

Article

Rate-Dependent Hysteresis Compensation and Prescribed Performance-Based Composite Sliding Mode Control Strategy for Giant Magnetostrictive Actuators

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Abstract

The inherent nonlinear hysteresis characteristics of giant magnetostrictive actuators are significantly influenced by the frequency of the input signal, and this rate-dependent effect poses severe challenges to high-precision modeling and control. To address this issue, this paper proposes an improved rate-dependent Prandtl–Ishlinskii (PI) model, which incorporates a rate-dependent envelope function with asymmetric left–right thresholds, and additional rate-dependent nonlinear terms to more accurately characterize the frequency-dependent hysteresis behavior of giant magnetostrictive actuators (GMAs). Based on this model, its inverse model is analytically constructed to achieve feedforward compensation, thereby substantially mitigating the influence of hysteresis nonlinearity. On this compensation basis, a sliding mode control strategy with prescribed performance is designed to guarantee global closed-loop stability, and to satisfy the predefined transient and steady-state performance on the tracking error. Finally, verification tests are conducted on a GMA experimental platform, and the results demonstrate that the proposed modeling and composite control scheme can effectively suppress rate-dependent hysteresis disturbances, achieving fast system response and desirable steady-state accuracy, thus validating the feasibility and engineering practicality of the method.

Keywords: rate-dependent hysteresis; modified PI model; prescribed performance sliding mode control; giant magnetostrictive actuator

1. Introduction

The requirements of ultra-precision machining and drive control are increasingly difficult to meet using traditional materials. As a result, micro/nano-drive systems fabricated from smart materials have emerged and play a vital role [1–3]. Hysteresis is a nonlinear phenomenon observed in a variety of fields. In actuators employing smart materials, such as GMA and piezoelectric actuators, a loop-shaped nonlinear hysteresis occurs between the input excitation and the output displacement [4,5]. In mechanisms such as motors or transmission systems, hysteresis phenomena, including motion stalling and response delays, may also arise due to factors such as friction or backlash [6].

Two primary categories of mainstream models exist in hysteresis nonlinearity modeling: physical models and phenomenological models. Physical models are constructed based on the physical properties of the system, with prominent examples including the Jiles–Atherton model [7,8] and the Duhem model [9,10]. Phenomenological models, on



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the other hand, are developed using the actual input–output data of the system, and their parameters generally lack specific physical meanings. Representative phenomenological models include the Preisach model [11,12], the Krasnoselskii–Pokrovskii (KP) model [13], and the PI model [14,15]. The PI model is widely used for modeling actuators made from smart materials, such as piezoelectric ceramics, giant magnetostrictive actuators, and pneumatic artificial muscles [16,17]. It is formulated by the weighted superposition of various play operators, each defined by a distinct threshold. A key advantage of the PI model is its simple mathematical expression, which contributes to its strong adaptability in practical applications. When a control system involves hysteresis nonlinearities introduced by smart material-based actuators, these nonlinearities can adversely affect the system, degrade its performance and lead to inaccuracies or oscillations. Some research approaches utilize improved neural network controllers to adaptively tune controller parameters online, aiming to mitigate these effects and achieve satisfactory control performance [18]. A more common methodology involves constructing an inverse model of hysteresis, which is then employed as a compensator within the control loop to reduce or eliminate the detrimental impact of hysteresis on the system [19–21].

The hysteresis nonlinearity of GMA is influenced by various factors, such as input signal frequency, ambient temperature, and load magnitude, making it difficult for the classical PI model to provide accurate descriptions. Therefore, researchers have proposed several modified PI models. For GMA systems, rate-dependent asymmetric PI formulations have also been explored to improve hysteresis representation under varying excitation rates [22]. The approach of decomposing rate-dependent hysteresis into a rate-independent part in series with a linear dynamic component led to the model presented in [23]. However, this approach not only increases model complexity but also results in a poor match between the linear rate-dependent component and the nonlinear hysteresis loop. Reference [24] introduced a rate-dependent PI model composed of a dynamic envelope function and dynamic weights, and its effectiveness was validated experimentally. Nevertheless, this method employs static operator thresholds, overlooking the influence of threshold variations under different frequencies on the model performance. The aforementioned studies have not fully addressed the issue of rate-dependent hysteresis in GMAs. Furthermore, based on extensive experimental results, this study reveals that the operator threshold can be more appropriately divided into distinct left and right thresholds to better capture the asymmetric rate-dependent hysteresis characteristics. Accordingly, this paper proposes a novel rate-dependent PI model, which consists of a rate-dependent envelope function, asymmetric left and right rate-dependent thresholds, and a rate-dependent nonlinear term added to the operator component.

Subsequently, an inverse rate-dependent PI model (hereafter, the inverse model) is constructed and employed to compensate for hysteresis. However, since the inverse model typically cannot completely eliminate the influence of hysteresis, a compensation error often remains. To further reduce this error and enhance robustness against residual hysteresis and disturbances, a closed-loop robust control scheme should be adopted [25]. Reference [26] proposed a closed-loop control scheme integrated with prescribed performance control, which guarantees the preset tracking performance and has been validated on an experimental platform. Reference [27] introduced a closed-loop scheme combining a predictive observer and prescribed performance control, which increases image feedback frequency and strictly ensures that feature points remain within the camera's field of view. Prescribed performance control has also been extended to nonlinear systems involving actuator hysteresis, where an analytical inverse of the asymmetric PI model is incorporated to mitigate hysteresis effects while guaranteeing predefined tracking performance [28]. Building on the above references and aiming to achieve rapid error convergence and

reduced steady-state error in the giant magnetostrictive actuation platform, this paper adopts a prescribed-performance sliding mode control scheme with distinct upper and lower bounds. This control strategy integrates the concept of prescribed performance with sliding mode control, endowing the controller with strong robustness and the ability to further diminish steady-state error, thereby enabling it to adapt well to system uncertainties and external disturbances [29,30]. A series of experiments were conducted on a giant magnetostrictive actuation platform. The results confirm the efficacy of the developed rate-dependent PI model, its inverse, and the prescribed-performance sliding mode control. Furthermore, they verify that the control scheme achieves global stability while maintaining excellent transient and steady-state performance.

2. Modeling of Rate-Dependent Hysteresis Characteristics

The giant magnetostrictive actuator operates based on the magnetostrictive effect of its core component, a magnetostrictive rod. A cross-sectional view of the GMA is presented in Figure 1, with the labeled components as follows: 1. output rod, 2. disc spring, 3. permanent magnet, 4. driving coil, 5. magnetostrictive rod, and 6. housing. The device operates as follows: first, a drive current in the coil generates a magnetic field. Then, due to the magnetostrictive effect, the rod elongates or contracts, generating the output displacement.

