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A High-Strain-Rate Viscohyperelastic Constitutive Framework for Soft Biological Tissues: A Multi-Tissue Evaluation

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Abstract

Viscous effects play an important role in the mechanical characterization of soft biological tissues under high-strain-rate loading. Accurate modeling of these behaviors is important for impact biomechanics, injury prediction, and crash safety analysis, in which biological tissues may experience high-strain-rate deformation. To describe the dynamic mechanical responses of soft tissues, a reliable constitutive framework is therefore needed to represent the dynamic response of soft tissues under high-strain-rate loading. The objective of this study is to develop and evaluate a viscohyperelastic constitutive framework for describing the dynamic compressive responses of multiple soft tissues. The proposed formulation is constructed within a continuum mechanics framework, in which the viscous contribution is expressed using objective invariant functions, namely J_2 , J_6 , and J_7 . The developed analytical formulations are calibrated against high-strain-rate experimental data from different soft biological tissues, namely porcine meniscus, bovine liver, and ovine brain tissues. To find the material model parameters, genetic algorithm optimization is used to identify the material parameters and assess the robustness of the fitting procedure. In order to assess the robustness of the proposed constitutive framework across different loading rates, a multi-objective optimization strategy is used to calibrate the model parameters by fitting multiple strain-rate-dependent responses at the same time. This approach enables the model predictive capability to be evaluated over a range of high-strain-rate conditions. These results show that the proposed framework can reasonably describe the nonlinear and rate-dependent mechanical responses of different soft tissues under dynamic compression.

Keywords: high-strain rate; viscohyperelasticity; soft tissues; material characterization; optimization



Academic Editors: Zelin Liu, Teng Su and Senlin Xie

Received: 19 May 2026

Revised: 18 June 2026

Accepted: 24 June 2026

Published: 1 July 2026

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1. Introduction

Soft biological tissues show strongly nonlinear and rate-dependent mechanical responses under dynamic loading conditions [1]. Accurate modeling of these behaviors is important for impact biomechanics, injury prediction, and crash safety analysis, where biological materials may undergo high-strain-rate deformation. During traffic accidents, sudden impact, restraint loading, and contact with interior vehicle structures deform soft tissues within a very short time [2]. Such rapid loading may stretch muscles, tendons, ligaments, liver, brain, and other soft tissues beyond their physiological limits and lead to mechanical damage [2]. Therefore, characterizing the high-strain-rate viscohyperelastic behavior of soft tissues is important for crash injury prediction and computational impact biomechanics.

From a constitutive modeling perspective, soft tissues are commonly treated as large-deformation materials [3] because they usually undergo a large strain process before failure. Hyperelastic models are useful for describing the nonlinear elastic response under finite deformation. However, hyperelasticity alone cannot capture the strain-rate effect. During dynamic or impact loading, the material stiffness of soft tissues often changes significantly. Therefore, viscohyperelastic formulations are often introduced to describe the soft tissue viscous effects and allow the constitutive model to represent both finite deformation and rate-dependent behavior.

In finite deformation viscoelasticity, the viscous response can be formulated through a dissipation potential [4]. To satisfy material frame indifference, such a potential should be constructed using objective scalar quantities [5], such as invariants of the right Cauchy–Green tensor and its material time derivative. This invariant-based formulation avoids direct dependence on a particular coordinate system. It provides a systematic way to derive stress responses under finite deformation. Therefore, the selection of appropriate invariant terms is an important and first step in developing viscohyperelastic constitutive models for biological tissues.

In recent years, several viscous potential forms have been reported to model soft tissues under high strain rates. Those soft tissue dissipation potential models are shown in Table 1. The linear dissipation potential for porcine skin [6] and porcine lung [7] soft tissues is the primary and most straightforward approach to model the viscous effect. However, it is more natural and general to use nonlinear dissipation potential models, for example, on human [8–10], porcine [11], and bovine [12–14]. Among those nonlinear models, only Sista et al. [12] applied them to high strain rates. However, the fixed power of 0.4 on $(I_1 - 3)$ limits the flexibility of the model. In addition, most existing formulations were developed or validated for a single biological tissue type.

Table 1. Soft tissue material model viscous potential [15].

Reference	Soft Tissue	Strain Rate (s^{-1})	Material Model Expression
Comley et al. [6]	Porcine skin	1500–6300	Prony series
Sanborn et al. [7]	Porcine lung	1300–3000	Prony series
Sista et al. [12]	Bovine brain	1000–3000	$\eta_1(I_1 - 3)^{0.4} J_2^{\eta_2}$
Kulkarni et al. [8]	Porcine and human brain	0.1–90	$\frac{1}{2}\mu_2(I_1 - 3)^{\eta_1} J_2$
Limbert et al. [9]	Human knee and tendons	0.00012–0.5	$\eta_1(I_1 - 3)J_2$
Vogel et al. [11]	Porcine intervertebral disc	0.0005–0.13	$(I_1 - 3)^2[\tilde{J} + \tau(e^{-\frac{\tilde{J}}{\tau}}) - 1]$ with $\tilde{J} = \eta_1 J_1 + \eta_2J_2$
Yousefi et al. [13]	Bovine tongue	0.0146–0.066	$C_3(I_1 - 3)^{c_4} J_2$
Roan et al. [14]	Bovine liver	0.001–0.04	$(I_1 - 3)J_2 + (I_1 - 3)J_3^2$
Pioletti et al. [10]	Human ACL	NA	$\frac{\eta_2}{4}(I_1 - 3)J_2$

Although the studies summarized in Table 1 provide useful constitutive forms for different soft tissues, several limitations remain. First, many reported models were developed or validated for a specific tissue type and this makes it difficult to assess their general applicability across different biological materials. Second, some formulations were mainly evaluated under quasi-static or low-rate loading conditions [9,11,13,14]. This means that their performance under high-strain-rate deformation remains an open question. Third, several viscous potentials use fixed powers or relatively simple invariant combinations [9,12–14,16], which may restrict the flexibility of the model when fitting tissues with different levels of nonlinearity and strain-rate sensitivity. In addition, some reported studies calibrated material parameters separately for each strain rate [17,18], which may limit the evaluation of model performance across different loading rates using a unified

parameter set. Thus, these limitations motivate the development and evaluation of a more flexible invariant-based framework across multiple soft tissue datasets.

However, to the best of our knowledge, no prior study has adopted a better flexible viscohyperelastic formulation to evaluate multiple soft biological tissues. To address this problem, this study uses experimental data from different tissue types to evaluate the general applicability of the proposed framework based on the invariants J_2 , J_6 , and J_7 . Experimental datasets from porcine meniscus, ovine brain, and bovine liver tissues are used to assess the general applicability of the framework. In this study, a multi-objective optimization strategy [19] is employed to calibrate the model parameters by fitting responses at multiple strain rates simultaneously. In this way, a single parameter set is identified for each tissue and each constitutive form, allowing the capability of the model to represent rate-dependent behavior across different loading conditions to be evaluated. To be specific, the nominal stress–stretch responses predicted by the proposed formulations are compared with experimental data to demonstrate the performance of the framework.

The remainder of this paper is organized as follows. Section 2 reviews the continuum mechanics background, introduces the invariant-based viscohyperelastic formulation, derives the nominal stress–stretch relations, and describes the experimental datasets and optimization procedure. Section 3 presents the calibration results for porcine meniscus, ovine brain, and bovine liver tissues under different strain-rate conditions. Section 4 discusses the model performance, parameter trends, advantages, and limitations of the proposed framework. Finally, Section 5 summarizes the main conclusions and outlines possible directions for future work.

