

Article

Integrated Design Optimization of Aerodynamic Shape and Flow Control Parameters for Co-Flow Jet Airfoils

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Abstract

Co-Flow Jet (CFJ) technology is an effective active flow control method for improving aerodynamic performance by enhancing circulation and suppressing flow separation. However, existing CFJ airfoil optimization studies mainly focus on either aerodynamic shape or flow control parameters separately, limiting the exploitation of their coupled effects. This study proposes an integrated optimization framework that simultaneously considers airfoil geometry and CFJ parameters. A combined parameterization method is developed by integrating Class Shape Transformation (CST) for aerodynamic shape representation and adaptive interpolation for CFJ shroud construction. Based on the unified design space, a multi-objective optimization framework using a Kriging surrogate model is established to efficiently optimize the coupled design variables. The proposed method is applied to a NACA6415-based CFJ airfoil, generating 35 Pareto-optimal solutions. Compared with the baseline configuration, the lift-oriented design increases the lift coefficient from 1.53 to 2.26 (47.7%), while the energy-efficiency-oriented design improves the effective lift-to-drag ratio from 37.97 to 75.84. The integrated optimization also outperforms independent shape or flow-control optimization by capturing the nonlinear coupling effects between aerodynamic geometry and jet parameters. This framework provides an effective strategy for the integrated design of CFJ airfoils and other active flow control configurations.

Keywords: Co-Flow Jet; combined parameterization; airfoil; multi-objective optimization; Pareto solution set; aerodynamic optimization design

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1. Introduction

Active flow control has attracted increasing attention in aerodynamic design because of its adjustability and flexibility in improving lift, reducing drag, delaying stall, and expanding the operating envelope of airfoils and wings. Compared with passive flow control methods, active flow control introduces external energy or momentum into the flow field and can be adjusted according to different operating conditions. Typical active flow control techniques include blowing/suction control, synthetic jets, plasma actuation, and co-flow jet (CFJ) control.

Representative non-CFJ active flow control technologies have been widely investigated and integrated with aerodynamic optimization. In terms of blowing/suction

control, Huang et al. [1] used a genetic algorithm to optimize a dual-jet system on a NACA 0012 airfoil, demonstrating that synergistic optimization of blowing/suction parameters can effectively suppress flow separation at high angles of attack. To suppress shock-induced separation in transonic environments, Abramova et al. [2] numerically optimized tangential blowing parameters to enhance boundary-layer energy and improve supercritical airfoil performance. Concerning synthetic jets, Duvigneau et al. [3] optimized control parameters for stall control of a NACA 0015 airfoil, achieving a 52% increase in maximum lift over the baseline. Tousi et al. [4] utilized automated simulation workflows and multi-objective genetic algorithms to investigate the flow evolution of an SD7003 airfoil under synthetic jet control. In terms of plasma actuation, Zong et al. [5] optimized the electrical parameters of plasma actuators using intelligent algorithms, achieving significant zero-lift drag reduction and highlighting the role of unidirectional unsteady excitation in improving aerodynamic efficiency. More recently, Karthikeyan and Harish [6] reviewed advances in plasma-actuator-based flow control, including transition delay, lift enhancement, drag reduction, and separation control. In addition, recent studies have combined plasma actuation with passive devices to form hybrid flow control strategies. For example, the integrated use of a dielectric barrier discharge (DBD) plasma actuator and a Gurney flap has been shown to enhance the aerodynamic performance of a NACA4412 airfoil [7]. Hybrid configurations combining cavity shape and DBD plasma actuation [8], as well as Gurney flap orientation and plasma actuation [9], have further demonstrated the potential of coupling passive geometric modifications with active actuation. These studies indicate that active flow control is gradually evolving from single-parameter control toward the coupled and hybrid optimization of aerodynamic geometry and control parameters.

Co-flow jet (CFJ) technology, first proposed by Zha et al. [10], is another promising active flow control method for improving aerodynamic lift and reducing drag. The fundamental principle of CFJ involves establishing a Zero-Net-Mass-Flux (ZNMF) circulation by setting an injection slot near the leading edge and a suction slot near the pressure recovery region of the airfoil suction surface. Driven by an internal pump, this process induces intense energy mixing between the jet and the freestream, thereby augmenting airfoil circulation and suppressing flow separation [11–13]. Due to its broad operating range, significant enhancement of stall margin, and high energy efficiency [14–16], CFJ airfoils have attracted widespread attention in various fields, including fixed-wing aircraft [17], rotorcraft [18], propellers [19], and wind turbine blades [20], demonstrating substantial potential for engineering applications.

Extensive research has been conducted on CFJ technology, covering flow mechanisms, parametric studies, and optimization design. In terms of flow mechanisms, Zha et al. revealed the physical mechanisms of CFJ through numerical simulations [21] and wind tunnel experiments [22], highlighting that turbulent mixing between the jet and the freestream is critical for energy transfer. Im et al. [23] used Large Eddy Simulation (LES) to investigate the flow-field structure of CFJ airfoils at high angles of attack, verifying their effectiveness in suppressing large-scale flow separation. For parametric influence, Wang et al. [24] investigated the effects of CFJ slot parameters, including the locations and sizes of the injection and suction slots, on airfoil aerodynamic characteristics. Lefebvre et al. [25,26] systematically studied the effects of the momentum coefficient and airfoil thickness on power consumption and aerodynamic efficiency, providing an important metric for evaluating CFJ energy efficiency. Xu et al. [27] further elucidated the interaction among jet parameters under low-Reynolds-number separated flow conditions, while Song et al. [28] compared discrete and continuous CFJ configurations and clarified the lift-enhancement mechanism associated with three-dimensional vortex structures.

In terms of CFJ optimization design, Zha and co-workers established the early parametric and configuration-design foundation of CFJ airfoils. Subsequent studies further optimized different aspects of CFJ configurations. Jiang et al. [29] performed aerodynamic shape optimization using a Multi-Island Genetic Algorithm. Payne et al. [30] achieved system-level optimization by decomposing external flow fields and internal ducts. Wang et al. [31] proposed a CST-EGO multi-parameter optimization method for the synergistic design of the shroud curve and jet parameters. These studies have significantly improved CFJ aerodynamic performance and clarified the influence of individual geometric or control parameters, particularly those related to the CFJ shroud and slot configuration.

However, most existing CFJ airfoil optimization studies still follow a sequential or partially coupled design routine: a baseline airfoil is first selected, the CFJ shroud and slots are then constructed, and finally the jet or local shroud parameters are optimized. In such approaches, the global aerodynamic shape of the airfoil and the CFJ flow-control parameters are not treated as intrinsic variables within a unified design space. As a result, the nonlinear coupling between airfoil geometry, surface pressure distribution, jet mixing, and energy consumption may not be fully exploited. Zhang and He [32] pointed out that the performance gains obtained by simultaneously optimizing aerodynamic geometry and flow-control parameters can be significantly greater than those achieved by applying either discipline independently. Dai et al. [33] further demonstrated, through the integrated optimization of a jet-controlled supercritical airfoil, that combining jet parameters with airfoil geometry can substantially improve aerodynamic efficiency.

Addressing this research gap, the present study proposes an integrated optimization framework for CFJ airfoils by simultaneously considering aerodynamic shape parameters and CFJ flow-control parameters. In contrast to conventional CFJ optimization studies that mainly optimize the shroud, slot, or jet parameters based on a fixed baseline airfoil, the present method constructs a unified design space that includes both CST-based airfoil geometry variables and CFJ parameter variables. This enables the coupling between aerodynamic shape and active flow control to be captured during the optimization process.

Current CFJ airfoil design generally follows a conventional routine: selecting a baseline airfoil, arranging surface ducts, and then optimizing only the jet control parameters. Whether through early parametric studies [24–27] or recent multi-parameter optimization [29–31], the baseline airfoil parameters and CFJ parameters have not been treated as a unified synergistic system. In this conventional approach, CFJ serves merely as an add-on control means. On one hand, the CFJ shroud is usually constrained by the baseline geometry. Whether using traditional geometric translation and rotation [11] or recent parameterized shroud construction [31], the shroud is built as an auxiliary segment, lacking a unified parameterization with the baseline. During optimization, variations in slot location or size often lead to curve discontinuities or geometric distortion, compromising geometric consistency and design controllability. On the other hand, the lack of a unified design space for aerodynamic shape and flow control parameters ignores the significant coupling between them; the overall airfoil shape directly influences surface pressure distribution and jet mixing [25], while control parameters conversely affect aerodynamic characteristics. Separating these parameters fails to fully exploit the synergistic potential of CFJ, limiting further performance improvements.

To address these issues, this paper proposes an integrated optimization design method for CFJ airfoils based on combined parameterization. The core philosophy is to model and optimize aerodynamic shape parameters and flow control parameters simultaneously, transforming CFJ parameters from auxiliary control variables into intrinsic design drivers. First, the Class Shape Transformation (CST) technique is

employed to achieve global control over the baseline aerodynamic shape. Second, a linear interpolation strategy is introduced during the construction of the CFJ airfoil to ensure the shroud curve is adaptively associated with the baseline, facilitating a unified description and ensuring geometric continuity. Based on this, a joint design space incorporating CST parameters, CFJ geometric parameters, and jet control parameters is constructed. Global optimization is subsequently conducted using a multi-objective surrogate model framework to achieve high-efficiency design.

Compared with existing methods, the proposed approach avoids the limitation of seeking optimal CFJ parameters on a fixed airfoil. By constructing a unified design space through combined parameterization, this method realizes the synergistic design of aerodynamic shape and flow control parameters, ensuring geometric rationality while maximizing the aerodynamic potential of CFJ. This synergistic design philosophy is not only applicable to CFJ airfoils but also adaptable to other active flow control configurations, such as suction, blowing, and plasma actuation, providing a new technical reference for the systematic optimization of active flow control airfoils.

2. Co-Flow Jet Technology

Co-Flow Jet (CFJ) technology is an advanced active flow control technique characterized by zero-net-mass-flux (ZNMF), first proposed by Zha's research team [10] at the University of Miami in 2004. Compared to conventional airfoils, a CFJ airfoil features an integrated internal pump and duct system. An injection slot is positioned near the leading-edge suction region on the upper surface, while a suction slot is situated in the pressure recovery region toward the trailing edge. The internal pump extracts fluid from the suction slot, pressurizes it, and re-injects it tangentially along the airfoil surface via the injection slot. Throughout this process, the mass flow rate at the suction slot is precisely balanced with that at the injection slot, completing a closed-loop jet cycle without the addition of net mass; consequently, it is classified as a zero-net-mass-flux active flow control technology. A schematic diagram of its operating principle is shown in Figure 1.

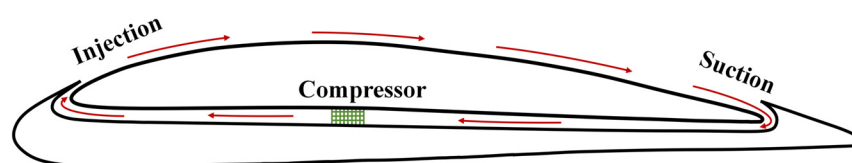


Figure 1. Operating principle of Co-Flow Jet airfoil.

The fundamental principle of CFJ airfoils lies in the intense turbulent mixing facilitated by the unstable shear layer between the jet and the freestream. Through turbulent diffusion, the kinetic energy of the jet is continuously and efficiently transferred to the freestream. This mechanism energizes the main flow with sufficient kinetic energy to overcome severe adverse pressure gradients, thereby maintaining flow attachment at high angles of attack and significantly extending the stall margin.

Furthermore, the jet substantially increases the velocity over the airfoil's upper surface, which subsequently enhances circulation and lift. Simultaneously, the suction at the trailing-edge slot further accelerates the flow across the upper surface. From a momentum balance perspective, since the flow velocity at the trailing edge exceeds the freestream velocity, a forward thrust component is provided to the airfoil. This reduces drag and potentially achieves negative drag (thrust) at low angles of attack. Consequently, CFJ airfoils offer the following advantages [14]:

1. Effective suppression of flow separation: The jet momentum significantly delays or eliminates flow separation, leading to enhanced lift coefficients even at high angles of attack.
2. Significant drag reduction: The momentum exchange allows for substantial drag reduction, with the potential to generate negative drag (thrust) at low angles of attack.
3. Improved stall margin: The technology notably extends the stall angle, providing a broader operational envelope for flight stability.
4. High energy efficiency: The system is characterized by high energy utilization, with minimal thrust loss in the propulsion system.
5. Broad applicability: The CFJ concept is versatile and can be adapted to virtually any baseline airfoil geometry.
6. Full-mission capability: The flow control mechanism remains effective throughout all flight phases instead of being restricted to takeoff and landing operations.

