

## Article

# Cooperative Guidance Law for Targets Maneuvering in 3D with Multiple Constraints

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## Abstract

This study focuses on the situation where multiple vehicles simultaneously intercept a maneuvering target with desired line-of-sight (LOS) angles and impact time in a three-dimensional environment. A predefined-time cooperative guidance law is proposed, which adapts to the switching communication topologies among the vehicles. To accomplish the task of adjusting the impact time by leveraging the multi-agent consensus principle, a novel adaptive predefined-time consensus protocol is designed in the LOS direction, which allows multiple vehicles to intercept the target at the desired impact time, while ensuring that the impact times of all vehicles converge within a predefined time. In the LOS normal direction, based on the error transformation function, a novel predefined-time non-singular fast terminal sliding mode guidance law is proposed to guarantee the convergence of the LOS angular rate within the predefined time, achieving the simultaneous interception of the target by multiple vehicles at different terminal angles and eliminating singularities. At the same time, the tracking error satisfies the prescribed performance constraint. In particular, for scenarios where the target acceleration is unknown, a predefined-time external state observer is employed to estimate disturbances and compensate within the guidance law. The simulation results validate the effectiveness and robustness of the proposed method.

**Keywords:** cooperative guidance law; predefined-time consensus; switching topologies; impact time constraint; terminal angle constraint; maneuvering target; prescribed performance

## 1. Introduction

With the continuous development of multi-vehicle systems, modern complex systems are evolving toward information-driven and intelligent architectures. Single-vehicle guidance systems are often limited in performance, leading to reduced accuracy. Multi-vehicle cooperative strategies improve system performance and robustness, so cooperative guidance has drawn much attention recently [1,2]. Also, to enhance lethality, vehicles often need to hit the vulnerable parts of targets at the desired angles and the expected impact time. Thus, studying high-performance cooperative guidance laws with multiple constraints is crucial in engineering applications.

Research on multi-vehicle cooperative guidance can be broadly classified into two categories. The first category focuses on specifying the cooperative impact time before vehicles launch. By controlling each vehicle, multiple vehicles can intercept the target simultaneously. The main design methods for guidance laws include proportional navigation guidance (PNG) law [3], optimal guidance law [4], sliding mode guidance law [5], and



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convex optimization [6]. The guidance laws allow multiple vehicles to intercept simultaneously, but there is no information exchange during guidance. External disturbances make it difficult to accurately specify the impact time, and dealing with highly maneuverable targets reduces guidance accuracy [7].

To boost multi-vehicle cooperative guidance robustness, the second method applies multi-agent consensus theory. A communication protocol is introduced to transmit vehicle information and make adjustments in real time, enabling the simultaneous interception of the target without pre-setting impact time. In [8], for three-dimensional (3D) cooperative guidance of maneuvering targets, considering vehicle communication, a robust cooperative guidance law was proposed to ensure cooperative performance under system failures. Most studies assume ideal vehicle communication with a fixed topology. In reality, this is unstable due to equipment and interference, leading to switching communication topologies that affect cooperative performance [9]. In [10], a time-varying leader–follower cooperative guidance law for maneuvering targets in formation with switching communication topologies was designed; however, terminal angle constraints were ignored. Therefore, it is important to study cooperative guidance laws with switching topologies and angle constraints.

In certain complex scenarios, to optimize the effectiveness of cooperative engagement, vehicles are required to intercept the target with a constrained terminal angle and an adjustable impact time [11]. At present, there are only a few studies on guidance laws considering impact time constraints. In [7], a time-consistent algorithm was designed using the consistency error function and fixed time theory, which realizes terminal-angle-constrained interception at a prescribed arrival time. However, the method does not consider the interception of maneuvering targets. In [12], a collision-time-constrained cooperative guidance law combining PNG and super-twisting theory was proposed in 2D, but its performance may degrade in 3D scenarios. In [13], an adaptive cooperative guidance law was proposed to enable vehicles to achieve the interception of the target with both arrival time and terminal angle constraints.

In practical scenarios, terminal guidance time is short, and traditional asymptotic stability methods are insufficient [14]. Studying fast consensus of arrival time among multiple vehicles can improve cooperative performance. In [15], based on finite-time consensus theory, a 3D cooperative guidance law considering terminal angle constraints was designed. In [16], when radial velocity was unknown, a 3D cooperative guidance with fixed-time convergence and fixed topologies was proposed. However, terminal angle constraints were not considered. In [17], the concept of predefined-time stability was proposed, where the settling time can be adjusted as a controller gain to achieve predefined-time convergence. This theory has been widely applied in various fields, such as attitude control [18] and a transport robot [19]. In [20], considering 3D coupled kinematics, a guidance law with predefined-time impact angle constraints was proposed. In [21], a predefined-time cooperative guidance law considering switching topologies was designed for simultaneous interception in a 2D environment. However, it does not consider angle constraints.

In the guidance process, constraining the convergence of the LOS angle can be formulated as a state-performance-constrained control problem. However, in existing studies on terminal angle guidance, the tracking process is rarely explicitly constrained. In [22,23], prescribed performance control (PPC) theory was proposed based on a pre-designed performance function. It essentially employs a transformation function to map the constrained tracking error into an unconstrained space, thereby ensuring that system states evolve within predefined performance bounds. PPC is widely used in space attitude control [24]. In [25], the prescribed performance function (PPF) was used in a multi-vehicle intercept-

tion problem, and a guidance law was proposed. However, the convergence time of the designed performance constraint function is implicit.

Existing studies seldom address 3D cooperative guidance with simultaneous consideration of impact time constraints, terminal angle constraints, switching communication topologies, predefined-time convergence, prescribed performance, and interception of maneuvering targets in a unified framework. Motivated by the above challenges, this paper designs a non-singular predefined-time convergent cooperative guidance law for multi-vehicle interception of maneuvering targets under multiple constraints, applicable to switching communication topologies. Main contributions are:

- (1) This study investigates a cooperative guidance strategy with multiple constraints in 3D space, considering a complex nonlinear kinematic model under switching communication topologies. Since inter-vehicle communication is inherently dynamic in real-world scenarios, the proposed approach provides a more practical and realistic framework for cooperative guidance applications.
- (2) To address the cooperative guidance problem, a novel adaptive predefined-time consensus protocol is proposed in the LOS direction based on predefined-time convergence theory and the multi-agent consensus principle. This protocol enables multiple vehicles to achieve the simultaneous interception of maneuvering targets at a prescribed impact time.
- (3) To handle terminal angle constraints, a predefined-time non-singular terminal sliding mode guidance law is developed in the LOS normal direction using prescribed performance functions. It enables multiple vehicles to achieve simultaneous interception at different terminal angles while satisfying prescribed performance constraints. In addition, it removes singularities and alleviates chattering in sliding-mode control.
- (4) To address the scenario where the target acceleration is unknown, a novel predefined-time external state observer is proposed to estimate the target acceleration and compensate for disturbances in the guidance law.
- (5) In the proposed cooperative guidance method, the convergence time is an explicitly tunable parameter, and all error states are guaranteed to converge within the predefined time.

The structure of this paper is as follows. Section 2 presents the preliminaries and the problem formulation of cooperative engagement dynamics. Section 3 presents the cooperative guidance law and predefined-time stability analysis. In Section 4, the numerical simulation results validate the proposed method. Section 5 is the Conclusion Section of this paper.

## 2. Preliminaries and Problem Formulation

### 2.1. Preliminaries

In the field of cooperative guidance, graph theory is used to represent a communication topology consisting of multiple vehicles, where the vehicles exchange their state information. The communication network can be represented by an  $n$ -order directed graph  $G = (v, \zeta, A)$ ,  $v = \{v_1, v_2, \dots, v_n\}$  is the set of nodes in the graph,  $\zeta \subseteq v \times v$  is the set of connecting lines in the graph, matrix  $A = [a_{ij}] \in \mathbf{R}^{n \times n}$  represents the weighted connectivity matrix; when vehicle  $i$  receives information from vehicle  $j$ , then  $a_{ij} > 0$ ; otherwise,  $a_{ij} = 0$ . If  $G$  is an undirected graph, then we have  $a_{ij} = a_{ji}$ .

**Lemma 1** ([26]). *The Laplacian matrix  $L_G = [l_{ij}] \in \mathbf{R}^{n \times n}$ , corresponding to the undirected graph  $G$ , is defined by*

$$l_{ij} = \begin{cases} \sum_{k=1, k \neq i}^n a_{ik}, & j = i \\ -a_{ij}, & j \neq i \end{cases} \quad (1)$$

and, if the undirected graph  $G$  is connected,  $L_G$  has the following properties:

- (1)  $L_G \mathbf{1}_n = \mathbf{0}_n$ , i.e.,  $\mathbf{1}_n = [1, \dots, 1]^T$ .
- (2)  $\lambda_2(L_G) > 0$  is the algebraic connectivity of  $L_G$ .
- (3) For any  $\mathbf{x} = [x_1, x_2, \dots, x_n]^T \in \mathbb{R}^n$ , there is  $\mathbf{x}^T L_G \mathbf{x} = \frac{1}{2} \sum_{i=1}^n \sum_{j=1}^n a_{ij} (x_j - x_i)^2$ .
- (4) For any  $\mathbf{x} = [x_1, x_2, \dots, x_n]^T \in \mathbb{R}^n$  that satisfies  $\mathbf{1}^T \mathbf{x} = 0$ , then  $\mathbf{x}^T L_G \mathbf{x} \geq \lambda_2(L_G) \mathbf{x}^T \mathbf{x}$ .

**Lemma 2** ([27]). For  $x_1, x_2, \dots, x_n \geq 0$ , if  $0 < p \leq 1$ , then

$$\begin{cases} \sum_{i=1}^n x_i^p \geq (\sum_{i=1}^n x_i)^p, & 0 < p \leq 1 \\ \sum_{i=1}^n x_i^p \geq n^{1-p} (\sum_{i=1}^n x_i)^p, & p \geq 1 \end{cases} \quad (2)$$

**Notation 1.** Throughout this paper,  $\|\cdot\|$  signifies the Euclidean norm. For a vector  $\mathbf{x} = [x_1, \dots, x_n]^T$  and  $k \in \mathbb{R}$ , define  $\text{sig}^k(\mathbf{x}) = [|x_1|^k \text{sign}(x_1), \dots, |x_n|^k \text{sign}(x_n)]^T$ , where  $\text{sign}(\cdot)$  represents the sign function.

**Definition 1** ([17]). Consider a nonlinear system defined by the following equation:

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \lambda), \quad \mathbf{x}(0) = \mathbf{x}_0 \quad (3)$$

where  $\mathbf{x} \in \mathbb{R}^n$ ,  $\mathbf{f}(\mathbf{x}(t)) : \mathbb{R}^n \rightarrow \mathbb{R}^n$  is a nonlinear function;  $\lambda \in \mathbb{R}^n$  are tunable parameters. System (3) exhibits predefined-time stability with convergence time  $T_c$  if  $T(\mathbf{x}_0) \leq T_c < \infty$  for any initial condition  $\forall \mathbf{x}_0 \in \mathbb{R}^n$ .

**Lemma 3** ([28]). Given system (3), stability is guaranteed if there exists a continuous Lyapunov function  $V(\mathbf{x})$ , if it satisfies the following conditions:

$$\dot{V} \leq -\frac{\pi}{\eta T_c \sqrt{\alpha\beta}} \left( \alpha V^{(1-\frac{\eta}{2})} + \beta V^{(1+\frac{\eta}{2})} + 2\sqrt{\alpha\beta} V \right) \quad (4)$$

where  $0 < \eta < 1$ ,  $\alpha, \beta, T_c > 0$ , and the system exhibits predefined-time stability with  $T_c$ .

**Lemma 4** ([28]). Given system (3), it satisfies the following conditions:

$$\dot{V} \leq -\frac{\pi}{\eta T_c \sqrt{\alpha\beta}} \left( \alpha V^{(1-\frac{\eta}{2})} + \beta V^{(1+\frac{\eta}{2})} + 2\sqrt{\alpha\beta} V \right) + \Delta \quad (5)$$

Based on Lemma 3, if  $\Delta > 0$ , then the system is stable within the predefined time  $\sqrt{2}T_c$ , and the residual set of the solution of the system (3) is  $\mathbf{x} \in \left\{ V(\mathbf{x}) \leq \min \left\{ \left( \frac{2\Delta\eta\sqrt{\alpha\beta}}{\alpha\pi} \right)^{\frac{2}{2-\eta}}, \left( \frac{2\Delta\eta\sqrt{\alpha\beta}}{\beta\pi} \right)^{\frac{2}{2+\eta}} \right\} \right\}$

**Lemma 5** ([29]). For the system (3), it satisfies the following conditions:

$$\dot{V} \leq -\frac{\gamma}{T_c} (\alpha V^p + \beta V^q)^l \quad (6)$$

where  $l, p, q, \alpha, \beta$  are positive gains, and  $lp < 1, lq > 1$ , and  $\gamma$  is

$$\gamma = \frac{\Gamma(p_n)\Gamma(q_n)}{\alpha^l \Gamma(l)(q-p)} \left( \frac{\alpha}{\beta} \right)^{p_n}$$

where  $p_n = \frac{1-lp}{q-p}$ ,  $q_n = \frac{lq-1}{q-p}$ , and  $\Gamma(\cdot)$  is the Gamma function, which is defined as  $\Gamma(x) = \int_0^{+\infty} e^{-t} t^{x-1} dt$ . The system (3) is predefined-time stable;  $T_c$  is the predefined convergence time.

