

Article

Current-Feedback RST Thrust Regulation Within a Nested Saturation Attitude Loop: Design, Embedded Realisation and Characterisation on 8-Bit Hardware

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Abstract

Advanced quadrotor control is validated predominantly in simulation or on 32-bit floating-point flight computers, leaving open whether comparable architectures are deployable on the low-cost 8-bit hardware common to small-scale platforms. This paper reports the design, embedded realisation and experimental characterisation of a two-level discrete-time architecture for a 1.56 kg quadrotor implemented entirely in bare-metal C on an ATmega 2560. The actuator level departs from open-loop commanding of the electronic speed controllers: a discrete-time RST regulator closes a thrust loop around the bus current through a fourth-order calibration whose coefficients are reported, fitted to 615 bench points with a coefficient of determination of 0.998. The supervisory level combines proportional-derivative regulation of altitude and yaw with a nested saturation law for roll and pitch. The synthesised inner loop attains a modulus margin of 0.43 and a delay margin of 0.67 samples, tolerating approximately 43 percent multiplicative plant uncertainty. Bench records give a measured inner-loop rise time of 60 ms, which against the measured outer-loop rise time of 310 ms corresponds to a bandwidth separation of 5.2 to 1. Attitude experiments give rise times of 0.31 and 0.32 s and a residual roll offset of 2.04 degrees. The inner loop is found to exhibit approximately 49 percent overshoot despite a damping specification of 0.7, and this rather than time-scale separation is identified as the likely origin of the measured attitude transients. No comparative performance claim against conventional architectures is made.

Keywords: quadrotor UAV; RST control; sensorless thrust estimation; nested saturation; embedded control; real-time implementation; low-cost embedded platform; system identification; robustness margins



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1. Introduction

Unmanned Aerial Systems (UASs) have emerged as critical enablers of autonomous operations across a wide spectrum of industrial and societal applications. Their agility, manoeuvrability and relatively low acquisition cost render them well suited to missions spanning large or hazardous areas, including infrastructure inspection, environmental monitoring and automated maintenance [1]. They are effective in dynamic operational environments encompassing precision agriculture, public security and critical infrastructure management [2], and in governmental and humanitarian contexts they support monitoring, reconnaissance and hazard assessment while reducing personnel exposure in hazardous environments [3,4]. After natural disasters, aerial platforms enable rapid mapping and damage assessment in support of search and rescue [5], and their growing role in logistics, topographic mapping and industrial leak detection underlines their integration across multiple sectors [6]. When coupled with real-time control systems these platforms attain high degrees of operational autonomy, which in turn raises substantive questions of safety assurance, regulatory compliance and public acceptance [7].

Quadrotors exhibit nonlinear dynamics in which translational and rotational degrees of freedom are inherently coupled, so accurate modelling is a prerequisite for any advanced control design [8]. The governing equations are commonly derived using the Newton–Euler formalism, which accounts systematically for aerodynamic forces, gravitational effects and gyroscopic interactions [9]. Three broad families of control strategy are conventionally distinguished. Linear methods such as PID and Linear–Quadratic Regulators are widely employed for their analytical tractability and ease of implementation, but their performance degrades in strongly nonlinear or heavily disturbed regimes [10], and their effectiveness depends on the accuracy of the underlying plant model, which is rarely guaranteed under realistic conditions [9]. Classical nonlinear methods, including backstepping and Sliding Mode Control, offer improved robustness against model uncertainty and internal coupling [11], and are better suited to multivariable architectures [12]. Model Predictive Control achieves trajectory tracking with formal constraint satisfaction [13], although its computational cost limits deployment on lightweight embedded platforms. Learning-based strategies adapt to changing conditions and improve decision-making under uncertainty [14], but remain computationally intensive and raise non-trivial questions of closed-loop stability, convergence and real-time reliability on resource-constrained hardware [15]. Hybrid [16] and fault-tolerant architectures [17], cooperative multi-agent schemes [18] and sliding-mode observer designs for fixed-wing configurations [19] extend this landscape further.

Because the innermost loop of the architecture proposed here is a sampled-data motor regulation problem, recent advances in electric drive control bear directly on it. Ahmad et al. [20] address sampled-data multi-objective control for in-wheel-motor-driven electric vehicles under stochastically varying sampling periods, deriving stability criteria that hold when the sampling instant is not deterministic, a concern shared by any embedded controller whose loop period is subject to interrupt jitter. Shi et al. [21] observe that conventional proportional–integral–derivative and model predictive schemes struggle to regulate an in-wheel motor precisely under nonlinearity and unknown disturbance, and propose a multi-step compensation predictive functional controller that reduces mean squared error by 20.01 percent relative to nonlinear predictive functional control. Both approaches rely on direct measurement of the regulated mechanical variable and on computational budgets characteristic of automotive-grade processors. The constraint addressed here differs in kind. On a small quadrotor the regulated variable, rotor thrust, cannot be measured in flight without a force transducer that the mass and cost budget will not admit, and the available processor is an 8-bit fixed-point device without a floating-point unit. The

present work therefore takes the complementary route of reconstructing thrust from the bus current and regulating that estimate with a low-order polynomial controller whose cost is a fixed handful of multiply–accumulate operations per sample.

The accuracy of the plant model used in controller synthesis governs closed-loop performance. Physical parameters such as moments of inertia, aerodynamic coefficients and actuator characteristics are seldom precisely known a priori and may vary with flight condition or hardware configuration, so parametric identification has become an essential step in model-based design [22]. Frequency-domain analysis [23] and recursive least squares algorithms are widely applied to quadrotor dynamics estimation, and adaptive observer-based approaches have been proposed for real-time parameter estimation subject to closed-loop stability constraints [24]. At the actuator level in particular, Asadi [25] shows that parity-space residuals processed by recursive least squares can identify partial rotor faults from actuator-level signals, which situates actuator instrumentation as a resource for diagnosis as well as for regulation.

Three gaps persist across this literature. First, the great majority of quadrotor control studies command the actuator in open loop: a duty cycle or angular-rate setpoint is issued and the delivered thrust is assumed to follow, so that the effective actuator gain of the attitude loop depends on battery state of charge, propeller condition and electronic speed controller temperature. Rejection of this dependence is left implicitly to the outer loop, and its magnitude is rarely quantified. Second, advanced schemes are validated predominantly in simulation or on 32-bit floating point flight computers, leaving open whether comparable structures are realisable on the 8-bit fixed-point hardware that dominates low-cost and educational platforms. Third, reported hardware validations seldom quantify the discrepancy between simulated and measured transients, so the practical cost of the modelling simplifications on which the design rests remains unknown to the reader. This paper addresses the three together.

The contributions are as follows: (1) a discrete-time RST thrust regulator closed on electronic speed controller bus current, with the fourth-order current-to-thrust calibration reported in full, together with the conditioned realisation required because the synthesised controller polynomial is itself open-loop unstable; (2) a complete bare-metal C implementation on an 8-bit ATmega 2560; (3) a quantified characterisation of the resulting design—robustness margins computed on the identified plant, a stated validity envelope for the current-based thrust estimate, and the selection criteria for every controller parameter; and (4) an accounting of the discrepancy between simulated and measured transients, including a measurement of the inner-loop step response that locates the discrepancy in the actuator loop rather than in the cascade structure.

We claim novelty for none of the control laws themselves. The RST realisation is standard two-degree-of-freedom polynomial control, and the nested saturation law is that of Teel [26] as extended to underactuated aerial vehicles by Lozano and co-workers [27]. The contribution claimed is the architectural choice of what those loops are closed around, together with the characterisation of what that choice costs and delivers.

We also state plainly what is not claimed. This paper makes no claim of superiority over conventional cascade architectures. Establishing such a claim would require a controlled hardware comparison under matched disturbance and supply conditions, which is identified in Section 5.6 as the principal outstanding validation and among the directions for future work set out in Section 6.

The remainder of this article is organised as follows. Section 2 presents the quadrotor model and its linearisation. Section 3 develops the actuator-level RST controller, its calibration, its conditioned realisation and its robustness margins. Section 4 develops the supervisory nested saturation strategy and states the selection criteria for the controller

parameters. Section 5 reports the experimental characterisation and the limitations of the study. Section 6 concludes.

2. Quadrotor Modeling

2.1. General Structure and Flight Principle

The quadrotor shown in Figure 1 adopts the standard cross configuration widely employed in small-scale UAVs. Each of the four rotors generates an upward thrust; by differentially modulating rotor angular velocities the system produces controlled net forces and moments, enabling motion in three-dimensional space [28].