2.1. Aerodynamic Analysis of Co-Flow Jet Airfoils

When performing aerodynamic analysis on airfoils employing Co-Flow Jet (CFJ) technology, it is essential to consider both the conventional aerodynamic forces, such as surface pressure and skin friction, and the jet reaction force generated at the injection and suction slots. According to the principle of momentum conservation, the force exerted by the jet is equivalent to the sum of the momentum flux difference and the static pressure difference between the injection and suction slots. Therefore, the jet reaction force can be determined based on the flow parameters at both slots. Assuming the freestream direction is aligned with the positive x-axis, the simplified model, which excludes the internal pump and duct system, is illustrated in the figure below:

In Figure 2, the subscript “1” denotes the injection slot, while the subscript “2” denotes the suction slot. V_j represents the average jet velocity, α denotes the angle of attack, and F_x and F_y represent the components of the conventional aerodynamic forces acting on the airfoil surface in the x and y directions, respectively.

The jet reaction force F_{jet} is calculated as follows:

$$F_{jet} = \dot{m}_2 V_2 + p_2 A_2 - (\dot{m}_1 V_1 + p_1 A_1) \quad (1)$$

where the subscripts “1” and “2” denote the injection and suction slots, respectively. $\dot{m}$ represents the mass flow rate, V is the average jet velocity, p is the static pressure, and A is the cross-sectional area of the slot. Given that the system operates under the zero-net-mass-flux (ZNMF) condition, the mass flow rate satisfies $\dot{m}_1 = \dot{m}_2 = \dot{m}$. Consequently, the equation can be simplified to:

$$F_{jet} = \dot{m}(V_2 - V_1) + (p_2 A_2 - p_1 A_1) \quad (2)$$

Therefore, the total lift force L and drag force D acting on the airfoil are expressed as:

$$L = F_y + (F_{jet})_y \quad (3)$$

$$D = F_x + (F_{jet})_x \quad (4)$$

The corresponding lift coefficient C_l and drag coefficient C_d can be expressed as:

$$C_l = \frac{L}{\frac{1}{2} \rho_{\infty} V_{\infty}^2 S} \quad (5)$$

$$C_d = \frac{D}{\frac{1}{2} \rho_{\infty} V_{\infty}^2 S} \quad (6)$$

where ρ_{∞} represents the freestream density, V_{∞} denotes the freestream velocity, and S represents the reference area.

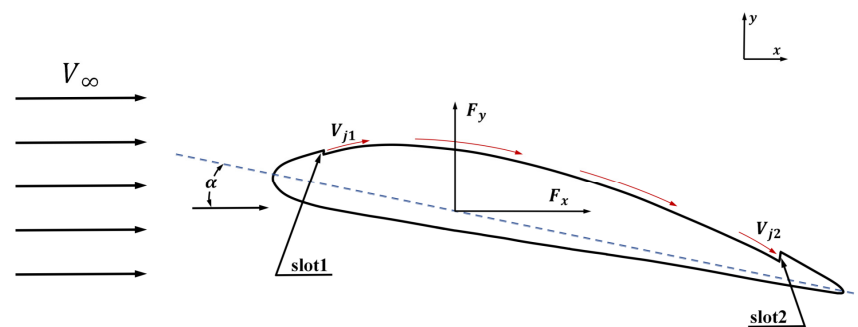


Figure 2. Simplified models of Co-Flow Jet airfoil.

2.2. Key Parameters of CFJ Airfoils

For CFJ airfoils, the jet momentum coefficient C_{μ} is a critical parameter used to quantify the intensity of the jet flow. It is defined as:

$$C_{\mu} = \frac{\dot{m} V_j}{\frac{1}{2} \rho_{\infty} V_{\infty}^2 S} \quad (7)$$

In practical numerical simulations, a constant mass flow rate is typically prescribed at the injection and suction slots. However, the actual momentum coefficient often fluctuates with changes in the angle of attack rather than remaining constant. Therefore, the nominal jet momentum coefficient C_{μ}^* is defined as:

$$C_{\mu}^* = \frac{\dot{m} V_{j1}^*}{\frac{1}{2} \rho_{\infty} V_{\infty}^2 S} \quad (8)$$

$$V_{j1}^* = \frac{\dot{m}}{\rho_{\infty} A_{j1}} \quad (9)$$

The mass flow rate per unit span is defined as:

$$M = \frac{\dot{m}}{l} \quad (10)$$

where l denotes the wingspan. For conventional airfoils, the aerodynamic efficiency is defined as the ratio of lift to drag:

$$\left(\frac{L}{D} \right) = \frac{L}{D} = \frac{C_l}{C_d} \quad (11)$$

However, for CFJ airfoils, the energy consumption of the propulsion system must be accounted for; therefore, a correction to the aerodynamic coefficients is required. According to Reference [12], the power consumed by the air pump is treated as an equivalent drag. Consequently, the effective drag D_e and the effective drag coefficient $(C_d)_e$ are defined as follows:

$$D_e = D + \frac{P}{V_\infty} \quad (12)$$

$$(C_d)_e = C_d + P_c \quad (13)$$

The corrected aerodynamic efficiency, represented as the effective lift-to-drag ratio is expressed as follows:

$$\left(\frac{L}{D}\right)_e = \frac{L}{D_e} = \frac{L}{D + \frac{P}{V_\infty}} = \frac{C_l}{C_d + P_c} \quad (14)$$

where P is the power consumed by the air pump for compressing the gas inside the airfoil, and P_c is the power coefficient. They are defined as follows:

$$P = \dot{m}(H_{t1} - H_{t2}) = \frac{\dot{m}C_p T_{t2}}{\eta} \left(\Gamma^{\frac{\gamma-1}{\gamma}} - 1 \right) \quad (15)$$

$$P_c = \frac{P}{\frac{1}{2} \rho_\infty V_\infty^3 S} \quad (16)$$

where H_{t1} and H_{t2} denote the total enthalpy at the injection and suction slots, respectively; C_p represents the specific heat at constant pressure; γ is the ratio of specific heats. The efficiency of the air pump, η , is taken as 1.0 in the baseline optimization to focus on the aerodynamic coupling between the airfoil geometry and CFJ parameters, which is consistent with previous CFJ numerical studies using an ideal compressor efficiency [24]. Practical micro-compressors for aerospace applications can achieve efficiencies of approximately 85%, and the efficiency may be even higher for larger-scale applications [34]. Therefore, the influence of realistic compressor efficiency on the energy-related performance is further examined in Section 5.2. T_2 denotes the total temperature at the suction slot, and Γ represents the total pressure ratio between the injection and suction slots, defined as:

$$\Gamma = \frac{P_{01}}{P_{02}} \quad (17)$$

where P_{01} and P_{02} are the total pressures at the injection and suction slots, respectively.

3. Numerical Simulation Methodology for Co-Flow Jet Airfoils

3.1. Governing Equations

Currently, Computational Fluid Dynamics (CFD) numerical simulations are primarily based on solving the Navier–Stokes (N-S) equations. These methods can be categorized into Direct Numerical Simulation (DNS), Large Eddy Simulation (LES), and Reynolds-Averaged Navier–Stokes (RANS) approaches. Among these, the RANS method

does not solve the instantaneous N-S equations directly, but instead solves the time-averaged turbulent flow equations. Due to its relatively low computational cost and its ability to meet the precision requirements for most engineering applications, RANS has been widely adopted and is the method employed in this study. The integral form of the governing equations is expressed as follows:

$$\frac{\partial}{\partial t} \iiint_{\Omega} \bar{Q} dv + \oint_{\partial\Omega} \vec{F} \cdot \vec{n} ds - \oint_{\partial\Omega} \vec{F}_v \cdot \vec{n} ds = 0 \quad (18)$$

where t denotes time, Ω is the control volume, $\bar{Q}$ represents the vector of conserved variables, $\partial\Omega$ is the boundary of the control volume, $\vec{F}$ is the inviscid flux, $\vec{n}$ is the unit outward normal vector to the control volume boundary, and $\vec{F}_v$ is the viscous flux.

3.2. Turbulence Model

The RANS method is based on the concept of modeling turbulent flow, which consists of numerous vortices with varying structures and scales. It decomposes the instantaneous turbulent motion into mean and fluctuating components, where the Reynolds stress tensor represents the impact of turbulent fluctuations on the mean flow. These Reynolds stresses are typically determined by closure models derived from turbulence theories and experimental data. In this study, the four-equation intermittency transition shear stress transport model (γ -Transition SST $k-\omega$ model) is employed. A brief introduction to this model is provided below.

The γ -Transition SST $k-\omega$ model [35] is a four-equation turbulence model that integrates a $\gamma-\overline{Re_{\theta t}}$ transition model with the two-equation SST $k-\omega$ model. The transport equations of the $\gamma-\overline{Re_{\theta t}}$ transition model consist of the intermittency factor transport equation and the transition momentum-thickness Reynolds number transport equation. Their specific expressions are given as follows:

Transport equation for the intermittency factor (γ):

$$\frac{\partial(\rho\gamma)}{\partial t} + \frac{\partial(\rho U_j \gamma)}{\partial x_j} = P_\gamma - E_\gamma + \frac{\partial}{\partial x_j} \left[\left(\mu + \frac{\mu_t}{\sigma_f} \right) \frac{\partial \gamma}{\partial x_j} \right] \quad (19)$$

where t denotes time; ρ is the fluid density; γ is the intermittency factor; U_j is the velocity component; P_γ and E_γ are the production and destruction terms for the intermittency factor, respectively; μ is the laminar dynamic viscosity; μ_t is the turbulent eddy viscosity; and σ_f is the diffusion constant.

Transport equation for the transition momentum-thickness Reynolds number ($\overline{Re_{\theta t}}$):

$$\frac{\partial(\rho \overline{Re_{\theta t}})}{\partial t} + \frac{\partial(\rho U_j \overline{Re_{\theta t}})}{\partial x_j} = P_{\theta t} + \frac{\partial}{\partial x_j} \left[\sigma_{\theta t} (\mu + \mu_t) \frac{\partial \overline{Re_{\theta t}}}{\partial x_j} \right] \quad (20)$$

where $\overline{Re_{\theta t}}$ is the transition momentum-thickness Reynolds number; $P_{\theta t}$ is the source term; and $\sigma_{\theta t}$ is the diffusion coefficient.

3.3. Boundary Conditions

Appropriate boundary conditions are fundamental to ensuring that the Navier-Stokes equations satisfy the requirements for a well-posed problem. Inappropriate

boundary conditions may result in convergence failure and inaccurate flow field predictions. The primary boundary conditions employed in this study are briefly described below.

1. Far-field Boundary Conditions

The far-field boundaries are implemented using free-stream boundary conditions based on Riemann invariants. The local one-dimensional Riemann invariant is expressed as:

$$R^- = v_n - \frac{2a}{\gamma-1} \quad (21)$$

$$R^+ = v_n + \frac{2a}{\gamma-1} \quad (22)$$

where v_n denotes the normal component of the velocity at the far-field boundary, and a represents the speed of sound.

For subsonic inflow and outflow boundaries, the values are determined by a combination of free-stream calculations and interior extrapolation:

$$R^- = v_n - \frac{2a}{\gamma-1} = v_{n\infty} - \frac{2a_\infty}{\gamma-1} \quad (23)$$

$$R^+ = v_n + \frac{2a}{\gamma-1} = v_{ne} + \frac{2a_e}{\gamma-1} \quad (24)$$

where the subscript “ ∞ ” denotes free-stream values, and the subscript “ e ” indicates values extrapolated from the interior of the computational domain. By operating on these Riemann invariants, the speed of sound and the normal velocity component at the far-field boundary can be derived as:

$$a = \frac{\gamma-1}{4} (R^+ - R^-) \quad (25)$$

$$v_n = \frac{1}{2} (R^+ + R^-) \quad (26)$$

For inflow boundaries ($v_n < 0$), the entropy and tangential velocity at the boundary points are set to their free-stream values. For outflow boundaries ($v_n > 0$), the entropy and tangential velocity are obtained by extrapolation from the interior of the computational domain. Thus, all flow variables at the far-field boundary are determined.

2. Wall Boundary Conditions

The solid wall surfaces are treated as no-slip, adiabatic walls. The boundary conditions are expressed as follows:

$$u = v = w = 0, \quad \frac{\partial T}{\partial n} = 0 \quad (27)$$

where u, v, w are the velocity components in the Cartesian coordinate system, T is the temperature, and n denotes the normal direction to the wall surface.

3. Inlet/Outlet Boundary Conditions

For the CFJ airfoil, mass flow inlet and mass flow outlet boundary conditions are applied to the injection and suction slots, respectively. By prescribing identical mass flow rates and achieving iterative convergence, the zero-net-mass-flux requirement for the CFJ airfoil is satisfied.

For the mass flow inlet boundary, when a total mass flow rate is specified, the uniform mass flow rate at each face of the boundary is calculated as:

$$\dot{m} = \dot{m}_{total,spec} \frac{|a|}{A} \quad (28)$$

where a denotes the outward grid area vector, and A represents the total boundary area.

The static pressure at the boundary is extrapolated from the interior of the domain:

$$P_s = P_s^{ext} \quad (29)$$

where the superscript “*ext*” indicates that the value is extrapolated from the adjacent grid cell.

