

## Article

# Velocity-Free Particle Swarm Optimization for Image Segmentation Retaining Velocity Features

Muhammad Waseem Asghar <sup>1,\*</sup>  and Mujahid Abbas <sup>1,2</sup> <sup>1</sup> Department of Mechanical Engineering Science, University of Johannesburg, Johannesburg 2092, South Africa; mujahida@uj.ac.za<sup>2</sup> Department of Medical Research, China Medical University, Taichung 404328, Taiwan

\* Correspondence: waseema@uj.ac.za

## Abstract

The purpose of this paper is to introduce a Particle Swarm Optimization (PSO) algorithm that eliminates the velocity vector while using only the personal best and global best positions to guide the search. An additional directional term is incorporated into the proposed algorithm to achieve a better balance between exploration and exploitation. Moreover, the strong convergence of the proposed algorithm is established. The proposed algorithm is applied to optimize several benchmark functions and is compared with well-known existing methods in terms of convergence speed, robustness, and CPU time. Finally, the algorithm is applied to image segmentation and evaluated on the Berkeley Segmentation Dataset. The obtained results are compared with those of several existing algorithms to demonstrate the effectiveness of the proposed approach.

**Keywords:** Particle Swarm Optimization; convergence; image segmentation; swarm intelligence**MSC:** 90C59; 65K10; 68U10; 68T20

## 1. Introduction and Preliminaries

In computer vision, image segmentation plays a crucial role in image processing and pattern recognition. Due to its wide range of applications in medical imaging, self-driving vehicles, satellite image analysis, face recognition systems, and security surveillance, it has attracted considerable attention from researchers in recent years. For further details, see ([1–6]). In image segmentation, an image is divided into different regions to make it easier to understand and analyze. Each region usually represents an object or a specific part of the image, such as a person, a car, or an aircraft. This enables computers to identify and understand the contents of an image in a way that is similar to how humans naturally recognize objects when viewing a picture.

Obtaining accurate and efficient image segmentation is still a challenging task, especially for complex images. Existing approaches, such as clustering and thresholding, have been widely used to address this problem. However, each method has its own limitations, including sensitivity to noise and difficulty in handling overlapping objects. On the other hand, metaheuristic algorithms have been widely applied to solve complex and nonconvex optimization problems in high-dimensional spaces across various fields, including science, engineering, industry, and image processing. Metaheuristic algorithms can generally be classified into two categories: nature-inspired and non-nature inspired algorithms. One of the most widely used nature-inspired metaheuristic algorithms is the Particle Swarm



Academic Editor: Alicia Cordero

Received: 6 June 2026

Revised: 19 July 2026

Accepted: 27 July 2026

Published: 1 August 2026

**Copyright:** © 2026 by the authors.

Licensee MDPI, Basel, Switzerland.

This article is an open access article distributed under the terms and conditions of the [Creative Commons Attribution \(CC BY\)](https://creativecommons.org/licenses/by/4.0/) license.

Optimization (PSO) algorithm. It is a modern gradient-free optimization technique that has been successfully applied to many optimization problems. The PSO algorithm has found numerous applications in image processing, feature selection, network optimization, machine learning, and engineering design optimization. It is a population-based stochastic optimization algorithm inspired by the social behavior of bird flocks and fish schools. In PSO, a collection of candidate solutions is called a swarm, and each candidate solution is referred to as a particle. The PSO is an evolutionary algorithm introduced by Eberhart and Kennedy [7] that utilizes the individual potential to converge on the optimal solution. In PSO, each solution (particle) has knowledge of three components, namely, its velocity, its own best position achieved so far (personal best or p-best), and the best position among all particles in the swarm (global best or g-best). In each iteration, every particle updates its velocity in a way that moves its new position closer to both its p-best and the g-best.

The velocity and the updated position of a particle in PSO can be given as follows:

$$\begin{aligned} v_{n+1}^{ij} &= \omega v_n^{ij} + c_1 r_1 (p_{best}^{ij} - x_n^{ij}) + c_2 r_2 (g_{best}^j - x_n^{ij}), \\ x_{n+1}^{ij} &= x_n^{ij} + v_{n+1}^{ij}. \end{aligned} \quad (1)$$

where  $v^{ij}$  is the velocity vector of the  $i$ -th particle in the  $j$ -th dimension, that is, the  $j$ -th component of the velocity vector, whereas  $x^{ij}$  stands for the  $j$ -th component of the current position of the  $i$ -th particle. The constant  $\omega$  is the inertia weight that controls the influence of the previous velocity. The constants  $c_1$  and  $c_2$  are the cognitive and social acceleration coefficients, respectively, whereas  $r_1, r_2 \in (0, 1)$  are independent uniformly distributed random numbers. Together, these parameters determine the influence of the personal-best and global-best positions on the movement of each particle.

The PSO algorithm has been used by many authors in different fields, for instance, in soft-sensors [8], for solving the capacitor placement issues [9], for estimation of voltage stability in power distribution systems [10,11], for load forecasting [12], in permutation sequencing problems [13] and in chaotic dynamical systems [14]. Moreover, different versions of PSO algorithms have been developed by many authors; for instance, see [15–18]. The useful versions of the PSO algorithm that were proposed and extensively used are modified Particle Swarm Optimization (MPSO) [19], adaptive Particle Swarm Optimization and fuzzy C-mean (APSO-F) [20], and the combined Particle Swarm Optimization algorithm (CPSO) [21]. Continuing in this direction, a PSO algorithm without velocity (PSWV) [22] was proposed, in which a particle updates its position in the current iteration with the random linear combination of p-best and g-best of its previous iteration. The PSWV is given as follows:

$$x_{n+1}^i = r_1 b_1 p_n^i + r_2 b_2 g_n. \quad (2)$$

Here, we present, compare, and review some PSO variants and identify the research gaps in Table 1.

**Table 1.** Comparison of selected PSO-type algorithms.

| Authors                        | PSO Type                       | Main Contribution                                                                     | Limitations                                                                                                                       |
|--------------------------------|--------------------------------|---------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------|
| Kennedy & Eberhart (1995) [23] | Canonical-PSO (velocity-based) | Introduces velocity + position updates, cognitive and social terms; foundational PSO. | Velocity term introduces oscillations; parameters (inertia, $c_1, c_2$ ) need tuning; limited theoretical convergence guarantees. |

Table 1. Cont.

| Authors                               | PSO Type                                     | Main Contribution                                                                                                                                  | Limitations                                                                                                                                                                                          |
|---------------------------------------|----------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Clerc & Kennedy (2002) [24]           | Constriction factor PSO                      | Introduces constriction coefficient to ensure stability.                                                                                           | Improves stability but still relies on velocity tuning; lacks adaptations for velocity-free formulations and rigorous proofs in infinite-dimensional spaces.                                         |
| Kennedy (2003) [25]                   | Bare-bones-PSO (velocity-free)               | Removes explicit velocity; samples new positions from a Gaussian distribution based on $p_{best}$ and $g_{best}$ .                                 | Highly stochastic; sensitive to sampling variance; can lack directed search (may need jump strategies or adaptive variance tuning).                                                                  |
| El-Sherbiny (2011) [22]               | PSO without velocity (PSWV)                  | Replaces velocity update by a linear combination of $p_{best}$ and $g_{best}$ (direct position update); reports faster convergence in experiments. | Lacks deep theoretical convergence analysis; the proof of convergence holds only in the special case where the $g_{best}$ does not change across iterations, i.e., the $g_{best}$ is iteration free. |
| Ang et al. (2020) [26]                | Constrained multi-swarm PSO without velocity | Uses multiple swarms and constraint handling while discarding the velocity term to improve feasibility search.                                     | Improvement of the constraint increases the complexity; scalability and theoretical analysis of velocity-less dynamics remain limited.                                                               |
| Bare-bones and BBPSO variants [27–29] | Sampling-based extensions                    | Many works extend sampling (jumps, deep-memory variants) to improve exploitation/exploration balance.                                              | Sampling-based updates often require additional mechanisms (jumps, memory) to prevent the algorithm from getting stuck in one place without making further progress toward a better solution.        |
| Gad, A. G. (2022) [30]                | Surveys and theory                           | Theoretical analyses and surveys discuss velocity interpretation, convergence and many hybrids.                                                    | Research gap in rigorously analyzing algorithms that remove the velocity vector yet provably retain momentum behavior; proofs needed in Hilbert spaces.                                              |

In view of the slow convergence and premature convergence of particle swarm-based algorithms in high-and infinite-dimensional search spaces, a natural question arises: Is it possible to develop a velocity free optimization algorithm that converges to the global optimum while requiring fewer iterations and less CPU time than the existing methods? Furthermore, can such an algorithm improve the balance between exploration and exploitation by incorporating an effective directional search mechanism?

To answer the above question, we propose an efficient Particle Swarm Optimization algorithm called the Adaptive Accelerated Particle Swarm Optimization Without Velocity (AA-PSWV) algorithm. The proposed algorithm does not explicitly use the velocity vector; instead, it introduces a directional search component based on the personal-best and global-best positions to improve the balance between exploration and exploitation. The proposed algorithm is applicable to solving complex optimization problems involving both convex and non-convex functions. To demonstrate its effectiveness, we evaluate the algorithm on several benchmark functions and apply it to color image segmentation. The experimental results show that the proposed algorithm performs better than the existing methods in terms of convergence speed and solution quality.

This paper is organized as follows. Section 1 presents the introduction, preliminaries, and related literature. Section 2 introduces the proposed algorithm and establishes its strong convergence. Section 3 presents the experimental results on benchmark functions and compares the proposed algorithm with existing methods. In Section 4, the proposed

algorithm is applied to image segmentation and its performance is compared with several existing algorithms. Finally, Section 5 concludes the paper.

Throughout this paper,  $\mathbb{R}^d$  denotes the  $d$ -dimensional Euclidean space,  $x^i \in \mathbb{R}^d$ ,  $\mathbb{R}$  denotes the set of real numbers,  $\mathbb{N}$  denotes the set of natural numbers,  $n \in \mathbb{N}$  represents the iteration number, and  $N$  denotes the population size.

The following concepts are required to establish the convergence result:

**Definition 1** ([31]). Let  $C \subset \mathbb{R}^n$  be a nonempty set. The convex hull  $\text{conv}(C)$  of  $C$  is defined as the smallest convex set containing  $C$  and is given as follows:

$$\text{conv}(C) = \left\{ \sum_{i=1}^k \lambda_i x_i : x_i \in C, \lambda_i \geq 0, \sum_{i=1}^k \lambda_i = 1, k \in \mathbb{N} \right\}.$$

**Lemma 1** ([31]). Let  $C = \{x_1, x_2, \dots, x_m\} \subset \mathbb{R}^n$  be a finite set. Then the convex hull of  $C$  ( $\text{conv}(C)$ ) is a bounded set.

**Proof.** By definition, every point  $y \in \text{conv}(C)$  can be expressed as a convex combination of the points in  $C$ :

$$y = \sum_{i=1}^m \lambda_i x_i, \quad \text{where } \lambda_i \geq 0 \text{ and } \sum_{i=1}^m \lambda_i = 1.$$

As  $C$  is finite, there exists  $M > 0$  such that  $\|x_i\| \leq M$  for all  $i = 1, \dots, m$ . Note that

$$\|y\| = \left\| \sum_{i=1}^m \lambda_i x_i \right\| \leq \sum_{i=1}^m \lambda_i \|x_i\| \leq \sum_{i=1}^m \lambda_i M = M.$$

Hence,  $\text{conv}(C)$  is bounded.  $\square$

#### Novelty and Research Gap of the Proposed AA-PSWV Algorithm

Although numerous variants of PSO have been developed over the past decades, most existing algorithms either rely explicitly on the velocity vector or completely eliminate its influence. Classical PSO employs both position and velocity updates to balance exploration and exploitation. However, the velocity equation introduces additional control parameters and may lead to oscillatory particle trajectories. To improve the stability of PSO, several velocity-free variants have been proposed, such as Bare-Bones PSO (BBPSO) and Particle Swarm Optimization Without Velocity (PSWV).

BBPSO completely removes the velocity equation and generates new particle positions by sampling from a Gaussian probability distribution defined by the personal and global best positions. Although this stochastic strategy simplifies the algorithm, the search direction is entirely determined by random sampling, resulting in limited directional guidance and slower convergence in some optimization problems.

On the other hand, PSWV replaces the velocity update equation with a linear combination of the personal-best and global-best positions. Although this approach simplifies the update rule, it does not explicitly include the directional information that is naturally provided by the velocity vector. As a result, the particles are guided only by the weighted combination of the personal-best and global-best positions, which may reduce the exploration capability and increase the risk of premature convergence, especially for multimodal optimization problems.

To address this limitation, the proposed AA-PSWV algorithm introduces an additional directional component,

$$\gamma_n (p_n^i - g_n),$$

into the position update equation. Unlike the conventional velocity vector, this term does not require maintaining or updating an additional velocity variable. Instead, it utilizes the relative displacement between the personal best and the global best to preserve the directional movement of the particles. Consequently, the proposed algorithm maintains the simplicity of velocity-free optimization while retaining the directional search characteristics that are naturally present in the classical PSO.

From a theoretical perspective, the proposed directional term provides a deterministic mechanism for guiding the search without introducing an additional state variable. From a practical perspective, this modification improves the balance between exploration and exploitation, accelerates convergence, and enhances the optimization performance on both benchmark functions and image segmentation problems. Therefore, the proposed AA-PSWV algorithm bridges the gap between classical velocity-based PSO and existing velocity-free PSO algorithms by incorporating an explicit directional search component while maintaining a simple velocity-free update rule.

Now, the main contributions of this work are summarized as follows:

- A novel Adaptive Accelerated Particle Swarm Optimization Without Velocity (AA-PSWV) algorithm is proposed by introducing a directional search component while avoiding the explicit use of the velocity vector.
- A theoretical convergence analysis of the proposed algorithm is established in Hilbert spaces under suitable assumptions.
- The computational complexity of the proposed AA-PSWV algorithm is analyzed and compared with the classical PSO and PSWV algorithms.
- The effectiveness of the proposed algorithm is validated on several benchmark optimization problems and an image segmentation application.
- Statistical significance analysis is performed to demonstrate that the proposed algorithm provides meaningful improvements over the existing PSWV algorithm.

