



Article Study on Hysteresis Model of Welding Material in Unstiffened Welded Joints of Steel Tubular Truss Structure

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Featured Application: The main purpose of this study is to present a damage constitutive model of welded joints in steel structures, which can provide a basis for the precise design of steel structure joints. It can be popularized and applied in steel structure building, hydraulic steel structure pump house, dike and so on in field of civil engineering and hydraulic engineering. It also can be used for reference in the field of mechanical manufacturing.

Abstract: The weld form of intersecting joints in a steel tubular truss structure changes with the various intersecting curves. As the key role of joints in energy dissipation and seismic resistance, the weld is easy to damage, as a result the constitutive behavior of the weld is different from that of the base metal. In order to define the cumulative damage characteristic and study the constitutive behavior of welded metal with the influence of damage accumulation, low-cycle fatigue tests were carried out to evaluate overall response characteristics and to quantify variation of cyclic stress amplitude, unloading stiffness and energy dissipation capacity. The results show that the cyclic softening behavior of welding materials is apparent, however, the steel shows hardening behavior with the increase of cyclic cycles, while the cyclic stress amplitude, unloading stiffness, and energy dissipation capacity. Based on the Ramberg–Osgood model and introducing the damage variable *D*, a hysteretic model of welding material with the effect of damage accumulation was established, including an initial loading curve, cyclic stress-strain curve, and hysteretic curve model. Further, the evolution equation of *D* was also built. The parameters reflecting the damage degradation were fitted by the test data, and the simulation results of the model were proved to be in good agreement with the test results.

Keywords: damage accumulation; unstiffened welded joints of steel tube truss; welding material; low-fatigue test; evolution equation of damage accumulation; hysteretic model; parameter fitting

1. Introduction

Since steel tubular truss structures have fluent joint construction, favorable seismic behavior and a handsome visual effect, they have been widely used and researched in large public buildings such as stadiums, airports and bridges [1]. For example, 48 railway truss bridges were used as case studies by Khademi [2,3] to study hybrid structural analysis and the procedures that were developed to enhance the load rating of railway truss bridges. There are also many other investigations for the behavior of trusses [4–12]. Still, the energy dissipation of a steel tubular truss structure depends mainly on the welded joint under earthquake. The intersecting line of the joint is a continuous spatial curve, which leads to the weld form needing constant change, so the welding process usually needs to be

cyclic are prone to damage accumulation, which is observed to be a process that can cause immediate failure when the load is not large enough [13]. It should be pointed out that damage accumulation has such an important impact on the performance and service life of structure [14–19], and even several studies have focused on the damage detection of the construction structure and material [20–24], but there is little consideration of the weld damage accumulation in the study of seismic performance of steel tubular truss structure joints [25–28], and the difference of the constitutive relation between the welding material and base metal is often neglected. For instance, the research results of Wang [29] revealed that the welding material and its parent material of 304 L steel have different hardening laws and damage evolution laws under cyclic loading. In addition, the size and shape of the yield surface of material will change continuously under cyclic loading, which is quite different from the constitutive behavior under a monotonic load. If the stress and strain distribution of the large cyclic deformation is calculated only by the plastic model, such as the perfect elastic-plastic and bilinear model that is always used for monotonous loading, it will result in a greater difference from the actual process.

It is critical to find a constitutive model that can be used to describe the cyclic characteristics of welding materials. Shen [30,31] put forward a hysteresis model of steel, which has been considered as the damage accumulation of steel under cyclic loads. The model has good applicability but did not consider the effect of nonlinear hardening. A constitutive model of piezoelectric materials with damage is presented by Sun et al. [32], meanwhile, the evolution for damage accumulation is developed by using the finite difference method and the Newmark scheme. Shi et al. [33] conducted the cyclic experiments of Q235B and Q345B structural steel, and a hysteretic model of structural steel under cyclic loading was proposed. The model has proven to be effective and can be used in finite element analysis. Van Do et al. [34] derived a cyclic constitutive model about a super duplex stainless steel, the model was established based on the Chaboche and the Burlet and Cailletaud model, which can reflect the nonlinear kinematic hardening rule. Wang et al. [35] studied the cyclic behavior of law yield point steels (LYP100 and LYP160), then the Ramberg–Osgood model was used to fit the cyclic skeleton curves of low yield point (LYP) and the parameters for the combined hardening model were obtained. The form of the Ramberg–Osgood relationship was also verified and can be used to define the cyclic stress-strain curves of structural carbon steel by Nip et al. [36]. Besides, the difference between the Chaboche model and the Ramberg–Osgood model in simulating and analyzing the cyclic loading characteristics of low alloy 42CrMo steel was compared and analyzed by Basan et al. [37], which showed that the two models have similar results on the premise of stable material performance. In recent years, the Ramberg–Osgood model has been adopted in other research to describe the stress-strain constitutive relation of metallic materials [38-42].

In conclusion, numerous previous studies have shown that the Ramberg–Osgood model [43] has good applicability in describing the cyclic stress-strain relationship of metallic materials, but little related research has taken into account the damage accumulation in the hysteretic model and constitutive model. On the other hand, the study showed that there are obvious differences in the cyclic characteristics between the welding material and its parent material. For weld, Corigliano et al. [44] investigated the behavior of welded T-joints under low-cycle fatigue though tests and nonlinear finite element analysis, which has contributed to the predicting model of fracture and fatigue behavior for welded joints. Some other studies [45–50] relating to the constitutive model of the different materials of a weld also should be mentioned, but the research on the cyclic constitutive model for the welding material of steel tubular truss structure joints has still not been fully carried out, and there is also a lack of model parameters that can reflect the damage deterioration of the welding material. Thus, the cumulative damage evolution rule [51–55] of welding materials also should be paid attention to. Consequently, an experimental investigation was performed in this research on the welding material of steel tubular truss structure joints under law cyclic loading. The experimental details are presented and the observations of test, the stress-strain responses, as well as the hysteretic properties and energy dissipation capacity, were discussed. The cumulative evolution law of the welding material

was obtained through experiments. Based on the experimental results, a damage cumulative evolution equation of welding material was established, and a hysteresis model of welding material considering damage cumulative effects was constructed. Then, the damage parameters of the hysteresis model were fitted by the experimental data. Finally, the results calculated by the hysteretic model were compared with the experimental results.

