Frequency Domain Design Method of the Aeroengine Fuel Servo Constant Pressure Difference Control System with High Performance

Wenshuai Zhao 1,*, Xi Wang 1, Yifu Long 1, Zhenhua Zhou 2 and Linhang Tian 2

1 School of Energy and Power Engineering, Beijing University of Aeronautics and Astronautics, Beijing 100191, China
2 AECC Guizhou Honglin Aeroengine Control Technology Co., Ltd., Guiyang 550009, China
* Correspondence: m13716752318_1@163.com

Abstract: The constant pressure difference regulating mechanism is widely used in aeroengine fuel servo metering systems, and it almost decides the metering precision. However, the design theory and design method of the available constant pressure difference regulating mechanism are unclear, and it is difficult to follow the high stability, high accuracy, and high robustness requirements of the modern aeroengine fuel servo metering system. In this paper, the design theory of the constant pressure difference regulating mechanism is revealed, and it indicates that it consists of two basic control units: a state feedback stabilization controller to ensure the asymptotic stability and disturbance rejection performance; and a servo and feed-forward compensator to ensure the asymptotic tracking ability. In addition, based on the frequency domain analysis method, the decisive influences about the control gains of the two control units on the dynamic performance and stability are analyzed. On this basis, a frequency domain design method of the two core control gains is proposed to complete the design task of the closed-loop system. The simulation results show that, under the adverse conditions of 1 MPa strong step disturbance of the inlet pressure and 50 mm² strong step disturbance of the variable inlet flow area, the steady-state working range of the controlled pressure difference meets $0.92 \pm 0.01$ MPa, the steady-state error is not more than 1%, the regulation time is not more than 0.01 s, the dynamic overshoot is not more than 10%, and the designed phase margin is more than 70°.

Keywords: constant pressure difference control system; stabilization controller; servo and feed-forward compensator; frequency domain design

1. Introduction

The aeroengine control system is gradually transitioning from the hydraulic mechanical system to the digital electronic system. However, regardless of the hydraulic mechanical system or the digital electronic system, the fuel metering device primarily uses the constant pressure difference metering method, and the metering accuracy depends on the design performance of the constant pressure difference regulating mechanism. If the constant pressure difference regulating mechanism performs badly, it will cause aeroengine instability, such as speed and thrust swing, turbine overtemperature, and even compressor stall and surge [1]. With the proposal of the high stability, high accuracy, and high robustness performance requirements of the modern aeroengine control system, however, the traditional design methods cannot solve this high-tech problem because the design theory and design method of the available constant pressure difference regulating mechanism are unclear. Hence, the analysis and design research of the system has become a more core issue.

Research on the constant pressure difference regulating mechanism has mainly focused on modeling, simulation, and stability analysis. In the early stage, basis on the classical control theory, the dynamic equations and the transfer function block diagram were established. Meanwhile, the frequency domain model was obtained, and the static and dynamic characteristics were analyzed [2–4]. These works provide research basis,
yet the frequency domain dynamic model is very complex, and they do not reveal the
design theory as well as the relationship between the design parameters and the system
performance. Additionally, there is a lack of test and simulation works. In recent years, with
the development of simulation technology, research mainly focuses on dynamic modeling
and simulation analysis, but the reports of theoretical research are still rare. For instance,
the influences of the spring stiffness, hole diameter, and other parameters on the system
performance are analyzed based on AMESim [5–9]. Unfortunately, these research works
are carried out only on nonlinear models. Due to the lack of theoretical analysis to guide
the research process, the main research method is the trial-and-error method, which is
operated by changing the design parameters to explore the system performance, which
is inefficient. Meanwhile, the influences of the oil return orifice profile structure on the
system characteristics are deeply researched, and the study results show that the profile
structure is related to the control gain of the system [10–12]. However, these researches
do not involve system analysis and design but only provides guidance for the design of
the orifice profile, which has limitations. Moreover, the stability conditions of the system
are analyzed based on the transfer function models [13]. Problematically, the model is
too simplified, and it lacks simulation or experimental verification, so the effectiveness
of the method should be verified. Comfortingly, an optimal design of the variable orifice
is studied by physical test, and the test results show that the variable orifice can bring
better performance [14]. Although this study does not involve the design theory analysis,
it is still instructive. In addition, based on the CFD simulation, the influences of the flow
force on the balance of the motion valve are researched [15–18]. Indeed, the results show
that the flow forces do have impacts on the dynamic performance of the system, but the
impacts are not decisive. Additionally, there are some reports about the structure design of the
pressure valves and the effect of the hysteresis on the speed fluctuation by physical test [19,20].
Nevertheless, this research lacks clear theoretical analysis and discussion of the results.

Although these researches are valuable, they are confined to classic analysis methods
or to relying on simulation works, causing unclear design analysis results and inefficient
guidance measures, and they are unable to realize the efficient design of the system. To solve
the design problem, this paper firstly adopts the modern control theory to analyze the
design theory of the system and proposes efficient guidance measures and design methods.
The novelty of the work includes:

1. Firstly, this paper adopts the linear incremental analysis method, which is based on
the state space theory, and this method successfully reveals the design theory of the
constant pressure difference regulating mechanism. The results clearly show that the
system has three elements: the controlled object, the stabilization controller, and the
servo and feed-forward compensator.

2. Secondly, the precise state space models and frequency domain models of the system are
established. On the basis concerning the advantage of frequency domain analysis methods,
the accurate influences of the design parameters on the dynamic performance and stability
of the system are analyzed, and the effective guidance measures are provided.

3. Finally, a frequency domain design method of the core parameters is proposed, which
includes the stabilization control gain and the servo control gain, and the method is
proven to solve the design work efficiently and accurately.

The structure of this paper is as follows. In Section 2, the design theory and the com-
opositions of the controller are provided, and the dynamic models are derived. In Section 3,
the influences of the control gains on the dynamic performance and stability are clarified.
In Section 4, the frequency domain design methods are provided. In Section 5, a design
example is established. In Section 6, the conclusions are presented.

2. Design Theory and Dynamic Equations

Generally, the fuel flow metering formula of the fuel metering system is expressed as
$Q = C_d A \sqrt{2\Delta P / \rho}$, and the metering principle is: ensure the pressure difference $\Delta P$ is a
designed value, then determine the required fuel flow by controlling the flow area $A$ of the
metering valve [13]. Independently, the flow area $A$ of the metering valve is controlled by a position control system, and, of course, the pressure difference $\Delta P$ is controlled by the constant pressure difference regulating mechanism. The structure of the constant pressure difference regulating mechanism is shown in Figure 1.

![Figure 1. Structure diagram of the constant pressure difference regulating mechanism.](image)

Where $P_S$ is the inlet pressure, $P_C$ is the controlled pressure, $P_Z$ is the regulating pressure, $P_O$ is the ejection pressure, $P_T$ is the return pressure, $A_f$ is the variable inlet flow area, $A_C$ is the variable compensated flow area, $A_Z$ is the variable regulating flow area, $A_1$ is the fixed inlet flow area, $A_2$ is the fixed ejection flow area, $A_3$ is the fixed outlet flow area, $V_C$ is the volume of the controlled chamber, $V_Z$ is the volume of the regulating chamber, $V_O$ is the volume of the ejection chamber, $x_y$ is the displacement of the compensated motion valve, and $x_z$ is the displacement of the regulating motion valve.

Illustratively, the controlled object is constructed by the fuel output path, which is represented by the blue arrows, and the fuel flow metering formula is expressed as $Q = C_d A_f \sqrt{2(P_S - P_C)/\rho}$. Generally, the regulation processes are that when the controlled pressure $P_C$ changes because of the disturbance of the inlet pressure $P_S$ or the flow area $A_f$, the compensated motion valve senses change of the pressure difference $(P_S - P_C)$ to regulate the compensated flow area $A_C$. Then, acting on the fuel regulating path, which is represented by the green arrows, the regulating pressure $P_Z$ changes, and the regulating motion valve senses change of the pressure difference $(P_Z - P_C)$ to regulate the regulating flow area $A_Z$, which is the control input of the controlled object, realizing the regulation of the controlled pressure $P_C$ [21–26].

2.1. Design Theory

The constant pressure difference regulating mechanism is a typical closed-loop servo tracking and disturbance rejection system. Its design objective is to ensure the controlled pressure increment $\Delta P_C$ servo tracks the inlet pressure increment $\Delta P_S$ and reject the disturbances caused by the variable inlet flow area increment $\Delta A_f$, and ensure that the controlled pressure increment difference $(\Delta P_S - \Delta P_C)$ is zero. Its design diagram is shown in Figure 2.