## 2. Proposed Algorithm and Its Convergence

It is worth mentioning that the existing PSWV algorithm does not explicitly incorporate a directional search mechanism. In the proposed AA-PSWV algorithm, an additional difference vector is introduced as a directional search component to guide the particles more effectively during the optimization process. This component directs the particles along the relative direction between their personal-best and global-best positions, thereby improving the balance between exploration and exploitation. As a result, the proposed algorithm can accelerate convergence while maintaining a simple velocity-free update rule and allowing finer adjustments through small directional steps.

The proposed algorithm (called it AA-PSWV) is defined by using the random convex combination of the sequences of parameters with three vectors, which is given as follows:

$$x_{n+1}^i = \alpha_n p_n^i + \beta_n g_n + \gamma_n (p_n^i - g_n), \quad (3)$$

where  $\alpha_n, \beta_n, \gamma_n \in (0, 1)$  are social coefficients that allow adjusting the balance between exploration and exploitation in the search space with  $\alpha_n + \beta_n + \gamma_n = 1$  and  $\lim_{n \rightarrow \infty} \gamma_n = 0$ .

The small values of  $\gamma_n$  guide the particles to refine their search in the current region. Moreover,  $p_n^i$  and  $g_n$  are the personal best (p-best) and global best (g-best) in the  $n$ -th iteration. Note that, if we set  $\gamma_n = 0$  in (3), then AA-PSWV reduces to the PSWV defined in (2). The parameters  $\alpha_n$ ,  $\beta_n$ , and  $\gamma_n$  control the balance between exploration and exploitation. The coefficients  $\alpha_n$  and  $\beta_n$  determine the influence of the personal-best and global-best positions, respectively, while the directional coefficient  $\gamma_n$  preserves the velocity effect without explicitly introducing a velocity vector. The advantages of the AA-PSWV

algorithm are that it maintains the velocity vector's features and allows particles to move in a suitable direction, converge faster than the existing version of the PSO algorithms due to its inertial factor, and avoid converging with the local optima, which lacks in (2). Moreover, for (2), the convergence was demonstrated only for the special case in which  $g_{best}$  stays unchanged during iterations; see [22].

**Remark 1.** *Although the update rule of the proposed AA-PSWV algorithm can be algebraically rewritten as*

$$x_{n+1}^i = (\alpha_n + \gamma_n)p_n^i + (\beta_n - \gamma_n)g_n,$$

*its novelty does not rely on this algebraic representation. The proposed algorithm is constructed by explicitly introducing the directional search component  $\gamma_n(p_n^i - g_n)$ , which represents the relative direction between the personal-best and global-best positions. This directional component is controlled independently through the parameter  $\gamma_n$ , whose magnitude gradually decreases during the optimization process. Consequently, the proposed algorithm provides an additional mechanism to regulate directional search behavior while preserving the simplicity of a velocity-free update. Therefore, the proposed AA-PSWV algorithm is conceptually different from the conventional PSWV algorithm, where only a weighted linear combination of the personal-best and global-best positions is employed without an explicit directional interpretation. Moreover, the additional term  $\gamma_n(p_n^i - g_n)$  should not be interpreted as a replacement for the velocity vector. Instead, it introduces an explicit directional search component based on the relative displacement between the personal-best and global-best positions.*

#### Computational Complexity Analysis

Let  $N$  denote the swarm size,  $d$  the problem dimension, and  $T$  the maximum number of iterations.

For the classical PSO algorithm, each particle updates its velocity and position in every iteration, requiring  $\mathcal{O}(d)$  operations. Consequently, the computational complexity per iteration is  $\mathcal{O}(Nd)$ , and the overall computational complexity is

$$\mathcal{O}(NTd).$$

Similarly, the PSWV algorithm updates only the particle positions using the personal-best and global-best information. Since each position update also requires  $\mathcal{O}(d)$  operations, the overall computational complexity remains

$$\mathcal{O}(NTd).$$

For the proposed AA-PSWV algorithm, the additional directional term

$$\gamma_n(p_n^i - g_n)$$

requires only one vector subtraction and one scalar–vector multiplication, both of which require  $\mathcal{O}(d)$  operations. Therefore, the position update for each particle still requires  $\mathcal{O}(d)$  operations. Hence, the overall computational complexity of the proposed algorithm is also

$$\mathcal{O}(NTd).$$

Therefore, although the proposed AA-PSWV algorithm introduces an additional directional component to preserve the velocity effect, it does not increase the asymptotic computational complexity compared with the classical PSO and PSWV algorithms. This comparison can be seen in Table 2.

**Table 2.** Computational complexity comparison.

| Algorithm | Position Update             | Overall Complexity |
|-----------|-----------------------------|--------------------|
| PSO       | Velocity + Position         | $\mathcal{O}(NTd)$ |
| PSWV      | Position Only               | $\mathcal{O}(NTd)$ |
| AA-PSWV   | Position + Directional Term | $\mathcal{O}(NTd)$ |

Below, the optimal all g-best value is denoted by  $g^*$ .

Here we present the convergence result of our proposed algorithm as follows:

**Theorem 1.** Suppose that an objective function  $f : \mathbb{R}^d \rightarrow \mathbb{R}$  is continuous and bounded below. If  $\{x_n^i\}$  is a sequence defined by the AA-PSWV algorithm (3), then  $\{x_n^i\}$  converges strongly to  $g^*$ , provided that  $f$  has a unique global minimizer.

For the sake of simplicity, we split our proof into the following lemmas.

**Lemma 2.** Suppose that a swarm has  $N$  particles with initial positions  $x_0^1, \dots, x_0^N \in \mathbb{R}^d$ , and  $C := \text{conv}\{x_0^1, \dots, x_0^N\} \subset \mathbb{R}^d$ . Moreover, the coefficients of  $\{x_{n+1}^i\}$  satisfy the following relation:

$$a_n := \alpha_n + \gamma_n \geq 0, \quad b_n := \beta_n - \gamma_n \geq 0, \quad a_n + b_n = 1, \quad \forall n.$$

Then the sequences  $\{x_n^i\}, \{p_n^i\}, \{g_n\}$  given in (3) are bounded.

**Proof.** Take  $\alpha_n + \beta_n + \gamma_n = 1$  in (3); then, by setting  $a_n := \alpha_n + \gamma_n, b_n := \beta_n - \gamma_n$  in (3), we obtain  $a_n + b_n = 1$ , and the algorithm (3) reduces to

$$x_{n+1}^i = a_n p_n^i + b_n g_n, \quad (4)$$

with

$$p_{n+1}^i = \begin{cases} p_n^i, & f(p_n^i) \leq f(x_{n+1}^i), \\ x_{n+1}^i, & f(x_{n+1}^i) < f(p_n^i), \end{cases} \quad (5)$$

and  $g_n \in \{p_n^1, \dots, p_n^N\}$ , where  $f : \mathbb{R}^d \rightarrow \mathbb{R}$  is an objective function to be minimized. We now show that  $x_n^i, p_n^i, g_n \in C, \forall n, i$ . We shall use induction to prove it.

If we set  $n = 0$ ; then, by definition, we have  $x_0^i \in C$ . Choose  $p_0^i = x_0^i$ , which gives that  $p_0^i \in C$ . Also,  $g_0 \in \{p_0^1, \dots, p_0^N\} \subset C$ .

Assume  $p_n^i \in C$  and  $g_n \in C$  for all  $i$ . As  $a_n, b_n \geq 0$  and  $a_n + b_n = 1, x_{n+1}^i$  in (4), being a convex combination of two points of  $C$ , belongs to  $C$ .

By the update rule, we have

$$p_{n+1}^i \in \{p_n^i, x_{n+1}^i\} \subset C.$$

Finally,

$$g_{n+1} = \arg \min_i f(p_{n+1}^i)$$

is one of the  $p_{n+1}^i$  values and hence  $g_{n+1} \in C$ . Thus,  $x_n^i, p_n^i, g_n \in C, \forall n, i$ . As  $C$  is bounded, there exists  $M < \infty$  such that for all  $n, i, \|x_n^i\| \leq M, \|p_n^i\| \leq M$  and  $\|g_n\| \leq M$ . Hence,  $\{x_n^i\}, \{p_n^i\}, \{g_n\}$  are bounded.  $\square$

**Assumption 1.** Suppose that the search space  $D$  is nonempty, closed, and bounded, and let  $g^*$  be a global minimizer of the objective function  $f$  over  $D$ . Assume that the swarm satisfies the following exploration property:

For every  $\varepsilon > 0$ , there exists a particle index  $i$  and an iteration  $N(\varepsilon) \in \mathbb{N}$  such that

$$x_{N(\varepsilon)}^i \in B(g^*, \varepsilon),$$

where

$$B(g^*, \varepsilon) = \{x \in D : \|x - g^*\| < \varepsilon\}.$$

That is, during the optimization process, at least one particle visits every neighborhood of the global minimizer.

**Lemma 3.** Under the assumptions of Theorem 1, if  $\{g_n\}$  is global best sequence generated by (3), then  $f(g_n) \rightarrow f(g^*)$  and  $g_n \rightarrow g^*$ .

**Proof.** Since

$$g_n = \arg \min_{i=1, \dots, N} f(p_n^i),$$

where  $p_n^i$  is the personal best of particle  $i$  at iteration  $n$ :

$$p_n^i = \arg \min_{0 \leq k \leq n} f(x_k^i).$$

Then the global best fitness value is

$$f(g_n) = \min_{1 \leq i \leq N} f(p_n^i).$$

It follows from (5) that  $f(p_{n+1}^i) \leq f(p_n^i)$ . Hence,

$$f(g_{n+1}) = \min_i f(p_{n+1}^i) \leq \min_i f(p_n^i) = f(g_n).$$

So we have  $f(g_{n+1}) \leq f(g_n)$ ,  $\forall n$ ; hence,  $\{f(g_n)\}$  is a non-increasing sequence as  $f(g^*) \leq f(g_n)$ . Thus,  $\{f(g_n)\}$  is bounded below a monotone decreasing sequence. Assume that there exists  $L \geq f(g^*)$  such that  $f(g_n) \rightarrow L$ , as  $n \rightarrow \infty$ .

Now, we claim that  $f(g^*) = L$ . Assume on the contrary that  $\lim_{n \rightarrow \infty} f(g_n) = L > f(g^*)$ , where  $g^*$  is the global minimizer. Then, by the definition of a global minimum, there exists at least one point  $x^*$  such that  $f(x^*) = f(g^*)$ . By Assumption 1, for every  $\varepsilon > 0$ , there exists an iteration  $N(\varepsilon)$  and a particle  $x_{N(\varepsilon)}^i$  satisfying

$$x_{N(\varepsilon)}^i \in B(g^*, \varepsilon).$$

Since  $f$  is continuous, there exists  $\delta > 0$  such that

$$\|x - g^*\| < \delta \implies |f(x) - f(g^*)| < \varepsilon.$$

Hence,

$$f(x_{N(\varepsilon)}^i) < f(g^*) + \varepsilon.$$

According to the updating rule of the personal best,

$$f(p_{N(\varepsilon)}^i) \leq f(x_{N(\varepsilon)}^i) < f(g^*) + \varepsilon.$$

Since the global best is selected as

$$g_{N(\varepsilon)} = \arg \min_{1 \leq j \leq M} f(p_{N(\varepsilon)}^j),$$

it follows that

$$f(g_{N(\varepsilon)}) \leq f(p_{N(\varepsilon)}^i) < f(g^*) + \varepsilon.$$

Because  $\varepsilon > 0$  is arbitrary,

$$\lim_{n \rightarrow \infty} f(g_n) = f(g^*).$$

Since  $\{g_n\}$  is decreasing and convergent to the minimizer of  $f$ , let  $g_n \rightarrow x^*$ . As  $f$  has a unique global minimizer,

$$f(x^*) = f(g^*) \quad \text{implies that} \quad x^* = g^*.$$

Thus, convergence in values implies convergence in arguments. That is,  $g_n \rightarrow g^*$ .  $\square$

**Remark 2.** Assumption 1 is commonly adopted in the convergence analysis of swarm intelligence algorithms. It characterizes the ability of the swarm to continue exploring the feasible search region so that the neighborhood of the global optimum is eventually visited. In practice, this assumption is promoted through swarm diversity, random initialization, and the stochastic nature of the optimization process. Once a particle reaches a neighborhood of the global minimizer, the personal-best and global-best updating mechanisms ensure that the best-so-far solution is retained throughout the subsequent iterations.

**Lemma 4.** If the assumptions of Theorem 1 are satisfied, then the sequence  $\{x_n^i\}$  defined in (3) converges strongly to  $g^*$ .

