

Improved Calculation of Load and Resistance Factors Based on Third-Moment Method

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Abstract: Load and resistance factor design (LRFD) is widely used in building codes for reliability design. In the calculation of load and resistance factors, the third-moment method (3M) has been proposed to overcome the shortcomings (e.g., inevitable iterative computation, requirement of probability density functions (PDFs) of random variables) of other methods. With the existing 3M method, the iterative is simplified to one computation, and the PDFs of random variables are not required. In this paper, the computation of load and resistance factors is further simplified to no iterations. Furthermore, the accuracy of the proposed method is proved to be higher than the existing 3M methods. Additionally, with the proposed method, the limitations regarding applicable range in the existing 3M methods are avoided. With several examples, the comparison of the existing 3M method, the ASCE method, the Mori method, and the proposed method is given. The results show that the proposed method is accurate, simple, safe, and saves material.

Keywords: load and resistance factors; third-moment method; reliability; iteration computation

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1. Introduction

Load and resistance factor design (LRFD), as a rational probabilistic approach based on reliability to manage uncertainties in the building process, is widely employed in structural engineering [1–7]. In the past three decades, several methods for determining load and resistance factors have been developed, of which the first order reliability method (FORM) [8] was frequently used to account for stochastic fracture problems [9–11]. FORM is suggested to observe reliability accurately and generally. However, the design point must be firstly determined, and derivative-based iterations have to be used. Furthermore, the probability density functions (PDFs) of random variables are required. Thus, reliability calculated based on FORM is inefficient.

In order to improve the computational efficiency and simultaneously guarantee accurate results, several simplified methods were proposed. According to the Mori method [12], all random variables are assumed to have known PDFs and are transferred into lognormal random variables. The results calculated by applying the Mori method were proved to be highly accurate. In the ASCE method [13], the load factors are well approximated by a simple function, which is valid for a broad range of common probability distributions used to model structural loads. Moment-based methods, being sufficiently accurate and efficient, have been widely used to assess the reliability analytically [14–18]. For example, the existing third-moment (3M) method [19] shows no need for the design point or any assumption of the PDFs of random variables. In this method, the iterative

computation of the target mean resistance is simplified to one computation. Then, the load and resistance factors can be easily calculated.

In this paper, the applications of the existing methods (including the ASCE method, the existing 3M method, and the Mori method) are inspected, and a simpler method is proposed. Compared with the existing 3M method [19], in which iterative-based computation is still inevitable, this paper provides a simplified method for the computation of target mean resistance without iteration, by improving the accuracy of computing the mean target resistance. Besides, the comparison of the ASCE method, the existing 3M method, and the proposed method is obtained by employing the load combination and example in the ASCE Standard 7–10 (2.3.2) [13]. The result shows that the ASCE method is easy and safe, but a waste of material.

Generally, the difficulties with the existing methods can be summarized as: (1) for the existing 3M method, iterative-based computation is required. (2) There is a mathematical limitation for the existing 3M method, and its calculation formula is complicated. (3) For the ASCE method, the accuracy of calculation needs to be improved. (4) For the Mori method, the PDFs of some random variables are not always provided, resulting in calculation difficulties. Additionally, the random variables of the Mori method are required to be independent and obey the lognormal distribution.

In light of the aforementioned analysis, it is necessary to develop a simple and accurate method. Therefore, in this paper, a new method for the calculation of load and resistance factors is proposed. The contribution of this paper is mainly reflected in the following points: (1) the calculation processes of three existing methods are summarized. (2) A new model is proposed to avoid the mathematical limitation and to overcome the shortcoming of iterative computation. (3) The feasibility of calculating target mean resistance without iteration of the proposed method is investigated. (4) Based on an example (ASCE 7–10, C2.3.6), the accuracy of the proposed method is investigated. (5) Nine cases with different distribution types are applied to verify the precision of the proposed method.

2. Computation Procedure of Load and Resistance Factors

2.1. Determination of Load and Resistance Factors

Based on the LRFD format, the performance function in structural design can be expressed as:

$$\phi R_n \geq \sum \gamma_i Q_{ni} \quad (1)$$

where ϕ is the resistance factor, γ_i is the partial load factor applied to load, R_n is the nominal value of the resistance, and Q_{ni} is the nominal value of the load.

In reliability-based design, the load and resistance factors ϕ and γ_i should be determined with a specified reliability, namely target reliability. Therefore, the computation of Equation (1) should be achieved by a performance function, as presented below:

$$G(X) = R - \sum Q_i \quad (2)$$

where R and Q_i are random variables representing uncertainty in the resistance and load effects, respectively.

For a given target reliability, β_T , or target failure probability, P_{fT} , Equation (2) can be expressed as:

$$\beta \geq \beta_T \text{ or } P_f \leq P_{fT} \quad (3)$$

where β and P_f are the reliability and the failure probability, respectively.

If R and Q_i are mutually independent normal random variables, the second-moment (2M) method is correct and the design formula is expressed as:

$$\beta_{2M} \geq \beta_T \quad (4)$$

$$\beta_{2M} = \frac{\mu_Z}{\sigma_Z} \quad (5a)$$

$$\mu_Z = \mu_R - \sum \mu_{Q_i}, \sigma_Z = \sqrt{\sigma_R^2 + \sum \sigma_{Q_i}^2} \quad (5b)$$

where β_{2M} is the 2M reliability index; μ_Z and σ_Z are the mean value and standard deviation of the performance function $G(X)$, respectively; μ_R and σ_R are the mean value and standard deviation of R , respectively; μ_{Q_i} and σ_{Q_i} are the mean value and standard deviation of Q_i , respectively.

