

Article

High Signal-to-Noise Ratio Method Without Phase Deviation for X-Ray Pulsar Profile Acquisition

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Abstract

High-quality X-ray pulsar observation profiles are vital for investigating both their physical properties and navigation applications. Conventional profile extraction relies on epoch folding, whose performance is constrained by observation duration and bin size, often leading to poor-quality profiles or even failure under extremely low-photon conditions. This paper proposes a novel method that directly extracts high-quality profile frequency spectra merely by statistical analysis of photon sequences followed by the reconstruction of time domain waveforms. Monte Carlo simulations and real observational data demonstrate that the proposed method exhibits higher correlation coefficients and signal-to-noise ratios than those obtained using traditional epoch folding, and also outperforms the Fourier-series-based frequency cutoff method. Moreover, comparable profile quality can be achieved using an order of magnitude fewer photons than required by epoch folding. The lower the photon count, the more significant the improvement, making the method especially suitable for small-area detectors and resource-constrained observation scenarios.

Keywords: pulsar; X-ray pulsar navigation; observation profiles; frequency spectrum

1. Introduction

A pulsar is a neutron star with a super-strong magnetic field, ultra-high density, and high-speed rotation [1,2], and it has excellent rotational stability and can radiate multiple outward signal frequency bands [3,4]. Pulsars that can radiate signals in the X-ray frequency band are called X-ray pulsars [5]. They serve as both natural laboratories for extreme astrophysical phenomena and as interstellar beacons for deep-space navigation [6–8]. By providing precise time, position, and velocity information, X-ray pulsars can serve as spatiotemporal references for spacecraft, a capability known as X-ray pulsar navigation (XPNAV) [9,10]. They also enable the formation of pulsar timing arrays, complement gravitational wave detection in the radio band [11,12], and function as cosmic high-energy probes for studying extreme accretion and magnetospheric physics in the universe's high-energy environments [13,14].

Whether for XPNAV, astrophysical observations, or other applications, successful implementation relies on the accurate extraction of information from pulsar signals. As a fundamental manifestation of pulsar physical properties, the acquisition of high-quality pulse observation profiles is a critical component of these processes [15,16]. Epoch folding

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is currently the primary method for obtaining observation profiles; however, the profile quality is limited by factors such as observation time and bin size. Therefore, many scholars have conducted extensive research on noise reduction in observation profiles based on epoch folding.

Hanson et al. systematically analyzed the noise characteristics in pulsar observations and their impact on the profiles [17]; Su et al. applied the theory of compressive sensing to the reconstruction of signal profiles [18]; and Zhang et al. evaluated the quality of signal profiles using entropy as an indicator and proposed a pulse profile accumulation method based on minimum entropy [19]. Another study used Gaussian functions to fit the recovered signal profile and improve its quality [20]. In terms of using wavelet methods for denoising, Liu et al. and Xue et al. conducted extensive research on pulse profile denoising methods based on wavelet transform, and they successively proposed methods such as Daubechies wavelet, lifting wavelet, non-sampling wavelet and wavelet packet [21–24]. Hu et al. proposed a wavelet denoising method based on non-normalized Haar wavelets and S-type threshold functions [25]. Based on empirical mode decomposition (EMD), some studies proposed a noise mode unit prediction-based EMD method, a modal unit proportion shrinkage-based EMD method, a kurtosis test-windowed EMD denoising method, and variational mode decomposition (VMD) denoising methods to further improve signal denoising effectiveness and enhance algorithm autonomy [26–28]. In addition, Wang et al. proposed a kernel regression pulsar profile denoising method based on the maximum correlation entropy criterion, which reduces the dependence on human experience in parameter selection and improves the autonomy of the method [29]. Song et al. proposed an optimized kernel regression profile denoising and an optimized kernel probability density profile construction method [30]; Li et al. proposed a profile recovery method for variable packaging segment pulsars based on equal photon distribution [31]; and Song et al. achieved denoising by truncating the frequency spectrum of the profile through epoch folding [32]. In addition, machine learning-based signal denoising methods have been investigated in several studies [33–35]; however, their practical engineering deployment remains challenging.

The above studies on noise reduction processing are all based on the profiles obtained through epoch folding; however, the requirement of the infinitesimal bin size resulting from epoch folding limits the accuracy of the pulsar information. In addition, denoising methods such as discrete wavelet transform will introduce a certain degree of phase shift [36], which strongly affects the application of pulsars in navigation. This study analyzes the characteristics of X-ray pulsar radiation signals, breaks through the traditional approach of obtaining information through epoch folding, and proposes a high-precision observation profile acquisition method that directly obtains spectra from photon sequences. The remainder of this paper is organized as follows: Section 2 analyzes the characteristics of photon statistical sequences and the limitations of epoch folding; Section 3 focuses on the theoretical derivation of this method; and Section 4 evaluates the performance of the proposed method using Monte Carlo simulation data as well as observational data from the Neutron Star Interior Composition Explorer (NICER) and the Insight Hard X-ray Modulation Telescope (Insight-HXMT) for multiple pulsars observed over different observation durations. In addition, Section 4 analyzes the simulation results of the proposed method and epoch folding as applied to XPNAV.

2. Analysis of the Radiation Signal Characteristics of X-Ray Pulsars

2.1. Analysis of the Statistical Sequence Characteristics of Photons

Pulsars are extremely distant from Earth, and by the time their X-ray radiation reaches the vicinity of Earth, the signal has attenuated into discrete X-ray photons. Therefore, the signals received by the detector are random time-photon sequences that can be

modeled using a nonhomogeneous Poisson process. If using $\lambda(t)$ to represent the density function of pulsar-emitted photons, the number of photons ($N_{s,e}$) received by the detector during the observation time follows a Poisson distribution with the parameter of $\int_{t_s}^{t_e} \lambda(\tau) d\tau$, and the expression is given by Equation (1).

$$P(N_{s,e} = k) = \left[\left(\int_{t_s}^{t_e} \lambda(\tau) d\tau \right)^k e^{-\int_{t_s}^{t_e} \lambda(\tau) d\tau} \right] / k! \quad (1)$$

$$\lambda(t) = \lambda_b + \lambda_s h(\phi(t))$$

where λ_b is the X-ray background radiation intensity; λ_s is the X-ray radiation intensity of the pulsar; $\phi(t)$ is the phase model of the time of arrival, that is, the pulse phase corresponding to the observed signal at time t ; and $h(\phi)$ is the standard density function (later referred to as the standard profile), which satisfies:

$$\int_0^1 h(\phi) d\phi = 1, \phi \in [0, 1), \min_{\phi} h(\phi) = 0 \quad (2)$$

Notably, $h(\phi)$ has periodicity

$$h(\phi(t + T_p)) = h(\phi(t)) \quad (3)$$

where T_p is the period of the pulsar radiation signal.

It can be seen that the photon sequence emitted by X-ray pulsars exhibits the characteristics of a cyclo-stationary process, allowing processing methods developed for generalized stationary signals to be broadly applied [37].

2.2. Limitations of Traditional Epoch Folding

The process of recovering the density function of X-ray pulsars from the received photon time series by counting photons is generally referred to as epoch folding, as shown in Figure 1. If this process involves folding time segments instead of phases, it is called periodic folding. And the principles for both are essentially the same.

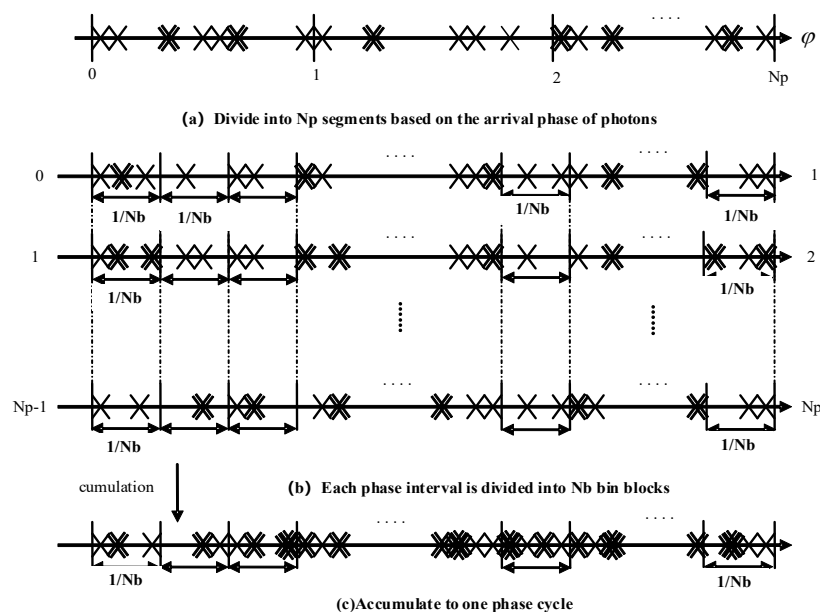


Figure 1. Principle of Epoch Folding.

