

Article

Optimization of the Location of Piezoelectric Patches Bonded on a Rotor Shaft Surface Using an Iterative Optimization Framework

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Abstract

This paper presents an optimization-based framework for active vibration control of rotor bearing systems using external surface-bonded piezoelectric patches. The rotor bearing system is modelled using the Finite Element Method (FEM), enabling the coupling between the shaft and the flexible piezoelectric actuators. A Linear Quadratic Regulator (LQR) is adopted to achieve optimal feedback control considering the balance between vibration reduction and control effort. The central contribution of this work is a comprehensive actuator placement optimization of the axial and angular position of the piezoelectric patches along the shaft. Firstly, axial positions are selected by maximizing a multi-modal weighted Modal Strain Energy (MSE) criterion over a selected number of bending modes. In the second stage, which constitutes the main novelty of this work, the angular position of each pair of bonded piezoelectric patches is optimized. Each piezoelectric pair generates control moments at each extremity of the patch. The influence of the angular separation between independent piezoelectric pairs bonded at different axial locations is investigated through an iterative optimization framework. The optimized actuator placements are subsequently employed within an LQR-based active vibration control framework. The parameters of the controller are selected using a Genetic Algorithm (GA). Numerical simulations are performed on a bi-disk flexible rotor bearing system. The results of the numerical simulations demonstrate that the combined axial-circumferential optimization significantly enhances the controllability of the rotor system and improves the multimodal vibration suppression capability, achieving an improvement of approximately 93%. The proposed methodology offers a physically meaningful and computationally efficient framework, guaranteeing symmetric and effective vibration control.

Keywords: rotor bearing system; piezoelectric pairs; active vibration control; modal strain energy (MSE); circumferential optimization; axial position; angular separation; Gramian controllability; LQR controller; genetic algorithm; finite element method; iterative optimization framework

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1. Introduction

Rotor bearing systems are widely used in rotating machinery such as turbines, compressors, electric motors, jet engines, and other high-speed rotating systems.

With the increased need for low-weight and flexible rotors, the problem of vibration reduction has become an important part of today's rotor systems design. Vibration can cause various problems in machine operation, such as the phenomenon of instabilities, resonances, mechanical wear, lack of accuracy, etc. Hence, there is currently a great need for research on vibration suppression approaches.

Among smart material technologies, piezoelectric actuators have emerged as highly effective transducers due to their lightweight characteristics, fast dynamic response, compact size, and strong electromechanical coupling properties [1–7]. These properties make piezoelectric actuators particularly suitable for active vibration control, where they are widely used to generate control forces for suppressing structural and rotor vibrations [8–14].

Surface-mounted piezoelectric actuators generate bending moments in the structure, enabling the attenuation of undesired vibrations. The efficiency of an active vibration control scheme highly relies not only on the choice of control algorithm but also on the proper placement of the actuators. Actuator misplacement may cause severe deterioration in the control effectiveness and the attainable performance of vibration suppression. Therefore, the optimization of actuator placement has received increasing attention in recent years, particularly in the context of smart structures and rotor systems, where it plays a key role in enhancing control performance and vibration suppression efficiency [15–17]. Several studies have investigated piezoelectric actuator placement strategies using different optimization approaches. For instance, Sun et al. [12] established an electromechanical coupling model for rotating carbon nanotube reinforced composite (CNTRC) cylindrical shells and implemented a Linear Quadratic Regulator (LQR) closed-loop controller for active vibration suppression. Furthermore, they optimized the layout and angular distribution of the piezoelectric patches by performing a comparative performance analysis of five typical configurations, concluding that a smaller number of patches in the circumferential direction provides superior vibration suppression when the total area of the patches remains constant. Mojtaba Biglar et al. [18] analysed the active vibration control of cylindrical shells, specifically focusing on the optimization of the axial and angular positions of surface-attached piezoelectric transducers. They argued that identifying the optimal axial and angular positions are essential because traditional random placement results in poor modal observability and inefficient energy transmission. The authors implemented the Genetic Algorithm (GA) to determine configurations that maximize the controllability Gramians for targeted vibration modes. This optimization process is designed to ensure that actuators and sensors are placed where they can provide the most effective damping while minimizing spillover effects into residual modes. This research ultimately demonstrates that transducers situated in these optimal angular and axial positions suppress structural vibrations significantly faster and more effectively than non-optimized configurations. Zaho et al. [19] considered multimodal vibration control for smart rotating cylindrical shells under circumferential moving loads by developing a coupled electro-mechanical model that incorporates Coriolis and centrifugal forces. They implemented a model reduction technique to focus on flexural vibrations, which decreased the degrees of freedom from 10 to 2 per mode to facilitate the design of an LQR closed-loop controller. To ensure control efficiency across a specified rotational speed range, the authors utilized a mini-max QPSO algorithm to optimize piezoelectric placement based on the system controllability index. Their findings highlight that the intersection points of traveling wave frequencies significantly affect controllability, and numerical results confirm that the optimized layouts successfully suppress vibrations in both body-fixed and inertial coordinate systems.

Brand and Cole [20] addressed the challenge of loss of controllability in rotating thin-walled shell caused by frequency-crossings at specific rotational speeds. To ensure effec-

tive vibration suppression across a wide operating range, they proposed a mini-max optimization approach that maximizes system controllability. This is achieved using the Particle Swarm Optimization (PSO) algorithm to determine optimal surface-mounted actuator and sensor placement. Their strategy was validated through numerical studies and experimental tests, demonstrating that it successfully maintains control performance. Mingming et al. [21] proposed an integrated vibration suppression scheme for flywheel energy storage system that utilizes a novel active-passive support architecture featuring a damping ring. This system addresses radial structural coupling, unbalanced excitation, and high-speed gyroscopic effects by deriving a 4-degree-of-freedom inverse system decoupling controller. By transforming the strongly coupled system into independent pseudo-linear subsystems, the scheme enables a segmented control strategy that applies unbalance compensation in the subcritical stage, continuous variable stiffness control in the critical stage, and complete gyroscopic decoupling in the supercritical stage. Simulation results validate that this approach reduces average steady-state tracking errors by 99.86% and resonance peaks by 81.23%, ensuring stable operation across a full speed range of 100 to 20,000 rpm. Ultimately, the proposed architecture reduces critical resonance peaks by 94.72% compared to pure mechanical bearing systems. Mohammad Jafari et al. [22] developed an optimization method to control vibrations in rotating conical shells by determining the ideal axial position of piezoelectric patches. The method focuses on minimizing the system's settling time as the objective function while adhering to static, stability, and dynamic constraints. By comparing Sequential Quadratic Programming (SQP) and Pattern Search (PS) algorithms, the authors found that Pattern Search more reliably identifies absolute extremum points for patch placement. Their optimization results concluded that the most effective vibration suppression occurs when patches are located between the middle and the base of the cone. Consequently, the previously used approaches may yield precise optimization solutions, they typically demand extensive calculations involving evaluation of the dynamic system, which is costly and time-consuming, especially in the case of larger scale finite element rotor systems. To overcome these limitations, energy-based methodologies for optimal placement are chosen to be more computationally efficient and physically meaningful [23–25]. Several studies have applied the Modal Strain Energy (MSE) criterion for actuator placement in plate structures. Porter et al. [26] demonstrated a methodology for predicting structural damping in large-scale composite structures by combining the MSE approach with vibration-based measurements of small material coupons. By validating this method against a composite beam, they achieved a significant reduction in prediction error to an average of 5.2% for the first five vibration modes, representing a substantial improvement over previous literature where errors often reached 50%. The study highlights that non-contact vibration testing provides more reliable material loss factors than traditional methods like Dynamic Mechanical Analysis (DMA). Furthermore, they introduced a novel integration of MSE with Geometrically Exact Beam Theory (GEBT), which offers a roughly 100-fold increase in computational speed, reducing simulation evaluation time from 185 s to approximately 2 s, making it highly effective for early-stage conceptual design. This approach enables a significant reduction in development time and cost by allowing accurate prediction of vibration behavior prior to full-scale prototyping.

Building upon existing studies, the present work proposes a physically guided piezoelectric actuator placement strategy that differs fundamentally from conventional approaches. In the existing literature, actuator placement optimization is generally addressed in a partial manner. For plate structures, most studies focus primarily on the axial or longitudinal positioning of bonded piezoelectric patches, whereas for cylindrical or shells, the optimization is typically restricted to the circumferential distribution. Consequently, these two aspects are commonly investigated independently.

In contrast, the present study introduces a unified framework for flexible rotor bearing systems, in which both the axial and circumferential placements of piezoelectric actuator pairs are considered simultaneously. The axial locations are determined using the modal strain energy criterion, while the circumferential arrangement is investigated through a structured parametric analysis based on the separation angle between the actuator pairs. This optimization framework enables an efficient exploration of piezoelectric actuator placement, while avoiding the complexity associated with high-dimensional optimization methods. As a result, the proposed methodology considerably reduces computational cost compared with conventional multidimensional optimization techniques, while maintaining an effective and physically meaningful piezoelectric actuator placement strategy.

2. Finite Element Modelling of the Rotor Bearing System

2.1. Description of the Controlled Rotor Bearing System

To mitigate the vibration of rotating machinery that degrades operational performance, a design of Active Vibration Control (AVC) system is proposed in this paper. An AVC system typically consists of three main components: a controller, sensors, and actuators.

In general, AVC systems employ actuators and sensors based on smart materials. Among these, piezoelectric materials are particularly attractive due to their low mass and robustness under harsh operating conditions. Piezoelectric materials exhibit a direct effect, whereby mechanical stress induces an electric charge, and an inverse effect, whereby an applied electric field generates mechanical strain.

In the present work, actuation is achieved using piezoelectric patches bonded to the external surface of the tubular rotating shaft. To conform to the cylindrical geometry of the shaft and avoid the creation of planar bonding surfaces, flexible piezoelectric patches are required. Such patches, suitable for curved surfaces, are commonly manufactured using composite or multilayer structures combining piezoelectric materials and polymer layers, which ensure sufficient flexibility, electrical insulation, and mechanical stability. The piezoelectric patches are used exclusively as actuators, exploiting the inverse piezoelectric effect. When subjected to an applied electric field, the bonded patches generate axial forces. Since the patches are positioned away from the shaft neutral axis, these forces induce bending moments at the patch extremities. One or more piezoelectric patches may be bonded to the shaft surface of the rotor bearing system. The actuation introduced by the patches produces a secondary vibration field that interacts with the primary vibration response of the rotor system, leading to vibration attenuation through destructive interference.

The rotor bearing system considered, shown in Figure 1, consists of a flexible tubular shaft with a circular cross-section supported by two flexible bearings located at its ends. The stiffness and damping coefficients of the bearings are assumed to be independent of the rotational speed. Several unbalanced disks are mounted at selected positions along the shaft, and the resulting unbalance forces are assumed to be the sole excitation sources. Proximity sensors are used to measure the lateral vibration response of the rotor at selected locations. Based on the acquired vibration signals, the LQR controller determines the required control inputs and produces the corresponding actuation voltages. These voltages are delivered to the piezoelectric actuators mounted on the rotating shaft via a slip ring mechanism, allowing the active control system to generate the forces necessary to counteract external disturbances and suppress vibrations.

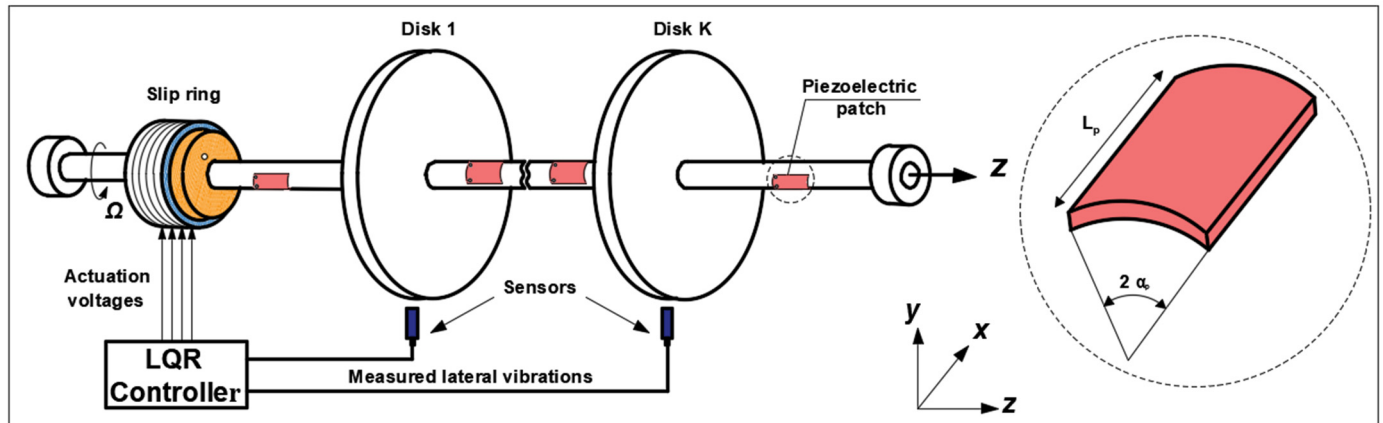


Figure 1. Model of the Active Vibration Control (AVC) rotor bearing system with piezoelectric patches, including actuator geometry.

