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Transformation of Discarded Pumpkin into High-Value Powder: A Drying Process Model for Functional Food Ingredients

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Abstract: Large quantities of pumpkins, rich in valuable nutrients, are lost due to superficial imperfections or size variations. This study explores a solution: transforming this unused resource into a highly functional food ingredient—pumpkin powder obtained from dehydration. This study emphasizes the importance of a detailed particle-level mathematical model in dehydrator design and operation, particularly for drying conditions using air at temperatures between 333 K and 353 K. The model investigates the effect of sample geometry on the moisture reduction rate and the product quality. Here, a model considers mass and energy transport, including the shrinkage ratio of the samples. The results effectively demonstrate the deformation, moisture content, and temperature evolution within the samples throughout the drying process. The findings reveal that both the drying temperature and initial sample geometry significantly influence the moisture loss rate, the final product texture, and the powder's absorption capacity. Notably, the nutritional composition (except for lipids) remains largely unaffected by the drying process. Additionally, the bulk and compacted densities of the powders decrease with increasing temperature. These insights not only illuminate the performance of the drying process but also provide valuable knowledge regarding the dehydrated product's technological behavior and potential functionalities within various food applications.

Keywords: agricultural waste valorization; heat and mass transport; numerical simulation; vegetable shrinkage; techno-functional properties

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1. Introduction

The agricultural sector generates significant volumes of underutilized or discarded produce due to imperfect size, blemishes, and spoilage after harvest. Despite their potential nutritional value, these discarded pumpkins often fail to enter commercial markets, leading to economic losses and food waste.

Pumpkins, *Cucurbita moschata*, have a wealth of dietary fiber, carotenoids, and vitamins [1,2]. They have an important beta-carotene content, which converts into vitamin

A in humans and lends their vibrant yellow or orange color [3,4]. Studies suggest beta-carotene consumption may lower the risk of eye problems, cancer, and skin diseases [4,5].

On the other hand, the deterioration of fresh products, including fruits and vegetables, is a microbiological process that gradually alters the food's color, texture, flavor, nutritional qualities, and edibility. The consumption of spoiled food can lead to illnesses and, in severe cases, even death [6]. Drying, particularly convective drying, is a powerful ally in safeguarding food quality, especially those containing high moisture. This technique effectively hinders the enzymatic activity and microbial growth, ensuring the shelf life of food [7–9]. Dehydration unlocks the potential of these vegetables, transforming them into versatile flours. Research has convincingly shown that these flours significantly elevate the nutritional value of processed foods [10,11], opening a new avenue for fortifying bakery products, a cornerstone food in child nutrition programs, elderly care facilities, and low-income communities. During pumpkin processing, peels are often discarded as waste. However, these peels contain valuable nutrients like protein and fiber, making them a potential food source. To address this issue, Román et al. [12] investigated dehydrating pumpkin peels using a fluidized bed contactor. Their study found that, while higher drying temperatures decrease the drying time and exergy efficiency, they also lead to a greater environmental impact. Considering this trade-off, the authors recommend drying at 70 °C. This temperature offers a favorable balance between faster drying times due to the increased diffusion and mass transfer, and a minimal impact on nutrient content.

Fruits and vegetables pose unique challenges in dehydration. Their intricate cellular structure, characterized by varying pore sizes and cell orientations, presents a remarkable obstacle to understanding the drying process [13]. This structural complexity encompasses cellular tissues, intercellular spaces, and cell walls, each harboring distinct water behaviors. Understanding the intricate interplay between these diverse components during drying remains a scientific frontier.

Vegetable drying involves heat and mass transfer, leading to vegetable shrinkage (size and shape changes) [14]. Water exiting the pores creates pressure that pulls on the solid structure, causing shrinkage [15,16]. This contraction slows moisture removal due to less surface area being available for evaporation [17]. Consequently, shrinkage can worsen porosity, alter microstructure [18], hinder mass transfer rate [19], and affect rehydration patterns [20–22].

Chandramohan [23] reported that 29% of academic papers on food drying use empirical, semi-empirical, or analytical approaches, often solving equations to model Fick's second law. However, these models fail to explain heat and mass transfer mechanisms. Numerical models, which predict temperature and moisture distributions, are better at identifying changes like color and nutrient degradation. Katekawa and Silva [24] noted that most studies overlook the coupling between transport phenomena and shrinkage. Sandoval-Torres et al. [9] developed a 1D model for potato slice drying, accurately reflecting experimental data but not addressing other food geometries. Agrawal and Methekar [25] created a model for pumpkin particles, showing qualitative agreement with the literature but not accounting for shrinkage.

Numerical models excel at predicting the evolving temperature and moisture distributions within food during drying, allowing us to identify undesirable temperature-related changes like color and nutrient degradation [25,26].

On the other hand, the conversion of residual waste into value-added biochar is relevant. In this sense, several authors have studied this approach, such as Nan et al. [27], who investigated the preparation of ball-milled biochar for activating peroxydisulfate (PDS) to degrade tetracycline hydrochloride (TCH). They found that ball-milling enhances biochar's electron transport capabilities, thereby improving PDS activation for TCH degradation, and they detailed the degradation pathways and mineralization of TCH. Zalazar-García et al. [28] evaluated the pyrolysis of 12 types of bio-waste using the Cape Open to Cape Open Simulator (COCO) software. The study predicted the yields of

biochar, bio-oil, and gas, and calculated the energy, water consumption, and CO₂ emissions for each type of bio-waste. Significant variations in resource consumption and environmental impact were identified based on the type of bio-waste and pyrolysis conditions. Fernandez et al. [29] examined the co-pyrogasification kinetics of municipal solid waste (MSW). Their findings reveal that MSW is a solid matrix that is more easily treated on thermochemical platforms, with kinetic and thermodynamic parameters favoring its processing.

Previous research has highlighted the significance of temperature and sample geometry on the drying kinetics of various food materials [30–33]. It is well documented that higher drying temperatures generally lead to faster moisture removal and shorter drying times. However, the magnitude of this effect can vary significantly based on the geometry of the sample. For instance, samples with larger surface areas exposed to hot air, such as pumpkin cubes, tend to exhibit more rapid moisture loss compared to samples with smaller surface areas, such as pumpkin puree slices.

This highlights the necessity of including comprehensive modeling to support or refute findings, providing a clear context for the results. Thus, this study shows that particle-level modeling, which incorporates the effects of shrinkage, successfully addresses the limitations of empirical models. By accurately predicting the behavior of different geometries and capturing the complex phenomena occurring during drying, this approach offers a more robust and reliable method for optimizing the drying processes. Consequently, particle-level modeling allows for better control and efficiency in the drying process, ultimately leading to higher-quality final products.

Thus, the current literature presents limited studies [25,34] on the effect of sample geometry and air temperature on techno-functional and physicochemical properties. Several research gaps demand further development in these areas. These findings highlight the importance of a particle-level mathematical model for effectively designing and operating dehydrators. Therefore, this work aimed to develop a drying model that involves transport phenomena in its equations. The coupled balances of mass and thermal energy with the contraction model were solved using the finite element method. The geometries considered in this study for the pumpkin samples were the pumpkin cube (PC) and the pumpkin puree slice (PP). These equations were incorporated into COMSOL Multiphysics 5.6 software to simulate air convection drying at 333, 343, and 353 K. This software allows the inclusion of the geometry to be modeled and the system of partial differential equations without the need for programming. The obtained results were validated with the respective convective drying experiments. Finally, the techno-functional and physicochemical properties of the obtained pumpkin cube powder (PPC) and pumpkin puree powder (PPP) were measured. Figure 1 shows a logical diagram of this research.

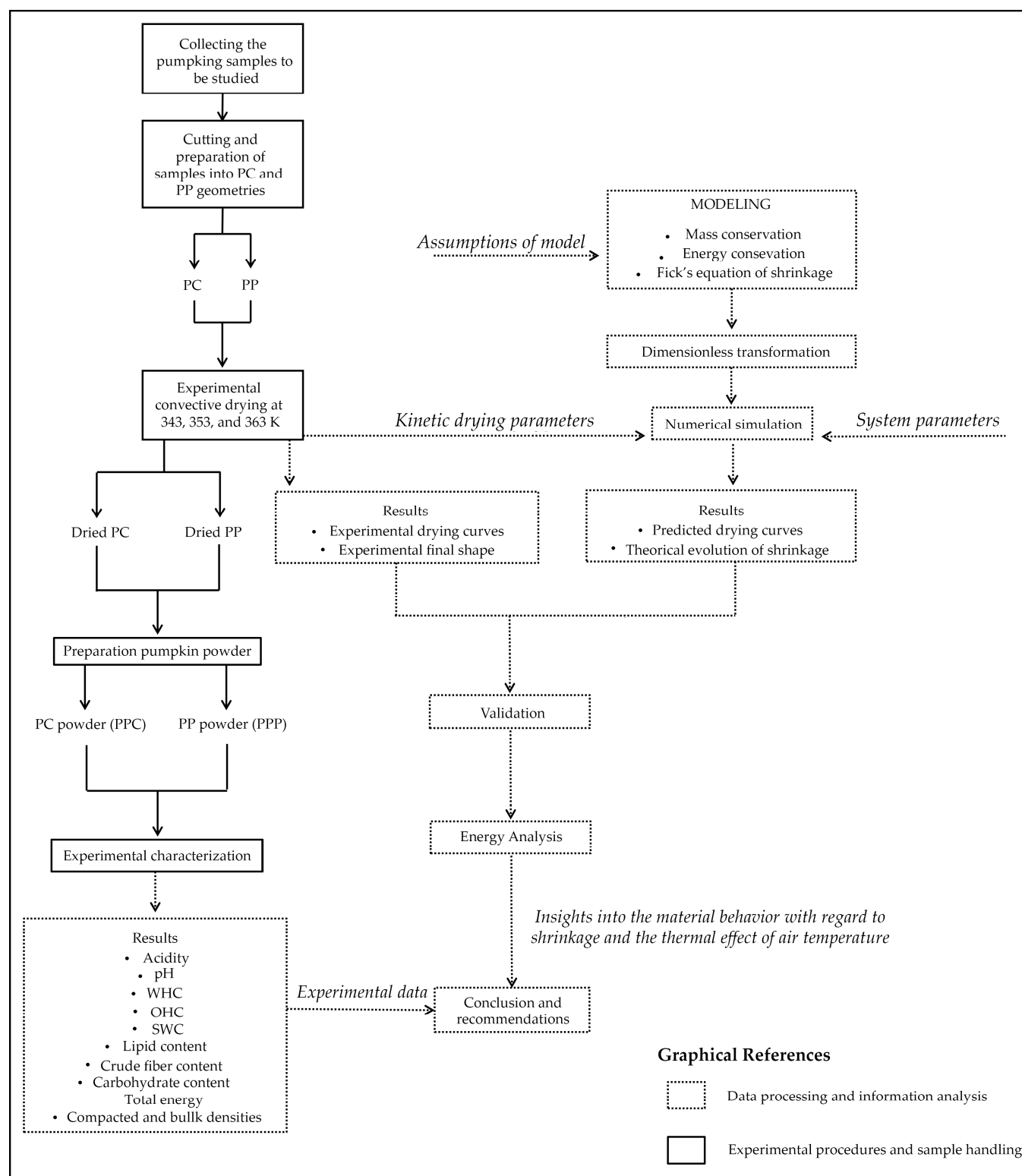


Figure 1. Logic diagram of the present work.

2. Materials and Methods

2.1. Sample Preparation

Samples of pumpkins, *Cucurbita moschata*, were collected from the Iglesia Department, located in the Province of San Juan, Argentina. This variety has an intense orange rind and pulp and an exquisite sweet flavor due to its high sugar content. Fresh pumpkins were washed and peeled. The pulp was cut in two different geometries: (A) PC

(1.5 cm³) and (B) PP, the pulp was crushed in a stainless steel mill until form a pumpkin paste (TecnoDalvo, model TDMC). Subsequently, the samples were kept refrigerated at 277 K until undergoing further drying, within 24 h.

2.2. Drying Procedure

The drying experiments of pumpkin pulp were carried out in a convective drying oven at three temperatures (333, 343, and 353 K). The drying temperature of 333 K is a vastly referenced value in the food technology literature for drying processes [8,35]. In addition, a higher temperature (353 K) was used in the present study to reduce the drying process times and to evaluate whether this temperature influences the nutritional and techno-functional characteristics of the powder obtained.

