

Article

Differential-Flatness-Based Trajectory Planning for Direct Helicopter Shipboard Landing with Geometry-Aware Final-Landing Constraints

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Abstract

Autonomous helicopter shipboard landing requires the generation of dynamically feasible trajectories that can accurately match the time-varying motion of a moving deck while maintaining desirable near-deck geometric and kinematic characteristics. This paper proposes a differential-flatness-based trajectory planning method for single-phase direct shipboard landing from a cruising state in the vicinity of the ship. The landing trajectory is planned in the flat-output space of a simplified differentially flat helicopter model and parameterized using the minimum-control-effort trajectory representation, MINCO, together with an unconstrained optimization formulation. To improve the final landing process, a set of geometry-aware final-landing constraints is developed by explicitly accounting for the relative pose, relative motion, and near-deck geometry between the helicopter and the moving deck. These constraints enforce terminal motion-state matching and regulate the landing-gear/deck geometry, relative tangential velocity, and relative tilt angle in the near-deck region. Comprehensive simulation studies, including ablation and parameter-sensitivity analyses, computational comparisons, Monte Carlo evaluations, and closed-loop validation with a full nonlinear UH-60 model, demonstrate that the proposed method can efficiently generate trackable shipboard landing trajectories with improved terminal consistency and pre-touchdown geometric and kinematic regulation. Additional closed-loop sensitivity tests show that the tested trajectories remained executable under moderate high-level gain variations, while accurate terminal horizontal alignment remains dependent on deck-motion prediction accuracy.

Keywords: helicopter shipboard landing; trajectory planning; differential flatness; geometry-aware final-landing constraints; MINCO



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1. Introduction

Autonomous shipboard landing is a key enabling technology for extending unmanned helicopters from land-based operations to maritime missions. By enabling reliable recovery on moving ship decks, it enhances the operational autonomy and maritime deployment capability of unmanned helicopters. Nevertheless, helicopter shipboard landing remains a highly challenging task. The vehicle must complete an accurate recovery maneuver within a limited time and confined deck space while being affected by ship-motion-induced deck movement and the unsteady ship airwake [1–4]. These factors make shipboard landing a safety-critical and highly constrained operation, in which trajectory planning plays a crucial role. The planned trajectory not only provides the reference motion for downstream

tracking control but also determines whether the vehicle can satisfy the time-varying touchdown conditions imposed by the moving deck while maintaining reasonable margins during the final recovery process [3–5]. However, dedicated studies on shipboard landing trajectory generation for unmanned helicopters remain relatively limited.

Existing trajectory-generation methods for helicopter shipboard landing can be broadly divided into rule-based or simply parameterized methods and optimization-based methods. Rule-based designs inspired by pilot experience or bio-inspired principles have been used to construct landing trajectories from simple geometric or kinematic variables [6–9]. In addition, simple parameterized trajectories have been optimized under kinematic constraints to generate shipboard landing trajectories [5]. These methods are usually computationally efficient because of their simple trajectory representations. However, they have limited capability in handling complex nonlinear constraints, such as helicopter dynamic constraints, attitude and angular-motion constraints, and near-deck geometric constraints. Consequently, the generated trajectories may not be explicitly guaranteed to be dynamically feasible, and their quality can depend strongly on the downstream tracking controller.

Optimal-control-based trajectory generation provides a systematic way to incorporate nonlinear dynamics, path constraints, and terminal conditions into the planning problem. In a broad sense, model predictive control (MPC)-based approaches can also be regarded as belonging to this category. Such continuous-time optimal-control problems are commonly converted into finite-dimensional constrained nonlinear programming problems through direct transcription methods, including multiple shooting and direct collocation, in which the system dynamics are enforced at discrete nodes or collocation points [10,11]. Constrained optimal-control and MPC-based methods have been investigated for helicopter shipboard landing tasks [1,6,12–15]. These studies have considered finite-, infinite-, shrinking-, and variable-horizon formulations to handle moving-deck tracking, landing-time selection, system constraints, and environmental disturbances. More recently, shrinking-horizon MPC combined with disturbance-feedback control has been investigated for time-constrained helicopter deck landing under wind disturbances [16]. These methods provide general constraint-handling capability and, for MPC, enable feedback-based trajectory updating. However, their computational burden and constraint accuracy depend strongly on the complexity of the internal dynamic model, the prediction horizon, and the temporal transcription resolution. Directly incorporating a full nonlinear helicopter model can result in substantial online computation, whereas linearized or simplified internal models improve computational efficiency at the cost of model fidelity. These tradeoffs motivate trajectory-generation frameworks that retain rich constraint-handling capability while reducing the dimension and computational burden of the optimization.

Experimental progress has also been reported for autonomous moving-deck landing on smaller VTOL UAV platforms. Vision-based autonomous landing systems have been demonstrated through flight tests on moving platforms, while recent MPC-based trajectory-generation methods have achieved real-time onboard landing on moving surface vessels with changing deck inclination [17,18]. These studies provide valuable evidence of perception–control integration and onboard real-time implementation. Nevertheless, their vehicle scale, dynamic characteristics, and control architecture differ substantially from those of a full-scale conventional helicopter. They are therefore complementary to the present study, which focuses on computationally efficient trajectory generation for a full-scale conventional helicopter and on refined geometric and kinematic treatment of the final landing process.

Differential flatness provides an effective way to reduce the complexity of dynamic feasibility constraints. For a differentially flat system, the system state and control input

can be expressed as smooth functions of the flat outputs and a finite number of their derivatives [19,20]. This property allows the nonlinear dynamics to be represented by an equivalent chain-of-integrators form so that trajectory optimization can be performed directly over smooth parameterized flat-output trajectories. Meanwhile, vehicle motion states, including attitude and angular velocity, can be reconstructed from the planned trajectory through the differential-flatness transformation. Therefore, differential flatness is particularly suitable for coupled position-and-attitude trajectory planning on SE(3) [21,22]. It has been widely used for aerial vehicle trajectory generation, especially for multirotor systems, and has demonstrated excellent performance in various mission scenarios [21–31].

For helicopter shipboard landing, Zhao et al. [3,4] investigated differential-flatness-based trajectory generation and demonstrated promising performance. Their formulation explicitly decomposes the recovery maneuver into an approach phase and a landing phase, with an intermediate relative-hover condition above the deck. In the landing phase, the terminal conditions primarily prescribe the predicted deck-related horizontal position, velocity, and yaw states, together with the vertical position and velocity determined from the predicted deck heave. A scalar altitude-based clearance condition is imposed during descent, while a terminal deck-heave-rate condition is used for gentle touchdown. Although these conditions effectively regulate the principal motion states, they do not explicitly model the landing-gear/deck geometry or enforce full deck-relative attitude and angular-velocity matching, near-deck tangential-relative-velocity limitation, or relative-tilt regulation.

The present study also builds on our previous helicopter maneuvering-flight framework [32]. That work developed a simplified differentially flat helicopter model and its flatness transformation and established a trajectory-generation and tracking framework employing the approximate no-sideslip reduction, MINCO-based trajectory parameterization, an unconstrained optimization treatment, and a two-layer trajectory-tracking controller. These modeling, parameterization, optimization, and control elements are adopted rather than newly proposed in the present study.

In this paper, we focus on near-ship recovery within a finite ship-motion forecast horizon and aim to generate a single-phase direct landing trajectory from a cruising state in the vicinity of the ship to touchdown. This formulation removes the need for a prescribed intermediate hover or phase-transition state, thereby improving the planning flexibility of the recovery maneuver. Methodologically, the trajectory is planned in the flat-output space of a simplified differentially flat helicopter model, and the minimum-control-effort trajectory representation, MINCO [22], is adopted as an efficient trajectory parameterization. More importantly, a set of geometry-aware final-landing constraints (GFLCs) is developed to explicitly characterize the relative geometric and kinematic relationship between the helicopter and the moving deck during the final landing process. The proposed GFLCs enforce touchdown motion-state consistency and regulate the landing-gear/deck geometry, relative tangential velocity, and relative tilt angle in the near-deck region, thereby promoting refined position-and-attitude matching and improved pre-touchdown geometric and kinematic behavior. Together with an unconstrained reformulation of the optimization problem, the proposed framework enables efficient generation of single-phase direct landing trajectories under the adopted numerical planning criteria.

The main contributions of this work are summarized as follows:

1. A shipboard-specific trajectory-planning formulation based on differential flatness is developed for single-phase direct landing from a cruising state in the vicinity of the ship. The formulation generates one continuous trajectory to touchdown without prescribing an intermediate relative-hover or phase-transition state and incorporates the moving-deck quiescent-period window and deck-consistent terminal conditions.

2. A set of geometry-aware final-landing constraints is developed to refine the terminal and near-deck treatment of helicopter shipboard landing. Beyond conventional terminal motion-state conditions and scalar altitude-based deck-clearance constraints, the proposed GFLCs enforce refined terminal attitude and angular-velocity matching and regulate the landing-gear/deck geometry, tangential relative velocity, and relative tilt angle during the near-deck process.
3. Comprehensive simulation studies, including trajectory generation, GFLC ablation and parameter-sensitivity analyses, controlled computational comparisons, closed-loop validation with the full nonlinear model, Monte Carlo evaluations with dense sampled-time constraint post-processing, and bounded sensitivity tests for deck-motion prediction timing mismatch and controller tuning, are conducted to assess planning effectiveness, computational efficiency, and closed-loop executability under the modeled conditions.

The rest of this paper is organized as follows. Section 2 introduces the helicopter model, the shipboard operating environment, and the geometric models used for final-landing constraint design. Section 3 presents the trajectory optimization formulation and the design of the proposed GFLCs. Section 4 reports the simulation studies, including trajectory-generation results, GFLC ablation and parameter-sensitivity analyses, comparisons with an adapted time-discretized NLP formulation, closed-loop validation with the full nonlinear model, Monte Carlo evaluations with dense sampled-time constraint post-processing, and closed-loop sensitivity tests. Section 5 concludes the paper.

2. System and Environment Modeling

This section summarizes the modeling elements required for trajectory generation, final-landing constraint design, and closed-loop validation. The detailed helicopter modeling and differential-flatness analysis have been presented in our previous work [32]. The nonlinear UH-60 simulation model used for closed-loop validation has been validated against the flight-test data reported by Cross et al. [33], supporting its use as the validation model in this study.

2.1. Coordinate Systems

As shown in Figure 1, the world frame $\mathcal{F}^W : \{O_W, x_W, y_W, z_W\}$ is taken as the inertial frame and follows the North-East-Down (NED) convention. The helicopter body frame $\mathcal{F}^B : \{O_B, x_B, y_B, z_B\}$ is attached to the helicopter center of mass and follows the Forward-Right-Down convention. The deck-fixed frame $\mathcal{F}^D : \{O_D, x_D, y_D, z_D\}$ is attached to the center of the landing deck, with its x_D -axis pointing toward the bow of the ship. The canonical basis vectors are defined as $e_1 = [1, 0, 0]^T$, $e_2 = [0, 1, 0]^T$, and $e_3 = [0, 0, 1]^T$.

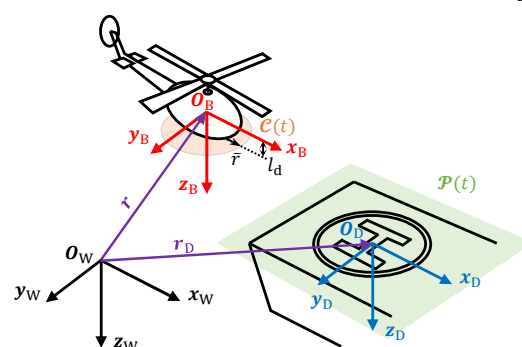


Figure 1. Coordinate frames and schematic illustration of the helicopter landing-gear plane and the deck plane.

2.2. Helicopter Dynamics and Differential Flatness

A UH-60 helicopter model is used in this study. In compact rigid-body form, the helicopter translational and rotational dynamics can be written as

$$\begin{aligned}\dot{\mathbf{r}} &= \mathbf{v}, \\ \dot{\mathbf{v}} &= \mathbf{g} + \frac{1}{m}\mathbf{R}\mathbf{f}, \\ \dot{\mathbf{R}} &= \mathbf{R}\hat{\boldsymbol{\omega}}, \\ \dot{\boldsymbol{\omega}} &= \mathbf{J}^{-1}(\boldsymbol{\tau} - \hat{\boldsymbol{\omega}}\mathbf{J}\boldsymbol{\omega}),\end{aligned}\quad (1)$$

where $\mathbf{r} = [X, Y, Z]^T$, $\mathbf{v} = [v_x, v_y, v_z]^T$, $\mathbf{R}$, and $\boldsymbol{\omega} = [p, q, r]^T$ denote the position, velocity, attitude, and body-frame angular velocity of the helicopter, respectively. $\mathbf{R}$ is parameterized by the Z-Y-X Euler angles ψ , θ , and ϕ . The vector $\mathbf{g}$ denotes the gravitational-acceleration vector. The vectors $\mathbf{f}$ and $\boldsymbol{\tau}$ denote the resultant aerodynamic force and moment, and m and $\mathbf{J}$ are the helicopter mass and inertia matrix.

For trajectory planning, the full nonlinear helicopter model is simplified into a differentially flat rigid-body model with state and input

$$\mathbf{x}_v = (\mathbf{r}, \mathbf{v}, \mathbf{R}, \boldsymbol{\omega}), \quad \mathbf{u}_v = (T_{MR}, \boldsymbol{\tau}), \quad (2)$$

where T_{MR} is the axial thrust of the main rotor. The simplified model is differentially flat, with flat output $\mathbf{z} = [\mathbf{r}^T, \psi]^T$ and flatness order $s = 4$. The main applicability limitation of the flatness property is that the roll and pitch angles should satisfy $|\phi| < \pi/2$ and $|\theta| < \pi/2$, respectively, to avoid singularity [32].

The fidelity of the simplified planning model and its differential-flatness transformation was quantitatively evaluated in our previous work [32]. Across 30 steady-state benchmark trajectories, the median reconstruction errors relative to the corresponding trim states of the full nonlinear helicopter model were 0.209° for roll, 0.682° for pitch, $0.192^\circ/\text{s}$ for roll rate, and $0.046^\circ/\text{s}$ for pitch rate, indicating limited attitude and angular-rate discrepancies over the tested flight conditions.

To reduce the trajectory-generation variables, an approximate no-sideslip condition is imposed:

$$\psi = \text{atan2}\left(e_2^T \dot{\mathbf{r}}, e_1^T \dot{\mathbf{r}}\right). \quad (3)$$

This relation is used only to determine the reference yaw angle in the trajectory-planning model and is not imposed on the full nonlinear helicopter dynamics used for closed-loop validation.