Substituting Equation (5a,b) in Equation (4), the load and resistance factors can be expressed as:

$$\mu_R (1 - \alpha_R V_R \beta_T) \geq \sum \mu_{Q_i} (1 + \alpha_{Q_i} V_{Q_i} \beta_T) \quad (6)$$

Comparing Equation (6) with Equation (1), the load and resistance factors can be expressed as:

$$\phi = (1 - \alpha_R V_R \beta_T) \frac{\mu_R}{R_n} \quad (7a)$$

$$\gamma_i = (1 + \alpha_{Q_i} V_{Q_i} \beta_T) \frac{\mu_{Q_i}}{Q_{ni}} \quad (7b)$$

where V_R and V_{Q_i} are the coefficient of variation for R and Q_i , respectively; α_R and α_{Q_i} are the sensitivity coefficients of R and Q_i , respectively, where:

$$\alpha_{Q_i} = \frac{\sigma_{Q_i}}{\sigma_Z}, \alpha_R = \frac{\sigma_R}{\sigma_Z} \quad (8)$$

As introduced above, the 2M method is based on the assumption that all of the variables obey normal distribution and are independent of each other. When R and Q_i are other random variables, the 2M reliability computed by Equation (5a) is inaccurate. Therefore, other methods were proposed. Typically, the FORM is widely implemented, in which the load and resistance factors can be obtained as:

$$\phi = \frac{R^*}{R_n}, \gamma_i = \frac{Q_i^*}{Q_{ni}} \quad (9)$$

where R^* and Q_i^* are the values of the variables R and Q_i , respectively, at the design point of the FORM.

2.2. Existing 3M Method for the Computation of Load and Resistance Factors

R^* and Q_i^* are obtained using derivative-based iterations, but explicit expressions of R^* and Q_i^* are not available. Some simplifications have been provided to simplify iterative computations. In the existing 3M method based on 3P-lognormal distribution [19], the two-step recursive optimization is used to avoid iterative computation. The process of calculation is presented as follows.

In the existing paper, the third-moment (3M) reliability index is employed as the reliability index for the performance function of Equation (3). Substituting the 3M reliability index in the design format obtains:

$$\beta_{3M} \geq \beta_T \quad (10)$$

where the 3M reliability index, β_{3M} , is expressed as [19]:

$$\beta_{3M} = -\frac{\alpha_{3Z}}{6} - \frac{3}{\alpha_{3Z}} \ln(1 - \frac{1}{3} \alpha_{3Z} \beta_{2M}) \quad (11)$$

$$\alpha_{3Z} = \frac{1}{\sigma_Z^3} (\alpha_{3R} \sigma_R^3 - \sum \alpha_{3i} \sigma_{Q_i}^3) \quad (12)$$

where α_{3Z} , α_{3R} , and α_{3S_i} are the skewness of $G(X)$, R , and S_i , respectively.

The skewness of random variable can be calculated by:

$$\alpha_3 = \frac{E(X - \mu_x)^3}{\sigma_x^3} \quad (13)$$

where μ_x and σ_x are the mean value and standard deviation of random variable X , respectively.

Substituting Equation (11) into Equation (10), one obtains:

$$\beta_{2M} \geq \frac{3}{\alpha_{3Z}} \left\{ 1 - \exp \left[\frac{\alpha_{3Z}}{3} \left(-\beta_T - \frac{\alpha_{3Z}}{6} \right) \right] \right\} \quad (14)$$

Denoting the right-hand side of Equation (14) as β_{2T} , one obtains:

$$\beta_{2M} \geq \beta_{2T} \quad (15)$$

$$\beta_{2T} = \frac{3}{\alpha_{3Z}} \left\{ 1 - \exp \left[\frac{\alpha_{3Z}}{3} \left(-\beta_T - \frac{\alpha_{3Z}}{6} \right) \right] \right\} \quad (16)$$

Equation (15) is equivalent to Equation (10). It indicates that if β_{2M} is at least equal to β_{2T} , β_{3M} will be at least equal to β_T , and the reliability-based design conditions will be satisfied. Hence, β_{2T} and β_T can be regarded as the target values of β_{2M} and β_{3M} , respectively.

Since Equation (15) is equivalent to Equation (10), except that the right-hand side is β_{2T} , the load and resistance factors can be easily obtained by substituting β_T in Equation (6) with β_{2T} . The design formula becomes:

$$\mu_R (1 - \alpha_R V_R \beta_{2T}) \geq \sum \mu_{Q_i} (1 + \alpha_{Q_i} V_{Q_i} \beta_{2T}) \quad (17)$$

and the load and resistance factors are obtained as:

$$\phi = (1 - \alpha_R V_R \beta_{2T}) \frac{\mu_R}{R_n} \quad (18a)$$

$$\gamma_i = (1 + \alpha_{Q_i} V_{Q_i} \beta_{2T}) \frac{\mu_{Q_i}}{Q_{ni}} \quad (18b)$$

At the limit state, according to Equation (5a,b), one obtains:

$$\mu_{RT} = \sum \mu_{Q_i} + \beta_{2T} \sigma_Z \quad (19)$$

The initial value, μ_{R_0} , is expressed as [19]:

$$\mu_{R_0} = \sum \mu_{Q_i} + \sqrt{\beta_T^{3.5} \sum \sigma_{Q_i}^2} \quad (20)$$

Based on the discussion above, the target mean resistance calculated using two-step iteration is expressed as:

$$\mu_{RT} = \sum \mu_{Q_i} + \beta_{2T_0} \sigma_{Z_0} \quad (21)$$

where μ_{RT} is the target mean resistance; β_{2T_0} is the original target 2M reliability; σ_{Z_0} is the original standard deviation of $G(X)$; σ_{Z_0} and β_{2T_0} are obtained using Equations (5a,b) and (16).

The steps for determining the load and resistance factors using this method are shown below:

Calculate μ_{R_0} using Equation (20).

Calculate σ_{Z_0} , α_{3Z_0} , and β_{2T_0} using Equations (5a,b), (12), and (16), respectively.

Calculate μ_{RT} with Equation (21).

Repeat step b with μ_{rt} to determine the values of σ_z , α_{3Z} , and β_{2T} , which are used to calculate α_R and α_{Qi} with Equation (8).

