applied sciences

MDPI

Article

Flocking of Multi-Agent System with Nonlinear Dynamics via Distributed Event-Triggered Control

Yanhua Shen [1], **Zhengmin Kong** [2,*] and **Li Ding** [2]

[1] School of Materials Science and Engineering, South China University of Technology, Guangzhou 510641, China; shenyh@scut.edu.cn

[2] School of Electrical Engineering and Automation, Wuhan University, Wuhan 430072, China; liding@whu.edu.cn

* Correspondence: zmkong@whu.edu.cn; Tel.: +86-177-8648-6625

Received: 23 February 2019; Accepted: 26 March 2019; Published: 29 March 2019

check for updates

Abstract: In this paper, a distributed event-triggered control strategy is proposed to investigate a flocking problem in a multi-agent system with Lipschitz nonlinear dynamics, where triggering conditions are proposed to determine the instants to update the controller. A distributed event-triggered control law with bounded action function is proposed for free flocking. It is proved that the designed event-triggered controller ensures a group of agents reach stable flocking motion while preserving connectivity of the communication network. Lastly, simulations are provided to verify the effectiveness of the theoretical results.

Keywords: multi-agent system; distributed event-triggered control; flocking; nonlinear dynamics

1. Introduction

Event-triggered control (ETC) is a kind of control strategy in which the controller is actuated by the occurrence of a specific event [1–3]. ETC has been widely used for the control of discrete-event systems, Petri Nets, and finite-state machines [4]. The possibility of reducing control costs and saving resources makes ETC appealing in resource-limited control systems. For example, the application of the ETC strategy in multi-agent system (MAS) equipped with microprocessors, such as physically distributed sensor/actuator networks [5], can lower the power consumption and prolong the lifetime of networks. Given these advantages, increasing attention has been paid to event-based control and communication [6–13]. Within these studies, the practices of ETC have been carried out on the general sampled-data systems [6], sensor systems [7–9], signal source localization and navigation of multiple robots [10–12], and human mobility analysis [13].

Several results related to diverse ETC strategies have been reported in recent literature concerning multi-agent cooperative control [14–23], which involves the consensus, the synchronization, etc. In [16–19], distributed ETC formulation was established to study first-order consensus. In [20,21], distributed ETC scheme was presented to study MAS with general linear dynamics. In [22,23], a synchronization problem was investigated by using distributed ETC strategy.

Flocking, which is a conspicuous behavior in life, can be regarded as another form of cooperative control in MAS. The research on flocking problems in MAS has attracted increasing attention (Refs. [24–29]) due to its wide engineering applications, such as massive distributed sensing using mobile sensor networks, automated parallel delivery of payloads, and cooperative control of unscrewed air vehicles [30,31]. In recent years, the application of ETC to study flocking problems has been a hot topic for the research community. Distributed ETC, as a new control strategy, is more in line with the biological characteristics of the interaction and decision-making behavior. In addition, it may potentially reduce the control costs compared to continuous-time flocking control of artificial-intelligence-based MAS. Recently, some distributed ETC algorithms on the flocking problem have been proposed [32–35]. Among these works, it is assumed that there is a virtual leader to be tracked in the flocking motion. For example, leader-follower flocking based on distributed event-triggered hybrid control has been studied in our previous works [32,33]. Generally, the flocking motion is easier to achieve when all agents are able to communicate with the leader [34], while for the free (leaderless) flocking problem, limited communication and only local information pose a challenge for the coordination among agents.

Motivated by the above consideration, a distributed ETC strategy is proposed to investigate the free flocking problem for MAS in this paper. In the proposed algorithm, we consider only the information of neighbors and no global information is required, which is different from [35]. Both the relative position and the relative velocity information are updated at discrete instants. Distributed triggering conditions are established to execute the update of both the position and velocity information in the controller. The contributions of this paper are as follows. Firstly, in contrast to the abovementioned works, the free flocking motion considering the Lipschitz nonlinear dynamics of each agent is investigated. Secondly, distributed ETC strategy has been introduced to realize flocking motion for MAS. Distributed ETC law with bounded action function is proposed. Thirdly, it is proved that the designed ETC strategy can guarantee a group of agents to achieve stable flocking motion.

The rest of the paper is organized as follows. In Section 2, notations, preliminaries on graph theory, and the problem formulation are given. The main results of flocking in MAS with distributed ETC are presented in Section 3. In Section 4, simulations are presented to validate the theoretical results. Conclusions are drawn in Section 5.

2. Preliminaries and Problems Formulation

2.1. Notation

Throughout this paper, the following notations will be used. Denote $\mathbb{R}$ as the set of real number, $\mathbb{R}_{\geq 0}$ as the set of nonnegative real number, $\mathcal{Z}_{\geq 0}$ as the set of all positive integer and $\mathcal{Z}$ as the set of nonnegative integers. I_N is the identity matrix with order N and 1_N is the column vector of order n with all entries equal to one. $\mathbb{R}^n$ is the set of real vectors with dimension n and $\mathbb{R}^{n\times n}$ is the $n \times n$ real matrix space. In addition, we use notation $\|\cdot\|$ denotes the spectral norm of a matrix and $\cup$ to denote the logical operator AND. Kronecker product is denoted by $\otimes$.

2.2. Preliminaries

Let $\mathcal{G} = \{\mathcal{V}, \mathcal{E}, \mathcal{A}\}$ be a weighted undirected graph with the set of vertices $\mathcal{V} = \{1, 2, \cdots, N\}$, and the set of edges is denoted by $\mathcal{E} \subseteq \{(i, j) : i, j \in \mathcal{V}, j \neq i\}$. The weighted adjacency matrix $\mathcal{A} = (a_{ij})_{N\times N}$, where $a_{ij} \in (0, 1]$ if $(i, j) \in \mathcal{E}$, otherwise, $a_{ij} = 0$. An edge denoted by the pair (j, i) represents a communication link from agent j to agent i. A path from vertex i to j is a sequence of edges, $(i, k_1), (k_1, k_2), \cdots, (k_l, j)$ with distinct vertices $k_\iota, \iota = 1, 2, \cdots, l$. An undirected graph is called connected if there is a path between each pair of distinct vertices. Let $\mathcal{D} = (d_{ij})_{N\times N}$ represent the degree matrix which is a diagonal matrix with entries $d_i = \sum_{j=1, j\neq i}^{N} a_{ij}$. The Laplacian matrix of graph $\mathcal{G}$ is defined as $L = (l_{ij})_{N\times N} = \mathcal{D} - \mathcal{A}$. Then it has some properties [25] as follows

(i) The eigenvalues of L satisfy $0 = \lambda_1(L) \leq \lambda_2(L) \leq \cdots \leq \lambda_N(L)$, if $\mathcal{G}$ is connected, one has

$$\lambda_2(L) = \min_{z \perp 1_n} \frac{z^T L z}{\|z\|^2} > 0.$$

(ii) L is a positive semi-definite matrix that satisfies the following sum-of-squares (SOS) property:

$$z^T L z = \frac{1}{2} \sum_{i,j\in\mathcal{E}} a_{ij}(z_j - z_i)^2, \ \ z \in \mathbb{R}^n.$$

Lemma 1. *[36] Suppose $\mathcal{G}$ be an undirected graph of order N, and $\mathcal{G}_1$ is a graph generated by adding some edge(s) into the graph $\mathcal{G}$. Then, $\lambda_2(L(\mathcal{G}_1)) \geq \lambda_2(L(\mathcal{G}))$, where $L(\mathcal{G})$ and $L(\mathcal{G}_1)$ are the symmetric Laplacian matrixes of graphs $\mathcal{G}$ and $\mathcal{G}_1$, respectively.*

2.3. System Model

Consider a group of N agents moving in n dimensional space, the dynamics of agent i is described by

$$\begin{cases} \dot{x}_i(t) = v_i(t), \\ \dot{v}_i(t) = f(v_i(t)) + u_i(t), \quad i = 1, 2, \cdots, N, \end{cases} \tag{1}$$

where $x_i \in \mathbb{R}^n$, $v_i \in \mathbb{R}^n$ and $u_i \in \mathbb{R}^n$ are the position, the velocity and the control input of the i^{th} agent, respectively. The function $f(v_i(t)) \in \mathbb{R}^n$ is the nonlinear dynamics of agent i, which is Lipschitz continuous.

