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Fixed-Time Disturbance Observer-Based Connectivity-Preserving Formation Control for High-Order Multi-Agent Systems with Actuator Faults

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Abstract

This paper investigates connectivity-preserving formation control for high-order multi-agent systems under switching distance-dependent topologies, actuator faults and external disturbances. The admissible interaction edges are selected by a switching signal, while the actual edge weights vary continuously with inter-agent distances. To avoid unsafe jumps of algebraic connectivity at switching instants, a connectivity-certified switching condition is introduced. Between switching instants, an algebraic-connectivity-dependent barrier signal is incorporated into a recursive formation controller. A fixed-time extended state observer is designed to estimate the matched lumped uncertainty caused by actuator loss of effectiveness, additive faults and external disturbances. Explicit ultimate estimation-error bounds and a fixed convergence-time upper bound are derived. The effects of command filtering and smooth actuator saturation used in the numerical implementation are further represented by bounded implementation residuals in the stability analysis. Theoretical results establish boundedness of all closed-loop signals, uniform ultimate boundedness of formation errors and preservation of a prescribed connectivity margin under the stated hybrid safety conditions. Numerical simulations with five planar followers illustrate formation tracking, connectivity preservation, fault-tolerant disturbance rejection and observer estimation performance.

Keywords: multi-agent systems; formation control; connectivity preservation; fixed-time extended state observer; actuator faults; switching topology



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1. Introduction

Formation control of multi-agent systems (MASs) is a fundamental problem in cooperative control, with applications in unmanned vehicles, mobile robots, sensor networks and cooperative transportation systems [1,2]. Adaptive and learning-based formation strategies have also been developed to improve parameter adjustment and tracking performance under uncertainty [3]. For practical robotic platforms, first-order and second-order models may be insufficient to represent position–velocity–acceleration coordination. High-order models therefore provide a useful description of cooperative motion [4,5]. When actuator faults and external disturbances are present, however, tracking errors may increase, and distance-dependent communication links may be weakened or lost.

Finite-range communication introduces a direct coupling between motion and network connectivity. Fixed or binary graphs cannot describe the gradual attenuation of a communication link as two agents move apart. Distance-dependent weighted graphs are more appropriate because the link weight is strong in the near region, decreases continuously in a fading region and becomes zero outside the communication radius [6–9]. In addition, admissible links may switch because of channel scheduling, obstacle occlusion, sensing limitations or energy-saving policies. A rigorous connectivity analysis must therefore address both the continuous variation in edge weights and the discontinuous change in the active graph.

Connectivity-preserving methods based on potential functions, prescribed performance or distance constraints commonly protect selected links [10–12]. Algebraic-connectivity-based methods instead regulate the second smallest eigenvalue of the Laplacian matrix and allow noncritical links to disappear or reappear as long as the graph remains connected [6–8]. Nevertheless, most available results consider relatively low-order agents, fixed topologies or fault-free actuators. For switching weighted graphs, a continuous barrier argument alone cannot prevent the algebraic connectivity from jumping below its prescribed limit at a switching instant. This issue motivates an explicit post-switch connectivity certificate.

Actuator faults and matched disturbances provide another difficulty. Disturbance observers and extended state observers can estimate lumped uncertainties and support fault-tolerant compensation [13–15]. Fixed-time designs are attractive because the convergence-time upper bound is independent of the initial estimation error [16,17]. Related fixed-time tracking methods have been developed for robotic systems with uncertainties, input saturation and prescribed performance [18,19]. Recent adaptive designs have also considered actuator faults, input delay, command filters and unknown control gains [20,21]. Fault-tolerant time-varying formation tracking under actuator faults and switching topologies has also been investigated [22], while FTESO-based fixed-time control and finite-time-ESO-based prescribed-performance formation control have been developed for high-order MASs [23,24]. These studies provide useful tools, but they do not simultaneously address fixed-time disturbance estimation, high-order formation dynamics, switching distance-dependent links and algebraic-connectivity preservation.

Motivated by these observations, this paper develops a fixed-time extended state observer-based connectivity-preserving formation controller for high-order MASs with actuator faults and external disturbances. The main contributions are summarized as follows:

- A switching distance-dependent graph model is equipped with a connectivity-certified switching condition. A candidate topology is accepted only when its post-switch algebraic connectivity has a prescribed positive safety margin, thereby excluding unsafe discontinuous jumps.
- An algebraic-connectivity-dependent barrier signal is incorporated into a recursive high-order controller. The continuous evolution between switching instants is treated separately from the jump condition, and the centralized connectivity-evaluation architecture and its computational complexity are stated explicitly.
- A fixed-time extended state observer is developed for the matched uncertainty generated by actuator loss of effectiveness, additive faults and external disturbances. Explicit ultimate estimation bounds and a fixed convergence-time upper bound are derived. The command filters and smooth actuator saturation used in the numerical implementation are included in the stability analysis through bounded residual terms.

The remainder of the paper is organized as follows: Section 2 formulates the switched fault-tolerant formation problem. Section 3 presents the observer, connectivity barrier,

recursive controller and stability analysis. Section 4 reports the numerical results. Section 5 concludes the paper.

2. Problem Formulation

2.1. Switching Distance-Dependent Topology

Consider N followers indexed by $\mathcal{I} = \{1, \dots, N\}$. The admissible communication topology is selected from a finite graph set $\{\mathcal{G}_1, \dots, \mathcal{G}_M\}$, where

$$\mathcal{G}_\ell = (\mathcal{I}, \mathcal{E}_\ell), \quad \ell \in \{1, \dots, M\},$$

and $\mathcal{E}_\ell$ is the admissible edge set of the ℓ th graph. Let $\omega(t) : [0, \infty) \rightarrow \{1, \dots, M\}$ be a right-continuous piecewise-constant switching signal. If edge (i, j) is admissible under $\mathcal{G}_{\omega(t)}$, then $\bar{a}_{ij}^{\omega(t)} = 1$; otherwise, $\bar{a}_{ij}^{\omega(t)} = 0$.

For finite-range communication, let $d_{ij}(t) = \|\xi_{i,1}(t) - \xi_{j,1}(t)\|$. The distance-dependent edge weight is

$$a_{ij}(t) = \bar{a}_{ij}^{\omega(t)} \chi(d_{ij}(t)), \quad \chi(d) = \begin{cases} 1, & 0 \leq d < \rho R, \\ \frac{1}{2} \left[1 + \cos \left(\pi \frac{d/R - \rho}{1 - \rho} \right) \right], & \rho R \leq d < R, \\ 0, & d \geq R, \end{cases} \quad (1)$$

where $R > 0$ is the communication radius and $\rho \in (0, 1)$ determines the fading region. Let $A(t) = [a_{ij}(t)]$ and

$$L(t) = D(t) - A(t), \quad D(t) = \text{diag} \left\{ \sum_{j=1}^N a_{ij}(t) \right\}. \quad (2)$$

The algebraic connectivity is denoted by $\lambda_2(t) = \lambda_2(L(t))$.

Proposition 1. For an undirected weighted graph, the graph is connected if and only if $\lambda_2(L(t)) > 0$ [1,6].

