

Review

Technological Aspects in Manufacturing of Non-Circular Gears

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Abstract: The construction and manufacture of non-circular gears is one of the most difficult issues in machine technology. Due to the limited technological capabilities of traditional machines and tools, past manufacturing methods were unable to calculate the assumed shape of the tooth line and its profile. Modern manufacturing methods using computer numerical control allow for the generation of non-circular gears with free-form tooth profiles and curvilinear teeth in their lengthwise direction. These methods are also used to manufacture internal gears. This article provides a review of the methods of machining non-circular gears and presents the results of the research on the Fellows method using a shaper cutter, as well as the wire electrical discharge machining method.

Keywords: non-circular gears; wire electrical discharge (WEDM) method; Fellows method

1. Introduction

The idea of non-circular gears dates from the early history of engineering. Leonardo da Vinci sketched some spatial versions of non-circular gears. In the 18th century, non-circular gears were used in flow pumps (Figure 1), clocks, music boxes, toys, and other devices. At the end of the 19th century, Franz Reuleaux ordered a series of non-circular gear-type mechanisms from the Gustav Voigt Mechanische Werkstatt in Berlin. Those mechanisms were designed to be used in the study of kinematics. Gears manufactured at that time had simplified shapes, which led to inappropriate meshing conditions, as shown, in Figure 2.

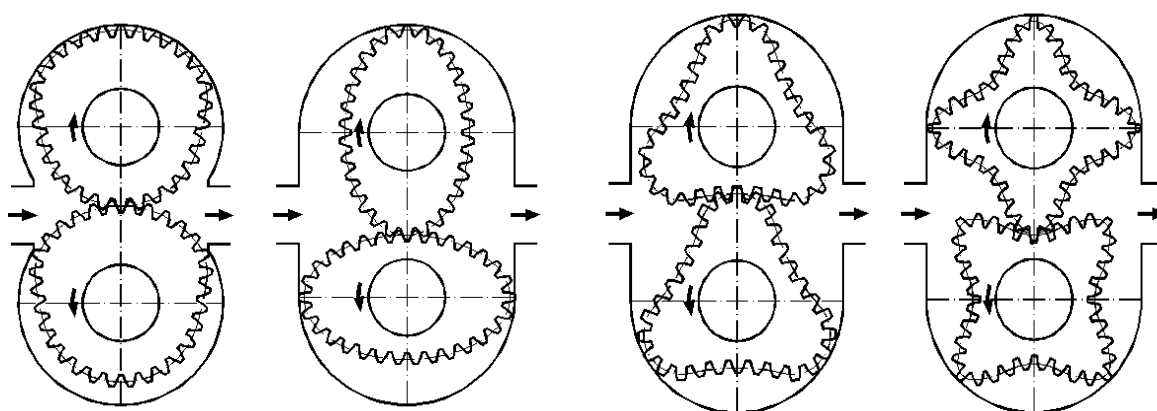


Figure 1. Examples of flow pumps using non-circular gears.

At present, non-circular gears and toothed cams are used in textile machines, non-linear potentiometer drives, continuously variable transmissions, transportation lines, Geneva indexing devices, and other mechanisms. Although modern computer numerical control (CNC)-controlled machines enable the manufacture of complex shapes, appropriate non-circular gear design methods are not readily available and usually the simplest elliptical gears are used.

Many academics around the world are interested in non-circular gears, including researchers working in Sweden, Russia, USA, Hungary, and Poland. Regardless, development in the field has been slow due to the complicated nature of equations describing non-circular gears [1–3]. This complexity has made the analysis of non-circular gears virtually impossible without the aid of computer technology [4–6].

Compared with standard gears, the design and manufacture of non-circular gears with parallel axes involve several additional problems. The most important of those problems are:

- the calculation of roll lines for both members that work properly together, i.e., rolling without slipping [7,8], and
- the calculation of cooperating tooth forms for which the meshing pole and reference rack's roll lines change their positions over time. Additionally, in non-circular gears, each tooth's flank has an independent baseline [9].

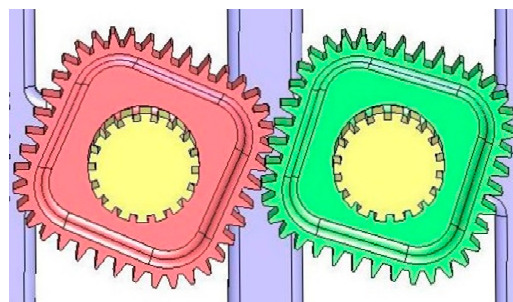


Figure 2. Historical model of non-circular gear [10].