For the outlet boundary, when a total mass flow rate is specified, the velocity at the boundary face is calculated as follows:

$$v = v^{ext} + \frac{x_i}{\rho} n; \quad n = \frac{a}{|a|} \quad (30)$$

where ρ is the density calculated at the boundary face, a is the outward grid area vector, and x_i is the boundary mass flux correction factor calculated for the outlet boundary i based on the mass flow rate requirement:

$$x_i = \frac{\dot{m}_{i,spec} - \dot{m}_{i,total}}{\sum_{outlet\ i\ faces} |a|} \quad (31)$$

The total mass flow rate $\dot{m}_{i,total}$ passing through the outlet boundary i is calculated as:

$$\dot{m}_{i,total} = \sum_{outlet\ i\ faces} \rho (v^{ext} \cdot a) \quad (32)$$

3.4. Numerical Simulation Approach Validation

To validate the accuracy of the mass flow inlet and outlet boundary conditions employed for the CFJ airfoil injection and suction slots, the CFJ6415 airfoil, which is based on the NACA6415 profile, was selected for validation against available wind tunnel experimental data. The wind tunnel test was conducted by Dano et al. [36] at the University of Miami. The geometric parameters of the CFJ6415 airfoil are summarized in Table 1.

Table 1. Geometric parameters of the CFJ6415 airfoil.

Parameter	Value
Reference chord length c (m)	0.3048
Injection location	7.5% c
Injection slot size	0.65% c
Suction location	88.5% c
Suction slot size	1.3% c

The computational operating conditions are summarized in Table 2.

Table 2. Computational conditions.

Parameter	Value
Mach number	0.03
Reynolds number	2.078×10^5

Angle of attack range	0°~25°
Jet mass flow rate (kg/s)	0.03

When performing two-dimensional CFD calculations for the CFJ airfoil, the reference span is defaulted to a unit span of 1 m. However, as the actual span used in the wind tunnel experiment was 0.5906 m, the jet mass flow rate was adjusted to 0.0508 kg/s based on mass flow scaling. The computational mesh for the CFJ6415 airfoil, which is illustrated in Figure 3, consists of approximately 95,000 cells with $y^+ < 1$ maintained across the airfoil surface. A hybrid meshing strategy was adopted by utilizing structured grids near the airfoil surface and unstructured grids in the far-field, with a structured grid growth rate of 1.1.

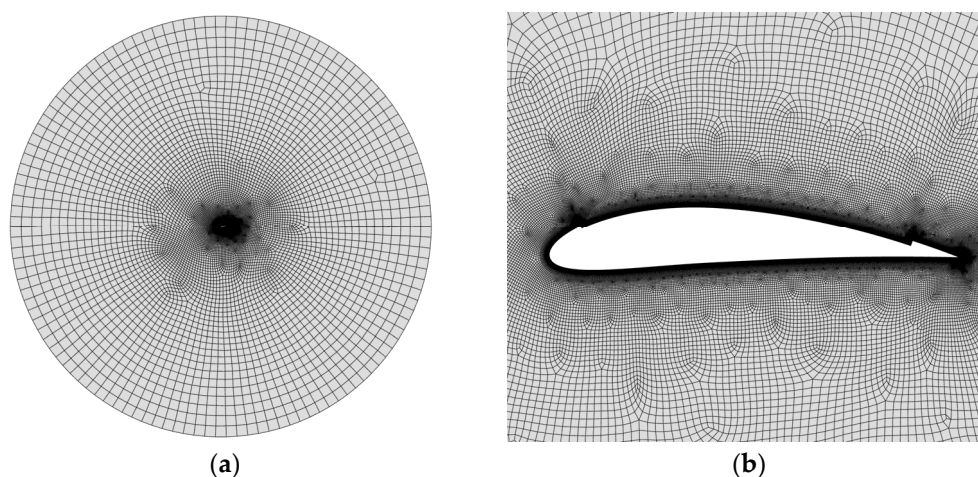


Figure 3. Mesh for the CFJ6415 airfoil. (a) Far-field mesh; (b) Near-field mesh.

Figure 4 presents a comparison of the lift and drag coefficients for the CFJ6415 airfoil. The current numerical results are compared with both the wind tunnel experimental data and the numerical simulations performed by Lefebvre [37] in 2016. As illustrated in the figure, the predicted stall angle of attack obtained by the present numerical method is slightly lower, which is consistent with the trend observed in Lefebvre's RANS-based simulation. The lift coefficients are accurately predicted at low angles of attack; however, discrepancies emerge at higher angles of attack. This behavior is attributed to the inherent limitations of RANS methods in accurately capturing large-scale flow separation. The calculated drag coefficients deviate only slightly from the experimental data and follow the same trends with relatively small errors. Based on the aforementioned analysis, subsequent optimization research in this study will be conducted within the linear range of 0°~15° angle of attack to ensure the reliability of the computational results and the derived conclusions.

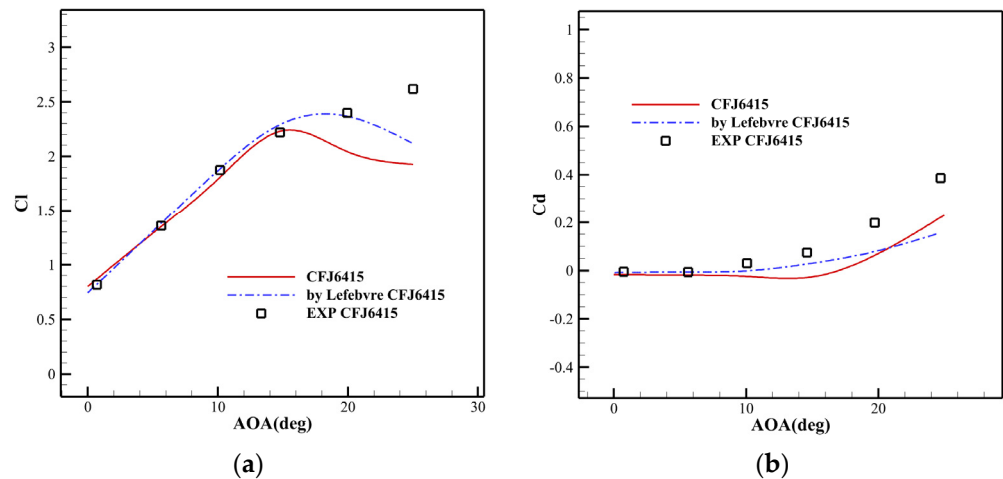


Figure 4. Validation of the present numerical method for the CFJ6415 airfoil under the wind-tunnel conditions of $Ma = 0.03$, $Re = 2.078 \times 10^5$, and $AOA = 0\text{--}25^\circ$: (a) Lift coefficient; (b) Drag coefficient.

4. Optimization Design Methodology

4.1. Integrated Parametrization Method for CFJ Airfoils

The aerodynamic performance of a CFJ airfoil is highly dependent on the synergistic matching between its geometric configuration and jet control parameters. To achieve efficient and precise design of CFJ airfoils, this study proposes an integrated parametrization method. This method is designed to resolve the parameter coupling issues between the baseline airfoil and the sunken channel geometry typically encountered in conventional approaches. Consequently, the representation capability of the design space and the overall optimization efficiency are significantly enhanced.

As shown in Figure 5, the baseline airfoil for the CFJ configuration is parameterized using the CST (Class function/Shape function Transformation) method [38–40].

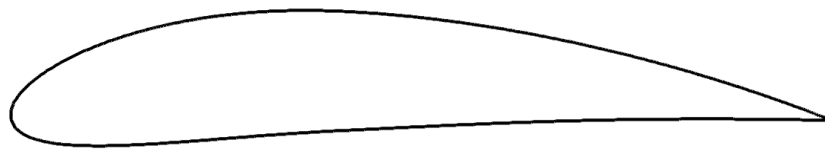


Figure 5. Baseline airfoil.

The CST method describes the geometric characteristics of an airfoil with high accuracy using a minimal number of design variables by multiplying a class function by a shape function and adding a trailing-edge thickness term. The mathematical expressions are as follows:

For the upper surface:

$$y_u = C(x) \cdot S_u(x) + x \cdot y_{TEu} \quad (33)$$

For the lower surface:

$$y_l = C(x) \cdot S_l(x) + x \cdot y_{TEl} \quad (34)$$

where y_u is the y -coordinate of the upper surface; x is the surface x -coordinate; $C(x)$ is the class function; $S_u(x)$ is the shape function for the upper surface; y_{TEu} is the y -coordinate of the trailing edge on the upper surface; y_l is the y -coordinate of

the lower surface; $S_l(x)$ is the shape function for the lower surface; and y_{TEl} is the y -coordinate of the trailing edge on the lower surface.

The class function $C(x)$ is defined as:

$$C(x) = x^{N_1} \cdot (1-x)^{N_2} \quad (35)$$

where N_1 and N_2 are fixed constants set to 0.5 and 1.0, respectively, representing the first and second exponents.

The shape functions $S_u(x)$ and $S_l(x)$ are expanded using Bernstein polynomials, expressed as:

$$S_u(x) = \sum_{i=0}^N A_{ui} \cdot S_i(x) \quad (36)$$

$$S_l(x) = \sum_{i=0}^N A_{li} \cdot S_i(x) \quad (37)$$

where $S_i(x)$ denotes the i -th order Bernstein polynomial, and A_{ui} and A_{li} are the undetermined coefficients. The CST parametrization variables can be solved by fitting known airfoil coordinate points using the least-squares method, thereby achieving a complete geometric description of the baseline airfoil.

Upon completing the parameterization of the baseline airfoil, specific design variables unique to the CFJ configuration are introduced to construct the CFJ airfoil, as illustrated in Figure 6. These variables include the injection location c_1 , injection slot size l_1 , suction location c_2 , and suction slot size l_2 . For clarity, the injection slot, the translated suction-surface segment formed by suction surface translation, and suction slot are highlighted in blue, red, and green, respectively.

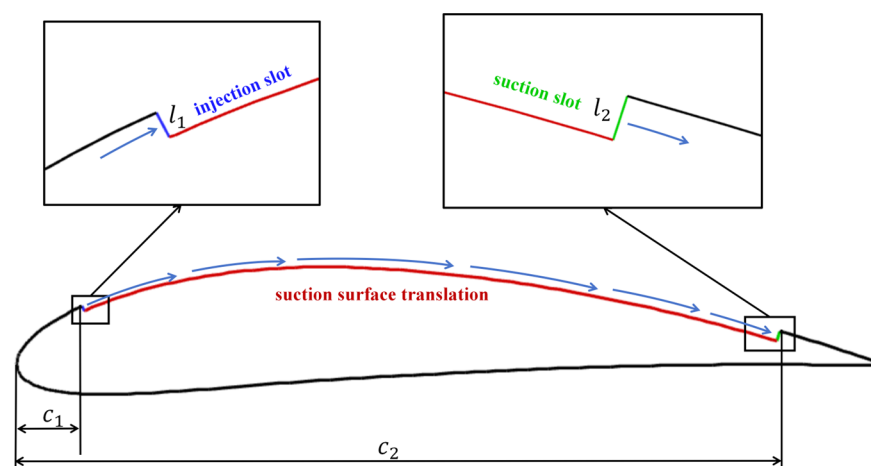


Figure 6. Geometric parameterization of the CFJ airfoil. The injection slot, translated suction-surface segment, and suction slot are highlighted in blue, red, and green, respectively.

To establish an adaptive correlation between the sunken section and the baseline airfoil, this study proposes an adaptive modeling method based on linear interpolation. Specifically, for the airfoil segment located between the injection and suction slots, the original surface points are shifted inward along the normal direction, with the displacement d_c determined by the following equations:

$$t_c = \frac{c}{k-1} \quad (38)$$

$$d_c = t_c \cdot L_1 + (1-t_c) \cdot L_2 \quad (39)$$

where c denotes the index of the airfoil point starting from the suction slot, with $c = 0, 1, 2, \dots, (k-1)$; k represents the total number of airfoil points between the suction and injection slots; t_c is the movement coefficient for the c -th airfoil point; L_1 and L_2 represent the design values for the injection slot size l_1 and the suction slot size l_2 , respectively; and d_c denotes the normal shift distance for the c -th airfoil point.

Through this linear mapping, the sunken airfoil curve is adaptively correlated with the baseline airfoil, thereby avoiding geometric distortion problems often encountered in traditional methods due to independent fitting.

Ultimately, the CST parametrization variables, CFJ geometric design variables, and jet control parameters are integrated into a unified design space, forming a comprehensive parametrized model. This model not only facilitates a holistic characterization of the CFJ airfoil's geometric and flow features but also provides a high-fidelity input foundation for subsequent multi-objective optimization design.

4.2. Efficient Surrogate-Based Aerodynamic Optimization Method

The aerodynamic optimization design of the CFJ airfoil is conducted using SurroOpt [41], which is a multi-objective and multi-constraint optimization software developed independently by our research group. The fundamental framework of this optimization process is illustrated in Figure 7.

First, an initial set of sample points is selected from the design space using a design of experiments (DOE) method. High-fidelity CFD simulations are then performed to obtain the aerodynamic response values for these samples, which are used to construct an initial Kriging surrogate model. Second, based on the surrogate model, sub-optimization problems are constructed according to specific infill criteria and solved using traditional optimization algorithms to identify new candidate sample points. Finally, high-fidelity CFD calculations are performed for these new points, and the results are integrated into the existing sample set to update the surrogate model. This iterative process continues until the prescribed convergence criterion or sampling budget is reached. A detailed description of this methodology can be found in references [42–45].