Determine the load and resistance factors with Equation (18a,b).

The shortcomings of steps a–e is that the iterative calculation of σ_z , α_{3Z} , and β_{2T} is inevitable. In addition, Equation (16) is complicated. When Equation (11) is used for the calculation of 3M reliability, there is a mathematical limitation, presented as:

$$1 - \frac{1}{3}\alpha_{3Z}\beta_{2M} > 0 \quad (22)$$

where:

$$\alpha_{3Z} < \frac{3}{\beta_{2M}} \quad (23)$$

2.3. ASCE Method for the Computation of Load and Resistance Factors

In the ASCE method, load factors are well approximated by:

$$\gamma_Q = (\mu_Q / Q_n)(1 + \alpha_Q \beta V_Q) \quad (24)$$

where β is the reliability index; α_Q is a sensitivity coefficient that is approximately equal to 0.8 when Q is a principal action and 0.4 when Q is a companion action. This approximation is valid for a broad range of common probability distributions used to model structural loads. The load factor is an increasing function of the bias in the estimation of the nominal load, the variability in the load, and the target reliability index.

Similarly, the resistance factor that is consistent with the aforementioned load factors are well approximated for most materials by:

$$\phi = (\mu_R / R_n) \exp(-\alpha_R \beta V_R) \quad (25)$$

where α_R is the sensitivity coefficient equal approximately to 0.7.

2.4. Mori Method for the Computation of Load and Resistance Factors

When R and Q_i are independent and the lognormal distribution is used to describe all random variables, the performance function is expressed as:

$$g(X) = \ln R - \ln Q_i \quad (26)$$

The function of reliability index can be interpreted as:

$$\beta_{2M} = \frac{\lambda_R - \lambda_Q}{\sqrt{\zeta_R^2 + \zeta_Q^2}} \quad (27)$$

where λ_R and λ_Q are the mean of $\ln R$ and $\ln Q$, respectively; ζ_R and ζ_Q are the variance of $\ln R$ and $\ln Q$, respectively.

Using the coefficient of variation (V), λ and ζ can be calculated from the following formula:

$$\zeta^2 = \ln(1 + V^2) \quad (28)$$

$$\lambda = \ln \frac{\mu}{\sqrt{1 + V^2}} \quad (29)$$

Substituting Equation (27) into Equation (4), the load and resistance factors can be expressed as:

$$\lambda_R - \alpha_R \zeta_R \beta_T > \sum (\lambda_{Qi} + \alpha_{Qi} \zeta_{Qi} \beta_T) \quad (30)$$

Substituting Equations (28) into (30), the load and resistance factors can be expressed as:

$$\gamma_i = \frac{1}{\sqrt{1 + V_{Qi}^2}} \exp(\alpha_{Qi} \zeta_{Qi} \beta_T) \frac{\mu_{Qi}}{Q_{mi}} \quad (31)$$

$$\phi = \frac{1}{\sqrt{1+V_R^2}} \exp(-\alpha_R \zeta_R \beta_T) \frac{\mu_R}{R_n} \quad (32)$$

α_{Qi} and α_R are determined by the iteration computation, as shown by the calculation steps below.

α_{Qi} and α_R are the relative weight coefficient of the corresponding loads and resistance, which are expressed as:

$$\alpha_{Qi} = \alpha_{Qi}^* \cdot u \quad (33)$$

$$\alpha_R = \alpha_R^* \cdot u \quad (34)$$

α_{Qi}^* , α_R^* , and u are determined by the following functions:

$$\alpha_{Qi}^* = \frac{c_i \cdot \xi_{Qi}}{\sqrt{\xi_R^2 + \sum_j (c_j \cdot \xi_{Qj})^2}} \quad (35)$$

$$\alpha_R^* = \frac{\xi_R}{\sqrt{\xi_R^2 + \sum_j (c_j \cdot \xi_{Qj})^2}} \quad (36)$$

$$u = \frac{1.05}{1 - \{1 - \sqrt{\alpha_R^{*2} + (\max_i \alpha_{Qi}^*)^2}\} \cdot \Phi\left(\frac{\max_i V_{Qi} - 0.6}{0.6}\right)} \quad (37)$$

where Φ denotes the cumulative distribution function of the standard normal variable.

c_i is the ratio of the average of the sum of Q_i to the average of Q_i , which is expressed as:

$$c_i = \frac{\exp(\lambda_{Qi}^* + \frac{1}{2} \xi_{Qi}^2) \cdot \frac{Q_{ni}}{G_n} \frac{\mu_{Qi}}{Q_{ni}}}{\sum \{ \exp(\lambda_{Qj}^* + \frac{1}{2} \xi_{Qj}^2) \cdot \frac{Q_{nj}}{G_n} \frac{\mu_{Qj}}{Q_{nj}} \}} \quad (38)$$

where λ_{Qi}^* is the mean of $\ln(Q_i / \mu_{Qi})$; G_n is the nominal value of resistance; Q_{ni} is the nominal value of load Q_i .

When R and Q_i are independent but do not follow the lognormal distribution, the distribution types are first required to be transformed into lognormal ones. Then, the calculated results can be acquired based on Equations (26)–(38).

3. Proposition of the New Method

3.1. Computation Process of the Proposed Method

In order to avoid the mathematical limitation of Equation (11), a new model is proposed [20] as:

$$\beta_{3M-new} = \frac{1}{3} \beta_{2M} \left[2 + e^{\frac{1}{2} \alpha_{3Z} (\beta_{2M} - \frac{1}{\beta_{2M}})} \right] \quad (39)$$

Obviously, Equation (39) is simpler than Equation (11) without the limitation of applicable range. Since the mathematical inverse function of Equation (39) is nonexistent, an approximate function for β_{2T} is proposed as:

$$\beta_{2T} = \frac{3.8}{\alpha_{3Z}} \left[1 - e^{\frac{-\alpha_{3Z} \beta_T}{3.8}} \right] \quad (40)$$

In order to investigate the relativity of Equations (39) and (40), the inverse function of Equation (39) is introduced as:

$$\beta_T = -\ln\left(1 - \frac{\alpha_{3Z}\beta_{2T}}{3.8}\right) \times \frac{3.8}{\alpha_{3Z}} \quad (41)$$

Only when the results of Equations (39) and (41) are similar, can Equation (40) be assigned as the inverse function of Equation (39). Figure 1 shows the calculated results of Equations (39) and (41), indicating that Equation (39) is in great agreement with Equation (41), with errors certified to be below 7%. Therefore, Equation (40) can be employed as the inverse function of Equation (39).