Mass loss (moisture loss) was recorded every 30 min until a moisture content of less than 15% was achieved by CODEX Standard [36] and the tests were carried out using the experimental device described by Roman et al. [37]. The tests were performed in triplicate and the average weight loss was reported. The initial moisture (AOAC Method 925.40/00) was determined by an infrared moisture analyzer (Radwag PMR50) with a halogen energy source at a temperature of 378 K [38].

2.3. Determination of Kinetic Parameters

The kinetic parameters of the pumpkin pulp drying process were determined using the Dincer and Dost model (Equation (1)) [39,40], which was applied by Riveros-Gomez et al. [8]. These parameters are needed for incorporation into the phenomenological model in the following section.

$$MR = G \exp(-Sdr t) \quad (1)$$

Effective diffusivity (D_{eff}) is an important parameter considering moisture transfer in drying. A simple diffusion model based on Fick's second law was used to obtain this coefficient, as described in Riveros-Gómez et al. [8].

The activation energy (E_a) during pumpkin drying in a convective dryer was calculated by determining the slope of the linearized Arrhenius equation (Equation 2) as described by Onwude et al. [41]:

$$D_{\text{eff}} = D_0 \exp\left(-\frac{E_a}{RT}\right) \quad (2)$$

2.4. Mathematical Model

The main assumption for the model is given in Appendix A.1.

During convective drying, heat is transferred from the air entering the food, while moisture moves from the interior of the solid matrix to the surface by diffusion. This moisture then evaporates from the surface to the drying air. Fick's law describes mass transport in a medium initially lacking chemical equilibrium. Table 1 presents the mass and energy balances for CP and PP geometries, while Table 2 details the boundary condition equations. Moisture loss during drying, governed by Fick's law, models its spatial and temporal distribution within the pumpkin sample (Equations (3)–(8)). Shrinkage terms are included in the second and fourth terms of Equation (3) and the second term of Equation (6). The mathematical development of these shrinkage terms is detailed in Appendix A. Temporal length variable L is defined in Equations (4) and (7).

For energy conservation, Fourier's equations (Equations (9) and (10)) model heat transport by conduction, the primary mechanism in food drying, incorporating terms for the thermal effect on material shrinkage. The dimensionless transformation of mass conservation, energy conservation, and shrinkage equations is detailed in Appendix A.2. Previous studies have applied non-dimensional transformations [9,42–45]. Using non-dimensional variables allows a differential equation system integrating mass, energy, and shrinkage conservation equations. This approach is vital for numerical simulations,

ensuring scale-independent results, and simplifying the application of initial or boundary conditions in COMSOL Multiphysics, leading to an efficient numerical solution process.

Figure 2a shows the symmetry along the vertical and horizontal lines through the cube center, with water flow directed outward from the surface, as indicated by blue arrows. Shrinkage occurs in opposition to the moisture flow due to pressure on the cube walls. Initial conditions for solving the system of equations (Equations (11)–(13)) represent moisture and heat mass flow in the x and y directions, respectively. Equations (14) and (15) represent moisture and heat mass flow along the radial direction.

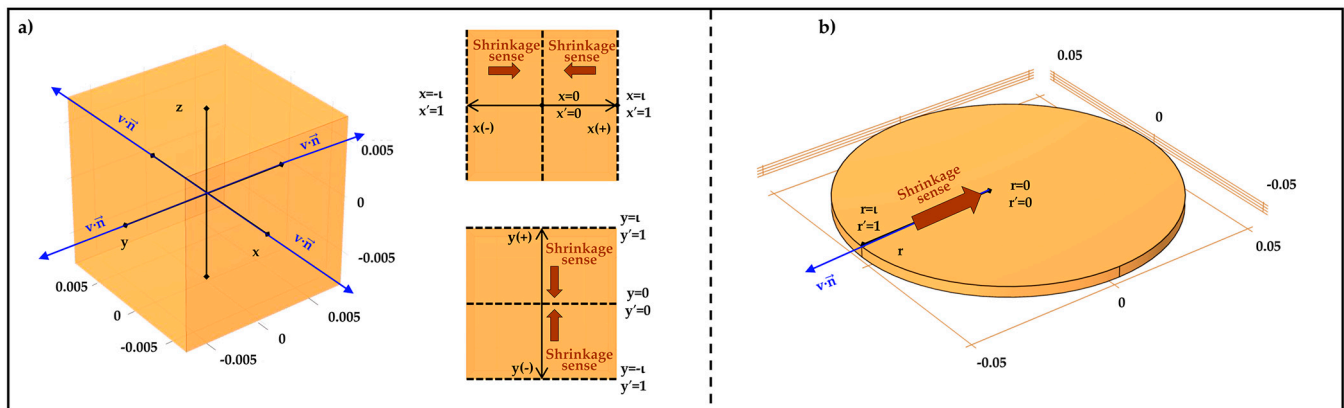


Figure 2. Boundary conditions established in modeling for (a) PC and (b) PP.

Table 1. Conservation and shrinkage equations.

Mass Conservation

Applied to the PC geometry:

$$\left. \begin{aligned} Q_s \frac{\partial W}{\partial t} &= \frac{\partial}{\partial x} \left(D_{\text{eff}} Q_s \frac{\partial W}{\partial x} \right) + Q_s L_x \frac{\partial W}{\partial x} \\ Q_s \frac{\partial W}{\partial t} &= \frac{\partial}{\partial y} \left(D_{\text{eff}} Q_s \frac{\partial W}{\partial y} \right) + Q_s L_y \frac{\partial W}{\partial y} \end{aligned} \right\} \text{Equation of mass balance for 2D (PC geometry)} \quad (3)$$

$$L_x = \frac{\partial x}{\partial t}, \quad L_y = \frac{\partial y}{\partial t} \quad (4)$$

$$\left. \begin{aligned} x' &= \frac{x}{l(t)}; 0 < x' < 1 \\ y' &= \frac{y}{l(t)}; 0 < y' < 1 \end{aligned} \right\} \text{Dimensionless variables for 2D (PC geometry)} \quad (5)$$

Applied to the PP geometry:

$$Q_s \frac{\partial W}{\partial t} = \frac{\partial}{\partial r} \left(D_{\text{eff}} Q_s \frac{\partial W}{\partial r} \right) + Q_s L_r \frac{\partial W}{\partial r} \quad (6)$$

$$L_r = \frac{\partial r}{\partial t} \quad (7)$$

$$r' = \frac{r}{l(t)}; 0 < r' < 1 \left\} \text{Dimensionless variables for 1D (PP geometry)} \quad (8)$$

Energy conservation

Applied to the PC geometry:

$$\left. \begin{aligned} Q_s C_p \frac{\partial T}{\partial t} &= \frac{\partial}{\partial x} \left(k \frac{\partial T}{\partial x} \right) + Q_s C_p L_x \frac{\partial T}{\partial x} \\ Q_s C_p \frac{\partial T}{\partial t} &= \frac{\partial}{\partial y} \left(k \frac{\partial T}{\partial y} \right) + Q_s C_p L_y \frac{\partial T}{\partial y} \end{aligned} \right\} \text{Equation of energy balance for 2D (PC geometry)} \quad (9)$$

Applied to the PP geometry:

$$Q_s C_p \frac{\partial T}{\partial t} = \frac{\partial}{\partial r} \left(k \frac{\partial T}{\partial r} \right) + Q_s C_p L_r \frac{\partial T}{\partial r} \quad (10)$$

Table 2. Boundary condition equations.

Initial conditions	
	$\begin{aligned} W &= W_0 \\ T &= T_0 \\ t &= t_0 \end{aligned} \quad (11)$
Symmetry considerations	
Applied to the PC geometry:	
	$\left. \begin{aligned} \vec{n} \left(D_{\text{eff}} Q_s \frac{\partial W}{\partial x} \right) &= h_m (C_s - C_a) \\ \vec{n} \left(D_{\text{eff}} Q_s \frac{\partial W}{\partial y} \right) &= h_m (C_s - C_a) \end{aligned} \right\} \text{Mass flux for 2D} \quad (12)$
	$\left. \begin{aligned} \vec{n} \left(-k \frac{\partial T}{\partial x} \right) &= h_c (T_a - T) - \Delta H_{\text{vap}} D_{\text{eff}} Q_s \frac{\partial W}{\partial x} \\ \vec{n} \left(-k \frac{\partial T}{\partial y} \right) &= h_c (T_a - T) - H_{\text{vap}} D_{\text{eff}} Q_s \frac{\partial W}{\partial y} \end{aligned} \right\} \text{Heat flux for 2D} \quad (13)$
Applied to the PP geometry:	
	$\vec{n} \left(D_{\text{eff}} Q_s \frac{\partial W}{\partial r} \right) = h_m (C_s - C_a) \quad \left. \right\} \text{Mass flux} \quad (14)$
	$\vec{n} \left(-k \frac{\partial T}{\partial r} \right) = h_c (T_a - T) - H_{\text{vap}} D_{\text{eff}} Q_s \frac{\partial W}{\partial r} \quad \left. \right\} \text{Heat flux} \quad (15)$

2.5. Numerical Simulation in COMSOL Multiphysics: Equations, Method, and Parameters

To enter the dimensionless mass and energy conservation equations, as well as Fick's equation of shrinkage (as a system of partial differential equations) into COMSOL Multiphysics, the partial differential equation (PDE) domain interface was used to define the domain addressed here. The ordinary differential equation (ODE) interface was used for Fick's equation of shrinkage. The mass and energy flow source equations (Equations (14) and (15)) were also transformed into dimensionless form. The equations entered in COMSOL Multiphysics are presented in Table 3.

Auxiliary nodes were also employed, which allow for the addition of equations and constraints to the mathematical model. In this regard, the initial conditions of the dimensionless variables in Equation (11) were used. To introduce these conditions, COMSOL Multiphysics allows adding flow equations [46].

- a. Zero-flux boundary condition is represented by the following equation:

$$\vec{n}J = 0 \quad (16)$$

- b. The flux/source boundary condition in COMSOL is represented by the following equation:

$$-\vec{n}J = g - qu \quad (17)$$

For condition (a), the flux is zero when the dimensionless variable x' is equal to zero. As established in the assumptions, the flux is zero at the center of the sample. This is valid for the two studied geometries.

Table 3. Domain equations.

Domain	Mathematical Dimensionless Expression	Unknowns
Mass conservation	PC geometry	(18) W'
	$\left. \begin{aligned} \frac{\partial W_x'}{\partial \tau} &= \frac{D_{eff}}{S^2} \frac{\partial}{\partial x'} \left(\frac{W'}{\partial x'} \right) + \frac{x'}{S} \frac{\partial S}{\partial \tau} \frac{\partial W'}{\partial x'} \\ \frac{\partial W_y'}{\partial \tau} &= \frac{D_{eff}}{S^2} \frac{\partial}{\partial y'} \left(\frac{W'}{\partial y'} \right) + \frac{y'}{S} \frac{\partial S}{\partial \tau} \frac{\partial W'}{\partial y'} \end{aligned} \right\}$	
	PP geometry:	
	$\frac{\partial W_r'}{\partial \tau} = \frac{D_{eff}}{S^2} \frac{\partial}{\partial r'} \left(\frac{W'}{\partial r'} \right) + \frac{x'}{S} \frac{\partial S}{\partial \tau} \frac{\partial W'}{\partial r'}$	
Energy conservation	PC geometry	(19) T'
	$\left. \begin{aligned} \frac{\partial T_x'}{\partial \tau} &= \frac{Le}{S} \frac{\partial^2 T'}{\partial x'^2} + \frac{x'}{S} \frac{dS}{d\tau} \frac{\partial T'}{\partial x'} \\ \frac{\partial T_y'}{\partial \tau} &= \frac{Le}{S} \frac{\partial^2 T'}{\partial y'^2} + \frac{y'}{S} \frac{dS}{d\tau} \frac{\partial T'}{\partial y'} \end{aligned} \right\}$	
	PP geometry:	
	$\frac{\partial T_r'}{\partial \tau} = \frac{Le}{S} \frac{\partial^2 T'}{\partial r'^2} + \frac{r'}{S} \frac{dS}{d\tau} \frac{\partial T'}{\partial r'}$	
Fick's equation of shrinkage	For PC geometry:	(20) S'
	$\left. \begin{aligned} \frac{dS_x}{d\tau} &= \frac{q_s}{q_w} \frac{D_{eff}}{S} (W_0 - W_e) \frac{\partial W'}{\partial x'} \\ \frac{dS_y}{d\tau} &= \frac{q_s}{q_w} \frac{D_{eff}}{S} (W_0 - W_e) \frac{\partial W'}{\partial y'} \end{aligned} \right\}$	
	For PP geometry:	
	$\frac{dS_r}{d\tau} = \frac{q_s}{q_w} \frac{D_{eff}}{S} (W_0 - W_e) \frac{\partial W'}{\partial r'}$	

For (b), the flux/source boundary condition (in this case, q_u is equal to zero) represents a boundary condition for a drying problem, where $\vec{n}$ is the normal vector to the surface of the sample, J is the mass flux, and g is the rate of moisture extraction from the sample. The q coefficient simplifies the implementation of a Robin boundary condition by including a term on the form q_u , where u is the dependent variable. This boundary condition indicates that the amount of mass leaving the sample through its surface is equal to the rate of mass extraction. The negative sign indicates that mass is leaving the sample and therefore the mass flux J must be in the opposite direction to the normal vector $\vec{n}$. In the case of heat transfer, the flux boundary condition represents the boundary condition, where $\vec{n}$ is the normal vector to the surface of the sample, J is here the heat flux (instead of the mass flux), and g is the rate of heat input rate from the sample. Sandoval-Torres et al. [9] found mathematical expressions for flux constraint (Equations (21)–(23)). Table 4 shows the constrain equations.

