

Analysis and Design of Rateless Two-way Relay Networks Based on a Multiply-and-Forward Scheme

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Abstract: In this paper, we propose a rateless, three-stage, two-way multiply-and-forward (MF) relaying system over the Rayleigh flat fading channel, where two source nodes communicate with each other through a relay node and all nodes work on half-duplex and time-division mode. We thoroughly analyze the signals during all three stages in the proposed MF system and derive the closed-form symbol error rate (SER) expressions for an uncoded MF-two-way relay network (MF-TWRN). Furthermore, we provide the equivalent point-to-point fading channel model, which is employed to carry out the asymptotic performance analysis. We finally put forth an optimization model for the MF-TWRN with fountain codes. Simulation results show that our optimized degree distribution can provide outstanding performance for the MF-TWRN compared to those in the literature.

Keywords: two-way relaying; multiply-and-forward; fountain codes; optimization

1. Introduction

Over the past few decades, the two-way relay network (TWRN) [1–13] has aroused great interest due to the overwhelming development of wireless cellular systems, especially when one-way relaying is of low spectral efficiency because of the half-duplex constraint. In a TWRN system, two source nodes exchange information through a relay node. A TWRN can be realized in three or even two stages with the help of network coding technologies. In the three-stage scheme, source nodes send their corresponding messages to the relay in the first two stages, respectively. In the third stage, the relay processes received signals with either a decode-and-forward (DF) approach [1–6] or an amplify-and-forward approach [7–13], and then broadcasts back to the source nodes. In [14], Xu et al. proposed a multiply-and-forward (MF) approach, in which the relay multiplies the signals sent from different source nodes. More importantly, MF has shown a significant performance increase compared to AF in a three-stage TWRN system [14–17].

To ensure reliable data transmission in a large-scale network, fountain codes [18–20] were proposed and have been well investigated in binary erasure channels and noisy channels [21,22]. Some works have been carried out on designing well-performing degree distributions in noisy channels [22–24]. Nguyen et al. proposed the concept of systematic Luby transform (SLT) codes and put forth the truncated robust soliton degree distribution for SLT codes in wireless channels [23]. Xu et al. [24] extended the work of Etesami et al. [22] for raptor codes in memoryless binary symmetric channels to study systematic LT codes in binary input additive white Gaussian noise (BIAWGN) channels and analyzed the asymptotic bit error rate (BER) performance using a Gaussian approximation technique.

To take advantage of relaying systems and rateless codes, we therefore consider employing fountain codes, a class of rateless codes in the TWRN for channel coding at source nodes in this paper. Zhang et al. [25] analyzed a three-stage rateless coded protocol for a DF-TWRN and proposed some

related optimization models for rateless codes used at source nodes. Since the relay carries out decoding operations in the DF approach, which is more complex than AF and MF, we focus on fountain-coded two-way MF relaying in this paper. Specifically, we consider the SLT-coded MF-TWRN over Rayleigh flat fading channels. Furthermore, we will investigate BER performance of our proposed MF-TWRN and its corresponding lower bound on BER. In addition, an optimization model will be derived to generate the well performing degree distribution for SLT codes in the MF-TWRN. Our proposed rateless relaying scheme can lead to heuristic designs of TWRN extending the range of wireless transmission, such as in beyond-5G cellular systems and wireless ad hoc networks.

The rest of this paper is organized as follows. Section 2 introduces the system model of three-stage TWRN and Section 3 derives and compares performance of uncoded TWRN with MF and AF schemes. In Section 4, we establish the equivalent point-to-point channel model, and further study BER performance as well as the optimization method for SLT-coded MF-TWRN. Some numerical results are shown in Section 5, and Section 6 concludes the paper.

The key variables and notations used in this paper are summarized in Table A1.

2. System Model

Consider a time-division TWRN, as shown in Figure 1, where two source nodes A and B communicate with each other through a relay node R. Note that all nodes work on half-duplex mode, in which they cannot transmit and receive signals simultaneously. Let $\mathbf{s}_A$ and $\mathbf{s}_B$ denote the source bit vectors of length k , respectively. In this paper, we employ SLT codes for rateless coding and the corresponding output bits are represented as $\mathbf{c}_A$ and $\mathbf{c}_B$. We refer readers to [24] and references therein for details about SLT encoding and decoding algorithms. Using BPSK(Binary Phase Shift Keying) modulation, the encoded bits transmitted from the source nodes to the relay node are modulated as

$$\mathbf{x}_A = 1 - 2\mathbf{c}_A \quad (1)$$

and

$$\mathbf{x}_B = 1 - 2\mathbf{c}_B \quad (2)$$

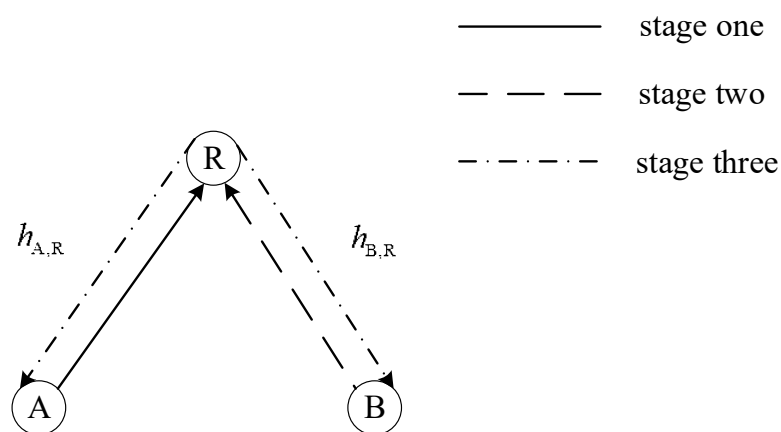


Figure 1. System model of a two-way relay network.

Let the distance between source node i and relay node R denoted by $d_{i,R}$, $i \in \{A, B\}$. Thus, the large-scale path loss is denoted by $d_{i,R}^{-\eta}$, where η represents the path loss exponent. Assume the channels are Rayleigh-distributed, reciprocal and time-invariant during one round of message exchange between nodes A and B. More importantly, assume channel state information is perfectly known to all receivers of nodes A, B and R, therefore, we will not consider any phase rotation that could cause the change of symbol signs during the transmission, for the simplicity of analysis. Denote the small-scale fading

coefficients of each channel by $g_{A,R}$ and $g_{B,R}$, respectively, that follow the same Rayleigh distribution, i.e.,

$$f(g) = \frac{g}{\sigma_R^2} \exp\left(-\frac{g^2}{2\sigma_R^2}\right), g \geq 0 \quad (3)$$

where σ_R^2 represents a crucial parameter in the Rayleigh density and we denote the density by $\text{Rayleigh}(\sigma_R^2)$. in this paper. The fading coefficients $h_{A,R}$ and $h_{B,R}$ are therefore denoted by

$$h_{i,R} = g_{i,R} d_{i,R}^{-\eta} \quad (4)$$

with $h_{i,R}^2$ following the exponential distribution

$$f(h_{i,R}^2) = \frac{d_{i,R}^{2\eta}}{2\sigma_R^2} e^{-\frac{d_{i,R}^{2\eta}}{2\sigma_R^2} h_{i,R}^2} \quad (5)$$

where $i \in \{A, B\}$. Assume all nodes transmit signals with the same power of P and Gaussian noise at each node follows the same density with mean zero and variance σ_n^2 , denoted by $\mathcal{N}(0, \sigma_n^2)$. Using the definition of transmit power and noise, here we can define the transmit signal-to-noise ratio (SNR) as $\gamma = P/\sigma_n^2$, for all nodes in a TWRN.

