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A Superellipse-Based Gradient-Free Topology Optimization Method with Application to Magnetic Actuator Design

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Abstract

Structural optimization plays a crucial role in enhancing the performance of magnetic actuators. Traditional design approaches, such as parametric scanning, are limited by their reliance on empirical geometries. Meanwhile, widely used topology optimization techniques—including the Solid Isotropic Material with Penalization (SIMP) method and level-set methods—often encounter difficulties such as a large number of design variables, high computational cost, and unclear structural boundaries. To overcome these limitations, this paper proposes a novel gradient-free topology optimization method based on superellipses for designing magnetic actuator yokes. The proposed approach offers three key benefits: (1) It requires very few design variables, with each superellipse described by only seven parameters, thereby reducing the dimensionality of the design space and simplifying the optimization problem. (2) It yields clear and smooth structural boundaries without the need for post-processing. (3) It operates without gradient information, employing stochastic algorithms such as genetic algorithms that rely solely on objective function evaluations. A case study on yoke optimization demonstrates that our method achieves magnetic force output comparable to or better than the SIMP method, but with significantly fewer variables and a simpler implementation. This work provides an efficient and new tool for the conceptual design of magnetic actuators and related electro-magnetic devices.

Keywords: gradient-free; magnetic actuator; superellipse; topology optimization

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1. Introduction

Magnetic actuators are fundamental devices that utilize magnetic fields to control torque and motion. As critical components in advanced engineering systems—such as rail transit, electric vehicles, and precision manufacturing [1,2]—their performance is of great significance. Against the backdrop of global trends toward industrial intelligence and low-carbon development, enhancing the overall performance of magnetic actuators has become increasingly imperative. A major engineering challenge lies in maximizing magnetic force under strict volume constraints while ensuring manufacturability [3]. Hence, developing efficient structural optimization methods to overcome the limitations of conventional design approaches holds substantial practical importance for advancing magnetic actuator technology.

Currently, the structural optimization of magnetic actuators is primarily carried out through two approaches: conventional parametric scanning and modern topology optimization. Parametric scanning enhances performance by adjusting the dimensional parameters of predefined geometric shapes. However, this method essentially performs local modifications to empirically designed initial structures, and its design space is inherently limited. Consequently, it cannot transcend conventional design paradigms to achieve disruptive performance improvements, making it difficult to meet the increasingly stringent demands of modern industrial systems [4].

In contrast, topology optimization seeks the optimal material distribution within a design domain without prior empirical structures. This approach enables the generation of innovative configurations and offers a new paradigm for the conceptual design of magnetic actuators [5]. For instance, Qi Wang et al. applied topology optimization to design a unilateral magnetic core, which significantly improved the homogeneity of the magnetic field in a remote region [6]. Capellari et al. further demonstrated the value of topology optimization in lightweight magnetic structures. They introduced a multiscale method for sensor deployment on lightweight magnetic components, where topology-optimized structures provided the foundation for high-sensitivity sensing [7].

Current mainstream topology optimization methods for magnetic actuators fall into two categories: gradient-dependent and gradient-free approaches. The Solid Isotropic Material with Penalization (SIMP) method, a representative gradient-dependent technique, is widely adopted due to its intuitiveness [8,9]. For example, Philipp Seebacher employed SIMP to optimize a magnetic actuator considering ferromagnetic material non-linearity, demonstrating its applicability in electromagnetic devices [10]. However, the SIMP method exhibits several notable limitations: (1) The number of design variables scales with finite element mesh size, causing a “dimensionality curse” and reducing computational efficiency; (2) It requires complex sensitivity analysis, typically through the adjoint method, which becomes particularly cumbersome for complex objective functions [11]; (3) Optimized results often contain gray elements and jagged boundaries. While sensitivity filtering and boundary interpolation can alleviate these issues, they introduce artificial parameters that compromise optimization accuracy and complicate post-processing [12,13].

The level-set method, another gradient-based topology optimization technique, is powerful and widely used because of its ability to describe complex evolving boundaries and topological changes. This approach can generate smooth and clear topological configurations [14,15]. For instance, Seungmin Jeong et al. applied the level-set method to optimize a magnetic actuator, achieving precise magnetic field control and improved force output [16]. However, it generally involves a large number of design variables and requires additional numerical treatments, such as solving complex Hamilton–Jacobi equations, deriving sensitivity functions for boundary evolution, and performing reinitialization and regularization during the process. These requirements increase the difficulty of implementation [16,17].

In fact, both the SIMP method and the level-set method belong to gradient-based topology optimization methods and therefore share several common characteristics. For example, they generally involve a large number of design variables and require complex sensitivity analysis, typically performed using the adjoint method. This process can become particularly cumbersome when dealing with complex objective functions.

To overcome the challenges of gradient calculation, gradient-free topology optimization methods have gained increasing research attention. These methods achieve topological evolution by adjusting parameters of basic geometric components. For example, Meng Xia et al. proposed a gradient-free approach using projective transformations and Boolean

operations. In their method, triangular, rectangular, and circular components are deformed and combined to optimize the yoke structure of magnetic actuators [11]. This approach simplifies the optimization process, but the final topology remains dependent on the initial selection of basic components. Igarashi et al. developed a robust covariance matrix adaptation evolution strategy for magnetic device optimization, which adjusts topology by modifying Gaussian function distributions [18]. Although this method eliminates gradient dependence, it still requires tuning numerous parameters, increasing computational burden. Additionally, it often suffers from checkerboard patterns and strong dependence on the initial design.

To overcome the aforementioned limitations, this paper proposes a gradient-free topology optimization method based on superellipses, and applies it to the yoke design of a magnetic actuator. The proposed method offers three key advantages:

- (1) It uses very few design variables. Each superellipse is defined by only seven parameters and exhibits exceptional deformability, and complex topologies can be achieved with a small number of superellipses. This avoids the dimensionality curse associated with SIMP.
- (2) It produces clear and smooth boundaries. Material interfaces are explicitly described by superellipse curves, eliminating the need for filtering or interpolation. This ensures geometric accuracy and reduces artificial parameter effects.
- (3) It is fully gradient-free. The optimization uses stochastic algorithms such as the genetic algorithm. Only objective function evaluation (magnetic force) is required, with no need for sensitivity analysis or specialized finite element code.

After introducing the proposed superellipse-based method, it is necessary to distinguish it from existing gradient-free topology optimization methods, particularly those based on projection transformation and predefined geometric components. In projection-transformation-based approaches, the topology is generally constructed by deforming and combining several basic primitives, such as triangles, rectangles, and circles. Although these methods avoid gradient calculations and simplify the optimization procedure, the final topology remains strongly dependent on the initial type, number, and arrangement of the selected primitives. In contrast, the proposed method employs superellipses to explicitly represent material boundaries. Owing to their high geometric flexibility, superellipses can continuously evolve among circular, elliptical, rectangular-like, and anisotropic profiles by adjusting a compact set of parameters. In this work, each superellipse is described by only seven design variables, enabling complex topologies to be represented with fewer parameters while maintaining smooth and clear boundaries. Therefore, compared with existing gradient-free methods based on projection transformation or simple geometric primitives, the proposed superellipse-based representation offers stronger deformation capability and higher parameter efficiency for magnetic actuator yoke design.

