

## Article

# Adaptive Sliding Mode Control Based on a Peak-Suppression Extended State Observer for Angle Tracking in Steer-by-Wire Systems

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## Abstract

To address the degradation of angle tracking performance in steer-by-wire (SBW) systems caused by external disturbances and parameter uncertainties, this paper proposes a composite control strategy integrating adaptive sliding mode control (ASMC) and a peak-suppression extended state observer (PSESO). Firstly, a novel sliding mode reaching law is designed, which incorporates a dynamic adaptive gain function to achieve real-time adjustment of the control gain. This approach accelerates the reaching speed while effectively mitigating chattering. Secondly, to enhance the disturbance rejection capability of the system, a PSESO is developed to estimate the lumped disturbance in the SBW system in real time. By dynamically adjusting the observer bandwidth, the peak phenomenon in state estimation is suppressed, thereby avoiding saturation of the control signal. The disturbance estimate from the PSESO is then fed forward as a compensation term into the adaptive sliding mode (ASM) controller, forming a composite ASMC+PSESO controller that enables active compensation and suppression of disturbances. Finally, the proposed composite control strategy is validated through both simulations and experiments. Experimental results demonstrate that under sinusoidal signal tracking conditions, the proposed method reduces the maximum tracking error, the mean absolute error, and the integral absolute error by 64.4%, 74.2%, and 73.1%, respectively, compared to traditional sliding mode control (TSMC). These results fully underscore its superiority in angle tracking control and disturbance rejection for SBW systems.

**Keywords:** steer-by-wire; adaptive sliding mode control; extended state observer; angle tracking; disturbance rejection



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## 1. Introduction

With the rapid advancement of automotive technology, the steer-by-wire (SBW) system, serving as a critical actuator for advanced driver-assistance systems (ADAS), has emerged as an important research topic in the field of automotive engineering [1–3]. Unlike conventional mechanical steering systems, the SBW system eliminates the intermediate shaft and achieves complete mechanical decoupling between the upper and lower modules, enabling steering functionality via drive signals transmitted through communication buses. This architecture fundamentally enhances the design flexibility and versatility of the vehicle steering system, which not only contributes to improved handling stability and

active safety, but also provides essential underlying support for high-precision trajectory tracking in autonomous driving applications [4,5].

However, due to the absence of a mechanical linkage for direct coupling in SBW systems, the angular displacement output of the steering actuator entirely relies on the precise regulation of control algorithms [6]. High-precision tracking control of the front-wheel steering angle is crucial for achieving superior steering performance. Its accuracy directly determines the vehicle's path-following capability and driving stability, particularly under extreme operating conditions such as high-speed maneuvers, steering on low friction coefficient road surfaces, and emergency obstacle avoidance [7]. Nevertheless, SBW systems are susceptible to various uncertainties during the control process, including variations in motor parameters, unmodeled dynamics, and changes in tire self-aligning torque [8]. These disturbance factors are mutually coupled, resulting in a system that exhibits strong nonlinearity and significant disturbance characteristics, thereby constituting a complex uncertain system. These compounded uncertainties can substantially degrade the tracking accuracy of the front-wheel angle and the robustness of the system, potentially even leading to instability [9,10]. Therefore, investigating and designing a control strategy with strong disturbance rejection capabilities is of great importance for enhancing the overall performance of SBW systems.

To address the aforementioned challenges, numerous control strategies have been proposed by researchers, such as proportional-integral-derivative (PID) control [11,12], model predictive control (MPC) [8,13], and sliding mode control (SMC) [14,15]. PID control has been widely adopted in early control systems owing to its simple structure and ease of parameter tuning. However, limited by its linear control architecture, this method struggles to effectively handle the strong nonlinear characteristics of the system, particularly under extreme operating conditions where tracking errors increase significantly. MPC exhibits notable advantages in dealing with multivariable and nonlinear constraints. Nevertheless, its performance heavily relies on the accuracy of the system model. Furthermore, the substantial computational burden of MPC makes it difficult to meet the stringent requirements of real-time vehicle control. As a powerful robust control methodology, SMC was established in the seminal works of Utkin et al. [16,17]. In comparison with other control methods, SMC offers prominent advantages such as low dependence on model accuracy, strong robustness, and ease of engineering implementation [18]. As a result, SMC has become an important control methodology in SBW systems.

However, traditional sliding mode control (TSMC) also suffers from inherent limitations. The discontinuous term in its control law can induce high-frequency chattering, which not only reduces control precision but may also excite harmful mechanical vibrations, thereby accelerating actuator wear [19]. To improve its control performance, scholars have proposed various enhancement strategies, such as sliding mode reaching laws [20–23], higher-order SMC [24], and terminal SMC [25]. Among these, the reaching law approach designs an independent differential equation for the system's reaching mode, directly shaping the convergence process of the state trajectory, which effectively suppresses chattering [26]. Rohith [20] proposed a fractional-power reaching law that introduces a fractional-order proportional term. Experimental results demonstrated that this method effectively reduces chattering while maintaining reaching time and robustness comparable to conventional reaching laws. Kim et al. [21] introduced a nonlinear exponential gain term related to the sliding surface into the traditional exponential reaching law and replaced the sign function with a saturation function to limit control input, though at the expense of convergence speed. Xu et al. [22] developed a composite sliding mode reaching law incorporating a terminal attractor and an adaptive function. Experimental results indicated improved dynamic performance, albeit with increased algorithmic complexity. Zhang et al. [23] proposed

an exponential function-based reaching law that effectively suppresses high-frequency chattering. However, under extreme operating conditions, the controller's performance degrades, requiring parameter retuning.

The aforementioned improved SMC methods can enhance the dynamic characteristics of the system to some extent. However, when dealing with rapidly varying and high-amplitude compound disturbances, maintaining robustness often requires the use of large switching gains, which may exacerbate chattering and even lead to system instability [27]. Disturbance estimation and compensation, as a means to enhance robustness, has a long history in control theory. Early concepts like the disturbance observer (DOB) [28] laid the groundwork. The extended state observer (ESO), a pivotal advancement formalized by Han [29] within the active disturbance rejection control (ADRC) framework, provides a model-free approach to estimate and cancel 'total disturbance'. Integrating an ESO with SMC has been recognized as an effective solution to this issue [30]. Without relying on an accurate system model, the ESO can estimate the lumped disturbances in real time and feed the estimated values back into the control law for compensation, thereby enabling robust performance with relatively lower gains [31]. In recent years, ESO has been widely applied in various control fields such as robotic systems, SBW systems, and motor servo control due to its advantages. For instance, Zhao et al. [32] designed a nonlinear prescribed-time ESO to estimate both the states and disturbances of a robotic system, which achieves theoretical preset-time convergence, though its complex structure increases the computational burden. Sun et al. [33] developed an ESO to observe the states and lumped disturbances in permanent magnet synchronous motors (PMSM), while Sun et al. [34] applied an ESO to estimate aperiodic disturbances in an SBW system and updated the control law in real time, thereby improving steering angle tracking accuracy. However, when there is a discrepancy between the initial state of the ESO and the actual initial operating state of the system, significant transient estimation errors may occur during the initial phase, leading to a peak phenomenon in the observed values. If such peak values are compensated into the control system, they may exceed the physical limits of the actuator and cause undesirable impacts. To improve disturbance estimation performance, methods such as the uncertainty and disturbance estimator (UDE) and cascaded high-gain observers (CHO) have been proposed. UDE actively estimates and compensates for matched or mismatched disturbances by incorporating a filter into the control law, effectively suppressing high-frequency disturbances and enhancing robustness [35,36]. However, UDEs are also susceptible to the initial peaking phenomenon. CHOs employ multiple layers of observers to progressively estimate states and disturbances, which can theoretically achieve peaking reduction through distributed gains [37]. Nevertheless, this architecture increases the observer order and computational complexity. In the field of soft robotics, Shao et al. [38] achieved high-precision and robust trajectory tracking control by integrating an adaptive fractional-order sliding mode controller with a nonlinear disturbance observer. However, the core focus of their method lies in enhancing the dynamic regulation capability of the controller. In SBW systems, existing research has often paid insufficient attention to the initial estimation peak phenomenon of observers. Therefore, this paper proposes a peak-suppression ESO (PSESO), which dynamically adjusts the observer bandwidth to effectively suppress the initial peak phenomenon, further enhancing the control performance.