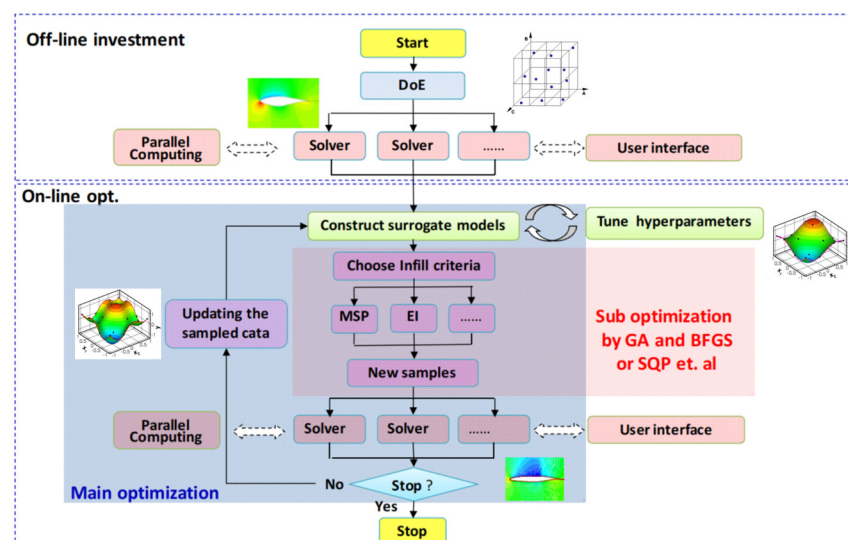


Figure 7. Flowchart of SurroOpt, a generic optimization code [41].

5. Optimization Design of the Co-Flow Jet Airfoil

5.1. Mathematical Model and Parameter Settings of Optimization

In this section, the NACA6415 airfoil is selected as the baseline airfoil. The rationale for this selection is based on validation feasibility and geometric applicability.

First, the NACA6415-based CFJ configuration has been widely investigated in previous studies, ranging from wind-tunnel experiments using discrete jets [36] and numerical validations against experimental data [24], to three-dimensional wing studies [46] and comparisons between open-slot and discrete configurations [47]. The availability of these complementary experimental and numerical datasets provides a reliable basis for the quantitative validation of the present numerical method, ensuring the comparability and reproducibility of the optimization results.

Second, regarding geometric applicability, the NACA6415 airfoil possesses a relative thickness of 15%. This thickness offers sufficient geometric space to integrate the CFJ injection and suction slots, as well as the internal recirculation passages, while avoiding the aerodynamic penalties associated with unusually thick reference profiles [46].

Following the integrated parametrization method described in Section 4.1, a baseline CFJ airfoil is constructed with a reference chord length C of 0.3048 m. The specific geometric parameters for this baseline CFJ airfoil are summarized in Table 3. This configuration is designated as “CFJ-Baseline,” where the NACA6415 profile serves as the foundational geometry from which the CFJ-Baseline is derived. Based on this configuration, optimization research for the Co-Flow Jet airfoil is conducted using the integrated parametrization method.

The NACA6415 baseline airfoil is parameterized using an 8th-order CST method, yielding 18 CST parametrization variables. Building upon this, a comprehensive three-category design space is constructed by combining these CST variables with the specific CFJ geometric parameters and jet control parameters. The design operating conditions are summarized in Table 4.

Table 3. Geometric parameters of the CFJ-Baseline airfoil.

Parameter	Value
Injection location c_1	7.5% C
Injection slot size l_1	0.65% C
Suction location c_2	88.5% C
Suction slot size l_2	1.3% C

Table 4. Design operating conditions.

Parameter	Value
Mach number	0.15
Reynolds number	1.026×10^6
Angle of attack	5°
Jet mass flow rate (kg/s)	0.3596

To quantify the mesh-induced discretization uncertainty under the design operating condition, a grid convergence study was conducted for the CFJ-Baseline configuration at $Ma = 0.15$, $Re = 1.026 \times 10^6$, $AOA = 5^\circ$, and a jet mass flow rate of 0.3596 kg/s. Three systematically refined meshes containing 56,411, 79,117, and 112,988 cells were examined. The Grid Convergence Index (GCI) procedure based on Richardson extrapolation, with a

safety factor of 1.25, was employed to estimate the uncertainty associated with spatial discretization [48].

As summarized in Table 5, the asymptotic range check ratios R for C_l , C_d , and P_c are 1.0007, 1.0082, and 0.9912, respectively. All values satisfy $0.8 \leq R \leq 1.2$, indicating that the three-grid solutions are within the asymptotic convergence range. The fine-grid GCI values are 0.0142% for C_l , 0.2543% for C_d , and 0.3925% for P_c . Therefore, the mesh-induced discretization uncertainties of all key aerodynamic quantities are below 0.4%. Accordingly, the mesh containing 112,988 cells was adopted for all subsequent optimization simulations.

Table 5. Grid convergence and mesh-induced discretization uncertainty assessment for the CFJ-Baseline configuration under the design operating condition.

Mesh Level	Number of Cells	C_l	C_d	P_c
Coarse	56411	1.525	−0.0471	0.0932
Medium	79117	1.531	−0.0489	0.0904
Fine	112988	1.532	−0.0493	0.0896
R	—	1.0007	1.0082	0.9912
GCI(%)	—	0.0142	0.2543	0.3925

The design space for the present study is defined as follows:

$$0.4\%c \leq l_1 \leq 0.65\%c \quad (40)$$

$$1.3\%c \leq l_2 \leq 1.6\%c \quad (41)$$

Additionally, the CST parameterization variables are allowed to fluctuate within $\pm 25\%$ of their initial values, where c represents the reference chord length (0.3048 m). This range is chosen based on empirical evidence and established studies. It provides a sufficient design space for optimization while preventing severe geometric distortions that could compromise CFD convergence [49].

As indicated by Equations (8) and (9), the nominal jet momentum coefficient is influenced by the injection slot size. In the present study, the jet mass flow rate $\dot{m}$ is fixed at 0.3596 kg/s under the design operating condition. This value is derived from the CFJ-Baseline configuration by setting its nominal jet momentum coefficient to 0.12, which is a moderate value within the commonly used range of CFJ experimental and numerical studies [25,26,36,37].

During optimization, the jet mass flow rate is kept constant, while the injection slot size is allowed to vary within $0.4\%c \leq l_1 \leq 0.65\%c$. The upper bound of l_1 corresponds to the injection slot size of the CFJ-Baseline configuration. Since a larger injection slot would reduce the jet velocity and the nominal jet momentum coefficient under a fixed mass flow rate, l_1 is not allowed to exceed the baseline value. The lower bound of $0.4\%c$ allows a moderate increase in jet intensity while avoiding an excessively small injection slot. This setting is consistent with previous CFJ parameter studies, which showed that increasing the nominal jet momentum coefficient generally enhances lift, whereas excessive jet intensity may increase power consumption [24–26].

Therefore, with the jet mass flow rate fixed at 0.3596 kg/s under the design operating condition, the design space for the nominal jet momentum coefficient $C_{\mu,opt}^*$ is derived as:

$$C_{\mu,baseline}^* \leq C_{\mu,opt}^* \leq 1.625C_{\mu,baseline}^* \quad (42)$$

where $C_{\mu, \text{baseline}}^* = 0.12$ is the nominal jet momentum coefficient for the CFJ-Baseline airfoil under the design operating condition, calculated according to Equation (8). The factor 1.625 is obtained from the ratio between the baseline injection slot size and the minimum injection slot size, namely $0.65\%c/0.4\%c = 1.625$.

The mathematical model for the multi-objective optimization problem is formulated as follows:

$$\begin{aligned} \max \quad & \text{obj}_1 = C_l / C_{l, \text{baseline}} \\ & \text{obj}_2 = \left(\frac{L}{D} \right)_e / \left(\frac{L}{D} \right)_{e, \text{baseline}} \\ \text{s.t.} \quad & C_l > C_{l, \text{baseline}} \\ & C_d < C_{d, \text{baseline}} \\ & t > 0.95t_{\text{baseline}} \end{aligned} \quad (43)$$

where $C_{l, \text{baseline}}$, $C_{d, \text{baseline}}$ and $\left(\frac{L}{D} \right)_{e, \text{baseline}}$ are the lift coefficients, drag coefficients and effective lift-to-drag ratio of the CFJ-Baseline airfoil under the design operating condition, respectively; t_{baseline} is the thickness of the NACA6415 airfoil used to construct the CFJ-Baseline; and t is the thickness of the underlying baseline airfoil corresponding to the CFJ configuration during the optimization process.

The SurroOpt optimization software was used to construct the initial Kriging surrogate model with 40 sample points. In each iteration, 24 new sample points were added in parallel using a hybrid infill strategy combining the Multi-objective Selection Procedure (MSP) and Expected Improvement (EI) criteria, with a total of 1000 samples calculated. In the surrogate-based optimization framework, the Kriging model is used to guide the adaptive sampling process, and each newly selected sample is evaluated by CFD before being added to the sample database for model updating [41,50]. Therefore, the final Pareto solutions reported in this study are selected from the CFD-evaluated sample database rather than from the predicted solutions of the surrogate model.

During the adaptive sampling process, closely spaced sample points in the design space may result in an ill-conditioned correlation matrix of the Kriging model. To improve the numerical stability of the correlation matrix, a small diagonal regularization parameter was introduced during model construction [42]. Although an ideal Kriging model has an exact interpolation property at the training samples, the introduction of this regularization term slightly relaxes this property. Therefore, an a posteriori consistency assessment was conducted for the final regularized Kriging model using the complete database of 1000 CFD-evaluated samples.

5.2. Optimization Results and Discussion

In this section, the multi-objective optimization results of the Co-Flow Jet airfoil are evaluated within a normalized framework to eliminate the scale disparity between different objective functions. The hybrid infill strategy combining the Multi-objective Selection Procedure (MSP) and Expected Improvement (EI) criteria utilizes 40 initial database samples and 24 parallel infill points per iteration, enabling a dense exploration of both feasible and infeasible boundaries.

To evaluate the convergence of the multi-objective optimization process, the normalized hypervolume (HV) indicator of the non-dominated solution set [51] was monitored throughout the iterative optimization, as shown in Figure 8. The optimization starts with 40 initial samples, and 24 new samples are added in each iteration. Therefore,

the cumulative sample count at the k -th iteration is $N = 40 + 24k$, where k denotes the iteration number. Accordingly, the 40th iteration corresponds to 1000 CFD-evaluated samples.

As shown in Figure 8, the normalized HV increases rapidly from 0.173249 at the initial stage to 0.582317 after the first iteration, indicating a significant improvement in the Pareto front at the early optimization stage. The HV further increases to 0.622232 by the 4th iteration, reaching approximately 97.97% of the final value. Thereafter, the growth rate becomes much smaller, and the curve gradually approaches a stable plateau. From the 34th to the 40th iteration, the HV increases only from 0.634482 to 0.635134, corresponding to a relative change of approximately 0.10%. This indicates that the Pareto front becomes sufficiently stable under the present sampling budget of 1000 samples.

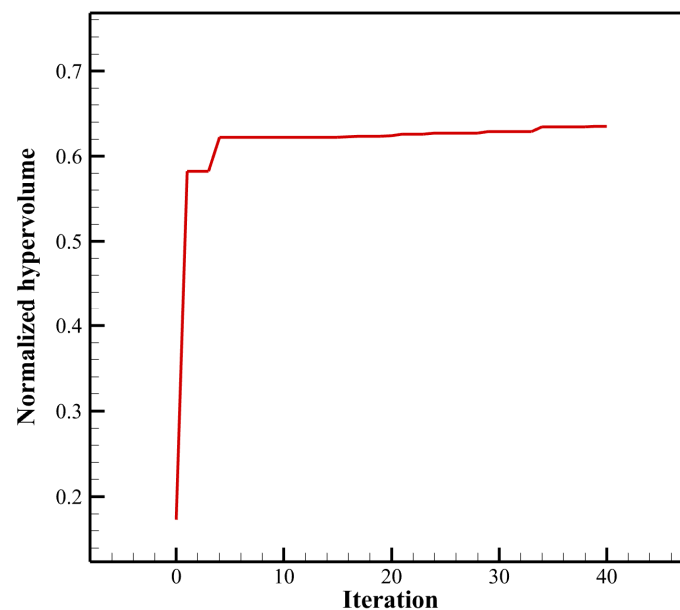


Figure 8. Evolution of the normalized hypervolume indicator during the iterative multi-objective optimization process. The cumulative sample count is $N = 40 + 24k$, where k is the iteration number.

Figure 9 shows the distribution of all samples generated during the iterative two-objective optimization process in the objective space. The green diamonds represent feasible samples that satisfy all prescribed constraints, whereas the blue circles denote infeasible samples that violate at least one constraint. The red squares represent the Pareto solutions identified by non-dominated sorting of the feasible samples. These non-dominated solutions constitute the approximate Pareto front and are highlighted to distinguish the optimal trade-off designs from the remaining samples.