Assumption 1. *The nonlinear function $f(z)$ is Lipschitz in z over the time, which means that there exists a positive constant ρ, such that $f(z)$ satisfies*

$$\|f(z_1) - f(z_2)\| \leq \rho \|z_1 - z_2\|, \quad \forall z_1, z_2 \in \mathbb{R}^n. \tag{2}$$

Supposed that all the agents have the same sensing radius $r > 0$, then the neighboring set of agent i is defined as $\mathcal{N}_i(t) = \{j | \|x_i(t) - x_j(t)\| < r, j = 1, 2, \ldots, N, j \neq i\}$. A minimum allowable distance r_0 (we also called collision distance) is considered in the model since that the size of agents cannot be ignored usually. Let $\tau \in (0, r)$ be a given constant. Then, the graph $\mathcal{G}(t)$ is a dynamic undirected graph with a time-varying set of links $\mathcal{E}(t)$ such that

(i) Initial links are generated by $\mathcal{E}(0) = \{(i,j) | r_0 < \|x_i(0) - x_j(0)\| < r\}$;

(ii) If $(i,j) \notin \mathcal{E}(0)$ and $\|x_i(t) - x_j(t)\| \leq r - \tau$, then (i,j) is a new link to be added to $\mathcal{E}(t)$. It was called hysteresis effect and τ is the hysteresis distance, which is crucial in preserving connectivity of the network [26,37];

(iii) If $\|x_i(t) - x_j(t)\| \geq r$, then $(i,j) \notin \mathcal{E}(t)$.

The neighboring set of agent i is divided into four regions, named collision region, separation region, alignment region and attraction region (see Figure 1), in which $r > r - \tau > \bar{d} \geq \underline{d} > r_0$. If $\underline{d} \leq \|x_{ij}(t)\| \leq \bar{d}$, it is said that agent i and agent j are in desired distance.

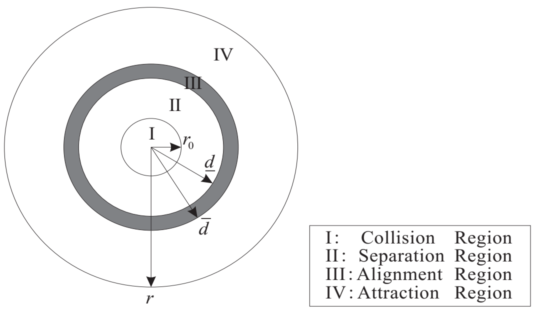

Figure 1. The neighboring set of agent i.

2.4. Problem Statement

In this paper, flocking is investigated. Due to the fact that the sensing abilities of each agent is limited, the control law, except for achieving desired flocking motion, it should also guarantee for all $t \in \mathcal{R}_{\geq 0}$ that i) the agents avoid collision with each other; ii) the communication graph $\mathcal{G}$ is connected.

Definition 1. *Flocking motion is said to be achieved asymptotically for MAS (1), if for any initial condition,*

$$\lim_{t\to\infty} \underline{d} \le \|x_i(t) - x_j(t)\| \le \bar{d},$$
$$\lim_{t\to\infty} \|v_i(t) - v_j(t)\| = 0, \quad \forall (i,j) \in \mathcal{E},$$

3. Flocking via Distributed ETC

In this section, a systematic solution to flocking problem is introduced. Our overall approach is to design a distributed action function for each agent that captures all the desired control specifications. The action function will then be implemented into the distributed controller of each agent.

3.1. Design of Action Function

Define $\phi(\cdot)$ as a bounded pairwise action function, which is given by

$$\phi(z) = \begin{cases} \alpha_1 (z - \underline{d})\, e^{\frac{(z-\underline{d})^2}{\beta_1}}, & z \in (r_0, \underline{d}) \\ \alpha_2 (z - \bar{d})\, e^{\frac{(z-\bar{d})^2}{\beta_2}}, & z \in (\bar{d}, r] \\ 0, \quad \text{otherwise}, \end{cases} \tag{3}$$

and the pairwise bounded potential function $\psi(z) : \mathbb{R}_{\ge 0} \to \mathbb{R}_{\ge 0}$ is defined correspondingly as

$$\psi(z) = \begin{cases} \int_{\underline{d}}^{z} \phi(s) ds, z \in (0, \underline{d}) \\ 0, z \in [\underline{d}, \bar{d}] \\ \int_{\bar{d}}^{z} \phi(s) ds, z \in (\bar{d}, \infty), \end{cases} \tag{4}$$

where the parameters $\alpha_1, \beta_1, \alpha_2, \beta_2 > 0$ are chosen properly, such that $\psi(r_0) \ge Q^*, \psi(r) \ge Q^*$ (see Figure 2), and

$$Q^* = Q_0 + \left\{ \frac{N(N-1)}{2} - m_0 \right\} \psi(r - \tau), \tag{5}$$

where m_0 is the number of edges in graph $\mathcal{G}(0)$, $Q_0 > 0$ is associated with the initial states of all agents, which will be defined later.

Definition (3) indicates that the action function $\phi(z) < 0$ when $z \in (r_0, \underline{d})$, $\phi(z) > 0$ when $z \in (\bar{d}, r]$ and $\phi(z) = 0$ otherwise. The potential function $\psi(z)$ is the integral of ϕ from $\underline{d}$ to z when $z \in (0, \underline{d})$, and from $\bar{d}$ to z when $z \in (\bar{d}, \infty)$, which implies that $\psi(z)$ is positive. Moreover, it decreases with the increase of z when $z \in (0, \underline{d})$ and increases with the increase of z when $z \in (\bar{d}, \infty)$. Obviously, the potential function $\psi(z)$ reaches its minimum value 0 when $z \in [\underline{d}, \bar{d}]$. Condition (5) states that the potential function will be sufficiently large when the distance between two agents' approaches r_0 or r, which will be used to ensure that collisions are avoided and edges among agents are preserved.

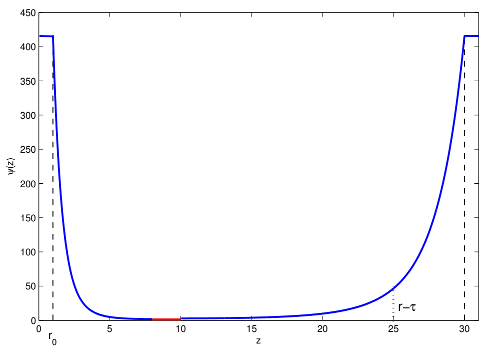

Figure 2. The potential function $\psi(z)$.