Based on the above analysis, this paper proposes a composite control strategy integrating adaptive sliding mode control (ASMC) with a PSESO for angle tracking control of SBW systems. Notably, compared with MPC [8], which requires online solution of optimization problems, and high-order SMC [24], which often involves complex structures and high-order state derivatives, the ASMC + PSESO composite strategy proposed in this

paper features control laws and observers composed of explicit algebraic operations and differential equation updates. With a well-defined structure and no iterative optimization loops, it not only ensures high precision and strong robustness but also inherently features lower computational complexity, making it more suitable for real-time critical automotive embedded systems. The main contributions of this work are summarized as follows:

(1) A novel sliding mode reaching law is proposed. This reaching law incorporates a dynamic adaptive gain function to achieve self-adjusting control gains based on the system states, and utilizes a segmented function to replace the traditional switching term, thereby accelerating convergence while effectively suppressing chattering.

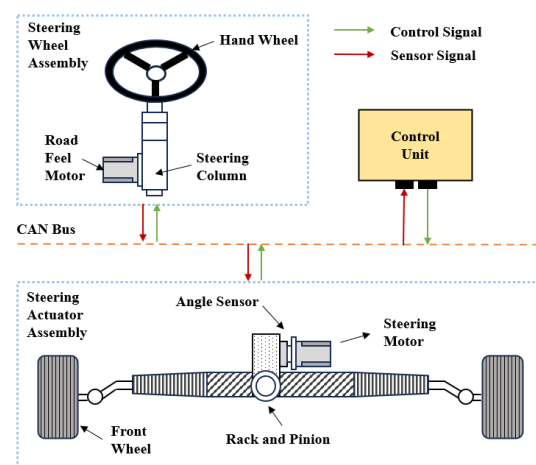
(2) A PSESO is designed to estimate the lumped disturbances in real time. Through a feedforward compensation mechanism, the angle tracking accuracy of the SBW system is significantly improved. Moreover, a time-varying bandwidth strategy is introduced in the PSESO to suppress the initial peak phenomenon in observation, further enhancing the control performance.

The remainder of this paper is organized as follows. Section 2 establishes the mathematical model of the SBW system. Section 3 presents the design of the sliding mode controller based on the proposed adaptive sliding mode reaching law (ASMRL). Section 4 describes the design procedure of the PSESO and develops the composite controller combining ASMC and PSESO. Simulation and experimental results along with corresponding analysis are provided in Section 5. Finally, Section 6 concludes the paper.

## 2. Structure and Mathematical Model of the SBW System

### 2.1. Architecture and Operating Principle of the SBW System

A schematic diagram of the SBW system structure is shown in Figure 1. The system primarily consists of three core modules: the steering wheel unit, the steering actuator unit, and the electronic control unit (ECU). Upon receiving a steering angle command from either the driver or an autonomous driving system, the ECU processes the instruction based on its embedded control strategy and generates corresponding motor drive signals. These signals are transmitted via the controller area network (CAN) to the steering actuator unit, which precisely controls the wheel steering angle through the drive motor.



**Figure 1.** Schematic diagram of the SBW system structure.

### 2.2. Mathematical Modeling of the SBW System

The mathematical model of the steering system is established as follows: [34]

$$J\ddot{\delta}_f + B\dot{\delta}_f + T_f + T_a = \kappa_\theta T_e \quad (1)$$

where  $J$  represents the equivalent moment of inertia of the system,  $B$  denotes the equivalent damping coefficient,  $\kappa_\theta$  is the transmission ratio of the steering system,  $T_f$  signifies the friction torque inherent in the steering system,  $\delta_f$  indicates the steering angle, and  $T_e$  refers to the output torque of the steering motor.

Assuming uniform tire pressure distribution and a small slip angle ( $<4^\circ$ ), the self-aligning torque  $T_a$  can be described as: [39,40]

$$\begin{cases} T_a = (l_p + l_c)F_{yf} \\ F_{yf} = -C_{rf}(\beta + \frac{\omega l_f}{V} - \delta_f) \end{cases} \quad (2)$$

where  $l_p$  denotes the pneumatic trail, representing the distance between the tire center of pressure and the point of application of the lateral force;  $l_c$  indicates the mechanical trail, defined as the distance from the tire center of pressure to the tire rotation point on the ground due to the caster angle;  $F_{yf}$  is the lateral force of the front tire;  $C_{rf}$  represents the cornering stiffness of the front tire;  $\beta$  refers to the sideslip angle at the center of mass;  $\omega$  denotes the yaw rate;  $l_f$  is the distance from the front axle to the center of mass; and  $V$  indicates the longitudinal vehicle velocity.

The friction torque  $T_f$  can be expressed as:

$$T_f = F_\omega \text{sgn}(\dot{\delta}_f) \quad (3)$$

where  $F_\omega$  denotes the coulomb friction constant.

This paper employs a PMSM as the steering actuator. The electromagnetic torque equation of the PMSM is expressed as follows:

$$T_e = \frac{3}{2} P_n \psi_f i_q \quad (4)$$

where  $P_n$  denotes the number of pole pairs,  $i_q$  represents the q-axis stator current, and  $\psi_f$  signifies the permanent magnet flux linkage.

Let  $f_d = T_f + T_a$ ; then, Equation (1) can be rewritten as:

$$J\ddot{\delta}_f + B\dot{\delta}_f + f_d = \kappa_\theta T_e \quad (5)$$

In practice, factors such as component aging, thermal expansion and contraction, and continuous vibration may cause parameter variations. However, the magnitude of such variations remains confined within a deterministic bound. The uncertainties associated with the parameters of the SBW system can be described as follows: [10]

$$\begin{cases} |J - J_0| < \Delta J \\ |B - B_0| < \Delta B \\ |\psi_f - \psi_{f0}| < \Delta \psi_f \end{cases} \quad (6)$$

where  $J_0$ ,  $B_0$ , and  $\psi_{f0}$  denote the nominal values of the system parameters.

By combining Equations (4) and (5) and explicitly introducing parameter perturbation terms, we obtain the mathematical model of the SBW system that accounts for parameter uncertainties as:

$$\ddot{\delta}_f = (P + \Delta P)i_q - (Q + \Delta Q)\dot{\delta}_f - (R + \Delta R)f_d \quad (7)$$

$$P = \frac{3\kappa_\theta P_n \psi_f}{2J}, Q = \frac{B}{J}, R = \frac{1}{J} \quad (8)$$

where  $\Delta J$ ,  $\Delta B$ , and  $\Delta R$  represent the variations in the system parameters.

Two disturbances,  $d_1$  and  $d_2$ , are introduced to represent the system uncertainties, which can be expressed as follows:

$$\begin{cases} d_1 = P(i_q - i_q^*) - Q\dot{\delta}_f - Rf_d \\ d_2 = \Delta P i_q - \Delta Q \dot{\delta}_f - \Delta R f_d \end{cases} \quad (9)$$

Finally, Equation (7) can be rewritten as:

$$\ddot{\delta}_f = P i_q^* + d_1 + d_2 \quad (10)$$

Equation (10) represents a comprehensive model of the SBW system incorporating all system uncertainties, where  $d_1$  accounts for time-varying unknown disturbances such as external interference, friction torque, and current tracking deviations, while  $d_2$  represents perturbations induced by variations in system parameters. In this paper, the disturbances  $d_1$  and  $d_2$  are aggregated into a lumped disturbance, denoted as  $d$ , such that  $d = d_1 + d_2$ .

**Assumption 1.** The lumped disturbance  $d$  satisfies  $|d| \leq D_1$ , and its time derivative is bounded by  $|\dot{d}| \leq D_2$ , where  $D_1$  and  $D_2$  are unknown positive constants.

### 3. Controller Design

This section begins with an analysis of the conventional sliding mode reaching law. To address its limitations, a novel sliding mode reaching law is designed. Using Lyapunov stability theory, the finite-time stability of the system states under this reaching law is rigorously proven. Simulation results are provided to demonstrate its comprehensive performance. Finally, based on the proposed reaching law, a sliding mode controller with disturbance rejection capability is developed for angle tracking in the SBW system.

#### 3.1. Analysis of the Traditional Exponential Reaching Law (TERL)

Among conventional reaching laws, TREL exhibits a significantly faster convergence rate. The TREL can be expressed as:

$$\dot{s} = -\varepsilon \operatorname{sgn}(s) - ks \quad (11)$$

where  $\varepsilon$  and  $k$  are positive constants,  $s$  denotes the sliding surface function. The constant rate term  $\varepsilon \operatorname{sgn}(s)$  ensures that when  $s$  approaches zero, the reaching speed remains at  $\varepsilon$  rather than zero, thereby guaranteeing that the system states reach the sliding manifold within a finite time.