Assuming that the overall observation time T_{obs} can be divided into N_p observation periods, the fragment size of each bin is T_b , where $c(t_i)$ is the number of detected photons within the i_{th} bin and t_i is the center of the bin.

According to the Poisson distribution theorem, the mean and variance of the random variable $c(t_i)$ are as follows:

$$E[c(t_i)] = \text{var}[c(t_i)] = \lambda(t_i)T_b \quad (4)$$

Notably, the fragment size of the bin must be sufficiently small or tend towards being infinitely small for Equation (4) to hold. Thus, the density function obtained by epoch folding tends towards the true density function. During a long observation time, the observation density function can be expressed as

$$\hat{\lambda}(t_i) = \lambda(t_i) + \hat{n}(t_i) \quad (5)$$

where $\lambda(t_i)$ is the true density function and $\hat{n}(t_i)$ is the unrelated noise of epoch folding. According to the independent increment and central limit theorem of the Poisson process, Emadzadeh et al. derived the mean and variance when $N_p \rightarrow \infty$, as in Equation (6) [15].

$$\begin{aligned} E[\hat{n}(t_i)] &= 0 \\ E[\hat{\lambda}(t_i)] &= \lambda(t_i) \\ \text{var}[\hat{n}(t_i)] &= \text{var}[\hat{\lambda}(t_i)] = \frac{1}{N_p T_b} \lambda(t_i) = \frac{N_b}{T_{obs}} \lambda(t_i) \end{aligned} \quad (6)$$

From Equations (4) and (6), the principle of epoch folding requires that the fragment size (T_b) of each bin be sufficiently small, and when the overall observation duration T_{obs} is fixed, it is necessary to minimize the number of bins (N_b) to minimize the noise introduced by epoch folding. However, because the multiplication of the two ($T_b \times N_b = T_p$) is a fixed-value pulsar period, a contradiction exists in the epoch folding itself. In addition, if the value selection of N_b is too small, the resolution of the obtained profile will be low, and the search deviation of the profile peak point will be large, resulting in phase distortion. If a larger value is selected, the noise characteristics will be highlighted and false peaks will be generated. Therefore, from experience in practical operations, balancing the results typically helps resolve this issue.

3. High Signal-to-Noise Pulse Observation Profile Acquisition Method

3.1. Construction of λ Statistical Sequences

As analyzed in Section 2.1, if the photon statistical sequence detected exhibits cyclo-stationary characteristics, then its first-order statistic is also a cyclo-stationary stochastic process. Accordingly, we can use relevant methods for generalized stationary signals to process and analyze them.

According to estimation theory, taking the arithmetic mean of a set of uncorrelated observations for a random variable is considered an unbiased estimate of the variable's mean, achieving the expected value with reduced variance. However, many uncorrelated instances of random processes, such as pulsar radiation signals, are challenging to implement, as only a single fixed-length implementation is typically available in practice. To address this, as illustrated in Figure 2, Bartlett divided the data into K non-overlapping subsequences of length L . The periodogram of each subsequence is then averaged to provide an unbiased estimate of the overall periodogram. By doing so, Bartlett reduced the variance in the periodogram to $1/K$ of its original value, but increased its resolution to K times its original value.

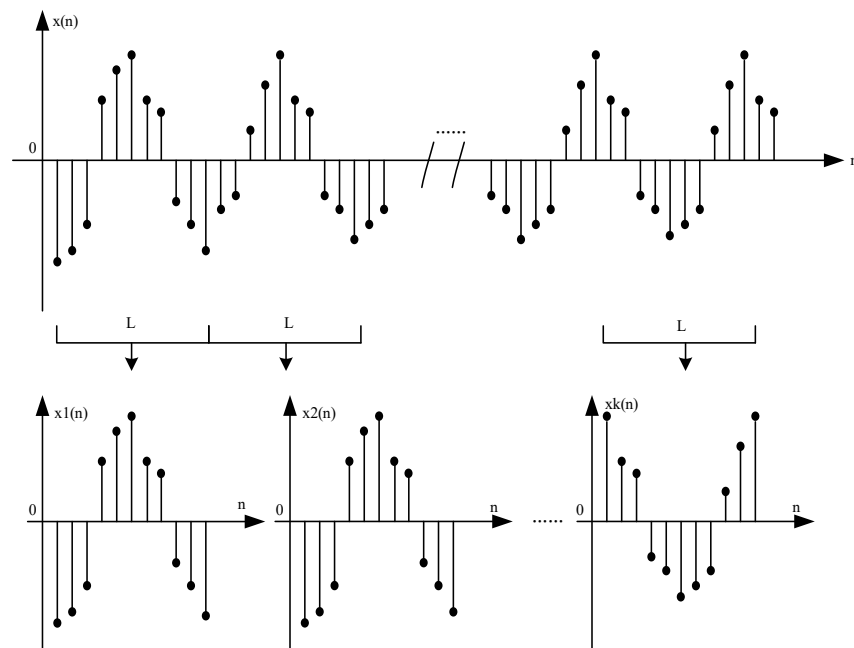


Figure 2. Bartlett method for segmenting of $x(n)$.

Similar to the improved Bartlett periodogram method, the photon sequence with an observation time of T_{obs} can be divided into N_p non-overlapping time segments with a length of T_p that meets the requirements of $T_{obs} = T_p \times N_p$. This is equivalent to dividing the statistic sequence with the length of N into N_p non-overlapping subsequences with a length of N_b . Each subsequence is given as follows:

$$x_i(n) = x(n + iN_b) \quad n = 0, 1, \dots, N_b - 1, \quad i = 0, 1, \dots, N_p - 1 \quad (7)$$

According to the theory of the periodogram method, if both N_b and N_p tend to infinity when N approaches infinity, the power spectrum of the statistic sequence has consistent estimation [38]. At this time, we refer to this statistical sequence as the λ sequence.

In practical situations, considering that the length of the observed signal T_{obs} and the period of the pulsar T_p are fixed, the number of segments N_p into which sequences of the time of arrival can be divided is fixed. Therefore, N can only be maximized by increasing the value of N_b as much as possible. However, N_b cannot be increased indefinitely and should be selected by jointly considering the effective timing resolution of the detector system, photon statistics, and available computational resources.

Compared with the bin numbers typically adopted in conventional epoch folding (128–1024), the proposed λ -sequence-based method generally employs a substantially larger value of N_b . Under the timing capabilities of current X-ray detectors and typical observation conditions, N_b is typically chosen within the range of $2^{15} \sim 2^{20}$, providing a good balance among spectral resolution, statistical reliability, and computational efficiency.

Although the λ sequence is defined in the time domain, it does not directly reflect the pulse profile. Figure 3 highlights this distinction using Crab pulsar data. Figure 3a shows the λ sequence constructed from 244 pulse cycles, while Figure 3b presents the corresponding pulse profile obtained through epoch folding. The comparison clearly demonstrates that the λ sequence is not equivalent to epoch folding, and the correlation charac-

teristics of the pulse profile cannot be directly observed from the λ sequence itself. The primary purpose of statistically constructing the λ sequence is to meet the need to acquire high-precision spectra. At the same time, it is also implemented to transform the non-uniformly spaced photon arrival time series into a uniformly spaced statistical sequence. It should be clarified that the λ sequence does not rely on strict periodic segmentation of photon arrivals. Instead, it is obtained through discretization of the statistical photon-counting process into time bins.