3.2. Controller Design and Stability Analysis

A distributed ETC law using relative position and velocity information is proposed to realize flocking motion, which is given by

$$u_i(t) = u_i^x(t) + u_i^v(t), \tag{6}$$

in which

$$\begin{aligned} u_i^x(t) &= -\sum_{j\in\mathcal{N}_i(t_k^i)} \phi\left(\left\|x_i(t_k^i) - x_j(t_k^i)\right\|\right)\mathbf{n}_{ij}(t_k^i), \\ u_i^v(t) &= -c\sum_{j\in\mathcal{N}_i(t_k^i)} a_{ij}(t_k^i)\left(v_i(t_k^i) - v_j(t_k^i)\right), \quad t\in[t_k^i, t_{k+1}^i), \end{aligned}$$

where $c > 0$ is a constant coupling gain, $\mathbf{n}_{ij}(t) = (x_i(t) - x_j(t))/\left\|x_i(t) - x_j(t)\right\|$, $\phi(\cdot)$ is the action function defined in (3) and $t_k^i, k \in \mathcal{Z}$ is the event-triggered controller update time sequence of agent i.

Denote the average position and the average velocity of all the agents by $\bar{x}(t) = 1/N\sum_{j=1}^N x_j(t)$ and $\bar{v}(t) = 1/N\sum_{j=1}^N v_j(t)$, respectively. Then the position and velocity difference between agent i and the average are defined as $\tilde{x}_i(t) = x_i(t) - \bar{x}(t)$ and $\tilde{v}_i(t) = v_i(t) - \bar{v}(t)$, respectively.

Let $q_i^x(t) = \sum_{j\in\mathcal{N}_i(t)}\phi\left(\left\|\tilde{x}_i(t) - \tilde{x}_j(t)\right\|\right)\mathbf{n}_{ij}(t)$, $q_i^v(t) = \sum_{j\in\mathcal{N}_i(t)}\left(\tilde{v}_i(t) - \tilde{v}_j(t)\right)$. For agent i, one can see that u_i^x and u_i^v are updated at time t_k^i and held a constant between t_k^i and t_{k+1}^i. Then, we define the position and the velocity error of each agent i as the combined measurements between the latest sampled value and current value, which are

$$\begin{aligned} e_i^x(t) &= q_i^x(t_k^i) - q_i^x(t), \\ e_i^v(t) &= q_i^v(t_k^i) - q_i^v(t). \end{aligned} \tag{7}$$

Let $\bar{u}(t) = 1/N\sum_{i=1}^N u_i(t)$, then one has

$$\dot{\bar{u}}(t) = \frac{1}{N}\sum_{i=1}^N \dot{u}_i(t) = \frac{1}{N}\sum_{i=1}^N \left(u_i^x(t) + u_i^v(t)\right),$$

where

$$\sum_{i=1}^N u_i^x(t) = \sum_{i=1}^N\left(-\sum_{j\in\mathcal{N}_i(t_k^i)}\phi\left(\left\|x_i(t_k^i) - x_j(t_k^i)\right\|\right)\mathbf{n}_{ij}(t_k^i)\right),$$

and

$$\begin{aligned} \sum_{i=1}^N u_i^v(t) &= \sum_{i=1}^N\left(-c\sum_{j\in\mathcal{N}_i(t_k^i)} a_{ij}(t_k^i)\left(v_i(t_k^i) - v_j(t_k^i)\right)\right) \\ &= -\left(I_n\otimes 1_N^T\right)(c\otimes L)\,v(t_k^i) \\ &= -\left(cI_n\otimes 1_N^T L\right)v(t_k^i). \end{aligned}$$

For $\forall(i,j)\in\mathcal{E}(t_k^i)$, one has $\phi\left(\left\|x_i(t_k^i) - x_j(t_k^i)\right\|\right) = \phi\left(\left\|x_j(t_k^i) - x_i(t_k^i)\right\|\right)$ and $\mathbf{n}_{ij}(t_k^i) = -\mathbf{n}_{ji}(t_k^i)$, then $\sum_{i=1}^N u_i^x(t) = 0$. Besides, for a connected undirected graph $\mathcal{G}$, one has $1_N^T L = 0 \Rightarrow \sum_{i=1}^N u_i^v(t) = 0$. Thus, $\dot{\bar{u}}(t) \equiv 0$. Then, the MAS (1) can be rewritten as

$$\begin{cases} \dot{\tilde{x}}_i(t) = \tilde{v}_i(t), \\ \dot{\tilde{v}}_i(t) = f(v_i, t) - \frac{1}{N}\sum\limits_{j=1}^N f(v_j, t) + u_i(t), \end{cases} \tag{8}$$

where

$$u_i(t) = -\left(\sum_{j\in\mathcal{N}_i(t)} \phi(\|\tilde{x}_i(t)-\tilde{x}_j(t)\|)\mathbf{n}_{ij}(t) + e_i^x(t)\right) - c\left(\sum_{j\in\mathcal{N}_i(t)} a_{ij}(t)(\tilde{v}_i(t)-\tilde{v}_j(t)) + e_i^v(t)\right).$$

Now, we are ready to define the event-triggered controller update time sequence. The time instant t_k^i for each agent i is updated by

$$t_{k+1}^i = \inf\left\{t > t_k^i : g_1\left(e_i^x(t), e_i^v(t), \tilde{v}_i(t)\right) \geq 0\right\}, \tag{9}$$

where

$$\begin{aligned} g_1\left(e_i^x(t), e_i^v(t), \tilde{v}_i(t)\right) =& \frac{1}{2a_1}\|e_i^x(t)\|^2 + \frac{c}{2a_2}\|e_i^v(t)\|^2 - \eta\|\tilde{v}_i(t)\|^2 \\ \eta =& \theta\left(c\lambda_2\left(L(0)\right) - \left(2\rho + \frac{a_1 + ca_2}{2}\right)\right), \end{aligned} \tag{10}$$

with $0 < \theta < 1$, constants a_1, a_2 and c satisfy $c > \left(2\rho + (a_1 + ca_2)/2\right)/\lambda_2(L(0))$ and ρ is defined in (2). The condition $g_1\left(e_i^x(t), e_i^v(t), \tilde{v}_i(t)\right) \geq 0$ is called the triggering condition. Without loss of generality, we assume $t_0^i = 0, \forall i$.

Remark 1. *It must be pointed out that the triggering condition (10) of each agent is associated with the average position and velocity of all agents. However, since*

$$\begin{aligned} &\sum_{j\in\mathcal{N}_i(t)} \phi\left(\|\tilde{x}_i(t)-\tilde{x}_j(t)\|\right)\mathbf{n}_{ij}(t) \\ =& \sum_{j\in\mathcal{N}_i(t)} \phi\left(\|(\tilde{x}_i(t)-\bar{x}(t)) - (\tilde{x}_j(t)-\bar{x}(t))\|\right)\mathbf{n}_{ij}(t) \\ =& \sum_{j\in\mathcal{N}_i(t)} \phi\left(\|x_i(t)-x_j(t)\|\right)\mathbf{n}_{ij}(t) \end{aligned}$$

and similarly

$$\sum_{j\in\mathcal{N}_i(t)} \left(\tilde{v}_i(t)-\tilde{v}_j(t)\right) = \sum_{j\in\mathcal{N}_i(t)} \left(v_i(t)-v_j(t)\right),$$

one can see that the average position and velocity are not needed for the detection of triggering condition (10). Therefore, the proposed ETC strategy is in a distributed manner.

