Research on Real-Time Monitoring and Performance Optimization of Suspension System in Maglev Train

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Abstract: With the success of the commercial operation of the maglev train, the demand for real-time monitoring and high-performance control of the maglev train suspension system is also increasing. Therefore, a framework for performance monitoring and performance optimization of the maglev train suspension system is proposed in this article. This framework consists of four parts: plant, feedback controller, residual generator, and dynamic compensator. Firstly, after the system model is established, the nominal controller is designed to ensure the stability of the system. Secondly, the observer-based residual generator is identified offline based on the input and output data without knowing the accurate model of the system, which avoids the interference of the unmodeled part. Thirdly, the control performance is monitored and evaluated in real time by analyzing the residual and executing the judgment logic. Fourthly, when the control performance of the system is degraded or not satisfactory, the dynamic compensator based on the residual is updated online iteratively to optimize the control performance. Finally, the proposed framework and theory are verified on the single suspension experimental platform and the results show the effectiveness.

Keywords: suspension system; data-driven residual generator; performance degradation; performance optimization

1. Introduction

As a new type of rail transportation, the maglev train has acquired attention at home and abroad due to its low noise, low energy consumption, and high comfort [1]. In addition, the commercial operation of high-speed maglev trains is also proceeding as scheduled [2], and maglev trains are gradually becoming a promising transportation tool.

For actual automation engineering systems, control performance monitoring has always been a part of the operation of the system. By measuring and monitoring the performance of system elements and equipment, the operation of the system can be obtained, and maintenance plans can be made to reduce the life cycle cost of the system [3–5]. In order to perform performance monitoring, engineers usually pre-design a performance monitoring and evaluation unit, which evaluates the control performance based on the measurement values obtained by the sensors. Model-based control performance monitoring is widely used in industrial automation control systems and has always been favored [6–8]. However, the model-based control performance monitoring program requires prior knowledge of physics and mathematics, which reduces the robustness to the uncertainty of the model to a certain extent. With the development of data network technology, data acquisition systems are widely used in industrial automation systems, and more and more process data can be collected and stored. Data-driven process monitoring programs have been widely used in many complex industries in recent years, for example [9–11]. This scheme can directly extract the necessary process information from a large amount of process data, thus saving the complicated modeling process, simple in form and low design workload [12–16]. However, for large-scale systems, the number and types of sensors are very large, a large amount of process data will be generated during operation, which undoubtedly increases...
the difficulty of performance monitoring and evaluation. Therefore, it is necessary to establish a unified dimension based on measurement data, and the monitoring and evaluation of system performance can be realized based on this dimension.

Meanwhile, similarly to other automation systems for Industry 4.0 [17], maglev trains are equipped with highly intelligent components and equipment, so they also face the problem of ensuring high control performance during the entire operation [18]. Taking the suspension system of the maglev train as an example, the nominal controller of suspension system in the maglev train is basically a PID controller. Although the PID controller ensures the stable suspension of the maglev train, and there is a certain stability margin to counter interference and other factors. In actual operation, maglev trains are often subject to various environmental conditions (such as track irregularities, load fluctuations caused by passengers getting on and off the train, excessive temperature of electromagnets [2,18–22], etc.), the control performance of the PID controller is not ideal. If no active and effective measures are taken, the control performance of the suspension system is very likely to be degraded, causing the train to deviate from the operating point or even fail. Although there are many algorithms for the online adjustment of PID parameters, in actual engineering, online adjustment of PID parameters is likely to cause system instability [23–26]. In addition, the controllers of most systems are encapsulated inside the system, and it is difficult to change easily once it has been determined. Therefore, it is an urgent problem to be solved in the commercial operation of the maglev train to improve the controller of the suspension system and optimize the control performance of the system in the operation of the maglev train.

Strongly motivated by the above discussion, this article establishes a real-time performance monitoring and performance optimization framework for suspension system of maglev train to achieve performance optimization and higher control performance, and provides a data-driven solution to its implementation.

