm 0.5\%$ FS. The industrial computer used is an Advantech IPC-610 L (Advantech Co., Ltd., Taipei, Taiwan), while the input and output data acquisition cards are PCI-1713U (Advantech Co., Ltd., Taipei, Taiwan) and PCI-1720U (Advantech Co., Ltd., Taipei, Taiwan), respectively, with a sampling frequency set to 1000 Hz. The design parameters of the hydraulic cylinder are consistent with those used in the simulations. To simulate the load mass, weights are added to the top of the suspension, with each weight having a mass of 20 kg. The self-weight of the independent suspension test rig is approximately 40 kg. These load effects, together with mechanical disturbances arising from unmodeled friction forces and external influences, are not explicitly modeled in the controller design and are regarded as

unknown lumped disturbances, which are estimated online by the ESO. The experimental parameters are consistent with the simulation settings, and experimental investigations are conducted to verify the effectiveness of the proposed controller.

5.2. Step Response Experimental Results

The step response characteristics are an important criterion for evaluating the rapidity and disturbance rejection capability of a controller. To further verify the practical control performance of the proposed BSO-ESO-SMC controller in a hydraulic system, experimental tests were carried out under step force commands with different amplitudes. The step response experiments were conducted under the same conditions as the simulations. The initial output force was set to 0 N, and step force inputs of 5000 N and 8000 N were applied at 0.1 s, respectively. The total duration of the test was 0.5 s. The corresponding experimental results are shown in Figures 13 and 14.

As shown in Figure 13, the step response performance of the hydraulic servo system under a 5000 N force command is experimentally evaluated. With the proposed BSO-ESO-SMC controller, the system output force responds rapidly after the step input is applied at 0.1 s, exhibiting a peak value of approximately 5450 N, which corresponds to an overshoot of about 9%. The response remains smooth and converges quickly to the desired force without noticeable oscillations. In contrast, the peak output forces obtained with the ESO-SMC and PID controllers reach approximately 5900 N and 6550 N, resulting in overshoots of 18% and 31%, respectively. Moreover, the settling time of the BSO-ESO-SMC controller is about 0.04 s, which is reduced by approximately 75.00% and 69.23% compared with those of the PID controller (0.16 s) and the ESO-SMC controller (0.13 s). These experimental results demonstrate that the proposed BSO-ESO-SMC controller achieves superior dynamic performance in terms of overshoot suppression and response speed.

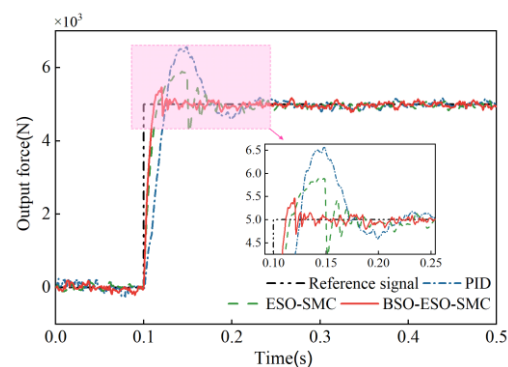


Figure 13. Experimental tracking under the 5000 N step input.

As shown in Figure 14, a comparative experimental study is conducted under an 8000 N step force command to further evaluate control performance. With the proposed BSO-ESO-SMC controller, the output force reaches a peak value of approximately 8700 N, corresponding to an overshoot of 8.75%, and the response time is about 0.07 s. In comparison, the ESO-SMC and PID controllers exhibit peak output forces of approximately 8900 N and 10500 N, resulting in overshoots of 11.25% and 31.25%, respectively, and both responses show evident oscillations and time delay. The response times of the PID and ESO-SMC controllers are measured to be 0.17 s and 0.10 s, respectively. Consequently, the response speed of the BSO-ESO-SMC controller is improved by 58.82% and 30.00% relative to the PID and ESO-SMC controllers. These results indicate that, even under high-amplitude step commands, the proposed controller effectively suppresses overshoot while maintaining fast response and stable convergence, demonstrating strong adaptability to varying operating conditions and robust control performance.

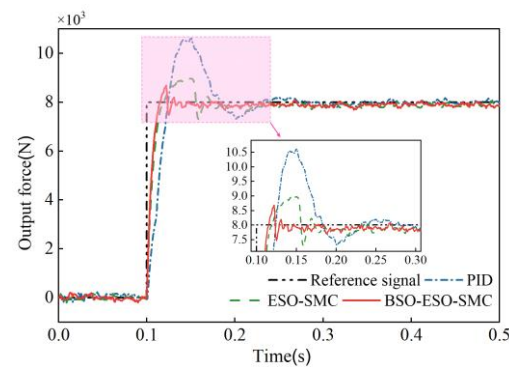


Figure 14. Experimental tracking under the 8000 N step input.

