

Article

Software for Estimation of Stochastic Model Parameters for a Compacting Reservoir

Wojciech T. Witkowski *  and Ryszard Hejmanowski 

The Department of Mining Surveying and Environmental Engineering, AGH University of Science and Technology, 30-059 Krakow, Poland; hejman@agh.edu.pl

* Correspondence: wwitkow@agh.edu.pl

Received: 14 April 2020; Accepted: 6 May 2020; Published: 8 May 2020



Abstract: The paper presents a computer program called SubCom v1.0 for determining mathematical model parameters of compaction layers in areas of oil, gas or groundwater extraction. A stochastic model based on the influence function was used to model compaction and subsidence. Estimation of the model parameters was based on solving the inverse problem. Two model parameters were determined: the compaction coefficient C_m of reservoir rocks, and the parameter $tg\beta$, which indirectly describes the mechanical properties of the overburden. The calculations were performed on leveling measurements of land subsidence, as well as on the geometry of the compaction layer and pressure changes in aquifers. The estimation of model parameters allows the prediction of surface deformations due to planned fluid extraction. An algorithm with a graphical user interface was implemented in the Scilab environment. The use of SubCom v1.0 is presented using the case of an underground hard coal mine. Water drainage from rock mass accompanying coal extraction resulted in compaction of the aquifer, which in turn led to additional surface subsidence. As a result, a subsidence trough occurred with a maximum subsidence of 0.56 m.

Keywords: reservoir compaction; subsidence modeling; aquifer; rock-mass mechanics; water pumping; Gauss-Newton method

1. Introduction

The compaction of porous geological layers in oil, gas or groundwater extraction sites leads to the formation of a subsidence trough on the land surface. Among the best-known examples of such phenomena are large areas in The Netherlands, e.g., the Groningen gas field [1], Mexico, e.g., Mexico City [2,3], or Italy, e.g., Venice [4]. Land subsidence due to compaction has also been observed in mining drainage sites and is mainly reported in deep, solid, mineral mine sites [5–7]. Changes on the surface may be registered with leveling surveys, GNSS, or InSAR [4,8]. Analysis of measurement results indirectly describes the properties of reservoir rocks [9–11]. By comparing such data with rock mass geology, a connection may be found between the cause of the movement and its outcomes on the surface [12–14]. The parameters can be determined with a selected mathematical model, and a tool for predicting the future consequences of mining activity may be developed. Appropriately performed subsidence modeling will provide safer engineering structures [15] and surface infrastructure [16]. The compaction phenomenon may be described with a poro-elasticity model which establishes correlations between the reservoir skeleton and pressure changes in the fluid [17,18]. This results in a deformation field on the land surface in the form of a subsidence trough. Various groups of mathematical models (empirical, analytical or theoretical) may be used to predict compaction and surface subsidence [8–11,19], but they differ in computational complexity, the number of the required parameters, and the time necessary to make a rock mass model [20]. Analytical models use a limited number of parameters which each have a physical interpretation and may be seen as a compromise

with the theoretical models [13,20,21] that have been described in our research. Analytical methods may potentially serve as a useful tool, particularly in the case of operating mines. Additionally, these methods have shown that the results of analytical models may indicate a similar land subsidence prediction accuracy as the results of numerical modeling. One of these analytical methods is called a stochastic model or an influence function model [13,22–25]. Models of this type first appeared in mathematical applications, statistics and applied research decades ago, but they are widely recognized today due to their relatively small number of parameters and high reliability. However, like all prediction models, they require the definition of parameters which control the calculation process. Researchers who model compaction-induced surface movements agree that evaluation of model parameters is very important for the accuracy and reliability of forecasts [10,11,26–28]. Frequently, the parameters cannot be acquired directly from in-situ tests or sampling, while at other times the density of samples is insufficient [29]. In such a situation, the presented model and parametrization method may be an alternative to existing solutions. This is the research gap that is filled by this study.

The objectives comprise the following:

- verify the efficacy of a parameter estimation method based on a stochastic model;
- implement an algorithm in open-source software;
- model subsidence due to dewatering in an underground coal mine.

The article is structured as follows: The model parameters were established based on the influence function model (Section 2). The proposed software was tested in an underground coal mine where the compaction effect due to the pumping of water and its outcome in the form of extensive subsidence due to the drainage trough have been observed (Section 3). This example formed the basis of the discussion concerning the operation of the software (Section 4). The obtained results are discussed in the last chapter and show the applicability of this tool in areas where other raw materials are extracted, e.g., oil, gas or water (Section 5).

2. Compaction and Subsidence Model

The group of analytical methods also includes models based on Geertsma's nucleus of strain concept [30], and models based on the influence function have been proposed by other researchers [13,14,22,23]. Geertsma proposed a model based on elasticity theory. This concept was created for modeling land subsidence above Groningen reservoir [14]. The similarities between Geertsma's model and the model based on the influence function are discussed in [14,31]. Nevertheless, the influence function model was created for forecasting land subsidence caused by the operation of underground coal mines [32] and has been applied in many countries around the world [23,24,33,34]. These methods were developed by successive researchers and are presently used, for instance, to predict deformations induced by the production of oil and gas [13,21,25] or by underground water [35]. The obtained results do not differ much from the results of numerical modeling, and their applicability is much simpler [20,36].