**Proof.** First, we prove that  $\|p_n^i - g_n\| \rightarrow 0$  as  $n \rightarrow \infty$ .

Assume on the contrary that  $\|p_n^i - g_n\| \not\rightarrow 0$ . Then there exists  $\varepsilon > 0$  and a subsequence  $\{n_k\}$  of natural numbers such that

$$\|p_{n_k}^i - g_{n_k}\| \geq \varepsilon \quad \text{for all } k.$$

Note that  $\{p_{n_k}^i\}$  and  $\{g_{n_k}\}$  are bounded sequences. Suppose that  $\{p_{n_{k_\ell}}^i\}$  and  $\{g_{n_{k_\ell}}\}$  are convergent sub-sequences; that is,

$$p_{n_{k_\ell}}^i \rightarrow \bar{p}, \quad g_{n_{k_\ell}} \rightarrow \bar{g}$$

with

$$\|\bar{p} - \bar{g}\| \geq \varepsilon > 0.$$

as  $\ell \rightarrow \infty$ . By the continuity of  $f$  and the assumption of convergence of functional values, we have

$$f(\bar{p}) = \lim_{\ell \rightarrow \infty} f(p_{n_{k_\ell}}^i) = f(g^*), \quad f(\bar{g}) = \lim_{\ell \rightarrow \infty} f(g_{n_{k_\ell}}) = f(g^*),$$

which further implies that  $\bar{p} = \bar{g}$  and hence contradicts  $\|\bar{p} - \bar{g}\| \geq \varepsilon > 0$ . Thus,

$$\|p_n^i - g_n\| \rightarrow 0, \quad \text{as } n \rightarrow \infty. \quad (6)$$

From (4), we have

$$x_{n+1}^i = a_n p_n^i + b_n g_n, \quad (7)$$

where  $a_n + b_n = 1$ , so (7) reduces to

$$x_{n+1}^i = (1 - b_n) p_n^i + b_n g_n. \quad (8)$$

We now show that  $\|x_n^i - g_n\| \rightarrow 0$ , as  $n \rightarrow \infty$ . Note that

$$\begin{aligned}\|x_{n+1}^i - g_n\| &= \|(1 - b_n)p_n^i + b_ng_n - g_n\| \\ &= \|(1 - b_n)(p_n^i - g_n)\| \\ &= (1 - b_n)\|p_n^i - g_n\| \\ &\leq \|p_n^i - g_n\|.\end{aligned}\tag{9}$$

By (6) and by taking the limit as  $n \rightarrow \infty$ , the above inequality gives

$$\|x_{n+1}^i - g_n\| \rightarrow 0.$$

The result now follows from Lemma 3.  $\square$

**Remark 3.** The convergence theorem is established under the assumption that the objective function admits a unique global minimizer. This assumption is commonly adopted in the theoretical analysis of optimization algorithms since it guarantees the uniqueness of the limit point and simplifies the convergence proof.

However, the proposed AA-PSWV algorithm is not limited to optimization problems with a unique global minimizer. In many practical applications, such as multimodal benchmark functions and image segmentation problems, multiple global minimizers or several local minimizers may exist. In such cases, the update mechanism of the proposed algorithm remains unchanged, and the swarm continues to search for high-quality solutions using the personal-best and global-best information. Consequently, the algorithm may converge to any one of the global minimizers, depending on the initial distribution of the particles and the search path followed during the optimization process. Therefore, the uniqueness assumption is required only to establish the theoretical convergence result. It does not limit the practical applicability of the proposed AA-PSWV algorithm, which can be effectively applied to optimization problems with multiple global or local optima.

**Remark 4.** Suppose that the objective function  $f : D \rightarrow \mathbb{R} \cup \{+\infty\}$  is proper, lower-semicontinuous, and bounded below on the closed convex set  $D$  of a Hilbert space  $\mathcal{H}$ . If every weak cluster point  $\bar{g}$  of the sequence  $\{g_n\}$  satisfies  $f(\bar{g}) = L$  and  $f$  is strictly convex and coercive on  $D$ , then the result given in Theorem 1 also holds.

### 3. Optimization Experiments and Results

In this section, we present the numerical experiments to support our proposed AA-PSWV algorithm. The codes were written in MATLAB (version R2018a) and performed on a personal computer with a DESKTOP AMD A6-6310 APU with AMD Radeon R4 Graphics at 1.80 GHz, 512Gb SSD and 16.00 Gb RAM.

We compare the AA-PSWV algorithm with the existing comparable Particle Swarm algorithms, such as two forms of the PSO algorithm (PSO1 and PSO2) defined in [32], two versions of PSWV (PSWV1 and PSWV2) defined in [22], two forms of Modified PSO (MPSO1 and MPSO2) defined in [21], and two forms of the Combined PSO algorithm (CPSO1 and CPSO2) from [19]. For this purpose, we use the five well-known benchmark functions given in Table 3 as used in [22,32–34]. Further details of the benchmark function are given in Table 3.

Two cases were taken in order to test the performance, as used in [13,22,24,32,33].

In case I, set  $a = 0.6$ ,  $b_1 = b_2 = b = 1.7$  and  $c_1 = c_2 = 0.5882$  for (2) and in case II; set  $a = 0.729$ ,  $b = 1.494$  and  $c_1 = c_2 = 0.2225$  for (2) as given in [22].

Further two sets of experiments are performed in light of the test conditions given above. The results of the first experiment are given in Tables 4–8. In the first experiment, we used the stopping condition  $1 \times 10^{-2}$ , a maximum number of iterations of 2000, 20 runs

with random initial particles, and the population size  $N \in \{5, 10, 15, 20, 30\}$  to find the average number of iterations, the CPU time, the expected number of function evaluations, and the success rate (which is the number of runs needed to attain the optimum below the target) of the required iteration. The number of function evaluations required on average to converge is obtained using the following formula:

$$\text{Ex. No. of Fn. Evaluation} = \frac{\text{Avg. No. of Iterations} \times N}{\text{Success Rate}}.$$

Note from Table 5 that none of the algorithms reaches the optimal solution within the first 2000 iterations. In fact, optimizing the Rosenbrock function is challenging because it is a non-convex function with a narrow, curved valley. To obtain its optimum using gradient-free optimization methods, careful parameter tuning, an appropriate swarm size ( $N$ ), and proper initialization are required. In this experiment, the algorithms do not converge within 2000 iterations because the stopping criterion is very strict. Therefore, in the second experiment, we relaxed the stopping criterion from  $F_i(x) \leq 1 \times 10^{-2}$  to  $F_i(x) \leq 100$ . Under this relaxed criterion, all algorithms converged for the Rosenbrock function. The corresponding experimental results are presented in Tables 6 and 9–13.

**Table 3.** Benchmark Functions.

| Test Function                                                                                                     | Dim | Range<br>[ $x_{min}$ , $x_{max}$ ] | $F_{min}$ |
|-------------------------------------------------------------------------------------------------------------------|-----|------------------------------------|-----------|
| $F_0(x) = \sum_{i=1}^d x_i^2$ (Sphere)                                                                            | 30  | $[-100, 100]^d$                    | 0         |
| $F_1(x) = \sum_{i=1}^{d-1} [100(x_{i+1} - x_i^2)^2 + (x_i - 1)^2]$ (Rosenbrock)                                   | 30  | $[-30, 30]^d$                      | 0         |
| $F_2(x) = \sum_{i=1}^d [x_i^2 - 10 \cos(2\pi x_i) + 10]$ (Rastrigin)                                              | 30  | $[-5.12, 5.12]^d$                  | 0         |
| $F_3(x) = \frac{1}{4000} \sum_{i=1}^d x_i^2 - \prod_{i=1}^D \cos\left(\frac{x_i}{\sqrt{i}}\right) + 1$ (Griewank) | 30  | $[-600, 600]^d$                    | 0         |
| $F_4(x) = 0.5 + \frac{\sin^2(\sqrt{x_1^2 + x_2^2}) - 0.5}{(1 + 0.001(x_1^2 + x_2^2))^2}$ (Schaffers)              | 2   | $[-100, 100]^d$                    | 0         |

**Remark 5.** From Table 4, note that PSO1, PSO2, MPSO1, and MPSO2 fail to achieve the goal in 2000 iterations for different cases regarding the population. However, the proposed AA-PSWV converges faster toward the goal in fewer steps, and its CPU time is also less than that of the other comparable methods. Similar findings can be noted from Tables 6–8. This shows that AA-PSWV is more efficient than all versions of PSO and even PSWV, regardless of the population and the values of the parameters. Similarly, if we relax the stopping criterion as  $F_i(x) \leq 100$  in 2000 iterations, then the same behavior is observed, which also shows the clear difference in the number of iterations and the CPU time even as compared to PSWV, which has been known as an efficient algorithm among all PSO versions till now. For each test function, in every case, regardless of the parameters and the population size, our proposed approach performs well. Moreover, AA-PSWV guides the particles to move in suitable direction with the help of p-best and g-best without an explicit presence of the velocity vector, whereas PSWV fails to perform well. Also, the inertial term in our proposed algorithm accelerates the convergence in all aspects, for instance, robustness, number of iterations and CPU time.

**Remark 6.** Figures 1–5 show the fitness values of all test functions in 50 iterations. One can see that for all test functions with different population sizes, the AA-PSWV algorithm converges to the optimal solution in fewer steps than the other existing algorithms, which means that it converges faster and is more robust than the other existing approaches. This shows that the quality of the solution to this problem attained by the AA-PSWV is much better than that of the other algorithms.

**Table 4.** The convergence for the  $F_0$  Sphere function.

| No. of Particles | Alg. Ver. | Success Rate | Avg. No. of Iterations | Ex. No. of Fn. Evaluation | CPU Time |
|------------------|-----------|--------------|------------------------|---------------------------|----------|
| 5                | PSWV1     | 1.00         | 8.00                   | 40.00                     | 0.005219 |
|                  | PSWV2     | 1.00         | 8.80                   | 44.00                     | 0.003440 |
|                  | PSO1      | 0.00         | 2000.00                | 10,000.00                 | 0.465351 |
|                  | PSO2      | 0.00         | 2000.00                | 10,000.00                 | 0.451910 |
|                  | MPSO1     | 0.00         | 2000.00                | 10,000.00                 | 0.473626 |
|                  | MPSO2     | 0.00         | 2000.00                | 10,000.00                 | 0.469114 |
|                  | CPSO1     | 1.00         | 77.80                  | 389.00                    | 0.023009 |
|                  | CPSO2     | 1.00         | 94.60                  | 473.00                    | 0.021806 |
|                  | AA-PSWV   | 1.00         | 6.45                   | 32.25                     | 0.003147 |
| 10               | PSWV1     | 1.00         | 7.20                   | 72.00                     | 0.003191 |
|                  | PSWV2     | 1.00         | 7.40                   | 74.00                     | 0.003112 |
|                  | PSO1      | 0.00         | 2000.00                | 20,000.00                 | 0.617328 |
|                  | PSO2      | 0.40         | 1584.13                | 15,841.25                 | 0.566938 |
|                  | MPSO1     | 0.00         | 2000.00                | 20,000.00                 | 0.630745 |
|                  | MPSO2     | 0.00         | 2000.00                | 20,000.00                 | 0.620235 |
|                  | CPSO1     | 1.00         | 79.70                  | 797.00                    | 0.025225 |
|                  | CPSO2     | 1.00         | 99.00                  | 990.00                    | 0.031313 |
|                  | AA-PSWV   | 1.00         | 5.95                   | 59.50                     | 0.003102 |
| 15               | PSWV1     | 1.00         | 6.05                   | 90.75                     | 0.003238 |
|                  | PSWV2     | 1.00         | 6.20                   | 93.00                     | 0.003536 |
|                  | PSO1      | 0.25         | 562.40                 | 8436.00                   | 0.664033 |
|                  | PSO2      | 0.75         | 700.27                 | 10,504.00                 | 0.417813 |
|                  | MPSO1     | 0.00         | 2000.00                | 30,000.00                 | 0.831840 |
|                  | MPSO2     | 0.00         | 2000.00                | 30,000.00                 | 0.823852 |
|                  | CPSO1     | 1.00         | 77.15                  | 1157.25                   | 0.031649 |
|                  | CPSO2     | 1.00         | 101.15                 | 1517.25                   | 0.042257 |
|                  | AA-PSWV   | 1.00         | 5.75                   | 86.25                     | 0.002933 |
| 20               | PSWV1     | 1.00         | 5.70                   | 114.00                    | 0.004187 |
|                  | PSWV2     | 1.00         | 6.25                   | 125.00                    | 0.004625 |
|                  | PSO1      | 0.70         | 597.79                 | 11,955.71                 | 0.517700 |
|                  | PSO2      | 1.00         | 525.25                 | 10,505.00                 | 0.267038 |
|                  | MPSO1     | 0.00         | 2000.00                | 40,000.00                 | 1.061701 |
|                  | MPSO2     | 0.00         | 2000.00                | 40,000.00                 | 1.024826 |
|                  | CPSO1     | 1.00         | 77.90                  | 1558.00                   | 0.040348 |
|                  | CPSO2     | 1.00         | 98.40                  | 1968.00                   | 0.050497 |
|                  | AA-PSWV   | 1.00         | 5.45                   | 109.00                    | 0.003874 |
| 30               | PSWV1     | 1.00         | 4.85                   | 145.50                    | 0.004346 |
|                  | PSWV2     | 1.00         | 5.75                   | 172.50                    | 0.005049 |
|                  | PSO1      | 0.95         | 354.16                 | 10,624.74                 | 0.311021 |
|                  | PSO2      | 1.00         | 365.20                 | 10,956.00                 | 0.260664 |
|                  | MPSO1     | 0.00         | 2000.00                | 60,000.00                 | 1.451679 |
|                  | MPSO2     | 0.00         | 2000.00                | 60,000.00                 | 1.479670 |
|                  | CPSO1     | 1.00         | 77.55                  | 2326.50                   | 0.056099 |
|                  | CPSO2     | 1.00         | 106.45                 | 3193.50                   | 0.083540 |
|                  | AA-PSWV   | 1.00         | 4.45                   | 143.50                    | 0.004853 |

