

Article

Design and Optimization of a Two-Stage Magnetically Geared Machine Comprising Radial-Flux and Axial-Flux Magnetic Gears

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Abstract

Drive systems for robot joints must provide a high gear ratio within limited axial space. When two magnetic gears are axially stacked to form a two-stage transmission, the axial lengths of the individual stages accumulate. This paper therefore proposes a magnetically geared machine (MGM) comprising a radial-flux magnetic gear and an axial-flux magnetic gear. The permanent-magnet synchronous motor and the radial-flux magnetic gear occupy the inner space of the axial-flux magnetic gear, while shared rotors connect the three electromagnetic components. For this topology, the magnetic field modulation and torque relationships are derived, and the main design parameters are determined through two-stage optimization and three-dimensional transient finite-element analysis. The results show that the air gaps of both magnetic gears contain the required working harmonics and that the steady-state torques of the three rotors follow the two-stage transmission relationship. The optimized design achieves an overall gear ratio of 84.64 and a maximum transferable torque of 1098.97 N m. At this operating point, the volumetric torque density based on the overall cylindrical envelope volume is 328.00 N m L⁻¹.

Keywords: magnetically geared machine (MGM); two-stage transmission; robot joint; high torque density

1. Introduction

Drive systems for industrial robot joints must integrate the motor, reduction mechanism, sensors, and drive electronics within a constrained joint volume while delivering high torque at low speed [1]. Because these components share the joint envelope, increasing the motor dimensions is constrained; axial span and volumetric torque density therefore become coupled design requirements. A mechanical gearbox reduces the torque required from the motor, but contact transmission introduces friction, wear, and lubrication requirements [2]. Magnetic gears provide non-contact speed reduction through magnetic field modulation [3]. A magnetically geared machine (MGM) integrates a magnetic gear with a permanent-magnet synchronous motor (PMSM), reducing the number of independent coupling components [4]. Shaft-coupled motor–magnetic-gear optimization has also been reported [5]. An integrated Halbach magnetic-geared permanent-magnet motor provides another form of combined electromagnetic conversion and magnetic gearing [6].

Field-modulated magnetic-gear topologies, applications, and design challenges have been reviewed in [7]. Magnetic gears can be classified as radial-flux and axial-flux configurations according to the main-flux direction [8]. Axial-field magnetic gears provide a disk-shaped alternative to radial-flux coaxial configurations [9]. Halbach-array axial-flux magnetic gears



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can use a relatively large effective torque radius [10]. Axial-flux magnetically geared generators have also been investigated [11]. Flux-focusing variants extend this arrangement to magnetically geared motors [12]. Axial-force characteristics have been evaluated for integer and fractional gear ratios [13]. Within the examined design space for a surface-permanent-magnet radial-flux coaxial magnetic gear, the maximum transferable torque decreased when the single-stage gear ratio exceeded approximately 10 [14]. Increasing the pole-pair numbers of a single stage therefore cannot maintain both a high gear ratio and high torque capability. Connecting two magnetic-gear stages in series allows their gear ratios to multiply without further increasing the ratio of either stage [14].

When two complete magnetic gears are arranged sequentially in the axial direction, the axial spans of their rotors, air gaps, and supporting structures accumulate. In an experimentally evaluated dual-stage coaxial magnetic gear with an overall gear ratio of 63.3, the volumetric torque density decreased from 268 N m L^{-1} for the first stage to 228.6 N m L^{-1} for the complete assembly [15]. The corresponding reduction of 14.7% indicates the system-level volume penalty associated with the second stage and its supporting structure. A previously reported MGM reduced the number of independent rotating components by sharing the high-speed rotor between a vernier machine and a coaxial magnetic gear [16]. This configuration did not include an axial-flux magnetic gear and therefore did not use its inner space for another coaxial transmission stage. A radial-flux cycloidal magnetic gear, an axial-flux coaxial magnetic gear, and a motor have also been combined in a single system [17]. A further cycloidal magnetic gear combined axial and radial flux paths within one structure [18]. In these radial-axial configurations, the radial component operates through cycloidal motion, and the rotating components and interstage connections differ from those of two coaxial magnetic gears. The reviewed configurations therefore do not establish a continuous two-stage torque-transmission path by placing a coaxial radial-flux magnetic gear within the inner space of a coaxial axial-flux magnetic gear. In a serial arrangement, the occupied axial span is the sum of the two stage spans and their interstage support.

Accordingly, this paper presents an MGM comprising radial-flux and axial-flux coaxial magnetic gears. The PMSM and radial-flux magnetic gear occupy the inner space of the axial-flux magnetic gear. A shared permanent-magnet rotor connects the PMSM to the radial-flux magnetic gear, while a shared back iron connects the two magnetic gears. The shared components connect the main and leakage flux paths of the two stages, so their magnetic circuits cannot be treated as independent. The magnetic-gear dimensions determine the maximum transferable torque and overall cylindrical envelope volume, whereas the stator dimensions determine the rated drive capability. The reviewed studies do not report a staged optimization that first determines magnetic-gear geometry from volumetric torque density and maximum transferable torque and then selects the stator for rated operation under the resulting envelope. These coupled design requirements are addressed by first optimizing the two magnetic gears for volumetric torque density and then optimizing the stator under the rated-torque constraint. The main magnetic circuit, magnetic field modulation of both stages, and the motion and torque-transmission relationships of the three rotors are derived, and three-dimensional transient finite-element analysis is used to evaluate the working harmonics and steady-state torque transmission.

2. Machine Configuration and Operating Principle

2.1. Machine Configuration and Torque Transmission

Figure 1 shows the overall machine configuration. The 3-D cutaway view in Figure 1a shows the coaxial arrangement of the outer-rotor PMSM, radial-flux magnetic gear, and axial-flux magnetic gear. In the top view (Figure 1b), the PMSM is at the center, surrounded

by the radial-flux magnetic gear, with the axial-flux magnetic gear occupying the outer annular region. The axial cross-section (Figure 1c) shows that both the PMSM and radial-flux magnetic gear lie within the inner space of the axial-flux magnetic gear.

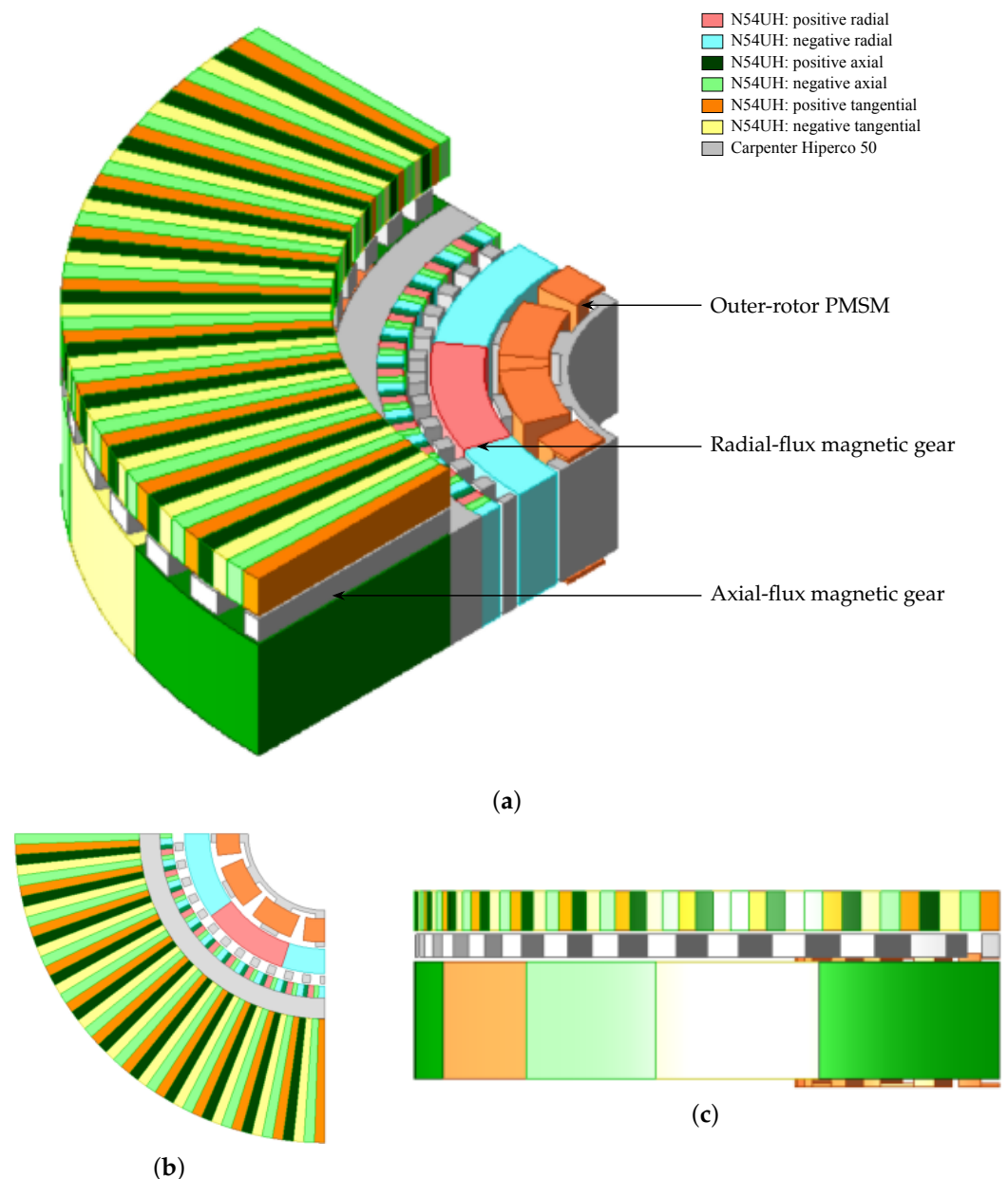


Figure 1. Overall configuration of the MGM. (a) 3-D cutaway view. (b) Top view. (c) Axial cross-section.

The PMSM produces electromagnetic torque on its outer rotor, which also serves as the high-speed rotor of the radial-flux magnetic gear. The radial-flux magnetic gear transmits this torque to the intermediate rotor. This rotor comprises the low-speed rotor of the radial-flux magnetic gear and the high-speed rotor of the axial-flux magnetic gear, enabling torque transmission between the two stages. After the second-stage reduction, the torque is output through the low-speed rotor of the axial-flux magnetic gear.