2.2. The Rotor Dynamic Equations

The rotor is modelled using the finite element method based on the Euler-Bernoulli beam theory. This formulation takes into consideration bending deformation only while ignoring the shear deformation effect. The lateral displacement field within each beam element is approximated using cubic Hermite shape functions, which ensure continuity between adjacent elements. Each shaft node possesses four degrees of freedom, namely two lateral translations and two bending rotations $(x, y, \varphi_x, \varphi_y)$. The generalized global displacement vector $\{q(t)\}$ includes the lateral translations and rotations of all nodes.

After assembly of shaft, disks, and bearings element matrices, the global system of equations of motion of the controlled rotor system is written as follows [27]

$$[M]\{\ddot{q}(t)\} + ([C] + \Omega[G])\{\dot{q}(t)\} + [K]\{q(t)\} = \{f_{un}\} + [B_c]\{U\} \quad (1)$$

where Ω denotes the rotational speed. $[M]$, $[C]$, $[G]$ and $[K]$ are the mass, damping, the gyroscopic and stiffness matrices, respectively. These matrices are assembled from shaft, disk, and bearing contributions:

$$\begin{aligned} [M] &= [M_s] + [M_d] \\ [G] &= [G_s] + [G_d] \\ [C] &= [C_s] + [C_b] \\ [K] &= [K_s] + [K_b] \end{aligned} \quad (2)$$

where $[M_s]$ and $[M_d]$ are the mass matrices of the shaft and the disks, respectively; $[C_b]$ and $[K_b]$ are the viscous damping and stiffness matrices of the bearings; $[K_s]$ is the stiffness matrix of the shaft; $[C_s]$ is the viscous damping matrix of the shaft; $[G_s]$ and $[G_d]$ are the gyroscopic contributions from the shaft and the disks, respectively. The mass and stiffness of the piezoelectric patches are neglected. A stiffness-proportional damping model is adopted in this paper. Therefore, the damping matrix is expressed as

$$[C] = \beta[K] \quad (3)$$

where β denotes the damping coefficient. This modelling approach is commonly used in finite element rotor dynamic analyses, even though the damping mechanism may be more complex.

The unbalance force vector $\{f_{un}\}$, for a rotor having K disks fixed to the shaft is expressed as follows

$$\{f_{un}\} = \sum_{k=1}^K m_k \varepsilon_k \{e\}_k \Omega^2 e^{j(\Omega t + \beta_k)} \quad (4)$$

where β_k is the phase angle of k^{th} disk measured from the reference angular position fixed to the rotor. Each disk k , has a mass m_k and an eccentricity ε_k . When an unbalance force is applied on a disk k located at node i , this excitation acts only on the translational degrees of freedom of this node. Therefore, the corresponding position vector $\{e\}_k$ is an $(ndof \times 1)$ vector containing only two non-zero components. The $((i-1) \times 4 + 1)$ component is equal to 1 whereas the $((i-1) \times 4 + 2)$ component is equal $-j$. These components correspond respectively to the translational DOF in the x and y directions of node i .

When piezoelectric actuators are present, the external control force vector can be decomposed according to $[B_c]\{U\}$, where the control input vector is denoted by $\{U\}$, and the matrix $[B_c]$ represents the input distribution that specifies the actuator placement. This matrix maps the actuator-induced effects into the global degrees of freedom of the finite element model. Specifically, the distributed bending moments generated by the piezoelectric patches are first represented at the element level and subsequently transformed into equivalent nodal moments, which are assembled into the global force vector through standard finite element procedures.

2.3. Piezoelectric Patch Pair Modelling

The piezoelectric patch is assumed to have a rectangular shape and is bonded to the outer surface of the rotor shaft, shown in Figure 1. The geometric and physical characteristics of the piezoelectric patch and the shaft are defined through several parameters. The patch length L_p represents the axial extent of the patch along the shaft, while the patch width w_p corresponds to its arc length along the circumferential direction. The patch thickness h defines its radial dimension. From these parameters, the enclosure angle α represents the angular span of the patch on the shaft surface. The patch is assumed to be perfectly bonded to the shaft surface, ensuring full strain transfer between the rotor and the actuator. The bonding layer is considered infinitely thin, and its thickness is neglected in the designed model. These assumptions introduce limitations by neglecting effects such as adhesive compliance, local stiffness and mass variations, and electromechanical losses, which may affect the magnitude of the actuation. However, these effects do not significantly influence the spatial distribution of the dynamic response and therefore have a limited impact on the identification of optimal actuator locations.

Consider a pair of diametrically opposite patches centered at angular positions θ and $\theta + \pi$, respectively, measured from a phase reference fixed on the rotor. Diametrically opposed patches are adopted to provide balanced actuation. Each diametrically opposed actuator pair is described by a single angular design variable $\theta_k \in [0, \pi]$. The opposite patch angular location is therefore $\theta_k + \pi (\text{mod } 2\pi)$. This configuration generates pure bending moments while preventing the development of unwanted axial forces or torsional deformation.

The actuation mechanism of the piezoelectric patches is based on the inverse piezoelectric effect. When a voltage is applied, an electric field is generated within the piezoelectric patch, inducing an axial strain. Due to the bonding between the patches and the rotor surface, the free deformation of the patch is constrained by the shaft. This constraint creates a strain incompatibility between the piezoelectric patch and the shaft, which results in internal stresses at the interface. The actuation strain induced in the shaft by a pair of patches is expressed as

$$\varepsilon_s = \kappa \frac{d_{31}}{h} V_k \quad (5)$$

where d_{31} and h are the piezoelectric strain constant and the thickness of piezoelectric patch, respectively. The factor κ is obtained using the force equilibrium of the shaft and the piezoelectric patches. V_k is the actuation voltage of the k^{th} piezoelectric pair. Finally, bending moments, which are proportional to the applied actuation voltage, are generated about the x and y directions, as illustrated in Figure 2.

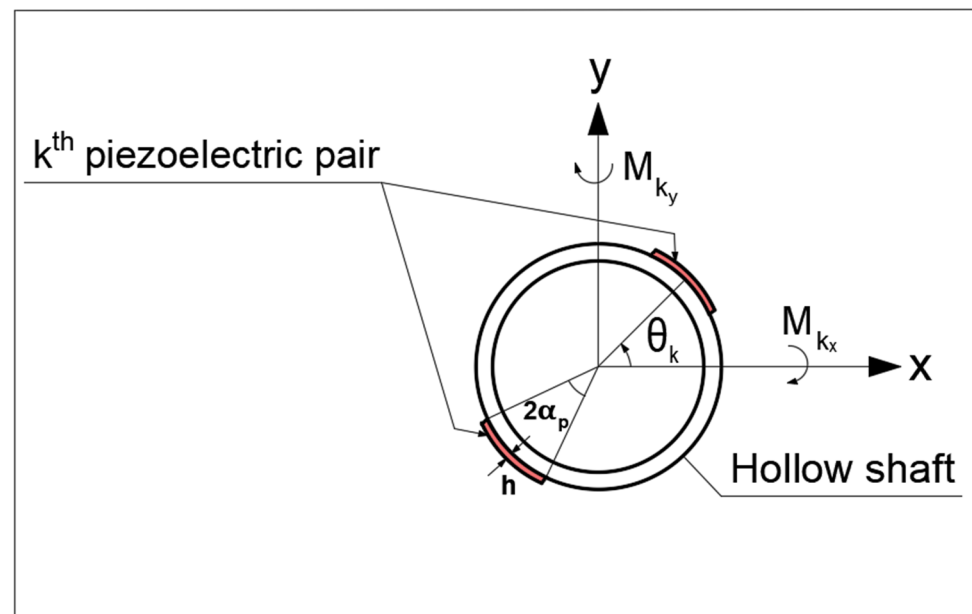


Figure 2. Cross-sectional view of the rotating shaft showing the placement of a pair of piezoelectric patches and associated control moments.

The orientation of the generated moment is governed by the angular position of the piezoelectric pair. Let θ_k denote the angular position of the center of the k^{th} piezoelectric pair measured with respect to a fixed angular position and let $M_{p,k}$ represent the amplitude of the generated moment by a pair of opposite piezoelectric patches, which can be written as:

$$M_k = M_{p,k} e^{i(\Omega t + \theta_k)} \quad (6)$$

This formulation provides a concise description of both the magnitude and orientation of the moment in the transverse plane. In addition, since the generated moment is proportional to the applied voltage V_k , it can be expressed as:

$$M_{p,k} = \delta_p V_k \quad (7)$$

where δ_p is the electromechanical coupling coefficient of the piezoelectric pair between the applied voltage and the resulting moment is expressed as follows [28,29]

$$\delta_p = \left[\frac{4 \mu E_p d_{31} [(r_o + h)^3 - r_o^3] \sin \alpha_p}{3 (1 + \mu) h} \right] \quad (8)$$

where $\mu = \frac{E_s I_s}{E_p I_p}$ and E_p, E_s are the Young's modulus of the piezoelectric patch and the rotor shaft respectively. I_s and I_p are the area moment of inertia of the shaft and the piezoelectric patch respectively. The angle of enclosure of the patch is $2\alpha_p$ as shown in Figure 1. The dimensions characterizing the section of the shaft r_i and r_o are, respectively, the inner and outer radius of the tubular shaft and $r_o = r_i + h$, where h is the thickness of the patch.

Taking the real and imaginary parts corresponds to the bending moments in the two orthogonal transverse directions for the k^{th} pair of patches.

$$\begin{cases} M_{k_x} = \Re(M_k) = \delta_p V_k \cos(\Omega t + \theta_k) \\ M_{k_y} = \Im(M_k) = \delta_p V_k \sin(\Omega t + \theta_k) \end{cases} \quad (9)$$

When N_p piezoelectric pairs are used, the total bending moment is obtained by superposition:

$$M_{tot} = \sum_{k=1}^{N_p} M_k = \sum_{k=1}^{N_p} \delta_p V_k e^{j\theta_k} \quad (10)$$

2.4. State-Space Representation of the Rotor Dynamic Model

To apply active vibration control, the equation of motion of the rotor bearing system defined in Equation (1), is written in a state-space form, which provides a framework for controller design as follows [30]:

$$\{\dot{X}\} = [A]\{X\} + [B]\{u\} + \{D\}, \quad \{Y\} = [C_y]\{X\} \quad (11)$$

$$\text{where } \{X\} = \begin{Bmatrix} q \\ \dot{q} \end{Bmatrix}; \quad \{u\} = \begin{Bmatrix} V_1 \\ \vdots \\ V_{N_p} \end{Bmatrix}; \quad [A] = \begin{bmatrix} 0 & [I] \\ -[M]^{-1}[K] & -[M]^{-1}(\Omega[G] + [C]) \end{bmatrix};$$

$$[B] = \begin{bmatrix} 0 \\ -[M]^{-1}[B_c] \end{bmatrix} \text{ and } \{D\} = \begin{Bmatrix} 0 \\ -[M]^{-1}\{F_u\} \end{Bmatrix}$$

Where $[A]$, $[B]$, $\{u\}$ and $\{D\}$ are the state matrix, the input matrix, the actuation voltage vector, and the excitation vector, respectively. Their corresponding dimensions are $(m \times m)$, $(m \times N_p)$, $(N_p \times 1)$, and $(m \times 1)$ in the same order. The variables m and N_p denote, respectively, the number of states ($m = 2 \times ndof$) and the number of control voltage inputs.