Overall, the experimental results obtained under step force commands with different amplitudes demonstrate that the proposed BSO-ESO-SMC strategy achieves the smoothest dynamic response and the fastest convergence among all the tested controllers. These results provide strong evidence of the effectiveness and reliability of the proposed scheme in addressing practical engineering challenges that require high control accuracy and high stability.

5.3. Sine Wave Response Experimental Results

To further validate the dynamic tracking performance of the controller on the actual hydraulic system, sine wave tracking experiments were conducted under conditions corresponding to the simulations. The input signal frequencies were set to 0.5 Hz, 1 Hz, and 2 Hz, with an amplitude of 5000 N. The system's tracking curves and error curves under the experimental inputs are presented in Figures 15–20. Specifically, the tracking and error curves for the 0.5 Hz signal are shown in Figures 15 and 16, for the 1 Hz signal in Figures 17 and 18, and for the 2 Hz signal in Figures 19 and 20.

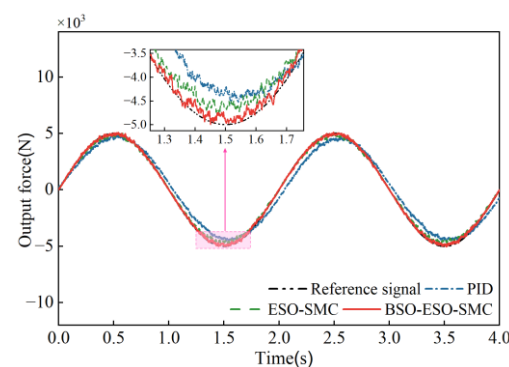


Figure 15. Experimental force tracking performance at a 0.5 Hz sinusoidal signal.

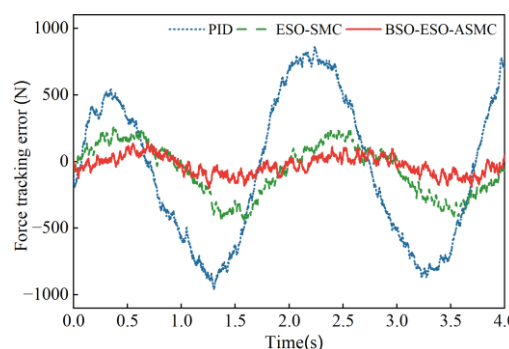


Figure 16. Experimental force tracking error under the 0.5 Hz sinusoidal input.

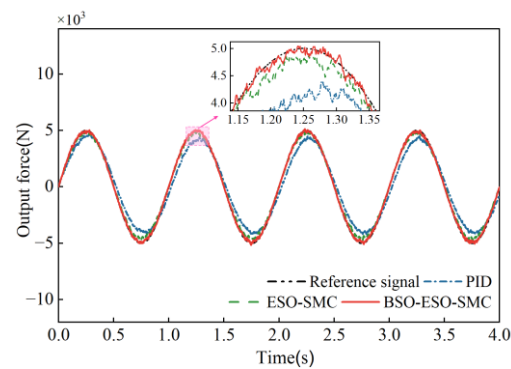


Figure 17. Experimental force tracking performance under the 1 Hz sinusoidal input.

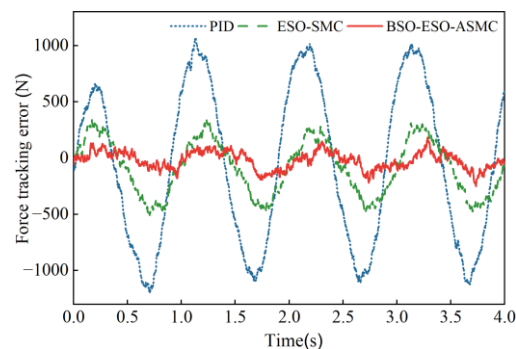


Figure 18. Experimental force tracking error under the 1 Hz sinusoidal input.

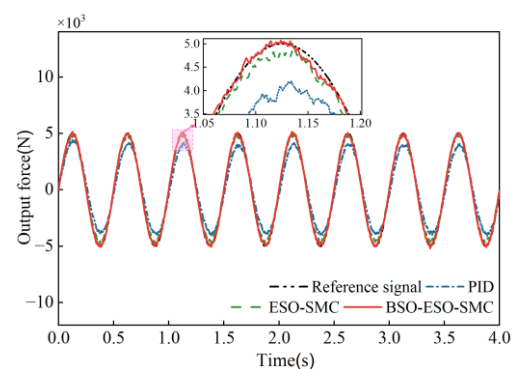


Figure 19. Experimental force tracking performance under the 2 Hz sinusoidal input.