![Figure 2. Design block diagram of the constant pressure difference regulating mechanism.](image)
There are three basic regulation processes, as follows:

1. Firstly, $\Delta P_S$ and $\Delta A_I$ are the disturbance inputs, and the disturbances caused by them are rejected through the regulation function of the stabilization controller.
2. Secondly, $\Delta P_S$ is the reference input, and the controlled pressure increment $\Delta P_C$ servo tracks it through the regulation function of the servo compensator.
3. Thirdly, $\Delta P_S$ is the feed-forward input, and the regulation ability of the system is improved, and the steady error is reduced.

2.2. Composition of the Controllers

The composition of the stabilization controller and the servo and feed-forward compensator is shown in Figure 3, and the derivation processes are shown in the following subsections.

![Figure 3. Composition of the controllers.](image)

Where:

1. $G_{fp}$ is the dynamic matrix of the controlled object.
2. $G_{cv}$ is the dynamic matrix of the regulating motion valve, and $K_Z$ is the generalized stabilization control gain, and they construct the stabilization controller.
3. $G_{cv}$ is the dynamic matrix of the compensated motion valve, $K_C$ is the generalized servo control gain, and $C_{cfp}$ is the dynamic matrix of the feed-forward compensated flow path, and they construct the servo and feed-forward compensator.

2.2.1. Controlled Object

The pressure-flow nonlinear dynamic equations of the controlled chamber and the ejection chamber are:

$$\frac{dP_S}{dt} = \frac{B}{V_C} \left( C_{q}A_I + C_{q}A_1 \right) \sqrt{\frac{2(P_S - P_C)}{\rho}} - C_{q}A_Z \sqrt{\frac{2(P_C - P_O)}{\rho}} + A_q \Delta x_y - A_{2x} \Delta z} \tag{1}$$

$$\frac{dP_O}{dt} = \frac{B}{V_O} \left( C_{q}A_Z \sqrt{\frac{2(P_C - P_O)}{\rho}} - C_{q}A_Z \sqrt{\frac{2(P_C - P_O)}{\rho}} \right) \tag{2}$$

where $B$ is the oil bulk modulus, $\rho$ is the oil density, and $C_q$ is the flow coefficient.

The calculation equation of the flow coefficient is:

$$C_q = C_{q_{\text{max}}} \cdot \tanh\left( \sqrt{\frac{8|\Delta P|}{\rho}} \cdot \frac{\rho \cdot d_h}{Nu \cdot l_{\text{max}}} \right) \tag{3}$$

where $C_{q_{\text{max}}}$ is the maximum flow coefficient, $d_h$ is the hydraulic diameter, $Nu$ is the absolute viscosity, and $l_{\text{max}}$ is the critical flow number.

The pressure-flow linear dynamic differential equations can be described as:

$$\Delta \dot{P}_C = \frac{B}{V_C} \cdot \left(-(K_{p_f} + K_{p_Z}) \cdot \Delta P_C + K_{p_Z} \cdot \Delta P_O + K_{A_f} \cdot \Delta A_I + K_{p_f} \cdot \Delta P_S - K_{A_Z} \cdot \Delta A_Z + A_{2y} \cdot \Delta x_y - A_2x \cdot \Delta z} \tag{4}$$

$$\Delta \dot{P}_O = \frac{B}{V_O} \cdot \left(-(K_{p_Z} + K_{p_T}) \cdot \Delta P_O + K_{p_Z} \cdot \Delta P_C + K_{A_Z} \cdot \Delta A_Z \right) \tag{5}$$
where \( K_{AI} = C_{q1}\sqrt{\frac{2P_I-P_T}{\rho}} \), \( K_{AZ} = C_{q2}\sqrt{\frac{2P_I-P_T}{\rho}} \), \( K_{PZ} = C_{q2}A_Z\sqrt{\frac{1}{2P_I-P_T-P_J}} \), \( K_{PT} = C_{q0}A_2\sqrt{\frac{1}{2P_I-P_T-P_J}} \), \( K_{Pf} = C_{q0}A_1\sqrt{\frac{1}{2P_I-P_T-P_J}} \),

Then, the state space model of the controlled object is:

\[
\dot{x} = Ax + Bu + Eu
\]
\[
y = Cx + Du
\]

where \( x = [\Delta P_C \ \Delta P_0]^T \), \( u = [\Delta A_Z \ \Delta x_y \ \Delta x_z]^T \), \( w = [\Delta A_I \ \Delta P_Z]^T \), \( y = [\Delta P_C] \).

\[
A = \begin{bmatrix}
-\frac{B}{\rho C} (K_{Pf} + K_{PZ}) & \frac{B}{\rho C} K_{PZ} \\
\frac{B}{\rho C} K_{AZ} & -\frac{B}{\rho C} (K_{PZ} + K_{PT})
\end{bmatrix}
\]

\[
B = \begin{bmatrix}
\frac{B}{\rho C} K_{AZ} & \frac{B}{\rho C} A_y & -\frac{B}{\rho C} A_{xx} \\
0 & 0 & 0
\end{bmatrix},
C = [1 \ 0], D = [0 \ 0 \ 0]
\]

2.2.2. Stabilization Controller

A local coordinate for the regulating motion valve is established by taking the steady-state working point \( x_0 = 0 \) as the local coordinate origin, \( x_z \) as the relative displacement, and the arrow direction as the positive direction.

The motion nonlinear dynamic equation of the regulating motion valve is:

\[
A_{2x} \Delta P_C - A_{2x} \Delta P_Z - M_z \ddot{x}_z - K_{f2} \dot{x}_z - K_2 x_z - F_{I2} = 0
\]

where \( M_z \) is the mass, \( K_{f2} \) is the viscous friction coefficient, \( K_2 \) is the spring stiffness, \( A_{2x} \) is the pressure bearing area, and \( F_{I2} \) is the initial spring force.

The motion linear dynamic differential equation can be described as:

\[
A_{2x} (\Delta P_C - \Delta P_Z) - M_z \Delta \ddot{x}_z - K_{f2} \Delta \dot{x}_z - K_2 \Delta x_z = 0
\]

A function \( A_Z = f_Z(x_{uz}) \) is used to express the geometry design of the regulating motion valve orifice. According to its linearized gain characteristic \( \Delta A_Z = \frac{df_Z}{dx_{uz}} \Delta x_{uz} \) and \( \Delta x_{uz} = \Delta x_z \), the gain control law of the stabilization controller is indirectly expressed as:

\[
\Delta A_Z = K_Z \Delta x_z
\]

where \( K_Z = \frac{df_Z}{dx_{uz}} \), and it is the generalized stabilization control gain.

Then, the state space model of the stabilization controller is:

\[
\begin{align*}
\dot{x}_{s1} &= A_{s1} x_{s1} + B_{s1} y + E_{s1} r_{s1} \\
y_{s1} &= C_{s1} x_{s1} + D_{s1} y + H_{s1} r_{s1}
\end{align*}
\]

where \( x_{s1} = [\Delta x_z \ \Delta \dot{x}_z]^T, r_{s1} = [\Delta P_Z], y_{s1} = [\Delta A_Z] \),

\[
A_{s1} = \begin{bmatrix}
0 & K_{f2} \\
-\frac{K_2}{M_z} & -\frac{K_{f2}}{M_z}
\end{bmatrix}, B_{s1} = \begin{bmatrix}
\frac{A_{xx}}{M_z} \\
-\frac{A_{xx}}{M_z}
\end{bmatrix}, E_{s1} = \begin{bmatrix}
0 \\
-\frac{A_{xx}}{M_z}
\end{bmatrix}, C_{s1} = [ K_Z \ 0 ], D_{s1} = [0], H_{s1} = [0]
\]

2.2.3. Servo and Feed-Forward Compensator

A local coordinate for the compensated motion valve is established by taking the steady-state working point \( x_{s0} = 0 \) as the local coordinate origin, \( x_y \) as the relative displacement, and the arrow direction as the positive direction.
The motion nonlinear dynamic equation of the compensated motion valve is:

\[ A_y P_5 - A_y P_C - M_y \ddot{x}_y - K_{f1} \dot{x}_y - K_1 x_y - F_{L1} = 0 \] (13)

where \( M_y \) is the mass of the motion valve, \( K_{f1} \) is the viscous friction coefficient, \( K_1 \) is the spring stiffness, \( A_y \) is the pressure bearing area, and \( F_{L1} \) is the initial spring force.

