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Scale Effect and Correlation between Uniaxial Compressive Strength and Point Load Index for Limestone and Travertine

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Abstract: Determining the uniaxial compressive strength of intact rock is the primary objective of a geomechanical project, and a reliable estimate in the early phases saves time and costs for more sophisticated laboratory tests. The problem is knowing which of the correlations between the resistance to uniaxial compression and point load index are reliable, those that cover one or several types of rock (depending on the type of statistical adjustment). In this work, they were evaluated with respect to limestone and travertine from experimental results, and the statistical models of the scale effect of the point load index were determined, and the uniaxial compressive strength being estimated from correlations reported in literature. The limestone model was ascending (strength increases as diameter increases), while the travertine model was descending (strength decreases as diameter increases), obtaining similar exponents for the scale effect equations modeled from the uniaxial compressive strength and point load index in both cases.

Keywords: scale effect; point load index; uniaxial compressive strength; limestone; travertine



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1. Introduction

Determining the compressive strength of a rock mass is one of the main objectives that geomechanical design projects address, because it depends not only on the rock material (intact rock), but also on the discontinuities that separate the intact rock blocks. The main parameter used to represent the strength of the intact rock is the uniaxial compressive strength (σ_c), and it is determined in the laboratory using cylindrical specimens according to the standards of American Society for Testing and Materials (ASTM) [1,2] and International Society for Rock Mechanics and Rock Engineering (ISRM) [3].

In the same way that it can be determined in the laboratory directly, indirect methods have also been developed to estimate σ_c for almost types of rock in general, and limestone and travertine in particular, because it can be correlated experimentally with its index properties. In this way, predictive models emerge that experimentally relate σ_c and porosity [4–9], σ_c and density [7,10,11], and σ_c and the point load index [3,10,12–22] or the number of rebounds of the Schmidt hammer [10,18,23–25], which will be seen throughout this work.

This work emphasizes the effect of geometry of the specimens, due to its importance in the magnitude of the value of σ_c , since both ASTM [1,2] and ISRM [3] recommend cylindrical specimens with a diameter (d) between 50 and 54 mm—reference value according to the scale effect [26]—and a height/diameter ratio (H/d) of between 2 and 3 [27–29].

Both d and H/d are very important in the experimental correlations because these are parameters directly related to the values used in the least squares adjustment and

linear regression analysis, so the d of the specimen tested under uniaxial compressive stress will be the reference of the predictive model. Therefore, if the values of σ_c obtained from specimens of 48 mm diameter are correlated by means of a simple (or multiple) regression analysis with those corresponding to any index property, the predictive model will estimate a value of σ_c of 48 mm and not 50 mm. Between 48 and 50 mm there is almost no difference, but smaller diameters are also used in research, such as 21 mm [30], for example.

For this reason, González et al. [31] and Saldaña et al. [32] used P wave velocity values (v_p), measured in cylindrical specimens of 31 mm diameter, in experimental correlations determined for other diameters, thus estimating an erroneous σ_c value. This led them to the generation of their own experimental predictive models.

Finally, the objective of this research is to determine the experimental correlation between σ_c and I_s for limestone and travertine, for diameters of 31 mm and 50 mm, from the experimental $I_{s(50)}$ and $\sigma_{c(50)}$ estimated from the literature referring to the correlations between σ_c and I_s . There was a special emphasis on the geometry of the specimen tested for each of the tests, which led to studying the scale effect for both rocks because (most of) the correlations referred to the 50 mm diameter.

1.1. Correlation between Uniaxial Compressive Strength and Point Load Index

The point load index is an indirect way of estimating the σ_c of rock, and there is a wide range of correlations for this. During the early days of this test, it was determined based on the uncorrected point load index (I_s) [12,13], but later the use of the point load index corrected for a diameter of 50 mm ($I_{s(50)}$) was widespread due to the publication of the scale effect of the point load index by Brook [16], analogous to that of Hoek and Brown [33,34] for the σ_c from cylindrical specimens of intact rock.

Table 1 shows the different correlations for limestone and travertine, from which the dimensions of the test specimens were extracted to determine σ_c and the corresponding ones to determine I_s or $I_{s(50)}$. The point load tests were mainly carried out with diametrically loaded cylinders (Equations (1)–(5), (10), (15) and (16)), but also with axially and diametrically loaded cylinders (Equation (9)) or with regular blocks (Equations (11)–(13)). Another case is the correlations obtained from the results of other publications, such as that of Brook [16] (Equation (8)), analogous to his method of the resistance index for a minimum area of 500 mm² (T_{500}) (Equation (7)) or those that do not define the geometry or the type of test (Equations (6) and (14)).

It can be observed that most correlations are linear and pass through the origin of the coordinates, but both ASTM (Equation (18)) and ISRM (Equation (17)) do not clarify the cutoff point with the horizontal axis: ASTM [22] indicates that, if not there are experimental values of σ_c , I_s multiplied by 23 to estimate the uniaxial compressive strength at a diameter of 50 mm ($\sigma_{c(50)}$) based on Bieniawski [13], while the ISRM [3] indicates that $I_{s(50)}$ must be multiplied by a value between 20 and 25, and can vary between 16 and 60 for anisotropic rocks.

Table 1. Predictive models of uniaxial compressive strength (MPa) from point load index (MPa).

Correlation	R^2	Rock Type	Geometry				Reference	Equation
			σ_c	PLT *				
				Axial	Diametral	Blocks or Slumps		
$\sigma_{c(54)} = 14 I_S + 2.9$	0.92	Basalt, diabase, dolomite, gneiss, granite, limestone, marble, quartzite, rock salt, sandstone, schist, siltstone, tuff	d = 54 mm H/d = 2	-	d = 54 mm H/d = 2	-	Deere and Miller [10]	(1)
$\sigma_{c(38)} = 23,7 I_S$	0.88	Dolerite, sandstone	d = 38 mm H/d = 2	-	d = 38 mm (50 mm corrected)/L $\geq$ 0.7D/H/d > 1.4	-	Broch and Franklin [12]	(2)
$\sigma_{c(54)} = 24 I_S$	-	Norite, quartzite, sandstone	d = 54 mm	-	d = 54 mm/L > 0.7D	-	Bieniawski [13]	(3)
$\sigma_{c(42)} = 21 I_S$	-		d = 42 mm		d = 42 mm/L > 0.7D			(4)
$\sigma_{c(21.5)} = 18 I_S$	-		d = 21.5 mm		d = 21.5 mm/L > 0.7D			(5)
$\sigma_{c(38)} = (22 - 27) I_{S(50)}$	-	Dolomite, limestone, sandstone	d = 38 mm H = 76 mm	d = 50 mm (does not specify the type of test)			Al-Jassar [14]	(6)
$\sigma_c = 12,5 T_{500}$	-	Basalt, diabase, dolerite, norite, quartzite, sandstone, etc.	The data used to correlate σ_c and T_{500} are those of Broch and Franklin [12], Bieniawski [13], D'Andrea et al. [35], and Deere [36]				Brook [15]	(7)
$\sigma_c = 22 I_{S(50)}$	-						Brook [16]	(8)
$\sigma_{c(50)} = 26.5 I_{S(50)}$	-	Limestone	d = 50 mm H = 100 mm	d = 76-92 mm corrected to 50 mm with Broch and Franklin [12]		-	Hawkins and Olver [17]	(9)
$\sigma_{c(54)} = 23 I_{S(54)} + 13$	0.94	Dolomite, gneiss, limestone, marble, sandstone	d = 54 mm H/d = 2	-	d = 50 mm H/d = 1.1	-	Cargill and Shakoor [18]	(10)
$\sigma_c = 12.7 I_{S(50)}$	-	Dolomitic limestone	$5 \times 5 \times 10 \text{ cm}^3$ ($d_e = 56.4 \text{ mm}$) D/W = 1	-	-	$10 \times 10 \times 20 \text{ cm}^3$ ($d_e = 112.8 \text{ mm}$)	Quinta-Ferreira and Machado [19]	(11)
$\sigma_c = 22 I_{S(50)}$	-	Micritic limestone				$5 \times 5 \times 10 \text{ cm}^3$ ($d_e = 56.4 \text{ mm}$)		(12)
$\sigma_c = 18 I_{S(50)}$	-	Mean correlation				$2.5 \times 2.5 \times 5 \text{ cm}^3$ ($d_e = 28.2 \text{ mm}$) all corrected to d = 50 mm		(13)
$\sigma_{c(54)} = 21.9 I_{S(50)}$	-	Limestone	d = 54 mm (most) d = 75 mm (rest)	-	-	-	Rusnak and Mark [20]	(14)
$\sigma_{c(54)} = 20 I_{S(50)}$	0.40	Calcareous-marly, limestone, marlstone, sandstone, quartzitic-greywacke	d = 54 mm H/d = 2–2.5	-	d = 50 mm H = 110 mm	-	Tsiambaos and Sabatakakis [21]	(15)
$\sigma_{c(54)} = 7.3 I_{s(50)}^{1.71}$	0.82							(16)
$\sigma_c = (20\text{--}25) I_{(50)}$	-	-	-	$0.3 < L/D < 1$	$L/D > 1$ L = 0.5D	$0.3 < D/W < 1$ L/W > 0.5	ISRM [3]	(17)
$\sigma_c = (18\text{--}24.5) I_S$	-	-	-	$30 < d < 85 \text{ mm}$ H/d = 0.3–1 $0.3W > D > W$	$30 < d < 85 \text{ mm}$ H/d > 1 L = 0.5	$0.3 < D/W < 1$ L/W > 0.5	ASTM [22]	(18)

* Information in the literature about the geometry of the specimens/slumps used in the point load test to determine the point load index [3,22]. It must be taken into account that D is the height of the failure cross section, W is the width of the failure cross section, P is the failure load, L is the distance between the failure cross section and the two free faces parallel to it, d is the diameter of the cylindrical specimen, and d_e is the equivalent diameter of blocks and slumps.