To validate the proposed method, a case study on yoke topology optimization for a magnetic actuator is carried out and compared in detail with the mainstream SIMP method. Results show that our proposed method achieves comparable or even superior magnetic force output, while using fewer design variables and a simpler implementation process. Furthermore, this study does not seek to replace existing topology optimization approaches such as SIMP or level-set methods. Instead, it offers an efficient alternative tool for the conceptual design of magnetic actuators and other electromagnetic devices.

2. Materials and Methods

2.1. Magnetic Actuator Model

Figure 1 illustrates the magnetic actuator model analyzed in this work. The coil is wound around the yoke to generate a magnetic field in the armature region, thereby producing a magnetic force in the x -direction. This force actuates the armature, enabling it to function as a locking device. The coil carries an electric current density of 2 A/mm² along the z -direction. The relative magnetic reluctivity is set to 0.001 for the yoke and 1 for air. To maximize the magnetic force of the armature, the material distribution of the yoke within the design domain is optimized under a volume fraction constraint of 0.6. Since a two-dimensional model is considered, the volume constraint is expressed as an area constraint.

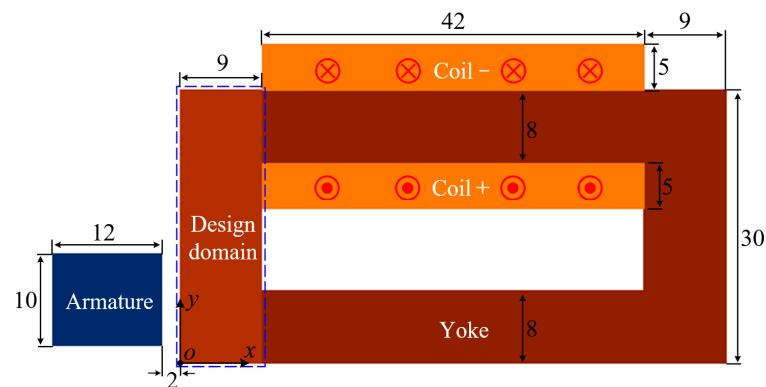


Figure 1. Schematic of the magnetic actuator. The goal is to optimize the distribution of the yoke in the design domain to maximize the magnetic force of the armature. (Unit: mm).

Typically, the finite element method is employed to compute the magnetic field and magnetic force in magnetic actuator devices. For the 2D Cartesian configuration shown in Figure 1, the variation in magnetic vector potential $\mathbf{A}$ along the z -direction can be neglected. Therefore, we consider the following governing equation:

$$\nabla \times \left(\frac{1}{\mu} \nabla \times \mathbf{A} \right) = \mathbf{J} \quad (1)$$

where μ represents the permeability and $\mathbf{J}$ denotes the current density through the coil. In this 2D model, the current density has only a z -component J_z , consequently, the magnetic vector potential $\mathbf{A}$ possesses only a z -component A_z ($A_x = A_y = 0$). Applying the Galerkin technique [19], the linear finite element equation can be expressed as follows:

$$\mathbf{K} \mathbf{A}_z = \mathbf{B} \quad (2)$$

where $\mathbf{K}$ represents the global stiffness matrix and $\mathbf{B}$ denotes the right-hand-side vector. These are computed as follows:

$$\begin{cases} \mathbf{K} = \iint_s v \left(\frac{\partial A_z}{\partial x} \frac{\partial N}{\partial x} + \frac{\partial A_z}{\partial y} \frac{\partial N}{\partial y} \right) dx dy \\ \mathbf{B} = \iint_s J_z N dx dy \end{cases} \quad (3)$$

where N is the shape function of the finite element. To efficiently calculate the magnetic force of the armature, the Maxwell's stress tensor method is applied, and the general magnetic force can be formulated as follows:

$$\mathbf{F} = \int_s \mathbf{T} \cdot \mathbf{n} ds \quad (4)$$

where $\mathbf{T}$ is the Maxwell stress tensor and $\mathbf{n}$ is the outward normal vector to the surface S . For the 2D magnetic actuator model shown in Figure 1, the surface integral in Equation

(4) reduces to a closed line integral [20]. Accordingly, the magnetic force can be simplified as follows:

$$\mathbf{F} = \oint_l \left[(v_0 B_n B_t) \mathbf{t} + \frac{1}{2} (v_0 B_n^2 - v_0 B_t^2) \mathbf{n} \right] dl \quad (5)$$

where B_t and B_n denote the tangential and normal components of the magnetic flux density, with $\mathbf{t}$ and $\mathbf{n}$ being the corresponding tangential and normal unit vectors along the integration line. The integration path in Equation (5) is a closed loop enclosing the armature. Although the complete magnetic force formulation is provided, the optimization objective in this study is specifically the x -component of the magnetic force, F_x .

The optimization objective for the magnetic actuator is to determine the optimal yoke distribution within the design domain that satisfies the area fraction constraint while maximizing the magnetic force acting on the armature. Topology optimization provides a powerful approach to identify the optimal material layout for a given design objective without relying on prior structural experience. Among existing methods, the SIMP method is widely adopted, largely due to its natural compatibility with the finite element method. To better highlight the advantages of our proposed gradient-free method, we first introduce the traditional SIMP framework.

2.2. Traditional SIMP Method

The core idea of the SIMP method is to assign a pseudo-density to each element within the design domain, which corresponds directly to the finite element mesh. Let ρ_e denote the pseudo-density of element e in the design domain, with $\rho_e \in [0, 1]$. A value of $\rho_e = 0$ represents air, while $\rho_e = 1$ corresponds to yoke material. Thus, the optimal material distribution is achieved by optimizing the set of pseudo-densities across all elements in the design domain.

When the pseudo-density approaches 0.5, such intermediate values lack a clear physical interpretation and present manufacturing difficulties. To promote a clear 0–1 distribution, a material interpolation scheme based on a smoothstep function is employed. The interpolated density for element e is shown below:

$$I(\rho_e) = (3 - 2\rho_e) \cdot \rho_e^2 \quad (6)$$

As shown in Equation (6), the smoothstep function controls the rate at which intermediate densities converge to 0 or 1, thereby making the final design manufacturable. Figure 2 illustrates the variation in the interpolated density $I(\rho_e)$ with respect to the initial element density ρ_e . It can be observed that the gradient of the interpolation curve reaches its maximum value of 1.5 when $\rho_e = 0.5$, while the gradient becomes zero at $\rho_e = 0$ or 1. During optimization iterations, the sensitivity of the objective function with respect to element density is relatively high when ρ_e is near 0.5, causing instability in the objective value until the density approaches either 0 or 1. Thus, this interpolation scheme effectively drives element densities toward discrete 0–1 values. Notably, when ρ_e equals 0 or 1, the interpolated density remains strictly 0 or 1, thereby ensuring physical realizability and consistency between the interpolated and actual material distributions.

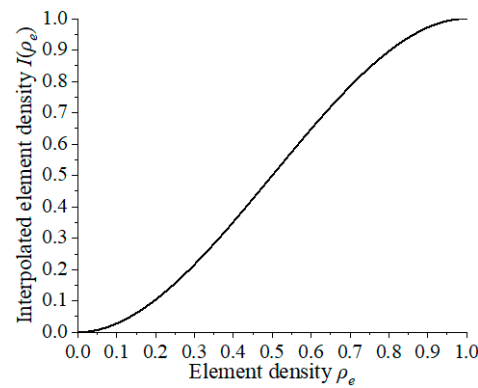


Figure 2. Interpolated density changes with respect to the element density.