2.2. Electromagnetic Component Configuration and Main Flux Paths

The magnetic equivalent circuit uses lumped reluctances to represent the principal magnetic paths of the three electromagnetic components while retaining the leakage branches [19]. The permanent magnets are described using their recoil permeability. For

the ferromagnetic components, the secant permeability is iteratively updated from the B – H relation of Hiperco 50 to represent its average nonlinear response. The model calculates the coupled main flux and dominant working harmonics of the two magnetic gears. It does not resolve the spatial distributions of local saturation, three-dimensional fringing flux, or rotor eddy-current reaction. The magnetic reluctance R_i of the i th magnetic-circuit section is

$$R_i = \frac{l_i}{\mu_0 \mu_{r,i} A_i} \quad (1)$$

where l_i , A_i , and $\mu_{r,i}$ are the mean magnetic path length, cross-sectional area, and relative permeability, respectively, and μ_0 is the permeability of free space. Leakage flux paths are shown separately in the MECs. The analytical expressions describe the total reluctance and flux of each main magnetic circuit. For the ferromagnetic sections, the secant relative permeability at iteration k is obtained from the material curve as

$$\mu_{r,fe}^{(k)} = \frac{B_{fe}^{(k)}}{\mu_0 H_{fe}^{(k)}} \quad (2)$$

where $B_{fe}^{(k)}$ and $H_{fe}^{(k)}$ are the flux density and magnetic field strength of the ferromagnetic section at iteration k , respectively. The value of $H_{fe}^{(k)}$ is obtained from the Hiperco 50 B – H curve. The updated permeability is used to update the corresponding reluctances before the coupled flux equations are solved again.

The configuration and simplified MEC of the outer-rotor PMSM are shown in Figure 2.

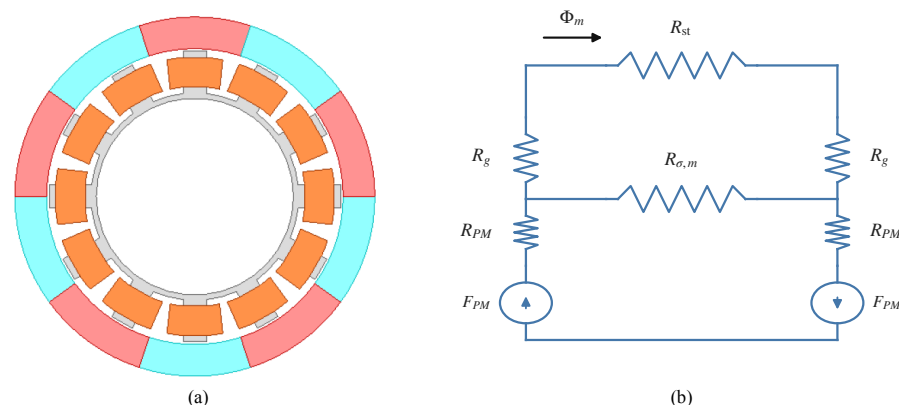


Figure 2. Outer-rotor PMSM. (a) Configuration. (b) Main circuit and leakage flux path.

In Figure 2b, R_{PM} , R_g , R_{st} , and $R_{\sigma,m}$ denote the PM reluctance, air-gap reluctance, stator-core reluctance, and motor leakage reluctance, respectively. The total reluctance of the main flux path is

$$R_{main,m} = 2R_{PM} + 2R_g + R_{st} \quad (3)$$

The main flux is

$$\Phi_m = \frac{2F_{PM}}{2R_{PM} + 2R_g + R_{st}} \quad (4)$$

where F_{PM} is the equivalent magnetomotive force (MMF) of a single PM.

The configuration and simplified MEC of the radial-flux magnetic gear are shown in Figure 3.

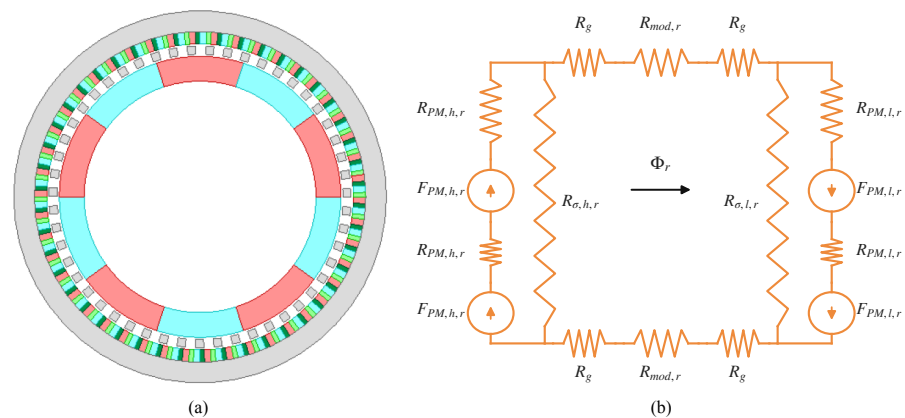


Figure 3. Radial-flux magnetic gear. (a) Configuration. (b) Main and leakage-flux paths.

The subscripts h , l , and r denote the high-speed side, low-speed side, and radial-flux magnetic gear, respectively. $R_{mod,r}$ is the equivalent reluctance of one ferromagnetic pole piece, whereas $R_{\sigma,h,r}$ and $R_{\sigma,l,r}$ denote the reluctances of the leakage-flux paths on the high- and low-speed sides, respectively. The total reluctance of the main flux path is

$$R_{main,r} = 2R_{PM,h,r} + 4R_g + 2R_{mod,r} + 2R_{PM,l,r} \quad (5)$$

The main flux is

$$\Phi_r = \frac{2F_{PM,h,r} + 2F_{PM,l,r}}{2R_{PM,h,r} + 4R_g + 2R_{mod,r} + 2R_{PM,l,r}} \quad (6)$$

where $F_{PM,h,r}$ and $F_{PM,l,r}$ are the equivalent MMFs of a single PM on the high-speed and low-speed sides, respectively.

The configuration and simplified MEC of the axial-flux magnetic gear are shown in Figure 4.

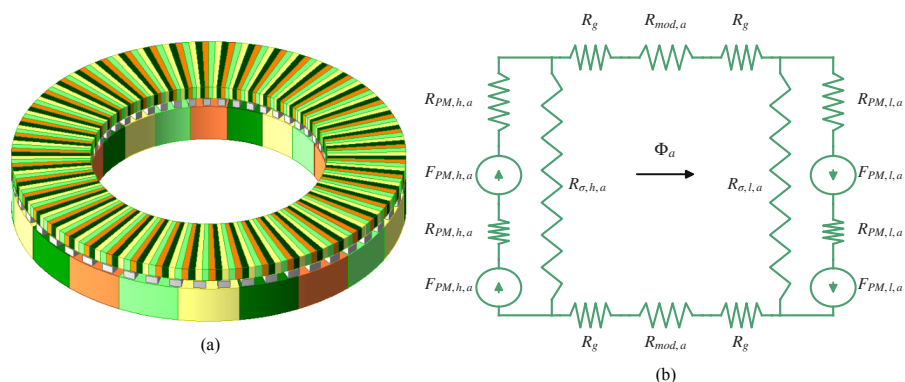


Figure 4. Axial-flux magnetic gear. (a) Configuration. (b) Main and leakage-flux paths.

The subscript a denotes the axial-flux magnetic gear. $R_{mod,a}$ is the equivalent reluctance of one ferromagnetic pole piece, whereas $R_{\sigma,h,a}$ and $R_{\sigma,l,a}$ denote the reluctances of the leakage-flux paths on the high- and low-speed sides, respectively. The total reluctance of the main flux path is

$$R_{main,a} = 2R_{PM,h,a} + 4R_g + 2R_{mod,a} + 2R_{PM,l,a} \quad (7)$$

The main flux is

$$\Phi_a = \frac{2F_{PM,h,a} + 2F_{PM,l,a}}{2R_{PM,h,a} + 4R_g + 2R_{mod,a} + 2R_{PM,l,a}} \quad (8)$$

where $F_{PM,h,a}$ and $F_{PM,l,a}$ denote the equivalent MMFs of one PM on the high- and low-speed sides, respectively.

2.3. Shared Components and Overall Magnetic Circuit

Each electromagnetic component forms a local main-flux path, but the shared components connect these paths. The shared permanent-magnet rotor links the PMSM and radial-flux magnetic gear, whereas the shared back iron links the radial-flux and axial-flux magnetic gears. The main and leakage fluxes must therefore be considered together when the complete-machine magnetic circuit is formulated. As shown in Figure 5a, the PMSM outer rotor also serves as the high-speed rotor of the radial-flux magnetic gear. On the motor side, the main flux crosses the air gap and closes through the stator core. On the magnetic-gear side, it passes through the high-speed-side air gap, stationary modulation ring, and low-speed-side air gap before closing through the low-speed rotor. In Figure 5b, the two main flux paths share the permanent-magnet branch represented by F_{PM} and R_{PM} , whereas $R_{\sigma,m}$ and $R_{\sigma,h,r}$ denote the corresponding leakage-flux paths. Hence,

$$\Phi_{PM} = \Phi_m + \Phi_r + \Phi_{\sigma,m} + \Phi_{\sigma,h,r} \quad (9)$$

The flux distribution is governed by the reluctances of the two main paths and the associated leakage paths.

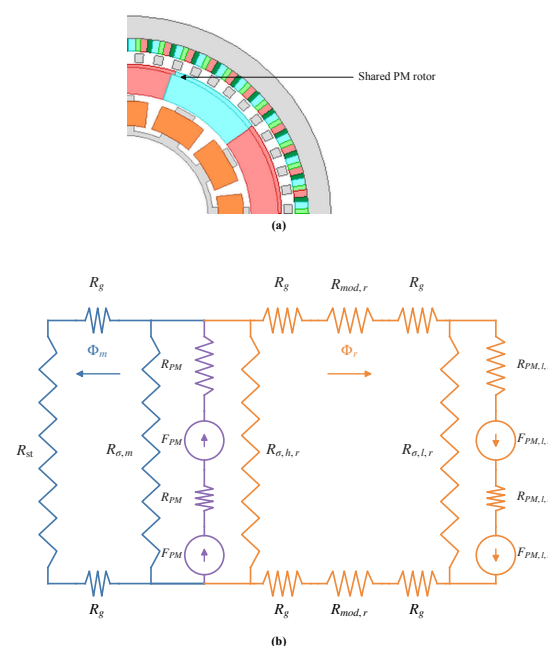


Figure 5. Rotor shared by the PMSM and the radial-flux magnetic gear. (a) Structural arrangement. (b) MEC.

As shown in Figure 6a, the low-speed rotor of the radial-flux magnetic gear and the high-speed rotor of the axial-flux magnetic gear are connected by a shared back iron. This connection closes the permanent-magnet branches of the two gears and provides the magnetic path between them. In Figure 6b, R_b represents the reluctance of each back-

iron section. On both sides, the PM reluctance and the reluctance of the corresponding leakage-flux path form parallel branches. The resulting equivalent reluctances are

$$\mathcal{R}_{l,r} = \frac{2R_{PM,l,r}R_{\sigma,l,r}}{2R_{PM,l,r} + R_{\sigma,l,r}} \quad \mathcal{R}_{h,a} = \frac{2R_{PM,h,a}R_{\sigma,h,a}}{2R_{PM,h,a} + R_{\sigma,h,a}} \quad (10)$$

The total reluctance of the path through the shared back iron is

$$R_{eq,b} = \mathcal{R}_{l,r} + 2R_b + \mathcal{R}_{h,a} \quad (11)$$

The radial-side main flux enters the axial side through the upper section of the shared back iron and returns through the lower section. The leakage fluxes on the two sides are denoted $\Phi_{\sigma,l,r}$ and $\Phi_{\sigma,h,a}$, respectively. Magnetic-flux continuity gives

$$\Phi_r = \Phi_b + \Phi_{\sigma,l,r} \quad \Phi_a = \Phi_b + \Phi_{\sigma,h,a} \quad (12)$$

where Φ_b denotes the main flux through the shared back iron.