2. Experimental Study

2.1. Specimen Design

There are butt welds and fillet welds in the welded joints of the steel tubular truss structures, as illustrated in Figure 1. For instance, there are butt welds near the crown points, fillet welds around the heel point, and fillet welds around the saddle point. According to the two kinds of weld forms and welding process, the model of T type fillet welds and butt welds were welded by the advanced welders, which fully penetrated the Q235 and Q345 law carbon steel produced in accordance with Chinese standard [56] that were used as the base material. The welding process satisfied the requirements prescribed in the Chinese national standard for the welding of a steel structure (JGJ81-2002) [57]. Then, the standard specimens of weld metal, steel in heat-affected zone, and steel before welding were extracted from the model. The butt weld and the fillet weld model as the sampling position of the standard specimens are shown in Figure 2. The steel specimens are extracted in vertical and parallel to rolling directions to study whether the rolling direction has an effect on the cyclic characteristics of the specimens. The specific dimensions of the specimens are depicted in Figure 3a. The specimens were marked based on the steel strength grade, welding type, and loading pattern, as presented in Table 1, where the labeled numbers of (1)–(3) in each group were designed for different loading patterns.

Table 1. Labels of the specimens of low cycle fatigue test.

	Steel Specimen before Welding Welding Material			Steel of Heat-Effected Zone						
Type for Base Metal	Parallel Rolling	ng Vertical Rolling Data Market		Vertical Rolling Fillet Weld of T Type		Fillet Weld of T Type		D	Fillet Weld of T Type	
Dase Wietai	Direction	Direction	Butt weld	Left	Right	Butt weld	Left	Right		
Q235	SA1 (1)–(3)	SA2 (1)–(3)	WA1 (1)–(3)	WA2 (1)–(3)	WA3 (1)–(3)	HA1 (1)–(3)	HA2 (1)–(3)	HA3 (1)–(3)		
Q345	SB1 (1)–(3)	SB2 (1)–(3)	WB1 (1)–(3)	WB2 (1)–(3)	WB3 (1)–(3)	HB1 (1)–(3)	HB2 (1)–(3)	HB3 (1)–(3)		



Figure 1. Weld distribution of intersecting joints in steel tubular truss structure.



Figure 2. Acquisition of the standard specimen. (**a**) Sampling of fillet weld(mm); (**b**) Sampling of butt weld (mm).



Figure 3. Experimental specimen and setup. (**a**) Dimensions of specimen (mm); (**b**) Experimental setup and instrumentation.

2.2. Test Procedure

The specimens were loaded by an electro-hydraulic servo fatigue testing machine (DPL-9010, DEXKC, Chengdu, China), as shown in Figure 3b. Low cyclic loads were applied to test the cyclic behavior of the specimens. And the loading was realized by a hydraulic servo drive technology and controlled by computer. As the rapid development of deformation after yielding, strain controlled loading was adopted though the measured displacement data from the extensometer over the gage length (see Figure 3a,b). The gage length and the elongation of the extensometer are 25 mm and 10% respectively, therefore, the strain values could be obtained from the displacements divided by 25 mm. The specimens were loaded until fracture, although the extensometer was dismantled and replaced by the displacement control when the wide fatigue cracks appeared in the specimen or the stress amplitude attenuated to around 50%, so as to prevent the extensometer from being damaged caused by the sudden breaking of the specimen. On the other hand, the stress could be obtained from the load divided by the section area of the standard specimen. After the test, the stress-strain data were outputted to depict the hysteretic curves.

The loading patterns, which are described in Figure 4, for weld metal, steel in the heat-affected zone and steel before welding were imposed. As showed in Figure 4, the tensile and compress cyclic

loads were applied in the form of symmetrical triangular waves. The loading rate is 0.5%, and the strain ratio is $-1(\varepsilon_{max}/\varepsilon_{min} = -1)$. In order to study the effect of the loading strain amplitude on the hysteretic model, the loading strain amplitude of each group of specimens ranges from 0.2% to 0.5%, that is, the loading strain amplitudes of the specimens numbered (1), (2), (3) are 0.25%, 0.35% and 0.45%, for specimens of weld, steel, and the heat-effected zone. As a result, Figure 4a corresponds to all No. (1) specimens, Figure 4b corresponds to all No. (2) specimens, and Figure 4c corresponds to all No. (3) specimens. Table 2 summarizes the primary mechanical parameters of the specimens, which were obtained by uniaxial tensile test (averaged over three specimens).



Figure 4. Cyclic loading patterns. (**a**) For all No. (1) specimens; (**b**) For all No. (2) specimens; (**c**) For all No. (3) specimens.

Specimens	fy/MPa	f _u /MPa	ϵ_y /%	$\varepsilon_{\rm u}$ /%	E/GPa
SA1	268.8	430.1	0.15	16.3	207.1
SA2	268.1	429.7	0.15	16.4	195.7
WA1	391.7	497.6	0.16	13.0	239.7
WA2	401.6	486.8	0.17	12.7	233.6
WA3	402.3	497.8	0.17	11.8	213.0
HA1	254.7	433.5	0.13	15.8	223.3
HA2	259.2	423.6	0.13	15.5	203.6
HA3	255.4	422.9	0.13	15.6	204.5
SB1	365.6	532.6	0.16	16.0	216.1
SB2	385.1	540.1	0.15	15.2	217.9
WB1	420.1	498.1	0.15	12.7	235.5
WB2	426.3	508.5	0.17	11.7	251.0
WB3	431.5	525.4	0.17	11.3	217.6
HB1	365.2	531.0	0.16	16.0	235.0
HB2	357.8	530.5	0.15	15.3	225.1
HB3	358.5	527.6	0.16	15.2	218.3

Table 2. Mechanical parameters under monotonic loading.

3. The Test Results and Discussion

3.1. Failure Model and Damage Processes

Figure 5 shows three stages of cumulative damage of welded specimens under low-cycle cyclic loading. The first stage that can be observed is the initial stage of damage process, which was characterized by the initiation of cracks on the surface of the specimen. As the loading cycles increased, micro cracks could be observed on the surface of the specimen, as shown in Figure 5a. This stage was typically short in duration, and the crack occurred in the slip band of the micro-defect or weak position of the material, which then developed internally along the direction of the maximum shear stress (45° with the principal stress). As shown in Figure 5b, with the increase of loading cycles, the damage accumulated continually and the specimen gradually entered into the second stable stage, in which the crack began to widen and the direction of the crack propagation gradually turned perpendicular to the principal stress. This stage has the longest duration and the crack developed relatively slow,

which is considered to be the main stage of damage accumulation. The last stage is the failure stage, as shown in Figure 5c, where the specimens were fractured at the maximum main crack.