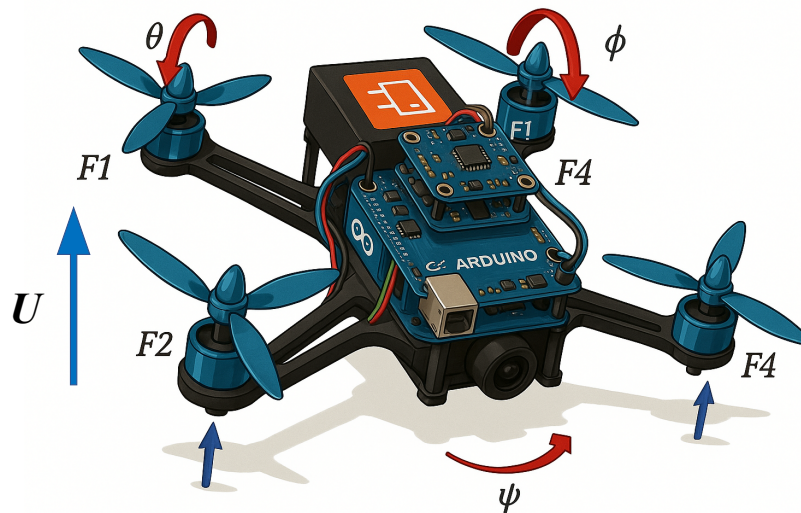


Figure 1. General structure of the quadrotor platform used to derive the mathematical model.

2.2. Mathematical Model

The quadrotor is modelled as a symmetric rigid body whose dynamics are governed by coupled translational and rotational motion under gravity and rotor-generated thrust [29].

2.2.1. Translational Motion

Applying Newton's second law to the centre of mass in the inertial frame,

$$m\ddot{\mathbf{r}} = R(\eta)\mathbf{F}_B - mg\hat{z} \quad (1)$$

where $\mathbf{r} = [x, y, z]^\top$ is the position vector in the inertial frame; $\mathbf{F}_B = [0, 0, f_T]^\top$ is the total aerodynamic thrust in the body-fixed frame; $R(\eta) \in SO(3)$ is the rotation matrix mapping body-frame quantities to the inertial frame; and $\eta = [\phi, \theta, \psi]^\top$ is the vector of Euler angles. Each rotor i produces a thrust $f_i = k\omega_i^2$ proportional to the square of its angular velocity, and the total thrust is

$$f_T = \sum_{i=1}^4 f_i. \quad (2)$$

2.2.2. Rotational Motion

The rotational dynamics follow the Euler equations for a rigid body,

$$I\dot{\boldsymbol{\omega}} = \mathbf{M} - \boldsymbol{\omega} \times (I\boldsymbol{\omega}) \quad (3)$$

where $\boldsymbol{\omega} = [p, q, r]^\top$ is the body-frame angular velocity and $\mathbf{M} = [M_x, M_y, M_z]^\top$ is the net aerodynamic torques arising from differential rotor forces.

2.2.3. Rotor Force-to-Wrench Mapping

The mapping from individual rotor forces to total thrust and body torques is given by the allocation matrix Q :

$$\begin{bmatrix} f_T \\ M_x \\ M_y \\ M_z \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & -l & 0 & l \\ -l & 0 & l & 0 \\ K_{TM} & -K_{TM} & K_{TM} & -K_{TM} \end{bmatrix}}_Q \begin{bmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \end{bmatrix} \quad (4)$$

where l is the distance from the centre of mass to each rotor and K_{TM} the torque-to-thrust coefficient.

2.3. Model Linearization

Linearisation proceeds in four steps.

First, the hover equilibrium is defined by $\mathbf{r} = \dot{\mathbf{r}} = \mathbf{0}$, $\eta = \mathbf{0}$, $\boldsymbol{\omega} = \mathbf{0}$ and $f_T = mg$, at which the nonlinear model of Equations (1)–(3) satisfies $\dot{\mathbf{x}} = \mathbf{0}$.

Second, the state and input are decomposed as $\mathbf{x} = \mathbf{x}_e + \delta\mathbf{x}$ and $\mathbf{u} = \mathbf{u}_e + \delta\mathbf{u}$, and the vector field is expanded to first order about the equilibrium, so that $A = \partial f / \partial \mathbf{x}$ and $B = \partial f / \partial \mathbf{u}$ evaluated at $(\mathbf{x}_e, \mathbf{u}_e)$.

Third, the rotation matrix is approximated as $R(\eta) \approx I + S(\eta)$, with $S(\cdot)$ the skew-symmetric operator. This yields the coupling block in which lateral acceleration is proportional to attitude deviation.

Fourth, the Euler kinematic map relating $\dot{\eta}$ to $\boldsymbol{\omega}$ reduces to the identity at $\eta = \mathbf{0}$, so that $\dot{\eta} = \boldsymbol{\omega}$ to first order.

Two classes of nonlinear term are discarded. The gyroscopic term $\boldsymbol{\omega} \times (I\boldsymbol{\omega})$ is quadratic in the angular-velocity perturbation and therefore contributes nothing at first order. The trigonometric terms are retained to first order only, so that $\sin \phi \approx \phi$, $\cos \phi \approx 1$ and $\tan \phi \approx \phi$. The resulting model is valid for attitude deviations within approximately $\pm 15^\circ$, beyond which the neglected terms exceed 10 percent of those retained. The nonlinear model of Section 2.2, and not its linearisation, is used for simulation; the linearised form serves only for controller synthesis and stability analysis.

2.3.1. State-Space Representation

The state and input vectors are

$$\mathbf{x} = [x \quad y \quad z \quad \dot{x} \quad \dot{y} \quad \dot{z} \quad \phi \quad \theta \quad \psi \quad p \quad q \quad r]^\top, \quad \mathbf{u} = [u_1 \quad u_2 \quad u_3 \quad u_4]^\top$$

where u_1 denotes the total thrust deviation from the hover value and u_2 to u_4 the body torques about the roll, pitch and yaw axes. The linearised model takes the standard continuous-time form

$$\dot{\mathbf{x}} = A\mathbf{x} + B\mathbf{u}. \quad (5)$$

2.3.2. Linearized System Matrices

With the ordering above, the system and input matrices are

$$A = \begin{bmatrix} \mathbf{0}_3 & \mathbf{I}_3 & \mathbf{0}_3 & \mathbf{0}_3 \\ \mathbf{0}_3 & \mathbf{0}_3 & G & \mathbf{0}_3 \\ \mathbf{0}_3 & \mathbf{0}_3 & \mathbf{0}_3 & \mathbf{I}_3 \\ \mathbf{0}_3 & \mathbf{0}_3 & \mathbf{0}_3 & \mathbf{0}_3 \end{bmatrix}, \quad G = \begin{bmatrix} 0 & -g & 0 \\ g & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad (6)$$

$$B = \begin{bmatrix} \mathbf{0}_{3 \times 4} \\ B_f \\ \mathbf{0}_{3 \times 4} \\ B_\tau \end{bmatrix}, \quad B_f = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \frac{1}{m} & 0 & 0 & 0 \end{bmatrix}, \quad B_\tau = \begin{bmatrix} 0 & \frac{1}{I_x} & 0 & 0 \\ 0 & 0 & \frac{1}{I_y} & 0 \\ 0 & 0 & 0 & \frac{1}{I_z} \end{bmatrix} \quad (7)$$

so that $A \in \mathbb{R}^{12 \times 12}$ and $B \in \mathbb{R}^{12 \times 4}$. The block G encodes the underactuation of the platform: lateral acceleration is available only through attitude deviation, which is what makes the cascaded structure of Sections 3 and 4 necessary rather than merely convenient.

Open-loop behaviour. The matrix A of Equation (6) has all twelve eigenvalues at the origin and rank 8, so it is a Jordan chain of integrators rather than a diagonalisable matrix. The open-loop linearised system is consequently not asymptotically stable: any attitude perturbation integrates without restoring action, and a bounded torque disturbance produces unbounded lateral displacement. This establishes the need for active closed-loop feedback analytically, without requiring a numerical demonstration of divergence.

3. Actuator-Level Control: Brushless Motor Command

Quadrotor control is complicated by inherently unstable and strongly nonlinear coupled dynamics [30]. A stratified architecture is therefore adopted: the supervisory level manages attitude and altitude, while the actuator level regulates the individual motors that produce the required thrust and torques [31]. This section develops the discrete-time RST strategy applied to the BLDC motors of the platform [32].

3.1. Thrust Estimation from Bus Current

Each rotor is driven by a BLDC motor with a dedicated electronic speed controller (ESC) [33]. Standard ESCs operate in open loop and provide no feedback on the thrust actually generated, which limits control accuracy under external disturbance or varying load. A closed-loop strategy is therefore introduced, based on measurement of the current drawn by each ESC, which serves as an indirect and readily obtainable estimate of the thrust produced by the corresponding motor [34].