Table 4. Constraint equations.

Flux Constraint	Mathematical Dimensionless Expression	Independent Variable Value for Specified Boundary Condition
Zero-flux mass	$\left. \begin{aligned} -\frac{D_{eff}}{S^2} \frac{\partial W'_x}{\partial x'} &= 0 \\ -\frac{D_{eff}}{S^2} \frac{\partial W'_y}{\partial y'} &= 0 \end{aligned} \right\} \quad (21)$ For PP geometry: $-\frac{D_{eff}}{S^2} \frac{\partial W'_r}{\partial r'} = 0$ For PC geometry: $\left. \begin{aligned} -\frac{D_{eff}}{S^2} \frac{\partial W'_x}{\partial x'} &= S \frac{Bi_m}{Q_s} \frac{C_a}{W_0 - W_e} \left(\frac{C_s}{C_a} - 1 \right) \\ -\frac{D_{eff}}{S^2} \frac{\partial W'_y}{\partial y'} &= S \frac{Bi_m}{Q_s} \frac{C_a}{W_0 - W_e} \left(\frac{C_s}{C_a} - 1 \right) \end{aligned} \right\} \quad (22)$ For PP geometry: $-\frac{D_{eff}}{S^2} \frac{\partial W'_r}{\partial r'} = S \frac{Bi_m}{Q_s} \frac{C_a}{W_0 - W_e} \left(\frac{C_s}{C_a} - 1 \right)$	$\begin{aligned} x' &= 0 \\ y' &= 0 \\ r' &= 0 \end{aligned}$ $\begin{aligned} x' &= 1 \\ y' &= 1 \\ r' &= 1 \end{aligned}$
Zero-flux energy	$\left. \begin{aligned} \frac{D_{eff}}{Le^2} \frac{\partial T'_x}{\partial x'} &= 0 \\ \frac{D_{eff}}{Le^2} \frac{\partial T'_y}{\partial y'} &= 0 \end{aligned} \right\} \quad (23)$ For PP geometry: $\frac{D_{eff}}{Le^2} \frac{\partial T'_r}{\partial r'} = 0$	$\begin{aligned} x' &= 0 \\ y' &= 0 \\ r' &= 0 \end{aligned}$
Flux/source Energy	$\left. \begin{aligned} \frac{D_{eff}}{Le^2} \frac{\partial T'_x}{\partial x'} &= (1-T') S Bi_T - \frac{\Delta H'_{vap} D_{eff}}{Le} \frac{Q_w}{Q_s} \frac{\partial W'}{\partial x'} \\ \frac{D_{eff}}{Le^2} \frac{\partial T'_y}{\partial y'} &= (1-T') S Bi_T - \frac{\Delta H'_{vap} D_{eff}}{Le} \frac{Q_w}{Q_s} \frac{\partial W'}{\partial y'} \end{aligned} \right\} \quad (24)$ For PP geometry: $\frac{D_{eff}}{Le^2} \frac{\partial T'_r}{\partial r'} = (1-T') S Bi_T - \frac{\Delta H'_{vap} D_{eff}}{Le} \frac{Q_w}{Q_s} \frac{\partial W'}{\partial r'}$	$\begin{aligned} x' &= 1 \\ y' &= 1 \\ r' &= 1 \end{aligned}$

2.6. Numerical Solution Methodology and Parameters

To define the terms of the differential equations shown in the previous sections, it is necessary to introduce parameters related to the solid matrix (pumpkin) and the conditions established in the drying system. Table 5 shows the parameters used in the model.

Table 5. Model input parameters.

Parameter	Expression	Reference
Arrhenius pre-exponential factor D_0 (m ² /s)	Calculated for temperature range (333–353 K)	
PC geometry	$D_0 = 10^{-5}$	Calculated from experimental data
PP geometry	$D_0 = 5^{-4}$	Calculated from experimental data
Activation energy E_a (kJ/mol)	Calculated for temperature range (333–353 K)	
PC geometry	$E_a = 26.19$	Calculated from experimental data

PP geometry	$E_a = 37.17$	Calculated from experimental data
Mass Biot number, dimensionless	$Bi_m = \frac{h_m l_0}{D_{eff}}$ (25)	[8]
Thermal Biot number, dimensionless	$Bi_T = \frac{h_c l_0}{k}$ (9)	[9]
Latent heat of vaporization (kJ/kg)	$\Delta H_{vap} = 2501.3 - 2.301T_{sat} - 0.00142T_{sat}^2$ (26)	[15]
Heat transfer coefficient (W/m ² K) estimated between (48.8–59.3)	$h_c = \frac{Nu \cdot K_{air}}{l_0}$ (27)	[47]
Mass transfer coefficient (m/s)	$Nu = 0.664Re^{1/2}Pr^{1/3} \quad Pr < 1$	
Analogy dimensionless number (forced convection flux and flat plate with laminar flux) estimated between (0.11–0.124)	$h_m = \frac{Sh D_{eff a}}{l_0}$ (28)	[25]
Saturated water vapor pressure (Pa)	$Sh = 0.664Re^{1/2}Sc^{1/3} \quad 0.6 \leq Sc \leq 50$	
Water vapor concentration at the surface of the sample (kg/m ³)	$P_{vsat} = 100 \exp \left[20.9006 - \frac{5204.9}{T_{sat}} \right]$ (29)	[48]
Thermal conductivity (W/(m K))	$C_s = 2.166 \times 10^{-3} \left(\frac{P_{vsat}}{T_{sat}} \right)$ (30)	[49]
Specific heat C_p (J/(kg K))	$k = 0.149 + 0.493 \left(\frac{W}{1+W} \right)$ (31)	[50]
Solid density on a dry basis (kg/m ³)	$C_p = 1.26 + 2.93 \left(\frac{W}{1+W} \right)$ (32)	[51]
Density liquid water (kg/m ³)	$Q_s = 1040$ (33)	[52]
Initial moisture content of PC sample (kg water/kg dry matter)	$Q_w = 1000 \text{ kg/m}^3$	[53]
Initial moisture content of the PP sample (kg water/kg dry matter)	W_0 8.27 (333 K), 9.20 (343 K) and 9.20 (353 K)	Experimentally determined
	W_0 9.00 (333 K), 8.52 (343 K) and 9.00 (353 K)	Experimentally determined

The MUMPS solver in COMSOL was used to solve the systems of partial differential equations. The time step used was 60 s, and we set the relative tolerance to 0.01 to control the unknowns. This scalar parameter was used to obtain accurate and efficient numerical solutions.

2.7. Graphical Representation of the Sample Shrinkage

A method was proposed to visualize the deformation of the PC sample concerning its axes of symmetry. This was achieved by plotting the variation in the sample's characteristic length over time, as provided by the COMSOL simulation. This approach allows the observation of how the material deformed over time, as well as the distribution of stresses and strains within. However, the final shape of the puree pumpkin slice could not be predicted by the COMSOL model due to the formation of cracks and voids.

To validate the simulation results, the predicted values of S of the circular slices using the characteristic lengths at $t = 7$ h (time in which equilibrium moisture is reached at 353 K), for consistency across different temperatures, were calculated and compared from the experimental areas obtained from image processing (via area calculation using ImageJ software version 1.8.0) of the puree pumpkin samples.

All images were taken at the initial time and at 7 h, which corresponds to the time when the PP samples reached their final equilibrium moisture content after drying.

2.8. Preparation and Characterization of Pumpkin Powder

A simple random sample of three samples of pumpkin peel and dried pumpkin pulp was taken, ($\times 2$ geometries $\times 3$ temperatures $\times 500$ g/sample) and used for analyses. To obtain PPC and PPP, the dry samples at different temperatures were ground separately in a stainless-steel knife mill (TecnoDalvo, model TDMC). Subsequently, the powders were sieved to obtain a grain size of 0.10–0.20 mm. The samples were stored in the dark in sealed plastic bags at room temperature until analysis, within 2 months.

Each powder sample (2.0 g) was weighed into a 100 mL beaker and 40 mL of distilled water was added and stirred for 30 min. The mixture was then filtered and the pH of the filtrate was measured (AOAC Method 10.041/84) [38]. The total titratable acidity of the powder sample was determined in the filtrate by titration with NaOH (0.1 N), using five drops of phenolphthalein indicator until the mixture turned pink. The total titratable acidity of the pumpkin powder sample was informed as grams of citric acid/100 g sample (acid factor used: 0.064) (AOAC Method 942.15/90) [38]. The techno-functional properties, the water holding capacity (WHC) and oil holding capacity (OHC), were determined in the dry pumpkin samples, following the techniques described by Capossio et al. [54]. For the determination of WHC, pumpkin powder samples (0.5 g) were hydrated with excess water in 15 mL Falcon tubes for 24 h and then centrifuged at $2000\times g$ for 30 min. The supernatant was removed and water retention was expressed as grams of water retained per gram of pumpkin powder sample on a dry weight basis (dwb; g water/g sample dwb). The OHC analyses were performed by mixing 0.5 g of the sample with sunflower oil (10 mL) in 15 mL Falcon tubes. After 24 h, tubes were centrifuged at $2000\times g$ for 30 min, the supernatant was eliminated and oil retention was reported as grams of oil held per gram of pumpkin powder sample on a dry weight basis (g oil/g sample dwb). The swelling capacity (SWC) was calculated according to the method reported by Robertson et al. [55]. The pumpkin powder sample (100 mg dry weight) was hydrated in distilled water (10 mL) during 18 h, then the bed volume was documented and expressed as volume per gram of pumpkin powder sample on a dry weight basis (mL/g sample dwb).

The moisture of the dry samples was determined as described in the previous section for fresh samples. The ash content was analyzed by AOAC Method 923.03/90 [38]. The lipid content was determined via Soxhlet extraction (AOAC Method 920.39/90) and protein via the Kjeldahl method (AOAC Method 960.52/90) [38]. Moreover, the crude fiber content (ASTM Method D1104–56) was determined [56]. Subsequently, carbohydrates were calculated by the difference method reported by Campuzano et al. [57]. In addition, total energy was estimated using Atwater conversion factors, considering 4 kcal/g for carbohydrate, 4 kcal/g for protein, and 9 kcal/g for lipids, as described by Baldán et al. [58]. In addition, bulk density was measured by pouring approximately 20 mL of powder into a 50 mL graduated cylinder without tapping. The mass and volume of the powder were recorded and calculated as the relation between the weight of the pumpkin powder, and the volume of the powder without tapping, while compacted density was considered the weight of the pumpkin powder, and the volume of the powder after tapping. Then, the Hausner ratio was calculated as the ratio between the compact density and the bulk density [59].

2.9. Energy Analysis

The energy required to dry 1 kg of pumpkin pulp is estimated by calculating the specific energy consumption (SEC) using Equation (34) [60]:

$$SEC = (C_{pa} + C_{pv}h_v) \frac{q_{air}t(T_{in}-T_{am})}{m_v V_h} \quad (34)$$

A power plant supplies electricity for the experiments.

2.10. Emissions of CO₂

CO₂ emissions were estimated following the guidelines of Climate Transparency [61]. For Argentina, the CO₂ emission factor for electricity is 0.3583 kg CO₂/kWh. Therefore, the CO₂ emissions associated with drying 1 kg of pumpkin pulp are equal to [35]:

$$\text{CO}_2\text{emissions} = 0.3583 \times \frac{\text{kg}_{\text{CO}_2}}{\text{kWh}} \quad (35)$$

2.11. Statistical Analysis

All analyses were performed in triplicate and data were reported as mean $\pm$ standard deviation (SD). The results were analyzed by unidirectional ANOVA and the significant differences between the mean values were determined by the Tuckey test ($p < 0.05$) using the InfoStat software [8]. Pearson's correlation analysis was used to determine statistical significance.