The main contributions of this article are:
1. A framework for performance monitoring and performance optimization of the suspension system in maglev train is proposed. The framework consists of a nominal controller, a residual generator and a dynamic compensator. The nominal controller is used to stabilize the system and achieve tracking performance. The residual generator realizes the performance monitoring of the system and the dynamic compensator is used to realize the performance compensation and recovery;
2. The observer-based residual generator is identified offline based on the data-driven method. The offline identification method does not need to know the accurate model of the system. During the operation of the system, the residual signal generated by the residual generator is monitored to distinguish disturbances and degradations of the suspension system, which provides strong support for performance evaluation and performance optimization;
3. A data-driven performance optimization algorithm for the suspension system is designed. The algorithm can optimize the control performance of the system online when the system performance is unsatisfactory or even degraded;
4. The validity of the proposed framework and algorithm is verified on a single suspension experimental platform.

The structure of this article is as follows. Section 2 introduces the preliminary knowledge about the residual generator based on data-driven method, and proposes the structure of real-time monitoring and performance optimization. Section 3 establishes the model of the suspension system and designs the nominal controller. Section 4 introduces the realization form of the residual generator and proposes the strategy of control performance evaluation. Section 5 proposes the parameters optimization method of the dynamic compensator. Section 6 verifies the proposed method through the single suspension system. Section 7 summarizes the full article.
2. Preliminaries

2.1. Data-Driven Residual Generator

Considering a general noisy linear time-invariant discrete system:

\[
\begin{align*}
x(k + 1) &= Ax(k) + Bu(k) + \omega(k) \\
y(k) &= Cx(k) + Du(k) + \nu(k)
\end{align*}
\]

Among them, \(x(k) \in \mathcal{R}^n\), \(u(k) \in \mathcal{R}^k\) and \(y(k) \in \mathcal{R}^m\) are the state variable, input variable and output variable of the system, respectively. \(\omega(k) \in \mathcal{R}^n\) is the process noise; \(\nu(k) \in \mathcal{R}^m\) is the measurement noise. Both obey the normal distribution and are independent of the input and state variables.

The input and output data model [27, 28] is defined as:

\[
\left[ \begin{array}{c}
U_{k,s} \\
Y_{k,s}
\end{array} \right] = \Psi_s \left[ \begin{array}{c}
U_{k,s} \\
X_k
\end{array} \right] + \left[ \begin{array}{c}
0 \\
H_{o,s}W_{k,s} + V_{k,s}
\end{array} \right]
\]

where \(s\) is the number of future data, \(s \geq n\). \(X_k, U_{k,s}\) and \(Y_{k,s}\) are the Hankel matrix of state, input and output, respectively [27]

\[
\Psi_s = \begin{bmatrix}
I & 0 \\
H_{o,s} & \Gamma_s
\end{bmatrix}; \quad H_{o,s} = \begin{bmatrix}
D & 0 & \cdots & 0 \\
CB & D & \cdots & 0 \\
\vdots & \ddots & \ddots & \vdots \\
CA^{s-1}B & \cdots & CB & D
\end{bmatrix};
\]

\[
\Gamma_s = \begin{bmatrix}
C \\
CA \\
\vdots \\
CA^s
\end{bmatrix}
\]

is the observability matrix of the system; \(W_{k,s}\) and \(V_{k,s}\) are the Hankel matrix of noise.

There exists \(\Psi_s^T\) satisfying \(\Psi_s^T \Psi_s = 0\). The data-driven residual is defined as:

\[
r(k) = g^T \Psi_s^T \left[ \begin{array}{c}
U_s \\
Y_s
\end{array} \right]
\]

where \(g\) is a parameter vector [29] and is the core of residual generator.

2.2. Controller Based on Residual Generator

Considering the definition of the residual generator (3) and the Youla parameterized form of controllers [30], all inner-stable controllers can be parameterized as:

\[
u(z) = u_c(z) + Q_c(z)r(z)
\]

Among them, \(u_c(z)\) is the output of any controller \(K_0\) that makes the linear time-invariant system (1) stable [31, 32] \(Q_c(z) \in RH_o\) is used to improve the robustness of the system and can be regarded as a residual-driven dynamic compensator [33, 34]. The expression of \(Q_c(z)\) is:

\[
Q_c(z) = C_r(sI - A_r)^{-1}B_r + D_r
\]

As a result, the proposed structure of real-time monitoring and performance optimization is shown in the Figure 1.
Figure 2. The single point suspension model.  