2.1. Principles of a Model Based on the Influence Function

The calculation model presented in the paper is based on the division of a geological structure containing fluid (oil, gas or water) into elementary cuboids, i.e., reservoir elements [13,14]. This aspect is discussed in more detail in Section 2.2. The suggested solution allows reservoir elements to be used in the form of cuboids with arbitrary shaped bases which increase the computational potential of the method [13]. In models based on the influence function, vertical displacements S are the result of the compaction of porous reservoir rocks and may be calculated with Equation (1):

$$S = C_m \{p_0 - p(t)\} M_0 \iint_A \frac{1}{r^2} \exp\left(-\pi \frac{R^2}{r^2}\right) dx dy \quad (1)$$

where $\Delta p = p_0 - p(t)$ —pore pressure decrease, M_0 —primary thickness of layer, A —geometry of reservoir element, $R^2 = (x - s)^2 + (y - t)^2$ —distance in the horizontal plane between a reservoir element (x, y) and a calculation point on the surface (s, t) and r —radius of influence, which takes the form of:

$$r = \frac{H}{\operatorname{tg}\beta}$$

and C_m —compaction coefficient, defined by the expression:

$$C_m = \frac{1}{dp} \frac{dz}{z} \quad (2)$$

where dp —change in pore pressure, z —initial height of the rock sample, dz —change in the height of the rock sample.

The compaction coefficient (2) is defined as a change in the height of a rock sample z with a corresponding drop in pore pressure dp [27]. In turn, the impact radius r in Equation (1) is associated with reservoir depth H and is directly associated with the mechanical properties of the rocks, which are expressed by parameter $\operatorname{tg}\beta$ (Figure 1) [37]. Both $\operatorname{tg}\beta$ and compaction coefficient C_m make up the set of basic parameters of the theoretical model. Their values should correspond with the local conditions in the rock mass. For this reason, determination of these parameters is a basic requirement for credible modeling of surface deformations.

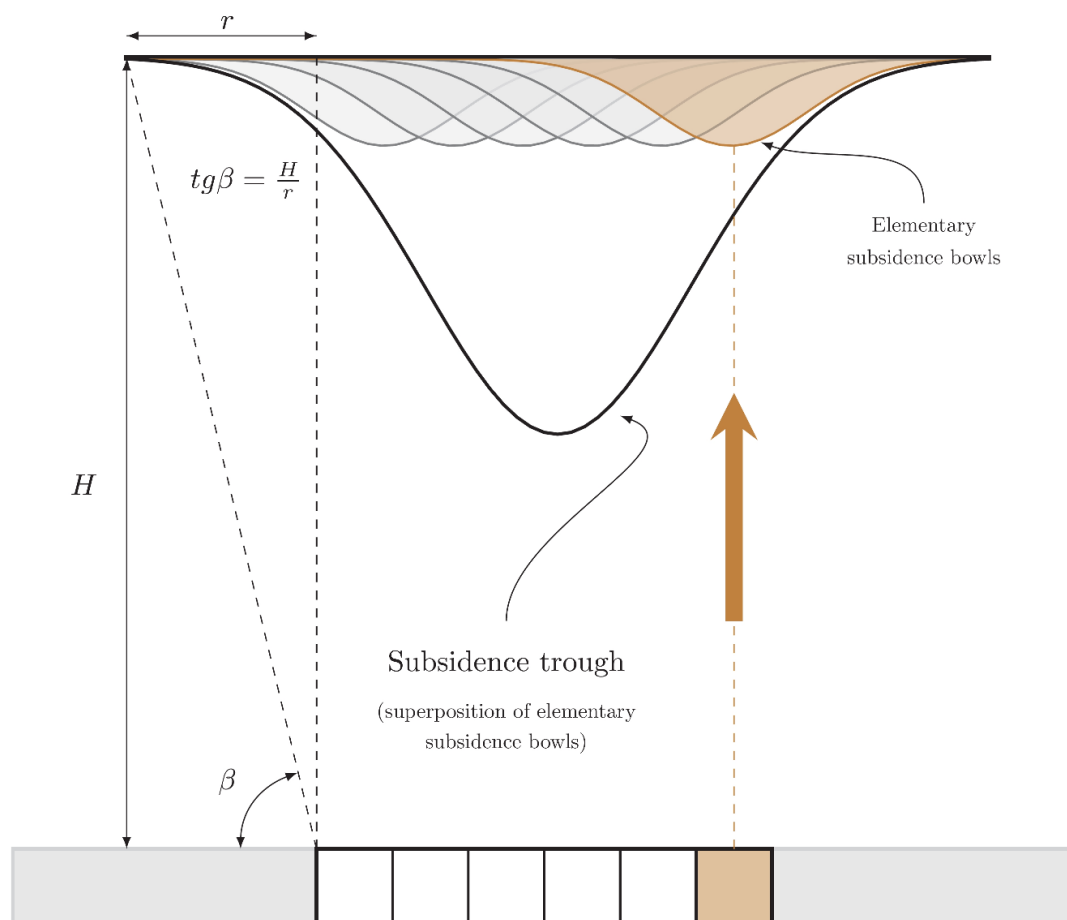


Figure 1. The idea of a computational model based on the influence function and graphical interpretation of the $\operatorname{tg}\beta$ parameter. The dark orange arrow visualizations the influence function, which represents the deformation process in rock mass.