The output matrix $[C_y]$ is assembled using binary entries (0 and 1), indicating which specific state variables are selected for measurement according to position of the sensors. The dimension of matrix $[C_y]$ is $(s \times 1)$. The variable s indicates the number of outputs of interest. $[B_c]$ is the global piezoelectric input matrix.

Multiple piezoelectric pairs can be bonded on the external surface of the shaft. Each column of $[B_c]$ represents the spatial distribution of forces induced by one piezoelectric pair, making $[B_c]$ a function of the angular position of all patch pairs. Each piezoelectric pair is supplied by an independent control voltage, allowing multiple-input active control. By superposition of all piezoelectric actuator contributions, the matrix $[B_c]$ is obtained as:

$$[B_c(\theta)] = [b_1(\theta_1)V_1 \quad b_2(\theta_2)V_2 \quad \dots \quad b_k(\theta_k)V_k \quad \dots \quad b_{N_p}(\theta_{N_p})V_{N_p}] \quad (12)$$

where each column b_k represents the global force distribution associated with k^{th} piezoelectric pair. The equivalent nodal force vector for the piezoelectric pair, accounting for angular position θ_k , is expressed as follows:

$$\{b_k^e\} = \delta_p \begin{Bmatrix} 0 \\ 0 \\ \cos(\Omega t + \theta_k) \\ \sin(\Omega t + \theta_k) \\ 0 \\ 0 \\ -\cos(\Omega t + \theta_k) \\ -\sin(\Omega t + \theta_k) \end{Bmatrix} \quad (13)$$

where δ_p is defined in Equation (8).

3. Control Theory

For applying active control, an LQR controller is employed to compute the optimal feedback gain for the proposed system.

The control law is defined as

$$\{u\} = -[K]\{X\} \quad (14)$$

where $[K]$ is obtained by minimizing the quadratic performance index [31]

$$J = \int_0^\infty (\{X\}^T [Q] \{X\} + \{u\}^T [R] \{u\}) dt \quad (15)$$

where $[Q]$ and $[R]$ are the state and control weighting matrices, respectively. The matrix $[Q]$ penalizes deviations of the system states from their desired values, whereas $[R]$ penalizes the control effort. The resulting optimal control formulation provides an effective compromise between vibration suppression and actuation requirements.

In this paper, the weighting matrices are selected in diagonal form and defined as follows

$$[Q] = q_c \times [I]_{(ndof \times ndof)}, \quad [R] = r_c \times [I]_{(N_p \times N_p)} \quad (16)$$

where the coefficients q_c and r_c are then refined to achieve the targeted closed-loop dynamic performance while minimizing control effort.

Thus, Equation (15) becomes

$$J = \int_0^\infty (q_c \|X\|^2 + r_c \|u\|^2) dt \quad (17)$$

The GA implementation begins by defining the chromosome structure. Each chromosome, shown in Figure 3, consists of two genes corresponding to the weighting coefficients of the matrices $[Q]$ and $[R]$, respectively.

After defining the chromosome structure, the coefficients are tuned to minimizing the performance index defined in Equation (15). The fitness function is then formulated to be minimized under the voltage constraints imposed by the operational limits of the piezoelectric actuators, and is defined as follows [32]:

$$S = q_c \left(\sum_t^T RMS(\{X(t)\})^2 \right) + r_c \left(\sum_t^T RMS(\{u(t)\})^2 \right) \quad (18)$$

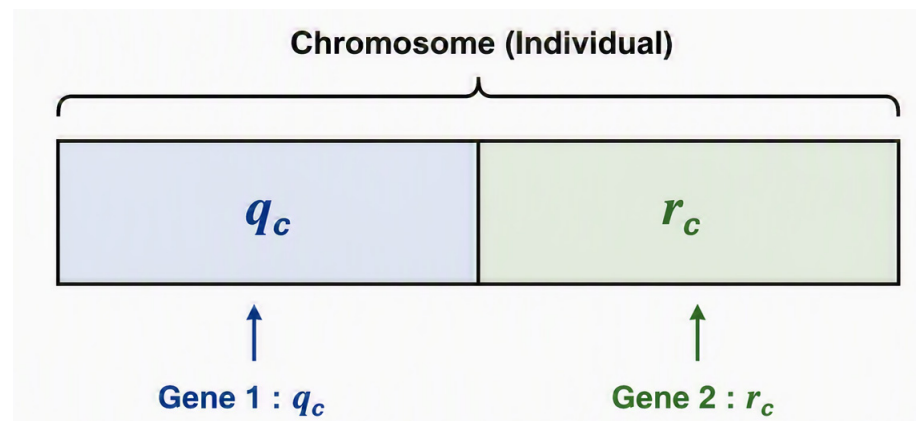


Figure 3. Chromosome representation used in the Genetic Algorithm (GA).

4. Piezoelectric Pair Placement

The effectiveness of the vibration control system depends on spatial distribution of actuators. Hence, the choice of both the axial and circumferential positioning of the piezoelectric actuators should be taken into consideration to maximize controllability of the rotor bearing system and improve vibration reduction.

4.1. Axial Position Optimization

Since the distribution of the vibration energy throughout the rotor geometry is not uniform, the locations of the actuators need to be selected where there is considerable deformation due to the vibration modes. In the current research, the optimal locations are selected using the multimodal MSE. The piezoelectric pairs will be bonded to the shaft elements having higher energy contribution.

The optimization of the axial placement of the piezoelectric patches begins with the discretization of the rotating shaft into finite elements. For a target mode shape r , the modal strain energy W_e^r stored within each shaft element e is expressed as follows:

$$W_e^r = \Re \left(\frac{1}{2} \{ \phi_e^r \}^T [K_e] \{ \phi_e^r \} \right) \quad (19)$$

where $[K_e]$ the elementary stiffness and $\{ \phi_e^r \}$ is the mode shapes at the given mode r .

The placement index γ_e^r normalizes the elemental energy against the total modal strain energy of the mode r :

$$\gamma_e^r = \frac{W_e^r}{W^r} \quad (20)$$

To ensure the active vibration control system can handle multiple modes simultaneously, aggregate the multimodal placement index expressed as follows γ_e

$$\gamma_e = \sum_{r=1}^{N_m} \omega_r \gamma_e^r \quad (21)$$

The number of piezoelectric pairs is limited by the available slip ring channels, so that the maximum number of independently controlled piezoelectric pairs cannot exceed the number of rings. Consequently, the shaft elements with the highest strain energy levels are selected as optimal axial positions for the piezoelectric pairs.

4.2. Circumferential Position and Separation Angle Between Independent Piezoelectric Pairs

Effective control due to active vibration control is achieved by positioning the piezoelectric actuators on the shaft along both the axial and circumferential directions. After selecting the optimal axial elements, the next step is to optimize the angular positions of the piezoelectric pairs on the circumference of the shaft, to achieve maximum effectiveness

in control in both transverse directions. Circumferential optimization is based on the spatial distribution of bending strains in the rotating shaft.

For a flexural rotor system, the shaft vibration occurs in two orthogonal transverse directions, namely the x and y directions. To investigate the influence of the circumferential distribution of multiple actuator pairs, different angular separations between independent pairs bonded on different shaft elements are considered.

Figure 4 shows two piezoelectric pairs distributed around the circumference of the external surface of the shaft.

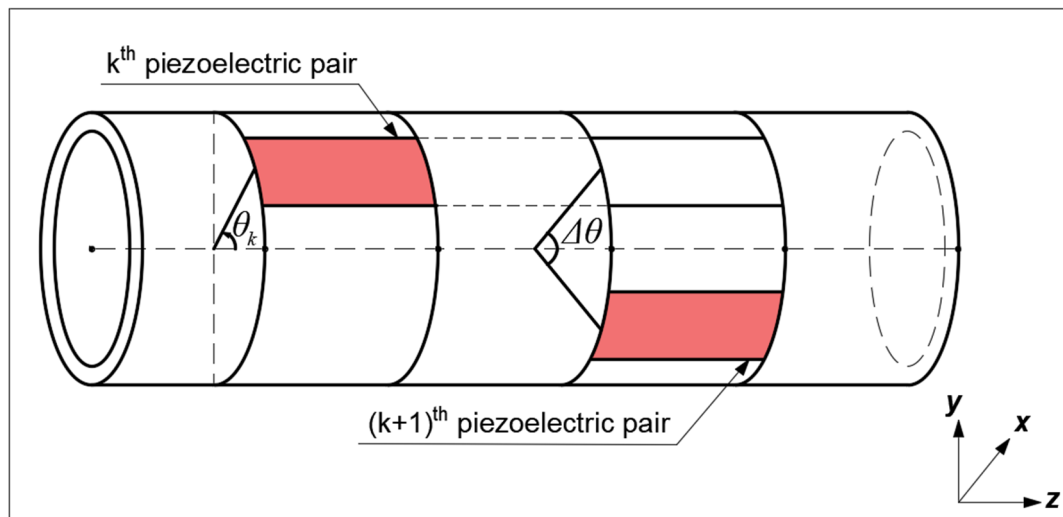


Figure 4. Circumferential placement of piezoelectric actuator pairs showing angular separation.

The separation angle between successive pairs k and $k + 1$, as shown in Figure 4, is expressed as follows:

$$\Delta\theta = |\theta_{k+1} - \theta_k| \quad (22)$$

To investigate the influence of the circumferential position of the piezoelectric pairs on the controllability of the system, the first piezoelectric pair is fixed at the reference angular position $\theta_k = 0^\circ$. The remaining pair is positioned at an angular separation $\Delta\theta$ with respect to the reference pair, as illustrated in Figure 4. Consequently, the global piezoelectric input matrix $[B_c]$ becomes a function of $\Delta\theta$, denoted $[B_c(\Delta\theta)]$:

$$[B_c(\Delta\theta)] = [b_1(\theta)V_1 \ b_2(\theta + \Delta\theta)V_2 \ \dots \ b_k(\theta + (k-1)\Delta\theta)V_k \ \dots \ b_{N_p}(\theta + (N_p-1)\Delta\theta)V_{N_p}] \quad (23)$$

The variation of the separation angle between piezoelectric pairs modifies the dynamic response of the system and directly affects the controllability Gramian, the dynamics of the system and the resulting control authority.

Since each piezoelectric pair possesses diametrically opposite symmetry, the angular configurations become repetitive beyond 180° . Therefore, the analysis is restricted to the interval:

$$0^\circ \leq \Delta\theta < 180^\circ \quad (24)$$

A parametric study over the angular separation $\Delta\theta$ is conducted. The influence of $\Delta\theta$ on the controllability and control performance of the rotor bearing system is subsequently investigated.

5. Performance Evaluation

5.1. Controllability Analysis

The optimal circumferential separation angle between independent piezoelectric pairs is primarily selected as the configuration maximizing the Gramian controllability index among the candidate configurations. This sequential optimization strategy enables the determination of circumferential actuator configurations combining high directional flexural deformation energy and strong dynamic controllability, thereby improving the active vibration suppression capability of the rotor bearing system. For the selected angular positions, the controllability properties are evaluated using the state-space model (Equation (11)). The controllability Gramian matrix is obtained from [33]:

$$[A][W_c] + [W_c][A]^T + [B][B]^T = 0 \quad (25)$$

where $[W_c]$ denotes the controllability Gramian matrix.

The controllability index is evaluated follows:

$$J_c = \log \det([W_c]) \quad (26)$$

The determinant of the controllability Gramian matrix provides a scalar measure that accounts for the controllability of all modes simultaneously. This index provides a quantitative measure of the overall control authority of the actuators. Higher values of J_c indicate that the system states can be more effectively influenced by the control inputs, reflecting improved controllability and enhanced actuation efficiency.

5.2. Closed-Loop Performance Evaluation

While the controllability index provides important information on actuator efficacy, it does not guarantee high system performance. The high controllability design might need much more effort to get the desired vibration attenuation effect, while a less controllable structure may produce the similar result without having to make such a considerable control effort. Also, the controllability measure does not give any insight about transient behavior and the required actuator voltage level.

Considering the above points, the dynamic analysis of the closed-loop system is carried out for all considered separation angles. Using the same control parameters, the rotor bearing system is simulated. The resulting dynamic responses are evaluated through three complementary criteria: vibration reduction performance, control effort, and actuator feasibility.

