

Article

Barrier–Energy-Driven Probabilistic Super-Twisting Missile Guidance with Composite FTDO–HESO for Maneuvering Target Interception

Hong Zhao, Zhanpeng Gao and Wenjun Yi *

National Key Lab of Transient Physics, Nanjing University of Science and Technology, Nanjing 210094, China; zhaohong@njust.edu.cn (H.Z.); gaozhanpeng@njust.edu.cn (Z.G.)

* Correspondence: yi_wj@njust.edu.cn

Abstract

This paper proposes a barrier-modulated probabilistic super-twisting guidance (BMPSTG) for robust three-dimensional maneuvering target interception under measurement uncertainty and packet loss. A stochastic barrier-adaptive mechanism inspired by tunneling dynamics is introduced to reshape the sliding variable evolution, enabling accelerated convergence and reduced chattering compared with conventional super-twisting schemes. To enhance disturbance reconstruction accuracy, a composite observer integrating a finite-time disturbance observer (FTDO) and a high-order extended state observer (HESO) is developed. The FTDO ensures fast transient estimation, while the HESO improves steady-state precision and robustness to noise. Simulation results demonstrate that the proposed FTDO–HESO structure outperforms both standalone FTDO and conventional FTDO–ESO configurations in terms of estimation accuracy and guidance performance. A probability-coupled gain adaptation strategy further adjusts control gains according to engagement states, improving robustness under aggressive target maneuvers. Monte Carlo simulations verify that the proposed method achieves high interception accuracy with stable convergence and reduced control effort in complex engagement scenarios.

Keywords: super-twisting guidance; barrier–energy modulation; composite disturbance observer; FTDO–HESO; finite-time stability; maneuvering target interception

1. Introduction

Achieving high-precision interception of maneuvering targets remains a fundamental objective in modern missile guidance system design. Proportional navigation guidance (PNG), proposed in the 1960s, has been extensively studied due to its simple structure, clear physical interpretation, and strong engineering practicality [1–5]. For nonmaneuvering or weakly maneuvering targets, PNG exhibits reliable capture capability and satisfactory interception accuracy.

However, its performance deteriorates significantly in highly dynamic engagement scenarios. It has been reported that PNG is sensitive to terminal distance, often generating excessively large commands near interception [6], which may lead to actuator saturation or unstable flight trajectories [7]. Moreover, classical guidance laws may produce acceleration commands exceeding vehicle maneuver limits, thereby restricting practical applica-

Academic Editor: Mingzhe Hou

Received: 22 May 2026

Revised: 25 June 2026

Accepted: 27 June 2026

Published: 30 June 2026

Copyright: © 2026 by the authors. Licensee MDPI, Basel, Switzerland. This article is an open access article distributed under the terms and conditions of the [Creative Commons Attribution \(CC BY\)](https://creativecommons.org/licenses/by/4.0/) license.

bility [8]. When confronting complex three-dimensional maneuvering targets, PNG is generally inferior to advanced adaptive or nonlinear guidance schemes [9].

Sliding mode control (SMC) has therefore attracted considerable attention in missile guidance due to its inherent robustness against parameter uncertainties and external disturbances [10–12]. Its strong disturbance rejection capability makes it suitable for high-dynamic interception scenarios. Nevertheless, classical SMC suffers from two major drawbacks: chattering induced by discontinuous switching functions and the difficulty of gain tuning in practical implementations. High-frequency switching may cause actuator wear and degrade system smoothness [13–16]. Although various adaptive strategies have been proposed to mitigate these issues, most approaches rely on deterministic gain adjustment and lack a systematic mechanism to accelerate convergence under complex disturbance conditions. Recent SMC-related studies provide useful reference points for positioning the proposed method. Gong et al. [17] developed a saturated super-twisting missile guidance law to suppress command saturation and chattering under acceleration constraints. Shtessel et al. [18] proposed an adaptive-gain super-twisting controller in which the gains are adjusted online to reduce conservatism under bounded disturbances. Gonzalez et al. [19] introduced variable-gain super-twisting control to improve robustness by deterministic gain variation. Behera et al. [20] developed periodic event-triggered SMC to reduce unnecessary control updates through triggering conditions. Different from these saturation-based, adaptive-gain, variable-gain, or event-triggered mechanisms, the proposed method maps a composite barrier–energy involving disturbance intensity, geometric coupling, historical fluctuation, and terminal urgency into a probabilistic escape measure, thereby reshaping the sliding variable and modulating the super-twisting gains in a risk-aware and history-dependent manner.

To better position the proposed barrier–energy probability mechanism, recent SMC-related studies and barrier-crossing theories are reviewed more specifically. Levant [21] developed gain-scheduled high-order sliding mode control, showing that gain variation can improve robustness for multi-input systems. Gong et al. [22] proposed a saturated super-twisting missile guidance law to reduce chattering and command saturation under acceleration constraints. Shtessel et al. [23] introduced an adaptive-gain super-twisting controller to reduce conservatism under bounded disturbances. Gonzalez et al. [24] further developed variable-gain super-twisting control, where robustness is improved through deterministic gain variation. Behera et al. [25] proposed periodic event-triggered SMC to reduce unnecessary control updates through triggering conditions. From the viewpoint of barrier-crossing theory, Kramers [26] and Hanggi et al. [27] showed that crossing rates are governed by the energy–barrier separation, while Razavy [28] summarized the WKB-type tunneling interpretation in which sub-barrier crossing probability depends exponentially on the barrier–energy gap. These studies motivate the present work to use a semi-classical tunneling-inspired exponential map as a bounded and physically interpretable risk measure, rather than an arbitrary monotone function, for reshaping the sliding variable and scheduling the super-twisting gains under maneuver uncertainty.

To exploit the complementary advantages of these observers, composite FTDO–ESO structures have been considered in recent studies [29,30]. Nevertheless, existing schemes rarely investigate high-order ESO integration for improving steady-state estimation precision while preserving fast transient response. ESO-based disturbance rejection and composite disturbance rejection control have been investigated in uncertain nonlinear systems [31,32]. Nevertheless, existing guidance-oriented observer schemes rarely integrate finite-time disturbance reconstruction with high-order ESO compensation to improve steady-state estimation precision while preserving fast transient response under measurement delay, packet loss, and noise. Recent studies on complex engineering reliability further show that uncertainty propagation, sub-system coupling, and data–physics fusion are es-

essential for dependable operation under incomplete or stochastic information. For example, cascade sampling-driven block mapping has been used for coupled reliability evaluation of turbine cooling systems [33], while generative data-physics fusion has been developed for hierarchical reliability assessment [34]. These works motivate the present formulation to treat maneuvering-target interception as a coupled uncertainty-handling problem, where probabilistic barrier modulation and composite disturbance reconstruction are introduced to improve guidance robustness under uncertain sensing and target maneuvers.

To address the above challenges, this paper proposes a barrier-energy-modulated super-twisting guidance framework combined with a composite FTDO–HESO disturbance observer. Instead of employing purely deterministic gain adaptation, a probabilistic barrier-based regulation mechanism inspired by tunneling dynamics is introduced. The sliding variable is interpreted through an energy–barrier interaction framework, where the dynamic barrier height is determined by guidance error magnitude, target maneuver intensity, and historical variation. A stochastic escape mechanism reshapes the sliding surface evolution, enabling accelerated convergence and reduced chattering in aggressive engagement scenarios.

Furthermore, a probability-coupled gain scheduling strategy is developed to adapt super-twisting gains according to engagement states such as relative distance, closing velocity, and terminal-phase dynamics. To enhance disturbance estimation performance under measurement uncertainty and packet loss, a composite observer integrating FTDO and high-order ESO (HESO) is constructed. The FTDO ensures rapid transient reconstruction, while the HESO improves steady-state accuracy and robustness against noise.

Monte Carlo simulations demonstrate that the proposed FTDO–HESO framework achieves superior disturbance reconstruction performance compared with standalone FTDO and conventional FTDO–ESO schemes. The overall guidance law achieves improved interception accuracy, faster convergence, and reduced control effort in complex three-dimensional maneuvering engagements.

The main contributions of this paper are summarized as follows:

1. A barrier-energy-modulated super-twisting guidance law with stochastic sliding variable reshaping for accelerated convergence and chattering reduction.
2. A dynamic barrier modeling mechanism incorporating disturbance intensity, historical evolution, and engagement state information.
3. A probability-coupled self-scheduling gain adaptation strategy for improved robustness under aggressive target maneuvers.
4. A composite FTDO–HESO disturbance observer that outperforms standalone FTDO and FTDO–ESO configurations in estimation precision and robustness.

The remainder of this paper is organized as follows. Section 2 formulates the three-dimensional engagement model. Section 3 presents the proposed barrier-adaptive super-twisting guidance law and its stability analysis. Section 4 develops the composite FTDO–HESO observer. Section 5 provides simulation results and comparative studies. Section 6 concludes the paper.

2. Model Architecture

Missile–Target Relative Equation

The 3D interceptor intercepts a moving target with the influence of many factors, and in order to simplify the relative motion equations, the interceptor and the moving target can be regarded as particles, and the signal transmission time lag of the interceptor's guide head, as well as the possible frame-loss problem and the observation noise influence, will be discussed in Section 5. At the same time, it is considered that the meteorological ele-

ments can be chosen to be ignored during the overall motion. As a result, the model of the projectile motion at this point can be derived as shown in the following figure:

As illustrated in Figure 1, O_Mxyz is the LOS coordinate system, i.e., line of sight. Let θ_M be the angle between the direction of the projectile velocity and the xoz plane in the LOS coordinate system, i.e., the velocity tilt angle. Let ψ_M be the angle between the water surface component of the projectile velocity and the xoy plane, i.e., the velocity yaw angle. V_M is the direction of the velocity of the projectile and is a vector. Let θ_L be the angle between the eye sight direction and the xoz plane in the LOS coordinate system, i.e., the azimuth angle; and let ψ_L be the angle between the water surface component of the eye sight direction and the xoy plane, i.e., the yaw angle. Similarly, θ_T is the target velocity tilt angle and ψ_T is the target velocity yaw angle.

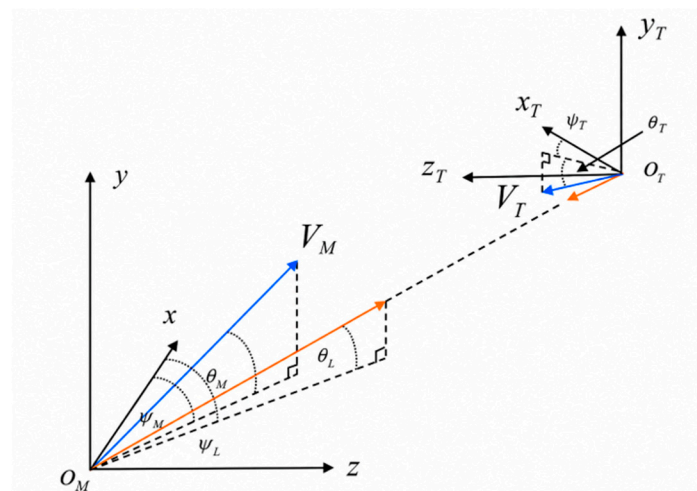


Figure 1. Geometry of missile and target engagement in collision-kill mode.

A kinetic analysis of the above system can be obtained:

$$\begin{bmatrix} \frac{dX_M}{dt} \\ \frac{dY_M}{dt} \\ \frac{dZ_M}{dt} \end{bmatrix} = \begin{bmatrix} V_M \cos \theta_L \cos \psi_L \\ V_M \sin \theta_L \\ -V_M \cos \theta_L \sin \psi_L \end{bmatrix} \quad (1)$$

In Equation (1), X_M , Y_M , and Z_M are the displacement components of the projectile in the LOS coordinate system as vectors, respectively. Then it can be further derived based on the interceptor kinematics equations as well as its dynamics vector equations:

$$\frac{d\vec{R}_M}{dt} = \vec{V}_M \quad (2)$$

$$\frac{d\vec{V}_M}{dt} = \frac{\delta \vec{V}_M}{dt} + \vec{\omega}_M \times \vec{V}_M \quad (3)$$

In Equation (2), R_M is the total displacement of the projectile as a vector. In Equation (3), ω_M is the vector of dynamic angular velocity, on the basis of which the target velocity, i.e., displacement, is introduced, and the following equation can be formed:

$$\vec{R} = \vec{R}_T - \vec{R}_M = \begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \begin{pmatrix} x_t - x_m \\ y_t - y_m \\ z_t - z_m \end{pmatrix} \quad (4)$$

In Equation (4), R is the relative position vector of the projectile. R_T is the total displacement of the target as a vector. X , Y , Z are the components of the specific relative position on each coordinate axis. x_t , y_t , z_t are the components of the current position of

the target. x_m , y_m , z_m are the components of the current position of the projectile, which can be obtained:

$$r = \|\vec{R}\| = \sqrt{X^2 + Y^2 + Z^2} \quad (5)$$

In the inertial frame $O-x_A y_A z_A$, let the relative position vector be $\mathbf{r} = [x, y, z]^T$ with magnitude $r = \|\mathbf{r}\| = \sqrt{x^2 + y^2 + z^2}$. To describe the LOS orientation unambiguously for arbitrary engagement geometries—including tail-chase and crossing scenarios where $x < 0$ —the elevation angle λ_ϵ and the azimuth angle λ_β are defined strictly via the four-quadrant arctangent as follows:

$$\lambda_\epsilon = \text{atan2}(y, \sqrt{x^2 + z^2}) \quad (6)$$

$$\lambda_\beta = \text{atan2}(z, x). \quad (7)$$

Remark 1. Using the conventional $\arctan(z/x)$ in Equation (7) would confine the azimuth to $(-\pi/2, \pi/2)$ and fail to resolve the second and third quadrants. During interception, the relative geometry often traverses $x < 0$ (e.g., tail-chase or high-aspect-angle engagement), where $\arctan(z/x)$ suffers a π -jump discontinuity and produces erroneous azimuth-rate singularities. The function $\text{atan2}(z, x)$ returns unique values in $(-\pi, \pi]$ according to the signs of both arguments, thereby guaranteeing geometric continuity. Although $\sqrt{x^2 + z^2} \geq 0$ makes $\arctan(y/\sqrt{x^2 + z^2})$ seemingly adequate for Equation (6), the unified adoption of atan2 ensures numerical robustness and implementation consistency.

The consistency of Equations (6) and (7) with the right-handed LOS coordinate frame is proved as follows. From the geometric definitions, the inertial components of $\mathbf{r}$ satisfy

$$\begin{aligned} x &= r \cos \lambda_\epsilon \cos \lambda_\beta, \\ y &= r \sin \lambda_\epsilon, \\ z &= r \cos \lambda_\epsilon \sin \lambda_\beta. \end{aligned} \quad (8)$$

Substituting Equation (8) into the LOS-frame relation $\mathbf{r}^L = [r, 0, 0]^T$ confirms that the x_L -axis aligns with the LOS, while y_L and z_L span the normal plane. Three limit cases further validate the axis consistency: (i) $\lambda_\epsilon = 0, \lambda_\beta = 0 \Rightarrow (x, y, z) = (r, 0, 0)$, i.e., $x_L \parallel x_A$; (ii) $\lambda_\epsilon = 0, \lambda_\beta = \pi/2 \Rightarrow (x, y, z) = (0, 0, r)$, i.e., $x_L \parallel z_A$; (iii) $\lambda_\epsilon = \pi/2 \Rightarrow (x, y, z) = (0, r, 0)$, i.e., $x_L \parallel y_A$. All cases conform to the right-hand rule, proving that the definitions are intrinsically consistent with the prescribed coordinate axes.