**Lemma 6** ([30]). Consider the following disturbance second-order nonlinear systems:

$$\begin{cases} \dot{x}_1(t) = x_2(t) \\ \dot{x}_2(t) = f(t) + u(t) + d(t) \end{cases} \quad (7)$$

where  $x_1(t) \in \mathbb{R}$  and  $x_2(t) \in \mathbb{R}$  are the system states,  $u(t)$  is the control input,  $f(t)$  is the known internal nonlinear term,  $d(t)$  is external disturbance. Assume that the external disturbance  $d(t)$  and its first-order derivatives  $\dot{d}(t)$  are bounded with  $|d(t)| \leq d_1$  and  $|\dot{d}(t)| \leq d_2$ , where  $d_1$  and  $d_2$  are positive constants.

For system (7), a predefined-time external state observer (PTESO) is designed as follows:

$$\begin{cases} \tilde{x}_1 = x_1 - \hat{x}_1 \\ \dot{\tilde{x}}_1 = x_2 + \Phi l_1 \zeta h_{ESO} \tilde{x}_1 + (1 - \Phi) k_1 \mu^{\frac{1}{3}} \text{sig}^{\frac{2}{3}}(\tilde{x}_1) \\ \dot{\tilde{x}}_2 = f + u + \hat{d} + \Phi l_2 \zeta^2 h_{ESO}^2 \tilde{x}_1 + (1 - \Phi) k_2 \mu^{\frac{2}{3}} \text{sig}^{\frac{1}{3}}(\tilde{x}_1) \\ \dot{\tilde{d}} = \Phi l_3 \zeta^3 h_{ESO}^3 \tilde{x}_1 + (1 - \Phi) k_3 \mu \text{sig}^0(\tilde{x}_1) \end{cases} \quad (8)$$

where  $h_{ESO} = a \sec(\frac{\pi}{2} \frac{t}{t_{f,ESO}})$  is a time-varying function,  $a > 0$ , and  $t_{f,ESO}$  is predefined time of PTESO.  $l_{1,2,3}, \mu$  and  $\zeta > 0$  are the parameters to be designed, meanwhile  $\mu \geq 0$ ,  $k_p = \alpha_p k_3^{\frac{p}{3}}$ ,  $\alpha_p > 0$ ,  $p = 1, 2, k_3 > 1$ ,  $\Phi(t)$  is the time-switching function of PTESO.

$$\Phi(t) = \begin{cases} 1, & t \in [0, t_{f,ESO}) \\ 0, & t \in [t_{f,ESO}, \infty) \end{cases} \quad (9)$$

where  $\hat{x}_1, \hat{x}_2, \hat{d}$  are the observations of  $x_1, x_2, d$ , respectively. For system (7) with PTESO (8), the error  $\tilde{d} = d - \hat{d}$  can achieve convergence to 0 within the predefined time  $t_{f,ESO}$ .

## 2.2. Problem Formulation

Concerning the issue of multiple vehicles' cooperative guidance in a 3D space. Figure 1 vividly illustrates the relative geometric relationship of interest, where  $M_i$  and  $T$  represent the  $i$ -th vehicle and the target, respectively.  $MX_I Y_I Z_I$ ,  $MX_L Y_L Z_L$  represent the inertial coordinate system and LOS coordinate system, respectively.  $V_n, \theta_n, \phi_n$  ( $n = M, T$ ) represent the velocity, flight-path angle and heading angle of the vehicle or target. The distance between the  $i$ -th vehicle and the target is denoted by  $R_i$ , while  $\theta_{L,i}$  and  $\phi_{L,i}$  represent the elevation and azimuth angles of the LOS coordinate system, respectively.

The 3D engagement kinematics can be described as [31]

$$\dot{R}_i = V_r \quad (10)$$

$$\dot{\theta}_{L,i} = V_{\theta_{L,i}} \quad (11)$$

$$-R_i \dot{\phi}_{L,i} \cos \theta_{L,i} = V_{\phi_{L,i}} \quad (12)$$

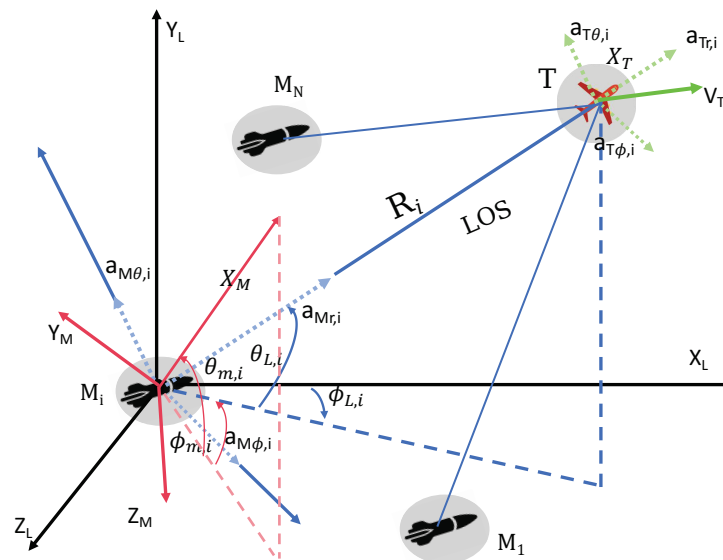
$$\ddot{R}_i - R_i \dot{\theta}_{L,i}^2 - R_i \dot{\phi}_{L,i}^2 \cos^2 \theta_{L,i} = a_{Tr,i} - a_{Mr,i} \quad (13)$$

$$R_i \ddot{\theta}_{L,i} + 2\dot{R}_i \dot{\theta}_{L,i} + \dot{R}_i \dot{\phi}_{L,i}^2 \sin \theta_{L,i} \cos \theta_{L,i} = a_{T\theta,i} - a_{M\theta,i} \quad (14)$$

$$-\dot{R}_i \ddot{\phi}_{L,i} \cos \theta_{L,i} - 2\dot{R}_i \dot{\phi}_{L,i} \cos \theta_{L,i} + 2R_i \dot{\theta}_{L,i} \dot{\phi}_{L,i} \sin \theta_{L,i} = a_{T\phi,i} - a_{M\phi,i} \quad (15)$$

where  $V_r, V_{\theta_{L,i}}, V_{\phi_{L,i}}$  are the relative velocities between the  $i$ th vehicle and the target in the LOS coordinate system. Moreover,  $a_T = [a_{Tr,i}, a_{T\theta,i}, a_{T\phi,i}]^T$  and  $a_M =$

$[a_{Mr,i}, a_{M\theta,i}, a_{M\phi,i}]^T$  denote the acceleration vectors of the target and  $i$ th vehicle in the LOS coordinate system, respectively.



**Figure 1.** Three-dimensional vehicle–target interception geometry.

**Assumption 1.** The target acceleration is unknown but bounded, continuous, and differentiable, and it is assumed that  $|\dot{a}_T| \leq L$ , where  $L$  represents a positive constant.

**Remark 1.** As demonstrated in [16], the coordinate transformation matrix from the inertial coordinate system  $OX_IY_IZ_I$  to the LOS frame  $OX_LY_LZ_L$  can be expressed as

$$C_I(\theta_L\phi_L) = \begin{bmatrix} \cos\theta_L\cos\phi_L & \sin\theta_L & -\cos\theta_L\sin\phi_L \\ -\sin\theta_L\cos\phi_L & \cos\theta_L & \sin\theta_L\sin\phi_L \\ \sin\phi_L & 0 & \cos\phi_L \end{bmatrix}$$

The following section details the coordinate transformation matrix from  $OX_IY_IZ_I$  to the vehicle (M) or target (T) velocity coordinate system  $OX_{Mv}Y_{Mv}Z_{Mv}$  and  $OX_{Tv}Y_{Tv}Z_{Tv}$ .

$$C_n(\theta_n\phi_n) = \begin{bmatrix} \cos\theta_n\cos\phi_n & \sin\theta_n & -\cos\theta_n\sin\phi_n \\ -\sin\theta_n\cos\phi_n & \cos\theta_n & \sin\theta_n\sin\phi_n \\ \sin\phi_n & 0 & \cos\phi_n \end{bmatrix} \quad (n = M, T)$$

This paper mainly addresses the problem of the simultaneous interception of a target by multiple vehicles under multiple constraints, which can be summarized as follows:

$$\lim_{t \rightarrow t_{f,i}} R_i = 0, \quad \lim_{t \rightarrow t_{f,i}} \theta_{L,i} - \theta_{LD,i} = 0, \quad \lim_{t \rightarrow t_{f,i}} \phi_{L,i} - \phi_{LD,i} = 0$$

where  $t_{f,i}$  is the impact time;  $\theta_{LD,i}$  and  $\phi_{LD,i}$  are the desired elevation angle and azimuth angle of the  $i$ th vehicle. To improve the maneuverability of the vehicles, it is necessary for the vehicles to intercept the target at the prescribed impact time  $t_d$ , i.e.,

$$t_{f,i} = t_{f,j} = t_d$$

For vehicle  $i, i = 1, 2, \dots, N$ , define  $x_{1i} = R_i$ ,  $x_{2i} = \dot{R}_i$ ,  $x_{3i} = [\theta_{L,i} - \theta_{LD,i}, \phi_{L,i} - \phi_{LD,i}]^T$ ,  $x_{4i} = [\dot{\theta}_{L,i}, \dot{\phi}_{L,i}]^T$ . Combining Equations (10)–(15), the three-dimensional guidance system is presented as follows:

$$\begin{cases} \dot{x}_{1,i} = x_{2,i} \\ \dot{x}_{2,i} = x_{1,i}\dot{\theta}_{L,i}^2 + x_{1,i}\dot{\phi}_{L,i}^2 \cos^2 \theta_{L,i} - a_{Mr,i} + a_{Tr,i} \\ \dot{x}_{3,i} = x_{4,i} \\ \dot{x}_{4,i} = A_i + B_i a_{M\lambda,i} + D_i \end{cases} \quad (16)$$

where

$$A_i = \begin{bmatrix} -\frac{2x_{2,i}\dot{\theta}_{L,i}}{x_{1,i}} - \dot{\phi}_{L,i}^2 \sin \theta_{L,i} \cos \theta_{L,i} \\ -\frac{2x_{2,i}\dot{\phi}_{L,i}}{x_{1,i}} + 2\dot{\theta}_{L,i}\dot{\phi}_{L,i} \tan \theta_{L,i} \end{bmatrix}, a_{M\lambda,i} = \begin{bmatrix} a_{M\theta,i} \\ a_{M\phi,i} \end{bmatrix}$$

$$B_i = \begin{bmatrix} -\frac{1}{x_{1,i}} & 0 \\ 0 & -\frac{1}{x_{1,i}\cos\theta_{L,i}} \end{bmatrix}, D_i = \begin{bmatrix} \frac{a_{T\theta,i}}{x_{1,i}} \\ -\frac{a_{T\phi,i}}{x_{1,i}\cos\theta_{L,i}} \end{bmatrix}$$

### 3. Cooperative Guidance Law Design

In this section, the cooperative guidance law is designed, consisting of two parts. Firstly, the LOS directional acceleration  $a_{Mr,i}$  enables multiple vehicles to intercept the target simultaneously within a predefined time and attack the target at the desired impact time  $t_d$ . Secondly, the normal acceleration  $a_{M\theta,i}$ ,  $a_{M\phi,i}$  enables interception at the desired LOS angle within a predefined time. A strict stability proof is provided for the cooperative guidance system. The flowchart of the guidance laws is clearly presented in Figure 2.