Choose constants

$$0 < \lambda_* < \lambda_c < \lambda_b < \lambda_s,$$

where λ_* is the required lower bound, λ_c is an operational safety margin, λ_b is the barrier activation threshold and λ_s is the post-switch safety level.

Assumption 1. The initial graph satisfies $\lambda_2(0) \geq \lambda_s$. The switching signal has a dwell time $\tau_d > 0$. At a candidate switching instant t_s , a new topology p is accepted only if

$$\lambda_2(L_p(\xi_1(t_s))) \geq \lambda_s. \quad (3)$$

Otherwise, the current topology is retained. During each continuous-flow interval, the supervisory connectivity condition

$$D^+ h_\lambda(t) \geq -\alpha_\lambda h_\lambda(t), \quad h_\lambda(t) = \lambda_2(t) - \lambda_c, \quad (4)$$

is satisfied whenever $\lambda_2(t) \leq \lambda_b$, where $\alpha_\lambda > 0$ and D^+ denotes the upper-right Dini derivative.

Condition (3) prevents an unsafe eigenvalue jump at a switching instant. Condition (4) is a continuous-flow certificate that can be checked by the same supervisory unit that eval-

uates λ_2 . By the comparison principle, it implies that $h_\lambda(t) \geq h_\lambda(t_s) \exp[-\alpha_\lambda(t - t_s)] \geq 0$ on every dwell interval.

Remark 1. The present design uses a supervisory connectivity-evaluation architecture rather than a fully distributed eigenvalue estimator. The supervisor reconstructs the active Laplacian from the current graph and communicated positions, computes λ_2 , checks (3) and (4) and broadcasts the scalar connectivity margin. Forming a sparse Laplacian requires $O(|\mathcal{E}|)$ operations. A dense eigenvalue decomposition has worst-case complexity $O(N^3)$, whereas sparse Lanczos iterations require approximately $O(K|\mathcal{E}|)$ operations for K iterations. Fully distributed algebraic-connectivity estimation is left for future work.

2.2. High-Order Dynamics with Actuator Faults

The i th follower is modeled as

$$\begin{aligned}\dot{\xi}_{i,r} &= \xi_{i,r+1}, & r &= 1, \dots, n-1, \\ \dot{\xi}_{i,n} &= u_i^a + d_i(t), \\ y_i &= \xi_{i,1},\end{aligned}\quad (5)$$

where $\xi_{i,r} \in \mathbb{R}^m$, $u_i^a \in \mathbb{R}^m$ is the actual actuator output, and $d_i(t)$ is a matched external disturbance. To avoid confusion with the distance-fading parameter ρ , the actuator effectiveness coefficient is denoted by $\mu_i(t)$. The actuator model is

$$u_i^a = \mu_i(t)u_i + \varphi_i(t), \quad 0 < \underline{\mu}_i \leq \mu_i(t) \leq \bar{\mu}_i, \quad (6)$$

where $\varphi_i(t)$ is an additive fault. Let v_i denote the nominal controller command and let the applied command be

$$u_i = \text{sat}_{u_{\max}}(v_i) = u_{\max} \tanh\left(\frac{v_i}{u_{\max}}\right). \quad (7)$$

Define

$$\Delta_i(t) = [\mu_i(t) - 1]u_i + \varphi_i(t) + d_i(t). \quad (8)$$

Then the highest-order channel becomes

$$\dot{\xi}_{i,n} = u_i + \Delta_i(t). \quad (9)$$

Assumption 2. The signals μ_i , $\dot{\mu}_i$, φ_i , $\dot{\varphi}_i$, d_i and $\dot{d}_i$ are bounded. Consequently, $\Delta_i(t)$ and $\dot{\Delta}_i(t)$ are bounded by unknown positive constants $\bar{\Delta}_i$ and $\bar{\dot{\Delta}}_i$. These bounds are used only in the analysis and are not required by the controller.

The boundedness of u_i follows directly from (7); therefore, the appearance of the input in (8) does not create a circular boundedness assumption.

Let $\xi_0(t) \in \mathbb{R}^m$ be the virtual leader, and let $h_i(t) \in \mathbb{R}^m$ be the formation offset. The desired position is $\xi_{i,d}(t) = \xi_0(t) + h_i(t)$, and the position error is

$$e_{i,1} = \xi_{i,1} - \xi_{i,d}. \quad (10)$$

Assumption 3. The virtual leader and formation offsets are sufficiently smooth and bounded together with their derivatives up to order n . The desired formation is feasible for the communication radius R [8,25,26].

3. Main Results

3.1. Fixed-Time Extended State Observer

Let $\hat{\xi}_{i,n}$ and $\hat{\Delta}_i$ be estimates of $\xi_{i,n}$ and Δ_i , respectively, and define

$$\tilde{\xi}_{i,n} = \xi_{i,n} - \hat{\xi}_{i,n}, \quad \tilde{\Delta}_i = \Delta_i - \hat{\Delta}_i.$$

The FTESO is

$$\begin{aligned} \dot{\hat{\xi}}_{i,n} &= u_i + \hat{\Delta}_i + l_{31}\tilde{\xi}_{i,n} + l_{32}\text{sig}^{\alpha_1}(\tilde{\xi}_{i,n}) + l_{33}\text{sig}^{\beta_1}(\tilde{\xi}_{i,n}), \\ \dot{\hat{\Delta}}_i &= l_{41}\tilde{\xi}_{i,n} + l_{42}\text{sig}^{\alpha_2}(\tilde{\xi}_{i,n}) + l_{43}\text{sig}^{\beta_2}(\tilde{\xi}_{i,n}), \end{aligned} \quad (11)$$

where

$$\text{sig}^q(x) = \text{sgn}(x)|x|^q, \quad q \in \{\alpha_1, \alpha_2, \beta_1, \beta_2\},$$

is applied componentwise, $0 < \alpha_2 < \alpha_1 < 1$, and $\beta_1, \beta_2 > 1$. The observer error dynamics are

$$\begin{aligned} \dot{\tilde{\xi}}_{i,n} &= \tilde{\Delta}_i - l_{31}\tilde{\xi}_{i,n} - l_{32}\text{sig}^{\alpha_1}(\tilde{\xi}_{i,n}) - l_{33}\text{sig}^{\beta_1}(\tilde{\xi}_{i,n}), \\ \dot{\tilde{\Delta}}_i &= \tilde{\Delta}_i - l_{41}\tilde{\xi}_{i,n} - l_{42}\text{sig}^{\alpha_2}(\tilde{\xi}_{i,n}) - l_{43}\text{sig}^{\beta_2}(\tilde{\xi}_{i,n}). \end{aligned} \quad (12)$$

Choose

$$V_{o,i} = \frac{1}{2}\|\tilde{\xi}_{i,n}\|^2 + \frac{1}{2\eta_o}\|\tilde{\Delta}_i\|^2, \quad \eta_o > 0.$$

After applying generalized Young inequalities to the cross terms, the observer gains are selected so that

$$\dot{V}_{o,i} \leq -a_o V_{o,i}^{\gamma_o} - b_o V_{o,i}^{\delta_o} + c_o \bar{d}_i^2, \quad a_o, b_o, c_o > 0, \quad 0 < \gamma_o < 1 < \delta_o. \quad (13)$$

A conservative sufficient tuning procedure is to choose $\eta_o = l_{41}$, select $l_{31} > 1/2$, and then increase the low-power gains l_{32}, l_{42} and the high-power gains l_{33}, l_{43} until the coefficients a_o and b_o obtained from the Young inequality bounds are positive. This condition is directly checkable after the gains are fixed and is less ambiguous than the original phrase “sufficiently large gains”.