Figure 5. Damage process of welded specimen in low cycle fatigue test. (**a**) Crack appeared in specimen; (**b**) Crack develop; (**c**) Specimen fractured at the maximum main crack.

3.2. Cyclic Behavior and Damage Analysis

The number of semi-cycles until the specimens were fractured are listed in Table 3, which are about the welding material and its base material of welded joints in the steel tubular truss structure. It can be revealed by Table 3 that the cyclic life of the welded metal is less than that of the base metal with the same working condition, which indicates that the speed of damage accumulation of the welding material is faster than that of the steel, in other words, the welding materials accumulate damage more easily than the base materials. The cycling life of the welding materials decreases with the increase of the loading strain amplitude. The higher the strain amplitude level, the faster the degradation rate of the specimens under cyclic loading, and resulting in earlier destruction. Table 3 also shows that the influence of the steel rolling direction on the cycling life of steel is mainly reflected in Q235 steels.

Spacimona		V	Velded S	pecimer	ns			Steel Sp	ecimens	•
Specimens	WA1	WA2	WA3	WB1	WB2	WB3	SA1	SA2	SB1	SB2
(1) (0.25%)	3091	3810	3430	4351	4205	4293	4321	3789	4766	4703
(2) (0.35%)	1865	1801	1831	2223	1992	1973	2109	1983	2525	2607
(3) (0.45%)	1089	994	977	1559	1165	1106	1301	976		1639

Table 3. Number of half cycles of welding material and steel.

The cyclic stress-strain (σ - ε) curves of welded specimens and steel specimens are shown in Figures 6 and 7, respectively, in which the abscissa axis represents the strain of the specimen, and the ordinate axis represents the stress. It can be seen that the stress-strain constitutive relation of the materials reflected by the hysteretic curves changes with the increase of cyclic cycles under cyclic loading. As can be studied analytically, each of the semi-cycles contains two main stages, that is, the loading stage with curvilinear segment and the unloading stage with approximate liner segment, respectively. There is an apparent difference between the curves of welded materials and steels. The welded material shows obvious damage degradation, that is, the stress amplitudes increase slightly only at the beginning of the loading, then decrease gradually with the increase of cyclic cycles, which indicate that the cyclic softening behavior of the welding material is apparent as the imposed displacement cycles increase, on the contrary, the steel shows cyclic hardening characteristics. During the cycling process, the stress level of the welded materials is higher than that of the steel specimens under the same working condition, but the cycling life of the welded material is less than that of the steel. This indicates that the welded material was hardened to a certain extent

due to the influence of complex factors such as the welding process with high temperature, but its ability of plastic deformation has been weakened, which makes the welded material more prone to damage accumulation and damage degradation than its base metal. Furthermore, it is suggested that the constitutive relation between the welded metal and its base metal should be considered distinguishingly and the damage accumulation should be considered in the cyclic nonlinear analysis of the direct welded joints of the tubular truss structures.

Figure 6 contains the hysteretic curves of the welding materials under different conditions of weld seam types, base metal types, and loading strain amplitudes. It can be seen from the curves that in addition to the degradation of the cyclic stress amplitude, the hysteretic curve of the welded metal shows a gradual "down" trend with the increase of the cyclic cycles, that is, the damage accumulation caused degradation of bearing capacity and unloading stiffness. Furthermore, the area around the hysteresis loop decreases with the increase of the cyclic cycles, which means that the damage accumulation can also lead to a decrease in energy dissipation capacity. In a word, under cyclic loading, the damage of the welded metal was expanding and accumulating, which is the main reason for the performance degradation until the final failure. For the existing constitutive models of metallic materials, the damage degradation behavior of welded materials has been seldom considered and the failure mechanism of materials and structures due to damage accumulation cannot be accurately revealed.



Figure 6. Cont.



Figure 6. Stress-strain (*σ*-*ε*) hysteresis curve of the welded specimen. (a) WA1 (1) $\varepsilon_{max} = 0.25\%$; (b) WA1 (2) $\varepsilon_{max} = 0.35\%$; (c) WA1 (3) $\varepsilon_{max} = 0.45\%$; (d) WA2 (1) $\varepsilon_{max} = 0.25\%$; (e) WA2 (2) $\varepsilon_{max} = 0.35\%$; (f) WA2 (3) $\varepsilon_{max} = 0.45\%$; (g) WA3 (1) $\varepsilon_{max} = 0.25\%$; (h) WA3 (2) $\varepsilon_{max} = 0.35\%$; (i) WA3 (3) $\varepsilon_{max} = 0.45\%$; (j) WB1 (1) $\varepsilon_{max} = 0.25\%$; (k) WB1 (2) $\varepsilon_{max} = 0.35\%$; (l) WB1 (3) $\varepsilon_{max} = 0.45\%$; (m) WB2 (1) $\varepsilon_{max} = 0.25\%$; (n) WB2 (2) $\varepsilon_{max} = 0.35\%$; (o) WB2 (3) $\varepsilon_{max} = 0.45\%$; (p) WB3 (1) $\varepsilon_{max} = 0.25\%$; (q) WB3 (2) $\varepsilon_{max} = 0.35\%$; (r) WB3 (3) $\varepsilon_{max} = 0.45\%$.



Figure 7. Cont.



Figure 7. Stress-strain (*σ*-ε) hysteresis curve of the steel (base metal) specimen.(**a**) SA1 (1) $\varepsilon_{max} = 0.25\%$; (**b**) SA1 (2) $\varepsilon_{max} = 0.35\%$; (**c**) SA1 (3) $\varepsilon_{max} = 0.45\%$; (**d**) SA2 (1) $\varepsilon_{max} = 0.25\%$; (**e**) SA2 (2) $\varepsilon_{max} = 0.35\%$; (**f**) SA2 (3) $\varepsilon_{max} = 0.45\%$; (**g**) SB1 (1) $\varepsilon_{max} = 0.25\%$; (**h**) SB1 (2) $\varepsilon_{max} = 0.35\%$; (**i**) SB2 (1) $\varepsilon_{max} = 0.25\%$; (**j**) SB2 (2) $\varepsilon_{max} = 0.35\%$; (**k**) SB2 (3) $\varepsilon_{max} = 0.45\%$.