The motion linear dynamic differential equation can be described as:

\[ A_y \cdot (\Delta P_5 - \Delta P_C) - M_y \cdot \ddot{x}_y - K_{f1} \cdot \dot{x}_y - K_1 \cdot \Delta x_y = 0 \] (14)

A function \( A_C = f_C(x_{uy}) \) is used to express the geometry design of the compensated motion valve orifice. According to its linearized gain characteristics \( \Delta A_C = \frac{df_C}{dx_{uy}} \cdot \Delta x_{uy} \) and \( \Delta x_{uy} = \Delta x_y \), the gain control law of the servo compensator is indirectly expressed as:

\[ \Delta A_C = K_C \cdot \Delta x_y \] (15)

where \( K_C = \frac{df_C}{dx_{uy}} \), and it is the generalized servo control gain.

The pressure-flow nonlinear dynamic equation of the regulating chamber is:

\[ \frac{dP_Z}{dt} = \frac{B}{V_Z} \left( C_{qc} A_C \sqrt{\frac{2(P_z - P_Z)}{\rho}} - C_{q3} A_3 \sqrt{\frac{2(P_z - P_T)}{\rho}} + A_Z \dot{x}_z \right) \] (16)

The pressure-flow linear dynamic differential equation can be described as:

\[ \Delta \dot{P}_Z = \frac{B}{V_Z} \left( -(K_{PY} + K_{PT2}) \cdot \Delta P_Z + K_{AY} \cdot \Delta A_C + K_{PY} \cdot \Delta P_5 + A_{ZX} \cdot \Delta \dot{x}_z \right) \] (17)

where \( K_{AY} = C_{qc} \sqrt{\frac{2(P_z - P_T)}{\rho}}, K_{PY} = C_{qc} A_C \sqrt{\frac{1}{2P_z - P_T}}, K_{PT2} = C_{q3} A_3 \sqrt{\frac{1}{2P_z - P_T}} \).

Then, the state space model of the servo and feed-forward compensator is:

\[ \begin{align*}
\dot{x}_{s2} &= A_{s2} x_{s2} + B_{s2} u_{s2} + E_{s2} r \\
y_{s2} &= C_{s2} x_{s2} + D_{s2} u_{s2} + H_{s2} r
\end{align*} \] (18)

where \( x_{s2} = [\Delta x_y \quad \Delta \dot{x}_y \quad \Delta P_Z]^T, u_{s2} = [\Delta P_C \quad \Delta \dot{x}_z]^T, r = [\Delta P_5], y_{s2} = [\Delta P_Z], \)

\[ A_{s2} = \begin{bmatrix}
0 & 1 & 0 \\
-\frac{B}{V_Z} K_{AY} - K_C & -\frac{B}{V_Z} K_{f1} & 0 \\
-\frac{B}{V_Z} & 0 & -\frac{B}{V_Z} (K_{PY} + K_{PT2})
\end{bmatrix}, B_{s2} = \begin{bmatrix}
\frac{A_y}{M_y} & 0 \\
-\frac{A_y}{M_y} & 0 \\
0 & \frac{B}{V_Z} A_{ZX}
\end{bmatrix}, E_{s2} = \begin{bmatrix}
0 \\
\frac{A_y}{M_y} \\
\frac{B}{V_Z} K_{PY}
\end{bmatrix} \]

3. Frequency Domain Analysis

Concerning the advantage of the explicit description between the designed parameters and the performance when analyzed by transfer functions, the frequency domain analysis methods are used to analyze the quantitative influence of the designed parameters on the performance of the system [27,28]. The design block diagram of the closed-loop controlled loop is shown in Figure 4.

### References

[27,28]
3.1. Design Analysis of the Stabilization Controller

The stabilization controller is used to ensure the asymptotic stability and disturbance rejection performance, and it acts on the inner loop. The analysis processes are as follows.

3.1.1. Calculation of the Open-Loop Transfer Function

The augmented open-loop state space model of the inner loop is:

\[
\begin{align*}
\dot{x}_a &= A_a x_a + B_a u_a + E_a w \\
y &= C_a x_a + D_a u_a
\end{align*}
\]

where \( x_a = [\Delta x_z \quad \Delta \dot{x}_z \quad \Delta P_C \quad \Delta P_O]^T \), \( u_a = [e_u \quad \Delta \dot{x}_y]^T \),

\[
A_a = \begin{bmatrix}
0 & -\frac{K_s}{M_s} & -\frac{K_f}{M_f} & 0 \\
-\frac{B}{V_c} & 0 & 0 & 0 \\
-\frac{B}{V_0} & 0 & 0 & 0 \\
-\frac{B}{V_0} & 0 & 0 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}, \\
B_a = \begin{bmatrix}
\Delta A_z \\
\Delta A_y
\end{bmatrix}, \\
E_a = \begin{bmatrix}
0 & 0 \\
0 & 0 \\
0 & 0
\end{bmatrix}, \\
C_a = [0 \quad 0 \quad 1 \quad 0], D_a = [0 \quad 0]
\]

Defining

\[
B_{a1} = \begin{bmatrix}
0 & -\frac{B}{M_s} & 0 & 0
\end{bmatrix}^T
\]

The open-loop transfer function of the inner loop is:

\[
L_a(s) = C_a (sI - A_a)^{-1} B_{a1}
\]

\[
= \frac{K_s \Delta A_z}{\left(s + \frac{B}{K_F} (K_P + K_Z)\right) \left(s + \frac{B}{K_P} \left(K_P + K_T\right)\right) - \frac{B}{V_0} K_Z \frac{\Delta A_y}{\left(s + \frac{B}{K_P} \left(K_P + K_T\right)\right)}}
\]

3.1.2. Calculation of the Corner Frequencies

The performance of the system is determined by the corner frequencies of the frequency domain curve, and analyzing the accurate influences of the design parameters on the corner frequencies is the core idea. According to Equation (23), the corner frequencies of the frequency domain curve are calculated as follows.
1. If \( K_2 > \frac{K_f^2}{2M_z} \), the second-order section is an underdamped oscillation link, and its poles are:

\[
p_{1,2} = -\frac{K_f}{2M_z} \pm j\sqrt{\frac{K_2}{M_z} - \left(\frac{K_f}{2M_z}\right)^2}
\]  

(24)

The corner frequencies are defined as:

\[
\omega_{1,2} = \frac{K_f}{2M_z}
\]  

(25)

2. If \( K_2 < \frac{K_f^2}{2M_z} \), the second-order section is an overdamped nonoscillation link, and its poles are:

\[
p_{1,2} = -\frac{K_f}{2M_z} \pm \sqrt{\left(\frac{K_f}{2M_z}\right)^2 - \frac{K_2}{M_z}}
\]  

(26)

The corner frequencies are defined as:

\[
\omega_{1,2} = \frac{K_f}{2M_z} \pm \sqrt{\left(\frac{K_f}{2M_z}\right)^2 - \frac{K_2}{M_z}}
\]  

(27)

The third and fourth poles are:

\[
p_{3,4} = -\left(\frac{B_{VC}}{V_C} (K_{PJ} + K_{PZ}) + \frac{B_{VO}}{V_O} (K_{PZ} + K_{PT})\right)
\]

\[\pm \sqrt{\left(\frac{B_{VC}}{V_C} (K_{PJ} + K_{PZ}) + \frac{B_{VO}}{V_O} (K_{PZ} + K_{PT})\right)^2 - \frac{B_{VC}}{V_C} \cdot \frac{B_{VO}}{V_O} (K_{PJ} \cdot K_{PZ} + K_{PJ} \cdot K_{PT} + K_{PZ} \cdot K_{PT})}
\]  

(28)

The corner frequencies are defined as:

\[
\omega_{3,4} = \frac{B_{VC}}{V_C} \cdot \frac{B_{VO}}{V_O} \cdot \frac{K_{PJ} \cdot K_{PZ} + K_{PJ} \cdot K_{PT} + K_{PZ} \cdot K_{PT}}{K_{PJ} + K_{PZ}}
\]

\[\pm \sqrt{\left(\frac{B_{VC}}{V_C} (K_{PJ} + K_{PZ}) + \frac{B_{VO}}{V_O} (K_{PZ} + K_{PT})\right)^2 - \frac{B_{VC}}{V_C} \cdot \frac{B_{VO}}{V_O} (K_{PJ} \cdot K_{PZ} + K_{PJ} \cdot K_{PT} + K_{PZ} \cdot K_{PT})}
\]  

(29)

The zero is:

\[
z_1 = -\frac{B}{V_O} \cdot K_{PT}
\]  

(30)

The corner frequency is defined as:

\[
\omega_5 = \frac{B}{V_O} \cdot K_{PT}
\]  

(31)

The open-loop gain of the inner loop is:

\[
K_a = \frac{K_{Z} \cdot A_{LZ} \cdot K_{AZ} \cdot K_{PT}}{[\{K_{PJ} + K_{PZ}\} \cdot (K_{PZ} + K_{PT}) - K_{PZ} \cdot K_{PZ}] \cdot \left(\frac{K_f}{2M_z}\right)}
\]  

(32)

The equations of the corner frequencies indicate that:

1. The stabilization control gain \( K_Z \) only affects the open-loop gain;
2. The spring stiffness \( K_2 \) affects \( \omega_1 \), \( \omega_2 \), and the open-loop gain,
3. The volume \( V_C \) of the controlled chamber and the volume \( V_O \) of the ejection chamber affect \( \omega_3 \), \( \omega_4 \), and \( \omega_5 \).
Designing the five corner frequencies and the open-loop gain is the key work. For example, to enhance the response performance and the disturbance rejection performance, the open-loop gain $K_a$ should be increased to make the magnitude curve move up, including:
1. Increasing the stabilization control gain $K_Z$;
2. Reducing the spring stiffness $K_Z$;
3. Increasing the pressure-bearing area $A_{ax}$.

