

Article

Algorithm for Determining the Strong Fuzzy Grade of a Hypergroup

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Abstract

In this paper, we present an algorithm for determining the strong fuzzy grade of a hypergroup, considering a particular class of hypergroups associated with genetics. A study is carried out depending on the cardinality of the initial hypergroup. The last paragraph of this paper contains the MATLAB R2018b program for the fuzzy grade for cardinals 3, 4, 5, 6, 8.

Keywords: join space; hypergroup; fuzzy grade; strong fuzzy grade

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1. Introduction

The theory of hyperstructures is a topic of algebra that is more than 90 years old and rich in applications, in particular in fuzzy, soft, and rough sets, lattices, cryptography, artificial intelligence (see [1,2]). In a hypergroupoid, the result of the “multiplication” of two elements is a set of elements rather than a single element.

The study of the connections between algebraic structures and fuzzy set theory has been a research direction since the 1970s, when Rosenfeld [3] introduced fuzzy groups. Later, algebraic hyperstructures induced by fuzzy sets were introduced and studied, as well as fuzzy hyperstructures.

First, let us present some basic definitions:

A hyperoperation on a non-empty set H is a function $\circ : H \times H \rightarrow \mathcal{P}^*(H)$, where $\mathcal{P}^*(H)$ is the set of non-empty subsets of H .

For subsets U, V of H , denote $U \circ V = \bigcup_{u \in U, v \in V} u \circ v$, and for $z \in H$ write $z \circ U$ and $U \circ z$ for $\{z\} \circ U$ and $U \circ \{z\}$, respectively.

The structure $(H, \circ)$ is a hypergroup if for all x, y, z of H we have

$$(x \circ y) \circ z = x \circ (y \circ z) \text{ and } z \circ H = H \circ z = H.$$

A hypergroup $(H, \circ)$ for which $x \circ y = H$ for all $x, y \in H$ is called a total hypergroup.

Various classes of hypergroups are known and studied. Regular reversible hypergroups have at least one identity and each element x has at least one inverse x' , such that if $x \in y \circ z$, then there exist the inverses y' of y and z' of z , for which $y \in x \circ z'$ and $z \in y' \circ x$. Complete hypergroups form a class of hypergroups, closed to groups, in which every hyperproduct of elements is a complete part. We say that a subset A of H is a complete part of H if all hyperproducts $\prod_{i=1}^n z_i$ of elements of H which have a non-empty intersection with

A are contained in A . Join spaces are another important class of commutative hypergroups. A commutative hypergroup $(H, \circ)$ is a join space if for all elements u, v, w, t, z of H ,

$$u \in v \circ z, w \in t \circ z \Rightarrow u \circ t \cap v \circ w \neq \emptyset.$$

Corsini began to study the join spaces induced by fuzzy sets. He associated a join space with a fuzzy set [4] and then a fuzzy set with a hypergroupoid/join space [5], constructing in this manner a sequence of join spaces and fuzzy sets, which ends if two consecutive join spaces are isomorphic. The length of this sequence is called the fuzzy grade. This indicates a point where the structure of the spaces essentially stabilizes. In this paper, we will call it the Corsini fuzzy grade to distinguish it from the strong fuzzy grade, which will be explained later. The study of this sequence offers the possibility of studying fuzzy sets from an algebraic point of view and also provides examples of join spaces on a certain set.

Now, let us present the algorithm introduced by Corsini in [5]. First, he associated a fuzzy set with every hypergroupoid $(H, \otimes)$, as follows:

$$\begin{aligned} \forall z \in H, Q(z) &= \{(x, y) \in H^2 \mid z \in x \otimes y\}, \\ q(z) &= |Q(z)|, \\ A(z) &= \sum_{(x, y) \in Q(z)} 1/|x \otimes y|, \\ \mu(z) &= A(z)/q(z). \end{aligned} \quad (1)$$

In [6], the above fuzzy set μ was called *grade fuzzy set* and was denoted by $\tilde{\mu}$.

On the other hand, a join space $H_1 = (H, \circ_\mu)$ can be associated with the fuzzy set μ as follows:

$$x \circ_\mu y = \{z \mid \min\{\mu(x), \mu(y)\} \leq \mu(z) \leq \max\{\mu(x), \mu(y)\}\}. \quad (2)$$

We repeat the process and obtain a sequence of join spaces $\{H_i\}_i$ and a sequence of fuzzy sets $\{\mu_i\}_i$. These sequences have been studied in many contexts, for instance, in several classes of hypergroups, such as hypergroups with partial scalar identities [7], non-complete 1-hypergroups [8], and complete hypergroups [9], but also in connection with hypergraphs [10,11], multivalued functions [12], and recently, genetics [13]. Studying these sequences can be useful for analyzing the properties of join spaces, particularly their structure. Actually, the sequences are a hierarchical construction of associated join spaces, built step by step from a hypergroup or just a hypergroupoid.

In [14], the fuzzy grade is calculated for hypergroups associated with hypergraphs with the help of MATLAB. In the papers [13,14], the authors declare that they calculated the fuzzy grade of the corresponding hyperstructures, but in reality, the condition they used to stop the sequence of join spaces and fuzzy sets was that the last two join spaces had to be exactly the same or the latest two fuzzy sets had to have the same order.

The fuzzy grade of the starting hypergroup is the minimum natural number i for which $H_i \cong H_{i+1}$, while the strong fuzzy grade is the minimum natural number j for which $H_j = H_{j+1}$. It is easy to see that for the same hyperstructure, the strong fuzzy grade *s.f.g.* is greater than or equal to the fuzzy grade *f.g.*

According to [15], if we consider the equivalence relation R_i on $H_i = (H, \circ_{\mu_i})$ defined by $xR_iy \Leftrightarrow \mu_i(x) = \mu_i(y)$, then it follows that R_i is a regular equivalence and a congruence in H_i . The quotient set H_i/R_i consists of equivalence classes $\{C_j^i\}_{1 \leq j \leq r_i}$ with cardinals k_j for all j , $1 \leq j \leq r_i$. Therefore, with each join space H_i we can associate an r_i -tuple $(k_1, k_2, \dots, k_{r_i})$. In [15], the authors established a sufficient condition in order to have two consecutive associated join spaces of a sequence be non-isomorphic. They also found a

property of the quotient hypergroup determined by the join space H_i and the relation associated with the fuzzy set $\tilde{\mu}$ and determined a necessary and sufficient condition for the join space H_i in order for it to be a reduced hypergroup. In [9], the authors provide a method for constructing a finite hypergroup with the strong fuzzy grade equal to a given natural number.

In this paper, we consider the above construction for a class of hypergroups which has been obtained in connection with genetics (see [16–18]) and analyze the strong fuzzy grade of these hypergroups. This study is a continuation of the paper [13]. The last section of this paper contains the MATLAB programs for the strong fuzzy grade for cardinals 3, 4, 5, 6, 8.

2. The Fuzzy Grade of a Genetic Hypergroup

We apply Corsini's construction (1) for the next join space, which is connected with a genetic inheritance (see [13]). This hypergroup is not complete, but it is regular and reversible.

We set $n \geq 3$ and $H = \{z_1, z_2, \dots, z_n\}$.

We consider the hyperoperation on H given in Table 1.

Table 1. The multiplication table of $\circ$.

$\circ$	z_1	z_2	z_3	$\dots$	z_n
z_1	H	H	H	$\dots$	H
z_2	H	$\{z_2, z_3, \dots, z_n\}$	$\{z_2, z_3, \dots, z_n\}$	$\dots$	$\{z_2, z_3, \dots, z_n\}$
z_3	H	$\{z_2, z_3, \dots, z_n\}$	$\{z_3, \dots, z_n\}$	$\dots$	$\{z_3, \dots, z_n\}$
$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$
z_n	H	$\{z_2, z_3, \dots, z_n\}$	$\{z_3, \dots, z_n\}$	$\dots$	$\{z_n\}$

For all $1 \leq i \leq j \leq k \leq n$, we have $z_i \circ (z_j \circ z_k) = (z_i \circ z_j) \circ z_k = \{z_i, z_{i+1}, \dots, z_n\}$.

Moreover, $(H, \circ)$ is a join space. The idea of considering such a hypergroup comes from genetics; more precisely, it is associated with a type of non-Mendelian inheritance, namely Epistasis [16,18].

We have $q_1(z_i) = |Q_1(z_i)|$ for all $i \in \{1, 2, \dots, n\}$. By calculation, for all natural numbers k , $1 \leq k \leq n$,

$$q_1(z_k) = 2nk - k^2. \quad (3)$$

Indeed,

$$Q_1(z_1) = \{(z_1, z), (z, z_1) \mid z \in H\} \text{ whence } q_1(z_1) = 2n - 1.$$

$$Q_1(z_2) = Q_1(z_1) \cup \{(z_2, z), (z, z_2) \mid z \in H\} \text{ whence } q_1(z_2) = 4n - 4.$$

$$Q_1(z_3) = Q_1(z_2) \cup \{(z_3, z), (z, z_3) \mid z \in H\} \text{ whence } q_1(z_3) = 6n - 9.$$

$$Q_1(z_k) = Q_1(z_{k-1}) \cup \{(z_k, z), (z, z_k) \mid z \in H\} \text{ whence } q_1(z_k) = 2nk - k^2.$$

$$\text{Finally, } q_1(z_n) = n^2.$$

For all $x \in H$, we have

$$\mu_1(z) = \alpha_1(z) / q_1(z). \quad (4)$$

Theorem 1 ([13]). For $1 \leq i < j \leq n$, we obtain $\mu_1(z_i) < \mu_1(z_j)$.