One of the ways of determining parameters ($C_m, tg\beta$) for local conditions is the Gauss-Newton method for nonlinear equation systems. A Taylor series expansion is used to express the original nonlinear equation in linear form, which is the key concept of this technique. An algorithm minimizes the sum of the squares of the residuals between the data and the model. This method has been commonly used to evaluate determined parameters when a sufficient amount of data is available whose values have a relatively low degree of uncertainty [10,38–41]. By comparing a single empirical observation with a value determined on the basis of the assumed model, the value of error δ for the i th observation is obtained. The least square method assumes the minimization of the function of error E for n observations (3):

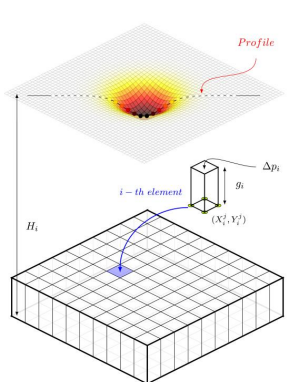
$$E = \sum_{i=1}^n \delta_i \delta_i \rightarrow \text{minimum} \quad (3)$$

The Gauss-Newton algorithm for the model for Equation (1) was implemented. Owing to the nonlinear equation of the theoretical function, the system of observation equations needs to be linearized, as shown in the Appendix A. The solution requires the assumption of initial values and approximate parameters ($C_{m0}, tg\beta_0$) for which the increment of ($\Delta C_m, \Delta tg\beta$) is determined. The correct choice of initial parameter values determines the convergence of the solution and the calculation process. The correct application of the least square approach assumes that the testing of outliers in subsidence results has already been performed.

2.2. Program Framework

The presented solution was implemented in the Scilab environment (version 5.5.2); the result was the GUI-based SubCom v1.0 program (Supplementary Materials). Data on vertical surface displacements (e.g., subsidence), the geometry of the aquifer and changes in pore pressure had to be obtained in order to determine the model parameters (Table 1). The first set of data contained the number of points (Id), their coordinates (X, Y) and the value of vertical displacements (S). Vertical movements were determined in the subsidence trough profile, which is useful for analyzing results. Another set consisted of geometrical data about the reservoir and data about the pressure drop in a given reservoir element. Such an element had ascribed local values: pressure decrease (Δp_i), thickness (M_i), depth (H_i) and coordinates describing the arbitrary geometry of a single element (X_i^j, Y_i^j).

Table 1. Required input data for the SubCom v1.0 program.

Type of Data	Symbol	Description	Data visualization
Profile	Id, X, Y, S	Information about surface subsidence on the profile	
Aquifer	X_i^j, Y_i^j	The geometry of a single aquifer element in the same coordinate system as the profile	
	H_i	Depth	
	M_i	The thickness of a single aquifer element	
	Δp_i	Pressure decrease in the aquifer, determined for the bottom of the aquifer element	

The created graphical user interface simplifies the process of managing the software's functionality (Figure 2) [42]. The SubCom_v1_0.sce script loads the functions and launches the main window (Figure 3). All plots and graphical presentations are performed with the use of functions in the Scilab environment. The correctness of the prepared data can be verified by visualizing them with the use

of *drawdata.sci*. An error at this stage indicates a problem with the input data. The calculations are based on the initial values of the analyzed parameters. Firstly, the program calculates the theoretical values of vertical deformations with the *calculation.sci* function; subsequently, a plot showing the fit of the subsidence trough with the theoretical values is obtained (Figure 2). The problem of determining the parameters was solved on the basis of the Gauss-Newton method with a system of observation equations that is described in the Appendix A.