In the finite element analysis of the magnetic actuator, the distinction between air and yoke elements lies in their relative magnetic reluctivity. This property is defined by the element density during the optimization process. The relative magnetic reluctivity for element e is expressed as follows:

$$v^e(\rho_e) = v_0 + I(\rho_e) \cdot (v_m - v_0) \quad (7)$$

where v_0 and v_m represent the relative magnetic reluctivity of air and yoke, respectively. Considering the huge number of variables (densities of all elements in the design domain) to be optimized, the sensitivity of the objective function with respect to the density of each element should be derived. Thus, the gradient-based algorithms can be applied. The adjoint method is typically employed to compute these sensitivities efficiently [21]. Since the finite element model in Equation (2) constitutes a linear system, the sensitivity can be derived using the following expression:

$$\frac{dF_x}{d\rho_e} = \frac{\partial F_x}{\partial \rho_e} + \lambda^T \left(\frac{d\mathbf{B}}{d\rho_e} - \frac{d\mathbf{K}}{d\rho_e} \mathbf{A}_z \right) \quad (8)$$

where λ is the adjoint vector, and can be obtained below:

$$\mathbf{K}^T \lambda = \left(\frac{\partial F_x}{\partial \mathbf{A}_z} \right)^T \quad (9)$$

In Equation (8), when solving $d\mathbf{K}/d\rho_e$, we need to calculate $dv^e(\rho_e)/d\rho_e$, which is easy to obtain based on Equation (7). Moreover, to obtain $\partial F_x/\partial \mathbf{A}_z$, the chain rule is used, as follows:

$$\frac{\partial F_x}{\partial \mathbf{A}_z} = \frac{\partial F_x}{\partial \mathbf{B}} \cdot \frac{\partial \mathbf{B}}{\partial \mathbf{A}_z} \quad (10)$$

where $\partial F_x/\partial \mathbf{B}$ can be easily obtained based on the objective function (Equation (5)), and $\partial \mathbf{B}/\partial \mathbf{A}_z$ can be solved based on the relationship $\mathbf{B} = \nabla \times \mathbf{A}_z$.

The SIMP method is prone to checkerboard patterns, which render the optimized topology unmanufacturable. To mitigate this issue, a filtering technique is generally incorporated into the optimization process [22]. For example, the sensitivity filter modifies the sensitivities $dF_x/d\rho_e$ as follows:

$$\widehat{\frac{dF_x}{d\rho_e}} = \frac{1}{\max(\gamma, \rho_e) \sum_{i \in N_e} H_{ei}} \sum_{i \in N_e} H_{ei} \rho_i \frac{dF_x}{d\rho_i} \quad (11)$$

where N_e corresponds to the number of elements whose center-to-center distance $\Delta(e, i)$ to element e is smaller than the preset filter radius $r_{\min}$. The term γ is a small positive number (0.001) to avoid division by zero. H_{ei} is a weight factor, and can be defined as:

$$H_{ei} = \max(0, r_{\min} - \Delta(e, i)) \quad (12)$$

The preceding content outlines the fundamental principles of the topology optimization using the SIMP method. A major challenge of this approach lies in the need to derive the sensitivity of the objective function with respect to each element density. This requirement entails low-level modifications to the finite element codes, thereby complicating implementation and increasing computational cost [11]. Furthermore, any change in the objective function necessitates re-derivation of the sensitivity, limiting the generality and flexibility of the method. Another limitation is the presence of elements with intermediate densities along material boundaries in the optimized results. Additional post-processing steps, such as boundary interpolation, are often required to obtain a clear and manufacturable geometry [23]. Topology optimization results for the magnetic actuator based on the SIMP method will be presented and discussed in Section 3.

2.3. Proposed Topology Optimization Method

In essence, topology optimization aims to determine the optimal material distribution that satisfies given constraints while maximizing the objective function. While topology can be represented using finite element meshes, it can also be described by combinations of basic geometric components controlled by a small set of parameters. By adopting the latter representation, the topology optimization problem is transformed into a parametric optimization task. This approach eliminates the need for complex finite element programming, enabling the use of efficient stochastic search algorithms.

Various geometric primitives, such as rectangles, circles, and triangles, can be used to construct complex topological configurations. In this work, to achieve rich topological variations with few control parameters, the superellipse [24] is adopted as the basic building block. As a generalized form of the ellipse, the superellipse can represent a wide range of shapes, including asteroids, circles, squircles, and rectangles, through simple adjustments of its control parameters. The algebraic expression of a standard superellipse is given by:

$$\left| \frac{x}{a} \right|^{\frac{2}{\varepsilon}} + \left| \frac{y}{b} \right|^{\frac{2}{\varepsilon}} = 1 \quad (13)$$

where x and y are the coordinates of points on the superellipse; a and b are the semi-diameters, which can control the size of the superellipse; ε is the parameter of squareness, which is related to the shape of the superellipse.

Based on Equation (13), the size of the superellipse can be adjusted by changing the semi-diameters a and b . The shape can be varied by adjusting the value of squareness ε . Figure 3 shows the superellipses under different a , b , and ε . In Figure 3a, a rectangular is obtained when $a = 2$, $b = 1$, and $\varepsilon = 0$. If b is the same as a , then a standard square can be obtained, as shown in Figure 3b. Particularly, the value of ε can significantly influence the shape of the superellipse. If the range of ε is between 0 and 1, then a squircle is acquired, as shown in Figure 3c, where $\varepsilon = 0.5$. With the increase in ε , the four corners of the rounded square gradually shrink, and a standard circle can be obtained when $\varepsilon = 1$, as presented in Figure 3d. It is interesting that a different square can be obtained when $\varepsilon = 2$, as shown in Figure 3e. With the further increase in ε , the corners of the superellipse become more acute, as presented in Figure 3f, where $\varepsilon = 3$.

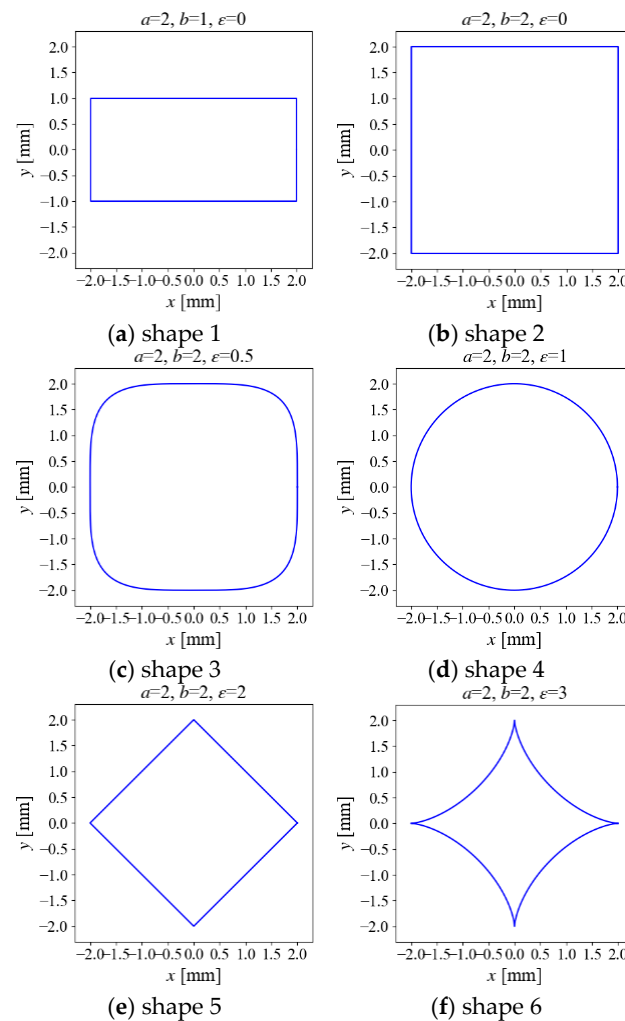


Figure 3. Superellipses under different control parameters a and b .