For $n = 2^s$, the above join space $(H, \circ)$ is an example of a hypergroup that satisfies the conditions of Theorem 6.1 [19], which states the following:

Theorem 2 ([19]). Let H be the finite hypergroupoid $H = \{z_1, z_2, \dots, z_n\}$, with $n = 2^s$, and s a nonzero natural number, which verifies the relation

$$\mu_1(z_1) < \mu_1(z_2) < \dots < \mu_1(z_n),$$

i.e., the n -tuple associated with H is $(1, 1, \dots, 1)$. Then, the join space H_{s+1} is a total hypergroup, and by consequence $f.g.(H) = s + 1$.

In the proof of this theorem, it is checked that $\mu_2(z_i) = \mu_2(z_{n+1-i})$ for all $i \in \{1, 2, \dots, n/2\}$ and for all $i < j \leq n/2$, $\mu_2(z_i) > \mu_2(z_j)$. Thus, the join space $H_2 = (H, \circ_{\mu_2})$ is associated with the r_2 -tuple $(2, 2, \dots, 2)$, where $r_2 = 2^{s-1}$.

Continuing in this manner, the join space $H_3 = (H, \circ_{\mu_3})$ is associated with the r_3 -tuple $(4, 4, \dots, 4)$, with $r_3 = 2^{s-2}$, and so on. The join space $H_s = (H, \circ_{\mu_s})$ is associated with the pair $(2^{s-1}, 2^{s-1})$. The join space H_{s+1} is a total hypergroup and $f.g.(H) = s + 1$.

Example 1. Consider $n = 8 = 2^3$ for hypergroup $(H, \circ)$, and we obtain the following:

$\mu_1(z_1) < \mu_1(z_2) < \mu_1(z_3) < \mu_1(z_4) < \mu_1(z_5) < \mu_1(z_6) < \mu_1(z_7) < \mu_1(z_8)$, corresponding to $(1, 1, 1, 1, 1, 1, 1, 1)$,

$\mu_2(z_1) = \mu_2(z_8) > \mu_2(z_2) = \mu_2(z_7) > \mu_2(z_3) = \mu_2(z_6) > \mu_2(z_4) = \mu_2(z_5)$, corresponding to $(2, 2, 2, 2)$,

$\mu_3(z_1) = \mu_3(z_8) = \mu_3(z_4) = \mu_3(z_5) > \mu_3(z_2) = \mu_3(z_7) = \mu_3(z_3) = \mu_3(z_6)$, corresponding to $(4, 4)$,

$\mu_4(z_1) = \mu_4(z_2) = \mu_4(z_3) = \mu_4(z_4) = \mu_4(z_5) = \mu_4(z_6) = \mu_4(z_7) = \mu_4(z_8)$, corresponding to the total hypergroup.

Hence, the fuzzy grade of $(H, \circ)$ is 4.

We continue the algorithm by obtaining the join spaces H_i and fuzzy functions μ_i in order to establish the fuzzy degree in the other situations where $n \neq 2^s$.

With (2), we obtain a join space $H_1 = (H, \circ_1)$ such that for all $z_i \in H$ we have

$z_i \circ_1 z_i = \{z_i\}$ and for all $1 \leq i < j \leq n$ we have $z_i \circ_1 z_j = z_j \circ_1 z_i = \{z_i, z_{i+1}, \dots, z_j\}$.

Hence, we obtain Table 2.

Table 2. The multiplication table of $\circ_1$.

$\circ_1$	z_1	z_2	z_3	$\dots$	z_n
z_1	$\{z_1\}$	$\{z_1, z_2\}$	$\{z_1, z_2, z_3\}$	$\dots$	H
z_2		$\{z_2\}$	$\{z_2, z_3\}$	$\dots$	$\{z_2, z_3, \dots, z_n\}$
z_3			$\{z_3\}$	$\dots$	$\{z_3, \dots, z_n\}$
$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$
z_n				$\dots$	$\{z_n\}$

Note that all hypergroups we consider in this paper are commutative, so we can only write in the table the elements above the main diagonal.

Applying (1), we obtain the following:

$$q_2(z_1) = 2n - 1, \quad q_2(z_2) = 4n - 5,$$

$$q_2(z_3) = 6n - 13, \quad q_2(z_4) = 8n - 25 \text{ and for } 1 \leq k \leq n,$$

$$q_2(z_k) = 2 \cdot k \cdot (n - k + 1) - 1. \quad (5)$$

$$\text{In particular, } q_2(z_n) = 2n - 1 = q_2(z_1), \quad q_2(z_{n-1}) = q_2(z_2) = 4(n - 1) - 1$$

$$q_2(z_{n-2}) = q_2(z_3) = 2 \cdot 3 \cdot (n - 2) - 1.$$

If $n = 2s$, where s is a natural number, $s \geq 2$, then

$$q_2(z_s) = q_2(z_{s+1}) = 2s(s + 1) - 1. \quad (6)$$

If $n = 2s + 1$, where s is a natural number, $s \geq 1$, then

$$q_2(z_s) = q_2(z_{s+2}) = 2s(s + 2) - 1 \quad (7)$$

and

$$q_2(z_{s+1}) = 2(s+1)^2 - 1. \quad (8)$$

For any $z \in H$ we have

$$\mu_2(z) = A_2(z)/q_2(z). \quad (9)$$

Theorem 3 ([13]). (i) For $n = 2s$ we have the following:

$$\mu_2(z_1) = \mu_2(z_{2s}), \mu_2(z_2) = \mu_2(z_{2s-1}), \dots, \mu_2(z_s) = \mu_2(z_{s+1})$$

$$\text{and } \mu_2(z_1) > \mu_2(z_2) > \dots > \mu_2(z_s).$$

(ii) For $n = 2s + 1$ we have the following:

$$\mu_2(z_1) = \mu_2(z_{2s+1}), \mu_2(z_2) = \mu_2(z_{2s}), \dots, \mu_2(z_s) = \mu_2(z_{s+2})$$

$$\text{and } \mu_2(z_1) > \mu_2(z_2) > \dots > \mu_2(z_s) > \mu_2(z_{s+1}).$$

Hence, for $n = 2s$ the associated s -tuple is $(2, 2, \dots, 2)$, and for $n = 2s + 1$ the associated $(s + 1)$ -tuple is $(2, 2, \dots, 2, 1)$.

Applying (2) again, we obtain the join space $H_2 = (H, \circ_2)$, where $\circ_2 = \circ_{\mu_2}$. For all $z_i \in H$ we have $z_i \circ_2 z_i = \{z_i, z_{n+1-i}\}$. Moreover, for all $1 \leq i < j \leq n$ we have $z_i \circ_2 z_j = z_j \circ_2 z_i = \{z_i, z_{n+1-i}, z_{i+1}, z_{n-i}, \dots, z_j, z_{n+1-j}\}$.

Denote $\hat{z}_i = \{z_i, z_{n+1-i}\}$. Hence,

$$z_i \circ_2 z_i = \hat{z}_i, \quad z_i \circ_2 z_j = z_j \circ_2 z_i = \hat{z}_i \cup \hat{z}_{i+1} \cup \dots \cup \hat{z}_j.$$

In what follows, we analyze separately a case where n is even and a case where n is odd.

I. Case $n = 2s$.

This allows us to obtain Table 3.

Table 3. The multiplication table for $n = 2s$.

$\circ_2$	z_1	z_2	$\dots$	z_s	z_{s+1}	$\dots$	z_{2s-1}	z_{2s}
z_1	$\hat{z}_1$	$\hat{z}_1 \cup \hat{z}_2$	$\dots$	$\hat{z}_1 \cup \dots \cup \hat{z}_s$	$\hat{z}_1 \cup \dots \cup \hat{z}_s$	$\dots$	$\hat{z}_1 \cup \hat{z}_2$	$\hat{z}_1$
z_2		$\hat{z}_2$	$\dots$	$\hat{z}_2 \cup \dots \cup \hat{z}_s$	$\hat{z}_2 \cup \dots \cup \hat{z}_s$	$\dots$	$\hat{z}_2$	$\hat{z}_1 \cup \hat{z}_2$
$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$
z_s				$\hat{z}_s$	$\hat{z}_s$	$\dots$	$\dots$	$\hat{z}_1 \cup \dots \cup \hat{z}_s$
z_{s+1}				$\hat{z}_s$	$\hat{z}_s$	$\dots$	$\dots$	$\hat{z}_1 \cup \dots \cup \hat{z}_s$
$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$
z_{2s}	$\hat{z}_1$	$\hat{z}_1 \cup \hat{z}_2$		$\hat{z}_1 \cup \dots \cup \hat{z}_s$	$\hat{z}_1 \cup \dots \cup \hat{z}_s$		$\hat{z}_1 \cup \hat{z}_2$	$\hat{z}_1$

We analyze the following cases:

(i) If $s = 2k$ is even, i.e., $n = 4k$, then

$$\mu_3(z_1) = \mu_3(z_s), \mu_3(z_2) = \mu_3(z_{2k-1}), \dots, \mu_3(z_k) = \mu_3(z_{k+1}).$$

In this case, the associated k -tuple of H_3 is $(4, 4, \dots, 4) = (2^2, 2^2, \dots, 2^2)$.