2. Materials and Methods

In this section, the fundamentals of continuum mechanics are first reviewed. Then, the derivatives of the novel nominal stress vs. stretch relations are presented. This is followed by the review of three experiments to be used as the benchmark cases. Finally, the multi-objective strategy is shown.

2.1. Fundamentals of Continuum Mechanics

In continuum mechanics, a material point is described by $\mathbf{X}$ in the initial configuration at time $t = 0$, whereas the vector $\mathbf{x}$ represents the position of the same material point in the spatial or Eulerian coordinate system after deformation at time t .

$$\mathbf{x} = \mathbf{x}(\mathbf{X}, t) \quad (1)$$

In finite deformation theory, the deformation gradient $\mathbf{F}$ characterizes the transformation from the undeformed material configuration to the current spatial configuration. Mathematically, it is defined as the gradient of the current position vector with respect to the reference material coordinates as shown in Equation (2). The determinant of the deformation gradient, $\det(\mathbf{F})$, quantifies the volumetric change associated with the deformation. In other words, it relates the current differential volume element dv to the corresponding reference volume element dV in Equation (3).

$$\mathbf{F} = \frac{\partial \mathbf{x}}{\partial \mathbf{X}} \quad (2)$$

$$J = \det(\mathbf{F}) = \frac{v}{V} \quad (3)$$

The right Cauchy–Green tensor, $\mathbf{C}$, is defined as in Equation (4). In finite deformation analysis, the Green–Lagrangian strain tensor— $\mathbf{E}$, in Equation (5)—is commonly used to provide an objective measure of strain in the reference configuration.

$$\mathbf{C} = \mathbf{F}^T \mathbf{F} \quad (4)$$

$$\mathbf{E} = \frac{1}{2}(\mathbf{C} - \mathbb{I}) \quad (5)$$

By taking the material time derivative of Equation (4), the rate form of the right Cauchy–Green deformation tensor can be obtained.

$$\dot{\mathbf{C}} = \dot{\mathbf{F}}^T \mathbf{F} + \mathbf{F}^T \dot{\mathbf{F}} = 2\mathbf{F}^T \mathbf{d}\mathbf{F} \quad (6)$$

This rate $\dot{\mathbf{C}}$ can be expressed in terms of deformation rate tensor $\mathbf{d}$. The deformation rate tensor is a symmetric expression in Equation (7). The corresponding antisymmetric part is called the spin tensor in Equation (8). The velocity gradient is defined in Equation (9), where $\mathbf{v}(\mathbf{x}, \mathbf{t})$ is the spatial velocity field.

$$\mathbf{d} = \frac{1}{2}(\mathbf{l} + \mathbf{l}^T) \quad (7)$$

$$\mathbf{w} = \frac{1}{2}(\mathbf{l} - \mathbf{l}^T) \quad (8)$$

$$\mathbf{l} = \frac{\partial \mathbf{v}(\mathbf{x}, \mathbf{t})}{\partial \mathbf{x}} \quad (9)$$

In order to preserve material frame indifference in the constitutive model [4], the strain energy density function is formulated using invariant quantities rather than quantities that depend on the observer. Specifically, the principal invariants of the right Cauchy–Green tensor $\mathbf{C}$ are adopted. These invariants are denoted by I_k ($k = 1, 2, 3$) in Equations (10)–(12).

$$I_1(\mathbf{C}) = \text{tr}(\mathbf{C}) \quad (10)$$

$$I_2(\mathbf{C}) = \frac{1}{2}((\text{tr}(\mathbf{C}))^2 - \text{tr}(\mathbf{C}^2)) \quad (11)$$

$$I_3(\mathbf{C}) = \det(\mathbf{C}) = J^2 \quad (12)$$

Their derivatives with respect to $\mathbf{C}$ are expressed in Equations (13)–(15).

$$\frac{\partial I_1}{\partial \mathbf{C}} = \mathbf{I} \quad (13)$$

$$\frac{\partial I_2}{\partial \mathbf{C}} = I_1 \mathbf{I} - \mathbf{C} \quad (14)$$

$$\frac{\partial I_3}{\partial \mathbf{C}} = I_3 \mathbf{C}^{-1} \quad (15)$$

In a similar manner, objectivity is also maintained for the rate quantity $\dot{\mathbf{C}}$. These invariants are denoted by J_k ($k = 1, \dots, 7$) from Equation (16) to Equation (22).

$$J_1 = \mathbf{I} : \dot{\mathbf{C}} = \text{tr}(\dot{\mathbf{C}}) \quad (16)$$

$$J_2 = \frac{1}{2}(\mathbf{I} : \dot{\mathbf{C}}^2) = \frac{1}{2}\text{tr}(\dot{\mathbf{C}}^2) \quad (17)$$

$$J_3 = \det(\dot{\mathbf{C}}) \quad (18)$$

$$J_4 = \mathbf{I} : (\mathbf{C}\dot{\mathbf{C}}) = \text{tr}(\mathbf{C}\dot{\mathbf{C}}) \quad (19)$$

$$J_5 = \mathbf{I} : (\mathbf{C}^2\dot{\mathbf{C}}) = \text{tr}(\mathbf{C}^2\dot{\mathbf{C}}) \quad (20)$$

$$J_6 = \mathbf{I} : (\mathbf{C}\dot{\mathbf{C}}^2) = \text{tr}(\mathbf{C}\dot{\mathbf{C}}^2) \quad (21)$$

$$J_7 = \mathbf{I} : (\mathbf{C}^2\dot{\mathbf{C}}^2) = \text{tr}(\mathbf{C}^2\dot{\mathbf{C}}^2) \quad (22)$$

Their derivatives with respect to the rate quantity are presented as follows from Equation (23) to Equation (29).

$$\frac{\partial J_1}{\partial \dot{\mathbf{C}}} = \mathbf{I} \quad (23)$$

$$\frac{\partial J_2}{\partial \dot{\mathbf{C}}} = \dot{\mathbf{C}} \quad (24)$$

$$\frac{\partial J_3}{\partial \dot{\mathbf{C}}} = J_2 \mathbf{I} - J_1 \dot{\mathbf{C}} \quad (25)$$

$$\frac{\partial J_4}{\partial \dot{\mathbf{C}}} = \mathbf{C} \quad (26)$$

$$\frac{\partial J_5}{\partial \dot{\mathbf{C}}} = \mathbf{C}^2 \quad (27)$$

$$\frac{\partial J_6}{\partial \dot{\mathbf{C}}} = \mathbf{C}\dot{\mathbf{C}} + \dot{\mathbf{C}}\mathbf{C} \quad (28)$$

$$\frac{\partial J_7}{\partial \dot{\mathbf{C}}} = \mathbf{C}^2\dot{\mathbf{C}} + \dot{\mathbf{C}}\mathbf{C}^2 \quad (29)$$

If the loading is only in the third direction, the uniaxial expressions of deformation gradient, Green–Lagrangian strain tensor, and Green–Lagrangian strain tensor rate are shown in Equations (30)–(32). Its invariant are expressed in from Equation (33) to Equation (40).