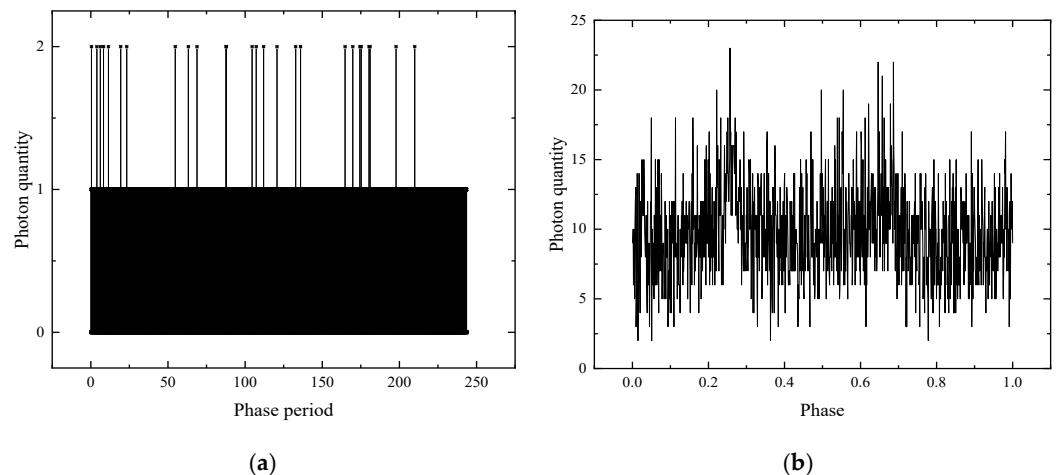


Figure 3. Comparison of λ sequence and epoch folding statistics: (a) λ sequence; (b) epoch folding.

3.2. Spectral Acquisition of Observation Profiles Using λ Statistical Sequences

Due to the fact that the λ sequence is a non-stationary random signal, we can also draw inspiration from the Bartlett periodogram method to derive the spectrum of the λ sequence.

Calculate the spectrum of each subsequence in Equation (7) and take its average:

$$\hat{X}(e^{j\omega}) = \frac{1}{N_p} \sum_{i=0}^{N_p-1} \left[\sum_{n=0}^{N_b-1} x(n + iN_b) e^{-j\omega n} \right] = \frac{1}{N_p} \sum_{i=0}^{N_p-1} X_{per}^{(i)}(e^{j\omega}) \quad (8)$$

where $X_{per}^{(i)}(e^{j\omega})$ is the spectrum of the i_{th} subsequence, and $\hat{X}(e^{j\omega})$ is the estimated spectrum value of the λ sequence. $\hat{X}(e^{j\omega})$ is a progressive unbiased estimation of $X(e^{j\omega})$, analogous to a periodogram.

Then the computational frequency resolution of the sequence spectrum can be expressed as follows:

$$Res[\hat{X}(e^{j\omega})] = 0.89 \frac{2\pi}{N_b} \quad (9)$$

where coefficient 0.89 is due to dividing the subsequences according to the length of the period T_p , which is equivalent to adding a rectangular window to the original signal. Furthermore, the spectrum of each subsequence is calculated based on a sequence of length N_b and we can improve spectral visibility by increasing N_b .

As mentioned earlier, the arrival process of X-ray pulsar photons can be modeled as a nonhomogeneous Poisson process (NHPP) with a periodic intensity function. The λ sequence is defined as the sequence of photon counts in a series of non-overlapping time bins obtained by discretizing the observation interval. Although the expected photon count in each time bin is periodically modulated by the pulsar signal, the photon counts in different non-overlapping time bins remain statistically independent because of the independent-increment property of the NHPP. Accordingly, in the derivation of the spectral variance, the covariance terms are neglected, and only the dominant variance term is re-

tained, yielding an asymptotic approximation of the spectral variance. It should be emphasized that this derivation does not assume strict statistical independence between successive pulse periods. Instead, it is based on the independent-increment property of the NHPP, while the variance expression itself is an asymptotic approximation obtained by neglecting higher-order covariance terms. Such an approximation is widely adopted in asymptotic spectral estimation, where the influence of covariance terms diminishes with increasing temporal resolution and sample size, thereby ensuring the validity of the asymptotic variance expression [38]. Under these conditions, when the T_{obs} is sufficient, the estimated variance in the spectrum for the λ sequence is approximately

$$Var[\hat{X}(e^{j\omega})] \approx \frac{1}{N_p} Var[X_{per}^{(i)}(e^{j\omega})] = \frac{T_p}{T_{obs}} Var[X_{per}^{(i)}(e^{j\omega})] \quad (10)$$

Equations (9) and (10) indicate that their computational frequency resolution is inversely proportional to N_b , and their variance is inversely proportional to N_p .

Considering that Equation (8) needs to calculate spectrums N_p times, we can swap its summation order and transform it into:

$$\hat{X}(e^{j\omega}) = \frac{1}{N_p} \sum_{i=0}^{N_p-1} \left[\sum_{n=0}^{N_b-1} x(n+iN_b) e^{-j\omega n} \right] = \sum_{n=0}^{N_b-1} \left[\frac{1}{N_p} \sum_{i=0}^{N_p-1} x(n+iN_b) \right] e^{-j\omega n} = \sum_{n=0}^{N_b-1} [\bar{x}(n) e^{-j\omega n}] \quad (11)$$

where $\bar{x}(n)$ is the mean of the λ sequence. Therefore, Equation (11) only needs to calculate the frequency spectrum once, effectively reducing the computational cost.

According to Equations (9) and (10), when N approaches infinity and both N_b and N_p tend to infinity, $\hat{X}(e^{j\omega})$ will be a consistent estimate of $X(e^{j\omega})$. It is important to clarify that N denotes the length of the λ sequence, rather than the number of photons received. And we are not increasing the observational data to make N approach infinity.

In other words, when establishing the λ sequence, increasing N_b as much as possible effectively increases the sampling rate, which can significantly reduce spectral aliasing of noise. At the same time, for quantization noise (referred to as folding noise in this study), the total quantization noise power remains constant. Originally distributed evenly across the 0 to $f_s/2$ frequency band, the quantization noise is now spread over the 0 to $N_b/2$ frequency band, pushing most of it into the high-frequency region outside the signal band.

3.3. Acquisition of the Time-Domain Pulse Profile

In Section 3.2, we greatly improved the sampling frequency by adjusting N_b . We obtained the high-quality spectrum of the λ sequence, which also possesses undistorted phase information characteristics. According to the Nyquist sampling theorem, if the sampling frequency is twice as high as the highest frequency of the signal, the original signal can be recovered from the spectrum. Therefore, the time-domain profile can be recovered theoretically by using the spectrum in Section 3.2, which only needs to be larger than the signal bandwidth.

Then we use the inverse Fourier transform to obtain pulse profiles:

$$\hat{x}(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} \hat{X}(e^{j\omega}) e^{j\omega n} d\omega \quad n = 0, 1, 2, \dots, N_b - 1 \quad (12)$$

But for the convenience of computer processing, the study uses Equation (13) to obtain pulse profiles. Since the inverse Fourier transform does not introduce a phase shift, the profile obtained from Equation (13) retains a high signal-to-noise ratio (SNR) and no loss of phase information.

$$\hat{x}(n) = \frac{1}{N_b} \left[\hat{X}(0) + 2 \operatorname{real} \left(\sum_{k=1}^M \hat{X}(k) e^{2\pi j n k / N_b} \right) \right] \quad n = 0, 1, 2, \dots, N_b - 1 \quad (13)$$

In this section, we use the data measured by the NICER satellite to analyze the spectral characteristics of the λ sequences of PSR J1939+2134, PSR J1513-5908, PSR J0540-6919, and PSR J0534+2200, where the standard λ sequences are obtained using all the data packets from 2017 to 2020, and the cumulative λ sequences are obtained by some data packets. Figure 4 shows that the frequency band of the λ sequence is finite and convergent, and the frequency spectrum is concentrated in the low frequency range, but the noise is distributed throughout the entire frequency band and can be regarded as Gaussian white noise.