The statistical coefficients to evaluate the fit of the mathematical model with the experimental data were Chi-square (χ^2), the sum of squared errors (SSE), and the square root of squared errors (RMSE), those with the lowest values of χ^2 , SSE and RMSE [62] represent the best fit. In turn, to find the kinetic constants, the values of the constants were iterated using the Microsoft Excel 2019 Solver complementary program, until finding those that make the statistical parameters are minimal.

3. Results and Discussion

The drying kinetics of two sample geometries were evaluated at different drying air temperatures. A phenomenological mathematical model was used to simulate the drying kinetics, and the results obtained by simulation were compared with experimental data.

3.1. Mathematical Model: Drying Curves for PC and CC Geometries

The drying curves (Figure 3) showed that as the air temperature increases, the moisture reduction rate becomes sharper for both forms of pumpkin.

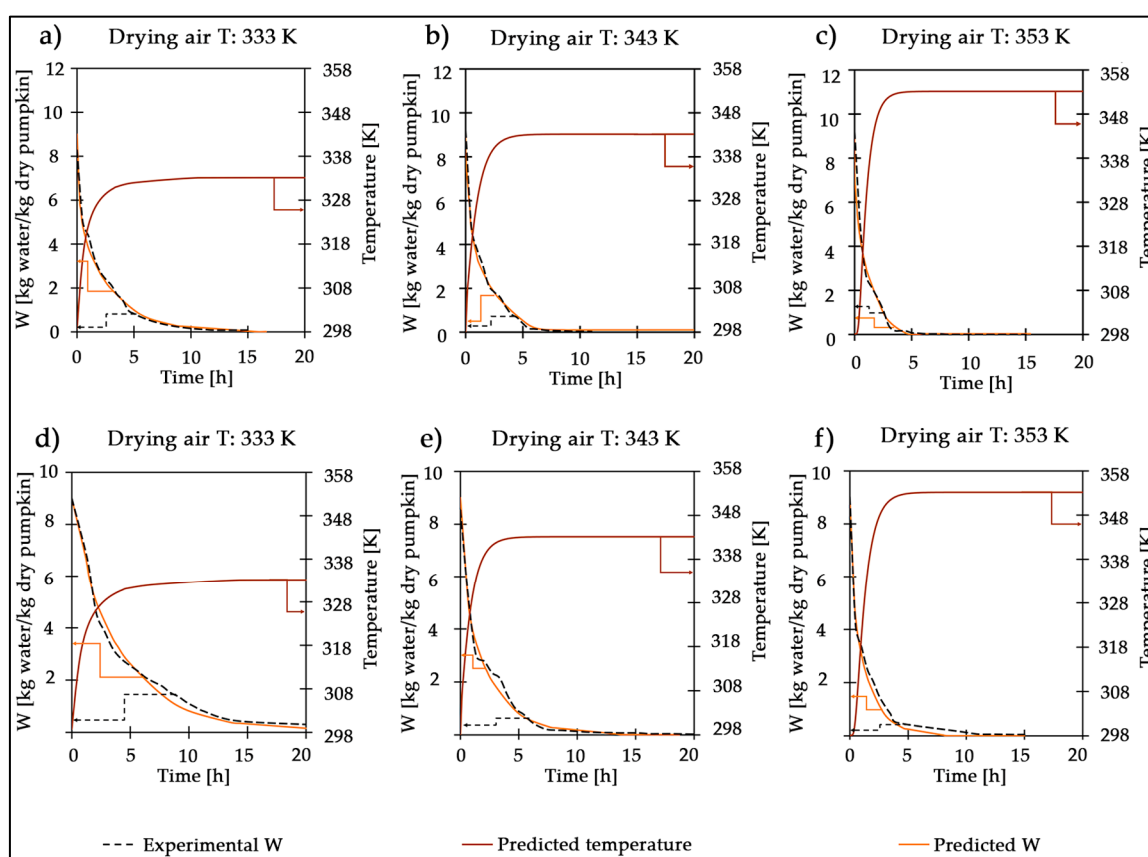


Figure 3. Model predicted and measured values of W and the model predicted values of T vs. time for PC and PP geometry at 333, 343, and 353 K of drying air temperature.

This behavior is consistent with previous findings reported by various authors [9,16,25]. Nevertheless, PC geometry had a more pronounced trend than PP geometry puree slices.

For PC geometry, as air temperature increases, the predicted moisture reduction curve shows a progressively steeper slope, indicating that the reduction in moisture increases more rapidly because PC geometry samples have a larger surface area exposed to hot air, which facilitates the transfer of water from the interior of the solid to the surface. As a result, as the temperature increases, the moisture transfer rate also increases. Figure 3a–c compare the actual moisture content measured in experiments (experimental) with the values predicted by numerical simulation (predicted) under studied drying conditions. The model accurately reflects the drying behavior (kinetics) for all the experiments conducted. These figures clearly show that air temperature has a significant impact on drying speed. As expected, higher drying temperatures lead to faster moisture removal and shorter drying times. This is because the temperature difference between the fresh sample (PC) and the surrounding air is greater at higher temperatures.

In the later stages of drying, the rate of moisture removal slows down (falling rate period). This is because the moisture inside the PC needs to travel to the surface before it can evaporate. The speed of this moisture movement is controlled by the rate of diffusion (movement) through the product itself. Finally, as the drying process nears completion, the moisture profile flattens out, and the moisture content gradually reaches the equilibrium level for the pumpkin sample.

In the case of PP geometry, the tendency for the moisture reduction rate to increase with temperature is less pronounced than in the case of PC geometry because the puree slices have a smaller surface area exposed to hot air, which hinders the transfer of moisture to the surface. Therefore, as the temperature increases, the moisture transfer rate does not increase as rapidly as in the case of PC geometry. Similar findings were reported by

Balzarini et al. [63], who studied the drying treatment on the quality attributes of chicory root cubes.

In addition, due to the significant shrinkage that occurs during the drying process, the PP geometry samples tend to crack, resulting in the formation of voids in the transfer zone. This effect reduces the mass and heat transfer area, affecting the overall performance of the drying process. Although cracks or gaps formed in the pumpkin puree slices may increase the effective moisture transfer surface area, they also decrease the thickness of the material in the voids, which reduces the heat transfer area. In addition, at the onset of drying, the gradient of available free moisture concentration was higher, leading to a greater drying rate. In addition, when there is a high initial moisture content, the initial drying rate is consistently higher because of the vapor pressure gradient, which diminishes as the drying process continues. This initial increase in the drying rate can also be attributed to the opening of the physical structure, allowing for faster evaporation and water transport. Similarly, Yuan et al. [64], Wang et al. [65], and Das Purkayastha et al. [66] observed comparable alterations in the drying rate curves during the thin-layer drying of apple pomace. A constant drying rate phase was not detected (Figure 3d–f). Therefore, it can be inferred that the drying process in this study was entirely within the falling rate period. This observation aligns with the findings of Cano-Chauca et al. [67] and Das Purkayastha et al. [66]. The falling rate period suggests greater resistance to heat and mass transfer through the internal cells, as well as an increase in the thickness of the wrinkled and shrunken skin [66].

Previous studies have documented how the geometry of food material, including its shapes and sizes, affects drying rates [30–33]. For instance, Garau et al. [30] and Panyawong and Devahastin [31] developed linear contraction models for different food materials, but it is important to note that these materials also exhibit elastic, hyper elastic, elastoplastic, or viscoelastic properties, implying that food contraction models are predominantly nonlinear. Moreover, complex phenomenology during drying affect food geometry in ways that empirical models cannot fully capture [68].

This highlights the necessity of including comprehensive modeling to support or refute findings, providing a clear context for the results. Thus, this study shows that particle-level modeling, which incorporates the effects of shrinkage, successfully addresses the limitations of empirical models. By accurately predicting the behavior of different geometries and capturing the complex phenomena occurring during drying, this approach offers a more robust and reliable method for optimizing drying processes. Consequently, particle-level modeling allows for the better control and efficiency in the drying process, ultimately leading to higher-quality final products.

It is necessary to consider both the advantages and disadvantages of each sample geometry (cube or slice) before deciding which geometry is optimal for a given drying process. The use of PC geometry may be preferable due to their more compact structure and lower susceptibility to void formation, which may provide better mass and thermal transfer efficiency. Furthermore, shrinkage during the drying process is a critical factor to consider as it can affect both the efficiency of mass and heat transfer as well as the quality of the final product. Therefore, achieving these goals requires the careful control of the temperature and velocity of the hot air flux, as well as carefully selecting the geometry of the material to be dried.

Ultimately, the choice of geometry will depend on the process-specific needs and the desired characteristics of the final product.

In terms of specific requirements, this study aims to evaluate how these drying characteristics affected the techno-functional properties of the powders obtained from PC and PP geometries. Therefore, the following sections will address the analysis of the techno-functional properties of the powders.

3.2. Graphical Representation of the Sample Shrinkage

The results obtained from the graphical representation of the sample shrinkage provide valuable insights into the behavior of the material during the drying process. As shown in Figure 4a, the simulation accurately captured the experimental observations of the PC sample's shape change over time, as evidenced by the close match between the theoretical and experimental geometries. The plot of the characteristic length variation with time allowed us to observe the sample's deformation over time, as well as the distribution of stresses and strains within the material. It is worth noting that the simulation was able to predict the final shape of the PC sample, which provides confidence in the accuracy of the simulation results.

In Figure 4b, the simulation results for the circular slice sample are presented. The simulation performed well in predicting the time-evolving characteristic length values, but it failed to capture the final shape of the puree pumpkin slice due to the formation of cracks and voids. This limitation is likely a consequence of the model not accounting for the mechanical stresses that develop during drying. As Aguilera and Stanley [69] pointed out, vegetable contraction during drying induces a stiffening phenomenon. This occurs because the surface dries faster than the interior, leading to internal tension and the formation of cracks and voids. The inability of the current model to predict this phenomenon explains the discrepancy between the predicted and observed final shapes.

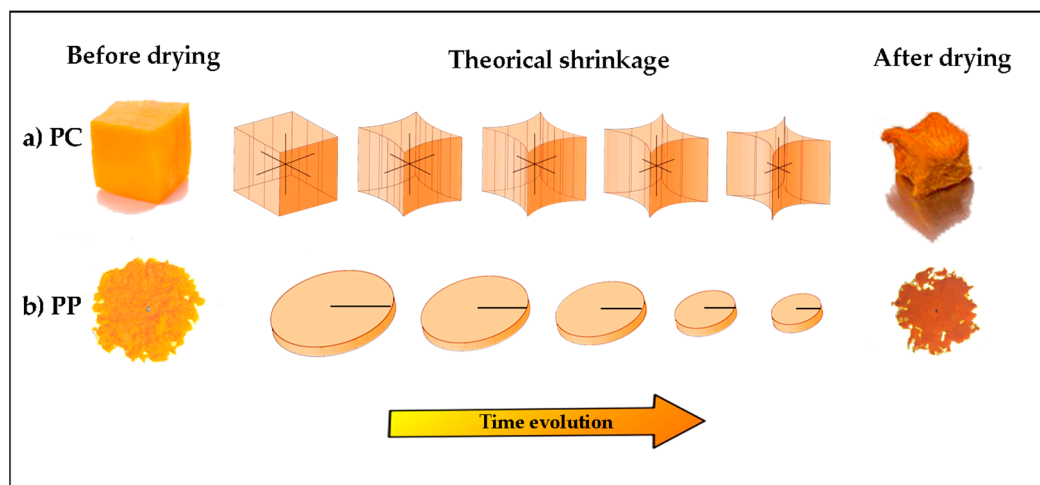


Figure 4. Theoretical and experimental shrinkage evolution over time for (a) PC and (b) PP geometry.

The results of the length calculations from images indicated that the contraction increases as the temperature increases, which is consistent with the formation of voids and the reduction in the cross-sectional area of the sample. The relative percentage errors were calculated as 5.32%, 3.05%, and 6.64% for 333, 343, and 353 K temperatures, respectively. Despite the limitation (the inability of the current model to predict cracks and voids), the simulation results could still be validated by comparing the predicted and experimental values of S of the circular slices after drying. The close agreement between the theoretical and experimental areas provides strong evidence that the simulation accurately captured the overall drying process, excluding the effects of mechanical stress and crack formation.