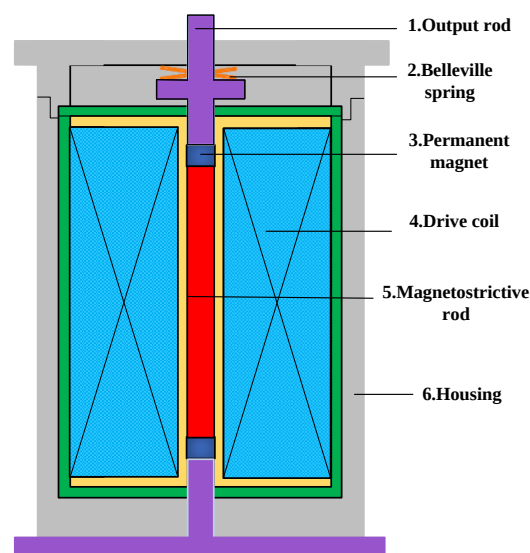


Figure 1. Structural diagram of the GMA.

2.1. Classical PI Model

The rate-dependent modified PI model is developed from the classical PI model. Since its introduction, the PI model has been widely adopted. It was first applied by Prandtl in 1928 to characterize hysteresis phenomena and is constructed through the weighted superposition of different play operators [31,32], as illustrated in Figure 2.

The figure illustrates the operator, where $v(t)$ is the input, $F_r[v](t)$ is the output, and r denotes the non-negative threshold ($r \geq 0$).

For completeness, the classical play operator is recursively defined as

$$F_r[v](0) = \max(v(0) - r, \min(v(0) + r, w_0)),$$

$$F_r[v](t) = \max(v(t) - r, \min(v(t) + r, F_r[v](t_i))), \quad t_i \leq t < t_{i+1},$$

where w_0 is the initial state of the operator. In contrast, the rate-dependent operator introduced below uses rate-dependent envelope functions and asymmetric left and right thresholds. The PI model based on the play operator can be expressed as

$$u_{\text{classics}}(t) = \Pi[v]_{\text{classics}}(t) = p_0 v(t) + \int_0^R p(r) F_r[v](t) dr. \quad (1)$$

In (1), $u_{\text{classics}}(t)$ is the model output, and $\Pi[v]_{\text{classics}}(t)$ denotes the nonlinear PI operator. The term $p_0 v(t)$ is the linear part of the model, where $p_0 > 0$. The nonlinear part is constructed from a weighted superposition of play operators, where $p(r)$ is the weight density, $r \in [0, R]$, and $F_r[v](t)$ is the play operator.

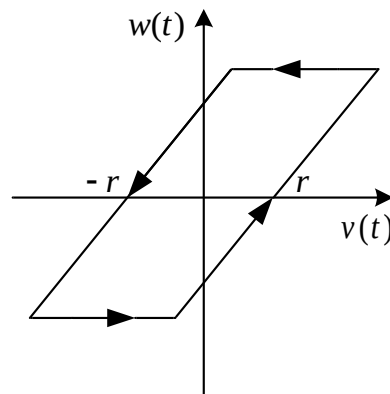


Figure 2. Schematic diagram of the play operator.

2.2. Rate-Dependent PI Model

The classical PI model has difficulty accurately characterizing the rate-dependent hysteresis nonlinearities in GMA. Therefore, certain improvements to the classical PI model are necessary to meet these requirements. Several studies have shown that the hysteresis characteristics of GMA vary with the frequency of the input signal, manifesting as an increased phase difference between the input and output signals and a decreased output displacement. Similar rate-dependent variations, including the rotation and expansion of hysteresis loops, have also been reported in piezoelectric actuators, further demonstrating the necessity of incorporating rate-dependent characteristics into hysteresis models [33]. To accommodate these variations, the envelope function within the operator must be rate-dependent, the threshold must be rate-dependent, and the nonlinear term added to the operator component must also be rate-dependent. These three elements are the most critical components constituting the rate-dependent PI model. A schematic diagram of a single rate-dependent play operator is shown in Figure 3.

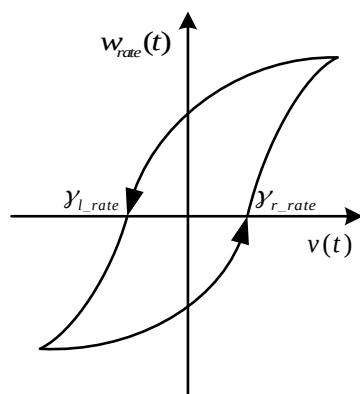


Figure 3. Schematic diagram of the rate-dependent play operator.

The operator depicted in the figure is defined by the input $v(t)$, the output $F_{\text{rate}}[v](t)$, and the left and right envelope functions, denoted by $\gamma_{l,\text{rate}}$ and $\gamma_{r,\text{rate}}$, respectively.

The rate-dependent PI model based on the play operator can be expressed as

$$u(t) = \Pi[v](t) = G_{v(t)} + \int_0^R p_{\text{rate}} F_{\text{rate}}[v](t) dr. \quad (2)$$

In (2), $u(t)$ is the model output, $G_{v(t)}$ is a nonlinear function of the input signal, and the integral term represents the weighted superposition of rate-dependent play operators. In practical implementation, the integral is approximated using $K = 11$ discrete operators:

$$u[k] = G_{v[k]} + \sum_{m=0}^{10} p_{m,\text{rate}}[k] F_{m,\text{rate}}[v][k],$$

where $p_{m,\text{rate}}[k]$ includes the quadrature weight associated with the m th operator. The input rate is evaluated using

$$\dot{v}[k] \approx \frac{v[k] - v[k-1]}{T_s},$$

where T_s is the sampling period. This implementation requires 11 operator updates and one weighted summation at each sampling instant; therefore, its online computational complexity is $O(K)$ per sample. The rate-dependent play operator is defined by

$$F_{\text{rate}}[v](0) = f_{\text{rate}}(v(0), 0), \quad (3)$$

$$F_{\text{rate}}[v](t) = f_{\text{rate}}(v(t), F_{\text{rate}}[v](t_i)), \quad (4)$$

where $t_i \leq t < t_{i+1}$ and $t_0 = 0$.

$$f_{\text{rate}}(v, w_{\text{rate}}) = \max\left(\gamma_{r,\text{rate}} - \frac{m+1}{K} r_{r,\text{rate}}, \min\left(\gamma_{l,\text{rate}} + \frac{m+1}{K} r_{l,\text{rate}}, w_{\text{rate}}\right)\right). \quad (5)$$