In the TRWN that we will discuss in the following sections, it takes three stages to exchange messages between two source nodes:

- a) node A sends modulated symbols $\mathbf{x}_A$ to node R and the received signal vector is given by

$$\mathbf{y}_{AR} = h_{A,R} \sqrt{P} \mathbf{x}_A + \mathbf{n}_{AR} \quad (6)$$

- b) node B sends modulated symbols $\mathbf{x}_B$ to node R and the received signal vector is given by

$$\mathbf{y}_{BR} = h_{B,R} \sqrt{P} \mathbf{x}_B + \mathbf{n}_{BR} \quad (7)$$

- c) node R processes the signals received from A and B and broadcasts to nodes A and B with the signal

$$\mathbf{x}_R = \rho R(\mathbf{y}_{AR}, \mathbf{y}_{BR}) \quad (8)$$

where ρ is a factor to ensure power normalization and function $R(\cdot, \cdot)$ represents the processing of received signals at node R, such as DF and AF. Then, the received signals at nodes A and B can be represented as

$$\mathbf{y}_{RA} = h_{A,R} \sqrt{P} \mathbf{x}_R + \mathbf{n}_{RA} \quad (9)$$

and

$$\mathbf{y}_{RB} = h_{B,R} \sqrt{P} \mathbf{x}_R + \mathbf{n}_{RB} \quad (10)$$

Finally, the received signals are processed at each source node to recover source bits from the opposite one. Note that all elements in $\mathbf{n}_{iR}$ and $\mathbf{n}_{Ri}$ are i.i.d (independent and identically distributed). Gaussian variables following $\mathcal{N}(0, \sigma_n^2)$, $i \in \{A, B\}$. In addition, the duration of each exchange round is variant since SLT codes are rateless, i.e., each round comes to an end when both transmitters receive acknowledge from the corresponding destinations.

3. Analysis on Uncoded TWRN

3.1. AF Scheme

In the traditional AF scheme, relay node R adds the received signals from nodes A and B together at the third stage and we have

$$\mathbf{x}_R = \rho_{AF}(\mathbf{y}_{AR} + \mathbf{y}_{BR}) \quad (11)$$

where ρ_{AF} denotes the normalization factor in the AF scheme, so that

$$\rho_{AF}^2 E[\|\mathbf{y}_{AR} + \mathbf{y}_{BR}\|_2^2] = 1 \quad (12)$$

Substituting (6) and (7) into (12), we can further derive

$$\rho_{AF} = \sqrt{\frac{1}{(|h_{A,R}|^2 + |h_{B,R}|^2)P + 2\sigma_n^2}} \quad (13)$$

Due to the similarity of signal processing at nodes A and B, we take source node A for example to describe the processing in details. Employing (6), (7), (9) and (11), the received signal at node A is given by

$$\mathbf{y}_{RA} = \rho_{AF} h_{A,R}^2 P \mathbf{x}_A + \rho_{AF} h_{A,R} h_{B,R} P \mathbf{x}_B + \rho_{AF} h_{A,R} \sqrt{P} \mathbf{n}_R + \mathbf{n}_{RA} \quad (14)$$

where $\mathbf{n}_R$ represents the sum of white Gaussian distributed noise $\mathbf{n}_{AR}$ and $\mathbf{n}_{BR}$, which implies that all elements in $\mathbf{n}_R$ are independently and identically distributed by $\mathcal{N}(0, 2\sigma_n^2)$. Given that the channel state information is well known to both receivers, node A can subtract its own source signal from the received one and get

$$\mathbf{y}'_{RA} = \rho_{AF} h_{A,R} h_{B,R} P \mathbf{x}_B + \rho_{AF} h_{A,R} \sqrt{P} \mathbf{n}_R + \mathbf{n}_{RA} \quad (15)$$

For a coded system, node A can further employ the belief propagation (BP) algorithm to decode the source information $\mathbf{x}_B$ from (15). Moreover, the received SNR at node A derived from (15) is given by

$$\gamma_A^{AF} = \frac{h_{A,R}^2 h_{B,R}^2 P^2}{(3h_{A,R}^2 P + h_{B,R}^2 P + 2\sigma_n^2)\sigma_n^2} \quad (16)$$

Similarly, the received SNR at node B is given by

$$\gamma_B^{AF} = \frac{h_{A,R}^2 h_{B,R}^2 P^2}{(h_{A,R}^2 P + 3h_{B,R}^2 P + 2\sigma_n^2)\sigma_n^2} \quad (17)$$

With the given channel state information $h_{A,R}$ and $h_{B,R}$, the symbol error ratio (SER) for an uncoded BPSK-modulated symbol can be yielded according to the received SNR [26], taking source symbol $\mathbf{s}_B$ at node A, for example:

$$P_{SER,A}^{h_{A,R}/h_{B,R}} = \frac{1}{\pi} \int_0^{\pi/2} \exp\left(-\frac{\gamma_A^{AF}}{2 \sin^2 \theta}\right) d\theta \quad (18)$$

Furthermore, the mean SER can be derived considering Rayleigh fading channels, i.e.,

$$P_{SER,A}^{AF} = \frac{1}{\pi} \int_0^{\pi/2} M_{\gamma_A^{AF}}\left(\frac{1}{2 \sin^2 \theta}\right) d\theta \quad (19)$$

where $M_{\gamma_A^{AF}}(\tau)$ denotes the moment generating function (MGF) of variable γ_A^{AF} , i.e.,

$$M_{\Gamma}(\tau) = \int_{-\infty}^{+\infty} \exp(-\Gamma\tau) p(\Gamma) d\Gamma \quad (20)$$

and $p(\Gamma)$ denotes the probability density function of Γ . In particular, Lemma 1 introduces a more simplified expression of $M_\Gamma(\tau)$ in the high SNR regime.

Lemma 1. ([27]) When Γ can be expressed as the harmonic mean of two exponential distributed random variables with parameters p_1 and p_2 , respectively, the MGF in the sufficiently high SNR regime can be approximated as

$$M_\Gamma(\tau) \approx \frac{1}{\tau}(p_1 + p_2) \quad (21)$$

Accordingly, the mean SER of a BPSK-modulated AF-TWRN is introduced in the following theorem.