Define $\tilde{x}_{ij} = \tilde{x}_i - \tilde{x}_j$, the constant Q_0 in (5) is given as

$$Q_0 = \frac{1}{2}\sum_{i=1}^{N}\left\{\sum_{j\in\mathcal{N}_i(0)} \psi(\|\tilde{x}_{ij}(0)\|) + \tilde{v}_i^T(0)\tilde{v}_i(0)\right\}, \tag{11}$$

then the following results are obtained.

Theorem 1. *Consider the MAS of N agents with dynamics expressed by (1). The controller for each agent is given by (6), which is updated by (9). Suppose that the network is initially connected and Q_0 is bounded. Then, the following statements hold.*

(i) $\mathcal{G}(t)$ *is connected and no collisions occur for* $\forall t \geq 0$*;*

(ii) Flocking motion is achieved asymptotically.

Proof. Define an energy-like Lyapunov function as

$$Q(t) = \frac{1}{2}\sum_{i=1}^{N}\left\{\sum_{j\in\mathcal{N}_i(t)} \psi(\|\tilde{x}_{ij}(t)\|) + \tilde{v}_i^T(t)\tilde{v}_i(t)\right\}. \tag{12}$$

Denote the increasing topology switching time sequence as $\hat{t}_{k'}, k' \in \mathcal{Z}^+$, then $\{\hat{t}_{k'}\}$ are the set of discontinuous points of Q. The increasing controller update time sequence of all the agents is denoted by $\tilde{t}^i_{k''} = \cup_{i=1}^N t^i_k, k'' \in \mathcal{Z}$. Let $T_k = \hat{t}_{k'} \cup \tilde{t}_{k''}, k \in \mathcal{Z}$, where $T_0 < T_1 < \cdots$. Without loss of generality, we assume $\hat{t}_0 = 0, T_0 = 0$. Taking the time derivative of $Q(t)$ on time interval $[T_0, T_1)$ gives

$$\begin{aligned}
\dot{Q}(t) = & \sum_{i=1}^{N}\tilde{v}_i^T(t)\sum_{j\in\mathcal{N}_i(t)}\phi(\|\tilde{x}_{ij}(t)\|)\mathbf{n}_{ij}(t) + \sum_{i=1}^{N}\tilde{v}_i^T(t)\Big\{f(v_i(t)) \\
& - \frac{1}{N}\sum_{j=1}^{N}f(v_j(t)) - \sum_{j\in\mathcal{N}_i(t)}\phi(\|\tilde{x}_{ij}(t)\|)\mathbf{n}_{ij}(t) - e_i^x(t) \\
& - c\sum_{j\in\mathcal{N}_i(t)}a_{ij}(t)(\tilde{v}_i(t) - \tilde{v}_j(t)) - ce_i^v(t)\Big\} \\
= & \sum_{i=1}^{N}\tilde{v}_i^T(t)[f(v_i(t)) - f(\bar{v}(t))] + \sum_{i=1}^{N}\tilde{v}_i^T(t)\Big[f(\bar{v}(t)) - \frac{1}{N}\sum_{j=1}^{N}f(v_j(t))\Big] \\
& - \sum_{i=1}^{N}\tilde{v}_i^T(t)e_i^x(t) - c\sum_{i=1}^{N}\tilde{v}_i^T(t)\sum_{j\in\mathcal{N}_i(t)}a_{ij}(t)(\tilde{v}_i(t) - \tilde{v}_j(t)) - c\sum_{i=1}^{N}\tilde{v}_i^T(t)e_i^v(t) \\
\leq & \rho\sum_{i=1}^{N}\tilde{v}_i^T(t)\tilde{v}_i(t) + \sum_{i=1}^{N}\left\|\tilde{v}_i^T(t)\right\|\frac{1}{N}\rho\sum_{j=1}^{N}\|\tilde{v}_j(t)\| - \sum_{i=1}^{N}\tilde{v}_i^T(t)e_i^x(t) \\
& - c\sum_{i=1}^{N}\tilde{v}_i^T(t)\sum_{j\in\mathcal{N}_i(t)}a_{ij}(t)(\tilde{v}_i(t) - \tilde{v}_j(t)) + c\sum_{i=1}^{N}\tilde{v}_i^T(t)e_i^v(t)) \\
\leq & \sum_{i=1}^{N}\Big\{2\rho\|\tilde{v}_i(t)\|^2 - \tilde{v}_i^T(t)e_i^x(t) - c\lambda_2(L)\|\tilde{v}_i(t)\|^2 - c\tilde{v}_i^T(t)e_i^v(t)\Big\}.
\end{aligned}$$

Please note that the inequality $|xy| \leq ax^2/2 + y^2/2a$ holds for $\forall a > 0$. According to the properties of Laplacian matrix, one has $\sum_{i=1}^{N}\tilde{v}_i^T\sum_{j\in\mathcal{N}_i(t)}a_{ij}(t)(\tilde{v}_i(t) - \tilde{v}_j(t)) \geq \sum_{i=1}^{N}\lambda_2(L(0))\|\tilde{v}_i(t)\|^2$, then

$$\begin{aligned}
\dot{Q} \leq & \sum_{i=1}^{N}\Big\{2\rho\|\tilde{v}_i(t)\|^2 + \frac{a_1}{2}\tilde{v}_i^T(t)\tilde{v}_i(t) + \frac{1}{2a_1}e_i^{xT}(t)e_i^x(t) - c\lambda_2(L)\|\tilde{v}_i(t)\|^2 \\
& + c\left(\frac{a_2}{2}\tilde{v}_i^T(t)\tilde{v}_i(t) + \frac{1}{2a_2}e_i^{vT}(t)e_i^v(t)\right)\Big\} \\
= & \sum_{i=1}^{N}\Big\{(2\rho + \frac{a_1 + ca_2}{2} - c\lambda_2(L))\|\tilde{v}_i(t)\|^2 + \frac{1}{2a_1}\|e_i^x(t)\|^2 + \frac{c}{2a_2}\|e_i^v(t)\|^2\Big\}
\end{aligned}$$

Since a_1, a_2 and c satisfy $c > (2\rho + (a_1 + ca_2)/2)/\lambda_2(L)$, enforcing the triggering condition (10), one has

$$\dot{Q}(t) \leq (\theta - 1)\left(c\lambda_2(L(0)) - \left(2\rho + \frac{a_1 + ca_2}{2}\right)\right)\|\tilde{v}(t)\|^2 < 0,$$

which implies that $Q(t) < Q(T_1), \forall t \in [T_0, T_1)$.