The resulting Pareto optimal solution set consists of 35 optimized airfoils, which represent a non-dominated collection of designs derived from varying weightings of the normalized lift coefficient (obj1) and the normalized effective lift-to-drag ratio (obj2). Figure 10 presents the finalized approximate Pareto front, from which three representative optimized airfoils, labeled as Opt1, Opt2, and Opt3, are selected for detailed aerodynamic analysis.

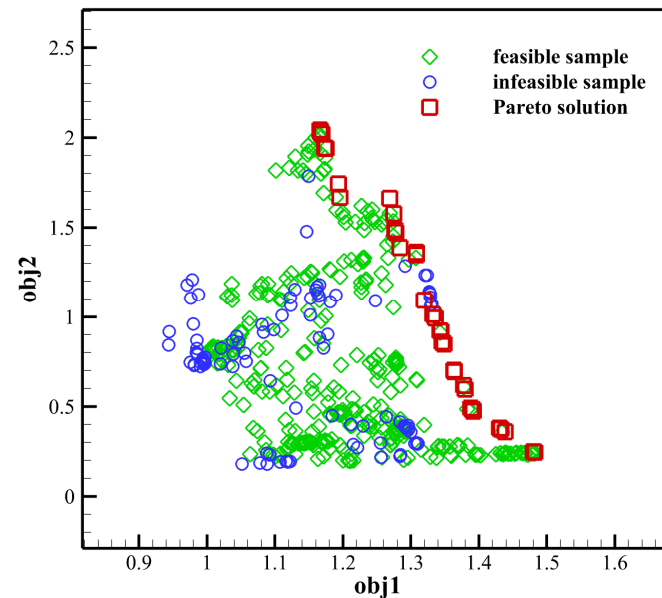


Figure 9. Distribution of feasible samples, infeasible samples, and Pareto solutions in the two-objective optimization space.

Based on the Pareto solutions identified in Figure 9, three representative optimized airfoils, denoted as Opt1, Opt2, and Opt3, are selected from different regions of the Pareto front for detailed analysis. Opt1 is selected from the lower-right part of the Pareto front and represents a lift-oriented solution with the largest obj1 value. Opt2 is selected from the intermediate region and represents a balanced trade-off between lift enhancement and effective aerodynamic efficiency. Opt3 is selected from the upper-left part of the Pareto front and represents an efficiency-oriented solution with the largest obj2 value. These three airfoils are used as typical designs to illustrate different design preferences within the Pareto-optimal solution set.

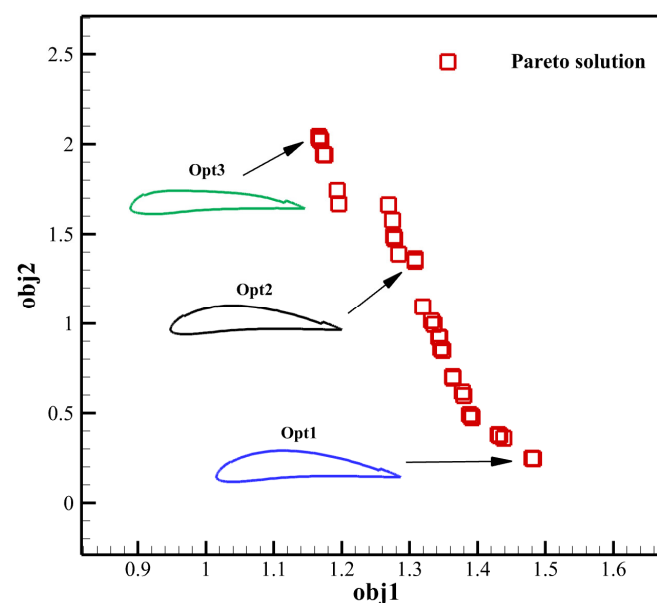


Figure 10. Pareto front and three representative airfoils selected from it.

Opt1 is located at the lower-right extremity of the Pareto front where obj1 reaches its maximum extension, which explicitly targets maximum lift enhancement. Opt3 is situated at the upper-left limit where obj2 is prioritized, representing a high energy efficiency

configuration. Opt2 is selected from the central inflection region of the front, offering a well-balanced compromise between lift enhancement and energy consumption. The specific performance metrics of the baseline and optimized airfoils under the design operating condition are summarized in Table 6. Their geometric configurations are compared in Figure 11.

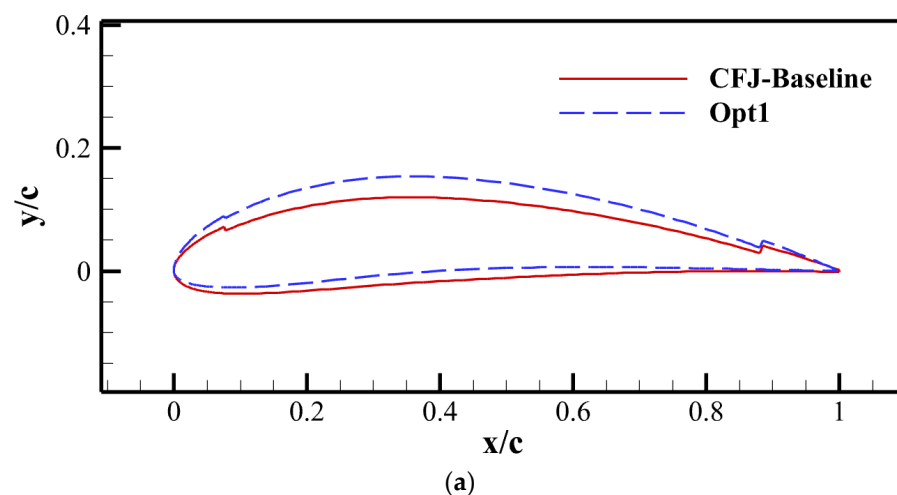
As shown in Table 6, the optimized CFJ configurations exhibit distinct trade-offs among lift performance, aerodynamic drag, and power consumption. Although Opt1 has a high lift coefficient and a low aerodynamic drag coefficient, its larger power coefficient leads to an increase in the effective drag coefficient. As a result, the effective lift-to-drag ratio of Opt1 is 9.27, which is lower than that of the CFJ-Baseline configuration. This relatively low effective lift-to-drag ratio indicates that Opt1 is not an efficiency-oriented design. Instead, it corresponds to the lift-oriented extreme of the Pareto front, where lift enhancement is achieved at the expense of increased power consumption. Therefore, Opt1 should be regarded as a candidate for operating conditions where lift is prioritized, rather than for long-endurance operation where efficiency is the primary requirement.

Opt2 demonstrates improvements in both lift and drag coefficients compared to the baseline while maintaining the power coefficient at a relatively low level, yielding an effective lift-to-drag ratio of 50.69. This configuration achieves an excellent balance between lift enhancement and energy consumption, demonstrating robust engineering feasibility as a well-rounded optimization result.

Although the lift enhancement of Opt3 is relatively limited, it exhibits the lowest power and effective drag coefficients. It achieves an effective lift-to-drag ratio of 75.84, which is nearly double that of the baseline configuration, thereby demonstrating superior energy efficiency.

Table 6. Comparison of aerodynamic performance for the baseline and optimized profiles.

	C_l	$\left(\frac{L}{D}\right)_e$	C_d	P_c	$(C_d)_e$	l_1	l_2	t	C_μ^*
CFJ-Baseline	1.53	37.97	−0.0493	0.0896	0.0403	0.65% c	1.30% c	15.0% c	0.120
Opt1	2.26	9.27	−0.0890	0.3332	0.2442	0.40% c	1.30% c	16.6% c	0.195
Opt2	2.00	50.69	−0.0506	0.0901	0.0396	0.65% c	1.51% c	16.6% c	0.120
Opt3	1.78	75.84	−0.0612	0.0847	0.0235	0.65% c	1.60% c	14.4% c	0.120



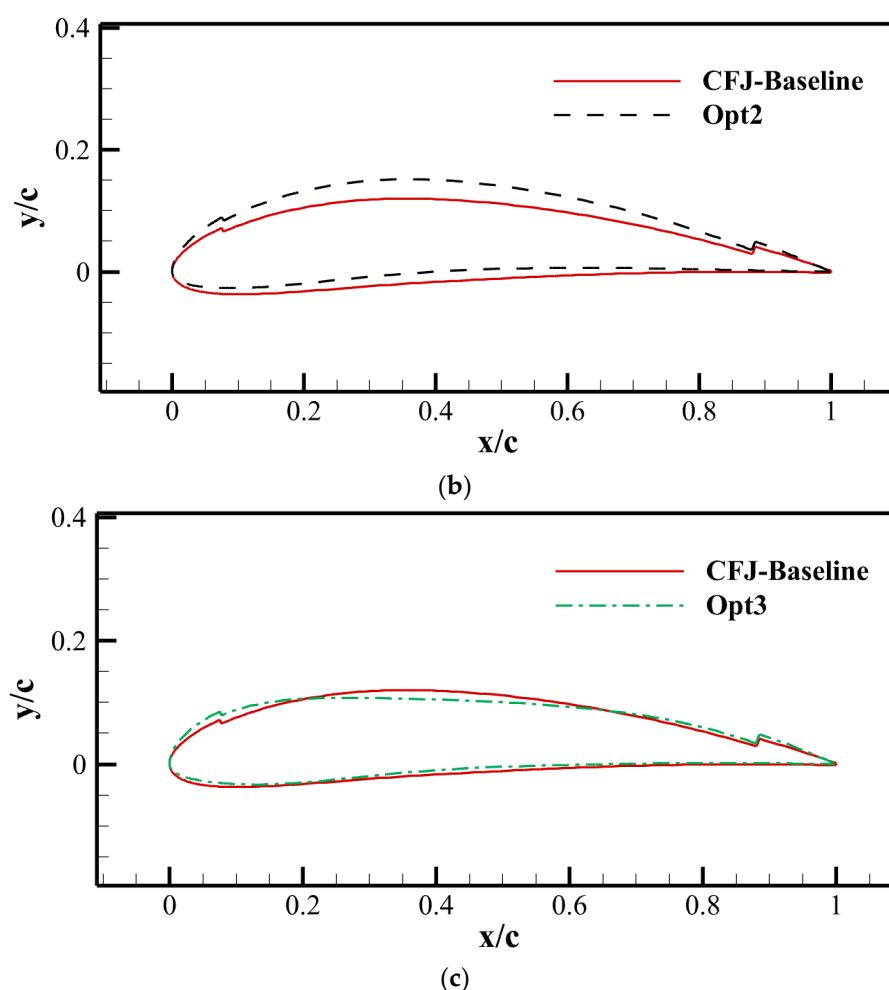


Figure 11. Three optimized airfoils selected from the Pareto front obtained by a multi-objective optimization. (a) Comparison of Opt1 and baseline airfoil; (b) Comparison of Opt2 and baseline airfoil; (c) Comparison of Opt3 and baseline airfoil.

To quantify the consistency between the final regularized Kriging model and the high-fidelity sample database, the final model was reconstructed using all 1000 CFD-evaluated samples, and the predicted normalized objective values were compared with their corresponding CFD values at each sampled design. The absolute relative prediction deviation for the j -th objective was defined as $|\varepsilon_j| = |(f_j - f_{j,CFD})/f_{j,CFD}| \times 100\%$, where f_j and $f_{j,CFD}$ denote the Kriging-predicted value and the corresponding CFD-evaluated value, respectively. Across the complete sample database, the maximum absolute relative deviations are 1.60% for obj1 and 2.61% for obj2. Because these samples constitute the database used to construct the final model, this assessment is interpreted as an a posteriori consistency check of the regularized surrogate rather than an independent out-of-sample validation.

The prediction consistency at the three representative Pareto solutions is further examined in Table 7. For Opt1, Opt2, and Opt3, the maximum absolute relative deviations are 0.46% for obj1 and 0.90% for obj2. These small differences demonstrate that the final regularized Kriging model remains closely consistent with the direct CFD responses near the selected Pareto solutions. Nevertheless, the Pareto-front construction, optimization-convergence assessment, and aerodynamic-performance analyses reported in this study are all based on CFD-evaluated samples rather than surrogate-only predictions.

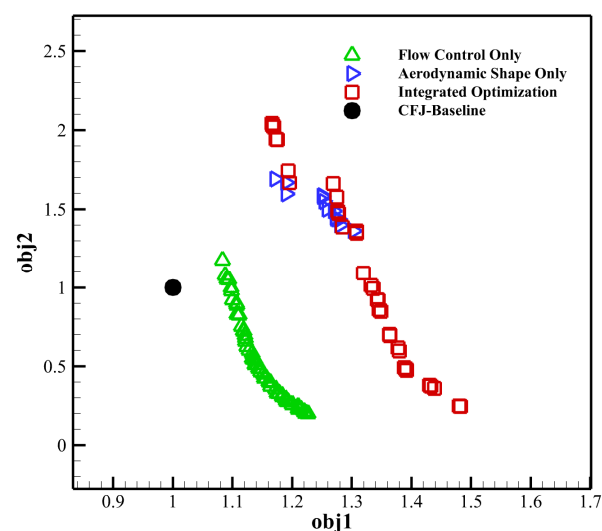
Table 7. Comparison between the final Kriging predictions and CFD objective values for the representative Pareto solutions.