(ii) If $s = 2k + 1$ is odd, i.e., $n = 4k + 2$, then

$$\mu_3(z_1) = \mu_3(z_{2k+1}), \mu_3(z_2) = \mu_3(z_{2k}), \dots, \mu_3(z_k) = \mu_3(z_{k+2}).$$

The associated $(k + 1)$ -tuple of H_3 is $(4, 4, \dots, 4, 2) = (2^2, 2^2, \dots, 2^2, 2)$.

The cardinal 2 corresponds to $\mu_3(z_{k+1}) = \mu_3(z_{3k+2})$, since we have $\mu_2(z_{t_1}) = \mu_2(z_{t_2})$ for all t_1, t_2 , for which $t_1 + t_2 = 2s + 1$, according to Theorem 2.3.

Applying (i) s times and then (ii), we obtain the following:

Theorem 4. Let $n = 2^s k$ where $2 \nmid k$.

Then, the associated k -tuple of H_{s+1} is $(2^s, 2^s, \dots, 2^s)$ and the associated $(k+1)/2$ -tuple of H_{s+2} is $(2^{s+1}, 2^{s+1}, \dots, 2^{s+1}, 2^s)$.

Example 2. For $n = 6$, we have the following:

$\mu_1(z_1) < \mu_1(z_2) < \mu_1(z_3) < \mu_1(z_4) < \mu_1(z_5) < \mu_1(z_6)$, corresponding to $(1, 1, 1, 1, 1, 1)$,
 $\mu_2(z_1) = \mu_2(z_6) > \mu_2(z_2) = \mu_2(z_5) > \mu_2(z_3) = \mu_2(z_4)$, corresponding to $(2, 2, 2)$,
 $\mu_3(z_1) = \mu_3(z_6) = \mu_3(z_3) = \mu_3(z_4) > \mu_3(z_2) = \mu_3(z_5)$, corresponding to $(4, 2)$,
 $\mu_4(z_1) = \mu_4(z_6) = \mu_4(z_3) = \mu_4(z_4) < \mu_4(z_2) = \mu_4(z_5)$, corresponding to $(4, 2)$.
Hence, the fuzzy grade of $(H, \circ)$ is 3.

Remark 1. If in H_i for all elements z_j, z_l for which $\mu_i(z_j) < \mu_i(z_l)$ we have $\mu_{i+1}(z_j) > \mu_{i+1}(z_l)$, then $H_i \cong H_{i+1}$.

II. Case $n = 2s + 1$.

Clearly, if $n = 2s + 1$, then $z_{s+1} = \{z_{s+1}\}$. Notice that this is the only class containing only one element.

We can thus obtain Table 4.

Table 4. The multiplication table for $n = 2s + 1$.

o_2	z_1	z_2	$\dots$	z_s	z_{s+1}	z_{s+2}	$\dots$	z_{2s+1}
z_1	$\hat{z}_1$	$\hat{z}_1 \cup \hat{z}_2$	$\dots$	$\hat{z}_1 \cup \dots \cup \hat{z}_s$	$\hat{z}_1 \cup \dots \cup z_{s+1}$	$\hat{z}_1 \cup \dots \cup \hat{z}_s$	$\dots$	$\hat{z}_1$
z_2		$\hat{z}_2$	$\dots$	$\hat{z}_2 \cup \dots \cup \hat{z}_s$	$\hat{z}_2 \cup \dots \cup z_{s+1}$	$\hat{z}_2 \cup \dots \cup \hat{z}_s$	$\dots$	$\hat{z}_1 \cup \hat{z}_2$
$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$
z_s				$\hat{z}_s$	$\hat{z}_s \cup z_{s+1}$	$\hat{z}_s$	$\dots$	$\hat{z}_1 \cup \dots \cup \hat{z}_s$
z_{s+1}					z_{s+1}	$\hat{z}_s \cup \hat{z}_{s+1}$	$\dots$	$\hat{z}_1 \cup \dots \cup \hat{z}_s$
$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$
z_{2s+1}								$\hat{z}_1$

By calculation, we finally obtain that

$$\mu_3(z_{s+1}) > \mu_3(z_s) > \dots > \mu_3(z_2) > \mu_3(z_1).$$

The associated $(s+1)$ -tuple of H_2 is $(2, 2, \dots, 2, 1)$, as for H_2 .

Example 3. For $n = 5$ we have the following (see [13]): $\mu_1(z_1) < \mu_1(z_2) < \mu_1(z_3) < \mu_1(z_4) < \mu_1(z_5)$, corresponding to $(1, 1, 1, 1, 1)$,

$\mu_2(z_1) = \mu_2(z_5) > \mu_2(z_2) = \mu_2(z_4) > \mu_2(z_3)$, corresponding to $(2, 2, 1)$,

$\mu_3(z_1) = \mu_3(z_5) < \mu_3(z_2) = \mu_3(z_4) < \mu_3(z_3)$, corresponding to $(2, 2, 1)$.

Hence, the fuzzy grade of $(H, \circ)$ is 2.

Example 4. For $n = 7$ we have the following (see [13]):

$\mu_1(z_1) < \mu_1(z_2) < \mu_1(z_3) < \mu_1(z_4) < \mu_1(z_5) < \mu_1(z_6) < \mu_1(z_7)$, corresponding to $(1, 1, 1, 1, 1, 1, 1)$,

$\mu_2(z_1) = \mu_2(z_7) > \mu_2(z_2) = \mu_2(z_6) > \mu_2(z_3) = \mu_2(z_5) > \mu_2(z_4)$, corresponding to $(2, 2, 2, 1)$,

$\mu_3(z_2) = \mu_3(z_6) < \mu_3(z_1) = \mu_3(z_7) < \mu_3(z_3) = \mu_3(z_5) < \mu_3(z_4)$, corresponding to $(2, 2, 2, 1)$,

$\mu_4(z_1) = \mu_4(z_7) < \mu_4(z_2) = \mu_4(z_6) < \mu_4(z_3) = \mu_4(z_5) < \mu_4(z_4)$, corresponding to $(2, 2, 2, 1)$.

We obtain $H_2 = H_4$ and $H_3 = H_5$, but $H_2 \cong H_3$. In this case, there is no strong fuzzy grade, but $f.g.(H) = 2$.

Now we will consider a new example, for $n = 3$, in order to verify the results obtained using the MATLAB program of Section 3. Notice that for other values of n , the results obtained using the MATLAB program are verified through direct calculations.

Example 5. We present the algorithm for $n = 3$, starting with the Table 5:

Table 5. The multiplication table of $\circ$.

$\circ$	z_1	z_2	z_3
z_1	$\{z_1, z_2, z_3\}$	$\{z_1, z_2, z_3\}$	$\{z_1, z_2, z_3\}$
z_2	$\{z_1, z_2, z_3\}$	$\{z_2, z_3\}$	$\{z_2, z_3\}$
z_3	$\{z_1, z_2, z_3\}$	$\{z_2, z_3\}$	$\{z_3\}$

We have $H = \{z_1, z_2, z_3\}$ and $(H, \circ)$ is the starting hypergroup.

By calculation, we obtain $q_1(z_1) = 5$, $q_1(z_2) = 8$, $q_1(z_3) = 9$;

$A_1(z_1) = 5/3$, $A_1(z_2) = 5/3 + 3/2$, $A_1(z_3) = 5/3 + 3/2 + 1$, hence

$\mu_1(z_1) = 5/15 = 0.333$, $\mu_1(z_2) = 3.166/8 = 0.395$, $\mu_1(z_3) = 4.166/9 = 0.462$. At the second step, we obtain Table 6.

Table 6. The multiplication table of $\circ_1$.

$\circ_1$	z_1	z_2	z_3
z_1	$\{z_1\}$	$\{z_1, z_2\}$	$\{z_1, z_2, z_3\}$
z_2	$\{z_1, z_2\}$	$\{z_2\}$	$\{z_2, z_3\}$
z_3	$\{z_1, z_2, z_3\}$	$\{z_2, z_3\}$	$\{z_3\}$

We obtain $q_2(z_1) = 5 = q_2(z_3)$, $q_2(z_2) = 7$;

$A_2(z_1) = 1 + 2/2 + 2/3 = 2.666 = A_2(z_3)$, $A_2(z_2) = 1 + 4/2 + 2/3 = 3.666$, hence $\mu_2(z_1) = 0.533 = \mu_2(z_3)$, $\mu_2(z_2) = 0.523$.

At the next step, we obtain $\circ_2$ of Table 7.

Table 7. The multiplication table of $\circ_2$.