$$\mathbf{F} = \begin{bmatrix} \lambda^{-\frac{1}{2}} & 0 & 0 \\ 0 & \lambda^{-\frac{1}{2}} & 0 \\ 0 & 0 & \lambda \end{bmatrix} \quad (30)$$

$$\mathbf{C} = \begin{bmatrix} \lambda^{-1} & 0 & 0 \\ 0 & \lambda^{-1} & 0 \\ 0 & 0 & \lambda^2 \end{bmatrix} \quad (31)$$

$$\dot{\mathbf{C}} = \begin{bmatrix} -\lambda^{-2}\dot{\lambda} & 0 & 0 \\ 0 & -\lambda^{-2}\dot{\lambda} & 0 \\ 0 & 0 & 2\lambda\dot{\lambda} \end{bmatrix} \quad (32)$$

$$I_1 = \lambda^2 + 2\lambda^{-1} \quad (33)$$

$$J_1 = 2(\lambda - \lambda^{-2})\dot{\lambda} \quad (34)$$

$$J_2 = (2\lambda^2 + \lambda^{-4})\dot{\lambda}^2 \quad (35)$$

$$J_3 = 2\lambda^{-3}\dot{\lambda}^3 \quad (36)$$

$$J_4 = 2(\lambda^3 - \lambda^{-3})\dot{\lambda} \quad (37)$$

$$J_5 = 2(\lambda^5 - \lambda^{-4})\dot{\lambda} \quad (38)$$

$$J_6 = 2(2\lambda^4 + \lambda^{-5})\dot{\lambda}^2 \quad (39)$$

$$J_7 = 2(2\lambda^6 + \lambda^{-6})\dot{\lambda}^2 \quad (40)$$

2.2. Constitutive Modeling

To increase the flexibility of the viscous response, a power term η_3 is introduced on J_2 , J_6 , and J_7 to generalize viscosity behavior. In addition, to adjust the overall response and make sure there is no strain energy when there is no deformation, $(I_1 - 3)$ needs to be placed before J_2 , J_6 , and J_7 . To scale the response, a scaling parameter η_1 is introduced before $(I_1 - 3)^{\eta_2} J_k$ ($k = 2, 6, 7$). Then, the novel viscosity strain energy functions are structured as in Equations (41)–(43), where the upper script pv stands for the power term for the viscosity effect and 2, 6, and 7 represent J_2 , J_6 , and J_7 , respectively. In this formulation, η_1 , η_2 , and η_3 are phenomenological parameters identified at the macroscopic level. The parameter η_1 acts as a scaling parameter for the viscous contribution and controls the overall magnitude of the viscous stress response. Mathematically, it works as a scaling factor to amplify the viscous potential or viscous stress response. η_2 controls the contribution associated with the deformation-dependent term $(I_1 - 3)$, while η_3 governs the power dependence on the selected rate invariant J_k . Therefore, the nonlinear rate-dependent response is governed by the combined effect of η_2 and η_3 , rather than by either parameter alone.

The details of invariant selection to narrow down J_2 , J_6 , and J_7 are shown in the previous work by Long et al. [15]. A concise review of this selection process is also provided in Appendix A.1.

$$W(\mathbf{C}, \dot{\mathbf{C}})^{pv2} = \eta_1 (I_1 - 3)^{\eta_2} J_2^{\eta_3} \quad (41)$$

$$W(\mathbf{C}, \dot{\mathbf{C}})^{pv6} = \eta_1 (I_1 - 3)^{\eta_2} J_6^{\eta_3} \quad (42)$$

$$W(\mathbf{C}, \dot{\mathbf{C}})^{pv7} = \eta_1 (I_1 - 3)^{\eta_2} J_7^{\eta_3} \quad (43)$$

Following the procedure in [15], the nominal stress expressions are derived as

$$\begin{aligned} \mathbf{S}^{pv2} &= 2 \frac{\partial W^{pv2}}{\partial \dot{\mathbf{C}}} - p \mathbf{C}^{-1} \\ &= 2\eta_1 (I_1 - 3)^{\eta_2} \eta_3 J_2^{\eta_3-1} \frac{\partial J_2}{\partial \dot{\mathbf{C}}} - p \mathbf{C}^{-1} \\ &= 2\eta_1 (I_1 - 3)^{\eta_2} \eta_3 J_2^{\eta_3-1} \dot{\mathbf{C}} - p \mathbf{C}^{-1} \\ \mathbf{N}^{v2} &= \mathbf{S}^{v2} \mathbf{F}^T - p \mathbf{F}^{-1} \\ &= 2\eta_1 (I_1 - 3)^{\eta_2} \eta_3 J_2^{\eta_3-1} \dot{\mathbf{C}} \mathbf{F}^T - p \mathbf{F}^{-1} \\ &= 2\eta_1 (I_1 - 3)^{\eta_2} \eta_3 J_2^{\eta_3-1} \begin{bmatrix} -\lambda^{-2} \dot{\lambda} & 0 & 0 \\ 0 & -\lambda^{-2} \dot{\lambda} & 0 \\ 0 & 0 & 2\lambda \dot{\lambda} \end{bmatrix} \begin{bmatrix} \lambda^{-\frac{1}{2}} & 0 & 0 \\ 0 & \lambda^{-\frac{1}{2}} & 0 \\ 0 & 0 & \lambda \end{bmatrix} - p \begin{bmatrix} \lambda^{\frac{1}{2}} & 0 & 0 \\ 0 & \lambda^{-\frac{1}{2}} & 0 \\ 0 & 0 & \lambda^{-1} \end{bmatrix} \end{aligned} \quad (44)$$

$$\begin{aligned} \mathbf{S}^{pv6} &= 2 \frac{\partial W^{pv6}}{\partial \dot{\mathbf{C}}} - p \mathbf{C}^{-1} \\ &= 2\eta_1 (I_1 - 3)^{\eta_2} \eta_3 J_6^{\eta_3-1} \frac{\partial J_6}{\partial \dot{\mathbf{C}}} - p \mathbf{C}^{-1} \\ &= 2\eta_1 (I_1 - 3)^{\eta_2} \eta_3 J_6^{\eta_3-1} (\mathbf{C} \dot{\mathbf{C}} + \dot{\mathbf{C}} \mathbf{C}) - p \mathbf{C}^{-1} \\ \mathbf{N}^{v6} &= \mathbf{S}^{v6} \mathbf{F}^T - p \mathbf{F}^{-1} \\ &= 2\eta_1 (I_1 - 3)^{\eta_2} \eta_3 J_6^{\eta_3-1} (\mathbf{C} \dot{\mathbf{C}} + \dot{\mathbf{C}} \mathbf{C}) \mathbf{F}^T - p \mathbf{F}^{-1} \\ &= 2\eta_1 (I_1 - 3)^{\eta_2} \eta_3 J_6^{\eta_3-1} \begin{bmatrix} -\lambda^{-3} \dot{\lambda} & 0 & 0 \\ 0 & -\lambda^{-3} \dot{\lambda} & 0 \\ 0 & 0 & 2\lambda^3 \dot{\lambda} \end{bmatrix} \begin{bmatrix} \lambda^{-\frac{1}{2}} & 0 & 0 \\ 0 & \lambda^{-\frac{1}{2}} & 0 \\ 0 & 0 & \lambda \end{bmatrix} - p \begin{bmatrix} \lambda^{\frac{1}{2}} & 0 & 0 \\ 0 & \lambda^{-\frac{1}{2}} & 0 \\ 0 & 0 & \lambda^{-1} \end{bmatrix} \end{aligned} \quad (45)$$