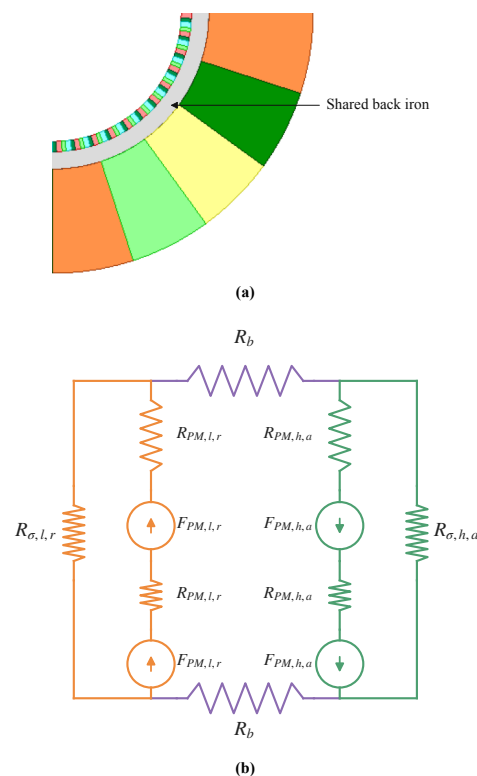


Figure 6. Backiron shared by the radial-flux and axial-flux magnetic gears. (a) Structural arrangement. (b) MEC.

Figure 7 shows the machine assembly. The high-speed rotor is shared by the PMSM and the radial-flux magnetic gear. The shared back iron connects the low-speed rotor of the radial-flux magnetic gear to the high-speed rotor of the axial-flux magnetic gear, forming the intermediate rotor. The low-speed rotor of the axial-flux magnetic gear is the machine's low-speed rotor. The PMSM stator and both modulation rings remain stationary.

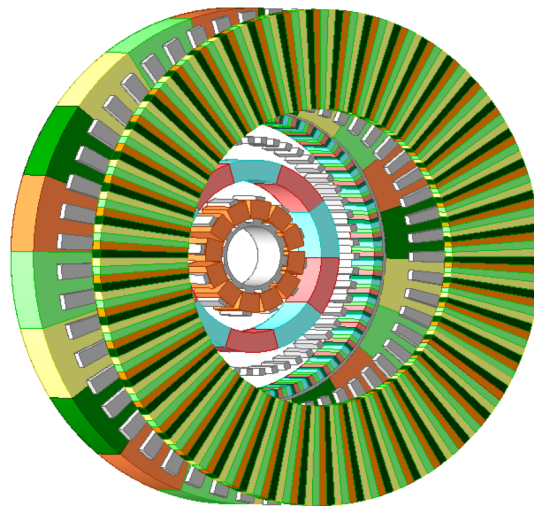


Figure 7. Exploded view of the machine.

The two shared-component relations are combined in Figure 8. The shared permanent-magnet branch couples the PMSM and radial-flux magnetic-gear loops, and the shared back-iron branch couples the radial-flux and axial-flux magnetic-gear loops. Figure 8 therefore represents the complete-machine MEC rather than three independent circuits. The loop directions follow the MMF arrows shown in the figure. Φ_m , Φ_r , Φ_b , and Φ_a denote the loop fluxes of the PMSM, radial-flux magnetic gear, shared-back-iron connection, and axial-flux magnetic gear, respectively.

$$M = \begin{bmatrix} \left(R_{st} + 2R_g \right) + 2R_{PM} & 2R_{PM} & 0 & 0 \\ 2R_{PM} & \left(4R_g + 2R_{mod,r} \right) + 2R_{PM} + 2R_{PM,l,r} & 2R_{PM,l,r} & 0 \\ 0 & 2R_{PM,l,r} & \left(2R_b + 2R_{PM,l,r} \right) + 2R_{PM,h,a} & 2R_{PM,h,a} \\ 0 & 0 & 2R_{PM,h,a} & \left(4R_g + 2R_{mod,a} \right) + 2R_{PM,h,a} + 2R_{PM,l,a} \end{bmatrix}$$

The corresponding MMF vector is

$$f = \begin{bmatrix} 2F_{PM} \\ 2F_{PM} + 2F_{PM,l,r} \\ 2F_{PM,l,r} + 2F_{PM,h,a} \\ 2F_{PM,h,a} + 2F_{PM,l,a} \end{bmatrix}$$

The reluctance matrix, loop-flux vector, and MMF vector satisfy The off-diagonal terms in the complete-machine reluctance matrix arise from the shared permanent-magnet and shared back-iron branches shown in Figures 5 and 6.

$$M \begin{bmatrix} \Phi_m \\ \Phi_r \\ \Phi_b \\ \Phi_a \end{bmatrix} = f$$

Cramer's rule gives

$$\Phi_m = \frac{\det(M_m)}{\det(M)} \quad \Phi_r = \frac{\det(M_r)}{\det(M)} \quad \Phi_b = \frac{\det(M_b)}{\det(M)} \quad \Phi_a = \frac{\det(M_a)}{\det(M)} \quad (13)$$

Here, M_m , M_r , M_b , and M_a are obtained by replacing the columns of the coefficient matrix M associated with Φ_m , Φ_r , Φ_b , and Φ_a by the MMF vector f , respectively. Under a one-dimensional approximation of the main magnetic circuits, the two air gaps connected in series within each branch carry the same flux. The air-gap main fluxes are therefore defined as

$$\Phi_{g,m} = \Phi_m \quad \Phi_{g,r} = \Phi_r \quad \Phi_{g,a} = \Phi_a \quad (14)$$

The PMSM and the two magnetic gears each form a closed main flux path and are magnetically coupled through the shared permanent-magnet and back-iron branches.

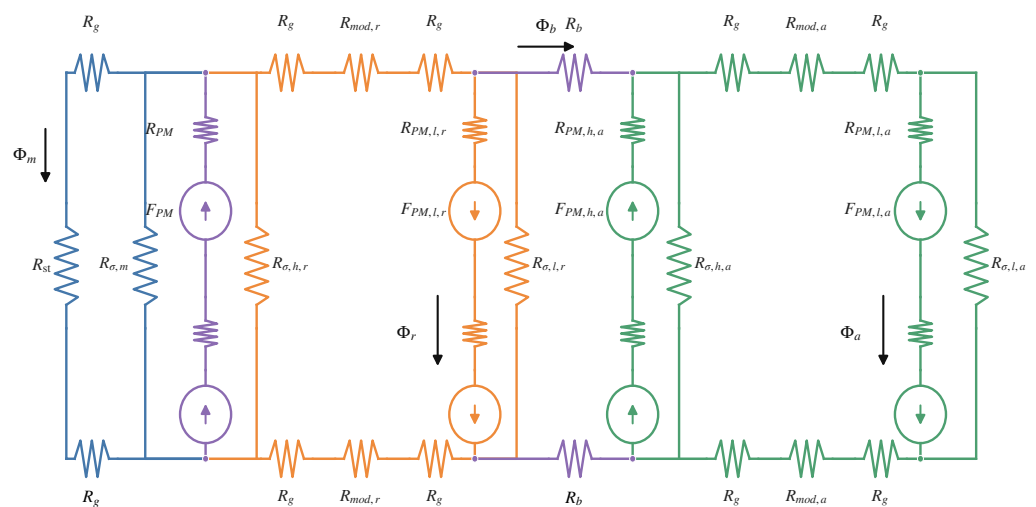


Figure 8. Magnetic equivalent circuit of the complete machine.

2.4. Air-Gap Magnetic Fields, Rotor Torques, and Equations of Motion

The average air-gap flux densities are

$$\bar{B}_{PM,m} = \frac{\Phi_m}{A_{g,m}} \quad \bar{B}_{j,r} = \frac{\Phi_r}{A_{g,j,r}} \quad \bar{B}_{j,a} = \frac{\Phi_a}{A_{g,j,a}} \quad j \in \{h, l\} \quad (15)$$

where $j \in \{h, l\}$ identifies the high- or low-speed side. $A_{g,m}$ is the effective area of the PMSM air gap, whereas $A_{g,j,r}$ and $A_{g,j,a}$ are the effective areas of the corresponding air gaps in the radial-flux and axial-flux magnetic gears.

2.4.1. High-Speed Rotor

The motor air gap lies between the PMSM stator and the high-speed rotor. The armature field and the high-speed rotor have the same pole-pair number:

$$p_m = p_{h,r} \quad (16)$$

where p_m is the pole-pair number of the armature field. The no-load motor air-gap flux density is

$$B_{PM,m}(\vartheta, t) = \bar{B}_{PM,m} \sum_{v=1,3,5,\dots} c_{v,m} \cos[v p_{h,r}(\vartheta - \varphi_1(t))] \quad (17)$$

where ϑ is the spatial mechanical angle in the stator reference frame, $\varphi_1(t)$ is the mechanical angular position of the high-speed rotor, $p_{h,r}$ is its pole-pair number, ν is the space-harmonic order, and $c_{\nu,m}$ is the corresponding normalized coefficient.

Throughout the field analysis, ϑ and $\varphi_i(t)$ denote mechanical angles. Electrical phase angles are written as products of the relevant mechanical angles and pole-pair numbers, such as $p_{h,r}[\vartheta - \varphi_1(t)]$.

The armature-reaction flux density is

$$B_{w,m}(\vartheta, t) = F_w(\vartheta, t) \Lambda_m(\vartheta) \quad (18)$$

where F_w denotes the armature-winding MMF and Λ_m denotes the specific permeance of the motor air gap.

The resulting motor air-gap flux density is

$$B_m(\vartheta, t) = B_{PM,m}(\vartheta, t) + B_{w,m}(\vartheta, t) \quad (19)$$

According to the Maxwell stress tensor, the electromagnetic torque exerted by the PMSM on the high-speed rotor is [20]

$$T_m = \int_{S_{g,m}} \frac{r B_{n,m} B_{t,m}}{\mu_0} dS \quad (20)$$

where $S_{g,m}$ is a cylindrical integration surface in the motor air gap and coaxial with the rotor axis. $B_{n,m}$ and $B_{t,m}$ are the normal and tangential flux-density components on this surface, respectively; r is the radial distance from the integration point to the rotor axis; and μ_0 is the permeability of free space.