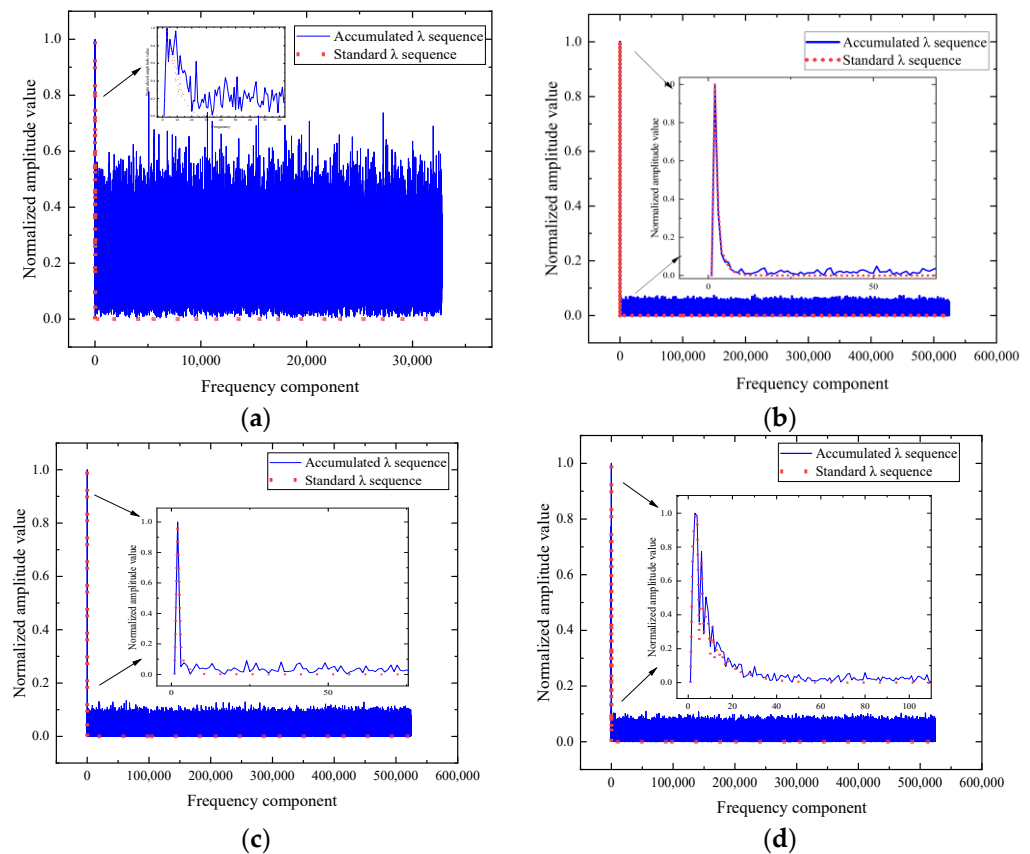


Figure 4. Spectral characteristics of the λ sequence of universal pulsars: (a) PSR J1939+2134; (b) PSR J1513-5908; (c) PSR J0540-6919; (d) PSR J0534+2200.

Based on the spectral characteristics of the λ sequence, the signal and noise can be separated by intercepting the first M harmonic components, and then the SNR of the time-domain profile can be improved. An appropriate value of M is beneficial for SNR separation. If the value is too small, critical information may be lost, whereas if the value is too large, high-frequency noise cannot be effectively removed.

To address the issue of potential local optima when estimating M using a finite sample of size n , Hart [39] proposes a cost function for solving M based on the criterion of minimizing the mean integrated squared error (MISE) as follows:

$$\hat{M} = \arg \max_M H(f, \hat{f}_M) \quad (14)$$

$$H(f, \hat{f}_M) = \frac{(n+1)}{n} \sum_{k=1}^M |\phi(k)|^2 - \frac{M}{n} \quad (15)$$

$$E|\hat{\phi}(k)|^2 = |\phi(k)|^2 + \frac{1 - |\phi(k)|^2}{n} \quad (16)$$

where $\phi(k)$ is the k -th harmonic component of the λ sequence spectrum and $\hat{\phi}(k)$ is the estimated value of $\phi(k)$.

Considering the extremely low SNR of the high-order part of the cumulative λ sequence, a penalty factor c (usually taken as a constant greater than 1) is introduced for the high-order term of Equation (15), and an unbiased estimate of $H(f, \hat{f}_M)$ is obtained, as shown in Equation (17)

$$\eta(M) = \frac{(n+1)}{(n-1)} \sum_{k=1}^M |\hat{\phi}(k)|^2 - \frac{2cM \log(\log n)}{n-1} \quad (17)$$

This study uses the value calculated by Equation (17) to determine M and c is set to 2.

4. Result Validation and Analysis

4.1. Numerical Simulation Verification and Analysis

To demonstrate the correctness and superiority of the method proposed in this article, a computer with an Intel (R) Core (TM) i7-9700 @ 3.00 GHz CPU and MATLAB 2023a was used to compare the proposed method with the epoch folding method via Monte Carlo simulation experiments. Two typical pulsars were selected: a Crab pulsar with a high flow rate and a PSR B1937+21 pulsar with a low flow rate. The orbit parameters of the spacecraft used in the simulation are listed in Table 1, and the pulsar parameters are listed in Table 2. The method proposed by Reference [40] was adopted to generate photon sequences at spacecraft locations, and the Monte Carlo simulation was conducted 200 times.

Table 1. The orbital elements of a spacecraft.

Orbital Elements	Semimajor Axis (km)	Eccentricity	Inclination (°)	Argument of Perigee (°)	Right Ascension of the Ascending Node (°)	True Anomaly (°)
value	6979.14	0.05	28.5	30	10	0

Table 2. Characteristic parameters of simulated pulsars.

Pulsar	Rotation Frequency (s ⁻¹)	Frequency First-Order Derivative (s ⁻²)	Frequency Second-Order Derivative (s ⁻³)	Signal Source Flow (ph×s ⁻¹ ×cm ⁻²)	Background Traffic (ph×s ⁻¹ ×cm ⁻²)	Detector Area (cm ²)
Crab	29.9516010684378	−3.77726 × 10 ⁻¹⁰	8.18 × 10 ⁻²¹	1.54	13.86	1000
PSR B1937+21	641.928263870956	−4.33167 × 10 ⁻¹⁴	5.85 × 10 ⁻³⁰	0.499	4.4910	10,000

The time conversion model in Reference [41] is used to convert photon sequences at the spacecraft to photon sequences at the Solar System barycenter (SSB), and the observed profiles at the SSB are obtained using these methods. In addition to the conventional epoch folding method, this study also adopts the Fourier-series-based frequency cutoff method reported in Reference [32] as a comparison approach, hereafter referred to as Method 2, in which the M is determined according to Equation (16). In both Method 2 and the proposed method, the number of bins is 1024 during time-domain reconstruction. This choice facilitates smooth interpolation of the time-domain profile by zero-padding beyond the retained harmonic components. In contrast, epoch folding is performed with 256 bins, since increasing the number of bins in epoch folding exacerbates folding noise and degrades profile quality.

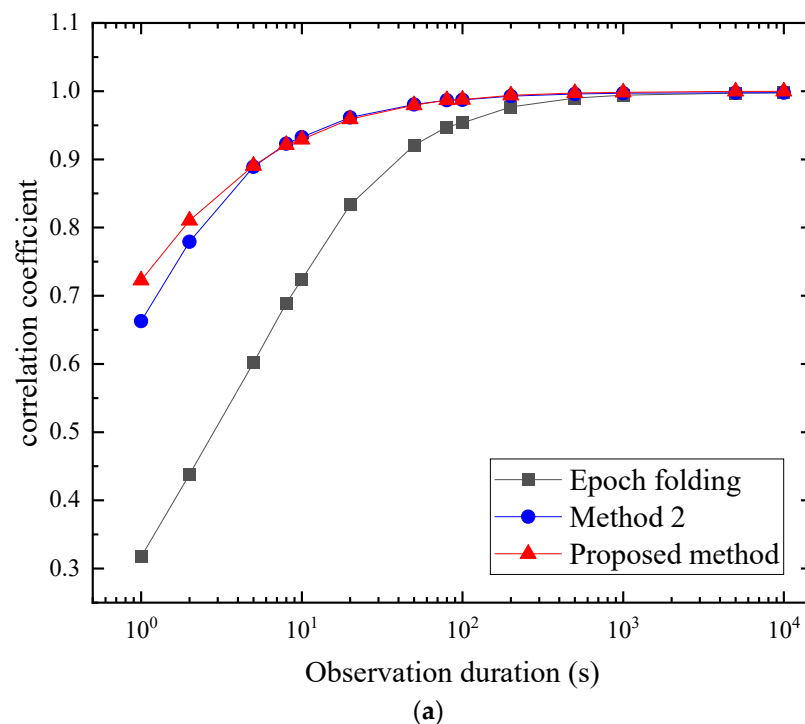
The Pearson correlation coefficient obtained from Equation (18) was used to compare the overlap between the observed profiles obtained by observed profile and the standard profile, and the SNRs of the observed profiles obtained by the three methods were compared using Equation (19). The results are shown in Figures 5 and 6.

$$R_{\text{pearson}} = \frac{\sum_{k=1}^N (s_k - \bar{s})(w_k - \bar{w})}{\sqrt{\sum_{k=1}^N (s_k - \bar{s})^2} \sqrt{\sum_{k=1}^N (w_k - \bar{w})^2}} \quad (18)$$

$$\text{SNR} = 10 \log_{10} \frac{P_{w_k}}{P_{s_k - w_k}} \quad (19)$$

where s_k is the observed profile, p_k is the standard profile, $\bar{s}$ and $\bar{p}$ are respectively the means of s_k and p_k , P_{w_k} and $P_{s_k - w_k}$ are respectively the power values of the observed profile and noise (subtracting the standard profile from the observed profile).