With $\mathbf{r}^{[s-1]} := (\mathbf{r}_1^{[s-1]T}, \mathbf{r}_2^{[s-1]T}, \mathbf{r}_3^{[s-1]T})^T$, $\mathbf{r}_i^{[s-1]} := (r_i, \dot{r}_i, \dots, r_i^{(s-1)})^T$, $i = 1, 2, 3$, and $\mathbf{v}_c := \mathbf{r}^{(s)}$, the planning model is written in Brunovsky form as

$$\dot{\mathbf{r}}^{[s-1]} = \mathbf{A}_z \mathbf{r}^{[s-1]} + \mathbf{B}_z \mathbf{v}_c, \quad (4)$$

where $\mathbf{A}_z = \text{blkdiag}(\mathbf{A}_s, \mathbf{A}_s, \mathbf{A}_s)$ and $\mathbf{B}_z = \text{blkdiag}(\mathbf{B}_s, \mathbf{B}_s, \mathbf{B}_s)$, with $\mathbf{A}_s$ and $\mathbf{B}_s$ being the standard companion matrix and input vector of an s th-order integrator chain, respectively. In this study, Equation (4) is used for trajectory planning, whereas the full UH-60 nonlinear model built upon Equation (1) is used for closed-loop validation.

2.3. Ship Kinematics

The ship-motion data are taken from the publicly available NATO Generic Destroyer (NATO-GD) dataset [34]. By combining the ship-motion data with the geometric information of the NATO-GD, the landing-deck state is obtained as

$$\mathbf{x}_D = (\mathbf{r}_D, \mathbf{v}_D, \mathbf{R}_D, \boldsymbol{\omega}_D), \quad (5)$$

where $\mathbf{r}_D$, $\mathbf{v}_D$, $\mathbf{R}_D$, and $\boldsymbol{\omega}_D$ denote the deck position, velocity, attitude, and angular velocity, respectively. Following existing studies on ship-motion prediction [35], the ship motion is assumed to be available within a 15 s prediction window.

Helicopter shipboard landing is planned to be completed within a quiescent period of the deck motion, defined as a continuous time interval during which the deck motion remains sufficiently small in both magnitude and rate. Motivated by real-time ship-motion criteria for maritime helicopter operations [35–37], the criteria adopted in this study are

$$\begin{aligned} |\Delta Z_D| &\leq 0.3 \text{ m}, \quad |v_{D_z}| = |\mathbf{e}_3^T \mathbf{v}_D| \leq 0.2 \text{ m/s}, \\ |\phi_D| &\leq 2^\circ, \quad |\dot{\phi}_D| \leq 1^\circ/\text{s}, \quad |\theta_D| \leq 2^\circ, \quad |\dot{\theta}_D| \leq 1^\circ/\text{s}, \\ T_{\text{ub}} - T_{\text{lb}} &\geq 4 \text{ s}, \end{aligned} \quad (6)$$

where $\Delta Z_D = \mathbf{e}_3^T \mathbf{r}_D - Z_{D,\text{base}}$ denotes the deck heave displacement relative to the baseline vertical deck position $Z_{D,\text{base}}$, and T_{lb} and T_{ub} denote the lower and upper bounds of the quiescent period, respectively. If no continuous interval satisfying the prescribed quiescent-period criteria is identified within the prediction horizon, landing-trajectory generation is not initiated. The helicopter is instead expected to maintain its waiting condition until an admissible interval is detected from an updated ship-motion prediction.

2.4. Aerodynamic Environment

The aerodynamic environment includes ship-airwake effects and deck/sea-surface ground effect. Ship airwake is a major aerodynamic factor affecting ship-helicopter operations [38]. In this study, the time-varying deck-region airwake velocity $\mathbf{v}_{w,\text{deck}}(t) = [u_{w,\text{deck}}(t), v_{w,\text{deck}}(t), w_{w,\text{deck}}(t)]^T$ is directly obtained from the wind-tunnel-based NATO-GD airwake dataset [34], rather than generated by an independently developed deck-region airwake model. To approximate the spatial variation of the airwake away from the deck region, the local airwake acting on the helicopter is modeled as

$$\mathbf{v}_w(t) = f_C(t) \mathbf{v}_{w,\text{deck}}(t), \quad (7)$$

with the Gaussian-type spatial attenuation factor

$$f_C(t) = \exp \left(-\frac{(\Delta r_x^Y(t))^2}{2\sigma_x^2} - \frac{(\Delta r_y^Y(t))^2}{2\sigma_y^2} - \frac{(\Delta r_z^Y(t))^2}{2\sigma_z^2} \right), \quad (8)$$

where $\Delta \mathbf{r}^Y(t) = [\Delta r_x^Y(t), \Delta r_y^Y(t), \Delta r_z^Y(t)]^T$ denotes the relative position between the helicopter and the target landing point in the ship-yaw frame $\mathcal{F}^Y$. The axes of $\mathcal{F}^Y$ are given by $\mathbf{x}_Y = [\cos \psi_D, \sin \psi_D, 0]^T$, $\mathbf{y}_Y = [-\sin \psi_D, \cos \psi_D, 0]^T$, and $\mathbf{z}_Y = [0, 0, 1]^T$. The Gaussian-type attenuation model is adopted as a computationally efficient approximation for extending the deck-region airwake data to the instantaneous helicopter position.

The ground effect is modeled by correcting the main-rotor induced velocity [39], i.e., $v_{i_{\text{mod}}} = f_G v_i$, with

$$f_G = 1 - \frac{R^2}{16 h_R^2}, \quad (9)$$

where R is the main-rotor radius and h_R is the distance from the main rotor to the sea surface or the landing deck. The full time-varying airwake velocity is not explicitly incorporated into the dynamic model used for trajectory optimization. Instead, the spatial attenuation factor $f_C(t)$ is used to tighten the control-related bounds near the deck, where the airwake influence becomes stronger, thereby reserving additional control margin for closed-loop execution. This treatment avoids the need to predict the future time history of the unsteady airwake and preserves the computational efficiency of the planner. The time-varying NATO-GD airwake sequence is explicitly applied to the full nonlinear helicopter model during closed-loop validation. In the closed-loop Monte Carlo evaluation, the retained time-varying airwake sequence is additionally scaled by a uniformly distributed factor between 0.8 and 1.2 to assess sensitivity to airwake-intensity variation.

2.5. Geometric Modeling

To support GFLC design, the helicopter landing-gear plane and the half-space above the deck are modeled explicitly. Inspired by the work of Ji et al. [26], the helicopter landing-gear plane is modeled as a disc:

$$\mathcal{C}(t) = \left\{ \mathbf{x} = \mathbf{R}(t) \mathbf{B} \bar{\mathbf{r}} \bar{\mathbf{u}} + \mathbf{c}(t) \mid \|\bar{\mathbf{u}}\| \leq 1, \bar{\mathbf{u}} \in \mathbb{R}^2 \right\}, \quad (10)$$

where $\mathbf{B} = [\mathbf{e}_1, \mathbf{e}_2] \in \mathbb{R}^{3 \times 2}$ is the basis matrix of the landing-gear plane, $\bar{\mathbf{r}}$ is the disc radius, and $\mathbf{c}(t) = \mathbf{r}(t) + l_d \mathbf{R}(t) \mathbf{e}_3$ is the disc center, with l_d denoting the distance from the helicopter center of mass to the landing-gear plane.

The half-space above the deck is represented as

$$\mathcal{P}(t) = \left\{ \mathbf{x} \in \mathbb{R}^3 \mid \mathbf{z}_D^T \mathbf{x} \leq \mathbf{z}_D^T \mathbf{r}_D \right\}, \quad (11)$$

where $\mathbf{z}_D(t) = \mathbf{R}_D(t) \mathbf{e}_3$ is the normal vector of the deck plane.

The present study focuses on pre-touchdown trajectory generation and the regulation of the relative geometry and motion between the helicopter and the moving deck. Accordingly, the landing gear is represented by a disc and the deck by a half-space, providing a compact geometric description for formulating the landing-gear/deck non-penetration and alignment constraints while maintaining the computational tractability of the trajectory-optimization problem. This abstraction does not account for finite deck boundaries, detailed landing-gear geometry, or three-dimensional clearance and collision constraints between other helicopter components and ship structures. Post-touchdown contact phenomena, including impact loads and friction, are also beyond the scope of the present study.

3. Planning for Shipboard Landing

3.1. Problem Formulation

The objective of trajectory planning is to generate a smooth and dynamically feasible single-phase direct landing trajectory from a cruising state in the vicinity of the ship to touchdown. The planned trajectory should satisfy the quiescent-period constraint, conventional flight-feasibility constraints, and the proposed geometry-aware final-landing

constraints. Based on the differential flatness of the simplified helicopter model, the trajectory planning problem is formulated as follows:

$$\min_{\mathbf{r}(t), T} J_o = \int_0^T \left\| \mathbf{r}^{(s)}(t) \right\|_W^2 dt + \rho_T(T), \quad (12a)$$

$$\text{s.t.} \quad T_{lb} < T < T_{ub}, \quad (12b)$$

$$\dot{\mathbf{r}}^{[s-1]} = \mathbf{A}_z \mathbf{r}^{[s-1]} + \mathbf{B}_z \mathbf{r}^{(s)}, \quad \forall t \in [0, T], \quad (12c)$$

$$\mathbf{r}^{[s-1]}(0) = \mathbf{r}_0^{[s-1]}, \quad (12d)$$

$$\mathbf{r}(T) = \mathbf{r}_D(T) - l_d \mathbf{z}_D(T), \quad (12e)$$

$$\mathbf{v}(T) = \mathbf{v}_D(T) - l_d \mathbf{R}_D(T) \hat{\boldsymbol{\omega}}_D(T) \mathbf{e}_3, \quad (12f)$$

$$\mathbf{R}(T) = \mathbf{R}_D(T), \quad (12g)$$

$$\boldsymbol{\omega}(T) = \boldsymbol{\omega}_D(T), \quad (12h)$$

$$\|\mathbf{v}(t)\| \leq v_{\max}, \quad \forall t \in [0, T], \quad (12i)$$

$$|\phi(t)| \leq \phi_{\max}, \quad |\theta(t)| \leq \theta_{\max}, \quad \forall t \in [0, T], \quad (12j)$$

$$\|\boldsymbol{\omega}(t)\| \leq \omega_{\max}, \quad \forall t \in [0, T], \quad (12k)$$

$$T_{MR_{\min}}(t) \leq T_{MR}(t) \leq T_{MR_{\max}}(t), \quad \forall t \in [0, T], \quad (12l)$$

$$\|\boldsymbol{\tau}(t)\| \leq \tau_{\max}(t), \quad \forall t \in [0, T], \quad (12m)$$

$$\mathbf{e}_3^T \mathbf{r}(t) \leq Z_{\max}, \quad \forall t \in [0, T], \quad (12n)$$

$$\mathcal{C}(t) \subseteq \mathcal{P}(t), \quad \text{if } \|\mathbf{r}(t) - \mathbf{r}_D(t)\| \leq \bar{d}, \quad (12o)$$

$$\|\mathbf{v}_t(t)\| \leq v_{t_{\max}}(t), \quad \text{if } \|\mathbf{r}(t) - \mathbf{r}_D(t)\| \leq \bar{d}, \quad (12p)$$

$$\zeta(t) \leq \zeta_{\max}(t), \quad \text{if } \|\mathbf{r}(t) - \mathbf{r}_D(t)\| \leq \bar{d}, \quad (12q)$$

where $\mathbf{W} \in \mathbb{R}^{3 \times 3}$ is a positive diagonal weighting matrix for the control-effort term, and $\rho_T(T)$ denotes the flight-time penalty. The constraints in Equation (12) can be divided into three groups. First, Equations (12b)–(12d) specify the flight-time window, the flatness-induced system equation, and the initial flat-output state. Second, Equations (12i)–(12n) impose conventional flight-feasibility constraints, including velocity, attitude, angular-velocity, control-input, and sea-surface clearance constraints. Third, Equations (12e)–(12h) and Equations (12o)–(12q) constitute the proposed geometry-aware final-landing constraints.

The GFLCs consist of two complementary parts. The first part enforces terminal deck-state consistency, including position, velocity, attitude, and angular-velocity matching at touchdown. The second part introduces near-deck constraints on the landing-gear/deck geometry, tangential relative velocity, and relative tilt angle. Unlike conventional box constraints on flight states or control inputs, these constraints explicitly account for the relative pose, relative motion, and near-deck geometry between the helicopter and the moving deck, thereby shaping the final landing process before touchdown.

The trajectory is parameterized using $\mathfrak{T}_{\text{MINCO}}$ with $s = 4$. In this case, each trajectory segment is a seventh-degree polynomial, which inherently satisfies the Brunovsky-form system equation in Equation (12c). The energy term in Equation (12a) can also be evaluated analytically under this polynomial representation. For an M -piece trajectory, the MINCO parameters include the segment durations $\mathbf{T} \in \mathbb{R}_{>0}^M$ and the $N = M - 1$ intermediate waypoints $\mathbf{q} \in \mathbb{R}^{3 \times N}$. To reduce the number of temporal optimization variables, a uniform time allocation is adopted, so that all segment durations are determined by the total flight time T . Therefore, the optimization variables reduce to T and $\mathbf{q}$, enabling joint optimization of the landing time and spatial trajectory.

3.2. Flight Time Constraint

The flight time T is optimized within the quiescent-period window $T \in (T_{lb}, T_{ub})$. To remove this bound constraint, an unconstrained variable $\xi \in \mathbb{R}$ is introduced through the diffeomorphic transformation

$$\xi = \ln\left(\frac{T - T_{lb}}{T_{ub} - T}\right) \Leftrightarrow T = \frac{T_{ub}e^{\xi} + T_{lb}}{e^{\xi} + 1}. \quad (13)$$

Thus, the bounded flight-time variable can be replaced by the unconstrained variable ξ in the numerical optimization.