Configuration	Sample No.	CFD obj1	Kriging obj1	Absolute Deviation of obj1	CFD obj2	Kriging obj2	Absolute Deviation of obj2
Opt1	831	1.477124	1.480896	0.26%	0.244140	0.244491	0.14%
Opt2	953	1.307190	1.308472	0.10%	1.335001	1.347045	0.90%
Opt3	983	1.163399	1.168757	0.46%	1.997366	2.014897	0.88%

To evaluate the specific contributions of different categories of design variables, two comparative optimization cases were conducted within the same dual-objective framework under the identical normalization criteria. The first case, labeled “Aerodynamic Shape Only”, restricts the design space to the 18 CST parametrization variables while fixing the jet parameters. The second case, labeled “Flow Control Only”, is confined to the dimensions of the injection and suction slots.

The resulting Pareto front comparison is illustrated in Figure 12. The Pareto front of the integrated design space demonstrates an absolute and comprehensive superiority over both independent sub-spaces. Across the entire range, the integrated optimization successfully pushes the aerodynamic performance boundary far beyond the limits of the decoupled domains, achieving both higher lift and superior efficiency simultaneously.

Specifically, the “Flow Control Only” front is strictly bottlenecked by the frozen baseline profile, locking the maximum achievable lift performance below an obj1 value of 1.23. Although the geometry-focused “Aerodynamic Shape Only” setup shows competitive trade-offs in the mid-efficiency region, it completely fails to expand into the high-lift region, exhibiting a sudden performance termination when obj1 exceeds 1.32. By contrast, the integrated design space fully overcomes these structural boundaries, expanding obj1 to approximately 1.48 while maintaining a significantly higher obj2 curve. This uniform dominance confirms that the strong non-linear coupling between airfoil geometry and flow control parameters can only be captured through the integrated approach, which is essential for identifying improved Pareto-optimal solutions within the predefined integrated design space.

**Figure 12.** Pareto front comparison for the flow control-only case, shape-only case, and the integrated case.

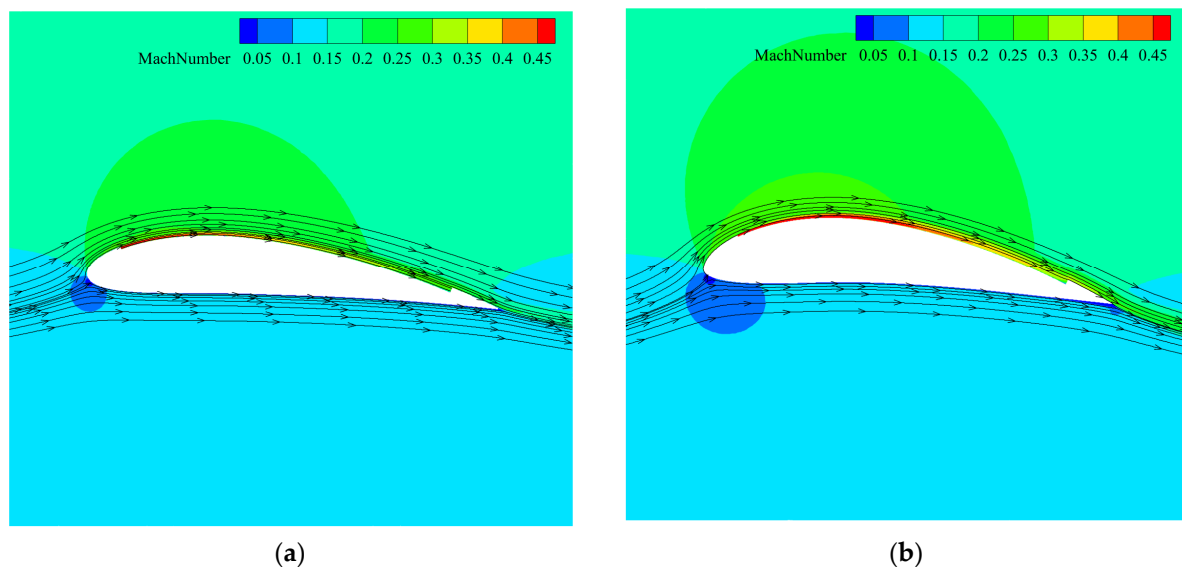
As shown in Figures 13 and 14, which present the Mach number contours, streamlines, and pressure coefficient (C_p) distribution plots for both the baseline and optimized airfoils, it is evident that the baseline airfoil exhibits a limited acceleration region on its upper surface. The flow control effect is relatively weak, as indicated by a smaller suction peak in the C_p distribution, reflecting limited airflow acceleration and a flow state that, while balanced, lacks significant lift enhancement capability.

Opt1 generates a large high-speed region on the upper surface, resulting in a substantial increase in the lift coefficient. As illustrated in the C_p distribution, although Opt3 captures a higher localized peak right at the injection location, Opt1 maintains the highest and most sustained low-pressure platform across the chord from $x/c = 0.1$ to 0.8 . However, the continuous momentum input required to maintain such high acceleration leads to a substantial increase in energy consumption.

Opt2 successfully enhances the upper-surface acceleration region while maintaining flow attachment; with a moderate suction peak and sustained low pressure across the mid-chord, it achieves a superior balance between aerodynamic performance and power consumption.

Opt3 demonstrates an intense velocity acceleration concentrated near the leading-edge injection slot, creating the highest localized suction peak in the C_p distribution around $x/c = 0.08$. Downstream of this peak, the airflow acceleration tapers off uniformly along the upper surface, maintaining stable flow with minimal momentum input, thereby achieving the lowest effective drag coefficient and the highest effective lift-to-drag ratio.

These results demonstrate that an optimal CFJ configuration and jet input can achieve an effective trade-off between aerodynamic performance and energy consumption by coordinating the Mach number distribution, flow field structure, and pressure gradient variations.



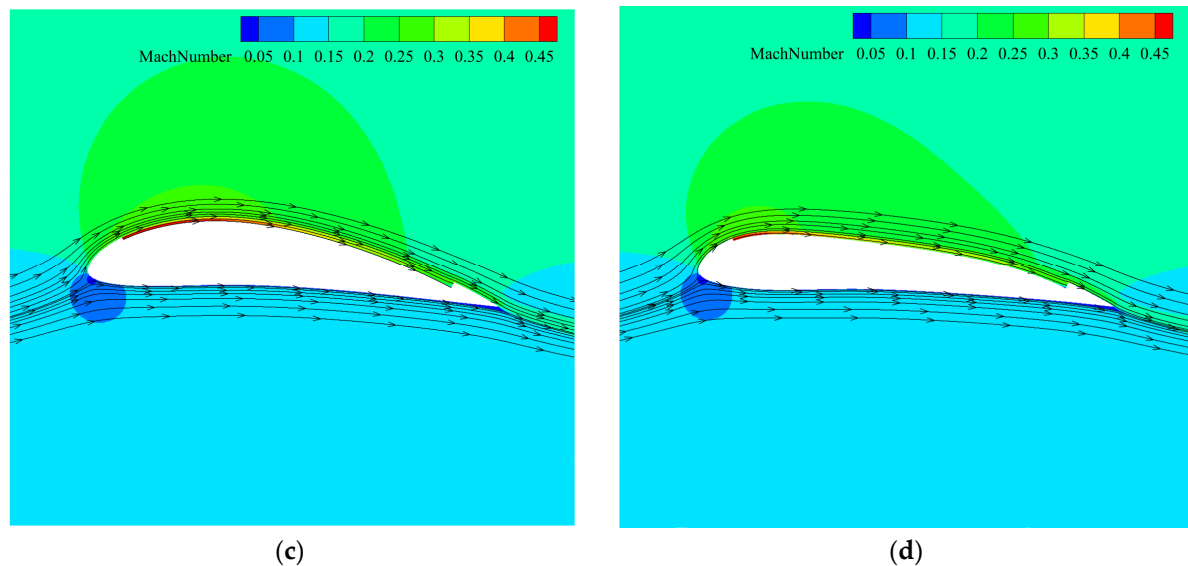


Figure 13. Mach number contours and streamlines for the baseline and optimized airfoils under design operating conditions. (a) CFJ-Baseline; (b) Opt1; (c) Opt2; (d) Opt3.

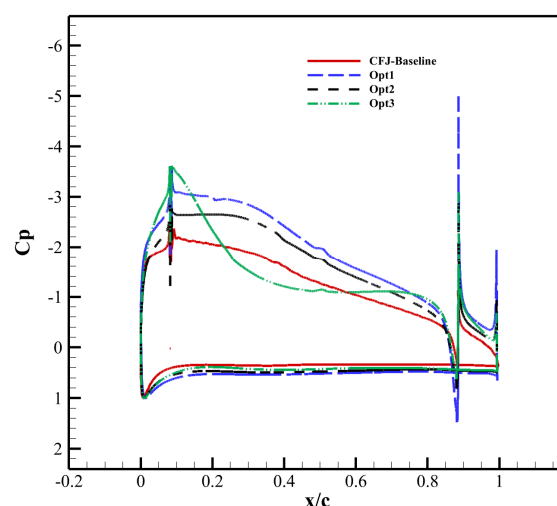


Figure 14. Comparison of pressure coefficient distributions between the baseline and optimized airfoils.

To further examine the boundary-layer behavior associated with the extended low-pressure region of Opt1, velocity profiles normal to the upper surface were extracted at three chordwise stations, namely station A at $x/c = 26.2\%$, station B at $x/c = 52.5\%$, and station C at $x/c = 78.7\%$, as shown in Figure 15. Here, H denotes the wall-normal distance from the airfoil surface, and Vel denotes the local velocity magnitude.

As shown in Figure 15, Opt1 exhibits significantly higher velocity levels near the upper surface than the CFJ-Baseline, Opt2, and Opt3 at all three chordwise stations. This tendency is particularly pronounced at stations A and B, where the velocity peak and the near-wall velocity level of Opt1 are significantly higher than those of the other configurations. At station C, although the difference becomes smaller, Opt1 still maintains a higher velocity level over the upper surface. These results indicate that the extended low-pressure region of Opt1 is accompanied by stronger flow acceleration.

The higher boundary-layer velocity of Opt1 is consistent with its lift-oriented design feature. Stronger acceleration over the suction surface enhances the pressure difference between the upper and lower surfaces and contributes to the increase in lift coefficient.

However, this improvement is achieved with stronger CFJ actuation and higher power consumption, which is consistent with the increased power coefficient and the reduced effective lift-to-drag ratio of Opt1 discussed above.

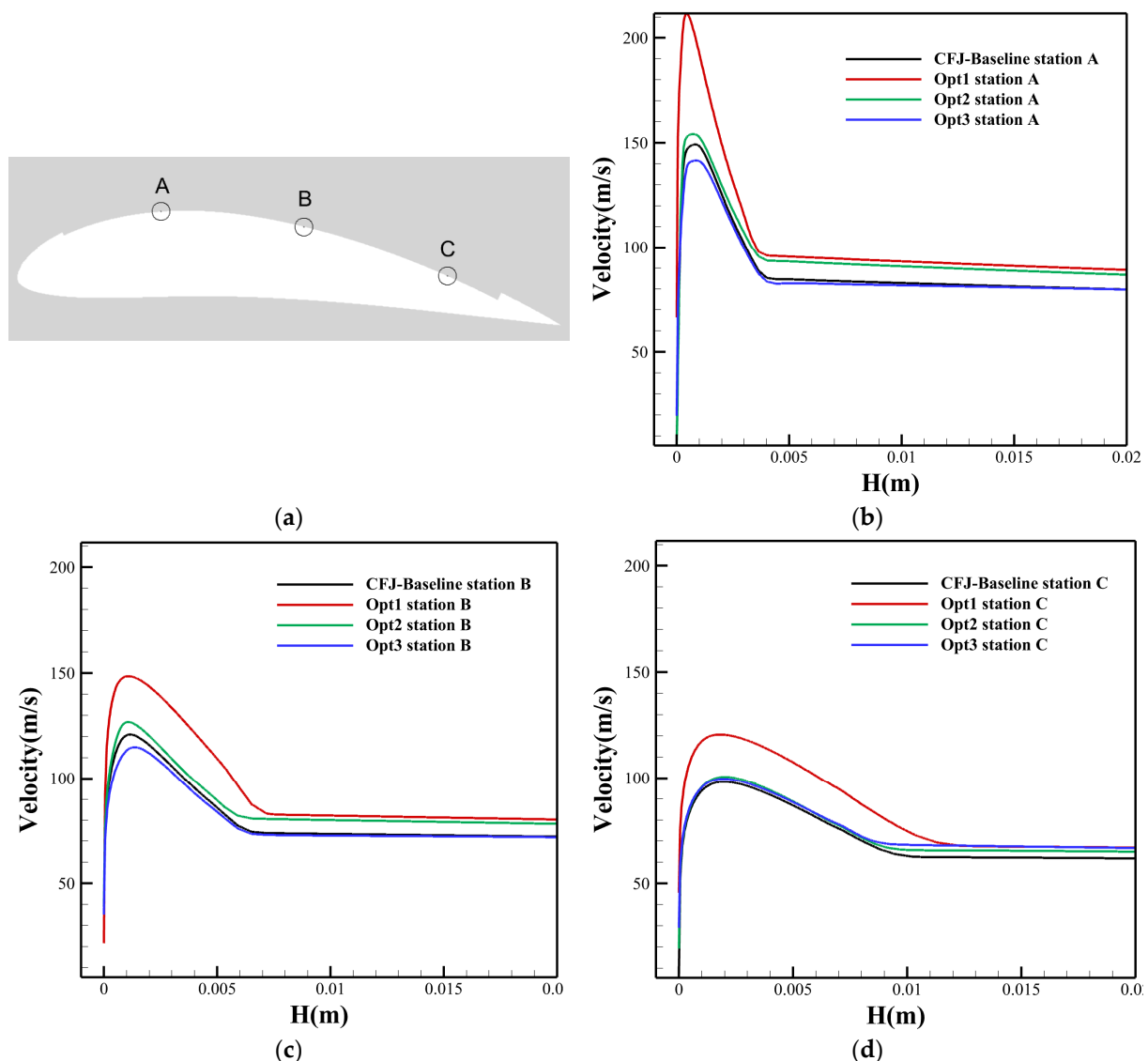


Figure 15. Boundary-layer velocity profiles on the upper surface of the CFJ airfoils: (a) locations of the three extraction stations; (b) station A at $x/c = 26.2\%$; (c) station B at $x/c = 52.5\%$; (d) station C at $x/c = 78.7\%$. H denotes the wall-normal distance from the airfoil surface, and Velocity denotes the local velocity magnitude.