The above analysis demonstrates that the standard superellipse offers a compact and versatile representation for various basic topological shapes. However, its deformability remains limited. For instance, it cannot adequately capture curvature variations with different bending rates along the x - and y -axes, as required in certain curve styles such as lead-in and tension profiles. To enhance its geometric flexibility, three enhancement strategies are introduced in the following section.

First, to enhance geometric flexibility, distinct exponential parameters can be assigned to the two semi-diameters, thereby enabling the representation of more complex topological configurations. A more general form of the superellipse can be expressed as follows:

$$\left| \frac{x}{a} \right|^{\frac{2}{p}} + \left| \frac{y}{b} \right|^{\frac{2}{q}} = 1 \quad (14)$$

where p and q replace the parameter ε in Equation (13) in the orthogonal directions. As a result, the coordinates of point (x, y) on the generalized superellipses can be expressed as:

$$\begin{cases} x(\theta) = |\cos(\theta)|^p \cdot a \cdot \text{sgn}(\cos(\theta)) \\ y(\theta) = |\sin(\theta)|^q \cdot b \cdot \text{sgn}(\sin(\theta)) \end{cases}, \theta \in [0, 2\pi] \quad (15)$$

where $\text{sgn}()$ is the sign function. Therefore, the generalized superellipse curve can be readily constructed using Equation (15). Furthermore, to address directional constraints in its deformation capability, a rotation parameter is incorporated. Let α denote the rotation

angle. The coordinates (x, y) of a point on the generalized superellipse after rotation are given by Equation (16):

$$\begin{bmatrix} x_1(\theta) \\ y_1(\theta) \end{bmatrix} = \begin{bmatrix} \cos(\alpha) & -\sin(\alpha) \\ \sin(\alpha) & \cos(\alpha) \end{bmatrix} \times \begin{bmatrix} |\cos(\theta)|^p \cdot a \operatorname{sgn}(\cos(\theta)) \\ |\sin(\theta)|^q \cdot b \operatorname{sgn}(\sin(\theta)) \end{bmatrix}, \quad \theta \in [0, 2\pi] \quad (16)$$

Thus, the introduction of a single rotation parameter α significantly enriches the topological variety that can be represented. In the current formulation, the superellipse is centered at the origin $(0, 0)$. To enable the representation of topologies at arbitrary locations within the design domain, the superellipse must be translatable. By introducing two additional parameters (x_0, y_0) to control the center position, the coordinates (x, y) of a point on the fully defined superellipse are given by:

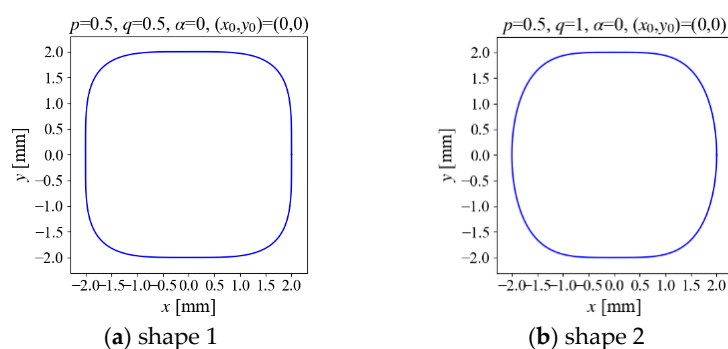
$$\begin{bmatrix} x_2(\theta) \\ y_2(\theta) \end{bmatrix} = \begin{bmatrix} \cos(\alpha) & -\sin(\alpha) \\ \sin(\alpha) & \cos(\alpha) \end{bmatrix} \times \begin{bmatrix} |\cos(\theta)|^p \cdot a \operatorname{sgn}(\cos(\theta)) \\ |\sin(\theta)|^q \cdot b \operatorname{sgn}(\sin(\theta)) \end{bmatrix} + \begin{bmatrix} x_0 \\ y_0 \end{bmatrix}, \quad \theta \in [0, 2\pi] \quad (17)$$

In summary, each improved superellipse in this work is defined by only seven parameters: a, b, p, q, α, x_0 , and y_0 . This compact set of parameters enables the superellipse to represent a wide variety of geometric shapes through flexible deformation. To further enhance its representational capability, multiple superellipses can be combined using Boolean union operations. The resulting topology is formed by taking the union of all individual superellipse regions, allowing the generation of highly complex and diverse topological configurations.

Figure 4 builds upon the geometric foundation established in Figure 3 by demonstrating various topologies generated using the improved superellipse formulation. In all examples, the semi-diameters remain fixed at $a = b = 2$. Figure 4a shows the base configuration with $p = q = 0.5$. By contrast, Figure 4b illustrates how the shape changes when q is increased to 1, creating visible asymmetry between the x - and y -direction profiles. This directional difference becomes more pronounced in Figure 4c, where q is further increased to 3.

Figure 4d applies a rotation of $\alpha = \pi/4$ to the configuration from Figure 4c. Figure 4e then shows the effect of translating the center coordinates from $(0, 0)$ to $(2, 0)$. Finally, Figure 4f presents the union of two superellipses, specifically, those from Figure 4d,e, with the resulting composite topology outlined in solid lines. This example confirms that Boolean operations involving multiple superellipses can successfully generate complex topological patterns.

In summary, each enhanced superellipse is controlled by only seven parameters yet capable of generating diverse geometric configurations. Through Boolean operations, complex topological structures can be achieved with remarkably few parameters, enabling compact representation of sophisticated designs while maintaining computational efficiency and manufacturing feasibility.



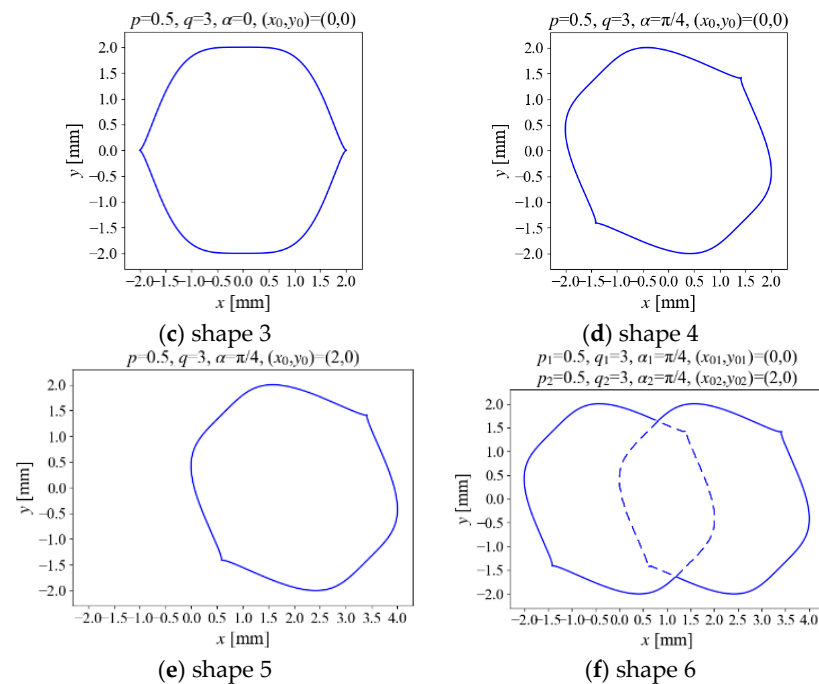


Figure 4. Superellipses under different control parameters p , q , α , x_0 and y_0 . The final topology is outlined by the solid lines.

