




# Algorithms for Reliable Estimation, Identification and Control

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The two-part Special Issue “Algorithms for Reliable Estimation, Identification and Control” deals with the optimization of feedforward and feedback controllers with respect to predefined performance criteria as well as the state and parameter estimation for systems with uncertainty. The developed methods focus both on offline applications during system design and on the online application for the actual implementation in real-time capable environments.

In particular, the enhancement and verification of the robustness of control and state estimation procedures concerning external disturbances and uncertain parameters are widespread aspects of current research activities. The same holds for the reliable estimation of nonmeasurable system states and the identification of parameters based on uncertain measurements. Possible applications of related optimization algorithms can be found not only in the frame of a control and estimator synthesis, but also in the field of reliable modeling and model-based analysis of measured data.

This Special Issue is a platform for novel algorithms in the frame of reliable and optimal estimation and control. Moreover, application-oriented aspects highlighting the practical applicability of theoretical approaches are included.

The first part of this Special Issue starts with a contribution by S. Romig, L. Jaulin, and A. Rauh on *Using Interval Analysis to Compute the Invariant Set of a Nonlinear Closed-Loop Control System* [1], in which novel interval arithmetic techniques are developed for a rigorous enclosure of positively invariant sets of nonlinear closed-loop structures. To obtain desired invariant sets in a rigorous manner, sliding mode controllers are implemented with the help of a suitably chosen Gröbner basis that is employed to solve Bézout’s identity. The associated stability proof is carried out through control Lyapunov functions, where the enclosure approach for invariant sets exploits an algorithm for a set-based inversion of nonlinear mappings.

N. Keshtkar and K. Röbenack present their work on *Unstructured Uncertainty Based Modeling and Robust Stability Analysis of Textile-Reinforced Composites with Embedded Shape Memory Alloys* [2], in which mathematical models are developed for the deflection control of a textile-reinforced composite. The mathematical modeling of the system comprises a systematic parameter identification and an unstructured uncertainty representation. Thereafter, robust proportional–integral controllers are designed for the composite deflection on the basis of a robust stability analysis. The experimental results demonstrate the practical applicability of the proposed technique.

The article *Predictive Path Following and Collision Avoidance of Autonomous Connected Vehicles* [3] by M. Abdelaal and S. Schön deals with the control synthesis of multi-agent robot systems, where each agent is described by a nonlinear bicycle model. The presented optimization approach is based on the aims to follow the planned path—represented by a Bézier curve—and to guarantee collision avoidance among the networked vehicles.



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A validation of the proposed algorithm is performed with the help of a MATLAB simulation that makes use of the ACADO toolkit.

A. Rauh, W. Frenkel, and J. Kersten derive a *Kalman Filter-Based Online Identification of the Electric Power Characteristic of Solid Oxide Fuel Cells Aiming at Maximum Power Point Tracking* [4]. This approach is validated by means of real laboratory measurements and not only forms the basis for a filter-based identification of the hydrogen mass flow and electric current dependencies of the electrochemical behavior of solid oxide fuel cells but also helps to estimate the information that is necessary to maximize the system's fuel efficiency while simultaneously guaranteeing that the operating point remains in the desired domain of Ohmic polarization.

A methodological discussion on the analysis of *Observability of Uncertain Nonlinear Systems Using Interval Analysis* [5] is presented by T. Paradowski, S. Lerch, M. Damaszek, R. Dehnert, and B. Tibken. This research is especially challenging due to the fact that for nonlinear dynamic systems, in contrast to linear ones, the required number of time derivatives of the system outputs, from which the state can be reconstructed uniquely, is not a priori bounded. The proposed algorithm does not only allow for identifying those domains in the state space that are guaranteed to be observable by a given sensor configuration but also to detect domains in which states are not distinguishable.

