

Article

Inforpower: Quantifying the Informational Power of Probability Distributions

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Abstract

In many scientific and engineering fields (e.g., measurement science), a probability density function often models a system comprising a signal embedded in noise. Conventional measures, such as the mean, variance, entropy, and informity, characterize signal strength and uncertainty (or noise level) separately. However, the true performance of a system depends on the interaction between signal and noise. In this paper, we propose a novel measure, called “inforpower”, for quantifying the system’s informational power that explicitly captures the interaction between signal and noise. We also propose a new measure of central tendency, called “information-energy center”. Closed-form expressions for inforpower and information-energy center are provided for ten well known continuous distributions. Moreover, we propose a maximum inforpower criterion, which can complement the Akaike information criterion (AIC), the minimum entropy criterion, and the maximum informity criterion for selecting the best distribution from a set of candidate distributions. Two examples (synthetic Weibull distribution data and Tana River annual maximum streamflow) are presented to demonstrate the effectiveness of the proposed maximum inforpower criterion and compare it with existing goodness-of-fit criteria.

Keywords: informity; information-energy center; informational power; model selection; signal–noise interaction

1. Introduction

Consider a continuous random variable Y with probability density function (PDF) $p(Y)$. The mean of Y is

$$\mu_M = \mathbb{E}_p[Y] = \int yp(y)dy, \quad (1)$$

which is the expected value under $p(Y)$. That is, μ_M is the weighted average of all possible values the random variable Y can take, where the weights are determined by the PDF $p(Y)$. It is often interpreted as the center of mass of the distribution.

The variance of Y is

$$\text{Var}(Y) = \mathbb{E}_p[(Y - \mu_M)^2] = \int (y - \mu_M)^2 p(y)dy = \mathbb{E}_p[Y^2] - \mu_M^2, \quad (2)$$

which quantifies the spread or dispersion of Y about its mean.

In information theory, Shannon entropy measures the uncertainty represented by a probability distribution. The continuous (i.e., differential) entropy is defined as [1].

$$H(p) = - \int p(y) \log p(y) dy = -\mathbb{E}_p[\log p(Y)]. \quad (3)$$



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Recently, Huang [2] introduced informity theory, in which, informity measures the certainty represented by a probability distribution and acts as the complementary dual of entropy [3]. The continuous informity is defined as [2].

$$\beta(p) = \int p(y)^2 dy = \int [p(y)]p(y) dy = \mathbb{E}_p[p(Y)]. \quad (4)$$

In many practical applications, a PDF describes a system comprising a signal component and a noise component. The mean μ_M naturally quantifies signal strength (or systematic effect), whereas variance $\text{Var}(Y)$, entropy $H(p)$, and informity $\beta(p)$ each characterize different aspects of noise or uncertainty. However, these measures have limitations for assessing the overall performance or informational power of a system. Mean is outlier-sensitive and undefined for some distributions (e.g., Cauchy distribution); variance, entropy, and informity are translation-invariant, ignoring signal magnitude shifts. Importantly, although these measures separately quantify signal strength and noise level (or uncertainty), they do not capture their interaction, which is often critical for assessing the overall performance or informational power of a system.

This paper introduces a new measure termed inforpower that explicitly incorporates the interplay between signal and noise, thereby providing a more comprehensive quantification of a distribution's informational power. The remainder of the paper is organized as follows: Section 2 formally defines inforpower; Section 3 interprets its meaning and properties; Section 4 presents inforpower for ten well known continuous distributions; Section 5 develops an inforpower-based metric and illustrates its use in practical applications; Section 6 offers discussion and conclusions.

2. Definition and Basic Properties of Inforpower

From an information-theoretic perspective [4], a realized value y of a continuous random variable represents information (signal), while the probability density $p(y)$ quantifies the certainty (inverse of local noise or uncertainty) associated with that value. The product $yp(y)$ therefore captures effective information after accounting for certainty weighting.

Definition 1 (Effective information). *Let Z be the random variable representing effective information*

$$Z = Yp(Y). \quad (5)$$

For any realized value y , the corresponding effective information is $z = yp(y)$.

Definition 2 (Inforpower). *Let ϕ denote inforpower of a continuous probability distribution. It is defined as the expected value of the effective information Z*

$$\phi = \mathbb{E}_p[Z] = \int zp(y)dy = \int yp(y)^2 dy. \quad (6)$$

The definition of inforpower reflects the interaction between signal and noise. The product $yp(y)$ is the signal y weighted by its local certainty $p(y)$, and squaring $p(y)$ quadratically amplifies contributions of high-certainty (low-noise) regions. Thus, inforpower ϕ directly measures the effective signal strength after noise interference: it rewards distributions where strong signals (large $|y|$) coincide with high certainty (large $p(y)$), while penalizing cases where signal is diluted by uncertainty or spread across low-certainty regions. In this way, ϕ explicitly captures the interaction between signal magnitude and noise level.

