Precise Measurement of the Surface Shape of Silicon Wafer by Using a New Phase-Shifting Algorithm and Wavelength-Tuning Interferometer

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Abstract: In wavelength-tuning interferometry, the surface profile of the optical component is a key evaluation index. However, the systematic errors caused by the coupling error between the higher harmonics and phase shift error are considerable. In this research, a new $10N-9$ phase-shifting algorithm comprising a new polynomial window function and a DFT is developed. A new polynomial window function is developed based on characteristic polynomial theory. The characteristic of the new $10N-9$ algorithm is represented in the frequency domain by Fourier description. The phase error of the new algorithm is also discussed and compared with other phase-shifting algorithms. The surface profile of a silicon wafer was measured by using the $10N-9$ algorithm and a wavelength-tuning interferometer. The repeatability measurement error across 20 experiments was 2.045 nm, which indicates that the new $10N-9$ algorithm outperforms the conventional phase-shifting algorithm.

Keywords: fringe analysis; interferometry; surface shape; wavelength tuning; phase error; nondestructive testing

1. Introduction

In the semiconductor industry, high-purity and high-quality polished silicon wafers are the best and most widely used material for the manufacturing of integrated circuits (ICs). Silicon wafers will still be the most basic and important functional material of the IC industry for the next several decades. This kind of material has high quality characteristics; for example, the current passes the wafer more quickly than other conductors and has excellent mobility under high temperature. Therefore, silicon wafers have a broad range of uses not only in semiconductors but also in microlithography and micro-electromechanical systems, even in tire pressure sensor systems.

One of the basic parameters for measuring the performance of a silicon wafer is the surface profile. Therefore, the surface profile must be accurately measured [1]. Generally, the surface shape measurement methods can be divided into non-optical methods and optical methods. Non-optical methods include inductance methods [2], capacitance methods [3], atomic force microscopy (AFM), scanning probe microscopy (SPM), and scanning electron microscopy (SEM), among others. For testing large aperture optical devices, the limitations of non-optical methods are their nonlinearity and narrow dynamic range; thus, they do not meet the requirements of surface shape measurement. Optical methods mainly include confocal microscopy [4], structured light microscopy [5], and optical interferometry [6–15]. Optical interferometry outperforms other methods not only because it is a non-contact measurement method but because optical interferometry has a broad measurement range. Optical interferometry involves low-coherence interferometry [6–9], monochromatic interferometry [10–12], and spectral interferometry [13–15]. Spectral interferometry includes spectrally resolved interferometry [13] and wavelength-tuning interferometry [14,15]. There are many other interference approaches to shape...
measurements. Günther et al. [16] utilized a non-incremental interferometer based on two mutually tilted interference fringe systems to realize precise distance measurement of fast-moving rough surfaces. By using the specialized imaging acousto-optical tunable filter, Machikhin et al. [17] achieved spectral selection of light in the output arm of an interferometer. Kato et al. [18] developed a no-scanning 3D measurement method with high-precision, wide-range, and ultrafast time-resolution. Wavelength-tuning Fizeau interferometry has demonstrated better performance in the measurement of the surface profile [19–23] than other optical interferometry methods because of its ability for separating overlapped interference signals in frequency space. In the measurement process of using a wavelength-tuning Fizeau interferometer, the phase difference between the sample surface and the reference surface is changed step by step, the interferograms are captured by the CCD (Charge Coupled Device) camera at each step, and the phase distribution can then be obtained.

When measuring the surface profile of silicon wafers with a wavelength-tuning phase-shifting interferometer, several kinds of errors may be generated:

- phase shift errors, produced during phase shift;
- harmonics, resulting from high reflectivity; and
- coupling errors, generated between harmonics and phase shift error.

Except for the phase shift error and harmonics, the coupling errors must be considered in order to obtain high measurement accuracy [24].