3.3. Variation of Cyclic Stress Amplitude

As seen in Figure 8, obtained through Figures 6 and 7 are curves about the variation of the cyclic stress amplitudes with increases to the cyclic cycles of the welding materials and steels, in which the non-dimensional parameter η was obtained from the numbers of semi-cycles in a certain time divided by the numbers of semi-cycles at the end of the test. Each picture in Figure 8 contains three stress amplitude- η curves that were obtained from three loading conditions with different strain amplitudes, which can be realized from Table 1 and Figure 4. Therefore, the influence of weld form, base material type, and loading mode on the damage accumulation of the welding materials will be compared and analyzed.

As shown in Figure 8a–f, the degradation of the cyclic stress amplitude is basically divided into three stages, corresponding to the three stages of crack development in a low-cycle fatigue test. The stress amplitude degenerated rapidly in the first stage, which has a number of cycles less than 20%, corresponding to the initial stage of fatigue crack development. When the numbers of cycle reached about 20%, the stress amplitude decreased steadily, and corresponded to the stable stage of fatigue crack development, which is the longest stage of stress amplitude degradation. Finally, the stress amplitude degenerated sharply and the specimens were fractured rapidly when the number of cycles reached about 80% or 90%. Loading strain amplitude has a certain effect on the damage degradation rate. As we can see, there are three curves in each picture of Figure 8a–f, and the loading train amplitude of the curves increased in turn. It indicates that the cyclic stress amplitude increased with the increasing of the controlled strain amplitude. Moreover, the specimens with larger controlled strain amplitude degenerated faster in the failure stage. All of the above indicate that the larger the controlled loading strain amplitude, the faster the rate of damage accumulation.

The influence of type of specimens also should be discussed. As shown in Figure 8g–h, the stress amplitude- η curves of WA and WB welding specimens are compared at the same time. It can be concluded that the stress amplitude of the welding materials with Q235 base metal degenerated faster than that of the Q345 base metal, and the curves of WB specimens are more concentrated than those of WA specimens, indicating that the influence of the loading strain amplitude in the rate of damage accumulation is more significant for welding materials with a Q235 base steel. For the influence of the welding mode, the welded form of No. (1) specimens with the Q235 and Q345 base metal is a butt

weld (the dotted line in Figure 8g–h), whose stress amplitudes are lower than that of the fillet weld specimens under the same working conditions. This can be attributed to the fact that the structure and welding process of the fillet welds are more complex than the butt welds, resulting in more damage accumulation than the fillet welds.



Figure 8. Stress amplitude-dimensionless (σ_m - η) half cycle of welding material and steel. (a) WA1; (b) WA2; (c) WA3; (d) WB1; (e) WB2; (f) WB3; (g) WA; (h) WB; (i) SA1; (j) SB2.

3.4. EnergyDissipation Behavior

Referring to the consideration of energy dissipation in the Park-Ang damage cumulative model [58], the cumulative energy dissipation is dealt with in the dimensionless method, and the energy dissipation capacity is reflected by the cumulative energy dissipation coefficient ξ_a . The expression is shown in Equation (1) [58]. E_u is the ultimate hysteretic energy under monotonic loading, which can be calculated from the area enclosed by the stress-strain curves of monotonic tensile test and the coordinate axis. E_i is the actual dissipated energy of the No. *i* semi-cycle, which can be calculated

from the area enclosed by the cyclic stress-strain hysteretic curves of cyclic loading test and the coordinate axis. Then, ξ_a - η curve of welded material is depicted in Figure 9, it can be seen that the slope of the curves decreases with the increase of cyclic cycles, which means that the rate of cumulative energy dissipation decreases gradually, in other words, the enclosing area of the hysteretic loops decreases with the increase of cyclic cycles, and the energy dissipation capacity decreases gradually. It is due to the continuous damage accumulation of the welding materials under cyclic loading. In a word, the cumulative damage rule of the welding material can be discussed through cumulative energy dissipation behavior is consistent with the law of stress amplitude degradation. That is, with the continuous development of damage accumulation under cyclic loading, the amplitude of cyclic stress, unloading stiffness, and energy dissipation capacity of the welded metal, these degenerate gradually until failure occurs, and compared with the base metal, the welded metal is more prone to damage accumulation.



Figure 9. $\xi_a - \eta$ curve of welding material. (a) WA1 $\xi_a - \eta$; (b) WA2 $\xi_a - \eta$; (c) WA3 $\xi_a - \eta$; (d) WB1 $\xi_a - \eta$; (e) WA2 $\xi_a - \eta$; (f) WA3 $\xi_a - \eta$.

4. An Evolution Equation of Damage Accumulation for Welding Materials

Damage variable *D* is used to represent the damage degree of material in damage theory [59]. Meanwhile, the experimental results show that the macroscopic mechanical behavior of the damage of the welding material can be expressed in both energy and deformation. Therefore, in order to describe the microscopic structural damage mechanism of the material in macroscopic mechanics, the evolution equation of the damage variable *D* of the welding material under cyclic loading is established based on the experimental results. As shown in Equation (2), the Park-Ang [58] model is suitable for structural damage cumulative analysis, in which δ_m and δ_u is the maximum deformation with loading and ultimate deformation of material respectively, $\int dE$ is the cumulative dissipation of plastic energy, $F_y \delta_u$ is equivalent to the ultimate hysteretic energy under the monotonic load of perfect elastic-plastic condition.

Based on the above model, an evolution equation of damage variable D (2) for calculating the cumulative damage of the welding material is established, as seen in Equation (3), in which the effects

of plastic strain and cumulative plastic energy dissipation on the damage accumulation of the welding material are considered comprehensively. In Equation (3), ε_m^p is the maximum plastic strain in cyclic process, ε_u^p is the ultimate plastic strain of materials that can be obtained from the uniaxial tensile test, E_i is the plastic deformation energy dissipation of No. *i* semi-cycles, which is equal to the area of the hysteresis loop in numerical value, E_u is the ultimate hysteretic energy under monotonic loading, which can be calculated from the area enclosed by the stress-strain curves of the monotonic tensile test and the coordinate axis, λ is a parameter means the weight of cumulative plastic energy dissipation, which can be calculated by the cyclic loading test results. Compared with Equation (2), the energy part of the model in Equation (3) is no longer confined to the perfect elastic-plastic condition, and the fact that the value of damage variable *D* is always less than or equal to 1 until fractured is ensured.

$$D = \frac{\delta_m}{\delta_u} + \beta \frac{\int dE}{F_y \delta_u} \tag{2}$$

$$D = (1 - \lambda)\frac{\varepsilon_{m}^{p}}{\varepsilon_{u}^{p}} + \sum_{i=n_{1}}^{n} \lambda \frac{E_{i}}{E_{u}}$$
(3)