The effectiveness of each separation angle in reducing rotor vibrations is evaluated based on the orbit response at a selected node of the shaft. The vibration level is quantified using the root mean square (RMS) value of the orbit, computed from the lateral displacements in the orthogonal directions x and y . The RMS orbit is defined as:

$$R_{RMS} = \sqrt{\frac{1}{T} \int_0^T (x^2(t) + y^2(t)) dt} \quad (27)$$

where T is the observation time interval.

To assess the performance of the control system, the vibration reduction ratio is introduced by comparing the controlled and uncontrolled responses:

$$R_v = \frac{R_{RMS}^{uncon} - R_{RMS}^{ctrl}}{R_{RMS}^{uncon}} \times 100 \quad (28)$$

where R_{RMS}^{unc} and R_{RMS}^{ctrl} represent the RMS orbit values before and after applying the control, respectively. A larger value of R_v indicates a more effective reduction of vibrations.

While the vibration reduction ratio R_v provides a direct measure of the attenuation rate, it does not account for the actuation effort required to achieve this performance. In practical applications, it is essential to evaluate the efficiency of the control strategy by considering both vibration suppression and the required control effort, which must re-

spect the allowable voltage range of the piezoelectric patches imposed by the manufacturer. Thus, the optimal angular separation should be selected to satisfy the best compromise between control authority, reduction of vibration, control effort, and actuator operating constraints. For this purpose, a performance index J_{ratio} is introduced as:

$$J_{ratio} = \frac{R_v}{\sum_{k=1}^{N_p} PP_k} \quad (29)$$

where $\sum_{k=1}^{N_p} PP_k$ represents the total peak-to-peak voltage applied to all used piezoelectric pair. This formulation enables a comprehensive assessment of control authority by favoring configurations that maximize vibration reduction while minimizing control effort.

The control input is constrained within prescribed bounds (V_{min}, V_{max}), reflecting the physical voltage limits of the actuators and ensuring admissible operation. Consequently, a configuration is deemed feasible only if it satisfies both the imposed control bounds and the manufacturer-specified voltage range to prevent saturation.

6. Flowchart of Methodology

Figure 5 illustrates a schematic block diagram of the proposed active control strategy, outlining the complete control framework employed in this study.

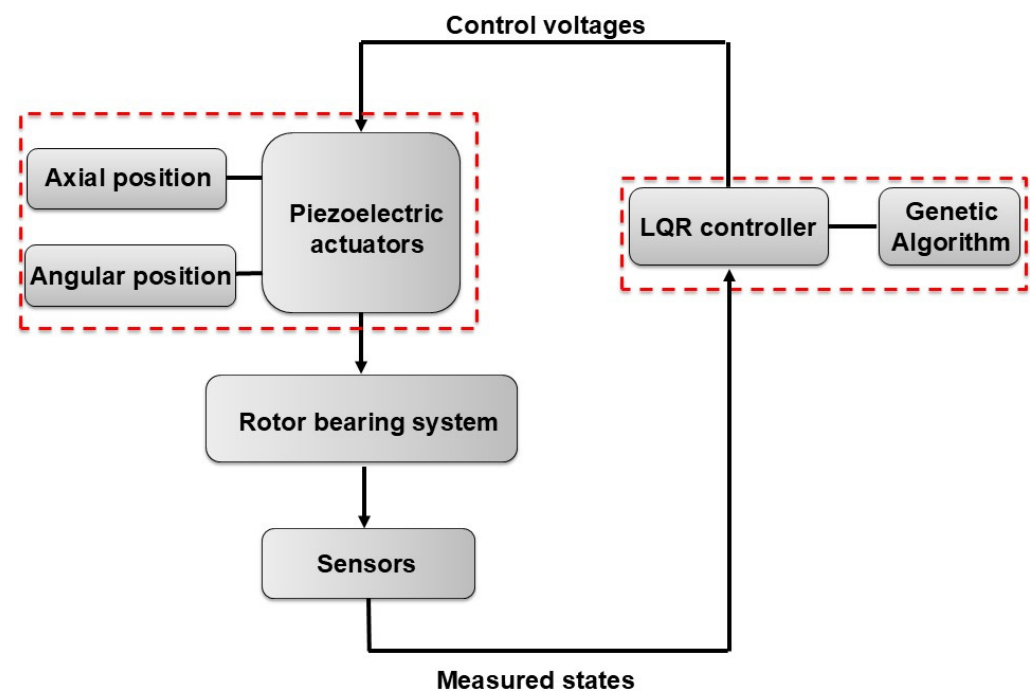


Figure 5. Block diagram of the proposed active vibration control strategy.

7. Numerical Example

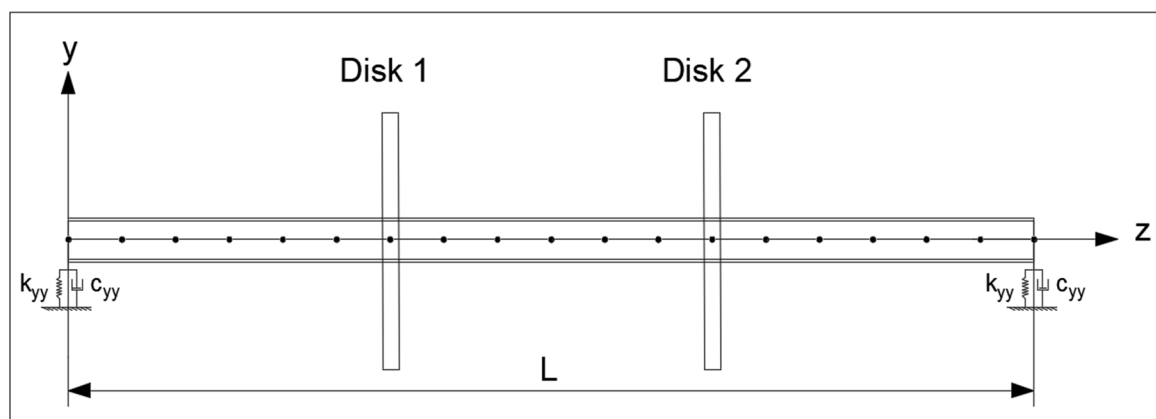
7.1. Description of the Designed System

The rotor bearing system considered in this paper is adopted from the model proposed by Braham et al. [34] and is used here as a numerical example. The system, as depicted in Figure 6, consists of a tubular steel shaft supported at both ends by flexible bearings, with two symmetrically and equally spaced disks mounted along the shaft. The corresponding properties of the system are summarized in Table 1.

Table 1. Physical properties of the rotor bearing system.

Parameter	Value
Length of shaft L	1.1 m
Outer diameter of the shaft	24 mm
Inner diameter of the shaft	22 mm
Young's modulus	210 GPa
Shaft material density	7810 Kg/m ³
Outer diameter of each disc	150 mm
Inner diameter of each disc	24 mm
Thickness of each disc	25 mm
Eccentricity of each disc	2 μ m
Unbalanced phase of disk 1	0
Unbalanced phase of disk 2	$-\pi/2$
Bearing stiffness k_{xx}, k_{yy}	1×10^6 N/m
Bearing damping c_{xx}, c_{yy}	100 N.s/m

Both bearings have identical stiffness and identical damping in both x and y directions, resulting in a symmetric rotor bearing system. This assumption enables a clearer assessment of piezoelectric pair placement effects, while avoiding bias introduced by asymmetric boundary conditions. The studied rotor bearing system is modelled using the open-source MATLAB R2023a rotor dynamics toolbox developed by Friswell et al. [35].

**Figure 6.** Finite element model of the rotor bearing system, showing the shaft discretization.

Over the operating range of 0 – 5000 RPM, the rotor bearing system displays three forward critical speeds, located close to 780 RPM, 3150 RPM and 5000 RPM.

7.2. Control Parameters

In this numerical example, the control is governed by LQR. The efficiency of the LQR controller depends heavily on the selection of the state and control weighting matrices, $[Q]$ and $[R]$. In this work, the optimal values for these matrices are determined using a GA optimization procedure, following the methodology established by Brahem et al. [32].

The GA is employed to search the optimal scalar weighting factors q_c and r_c by minimizing a multi-objective function. This objective function is designed to maximize vibration reduction of the motion of the rotor bearing system, while simultaneously constraining the control effort.

For the numerical simulations, the control weighting coefficient r_c is maintained at a fixed value of 1. So that, the optimization procedure is applied only to the state weighting coefficient q_c .

Figure 7 illustrates the evolution of the objective function S , defined in Equation (18), as a function of the number of generations during the GA optimization process. Both the average and the best values of the objective function are presented.

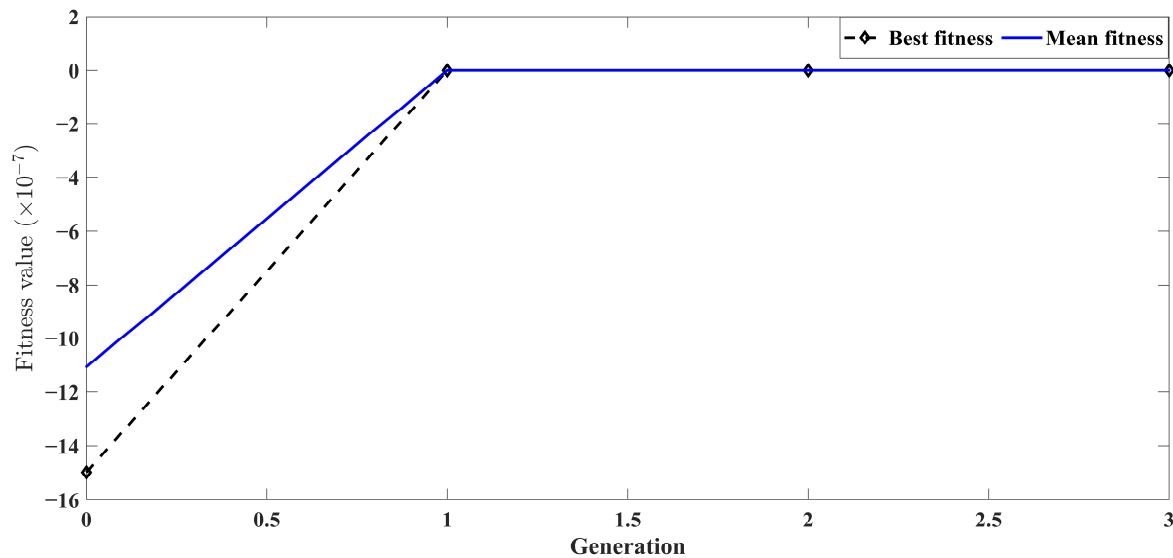


Figure 7. Fitness function evolution during the optimization process.

Figure 7 shows that the value of S undergoes a noticeable increase during the first generation, followed by a gradual stabilization, indicating convergence of the algorithm.

The optimal solution corresponds to the minimum value of the objective function while satisfying the imposed voltage constraints on the control inputs. In the present case, the best value obtained is equal to $1 \times e^9$. Consequently, the optimal weighting matrices are defined as follows:

$$[Q] = 1 \times e^9 \times [I]_{(76 \times 76)}, \quad [R] = [I]_{(4 \times 4)} \quad (30)$$

7.3. Piezoelectric Pair Placement Results

The active control elements used in this paper are DuraAct P-876.A11 piezoelectric patches (PI Ceramic GmbH, Lederhose, Germany). Each patch has a total thickness of 0.4 mm, including a 0.1 mm piezoceramic layer, and a width of 35 mm. The actuator length of 0.061 m corresponds to the distance between nodes i and $i + 1$ in the finite element model. Due to their high flexibility, these patches can be bonded to curved surfaces with a minimum radius of 12 mm; this requirement is satisfied by the 24 mm diameter shaft used in this study. When mounted, the patch covers an enclosure angle 2α of approximately 167° . The piezoelectric material is characterized by a Young's modulus of 16.4 GPa.

Given their lightweight construction of 2.1 g, the mass contribution of the patches to the global rotor system is negligible and does not significantly alter the overall dynamic behavior. These specifications ensure both high structural flexibility and strong electro-mechanical coupling.

Specifically, the optimization ensures that the required control voltage does not exceed the $-50V$ to $+200V$ operating range of the DuraAct P-876.A11 patches, imposed by its manufacturer.

7.3.1. Axial Position

The first three bending modes are considered; these modes are expected to dominate the dynamic response of the rotor bearing system. Higher-order modes are neglected because their contribution to the vibration response within the operating range considered is insignificant. Moreover, when higher modes are considered, they increase the computational cost and complicate the controller design.