Differentiating $r^2 = x^2 + y^2 + z^2$ with respect to time gives the exact range rate $\dot{r} = \frac{x\dot{x} + y\dot{y} + z\dot{z}}{r}$. Differentiating $\tan \lambda_\beta = z/x$ and substituting Equation (8) to eliminate the trigonometric terms yields the azimuthal angular velocity

$$\dot{\lambda}_\beta = \frac{x\dot{z} - z\dot{x}}{x^2 + z^2}. \quad (9)$$

The accuracy of Equation (9) is cross-verified by an independent derivation: directly differentiating $x = r \cos \lambda_\epsilon \cos \lambda_\beta$ and $z = r \cos \lambda_\epsilon \sin \lambda_\beta$ and eliminating $\dot{r}$ and $\dot{\lambda}_\epsilon$ reproduce exactly the same expression, confirming the absence of algebraic or sign errors. The denominator $x^2 + z^2 = r^2 \cos^2 \lambda_\epsilon$ remains strictly positive throughout the engagement because the zenith/nadir singularities ($\lambda_\epsilon = \pm\pi/2$) are excluded by the physical interception geometry.

On this basis, in Figure 1, the following equation can be structured using geometric relations:

$$\theta_L = \text{atan2}\left(Y, \sqrt{X^2 + Z^2}\right) \quad (10)$$

$$\psi_L = \text{atan2}(Z, X) \quad (11)$$

where $\vec{R}_{oxy} = [X, 0, Z]^T$, $\vec{X} = [r, 0, 0]^T$, so that $D = \sqrt{X^2 + Z^2}$, which is obtained by taking into Equation (10):

$$\theta_L = \text{atan2}\left(\frac{Y}{D}\right) \quad (12)$$

Differentiating the atan2 form in Equation (12), and noting that $D > 0$, gives the same derivative as $\arctan(Y/D)$:

$$\dot{\theta}_L = \frac{1}{1 + \left(\frac{Y}{D}\right)^2} \cdot \frac{D \cdot \dot{Y} - Y \cdot \dot{D}}{D^2} \quad (13)$$

where $\dot{D} = (X\dot{X} + Z\dot{Z})/D = (XV_X + ZV_Z)/D$ follows from $D^2 = X^2 + Z^2$. Substituting $\dot{D}$ and simplifying with $D^2 + Y^2 = r^2$ yield

$$\dot{\theta}_L = \frac{D^2}{D^2 + Y^2} \cdot \frac{D \cdot V_Y - Y \cdot \frac{XV_X + ZV_Z}{D}}{D^2} \quad (14)$$

$$\dot{\theta}_L = \frac{(X^2 + Z^2)V_Y - Y(XV_X + ZV_Z)}{r^2\sqrt{X^2 + Z^2}} \quad (15)$$

In Equation (13), Y/D is dimensionless, while the factor $(D\dot{Y} - Y\dot{D})/D^2$ carries dimensions $[L \cdot (LT^{-1})]/[L^2] = [T^{-1}]$, which is the correct physical dimension of an angular rate. In the fully expanded form (15), the numerator has dimensions $[L^2] \cdot [LT^{-1}] = [L^3T^{-1}]$ and the denominator has dimensions $[L^2] \cdot [L] = [L^3]$; their ratio therefore yields $[T^{-1}]$. Thus, every geometric term in Equations (13)–(15) is dimensionally homogeneous.

Similarly, the angular rate of yaw in the LOS direction follows from the planar azimuth relation

$$\frac{d\psi_L}{dt} = \frac{d}{dt} \left[\text{atan2}\left(\frac{Z}{X}\right) \right] \quad (16)$$

Differentiating Equation (16) and inserting the velocity components give

$$\dot{\psi}_L = \frac{1}{1 + \left(\frac{Z}{X}\right)^2} \cdot \frac{X \cdot \dot{Z} - Z \cdot \dot{X}}{X^2} \quad (17)$$

$$\dot{\psi}_L = \frac{X^2}{X^2 + Z^2} \cdot \frac{X \cdot V_Z - Z \cdot V_X}{X^2} \quad (18)$$

$$\dot{\psi}_L = \frac{X \cdot V_Z - Z \cdot V_X}{X^2 + Z^2} \quad (19)$$

The numerator in Equation (19) has dimensions $[L] \cdot [LT^{-1}] = [L^2T^{-1}]$, while the denominator has dimensions $[L^2]$; hence $[\dot{\psi}_L] = [T^{-1}]$.

The LOS angular velocity vector is constructed from the yaw and pitch angular rates as

$$\omega_L = [\dot{\psi}_L \sin \theta_L, \dot{\psi}_L \cos \theta_L, \dot{\theta}_L]^T. \quad (20)$$

Construct the LOS coordinate system into an elastic eye coordinate system transformation matrix, i.e.,

$$T_{G2L} = R_Z(\psi_L) \cdot R_Y(\theta_L) \quad (21)$$

$$T_{G2L} = \begin{bmatrix} \cos \psi_L \cos \theta_L & \sin \theta_L & -\cos \theta_L \sin \psi_L \\ -\cos \psi_L \sin \theta_L & \cos \theta_L & \sin \psi_L \sin \theta_L \\ \sin \psi_L & 0 & \cos \psi_L \end{bmatrix} \quad (22)$$

The vector form of the angular velocity of rotation of the projectile in the coordinate system of the shell and the target is brought into Equation (22) as follows:

$$\begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} = T_{G2L} \begin{bmatrix} 0 \\ \dot{\psi}_L \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_L \end{bmatrix} = \begin{bmatrix} \dot{\psi}_L \sin \theta_L \\ \dot{\psi}_L \cos \theta_L \\ \dot{\theta}_L \end{bmatrix} \quad (23)$$

ω_x , ω_y , ω_z are the components of the rotational angular velocity on the coordinate axes, and the following relationship exists between the absolute and partial derivatives of the velocity:

$$\dot{R}_L = \frac{\delta R_L}{\delta t} + \omega_L \cdot R_L \quad (24)$$

Bringing Equation (24) into Equation (23) in vector form in the LOS coordinate system in terms of coordinate components gives:

$$\begin{bmatrix} V_x \\ V_y \\ V_z \end{bmatrix} = \begin{bmatrix} \dot{r} \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} \dot{\psi}_L \sin \theta_L \\ \dot{\psi}_L \cos \theta_L \\ \dot{\theta}_L \end{bmatrix} \times \begin{bmatrix} r \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} \dot{r} \\ r\dot{\theta}_L \\ -r\dot{\psi}_L \cos \theta_L \end{bmatrix} \quad (25)$$

To summarize, it can be seen from Equations (10), (11) and (25) that there is a coupling phenomenon between the pitch and yaw channels in the equations of motion of the 3D interceptor, for which consideration is given to the introduction of a decoupling compensated control rate to constrain the decoupling control.

3. Barrier–Energy-Driven Probabilistic Super-Twisting Guidance Law

3.1. Reformulation of the Three-Dimensional Engagement Dynamics

Summarizing the above, it can be seen that the whole guidance law architecture has a certain three-dimensional geometric coupling effect phenomenon, and the *SMC* design has a parameter uncertainty problem and local optimization problem, and the *PNG* design has the problem of excessive maneuvering and singularity due to the gain problem in the final guidance stage. Considering the architecture of the multi-method organic fusion guidance law, it can cope with the end-guidance scenario of high-speed maneuvering targets under the premise of fully guaranteeing high accuracy and strong robustness.

The design of the sliding mode surface is the basis of the whole guidance law, which directly determines the stability and convergence characteristics of the control system. Considering that the zeroed line-of-sight angular rate is a sufficiently necessary condition to realize precise interception, and in order to subsequently introduce adaptive adjustment gain, it is considered to introduce the relative distance between the bullet and the eye as the basis of the sliding mode surface, and the pitch and yaw channels are designed separately for preliminary decoupling, which also facilitates the subsequent design of the controllers separately.

According to Equation (25) the sliding mode surface can be designed as below:

$$\begin{cases} s_y = r\dot{\theta}_L \\ s_z = -r\dot{\psi}_L \cos \theta_L \end{cases} \quad (26)$$

The quantities $s_y(t)$ and $s_z(t)$ represent the tangential components of relative angular momentum in the LOS frame. Under standard engagement kinematics, their dynamics can be written in compact form as:

$$\dot{s}_i(t) = f_i(t) + u_i(t) + d_i(t), i \in \{y, z\} \quad (27)$$

The compact dynamics in Equation (27) involve three key terms whose physical meanings are rigorously delineated below:

- (1) Control input $u_i(t)$. The variable $u_i(t)$ denotes the equivalent control input for the i -th channel ($i = y$ for the pitch/elevation transverse channel and $i = z$ for the yaw/azimuth transverse channel). It is defined as the projection of the actual missile command acceleration vector a_M onto the corresponding axis of the LOS coordinate frame; that is, $u_i(t) = e_i^T a_M^{\text{LOS}}(t)$, where $a_M^{\text{LOS}}(t)$ is obtained by transforming the physical missile acceleration command from the guidance frame to the LOS frame

via the rotation sequence in Equations (21)–(25). Consequently, $u_i(t)$ is the direct control variable synthesized by the guidance law and is linearly related to the physical acceleration command generated by the missile autopilot.

- (2) Geometric coupling term $f_i(t)$. This term encompasses all known, state-dependent nonlinearities arising from the three-dimensional engagement geometry. As evident from Equations (10), (11) and (25), $f_i(t)$ comprises the coupled Coriolis-like and centrifugal terms induced by the rotation of the LOS frame (e.g., cross-products of the LOS angular rates) as well as the kinematic coupling between the pitch and yaw channels. Importantly, $f_i(t)$ is a deterministic function of the measurable states (LOS angles and their rates) and therefore differs fundamentally from unknown external disturbances.
- (3) Lumped disturbance $d_i(t)$. The term $d_i(t)$ aggregates all unknown and uncertain factors affecting the i -th channel and is explicitly decomposed as $d_i(t) = a_{T,i}^{\text{LOS}}(t) + \Delta_i(t)$, where $a_{T,i}^{\text{LOS}}(t)$ is the projection of the target maneuver acceleration onto the i -th LOS axis, and $\Delta_i(t)$ subsumes model uncertainties, unmodeled higher-order dynamics, aerodynamic parameter variations, observer residual errors, and measurement perturbations. Thus, $d_i(t)$ is strictly distinguished from $f_i(t)$: the former captures unknown target maneuvers and model uncertainties, whereas the latter represents the known geometric coupling inherent to the spherical LOS dynamics.

Definition 1. The disturbance $d_i(t)$ is bounded and its time derivative is bounded, i.e.,

$$|d_i(t)| \leq D_i, |\dot{d}_i(t)| \leq \bar{D}_i \quad (28)$$

This definition is standard in super-twisting analysis and guarantees second-order sliding feasibility.

3.2. Baseline Super-Twisting Guidance Structure

From Equation (27), we can further consider the classical super-twisting sliding mode control law:

$$\begin{aligned} u_i(t) &= -k_{1,i} |s_i|^{\frac{1}{2}} \text{sgn}(s_i) + z_i(t), \\ \dot{z}_i(t) &= -k_{2,i} \text{sgn}(s_i), \end{aligned} \quad (29)$$

where: $k_{1,i} > 0$ and $k_{2,i} > 0$ are constant gains, $\text{sgn}(\cdot)$ the sign function. $k_{1,i} > 2D_i$, $k_{2,i} > D_i$, finite-time convergence of $s_i(t)$ to zero is guaranteed. However, this structure suffers from two practical limitations:

1. Gain conservatism:

To guarantee robustness under worst-case disturbances, large gains must be chosen, leading to excessive control effort.

2. Uniform deterministic correction:

The switching mechanism applies identical correction intensity regardless of engagement phase, error history, or disturbance evolution.

Therefore, a mechanism is needed to modulate convergence intensity dynamically, reduce control effort in low-disturbance phases, accelerate convergence under aggressive maneuvers, and preserve strict stability guarantees.

Classical adaptive gain control (AGC) methods—such as gain-scheduling super-twisting or boundary-layer sliding mode schemes—attempt to mitigate the above issues by adjusting the switching gain as a deterministic function of the instantaneous sliding error, e.g., $k = k(\|s\|, \dot{s})$. However, such adaptation remains inherently memoryless and threshold-driven: it reacts only to the current magnitude of s without systematic retention of disturbance history or engagement-phase context, and once activated, it applies

correction uniformly. Consequently, classical AGC cannot distinguish between a transient perturbation that requires mild intervention and a sustained aggressive maneuver that demands high-energy counteraction, thereby suffering from either excessive conservatism or insufficient robustness.

Therefore, a fundamentally different mechanism is needed to modulate convergence intensity dynamically: reduce control effort in low-disturbance phases, accelerate convergence under aggressive maneuvers, and preserve strict stability guarantees, all while overcoming the memoryless, uniform-correction limitation of classical AGC.

3.3. Barrier–Energy-Driven Probabilistic Sliding Dynamics

To address the above limitations, we reinterpret the sliding variable evolution as an energy–barrier interaction process within a nonlinear potential landscape.

It is emphasized that this reinterpretation is fundamentally distinct from classical adaptive gain control. Conventional AGC scales the controller gain *externally* based on a deterministic threshold of $\|s\|$; by contrast, the proposed barrier–energy mechanism modulates the sliding dynamics *internally* by reshaping the potential landscape through which the state trajectory evolves. As a result, the correction action is no longer triggered by a binary threshold but is probabilistically coupled to the escape likelihood from a dynamically evolving energy well, thereby integrating disturbance intensity, historical evolution, and engagement geometry into the modulation decision.

3.3.1. Normalized Sliding Energy

Define the normalized sliding magnitude:

$$\bar{s}_i = \frac{|s_i|}{1 + |s_i|} \quad (30)$$

Define the instantaneous sliding energy as:

$$E_i(t) = \alpha_1 \bar{s}_i^2 + \alpha_2 \bar{h}_i + \alpha_3 \bar{d}_i \quad (31)$$

where: $\bar{h}_i$ is the normalized historical variance of s_i , $\alpha_1, \alpha_2, \alpha_3 > 0$ are weighting coefficients. The normalized disturbance indicator used in the simulations is defined as

$$\bar{d}_i(t) = \frac{|\hat{d}_i^{LOS}(t)|}{g_0 + |\hat{d}_i^{LOS}(t)|}, \quad i = y, z,$$

where $g_0 = 9.81, \text{m/s}^2$. The reconstructed LOS-frame disturbance is obtained from the HESO third-state estimate as

$$\hat{d}_i^{LOS}(t) = e_i^T C_G^L(t) \hat{x}_3(t).$$

Thus, $\bar{d}_i(t)$ is not computed from an observer residual ratio but from the reconstructed LOS-frame disturbance acceleration.

3.3.2. Dynamic Barrier Function

We define a time-varying barrier function:

$$\Delta V_i(t) = \beta_0 + \beta_1 \bar{d}_i + \beta_2 \bar{c}_i + \beta_3 \bar{h}_i + \beta_4 \bar{t}_i \quad (32)$$

where $\bar{d}_i(t)$ is the normalized reconstructed-disturbance indicator defined after Equation (31), $\bar{c}_i(t)$ is the normalized cross-channel coupling intensity, $\bar{h}_i(t)$ is the normalized historical fluctuation, and $\bar{t}_i(t)$ is the normalized terminal-urgency factor.