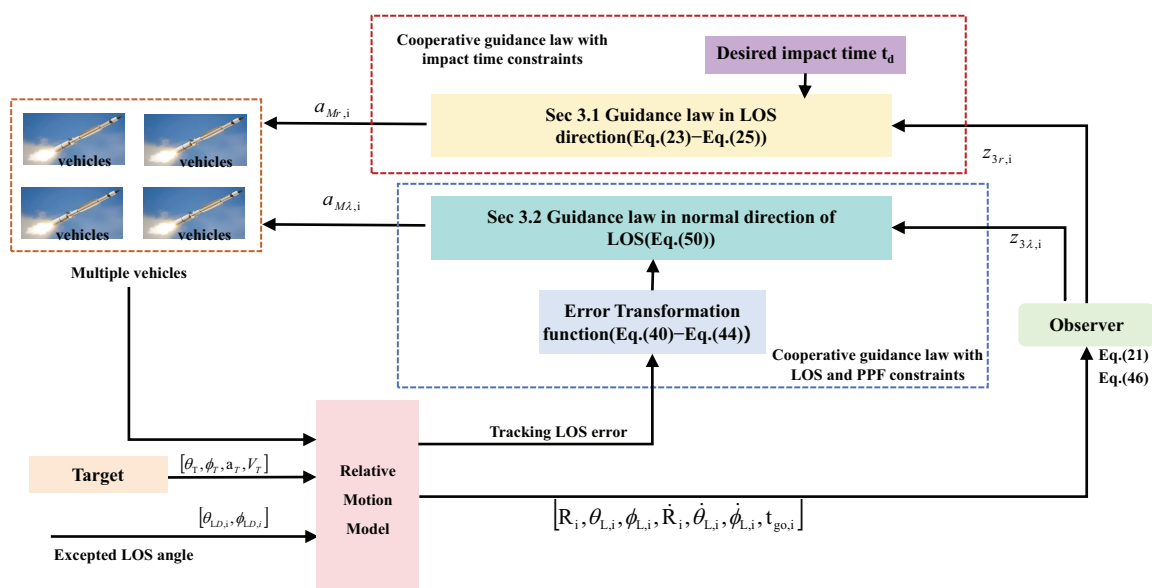


Figure 2. Guidance law flowchart.

#### 3.1. Guidance Law in the LOS Direction with Desired Impact Time

For vehicle  $i, i = 1, 2, \dots, N$ , the impact time  $t_{fi}$  of the  $i$ th vehicle is given by

$$t_{f,i} = t_{go,i} + t \quad (17)$$

where  $t_{go,i}$  is the time-to-go estimate, i.e., the remaining time before the target is intercepted.  $t_{go,i}$  is estimated by

$$t_{go,i} = -\frac{R_i}{\dot{R}_i} = -\frac{x_{1,i}}{x_{2,i}} \quad (18)$$

It can be seen from Equation (17) that  $t_{f,i} - t_{f,j} = t_{go,i} - t_{go,j}$ . Therefore, if  $t_{go,i}$  converges to unity,  $t_{f,i}$  also reaches consensus.

Taking the time derivative of  $t_{f,i}$ , we obtain:

$$\begin{aligned} \dot{t}_{f,i} &= \dot{t}_{g0,i} + 1 \\ &= \frac{x_{1,i}^2 \dot{\theta}_{L,i}^2}{x_{2,i}^2} + \frac{x_{1,i}^2 \phi_{L,i}^2 \cos^2 \theta_{L,i}}{x_{2,i}^2} - \frac{x_{1,i}}{x_{2,i}^2} a_{Mr,i} + \frac{x_{1,i}}{x_{2,i}^2} a_{Tr,i} \end{aligned} \quad (19)$$

According to Assumption 1, there exists a positive constant  $L_{r,i}$  such that the target acceleration in the LOS direction,  $a_{Tr,i}$ , satisfies

$$|\dot{a}_{Tr,i}| \leq L_{r,i} \quad (20)$$

Firstly, based on Lemma 6, Equation (19) is constructed to estimate the target acceleration  $a_{Tr,i}$ :

$$\begin{cases} e_{1r,i} &= x_1 - z_{1r,i} \\ \dot{z}_{1r,i} &= z_{2r,i} + \Phi_r l_{1,r} \zeta_r h_{ESO,r}(e_{1r,i}) + (1 - \Phi_r) k_{1,r} \mu_r^{\frac{1}{3}} \text{sig}^{\frac{2}{3}}(e_{1r,i}) \\ \dot{z}_{2r,i} &= \Phi_r l_{2,r} \zeta_r^2 h_{ESO,r}^2(e_{1r,i}) + (1 - \Phi_r) k_{2,r} \mu_r^{\frac{2}{3}} \text{sig}^{\frac{1}{3}}(e_{1r,i}) \\ &\quad + x_{1,i} \dot{\theta}_{L,i}^2 + x_{1,i} \phi_{L,i}^2 \cos^2 \theta_{L,i} - a_{Mr,i} + z_{3r,i} \\ \dot{z}_{3r,i} &= \Phi_r l_{3,r} \zeta_r^3 h_{ESO,r}^3(e_{1r,i}) + (1 - \Phi_r) k_{3,r} \mu_r \text{sig}^0(e_{1r,i}) \end{cases} \quad (21)$$

where  $z_{1r,i}, z_{2r,i}, z_{3r,i}$  are the estimates of  $x_{1,i}, x_{2,i}$  and  $a_{Tr,i}$ , respectively. The parameters  $h_{ESO,r}, l_{1,2,3,r}, \mu_r, k_{p,r}, p = 1, 2, 3, \Phi_r$  and  $\zeta_r$  are design parameters given in Lemma 6, and  $\mu_r > 0, \Phi_r$  is the switching function of PTESO with  $\Phi_r = 1$ , when  $t \in [0, T_{fr,ESO})$ ; otherwise,  $\Phi_r = 0$ . Then,  $x_{1,i}, x_{2,i}$  and  $a_{Tr,i}$  can be estimated by  $z_{1r,i}, z_{2r,i}$  and  $z_{3r,i}$ , respectively, within a predefined time  $T_{fr,ESO}$ .

From Equation (17), it can be seen that the impact time of the vehicle is related to  $a_{Mr,i}$  in the LOS direction. Therefore, when considering the impact time constraint, only  $a_{Mr,i}$  needs to be designed.

The error between the desired impact time  $t_d$  and the actual time  $t_{f,i}$  of the vehicle is defined as

$$e_i = t_{f,i} - t_d \quad (22)$$

It follows from Equation (22) that when  $t_{f,i}$  reaches consensus,  $e_i$  also achieves consensus. Then, the guidance law can be designed as:

$$a_{Mr,i} = x_{1,i} \dot{\theta}_{L,i}^2 + x_{1,i} \phi_{L,i}^2 \cos^2 \theta_{L,i} + z_{3r,i} + \frac{x_{2,i}^2}{x_{1,i}} u_{Mr,i} \quad (23)$$

where  $z_{3r,i}$  is the estimation of the target acceleration  $a_{Tr,i}$  in the LOS direction, obtained by the PTESO (21), and

$$\begin{aligned} u_{Mr,i} &= \frac{\pi}{2\eta_r T_r \tilde{\lambda}_2 \sqrt{\alpha_r \beta_r}} \left[ \alpha_r N^{\frac{\eta_r}{2}} \text{sig}^{(1-\eta_r)} \left( \sum_{j=1}^N a_{ij} (t_{f,j} - t_{f,i}) \right) \right. \\ &\quad \left. + \beta_r \text{sig}^{(1+\eta_r)} \left( \sum_{j=1}^N a_{ij} (t_{f,j} - t_{f,i}) \right) + 2\sqrt{\alpha_r \beta_r} \left( \sum_{j=1}^N a_{ij} (t_{f,j} - t_{f,i}) \right) \right] \end{aligned} \quad (24)$$

$$\begin{aligned} &+ \omega_{r,i} \text{sig}(t_{f,i} - t_d) \\ \dot{\omega}_{r,i} &= \zeta |t_{f,i} - t_d|^2 \end{aligned} \quad (25)$$

where  $\alpha_r > 0, \beta_r > 0, T_r > 0$  and  $0 < \eta_r < 1$ ;  $N$  denotes the number of vehicles.  $\tilde{\lambda}_2$  is the minimum eigenvalue of the communication topology matrix  $L$ . If the vehicle commu-

nication topology is time-varying, then  $\tilde{\lambda}_2 = \min[\lambda_2(L_1), \lambda_2(L_2), \dots, \lambda_2(L_m)]$ ; otherwise,  $\tilde{\lambda}_2 = \lambda_2(L)$ .

**Remark 2.** The communication topology is allowed to switch arbitrarily, and each topology is connected at any time instant.

**Theorem 1.** For system (19) and PTESO (21), based on the guidance law (23)–(25), the time error  $e_i$  is guaranteed to reach consensus within a predefined time, and the vehicle achieves interception of the target at the desired time  $t_d$ .

**Proof.** According to the PTESO (21), the estimation error can be described by

$$\begin{aligned}\dot{e}_{1r,i} &= e_{2r,i} - \Phi_r l_{1,r} \zeta_r h_{ESO,r} e_{1r,i} - (1 - \Phi_r) k_{1,r} \mu_r^{\frac{1}{3}} \text{sig}^{\frac{2}{3}}(e_{1r,i}) \\ \dot{e}_{2r,i} &= e_{3r,i} - \Phi_r l_{2,r} \zeta_r^2 h_{ESO,r}^2 e_{1r,i} - (1 - \Phi_r) k_{2,r} \mu_r^{\frac{2}{3}} \text{sig}^{\frac{1}{3}}(e_{1r,i}) \\ \dot{e}_{3r,i} &= a_{Tr,i} - \Phi_r l_{3,r} \zeta_r^3 h_{ESO,r}^3 e_{1r,i} - (1 - \Phi_r) k_{3,r} \mu_r \text{sig}^0(e_{1r,i})\end{aligned}\quad (26)$$

where  $e_{1r,i} = x_{1,i} - z_{1r,i}$ ,  $e_{2r,i} = x_{2,i} - z_{2r,i}$ ,  $e_{3r,i} = a_{Tr,i} - z_{3r,i}$ . Based on Lemma 6, the estimation error converges to zero within a predefined time  $T_{fr,ESO}$ , and  $T_{fr,ESO}$  is the switching time of  $\Phi_r$ .

Substituting the guidance law  $a_{Mr,i}$  in (23) into (22), we obtain:

$$\begin{aligned}\dot{e}_i &= \dot{t}_{f,i} = -\frac{\pi}{2\eta_r T_r \tilde{\lambda}_2 \sqrt{\alpha_r \beta_r}} \left[ \alpha_r N^{\frac{\eta_r}{2}} \text{sig}^{(1-\eta_r)} \left( \sum_{j=1}^N a_{ij} (t_{f,j} - t_{f,i}) \right) \right. \\ &\quad \left. + \beta_r \text{sig}^{(1+\eta_r)} \left( \sum_{j=1}^N a_{ij} (t_{f,j} - t_{f,i}) \right) + 2\sqrt{\alpha_r \beta_r} \left( \sum_{j=1}^N a_{ij} (t_{f,j} - t_{f,i}) \right) \right] \\ &\quad - \omega_{r,i} \text{sig}(t_{f,i} - t_d) + \frac{x_{1,i}}{x_{2,i}^2} (a_{Tr,i} - z_{3r,i})\end{aligned}\quad (27)$$

The Lyapunov function is established:

$$V_r = \frac{1}{2} \sum_i^N e_i^2 + \frac{1}{2\zeta} \sum_i^N \omega_{r,i}^2 \quad (28)$$

Taking the derivative of  $V_r$ , we obtain:

$$\begin{aligned}\dot{V}_r &= \sum_{i=1}^N e_i \dot{e}_i + \frac{1}{\zeta} \sum_{i=1}^N \omega_{r,i} \dot{\omega}_{r,i} \\ &= -\frac{\pi}{2\eta_r T_r \tilde{\lambda}_2 \sqrt{\alpha_r \beta_r}} \left[ \alpha_r \sum_{i=1}^N e_i N^{\frac{\eta_r}{2}} \text{sig}^{(1-\eta_r)} \left( \sum_{j=1}^N a_{ij} (t_{f,j} - t_{f,i}) \right) \right. \\ &\quad \left. + \beta_r \sum_{i=1}^N e_i \text{sig}^{(1+\eta_r)} \left( \sum_{j=1}^N a_{ij} (t_{f,j} - t_{f,i}) \right) + 2\sqrt{\alpha_r \beta_r} \sum_{i=1}^N e_i \left( \sum_{j=1}^N a_{ij} (t_{f,j} - t_{f,i}) \right) \right] \\ &\quad - \omega_{r,i} \sum_{i=1}^N |e_i|^2 + \frac{x_{1,i}}{x_{2,i}^2} (a_{Tr,i} - z_{3r,i}) + \sum_{i=1}^N \omega_{r,i} |e_i|^2 \\ &\leq -\frac{1}{2} \frac{\pi}{2\eta_r T_r \tilde{\lambda}_2 \sqrt{\alpha_r \beta_r}} \left[ \alpha_r N^{\frac{\eta_r}{2}} \left( \sum_{i=1}^N \sum_{j=1}^N a_{ij}^{\frac{\eta_r}{2}-1} (e_{f,j} - e_{f,i})^2 \right)^{1-\frac{\eta_r}{2}} \right. \\ &\quad \left. + \beta_r N^{-\frac{\eta_r}{2}} \left( \sum_{i=1}^N \sum_{j=1}^N a_{ij}^{-\frac{\eta_r}{2}-1} (e_{f,j} - e_{f,i})^2 \right)^{1+\frac{\eta_r}{2}} + 2\sqrt{\alpha_r \beta_r} \left( \sum_{i=1}^N \sum_{j=1}^N a_{ij} (t_{e,j} - t_{e,i})^2 \right) \right]\end{aligned}\quad (29)$$