Here, $0 = t_0 < t_1 < \dots < t_N = T$ is a partition determined by the variation in the input $v(t)$, such that $v(t)$ is monotone on each sub-interval $[t_i, t_{i+1}]$. For a fixed input rate $q = |\dot{v}(t)|$, the left and right envelope functions $\gamma_{l,\text{rate}}(v, q)$ and $\gamma_{r,\text{rate}}(v, q)$ are required to be continuous and increasing with respect to v , following the generalized play-operator formulation in [34]. The symbol K denotes the number of discrete operators; in this study, $K = 11$, and $m \in \{0, 1, \dots, 10\}$ is the operator index. The variables $r_{l,\text{rate}}$ and $r_{r,\text{rate}}$ are the rate-dependent left and right thresholds, respectively. The left and right envelope functions are expressed as

$$\begin{cases} \gamma_{l,\text{rate}}(v, q) = (x_1 + x_2 \sin(x_3 v + x_4)) \sin v + x_5 + x_6 q v, \\ \gamma_{r,\text{rate}}(v, q) = (x_7 + x_8 \sin(x_9 v + x_{10})) \sin v + x_{11} + x_{12} q v, \end{cases} \quad q = |\dot{v}(t)|. \quad (6)$$

where $q = |\dot{v}(t)|$ denotes the magnitude of the input rate, and $x_1, \dots, x_{12}$ are parameters to be identified. The twelve parameters in (6) specify the two envelope functions and are independent of the number K of play operators. To satisfy the generalized play-operator assumptions, the identified parameters and the operating range must ensure

$$\frac{\partial \gamma_{l,\text{rate}}(v, q)}{\partial v} \geq 0, \quad \frac{\partial \gamma_{r,\text{rate}}(v, q)}{\partial v} \geq 0,$$

thereby satisfying the monotonicity requirement of the generalized play operator.

The hysteresis characteristics of the GMA become more pronounced at higher frequencies. Therefore, the original fixed threshold r can be modified into dynamic thresholds

to more accurately characterize the hysteresis behavior at elevated frequencies. Through extensive experimentation, it has been observed that using distinct thresholds for the rising and falling segments of the operator yields better modeling performance. Consequently, the original threshold r is improved to be asymmetric, denoted as $r_{l,\text{rate}}$ and $r_{r,\text{rate}}$, whose expressions are given by

$$\begin{cases} r_{l,\text{rate}} = a_1 \ln(b_1 + c_1 |\dot{v}(t)|^{d_1}) + e_1, \\ r_{r,\text{rate}} = a_2 \ln(b_2 + c_2 |\dot{v}(t)|^{d_2}) + e_2. \end{cases} \quad (7)$$

The parameters are identified subject to the constraints $a_i \geq 0$, $c_i \geq 0$, and $b_i \geq 1$, where $i = 1, 2$; the offset parameters e_i are allowed to be signed constants.

The GMA output displacement is inversely related to the frequency of the input signal. The linear term of the classical PI model cannot satisfy this characteristic. Therefore, this linear term can be improved to a rate-dependent nonlinear term. Its mathematical expression is

$$G_{v(t)} = a_3 \ln(b_3 + c_3 |\dot{v}(t)|) + d_3 + e_3 |v(t)|v(t), \quad (8)$$

where $b_3 > 0$, $c_3 > 0$, and a_3, d_3, e_3 are constants.

The weight of the m th rate-dependent play operator is denoted by $p_{m,\text{rate}}$ and is defined as

$$p_{m,\text{rate}} = a_4 e^{-b_4(m+1)} + c_4, \quad (9)$$

where $a_4 > 0$, $c_4 > 0$, $K = 11$ is the number of discrete operators, and $m = 0, 1, \dots, 10$.

3. Prescribed Performance Controller Design

Consider a dynamic nonlinear system consisting of a hysteretic nonlinear part and a linear part, expressed as

$$a\ddot{y}(t) + b\dot{y}(t) + cy(t) = u(t) + d(t), \quad u(t) = \Pi[v](t). \quad (10)$$

Here, $y(t)$ is the system output, and a , b , and c are known positive constants. The hysteretic nonlinearity has $v(t)$ as its input and $u(t)$ as its output. The term $d(t)$ captures bounded uncertainties or disturbances, satisfying $|d(t)| \leq d_m$.

The complete control scheme is illustrated in Figure 4.

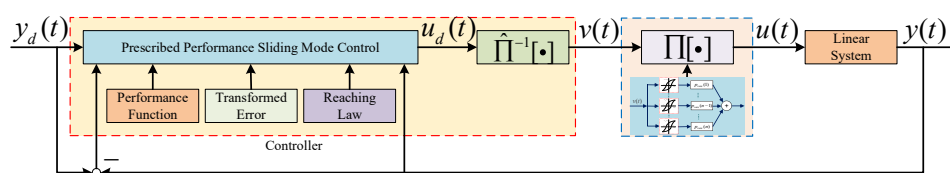


Figure 4. Control scheme.

The following standard assumptions are made throughout this article.

Assumption 1. $d(t)$ represents system uncertainty or external disturbance. Its specific value is unknown but is bounded, satisfying $|d(t)| \leq d_m$.

Assumption 2. The desired signal $y_d(t)$ and its n th derivative are continuous, where n is a bounded positive integer.

3.1. Rate-Dependent PI Inverse Model Construction

Due to the highly nonlinear characteristics of the controlled plant, which introduce difficulties in control, this paper adopts a strategy of constructing the inverse of the hysteresis nonlinearity to mitigate its impact on the system. Traditional inverse and forward models can only capture hysteresis nonlinearity with a single characteristic and are unable to represent hysteresis behavior at different frequencies. To address this issue, this paper constructs a rate-dependent inverse PI model based on the classical inverse PI model.

Given that both the classical PI model and the rate-dependent PI model proposed in this paper are constructed from weighted superpositions of operators, the inverse model is likewise represented by weighted inverse operators. The implementation uses the same $K = 11$ operators:

$$v(t) = \hat{\Pi}^{-1}[u](t) = G'_{u(t)} + \int_0^R p'_{\text{rate}} F'_{\text{rate}}[u](t) dr. \quad (11)$$

where $v(t)$ is the output of the inverse model, $G'_{u(t)}$ is a nonlinear function of $u(t)$, and the integral term is the weighted superposition of rate-dependent inverse play operators.