1.2. Correlation between Uniaxial Compressive Strength and the Number of Rebounds of the Schmidt Hammer

The number of rebounds of the Schmidt hammer is another index property that provides information on rock strength. There are two types of hammer depending on their impact energy, *L*-type (0.735 Nm) and *N*-type (2.207 Nm). The *L*-type being is chosen for laboratory correlations due to its lower impact energy since, in this way, the specimens are not broken before the uniaxial compressive strength test, and thus a simple regression analysis can be carried out, since σ_c and R_L come from the same source.

Likewise, there is a correlation between the readings of both hammers (Equation (19)), proposed by Aydin and Basu [37] and collected by the ISRM [38], whose reliability is very high when the number of rebounds of the *L*-type hammer (R_L) is greater than 30, and the number of rebounds of the hammer *N*-type (R_N) is greater than 40:

$$R_N = 1.0646R_L + 6.3673. \quad (19)$$

On the other hand, the ASTM [39] does not distinguish between types of hammer, nor does it recommend an estimate of σ_c , while the ISRM [38] indicates the corrections for each hammer as a function of the hammer's position with respect to the horizontal (due to the effect of gravity) and proposes potential and exponential models to estimate σ_c .

Table 2 shows the main experimental correlations, where three models are exponential (Equations (20), (21), and (23)) and two linear (Equations (22) and (24)). The exponential equations multiply R_L or R_N and the specific weight to obtain a better adjustment coefficient (R^2), as pointed out by Deere and Miller [10].

1.3. Scale Effect

Weibull [40] assumed that every solid contains imperfections or defects (microcracks) that diminish the strength of the material due to rupture taking place “as soon as they [weak places] fall within a volume subjected to the stress”. This means that, for two cylinders of different size (but identical shape), the probability of the failure of the large specimen is greater than that of the small one, because it contains more defects, and the strength of the large sample is less than that of the small one [26]. Equation (25) is the generalized modification by which Weibull [41] introduced the positive constant m for a better statistical fit, where σ_1 and σ_2 are the strength of the two specimens and V_1 and V_2 the respective volumes, according to Bieniawski [42].

$$m \log \left(\frac{\sigma_1}{\sigma_2} \right) = \log \left(\frac{V_2}{V_1} \right) \quad (25)$$

In the case of cylindrical specimens with identical shapes and a constant H/d ratio, the volume can be replaced by d , hence Equations (26) and (27), proposed by Hoek and Brown [33,34] and Brook [16], respectively. It can be seen that the constant $1/m$ is now called k (Figure 1), so in the case of Hoek and Brown [33,34] the value of k_1 is 0.18 and, in that of Brook [16], k_2 can be approximated to 0.45. Yoshinaka et al. [43] reported k_1 values between 0.1 and 0.3 for hard and homogeneous rocks, while for weathered rocks they are between 0.3 and 0.9. Both correlations refer to the relationship between the strength of the specimen at any diameter (σ_c , I_s) and another at a 50 mm diameter ($\sigma_{c(50)}$, $I_{s(50)}$).

$$\frac{\sigma_c}{\sigma_{c(50)}} = \left(\frac{50}{d} \right)^{k_1} \quad (26)$$

$$\frac{I_s}{I_{s(50)}} = \left(\frac{50}{d} \right)^{k_2} \quad (27)$$

Table 2. Predictive models of simple compressive strength (MPa) from the number of rebounds of the Schmidt hammer.

Correlation	R^2	Rock Type	Geometry σ_c	Geometry R *	Reference	Equation
$\sigma_{c(54)} = 9.97e^{0.02R_L \rho_{ap}}$	-	Basalt, diabase, dolomite, gneiss, granite, limestone, marble, quartzite, rock salt, sandstone, schist, siltstone, tuff	d = 54 mm H/d = 2	d = 54 mm H/d = 2	Deere and Miller [10]	(20)
$\sigma_{c(54)} = e^{0.018R_L \rho_{ap} + 2.9}$	-	Dolomite, gneiss, limestone, marble	d = 54 mm H/d = 2	d = 54 mm $2 < H/d < 2.3$	Cargill and Shakoor [18]	(21)
$\sigma_{c(52)} = 4.29R_L - 67.52$	0.92	Dolomite, limestone, marble	d = 52 mm d = 38 mm	Blocks $25 \times 25 \times 20 \text{ cm}^3$	Sachpazis [23]	(22)
$\sigma_{c(33)} = 6.97e^{0.014R_N \rho_{ap}}$	0.92	Diabase, dolomite, marl, sandstone, serpentine, tuff	d = 33 mm H/d = 2	-	Kahraman [24]	(23)
$\sigma_{c(50)} = \frac{6222}{88.15 - R_L} - 70.38$	0.92	Granite, limestone, sandstone	d = 50 mm H = 100 mm	$10 \leq R_L \leq 70$	Wang and Wan [25]	(24)

* Information in the literature about the geometry of the specimens used to determine the number of rebounds of the Schmidt hammer.

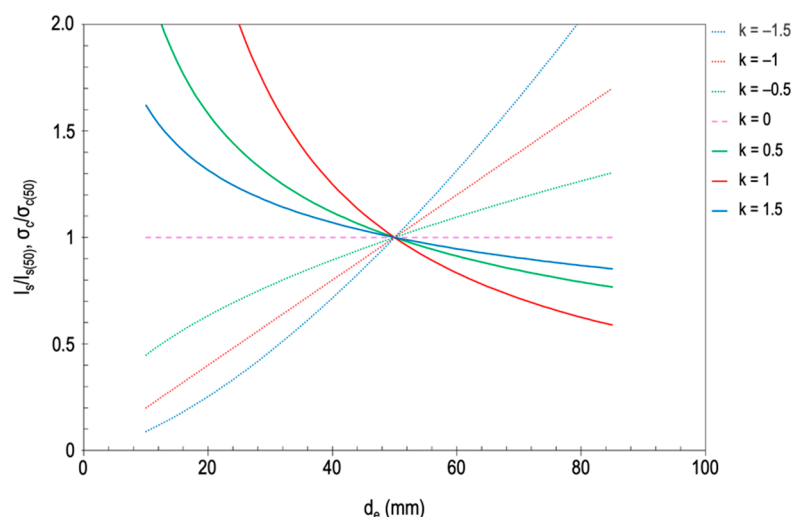


Figure 1. Effect of the k value (k_1 and k_2 for Equations (26) and (27), respectively) on the scale effect of rock strength. By definition, $k > 0$ [41], but negative k values have been added to represent possible ascending models (adapted from Wang et al. [26]).

The models of Hoek and Brown [33,34] and Brook [16] are considered descending, since σ_c decreases with increasing d . However, the model of Hoek and Brown [33,34] only took into account one sedimentary rock (limestone) of the 10 that it used to generate the experimental model, and also diameters greater than 50 mm. This fact did not go unnoticed by Hawkins [44], who pointed out that the model of Hoek and Brown [33,34] did not adapt to their experimental results obtained from five types of limestone and two of sandstone. To determine σ_c he tested cylindrical specimens with d values ranging from 12.5 to 150 mm.

Hawkins's [44] discovery was no longer so much that his results did not fit a descending model with a k_1 of approximately 0.18, but that for a diameter less than 40 mm and greater than 60 mm, the results of σ_c were inferior to those obtained within that interval (mainly 54 mm). In a σ_c - d graph, this means that for values of diameter less than 40 mm the model is ascending, and the model is descending for values greater than 60 mm. This phenomenon showed that the model of Hoek and Brown [33,34] was not applicable to all types of rock, being able to follow an ascending, descending, or mixed model.

From the energetic point of view, Bazant [45] proposed an alternative descending model to Weibull's [41], based on Griffith's [46] theory on the creation and propagation of cracks, which is summarized in two assumptions: (i) when a crack begins to propagate, it is expected that there will be a greater release of energy in the larger specimen under the same stress [47], since it stores more energy than the small one; and (ii) this greater release of energy means that less effort is needed to initiate a crack in larger samples [48]. This model was called the scale effect law (SEL) (Equation (28)):

$$\sigma_N = \frac{B f_t}{\sqrt{1 + \left(\frac{d}{\lambda d_0}\right)}}, \quad (28)$$

where σ_N is the nominal resistance (MPa), B and λ are the dimensionless constants of the material, f_t is the strength of a sample with an insignificant size (MPa), d is the diameter of the specimen (mm), and d_0 is the maximum diameter of the aggregate (mm).

Nine years later, Bazant [49] proposed a new law to describe ascending models, in which he incorporated fractals (through the fractal dimension d_f) into the energy required to produce the fracture on concrete, rock, and ceramic. This is the so-called fractal fracture scale effect law (FFSEL) (Equation (29)):

$$\sigma_N = \frac{\sigma_0 d^{(d_f-1)/2}}{\sqrt{1 + \left(\frac{d}{\lambda d_0}\right)}} \quad (29)$$

Finally, Masoumi et al. [50] proposed the unified scale-effect law (USEL) (Equation (30)), which is the intersection (d_i) between the ascending model (FFSEL) and the descending model (SEL), which is applied when there are mixed cases such as limestone and sandstone described by Hawkins [44]. The intersection d_i is the diameter of the model-change (ascending to descending) in millimeters.

$$d_i = \left(\frac{B f_t}{\sigma_0} \right)^{2/(d_f-1)} \quad (30)$$

2. Materials and Methods

2.1. Origin of Limestone and Travertine

The limestone belongs to a quarry near Antofagasta (Chile) and dates from the Lower Cretaceous era, between 66 and 100 Ma old. It is a fine limestone rock with a variable CaCO_3 content between 78% and 90% [31].

The travertine belongs to a quarry located in Calama, 200 km northeast of Antofagasta (Chile). It belongs to the Chiu-Chiu formation, the youngest within the Calama basin, with an age between 2.5 and 0.5 Ma, from the Upper Pliocene [32].