The steady-state relationship between the current $I_{di}(k)$ drawn by motor i and its generated thrust $T_i(k)$ is approximated by a fourth-degree polynomial regression:

$$T_i(k) = a_4 I_{di}^4(k) + a_3 I_{di}^3(k) + a_2 I_{di}^2(k) + a_1 I_{di}(k) + a_0 \quad (8)$$

Calibration. The coefficients were determined by least squares from a bench record of 615 points spanning the full range of the 10-bit converter, with the raw converter count as regressor and the load-cell reading as response. Expressed with thrust in newtons and I_{di} as the raw converter count, the fitted coefficients are those of Table 1. The fit attains a coefficient of determination of 0.9982 with a root mean square residual of 0.065 N, which is 1.7 percent of the per-motor hover thrust. The coefficients are reported here so that the estimator can be reproduced.

Units. The bench transducer used for this calibration outputs gram-force. All thrust quantities in this paper are reported in newtons, obtained by multiplying the transducer reading by $g/1000$ with $g = 9.81 \text{ m/s}^2$. For reference, the per-motor thrust at hover for the 1.56 kg airframe of Table A1 is $mg/4 = 3.83 \text{ N}$, equivalently 390 gram-force, and the bench sweep spans 0.07 to 1.47 times this value.

Validity envelope. Equation (8) is a static calibration and its coefficients are conditional on the operating state of the power train, in particular on bus voltage, ambient temperature and propeller condition. Propeller replacement or appreciable wear requires recalibration, and a protocol is given in Section 6.

Table 1. Fitted coefficients of the current-to-thrust calibration of Equation (8), with thrust in newtons and I_{di} the raw 10-bit converter count. Fitted to 615 bench points spanning counts 409 to 1023.

Coefficient	Value	Fit Quality
a_4	$-2.848196 \times 10^{-11}$	Coefficient of determination $R^2 = 0.9982$ RMS residual = 0.065 N equivalently 1.7 percent of hover thrust
a_3	$+6.987492 \times 10^{-8}$	
a_2	-5.131290×10^{-5}	
a_1	$+1.698184 \times 10^{-2}$	
a_0	-1.817314	

Why exactness is not required. The current-based loop is not the outermost loop, and this materially relaxes the accuracy demanded of Equation (8). Attitude is closed on inertial measurement, and the four ESCs share a common supply bus. Estimation error therefore decomposes into two components with different consequences. Common-mode error, arising from bus-voltage droop, air density or ambient temperature, perturbs all four estimates in the same direction. It appears as a total thrust bias, is absorbed by the altitude loop where altitude sensing is present, and does not perturb attitude. Differential error, arising from propeller wear or controller-to-controller mismatch, does reach the attitude channel, but such errors vary slowly and are rejected by the integral action of the outer loop at the cost of a small steady offset. Section 5.5 identifies this mechanism as a candidate explanation for the residual roll offset measured on the roll channel.

A structurally better regressor. From momentum theory the induced power of a hovering rotor is $P_i = T^{3/2} / \sqrt{2\rho A_d}$, with A_d being the disc area, so that $T \propto P^{2/3}$ where $P = VI$ is the electrical input power. A regressor in electrical power rather than in current alone is therefore insensitive to bus-voltage variation to first order, because the voltage enters explicitly. This refinement is identified as future work in Section 6.

3.2. Identification of the Motor Model

To synthesise the RST controller, the dynamic model of the motor subsystem was determined experimentally. Step-amplitude duty-cycle signals were applied to the ESC and the corresponding responses recorded across a representative range of operating conditions [35]. The measured record and the excitation applied are shown in Figure 2, and the fit of the identified model to that record is shown in Figure 3.

The identification record comprises 1302 samples acquired at a 10 ms interval over 15.26 s, with the duty cycle stepped over the range 20 to 70 percent and the resulting thrust spanning 0.29 to 5.63 N, that is, up to 1.47 times the per-motor hover value.

Transfer function estimation used `tfest` with a second-order denominator and a first-order numerator carrying a one-sample transport delay, which is the structure consistent with the polynomials reported below. The resulting discrete-time transfer function is

$$G(q^{-1}) = \frac{9.089 q^{-1} - 7.88 q^{-2}}{1 - 0.7838 q^{-1} + 0.01384 q^{-2}} \quad (9)$$

which is written throughout as $G(q^{-1}) = B(q^{-1})/A(q^{-1})$.

The fit metric, defined. The goodness-of-fit figure reported by the System Identification Toolbox is the normalised root mean square error criterion

$$\text{FIT} = 100 \left(1 - \frac{\|y - \hat{y}\|}{\|y - \bar{y}\|} \right) \text{ percent}, \quad (10)$$

and not a coefficient of determination. The value of 88 percent obtained for the output-error estimate of Equation (9) therefore means that the residual norm is 12 percent of the norm of the centred measurement, which is a stricter criterion than R^2 on the same data.

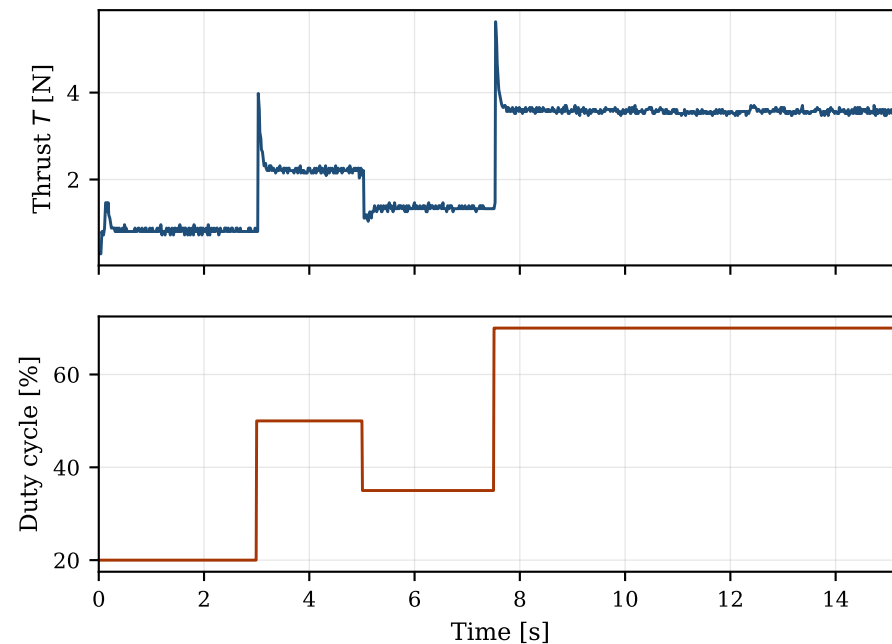


Figure 2. System identification record: Measured thrust (**upper** panel) under the step-amplitude duty-cycle excitation (**lower** panel). A total of 1302 samples at a 10 ms interval over 15.26 s. Thrust in newtons, converted from the gram-force output of the bench transducer.

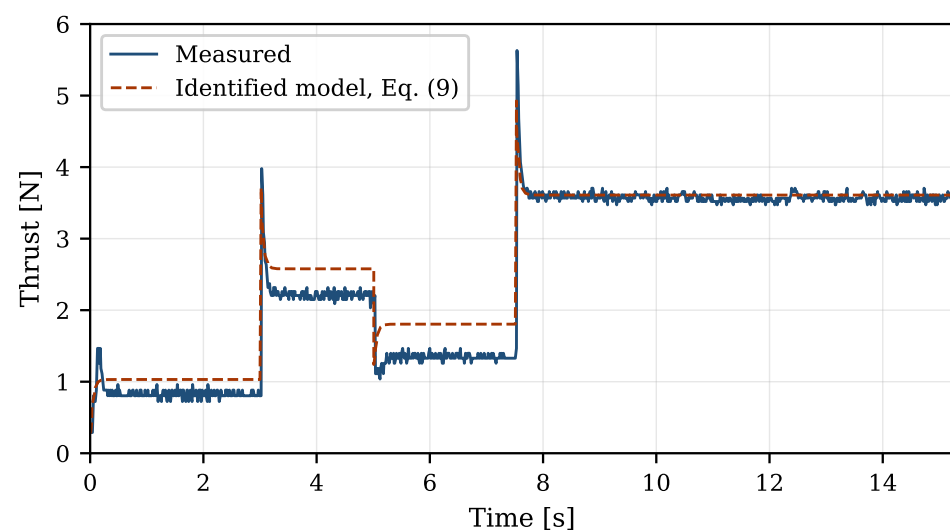


Figure 3. Comparison between the measured thrust and the output simulated by the identified model of Equation (9) under the same duty-cycle excitation. Thrust in newtons throughout.