Furthermore, the comparison between the experimental and predicted values of S using the characteristic length values provided insights into the material behavior during the drying process. In this sense, as depicted in Figure 4a for PC samples, shrinkage is evident through the change in sample volume. The developed model effectively reproduces the shrinkage deformation experienced during drying. In line with these observations, the model demonstrates that, during the initial drying stages (for W values between 6 and 3 kg water/kg dry pumpkin and t values between 0.5 and 2 h, as exemplified in Figure 3a, small size variations are observed in the cube samples,

corresponding to the lowest rate period. During this period, the primary process is the evaporation of surface water. However, during the falling rate period (with W less than 2.5 kg water/kg dry pumpkin), significant volume variations are observed in all experimental trials, attributed to the molecular movement of water during the diffusion process.

Similarly to observations in apple slices [64], the simulations captured a larger initial deformation during drying due to rapid moisture loss (Figure 4). The observed increase in contraction with increasing temperature (mentioned earlier) aligns with findings for apple slices and Figure 3 results, where higher drying temperatures resulted in greater stresses and shrinkage [64].

3.3. Obtaining Dried Pumpkin Powder and Corresponding Characterization

The average weight of the fruits was 1181.67 ± 140.12 g and they were 20.08 ± 1.26 cm high, with a diameter between 8.48 ± 1.02 and 8.39 ± 1.04 cm in the upper part and between 11.25 ± 0.82 and 11.26 ± 0.84 cm in the area of the seminal cavity. The fruits were examined to verify their good condition, washed, and then shaped into puree and cubes.

The composition of the pumpkin is given by 69% of pulp, 27% of peel, and the remaining 4% by the seeds. The yields obtained show that the pulp represents a considerable percentage of the pumpkin that does not meet sales standards but does meet consumption standards, since it is a source of important compounds and is usually discarded.

In the temperature range studied, between 333 and 353 K, the acidity values for PPP and PPC showed a significant increase when the temperature rose, from 1.03 to 1.57 and from 1.30 to 1.57 g citric acid/100 g dwb, respectively. On the other hand, pH values decreased significantly from 6.70 to 6.55 for PPP and from 6.63 to 6.17 for PPC with the increasing drying temperature showing an inversely proportional correlation between acidity and pH results. Similar trends were reported by Zzaman et al. [70] during the drying of untreated pineapple at 323, 328, and 333 K and by Capposio et al. [54] during the drying of watermelon peel for the temperature range coincident with this study. The increase in acidity and decrease in pH may be attributed to the organic compound decomposition by heat effect, which caused the release of organic acids into the vegetal matrix [71]. In addition, the evaporation of water during drying concentrates these acids, increasing the effect of pH decrease and acidity increase. The obtained values WHC, OHC, and SWC for PPC and PPP are shown in Figure 5.

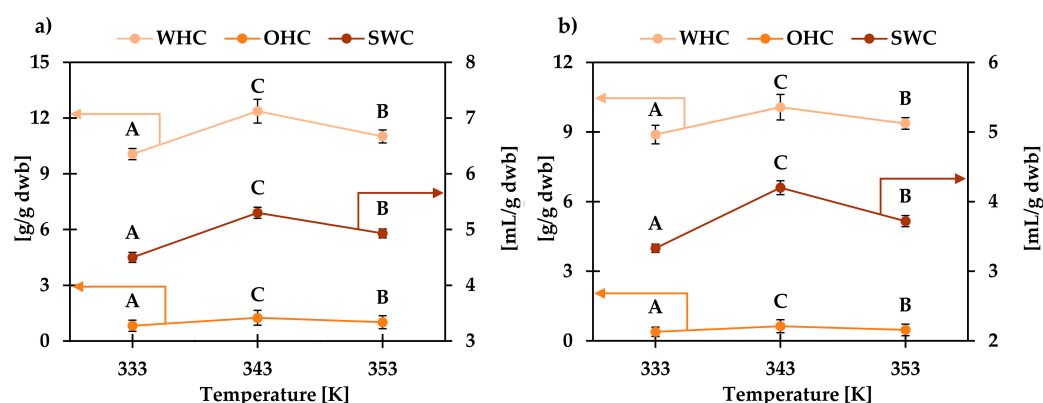


Figure 5. WHC, OHC, and SWC of (a) PPC and (b) PPP dried at 333, 343, and 353 K.

Analyzing the initial geometry of the samples, it can be seen that the PPC presents significantly higher values than those corresponding to PPP for WHC, OHC, and SWC. In this sense, WHC, OHC, and SWC values had a dramatic increase from 333 to 343 K, and a gradual decrease was detected from 343 to 353 K, showing significant differences for the three temperatures analyzed. The maximum values were obtained at 343 K for PPC, being

12.37 ± 0.64 g water/g dwb, 1.25 ± 0.03 g oil/g dwb, and 5.30 ± 0.20 mL/g dwb for WHC, OHC, and SWC, respectively. Capposio et al. [54] dried the watermelon peel between 333 and 373 K and Garau et al. [72] dehydrated the orange between 303 and 363 K, both studies also reported the maximum values of absorption capacities (WHC, OHC, and SWC) at intermediate temperatures of the studied ranges.

The results obtained above were considered to select the powders to continue with the evaluation of the physicochemical properties of the dehydrated products.

Comparing the drying times at 333, 343, and 353 K required to achieve a moisture content below 15% (equivalent to 0.18 kg of water/kg of dry solid), it was found that the time was significantly longer (more than 40%) at the lower temperature. In this sense, when considering the process performance, longer drying times lead to higher energy consumption, higher operating costs, the possible deterioration of product quality (changes in nutrients, texture, flavor, and color, as well as the formation of undesirable compounds, and lower production capacity) [73–75]. For this reason, the characterization studies were performed in the PPC and PPP samples at 343 and 353 K.

Regarding the shrinkage effect, it is important to note that it can affect both mass and heat transfer efficiency as well as the quality of the final product. Shrinkage during the drying process can cause cracks and voids in the sample, which affects the quality of the final product by altering its texture. The increase in WHC, OHC, and SWC from 333 to 343 K in the first instance could be because an increase in the drying air temperature can affect the structure and physical properties of the pumpkin samples, including their ability to retain water and oil. As the temperature increases, the pores and fibers of the materials open up, increasing their capacity for the absorption and retention of liquids. However, temperatures above 343 K can cause structure breakage (greater shrinkage than at lower temperatures), and under these conditions, the dried material cannot hold any water or oil.

The physicochemical properties of PPC at 343 and 353 K are shown in Table 6. From the results obtained, it can be seen that some components are not influenced by drying temperature (moisture, ash, protein, carbohydrates, and total energy), while others are significantly influenced by this parameter (lipids and crude fiber).

Table 6. Physicochemical properties of pumpkin powder obtained from cubes, dried at 343 and 353 K.

Chemical Property	343 K	353 K
Moisture content [%]	5.0 ± 0.6^a	5.4 ± 0.5^a
Ash content [g/100 g dwb]	8.26 ± 0.15^a	8.01 ± 0.08^a
Lipids [g/100 g dwb]	0.47 ± 0.02^b	0.33 ± 0.01^a
Protein [g/100 g dwb]	10.35 ± 0.30^a	10.87 ± 0.72^a
Crude fibers [g/100 g dwb]	7.01 ± 0.08^a	13.58 ± 0.42^b
Total carbohydrates [g/100 g dwb]	76.0 ± 0.2^a	75.4 ± 0.8^a
Total energy [kcal/100 g dwb]	349 ± 2^a	348 ± 3^a

ANOVA. The same letters indicate non-significant differences between pumpkin powder samples ($p > 0.05$).

The crude fiber content represents 20–50% of the total dietary fiber [76]. The crude fiber content obtained in this study was augmented by more than 90% on average, with an increasing drying temperature from 343 to 353 K. Similar values were reported by Guiné et al. [77], 7.85 ± 0.09 and 9.69 ± 0.01 (expressed as a percentage of dry mass), in pumpkin (*Cucurbita maxima*) pulp samples dried at 303 and 343 K, respectively.

The crude fiber content is a complex mixture of components, including cellulose, hemicellulose, and lignin, among others, and its increase during drying can be attributed to the effect of heat on cellulose (one of its main components) could cause changes in its crystalline structure and intramolecular bonds [78], making the crude fiber more

accessible and susceptible to extraction and quantification using the ASTM Method D1104–56 used in this study.

On the other hand, although the variation in lipid content was significant, this product is very poor in this component. When comparing these results with those obtained by Guiné et al. [77] and López Mejía et al. [79] for pumpkin (*Cucurbita maxima*), it is possible to observe that, in the case of this work, the dehydrated samples presented slightly lower lipid content.

The lipids content decreased with temperature, showing significant differences for the pumpkin powder obtained at 343 (0.47 ± 0.02 g/100 g dwb) and 353 K (0.33 ± 0.01 g/100 g dwb), respectively. In this sense, the increasing drying temperatures can lead to the degradation or oxidation of the lipid content, resulting in its reduction.

Bulk density is an important property when considering the package; it depends on the size and distribution of the particle and is closely related to other physicochemical properties [80]. The values determined in this study for the bulk density of pumpkin powder produced at 343 and 353 K were 0.658 ± 0.005 and 0.616 ± 0.009 g/mL, respectively. Similar values were found in the bibliography for powders of different vegetable origins. Chinma et al. [81] reported a bulk density of 0.69 ± 0.03 g/mL for ungerminated moringa seed powder. Miquilena et al. [82] obtained bulk densities in a range of 0.626–0.631 g/mL for powders of Quinchoncho and three different varieties of beans. The values determined in this study for the compacted density were 0.809 ± 0.004 and 0.758 ± 0.016 g/mL for the pumpkin powder produced at 343 and 353 K, respectively. It could be seen that the bulk and compacted density decreased significantly ($p \leq 0.05$) with increasing drying temperature. In contrast, the values obtained for the Hausner ratio were coincident for both, 343 and 353 K (1.23 ± 0.01). The pumpkin powders analyzed could be classified as medium-flowing powder taking into account the Hausner ratio value between 1.1 and 1.25 [59].

The results suggest that the operating conditions in convective drying can influence particle size and distribution, which in turn affects the bulk density of the powder. The reduction in bulk density observed with the increasing drying temperature suggests that convective drying leads to a more porous structure (easier to break or grind). This fact may have implications for packaging efficiency, as a lower bulk density allows for the more efficient utilization of packaging space. Moreover, the compacted density of the powder, which is also influenced by convective drying, affects its flow properties. The decrease in compacted density with an increasing drying temperature indicates better powder flowability. This fact is crucial for powder handling and processing processes, as good flowability facilitates the uniform mixing, filling, and conveying of the powder. However, higher temperatures cause a decrease in WHC, OHC, and SWC properties, so it will be desirable to consider these changes during the convective drying process to ensure the preservation of the functional characteristics and quality of the resulting pumpkin powder.

3.4. SEC and CO₂ Emissions

Energy consumption can be reduced by shortening the blanching time, as the energy requirements for this process are significant [7]. At different temperatures (333 K, 343 K, and 353 K), the specific energy consumption (SEC) values were 3911.44 kW/kg, 3592.85 kW/kg, and 3502.97 kW/kg, respectively (Table 7). The decrease in SEC with the increase in temperature indicates that less energy is required as the drying process temperature rises, which also results in shorter drying times. This increase in temperature and the corresponding reduction in drying time led to lower CO₂ emissions. The CO₂ emissions associated with these energy consumptions were 1401.47 kg CO₂/kW, 1287.32 kg CO₂/kW, and 1255.11 kg CO₂/kW, respectively, demonstrating a clear relationship between improved energy efficiency and reduced environmental impact [35].

Table 7. SEC values and CO₂ emissions for PPC samples at different temperatures.

	333 K	343 K	353 K
SEC [kW/kg]	3911.44	3592.85	3502.97
CO ₂ emissions [kg CO ₂ /kW]	1401.47	1287.32	1255.11

3.5. Innovative Contributions and Originality of This Research

To provide a comprehensive overview of the key findings and their alignment with the existing literature, Table 8 summarizes the impact of temperature, the falling rate period, geometry and surface area, shrinkage effects, and particle-level modeling in this study compared to those in previous studies. This comparison highlights both the similarities and differences, offering valuable context for the observed drying kinetics and shrinkage behavior of pumpkin samples.

Table 8. Innovative contributions of this research.