Proposition 2. For any $\theta_o \in (0, 1)$, define

$$V_{o,\infty} = \max \left\{ \left(\frac{c_o \bar{d}_i^2}{\theta_o a_o} \right)^{1/\gamma_o}, \left(\frac{c_o \bar{d}_i^2}{\theta_o b_o} \right)^{1/\delta_o} \right\}. \quad (14)$$

Then the observer errors enter the set $\{V_{o,i} \leq V_{o,\infty}\}$ within

$$T_{o,\max} \leq \frac{1}{(1-\theta_o)a_o(1-\gamma_o)} + \frac{1}{(1-\theta_o)b_o(\delta_o-1)}, \quad (15)$$

which is independent of the initial observer errors. After this time,

$$\|\tilde{\xi}_{i,n}\| \leq \sqrt{2V_{o,\infty}}, \quad \|\tilde{\Delta}_i\| \leq \sqrt{2\eta_o V_{o,\infty}}. \quad (16)$$

Proof. Outside the set defined by (14),

$$c_o \bar{d}_i^2 \leq \theta_o \left(a_o V_{o,i}^{\gamma_o} + b_o V_{o,i}^{\delta_o} \right).$$

Substituting this inequality into (13) gives

$$\dot{V}_{o,i} \leq -(1 - \theta_o)a_o V_{o,i}^{\gamma_o} - (1 - \theta_o)b_o V_{o,i}^{\delta_o}.$$

Integrating the high-power term over $V_{o,i} \geq 1$ and the low-power term over $0 < V_{o,i} \leq 1$ yields (15) [16,17]. The componentwise bounds follow from the definition of $V_{o,i}$. $\square$

Remark 2. The observer parameters have different effects. The gain l_{31} increases linear damping. The low-power gains l_{32} and l_{42} mainly improve the local convergence rate, whereas l_{33} and l_{43} accelerate the decay of large initial errors. Increasing all observer gains generally shortens the transient but increases sensitivity to measurement noise. Because measurement noise is not included in the present model, the manuscript does not claim noise robustness.

3.2. Connectivity-Preserving Barrier and Hybrid Safety Conditions

Define the theoretical barrier

$$B(\lambda_2) = \begin{cases} 0, & \lambda_2 \geq \lambda_b, \\ \frac{\kappa_b}{(\lambda_2 - \lambda_*)^2}, & \lambda_* < \lambda_2 < \lambda_b, \end{cases} \quad (17)$$

where $\kappa_b > 0$. No saturation is used in the theoretical definition. Under Assumption 1, $\lambda_2(t) \geq \lambda_c > \lambda_*$; therefore

$$0 \leq B(\lambda_2(t)) \leq \bar{B} := \frac{\kappa_b}{(\lambda_c - \lambda_*)^2}. \quad (18)$$

Remark 3. The numerical code retains a large upper limiter $B_{\max} = 2$ only as a floating-point protection. In the reported simulation, $\min \lambda_2 = 0.42472$, $\lambda_* = 0.12$ and $\kappa_b = 0.018$, which give a maximum evaluated barrier of approximately 0.194. Hence, the numerical limiter is inactive and the implemented barrier coincides with (17) along the reported trajectory.

3.3. Recursive Controller and Implementation Residuals

Let $\alpha_{i,0} = \zeta_{i,d}$ and define

$$e_{i,1} = \zeta_{i,1} - \alpha_{i,0}, \quad e_{i,r} = \zeta_{i,r} - \alpha_{i,r-1}, \quad r = 2, \dots, n. \quad (19)$$

The ideal virtual controllers are

$$\alpha_{i,1} = \dot{\alpha}_{i,0} - [k_1 + B(\lambda_2)]e_{i,1}, \quad (20)$$

and, for $r = 2, \dots, n-1$,

$$\alpha_{i,r} = \dot{\alpha}_{i,r-1} - k_r e_{i,r} - e_{i,r-1}. \quad (21)$$

The nominal final command is

$$v_i^* = \dot{\alpha}_{i,n-1} - k_n e_{i,n} - e_{i,n-1} - \hat{\Delta}_i - \chi_b B(\lambda_2) e_{i,1}. \quad (22)$$

In the numerical implementation, first-order command filters are used to avoid repeated differentiation:

$$\dot{\zeta}_{i,r} = -\omega_{fr}(\zeta_{i,r} - \alpha_{i,r}), \quad r = 1, \dots, n-1. \quad (23)$$

Let $\varepsilon_{i,r}^f = \zeta_{i,r} - \alpha_{i,r}$ denote the filter error, and let

$$\varepsilon_i^u = \text{sat}_{u_{\max}}(v_i^*) - v_i^*$$

denote the saturation residual.

Assumption 4. The command filters are initialized consistently, and their errors satisfy $\|\varepsilon_{i,r}^f\| \leq \bar{\varepsilon}_f$. The saturation residual satisfies $\|\varepsilon_i^u\| \leq \bar{\varepsilon}_u$ over the operating region. These bounds are known for analysis but are not used in the online controller.

Remark 4. The recursive gains k_r determine the negative quadratic terms in the Lyapunov derivative. Larger values generally accelerate error decay but increase control effort. Decreasing k_2 or k_3 reduces recursive damping, enlarges the theoretical ultimate bound and lengthens the transient. The barrier coefficient κ_b determines how rapidly connectivity feedback grows near the boundary, while $\lambda_b - \lambda_*$ determines how early the barrier is activated. Larger command-filter bandwidths reduce filter lag but may increase numerical and measurement-noise sensitivity.

3.4. Stability Analysis

Theorem 1. Consider the high-order MAS in (5) under Assumptions 1–4. Suppose that the FTESO gains satisfy (13), the recursive gains are positive, and

$$k_n > \frac{1}{2} + \frac{1}{2}\chi_b^2\bar{B} + \varepsilon_k \quad (24)$$

for some $\varepsilon_k > 0$. Then all closed-loop signals are bounded, the formation errors are uniformly ultimately bounded, and

$$\lambda_2(t) \geq \lambda_c > \lambda_*$$

for all $t \geq 0$. Command-filter and saturation residuals enlarge the ultimate formation-error bound but do not alter the conclusion, provided that Assumption 4 holds.