$\circ_2$	z_1	z_2	z_3
z_1	$\{z_1, z_3\}$	$\{z_1, z_2, z_3\}$	$\{z_1, z_3\}$
z_2	$\{z_1, z_2, z_3\}$	$\{z_2\}$	$\{z_1, z_2, z_3\}$
z_3	$\{z_1, z_3\}$	$\{z_1, z_2, z_3\}$	$\{z_1, z_3\}$

We obtain $q_3(z_1) = 8 = q_3(z_3)$, $q_3(z_2) = 5$;

$A_3(z_1) = 4/2 + 4/3 = 3.333 = A_3(z_3)$, $A_3(z_2) = 1 + 4/3 = 2.333$, hence

$\mu_3(z_1) = 0.416 = \mu_3(z_3)$, $\mu_3(z_2) = 0.466$.

At the next step, we obtain the same hyperoperation, which means that $\circ_3 = \circ_2$, whence it follows that $s.f.g(H) = 2$.

3. MATLAB Program for Obtaining the Strong Fuzzy Grade

In this section, we present the R2018b program used to obtain the strong fuzzy grade for a hypergroup, in particular for the hypergroup H defined in the above section. Moreover, we calculate the strong fuzzy grade for cardinals 3, 4, 5, 6, 8 of H . We use some functions to obtain this program: a function to generate the hyperoperation, then a function to obtain the fuzzy set from the given hyperoperation, then a third function to generate a new join space using the fuzzy set, and then a final function to call the first three functions. The last three functions are valid for any finite hypergroup. Only the first function is characteristic of the particular case of hypergroup studied in this paper.

The first function is used to generate a hyperoperation as described in Table 1. For example, for $n = 3$ we obtain Table 8, which is isomorphic to $(\{z_1, z_2, z_3\}, \circ)$ of Table 5.

This table can be obtained as follows: we type

```
H=generate_H(3)
```

in the command window, and we can get

```
>> H=generate_H(3)
```

```
H(:, :, 1) =
```

```
1    1    1
1    0    0
1    0    0
```

```
H(:, :, 2) =
```

```
2    2    2
2    2    2
2    2    0
```

```
H(:, :, 3) =
```

```
3    3    3
3    3    3
3    3    3
```

and then we obtain the multiplication table of $\otimes_3$, presented in the following as Table 8.

Table 8. The multiplication table of $\otimes_3$.

$\otimes_3$	1	2	3
1	1, 2, 3	1, 2, 3	1, 2, 3
2	1, 2, 3	2, 3	2, 3
3	1, 2, 3	2, 3	3

The second function can be used to get a fuzzy set from a hyperoperation. For example, if we type

```
mu=h2(H)
```

in the command window, we can get

```
>> mu=h2(H)
```

```
mu =
```

```
0.333333333333333    0.395833333333333    0.462962962962963
```

The third function can be used to generate a new join space by a fuzzy set. For example, if we type

```
H1 = h3(mu)
```

in the command window, we can get

```
>> H1 = h3(mu)
```

```
H1(:, :, 1) =
```

```
1    1    1
1    0    0
1    0    0
```

```
H1(:, :, 2) =
```

```
0    2    2
2    2    2
2    2    0
```

```
H1(:, :, 3) =
```

0	0	3
0	0	3
3	3	3

The fourth function is obtained by calling the first three functions. If we want to apply the fourth function, we first need to save the first three functions as individual m-files named *generate_H.m*, *h2.m*, and *h3.m*, respectively.

The results of the application of these programs can be found after the codes, from which we know that the strong fuzzy grade of H with $n = 3, 4, 5, 6, 8$ is 2, 3, 2, 3, 4, respectively, which covers the results of [13]. As for $n = 7$, Leoreanu et al. showed in Example 7.4 of [13] that there is no such strong fuzzy grade.

Table 9 lists the strong fuzzy grades for H with the cardinals 3, 4, 5, 6, 8 of Table 1, respectively.

Table 9. The strong fuzzy grades for H with the cardinals 3, 4, 5, 6, 8 of Table 1, respectively.

the cardinality of H	3	4	5	6	8
strong fuzzy degree	2	3	2	3	4

The MATLAB programs are as follows.

```

%%The first function
function H = generate_H(n)
    H = zeros(n, n, n); % Initialize an all-zero 3D array
    for i = 1:n
        for j = 1:n
            for k = 1:n
                if k >= min(i, j)
                    H(i, j, k) = k;
                end
            end
        end
    end
end

.....

%%The second function
function mu=h2(H)
% Calculate the fuzzy degree of elements in H
n = size(H, 1);
mu=zeros(1,n);
for x=1:n
    q=0;mu1=0;
    for i=1:n
        for j=1:n
            flag=0;p=0;
            for k=1:n
                if H(i,j,k)~=0
                    p=p+1;
                end
                if x==H(i,j,k)
                    q=q+1;
                    flag=1;
                end
            end
        end
    end
end

```

```

        end
        if flag==1
            mu1=mu1+1/p;
        end
    end
    end
    if q~=0
        mu(x)=mu1/q;
    end
end

.....

%%The third function
function H1 = h3(mu)
% to generate a new join space H1 by a fuzzy set mu
% H1(i,j,k) = k, if mu(k) is between mu(i) and mu(j); H1(i,j,k)=0,
% otherwise

if ~isvector(mu)
    error('Input mu must be a vector.');
```

end

```

n = numel(mu);
H1 = zeros(n, n, n);

for i = 1:n
    for j = 1:n
        for k = 1:n
            if mu(k) >= min(mu(i), mu(j)) && mu(k) <= max(mu(i), mu(j))
                H1(i, j, k) = k;
            end
        end
    end
end
end
end

.....

%The fourth function
function grad = h4(n)
    H = generate_H(n); %To generate H
    mu = h2(H); %To generate a fuzzy set mu corresponding to H
    grad = 1;
    iteration = 0;
    max_iterations = 50;

    %%%Note 1: the following codes are designed for display the entries of H
    for i = 1:size(H, 1)
        for j = 1:size(H, 2)
            % Extract (i,j,:) and convert it to a row vector
            slice = nonzeros(reshape(H(i,j,:), 1, []));
            % Display results
            fprintf('H(%d,%d,:) = [', i, j);
            fprintf('%d ', slice);
            fprintf(']\n');
```

```

        end
    end
    %%%The end of Note 1

    %To display the results of mu
    fprintf('mu1 =[ ');
    fprintf('%.4f ', mu);
    fprintf(']\n');

    while iteration < max_iterations
        H1 = h3(mu);
        mu1 = round(h2(H1),5); %This line can calculate %n=3,4,5,6,8
        for i = 1:size(H1, 1)
            for j = 1:size(H1, 2)
                % Extract (i,j,:) and convert it to a row vector
                slice = nonzeros(reshape(H1(i,j,:), 1, []));
                % Display results
                fprintf('H%d(%d,%d,:) = [', grad, i, j);
                fprintf('%d ', slice);
                fprintf(']\n');
            end
        end
        %To display the results of all mu
        fprintf('mu%d =[ ', grad+1);
        fprintf('%.4f ', mu1);
        fprintf(']\n');

        [A, ind] = sort(mu);
        [B, ind1] = sort(mu1);
        if ind==ind1;
            break;
        end

        H = H1;
        mu = mu1;
        grad = grad + 1;
        iteration = iteration + 1;
    end

    if iteration >= max_iterations
        warning('Reached the maximum iteration count of 50.');
```

```

    end
    %To display the fuzzy grade of H
    fprintf('The fuzzy grade of H with n=%d is %d ', size(H,1), grad);
end
.....

```

To measure the execution time of the h4 function for different values on n , we make the following program, from which we obtain Figure 1.

```

% Measure execution time of h4 function for different n values
n_values = [3, 4, 5, 6, 8];
num_trials = 5; % Number of trials for each n value
time_results = zeros(length(n_values), num_trials);

for i = 1:length(n_values)

```

```

        n = n_values(i);
        fprintf('Testing n = %d...\n', n);

        for trial = 1:num_trials
            tic;
            [~, ] = evalc('h4(n)');
            time_results(i, trial) = toc;
            fprintf('Trial %d: %.4f seconds\n', trial, time_results(i, trial));
        end
    end

    % Calculate average times
    avg_times = mean(time_results, 2);

    % Create and display results table
    result_table = table(n_values', avg_times, ...
        'VariableNames', {'n', 'AverageTime_seconds'});
    disp(result_table);

    % Plot results
    figure;
    ax = gca;

    % Set axis properties
    set(ax, 'XColor', [0 0 0], 'YColor', [0 0 0], ...
        'Color', 'white', ...
        'GridColor', [0.8 0.8 0.8], ...
        'GridAlpha', 0.5);

    % Plot data
    plot(n_values, avg_times, '-o', ...
        'LineWidth', 2, ...
        'MarkerSize', 8, ...
        'MarkerEdgeColor', 'b', ...
        'MarkerFaceColor', 'w');

    % Set labels with mathematical n symbol
    xlabel('$n$ value', 'FontSize', 11, 'Interpreter', 'latex');
    ylabel('Average Execution Time (seconds)', 'FontSize', 11);
    title('Execution Time vs $n$ Value', 'FontSize', 12, 'Interpreter', 'latex');

    % Set axis limits
    xlim([2, max(n_values)+1]);
    ylim([0, max(avg_times)*1.1]);

    % Set x-axis ticks
    set(ax, 'XTick', 2:max(n_values));

    % Add grid
    grid on;

    % Add data labels (removed 's' unit and adjusted position)
    for i = 1:length(n_values)
        text(n_values(i) - 0.1, avg_times(i) + max(avg_times)*0.02, ...
            % Moved left by 0.1 and up by 2% of max value

```

```

        sprintf('%.4f', avg_times(i)), ...
        'FontSize', 9, ...
        'VerticalAlignment', 'bottom', ...
        'HorizontalAlignment', 'right');
    end

    % Set figure background
    set(gcf, 'Color', 'white');

    % Save results
    writetable(result_table, 'execution_times.csv');
    print('execution_time_plot.png', '-dpng', '-r300');

```

This algorithm exhibits significant bottlenecks in computational efficiency when handling large-scale problems, reflected mainly in the extremely high time complexity of the core function $h2$ of $O(n^4)$. As the size n increases, the computation time grows superlinearly, with experimental data showing a seven-fold increase in the computation time when n increases from 3 to 8. This exponential growth in complexity renders the algorithm impractical beyond certain scales, making parallel computing essential for improving computational efficiency.