The following theorems reveal important structural properties of inforpower.

Theorem 1. If $p(Y)$ is a location-scale family of distributions and is symmetric about its location parameter a (e.g., the mean μ or the location of symmetry), then,

$$\phi = a\beta(p), \quad (7)$$

where $\beta(p) = \int p(y)^2 dy$ is the continuous informity.

Proof of Theorem 1. The PDF for a generalized location-scale family of distributions is

$$p(y) = \frac{1}{b} f\left(\frac{y-a}{b}\right), \quad b > 0, \quad (8)$$

where a is the location parameter; b is the scale parameter.

Its inforpower is

$$\phi = \int y p(y)^2 dy = \int y \frac{1}{b^2} f\left(\frac{y-a}{b}\right)^2 dy. \quad (9)$$

Make the change in variable $u = (y-a)/b$, then $dy = bdu$. Substituting yields

$$\phi = \frac{1}{b} \int (bu+a) f(u)^2 du = \int u f(u)^2 du + \frac{a}{b} \int f(u)^2 du. \quad (10)$$

When the distribution is symmetric about a , the standardized density $f(u)$ is symmetric about zero. Consequently, $f(u)^2$ is even, and $uf(u)^2$ is odd, so

$$\int u f(u)^2 du = 0. \quad (11)$$

Thus,

$$\phi = \frac{a}{b} \int f(u)^2 du = a \int p(y)^2 dy = a\beta(p). \quad (12)$$

This establishes Theorem 1. $\square$

Theorem 2. If $p(Y)$ is a scale-shape family of distributions supported on $[y_0, \infty)$, where $y_0 \geq 0$, then ϕ is scale-invariant.

Proof of Theorem 2. The PDF of a generalized scale-shape family of non-negative distributions is

$$p(y) = \frac{1}{b} f\left(\frac{y}{b}, c\right), \quad b > 0, \quad (13)$$

where c is the shape parameter.

Its inforpower is

$$\phi = \int y p(y)^2 dy = \int y \frac{1}{b^2} f\left(\frac{y}{b}, c\right)^2 dy. \quad (14)$$

Make the change in variable $v = y/b$, then $dy = b dv$. Substituting yields

$$\phi = \int v f(v, c)^2 dv, \quad (15)$$

which depends only on the shape parameter c and is independent of the scale parameter b . If the distribution has no shape parameter (e.g., standard exponential distribution), ϕ is an absolute constant. This proves scale invariance and establishes Theorem 2. $\square$

3. Interpretation of Inforpower

3.1. Inforpower as a Measure of Informational Power

Inforpower ϕ is the expected value of the effective information $Z = Yp(Y)$. Therefore, it quantifies the informational power (effective signal or performance) of the system described by the distribution p . To illustrate this, consider the normal distribution $Y \sim N(\mu, \sigma^2)$. Its inforpower is

$$\phi = \frac{\mu}{2\sigma\sqrt{\pi}} \propto \frac{\mu}{\sigma}. \quad (16)$$

Here μ represents the signal strength and σ represents the noise level. Therefore, μ/σ is the classical signal-to-noise ratio (SNR), a standard performance metric in signal processing, detection theory, and measurement science. Thus, inforpower is proportional to SNR and directly measures the degree to which the original signal is preserved after being degraded by noise. In this sense, ϕ serves as a natural and distribution-based indicator of a system's informational power.

3.2. Inforpower as the Standardized Center of Information Energy

Inforpower admits an elegant energy-based reinterpretation. Rewrite ϕ as

$$\phi = \beta(p) \int y \frac{p(y)^2}{\beta(p)} dy = \beta(p) \int y \frac{p(y)^2}{\int p(y)^2 dy} dy. \quad (17)$$

Define the normalized function

$$g(y) = \frac{p(y)^2}{\int p(y)^2 dy} = \frac{p(y)^2}{\beta(p)}. \quad (18)$$

The function $g(y)$ is a valid probability density function, called the information-energy density function. This name is consistent with Onicescu [5], who named the quantity $\int p(y)^2 dy$ as the information energy.