5. A Hysteresis Model with Damage Accumulation of Welding Materials

5.1. Basic Requirements of the Model

The damage cumulative evolution law of the welding material under cyclic loading was revealed by the experimental study. That is, with the increase of cyclic loading cycles, its plastic strain will be accumulated, and the cyclic stress amplitude, stiffness and energy dissipation capacity will be degenerated with damage accumulation. The ideal elastic-plastic, bilinear or isotropic and dynamic hardening models without considering the effect of damage accumulation cannot simulate the process of plastic strain accumulation and damage degradation, for which, the hysteretic curves will only follow the isotropic or isotropic hardening path, and there is no degradation of amplitude and stiffness. For the elastic-plastic model, the hysteresis curves are basically identical while the loading amplitude remains unchanged, and the curves are linear in the hardening and plastic stage, as shown in Figure 10a, which is different from the constitutive behavior of the welded materials under cyclic large deformation, and is not consistent with the damage degradation law of welded materials under cyclic loading. Therefore, as shown in Figure 6b, a hysteretic model can used for welding material, which considering the effect of damage accumulation, should be constructed to satisfy the following requirements: (1) it can reflect the effect of damage accumulation under cyclic loading; (2) every semi-cyclic hysteretic curve calculated by the hysteretic model will change constantly, and the expression of the curve should be recursive; (3) it can reflect the nonlinear strengthening.



Figure 10. Hysteresis curves under different stress-strain constitutive relations. (a) Bilinear model; (b) Basic requirements of the hysteresis model.

The expression of the initial loading curve is shown in Equation (4), which is established though the uniaxial tensile test for a welded material and the curve is depicted in Figure 11a, in which the stages named I, II, and III correspond to the formulas in Equation (4), respectively. The constitutive equation contains the effect of damage, and the parameters will be fitted by the uniaxial tensile test results obtained before the cyclic test. Figure 11b is the fitting result of the WA1 specimen. And the results of the other specimens are shown in Table 4.

$$\begin{cases} \sigma = E\varepsilon \qquad ; \varepsilon \le \varepsilon_{y1} \\ \sigma = \sigma_y \qquad ; \varepsilon_{y1} < \varepsilon \le \varepsilon_{y2} \\ \sigma = [1 - (D_C \frac{\varepsilon - \varepsilon_0}{\varepsilon_C - \varepsilon_0})] K(\varepsilon + b)^m \quad ; \varepsilon > \varepsilon_{y2} \end{cases}$$
(4)

Base Metal	Welding Material	E/Gpa	σ_y /MPa	K/Gpa	ı <i>m</i>	$\varepsilon_{y1}/\%$	$\varepsilon_{y2}(\varepsilon_0)/\%$	$\varepsilon_c/\%$	b	$D_C/\%$
Q235	Butt weld	239.70	391.70	2.73	0.77	0.16	1.50	21.25	0.065	59.24
	Fillet weld	223.30	401.94	1.82	0.54	0.18	1.50	21.34	0.046	50.03
Q345	Butt weld	247.71	420.07	2.52	0.65	0.17	1.70	17.71	0.052	57.64
	Fillet weld	241.07	433.92	2.17	0.60	0.18	1.70	16.97	0.044	53.11

Table 4. Parameter values of initial loading curve.

III 500 400 σ 005م/MPa 200 100 Formula fitting Tset 0 0.15 0.10 0.20 0.25 0.05 $\varepsilon_{y2}(\varepsilon_0)$ ε_{y1} E_u ε, ε ε

Figure 11. Fitting results of the initial loading curve. (**a**) The schematic diagram of the initial loading curve; (**b**) Fitting result of the WA1 specimen.

5.3. Cyclic Stress-Strain Curve Based on Ramberg–Osgood Model

(a)

The stress-strain hysteretic curves obtained from different loading strain amplitudes are placed in the same coordinate system, and the maximum and minimum stress peaks are connected to obtain the cyclic stress-strain curves, just as depicted in Figure 12, which does not represent the true stress-strain path of the welded material under cyclic loading, but is a cyclic stress-strain curve under steady state. Researches [33,60,61] show that the cyclic stress-strain curves of steady-state are different from those under monotonic loading, the cyclic stress-strain curve can be described by the Ramberg–Osgood [43] model approximately, as presented in Equation (5). E_0 and K_0 are the initial elastic modulus and hardening coefficients, respectively, which can be fitted by the Low-cycle fatigue test of the welding materials. The parameters of Equation (5) are summarized in Table 5.

$$\varepsilon = \begin{cases} \frac{\sigma}{E_0} + \left(\frac{\sigma}{K_0}\right)^{n_0} & ; \sigma > 0\\ \frac{\sigma}{E_0} - \left(\frac{-\sigma}{K_0}\right)^{n_0} & ; \sigma \le 0 \end{cases}$$
(5)

(b)

Got the logarithm of Equation (5), just as Equation (6), then, let $y = \ln(\varepsilon - \frac{\sigma}{E_0})$, $x = \ln \sigma$, $b = -n_0 \ln K_0$, Equation (6) will transform to Equation (7), thus, linear fitting can be performed.

$$\ln(\varepsilon - \frac{\sigma}{E_0}) = n_0(\ln \sigma - \ln K_0)$$
(6)

$$y = n_0 x + b \tag{7}$$



Figure 12. Schematic diagram of cyclic stress-strain curve.

Table 5. Parameters of the cyclic stress-strain curves for welding materials.

With Q235 Base Metal				Wi	th Q345 B	ase Metal	
Specimens	E ₀ /GPa	K ₀ /MPa	<i>n</i> ₀	Specimens	E ₀ /GPa	K ₀ /MPa	n_0
WA1	239.700	1266	4.784	WB1	247.710	953	8.099
WA2	223.000	1331	5.531	WB2	241.070	939	8.610
WA3	223.000	1466	6.203	WB3	241.070	885	9.272

5.4. A Model of Hysteretic Curve with Damage Accumulation

The Ramberg–Osgood model has proven to be a good description of the cyclic stress-strain relationship of the metallic materials by many studies, and the Masing criterion shows that the stress-strain path under cyclic loading can be described by amplifying 1 time the cyclic stress-strain curve in a steady state [62–65], such as explained in Figure 13. Based on the theory, and considering the effect of the damage, the evolution equation of damage variable D is introduced to construct a model of the hysteretic curve for the welding material. As shown in Figure 14, a hysteresis model with damage accumulation of welding materials is presented, in which the No. n semi-cycle is shown as an example to describe the hysteresis criterion.