Remark 2. The barrier height $\Delta V_i(t)$ acts as a risk-adaptive stability threshold rather than a static constraint. The positive weights $\beta_j > 0$ imply that an increase in disturbance intensity $\bar{d}_i$, cross-channel coupling $\bar{c}_i$, historical fluctuation $\bar{h}_i$, or terminal urgency $\bar{t}_i$ all raise the required acti-

vation energy for sliding variable reshaping. This design intentionally elevates the crossing threshold when the engagement is anticipated to be more aggressive or uncertain, thereby:

1. *Preventing over-control and chattering:* A higher barrier suppresses the probabilistic modulation term $P_i(t)$ when the sliding energy E_i is below ΔV_i , filtering out high-frequency responses to transient disturbances.
2. *Suppressing impact propagation:* During abrupt maneuvers (large $\bar{d}_i$ or $\bar{c}_i$), the raised barrier temporarily insulates the sliding surface from excessive gain adjustments. It should be emphasized that a larger barrier does not mean slower convergence. Instead, it prevents unnecessary high-gain responses when the sliding energy is still below the barrier, while Equation (33) rapidly increases the escape probability once E_i exceeds ΔV_i . Hence, the barrier height acts as a risk-adaptive selectivity threshold coordinated with the probability-coupled gain law.
3. *Preserving deterministic convergence:* Once the accumulated sliding energy genuinely exceeds the barrier ($E_i \geq \Delta V_i$), the escape probability $P_i(t)$ rapidly saturates toward 1 according to Equation (33), and the coupled gain adaptation in Equation (35) restores aggressive correction capability.

Therefore, the barrier height dynamically modulates the trade-off between robustness and control impact: it becomes more “selective” under harsh conditions, allowing control actions only when the state discrepancy is sufficiently severe to justify a cross-barrier escape.

3.3.3. Probabilistic Escape Mechanism

Define the probabilistic modulation term:

$$P_i(t) = \begin{cases} \exp(-2a_i\sqrt{\Delta V_i - E_i}), & E_i < \Delta V_i \\ 1 - \exp(-(E_i - \Delta V_i)), & E_i \geq \Delta V_i \end{cases} \quad (33)$$

where $a_i(t)$ is the adaptive barrier-width coefficient used in the sub-barrier probability. In the simulations, it is updated online according to $a_i(t) = \frac{a_0}{1+a_s\bar{s}_i(t)+a_d\bar{d}_i(t)}$, $a_0 = 0.7$, $a_s = 0.7$, $a_d = 0.2$. Since $\bar{s}_i(t), \bar{d}_i(t) \in [0,1]$, $a_i(t)$ is bounded as $0.368 < a_i(t) \leq 0.7$.

Thus, $a_i(t)$ is not a constant. Under benign conditions, $a_i(t)$ remains close to 0.7, while larger sliding magnitude or reconstructed disturbance reduces $a_i(t)$, increasing the sensitivity of the tunneling probability to the energy–barrier separation.

The function $P_i(t) \in (0,1)$ is continuous, bounded, and monotone with respect to the energy–barrier separation. The exponential form is selected not as a literal quantum model but as a WKB-inspired risk map: it preserves a nonzero sub-barrier activation probability, provides high sensitivity near $E_i \approx \Delta V_i$, and rapidly saturates after E_i exceeds ΔV_i . A generic bounded monotone function could provide numerical saturation but would not encode this energy-gap-dependent crossing interpretation.

3.3.4. Probabilistic Sliding Variable Reshaping

The reshaped sliding variable is defined as:

$$\tilde{s}_i = (1 - \gamma_i)s_i + \gamma_i s_i e^{-\rho_i P_i} \quad (34)$$

where: $\gamma_i \in (0,1)$. This transformation ensures: $|\tilde{s}_i| \leq |s_i|$ and strictly reduces magnitude when P_i is large.

3.3.5. Probability-Coupled Gain Adaptation

The super-twisting gains are modulated as:

$$\begin{aligned} k_{1,i}(t) &= k_{1,0}(1 + \eta_1(1 - P_i) + \eta_2\delta_i) \\ k_{2,i}(t) &= k_{2,0}(1 + \mu_1(1 - P_i) + \mu_2\delta_i) \end{aligned} \quad (35)$$

where: $\delta_i \in \{0,1\}$ indicates an escape activation event. Thus, gains increase automatically when escape probability is low and reduce when sliding convergence is progressing efficiently.

Remark 3. Classical adaptive STG or SMC schemes typically increase gains whenever $\|s\|$ exceeds a prescribed threshold and decrease them otherwise. This deterministic switching is uniformly applied across the entire engagement, meaning that a large $\|s\|$ in the mid-course phase and a large $\|s\|$ during terminal homing trigger identical gain escalation. By contrast, the probability-coupled adaptation in (35) integrates the normalized sliding energy $\mathcal{E}_s$ in (31) and the dynamic barrier $\mathcal{B}_s$ in (32), which embed both instantaneous disturbance magnitude and its historical variance. Consequently, the same instantaneous $\|s\|$ may yield markedly different gain levels depending on the engagement context, enabling targeted correction that avoids the persistent over-actuation typical of memoryless AGC.

3.3.6. Final Control Law

Combining Equations (34) and (35) can obtain the final barrier–energy-driven super-twisting guidance law:

$$\begin{aligned} u_i &= -k_{1,i}(t) |\tilde{s}_i|^{\frac{1}{2}} \operatorname{sgn}(\tilde{s}_i) + z_i + \phi_i \\ \dot{z}_i &= -k_{2,i}(t) \operatorname{sgn}(\tilde{s}_i) \end{aligned} \quad (36)$$

where ϕ_i is a bounded feedforward compensation term generated during escape activation.

The proposed barrier–energy-driven probabilistic structure preserves the second-order sliding architecture while introducing a nonlinear adaptive modulation mechanism that reshapes sliding evolution according to engagement conditions.

In the next sub-section, we rigorously establish closed-loop stability properties and derive sufficient conditions ensuring fixed-time convergence independent of initial conditions.

3.4. Closed-Loop Stability Analysis

In this sub-section, the finite-time stability of the proposed barrier–energy-driven probabilistic super-twisting guidance law is rigorously established. The analysis is conducted for each lateral channel $i \in \{y, z\}$. For simplicity, the subscript i is omitted where no ambiguity arises.

3.4.1. Closed-Loop Dynamics Under Probabilistic Modulation

From the sliding reformulation, the dynamics of the sliding variable can be written as

$$\dot{s}(t) = f(t) + u(t) + d(t) \quad (37)$$

where: $s(t) \in \mathbb{R}$ is the sliding variable, $d(t)$ is a lumped disturbance including target maneuvers, modeling uncertainties, observer residuals, and measurement perturbations:

$$|d(t)| \leq D, |\dot{d}(t)| \leq \bar{D} \quad (38)$$

This assumption is standard in super-twisting control and guarantees second-order sliding feasibility.

The proposed probabilistic barrier mechanism reshapes the sliding variable as

$$\tilde{s}(t) = \Psi(s(t)) \quad (39)$$

where the reshaping function satisfies

$$\Psi(0) = 0, |\tilde{s}| \leq |s|, \operatorname{sgn}(\tilde{s}) = \operatorname{sgn}(s) \quad (40)$$

The control law is given by:

$$\begin{aligned}\dot{s} &= -k_1(t) |\tilde{s}|^{\frac{1}{2}} \operatorname{sgn}(s) + z \\ \dot{z} &= -k_2(t) \operatorname{sgn}(s)\end{aligned}\quad (41)$$

The gain adaptation law guarantees bounded positive gains:

$$k_1(t) \in [k_{1,\min}, k_{1,\max}], k_2(t) \in [k_{2,\min}, k_{2,\max}] \quad (42)$$

With:

$$k_{1,\min} > 2D, k_{2,\min} > D \quad (43)$$

Define the lumped disturbance:

$$\Delta(t) = f(t) + d(t) \quad (44)$$

In Equation (44), it satisfies:

$$|\Delta(t)| \leq \bar{\Delta} \quad (45)$$

The closed-loop system becomes:

$$\begin{aligned}\dot{s} &= -k_1(t) |\tilde{s}|^{\frac{1}{2}} \operatorname{sgn}(s) + z + \Delta(t) \\ \dot{z} &= -k_2(t) \operatorname{sgn}(s)\end{aligned}\quad (46)$$

3.4.2. Gain Lower Bounds and Bounded Modulation

In Equation (42), the probabilistic gain adaptation law satisfies:

$$\begin{aligned}k_1(t) &\geq k_{1,\min} > 0 \\ k_2(t) &\geq k_{2,\min} > 0\end{aligned}\quad (47)$$

Moreover, the gains are upper bounded:

$$k_1(t) \leq k_{1,\max}, k_2(t) \leq k_{2,\max} \quad (48)$$

Therefore, despite stochastic modulation, gains remain within a compact positive interval. This property is essential to preserve second-order sliding mode convergence.

3.4.3. Lyapunov Function Construction

Theorem 1. Summarize Equations (44)–(46), then the origin $(s, z) = (0, 0)$ of the closed-loop system is globally fixed-time stable. Moreover, the settling time admits an upper bound independent of initial conditions.

Proof. Consider the Lyapunov function:

$$V(s, z) = \frac{2}{3} k_{1,\min} |s|^{\frac{3}{2}} + \frac{1}{2} z^2 \quad (49)$$

This function V is: positive definite, radially unbounded and continuously differentiable for $s \neq 0$. Its time derivative along system trajectories is

$$\dot{V} = k_{1,\min} |s|^{\frac{1}{2}} \dot{s} \operatorname{sgn}(s) + z \dot{z} \quad (50)$$

Substituting closed-loop dynamics:

$$\dot{V} = k_{1,\min} |s|^{\frac{1}{2}} \left(-k_1(t) |\tilde{s}|^{\frac{1}{2}} \operatorname{sgn}(\tilde{s}) + z + \Delta \right) \operatorname{sgn}(s) - k_2(t) z \operatorname{sgn}(\tilde{s}) \quad (51)$$

Because $\operatorname{sgn}(\tilde{s}) = \operatorname{sgn}(s)$, we obtain:

$$\dot{V} = -k_{1,\min} k_1(t) |s|^{\frac{1}{2}} |\tilde{s}|^{\frac{1}{2}} + k_{1,\min} |s|^{\frac{1}{2}} z + k_{1,\min} |s|^{\frac{1}{2}} \Delta - k_2(t) |z| \quad (52)$$

Using $|\tilde{s}| \leq |s|$, it follows that:

$$|s|^{\frac{1}{2}} |\tilde{s}|^{\frac{1}{2}} \geq |s| \quad (53)$$

Using gain lower bounds:

$$\dot{V} \leq -k_{1,min}k_{1,min}|s| + k_{1,min}|s|^{\frac{1}{2}}|z| + k_{1,min}|s|^{\frac{1}{2}}\bar{\Delta} - k_{2,min}|z| \quad (54)$$

□

3.4.4. Fixed-Time Convergence

Theorem 2. Summarize Equation (43), where $V(s, z) = \frac{2}{3}k_{1,min}|s|^{3/2} + \frac{1}{2}z^2$ is the Lyapunov function defined in Theorem 1, and $\bar{\Delta}$ denotes the upper bound of the lumped disturbance.

If the gain parameters satisfy:

$$k_{1,min}^2 > 2\bar{\Delta}, k_{2,min} > \frac{k_{1,min}^2}{2\epsilon} \quad (55)$$

For some $\epsilon > 0$, then the equilibrium point $(s, z) = (0, 0)$ of the closed-loop system is globally fixed-time stable. Moreover, the settling time is bounded by a constant independent of the initial conditions.

Proof. From inequality (54), the cross term $k_{1,min}|s|^{1/2}|z|$ can be bounded using Young's inequality:

$$|s|^{\frac{1}{2}}|z| \leq \frac{\epsilon}{2}|z|^2 + \frac{1}{2\epsilon}|s| \quad (56)$$

where $\epsilon > 0$ is an arbitrary constant.

Substituting (56) into (54) yields:

$$\dot{V} \leq -c_1|s| - c_2|z| + c_3|s|^{\frac{1}{2}} \quad (57)$$

For positive constants c_1, c_2 . Since:

$$|s| \geq \alpha V^{\frac{3}{2}}, |z| \geq \beta V^{\frac{1}{2}} \quad (58)$$

We derive:

$$\dot{V} \leq -\alpha V^{\frac{3}{2}} - \beta V^{\frac{1}{2}} \quad (59)$$

This differential inequality satisfies the standard fixed-time stability condition:

$$\dot{V} \leq -\alpha V^p - \beta V^q \quad (60)$$

The differential inequality (59) satisfies the standard fixed-time stability condition for nonlinear systems. Therefore, under ideal continuous-time assumptions, the origin is globally fixed-time stable. In practice, due to discretization, switching delays, and numerical integration effects, the trajectories converge to a small neighborhood of the origin, which is referred to as practical bounded convergence:

$$0 < p < 1, q > \frac{1}{2} \quad (61)$$

By standard fixed-time stability theory, the settling time satisfies:

$$T \leq \frac{1}{\alpha(1-p)} + \frac{1}{\beta(q-1)} \quad (62)$$

Since the bound does not depend on the initial conditions of $s(0)$ and $z(0)$, the system converges to the origin within a fixed time independent of the initial state.

Therefore: $s(t) \rightarrow 0$ and $z(t) \rightarrow 0$ for all $t \geq T$. This completes the proof. □

Remark 4. The barrier–energy probability differs from a deterministic bounded adaptive scheduler in both information structure and scheduling mechanism. A deterministic scheduler using the same normalized states reacts only after the sliding variable crosses a prescribed threshold. By contrast, the proposed probability law maps the barrier–energy separation into an escape probability, thereby

incorporating instantaneous error, disturbance intensity, historical fluctuation, and terminal urgency into a single risk measure. This enables pre-emptive gain adjustment before the sliding variable reaches a hard switching boundary. Moreover, deterministic switching or saturation may introduce nonsmooth gain transitions near the boundary, whereas the probability-coupled law provides a bounded and continuously reshaped modulation of the gains. Thus, the barrier–energy probability is not merely a randomized version of a deterministic scheduler; it provides a risk-sensitive performance-shaping mechanism under the same stability margin.

Remark 5. The role of Bernoulli activation is not to relax the stability condition but to provide a risk-weighted duty-cycle mechanism for performance shaping. Let $\sigma_i(t) \in \{0,1\}$ be generated conditionally on the current engagement information F_t according to $P_r\{\sigma_i(t) = 1|F_t\} = P_i(t)$. Then $E[\sigma_i(t)|F_t] = P_i(t)$, and the conditional expected gains satisfy $E[k_{1,i}(t)|F_t] = k_{1,0}[1 + \eta_1(1 - P_i(t)) + \eta_2 P_i(t)]$, $E[k_{2,i}(t)|F_t] = k_{2,0}[1 + \mu_1(1 - P_i(t)) + \mu_2 P_i(t)]$. Thus, although each sample path still uses an admissible binary activation, the average correction intensity varies continuously with the barrier–energy separation through $P_i(t)$. A deterministic hard scheduler, such as $\sigma_i D = 1\{E_i \geq \Delta V_i\}$, changes its activation state whenever $E_i - \Delta V_i$ crosses zero and may therefore generate relay-type boundary dithering under measurement noise, delay, and target maneuver oscillation. The Bernoulli mechanism spreads the high-gain activations in time with duty cycle $P_i(t)$, reducing deterministic phase locking between the target maneuver and the gain-switching instants. Therefore, the proposed mechanism is not claimed to dominate every possible deterministic scheduler in every realization; rather, under the same pathwise finite-time stability margin, it provides a stochastic performance-shaping layer that can reduce persistent switching synchronization and allocate high-gain control effort according to the estimated escape risk.