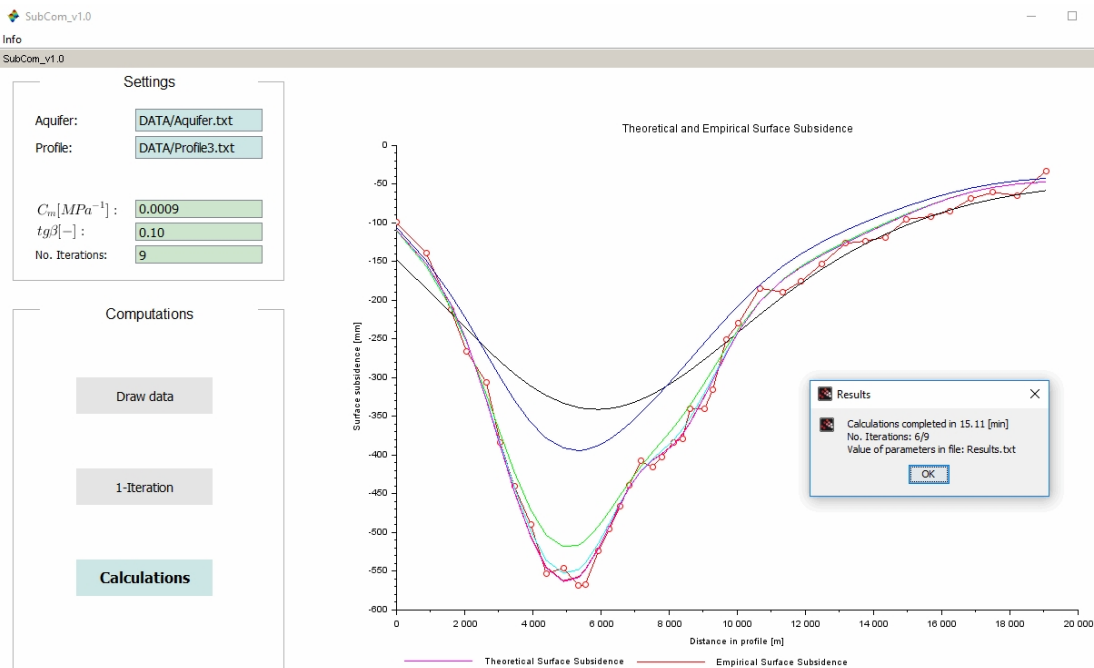


Figure 2. The main window of SubCom v1.0, showing the results of computation.

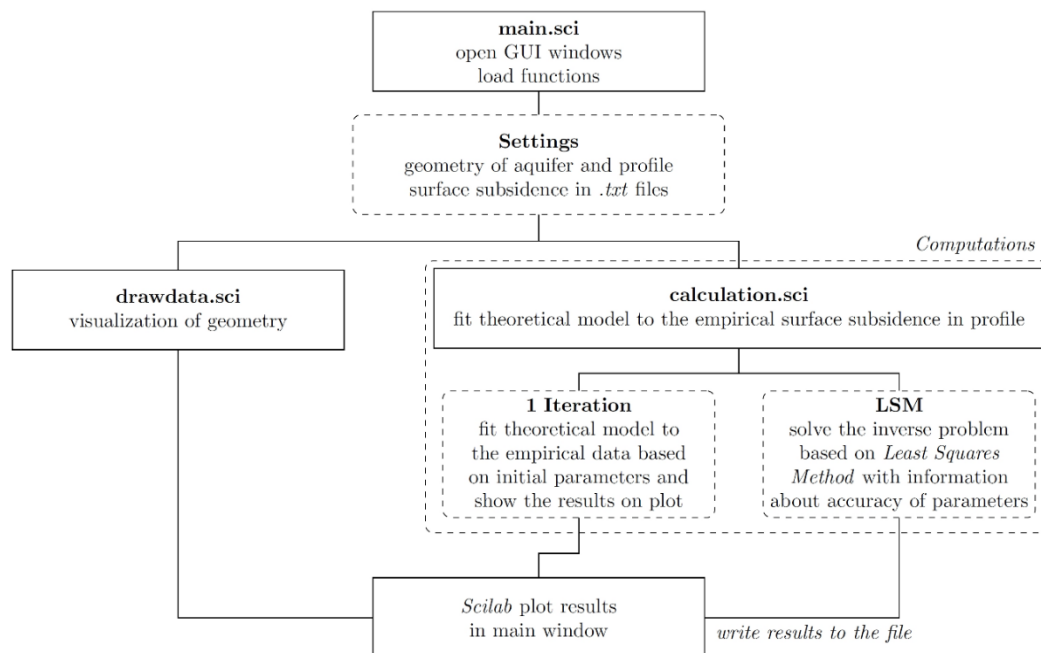


Figure 3. The framework of the SubCom v1.0 program.

2.3. Calculations for Testing Data

In the first stage, it was decided to validate the results of the implemented solution. The goal was to check the correctness and convergence of the calculations performed. Testing was performed for modeling data. The modeling values of subsidence were disturbed by Gaussian noise. The initial parameter values were changed, and the optimal obtained results were checked (Figure 4). Regardless of the initial calculation parameters, the same final values were obtained for each sample test. In the initial iterations, the convergence of the computational process was stronger than in the subsequent iterations due to a small change in the accepted corrections to the determined values of the parameters. The test confirms the correctness of the algorithm and its implementation in the Scilab environment.