Aspect	In this Study	Similarities and Differences	Referenced Studies
Impact of temperature	Higher temperatures accelerate the drying rate, leading to greater moisture removal and shorter drying times	Similar observation	Balzarini et al. [63]; Yuan et al. [64]; Wang et al. [65]; Das Purkayastha et al. [66]
Falling rate period	No constant rate phase was identified in the drying process; the entire drying process was in the falling rate period	Similar observation	Cano-Chauca et al. [67]; Das Purkayastha et al. [66]
Geometry and surface area	Differences between PC and PP geometries; smaller surface area in PP reduces the moisture transfer rate	No study emphasized the difference in geometry and moisture transfer rate	-
Shrinkage effects	Significant shrinkage and crack formation in PP affect the area of mass and heat transfer	Drying quality was mentioned but not delved into the effects of shrinkage and cracking	Balzarini et al. [63]
Particle-level modeling	Incorporates shrinkage effects and captures complex phenomena, surpassing limitations of empirical models	There was a focus on linear and empirical contraction models, but they did not fully capture complex phenomena	Garau et al. [30]; Panyawong and Devahastin [31]

4. Conclusions

This study on the drying kinetics of pumpkin cubes and purees by convective drying using air at different temperatures (333, 343, and 353 K) found that controlling temperature and sample geometry is necessary for achieving efficient mass and heat transfer conditions, and a high-quality final product. The model developed here accurately predicted the moisture reduction rate of pumpkin samples during drying. The sample geometry and the air temperature affected the moisture reduction rate and the formation of voids. Shrinkage during the drying process can cause cracks and voids in the sample, affecting the quality of the final product by altering its texture. The increase in drying air temperature can affect the structure and physical properties of the pumpkin samples, including their ability to retain water and oil. Increasing drying air temperature affects pumpkin sample structure and properties like water and oil retention. Lower SEC with higher temperatures suggests better energy efficiency without compromising product quality. Optimizing drying temperature for lower SEC can shorten the drying times and reduce CO₂ emissions, preserving the pumpkin's technological and functional qualities. Temperatures above 343 K can cause greater shrinkage, resulting in a dried material that cannot hold any water or oil. In summary, this study highlights the relevance of considering the shape of pumpkin samples during the processing of dehydrated powders. The results showed significant changes in liquid absorption properties and transport characteristics, suggesting that the sample geometry and the air temperature

influence the techno-functional properties of the final product. These findings open new perspectives for the performance enhancement of processing pumpkin powders and their use as food additives for vulnerable populations. On the other hand, particle-level modeling allows for a detailed analysis of how heat and moisture are distributed and transported within each particle during the drying. In this context, all considerations and empirical correlations can vary significantly with the behavior of the vegetable or plant material. Consequently, there could be potential limitations in the applicability of the model. As discussed earlier, the geometric shape of the sample significantly influences its behavior during drying. Therefore, the application of the model, the availability of technology, and the consistency in maintaining the shape, among other factors, need to be evaluated. For future work, it is proposed to utilize the information obtained in this study from particle-level modeling to scale the drying process from the laboratory to the industrial scale. This will involve designing equipment and processes, evaluating efficiency and product quality on a larger scale, and conducting sensitivity analyses and process simulations.

Author Contributions: I.M.: Conceptualization, Methodology, Investigation, Writing—original draft, Software. D.Z.-G.: Conceptualization, Methodology, Investigation, Writing—original draft, Software. Y.B.: Methodology, Formal analysis. M.C.R.: Conceptualization, Methodology, Formal analysis, Investigation. A.F.: Conceptualization, Formal analysis, Funding acquisition. M.P.F.: Conceptualization, Investigation, Formal analysis, Writing—original draft. A.P.B.: Formal analysis, Software. G.M.: Conceptualization, Resources, Formal analysis, Writing—review and editing, Supervision, Project administration, Funding acquisition. R.R.: Conceptualization, Resources, writing—review and editing, Supervision, Project administration, Funding acquisition. All authors have read and agreed to the published version of the manuscript.

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Data Availability Statement: The datasheets generated during and/or analyzed during the current study are available from the corresponding author upon reasonable request. The characteristics of used materials are also available on request.

Conflicts of Interests: The authors declare no competing interests.

Declarations: Ethics approval was not required for the present investigation. It does not involve human participants and/or animals. We should not be on Institutional Review Board Statement.

Abbreviation

dw _b	Dry weight basis
OHC	Oil holding capacity
PC	Pumpkin cubes
PPC	Pumpkin powders from cubes
PP	Pumpkin puree
PPP	Pumpkin powders from puree
RMSE	Squared root squared errors
SSE	Sum of squared errors
SWC	Swelling capacity
WHC	Water holding capacity

Nomenclature

Symbols	
$\vec{A}$	Surface vector (m ²)

Bi_T	Thermal Biot number (dimensionless)
B_m	Mass Biot number (dimensionless)
C_a	Concentration of water vapor in the air (kg/m ³)
C_p	Solid (pumpkin) specific heat (kJ/(kg K))
C_s	Concentration of water vapor at the surface of the sample (kg/m ³)
D_0	Arrhenius pre-exponential factor (m ² /s)
D_{eff}	Effective diffusion coefficient (m ² /s)
E_a	Activation energy (kJ/mol)
G	Lag factor (dimensionless)
g	Heat input rate (flux/source boundary condition) (dimensionless)
h_m	Convective mass transfer coefficient (m/s)
h_c	Heat transfer coefficient (W/m ² K)
J	Gradient term of flux/source boundary condition (dimensionless)
k	Thermal conductivity (W/m K)
L	Length change with respect to time (m)
Le	Lewis number (dimensionless)
MR	Moisture ratio, dimensionless
$\vec{n}$	Unit vector normal to the transfer area (dimensionless)
Nu	Nusselt dimensionless number (dimensionless)
P_{vsat}	Saturated water vapor pressure (dimensionless)
Pr	Prandtl dimensionless number (dimensionless)
q	Coefficient of a Robin boundary condition of COMSOL (dimensionless)
r	R-coordinate (m)
R	Universal gas constant (kJ /mol K)
Re	Reynolds number (dimensionless)
S_{dr}	Drying rate constant (1/s)
S	Shrinkage ratio (dimensionless)
Sc	Schmidt dimensionless number (dimensionless)
Sh	Sherwood dimensionless number (dimensionless)
t	Time (s)
T	Surface temperature (K)
U	Term of the dependent variable of boundary condition (the units of U in the boundary condition depend on the considered variable)
V	Velocity module (m/s)
V_s	Volume of the food sample (m ³)
x	X coordinate (m)
y	Y coordinate (m)
z	Z coordinate (m)
W	Specific moisture content (kg water/kg dry matter)
ΔH_{vap}	Water vaporization latent heat of (kJ/kg)
<i>Greek letters</i>	
α	Thermal diffusivity of the material (m ² /s)
ι	Pumpkin sample thickness length (m)
ρ_s	Solid (pumpkin) density on a dry basis (kg /m ³)
ρ_w	Water density (kg/m ³)
χ^2	Chi-squared (dimensionless)
τ	Fourier number (dimensionless time), (dimensionless)
<i>Superscripts and subscripts</i>	
'	Prime symbol indicates the dimensionless version of the variable or parameter
0	Initial condition
a	Relative to the air
e	Equilibrium condition
r	Relative to the r coordinate direction

sat	Relative to the saturated vapor
x	Relative to the x coordinate direction
y	Relative to the y coordinate direction
z	Relative to the z coordinate direction

Appendix A

Appendix A.1. Fick's Equation of Shrinkage

The following assumptions were adopted during the development of the drying model:

1. The equations representing the transport mechanisms had domains in the x and y variables for PC (2D) and in the r variable for PP (1D-axisymmetric).
2. Shrinkage occurs on the x and y axes for PC, and the r axis for PP.
3. The moisture content is homogeneously distributed throughout the volume of the cube or area of puree and slices, respectively, at the initial time.
4. The effective diffusion coefficient is a function of two variables, namely moisture content and temperature.
5. Shrinkage occurs at the interface between the solid and air.
6. Water transport occurs only by diffusive phenomena.
7. Air flux is uniform throughout the convective dryer and drying occurs at isothermal conditions.
8. To model the deformation of PC, the lengths of the x and y axes, which form the structure, were shortened in length ι (see Figure 2a). Thus, the cube walls attached to these axes contract causing the volume reduction.
9. Similarly, for PP samples, the radius to be shortening was considered (see Figure 2b). Although the PP samples do not symmetrically contract (because the tissue structures were broken and decreased in size) were modeled considering the equivalent transfer area calculated from r.

Considering the volume of the food sample, it can be expressed as:

$$V_s = A\iota \quad (A1)$$

Since ι decreases as time progresses due to the outgoing water flow through the walls of the food sample (Figure 2), a variation in volume V_s (decrease) will occur, causing contraction.

The variation in volume with respect to time can then be expressed as follows:

$$\frac{d}{dt} \int dV_s = \frac{dV_s}{dt} = \frac{d(A\iota)}{dt} = A \frac{d\iota}{dt} \quad (A2)$$

where the last term considers that the transfer area remains constant.

On the other hand, Reynolds's transport theorem states that $\left(\frac{dV}{dt}\right)$ can be expressed as the dot product between $\vec{v}\vec{n}$ and $d\vec{S}$ [9]. Here, $\vec{n}$ is the unit vector perpendicular to the area, defining the direction and velocity module $\vec{v}$ with which water diffuses through the walls of the sample, and $d\vec{A}$ represents the differential surface vector (Figure 2). $d\vec{A}$ and $\vec{v}\vec{n}$ have the same direction.

$$\frac{d}{dt} \int V_s = \frac{dV}{dt} = \int \vec{v}\vec{n} \cdot d\vec{A} = \vec{v}\vec{n}A \quad (A3)$$

If Fick's law is expressed in terms of the amount of water (i.e., in the x direction) that diffuses through the sample, Equation (A4) is obtained:

$$J = -Q_s D_{eff} \frac{\partial W}{\partial x} \quad (A4)$$

Expressing J in terms of $\vec{v}\vec{n}$ and ρ_s , Equation (A5), and solve for $\vec{v}\vec{n}$, Equation (A6) is obtained. Equation (A5) represents the kilograms of water per kilogram of dry food or solid that passes through a section of one square meter per second.

$$\vec{v}\vec{n}Q_w = Q_s D_{\text{eff}} \frac{\partial W}{\partial x} \quad (\text{A5})$$

$$\vec{v}\vec{n} = \frac{Q_s}{Q_w} D_{\text{eff}} \frac{\partial W}{\partial x} \quad (\text{A6})$$

Substituting the value of $\vec{v}\vec{n}$ from Equation (A6) and the value of $\left(\frac{dV}{dt}\right)$ from Equation (A2) into Equation (A3):

$$A \frac{d\iota}{dt} = \frac{Q_s}{Q_w} D_{\text{eff}} \frac{\partial W}{\partial x} A \quad (\text{A7})$$

Simplifying the area A , Equation (A8) results:

$$\frac{d\iota}{dt} = \frac{Q_s}{Q_w} D_{\text{eff}} \frac{\partial W}{\partial x} \quad (\text{A8})$$

Equation (A8) represents the contraction term as a function of the moisture content. The variable ι represents the characteristic length. The term $\left(\frac{d\iota}{dt}\right)$ reflects the change over time. If the sample shrinks or expands during drying, ι also changes. The term $\left(\frac{x'}{\iota}\right)$ represents the relationship between the current position x' and the characteristic length ι of the sample.

The term $\left(\frac{d\iota}{dt}\right)$ represents the rate at which the characteristic length ι of the sample changes over time. In mass conservation expression, this term is multiplied by the partial derivative of water concentration in the direction of mass transfer $\left(\frac{\partial W}{\partial x}\right)$ to obtain the total contribution of the change in sample length to mass transfer.

Appendix A.2. Dimensionless Transformation of Equation

The equations governing the transport phenomena in drying processes are typically so complex that exact solutions are often impossible. As a result, empirical relationships must be established between variables, rather than relying on analytical solutions. When dealing with more than two variables, such as in this case, involving moisture content, temperature, and shrinkage, the resulting correlations can become complex. However, by logically grouping variables, correlations can be considerably simplified.

