

Article

Earth-Point Stabilization Performance Using Asynchronous Star Tracker and Angular Velocity Sensor Measurements

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Abstract

The paper proposes two attitude motion determination algorithms based on an extended Kalman filter dealing with asynchronous measurements of the star tracker and angular velocity sensor. The satellite control system tracks a complex angular velocity profile during the Earth-point stabilization of the remote sensing camera axis. The star tracker measurement accuracy decreases with higher satellite angular velocity; when a certain angular velocity value threshold is exceeded, the star tracker measurements might be unavailable. These star tracker aspects, along with the variable bias of the angular velocity sensor, are addressed by the developed algorithms. The first algorithm uses kinematic relations; it estimates the attitude quaternion and the angular velocity sensor bias. It does not require information on satellite inertia parameters and on current control torque; it is characterized by low computational burden, though the angular velocity estimation accuracy is limited by the standard deviation of the sensor random noise. The second algorithm is based on both kinematic and dynamic motion equations: it estimates the attitude quaternion, angular velocity sensor bias, and angular velocity vector as well. The satellite tensor of inertia, reaction wheels' parameters, and history of control inputs are required for the state vector estimation. The performance of these algorithms is compared under the scenario of Earth-point stabilization attitude motion, taking into account different levels of angular velocity measurement errors. It is obtained that the algorithm based on kinematic equations only is characterized by lower stabilization and estimation accuracies compared to the algorithm based on both kinematic and dynamic motion equations, though the latter is significantly more computationally complex. The influence of the state vector estimation errors on the Earth-point stabilization accuracy is studied. The paper contribution is an algorithm performance study considering star-tracker accuracy degradation with angular velocity during the Earth-point flyby.

Keywords: attitude stabilization; attitude determination; star tracker; extended Kalman filter; angular velocity sensor; asynchronous measurements

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1. Introduction

The accuracy of the satellite stabilization directly affects the quality of images obtained by remote sensing satellites. In the case that the observing Earth point shifts more than the pixel size on the camera sensor during the exposure time, the resulting image is blurred. The characteristics of the on-board camera define the requirements on the accuracy of the satellite attitude determination and control system (ADCS). High performance

of the attitude motion control for remote sensing satellites is usually provided by reaction wheels-based systems, while real-time attitude motion estimation is obtained via processing of the star tracker and angular velocity sensor measurements. Since the system state vector estimation is an input for the control system, the estimation errors influence the motion stabilization errors along with control implementation errors and disturbance torques acting on the satellite. Thus, the study of ADCS performance is a crucial step in remote sensing satellite mission design.

A problem of satellite attitude real-time determination using star tracker and angular velocity sensor measurements is well studied in the literature [1–6], though there are still several issues to be dealt with. First of all, it involves asynchronous measurements, when information on the attitude quaternion and on the angular velocity vector is available onboard at different time moments and received with different frequencies. In addition, attitude and angular velocity estimations are required at the time moments of the control inputs calculation, which in general does not match the time of any measurements. The second issue is a dependence of star tracker measurements accuracy on the satellite angular velocity: at higher angular rates the accuracy degrades [7], and attitude measurements might even become unavailable when a certain angular velocity value threshold is exceeded [8]. This issue is especially essential for remote sensing satellites: when the camera optical axis tracks the point of interest on the Earth's surface, the angular velocity increases significantly during the flyby. The third issue is the variable angular velocity sensor bias: it may significantly drift over time, and it must be estimated in real time.

Many papers are devoted to the processing of the images of the star trackers in different conditions and scenarios [9–11]. The paper [12] proposes an algorithm dealing with images with a large number of dense stars to provide robust and real-time star identification. A star identification algorithm based on dynamic distance ratio matching is developed in [13], where the angles between any neighboring stars and the distance ratio between neighboring stars are used for the matching criteria calculations. A masked distance map is designed in [14] to provide a shortlist of stars for the 'lost-in-space' mode for star identification algorithms for CubeSat applications. In many papers, the star tracker measurements are directly used for control calculation, disregarding the measurement delays. This approach can be acceptable in the case of low angular velocity or three-axis stabilization in an inertial reference frame. At high angular velocity, the delay must be taken into account in measurement processing. In paper [15], a time-delayed integration technique is applied to provide autonomous spacecraft attitude estimates at 10 Hz and at spacecraft angular rates up to 10 rpm. A set of different methods are considered for incorporating delayed measurements in the attitude filter, and their performance is evaluated for the case of Gokturk-2 satellite [16]. In paper [17], the angular velocity estimation is calculated by application of a least-squares algorithm for a linearized system using star tracker measurements only, though the estimation time interval is restricted by a certain value depending on the angular velocity. The restriction on low angular velocity is overcome in [18] by considering the q-method and quaternion kinematic equations. Taking into account measurement delays is especially crucial for the star trackers with rolling shutter exposure when each row of the image is acquired during satellite-controlled motion [8]. The paper [19] proposes the application of particle swarm optimization for star tracker angular velocity estimation to correct the star centroid position in the image. Beyond the challenges of asynchronous and rate-dependent measurements, on-orbit thermal deformations can introduce systematic inter-boresight angle shifts in star trackers, necessitating temperature-dependent calibration and multi-sensor data fusion for high-precision angular velocity reconstruction [20]. To mitigate the degradation of star tracker measurement accuracy under high angular rates and accelerations, advanced centroid localization methods based

on motion trajectory modeling and particle swarm optimization have been developed to extend the operational range of star trackers during agile maneuvers [21].

The paper proposes two attitude determination algorithms based on the extended Kalman filter, which is able to deal with asynchronous measurements, dependence of star tracker measurement accuracy on the satellite angular velocity, and angular velocity sensor bias. The first algorithm uses kinematic relations; it estimates only the angular velocity sensor bias and attitude quaternion. It does not require information on satellite parameters; it is characterized by less computational burden, though the angular velocity estimation accuracy is limited by the standard deviation of the sensor random noise. The second algorithm is based on both kinematic and dynamic motion equations: it estimates angular velocity sensor bias, angular velocity, and attitude quaternion. The satellite tensor of inertia, reaction wheels parameters, and history of control inputs are required for state vector estimation. The main contribution of this paper is the proposed algorithms' performance study considering star-tracker accuracy degradation with angular velocity during the stabilization of the camera axis to the fixed Earth-point.

This paper has the following structure. First, the mathematical description of the proposed algorithms is provided. The reference motion construction for Earth-point tracking by the optical axis is explained. The numerical study results of the performance of the two algorithms are provided; their accuracy is compared and discussed.

2. Preliminaries

2.1. Equations of Motion and Coordinate Systems

There are several coordinate systems utilized in this paper:

- $Ox_1x_2x_3$ —Inertial Frame (IF), origin O is located in the Earth center of mass, axes coincide with the Geocentric Celestial Reference Frame.
- $OX_1X_2X_3$ —Greenwich Frame (GF), OX_3 corresponds to the Earth rotation axis, OX_1 points to the Greenwich meridian, OX_2 complements this system to the right-handed Cartesian. This system is close to the International Terrestrial Reference Frame. For simplicity, we suppose that the transition from IF to GF is described by rotation along the Ox_3 axis with a constant Earth angular velocity ω_E .
- $Sy_1y_2y_3$ —Body Frame (BF), origin S is located in the spacecraft center of mass, axes are fixed with respect to the satellite.
- $SY_1Y_2Y_3$ —Reference Frame (RF), corresponds to the necessary (or reference) attitude motion, which is dictated by mission requirements. Main goal of the control law is to ensure that BF and RF coincide.

Satellite's equations of motion take into account both translational and attitude motion:

$$\begin{aligned}\dot{\Lambda} &= \frac{1}{2} \Lambda \circ \omega, \\ \dot{\mathbf{r}} &= \mathbf{V}, \quad \dot{\mathbf{V}} = -\mu \frac{\mathbf{r}}{r^3}, \\ \dot{\omega} &= \mathbf{J}^{-1} \left(-\omega \times \mathbf{J} \omega + 3 \frac{\mu}{r^3} \mathbf{e}_r \times \mathbf{J} \mathbf{e}_r + \mathbf{M}_{dist} + \mathbf{M}_{ctrl} \right).\end{aligned}\tag{1}$$

Here $\mathbf{r}, \mathbf{V}$ are the satellite's radius-vector and velocity, μ is the Earth gravity parameter, $\Lambda = [\lambda_0, \boldsymbol{\lambda}]$ is the satellite attitude quaternion (λ is its scalar part, $\boldsymbol{\lambda}$ is its vector part expressed in BF), ω is the absolute angular velocity expressed in BF, $\mathbf{J}$ is the tensor of inertia, $\mathbf{e}_r = \mathbf{r}/r$ and is given in BF, $\mathbf{M}_{dist}$ is disturbance torque affecting the

satellite (e.g., atmospheric drag torque, solar radiation pressure torque etc.), $\mathbf{M}_{ctrl}$ is the control torque, “ $\times$ ” denotes cross product and the skew-symmetric matrix of cross-product is introduced:

$$\mathbf{a} \times \mathbf{b} = \begin{pmatrix} a_2 b_3 - a_3 b_2 \\ a_3 b_1 - a_1 b_3 \\ a_1 b_2 - a_2 b_1 \end{pmatrix} = [\mathbf{a}]_{\times} \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix},$$

$$[\mathbf{a}]_{\times} = \begin{pmatrix} 0 & -a_3 & a_2 \\ a_3 & 0 & -a_1 \\ -a_2 & a_1 & 0 \end{pmatrix}.$$

Quaternion product is denoted “ $\circ$ ”:

$$\mathbf{Q} \circ \mathbf{N} = \begin{pmatrix} q_0 \\ \mathbf{q} \end{pmatrix} \circ \begin{pmatrix} n_0 \\ \mathbf{n} \end{pmatrix} = \begin{pmatrix} q_0 n_0 - (\mathbf{q}, \mathbf{n}) \\ q_0 \mathbf{n} + n_0 \mathbf{q} + \mathbf{q} \times \mathbf{n} \end{pmatrix},$$

and $(\cdot, \cdot)$ denote the dot product.