When the material topology is represented by superellipses, the optimization process becomes straightforward and highly compatible with common stochastic search algorithms such as the genetic algorithm. These algorithms efficiently search for the optimal control parameters that maximize the objective function. During optimization, each parameter-defined topology can be analyzed using the finite element method. One approach involves remeshing the entire domain for each new topology, where superellipse boundaries explicitly define material interfaces. However, this remeshing process significantly increases computational time.

The ranges of each parameter of the superellipse are selected according to the design domain dimensions. On the one hand, for each superellipse, in order to improve its deformation ability, and find the optimal material distribution in the given design domain, its maximum size should be close to the design domain size. On the other hand, the variation ranges should not exceed the design domain range too much, so as to reduce the wasting of the searching space. For the parameters of a superellipse, a and b are the semi-diameters, which can control the size of the superellipse; p and q are the parameters of squareness, which is related to the shape of the superellipse; α can change its rotation angle; and (x_0, y_0) represents its center position. As a result, for a rectangular design domain of size $L_1 \times L_2$, the ranges of each parameter are presented in Table 1.

Table 1. Ranges of parameters for each superellipse in a rectangular design domain of size $L_1 \times L_2$.

Parameters	Ranges
a	$(0, \min(L_1, L_2)]$
b	$(0, \max(L_1, L_2)]$
p	$[0.1, 4]$
q	$[0.1, 4]$
α	$[0, 2\pi]$
x_0	$[-L_1, 2L_1]/[\text{mm}]$
y_0	$[-L_2, 2L_2]/[\text{mm}]$

As an alternative, a fixed initial mesh can be employed throughout the optimization. In this approach, material properties are assigned based on the spatial relationship between element centers and superellipse regions. Specifically, if an element's center lies inside any superellipse, it is assigned to material 1; otherwise, it is assigned to material 2. In this work, we adopt the latter strategy using the same initial mesh as in the SIMP method to ensure consistent comparison, thereby implementing the superellipse-based topology optimization efficiently.

The proposed superellipse-based topology optimization offers distinct advantages over the conventional SIMP method. First, the material boundaries represented by superellipses are inherently clear and explicit, eliminating non-manufacturable intermediate density elements. Consequently, additional post-processing steps such as sensitivity filtering or boundary interpolation are no longer required. Moreover, by transforming topology optimization into a parametric search problem, the method avoids the computationally expensive derivation of sensitivity fields with respect to each finite element, which is a major bottleneck in SIMP. This gradient-free nature enables the handling of complex objective functions without reformulation. Another practical benefit is the elimination of complex finite element programming. Optimization can be performed by iteratively calling commercial finite element software, significantly simplifying implementation. Furthermore, the parameter-based search structure naturally supports parallel computation. This allows for straightforward acceleration when dealing with complex models, such as those involving nonlinearity or multiscale materials.

The topology optimization results based on the proposed method and comparison results will be presented in Section 3.

3. Results

This section presents and compares the optimization results of the magnetic actuator, obtained using both the traditional SIMP method and the proposed superelliptic-based approach.

3.1. Results of SIMP Method

For the yoke distribution optimization using the SIMP method, the design domain (Figure 1) was discretized into 1864 triangular elements. A filter radius of 1 mm was employed to prevent checkerboard patterns. The element densities were efficiently updated using the gradient-based Method of Moving Asymptotes (MMA) [25].

Let N_t denote the total number of triangular elements in the design domain. The optimization problem for the SIMP method is then formulated as:

$$\begin{aligned}
 &\text{find:} && \boldsymbol{\rho} = (\rho_1, \rho_2, \dots, \rho_{N_t}) \\
 &\text{maximize:} && F_x = \oint_l \left[(v_0 B_n B_t) \mathbf{t} + \frac{1}{2} (v_0 B_n^2 - v_0 B_t^2) \mathbf{n} \right]_{\mathbf{x}} dl \\
 &\text{s.t.:} && 0 \leq \rho_e \leq 1, \quad (e = 1 \text{ to } N_t) \\
 &&& \sum_{e=1}^{N_t} \rho_e s_e \leq 0.6 \sum_{e=1}^{N_t} s_e
 \end{aligned} \tag{18}$$

where s_e denotes the area of triangular element e within the design domain. An area fraction constraint of 0.6 is imposed. Figure 5 illustrates the variation in the magnetic force along the x -direction (F_x) during the SIMP optimization process, which was run for 100 iterations. The convergence behavior of the iteration curve indicates that the magnetic force F_x stabilizes after approximately 40 iterations, beyond which no significant change is observed.

Figure 5 also shows several intermediate designs of the yoke. In these images, the black regions represent the yoke material, while white areas correspond to air. For instance, at iteration 0, all elements are assigned the uniform initial density of 0.6, resulting in a gray design domain. As the element densities are updated according to the optimization objective, a preliminary yoke distribution begins to emerge. As iterations proceed, the structural boundaries become increasingly distinct, and the yoke topology gradually stabilizes into its final configuration.

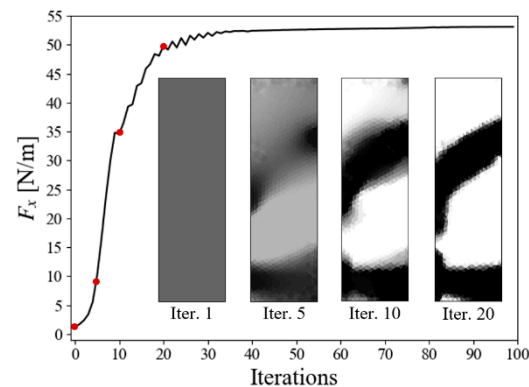


Figure 5. Iterative history of the objective. Inserts show the intermediate designs, and the area fractions is 0.6.

Figure 6 presents the optimized magnetic actuator obtained after 100 iterations using the SIMP method under an area fraction constraint of 0.6. The final design yields a yoke area of approximately 120.47 mm², corresponding to an area fraction of 0.45, which adheres to the prescribed limit. The resulting magnetic force in the x -direction is 53.09 N/m. As illustrated in Figure 6, the optimized yoke topology effectively guides the magnetic flux toward the armature. This concentration consequently increases the magnetic field strength around the armature and enhances the magnetic force F_x .

However, the above analysis reveals several inherent drawbacks of the SIMP method for this magnetic actuator optimization:

- (1) **High Computational Cost:** The method requires optimizing a large number of variables, which is computationally expensive. For instance, the model in Figure 5 with 1864 elements entails an equal number of design variables, leading to low optimization efficiency.
- (2) **Implementation Complexity:** Deriving the sensitivity of the magnetic force with respect to the element density requires an adjoint variable analysis. This analysis is often difficult to implement in practice, as it typically requires access to and modification of the finite element solver's source code.
- (3) **Unclear Structural Boundaries:** The optimized design suffers from the presence of elements with intermediate densities around the yoke boundaries, resulting in blurred interfaces, as evident in Figure 6. To obtain clear and smooth boundaries suitable for direct conversion to a manufacturable CAD model, additional post-processing steps, such as a black-and-white projection filter and boundary interpolation, are necessary, increasing the overall effort.