3.4.5. Influence of Probabilistic Reshaping

The reshaping function satisfies:

$$|\tilde{s}| \leq |s|, \text{sgn}(\tilde{s}) = \text{sgn}(s) \quad (63)$$

Therefore, it does not alter sign structure nor reduce effective control direction. It only scales magnitude. Since gains have positive lower bounds, convergence conditions remain satisfied. Thus, probabilistic modulation preserves fixed-time stability.

3.4.6. Closed-Loop Boundedness with Observer Dynamics

The composite observer error dynamics are linear and exponentially stable. Let observer error be $e_o(t)$. Then:

$$\dot{e}_o = A e_o + w(t) \quad (64)$$

where A is Hurwitz. Therefore:

$$e_o(t) \in \mathcal{L}_\infty. \quad (65)$$

Since e_o enters the system as part of $d(t)$, boundedness is preserved. Thus the entire closed-loop system is:

- Fixed-time stable,
- Globally bounded,
- Robust to bounded disturbance derivatives,
- Robust to probabilistic modulation.

To facilitate a system-level understanding of the interdependencies among the barrier–energy modulation, probabilistic sliding variable reshaping, and adaptive gain scheduling derived above, a graphical abstract summarizing the complete information flow and logic architecture of the proposed BMPSTG law is provided in Figure 2. This modular block diagram distills the ten-plus coupled equations presented in Section 3 into

an intuitive physical logic chain, clarifying how raw engagement measurements are transformed into executable missile acceleration commands.

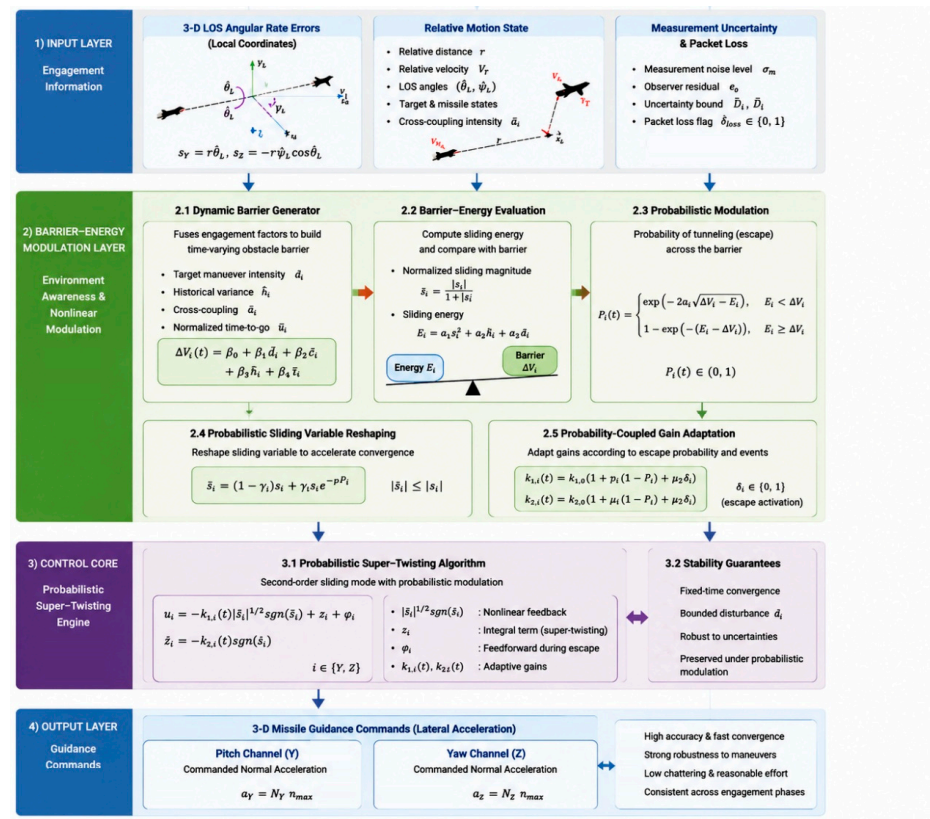


Figure 2. Graphical Abstract: Overall Architecture of the BMPSTG Law.

4. Observer Model Architecture

Consider the target position vector $P_T = [x_T, y_T, z_T]^T$ as the output available from the seeker/radar measurement channel. This does not mean that the true target position is exactly known by the observer. In the ideal observer derivation, P_T is used as the measured output to simplify the observer structure, while in practical implementation the available signal is a noisy, delayed, packet-loss-affected, and sample-and-hold updated measurement, as formalized later in Equation (84).

$$\begin{cases} \dot{P}_T = v_T \\ \dot{v}_T = a_T + d(t) \end{cases} \quad (66)$$

In the ideal observer model, the target position P_T is used as the measured signal. In the practical implementation, however, the observer input may be affected by measurement noise, time delay, packet loss, and sample-and-hold updating. These nonideal effects are explicitly modeled in Section 4.4 and are absorbed into the implementation-induced residual bound.

Applicability of this assumption. The above assumption should be interpreted as an output-availability assumption rather than a perfect-state-measurement assumption. It is applicable to active radar, imaging seekers, or fused tracking systems that can provide target/relative position information after preprocessing. If only bearing or LOS-angle measurements are available, an additional range/position reconstruction module is required before applying the proposed observer. The resulting reconstruction error can be incorporated into the implementation-induced residual bound L_i^{imp} in Section 4.4. Therefore, bounded measurement imperfection does not invalidate the observer stability result but enlarges the ultimate estimation-error bound and may degrade terminal accuracy.

Complete loss of position observability or unbounded sensing errors is beyond the scope of the present formulation.

4.1. FTDO Architecture

The composite observer contains an FTDO module and ESO module, where the FTDO is used for fast and accurate estimation of target velocity. Designed independently for each channel, the following auxiliary variables are defined for channel x as an example:

$$\begin{cases} \dot{e}_1 = x_T - z_1 \\ z_1 = \hat{x}_T \\ z_2 = \hat{v}_{T_x} \end{cases} \quad (67)$$

In Equation (67), z_1 is the estimated value of the position component in the x -axis direction, z_2 is the estimated value of the velocity component in the x -axis direction, and $\dot{e}_1$ is the estimated error value of the position component in the x -axis direction. The FTDO dynamic equation is structured on the basis of Equation (67):

$$\begin{cases} \dot{z}_1 = z_2 - \lambda_1 \cdot |e_1|^{\alpha_1} \text{sign}(e_1) \\ \dot{z}_2 = -\lambda_2 \cdot |e_1|^{\alpha_2} \text{sign}(e_1) + \hat{f}_x \end{cases} \quad (68)$$

In Equation (68), $\lambda_1, \lambda_2 > 0$, are the gain coefficients of the FTDO. α_1, α_2 is the power parameter of FTDO, which directly determines the convergence rate of observation error and the upper bound of convergence time. Satisfying $\alpha_2 = 2\alpha_1 - 1$, consider the value range of $\alpha_1 \in [0.7, 0.9]$, $\alpha_2 \in [0.4, 0.8]$. $\hat{f}_x$ is the perturbation estimate of the HESO and is the coupling term.

4.2. ESO Architecture

In the ESO module, the target acceleration and the unmodeled dynamics are considered as total perturbations to be estimated as expansion states. That is, the expansion state is defined as:

$$\begin{cases} x_1 = x_T \\ x_2 = v_{T_x} \\ x_3 = f_x \end{cases} \quad (69)$$

On the basis of Equation (69), the ESO dynamic equations are structured:

$$\begin{cases} e = x_1 - \hat{x}_1 \\ \dot{\hat{x}}_1 = \hat{x}_2 - \beta_1 e \\ \dot{\hat{x}}_2 = \hat{x}_3 - \beta_2 e + \hat{f}_x \\ \dot{\hat{x}}_3 = -\beta_3 e \end{cases} \quad (70)$$

In Equation (70), $\beta_1, \beta_2, \beta_3 > 0$ is the ESO gain. Its value is determined by the bandwidth:

$$\begin{cases} \beta_1 = 3\omega_0 \\ \beta_2 = 3\omega_0^2 \\ \beta_3 = 3\omega_0^3 \end{cases} \quad (71)$$

4.3. Coupling Mechanism of FTDO and HESO

From Equations (70) and (68), the perturbation estimate of the ESO is fed back to the FTDO, where the coupling term is:

$$\hat{f}_x = \hat{x}_3 \quad (72)$$

Combining Equation (66) with Equation (72) gives the final output as:

$$\hat{x}_2^{final} = \gamma \cdot z_2 + (1 - \gamma) \cdot \hat{x}_2 \quad (73)$$

In Equation (73), $\gamma \in [0, 1]$ denotes the FTDO-side fusion weight, while $1 - \gamma$ denotes the HESO-side weight. In the observer-comparison simulations, γ is not retuned case by case but is selected as a fixed phase-scheduled function, $\gamma(t) = wF(t) = \exp(-\kappa t)$, $1 - \gamma(t) = wH(t)$, $\kappa = 0.50 \text{ s}^{-1}$. This choice reflects the complementary roles of

the two observers: FTDO is assigned a large weight at the initial transient stage to provide fast reconstruction, whereas HESO gradually dominates after sufficient measurement history is accumulated. Since $wF(5.0) = 0.082$ and $wF(6.4) = 0.041$, the maneuver-critical interval $5.0 \leq t < 6.4$ s and the terminal phase are mainly governed by the HESO component. Therefore, the fusion rule is a fixed phase-scheduling strategy rather than a run-by-run optimized parameter.

4.4. Proof of Convergence of Composite Observer

In summary, combining Equations (62)–(69) gives the complete composite observer as:

$$\left\{ \begin{array}{l} FTDO : \begin{cases} \dot{z}_1 = z_2 - \lambda_1 \cdot |e_1|^{\alpha_1} \text{sign}(e_1) \\ \dot{z}_2 = -\lambda_2 \cdot |e_1|^{\alpha_2} \text{sign}(e_1) + \hat{f}_x \end{cases} \\ ESO : \begin{cases} \hat{x}_1 = \hat{x}_2 - \beta_1 e \\ \dot{\hat{x}}_2 = \hat{x}_3 - \beta_2 e + \hat{f}_x \\ \dot{\hat{x}}_3 = -\beta_3 e \end{cases} \\ Coupling : \hat{f}_x = \hat{x}_3 \\ Fusion : \hat{x}_2^{final} = \gamma \cdot z_2 + (1 - \gamma) \cdot \hat{x}_2 \end{array} \right. \quad (74)$$

Let $e_2 = x_2 - z_2$, then the error dynamics equation is:

$$\begin{cases} \dot{e}_1 = e_2 - \lambda_1 \cdot |e_1|^{\alpha_1} \text{sign}(e_1) \\ \dot{e}_2 = \dot{f}_x - \dot{z}_2 = \dot{f}_x + \lambda_2 \cdot |e_1|^{\alpha_2} \text{sign}(e_1) - \dot{\hat{f}}_x \end{cases} \quad (75)$$

Theorem 3. Assuming that f_x is bounded, and that $\dot{f}_x$ must be bounded, and that its derivatives must also be bounded, then the following exists:

$$|\dot{f}_x - \dot{\hat{f}}_x| \leq M \quad (76)$$

There must exist a fixed time $T_1 > 0$ such that for any initial conditions $e_1(0), e_2(0)$, the errors $e_1(t), e_2(t)$ converge to zero after time $t \geq T_1$.

Proof. To summarize, consider the establishment of the Lyapunov function:

$$V_1 = \frac{1}{2} e_1^2 + \frac{1}{2} e_2^2 \quad (77)$$

Derivation of Equation (77) yields:

$$\begin{aligned} \dot{V}_1 &= e_1 \dot{e}_1 + e_2 \dot{e}_2 \\ &= e_1 [e_2 - \lambda_1 \cdot |e_1|^{\alpha_1} \text{sign}(e_1)] + e_2 [\dot{f}_x + \lambda_2 \cdot |e_1|^{\alpha_2} \text{sign}(e_1) - \dot{\hat{f}}_x] \\ &= -\lambda_1 |e_1|^{\alpha_1+1} + e_1 e_2 + \lambda_2 e_2 |e_1|^{\alpha_1} \text{sign}(e_1) + e_2 (\dot{f}_x - \dot{\hat{f}}_x) \end{aligned} \quad (78)$$

Using Young's inequality, it can be obtained that:

$$\begin{aligned} |e_1 e_2| &\leq \frac{1}{2} \epsilon_1 |e_1|^2 + \frac{1}{2\epsilon_1} |e_2|^2 \\ |\lambda_2 e_2 |e_1|^{\alpha_2} \text{sign}(e_1)| &\leq \frac{1}{2} \epsilon_2 \lambda_2^2 |e_1|^{2\alpha_2} + \frac{1}{2\epsilon_2} |e_2|^2 \\ \left| e_2 (\dot{f}_x - \dot{\hat{f}}_x) \right| &\leq \frac{1}{2} \epsilon_3 M^2 + \frac{1}{2\epsilon_3} |e_2|^2 \end{aligned} \quad (79)$$

Choosing the appropriate $\epsilon_1, \epsilon_2, \epsilon_3$, it can be obtained that:

$$\dot{V}_1 \leq -c_1 V_1^{\frac{\alpha_1+1}{2}} - c_2 V_1^{\frac{2\alpha_2+1}{2}} + c_3 \quad (80)$$

Since $\alpha_2 = 2\alpha_1 - 1$, there is:

$$\frac{2\alpha_2 + 1}{2} = \frac{2(2\alpha_1 - 1) + 1}{2} = \frac{4\alpha_1 - 1}{2} > 1 \quad (81)$$

According to Lyapunov stability theory, the system is finite-time asymptotically time stable. The proof is complete. $\square$

Let $\tilde{x}_1 = x_1 - \hat{x}_1$, $\tilde{x}_2 = x_2 - \hat{x}_2$, $\tilde{x}_3 = f_x - \hat{x}_3$. Then the error dynamics can be expressed as:

$$\begin{cases} \dot{\tilde{x}}_1 = \tilde{x}_2 - \beta_1 \tilde{x}_1 \\ \dot{\tilde{x}}_2 = \tilde{x}_3 - \beta_2 \tilde{x}_1 \\ \dot{\tilde{x}}_3 = \dot{f}_x - \beta_3 \tilde{x}_1 \end{cases} \quad (82)$$

Then Equation (82) can be written in matrix form:

$$\dot{\tilde{x}} = A\tilde{x} + Bw_i(t) \quad (83)$$

The implemented measurement for the i -th channel is modeled as

$$\begin{aligned} y_{i(t)} &= y_{i,k}, \quad t \in [t_k, t_{k+1}), \\ y_{i,k} &= \gamma_k [p_{T,i(t_k-\tau_k)+v_{i,k}}] + (1-\gamma_k)y_{i,k-1}, \end{aligned} \quad (84)$$

where $t_k = kT_s$, $|v_{i,k}| \leq \bar{v}_i$, $0 \leq \tau_k \leq \bar{\tau}$, and $\gamma_k \in \{0, 1\}$. If $|v_{T,i(t)}| \leq \bar{v}_i$ and the maximum consecutive packet-loss number is N_l , then

$$|\eta_{y,i(t)}| \leq \bar{v}_i + \bar{v}_i[\bar{\tau} + (N_l + 1)T_s] \triangleq \bar{\eta}_{y,i}. \quad (85)$$

The residual input is decomposed as

$$w_{i(t)} = \dot{f}_{i(t)} + r_{y,i(t)} + r_{sgn,i(t)} + r_{aux,i(t)}, \quad (86)$$

and satisfies

$$|w_{i(t)}| \leq L_{f,i} + \beta_{3,i}\bar{\eta}_{y,i} + \bar{c}_{sgn,i}\varepsilon_s + \bar{r}_{aux,i} \triangleq L_i^{imp}. \quad (87)$$

Within Equation (83):

$$A = \begin{bmatrix} -\beta_1 & 1 & 0 \\ -\beta_2 & 0 & 1 \\ -\beta_3 & 0 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \quad (88)$$

Theorem 4. The ESO error system is exponentially input-to-state stable if A is Hurwitz and the implementation-induced residual input satisfies $|w_{i(t)}| \leq L_i^{imp}$.