Theorem 1. In the high SNR regime, when source nodes transmit uncoded BPSK symbols in a three-stage AF-TWRN over the flat Rayleigh fading channel, the mean SER of AF-TWRN is derived as

$$P_{\text{SER,AF}} = \frac{d_{\text{A,R}}^{2\eta} + d_{\text{B,R}}^{2\eta}}{2\gamma\sigma_{\text{R}}^2} \quad (22)$$

Proof: Let us approximate the received SNR $\gamma_{\text{A}}^{\text{AF}}$ at node A in the high SNR regime as

$$\begin{aligned} \gamma_{\text{A}}^{\text{AF}} &\approx \frac{h_{\text{A,R}}^2 h_{\text{B,R}}^2 P^2}{(3h_{\text{A,R}}^2 P + h_{\text{B,R}}^2 P)\sigma_{\text{n}}^2} \\ &= \frac{1}{3} \cdot \frac{3h_{\text{A,R}}^2 \gamma h_{\text{B,R}}^2 \gamma}{3h_{\text{A,R}}^2 \gamma + h_{\text{B,R}}^2 \gamma} \end{aligned} \quad (23)$$

It is clear that $\gamma_{\text{A}}^{\text{AF}}$ in (23) is the harmonic mean of $3h_{\text{A,R}}^2 \gamma$ and $h_{\text{B,R}}^2 \gamma$. Since $3h_{\text{A,R}}^2 \gamma$ and $h_{\text{B,R}}^2 \gamma$ follow exponential distributions with parameters $d_{\text{A,R}}^{2\eta}/(6\gamma\sigma_{\text{R}}^2)$ and $d_{\text{B,R}}^{2\eta}/(2\gamma\sigma_{\text{R}}^2)$ respectively, the MGF of $\gamma_{\text{A}}^{\text{AF}}$ can be well approximated by

$$M_{\gamma_{\text{A}}^{\text{AF}}}(\tau) \approx \frac{3}{\tau} \left(\frac{d_{\text{A,R}}^{2\eta}}{6\gamma\sigma_{\text{R}}^2} + \frac{d_{\text{B,R}}^{2\eta}}{2\gamma\sigma_{\text{R}}^2} \right) = \frac{d_{\text{A,R}}^{2\eta} + 3d_{\text{B,R}}^{2\eta}}{2\gamma\sigma_{\text{R}}^2 \tau} \quad (24)$$

according to Lemma 1.

Substituting (24) into (19), we can yield the mean SER at node A as

$$P_{\text{SER,A}}^{\text{AF}} = \frac{1}{\pi} \int_0^{\pi/2} \frac{d_{\text{A,R}}^{2\eta} + 3d_{\text{B,R}}^{2\eta}}{\gamma\sigma_{\text{R}}^2} \sin^2 \theta d\theta = \frac{d_{\text{A,R}}^{2\eta} + 3d_{\text{B,R}}^{2\eta}}{4\gamma\sigma_{\text{R}}^2} \quad (25)$$

Similarly, the mean SER at node B is given by

$$P_{\text{SER,B}}^{\text{AF}} = \frac{3d_{\text{A,R}}^{2\eta} + d_{\text{B,R}}^{2\eta}}{4\gamma\sigma_{\text{R}}^2} \quad (26)$$

According to the results in (25) and (26), the mean SER of a BPSK-modulated AF-TWRN is given by

$$P_{\text{SER,AF}} = \frac{1}{2} (P_{\text{SER,A}}^{\text{AF}} + P_{\text{SER,B}}^{\text{AF}}) = \frac{d_{\text{A,R}}^{2\eta} + d_{\text{B,R}}^{2\eta}}{2\gamma\sigma_{\text{R}}^2} \quad (27)$$

□

3.2. MF Scheme

The relay node will amplify the noise in the AF scheme, which may cause severe error expansion problems, though the processing of the AF scheme is relatively simple. Therefore, this section presents a more efficient MF scheme and the performance in the flat Rayleigh fading channel is studied.

In the MF scheme, relay node R multiplies signals from different source nodes before broadcasting. Therefore, at the third stage, the signal $\mathbf{x}_R$ processed by relay node R is shown as

$$\mathbf{x}_R = \rho_{MF}(\mathbf{y}_{AR} \circ \mathbf{y}_{BR}) \quad (28)$$

where $\circ$ represents the Hadamard product of vectors, which is a product operation performed on corresponding elements in vectors of the same dimensions, and ρ_{MF} denotes the power normalization factor, so that

$$\rho_{MF}^2 E[\|\mathbf{y}_{AR} \circ \mathbf{y}_{BR}\|_2^2] = 1 \quad (29)$$

Substituting (6) and (7) into (29), then we have

$$\rho_{MF} = \sqrt{\frac{1}{(h_{A,R}^2 P + \sigma_n^2)(h_{B,R}^2 P + \sigma_n^2)}} \quad (30)$$

Hence, in the MF scheme, the signal received from relay node R at node A can be described as

$$\begin{aligned} \mathbf{y}_{RA} = & \mathbf{n}_{RA} + \rho_{MF} h_{A,R}^2 h_{B,R} P^{\frac{3}{2}} \mathbf{x}_A \circ \mathbf{x}_B + \rho_{MF} h_{A,R}^2 P \mathbf{x}_A \circ \mathbf{n}_{BR} \\ & + \rho_{MF} h_{A,R} h_{B,R} P \mathbf{x}_B \circ \mathbf{n}_{AR} + \rho_{MF} h_{A,R} \sqrt{P} \mathbf{n}_{AR} \circ \mathbf{n}_{BR} \end{aligned} \quad (31)$$

To eliminate its own source signal $\mathbf{x}_A$ of node A, simply multiply $\mathbf{y}_{RA}$ with the conjugate signal of $\mathbf{x}_A$ and we have

$$\begin{aligned} \mathbf{y}'_{RA} = & \rho_{MF} h_{A,R}^2 h_{B,R} P^{\frac{3}{2}} \mathbf{x}_B + \rho_{MF} h_{A,R}^2 P \mathbf{n}_{BR} + \rho_{MF} h_{A,R} h_{B,R} P \mathbf{x}_B \circ \mathbf{n}_{AR} \circ \mathbf{x}'_A \\ & + \rho_{MF} h_{A,R} \sqrt{P} \mathbf{n}_{AR} \circ \mathbf{n}_{BR} \circ \mathbf{x}'_A + \mathbf{n}_{RA} \circ \mathbf{x}'_A \end{aligned} \quad (32)$$