Proof. For each follower, define

$$V_{c,i} = \frac{1}{2} \sum_{r=1}^n e_{i,r}^T e_{i,r}. \quad (25)$$

Using (20)–(22), the adjacent recursive cross terms cancel. With the command-filter and saturation residuals collected in $\varepsilon_i^{\text{imp}}$, the derivative satisfies

$$\begin{aligned} \dot{V}_{c,i} = & -[k_1 + B(\lambda_2)]\|e_{i,1}\|^2 - \sum_{r=2}^n k_r \|e_{i,r}\|^2 \\ & + e_{i,n}^T \tilde{\Delta}_i - \chi_b B(\lambda_2) e_{i,n}^T e_{i,1} + e_{i,n}^T \varepsilon_i^{\text{imp}}. \end{aligned} \quad (26)$$

Young's inequality gives

$$\begin{aligned} e_{i,n}^T \tilde{\Delta}_i & \leq \frac{1}{2} \|e_{i,n}\|^2 + \frac{1}{2} \|\tilde{\Delta}_i\|^2, \\ -\chi_b B e_{i,n}^T e_{i,1} & \leq \frac{1}{2} B \|e_{i,1}\|^2 + \frac{1}{2} \chi_b^2 B \|e_{i,n}\|^2, \end{aligned}$$

and, for any $\epsilon_u > 0$,

$$e_{i,n}^T \varepsilon_i^{\text{imp}} \leq \frac{\epsilon_u}{2} \|e_{i,n}\|^2 + \frac{1}{2\epsilon_u} \|\varepsilon_i^{\text{imp}}\|^2.$$

Using (18) and (24), there exist constants $c_e, c_f, c_u > 0$ such that

$$\dot{V}_c \leq -c_e V_c + \frac{1}{2} \sum_{i=1}^N \|\tilde{\Delta}_i\|^2 + c_f \bar{\epsilon}_f^2 + c_u \bar{\epsilon}_u^2, \quad (27)$$

where $V_c = \sum_i V_{c,i}$. After $T_{o,\max}$, Proposition 2 gives

$$\|\tilde{\Delta}_i\|^2 \leq 2\eta_o V_{o,\infty}.$$

Hence,

$$\dot{V}_c \leq -c_e V_c + N\eta_o V_{o,\infty} + c_f \bar{\epsilon}_f^2 + c_u \bar{\epsilon}_u^2.$$

The comparison lemma establishes boundedness and uniform ultimate boundedness of all recursive errors.

Connectivity remains to be addressed. At each accepted switching instant, (3) gives $\lambda_2(t_s^+) \geq \lambda_s > \lambda_c$. During a continuous-flow interval, (4) gives

$$h_\lambda(t) \geq h_\lambda(t_s^+) e^{-\alpha_\lambda(t-t_s)} \geq 0.$$

Therefore, $\lambda_2(t) \geq \lambda_c > \lambda_*$ throughout the interval. Combining the jump and flow conditions proves connectivity preservation for all $t \geq 0$. $\square$

Remark 5. The theorem uses sufficient conditions. The post-switch certificate may reject a topology that would in fact remain safe, the flow certificate requires supervisory evaluation of λ_2 , and repeated Young inequality bounds generally overestimate the actual ultimate error. These conditions are conservative but make the hybrid connectivity guarantee explicit. The present theorem does not cover discontinuous faults with unbounded derivatives or measurement noise.

4. Simulation

Numerical simulations are conducted with five followers moving on the x - y plane. The plant is the third-order case of (5):

$$\dot{\xi}_{i,1} = \xi_{i,2}, \quad \dot{\xi}_{i,2} = \xi_{i,3}, \quad \dot{\xi}_{i,3} = u_i + \Delta_i(t).$$

The actuator effectiveness is a smooth time-varying coefficient in $[0.68, 1.02]$, and the additive actuator fault is smoothly activated after $t = 3$ s. The fault is therefore time-varying but not discontinuous, consistently with Assumption 2. The simulation horizon is $T = 10$ s, and the integration step is 0.002 s.

The virtual leader trajectory is $\xi_0(t) = [0.18t, 0.45 \sin(0.30t)]^T$. The desired offsets are generated by $h(t) = C_f q(t)$, where $q(t) = 0.60 + 0.10 \cos(0.20t)$ and

$$C_f = \begin{bmatrix} -0.85 & -0.30 & 0.35 & 0.80 & 0.00 \\ 0.15 & 0.65 & 0.70 & 0.20 & -0.55 \end{bmatrix}.$$

The initial positions are $\xi_{1,1}(0) = [-0.80, 0.38]^T$, $\xi_{2,1}(0) = [-0.15, -0.68]^T$, $\xi_{3,1}(0) = [0.55, -0.54]^T$, $\xi_{4,1}(0) = [1.00, 0.12]^T$ and $\xi_{5,1}(0) = [-1.05, -0.18]^T$. The initial velocity-like states are $\xi_{1,2}(0) = [0.03, -0.01]^T$, $\xi_{2,2}(0) = [-0.02, 0.02]^T$, $\xi_{3,2}(0) = [0.02, -0.01]^T$, $\xi_{4,2}(0) = [-0.015, 0.015]^T$ and $\xi_{5,2}(0) = [0.02, -0.02]^T$, while $\xi_{i,3}(0) = [0, 0]^T$.

The communication parameters are $R = 2.20$, $\rho = 0.65$, $\lambda_* = 0.12$ and $\lambda_b = 0.55$. The barrier parameters are $\kappa_b = 0.018$, $B_{\max} = 2.0$ and $\chi_b = 0.08$. The switching dwell time is 2.0 s. The admissible edge sets are

$$\mathcal{E}_1 = \{(1,2), (2,3), (3,5), (1,4), (4,5)\},$$

$$\mathcal{E}_2 = \{(1,2), (2,3), (3,4), (4,5), (1,5)\},$$

and

$$\mathcal{E}_3 = \{(1,3), (3,5), (5,4), (4,2), (2,1)\}.$$

The controller gains are $k_1 = 2.15$, $k_2 = 3.00$ and $k_3 = 3.10$. The command-filter bandwidths are $\omega_{f1} = 12.0$ and $\omega_{f2} = 14.0$, and the smooth actuator limit is 32. The FTESO gains are $l_{31} = 14.0$, $l_{32} = 4.0$, $l_{33} = 1.0$, $l_{41} = 38.0$, $l_{42} = 6.0$ and $l_{43} = 1.2$, with $\alpha_1 = 0.65$, $\beta_1 = 1.35$, $\alpha_2 = 0.45$ and $\beta_2 = 1.25$.

The recursive gains were selected first to obtain an adequately damped nominal formation response. The observer linear gain was then increased until the initial estimation transient was sufficiently short, after which the fractional-power gains were adjusted to reduce the remaining estimation error without generating excessive control oscillations. Finally, κ_b and λ_b were chosen so that connectivity feedback was inactive far from the boundary and increased smoothly as λ_2 decreased. This sequential procedure is consistent with the parameter effects discussed in Remark 4.

A baseline controller without FTESO compensation and without the active connectivity barrier is tested under the same plant, switching sequence, faults and initial conditions. This combined reduced controller provides a same-model reference. It does not isolate the individual numerical contribution of the observer and barrier, and the revised manuscript does not claim that it represents every state-of-the-art method.