Figures 5 and 6 show the correlation coefficients and SNRs of the observation profiles obtained using the three methods for different observation durations. Figures 7 and 8 present the observation profiles obtained using epoch folding and the proposed method for different observation durations. Figures 5a and 6a show the profile correlation coefficients of the Crab and B1937 +21 pulsars, respectively. Figures 5b and 6b show the profile SNR of the Crab and B1937+21 pulsars, respectively.



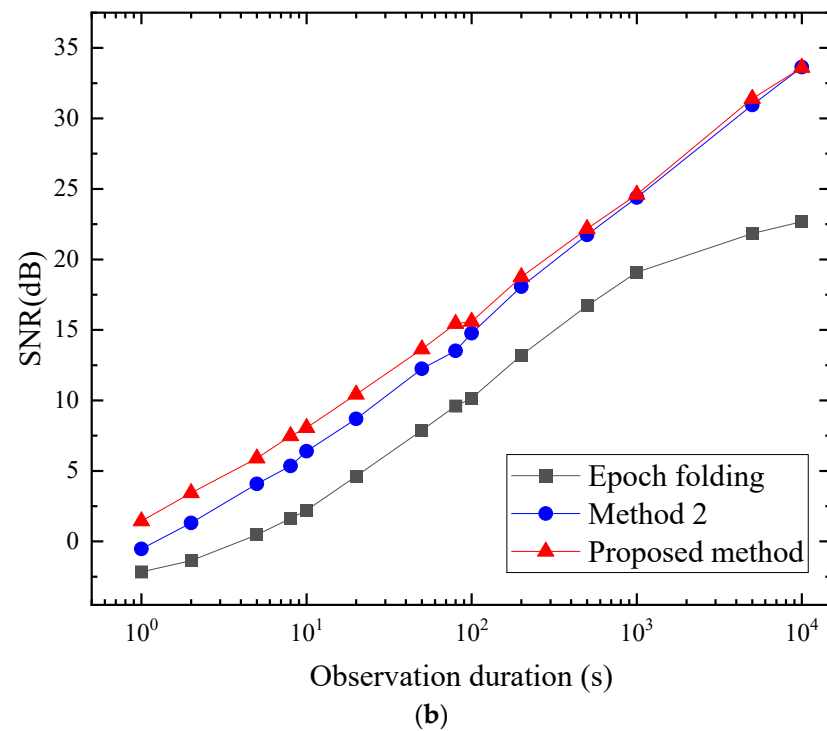
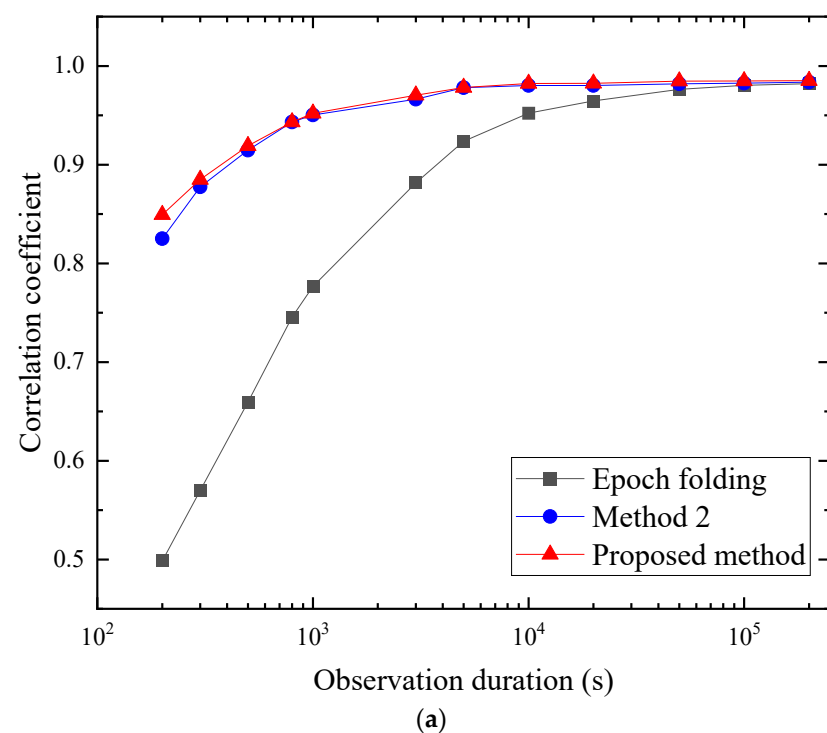


Figure 5. Comparison of the Crab pulsar observation profile quality obtained by the three methods: (a) correlation coefficient variation with observation duration; (b) SNR variation with observation duration.



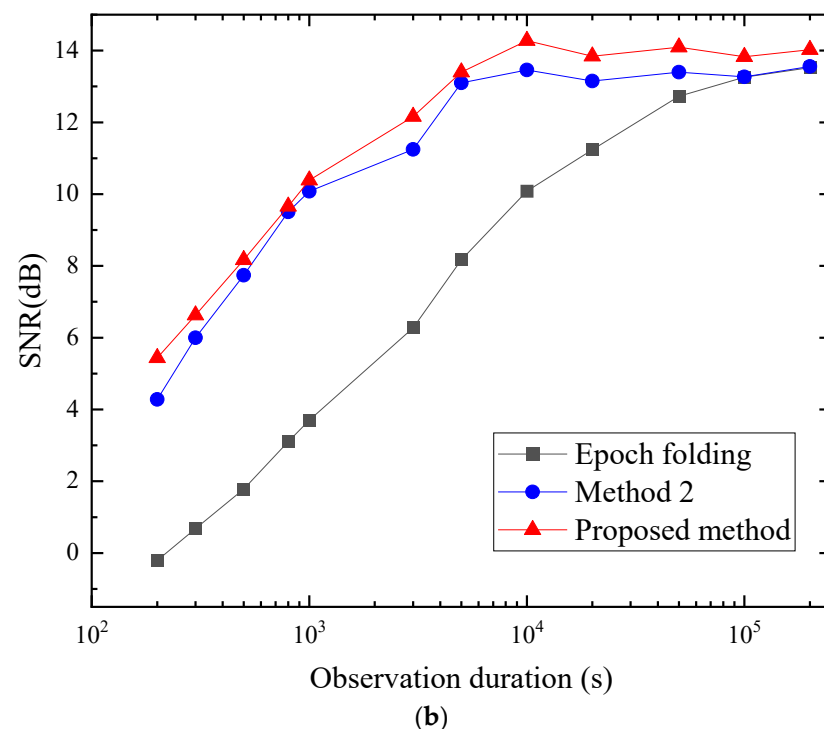


Figure 6. Comparison of the PSR B1937+21 observation profile quality obtained by the three methods: (a) correlation coefficient variation with observation duration; (b) SNR variation with observation duration.

Figure 7 compares the observed profiles of the Crab pulsar, and Figure 8 compares the observed profiles of the B1937+21 pulsar. According to the results, the profiles obtained using the proposed method exhibit higher quality than those obtained by epoch folding under the same observation duration, with the highest correlation coefficient increasing by 0.41 and the maximum SNR improving by 6.689 dB. Moreover, under the same observation duration, the proposed method also outperforms Method 2, achieving an increase of 0.024 in the maximum correlation coefficient and an improvement of 2.126 dB in the maximum SNR. Furthermore, to obtain observational profiles with the same SNR and correlation coefficient, the proposed method requires approximately one order of magnitude less observation time than epoch folding.