When  $s > 0$ , the following expression can be derived from Equation (11):

$$\dot{s} = -\varepsilon - ks \quad (12)$$

Solving Equation (12) yields:

$$s(t) = [s(0) + \frac{\varepsilon}{k}]e^{-\lambda t} - \frac{\varepsilon}{k} \quad (13)$$

where  $s(0)$  denotes the initial condition of the sliding surface.

The time  $t_a$  required to reach the sliding surface can be determined from Equation (13) as:

$$t_a = \frac{1}{k} \left\{ \ln[s(0) + \frac{\varepsilon}{k}] - \ln \frac{\varepsilon}{k} \right\} \quad (14)$$

As evidenced by Equation (14), the response speed and steady-state accuracy of the TREL heavily depend on the coordinated design of parameters  $k$  and  $\varepsilon$ . It is noteworthy

that due to the presence of the discontinuous function  $\text{sgn}(s)$ , although increasing  $\varepsilon$  can reduce the reaching time, an excessively high gain will excite high-frequency switching when  $s$  approaches zero, thereby amplifying chattering and compromising the stability of the overall control system. Furthermore, the convergence rate of the linear term  $ks$  is constrained by its linear growth characteristic, preventing nonlinear acceleration. Moreover, since all adjustable parameters are fixed gains, the TERL lacks adaptability to varying external conditions. To address the above limitations of the TERL, this paper proposes an adaptive sliding mode reaching law (ASMRL).

### 3.2. Design of ASMRL

The proposed ASMRL incorporates a dynamic gain term and a power term, which substantially resolves the inherent conflict between chattering suppression and convergence rate associated with fixed gains. The specific design process is elaborated as follows.

(1) An adaptive gain function  $f(s, x)$  is introduced into the constant rate reaching term. The function is designed as follows:

$$\begin{cases} f(s, x) = \frac{\lambda}{\varepsilon + (1 - \varepsilon)e^{-\delta(|s| + \gamma|x|)}} \\ \lim_{t \rightarrow \infty} |x| = 0, \lambda > 0, 0 < \varepsilon < 1, \gamma > 0 \end{cases} \quad (15)$$

where  $x$  represents the state variable of the system, and  $\lambda$ ,  $\varepsilon$ , and  $\gamma$  are tuning parameters.

By incorporating the system state  $x$ , the function  $f(s, x)$  enables the control gain to adapt in real time based on the system dynamics, thereby enhancing robustness against uncertainties. When the system is far from the sliding surface, both  $s$  and  $x$  are large, yielding  $f(s, x) \approx \lambda/\varepsilon > \lambda$ . This elevated gain provides robust control authority to ensure rapid convergence during transient maneuvers, such as emergency steering, thereby enhancing the vehicle's transient stability and response speed. As the system approaches the equilibrium point,  $s$  and  $x$  become small, leading to  $f(s, x) \approx \lambda$ , which reduces the reaching speed. The attenuation of the control gain suppresses the high-frequency chattering inherent to the discontinuous switching in SMC. This suppression minimizes undesirable mechanical vibrations in SBW actuators, leading to reduced component wear. Therefore, the introduction of the adaptive gain function shortens the reaching time during large deviations while simultaneously reducing high-frequency chattering.

(2) A piecewise function is defined as follows:

$$G(s) = \begin{cases} \text{sgn}(s), |s| \geq \sigma \\ \tanh(\mu s), |s| < \sigma \end{cases} \quad (16)$$

where  $\mu = 2\pi/\sigma$ ,  $\sigma > 0$  represents the thickness of the boundary layer.

The function  $G(s)$  is employed to replace the signum function  $\text{sgn}(s)$  in the exponential reaching term. Outside the boundary layer ( $|s| \geq \sigma$ ), the signum function is retained to enable the system states to approach the sliding surface at the maximum rate. Inside the boundary layer ( $|s| < \sigma$ ), the smooth hyperbolic tangent function is adopted to replace the discontinuous switching term, which effectively reduces the high-frequency chattering near the equilibrium point caused by the switching behavior in TSMC. Moreover, owing to its smoothness, this function inherently filters out high-frequency noise in disturbance rejection, preventing the amplification of disturbances and thereby further enhancing the robustness of the overall control system. Therefore, compared with SMC laws using only  $\tanh(s)$  or only  $\text{sgn}(s)$ , the introduction of the piecewise function further shortens the reaching time while effectively mitigating chattering.

(3) A power function term  $|x|^\eta$  is introduced, where  $\eta$  is a design parameter to be determined and satisfies  $\eta > 0$ . When the system state is far from the sliding surface,

the power term provides stronger nonlinear convergence rates, overcoming the speed limitation of traditional linear terms. As the system state approaches the sliding surface, the influence of the power term diminishes, leading to reduced reaching speed and chattering, and ultimately converging to zero. Thus, chattering is suppressed without sacrificing the reaching speed.

Finally, the ASMRL is proposed as follows:

$$\dot{s} = -f(s, x)G(s) - k|x|^\eta s \quad (17)$$

where  $\eta > 0, k > 0$ .

To verify the stability of the proposed ASMRL, consider the actual reaching dynamics in the presence of  $d$ :

$$\dot{s} = -f(s, x)G(s) - k|x|^\eta s + d \quad (18)$$

A Lyapunov function candidate is chosen as  $V = s^2/2$ . Its time derivative along the system trajectories is:

$$\dot{V} = s\dot{s} = -sf(s, x)G(s) - k|x|^\eta s^2 + sd \quad (19)$$

When  $|s| \geq \sigma$ ,  $G(s) = \text{sgn}(s)$ , so  $-sf(s, x)\text{sgn}(s) = -f(s, x)|s|$ , from the definition of the adaptive gain function in (15), it follows that  $f(s, x) \geq \lambda$ . Substituting this bound and applying the disturbance bound  $|d| \leq D_1$ , we obtain  $\dot{V} \leq -\lambda|s| - k|x|^\eta s^2 + |s|D_1$ . By selecting  $\lambda > D_1$ ,  $\dot{V} \leq 0$  holds. When  $|s| < \sigma$ ,  $G(s) = \tanh(\mu s)$  and  $f(s, x) = \lambda$ . Using the inequality  $\tanh(\mu s) \geq \mu^2 s^2 / (1 + \mu\sigma)$ , which holds for  $|s| < \sigma$ , and again invoking the disturbance bound, leads to

$$\dot{V} \leq -\lambda \frac{\mu^2 s^2}{1 + \mu\sigma} - k|x|^\eta s^2 + |s|D_1 \quad (20)$$

Since  $|s| < \sigma$ , the term  $|s|D_1$  can be conservatively bounded as  $|s|D_1 < (D/\sigma)s^2$ . Consequently,

$$\dot{V} \leq -\left(\lambda \frac{\mu^2}{1 + \mu\sigma} - \frac{D_1}{\sigma}\right) - k|x|^\eta s^2 \quad (21)$$

To maintain  $\dot{V} \leq 0$ , the parameters must satisfy  $\lambda\mu^2/(1+\mu\sigma) > D_1/\sigma$ . Therefore, for all  $s \neq 0$ ,  $\dot{V} \leq 0$ , and  $\dot{V} = 0$  if and only if  $s = 0$ . The proposed ASMRL satisfies the sliding mode reaching condition in the presence of bounded disturbances and parameter uncertainties.

To evaluate the performance of the ASMRL, the state equation of a controllable system is given as follows:

$$\ddot{\theta}(t) = -f(\theta, t) + hu(t) + d(t) \quad (22)$$

where  $\theta$  represents the angular signal,  $f(\theta, t) = 25\dot{\theta}$ ,  $u(t)$  is the control input,  $d(t) = 15\sin(\pi t)$  denotes the external disturbance, and  $h = 133$  is the system gain.