Equation (15) shows that $\bar{B}_{h,r}$ is determined by the main-circuit flux Φ_r . Magnetic field modulation in the radial-flux magnetic gear is described using an MMF-permeance model [21,22]. The working harmonics are determined by products between the PM MMF and air-gap-permeance harmonics [23]. The PM MMF of the high-speed rotor and the air-gap permeance introduced by the stationary modulation ring are expanded as

$$F_{h,r}(\vartheta, t) = \sum_{\nu=1,3,5,\dots} F_{\nu,h,r} \cos[\nu p_{h,r}(\vartheta - \varphi_1(t))] \quad (21)$$

$$\Lambda_r(\vartheta) = \Lambda_{0,r} + \sum_{k=1}^{\infty} \Lambda_{k,r} \cos(k N_{s,r} \vartheta) \quad (22)$$

where $F_{\nu,h,r}$ is the ν th Fourier coefficient of the PM MMF of the high-speed rotor, $\Lambda_{0,r}$ is the constant component of the air-gap permeance, $\Lambda_{k,r}$ is its k th Fourier coefficient, and $N_{s,r}$ is the number of ferromagnetic pole pieces in the radial modulation ring. The resulting high-speed-side air-gap flux density of the radial-flux magnetic gear is

$$\begin{aligned} B_{h,r}(\vartheta, t) = & \Lambda_{0,r} \sum_{\nu=1,3,\dots} F_{\nu,h,r} \cos[\nu p_{h,r}(\vartheta - \varphi_1(t))] \\ & + \frac{1}{2} \sum_{\nu=1,3,\dots} \sum_{k=1}^{\infty} \sum_{\sigma=\pm 1} F_{\nu,h,r} \Lambda_{k,r} \cos[(k N_{s,r} + \sigma \nu p_{h,r})\vartheta - \sigma \nu p_{h,r} \varphi_1(t)] \end{aligned} \quad (23)$$

The product of the fundamental PM MMF and first permeance harmonic is

$$\begin{aligned} & F_{1,h,r} \Lambda_{1,r} \cos[p_{h,r}(\vartheta - \varphi_1(t))] \cos(N_{s,r} \vartheta) \\ & = \frac{F_{1,h,r} \Lambda_{1,r}}{2} \left\{ \cos[(N_{s,r} - p_{h,r})\vartheta + p_{h,r} \varphi_1(t)] + \cos[(N_{s,r} + p_{h,r})\vartheta - p_{h,r} \varphi_1(t)] \right\} \end{aligned} \quad (24)$$

The pole-pair numbers satisfy

$$p_{l,r} = N_{s,r} - p_{h,r}$$

According to the Maxwell stress tensor, the torque exerted by the radial-flux magnetic gear on the high-speed rotor is

$$T_{h,r} = \int_{S_{g,h,r}} \frac{r B_{n,h,r} B_{t,h,r}}{\mu_0} dS \quad (25)$$

where $S_{g,h,r}$ is a cylindrical integration surface in the high-speed-side air gap and coaxial with the rotor axis. $B_{n,h,r}$ and $B_{t,h,r}$ are the normal and tangential flux-density components on this surface, respectively.

Accordingly, the equation of motion of the high-speed rotor is

$$J_1 \dot{\omega}_1 = T_m + T_{h,r} \quad (26)$$

where J_1 is the moment of inertia of the high-speed rotor and ω_1 is its mechanical angular velocity. The positive torque direction coincides with the positive direction of ω_1 . Under steady-state operation at a prescribed speed, the angular acceleration vanishes. Hence,

$$T_m + T_{h,r} = 0 \quad (27)$$

2.4.2. Intermediate Rotor

The intermediate rotor comprises the low-speed rotor of the radial-flux magnetic gear and the high-speed rotor of the axial-flux magnetic gear, which are rigidly connected through the shared back iron. Its mechanical angular position and mechanical angular velocity are denoted by $\varphi_2(t)$ and ω_2 , respectively.

Based on the magnetic field modulation relation in Equation (24), the radial air-gap flux density adjacent to the intermediate rotor is

$$B_{l,r}(\vartheta, t) = \bar{B}_{l,r} \sum_{v=1,3,5,\dots} c_{v,l,r} \cos[v p_{l,r}(\vartheta - \varphi_2(t))] + \frac{F_{1,h,r} \Lambda_{1,r}}{2} \cos[p_{l,r} \vartheta + p_{h,r} \varphi_1(t)] \quad (28)$$

where $c_{v,l,r}$ is the normalized coefficient of the v th space harmonic. According to the Maxwell stress tensor, the torque exerted by the radial-flux magnetic gear on the intermediate rotor is

$$T_{l,r} = \int_{S_{g,l,r}} \frac{r B_{n,l,r} B_{t,l,r}}{\mu_0} dS \quad (29)$$

where $S_{g,l,r}$ is a cylindrical integration surface in the radial air gap and coaxial with the rotor axis. $B_{n,l,r}$ and $B_{t,l,r}$ are the normal and tangential flux-density components on this surface, respectively.

For the axial-flux magnetic gear, the pole-pair numbers satisfy

$$p_{h,a} = N_{s,a} - p_{l,a}$$

where $N_{s,a}$ is the number of ferromagnetic pole pieces in the axial modulation ring. The axial air-gap flux density adjacent to the intermediate rotor is

$$B_{h,a}(\vartheta, t) = \bar{B}_{h,a} \sum_{v=1,3,5,\dots} c_{v,h,a} \cos[v p_{h,a}(\vartheta - \varphi_2(t))] + \frac{F_{1,l,a} \Lambda_{1,a}}{2} \cos[p_{h,a} \vartheta + p_{l,a} \varphi_3(t)] \quad (30)$$

where $c_{v,h,a}$ is the normalized coefficient of the v th space harmonic. $F_{1,l,a}$ and $\Lambda_{1,a}$ are the amplitudes of the fundamental PM MMF and first air-gap-permeance harmonic, respectively; $\varphi_3(t)$ is the mechanical angular position of the low-speed output rotor.

According to the Maxwell stress tensor, the torque exerted by the axial-flux magnetic gear on the intermediate rotor is

$$T_{h,a} = \int_{S_{g,h,a}} \frac{r B_{n,h,a} B_{t,h,a}}{\mu_0} dS \quad (31)$$

where $S_{g,h,a}$ is an annular integration surface in the axial air gap. $B_{n,h,a}$ and $B_{t,h,a}$ are the normal and tangential flux-density components on this surface, respectively.

Accordingly, the equation of motion of the intermediate rotor is

$$J_2 \dot{\omega}_2 = T_{l,r} + T_{h,a} \quad (32)$$

where J_2 is the moment of inertia of the intermediate rotor. The positive torque direction coincides with the positive direction of ω_2 . Under steady-state operation at a prescribed speed,

$$T_{l,r} + T_{h,a} = 0 \quad (33)$$

The torque exerted by the radial-flux magnetic gear is balanced by the oppositely directed torque exerted by the axial-flux magnetic gear. Torque is thereby transferred from the radial-flux magnetic gear to the axial-flux magnetic gear through the intermediate rotor.

2.4.3. Low-Speed Rotor

The low-speed rotor serves as the output rotor of the axial-flux magnetic gear. The stationary modulation ring faces one side across the axial air gap, while the load is connected to the opposite side. Its mechanical angular position and mechanical angular velocity are denoted by $\varphi_3(t)$ and ω_3 , respectively. The pole-pair numbers satisfy

$$p_{l,a} = N_{s,a} - p_{h,a}$$

Accordingly, the axial air-gap flux density adjacent to the low-speed rotor is

$$B_{l,a}(\vartheta, t) = \bar{B}_{l,a} \sum_{v=1,3,5,\dots} c_{v,l,a} \cos[v p_{l,a}(\vartheta - \varphi_3(t))] + \frac{F_{1,h,a} \Lambda_{1,a}}{2} \cos[p_{l,a} \vartheta + p_{h,a} \varphi_2(t)] \quad (34)$$

where $c_{v,l,a}$ is the normalized coefficient of the v th space harmonic, and $F_{1,h,a}$ is the amplitude of the fundamental PM MMF of the high-speed rotor of the axial-flux magnetic gear.

According to the Maxwell stress tensor, the torque exerted by the axial-flux magnetic gear on the low-speed rotor is

$$T_{l,a} = \int_{S_{g,l,a}} \frac{r B_{n,l,a} B_{t,l,a}}{\mu_0} dS \quad (35)$$

where $S_{g,l,a}$ is an annular integration surface in the axial air gap. $B_{n,l,a}$ and $B_{t,l,a}$ are the normal and tangential flux-density components on this surface, respectively.

Accordingly, the equation of motion of the low-speed rotor is

$$J_3 \dot{\omega}_3 = T_{l,a} - T_L \quad (36)$$

where J_3 is the moment of inertia of the low-speed rotor, and T_L is the load torque acting opposite to the positive direction of ω_3 . Under steady-state operation at a prescribed speed,

$$T_{l,a} = T_L \quad (37)$$

2.4.4. Two-Stage Speed Relationships and Maximum Transferable Torque

During synchronous operation, the working harmonic in Equation (28) rotates synchronously with the radial PM field of the intermediate rotor. Therefore,

$$p_{h,r}\omega_1 + p_{l,r}\omega_2 = 0 \quad (38)$$

The gear ratio of the radial-flux magnetic gear is

$$G_r = -\frac{\omega_1}{\omega_2} = \frac{p_{l,r}}{p_{h,r}} \quad (39)$$

Similarly, the working harmonic in Equation (34) rotates synchronously with the axial PM field of the low-speed rotor. Therefore,

$$p_{h,a}\omega_2 + p_{l,a}\omega_3 = 0 \quad (40)$$

The gear ratio of the axial-flux magnetic gear is

$$G_a = -\frac{\omega_2}{\omega_3} = \frac{p_{l,a}}{p_{h,a}} \quad (41)$$

The high-speed and intermediate rotors rotate in opposite directions, as do the intermediate and low-speed rotors. The overall gear ratio is

$$G_k = \left| \frac{\omega_1}{\omega_3} \right| = G_r G_a = \frac{p_{l,r} p_{l,a}}{p_{h,r} p_{h,a}} \quad (42)$$

Because Equations (39)–(42) are determined solely by the pole-pair and modulation-pole numbers, the two stage ratios and the overall gear ratio remain unchanged as long as synchronous operation is maintained. Magnetic saturation affects the reluctance, flux, and working-harmonic amplitudes but does not change the gear ratios determined by the pole-pair and modulation-pole numbers under synchronous operation.

With both modulation rings stationary and losses neglected, the radial-flux and axial-flux magnetic gears satisfy, respectively,

$$T_{h,r}\omega_1 + T_{l,r}\omega_2 = 0 \quad (43)$$

$$T_{h,a}\omega_2 + T_{l,a}\omega_3 = 0 \quad (44)$$

Combining the power-balance equations with the synchronous relationships gives the torque ratios of the two stages:

$$\frac{|T_{l,r}|}{|T_{h,r}|} = G_r \quad \frac{|T_{l,a}|}{|T_{h,a}|} = G_a \quad (45)$$

Expressing the maximum transferable torque of each magnetic gear at the low-speed output gives

$$T_{\max} = \min [G_a (T_{l,r})_{\max}, (T_{l,a})_{\max}] \quad (46)$$

The maximum transferable torque of the machine is therefore determined by the smaller of the two terms in Equation (46).