3.3. Initial State Constraint

In the flat-output representation, the trajectory state is given by $\mathbf{r}^{[s-1]}$, which consists of $\mathbf{r}$ and its derivatives up to order $s - 1 = 3$, i.e., position, velocity, acceleration, and jerk. The helicopter is assumed to start the landing maneuver from a trimmed steady forward-flight cruise condition. Therefore, the initial conditions are specified as

$$\mathbf{r}(0) = \mathbf{r}_0, \quad \mathbf{v}(0) = \mathbf{v}_0, \quad \mathbf{a}(0) = \mathbf{0}, \quad \mathbf{j}(0) = \mathbf{0}. \quad (14)$$

3.4. Kinematic Constraints

The velocity constraint in Equation (12i) is enforced through a continuous-time penalty. Since the direct use of the max function introduces nonsmoothness, the following C^2 -smooth approximation is adopted:

$$\mathcal{L}_{\mu}(x) = \begin{cases} 0, & x \leq 0, \\ (\mu - x/2)(x/\mu)^3, & 0 < x \leq \mu, \\ x - \mu/2, & x > \mu, \end{cases} \quad (15)$$

where $\mu > 0$ is the smoothing parameter. The velocity-constraint penalty is then defined as

$$I_{\mathcal{G}_v} = \int_0^T \mathcal{G}_v(t) dt, \quad \mathcal{G}_v(t) = \mathcal{L}_{\mu}(\|\mathbf{v}(t)\|^2 - v_{\max}^2). \quad (16)$$

The attitude and angular-velocity constraints in Equations (12j) and (12k) can be evaluated through the differential-flatness transformation. However, to avoid repeated reconstruction of the full attitude and angular velocity during optimization, simplified non-aggressive-maneuver approximations are adopted following Zhao et al. [4]:

$$\left\| [a_x(t), a_y(t)]^T \right\| \leq g \gamma_{\max}, \quad \forall t \in [0, T], \quad (17)$$

$$\left\| [j_x(t), j_y(t)]^T \right\| \leq g \dot{\gamma}_{\max}, \quad |\dot{\psi}(t)| \leq \dot{\psi}_{\max}, \quad \forall t \in [0, T], \quad (18)$$

where $\gamma_{\max} = \phi_{\max} = \theta_{\max}$ and $\dot{\gamma}_{\max} = \dot{\phi}_{\max} = \dot{\theta}_{\max}$. These expressions are used only as computational surrogates for evaluating the attitude and angular-velocity constraints during optimization. The complete attitude and angular velocity are subsequently reconstructed through the differential-flatness transformation.

The corresponding penalty functions are

$$I_{\mathcal{G}_{\phi,\theta}} = \int_0^T \mathcal{G}_{\phi,\theta}(t) dt, \quad (19)$$

$$I_{\mathcal{G}_{\omega}} = \int_0^T \mathcal{G}_{\omega}(t) dt, \quad (20)$$

with

$$\mathcal{G}_{\phi,\theta}(t) = \mathcal{L}_{\mu} \left(\left\| [a_x(t), a_y(t)]^T \right\|^2 - (g\gamma_{\max})^2 \right),$$

$$\mathcal{G}_{\omega}(t) = \mathcal{L}_{\mu} \left(\left\| [j_x(t), j_y(t)]^T \right\|^2 - (g\dot{\gamma}_{\max})^2 \right) + \mathcal{L}_{\mu} \left(|\dot{\psi}(t)|^2 - \dot{\psi}_{\max}^2 \right).$$

3.5. Control Input Constraints

The control inputs T_{MR} and τ can also be reconstructed from the planned trajectory through the differential-flatness transformation. To reduce the computational burden during optimization, their constraints are enforced through approximate flat-output-derivative bounds:

$$a_{\text{aero}_{\min}}(t) \leq \|a_{\text{aero}}(t)\| \leq a_{\text{aero}_{\max}}(t), \quad \forall t \in [0, T], \quad (21)$$

$$\left\| [s_x(t), s_y(t)]^T \right\| \leq g \dot{\gamma}_{\max}(t), \quad |\dot{\psi}(t)| \leq \dot{\psi}_{\max}(t), \quad \forall t \in [0, T], \quad (22)$$

where $a_{\text{aero}} = a - g$ approximates the acceleration generated by the main-rotor thrust, and $s = r^{(4)}$ denotes the snap of the planned trajectory. The moment constraint is approximated by limiting the corresponding angular-acceleration-related quantities under non-aggressive maneuvering assumptions. These approximate flat-output-derivative bounds are used only as computational surrogates for evaluating the control-related constraints during optimization. The corresponding virtual control inputs are subsequently reconstructed through the differential-flatness transformation, and the resulting trajectories are evaluated using the full nonlinear helicopter model in closed-loop simulations.

To preserve control margin near the deck, where ship-airwake effects become more significant, the bounds are tightened according to the airwake attenuation factor $f_C(t)$:

$$a_{\text{aero}_{\min}}(t) = a_{\text{aero}_{\text{act},\min}} + \Delta a_{\text{aero}_{\max}} f_C(t), \quad (23)$$

$$a_{\text{aero}_{\max}}(t) = a_{\text{aero}_{\text{act},\max}} - \Delta a_{\text{aero}_{\max}} f_C(t), \quad (24)$$

$$\dot{\gamma}_{\max}(t) = \dot{\gamma}_{\text{act},\max} - \Delta \dot{\gamma}_{\max} f_C(t), \quad (25)$$

$$\dot{\psi}_{\max}(t) = \dot{\psi}_{\text{act},\max} - \Delta \dot{\psi}_{\max} f_C(t). \quad (26)$$

The corresponding penalty functions are defined as

$$I_{\mathcal{G}_{T_{MR}}} = \int_0^T \mathcal{G}_{T_{MR}}(t) dt, \quad (27)$$

$$I_{\mathcal{G}_{\tau}} = \int_0^T \mathcal{G}_{\tau}(t) dt, \quad (28)$$

where

$$\mathcal{G}_{T_{MR}}(t) = \mathcal{L}_{\mu} \left(\|a_{\text{aero}}(t)\|^2 - (a_{\text{aero}_{\max}}(t))^2 \right) + \mathcal{L}_{\mu} \left((a_{\text{aero}_{\min}}(t))^2 - \|a_{\text{aero}}(t)\|^2 \right),$$

$$\mathcal{G}_{\tau}(t) = \mathcal{L}_{\mu} \left(\left\| [s_x(t), s_y(t)]^T \right\|^2 - (g\dot{\gamma}_{\max}(t))^2 \right) + \mathcal{L}_{\mu} \left(|\dot{\psi}(t)|^2 - (\dot{\psi}_{\max}(t))^2 \right).$$

3.6. Sea-Surface Clearance Constraint

To avoid contact with the sea surface during flight, the helicopter center of mass is required to remain at least l_d above sea level. Accordingly, $Z_{\max}$ in Equation (12n) is set to $-l_d$, and the corresponding penalty function is

$$I_{\mathcal{G}_Z} = \int_0^T \mathcal{G}_Z(t) dt, \quad \mathcal{G}_Z(t) = \mathcal{L}_{\mu} \left(e_3^T r(t) + l_d \right). \quad (29)$$

3.7. Geometry-Aware Final-Landing Constraints

The conventional feasibility constraints introduced above mainly restrict the flight envelope and control-input ranges of the planned trajectory. However, they do not explicitly characterize how the helicopter approaches the moving deck and reaches the touchdown condition during the final landing process. For shipboard landing, terminal position and velocity matching alone is insufficient, because the helicopter and the deck are both moving rigid bodies. The touchdown quality is also affected by the relative attitude, angular motion, tangential relative velocity, and near-deck geometry between the landing-gear plane and the deck plane.

To address this issue, the geometry-aware final-landing constraints (GFLCs) are introduced to shape the final landing process from both terminal motion-state and near-deck geometric perspectives. The GFLCs consist of two complementary components. The first component enforces terminal motion-state consistency in both translational and rotational degrees of freedom, including position, velocity, attitude, and angular-velocity matching at touchdown. The second component regulates the near-deck behavior before touchdown by constraining the landing-gear/deck geometry, relative tangential velocity, and relative tilt angle. Accordingly, the present study addresses pre-touchdown geometric and kinematic regulation and terminal motion-state alignment, rather than a complete assessment of practical landing safety. This geometric and kinematic interpretation is consistent with existing ship-deck landing evaluation work, where touchdown position, velocity, and attitude relative to the ship deck are used as objective evaluation quantities [40].

3.7.1. Terminal State Constraint

The first component of the GFLCs enforces terminal motion-state consistency between the helicopter and the moving deck. Unlike conventional terminal constraints that prescribe only the touchdown position or velocity, the proposed terminal constraints require the helicopter position, velocity, attitude, and angular velocity to be consistent with the corresponding deck motion state at touchdown.

Specifically, the terminal position and velocity of the helicopter are determined by the deck pose and rigid-body kinematics as specified in Equations (12e) and (12f). The terminal attitude and angular velocity are further constrained by

$$\mathbf{R}(T) = \mathbf{R}_D(T), \quad (30)$$

$$\boldsymbol{\omega}(T) = \boldsymbol{\omega}_D(T). \quad (31)$$

These constraints make the terminal state consistent with the moving deck not only in translation, but also in attitude and angular motion.

Since the MINCO trajectory with $s = 4$ requires boundary conditions up to jerk, the terminal acceleration $\mathbf{a}(T)$ and jerk $\mathbf{j}(T)$ must also be determined. In this work, they are obtained through the inverse direction of the differential-flatness relationship by prescribing the terminal state $\mathbf{x}_v(T) = (\mathbf{r}(T), \mathbf{v}(T), \mathbf{R}(T), \boldsymbol{\omega}(T))$ and the terminal control input $\mathbf{u}_v(T) = (T_{MR}(T), \boldsymbol{\tau}(T))$ of the simplified dynamic model. The terminal moment is determined from Euler's rotational equation,

$$\boldsymbol{\tau}(T) = \mathbf{J}\dot{\boldsymbol{\omega}}(T) + \dot{\boldsymbol{\omega}}(T)\mathbf{J}\boldsymbol{\omega}(T), \quad (32)$$

under the assumption of zero angular acceleration, i.e., $\dot{\boldsymbol{\omega}}(T) = \mathbf{0}$. The terminal main-rotor thrust is required to satisfy the thrust bound in Equation (12l). To remove this bound constraint during optimization, an unconstrained variable $\eta \in \mathbb{R}$ is introduced:

$$T_{MR}(T) = T_{MR_m} + T_{MR_r} \sin(\eta), \quad (33)$$

where

$$T_{MR_m} = \frac{T_{MR_{\max}}(T) + T_{MR_{\min}}(T)}{2}, \quad T_{MR_r} = \frac{T_{MR_{\max}}(T) - T_{MR_{\min}}(T)}{2}. \quad (34)$$

The resulting terminal acceleration and jerk are then used as the terminal boundary conditions of the MINCO trajectory.

3.7.2. Near-Deck Constraints

The second component of the GFLCs regulates the near-deck behavior before touchdown. In this work, final landing does not denote a separately planned phase. Rather, it refers to the terminal near-deck portion of the single planned trajectory, from the moment the helicopter enters the prescribed near-deck region $\|r(t) - r_D(t)\| \leq \bar{d}$ to the planned touchdown instant. Accordingly, the near-deck constraints are activated only within this region, where $\bar{d}$ is the prescribed near-deck activation distance. Their purpose is not to impose general flight-envelope limits, but to shape the relative geometry and relative motion between the helicopter and the deck during the final landing process.

The near-deck contact-avoidance constraint prevents the landing-gear plane from penetrating the deck plane before the planned touchdown instant. Based on the definitions of $\mathcal{C}(t)$ and $\mathcal{P}(t)$, the condition $\mathcal{C}(t) \subseteq \mathcal{P}(t)$ is equivalent to

$$\bar{r} \left\| \mathbf{B}^T \mathbf{R}^T(t) \mathbf{z}_D(t) \right\| + \mathbf{z}_D^T(t) \mathbf{c}(t) - \mathbf{z}_D^T(t) \mathbf{r}_D(t) \leq 0. \quad (35)$$

To avoid introducing an explicit logic condition, a C^2 -smooth activation function is used:

$$\mathcal{L}_\varepsilon(x) = \begin{cases} 0, & x \leq -\varepsilon, \\ 6x_\varepsilon^5 - 15x_\varepsilon^4 + 10x_\varepsilon^3, & |x| < \varepsilon, \\ 1, & x \geq \varepsilon, \end{cases} \quad x_\varepsilon = \frac{x + \varepsilon}{2\varepsilon}, \quad (36)$$

where $\varepsilon > 0$ determines the width of the transition region. The near-deck activation factor is then defined as

$$\sigma_D(t) = \mathcal{L}_\varepsilon\left(\bar{d}^2 - \|r(t) - r_D(t)\|^2\right). \quad (37)$$

Thus, $\sigma_D(t)$ approaches one when the helicopter is inside the near-deck region and approaches zero when it is outside this region.

The corresponding penalty function is defined as

$$I_{\mathcal{G}_{\mathcal{C},\mathcal{P}}} = \int_0^T \mathcal{G}_{\mathcal{C},\mathcal{P}}(t) dt, \quad (38)$$

where

$$\mathcal{G}_{\mathcal{C},\mathcal{P}}(t) = \mathcal{L}_\mu\left(\bar{r} \left\| \mathbf{B}^T \mathbf{R}^T(t) \mathbf{z}_D(t) \right\| + \mathbf{z}_D^T(t) \mathbf{c}(t) - \mathbf{z}_D^T(t) \mathbf{r}_D(t)\right) \sigma_D(t). \quad (39)$$

The relative velocity between the helicopter center of mass and the deck-consistent touchdown reference point is defined as

$$\mathbf{v}_{\text{rel}}(t) = \mathbf{v}(t) - (\mathbf{v}_D(t) - l_d \mathbf{R}_D(t) \hat{\boldsymbol{\omega}}_D(t) \mathbf{e}_3). \quad (40)$$

The relative velocity can be decomposed into the deck-normal and deck-tangential components as

$$\mathbf{v}_{\text{rel}}(t) = \mathbf{v}_n(t) + \mathbf{v}_t(t), \quad \mathbf{v}_n(t) = \left(\mathbf{v}_{\text{rel}}^T(t) \mathbf{z}_D(t) \right) \mathbf{z}_D(t), \quad \mathbf{v}_t(t) = \mathbf{v}_{\text{rel}}(t) - \mathbf{v}_n(t). \quad (41)$$

The relative tangential-velocity constraint reduces the tendency of tangential sliding or scraping contact during touchdown:

$$\|\mathbf{v}_t(t)\| \leq v_{t\max}(t). \quad (42)$$

The relative tilt angle between the helicopter and the deck is defined as

$$\zeta(t) = \arccos(\mathbf{z}_B^T(t)\mathbf{z}_D(t)), \quad \mathbf{z}_B(t) = \mathbf{R}(t)\mathbf{e}_3. \quad (43)$$

The relative tilt-angle constraint promotes attitude alignment between the helicopter and the moving deck before touchdown:

$$\zeta(t) \leq \zeta_{\max}(t). \quad (44)$$

The allowable bounds of the relative tangential velocity and relative tilt angle are tightened as the helicopter approaches the target touchdown position:

$$v_{t\max}(t) = v_{t\max} - \Delta v_{t\max} g_t(t), \quad (45)$$

$$\zeta_{\max}(t) = \zeta_{\max} - \Delta \zeta_{\max} g_t(t), \quad (46)$$

where

$$g_t(t) = \exp\left(-\frac{\|\mathbf{r}(t) - (\mathbf{r}_D(t) - l_d \mathbf{z}_D(t))\|^2}{2\sigma_t^2}\right), \quad (47)$$

where σ_t controls the spatial range of the tightening effect.