Modern manufacturing methods allow for the generation of non-circular bevel gears with free-form tooth profiles and curvilinear teeth in their lengthwise direction (Figure 3) [10]. These methods are also used in the manufacture of internal gears (Figure 4) [4].

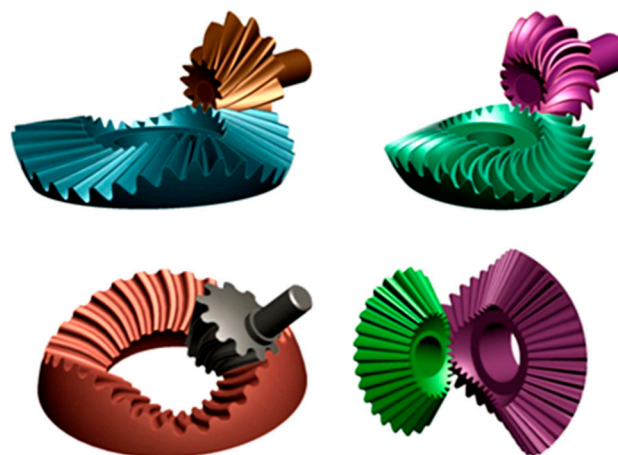


Figure 3. Examples of non-circular bevel gears with free-form tooth profiles and curvilinear teeth in their lengthwise direction [11].

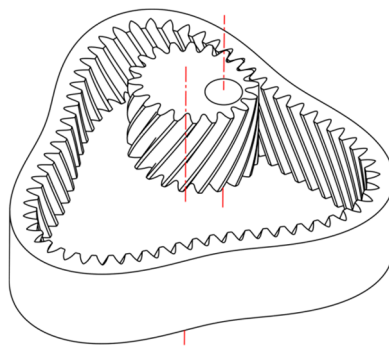


Figure 4. Example of non-circular bevel internal gears [4].

2. Methods of Manufacturing Non-Circular Gears

Basic manufacturing methods of non-circular gears are shown in Figure 5.

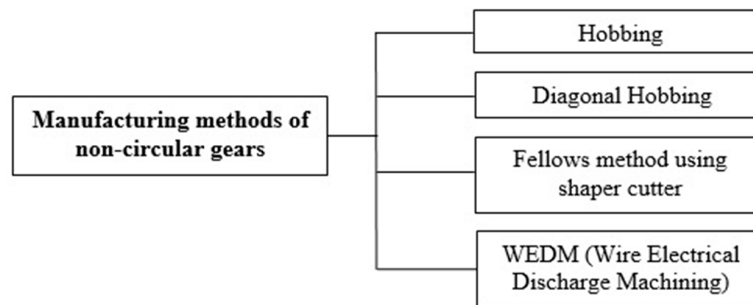


Figure 5. Manufacturing methods of non-circular gears.

3. Diagonal Hobbing of a Non-Circular Gear

Previous studies [12,13] described a strategy for machining non-circular gears using the diagonal hobbing method.

When hobbing non-circular helical gears, issues affecting the cutter teeth of the hob include uneven load and wear. To solve these problems, a simultaneous six-axis hobbing model was developed based on diagonal hobbing (Figure 6). Linkage models and their strategies were verified by hobbing testing. Experimental results showed that diagonal hobbing has several advantages over non-diagonal hobbing. Specifically, the hob life is 3.68 times longer with the diagonal hobbing method than with non-diagonal hobbing, and the micro-topography in the surface of the teeth is more stable during transmission with the diagonal hobbing method.