**Table 5.** The convergence for the  $F_1$  Rosenbrock function.

| No. of Particles | Alg. Ver. | Success Rate | Avg. No. of Iterations | Ex. No. of Fn. Evaluations | CPU Time |
|------------------|-----------|--------------|------------------------|----------------------------|----------|
| 5                | PSWV1     | 0.00         | 2000.00                | 10,000.00                  | 0.485937 |
|                  | PSWV2     | 0.00         | 2000.00                | 10,000.00                  | 0.485081 |
|                  | PSO1      | 0.00         | 2000.00                | 10,000.00                  | 0.519622 |
|                  | PSO2      | 0.00         | 2000.00                | 10,000.00                  | 0.514630 |
|                  | MPSO1     | 0.00         | 2000.00                | 10,000.00                  | 0.555462 |
|                  | MPSO2     | 0.00         | 2000.00                | 10,000.00                  | 0.531380 |
|                  | CPSO1     | 0.00         | 2000.00                | 10,000.00                  | 0.562962 |
|                  | CPSO2     | 0.00         | 2000.00                | 10,000.00                  | 0.639282 |
|                  | AA-PSWV   | 0.00         | 2000.00                | 10,000.00                  | 0.448315 |
| 10               | PSWV1     | 0.00         | 2000.00                | 20,000.00                  | 0.777117 |
|                  | PSWV2     | 0.00         | 2000.00                | 20,000.00                  | 0.719352 |
|                  | PSO1      | 0.00         | 2000.00                | 20,000.00                  | 0.845428 |
|                  | PSO2      | 0.00         | 2000.00                | 20,000.00                  | 0.744612 |
|                  | MPSO1     | 0.00         | 2000.00                | 20,000.00                  | 0.754441 |
|                  | MPSO2     | 0.00         | 2000.00                | 20,000.00                  | 0.743743 |
|                  | CPSO1     | 0.00         | 2000.00                | 20,000.00                  | 0.737817 |
|                  | CPSO2     | 0.00         | 2000.00                | 20,000.00                  | 0.728343 |
|                  | AA-PSWV   | 0.00         | 2000.00                | 20,000.00                  | 0.715246 |
| 15               | PSWV1     | 0.00         | 2000.00                | 30,000.00                  | 0.955884 |
|                  | PSWV2     | 0.00         | 2000.00                | 30,000.00                  | 0.963885 |
|                  | PSO1      | 0.00         | 2000.00                | 30,000.00                  | 1.003089 |
|                  | PSO2      | 0.00         | 2000.00                | 30,000.00                  | 1.017170 |
|                  | MPSO1     | 0.00         | 2000.00                | 30,000.00                  | 1.074654 |
|                  | MPSO2     | 0.00         | 2000.00                | 30,000.00                  | 1.005678 |
|                  | CPSO1     | 0.00         | 2000.00                | 30,000.00                  | 0.988245 |
|                  | CPSO2     | 0.00         | 2000.00                | 30,000.00                  | 0.942876 |
|                  | AA-PSWV   | 0.00         | 2000.00                | 30,000.00                  | 0.963811 |
| 20               | PSWV1     | 0.00         | 2000.00                | 40,000.00                  | 1.225832 |
|                  | PSWV2     | 0.00         | 2000.00                | 40,000.00                  | 1.240625 |
|                  | PSO1      | 0.00         | 2000.00                | 40,000.00                  | 1.274047 |
|                  | PSO2      | 0.00         | 2000.00                | 40,000.00                  | 1.263082 |
|                  | MPSO1     | 0.00         | 2000.00                | 40,000.00                  | 1.268667 |
|                  | MPSO2     | 0.00         | 2000.00                | 40,000.00                  | 1.272386 |
|                  | CPSO1     | 0.00         | 2000.00                | 40,000.00                  | 1.257570 |
|                  | CPSO2     | 0.00         | 2000.00                | 40,000.00                  | 1.281375 |
|                  | AA-PSWV   | 0.00         | 2000.00                | 40,000.00                  | 1.217505 |
| 30               | PSWV1     | 0.00         | 2000.00                | 60,000.00                  | 1.742664 |
|                  | PSWV2     | 0.00         | 2000.00                | 60,000.00                  | 1.692003 |
|                  | PSO1      | 0.00         | 2000.00                | 60,000.00                  | 1.772340 |
|                  | PSO2      | 0.00         | 2000.00                | 60,000.00                  | 1.776244 |
|                  | MPSO1     | 0.00         | 2000.00                | 60,000.00                  | 1.795384 |
|                  | MPSO2     | 0.00         | 2000.00                | 60,000.00                  | 1.790213 |
|                  | CPSO1     | 0.00         | 2000.00                | 60,000.00                  | 1.772600 |
|                  | CPSO2     | 0.00         | 2000.00                | 60,000.00                  | 1.760596 |
|                  | AA-PSWV   | 0.00         | 2000.00                | 60,000.00                  | 1.717250 |

**Table 6.** The convergence for the  $F_2$  Rastrigin function.

| No. of Particles | Alg. Ver. | Success Rate | Avg. No. of Iterations | Ex. No. of Fn. Evaluations | CPU Time |
|------------------|-----------|--------------|------------------------|----------------------------|----------|
| 5                | PSWV1     | 1.00         | 7.85                   | 39.25                      | 0.002483 |
|                  | PSWV2     | 1.00         | 8.45                   | 42.25                      | 0.002578 |
|                  | PSO1      | 0.00         | 2000.00                | 10,000.00                  | 0.447599 |
|                  | PSO2      | 0.00         | 2000.00                | 10,000.00                  | 0.443609 |
|                  | MPSO1     | 0.00         | 2000.00                | 10,000.00                  | 0.458314 |
|                  | MPSO2     | 0.00         | 2000.00                | 10,000.00                  | 0.449423 |
|                  | CPSO1     | 1.00         | 110.75                 | 553.75                     | 0.024974 |
|                  | CPSO2     | 1.00         | 150.50                 | 752.50                     | 0.034000 |
|                  | AA-PSWV   | 1.00         | 6.80                   | 34.00                      | 0.001934 |
| 10               | PSWV1     | 1.00         | 6.80                   | 68.00                      | 0.002942 |
|                  | PSWV2     | 1.00         | 7.30                   | 73.00                      | 0.003034 |
|                  | PSO1      | 0.00         | 2000.00                | 20,000.00                  | 0.667150 |
|                  | PSO2      | 0.00         | 2000.00                | 20,000.00                  | 0.666455 |
|                  | MPSO1     | 0.00         | 2000.00                | 20,000.00                  | 0.683919 |
|                  | MPSO2     | 0.00         | 2000.00                | 20,000.00                  | 0.673501 |
|                  | CPSO1     | 1.00         | 113.20                 | 1132.00                    | 0.037531 |
|                  | CPSO2     | 1.00         | 171.65                 | 1716.50                    | 0.056658 |
|                  | AA-PSWV   | 1.00         | 6.05                   | 60.50                      | 0.002479 |
| 15               | PSWV1     | 1.00         | 5.70                   | 85.50                      | 0.003572 |
|                  | PSWV2     | 1.00         | 6.65                   | 99.75                      | 0.003500 |
|                  | PSO1      | 0.00         | 2000.00                | 30,000.00                  | 0.888195 |
|                  | PSO2      | 0.00         | 2000.00                | 30,000.00                  | 0.886960 |
|                  | MPSO1     | 0.00         | 2000.00                | 30,000.00                  | 0.909952 |
|                  | MPSO2     | 0.00         | 2000.00                | 30,000.00                  | 0.900293 |
|                  | CPSO1     | 1.00         | 124.35                 | 1865.25                    | 0.055570 |
|                  | CPSO2     | 1.00         | 152.40                 | 2286.00                    | 0.068353 |
|                  | AA-PSWV   | 1.00         | 5.90                   | 88.50                      | 0.003419 |
| 20               | PSWV1     | 1.00         | 5.60                   | 112.00                     | 0.003951 |
|                  | PSWV2     | 1.00         | 6.55                   | 131.00                     | 0.004484 |
|                  | PSO1      | 0.00         | 2000.00                | 40,000.00                  | 1.110467 |
|                  | PSO2      | 0.00         | 2000.00                | 40,000.00                  | 1.120797 |
|                  | MPSO1     | 0.00         | 2000.00                | 40,000.00                  | 1.134213 |
|                  | MPSO2     | 0.00         | 2000.00                | 40,000.00                  | 1.130034 |
|                  | CPSO1     | 1.00         | 126.20                 | 2524.00                    | 0.075543 |
|                  | CPSO2     | 1.00         | 155.15                 | 3103.00                    | 0.105584 |
|                  | AA-PSWV   | 1.00         | 5.35                   | 107.00                     | 0.003893 |
| 30               | PSWV1     | 1.00         | 5.40                   | 162.00                     | 0.005555 |
|                  | PSWV2     | 1.00         | 5.45                   | 163.50                     | 0.005636 |
|                  | PSO1      | 0.00         | 2000.00                | 60,000.00                  | 1.586913 |
|                  | PSO2      | 0.00         | 2000.00                | 60,000.00                  | 1.564204 |
|                  | MPSO1     | 0.00         | 2000.00                | 60,000.00                  | 1.611953 |
|                  | MPSO2     | 0.00         | 2000.00                | 60,000.00                  | 1.585439 |
|                  | CPSO1     | 1.00         | 125.45                 | 3763.50                    | 0.099975 |
|                  | CPSO2     | 1.00         | 143.25                 | 4297.50                    | 0.112078 |
|                  | AA-PSWV   | 1.00         | 5.35                   | 166.50                     | 0.005342 |

**Table 7.** The convergence for the  $F_3$  Griewank function.

| No. of Particles | Alg. Ver. | Success Rate | Avg. No. of Iterations | Ex. No. of Fn. Evaluations | CPU Time |
|------------------|-----------|--------------|------------------------|----------------------------|----------|
| 5                | PSWV1     | 1.00         | 8.45                   | 42.25                      | 0.002975 |
|                  | PSWV2     | 1.00         | 9.65                   | 48.25                      | 0.003114 |
|                  | PSO1      | 0.00         | 2000.00                | 10,000.00                  | 0.520263 |
|                  | PSO2      | 0.00         | 2000.00                | 10,000.00                  | 0.524240 |
|                  | MPSO1     | 0.00         | 2000.00                | 10,000.00                  | 0.533807 |
|                  | MPSO2     | 0.00         | 2000.00                | 10,000.00                  | 0.522330 |
|                  | CPSO1     | 1.00         | 99.95                  | 499.75                     | 0.026770 |
|                  | CPSO2     | 1.00         | 116.25                 | 581.25                     | 0.031340 |
|                  | AA-PSWV   | 1.00         | 6.95                   | 34.75                      | 0.002345 |
| 10               | PSWV1     | 1.00         | 7.40                   | 74.00                      | 0.003537 |
|                  | PSWV2     | 1.00         | 7.50                   | 75.00                      | 0.003637 |
|                  | PSO1      | 0.00         | 2000.00                | 20,000.00                  | 0.795547 |
|                  | PSO2      | 0.10         | 1268.00                | 12,680.00                  | 0.767483 |
|                  | MPSO1     | 0.00         | 2000.00                | 20,000.00                  | 0.817414 |
|                  | MPSO2     | 0.00         | 2000.00                | 20,000.00                  | 0.807430 |
|                  | CPSO1     | 1.00         | 88.30                  | 883.00                     | 0.035874 |
|                  | CPSO2     | 1.00         | 131.30                 | 1313.00                    | 0.052688 |
|                  | AA-PSWV   | 1.00         | 6.35                   | 63.50                      | 0.003210 |
| 15               | PSWV1     | 1.00         | 6.35                   | 95.25                      | 0.004420 |
|                  | PSWV2     | 1.00         | 6.90                   | 103.50                     | 0.004584 |
|                  | PSO1      | 0.00         | 2000.00                | 30,000.00                  | 1.072451 |
|                  | PSO2      | 0.20         | 709.50                 | 10642.50                   | 0.953638 |
|                  | MPSO1     | 0.00         | 2000.00                | 30,000.00                  | 1.116286 |
|                  | MPSO2     | 0.00         | 2000.00                | 30,000.00                  | 1.095110 |
|                  | CPSO1     | 1.00         | 104.50                 | 1567.50                    | 0.057914 |
|                  | CPSO2     | 1.00         | 134.70                 | 2020.50                    | 0.073945 |
|                  | AA-PSWV   | 1.00         | 6.10                   | 91.50                      | 0.003992 |
| 20               | PSWV1     | 1.00         | 6.00                   | 120.00                     | 0.005255 |
|                  | PSWV2     | 1.00         | 6.75                   | 135.00                     | 0.005695 |
|                  | PSO1      | 0.10         | 434.00                 | 8680.00                    | 1.254235 |
|                  | PSO2      | 0.30         | 491.83                 | 9836.67                    | 1.051119 |
|                  | MPSO1     | 0.00         | 2000.00                | 40,000.00                  | 1.421120 |
|                  | MPSO2     | 0.00         | 2000.00                | 40,000.00                  | 1.390467 |
|                  | CPSO1     | 1.00         | 83.80                  | 1676.00                    | 0.058331 |
|                  | CPSO2     | 1.00         | 134.95                 | 2699.00                    | 0.093391 |
|                  | AA-PSWV   | 1.00         | 5.90                   | 118.00                     | 0.005229 |
| 30               | PSWV1     | 1.00         | 5.70                   | 171.00                     | 0.007531 |
|                  | PSWV2     | 1.00         | 6.15                   | 184.50                     | 0.008061 |
|                  | PSO1      | 0.60         | 334.08                 | 10,022.50                  | 0.992523 |
|                  | PSO2      | 0.25         | 396.80                 | 11,904.00                  | 1.535363 |
|                  | MPSO1     | 0.00         | 2000.00                | 60,000.00                  | 2.015974 |
|                  | MPSO2     | 0.00         | 2000.00                | 60,000.00                  | 1.968755 |
|                  | CPSO1     | 1.00         | 98.20                  | 2946.00                    | 0.096358 |
|                  | CPSO2     | 1.00         | 131.40                 | 3942.00                    | 0.128582 |
|                  | AA-PSWV   | 1.00         | 5.55                   | 166.50                     | 0.006786 |