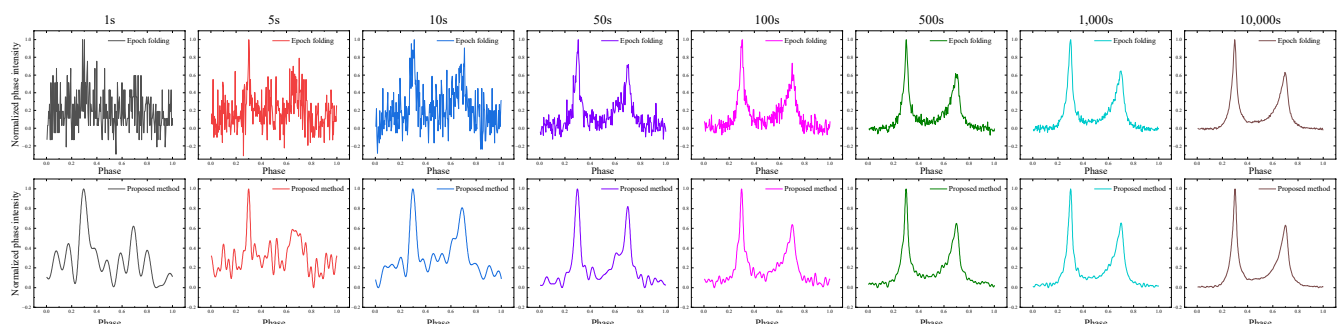


Figure 7. Comparison of the observation profiles of Crab pulsar obtained by epoch folding and the proposed method at different observation durations (1 s, 5 s, 10 s, 50 s, 100 s, 500 s, 1000 s, and 10,000 s).

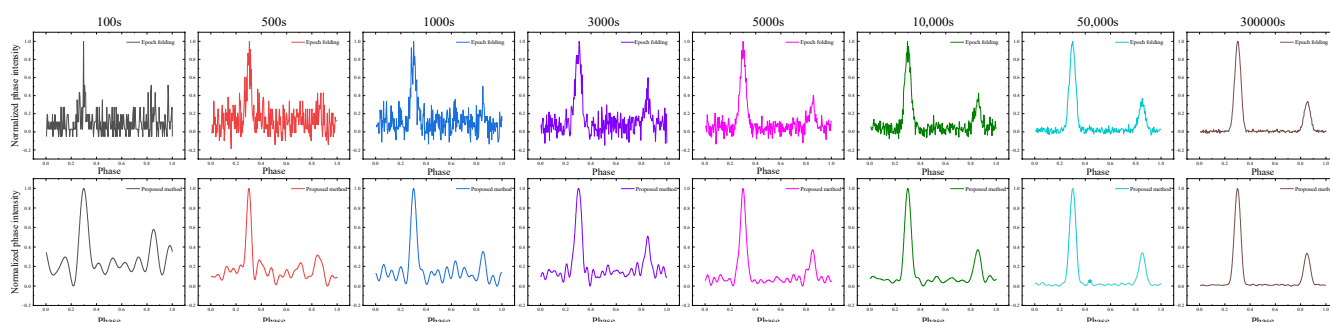


Figure 8. Comparison of the observation profiles of PSR B1937+21 obtained by epoch folding and the proposed method at different observation durations (100 s, 500 s, 1000 s, 3000 s, 5000 s, 10,000 s, 50,000 s, and 300,000 s).

4.2. Validation and Analysis Using Real Observational Data

To further validate the proposed method, the measured photon data from the current in-orbit X-ray detection satellites NICER and Insight-HXMT were converted to photons at the SSB, and the observed profiles were obtained using the three methods. Similarly, the quality of the profiles was compared and analyzed using Equations (18) and (19).

Validation analysis was conducted using the following: the ni1070020349 (PSR B1937+21) and ni0020020106-16 merged with ni1020020101-05 (PSR B1509-58) of the NICER satellite [42], and the P0111605063 LE (crab pulsar) data package of the Insight-HXMT satellite [43]. The observed profiles are presented in Figure 9. Figure 9a compares the PSR B1937+21 profiles obtained using a data packet of 30,000 photons. The profile obtained using the proposed method achieved an SNR of 6.7188 dB and a correlation coefficient of 0.8874. In comparison, Method 2 achieved an SNR of 5.8279 dB and a correlation coefficient of 0.8831, while epoch folding yielded an SNR of 1.7534 dB and a correlation coefficient of 0.6198. Figure 9b compares the PSR B1509-58 profiles obtained using a data packet of 450,000 photons. The profile obtained using the proposed method achieved an SNR of 21.9147 dB and a correlation coefficient of 0.9971. In comparison, Method 2 achieved an SNR of 21.8343 dB and a correlation coefficient of 0.9967, while epoch folding yielded an SNR of 12.5707 dB and a correlation coefficient of 0.9743. Figure 9c compares the Crab pulsar profiles obtained using a data packet of 600,000 photons. The profile obtained using the proposed method achieved an SNR of 15.5155 dB and a correlation coefficient of 0.9874. In comparison, Method 2 achieved an SNR of 15.1836 dB and a correlation coefficient of 0.9804, while epoch folding yielded an SNR of 12.3222 dB and a correlation coefficient of 0.9717. From the results, it can be intuitively observed that for the measured data of different pulsars with different observation durations, the observation profiles obtained using the proposed method were better than those obtained by epoch folding and Method 2, and the effect was more prominent when the number of photons was small.

Owing to the discontinuity of the satellite observation data, the quality of the observation profiles obtained by the two methods was compared based on the number of photons rather than the observation duration. Taking the Crab pulsar as an example, the analysis was validated using 3000–1,000,000 photons from the ni1011010101 data packet of the NICER satellite. The correlation coefficient and SNR variations with the number of photons are shown in Figure 10, and the observed profiles obtained are compared in Figure 11.

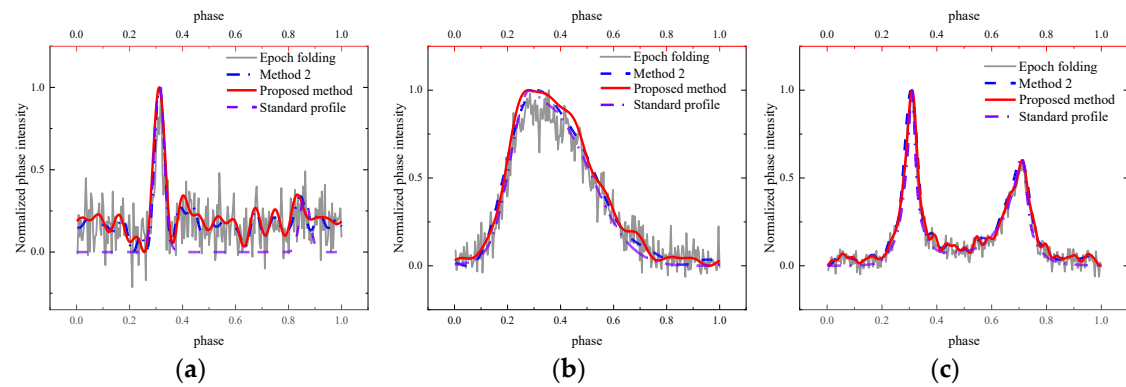


Figure 9. The observation profiles obtained by the three methods for different pulsars: (a) PSR B1937+21; (b) PSR B1509-58; (c) Crab pulsar.