In physical analogy, on one hand, $p(y)$ behaves like mass density (amplitude), $p(y)dy$ is the differential mass element, and the ordinary mean $\mu_M = \mathbb{E}_p(Y)$ is the center of mass. On the other hand, $p(y)^2$ behaves like energy density (energy $\propto$ amplitude²), and $g(y)dy$ is the differential information-energy element. Thus, the expected value of Y under $g(y)$ is the center of information energy

$$\mu_E = \mathbb{E}_g(Y) = \int yg(y)dy = \frac{\phi}{\beta(p)}. \quad (19)$$

Consequently, inforpower can be expressed as the product of informity and information-energy center

$$\phi = \beta(p)\mu_E. \quad (20)$$

Dimensional analysis confirms the following consistency: $\beta(p)$ has dimension $[y]^{-1}$, μ_E has dimension $[y]$, and their product ϕ is dimensionless, which matches its interpretation as a pure performance index. Thus, inforpower ϕ is the standardized center of information energy. This dual view (the SNR interpretation in Section 3.1 and the energy-center interpretation here) reinforces ϕ as a powerful, unified measure of informational power or performance of the system.

For symmetric distributions (e.g., normal, Laplace, logistic, and Cauchy distributions), $g(y)$ inherits the symmetry of $p(y)$, so the information-energy center μ_E coincides with the ordinary mean μ_M (mass center). For right-skewed distributions (e.g., exponential, gamma,

Weibull, and lognormal distributions), however, squaring the probability density function disproportionately amplifies the high-density peak (near the mode) while suppressing the long right tail. Consequently, μ_E shifts to the left, towards the region of higher probability density. This can be seen from the computations in Section 4 below. Thus, μ_E can be considered a new, robust measure of central tendency, representing the location where the signal is most reliably concentrated under the energy perspective.

The concept of information-energy center extends naturally to discrete distributions; details are provided in the Appendix A.

4. Inforpower for Ten Well-Known Continuous Distributions

Table 1 presents the PDF, infority $\beta(p)$, inforpower ϕ , ordinary mean (center of mass) μ_M , and center of information energy μ_E for ten widely used continuous distributions. The first four (normal, Laplace, logistic, and Cauchy) have support on $(-\infty, \infty)$. The next five (exponential, Gamma, Rayleigh, Weibull, and lognormal) have support on $[0, \infty)$. The Pareto distribution has support $[y_m, \infty)$, where $y_m > 0$ is the minimum possible value.

Table 1. The PDF, infority, inforpower, mass center (ordinary mean), and energy center of ten well known probability distributions.

Distribution	PDF	Infority $\beta(p)$	Inforpower ϕ	Mass Center $\mu_M = \mathbb{E}_p(Y)$	Energy Center $\mu_E = \mathbb{E}_g(Y)$
Normal	$\frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{y-\mu}{\sigma}\right)^2\right]$	$\frac{1}{2\sigma\sqrt{\pi}}$	$\frac{\mu}{2\sigma\sqrt{\pi}} = \mu\beta(p)$	μ	μ
Laplace	$\frac{1}{2b} \exp\left(-\frac{ y-\mu }{b}\right)$	$\frac{1}{4b}$	$\frac{\mu}{4b} = \mu\beta(p)$	μ	μ
Logistic	$\frac{\exp\left(-\frac{y-\mu}{s}\right)}{s[1+\exp\left(-\frac{y-\mu}{s}\right)]^2}$	$\frac{1}{6s}$	$\frac{\mu}{6s} = \mu\beta(p)$	μ	μ
Cauchy	$\frac{1}{\pi\gamma\left[1+\frac{1}{\gamma^2}(y-y_0)^2\right]}$	$\frac{1}{2\pi\gamma}$	$\frac{y_0}{2\pi\gamma} = y_0\beta(p)$	undefined	y_0
Exponential	$\lambda\exp(-\lambda y)$	$\frac{\lambda}{2}$	$\frac{1}{4}$	$\frac{1}{\lambda}$	$\frac{1}{2\lambda}$
Gamma	$\frac{\lambda^\alpha}{\Gamma(\alpha)} y^{\alpha-1} \exp(-\lambda y)$	$\frac{\lambda\Gamma(2\alpha-1)}{2^{(2\alpha-1)}\Gamma(\alpha)^2}$	$\frac{\Gamma(2\alpha)}{2^{2\alpha}\Gamma(\alpha)^2}$	$\frac{\alpha}{\lambda}$	$\frac{\Gamma(2\alpha)}{2\lambda\Gamma(2\alpha-1)}$
Rayleigh	$\frac{y}{\sigma^2} \exp\left(-\frac{y^2}{2\sigma^2}\right)$	$\frac{\sqrt{\pi}}{4\sigma}$	$\frac{1}{2}$	$\sigma\sqrt{\frac{\pi}{2}}$	$\frac{2\sigma}{\sqrt{\pi}}$
Weibull	$\frac{k}{\lambda^k} y^{k-1} \exp\left(-\frac{y^k}{\lambda^k}\right)$	$\frac{k}{\lambda} 2^{-\frac{2k-1}{k}} \Gamma\left(\frac{2k-1}{k}\right)$	$\frac{k}{4}$	$\lambda\Gamma\left(1+\frac{1}{k}\right)$	$\frac{2^{\frac{2k-1}{k}}\lambda}{4\Gamma\left(\frac{2k-1}{k}\right)}$
Lognormal	$\frac{1}{y\sigma\sqrt{2\pi}} \exp\left(-\frac{(\ln y - \mu)^2}{2\sigma^2}\right)$	$\frac{1}{2\sigma\sqrt{\pi}} \exp\left(-\mu + \frac{\sigma^2}{4}\right)$	$\frac{1}{2\sigma\sqrt{\pi}}$	$\exp\left(\mu + \frac{\sigma^2}{2}\right)$	$\exp\left(\mu - \frac{\sigma^2}{4}\right)$
Pareto	$\begin{cases} \frac{\alpha y_m^\alpha}{y^{\alpha+1}} & y \geq y_m \\ 0 & y < y_m \end{cases}$	$\frac{\alpha^2}{2\alpha+1} \frac{1}{y_m}$	$\frac{\alpha}{2}$	$\begin{cases} \infty & \alpha \leq 1 \\ \frac{\alpha y_m}{\alpha-1} & \alpha > 1 \end{cases}$	$\frac{2\alpha+1}{2\alpha} y_m$