The corresponding penalty functions are

$$I_{\mathcal{G}_{v_t}} = \int_0^T \mathcal{G}_{v_t}(t) dt, \quad (48)$$

$$I_{\mathcal{G}_{\zeta}} = \int_0^T \mathcal{G}_{\zeta}(t) dt, \quad (49)$$

where

$$\mathcal{G}_{v_t}(t) = \mathcal{L}_{\mu}(\|\mathbf{v}_t(t)\|^2 - (v_{t\max}(t))^2)\sigma_D(t), \quad (50)$$

$$\mathcal{G}_{\zeta}(t) = \mathcal{L}_{\mu}(\zeta^2(t) - (\zeta_{\max}(t))^2)\sigma_D(t). \quad (51)$$

In summary, the terminal component of the GFLCs enforces deck-state consistency at touchdown, whereas the near-deck component shapes the relative geometry and relative motion before touchdown. These constraints are geometry-aware because they explicitly depend on the deck attitude, deck normal, landing-gear geometry, and helicopter-deck relative motion, rather than only on state-wise upper or lower bounds. Therefore, the GFLCs provide a dedicated final-landing constraint design for single-phase direct shipboard landing.

3.8. Implementation of Trajectory Optimization

The continuous-time trajectory constraints are incorporated into the objective function through smooth penalty terms. Specifically, the overall penalty function is constructed as

$$I_G = \sum_{\mathcal{G}_i \in \mathbb{G}} \chi_{\mathcal{G}_i} I_{\mathcal{G}_i}, \quad \mathbb{G} = \{\mathcal{G}_v, \mathcal{G}_{\phi, \theta}, \mathcal{G}_{\omega}, \mathcal{G}_{T_{MR}}, \mathcal{G}_{\tau}, \mathcal{G}_Z, \mathcal{G}_{C, \mathcal{P}}, \mathcal{G}_{v_t}, \mathcal{G}_{\zeta}\}, \quad (52)$$

where $I_{\mathcal{G}_i}$ denotes the penalty associated with the constraint function $\mathcal{G}_i$, and $\chi_{\mathcal{G}_i} > 0$ is the corresponding penalty weight. The penalized objective function is then written as

$$J = J_o + I_{\mathcal{G}}. \quad (53)$$

Since the penalty terms are defined as integrals along the trajectory, they are evaluated numerically by sampling each trajectory segment. The overall penalty is approximated, using the trapezoidal rule, as

$$I_{\mathcal{G}} \approx \sum_{i=1}^M \frac{T_i}{\kappa_i} \sum_{j=0}^{\kappa_i} \bar{\omega}_j \sum_{\mathcal{G}_k \in \mathbb{G}} \chi_{\mathcal{G}_k} \mathcal{G}_k(t_{i,j}), \quad (54)$$

where i indexes the trajectory segment, κ_i is the number of sampling intervals in the i -th segment, and $\bar{\omega}_j$ are the trapezoidal weights, with $\bar{\omega}_0 = \bar{\omega}_{\kappa_i} = 1/2$ and $\bar{\omega}_j = 1$ otherwise. The sampling time is given by $t_{i,j} = t_{i-1} + jT_i/\kappa_i$, where t_{i-1} is the start time of the i -th segment. Since uniform time allocation is adopted, $T_i = T/M$ for all segments.

Because the smooth penalties are evaluated at a finite set of quadrature nodes, optimizer convergence and a small integrated penalty do not by themselves guarantee exact continuous-time constraint satisfaction. An independent dense sampled-time post-processing check is therefore reported in Section 4.6.

Under the MINCO parameterization, the original planning variables are the total flight time T and the intermediate waypoints $\mathbf{q}$. The bounded flight time is transformed into the unconstrained variable ζ through Equation (13). In addition, the auxiliary variable η is introduced to determine the terminal main-rotor thrust through Equation (33). Therefore, the trajectory optimization problem is reformulated in terms of the unconstrained variables

$$\zeta \in \mathbb{R}, \quad \mathbf{q} \in \mathbb{R}^{3 \times N}, \quad \eta \in \mathbb{R}. \quad (55)$$

The resulting unconstrained optimization problem is solved numerically to obtain the landing time, intermediate waypoints, and terminal thrust parameter.

The optimization is initialized using a seventh-degree polynomial trajectory. The initial flight time is set to the midpoint of the quiescent period, $T_{\text{ini}} = (T_{\text{ub}} + T_{\text{lb}})/2$. The polynomial coefficients are determined from the initial trimmed cruise condition and the terminal deck-consistent boundary conditions. If the resulting initial trajectory violates basic motion-state constraints in the sampled feasibility check, the initial flight time is adjusted within the quiescent-period window and the polynomial initialization is recomputed. Once a feasible initial trajectory is obtained, ζ_{ini} is computed from Equation (13), the initial intermediate waypoints are obtained by uniformly sampling the polynomial trajectory, and the terminal-thrust variable is initialized as $\eta_{\text{ini}} = 0$.

No warm-start strategy or reuse of previously optimized solutions across different scenarios was employed in the reported optimization tests.

The key trajectory-planning parameters and numerical optimization settings are summarized in Tables 1 and 2, respectively. Unless otherwise stated, the same parameter settings are used throughout the simulation studies.

The conventional flight-state and control-related bounds listed in Table 1 are adopted as numerical design bounds for the trajectory-planning problem rather than as certified operational limits of the UH-60. Their purpose is to define a bounded search domain for the shipboard-landing scenarios considered in this study, exclude trajectories involving undesirably large flight states or control demands, and reserve additional control margin near the deck through the position-dependent tightening terms. The resulting trajectories are subsequently evaluated using the full nonlinear

UH-60 model in the closed-loop simulations. The objective function and penalty parameters in Table 2 were selected to balance trajectory smoothness, agility, constraint satisfaction, and numerical convergence.

Table 1. Key parameters used in the trajectory-planning method.

Parameter	Value
<i>Trajectory discretization</i>	
Number of MINCO pieces, M	6
Sampling intervals per piece, κ_i	4
<i>Flight-feasibility bounds</i>	
Maximum velocity, $v_{\max}$	40 m/s
Maximum roll and pitch angles, $\gamma_{\max}$	58.5 °
Maximum roll and pitch rates, $\dot{\gamma}_{\max}$	35 °/s
Maximum yaw rate, $\dot{\psi}_{\max}$	40 °/s
Aerodynamic-acceleration bounds, $a_{\text{aeroact,min}}$ and $a_{\text{aeroact,max}}$	(4.9, 14.7) m/s ²
Maximum roll and pitch angular acceleration, $\ddot{\gamma}_{\text{act,max}}$	80 °/s ²
Maximum yaw angular acceleration, $\ddot{\psi}_{\text{act,max}}$	100 °/s ²
Aerodynamic-acceleration margin, $\Delta a_{\text{aero,max}}$	2.94 m/s ²
Roll and pitch angular-acceleration margin, $\Delta \dot{\gamma}_{\max}$	40 °/s ²
Yaw angular-acceleration margin, $\Delta \ddot{\psi}_{\max}$	50 °/s ²
<i>GFLC and airwake parameters</i>	
Landing-gear disc radius, $\bar{r}$	3 m
Distance from the center of mass to the landing-gear plane, l_d	3 m
Near-deck activation distance, $\bar{d}$	20 m
Activation smoothing parameter, ε	4 m ²
Maximum relative tangential velocity, $v_{t,\max}$	20 m/s
Tangential-velocity tightening, $\Delta v_{t,\max}$	18 m/s
Maximum relative tilt angle, $\zeta_{\max}$	50 °
Tilt-angle tightening, $\Delta \zeta_{\max}$	45 °
Spatial tightening scale, σ_t	2 m
Airwake attenuation scales, σ_x , σ_y , and σ_z	(80, 20, 15) m

The nominal GFLC parameters were selected according to their respective functional roles rather than as uniquely optimal values. The activation distance $\bar{d}$ determines when the near-deck geometric and kinematic regulation begins, whereas σ_t localizes the strongest tightening effect to the immediate touchdown neighborhood. The parameters $\Delta v_{t,\max}$ and $\Delta \zeta_{\max}$ determine the amounts by which the general relative-motion bounds are tightened near the target touchdown position. The GFLC penalty weights were selected to give the near-deck constraints sufficient numerical priority relative to the trajectory-smoothness and flight-time objectives without introducing unnecessarily large penalty scaling. The airwake spatial scales determine the region over which the airwake-related planning margins vary around the deck. The quiescent-period bounds are obtained from the prescribed deck-motion criteria and are therefore scenario dependent rather than independently tuned. The sensitivity of these parameter choices is examined in Section 4.3.

Table 2. Numerical optimization settings of the trajectory-planning method.

Setting	Value
<i>Objective and penalty parameters</i>	
Control-effort weighting matrix, W	diag(100, 100, 100)
Flight-time penalty, $\rho_T(T)$	500T
Penalty smoothing parameter, μ	1×10^{-2}
Velocity-constraint penalty weight, $\chi_{\mathcal{G}_v}$	1×10^6
Attitude-constraint penalty weight, $\chi_{\mathcal{G}_{\phi,\theta}}$	1×10^6
Angular-velocity-constraint penalty weight, $\chi_{\mathcal{G}_\omega}$	1×10^6
Main-rotor-thrust-constraint penalty weight, $\chi_{\mathcal{G}_{TMR}}$	1×10^6
Moment-constraint penalty weight, $\chi_{\mathcal{G}_\tau}$	1×10^6
Sea-surface-clearance penalty weight, $\chi_{\mathcal{G}_z}$	1×10^{10}
Landing-gear/deck-geometry penalty weight, $\chi_{\mathcal{G}_{c,p}}$	1×10^8
Tangential-velocity penalty weight, $\chi_{\mathcal{G}_{v_t}}$	1×10^6
Relative-tilt-angle penalty weight, $\chi_{\mathcal{G}_\zeta}$	1×10^{10}
<i>Solver settings</i>	
MATLAB release	R2024a
Solver	fminunc
Algorithm	quasi-newton
OptimalityTolerance	1×10^{-3}
StepTolerance	1×10^{-6}
FunctionTolerance	1×10^{-3}
MaxIterations	1×10^5
MaxFunctionEvaluations	1×10^6

4. Simulation Studies

This section evaluates the proposed trajectory planning method through a sequence of simulation studies. First, trajectory-generation results are presented to assess the proposed method in representative shipboard landing scenarios under the adopted numerical planning criteria. Then, an ablation study is conducted to examine the role and effectiveness of the proposed GFLCs. A one-at-a-time sensitivity analysis is subsequently performed to examine the influence of the principal GFLC-related parameters. Next, the proposed planning formulation is compared with an adapted time-discretized NLP formulation based on Zhao et al. [4]. Closed-loop validation with the full nonlinear helicopter model is then performed to assess the closed-loop executability of the planned trajectories under the modeled conditions. Planning-level and closed-loop Monte Carlo studies provide statistical evaluations under randomized planning and execution conditions, while dense sampled-time post-processing further quantifies the residuals associated with finite-node constraint enforcement. Finally, additional closed-loop sensitivity tests examine the effects of deck-motion prediction timing mismatch and moderate variations of the high-level tracking gains.

4.1. Trajectory Generation for Helicopter Shipboard Landing

This subsection evaluates whether the proposed method can generate single-phase direct landing trajectories satisfying the adopted numerical planning criteria from a cruising state in the vicinity of the ship. In all test scenarios, the helicopter starts from the same trimmed cruising condition, with initial position $r_0 = [0, 0, -50]^T$ m, velocity $v_0 = [30, 0, 0]^T$ m/s, and heading $\psi_0 = 0^\circ$. Within the 15 s ship-motion prediction horizon, a quiescent period is identified according to Equation (6), and trajectory planning is performed to complete touchdown within this interval.

The ship-motion data are taken from the NATO-GD dataset [34]. Two sea states and two wind/wave directions are considered: Sea State 4 with wave height 2 m, wave period 10 s, and relative wind speed 25 kn; and Sea State 5 with wave height 4 m, wave period 12 s, and relative wind speed 40 kn. The wind/wave directions are 0° (head) and 30° (Green 30), and the ship forward speed is 15 kn in all cases. In addition, three initial deck configurations are used:

$$\begin{aligned} [r_{D_0}^T, \psi_{D_0}]^T &= [120 \text{ m}, 0 \text{ m}, -7.2 \text{ m}, 0^\circ]^T, \\ &[103.92 \text{ m}, 60 \text{ m}, -7.2 \text{ m}, 30^\circ]^T, \\ &[84.85 \text{ m}, 84.85 \text{ m}, -7.2 \text{ m}, 90^\circ]^T. \end{aligned}$$

Here, $Z_{D,\text{base}} = -7.2 \text{ m}$ is the baseline deck height above the sea surface [34]. Combining the four ship-motion cases with the three initial deck configurations gives 12 representative landing scenarios.

The proposed method successfully generated single-phase direct landing trajectories satisfying the adopted numerical planning criteria in all 12 test scenarios. Figure 2 shows representative planning results under Sea State 5 and a wave/wind direction of 30° , with the three initial deck configurations denoted as Cases 1–3. The results include the ground tracks, altitude, speed, and attitude profiles. The attitude profiles shown in Figure 2c are the nominal attitudes reconstructed from the planned trajectory through the differential-flatness transformation.

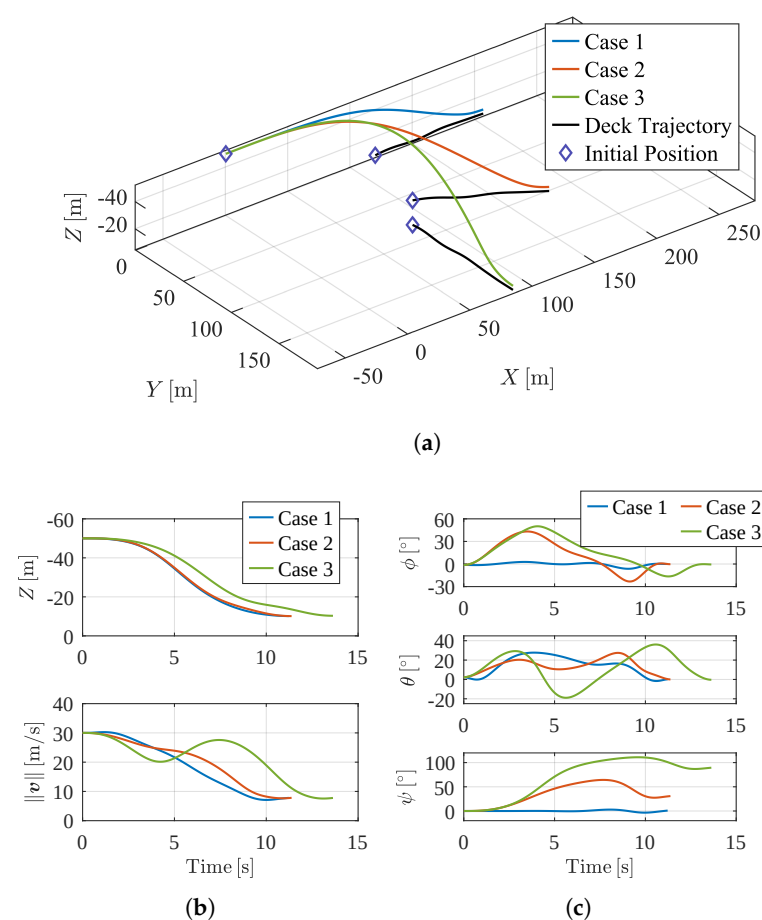


Figure 2. Optimized helicopter shipboard landing trajectories. (a) Ground tracks r . (b) Altitude Z and speed $\|v\|$ profiles. (c) Nominal attitude ϕ, θ, ψ profiles reconstructed through the differential-flatness transformation.