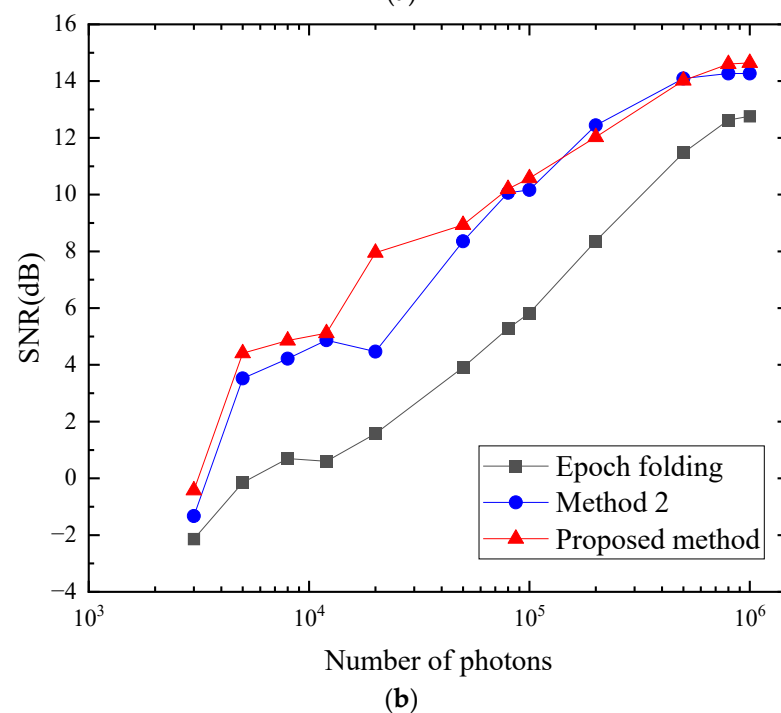
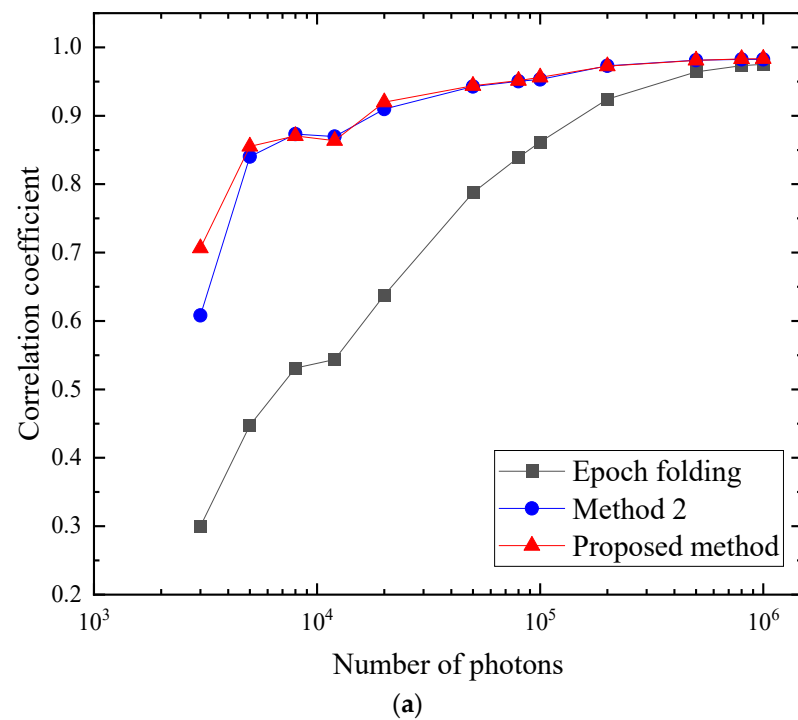


Figure 10. Comparison of the quality of the observation profiles obtained by three methods using ni1011010101 package: (a) Correlation coefficient variation with the number of photons; (b) SNR variation with the number of photons.

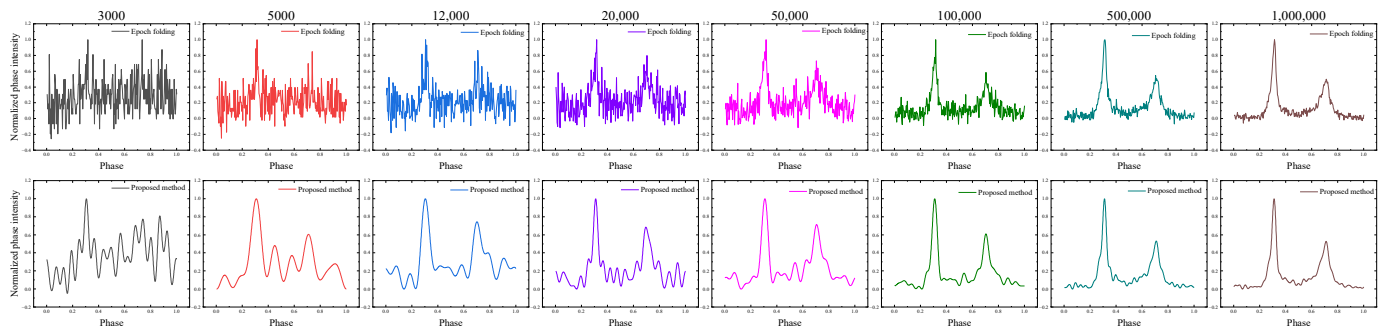


Figure 11. Comparison of the Crab pulsar observation profiles obtained by the three methods using the ni1011010101 package with different numbers of photons (3000, 5000, 12,000, 20,000, 50,000, 100,000, 500,000, and 1,000,000).

Based on the results, the trend of the curves shown in Figure 10 is consistent with that observed in the simulation results in Figure 5. Relative to epoch folding, the proposed method increases the highest correlation coefficient by 0.4 and improves the maximum SNR by as much as 6.38 dB. In addition, compared with Method 2, the proposed method achieves an increase of 0.098 in the maximum correlation coefficient and an improvement of 3.49 dB in the maximum SNR. Similarly, to obtain observational profiles with the same SNR and correlation coefficient, the required number of photons can be reduced by approximately one order of magnitude. As the photon count increases, the performance of the three methods becomes increasingly similar.

4.3. Analysis of Navigation Applications

To further evaluate the performance of the proposed method in navigation applications, this section conducts simulation-based verification using the XPNNAV process shown in Figure 12. Except for the different profile acquisition methods, the remaining processing steps are identical. The photon sequence simulation and time conversion models follow the approaches described in References [40] and [41], respectively. Phase estimation is performed using the cross-correlation method, and state estimation employs an unscented Kalman filter (UKF).

Taking one navigation result in geostationary orbit as an example, Figure 13 presents the navigation results obtained using profiles acquired by the proposed method and epoch folding. Three pulsars are employed for navigation: the Crab pulsar, PSR B1937+21, and PSR B1821–24. The process noise covariance matrix Q of the UKF is set as a diagonal matrix with all diagonal elements equal to 1×10^{-4} . The measurement noise covariance R is set to the Cramér–Rao lower bound (CRLB) of the time-of-arrival (TOA) estimation for the observed pulsars. The initial state error is given as $[10,000 \text{ m}, 10,000 \text{ m}, 10,000 \text{ m}, 10 \text{ m/s}, 10 \text{ m/s}, 10 \text{ m/s}]^T$. As shown in Figure 13, each navigation filtering update is typically performed over a single observation interval of 300 s, with a detector area of 1 m^2 , under which the epoch-folded profile yields a position error of 1794.38 m and a velocity error of 0.192 m/s. When the detector area is reduced to 0.1 m^2 , the profile obtained using the proposed method yields a position error of 2113.29 m and a velocity error of 0.524 m/s for the same 300 s observation duration. In contrast, under identical conditions, the epoch-folded profile results in a position error of 2878.42 m and a velocity error of 0.662 m/s.

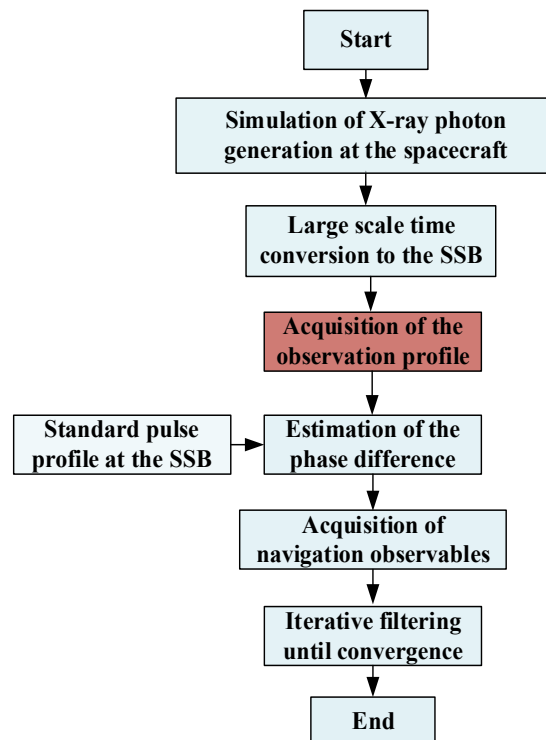
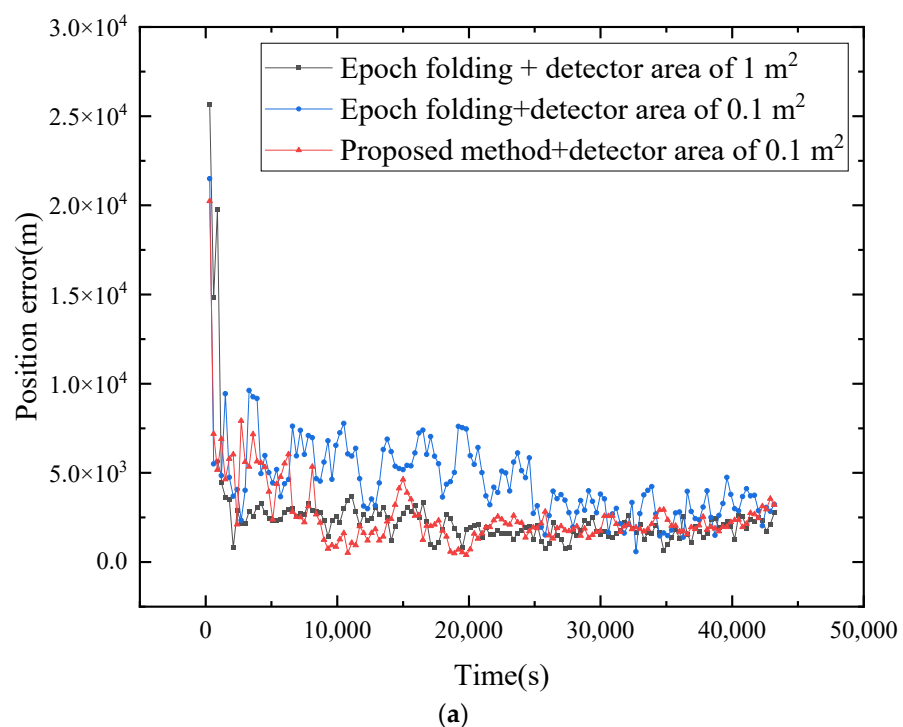


Figure 12. Flowchart of current mainstream X-ray pulsar navigation methods.