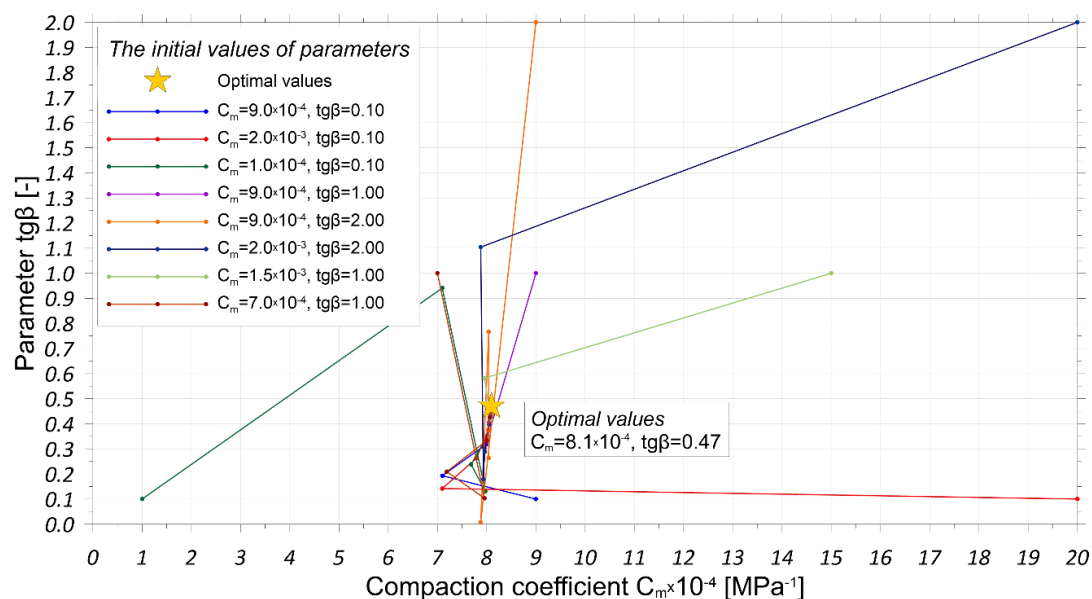


Figure 4. Determination of optimal parameter values related to different initial conditions. The testing was carried out for modeling data; optimal parameters are represented by a star on the graph. The iterations are represented by colored dots on each track.

3. Study Area

The presented tool was used on the example of an underground coal mine where aquifer draining resulted in subsidence caused by rock mass compaction.

3.1. Geological Background

There is an underground hard coal mine in eastern Poland where coal has been extracted from Carboniferous beds since the 1980s (Figure 5). These strata are covered by Jurassic, Cretaceous and Quaternary horizons with a total thickness of 500–1400 m. The Carboniferous sediments are directly overlaid by the Jurassic strata, which is 80–120 m thick and consists of sand-limestone sediments from the middle Jurassic period and carbonate sediments from the Upper Jurassic period. The Cretaceous rocks form an almost impervious layer with an average thickness of 470 m. These are mainly marl and limestone strata. The Quaternary sediments generally consist of sandy, clayey material with a thickness of several up to about ten meters [43,44].

The main source of water inflow to the mine is fractures in the Jurassic strata, which consists of carbonate and sandy sediments that are 80 m to 120 m thick. The average water mineralization is about 1292–1583 mg/dm³ with an average water hardness in the range 0.32–0.78 °n. These are very soft waters of the Cl-HCO₃-Na type, whose characteristics should be associated with the fractures and 11–23% porosity. The filtration coefficient of this layer is 10^{−8} m/s to 10^{−5} m/s, with an average of 10^{−6} m/s in the mine area. The primary pressure acting on the top of the Carboniferous layer amounts to approx.

7.5 MPa [43]. The depression cone was monitored with 44 piezometric boreholes located in the vicinity of the mine (Figure 5). Surface deformations could be observed along with the development of the depression cone (Figure 5).

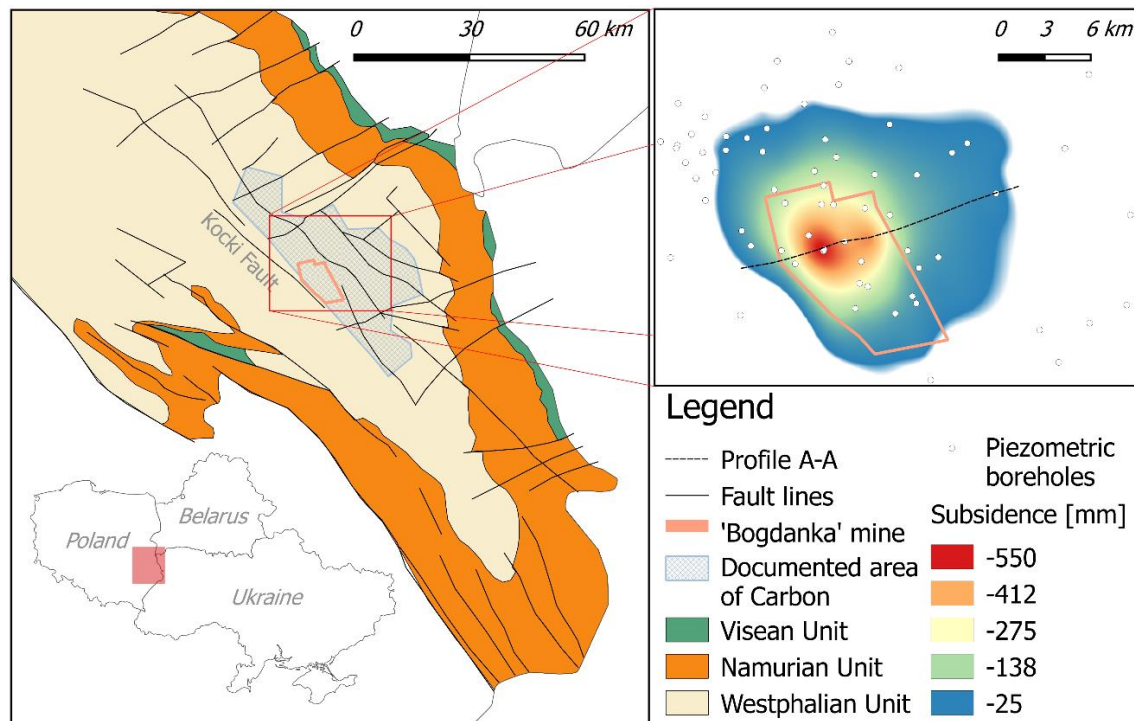


Figure 5. Location of the hard coal mine and the regional tectonics of the area. Piezometric boreholes and a large dewatering trough in the deep extraction area are shown. Condition as of 2011.