To justify the inverse weights, define the cumulative slope of the forward PI mapping as

$$S_i = \sum_{j=1}^i p_{\text{rate},j}.$$

For a monotone branch on which the first i play operators are active, the local slope of the forward mapping is S_i . The corresponding local slope of the inverse mapping is therefore $1/S_i$, so that

$$\sum_{j=1}^i p'_{\text{rate},j} = \frac{1}{S_i}.$$

For $i = 1$, this gives $p'_{\text{rate},1} = 1/p_{\text{rate},1}$. For $i \geq 2$, subtracting two adjacent cumulative slopes gives

$$p'_{\text{rate},i} = \frac{1}{S_i} - \frac{1}{S_{i-1}} = -\frac{p_{\text{rate},i}}{S_i S_{i-1}}.$$

Thus, the inverse-weight relationship is

$$\begin{cases} p'_{\text{rate},1} = \frac{1}{p_{\text{rate},1}}, \\ p'_{\text{rate},i} = \frac{-p_{\text{rate},i}}{\left(\sum_{j=1}^i p_{\text{rate},j}\right) \left(\sum_{j=1}^{i-1} p_{\text{rate},j}\right)}, \quad i = 2, 3, \dots, 11. \end{cases} \quad (12)$$

This relationship is well defined when the cumulative forward slopes S_i are nonzero and the forward mapping satisfies the monotonicity assumptions required by the PI inverse construction. It is used here to obtain a feedforward inverse compensation model; the residual compensation error is subsequently handled by the closed-loop controller [19].

3.2. Prescribed Performance Function

The tracking error of the actuation platform is defined as

$$e(t) = y(t) - y_d(t), \quad (13)$$

where $y_d(t)$ is the reference signal.

A positive definite, decreasing, and smooth performance function $\mu(t)$ is introduced:

$$\mu(t) = (\mu_0 - \mu_\infty)e^{-\alpha_0 t} + \mu_\infty. \quad (14)$$

It can be verified that $\mu(t)$ satisfies the following conditions:

$$\lim_{t \rightarrow 0} \mu(t) = \mu_0, \quad \lim_{t \rightarrow \infty} \mu(t) = \mu_\infty, \quad \mu(0) = (\mu_0 - \mu_\infty)e^{-\alpha_0 \cdot 0} + \mu_\infty = \mu_0.$$

Here, $\mu_0 > \mu_\infty > 0$ and $\alpha_0 > 0$ are the prescribed performance parameters set by the designer. The parameters μ_0 , μ_∞ , and α_0 represent the initial value of the performance function, the maximum steady-state error bound, and the convergence rate, respectively.

The prescribed performance function constrains the tracking error within the following predefined boundaries:

$$-\underline{\delta}\mu(t) < e(t) < \bar{\delta}\mu(t), \quad \forall t > 0. \quad (15)$$

Here, the boundary parameters $\underline{\delta} > 0$ and $\bar{\delta} > 0$ are set by the designer. The lower-bound $-\underline{\delta}\mu(t)$ constrains the error undershoot, while the upper-bound $\bar{\delta}\mu(t)$ constrains the overshoot.

As evidenced by (14) and (15), the appropriate selection of parameters can guarantee transient and steady-state performance for the tracking error. The design philosophy of the prescribed performance controller is to transform the constrained tracking error into the unconstrained error signal $z(t)$. Here, $\lambda: \mathbb{R} \rightarrow (-\underline{\delta}, \bar{\delta})$ is a smooth, strictly increasing mapping, and $z(t) \in L_\infty$ means that the transformed error is essentially bounded; i.e., $\sup_{t \geq 0} |z(t)| < \infty$. The mapping is characterized by

- 1) $-\underline{\delta} < \lambda(z) < \bar{\delta}, \quad \forall z \in \mathbb{R};$
- 2) $\lim_{z \rightarrow +\infty} \lambda(z) = \bar{\delta}, \quad \lim_{z \rightarrow -\infty} \lambda(z) = -\underline{\delta}.$

Based on the properties of $\lambda(z)$, condition (15) can be rewritten as

$$e(t) = \mu(t)\lambda(z), \quad (16)$$

where this paper selects

$$\lambda(z) = \frac{\bar{\delta}e^z - \underline{\delta}e^{-z}}{e^z + e^{-z}}. \quad (17)$$

Consequently, $-\underline{\delta} < \lambda(z) < \bar{\delta}$. Based on $\lambda(z)$, the transformed error z can be calculated as

$$z = \lambda^{-1}\left(\frac{e}{\mu}\right) = \frac{1}{2} \ln\left(\frac{\lambda + \underline{\delta}}{\bar{\delta} - \lambda}\right) = \frac{1}{2} \ln\left(\frac{e + \underline{\delta}\mu}{\bar{\delta}\mu - e}\right). \quad (18)$$

By performing an equivalent error transformation to obtain z and designing a controller to ensure that z remains bounded, the tracking error e of the giant magnetostrictive actuation platform can be controlled within the predefined boundary (15).

Theorem 1. *If the transformed error z is bounded such that $|z| \leq z_m$ for some positive constant z_m , then the tracking error defined in (13) is guaranteed to remain within the preset performance boundaries.*

3.3. Prescribed Performance Sliding Mode Controller

To design the controller, we calculate the derivative of the equivalent error z as

$$\dot{z} = \frac{1}{2\mu} \left(\frac{1}{\lambda + \underline{\delta}} - \frac{1}{\lambda - \bar{\delta}} \right) \left(\dot{e} - \frac{e\dot{\mu}}{\mu} \right). \quad (19)$$

To avoid repeating the common factor in the following derivatives and controller equations, define

$$\rho = \frac{1}{2\mu} \left(\frac{1}{\lambda + \underline{\delta}} - \frac{1}{\lambda - \bar{\delta}} \right).$$

Here, ρ is a bounded variable satisfying $0 < \rho \leq \rho_m$, where ρ_m is a positive constant. This notation distinguishes the auxiliary variable from the play-operator threshold r . The derivative of ρ is given by

$$\begin{aligned} \dot{\rho} = & -\frac{\dot{\mu}}{2\mu^2} \left[\frac{1}{\lambda + \underline{\delta}} - \frac{1}{\lambda - \bar{\delta}} \right] \\ & + \frac{\dot{e}\mu - e\dot{\mu}}{2\mu^3} \left[-\frac{1}{(\lambda + \underline{\delta})^2} + \frac{1}{(\lambda - \bar{\delta})^2} \right]. \end{aligned} \quad (20)$$

From this, we can further obtain

$$\ddot{z} = \dot{\rho} \left(\dot{e} - \frac{e\dot{\mu}}{\mu} \right) + \rho \left(\ddot{e} - \frac{e\ddot{\mu}}{\mu} - \frac{\dot{e}\dot{\mu}}{\mu} + \frac{e\dot{\mu}^2}{\mu^2} \right). \quad (21)$$