To identify the shaft elements that contribute most significantly to the system dynamics, the placement index γ_e^r , defined in Equation (20), is evaluated for the first three bending modes, denote γ_e^1 , γ_e^2 and γ_e^3 . The corresponding natural frequencies are approximately 13.03 Hz, 52.5 Hz, and 80 Hz, respectively. Figure 8 illustrates the distribution of the placement index γ_e^r along the shaft elements for the first three vibration modes. The placement index quantifies the contribution of each finite element to the modal strain energy and, consequently, its suitability for piezoelectric patch placement.

The placement index γ_e^1 , presented in Figure 8a and corresponds to the first bending mode, reaches its maximum of approximately 1.15 at the shaft center, indicating locations where the modal strain energy is highly concentrated. This suggests that piezoelectric patches placed near the mid-span can achieve the highest control effectiveness for suppressing the first bending vibration mode.

Figure 8b shows the distribution of the placement index γ_e^2 , corresponding to the second mode. The shaft elements (5 and 14) associated with the highest placement index values reaching approximately 0.09, the most favorable locations for controlling vibrations related to the second mode. As shown in Figure 8c, the placement index γ_e^3 , associated with the third bending mode, exhibits a more distributed spatial pattern than those observed for the first and second modes in Figure 8a,b. Several local maxima are identified, corresponding to regions of significant modal deformation. The highest values occur at elements 9 and 10, where the placement index reaches approximately 0.08, while elements 3 and 16 exhibit slightly lower values of about 0.075. In contrast to the lower-order modes, the third mode is characterized by a more complex deformation shape, leading to multiple candidate locations for piezoelectric actuator placement.

Consequently, Figure 8 demonstrates that the optimal patch locations depend strongly on the vibration mode considered. Although the placement index distributions corresponding to the individual modes have given important insights into the optimal locations of the piezoelectric pairs, the design of the actuators according to only one mode will not guarantee satisfactory results at a range of speeds. In fact, it can be expected that the rotor will run close to several bending modes. Consequently, a multimodal placement index γ_e , defined in Equation (21), is adopted to simultaneously account for the influence of the first three bending modes. The resulting multimodal placement index is as follows:

$$\gamma_e = \varpi_1 \gamma_e^1 + \varpi_2 \gamma_e^2 + \varpi_3 \gamma_e^3 \quad (31)$$

The weighting coefficients are selected as $\varpi_1 = 0.6$, $\varpi_2 = 0.2$ and $\varpi_3 = 0.2$ giving greater importance to the first bending mode due to its dominant contribution to the overall vibration response as shown in Figure 8a, while preserving the influence of the second and third modes in the optimization process. These coefficients are empirically tuned through preliminary simulations to provide a balanced compromise between the contributions of the three dominant modes considered in the control design.

The resulting weighted placement index distribution is presented in Figure 9, which serves for the final selection axial position of the piezoelectric pair.

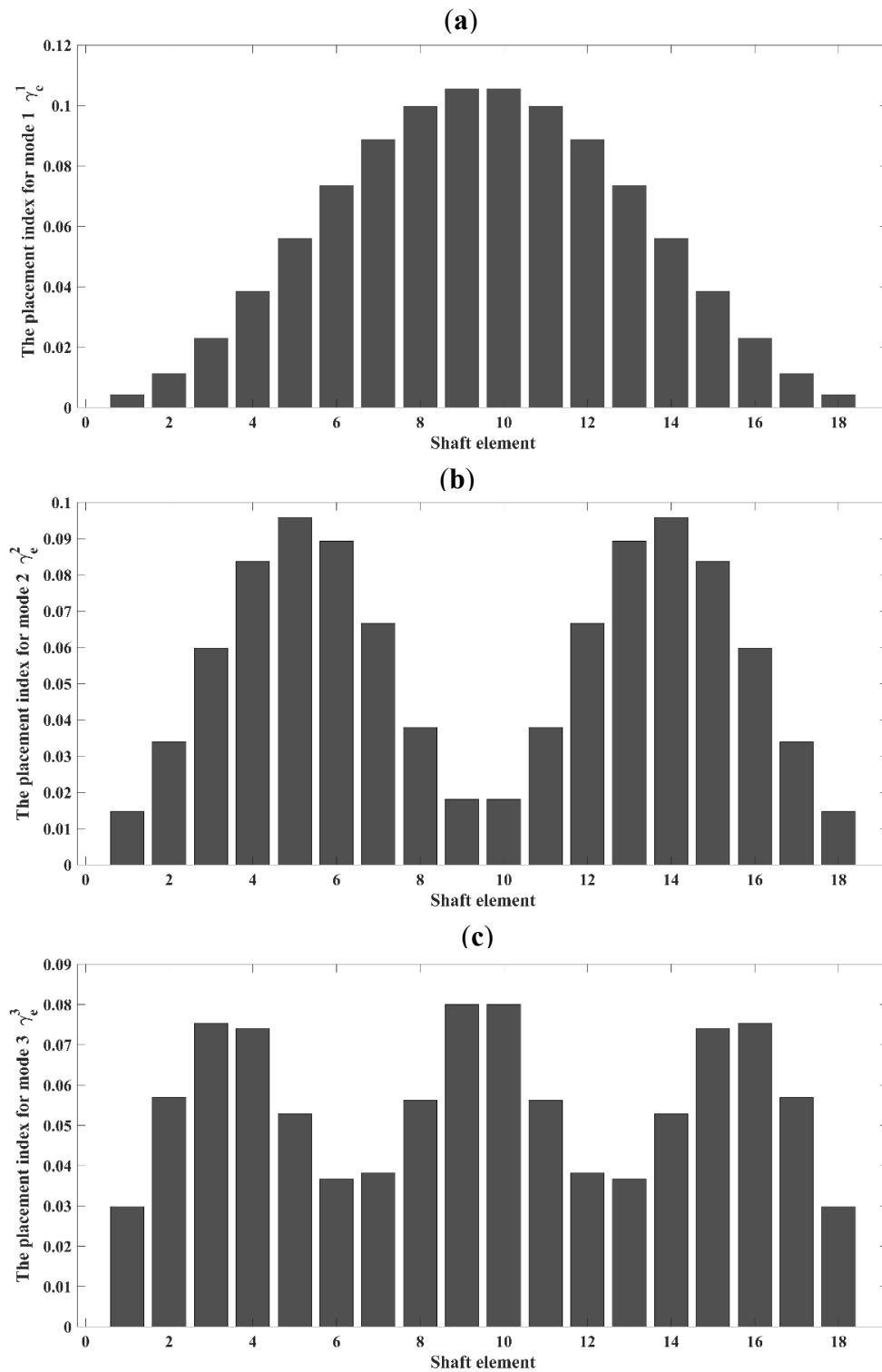


Figure 8. The placement index for three first modes: (a) mode 1; (b) mode 2 and (c) mode 3.

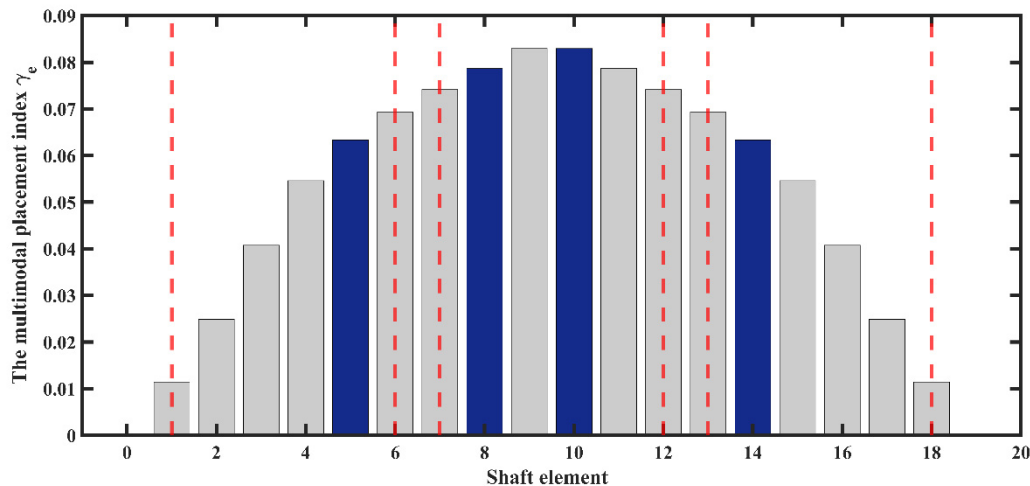


Figure 9. Multimodal placement index combining the first three bending modes, with red dashed lines indicating constrained elements.

Figure 9 shows the multimodal placement index, which is computed from the first three bending modes, weighted with the selected weighting factors. Several shaft elements exhibit elevated index values, clustering around 0.08, which suggests these are promising axial positions for bonding the piezoelectric pairs. Nevertheless, the optimal location must respect a few practical and geometric constraints which are presented with vertically dashed red line in Figure 9. First, piezoelectric patches cannot be bonded at the bearing nodes (nodes 1 and 19), since these locations correspond to the support regions of the rotor. Second, patches are not permitted on the nodes where the disks are mounted, nor on the nodes immediately preceding the disk locations, to avoid interference with the disk assemblies and to preserve the structural integrity of the system. Moreover, when several sets of piezoelectric patches are used, it is required that there is at least one shaft element between successive two independent piezoelectric pairs. Such a restriction avoids overlapping of the actuator regions and provides adequate space for installation.

Based on Figure 9, only four patch pairs are retained. This choice is motivated by the Pareto principle, whereby the selected locations correspond to the highest placement index values, ranging from 0.060 to 0.085. The blue bars in Figure 9 denote the selected axial positions (5, 8, 10 and 14), which are determined by selecting the positions with the highest multimodal placement index values from among all possible candidates that also satisfy the imposed constraint.

7.3.2. Angular Position

While the axial positions, where the piezoelectric pairs are bonded, are fixed at the selected shaft elements (5, 8, 10 and 14), the angular position of these pairs constitutes a very significant aspect of their positioning from the standpoint of vibration suppression ability. The angle by which the pairs are separated from each other defines the orientation of the active control forces applied to the shaft, affecting the controllability of the lateral bending vibrations. Consequently, the angular placement is parameterized using a single separation angle, which serves as a physically meaningful descriptor of the actuator configuration. This choice allows the dominant effects of angular positioning to be captured while preserving structural symmetry and significantly reducing the number of design variables. Although a fully decoupled optimization of individual angular positions for each bonded piezoelectric pair is feasible, it results in a complex problem with increased computational cost. The proposed approach therefore provides a structured and physi-

cally guided framework for piezoelectric pair placement, while remaining extensible to more general formulations if required.

In this numerical study, the considered separation angles $\Delta\theta$ are as follows:

$$\Delta\theta = \{0^\circ, 15^\circ, 30^\circ, 45^\circ, 60^\circ, 75^\circ, 90^\circ\} \quad (32)$$

Having defined the candidate separation angles, a parametric analysis is performed to identify the angular configuration that provides the most effective vibration suppression. The objective is to determine the separation angle that maximizes the control authority of the piezoelectric actuators, reducing vibration and respecting the operational range of voltage of the used piezoelectric patches, while maintaining the previously selected axial locations.

Each configuration is then analyzed using different performance criteria based on Gramian controllability, vibration reduction, and control effort.

7.4. Gramian Controllability

The influence of the angular separation angle on the Gramian controllability index is investigated. The controllability index is calculated for each tested separation angle. The resulting values of J_c are shown in Figure 10.