The system model in Figure 2 is:

\[
\begin{align*}
\dot{x}(k+1) &= A x(k) + B u(k) + E \sigma(k) \\
\dot{y}(k) &= C x(k) + D u(k) + \sigma(k)
\end{align*}
\]

Structure of real-time monitoring and performance optimization.

The dynamic compensator based on the residual generator has the following advantages:

- The residual generator is integrated in the system and residual signal is generated in the operation, which can be used for process monitoring and further optimization of the controller;
- \( K_0 \) guarantees the stability of the closed-loop system. When \( Q_c(z) \) is stable, the design of \( Q_c(z) \) does not affect the stability of the system;
- When the system is subjected to unmodeled disturbances that cause changes in the input structure, \( Q_c(z) \) will be activated to compensate system disturbance. When the system performance is degraded, \( Q_c(z) \) is updated iteratively to restore the performance loss of the system.

3. Modelling and Nominal Controller of Suspension System

The article takes the suspension system of the maglev train as an example to discuss the optimization of control performance based on data-driven methods. The model of the suspension system is the prerequisite for the design of nominal controller.

3.1. Model of Suspension System

Each carriage of the maglev train has five bogies, each bogie has two suspension modules, and each suspension module has two suspension units, so the entire suspension system is composed of twenty sets of single suspension systems controlled independently [35]. The single suspension system is composed of a suspension control box, a set of sensors and an electromagnet. Taking a suspension point as an example, the single suspension point of a medium-speed maglev train is simplified to obtain the single point suspension model, as shown in Figure 2.
The system model in Figure 2 is:

\[
\begin{aligned}
\frac{m_B d^2z}{dt^2} &= m_B g - F(i, z) \\
F(i, z) &= K \left( \frac{i}{2} \right)^2 \\
u &= Ri + \frac{2K}{z_0^2} d_i dt - \frac{2K_i_0}{z_0^2} \cdot dz dt \\
m_B g - F(i_0, z_0) &= 0
\end{aligned}
\]

(6)

where \(m_B\) is the equivalent mass of the electromagnet; \(F(i, z)\) is the electromagnetic attraction exerted on the electromagnet; \(i\) is the current in the electromagnet coil; \(z\) is the gap between the electromagnet and the track and \(i\) and \(z\) are both variables that change over time; \(g\) is the acceleration of gravity; \(K = \frac{\mu_0 A N^2}{4} \); \(N\) is the number of turns of the solenoid winding, \(A\) is the pole area of the iron core, and \(\mu_0\) is the vacuum permeability rate; \(i_0\) and \(z_0\) are the coil current and the gap at equilibrium point, respectively.

The model of the suspension system consists of a set of nonlinear equations and it is difficult to design the nominal controller directly. Considering that during the operation, the current and gap of the electromagnet fluctuate up and down at the equilibrium point \((i_0, z_0)\), \(F(i, z)\) can be expanded by Taylor series at the equilibrium point, and higher-order terms can be ignored, and the system model is linearized as [36]:

\[
\begin{aligned}
\frac{m_B \ddot{z}}{dt^2} &= \frac{2K_i_0^2}{z_0^2} z - \frac{2K_i_0}{z_0^2} i \\
u &= Ri + \frac{2K}{z_0^2} i
\end{aligned}
\]

(7)