(1) If $T_1 = \hat{t}_1$, Q is continuous and $T_1^+ = T_1^- < Q^*$;

(2) If $T_1 = \tilde{t}_1$, which implies the first topology switching occurs at instant T_1. Since $Q(t) < Q(T_0), \forall t \in [T_0, T_1)$, from the definition of the potential function, one has $Q_0 < \psi(r)$, which implies that no edge will tend to r for $t \in [T_0, T_1)$. Therefore, new edges must be added into the network at

instant T_1. For a system consists of N agents, there are at most $N(N-1)/2$ edges. At the initial instant t_0, the system contains m_0 edges, then $Q(\hat{t}_1^+) \leq Q_0 + (N(N-1)/2 - m_0)\psi(\|r-\tau\|) < Q^*$, no edges are lost. In addition, since $\psi(\|r_0\|) \geq Q^*$, we can also derive that no collision occurs during $t \in [t_0, \hat{t}_1)$.

Similarly, taking the time derivative of $Q(t)$ on $\forall t \in [T_{k''}, T_{k''+1})$, one can have $\dot{Q}(t) \leq 0$, which implies $Q(t)$ is decreasing monotonously as far as the topology of the network keeps fixed; thus, no edge-distance will tend to r for $t \in [T_{k''}, T_{k''+1})$, which means no edges will be lost at time $\hat{t}_{k'}$. Then by Lemma 1, $\lambda_2(L(0)) \leq \lambda_2(L(t))$ for $\forall t \geq 0$. Thus, the triggering function (10) is available in the whole process. As the network is initially connected, $\mathcal{G}(t)$ will be connected for $\forall t \geq 0$. Furthermore, from the definition of potential function, $\psi(r_0) \geq Q^*$, we can deduce no edge-distance will tend to r_0 for $\forall t \geq 0$ similar to the above analysis, thus collisions are avoided during the whole process. This completes the proof of part i).

Next, we prove part ii). Assume that there are m_k new edges that can be added into $\mathcal{G}(t)$ at $\hat{t}_{k'}$. As no edges are lost $\forall t \geq 0$, one has $m_k \leq N(N-1)/2 - m_0$, the number of switching times of the system (1) is finite, thus the topology of graph $\mathcal{G}$ becomes fixed eventually. Denote the last topology switching instant as $\hat{t}_{k^*}$, then $Q(t)$ is continuous and monotonously decreasing for $t \in [\hat{t}_{k^*}, \infty)$. Hence, the set $\Omega := \{\tilde{x}(t) \in D, \tilde{v}(t) \in \mathbb{R}^{Nn} | Q(\tilde{x}(t), \tilde{v}(t)) \leq Q^*\}$ is positive invariant, where

$$D = \left\{ \tilde{x}(t) \in \mathbb{R}^{N^2 n} |\, \|\tilde{x}(t)\| \in \left[\min\left\{ \psi^{-1}(Q^*) \right\}, \right.\right.$$
$$\left.\left. \max\left\{ \psi^{-1}(Q^*) \right\} \right], \forall i,j \in \mathcal{E}(t), t \in [\hat{t}_{k^*}, \infty) \right\},$$

$\tilde{x} = (\tilde{x}_{11}, \cdots, \tilde{x}_{1N}, \cdots, \tilde{x}_{N1}, \cdots, \tilde{x}_{NN})^T \in \mathbb{R}^{N^2 n}$ and $\tilde{v} = (\tilde{v}_1, \cdots, \tilde{v}_N)^T \in \mathbb{R}^{Nn}$. Since $\mathcal{G}(t)$ is connected for $t \geq 0$, one has $\|\tilde{x}_{ij}(t)\| \leq (N-1)r, \forall i,j \in \mathcal{E}(t)$, besides, one has $\tilde{v}_i^T(t)\tilde{v}_i(t) \leq 2Q^* \Rightarrow \|\tilde{v}_i(t)\| \leq \sqrt{2Q^*}$, and thus the set Ω is compact. It follows from Lasalle's invariance principle that if the initial condition lies in Ω, its trajectories will converge to the largest invariant set inside the region $S = \{\tilde{x}(t) \in \mathbb{R}^{N^2 n}, \tilde{v}(t) \in \mathbb{R}^{Nn} | \dot{Q}(\tilde{x}(t), \tilde{v}(t)) = 0\}$. Thus, the MAS converges asymptotically to a fixed configuration corresponding to the extreme of the global potential $Q(t)$. Please note that not all solutions of (1) converge to local minima. However, anything but the local minima is an unstable equilibrium [25]. Thus, every final configuration locally minimizes each agent's global potential, which implies $\|x_{ij}\| \in [\underline{d}, \bar{d}], \forall (i,j) \in \mathcal{E}$.Besides, from (12), one has $\dot{Q}(t) = 0$ if and only if $\tilde{v}_1(t) = \tilde{v}_2(t) = \cdots = \tilde{v}_N(t)$, which is equivalent to $\|v_i(t) - v_j(t)\| = 0, \forall (i,j) \in \mathcal{E}$. Thus, the flocking motion is achieved. □

Remark 2. *Since the potential function $\psi(\cdot)$ is bounded, we can also have that the action function $\phi(\cdot)$ is bounded. Thus, there exists a constant $M > 0$, such that $\|\phi(z)\| \leq M, \forall z \in (0, \infty)$.*

Let $\tilde{e}_i(t) = 1/2a_1\|e_i^x(t)\|^2 + c/2a_2\|e_i^v(t)\|^2$. Combining (7) with inequality (10), we can get that $\|\tilde{e}_i(t)\|$ decreasing to $\theta_1 = 1/(1+\sqrt{\eta})\,\|\tilde{v}_i\,(t_k^i)\|$ and $\|\tilde{e}_i(t)\|$ increasing to $\theta_2 = 1/(1-\sqrt{\eta})\,\|\tilde{v}_i\,(t_k^i)\|$ are the two "worst" cases when event occurs. Thus, t_k^i exists when $\tilde{v}_i(t_k^i) \neq 0$.

From (7), one has $\dot{e}_i^x\,(t) = -\dot{q}_i^x(t), \dot{e}_i^v\,(t) = -\dot{q}_i^v(t)$. From (7), one has

$$\frac{d}{dt}\|e_i^x(t)\| = \frac{d}{dt}\|q_i^x\,(t)\|$$
$$\leq \sum_{j \in \mathcal{N}_i(t)} \frac{d\,\|\phi\,(\|x_i(t) - x_j(t)\|)\,\mathbf{n}_{ij}(t)\|}{dt}$$
$$\leq N\varrho\,\|\tilde{v}_i\,(t)\|$$

where $\varrho = \max\limits_{\alpha_1,\alpha_2,\beta_1,\beta_2} \left\{ \alpha_1 e^{\frac{r^2}{\beta_1}} \left(\frac{\beta_1+2r^2}{\beta_1}\right), \alpha_2 e^{\frac{r^2}{\beta_2}} \left(\frac{\beta_2+2r^2}{\beta_2}\right) \right\}$.