Note that the main goal of this paper is the investigation of attitude determination algorithms, so such a simple model of orbital motion (central Newtonian field, simple rotation between IF and GF) is sufficient.

2.2. Conventional Kalman Filters

Kalman filter is a recursive algorithm that uses a dynamic system model and sensor measurements to estimate the system state vector [22]. First, consider the classical discrete Kalman Filter. Let the state vector estimation and its covariance matrix be denoted $\hat{\mathbf{x}}$ and $\mathbf{P}$, respectively. The measurements are available at time moments t_k , and it is required to obtain state vector estimation at the same moments $\hat{\mathbf{x}}_k = \hat{\mathbf{x}}(t_k)$. The discrete Kalman filter consists of two stages: prediction and correction (see Figure 1). Consider that for t_{k-1} the state vector estimation $\hat{\mathbf{x}}_{k-1}^+$ and its covariance matrix $\mathbf{P}_{k-1}^+$ are given, and our goal is to provide an estimation of the state vector $\hat{\mathbf{x}}_k^+$ for time point t_k . First, it is necessary to propagate $\hat{\mathbf{x}}_{k-1}^+$ to the current moment t_k , thus obtaining the «a priori» estimation $\hat{\mathbf{x}}_k^-$. Then it is necessary to process the measurement vector $\mathbf{z}_k$ to obtain the «a posteriori» estimation $\hat{\mathbf{x}}_k^+$. The covariance matrix $\mathbf{P}_k^-$ is calculated using the Riccati equation, and after measurement processing, the value $\mathbf{P}_k^+$ is obtained.

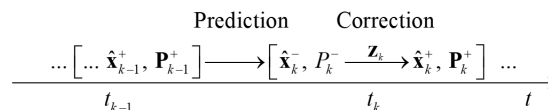


Figure 1. Kalman filter scheme.

Extended Kalman filter (EKF) is applied if the motion equations and/or measurement model are nonlinear as follows:

$$\dot{\mathbf{x}}(t) = \mathbf{f}(\mathbf{x}, t) + \mathbf{G}\mathbf{w}(t), \quad (2)$$

$$\mathbf{z}(t) = \mathbf{h}(\mathbf{x}, t) + \mathbf{v}(t). \quad (3)$$

Here $\mathbf{w}(t)$ is the normally distributed system noise vector with covariance matrix $\mathbf{D}$, $\mathbf{G}$ is the system noise influence matrix, $\mathbf{v}(t)$ is a normally distributed measurement vector with covariance matrix $\mathbf{R}$. To apply the EKF, the nonlinear functions

$\mathbf{f}(\mathbf{x}, t)$ and $\mathbf{h}(\mathbf{x}, t)$ are linearized in the vicinity of the current state vector estimation [23]. The matrices of dynamics and measurements are calculated as follows:

$$\mathbf{H}_k = \left. \frac{\partial \mathbf{h}(\mathbf{x}, t)}{\partial \mathbf{x}} \right|_{\mathbf{x}=\hat{\mathbf{x}}_k^-, t=t_k}, \quad \mathbf{F}_k = \left. \frac{\partial \mathbf{f}(\mathbf{x}, t)}{\partial \mathbf{x}} \right|_{\mathbf{x}=\hat{\mathbf{x}}_k^-, t=t_k}. \quad (4)$$

The EKF prediction stage is described by the following:

$$\begin{aligned} \hat{\mathbf{x}}_k^- &= \hat{\mathbf{x}}_{k-1}^+ + \int_{t_{k-1}}^{t_k} \mathbf{f}(\hat{\mathbf{x}}_{k-1}^+, t) dt; \\ \mathbf{P}_k^- &= \mathbf{\Phi}_k \mathbf{P}_{k-1}^+ \mathbf{\Phi}_k^T + \mathbf{Q}_k, \end{aligned} \quad (5)$$

where

$$\begin{aligned} \mathbf{Q}_k &= \int_{t_{k-1}}^{t_k} \mathbf{\Phi}_k \mathbf{G} \mathbf{D} \mathbf{G}^T \mathbf{\Phi}_k^T dt, \\ \mathbf{\Phi}_k &= e^{\mathbf{F}_k(t_k - t_{k-1})}. \end{aligned}$$

The correction stage:

$$\begin{aligned} \mathbf{K}_k &= \mathbf{P}_k^- \mathbf{H}_k^T (\mathbf{H}_k \mathbf{P}_k^- \mathbf{H}_k^T + \mathbf{R}_k)^{-1}, \\ \hat{\mathbf{x}}_k^+ &= \hat{\mathbf{x}}_k^- + \mathbf{K}_k [\mathbf{z}_k - \mathbf{h}(\hat{\mathbf{x}}_k^-, t_k)], \\ \mathbf{P}_k^+ &= [\mathbf{E} - \mathbf{K}_k \mathbf{H}_k] \mathbf{P}_k^-. \end{aligned} \quad (6)$$

For EKF-based algorithms using different sensors sets and with different state vectors it is essential to define matrix of dynamics $\mathbf{F}$, measurement matrix $\mathbf{H}$, system noise influence matrix $\mathbf{G}$, and also set the covariance matrix of measurement noise $\mathbf{R}$ and system noise $\mathbf{D}$.

Note that initially EKFs were developed for dynamical systems on $\mathbb{R}^n$, but attitude motion belongs to a manifold $\text{SO}(3)$. Hence, we can either utilize three independent variables to describe the attitude (e.g., Euler angles) and face singularities in kinematics, or adopt quaternions for attitude description. The latter way does not have singularities, but imposes constraints on quaternion components:

$$|\boldsymbol{\Lambda}| = \sqrt{\lambda_0^2 + \lambda_1^2 + \lambda_2^2 + \lambda_3^2} = 1.$$

It means that although the state vector of angular motion consists of seven components $\mathbf{x} = (\boldsymbol{\Lambda}^T, \boldsymbol{\omega}^T)^T$, only six of them would be independent. That problem might be resolved in the following way: an absolute quaternion can be represented as a composition of two quaternions

$$\boldsymbol{\Lambda} = \boldsymbol{\Lambda}_0 \circ \delta \boldsymbol{\Lambda}.$$

Here $\delta \boldsymbol{\Lambda}$ is considered to be a small addition to the quaternion $\boldsymbol{\Lambda}_0$, $\delta \boldsymbol{\Lambda}$ is expressed in deviated from the BF reference frame, so

$$\delta \boldsymbol{\Lambda} = \left(\sqrt{1 - (\delta \boldsymbol{\lambda})^2}, \delta \boldsymbol{\lambda}^T \right)^T.$$

Since $\delta \boldsymbol{\Lambda}$ represents small rotation, $|\delta \boldsymbol{\lambda}| \ll 1$ and scalar part of $\delta \boldsymbol{\Lambda}$ would be represented by $\delta \lambda_0 = 1 + O(|\delta \boldsymbol{\lambda}|^2)$, i.e., with the accuracy up to the second order infinitesimals it equals 1. This way $\delta \boldsymbol{\lambda}$ can be considered as three independent variables.

3. Proposed Algorithms Description

3.1. Kinematic EKF

Consider a kinematic EKF based on a star tracker and a gyro. This way, we do not take into account satellite dynamics, and only quaternion and gyro bias are estimated, i.e.,

$$\mathbf{x} = \begin{bmatrix} \boldsymbol{\lambda} \\ \Delta\boldsymbol{\omega} \end{bmatrix}.$$

The angular velocity estimation $\hat{\boldsymbol{\omega}}$ is calculated as the angular velocity sensor measurements $\tilde{\boldsymbol{\omega}}^{meas}$ minus the estimation of the sensor bias $\Delta\hat{\boldsymbol{\omega}}$:

$$\hat{\boldsymbol{\omega}} = \tilde{\boldsymbol{\omega}}^{meas} - \Delta\hat{\boldsymbol{\omega}}.$$

The angular velocity sensor model is as follows:

$$\begin{aligned} \tilde{\boldsymbol{\omega}}^{meas} &= \boldsymbol{\omega} + \Delta\boldsymbol{\omega} + \mathbf{v}_{\boldsymbol{\omega}}, \\ \Delta\dot{\boldsymbol{\omega}} &= \mathbf{v}_{\Delta\dot{\boldsymbol{\omega}}}, \end{aligned} \quad (7)$$

where $\boldsymbol{\omega}$ is the real angular velocity vector, $\Delta\boldsymbol{\omega}$ is the actual angular velocity sensor bias, $\Delta\dot{\boldsymbol{\omega}}$ is the rate of bias change, $\mathbf{v}_{\boldsymbol{\omega}}$ and $\mathbf{v}_{\Delta\dot{\boldsymbol{\omega}}}$ are normally distributed sensor and bias change noise with covariance matrix $\mathbf{D}_{\mathbf{v}_{\boldsymbol{\omega}}}$ and $\mathbf{D}_{\mathbf{v}_{\Delta\dot{\boldsymbol{\omega}}}}$.

Star tracker measurement model $\boldsymbol{\Lambda}^{meas}$ is as follows:

$$\boldsymbol{\Lambda}^{meas} = \boldsymbol{\Lambda} \circ \Delta\boldsymbol{\Lambda},$$

where $\boldsymbol{\Lambda}$ is the real quaternion value, $\Delta\boldsymbol{\Lambda}$ is the star tracker measurement error. Again, since star tracker measurement errors are rather small, with the accuracy up to the second-order infinitesimals $\Delta\boldsymbol{\Lambda} \approx (1, \Delta\boldsymbol{\lambda}^T)^T$. Vector part $\Delta\boldsymbol{\lambda}$ is considered to be normally distributed vector with zero mean value and covariance matrix $\mathbf{R}_{\Delta\boldsymbol{\lambda}}$.