A well-designed phase-shifting algorithm can strongly improve the accuracy of the measurement. Therefore, various algorithms [25–39] have been developed with the ability to compensate for the systematic errors in the measurement. Schwider and Hariharan developed a five-sample algorithm [25,32] that can compensate for the phase shift miscalibration. However, this algorithm cannot compensate for coupling errors. Schmit and Creath’s five- and six-sample [27] algorithms effectively eliminate the phase shift nonlinearity by the extended averaging method. Zhao et al. [28] developed a six-sample algorithm insensitive to the second harmonic component. Other phase-shifting algorithms can be designed using Fourier representation [26], data-sampling windows [29,31], or the theory of characteristic polynomial [30,34,38,39]. Surrel [30] developed the $2N − 1$ algorithm by using characteristic polynomial theory, which can compensate for both coupling errors and phase shift miscalibration. However, the $2N − 1$ algorithm did not have the ability to compensate for the nonlinearities of the phase shift error. Hibino et al. designed a 19-sample algorithm with an ability to compensate for the coupling error [33,35]. A new three-step phase-shifting algorithm with a faster speed than the traditional one was presented by Huang et al. [32] by changing the arctangent function to a simple intensity ratio function. To reduce errors and eliminate ghosts, Kakue et al. [37] presented a new algorithm that can improve the quality of reconstructed images by applying parallel four-step phase-shifting holography to the hologram. We designed the $5N − 4$ algorithm [38] by placing the fifth multiple roots on the complex-unit circle. The $5N − 4$ algorithm has the ability to compensate for the third-order nonlinearity of phase shifts, but the errors occurring from the residual phase shift errors and coupling errors were observed clearly in the generated surface profile. To obtain more accurate measurements, a new algorithm with a stronger ability to compensate for the high-order nonlinearity of phase-shift errors and coupling errors should be developed.

In this research, a novel $10N − 9$ phase-shifting algorithm was derived. The new algorithm utilizes the Discrete Fourier Transform function and polynomial window function. The proposed algorithm enables the measurement of silicon wafers with high reflectivity [39]. The characteristic of the new $10N − 9$ algorithm is represented in the frequency domain by Fourier description [40]. The phase errors were calculated, and the results indicate that the new $10N − 9$ algorithm has the smallest value of phase error, compared with other conventional algorithms. Finally, the repeatability measurement error by calculation across 20 experiments was 2.045 nm, which indicates that the novel $10N − 9$ algorithm outperforms the conventional phase-shifting algorithm.
2. Wavelength-Tuning Interferometry

To show the optical path, the optical setup of the wavelength-tuning Fizeau interferometer, used for the measurement of the surface profile of highly reflective silicon wafers, is shown in Figure 1.

![Figure 1. The setup of the wavelength-tuning Fizeau interferometer. PBS, Polarizing beam splitter; QWP, Quarter wave plate; CCD, Charge Coupled Device.](image)

A wavelength-tuning phase-shifting Fizeau interferometer has much better anti-air turbulence ability than other conventional interferometers. The light source is a tunable diode laser with a Littman external cavity [41] (Newport Inc., NewFocus TLB-6804, mode-hop-free fine-frequency tuning range > 120 GHz), which can scan the wavelength linearly from 632.5 to 640 nm. The light was transmitted out from the laser source and was then divided into two beams in the beam splitter: one was incident to the interferometer and the other was transmitted to a wavelength meter. In the interferometer, the collimated laser beam illuminated the object (silicon wafer) and reference surface. The beams reflected from silicon wafers and the reference surfaces combine and generate the interferograms on the screen and are acquired by a Charge Coupled Device (CCD) (The Imaging Source, DMK 33G445) camera and processed by an Analog to Digital Converter (ADC) (The Imaging Source, DFG/USB2pro). The technical parameters of the CCD are shown in Table 1. The PC processor is an Intel® Core i7-6900K with an NVIDIA GeForce RTX 2080Ti graphic card and 32 GB of RAM.

<table>
<thead>
<tr>
<th>Items</th>
<th>Technical Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dynamic range</td>
<td>12 bit</td>
</tr>
<tr>
<td>Resolution</td>
<td>1280 × 960 pixels</td>
</tr>
<tr>
<td>Frame rate</td>
<td>30 fps</td>
</tr>
<tr>
<td>Pixel size</td>
<td>Vertical/horizontal: 3.75 μm</td>
</tr>
<tr>
<td>Sensor</td>
<td>SONY ICX 445ALA</td>
</tr>
<tr>
<td>Shutter</td>
<td>10 μs–30 s</td>
</tr>
<tr>
<td>Gain</td>
<td>0–30.39 dB</td>
</tr>
</tbody>
</table>

In the wavelength-tuning Fizeau interferometry, significant interferences appear as a result of multi-reflections between the sample surface and the reference surface. Figure 2 shows the multi-reflections between the reference surface and the sample surface with reflectivities $r_1$ and $r_2$.