Besides, one has

$$\begin{aligned}\frac{d}{dt}\left\|q_i^v(t)\right\| \le & \sum_{j\in\mathcal{N}_i(t)} \frac{d}{dt}\left\{\left\|f(v_i(t))-f(v_j(t))\right\|+\left\|u_i(t)-u_j(t)\right\|\right\} \\ \le & \sum_{j\in\mathcal{N}_i(t)} \left\{\rho\left(\left\|\tilde{v}_i(t)\right\|+\left\|\tilde{v}_j(t)\right\|\right)\right. \\ & \left.+\left\|q_i^x\left(t_k^i\right)+q_j^x\left(t_k^i\right)\right\|+c\left\|q_i^v\left(t_k^i\right)+q_j^v\left(t_k^i\right)\right\|\right\}\end{aligned}$$

where

$$\left\|q_i^x\left(t_k^i\right)\right\| = \left\|\sum_{j\in\mathcal{N}_i(t_k^i)} \phi\left(\left\|\tilde{x}_i(t_k^i)-\tilde{x}_j(t_k^i)\right\|\right)\mathbf{n}_{ij}(t_k^i)\right\| \le NM,$$

$$\left\|q_i^v\left(t_k^i\right)\right\| = \left\|\sum_{j\in\mathcal{N}_i(t_k^i)} \left(\tilde{v}_i(t_k^i)-\tilde{v}_j(t_k^i)\right)\right\| \le N\left\|\tilde{v}_i(t_k^i)\right\|+\sum_{j\in\mathcal{N}_i(t_k^i)}\left\|\tilde{v}_j(t_k^i)\right\|.$$

According to Equation (12), one has $\|\tilde{v}_i(t)\| \le \sqrt{2Q(t)} \le \sqrt{2Q^*}, \forall t \ge 0, \forall i$*, then we can conclude that*

$$\frac{d}{dt}\left\|e_i^v(t)\right\| \le 2N\left(NM+(\rho+cN)\sqrt{2Q^*}\right).$$

Then, according to [18] Lemmas 2, 3, and 4, we can conclude that if $\tilde{v}_i(t_k^i) \neq 0$*, no Zeno triggering occurs for all* $t \in [t_k^i, t_{k+1}^i)$*.*

Theorem 1 demonstrates that flocking motion can be achieved for MAS (1) with ETC, where each agent has nonlinear dynamics. For the case that agents have no nonlinear dynamics, which means $f(z) \equiv 0$. Since $\dot{\bar{v}}(t) = 1/N\sum_{i=1}^N \dot{v}_i(t) \equiv 0$, $\bar{v}(t) \equiv 1/N\sum_{i=1}^N v_i(0) = \bar{v}(0)$. The velocity of all agents will converge to the initial average $\bar{v}(0)$ asymptotically.

Corollary 1. *Consider the MAS of N agents with dynamics expressed by (1), where* $f(v,t) \equiv 0$*. The control input is given by (6), in which the controller update time instants* t_k^i *for each agent i is triggered if and only if*

$$\frac{1}{a_1}\|e_i^x\|^2+\frac{c}{a_2}\|e_i^v\|^2 > \theta\left(2c\lambda_2(L)-(a_1+ca_2)\right)\|\tilde{v}_i\|^2,$$

is satisfied, where the constants $0<\theta<1$, a_1, a_2 *and c satisfy* $2c\lambda_2(L(0))-(a_1+ca_2)>0$*. Suppose that the network is initially connected and* Q_0 *is bounded. Then, the following statements hold.*

(i) $\mathcal{G}(t)$ *is connected and no collisions occur for* $\forall t \ge 0$*;*
(ii) Flocking motion is achieved asymptotically.

4. Simulations

In this section, numerical examples are given to validate the effectiveness of the theoretical results.

Consider a group of 4 mobile robots [38] moving in 3-dimensional space, the dynamics of each robot is expressed in (1), where the nonlinear dynamic function $f\left(v_i^x, v_i^y, v_i^z, t\right)$ satisfies the Lorenz equation

$$\begin{pmatrix}\dot{v}_i^x\\ \dot{v}_i^y\\ \dot{v}_i^z\end{pmatrix} = 0.01 * \begin{pmatrix}10\left(v_i^y-v_i^x\right)\\ 28v_i^x-v_i^xv_i^z-v_i^y\\ v_i^xv_i^y-\frac{8}{3}v_i^z\end{pmatrix},$$

where v_i^x, v_i^y and v_i^z are the x axis, y axis, and z axis velocity component of agent i. The initial position and velocity of each agent is chosen randomly from the box $[-15,15]\times[-15,15]\times[-15,15]$ and $[-10,10]\times[-10,10]\times[-10,10]$, respectively. For each agent, the radius of the Dead zone (Collision region) is chosen as $r_0 = 1$, the radius of Controlled zone (Separation, Alignment, Attraction Region) is chosen as $r = 15$, which is the communication radius of each agent. If $\|x_{ij}\| > 15$, the connection between two agents will be lost. Besides, the following parameters remain fixed throughout all simulations: $\tau = 1, \underline{d} = 5, \bar{d} = 8$ and $\theta = \theta' = 0.9$.

The action function $\phi(\cdot)$ is designed as

$$\phi(\|x_{ij}\|) = \begin{cases} 6(\|x_{ij}\| - 5)e^{\frac{(\|x_{ij}\|-5)^2}{3}}, & \|x_{ij}\| \in (1,5) \\ 4(\|x_{ij}\| - 8)e^{\frac{(\|x_{ij}\|-8)^2}{8}}, & \|x_{ij}\| \in (8,15] \\ 0, \quad \text{otherwise.} \end{cases}$$

For the flocking, the initial algebraic connectivity $\lambda_2(L(0)) = 1$. Choosing $\rho = 0.5$, constants $a_1 = 1, a_2 = 0.5$, the control gain $c = 4$ such that $c > (2\rho + (a_1 + ca_2)/2) / \lambda_2(L(0))$.

The simulation results for flocking of MAS applying algorithm (6) are shown in Figures 3 and 4. Figure 3a shows the initial states of the agents, where the solid lines represent the links between agents, x_{i1}, x_{i2} and x_{i3} are the position component of agent i and the red solid lines with arrow represent the direction of velocity. Figure 3b presents the configuration and velocities of the group of agents at $t = 50$ s. It can be seen that the communication network is fully connected, which means any agent can communicate with the others. Also, the velocities of 4 agents are almost same at $t = 50$ s. Figure 3c depicts the motion trajectories of all agents from $t = 0$ s to $t = 50$ s. Here, it must be stated that by observing the moving trajectories of all agents from different angle of view, we found no collision occurs during the whole process. Besides, the algebraic connectivity of the system is always positive, which implies that the network is connected during the whole process. Figure 3d shows the convergence of velocity, from which we can see that all the agents eventually achieve the same velocity. However, it can be seen from the subfigure that there are some vibrations during the convergence process of velocity, for example, at about $t = 35.5$ s. It is caused by new edges being added into the network. From Figure 3b–d, it is observed that the free flocking motion is achieved asymptotically with our ETC algorithm (6). Also, $\forall t \geq 0$, the communication $\mathcal{G}(t)$ is connected and no collisions occur along the motion of agents. Thus, the results of Theorem 1 is illustrated by these numerical examples.

In Figure 4, the controller update time instants of each agent for the first 5s are marked with $*$. It can be observed that the controller update times are heterogeneous for the 4 agents, which implies that the controller is actuated asynchronously. Seemingly, the agent 4 has less controller times compared with the other 3 agents. Moreover, the time interval between two controlling actions is aperiodic for each agent. The asynchronous and aperiodic controlling action is two basic characteristics for the ETC, which is actuated by the occurrence of a specific event (the conditions (9) and (10) in our algorithm). Within these characteristics, it is possible for the ETC to reduce control costs and save resources, which effect is related to the initial value of the system. This is actually the charm of ETC in the resource-limited systems and one of our motivations for the flocking problem in this paper.