2.2. Point Load Test

To carry out the test following the suggestions of ASTM [22] and ISRM [3], irregular fragments that meet the specific geometric conditions were selected: the height/width (D/W) ratio of the cross section was between 0.3 and 1. The calculation of the equivalent diameter (d_e) was carried out according to Equation (31), where D is the distance between the conical tips of the point-load testing machine (height of the cross section) and W the average of the maximum and minimum width of the loaded cross section.

$$d_e^2 = \frac{4WD}{\pi} \quad (31)$$

The ISRM [3] suggests determining the σ_c by multiplying the $I_{s(50)}$ by a factor F that varies between 20 and 25 when the rock does not present anisotropy, as shown in Equation (32), where d_e is the equivalent diameter (mm), P is the failure load (kN), k_2 is the coefficient of the scale factor, and σ_c is the uniaxial compressive strength (MPa).

This value of k_2 is obtained experimentally by a least squares fit in the $P-(d_e)^2$ graph, discarding all the values that do not belong to the line (but they are not discarded for subsequent calculations of $I_{s(50)}$, just to determine where). However, when there are insufficient experimental values, it is suggested that k_2 takes a value of -0.45 .

$$\sigma_c = F I_{s(50)} = F \left[\left(\frac{d_e}{50} \right)^{-k_2} I_s \right] 10^3 = F \left[\left(\frac{d_e}{50} \right)^{-k_2} \frac{P}{d_e^2} \right] 10^3 \quad (32)$$

On the other hand, ASTM [22] suggests that to estimate σ_c when reliable k_2 data are not available, it is recommended to take a value from Table 3, based on Bieniawski [13], according to the chosen d (generally 50 mm). When F is 23, it is considered a 50 mm diameter, as presented in Equation (18).

Table 3. Values of F for different diameters according to American Society for Testing and Materials (ASTM) [22].

Diameter, d (mm)	F
21.5	18
30	19
42	21
50	23
54	24
60	24.5

2.3. Schmidt Hammer Test

This test is usually performed on the same specimen so that R comes from the same source when performing linear regression analyzes. However, in this case, they were carried out on perfectly parallelepipedic plates of more than 70 mm thick (they are more than 54 mm of the cylindrical specimens used in the correlations of Table 2).

The hammer used was N -type and, as indicated by the ISRM [38], 10 readings were made on each surface greater than 1 m^2 , in a vertical downward position, later correcting its value according to the normalization abacus proposed by Basu and Aydin [51], according to the position of the hammer with respect to the horizontal (and the force of gravity).

2.4. Uniaxial Compressive Strength Test

The tests were carried out using a 300 t CONTROLS servo-controlled press, model 50-C52Z00 + MCC8 50-C8422/M. According to ASTM [1], the loading speed was set to $0.2 \text{ MPa} \cdot \text{s}^{-1}$ so that the failure of the specimen occurred between 5 and 10 min after the test started.

The specimen specifications, according to ASTM [2], were an H/d ratio of 2 and the polishing of the basal faces for a better transmission of the force applied by the press.

2.5. Density and Porosity Test

A hydrostatic balance was used to apply the Archimedes thrust principle, collected by the ISRM [3]. The dry mass (m_{dry}) was determined after having the specimens in an oven at 105°C for 24 h, and the saturated mass (m_{sat}) after 48 h immersed in water, while the immersed mass (m_{sub}) was obtained by immersing them already saturated to consider the specimen as a solid. The dry density (ρ_{dry}), saturated density (ρ_{sat}), and effective porosity (n_{eff}) were calculated from Equations (33)–(35), respectively:

$$\rho_{dry} = \frac{m_{dry} \rho_w}{m_{sat} - m_{sub}}, \quad (33)$$

$$\rho_{sat} = \frac{m_{sat} \rho_w}{m_{sat} - m_{sub}}, \quad (34)$$

$$n_{eff} = \frac{m_{sat} - m_{dry}}{m_{sat} - m_{sub}} 100, \quad (35)$$

where m_{dry} is the dry mass (g), m_{sat} is the mass of the saturated specimen (g), m_{sub} is the mass of the immersed specimen (g), and ρ_w is the density of water ($\text{g} \cdot \text{cm}^{-3}$).

3. Results and Discussion

The results of d , H , n_{eff} , ρ_{dry} , ρ_{sat} , and σ_c have already been published by González et al. [31] and Saldaña et al. [32], while those for the point load index (I_s) and the N -type Schmidt hammer rebounds (R_N) are new. The results of the simple regression analysis for limestone and travertine are presented below, ending with an estimate of the scale effect of σ_c for both types of rock, in which only the Weibull models are taken into account (Equations (26) and (27)). This decision was taken because only point load index data are

available, and there is no information about the change of the trend of the curve for diameters unknown. Finally, from the estimated values of $\sigma_{c(50)}$ and $I_{s(50)}$, the correlations between both properties are made.

3.1. Intact Rock Properties

3.1.1. Limestone

A total of 19 saturated specimens of 31 mm of d and an H/d ratio of 2 were tested, whose results for uniaxial compressive strength, density, and porosity are presented in Table 4. The compressive strength test was performed with saturated specimens, although the results did not differ from those with dry specimens.

Table 4. Results of the uniaxial compressive strength, density, and porosity tests on limestone.

Specimen	d (mm)	H (mm)	n_{eff} (%)	ρ_{sat} (g·cm ⁻³)	σ_c (MPa)
Limestone 1	30.830	60.548	2.46	2.66	76.30
Limestone 2	30.853	60.335	3.15	2.62	41.37
Limestone 3	30.968	60.135	2.26	2.63	77.59
Limestone 4	30.860	60.588	1.95	2.64	106.19
Limestone 5	30.923	60.345	2.55	2.61	77.10
Limestone 6	30.858	60.718	2.46	2.65	91.43
Limestone 7	30.933	60.395	2.43	2.62	55.93
Limestone 8	30.193	60.650	2.29	2.64	73.14
Limestone 9	30.843	59.855	3.27	2.60	49.24
Limestone 10	30.853	60.518	2.95	2.61	42.15
Limestone 11	30.975	60.155	2.26	2.68	61.24
Limestone 12	30.808	60.223	2.32	2.63	94.32
Limestone 13	30.915	60.458	2.77	2.62	50.80
Mean	30.832	60.379	2.55	2.63	68.99
SD	0.199	0.241	0.384	0.02	20.77

The average value of ρ_{sat} was 2.63 g·cm⁻³, close to the 2.7 g·cm⁻³ found by Goodman [52] and within the range between 2.37 and 2.7 g·cm⁻³, collected by Lama and Vutukuri [53]. The average n_{eff} (2.55%) was considered low, according to IAEG [54], and the 68.99 MPa of σ_c was within the range of 24–290 MPa collected by Zhang [55], and that of Ramírez-Oyanguren and Alejano [56], from 50 to 200 MPa. Regardless of the range, neither the H/d ratio of the specimen nor d is specified in the case of being cylindrical, so these historical data can be collected from different geometries and degrees of weathering.

The results of the point load test are presented in Table 5, where the values of $I_{s(50)}$ were determined from Equation (27), with a k_2 of -0.3126 , obtained experimentally from Figure 2. This suggests that the model is ascending, although it cannot be confirmed whether the decrease in I_s begins in the range of 50–60 mm of d_e , due to a lack of data, as occurs with the limestone of Hawkins [44].

Figure 2a shows the $P-(d_e)^2$ curve, the one designated to determine $I_{s(50)}$, but since data that were part of the same line cannot be discarded [3,22], it was decided to use the I_s-d_e curve (Figure 2b) as a reference, obtaining values of 4.09 and 3.98 MPa, respectively. The average value of these two results (4.03 MPa) was used to determine the curves of the scale effect, $I_s/I_{s(50)}-50/d_e$ (Figure 2c), according to Equation (27), and the most common $I_s/I_{s(50)}-d_e$ (Figure 2d).

3.1.2. Travertine

Analogously to limestone, a total of 29 dry specimens of 31 mm of d and the same H/d ratio of 2 were tested, whose results for compressive strength (with the dry specimen), density, and porosity are presented in Table 6.

Table 5. Results of the point load test on limestone.

Slump	D (mm)	W (mm)	d_e (mm)	P (kN)	I_s (MPa)	$I_{s(50)}$ (MPa)
Limestone 1	18	55.38	35.6	3.598	2.83	2.55
Limestone 2	22	57.4	40.1	7.135	4.44	4.14
Limestone 3	22	64.18	42.4	5.175	2.88	2.73
Limestone 4	22	57.95	40.3	5.49	3.38	3.16
Limestone 5	30	32	35.0	5.459	4.47	3.99
Limestone 6	29	56.3	45.6	9.376	4.51	4.38
Limestone 7	30	49.75	43.6	9.459	4.98	4.77
Limestone 8	42	69.28	60.9	14.423	3.89	4.14
Limestone 9	38	67.13	57.0	15.2	4.68	4.88
Limestone 10	40	52.35	51.6	12.662	4.75	4.80
Limestone 11	39	55.9	52.7	12.845	4.63	4.70
Limestone 12	30	53.48	45.2	7.235	3.54	3.43
Limestone 13	22	48.05	36.7	6.614	4.91	4.46
Limestone 14	39	43.1	46.3	9.152	4.28	4.17
Limestone 15	42	43.15	48.0	5.776	2.50	2.47
Limestone 16	23	47.18	37.2	3.512	2.54	2.32
Limestone 17	31	39.43	39.5	3.848	2.47	2.30
Limestone 18	21	42.03	33.5	4.492	4.00	3.53
Limestone 19	27	41.65	37.8	6.514	4.55	4.17