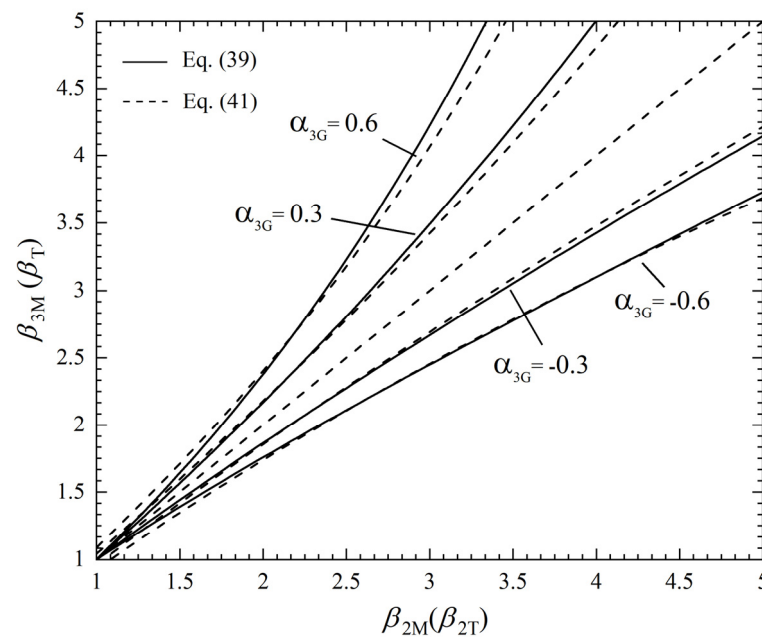


Figure 1. Precision testing of Equation (39).

To overcome the shortcoming of iterative computation, the following formula, with better convergence, is proposed to replace Equations (19) and (20):

$$\mu_{RT-\text{new}} = \sum \mu_{S_i} + \left(\beta_T + \frac{1}{2\beta_T} \right)^{1.7} \sqrt{\sum \sigma_{S_i}^2} \quad (42)$$

The steps for determining the load and resistance factors using the new method are shown below:

1. Calculate μ_{RT} using Equation (42).
2. Calculate σ_Z , α_{3Z} , and β_{2T} using Equations (5a,b), (12) and (40), respectively.

Then, calculate α_R and α_{Qi} with Equation (8).

3. Determine the load and resistance factors with Equation (18a,b).

In the proposed method, there is no need for step c and d in the existing 3M method, that is, iterative calculation is unnecessary. Besides, there is no mathematical limitation in Equation (39).

3.2. Accuracy of the Zero-Iterative-Based Calculation

In order to investigate the feasibility of calculating target resistance without iteration, the following performance function is applied:

$$G(X) = R - (D + L + S) \quad (43)$$

where R , D , L , and S are the resistance, dead load, live load, and snow load, respectively. The details (the mean value, μ_{s_i} , coefficient of variation, V_i , and skewness, α_{3i}) of the basic variables are shown in Table 1.

Table 1. Basic random variables for Equation (43).

RVs	μ_{Q_i}	V_i	α_{3i}
R	-	0.2	0.608
G	1	0.1	0
Q	0.5	0.59	1.18
S	0.47	0.21	1.14

μ_{R_0} can be computed using either the proposed method or the existing method introduced in this paper, but with different convergence speeds to obtain μ_{RT} . The convergence rates of two methods are shown in Table 2. As presented in Table 2, it is evident that the convergence rate of the proposed formula is faster than the existing one. Besides, compared with μ_{R_0} computed by Equation (20), the other computed by Equation (42) with the value of 3.27 is far closer to the convergence value of 3.32. In order to simplify the calculated process, μ_{RT0} calculated by Equation (42) is directly regarded as the target mean resistance, which means iteration is not required in the proposed method.

Table 2. Convergence of the existing formula and the proposed formula.

Method	μ_{RT0}	μ_{RT1}	μ_{RT2}	μ_{RT3}	μ_{RT4}	μ_{RT5}
Existing formula	3.07	3.25	3.30	3.31	3.32	3.32
Proposed formula	3.27	3.31	3.32	3.32	3.32	3.32

μ_{RTk} means the k th iteration value of the target mean resistance.

To investigate the feasibility of the simplified process, μ_{RT} was obtained by applying the proposed method and the existing methods were used to compare to the convergence value, which served as an accurately calculated result, as shown in Figure 2. In the case of the target reliability index being less than 2.5, the calculated result of the proposed method is slightly less than the convergence value and extremely close to that of the existing method, indicating that the accuracy is acceptable. When β_T is greater than 2.5, μ_{RT} computed by the proposed method is greater than the existing method and the convergence value, but with errors in the acceptable range, which implies that buildings designed with the proposed method are relatively safer. Therefore, the proposed zero-iterative-based calculation is feasible along with the acceptable calculated results.