### 3.1.3. Influence of the Stabilization Control Gain on the Frequency Domain Performance

Assuming $K_2 < \frac{K_2^2}{4M_0}$ and only changing $K_Z$, by calculating the magnitude of the parameters $\omega_1, \omega_2, \omega_3, \omega_4, \omega_5$, and $K_Z$, the bode diagram curves are shown in Figure 5.

![Figure 5. Bode diagram curves under different stabilization control gains.](image)

When the stabilization control gain $K_Z$ increases, the magnitude curve moves up, and the phase curve remains unchanged, then:
1. The steady-state gain $K_a$ increases according to Equation (32), and the steady error caused by disturbances is reduced, bringing a better disturbance rejection performance;
2. The crossover frequency $\omega_c$ increases, and the settling time is reduced, bringing a faster response performance;
3. The slope of the magnitude curve at the crossover frequency increases, and the damping ratio decreases, bringing a bigger overshoot;
4. The phase margin decreases, bringing a worse robustness performance.

### 3.1.4. Stability Analysis

The characteristic polynomial of the inner closed-loop system is:

$$d(s) = \left( s + \frac{B}{V_c} (K_{Pj} + K_{PZ}) \right) \cdot \left( s + \frac{B}{V_O} (K_{PZ} + K_{PT}) \right) - \frac{B}{V_c} K_{PZ} \cdot \frac{B}{V_O} K_{PZ} \cdot \left( s^2 + \frac{K_Z}{M_c} s + \frac{K_Z}{M_c} \right)$$

$$+ \frac{d_{x2}}{A_{x2}} \cdot \frac{B}{V_c} K_{AZ} \cdot \left( s + \frac{B}{V_O} K_{PT} \right)$$

(33)

The equation is converted to:

$$d(s) = a_0 s^4 + a_1 s^3 + a_2 s^2 + a_3 s + a_4$$

(34)

According to the stability conditions of the fourth-order system: all the coefficients of the characteristic polynomial should be positive, expressed as $a_i > 0$, $i = 0, 1, 2, 3, 4$, and

$$a_1 a_2 - a_0 a_3 > 0$$

(35)

$$a_3 (a_1 a_2 - a_0 a_3) - a_4 a_1^2 > 0$$

(36)
There is a stabilization control gain extremum that makes the inner loop system asymptotically stable, expressed as $K_{Z\text{max}}$. Then, the stability condition of the inner loop is:

$$K_Z < K_{Z\text{max}} \quad (37)$$

### 3.2. Design Analysis of the Servo and Feed-Forward Compensator

The servo and feed-forward compensator is used to ensure the asymptotic tracking ability, and it acts on the series loop. The analysis processes are as follows.

#### 3.2.1. Calculation of the Open-Loop Transfer Function

Defining

$$B_{33} = \begin{bmatrix} 0 & \frac{A_y}{M_y} & 0 \end{bmatrix}^T \quad (38)$$

Then, the open-loop transfer function of the servo compensator from $e$ to $\Delta P_Z$ is:

$$K_S(s) = C_{s2} (sI - A_{s2})^{-1} B_{33} = \frac{K_C \cdot \frac{A_y}{M_y} \cdot B_{VZ} \cdot K_{AY}}{(s + \frac{B_{VZ}}{V_Z} (K_{PY} + K_{PT2})) \cdot \left( s^2 + \frac{K_{f1}}{M_y} s + \frac{K_1}{M_y}\right)} \quad (39)$$

#### 3.2.2. Calculation of the Corner Frequencies

According to Equation (39), the corner frequencies of the frequency domain curve are calculated as follows.

1. If $K > \frac{K_{f1}^2}{4M_y}$, the second-order section is an underdamped oscillation link, and its poles are:

   $$p_{1,2} = -\frac{K_{f1}}{2M_y} \pm j \sqrt{\frac{K_1}{M_y} - \left( \frac{K_{f1}}{2M_y} \right)^2} \quad (40)$$

   The corner frequencies are defined as:

   $$\omega_{1,2} = \frac{K_{f1}}{2M_y} \quad (41)$$

2. If $K < \frac{K_{f1}^2}{4M_y}$, the second-order section is an overdamped nonoscillation link, and its poles are:

   $$p_{1,2} = -\frac{K_{f1}}{2M_y} \pm \sqrt{\left( \frac{K_{f1}}{2M_y} \right)^2 - \frac{K_1}{M_y}} \quad (42)$$

   The corner frequencies are defined as:

   $$\omega_{1,2} = \frac{K_{f1}}{2M_y} \pm \sqrt{\left( \frac{K_{f1}}{2M_y} \right)^2 - \frac{K_1}{M_y}} \quad (43)$$

   The third pole is:

   $$p_3 = -\frac{B_{VZ}}{V_Z} (K_{PY} + K_{PT2}) \quad (44)$$

   The corner frequency is defined as:

   $$\omega_3 = \frac{B_{VZ}}{V_Z} (K_{PY} + K_{PT2}) \quad (45)$$
The open-loop gain of the servo compensator is:

\[ K_c = \frac{K_C \cdot \frac{A_y}{M_y} \cdot K_{AY}}{(K_{PY} + K_{PT2}) \cdot \frac{K}{M_y}} \]  

(46)

The equations of the corner frequencies indicate that:

1. The control gain \( K_C \) only affects the open-loop gain;
2. The spring stiffness \( K \) affects \( \omega_1 \), \( \omega_2 \), and the open-loop gain;
3. The volume \( V_Z \) of the regulating chamber affects \( \omega_3 \).

Designing the three corner frequencies and the open-loop gain is the key work. For example, to enhance the servo tracking performance of the system, the open-loop gain \( K_c \) should be increased to make the magnitude curve move up, or the steady state gain of the feed-forward compensator should be increased, including:

1. Increasing the control gain \( K_C \);
2. Reducing the spring stiffness \( K_1 \);
3. Increasing the pressure-bearing area \( A_y \);
4. Increasing the compensated flow area \( A_C \).

To enhance the stability margin of the system, the corner frequency \( \omega_1 \) and \( \omega_2 \) should be increased to make the phase curve move right, or the open-loop gain \( K_c \) should be decreased to make the magnitude curve move down. In addition to adopting the measures contrary to the above measures, also included:

1. Reducing the mass \( M_y \) of the compensated motion valve;
2. Increasing the viscous friction coefficient \( f_1 \) of the compensated motion valve.

3.2.3. Influence of the Servo Control Gain on the Frequency Domain Performance

Assuming \( K < \frac{K^2_{f1}}{M_y} \) and only changing \( K_C \), the bode diagram curves are shown in Figure 6.

![Figure 6. Bode diagram curves under different servo control gains.](image)

When the control gain \( K_C \) increases, the magnitude curve moves up, and the phase curve remains unchanged, then:

1. The open loop gain \( K_c \) increases according to Equation (46), and the steady error caused by reference input is reduced, bringing a better servo tracking performance;
2. The crossover frequency \( \omega_c \) increases, and the settling time is reduced, bringing a faster response performance;
3. The slope of the magnitude curve at the crossover frequency increases, and the damping ratio decreases, bringing a bigger overshoot;
4. The phase margin decreases, bringing a worse robustness performance.

3.3. Calculation of the Transfer Functions

It is inconvenient to calculate the output and error transfer functions by directly serializing the transfer functions obtained above, because there are coupling problems, and the simpler calculation method of the transfer functions are as follows.