**Table 8.** The convergence for the  $F_4$  Schaffers function.

| No. of Particles | Alg. Ver. | Success Rate | Avg. No. of Iterations | Ex. No. of Fn. Evaluations | CPU Time |
|------------------|-----------|--------------|------------------------|----------------------------|----------|
| 5                | PSWV1     | 1.00         | 30.45                  | 152.25                     | 0.006968 |
|                  | PSWV2     | 1.00         | 26.90                  | 134.50                     | 0.005987 |
|                  | PSO1      | 0.45         | 43.44                  | 217.22                     | 0.239377 |
|                  | PSO2      | 0.95         | 64.89                  | 324.47                     | 0.034815 |
|                  | MPSO1     | 0.40         | 27.75                  | 138.75                     | 0.263418 |
|                  | MPSO2     | 0.55         | 43.82                  | 219.09                     | 0.201019 |
|                  | CPSO1     | 1.00         | 46.65                  | 233.25                     | 0.010441 |
|                  | CPSO2     | 1.00         | 62.10                  | 310.50                     | 0.013964 |
|                  | AA-PSWV   | 1.00         | 16.60                  | 83.00                      | 0.003892 |
| 10               | PSWV1     | 1.00         | 10.15                  | 101.50                     | 0.003729 |
|                  | PSWV2     | 1.00         | 15.45                  | 154.50                     | 0.005414 |
|                  | PSO1      | 1.00         | 36.15                  | 361.50                     | 0.012022 |
|                  | PSO2      | 1.00         | 36.90                  | 369.00                     | 0.012355 |
|                  | MPSO1     | 0.80         | 26.06                  | 260.63                     | 0.133350 |
|                  | MPSO2     | 1.00         | 31.75                  | 317.50                     | 0.010606 |
|                  | CPSO1     | 1.00         | 37.25                  | 372.50                     | 0.012227 |
|                  | CPSO2     | 1.00         | 43.10                  | 431.00                     | 0.013980 |
|                  | AA-PSWV   | 1.00         | 9.85                   | 98.50                      | 0.003594 |
| 15               | PSWV1     | 1.00         | 9.15                   | 137.25                     | 0.004337 |
|                  | PSWV2     | 1.00         | 13.40                  | 201.00                     | 0.006113 |
|                  | PSO1      | 1.00         | 23.60                  | 354.00                     | 0.010295 |
|                  | PSO2      | 1.00         | 26.15                  | 392.25                     | 0.011523 |
|                  | MPSO1     | 0.95         | 19.00                  | 285.00                     | 0.049764 |
|                  | MPSO2     | 1.00         | 23.00                  | 345.00                     | 0.010125 |
|                  | CPSO1     | 1.00         | 33.45                  | 501.75                     | 0.014343 |
|                  | CPSO2     | 1.00         | 34.05                  | 510.75                     | 0.014670 |
|                  | AA-PSWV   | 1.00         | 8.00                   | 120.00                     | 0.003987 |
| 20               | PSWV1     | 1.00         | 7.35                   | 147.00                     | 0.004814 |
|                  | PSWV2     | 1.00         | 8.70                   | 174.00                     | 0.004914 |
|                  | PSO1      | 1.00         | 25.60                  | 512.00                     | 0.014197 |
|                  | PSO2      | 1.00         | 20.85                  | 417.00                     | 0.011648 |
|                  | MPSO1     | 1.00         | 14.85                  | 297.00                     | 0.008347 |
|                  | MPSO2     | 1.00         | 23.15                  | 463.00                     | 0.012953 |
|                  | CPSO1     | 1.00         | 21.25                  | 425.00                     | 0.011606 |
|                  | CPSO2     | 1.00         | 27.55                  | 551.00                     | 0.015029 |
|                  | AA-PSWV   | 1.00         | 7.05                   | 141.00                     | 0.004305 |
| 30               | PSWV1     | 1.00         | 5.70                   | 171.00                     | 0.004930 |
|                  | PSWV2     | 1.00         | 7.85                   | 235.50                     | 0.006339 |
|                  | PSO1      | 1.00         | 18.60                  | 558.00                     | 0.014520 |
|                  | PSO2      | 1.00         | 23.10                  | 693.00                     | 0.017820 |
|                  | MPSO1     | 1.00         | 14.85                  | 445.50                     | 0.011473 |
|                  | MPSO2     | 1.00         | 18.50                  | 555.00                     | 0.014192 |
|                  | CPSO1     | 1.00         | 23.15                  | 694.50                     | 0.017603 |
|                  | CPSO2     | 1.00         | 19.15                  | 574.50                     | 0.015320 |
|                  | AA-PSWV   | 1.00         | 4.90                   | 147.00                     | 0.004470 |

**Table 9.** The convergence for the  $F_0$  Sphere function.

| No. of Particles | Alg. Ver. | Success Rate | Avg. No. of Iterations | Ex. No. of Fn. Evaluations | CPU Time |
|------------------|-----------|--------------|------------------------|----------------------------|----------|
| 5                | PSWV1     | 1.00         | 8.00                   | 40.00                      | 0.007793 |
|                  | PSWV2     | 1.00         | 8.80                   | 44.00                      | 0.003646 |
|                  | PSO1      | 0.00         | 2000.00                | 10,000.00                  | 0.444228 |
|                  | PSO2      | 0.00         | 2000.00                | 10,000.00                  | 0.435246 |
|                  | MPSO1     | 0.00         | 2000.00                | 10,000.00                  | 0.441387 |
|                  | MPSO2     | 0.00         | 2000.00                | 10,000.00                  | 0.436264 |
|                  | CPSO1     | 1.00         | 77.80                  | 389.00                     | 0.020856 |
|                  | CPSO2     | 1.00         | 94.60                  | 473.00                     | 0.022653 |
|                  | AA-PSWV   | 1.00         | 6.45                   | 32.25                      | 0.003270 |
| 10               | PSWV1     | 1.00         | 7.20                   | 72.00                      | 0.003194 |
|                  | PSWV2     | 1.00         | 7.40                   | 74.00                      | 0.002898 |
|                  | PSO1      | 0.00         | 2000.00                | 20,000.00                  | 0.630697 |
|                  | PSO2      | 0.40         | 1584.13                | 15,841.25                  | 0.584110 |
|                  | MPSO1     | 0.00         | 2000.00                | 20,000.00                  | 0.651225 |
|                  | MPSO2     | 0.00         | 2000.00                | 20,000.00                  | 0.642951 |
|                  | CPSO1     | 1.00         | 79.70                  | 797.00                     | 0.026492 |
|                  | CPSO2     | 1.00         | 99.00                  | 990.00                     | 0.031899 |
|                  | AA-PSWV   | 1.00         | 5.95                   | 59.50                      | 0.002366 |
| 15               | PSWV1     | 1.00         | 6.05                   | 90.75                      | 0.003473 |
|                  | PSWV2     | 1.00         | 6.20                   | 93.00                      | 0.003778 |
|                  | PSO1      | 0.25         | 562.40                 | 8436.00                    | 0.683446 |
|                  | PSO2      | 0.75         | 700.27                 | 10,504.00                  | 0.427944 |
|                  | MPSO1     | 0.00         | 2000.00                | 30,000.00                  | 0.863066 |
|                  | MPSO2     | 0.00         | 2000.00                | 30,000.00                  | 0.851532 |
|                  | CPSO1     | 1.00         | 77.15                  | 1157.25                    | 0.033509 |
|                  | CPSO2     | 1.00         | 101.15                 | 1517.25                    | 0.043372 |
|                  | AA-PSWV   | 1.00         | 5.75                   | 86.25                      | 0.003274 |
| 20               | PSWV1     | 1.00         | 5.70                   | 114.00                     | 0.003859 |
|                  | PSWV2     | 1.00         | 6.25                   | 125.00                     | 0.004621 |
|                  | PSO1      | 0.70         | 597.79                 | 11,955.71                  | 0.526893 |
|                  | PSO2      | 1.00         | 525.25                 | 10,505.00                  | 0.274402 |
|                  | MPSO1     | 0.00         | 2000.00                | 40,000.00                  | 1.078562 |
|                  | MPSO2     | 0.00         | 2000.00                | 40,000.00                  | 1.072120 |
|                  | CPSO1     | 1.00         | 77.90                  | 1558.00                    | 0.041997 |
|                  | CPSO2     | 1.00         | 98.40                  | 1968.00                    | 0.052430 |
|                  | AA-PSWV   | 1.00         | 5.45                   | 109.00                     | 0.003652 |
| 30               | PSWV1     | 1.00         | 4.85                   | 145.50                     | 0.004601 |
|                  | PSWV2     | 1.00         | 5.75                   | 172.50                     | 0.005237 |
|                  | PSO1      | 0.95         | 354.16                 | 10,624.74                  | 0.318516 |
|                  | PSO2      | 1.00         | 365.20                 | 10,956.00                  | 0.267689 |
|                  | MPSO1     | 0.00         | 2000.00                | 60,000.00                  | 1.529475 |
|                  | MPSO2     | 0.00         | 2000.00                | 60,000.00                  | 1.504683 |
|                  | CPSO1     | 1.00         | 77.55                  | 2326.50                    | 0.056753 |
|                  | CPSO2     | 1.00         | 106.45                 | 3193.50                    | 0.076945 |
|                  | AA-PSWV   | 1.00         | 4.45                   | 163.50                     | 0.003977 |

**Table 10.** The convergence for the  $F_1$  Rosenbrock function.

| No. of Particles | Alg. Ver. | Success Rate | Avg. No. of Iterations | Ex. No. of Fn. Evaluations | CPU Time |
|------------------|-----------|--------------|------------------------|----------------------------|----------|
| 5                | PSWV1     | 1.00         | 4.70                   | 23.50                      | 0.002092 |
|                  | PSWV2     | 1.00         | 4.95                   | 24.75                      | 0.001986 |
|                  | PSO1      | 0.00         | 2000.00                | 10,000.00                  | 0.494766 |
|                  | PSO2      | 0.00         | 2000.00                | 10,000.00                  | 0.491189 |
|                  | MPSO1     | 0.00         | 2000.00                | 10,000.00                  | 0.503515 |
|                  | MPSO2     | 0.00         | 2000.00                | 10,000.00                  | 0.515550 |
|                  | CPSO1     | 1.00         | 53.40                  | 267.00                     | 0.014269 |
|                  | CPSO2     | 1.00         | 64.85                  | 324.25                     | 0.017429 |
|                  | AA-PSWV   | 1.00         | 3.80                   | 19.00                      | 0.001566 |
| 10               | PSWV1     | 1.00         | 3.80                   | 38.00                      | 0.002137 |
|                  | PSWV2     | 1.00         | 4.30                   | 43.00                      | 0.002372 |
|                  | PSO1      | 0.00         | 2000.00                | 20,000.00                  | 0.752220 |
|                  | PSO2      | 0.15         | 1400.33                | 14,003.33                  | 0.718594 |
|                  | MPSO1     | 0.00         | 2000.00                | 20,000.00                  | 0.770855 |
|                  | MPSO2     | 0.00         | 2000.00                | 20,000.00                  | 0.755927 |
|                  | CPSO1     | 1.00         | 50.75                  | 507.50                     | 0.020348 |
|                  | CPSO2     | 1.00         | 81.80                  | 818.00                     | 0.031838 |
|                  | AA-PSWV   | 1.00         | 3.30                   | 33.00                      | 0.001919 |
| 15               | PSWV1     | 1.00         | 3.45                   | 51.75                      | 0.002674 |
|                  | PSWV2     | 1.00         | 4.00                   | 60.00                      | 0.002924 |
|                  | PSO1      | 0.60         | 586.08                 | 8791.25                    | 0.587981 |
|                  | PSO2      | 0.65         | 858.46                 | 12,876.92                  | 0.636240 |
|                  | MPSO1     | 0.00         | 2000.00                | 30,000.00                  | 1.043585 |
|                  | MPSO2     | 0.00         | 2000.00                | 30,000.00                  | 1.027192 |
|                  | CPSO1     | 1.00         | 51.25                  | 768.75                     | 0.026758 |
|                  | CPSO2     | 1.00         | 65.30                  | 979.50                     | 0.033848 |
|                  | AA-PSWV   | 1.00         | 3.40                   | 51.00                      | 0.002532 |
| 20               | PSWV1     | 1.00         | 3.15                   | 63.00                      | 0.003128 |
|                  | PSWV2     | 1.00         | 3.40                   | 68.00                      | 0.003089 |
|                  | PSO1      | 0.60         | 495.83                 | 9916.67                    | 0.706872 |
|                  | PSO2      | 0.70         | 600.50                 | 12,010.00                  | 0.656596 |
|                  | MPSO1     | 0.00         | 2000.00                | 40,000.00                  | 1.321812 |
|                  | MPSO2     | 0.00         | 2000.00                | 40,000.00                  | 1.305156 |
|                  | CPSO1     | 1.00         | 54.15                  | 1083.00                    | 0.036372 |
|                  | CPSO2     | 1.00         | 70.40                  | 1408.00                    | 0.045470 |
|                  | AA-PSWV   | 1.00         | 3.10                   | 62.00                      | 0.002994 |
| 30               | PSWV1     | 1.00         | 2.75                   | 82.50                      | 0.004135 |
|                  | PSWV2     | 1.00         | 3.20                   | 96.00                      | 0.003971 |
|                  | PSO1      | 0.50         | 372.40                 | 11,172.00                  | 1.073257 |
|                  | PSO2      | 0.75         | 415.60                 | 12,468.00                  | 0.733912 |
|                  | MPSO1     | 0.00         | 2000.00                | 60,000.00                  | 1.861617 |
|                  | MPSO2     | 0.00         | 2000.00                | 60,000.00                  | 1.842922 |
|                  | CPSO1     | 1.00         | 52.30                  | 1569.00                    | 0.048205 |
|                  | CPSO2     | 1.00         | 78.60                  | 2358.00                    | 0.072040 |
|                  | AA-PSWV   | 1.00         | 2.85                   | 85.50                      | 0.004127 |