The kinematic relations are as follows:

$$\dot{\boldsymbol{\Lambda}} = \frac{1}{2} \boldsymbol{\Lambda} \circ \boldsymbol{\omega} = \frac{1}{2} \begin{pmatrix} 0 & -\boldsymbol{\omega}^T \\ \boldsymbol{\omega} & [\boldsymbol{\omega}]_{\times} \end{pmatrix} \begin{pmatrix} \lambda_0 \\ \boldsymbol{\lambda} \end{pmatrix}, \quad (8)$$

For EKF, it is required to linearize the motion equations to obtain the following form:

$$\frac{d}{dt} \delta\mathbf{x}(t) = \mathbf{F}(t) \delta\mathbf{x}(t) + \mathbf{G}\mathbf{w}. \quad (9)$$

Here $\delta\mathbf{x}(t)$ is a small increment in the state vector. Satellite attitude can be represented by the following relation:

$$\boldsymbol{\Lambda} = \boldsymbol{\Lambda}_0 \circ \delta\boldsymbol{\Lambda}. \quad (10)$$

Consider linearization of the kinematic relation (8) in the vicinity of the quaternion $\boldsymbol{\Lambda}_0$ and angular velocity $\boldsymbol{\omega}_0$ expressed in BF. Since $\delta\boldsymbol{\Lambda}$ is expressed in the deviated from BF reference frame, the incremented angular velocity $\boldsymbol{\omega}$ is also expressed in the deviated from BF reference frame according to the following expression:

$$\boldsymbol{\omega} = \delta\tilde{\boldsymbol{\Lambda}} \circ \boldsymbol{\omega}_0 \circ \delta\boldsymbol{\Lambda} + \delta\boldsymbol{\omega}, \quad (11)$$

where $\delta\tilde{\boldsymbol{\Lambda}}$ is the conjugate quaternion, $\delta\boldsymbol{\omega}$ is the angular velocity increment expressed in deviated from BF reference frame. Taking the derivative of the (10) and substituting (8) and (11), one obtains:

$$\begin{aligned}
\dot{\Lambda} &= \dot{\Lambda}_0 \circ \delta\Lambda + \Lambda_0 \circ \delta\dot{\Lambda} = \\
&= \frac{1}{2} \Lambda_0 \circ \delta\Lambda \circ (\delta\tilde{\Lambda} \circ \omega_0 \circ \delta\Lambda + \delta\omega) = \\
&= \frac{1}{2} \Lambda_0 \circ \omega_0 \circ \delta\Lambda + \frac{1}{2} \Lambda_0 \circ \delta\Lambda \circ \delta\omega.
\end{aligned} \tag{12}$$

Applying relation $\dot{\Lambda}_0 = \frac{1}{2} \Lambda_0 \circ \omega_0$ and multiplying the right and left parts of the expression by $\tilde{\Lambda}_0$ from the left side, the relation for the $\delta\dot{\Lambda}$ is obtained:

$$\delta\dot{\Lambda} = \frac{1}{2} \delta\Lambda \circ \delta\omega.$$

Leaving the first-order terms, the relation for $\delta\dot{\Lambda}$ is following:

$$\delta\dot{\Lambda} = \begin{bmatrix} 1 \\ \delta\lambda \end{bmatrix} \circ \begin{bmatrix} 0 \\ \delta\omega \end{bmatrix} = \begin{bmatrix} 0 \\ \frac{1}{2} \delta\omega \end{bmatrix}.$$

Since the angular velocity vector is not a part of the state vector, its small increment $\delta\omega = \omega - \hat{\omega}$ is defined using the small increment of sensor bias as follows:

$$\delta\omega = -(\delta\Delta\omega + \mathbf{v}_\omega). \tag{13}$$

Then, the linearized kinematic equations have the form:

$$\delta\dot{\lambda} = -\frac{1}{2}(\delta\Delta\omega + \mathbf{v}_\omega).$$

Taking into account the bias model the dynamical matrix and system noise influence matrix are as follows:

$$\mathbf{F} = \begin{bmatrix} \mathbf{0}_{3 \times 3} & -\frac{1}{2} \mathbf{E}_{3 \times 3} \\ \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 3} \end{bmatrix}_{6 \times 6}, \quad \mathbf{G} = \begin{bmatrix} -\frac{1}{2} \mathbf{E}_{3 \times 3} \\ \mathbf{E}_{3 \times 3} \end{bmatrix}.$$

Measurement matrix is

$$\mathbf{H} = [\mathbf{E}_{3 \times 3} \quad \mathbf{0}_{3 \times 3}].$$

The next problem to be dealt with is the asynchrony of the measurements from different sensors and their delays, i.e., attitude control commands must be given based on the measurements from the past moments. Here we consider only angular velocity sensors (AVS) and the star tracker (ST). Due to the high sampling rate of the AVS, its delays can be neglected; however, ST delays might be rather high, up to 0.1–0.5 s. In the case of precise attitude pointing, control commands must be updated at least 20–50 times per second, so a delay of even 0.1 s might affect the performance of the attitude control system.

Note that the correction procedure of the Kalman Filter might be performed only at the moments when measurements are received. Hence, if we want to update control commands at a given time, only predicted estimation would be available. Let us consider the procedure of estimation in more detail. Let t_{k-1}^{ctrl} be the moment of the previous state vector estimation $\hat{\mathbf{x}}_{k-1}^-$ (note “-” as the upper index: there were no ST measurements at this moment, so correction could not be performed). Suppose that at the interval between control calculations $[t_{k-1}^{ctrl}, t_k^{ctrl}]$ there are many time moments of angular velocity measurements t_i^{AVS} and a time moment of receiving the star tracker measurement $t_j^{ST\ received}$ with a time stamp of measurement $t_j^{ST} < t_j^{ST\ received}$. All these time moments are different, and

they are schematically presented in Figure 2. Also, assume that for the previous star tracker measurement moment t_{j-1}^{ST} the estimation $\hat{\mathbf{x}}_{j-1}^+$ and its covariance matrix $\mathbf{P}_{j-1}^+$ are available.

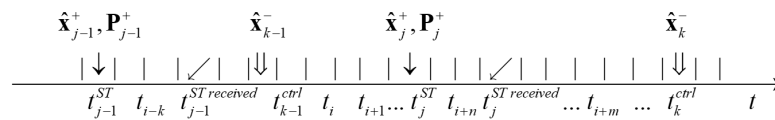


Figure 2. Time diagram for kinematic EKF.

Depending on whether interval $[t_{k-1}^{ctrl}, t_k^{ctrl}]$ contains newly received ST measurements, there are two ways to obtain an estimation at t_k^{ctrl} (see Figure 3). First, consider the case when no new ST measurements were obtained:

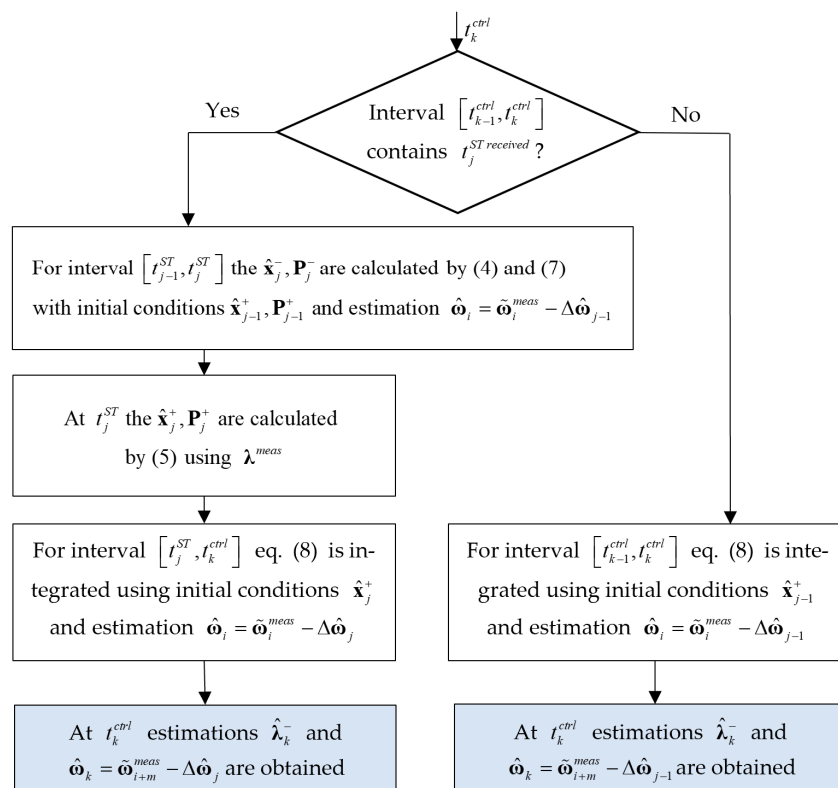


Figure 3. Block-scheme of the proposed processing procedure using kinematic EKF.