GPU acceleration offers unique advantages for such problems, particularly for processing the algorithm's regular matrix operations. Since the algorithm involves numerous independent three-dimensional/four-dimensional array operations, these computations can be perfectly mapped to a GPU's parallel architecture. The thousands of computing cores in GPUs can simultaneously process matrix element operations, while their high-bandwidth memory effectively alleviates bottlenecks in large-scale data access. Theoretical analysis indicates that when $n > 16$, the GPU solution is expected to achieve 10–100× performance improvements.

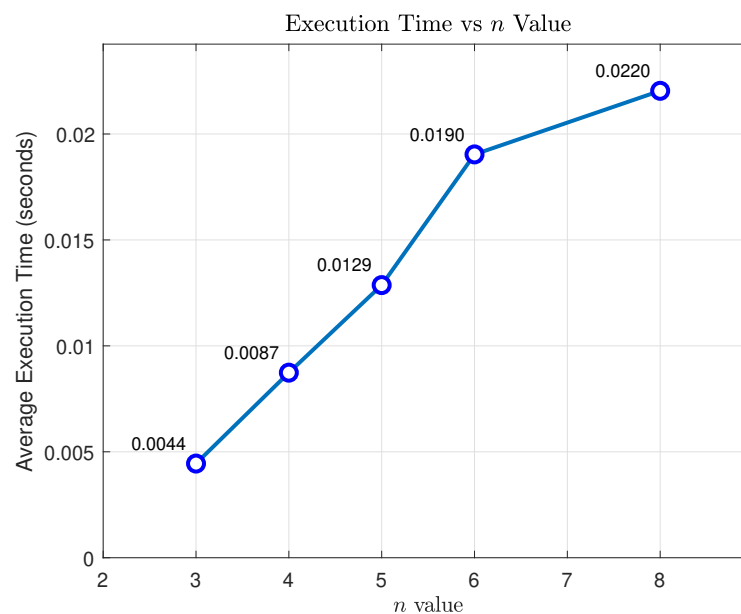


Figure 1. Execution time versus n value.

However, several challenges remain for effective GPU acceleration of this algorithm. Conditional branching statements in the algorithm may impact GPU execution efficiency, and frequent CPU–GPU data transfers introduce additional overhead. Furthermore, the current algorithm design's memory access patterns are not fully optimized, potentially limiting the GPU's memory bandwidth advantages. These factors require targeted optimization during implementation to achieve the desired acceleration results.

We make the following remark to interpret the obtained results: for example, $H(1, 1, :) = [1 \ 2 \ 3]$ means that we have $1 \circ 1 = \{1, 2, 3\}$ at position (1, 1) of the multiplication table. After presenting

each table corresponding to the join space H_i in the sequence, the fuzzy function μ_i is calculated, and we indicate when the sequence stops.

The results of applying our programs are shown below.

```

>> h4(3);
H(1,1,:) = [1 2 3]
H(1,2,:) = [1 2 3]
H(1,3,:) = [1 2 3]
H(2,1,:) = [1 2 3]
H(2,2,:) = [2 3]
H(2,3,:) = [2 3]
H(3,1,:) = [1 2 3]
H(3,2,:) = [2 3]
H(3,3,:) = [3]
mu1 =[0.3333 0.3958 0.4630]
H1(1,1,:) = [1]
H1(1,2,:) = [1 2]
H1(1,3,:) = [1 2 3]
H1(2,1,:) = [1 2]
H1(2,2,:) = [2]
H1(2,3,:) = [2 3]
H1(3,1,:) = [1 2 3]
H1(3,2,:) = [2 3]
H1(3,3,:) = [3]
mu2 =[0.5333 0.5238 0.5333]
H2(1,1,:) = [1 3]
H2(1,2,:) = [1 2 3]
H2(1,3,:) = [1 3]
H2(2,1,:) = [1 2 3]
H2(2,2,:) = [2]
H2(2,3,:) = [1 2 3]
H2(3,1,:) = [1 3]
H2(3,2,:) = [1 2 3]
H2(3,3,:) = [1 3]
mu3 =[0.4167 0.4667 0.4167]
H3(1,1,:) = [1 3]
H3(1,2,:) = [1 2 3]
H3(1,3,:) = [1 3]
H3(2,1,:) = [1 2 3]
H3(2,2,:) = [2]
H3(2,3,:) = [1 2 3]
H3(3,1,:) = [1 3]
H3(3,2,:) = [1 2 3]
H3(3,3,:) = [1 3]
mu4 =[0.4167 0.4667 0.4167]
The strong fuzzy degree of H with
n=3 is 2.
.....
>> h4(4);
H(1,1,:) = [1 2 3 4]
H(1,2,:) = [1 2 3 4]
H(1,3,:) = [1 2 3 4]
H(1,4,:) = [1 2 3 4]
H(2,1,:) = [1 2 3 4]
H(2,2,:) = [2 3 4]
H(2,3,:) = [2 3 4]
H(3,1,:) = [1 2 3 4]
H(3,2,:) = [2 3 4]
H(3,3,:) = [3 4]
H(3,4,:) = [3 4]
H(4,1,:) = [1 2 3 4]
H(4,2,:) = [2 3 4]
H(4,3,:) = [3 4]
H(4,4,:) = [4]
mu1 =[0.2500 0.2847
0.3278 0.3698]
H1(1,1,:) = [1]
H1(1,2,:) = [1 2]
H1(1,3,:) = [1 2 3]
H1(1,4,:) = [1 2 3 4]
H1(2,1,:) = [1 2]
H1(2,2,:) = [2]
H1(2,3,:) = [2 3]
H1(2,4,:) = [2 3 4]
H1(3,1,:) = [1 2 3]
H1(3,2,:) = [2 3]
H1(3,3,:) = [3]
H1(3,4,:) = [3 4]
H1(4,1,:) = [1 2 3 4]
H1(4,2,:) = [2 3 4]
H1(4,3,:) = [3 4]
H1(4,4,:) = [4]
mu2 =[0.4524 0.4394
0.4394 0.4524]
H2(1,1,:) = [1 4]
H2(1,2,:) = [1 2 3 4]
H2(1,3,:) = [1 2 3 4]
H2(1,4,:) = [1 4]
H2(2,1,:) = [1 2 3 4]
H2(2,2,:) = [2 3]
H2(2,3,:) = [2 3]
H2(2,4,:) = [1 2 3 4]
H2(3,1,:) = [1 2 3 4]
H2(3,2,:) = [2 3]
H2(3,3,:) = [2 3]
H2(3,4,:) = [1 2 3 4]
H2(4,1,:) = [1 4]
H2(4,2,:) = [1 2 3 4]
H2(4,3,:) = [1 2 3 4]
H2(4,4,:) = [1 4]
mu3 =[0.3333 0.3333
0.3333 0.3333]
H3 is the total hypergroup.
mu4 =[0.2500 0.2500

```

0.2500 0.2500]

The strong fuzzy grade of H
with n=4 is 3.