Independent confirmation of the model. As a check on Equation (9), an equation-error model of the same order was estimated independently from the identification record by ordinary least squares. This gives a denominator $1 - 0.7894q^{-1} + 0.0095q^{-2}$ against the $1 - 0.7838q^{-1} + 0.01384q^{-2}$ of Equation (9), a dominant pole at $z = 0.777$ against 0.766, and a static gain of 5.05 against 5.26. The agreement in pole location and static gain confirms that Equation (9) describes the recorded dynamics, notwithstanding that the two criteria distribute the numerator coefficients differently.

What the model does not capture. The same check shows that a single linear model does not account for the record uniformly across the excitation range. The equation-error

fit over the whole record is 75 percent by the criterion of Equation (10), and the residuals are significantly correlated at short lags. This is consistent with the known nonlinearity of thrust against duty cycle, thrust varying approximately with the square of rotor speed, over a sweep as wide as 20 to 70 percent. We therefore do not claim that the residuals are white, and the adequacy of the model is argued in Section 3.5 on the different ground of closed-loop robustness margin rather than on residual whiteness.

3.3. Synthesis of the RST Controller

RST synthesis begins with a desired closed-loop characteristic polynomial derived from target performance specifications [36]. A second-order reference model with damping ratio $\xi = 0.7$ was specified, giving the dominant polynomial

$$P_D(q^{-1}) = 1 - 1.853 q^{-1} + 0.8633 q^{-2}. \quad (11)$$

The closed-loop characteristic polynomial is properly written as the product $P(q^{-1}) = P_D(q^{-1}) P_F(q^{-1})$, where P_F carries the auxiliary poles introduced by the degree of the Bezout solution. With $\deg S = \deg R = 2$ the product $AS + BR$ has degree four, so two auxiliary poles are placed at the origin.

Solving the Bezout identity for the RST polynomials yields

$$S(q^{-1}) = 1 - 2.4186 q^{-1} + 1.4186 q^{-2} \quad (12)$$

$$R(q^{-1}) = 0.1485 - 0.1425 q^{-1} + 0.0025 q^{-2} \quad (13)$$

$$T = t_0 = R(1) = r_0 + r_1 + r_2 = 0.0085 \quad (14)$$

where T is the scalar prefilter ensuring unit DC gain in the reference tracking path.

Two structural properties of these polynomials govern the implementation.

First, S contains the integrator required for zero steady-state error, but it also contains an unstable root:

$$S(q^{-1}) = (1 - q^{-1})(1 - 1.4186 q^{-1}), \quad (15)$$

whose second factor has a root at $z = 1.4186$, outside the unit disc. The controller is therefore open-loop unstable and is stabilised only by feedback.

Second, because $S(1) = 0$, evaluating the Bezout identity at $q = 1$ gives $B(1)R(1) = P(1)$ exactly, so the choice $T = R(1)$ delivers unit DC gain by construction rather than by numerical coincidence: $H_{cl}(1) = B(1)T/P(1) = 1$.

The nominal control law is

$$u_c(k) = -s_1 u(k-1) - s_2 u(k-2) + T T_d(k) - r_0 \hat{T}(k) - r_1 \hat{T}(k-1) - r_2 \hat{T}(k-2) \quad (16)$$

where $T_d(k)$ is the desired thrust reference and $\hat{T}(k)$ the estimated thrust from Equation (8).

Because of the unstable root in Equation (15), a recursion driven by the computed control u_c diverges whenever the pulse-width modulation command saturates. A conditioned realisation is therefore required, in which the command is clipped to the admissible actuator range and the clipped value, not the computed one, is fed back into the recursion at the following sample,

$$u(k) = \text{sat}_{[u_{\min}, u_{\max}]}(u_c(k)), \quad (17)$$

with $u(k-1)$ and $u(k-2)$ in Equation (16) taken from Equation (17). This is a necessary rather than an optional refinement: without it the inner loop is not implementable on saturating hardware. Section 5.5 returns to this point in connection with the measured inner-loop transient.

3.4. Closed-Loop Stability

Proposition 1 (RST Closed-Loop Stability). Let $G(q^{-1}) = B(q^{-1})/A(q^{-1})$ be the identified plant. If $R(q^{-1})$, $S(q^{-1})$ and the scalar T are obtained by solving the Bezout identity

$$A(q^{-1})S(q^{-1}) + B(q^{-1})R(q^{-1}) = P(q^{-1}), \quad (18)$$

with P chosen so that all roots of the corresponding z -domain polynomial lie strictly inside the unit disc, then the closed loop is bounded-input bounded-output stable and tracks a constant reference with zero steady-state error.

Proof. The transfer function from reference T_d to estimated output $\hat{T}$ is

$$H_{cl}(q^{-1}) = \frac{B(q^{-1})T}{A(q^{-1})S(q^{-1}) + B(q^{-1})R(q^{-1})} = \frac{B(q^{-1})T}{P(q^{-1})}. \quad (19)$$

By construction the characteristic polynomial is exactly P . Since its roots are placed inside the unit disc, all closed-loop poles satisfy $|z_i| < 1$, which is necessary and sufficient for bounded-input bounded-output stability of a linear discrete-time system [36]. Zero steady-state error follows from $T = R(1)$ together with $S(1) = 0$, which give $H_{cl}(1) = B(1)R(1)/P(1) = 1$ by Equation (18) evaluated at $q = 1$. $\square$

Remark 1. Numerical verification of the design. Expanding Equation (18) with the identified plant of Equation (9) and the polynomials of Equations (12) and (13) gives coefficients 1, -1.85268 , $+0.86278$, $+2.50 \times 10^{-4}$ and -6.66×10^{-5} in ascending powers of q^{-1} , reproducing P_D of Equation (11) to within 2.5×10^{-4} . The four closed-loop poles are $z_{1,2} = 0.9264 \pm 0.0697j$, with modulus 0.9290, together with two auxiliary poles of modulus below 0.009. All lie strictly inside the unit disc, so closed-loop stability of the RST motor loop is guaranteed for the computed polynomials.

3.5. Robustness Margins

Nominal stability is a weak guarantee for a design synthesised from an identified model, particularly one whose residuals are not white. The relevant question is how much plant error the loop tolerates, which is quantified by the sensitivity functions. Writing the output sensitivity as

$$S_{yp}(q^{-1}) = \frac{A(q^{-1})S(q^{-1})}{P(q^{-1})}, \quad (20)$$

the modulus margin is $\Delta M = 1/\|S_{yp}\|_{\infty}$, which is the shortest distance from the Nyquist plot of the open loop to the critical point and therefore bounds the multiplicative plant uncertainty the design absorbs [36].

Evaluated on the identified plant, the peak of $|S_{yp}|$ is 2.34 (+7.4 dB), occurring at the Nyquist frequency, so

$$\Delta M = 0.43 \quad (-7.4 \text{ dB}). \quad (21)$$

The open loop $L = BR/(AS)$ crosses unity gain at $\omega_c = 1.156$ rad per sample with a phase margin of 44.6° , a gain margin of 4.8 dB and a delay margin of 0.67 samples. These are collected in Table 2.

Three observations follow, and we state them plainly because two of them are unfavourable. First, $\Delta M = 0.43$ sits below the customary design target of 0.5, although above the value of 0.4 generally regarded as the minimum acceptable. Second, the margin nonetheless exceeds by a factor of 1.7 the 25 percent residual implied by the equation-error fit of 75 percent reported in Section 3.2, which is the more conservative of the two fit figures available, and this, rather than any claim about residual whiteness, is the sense in which the identified model is adequate for this design. Third, the delay margin of 0.67 samples

is small: less than one additional sample of latency would destabilise the nominal loop. Inertial sensor filtering delay, computational latency and pulse-width modulation transport delay all draw on that budget.

Table 2. Robustness margins of the synthesised RST loop, computed on the identified plant of Equation (9).

Quantity	Value	Customary Target	Interpretation
Peak $ S_{yp} $	2.34 (+7.4 dB)	≤ 2	Sensitivity peak at the Nyquist frequency
Modulus margin ΔM	0.43 (−7.4 dB)	≥ 0.5	Tolerates about 43 percent multiplicative uncertainty
Phase margin	44.6°	45 to 60°	At $\omega_c = 1.156$ rad per sample
Gain margin	4.8 dB	≥ 6 dB	Low but positive
Delay margin	0.67 samples	≥ 1 sample	The binding constraint

3.6. Control Architecture Representation

Figure 4 presents the hierarchical structure of the control system, showing the coordination between actuator-level motor loops, the attitude stabilisation layer and supervisory commands.