1. For the interval $[t_{j-1}^{ST}, t_j^{ST}]$ the kinematic relations (8) are integrated with initial conditions $\hat{\mathbf{x}}_{j-1}^+$, $\hat{\omega}_{j-1}^+$ using measurements $\tilde{\omega}_i^{meas}$ for estimation of the current angular velocity vector $\hat{\omega}_i = \tilde{\omega}_i^{meas} - \Delta\hat{\omega}_{j-1}$.
2. For the interval $[t_{j-1}^{ST}, t_j^{ST}]$ the covariance matrix is integrated with initial condition $\mathbf{P}_{j-1}^+$ using the estimation $\hat{\omega}_i = \tilde{\omega}_i^{meas} - \Delta\hat{\omega}_{j-1}$.
3. At time moment t_j^{ST} using the obtained measurement quaternion $\mathbf{q}^{meas}$ the estimation $\hat{\mathbf{x}}_j^+$ and its covariance $\mathbf{P}_j^+$ are calculated by (6).
4. For the interval $[t_j^{ST}, t_k^{ctrl}]$ the kinematic relations (8) are integrated using initial conditions $\hat{\mathbf{x}}_j^+$, $\hat{\omega}_j^+$, measurements $\tilde{\omega}_i^{meas}$ and bias estimation $\Delta\hat{\omega}_j$. So, by this work-

flow the estimations $\hat{\mathbf{q}}_k^-$ and $\hat{\boldsymbol{\omega}}_k = \tilde{\boldsymbol{\omega}}_{i+m}^{meas} - \Delta\hat{\boldsymbol{\omega}}_j$ (here $\tilde{\boldsymbol{\omega}}_{i+m}^{meas}$ is the last measurement before the control calculation) are obtained.

When the interval $[t_{k-1}^{ctrl}, t_k^{ctrl}]$ does not contain new star tracker measurements, the quaternion $\hat{\lambda}_k^-$ is calculated using the integration of kinematic equations with initial conditions $\hat{\lambda}_{k-1}^-$, $\hat{\boldsymbol{\omega}}_{k-1}^-$ using the measurements $\tilde{\boldsymbol{\omega}}_i^{meas}$ and bias estimation $\Delta\hat{\boldsymbol{\omega}}_{j-1}$. The covariance matrix in this case is not calculated and not updated.

Note that such an approach requires several integrations of kinematic equations of motion for the same time interval, e.g., before and after correction. However, this integration is rather simple and does not require much computational time.

3.2. Dynamic EKF

Now, consider the dynamic EKF. This filter estimates attitude, angular velocity and AVS bias:

$$\mathbf{x} = \begin{bmatrix} \lambda \\ \boldsymbol{\omega} \\ \Delta\boldsymbol{\omega} \end{bmatrix}.$$

Dynamic EKF takes into account equations of motion (1). Control torque is supposed to be implemented using the set of reaction wheels, so $\mathbf{M}_{ctrl} = -\dot{\mathbf{H}} - \boldsymbol{\omega} \times \mathbf{H}$ where $\mathbf{H}$ is the total angular momentum of reaction wheels. The dynamic Euler equations are then

$$\mathbf{J}\dot{\boldsymbol{\omega}} = -\boldsymbol{\omega} \times \mathbf{J}\boldsymbol{\omega} + 3\frac{\mu}{r^3}\mathbf{e}_r \times \mathbf{J}\mathbf{e}_r + -\dot{\mathbf{H}} - \boldsymbol{\omega} \times \mathbf{H} + \mathbf{M}_{dist}, \quad (14)$$

Model of the disturbance torque in EKF is supposed to be a normally distributed random vector with zero mean and covariance matrix $\mathbf{E}_{3 \times 3} \sigma_d^2$.

Angular velocity is included in the estimated state vector; hence $\boldsymbol{\omega}$ is related to the estimation $\hat{\boldsymbol{\omega}}$ as follows:

$$\boldsymbol{\omega} = \mathbf{B}\hat{\boldsymbol{\omega}} + \delta\boldsymbol{\omega}, \quad (15)$$

where $\mathbf{B}$ is transition matrix from Estimate Frame to Body Frame. Also, we assume that matrix kinematics are satisfied:

$$\dot{\mathbf{B}} = -[\delta\boldsymbol{\omega}]_{\times} \mathbf{B}. \quad (16)$$

Then, taking the derivative of (15), one can obtain:

$$\dot{\boldsymbol{\omega}} = -[\delta\boldsymbol{\omega}]_{\times} \mathbf{B}\hat{\boldsymbol{\omega}} + \mathbf{B}\dot{\hat{\boldsymbol{\omega}}} + \delta\dot{\boldsymbol{\omega}}. \quad (17)$$

Substituting (15) and (17) in motion Equation (14), and taking into account that $\mathbf{B} \approx \mathbf{E} - 2[\delta\lambda]_{\times}$

$$\begin{aligned} \delta\dot{\boldsymbol{\omega}} = & \mathbf{J}^{-1} \left(2[\mathbf{J}\hat{\boldsymbol{\omega}}]_{\times} [\hat{\boldsymbol{\omega}}]_{\times} \delta\lambda - 2[\hat{\boldsymbol{\omega}}]_{\times} \mathbf{J}[\hat{\boldsymbol{\omega}}]_{\times} \delta\lambda - \right. \\ & - [\hat{\boldsymbol{\omega}}]_{\times} \mathbf{J}\delta\boldsymbol{\omega} + [\mathbf{J}\hat{\boldsymbol{\omega}}]_{\times} \delta\boldsymbol{\omega} + \frac{6\mu}{R^3} [\hat{\mathbf{e}}_r]_{\times} \mathbf{J}[\hat{\mathbf{e}}_r]_{\times} \delta\lambda - \\ & - \frac{6\mu}{R^3} [\mathbf{J}\hat{\mathbf{e}}_r]_{\times} [\hat{\mathbf{e}}_r]_{\times} \delta\lambda + 2[\mathbf{H}]_{\times} [\hat{\boldsymbol{\omega}}]_{\times} \delta\lambda + [\mathbf{H}]_{\times} \delta\boldsymbol{\omega} \left. \right) - \\ & - [\hat{\boldsymbol{\omega}}]_{\times} \delta\boldsymbol{\omega} - 2[\dot{\hat{\boldsymbol{\omega}}}]_{\times} \delta\lambda. \end{aligned}$$

It is supposed that during the interval between the control calculations the angular momentum derivative $\dot{\mathbf{H}}$ is constant, i.e., it is not dependent on the current state vector; then matrix $\mathbf{F}$ after the linearization and matrix $\mathbf{G}$ are as follows:

$$\mathbf{F} = \begin{bmatrix} \mathbf{0}_{3 \times 3} & \frac{1}{2} \mathbf{E}_{3 \times 3} & \mathbf{0}_{3 \times 3} \\ (\mathbf{F}_\lambda)_{3 \times 3} & (\mathbf{F}_\omega)_{3 \times 3} & \mathbf{0}_{3 \times 3} \\ \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 3} \end{bmatrix}_{9 \times 9}, \quad \mathbf{G} = \begin{bmatrix} \mathbf{0}_{3 \times 3} \\ \mathbf{J}^{-1} \mathbf{E}_{3 \times 3} \\ \mathbf{E}_{3 \times 3} \end{bmatrix}_{9 \times 6}.$$

Here

$$\begin{aligned} \mathbf{F}_\lambda &= \mathbf{J}^{-1} \left(2[\mathbf{J}\dot{\boldsymbol{\omega}}]_\times [\hat{\boldsymbol{\omega}}]_\times - 2[\hat{\boldsymbol{\omega}}]_\times \mathbf{J}[\dot{\boldsymbol{\omega}}]_\times + \frac{6\mu}{R^3} [\hat{\mathbf{e}}_r]_\times \mathbf{J}[\hat{\mathbf{e}}_r]_\times - \right. \\ &\quad \left. - \frac{6\mu}{R^3} [\mathbf{J}\hat{\mathbf{e}}_r]_\times [\hat{\mathbf{e}}_r]_\times + 2[\mathbf{H}]_\times [\hat{\boldsymbol{\omega}}]_\times \right) - 2[\dot{\hat{\boldsymbol{\omega}}}]_\times, \\ \mathbf{F}_\omega &= \mathbf{J}^{-1} \left(-[\hat{\boldsymbol{\omega}}]_\times \mathbf{J} + [\mathbf{J}\dot{\boldsymbol{\omega}}]_\times + [\mathbf{H}]_\times \right) - [\dot{\hat{\boldsymbol{\omega}}}]_\times. \end{aligned}$$

Since the star tracker measurements and angular velocity measurements correspond to different time moments, their processing by EKF is performed by different measurement matrices. When the star tracker measurement is processed, the following matrix is used:

$$\mathbf{H}_{ST} = [\mathbf{E}_{3 \times 3} \quad \mathbf{0}_{3 \times 3} \quad \mathbf{0}_{3 \times 3}]_{3 \times 9}.$$

When the angular velocity measurement is available, the measurement matrix is as follows:

$$\mathbf{H}_{AVS} = [\mathbf{0}_{3 \times 3} \quad \mathbf{E}_{3 \times 3} \quad \mathbf{E}_{3 \times 3}]_{3 \times 9}.$$

Again, estimation of the state vector is performed only at moments t_k^{ctrl} when control torque must be calculated. Assume that estimation $\hat{\mathbf{x}}_{k-1}^-, P_{k-1}^-$ is available at t_{k-1}^{ctrl} . The time diagram is presented in Figure 4.

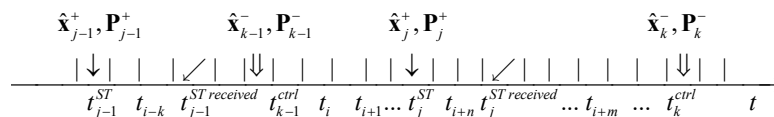


Figure 4. Time diagram for dynamic EKF.