To further assess the statistical significance of the results, 100 Monte Carlo simulations were performed for each method under different detector areas within the navigation framework. The mean and standard deviation of the position and velocity errors were computed, and the results are summarized in Table 3.



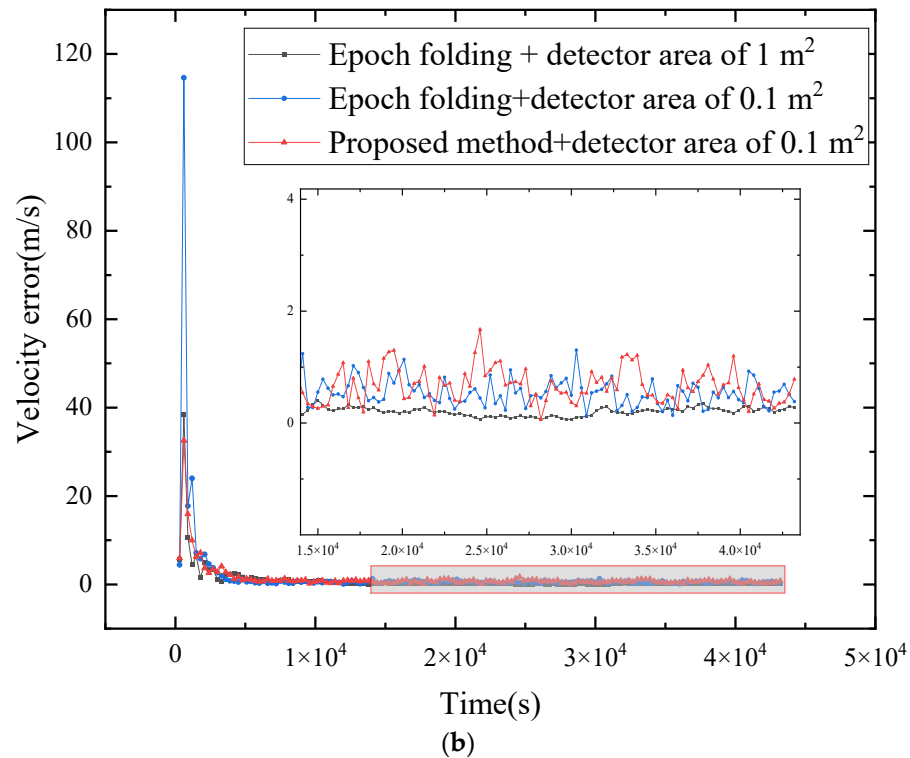


Figure 13. Comparison of Navigation Performance Using Profiles Obtained by the Proposed Method and Epoch Folding: (a) position error; (b) speed error.

Table 3. Statistical comparison of navigation performance over 100 Monte Carlo simulations.

Profile Acquisition Method	Detector Area (m ²)	Mean Position Error (m)	Std. Position Error (m)	Mean Velocity Error (m/s)	Std. Velocity Error (m/s)
the proposed method	0.1	2105.25	61.72	0.5332	0.0128
Epoch folding	0.1	2812.89	73.45	0.6541	0.0153
Epoch folding	1	1965.32	59.17	0.3256	0.0131

These results indicate that, for small detector areas corresponding to low photon counts, the proposed method provides higher navigation accuracy than epoch folding. This improvement arises because epoch folding sometimes fail to extract effective profiles for low-flux pulsars such as PSR B1937+21 and PSR B1821–24 under photon-limited conditions. Nevertheless, both methods exhibit degraded navigation performance at a detector area of 0.1 m² compared with 1 m².

5. Discussion

In this study, a new profile acquisition method is proposed that recovers time-domain waveforms by directly estimating high-quality spectra from photon arrival sequences and subsequently extracting the effective spectral bands.

Numerical simulations and real observational data are employed to validate the effectiveness of the proposed method. Photon arrival sequences from the Crab pulsar and the millisecond pulsar PSR B1937+21 are numerically simulated. For different observation durations, the profiles obtained using the proposed method exhibit higher correlation coefficients and signal-to-noise ratios than those obtained using traditional epoch folding, and also outperform Method 2 (Figures 5–8). Furthermore, measured photon data from the currently operating X-ray observatories NICER and Insight-HXMT confirm that the observation profiles generated by the proposed method are superior to those obtained

using epoch folding and Method 2. This advantage becomes increasingly pronounced under low-photon-count conditions (Figures 9–11).

Based on the proposed method, profiles of comparable quality can be obtained using fewer photons. XPNAV is typically performed with an observation duration of 300 s and a detector area of 1 m². Under these conditions, the photon count is sufficient, and the profile quality achieved by the proposed method is comparable to that obtained using epoch folding, resulting in consistent navigation performance. The advantages of the proposed method become fully evident only when the detector area is small or observational resources are limited. When the proposed method is applied to XPNAV with an observation duration of 300 s and a reduced detector area of 0.1 m², the navigation position error is reduced by 707.64 m compared with profile acquisition using epoch folding. It is important to note that XPNAV involves a highly nonlinear process. Improvements in profile quality are positively correlated with enhancements in navigation performance; however, the resulting navigation accuracy gains are not necessarily of the same order of magnitude.

6. Conclusions

This study developed a new method for obtaining profiles by obtaining high-precision spectra without phase deviation directly from photon sequences. The approach was validated using both simulated data and observational data from NICER and Insight-HXMT. The results demonstrate that the proposed method outperforms conventional epoch folding in terms of profile quality; it also surpasses Method 2, and achieves equivalent SNR and correlation coefficient using approximately one order of magnitude fewer photons than epoch folding. For example, using only 30,000 photons from NICER observations of PSR B1937+21, the proposed method achieves an SNR improvement of 4.9654 dB and an increase of 0.2676 in the correlation coefficient, compared with 1.7534 dB and 0.6198, respectively, obtained using epoch folding. In comparison with Method 2, it achieves an SNR improvement of 0.8909 dB and a correlation coefficient increase of 0.0043, versus 5.8279 dB and 0.8831, respectively. And for the Crab pulsar data from NICER, the method achieved a correlation coefficient of 0.8552 and an SNR of 4.4096 dB using just 5000 photons, whereas epoch folding required approximately 80,000 photons to reach comparable quality. Its advantage becomes especially prominent under low-photon conditions, making it highly suitable for small-area detectors and resource-constrained observation scenarios in practical detection tasks. Using a geostationary orbit as an example, for an observation duration of 300 s and a detector area of 0.1 m², the profile obtained using the proposed method reduces the navigation position error by 707.64 m compared with that obtained using epoch folding-based profiles (2812.89 m). To be sure, obtaining the optimal truncation frequency remains a highly valuable research topic. For different pulsars and observation durations, studying the adaptive optimal truncation frequency determination method will improve the separation of signals and noise in the frequency domain, thereby further improving the results of the proposed method.

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Conflicts of Interest: Author Weimin Bao was employed by China Aerospace Science and Technology Corporation. The remaining authors declare that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

Abbreviations

The following abbreviations are used in this manuscript:

XPNAV	X-ray pulsar navigation
EMD	empirical mode decomposition
VMD	variational mode decomposition
NICER	Neutron Star Interior Composition Explorer
Insight-HXMT	Insight Hard X-ray Modulation Telescope
SSB	Solar System barycenter
MISE	mean integrated squared error
SNR	signal-to-noise ratio

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