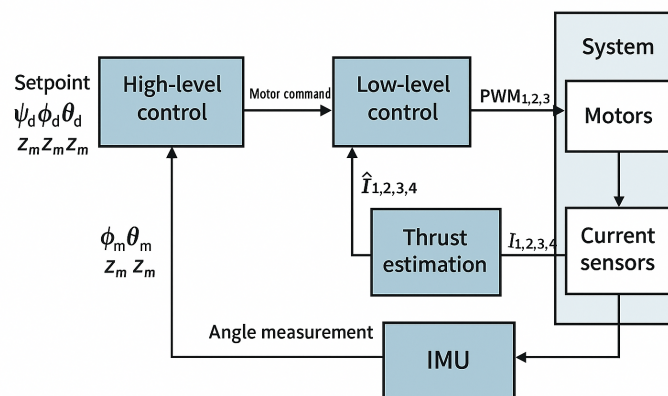


Figure 4. Overall structure of the proposed hierarchical quadrotor control architecture.

Figure 5 shows the closed-loop configuration incorporating bus current feedback for thrust estimation, which enables real-time adjustment of motor thrust from electrical measurement without a dedicated mechanical force sensor.

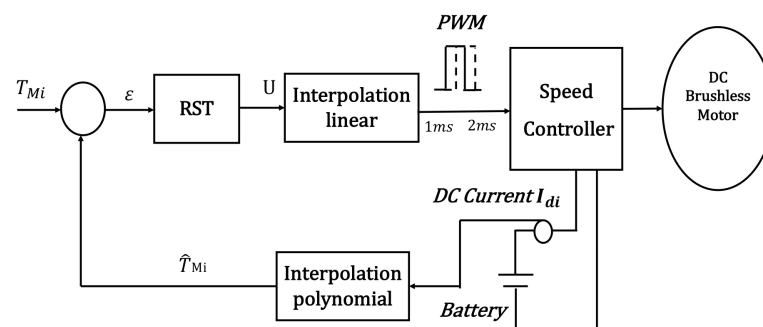


Figure 5. Closed-loop motor control system incorporating DC current feedback for real-time thrust estimation.

3.7. Bench Characterisation of the Inner Loop

The controller was implemented and tested on the BLDC motor test bench. Figure 6 presents the measured thrust response alongside the reference.

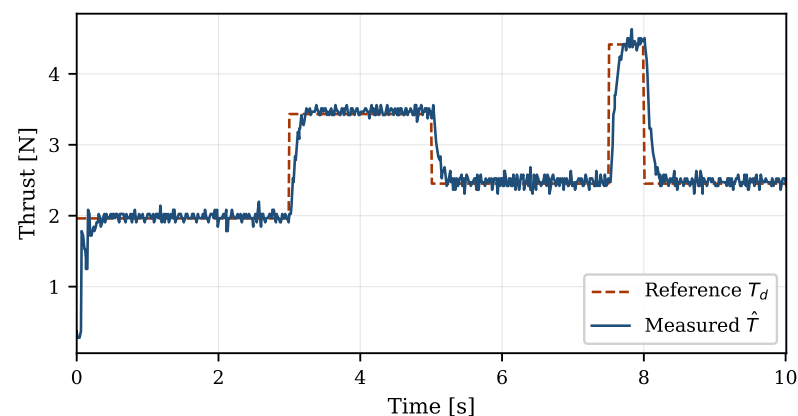


Figure 6. Experimental validation of the RST controller on the BLDC motor test bench: Measured thrust against reference trajectory. Ordinate in newtons, converted from the gram-force output of the bench transducer. A total of 854 samples at a 10 ms interval over 10.01 s.

The record comprises 854 samples acquired at a 10 ms interval over 10.01 s, with the thrust reference stepped between 1.96 and 4.41 N, that is, between 0.51 and 1.15 times the per-motor hover value. The loop tracks the reference at every step and returns to it without offset, confirming the integrator action established in Section 3.3.

Two reference steps in the record are large enough for transient metrics to be extracted reliably. Analysed from the recorded samples, the inner loop attains a rise time from 10 to 90 percent of 50 ms on the step from 1.97 to 2.47 N and 70 ms on the step from 3.51 to 4.26 N, giving a mean measured inner-loop rise time of 60 ms. The same two steps exhibit substantial transient overshoot, 48.7 percent on the larger and better conditioned of the two. Section 5.5 takes up the significance of this figure, which is considerably larger than the approximately 5 percent that a damping ratio of $\zeta = 0.7$ would imply and which we regard as the most important quantitative finding of the experimental campaign.

4. Supervisory Control: Attitude and Altitude

While the actuator-level RST controller regulates thrust for each motor, a supervisory strategy is required to stabilise attitude and position in three-dimensional space [37]. This section presents the nested saturation scheme developed for the coupled rotational and translational dynamics.

4.1. Altitude and Yaw Control

Altitude control regulates the total thrust through a proportional–derivative feedback law:

$$r_1 = -a_{z1} \dot{z} - a_{z2} (z - z_d) \quad (22)$$

where z_d is the desired altitude and a_{z1} , a_{z2} are positive gains. The resulting vertical acceleration command is

$$\ddot{z} = \frac{r_1}{m} \quad (23)$$

and, incorporating the coupling effect of roll and pitch, the total thrust command passed to the actuator level is

$$u = \frac{r_1 + mg}{\cos \phi \cos \theta}. \quad (24)$$

Yaw control uses a structurally analogous scheme:

$$\ddot{u}_\psi = -a_{\psi 1} \dot{\psi} - a_{\psi 2} (\psi - \psi_d) \quad (25)$$

corresponding to the second-order closed-loop transfer function

$$\frac{\Psi(s)}{\Psi_d(s)} = \frac{a_{\psi 2}}{s^2 + a_{\psi 1} s + a_{\psi 2}}. \quad (26)$$

The gains of each channel follow directly from a second-order specification: for a chosen natural frequency ω_n and damping ratio ζ , Equation (26) gives $a_{\psi 2} = \omega_n^2$ and $a_{\psi 1} = 2\zeta\omega_n$, and identically for the altitude channel through Equation (23). The natural frequency is bounded above by the requirement that the supervisory channel remain slow relative to the actuator loop characterised in Section 3.7. Figure 7 shows the corresponding block diagram.

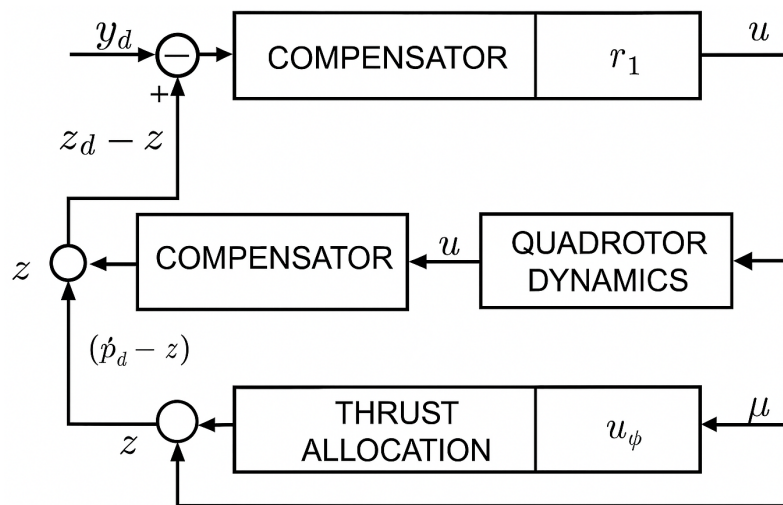


Figure 7. Block diagram of the altitude and yaw control loops within the hierarchical control architecture.

4.2. Roll and Pitch Control via Nested Saturation

Lateral position control along x and y , and the associated regulation of roll and pitch, use a nested saturation approach [38], which applies a hierarchical cascade of saturation functions to bound intermediate variables and to provide robustness against disturbance and actuator limits [39].

The lateral acceleration dynamics from the linearised model are

$$\ddot{x} = -g \tan \theta \quad (27)$$

$$\ddot{y} = g \tan \phi \quad (28)$$

which under the small-angle approximation simplify to

$$\ddot{x} = -g \theta \quad (29)$$

$$\ddot{y} = g \phi. \quad (30)$$

The nested saturation law for the roll channel, with an analogous structure for pitch, is

$$\ddot{u}_\phi = -\sigma_{\phi 1} \left(\dot{\phi} + \sigma_{\phi 2} \left(v_1 + \sigma_{\phi 3} \left(v_2 + \sigma_{\phi 4} (v_3) \right) \right) \right) \quad (31)$$

where the nested variables are

$$v_1 = \dot{\phi} + \phi \quad (32)$$

$$v_2 = \dot{\phi} + 2\phi + \frac{\dot{y}}{g} \quad (33)$$

$$v_3 = \dot{\phi} + 3\phi + \frac{3\dot{y}}{g} + \frac{y}{g}. \quad (34)$$

The form above follows the canonical construction for the four-integrator chain, in which the normalisation by g renders the position and velocity contributions commensurate with the angular terms, as required for the saturation levels to be comparable quantities. Figure 8 shows the saturation function applied at each level of the cascade.