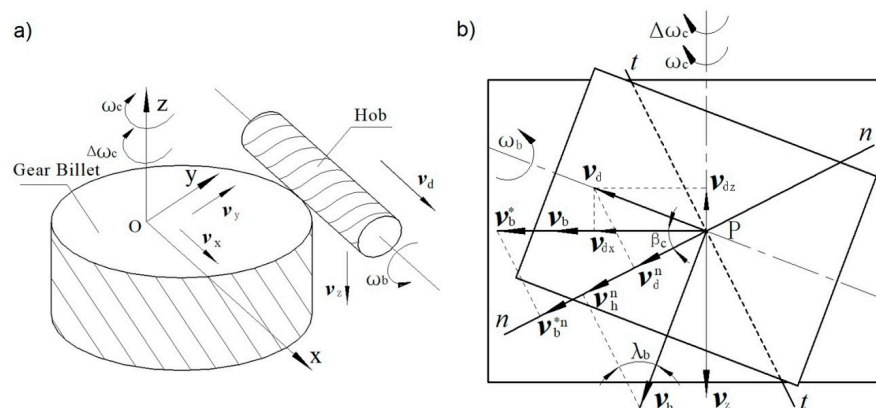


Figure 6. (a) Schematic diagram of the hobbing process; (b) common tangent plane of hobbing. ω_c : angular velocity, $\Delta\omega_c$: extra angular velocity, v_d : diagonal feed rate, v : velocity, ω_b : lead angle, β_c : helical angle [12].

A schematic diagram and a kinematic representation of this hobbing method are shown in Figure 7. In addition, the amount of wear observed with diagonal hobbing was smaller than that observed with non-diagonal hobbing (Figure 8).

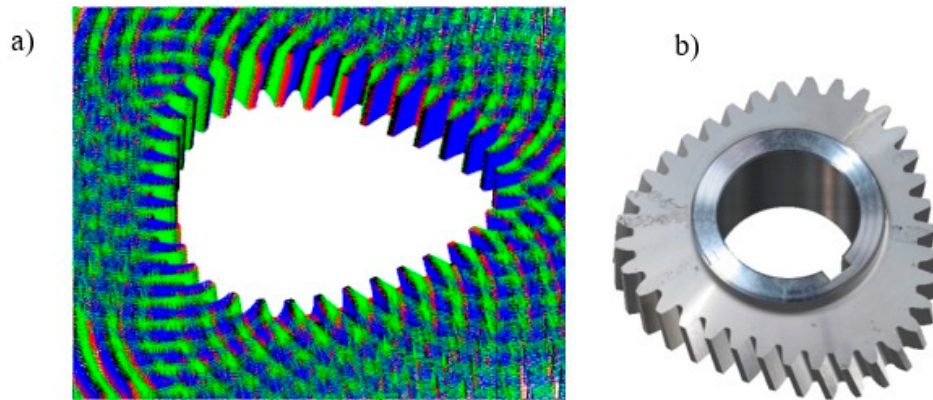


Figure 7. (a) Virtual hobbing of a non-circular helical gear based on the diagonal hobbing process; (b) 3-order elliptic helical gear hobbled using diagonal hobbing [12].

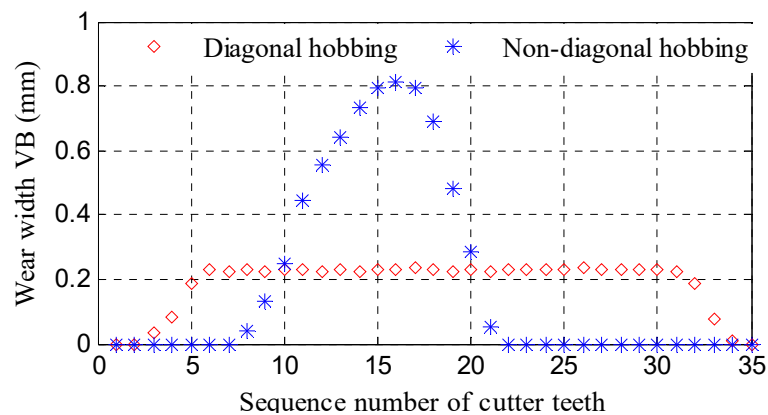


Figure 8. Wear width of flank surface for two kinds of hobbing [12].

4. Machining of a Non-Circular Gear Using the Fellows Method and a Shaper Cutter

The Warsaw University of Technology developed an innovative method of forming non-circular gears using the Fellows method [14,15]. This method is used in situations in which other methods do not allow the machining of a concave profile [16,17].

A kinematic model of teeth shaping using this method is shown in Figure 9.