As discussed by W. Frenkel, A. Rauh, J. Kersten, and H. Aschemann in their article on *Experiments-Based Comparison of Different Power Controllers for a Solid Oxide Fuel Cell Against Model Imperfections and Delay Phenomena* [6], the actual electric power of high-temperature fuel cells is influenced by numerous effects such as gas mass flow variations, electric currents, and temperature changes at the gas supply manifolds as well as in the interior of a fuel cell stack. To reliably compensate for these influence factors as well as to reduce the effect of lag dynamics caused by long gas supply tubes, standard proportional–integral controllers are replaced with internal model, linear state-feedback, and sliding mode control approaches. These advanced approaches are compared in simulations and experiments. For all cases, state observers are derived which allow for a model-based identification of nonmeasurable temporal derivatives of the electric power as well as the quantification of external disturbances and possible deviations of simplified models from reality.

In the second part of this Special Issue, A. Rauh and J. Kersten study the *Transformation of Uncertain Linear Systems with Real Eigenvalues into Cooperative Form: The Case of Constant and Time-Varying Bounded Parameters* [7] to perform a computationally efficient reachability analysis of linear time-invariant and linear time-varying continuous-time systems with the help of interval methods. The proposed change of coordinates allows for a reduction in both the computational complexity and the pessimism of the computed state bounds by transforming uncertain system models into a cooperative form so that crisp system representations are obtained for the lower and upper bounds of all components of the (transformed) state trajectories.

The article *Union and Intersection Operators for Thick Ellipsoid State Enclosures: Application to Bounded-Error Discrete-Time State Observer Design* [8] by A. Rauh, A. Bourgois, and L. Jaulin has a special focus on discrete-time dynamic systems given in a quasi-linear discrete-time state-space representation. The proposed techniques for propagating uncertainty domains have the advantage over existing approaches that the online solution of linear matrix inequalities is avoided so that the presented enclosure techniques as well as intersection and union operators can be included directly in predictor–corrector-type state estimators for dynamic systems with fast sampling rates. Moreover, the suggested thick ellipsoid set representation provides guaranteed outer bounds of reachable state domains in combination with inner bounds that are guaranteed to be fully included in the reachable sets. Hence, the distance between these two bounds can serve as an indicator for overestimation that occurs unavoidably if set-valued estimation approaches are implemented for nonlinear dynamic systems.

M. Louédec and L. Jaulin discuss in their paper *Interval Extended Kalman Filter—Application to Underwater Localization and Control* [9] that the quasi-standard Extended

Kalman Filter may deteriorate significantly in navigation systems if the true and estimated states deviate significantly from each other. To bridge the gap between such (probably unreliable) stochastic approaches and less precise but guaranteed set-valued alternatives, the use of an interval algorithm is suggested in this paper to estimate suitable linearization points for the Kalman filter. This combination leads to a significant enhancement of the localization accuracy which is an indispensable prerequisite for the implementation of reliable closed-loop control procedures in marine applications.

As discussed by A. Rauh, R. Dehnert, S. Romig, S. Lerch, and B. Tibken in their article *Iterative Solution of Linear Matrix Inequalities for the Combined Control and Observer Design of Systems with Polytopic Parameter Uncertainty and Stochastic Noise* [10], the classical separation principle of control and observer design no longer holds if dynamic systems are simultaneously subject to bounded parameter uncertainty and stochastic process and measurement noise. Then, even for linear dynamics, control and observer gains need to be determined simultaneously so that the closed-loop stability can be ensured. To solve this, this paper proposes a novel iterative solution procedures of matrix inequalities which express stability requirements by suitable Lyapunov function candidates and simultaneously allow for a minimization of the sensitivity of the closed-loop control structure against stochastic noise. The efficiency of the proposed solution approach is demonstrated by robustly stabilizing the Zeeman catastrophe machine along the unstable branch of its bifurcation diagram.