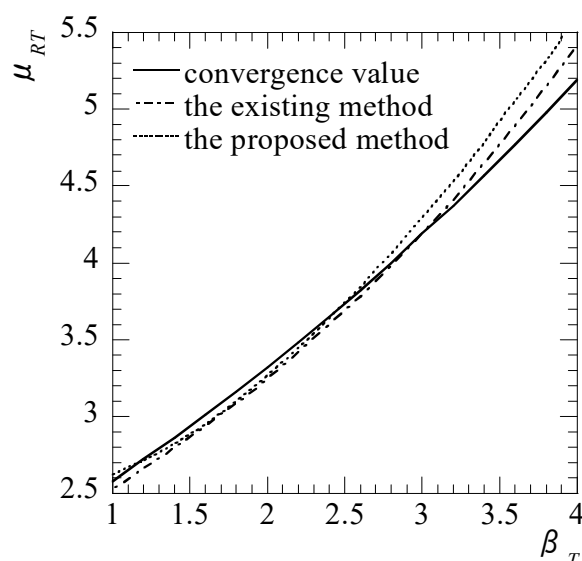


Figure 2. Target mean resistance.

4. Comparison of the ASCE Method, the Existing 3M Method, the Proposed Method, and the Mori Method

In order to investigate the accuracy of the proposed method further, the following example is considered (ASCE 7–10, C2.3.6) [13]:

$$G(X) = R - D - L - S \quad (44)$$

where R , D , L , and S are the resistance, dead load, live load, and snow load, respectively.

The load combination is the same as ASCE 7–10 (combination 2 in 2.3.2). The details (the mean value, coefficient of variation, and skewness) of the basic variables are shown in Table 3. The distribution types are also provided in Table 3 for the Monte-Carlo (MC) simulation in Section 4.

Table 3. Basic random variables for Equation (44).

RVs	μ_{Q_i} / D_n	V_i	$\sigma_{Q_i} = \mu_{Q_i} / D_n$	α_{3i}	μ_R / R_n or μ_{Q_i} / Q_{in}	Q_{in} / D_n	Distribution
R	–	0.09	–	0.27	1.06	–	Lognormal
D	1	0.25	0.25	0	1.0	1	Normal
L	0.175	0.59	0.103	1.18	0.35	0.5	Gamma
S	0.6874	0.21	0.144	1.14	0.982	0.7	Gumbel

Results Comparison of Four Methods

The results of load and resistance factors obtained using different methods are listed in Table 4. The results show that the resistance factors, ϕ , are in great agreement. The results of the dead load factor, γ_D , are similar, while the live load factor, γ_L , determined using the Mori method is obviously greater than those determined using the other methods, and the snow load factor, γ_S , obtained using the Mori method is the smallest one compared to the results obtained using other methods.

Table 4. Results of load and resistance factors calculated using different methods.

	Φ	γ_D	γ_L	γ_S
ASCE method	0.877	1.600	0.598	1.229
Existing 3M method	0.865	1.439	0.500	1.191

Proposed method	0.848	1.383	0.481	1.164
Mori method	0.924	1.328	1.352	0.444

In order to compare the accuracy of the four methods, the MC simulation (100,000 times when $\beta_T \leq 4$ and 1,000,000 times when $\beta_T > 4$) is used to calculate the reliability index. For the MC simulation, the calculation of μ_R/D_n is necessary (e.g., the proposed method):

$$\frac{\mu_R}{D_n} = \frac{\mu_R}{R_n} \cdot \frac{R_n}{D_n} = \frac{\mu_R}{R_n} \cdot \frac{1}{\phi} \left(\gamma_D \cdot \frac{D_n}{D_n} + \gamma_L \cdot \frac{L_n}{D_n} + \gamma_S \cdot \frac{S_n}{D_n} \right) = 3.05$$

With the values of μ_R/D_n , μ_D/D_n , μ_L/D_n , and μ_S/D_n , the results of the MC simulation are shown in Table 5; reliability of 3.01 of the proposed method is closest to the target reliability of 3.0. Therefore, the proposed method is considered accurate enough. The existing method is also accurate and safe, but with some limitations, and iterative computation is inevitable. The ASCE method is simpler, but the reliability computed by this method is much greater than the target reliability, which is safe but wastes a lot of structural materials. Conversely, the reliability calculated by the Mori method is much smaller than the target reliability, indicating that the design is not safe.

Table 5. Reliability of MC simulation with different methods.

	ASCE Method	Existing 3M Method	Proposed Method	Mori Method
β	3.53	3.08	3.01	2.656

In ASCE 7–10 (1.5.1), structures are classified into four different risk categories. For different risk categories and damage types, the acceptable reliability indexes ($\beta_T=2.5\text{--}4.5$) are provided in ASCE 7–10 C1.3.1 for a 50-year service period. In this paper, $\beta_T=1.0\text{--}4.5$ is chosen to analyze the application of four different methods.

As shown in Figure 3, load and resistance factors calculated using the existing method, the ASCE method, the proposed method, and the Mori method are inconsistent. In addition, the difference increases with the increase in the target reliability, β_T . For resistance factor, the difference among the four methods is slight.

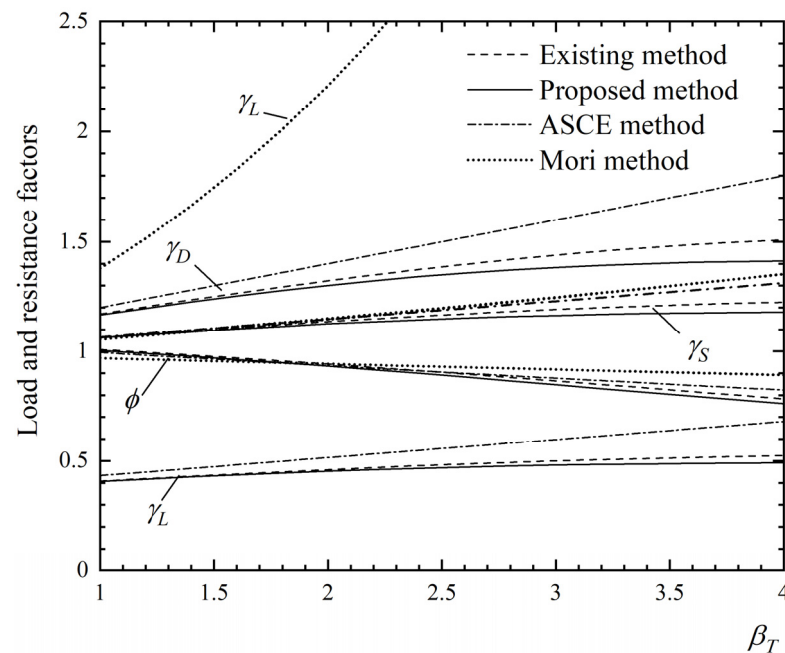


Figure 3. Load and resistance factors calculated using different methods for $\beta_T = 1.0\text{--}4.0$.