3.2. Nominal Controller

From the above analysis, the electromagnet is a first-order inertial link and the time constant is \(T_i = \frac{2K}{\mu_0 i_0} [37]\), so there is a large delay between the solenoid coil current \(i\) and the control voltage \(u\). Current feedback is introduced to reduce the time constant of this link. After introducing the current loop and determining the appropriate parameters, the corrected electromagnet can be approximated as a proportional link with a gain equal to 1 in the response frequency band of the system, which means that \(u = i\). The state space equation of the system is degraded into a second-order differential equation, and the state space expression is:

\[
\begin{aligned}
\begin{bmatrix} \dot{z} \\ \ddot{z} \end{bmatrix} &= \begin{bmatrix} 0 & 1 \\ \frac{2K_i_0^2}{z_0^2 m_B} & 0 \end{bmatrix} \begin{bmatrix} z \\ \dot{z} \end{bmatrix} + \begin{bmatrix} 0 \\ -\frac{2K_i_0}{z_0^2 m_B} \end{bmatrix} u \\
y &= \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} z \\ \dot{z} \end{bmatrix}
\end{aligned}
\]

(8)

If (8) is completely controllable and observable, a stable state feedback controller \(K_0\) can be further designed. The minimum realization of the state feedback controller [38] is:

\[
\begin{aligned}
x_c(k + 1) &= A_c x_c(k) + B_c e(k) \\
u_c(k) &= C_c x(k) + D_c e(k) \\
e(k) &= \omega(k) - y(k)
\end{aligned}
\]

(9)

where \(A_c, B_c, C_c\) and \(D_c\) are matrices of appropriate dimensions. \(x_c\) is the state variable of the controller; \(\omega(k)\) is the system given value; \(e(k)\) is the deviation, and \(u_c\) is the output of the controller.

Therefore, the structure of the feedback control of the system is as shown in Figure 3.
4. Real-Time Performance Monitoring and Performance Evaluation

In order to ensure the high control performance of the suspension system, combining the nominal controller $K_0$, the residual generator $r(k)$ and the dynamic compensator $Q_c(z)$ based on the residual drive, this section proposes a new controller framework and its implementation of a suspension system in the maglev train.

4.1. Problem Formulation

This article mainly considers the feedback control loop of the suspension unit as shown in Figure 1. After the current loop is set, the suspension unit is unstable in the second-order open loop. By designing the nominal controller, the stable performance and tracking performance of the system are guaranteed. However, maglev trains are subject to multiple disturbances during operation, such as changes in environmental conditions (temperature, humidity, etc.), track irregularities, aging of IGBTs, and changes in load caused by passengers getting on and off the train. These changes and disturbances will lead to the control performance of the suspension unit. A certain degree of degradation will occur, affecting the stable operation of the maglev train. Therefore, the performance optimization after the controller performance of the suspension system is degraded is the primary concern.

In this article, we will use the input and output data of the actual system during normal operation to identify the residual generator of the system offline and monitor the performance of the suspended unit online. In addition, the optimization of the controller during the operation phase of the maglev train will be introduced based on the monitoring results.

4.2. Realization of Data-Driven Residual Generator

Let $\Psi_{+,s}^\perp = \begin{bmatrix} \Psi_{s,s}^\perp & \Psi_{s,y}^\perp \end{bmatrix}$, $\Psi_{s,y}^\perp \in \mathbb{R}^{((s+1)m-n) \times (s+1)m}$ [39], then:

$$\Psi_{s,y}^\perp H_{0,s} = 0, \Psi_{s,s}^\perp = -\Psi_{s,y}^\perp H_{0,s}$$

Consider a single-input and single-output system, that is, $k_u = m = 1$, we can get:

$$\begin{cases} a_s = \begin{bmatrix} a_{s,0} & a_{s,1} & \cdots & a_{s,s} \end{bmatrix} \in \mathbb{R}^{s+1}, s = n \\ \beta_s = \begin{bmatrix} \beta_{s,0} & \beta_{s,1} & \cdots & \beta_{s,s} \end{bmatrix} \in \mathbb{R}^{s+1} H_{0,s} \end{cases}$$

The observer-based residual generator is constructed that:

$$\begin{cases} x_o(k+1) = A_o x_o(k) + B_o u(k) + L_o y(k) \\ r(k) = g y(k) - C_o x_o(k) - D_o u(k) \end{cases}$$

where $A_o = \begin{bmatrix} 0 & 0 & \cdots & 0 \\ 1 & 0 & \cdots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \cdots & 1 & 0 \end{bmatrix}$, $B_o = \begin{bmatrix} L_o \\ -g \end{bmatrix}$, $D_o = \beta_s^T$.