In this study, the variables are made dimensionless using the quantities of time, represented by the Fourier number τ , moisture content denoted as W' , temperature T' , and shrinkage S , which is the solid–air interface position (Equation (A9)). Equation (A10) represents the mass conservation and Equation (A11) represents Equation (A10) in terms of dimensionless variables.

$$\tau = \frac{D_0 t}{\iota_0^2}; \quad W' = \frac{W - W_e}{W_0 - W_e}; \quad T' = \frac{T - T_0}{T_a - T_0}; \quad \tau = \frac{\iota}{\iota_0^2} \quad (\text{A9})$$

Mass conservation

For 2D (PC geometry):

$$\left\{ \begin{aligned} \frac{\partial W_x}{\partial t} &= \frac{1}{\iota^2} \frac{\partial}{\partial x'} \left(D_{\text{eff}} \frac{\partial W}{\partial x'} \right) + \frac{x'}{\iota} \frac{d\iota}{dt} \frac{\partial W}{\partial x'} \\ \frac{\partial W_y}{\partial t} &= \frac{1}{\iota^2} \frac{\partial}{\partial y'} \left(D_{\text{eff}} \frac{\partial W}{\partial y'} \right) + \frac{y'}{\iota} \frac{d\iota}{dt} \frac{\partial W}{\partial y'} \end{aligned} \right\} \quad (\text{A10})$$

For 1D (PP geometry):

$$\frac{\partial W}{\partial t} = \frac{1}{t^2} \frac{\partial}{\partial r'} \left(D_{\text{eff}} \frac{\partial W}{\partial r'} \right) + \frac{r'}{t} \frac{d}{dt} \frac{\partial W}{\partial r'} \quad (\text{A11})$$

To obtain the dimensionless transformation, it is necessary to express $\left(\frac{\partial W}{\partial t}\right)$ in terms of the dimensionless variables defined in (A9), it yields in Equation (A11)

$$\begin{aligned} \frac{\partial W}{\partial t} &= \frac{\partial [W'(W_0 - W_e) + W_e]}{\partial \left(\frac{\tau t_0^2}{D_0} \right)} \\ \frac{\partial W}{\partial t} &= \frac{\partial [W'(W_0 - W_e) + W_e]}{\frac{t_0^2}{D_0} \partial \tau} = D_0 \frac{\partial [W'(W_0 - W_e) + W_e]}{t_0^2 \partial \tau} = \frac{D_0}{t_0^2} \left[\frac{\partial W'(W_0 - W_e)}{\partial \tau} + \frac{\partial W_e}{\partial \tau} \right] = \frac{D_0}{t_0^2} (W_0 - W_e) \frac{\partial W'}{\partial \tau} \end{aligned} \quad (\text{A12})$$

$$\frac{\partial W}{\partial t} \frac{t_0^2}{D_0 (W_0 - W_e)} = \frac{\partial W'}{\partial \tau}$$

Analyzing Equation (A11), in order to find the dimensionless expression of the mass variation with respect to the Fourier number, $\left(\frac{\partial W}{\partial t}\right)$ Equation (A10) must be multiplied on both sides by the factor $\left(\frac{t_0^2}{D_0 (W_0 - W_e)}\right)$.

The solution of multiplying both terms on the right-hand side of Equations (A10) and (A11) by the factor $\left(\frac{t_0^2}{D_0 (W_0 - W_e)}\right)$ is shown in Equations (A13)–(A30). The mathematical development was carried out for the x' dimensionless direction of the pumpkin cube (PC) geometry. The direction along the y' axis for PC geometry and in the radial direction for pumpkin puree (PP) geometry are obtained analogously by operating similarly.

The first term on the right-hand side of Equations (A10) and (A11) in terms of the dimensionless variables for 2D (PC geometry):

$$\begin{aligned} \frac{1}{t^2} \frac{\partial}{\partial x'} \left(D_{\text{eff}} \frac{\partial W}{\partial x'} \right) &= \frac{1}{(S t_0)^2} \frac{\partial}{\partial x'} \left\{ \frac{D_{\text{eff}}' D_0 \partial [W'(W_0 - W_e) + W_e]}{\partial x'} \right\} \\ \frac{1}{t^2} \frac{\partial}{\partial y'} \left(D_{\text{eff}} \frac{\partial W}{\partial y'} \right) &= \frac{1}{(S t_0)^2} \frac{\partial}{\partial y'} \left\{ \frac{D_{\text{eff}}' D_0 \partial [W'(W_0 - W_e) + W_e]}{\partial y'} \right\} \end{aligned} \quad (\text{A13})$$

For 1D (PP geometry):

$$\frac{1}{t^2} \frac{\partial}{\partial r'} \left(D_{\text{eff}} \frac{\partial W}{\partial r'} \right) = \frac{1}{(S t_0)^2} \frac{\partial}{\partial r'} \left\{ \frac{D_{\text{eff}}' D_0 \partial [W'(W_0 - W_e) + W_e]}{\partial r'} \right\} \quad (\text{A14})$$

The mathematical development (dimensionless first term) for the x' direction of the PC geometry:

$$\frac{1}{(S t_0)^2} \frac{\partial}{\partial x'} \left\{ \frac{D_{\text{eff}}' D_0 \partial [W'(W_0 - W_e) + W_e]}{\partial x'} \right\} \frac{t_0^2}{D_0 (W_0 - W_e)} \quad (\text{A15})$$

Applying the distributive property of partial derivative, it yields:

$$\frac{1}{(S_{i_0})^2} \frac{\partial}{\partial x'} \left\{ \frac{D_{eff}' D_0 \partial [W'(W_0 - W_e) + W_e]}{\partial x'} \right\} \frac{i_0^2}{D_0 (W_0 - W_e)} \quad (A16)$$

where $\left(\frac{\partial W_e}{\partial x'} = 0\right)$

$$\frac{1}{S^2 i_0^2} \frac{\partial}{\partial x'} D_{eff}' D_0 \left[\frac{\partial W' (W_0 - W_e)}{\partial x'} \right] \frac{i_0^2}{D_0 (W_0 - W_e)} = \frac{1}{S^2 i_0^2} \frac{\partial}{\partial x'} D_{eff}' D_0 (W_0 - W_e) \frac{\partial W'}{\partial x'} \frac{i_0^2}{D_0 (W_0 - W_e)} \quad (A17)$$

Simplifying the factor $(W_0 - W_e)$, D_0 , and i_0^2 and rearranging the equation, knowing that diffusivity is isotropic (does not vary with x'), yields:

$$\frac{1}{S^2} \frac{\partial}{\partial x'} \left(D_{eff}' \frac{\partial W'}{\partial x'} \right) = \frac{D_{eff}'}{S^2} \frac{\partial}{\partial x'} \left(\frac{\partial W'}{\partial x'} \right) = \frac{D_{eff}'}{S^2} \frac{\partial^2 W'}{\partial x'^2} \quad (A18)$$

The first dimensionless term of the right-hand side of Equation (A10) for PC geometry:

$$\left. \begin{aligned} \frac{D_{eff}'}{S^2} \frac{\partial}{\partial x'} \left(\frac{W'}{\partial x'} \right) &= \frac{D_{eff}'}{S^2} \frac{\partial^2 W'}{\partial x'^2} \\ \frac{D_{eff}'}{S^2} \frac{\partial}{\partial y'} \left(\frac{W'}{\partial y'} \right) &= \frac{D_{eff}'}{S^2} \frac{\partial^2 W'}{\partial y'^2} \end{aligned} \right\} \quad (A19)$$

For 1D (PP geometry, Equation (A11)):

$$\frac{D_{eff}'}{S^2} \frac{\partial}{\partial r'} \left(\frac{W'}{\partial r'} \right) \quad (A20)$$

The second term on the right-hand side of Equation (A10) in terms of the dimensionless variables for 2D (PC geometry):

$$\left. \begin{aligned} \frac{x'}{i} \frac{\partial i}{\partial t} \frac{\partial W}{\partial x'} &= \frac{\partial i x'}{\partial t} \frac{\partial W}{\partial x' i} = \frac{\partial S_{i_0} x'}{\partial \frac{\tau i_0^2}{D_0}} \frac{\partial [W'(W_0 - W_e) + W_e]}{\partial x' S_{i_0}} \\ \frac{y'}{i} \frac{\partial i}{\partial t} \frac{\partial W}{\partial y'} &= \frac{\partial i y'}{\partial t} \frac{\partial W}{\partial y' i} = \frac{\partial S_{i_0} y'}{\partial \frac{\tau i_0^2}{D_0}} \frac{\partial [W'(W_0 - W_e) + W_e]}{\partial y' S_{i_0}} \end{aligned} \right\} \quad (A21)$$

For 1D (PP geometry, Equation (A11)):

$$\frac{r'}{i} \frac{\partial i}{\partial t} \frac{\partial W}{\partial r'} = \frac{\partial i r'}{\partial t} \frac{\partial W}{\partial r' i} = \frac{\partial S_{i_0} r'}{\partial \frac{\tau i_0^2}{D_0}} \frac{\partial [W'(W_0 - W_e) + W_e]}{\partial r' S_{i_0}} \quad (A22)$$

The mathematical development (dimensionless second term) for the x' direction of the PC geometry:

$$\frac{x'}{i} \frac{\partial i}{\partial t} \frac{\partial W}{\partial x'} \frac{i_0^2}{D_0 (W_0 - W_e)} = \frac{\partial i x'}{\partial t} \frac{\partial W}{\partial x' i} \frac{i_0^2}{D_0 (W_0 - W_e)} = \frac{\partial S_{i_0} x'}{\partial \frac{\tau i_0^2}{D_0}} \frac{\partial [W'(W_0 - W_e) + W_e]}{\partial x' S_{i_0}} \frac{i_0^2}{D_0 (W_0 - W_e)} \quad (A23)$$

Applying the distributive property of the partial derivative yields:

$$\frac{\partial(S_{l_0})x'}{\partial \frac{\tau l_0^2}{D_0}} \left[\frac{\partial W'(W_0-W_e)}{\partial x' S_{l_0}} + \frac{\partial W_e}{\partial x' S_{l_0}} \right] \frac{l_0^2}{D_0(W_0-W_e)} \quad (A24)$$

where $\left(\frac{\partial W_e}{\partial x'}=0\right)$

$$\begin{aligned} & \frac{\partial(S_{l_0})x'}{\partial \frac{\tau l_0^2}{D_0}} \left[\frac{1}{S_{l_0}} (W_0-W_e) \frac{\partial W'}{\partial x'} \right] \frac{l_0^2}{D_0(W_0-W_e)} = \\ & \frac{l_0 \partial S x'}{\frac{l_0^2}{D_0} \partial \tau} \left[\frac{1}{S_{l_0}} (W_0-W_e) \frac{\partial W'}{\partial x'} \right] \frac{l_0^2}{D_0(W_0-W_e)} = \frac{D_0 l_0 \partial S x'}{l_0^2 \partial \tau} \left[\frac{1}{S_{l_0}} (W_0-W_e) \frac{\partial W'}{\partial x'} \right] \frac{l_0^2}{D_0(W_0-W_e)} \end{aligned} \quad (A25)$$

Simplifying the factor (W_0-W_e) , D_0 , and l_0^2 and rearranging the above equation yields:

$$\frac{D_0 l_0 \partial S}{l_0^2 S_{l_0}} (W_0-W_e) \frac{x'}{\partial \tau} \frac{\partial W'}{\partial x'} \frac{l_0^2}{D_0(W_0-W_e)} = \frac{\partial S x'}{S} \frac{\partial W'}{\partial \tau \partial x'} = \frac{x'}{S} \frac{\partial S \partial W'}{\partial \tau \partial x'} \quad (A26)$$

The second dimensionless term of the right-hand side of Equation (A10) for PC geometry:

$$\left\{ \begin{array}{l} \frac{x'}{S} \frac{\partial S \partial W'}{\partial \tau \partial x'} \\ \frac{y'}{S} \frac{\partial S \partial W'}{\partial \tau \partial y'} \end{array} \right\} \quad (A27)$$

For PP geometry (Equation (A11)):

$$\left\{ \frac{r'}{S} \frac{\partial S \partial W'}{\partial \tau \partial r'} \right\} \quad (A28)$$

Finally, the equation is assembled with all its dimensionless terms to express the mass conservation (Equations (A29) and (A30)):

For PC geometry:

$$\left\{ \begin{array}{l} \frac{\partial W_x'}{\partial \tau} = \frac{D_{eff}'}{S^2} \frac{\partial}{\partial x'} \left(\frac{W'}{\partial x'} \right) + \frac{x'}{S} \frac{\partial S \partial W'}{\partial \tau \partial x'} \\ \frac{\partial W_y'}{\partial \tau} = \frac{D_{eff}'}{S^2} \frac{\partial}{\partial y'} \left(\frac{W'}{\partial y'} \right) + \frac{y'}{S} \frac{\partial S \partial W'}{\partial \tau \partial y'} \end{array} \right\} \quad (A29)$$

For PP geometry:

$$\frac{\partial W_r'}{\partial \tau} = \frac{D_{eff}'}{S^2} \frac{\partial}{\partial r'} \left(\frac{W'}{\partial r'} \right) + \frac{r'}{S} \frac{\partial S \partial W'}{\partial \tau \partial r'} \quad (A30)$$

Energy conservation

Similarly, to obtain the dimensionless transformation of the energy conservation Equations (A31) and (A32), it is necessary to express that, in terms of the dimensionless variables defined in (A9), which yields Equation (A36).