Define  $W_B = \sum_{i=1}^N b_{ij}(e_{f,j} - e_{f,i})^2$ ,  $W_C = \sum_{i=1}^N c_{ij}(e_{f,j} - e_{f,i})^2$ , and  $W_D = \sum_{i=1}^N d_{ij}(e_{f,j} - e_{f,i})^2$ , where  $b_{ij} = a_{ij}^{\frac{\eta_r}{2}-1}$ ,  $b_{ij} = a_{ij}^{-\frac{\eta_r}{2}-1}$  and  $c_{ij} = a_{ij}$ . According to Lemma 1, then we can obtain  $W_B = 2e^T L_B e$ ,  $W_C = 2e^T L_C e$ , and  $W_D = 2e^T L_D e$ , where  $L_B, L_C$ , and  $L_D$  are the graph Laplacians of  $G(B), G(C)$ , and  $G(D)$ . Because the graph  $G(B), G(C)$ , and  $G(D)$  are connected, then

$$W_B \geq 2\lambda_2(L_B)e^T e, \quad W_C \geq 2\lambda_2(L_C)e^T e, \quad W_D \geq 2\lambda_2(L_D)e^T e \quad (30)$$

Define  $\Omega(e_i) = \frac{1}{2} \sum_{i=1}^N e_i^2$ , then (29) can be reduced to

$$\begin{aligned} \dot{V}_{r,2} \leq & -\frac{1}{2} \frac{\pi}{2\eta_r T_r \tilde{\lambda}_2 \sqrt{\alpha_r \beta_r}} \left[ \alpha_r N^{\frac{\eta_r}{2}} (4\lambda_2 \Omega(e_i))^{1-\frac{\eta_r}{2}} \right. \\ & \left. + \beta_r N^{-\frac{\eta_r}{2}} (4\lambda_2 \Omega(e_i))^{1+\frac{\eta_r}{2}} + 2\sqrt{\alpha_r \beta_r} (4\lambda_2 \Omega(e_i)) \right] \end{aligned} \quad (31)$$

where  $\lambda_2 = \min[\lambda_2(L_B), \lambda_2(L_C), \dots, \lambda_2(L_D)]$ . Based on (28) and  $\Omega(e_i)$ , then

$$\begin{cases} V_{r,2}^{1-\frac{\eta_r}{2}} = \left[ \Omega(e_i) + \frac{1}{2\zeta} \sum_i^N \omega_{r,i}^2 \right]^{1-\frac{\eta_r}{2}} \\ V_{r,2}^{1+\frac{\eta_r}{2}} = \left[ \Omega(e_i) + \frac{1}{2\zeta} \sum_i^N \omega_{r,i}^2 \right]^{1+\frac{\eta_r}{2}} \end{cases} \quad (32)$$

Then, based on Lemma 2, we can obtain

$$[\Omega(e_i)]^{1-\frac{\eta_r}{2}} \geq V_{r,2}^{1-\frac{\eta_r}{2}} - \left[ \frac{1}{2\zeta} \sum_i^N \omega_{r,i}^2 \right]^{1-\frac{\eta_r}{2}} \quad (33)$$

$$[\Omega(e_i)]^{1+\frac{\eta_r}{2}} \geq 2^{\frac{\eta_r}{2}} V_{r,2}^{1+\frac{\eta_r}{2}} - \left[ \frac{1}{2\zeta} \sum_i^N \omega_{r,i}^2 \right]^{1+\frac{\eta_r}{2}} \quad (34)$$

Combining Equations (31), (33) and (34), then

$$\begin{aligned} \dot{V}_{r,2} \leq & -\frac{1}{2} \frac{\pi}{2\eta_r T_r \tilde{\lambda}_2 \sqrt{\alpha_r \beta_r}} \left[ \alpha_r N^{\frac{\eta_r}{2}} (4\lambda_2 V_{r,2})^{1-\frac{\eta_r}{2}} \right. \\ & \left. + \beta_r N^{-\frac{\eta_r}{2}} (4\lambda_2 V_{r,2})^{1+\frac{\eta_r}{2}} + 2\sqrt{\alpha_r \beta_r} (4\lambda_2 V_{r,2}) \right] + \Phi \end{aligned} \quad (35)$$

where  $\Phi = \frac{1}{2} \frac{\pi}{2\eta_r T_r \tilde{\lambda}_2 \sqrt{\alpha_r \beta_r}} \left[ \alpha_r N^{\frac{\eta_r}{2}} (4\lambda_2 \nu)^{1-\frac{\eta_r}{2}} + \beta_r N^{-\frac{\eta_r}{2}} (4\lambda_2 \nu)^{1+\frac{\eta_r}{2}} + 2\sqrt{\alpha_r \beta_r} (4\lambda_2 \nu) \right]$ , and  $\nu = \frac{1}{2\zeta} \sum_i^N \omega_{r,i}^2$ , thus  $\Phi > 0$ .

Define  $W_2 = 4\lambda_2(L) V_{r,2}$ , then take the derivative of  $W_2$ ,

$$\begin{aligned} \dot{W}_2 \leq & -\frac{\pi}{\eta_r T_r \sqrt{\alpha_r \beta_r}} \left[ \alpha_r N^{\frac{\eta_r}{2}} W_2^{1-\frac{\eta_r}{2}} + \beta_r N^{-\frac{\eta_r}{2}} W_2^{1+\frac{\eta_r}{2}} + 2\sqrt{\alpha_r \beta_r} W_2 \right] + \hat{\Phi} \\ = & -\frac{\pi}{\eta_r T_r \sqrt{\tilde{\alpha}_r \tilde{\beta}_r}} \left[ \tilde{\alpha}_r W_2^{1-\frac{\eta_r}{2}} + \tilde{\beta}_r W_2^{1+\frac{\eta_r}{2}} + 2\sqrt{\tilde{\alpha}_r \tilde{\beta}_r} W_2 \right] + \hat{\Phi} \end{aligned} \quad (36)$$

where  $\tilde{\alpha}_r = \alpha_r N^{\frac{\eta_r}{2}}$ ,  $\tilde{\beta}_r = \beta_r N^{-\frac{\eta_r}{2}}$ ,  $\hat{\Phi} = 4\lambda_2 \Phi$ , and  $\hat{\Phi} > 0$ . Based on Lemma 4,  $V_{r,2}$  is predefined-time stable, and the residual set of the solution is  $e_i \in \left\{ V(x) \leq \min \left\{ \left( \frac{2\hat{\Phi}\eta_r \sqrt{\tilde{\alpha}_r \tilde{\beta}_r}}{\tilde{\alpha}_r \pi} \right)^{\frac{2}{2-\eta_r}}, \left( \frac{2\hat{\Phi}\eta_r \sqrt{\tilde{\alpha}_r \tilde{\beta}_r}}{\tilde{\beta}_r \pi} \right)^{\frac{2}{2+\eta_r}} \right\} \right\}$

As a result, combining (35) and (36), the error  $e_i$  between the desired impact time and the actual time of the vehicles converges to a small residual set within the predefined time  $T_{r,1} = \sqrt{2}T_r$ , and the  $t_{f,i}$  of the multi-vehicles converges to unity within the predefined

time  $\tilde{T}_r = T_{r,1} + T_{fr,ESO}$ . By appropriately selecting the design parameters, the size of the residual set can be made arbitrarily close to zero. Theorem 1 is proven.  $\square$

### 3.2. Guidance Law in the Normal Direction of LOS with PPF

In this section, a guidance law  $\mathbf{a}_{M\lambda,i}$  is designed, which enables vehicles to intercept the target at desired LOS angles to achieve the maximum destruction effect on the target.

From Equation (16), the guidance model for the LOS angle constraint is

$$\begin{cases} \dot{\mathbf{x}}_{3,i} = \mathbf{x}_{4,i} \\ \dot{\mathbf{x}}_{4,i} = \mathbf{A}_i + \mathbf{B}_i \mathbf{a}_{M\lambda,i} + \mathbf{D}_i \end{cases} \quad (37)$$

To guarantee the convergence of the LOS angle error within the preset steady-state error range, the prescribed performance function (PPF) with predefined time convergence is designed in the following form

$$\rho_j(t) = \begin{cases} \left| \frac{T_c - t}{T_c} \right|^\kappa \text{sign}\left(\frac{T_c - t}{T_c}\right) (\rho_{0,j} - \rho_{\infty,j}) + \rho_{\infty,j}, & \text{if } t \leq T_c, \\ \rho_{\infty,j} & \text{if } t > T_c \end{cases} \quad (38)$$

where  $j = (\theta_L, \phi_L)$ ,  $\kappa > 2$ ,  $\rho_{0,j}$  is the initial value of the PPF satisfying  $\rho_{0,j} \geq |x_{3,j}(0)|$ ,  $x_{3,j}(0)$  denotes the initial value of the tracking error,  $\rho_{\infty,j}$  denotes the steady state error.  $T_c$  is the predefined convergence time;  $\kappa$  denotes the speed of convergence.

The system error is guaranteed by the prescribed performance controller (PPC) to satisfy the following equation:

$$\begin{cases} -\iota_j \rho_j(t) < x_{3,j}(t) < \rho_j(t), & \text{if } x_{3,j}(0) \geq 0, \\ -\rho_j(t) < x_{3,j}(t) < \iota_j \rho_j(t), & \text{if } x_{3,j}(0) < 0 \end{cases} \quad (39)$$

where  $0 < \iota_j < 1$ , and  $\iota_j \rho_j(t)$  represents the maximum amount of overshoot.

Inspired by [23], the error transformation function is

$$h_{1,j}(t) = \begin{cases} \ln\left(\frac{\iota_j + z_j(t)}{\iota_j(1 - z_j(t))}\right), & \text{if } x_{3,j}(0) \geq 0, \\ \ln\left(\frac{\iota_j(1 + z_j(t))}{\iota_j - z_j(t)}\right), & \text{if } x_{3,j}(0) < 0 \end{cases} \quad (40)$$

where  $z_j(t) = \frac{x_{3,j}(t)}{\rho_j(t)}$ .

Taking the time derivative of  $h_{1,j}(t)$  in Equation (40),

$$\dot{h}_{1,j}(t) = \begin{cases} \frac{1 + \iota_j}{(\iota_j + z_j(t))(1 - z_j(t))\rho_j(t)} (\dot{x}_{3,j}(t) + \psi_j(t)x_{3,j}(t)), & \text{if } x_{3,j}(0) \geq 0, \\ \frac{1 + \iota_j}{(\iota_j - z_j(t))(1 + z_j(t))\rho_j(t)} (\dot{x}_{3,j}(t) + \psi_j(t)x_{3,j}(t)), & \text{if } x_{3,j}(0) < 0 \end{cases} \quad (41)$$

where  $\psi_j(t) = -\frac{\dot{\rho}_j(t)}{\rho_j(t)}$ , and  $\dot{h}_{1,j}(t)$  can be rewritten:

$$\begin{aligned} \dot{h}_{1,j}(t) &= \omega_j(t) h_{2,j}(t) \\ \omega_j(t) &= \begin{cases} \frac{1 + \iota_j}{(\iota_j + z_j(t))(1 - z_j(t))\rho_j(t)}, & \text{if } x_{3,j}(0) \geq 0, \\ \frac{1 + \iota_j}{(\iota_j - z_j(t))(1 + z_j(t))\rho_j(t)}, & \text{if } x_{3,j}(0) < 0 \end{cases} \end{aligned} \quad (42)$$

where  $h_{2,j}(t) = \dot{x}_{3,j}(t) + \psi_j(t)x_{3,j}(t)$ . Take the time derivative of  $h_{2,j}(t)$ .

$$\dot{h}_{2,j}(t) = \ddot{x}_{3,j}(t) + \dot{\psi}_j(t)x_{3,j}(t) + \psi_j(t)\dot{x}_{3,j}(t) \quad (43)$$

Based on the error transformation function Equations (40)–(43), the guidance model (37) can be rewritten:

$$\begin{cases} \dot{h}_{1,i} = \omega_i h_{2,i} \\ \dot{h}_{2,i} = A_i + B_i a_{M\lambda,i} + D_i + \dot{\psi}_i(t)x_{3,i} + \psi_i(t)\dot{x}_{3,i} \end{cases} \quad (44)$$

where  $\omega_i = [\omega_{\theta_{L,i}}, \omega_{\phi_{L,i}}]^T$ ;  $\psi_i(t) = [\psi_{\theta_{L,i}}, \psi_{\phi_{L,i}}]^T$ .