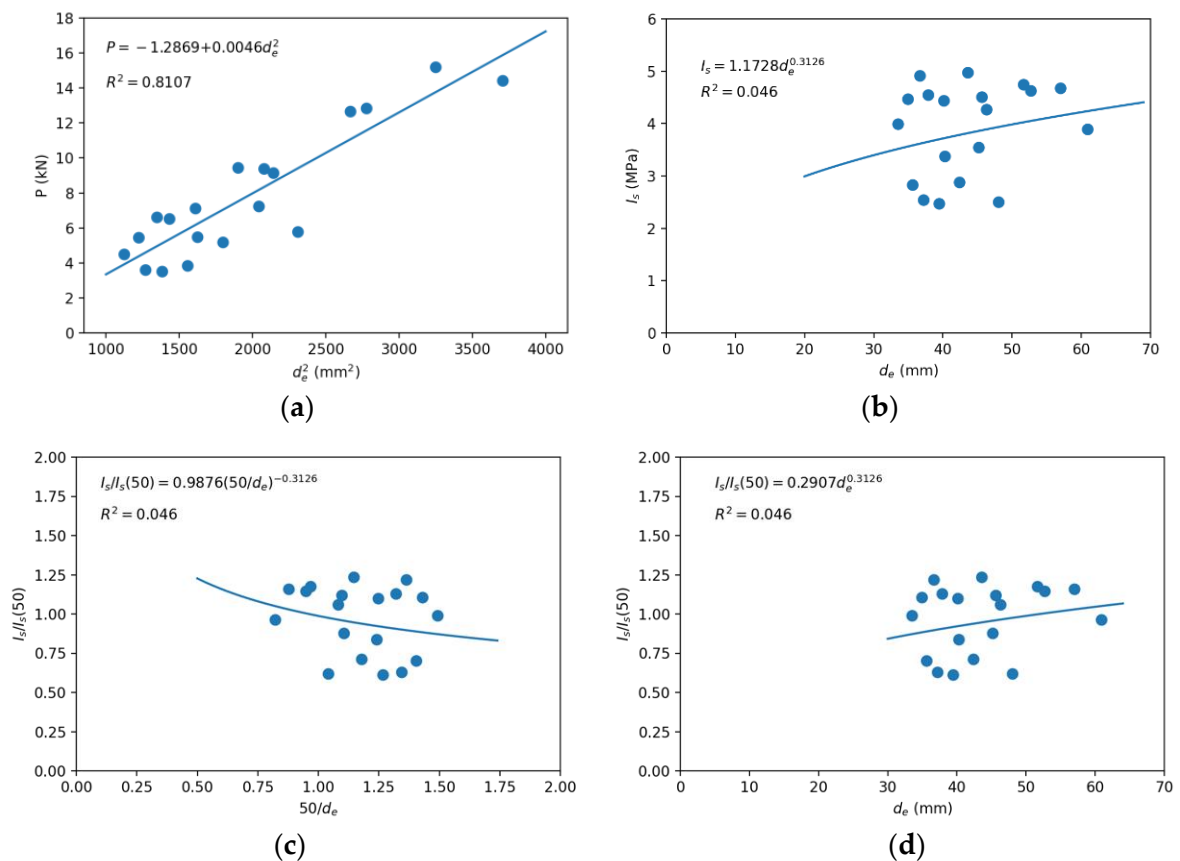
**Figure 2.** Scale effect for the point load index of the limestone: (a) quadratic fit of P – d_e and (b) potential fit of I_s – d_e , to determine the referential value of $I_{s(50)}$; (c) potential fit of $I_s/I_{s(50)}$ – $50/d_e$ and (d) $I_s/I_{s(50)}$ – d_e to determine k_2 .

Table 6. Results of the uniaxial compressive strength, density, and porosity tests on travertine.

Specimen	d (mm)	H (mm)	n_{eff} (%)	ρ_{dry} (g·cm ⁻³)	σ_c (MPa)
Travertine 1	31.090	59.470	3.65	2.48	75.28
Travertine 2	30.970	59.550	4.09	2.37	74.95
Travertine 3	31.000	60.350	4.44	2.37	97.55
Travertine 4	30.750	60.350	4.58	2.36	89.79
Travertine 5	31.050	60.160	3.58	2.38	54.17
Travertine 6	31.080	60.310	4.57	2.47	78.43
Travertine 7	31.170	59.980	4.70	2.41	93.17
Travertine 8	31.118	59.688	4.37	2.48	74.79
Travertine 9	31.070	59.780	7.19	2.41	78.84
Travertine 10	31.020	59.800	6.65	2.40	85.98
Travertine 11	31.175	61.613	4.92	2.47	63.78
Travertine 12	31.213	62.450	4.71	2.46	104.00
Travertine 13	31.200	62.900	3.83	2.45	121.98
Travertine 14	31.200	61.950	4.00	2.41	81.91
Travertine 15	31.187	62.000	4.12	2.44	94.61
Travertine 16	31.250	61.663	4.61	2.41	88.86
Travertine 17	31.163	61.938	8.01	2.34	85.65
Travertine 18	31.200	61.950	7.34	2.40	50.89
Travertine 19	31.300	62.650	3.43	2.42	98.26
Travertine 20	31.250	61.925	3.68	2.45	84.71
Travertine 21	31.150	62.450	5.42	2.37	81.05
Travertine 22	31.150	62.350	2.32	2.41	123.38
Travertine 23	31.220	62.300	4.19	2.37	90.77
Travertine 24	31.100	62.650	4.71	2.45	88.17
Travertine 25	31.200	62.250	1.05	2.49	115.430
Travertine 26	31.400	62.100	0.99	2.47	129.170
Travertine 27	31.400	62.200	0.77	2.46	115.560
Travertine 28	31.200	62.700	0.72	2.50	125.870
Travertine 29	31.250	62.500	0.70	2.43	112.870
Mean	31.128	61.259	4.05	2.43	91.72
SD	0.116	1.192	1.924	0.044	20.44

Table 6 shows an average value for ρ_{dry} of 2.43 g·cm⁻³, within the range between 2.34 and 2.51 g·cm⁻³ found by authors such as García del Cura et al. [57]. The mean value of n_{eff} was 4.05%, within the range of 2.8–34.7% obtained by Soete et al. [58] for “plugs” of 3.81 cm in diameter, which, on the one hand, is very wide due to the irregularity of the distribution of the pores in this type of rock. On the other hand, it is close to the lower limit of the range obtained by García del Cura et al. [57] for 4 × 4 × 4 cm³ cubes, whose experimental range was 5.18–24.99%. The n_{eff} value (3.47%) is considered low, according to IAEG [54]. Above 91.72 MPa of σ_c , it does not fall within a generalized range, as in the case of limestone, since travertine is not a very common rock. In any case, the value of $\sigma_{c(31)}$ is well above the range 30–60 MPa for d between 43 and 54 mm [59–61].

Regarding the point load test, the results are shown in Table 7. The values of $I_{s(50)}$ were determined from Equation (27), with a coefficient k_2 of +1.160 obtained experimentally from Figure 3. In this case, it is confirmed that the model is descending, although it is true that the minimum was 44 mm, so it cannot be ruled out that for lower values the model is ascending.

Table 8 shows the 30 corrected R_N measurements taken on the three plates 7 cm thick and more than 1 m² in surface area. The corrected results do not fall within any range, since no bibliographic reference has been found in this regard.

Table 7. Results of the point load test on travertine.

Slump	D (mm)	W (mm)	d_e (mm)	P (kN)	I_s (MPa)	$I_{s(50)}$ (MPa)
Travertine 30	46	74.5	66.1	16.6	3.80	4.15
Travertine 31	57	99	84.8	22.8	3.17	3.74
Travertine 32	36	47.5	46.7	12.2	5.60	5.48
Travertine 33	45	56	56.6	15.7	4.89	5.09
Travertine 34	41	78.1	63.9	14.1	3.46	3.73
Travertine 35	48	64.5	62.8	16.3	4.14	4.44
Travertine 36	39	101	70.8	12.4	2.47	2.76
Travertine 37	42	53.5	53.5	12.3	4.30	4.39
Travertine 38	39	59.5	54.4	16.3	5.52	5.66
Travertine 39	32.2	61.3	50.1	12.4	4.93	4.94
Travertine 40	39	67.9	58.1	15.6	4.63	4.85
Travertine 41	36	74	58.2	15.9	4.69	4.92
Travertine 42	43	70.8	62.3	13.1	3.38	3.62
Travertine 43	25	80.6	50.7	10.7	4.17	4.19

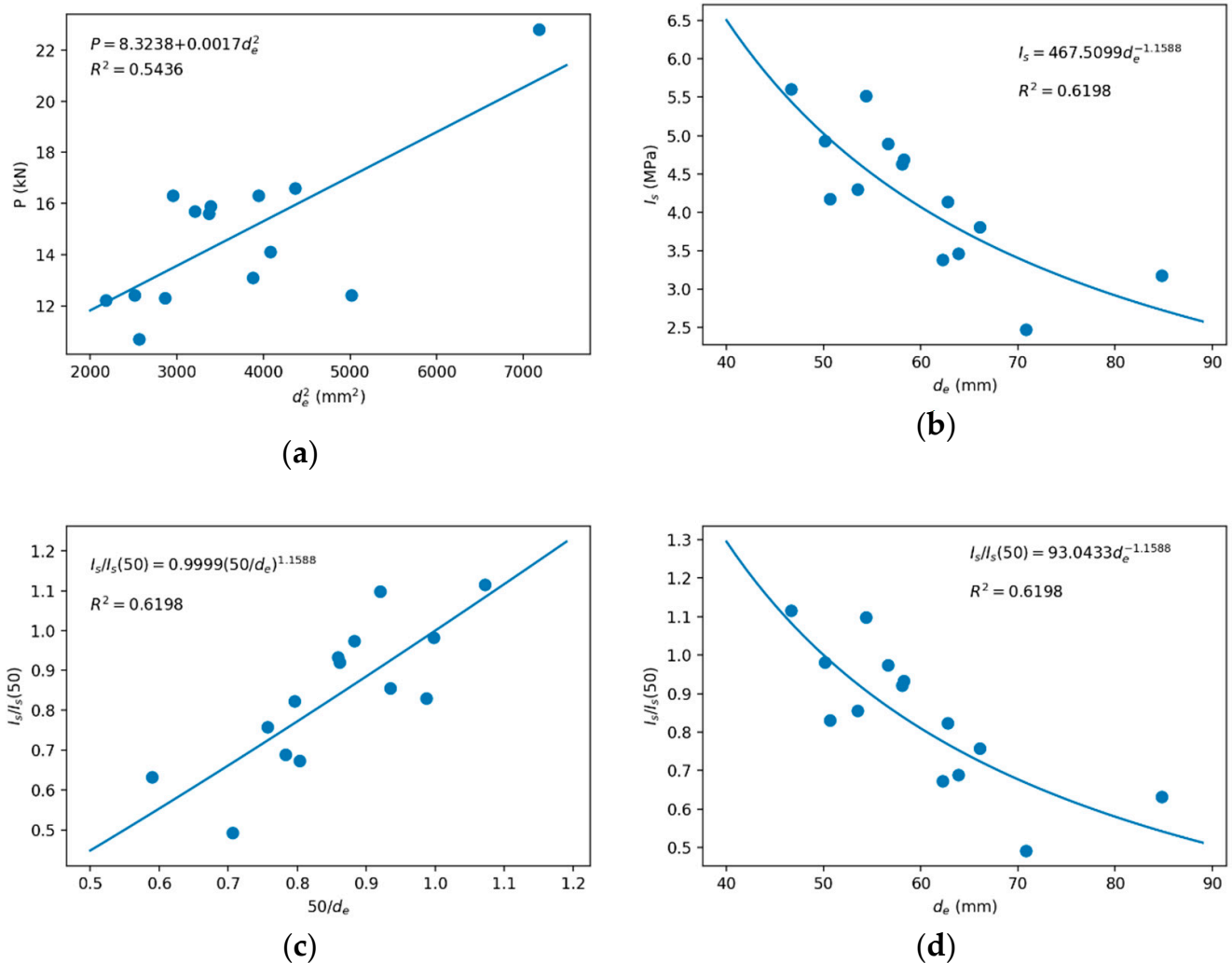
**Figure 3.** Scale effect for the point load index of the travertine: (a) quadratic fit of P – d_e and (b) potential fit of I_s – d_e , to determine the referential value of $I_{s(50)}$; (c) potential fit of $I_s/I_{s(50)}$ – $50/d_e$ and (d) $I_s/I_{s(50)}$ – d_e to determine k_2 .