Figures 1–4 show the basic formation response. The active graph switches among connected but non-complete topologies, so all-to-all communication is not required. In Figure 2, the third coordinate is time rather than a physical z -position. The followers approach the prescribed moving pattern, while the signed position errors in Figure 3 converge to small neighborhoods of zero. The higher-order recursive errors in Figure 4 exhibit larger initial transients because they contain filtered virtual-control derivatives, but remain bounded and decrease after the initial adjustment.

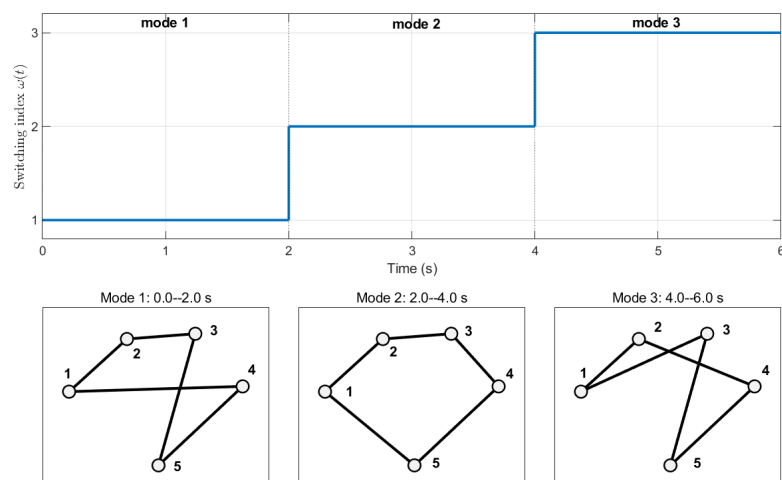


Figure 1. Switching communication topology and switching signal.

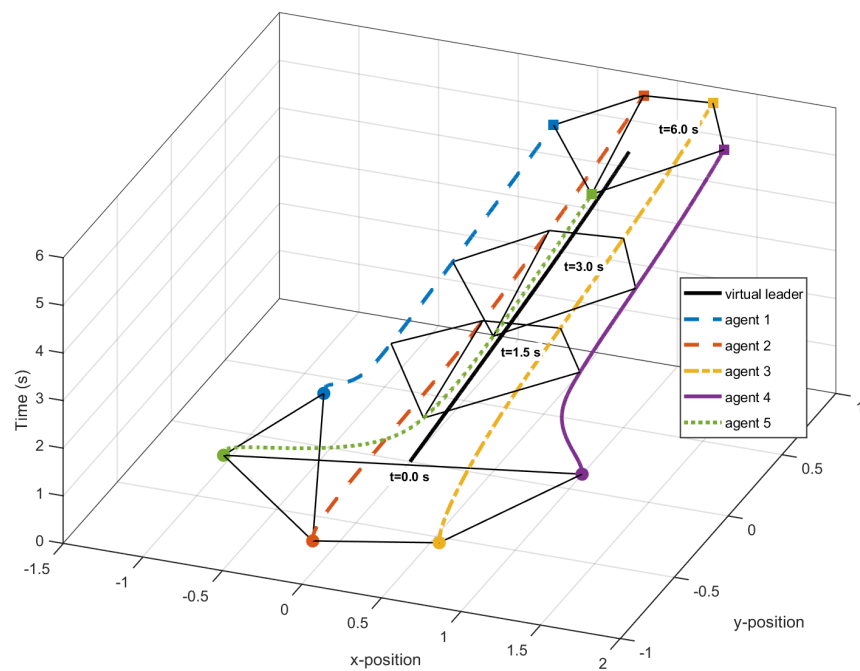


Figure 2. Formation trajectories in the x – y –time space.

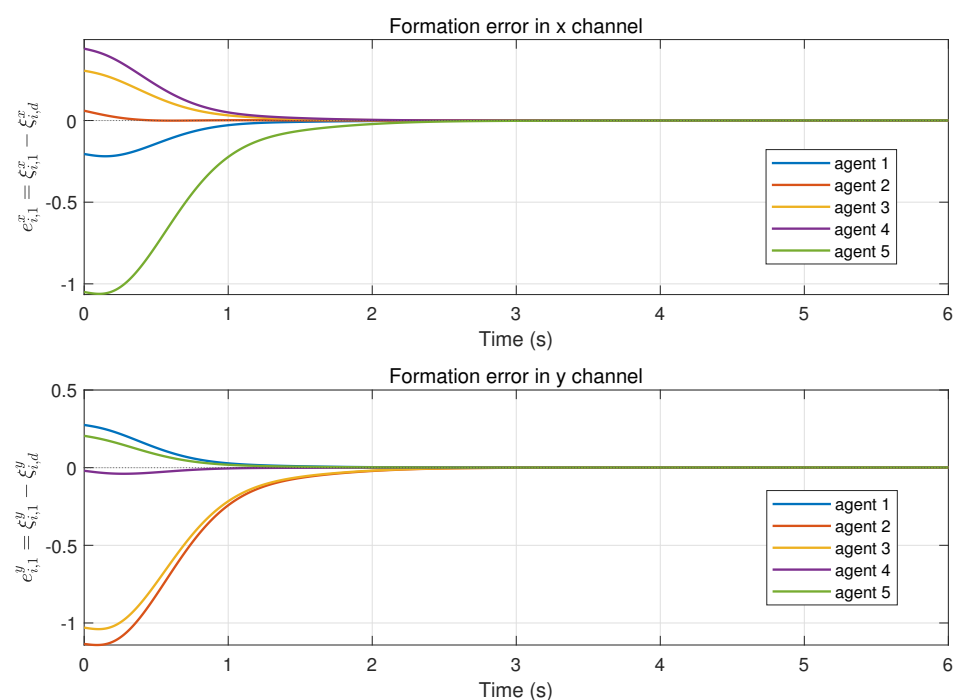


Figure 3. Signed formation errors $e_{i,1}$ of all followers.

Figures 5 and 6 describe the connectivity behavior. The minimum algebraic connectivity is 0.42472, which is greater than $\lambda_* = 0.12$. The representative edge weights vary because of switching and distance attenuation, but the graph remains connected. The maximum barrier value evaluated from the recorded minimum eigenvalue is approximately 0.194, well below the numerical protection value $B_{\max} = 2.0$; hence, the numerical barrier limiter is not activated. Because the minimum eigenvalue remains far above the required bound, this case verifies connectivity with a positive margin but should not be interpreted as a near-disconnection stress test.