As Table 1 shows, for the symmetric normal, Laplace, and logistic distributions, $\phi = \mu\beta(p)$. For the symmetric Cauchy distribution (location parameter y_0), $\phi = y_0\beta(p)$. These results confirm Theorem 1: when the distribution is symmetric about its location parameter a , inforpower reduces to $\phi = a\beta(p)$.

For the remaining six distributions, which are asymmetric and supported on $[0, \infty)$ or $[y_m, \infty)$, the expressions for ϕ contain no scale parameter. Specifically, for the exponential and Rayleigh distributions (one-parameter families with fixed shape), ϕ is a numerical constant; for the Gamma, Weibull, lognormal, and Pareto distributions, ϕ depends solely on the shape parameter and is invariant to scale parameter. These observations fully confirm Theorem 2.

Figures 1–5 display, for the exponential, Gamma, Rayleigh, Weibull, and lognormal distributions, respectively, both the probability density function $p(y)$ and the corresponding

information-energy density function $g(y) = p(y)^2 / \beta(p)$. The figures also mark the mode, median, ordinary mean μ_M (center of mass), and energy center μ_E .

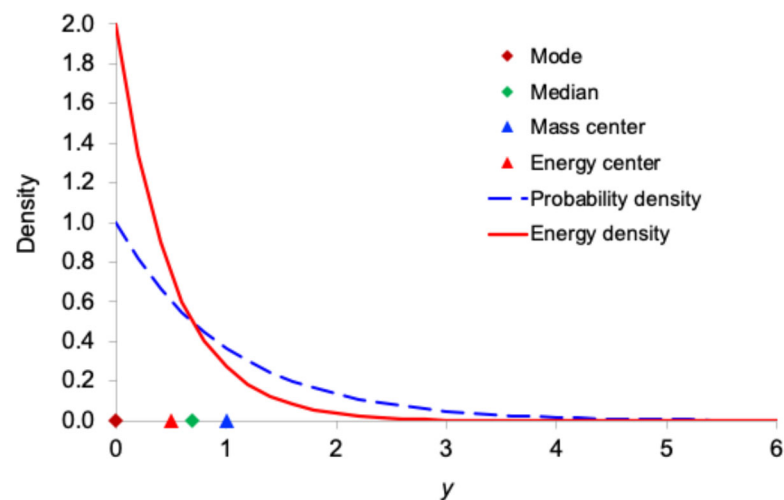


Figure 1. Comparison of the probability density function and the information-energy density function for the exponential distribution with $\lambda = 1$.