**Table 11.** The convergence for the  $F_2$  Rastrigin function.

| No. of Particles | Alg. Ver. | Success Rate | Avg. No. of Iterations | Ex. No. of Fn. Evaluations | CPU Time |
|------------------|-----------|--------------|------------------------|----------------------------|----------|
| 5                | PSWV1     | 1.00         | 3.20                   | 16.00                      | 0.001369 |
|                  | PSWV2     | 1.00         | 3.60                   | 18.00                      | 0.001274 |
|                  | PSO1      | 0.00         | 2000.00                | 10,000.00                  | 0.460312 |
|                  | PSO2      | 0.00         | 2000.00                | 10,000.00                  | 0.456073 |
|                  | MPSO1     | 0.00         | 2000.00                | 10,000.00                  | 0.467511 |
|                  | MPSO2     | 0.00         | 2000.00                | 10,000.00                  | 0.461227 |
|                  | CPSO1     | 1.00         | 44.10                  | 220.50                     | 0.010920 |
|                  | CPSO2     | 1.00         | 74.55                  | 372.75                     | 0.017625 |
|                  | AA-PSWV   | 1.00         | 2.90                   | 14.50                      | 0.001234 |
| 10               | PSWV1     | 1.00         | 2.90                   | 29.00                      | 0.001619 |
|                  | PSWV2     | 1.00         | 3.15                   | 31.50                      | 0.001749 |
|                  | PSO1      | 0.05         | 372.00                 | 3720.00                    | 0.647051 |
|                  | PSO2      | 0.20         | 420.25                 | 4202.50                    | 0.572269 |
|                  | MPSO1     | 0.00         | 2000.00                | 20,000.00                  | 0.705755 |
|                  | MPSO2     | 0.00         | 2000.00                | 20,000.00                  | 0.705827 |
|                  | CPSO1     | 1.00         | 39.65                  | 396.50                     | 0.014565 |
|                  | CPSO2     | 1.00         | 50.50                  | 505.00                     | 0.018062 |
|                  | AA-PSWV   | 1.00         | 2.50                   | 25.00                      | 0.001564 |
| 15               | PSWV1     | 1.00         | 2.15                   | 32.25                      | 0.007040 |
|                  | PSWV2     | 1.00         | 2.70                   | 40.50                      | 0.002490 |
|                  | PSO1      | 0.05         | 202.00                 | 3030.00                    | 0.925150 |
|                  | PSO2      | 0.10         | 177.00                 | 2655.00                    | 0.829400 |
|                  | MPSO1     | 0.00         | 2000.00                | 30,000.00                  | 0.936750 |
|                  | MPSO2     | 0.10         | 86.00                  | 1290.00                    | 0.841609 |
|                  | CPSO1     | 1.00         | 45.20                  | 678.00                     | 0.021714 |
|                  | CPSO2     | 1.00         | 63.70                  | 955.50                     | 0.030422 |
|                  | AA-PSWV   | 1.00         | 2.20                   | 33.00                      | 0.001938 |
| 20               | PSWV1     | 1.00         | 1.95                   | 39.00                      | 0.001848 |
|                  | PSWV2     | 1.00         | 2.20                   | 44.00                      | 0.001949 |
|                  | PSO1      | 0.10         | 184.00                 | 3680.00                    | 1.040838 |
|                  | PSO2      | 0.15         | 185.67                 | 3713.33                    | 0.989752 |
|                  | MPSO1     | 0.05         | 47.00                  | 940.00                     | 1.120738 |
|                  | MPSO2     | 0.20         | 83.50                  | 1670.00                    | 0.940189 |
|                  | CPSO1     | 1.00         | 39.90                  | 798.00                     | 0.024225 |
|                  | CPSO2     | 1.00         | 73.90                  | 1478.00                    | 0.043283 |
|                  | AA-PSWV   | 1.00         | 1.95                   | 39.00                      | 0.001849 |
| 30               | PSWV1     | 1.00         | 1.95                   | 58.50                      | 0.002703 |
|                  | PSWV2     | 1.00         | 1.90                   | 57.00                      | 0.002826 |
|                  | PSO1      | 0.30         | 155.50                 | 4665.00                    | 1.159757 |
|                  | PSO2      | 0.40         | 175.25                 | 5257.50                    | 1.013500 |
|                  | MPSO1     | 0.00         | 2000.00                | 60,000.00                  | 1.658604 |
|                  | MPSO2     | 0.30         | 63.50                  | 1905.00                    | 1.155229 |
|                  | CPSO1     | 1.00         | 47.90                  | 1437.00                    | 0.040383 |
|                  | CPSO2     | 1.00         | 69.60                  | 2088.00                    | 0.056899 |
|                  | AA-PSWV   | 1.00         | 2.00                   | 60.00                      | 0.002816 |

**Table 12.** The convergence for the  $F_3$  Griewank function.

| No. of Particles | Alg. Ver. | Success Rate | Avg. No. of Iterations | Ex. No. of Fn. Evaluations | CPU Time |
|------------------|-----------|--------------|------------------------|----------------------------|----------|
| 5                | PSWV1     | 1.00         | 7.10                   | 35.50                      | 0.002644 |
|                  | PSWV2     | 1.00         | 8.35                   | 41.75                      | 0.002913 |
|                  | PSO1      | 0.00         | 2000.00                | 10,000.00                  | 0.537290 |
|                  | PSO2      | 0.00         | 2000.00                | 10,000.00                  | 0.534323 |
|                  | MPSO1     | 0.00         | 2000.00                | 10,000.00                  | 0.550312 |
|                  | MPSO2     | 0.00         | 2000.00                | 10,000.00                  | 0.539064 |
|                  | CPSO1     | 1.00         | 77.55                  | 387.75                     | 0.021647 |
|                  | CPSO2     | 1.00         | 110.70                 | 553.50                     | 0.029957 |
|                  | AA-PSWV   | 1.00         | 5.85                   | 29.25                      | 0.002127 |
| 10               | PSWV1     | 1.00         | 6.10                   | 61.00                      | 0.003247 |
|                  | PSWV2     | 1.00         | 6.50                   | 65.00                      | 0.003358 |
|                  | PSO1      | 0.00         | 2000.00                | 20,000.00                  | 0.820719 |
|                  | PSO2      | 0.10         | 1754.00                | 17,540.00                  | 0.808729 |
|                  | MPSO1     | 0.00         | 2000.00                | 20,000.00                  | 0.851409 |
|                  | MPSO2     | 0.00         | 2000.00                | 20,000.00                  | 0.835961 |
|                  | CPSO1     | 1.00         | 78.60                  | 786.00                     | 0.033633 |
|                  | CPSO2     | 1.00         | 119.85                 | 1198.50                    | 0.051205 |
|                  | AA-PSWV   | 1.00         | 5.55                   | 55.50                      | 0.002986 |
| 15               | PSWV1     | 1.00         | 5.40                   | 81.00                      | 0.004024 |
|                  | PSWV2     | 1.00         | 5.75                   | 86.25                      | 0.004012 |
|                  | PSO1      | 0.25         | 653.00                 | 9795.00                    | 0.912433 |
|                  | PSO2      | 0.30         | 529.33                 | 7940.00                    | 0.860573 |
|                  | MPSO1     | 0.00         | 2000.00                | 30,000.00                  | 1.171640 |
|                  | MPSO2     | 0.00         | 2000.00                | 30,000.00                  | 1.138708 |
|                  | CPSO1     | 1.00         | 73.80                  | 1107.00                    | 0.042506 |
|                  | CPSO2     | 1.00         | 117.75                 | 1766.25                    | 0.067150 |
|                  | AA-PSWV   | 1.00         | 5.00                   | 75.00                      | 0.003743 |
| 20               | PSWV1     | 1.00         | 5.10                   | 102.00                     | 0.004526 |
|                  | PSWV2     | 1.00         | 5.65                   | 113.00                     | 0.005133 |
|                  | PSO1      | 0.60         | 373.83                 | 7476.67                    | 0.786502 |
|                  | PSO2      | 0.70         | 431.36                 | 8627.14                    | 0.670833 |
|                  | MPSO1     | 0.00         | 2000.00                | 40,000.00                  | 1.470418 |
|                  | MPSO2     | 0.00         | 2000.00                | 40,000.00                  | 1.440206 |
|                  | CPSO1     | 1.00         | 78.10                  | 1562.00                    | 0.057028 |
|                  | CPSO2     | 1.00         | 114.35                 | 2287.00                    | 0.082022 |
|                  | AA-PSWV   | 1.00         | 5.10                   | 103.00                     | 0.004591 |
| 30               | PSWV1     | 1.00         | 5.00                   | 150.00                     | 0.006441 |
|                  | PSWV2     | 1.00         | 4.90                   | 147.00                     | 0.006384 |
|                  | PSO1      | 0.85         | 310.94                 | 9328.24                    | 0.559940 |
|                  | PSO2      | 0.80         | 314.88                 | 9446.25                    | 0.647068 |
|                  | MPSO1     | 0.00         | 2000.00                | 60,000.00                  | 2.088914 |
|                  | MPSO2     | 0.00         | 2000.00                | 60,000.00                  | 2.039662 |
|                  | CPSO1     | 1.00         | 86.60                  | 2598.00                    | 0.087385 |
|                  | CPSO2     | 1.00         | 118.10                 | 3543.00                    | 0.122022 |
|                  | AA-PSWV   | 1.00         | 4.80                   | 144.00                     | 0.006088 |

**Table 13.** The convergence for the  $F_4$  Schaffers function.

| No. of Particles | Alg. Ver. | Success Rate | Avg. No. of Iterations | Ex. No. of Fn. Evaluations | CPU Time |
|------------------|-----------|--------------|------------------------|----------------------------|----------|
| 5                | PSWV1     | 1.00         | 47.90                  | 239.50                     | 0.011015 |
|                  | PSWV2     | 1.00         | 100.85                 | 504.25                     | 0.021491 |
|                  | PSO1      | 0.10         | 417.50                 | 2087.50                    | 0.396452 |
|                  | PSO2      | 0.15         | 549.33                 | 2746.67                    | 0.388932 |
|                  | MPSO1     | 0.00         | 2000.00                | 10,000.00                  | 0.437782 |
|                  | MPSO2     | 0.00         | 2000.00                | 10,000.00                  | 0.444153 |
|                  | CPSO1     | 1.00         | 349.55                 | 1747.75                    | 0.076902 |
|                  | CPSO2     | 0.95         | 341.32                 | 1706.58                    | 0.092889 |
|                  | AA-PSWV   | 1.00         | 19.10                  | 95.50                      | 0.004812 |
| 10               | PSWV1     | 1.00         | 23.60                  | 236.00                     | 0.008156 |
|                  | PSWV2     | 1.00         | 25.40                  | 254.00                     | 0.008492 |
|                  | PSO1      | 0.15         | 754.33                 | 7543.33                    | 0.580416 |
|                  | PSO2      | 0.15         | 377.67                 | 3776.67                    | 0.564198 |
|                  | MPSO1     | 0.10         | 49.50                  | 495.00                     | 0.597339 |
|                  | MPSO2     | 0.30         | 272.50                 | 2725.00                    | 0.482878 |
|                  | CPSO1     | 1.00         | 170.10                 | 1701.00                    | 0.054479 |
|                  | CPSO2     | 1.00         | 277.25                 | 2772.50                    | 0.088487 |
|                  | AA-PSWV   | 1.00         | 12.90                  | 129.00                     | 0.004493 |
| 15               | PSWV1     | 1.00         | 19.25                  | 288.75                     | 0.008415 |
|                  | PSWV2     | 1.00         | 28.10                  | 421.50                     | 0.012059 |
|                  | PSO1      | 0.30         | 542.00                 | 8130.00                    | 0.652370 |
|                  | PSO2      | 0.40         | 603.13                 | 9046.88                    | 0.605859 |
|                  | MPSO1     | 0.20         | 69.25                  | 1038.75                    | 0.691719 |
|                  | MPSO2     | 0.25         | 118.40                 | 1776.00                    | 0.650918 |
|                  | CPSO1     | 1.00         | 127.95                 | 1919.25                    | 0.054582 |
|                  | CPSO2     | 1.00         | 219.45                 | 3291.75                    | 0.092138 |
|                  | AA-PSWV   | 1.00         | 13.35                  | 200.25                     | 0.006191 |
| 20               | PSWV1     | 1.00         | 15.05                  | 301.00                     | 0.008403 |
|                  | PSWV2     | 1.00         | 17.85                  | 357.00                     | 0.009993 |
|                  | PSO1      | 0.45         | 280.44                 | 5608.89                    | 0.647572 |
|                  | PSO2      | 0.45         | 476.44                 | 9528.89                    | 0.702622 |
|                  | MPSO1     | 0.10         | 104.50                 | 2090.00                    | 0.952918 |
|                  | MPSO2     | 0.35         | 202.86                 | 4057.14                    | 0.726957 |
|                  | CPSO1     | 1.00         | 137.40                 | 2748.00                    | 0.071993 |
|                  | CPSO2     | 1.00         | 227.55                 | 4551.00                    | 0.119085 |
|                  | AA-PSWV   | 1.00         | 11.35                  | 227.00                     | 0.006532 |
| 30               | PSWV1     | 1.00         | 11.90                  | 357.00                     | 0.009439 |
|                  | PSWV2     | 1.00         | 14.80                  | 444.00                     | 0.011543 |
|                  | PSO1      | 0.30         | 247.33                 | 7420.00                    | 1.132836 |
|                  | PSO2      | 0.70         | 297.86                 | 8935.71                    | 0.590960 |
|                  | MPSO1     | 0.50         | 182.10                 | 5463.00                    | 0.804520 |
|                  | MPSO2     | 0.40         | 163.00                 | 4890.00                    | 0.927614 |
|                  | CPSO1     | 1.00         | 160.90                 | 4827.00                    | 0.116947 |
|                  | CPSO2     | 1.00         | 120.70                 | 3621.00                    | 0.088102 |
|                  | AA-PSWV   | 1.00         | 9.60                   | 288.00                     | 0.007911 |

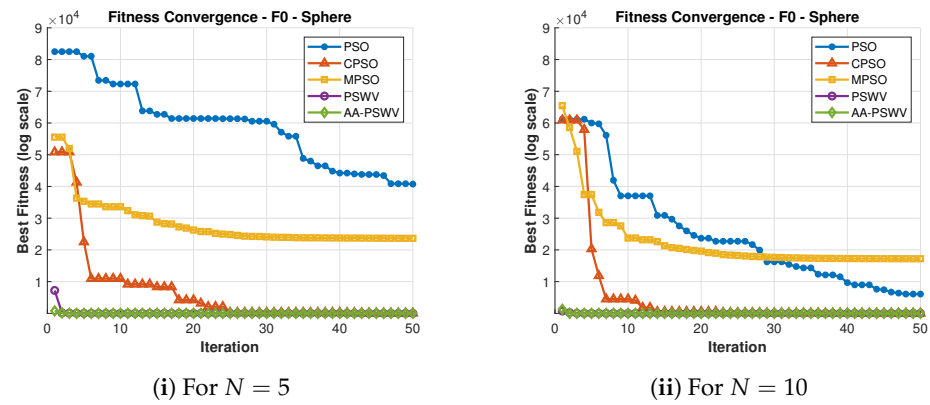


Figure 1. Comparisons of fitness for the Sphere function  $F_0$ .