Proof. Since A is Hurwitz, there exists a positive definite matrix P satisfying the Lyapunov equation:

$$A^T P + PA = -Q \quad (89)$$

where $Q > 0$. Consider the Lyapunov function as:

$$V_2 = \tilde{x}^T P \tilde{x} \quad (90)$$

Derivation of Equation (82) gives:

$$\begin{aligned} \dot{V}_2 &= \dot{\tilde{x}}^T P \tilde{x} + \tilde{x}^T P \dot{\tilde{x}} \\ &= (A\tilde{x} + Bw_{i(t)})^T P \tilde{x} + \tilde{x}^T P (A\tilde{x} + Bw_{i(t)}) \\ &= \tilde{x}^T (A^T P + PA) \tilde{x} + 2\tilde{x}^T P B w_{i(t)} \\ &\leq -\lambda_{\min}(Q) \|\tilde{x}\|^2 + 2\|PB\| L_i^{imp} \|\tilde{x}\| \end{aligned} \quad (91)$$

Thus, it can be obtained that:

$$\dot{V}_2 \leq -\frac{\lambda_{\min}(Q)}{2\lambda_{\max}(P)} V_2 + \frac{2\|PB\| L_i^{imp}}{\lambda_{\min}(\sqrt{P})} V_2 \quad (92)$$

From Equation (92), $\dot{V}_2 < 0$ whenever $\|\tilde{x}_i\| > \frac{2\|PB\| L_i^{imp}}{\lambda_{\min}(Q)}$.

Therefore, the ESO estimation error is ultimately bounded by

$$\lim_{\sup t \rightarrow \infty} \|\tilde{x}_i\| \leq \frac{2\|PB\| L_i^{imp}}{\lambda_{\min}(Q)}. \quad (93)$$

This explicitly shows that the residual set is proportional to the implementation-induced bound L_i^{imp} , which aggregates measurement noise, delay, packet loss, sign regularization, sample-and-hold, and auxiliary-compensation errors. The proof is complete.

Therefore, the ESO estimation error is ultimately bounded by

$$\dot{V}_2 \leq -\frac{\lambda_{\min}(Q)}{2\lambda_{\max}(P)}V_2 + \frac{2\|PB\|L_i^{imp}}{\lambda_{\min}(\sqrt{P})}V_2 \quad (94)$$

□

Theorem 5. *The composite observer as a whole is stable and there exists a time $T > 0$ and a positive constant c such that, for $t \geq T$, there is:*

$$\|e_1(t)\| + \|e_2(t)\| + \|\tilde{x}_1(t)\| + \|\tilde{x}_2(t)\| + \|\tilde{x}_3(t)\| < cL_i^{imp} \quad (95)$$

Proof. Consider the Lyapunov function for composite systems:

$$V = V_1 + \mu V_2 \quad (96)$$

In Equation (96) $\mu > 0$ is the coupling parameter to be determined. The derivation is obtained:

$$\dot{V} = \dot{V}_1 + \mu \dot{V}_2 \quad (97)$$

Combining Equation (80) with Equation (92), it can be obtained that:

$$\dot{V} \leq -c_1V_1^{\frac{\alpha_1+1}{2}} - c_2V_1^{\frac{2\alpha_2+1}{2}} - \mu\frac{\lambda_{\min}(Q)}{2\lambda_{\max}(P)}V_2 + \mu\frac{2\|PB\|L_i^{imp}}{\lambda_{\min}(\sqrt{P})}V_2 + c_3 \quad (98)$$

Since FTDO converges in fixed time, there exists $T_1 > 0$ such that, for $t \geq T_1$, there is $e_1 \approx 0, e_2 \approx 0$. At this point, the coupling term $\hat{f}_x = \hat{x}_3$ provides accurate compensation for perturbations, and the ESO error system simplifies to:

$$\dot{\tilde{x}} = A\tilde{x} + B(\dot{\hat{f}}_x - \hat{\dot{f}}_x) \quad (99)$$

Since FTDO has converged, $\dot{\hat{f}}_x - \hat{\dot{f}}_x \approx 0$, ESO reaches its optimal estimation performance. Choosing a sufficiently large μ ensures that $\dot{V} < 0$ when V is large enough so that the composite system is consistently eventually boundedly stable. □

To provide a system-level visualization of how the complementary capabilities of the FTDO and HESO are orchestrated under delayed and lossy measurements, a graphical abstract summarizing the dual-channel cooperative architecture and information fusion logic of the composite observer is presented in Figure 3. This modular block diagram distills the multi-order observer designs, finite-time convergence analyses, and consistency boundary conditions established in Section 4 into an intuitive signal-flow representation, enabling readers to readily comprehend the complete disturbance reconstruction chain.

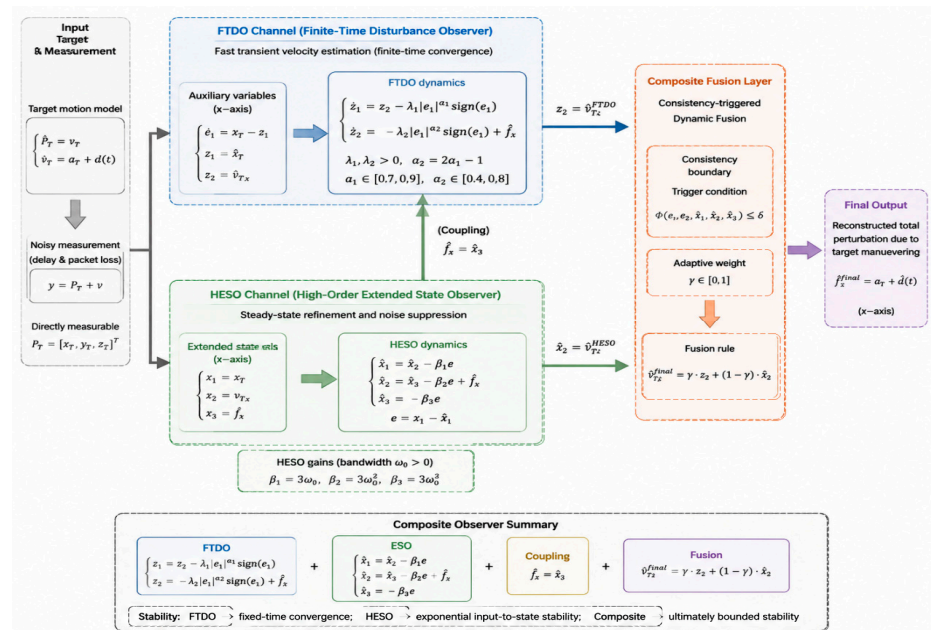


Figure 3. Graphical Abstract: FTDO–HESO Composite Observer Architecture.

5. Simulation Results

5.1. Simulation Results and Mechanism Analysis of the BMPSTG Guidance Law

In summary, in this section, the guidance law comparison ablation experiment will be constructed to utilize multiple metrics to verify that the BMPSTG has high accuracy, and the total cumulative energy is calculated to verify that the energy dissipation of BMPSTG will be different compared to that of the PNG with SMC and super-twisting guidance (STG).

The gravitational acceleration is set to $g_0 = 9.81 \text{ m/s}^2$. The simulation step is selected as $1 \times 10^{-4} \text{ s}$, while the guidance command update period is set to $1 \times 10^{-2} \text{ s}$. The total simulation duration is 20 s, and the guidance computation is terminated when the relative distance decreases below 100 m.

The initial missile state is defined as:

$$X_M(0) = [x_M(0) \ y_M(0) \ z_M(0) \ V_M(0) \ \theta_M(0) \ \psi_M(0)]^T \quad (100)$$

With:

$$X_M(0) = [0 \ 0 \ 0 \ 1800 \ 47^\circ \ -4^\circ]^T \quad (101)$$

where the position components are expressed in meters, the velocity is in meters per second, and the flight path and heading angles are in degrees.

The initial target state is given by:

$$X_T(0) = [x_T(0) \ y_T(0) \ z_T(0) \ V_T(0) \ \theta_T(0) \ \psi_T(0)]^T \quad (102)$$

With:

$$X_T(0) = [25000 \ 25000 \ 0 \ 1000 \ -45^\circ \ -175^\circ]^T \quad (103)$$

In Figure 4, compared with PNG, SMC, and STG, the proposed BMPSTG generates a smoother interception trajectory with a larger radius of curvature, rather than directly following the target's maneuvering path. This indicates that the proposed method actively reconstructs the interception geometry instead of passively tracking the target motion.

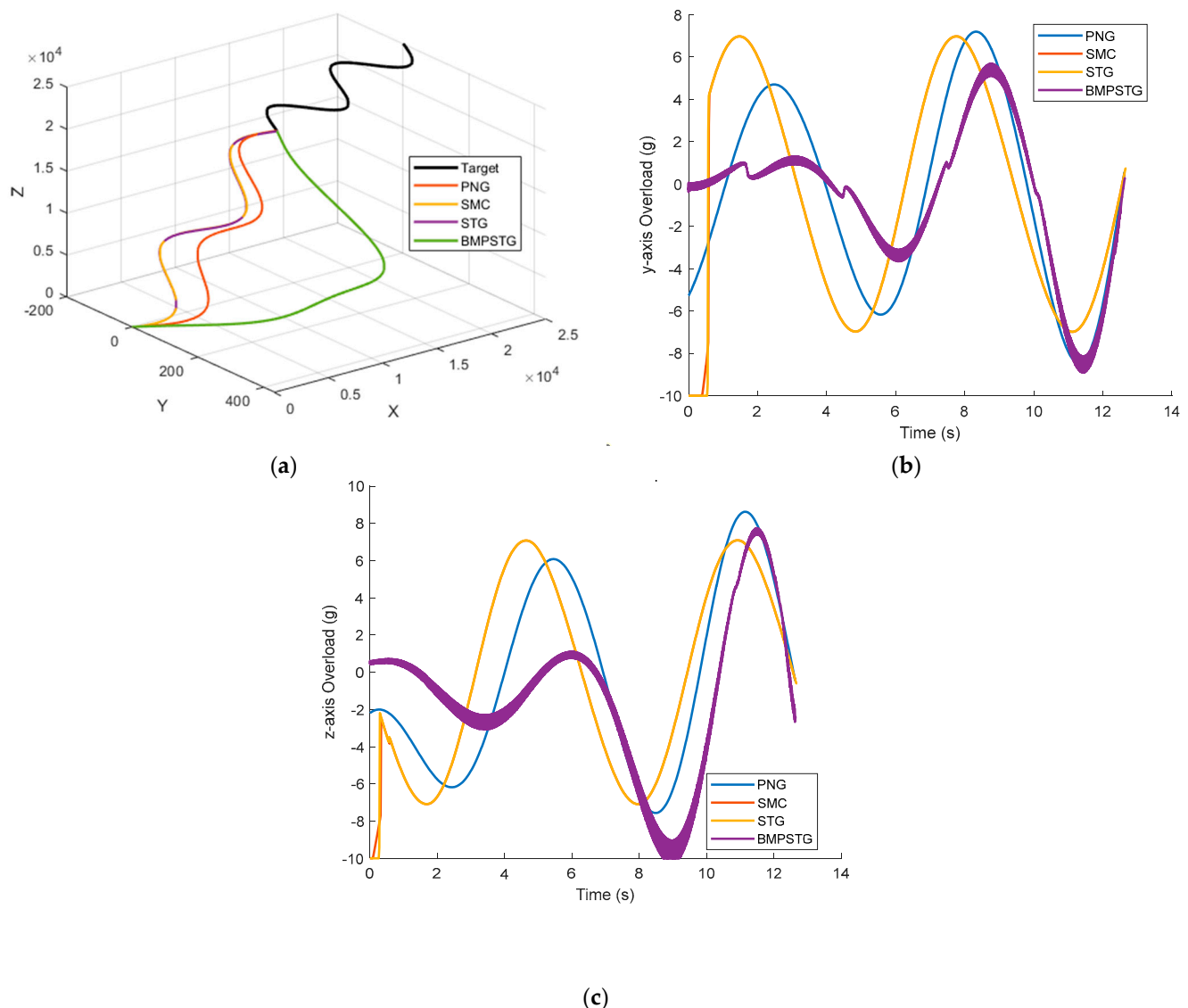


Figure 4. Comparison of 3D Interception Trajectories and Normal Acceleration Responses for Four Guidance Laws. (a) 3D Trajectories of Target and Four Guidance Laws; (b) y-axis Overload Comparison; (c) z-axis Overload Comparison.

Meanwhile, oscillations appear in both the y-axis and z-axis overload responses of BMPSTG, which mainly originate from the stochastic tunneling-triggered compensation, adaptive gain adjustment, and the intrinsic oscillatory behavior of sliding mode convergence under sinusoidal target maneuvers. However, the overload amplitude remains significantly smaller than those of the other three guidance laws, showing that BMPSTG effectively suppresses excessive control effort while maintaining convergence.

In particular, the z-axis overload becomes relatively large during the mid-course phase because the azimuth correction dominates the interception geometry reconstruction, whereas the terminal overload remains small due to the reduced line-of-sight angular rate and the smooth convergence property of the tunneling-based adaptive mechanism.