According to Assumption 1, there is a positive constant  $L_{\lambda,i}$  for which external disturbance  $D_i$  satisfies

$$|\dot{D}_i| \leq L_{\lambda,i} \quad (45)$$

Then, based on Lemma 6 and Equation (37), a PTESO is designed to estimate  $D_i$

$$\begin{cases} e_{1\lambda,i} = x_{3,i} - z_{1\lambda,i} \\ \dot{z}_{1\lambda,i} = z_{2\lambda,i} + \Phi_\lambda l_{1,\lambda} \zeta_\lambda h_{ESO,\lambda} e_{1\lambda,i} + (1 - \Phi_\lambda) k_{1,\lambda} \mu_\lambda^{\frac{1}{3}} \text{sig}^{\frac{2}{3}}(e_{1\lambda,i}) \\ \dot{z}_{2\lambda,i} = \Phi_\lambda l_{2,\lambda} \zeta_\lambda^2 h_{ESO,\lambda}^2 e_{1\lambda,i} + (1 - \Phi_\lambda) k_{2,\lambda} \mu_\lambda^{\frac{2}{3}} \text{sig}^{\frac{1}{3}}(e_{1\lambda,i}) \\ \quad + A_i + B_i a_{M,i} + z_{3\lambda,i} \\ \dot{z}_{3\lambda,i} = \Phi_\lambda l_{3,\lambda} \zeta_\lambda^3 h_{ESO,\lambda}^3 e_{1\lambda,i} + (1 - \Phi_\lambda) k_{3,\lambda} \mu_\lambda \text{sig}^0(e_{1\lambda,i}) \end{cases} \quad (46)$$

where  $z_{1\lambda,i}, z_{2\lambda,i}, z_{3\lambda,i}$  are the estimations for  $x_{3,i}, x_{4,i}$  and  $D_i$ . The parameters  $h_{ESO,\lambda}, l_{1,2,3,\lambda}, \eta_\lambda, k_{p,\lambda}, p = 1, 2, 3, \Phi_\lambda$  and  $\zeta_\lambda$  are the parameters to be designed according to Lemma 6,  $\Phi_\lambda$  is the swithcing function of PTESO with  $\Phi_\lambda = 1$ , when  $t \in [0, T_{f\lambda,ESO})$ , otherwise  $\Phi_\lambda = 0$ .

To make the system Equation (44) coverage to zero within a predefined time, a predefined-time non-singular terminal sliding mode (PTNTSM) is designed as

$$S_i = h_{2,i} + \frac{\pi}{2\lambda(\omega_i)\eta_\lambda T_\lambda \sqrt{\alpha_\lambda \beta_\lambda}} \left( \alpha_\lambda F(h_{1,i}) + 2^{\frac{\eta_\lambda}{2}} \beta_\lambda \text{sig}^{1+\eta_\lambda}(h_{1,i}) + 2\sqrt{\alpha_\lambda \beta_\lambda} h_{1,i} \right) \quad (47)$$

where  $S_i = [s_{1,i}, s_{2,i}]^T$ ,  $\lambda(\omega_i)$  is the minimum eigenvalue of  $\omega_i$ , and  $\alpha_\lambda > 0, \beta_\lambda > 0, 0 < \eta_\lambda < 1, T_\lambda > 0, F(h_{1,i}) = [f(h_{1\theta_{L,i}}), f(h_{1\phi_{L,i}})]^T$

$$f(x_i) = \begin{cases} |x_i|^{1-\eta_\lambda} \text{sign}(x_i), & |x_i| > \delta \\ r_1 x_i + r_2 x_i^2 \text{sign}(x_i), & |x_i| \leq \delta \end{cases}$$

where  $r_1 = (1 + \eta_\lambda)\delta^{-\eta_\lambda}$ , and  $r_2 = -\eta_\lambda \delta^{-(1+\eta_\lambda)}$ ,  $\delta$  is a small normal number.

Taking the time derivative of the PTNTSM,

$$\dot{S}_i = \dot{h}_{2,i} + \frac{\pi}{2\lambda(\omega_i)\eta_\lambda T_\lambda \sqrt{\alpha_\lambda \beta_\lambda}} \left( \alpha_\lambda \dot{F}(h_{1,i}) + 2^{\frac{\eta_\lambda}{2}} \beta_\lambda (1 + \eta_\lambda) h_{1,i}^{\eta_\lambda} \dot{h}_{1,i} + 2\sqrt{\alpha_\lambda \beta_\lambda} \dot{h}_{1,i} \right) \quad (48)$$

where  $\dot{F}(h_{1,i}) = [\dot{f}(h_{1\theta_{L,i}}), \dot{f}(h_{1\phi_{L,i}})]^T$ .

$$\dot{f}(x_i) = \begin{cases} (1 - \eta_\lambda)|x_i|^{-\eta_\lambda} \dot{x}_i, & |x_i| > \delta \\ r_1 \dot{x}_i + 2r_2 |x_i| \dot{x}_i, & |x_i| \leq \delta \end{cases}$$

A predefined-time sliding mode reaching law is designed to drive the system onto the PTNTSM within a predefined time:

$$\dot{S}_i = -\frac{\gamma_s}{T_s} (\alpha_s |S_i|^{p_s} + \beta_s |S_i|^{q_s})^{l_s} \text{sign}(S_i) \quad (49)$$

where  $\alpha_s > 0$ ,  $\beta_s, p_s > 0$ ,  $q_s > 0$ ,  $l_s > 0$  also need to satisfy the conditions  $l_s p_s < 1$ ,  $l_s q_s > 1$ ;  $\gamma_s$  is defined the same as in Lemma 5.

$$\begin{aligned} a_{M\lambda,i} = & -B_i^{-1} \left[ A_i + \frac{\gamma_s}{T_s} (\alpha_s |S_i|^{p_s} + \beta_s |S_i|^{q_s})^{l_s} \text{sign}(S_i) + z_{3\lambda,i} \right. \\ & + \frac{\pi}{2\lambda(\omega_i)\eta_\lambda T_\lambda \sqrt{\alpha_\lambda \beta_\lambda}} \left( \alpha_\lambda \dot{F}(h_{1,i}) + 2^{\frac{\eta_\lambda}{2}} \beta_\lambda (1 + \eta_\lambda) h_{1,i}^{\eta_\lambda} \dot{h}_{1,i} + 2\sqrt{\alpha_\lambda \beta_\lambda} \dot{h}_{1,i} \right) \\ & \left. + \dot{\psi}_i(t)x_{3,i} + \psi_i(t)\dot{x}_{3,i} \right] \end{aligned} \quad (50)$$

**Theorem 2.** Under the guidance law (50)  $a_{M\lambda,i}$  and PTESO (46), the LOS angle error and LOS angular rate can achieve convergence to zero within predefined time, i.e., multiple vehicles are capable of intercepting the target at the desired LOS angles. Meanwhile, the error  $x_{3,i}$  conforms to the prescribed performance constraint that is presented in (39).

**Proof.** Owing to the structural similarity between PTESO (46) and (26), the detailed proof of the former is omitted for brevity. Based on Lemma 6, the error of estimation converges within a predefined time  $T_{f\lambda,ESO}$ .

The Lyapunov function is defined as follows:

$$V_{s,i} = S_i \text{sign}(S_i) \quad (51)$$

Substituting the guidance law  $a_{M\lambda,i}$  (50) to (48). Take the derivative of  $V_{s,i}$ :

$$\begin{aligned} \dot{V}_{s,i} &= \dot{S}_i \text{sign}(S_i) \\ &= \left[ A_i + B_i a_{M\lambda,i} + D_i + \dot{\psi}_i(t)x_{3,i} + \psi_i(t)\dot{x}_{3,i} + \frac{\pi}{2\lambda(\omega_i)\eta_\lambda T_\lambda \sqrt{\alpha_\lambda \beta_\lambda}} \right. \\ &\quad \cdot \left( \alpha_\lambda \dot{F}(h_{1,i}) + 2^{\frac{\eta_\lambda}{2}} \beta_\lambda (1 + \eta_\lambda) h_{1,i}^{\eta_\lambda} \dot{h}_{1,i} + 2\sqrt{\alpha_\lambda \beta_\lambda} \dot{h}_{1,i} \right) \Big] \text{sign}(S_i) \\ &= \left[ -\frac{\gamma_s}{T_s} (\alpha_s |S_i|^{p_s} + \beta_s |S_i|^{q_s})^{l_s} \text{sign}(S_i) - (D_i - z_{3\lambda,i}) \right] \text{sign}(S_i) \end{aligned} \quad (52)$$

When  $t > T_{f\lambda,ESO}$ , the error  $D_i - z_{3\lambda,i} = 0$ . Equation (52) can be rewritten

$$\begin{aligned} \dot{V}_{s,i} &\leq -\frac{\gamma_s}{T_s} (\alpha_s |S_i|^{p_s} + \beta_s |S_i|^{q_s})^{l_s} \text{sign}(S_i) \\ &\leq -\frac{\gamma_s}{T_s} (\alpha_s V_{s,i}^{p_s} + \beta_s V_{s,i}^{q_s})^{l_s} \end{aligned} \quad (53)$$

Therefore, based on Lemma 5, Equation (44) can achieve convergence to the sliding-mode manifold  $S_i = 0$  within the predefined time  $T_s$ .

Once the system reaches the sliding mode manifold (47), i.e.,  $S_i = 0$ , then in the sliding stage, we get

$$h_{2,i} = -\frac{\pi}{2\lambda(\omega_i)\eta_\lambda T_\lambda \sqrt{\alpha_\lambda \beta_\lambda}} \left( \alpha_\lambda F(h_{1,i}) + 2^{\frac{\eta_\lambda}{2}} \beta_\lambda \text{sig}^{1+\eta_\lambda}(h_{1,i}) + 2\sqrt{\alpha_\lambda \beta_\lambda} h_{1,i} \right) \quad (54)$$

Define the Lyapunov function as

$$V_{\lambda,i} = h_{1,i}^T h_{1,i} \quad (55)$$

(1) Case 1: If  $|h_{1,i}| > \delta_i$ , taking the derivative of  $V_{\lambda,i}$ , and based on Lemma 2,

$$\begin{aligned}
\dot{V}_{\lambda,i} &= 2\mathbf{h}_{1,i}^T \dot{\mathbf{h}}_{1,i} \\
&= -\mathbf{h}_{1,i}^T \frac{\pi \boldsymbol{\omega}_i}{\lambda(\boldsymbol{\omega}_i) \eta_\lambda T_\lambda \sqrt{\alpha_\lambda \beta_\lambda}} \left( \alpha_\lambda \operatorname{sig}^{1-\eta_\lambda}(\mathbf{h}_{1,i}) + 2^{\frac{\eta_\lambda}{2}} \beta_\lambda \operatorname{sig}^{1+\eta_\lambda}(\mathbf{h}_{1,i}) + 2\sqrt{\alpha_\lambda \beta_\lambda} \mathbf{h}_{1,i} \right) \\
&\leq -\frac{\pi}{\eta_\lambda T_\lambda \sqrt{\alpha_\lambda \beta_\lambda}} \left( \alpha_\lambda \left( \sum_{j=1}^2 h_{1,i}^2 \right)^{1-\frac{\eta_\lambda}{2}} + \beta_\lambda \left( \sum_{j=1}^2 h_{1,i}^2 \right)^{1+\frac{\eta_\lambda}{2}} + 2\sqrt{\alpha_\lambda \beta_\lambda} \left( \sum_{j=1}^2 h_{1,i}^2 \right) \right) \\
&= -\frac{\pi}{\eta_\lambda T_\lambda \sqrt{\alpha_\lambda \beta_\lambda}} \left( \alpha_\lambda V_{\lambda,i}^{1-\frac{\eta_\lambda}{2}} + \beta_\lambda V_{\lambda,i}^{1+\frac{\eta_\lambda}{2}} + 2\sqrt{\alpha_\lambda \beta_\lambda} V_{\lambda,i} \right)
\end{aligned} \tag{56}$$

According to Lemma 3, the system state can achieve convergence within the predefined time  $T_\lambda$ .