![Figure 2. The reflections between surfaces.](image)
The signal irradiance detected by a CCD camera is given by

\[ I(\alpha_k) = \sum_{k=1}^{\infty} A_m \cos(\varphi_m - \alpha_k) \]
\[ = I_0 + I_0 \gamma_1 \cos(\varphi_1 - \alpha) + I_0 \gamma_2 \cos(\varphi_2 - 2\alpha) + \ldots, \]  
(1)

where \( \alpha_k \) is the phase shift amount, \( I_0 \) is the DC component, and \( A_m \) and \( \varphi_m \) are the amplitude and phase of the \( k \)th harmonic component, respectively. The parameters that are given by

\[ I_0 = \frac{r_1 + r_2 - 2r_1r_2}{1 - r_1r_2}, \]
(2)

\[ \gamma_1 = \frac{2(1 - r_1)(1 - r_2)}{r_1 + r_2 - 2r_1r_2} \sqrt{r_1r_2}, \]
(3)

\[ \gamma_2 = -\gamma_1 \sqrt{r_1r_2}, \]
(4)

\[ \gamma_3 = \gamma_1 r_1r_2. \]
(5)

where \( r_1 \) is the reflectivity of the reference surface and \( r_2 \) is the reflectivity of the sample surface. For an \( M \)-sample algorithm, the phase \( \varphi_1 \) of the fundamental signal can be expressed as

\[ \varphi_1 = \arctan \left( \frac{\sum_{k=1}^{M} b_k I(\alpha_k)}{\sum_{k=1}^{M} a_k I(\alpha_k)} \right) \]  
(6)

The reference phases can be divided into \((M - 1)\) intervals. The phase shift value \( \alpha = 2\pi/N \) (\( N \) is an integer), \( a_k \) and \( b_k \) are the \( k \)th sampling amplitude, and \( I_k \) is the \( k \)th sampled irradiance \((I_k = I(\alpha_k))\) defined in Equation (1). The nonlinear relation between the \( a_k \) value and the phase shift parameter is clearly present when the phase shift is nonlinear, which can be expressed as \([42]\)

\[ \alpha_k = \alpha_{0k} \left[ 1 + \varepsilon(\alpha_{0k}) \right] \]
\[ = \alpha_{0k} \left[ 1 + \varepsilon_0 + \varepsilon_1 \frac{\alpha_{0k}}{\pi} + \varepsilon_2 \left( \frac{\alpha_{0k}}{\pi} \right)^2 + \ldots + \varepsilon_p \left( \frac{\alpha_{0k}}{\pi} \right)^p \right], \]  
(7)

where \( \alpha_{0k} = (2\pi/N)[k - (M + 1)/2] \) is the unperturbed phase shift value and \( \varepsilon_p \) is the error coefficient that is assumed to be much smaller than unity and be spatially non-uniform. \( \varepsilon_1 \) is the coefficient of the phase shift miscalibration. \( \varepsilon_2 \) is the coefficient of the first-order nonlinearity. \( \varepsilon_3 \) is the coefficient of the second-order nonlinearity.

The phase error \( \Delta\varphi \) is the function of \( A_m \) and \( \varepsilon_p \). The function after Taylor expansion is shown below:

\[ \Delta\varphi = \left( A_m \right) + \left( \varepsilon_p \right) + \left( A_m \varepsilon_p \right), \]
(8)

where \( o(A_m) \), \( o(\varepsilon_p) \), and \( o(A_m\varepsilon_p) \) are the harmonics error, phase shift error, and coupling error, respectively. For the measurement of high reflective silicon wafer, the coupling error value could significantly influence the phase distribution.