Figure 4 displays the comparison of reliability calculated by different methods for $\beta_T = 1.0\text{--}4.5$. The result of the ASCE method is obviously greater than the other three methods and much greater than the target reliability, while the results of the existing method and the proposed method are close to the target reliability. The reliability calculated with the proposed method is closest to the β_T , although it is a little smaller than β_T when β_T is great.

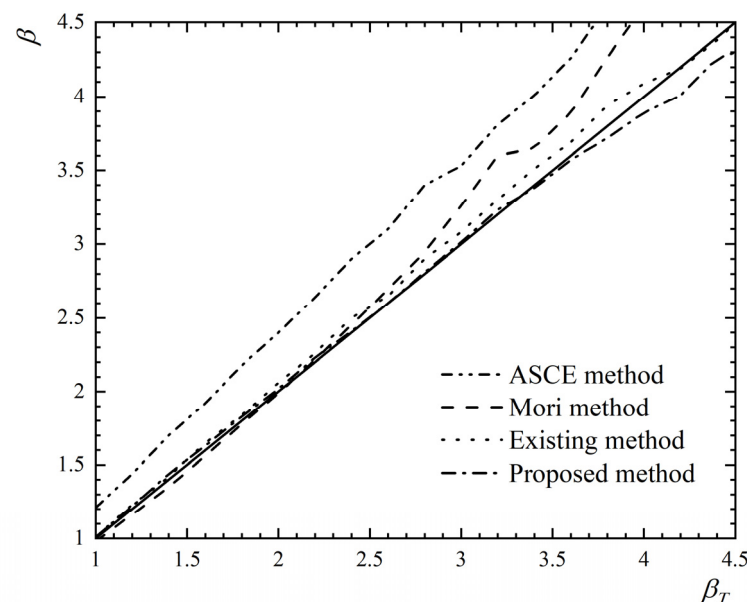


Figure 4. Reliability calculated with different methods for $\beta_T = 1.0\text{--}4.5$.

In order to investigate the sensitivity of load in the four methods, the influence of the snow load ($S_n/D_n = 0.5\text{--}2.5$) and dead load ($L_n/D_n = 0.5\text{--}2.5$) is considered. As shown in Figure 5, the results of the Mori method are greater than β_T when $S_n/D_n = 0.5\text{--}2.2$ and the error is stable. When $S_n/D_n = 2.3\text{--}2.5$, the results become much smaller than β_T . The results of the other three methods are much greater than the target reliability 3.0, in which the result of the proposed method is relatively close to 3.0.

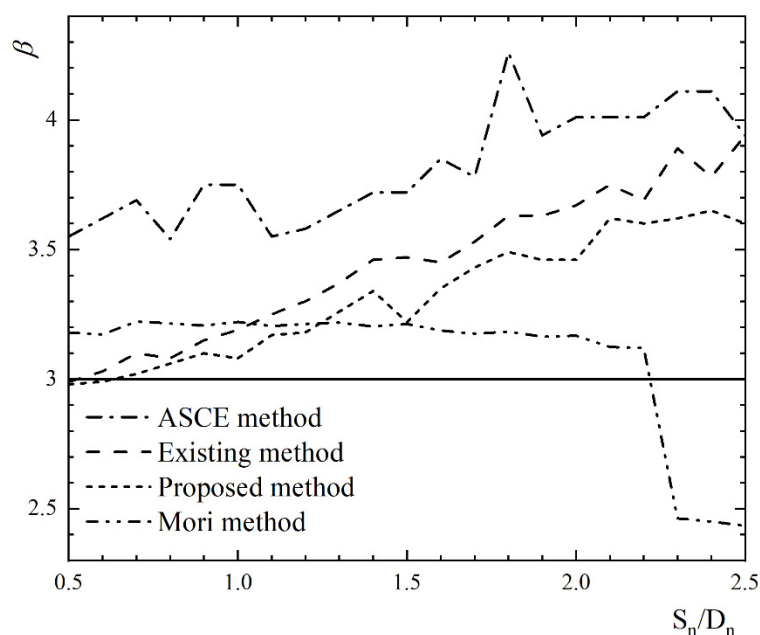


Figure 5. Reliability calculated by different methods for $S_n/D_n = 0.5\text{--}2.5$ ($\beta_T=3$).

In Figure 6, the results of the existing method and the proposed method are much closer to β_T , compared to the other two methods. In the case of $L_n/D_n = 0.5\text{--}1.4$, the reliability calculated using both methods are greater than β_T , and the error of the proposed method is relatively smaller than that of the existing method. In the case of $L_n/D_n = 1.5\text{--}2.5$, the results of the proposed method are slightly smaller than β_T , while the reliability calculated using the existing method is slightly greater than β_T . For the ASCE method, the reliability increases to a peak when L_n/D_n reaches 0.6, then falls as L_n/D_n increases, especially from 0.6–0.7. The results of the Mori method increase initially in the case of $L_n/D_n = 0.5\text{--}1.2$, and then drop gently until $L_n/D_n = 2.2$. Afterwards, the reliability inclines sharply from 2.2–2.3, but keeps close to β_T from 2.4–fig2.5.