$C_o = \begin{bmatrix} 0 & \cdots & 0 & 1 \end{bmatrix}$. Specific steps are in [40].
4.3. Performance Evaluation and Classification

During actual operation, the operation of the system often goes through three stages, normal operation, degradation and failure. Among them, the degradation can be divided into three stages, that is, the tiny degradation that can return to normal, the medium degradation that can cause poor control performance but does not cause failure, and the severe degradation that is prone to failure. Different measures should be taken in time for different performance degradation to prevent major safety accidents. Taking the maglev system as an example, maglev trains are often affected by multiple uncertain factors (for example: degradation/failure of the component, wearing out of the windings, aging of the IGBTs, variation of the load, demagnetization of the magnet caused by collision, changing of the operation point, humidity), resulting in the degradation of system performance. Therefore, it is necessary to classify the degradation of the system and adopt different measures.

According to the residual generator constructed in Section 3.2, the thresholds are set by analyzing the residual signal during normal operation and without degradation. According to the thresholds $R_{th1}$ and $R_{th2}$, the degradation of the suspension system is divided into three levels [38]:

- **Tiny degradation:** $-R_{th1} < r(k) < R_{th1}$. The degradation may be caused by parameter changes or external interference. Through the feedback control of the nominal controller, the train can still run safely and stably without any additional measures;
- **Medium degradation:** $-R_{th2} < r(k) \leq -R_{th1}$ or $R_{th1} \leq r(k) < R_{th2}$. This may be caused by changes in the component parameters of the suspension system (such as large load fluctuations, track irregularities, etc.). If medium degradation occurs, there is no need to stop the maglev train for overhaul, and one only needs to activate the online performance optimization algorithm;
- **Severe degradation:** $r(k) \leq -R_{th2}$ or $r(k) \geq R_{th2}$. This may be caused by component failure of the suspension system. At this stage, the train is very prone to breakdowns. At this time, the maglev train should be stopped and overhauled as soon as possible to prevent major safety accidents.

5. Control Performance Optimization Architecture

5.1. Quadratic Performance Index

Although the nominal controller, based on the simplified model, can realize the stable suspension of the suspension system, when the suspension system is medium degraded, the nominal controller alone cannot realize the performance optimization. According to the Youla, parameterized realization form of the controller based on the residual generator, $Q_c(z)$ can be regarded as a dynamic compensator based on the residual signals. When $Q_c(z)$ itself is stable, the robustness of the system can be further improved, and the performance optimization of the system can be achieved [39,40]. The design of the dynamic compensator $Q_c(z)$ and the iterative implementation based on the gradient descent are given below.

Considering the input standard type parameter [18] of the dynamic compensator $Q_c(z)$ as shown in (13):

$$\theta_r = \begin{bmatrix} \theta_{AB,r} \\ \theta_{C,r} \\ \theta_{D,r} \end{bmatrix}$$  \hspace{1cm} (13)

Among them, $\theta_{AB,r}$ is the parameterization of $A_r$ and $B_r$; $\theta_{C,r}$ and $\theta_{D,r}$ are the parameterization of $C_r$ and $D_r$, respectively.

The quadratic performance indicator is defined as [32]:

$$J = \frac{1}{2N_r} \sum_{k=k_0}^{k_0+N_r-1} \left[ e(k)^T W_e(k) e(k) + u(k)^T W_u(k) u(k) \right]$$  \hspace{1cm} (14)
where \( W_r(k) \) and \( W_u(k) \) are the weight value of error and input, respectively, which are constants; \( k_0 \) is the specified start time of the calculation; \( N_r \) is the data length. The optimization of the dynamic compensator \( Q_c(z) \) is transformed to find the \( \theta_r \) that minimizes the performance index \( J \). To solve this problem, the gradient descent [41] can be used to change \( \theta_r \) along the direction of the negative gradient of \( J \) to \( \theta_r \), namely:

\[
\theta_r(i + 1) = \theta_r(i) - \lambda \nabla J(i)
\]