3. Derivation of the 10N−9 Phase-Shifting Algorithm

3.1. Characteristic Polynomial Theory

Surrel [30] designed a simple method to derive new algorithm. The derived algorithm not only satisfies the phase measurement requirements but also is insensitive to harmonics and phase shift miscalibration. With this method, arbitrary and custom algorithms can be generated without complicated calculations. For an \( M \)-sample algorithm, the characteristic polynomial is defined as
where $x = \exp(ima)$, $\alpha = 2\pi/N$, and $i$ is an imaginary unit. In this method, the multiplicities and locations of the roots of the characteristic polynomial determine that the algorithm is not sensitive to harmonic and phase shift error.

3.2. Design Process of the 10N − 9 Algorithm

For achieving insensitivity to the $k$th harmonic component in the intensity signal, a single root should be located in a characteristic diagram except for the fundamental signal $m = 1$, based on Surrel’s theory. However, the synchronous detection algorithm [43] cannot compensate for the phase shift errors and coupling errors. To be insensitive to the phase shift miscalibration, the complex number $x = \exp(-ia)$ should be double roots of characteristic polynomial, which is the $N + 1$ algorithm derived by Surrel [44]. By placing double roots at the complex number $\exp(ima)$ ($m = 2, 3, 4$) on the characteristic diagram, the generated $2N − 1$ algorithm is insensitive to both phase shift miscalibration and the coupling errors.

To obtain high-precision measurement results, it is very important to suppress the harmonics while compensating for the nonlinearity in phase shift error [24]. By placing 10 multiple roots on the characteristic diagram, a novel $10N − 9$ algorithm was generated, as shown in Figure 3. The new $10N − 9$ algorithm can compensate for up to eighth-order nonlinearity of phase shift errors $o(\varepsilon_8)$ and coupling errors $o(A_m\varepsilon_8)$.

\[
P(x) = \sum_{k=0}^{M-1} (a_k + ib_k)x^k, \tag{9}\]

Figure 3. Characteristic diagram of the $10N − 9$ algorithm.

The characteristic polynomial of the $10N − 9$ algorithm can be described as

\[
P(x) = [P_{\text{sync}}(x)]^{10} = (1 + x + x^2 + x^3 + \cdots + x^{N-1})^{10} = \sum_{k=1}^{10N-9} w_k x^k. \tag{10}\]

where $P_{\text{sync}}$ is the characteristic polynomial of synchronous detection.

The sampling amplitudes of the $10N − 9$ algorithm are defined as

\[
a_k = \frac{2}{N}w_k \cos \frac{2\pi}{N} [k - (5N - 4)]. \tag{11}\]
\[ b_k = \frac{2}{N} w_k \sin \frac{2\pi}{N} [k - (5N - 4)]. \]  

where \( w_k \) represents the polynomial window functions, which are described in Appendix A. Figure 4 represents the shape of the window function.

**Figure 4.** Shape of polynomial window function.

### 3.3. Fourier Representation of the 10N – 9 Algorithm

By using the Fourier representation [40] of the sample amplitude \( a_k \) and \( b_k \), the new algorithm can be better understood and visualized. For an \( M \)-sample algorithm, the sampling functions \( f_1 \) and \( f_2 \) in time domain can be described as

\[
f_1(a) = \sum_{k=1}^{M} b_k \delta(a - a_k),
\]

\[
f_2(a) = \sum_{k=1}^{M} a_k \delta(a - a_k),
\]

where \( a_k \) and \( b_k \) are the sample amplitude and \( \delta(a) \) is the Dirac delta function.

The sampling functions in the frequency domain can be described by using Fourier transform as

\[
F_1(v) = \sum_{k=1}^{M} b_k \exp(-ia_k v),
\]

\[
F_2(v) = \sum_{k=1}^{M} a_k \exp(-ia_k v),
\]

where \( v \) is the frequency variable and \( i \) is an imaginary unit. \( F_1 \) is purely imaginary and \( F_2 \) represents purely real functions based on the asymmetrical and symmetrical properties of the sampling amplitude and phase shift parameter [45].