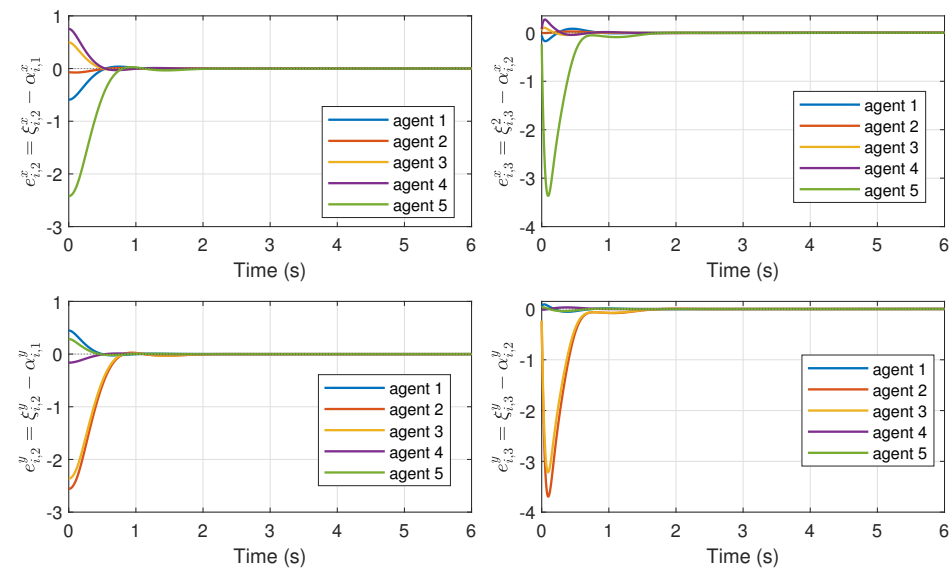


Figure 4. Recursive errors $e_{i,2}$ and $e_{i,3}$.

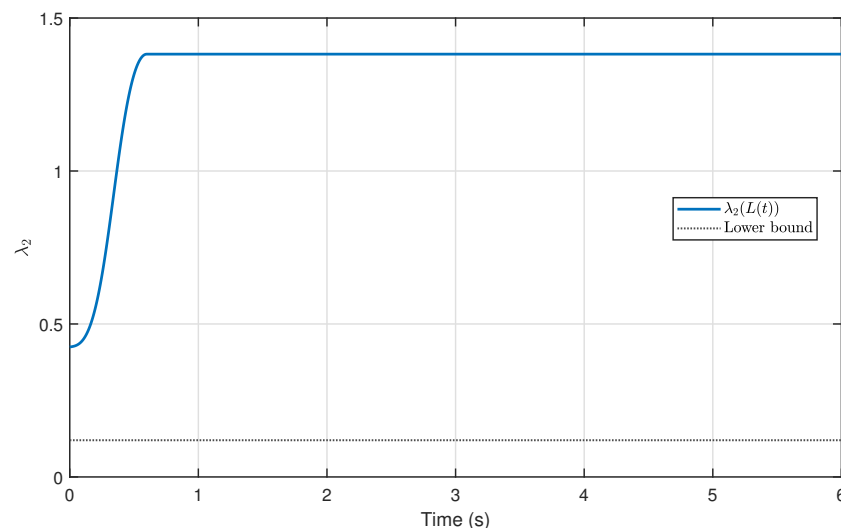


Figure 5. Evolution of algebraic connectivity.

Figure 7 shows that the estimation errors decrease rapidly during the initial transient and subsequently remain in a small neighborhood of zero, which is consistent with the fixed-time ultimate-boundedness result. A single simulation trajectory cannot establish the worst-case uniform time bound in (15); therefore, the figure is used as an illustration rather than as a numerical proof of $T_{o,max}$. Figure 8 shows that the applied inputs remain bounded, with the maximum value below the smooth limit of 32. Figure 9 shows that the complete controller produces smaller average formation errors than the combined reduced controller under the same operating conditions.

Table 1 shows that the complete method decreases the RMS formation error from 0.209 to 0.186 and the final mean error from 0.012 to 0.001. Both cases remain connected in this nominal scenario, so the table supports the combined tracking and disturbance-compensation benefit but does not quantify the isolated contribution of the barrier. The RMS-ESO value of 0.125 is consistent with the bounded estimation-error curves.

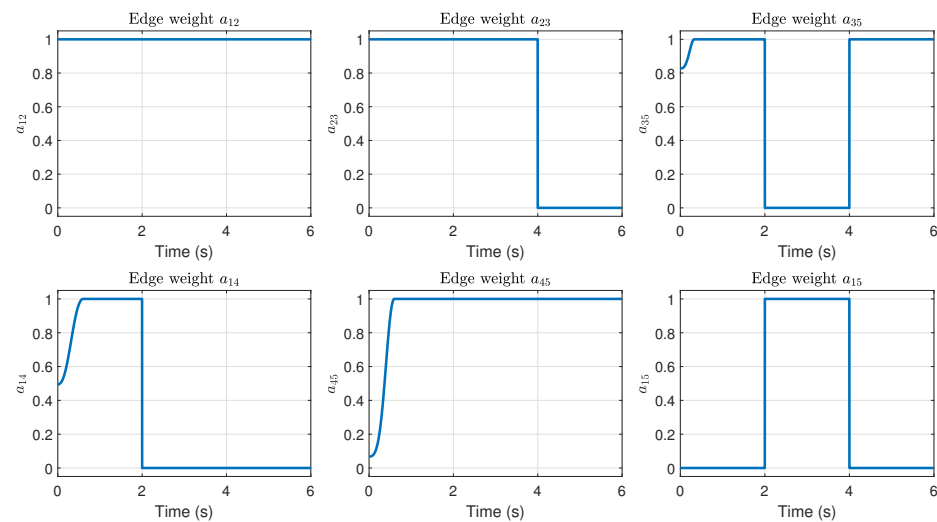


Figure 6. Evolution of representative communication edge weights.

The present simulation does not include discontinuous faults, measurement noise, a near-disconnection test, separate observer/barrier ablations or a larger network. Adding these tests without changing the plant assumptions and observer analysis would create an inconsistency between the numerical and theoretical sections. Accordingly, the claims have been narrowed to the smooth-fault, noise-free scenario analyzed in this paper.

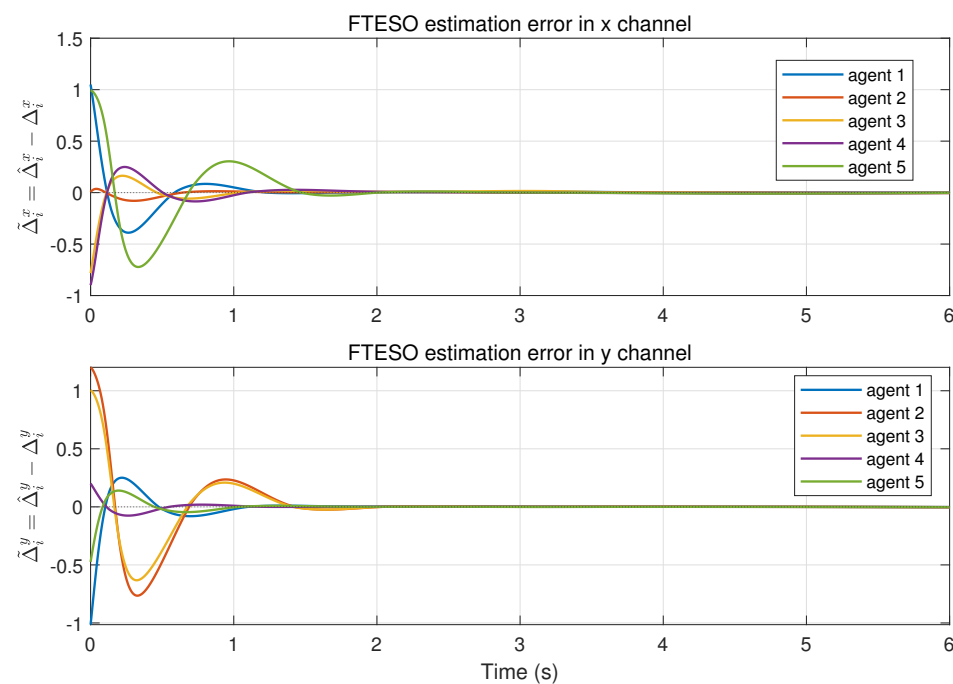


Figure 7. FTESO estimation errors of all followers.