$$\begin{aligned}
 \mathbf{S}^{\text{pv}7} &= 2 \frac{\partial W^{\text{pv}7}}{\partial \dot{\mathbf{C}}} - p \mathbf{C}^{-1} \\
 &= 2\eta_1(I_1 - 3)^{\eta_2} \eta_3 J_7^{\eta_3-1} \frac{\partial J_7}{\partial \dot{\mathbf{C}}} - p \mathbf{C}^{-1} \\
 &= 2\eta_1(I_1 - 3)^{\eta_2} \eta_3 J_7^{\eta_3-1} (\mathbf{C}^2 \dot{\mathbf{C}} + \dot{\mathbf{C}} \mathbf{C}^2) - p \mathbf{C}^{-1} \\
 \mathbf{N}^{\text{v}7} &= \mathbf{S}^{\text{v}7} \mathbf{F}^T - p \mathbf{F}^{-1} \\
 &= 2\eta_1(I_1 - 3)^{\eta_2} \eta_3 J_7^{\eta_3-1} (\mathbf{C}^2 \dot{\mathbf{C}} + \dot{\mathbf{C}} \mathbf{C}^2) \mathbf{F}^T - p \mathbf{F}^{-1} \\
 &= 2\eta_1(I_1 - 3)^{\eta_2} \eta_3 J_7^{\eta_3-1} \begin{bmatrix} -\lambda^{-4} \dot{\lambda} & 0 & 0 \\ 0 & -\lambda^{-4} \dot{\lambda} & 0 \\ 0 & 0 & 2\lambda^5 \dot{\lambda} \end{bmatrix} \begin{bmatrix} \lambda^{-\frac{1}{2}} & 0 & 0 \\ 0 & \lambda^{-\frac{1}{2}} & 0 \\ 0 & 0 & \lambda \end{bmatrix} - p \begin{bmatrix} \lambda^{\frac{1}{2}} & 0 & 0 \\ 0 & \lambda^{-\frac{1}{2}} & 0 \\ 0 & 0 & \lambda^{-1} \end{bmatrix}
 \end{aligned} \tag{46}$$

By forcing $N_{11} = N_{22}$, the nominal stress from Equations (47)–(49) are expressed as

$$N_{33}^{\text{pv}2} = 2\eta_1(\lambda^2 + 2\lambda^{-1} - 3)^{\eta_2} \eta_3 (2(2\lambda^2 + \lambda^{-4})\dot{\lambda}^2)^{\eta_3-1} (2\lambda^2 + \lambda^{-4})\dot{\lambda} \tag{47}$$

$$N_{33}^{\text{pv}6} = 4\eta_1(\lambda^2 + 2\lambda^{-1} - 3)^{\eta_2} \eta_3 (2(2\lambda^4 + \lambda^{-5})\dot{\lambda}^2)^{\eta_3-1} (2\lambda^4 + \lambda^{-5})\dot{\lambda} \tag{48}$$

$$N_{33}^{\text{pv}7} = 4\eta_1(\lambda^2 + 2\lambda^{-1} - 3)^{\eta_2} \eta_3 (2(2\lambda^6 + \lambda^{-6})\dot{\lambda}^2)^{\eta_3-1} (2\lambda^6 + \lambda^{-6})\dot{\lambda}. \tag{49}$$

It is noted that the total nominal stress–stretch response is the summation of quasi-static responses and purely viscohyperelastic responses (Equations (47)–(49)). The quasi-static nominal stress–stretch response is documented in Appendix A.2. The total nominal stress–stretch response is used to fit the experimental datasets.

2.3. Review of Experimental Data

In this section, three experiments for porcine meniscus [17], ovine brain [18], and bovine liver [20] are reviewed. The specimen preparation and experiment setups are recalled in detail. The experimental data at high strain rates from each work were used to fit parameters in Equations (47)–(49) for η_1 , η_2 , and η_3 .

2.3.1. Case 1: Porcine Meniscus [17]

The experimental data used in the present work are from the dynamic compression study conducted by Juang et al. [17]. In that study, porcine knee tissues were obtained from a slaughterhouse and stored at -30 Celsius. The specimens were thawed before the experiments. The experiment was completed within 72 h after euthanasia. The cylindrical meniscus samples were prepared using a scalpel. The specimen height-to-diameter ratio was maintained between 0.2 and 0.3. Dynamic unconfined compression experiments were performed using a Split Hopkinson Pressure Bar (SHPB) at strain rates of 690 s^{-1} , 980 s^{-1} , 1100 s^{-1} , 1380 s^{-1} , and 1560 s^{-1} . The nominal stress–stretch curves used for model calibration were extracted from the unconfined compression results reported by Juang et al. [17]. Overall, the reported curves show an approximately linear relationship between nominal stress and stretch over the selected deformation range. However, the responses associated with the 980 s^{-1} and 1100 s^{-1} cases appear to be highly overlapped. This indicates a possible uncertainty or inconsistency in at least one of these two datasets. For this reason, these two strain-rate cases were excluded from the parameter fitting procedure. As a result, the material parameters were identified using the 690 s^{-1} , 1380 s^{-1} , and 1560 s^{-1} responses in this study. It should also be noted that Juang et al. [17] did not provide the number of tested specimens, the averaged experimental response, or the variation range for each strain-rate condition. Moreover, below a stretch ratio of approximately 0.9, the response at 1380 s^{-1} shows a softening phenomenon. A similar pattern can also be observed in the 690 s^{-1} curve. This behavior may be associated with irreversible deformation, damage, or other inelastic effects at relatively large compressive strains. Since the present work

focuses on viscohyperelastic behavior, only the stretch interval from 1.0 to 0.9 was used for material model parameter identification.

2.3.2. Case 2: Ovine Brain [18]

The fresh ovine brains were obtained from lambs at least ten months old. After slaughter, the brains were immediately removed and stored in phosphate-buffered saline to prevent dehydration. They were transported to the laboratory under cold and non-frozen conditions to reduce tissue degradation. All experiments were performed within 7 to 14 h after slaughter. Unconfined uniaxial compression experiments were performed. Specimens were excised from the frontal and parietal lobes of both hemispheres following the protocol of Miller et al. [21]. Cylindrical samples were prepared using a sharp-edged steel pipe and surgical scalpels with dimensions of 15 ± 3.5 mm in height and 25 ± 1.5 mm in diameter. The experiment was carried out at room temperature (approximately 22 Celsius) using a BOSE 3200 ElectroForce testing machine. Surgical lubricant was applied on the compression platens to reduce friction and better approximate unconfined compression conditions. Each sample was compressed to 30% strain at six strain rates: 0.0667 s^{-1} , 3.33 s^{-1} , 6.667 s^{-1} , 33.33 s^{-1} , 66.667 s^{-1} , and 200 s^{-1} . The sample numbers for these strain-rate groups were 5, 6, 5, 5, 6, and 5, respectively.

2.3.3. Case 3: Bovine Liver [20]

The experimental data used in this study were obtained from the high-rate compression tests on bovine liver tissue reported in [20]. In that study, three fresh bovine livers were collected from 18-month-old steers shortly after slaughter. The livers were transported to the laboratory in Krebs solution to prevent dehydration and tissue degradation. All specimen tests were conducted within 3 h after slaughter. For each strain-rate condition, 15 specimens were tested with 5 samples prepared from each liver. The specimens were harvested from the right lobe of the liver. Large veins and the outer membrane were avoided to improve sample consistency. Ring-shaped samples were prepared using surgical scalpels, trephine blades, a punch, and an aluminum template. Each specimen had an outer diameter of 10 mm, an inner diameter of 4.7 mm, and a thickness of approximately 1.7 mm. Two sampling orientations were considered: D1, where the loading direction was parallel to the liver surface and aligned with the main venous direction, and D2, where the loading direction was perpendicular to the liver surface. The small specimen thickness was selected to promote dynamic stress equilibrium during high-rate loading, while the ring-shaped geometry was used to reduce radial inertia effects. A thin layer of vegetable oil was applied to the contact surfaces to minimize friction. Dynamic compression tests were performed using a modified Kolsky bar, which is also known as a Split Hopkinson Pressure Bar. Since liver tissue has low strength and low mechanical impedance, a hollow aluminum transmission bar, sensitive semiconductor strain gauges, and quartz-crystal force transducers were used to improve the accuracy of transmitted force measurement. In the experiment, there are seven strain rates: 0.01 s^{-1} , 1 s^{-1} , 10 s^{-1} , 100 s^{-1} , 1000 s^{-1} , 2000 s^{-1} , and 3000 s^{-1} . Since there is no significant difference between D1 and D2 reported in [20], only the experimental data for D1 was used in this study.