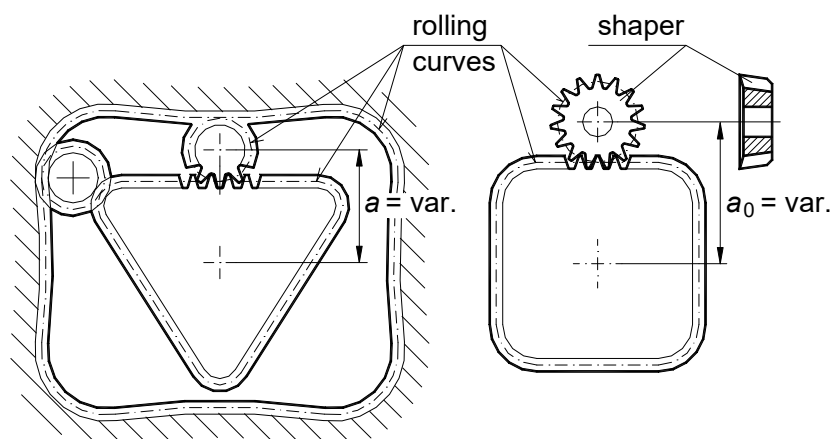


Figure 9. Examples of technological transmission for a shaper and a non-circular gear [14,15].

Let us consider two curves rolling on each other without sliding (Figure 10). The first curve is rigidly connected with the coordinate system x_1, O_1, y_1 rotating with constant angular velocity by the angle φ_1 . The second curve is rigidly connected with the coordinate system x_2, O_2, y_2 rotating with variable angular velocity by the angle φ_2 . Both curves may be described in their own systems using variable parameters ϑ_1, ϑ_2 as:

$$\left. \begin{array}{l} x_1 = x_1(\vartheta_1) \\ y_1 = y_1(\vartheta_1) \end{array} \right\} \text{or } \bar{r}_1 = \begin{bmatrix} x_1(\vartheta_1) \\ y_1(\vartheta_1) \end{bmatrix} \quad (1)$$

$$\left. \begin{array}{l} x_2 = x_2(\vartheta_2) \\ y_2 = y_2(\vartheta_2) \end{array} \right\} \text{or } \bar{r}_2 = \begin{bmatrix} x_2(\vartheta_2) \\ y_2(\vartheta_2) \end{bmatrix} \quad (2)$$

and vector $\begin{bmatrix} a \\ 0 \end{bmatrix}$ represents the center distance in the transmission.

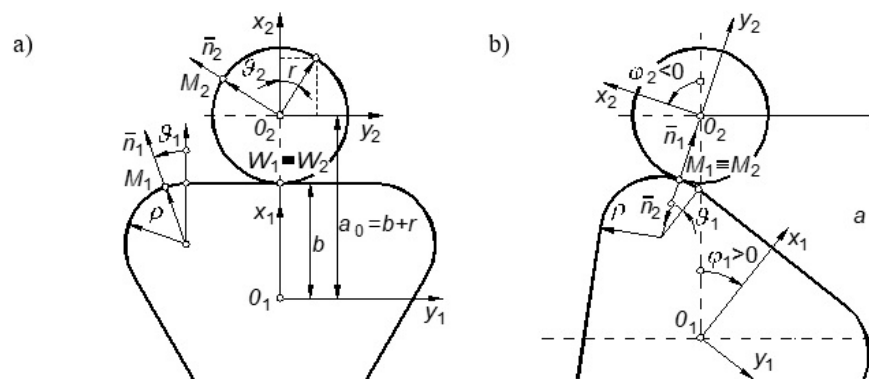


Figure 10. The roll curves of a hydraulic motor: (a) At the beginning ($\varphi_1 = 0$); (b) after rotation of the triangle rotor by any angle φ_1 [14,15].

In the general case, the center distance a is also variable (Figure 9), although in a specific case, it can be constant. The task is to compute the kinematic functions $\varphi_2 = f_1(\varphi_1)$ and $a_2 = f_2(\varphi_1)$ describing the relative motions (rotational and rectilinear) of both gears.

To solve this task, let us consider any two points M_1, M_2 being in contact. Let us take as an example the part of a hydraulic motor transmission consisting of a toothed triangle rotor (with rounded corners) and a circular satellite. The roll curves are shown in Figure 10.