The error sliding surface is defined as

$$s = \Lambda z + \dot{z}, \quad (22)$$

where $\Lambda > 0$ is a constant. Further calculation yields the derivative of s :

$$\dot{s} = \psi + \frac{\rho}{a} (u + d - b\dot{y} - cy), \quad (23)$$

where

$$\psi = (\Lambda\rho + \dot{\rho}) \left(\dot{e} - \frac{e\dot{\mu}}{\mu} \right) + \rho \left[-\ddot{y}_d - \frac{e\ddot{\mu}}{\mu} - \frac{\dot{e}\dot{\mu}}{\mu} + \frac{e\dot{\mu}^2}{\mu^2} \right].$$

To obtain a stable reaching process while retaining the plant scaling in (10), the sliding-mode reaching law is selected as

$$a\dot{s} = -k_1s - k_2 \tanh(s), \quad (24)$$

where both k_1 and k_2 are positive constants. This design ensures a sufficiently fast reaching speed when s is far from zero and a slower reaching speed when s approaches zero, effectively suppressing chattering.

Based on (23) and (24), the prescribed performance controller is designed as

$$u = b\dot{y} + cy + \frac{1}{\rho} (-a\psi - k_1s - k_2 \tanh(s)). \quad (25)$$

The following theorem establishes boundedness of the resulting closed-loop system.

4. Stability Analysis

Theorem 2. Consider the nonlinear system (10) with the controller given by (24) and (25). Then, provided that the initial state satisfies the prescribed performance constraints, the tracking error is confined to the prescribed performance boundaries, and all signals in the closed-loop system remain bounded.

The proof is as follows. Substituting (25) into (23) yields

$$a\dot{s} = -k_1s - k_2 \tanh(s) + \rho d. \quad (26)$$

To prove global boundedness, consider the following Lyapunov function:

$$V = \frac{1}{2}as^2. \quad (27)$$

Differentiating V with respect to time gives

$$\begin{aligned}\dot{V} &= as\dot{s} \\ &= -k_1s^2 - k_2s \tanh(s) + s\rho d \\ &\leq -k_1s^2 - k_2s \tanh(s) + |s|\rho_m d_m \\ &\leq -k_1s^2 - k_2s \tanh(s) + s \tanh(s)\rho_m d_m + \delta_0\rho_m d_m.\end{aligned}\quad (28)$$

where δ_0 is a constant satisfying $\delta_0 \geq 0.2785$. If $k_2 > \rho_m d_m$, then

$$\begin{aligned}\dot{V} &\leq -k_1s^2 + \delta_0\rho_m d_m \\ &\leq -\alpha V + \beta,\end{aligned}\quad (29)$$

where $\alpha = \frac{2k_1}{a}$ and $\beta = \delta_0\rho_m d_m$ are positive constants. Solving (29) yields $V(t) \leq V(0)e^{-\alpha t} + \frac{\beta}{\alpha}$. Hence, s is uniformly ultimately bounded, which implies that z remains bounded. According to Theorem 1, the tracking error remains within the predefined performance boundaries.

5. Experimental Results

5.1. Experimental Setup

An experimental platform was established based on the system description shown in Figure 5. The current applied to the actuator is generated by a power amplifier (model LYG-10150) produced by Nantong Longyi Electronics Technology Co., Ltd. (Nantong, Jiangsu, China), with a maximum output voltage of ± 150 V and a maximum output current of ± 10 A.

The actuator employs a custom-made GMA with a maximum input AC current of 5 A, an output displacement range of 0–25 μm , an operating bandwidth of 0–300 Hz, and a maximum output force of 100 N. The magnetostrictive rod used in the custom-made GMA is made of Terfenol-D, a Tb–Dy–Fe-based giant magnetostrictive material.

The displacement is measured using a capacitive sensor (model CPL-190 from Lion Precision, Shoreview, MN, USA) with an output voltage range of ± 10 V, a measurement range of ± 125 μm , a bandwidth of 15 kHz, and a resolution of 5 nm. Real-time data processing and transmission are handled by a MicroLabBox (model MLBX_1302T) developed by dSPACE GmbH (Paderborn, Germany), using the Control Development Package (CDP), including the Real-Time Interface (RTI) and ControlDesk software, version R2020B. The system provides 24 analog input channels and 16 analog output channels, both with 16-bit resolution, and an input/output voltage range of ± 10 V per channel.

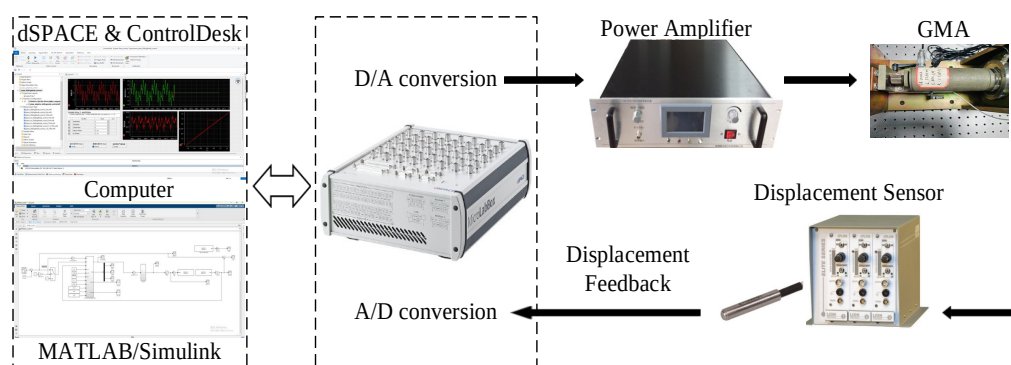


Figure 5. Experimental platform.

5.2. Experimental Results of Rate-Dependent Hysteresis Modeling with Inverse Compensation

The rate-dependent PI model and its inverse model were identified using a genetic algorithm [35,36]. The algorithm was implemented using the Global Optimization Toolbox in MATLAB R2021a (MathWorks, Natick, MA, USA) and supports parallel computing to reduce the identification time. Its main advantage is its global search capability, which helps reduce the risk of becoming trapped in local optima.

For parameter identification, the genetic-algorithm fitness function was defined as the sum of squared errors:

$$F = \sum_{k=1}^N (y_p(k) - y_t(k))^2, \quad (30)$$

where y_p denotes the model output, y_t denotes the measured GMA output, and N is the total number of data points. The identified parameters of the rate-dependent PI model and its inverse are listed in Tables 1 and 2, respectively.