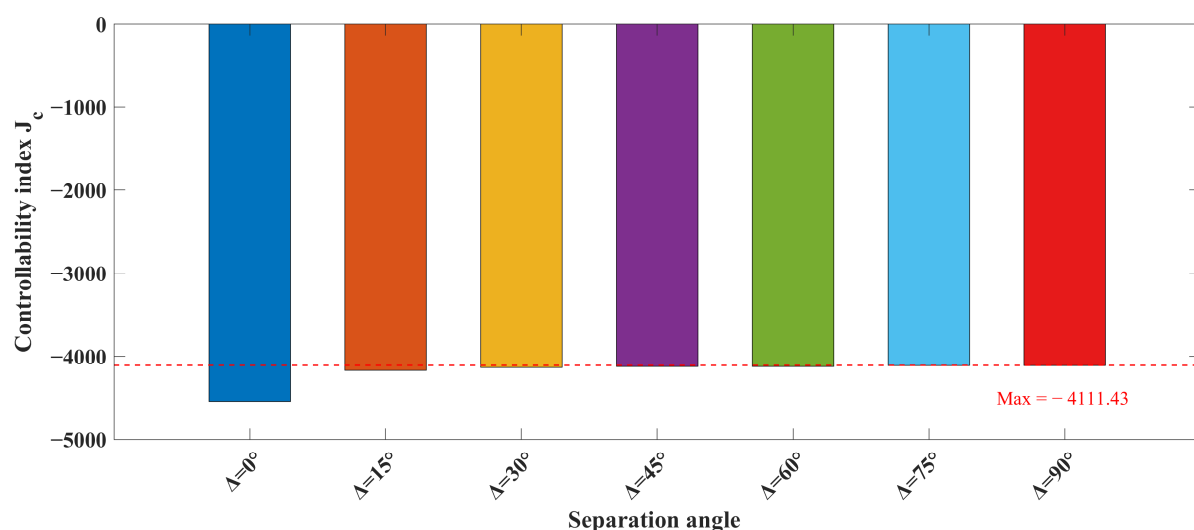


Figure 10. Variation of the controllability index as a function of the angular separation angle.

As illustrated in Figure 10, the controllability index exhibits a dependence on the angular separation angle. This demonstrates that the angular position of the piezoelectric pairs affects the degree of control authority that can be exerted on the rotor bearing system. The controllability index reaches its maximum value at a separation angle $\Delta\theta$ of 90° , suggesting that this configuration provides the most effective coupling between piezoelectric pair and the shaft and, consequently, the highest level of controllability. A larger J_c means a larger Gramian determinant and therefore better controllability.

7.5. Closed-Loop Response

While the controllability index provides insight into the theoretical control authority of each separation angle configuration, its practical effectiveness must also be assessed in terms of vibration reduction. Therefore, the influence of the separation angle on the vibration response of the controlled rotor bearing system is investigated. Figure 11 presents the superposed controlled rotor orbits corresponding to the different separation angles.

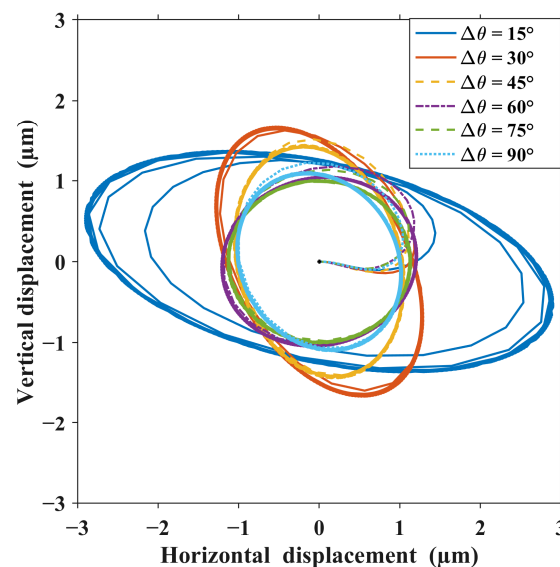


Figure 11. Effect of the angular separation angle on the controlled rotor orbit response.

Figure 11 shows a clear variation in the orbit size when the separation angle varies, indicating that the angular position of the piezoelectric pairs significantly affects the vibration response of the rotor bearing system. The figure reveals that for low separation angle ($\Delta\theta = 15^\circ$), the orbit is significantly elongated along the x direction, indicating inadequate control in the orthogonal direction. As the separation angle increases to 30° and 45° , the orbit exhibits pronounced vertical elongation, reflecting a strong anisotropy in the system response. For larger separation angles (60° and 75°), the orbit progressively contracts and approaches a quasi-circular shape, indicating a partial transition toward isotropic behavior, although slight anisotropy persists. The fully developed isotropic response, characterized by a nearly perfect circular orbit with reduced amplitudes in both directions, revealing an optimal distribution of control effort, is achieved for the orthogonal configuration ($\Delta\theta = 90^\circ$).

Consequently, the reduction in orbit size for separation angles ($\Delta\theta = 60^\circ, 75^\circ, 90^\circ$) demonstrates a more effective interaction between the piezoelectric pairs and vibrations of the rotor bearing system. Conversely, larger orbit trajectories, corresponding to the separation angles ($\Delta\theta = 15^\circ, 30^\circ, 45^\circ$), indicate a less efficient coupling between piezoelectric pairs and the rotor bearing system. These results confirm that the angular position of piezoelectric pairs constitutes a key design parameter in determining the effectiveness of the active vibration control system.

To further assess the impact of the angular position, the voltage response, generated by the LQR controller and sent to the piezoelectric pairs, is analyzed for each configuration. Figure 12 illustrates the actuation voltage response obtained for the different separation angles. For each separation angle, the peak-to-peak actuation voltage is evaluated for each piezoelectric pair. In subfigure (b), the voltage limits for all actuator pairs are displayed, illustrating the global maximum ($V_{\max}$) and minimum ($V_{\min}$) bounds.

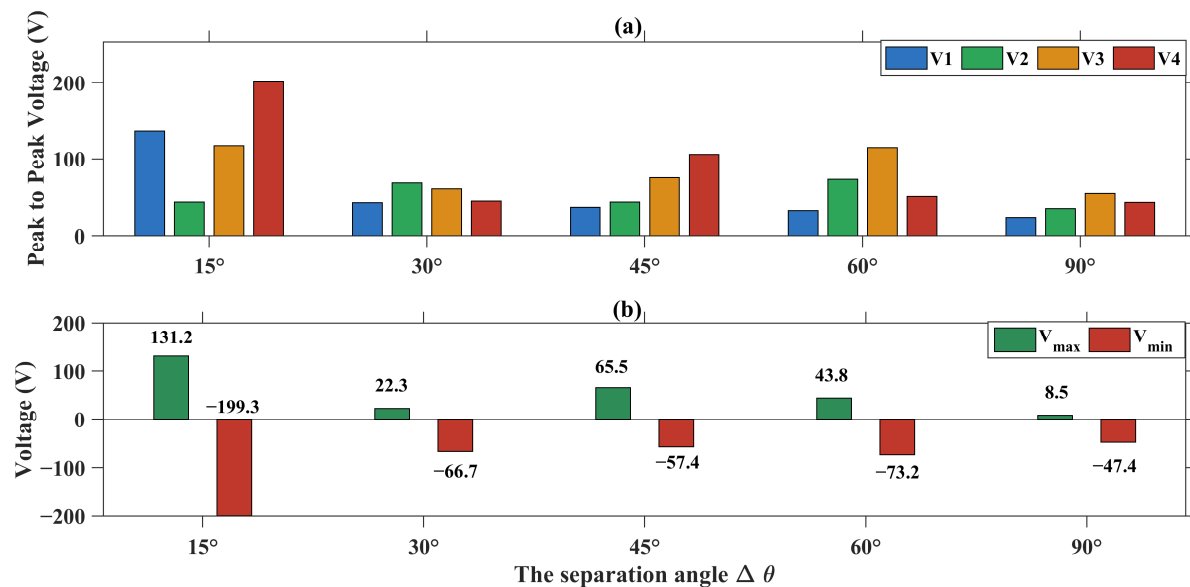


Figure 12. Influence of the angular position on the control effort: (a) Peak-to-Peak voltage PP_k ; (b) Bounds of voltage amplitudes ($V_{\max}$, $V_{\min}$).

The voltage metric analysis, presented in Figure 12, demonstrates a pronounced dependence of the control effort to the angular position of the piezoelectric pairs. The maximum peak-to-peak voltage PP_k exhibits a substantial decrease, dropping from approximately 202 V (observed for V_4) at $\Delta\theta = 15^\circ$ to about 54.5 V (for V_3) at $\Delta\theta = 90^\circ$, representing an overall reduction of nearly 73% in control effort. Furthermore, when the separation angle increases, it leads to a homogeneous distribution of actuation voltages among all pairs, reflecting an improved balance in control effort, contrary to the distribution observed at 15° . In addition, from a constraint perspective, only the configuration at 90° satisfies the admissible voltage limits ($-50 \text{ V} \leq V \leq 200 \text{ V}$), whereas all other configurations exhibit violations. The most critical case occurs at the separation angle equal to 15° , where the minimum voltage exceeds the lower bound by 149.3 V, highlighting a severe departure from feasible operating conditions. This causes actuator saturation, which impairs control performance. These quantitative results demonstrate that increasing the separation angle leads to reduced actuation demand, improved uniformity, and compliance with voltage constraints. Thus, the separation angle $\Delta\theta = 90^\circ$ is the optimal configuration.

To assess the balance between vibration reduction and control effort, the evolution of the performance index J_{ratio} is examined as a function of the separation angle in Figure 13.

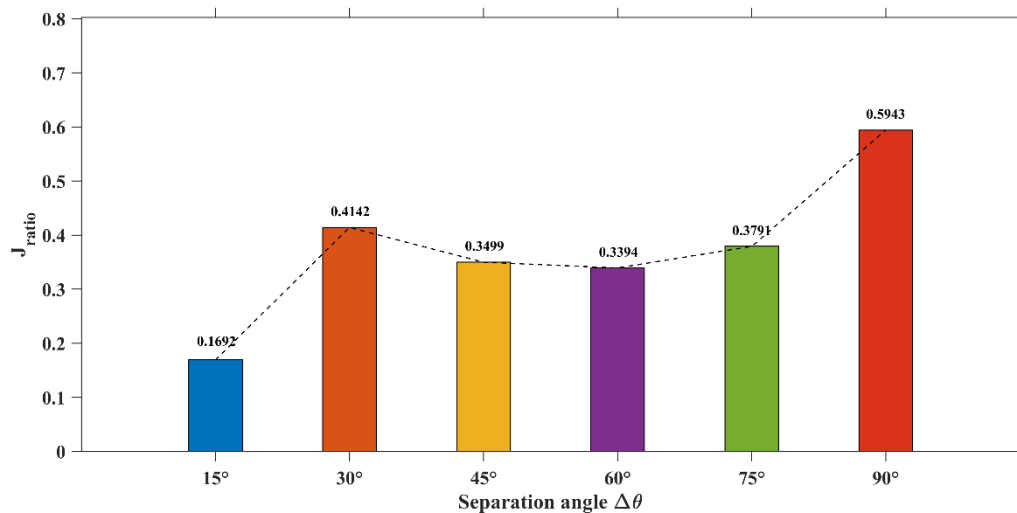


Figure 13. Variation of the performance index J_{ratio} as a function of the separation angle, with the dashed line highlighting the trend.

The results of Figure 13 show a significant increase from approximately 0.16 at $\Delta\theta = 15^\circ$ to a maximum value of almost 0.59 at $\Delta\theta = 90^\circ$, corresponding to an improvement of approximately 73%. Intermediate configurations ($\Delta\theta = 30^\circ, 45^\circ, 60^\circ, 75^\circ$), where J_{ratio} remains in a relatively narrow range (0.37 – 0.41), reflecting moderate efficiency. A pronounced peak appears at $\Delta\theta = 90^\circ$.

Consequently, the evolution of the performance index J_{ratio} reflects the efficiency of the coupling between the piezoelectric pairs and the rotor bearing system. The low values observed at small separation angles ($\Delta\theta = 15^\circ, 30^\circ, 45^\circ, 60^\circ, 75^\circ$) indicate poor alignment between control forces and the dominant vibration directions, resulting in inefficient use of control energy. As the angle increases, the actuators become progressively better aligned with the modal deformation patterns, enhancing the coupling between actuation voltages and rotor dynamic response. This leads to improved vibration reduction for a reduced voltage demand. The maximum value obtained at $\theta = 90^\circ$ corresponds to an optimal configuration in which actuator forces are distributed along the principal directions of motion, ensuring balanced and highly efficient vibration suppression.

Based on the proposed parametric study, the separation angle $\theta = 90^\circ$ is selected for the subsequent investigations. This configuration provides the highest performance, simultaneously achieving the lowest vibration amplitudes, the most stable orbit shape, and the minimum and well-balanced voltage requirements within the admissible limits. In addition, it yields the highest efficiency in terms of vibration reduction relative to control effort, as reflected by the maximum value of the performance index J_{ratio} . Therefore, the separation angle $\Delta\theta = 90^\circ$ is identified as the optimal actuator configuration.