The desired angular signal  $\theta_d$  is set to  $\sin(t)$ , and the control error is defined as  $e = \theta_d - \theta$ . The sliding surface function is selected as follows:

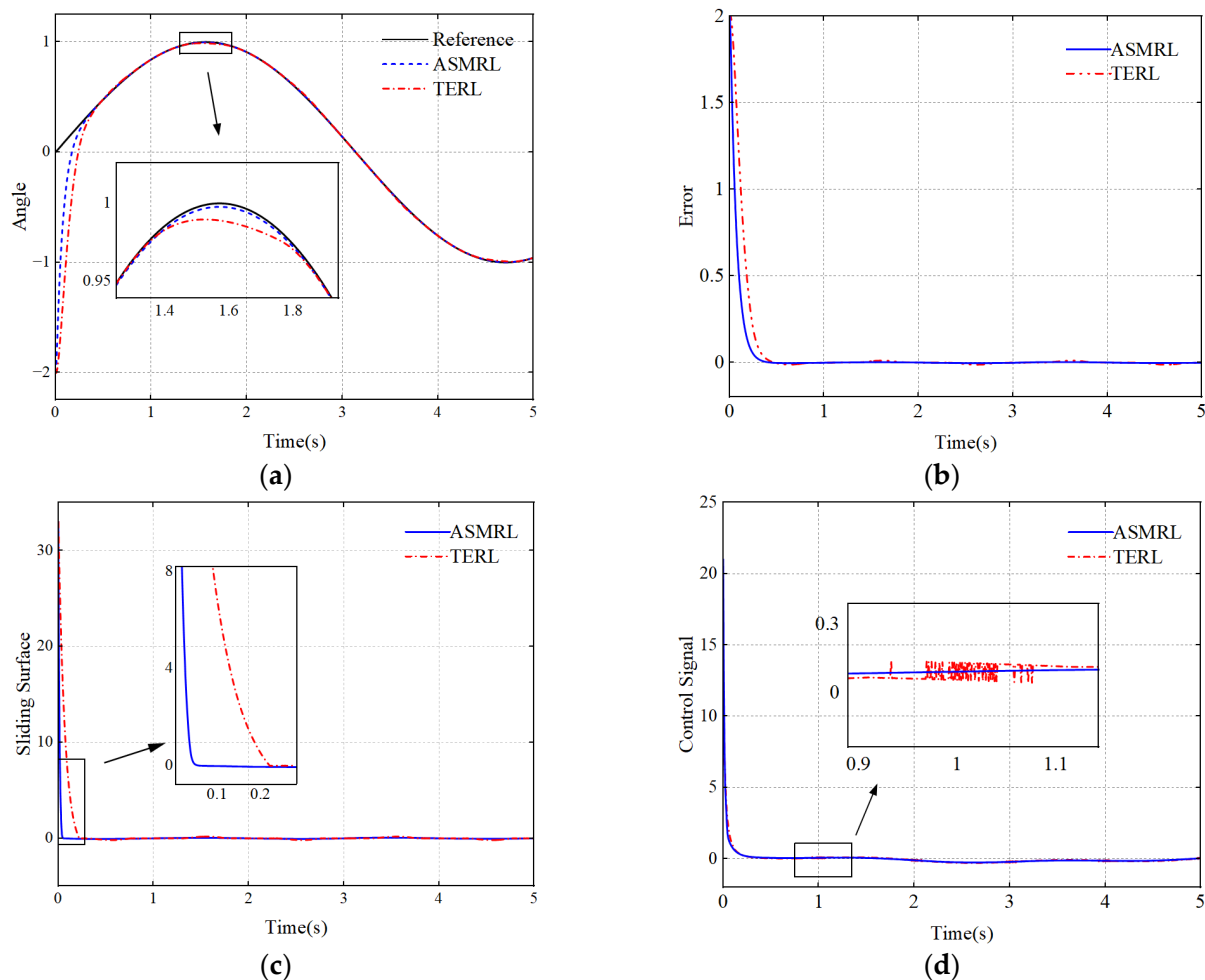
$$s = \dot{e} + ce \quad (23)$$

where  $c$  is the sliding surface coefficient, satisfying  $c > 0$ .

By integrating Equations (17), (22) and (23), the ASMRL-based sliding mode controller can be synthesized as follows:

$$u = \frac{1}{h}(f(s, x)G(s) + k|x|^\eta s + c\dot{e} + \ddot{\theta}_d + f(\theta, t)) \quad (24)$$

A simulation model of the corresponding system was developed using MATLAB/Simulink (R2024b) to compare the control performance between the TERL and the proposed ASMRL. The control parameters were configured as follows:  $c = 25$ ,  $\lambda = 70$ ,  $k = 15$ ,  $\varepsilon = 0.3$ ,  $\delta = 2$ ,  $\eta = 1.6$ ,  $\gamma = 5$ ,  $\sigma = 0.2$ . The initial state of the controlled system was set to  $x = [x_1, x_2] = [-2, -2]$ . The simulation results are shown in Figure 2.



**Figure 2.** Comparison of the control performance: (a) tracking performance; (b) tracking error; (c) performance during the reaching phase; (d) control signal.

As observed from Figure 2a,b, when the initial state of the system significantly deviates from the given reference signal and in the presence of external disturbances, the proposed ASMRL achieves faster tracking of the reference signal and exhibits a smaller steady-state error compared to the TERL. Figure 2c demonstrates that the ASMRL effectively shortens the reaching time. Furthermore, it can be seen from Figure 2d that the control signal generated by the ASMRL is smoother, effectively suppressing chattering. Therefore, the proposed ASMRL successfully resolves the inherent conflict between reaching speed and chattering in the system.

### 3.3. Design of an Angle Tracking Controller Based on ASMC

The tracking error for the front-wheel steering angle is defined as follows:

$$e_\delta = \delta_f^* - \delta_f \quad (25)$$

where  $\delta_f^*$  denotes the reference steering angle and  $\delta_f$  represents the actual steering angle.

The first and second derivatives of the front wheel steering angle tracking error are given as follows:

$$\dot{e}_\delta = \dot{\delta}_f^* - \dot{\delta}_f \quad (26)$$

$$\ddot{e}_\delta = \ddot{\delta}_f^* - \ddot{\delta}_f \quad (27)$$

Combining Equations (10) and (27), it can be concluded that:

$$\ddot{e}_\delta = \ddot{\delta}_f^* - P i_q^* - d \quad (28)$$

The design of the ASMC involves two key steps: the construction of an appropriate sliding surface, followed by the formulation of the control output utilizing the designed sliding mode reaching law.

The sliding surface function is designed as:

$$s = c e_\delta + \dot{e}_\delta \quad (29)$$

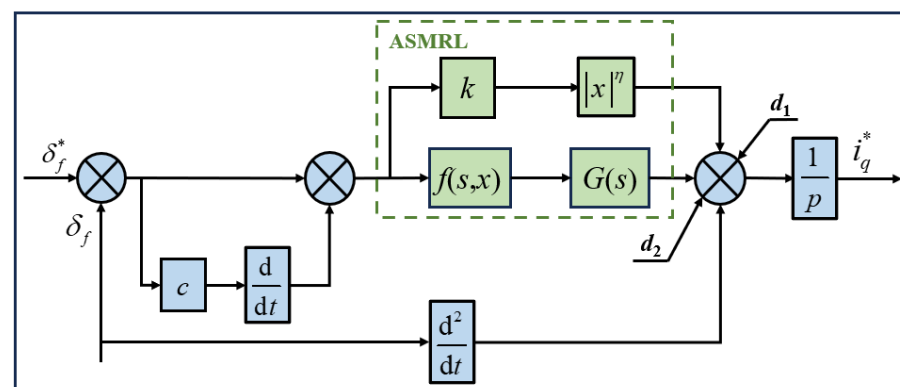
The time derivative of the sliding surface function is given by:

$$\dot{s} = c \dot{e}_\delta + \ddot{e}_\delta \quad (30)$$

Combining the ASMRL (17) with Equations (28) and (30), the control output of the ASMC-based angle tracking controller for the SBW system can be obtained as:

$$i_q^* = \frac{1}{P} (c \dot{e}_\delta + \ddot{\delta}_f^* - d + f(s, x) G(s) + k |x|^\eta s) \quad (31)$$

Figure 3 shows the block diagram of the proposed adaptive sliding mode (ASM) controller.



**Figure 3.** Block diagram of the proposed ASM controller structure.

Compared with the traditional sliding mode controller, the proposed method can reduce system chattering and improve the dynamic performance of the SBW system. However, the presence of lumped disturbance  $d$  in (31) may introduce deviations in the control signal. This implies that a larger control gain must be selected to compensate for these deviations, which in turn exacerbates system chattering and may compromise the steering angle tracking accuracy of the SBW system. Therefore, further improvements to the controller will be presented in the next section.

#### 4. Design of an ASMC and PSES Integrated Controller

As mentioned earlier, the presence of unknown disturbances and parametric uncertainties may lead to degradation in control performance. In this section, a PSES is further

constructed to estimate these unknown disturbances and parametric uncertainties. The estimated values are then incorporated as a feedforward compensation term into Equation (31), thereby enhancing the angle tracking accuracy of the SBW system.