Based on (32), we can observe that all the terms are noise-related except the one containing $\mathbf{x}_B$. The noise terms are with mean zero and variance $(1 + \rho_{MF}^2 h_{A,R}^4 P^2 + \rho_{MF}^2 h_{A,R}^2 h_{B,R}^2 P^2 + \rho_{MF}^2 h_{A,R}^2 P \sigma_n^2) \sigma_n^2$. Since the product of two Gaussian noise is trivial when the noise variance σ_n^2 is much less than the transmit power P , the noise terms can be roughly approximated as Gaussian in the high SNR. BP decoding algorithm is then utilized to recover the original symbol $\mathbf{s}_B$. For $\mathbf{y}'_{RA}$ in (32), the received SNR at source node A in the MF scheme is defined as

$$\begin{aligned} \gamma_A^{MF} &= \frac{\rho_{MF}^2 h_{A,R}^4 h_{B,R}^2 P^3}{(1 + \rho_{MF}^2 h_{A,R}^4 P^2 + \rho_{MF}^2 h_{A,R}^2 h_{B,R}^2 P^2 + \rho_{MF}^2 h_{A,R}^2 P \sigma_n^2) \sigma_n^2} \\ &= \frac{h_{A,R}^4 h_{B,R}^2 P^3}{(h_{A,R}^4 P^2 + 2h_{A,R}^2 h_{B,R}^2 P^2 + 2h_{A,R}^2 P \sigma_n^2 + h_{B,R}^2 P \sigma_n^2 + \sigma_n^4) \sigma_n^2} \end{aligned} \quad (33)$$

Similarly, the received SNR at source node B is given by

$$\gamma_B^{MF} = \frac{h_{A,R}^2 h_{B,R}^4 P^3}{(h_{B,R}^4 P^2 + 2h_{A,R}^2 h_{B,R}^2 P^2 + 2h_{B,R}^2 P \sigma_n^2 + h_{A,R}^2 P \sigma_n^2 + \sigma_n^4) \sigma_n^2} \quad (34)$$

Then, the mean SER of MF-TWRN with BPSK modulation is proposed in Theorem 2.

Theorem 2. In the high SNR regime, when source nodes transmit uncoded BPSK symbols in a three-stage MF-TWRN over the flat Rayleigh fading channel, the mean SER of MF-TWRN is derived as

$$P_{\text{SER, MF}} = \frac{3(d_{\text{A,R}}^{2\eta} + d_{\text{B,R}}^{2\eta})}{8\gamma\sigma_{\text{R}}^2} \quad (35)$$

Proof: For the convenience of theoretical analysis, an approximation is performed on (33) to simplify the complicated expression of received SNR in the MF scheme. In the high SNR regime, the quadratic term of power P is much larger than the linear term so that the received SNR at source node A can be approximated as

$$\gamma_{\text{A}}^{\text{MF}} \approx \frac{h_{\text{A,R}}^2 h_{\text{B,R}}^2 P}{(h_{\text{A,R}}^2 + 2h_{\text{B,R}}^2)\sigma_{\text{n}}^2} \quad (36)$$

in which the linear term of P and noise term are ignored. By means of the harmonic mean, (36) can be reformed as

$$\gamma_{\text{A}}^{\text{MF}} = \frac{1}{2} \cdot \frac{h_{\text{A,R}}^2 \gamma \cdot 2h_{\text{B,R}}^2 \gamma}{h_{\text{A,R}}^2 \gamma + 2h_{\text{B,R}}^2 \gamma} \quad (37)$$

As the SER derived in (19) for the AF scheme, the mean SER at source node A in the flat Rayleigh fading channel for the MF scheme is given by

$$P_{\text{SER,A}}^{\text{MF}} = \frac{1}{\pi} \int_0^{\pi/2} M_{\gamma_{\text{A}}^{\text{MF}}} \left(\frac{1}{2 \sin^2 \theta} \right) d\theta \quad (38)$$

Based on (37), the MGF of $\gamma_{\text{A}}^{\text{MF}}$ in the high SNR regime can be approximately expressed by

$$M_{\gamma_{\text{A}}^{\text{MF}}}(\tau) \approx \frac{2d_{\text{A,R}}^{2\eta} + d_{\text{B,R}}^{2\eta}}{2\gamma\sigma_{\text{R}}^2 \tau} \quad (39)$$

Substituting (39) into (38), the mean SER at source node A in the MF scheme is derived as

$$P_{\text{SER,A}}^{\text{MF}} = \frac{1}{\pi} \int_0^{\pi/2} \frac{2d_{\text{A,R}}^{2\eta} + d_{\text{B,R}}^{2\eta}}{\gamma\sigma_{\text{R}}^2} \sin^2 \theta d\theta = \frac{2d_{\text{A,R}}^{2\eta} + d_{\text{B,R}}^{2\eta}}{4\gamma\sigma_{\text{R}}^2} \quad (40)$$

Similarly, the mean SER at source node B in the MF scheme is derived as

$$P_{\text{SER,B}}^{\text{MF}} = \frac{d_{\text{A,R}}^{2\eta} + 2d_{\text{B,R}}^{2\eta}}{4\gamma\sigma_{\text{R}}^2} \quad (41)$$

Therefore, the mean SER of a BPSK-modulated MF-TWRN is given by

$$P_{\text{SER, MF}} = \frac{1}{2} (P_{\text{SER,A}}^{\text{MF}} + P_{\text{SER,B}}^{\text{MF}}) = \frac{3(d_{\text{A,R}}^{2\eta} + d_{\text{B,R}}^{2\eta})}{8\gamma\sigma_{\text{R}}^2} \quad (42)$$

A comparison between (22) and (35) indicates

$$P_{\text{SER, MF}} < P_{\text{SER, AF}} \quad (43)$$

which shows that, with the same SNR, the performance of MF scheme proposed in this section is better than that of traditional AF scheme in the flat Rayleigh fading channel. $\square$

4. Fountain-coded MF-TWRN

Based on the analysis of three-stage TWRN with different relaying schemes, the performance of AF and MF has been compared by SER derivations in an uncoded system. In this section, asymptotic analysis of rateless coded MF-TWRN is further studied when SLT codes are employed at source nodes and then the corresponding optimization model is proposed to improve the performance of SLT-coded MF-TWRN.

4.1. Asymptotic Analysis

According to (32), the MF-TWRN channel from node B to node A is equivalent to a point-to-point fading channel shown in Figure 2. The BPSK-modulated symbol $\mathbf{x}_B$ is transmitted with unit power and the channel fading coefficient is given by $\rho_{MF} h_{A,R}^2 h_{B,R} P^{3/2}$. The equivalent Gaussian noise satisfies

$$\mathbf{n}_{eqv} = \rho_{MF} h_{A,R}^2 P \mathbf{n}_{BR} + \rho_{MF} h_{A,R} h_{B,R} P \mathbf{x}_B \circ \mathbf{n}_{AR} \circ \mathbf{x}'_A + \rho_{MF} h_{A,R} \sqrt{P} \mathbf{n}_{AR} \circ \mathbf{n}_{BR} \circ \mathbf{x}'_A + \mathbf{n}_{RA} \circ \mathbf{x}'_A \quad (44)$$

with mean zero and variance

$$\sigma_{n,eqv}^2 = \left(1 + \rho_{MF}^2 h_{A,R}^4 P^2 + \rho_{MF}^2 h_{A,R}^2 h_{B,R}^2 P^2 + \rho_{MF}^2 h_{A,R}^2 P \sigma_n^2\right) \sigma_n^2 \quad (45)$$

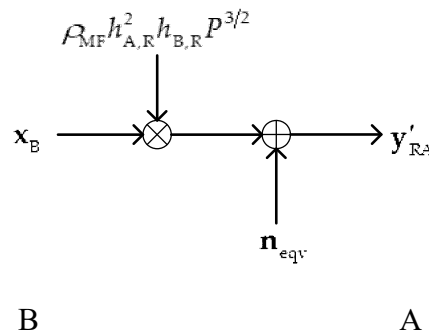


Figure 2. The equivalent point-to-point fading channel model from node B to node A.