Table 8. Corrected results of the Schmidt hammer test on travertine.

Sample	R_N (no.)									
	1	2	3	4	5	6	7	8	9	10
1	40	47	42	37	45	40	42	38	35	38
2	40	45	49	50	48	44	56	47	44	45
3	40	44	48	50	47	45	42	40	40	44

3.2. Correlation between Uniaxial Compressive Strength and Point Load Index without the Scale Effect

For this correlation, I_s was used instead of $I_{s(50)}$, because $I_{s(50)}$ depends on the scale effect through k_2 (Equation (27)), so it was preferred to carry out a correlation as free as possible from other statistical correlations. Likewise, σ_c comes from specimens with a d of 31 mm, so it would be appropriate to experimentally correlate $\sigma_{c(31)}$ with $I_{s(31)}$, but the practical application of $I_{s(31)}$ does not reach the level of importance and simplicity of I_s and $I_{s(50)}$.

With the linear correlation being $y = \beta_0 + \beta_1 \cdot x$, the intersection with the horizontal axis in this case was 0 ($\beta_0 = 0$). Finally, when the parameters do not come from the same source, the central limit theorem [62] is applied to the least squares fit, as is the case with $\sigma_{c(31)}$ and I_s .

3.2.1. Limestone

To correlate $\sigma_{c(31)}$ with I_s , 10 data points for each variable were randomly matched (without repetition) and a linear least squares adjustment was performed.

Taking the corresponding values from Tables 4 and 5 and carrying out 200 random samplings, the histogram in Figure 4a was obtained, which tended to follow a normal distribution with a mean value of β_1 of approximately 17 (Equation (36)). The descriptive statistics of the sampling are presented in Table 9, while the normality graph that verified that the data of the β_1 statistic were normally distributed is presented in Figure 4b.

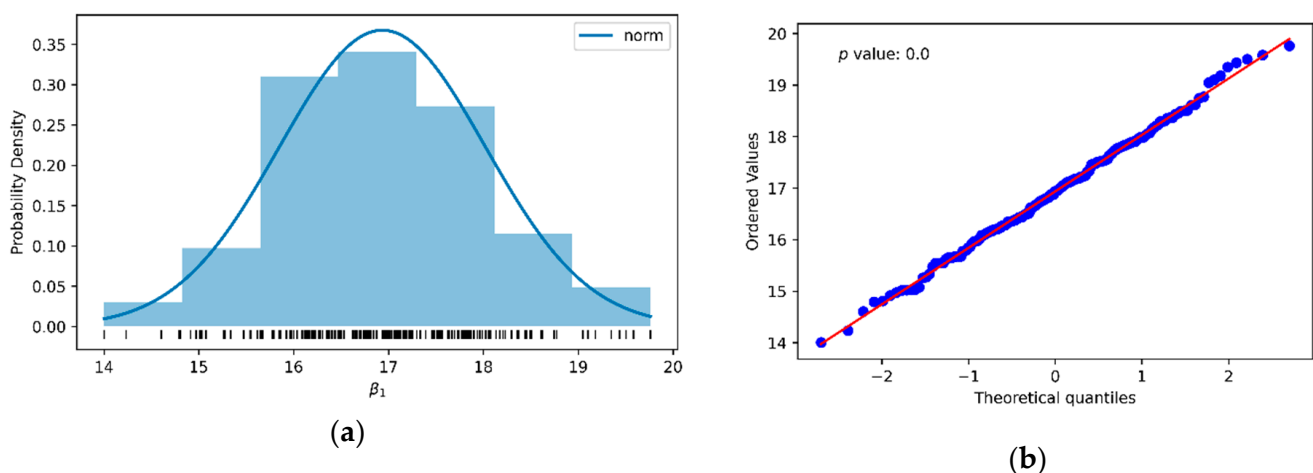


Figure 4. Histogram with distribution curve (a) and normal probability plot (b) of slope (β_1) between σ_c and I_s of the limestone, from the least squares fit of 10 pairs of data points selected from 200 random samples.

Table 9. Results of the normal distribution of the linear fit for the σ_c and I_s pairs of the limestone.

Parameter	N	Mean	Standard Deviation	Maximum	Minimum
β_1	200	16.93	1.09	19.76	14.00

The relative error between $\sigma_{c(31)}$ from direct and indirect methods was 3.7%. Furthermore, ASTM [22] suggests that $\beta_1 = 19$ to estimate $\sigma_{c(31)}$ from I_s , and the error was -7.6% .

$$\sigma_{c(31)} \approx 17 I_s \quad (36)$$

3.2.2. Travertine

As in the previous case, the sampling process was replicated from the data in Tables 3 and 4, obtaining the data distribution shown in the histogram in Figure 5a, with a mean of β_1 of approximately 21 (Equation (37)). The descriptive statistics of the sampling are presented in Table 10, while the normal probability graph shown in Figure 5b corroborated the normality of said statistic (β_1).

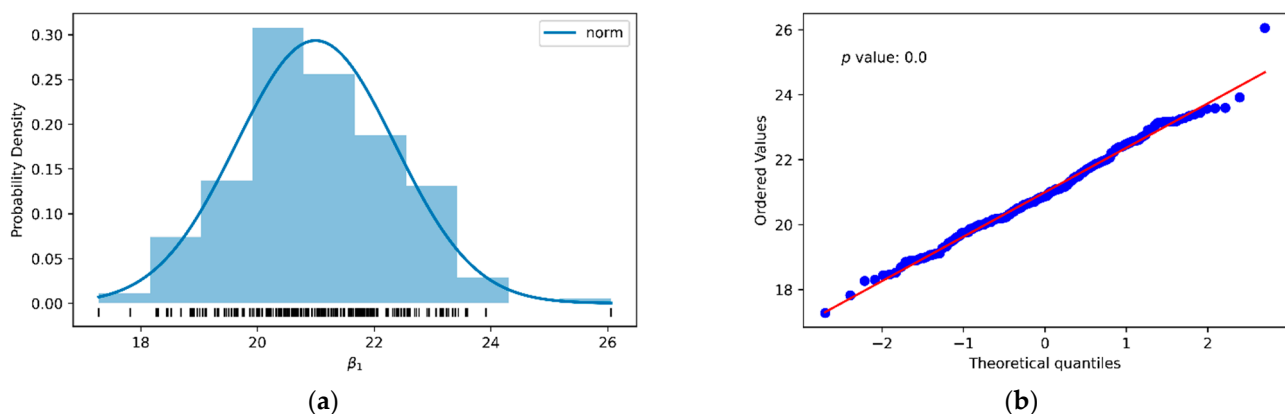


Figure 5. Histogram with distribution curve (a) and normal probability plot (b) of slope (β_1) between σ_c and I_s of the travertine, from the least squares fit of 10 pairs of data points selected from 200 random samples.

Table 10. Results of the normal distribution of the linear fit for the σ_c and I_s pairs of the travertine.

Parameter	N	Mean	Standard Deviation	Maximum	Minimum
β_1	200	21.00	1.36	26.06	17.28

Finally, Equation (37) reported a relative error of 12.5%, 91.72 MPa being the experimental $\sigma_{c(31)}$; if 19 [22] is used instead of 21, the error decreases to 3.3%.

$$\sigma_{c(31)} \approx 21 I_s \quad (37)$$

3.3. Scale Effect

3.3.1. Limestone

Based on the correlations presented in Table 1 regarding the point load index and σ_c , the scale effect curve σ_c-d was estimated for the limestone. The results of σ_c from all correlations are shown in Figure 6 as colored squares, while the experimental $\sigma_{c(31)}$ is represented by a red triangle. It can be clearly observed that the estimated values of σ_c generate an ascending model, like in Figure 2d ($k_2 = -0.3126$). Likewise, values that were outside the trend for d less than 50 mm were discarded, since in Figure 2d there are up to 60.9 mm included in the ascending model.

After discarding the lowest values of d in the range of 54–57 mm (Equations (1), (11), (13), (15) and (16)), we had Figure 7, whose adjustment by least squares gives the k_1 changed sign. Since k_1 is known, it allowed us to calculate the $\sigma_{c(50)}$ of the scale effect corresponding to Equation (27). The values of k_1 and $\sigma_{c(50)}$ were -0.2545 and 88.52 MPa, respectively.