#### 4.1. Analysis of ESO

As presented in Section 2.2, the lumped disturbance  $d$  is expressed as  $d = d_1 + d_2$ . The system dynamics (10) can be rewritten as:

$$\ddot{\delta}_f = P i_q^* + d \quad (32)$$

Based on assumption 1, the lumped disturbance  $d$  is extended as an additional state variable  $x_3$ , while the system control input and output are the q-axis current  $i_q^*$  and the front-wheel steering angle  $\delta_f$ , respectively. Let  $x_1 = \delta_f$ ,  $x_2 = \dot{\delta}_f$ ,  $x_3 = d$ , then Equation (32) can be rewritten as the following extended system:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = P i_q^* + d \\ \dot{x}_3 = \dot{d} \end{cases} \quad (33)$$

Based on Equation (33), an ESO is constructed as follows:

$$\begin{cases} \dot{z}_1 = \hat{x}_2 - \beta_1(z_1 - x_1) \\ \dot{z}_2 = \hat{x}_3 + P i_q^* - \beta_2(z_1 - x_1) \\ \dot{z}_3 = -\beta_3(z_1 - x_1) \end{cases} \quad (34)$$

where  $z_i$  ( $i = 1, 2, 3$ ) are the estimated values of  $x_i$ .  $\beta_1$ ,  $\beta_2$ , and  $\beta_3$  are the observer gains to be designed.

Let  $e_i = z_i - x_i$  ( $i = 1, 2, 3$ ), which represents the estimation error of the ESO. The vector differential equation of the estimation error system can be expressed as:

$$\dot{e}_i = A_e e_i + L \quad (35)$$

$$\text{where } e_i = \begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix}, A_e = \begin{bmatrix} -\beta_1 & 1 & 0 \\ -\beta_2 & 0 & 1 \\ -\beta_3 & 0 & 0 \end{bmatrix} \text{ is a Hurwitz matrix, } L = \begin{bmatrix} 0 \\ 0 \\ \dot{d} \end{bmatrix}.$$

The characteristic polynomial can be derived from Equation (35) as follows:

$$f(\lambda) = \lambda^3 + \beta_1 \lambda^2 + \beta_2 \lambda + \beta_3 = (\lambda + \omega_0)^3 \quad (36)$$

where  $\omega_0$  denotes the bandwidth of the ESO. The gains of the ESO (34) are designed as  $\beta_1 = 3\omega_0$ ,  $\beta_2 = 3\omega_0^2$ ,  $\beta_3 = \omega_0^3$ .

However, if the initial estimates of the ESO (34) significantly deviate from the actual initial states of the system, the high gains may induce a large and rapid peak in the estimation. This peak, when directly applied to the control system, may lead to control signal saturation and cause undesired actuator stress.

#### 4.2. Design of PSESO

To address the issue mentioned above, a PSESO is constructed based on (34) as follows:

$$\begin{cases} \dot{\hat{x}}_1 = \hat{x}_2 - \tau_1(\hat{x}_1 - x_1) \\ \dot{\hat{x}}_2 = \hat{x}_3 + P i_q^* - \tau_2(\hat{x}_1 - x_1) \\ \dot{\hat{x}}_3 = -\tau_3(\hat{x}_1 - x_1) \end{cases} \quad (37)$$

where  $\hat{x}_i$  ( $i = 1, 2, 3$ ) represent the estimated state values. The time-varying gains  $\tau_i$  are expressed as  $\tau_1 = 3\omega(t)$ ,  $\tau_2 = 3\omega^2(t)$ ,  $\tau_3 = \omega^3(t)$ , respectively. The time-varying bandwidth  $\omega(t)$  is generated by a second-order Butterworth filter [41], with Equation (38) serving as its input signal. The switching instant was determined through extensive simulation studies, ensuring that the transition is completed within the typical dynamic response period of the SBW system. The Butterworth filter is chosen for its maximally flat magnitude response in the passband, ensuring a smooth transition of the bandwidth during the switching phase and avoiding additional observation noise or oscillation due to abrupt bandwidth changes.

$$\omega_0(t) = \begin{cases} \omega_0^*, t \leq 0.3s \\ 3\omega_0^*, t > 0.3s \end{cases} \quad (38)$$

where  $\omega_0^*$  are parameters to be designed.

As indicated in Equation (38), during the initial phase ( $t \leq 0.3s$ ),  $\omega_0(t)$  remains relatively small. After a certain period ( $t > 0.3s$ ), it gradually increases until it converges to  $3\omega_0^*$ . This approach reduces the estimation peak caused by high gains without compromising observation accuracy.

Theoretically, the bandwidth of the ESO determines its trade-off between disturbance tracking speed and high-frequency noise suppression. A higher bandwidth results in faster disturbance estimation and reduced steady-state phase lag, but also amplifies measurement noise. After careful consideration, selecting  $\omega_0^* = 50$  achieves a favorable compromise between suppressing the initial peak and maintaining satisfactory estimation accuracy.

To verify the stability of the PSESO, the error vector of the PSESO is first defined as follows:

$$\Gamma = [\rho_1, \rho_2, \rho_3]^T = [\hat{x}_1 - x_1, \hat{x}_2 - x_2, \hat{x}_3 - x_3]^T \quad (39)$$

Combining Equations (33) and (37) yields:

$$\dot{\Gamma} = B_e \Gamma + H \quad (40)$$

$$\text{where } B_e = \begin{bmatrix} -\tau_1 & 1 & 0 \\ -\tau_2 & 0 & 1 \\ -\tau_3 & 0 & 0 \end{bmatrix} \text{ is a Hurwitz matrix, } H = \begin{bmatrix} 0 \\ 0 \\ \dot{d} \end{bmatrix}.$$

From Equation (40), it can be obtained that:

$$\Gamma = \int_{t_0}^t e^{(t-v)B_e} H(v) dv + e^{(t-t_0)B_e} \Gamma(t_0) \quad (41)$$

where  $t \geq t_0$ , with  $t_0$  being the initial time. The term  $e^{(t-t_0)B_e}$  satisfies the following relationship:

$$\|e^{(t-t_0)B_e}\| \leq \zeta e^{-\rho(t-t_0)} \quad (42)$$

where  $\zeta > 0$ ,  $\rho = n \min |\operatorname{Re}(\lambda_i(B_e))| = n\omega_0^*$ ,  $n$  is a constant lying between 0 and 1. Then, the following relationship holds for Equation (41), which serves as the stability condition for the PSESO:

$$\|\Gamma\| \leq \frac{\zeta}{n\omega_0^*} \sup_{t_0 \leq v \leq t} \|H(v)\| + \zeta e^{-n\omega_0^*(t-t_0)} \|\Gamma(t_0)\| \quad (43)$$

Based on the assumptions in the preceding text,  $\dot{d}$  is bounded. Consequently, the initial estimation error  $\|\Gamma(t_0)\|$  of the PSESO is bounded, and thus the term  $\zeta e^{-n\omega_0^*(t-t_0)} \|\Gamma(t_0)\|$  will eventually converge to zero. Moreover, the upper bound of  $\|H(v)\|$  is a small positive constant. Therefore, the convergence speed of  $\|\Gamma\|$  primarily depends on the adjustment of

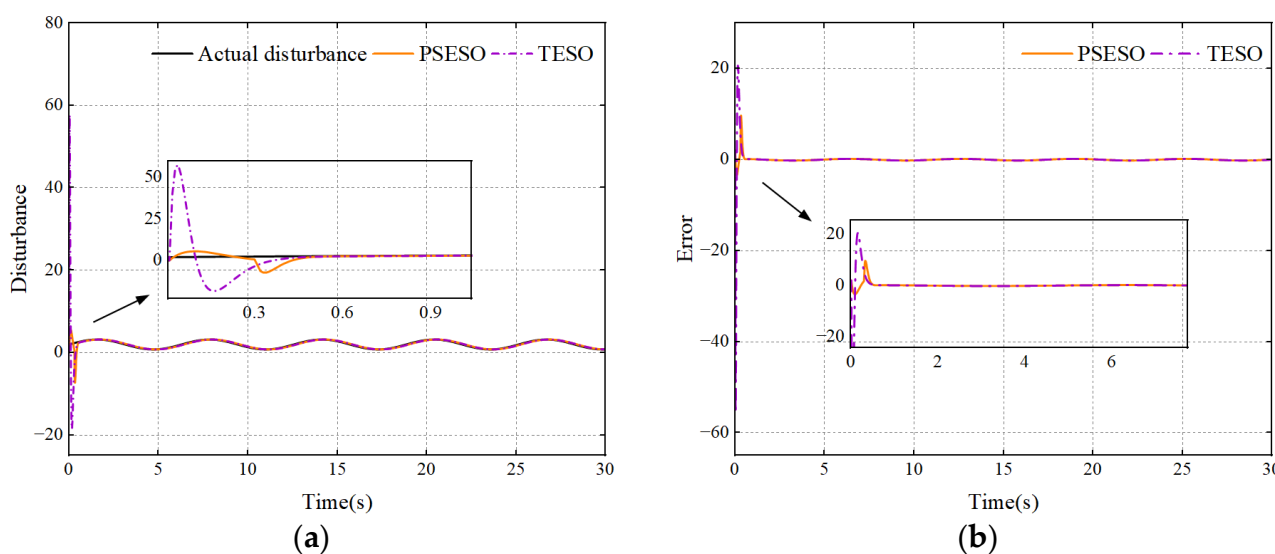
the parameter  $\omega_0^*$ . A larger value of  $\omega_0^*$  leads to a shorter convergence time to zero and a smaller steady-state error. Based on the above analysis, the disturbance estimation error of the PSESO is bounded, indicating that the system achieves finite-time stability.