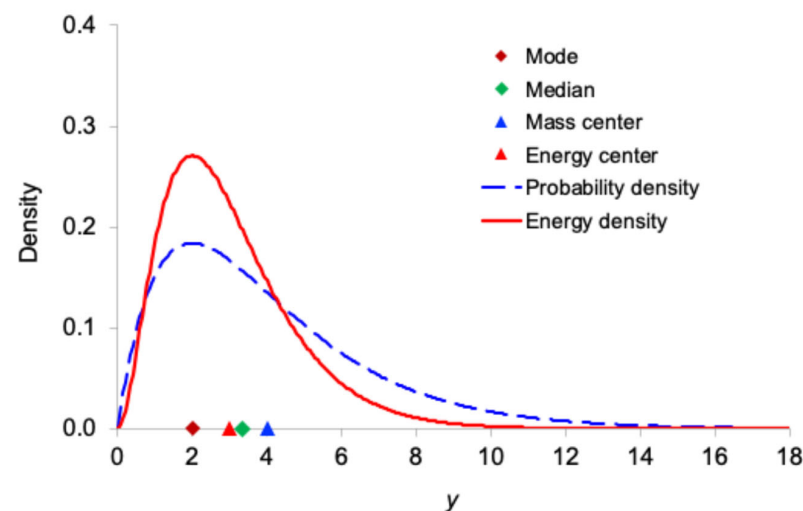


Figure 2. Comparison of the probability density function and the information-energy density function for the Gamma distribution with $\alpha = 2$ and $\lambda = 0.5$.

As expected, the information-energy density function $g(y)$ is visibly sharper and more concentrated around regions of high probability density than $p(y)$ itself, because squaring the density amplifies peaks and suppresses tails.

In all five right-skewed distributions, the locations satisfy the following consistent ordering:

$$\text{mode} < \mu_E(\text{energy center}) < \text{median} < \mu_M(\text{mass center}).$$

This systematic shift demonstrates that the center of information energy μ_E lies closer to the region of highest certainty (the mode) than does the ordinary mean. This provides further intuitive support for interpreting μ_E as the effective, certainty-weighted signal location and ϕ as a robust measure of the system's informational power.

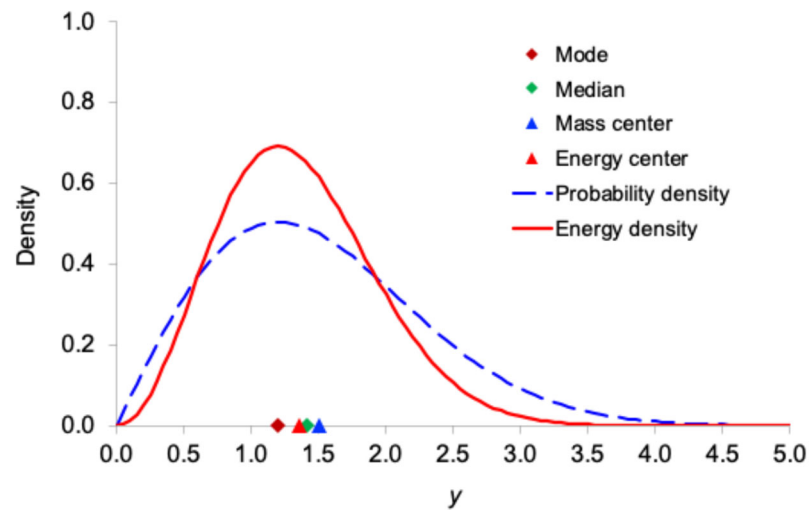


Figure 3. Comparison of the probability density function and the information-energy density function for the Rayleigh distribution with $\sigma = 1.2$.

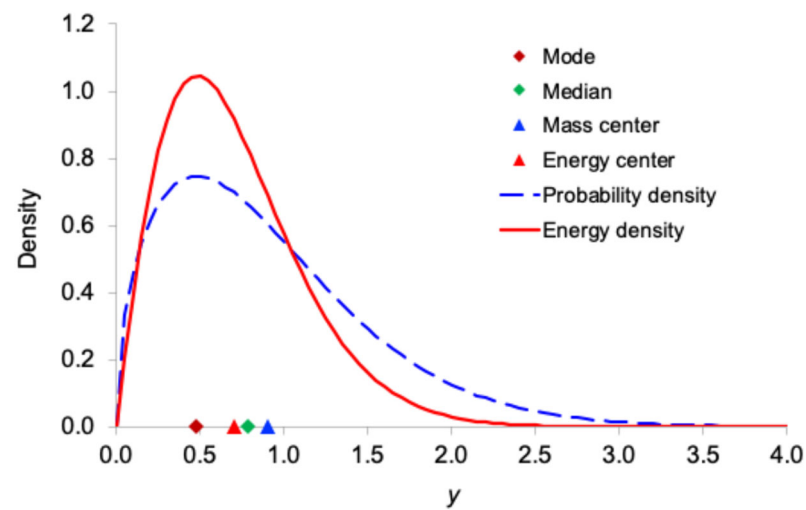


Figure 4. Comparison of the probability density function and the information-energy density function for the Weibull distribution with $k = 1.5$ and $\lambda = 1$.