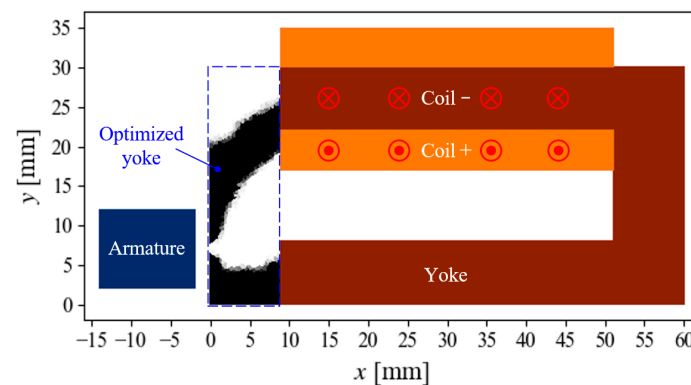


Figure 6. Optimized magnetic actuator based on the SIMP method, where the area fraction is 0.6. The magnetic force and area fraction for the yoke are 53.09 N/m and 0.45, respectively.

3.2. Results of Proposed Method

Here, the proposed superellipse-based method was applied to optimize the magnetic actuator yoke. The topology is parameterized by N_s superellipses, each defined by seven control parameters: a_i , b_i , p_i , q_i , α_i , x_{0i} , and y_{0i} (where $i = 1, 2, \dots, N_s$). Consequently, the total number of optimization variables is $7N_s$. A genetic algorithm was employed to solve this stochastic search problem.

The optimization problem necessitates the simultaneous consideration of the magnetic force F_x and the area fraction. Although inherently multi-objective, it is addressed here using a single-objective formulation for simplicity, integrating both performance and constraint into a unified function as follows:

$$E = W_1 \frac{F_{x_cal}}{F_{x_ref}} + W_2 \frac{e^{\beta(S_{ref} - S_{cal})}}{1 + e^{\beta(S_{ref} - S_{cal})}} \quad (19)$$

where F_{x_cal} and S_{cal} denote the computed magnetic force and yoke area, respectively. The reference value F_{x_ref} is derived from the SIMP optimization result, and $F_{x_ref} = 53.09$ N/m. The reference area S_{ref} is set to 162 mm², which is adopted to ensure the area fraction is below 0.6 to satisfy the area constraint. The second term in Equation (19) introduces a sigmoid function that penalizes deviations in the yoke area. This function yields a constant value close to 0 if $S_{cal} > S_{ref}$, and near 1 if $S_{cal} < S_{ref}$, thereby defining a safe operating region for areas below the reference. The parameter β , set to 1, acts as a scaling factor controlling the transition sharpness of the sigmoid function.

It should be noted that the objective function in Equation (19) consists of two terms: a magnetic force-related term and an area-related term. The weighting coefficient associated with the area-related term has a limited influence once the yoke area satisfies the prescribed constraint. This is because the sigmoid-based penalty function becomes saturated when the yoke area is below 162 mm², corresponding to an area fraction lower than 0.6. In this saturated region, the function value remains almost constant and its gradient is nearly zero. Consequently, increasing this weighting coefficient cannot continuously drive further area reduction. Therefore, the area-related term mainly acts as a threshold constraint, while the optimization is predominantly driven by the magnetic force once the constraint is satisfied. Based on this consideration, both weighting coefficients W_1 and W_2 are set to 1.

Thus, the superellipse-based topology optimization problem is expressed as:

$$\begin{aligned}
 &\text{find:} && [a_1, b_1, p_1, q_1, \alpha_1, x_{01}, y_{01}, \dots, a_{N_s}, b_{N_s}, p_{N_s}, q_{N_s}, \alpha_{N_s}, x_{0N_s}, y_{0N_s}] \\
 &\text{maximize:} && E = W_1 \frac{F_{x_cal}}{F_{x_ref}} + W_2 \frac{e^{\beta(S_{ref} - S_{cal})}}{1 + e^{\beta(S_{ref} - S_{cal})}} \\
 &\text{s.t.:} && a_{i_min} \leq a_i \leq a_{i_max} \\
 & && b_{i_min} \leq b_i \leq b_{i_max} \\
 & && p_{i_min} \leq p_i \leq p_{i_max} \\
 & && q_{i_min} \leq q_i \leq q_{i_max} \\
 & && x_{0i_min} \leq x_{0i} \leq x_{0i_max} \\
 & && y_{0i_min} \leq y_{0i} \leq y_{0i_max}, \quad (i = 1, 2, \dots, N_s)
 \end{aligned} \tag{20}$$

where each design variable is constrained within a predefined range. The same finite element mesh of 1864 triangular elements established for the SIMP method is reused here. A key advantage of the proposed superellipse-based method is the substantial reduction in the number of optimization parameters. In this implementation, the superellipses define the air regions. The material for any element whose centroid lies within a superellipse is assigned as air; otherwise, it is assigned as yoke material.

Prior to executing the genetic algorithm, the variation bounds for the superellipse control parameters must be defined. The center coordinates of a superellipse are constrained by the design domain dimensions. Given a rectangular domain of size $L_1 \times L_2$, the bounds for x_0 , y_0 , a , and b are set to $[-L_1, 2L_1]$, $[-L_2, 2L_2]$, $(0, \min(L_1, L_2)]$, and $(0, \max(L_1, L_2)]$, respectively. All superellipses share the same bounds. Based on the specific dimensions of the magnetic actuator yoke design domain, the resulting parameter ranges used in this study are listed in Table 2.

Table 2. Ranges of parameters in a superellipse for the magnetic actuator.

Parameters	Ranges
a	$(0, 9]/[\text{mm}]$
b	$(0, 30]/[\text{mm}]$
p	$[0.1, 4]$
q	$[0.1, 4]$
α	$[0, 2\pi]$
x_0	$[-9, 18]/[\text{mm}]$
y_0	$[-30, 60]/[\text{mm}]$

First, for comparative purposes, a single superellipse (i.e., $N_s = 1$) is used for topology optimization, resulting in a total of 7 design variables. A genetic algorithm is employed to optimize these parameters. The algorithm is configured with a population of 100 individuals per generation and is run for 120 generations. Selection is performed via roulette wheel selection, and crossover is implemented using a two-point method, with probabilities set to 0.7 for crossover and 0.02 for mutation. After 120 generations, the optimal yoke topology, corresponding to the highest objective function value, is obtained.

Figure 7 presents the optimized yoke topology and the superellipses at the same time. To improve the ability of superellipses to represent complex topology, for the design domain, the Boolean union of all superellipses represents the air region, and the remaining denotes the yoke region (black region).

Specifically, Figure 7a presents the optimized magnetic actuator design obtained using a single superellipse. A key distinction from the SIMP result (Figure 6) is the inherently clear and smooth boundary achieved directly by the proposed method. Furthermore, the optimized magnetic force in the x -direction is 52.32 N/m, a value very close to that achieved by the SIMP method. It is noteworthy that this competitive performance was

attained using only one superellipse, demonstrating the remarkable efficiency of the proposed topology optimization framework.