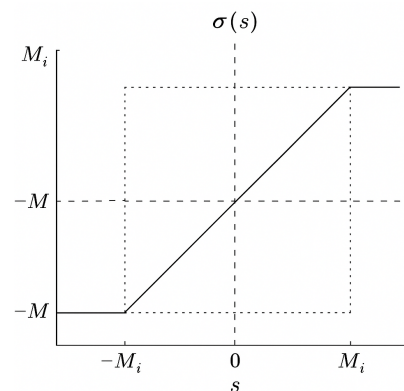


Figure 8. The saturation function $\sigma_i(\cdot)$ of level M_i applied at each level of the nested cascade of Equation (31) for roll and pitch.

4.3. Stability Analysis

Global asymptotic stability of the nested saturation scheme follows from the framework of Teel [26], extended to underactuated aerial vehicles by Lozano and co-workers [27]. We claim no novelty for the result and reproduce it here only to make explicit the condition that the implemented saturation levels must satisfy.

Theorem 1 (Global Asymptotic Stability of Nested Saturation Control, Lozano et al. [27]). Consider a chain of n integrators driven by

$$u = -\sigma_1(x_1 + \sigma_2(x_2 + \cdots + \sigma_n(x_n) \cdots)), \quad (35)$$

where $\sigma_i(\cdot)$ is a saturation of level $M_i > 0$. If

$$M_i > \sum_{j=i+1}^n M_j, \quad i = 1, \dots, n-1, \quad (36)$$

then the origin is a globally asymptotically stable equilibrium of the closed-loop system.

Remark 2. The lateral dynamics under the small-angle approximation, $\ddot{y} = g\phi$ and $\ddot{\phi} = \ddot{u}_\phi$, form a four-dimensional integrator chain in $[y, \dot{y}, \phi, \dot{\phi}]^\top$. Condition (36) is strictly stronger than the monotone ordering $M_{\phi 1} > M_{\phi 2} > M_{\phi 3} > M_{\phi 4}$, which does not by itself imply $M_{\phi 1} > M_{\phi 2} + M_{\phi 3} + M_{\phi 4}$. The construction procedure and the admissible range are given in Section 4.4. The altitude and yaw channels, governed by the second-order dynamics of Equations (22)–(26) with positive gains, are independently asymptotically stable by standard linear theory.

4.4. Controller Parameters and Selection Criteria

The procedure by which the controller parameters were obtained is given here, together with the numerical values of the actuator-level design and the admissible range for the saturation levels.

The actuator-level parameters are fixed by the reference model. A damping ratio of $\xi = 0.7$ was specified as the standard compromise giving approximately 5-percent step overshoot, and discretisation of the corresponding second-order reference model gives P_D of Equation (11). The polynomials S and R then follow as the minimum-degree Bezout solution with the integrator factor $(1 - q^{-1})$ imposed on S , and T follows as $R(1)$. Their values are those of Equations (12)–(14) and are collected in Table 3.

The supervisory gains follow from second-order channel specifications as described in Section 4.1, with the natural frequency bounded above by the requirement of adequate separation from the actuator loop.

The saturation levels are bounded above by actuator authority: the outermost level cannot demand more angular acceleration than differential thrust can deliver:

$$M_{\phi 1} \leq \frac{l(2\Delta T_{\max})}{I_x}, \quad (37)$$

where $\Delta T_{\max}$ is the per-motor thrust headroom above hover. The bench sweep of Section 3.2 reaches 5.63 N against a hover value of 3.83 N, giving $\Delta T_{\max} = 1.80$ N. With $l = 0.20$ m and $I_x = 1.61 \times 10^{-2}$ kg m² this yields $M_{\phi 1} \leq 44.8$ rad/s², and with $I_y = 1.56 \times 10^{-2}$ kg m² the corresponding pitch bound is $M_{\theta 1} \leq 46.3$ rad/s². Within that bound the levels are constructed downward from the innermost, each outer level chosen strictly greater than the sum of those inside it so that Equation (36) is satisfied by construction rather than assumed.

Table 3. Actuator-level controller parameters and the criteria fixing them.

Parameter	Value	Selection Criterion
ξ	0.7	Standard compromise giving about 5-percent step overshoot
$P_D(q^{-1})$	$1 - 1.853q^{-1} + 0.8633q^{-2}$	Discretisation of the second-order reference model
$S(q^{-1})$	$1 - 2.4186q^{-1} + 1.4186q^{-2}$	Bezout solution of minimum degree, integrator imposed
$R(q^{-1})$	$0.1485 - 0.1425q^{-1} + 0.0025q^{-2}$	Bezout solution
T	0.0085	$T = R(1)$, giving unit DC gain since $S(1) = 0$
$M_{\phi 1}$	≤ 44.8 rad/s ²	Equation (37) with $\Delta T_{\max} = 1.80$ N measured on the bench
$M_{\theta 1}$	≤ 46.3 rad/s ²	As above with I_y
$M_{\phi 2..4}, M_{\theta 2..4}$	See text	Constructed downward to satisfy Equation (36)

4.5. Stability of the Cascaded Architecture

The complete system is a cascade of the inner RST motor loops (Σ_1) and the outer nested saturation attitude and altitude controller (Σ_2).

Proposition 2 (Cascade Stability). *Let Σ_1 be the RST-controlled motor dynamics, which are bounded-input bounded-output stable by Proposition 1 and hence input-to-state stable with respect to the thrust setpoint, with finite gain γ_1 . Let Σ_2 be the nested saturation subsystem, which by Theorem 1 is asymptotically stable and whose output is bounded by $M_{\phi 1}$ through construction, with finite gain γ_2 . If the small-gain condition $\gamma_1\gamma_2 < 1$ holds, then the equilibrium of the*

interconnection is asymptotically stable over the region $|\phi|, |\theta| \leq 15^\circ$ on which the linearisation of Section 2.3 is valid.

Proof. Σ_1 is bounded-input bounded-output stable by Proposition 1 and is therefore input-to-state stable with respect to bounded setpoint commands [40]. Σ_2 produces bounded references because the saturation levels are finite by construction, so the interconnection signal is bounded a priori and finite escape time is precluded. Asymptotic stability of the interconnection then follows from the small-gain theorem applied to the two input-to-state stable subsystems, provided $\gamma_1\gamma_2 < 1$. The restriction to the stated region arises because Σ_2 is designed on the small-angle model of Equations (29) and (30); outside it the integrator-chain structure on which Theorem 1 depends no longer holds. $\square$

Remark 3. We draw attention to what this proposition does not claim. The statement is regional rather than global: a global claim would require an argument that does not assume boundedness of the trajectories it purports to establish. We also do not evaluate γ_1 and γ_2 numerically, so the small-gain condition should be read as a design requirement rather than as a verified certificate. In cascade design practice the requirement is met by ensuring adequate separation of the two loop bandwidths, and Section 5.5 reports the separation actually achieved.

5. Experimental Characterisation

The scope of the experimental claims should be stated at the outset. The platform is a constrained attitude rig carrying no altitude or position sensing: no barometer, rangefinder, optical-flow sensor, satellite navigation receiver or motion-capture reference. The altitude and yaw loops of Section 4.1 and the lateral loops of Section 4.2 are therefore analysed but not experimentally validated, and the lateral nested saturation law in particular feeds back y and $\dot{y}$, which the rig cannot measure. All experimental results below concern roll and pitch stabilisation about the hover equilibrium under external disturbance.

5.1. Experimental Setup

The platform comprises a quadrotor with BLDC motors and an embedded ATmega 2560 microcontroller; a motor characterisation test bench used during identification; a host PC for programming and monitoring; a digital oscilloscope for pulse-width modulation visualisation; a digital multimeter for battery voltage monitoring; and a precision levelling tool for inertial sensor calibration before each experiment. Figure 9 shows the configuration.

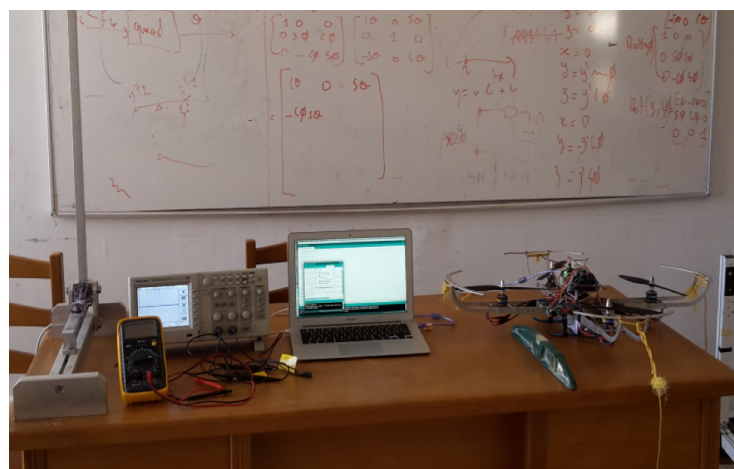


Figure 9. Experimental setup employed for real-time control validation.