3. Three-Dimensional Finite-Element Model and Two-Stage Optimization

3.1. Finite-Element Model, Design Variables, and Optimization Strategy

The PMs are N54UH Nd–Fe–B magnets [24]. The stator core and modulation pole pieces use Carpenter Hiperc 50 [25]. The corresponding nonlinear demagnetization data for N54UH and B – H data for Hiperc 50 are assigned in Maxwell. The nonlinear magnetization curve of Hiperc 50 used in the material model is shown in Figure 9. Figure 10 shows the three-dimensional transient finite-element model used for the simulations in Ansys Maxwell. The rotor speeds are prescribed according to the pole-pair relationships of the two magnetic gears. The reported torques are obtained from the periodic steady-state electromagnetic response under these prescribed-speed conditions; the model does not represent the coupled mechanical dynamics during load steps or speed transients. Identical material properties, boundary conditions, and solver settings are used for all designs. Eddy-current effects are enabled in the conductive PM and soft-magnetic regions of the three-dimensional transient model. The automatic Maxwell mesh setting was retained for all designs, so the same meshing procedure was applied throughout the optimization. Each transient solution covered 110.4 ms with a time step of 0.4 ms, and the reported average torques were calculated over this interval.

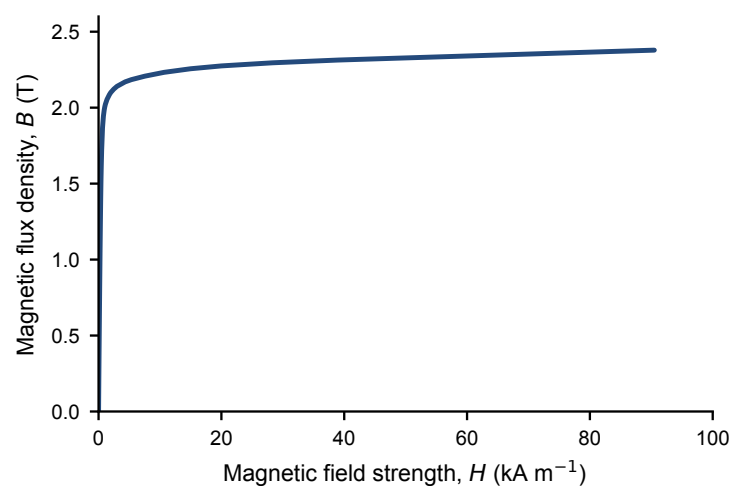


Figure 9. Normal magnetization curve of Hiperc 50 used in the material model.

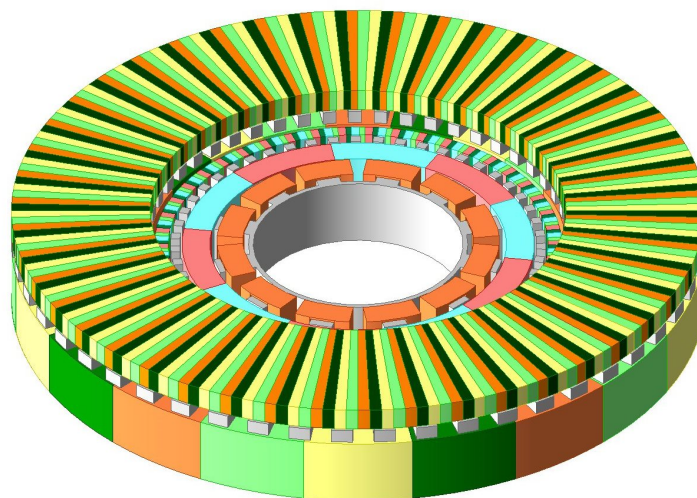


Figure 10. Three-dimensional finite-element model of the MGM.

The main simulation parameters are listed in Table 1.

Table 1. Main simulation parameters.

Symbol	Description	Value
D_{out}	Overall outer diameter	300 mm
g_m	PMSM air-gap length	0.7 mm
g_r	Air-gap length of the radial-flux magnetic gear	0.7 mm
g_a	Air-gap length of the axial-flux magnetic gear	0.7 mm
n_1	High-speed rotor speed	1000 r/min
n_2	Intermediate rotor speed	−108.696 r/min
n_3	Low-speed rotor speed	11.815 r/min
$p_{h,r}$	High-speed-rotor pole-pair number, radial-flux magnetic gear	5
$p_{l,r}$	Low-speed-rotor pole-pair number, radial-flux magnetic gear	46
$p_{h,a}$	High-speed-rotor pole-pair number, axial-flux magnetic gear	5
$p_{l,a}$	Low-speed-rotor pole-pair number, axial-flux magnetic gear	46
Z_s	Number of stator slots	12
N_c	Number of turns per coil	20
J_{rms}	RMS current-density constraint	3 A/mm ²
k_f	Slot fill factor	0.55

The prescribed rotor speeds and mechanical angular velocities are related by

$$\omega_i = \frac{2\pi n_i}{60} \quad i = 1, 2, 3$$

Figure 11 defines the geometric design variables of the MGM. The radial dimensions and axial heights are denoted by L_i and h_i , respectively. The symbols θ_1 – θ_4 denote dimensionless circumferential coefficients and are not angular coordinates. The bounds of the design variables are listed in Table 2.

Table 2. Design variables and bounds.

Symbol	Description	Bounds
L_1	Radial length of PMs in the axial-flux magnetic gear	30–90 mm
L_2	Radial thickness of outer PMs in the radial-flux magnetic gear	10–30 mm
L_3	Outer shared back-iron thickness	1.0–5.0 mm
L_4	Inner stator back-iron thickness	1.0–5.0 mm
L_5	Radial thickness of inner PMs in the radial-flux magnetic gear	6–18 mm
L_6	Stator-tooth radial length	8–24 mm
h_1	Axial thickness of the lower PM array	15–45 mm
h_2	Axial thickness of the upper PM array	5–15 mm
h_3	Axial modulation pole-piece thickness	3–9 mm
θ_1	Lower-PM pole-arc coefficient	0.25–0.75
θ_2	Upper-PM pole-arc coefficient	0.25–0.75
θ_3	Modulator-gap circumferential-width coefficient	0.30–0.90
θ_4	Stator-tooth circumferential-width coefficient	0.30–0.70

A two-stage optimization procedure is used to determine the geometric parameters of the MGM. The magnetic-gear geometries govern the maximum transferable torque and volumetric torque density, whereas the stator geometry affects the rated output torque and stator mass at a given current density. Optimizing both parameter sets simultaneously would enlarge the three-dimensional finite-element design space and combine objectives associated with different operating conditions. The first stage jointly optimizes the two magnetic gears to maximize the volumetric torque density at the maximum transferable torque

operating point. With the magnetic-gear parameters fixed, the second stage minimizes the combined mass of the stator core and windings subject to the rated-output-torque constraint.

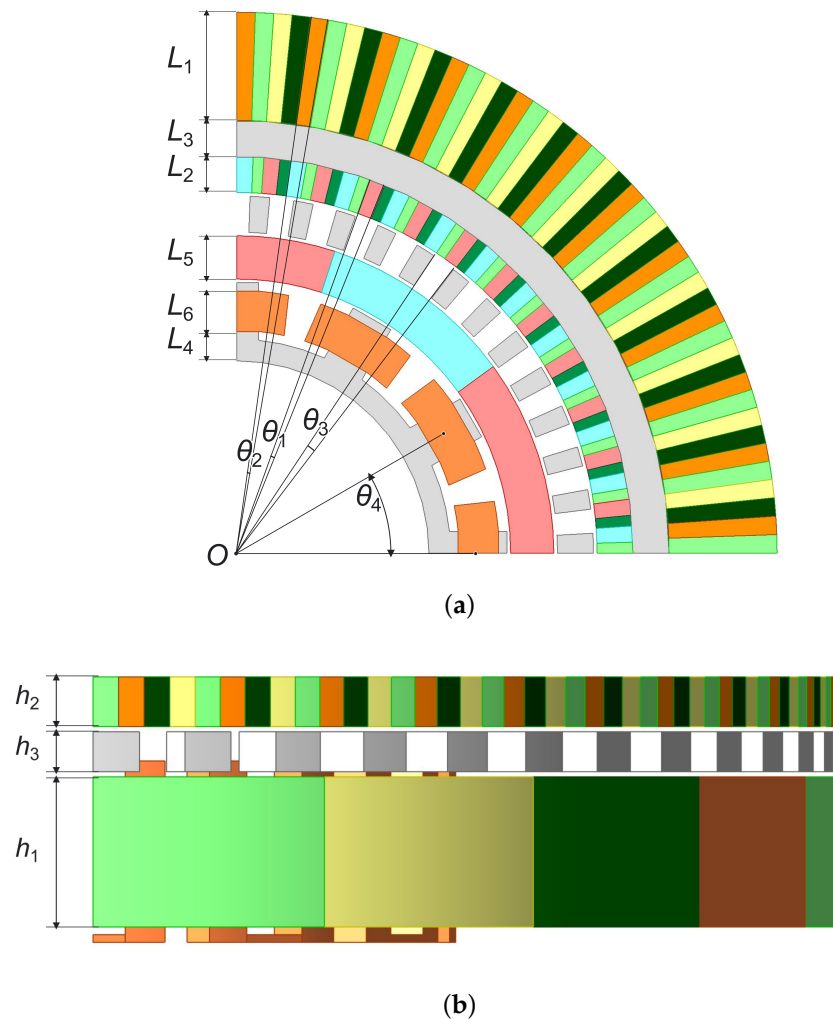


Figure 11. Geometric design variables of the MGM. (a) Radial and circumferential variables. (b) Axial variables.

3.2. Simultaneous Optimization of the Magnetic-Gear Design Variables

The first stage simultaneously optimizes ten continuous design variables of the two magnetic gears. These variables affect the PM dimensions, modulation pole-piece geometry, shared back-iron dimensions, and output torque. Each candidate design requires a three-dimensional finite-element evaluation. The resulting problem is therefore a bounded, single-objective, and computationally expensive black-box optimization problem.

To reduce the number of three-dimensional finite-element evaluations, the initial OLHS calculations were followed by an adaptive search based on local response estimation. For a candidate $\mathbf{x}$, its normalized Euclidean distance from the i th completed finite-element sample is

$$d_i(\mathbf{x}) = \left[\sum_{q=1}^{n_x} \left(\frac{x_q - x_{i,q}}{x_q^U - x_q^L} \right)^2 \right]^{1/2} \quad (47)$$

where n_x is the number of design variables. The neighborhood $\mathcal{N}_k(\mathbf{x})$ comprises the $k = 2n_x + 1$ completed samples with the smallest values of d_i . The distance weights, local response estimate, and local dispersion are

$$\begin{aligned} w_i(\mathbf{x}) &= \frac{[d_i(\mathbf{x}) + \epsilon]^{-2}}{\sum_{j \in \mathcal{N}_k(\mathbf{x})} [d_j(\mathbf{x}) + \epsilon]^{-2}} \\ \hat{\rho}_T(\mathbf{x}) &= \sum_{i \in \mathcal{N}_k(\mathbf{x})} w_i(\mathbf{x}) \rho_{T,i} \\ s_\rho(\mathbf{x}) &= \left[\sum_{i \in \mathcal{N}_k(\mathbf{x})} w_i(\mathbf{x}) [\rho_{T,i} - \hat{\rho}_T(\mathbf{x})]^2 \right]^{1/2} \end{aligned} \quad (48)$$

where ϵ is a small positive constant used to avoid division by zero. The improvement score is defined as

$$I(\mathbf{x}) = \frac{\hat{\rho}_T(\mathbf{x}) - \rho_{T,\text{best}}}{\rho_{T,\text{best}}} + \beta \frac{s_\rho(\mathbf{x})}{\rho_{T,\text{best}}} \quad \beta = 1 \quad (49)$$

The first term favors candidates with a high estimated volumetric torque density, whereas the second retains candidates in locally dispersed regions. Candidates were ranked by $I(\mathbf{x})$, and the highest-ranked candidate was evaluated by three-dimensional transient FEA. Only the resulting finite-element value was added to the completed sample set and used to update $\rho_{T,\text{best}}$.