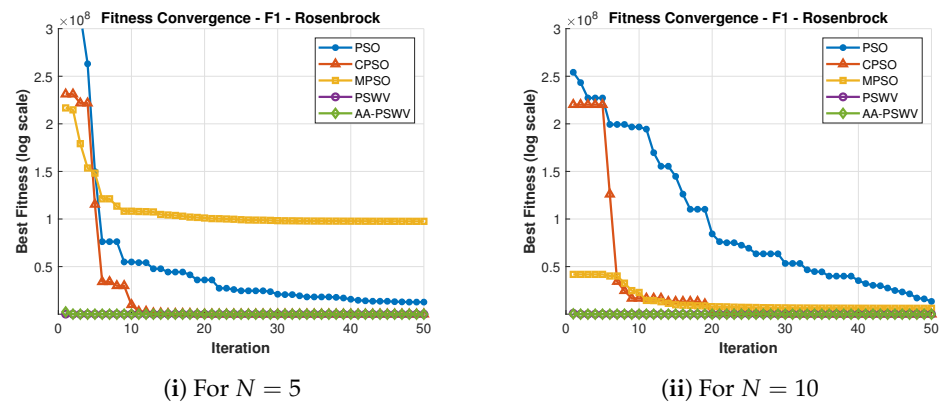


Figure 2. Comparisons of fitness for the Rosenbrock function  $F_1$ .

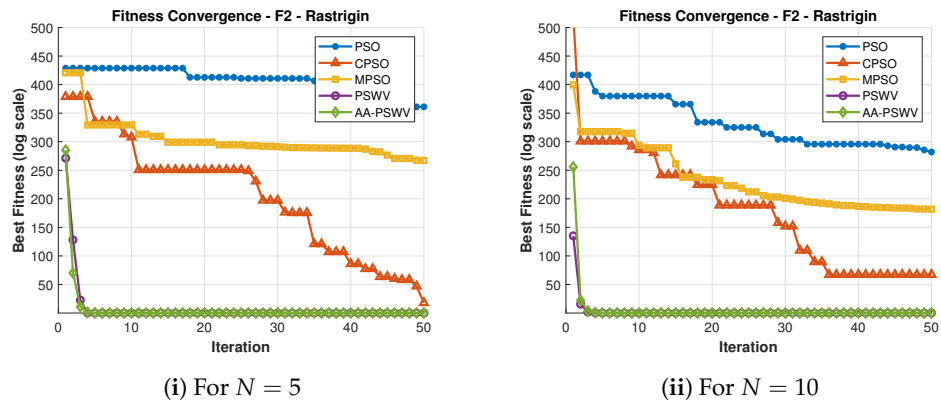


Figure 3. Comparisons of fitness for the Rastrigin function  $F_2$ .

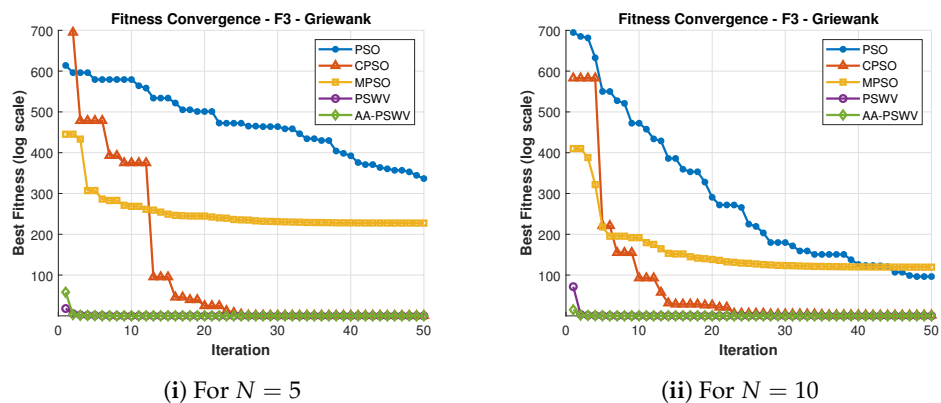
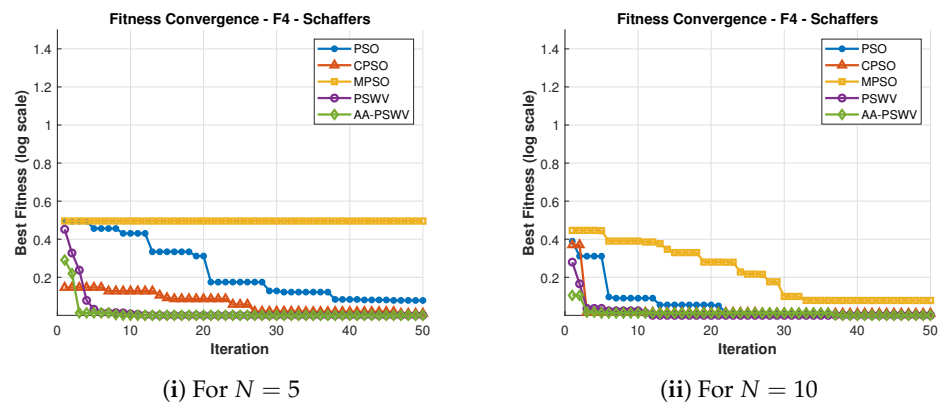


Figure 4. Comparisons of fitness for the Griewank function  $F_3$ .



**Figure 5.** Comparisons of fitness for the Schaffers function  $F_4$ .

#### Statistical Significance Analysis

To further verify whether the improvement achieved by the proposed AA-PSWV algorithm is statistically significant, a Wilcoxon signed-rank test was performed. Since swarm intelligence algorithms are stochastic in nature, each algorithm was executed independently for 30 runs under the same experimental settings. The significance level was fixed at  $\alpha = 0.05$ .

The null hypothesis states that there is no statistically significant difference between the performance of the proposed AA-PSWV algorithm and the PSWV algorithm. The obtained results are summarized in Table 14.

Since the obtained  $p$ -value is

$$p = 1.4078 \times 10^{-6} < 0.05,$$

the null hypothesis is rejected. Therefore, the proposed AA-PSWV algorithm achieves a statistically significant improvement over the PSWV algorithm. This confirms that the observed performance improvement is not due to random variation but is statistically supported.

**Table 14.** Wilcoxon signed-rank test comparing AA-PSWV and PSWV over 30 independent runs.

| Comparison      | Mean (AA-PSWV)        | Mean (PSWV)           | $p$ -Value              | Decision     |
|-----------------|-----------------------|-----------------------|-------------------------|--------------|
| AA-PSWV vs PSWV | $1.11 \times 10^{-4}$ | $4.16 \times 10^{-4}$ | $1.4078 \times 10^{-6}$ | Reject $H_0$ |

## 4. Application

As an application, we solve the image segmentation problem using the standard Berkeley Segmentation Dataset and Benchmark [35] and RGB real images. In the segmentation process, each image is first converted into an RGB data matrix, where each row represents a pixel. Clustering is then applied to group similar pixels based on their RGB values. After clustering, each pixel is assigned a cluster label, and the image is reconstructed by assigning the same color to all pixels belonging to the same cluster.

In PSO-based optimization algorithms, each particle represents a set of candidate cluster centers, and the fitness function measures how well these centers partition the image into different regions. During the optimization process, the particles update their positions using their personal-best and global-best information to search for better cluster centers. Finally, the optimized cluster centers are used to segment the image into meaningful regions.

In this experiment, we use the population size  $N = 10$ , the number of clusters used in APSOF [20] algorithms is 2 as in [20] because of comparison, the image size is  $[256, 256]$ ,

the maximum iterations is 50,  $\omega = 0.7$ ,  $b_1 = \alpha_n = 0.3$ ,  $b_2 = \beta_n = 0.45$  and  $\gamma_n = 0.35$ . We compared the proposed algorithm AA-PSWV with PSO [7], APSOF [20] and (PSWV) [22].

In Figure 6, the first two rows show the segmentation results for the human face image. It can be observed that the highlighted region produced by the proposed AA-PSWV algorithm is more similar to the original image. Moreover, the background is segmented more clearly than that obtained by the other comparable methods. In the third and fourth rows, the number displayed on the train is more clearly visible in the highlighted region produced by AA-PSWV. Similarly, in the fifth and sixth rows, the logo on the wing of the aircraft is more recognizable in the segmentation result obtained by the proposed algorithm.

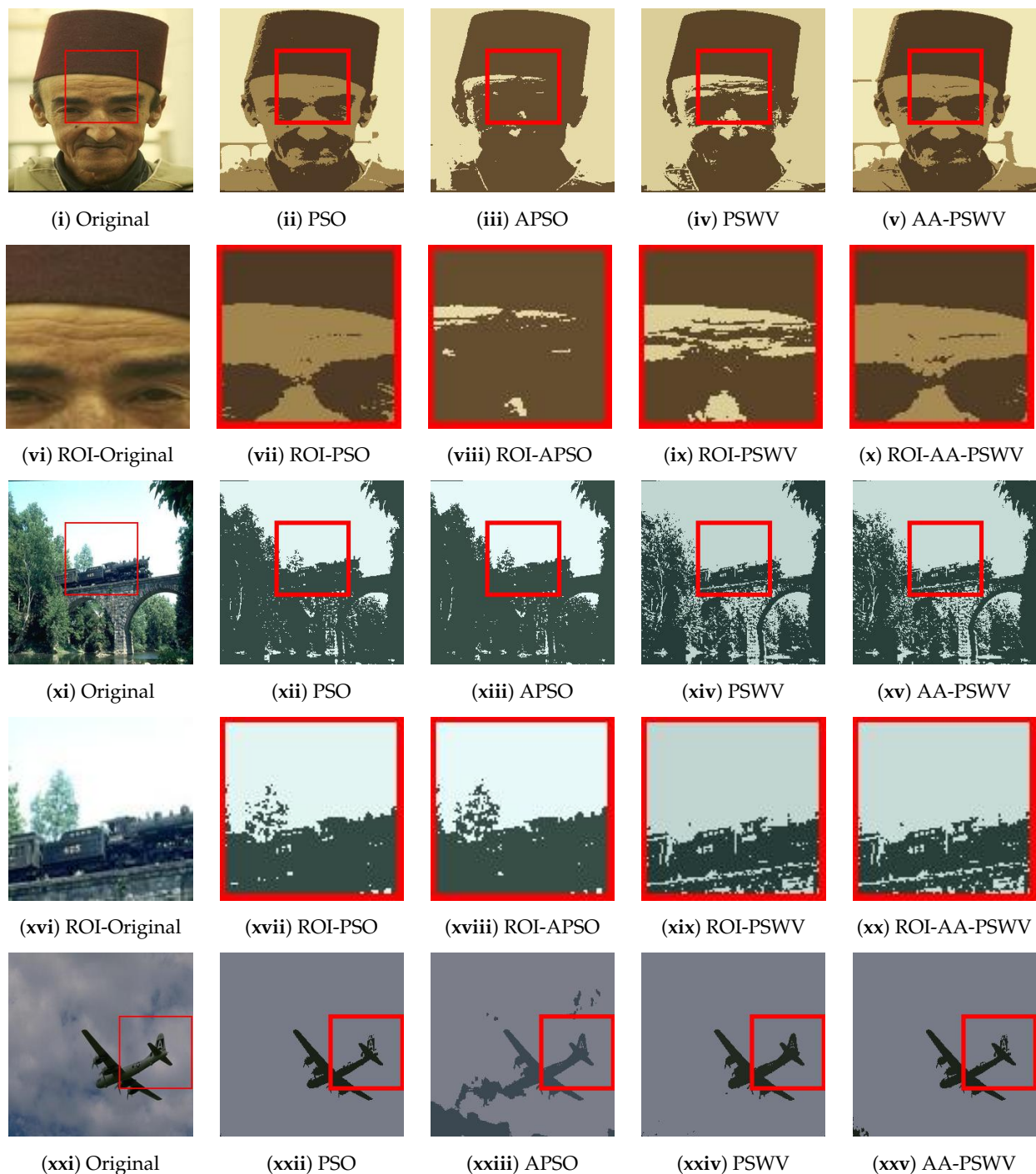
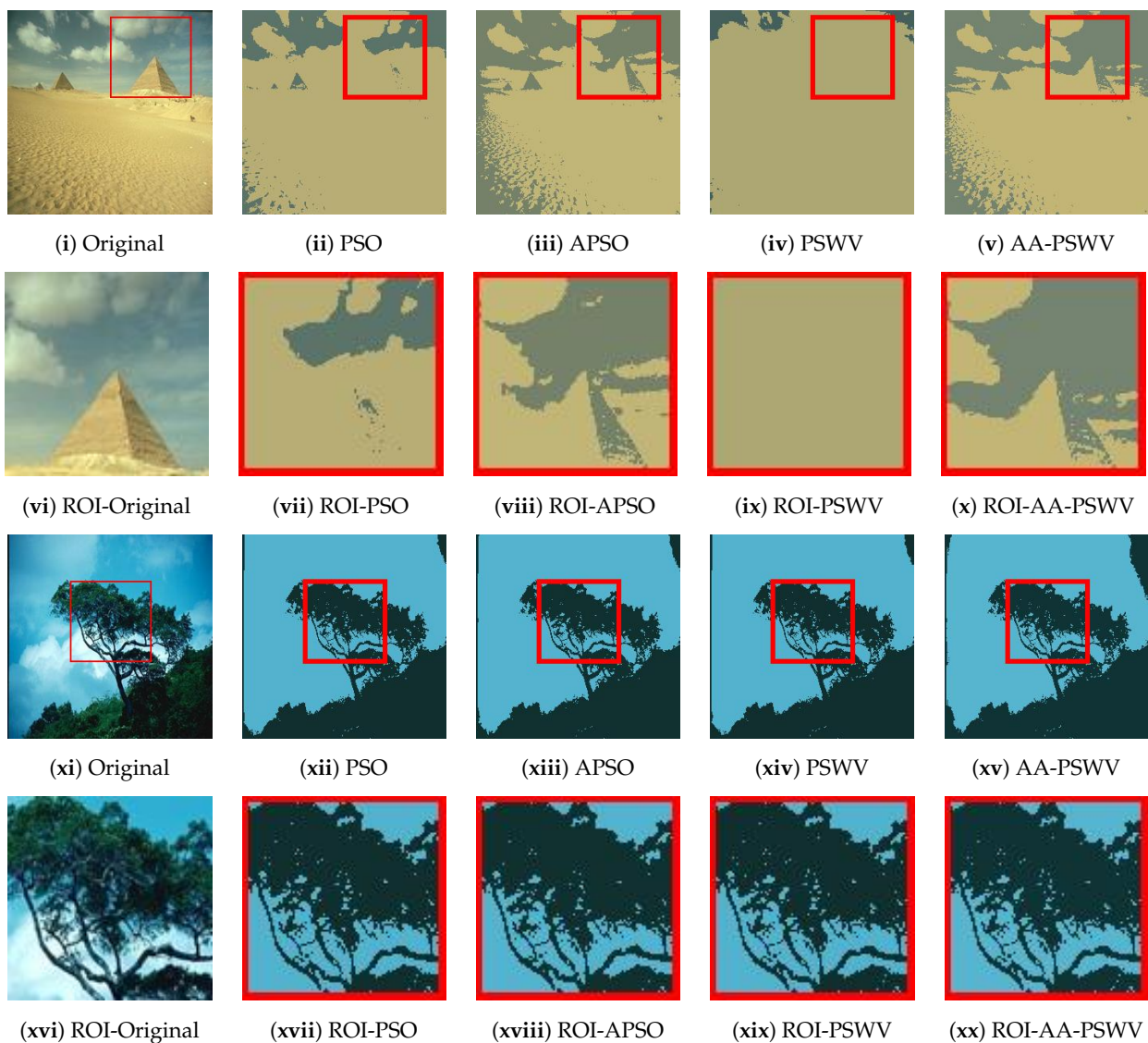