Table 1. Quantitative comparison between the proposed method and the reduced scheme.

Case	RMS- e_1	Final- e_1	MaxMean- e_1	$\min \lambda_2$	RMS-ESO	$\max u $
Proposed with FTESO/barrier	0.186	0.001	0.813	0.425	0.125	31.977
Without FTESO/barrier	0.209	0.012	0.814	0.425	—	20.965

Note: RMS- e_1 denotes the root-mean-square formation error, Final- e_1 denotes the final mean formation error, MaxMean- e_1 denotes the maximum mean formation error, and RMS-ESO denotes the root-mean-square FTESO estimation error.

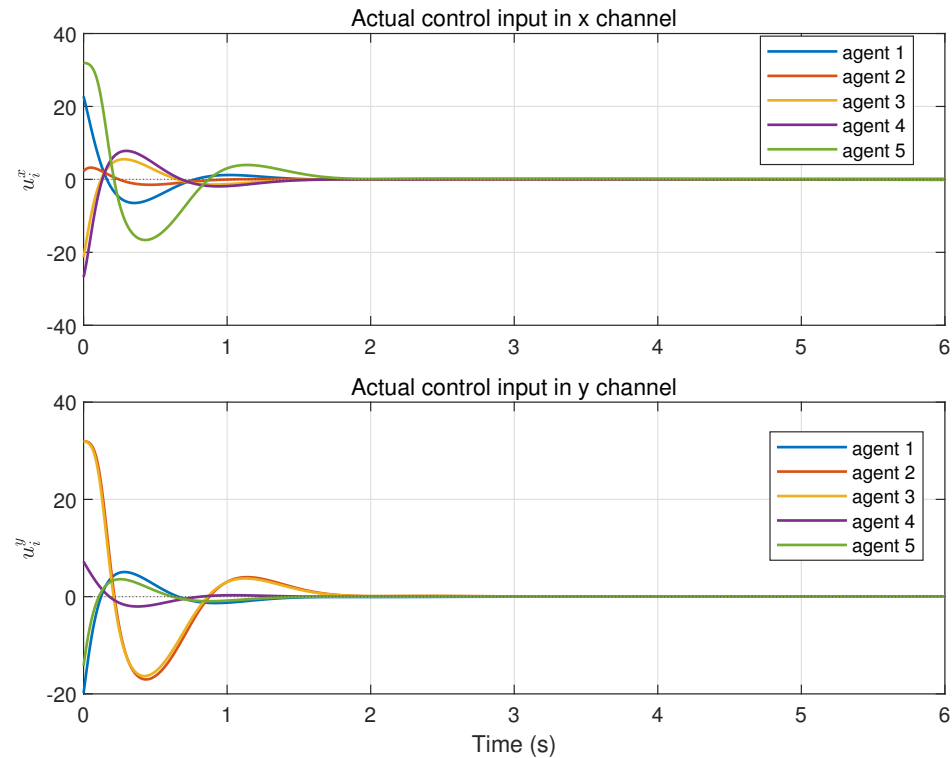


Figure 8. Actual control inputs.

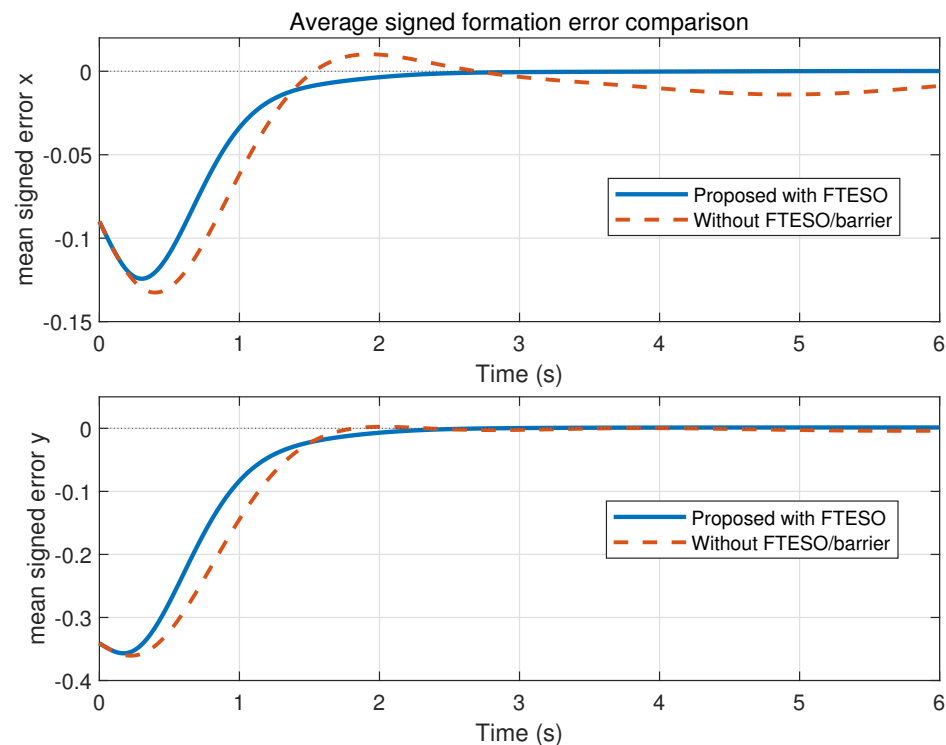


Figure 9. Comparison of average signed formation errors.

5. Conclusions

This paper has considered connectivity-preserving formation control for high-order MASs under switching distance-dependent topologies, actuator faults and external disturbances. A post-switch connectivity certificate was introduced to prevent unsafe discontinuous topology changes, while a continuous-flow certificate and an algebraic-connectivity-

dependent barrier were used between switching instants. A fixed-time extended state observer estimated the matched actuator-fault and disturbance term, and explicit ultimate estimation-error and convergence-time bounds were derived. Command-filter and smooth-saturation residuals were incorporated into the closed-loop analysis. The simulations illustrated bounded formation errors, positive algebraic connectivity, disturbance estimation and bounded control inputs for smoothly activated faults.

The sufficient hybrid safety conditions are conservative and require supervisory evaluation of algebraic connectivity. In addition, the current model does not cover discontinuous faults, measurement noise, isolated observer/barrier ablations or large-scale distributed eigenvalue estimation. These topics, together with experimental validation, constitute directions for future work. The fixed-time estimation and connectivity-certification ideas may also be adapted to secondary coordination in networked microgrids, but the corresponding electrical constraints and power-flow dynamics require a separate formulation [27].