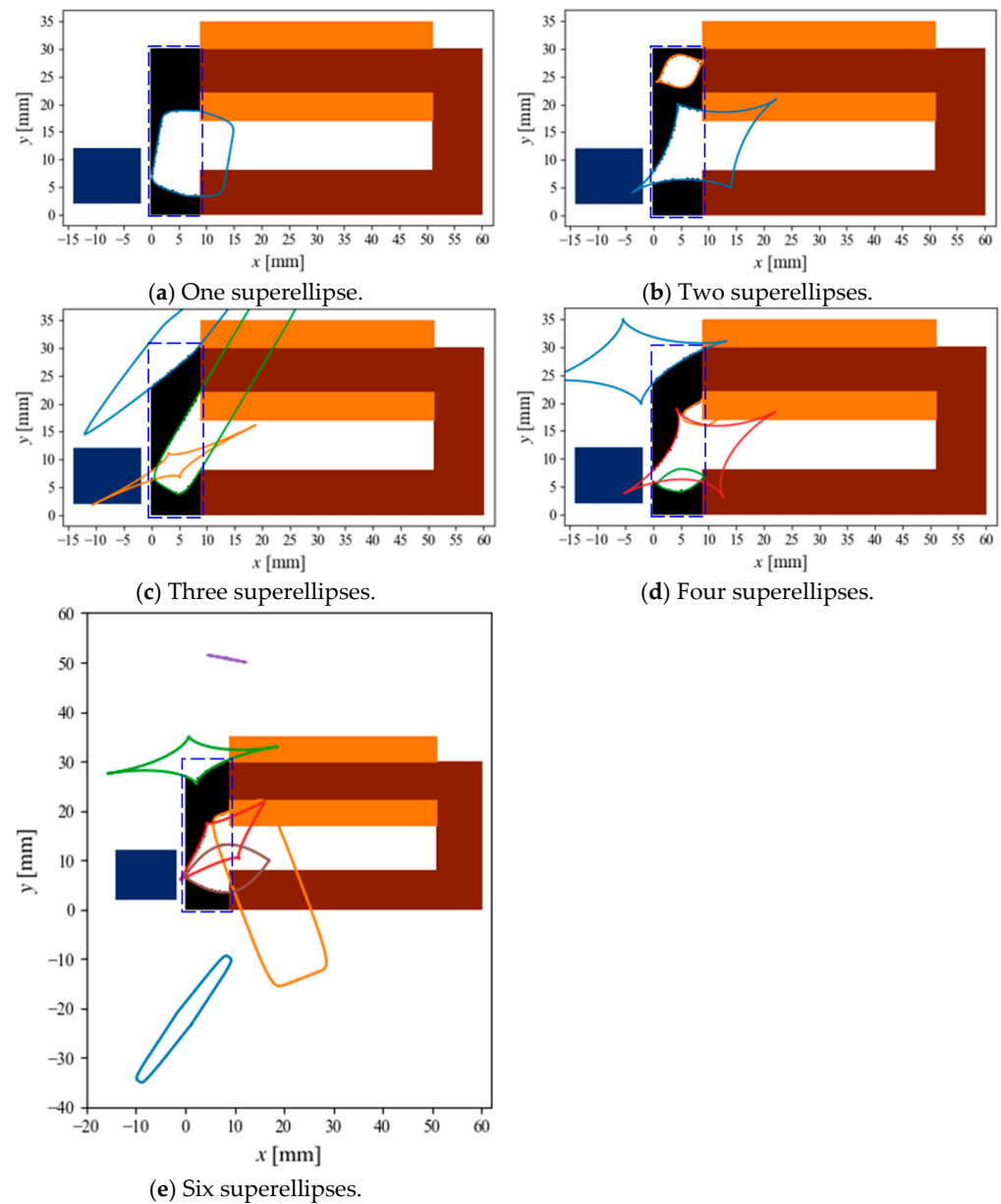


Figure 7. Topologies of the yoke in the magnetic actuator obtained using different numbers of superellipses. The black regions represent the yoke topologies, whose boundaries are described by the superellipses. (a) Yoke topology optimized using one superellipse. (b) Yoke topology optimized using two superellipses. (c) Yoke topology optimized using three superellipses. (d) Yoke topology optimized using four superellipses. (e) Yoke topology optimized using six superellipses.

To further refine the design, the topology optimization was conducted using an increased number of superellipses. Figure 7b shows the result for two superellipses, where an additional hole emerges in the yoke structure. The corresponding magnetic force F_x of 52.58 N/m is slightly higher than that achieved with a single superellipse. When three superellipses are employed, the convergence behavior of the average and best objective values (Figure 8) indicates that the solution stabilizes after 120 generations. The resulting topology, shown in Figure 7c, closely resembles the design obtained with the SIMP method. Moreover, the magnetic force reaches 53.82 N/m, which is approximately 1.4% higher than the SIMP result (53.09 N/m).

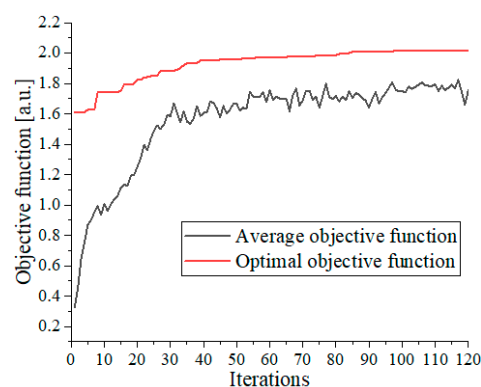


Figure 8. Iterative history of the average and optimal magnetic force when 3 superellipses are applied to the topology optimization.

It should be noted that the genetic algorithm parameters adopted in this study, including the population size, crossover probability, mutation probability, and maximum number of generations, were selected based on commonly used ranges in classical genetic algorithm studies and practical experience in genetic algorithm-based engineering optimization [26]. To further examine the robustness of the selected parameter settings, a comparative study was conducted by varying the population size, maximum number of generations, crossover probability, and mutation probability. Specifically, the corresponding parameters used in Figure 8 were changed to 150, 200, 0.8, and 0.04, respectively. The relative changes in all optimized objective function values are less than 0.2%, indicating that the proposed optimization framework is relatively robust to moderate variations in genetic algorithm parameters.

The optimized designs obtained with 4 and 6 superellipses are shown in Figures 7d and 7e, respectively. A comparison reveals that the topology remains largely unchanged, suggesting that the solution has converged in terms of structural layout. Particularly, as shown in Figure 7e, two superellipses are far away from the final optimized yoke boundaries, that is, there are redundant superellipses. Therefore, it can be concluded that three superellipses are adequate for this particular magnetic actuator design problem.

The relationship between the number of superellipses N_s and the resulting magnetic force F_x is detailed in Table 3. The data show that the magnetic force saturates once N_s reaches 3, establishing this as an adequate parametrization for the topology optimization in this study.

Table 3. Optimized magnetic force under different number of superellipses.

Number of Superellipses	Number of Parameters	Magnetic Force F_x [N/m]	Area Fraction
1	7	52.32	0.57
2	14	52.58	0.57
3	21	53.82	0.56
4	28	53.83	0.56
6	42	53.85	0.56

Since the topology optimization results are obtained using a fixed mesh for the design domain, jagged boundary errors may be introduced. Therefore, the effect of mesh density on the calculated magnetic force is analyzed. For the three-superellipse case shown in Figure 7c, the number of elements in the design domain is increased from 1864 to 3245 and 5005. The corresponding calculated magnetic forces are 53.63 N/m and 53.71 N/m, with

relative differences of 0.35% and 0.20%, respectively. These results indicate that the influence of mesh density on the magnetic force calculation is negligible.