The first-stage design vector is

$$\mathbf{x}_1 = [L_1, L_2, L_3, L_5, h_1, h_2, h_3, \theta_1, \theta_2, \theta_3]^T \quad (50)$$

where L_1 – L_3 and L_5 describe the radial dimensions of the PMs and shared back iron; h_1 – h_3 denote the axial thicknesses of the two PM arrays and modulation pole pieces; and θ_1 – θ_3 are the corresponding circumferential coefficients. L_4 , L_6 , and θ_4 are excluded from the first-stage optimization.

The volumetric torque density is defined as

$$\rho_T(\mathbf{x}_1) = \frac{T_{\text{max}}(\mathbf{x}_1)}{V_{\text{env}}(\mathbf{x}_1)} \quad V_{\text{env}} = \frac{\pi D_{\text{out}}^2}{4} (h_1 + h_2 + h_3 + 2g_a) \quad (51)$$

where V_{env} is the overall cylindrical envelope volume, D_{out} is the outer diameter, and g_a is the air-gap length of the axial-flux magnetic gear. The first-stage optimization problem is

$$\max_{\mathbf{x}_1} \rho_T(\mathbf{x}_1) \quad x_i^L \leq x_i \leq x_i^U \quad (52)$$

where x_i^L and x_i^U are the lower and upper bounds of the i th design variable, respectively.

The optimization procedure is shown in Figure 12. The first stage comprised 180 initial OLHS evaluations, 80 adaptive evaluations, and four final verification evaluations. After the initial sampling, subsequent points were selected using the local response estimate and improvement score described above. Each selected design was evaluated by three-dimensional finite-element analysis, and the cumulative best objective value was updated from these finite-element results. The search terminated when the relative improvement in the cumulative best value remained below 0.2% over 15 consecutive evaluations or when the evaluation budget was reached. The final design and its neighboring designs were re-evaluated by finite-element analysis.

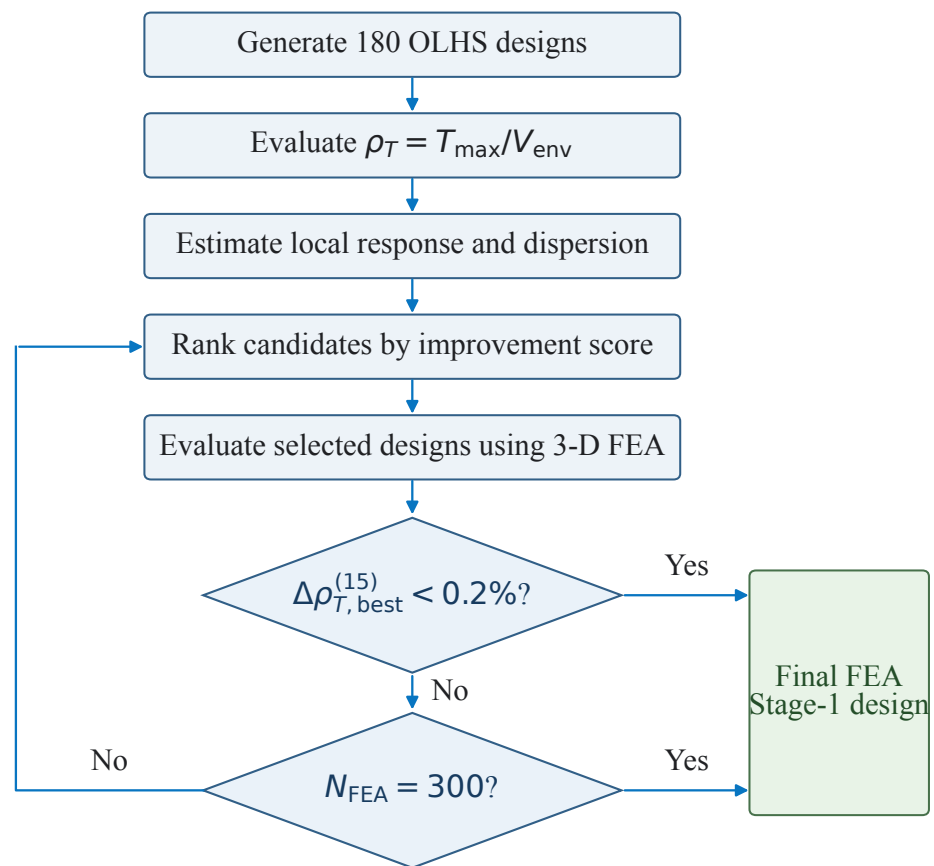


Figure 12. First-stage adaptive optimization procedure.

Figure 13 shows the first-stage finite-element samples and the cumulative best volumetric torque density. The initial OLHS sampling increased the cumulative best value from 270.88 N m/L to 308.94 N m/L. After adaptive infill, this value reached 327.25 N m/L near the 200th evaluation and increased to 328.00 N m/L at the 264th evaluation. The final design has a maximum transferable torque of 1098.97 N m and an overall cylindrical envelope volume of 3.3505 L.

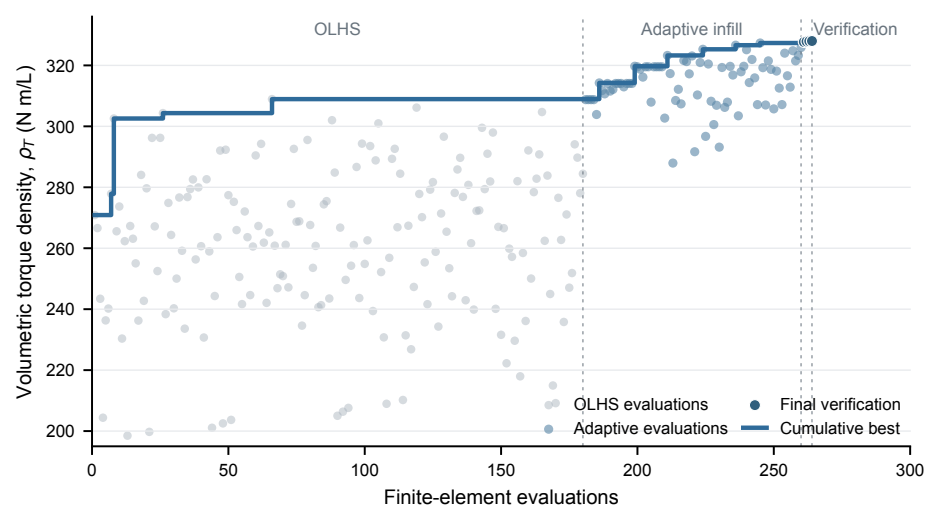


Figure 13. First-stage convergence of volumetric torque density.

3.3. Stator Optimization

The two optimization stages correspond to different parameter groups and operating objectives. The magnetic-gear dimensions determine the maximum transferable torque and volumetric torque density, whereas the stator dimensions determine the rated output capability and stator mass at the prescribed current density. Simultaneous optimization would combine these variables and operating objectives in one finite-element search. The magnetic-gear geometries obtained in the first optimization stage were held fixed in the second stage, in which only the stator geometry was varied. The design vector is

$$\mathbf{x}_2 = [L_4, L_6, \theta_4]^T \quad (53)$$

where L_4 is the inner stator back-iron thickness, L_6 is the stator-tooth radial length, and θ_4 is the stator-tooth circumferential-width coefficient.

The second-stage objective minimizes the combined mass of the stator core and windings

$$\min_{\mathbf{x}_2} m_{\text{st}}(\mathbf{x}_2) = m_{\text{iron}}(\mathbf{x}_2) + m_{\text{cu}}(\mathbf{x}_2) \quad (54)$$

The stator employs a 12-slot three-phase concentrated winding with 20 turns per coil. For each candidate design, the effective conductor area was calculated from the parameterized slot geometry and a slot fill factor of 0.55. The corresponding rated current was then determined at an RMS current density of 3 A/mm². The current therefore varied with the stator geometry, while the current-density constraint remained fixed throughout the second-stage optimization. The rated output torque was constrained by

$$g_T(\mathbf{x}_2) = T_{\text{rated}}(\mathbf{x}_2) - 0.5T_{\text{max}}^{(1)} \geq 0 \quad (55)$$

where $T_{\text{max}}^{(1)}$ is the maximum transferable torque obtained in the first optimization stage. Designs satisfying $g_T \geq 0$ were feasible and ranked by m_{st} ; infeasible designs were excluded from the mass ranking.

The second stage used the normalized-distance and nearest-neighbor definitions in Equations (47) and (48), with $n_x = 3$ and $k = 7$, to estimate the local stator mass and torque-constraint margin. Candidate designs with a nonnegative estimated constraint margin were ranked by estimated stator mass, while candidates nearest to a zero constraint margin were retained to refine the feasibility boundary. After 48 OLHS designs were evaluated by FEA, 24 additional designs were selected in low-mass feasible regions and near the constraint boundary. Each selected design was evaluated by finite-element analysis before the next selection step. Feasible designs were ranked by m_{st} , whereas infeasible samples were retained to update the constraint boundary.

A total of 74 FEA evaluations were completed, including 48 initial OLHS evaluations, 24 constraint-guided adaptive evaluations, one baseline design, and one final-design verification. Figure 14 presents all evaluated designs and the cumulative best feasible mass. The final design had $L_4 = 1.400$ mm, $L_6 = 11.400$ mm, and $\theta_4 = 0.500$. Its rated output torque was 547.07 N m, and the combined mass of the stator core and windings was 1.588 kg; these values were taken from the final verification calculation.

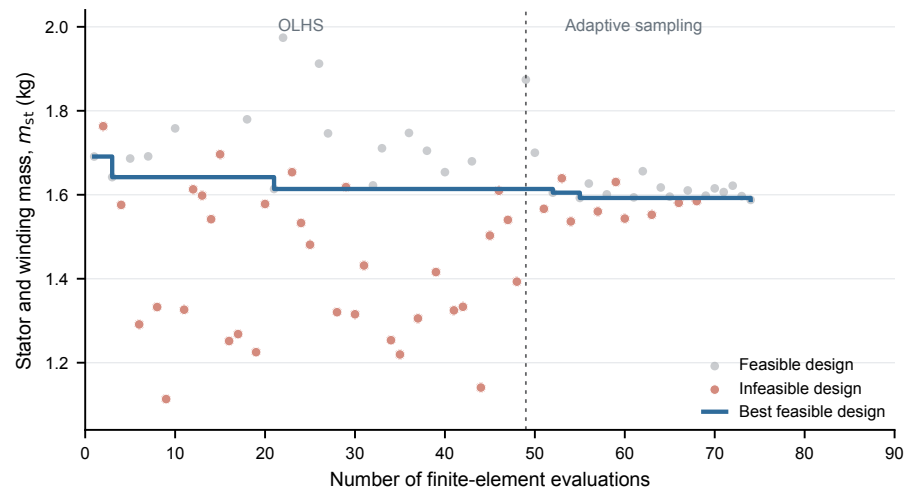


Figure 14. Second-stage convergence of minimum feasible stator mass.