Table 1. Identified parameters of the rate-dependent PI model.

Parameter	Estimate	Parameter	Estimate	Parameter	Estimate
x_i parameters					
x_1	0.148	x_2	0.639	x_3	0.491
x_4	0.110	x_5	0.205	x_6	−4.803
x_7	0.831	x_8	0.043	x_9	−0.135
x_{10}	0.532	x_{11}	−0.352	x_{12}	0.368
PI model coefficients					
a_1	0.464	b_1	2.850	c_1	0.544
d_1	0.176	e_1	−0.491	a_2	1.312
b_2	2.550	c_2	1.076	d_2	0.266
e_2	−0.596	a_3	0.088	b_3	0.010
c_3	0.228	d_3	0.091	e_3	−1.898
a_4	0.004	b_4	0.260	c_4	0.106

Table 2. Identified parameters of the rate-dependent inverse PI model.

Parameter	Estimate	Parameter	Estimate	Parameter	Estimate
x'_i parameters					
x'_1	3.193	x'_2	0.907	x'_3	−2.606
x'_4	−0.020	x'_5	−0.007	x'_6	−4.169
x'_7	5.190	x'_8	2.472	x'_9	−2.823
x'_{10}	−0.421	x'_{11}	−0.307	x'_{12}	2.019
Inverse PI model coefficients					
a'_1	1.912	b'_1	2.154	c'_1	2.508
d'_1	0.734	e'_1	−1.466	a'_2	1.827
b'_2	5.329	c'_2	1.165	d'_2	0.270
e'_2	−2.548	a'_3	−1.133	b'_3	0.611
c'_3	1.468	d'_3	−0.476	e'_3	0.149
Rate-dependent coefficients					
$p'_{\text{rate},1}$	9.003	$p'_{\text{rate},2}$	−4.531	$p'_{\text{rate},3}$	−1.514
$p'_{\text{rate},4}$	−0.760	$p'_{\text{rate},5}$	−0.459	$p'_{\text{rate},6}$	−0.309
$p'_{\text{rate},7}$	−0.223	$p'_{\text{rate},8}$	−0.171	$p'_{\text{rate},9}$	−0.136
$p'_{\text{rate},10}$	−0.111	$p'_{\text{rate},11}$	−0.094		

The evaluation of modeling accuracy employs the mean absolute error (MAE) and root mean square error (RMSE) as metrics, which are given by

$$\text{MAE} = \frac{1}{N} \sum_{k=1}^N |y_p(k) - y_t(k)|, \quad (31)$$

$$\text{RMSE} = \sqrt{\frac{1}{N} \sum_{k=1}^N (y_p(k) - y_t(k))^2}. \quad (32)$$

Here, y_p is the model output, y_t is the actual output data collected from the GMA, and N is the number of data points.

Figure 6 presents the hysteresis curves, depicting the relationship between the GMA input current (horizontal axis) and output displacement (vertical axis). The hysteresis loop obtained from the experimental data is shown in blue, while the red curve represents the rate-dependent PI model. The modeling results agree well with the measured GMA response over the tested 1–100 Hz range. Above 100 Hz, the actuator response becomes more nonlinear; this higher-frequency range is outside the validation scope of the present study. Accordingly, the proposed rate-dependent PI model is developed and validated for the 1–100 Hz operating range.

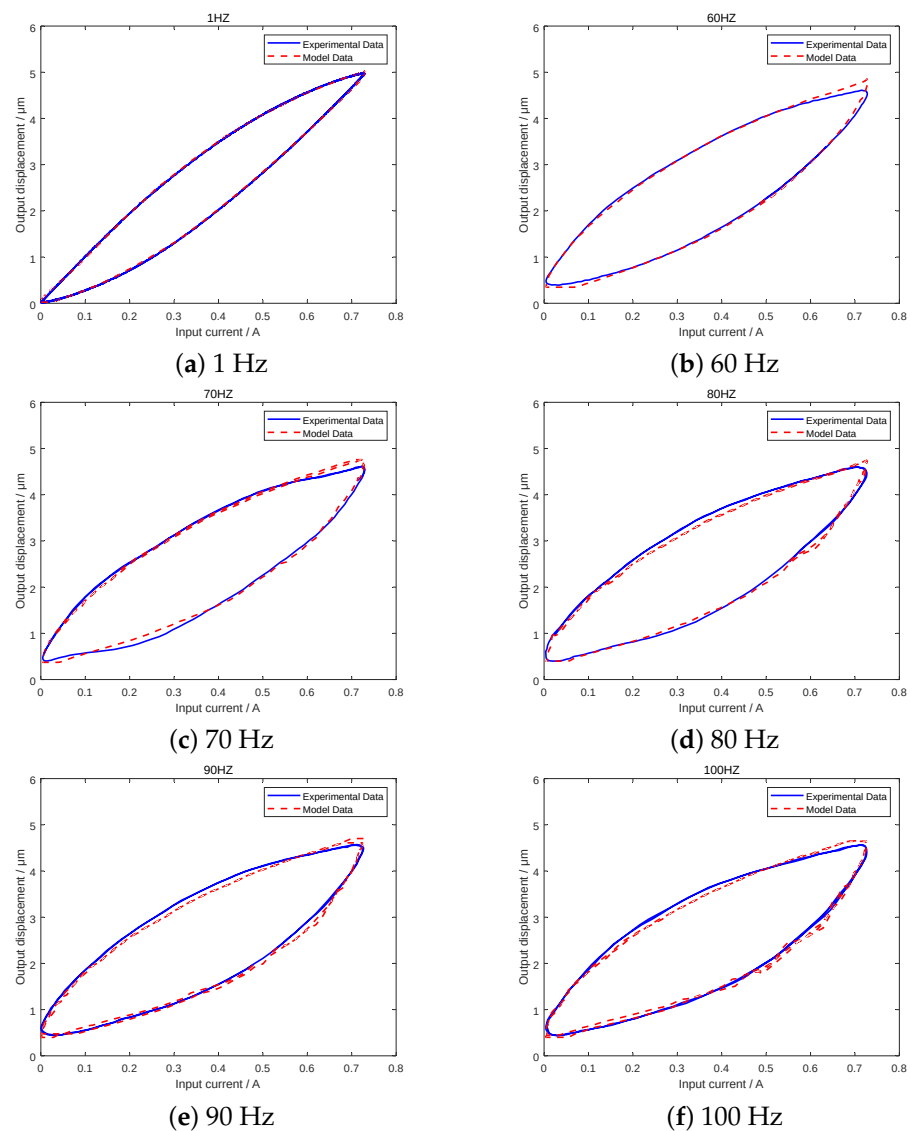


Figure 6. Performance of the rate-dependent PI model at different frequencies: (a) 1 Hz; (b) 60 Hz; (c) 70 Hz; (d) 80 Hz; (e) 90 Hz; (f) 100 Hz.