- (2) Case 2: If  $|\mathbf{h}_{1,i}| \leq \delta_i$ , where the state entered in  $|\mathbf{h}_{1,i}| \leq \delta_i$  within the predefined time  $T_\lambda$ , take the derivative of  $V_{\lambda,i}$ , and based on Lemma 2

$$\begin{aligned}
\dot{V}_{\lambda,i} &\leq -\frac{\delta_i^{-\eta_\lambda} \pi \alpha_\lambda}{\eta_\lambda T_\lambda \sqrt{\alpha_\lambda \beta_\lambda}} \left( \sum_{j=1}^2 h_{1,i}^2 + \eta_\lambda \left( \sum_{j=1}^2 h_{1,i}^2 - \delta_i^{-1} \sum_{j=1}^2 |\mathbf{h}_{1,i}|^3 \right) \right) \\
&\leq -\frac{\delta_i^{-\eta_\lambda} \pi \alpha_\lambda}{\eta_\lambda T_\lambda \sqrt{\alpha_\lambda \beta_\lambda}} \left( \sum_{j=1}^2 h_{1,i}^2 \right)
\end{aligned} \tag{57}$$

Hence, the error  $|\mathbf{h}_{2,i}|$  satisfies

$$|\mathbf{h}_{2,i}| \leq \frac{\pi}{2\lambda(\boldsymbol{\omega}_i) \eta_\lambda T_\lambda \sqrt{\alpha_\lambda \beta_\lambda}} \left( \alpha_\lambda \delta_i^{1-\eta_\lambda} + 2^{\frac{\eta_\lambda}{2}} \beta_\lambda \delta_i^{1+\eta_\lambda} + 2\sqrt{\alpha_\lambda \beta_\lambda} \delta_i \right) \tag{58}$$

Therefore, the transformed error  $|\mathbf{h}_{2,i}|$  achieves convergence to 0 within the predefined time  $\tilde{T}_\lambda = T_\lambda + T_s + T_{f\lambda,ESO}$ ; the system state of LOS angle error and LOS angular rate can converge within  $\tilde{T}_\lambda$ . This completes the proof of Theorem 2.  $\square$

In summary, based on guidance system (16), multiple vehicles are able to intercept the maneuvering target at desired LOS angles based on the designed non-singular predefined-time guidance law with prescribed performance constraint (50) in the normal direction of LOS; the designed guidance law (23) in the LOS direction allows vehicles to intercept the target with the desired impact time. Thus, the designed guidance law can achieve simultaneous interception of a maneuvering target by vehicles with multiple constraints.

**Remark 3.** Different from conventional terminal sliding-mode surfaces, the proposed predefined-time non-singular terminal sliding-mode surface does not contain negative fractional power terms of the error variables, thus avoiding singularities when the LOS angle error approaches zero. Moreover, continuous nonlinear power terms are adopted instead of the discontinuous sign function, thereby reducing the switching amplitude and alleviating chattering.

#### 4. Simulation

To validate the proposed 3D cooperative guidance law with predefined-time multiple constraints (3D-PfTCG), simulations were conducted under a switching topology. First, the efficacy of the proposed algorithm was substantiated with respect to different adjusted impact times. Then, its superiority was demonstrated by comparing it with the fixed-time cooperative guidance (FxTCG) method proposed in Ref [31].

The guidance laws designed in Equations (23) and (50) were used to intercept maneuvering targets. The key guidance parameters are presented below. The parameters in Equation (23) were selected as:

$$\alpha_r = 5, \quad \beta_r = 4, \quad \eta_r = 0.68, \quad T_r = 5 \quad (59)$$

In guidance law Equation (50), the parameters are chosen as:

$$\begin{aligned} \alpha_\lambda = 5, \quad \beta_\lambda = 4, \quad \eta_\lambda = 0.68, \quad T_\lambda = 20, \quad p_s = 0.5, \\ q_s = 3, \quad l_s = 1.5, \quad \alpha_s = 1, \quad \beta = \frac{1}{\alpha_s}, \quad T_s = 20, \quad \delta = 0.01 \end{aligned} \quad (60)$$

The parameters of the PTESO (21) and (46) are selected as follows:

$$\begin{aligned} l_{1,\lambda} = l_{1,r} = 6, \quad l_{2,\lambda} = l_{2,r} = 8, \quad l_{3,\lambda} = l_{3,r} = 0.2 \\ \zeta_\lambda = \zeta_r = 4, \quad \mu_r = 10, \quad \mu_\lambda = 0.5 \\ k_{3,r} = 11, \quad k_{3,\lambda} = 1.5, \quad k_{1,\lambda} = 3.34k_{3,\lambda}^{1/3} \\ k_{2,\lambda} = 5.3k_{3,\lambda}^{2/3}, \quad k_{1,r} = 3.34k_{3,r}^{1/3}, \quad k_{2,\lambda} = 5.3k_{3,r}^{2/3} \\ h_{ESO} = 0.1 \sec\left(\frac{\pi}{2} \frac{t}{t_{f,ESO}}\right), \quad T_{fr,ESO} = T_{f\lambda,ESO} = 5 \end{aligned} \quad (61)$$

The parameters of PPF (38) and (39) are selected as:

$$\kappa = 3, \quad T_c = 10, \quad \rho_0 = 20, \quad \rho_\infty = 0.1, \quad \iota = 0.9 \quad (62)$$

The acceleration of the vehicle is limited to  $20g$ , where  $g = 9.81 \text{ m/s}^2$ .

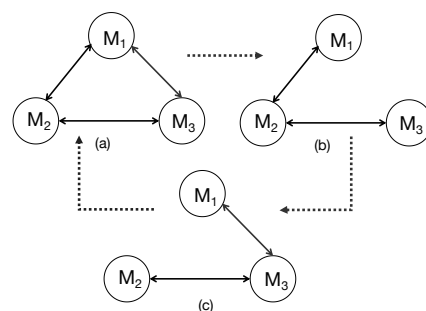
#### 4.1. Effectiveness and Robustness Verification of 3D-PfTCG

In the simulation scenario, a switching topology that better reflects practical scenarios was employed to validate the effectiveness and robustness of the proposed algorithm. The initial states of the vehicles and the target are detailed in Table 1. The target accelerations is given as:

$$a_{T_r} = 10 + 10 \sin(t) \text{ m/s}^2, \quad a_{T_\theta} = 10 \sin(0.7t) \text{ m/s}^2, \quad a_{T_\phi} = -30 \text{ m/s}^2$$

Figure 3 shows the communication topology among the vehicles. During the simulation, the topology changes every 1s, starting from the topology depicted in Figure 3a. The corresponding weighted coefficient matrix are as follows:

$$\mathbf{A}_a = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} \text{ and } \mathbf{A}_b = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \text{ and } \mathbf{A}_c = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$



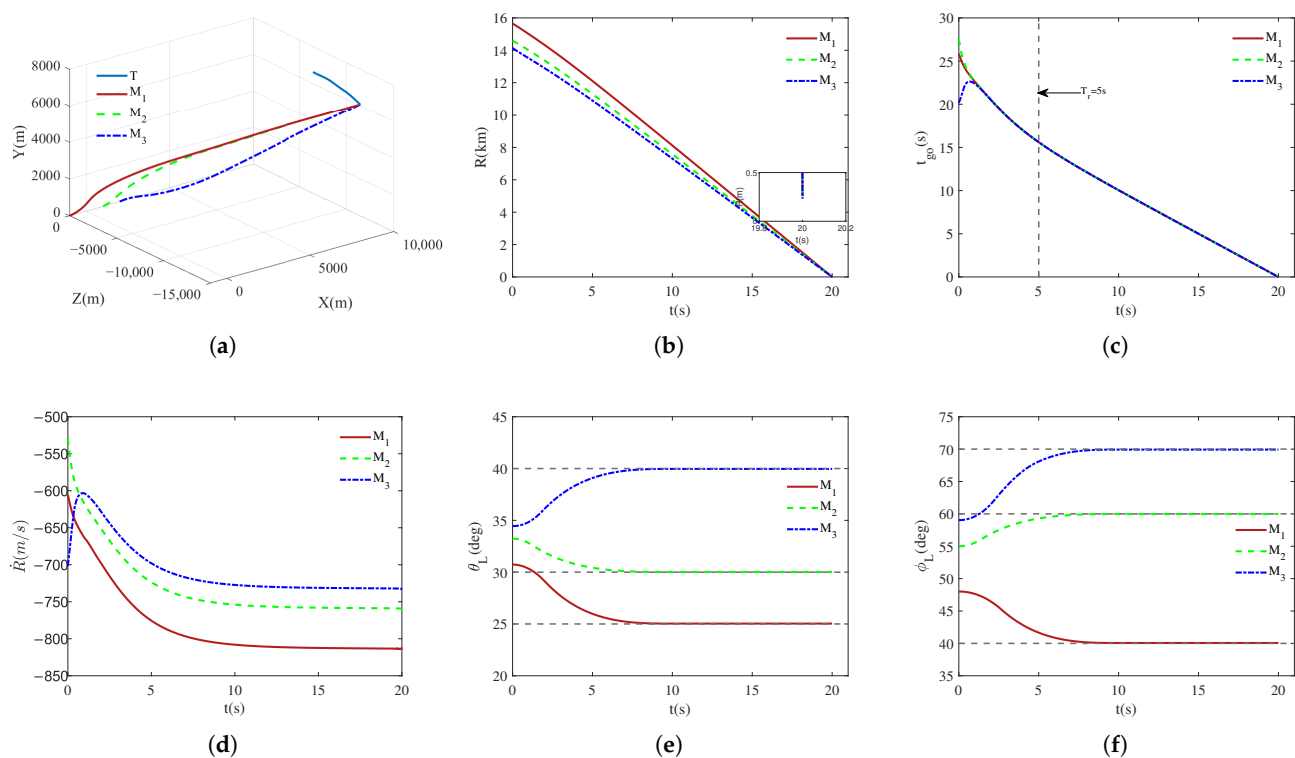
**Figure 3.** Switching communication topology between vehicles.

The desired impact times are  $T_d = 20$ ,  $T_d = 30$  and  $T_d = 35$ . The simulation outcomes are illustrated in Figure 4, Figure 5 and Figure 6, respectively. From Figures 4–6, multiple vehicles successfully intercept the target at the desired impact times; the miss distance

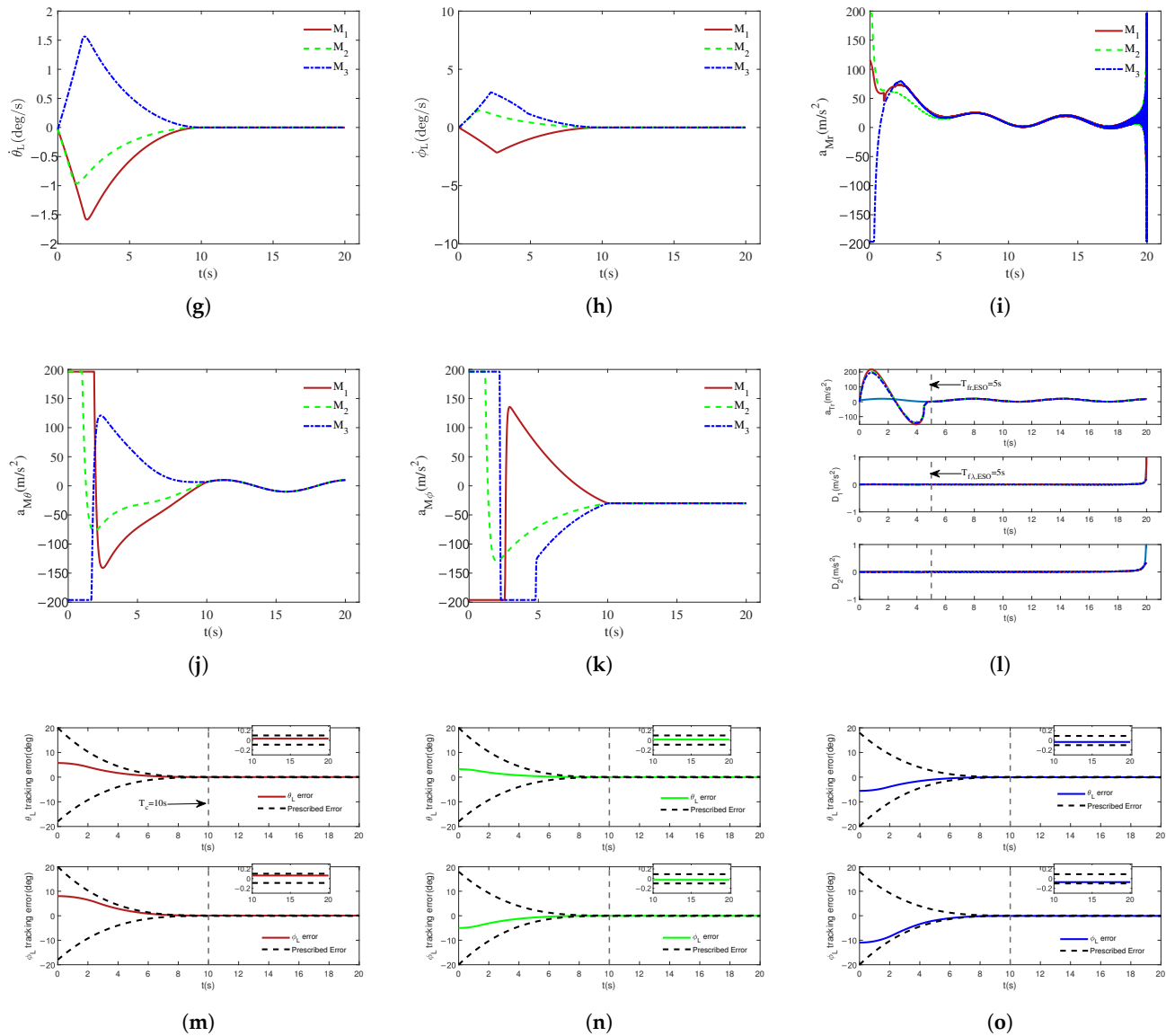
$R$  is 0. Meanwhile, Figures 4c,d, 5c,d and 6c,d illustrate the vehicles' time-to-go and radial rate. The time-to-go achieves consensus, while the radial rate converges within the predefined time. Figures 4e–h, 5e–h and 6e–h illustrate the evolution of the LOS angle and its angular rate. The LOS angles converge to the desired angle, while the LOS angular rate converges within the predefined time. Figures 4i–k, 5i–k and 6i–k show the acceleration of vehicles in three directions. It can be seen that, due to the topology switching, slight oscillations occur in the initial phase. These oscillations are caused by the communication topology switching. In practical scenarios, the switching frequency of the communication topology is not high, and the oscillation frequency of the cooperative guidance law is also reduced. Therefore, these oscillations do not significantly affect the vehicle guidance performance. Figures 4l, 5l and 6l show that the disturbance observer can converge to the actual values within the predefined time  $T_{fr,ESO}$ ,  $T_{f\lambda,ESO}$ , then the guidance law is compensated effectively, verifying the effectiveness of the PTESO. The results indicate that the proposed 3D-PfTCG can achieve the interception of maneuvering targets with high precision under the desired constraints. Figures 4m–o, 5m–o and 6m–o show the tracking errors of  $\theta_L$  and  $\phi_L$  under the performance constraint function, showing that the errors evolve within the desired performance bounds.