Figure 6. Cont.



**Figure 6.** Comparison of image segmentation results for human face, train and aircraft along with ROI after 30 iterations.

Now consider Figure 7. In the first two rows, the structure of the pyramid and the sky above it are more clearly preserved by the proposed AA-PSWV algorithm. Finally, in the last two rows, the background sky around the tree appears closer to the original image than the results produced by the other algorithms.



**Figure 7.** Comparison of image segmentation results for a pyramid and a tree along with ROI after 30 iterations.

**Remark 7.** Observe Figures 6 and 7 and note that when the segmented image is reconstructed by assigning distinct colors to each cluster, the proposed AA-PSWV performs well, and it yields more

*visually accurate segmentation results as compared to PSO, APSOF and PSWV. This shows the ability of AA-PSWV to handle complex image structures.*

#### *Limitation and Future Work*

The image segmentation experiments presented in this work are intended to demonstrate the practical applicability of the proposed AA-PSWV algorithm as an optimization technique. The evaluation is primarily based on visual quality and optimization performance. Although quantitative segmentation metrics such as the Dice coefficient, Jaccard index, Rand index, Variation of Information, and Boundary Displacement Error can provide further validation, their computation requires reliable ground-truth segmentations for all test images and additional experimental analysis. Incorporating such a comprehensive evaluation is beyond the scope of the present work and will be considered as an important direction for future research.

## 5. Conclusions

In this paper, we proposed a Particle Swarm Optimization algorithm equipped with a velocity-like behavior using only the p-best and g-best in each iteration, while excluding the explicit velocity vector. By Remarks 5 and 6, the proposed AA-PSWV algorithm is shown to be more efficient and applicable than other Particle Swarm variants for optimizing the benchmark functions in Table 3, with a reduced chance of premature convergence. Since updating the swarm particle with velocity is often computationally expensive and may lead to local optima, the proposed approach provides a simpler and more effective alternative. In contrast, in the PSWV algorithm [22], the estimated directions toward the optimum solution are unknown, which may also result in local convergence. The proposed AA-PSWV thus offers a better option compared with existing methods such as PSO [32], CPSO [19], APSOF [20], MPSO [21], and PSWV [22]. As an application, we applied our algorithm to the image segmentation problem, where Remark 7 demonstrates its effectiveness and efficiency against comparable algorithms.

**Author Contributions:** M.W.A. prepared the first draft, performed the experiments and coding, and M.A. analyzed the results, reviewed and revised the manuscript. All authors have read and agreed to the published version of the manuscript.

**Funding:** This article has been supported by the Faculty of Engineering and the Built Environment, University of Johannesburg, South Africa.

**Data Availability Statement:** The image datasets analyzed during the current study are publicly available in the Berkeley Segmentation Dataset (BSDS500/BSDS300), specifically in the “100 test color images” section, available at the following link: <https://www2.eecs.berkeley.edu/Research/Projects/CS/vision/bsds/BSDS300/html/dataset/images.html>, generated on 31 October 2003.

**Acknowledgments:** All authors are thankful to the reviewers for their useful comments, which helped to improve the presentation of the paper.

**Conflicts of Interest:** The authors declare no conflicts of interest.

## References

1. Bianco, S.; Cadene, R.; Celona, L.; Napoletano, P. Benchmark analysis of representative deep neural network architectures. *IEEE Access* **2018**, *6*, 64270–64277.
2. Caicedo, J.C.; Goodman, A.; Karhohs, K.W.; Cimini, B.A.; Ackerman, J.; Haghighi, M.; Heng, C.; Becker, T.; Doan, M.; McQuin, C. Nucleus segmentation across imaging experiments: The 2018 Data Science Bowl. *Nat. Methods* **2019**, *16*, 1247–1253.
3. Deng, J.; Guo, J.; Ververas, E.; Kotsia, I.; Zafeiriou, S. RetinaFace: Single-shot multi-level face localisation in the wild. In Proceedings of the IEEE/CVF Conference on Computer Vision and Pattern Recognition, Seattle, WA, USA, 13–19 June 2020; pp. 5203–5212.

4. Ghosh, A.; Das, M.K.; Das, R. Semantic segmentation using deep learning in autonomous driving: Recent trends and future outlook. *IEEE Access* **2021**, *9*, 108783–108796.
5. Razzak, M.I.; Naz, S.; Zaib, A. Deep learning for medical image processing: Overview, challenges and the future. In *Classification in BioApps: Automation of Decision Making*; Springer: Cham, Switzerland, 2017; pp. 323–350.
6. Zhong, L.; Hu, L.; Zhou, H. Deep learning based multi-temporal crop classification. *Remote Sens. Environ.* **2019**, *221*, 430–443.
7. Eberhart, R.; Kennedy, J. A new optimizer using particle swarm theory. In *Proceedings of the Sixth International Symposium on Micro Machine and Human Science (MHS'95)*; IEEE: Piscataway, NJ, USA, 1995; pp. 39–43.
8. Wang, H.; Qian, F. An improved particle swarm optimizer with behavior-distance models and its application in soft-sensor. In *Proceedings of the 7th World Congress on Intelligent Control and Automation*; IEEE: Piscataway, NJ, USA, 2008; pp. 4473–4478.
9. Oo, N.W. A comparison study on particle swarm and evolutionary particle swarm optimization using capacitor placement problem. In *Proceedings of the IEEE 2nd International Power and Energy Conference*; IEEE: Piscataway, NJ, USA, 2008; pp. 1208–1211.
10. El-Dib, A.A.; Youssef, H.K.M.; El-Metwally, M.M.; Osman, Z. Maximum loadability of power systems using hybrid particle swarm optimization. *Electr. Power Syst. Res.* **2006**, *76*, 485–492.
11. Naka, S.; Genji, T.; Yura, T.; Fukuyama, Y. A hybrid particle swarm optimization for distribution state estimation. *IEEE Trans. Power Syst.* **2003**, *18*, 60–68.
12. Wang, B.; Tai, N.-L.; Zhai, H.-Q.; Ye, J.; Zhu, J.-D.; Qi, L.-B. A new ARMAX model based on evolutionary algorithm and particle swarm optimization for short-term load forecasting. *Electr. Power Syst. Res.* **2008**, *78*, 1679–1685.
13. Tasgetiren, M.F.; Liang, Y.-C.; Sevkli, M.; Gencyilmaz, G. A particle swarm optimization algorithm for makespan and total flowtime minimization in the permutation flowshop sequencing problem. *Eur. J. Oper. Res.* **2007**, *177*, 1930–1947.
14. Liu, B.; Wang, L.; Jin, Y.-H.; Tang, F.; Huang, D.-X. Directing orbits of chaotic systems by particle swarm optimization. *Chaos Solitons Fractals* **2006**, *29*, 454–461.
15. Jiang, Y.; Yue, Q. An improved particle swarm optimization with new select mechanism. In *Proceedings of the First International Workshop on Knowledge Discovery and Data Mining (WKDD 2008)*; IEEE: Piscataway, NJ, USA, 2008; pp. 383–386.
16. Liu, J.; Fan, X.; Qu, Z. An improved particle swarm optimization with mutation based on similarity. In *Proceedings of the Third International Conference on Natural Computation (ICNC 2007)*, Haikou, China, 24–27 August 2007; IEEE: Piscataway, NJ, USA, 2007; pp. 824–828.
17. Wan, L.-Y.; Li, W. An improved particle swarm optimization algorithm with rank-based selection. In *Proceedings of the 2008 International Conference on Machine Learning and Cybernetics*; IEEE: Piscataway, NJ, USA, 2008; pp. 4090–4095.
18. Wang, H.; Liu, Y.; Wu, Z.; Sun, H.; Zeng, S.; Kang, L. An improved particle swarm optimization with adaptive jumps. In *Proceedings of the 2008 IEEE Congress on Evolutionary Computation (IEEE World Congress on Computational Intelligence)*; IEEE: Piscataway, NJ, USA, 2008; pp. 392–397.
19. El-Sherbiny, M.M. A modified algorithm for particle swarm optimization with constriction coefficient. *IJCI Int. J. Comput. Inf.* **2009**, *2*, 17–29.
20. Reddy, A.N.; Das, R. Color image segmentation using adaptive particle swarm optimization and fuzzy C-means. *arXiv* **2020**, arXiv:2004.
21. El-Sherbiny, M.M. A combined particle swarm optimization algorithm based on the previous global best and the global best positions. *IJCI Int. J. Comput. Inf.* **2007**, *1*, 11–22.
22. El-Sherbiny, M.M. Particle swarm inspired optimization algorithm without velocity equation. *Egypt. Inform. J.* **2011**, *12*, 1–8.
23. Kennedy, J.; Eberhart, R. Particle swarm optimization. In *Proceedings of ICNN'95—International Conference on Neural Networks*; IEEE: Piscataway, NJ, USA, 1995; Volume 4, pp. 1942–1948.
24. Clerc, M.; Kennedy, J. The particle swarm—Explosion, stability, and convergence in a multidimensional complex space. *IEEE Trans. Evol. Comput.* **2002**, *6*, 58–73.
25. Kennedy, J. Bare bones particle swarms. In *Proceedings of the 2003 IEEE Swarm Intelligence Symposium. SIS'03 (Cat. No. 03EX706)*; IEEE: Piscataway, NJ, USA, 2003; pp. 80–87.
26. Ang, K.M.; Lim, W.H.; Isa, N.A.M.; Tiang, S.S.; Wong, C.H. A constrained multi-swarm particle swarm optimization without velocity for constrained optimization problems. *Expert Syst. Appl.* **2020**, *140*, 112882.
27. Guo, J.; Zhou, G.; Yan, K.; Di Y.; Sato, Y.; He, Z.; Shi, B. A deep backtracking bare-bones particle swarm optimisation algorithm for high-dimensional nonlinear functions. *CAAI Trans. Intell. Technol.* **2025**, *10*, 1501–1520.
28. Solak, A.; Onat, A.; Kilinc, O. An improved bare bones particle swarm optimization algorithm based on sequential update mechanism and a modified structure. *IEEE Access* **2025**, *13*, 4789–4814.
29. Sun, Y.; Guo, J.; Yan, K.; Di Y.; Pan, C.; Shi, B.; Sato, Y. A deep memory bare-bones particle swarm optimization algorithm for single-objective optimization problems. *PloS ONE* **2023**, *18*, e0284170.
30. Gad, A.G. Particle swarm optimization algorithm and its applications: A systematic review. *Arch. Comput. Methods Eng.* **2022**, *29*, 2531–2561.
31. Boyd, S.; Vandenberghe, L. *Convex Optimization*; Cambridge University Press: Cambridge, UK, 2004.

32. Trelea, I.C. The particle swarm optimization algorithm: Convergence analysis and parameter selection. *Inf. Process. Lett.* **2003**, *85*, 317–325.
33. Boeringer, D.W.; Werner, D.H. Particle swarm optimization versus genetic algorithms for phased array synthesis. *IEEE Trans. Antennas Propag.* **2004**, *52*, 771–779.
34. Gazi, V.; Passino, K.M. Stability analysis of social foraging swarms. *IEEE Trans. Syst. Man Cybern. Part (Cybern.)* **2004**, *34*, 539–557.
35. Martin, D.; Fowlkes, C.; Tal, D.; Malik, J. A database of human segmented natural images and its application to evaluating segmentation algorithms and measuring ecological statistics. In *Proceedings of the Eighth IEEE International Conference on Computer Vision (ICCV 2001)*; IEEE: Piscataway, NJ, USA, 2001; pp. 416–423.

**Disclaimer/Publisher’s Note:** The statements, opinions and data contained in all publications are solely those of the individual author(s) and contributor(s) and not of MDPI and/or the editor(s). MDPI and/or the editor(s) disclaim responsibility for any injury to people or property resulting from any ideas, methods, instructions or products referred to in the content.