Since the power consumption of a practical CFJ system depends on the efficiency of the internal compressor, a parametric analysis was further conducted to evaluate the influence of compressor efficiency on the energy-related performance. In the baseline optimization, the compressor efficiency was set to $\eta = 100\%$ to focus on the aerodynamic coupling between the airfoil geometry and CFJ parameters, which is consistent with previous CFJ numerical studies [24]. Previous studies also reported that typical micro-compressors for aerospace applications can achieve efficiencies of approximately 85%, and the efficiency may be even higher for larger-scale applications [34]. Therefore, four compressor efficiencies, namely $\eta = 85\%$, 90%, 95%, and 100%, were considered in the present sensitivity analysis.

For a given CFJ configuration and a fixed nominal jet momentum coefficient, changing the compressor efficiency does not alter the external aerodynamic flow field. Therefore, the lift coefficient and aerodynamic drag coefficient remain unchanged. The influence of η is reflected in the power coefficient, effective drag coefficient, and effective lift-to-drag ratio. As shown in Figure 16, decreasing η increases the power coefficient and the effective drag coefficient, and consequently reduces the effective lift-to-drag ratio.

For all CFJ configurations, the effective lift-to-drag ratio decreases as the compressor efficiency decreases. This trend is most evident for Opt1 because it has the largest power coefficient and is therefore more sensitive to compressor efficiency. Nevertheless, Opt1 remains a lift-oriented configuration rather than an energy-efficiency-oriented design. For Opt2 and Opt3, the decrease in compressor efficiency also reduces the effective lift-to-drag ratio, but their relative advantages over the CFJ-Baseline are retained. In particular, Opt3 maintains the highest effective lift-to-drag ratio among all configurations even when η is reduced to 85%, indicating that its energy-efficiency-oriented characteristic remains valid under realistic compressor-efficiency assumptions.

These results indicate that the ideal compressor-efficiency assumption affects the absolute value of the effective lift-to-drag ratio but does not change the main performance ranking of the representative Pareto solutions.

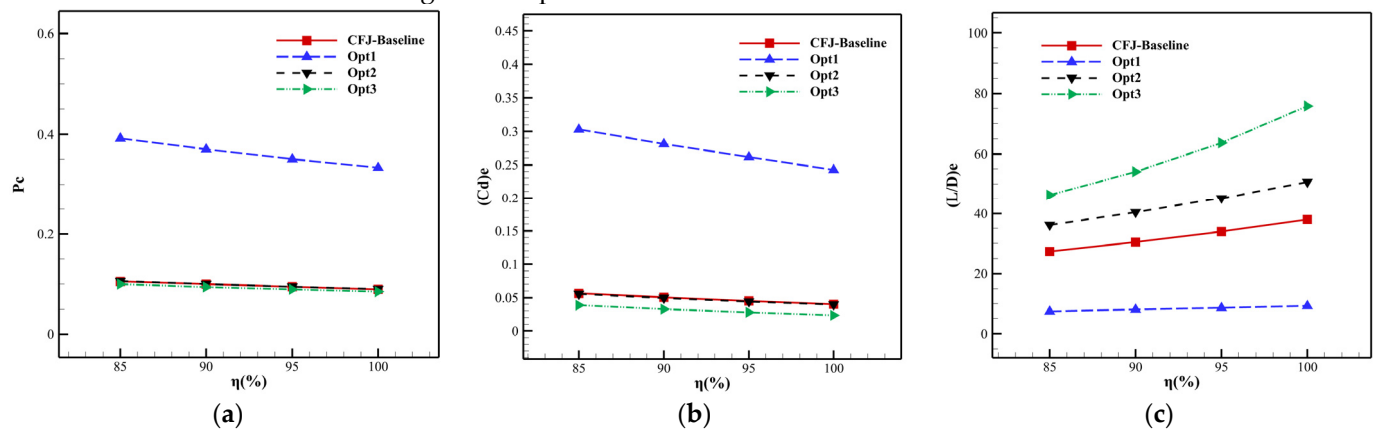


Figure 16. Influence of compressor efficiency on the energy-related performance of the CFJ-Baseline and optimized airfoils under the design operating condition: (a) power coefficient; (b) effective drag coefficient; (c) effective lift-to-drag ratio.

Given the inherent flexibility of active flow control, the full-envelope performance of the CFJ airfoil is determined by the coupling between its geometric configuration and jet intensity. For the optimization design proposed in this study, the primary focus is on its stability near the design operating condition. Therefore, its aerodynamic robustness is validated through numerical simulations across a specific range of angles of attack, while high-angle-of-attack conditions are managed by modulating the jet control intensity.

As shown in Figure 17, the variations in aerodynamic coefficients for both the baseline and optimized airfoils are presented across the angle of attack range of 2° – 8° . In summary, within this range, all optimized CFJ airfoils demonstrate excellent aerodynamic robustness and stable performance trends. Opt1 exhibits strong lift-enhancement control characteristics; Opt2 achieves a balanced trade-off between lift enhancement and energy consumption; and Opt3 demonstrates high-efficiency flow control advantages, characterized by the lowest effective drag and the highest effective lift-to-drag ratio. These results further indicate that a reasonable matching of CFJ geometric parameters and jet intensity can achieve stable and efficient aerodynamic performance near the design operating condition.

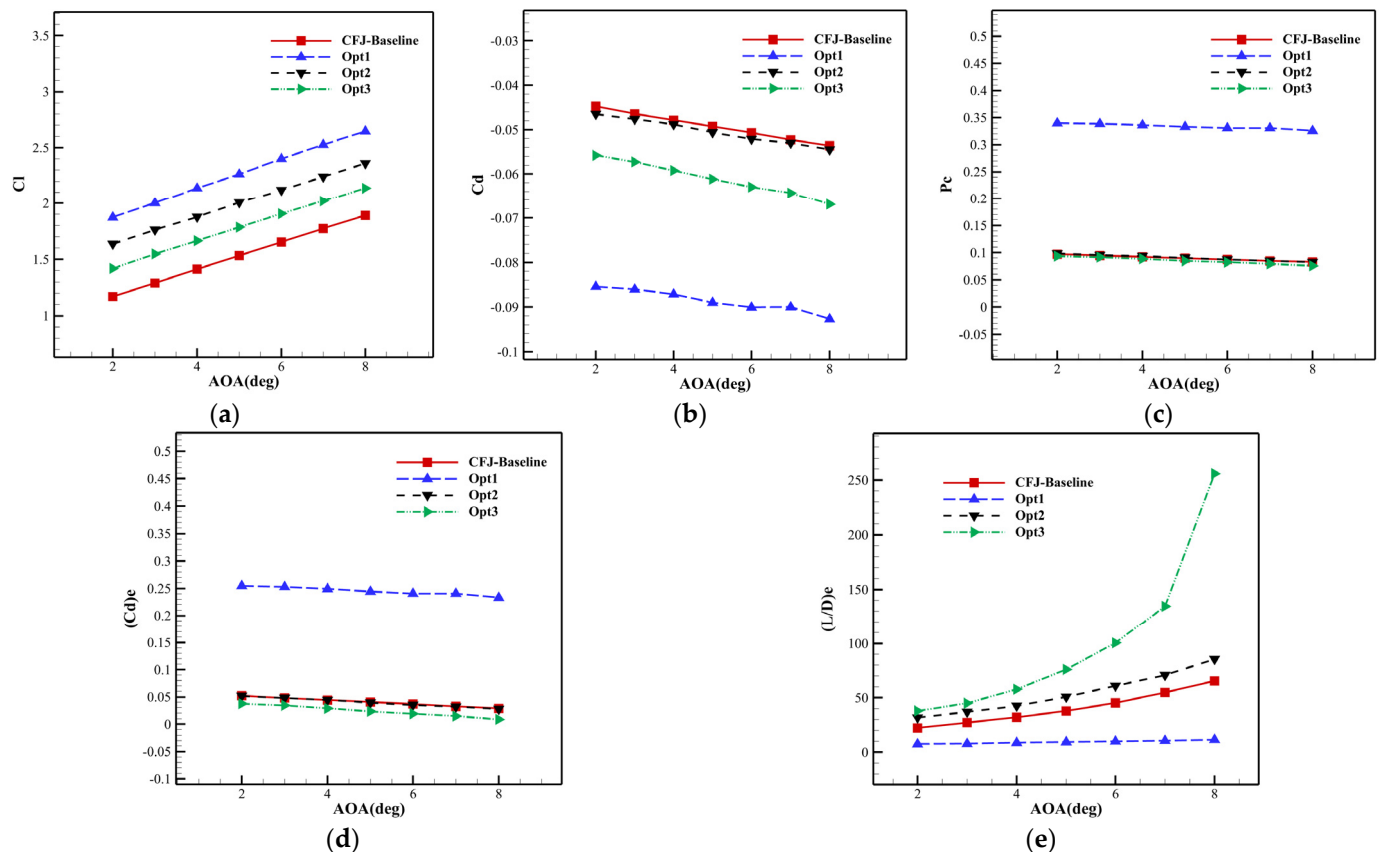


Figure 17. The aerodynamic performance variations with respect to the angle of attack. (a) Lift coefficient versus angle of attack; (b) Drag coefficient versus angle of attack; (c) Power coefficient versus angle of attack; (d) Effective drag coefficient versus angle of attack; (e) Effective lift-to-drag ratio versus angle of attack.

To further evaluate whether the optimized configurations retain their advantages beyond the single design point, additional off-design computations were performed by varying the Mach number and Reynolds number, respectively. In the first group of computations, the Mach number was varied from 0.10 to 0.25 while the Reynolds number was kept at the design value. In the second group, the Reynolds number was varied from 5.13×10^5 to 2.052×10^6 while the Mach number was kept at 0.15. The angle of attack was fixed at 5° in both groups. For each configuration, the nominal jet momentum coefficient was kept the same as that used in the optimization, so that the comparison reflects the off-design aerodynamic response of the optimized airfoil geometry and CFJ parameter combination.

The influence of Mach number on the aerodynamic performance is shown in Figure 18. Within the investigated Mach number range, the main performance ranking of the optimized configurations is maintained. Opt1 consistently provides the highest lift coefficient and the most pronounced drag reduction, confirming its lift-oriented characteristic. However, because of its much higher power coefficient, its effective lift-to-drag ratio remains low. By contrast, Opt2 and Opt3 maintain significantly higher effective lift-to-drag ratios than the CFJ-Baseline over the whole Mach number range. In particular, Opt3 remains the most energy-efficiency-oriented configuration, although its effective lift-to-drag ratio gradually decreases as the Mach number increases.

The influence of Reynolds number is shown in Figure 19. The optimized configurations also retain their characteristic performance trends over the Reynolds number range from 5.13×10^5 to 2.052×10^6 . Opt1 maintains the highest lift coefficient, while Opt2 and Opt3 continue to provide improved effective aerodynamic efficiency

compared with the CFJ-Baseline. The effective lift-to-drag ratio of Opt3 remains the highest among all configurations and shows a generally increasing trend with Reynolds number. These results indicate that the advantages of the representative optimized airfoils are not restricted to the original design point. Instead, the lift-oriented, balanced, and energy-efficiency-oriented characteristics of Opt1, Opt2, and Opt3 are retained within the examined Mach number and Reynolds number ranges.