Moreover, because the superellipse curve has an explicit analytical expression, the optimized yoke topology can be directly extracted without additional boundary fitting or approximation. As shown in Figure 9, the final yoke topology, whose boundaries are directly described by the three optimized superellipses, is obtained. Figure 9a shows the relative positions of the optimized superellipses and the design domain boundaries. Within the design domain, the superellipse regions represent air, while the remaining region represents the yoke. The final optimized yoke topology is shown in Figure 9b, where the black region denotes the yoke topology and the remaining region denotes the air region.

Based on the finite element method, the final yoke and air regions are meshed to accurately capture the boundaries. The calculated magnetic force and yoke area are approximately 53.65 N/m and 152.12 mm², respectively. In particular, the relative difference in magnetic force is only about 0.32%. Therefore, although the relatively coarse fixed mesh or grid may introduce certain errors, such errors are sufficiently small and can be neglected for the magnetic actuator model considered in this study. Moreover, the use of a fixed mesh simplifies the calculation and facilitates the optimization process.

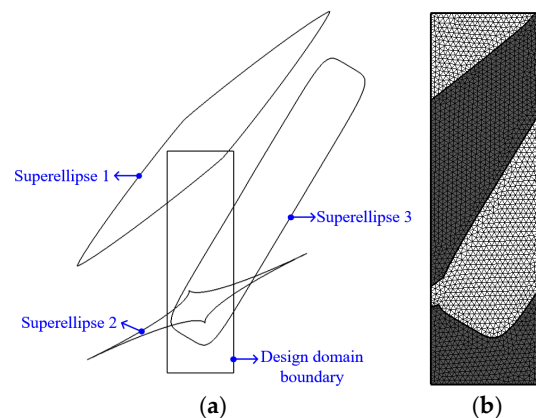


Figure 9. Yoke topology directly based on the three superellipses. (a) Relative positions of superellipses and design domain boundary. (b) Yoke topology based on the superellipses.

For other boundary-sensitive equipment, jagged boundary errors may not be negligible. In such cases, mature techniques, such as adaptive mesh refinement and remeshing [16,27], can be readily incorporated into the proposed superellipse-based topology optimization method. However, this issue is beyond the scope of the present study.

The magnetic flux path distribution for the optimal design (using three superellipses in Figure 7c) is presented in Figure 10. Here, the yoke (black), armature (blue), and flux paths (red) are distinctly shown. The results demonstrate that the obtained yoke topology serves to efficiently channel the magnetic flux into the armature region. This concentration consequently strengthens the local magnetic field and amplifies the magnetic force acting on the armature.

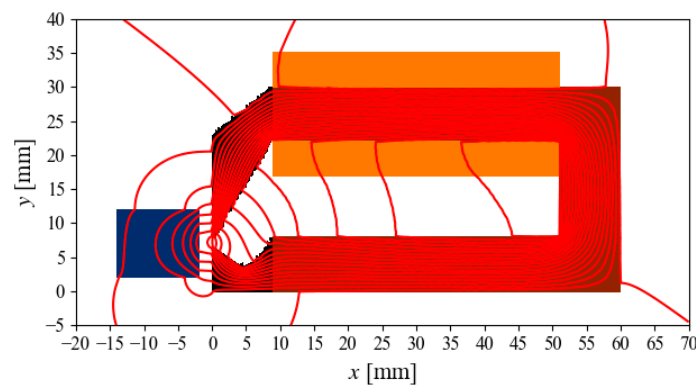


Figure 10. Magnetic flux paths distribution.

In order to better show the differences between the existing method and proposed new method, in the revised manuscript, Table 4 presents the optimization results based on different methods. We can see that our proposed superellipse-based topology optimization can get close or even higher electromagnetic performance compared with the popular SIMP method, but with a significantly smaller number of design variables and clearer and smoother yoke boundaries. Particularly, the proposed topology optimization method is gradient-free, and is convenient for complex electromagnetic equipment and objective function.

Table 4. Topology optimization results based on different methods.

Method	Number of Design Variables	Optimized Yoke Topology	Magnetic Force [N/m]	Area Fraction Less Than 0.6?	Boundaries
Traditional SIMP-based topology optimization	1864		53.09	Yes	Blurred and jagged boundaries
Proposed superellipse-based topology optimization	21		53.82	Yes	Clear and smooth boundaries

The preceding analysis demonstrates three distinct advantages of the proposed superellipse-based topology optimization over the conventional SIMP method:

- (1) **Inherently Clear Boundaries:** The structural boundaries are explicitly defined by smooth superellipse curves. This allows for the direct generation of a clean, well-defined yoke topology without requiring sensitivity filtering or additional boundary interpolation, thereby facilitating seamless conversion to a CAD model.
- (2) **Low dimensionality of the design space:** The method offers substantial parametric economy, with each superellipse requiring only seven control variables yet enabling rich topological deformations. In the present case, the proposed method uses only 21 variables (for 3 superellipses), achieving a 98.9% reduction in design variables compared to the 1864 required by the traditional SIMP approach, while still attaining a

higher magnetic force. This lower-dimensional parameterization can potentially improve search efficiency and reduce the optimization burden, especially for gradient-free algorithms such as the genetic algorithm used in this work.

- (3) **Gradient-Free Formulation:** As a non-gradient method, it entirely circumvents the need for deriving and implementing complex sensitivity formulas based on finite element analysis. This characteristic is particularly advantageous for problems with intricate objective functions, as it allows the optimization to be driven by straightforward stochastic search algorithms, such as genetic algorithms.

4. Discussion

The proposed superellipse-based topology optimization method offers several practical advantages over the conventional SIMP approach. It should be noted that as a gradient-free method relying on stochastic search, its optimization process is mathematically less straightforward than the elegant, gradient-based framework of SIMP. The primary aim of this work, however, is to provide a versatile and easy-to-implement alternative for the topology optimization of magnetic actuators. This method not only simplifies the implementation process but also serves as a valuable reference for optimizing other electromagnetic devices. Furthermore, by generalizing the representation from superellipse curves to superellipsoid surfaces, the proposed method can be seamlessly extended from 2D to 3D, showcasing significant potential for future applications.

5. Conclusions

This paper proposes a gradient-free topology optimization method based on superellipses for magnetic actuator design. Using the same initial finite element mesh as the SIMP method, which requires 1864 design variables, the proposed approach achieves a comparable design with only 3 superellipses (21 parameters), reducing the number of variables by 98.9%. This parametrization avoids the need for complex sensitivity derivation and enables the use of convenient stochastic optimizers, such as genetic algorithms, regardless of the objective function's complexity. Moreover, the resulting topology is directly represented as a CAD-ready geometry with clear and smooth boundaries, eliminating the need for additional post-processing that could compromise performance. Consequently, the optimized yoke obtained by the proposed method achieves a magnetic force approximately 1.4% higher than that of the SIMP-based design.

It should be noted that this work is not intended to completely replace existing topology optimization methods, such as the SIMP or level-set methods. Rather, it seeks to provide an alternative gradient-free topology optimization approach that facilitates the optimization process while achieving comparable performance.

Future work will focus on enhancing the efficiency of the parameter search process in topology optimization with more efficient search strategies, such as adaptive search strategies. Moreover, since the finite element analysis is relatively time-consuming during the optimization, the surrogate model is a potential prospective path to replace the finite element analysis and accelerate the optimization process. What is more, fabrication and experimental testing of the topology optimization results to further validate the proposed methods will be considered in future work.

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