They can be described as follows:

- Circular rolling curve of the satellite:

$$\left. \begin{array}{l} x_2 = r \cos \vartheta_2 \\ y_2 = r \sin \vartheta_2 \end{array} \right\} \quad (3)$$

$$\bar{n}_2 = \begin{bmatrix} n_{x2} \\ n_{y2} \end{bmatrix} = \begin{bmatrix} \cos \vartheta_2 \\ \sin \vartheta_2 \end{bmatrix} \quad (4)$$

- Rectilinear part of the triangle:

$$\left. \begin{array}{l} x_1 = b = \text{const} \\ y_1 = y_1(\text{equivalent of variable parameter } \vartheta_1) \end{array} \right\} \quad (5)$$

- Rounded corners of the triangle (point M_1 in Figure 4):

$$\left. \begin{aligned} x_1 &= b - \rho(1 - \cos \vartheta_1) \\ y_1 &= [(b - \rho)\operatorname{ctg} 30^\circ + \rho \sin \vartheta_1] \end{aligned} \right\} \quad (6)$$

$$\bar{n}_1 = \begin{bmatrix} n_{x1} \\ n_{y1} \end{bmatrix} = \begin{bmatrix} \cos \vartheta_1 \\ -\sin \vartheta_1 \end{bmatrix} \quad (7)$$

Equations of rolling curves and normal vectors are the starting point for determining the coordinates of tooth profiles [14].

5. Machining of a Non-Circular Gear Using the WEDM Method

An innovative method of modeling the machining of non-circular gears using the wire electrical discharge machining (WEDM) method was developed at the Warsaw University of Technology with the participation of the author of this article—Tadeusz Sałaciński [9,18]. García-Hernández et al. [19] also examined this problem.

In Figure 11, the non-circular gear in design is represented as two roll lines P1 and P2 with rotation centers at points O1 and O2, respectively. Those lines work together, rolling without slipping.

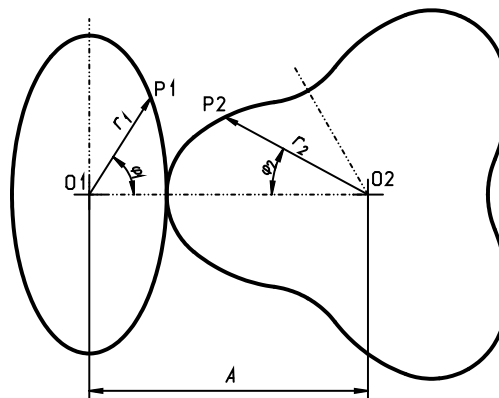


Figure 11. Roll lines representing a non-circular gear; P1, P2—roll lines, O1, O2—rotation centers.

The shape P1 is described by a polar equation as a function of radius $r_1(\varphi_1)$. The gear ratio is described by the function:

$$\nu(\varphi_1) = \frac{\omega_1(\varphi_1)}{\omega_2(\varphi_1)} = \frac{r_2(\varphi_1)}{r_1(\varphi_1)} \quad (8)$$

where $\omega_1(\varphi_1)$ and $\omega_2(\varphi_2)$ are angular velocity functions of the first and the second gears, respectively.

The axle distance A is constant:

$$A = r_1(\varphi_1) + r_2(\varphi_1) = \text{const} \quad (9)$$

The roll lines work together without slipping:

$$d\varphi_2 = \frac{r_1(\varphi_1)}{r_2(\varphi_1)} d\varphi_1 \quad (10)$$

Taking Equations (8) to (10) into account and integrating

$$\varphi_2 = \int_0^{\varphi_1} \frac{r_1(\varphi_1)}{A - r_1(\varphi_1)} d\varphi_1, \quad (11)$$

the integrating constant was found from the following condition: if $\varphi_1 = 0$, then $\varphi_2 = 0$.

Note that to continuously transfer the motion, the gear ratio function must be periodic and the gear's period T must be related to periods T_1 and T_2 of gears 1 and 2, respectively, as follows:

$$T = \frac{T_1}{n_2} = \frac{T_2}{n_1} \quad (12)$$

where n_1 and n_2 are natural numbers.

In the case shown in Figure 11, the period T of the gear ratio function $\nu(\varphi_1) = \pi$, $T_1 = 2\pi$, and $T_2 = 3\pi$; therefore, when $\varphi_1 = \pi$, then $\varphi_2 = 2/3\pi$. This renders:

$$\frac{2}{3}\pi = \int_0^\pi \frac{r_1(\varphi_1)}{A - r_1(\varphi_1)} d\varphi_1 \quad (13)$$

The value of the parameter A can be calculated from Equation (13). The $P2 = r_2(\varphi_1)$ can be calculated from Equations (9) and (10).