.....

```
>> h4(5);
H(1,1,:) = [1 2 3 4 5]
H(1,2,:) = [1 2 3 4 5]
H(1,3,:) = [1 2 3 4 5]
H(1,4,:) = [1 2 3 4 5]
H(1,5,:) = [1 2 3 4 5]
H(2,1,:) = [1 2 3 4 5]
H(2,2,:) = [2 3 4 5]
H(2,3,:) = [2 3 4 5]
H(2,4,:) = [2 3 4 5]
H(2,5,:) = [2 3 4 5]
H(3,1,:) = [1 2 3 4 5]
H(3,2,:) = [2 3 4 5]
H(3,3,:) = [3 4 5]
H(3,4,:) = [3 4 5]
H(3,5,:) = [3 4 5]
H(4,1,:) = [1 2 3 4 5]
H(4,2,:) = [2 3 4 5]
H(4,3,:) = [3 4 5]
H(4,4,:) = [4 5]
H(4,5,:) = [4 5]
H(5,1,:) = [1 2 3 4 5]
H(5,2,:) = [2 3 4 5]
H(5,3,:) = [3 4 5]
H(5,4,:) = [4 5]
H(5,5,:) = [5]
mu1 = [0.2000 0.2219 0.2484
0.2799 0.3087]
H1(1,1,:) = [1]
H1(1,2,:) = [1 2]
H1(1,3,:) = [1 2 3]
H1(1,4,:) = [1 2 3 4]
H1(1,5,:) = [1 2 3 4 5]
H1(2,1,:) = [1 2]
H1(2,2,:) = [2]
H1(2,3,:) = [2 3]
H1(2,4,:) = [2 3 4]
H1(2,5,:) = [2 3 4 5]
H1(3,1,:) = [1 2 3]
H1(3,2,:) = [2 3]
H1(3,3,:) = [3]
H1(3,4,:) = [3 4]
H1(3,5,:) = [3 4 5]
H1(4,1,:) = [1 2 3 4]
H1(4,2,:) = [2 3 4]
H1(4,3,:) = [3 4]
H1(4,4,:) = [4]
H1(4,5,:) = [4 5]
H1(5,1,:) = [1 2 3 4 5]
H1(5,2,:) = [2 3 4 5]
```

```
H1(5,3,:) = [3 4 5]
H1(5,4,:) = [4 5]
H1(5,5,:) = [5]
mu2 = [0.3963 0.3822
0.3765 0.3822 0.3963]
H2(1,1,:) = [1 5]
H2(1,2,:) = [1 2 4 5]
H2(1,3,:) = [1 2 3 4 5]
H2(1,4,:) = [1 2 4 5]
H2(1,5,:) = [1 5]
H2(2,1,:) = [1 2 4 5]
H2(2,2,:) = [2 4]
H2(2,3,:) = [2 3 4]
H2(2,4,:) = [2 4]
H2(2,5,:) = [1 2 4 5]
H2(3,1,:) = [1 2 3 4 5]
H2(3,2,:) = [2 3 4]
H2(3,3,:) = [3]
H2(3,4,:) = [2 3 4]
H2(3,5,:) = [1 2 3 4 5]
H2(4,1,:) = [1 2 4 5]
H2(4,2,:) = [2 4]
H2(4,3,:) = [2 3 4]
H2(4,4,:) = [2 4]
H2(4,5,:) = [1 2 4 5]
H2(5,1,:) = [1 5]
H2(5,2,:) = [1 2 4 5]
H2(5,3,:) = [1 2 3 4 5]
H2(5,4,:) = [1 2 4 5]
H2(5,5,:) = [1 5]
mu3 = [0.3000 0.3067
0.3482 0.3067 0.3000]
H3(1,1,:) = [1 5]
H3(1,2,:) = [1 2 4 5]
H3(1,3,:) = [1 2 3 4 5]
H3(1,4,:) = [1 2 4 5]
H3(1,5,:) = [1 5]
H3(2,1,:) = [1 2 4 5]
H3(2,2,:) = [2 4]
H3(2,3,:) = [2 3 4]
H3(2,4,:) = [2 4]
H3(2,5,:) = [1 2 4 5]
H3(3,1,:) = [1 2 3 4 5]
H3(3,2,:) = [2 3 4]
H3(3,3,:) = [3]
H3(3,4,:) = [2 3 4]
H3(3,5,:) = [1 2 3 4 5]
H3(4,1,:) = [1 2 4 5]
H3(4,2,:) = [2 4]
H3(4,3,:) = [2 3 4]
H3(4,4,:) = [2 4]
H3(4,5,:) = [1 2 4 5]
H3(5,1,:) = [1 5]
H3(5,2,:) = [1 2 4 5]
H3(5,3,:) = [1 2 3 4 5]
```

```

H3(5,4,:) = [1 2 4 5]
H3(5,5,:) = [1 5]
mu4 =[0.3000 0.3067 0.3482
0.3067 0.3000]
The strong fuzzy grade of H
with n=5 is 2.
.....

>> h4(6);
H(1,1,:) = [1 2 3 4 5 6]
H(1,2,:) = [1 2 3 4 5 6]
H(1,3,:) = [1 2 3 4 5 6]
H(1,4,:) = [1 2 3 4 5 6]
H(1,5,:) = [1 2 3 4 5 6]
H(1,6,:) = [1 2 3 4 5 6]
H(2,1,:) = [1 2 3 4 5 6]
H(2,2,:) = [2 3 4 5 6]
H(2,3,:) = [2 3 4 5 6]
H(2,4,:) = [2 3 4 5 6]
H(2,5,:) = [2 3 4 5 6]
H(2,6,:) = [2 3 4 5 6]
H(3,1,:) = [1 2 3 4 5 6]
H(3,2,:) = [2 3 4 5 6]
H(3,3,:) = [3 4 5 6]
H(3,4,:) = [3 4 5 6]
H(3,5,:) = [3 4 5 6]
H(3,6,:) = [3 4 5 6]
H(4,1,:) = [1 2 3 4 5 6]
H(4,2,:) = [2 3 4 5 6]
H(4,3,:) = [3 4 5 6]
H(4,4,:) = [4 5 6]
H(4,5,:) = [4 5 6]
H(4,6,:) = [4 5 6]
H(5,1,:) = [1 2 3 4 5 6]
H(5,2,:) = [2 3 4 5 6]
H(5,3,:) = [3 4 5 6]
H(5,4,:) = [4 5 6]
H(5,5,:) = [5 6]
H(5,6,:) = [5 6]
H(6,1,:) = [1 2 3 4 5 6]
H(6,2,:) = [2 3 4 5 6]
H(6,3,:) = [3 4 5 6]
H(6,4,:) = [4 5 6]
H(6,5,:) = [5 6]
H(6,6,:) = [6]
mu1 =[0.1667 0.1817
0.1994 0.2203 0.2443 0.2653]
H1(1,1,:) = [1]
H1(1,2,:) = [1 2]
H1(1,3,:) = [1 2 3]
H1(1,4,:) = [1 2 3 4]
H1(1,5,:) = [1 2 3 4 5]
H1(1,6,:) = [1 2 3 4 5 6]
H1(2,1,:) = [1 2]
H1(2,2,:) = [2]
H1(2,3,:) = [2 3]
H1(2,4,:) = [2 3 4]
H1(2,5,:) = [2 3 4 5]
H1(2,6,:) = [2 3 4 5 6]
H1(3,1,:) = [1 2 3]
H1(3,2,:) = [2 3]
H1(3,3,:) = [3]
H1(3,4,:) = [3 4]
H1(3,5,:) = [3 4 5]
H1(3,6,:) = [3 4 5 6]
H1(4,1,:) = [1 2 3 4]
H1(4,2,:) = [2 3 4]
H1(4,3,:) = [3 4]
H1(4,4,:) = [4]
H1(4,5,:) = [4 5]
H1(4,6,:) = [4 5 6]
H1(5,1,:) = [1 2 3 4 5]
H1(5,2,:) = [2 3 4 5]
H1(5,3,:) = [3 4 5]
H1(5,4,:) = [4 5]
H1(5,5,:) = [5]
H1(5,6,:) = [5 6]
H1(6,1,:) = [1 2 3 4 5 6]
H1(6,2,:) = [2 3 4 5 6]
H1(6,3,:) = [3 4 5 6]
H1(6,4,:) = [4 5 6]
H1(6,5,:) = [5 6]
H1(6,6,:) = [6]
mu2 =[0.3545 0.3404
0.3319 0.3319 0.3404 0.3545]
H2(1,1,:) = [1 6]
H2(1,2,:) = [1 2 5 6]
H2(1,3,:) = [1 2 3 4 5 6]
H2(1,4,:) = [1 2 3 4 5 6]
H2(1,5,:) = [1 2 5 6]
H2(1,6,:) = [1 6]
H2(2,1,:) = [1 2 5 6]
H2(2,2,:) = [2 5]
H2(2,3,:) = [2 3 4 5]
H2(2,4,:) = [2 3 4 5]
H2(2,5,:) = [2 5]
H2(2,6,:) = [1 2 5 6]
H2(3,1,:) = [1 2 3 4 5 6]
H2(3,2,:) = [2 3 4 5]
H2(3,3,:) = [3 4]
H2(3,4,:) = [3 4]
H2(3,5,:) = [2 3 4 5]
H2(3,6,:) = [1 2 3 4 5 6]
H2(4,1,:) = [1 2 3 4 5 6]
H2(4,2,:) = [2 3 4 5]
H2(4,3,:) = [3 4]
H2(4,4,:) = [3 4]
H2(4,5,:) = [2 3 4 5]
H2(4,6,:) = [1 2 3 4 5 6]
H2(5,1,:) = [1 2 5 6]

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```

H2(5,2,:) = [2 5]
H2(5,3,:) = [2 3 4 5]
H2(5,4,:) = [2 3 4 5]
H2(5,5,:) = [2 5]
H2(5,6,:) = [1 2 5 6]
H2(6,1,:) = [1 6]
H2(6,2,:) = [1 2 5 6]
H2(6,3,:) = [1 2 3 4 5 6]
H2(6,4,:) = [1 2 3 4 5 6]
H2(6,5,:) = [1 2 5 6]
H2(6,6,:) = [1 6]
mu3 =[0.2667  0.2619  0.2667
0.2667  0.2619  0.2667]
H3(1,1,:) = [1 3 4 6]
H3(1,2,:) = [1 2 3 4 5 6]
H3(1,3,:) = [1 3 4 6]
H3(1,4,:) = [1 3 4 6]
H3(1,5,:) = [1 2 3 4 5 6]
H3(1,6,:) = [1 3 4 6]
H3(2,1,:) = [1 2 3 4 5 6]
H3(2,2,:) = [2 5]
H3(2,3,:) = [1 2 3 4 5 6]
H3(2,4,:) = [1 2 3 4 5 6]
H3(2,5,:) = [2 5]
H3(2,6,:) = [1 2 3 4 5 6]
H3(3,1,:) = [1 3 4 6]
H3(3,2,:) = [1 2 3 4 5 6]
H3(3,3,:) = [1 3 4 6]
H3(3,4,:) = [1 3 4 6]
H3(3,5,:) = [1 2 3 4 5 6]
H3(3,6,:) = [1 3 4 6]
H3(4,1,:) = [1 3 4 6]
H3(4,2,:) = [1 2 3 4 5 6]
H3(4,3,:) = [1 3 4 6]
H3(4,4,:) = [1 3 4 6]
H3(4,5,:) = [1 2 3 4 5 6]
H3(4,6,:) = [1 3 4 6]
H3(5,1,:) = [1 2 3 4 5 6]
H3(5,2,:) = [2 5]
H3(5,3,:) = [1 2 3 4 5 6]
H3(5,4,:) = [1 2 3 4 5 6]
H3(5,5,:) = [2 5]
H3(5,6,:) = [1 2 3 4 5 6]
H3(6,1,:) = [1 3 4 6]
H3(6,2,:) = [1 2 3 4 5 6]
H3(6,3,:) = [1 3 4 6]
H3(6,4,:) = [1 3 4 6]
H3(6,5,:) = [1 2 3 4 5 6]
H3(6,6,:) = [1 3 4 6]
mu4 =[0.2083  0.2333  0.2083
0.2083  0.2333  0.2083]
H4(1,1,:) = [1 3 4 6]
H4(1,2,:) = [1 2 3 4 5 6]
H4(1,3,:) = [1 3 4 6]
H4(1,4,:) = [1 3 4 6]
H4(1,5,:) = [1 2 3 4 5 6]
H4(1,6,:) = [1 3 4 6]
H4(2,1,:) = [1 2 3 4 5 6]
H4(2,2,:) = [2 5]
H4(2,3,:) = [1 2 3 4 5 6]
H4(2,4,:) = [1 2 3 4 5 6]
H4(2,5,:) = [2 5]
H4(2,6,:) = [1 2 3 4 5 6]
H4(3,1,:) = [1 3 4 6]
H4(3,2,:) = [1 2 3 4 5 6]
H4(3,3,:) = [1 3 4 6]
H4(3,4,:) = [1 3 4 6]
H4(3,5,:) = [1 2 3 4 5 6]
H4(3,6,:) = [1 3 4 6]
H4(4,1,:) = [1 3 4 6]