2.4. Optimization

The root mean square error (RMSE) [22] is adopted as the objective function to evaluate the residual between the model prediction and the experimental response. Material parameters are identified using a two-step optimization procedure. First, a genetic algorithm [23–25] is employed to perform a global search and reduce the likelihood of convergence to a local minimum. The resulting parameter set is then further refined using the Nelder–Mead simplex algorithm [26]. Both algorithms are implemented through the

Python package SciPy-1.13.1 [27]. Since multiple strain-rate responses are fitted at the same time, a multi-objective optimization scheme [19] is used, where the objective contributions from different strain rates are weighted equally.

3. Results

The three cases demonstrate that the proposed viscohyperelastic framework can describe different high-rate compressive responses using the same general constitutive structure well. The porcine meniscus shows relatively high stress and approximately linear nominal stress–stretch behavior within the selected stretch range. The ovine brain tissue exhibits much lower stress magnitude and more nonlinearity than in the porcine meniscus case. The bovine liver tissue demonstrates significant nonlinear stiffening in the larger compression range. Despite these differences, the invariant-based viscosity formulations in Equations (47)–(49) provide reasonable agreement with the experimental responses for all three tissue types.

The curve fitting of multiple strain rates also demonstrates the capability of the proposed framework to represent rate-dependent behavior using a unified parameter set for each tissue and formulation. This multi-objective strategy avoids fitting each strain rate independently. Therefore, it provides a more direct evaluation of the model ability to capture strain-rate effects across multiple loading conditions.

3.1. Case 1: Porcine Meniscus

Figure 1 compares the experimental and analytical nominal stress–stretch responses of porcine meniscus tissue under strain rates of 690 s^{-1} , 1380 s^{-1} , and 1560 s^{-1} . The material model parameters, η_1 , η_2 , and η_3 , in the three proposed viscosity formulations, corresponding to the stress–strain relations in Equations (47)–(49), were optimized at the same time using the selected strain-rate-dependent responses. Overall, all three formulations provide good agreement with the experimental data within the stretch range from 1.0 to 0.9.

For this porcine meniscus case, the response at 690 s^{-1} remains much lower than those at 1380 s^{-1} and 1560 s^{-1} . This indicates a strong rate-dependent effect. The fitted curves agree well with the experimental data for all three strain rates. Among the three invariant-based formulations in Equations (41)–(43), the stress vs. strain predictions are generally similar. This suggests that J_2 , J_6 , and J_7 can all provide effective viscous contributions for representing the high-rate compressive response of porcine meniscus tissue. The identified material parameters are summarized in Table 2.

Table 2. Porcine meniscus material model parameters of Equations (47)–(49) at strain rates of 690 s^{-1} , 1380 s^{-1} , and 1560 s^{-1} for Figure 1. The unit of η_1 is MPa.

Strain Rate	η_1	η_2	η_3
Equation (47)	9.8963	0.3951	1.3563
Equation (48)	4.2971	0.4054	1.3559
Equation (49)	4.4791	0.4037	1.3552

3.2. Case 2: Ovine Brain

Figure 2 presents the nominal stress–stretch responses of ovine brain tissue at strain rates of 33.3 s^{-1} , 66.67 s^{-1} , and 200 s^{-1} . Compared with the porcine meniscus and bovine liver cases, the ovine brain tissue shows a much lower stress level. This is consistent with the naturally low stiffness of brain tissue. The stress response increases nonlinearly as the tissue is compressed.

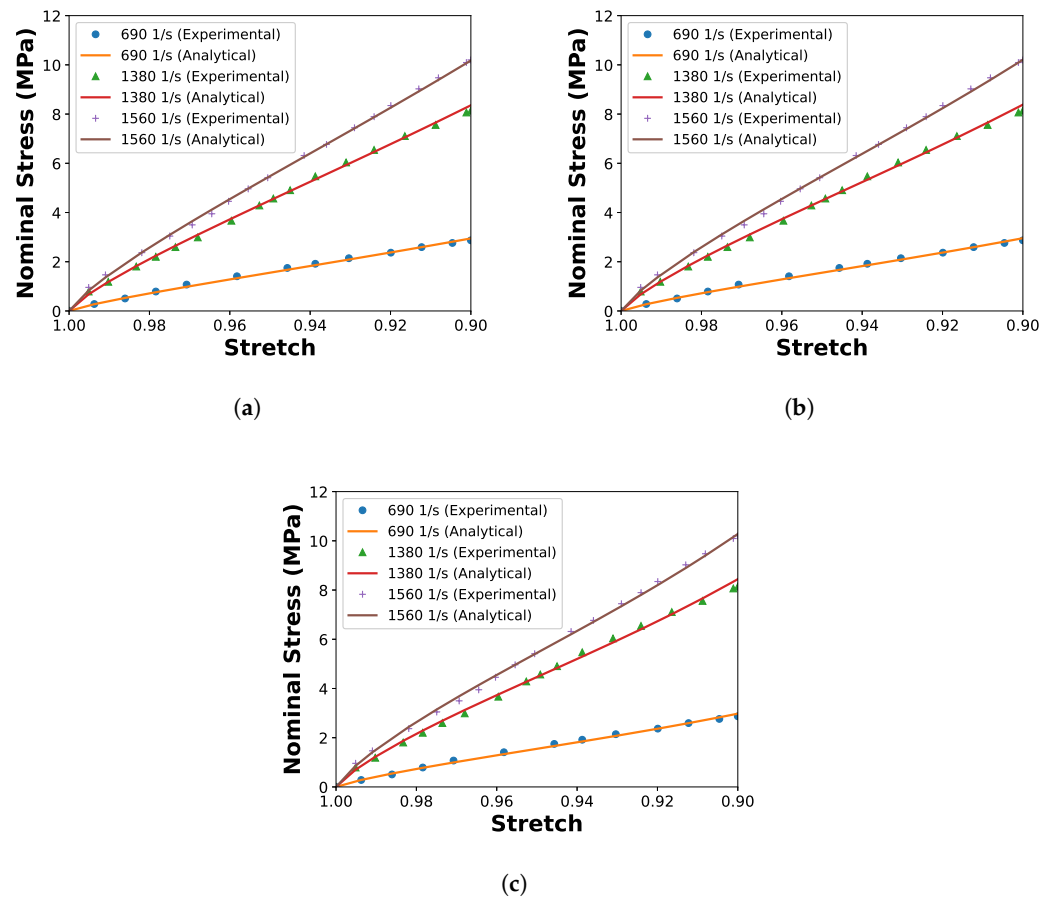


Figure 1. Porcine meniscus nominal stress vs. stretch response at strain rates of 690 s^{-1} , 1380 s^{-1} , and 1560 s^{-1} . (a) Equation (47). (b) Equation (48). (c) Equation (49).

The analytical results show that the proposed formulations, in Equations (47)–(49), can capture the overall nonlinear compressive behavior of the ovine brain tissue across the selected strain rates. The strain-rate effect is reflected by the increasing nominal stress from 33.3 s^{-1} to 200 s^{-1} . The calibrated curves follow the experimental data well over the full deformation range. However, some discrepancies are observed at the stretch level from 0.8 to 0.7, where stronger stress–strain nonlinearity is observed. This behavior may be related to the strong nonlinearity of brain tissue, variations in the experiment, etc.

The corresponding material parameters are listed in Table 3. The identified η_1 values are much smaller than those of the porcine meniscus case. This reflects a lower stress magnitude of brain tissue. The fitted η_3 values are below 1.0, which indicates a sublinear dependence of the viscous contribution on the selected rate invariant.