Similarly to the previous scenario, there are two possibilities (see Figure 5). The first one corresponds to the case when during $[t_{k-1}^{ctrl}, t_k^{ctrl}]$ new measurements from ST were received:

1. Find the closest control time moment t_n^{ctrl} such that $t_n^{ctrl} < t_j^{ST}$. Note that such an approach requires saving the values of $\hat{\mathbf{x}}_k^+, \mathbf{P}_k^+$ for a certain time interval (e.g., 1–10 s, depending on the ST delay).
2. For the time interval $[t_n^{ctrl}, t_j^{ST}]$ for each time moment of received angular velocity sensor measurements, t_i^{AVS} the estimation and matrix $\hat{\mathbf{x}}_i^+, \mathbf{P}_i^+$ are calculated using the measurement matrix $\mathbf{H}_{AVS}$.
3. At the time of the star tracker measurement t_j^{ST} the estimation and matrix $\hat{\mathbf{x}}_j^+, \mathbf{P}_j^+$ are calculated using the measurement matrix $\mathbf{H}_{ST}$.
4. For the time interval $[t_j^{ST}, t_k^{ctrl}]$ for each time moment t_i^{AVS} the calculation of $\hat{\mathbf{x}}_i^+, \mathbf{P}_i^+$ is performed using matrix $\mathbf{H}_{AVS}$.

5. Since the measurement time moments t_i^{AVS} and control calculation time t_k^{ctrl} are different, then for the interval from the last measurement t_i^{AVS} till t_k^{ctrl} the «a priori» values $\hat{\mathbf{x}}_k^-, \mathbf{P}_k^-$ are calculated.

Finally, if the time interval $[t_{k-1}^{ctrl}, t_k^{ctrl}]$ does not contain new star tracker measurements, then for each time moment t_i^{AVS} the calculation of $\hat{\mathbf{x}}_i^+, \mathbf{P}_i^+$ is performed using matrix $\mathbf{H}_{AVS}$, and for the interval from the last measurement t_i^{AVS} till t_k^{ctrl} the «a priori» values $\hat{\mathbf{x}}_k^-, \mathbf{P}_k^-$ are calculated.

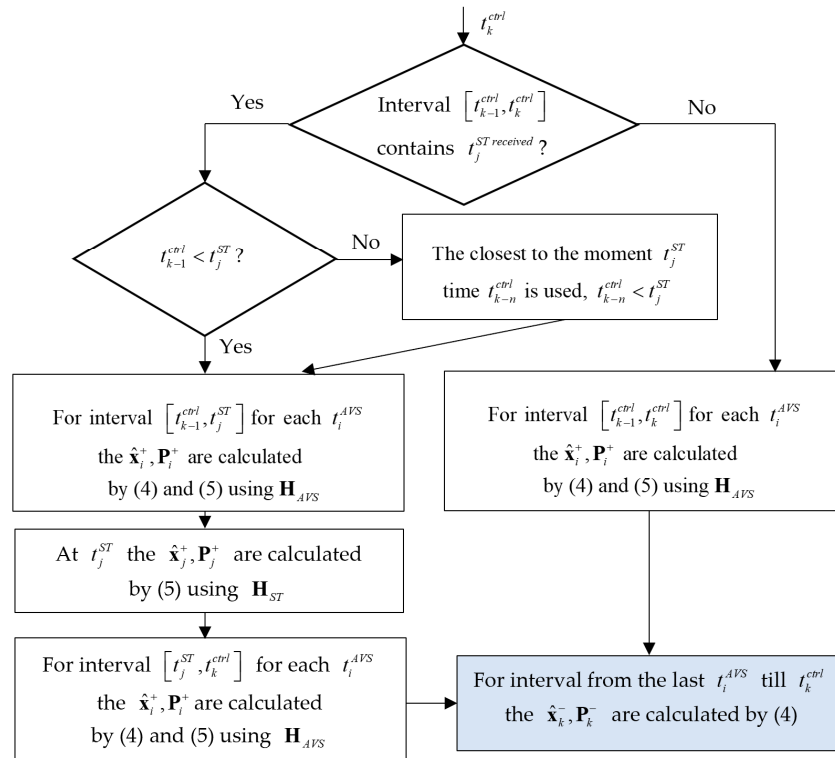


Figure 5. Block-scheme of the proposed processing procedure using dynamic EKF.

4. Reference Attitude Motion for Earth-Point Tracking

The efficiency of the proposed Kalman Filters is demonstrated using the Earth-point tracking scenario. The camera axis is directed along the vector $-\mathbf{e}_2$ of the Body Frame $Sy_1y_2y_3$. Since the camera must be aimed at a given point $\mathbf{R}_E$ at the Earth's surface, the reference position of $\mathbf{e}_2$ is described by

$$\mathbf{e}_2 = \frac{\boldsymbol{\rho}}{|\boldsymbol{\rho}|}.$$

Here $\boldsymbol{\rho} = \mathbf{r} - \mathbf{R}_E$, $\mathbf{r}$ is the satellite radius-vector. Another axis can be chosen rather arbitrarily, for example

$$\mathbf{e}_1 = \frac{\mathbf{e}_2 \times (\mathbf{r} \times \mathbf{V})}{|\mathbf{e}_2 \times (\mathbf{r} \times \mathbf{V})|} = \frac{\mathbf{h}}{|\mathbf{h}|},$$

where $\mathbf{V}$ is the satellite velocity. Finally, $\mathbf{e}_3 = \mathbf{e}_1 \times \mathbf{e}_2$. Reference angular velocity then can be obtained using Poisson equations $\dot{\mathbf{e}}_k = \boldsymbol{\omega}_{ref} \times \mathbf{e}_k$, which lead to

$$\boldsymbol{\omega}_{ref} = \frac{1}{2}(\mathbf{e}_1 \times \dot{\mathbf{e}}_1 + \mathbf{e}_2 \times \dot{\mathbf{e}}_2 + \mathbf{e}_3 \times \dot{\mathbf{e}}_3). \quad (18)$$

Equations for derivatives of basis vectors are as follows:

$$\begin{aligned} \dot{\mathbf{e}}_2 &= \frac{\dot{\boldsymbol{\rho}} - \mathbf{e}_2(\mathbf{e}_2, \dot{\boldsymbol{\rho}})}{|\boldsymbol{\rho}|}, \quad \dot{\boldsymbol{\rho}} = \mathbf{V} - \boldsymbol{\omega}_E \times \mathbf{R}_E, \\ \dot{\mathbf{e}}_1 &= \frac{\dot{\mathbf{h}} - \mathbf{e}_2(\mathbf{e}_2, \dot{\mathbf{h}})}{|\mathbf{h}|}, \quad \dot{\mathbf{h}} = \dot{\mathbf{e}}_2 \times (\mathbf{r} \times \mathbf{V}) + \mathbf{e}_2 \times \left(\mathbf{r} \times \frac{d\mathbf{V}}{dt} \right), \\ \dot{\mathbf{e}}_3 &= \dot{\mathbf{e}}_1 \times \mathbf{e}_2 + \mathbf{e}_1 \times \dot{\mathbf{e}}_2. \end{aligned}$$

Here $\boldsymbol{\omega}_E$ is the Earth angular velocity. Angular acceleration then can be found by taking the time derivative of Equation (18). However, it would require calculation of $\ddot{\mathbf{e}}_k$. Although they can be calculated, the resulting equation is rather bulky, and it would be simpler to calculate angular acceleration numerically

$$\dot{\boldsymbol{\omega}}_{ref} \approx \frac{\boldsymbol{\omega}_{ref}(t_k) - \boldsymbol{\omega}_{ref}(t_{k-1})}{t_k - t_{k-1}}.$$

Since timestep between consecutive control commands calculations is small (0.02–0.1 s) such an approach would give decent results.

We utilize a conventional Lyapunov-based attitude control law [24–26]:

$$\mathbf{M}_{ctrl} = -\mathbf{M}_{grav} + \boldsymbol{\omega} \times \mathbf{J}\boldsymbol{\omega} - \mathbf{J}(\boldsymbol{\omega}_{rel} \times \boldsymbol{\omega}_{ref}) + \mathbf{J}\dot{\boldsymbol{\omega}}_{ref} - k_\omega \mathbf{J}\boldsymbol{\omega}_{rel} - k_q \mathbf{J}\mathbf{q}_{rel} \text{sign}(\mathbf{q}_0^{rel}),$$

where $\boldsymbol{\omega}_{rel} = \boldsymbol{\omega} - \boldsymbol{\omega}_{ref}$ is the relative angular velocity, $\mathbf{Q}^{rel} = (q_0^{rel}, \mathbf{q}_{rel}^T)^T$ is the relative quaternion that describes transition from Reference Frame to the Body Frame, k_ω and k_q are positive control parameters. Such a control law ensures asymptotic stability of the required motion in the undisturbed case, so even in the presence of disturbances, it should still give satisfying results.

5. Numerical Performance Study

5.1. Simulation Parameters

Consider an example of algorithm performance using the scenarios of camera axis stabilization on the Earth point. All the simulation parameters are presented in Table 1.

The standard deviation σ_z of the star tracker measurements along the optical axis is higher than along the other two axes. In the simulations, it is assumed that the star tracker errors linearly depend on the angular velocity; the standard deviations for the case of no rotation and rotation with 1 deg/s along each axis are presented in Table 1. This assumption is based on experimental data obtained for the star tracker AZDK-1.5 by Azmerit company [27]. The angular velocity measurements delay is assumed to be constant, though the star tracker measurement delay is a sum of a constant delay τ and a random time-stamp noise with std σ_τ . Both delays are treated in numerical simulations as unknown values for the estimation algorithm. So, these delays result in additional error in measurements, but these values are not used in the prediction step of the EKF

Table 1. Simulation parameters.