For PC geometry:

$$\left. \begin{aligned} \frac{\partial T}{\partial t} &= \frac{1}{\tau^2} \frac{\partial}{\partial x'} \left(\alpha \frac{\partial T}{\partial x'} \right) + \frac{x'}{\tau} \frac{d\tau}{dt} \frac{\partial T}{\partial x'} \\ \frac{\partial T}{\partial t} &= \frac{1}{\tau^2} \frac{\partial}{\partial y'} \left(\alpha \frac{\partial T}{\partial y'} \right) + \frac{y'}{\tau} \frac{d\tau}{dt} \frac{\partial T}{\partial y'} \end{aligned} \right\} \quad (\text{A31})$$

For PP geometry:

$$\frac{\partial T}{\partial t} = \frac{1}{\tau^2} \frac{\partial}{\partial r'} \left(\alpha \frac{\partial T}{\partial r'} \right) + \frac{r'}{\tau} \frac{d\tau}{dt} \frac{\partial T}{\partial r'} \quad (\text{A32})$$

where α is the thermal diffusivity of pumpkin. Substituting the main variables (Equation (A9)) to apply the dimensionless transformation:

$$\frac{\partial [T'(T_a - T_0) + T_0]}{\partial \frac{\tau \tau_0^2}{D_0}} = \frac{\alpha}{(S \tau_0^2)} \frac{\partial^2 [T'(T_a - T_0) + T_0]}{\partial x'^2} + \frac{x'}{S \tau_0} \frac{d[S \tau_0]}{d \frac{\tau \tau_0^2}{D_0}} \frac{\partial [T'(T_a - T_0) + T_0]}{\partial x'} \quad (\text{A33})$$

Similarly, to the procedure used for the dimensionless transformation of the mass conservation equation, the energy equation will be multiplied by the factor $\left(\frac{\tau_0^2}{D_0(T_a - T_0)} \right)$ on both sides to isolate $\left(\frac{\partial T'}{\partial \tau} \right)$ and it yields:

$$\frac{\partial T}{\partial t} = \frac{\partial [T'(T_a - T_0) + T_0]}{\partial \frac{\tau \tau_0^2}{D_0}} = \frac{D_0}{\tau_0^2} \left[\frac{\partial T'(T_a - T_0)}{\partial \tau} + \frac{\partial T_0}{\partial \tau} \right]$$

$$\frac{\partial T_0}{\partial \tau} = 0$$

$$\frac{\partial T}{\partial t} = \frac{D_0}{\tau_0^2} \frac{\partial T'(T_a - T_0)}{\partial \tau} = (T_a - T_0) \frac{D_0}{\tau_0^2} \frac{\partial T'}{\partial \tau}$$

Rearranging the above equation yields:

$$\frac{\partial T'}{\partial \tau} = \frac{\partial T}{\partial t} \frac{\tau_0^2}{D_0(T_a - T_0)}$$

Analyzing the above equation, to find the dimensionless expression of the energy conservation with respect to the Fourier number, $\left(\frac{\partial T'}{\partial \tau} \right)$, Equation (A14) must be multiplied on both sides by the factor $\left(\frac{\tau_0^2}{D_0(T_a - T_0)} \right)$.

Similarly, the mathematical development was carried out for the x' dimensionless direction of the PC geometry. The direction along the y' axis for PC geometry and in the radial direction for PP geometry are obtained analogously by operating similarly.

The first term on the right-hand side of Equation (A14) in terms of the dimensionless variables for 2D (PC geometry):

$$\left. \begin{aligned} \frac{1}{\tau^2} \frac{\partial}{\partial x'} \left(\alpha \frac{\partial T}{\partial x'} \right) &= \frac{\alpha}{[S \tau_0^2]} \frac{\partial^2 [T'(T_a - T_0) + T_0]}{\partial x'^2} \\ \frac{1}{\tau^2} \frac{\partial}{\partial y'} \left(\alpha \frac{\partial T}{\partial y'} \right) &= \frac{\alpha}{[S \tau_0^2]} \frac{\partial^2 [T'(T_a - T_0) + T_0]}{\partial y'^2} \end{aligned} \right\}$$

For 1D (PP geometry):

$$\frac{1}{\tau^2} \frac{\partial}{\partial r'} \left(\alpha \frac{\partial T}{\partial r'} \right) = \frac{\alpha}{[S \tau_0^2]} \frac{\partial^2 [T'(T_a - T_0) + T_0]}{\partial r'^2}$$

The mathematical development (dimensionless first term) for the x' dimensionless direction of the PC geometry:

$$\frac{\alpha}{(S_{i_0}^2)} \frac{\partial^2 [T'(T_a - T_0) + T_0]}{\partial x'^2} \frac{i_0^2}{D_0(T_a - T_0)} = \frac{\alpha}{(S_{i_0}^2)} \left[\frac{\partial^2 [T'(T_a - T_0)]}{\partial x'^2} + \frac{\partial^2 T_0}{\partial x'^2} \right] \frac{i_0^2}{D_0(T_a - T_0)}$$

where $\left(\frac{\partial^2 T_0}{\partial x'^2} = 0\right)$.

$$\frac{\alpha}{(S_{i_0}^2)} (T_a - T_0) \frac{\partial^2 [T']}{\partial x'^2} \frac{i_0^2}{D_0(T_a - T_0)} = \frac{\alpha}{D_0 S} \frac{\partial^2 T'}{\partial x'^2}$$

where $\left(\frac{\alpha}{D_0} = Le\right)$.

In the convective drying of food, the Lewis number (Le) characterizes the relationship between the rate of vapor diffusion in the material and the rate of heat.

$$\frac{\alpha}{D_0 S} \frac{\partial^2 T'}{\partial x'^2} = \frac{Le}{S} \frac{\partial^2 T'}{\partial x'^2}$$

The first dimensionless term of the right-hand side of Equation (A31) for PC geometry:

$$\left. \begin{array}{l} \frac{Le}{S} \frac{\partial^2 T'}{\partial x'^2} \\ \frac{Le}{S} \frac{\partial^2 T'}{\partial y'^2} \end{array} \right\} \quad (A34)$$

For PP geometry:

$$\left. \frac{Le}{S} \frac{\partial^2 T'}{\partial r'^2} \right\} \quad (A35)$$

The second term on the right-hand side of Equation (A13) in terms of the dimensionless variables for 2D (PC geometry):

$$\left. \begin{array}{l} \frac{x'}{i} \frac{\partial i}{\partial t} \frac{\partial T}{\partial x'} = \frac{\partial x'}{\partial t} \frac{\partial T}{\partial x' i} = \frac{x'}{S_{i_0}} \frac{d[S_{i_0}]}{d \frac{\tau i_0^2}{D_0}} \frac{\partial [T'(T_a - T_0) + T_0]}{\partial x'} \\ \frac{y'}{i} \frac{\partial i}{\partial t} \frac{\partial T}{\partial y'} = \frac{\partial y'}{\partial t} \frac{\partial T}{\partial y' i} = \frac{y'}{S_{i_0}} \frac{d[S_{i_0}]}{d \frac{\tau i_0^2}{D_0}} \frac{\partial [T'(T_a - T_0) + T_0]}{\partial y'} \end{array} \right\} \quad (A36)$$

For 1D (PP geometry):

$$\frac{r'}{i} \frac{\partial i}{\partial t} \frac{\partial T}{\partial r'} = \frac{\partial r'}{\partial t} \frac{\partial T}{\partial r' i} = \frac{r'}{S_{i_0}} \frac{d[S_{i_0}]}{d \frac{\tau i_0^2}{D_0}} \frac{\partial [T'(T_a - T_0) + T_0]}{\partial r'} \quad (A37)$$

The mathematical development (dimensionless first term) for the x' direction of the PC geometry:

$$\frac{x'}{S_{i_0}} \frac{d[S_{i_0}]}{d \frac{\tau i_0^2}{D_0}} \frac{\partial [T'(T_a - T_0) + T_0]}{\partial x'} \frac{i_0^2}{D_0(T_a - T_0)} = \frac{x'}{S_{i_0}} \frac{i_0 D_0}{i_0^2} \frac{dS}{d\tau} \left[\frac{\partial [T'(T_a - T_0) + T_0]}{\partial x'} \right] \frac{i_0^2}{D_0(T_a - T_0)} = \frac{x'}{S} \frac{D_0}{i_0^2} \frac{d[S]}{d\tau} \left[(T_a - T_0) \frac{\partial T'}{\partial x'} + \frac{\partial T_0}{\partial x'} \right] \frac{i_0^2}{D_0(T_a - T_0)}$$

where $\left(\frac{\partial T_0}{\partial x'}=0\right)$.

Simplifying the factor (T_a-T_0) , D_0 , and ι_0^2 and rearranging the above equation yields:

$$\frac{x'}{S} \frac{dS}{d\tau} \frac{\partial T'}{\partial x'}$$

The second dimensionless term of the right-hand side of Equation (A31) for PC geometry:

$$\left\{ \begin{array}{l} \frac{x'}{S} \frac{dS}{d\tau} \frac{\partial T'}{\partial x'} \\ \frac{y'}{S} \frac{dS}{d\tau} \frac{\partial T'}{\partial y'} \end{array} \right\} \quad (A38)$$

For PP geometry (Equation (A32)):

$$\frac{\partial T_r'}{\partial \tau} = \frac{Le}{S} \frac{\partial^2 T'}{\partial r'^2} + \frac{r'}{S} \frac{dS}{d\tau} \frac{\partial T'}{\partial r'} \quad (A39)$$

Fick's equation of shrinkage

To obtain the dimensionless transformation of Equation (A8), it is necessary to express that in terms of the dimensionless variables defined in Equation (A9), which yields Equation (A40) for the x' dimensionless direction of the PC geometry:

$$\begin{aligned} \frac{dS \iota_0}{d\tau} \frac{\partial W'}{\partial x'} &= \frac{Q_s}{Q_w} \frac{D_{eff}}{S \iota_0} \frac{\partial [W'(W_0-W_e)+W_e]}{\partial x'} = \frac{D_0}{\iota_0^2} \frac{dS}{d\tau} = \frac{Q_s}{Q_w} \frac{D_{eff}}{S \iota_0} \left[(W_0-W_e) \frac{\partial W'}{\partial x'} + \frac{\partial W_e}{\partial x'} \right] \\ &= \frac{D_0}{\iota_0} \frac{dS}{d\tau} = \frac{Q_s}{Q_w} \frac{D_{eff}}{S} \left[(W_0-W_e) \frac{\partial W'}{\partial x'} + \frac{\partial W_e}{\partial x'} \right] \end{aligned} \quad (A40)$$

Solving $\left(\frac{dS}{d\tau}\right)$ for Equation (A40) results in:

$$\frac{dS}{d\tau} = \frac{Q_s}{Q_w} \frac{D_{eff}}{S} \left[(W_0-W_e) \frac{\partial W'}{\partial x'} + \frac{\partial W_e}{\partial x'} \right]$$

For PC geometry:

$$\left\{ \begin{array}{l} \frac{dS_x}{d\tau} = \frac{Q_s}{Q_w} \frac{D_{eff}}{S} (W_0-W_e) \frac{\partial W'}{\partial x'} \\ \frac{dS_y}{d\tau} = \frac{Q_s}{Q_w} \frac{D_{eff}}{S} (W_0-W_e) \frac{\partial W'}{\partial y'} \end{array} \right\} \quad (A41)$$

For PP geometry:

$$\frac{dS_r}{d\tau} = \frac{Q_s}{Q_w} \frac{D_{eff}}{S} (W_0-W_e) \frac{\partial W'}{\partial r'} \quad (A42)$$

General formulation for modeling in COMSOL

In general terms, in the COMSOL modeling of processes, partial differential equations are used. The software has a well-defined structure of partial differential equations (PDE) that contains accumulation terms, diffusion terms, and source terms, among others [47]. To enter the equations described in this work, a structure was arranged according to Sandoval et al. [9]:

$$\frac{\partial U}{\partial t} + \frac{\partial J}{\partial x'} = q \quad 0 \leq x' \leq 1 \quad (\text{A43})$$

where U is the accumulation term concerning the dimensionless Fourier number; J is the gradient term; q is the variation of moisture content or temperature as a function of contraction.

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