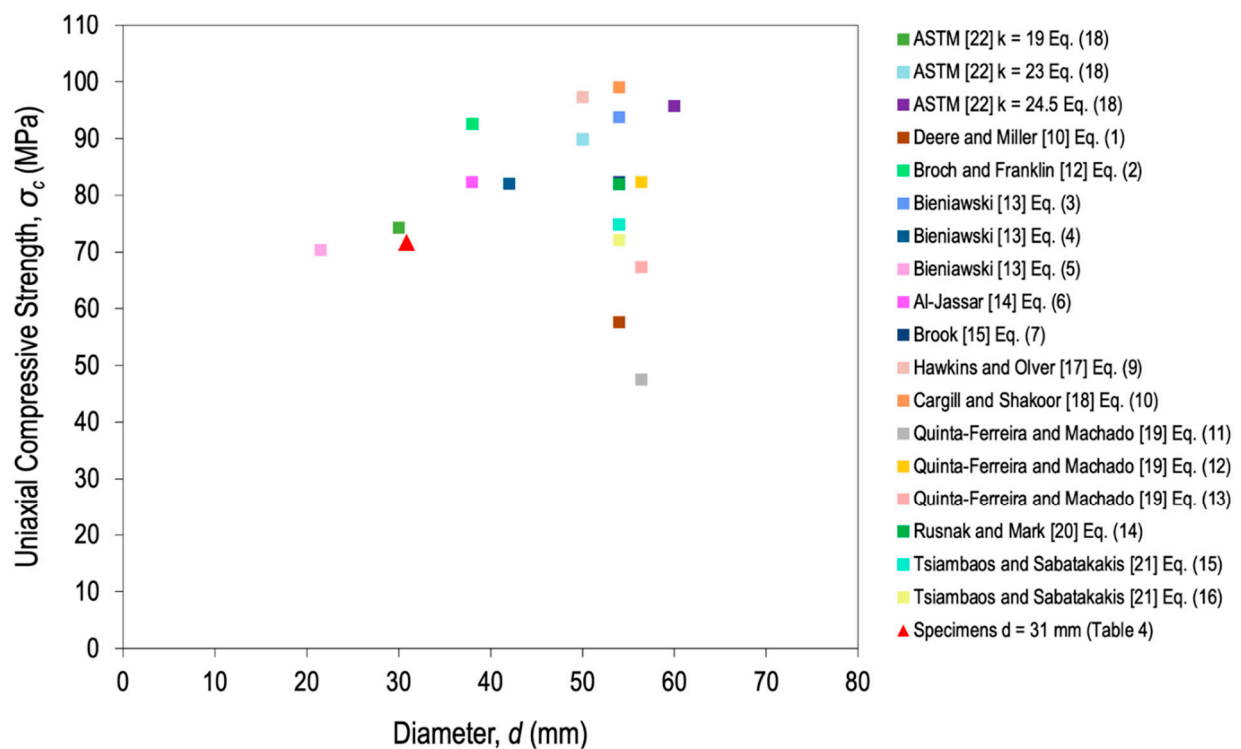


Figure 6. Estimation of σ_c from the correlations with I_s , $I_{s(50)}$, and $I_{s(50)}$ of Table 1 for limestone.

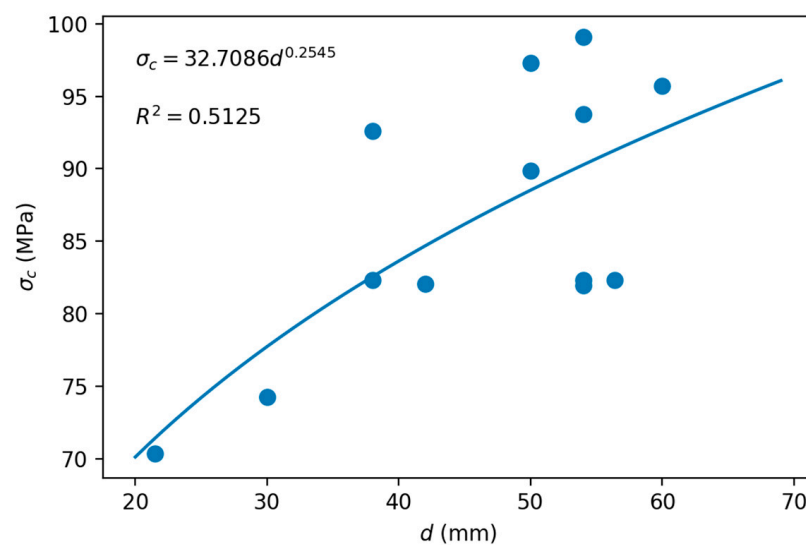


Figure 7. Least squares adjustment to estimate the $\sigma_{c(50)}$ of the scale effect of the limestone.

Based on the above, the scale effect curve is represented in Figure 8 and the ascending model is given by the following equation:

$$\frac{\sigma_c}{\sigma_{c(50)}} = \left(\frac{50}{d} \right)^{-0.2545}, \quad (38)$$

where the value of k_1 (-0.2545) does not correspond to the 0.18 proposed by Hoek and Brown [33,34], neither in sign nor in magnitude, since, as already indicated, the model of this curve was ascending (Figure 7) and not descending.

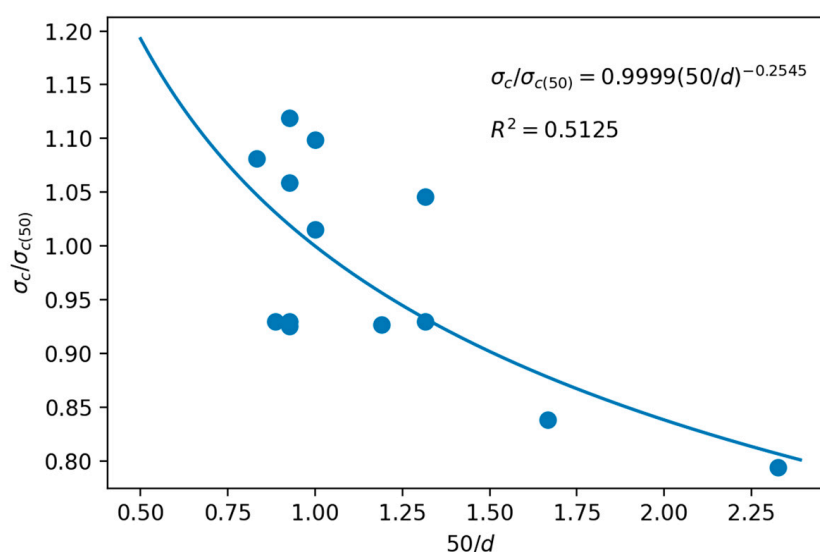


Figure 8. Scale effect curve from the estimated σ_c of the limestone.

Another issue is that the value of k_1 did not correspond to that of k_2 (-0.3126) obtained in the point load test. In this sense, neither Brook [16] nor Wang et al. [26] indicates that the two coefficients must be equal, but $\sigma_{c(50)}$ was estimated by multiplying I_s or $I_{s(50)}$ by a constant, so the exponent would not be modified. This suggests that, for the ascending model (Figure 7), the experimental k_1 value would be in a range close to the values of -0.3126 and -0.2545 .

3.3.2. Travertine

As with the limestone, the composition of travertine is calcium carbonate and, despite the fact that the structure is not the same (mainly due to the distribution of pores and, secondly, to the mineralogical composition being calcite and aragonite), the correlations presented in Table 1 were applied. The results obtained are plotted in Figure 9, in which it can be seen that the main trend of the curve was an ascending model again, but according to the scale effect of the point load index (Figure 3d), the model should be descending.

In accordance with the above, Figure 9 shows the area of the results of two investigations in which only travertine was used, those of Jamshidi et al. [60] and Ebdali et al. [61]. In them, σ_c intervals of 30–60 and 20–50 MPa were recorded, respectively, for a d of 54 mm in both cases. Despite the fact that the model must be descending, according to the experimental results of σ_c from the point load test, added to the results of Jamshidi et al. [60] and Ebdali et al. [61], these results support the theory of the descending model and not the ascending one. It has to be taken into account that limestone appeared in all the correlations but not for travertine.

In this sense, the curve of the scale effect of travertine (Figure 3d) began with a d_c of 46.7 mm, so it could not be ruled out with certainty that for lower values the model was ascending and it turned into a mixed (ascending-and-descending) model. Based on this, two curves would be estimated, but due to a lack of data and the dispersion of the existing ones, it was decided that it was too complex (due to uncertainty) and the ascending model would be estimated directly.

To try to shed more light on the trend of the curve, Figure 10 shows the results of σ_c estimated from the correlations with R_L and R_N in Table 5. The plotted points (colored squares) did not provide a clear trend (ascending or descending) if the experimental $\sigma_{c(31)}$ (red triangle) was taken into account as the reference value (91.72 MPa from Table 4). However, if it is supposed that the model must be descending, Equations (20) and (24)—proposed by Deere and Miller [10] and Wang and Wan [25], respectively—would provide reliability to the estimated curve of the scale effect.