Among them, \( \lambda \) is the step length of parameter update [42].

\[
\nabla J(i) = \frac{1}{N_r} \sum_{k=k_0}^{k_0+N_r} \left[ e(k)^T W_r(k) \frac{\partial e(k)}{\partial \theta_r} + u(k)^T W_u(k) \frac{\partial u(k)}{\partial \theta_r} \right]
\]

Therefore, when calculating the negative gradient vector \( \nabla J(i) \), it is necessary to obtain the partial differential of \( e(k) \) and \( u(k) \) to \( \theta_r \) first.

When the dynamic compensator \( Q_c(z) \) is activated, the output of the controller acting on the system is:

\[
u(k) = u_c(k) + u_r(k) = C_c x_c(k) + D_c e(k) + C_r x_r(k) + D_r r(k)
\]

\[
u(k) = u_c(k) + u_r(k) = C_c x_c(k) + D_c e(k) + C_r x_r(k) + D_r r(k)
\]

\[
C_c x_c(k) + D_c \omega(k) - y(k) + C_r x_r(k) + D_r r(k)
\]

\[
= C_c x_c(k) + D_c \omega(k) - g^{-1} r(k) - g^{-1} y_w(k) + C_r x_r(k) + D_r r(k)
\]

\[
= C_c x_c(k) + D_c \omega(k) - g^{-1} r(k) - g^{-1} C_o x_o(k) - g^{-1} D_o u(k)
\]

\[
+ C_r x_r(k) + D_r r(k)
\]

And then,

\[
(I + D_c g^{-1} D_o) u(k) = C_c x_c(k) + D_c [\omega(k) - g^{-1} C_o x_o(k)] + C_r x_r(k) + (D_r - D_c g^{-1}) r(k)
\]

Let \( (I + D_c g^{-1} D_o)^{-1} = D \), then:

\[
u(k) = DC_c x_c(k) + DD_c [\omega(k) - g^{-1} C_o x_o(k)] + DC_r x_r(k) + D (D_r - D_c g^{-1}) r(k)
\]

So, \( x_c(k + 1) \) can be written as:

\[
x_c(k + 1) = (A_c - B_c D_o DC_c) x_c(k) + B_c (D_o DD_c g^{-1} - I) x_o(k)
\]

\[
-B_c D_o DC_c x_r(k) - B_c [I + D_o D (D_r - D_c g^{-1})] r(k) + B_c (I - D_o DD_c) \omega(k)
\]

Let \( A_o + L_o g^{-1} C_o = A \) and \( B_o + g^{-1} D_o = B \), then:

\[
x_o(k + 1) = (A - BDD_c g^{-1} C_o) x_o(k) + BDC_c x_c(k) + BDC_r x_r(k)
\]

\[
+ [BD (D_r - D_c g^{-1}) + L_o g^{-1}] r(k) + BDD_o \omega(k) e(k)
\]

\[
= (I - D_c D_o) \omega(k) - [D_c D (D_r - D_c g^{-1}) + I] r(k) - D_o DC_c x_c(k)
\]

\[
- D_o DC_r x_r(k) + (D_o DD_c g^{-1} - I) C_o x_o(k)
\]

Differentiate the left and right sides of (19)–(21) to obtain the partial differential of \( e(k) \) and \( u(k) \) to \( \theta_r \).