Table 4 records the acquisition parameters of the experimental campaign, established from the recorded data files. The bench records carry a nominal 10 ms interval with a mean interval of 11.7 ms, the difference reflecting timestamp jitter in the host-side logging path rather than in the control loop itself.

Table 4. Acquisition parameters of the experimental records.

Record	Interval	Samples	Span
Motor identification	10 ms nominal	1302	15.26 s, duty 20 to 70 percent
Motor RST tracking	10 ms nominal	854	10.01 s, reference 1.96 to 4.41 N
Roll and pitch attitude	5 ms	up to 10,202	up to 51.0 s
Calibration sweep	static	615	converter counts 409 to 1023

5.2. Pitch Channel

Figure 10 presents the pitch angle response. The controller stabilises the pitch angle to the zero-degree equilibrium despite externally applied disturbances, with a transient during initial convergence before settling. The corresponding pulse-width modulation commands applied to motors M_2 and M_4 are shown in Figure 11.

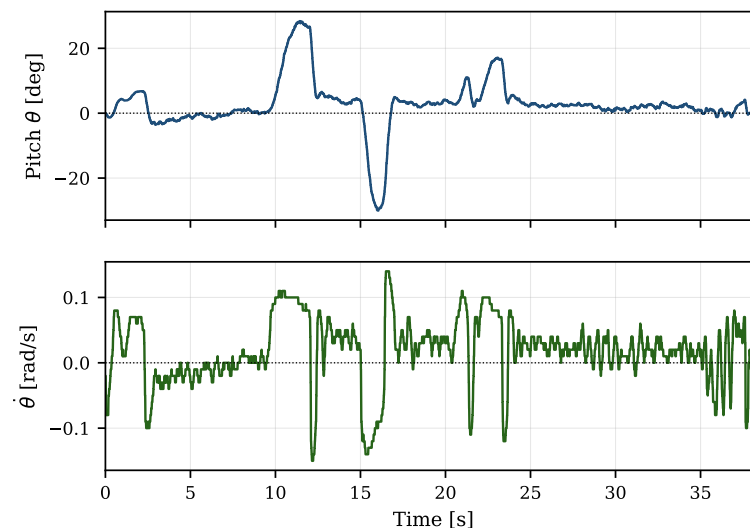


Figure 10. Pitch channel control performance: Angle time response (upper panel) and corresponding angular velocity (lower panel). A total of 7717 samples at a 5 ms interval over 38.6 s. The dotted line marks the zero-degree equilibrium.

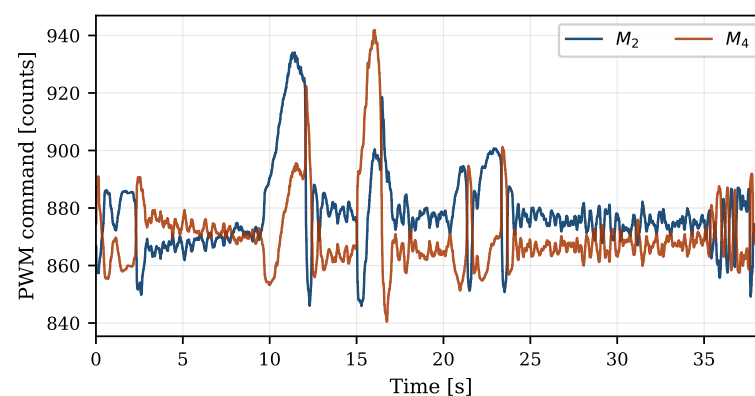


Figure 11. PWM command signals applied to motors M_2 and M_4 during pitch axis control.

5.3. Roll Channel

Figure 12 presents the roll angle response, showing stabilisation near the zero-degree equilibrium and compensation for externally applied disturbances. The corresponding commands applied to motors M_1 and M_3 are shown in Figure 13.

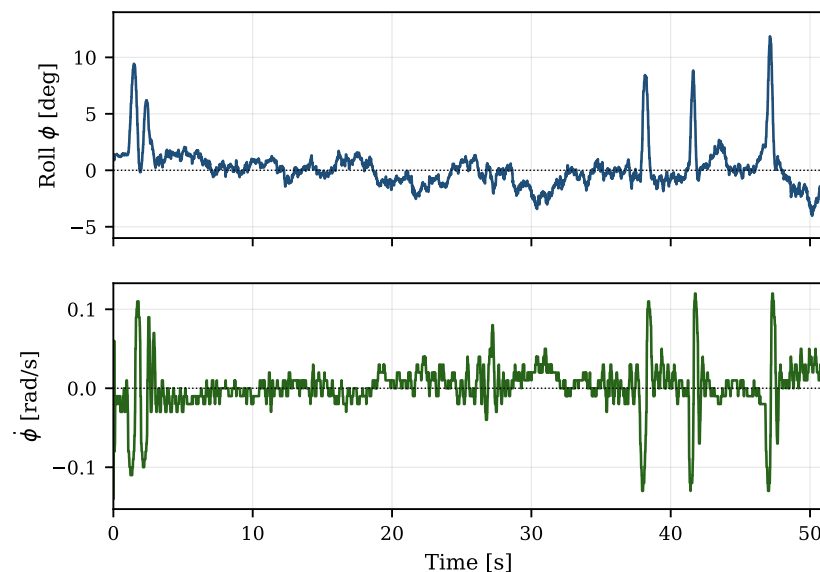


Figure 12. Roll channel control performance: Angle time response (upper panel) and corresponding angular velocity (lower panel). A total of 10,202 samples at a 5 ms interval over 51.0 s. The dotted line marks the zero-degree equilibrium.

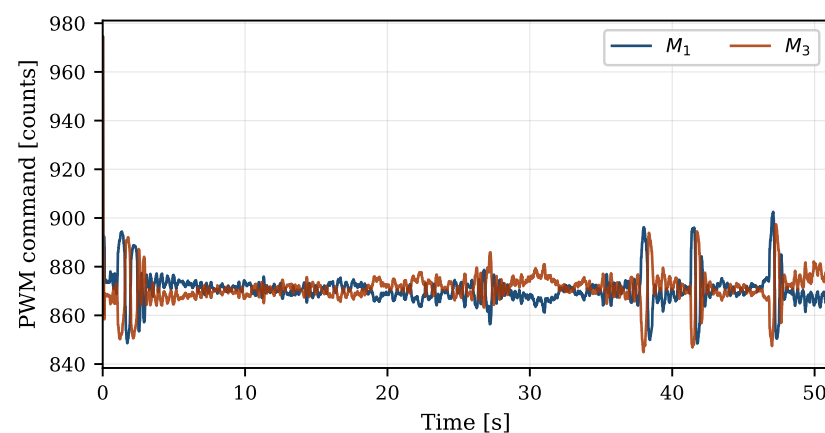


Figure 13. PWM command signals applied to motors M_1 and M_3 during roll axis control.

5.4. Quantitative Performance

Table 5 summarises the closed-loop performance metrics for both attitude channels. The residual roll offset is quantified from a dedicated 20.7 s balancing record of 4134 samples: the mean roll angle is -2.042° with a standard deviation of 0.212° and a peak-to-peak excursion of 1.35° . The corresponding pitch offset is not distinguishable from zero at the resolution of the instrumentation.

Table 5. Experimental closed-loop performance metrics for the roll and pitch attitude channels. Because the controller regulates to the zero-angle equilibrium and does not track a non-zero reference, the overshoot column reports disturbance-recovery overshoot: the counter-swing past equilibrium during recovery from an impulsive external disturbance, normalised by the peak disturbance-induced deviation. This is not step-reference overshoot.

Channel	Rise Time (s)	Settling Time (s)	Recovery Overshoot (%)	Steady-State Error (°)
Pitch (θ)	0.31	0.70	60	≈ 0
Roll (ϕ)	0.32	0.50	20	2.04

5.5. Discussion

Three observations follow from the records above, of which the second is the principal finding of this study.

Symmetric actuation is confirmed. The command patterns of Figures 11 and 13 show the symmetric differential actuation expected from the control design, and stability was preserved in every recorded trial: across the full experimental campaign, comprising more than 60 s of continuous roll and pitch operation with repeated manual disturbance injection, no divergence or sustained limit cycle was observed. We report this rather than describing the controller as robust, because robustness is a quantitative property and the appropriate figures are those of Table 2 together with the range of conditions just stated.