**Table 1.** Initial conditions between vehicle  $M_i$  and target  $T$  ( $i = 1, 2, 3, 4$ ).

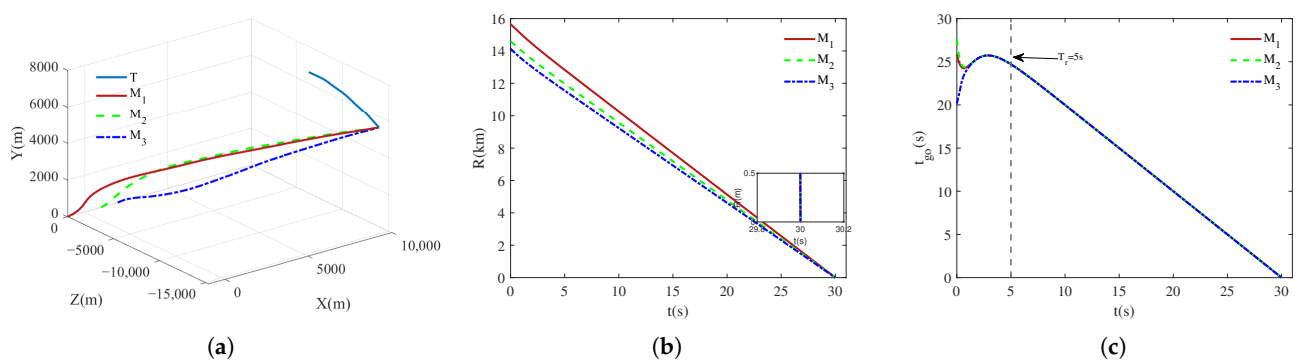
| $M_i/T$ | $(x, y, z)$<br>(km) | $V$<br>(m/s) | $\theta_{m/t}(0)$<br>( $^\circ$ ) | $\phi_{m/t}(0)$<br>( $^\circ$ ) | $\theta_{LD}$<br>( $^\circ$ ) | $\phi_{LD}$<br>( $^\circ$ ) |
|---------|---------------------|--------------|-----------------------------------|---------------------------------|-------------------------------|-----------------------------|
| $M_1$   | (−1,0,0)            | 680          | 60                                | 90                              | 25                            | 40                          |
| $M_2$   | (1,0,0)             | 600          | 60                                | 90                              | 30                            | 60                          |
| $M_3$   | (2,0,0)             | 780          | 60                                | 90                              | 40                            | 70                          |
| $T$     | (8,8,−10)           | 200          | 0                                 | 90                              |                               |                             |



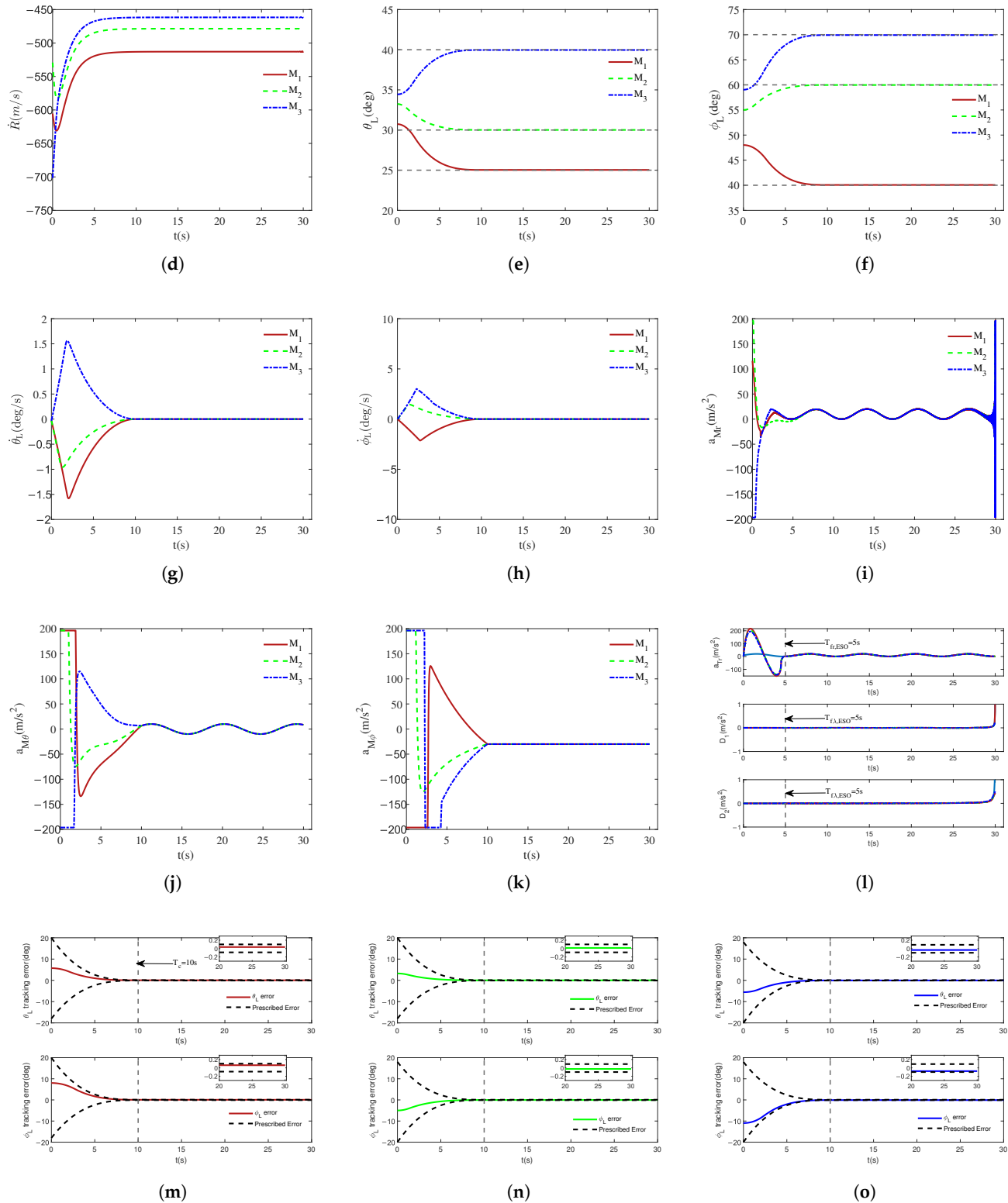
**Figure 4.** Cont.



**Figure 4.** The results for 3D-PfTCG with  $T_d = 20$ . (a) Trajectories of vehicles and target. (b) Relative vehicles–target distance. (c) Vehicles' time-to-go. (d) Radial vehicles–target velocities. (e) Elevation LOS angle. (f) Azimuth LOS angle. (g) Elevation LOS angle rates. (h) Azimuth LOS angle rates. (i) Vehicles' acceleration commands  $a_{Mr}$ . (j) Vehicles acceleration commands  $a_{M\theta}$ . (k) Vehicles' acceleration commands  $a_{M\phi}$ . (l) Actual values and estimations of disturbance. (m) M 1 LOS angle tracking error. (n) M 2 LOS angle tracking error. (o) M 3 LOS angle tracking error.



**Figure 5.** Cont.



**Figure 5.** The results for 3D-PfTCG with  $T_d = 30$ . (a) Trajectories of vehicles and target. (b) Vehicles-target relative distance. (c) Vehicles' time-to-go. (d) Vehicles-target radial velocities. (e) Elevation LOS angle. (f) Azimuth LOS angle. (g) Elevation LOS angle rates. (h) Azimuth LOS angle rates. (i) Vehicles' acceleration commands  $a_{Mr}$ . (j) Vehicles' acceleration commands  $a_{M\theta}$ . (k) Vehicles' acceleration commands  $a_{M\phi}$ . (l) Actual values and estimations of disturbance. (m) M 1 LOS angle tracking error. (n) M 2 LOS angle tracking error. (o) M 3 LOS angle tracking error.

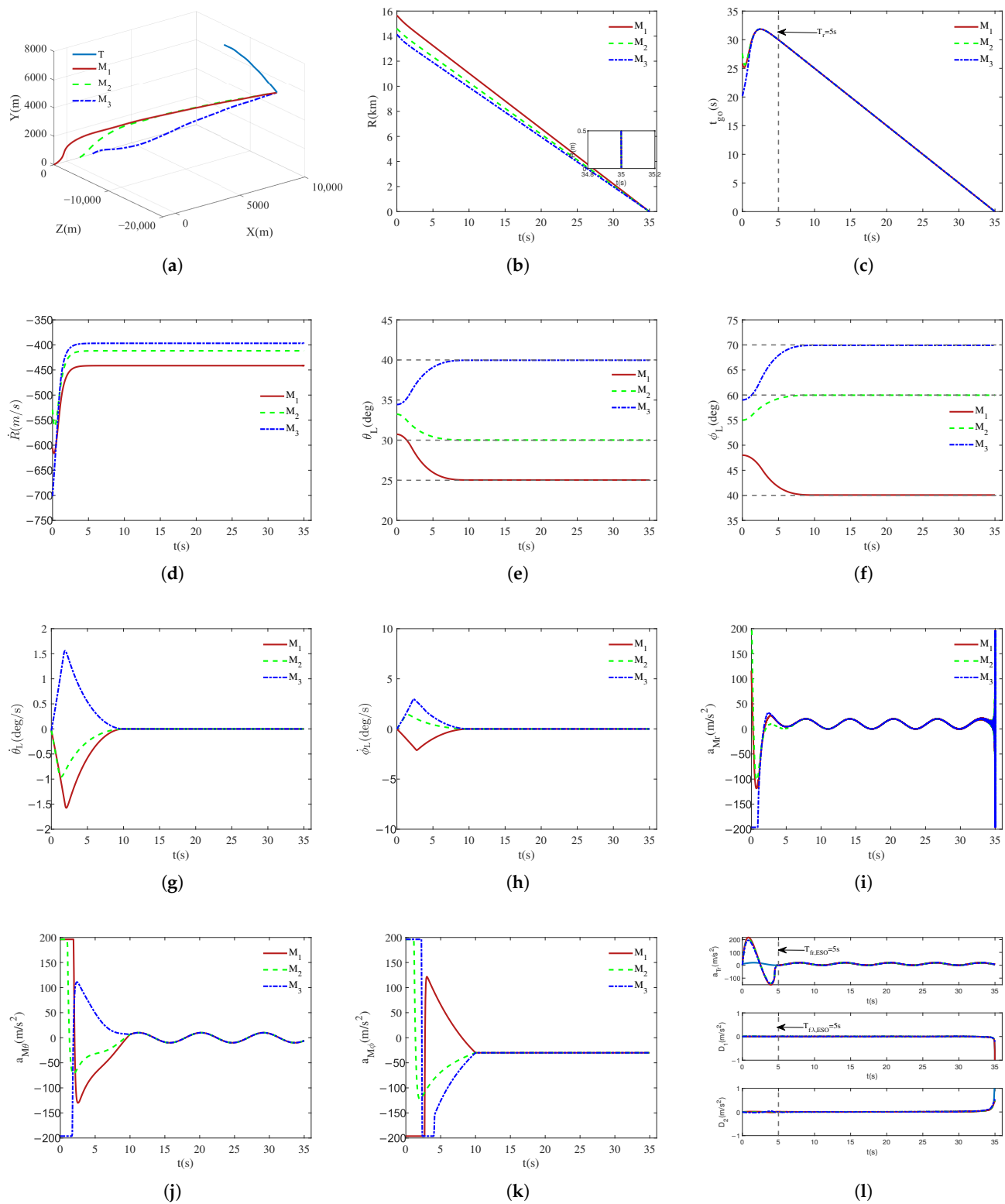
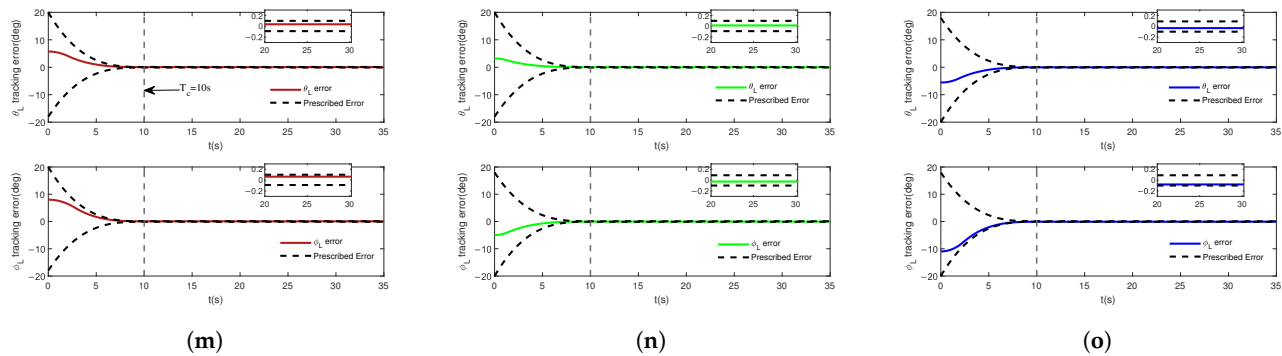


Figure 6. Cont.