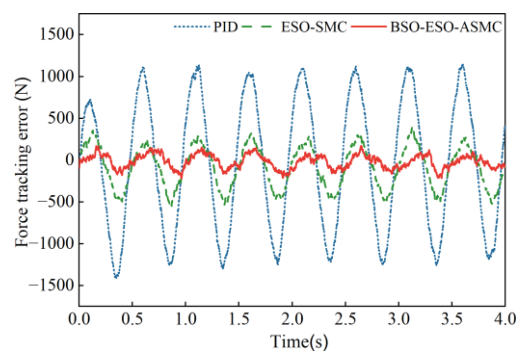


Figure 20. Experimental force tracking error under the 2 Hz sinusoidal input.

To quantitatively evaluate the sinusoidal tracking performance of each controller in the actual system, key indicators including the maximum tracking error, mean absolute error, and error standard deviation were calculated based on the experimental data, with

the results summarized in Table 3. The table also lists the percentage reductions in errors achieved by the ESO-SMC and BSO-ESO-SMC controllers relative to the PID controller, providing a clear illustration of the performance improvement.

In the experimental validation, as shown in Figures 15, 17, and 19, the BSO-ESO-SMCler consistently exhibits superior force tracking performance under sinusoidal inputs at 0.5 Hz, 1 Hz, and 2 Hz. The controller can rapidly and accurately reproduce the desired loading profiles, maintaining good response speed and stability across different frequencies. From the tracking error comparisons in Figures 16, 18, and 20, it is evident that the BSO-ESO-SMC controller achieves the smallest error fluctuations, with overall error levels significantly lower than those of the PID and ESO-SMC controllers. Experimental data further support this conclusion: as summarized in Table 3, the BSO-ESO-SMC controller outperforms the other controllers in terms of maximum error, mean absolute error, and error standard deviation, indicating superior tracking accuracy and robustness in the actual system. These experimental results are consistent with the simulation findings, fully demonstrating the effectiveness and reliability of the BSO-ESO-SMC control strategy in practical applications.

Table 3. Experimental results of tracking performance indicators under sinusoidal inputs at different frequencies.

Frequency	Controller	Me (N)	μ (N)	σ (N)
0.5 Hz	PID	958.97	483.71	249.96
	ESO-SMC	458.07 (52.23%↓)	172.30 (64.38%↓)	111.46 (55.40%↓)
	BSO-ESO-SMC	201.12 (79.03%↓)	62.58 (87.06%↓)	42.93 (82.83%↓)
1 Hz	PID	1195.64	628.48	318.33
	ESO-SMC	507.34 (57.57%↓)	220.25 (64.95%↓)	133.99 (57.91%↓)
	BSO-ESO-SMC	251.88 (78.93%↓)	66.20 (89.47%↓)	46.49 (85.40%↓)
2 Hz	PID	1412.19	706.72	364.82
	ESO-SMC	550.89 (60.99%↓)	225.00 (68.16%↓)	142.56 (60.92%↓)
	BSO-ESO-SMC	259.18 (81.67%↓)	71.12 (89.93%↓)	47.57 (86.96%↓)

6. Conclusions

This study addresses the core requirements for enhancing the performance of active hydraulic suspension systems in engineering vehicles. To tackle the challenges posed by system nonlinearity, time-varying parameters, and external disturbances in hydraulic servo systems, a force-tracking control method based on BSO-ESO-SMC is proposed and systematically evaluated through theoretical analysis, simulation, and experimental investigation.

From a theoretical perspective, a mathematical model of the hydraulic servo system was established, and the BSO-ESO-SMC controller was designed. Using Lyapunov stability theory, the stability of the closed-loop system was rigorously proven, providing a solid theoretical foundation for practical engineering implementation of the control strategy. Both simulation and experimental results consistently demonstrate that the BSO-ESO-SMC controller achieves superior tracking performance under inputs of various types and amplitudes. In step response tests, the controller exhibits the fastest response speed and minimal overshoot, with response times for 5000 N and 8000 N step commands being significantly shorter than those of the PID and ESO-SMC controllers, achieving improvements exceeding 50%. In sinusoidal tracking experiments at different frequencies, the BSO-ESO-SMC controller not only provides the highest agreement with the reference trajectories but also outperforms comparative methods across multiple quantitative metrics, including maximum error, mean absolute error, and error standard deviation, indicating superior accuracy and robustness.

Overall, the BSO-ESO-SMC strategy effectively estimates and compensates for internal and external disturbances, adapts to reference signals of varying amplitudes and frequencies, and significantly enhances force control precision, dynamic response speed, and disturbance rejection capability in hydraulic suspension systems. The consistency between simulation and experimental results fully validates the effectiveness and reliability of the proposed control approach in practical engineering applications. This provides a feasible solution for high-precision hydraulic servo control and offers clear engineering guidance for improving vehicle ride comfort, handling stability, and overall driving performance.

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