In the flat Rayleigh fading channel, coefficients $h_{A,R}$ and $h_{B,R}$ remain invariant during an information exchange between nodes A and B. Therefore, with known $h_{A,R}$ and $h_{B,R}$, the method we proposed in [24] can be used for the decoding and performance analysis of SLT codes in the channel model shown in Figure 2. With the received signal in (32), when the logarithm domain BP decoding algorithm is utilized for the decoder, the initial log-likelihood ratio (LLR) message is given by

$$\mathbf{z} = \frac{2\rho_{MF} h_{A,R}^2 h_{B,R} P^{3/2} \mathbf{y}'_{RA}}{\sigma_{n,eqv}^2} \quad (46)$$

When all-zero codewords are transmitted, the initial LLR message in (46) follows the symmetric Gaussian distribution $\mathcal{N}(2\gamma_A^{MF}, 4\gamma_A^{MF})$. Therefore, with known $h_{A,R}$ and $h_{B,R}$, the lower bound on BER at node A is given by [24]

$$P_{lb,MF,A}^{h_{A,R},h_{B,R}} = \frac{1}{12} e^{-\beta\epsilon - \gamma_A^{MF}/2} \left(e^{\beta\epsilon e^{-\gamma_A^{MF}/2}} - 1 \right) \quad (47)$$

where β represents the mean degree of degree distribution used in LT codes for source nodes. Employing Rayleigh density in (3), the mean lower bound on BER at node A can be calculated by means of

$$P_{lb,MF,A} = \int_0^{+\infty} \int_0^{+\infty} \frac{1}{12} e^{-\beta\epsilon - \gamma_A^{MF}/2} \left(e^{\beta\epsilon e^{-\gamma_A^{MF}/2}} - 1 \right) p(g_{A,R}) p(g_{B,R}) dg_{A,R} dg_{B,R} \quad (48)$$

while the mean lower bound at node B is given by

$$P_{lb,MF,B} = \int_0^{+\infty} \int_0^{+\infty} \frac{1}{12} e^{-\beta\epsilon - \gamma_B^{MF}/2} \left(e^{\beta\epsilon e^{-\gamma_B^{MF}/2}} - 1 \right) p(g_{A,R}) p(g_{B,R}) dg_{A,R} dg_{B,R} \quad (49)$$

Then, the mean lower bound on BER of SLT-coded MF-TWRN in the flat Rayleigh fading is derived as

$$P_{lb,MF} = \frac{P_{lb,MF,A} + P_{lb,MF,B}}{2} \quad (50)$$

Specifically, an approximation of the mean lower bound in a three-stage MF-TWRN with respect to high SNRs is proposed in Theorem 3.

Theorem 3. In the high SNR regime, with SLT coding and BPSK modulation, the mean lower bound on BER of the three-stage MF-TWRN in the flat Rayleigh fading channel is derived as

$$P_{lb,MF,HSNR} = \frac{(d_{A,R}^{2\eta} + d_{B,R}^{2\eta})\beta\epsilon e^{-\beta\epsilon}}{16\gamma\sigma_R^2} \quad (51)$$

Proof: With given $h_{A,R}$ and $h_{B,R}$, the lower bound on BER at node A can be approximated by [24]

$$P_{lb,MF,A,HSNR}^{h_{A,R},h_{B,R}} = \frac{1}{12} \beta\epsilon e^{-(\beta\epsilon + \gamma_A^{MF})} \quad (52)$$

Then, the MGF described in (39) can be used to derive the mean lower bound in the high SNR regime, i.e.,

$$\begin{aligned} P_{lb,MF,A,HSNR} &= \int_0^{+\infty} \int_0^{+\infty} \frac{1}{12} \beta\epsilon e^{-(\beta\epsilon + \gamma_A^{MF})} p(g_{A,R}) p(g_{B,R}) dg_{A,R} dg_{B,R} \\ &= \frac{1}{12} \beta\epsilon e^{-\beta\epsilon} M_{\gamma_A^{MF}}(1) \\ &= \frac{(2d_{A,R}^{2\eta} + d_{B,R}^{2\eta})\beta\epsilon e^{-\beta\epsilon}}{24\gamma\sigma_R^2} \end{aligned} \quad (53)$$

Similarly, the mean lower bound on BER at node B in the high SNR regime is described as

$$P_{lb,MF,B,HSNR} = \frac{(d_{A,R}^{2\eta} + 2d_{B,R}^{2\eta})\beta\epsilon e^{-\beta\epsilon}}{24\gamma\sigma_R^2} \quad (54)$$

Therefore, the mean lower bound on BER of SLT-coded three-stage MF-TWRN is given by

$$P_{lb,MF,HSNR} = \frac{P_{lb,MF,A,HSNR} + P_{lb,MF,B,HSNR}}{2} = \frac{(d_{A,R}^{2\eta} + d_{B,R}^{2\eta})\beta\epsilon e^{-\beta\epsilon}}{16\gamma\sigma_R^2} \quad (55)$$

□

Because the channels between two source nodes and the relay node are asymmetric, nodes A and B may decode the received signals correctly with different overheads. In order to consider the overall efficiency of information exchanges in the TWRN, the corresponding definition of throughput is introduced as the following.

Definition 1. During an information exchange, assume the overheads that nodes A and B need for successful decoding are ε_A and ε_B , respectively. Then, the throughput of TWRN, i.e., the number of useful information bits contained in a one-bit transmission, can be defined as

$$T = 1 / (1 + \varepsilon_{\max}) \quad (56)$$

where $\varepsilon_{\max} = \max(\varepsilon_A, \varepsilon_B)$.