3.2. Subsidence Due To Fluid Withdrawal

The first leveling surveys were performed in the mine area in the 1980s. Vertical deformations of the surface were already observed during the shaft drilling phase. In 1982, the maximum movements in this area reached approximately 0.1 m [5]. At present, deformations can be measured for the whole mine area. A detailed analysis of the leveling results was carried out in 2013 [45]. A more in-depth analysis revealed that the vertical surface movements were caused not only by coal mining operations, but also by draining of deeper geological strata. To determine the range of deformations due to drainage, a network of benchmarks from a national height grid network was located in the neighborhood of the mine. Vertical movements (1980–2011) were determined for 86 common points from the benchmark network. Maximum values of drainage-related movements were observed in the area of shafts, i.e., the main drainage points of the Jurassic strata. The maximum subsidence caused by drainage did not exceed 0.6 m (Figure 5).

4. Results

The SubCom v1.0 software may be used to determine the parameters of fluidal deposits (oil, gas or groundwater). The operation of this program was presented on the case of an underground mine where the compaction of the aquifer led to land-surface subsidence. The calculations were based on 2011 data and 2009 pressure changes in the Jurassic aquifer. The time lapse is due to a 2-year delay in the manifestation of the impact [7].

First, the aspect of the computational size of aquifer elements is discussed (Section 4.1). Finally, the implemented code is used for the case of the underground coal mine example (Section 4.2).

4.1. Dimension Testing Of Aquifer Elements

The assumed method of determining parameters that is referred to in Section 2 is based on dividing the reservoir into elements with ascribed pressure decrease and thickness. Therefore, suitable selection of reservoir element size is very important. The investigated reservoir was divided into cuboid elements whose bases measured 1000, 200 and 50 m (Figure 6). Calculations in profile A-A were conducted for the so-divided aquifer (Figure 6). The calculation results were compared on plots. The determined differences were up to 23 mm for 1000 m bases as compared to their 200 m and 50 m equivalents. The obtained differences in the bases (of 200 m and 50 m) did not exceed 2 mm at a maximum subsidence of 0.56 m with a relative error below 0.5%. Accordingly, it was assumed that the division into elements of the base measuring 200 m was sufficient and optimal for calculation efficiency.