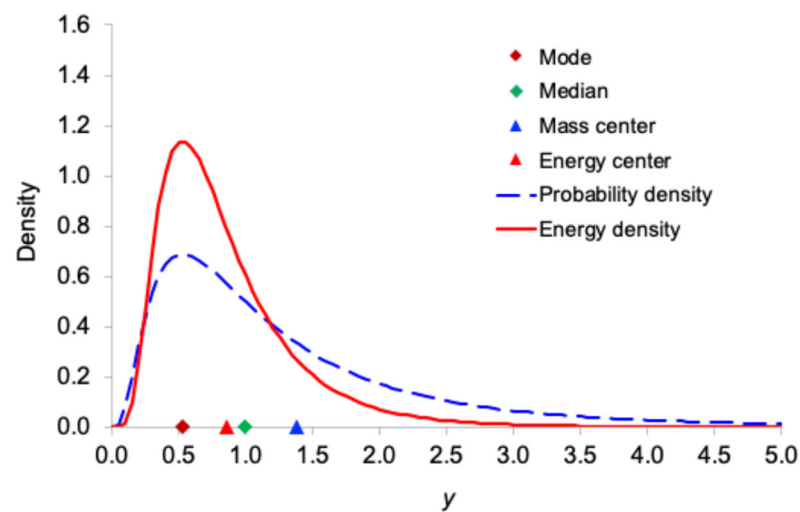


Figure 5. Comparison of the probability density function and the information-energy density function for the lognormal distribution with $\mu = 0$ and $\sigma = 0.8$.

5. The Inforpower Metric and Application Examples

Inforpower provides a metric to compare and select probability distributions that model the same data.

Maximum Inforpower Criterion. We propose the following model-selection rule, termed the maximum inforpower criterion: given a dataset and M candidate distributions $p_k(y)$ ($k = 1, \dots, M$) from the same family (i.e., sharing the same support), the preferred model is the one that maximizes inforpower

$$\text{best distribution} = \arg \max_k [\phi_k], \quad k = 1, 2, 3 \dots M, \quad (21)$$

where ϕ_k is the inforpower of the k -th candidate distribution. Here, the ‘preferred’ model or ‘best’ distribution is understood as the one providing the highest informational power under this criterion, analogous to minimizing AIC or entropy.

Example 1. *Synthetic Weibull distribution data.*

A sample of $n = 200$ observations was generated from a Weibull distribution with true parameters $k = 1.8$ and $\lambda = 1$ [6]. Xie [6] fitted exponential, Weibull, Gamma, and normal distributions and compared them using the Akaike information criterion (AIC) and Pearson’s Q-statistic. Huang [2,7] compared these candidate distributions using the maximum informity criterion and the minimum entropy criterion, respectively. Normal distribution was excluded from our analysis because its support differs from the other three (positive-support) candidates. The estimated parameters are as follows [6]: exponential distribution: $\hat{\lambda} = 1.167$; Weibull distribution: $\hat{k} = 1.662$ and $\hat{\lambda} = 0.9588$; and Gamma distribution: $\hat{\alpha} = 2.255$ and $\hat{\beta} = 2.6309$. Table 2 reports the performance of each distribution under five criteria.

Table 2. Evaluation of three candidate distributions using five metrics.

Distribution	AIC [6]	Q-Statistic [6]	Entropy (Nats) [7]	Informity [2]	Inforpower (This Study)
Exponential	340.3	48.1	0.846	0.5835	0.25
Weibull	276.6	7.07	0.680	0.5835	0.4155
Gamma	281.3	6.34	0.694	0.6010	0.4009

As shown in Table 2, the Weibull distribution ranks first under the AIC, minimum entropy, and maximum inforpower criteria, confirming that the true generating distribution is correctly identified by the inforpower metric. Although the Q-statistic and maximum informity criteria favor the Gamma distribution, Xie [6] noted that $\Delta\text{AIC} = 4.7$ between the Weibull and Gamma distributions is small and that the AIC “selects a good model which is not necessarily the true model.” Overall, the five criteria yield highly consistent conclusions.

Example 2. *Annual maximum streamflow of the Tana River, Kenya.*

Langat et al. [8] analyzed the historical flow data collected from the Tana River (one of the longest rivers in Kenya) between 1941 and 2016. One of their goals was to find the best statistical distribution to model annual maximum streamflow. They considered six candidate distributions and used the Akaike information criterion (AIC) for selection. We focused on the following three best-performing candidates they identified: the Gamma (Pearson

Type III), lognormal, and Weibull distributions. Table 3 shows the estimated parameters and AIC values, and our analysis results for the entropy, informity, and inforpower metrics.

Table 3. Estimated parameters and goodness-of-fit metrics for Tana River annual maximum streamflow.