Figure 5 presents the sampling functions \( iF_1 \) and \( F_2 \) of Surrel’s \( 2N - 1 \) algorithm. As mentioned above, this algorithm can compensate for not only the phase shift miscalibration but also the coupling error. However, the sidelobe still exists, with a value around 5.5883%. A high sidelobe amplitude indicates significant vulnerability to the adjacent signal. Therefore, this algorithm is not appropriate for measuring a highly reflective sample.

Figure 6 presents the sampling functions of the \( 10N - 9 \) algorithm. The sidelobe is 0.0007% compared with 5.5883% of Surrel’s \( 2N - 1 \) algorithm, which means that the \( 10N - 9 \) algorithm has a strong sidelobe suppression ability. The sampling functions of \( 10N - 9 \) have a zero gradient at relative frequency \( v = 1 \), which fulfills the requirements of the fringe contrast maximum condition [39]. The \( 10N - 9 \) algorithm can also compensate for the bias modulation because the sampling function has zero gradients at relative frequency \( v = 0 \). Moreover, the sampling functions of \( 10N - 9 \) have zero
3.4. Error Analysis

Research was done by de Groot and Hibino on the error analysis within the consideration of the coupling error [46,47]. In phase-shifting interferometry, de Groot found that the measurement accuracy can be influenced by harmonics, phase shift errors, or a combination of both. He derived a kind of analytical formula in order to check the ability to suppress both the phase shift errors and the coupling errors. In addition, to compensate for the coupling errors up to \( o(A_m c_8) \), the derivatives have zero gradients up to the eighth order.

\[
\sigma_m = \frac{1}{2 \sqrt{2}} \left| \frac{iF_2(v)}{F_2(v)} - 1 \right|, \tag{17}
\]

Figure 5. Sampling functions of Surrel’s \( 2N - 1 \) algorithm (\( N = 10 \)).

Figure 6. Sampling functions of \( 10N - 9 \) algorithm (\( N = 10 \)).
where \( F_1(v) \) and \( F_2(v) \) can be obtained by Equations (15) and (16), and \( i \) is an imaginary unit. The RMS error resulting from coupling error can be given by

\[
\sigma_c = \frac{1}{2} \sum_{m=2}^{\infty} \frac{\gamma_m}{\gamma_1} \sqrt{|iF_1(m\nu)|^2 + |F_2(m\nu)|^2},
\]

where \( \gamma_m \) can be obtained by Equations (2)–(5). Therefore, the entire RMS error can be given by

\[
\sigma = \sqrt{\sigma_m^2 + \sigma_c^2}.
\]

In Figure 7, we can see the trend of the RMS error (reference surface reflective \( r_1 = 4\% \); sample surface reflective \( r_2 = 30\% \)). When measuring a spherical surface by a wavelength-tuning interferometer, \( \varepsilon_0 \) is especially important, since the reference surface of the spherical is curved, and a non-uniform phase shift occurs along the axial translation in the field of view [48,49], thus the value of phase shift miscalibration \( \varepsilon_0 \) can be estimated to be around 0.3 [50].

![Figure 7. RMS error as a function of phase shift miscalibration (\( r_1 = 4\% \), \( r_2 = 30\% \)).](image)

Figure 7 also shows that a synchronous detection algorithm has the largest RMS error because of its inability to compensate for the coupling error. In comparison with Surrel’s \( 2N - 1 \), Hanayama’s \( 2N - 1 \) has a smaller RMS error because it is a modified and improved algorithm based on the theory of Surrel’s \( 2N - 1 \). Except for \( 10N - 9 \), Kim’s \( 5N - 4 \) algorithm outperforms the other three algorithms. Obviously, \( 10N - 9 \) has the smallest RMS error value (lower than 1 nm, about 0.7 nm) of all algorithms, which means that the \( 10N - 9 \) algorithm can more effectively suppress the RMS error than other conventional algorithms, even for a 30% phase shift miscalibration. Therefore, the \( 10N - 9 \) algorithm is appropriate for measuring the surface profile of a highly reflective silicon wafer.