Parameter	Value
Orbit height, km	500
Orbit inclination, deg	97.38
Earth point vector in Greenwich frame, km	$[-272.719, -6024.253, 2070.204]^T$

Standard deviation of star tracker measurements at zero angular velocity, arc sec	$\sigma_{x,y} = 10,$ $\sigma_z = 70$
Standard deviation of star tracker measurements at angular velocity 1 deg/s along each axis, arc sec	$\sigma_{x,y} = 25,$ $\sigma_z = 175$
Constant star tracker delay, s	$\tau = 5 \cdot 10^{-2}$
Std of time-stamp noise, s	$\sigma_\tau = 5 \cdot 10^{-3}$
Standard deviation of angular velocity measurements, deg/s	$\sigma_\omega = 1.25 \cdot 10^{-5}$
Standard deviation of derivative of AVS bias, deg/s ²	$\sigma_{\Delta\dot{\omega}} = 10^{-5}$
Angular velocity measurements delay, s	$5 \cdot 10^{-3}$
Inertia tensor, kg·m ²	$J = \text{diag}([1.67, 1.71, 0.93])$
Std of inertia tensor error, %	$\sigma_J = 5$
Std of principal axis directions, deg	$\sigma_{Oxyz} = 5$
Star tracker measurements frequency, Hz	5
Angular velocity sensor measurements frequency, Hz	50
Control calculation frequency, Hz	50

The change in star tracker measurement error with angular velocity is according to the following model:

$$\mathbf{R}_{\Delta\omega}(\omega) = \mathbf{R}_{\Delta\omega}^0 (1 + \alpha \|\omega\|),$$

where $\mathbf{R}_{\Delta\omega}^0$ is the covariance at zero rate and $\alpha = 1.5$ is derived from the two points (0 and 1 deg/s). It should be noted that random noise in the star tracker time-stamp of 5 ms results in additional error in the measurements up to about 18 arcsec.

For the algorithm parameters considered in the paper, the computational complexity of the dynamic EKF for iteration between the star tracker measurements is characterized by about 118,000 floating-point operations (FLOPs), while the kinematic EKF takes only about 26,000 FLOPs. So, dynamic EKF is 4.5 times more computationally complex than the kinematic EKF.

In numerical simulation, the satellite is affected by orbital disturbances, which are simplified as zero-mean Gaussian white noise with a specific amplitude. For the kinematic EKF the system noise covariance matrix and initial covariance matrix are:

$$\mathbf{D} = \begin{bmatrix} \sigma_\omega^2 \mathbf{E}_{3 \times 3} & 0_{3 \times 3} \\ 0_{3 \times 3} & (10^{-6} \text{ rad/s}^2)^2 \mathbf{E}_{3 \times 3} \end{bmatrix}, \quad \mathbf{P}(t=0) = \begin{bmatrix} (5 \text{ arc sec})^2 \mathbf{E}_{3 \times 3} & 0_{3 \times 3} \\ 0_{3 \times 3} & (10 \text{ arc sec/s})^2 \mathbf{E}_{3 \times 3} \end{bmatrix}.$$

For dynamic EKF, the system covariance matrix is different for the case of star tracker measurements and of angular velocity measurements:

$$\mathbf{D}_{ST} = \begin{bmatrix} (10^{-5} \text{ N} \cdot \text{m}) \mathbf{E}_{3 \times 3} & 0_{3 \times 3} \\ 0_{3 \times 3} & (10^{-6} \text{ rad/s}^2)^2 \mathbf{E}_{3 \times 3} \end{bmatrix},$$

$$\mathbf{D}_{AVS} = \begin{bmatrix} (10^{-5} \text{ N} \cdot \text{m}) \mathbf{E}_{3 \times 3} & 0_{3 \times 3} \\ 0_{3 \times 3} & (10^{-10} \text{ rad/s}^2)^2 \mathbf{E}_{3 \times 3} \end{bmatrix},$$

$$\mathbf{P}(t=0) = \begin{bmatrix} (10^{-4})^2 \mathbf{E}_{3 \times 3} & 0_{3 \times 3} & 0_{3 \times 3} \\ 0_{3 \times 3} & (10^{-4} \text{ rad/s})^2 \mathbf{E}_{3 \times 3} & 0_{3 \times 3} \\ 0_{3 \times 3} & 0_{3 \times 3} & (10^{-4} \text{ rad/s})^2 \mathbf{E}_{3 \times 3} \end{bmatrix},$$

The standard deviation of random disturbance torques is $\sigma_d = 10^{-6} \text{ N} \cdot \text{m}$ applied in the simulations. This value corresponds to the process-noise model used by the dynamic EKF. In the simulation, the control law uses estimated EKF attitude and angular velocity. Reaction-wheel momentum and its derivative are assumed perfectly known in the simulations. The inertia matrix error is modeled as a random perturbation of the true inertia tensor with a 5% standard deviation in each component. The principal axis misalignment is modeled as a random rotation of the body frame axes with a standard deviation of 5° . At each simulation, the value of the inertia tensor differs according to these parameters.

Figure 6 shows an example of star tracker measurements with noise parameters from Table 1, where the increase of errors with angular velocity is taken into account. The maximal angular velocity is achieved at time 300 s. The angular velocity vector components and the angular velocity sensor measurements with bias are presented in Figure 7.

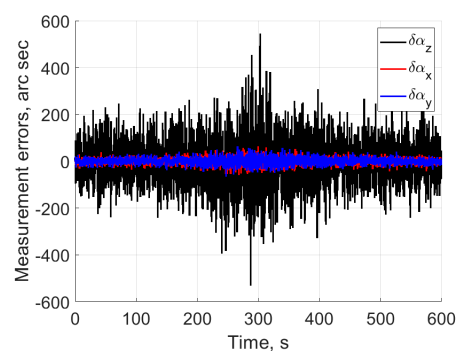


Figure 6. Star tracker measurement error (each curve corresponds to attitude measurement error along each axis of the satellite body-fixed reference frame).

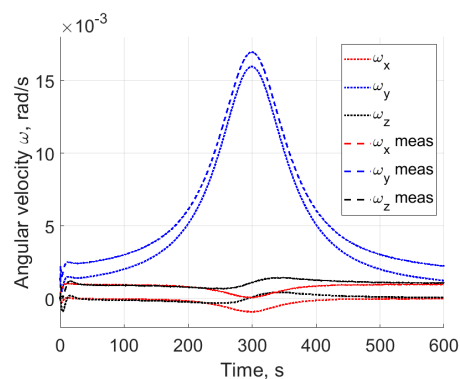


Figure 7. True angular velocity (dotted lines) and angular velocity sensor measurements with bias (dashed lines) in body-fixed reference frame. Red, blue, and black lines correspond to the components along the x, y, and z axes, respectively.

5.2. Kinematic EKF Performance Example

Figures 8–10 present the results of state vector estimation using kinematic EKF. Maximal attitude estimation errors are about 60 arc sec. The standard deviation of angular velocity estimation is close to the standard deviation of the angular velocity measurement error $1.3 \cdot 10^{-5} \text{ rad/s}$, and the maximal error of AVS bias estimation is about $2 \cdot 10^{-5} \text{ rad/s}$. Figure 11 presents the deviation angle of current attitude from the reference motion according to the control algorithm. These deviations are caused not only by the motion determination errors, but also by inertia tensor errors and by disturbances acting on the satellite.

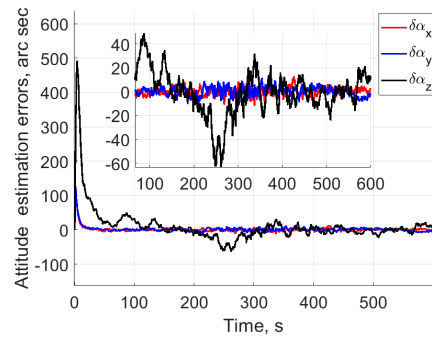


Figure 8. Attitude estimation error using kinematic EKF.

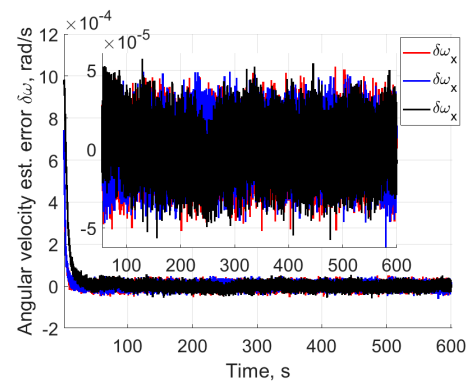


Figure 9. Angular velocity estimation error using kinematic EKF.

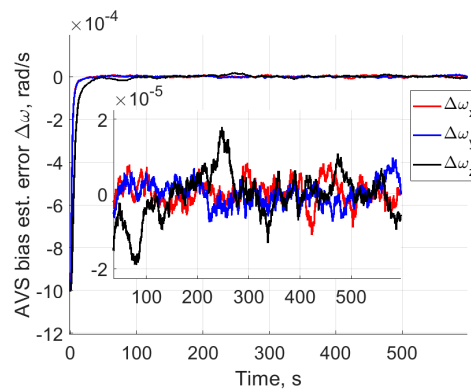


Figure 10. AVS bias estimation error using kinematic EKF.

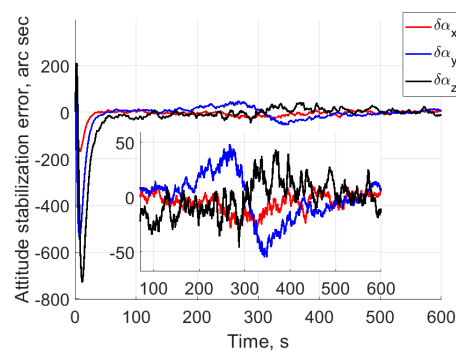


Figure 11. Errors of attitude stabilization according to the control algorithm.

Figures 12 and 13 show the actual errors in stabilization—the difference between the current body frame and the reference frame and between the angular velocities. During the flight over the Earth-point, the error is about 50 arc sec, which corresponds to an optical axis deviation of 120 m on the Earth surface.

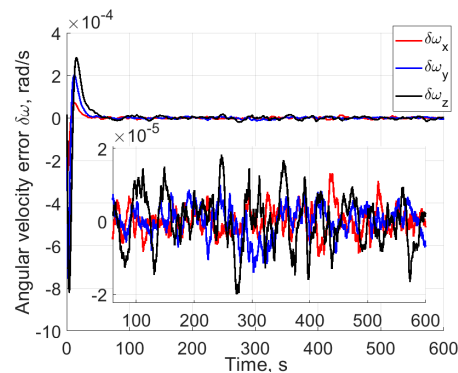


Figure 12. Actual angular velocity error during stabilization.