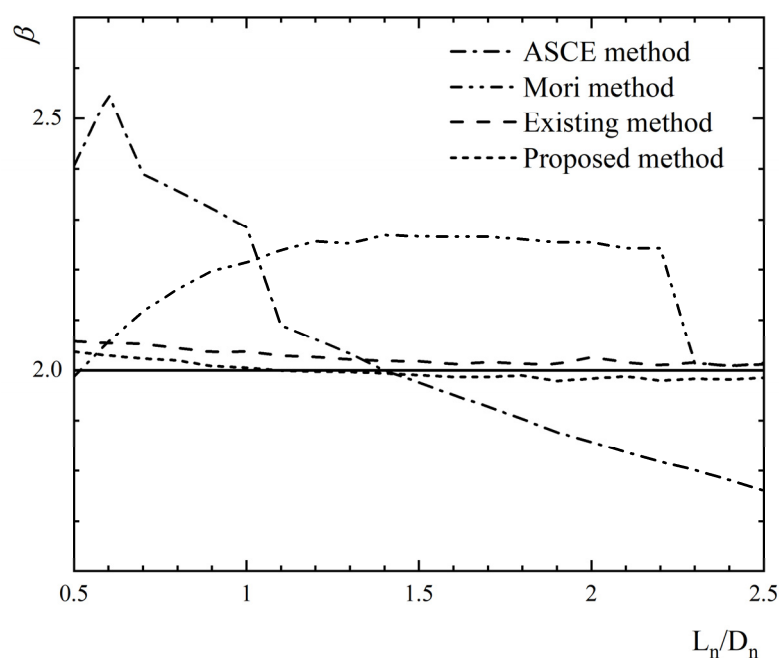


Figure 6. Reliability calculated by different methods for $L_n/D_n = 0.5\text{--}2.5$ ($\beta_T = 2$).

In light of Figures 4–6, 3M methods achieve higher precision compared to the ASCE method and the Mori method. In Figure 4, it can be seen that the reliabilities calculated by the proposed method and the existing 3M method are close to β_T when $\beta_T = 1.0\text{--}4.5$. In Figures 5 and 6, considering the effect of L_n/D_n and S_n/D_n , the reliability obtained using the existing method is moderately greater than β_T , but with the inevitable iterative computation for the target mean resistance. Moreover, the existing method cannot be universally deployed due to the limitation of Equation (11). Although reliability computation based on the proposed method might be slightly smaller than β_T in some cases, it has two intrinsic advantages: (1) No mathematical limitation; (2) No iteration calculation. Particularly, higher efficiency can be achieved when employing the proposed method for system reliability, and the error is also acceptable. Therefore, the proposed method is considered the most applicable for practical structural reliability design.

5. Influence of Distribution Type on the Result of the Proposed Method

In this example, several distribution types are assigned to observe their influence on the precision of the proposed method. Nine cases with different distribution types are applied, as presented in Table 6, along with different presumed resistances and loads listed in Table 7. Considering $\beta_T = 1.0, 2.0, 3.0, 4.0$, the reliability indices calculated using the proposed method are illustrated in Figure 7.

According to Figure 7a–i, it can be clearly seen that the reliability index β increases with the increase in β_T . For $\beta_T = 1.0\text{--}2.0$, the results are close to the target reliability in case 1 to 9. For $\beta_T = 3.0\text{--}4.0$, the results become smaller than β_T except case 7. It shows that a change in distribution type has a moderate effect on the accuracy of the proposed method when β_T is great, which needs to be further refined and perfected in future studies.

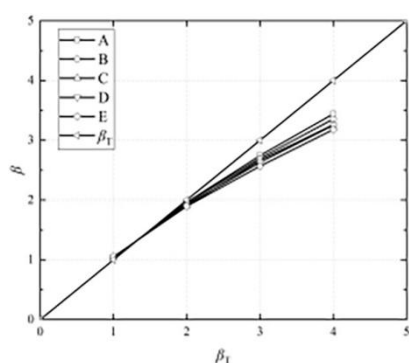
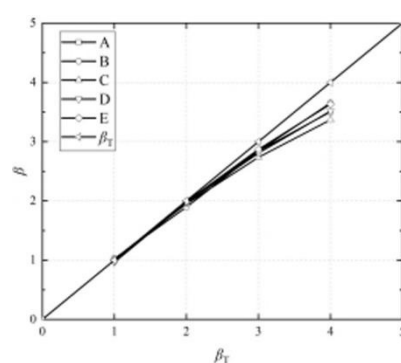
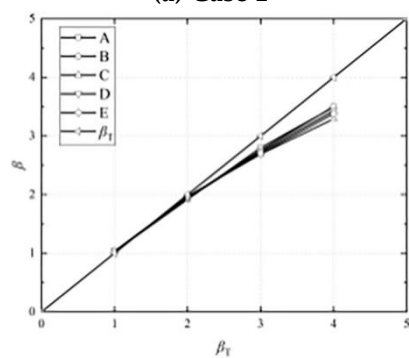
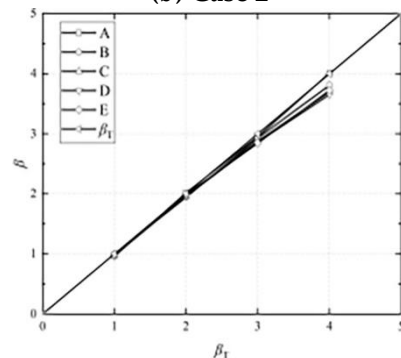
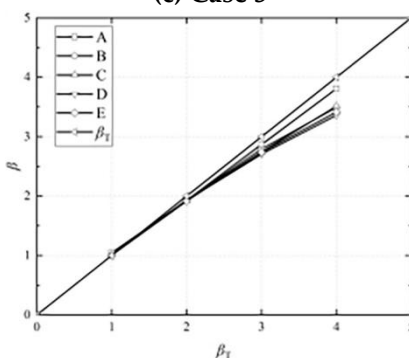
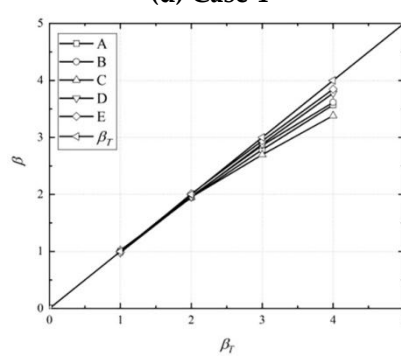
Table 6. Distribution types of the random variables.