Figure 13. Masing behavior.

Curve A_n - C_n - B_n is the loading curve of No. *n* semi-cycle, which is constructed by considering nonlinear loading. Point A_n is the starting point of the *n* semi-cycle loading curve, and B_n is the final point of the loading curve, but is the starting point of the *n* semi-cycle unloading curve. $\sigma_m^{D(n)}$ is the stress amplitude of the No. *n* semi-cyclic stress-strain curve and the ordinate of point B_n . The stress amplitude of each semi-cycle will be degraded with damage accumulating, so $\sigma_m^{D(n)}$ is thought of as the damage stress amplitude, which can be calculated by Equation (8) based on damage mechanics theory, σ_m is the initial stress amplitude, D_n is the cumulative damage variable after *n*th semi-cycle, which can be calculated by Equation (3), η and ξ are the damage parameters that can be fitted by the low-cycle fatigue test results of the welding material. Curve B_n - A_{n+1} is the unloading curve of No. *n* semi-cycle, and the unloading process is approximately elastic. E_n^D is the elastic modulus with damage accumulation, which in respect of D_n can be calculated by Equation (9) based on damage mechanics theory and Shen [30,31]. E_0 is the original elastic modulus, *g* and *h* are the damage parameters like η and ξ .

$$\sigma_m^{D(n)} = (\eta - \xi D_n)\sigma_m \tag{8}$$

$$E_n^D = (g - hD_n)E_0 \tag{9}$$



Figure 14. Hysteretic model of welding material considering cumulative damage under cyclic loading.

Overall, the hysteresis model considering damage accumulation of welding material is depicted in Figure 14, the expressions of the model are as follows:

(1) unloading curve (Linear)

$$\sigma_n = E_n^D \varepsilon_n + t_n \tag{10}$$

$$t_n = (\sigma_m^{D(n)} - E_n^D \varepsilon_B) \tag{11}$$

In Equations (10) and (11), σ_n and ε_n are stress and strain values of point at *n*th semi-cycle curves, t_n is the parameter for determining the shape of the curve, ε_B is the strain amplitude.

(2) loading curve (nonlinear)

The expression of the model is established based on the damage degenerating law, Ramberg–Osgood model and Masing behavior.

$$\varepsilon_{n} = \begin{cases} \frac{\sigma_{n} - \sigma_{m}^{D(n)}}{E_{n}^{D}} + \left(\frac{\sigma_{n} - \sigma_{m}^{D(n)}}{K'}\right)^{n'} + \varepsilon_{B} \quad ; (\sigma_{n} - \sigma_{m}^{D(n)}) > 0\\ \frac{\sigma_{n} - \sigma_{m}^{D(n)}}{E_{n}^{D}} - \left(\frac{\sigma_{m}^{D(n)} - \sigma_{n}}{K'}\right)^{n'} + \varepsilon_{B} \quad ; (\sigma_{n} - \sigma_{m}^{D(n)}) < 0 \end{cases}$$
(12)

In Equation (13), $\sigma_m^{D(n)}$ and E_n^D are degenerated according to Formulas (10) and (11), K' and n' can be fitted by the data of the low-cycle fatigue test. Equation (13) shows that, considering the effect of damage accumulation, the stress-strain paths of each semi-cycle determined by the model do not coincide completely even in the case of constant amplitude cycling. And the unloading stiffness, bearing capacity, and energy dissipation capacity are irreversibly degraded with the increase of the number of half cycles, which is consistent with the damage accumulation law of the welding materials reflected in the test.

In conclusion, the hysteresis model with damage accumulation for welding materials is as follows:

$$\varepsilon_{n} = \begin{cases} \frac{\sigma_{n} - \sigma_{m}^{D(n)}}{E_{n}^{D}} + \left(\frac{\sigma_{n} - \sigma_{m}^{D(n)}}{K_{n}^{D}}\right)^{n'} + \varepsilon_{B} \quad ;d|\sigma| \ge 0, (\sigma_{n} - \sigma_{m}^{D(n)}) \ge 0\\ \frac{\sigma_{n} - t_{n}}{E_{n}^{D}} \quad ;d|\sigma| 0 \qquad (13)\\ \frac{\sigma_{n} - \sigma_{m}^{D(n)}}{E_{n}^{D}} - \left(\frac{\sigma_{m}^{D(n)} - \sigma_{n}}{K_{n}^{D}}\right)^{n'} + \varepsilon_{B} \quad ;d|\sigma| \ge 0, (\sigma_{n} - \sigma_{m}^{D(n)}) \le 0 \end{cases}$$

As shown in Figure 14, the hysteresis model can be calculated by Equations (3), (8), (9) and (13), and the effect of damage accumulation is considered to start from the second semi-cycle. Damage degradation is considered by introducing the evolution equation of the damage variable *D*. The realization process of the hysteresis criterion is: the initial stress amplitude under cyclic loading is calculated by the cyclic stress-strain curve and the initial loading curve \rightarrow the damage variable *D*_n of each semi-cycle is calculated by Equation (3) \rightarrow the stress amplitude $\sigma_m^{D(n)}$ and the elastic modulus E_n^D are calculated by Equations (8) and (9) \rightarrow the hysteresis curve of the *n*th semi-cycle can be obtained by Equation (13) \rightarrow then calculate the (*n* + 1)th semi-cycle, and so on.

5.5. Parameter Fitting of the Model

5.5.1. The Parameter λ of EvolutionEquation of Damage Accumulation

The cumulative damage variable *D* will be valued 1 when the specimen is destroyed, and calculate the maximum plastic strain ε_m^p and the cumulative plastic energy dissipation in the cyclic process, then the parameter λ can be obtained use Equation (3), so the value of λ can fitaccording to Equation (14) by the test data, and the fitting results are presented in Table 6.

$$\lambda = \frac{1 - \frac{\varepsilon_m^p}{\varepsilon_u^p}}{\sum\limits_{n_1}^n \frac{E_i}{E_m} - \frac{\varepsilon_m^p}{\varepsilon_u^p}}$$
(14)

	Welding Material of Q235 Steel			Welding Material of Q345 Steel				
Specimens	Butt Weld	Fillet	Weld	Butt Weld	Fillet	Weld		
	WA1	WA2	WA3	WB1	WB2	WB3		
(1)	0.02529	0.02609	0.02871	0.02311	0.02156	0.02176		
(2)	0.02233	0.02538	0.02613	0.01905	0.01909	0.02433		
(3)	0.02905	0.03043	0.03280	0.02071	0.02326	0.02230		
Model Parameters	0.0256	0.0273	0.0294	0.0209	0.02130	0.0227		

Table 6. Values of Parameter λ .