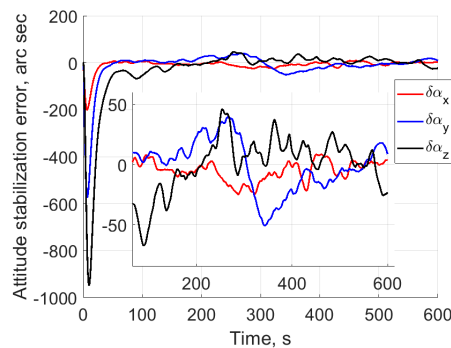


Figure 13. Actual attitude stabilization errors.

5.3. Dynamic EKF Performance Example

Consider an example of dynamic EKF performance. Figures 14–16 show the estimation errors of the attitude, angular velocity, and AVS bias. The maximum value of the attitude estimation error is about 30 arc sec; the angular velocity estimation errors are $2.5 \cdot 10^{-6}$ rad/s, which is 5 times less compared with kinematic EKF. The maximal error of AVS bias estimation is about $2 \cdot 10^{-6}$ rad/s; it is 10 times less compared with estimations by kinematic EKF.

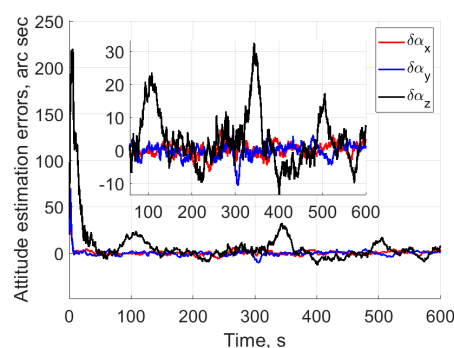


Figure 14. Attitude estimation error using dynamic EKF.

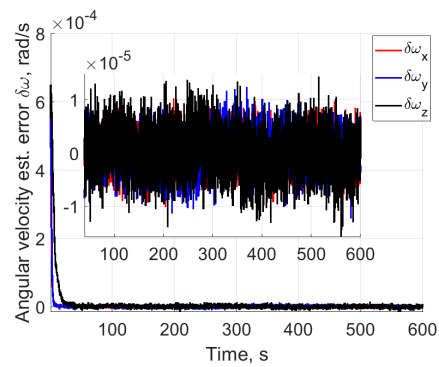


Figure 15. Angular velocity estimation error using dynamic EKF.

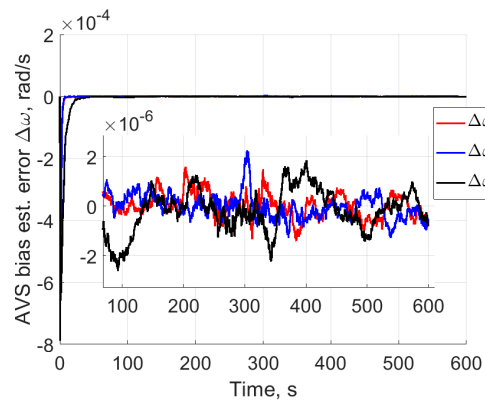


Figure 16. AVS bias estimation error using dynamic EKF.

Figures 17 and 18 show the actual stabilization errors and angular velocity errors during attitude motion. The maximal errors in attitude stabilization are about 40 arc sec, which is 10 arc sec less compared with kinematic EKF. The actual angular velocity error during stabilization for dynamic EKF is 2–3 times less than these errors for the kinematic EKF. This can be explained by a smoother angular velocity estimation in the case of dynamic EKF using the motion equations, taking into account the torques acting on the satellite.

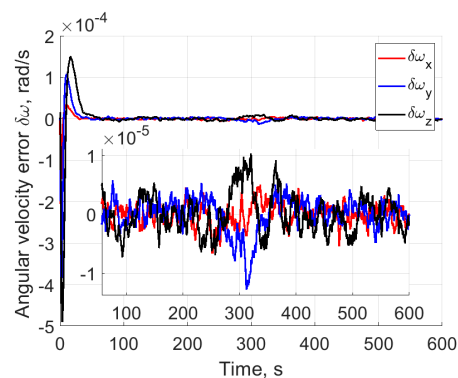


Figure 17. Actual angular velocity error during stabilization using dynamic EKF.

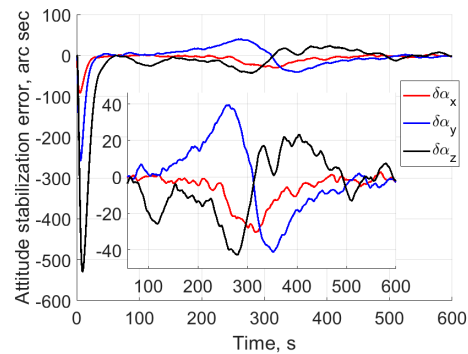


Figure 18. Actual attitude stabilization errors using dynamic EKF.

5.4. Case of Absence of Star Tracker Measurements

Consider the performance of the algorithms for the case when the star tracker is not able to provide the attitude measurements if an angular velocity exceeds 0.2 deg/s. In this section, it is assumed that the sensor provides degraded measurements up to 0.2 deg/s in the nominal mode; above 0.2 deg/s, the measurements are flagged as invalid and discarded. This angular velocity value threshold corresponds to the value obtained from the telemetry data containing measurements from the Azmerit AZDK-1.5 star tracker. Figure 19 shows an example of star tracker measurements during stabilization to the Earth point: there are no measurements from 175 s to 425 s. The angular velocity measurements profile is close to that presented in Figure 7.

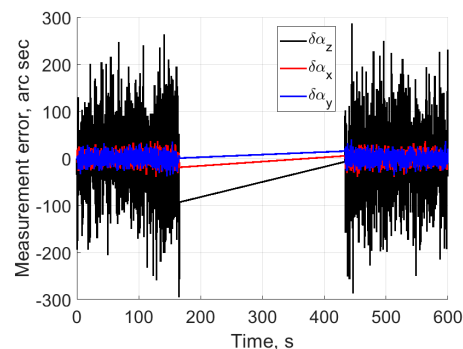


Figure 19. Star tracker measurements error, case of star tracker measurements absence.

Figures 20–22 present the results of kinematic EKF estimations. Due to the estimation errors of the AVS bias, the attitude estimation errors achieve 550 arc sec after 250 s of star tracker measurements absence. The AVS bias is not updated without new star tracker measurements, which results in considerable mean values of the bias causing the drift in attitude during kinematic relations integration. Figure 23 shows the actual stabilization error up to 600 arc sec, which corresponds to an optical axis deviation of 1.45 km on the Earth's surface.

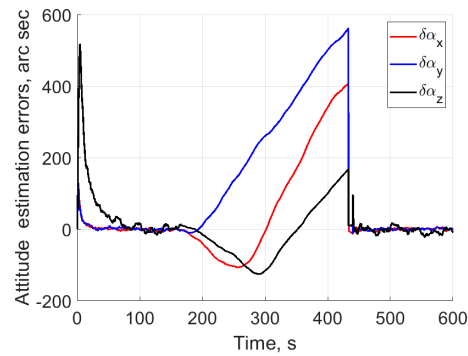


Figure 20. Attitude estimation error using kinematic EKF, case of absence of star tracker measurements.

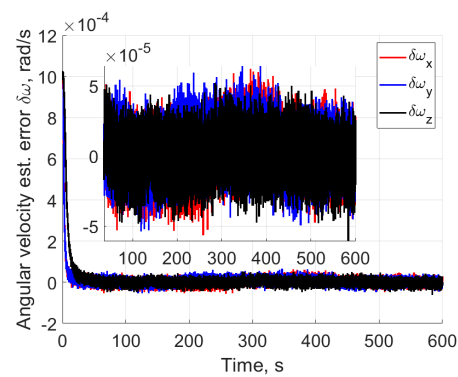


Figure 21. Angular velocity estimation error using kinematic EKF, case of absence of star tracker measurements.

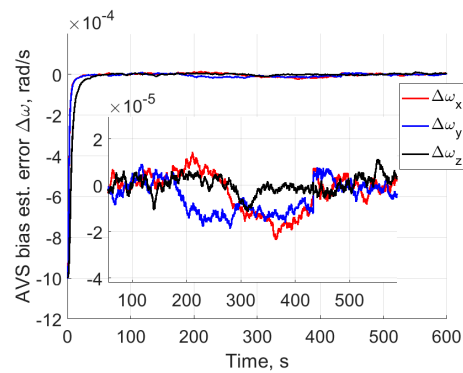


Figure 22. AVS bias estimation error using kinematic EKF, case of star tracker measurements absence.

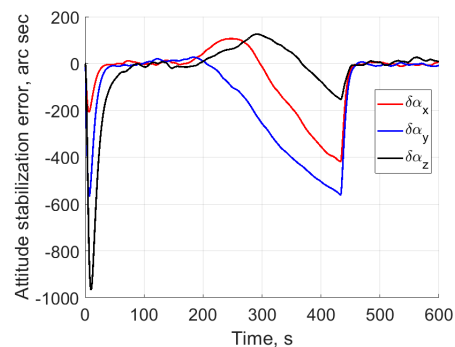


Figure 23. Attitude stabilization errors using kinematic EKF, case of star tracker measurements absence.