We assume there is roll line $P1$ for which a cooperating line $P2$ needs to be found that fulfils the requirements described above. The line $P1$ has been divided into i segments l_{1i} so that each of those segments can be represented as an Archimedes spiral with adequate precision (Figure 12).

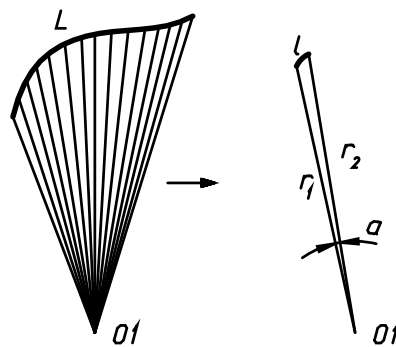


Figure 12. Roll line divided into segments.

An Archimedes spiral is described by:

$$r = k\alpha \quad (14)$$

Therefore:

$$\begin{aligned} \alpha_1 &= \frac{r_1}{k}; \alpha_2 = \frac{r_2}{k}; k = \frac{r_2 - r_1}{\alpha} \\ dl &= \sqrt{(dr)^2 + (r d\alpha)^2}; dr = d(k\alpha) = k d\alpha \\ dl &= \sqrt{(kd\alpha)^2 + (r d\alpha)^2} = \sqrt{(kd\alpha)^2 + (k\alpha d\alpha)^2} = k \sqrt{1 + \alpha^2} d\alpha \\ \int dl &= \frac{k}{2} \left[\alpha \sqrt{\alpha^2 + 1} + \ln(\alpha + \sqrt{\alpha^2 + 1}) \right] \\ l &= \int_{\alpha_1}^{\alpha_2} dl = \frac{k}{2} \left[\alpha_2 \sqrt{\alpha_2^2 + 1} + \ln(\alpha_2 + \sqrt{\alpha_2^2 + 1}) - \alpha_1 \sqrt{\alpha_1^2 + 1} - \ln(\alpha_1 + \sqrt{\alpha_1^2 + 1}) \right] \end{aligned} \quad (15)$$

If lines $P1$ and $P2$ are rolling together without slipping, and line $P2$ has been divided into i parts l_{2i} accordingly to $P1$, then, with reference to Equations (10), we get:

$$l_{1i} = l_{2i} \quad (16)$$

The equation of the line $P2$ is evaluated from Equations (9) to (16).

The next step is to calculate the tooth forms. For this purpose, the context analysis method was used. In this method, the system under examination is transformed through its states and data are collected, forming a context. Then the data that meet defined requirements are filtered out of the context.

The roll line of a non-circular gear is described by $L = r(\varphi)$. There is a reference rack guide S rolling over the gear's line without slipping. A reference rack's profile T is bound up with the S line (Figure 13). The coordinates of the lines' contact point P are:

$$P[x, y] = [r(\varphi) \cos(\varphi), r(\varphi) \sin(\varphi)] \quad (17)$$

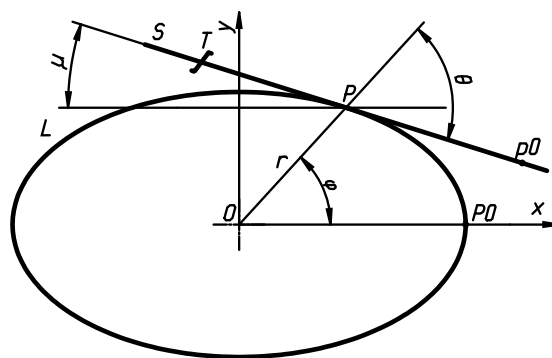


Figure 13. A reference rack guide over the roll line of a gear.

An angle between the $r(\varphi)$ radius and the reference rack's guide S is calculated from:

$$\theta = \arctan\left(\frac{r(\varphi)}{\frac{dr}{d\varphi}}\right) \quad (18)$$

So:

$$\mu = \varphi - \theta \quad (19)$$

If $\varphi = 0$ then $P0 = p0$. Since the guide S is rolling over the line L without slipping from point $P0$ to point P , the length of a segment l of line L between points $P0$ and P equals the length of a vector s between points $p0$ and P of the S line.