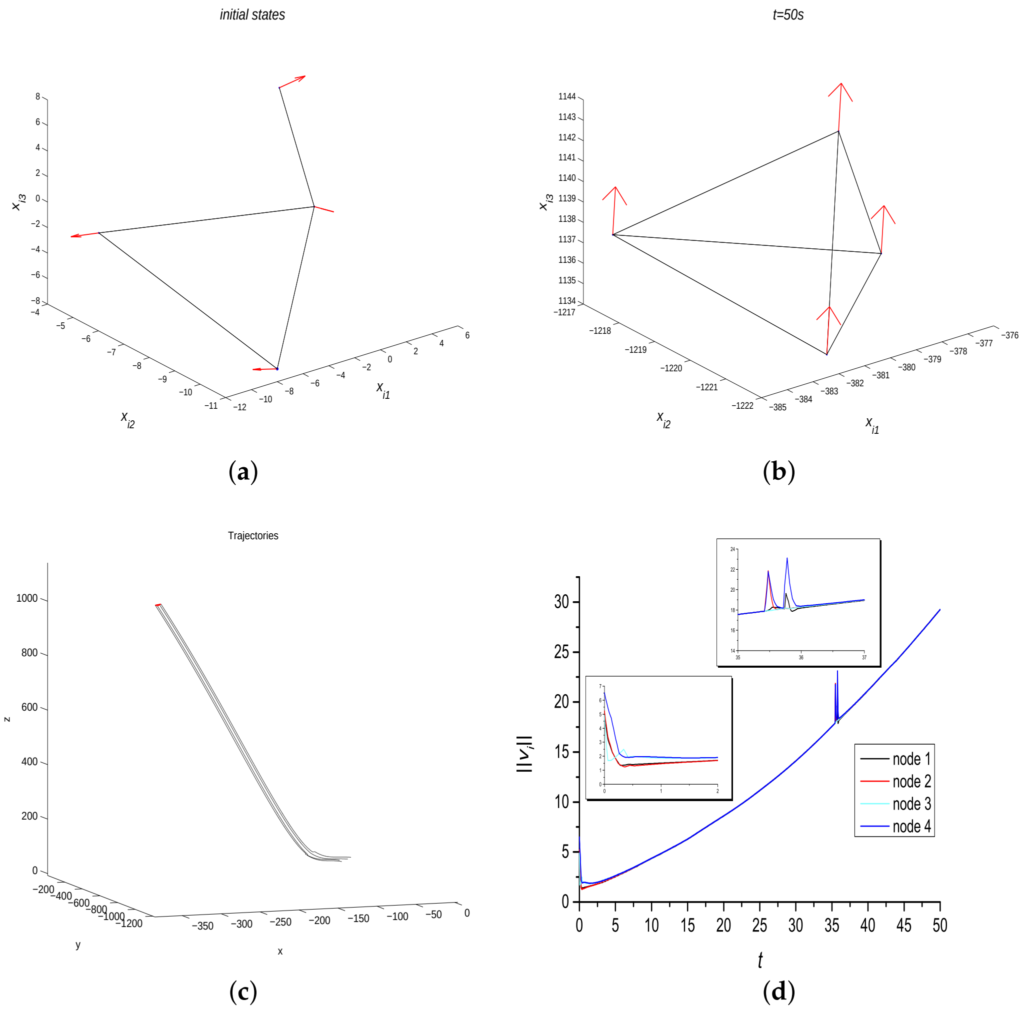

Figure 3. 3-D flocking for 4 agents applying algorithm (6). (**a**) Initial states; (**b**) states at $t = 50$ s; (**c**) trajectories of all agents; (**d**) velocity convergence.

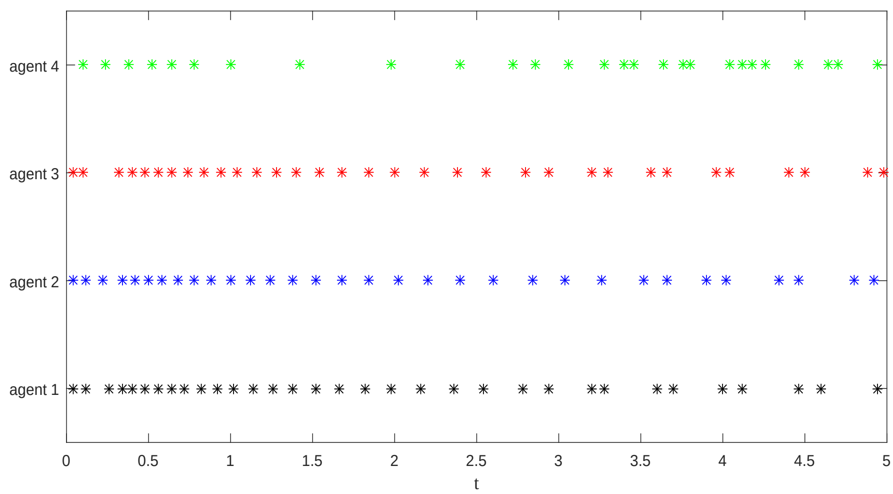

Figure 4. The controller update time instants of each agent with algorithm (6).

5. Conclusions

This paper investigated the flocking problem of MAS with distributed ETC strategy. Distributed ETC laws and triggering functions were proposed to determine the time instants to update the controller for each agent. The free flocking was studied. It was proved that the designed ETC strategy can guarantee the group of agents realize stable flocking motion asymptotically. Finally, simulations are provided to illustrate the theoretical results. In the future, the communication constraints and more efficient ETC design should be considered.

Author Contributions: Conceptualization, Y.S. and Z.K.; methodology, Y.S. and Z.K.; software Y.S.; validation Y.S.; data analysis, Y.S. and Z.K.; writing, Y.S., Z.K. and L.D.; supervision, Z.K. and L.D.

Funding: This research was funded by National Natural Science Foundation of China (61801518) and Hubei Provincial Natural Science Foundation of China (2017CFB661).

Conflicts of Interest: The authors declare no conflict of interest. The funders had no role in the design of the study; in the collection, analyses, and interpretation of data; in the writing of the manuscript, and in the decision to publish the results.