The relatively large roll and pitch excursions in the planned trajectories arise mainly from the rapid deceleration and direction adjustment required to transition from the initial cruising state to deck-aligned touchdown within a single continuous maneuver. These attitude variations reflect the intentional use of the helicopter's maneuvering capability to accomplish the direct-landing task, rather than indicating infeasible attitude commands. As clarified in Section 2.2, the differential-flatness analysis does not rely on a small-angle approximation, and its principal applicability conditions require $|\phi| < \pi/2$ and $|\theta| < \pi/2$ to avoid the Euler-angle singularity. The planned roll and pitch profiles shown in Figure 2 remain within this applicable domain. The curve exceeding 100° corresponds to the yaw angle ψ , which represents the helicopter heading about the vertical axis rather than the body tilt.

4.2. Ablation Study on the Geometry-Aware Final-Landing Constraints

To examine the necessity and effectiveness of the proposed GFLCs, an ablation study is conducted by removing the terminal motion-state matching constraints and the near-deck constraints from the trajectory optimization problem. The comparison is made from two perspectives: the motion-state consistency between the helicopter and the deck at the planned touchdown instant, and the near-deck geometric and kinematic behavior during the final landing process.

In the ablation setting, the terminal position and velocity are specified without explicitly accounting for the deck attitude and rigid-body rotational kinematics. Specifically, they are set to $\mathbf{r}(T) = \mathbf{r}_D(T) - l_d \mathbf{e}_3$ and $\mathbf{v}(T) = \mathbf{v}_D(T)$. The terminal attitude and angular velocity are not constrained to match those of the deck, and the terminal acceleration and jerk are directly set to $\mathbf{a}(T) = \mathbf{0}$ and $\mathbf{j}(T) = \mathbf{0}$. These settings represent a non-rotating steady-flight terminal condition and serve as the baseline without GFLCs.

In this planning-stage comparison, the planned touchdown instant refers to the terminal time of the optimized trajectory. The relative motion states between the helicopter and the deck at this instant are summarized in Table 3. In the table, ΔX , ΔY , and r_{xy} denote the planar position error in the deck frame $\mathcal{F}^D$, where $\Delta \mathbf{r} = \mathbf{r} - (\mathbf{r}_D - l_d \mathbf{z}_D)$, $\Delta X = \Delta \mathbf{r}^T \mathbf{R}_D \mathbf{e}_1$, $\Delta Y = \Delta \mathbf{r}^T \mathbf{R}_D \mathbf{e}_2$, and $r_{xy} = \|\Delta X, \Delta Y\|^T$. The attitude and angular-velocity errors are evaluated using the nominal quantities reconstructed from the planned trajectory through the differential-flatness transformation. The relative attitude and angular velocity are further shown in Figure 3.

Table 3. Comparison of relative motion states between the helicopter and the deck at the planned touchdown instant with and without GFLCs.

Terminal-State Error	Terminal-State Matching	
	With GFLCs Mean $\pm$ SD	w/o GFLCs Mean $\pm$ SD
$ \Delta X $ [m]	0.000 $\pm$ 0.000	0.010 $\pm$ 0.006
$ \Delta Y $ [m]	0.000 $\pm$ 0.000	0.013 $\pm$ 0.011
r_{xy} [m]	0.000 $\pm$ 0.000	0.019 $\pm$ 0.009
$\ \mathbf{v}_n\ $ [m/s]	0.000 $\pm$ 0.000	0.000 $\pm$ 0.000
$\ \mathbf{v}_t\ $ [m/s]	0.000 $\pm$ 0.000	0.014 $\pm$ 0.005
$ \Delta \phi $ [°]	0.20 $\pm$ 0.08	2.43 $\pm$ 0.32
$ \Delta \theta $ [°]	0.01 $\pm$ 0.02	2.93 $\pm$ 0.24
ζ [°]	0.21 $\pm$ 0.08	3.82 $\pm$ 0.32
$ \Delta p $ [°/s]	0.18 $\pm$ 0.02	0.17 $\pm$ 0.12
$ \Delta q $ [°/s]	0.18 $\pm$ 0.03	0.19 $\pm$ 0.06

Each entry reports the mean $\pm$ standard deviation (SD) over 12 test scenarios. Bold values indicate the smaller mean value in each row.

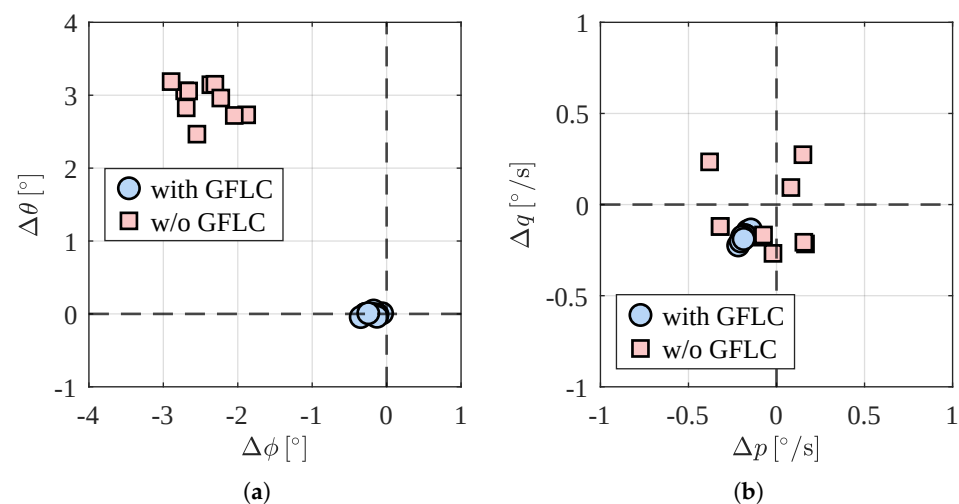


Figure 3. The relative attitude and relative body angular velocity between the helicopter and the deck at the planned touchdown instant for the planned trajectories in 12 test scenarios. (a) Relative roll angle $\Delta\phi$ and relative pitch angle $\Delta\theta$. (b) Relative roll angular velocity Δp and relative pitch angular velocity Δq .

The results in Table 3 and Figure 3 show that the terminal component of the GFLCs significantly improves the motion-state consistency between the helicopter and the deck at the planned touchdown instant. With GFLCs, the relative position and velocity errors are eliminated at the planning level. Without GFLCs, small but nonzero position and tangential-velocity errors remain because the deck attitude and rigid-body rotational kinematics are not explicitly considered in the terminal position and velocity conditions.

The improvement is more pronounced in the attitude variables. Without GFLCs, the relative roll and pitch errors are one to two orders of magnitude larger than those obtained with GFLCs, and the corresponding variability across different landing scenarios is also much greater. The relative angular-velocity errors remain small in both cases because the deck angular velocity is already limited during the quiescent period. Nevertheless, the results still indicate that the refined terminal constraints improve the consistency and repeatability of terminal motion-state matching.

The second part of the ablation study examines the final landing process before the planned touchdown instant. Figure 4 compares the trajectories with and without GFLCs for Case 3 in Section 4.1. The attitude quantities shown in the figure are again reconstructed from the planned trajectory through the differential-flatness transformation. In addition to the planned touchdown instant, the first geometric-contact instant is identified using the landing-gear/deck-plane contact condition and marked by triangle symbols.

As shown in Figure 4, the trajectories with and without GFLCs remain similar before the helicopter enters the near-deck region. It should be noted that ζ represents the relative tilt angle between the helicopter body z-axis and the deck normal, rather than an individual roll, pitch, or yaw angle. Its relatively large values during the earlier part of the maneuver result from the combined effects of the helicopter maneuvering attitude and the moving-deck attitude. The GFLCs are not intended to maintain helicopter–deck attitude alignment throughout the entire approach; instead, they progressively regulate the relative tilt after the helicopter enters the near-deck region. After entering this region, however, the trajectories with and without GFLCs exhibit markedly different behaviors. With GFLCs, the helicopter first adjusts its relative motion to approach the deck more consistently with the deck-normal direction, while its attitude is progressively aligned with the deck attitude. This behavior

reflects the effect of the near-deck contact-avoidance, tangential-velocity, and relative-tilt constraints in Equations (12o)–(12q).

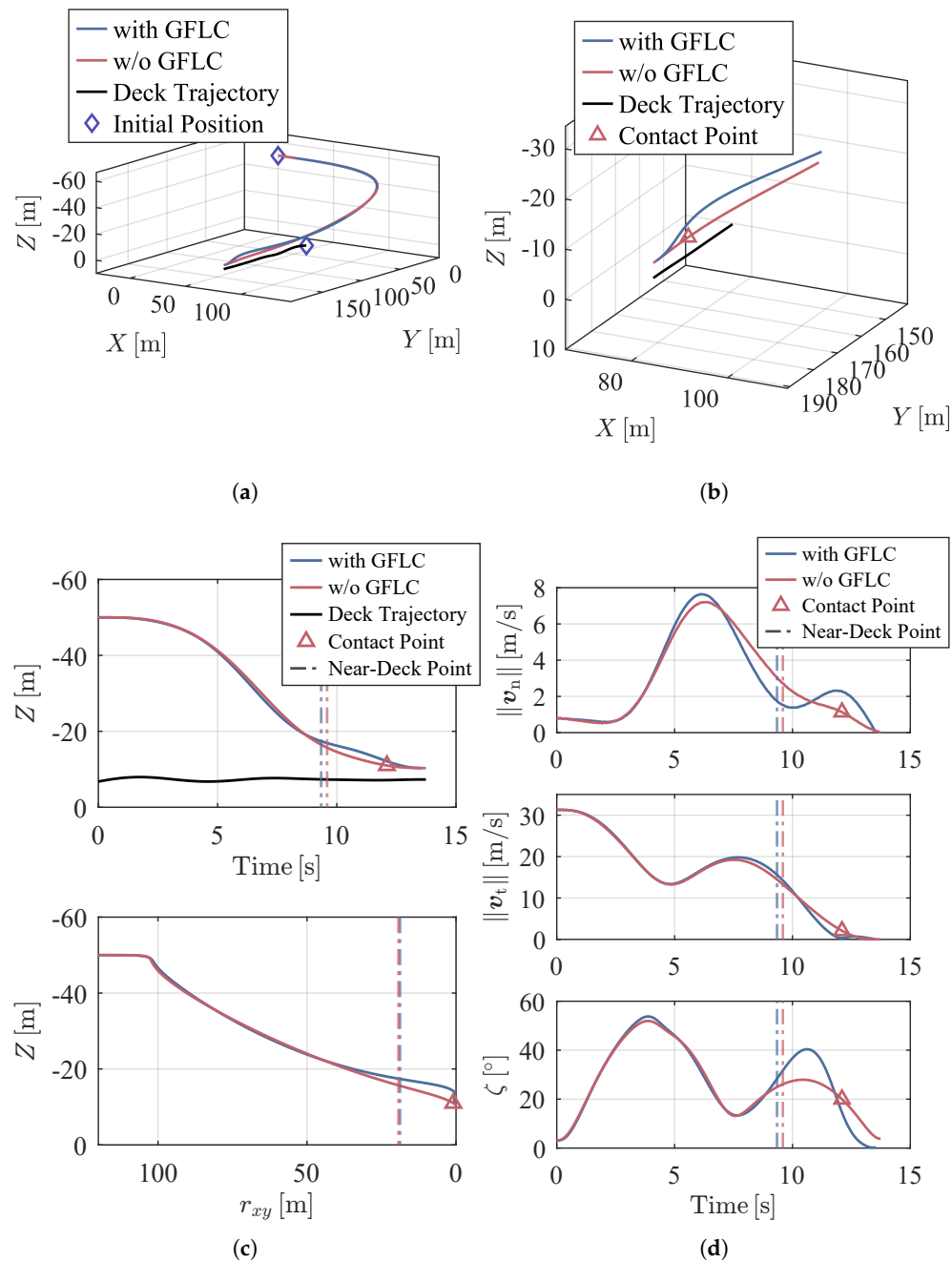


Figure 4. Optimized helicopter shipboard landing trajectories with and without GFLCs. The triangle markers on the curves denote the first geometric-contact instant identified by the landing-gear/deck-plane contact condition. The blue and red vertical dashed lines indicate the near-deck activation points for the trajectories with and without GFLCs, respectively. (a) Ground tracks r . (b) Ground tracks r in the near-deck region. (c) Altitude $Z(t)$ and $Z(r_{xy})$ profiles. (d) Relative deck-normal velocity $\|v_n\|$, relative deck-tangential velocity $\|v_t\|$, and relative tilt angle ζ profiles.

Without GFLCs, the trajectory does not explicitly regulate the near-deck relative geometry and relative motion. As a result, the helicopter approaches the deck in a more tangential manner and the first geometric contact occurs before the planned touchdown instant. At this premature-contact instant, the helicopter still has a planar position error of $r_{xy} = 0.900$ m, a relative tilt angle of $\zeta = 19.97^\circ$, a relative deck-normal velocity of

$\|v_n\| = 1.136 \text{ m/s}$, and a relative deck-tangential velocity of $\|v_t\| = 2.083 \text{ m/s}$. These values indicate an undesirable near-deck contact condition from a geometric and kinematic perspective. By contrast, no premature contact is detected before the planned touchdown instant when GFLCs are imposed in this representative case.

Overall, the ablation study demonstrates the twofold role of the proposed GFLCs. The terminal motion-state matching constraints improve the consistency of the helicopter and deck states at the planned touchdown instant, especially in relative attitude. The near-deck constraints further shape the final landing process by regulating the landing-gear/deck geometry, tangential relative velocity, and relative tilt angle before touchdown. These results demonstrate the importance of the proposed GFLCs for generating single-phase direct landing trajectories with improved terminal matching and pre-touchdown geometric and kinematic behavior.

4.3. Sensitivity Analysis of GFLC Parameters

A one-at-a-time sensitivity analysis was conducted using the representative Sea State 5, Green 30, Case 3 scenario. The trajectory discretization, initialization, solver settings, and all parameters not under examination were kept unchanged. The near-deck activation distance $\bar{d}$, tightening range σ_t , tightening magnitudes $\Delta v_{t_{\max}}$ and $\Delta \zeta_{\max}$, and airwake spatial scales were varied by $\pm 20\%$ from their nominal values. The three GFLC path-penalty weights were simultaneously scaled by factors of 0.5 and 2.0. In addition, the quiescent-period window was contracted about its midpoint to 80% of its nominal width. Together with the baseline case, 12 planning runs were evaluated. The activation-transition parameter ε was held fixed when $\bar{d}$ was varied.