If the line L is composed of i L_i segments according to the requirements listed above, the point P corresponds to L_j , and l_i lengths and k_i coefficients are found for each L_i segment from Equation (15), then:

$$\frac{dr}{d\varphi} = k_j |s| = \sum_{n=1}^i l_n. \quad (20)$$

To find the points of a non-circular gear tooth's form, the lines u_1 to u_z were introduced. The intersection points of the profiles T_j to T_k with the lines u_1 to u_z were calculated using analytic geometry. The points lying on a non-circular gear tooth's form were filtered out from all the points that were found. This procedure was repeated for all the flanks of the non-circular gear. The module of the non-circular gear is calculated using:

$$m = \frac{|L|}{\pi z} = \frac{\sum_{n=1}^i l_n}{\pi z} \quad (21)$$

where z is the number of teeth in the gear. An example model of a non-circular gear and a machined gear are shown in Figure 15. Figure 14 shows the consecutive positions of the roll line S_j to S_k and of the reference rack's profile T_j to T_k .

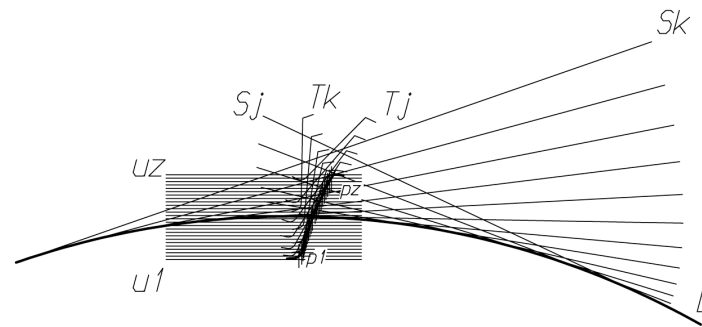


Figure 14. Evaluation of a tooth's form.

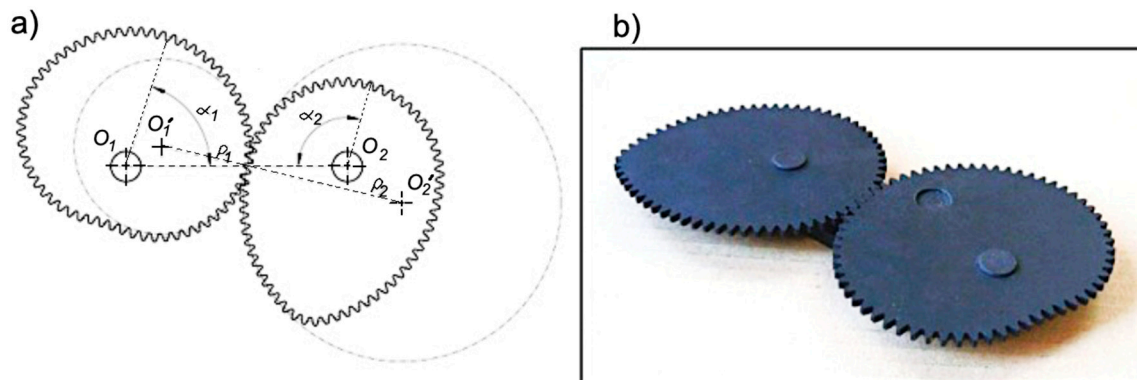


Figure 15. (a) Model of a non-circular gear; (b) machined gear.

This numerical model was used to create the manufacturing programs for a CNC-controlled wire electrical discharge machine. The gears were compared with a digital model using the coordinate measuring machine (CMM). A scan of the tooth flanks was performed and the scans were verified against the mathematical master using computer software developed because the CMM lacked the functionality.

The measuring reports showed that the form error r measured on various tooth heights h was less than 0.02 mm, which corresponds to class 8 of the Deutsches Institut für Normung (DIN) 3962 guidelines (Figure 16).