Figures 24–27 show performance of the dynamic EKF in the same scenario. According to Figure 26, the maximum estimation error in AVS bias is about $2 \cdot 10^{-6}$ rad/s: even in the absence of the star tracker measurements, the bias is updated by the dynamic EKF, though the errors steadily increase with time. Nevertheless, it can be concluded from Figure 27 that the stabilization error at the end of the interval of absence of the star tracker measurements is only 60 arc sec, which is almost 10 times smaller compared with kinematic EKF. This difference in the two EKF performances can be explained by more accurate attitude motion prediction in the case of using dynamical motion equations by dynamic EKF. In the absence of star tracker measurements, the kinematic EKF accumulates the attitude error directly from the bias in angular velocity sensor measurements, while dynamic EKF estimates the attitude taking into account the attitude motion equations.

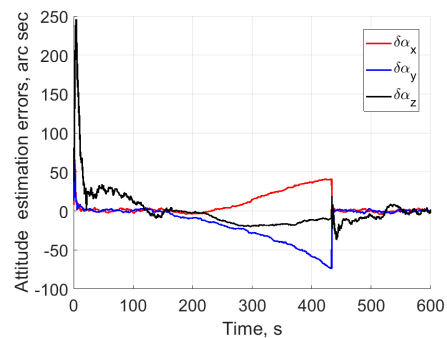


Figure 24. Attitude estimation error using dynamic EKF, case of star tracker measurements absence.

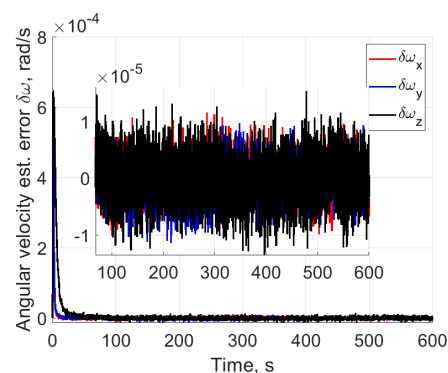


Figure 25. Angular velocity estimation error using dynamic EKF, case of star tracker measurements absence.

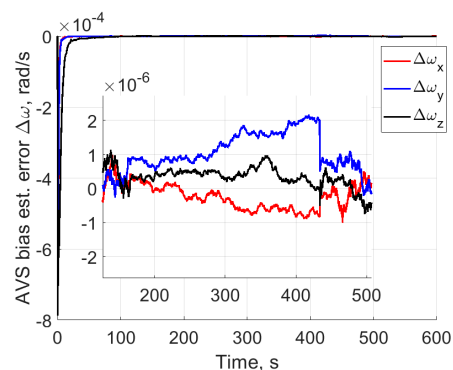
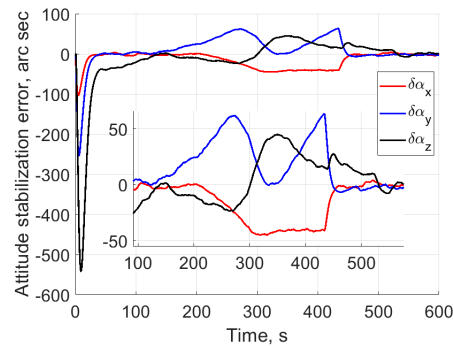
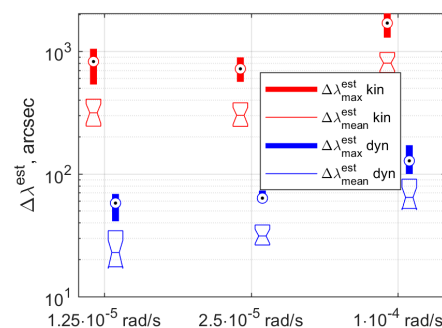


Figure 26. AVS bias estimation error using dynamic EKF, case of star tracker measurements absence.**Figure 27.** Attitude stabilization errors using dynamic EKF, case of star tracker measurements absence.

5.5. Algorithms' Characteristics Study

To study the characteristics of the algorithms, a numerical simulation of the motion was performed with different parameters of the angular velocity sensors over an interval of 200 s relative to the point of closest approach to the object of interest on the Earth's surface. The star tracker measurements are absent at an angular velocity of more than 0.2 deg/s, as in the previous Section. Monte-Carlo simulations with 100 independent runs for each scenario are performed using random initial conditions, disturbances, and measurement errors according to the parameters presented in Table 1.

Figures 28 and 29 present boxplots of the results of the calculations of the maximal and mean attitude estimation errors $\Delta\lambda_{\max}^{est}$ and angular velocity estimation error $\Delta\omega_{\max}^{est}$ for three different standard deviations σ_{avs} of the angular velocity sensor measurements. Figures 30 and 31 present the obtained maximal and mean values of attitude accuracy $\Delta\lambda_{\max}^{stab}$, as well as mean angular velocity error $\Delta\omega_{\max}^{stab}$ during stabilization at the point on the Earth's surface. As can be seen, with an angular velocity sensor with a standard deviation error at the level of 10^{-4} rad/s, which roughly corresponds to a sensor based on MEMS technology, the kinematic filter shows rather bad results: the maximal stabilization error of 1700 arc sec corresponds to the error of 4 km of the optical axis on the surface, and it is not acceptable for the remote sensing satellite. The dynamic EKF with the same data shows much better results with a maximal stabilization error of 100 arc sec.

**Figure 28.** Maximal and mean attitude estimation error using dynamic and kinematic EKF at different values of standard deviation error of angular velocity sensor.

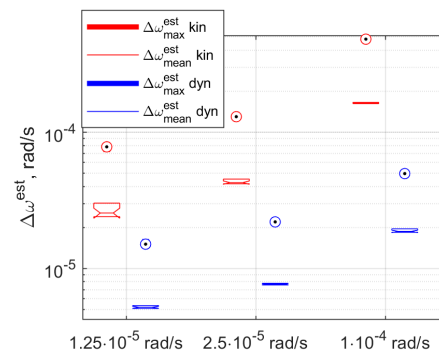


Figure 29. Maximal and mean angular velocity estimation error using dynamic and kinematic EKF at different values of standard deviation error of the angular velocity sensor.

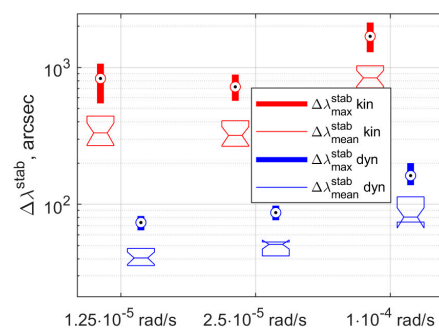


Figure 30. Maximal and mean attitude stabilization error using dynamic and kinematic EKF at different values of standard deviation error of angular velocity sensor.

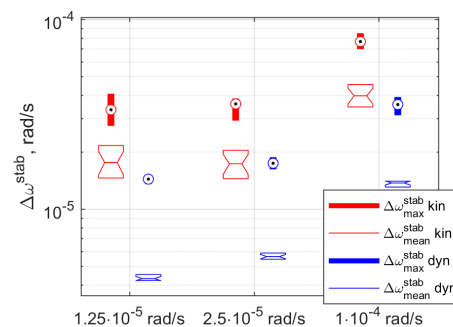


Figure 31. Maximal and mean attitude angular velocity error during stabilization using dynamic and kinematic EKF at different values of standard deviation error of angular velocity sensor.

Since the duration of the star tracker outage directly affects the relative performance of the two filters, Monte-Carlo simulations are carried out with different threshold values of angular velocity when the star tracker measurements are not available. The value of the standard deviation error of the angular velocity sensor is considered fixed as $\sigma_\omega = 1.25 \cdot 10^{-5}$ rad/s. The maximal attitude and angular velocity estimation errors using dynamic and kinematic EKF at different threshold values on angular velocity are presented in Figures 32 and 33. It can be seen that at each value of the threshold, the maximum error in attitude and angular velocity estimation is less for the dynamic EKF compared to the kinematic EKF.

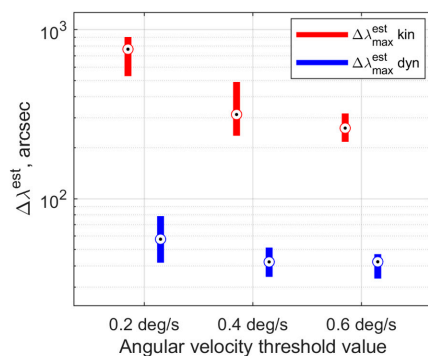


Figure 32. Maximal attitude estimation error using dynamic and kinematic EKF at different threshold values on angular velocity.

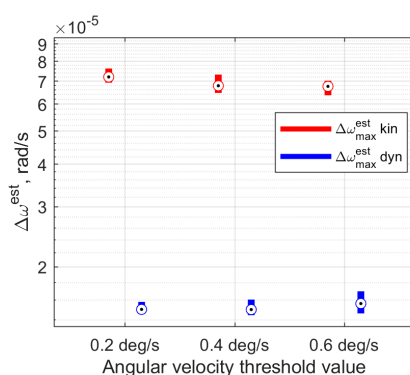


Figure 33. Maximal velocity estimation error using dynamic and kinematic EKF at different threshold values on angular velocity.

6. Conclusions

The two EKF-based attitude motion estimation algorithms for remote sensing satellites are developed in the paper. The attitude motion determination algorithms take into account the star tracker measurements delay; the measurements are assumed to be asynchronous, and the algorithms allow for the temporal absence of the star tracker measurements. The conducted numerical simulation results show that the dynamic EKF is superior to the kinematic EKF. This difference in performance can be explained by more accurate attitude motion prediction in the case of using dynamic motion equations by dynamic EKF. However, dynamic EKF requires more computational resources since its state vector includes the angular velocity vector. Moreover, dynamic EKF requires the knowledge of the satellite inertia tensor and the current reaction wheels' angular momentum and its derivative. As a result, the ADCS developers have to deal with a trade-off problem and choose between the complex but more accurate dynamic EKF and the simple but more sensitive to measurement errors kinematic EKF.

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