Table 3. Ovine brain material model parameters of Equations (47)–(49) at strain rates of 33.3 s^{-1} , 66.67 s^{-1} , and 200 s^{-1} for Figure 2. The unit of η_1 is MPa.

Strain Rate	η_1	η_2	η_3
Equation (47)	211.9293×10^{-6}	0.4680	0.7055
Equation (48)	109.4789×10^{-6}	0.3894	0.7022
Equation (49)	86.1830×10^{-6}	0.3019	0.6985

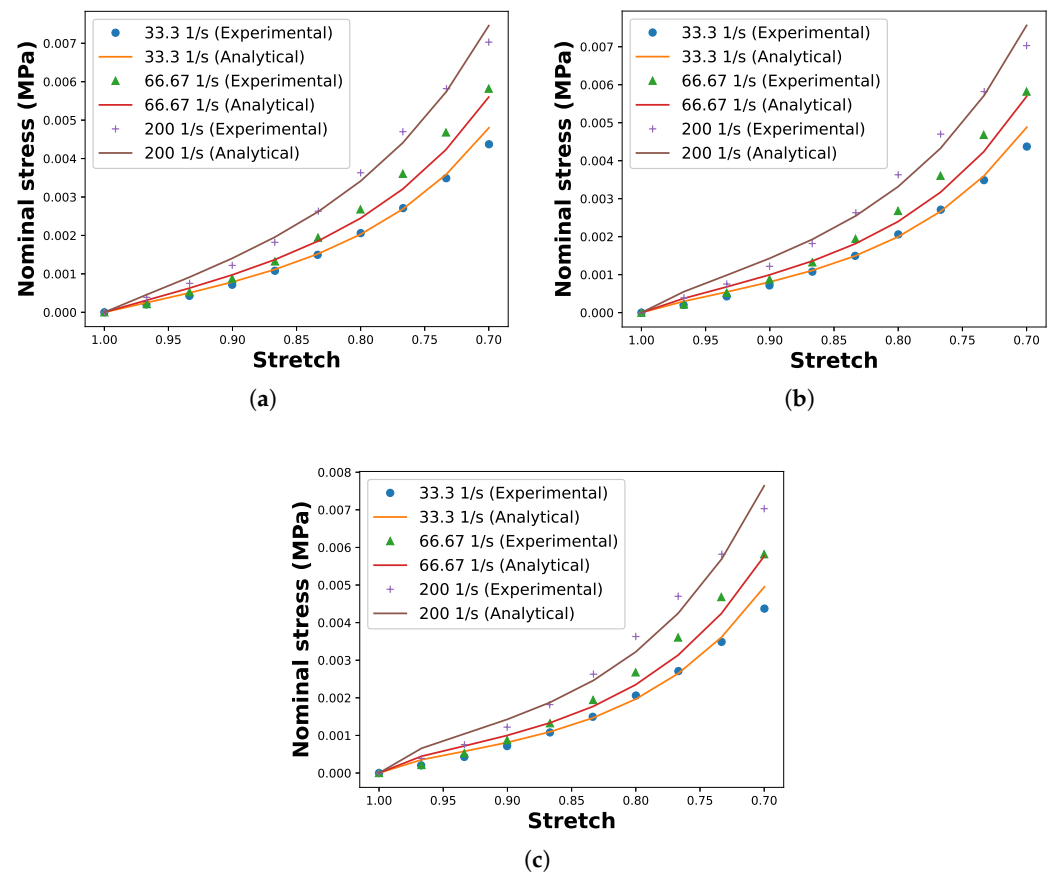


Figure 2. Ovine brain nominal stress vs. stretch response at strain rates of 33.3 s^{-1} , 66.67 s^{-1} , and 200 s^{-1} . (a) Equation (47). (b) Equation (48). (c) Equation (49).

3.3. Case 3: Bovine Liver

Figure 3 presents the comparison between experimental and analytical nominal stress–stretch curves for bovine liver tissue at strain rates of 1000 s^{-1} , 2000 s^{-1} , and 3000 s^{-1} . The bovine liver response shows a clear and highly nonlinear stiffening phenomenon under compression. The nominal stress increases gradually in the small deformation region and then rises significantly as the material deforms. With higher strain rates showing higher stress levels, the strain-rate dependence is also obvious.

The three proposed formulations, in Equations (41)–(43), can reproduce the general shape of the high-rate compressive response. In particular, the analytical curves capture both the nonlinear stress increase and the increase in stress among different strain-rate responses. Compared with the porcine meniscus case investigated in this study, the bovine liver response covers a larger deformation range and shows stronger nonlinear stiffening at high compression. Therefore, this case provides a more demanding evaluation of the proposed framework.

The fitted parameters for bovine liver are reported in Table 4. The identified η_2 values reach the upper bound of 1.0 for all three formulations, while η_3 becomes very small. This suggests that, for the bovine liver response, the optimization tends to use a nearly linear dependence on $(I_1 - 3)$ while decreasing the power dependence on the rate invariant.

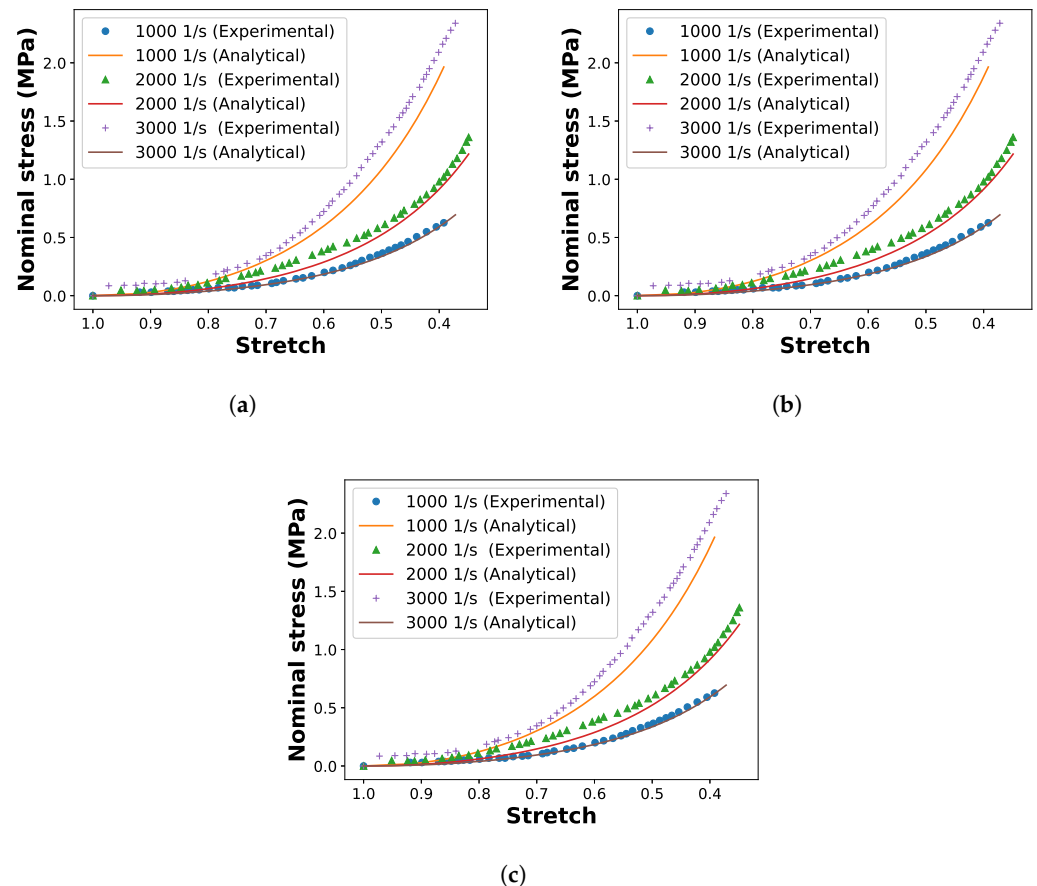


Figure 3. Bovine liver nominal stress vs. stretch response at strain rates of 1000 s^{-1} , 2000 s^{-1} , and 3000 s^{-1} . (a) Equation (47). (b) Equation (48). (c) Equation (49).