The performance degradation index shown in Figure 5d is computed from the same Monte Carlo data used in Figure 5a–c. For each guidance law m , let $e_{k,m}$, $J_{k,m}$, and $T_{k,m}$ denote the terminal error, accumulated control energy, and convergence time in the k -th Monte Carlo run. Their sample means are

$$\mu_e^m = \frac{1}{N_{MC}} \sum_{k=1}^{N_{MC}} e_{k,m} \quad (104)$$

$$\mu_j^m = \frac{1}{N_{MC}} \sum_{k=1}^{N_{MC}} J_{k,m} \quad (105)$$

$$\mu_T^m = \frac{1}{N_{MC}} \sum_{k=1}^{N_{MC}} \mu_{k,m}. \quad (106)$$

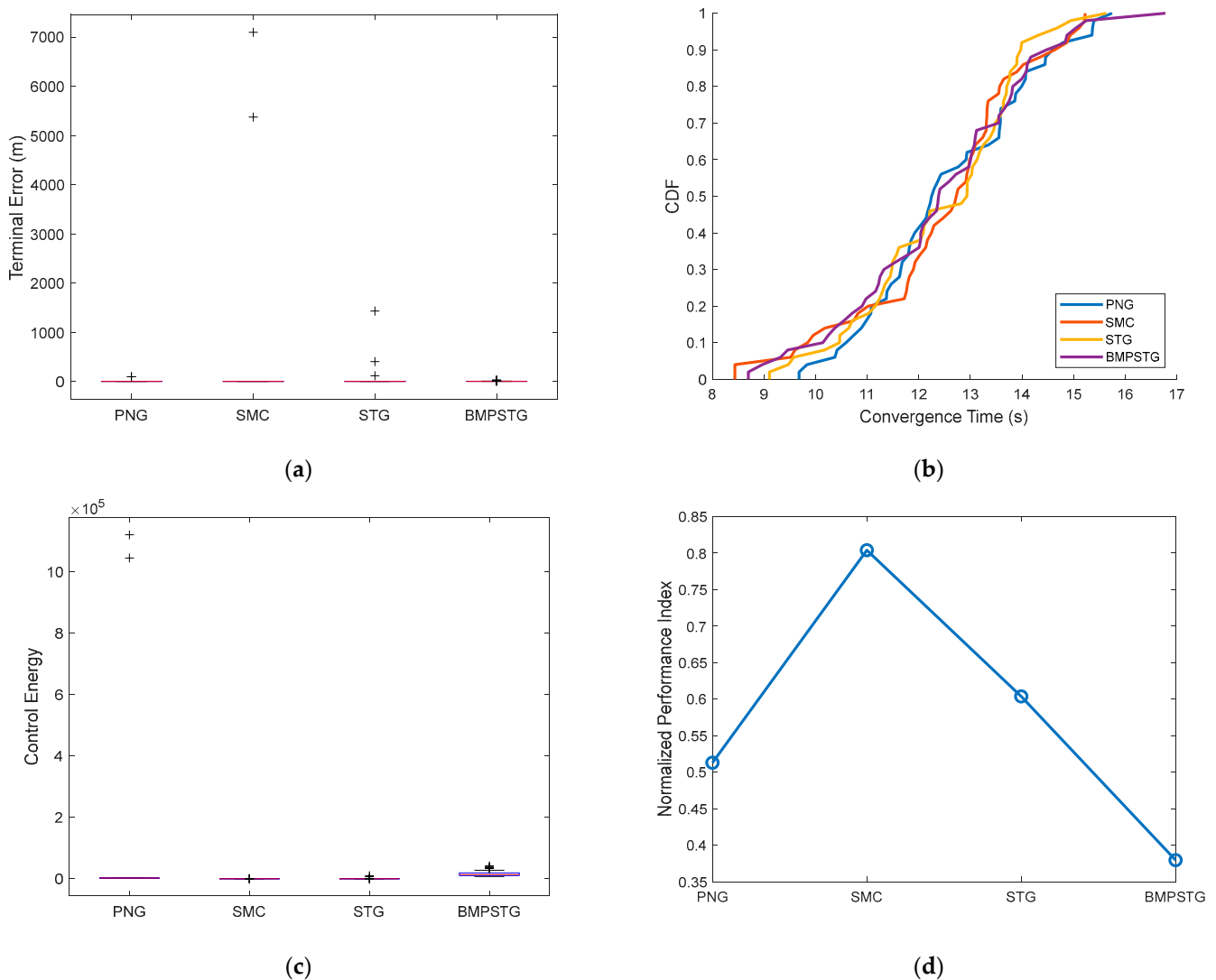


Figure 5. Monte Carlo Statistical Comparison of Terminal Error, Control Energy, Convergence Time, and Overall Performance for Four Guidance Laws. (a) Terminal Error Boxplot; (b) Control Energy Boxplot; (c) Convergence Time CDF; (d) Performance Degradation Curve.

Each cost metric is normalized by the maximum mean value among the four compared guidance laws:

$$\tilde{e}_m = \frac{\mu_e^m}{\max_l \mu_e^l} \quad (107)$$

$$\tilde{j}_m = \frac{\mu_j^m}{\max_l \mu_j^l} \quad (108)$$

$$\tilde{t}_m = \frac{\mu_t^m}{\max_l \mu_t^l}. \quad (109)$$

The aggregate degradation index is then defined as

$$D_m = 0.5\tilde{e}_m + 0.3\tilde{J}_m + 0.2\tilde{T}_m, \quad (110)$$

where $w_e = 0.5$, $w_J = 0.3$, and $w_T = 0.2$. These weights are fixed before comparison and reflect the design priority that terminal accuracy is primary, control energy is secondary, and convergence time is tertiary. Since all three terms are cost-type metrics, a smaller D_m indicates better overall performance.

Table 1 presents the parameter tuning and fair comparison protocol. To ensure a fair comparison, all parameters were tuned before the reported simulations and then kept fixed. PNG, SMC, STG, and BMPSTG were evaluated under the same engagement conditions, Monte Carlo samples, target maneuver law, integration step, guidance update period, termination criterion, and missile command limitation. The PNG navigation constant was selected within its conventional feasible range, while the SMC and STG gains were chosen to satisfy the same disturbance-rejection requirement determined by the target maneuver and geometric coupling bounds. BMPSTG uses the same STG robustness baseline and admissible gain bounds and only adds the barrier–energy probability and probability-coupled gain reshaping. For the observer comparison, FTDO, ESO, FTDO–ESO, and FTDO–HESO were tested under the same BMPSTG guidance law, noise distribution, delay buffer, packet-loss process, random seed, and command saturation; the composite observers retain the standalone observer gains and differ only in the listed fusion rule. No method was retuned for individual Monte Carlo runs or specific observer cases.

Table 1. Monte Carlo Simulation Conditions and Parameters.

Category	Parameter	Symbol	Value/Distribution	Unit
Fixed settings	Simulation step size	T_s	1.0×10^{-4}	s
	Guidance command update period	T_g	0.01	s
	Maximum simulation duration	t_{max}	20	s
	Termination distance threshold	r_{min}	100	m
	Missile initial state	$\mathbf{x}_{m0}$	Equation (95)	-
Monte Carlo setup	Random number generator seed	-	rng(MC)	
	Performance degradation weights	$\omega_1, \omega_2, \omega_3$	0.5, 0.3, 0.2	
Randomized parameter (per run)	Initial line-of-sight range		$25,000 + 3000 \cdot \mathcal{N}(0,1)$	m; zero-mean Gaussian
Nominal parameter	Nominal initial range		25000	m; deterministic baseline
	Target flight-path perturbation	$\Delta\theta_T$	$0.015 \mathcal{N}(0,1)$	rad
	Target heading perturbation	$\Delta\psi_T$	$0.015 \mathcal{N}(0,1)$	rad
	Target maneuver gain	m_g	$1 + 0.15 \mathcal{N}(0,1)$	-
	Target maneuver command	a_T	$m_g \cdot 7g0 \sin(t)$	m/s^2
	Sign regularization parameter	ε_s	0.95	-
	Barrier/guidance parameter	μ_1	1.2	-
	Power exponent	p	0.7	-

To comprehensively evaluate the performance of the proposed BMPSTG guidance law, 100 Monte Carlo simulations were conducted to compare PNG, SMC, STG, and BMPSTG in terms of terminal error, control energy, convergence time, and the overall performance degradation index. The statistical results demonstrate that BMPSTG exhibits clear superiority in interception accuracy, robustness, and overall performance.

First, as shown in the boxplot of terminal error, PNG exhibits an outlier of approximately 1132.806 m, indicating that, although it maintains a certain level of stability in most

cases, noticeable terminal deviation still occurs under some operating conditions. In contrast, the outliers of SMC and STG reach $2.83 \times 10^4 \sim 4.04 \times 10^4$ m and $2.93 \times 10^4 \sim 4.09 \times 10^4$ m, respectively, and these large values should be interpreted as failed-capture Monte Carlo outliers rather than as terminal miss distances after a successful homing process. In the Monte Carlo driver used for Figure 5, 100 runs are generated with $rng(1)$, and the initial range is randomized as $R_0 = 25000 + 3000\xi$ m, where $\xi \sim \mathcal{N}(0,1)$. Under some unfavorable large-range samples, the deterministic SMC and STG gains are kept fixed and constrained by the same command limitation, so the LOS sliding variables cannot converge before the maximum simulation time. The recorded terminal error in these cases is therefore the remaining relative distance at simulation termination or closest approach, which explains the 10^4 m-level outliers. Hence, the apparently low control energy of SMC/STG in these samples should not be interpreted as high efficiency but as a symptom of ineffective homing or failed capture, which are significantly larger than those of PNG and BMPSTG. This indicates that both SMC and STG are prone to severe divergence under random initial conditions and target maneuver disturbances, resulting in substantial dispersion of terminal performance. By comparison, the outliers of BMPSTG are confined to the range of 510.0293–1006.3895 m, which is not only significantly lower than those of SMC and STG, but also better than the extreme error level of PNG. These results indicate that BMPSTG effectively suppresses large-error events and achieves stronger robustness and higher terminal interception accuracy under uncertain conditions.

Second, from the boxplot of control energy, the median energy of PNG reaches 1.9247, with no upper whisker observed, indicating that PNG maintains a relatively high control output in most samples and thus exhibits a typical high-energy-consumption characteristic. Although the median control energies of SMC and STG are 0.5398 and 0.33198, respectively, both are lower than the 0.94514 of BMPSTG. However, when interpreted together with the terminal error statistics, such apparently low energy consumption does not imply higher control efficiency; rather, it mainly reflects the inability of these methods to sustain an effective and stable guidance process in a considerable number of cases. Therefore, such low energy should be regarded as an apparent low-consumption phenomenon under unstable conditions. In contrast, BMPSTG maintains robust interception performance and system stability while keeping the control energy substantially lower than that of PNG, indicating that it achieves a more reasonable balance between control effort and guidance accuracy. This empirical result corroborates the theoretical distinction drawn in Section 3.3: by replacing the memoryless, uniform correction of classical adaptive gain control with a probabilistic, context-sensitive barrier-energy modulation, BMPSTG allocates high control effort only when the dynamic barrier model predicts a genuine risk of divergence, rather than reactively escalating gains at every large instantaneous error.

Third, it should be emphasized that, while the theoretical framework establishes ideal fixed-time convergence to the origin (Theorem 2), the Monte Carlo simulation results presented here reflect practical bounded convergence due to discretization and numerical integration effects. Consequently, the term “effective convergence” used below refers to the trajectories entering and remaining within a predefined small bound, rather than reaching zero exactly. The convergence-time CDF further reveals the dynamic convergence characteristics of the four guidance laws. PNG and SMC start their statistical distribution from zero, whereas the effective convergence onset times of STG and BMPSTG are 9.5168 s and 9.0175 s, respectively, indicating that BMPSTG enters the effective convergence phase earlier than STG. Before 10.7987 s, BMPSTG remains in the most favorable position overall, showing stronger convergence capability in the early effective convergence stage. Although STG temporarily performs slightly better than BMPSTG over a limited time interval, BMPSTG regains its advantage as time evolves and becomes the best-

performing method again after 13.6547 s. This phenomenon suggests that BMPSTG does not merely pursue the fastest instantaneous convergence but instead optimizes the convergence process over the entire engagement through barrier-potential regulation, adaptive gain updating, and control law reconstruction. As a result, it achieves better stage-wise convergence quality as well as superior global convergence behavior.

Finally, the performance degradation curve provides further confirmation of the above observations. The degradation indices of PNG, SMC, STG, and BMPSTG are 0.5129, 0.8038, 0.6037, and 0.3796, respectively. According to the above definition, the degradation index is a weighted normalized cost composed of the mean terminal error, mean control energy, and mean convergence time. The percentage reduction of BMPSTG relative to a baseline method b is calculated as $R_b = \frac{D_b - D_{BMPSTG}}{D_b} \times 100\%$. Among all four guidance laws, BMPSTG achieves the minimum degradation index, corresponding to reductions of approximately 26.0%, 52.8%, and 37.1% compared with PNG, SMC, and STG, respectively. This demonstrates that BMPSTG achieves the best overall trade-off under multiple performance constraints. Its superiority is reflected not only in improvements of individual metrics but also in the systematic enhancement of the overall guidance quality.

In summary, the Monte Carlo statistical results clearly demonstrate that, compared with PNG, SMC, and STG, the proposed BMPSTG guidance law can significantly reduce the dispersion of terminal error, effectively suppress extreme unstable cases, and establish a better balance between control energy and convergence performance. In particular, its best result in the overall performance degradation index verifies that BMPSTG possesses stronger robustness, higher interception accuracy, and better engineering applicability for highly maneuvering target interception scenarios.

As shown in the Figure 6, the barrier heights in the Y- and Z-channels of BMPSTG almost completely overlap and evolve smoothly, indicating strong symmetry and continuous barrier regulation in both channels; the local rises and drops imply dynamic switching between stability preservation and cross-barrier accessibility. Meanwhile, both K_1 and K_2 remain under small but dense high-frequency adjustment for most of the engagement, and exhibit synchronized “peak–valley–peak” patterns around $t = 1.7088$, 4.5201 , and 7.5247 s, indicating that BMPSTG temporarily relaxes deterministic control at critical state-transition instants to suppress over-control and control shock, while restoring the gains before and after these instants to maintain convergence continuity. Correspondingly, the Y-channel tunneling probability rises sharply at these critical moments, whereas the Z-channel probability remains at a moderate level throughout, showing that high-probability tunneling is activated only when necessary. Overall, BMPSTG operates through a coupled closed-loop mechanism of barrier regulation, probability triggering, and gain reconfiguration, thereby enabling rapid escape from unfavorable states, reduced control impact, and smoother convergence.

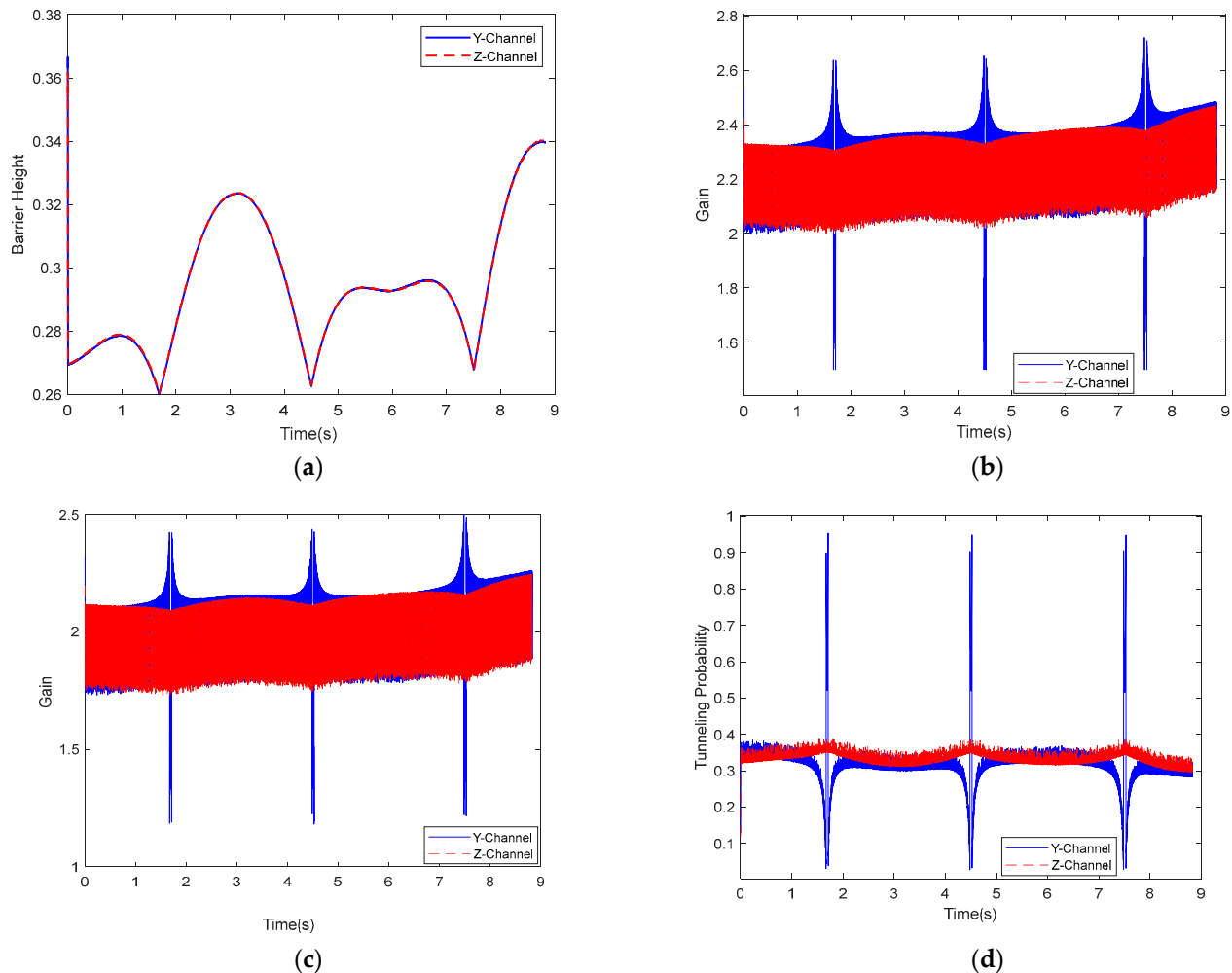


Figure 6. Coupled Time-Varying Evolution of Barrier Height, Adaptive Gains, and Tunneling Probability under BMPSTG Guidance. (a) Time-Varying Evolution of Barrier Heights in the Y- and Z-Channels under BMPSTG Guidance; (b) Time-Varying Response of the First-Stage Adaptive Gain K_1 in the Y- and Z-Channels under BMPSTG Guidance; (c) Time-Varying Response of the Second-Stage Adaptive Gain K_2 in the Y- and Z-Channels under BMPSTG Guidance; (d) Time-Varying Evolution of Tunneling Probability in the Y- and Z-Channels under BMPSTG Guidance.