As shown in Table 4, 10 of the 12 runs successfully generated trajectories satisfying the GFLC checks. All tested $\pm 20\%$ variations of the activation distance, tightening range, tightening magnitudes, and airwake spatial scales were successful. Across these successful perturbations, the optimized flight time differed from the baseline by no more than 0.216 s, the reported minimum geometric margin remained positive between 0.115 and 0.311 m, and no premature contact was detected. In the present representative scenario, varying $\bar{d}$ by $\pm 20\%$ did not change the resulting trajectory metrics, indicating that the result was not sensitive to the exact activation-distance value within this range.

Table 4. One-at-a-time sensitivity analysis of the GFLC-related parameters.

Parameter Variation	Successful Runs	Optimized Flight Time [s]	Minimum Geometric Margin [m]
Baseline	1/1	13.631	0.137
Activation distance $\bar{d}$: $0.8\text{--}1.2\times$	2/2	13.631–13.631	0.137–0.137
Tightening range σ_t : $0.8\text{--}1.2\times$	2/2	13.451–13.505	0.170–0.203
Tightening magnitudes $\Delta v_{t_{\max}}$ and $\Delta \zeta_{\max}$: $0.8\text{--}1.2\times$	2/2	13.415–13.591	0.196–0.311
GFLC path-penalty weights: $0.5\text{--}2.0\times$	1/2	13.618	0.119
Airwake spatial scales: $0.8\text{--}1.2\times$	2/2	13.652–13.675	0.115–0.178
Quiescent-window width: $0.8\times$	0/1	–	–

Ranges are reported over the successful runs. A successful run returned a valid trajectory satisfying the GFLC checks. A positive geometric margin indicates separation before the planned touchdown instant. No premature contact was detected in any successful run.

The two unsuccessful settings also clarify the parameter-selection requirements. Halving the GFLC path-penalty weights did not provide sufficient enforcement of the GFLC checks, whereas doubling the weights remained successful. This indicates that an adequate penalty magnitude is necessary, while the nominal values do not need to be tuned to a

single exact value. Contracting the quiescent-period window to 80% of its nominal width also failed to produce a GFLC-compliant trajectory. For this scenario, the nominal window was [8.75, 14.00] s, whereas the contracted window was [9.275, 13.475] s; its upper bound was below the nominal optimized landing time of 13.631 s. This result illustrates that the availability of a sufficiently long quiescent window is a feasibility condition rather than an arbitrary tuning choice. Therefore, the nominal parameters are not claimed to be globally optimal or insensitive over arbitrary ranges, but the successful perturbations show that they lie within a workable parameter region for the tested scenario.

4.4. Comparison with a Time-Discretized NLP Formulation

This subsection compares the proposed method with an adapted time-discretized NLP formulation based on the differential-flatness-based shipboard landing method of Zhao et al. [4]. The comparison is intended to examine the influence of different flatness-based model formulations, trajectory representations, constraint-handling strategies, and solution pipelines on the trajectory-generation performance. It is not intended as a pure benchmark between numerical solvers.

For a fair comparison, the method of Zhao et al. [4] is adapted to the single-phase direct-landing task considered in this paper. It should be noted that the original formulation of Zhao et al. employs phase-specific state-based terminal conditions and a scalar altitude-based deck-clearance condition, rather than the GFLCs proposed in this study. Because the incremental effect of the proposed GFLCs has already been isolated through the ablation study in Section 4.2, the same GFLCs are incorporated into the adapted implementation of Zhao et al. in the present comparison. Consequently, this subsection focuses on differences in the flatness-based model formulation, trajectory representation, constraint-handling strategy, and solution pipeline, rather than on differences in the final-landing constraint set. In the adapted implementation, the attitude and angular-velocity reconstruction from the trajectory and the determination of the terminal conditions follow the differential-flatness transformation proposed by Zhao et al. [4], while the remaining task settings and constraints are kept consistent with those of the proposed method. For the trajectory comparison, the proposed method uses $M = 6$ MINCO pieces and $\kappa_i = 4$, $i = 1, 2, \dots, M$, resulting in 24 sampling intervals. The same total number of discretization intervals is used in the adapted NLP formulation.

Trajectory planning tests are conducted over the same 12 landing scenarios. Figure 5 compares the landing processes for Case 3 in Section 4.1, and Table 5 reports the relative attitude and angular velocity between the helicopter and the deck at touchdown over all 12 test scenarios.

Table 5. Comparison of relative attitude and angular velocity between the helicopter and the deck at touchdown.

Terminal-State Error	Terminal-State Matching	
	Our Method Mean $\pm$ SD	Adapted Zhao et al. (NLP) Mean $\pm$ SD
$ \Delta\phi $ [°]	0.20 $\pm$ 0.08	0.52 $\pm$ 0.14
$ \Delta\theta $ [°]	0.01 $\pm$ 0.02	1.29 $\pm$ 0.12
ζ [°]	0.21 $\pm$ 0.08	1.40 $\pm$ 0.07
$ \Delta p $ [°/s]	0.18 $\pm$ 0.02	0.24 $\pm$ 0.17
$ \Delta q $ [°/s]	0.18 $\pm$ 0.03	0.25 $\pm$ 0.15

Each entry reports the mean $\pm$ standard deviation (SD) over 12 test scenarios. Bold values indicate the smaller mean value in each row.

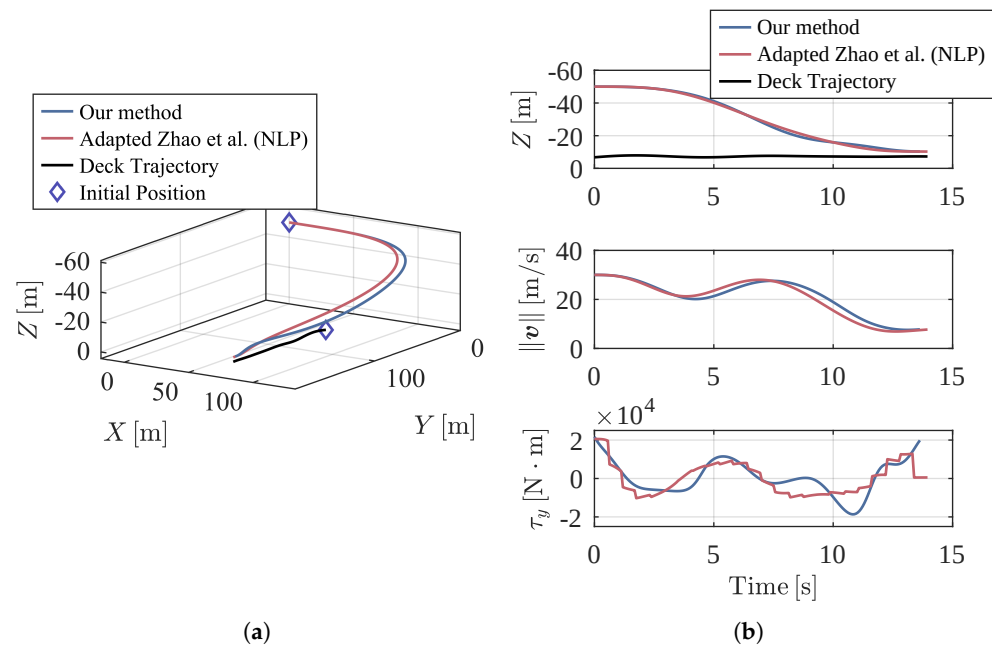


Figure 5. Optimized helicopter shipboard landing trajectories generated by different trajectory planning methods. (a) Ground tracks r . (b) Altitude Z , speed $\|v\|$, and aerodynamic-moment component τ_y profiles.

As shown in Figure 5, both methods generate single-phase landing trajectories satisfying the adopted numerical planning criteria, with similar altitude, speed, and task-duration profiles. The main difference observed in this representative case lies in the aerodynamic-moment component τ_y . Compared with the adapted time-discretized NLP formulation, the proposed MINCO-based formulation produces a more continuous τ_y profile with fewer abrupt variations. Since τ_y represents one of the reconstructed aerodynamic-moment components, this result suggests a smoother aerodynamic-load demand trend for the proposed formulation, which is favorable for trajectory executability and control-input smoothness.

The results in Table 5 show that the proposed method achieves more accurate attitude and angular-velocity matching at touchdown. The improvement is particularly evident in the pitch and relative tilt-angle errors. This difference can be largely attributed to the different flatness-based model formulations and trajectory representations used in the two methods. The simplified helicopter model adopted in the proposed method preserves more of the original model fidelity [32], making it more suitable for refined terminal-state constraint handling.

The computational efficiency of the two formulations is further compared. Both methods are implemented in MATLAB. The proposed method is formulated as an unconstrained optimization problem over the MINCO trajectory parameters and is solved using `fminunc`. The adapted method of Zhao et al. [4] is formulated as a constrained time-discretized NLP problem and is solved using `fmincon`. The mission settings, terminal conditions, constraint settings, and relevant optimization tolerances are kept consistent whenever applicable. Therefore, the comparison is made at the level of the overall planning formulation and solution pipeline, rather than as a pure benchmark between the two MATLAB optimizers.

The comparison is conducted over the same 12 landing scenarios under three discretization settings, corresponding to 24, 48, and 72 total intervals. For the proposed method, these settings are implemented with $M = 6$ and $\kappa_i \in \{4, 8, 12\}$, respectively, for all $i = 1, \dots, M$. Both methods were implemented in MATLAB R2024a and tested on a desktop computer with an Intel Core i7-14700KF 3.40 GHz processor. The computational times are summarized in Table 6.

Table 6. Comparison of optimization solve times.

Total Intervals	Optimization Solve Time [s]	
	Our Method Mean $\pm$ SD	Adapted Zhao et al. (NLP) Mean $\pm$ SD
24	0.670 $\pm$ 0.436	20.462 $\pm$ 3.920
48	0.766 $\pm$ 0.214	74.713 $\pm$ 20.968
72	1.157 $\pm$ 0.678	182.753 $\pm$ 82.694

Each entry reports the mean $\pm$ standard deviation (SD) over 12 test scenarios. Bold values indicate the smaller mean value in each row.

To eliminate the potential influence of optimizer selection, an additional controlled comparison was conducted at 24 total intervals. Both formulations were solved using `fmincon` with the SQP algorithm and identical convergence settings. The optimality, step, constraint, and function tolerances were set to 10^{-6} , 10^{-3} , 10^{-4} , and 10^{-6} , respectively, and the same maximum numbers of iterations and function evaluations were adopted. The mathematical formulations were otherwise kept unchanged: the proposed method retained the MINCO parameterization and smooth penalty objective, whereas the adapted method retained its time-discretized optimization variables and explicit nonlinear constraints.

The original end-to-end comparison in Table 6 shows that the proposed unconstrained optimization over the MINCO trajectory parameters requires substantially less computation time than the constrained time-discretized NLP formulation under all three discretization settings. Moreover, the solve time of the proposed method increases only mildly as the number of intervals grows, whereas the computational cost of the adapted NLP formulation increases markedly.

The controlled comparison in Table 7 further shows that this computational advantage remains when the numerical optimizer and convergence settings are unified. Using the same `fmincon` SQP solver, the proposed formulation required 26.08 ± 11.53 iterations and 1.265 ± 0.284 s, whereas the adapted time-discretized NLP required 82.92 ± 9.77 iterations and 30.695 ± 4.671 s. Thus, the proposed formulation remained approximately 24 times faster under the same solver settings. These results indicate that the observed computational advantage is not solely attributable to the original use of different MATLAB optimizers, but is also associated with the different trajectory-parameterization and constraint-handling structures of the two formulations. Overall, the proposed formulation improves terminal motion-state matching while substantially reducing the computational cost, supporting its potential for fast replanning within a finite ship-motion prediction window.

Table 7. Same-solver comparison at 24 total intervals.

Metric	Our Method	Adapted Zhao et al. (NLP)
Optimization iterations	26.08 $\pm$ 11.53	82.92 $\pm$ 9.77
Optimization solve time [s]	1.265 $\pm$ 0.284	30.695 $\pm$ 4.671

Each entry reports the mean $\pm$ standard deviation over the same 12 landing scenarios. Both formulations were solved using `fmincon` with the SQP algorithm and identical convergence settings. Bold values indicate the smaller mean value in each row.

4.5. Closed-Loop Validation of the Planned Trajectories

Closed-loop validation is conducted to assess whether the planned landing trajectories can be executed by the full nonlinear helicopter model under trajectory tracking control. The validation is not intended to introduce a new controller, but to examine the closed-loop executability of the trajectories generated by the proposed planning framework under the modeled conditions. The full nonlinear UH-60 model built upon Equation (1) is used, and trajectory tracking is achieved using the two-layer controller developed in our previous

work [32]. In this controller, an upper-level differential-flatness-based geometric tracking law generates the required motion-tracking commands, while a lower-level linear dynamic inversion (LDI) controller maps these commands to the physical rotor control inputs.

The 12 planned trajectories generated in Section 4.1 are used for closed-loop tests. Two validation settings are considered: one without aerodynamic-environment effects (w/o AE) and the other with aerodynamic-environment effects (with AE), including ship airwake and ground effect. The case without AE provides a clean validation of trajectory executability, whereas the case with AE evaluates tracking performance under additional aerodynamic disturbances. In both settings, all 12 closed-loop simulations are completed successfully, and the helicopter reaches the planned touchdown condition in each case, resulting in a success rate of 100%.

The trajectory-tracking errors are summarized in Table 8. The results show that accurate trajectory tracking is achieved in both validation settings. As expected, the AE case generally leads to larger position-tracking errors because of the additional aerodynamic disturbances, but the overall tracking errors remain limited.

Table 8. Trajectory-tracking errors in closed-loop shipboard landing validation.

Tracking Error	Tracking Performance	
	w/o AE Mean $\pm$ SD	With AE Mean $\pm$ SD
RMS E_p [m]	0.477 $\pm$ 0.068	0.609 $\pm$ 0.063
RMS E_ψ [$^\circ$]	0.33 $\pm$ 0.18	0.36 $\pm$ 0.14
max E_p [m]	0.775 $\pm$ 0.123	1.195 $\pm$ 0.124
max E_ψ [$^\circ$]	0.77 $\pm$ 0.42	0.84 $\pm$ 0.33

$E_p = \|\mathbf{r}_{\text{ref}} - \mathbf{r}\|$ denotes the position tracking error. $E_\psi = |\psi_{\text{ref}} - \psi|$ denotes the yaw tracking error. Each entry reports the mean $\pm$ standard deviation (SD) over 12 test scenarios. Bold values indicate the smaller mean value in each row.