Considering the characteristics of the GMA, the desired displacement at 1 Hz is $u_d(t) = 2.5\sin(2\pi ft) + 2.5$, where $f = 1$. As the frequency increases, the output displacement of the GMA decreases; correspondingly, both the amplitude and the bias (offset) of the desired signal also decrease. With the variation in frequency, values such as $f = 10, 20, 30, 40, 50, 60, 70, 80, 90, 100$ (unit: Hz) are considered.

Figure 7 shows the physical experiment result using the inverse model for compensation. In this figure, the horizontal axis corresponds to the desired displacement, and the vertical axis to the actual displacement.

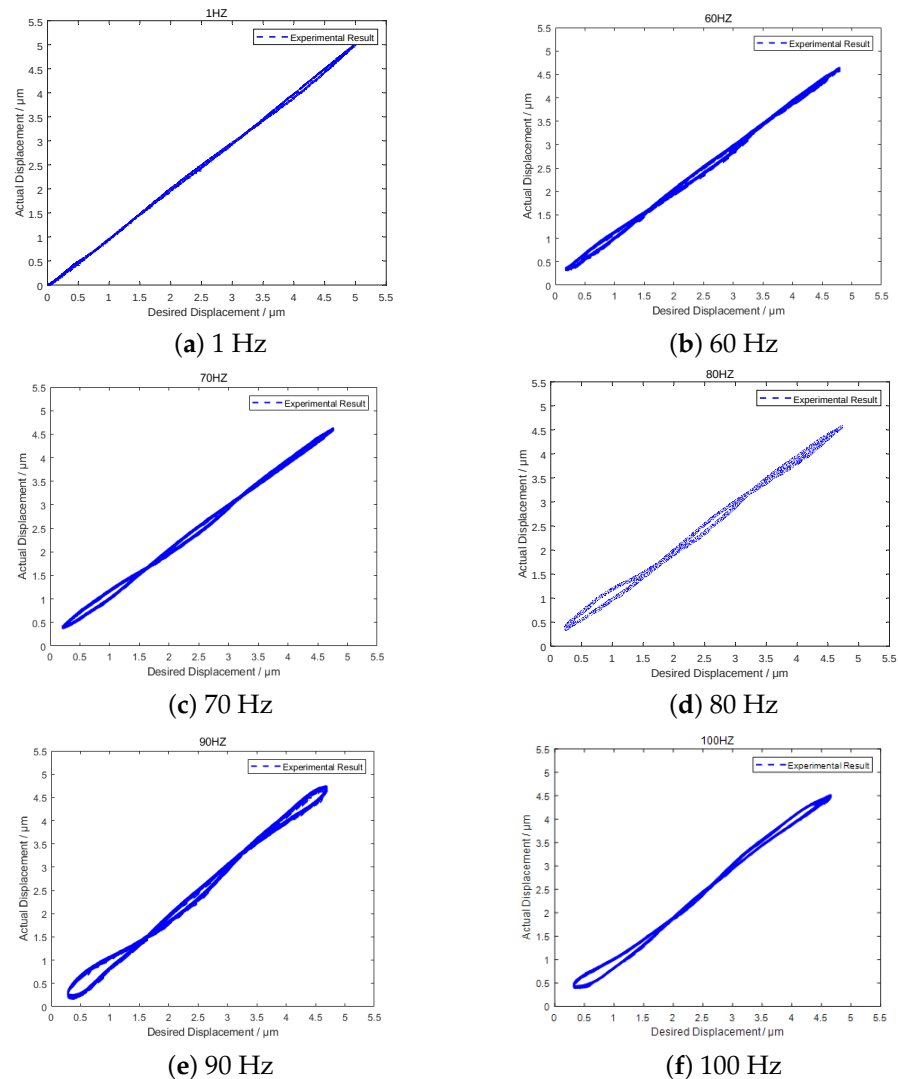


Figure 7. Effectiveness of feedforward inverse compensation at different frequencies: (a) 1 Hz; (b) 60 Hz; (c) 70 Hz; (d) 80 Hz; (e) 90 Hz; (f) 100 Hz.

The modeling errors of the rate-dependent PI model are listed in Table 3. Figure 7 and Table 4 demonstrate the practical effect of inverse model compensation. The results show that the rate-dependent PI inverse model designed in this paper has a significant effect, but the hysteresis characteristics are not completely eliminated. Therefore, a sliding mode control scheme with prescribed performance is applied to further reduce the error.

Table 3. Modeling errors of the rate-dependent PI model at different frequencies.

RMSE/μm	MAE/μm	Frequency/Hz
0.023	0.018	1
0.041	0.032	10

Table 3. *Cont.*

RMSE/ μm	MAE/ μm	Frequency/Hz
0.038	0.030	20
0.057	0.043	30
0.062	0.045	40
0.077	0.060	50
0.084	0.057	60
0.092	0.073	70
0.105	0.084	80
0.095	0.081	90
0.099	0.081	100

Table 4. Feedforward inverse compensation errors at different frequencies.

RMSE/ μm	MAE/ μm	Frequency/Hz
0.045	0.037	1
0.067	0.060	10
0.077	0.072	20
0.083	0.077	30
0.087	0.079	40
0.094	0.083	50
0.115	0.100	60
0.120	0.101	70
0.120	0.101	80
0.117	0.099	90
0.118	0.103	100

5.3. Experimental Results of Prescribed-Performance Sliding Mode Control

The objective of control is to ensure accurate tracking of the desired signal while satisfying the prescribed transient and steady-state performance. The desired signal at 1 Hz is $u_d(t) = 2.5 \sin(2\pi ft) + 2.5$, where $f = 1$. As the frequency increases, the output displacement of the GMA decreases; accordingly, both the amplitude and bias of the desired signal are reduced. The tests use $f = 10, 20, 30, 40, 50, 60, 70, 80, 90, 100$ Hz. Finally, the tracking performance for a mixed-frequency signal $u_d(t) = 1.25 \sin(2\pi t) + 0.625 \sin(20\pi t) + 0.625 \sin(40\pi t) + 2.5$ was also tested. The sampling period for all signals in this study is $T_s = 0.1$ ms, corresponding to a sampling frequency of 10 kHz. The prescribed performance function is selected as $\mu(t) = (\mu_0 - \mu_\infty)e^{-\alpha_0 t} + \mu_\infty$, with $\alpha_0 = 1$ and $\mu_0 = 6.25$. At 1 Hz, $\mu_\infty = 1$; as the frequency changes, μ_∞ gradually increases. The boundary parameters are set as $\underline{\delta} = 0.9$, $\bar{\delta} = 1$, $k_1 = 0.1$, and $k_2 = 0.001$. The model parameters are $a = 0.0002$, $b = 0.00091$, and $c = 0.932$. Figure 8 shows the physical experiment of sliding-mode control with prescribed performance. The tracking subplots display 10 cycles after the system reaches steady state, whereas the error subplots show the error from the initial time over a 10-s duration.