The frequency domain loop design block diagram of the system is shown in Figure 7.

Figure 7. Frequency domain loop design block diagram of the system.

The open-loop state space model of the system is:

\[
\begin{align*}
\dot{x}_p &= A_p \cdot x_p + B_p \cdot u_p + E_p \cdot w \\
y &= C_p \cdot x_p + D_p \cdot u_p + H_p \cdot w
\end{align*}
\]

where \( x_p = [\Delta x_y \quad \Delta x_y \quad \Delta P_Z \quad \Delta x_z \quad \Delta x_z \quad \Delta P_C \quad \Delta P_o] ^T \), \( u_p = [e] \), and

\[
A_p = \begin{bmatrix}
0 & 1 & 0 & 0 & 0 & 0 & 0 \\
-\frac{K_1}{M_1} & -\frac{K_2}{M_2} & 0 & 0 & 0 & 0 & 0 \\
\frac{B}{V_c} K_{AY} - K_C & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\]

\[
B_p = \begin{bmatrix}
0 & \frac{A_y}{M_0} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{B}{V_c} K_{PY} & 0 & 0 & \frac{B}{V_c} K_{AJ} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix} ^T
\]

\[
C_p = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\]

\( D_p = [0] \), \( H_p = [0 \quad 0] \)

The open-loop transfer function of the system is:

\[
L(s) = C_p (sI - A_p)^{-1} B_p
\]

Defining

\[
E_{p1} = \begin{bmatrix}
0 & 0 & 0 & 0 & \frac{B}{V_c} K_{AJ} & 0
\end{bmatrix} ^T
\]

\[
E_{p2} = \begin{bmatrix}
0 & 0 & \frac{B}{V_c} K_{PY} & 0 & \frac{B}{V_c} K_{PJ} & 0
\end{bmatrix} ^T
\]

1. The open-loop transfer function from \( \Delta A_j \) to \( \Delta P_C \) is \( G_A(s) = C_p (sI - A_p)^{-1} E_{p1} \).
2. The open-loop transfer function from \( \Delta P_S \) to \( \Delta P_C \) is \( G_P(s) = C_p (sI - A_p)^{-1} E_{p2} \).

The sensitivity function and complementary sensitivity function of the system are defined as:

\[
S = (I + L)^{-1}, T = (I + L)^{-1} \cdot L
\]
Then, the control output transfer function of the system is:

\[ \Delta P_C = (T + S \cdot G_P) \cdot \Delta P_S + S \cdot G_A \cdot \Delta A_J \]  

(52)

When the control error is expressed as \( e = \Delta P_S - \Delta P_C \), then it has:

\[ e = (S - S \cdot G_P) \cdot \Delta P_S - S \cdot G_A \cdot \Delta A_J \]  

(53)

4. Frequency Domain Design Method

Within the working range of the inlet pressure \( P_S \) and the variable inlet flow area \( A_J \), the design specifications of the steady-state and dynamic performance are:

1. Steady-state performance: the designed pressure difference is \( P_e \), and the phase margin is more than \( N^\circ \).
2. Dynamic performance: the regulating time is not more than \( t_s \), and the overshoot is not more than \( \sigma \).

4.1. Calculation of the Control Gains

The system can be regarded as a fixed system with multiple stable working points which are determined by the inlet pressure \( P_S \) and the variable inlet flow area \( A_J \), and each working point has a fixed control gain. Therefore, the design method of gain scheduling is appropriate. The core idea is to determine the steady-state parameters first, and then design the control gain. The design processes are as follows.

4.1.1. Calculation of the Stabilization Control Gain

The steady-state flow balance equation of the controlled object is described as:

\[
(C_{q_j} A_J + C_{q_1} A_1) V\sqrt{\frac{2(P_S - P_C)}{\rho}} = C_{q_2} A_Z V\sqrt{\frac{2(P_C - P_O)}{\rho}} = C_{q_3} A_2 V\sqrt{\frac{2(P_O - P_T)}{\rho}}
\]  

(54)

Assume the inlet pressure at a working point is \( P_{S,i} \) and the inlet flow area is \( A_{J,i} \). Since the design value of the pressure difference is \( P_e \), there are:

\[ P_{C,i} = P_{S,i} - P_e \]  

(55)

\[ P_{O,i} = \left( \frac{C_{q_j} A_{J,i} + C_{q_1} A_1}{C_{q_j} A_2} \right)^2 (P_{S,i} - P_{C,i}) + P_T \]  

(56)

\[ A_{Z,i} = \left( \frac{C_{q_j} A_{J,i} + C_{q_1} A_1}{C_{q_2} A_3} \right) \sqrt{\frac{(P_{S,i} - P_{C,i})}{(P_{C,i} - P_{O,i})}} \]  

(57)

Then, the parameter values \( K_{A_{J,i}}, K_{P_{J,i}}, K_{A_{Z,i}}, K_{P_{Z,i}}, \) and \( K_{P_{T,i}} \) are obtained, and the open-loop transfer function of the inner loop system \( L_{a,i}(s) \) is obtained.

According to Equation (37), there is a stabilization control gain extremum expressed as \( K_{Z_{max,i}} \), and it has:

\[ K_{Z,i} < K_{Z_{max,i}} \]  

(58)

Subsequently, an appropriate stabilization control gain value \( K_{Z,i} \) can be selected.

4.1.2. Calculation of the Servo Control Gain

The steady-state flow balance equation of the compensated flow path is described as:

\[
C_{q_C} A_C V\sqrt{\frac{2(P_S - P_Z)}{\rho}} = C_{q_f} A_3 V\sqrt{\frac{2(P_Z - P_T)}{\rho}}
\]  

(59)
Then, according to the steady-state balance condition of the regulating motion valve:

\[ 0 = A_{zx}P_{C,i} - A_{zx}P_{Z,i} - K_2x_{szd,i} \]  

(60)

There are:

\[ P_{Z,i} = P_{C,i} + \frac{K_2}{A_{zx}}x_{szd,i} \]  

(61)

\[ A_{C,i} = \frac{C_{q3,i}A_3}{C_{q,i}} \sqrt{\frac{(P_{Z,i} - P_T)}{(P_{S,i} - P_{Z,i})}} \]  

(62)

Then, the parameter values \( K_{AY,i} \), \( K_{PY,i} \), and \( K_{PT2,i} \) are obtained, and the transfer functions of the system \( L_i(s) \), \( e_i(s) \) are obtained. Additionally:

1. There is a servo control gain extremum that makes the system asymptotically stable, expressed as \( K_{Cmax,i} \), then it has:

\[ K_{C,i} < K_{Cmax,i} \]  

(63)

2. There is a minimum control gain value that makes the system within the phase margin constraint, expressed as \( K_{PH,i} \), then it has:

\[ K_{C,i} < K_{PH,i} \]  

(64)

3. There is a control-gain working range that makes the system within the dynamic performance constraint expressed as \( [K_{C,i}, K_{Ch,i}] \), then it has:

\[ K_{Cl,i} < K_{C,i} < K_{Ch,i} \]  

(65)

Subsequently, an appropriate servo control gain value \( K_{C,j} \) can be selected.

4.2. Geometry Design of the Orifices

4.2.1. The Regulating Motion Valve Orifice

The first spring compression of the regulating motion valve is designed as \( x_{szd,1} \), and the designed relationship between the orifice underlap increment \( \Delta x_{uz,i} \) and the area increment \( (A_{Z,i} - A_{Z,i-1}) \) is provided as:

\[ \Delta x_{uz,i} = \frac{(A_{Z,i} - A_{Z,i-1})}{K_{Z,i-1}}, \quad i = 2, 3, \ldots, N \]  

(66)

Then, the steady-state spring compression at the other working point is:

\[ x_{szd,i} = x_{szd,i-1} + \Delta x_{uz,i} \]  

(67)

where \( N \) is number of the selected steady-state working points.