The measured transients originate in the actuator loop, not in the cascade structure. It would be natural to attribute the large disturbance-recovery overshoot of Table 5 to insufficient time-scale separation between the two levels, since that is the usual cause of such behaviour in cascade designs. The records do not support that explanation. The measured inner-loop rise time of Section 3.7 is 60 ms, and the measured outer-loop rise time of Table 5 is 310 ms, giving a bandwidth separation of 5.2 to 1. This lies within the range of 5 to 10 that cascade design practice conventionally requires, so the separation is adequate and is not the limiting factor.

What the records do show is that the inner loop itself exhibits approximately 49 percent overshoot on a well-conditioned reference step, against the approximately 5 percent that its damping specification of $\zeta = 0.7$ would imply. An actuator loop that overshoots by half its step size will impose a corresponding transient on any outer loop it serves, which is consistent with the 60 percent and 20 percent recovery overshoot observed on the pitch and roll channels respectively. We therefore locate the discrepancy between simulated and measured transients in the actuator loop rather than in the cascade architecture.

Two candidate mechanisms are consistent with an inner-loop overshoot of this magnitude, and we distinguish them without at present being able to decide between them. The first is model error: the identification of Section 3.2 does not capture the record uniformly across the excitation range, the residuals being significantly correlated, so the plant seen by the controller near the operating point may differ from Equation (9) sufficiently to degrade the achieved damping. The second is realisation: as established in Equation (15) the controller polynomial S carries a root outside the unit disc, so the transient behaviour under actuator saturation depends critically on whether the conditioned realisation of Equation (17) is used, and an unconditioned recursion would produce exactly this class of transient. Distinguishing the two requires a bench experiment in which the realisation is varied with the plant held fixed, and this is identified as the first item of future work.

The residual offset is consistent with differential estimator error. The residual roll offset of 2.04° , against a pitch offset indistinguishable from zero, is consistent with a differential rather than a common-mode error in the thrust estimate, as anticipated in Section 3.1: a small asymmetry between the calibrations of the two roll-axis motors maps

directly to a steady attitude bias, whereas a supply-related error common to all four motors would not.

5.6. Limitations

Five limitations bound the claims made here, and we state them explicitly rather than distributing them through the discussion.

No comparison against a conventional baseline. The principal outstanding validation is a controlled hardware comparison against a conventional cascade controller under matched disturbance and supply conditions. The experimental platform was not available during the preparation of this work, so no such comparison was performed and none is claimed. Until it is carried out, the case for the additional complexity of the current-feedback inner loop rests on the structural argument of Section 3.1 and on the characterisation reported here, not on measured comparative performance, and the reader should weigh it accordingly.

Scope of experimental validation. Only roll and pitch stabilisation about the hover equilibrium has been demonstrated in hardware. Altitude, yaw and lateral position loops are analysed but not flown, because the rig carries no altitude or position sensing.

No non-zero reference tracking. The implementation regulates to zero attitude. Tracking a non-zero attitude reference requires a reference-shaping stage that has not been implemented, so the platform cannot at present execute the trajectory-following tasks that inspection or surveillance missions would demand.

Validity envelope of the thrust estimate. The calibration of Equation (8) is static and requires recalibration on propeller replacement or appreciable wear. Its coefficients are conditional on bus voltage and ambient temperature, and the sensitivity to these has not been characterised experimentally.

Computational budget not fully characterised. The acquisition parameters of Table 4 are established from the recorded data. A complete real-time budget, comprising worst-case interrupt service execution time, loop-period jitter measured at the processor, memory footprint and processor utilisation, is not reported here and is required before the real-time claim can be considered fully substantiated.

6. Conclusions

This paper has reported the design, embedded realisation and experimental characterisation of a two-level discrete-time control architecture for a 1.56 kg quadrotor implemented on an 8-bit ATmega 2560. At the actuator level a discrete-time RST regulator closes a thrust loop around a sensorless estimate reconstructed from electronic speed controller bus current, through a fourth-order calibration reported in full and fitted to 615 bench points with a coefficient of determination of 0.998. At the supervisory level, proportional–derivative regulation of altitude and yaw is combined with a nested saturation law whose admissible saturation levels are bounded by measured actuator authority.

The synthesised inner loop attains a modulus margin of 0.43, a phase margin of 44.6 degrees and a delay margin of 0.67 samples, tolerating multiplicative plant uncertainty of approximately 43 percent, which exceeds by a factor of 1.7 the residual implied by the more conservative of the two identification fit figures reported. Roll and pitch stabilisation was demonstrated under external disturbance, with rise times of 0.31 and 0.32 s and a residual roll offset of $2.04 \pm 0.21^\circ$.

The principal experimental finding concerns the origin of the discrepancy between simulated and measured transients. Bench records give a measured inner-loop rise time of 60 ms against a measured outer-loop rise time of 310 ms, a bandwidth separation of 5.2 to 1 which lies within conventional cascade design practice. Time-scale separation is

therefore not the limiting factor. The inner loop is instead found to overshoot by approximately 49 percent on a well-conditioned reference step, against the 5 percent implied by its damping specification, and this actuator-level transient rather than the cascade structure is identified as the likely origin of the attitude transients observed.

Five directions follow. First, and most importantly, a bench experiment distinguishing the two candidate mechanisms for the inner-loop overshoot, namely model error near the operating point and the realisation of the controller recursion under saturation, which requires only the motor test bench. Second, a controlled hardware comparison against a conventional cascade controller, which is the validation this study most needs. Third, replacing the current-only thrust regressor with one in electrical power, which momentum theory renders structurally insensitive to supply-voltage variation. Fourth, extending the thrust estimator to serve simultaneously as an actuator fault-detection residual, an application for which the parity-space and recursive least squares methods of [25] are directly applicable and for which the required instrumentation is already present. Fifth, adding altitude and position sensing so that the loops here analysed can be validated in flight, together with the reference-shaping stage needed for non-zero attitude tracking. Underlying the first and third of these is the broader question of how to quantify confidence in an identified actuator model whose residuals are known not to be white: where a fixed transfer function is replaced by a learned or adaptive representation, calibrated uncertainty estimates become a prerequisite for certifying closed-loop behaviour, and scalable Bayesian methods such as deep Gaussian processes [41] offer one route to such estimates.

The broader contribution is a demonstration that a two-level polynomial and saturated architecture, with its actuator loop closed on a sensorless thrust estimate, is realisable on 8-bit fixed-point hardware, together with a quantified account of its robustness margins and of where its measured behaviour departs from design. The control laws themselves are standard; what this study supplies is the architectural choice of what they are closed around, and an honest accounting of what that choice delivers and where it falls short.

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Conflicts of Interest: The authors declare no conflicts of interest.

Abbreviations

UAS	Unmanned aerial system
UAV	Unmanned aerial vehicle
BLDC	Brushless direct current (motor)
ESC	Electronic speed controller
IMU	Inertial measurement unit
PWM	Pulse-width modulation
RST	Two-degree-of-freedom polynomial controller (R, S, T polynomials)
PID	Proportional–integral–derivative
LQR	Linear–quadratic regulator
SMC	Sliding mode control
MPC	Model predictive control
RLS	Recursive least squares
PD	Proportional–derivative
BIBO	Bounded-input bounded-output
ISS	Input-to-state stable
NRMSE	Normalised root mean square error
z_d	Desired altitude
z	Measured altitude
ψ_d	Desired yaw angle
ψ	Measured yaw angle
$\tilde{u}_\psi$	Virtual control input for the yaw channel
r_1	Intermediate virtual control signal for altitude
u	Total thrust command
$\sigma(\cdot)$	Saturation function
M_i	Saturation threshold (linear-region limit)
$\hat{T}$	Estimated rotor thrust from bus current
ΔM	Modulus margin
S_{yp}	Output sensitivity function

Appendix A

Physical Parameters of the Quadrotor Model

Table A1. Physical parameters of the quadrotor system model.

Parameter	Symbol	Value [Unit]
Mass	m	1.56 [kg]
Distance, motor to CG	l	0.20 [m]
Gravitational acceleration	g	9.81 [m/s ²]
Moment of inertia (x -axis)	I_x	1.61×10^{-2} [kg·m ²]
Moment of inertia (y -axis)	I_y	1.56×10^{-2} [kg·m ²]
Moment of inertia (z -axis)	I_z	1.94×10^{-2} [kg·m ²]
Torque about ψ (yaw)	τ_ψ	3×10^{-3} [N·m]
Torque about θ (pitch)	τ_θ	3×10^{-3} [N·m]
Torque about ϕ (roll)	τ_ϕ	3×10^{-3} [N·m]
Per-motor thrust at hover	$T_h = mg/4$	3.83 [N]
Per-motor thrust headroom	$\Delta T_{\max}$	1.80 [N]

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