4. Experiment Results and Discussion

Experimental Results

A BK 7 plate (diameter: 10 cm; thickness: 5 mm) from the SCHOTT company with refractive index 1.515 at a wavelength of 633 nm was measured using the novel \( 10N - 9 \) algorithm and wavelength-tuning
Fizeau interferometer. The measurement was taken under lab-temperature $20.5 \pm 0.1 \, ^\circ\text{C}$. The silicon wafer was aligned vertically on a mechanical stage. The air gap distance between the silicon wafer and reference surface was set to $L = 10 \, \text{mm}$. The interferometer was placed on a stainless-steel table to isolate the interferometer from external vibration. To eliminate the temperature gradient of the BK 7 plate, it was placed in the laboratory for one day. We set CCD frame rate as 0.5 fps, auto exposure time, and auto gain to get high quality interferograms. Figure 8 presents the observed raw interferogram.

![Figure 8. Raw interferogram at a wavelength of 633 nm.](image)

The necessary wavelength-tuning range $\delta \lambda$ can be calculated as

$$
\delta \lambda = \frac{\lambda^2}{4\pi L} \delta \phi \approx 0.2024 \, \text{nm},
$$

(20)

The wavelength was scanned linearly from 632.8524 to 633.0551 nm and 91 interferograms were acquired by CCD camera. The wavelength values from start to end were accurately measured by the wavelength meter shown in Figure 1 with an accuracy value of $10^{-7}$ at a wavelength of 633 nm. For the whole wavelength-tuning process, the total phase was changed by $20\pi$, and each phase step was $\pi/5$. The recording time was 3 min. Figure 9 shows five consecutive interference fringe patterns chosen arbitrarily from 91 interferograms captured by CCD.

![Figure 9. Five consecutive interference fringe patterns captured by CCD.](image)

The phase $\phi$ can be calculated using the $10N - 9$ algorithm:
where $w_k$ is the polynomial window function, which is described in Appendix A, and $I_k$ is the intensity of the $k$th image defined by Equation (1).

The calculated surface profiles by phase unwrapping [51] are given in Figure 10, which presents the entire configuration of the silicon wafer. All 20 experiments were conducted successively within three days. To test the reliability of the proposed algorithm, the sample was measured using a FUJINON interferometer (G102, FUJIFILM Corporation). By using proposed new method, the obtained average peak-to-valley (PV) value of 20 experiments was 513.587 nm (Figure 10a), and the standard deviation was 2.33 nm. By using FUJINON interferometer, the obtained average peak-to-valley value was 524.376 nm (Figure 10b), and the standard deviation was 2.05 nm. The error map of the surface shape is shown in Figure 11. Table 2 shows the peak-to-valley and RMS values of the $10N - 9$ method and the FUJINON interferometer, respectively.

Figure 10. (a) Surface profile measured with new algorithm. (b) Surface profile measured with the FUJINON interferometer.

Figure 11. The error map of the surface shape.
Table 2. The comparison of different methods.

<table>
<thead>
<tr>
<th>Methods</th>
<th>PV (nm)</th>
<th>RMS (nm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>New $10N - 9$</td>
<td>513.587</td>
<td>2.05</td>
</tr>
<tr>
<td>FUJINON interferometer</td>
<td>524.376</td>
<td>2.33</td>
</tr>
<tr>
<td>Error</td>
<td>10.789</td>
<td>0.28</td>
</tr>
</tbody>
</table>

5. Discussion

In the previous sections, we demonstrate surface measurement using a new developed algorithm, which is preferable to conventional methods. As we can see in Section 3 and Figures 5 and 6, the new algorithm has a strong sidelobe suppression ability and can compensate for both the bias modulation and the coupling errors. As proposed by de Groot [46], the error caused by the combined effect of phase-shift miscalibration and the harmonics error is still one of the most frequently overlooked sources of error. Phase-shift miscalibration increase the sensitivity to high-order harmonics present in wavelength-tuning Fizeau interferometer. In this paper, the RMS phase error of phase shift miscalibration was also calculated with a value of 0.7 nm, which was the smallest error among all algorithms. The results demonstrated that the new algorithm can more strongly compensate for phase shift miscalibration in the presence of harmonics in comparison with other conventional algorithms, even for a phase shift miscalibration of $-30\%$ and high reflectivity (30%) sample surface. Based on Brophy’s theory [52], the quantization error of the CCD can be neglected with a total of 91 steps in measurement. In the experiment, since there was a nonlinearity of about $3\%$ in the PZT response, we applied a quadratic voltage increment to the PZT and the nonlinearity decrease to $1\%$ of the total phase shift. The standard deviation was 2.05 nm, which was caused by environmental variances in the laboratory, such as temperature variation and floor vibration. The errors can be reduced in a stable environment by using a properly performing laser source. The uncertainty of the reference surface shape was $\lambda/30 = 21$ nm. As a total, the uncertainty of the surface shape measurement was 23 nm.