5.5.2. The Parameters η , ξ , g, h, K' and n'

(1) Damage Parameters η , ξ , g, h

The parameters can be fitted according to Equations (8) and (9) by the results of low-cycle fatigue test. Equations (8) and (9) can be transformed into Equations (15) and (16), and the curves of $\frac{\sigma_m^{D(n)}}{\sigma_m} - D_n$ and $\frac{E_n^D}{E_0} - D_n$ of WB1(2) are presented in Figure 15 (an example). Therefore, the parameters can be fitted according to Figure 15, the values of the parameters of all welding specimens are summarized in Tables 7 and 8.

$$\frac{E_n^D}{E_0} = g - hD_n \tag{15}$$

$$\frac{\sigma_m^{D(n)}}{\sigma_m} = \eta - \xi D_n \tag{16}$$



Figure 15. Parameter fitting of WB1 (2). (a) Curve $\frac{\sigma_m^{D(n)}}{\sigma_m} - D_n$; (b) Curve $\frac{E_n^D}{E_0} - D_n$.

Specimens -			I			II		III	
			η_1	D_1	ξ2	η_2	D_2	ξ3	η_3
	WA1(1)	0.509	0.961	0.228	0.099	0.873	0.897	1.799	2.350
Butt weld of	WA1(2)	0.475	0.998	0.239	0.105	0.875	0.866	1.879	2.355
Q235 steel	WA1(3)	0.485	1.001	0.231	0.087	0.897	0.860	1.770	2.362
	Model Parameters	0.489	0.986	0.233	0.097	0.882	0.874	1.816	2.356
	WA2(1)	0.530	1.001	0.247	0.057	0.858	0.883	1.191	1.871
Left fillet weld of	WA2(2)	0.455	0.997	0.241	0.058	0.901	0.886	2.158	2.790
Q235 steel	WA2(3)	0.456	0.998	0.226	0.059	0.899	0.859	2.157	2.788
	Model Parameters	0.480	0.999	0.238	0.058	0.886	0.876	1.825	2.483
	WA3(1)	0.445	1.002	0.251	0.086	0.887	0.863	1.167	1.843
Right fillet weld of	WA3(2)	0.500	1.000	0.243	0.045	0.889	0.858	2.078	2.678
Q235 steel	WA3(3)	0.525	0.999	0.229	0.077	0.896	0.886	1.996	2.675
	Model Parameters	0.490	1.000	0.241	0.069	0.891	0.869	1.747	2.399
	WB1(1)	0.468	0.999	0.235	0.058	0.902	0.899	2.551	3.151
Butt weld of	WB1(2)	0.534	1.002	0.228	0.065	0.893	0.869	1.832	2.427
Q345 steel	WB1(3)	0.508	0.989	0.236	0.075	0.887	0.889	2.235	2.912
	Model Parameters	0.503	0.997	0.233	0.066	0.894	0.885	2.206	2.830
	WB2(1)	0.535	0.998	0.233	0.068	0.889	0.865	1.622	2.233
Left fillet weld of	WB2(2)	0.625	1.002	0.241	0.077	0.868	0.887	1.620	2.237
Q345 steel	WB2(3)	0.612	1.000	0.231	0.095	0.881	0.859	1.634	2.210
	Model Parameters	0.592	1.000	0.235	0.080	0.879	0.865	1.625	2.227
	WB3(1)	0.505	0.998	0.219	0.064	0.902	0.875	2.279	2.841
Right fillet weld of	WB3(2)	0.446	0.997	0.241	0.048	0.901	0.887	2.587	3.153
Q345 steel	WB3(3)	0.455	0.998	0.246	0.063	0.902	0.878	2.284	2.854
	Model Parameters	0.469	0.998	0.235	0.058	0.902	0.880	2.383	2.949

Table 7. Values of parameters η , ξ .

Table 8. Values of parameters *g*, *h*.

Enosi		I + II		III		
Speci	Specificity			D_2	h_3	<i>g</i> 3
	WA1(1)	0.045	0.991	0.889	2.083	2.466
Butt weld of	WA1(2)	0.024	0.982	0.881	1.694	2.433
Q235 steel	WA1(3)	0.024	0.989	0.879	1.668	2.453
	Model Parameters	0.031	0.987	0.883	1.815	2.446
	WA2(1)	0.020	0.997	0.875	1.751	2.553
Left fillet weld of	WA2(2)	0.021	0.998	0.882	1.751	2.503
Q235 steel	WA2(3)	0.020	0.997	0.879	1.709	2.513
	Model Parameters	0.020	0.997	0.879	1.737	2.523
	WA3(1)	0.056	1.000	0.887	1.345	2.134
Right fillet weld of	WA3(2)	0.020	0.997	0.886	1.384	2.196
Q235 steel	WA3(3)	0.113	1.000	0.886	0.981	1.796
	Model Parameters	0.063	0.999	0.886	1.225	2.044
	WB1(1)	0.109	0.999	0.890	1.519	2.269
Butt weld of	WB1(2)	0.111	1.000	0.882	1.807	2.499
Q345 steel	WB1(3)	0.103	1.002	0.886	1.445	2.196
	Model Parameters	0.107	1.000	0.886	1.509	2.321
	WB2(1)	0.113	0.997	0.879	1.005	1.825
Left fillet weld of	WB2(2)	0.094	0.999	0.881	1.083	1.878
Q345 steel	WB2(3)	0.104	1.001	0.877	1.054	1.883
	Model Parameters	0.104	0.999	0.879	1.047	1.862
	WB3(1)	0.083	0.999	0.857	1.198	1.956
Right fillet weld of	WB3(2)	0.057	1.000	0.887	2.365	3.043
Q345 steel	WB3(3)	0.089	0.999	0.890	1.789	2.513
	Model Parameters	0.076	0.999	0.878	1.784	2.504

(2) *K'* and *n'*

The Parameters K' and n' can be fitted through Equation (12) and the test results, which are presented in Table 9.