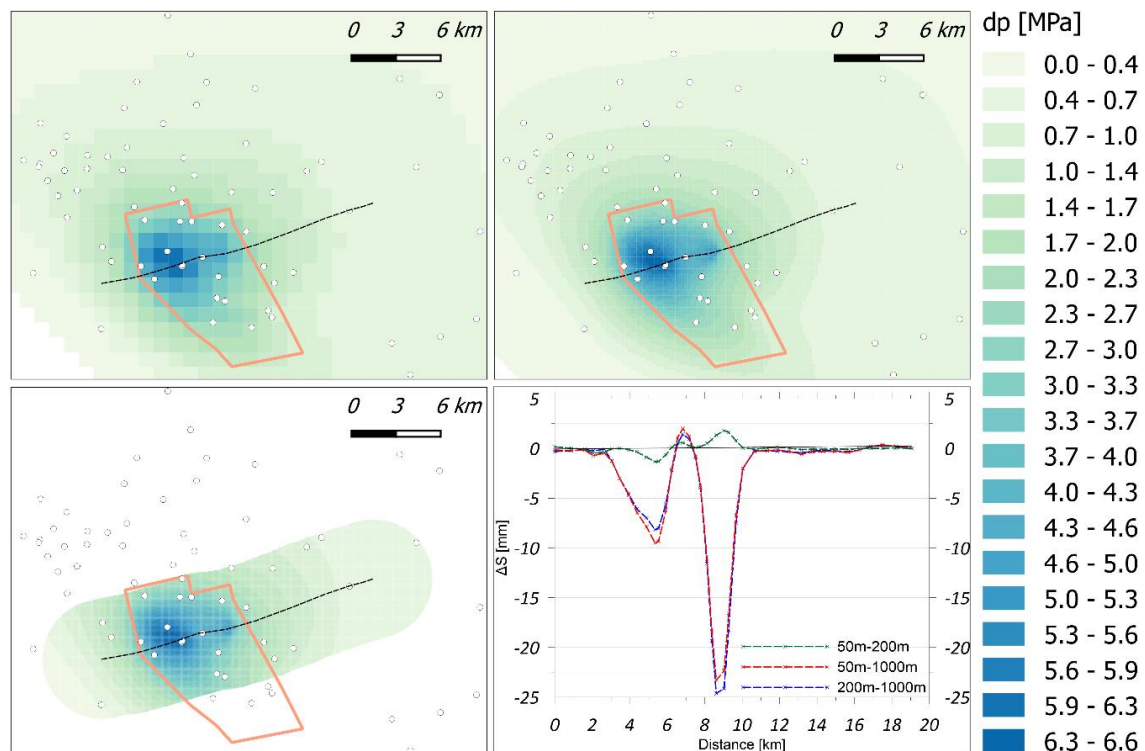


Figure 6. Division of aquifer into cuboid reservoir elements whose bases measured 1000 m, 200 m, and 50 m. Owing to the large number of base elements measuring 50 m, a buffer was used for profile A-A. Changes in reservoir element pressure are shown in the color scale.

4.2. Estimation Results of Compaction Coefficient C_m and $tg\beta$

The established algorithm allowed parameters C_m and $tg\beta$ to be determined. The estimation results in profile A-A and the deviation are shown in Figure 7. The compaction coefficient determined by the software for the Jurassic aquifer was $C_m = 8.0 \times 10^{-4} \pm 0.1 \text{ MPa}^{-1}$ at the range expressed by $tg\beta = 0.50 \pm 0.05$ at correlation coefficient R^2 of 0.77. According to Doornhof [46], the compaction coefficient for sandstone may oscillate between $5.0 \times 10^{-4} \text{ MPa}^{-1}$ and $15.0 \times 10^{-4} \text{ MPa}^{-1}$, which indicates high reliability of the obtained results. The $tg\beta$ value is responsible for the range of surface compaction. For instance, a value of $tg\beta$ of 0.10–0.20 was assumed in modeling for the gas field in Groningen, which is located at a depth of over 2000 m [47]. The determined value of $tg\beta = 0.50$ for the Jurassic strata in the coal mine at 500–600 m also seems credible.

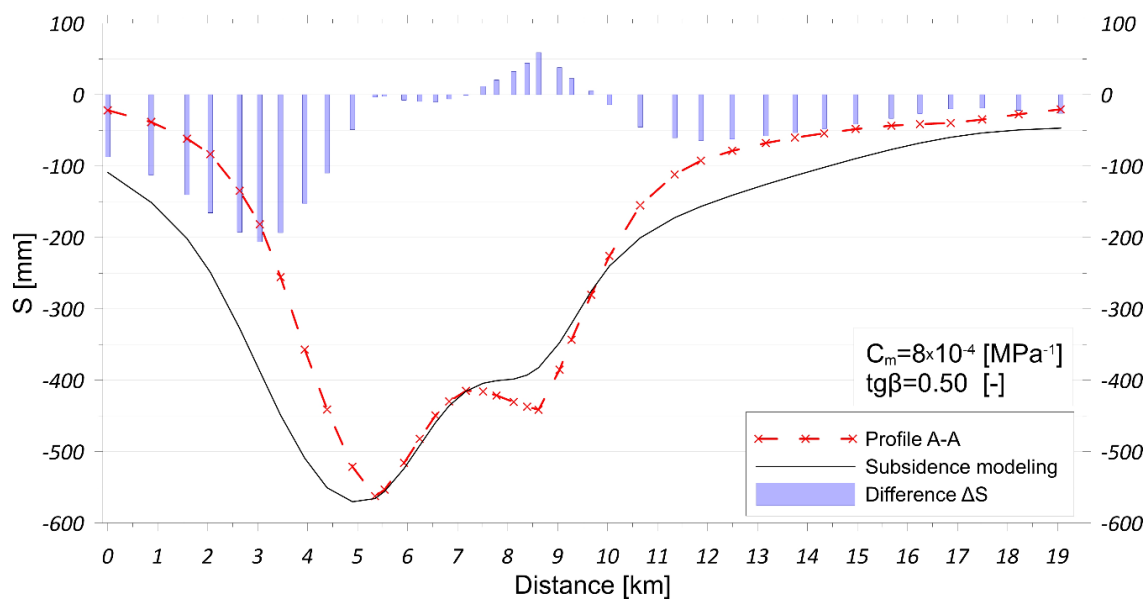


Figure 7. Fitting the model to the A-A dewatering trough profile with a plot of differences.

5. Discussion and Conclusions

The obtained fitting of the deformation field profile to the modeled values differs along the profile (Figure 7). The biggest discrepancy was observed in the initial part of the cross-section in the Kocki fault area (Figure 5), which constitutes a natural hydrogeological window for waters from the Jurassic horizon. This zone is responsible for the asymmetric shape of the depression cone (Figure 6), which may cause overestimation of vertical deformations in that area. The maximum values of the observed and modeled subsidence are similar, i.e., about 0.56 m, and they can be found in the mine shaft area where the aquifer depression was biggest. The modeled values are underestimated in the section between 7.5 km and 9.5 km of the profile. This is the second shaft drainage area of considerable depression in the reservoir. Due to the lack of piezometric boreholes in that area, the assumed pressure gradient may significantly differ from the real values, which are unknown. This could lead to underestimation of the observed vertical deformation values. The overestimation of modeled values in the eastern part of the mining area may stem from incomplete information about pressure gradients in that part. In the last part of the subsidence trough, where the modeled and observed values are similar, a piezometric well is located.