4.2.2. The Compensated Motion Valve Orifice

The above design processes are executed; then, the value pair \([A_{C,i}, K_{C,i}]\) can be obtained. The designed relationship between the orifice underlap increment \( \Delta x_{uy,i} \) and the area increment \( (A_{C,i} - A_{C,i-1}) \) is provided as:

\[ \Delta x_{uy,i} = \frac{(A_{C,i} - A_{C,i-1})}{K_{C,i-1}}, \quad i = 2, 3, \ldots, N \]  

(68)
4.3. Parameters Design of the Motion Valves

4.3.1. The Regulating Motion Valve
Assuming the initial spring compression is $x_{szd,0}$, if the initial orifice underlap of the regulating motion valve is $x_{uz,0}$, and the orifice underlap at the first working point is $x_{uz,1}$. Since $\Delta x_{uz} = \Delta x_{szd}$, then it has:

$$x_{szd,0} = x_{szd,1} - (x_{uz,1} - x_{uz,0})$$  \hfill (69)

4.3.2. The Compensated Motion Valve
The steady-state balance condition of the compensated motion valve is expressed as:

$$0 = A_y P_{S,i} - A_y P_{C,i} - K_{scd,i}$$  \hfill (70)

Then, the steady-state spring compression at the first working point is:

$$x_{scd,1} = \frac{A_y}{K} P_e$$  \hfill (71)

Assuming the initial spring compression is $x_{scd,0}$, if the initial orifice underlap of the compensated motion valve is $x_{uy,0}$, and the orifice underlap at the first working point is $x_{uy,1}$. Since $\Delta x_{uy} = \Delta x_{scd}$, then it has:

$$x_{scd,0} = x_{scd,1} - (x_{uy,1} - x_{uy,0})$$  \hfill (72)

5. Design Example
The known structural parameters of the constant pressure difference regulating mechanism are shown in Table 1.

<table>
<thead>
<tr>
<th>Parameter/Unit</th>
<th>Value</th>
<th>Parameter/Unit</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_y$/Kg</td>
<td>0.08</td>
<td>$M_z$/Kg</td>
<td>0.05</td>
</tr>
<tr>
<td>$K_1$/N/(m)</td>
<td>40,000</td>
<td>$K_2$/N/(m)</td>
<td>15,000</td>
</tr>
<tr>
<td>$K_f$/N/(m/s)</td>
<td>200</td>
<td>$K_f$/N/(m/s)</td>
<td>200</td>
</tr>
<tr>
<td>$d_y$/m</td>
<td>0.036</td>
<td>$d_z$/m</td>
<td>0.036</td>
</tr>
<tr>
<td>$A_1$/m$^2$</td>
<td>2.827433 $\times 10^{-7}$</td>
<td>$V_C$/m$^3$</td>
<td>$2 \times 10^{-6}$</td>
</tr>
<tr>
<td>$A_2$/m$^2$</td>
<td>1.900664 $\times 10^{-4}$</td>
<td>$V_Z$/m$^3$</td>
<td>$2 \times 10^{-6}$</td>
</tr>
<tr>
<td>$A_3$/m$^2$</td>
<td>6.2831852 $\times 10^{-6}$</td>
<td>$V_O$/m$^3$</td>
<td>$4.908739 \times 10^{-4}$</td>
</tr>
<tr>
<td>$\rho$/(Kg/m$^3$)</td>
<td>780</td>
<td>$B$/bar</td>
<td>17,000</td>
</tr>
<tr>
<td>$Nu$/Pas</td>
<td>0.051</td>
<td>$\nu_{mc}$</td>
<td>1000</td>
</tr>
<tr>
<td>$C_{jmax}$</td>
<td>0.7</td>
<td>$P_T$/bar</td>
<td>2</td>
</tr>
</tbody>
</table>

The working range of the inputs are:
1. The working range of the inlet pressure $P_S$ is [3, 9] MPa;
2. The working range of the variable inlet flow area $A_J$ is $[10, 240] \times 10^{-6}$ m$^2$.

The design objects are:
1. The designed pressure difference $P_e$ is $0.92 \pm 0.01$ MPa, and the phase margin is more than $70^\circ$;
2. The regulating time $t_s$ is not more than 0.01 s, and the overshoot is not more than 10%.

The design tasks are:
1. Design the stabilization control law $A_Z = f_Z(x_{uz})$;
2. Design the servo control law $A_C = f_C(x_{uy})$.  

Table 1. Structural parameters of the constant pressure difference regulating mechanism.
5.1. Dynamic Design of the First Working Point

5.1.1. Calculation of the Stabilization Control Gain

The inlet pressure at the first design point $P_{S,1}$ is 9 MPa, and the inlet flow area $A_{J,1}$ is $10 \times 10^{-6}$ m$^2$. According to Equations (55)–(57), there are:

$$P_{C,1} = P_{S,1} - \Delta P_e = 8.08 \text{ MPa} \quad (73)$$

$$P_{O,1} = (\frac{C_{qj,1}A_{j,1} + C_{q1,1}A_1}{C_{q0,1}A_2})^2 \cdot (P_{S,1} - P_{C,1}) + P_T = 0.20265028 \text{ MPa} \quad (74)$$

$$A_{Z,1} = (\frac{C_{qj,1}A_{j,1} + C_{q1,1}A_1}{C_{qz,1}}) \sqrt{\frac{(P_{S,1} - P_{C,1})}{(P_{C,1} - P_{O,1})}} = 3.4862689 \cdot 10^{-6} \text{ m}^2 \quad (75)$$

Then, the values of the parameters $K_{AJ,1}, K_{PJ,1}, K_{AZ,1}, K_{PZ,1}$, and $K_{PT,1}$ are obtained.

According to Equation (58), it has:

$$K_{Z,1} < K_{Z_{max},1} = 0.07609 \quad (76)$$

5.1.2. Calculation of the Servo Control Gain

The steady-state spring compression of the regulating motion valve at the first working point $x_{szd,1}$ is designed as 10 mm; then, according to Equation (61), it has:

$$P_{Z,1} = P_{C,1} + \frac{K_2}{A_{sx}} \cdot x_{szd,1} = 8.2273657 \text{ MPa} \quad (77)$$

Then, the parameter values $K_{AY,1}$, $K_{PY,1}$, and $K_{PT,2,1}$ are obtained, and the transfer functions of the system are obtained.

When selecting different $K_{Z,1}$, according to Equations (63)–(65), the values of the parameters $K_{C_{max,1}}, K_{PH,1}$, and $[K_{Cl,1}, K_{Ch,1}]$ can be obtained, as shown in Table 2.

<table>
<thead>
<tr>
<th>$K_{Z,1}$</th>
<th>$K_{C_{max,1}}$</th>
<th>$[K_{Cl,1}, K_{Ch,1}]$</th>
<th>$K_{PH,1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0005</td>
<td>4.61</td>
<td>(0.06, 3.30)</td>
<td>0.49</td>
</tr>
<tr>
<td>0.001</td>
<td>4.60</td>
<td>(0.03, 3.20)</td>
<td>0.49</td>
</tr>
<tr>
<td>0.005</td>
<td>4.56</td>
<td>(0.02, 3.20)</td>
<td>0.50</td>
</tr>
<tr>
<td>0.010</td>
<td>4.50</td>
<td>(0.02, 3.20)</td>
<td>0.50</td>
</tr>
<tr>
<td>0.015</td>
<td>4.44</td>
<td>(0.02, 3.20)</td>
<td>0.50</td>
</tr>
<tr>
<td>0.020</td>
<td>4.38</td>
<td>(0.01, 3.20)</td>
<td>0.50</td>
</tr>
</tbody>
</table>

Finally, the stabilization control gain $K_{Z,1}$ is designed as 0.01, and the servo control gain $K_{C,1}$ is designed as 0.05.

5.2. Dynamic Design of Other Working Points

Here, $4 \times 5$ steady-state working points are selected, as shown in Table 3.

<table>
<thead>
<tr>
<th>$P_S$ MPa</th>
<th>$A_J$ mm$^2$</th>
<th>$A_Z$ mm$^2$</th>
<th>$K_{Z_{max}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>10</td>
<td>7.1413178</td>
<td>0.276</td>
</tr>
<tr>
<td></td>
<td>30</td>
<td>21.258905</td>
<td>2.560</td>
</tr>
<tr>
<td></td>
<td>80</td>
<td>58.720918</td>
<td>26.510</td>
</tr>
<tr>
<td></td>
<td>160</td>
<td>138.75330</td>
<td>239.530</td>
</tr>
<tr>
<td></td>
<td>240</td>
<td>359.53286</td>
<td>5574.50</td>
</tr>
</tbody>
</table>
Table 3. Cont.