5.2. Comparison of Disturbance Reconstruction Performance Under Large-Maneuver Target Engagements

To further evaluate the applicability of the composite observers under measurement noise, delay, and packet loss, this sub-section compares FTDO, ESO, FTDO-ESO, and FTDO-HESO under the unified BMPSTG guidance law. Different from the previous case, the target here performs a more challenging large-amplitude lateral turning maneuver. It initially flies along its original heading, then suddenly undergoes a strong lateral turn in mid-course, and finally continues straight flight along the new heading. Such a scenario amplifies the influence of observation lag, state reconstruction error, and high-frequency noise on the guidance loop, thereby providing a more stringent benchmark for assessing the delay-suppression capability, disturbance reconstruction quality, and terminal guidance performance of different observer configurations.

As shown in Table 2, the observer comparison simulations use position measurements affected by delay, noise, and packet loss. The measurement noise is generated as zero-mean Gaussian noise with standard deviation $\sigma_{obs} = 6$ m. For the residual-bound calculation, it is represented by the practical bound $\bar{v}_i = 3\sigma_{obs} = 18$ m. The measurement

delay is implemented by a buffer of 80 integration steps, corresponding to $\tau_d = 0.08$ s. Together with the guidance update period $T_g = 0.01$ s, the prediction horizon used by FTDO/ESO is $\tau_c = 0.015$ s, whereas the HESO prediction horizon is $\tau_h = 0.020$ s. Packet loss is modeled as an independent Bernoulli process with probability $p_l = 0.04$. These implementation parameters are used to construct the residual bound L_i^{imp} in Section 4.4.

Table 2. Simulation Conditions and Implementation Parameters for Observer-Configuration Comparison.

Category	Parameter	Symbol	Value/Setting	Unit/Remark
Simulation setting	Integration step size	T_s	1.0×10^{-3}	s
	Guidance update period	T_g	T_s	-
	Maximum simulation time	T_{max}	30.0	s
	Random seed	-	rng(1, 'twister')	Reset before each observer case
Interception condition	Hit range	r_{hit}	10.0	m
	Break-after-minimum-range threshold	r_{br}	120.0	m
Initial condition	Missile initial state	X_{M0}	$[0, 0, 0, 2000, 2^\circ, 0^\circ]^T$	m, m/s, deg
	Target initial state	X_{T0}	$[26000, 25000, 0, 1000, 0^\circ, 180^\circ]^T$	m, m/s, deg
	Lateral acceleration command	a_{Tz}	$14g_0$ for $5.0 \leq t < 6.4$; otherwise 0	$a_{Ty} = 0$
Measurement noise	Position noise distribution	v_k	$\mathcal{N}(0, 6^2 I_3)$	m
	Practical noise bound	$\bar{v}_i$	$3\sigma_{obs} = 18$	m
Packet loss	Loss handling	-	Previous valid measurement is held	Sample-and-hold
Observer prediction	Common prediction horizon	τ	$\tau_d = 0.08$	s
Target maneuver	Maneuver start time	t_s	5.0	s
	Maneuver end time	t_e	6.4	s
Delay	Measurement-buffer delay	N_d	80	Samples
	Measurement-delay time	τ_d	0.08	s
	FTDO/ESO prediction horizon	τ_c	$N_d T_s + T_g = 0.015$	s
	HESO prediction horizon	τ_h	$\tau_c + 0.5 T_g = 0.020$	s
Packet loss	Packet-loss probability	p_l	0.04	Bernoulli loss
	Maximum consecutive packet losses	N_l	Finite simulation cap	-
FTDO	FTDO gains	λ_1, λ_2	50, 1500	Fixed for all channels
ESO	ESO gains	$\beta_1, \beta_2, \beta_3$	45, 600, 2500	Conventional ESO
HESO	HESO gains	$\beta_1^h, \beta_2^h, \beta_3^h$	90, 1800, 14,000	High-order ESO
HESO	Velocity moving-average window	N_{ma}	25	Samples
Fusion rule	FTDO + ESO fusion	-	$w_F = 0.55e^{-0.10t} + 0.35, w_E = 1 - w_F$	-
	FTDO + HESO fusion	-	$w_F = e^{-0.50t}, w_H = 1 - w_F$	-

As shown in Table 3, the four observers exhibit clearly different behaviors in the large-maneuver scenario. The standalone ESO achieves the best average estimation accuracy, with RMSE Position = 6.8765 and RMSE Velocity = 86.809, indicating that its high-gain estimation mechanism provides the most favorable full-time-domain smoothing effect. However, from the perspective of terminal guidance performance, FTDO–HESO yields the smallest miss distance, namely 7.5373 m, outperforming FTDO (9.9008 m), ESO

(9.5312 m), and FTDO–ESO (9.8426 m). This result suggests that FTDO–HESO does not necessarily minimize the global estimation RMSE, but it is more effective in suppressing lag and improving state usability during the terminal critical interval. Moreover, the interception time of FTDO–HESO is 24.304 s, which remains on the same order as the other methods. Therefore, the reduction in miss distance is not achieved by significantly prolonging the engagement but rather by a more effective state prediction and control allocation process. It should also be noted that FTDO–HESO produces the largest energy cost and CmdTV, reaching 5.9156×10^5 and 3.6263×10^5 , respectively. Therefore, CmdTV is interpreted here as a cumulative measure of total command activity over the full engagement, rather than as a direct indicator of local high-frequency chattering. The following discussion of chattering suppression refers only to the local Z-channel high-frequency oscillation during the critical maneuvering interval, not to a reduction in the global command total variation. Hence, its main advantage lies in trading a higher control effort for stronger terminal correction capability, smaller miss distance, and better delay-compensation effectiveness. In other words, the present results do not imply that FTDO–HESO is uniformly superior in every statistical metric; instead, they show that FTDO–HESO is more capable of converting reconstructed observation information into terminal interception accuracy in highly dynamic engagements, which is of greater engineering relevance.

Table 3. Quantitative Comparison of Observer Configurations Under Large-Maneuver Target Engagement.

Observer Name	RMSE Position	RMSE Velocity	Miss Distance	Intercept Time	Energy Cost	CmdTV
FTDO	21.204	113.97	9.9008	24.545	5.0085×10^5	2.1892×10^5
ESO	6.8765	86.809	9.5312	23.74	5.4019×10^5	3.0343×10^5
FTDO + ESO	18.815	105.16	9.8426	24.365	5.2082×10^5	2.6132×10^5
FTDO + HESO	18.289	114.35	7.5373	24.304	5.9156×10^5	3.6263×10^5

Figure 7a shows that all four observer configurations enable stable target interception under the large-maneuver scenario. Nevertheless, FTDO–HESO exhibits a more compact trajectory convergence after the abrupt target turn, and its terminal path clustering is more concentrated, which is consistent with the smallest miss distance reported in Table 3. Figure 7b indicates that the y-axis overload histories of the four observers remain broadly comparable, suggesting that their influence on BMPSTG is not significantly differentiated in this channel. In contrast, Figure 7c demonstrates that FTDO–HESO does not achieve the best full-time-domain average RMSE, but it delivers the smallest terminal miss distance and the most evident chattering suppression in the z-axis channel. This result indicates that, when the abrupt lateral turning maneuver induces rapid cross-channel state variation, FTDO–HESO is more effective in suppressing the high-frequency corrective actions caused by delayed observations and noise propagation. Since the Z-channel is usually more sensitive to sudden lateral target maneuvers in three-dimensional interception, this observation can be regarded as direct evidence that FTDO–HESO provides stronger delay compensation and more consistent state reconstruction. Taken together, Figure 7 and Table 3 suggest that, among the tested observer configurations, FTDO–HESO provides better terminal-guidance usability, although it does not minimize full-time-domain RMSE or cumulative command activity.

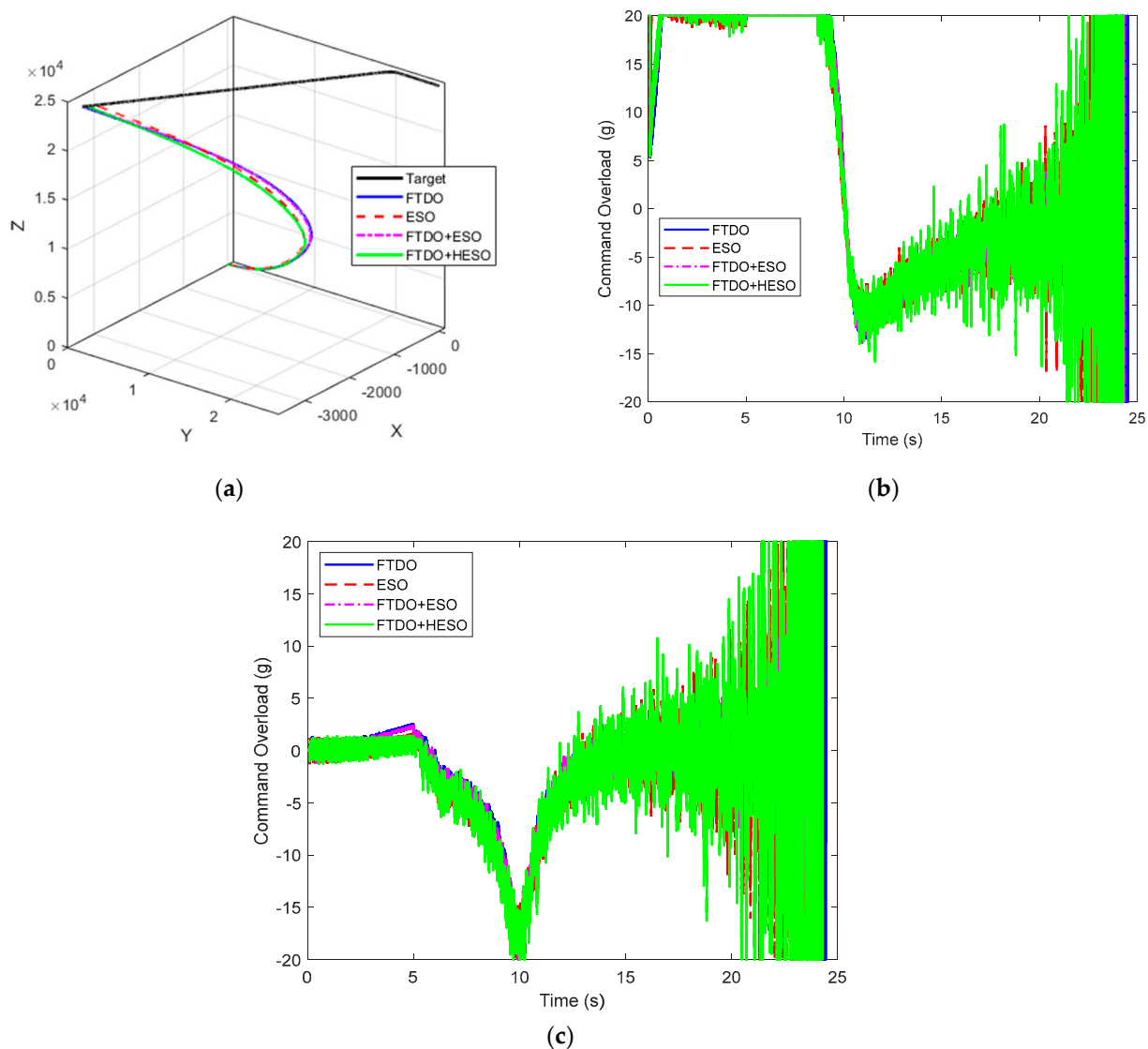


Figure 7. Trajectory and normal-overload comparison of different observer configurations under BMPSTG guidance. (a) 3D Engagement Trajectories; (b) Y-axis Normal Overload Histories; (c) Z-axis Normal Overload Histories.

Figure 8 illustrates the internal regulation mechanism of BMPSTG under FTDO–HESO. As shown in Figure 8a, the barrier height oscillates within approximately 0.3–0.5 from 0–5 s, indicating that the guidance loop maintains a moderate barrier level in the premaneuver stage so as to balance responsiveness and command smoothness. Around 5 s, when the target begins its abrupt lateral turn, the barrier height rises sharply to about 0.9 and remains elevated until 6.636 s. This behavior shows that the probabilistic barrier modulation mechanism successfully identifies the sudden engagement-state transition and strengthens the defensive response against strong maneuver-induced disturbances. Afterward, the barrier height decreases and fluctuates around 0.5, suggesting that the closed-loop system returns to a dynamically regulated state. Figure 8b,c show highly consistent scheduling patterns for K_1 and K_2 . From 0–5.109 s, both gains oscillate around relatively high baseline values. During the abrupt turning phase, they rapidly decay to lower levels with reduced fluctuation. In the terminal stage, both gains increase sharply again. This pattern indicates that the probability-coupled gain scheduling is not a monotonic gain-amplification process; rather, it adaptively switches between suppressing unnecessary high-frequency control activity and reinforcing terminal correction capability accord-

ing to the engagement condition. Figure 8d further reveals the channel-dependent evolution of the tunneling probability. Around the abrupt maneuver instant, the probability in the Y-channel rises rapidly, whereas the probability in the Z-channel first decreases briefly and then climbs to a higher peak. This indicates that the Z-channel initially experiences a stronger barrier constraint and subsequently triggers a more pronounced tunneling-like escape behavior as the state discrepancy accumulates. This interpretation is consistent with the smaller z-axis chattering and the more effective terminal correction observed in the overload histories. Overall, Figure 8 shows that the barrier modulation, probability evolution, and gain scheduling in BMPSTG form a coherent coordinated mechanism: the barrier is raised and the probability distribution is reallocated during the abrupt maneuver to suppress impact propagation, while the gains are re-enhanced in the terminal phase to improve correction capability, thus enabling stable convergence and improved interception accuracy in complex three-dimensional engagements.