Distribution	Parameter	AIC [8]	Entropy (Nats)	Informity (m^{-3}s)	Inforpower
Gamma	$\hat{\alpha} = 4.65$ $\hat{\lambda} = 0.0062$	1083.1	8.747	0.000886	0.5922
Lognormal	$\hat{\mu} = 6.51$ $\hat{\sigma} = 4.670$	1081.4	7.167	0.000950	0.6041
Weibull	$\hat{k} = 2.22$ $\hat{\lambda} = 849.40$	1089.5	7.194	0.000794	0.555

As shown in Table 3, among these three distributions, the lognormal distribution is the best according to all four goodness-of-fit metrics (minimum AIC, minimum entropy, maximum informity, and maximum inforpower). The Gamma distribution is the second best, while the Weibull is the worst.

Table 4 shows the percentage differences in metrics between the Weibull and lognormal distributions, computed as $(\text{Weibull} - \text{Lognormal})/\text{Lognormal} \times 100\%$.

Table 4. Percentage differences in metrics between the Weibull and lognormal distributions.

AIC	Entropy	Informity	Inforpower
0.157%	0.369%	−6.730%	−1.968%

Although the AIC and entropy metric detect only tiny differences (0.157% and 0.369%), the informity and inforpower metrics reveal substantially larger separations (−6.730% and −1.968%, respectively). This demonstrates that inforpower (and informity) can be considerably more sensitive than the traditional information criteria when distinguishing competing heavy-tailed distributions in hydrological and other positive-skewed applications.

These examples illustrate that the maximum inforpower criterion is intuitive, easy to compute, and either agrees with or usefully complements established model-selection tools such as the AIC, minimum entropy criterion, and maximum informity criterion.

6. Discussion and Conclusions

A distinctive and desirable feature of inforpower ϕ is its sensitivity to location shifts. Adding a constant a to the random variable $Y \rightarrow a + Y$ changes ϕ , reflecting an actual change in signal strength or systematic effect. In contrast, variance, differential entropy $H(p)$, and informity $\beta(p)$ are all translation-invariant. For example, in the normal distribution $N(\mu, \sigma^2)$, the entropy $\frac{1}{2} \log(2\pi e \sigma^2)$ and informity $\frac{1}{2\sigma\sqrt{\pi}}$ depend only on σ , not on the location μ . While translation invariance is useful when one wishes to measure pure spread or uncertainty, it is considered as a drawback in applications where it is necessary to take into account the values of the random variable along with their probabilities (or densities) [9].

Attempts to remedy the translation-invariance problem in related quantities have appeared in the literature. Belis and Guiasu [10] introduced weighted entropy: $H^w(p) = -\int yp(y)\log p(y)dy$. More recently, several authors [9,11,12] defined extropy for continuous distributions as $J(p) = -\frac{1}{2} \int p(y)^2 dy = -\frac{1}{2} \beta(p)$ and proposed a weighted extropy (restricted to nonnegative random variables) $J^w(p) = -\frac{1}{2} \int_0^\infty yp(y)^2 dy = -\frac{1}{2} \phi$.

Although weighted extropy is formally $-1/2$ times informity, the two measures arise from fundamentally different conceptual foundations. Extropy was originally motivated

as a “complementary dual” of entropy, a claim that has been critiqued on both theoretical and interpretative grounds [4]. Consequently, weighted extropy inherits these conceptual difficulties, whereas inforpower—defined directly as the expected effective information $\mathbb{E}_p[Yp(Y)]$ —possesses clear probabilistic, informational, and physical interpretations (Sections 3 and 4) without requiring an intermediate appeal to extropy.

Inforpower ϕ and the information-energy center $\mu_E = \phi / \beta(p)$ are both robust statistics. They exist even when ordinary moments fail: for the Cauchy distribution, the mean and variance are undefined, but ϕ and μ_E remain well defined, and ϕ equals the location parameter times μ_E . For right-skewed distributions on $[0, \infty)$, empirical evidence (Section 4) consistently places the energy center in the order: mode $< \mu_E < \text{median} < \text{mean}$, highlighting μ_E as an effective signal strength that lies closer to the region of highest probability density.

Computation of ϕ and μ_E requires an explicit PDF. In empirical studies, this is not a significant limitation: simply fit candidate distributions to the observed data (using maximum likelihood estimation or other estimation methods) and then evaluate ϕ on the fitted models. The maximum inforpower criterion (Section 5) thereby provides a simple model-selection rule that complements or even outperforms traditional tools (such as AIC). In addition to parametric approaches, non-parametric approaches such as kernel density estimation are possible but may introduce sensitivity to bandwidth selection.