To validate the performance of the PSESO, consider the following plant:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = h_0 u_0(t) + d_0(t) \\ \dot{x}_3 = \dot{d}_0(t) \end{cases} \quad (44)$$

where  $h_0 = 2$  is the control input gain,  $u_0(t) = 0.8\sin(2\pi t)$  is the control input, and  $d_0(t) = 2 + 1.2\sin(t)$  represents the external disturbance.

A corresponding simulation model was developed using MATLAB/Simulink, and a comparison was conducted with a traditional extended state observer (TESO) employing fixed bandwidth. The initial states of the plant were set to  $x_1(0) = 0.5$ ,  $x_2(0) = x_3(0) = 0$ , while the initial states of both observers were set to zero to create an initial deviation. Figure 4 shows the comparative simulation results of the PSESO and the TESO. As observed in Figure 4a,b, when the initial values of the observers deviate significantly from the actual initial states of the system, the PSESO effectively suppresses the peak phenomenon and exhibits a faster response speed, thereby avoiding control signal saturation and demonstrating greater suitability for SBW systems.



**Figure 4.** Performance comparison between PSESO and TESO: (a) disturbance estimation; (b) estimation error.

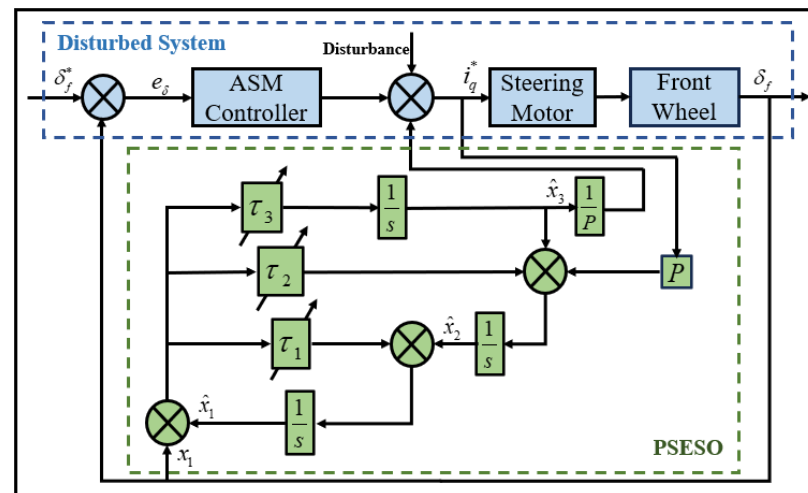
#### 4.3. PSESO-Based Composite Controller

Combining the ASMRL with the PSESO, the proposed angle tracking controller for the SBW system can be constructed as follows:

$$i_q^* = \frac{1}{P}(c\dot{e}_\delta + \ddot{\delta}_f - \hat{x}_3 + f(s, x)G(s) + k|x|^\eta s) \quad (45)$$

where  $\hat{x}_3$  is the estimated value of  $d$  provided by the PSESO.

Figure 5 presents the control block diagram of the proposed PSESO-based ASM controller designed for angle tracking control in the SBW system.



**Figure 5.** Block diagram of the proposed PSESO-based ASM controller architecture.

## 5. Simulation and Experimental Validation

To further validate the effectiveness of the proposed composite control algorithm for angle tracking control in the SBW system, comparative analyses are conducted in both simulation environments and an SBW hardware-in-the-loop (HIL) test bench. The proposed PSESO-based ASM controller is compared with both the standard ASM controller and a traditional sliding mode controller. The control performance is comprehensively evaluated using multiple performance metrics.

### 5.1. Simulation Validation

The controller parameters were initially set based on the system model and empirical guidelines. They were then adjusted and optimized through simulations, guided by key performance metrics as detailed later in this section. The final controller parameters were configured as follows:  $c = 20$ ,  $\lambda = 30$ ,  $k = 12$ ,  $\varepsilon = 0.2$ ,  $\delta = 2$ ,  $\eta = 1.6$ ,  $\gamma = 5$ ,  $\sigma = 0.2$ ,  $\omega_0^* = 50$ . The relevant parameters of the SBW system are listed in Table 1. Three controllers, namely the proposed method (ASMC + PSESO), the standard ASMC, and the TSMC, were tested under identical simulation environments and system parameters to ensure a fair comparison. All controller parameters were repeatedly optimized and tuned through experimentation to guarantee optimal performance. To comprehensively evaluate the superiority of the proposed method for angle tracking control in the SBW system, simulations were conducted under two typical test conditions.

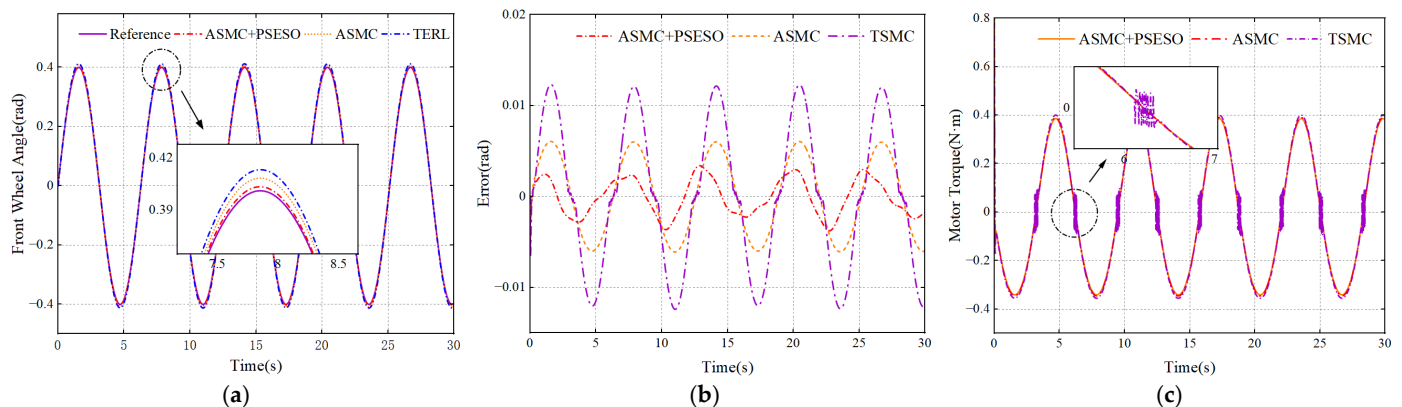
**Table 1.** Parameters of the SBW system.

| Parameters                                    | Symbol          | Value                           |
|-----------------------------------------------|-----------------|---------------------------------|
| Equivalent moment of inertia                  | $J$             | $3.6 \text{ kg}\cdot\text{m}^2$ |
| Equivalent damping coefficient                | $B$             | $12.9 \text{ Nms/rad}$          |
| Transmission ratio                            | $\kappa_\theta$ | 18                              |
| Pneumatic trail                               | $l_p$           | 0.028 m                         |
| Mechanical trail                              | $l_c$           | 0.04 m                          |
| Number of motor pole pairs                    | $P_n$           | 4                               |
| Permanent magnet flux linkage                 | $\psi_f$        | 0.05 Wb                         |
| Vehicle velocity                              | $V$             | 20 m/s                          |
| Distance from front axle to center of gravity | $l_f$           | 1.12m                           |

To provide a more intuitive comparison of the control performance of the three methods, this study further employed quantitative analysis. Specific performance evaluation metrics included the maximum error ( $\text{MAX} = \max(|e_\delta|)$ ), the mean absolute error

( $MAE = \frac{1}{n} \sum_{i=1}^n |e_{\delta}(i)|$ ), where  $n$  denotes the number of samples), the integral of absolute error (IAE =  $\int_0^t |e_{\delta}(\tau)| d\tau$ ), and the rise time (RT, defined as the time required for the step response to increase from 10% to 90% of its steady-state value).