<table>
<thead>
<tr>
<th>$P_S$ MPa</th>
<th>$A_J$ mm$^2$</th>
<th>$A_Z$ mm$^2$</th>
<th>$K_{Z_{\max}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>4.9691655</td>
<td>0.1328</td>
<td></td>
</tr>
<tr>
<td>30</td>
<td>14.750547</td>
<td>1.188</td>
<td></td>
</tr>
<tr>
<td>80</td>
<td>39.904931</td>
<td>10.328</td>
<td></td>
</tr>
<tr>
<td>160</td>
<td>85.546194</td>
<td>59.495</td>
<td></td>
</tr>
<tr>
<td>240</td>
<td>148.38949</td>
<td>235.570</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>4.0360839</td>
<td>0.0944</td>
<td></td>
</tr>
<tr>
<td>30</td>
<td>11.969912</td>
<td>0.834</td>
<td></td>
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<td>80</td>
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<td>160</td>
<td>67.213819</td>
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</tr>
<tr>
<td>240</td>
<td>109.70300</td>
<td>107.490</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>3.4862689</td>
<td>0.07609</td>
<td></td>
</tr>
<tr>
<td>30</td>
<td>10.334703</td>
<td>0.668</td>
<td></td>
</tr>
<tr>
<td>80</td>
<td>27.693258</td>
<td>5.255</td>
<td></td>
</tr>
<tr>
<td>160</td>
<td>57.160884</td>
<td>25.153</td>
<td></td>
</tr>
<tr>
<td>240</td>
<td>90.995231</td>
<td>71.128</td>
<td></td>
</tr>
</tbody>
</table>

1. The stabilization control gain extrema that make the inner loop system stable are shown in Table 3.
2. The design processes in Section 4.1 are executed; then, the value pair $[A_{Z,J}, K_{Z,J}]$ can be obtained, as shown in Table 4.

Table 4. The value pair of the stabilization controller at every working point.

<table>
<thead>
<tr>
<th>No.</th>
<th>$A_Z$ mm$^2$</th>
<th>$K_Z$</th>
<th>$\Delta x_{uc}$ mm</th>
<th>$A_C$ mm$^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3.4862689</td>
<td>0.0100</td>
<td>0</td>
<td>20.252534</td>
</tr>
<tr>
<td>2</td>
<td>4.0360839</td>
<td>0.0115</td>
<td>0.054981500</td>
<td>17.559553</td>
</tr>
<tr>
<td>3</td>
<td>4.9691655</td>
<td>0.0130</td>
<td>0.136119030</td>
<td>14.367214</td>
</tr>
<tr>
<td>4</td>
<td>7.1413178</td>
<td>0.0145</td>
<td>0.303207669</td>
<td>10.215733</td>
</tr>
<tr>
<td>5</td>
<td>10.334703</td>
<td>0.0130</td>
<td>0.523441131</td>
<td>20.364172</td>
</tr>
<tr>
<td>6</td>
<td>11.969912</td>
<td>0.0120</td>
<td>0.649226439</td>
<td>17.672847</td>
</tr>
<tr>
<td>7</td>
<td>14.750547</td>
<td>0.0130</td>
<td>0.880946022</td>
<td>14.490336</td>
</tr>
<tr>
<td>8</td>
<td>21.258905</td>
<td>0.0140</td>
<td>1.381588945</td>
<td>10.363465</td>
</tr>
<tr>
<td>9</td>
<td>27.693258</td>
<td>0.0130</td>
<td>1.841185588</td>
<td>20.652604</td>
</tr>
<tr>
<td>10</td>
<td>32.175269</td>
<td>0.0135</td>
<td>2.185955665</td>
<td>17.974097</td>
</tr>
<tr>
<td>11</td>
<td>39.904931</td>
<td>0.0125</td>
<td>2.758252220</td>
<td>14.812128</td>
</tr>
<tr>
<td>12</td>
<td>57.160884</td>
<td>0.0110</td>
<td>4.138999460</td>
<td>21.182637</td>
</tr>
<tr>
<td>13</td>
<td>58.720918</td>
<td>0.0125</td>
<td>4.280820733</td>
<td>10.781931</td>
</tr>
<tr>
<td>14</td>
<td>67.213819</td>
<td>0.0130</td>
<td>4.960252813</td>
<td>18.554773</td>
</tr>
<tr>
<td>15</td>
<td>85.546194</td>
<td>0.0135</td>
<td>6.370435505</td>
<td>15.482255</td>
</tr>
<tr>
<td>16</td>
<td>90.995231</td>
<td>0.0140</td>
<td>6.774067876</td>
<td>21.837585</td>
</tr>
<tr>
<td>17</td>
<td>109.70300</td>
<td>0.0150</td>
<td>8.110337090</td>
<td>19.275786</td>
</tr>
<tr>
<td>18</td>
<td>138.75330</td>
<td>0.0175</td>
<td>10.04702376</td>
<td>11.723734</td>
</tr>
<tr>
<td>19</td>
<td>148.38949</td>
<td>0.0190</td>
<td>10.59766319</td>
<td>16.368022</td>
</tr>
<tr>
<td>20</td>
<td>359.53286</td>
<td>0.0200</td>
<td>21.71047213</td>
<td>14.305249</td>
</tr>
</tbody>
</table>

3. The design processes in Section 4.1 are executed; then, the value pair $[A_{C,J}, K_{C,J}]$ can be obtained, as shown in Table 5.

Table 5. The value pair of the servo controller at every working point.

<table>
<thead>
<tr>
<th>$P_S$ MPa</th>
<th>$A_J$ mm$^2$</th>
<th>$A_C$ mm$^2$</th>
<th>$K_{C_{\max}}$</th>
<th>$K_{CI,K_{Ch}}$</th>
<th>$K_{PH}$</th>
<th>$K_C$</th>
<th>Margin/°</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>10.215733</td>
<td>3.79</td>
<td>(0.02, 1.50)</td>
<td>0.41</td>
<td>0.05</td>
<td>96.6</td>
<td></td>
</tr>
<tr>
<td>80</td>
<td>10.781931</td>
<td>3.74</td>
<td>(0.02, 0.55)</td>
<td>0.17</td>
<td>0.05</td>
<td>102.0</td>
<td></td>
</tr>
<tr>
<td>160</td>
<td>11.723734</td>
<td>3.56</td>
<td>(0.03, 0.45)</td>
<td>0.15</td>
<td>0.05</td>
<td>107.0</td>
<td></td>
</tr>
<tr>
<td>240</td>
<td>14.305249</td>
<td>2.74</td>
<td>(0.04, 0.50)</td>
<td>0.20</td>
<td>0.05</td>
<td>117</td>
<td></td>
</tr>
</tbody>
</table>
Table 5. Cont.

<table>
<thead>
<tr>
<th>( P_s ) MPa</th>
<th>( A_f ) mm(^2)</th>
<th>( A_C ) mm(^2)</th>
<th>( K_{\text{Cmax}} )</th>
<th>([K_{\text{CI}}, K_{\text{CH}}])</th>
<th>( K_{\text{PH}} )</th>
<th>( K_{\text{C}} )</th>
<th>Margin/( ^\circ )</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>14.367214</td>
<td>4.06</td>
<td>(0.02, 2.10)</td>
<td>0.46</td>
<td>0.05</td>
<td>101.0</td>
<td></td>
</tr>
<tr>
<td>30</td>
<td>14.490336</td>
<td>3.40</td>
<td>(0.02, 1.55)</td>
<td>0.27</td>
<td>0.05</td>
<td>98.3</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>80</td>
<td>14.812128</td>
<td>3.28</td>
<td>(0.02, 1.25)</td>
<td>0.18</td>
<td>0.05</td>
<td>102.0</td>
</tr>
<tr>
<td>160</td>
<td>15.482255</td>
<td>3.47</td>
<td>(0.03, 1.10)</td>
<td>0.15</td>
<td>0.05</td>
<td>106.0</td>
<td></td>
</tr>
<tr>
<td>240</td>
<td>16.368022</td>
<td>3.36</td>
<td>(0.03, 0.85)</td>
<td>0.13</td>
<td>0.05</td>
<td>97.3</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>17.559553</td>
<td>4.29</td>
<td>(0.02, 3.10)</td>
<td>0.49</td>
<td>0.05</td>
<td>104.0</td>
<td></td>
</tr>
<tr>
<td>30</td>
<td>17.672847</td>
<td>3.34</td>
<td>(0.02, 2.55)</td>
<td>0.28</td>
<td>0.05</td>
<td>99.9</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>80</td>
<td>17.974697</td>
<td>2.97</td>
<td>(0.02, 2.10)</td>
<td>0.17</td>
<td>0.05</td>
<td>103.0</td>
</tr>
<tr>
<td>160</td>
<td>18.554773</td>
<td>3.12</td>
<td>(0.03, 2.05)</td>
<td>0.14</td>
<td>0.05</td>
<td>104.0</td>
<td></td>
</tr>
<tr>
<td>240</td>
<td>19.275786</td>
<td>3.24</td>
<td>(0.03, 1.75)</td>
<td>0.12</td>
<td>0.05</td>
<td>87.0</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>20.252534</td>
<td>4.50</td>
<td>(0.02, 3.20)</td>
<td>0.50</td>
<td>0.05</td>
<td>106.0</td>
<td></td>
</tr>
<tr>
<td>30</td>
<td>20.364172</td>
<td>3.27</td>
<td>(0.02, 2.95)</td>
<td>0.29</td>
<td>0.05</td>
<td>102.0</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>80</td>
<td>20.652604</td>
<td>2.79</td>
<td>(0.02, 2.25)</td>
<td>0.17</td>
<td>0.05</td>
<td>104.0</td>
</tr>
<tr>
<td>160</td>
<td>21.182637</td>
<td>2.94</td>
<td>(0.03, 2.15)</td>
<td>0.14</td>
<td>0.05</td>
<td>105.0</td>
<td></td>
</tr>
<tr>
<td>240</td>
<td>21.837585</td>
<td>2.97</td>
<td>(0.03, 2.05)</td>
<td>0.10</td>
<td>0.05</td>
<td>79.6</td>
<td></td>
</tr>
</tbody>
</table>