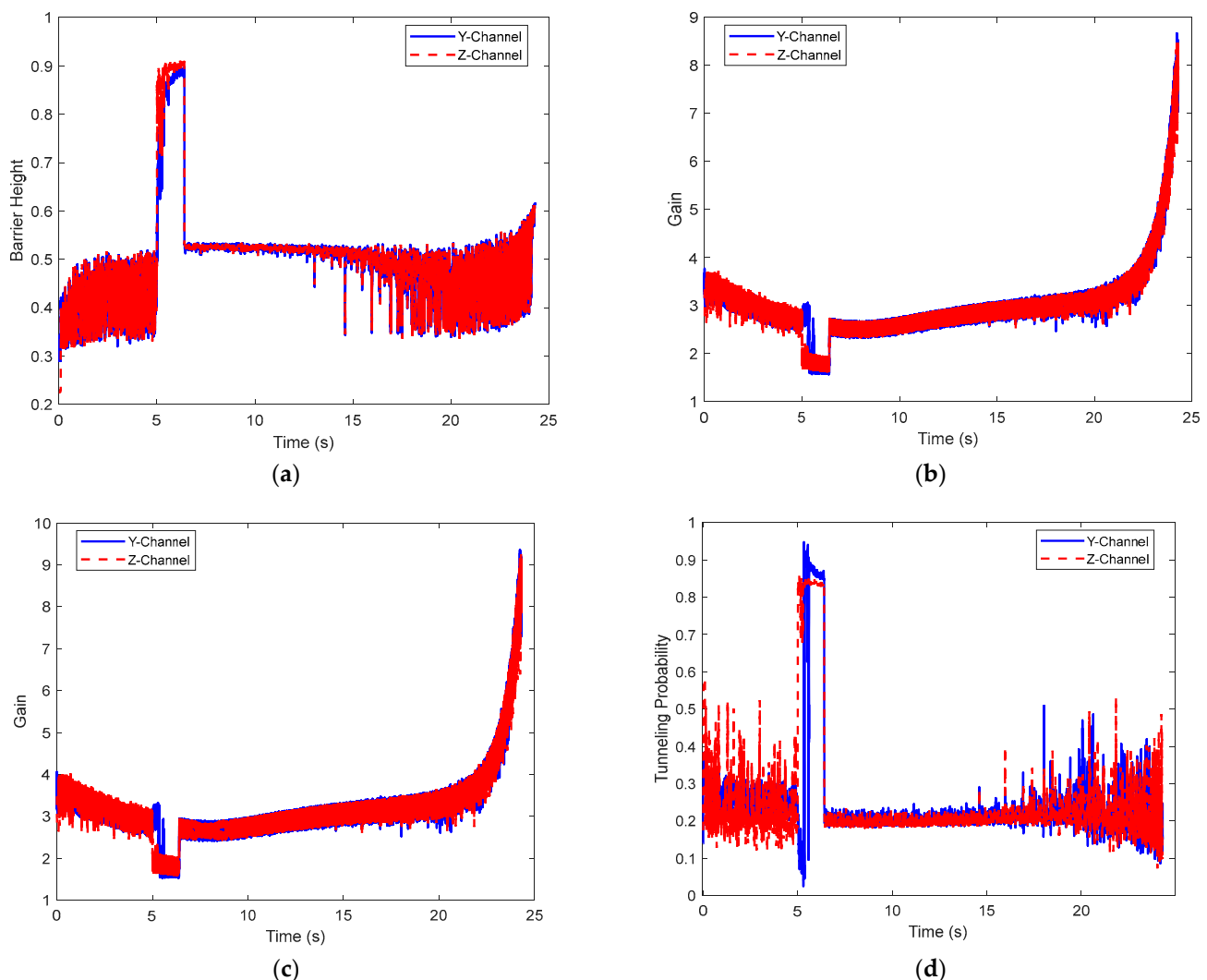


Figure 8. Evolution of Barrier–Energy and Probability–Coupled Adaptive Mechanisms in BMPSTG with FTDO–HESO. (a) Barrier Height Versus Time; (b) Probability–Coupled K_1 Gain scheduling; (c) Probability–Coupled K_2 Gain scheduling; (d) Tunneling Probability Versus Time.

In summary, under the unified BMPSTG guidance law, FTDO–HESO does not achieve the best full-time-domain average RMSE and does not minimize the cumulative command activity; however, it delivers the smallest terminal miss distance and a locally smoother z-axis response during the critical maneuvering interval. This demonstrates that

the composite FTDO–HESO observer is more advantageous in delay compensation, state-consistent reconstruction, and terminal effective control under large-maneuver target engagements. Meanwhile, the barrier–energy modulation, probability-coupled gain scheduling, and stochastic tunneling mechanism in BMPSTG exhibit a clear and meaningful cooperative behavior during strong target maneuvers, which provides effective support for fast convergence, chattering reduction, and terminal accuracy improvement.

6. Summary

This paper proposed a barrier–energy-driven probabilistic super-twisting guidance law for three-dimensional maneuvering target interception in the presence of measurement uncertainty, delay, and packet loss. By incorporating a tunneling-inspired stochastic barrier mechanism into the super-twisting framework, the proposed BMPSTG law achieved barrier modulation, probabilistic sliding variable reshaping, and adaptive gain reconfiguration, thereby improving convergence smoothness and reducing excessive control action. Unlike classical adaptive gain control methods that rely on memoryless threshold-driven gain switching, the proposed scheme modulates convergence intensity through an energy–barrier interaction process, enabling history-aware, phase-sensitive robustness without uniform conservatism.

To enhance disturbance reconstruction under uncertain observations, a composite FTDO–HESO observer was developed. The FTDO provided fast transient estimation, while the HESO improved steady-state accuracy and noise robustness. Their combination strengthened delay compensation and state-consistent reconstruction in highly dynamic engagement conditions.

Simulation results showed that, compared with PNG, SMC, and STG, BMPSTG yielded smoother interception trajectories, lower overload fluctuation, and a better overall compromise among interception accuracy, control effort, and convergence performance. Under the unified BMPSTG guidance law, FTDO–HESO further achieved the smallest miss distance among the tested observer configurations and produced a locally smoother z-axis response during the maneuver-critical interval, at the cost of higher cumulative command activity.

In summary, the proposed BMPSTG framework with composite FTDO–HESO disturbance reconstruction provides an effective and physically interpretable solution for robust interception of highly maneuvering targets. Future work will focus on stochastic finite-time stability analysis, online parameter tuning, and hardware-in-the-loop validation.

Author Contributions: Conceptualization, H.Z. and Z.G.; methodology, H.Z.; software, H.Z.; validation, H.Z., Z.G. and W.Y.; formal analysis, H.Z.; investigation, H.Z.; resources, W.Y.; data curation, Z.G.; writing—original draft preparation, H.Z.; writing—review and editing, Z.G.; visualization, W.Y.; supervision, W.Y. All authors have read and agreed to the published version of the manuscript.

Funding: This research received no external funding.

Institutional Review Board Statement: Not applicable.

Informed Consent Statement: Not applicable.

Data Availability Statement: Data are contained within the article. The data presented in this study can be requested from the authors.

Conflicts of Interest: The authors declare no conflicts of interest.

References

- Adler, F.P. Missile Guidance by Three-Dimensional Proportional Navigation. *J. Appl. Phys.* **1956**, *27*, 500–507. <https://doi.org/10.1063/1.1722411>.
- Kreindler, E. Optimality of proportional navigation. *AIAA J.* **1973**, *11*, 878–880.
- Shin, H.S.; Li, K.B. An Improvement in Three-Dimensional Pure Proportional Navigation Guidance. *IEEE Trans. Aerosp. Electron. Syst.* **2021**, *57*, 3004–3014. <https://doi.org/10.1109/TAES.2021.3067656>.
- Li, K.; Zhou, G. State Estimation with a Destination Constraint Imposed by Proportional Navigation Guidance Law. *IEEE Trans. Aerosp. Electron. Syst.* **2022**, *58*, 58–73. <https://doi.org/10.1109/TAES.2021.3094632>.
- Lee, Y.-L.; Chen, K.-Y.; Liao, S.-J. Using proportional navigation and a particle swarm optimization algorithm to design a dual mode guidance. *Comput. Electr. Eng.* **2016**, *54*, 137–146. <https://doi.org/10.1016/j.compeleceng.2016.01.025>.
- Byung Soo, K.; Jang Gyu, L.; Hyung Seok, H. Biased PNG law for impact with angular constraint. *IEEE Trans. Aerosp. Electron. Syst.* **1998**, *34*, 277–288. <https://doi.org/10.1109/7.640285>.
- Murtaugh, S.A.; Criel, H.E. Fundamentals of proportional navigation. *IEEE Spectr.* **1966**, *3*, 75–85. <https://doi.org/10.1109/MSPEC.1966.5217080>.
- Ranjan, A.; Hota, S.; Singh, N.K. Three-Stage Proportional Navigation for Intercepting Stationary Targets with Impact Angle Constraints. In Proceedings of the 2019 Sixth Indian Control Conference (ICC), Hyderabad, India, 18–20 December 2019; pp. 379–384.
- Liu, Y.; Li, K.; Chen, L.; Liang, Y. Novel augmented proportional navigation guidance law for mid-range autonomous rendezvous. *Acta Astronaut.* **2019**, *162*, 526–535. <https://doi.org/10.1016/j.actaastro.2019.05.031>.
- Zhou, D.; Mu, C.; Xu, W. Adaptive sliding-mode guidance of a homing missile. *J. Guid. Control Dyn.* **1999**, *22*, 589–594.
- Zhang, Q.Z.; Wang, Z.B.; Tao, F.; Sarker, B.R. On-line optimization design of sliding mode guidance law with multiple constraints. *Appl. Math. Model.* **2013**, *37*, 7568–7587. <https://doi.org/10.1016/j.apm.2013.02.030>.
- Song, J.; Song, S.; Zhou, H. Adaptive nonsingular fast terminal sliding mode guidance law with impact angle constraints. *Int. J. Control Autom. Syst.* **2016**, *14*, 99–114. <https://doi.org/10.1007/s12555-014-0155-8>.
- Behnamgol, V.; Vali, A.S.; Mohammadi, A. A new observer-based chattering-free sliding mode guidance law. *Proc. Inst. Mech. Eng. Part G J. Aerosp. Eng.* **2016**, *230*, 1486–1495.
- Zhu, C.; Mu, D. Design of head-pursuit guidance law based on sliding mode control. *IOP Conf. Ser. Mater. Sci. Eng.* **2019**, *563*, 042076. <https://doi.org/10.1088/1757-899X/563/4/042076>.
- Yeh, F.-K. Adaptive-sliding-mode guidance law design for missiles with thrust vector control and divert control system. *IET Control Theory Appl.* **2012**, *6*, 552–559.
- He, S.; Lin, D. Adaptive nonsingular sliding mode based guidance law with terminal angular constraint. *Int. J. Aeronaut. Space Sci.* **2014**, *15*, 146–152. <https://doi.org/10.5139/IJASS.2014.15.2.146>.
- Levant, A. Gain-scheduled high-order MIMO sliding mode control. In Proceedings of the 49th IEEE Conference on Decision and Control (CDC), Atlanta, GA, USA, 15–17 December 2010; pp. 5150–5155. <https://doi.org/10.1109/CDC.2010.5718194>.
- Gong, M.; Zhou, D.; Zou, X. Saturated super-twisting sliding mode missile guidance. *Chin. J. Aeronaut.* **2022**, *35*, 292–300. <https://doi.org/10.1016/j.cja.2021.11.008>.
- Shtessel, Y.; Taleb, M.; Plestan, F. A novel adaptive-gain supertwisting sliding mode controller: Methodology and application. *Automatica* **2012**, *48*, 759–769. <https://doi.org/10.1016/j.automatica.2012.02.024>.
- Gonzalez, T.; Moreno, J.A.; Fridman, L. Variable Gain Super-Twisting Sliding Mode Control. *IEEE Trans. Autom. Control* **2012**, *57*, 2100–2105. <https://doi.org/10.1109/TAC.2011.2179878>.
- Behera, A.K.; Bandyopadhyay, B.; Yu, X. Periodic event-triggered sliding mode control. *Automatica* **2018**, *96*, 61–72. <https://doi.org/10.1016/j.automatica.2018.06.035>.
- Kramers, H.A. Brownian motion in a field of force and the diffusion model of chemical reactions. *Physica* **1940**, *7*, 284–304. [https://doi.org/10.1016/S0031-8914\(40\)90098-2](https://doi.org/10.1016/S0031-8914(40)90098-2).
- Hanggi, P.; Talkner, P.; Borkovec, M. Reaction-rate theory: Fifty years after Kramers. *Rev. Mod. Phys.* **1990**, *62*, 251–341. <https://doi.org/10.1103/RevModPhys.62.251>.
- Razavy, M. *Quantum Theory of Tunneling*; World Scientific: Singapore, 2003.
- Fei, J.; Liu, L.; Chen, Y. Finite-Time Disturbance Observer of Active Power Filter With Dynamic Terminal Sliding Mode Controller. *IEEE J. Emerg. Sel. Top. Power Electron.* **2023**, *11*, 1604–1615. <https://doi.org/10.1109/JESTPE.2022.3222002>.

26. Mirzaei, M.J.; Aslmostafa, E.; Asadollahi, M.; Badamchizadeh, M.A. Robust adaptive finite-time stabilization control for a class of nonlinear switched systems based on finite-time disturbance observer. *J. Frankl. Inst.* **2021**, *358*, 3332–3352. <https://doi.org/10.1016/j.jfranklin.2021.02.010>.
27. Guo, B.-Z.; Zhao, Z.-L. On the convergence of an extended state observer for nonlinear systems with uncertainty. *Syst. Control Lett.* **2011**, *60*, 420–430. <https://doi.org/10.1016/j.sysconle.2011.03.008>.
28. Yang, X.; Huang, Y. Capabilities of extended state observer for estimating uncertainties. In Proceedings of the 2009 American Control Conference, St. Louis, MO, USA, 10–12 June 2009; pp. 3700–3705.
29. Yao, J.; Jiao, Z.; Ma, D. Adaptive Robust Control of DC Motors With Extended State Observer. *IEEE Trans. Ind. Electron.* **2014**, *61*, 3630–3637. <https://doi.org/10.1109/TIE.2013.2281165>.
30. Pu, Z.; Yuan, R.; Yi, J.; Tan, X. A Class of Adaptive Extended State Observers for Nonlinear Disturbed Systems. *IEEE Trans. Ind. Electron.* **2015**, *62*, 5858–5869. <https://doi.org/10.1109/TIE.2015.2448060>.
31. Talole, S.E.; Kolhe, J.P.; Phadke, S.B. Extended-State-Observer-Based Control of Flexible-Joint System With Experimental Validation. *IEEE Trans. Ind. Electron.* **2010**, *57*, 1411–1419. <https://doi.org/10.1109/TIE.2009.2029528>.
32. Zhang, Y.; Zhang, J.; Wang, L.; Su, J. Composite disturbance rejection control based on generalized extended state observer. *ISA Trans.* **2016**, *63*, 377–386. <https://doi.org/10.1016/j.isatra.2016.03.021>.
33. Song, L.-K.; Tao, F.; Choy, Y.; Yang, L.-C.; Wei, Y.-F. Cascade sampling-driven block mapping for coupled reliability evaluation of turbine cooling systems. *Mech. Syst. Signal Process.* **2025**, *236*, 113008. <https://doi.org/10.1016/j.ymssp.2025.113008>.
34. Song, L.-K.; Faes, M.G.R.; Tao, F. Generative data-physics fusion for hierarchical reliability assessment. *Reliab. Eng. Syst. Saf.* **2026**, *272*, 112642. <https://doi.org/10.1016/j.res.2026.112642>.

Disclaimer/Publisher's Note: The statements, opinions and data contained in all publications are solely those of the individual author(s) and contributor(s) and not of MDPI and/or the editor(s). MDPI and/or the editor(s) disclaim responsibility for any injury to people or property resulting from any ideas, methods, instructions or products referred to in the content.