References

1. Åström K.J.; Bernhardsson B. Comparison of periodic and event based sampling for first-order stochastic systems. *IFAC Proc. Vol.* **1999**, *32*, 5006–5011. [CrossRef]
2. Miskowicz, M. Send-On-Delta Concept: An Event-Based Data Reporting Strategy. *Sensors* **2006**, *6*, 49–63. [CrossRef]
3. Losada, M.G. *Contributions to Networked and Event-Triggered of Linear Systems*; Springer: Berlin, Germany, 2016.
4. Miskowicz, M. *Event-Based Control and Signal Processing*; CRC Press: Boca Raton, FL, USA, 2016.
5. Mazo, M.; Tabuada, P. Decentralized event-triggered control over wireless sensor/actuator networks. *IEEE Trans. Autom. Control* **2011**, *56*, 2456–2461. [CrossRef]
6. Santos, C.; Martínez-Rey, M.; Espinosa, F.; Gardel, A.; Santiso, E. Event-based sensing and control for remote robot guidance: An experimental case. *Sensors* **2017**, *17*, 2034. [CrossRef] [PubMed]
7. Acho, L.; Buenestado, P. A robust asynchronous sampled-data control design for nonlinear systems with actuator failures. *Appl. Sci.* **2017**, *7*, 875. [CrossRef]
8. Barrios-Avilés, J.; Rosado-Muñoz, A.; Medus, L.; Bataller-Mompeán, M.; Guerrero-Martínez, J. Less Data Same Information for Event-Based Sensors: A Bioinspired Filtering and Data Reduction Algorithm. *Sensors* **2018**, *18*, 4122. [CrossRef] [PubMed]
9. Thakur, S.; Abdul, S.; Chiu, H.Y.; Roy, R.B.; Huang. P.-Y.; Malwade. S.; Nursetyo, A.A.; Li, Y.-C. Artificial-intelligence-based prediction of clinical events among hemodialysis patients using non-contact sensor data. *Sensors* **2018**, *18*, 2833. [CrossRef]
10. Pan, L.; Lu, Q.; Yin, K.; Zhang, B. Signal Source Localization of Multiple Robots Using an Event-Triggered Communication Scheme. *Appl. Sci.* **2018**, *8*, 977. [CrossRef]
11. Hu, Y.; Lu, Q.; Hu, Y. Event-Based Communication and Finite-Time Consensus Control of Mobile Sensor Networks for Environmental Monitoring. *Sensors* **2018**, *88*, 2547. [CrossRef]
12. Socas, R.; Dormido, R.; Dormido, S. New control paradigms for resources saving: An approach for mobile robots navigation. *Sensors* **2018**, *18*, 281. [CrossRef] [PubMed]
13. Pérez-Torres, R.; Torres-Huitzil, C.; Galeana-Zapién, H. A Cognitive-Inspired Event-Based Control for Power-Aware Human Mobility Analysis in IoT Devices. *Sensors* **2019**, *19*, 832. [CrossRef] [PubMed]
14. Ding, D.R.; Wang, Z.D.; Shen, B.; Wei G.L. Event-triggered consensus control for discrete-time stochastic multi-agent systems: The input-to-state stability in probability. *Automatica* **2015**, *62*, 284–291. [CrossRef]
15. Wang, Z.D.; Ding, D.R.; Dong, H.L.; Shu, H.S. $H\infty$ consensus control for multi-agent systems with missing measurements: The finite-horizon case. *Syst. Control Lett.* **2013**, *62*, 827–836. [CrossRef]
16. Dimarogonas, D.V.; Frazzoli, E.; Johansson, K.H. Distributed event-triggered control for multi-agent systems. *IEEE Trans. Autom. Control* **2012**, *57*, 1291–1297. [CrossRef]
17. Wang, X.F.; Lemmon M. On event design in event-triggered feedback systems. *Automatica* **2011**, *47*, 2319–2322. [CrossRef]

18. Fan, Y.; Feng, G.; Wang, Y.; Song C. Distributed event-triggered control of multi-agent systems with combinational measurements. *Automatica* **2013**, *49*, 671–675. [CrossRef]
19. Seyboth, G.S.; Dimarogonas, D.V.; Johansson, K.H. Event-based broadcasting for multi-agent average consensus. *Automatica* **2013**, *49*, 245–252. [CrossRef]
20. Garcia, E.; Cao, Y.; Giua, A.; Casbeer, D. Decentralized event-triggered consensus with general linear dynamics. *Automatica* **2014**, *50*, 2633–2640. [CrossRef]
21. Yang, D.; Ren, W.; Liu, X.; Chen W. Decentralized event-triggered consensus for linear multi-agent systems under general directed graphs. *Automatica* **2016**, *69*, 242–249. [CrossRef]
22. Chen, Z.; Li, D.; Huang, Y.; Tang C. Event-triggered communication for time synchronization in WSNs. *Neurocomputing* **2016**, *177*, 416–426. [CrossRef]
23. Zhou, F.; Huang, Z.; Yang, Y.; Wang, J.; Li, L.; Peng, J. Decentralized event-triggered cooperative control for multi-agent systems with uncertain dynamics using local estimators. *Neurocomputing* **2017**, *237*, 388–396. [CrossRef]
24. Reynolds, C.W. Flocks, herds, and schools: A distributed behavioral model. *ACM SIGGRAPH Comput. Graph.* **1987**, *21*, 25–34. [CrossRef]
25. Saber, R.O.; Murray, R.M. Flocking for multi-agent dynamic systems: Algorithms and theory. *IEEE Trans. Autom. Control* **2006**, *51*, 401–419. [CrossRef]
26. Su, H.S.; Zhang, N.; Chen, M.Z.Q.; Wang, H.; Wang, X.F. Adaptive flocking with a virtual leader of multiple agents governed by locally Lipschitz nonlinearity. *Nonlinear Anal. Real World Appl.* **2013**, *14*, 798–806. [CrossRef]
27. Wen, G.; Duan, Z.; Su, H.; Chen, G.; Yu W. A Connectivity-preserving flocking algorithm for multi-agent dynamical systems with bounded potential function. *IET Control Theory Appl.* **2011**, *6*, 813–821. [CrossRef]
28. Yu, W.W.; Chen, G.R.; Cao, M. Distributed leader-follower flocking control for multi-agent dynamical systems with time-varying velocities. *Syst. Cont. Lett.* **2010**, *59*, 543–552. [CrossRef]
29. Mecholsky, N.A.; Edward, O.; Antonsen, T.M. Obstacle and predator avoidance in a model for flocking. *Physcia D Nonlinear Phenom.* **2010**, *239*, 988–996. [CrossRef]
30. Hwang, C.; Talipov, E.; Cha, H. Distributed geographic service discovery for mobile sensor networks. *Comput. Net.* **2011**, *55*, 1069–1082. [CrossRef]
31. Crowther, B. Flocking of autonomous unmanned air vehicles. *Aero-Naut. J.* **2003**, *107*, 99–109.
32. Yu, P.; Ding, L.; Liu, Z.W.; Guan, Z.H.; Hu, M.X. Flocking with a virtual leader based on distributed event-triggered hybrid control. In Proceedings of the 32nd Chinese Control Conference, Xi'an, China, 26–28 July 2013; pp. 6947–6952.
33. Yu, P.; Ding, L.; Liu, Z.W.; Guan, Z.H. Leader-follower flocking based on distributed event-triggered hybrid control. *Int. J. Robust Nonlinear Control* **2016**, *26*, 143–153. [CrossRef]
34. Fang, C.; Zhang, Y.; Huang, S. Distributed event-triggered flocking control of nonlinear multi-agent systems. In Proceedings of the 2017 Chinese Automation Congress (CAC), Jinan, China. 20–22 October 2017; pp. 2965–2970.
35. Sun, F.; Wang, R.; Zhu, W.; Li, Y. Flocking in nonlinear multi-agent systems with time-varying delay via event-triggered control. *Appl. Math. Comput.* **2019**, *350*, 66–77. [CrossRef]
36. Merris, R. Laplacian matrices of graph: A survey. *Linear Algebr. Appl.* **1994**, *197*, 143–176. [CrossRef]
37. Ji, M.; Egerstedt, M. Distributed coordination control of multiagent systems while preserving connectedness. *IEEE Trans. Robot.* **2007**, *23*, 693–703. [CrossRef]
38. Balch, T.; Arkin, R.C. Behavior-based formation control for multirobot teams. *IEEE Trans. Robot. Autom.* **1998**, *14*, 926–939. [CrossRef]