The optimal separation angle of $\Delta\theta = 90^\circ$ obtained in this study is physically consistent with the isotropic nature of the rotor bearing system, where identical stiffness properties in the orthogonal directions lead to symmetric vibration behavior. In such conditions, an orthogonal piezoelectric pair configuration ensures balanced control authority in both lateral x - and y -directions. However, for anisotropic systems, the optimal separation angle may deviate from 90° , as the piezoelectric angular position must align with the dominant modal deformation to maximize control efficiency.

8. Simulation Results with Optimal Piezoelectric Pair Placement

The effectiveness of the optimal piezoelectric configuration is highlighted in Figure 14, which compares the uncontrolled and controlled rotor orbits at the first critical speed

using the horizontal displacement of node 7, where the first disk is mounted on the rotor bearing system.

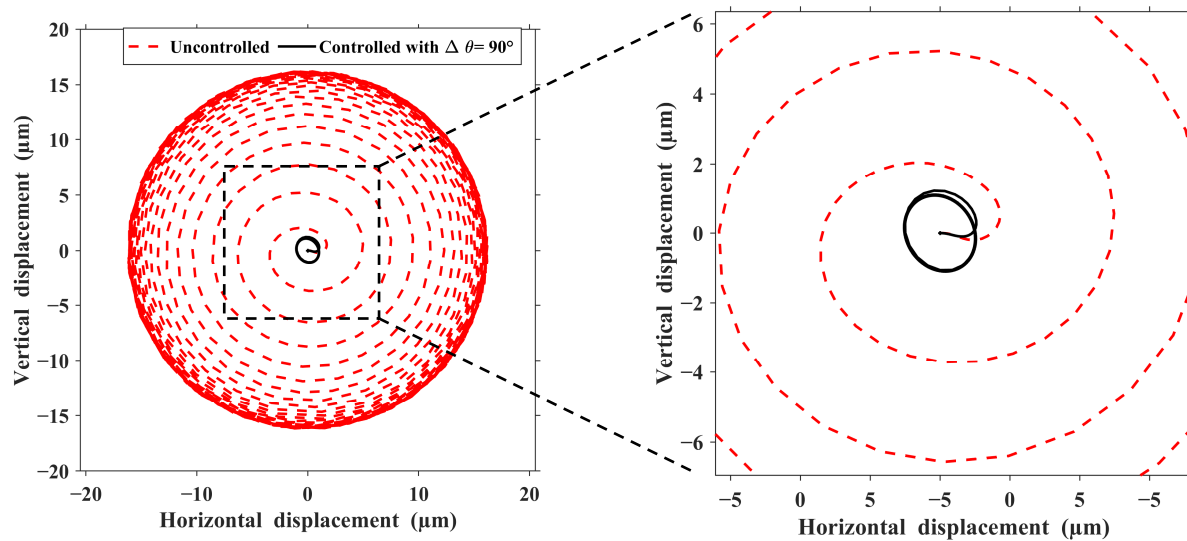


Figure 14. Uncontrolled and controlled rotor orbits at the first critical speed with separation angle $\Delta\theta = 90^\circ$.

Figure 14 shows that, before applying the control, the orbit exhibits a large circular trajectory with a radius of approximately $\pm 16.18 \mu\text{m}$, indicating strong vibration amplification at the first critical speed. When the control is applied with the optimal separation angle ($\Delta\theta = 90^\circ$), the orbit is reduced to a very small trajectory with a radius close to $\pm 1 \mu\text{m}$. This corresponds to a reduction of more than 93% in vibration amplitudes.

From a physical perspective, the large circular orbit in the uncontrolled case reflects the isotropic dynamic behavior of the rotor, where vibration energy is uniformly distributed in both lateral directions due to resonance. When applying the control, the piezoelectric pairs generate control forces that counteract the motion of the rotor bearing system. The optimal angular separation of 90° ensures that these control forces are effectively distributed along the two orthogonal directions, allowing balanced and efficient suppression of vibration. As a result, the radius of the orbit is significantly reduced, leading to a compact and centered trajectory. This demonstrates that the optimal actuator orientation maximizes the coupling with the rotor's modal response and enables an efficient conversion of actuation voltage into vibration reduction.

The RMS vibration amplitude at node x_7 is evaluated over a wide range of rotational speeds, as depicted in Figure 15, to compare different actuator placement strategies, including uncontrolled, random placement of piezoelectric pairs, axially optimized pairs, and combined axial-angular optimized configurations.

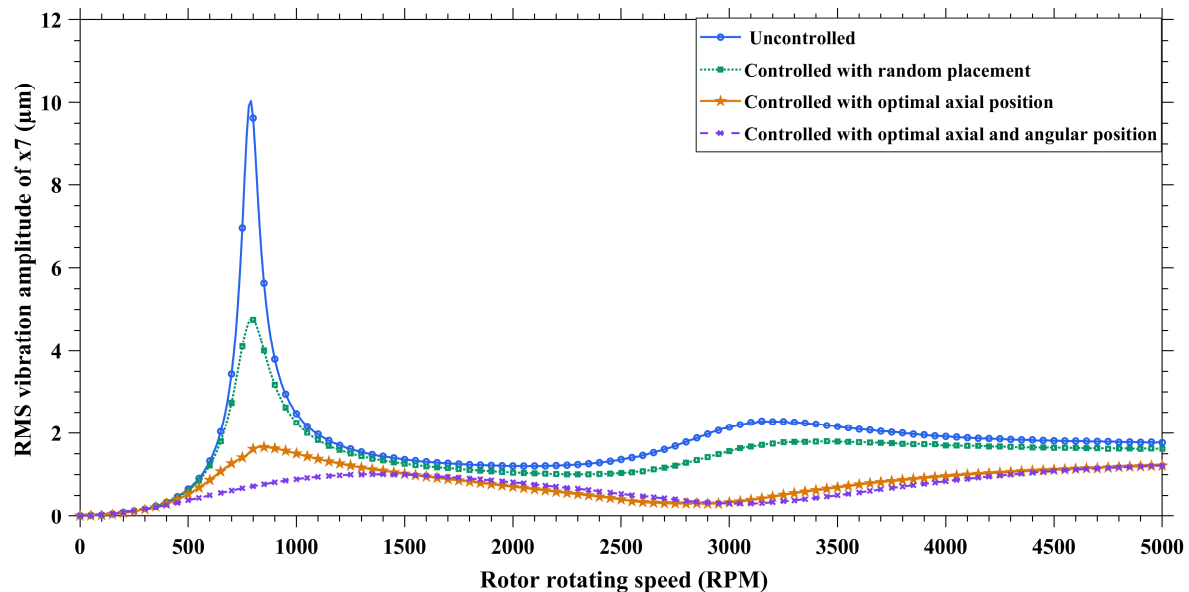


Figure 15. Steady-state RMS value of the horizontal displacement at node 7 over the rotor speed range, comparing the present results with the controlled responses reported in References [34] and [36].

The RMS vibration response, presented in Figure 15, exhibits a pronounced resonance peak at the first critical speed, located around 780 RPM , where the uncontrolled system reaches an amplitude of approximately $10 \text{ }\mu\text{m}$. The application of active control significantly attenuates these resonance peaks, with performance strongly dependent on the configuration and control strategy of the piezoelectric actuator pairs. The green curve corresponds to the configuration proposed by Brahem et al. [34], where piezoelectric pairs are bonded at the mid-span of the shaft using a fixed placement strategy. In this case, only the first bending mode is actively controlled. This approach reduces the first peak amplitude to approximately $4.7 \text{ }\mu\text{m}$ and the second peak to about $1.7 \text{ }\mu\text{m}$, showing limited effectiveness beyond the targeted mode. The brown curve represents the approach introduced by Brahem et al. [36], in which the axial positions of the piezoelectric pairs are optimized based on the modal strain energy criterion to control the first two bending modes. In this strategy, greater emphasis is placed on the first mode. This configuration leads to a more substantial reduction, with amplitudes reaching approximately $1.3 \text{ }\mu\text{m}$ at the first peak and $0.4 \text{ }\mu\text{m}$ at the second peak. Finally, the purple curve illustrates the results of the present work. Here, both axial and angular positions of the actuator pairs are simultaneously optimized, and the control strategy is extended to the first three bending modes. This comprehensive approach yields the most significant improvement, reducing the vibration amplitudes to approximately $0.71 \text{ }\mu\text{m}$ at the first peak and $0.32 \text{ }\mu\text{m}$ at the second peak. These results clearly demonstrate the enhanced effectiveness of the proposed configuration compared to existing strategies, highlighting the benefits of multimodal control combined with full spatial optimization of actuator placement.

This demonstrates the effectiveness of the proposed methodology in enhancing vibration attenuation compared to existing strategies. The percentage of vibration reduction obtained for different piezoelectric placement strategies at the first resonance peak corresponding to Figure 15, is presented in Figure 16, to quantitatively evaluate the effectiveness of each approach.

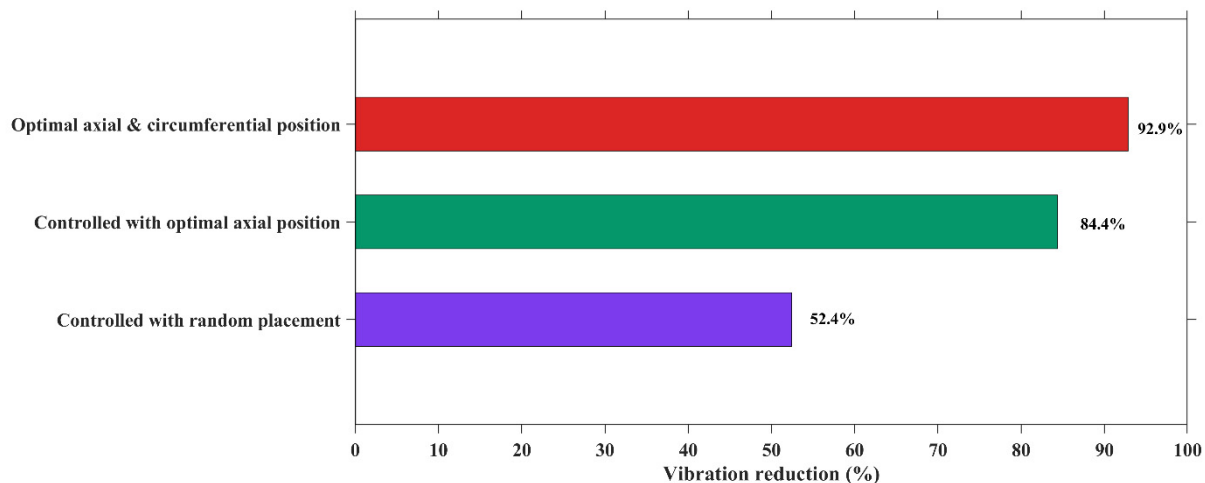


Figure 16. Vibration reduction percentage for different actuator placement strategies based on FRF analysis.

The results, presented in Figure 16, show a significant improvement in vibration reduction when optimizing simultaneously the axial and the angular position of the piezoelectric pairs. The configuration with random placement achieves a reduction of nearly 52%, indicating limited efficiency due to poor alignment with the vibration modes. Optimizing the axial positions increases the reduction close to 84%, corresponding to an improvement of approximately 32% compared to the random case. The best performance is obtained when both axial and circumferential positions are optimized, reaching a maximum reduction of approximately 93%, which represents an additional gain of about 9% over axial optimization and nearly 40% compared to random placement.

From a physical standpoint, this improvement between strategies can be explained by the enhanced modal interaction achieved as the number of controlled modes increases. In the random configuration, the actuators are not properly aligned with the modal deformation, leading to partial vibration cancellation. Axial optimization improves performance by placing the piezoelectric pairs in regions of high modal strain energy, enabling more efficient interaction with the dominant modes. The combined axial and circumferential optimization further enhance this effect by ensuring that actuator forces are both spatially and directionally aligned with the modal displacements. This results in a more effective transfer of energy from the control system into vibration suppression, leading to the highest reduction levels observed. Although the proposed numerical model demonstrates promising performance for active vibration suppression, its experimental validation on a dedicated rotor test rig equipped with bonded piezoelectric patches and real-time control hardware remains an important direction for future research, enabling quantitative comparison between numerical predictions and measured responses.