With given $h_{A,R}$ and $h_{B,R}$, the equivalent channel capacity between nodes A and B can be derived by substituting the received SNRs γ_A^{MF} and γ_B^{MF} into [18]

$$C = 1 - \frac{1}{2\sqrt{2\pi\gamma}} \int_{-\infty}^{+\infty} \log_2(1 + e^{-x}) \exp\left(-\frac{(x - 2\gamma)^2}{8\gamma}\right) dx \quad (57)$$

where C represents the BIAWGN channel capacity. Then, the capacities of equivalent channel between nodes A and B are shown as

$$C_A(h_{A,R}, h_{B,R}) = 1 - \frac{1}{2\sqrt{2\pi\gamma_A^{MF}}} \int_{-\infty}^{+\infty} \log_2(1 + e^{-\tau}) \exp\left(-\frac{(\tau - 2\gamma_A^{MF})^2}{8\gamma_A^{MF}}\right) d\tau \quad (58)$$

and

$$C_B(h_{A,R}, h_{B,R}) = 1 - \frac{1}{2\sqrt{2\pi\gamma_B^{MF}}} \int_{-\infty}^{+\infty} \log_2(1 + e^{-\tau}) \exp\left(-\frac{(\tau - 2\gamma_B^{MF})^2}{8\gamma_B^{MF}}\right) d\tau \quad (59)$$

where $C_A(h_{A,R}, h_{B,R})$ represents the equivalent channel capacity from node B to A, while $C_B(h_{A,R}, h_{B,R})$ represents the reverse one. Therefore, the theoretical upper bound on the throughput of MF-TWRN in an information exchange is given by

$$T \leq C_{\min}(h_{A,R}, h_{B,R}) \quad (60)$$

where

$$C_{\min}(h_{A,R}, h_{B,R}) = \min(C_A(h_{A,R}, h_{B,R}), C_B(h_{A,R}, h_{B,R})) \quad (61)$$

Consider Rayleigh distributed channel fading coefficients, the mean upper bound on throughput for MF-TWRN in the flat Rayleigh fading channel is represented by

$$T_{ub} = \int_0^{+\infty} \int_0^{+\infty} C_{\min}(h_{A,R}, h_{B,R}) p(g_{A,R}) p(g_{B,R}) dg_{A,R} dg_{B,R} \quad (62)$$

4.2. Optimization Design

In order to successfully decode the BPSK-modulated SLT codes in the noisy channel, the mean value of LLR messages should increase from iteration to iteration during BP decoding. Obviously, it is necessary to satisfy this condition for different received SNRs so that SLT-coded MF-TWRN could achieve good performance in the flat Rayleigh fading channel. Assume Γ denotes the set of received SNRs in MF-TWRN. For $\forall \gamma_0 \in \Gamma$, the aforementioned condition can be described as

$$\varepsilon \sum_{d=1}^{d_c} d\Omega_d f_d\left(\mu_Q^{(l-1)}\right) + 2\gamma_0 > \mu_Q^{(l-1)} \quad (63)$$

where $\mu_Q^{(l)}$ represents the mean of LLR messages in the l -th iteration, and

$$f_d\left(\mu_Q^{(l-1)}\right)=\varphi^{-1}\left(1-\left(1-\varphi\left(2\gamma_0\right)\right)\left(1-\varphi\left(\mu_Q^{(l-1)}\right)\right)^{d-1}\right) \quad (64)$$

where

$$\varphi(x)=\begin{cases} \exp(-0.4527x^{0.86}+0.0218), & 0 < x < 10 \\ \sqrt{\frac{\pi}{x}}\left(1-\frac{10}{x}\right) \exp\left(-\frac{x}{4}\right), & x \geq 10 \end{cases} \quad (65)$$

Inspired by the optimization model we have proposed for BPSK-modulated SLT codes in the BIAWGN channel [24], the optimization model of degree distribution for SLT-coded MF-TWRN in the flat Rayleigh fading channel is proposed in what follows, aiming at minimizing the average degree with main constraints introduced by (50) and (63):

$$\begin{aligned} \min \quad & \sum_{d=1}^{d_c} d\Omega_d \\ \text{s.t.} \quad & \varepsilon_0 \sum_{d=1}^{d_c} d\Omega_d f_d(\mu_i) + 2\gamma_0 > \mu_i, i = 1, \dots, N, \forall \gamma_0 \in \Gamma \\ & P_{\text{lb, MF}} \leq y_0 \\ & \sum_{d=1}^{d_c} \Omega_d = 1 \\ & \Omega_d \geq 0, d = 1, \dots, d_c \end{aligned} \quad (66)$$

where $\mu_i (i = 1, \dots, N)$ represents the N equidistant points in the interval $(0, \mu_0]$. The values of d_c , ε_0 , μ_0 , y_0 and N should be determined in advance, serving as control variables. For the practical use of this optimization model, discretization is generally performed on the set of received SNR, and we make use of a quasi-Newton method to iterate toward a solution of optimized degree distribution.

5. Simulation Results

In this section, we offer simulation results to validate our work on the design of rateless MF-TWRN. In our simulation, the transmit power of all nodes is normalized and the distances between the relay node and source nodes are identical with the value of 1. Moreover, assume the path loss exponent is of 2. The channel fading coefficients are with unit power, i.e., $g_{i,R} \sim \text{Rayleigh}(0.5)$, $i \in \{A, B\}$.

For an uncoded TWRN system with BPSK modulation, Figure 3 demonstrates the SER performance of two forwarding schemes, namely, MF and AF, over the flat Rayleigh fading channel, regarding theoretical and simulation results. The solid lines in Figure 3 depict numerical results of MF and AF, respectively, while the dotted lines are plotted using theoretical results in (22) and (35). It is implied that the numerical SERs can be approximated by theoretical ones, coinciding in the sufficiently high SNR regime especially. Just like the conclusion drawn in the Section 3, numerical results also show that the MF scheme outperforms the AF scheme in the flat Rayleigh channel.

Using the optimization model proposed in (66), Table A2 offers the optimized result with parameters $\sigma_n^2 = 10^{-2}$, $d_c = 200$, $y_0 = 5 \times 10^{-4}$ and $\varepsilon_0 = 5$, which is denoted by $\Omega_{\text{opt}}(x)$. The authors in [25] studied the design of fountain codes in a three-stage TWRN using joint network-channel coding. Their optimized distribution was obtained with the channel condition $\sigma_n^2 = 10^{-2}$ as well and is denoted by $\Omega_{\text{JNC}}(x)$ for short. In the flat Rayleigh fading channel with the transmit SNR of 20dB, Figures 4 and 5 demonstrate the BER performance of MF-TWRN at node A with different lengths of source information, when using the optimized distribution $\Omega_{\text{opt}}(x)$. It is clear that the BER decreases as the source length increases from 2000 to 4000, and approaches the theoretical lower bound especially in the high overhead regime. This observation made on Figures 4 and 5 also validate the theoretical analysis carried out in Section 4. To compare the overall performance of a TWRN system thoroughly, Figures 6–8 show the performance of MF-TWRN from the perspectives of frame-error-rate (FER) and

throughput, respectively. Figures 6 and 7 compare the mean FER of two source nodes using different optimized distributions and the comparison results imply that our optimized distribution $\Omega_{\text{opt}}(x)$ can improve the decoding performance of the whole TWRN system. Furthermore, Figure 8 describes the throughput of MF-TWRN using different degree distributions, in which the numerical results are obtained for the source length of 4000 while the theoretical results correspond to upper bounds on the throughput derived in (62). Along with the increase of the SNR, the actual throughput of TWRN approaches the theoretical upper bound. One can get a higher throughput using the optimized distribution $\Omega_{\text{opt}}(x)$ rather than $\Omega_{\text{JNC}}(x)$ proposed in [25], which means information exchanges can be successfully made with a smaller overhead. Especially in the low SNR regime, we can observe a 1dB gain with respect to the throughput. In other words, with the same transmit SNR, almost 20% of the overhead can be saved to ensure a less time-consuming information exchanges between source nodes. The comparison of MF and AF theoretical throughputs in Figure 8 is in accordance with what is offered in Figure 3, which validates the superiority of MF forwarding scheme over the flat Rayleigh fading channel.