The standard deviation was also calculated, as shown in Table 3, in comparison with other conventional algorithms. The standard deviation of synchronous detection algorithm shows the worst value among all because it cannot suppress the phase shift error and coupling error. Surrel’s $2N - 1$ and Hanayama’s $2N - 1$ can only suppress phase shift miscalibration $\varepsilon_0$. Kim’s $5N - 4$ algorithm has a much better repeatability. However, the novel $10N - 9$ shows the best performance because of the strong ability to suppress coupling errors.

Table 3. Standard deviations of algorithms.

<table>
<thead>
<tr>
<th>Phase-Shifting Algorithm</th>
<th>Standard Deviation (nm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Synchronous detection</td>
<td>27.61 [40]</td>
</tr>
<tr>
<td>Surrel’s $2N - 1$</td>
<td>8.93 [27]</td>
</tr>
<tr>
<td>Hanayama’s $2N - 1$</td>
<td>8.19 [31]</td>
</tr>
<tr>
<td>Kim’s $5N - 4$</td>
<td>4.14 [38]</td>
</tr>
<tr>
<td>New $10N - 9$</td>
<td>2.05</td>
</tr>
</tbody>
</table>

Even though the CCD we used in the experiments is low-noise device, the noise of the camera sensor is still a challenging work for researchers. In future work, the optimization of the exposure time and gain should still be considered. We should also focus on improving the insensitivity of the algorithm, improving the measurement speed, and using higher-quality equipment in order to improve the performance of the $10N - 9$ algorithm.

6. Conclusions

In this research, a novel $10N - 9$ phase-shifting algorithm was derived. The new algorithm utilizes the Discrete Fourier Transform function and polynomial window function. The $10N - 9$ algorithm
can be applied to the surface profile measurement of the highly reflective silicon wafer because of the strong suppression ability of the coupling errors occurring between the phase shift error and the higher harmonics. The performance of the 10N − 9 algorithm was represented in the frequency domain, and the sidelong value was also calculated and discussed in comparison with other conventional algorithms. The error analysis results show that the new 10N − 9 algorithm has a stronger error suppression ability than other algorithms. Finally, the surface profile of a four-inch silicon wafer was measured using the 10N − 9 algorithm and a wavelength-tuning Fizeau interferometer. According to the experimental results, the repeatability measurement error of the surface profile was 2.05 nm, and the entire measurement uncertainty was 23 nm, which indicates that the new 10N − 9 algorithm outperforms the conventional phase-shifting algorithm.