9. Conclusions

This paper investigates the optimal placement of piezoelectric actuator pairs for active vibration multimodal control in rotor bearing systems. The main contribution lies in the combined optimization of axial and angular positions, along with the introduction of a performance index that integrates vibration reduction and control effort. Based on a computationally efficient parametric study, the proposed approach avoids complex optimization procedures while accurately identifying the optimal configuration.

The results demonstrate that piezoelectric pair placement, both axial and angular, has a significant impact on control performance. The optimal configuration identified in

this study corresponds to axially positioned piezoelectric patch pairs, determined using the MSE approach to effectively control the first three vibration modes. Furthermore, a uniform angular separation of 90° between adjacent pairs yields the best steady-state performance. It reduces the amplitude of the first resonance to nearly 92.9% and up to 86% for the second resonance peak, while minimizing the actuation voltage demand.

From a physical perspective, the effectiveness of the control strategy is governed by the alignment between control forces and modal deformation patterns. Multimodal control further enhances performance by reducing both dominant directions of vibrations.

Overall, this work provides a robust theoretical framework for efficient piezoelectric pair placement and can be extended to anisotropic systems and more complex configurations. It emphasizes the importance of both axial and angular positions in achieving high control efficiency, while offering a computationally efficient and practical methodology suitable for engineering applications. These findings offer valuable insights into the design of smart rotor systems and open the way for future research, including the extension to anisotropic systems. Finally, experimental validation on real test rigs and the consideration of nonlinear effects, uncertainties, and actuator limitations would strengthen the practical application of the proposed approach. The present study is therefore a preliminary design stage, aimed at establishing a consistent and computationally efficient numerical framework that can serve as a basis for subsequent experimental implementation and validation.

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References

1. Jin, H.; Gao, X.; Ren, K.; Liu, J.; Qiao, L.; Liu, M.; Chen, W.; He, Y.; Dong, S.; Xu, Z.; Li, F. Review on Piezoelectric Actuators Based on High-Performance Piezoelectric Materials. *IEEE Trans. Ultrason. Ferroelectr. Freq. Control* **2022**, *69*, 3057–3069. <https://doi.org/10.1109/TUFFC.2022.3175853>. PMID: 35580107.
2. Jha, A.K.; Inman, D.J. Piezoelectric actuator and sensor models for an inflated toroidal shell. *Mech. Syst. Signal Process.* **2002**, *16*, 97–122. <https://doi.org/10.1006/mssp.2001.1442>.
3. Dong, H.; Zhu, Z.; Li, Z.; Li, M.; Chen, J. Piezoelectric Composites: State-of-the-Art and Future Prospects. *JOM* **2024**, *76*, 340–352. <https://doi.org/10.1007/s11837-023-06202-w>.
4. Zhou, X.; Wu, S.; Wang, X.; Wang, Z.; Zhu, Q.; Sun, J.; Huang, P.; Wang, X.; Huang, W.; Lu, Q. Review on piezoelectric actuators: Materials, classifications, applications, and recent trends. *Front. Mech. Eng.* **2024**, *19*, 6. <https://doi.org/10.1007/s11465-023-0772-0>.
5. Wu, T.; Hu, W.; Zhang, J.; Zhang, Z.; Cheng, C.; Wu, J.; Feng, Y. Research on Hysteresis Modeling and Nonlinear Compensation of Piezoelectric Actuators. *J. Vib. Eng. Technol.* **2025**, *13*, 52. <https://doi.org/10.1007/s42417-024-01623-6>.
6. Kovalovs, A.; Wesolowski, M.; Barkanov, E.; Gluhihs, S. Application of macro-fiber composite (MFC) as a piezoelectric actuator. *J. Vibroengineering* **2009**, *11*, 105.
7. Wang, Q.; Wu, N. A review on structural enhancement and repair using piezoelectric materials and shape memory alloys. *Smart Mater. Struct.* **2012**, *21*, 013001. <https://doi.org/10.1088/0964-1726/21/1/013001>.

8. Ray, M.C.; Reddy, J.N. Active control of laminated cylindrical shells using piezoelectric fiber reinforced composites. *Compos Sci. Technol.* **2005**, *65*, 1226–1236. <https://doi.org/10.1016/j.compscitech.2004.12.027>.
9. Hashemi, A.; Jang, J.; Hosseini-Hashemi, S. Smart Active Vibration Control System of a Rotary Structure Using Piezoelectric Materials. *Sensors* **2022**, *22*, 5691. <https://doi.org/10.3390/s22155691>. PMID: 35957246.
10. Wasim, M.; Li, X. Model predictive control of piezoelectric intelligent curved shell structure using reduced order model. *Eng. Res. Express* **2026**, *8*, 075501. <https://doi.org/10.1088/2631-8695/ae5449>.
11. Li, C.; Li, P.; Miao, X. Research on nonlinear vibration control of laminated cylindrical shells with discontinuous piezoelectric layer. *Nonlinear Dyn.* **2021**, *104*, 3247–3267. <https://doi.org/10.1007/s11071-021-06497-x>.
12. Sun, S.; Zhao, X.; Yang, Y.; Cao, D. Active vibration control of rotating carbon nanotube reinforced composite cylindrical shells via piezoelectric patches. *JVC/J. Vib. Control* **2025**, *31*, 3104–3125. <https://doi.org/10.1177/10775463241260081>.
13. Jalalnejhad, M. Active vibration control and optimized flutter suppression in missile tail structures using piezoelectric layers. *J. Braz. Soc. Mech. Sci. Eng.* **2026**, *48*, 337. <https://doi.org/10.1007/s40430-025-06274-6>.
14. Huang, C.; Liu, Y.; Zhai, J.; Chi, W.; Sun, X. Active Piezoelectric Control of Three-Dimensional Vibration in a Flexible Circular Shaft via a Fuzzy Adaptive PID Algorithm. *Actuators* **2026**, *15*, 226. <https://doi.org/10.3390/act15040226>.
15. Wang, S.; Jin, H.; Wang, Y.; Huo, J.; Liu, X. Optimizing piezoelectric actuator placement for enhanced vibration control using genetic algorithms. *Sci. Rep.* **2025**, *15*, 22340. <https://doi.org/10.1038/s41598-025-08651-6>. PMID: 40595377.
16. Magalhães, P.A.; Araújo, A.L.; Madeira, J.F.A. Multimodal optimization of piezoelectric patches number and placement for effective structural vibration reduction. *Acta Mech.* **2025**, *20*. <https://doi.org/10.1007/s00707-025-04351-4>.
17. Gong, J.; Wang, X. Piezoelectric actuator layout optimization for active vibration control considering fiber orientation. *Mech. Adv. Mater. Struct.* **2025**. <https://doi.org/10.1080/15376494.2025.2526803>.
18. Biglar, M.; Mirdamadi, H.R.; Danesh, M. Optimal locations and orientations of piezoelectric transducers on cylindrical shell based on gramians of contributed and undesired Rayleigh-Ritz modes using genetic algorithm. *J. Sound. Vib.* **2014**, *333*, 1224–1244. <https://doi.org/10.1016/j.jsv.2013.10.025>.
19. Zhao, X.; Sun, S.; Chu, S.; Cao, D. Multi-mode vibration control and optimization of smart rotating cylindrical shells subjected to circumferential moving loads. *Thin-Walled Struct.* **2025**, *211*, 113094. <https://doi.org/10.1016/j.tws.2025.113094>.
20. Brand, Z.; Cole, M.O.T. Mini-max optimization of actuator/sensor placement for flexural vibration control of a rotating thin-walled cylinder over a range of speeds. *J. Sound. Vib.* **2021**, *506*, 116105. <https://doi.org/10.1016/j.jsv.2021.116105>.
21. Hu, M.; Zeng, Y.; Li, D.; Luo, H.; Wei, J.; Liu, K. 4-DOF Full-Speed Range Vibration Suppression of an Active–Passive Supported Flywheel Rotor Based on Inverse System Decoupling. *Actuators* **2026**, *15*, 157. <https://doi.org/10.3390/act15030157>.
22. Jafari Niasar, M.; Jafari, A.A.; Irani Rahaghi, M.; Mohammadrezazadeh, S. Active control of free and forced vibration of a rotating FG cylindrical shell via FG piezoelectric patches. *Mech. Based Des. Struct. Mach.* **2024**, *52*, 3900–3924. <https://doi.org/10.1080/15397734.2023.2212297>.
23. Salem, M.A.M.; Kassem, M.; Amin, M.S.; Farag, H.M.; Osman, A. Energy-Based Optimal Placement of Piezoelectric Actuator on Smart Thin Plate. *J. Vib. Eng. Technol.* **2024**, *12*, 1813–1830. <https://doi.org/10.1007/s42417-023-00944-2>.
24. Bachmann, F.; Bergamini, A.E.; Ermanni, P. Optimum piezoelectric patch positioning: A strain energy-based finite element approach. *J. Intell. Mater. Syst. Struct.* **2012**, *23*, 1575–1591. <https://doi.org/10.1177/1045389X12447985>.
25. Gülbahçe, E.; Çelik, M. Investigation of piezoelectric patches placement according to the strain intensity. *Appl. Eng. Lett.* **2020**, *5*, 46–49. <https://doi.org/10.18485/aeletters.2020.5.2.2>.
26. Porter, J.H.; Mace, T.; Bortolotti, P.; Anderson, E.M.; Barnes, D.; Chetan, M.; Beach, R.; Hughes, S.; Davis, C.G.; Moldenhauer, B.J.; Rognerud, E.G.; Mac Donald, K.A.; Schwingshackl, C.W. Novel application of the modal strain energy technique for state-of-the-art damping predictions. *Compos Struct.* **2026**, *389*, 120394. <https://doi.org/10.1016/j.compstruct.2026.120394>.
27. Lalande, M.; Ferraris, G. *Rotordynamics Prediction in Engineering*, 2nd Ed.; Wiley: Hoboken, NJ, USA, 1997; 272p. <https://doi.org/10.1002/cnm.1630070310>.
28. Baz, A.; Kim, M. Active Modal Control of Vortex-Induced Vibrations of a Flexible Cylinder. *J. Sound Vib.* **1993**, *165*, 69–84. <https://doi.org/10.1006/jsvi.1993.1243>.
29. Lin, Y.H.; Chu, C.L. Comments on “Active modal control of vortex induced Vibrations of a flexible cylinder”. *J. Sound. Vib.* **1994**, *175*, 135–137. <https://doi.org/10.1006/jsvi.1994.1316>.
30. Krämer, E. Dynamics of Rotors and Foundations. In *Dynamics of Rotors and Foundations*, 1st Ed.; Springer: Berlin/Heidelberg, Germany, 1993; 384p. <https://doi.org/10.1007/978-3-662-02798-1>.
31. Bandyopadhyay, B.; Manjunath, T.C.; Umashathy, M. *Modeling, Control and Implementation of Smart Structure*, 1st Ed; Springer: Berlin/Heidelberg, Germany, 2007; 258p. <https://doi.org/10.1007/978-3-540-48394-6>.

32. Brahem, M.; Chouchane, M. GA tuned LQR controller for active vibration control of a rotor bearing system using piezoelectric actuators. *Eng. Res. Express* **2024**, *6*, 025562. <https://doi.org/10.1088/2631-8695/ad5499>.
33. Preumont, A. *Vibration Control of Active Structures*, 4th Ed.; Springer: Cham, Switzerland, 2018; Volume 246, 518p. <https://doi.org/10.1007/978-3-319-72296-2>.
34. Brahem, M.; Chouchane, M.; Amamou, A. Active vibration control of a rotor bearing system using flexible piezoelectric patch actuators. *J. Intell. Mater. Syst. Struct.* **2020**, *31*, 1284–1297. <https://doi.org/10.1177/1045389X20916804>.
35. Friswell, M.I.; Penny, J.E.T.; Garvey, S.D.; Lees, A.W. *Dynamics of Rotating Machines*; Cambridge University Press: Cambridge, UK, 2010; 542p. <https://doi.org/10.1017/CBO9780511780509>.
36. Maryam, B.; Mnaouar, C. Optimal placement of piezoelectric patches on the rotor shaft for vibration control using Modal Strain Energy method. *Eng. Res. Express* **2026**, *8*, 095508. <https://doi.org/10.1088/2631-8695/ae6027>.

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