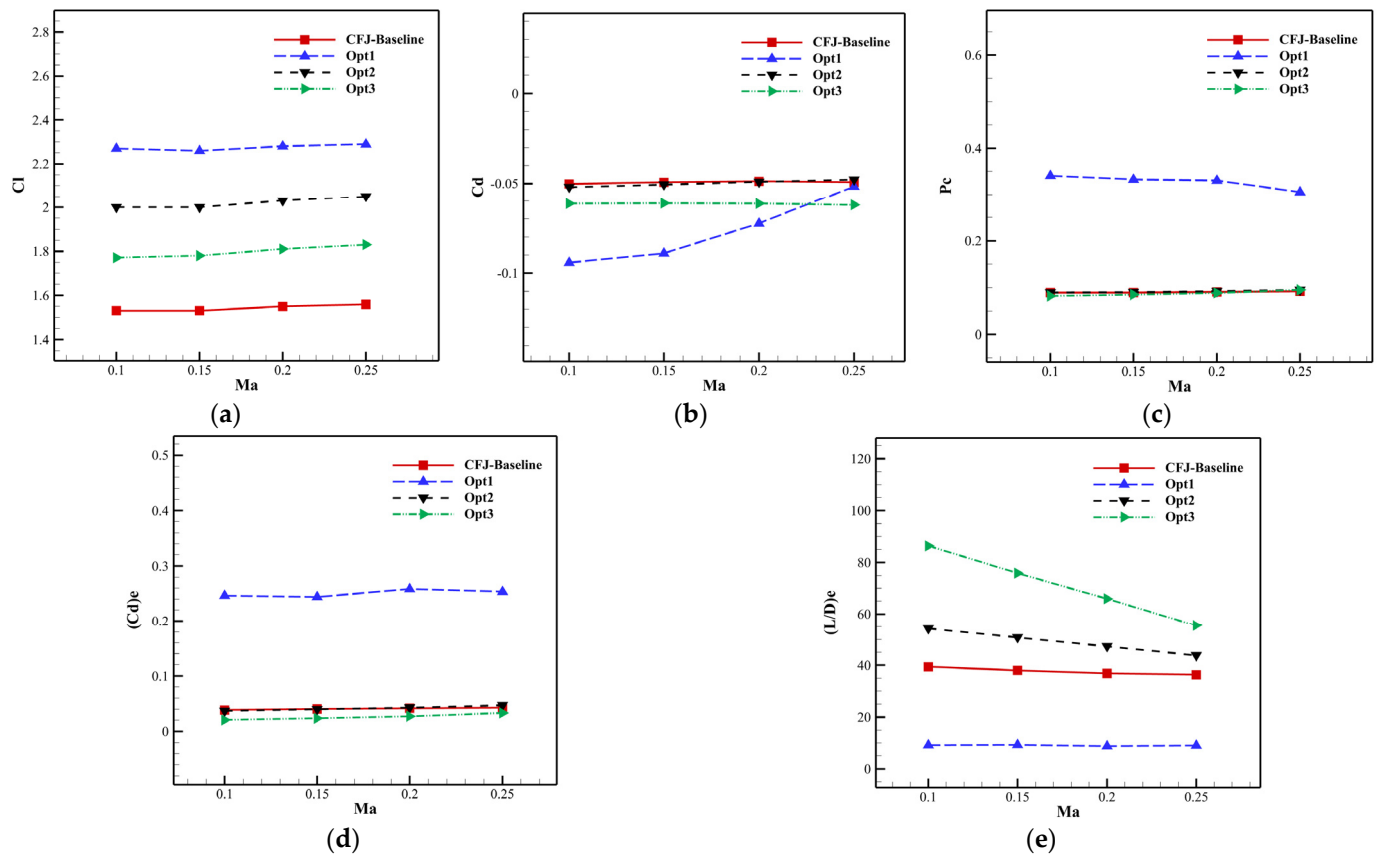
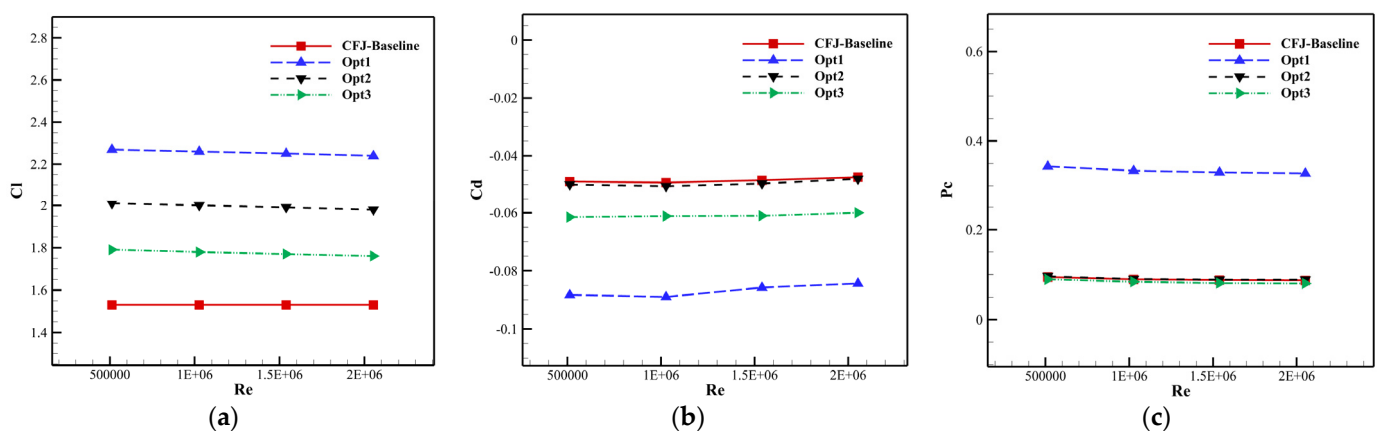


Figure 18. Influence of Mach number on the aerodynamic performance of the CFJ-Baseline and optimized airfoils at Reynolds number 1.026×10^6 and an angle of attack of 5° : (a) lift coefficient; (b) drag coefficient; (c) power coefficient; (d) effective drag coefficient; (e) effective lift-to-drag ratio.



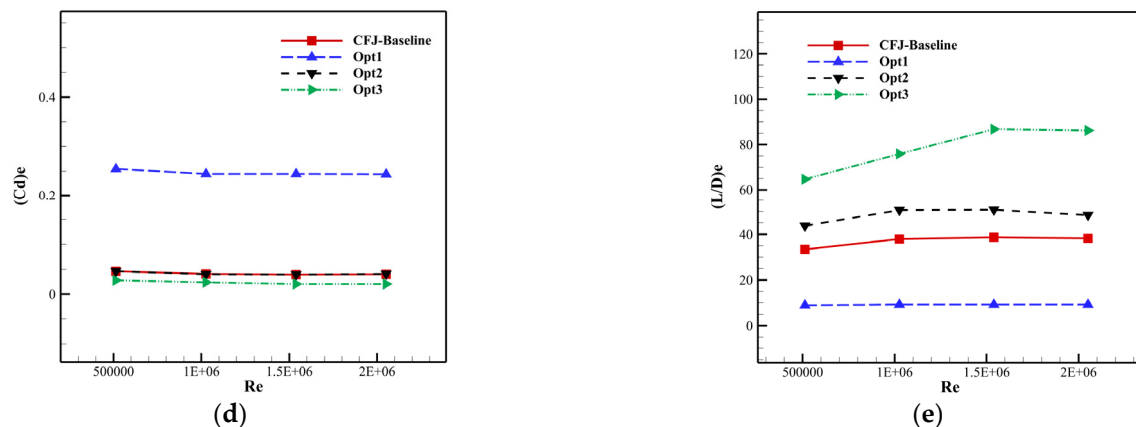


Figure 19. Influence of Reynolds number on the aerodynamic performance of the CFJ-Baseline and optimized airfoils at Mach number 0.15 and an angle of attack of 5°: (a) lift coefficient; (b) drag coefficient; (c) power coefficient; (d) effective drag coefficient; (e) effective lift-to-drag ratio.

Overall, the three optimization schemes represent distinct design priorities. Opt1 focuses on maximum lift enhancement, while Opt2 emphasizes a balance of comprehensive performance. In contrast, Opt3 pursues the maximization of energy efficiency. During the design optimization process, the coupling relationship between lift, drag, and power consumption should be considered holistically to select the appropriate CFJ airfoil based on specific mission requirements.

It should also be noted that the present optimization is performed in a two-dimensional airfoil setting. This configuration can be regarded as an idealized limiting case of an infinite-span, unswept, spanwise-uniform continuous CFJ configuration. Therefore, the present results are mainly intended to verify the feasibility and effectiveness of the proposed integrated optimization framework for simultaneously considering aerodynamic shape parameters and CFJ flow-control parameters.

For practical three-dimensional CFJ wings, additional effects may influence the transferability of the optimized parameters obtained from the two-dimensional study. These effects include finite-span end vortices, spanwise non-uniformity of the jet, three-dimensional vortex structures induced by discrete or non-uniform injection, chordwise interaction between the injection and suction channels, and wing-induced flow effects. Previous studies on discrete CFJ configurations have shown that streamwise and spanwise vortex structures can significantly enhance the mixing between the high-speed jet, the freestream, and the boundary layer, thereby affecting lift enhancement, drag reduction, and energy utilization [28]. Therefore, the optimized slot locations, slot sizes, airfoil geometry, and jet parameters obtained in the present two-dimensional study have limitations when applied to practical three-dimensional CFJ wings and require further verification.

The extension of the proposed integrated optimization framework to three-dimensional CFJ wings, including finite-span effects, spanwise jet distribution, internal-channel layout, and wing planform parameters, will be considered in future work. The optimized CFJ airfoils in this study can provide suitable initial profiles for CFJ wing optimization.

6. Conclusions

This work presents an integrated optimization framework based on combined parameterization for the aerodynamic shape and flow control parameters of CFJ airfoils. The key findings are summarized below:

1. A composite geometric representation method integrating CST parametrization and linear interpolation for the sunken section was established. This method enables global control of the baseline airfoil shape through CST parametrization, characterizes flow control geometric parameters via the positions and sizes of the injection/suction slots, and employs linear interpolation to achieve adaptive geometric correlation between the sunken section curve and the baseline airfoil, effectively resolving the geometric distortion problems of the sunken section inherent in traditional additive construction methods.
2. An integrated design space containing airfoil CST parameters, CFJ geometric design parameters, and jet control parameters was constructed, and an efficient surrogate-based aerodynamic optimization framework was established based on the SurroOpt software. By constructing an initial Kriging surrogate model through Latin Hypercube Sampling and adaptively updating it using the hybrid MSP and EI infill criteria, the collaborative optimization of multiple design variables was achieved, providing an effective technical means for the integrated design of CFJ airfoil aerodynamic shapes and flow control parameters.
3. Taking the NACA6415 airfoil as the foundation, 35 Pareto-optimal solutions were obtained through the integrated multi-objective optimization. The lift-oriented configuration Opt1 increases the lift coefficient from 1.53 to 2.26, corresponding to an improvement of 47.7%. The balanced configuration Opt2 provides a 30.7% increase in lift coefficient together with a 33.5% improvement in the effective lift-to-drag ratio, which reaches 50.69. The energy-efficiency-oriented configuration Opt3 increases the effective lift-to-drag ratio from 37.97 to 75.84 while maintaining a relatively low power coefficient. The flow-field, pressure-distribution, and boundary-layer velocity-profile analyses demonstrate that the different Pareto solutions achieve distinct compromises among upper-surface acceleration, circulation enhancement, aerodynamic drag, and power consumption. An a posteriori consistency assessment of the final regularized Kriging model was performed using the complete database of 1000 CFD-evaluated samples. The maximum absolute relative deviations are 1.60% for obj1 and 2.61% for obj2 over the complete database, and 0.46% for obj1 and 0.90% for obj2 at the representative Pareto solutions Opt1, Opt2, and Opt3. All Pareto solutions and aerodynamic-performance values reported in this study are obtained from direct CFD evaluations rather than surrogate-only predictions. Additional evaluations over angles of attack from 2° to 8°, Mach numbers from 0.10 to 0.25, Reynolds numbers from 5.13×10^5 to 2.052×10^6 , and compressor efficiencies from 85% to 100% show that the principal design characteristics of the representative solutions are retained within the examined two-dimensional low-speed ranges. Opt1 remains lift-oriented, Opt2 maintains a balanced performance compromise, and Opt3 remains the most energy-efficiency-oriented configuration. These results indicate that the advantages of the integrated optimization are not restricted to the single design condition, although further multi-point optimization and three-dimensional wing verification are required for practical applications.

In conclusion, the composite parametrization method enables integrated modeling and integrated optimization of the aerodynamic shape and flow control parameters for CFJ airfoils. CFJ parameters are accordingly transformed from auxiliary control variables into core design drivers of airfoil performance. This work offers a new design reference for the high-performance development of CFJ airfoils and can be further extended to the integrated parametric optimization of other active flow control strategies, including blowing, suction, and plasma actuation.

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Abbreviations

The following abbreviations are used in this manuscript:

CFJ	Co-Flow Jet
ZNMF	Zero-Net-Mass-Flux
CST	Class Shape Transformation
LES	Large Eddy Simulation
CFD	Computational Fluid Dynamics
N-S	Navier–Stokes
DNS	Direct Numerical Simulation
RANS	Reynolds-Averaged Navier–Stokes
MSP	Multi-objective Selection Procedure
EI	Expected Improvement

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