3.4. Optimization Results and Final Parameters

Table 3 lists the final structural parameters obtained from the two-stage optimization. The first stage determined the geometries of the two magnetic gears, and the second stage determined the stator geometry while holding the magnetic-gear parameters fixed.

Table 3. Final optimized structural parameters.

Symbol	Description	Final Value
L_1	Radial length of PMs in the axial-flux magnetic gear	76.0 mm
L_2	Radial thickness of outer PMs in the radial-flux magnetic gear	12.8 mm
L_3	Outer shared back-iron thickness	3.6 mm
L_4	Inner stator back-iron thickness	1.4 mm
L_5	Radial thickness of inner PMs in the radial-flux magnetic gear	9.6 mm
L_6	Stator-tooth radial length	11.4 mm
h_1	Axial thickness of the lower PM array	27.4 mm
h_2	Axial thickness of the upper PM array	9.6 mm
h_3	Axial modulation pole-piece thickness	9.0 mm
θ_1	Lower-PM pole-arc coefficient	0.50
θ_2	Upper-PM pole-arc coefficient	0.50
θ_3	Modulator-gap circumferential-width coefficient	0.50
θ_4	Stator-tooth circumferential-width coefficient	0.50

Table 4 summarizes the principal performance metrics at the maximum transferable torque and rated operating points. The reported torques are averages of the final transient torque time histories. The rated output torque is 49.78% of the maximum transferable torque.

Table 4. Performance at the maximum transferable torque and rated operating points.

Metric	Maximum Transferable Torque	Rated Operation
Output torque	1098.97 N m	547.07 N m
Normalized output torque	100%	49.78%
RMS phase current	—	13.4 A
Overall cylindrical envelope volume	3.3505 L	3.3505 L
Volumetric torque density	328.00 N m/L	—

4. Three-Dimensional Finite-Element Analysis

4.1. Flux-Density Distribution

The MEC in Section 2 establishes the main-flux paths and the coupling introduced by the shared components. Figure 15 shows the flux-density distribution of the complete machine at the maximum transferable torque operating point, allowing the calculated paths to be checked against the finite-element field.

The flux density extended continuously through the central drive region, the radial-flux magnetic gear, and the outer axial-flux magnetic gear. The resulting distributions followed the main flux paths represented by the MEC. In particular, the shared permanent-magnet rotor and shared back iron carried the flux linking adjacent electromagnetic components. Figure 16 resolves the local distributions using the same color scale as Figure 15.

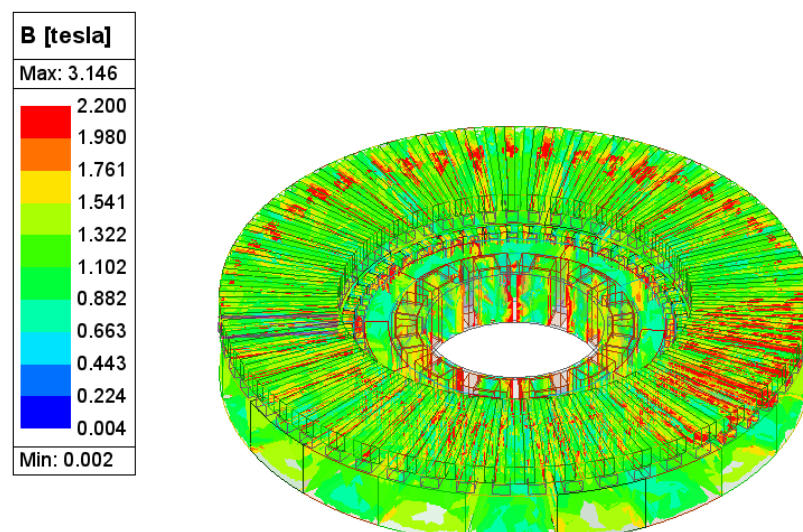


Figure 15. Sectional view of the machine flux-density distribution at the maximum transferable torque operating point.

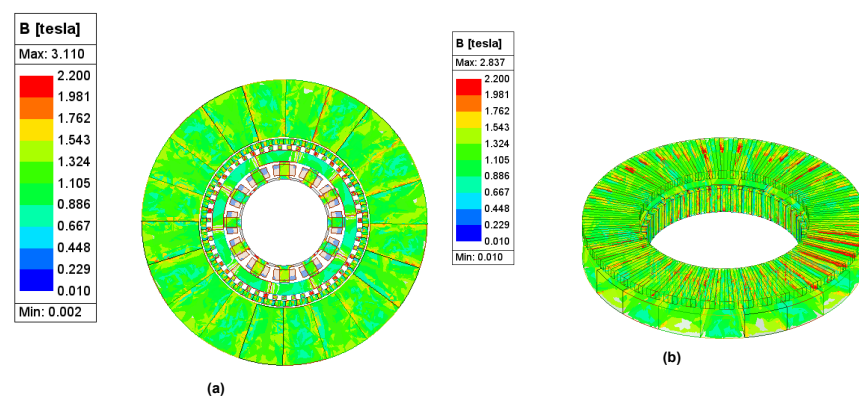


Figure 16. Flux-density distributions in the electromagnetic components. (a) PMSM and radial-flux magnetic gear. (b) Axial-flux magnetic gear.

In Figure 16a, the higher flux density occurred mainly near the PMs, ferromagnetic pole pieces, and stator teeth, following the main flux paths of the PMSM and radial-flux magnetic gear. In Figure 16b, the flux density varied periodically in the circumferential direction, with higher values near the PMs, ferromagnetic pole pieces, and changes in magnetic-path cross-section. Both distributions were consistent with the corresponding main paths in the MEC.

The point maxima of 3.110 T and 2.837 T in Figure 16 are confined to narrow regions near the edges of the PMs and modulation pole pieces and at abrupt changes in magnetic-path cross-section. They therefore represent localized flux concentration and local saturation rather than extensive saturation of the soft-magnetic components. The nonlinear B – H relation of Hiperco 50 was included in the finite-element model. Because pointwise extrema at sharp geometric edges can be sensitive to the local mesh, these maxima were not used as measures of the bulk magnetic loading or torque capability.

4.2. Air-Gap Working Harmonics and MEC–FEA Comparison

Section 2 derives the working-harmonic pole-pair relation from the product of the PM magnetomotive force and the air-gap permeance. Both magnetic gears use a pole-pair combination of 5/46 and 51 ferromagnetic pole pieces. Figure 17 presents the spatial spectra of the air-gap flux density on the high-speed and low-speed sides of the radial-flux and axial-flux magnetic gears. In both magnetic gears, modulation of the fifth-order source harmonic produced the 46th-order working harmonic, whose pole-pair number equals the difference between the number of ferromagnetic pole pieces and the high-speed-rotor pole-pair number. The MEC amplitudes are then compared with the corresponding finite-element spectral amplitudes using Equation (56).

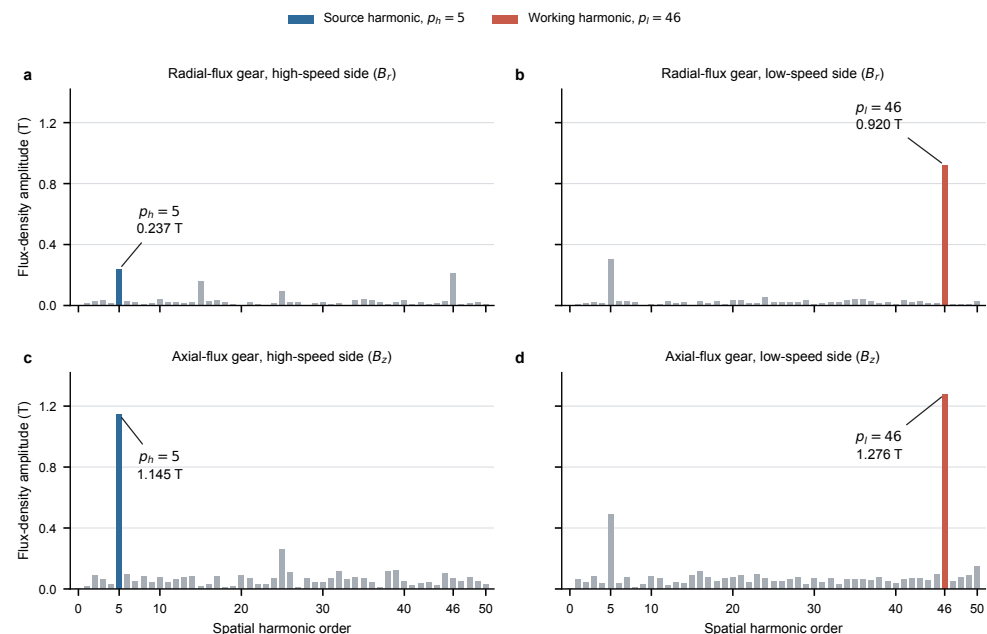


Figure 17. Air-gap flux-density spatial spectra. (a) Radial-flux gear, high-speed side. (b) Radial-flux gear, low-speed side. (c) Axial-flux gear, high-speed side. (d) Axial-flux gear, low-speed side.

The relative difference between the MEC and finite-element working-harmonic amplitudes was calculated as

$$\varepsilon_{B,j} = \frac{|\hat{B}_{\text{FEA},j} - \hat{B}_{\text{MEC},j}|}{\hat{B}_{\text{FEA},j}} \times 100\% \quad (56)$$

where $j \in \{r, a\}$ denotes the radial-flux or axial-flux magnetic gear, and $\hat{B}$ is the peak amplitude of the 46th-order working harmonic in the corresponding low-speed-side air gap. The MEC predicted 0.846 T and 1.231 T for the radial-flux and axial-flux magnetic gears, respectively. The corresponding finite-element values were 0.920 T and 1.276 T, giving relative differences of 8.04% and 3.52%. The comparison shows that the MEC captures the amplitudes of the dominant working harmonics. Because the MEC does

not explicitly resolve three-dimensional fringing flux, end leakage, or local saturation, discrepancies remain relative to the finite-element results.

4.3. Maximum Transferable Torque and Rated Operation

Section 2 obtains the electromagnetic torque from the air-gap normal and tangential flux-density components and establishes the steady-state balance of the three rotors. Figures 18 and 19 show the transient electromagnetic torques of the three rotors at the maximum transferable torque and rated operating points, respectively. Both operating points use the prescribed rotor speeds in Table 1; the torque waveforms therefore characterize periodic steady-state transmission rather than load-step or variable-speed response. The finite-element torque results verify the predicted transmission ratios and steady-state torque balances, whereas the quantitative amplitude comparison is performed for the dominant working harmonics.