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References

1. Olfati-Saber, R.; Fax, J.A.; Murray, R.M. Consensus and cooperation in networked multi-agent systems. *Proc. IEEE* **2007**, *95*, 215–233. [\[CrossRef\]](#)
2. Cao, Y.; Yu, W.; Ren, W.; Chen, G. An overview of recent progress in the study of distributed multi-agent coordination. *IEEE Trans. Ind. Inform.* **2013**, *9*, 427–438. [\[CrossRef\]](#)
3. Zhang, C.; Qin, W.; Fan, M.C.; Wang, T.; Shen, M.Q. A Q-learning-based parameters adaptive algorithm for formation tracking control of multi-mobile robot systems. *Complexity* **2022**, *2022*, 5093277. [\[CrossRef\]](#)
4. Rezaee, H.; Abdollahi, F. Consensus problem in high-order multiagent systems with Lipschitz nonlinearities and jointly connected topologies. *IEEE Trans. Syst. Man Cybern. Syst.* **2017**, *47*, 741–748. [\[CrossRef\]](#)
5. Li, K.; Hua, C.C.; You, X.; Guan, X.P. Distributed consensus control for nonlinear multiagent systems under directed graphs of dynamic frequency switches. *IEEE Trans. Autom. Control* **2021**, *66*, 841–848. [\[CrossRef\]](#)
6. Yang, P.; Freeman, R.; Gordon, G.; Lynch, K.; Srinivasa, S.; Sukthankar, R. Decentralized estimation and control of graph connectivity for mobile sensor networks. *Automatica* **2010**, *46*, 390–396. [\[CrossRef\]](#)
7. Griparic, K.; Polic, M.; Krizmancic, M.; Bogdan, S. Consensus-based distributed connectivity control in multi-agent systems. *IEEE Trans. Netw. Sci. Eng.* **2022**, *9*, 1264–1281. [\[CrossRef\]](#)
8. Cui, R.; Hua, C.; Li, Y.; Li, K.; Cui, H. Connectivity-preserving formation control of multiagent systems under time-varying topology. *IEEE Trans. Autom. Control* **2026**, 1–14. [\[CrossRef\]](#)
9. Krizmancic, M.; Bogdan, S. Adaptive connectivity control in networked multi-agent systems: A distributed approach. *PLoS ONE* **2024**, *19*, e0314642. [\[CrossRef\]](#) [\[PubMed\]](#)
10. Mehdifar, F.; Bechlioulis, C.P.; Hashemzadeh, F.; Baradarannia, M. Prescribed performance distance-based formation control of multi-agent systems. *Automatica* **2020**, *119*, 109086. [\[CrossRef\]](#)
11. Park, B.S.; Yoo, S.J. Connectivity-maintaining and collision-avoiding performance function approach for robust leader-follower formation control of multiple uncertain underactuated surface vessels. *Automatica* **2021**, *127*, 109501. [\[CrossRef\]](#)
12. Liu, Z.; Zhang, H.; Sun, J.; Wan, L. All agents connectivity-preserving and error-based cooperative learning control with data-filter memory-based event-triggered strategy. *IEEE Trans. Autom. Sci. Eng.* **2025**, *22*, 5566–5577. [\[CrossRef\]](#)
13. Han, J. From PID to active disturbance rejection control. *IEEE Trans. Ind. Electron.* **2009**, *56*, 900–906. [\[CrossRef\]](#)
14. Chen, W.H.; Yang, J.; Guo, L.; Li, S. Disturbance-observer-based control and related methods: An overview. *IEEE Trans. Ind. Electron.* **2016**, *63*, 1083–1095. [\[CrossRef\]](#)
15. Li, S.; Yang, J.; Chen, W.H.; Chen, X. *Disturbance Observer-Based Control: Methods and Applications*; CRC Press: Boca Raton, FL, USA, 2014.

16. Polyakov, A. Nonlinear feedback design for fixed-time stabilization of linear control systems. *IEEE Trans. Autom. Control* **2012**, *57*, 2106–2110. [[CrossRef](#)]
17. Parsegov, S.E.; Polyakov, A.E.; Shcherbakov, P.S. Fixed-time consensus algorithm for multi-agent systems with integrator dynamics. *IFAC Proc. Vol.* **2013**, *46*, 110–115. [[CrossRef](#)]
18. Liu, Z.; Zhang, O.; Gao, Y.; Zhao, Y.; Sun, Y.; Liu, J. Adaptive neural network-based fixed-time control for trajectory tracking of robotic systems. *IEEE Trans. Circuits Syst. II Express Briefs* **2023**, *70*, 241–245. [[CrossRef](#)]
19. Liu, Z.; Zhang, O.; Zhao, Y.; Zhu, Q.; Liu, J. Adaptive neural network-based fixed-time control for robots with input saturation and prescribed performance. *Nonlinear Dyn.* **2025**, *113*, 18229–18241. [[CrossRef](#)]
20. Mhemdi, A.; Kharrat, M.; Gargouri, A.; Mercorelli, P. Adaptive tracking control of nonstrict-feedback nonlinear systems with actuator faults and input delay via a fuzzy funnel approach. *Actuators* **2026**, *15*, 384. [[CrossRef](#)]
21. Alhazmi, H.; Kharrat, M.; Al-Jaser, A.; Mercorelli, P. Adaptive fixed-time prescribed performance command-filtered control for nonlinear systems with unknown control gains and actuator faults. *Mathematics* **2026**, *14*, 1781. [[CrossRef](#)]
22. Wu, X.; Guo, Z.; Liu, X.; Xie, T. Fault-tolerant time-varying formation tracking control for multi-agent systems with actuator faults and switching topologies. *IEEE Access* **2023**, *11*, 131140–131151. [[CrossRef](#)]
23. Zhang, C.; Zhang, G.; Han, W.; Lv, X.; Shi, Z. Distributed fixed-time control for high-order multi-agent systems with FTESO and feasibility constraints. *J. Frankl. Inst.* **2024**, *361*, 107219. [[CrossRef](#)]
24. Shi, Z.; Han, W.; Zhang, C.; Zhang, G. A modular prescribed performance formation control scheme of a high-order multi-agent system with a finite-time extended state observer. *Electronics* **2025**, *14*, 1783. [[CrossRef](#)]
25. Chai, X.; Liu, J.; Yu, Y.; Sun, C. Observer-based self-triggered control for time-varying formation of multi-agent systems. *Sci. China Inf. Sci.* **2021**, *64*, 132205. [[CrossRef](#)]
26. Deng, Y.; Zhu, W.; Zheng, H. Observer-based event-triggered time-varying formation control of linear multi-agent systems with distributed infinite delays. *Meas. Control* **2025**, *58*, 97–109. [[CrossRef](#)]
27. Ning, B.; Han, Q.-L.; Ding, L. Distributed finite-time secondary frequency and voltage control for islanded microgrids with communication delays and switching topologies. *IEEE Trans. Cybern.* **2021**, *51*, 3988–3999. [[CrossRef](#)] [[PubMed](#)]

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