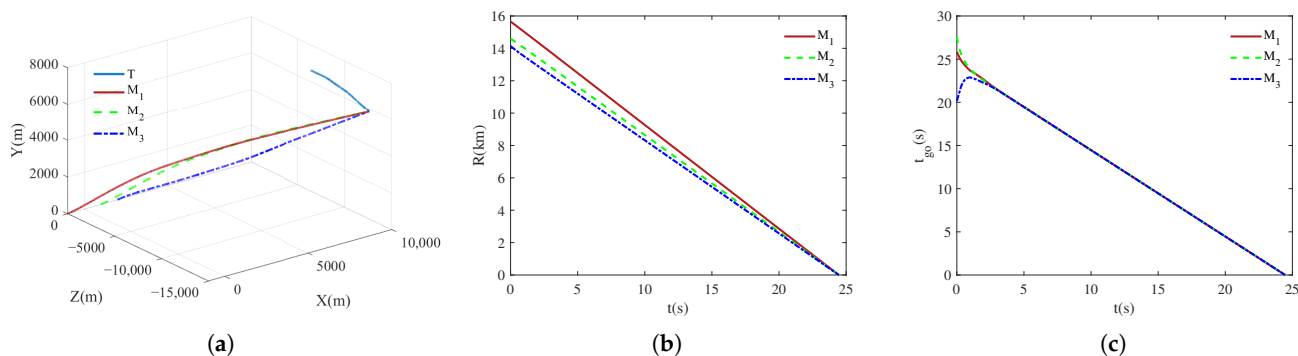


**Figure 6.** The results for 3D-PfTCG with  $T_d = 35$ . (a) Trajectories of vehicles and target. (b) Relative vehicles–target distance. (c) Vehicles’ time-to-go. (d) Radial vehicles–target velocities. (e) Elevation LOS angle. (f) Azimuth LOS angle. (g) Elevation LOS angle rates. (h) Azimuth LOS angle rates. (i) Vehicles’ acceleration commands  $a_{Mr}$ . (j) Vehicles’ acceleration commands  $a_{M\theta}$ . (k) Vehicles’ acceleration commands  $a_{M\phi}$ . (l) Actual values and estimations of disturbance. (m) M 1 LOS angle tracking error. (n) M 2 LOS angle tracking error. (o) M 3 LOS angle tracking error.

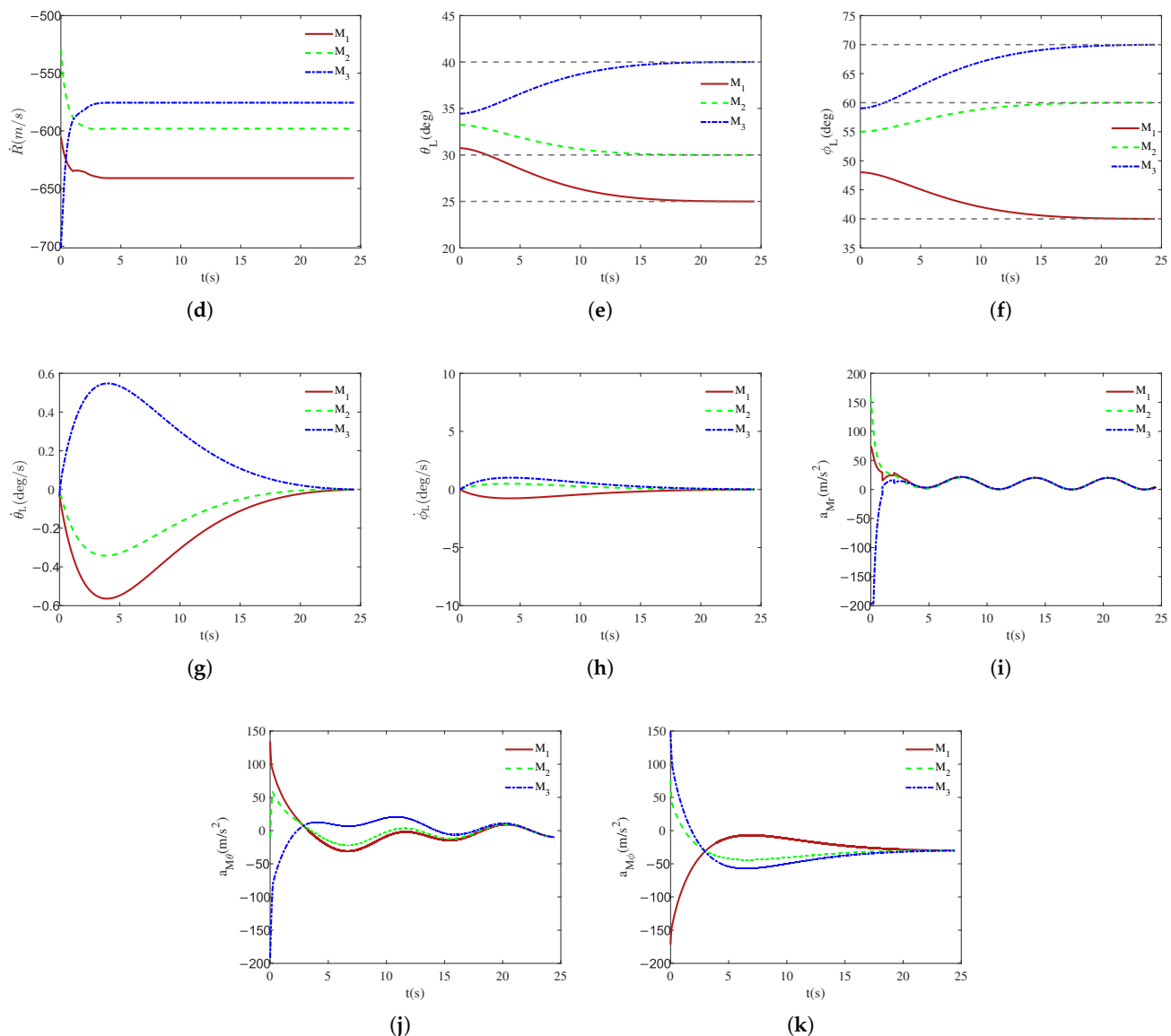
#### 4.2. Comparison with Other Reference

To illustrate the outstanding performance and superiority of the algorithm we presented, a comparative study was conducted with the fixed-time cooperative guidance (FxTCG) presented in Ref. [31]. The parameters of the guidance law were configured according to Ref. [31]. The scenarios involving the vehicles and the target are consistent with Table 1.

In Ref. [31], the convergence of the vehicles’ states depends on the initial conditions and parameter settings. The simulation results for the scenario are shown in Figure 7. The figure shows that the vehicles intercept the desired angles. By comparing Figures 4–7, the performance of the guidance law proposed in Ref. [31] is slightly inferior to the guidance law proposed in this paper. Specifically, compared to 3D-PfTCG, the time-to-go in Figure 7c takes longer to converge, indicating that the guidance system requires more time to adjust the flight trajectory to achieve cooperative target interception. Additionally, from Figure 7e–h, it can be seen that the LOS angle and angular rate converge just before intercepting the target. Compared to Figure 7e–h, the guidance law in Ref. [31] exhibits slower convergence. This suggests that the guidance system’s ability to track and adjust to the target is insufficient during the final approach phase, potentially affecting interception accuracy.



**Figure 7.** Cont.



**Figure 7.** The results for Ref. [31]. (a) Trajectories of vehicles and target. (b) Relative vehicles–target distance. (c) Vehicles’ time-to-go. (d) Radial vehicles–target velocities. (e) Elevation LOS angle. (f) Azimuth LOS angle. (g) Elevation LOS angle rates. (h) Azimuth LOS angle rates. (i) Vehicles’ acceleration commands  $a_{Mr}$ . (j) Vehicles’ acceleration commands  $a_{M\theta}$ . (k) Vehicles’ acceleration commands  $a_{M\phi}$ .

Table 2 presents the interception time, miss distance, and LOS angle errors for 3D-PfTCG and the guidance law from Ref. [31]. The results show that the impact time of Ref. [31] is 24.46 s, the mean vehicle distance is 0.274 m, and the mean errors of  $\theta_L$  and  $\phi_L$  are  $0.0272^\circ$  and  $0.0333^\circ$ , respectively. Therefore, the 3D-PfTCG proposed in this paper demonstrates superior performance compared to the algorithm in Ref. [31].

**Table 2.** Interception times, miss distances, error of angle for the two algorithms.

| Initial Condition | $M_i$ | Interception Time (s) | Miss Distance (m) | Error of $\theta_L$ ( $^\circ$ ) | Error of $\phi_L$ ( $^\circ$ ) |
|-------------------|-------|-----------------------|-------------------|----------------------------------|--------------------------------|
| $T_d = 20$        | $M_1$ | 20                    | 0.256             | 0.0167                           | 0.0330                         |
|                   | $M_2$ | 20                    | 0.239             | 0.0156                           | 0.0264                         |
|                   | $M_3$ | 20                    | 0.231             | 0.0242                           | 0.0217                         |
|                   | Mean  | 20                    | 0.242             | 0.0188                           | 0.0270                         |

Table 2. Cont.

| Initial Condition | $M_i$ | Interception Time (s) | Miss Distance (m) | Error of $\theta_L$ (°) | Error of $\phi_L$ (°) |
|-------------------|-------|-----------------------|-------------------|-------------------------|-----------------------|
| $T_d = 30$        | $M_1$ | 30                    | 0.256             | 0.0364                  | 0.0630                |
|                   | $M_2$ | 30                    | 0.249             | 0.0153                  | 0.0260                |
|                   | $M_3$ | 30                    | 0.231             | 0.0339                  | 0.0780                |
|                   | Mean  | 30                    | 0.245             | 0.0285                  | 0.0557                |
| $T_d = 35$        | $M_1$ | 35                    | 0.221             | 0.0363                  | 0.0134                |
|                   | $M_2$ | 35                    | 0.206             | 0.0152                  | 0.0156                |
|                   | $M_3$ | 35                    | 0.199             | 0.0312                  | 0.0317                |
|                   | Mean  | 35                    | 0.208             | 0.0276                  | 0.0202                |
| Ref. [31]         | $M_1$ | 24.46                 | 0.260             | 0.0318                  | 0.0323                |
|                   | $M_2$ | 24.46                 | 0.375             | 0.0267                  | 0.0220                |
|                   | $M_3$ | 24.46                 | 0.189             | 0.0230                  | 0.0456                |
|                   | Mean  | 24.46                 | 0.274             | 0.0272                  | 0.0333                |

## 5. Conclusions

With a 3D nonlinear guidance model as the theoretical basis, this paper proposes a predefined-time cooperative guidance law for vehicles intercept the maneuvering target with multiple constraints, under the switching communication topologies. In the LOS direction, an adaptive predefined-time consensus protocol is designed to manage the impact time of multiple vehicles, ensuring the simultaneous interception of the target at the desired impact time. In the normal direction of LOS, based on the error transformation function, a PTNTSM guidance law is employed to ensure that the LOS angles converge to their desired values within the predefined time; meanwhile, the error converges in accordance with the prescribed performance. To address the issue of the unmeasurable acceleration of the target, a PTESO is utilized for target estimation. Notably, the convergence time of the system is an explicit adjustable parameter. The simulation results validate the effectiveness, robustness, and superiority of the proposed algorithm.

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