When comparing symmetric distributions with the same estimated location $\hat{\mu}$ (e.g., normal, Laplace, logistic, and Student’s t distributions), the maximum inforpower criterion reduces to the maximum informity criterion. In this sense, the maximum informity criterion can be considered a special case of the maximum inforpower criterion. The maximum informity criterion (and the maximum inforpower criterion) can be used in measurement uncertainty analysis. For example, it has been used to select the best distribution from five candidate distributions (uniform, triangular, quadratic, raised cosine, and half-cosine distributions) to describe the calibration error of a measuring instrument [2].

The two examples presented represent two key scenarios: (i) recovery of a known ground-truth distribution (synthetic data) and (ii) discrimination among plausible but closely competing models in a real-world heavy-tailed dataset. They demonstrate the effectiveness of the maximum inforpower criterion in both controlled and practical settings. The synthetic Weibull distribution sample (Section 5, Example 1) demonstrates correct identification of the true underlying distribution among competing candidates, with the inforpower metric outperforming or matching existing metrics (AIC, entropy, informity). The real-world case of Tana River annual maximum streamflow (Section 5, Example 2) demonstrates the practical value of the method in hydrological extreme value modeling, where the inforpower metric exhibits higher sensitivity than the AIC when distinguishing nearly tied models (lognormal distribution versus Weibull distribution).

In summary, inforpower ϕ constitutes a principled, robust, and effective measure of informational power that naturally integrates signal strength and noise level. Unlike the traditional uncertainty measures such as variance and entropy, inforpower ϕ is sensitive to location shifts, exhibits scale invariance for non-negative distributions, and is well defined even when moments do not exist. The maximum inforpower criterion is easy to implement and provides a valuable new tool for distribution selection and performance assessment. Although current work primarily focuses on unimodal continuous distributions, inforpower and information-energy center can be easily extended to mixed distributions simply by performing calculations on the composite probability density function. Future research should explore these extensions, their behavior in multivariate settings, finite sample properties, potential bias correction, and their applications in machine learning, finance, and measurement science.

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Appendix A. Information-Energy Center for Discrete Distributions

Definition A1. For a discrete random variable X with probability mass function (PMF) $P(X)$, the center of information energy is defined as

$$\mu_E = \mathbb{E}_G(X) = \sum_{i=1}^N x_i \frac{P(x_i)^2}{\beta(P)} = \sum_{i=1}^N x_i G(X), \quad (\text{A1})$$

where $\beta(P) = \sum_{i=1}^N P(x_i)^2$ is the discrete informity [2], and $G(X)$ is called the information-energy function given by

$$G(X) = \frac{P(x_i)^2}{\beta(P)}. \quad (\text{A2})$$

Therefore, the center of information energy for a discrete distribution is the expected value of X under $G(X)$.

Example A1. Binomial distribution.

Consider the binomial distribution $X \sim B(n, p)$. Its probability mass function (PMF) is given by

$$P(k, n, p) = \Pr(X = k) = \binom{n}{k} p^k (1-p)^{n-k}, \quad (\text{A3})$$

where p is the success probability, n is the number of trials, $k = 0, 1, 2, \dots, n$, and

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}. \quad (\text{A4})$$

The discrete informity of the binomial distribution is given by

$$\beta(X) = \sum_{k=0}^n \binom{n}{k}^2 p^{2k} (1-p)^{2(n-k)}. \quad (\text{A5})$$

The information-energy function is given by

$$G(X) = \frac{P(x_i)^2}{\beta(P)} = \frac{\binom{n}{k}^2 p^{2k} (1-p)^{2(n-k)}}{\sum_{k=0}^n \binom{n}{k}^2 p^{2k} (1-p)^{2(n-k)}}. \quad (\text{A6})$$

Then, the center of information energy is given by

$$\begin{aligned}\mu_E &= \mathbb{E}_G(X = k) = \frac{\sum_{k=0}^n k \binom{n}{k}^2 p^{2k} (1-p)^{2(n-k)}}{\sum_{k=0}^n \binom{n}{k}^2 p^{2k} (1-p)^{2(n-k)}} \\ &= \frac{1}{\sum_{k=0}^n \binom{n}{k}^2 p^{2k} (1-p)^{2(n-k)}} \sum_{k=0}^n k \binom{n}{k}^2 p^{2k} (1-p)^{2(n-k)}.\end{aligned}\quad (\text{A7})$$

When $p = 0.5$, the distribution is symmetric; $\mu_E = \mu_M = 0.5n$. When $p < 0.5$, the distribution is slightly right-skewed; μ_E is slightly greater than μ_M . When $p > 0.5$, the distribution is slightly left-skewed; μ_E is slightly smaller than μ_M .

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