Table 4. Bovine liver material model parameters of Equations (47)–(49) at strain rates of 1000 s^{-1} , 2000 s^{-1} , and 3000 s^{-1} for Figure 3. The unit of η_1 is MPa.

Strain Rate	η_1	η_2	η_3
Equation (47)	4.7031×10^8	1.0000	9.5024×10^{-7}
Equation (48)	1.0651×10^{10}	1.0000	4.1960×10^{-8}
Equation (49)	1.3172×10^9	1.0000	3.3928×10^{-7}

4. Discussion

This study evaluated a high-strain-rate viscohyperelastic constitutive framework across three different soft biological tissues, namely porcine meniscus, ovine brain, and bovine liver. The main purpose was to assess whether the same invariant-based viscosity structure can be used to describe different dynamic compressive responses at different strain rates across multiple types of soft tissues. With tissue-specific parameter identification, the proposed material model demonstrates sufficient flexibility to describe the high-strain-rate compressive responses of different soft tissues evaluated in this work. Our results show that the proposed formulations in Equations (41)–(43) based on J_2 , J_6 , and J_7 can well describe the major rate-dependent and nonlinear features of the selected soft tissue datasets.

A key feature of the present framework is that the viscous contribution is formulated using objective invariants of $\mathbf{C}$ and $\dot{\mathbf{C}}$. This formulation preserves material frame indifference and avoids using tensor components that depend on the observer. From a mechanical perspective, the selected rate invariants, J_2 , J_6 , and J_7 , introduce different dependencies

on the stretch, λ , and stretch ratio rate, $\dot{\lambda}$. The invariant J_2 mainly reflects the magnitude of $\dot{\mathbf{C}}$, whereas J_6 and J_7 couple the rate quantity with the deformation state through $\mathbf{C}$ and $\mathbf{C}^2$, respectively. Therefore, J_6 and J_7 are expected to be more sensitive to the current deformation level, especially under larger compression. This feature is important for soft tissues because their rate-dependent response is influenced not only by loading rate but also by deformation state. The comparable fitting performance of the three formulations suggests that J_2 , J_6 , and J_7 can all represent the experimental rate dependence under uniaxial compression. However, their different parameter values indicate that these invariant forms should not be considered mechanically identical.

The porcine meniscus case shows that the proposed model can reproduce high-rate compression behavior over the selected stretch range from 1.0 to 0.9. Within this range, the experimental responses are approximately linear and the fitted curves successfully describe the stress increase with strain rate. The exclusion of the 980 s^{-1} and 1100 s^{-1} responses from calibration is reasonable because these two datasets appear to overlap strongly. This indicates uncertainty or other potential issues in the experiment. In addition, stretch values below 0.9 were not used because softening was observed in some curves. This choice is appropriate for a viscohyperelastic characterization, since softening at larger deformation may involve damage, irreversible deformation, or other inelastic mechanisms that are not explicitly included in the present formulation by Equations (41)–(43).

The ovine brain case provides a different type of evaluation because the stress magnitude is much lower than that of the meniscus and liver tissues. The successful fitting of this case indicates that the framework is not limited to relatively stiff tissue cases like the porcine meniscus. The fitted parameters for brain tissue are several orders of magnitude smaller in η_1 , which is consistent with the lower stiffness and lower nominal stress level. However, some mismatch can be observed at larger compression, where the stress response becomes more nonlinear. This suggests that certain additional terms or tissue-specific modifications may be needed if highly accurate fitting is required over a broader deformation range.

The bovine liver case is more challenging since its response displays strong nonlinear stiffening at large compressive deformation. The analytical curves reproduce the general rate-dependent behavior and the nonlinearity increases in stress. However, the optimized parameters show that η_2 reaches the upper bound and η_3 becomes very small. This may indicate that the current parameter bounds influence the final solution or the selected stress–strain relation reaches a limiting behavior for this dataset. Therefore, future studies should further examine the sensitivity of the fitted parameters to the prescribed parameter bounds and the selected optimization strategy.

The use of multi-objective optimization is another important aspect of this study. In previous studies on porcine meniscus [17] and ovine brain [18], material models were fitted separately for each individual strain rate. Although this approach can provide good agreement for a specific loading rate, it does not directly evaluate whether a single parameter set can represent the response across multiple strain rates. In contrast, the present study calibrates the material model parameters by fitting multiple strain-rate responses at the same time. This strategy provides a stricter test of the constitutive model because a single parameter set must describe the response across different strain rates. As a result, the fitted model can better reflect the overall rate-dependent behavior of each tissue rather than overfitting one individual strain rate condition. However, this multi-objective fitting method may also lead to a local mismatch for a specific curve, especially when experimental data contain uncertainty. When lower strain-rate curves (for example, two curves) are used for calibration and the highest strain-rate curve is reserved for prediction, the fitted parameters may differ from those obtained using all strain rates and the highest strain-rate curve may show a different fit result. This is expected because the optimized parameter set

depends on the selected calibration data. Therefore, parameter changes reflect differences in the fitting objective rather than inconsistency of the formulation.

It should be noted that the present results should not be interpreted as proof of a universal biological mechanism. Since only three soft-tissue datasets under uniaxial compression were evaluated, the model demonstrates feasibility and flexibility for the selected cases rather than universal applicability to all soft tissues or loading conditions.

In addition, several limitations should be noted. First, the present study relies on experimental data extracted from the literature. Therefore, the accuracy of the calibration depends on the quality, resolution, and completeness of the reported curves. Second, for some experimental datasets, information such as each specimen variation, averaged response, and standard deviation was limited or unavailable. This prevents a full statistical evaluation of the fitted parameters. Third, the current formulation focuses on viscohyperelastic behavior and does not explicitly include damage, permanent deformation, anisotropy, or tissue microstructural effects. These mechanisms may become important at larger deformations or under more complex loading conditions. Additionally, the present evaluation is based mainly on uniaxial compression data. Finally, the identified parameters should be interpreted as continuum-level parameters rather than direct microstructural constants. Although η_1 , η_2 , and η_3 can provide qualitative information about the macroscopic rate-dependent response, they cannot be directly assigned to a single microstructural component, such as collagen fibers, fluid content, cellular structure, etc. A direct microstructure-based interpretation would require additional experimental information, such as tissue composition, fiber orientation, fluid content, or microstructural imaging, which is beyond the scope of the present study.

Overall, our results demonstrate that the proposed high-strain-rate viscohyperelastic framework can provide a unified and flexible approach for modeling different soft biological tissues under strain rate in compression. The multi-tissue evaluation supports the invariant-based viscosity formulation. Therefore, the present results should be interpreted as a multi-tissue feasibility evaluation of the proposed invariant-based viscohyperelastic framework, rather than a complete validation of the model for all soft tissue loading conditions.

5. Conclusions and Further Work

In this study, a high-strain-rate viscohyperelastic constitutive framework was evaluated for porcine meniscus, ovine brain, and bovine liver tissues. The framework was also developed with potential applicability to other soft tissues. The models are based on the objective invariant-based viscous potentials constructed by J_2 , J_6 , and J_7 . The corresponding nominal stress–stretch relations were derived for uniaxial incompressible compression and calibrated using experimental data at multiple strain rates. Our results show that the proposed framework can reasonably describe the nonlinear and rate-dependent compressive responses of different soft tissues using the same general constitutive structure. The multi-objective optimization strategy enables curve fitting of multiple strain-rate responses. It provides a direct evaluation of the model's ability to capture rate-dependent behavior.