H4(4,2,:) = [1 2 3 4 5 6]
H4(4,3,:) = [1 3 4 6]
H4(4,4,:) = [1 3 4 6]
H4(4,5,:) = [1 2 3 4 5 6]
H4(4,6,:) = [1 3 4 6]
H4(5,1,:) = [1 2 3 4 5 6]
H4(5,2,:) = [2 5]
H4(5,3,:) = [1 2 3 4 5 6]
H4(5,4,:) = [1 2 3 4 5 6]
H4(5,5,:) = [2 5]
H4(5,6,:) = [1 2 3 4 5 6]
H4(6,1,:) = [1 3 4 6]
H4(6,2,:) = [1 2 3 4 5 6]
H4(6,3,:) = [1 3 4 6]
H4(6,4,:) = [1 3 4 6]
H4(6,5,:) = [1 2 3 4 5 6]
H4(6,6,:) = [1 3 4 6]
mu5 =[0.2083 0.2333 0.2083
0.2083 0.2333 0.2083]
The strong fuzzy grade of H
with n=6 is 3.

.....

>> h4(8);
H(1,1,:) = [1 2 3 4 5 6 7 8]
H(1,2,:) = [1 2 3 4 5 6 7 8]
H(1,3,:) = [1 2 3 4 5 6 7 8]
H(1,4,:) = [1 2 3 4 5 6 7 8]
H(1,5,:) = [1 2 3 4 5 6 7 8]
H(1,6,:) = [1 2 3 4 5 6 7 8]
H(1,7,:) = [1 2 3 4 5 6 7 8]
H(1,8,:) = [1 2 3 4 5 6 7 8]
H(2,1,:) = [1 2 3 4 5 6 7 8]
H(2,2,:) = [2 3 4 5 6 7 8]
H(2,3,:) = [2 3 4 5 6 7 8]
H(2,4,:) = [2 3 4 5 6 7 8]
H(2,5,:) = [2 3 4 5 6 7 8]
H(2,6,:) = [2 3 4 5 6 7 8]
H(2,7,:) = [2 3 4 5 6 7 8]
H(2,8,:) = [2 3 4 5 6 7 8]

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```

H(3,1,:) = [1 2 3 4 5 6 7 8]
H(3,2,:) = [2 3 4 5 6 7 8]
H(3,3,:) = [3 4 5 6 7 8]
H(3,4,:) = [3 4 5 6 7 8]
H(3,5,:) = [3 4 5 6 7 8]
H(3,6,:) = [3 4 5 6 7 8]
H(3,7,:) = [3 4 5 6 7 8]
H(3,8,:) = [3 4 5 6 7 8]
H(4,1,:) = [1 2 3 4 5 6 7 8]
H(4,2,:) = [2 3 4 5 6 7 8]
H(4,3,:) = [3 4 5 6 7 8]
H(4,4,:) = [4 5 6 7 8]
H(4,5,:) = [4 5 6 7 8]
H(4,6,:) = [4 5 6 7 8]
H(4,7,:) = [4 5 6 7 8]
H(4,8,:) = [4 5 6 7 8]
H(5,1,:) = [1 2 3 4 5 6 7 8]
H(5,2,:) = [2 3 4 5 6 7 8]
H(5,3,:) = [3 4 5 6 7 8]
H(5,4,:) = [4 5 6 7 8]
H(5,5,:) = [5 6 7 8]
H(5,6,:) = [5 6 7 8]
H(5,7,:) = [5 6 7 8]
H(5,8,:) = [5 6 7 8]
H(6,1,:) = [1 2 3 4 5 6 7 8]
H(6,2,:) = [2 3 4 5 6 7 8]
H(6,3,:) = [3 4 5 6 7 8]
H(6,4,:) = [4 5 6 7 8]
H(6,5,:) = [5 6 7 8]
H(6,6,:) = [6 7 8]
H(6,7,:) = [6 7 8]
H(6,8,:) = [6 7 8]
H(7,1,:) = [1 2 3 4 5 6 7 8]
H(7,2,:) = [2 3 4 5 6 7 8]
H(7,3,:) = [3 4 5 6 7 8]
H(7,4,:) = [4 5 6 7 8]
H(7,5,:) = [5 6 7 8]
H(7,6,:) = [6 7 8]
H(7,7,:) = [7 8]
H(7,8,:) = [7 8]
H(8,1,:) = [1 2 3 4 5 6 7 8]
H(8,2,:) = [2 3 4 5 6 7 8]
H(8,3,:) = [3 4 5 6 7 8]
H(8,4,:) = [4 5 6 7 8]
H(8,5,:) = [5 6 7 8]
H(8,6,:) = [6 7 8]
H(8,7,:) = [7 8]
H(8,8,:) = [8]
mu1 = [0.1250 0.1333
0.1427 0.1534 0.1657
0.1797 0.1950 0.2075]
H1(1,1,:) = [1]
H1(1,2,:) = [1 2]
H1(1,3,:) = [1 2 3]
H1(1,4,:) = [1 2 3 4]
H1(1,5,:) = [1 2 3 4 5]
H1(1,6,:) = [1 2 3 4 5 6]
H1(1,7,:) = [1 2 3 4 5 6 7]
H1(1,8,:) = [1 2 3 4 5 6 7 8]
H1(2,1,:) = [1 2]
H1(2,2,:) = [2]
H1(2,3,:) = [2 3]
H1(2,4,:) = [2 3 4]
H1(2,5,:) = [2 3 4 5]
H1(2,6,:) = [2 3 4 5 6]
H1(2,7,:) = [2 3 4 5 6 7]
H1(2,8,:) = [2 3 4 5 6 7 8]
H1(3,1,:) = [1 2 3]
H1(3,2,:) = [2 3]
H1(3,3,:) = [3]
H1(3,4,:) = [3 4]
H1(3,5,:) = [3 4 5]
H1(3,6,:) = [3 4 5 6]
H1(3,7,:) = [3 4 5 6 7]
H1(3,8,:) = [3 4 5 6 7 8]
H1(4,1,:) = [1 2 3 4]
H1(4,2,:) = [2 3 4]
H1(4,3,:) = [3 4]
H1(4,4,:) = [4]
H1(4,5,:) = [4 5]
H1(4,6,:) = [4 5 6]
H1(4,7,:) = [4 5 6 7]
H1(4,8,:) = [4 5 6 7 8]
H1(5,1,:) = [1 2 3 4 5]
H1(5,2,:) = [2 3 4 5]
H1(5,3,:) = [3 4 5]
H1(5,4,:) = [4 5]
H1(5,5,:) = [5]
H1(5,6,:) = [5 6]
H1(5,7,:) = [5 6 7]
H1(5,8,:) = [5 6 7 8]
H1(6,1,:) = [1 2 3 4 5 6]
H1(6,2,:) = [2 3 4 5 6]
H1(6,3,:) = [3 4 5 6]
H1(6,4,:) = [4 5 6]
H1(6,5,:) = [5 6]
H1(6,6,:) = [6]
H1(6,7,:) = [6 7]
H1(6,8,:) = [6 7 8]
H1(7,1,:) = [1 2 3 4 5 6 7]
H1(7,2,:) = [2 3 4 5 6 7]
H1(7,3,:) = [3 4 5 6 7]
H1(7,4,:) = [4 5 6 7]
H1(7,5,:) = [5 6 7]
H1(7,6,:) = [6 7]
H1(7,7,:) = [7]
H1(7,8,:) = [7 8]
H1(8,1,:) = [1 2 3 4 5 6 7 8]
H1(8,2,:) = [2 3 4 5 6 7 8]
H1(8,3,:) = [3 4 5 6 7 8]

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H1(8,4,:) = [4 5 6 7 8]
H1(8,5,:) = [5 6 7 8]
H1(8,6,:) = [6 7 8]
H1(8,7,:) = [7 8]
H1(8,8,:) = [8]
mu2 =[0.2957 0.2823
0.2720 0.2672 0.2672
0.2720 0.2823 0.2957]
H2(1,1,:) = [1 8]
H2(1,2,:) = [1 2 7 8]
H2(1,3,:) = [1 2 3 6 7 8]
H2(1,4,:) = [1 2 3 4 5 6 7 8]
H2(1,5,:) = [1 2 3 4 5 6 7 8]
H2(1,6,:) = [1 2 3 6 7 8]
H2(1,7,:) = [1 2 7 8]
H2(1,8,:) = [1 8]
H2(2,1,:) = [1 2 7 8]
H2(2,2,:) = [2 7]
H2(2,3,:) = [2 3 6 7]
H2(2,4,:) = [2 3 4 5 6 7]
H2(2,5,:) = [2 3 4 5 6 7]
H2(2,6,:) = [2 3 6 7]
H2(2,7,:) = [2 7]
H2(2,8,:) = [1 2 7 8]
H2(3,1,:) = [1 2 3 6 7 8]
H2(3,2,:) = [2 3 6 7]
H2(3,3,:) = [3 6]
H2(3,4,:) = [3 4 5 6]
H2(3,5,:) = [3 4 5 6]
H2(3,6,:) = [3 6]
H2(3,7,:) = [2 3 6 7]
H2(3,8,:) = [1 2 3 6 7 8]
H2(4,1,:) = [1 2 3 4 5 6 7 8]
H2(4,2,:) = [2 3 4 5 6 7]
H2(4,3,:) = [3 4 5 6]
H2(4,4,:) = [4 5]
H2(4,5,:) = [4 5]
H2(4,6,:) = [3 4 5 6]
H2(4,7,:) = [2 3 4 5 6 7]
H2(4,8,:) = [1 2 3 4 5 6 7 8]
H2(5,1,:) = [1 2 3 4 5 6 7 8]
H2(5,2,:) = [2 3 4 5 6 7]
H2(5,3,:) = [3 4 5 6]
H2(5,4,:) = [4 5]
H2(5,5,:) = [4 5]
H2(5,6,:) = [3 4 5 6]
H2(5,7,:) = [2 3 4 5 6 7]
H2(5,8,:) = [1 2 3 4 5 6 7 8]
H2(6,1,:) = [1 2 3 6 7 8]
H2(6,2,:) = [2 3 6 7]
H2(6,3,:) = [3 6]
H2(6,4,:) = [3 4 5 6]
H2(6,5,:) = [3 4 5 6]
H2(6,6,:) = [3 6]
H2(6,7,:) = [2 3 6 7]
H2(6,8,:) = [1 2 3 6 7 8]
H2(7,1,:) = [1 2 7 8]
H2(7,2,:) = [2 7]
H2(7,3,:) = [2 3 6 7]
H2(7,4,:) = [2 3 4 5 6 7]
H2(7,5,:) = [2 3 4 5 6 7]
H2(7,6,:) = [2 3 6 7]
H2(7,7,:) = [2 7]
H2(7,8,:) = [1 2 7 8]
H2(8,1,:) = [1 8]
H2(8,2,:) = [1 2 7 8]
H2(8,3,:) = [1 2 3 6 7 8]
H2(8,4,:) = [1 2 3 4 5 6 7 8]
H2(8,5,:) = [1 2 3 4 5 6 7 8]
H2(8,6,:) = [1 2 3 6 7 8]
H2(8,7,:) = [1 2 7 8]
H2(8,8,:) = [1 8]
mu3 =[0.2262 0.2197
0.2197 0.2262 0.2262
0.2197 0.2197 0.2262]
H3(1,1,:) = [1 4 5 8]
H3(1,2,:) = [1 2 3 4 5 6 7 8]
H3(1,3,:) = [1 2 3 4 5 6 7 8]
H3(1,4,:) = [1 4 5 8]
H3(1,5,:) = [1 4 5 8]
H3(1,6,:) = [1 2 3 4 5 6 7 8]
H3(1,7,:) = [1 2 3 4 5 6 7 8]
H3(1,8,:) = [1 4 5 8]
H3(2,1,:) = [1 2 3 4 5 6 7 8]
H3(2,2,:) = [2 3 6 7]
H3(2,3,:) = [2 3 6 7]
H3(2,4,:) = [1 2 3 4 5 6 7 8]
H3(2,5,:) = [1 2 3 4 5 6 7 8]
H3(2,6,:) = [2 3 6 7]
H3(2,7,:) = [2 3 6 7]
H3(2,8,:) = [1 2 3 4 5 6 7 8]
H3(3,1,:) = [1 2 3 4 5 6 7 8]
H3(3,2,:) = [2 3 6 7]
H3(3,3,:) = [2 3 6 7]
H3(3,4,:) = [1 2 3 4 5 6 7 8]
H3(3,5,:) = [1 2 3 4 5 6 7 8]
H3(3,6,:) = [2 3 6 7]
H3(3,7,:) = [2 3 6 7]
H3(3,8,:) = [1 2 3 4 5 6 7 8]
H3(4,1,:) = [1 4 5 8]
H3(4,2,:) = [1 2 3 4 5 6 7 8]
H3(4,3,:) = [1 2 3 4 5 6 7 8]
H3(4,4,:) = [1 4 5 8]
H3(4,5,:) = [1 4 5 8]
H3(4,6,:) = [1 2 3 4 5 6 7 8]
H3(4,7,:) = [1 2 3 4 5 6 7 8]
H3(4,8,:) = [1 4 5 8]
H3(5,1,:) = [1 4 5 8]
H3(5,2,:) = [1 2 3 4 5 6 7 8]
H3(5,3,:) = [1 2 3 4 5 6 7 8]