	R	S1	S2	S3
Case 1	Lognormal	Lognormal	Lognormal	Lognormal
Case 2	Lognormal	Normal	Lognormal	Lognormal
Case 3	Lognormal	Gamma	Lognormal	Lognormal
Case 4	Lognormal	Normal	Normal	Normal

Case 5	Lognormal	Gamma	Gamma	Gamma
Case 6	Lognormal	Gumbel	Lognormal	Lognormal
Case 7	Lognormal	Gumbel	Gumbel	Gumbel
Case 8	Lognormal	Weibull	Lognormal	Lognormal
Case 9	Lognormal	Weibull	Weibull	Weibull

Table 7. Basic random variable.

	μ_R	μ_{S1}	μ_{S2}	μ_{S3}	V_R	V_{S1}	V_{S2}	V_{S3}
A	-	10	10	10	0.15	0.35	0.4	0.1
B	-	10	6	4	0.15	0.35	0.4	0.1
C	-	10	9	1	0.15	0.35	0.4	0.1
D	-	10	4.6	0.8	0.15	0.35	0.4	0.1
E	-	10	3.6	0.7	0.15	0.35	0.4	0.1

**(a) Case 1****(b) Case 2****(c) Case 3****(d) Case 4****(e) Case 5****(f) Case 6**

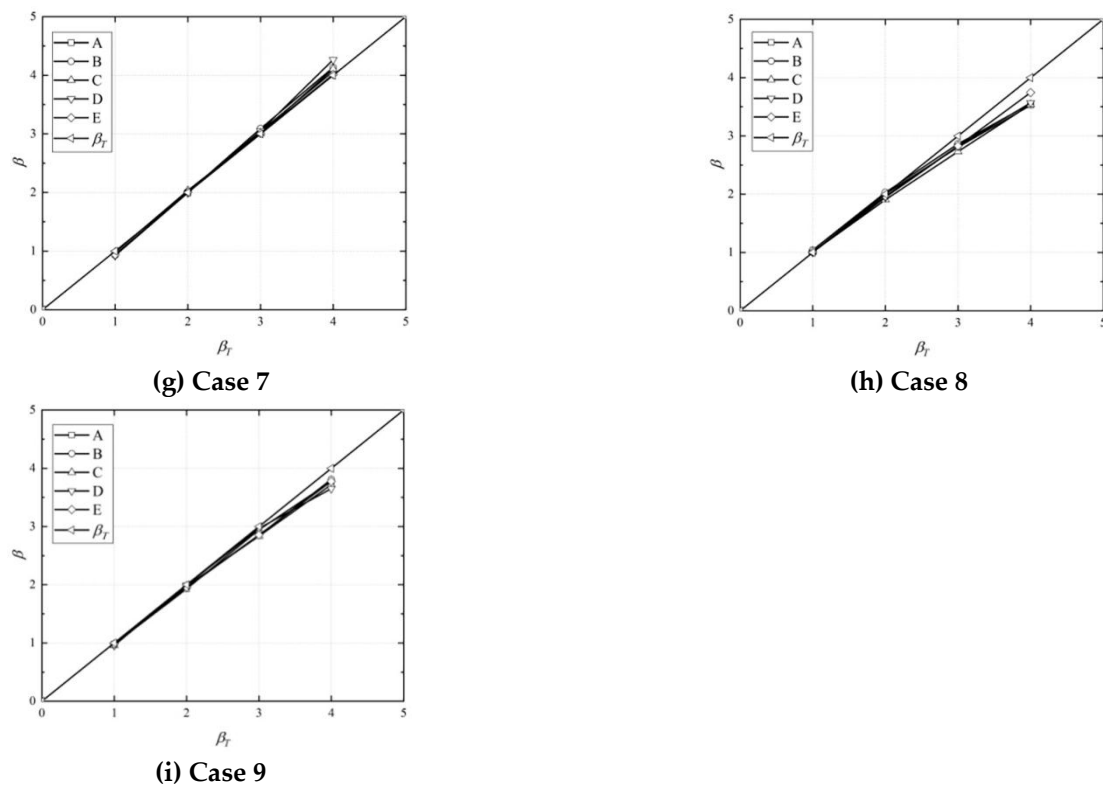


Figure 7. Reliability index (β) computed using the proposed method.

6. Conclusions

In this paper, a simplified third-moment method for determining the load and resistance factors is proposed. The following conclusions can be drawn:

1. The iteration calculation used in the existing 3M methods is cancelled. A new 3M method for the calculation of load and resistance factors is proposed with acceptable accuracy and applicable range. Different from other 3M methods, there is no mathematical limitation in the calculation process.
2. To investigate the accuracy of the proposed method, an example from ASCE 7–10 (C2.3.6) is considered. Comparing the results of four different methods with MC simulation (100,000 times), it can be concluded that the proposed method is considered accurate enough.
3. In order to verify the application of the new method, $\beta_T = 1.0\text{--}4.5$ is adopted. The result shows that the reliability index calculated by the proposed method is closest to the β_T , while the ASCE method and Mori method are much greater than the β_T .
4. The sensitivity of load in the new method is investigated (with snow load $S_n/D_n = 0.5\text{--}2.5$ and dead load $L_n/D_n = 0.5\text{--}2.5$). The result shows that the proposed method is the most accurate among all methods examined in this paper.
5. The influence of distribution type of variables is verified by nine cases. For $\beta_T = 1.0\text{--}2.0$, the reliability index calculated by the new method is very consistent with the target reliability. With the increase in the reliability index, the calculation error increases. Thus, further research is expected.

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