Model predictive control for unstable dynamic systems with uncertain parameters is a challenging task. Therefore, M. Fnadi and J. Alexandre dit Sandretto derive an interval-based control methodology in their paper *Experimental Validation of a Guaranteed Nonlinear Model Predictive Control* [11], which allows for tracking reference trajectories despite the aforementioned uncertainties within predefined tolerance bounds. Due to the computational complexity of the solution procedure, a two-stage approach is suggested. First, the robust and optimal control sequence (in the sense of the solution of a predictive control problem) is computed offline. Second, the input sequence is then applied to a real rotary inverse pendulum to demonstrate the successful stabilization of the system in the vicinity of the equilibrium point that would be unstable in a pure open-loop operation.

D. Gerbet and K. Röbenack develop *An Algebraic Approach to Identifiability* [12] in their publication. Although the problem of identifiability of nonlinear polynomial state-space systems has already been studied via input–output equations—requiring differential algebra—the authors propose an algebraic alternative that is based on distinguishability and observability. Employing techniques from algebraic geometry such as polynomial ideals and Gröbner bases, local as well as global results are derived. As such, this work provides complementary results to the article by T. Paradowski et al. [5] included in the first part of this Special Issue.

In their paper *Kleene Algebra to Compute Invariant Sets of Dynamical Systems* [13], T. Le Mézo, L. Jaulin, D. Massé, and B. Zerr aim at enclosing the greatest fixed point in Kleene algebra to compute inner and outer approximations of invariant-based sets for continuous-time nonlinear dynamical systems. The contribution of this article is to provide the required definitions and theorems to establish the link between the theory of invariant sets and the Kleene algebra that has not yet been done by other publications. Among other things, the proposed methodology can be employed to determine inner and outer approximations of control forward reach sets and to solve the task of robust path planning.

Signal temporal logic, as discussed by B. Finkbeiner, M. Fränzle, F. Kohn, and P. Kröger in their paper *A Truly Robust Signal Temporal Logic: Monitoring Safety Properties of Interacting Cyber-Physical Systems under Uncertain Observation* [14], is a linear-time temporal logic designed for classifying time-dependent signals originating from continuous-state or hybrid-state dynamical systems according to formal specifications. In this work, an existing analysis is extended by decomposing the error into an unknown yet fixed offset and an independent per-sample error. It can be shown that, in this setting, monitoring of temporal properties no longer coincides with collecting Boolean combinations of state predicates

evaluated in each time instant over the best-possible per-sample state estimates. Instead, the proposed monitoring approach can be genuinely more informative because it may infer determinate truth values for monitoring conditions that an interval-based evaluation remains inconclusive about. Suitable algorithms based on affine arithmetic and SAT modulo theory are further introduced in this paper.

The article *Experimental Validation of Ellipsoidal Techniques for State Estimation in Marine Applications* [15] by A. Rauh, Y. Gourret, K. Lagattu, B. Hummes, L. Jaulin, J. Reuter, S. Wirtensohn, and P. Hoher is an application-oriented extension of the second paper [8] from the second part of this Special Issue. It demonstrates the use of (thick) ellipsoid set representations for uncertainty modeling in the context of state and disturbance estimation of autonomous surface vessels. With the help of a predictor–corrector state estimator, computing guaranteed outer ellipsoidal state enclosures, it can be proven for the identified model of bounded uncertainties that the synchronized motion of two autonomous boats is guaranteed to be safe in the sense of a guaranteed collision avoidance. To perform this investigation, real measurements have been acquired in a dedicated experimental campaign.

This Special Issue is concluded with an article authored by C. Jaubertie and N. Verdière dealing with *Bounded-Error Parameter Estimation Using Integro-Differential Equations for Hindmarsh-Rose Model* [16]. In contrast to classical identification procedures, often involving derivatives of measured data and the requirement for suitably filtering measurements, the authors propose to reformulate the system into an integro-differential form during the pretreatment stage, which helps to significantly reduce sensitivity to measurement noise. This sensitivity reduction was achieved by the fact that the final identification model depends on integrals instead of derivatives of the sensor data. Finally, bounds on the system parameters to be estimated are determined with an algorithm for set inversion via interval analysis.

**Conflicts of Interest:** The authors declare no conflict of interest.

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