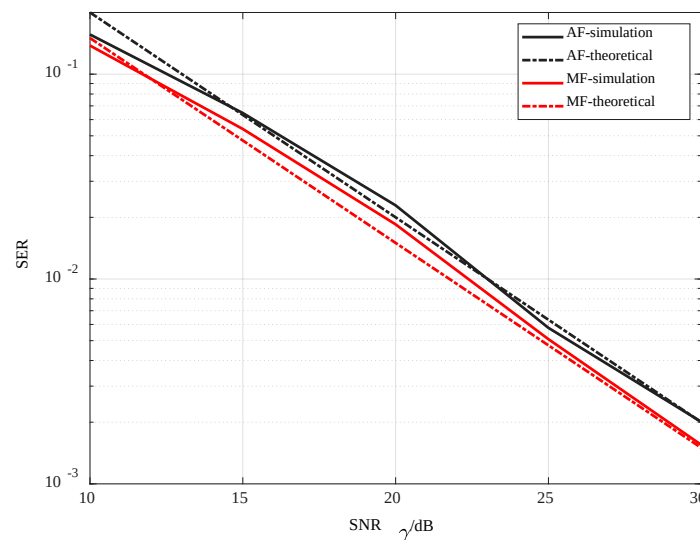


Figure 3. Symbol error rate (SER) performance comparison of different forwarding schemes over the flat Rayleigh fading channel.

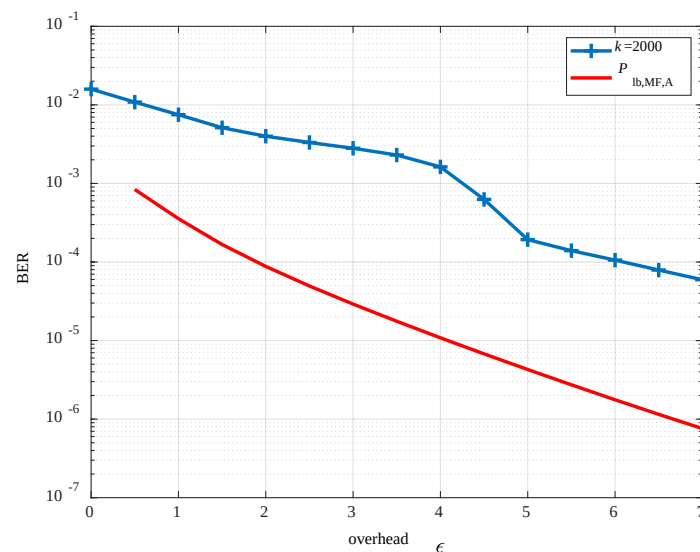


Figure 4. Bit error rate (BER) performance of source node A when using $\Omega_{\text{opt}}(x)$ with $k = 2000$.

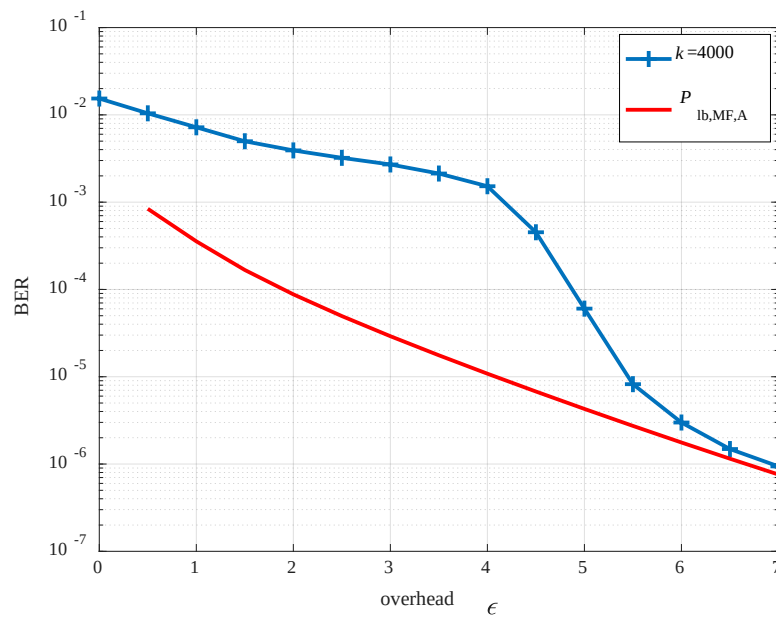


Figure 5. BER performance of source node A when using $\Omega_{\text{opt}}(x)$ with $k = 4000$.

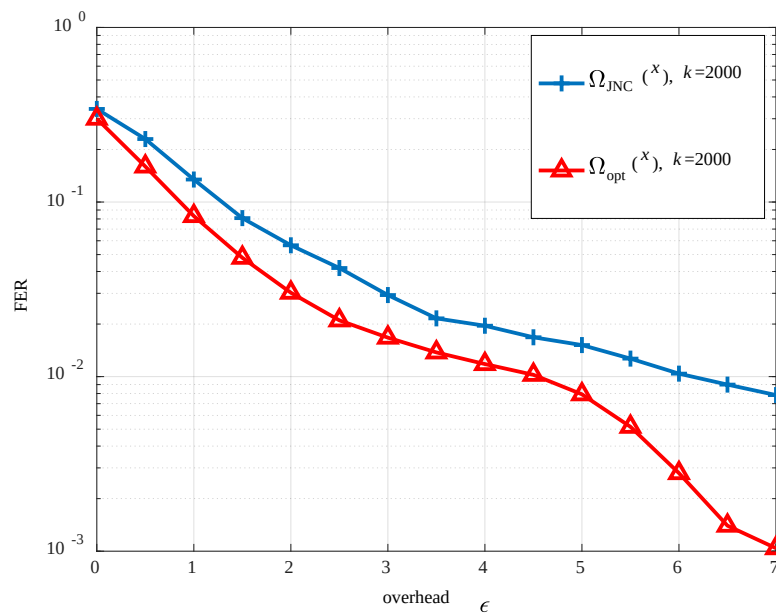


Figure 6. Frame error rate (FER) performance of systematic Luby transform (SLT)-coded multiply-and-forward two-way relay network (MF-TWRN) using different degree distributions with $k = 2000$.