5.3. Parameters Design of the Valves

5.3.1. Parameters Design of the Regulating Motion Valve

The designed initial orifice underlap of the regulating motion valve \( x_{uz,0} \) is 0.1 mm, and the designed orifice underlap at the initial working point \( x_{uz,1} \) is 0.1 mm. According to Equation (69), it has:

\[
x_{szd,0} = x_{szd,1} - (x_{uz,1} - x_{uz,0}) = 10 \text{ mm} \tag{78}
\]

5.3.2. Parameters Design of the Compensated Motion Valve

According to Equation (71), the steady-state spring compression at the first working point is:

\[
x_{scd,1} = \frac{A_f}{K} P_e = 23.411148 \text{ mm} \tag{79}
\]

The designed initial orifice underlap of the compensated motion valve \( x_{uy,0} \) is 0.1 mm, and the designed orifice underlap at the initial working point \( x_{uy,1} \) is 0.1 mm. According to Equation (72), it has:

\[
x_{scd,0} = x_{scd,1} - (x_{uy,1} - x_{uy,0}) = 23.411148 \text{ mm} \tag{80}
\]

5.3.3. Geometry Design of the Orifices

According to Equations (66) and (68), the geometry design of the orifices are shown in Table 6.

Table 6. Geometry design of the compensated orifice and the regulating orifice.

<table>
<thead>
<tr>
<th>( x_{uz} ) mm</th>
<th>( A_f ) mm(^2)</th>
<th>( x_{uy} ) mm</th>
<th>( A_C ) mm(^2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0.1</td>
<td>3.4862689</td>
<td>0.20431466</td>
<td>10.215733</td>
</tr>
<tr>
<td>0.154981500</td>
<td>4.0360839</td>
<td>0.2072693</td>
<td>10.363465</td>
</tr>
<tr>
<td>0.236119030</td>
<td>4.9691655</td>
<td>0.21563862</td>
<td>10.781931</td>
</tr>
<tr>
<td>0.403207669</td>
<td>7.1413178</td>
<td>0.23447468</td>
<td>11.723734</td>
</tr>
<tr>
<td>0.623441131</td>
<td>10.334703</td>
<td>0.28610498</td>
<td>14.305249</td>
</tr>
<tr>
<td>0.749226439</td>
<td>11.969912</td>
<td>0.28734428</td>
<td>14.367214</td>
</tr>
<tr>
<td>0.980946022</td>
<td>14.750547</td>
<td>0.28980672</td>
<td>14.490336</td>
</tr>
<tr>
<td>1.481588945</td>
<td>21.258905</td>
<td>0.29624256</td>
<td>14.812128</td>
</tr>
<tr>
<td>1.941185588</td>
<td>27.693258</td>
<td>0.3096451</td>
<td>15.482255</td>
</tr>
<tr>
<td>2.285955665</td>
<td>32.175269</td>
<td>0.32736044</td>
<td>16.368022</td>
</tr>
<tr>
<td>2.858523220</td>
<td>39.904931</td>
<td>0.35119106</td>
<td>17.559553</td>
</tr>
<tr>
<td>4.238999460</td>
<td>57.160884</td>
<td>0.35345694</td>
<td>17.672847</td>
</tr>
<tr>
<td>4.380820733</td>
<td>58.720918</td>
<td>0.35949394</td>
<td>17.974697</td>
</tr>
</tbody>
</table>
Additionally, the geometry relationship of the flow area and the underlap of the two orifices are shown in Figures 8a and 8b, respectively.

**Figure 8.** (a) Geometry design curve for the underlap and the flow area of the regulating orifice; (b) geometry design curve for the underlap and the flow area of the compensated orifice.

**5.4. Simulation and Discussion**

The simulation works are carried out on the nonlinear model as shown in Figure 9, which can be established based on AMESim, and the structural parameters and design parameters are set.

**Figure 9.** Nonlinear model of the constant pressure difference regulating mechanism.

**5.4.1. Simulation**

1. Within the working range of the inputs, set the variable inlet flow area input $A_J$ as 10 mm$^2$, 50 mm$^2$, 100 mm$^2$, 150 mm$^2$, 200 mm$^2$, and 240 mm$^2$, respectively, and give the inlet pressure step input signal as shown in Figure 10. The simulation results are shown in Figure 11.

2. Within the working range of the inputs, set the inlet pressure input $P_s$ as 3 MPa, 4 MPa, 5 MPa, 6 MPa, 7 MPa, 8 MPa, and 9 MPa, respectively, and give the variable inlet flow area step input signal, as shown in Figure 12. The simulation results are shown in Figure 13.
Evidently, the steady and dynamic performance all meet the design requirements. Especially, the steady-state error is very small, and the dynamic characteristics of most design points are consistent with the theoretical design because of the precise models, which verify that the proposed methods are correct and accurate.

However, according to the preceding step response curves, it can be found that, for the design points close to the working boundary, such as the inlet pressure input $P_S$ as 3 MPa and the inlet flow area input $A_J$ as 240 mm$^2$, the dynamic characteristics are very different from those design points far away from the working boundary. Especially, the settling time increases and the overshoot increases.
Figure 13. Step response curves of the inlet flow area disturbance.

This is a typical nonlinear phenomenon, and the reason can be explained from the following compensated flow area output curves, as shown in Figures 14 and 15.

Figure 14. Compensated flow area output curves of the inlet pressure disturbance.

Figure 15. Compensated flow area output curves of the inlet flow area disturbance.

Combining Figure 11 with Figure 14, the compensated flow area at 3 s and 7 s is far from the boundary value 0, and the system does not enter the nonlinear state; hence, the step response characteristics are consistent with the theoretical design. However, the compensated flow area at 11 s reaches the nonlinear boundary, and the controller loses the regulating ability, causing the settling time and the overshoot to increase.

Similarly, combining Figure 13 with Figure 15, the nonlinear characteristic of the compensated flow area also impacts the step response of the inlet flow area disturbance.
It follows that, in the design process of controllers, nonlinearity is also a very crucial factor, which needs to be paid enough attention to.

6. Conclusions

Based on the linear incremental analysis method, this paper reveals the design theory of the constant pressure difference regulating mechanism and proposes the frequency domain analysis and design methods of the system. The simulation results of the nonlinear model verify the accuracy of the proposed methods, and the conclusions are as follows:

1. Compared with the classic analysis method based on direct transfer function transformation, the linear incremental method is based on the state space, and it is clearer to reveal the design theory of the constant pressure difference regulating mechanism. Furthermore, the analysis method also clarifies the key design parameters, including the stabilization control gain and the servo control gain, which was not involved in previous research.

2. Concerning the advantage of the explicit description between the designed parameters and the performance when analyzed by the loop transfer functions, the frequency domain analysis method clearly describes the quantitative influence of the control gains on the frequency domain performance of the closed-loop system and provides more correct guidance for the design of the performance parameters, avoiding trial and error in simulation research.

3. The simulation results show that the steady and dynamic performance all meet the design requirements when utilizing the proposed frequency domain design method; especially, the steady-state error is very small, and the phase margin is very large, which are beyond the design requirements. Evidently, the proposed design methods are correct and accurate and can be widely applied in the analysis and predesign of other components of the fuel metering system, such as the constant pressure control system and the position control system.

Nevertheless, the proposed methods may not work as perfect as the simulation tests when implementing actual physical tests, which should be verified in the future research. Furthermore, it is also interesting to research the controller design method when considering the nonlinear characteristics, which will be explored in future works.

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Data Availability Statement: The data used to support the findings of this paper are contained in the text.

Conflicts of Interest: There is no conflict of interest.

References