The proposed method of parameter determination requires the establishment of reservoir elements with the ascribed geometrical information on Δp and H . The pressure gradient in a single element will affect the attained calculation accuracy. An example of a deposit divided into uniform elements with square bases is presented in Section 4. The possibility of using elements with various base sizes is very useful. As shown in the example (Figure 6), this is purposeful, especially in an area of strong local variability of pressure depletion (Δp). Such a solution is possible, and the presented software assumes that reservoir elements may vary in size. However, the proposed model does not assume chemical interactions such as non-equilibrated brines [48]. If it were possible to express the chemical interactions in space (as compaction in Equations (1) and (2)), the model could be expanded. The chemical interactions of rock weakening or chemical effects could be added to the model, and this could be a direction for future research.

The presented SubCom v1.0 software may be used to determine the parameters of a prediction model for areas where groundwater, oil, and gas are extracted. It applies the Scilab open-source environment, in which the graphical interface features of SubCom v1.0 software were created. This software may be used in all locations where a compaction process has been observed. For the sake of calculation, the reservoir has to be divided into reservoir elements which are ascribed a depth and pressure decrease. Initial data for the surface includes subsidence measurement results, which can

be obtained with methods such as classic leveling surveys, GNSS, or InSAR. The proposed solution assumes the interpretation of results based on the subsidence trough profile. However, the presented algorithm also allows calculations to be made in a grid of distributed points (3D); although this is not in the scope of the present paper, it may be a starting point for further work on the software and development by users.

Supplementary Materials: The source-code for SubCom v1.0 is available from <https://github.com/wwitkow/SubCom>.

Author Contributions: Research concept, W.T.W. and R.H.; methodology, W.T.W. and R.H.; software, W.T.W.; validation, W.T.W., writing—original draft preparation, W.T.W.; writing—review and editing, R.H. All authors have read and agreed to the published version of the manuscript.

Funding: This research was supported by the National Science Centre, Poland, grant number 2014/13/N/ST10/02845.

Conflicts of Interest: The authors declare no conflict of interest.

Appendix A

A system of observation equations is based on the function of the prediction model (1). The function (1) is nonlinear, therefore the equation needs to be linearized by developing Taylor's series. In the solution, only the linear part is assumed, and thus observation equations may be presented in the form (A1):

$$S^o - S^t = \frac{\partial S^t(C_m, tg\beta)}{\partial C_m} dC_m + \frac{\partial S^t(C_m, tg\beta)}{\partial tg\beta} dtg\beta + \delta \quad (A1)$$

where:

$$\frac{\partial S^t(C_m, tg\beta)}{\partial C_m} = \frac{\{p_0 - p(t)\}M_0}{H^2} tg^2\beta \iint_A \exp\left(-\pi \frac{(x-s)^2 + (y-t)^2}{H^2}\right) tg^2\beta dx dy \quad (A2)$$

$$\begin{aligned} \frac{\partial S^t(C_m, tg\beta)}{\partial tg\beta} &= 2 \frac{C_m \{p_0 - p(t)\}M_0}{H^2} tg\beta \left[\iint_A \exp\left(-\pi \frac{(x-s)^2 + (y-t)^2}{H^2}\right) tg^2\beta dx dy \right. \\ &\quad \left. - \pi \frac{tg^2\beta}{H^2} \iint_A ((x-s)^2 + (y-t)^2) \exp\left(-\pi \frac{(x-s)^2 + (y-t)^2}{H^2}\right) tg^2\beta dx dy \right] \end{aligned} \quad (A3)$$

where S^o —observed subsidence by measurement and S^t —theoretical value of subsidence.

The minimization of the objective function (3) may be realized iteratively by solving a system of matrix Equation (A4). It is necessary to establish the initial values and the approximate parameters ($C_{m0}, tg\beta_0$) for which changed values ($\Delta C_m, \Delta tg\beta$) are determined and added in successive iterations.

where:

$$A\hat{X} = L \rightarrow \hat{X} = (A^T A)^{-1} A^T L \quad (A4)$$

where:

$$A = \begin{bmatrix} \frac{\partial S_1^t(C_m, tg\beta)}{\partial C_m} & \frac{\partial S_1^t(C_m, tg\beta)}{\partial tg\beta} \\ \vdots & \vdots \\ \frac{\partial S_n^t(C_m, tg\beta)}{\partial C_m} & \frac{\partial S_n^t(C_m, tg\beta)}{\partial tg\beta} \end{bmatrix}, L = \begin{bmatrix} S_1^o - S_1^t \\ \vdots \\ S_n^o - S_n^t \end{bmatrix}, \hat{X} = \begin{bmatrix} \Delta C_m \\ \Delta tg\beta \end{bmatrix}.$$

By determining the variance-covariance matrix, the standard deviation of the obtained values ($\sigma_{C_m}, \sigma_{tg\beta}$) and correlation coefficient (R^2) may be defined.

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