```

H3(5,4,:) = [1 4 5 8]	H3(7,5,:) = [1 2 3 4 5 6 7 8]
H3(5,5,:) = [1 4 5 8]	H3(7,6,:) = [2 3 6 7]
H3(5,6,:) = [1 2 3 4 5 6 7 8]	H3(7,7,:) = [2 3 6 7]
H3(5,7,:) = [1 2 3 4 5 6 7 8]	H3(7,8,:) = [1 2 3 4 5 6 7 8]
H3(5,8,:) = [1 4 5 8]	H3(8,1,:) = [1 4 5 8]
H3(6,1,:) = [1 2 3 4 5 6 7 8]	H3(8,2,:) = [1 2 3 4 5 6 7 8]
H3(6,2,:) = [2 3 6 7]	H3(8,3,:) = [1 2 3 4 5 6 7 8]
H3(6,3,:) = [2 3 6 7]	H3(8,4,:) = [1 4 5 8]
H3(6,4,:) = [1 2 3 4 5 6 7 8]	H3(8,5,:) = [1 4 5 8]
H3(6,5,:) = [1 2 3 4 5 6 7 8]	H3(8,6,:) = [1 2 3 4 5 6 7 8]
H3(6,6,:) = [2 3 6 7]	H3(8,7,:) = [1 2 3 4 5 6 7 8]
H3(6,7,:) = [2 3 6 7]	H3(8,8,:) = [1 4 5 8]
H3(6,8,:) = [1 2 3 4 5 6 7 8]	mu4 =[0.1667 0.1667
H3(7,1,:) = [1 2 3 4 5 6 7 8]	0.1667 0.1667 0.1667
H3(7,2,:) = [2 3 6 7]	0.1667 0.1667 0.1667]
H3(7,3,:) = [2 3 6 7]	The strong fuzzy grade of H
H3(7,4,:) = [1 2 3 4 5 6 7 8]	with n=8 is 4.

4. Conclusions

In this paper, we present a MATLAB program for an algorithm that determines the strong fuzzy grade for small cardinal hypergroups: 3, 4, 5, 6, 8. For the hypergroup of cardinal 7, there is no strong fuzzy grade. In future research, we intend to obtain programs in order to calculate the strong fuzzy grade for hypergroups of large cardinals and to calculate the fuzzy grade, both for small and larger hypergroupoids, using GPU acceleration.

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