The actual relative motion states between the helicopter and the deck at the planned touchdown instant are summarized in Table 9. Different from the planning-stage results, where the attitude and angular velocity are reconstructed from the planned trajectory through the differential-flatness transformation, the quantities reported here are obtained from the actual closed-loop response of the full nonlinear helicopter model.

Table 9. Relative motion states between the helicopter and the deck at the planned touchdown instant in closed-loop validation.

Terminal-State Error	Terminal-State Matching	
	w/o AE Mean $\pm$ SD	With AE Mean $\pm$ SD
$ \Delta X $ [m]	0.137 $\pm$ 0.053	0.198 $\pm$ 0.073
$ \Delta Y $ [m]	0.061 $\pm$ 0.031	0.049 $\pm$ 0.028
r_{xy} [m]	0.156 $\pm$ 0.042	0.207 $\pm$ 0.069
$\ \mathbf{v}_n\ $ [m/s]	0.134 $\pm$ 0.097	0.170 $\pm$ 0.070
$\ \mathbf{v}_t\ $ [m/s]	0.163 $\pm$ 0.045	0.314 $\pm$ 0.111
$ \Delta\phi $ [$^\circ$]	0.92 $\pm$ 0.30	1.25 $\pm$ 0.57
$ \Delta\theta $ [$^\circ$]	1.03 $\pm$ 0.60	1.21 $\pm$ 0.85
ζ [$^\circ$]	1.42 $\pm$ 0.55	1.84 $\pm$ 0.81
$ \Delta p $ [$^\circ$ /s]	0.94 $\pm$ 0.65	0.64 $\pm$ 0.45
$ \Delta q $ [$^\circ$ /s]	2.88 $\pm$ 1.75	3.35 $\pm$ 2.18

Each entry reports the mean $\pm$ standard deviation (SD) over 12 test scenarios. Bold values indicate the smaller mean value in each row.

The results show that the planned trajectories with GFLCs lead to accurate terminal motion-state matching in both validation settings. The closed-loop errors are slightly larger

than their planning-stage counterparts in Table 3, mainly because of model simplification, tracking-control error, and, in the with-AE case, additional aerodynamic disturbances. Nevertheless, the overall terminal errors remain small, indicating that the GFLC-based planned trajectories can be effectively executed by the full nonlinear helicopter model.

Figure 6 further illustrates the closed-loop execution of the planned trajectory for Case 3 with AE. The actual attitude response follows the nominal attitude reconstructed from the planned trajectory with limited deviations, indicating that the differential-flatness-based reconstruction provides a feasible reference for the full nonlinear helicopter model. The relatively large roll and pitch variations have the same physical origin as the corresponding planned profiles in Figure 2, namely the rapid deceleration and direction adjustment required by the single-phase direct-landing maneuver. The angle exceeding 100° is the yaw angle ψ , representing the heading change rather than the helicopter body tilt. The small yaw-tracking errors summarized in Table 8 further show that this large heading variation can be accurately tracked by the full nonlinear helicopter model. Meanwhile, the corresponding physical control inputs remain bounded throughout the simulated maneuver. These results further support the closed-loop executability of the planned trajectory under the adopted full nonlinear helicopter model and tracking-controller settings.

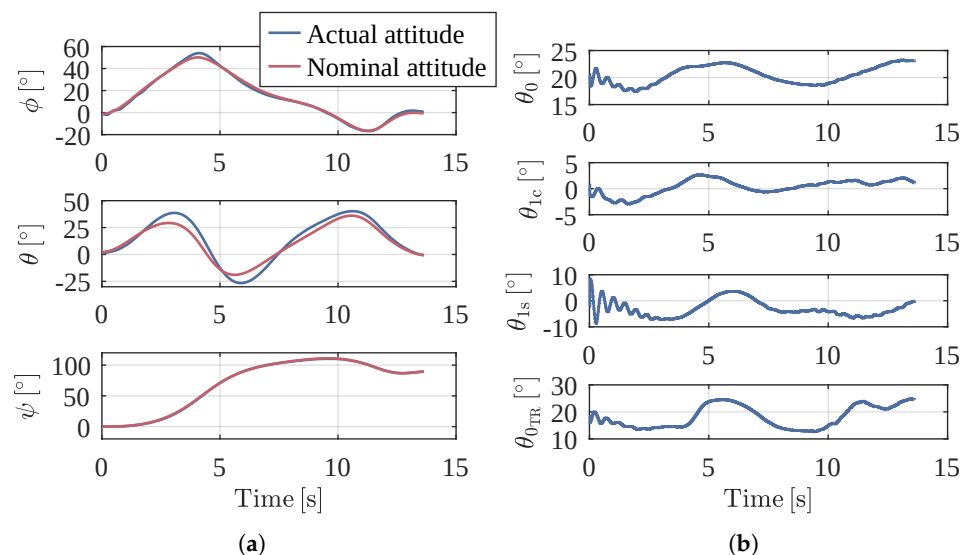


Figure 6. Closed-loop execution results for Case 3 with aerodynamic-environment effects. (a) Nominal and actual attitude profiles. (b) Control-input profiles.

To further examine whether the benefits of GFLCs remain in closed-loop execution, an additional validation is conducted using trajectories planned without GFLCs. The same ablation setting as in Section 4.2 is adopted. To isolate the influence of GFLCs from aerodynamic disturbances, only the case without AE is considered in this comparison.

Table 10 compares the closed-loop relative motion states at the planned touchdown instant with and without GFLCs. Both settings achieve small position and velocity errors at the planned terminal instant. However, the trajectories planned with GFLCs generally provide smaller planar position error, deck-normal relative velocity, relative tangential velocity, relative attitude error, and overall angular-velocity error. The most pronounced improvements appear in the pitch error and relative tilt angle, indicating that the terminal attitude-matching component of the GFLCs remains effective after closed-loop tracking.

Table 10. Comparison of relative motion states between the helicopter and the deck at the planned touchdown instant with and without GFLCs in closed-loop validation.

Terminal-State Error	Terminal-State Matching	
	With GFLCs Mean $\pm$ SD	w/o GFLCs Mean $\pm$ SD
$ \Delta X $ [m]	0.137 $\pm$ 0.053	0.198 $\pm$ 0.055
$ \Delta Y $ [m]	0.061 $\pm$ 0.031	0.030 $\pm$ 0.019
r_{xy} [m]	0.156 $\pm$ 0.042	0.201 $\pm$ 0.054
$\ v_n\ $ [m/s]	0.134 $\pm$ 0.097	0.256 $\pm$ 0.053
$\ v_t\ $ [m/s]	0.163 $\pm$ 0.045	0.180 $\pm$ 0.104
$ \Delta\phi $ [°]	0.92 $\pm$ 0.30	1.48 $\pm$ 0.68
$ \Delta\theta $ [°]	1.03 $\pm$ 0.60	4.67 $\pm$ 1.51
ζ [°]	1.42 $\pm$ 0.55	4.93 $\pm$ 1.57
$ \Delta p $ [°/s]	0.94 $\pm$ 0.65	0.86 $\pm$ 0.43
$ \Delta q $ [°/s]	2.88 $\pm$ 1.75	3.59 $\pm$ 1.58

Each entry reports the mean $\pm$ standard deviation (SD) over 12 test scenarios. Bold values indicate the smaller mean value in each row.

However, terminal-state matching at the planned touchdown instant alone does not fully characterize the pre-touchdown geometric and kinematic behavior of the landing process. As discussed in Section 4.2, the first geometric-contact instant must also be examined. Among the 12 closed-loop tests using trajectories planned without GFLCs, premature contact is detected in all cases, meaning that the first geometric-contact instant occurs before the planned touchdown instant. The corresponding relative motion states at the first geometric-contact instant are summarized in Table 11.

Table 11. Relative motion states at the first geometric-contact instant for the premature-contact cases without GFLCs.

Metric	Value Mean $\pm$ SD
r_{xy} [m]	1.375 $\pm$ 1.044
ζ [°]	18.96 $\pm$ 10.35
$\ v_n\ $ [m/s]	1.756 $\pm$ 0.407
$\ v_t\ $ [m/s]	2.662 $\pm$ 1.752

Each entry reports the mean $\pm$ standard deviation (SD) over the 12 closed-loop tests without GFLCs, in which premature contact is detected before the planned touchdown instant.

The results show that trajectories planned without GFLCs may lead to premature contact with relatively large planar position error, relative tilt mismatch, and non-negligible deck-normal and tangential relative velocities. These results indicate an undesirable near-deck contact condition from a geometric and kinematic perspective. Therefore, the closed-loop ablation results further confirm that GFLCs are important not only for terminal-state matching at the planned touchdown instant, but also for regulating the final landing process before touchdown.

Overall, the closed-loop validation supports the executability of the planned trajectories under the adopted full nonlinear helicopter model and tracking-controller settings. The results show that the proposed planning framework can generate trajectories that are not only trackable, but also exhibit improved terminal-state matching and pre-touchdown geometric and kinematic behavior during closed-loop execution.

Although the closed-loop simulations employ the full nonlinear helicopter model and map the tracking commands to physical rotor pitch inputs, the present validation does not explicitly model or evaluate detailed actuator dynamics, rate limits, or saturation margins. The baseline closed-loop tests assume that the prescribed deck-motion prediction

is available to the planner and controller. Section 4.7 provides only a bounded timing-offset sensitivity assessment; state-estimation errors, broader deck-motion prediction uncertainty beyond the prescribed timing-offset tests, and control-system delays remain outside the current validation. Therefore, the reported closed-loop results demonstrate the trackability and executability of the planned trajectories under the modeled conditions, rather than a complete assessment or certification of operational landing safety.

4.6. Monte Carlo Evaluation

To statistically assess the proposed planner under randomized initial conditions and the closed-loop executability of the generated trajectories under execution-stage perturbations, two Monte Carlo studies were conducted. The first study examines whether the proposed planner can consistently generate landing trajectories satisfying the adopted numerical criteria under randomized initial helicopter states and deck configurations, whereas the second study evaluates whether representative planned trajectories can be successfully tracked and executed by the full nonlinear UH-60 model under execution-stage perturbations.

At the planning level, the 12 representative scenarios introduced in Section 4.1 were used as the baseline cases. Each scenario was repeated 20 times, resulting in a total of 240 randomized planning trials. To represent variations in the initial conditions, mutually independent uniformly distributed perturbations were applied to the initial helicopter position, speed, and heading, as well as to the initial deck position and heading:

$$\begin{aligned}\delta X_0, \delta Y_0 &\sim \mathcal{U}(-2, 2) \text{ m}, & \delta Z_0 &\sim \mathcal{U}(-0.6, 0.6) \text{ m}, \\ \delta \|v_0\| &\sim \mathcal{U}(-0.4, 0.4) \text{ m/s}, & \delta \psi_0 &\sim \mathcal{U}(-1, 1)^\circ, \\ \delta X_{D_0}, \delta Y_{D_0} &\sim \mathcal{U}(-2, 2) \text{ m}, & \delta \psi_{D_0} &\sim \mathcal{U}(-1, 1)^\circ.\end{aligned}$$

Since the continuous-time constraints are incorporated into the objective function through smooth penalty terms, a trial was considered to satisfy the adopted numerical penalty criterion when the constraint penalty satisfied $I_G < 100$. Premature geometric contact before the planned touchdown instant was additionally examined using 25 uniformly spaced sampled nodes along each trajectory. A trial was classified as a successful planning run only if the optimization converged, the penalty criterion was satisfied, and no sampled premature contact was detected. The statistical results of the planning-level Monte Carlo trials are summarized in Table 12.

Table 12. Planning-level Monte Carlo results under randomized initial conditions.

Metric	Value
Number of planning trials	240
Converged optimization runs	240 (100.0%)
Runs satisfying the penalty criterion	223 (92.9%)
Runs with sampled premature contact	14 (5.8%)
Successful planning runs	214 (89.2%)
Optimization solve time [s]	1.107 ± 0.845 (5.214)
Terminal relative tilt $\zeta(T)$ [°]	0.188 ± 0.080 (0.351)

Continuous-valued results are reported as the mean $\pm$ standard deviation (SD), with the maximum value given in parentheses.

As shown in Table 12, all 240 optimization runs converged, indicating consistent numerical convergence of the proposed formulation under the considered randomized conditions. Among these runs, 223 satisfied the adopted penalty criterion, while 226 passed the original sampled-contact check. Overall, 214 trajectories, corresponding to 89.2% of all trials, satisfied both criteria and were therefore classified as successful planning outcomes.

Optimizer convergence and planning acceptance are distinct in this evaluation. Among the 26 candidates that did not satisfy the combined planning-success criterion, 9 failed only the sampled-contact check, 12 failed only the penalty criterion, and 5 failed both checks. For the 17 penalty-criterion failures, I_G ranged from 109.2 to 3.14×10^6 , with a median of 373.2. For the 14 sampled-contact failures, the maximum positive landing-gear/deck residual at the original sampling nodes had a mean of 0.494 mm, a median of 0.378 mm, and a maximum of 1.328 mm. Therefore, the reported 89.2% represents the first-attempt acceptance rate under the adopted numerical screening criteria, rather than the numerical convergence rate of the optimizer.

In the intended planning pipeline, optimizer convergence produces only a candidate trajectory. A candidate that fails either the adopted post-optimization penalty check or the sampled geometric check would be rejected and would not be sent to the tracking controller; the optimization would then be repeated from a reinitialized guess. If no acceptable trajectory were obtained, the landing maneuver would be deferred until an updated deck-motion prediction provided a new admissible planning window. Repeated recovery attempts are not included in the reported 89.2% first-attempt acceptance rate.

The optimization required an average solve time of 1.107 s, with a maximum of 5.214 s, demonstrating that the proposed planner maintained a relatively low computational cost under the randomized conditions. The terminal relative tilt remained small, with a mean value of 0.188° and a maximum value of 0.351° , indicating close attitude alignment between the helicopter and the moving deck at the planned touchdown instant.

To examine constraint behavior between the optimization quadrature nodes, the same 240 Monte Carlo trajectories were independently reevaluated on a dense grid containing 2400 uniform intervals and 2401 nodes over $[0, T]$, without replanning or changing any planning parameters. This grid is 100 times denser than the 24 integration intervals used during optimization and corresponds to an equivalent checking frequency of 171.4–218.5 Hz, with a mean of 199.7 Hz. For the landing-gear/deck constraint, the maximum pre-touchdown residual was defined as $g_{\max}^{\text{pre}} = \max_{t \in \mathcal{T}_{\text{dense}}^{\text{pre}}} g(t)$, where $\mathcal{T}_{\text{dense}}^{\text{pre}}$ contains the 2400 dense-grid nodes preceding the planned touchdown instant; the terminal node was excluded from this maximum.