Spee	cimens	ε _B /%	K'	n'
	WA1(1)	0.25	$2.26 imes 10^7$	0.546
TATA 1	WA1(2)	0.35	$2.65 imes 10^7$	0.499
WAI	WA1(3)	0.45	3.05×10^{7}	0.459
	Model Pa	rameters	$K'=3.95\times 10^9\varepsilon_B+1.27\times 10^7$	$n' = -43.5\varepsilon_B + 0.654$
	WA2(1)	0.25	$2.23 imes 10^7$	0.547
WA 2	WA2(2)	0.35	$2.64 imes10^7$	0.504
WAZ	WA2(3)	0.45	$3.05 imes 10^7$	0.461
	Model Pa	rameters	$K' = 4.10 \times 10^9 \varepsilon_B + 1.21 \times 10^7$	$n' = -43.0\varepsilon_B + 0.655$
	WA3(1)	0.25	$2.24 imes 10^7$	0.546
1474 2	WA3(2)	0.35	$2.64 imes 10^7$	0.505
WA3	WA3(3)	0.45	$3.04 imes 10^7$	0.464
	Model Pa	rameters	$K' = 4.00 \times 10^9 \varepsilon_B + 1.24 \times 10^7$	$n' = -41.0\varepsilon_B + 0.649$
	WB1(1)	0.25	$1.40 imes 10^7$	0.567
W/D1	WB1(2)	0.35	$1.63 imes 10^7$	0.521
VV D1	WB1(3)	0.45	$1.86 imes 10^7$	0.475
	Model Pa	rameters	$K'=2.30\times10^9\varepsilon_B+8.25\times10^6$	$n' = -46.0\varepsilon_B + 0.628$
	WB2(1)	0.25	1.41×10^7	0.566
WB2	WB2(2)	0.35	$1.63 imes 10^7$	0.521
WD2	WB2(3)	0.45	$1.85 imes 10^7$	0.476
	Model Pa	rameters	$K' = 2.25 \times 10^9 \varepsilon_B + 8.46 \times 10^6$	$n' = -45.0\varepsilon_B + 0.679$
	WB3(1)	0.25	1.40×10^{7}	0.566
WB2	WB3(2)	0.35	$1.64 imes 10^7$	0.520
vv D3	WB3(3)	0.45	$1.86 imes 10^7$	0.475
	Model Pa	rameters	$K' = 2.30 \times 10^9 \varepsilon_B + 8.28 \times 10^6$	$n' = -45.5\varepsilon_B + 0.680$

Table 9. Values of parameters *K*′ and *n*′.

5.6. Comparison between the Test Results and Results Calculated by the Proposal Model

The stress-strain hysteresis curves of the welded specimens are calculated by using the hysteresis model and considering the damage accumulation, constructed by this paper (Equation (13)). The results are compared with the experimental results, as shown in Figure 16, which contain the comparison results of any six semi-cycles referred to as twenty, forty, and eighty percent of the approximate cyclic life of each specimen. It is shown that the calculated results of the model agree well with the experimental curves. The comparison indicates that the model has a good applicability. It can be seen that the amplitude of cyclic stress, elastic modulus, and hysteretic energy decrease with the increase of cyclic cycles under the same loading condition, which is consistent with the experimental results. It shows that the damage accumulation evolution law of the welding materials can be reflected effectively by the hysteretic model considering the damage accumulation.



Figure 16. Cont.



Figure 16. Comparison of hysteretic curves calculated from welding hysteresis models and test results. (a) WA1(1) σ - ε ; (b) WA1(2) σ - ε ; (c) WA1(3) σ - ε ; (d) WA2(1) σ - ε ; (e) WA2(2) σ - ε ; (f) WA2(3) σ - ε ; (g) WA3(1) σ - ε ; (h) WA3(2) σ - ε ; (i) WA3(3) σ - ε ; (j) WB1(1) σ - ε ; (k) WB1(2) σ - ε ; (l) WB1(3) σ - ε ; (m) WB2(1) σ - ε ; (n) WB2(2) σ - ε ; (o) WB2(3) σ - ε ; (p) WB3(1) σ - ε ; (q) WB3(2) σ - ε ; (r) WB3(3) σ - ε .

6. Conclusions

In this research program, low-cycle fatigue experimental studies of welding material and steel welded joints in steel tubular truss structures were conducted under different loading conditions. The cyclic characteristics, variation of cyclic stress amplitude, and energy dissipation capacity were thoroughly discussed, with a focus on the law of damage accumulation of the welding material. Then, an evolution equation of damage accumulation for the welding material was obtained, and a hysteresismodel with damage accumulation was constructed. Based on this research of experiment and theory, the following conclusions can be drawn:

- (1) The cumulative damage process of welding materials can be divided into three stages, which are presented as the stage of crack initiation, the stage of crack propagation, and the stage the specimenis fractured, and when the specimensare destroyed along the maximum main crack.
- (2) As the imposed displacement cycles increased, the cyclic softening behavior of the welding materials of the welded joints in a tubular truss structure apparent, and the steel hardening behavior is shown. Furthermore, with the increase of cyclic cycles, the effect of damage accumulation on the welding materials is obvious, that is, the cyclic stress amplitude, unloading

stiffness, and energy dissipation capacity of the weldingmaterials degenerate gradually. Furthermore, the larger the controlled loading strain amplitude, the faster the rate of damage accumulation. The amplitude of the loading strain has a greater effect on the welding material of base metal Q235 than that of base metal Q345.

- (3) Based on the test results and Park–Ang model, an evolutionequation of damage accumulation for welding materials is established considering both energy and deformation comprehensively, in which the energy part is no longer confined to the perfect elastic-plastic condition. Then, the value of parameter *λ* is fitted by the test data.
- (4) It is revealed by the experimental research that the cyclic characteristic and constitutive behavior of the welding material is quite different from the base metal, and the constitutive model used for monotonic loading cannot simulate the damage degradation behavior of the welding materials under cyclic loading. Therefore, a hysteresismodel with damage accumulation of welding materials is constructed based on the Ramberg–Osgood model and the experimental results, which included the initial loading curve, cyclic stress-strain curve, and a model of the hysteretic curve. The model can reveal the effects of damage accumulation and the nonlinear constitutive relation of the plasticstage.
- (5) The damage parameters and model parameters η, ξ, g, h, K' and n' are fitted by the test results. The hysteretic curves calculated by the hysteretic model are compared with the test results, which show that the model has good applicability and the cumulative damage evolution law of the welding materials reflected by the model is basically consistent with the test results.

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