Case 1: sinusoidal signal. To validate the tracking performance of the proposed method under continuously varying steering angle commands, a sinusoidal signal was employed as the desired angle reference, set as  $\delta_f^* = 0.4 \sin t$ . The simulation time was set to 30 s. Figure 6 shows the simulation results under the sinusoidal reference.



**Figure 6.** Simulation results under sinusoidal signal: (a) angle tracking curve; (b) tracking error curve; (c) motor torque curve.

As observed in Figure 6a, all three controllers exhibit certain overshoot at the peak values. Among them, the ASMC + PSESO controller achieves the highest tracking accuracy and stability. The ASMC controller, lacking disturbance estimation and compensation, performs slightly worse than ASMC + PSESO but still outperforms the TSMC, further verifying the effectiveness of the proposed ASMR.

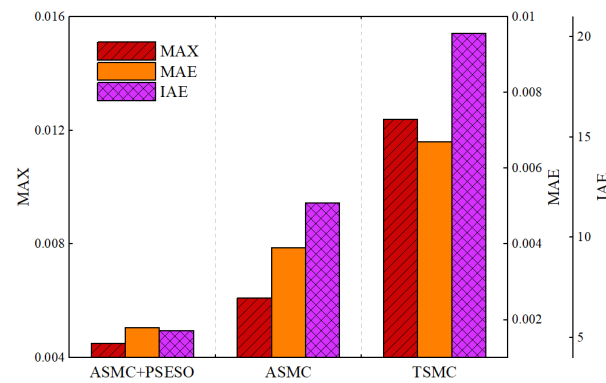
From Figure 6b, it can be seen that the composite ASMC + PSESO method yields the smallest tracking error with minimal fluctuation. The ASMC method ranks second, while the TSMC shows significantly larger errors, indicating its inferior performance under continuously varying steering conditions.

As illustrated in Figure 6c, the torque output of ASMC + PSESO is smooth with negligible chattering, which is beneficial for practical actuator application. In contrast, the TSMC exhibits noticeable chattering. This demonstrates the strong disturbance rejection and control stability of the proposed method.

Figure 7 and Table 2 summarize the tracking error metrics of the three control methods. Compared to ASMC and TSMC, the ASMC + PSESO approach achieves smaller values in MAX, MAE, and IAE. Specifically, the MAE of ASMC + PSESO is reduced by 53.8% and 73.1%, and the IAE is reduced by 54.4% and 73.5%, respectively, confirming the superiority of the proposed method.

**Table 2.** Tracking error metrics under sinusoidal signal.

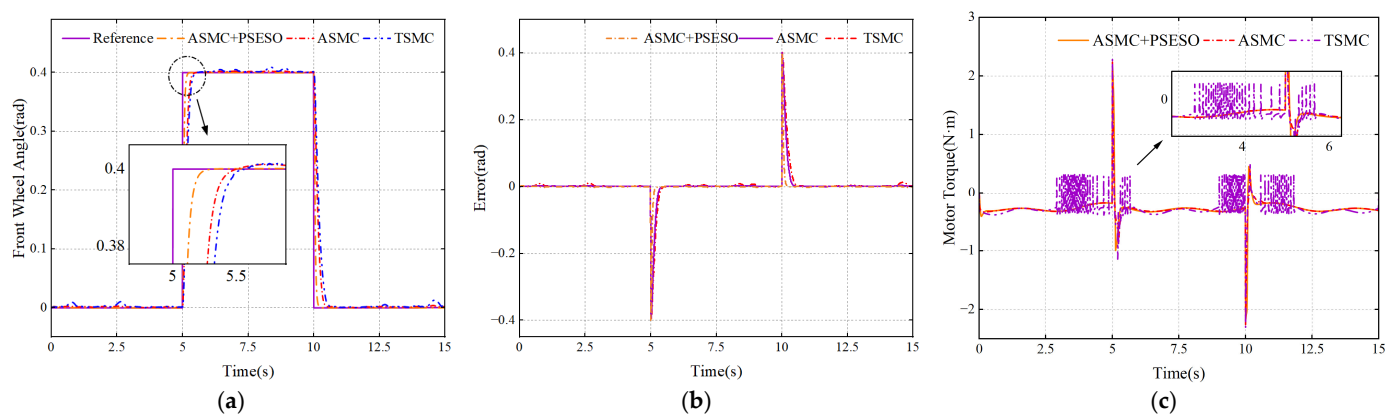
| Controllers  | MAX    | MAE    | IAE   |
|--------------|--------|--------|-------|
| ASMC + PSESO | 0.0041 | 0.0018 | 5.35  |
| ASMC         | 0.0061 | 0.0039 | 11.72 |
| TSMC         | 0.0124 | 0.0067 | 20.17 |



**Figure 7.** Comparison of tracking error metrics under sinusoidal signal.

The above simulation results indicate that the proposed method can achieve high-precision tracking of steering commands with excellent anti-interference performance, making it more suitable for SBW systems.

Case 2: step signal. The step signal simulates sudden steering commands issued by the driver or autonomous driving system in scenarios such as emergency obstacle avoidance and rapid lane changes. A step signal with an amplitude of 0.4 rad was used to evaluate the transient tracking performance of the proposed method. The simulation time was set to 15 s. Figure 8 shows the simulation results under the step signal.

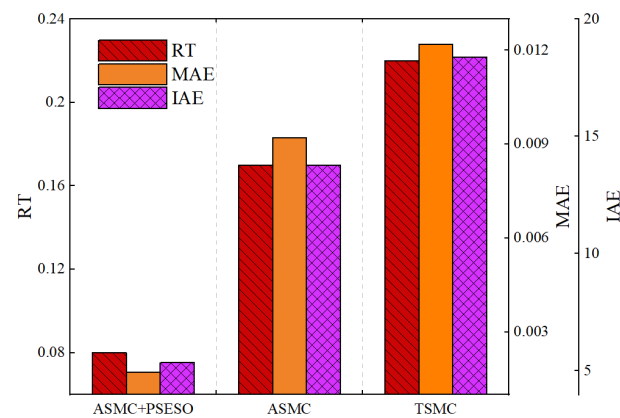


**Figure 8.** Simulation results under step signal: (a) Angle tracking curve; (b) Tracking error curve; (c) Motor torque curve.

As observed in Figure 8a,b, compared to ASMC and TSMC, the ASMC + PSESO method tracks the reference value more rapidly and smoothly with almost no overshoot. In contrast, due to the lack of disturbance compensation, both ASMC and TSMC exhibit minor fluctuations near the steady-state value. These results demonstrate the strong disturbance rejection and fast response of the proposed method.

From Figure 8c, it can be seen that in the presence of disturbances, the torque output of ASMC+PSESO remains smoother, further confirming its anti-interference capability. Additionally, while significant chattering is observed in TSMC, the ASMC + PSESO approach maintains excellent control smoothness.

Figure 9 and Table 3 provide a comparison of the tracking error metrics for the three control methods. Compared to ASMC and TSMC, the proposed method reduces the rise time by 52.9% and 63.6%, the MAE by 81.5% and 86%, and the IAE by 61.2% and 70.9%, respectively. These results validate the outstanding performance of the proposed method in tracking step-type steering angle commands.



**Figure 9.** Comparison of tracking error metrics under step signal.

**Table 3.** Tracking error metrics under step signal.

| Controllers  | RT   | MAE    | IAE   |
|--------------|------|--------|-------|
| ASMC + PSESO | 0.08 | 0.0017 | 5.22  |
| ASMC         | 0.17 | 0.0092 | 13.78 |
| TSMC         | 0.22 | 0.0122 | 28.39 |

In summary, the proposed PSESO-based ASM controller exhibits superior control performance and achieves high-precision angle tracking in the SBW system.