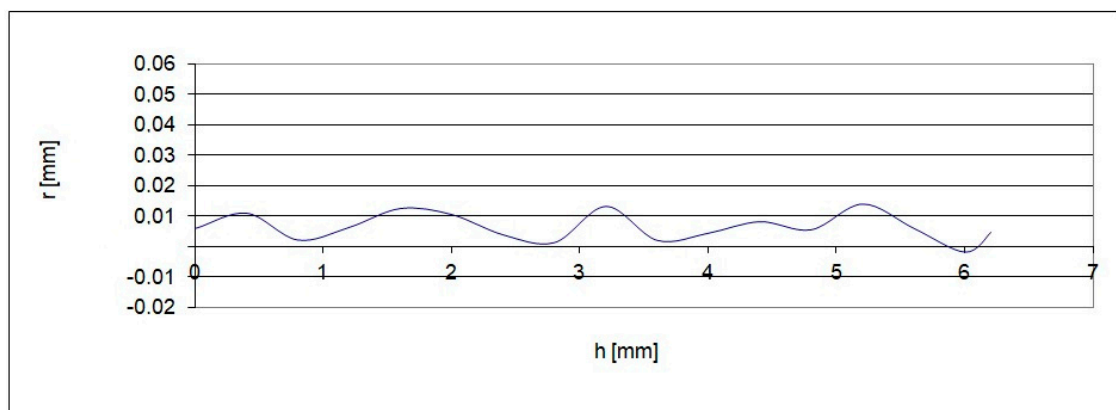


Figure 16. CMM (Coordinate Measuring Machine) measuring report.

As a result of machining non-circular gears on their faces, unfavorable burrs are formed. These can be removed with rotary brushes with ceramic fibers (Figure 17) [20–24].



Figure 17. Deburring of a non-circular gear with rotary brushes with ceramic fibers.

The problems associated with accuracy and machining times for the methods analyzed in this article are not sufficiently discussed in the literature. Liu et al. [25] assessed the accuracy of the machining of a non-circular gear (modulus (m) was 3, number of teeth (z) was 39, and tooth width (b) was 30 mm) using the hobbing method (Figure 18). The geometrical accuracy of the surface was observed to be at the level of 0.05–0.06 mm.

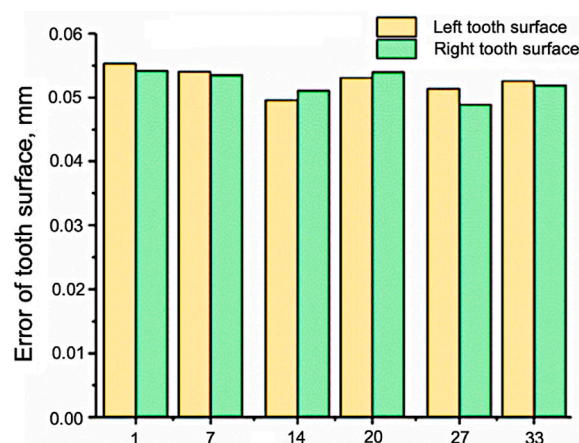


Figure 18. Errors of the tooth surface in the hobbing method [25].

In our opinion, the machining times for non-circular gears are comparable to those for classic circular gears. This will be the subject of further research.

6. Conclusions

This paper describes the technological aspects of different methods of manufacturing non-circular gears. The most efficient method is hobbing, but this method cannot be used in all cases (e.g., for concave tooth lines). Internal teeth are machined using the Fellows method or the WEDM method. The WEDM method enables the implementation of custom shapes for the undercut sections at the feet of the teeth and those sections can be different for each flank. Because the WEDM method is an expensive process, it is most often used for low volume production. For series production, other manufacturing methods, such as the Fellows method, may be beneficial. However, a disadvantage of this process is the need for unconventional machines and the complicated geometry of the tools. The meshing conditions in a non-circular technological gear are different in each phase of its generation, which causes the tool requirements to change. The main issues with this process are the form diameters, interference, and

changes in the parameters after the re-sharpening of a tool. For these reasons, WEDM may prove to be the only technology available that allows manufacturing of a proper non-circular gear.

With the development of technologies using computer numerical control (CNC), non-circular gears are increasingly used in practice and have been the subject of research for many years. This is evident from publications (e.g., [25–29]) that reference computer simulations and 3D modeling. Further research should examine the geometric and kinematic accuracy, as well as the strength, of non-circular gears manufactured through different methods.

The results presented here may form the groundwork for further investigations of properties particular to non-circular gears. Similar investigations have been conducted with spur, helical, and bevel gears. Examples of properties particular to non-circular gears include kinematics, applications of special-purpose gears, as well as issues related to strength, dynamics, and tribology.

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