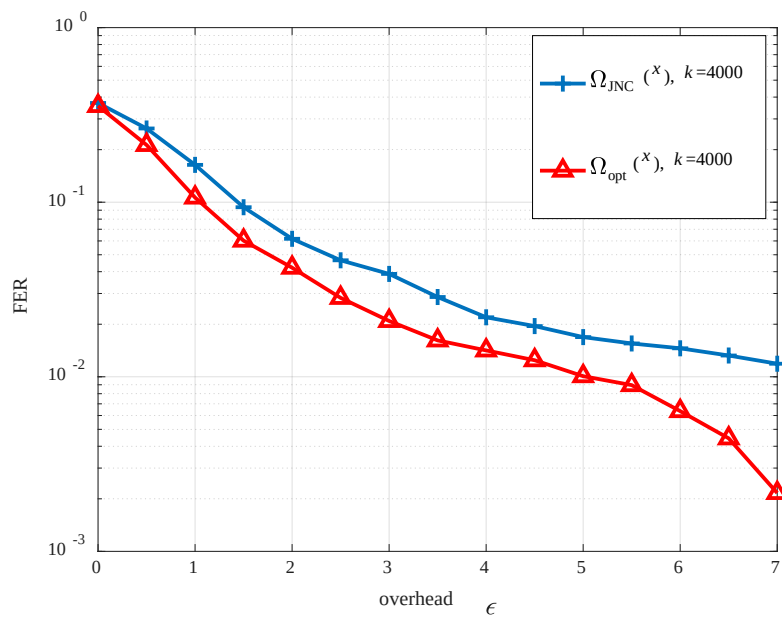


Figure 7. FER performance of SLT-coded MF-TWRN using different degree distributions with $k = 4000$.

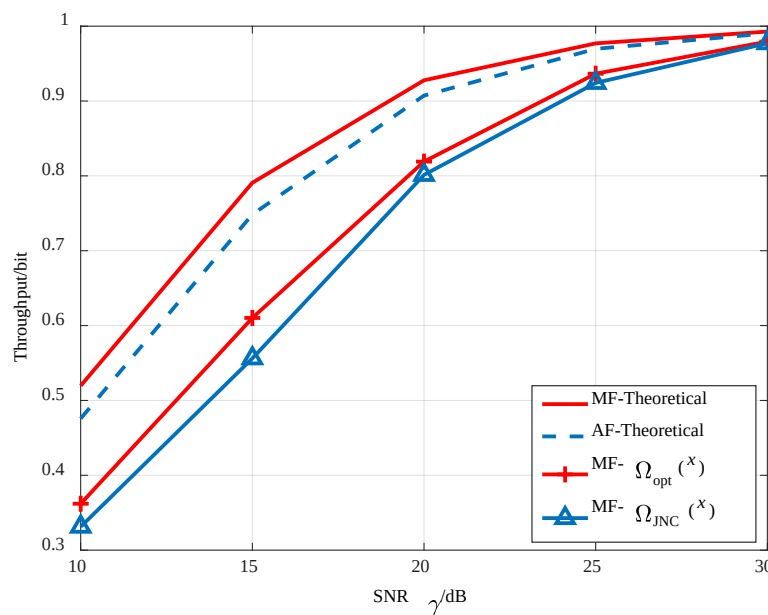


Figure 8. Throughputs of the SLT-coded TWRN system with different degree distributions.

6. Conclusions

In this paper, we investigated a fountain-coded MF-TWRN in the Rayleigh flat fading channel. Specifically, SLT codes are applied at both source nodes. The SERs of uncoded TWRN with different forwarding schemes were first derived and the theoretical analysis proved that our proposed MF-TWRN outperforms the traditional AF-TWRN. To analyze the BER performance of the proposed network, we first offered the equivalent channel model from the perspective of a point-to-point communication system. Using this equivalent channel model, we then studied the lower bound on BER in the MF-TWRN. The corresponding degree distribution was optimized for the Rayleigh flat fading channel. Through simulations, it is shown that our degree distribution has a significant BER performance increase for the proposed MF-TWRN compared to other degree distributions in the literature. In the future work, we will consider a more general application scenario, for example, asymmetric channels.

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Conflicts of Interest: The authors declare no conflict of interest.

Appendix A

Table A1. Summarization of Key Variables and Notations.

x	Source Vector
y	received vector
h	channel fading coefficient
d	distance between source and relay node
η	path loss exponent
γ	signal-to-noise ratio
P	transmit power
ρ	power normalization factor
$\mathcal{N}(\mu, \sigma^2)$	Gaussian density with mean μ and σ^2 variance
$E[\cdot]$	mathematical expectation
$\Omega(x)$	degree distribution
$M_{\Gamma}(\tau)$	Moment generating function of Γ
$\mathbf{A} \circ \mathbf{B}$	Hadamard product of vectors $\mathbf{A}$ and $\mathbf{B}$

Appendix B

Table A2. Optimized Degree Distribution for MF-TWRN by the Proposed Model.

d	Ω_d	d	Ω_d	d	Ω_d	d	Ω_d
1	0.0853	42	0.0002	74	0.0002	106	0.0002
2	0.2421	43	0.0002	75	0.0002	107	0.0002
3	0.0917	44	0.0002	76	0.0002	108	0.0002
4	0.1955	45	0.0002	77	0.0002	109	0.0002
5	0.0616	46	0.0002	78	0.0002	110	0.0002
6	0.0522	47	0.0002	79	0.0002	111	0.0002
7	0.0459	48	0.0002	80	0.0002	112	0.0002
8	0.0761	49	0.0002	81	0.0002	113	0.0002
9	0.0333	50	0.0002	82	0.0002	114	0.0002
10	0.0270	51	0.0002	83	0.0002	115	0.0002
11	0.0207	52	0.0002	84	0.0002	116	0.0002
12	0.0144	53	0.0002	85	0.0002	117	0.0002
13	0.0081	54	0.0002	86	0.0002	118	0.0002
14	0.0203	55	0.0002	87	0.0002	119	0.0002
24	0.0001	56	0.0002	88	0.0002	120	0.0002
25	0.0001	57	0.0002	89	0.0002	121	0.0002
26	0.0001	58	0.0002	90	0.0002	122	0.0002
27	0.0001	59	0.0002	91	0.0002	123	0.0002
28	0.0001	60	0.0002	92	0.0002	124	0.0002
29	0.0001	61	0.0002	93	0.0002	125	0.0001
30	0.0005	62	0.0002	94	0.0002	126	0.0001

Table A2. Cont.

d	Ω_d	d	Ω_d	d	Ω_d	d	Ω_d
31	0.0001	63	0.0002	95	0.0002	127	0.0001
32	0.0001	64	0.0002	96	0.0002	128	0.0001
33	0.0011	65	0.0002	97	0.0002	129	0.0001
34	0.0001	66	0.0002	98	0.0002	130	0.0001
35	0.0001	67	0.0002	99	0.0002	131	0.0001
36	0.0002	68	0.0002	100	0.0002	132	0.0001
37	0.0002	69	0.0002	101	0.0002	133	0.0001
38	0.0002	70	0.0002	102	0.0002	134	0.0001
39	0.0002	71	0.0002	103	0.0002	135	0.0001
40	0.0002	72	0.0002	104	0.0002	136	0.0001
41	0.0002	73	0.0002	105	0.0002	200	0.0008

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