The experimental data in Figure 8 and Table 5 show that the RMSE and MAE remain below $0.125 \mu\text{m}$ and $0.106 \mu\text{m}$, respectively, across the tested single- and mixed-frequency signals. These results demonstrate that the tracking error remains bounded within a small range under the adopted controller.

Table 5. Closed-loop control errors at different frequencies.

RMSE/ μm	MAE/ μm	Frequency/Hz
0.017	0.013	1
0.040	0.032	10
0.042	0.035	20
0.053	0.048	30
0.045	0.035	40
0.056	0.048	50
0.062	0.056	60

Table 5. Cont.

RMSE/ μm	MAE/ μm	Frequency/Hz
0.070	0.062	70
0.083	0.073	80
0.108	0.089	90
0.111	0.091	100
0.125	0.106	1/10/20

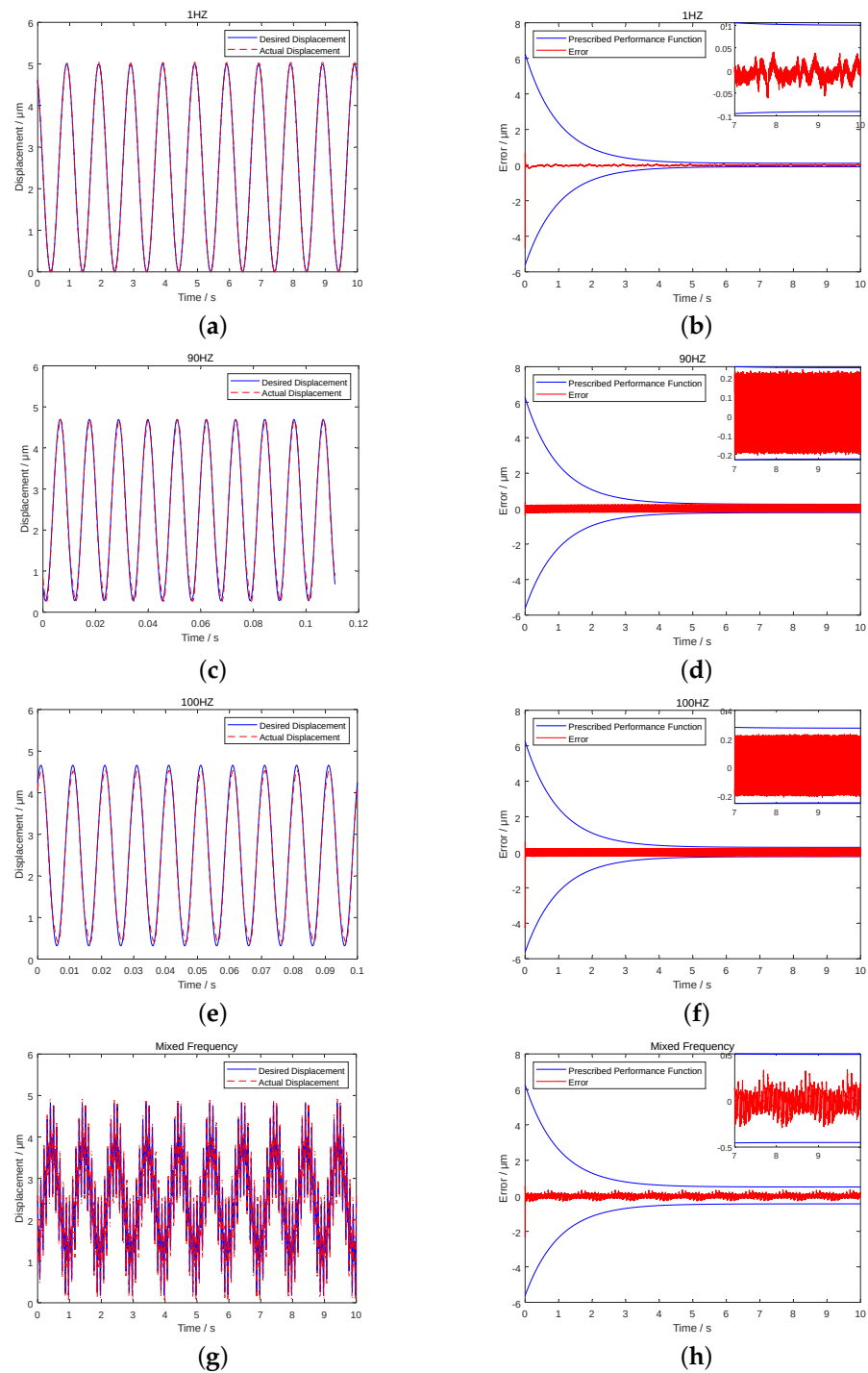


Figure 8. Tracking and error responses of the prescribed-performance sliding mode controller: (a) 1 Hz tracking; (b) 1 Hz error; (c) 90 Hz tracking; (d) 90 Hz error; (e) 100 Hz tracking; (f) 100 Hz error; (g) 1/10/20 Hz tracking; (h) 1/10/20 Hz error.

6. Conclusions

This paper addresses rate-dependent hysteresis modeling for nonlinear systems and the associated prescribed-performance sliding-mode control problem. A rate-dependent PI model is developed for GMA hysteresis. It consists primarily of rate-dependent envelope functions, distinct left and right rate-dependent thresholds, and a rate-dependent nonlinear term added to the operator component. A rate-dependent PI inverse model is then established to mitigate hysteresis over different frequencies. Finally, a prescribed-performance sliding-mode controller is adopted to achieve rapid error convergence and reduce the steady-state error while maintaining bounded closed-loop signals. Experiments on a giant magnetostrictive actuation platform demonstrate the feasibility and effectiveness of the proposed control scheme. The present validation is limited to the 1–100 Hz operating range and offline model identification. Future work will investigate online parameter adaptation and robustness under wider frequency, temperature, and load variations.

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