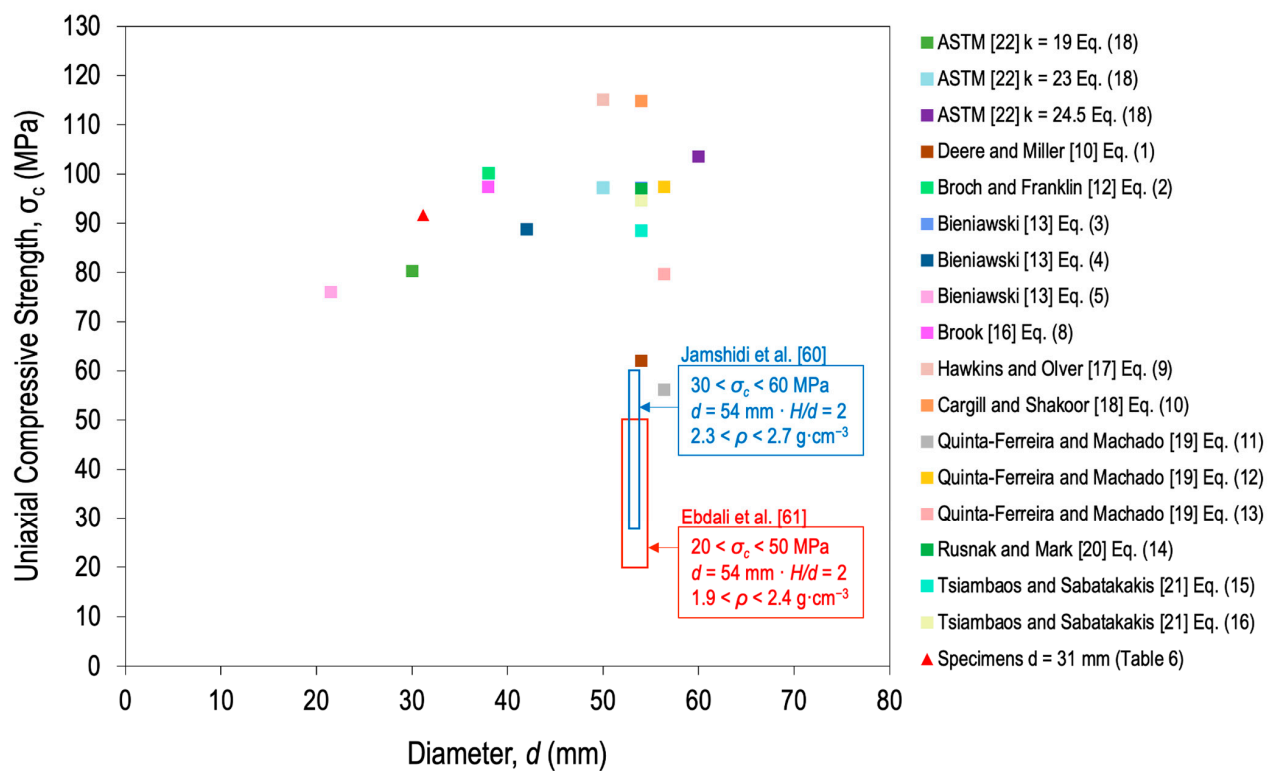


Figure 9. Estimation of σ_c from the correlations with I_s , $I_{s(50)}$, and $I_{s(50)}$ of Table 1 for travertine.

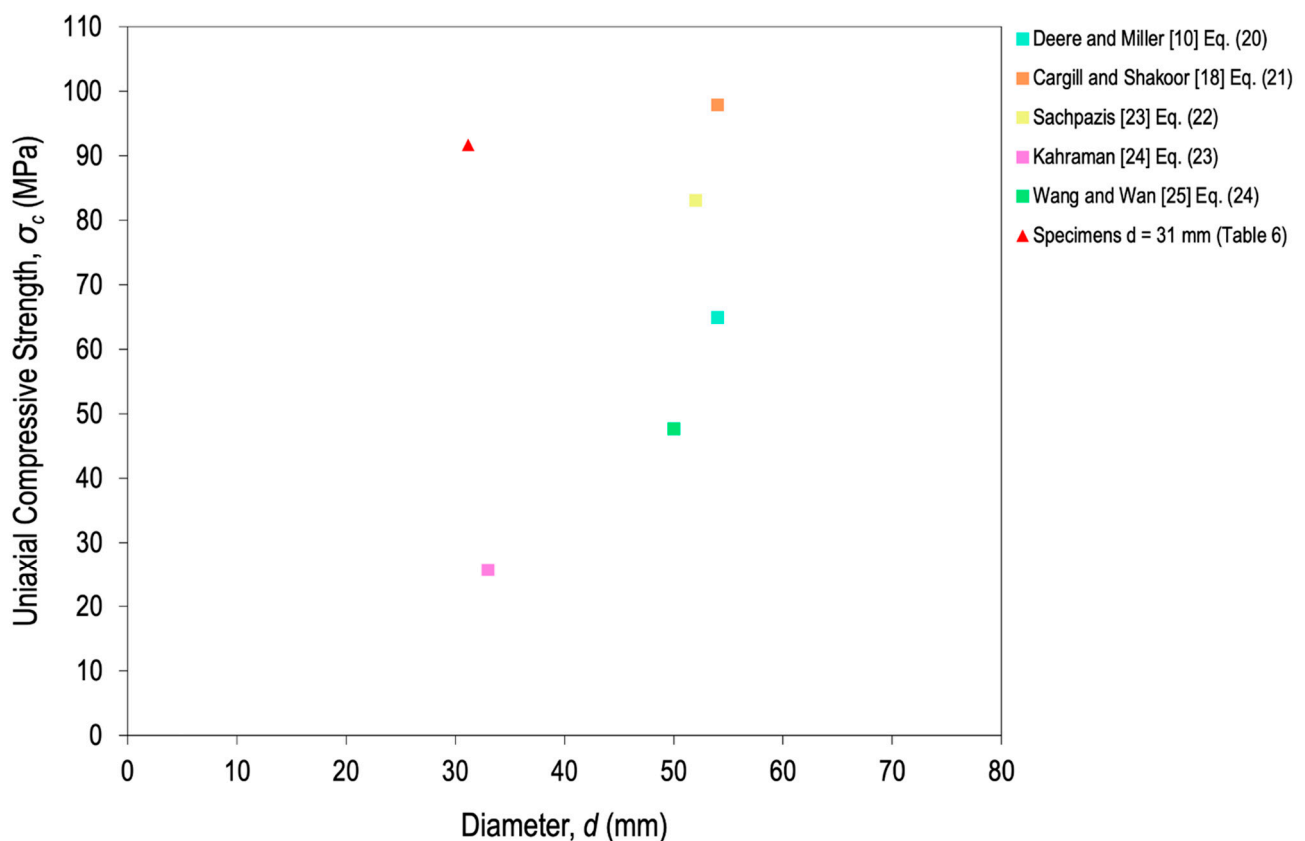


Figure 10. Estimation of σ_c from the correlations with R_N and R_L in Table 2 for travertine.

In Figure 11, all the pairs of points from Figures 9 and 10 were plotted, discarding those that are outside of the downward trend. The discarded values corresponded to the

correlations for the highest values of d , and are in the range of 54–60 mm. Therefore, the estimates of Equations (1), (2), (4), (6), (11), (19) and (21) were used, together with the $\sigma_{c(31)}$ value (91.72 MPa). After discarding values as necessary, a potential least squares adjustment was carried out, which yielded values of k_1 and $\sigma_{c(50)}$ of +0.858 and 71.70 MPa, respectively.

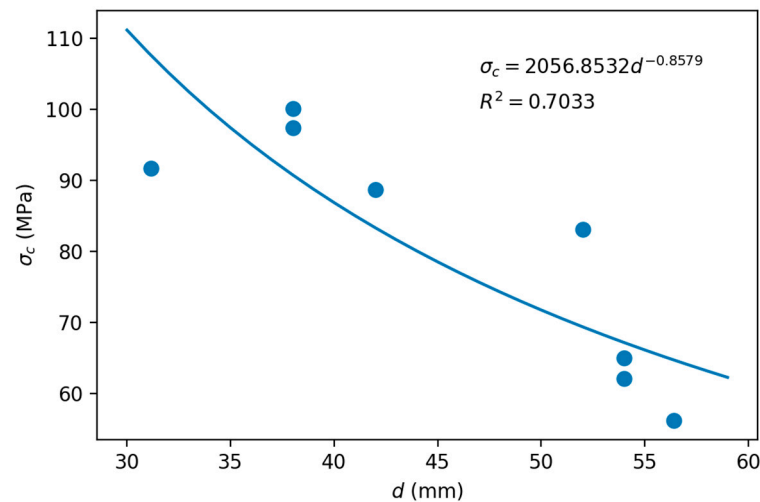


Figure 11. Least squares adjustment to estimate the $\sigma_{c(50)}$ of the travertine scale effect.

Finally, the scale effect curve, represented in Figure 12, is given by the following equation:

$$\frac{\sigma_c}{\sigma_{c(50)}} = \left(\frac{50}{d} \right)^{0.8579}, \quad (39)$$

where the value of k_1 (0.8579) differs from the 0.18 proposed by Hoek and Brown [33,34], and from the value of k_2 of +1.160 obtained in the point load test. As discussed in the case of limestone, Brook [16] and Wang et al. [26] do not indicate that the two coefficients should be equal, but it is assumed that the two coefficients should be similar. To support this assertion, it could be thought that the experimental k_1 value would have more weight due to its relatively low dispersion ($R^2 = 0.6198$), but in the absence of more data the experimental interval was between 0.8579 and 1.160.

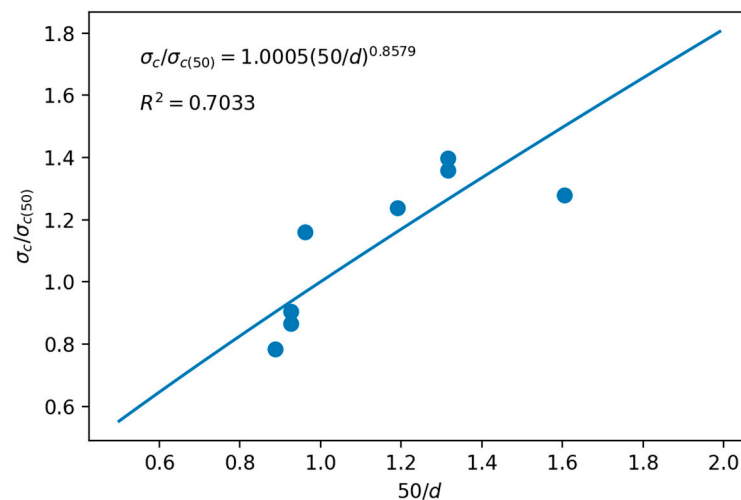


Figure 12. Scale effect curve for the σ_c of travertine.