Under the strict numerical condition $g_{\max}^{\text{pre}} \leq 10^{-9}$ m, positive residuals were detected in all 240 trajectories. Thus, the dense-grid results do not establish exact sampled-time non-penetration. The observed residual magnitudes had a mean, median, 95th percentile, and maximum of 10.680 mm, 10.640 mm, 17.514 mm, and 19.222 mm, respectively. The maximum value corresponds to approximately 0.64% of the landing-gear-disc radius $\bar{r} = 3$ m.

For all 240 trajectories, the maximum residual occurred in the sixth and final MINCO segment. The mean and median remaining times to the planned touchdown instant were only 0.0349 s and 0.0143 s, respectively. The residual-threshold statistics in Table 13 are therefore reported to quantify the scale and localization of the sampled constraint residuals. They do not replace the theoretical non-penetration condition $g(t) \leq 0$, provide a continuous-time guarantee, or constitute a detailed physical-contact safety certification.

The remaining planning constraints were evaluated on the same dense grid. Positive residuals were detected for the attitude-surrogate constraint in 5 of 240 trajectories, the upper aerodynamic-acceleration bound in 40 trajectories, the angular-acceleration-surrogate constraint in 5 trajectories, and the relative-tilt constraint in 35 trajectories. The corresponding maximum relative excesses, normalized by the relevant bounds, were 0.47%, 1.29%, 4.82%, and 6.53%, respectively. No positive residual was detected for the velocity, angular-velocity-surrogate, yaw-rate, lower aerodynamic-acceleration, yaw-acceleration, sea-surface, or relative tangential-velocity constraints. These results further show that

the finite-node smooth-penalty formulation provides approximate numerical constraint enforcement rather than exact continuous-time feasibility.

Table 13. Dense sampled-time post-processing of the original planning-level Monte Carlo trajectories.

Metric	Value
Dense checking grid	2400 intervals (2401 nodes)
Equivalent checking frequency [Hz]	199.7 (171.4–218.5)
Mean $g_{\max}^{\text{pre}}$ [mm]	10.680
Median $g_{\max}^{\text{pre}}$ [mm]	10.640
95th percentile of $g_{\max}^{\text{pre}}$ [mm]	17.514
Maximum $g_{\max}^{\text{pre}}$ [mm]	19.222
Trajectories with $g_{\max}^{\text{pre}} \leq 5$ mm	29 (12.1%)
Trajectories with $g_{\max}^{\text{pre}} \leq 10$ mm	111 (46.3%)
Trajectories with $g_{\max}^{\text{pre}} \leq 15$ mm	194 (80.8%)
Trajectories with $g_{\max}^{\text{pre}} \leq 20$ mm	240 (100.0%)
Maximum residual located in the final MINCO segment	240 (100.0%)
Time to touchdown at maximum residual [s]	0.0349/0.0143/0.3506

The equivalent checking frequency is reported as the mean, with the range in parentheses. The time to touchdown is reported as the mean/median/maximum. The residual thresholds are provided only as a numerical sensitivity description and do not represent demonstrated physical safety margins.

At the closed-loop level, the three planned trajectories corresponding to Cases 1–3 under Sea State 5 and Green 30 were selected for Monte Carlo validation. This condition represents the most severe ship-motion and aerodynamic-environment condition considered in this study. Each trajectory was executed 20 times using the full nonlinear UH-60 model, resulting in 60 closed-loop trials. The reference trajectories were kept fixed throughout the trials, and no replanning was performed. The ship-airwake and ground-effect models were enabled in all trials. For each case, the time history of the baseline airwake was retained, while its intensity was multiplied by a randomly sampled scaling factor.

Mutually independent uniformly distributed perturbations were applied to the actual initial helicopter state relative to the corresponding reference initial state:

$$\begin{aligned}
 \delta X_0, \delta Y_0, \delta Z_0 &\sim \mathcal{U}(-0.30, 0.30) \text{ m}, \\
 \delta v_{x_0}, \delta v_{y_0}, \delta v_{z_0} &\sim \mathcal{U}(-0.20, 0.20) \text{ m/s}, \\
 \delta \phi_0, \delta \theta_0, \delta \psi_0 &\sim \mathcal{U}(-1, 1)^\circ, \\
 \delta p_0, \delta q_0, \delta r_0 &\sim \mathcal{U}(-1, 1)^\circ/\text{s}, \\
 \lambda_w &\sim \mathcal{U}(0.8, 1.2),
 \end{aligned}$$

where the airwake velocity used in each trial was given by $v_w^{\text{MC}}(t) = \lambda_w v_w(t)$. A premature-contact check was also performed. A trial was classified as a successful closed-loop execution if the simulation was completed and no premature contact occurred before the planned touchdown instant. The closed-loop tracking and touchdown results are summarized in Table 14.

As shown in Table 14, all 60 closed-loop trials were completed without premature contact, corresponding to an observed successful execution rate of 100.0%. Despite the randomized initial-state errors and the airwake-intensity variations, the mean RMS position- and yaw-tracking errors remained limited to 0.453 m and 0.337° , respectively. The global maximum position- and yaw-tracking errors over all trials were 0.906 m and 3.904° , respectively.

Table 14. Closed-loop Monte Carlo results under randomized execution-stage perturbations.

Metric	Value
Number of closed-loop trials	60
Successful closed-loop executions	60 (100.0%)
Runs with premature contact	0 (0.0%)
RMS E_p [m]	0.453 ± 0.062
max E_p [m]	0.729 ± 0.114 (0.906)
RMS E_ψ [°]	0.337 ± 0.145
max E_ψ [°]	0.972 ± 0.745 (3.904)
Touchdown r_{xy} [m]	0.153 ± 0.049 (0.221)
Touchdown $\ v_n\ $ [m/s]	0.089 ± 0.046 (0.165)
Touchdown $\ v_t\ $ [m/s]	0.158 ± 0.040 (0.216)
Touchdown ζ [°]	1.489 ± 0.272 (1.874)

Continuous-valued results are reported as the mean $\pm$ standard deviation (SD), with the maximum value over all 60 trials given in parentheses where applicable.

At the planned touchdown instant, the planar position error, deck-normal relative velocity, deck-tangential relative velocity, and relative tilt angle remained limited across all trials. Their maximum values were 0.221 m, 0.165 m/s, 0.216 m/s, and 1.874° , respectively. These statistical results further indicate that the representative planned trajectories remain trackable and executable by the full nonlinear helicopter model under the considered initial-state and airwake-intensity perturbations.

Overall, the planning- and execution-level Monte Carlo studies, together with the dense sampled-time post-processing, provide complementary statistical evidence. At the planning level, all 240 optimization runs converged, and 214 trajectories (89.2%) satisfied the original combined acceptance criterion on the first attempt. The dense-grid analysis further quantified the residuals associated with finite-node constraint enforcement and showed that independent feasibility screening remains necessary before execution. At the execution level, all 60 representative closed-loop trials were completed without premature contact under the considered perturbations. These results characterize the numerical behavior of the planning formulation and the closed-loop executability of the representative planned trajectories within the tested perturbation ranges, without constituting a continuous-time feasibility or robust-safety guarantee.

4.7. Closed-Loop Sensitivity to Prediction and Controller-Tuning Variations

Additional closed-loop tests were conducted to examine whether the executability of the planned trajectories depends critically on exact deck-motion prediction and a single precisely tuned set of tracking gains. The tests used the three Sea State 5, Green 30 scenarios considered in the closed-loop Monte Carlo study. The nominal planned trajectories were retained without replanning. To introduce deck-motion prediction mismatch, the complete deck-state trajectory used during closed-loop execution was shifted according to $x_D^{\text{actual}}(t) = x_D^{\text{predicted}}(t + \Delta t_D)$, with $\Delta t_D = -0.2$ s and $+0.2$ s. The position, velocity, attitude, and angular-velocity components of the deck state were shifted consistently. No prediction-error compensation or online replanning was applied.

The controller-tuning tests retained the nominal trajectories, helicopter model, aerodynamic-environment models, and lower-level LDI settings. Only the active high-level position- and velocity-feedback gains, denoted here by P and D , were varied. Their nominal values, $P = D = 2.0$, were uniformly scaled by factors of 0.9 and 1.1. The resulting configurations therefore used $P = D = 1.8$ and $P = D = 2.2$, respectively. The test settings and the resulting trajectory-tracking and terminal relative-motion metrics are summarized in Table 15.

Table 15. Closed-loop sensitivity to deck-motion prediction timing mismatch and high-level tracking-gain variations.

Test Setting	RMS E_p [m]	r_{xy} [m]	$\ v_n\ $ [m/s]	$\ v_t\ $ [m/s]	ζ [°]
Nominal	0.447 ± 0.075	0.153 ± 0.059	0.089 ± 0.051	0.157 ± 0.049	1.48 ± 0.32
Deck timing offset, -0.2 s	0.446 ± 0.075	1.684 ± 0.075	0.102 ± 0.069	0.156 ± 0.074	1.43 ± 0.34
Deck timing offset, $+0.2$ s	0.444 ± 0.073	1.437 ± 0.071	0.103 ± 0.028	0.159 ± 0.024	1.53 ± 0.28
High-level gains, $0.9\times$	0.500 ± 0.083	0.170 ± 0.066	0.105 ± 0.060	0.176 ± 0.056	1.46 ± 0.28
High-level gains, $1.1\times$	0.402 ± 0.064	0.138 ± 0.053	0.078 ± 0.045	0.143 ± 0.041	1.52 ± 0.40

Each entry reports the mean $\pm$ standard deviation over three Sea State 5, Green 30 scenarios. All perturbed simulations were completed without premature geometric contact. The terminal relative states are evaluated with respect to the actual deck motion used during closed-loop execution.

All 12 perturbed closed-loop simulations were completed without premature geometric contact. Under the $\pm 10\%$ high-level feedback-gain variations, the terminal relative-motion metrics remained close to their nominal values. The RMS position-tracking error varied from 0.402 to 0.500 m, compared with 0.447 m under the nominal tuning, while the mean terminal horizontal relative-position error remained between 0.138 and 0.170 m. These results indicate that the closed-loop executability of the planned trajectories in the tested scenarios is not restricted to one exactly tuned set of high-level feedback gains.

The deck-motion timing mismatch produced a different effect. The RMS trajectory-tracking error remained nearly unchanged at 0.444–0.446 m, indicating that the helicopter continued to follow the nominal reference trajectory. However, the mean terminal horizontal relative-position error increased from 0.153 m in the nominal cases to 1.437–1.684 m under the ± 0.2 s timing offsets. The terminal normal velocity, tangential velocity, and relative tilt remained close to the nominal ranges. This distinction shows that accurate reference-trajectory tracking does not by itself compensate for an error in the predicted deck position. Therefore, the tests support closed-loop completion under the prescribed moderate timing mismatch, but also demonstrate that accurate terminal horizontal alignment remains sensitive to deck-motion prediction accuracy.

These tests constitute a bounded sensitivity assessment rather than a proof of robust stability or robust trajectory planning. The current framework does not explicitly compensate for deck-motion prediction errors and does not perform online replanning during execution.

5. Conclusions

In this paper, a differential-flatness-based trajectory planning method was proposed for single-phase direct helicopter shipboard landing from a cruising state in the vicinity of a moving ship. By exploiting the differential flatness of a simplified helicopter dynamic model, the landing trajectory was planned in the flat-output space and parameterized using MINCO. Together with an unconstrained reformulation, the proposed framework enables efficient generation of single-phase direct landing trajectories under the adopted numerical planning criteria without relying on a predefined phase decomposition of the recovery maneuver. A set of geometry-aware final-landing constraints was further developed to account explicitly for the relative pose, relative motion, and near-deck geometry between the helicopter and the moving deck. The GFLCs comprise terminal motion-state matching constraints and near-deck constraints on the landing-gear/deck geometry, relative tangential velocity, and relative tilt angle, thereby improving terminal consistency and pre-touchdown geometric and kinematic regulation.

In the 12 representative landing scenarios, the proposed planner successfully generated single-phase direct landing trajectories satisfying the adopted numerical planning crite-

ria. The ablation study demonstrated that the GFLCs improved helicopter–deck motion-state consistency at the planned touchdown instant and regulated the relative geometry and motion before touchdown. In the representative one-at-a-time parameter-sensitivity study, 10 of the 12 tested settings produced trajectories satisfying the GFLC checks. All tested $\pm 20\%$ variations of the activation distance, tightening parameters, and airwake spatial scales were successful, whereas the unsuccessful half-weight and 80%-window cases highlighted the need for sufficient penalty enforcement and adequate quiescent-window duration. Compared with the adapted time-discretized NLP formulation, the proposed method achieved more accurate terminal motion-state matching and substantially lower optimization solve time. In the controlled comparison using the same `fmincon` SQP solver and convergence settings, the proposed formulation remained approximately 24 times faster, indicating that the computational advantage was not solely attributable to the original use of different MATLAB optimizers.

Closed-loop validation using the full nonlinear UH-60 model, with and without aerodynamic-environment effects, supported the executability of the planned trajectories under the modeled conditions and the effectiveness of the GFLCs. At the planning level, all 240 randomized Monte Carlo runs converged, and 214 runs, corresponding to 89.2%, satisfied the combined planning-success criterion. An independent 2401-node post-processing check further quantified the residuals associated with finite sampled-time penalty enforcement. The maximum pre-touchdown landing-gear/deck residual was 19.222 mm, with the maxima of all trajectories located in the final MINCO segment. The same check detected limited positive residuals for four additional planning constraints, with the largest relative excess equal to 6.53%. These findings underscore the need for independent feasibility screening, followed by replanning or landing deferral when necessary, before a converged candidate is executed. At the execution level, all 60 randomized closed-loop trials were completed without premature contact. The planned trajectories also remained executable under $\pm 10\%$ variations of the active high-level position- and velocity-feedback gains in the tested scenarios. Under deck-motion timing offsets of ± 0.2 s, all simulations were completed without premature contact and the reference-trajectory tracking errors remained close to the nominal results; however, the terminal horizontal relative-position error increased markedly. This result highlights the importance of deck-motion prediction accuracy and motivates prediction-error-aware or online-replanning extensions. These tests provide a bounded sensitivity assessment rather than a robust-stability guarantee or an online prediction-error-compensation capability.

The proposed GFLCs are currently limited to pre-touchdown geometric and kinematic regulation and terminal motion-state alignment based on simplified landing-gear-disc and deck-half-space models. Finite-deck geometry, more complete three-dimensional clearance and collision constraints, and post-touchdown contact dynamics, including impact loads and friction, are beyond the scope of the present study. Detailed actuator dynamics and saturation margins, state-estimation errors, broader deck-motion prediction uncertainty beyond the prescribed timing-offset tests, and control-system delays are also not explicitly considered. In addition, the explicit incorporation of predicted unsteady airwake fields into trajectory planning remains to be investigated. Future work will extend the proposed framework to address these factors and will further evaluate its performance through higher-fidelity simulations and experimental validation.

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