Future work will focus on additional loading modes, including tension, shear, and multiaxial deformation and other types of soft tissue at high strain rates. In addition, the sensitivity of the genetic algorithm may need further investigation for higher performance.

Funding: This research received no external funding.

Informed Consent Statement: Informed consent for participation was obtained from all subjects involved in the study.

Data Availability Statement: The raw data supporting the conclusions of this article will be made available by the author on request.

Conflicts of Interest: The author declares no conflicts of interest.

Appendix A

Appendix A.1

The invariant selection process [15] is summarized as follows: To select which invariant form from Equation (34) to Equation (40) should be used, a simple example is introduced. In this example, the invariant is multiplied by $(I_1 - 3)$ to ensure zero viscous stress in the undeformed state. Following the procedure in [15] (as mentioned in the method section), their one-dimensional nominal stress expressions are shown in Table A1. As shown in this table, there is no strain-rate-related expression $\dot{\lambda}$ in J_1 , J_4 , and J_5 . The $\dot{\lambda}^2$ in J_3 makes the J_3 invariant-related nominal stress become positive in compression, which is not correct. Specifically, in compression, λ is between 0 and 1 with $\dot{\lambda}$ negative, which makes the nominal stress expression for $4(\lambda^2 + 2\lambda^{-1} - 3)(0.5\lambda^{-3} + \lambda^{-3})\dot{\lambda}^2$ positive. Thus, J_2 , J_6 , and J_7 are left to construct the viscous potentials.

Table A1. Viscous potentials and one-dimensional nominal stress expression [15].

Strain Energy Function	Nominal Stress Expression
$(I_1 - 3)J_1$	$2(\lambda^2 + 2\lambda^{-1} - 3)(\lambda - \lambda^{-2})$
$(I_1 - 3)J_2$	$2(\lambda^2 + 2\lambda^{-1} - 3)(2\lambda^2 + \lambda^{-4})\dot{\lambda}$
$(I_1 - 3)J_3$	$4(\lambda^2 + 2\lambda^{-1} - 3)(0.5\lambda^{-3} + \lambda^{-3})\dot{\lambda}^2$
$(I_1 - 3)J_4$	$2(\lambda^2 + 2\lambda^{-1} - 3)(\lambda^3 - \lambda^{-3})$
$(I_1 - 3)J_5$	$2(\lambda^2 + 2\lambda^{-1} - 3)(\lambda^5 - \lambda^{-4})$
$(I_1 - 3)J_6$	$4(\lambda^2 + 2\lambda^{-1} - 3)(2\lambda^4 + \lambda^{-5})\dot{\lambda}$
$(I_1 - 3)J_7$	$4(\lambda^2 + 2\lambda^{-1} - 3)(2\lambda^6 + \lambda^{-6})\dot{\lambda}$

Appendix A.2

The nominal stress vs. stretch responses under quasi-static loading for Case 1 [17], Case 2 [18], and Case 3 [20] are shown in Figure A1. For Case 1, since the quasi-static loading is not presented by Juang et al. [17], the average Yeoh model parameters from anterior, middle, and posterior, C_{10} and C_{20} , are applied as in the procedure by Long et al. [15,28]. For Case 2, the Yeoh model parameters are calibrated using the 0.00667 s^{-1} strain rate in the literature [18]. The parameters are based on the Yeoh model expression in Equation (A1). The Yeoh model material parameters are documented in Table A2. However, for Case 3 [20], the Ogden and Yeoh models fail to fit the experiment. Thus, the linear interpolation is used to access the quasi-static stress–strain response.

$$N_{33} = 2\left(\lambda - \frac{1}{\lambda^2}\right)(C_{10} + 2C_{20}(I_1 - 3) + 3C_{30}(I_1 - 3)^2) \quad (\text{A1})$$

Table A2. Material model parameters of Yeoh model for Case 1 (porcine meniscus) and Case 2 (ovine brain). The unit is MPa.

Case	C_{10}	C_{20}	C_{30}
Case 1	8.3498×10^{-1}	3.0237×10^{-1}	0.0000
Case 2	2.8332×10^{-4}	6.9382×10^{-4}	3.0616×10^{-4}

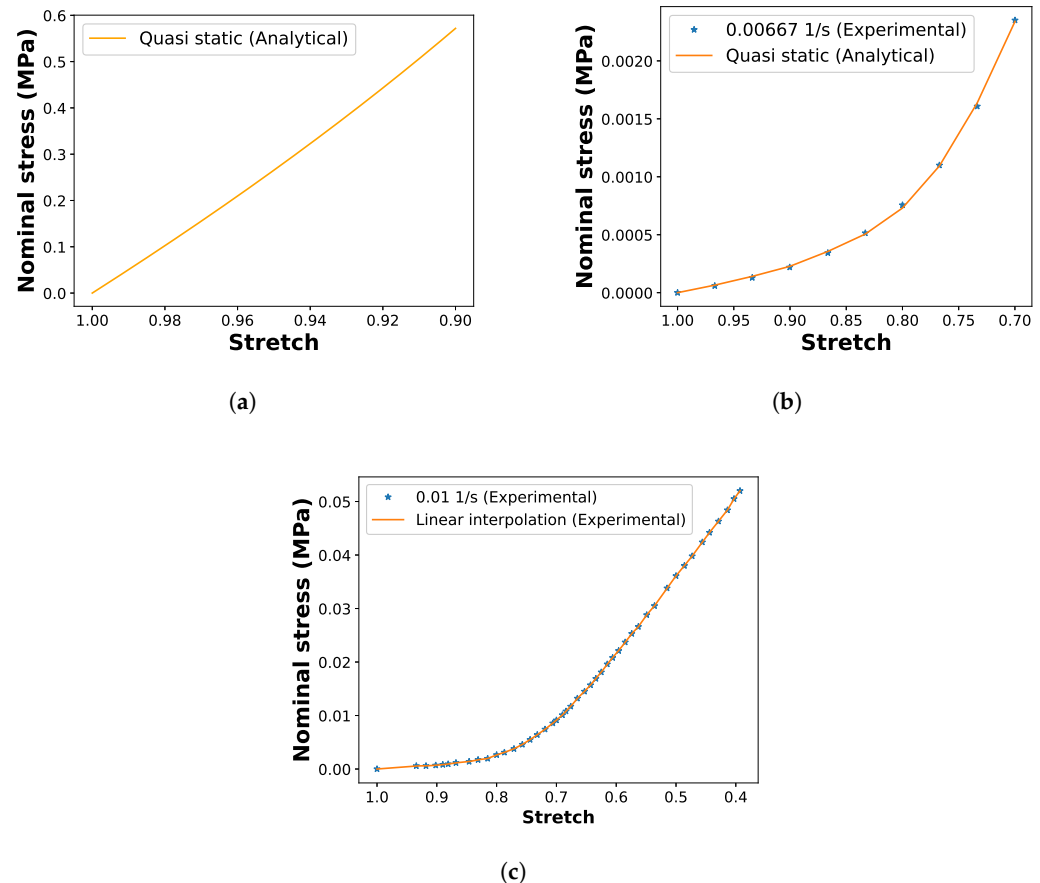


Figure A1. Nominal stress vs. stretch response for Case 1, Case 2, and Case 3 under quasi-static deformation. (a) Case 1: porcine meniscus. (b) Case 2: ovine brain. (c) Case 3: bovine liver.

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