3.4. Correlation between Uniaxial Compressive Strength and Point Load Index

3.4.1. Limestone

Finally, and because it is one of the reference equations in the point load test suggested by ISRM [3], the correlation between $\sigma_{c(50)}$ and $I_{s(50)}$ is determined from Equations (26) and (27) with k_1 of -0.2545 and k_2 of -0.3126 . Using the results of Tables 4 and 5 to determine both parameters, we proceeded to randomly combine 10 data pairs from each one of them to carry out a linear adjustment by ordinary least squares [62]. After 200 repetitions (Figure 13), the central limit theorem (Table 11) was applied to determine Equation (37). As can be seen, β_1 (20.6) was just above the lower limit of the 20–25 range suggested by the ISRM (Equation (17)), generating an error of -5.2 MPa (-6.8%) regarding Equation (8) by Brook [16], one of the most frequently used methods to estimate $\sigma_{c(50)}$ from $I_{s(50)}$.

$$\sigma_{c(50)} \approx 20,6 I_{s(50)} \quad (40)$$

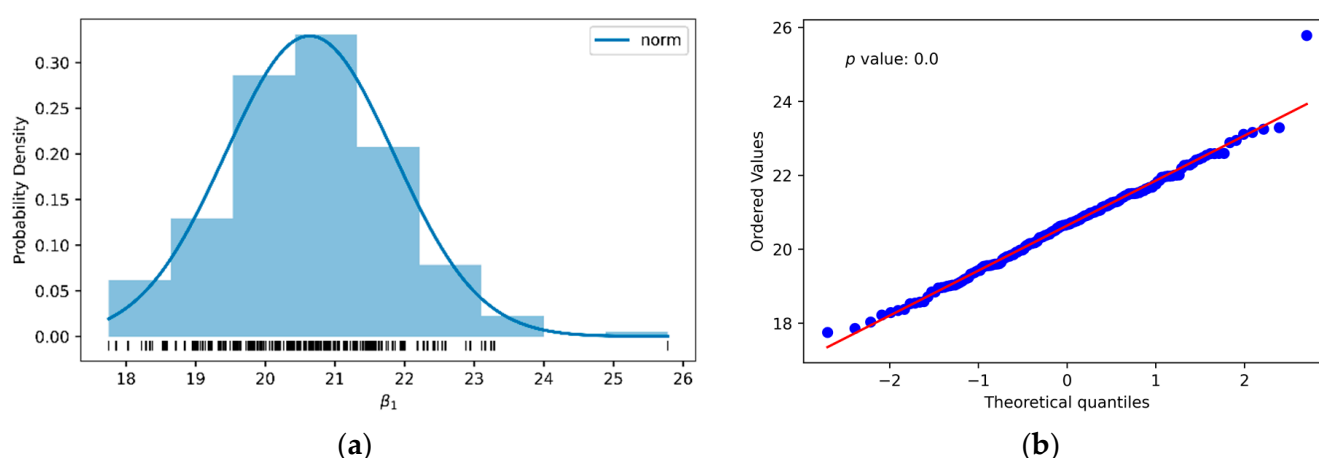


Figure 13. Histogram (a) and normal distribution graph (b) of the slope β_1 of the linear fit by ordinary least squares of 10 data pairs, of $\sigma_{c(50)}$ and $I_{s(50)}$ of the limestone, randomly selected from 200 random samples.

Table 11. Results of the normal distribution of the linear fit for the σ_c and $I_{s(50)}$ pairs of the limestone.

Parameter	N	Mean	Standard Deviation	Maximum	Minimum
β_1	200	20.64	1.21	25.78	17.75

3.4.2. Travertine

Finally, as with limestone, the travertine correlation between $\sigma_{c(50)}$ and $I_{s(50)}$ was determined due to its impact on the point load test, since the ISRM [3] indicates that β_1 was between 20 and 25 for rocks without apparent anisotropy. Then, from Equations (26) and (27), with k_1 of $+0.858$ and k_2 of $+1.160$ and the results of Tables 6 and 7, we proceeded to combine 10 pairs of random data from the two variables and to perform the linear least squares fit. After 200 repetitions (Figure 14) and applying the central limit theorem (Table 12), Equation (41) was determined. As can be seen in Table 12, β_1 (13, 4) was less than 20, the lower range suggested by the ISRM (Equation (17)), with the error with respect to Equation (8) of Brook [16] being -38.1 MPa (-64.2%).

$$\sigma_{c(50)} = 13,4 I_{s(50)} \quad (41)$$

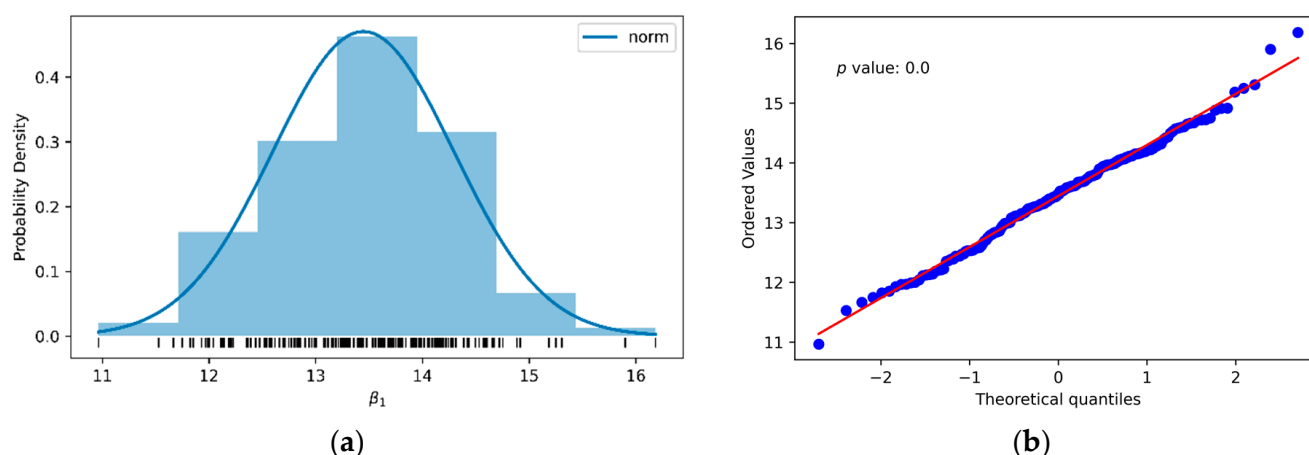


Figure 14. Histogram (a) and normal distribution graph (b) of the slope β_1 of the linear fit by ordinary least squares of 10 data pairs, of $\sigma_{c(50)}$ and $I_{s(50)}$ of the travertine, randomly selected from 200 random samples.

Table 12. Results of the normal distribution of the linear fit for the σ_c and I_s pairs of the travertine.

Parameter	N	Mean	Standard Deviation	Maximum	Minimum
β_1	200	13.445	0.85	16.18	10.96

4. Conclusions

After having carried out the tests of uniaxial compressive strength for 31-mm specimens, and point load with irregular samples of saturated limestone and dry travertine, the corresponding values of I_s and $I_{s(50)}$ were obtained to generate the scale effect models and correlate these parameters with $\sigma_{c(31)}$ and $\sigma_{c(50)}$, respectively.

For the correlations between $\sigma_{c(31)}$ and I_s , factors (β_1) of 17 and 21 were obtained for limestone and travertine, respectively, close to the 19 proposed by Bieniawski [13] for 30 mm (Table 3). The associated errors for both rocks are less than 3 MPa (3.5%) on average, which makes it reliable to estimate the $\sigma_{c(31)}$.

In the case of limestone, the scale effect turned out to have an ascending model (greater resistance at a larger diameter); however, due to the range of the diameter (33.5–66.9 mm), it was not possible to determine whether around 67 mm the model could change to a descending model. This uncertainty is based on the σ_c results obtained by Hawkins [44] for limestone (ascending-and-descending model), and on the scale effect graph plotted from the estimated values of bibliographic σ_c (Figure 6), which may change the trend to a descending model over 54 mm of diameter. Noting the dispersion of the results of I_s ($R^2 = 0.046$), it was decided to consider only the ascending model, obtaining coefficients k_1 and k_2 of -0.2545 and -0.3126 , respectively. The difference between both coefficients (0.06) generated a margin of error of 19% with respect to k_2 , the only experimental value used as a reference.

Regarding travertine, the experimental results of I_s determined a descending model for the scale effect, with a much lower dispersion than in the case of limestone ($R^2 = 0.6198$) at a range of 46.7–84.8 mm for the diameter. On the other hand, the trend of the curve of the scale effect from the bibliographic estimates of σ_c (Figure 9) was clearly upward. This reason was attributed to the fact that the bibliographic correlations were directly related to limestone and not to travertine, since, despite being practically 100% CaCO_3 , the crystalline structure was not the same (calcite versus calcite and aragonite), nor was the distribution of internal pores. Likewise, the test with the Schmidt hammer was carried out to estimate more values of σ_c and help with the downward trend, with Equations (20) and (24)—proposed by Deere and Miller [10] and Wang and Wan [25], respectively—that were close to the values published by Jamshidi et al. [60] and Ebdali et al. [61] for 54 mm, around 40 MPa.

Although a descending model was considered in its entirety, it could not be ruled out that, for values of diameters lower than 31 mm (the experimental reference value in Figure 9: $\sigma_{c(31)}$), the model was descending, having a mixed (ascending-and-descending) model with a d_i around 31 mm. For the descending model, which included the estimates of σ_c (from I_s , $I_{s(50)}$, and R_N) that favored this trend, coefficients k_1 and k_2 of +0.858 and +1.160, respectively, were obtained whose difference of 0.30 generates an error of 25.9% with respect to k_2 , the only experimental value.

Finally, $\sigma_{c(50)}$ and $I_{s(50)}$ were correlated for limestone and travertine, resulting in a factor (β_1) of 20.6 for the former, within the range of 20–25 suggested by the ISRM [3] for isotropic rocks, while for travertine, it was 13.4, below the lower limit. If it is taken into account that the value of β_1 generally used is 22 [16], it is only useful for limestone, since the average absolute error of −5 MPa (−6.8%) is acceptable, but not the −38.1 MPa (−64.2%) of travertine.

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