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Appendix A

\[ \begin{align*}
1 \leq k \leq N & \quad w_k = \frac{1}{525989} k(k+1)(k+2)(k+3)(k+4)(k+5)(k+6)(k+7)(k+8) \\
N + 1 \leq k \leq 2N & \quad w_k = \frac{1}{40320} k^9 + \left( \frac{4032}{120} - \frac{1069}{1920} \right) k^8 + \left( -\frac{1069}{120} - \frac{1091}{690} \right) k^7 + \left( \frac{1091}{690} - \frac{1069}{1920} \right) k^6 \\
& \quad + \left( -\frac{1069}{120} - \frac{1091}{690} \right) k^5 + \left( \frac{1091}{690} - \frac{1069}{1920} \right) k^4 \\
& \quad + \left( -\frac{1069}{120} - \frac{1091}{690} \right) k^3 + \left( \frac{1091}{690} - \frac{1069}{1920} \right) k^2 \\
& \quad + \left( -\frac{1069}{120} - \frac{1091}{690} \right) k + \left( \frac{1091}{690} - \frac{1069}{1920} \right) \\
& \quad 2N + 1 \leq k \leq 3N & \quad w_k = \frac{1}{1008} k^9 + \left( \frac{1008}{120} - \frac{1069}{1920} \right) k^8 + \left( -\frac{1069}{120} - \frac{1091}{690} \right) k^7 + \left( \frac{1091}{690} - \frac{1069}{1920} \right) k^6 \\
& \quad + \left( -\frac{1069}{120} - \frac{1091}{690} \right) k^5 + \left( \frac{1091}{690} - \frac{1069}{1920} \right) k^4 \\
& \quad + \left( -\frac{1069}{120} - \frac{1091}{690} \right) k^3 + \left( \frac{1091}{690} - \frac{1069}{1920} \right) k^2 \\
& \quad + \left( -\frac{1069}{120} - \frac{1091}{690} \right) k + \left( \frac{1091}{690} - \frac{1069}{1920} \right) \\
& \quad 3N + 1 \leq k \leq 4N & \quad w_k = \frac{1}{40320} k^9 + \left( \frac{4032}{120} - \frac{1069}{1920} \right) k^8 + \left( -\frac{1069}{120} - \frac{1091}{690} \right) k^7 + \left( \frac{1091}{690} - \frac{1069}{1920} \right) k^6 \\
& \quad + \left( -\frac{1069}{120} - \frac{1091}{690} \right) k^5 + \left( \frac{1091}{690} - \frac{1069}{1920} \right) k^4 \\
& \quad + \left( -\frac{1069}{120} - \frac{1091}{690} \right) k^3 + \left( \frac{1091}{690} - \frac{1069}{1920} \right) k^2 \\
& \quad + \left( -\frac{1069}{120} - \frac{1091}{690} \right) k + \left( \frac{1091}{690} - \frac{1069}{1920} \right)
\end{align*} \]
\[ w_2 = \frac{1}{800} k^9 + (-\frac{1}{72} N + \frac{1}{8}) k^8 + (\frac{20}{21} N^2 - \frac{3}{2} N + \frac{9}{480}) k^7 + (-\frac{1003}{432} N^3 + \frac{124}{90} N^2 - \frac{677}{108} N + \frac{9}{36}) k^6 \]
\[ + (\frac{4475}{288} N^4 - \frac{1055}{18} N^3 - \frac{22295}{288} N^2 - \frac{1256}{9} N + \frac{288}{3}) k^5 \]
\[ + (\frac{3893}{144} N^5 - \frac{22573}{72} N^4 - \frac{4082575}{576} N^3 + \frac{264785}{216} N + \frac{569}{8}) k^4 \]
\[ + (\frac{7755}{576} N^6 - \frac{18731}{1728} N^5 + \frac{2096125}{384} N^4 - \frac{5275}{8} N^3 + \frac{1309525}{96} N^2 - \frac{1246}{3} N + \frac{29531}{72}) k^3 \]
\[ + (-\frac{45485}{3072} N^7 + \frac{77555}{1944} N^6 - \frac{1704321}{256} N^5 + \frac{1870525}{384} N^4 - \frac{21805}{8} N^3 + \frac{61693}{4} N^2 - \frac{9939}{8} N + \frac{71}{2}) k^2 \]
\[ + (-\frac{185325}{576} N^8 - \frac{45485}{144} N^7 + \frac{2097005}{96} N^6 - \frac{807125}{384} N^5 + \frac{2391875}{64} N^4 - \frac{9389}{12} N^3 + \frac{103385}{48} N^2 - \frac{2344}{3} N + 14) k \]
\[ + (-\frac{47727311}{50288} N^9 + \frac{185325}{144} N^8 - \frac{4138125}{384} N^7 + \frac{2775595}{64} N^6 - \frac{20024325}{12} N^5 + \frac{268279}{8} N^4 - \frac{3185325}{90} N^3 + \frac{26435}{36} N^2 - \frac{56}{5} N) \]

The polynomial window function is symmetric for \( k = (10N - 9)/2 \).

References


5. Fu, Y.J.; Wang, Y.L.; Wan, M.T.; Wang, W. Three-dimensional profile measurement of the blade based on surface structured light. Optik 2013, 124, 3225–3229. [CrossRef]