### 5.2. Experimental Validation

To further validate the effectiveness of the proposed method, a HIL test bench for the SBW system was developed, and relevant tests were conducted. The configured SBW HIL test bench, as shown in Figure 10, primarily consists of an operation console, an SBW actuator, a power distribution cabinet, a prototype control system, and a software platform. The operation console comprises an upper computer equipped with a 13th Gen Intel Core i7-13700KF processor (3.4 GHz), 32 GB of baseband RAM, and an NVIDIA GeForce RTX 4070 graphics card. This setup provides substantial computational power for algorithm development, real-time simulation interfacing, and data visualization. The prototype control system is implemented based on rapid control prototyping (RCP). The RCP hardware is built around a dedicated microprocessor and its peripheral circuitry, housed in a dedicated enclosure. It features multiple analog-to-digital (A/D) and digital input/output (I/O) channels to interface with sensors and actuators. The system sampling frequency is set to 1 kHz. The power distribution cabinet supplies energy to the entire HIL test bench. A servo-electric cylinder is used to simulate the steering resistance torque. The HIL test bench adopts a modular design, allowing each module to operate independently or be flexibly integrated, with real-time adjustment and monitoring of key parameters. Various sensors are integrated into the test bench for real-time data acquisition of relevant parameters. The steering angle is measured by a 17-bit photoelectric encoder, which provides high-precision angular position feedback. Communication between modules is achieved via CAN bus.

A serpentine path steering test was carried out on the HIL test bench to simulate repeated obstacle avoidance scenarios in real vehicle operation. A sinusoidal signal was used as the desired angle command. The parameters of the SBW system and the controllers remained the same as those in the simulation. The test duration was 30 s. Figure 11 shows the comparative control performance under the serpentine path.

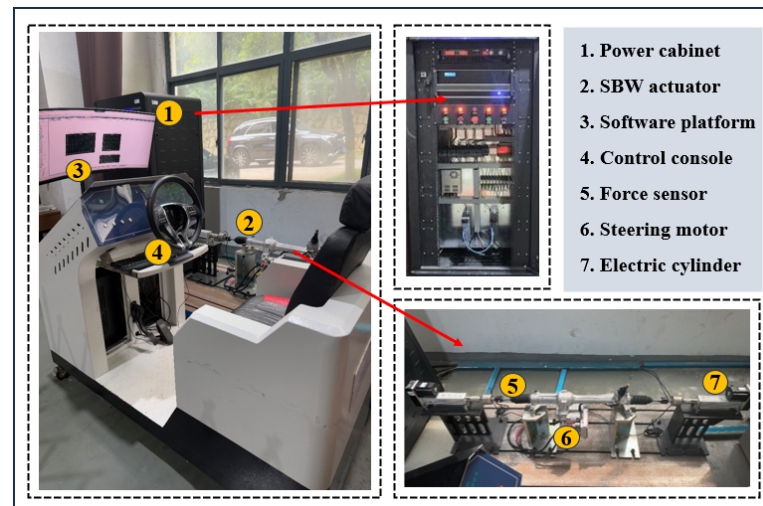


Figure 10. HIL test bench of the SBW system.

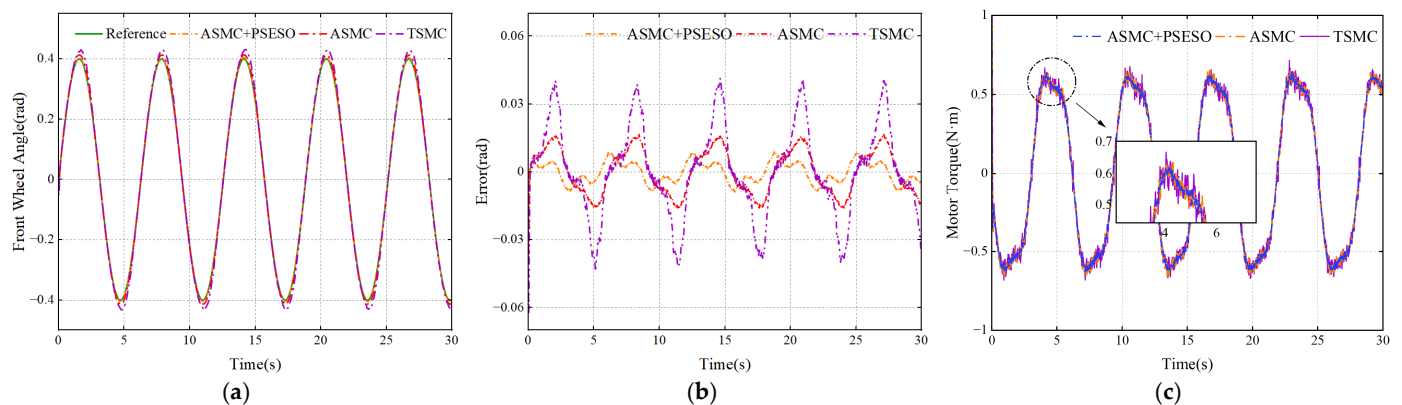


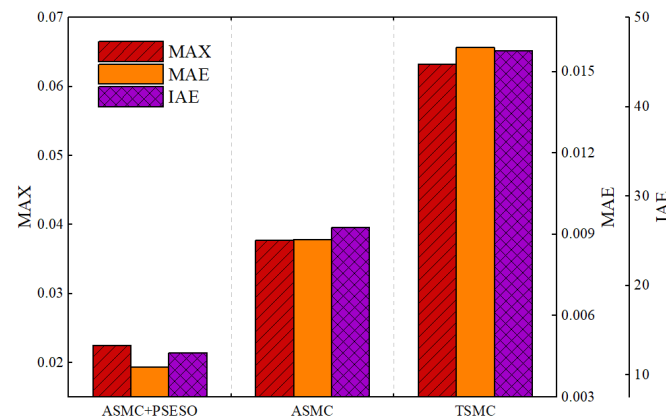
Figure 11. Experimental results of angle tracking under serpentine path: (a) tracking curve; (b) Tracking error curve; (c) motor torque curve.

As observed in Figure 11a,b, the angle tracking accuracy of ASMC + PSES0 is significantly higher than that of ASMC and TSMC, and it reaches the steady state more quickly. Compared with TSMC, ASMC reduces the tracking error, further verifying the control performance of the proposed ASML. Meanwhile, the presence of disturbances leads to a decrease in tracking accuracy. Thanks to the integration of PSES0, the control system is compensated effectively. From Figure 11c, it can be seen that ASMC + PSES0 achieves lower torque fluctuation compared to the other two methods, effectively reducing chattering. The experimental results demonstrate that the proposed method significantly improves the dynamic performance of the SBW system.

The MAX, MAE, and IAE values of the tracking errors were further calculated, and the results are summarized in Table 4. A bar chart based on these results is shown in Figure 12. It can be observed that all three error metrics of ASMC + PSES0 are superior to those of ASMC and TSMC. Specifically, the proposed ASMC + PSES0 method reduces the MAX by 40.3% and 64.4%, the MAE by 53.4% and 74.2%, and the IAE by 52.9% and 73.1%, respectively. The experimental results confirm that the proposed PSES0-based ASM controller exhibits excellent control performance and further enhances the dynamic characteristics of the SBW system.

**Table 4.** Tracking error metrics under serpentine path.

| Controllers  | MAX    | MAE    | IAE   |
|--------------|--------|--------|-------|
| ASMC + PSESO | 0.0225 | 0.0041 | 12.47 |
| ASMC         | 0.0377 | 0.0088 | 26.46 |
| TSMC         | 0.0632 | 0.0159 | 46.28 |

**Figure 12.** Comparison of tracking error metrics under serpentine path.

## 6. Conclusions

To address the angle tracking control problem of SBW systems under external disturbances and parameter uncertainties, this paper proposed a composite control strategy integrating adaptive ASMC with a PSESO. To overcome the inherent conflict between chattering and reaching speed in TSMC, a novel ASMRL was designed. By incorporating an adaptive gain function and a piecewise function, the ASMRL enhances the response speed of the control system and reduces chattering. Furthermore, a PSESO with time-varying bandwidth design was proposed to accurately estimate the lumped disturbance while suppressing the initial peak phenomenon. A feedforward compensation mechanism was introduced to further improve the angle tracking accuracy. Simulation and HIL experimental results demonstrate that the proposed method outperforms TSMC in both tracking accuracy and disturbance rejection capability. It effectively reduces steady-state error and chattering amplitude, thereby significantly improving the angle tracking performance of the SBW system. These improvements contribute to enhanced path-following capability and driving stability, which are critical for vehicle safety. In future work, a systematic comparative evaluation will be performed against representative UDE and CHO, to further validate the superiority of the proposed PSESO in terms of peak suppression and computational efficiency. Moreover, extensions to integrated vehicle control systems and fault-tolerant control in the presence of sensor or actuator faults will be explored to improve the practical applicability and functional safety of the proposed control scheme.

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