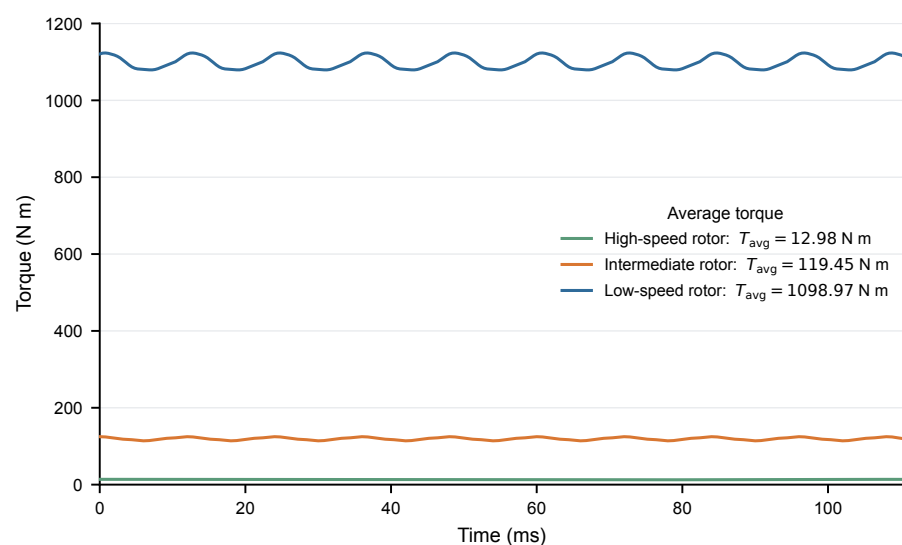


Figure 18. Rotor torques at the maximum transferable torque operating point.

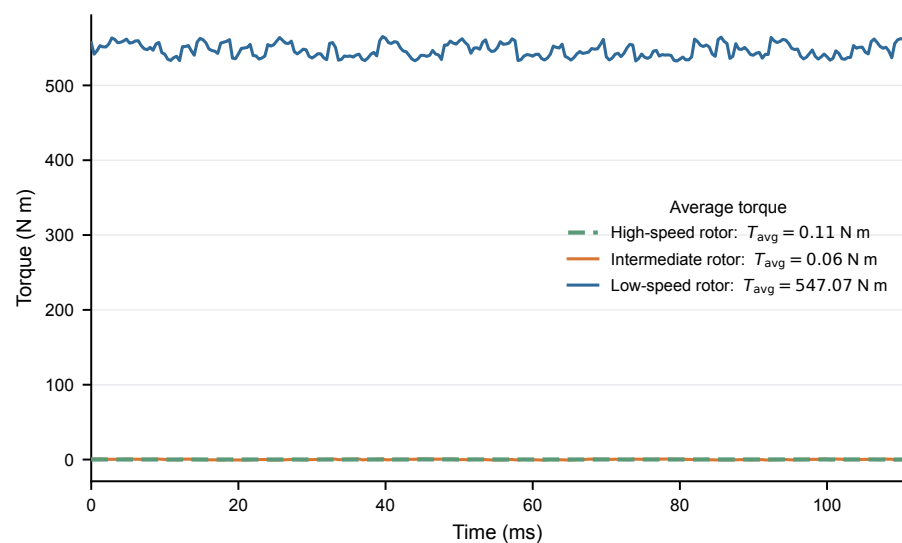


Figure 19. Rotor torques at the rated operating condition.

At the maximum transferable torque operating point, the electromagnetic torques of all three rotors varied periodically. The ratio of the intermediate-rotor average torque

magnitude to that of the high-speed rotor was 9.2, and the corresponding ratio between the low-speed and intermediate rotors was also 9.2, giving an overall torque ratio of 84.64. These results agree with Equations (45) and (42), indicating that torque was transmitted successively through the two magnetic gears. The torque ripple of the low-speed rotor was 4.12%.

Under the rated operating condition, the low-speed rotor produced an average output torque of 547.07 N m, corresponding to 49.78% of the maximum transferable torque, with a torque ripple of 6.49%. The average net torques of the high-speed and intermediate rotors remained close to zero, whereas the low-speed rotor delivered the rated output torque. Under prescribed-speed steady-state operation, the PMSM torque balanced the torque exerted by the radial-flux magnetic gear on the high-speed rotor. The torques exerted by the radial-flux and axial-flux magnetic gears on the intermediate rotor also balanced. These results agree with the steady-state torque relationships in Equations (27), (33), and (37).

The correspondence is therefore established at three levels: the flux-density distribution follows the coupled MEC paths, the air-gap spectra contain the working harmonics predicted by the modulation relations, and the rotor torque waveforms satisfy the motion-equation balances.

A time-step sensitivity check was performed for the final optimized design. Reducing the time step from 0.4 ms to 0.2 ms increased the average low-speed-rotor torque from 1098.970 N m to 1099.602 N m under the maximum transferable torque condition and from 547.070 N m to 547.520 N m under the rated condition. The corresponding relative differences were 0.057% and 0.082%, respectively, confirming the time-step convergence of the reported average torques.

At the rated operating point, the parameterized winding resistance and rated current gave a copper loss of 52 W, while the three-dimensional transient finite-element model gave an eddy-current loss of 8 W. Based on the low-speed-rotor speed and average output torque, the mechanical output power and the efficiency estimate based on these two modeled losses are

$$P_{\text{out}} = \frac{2\pi n_3 T_{\text{rated}}}{60} = 676.87 \text{ W}$$

$$\eta_{\text{Cu+eddy}} = \frac{P_{\text{out}}}{P_{\text{out}} + P_{\text{Cu}} + P_{\text{eddy}}} = 91.86\% \quad (57)$$

where P_{Cu} and P_{eddy} denote the winding copper loss and eddy-current loss, respectively. The resulting value of 91.86% includes only these two loss components and does not represent the complete overall machine efficiency because hysteresis, mechanical, and additional stray losses are excluded.

4.4. Discussion

Table 5 compares the present magnetically geared machine with representative published magnetic-gear configurations.

Table 5. Representative magnetic-gear configurations.

	D_{out} (mm)	V (L)	G	T_{max} (N m)	VTD (N m L ^{−1})
Baninajar et al. [26]	280	8.13	7.67	1796.8	221.1
Duan et al. [18]	177.5	1.19	20	20.9	17.6
Zhang et al. [27]	110	0.1901	29.45	17.83	93.8
Proposed MGM	300	3.3505	84.64	1098.97	328.00

D_{out} , V , G , T_{max} , and VTD denote the outer diameter, volume, gear ratio, maximum transferable torque, and volumetric torque density, respectively. When VTD was not reported in the cited study, it was calculated from the reported torque and the machine-envelope volume determined from the published overall dimensions.

The listed torque values retain the limiting-torque definitions adopted in the corresponding studies. The torque-density values are reported using each study's stated volume boundary; therefore, the table provides a contextual comparison rather than a strictly normalized ranking. Permanent-magnet volume and mass are not included because the selected studies do not use a common reporting boundary.

The flux-density distributions and air-gap spectra show that the PMSM, radial-flux magnetic gear, and axial-flux magnetic gear each established a corresponding magnetic field. The air gaps of both magnetic gears contained working harmonics satisfying the prescribed pole-pair relationships. Each magnetic gear provided a gear ratio of 9.2, giving an overall gear ratio of 84.64 without increasing either single-stage ratio.

The PMSM and radial-flux magnetic gear occupy the inner space of the axial-flux magnetic gear, avoiding the axial accumulation associated with stacking two complete magnetic gears. The volumetric torque density was calculated using the overall cylindrical envelope volume and therefore accounts for the space occupied by both the radial and axial arrangements. The steady-state rotor torques indicate that electromagnetic torque was transmitted through the intermediate rotor to the low-speed output rotor.

Together, the field and torque results are consistent with the main flux paths, gear ratios, and steady-state torque balances derived in Section 2. Through its coordinated radial and axial arrangement, the configuration therefore combines a high overall gear ratio with successive torque transmission from the high-speed rotor to the low-speed rotor while limiting the axial dimension.

The RMS current-density constraint limits the winding electrical loading during stator optimization. The efficiency calculated in this study includes copper and eddy-current losses but excludes hysteresis, mechanical, and additional stray losses. A complete assessment of continuous operation therefore requires these losses to be combined with the heat-transfer paths and cooling boundary conditions in a coupled electromagnetic–thermal model.

The present results are limited to the maximum transferable torque and rated operating points under prescribed-speed conditions. Dynamic torque coupling and harmonic evolution during load steps or speed transients require a coupled electromechanical model incorporating the rotor inertias, time-varying load, mechanical damping, and drive control, and are not evaluated here.

The quantitative MEC–FEA comparison defines the predictive scope of the reduced-order model for the dominant working harmonics. The transient finite-element results further confirm the required working harmonics and steady-state torque transmission under prescribed-speed conditions. However, no prototype experiments were performed. The present evidence therefore does not establish assembly tolerances, air-gap consistency, vibration, thermal behavior, or practical operating performance. The optimization was conducted for a fixed outer diameter, air-gap length, and pole-pair combination. The reported optimum therefore applies to the selected geometric and pole-pair condition. Sensitivity to the outer diameter, air-gap length, and pole-pair combination was not investigated and requires a separate parametric study.

5. Conclusions

The principal contribution of this study is the radial–axial spatial integration of two magnetic-gear stages within a common machine envelope. The PMSM and radial-flux magnetic gear occupy the inner space of the axial-flux magnetic gear, enabling two-stage transmission without axially stacking two complete magnetic gears. A shared permanent-magnet rotor integrates the PMSM with the radial-flux stage, while a shared back iron connects the radial-flux and axial-flux stages through the intermediate rotor. This arrangement forms a continuous torque-transmission path from the high-speed rotor to the

low-speed rotor. The coupled MEC describes the main-flux coupling introduced by the two shared components. The finite-element flux-density distributions agree with the predicted principal flux paths, while the field-modulation and rotor-motion equations establish the synchronous conditions and steady-state torque balances.

The two-stage optimization separates the maximum-transfer design of the magnetic gears from the rated-drive design of the stator. The first stage maximizes volumetric torque density under the maximum transferable torque condition. The second stage minimizes the combined mass of the stator core and windings under the rated-torque and current-density constraints. OLHS initial sampling and finite-element-driven adaptive search were used in both stages. The optimized machine provides an overall gear ratio of 84.64, a maximum transferable torque of 1098.97 N m, and a volumetric torque density of 328.00 N m/L. These results show that the proposed spatial integration can combine a high two-stage gear ratio with effective use of the common machine envelope. Within the contextual comparison in Table 5, the proposed MGM has the highest gear ratio and volumetric torque density among the selected configurations. This comparison remains subject to the different volume boundaries and operating definitions adopted in the cited studies.

Leakage flux and torque ripple remain performance limitations of the present configuration. The lumped MEC represents leakage through equivalent branches but does not resolve local three-dimensional leakage, fringing, or saturation distributions. The torque ripple is 4.12% under the maximum transferable torque condition and 6.49% under the rated operating condition. In addition, the present evidence is based on prescribed-speed three-dimensional transient finite-element analysis without prototype testing. Further work should examine leakage-flux and torque-ripple reduction, dynamic loading, coupled electromagnetic–thermal–mechanical behavior, assembly tolerances, and prototype-based experimental validation.

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