5.2. Iterative Update of Parameter \( \theta_r \)

Since the iterative update of the three parameters of \( \theta_r \) is very similar, this article only gives the update formula of \( \theta_{AB,r} \):

\[
\frac{\partial x_o(k+1)}{\partial \theta_{AB,r}(i)} = (A_c - B_c D_o DC_c) \frac{\partial x_c(k)}{\partial \theta_{AB,r}(i)} + B_c (D_o DD_c g^{-1} - I) \frac{\partial x_o(k)}{\partial \theta_{AB,r}(i)}
\]

\[
- B_c D_o DC_r \frac{\partial x_r(k)}{\partial \theta_{AB,r}(i)}
\]
\[
\frac{\partial x_0(k+1)}{\partial \theta_{AB,r}(i)} = \left(A - BDD_Cg^{-1}C_v\right)\frac{\partial x_0(k)}{\partial \theta_{AB,r}(i)} + BDc_v\frac{\partial x_c(k)}{\partial \theta_{AB,r}(i)} + BDc_r\frac{\partial x_r(k)}{\partial \theta_{AB,r}(i)}
\]

\[
\frac{\partial x_c(k+1)}{\partial \theta_{AB,r}(i)} = \frac{\partial A_r}{\partial \theta_{AB,r}(i)}x_r(k) + \frac{\partial A_r}{\partial \theta_{AB,r}(i)}r(k) + \frac{\partial B_r}{\partial \theta_{AB,r}(i)}r(k)
\]

\[
\frac{\partial u(k)}{\partial \theta_{AB,r}(i)} = DC_c\frac{\partial x_c(k)}{\partial \theta_{AB,r}(i)} - DD_cg^{-1}C_v\frac{\partial x_v(k)}{\partial \theta_{AB,r}(i)} + DC_r\frac{\partial x_r(k)}{\partial \theta_{AB,r}(i)}
\]

\[
\frac{\partial e(k)}{\partial \theta_{AB,r}(i)} = -D_cDC_r\frac{\partial x_c(k)}{\partial \theta_{AB,r}(i)} + \left(D_cDD_cg^{-1} - I\right)C_v\frac{\partial x_v(k)}{\partial \theta_{AB,r}(i)} - D_cDC_r\frac{\partial x_r(k)}{\partial \theta_{AB,r}(i)}
\]

\[
\theta_{AB,r}(i+1) = \theta_{AB,r}(i) - \frac{\kappa}{N_c} \sum_{k=0}^{k_0+N_c-1} \left(e(k)^T W_c(k) \frac{\partial \theta_{AB,r}(i)}{\partial \theta_{AB,r}(i)} + u(k)^T W_u(k) \frac{\partial \theta_{AB,r}(i)}{\partial \theta_{AB,r}(i)}\right)
\]

Through (22)–(26), \(\frac{\partial u(k)}{\partial \theta_{AB,r}(i)}\) and \(\frac{\partial e(k)}{\partial \theta_{AB,r}(i)}\) can be obtained, then parameters \(\theta_{AB,r}\) can be updated online iteratively according to (27).

The update equation of \(\theta_c\) and \(\theta_{D,r}\) can be obtained in the same way. Through the above equation, \(\theta\) is continuously updated iteratively during the operation of the system. When the performance index \(J\) meets a certain condition or reaches the maximum number of iterations, the online update stops and outputs the dynamic compensator \(Q_c(z)\).

6. Experimental Verification and Analysis
6.1. Experimental Device

Considering the single-point suspension system shown in Figure 2, the experimental device is shown in the Figure 4. The whole experimental platform is composed of an electromagnet, a chopper board, a control board, two power supplies, sensors and a semi-physical simulation platform. The sampling period of the single suspension system is 0.001 s.

![Experimental device](image)

Figure 4. Experimental device.

In each sampling period, the gap sensor obtains the suspension gap between the electromagnet and the track. Then the controller generates the input signal, and the chopper outputs the corresponding PWM wave. The suspension electromagnet generates different suspension forces according to the current, so that the electromagnet and the track can always maintain a stable gap of 4 mm.
According to the single suspension system model determined by (8), the stable feedback controller \( K_0 \) is determined as:

\[
\begin{align*}
    x_c(k + 1) &= \begin{bmatrix} 0.9987 & -0.0010 \\ -0.0010 & 0.9993 \end{bmatrix} x_c(k) + \begin{bmatrix} -1.0509 \\ -0.7974 \end{bmatrix} e(k) \\
    u_c(k) &= \begin{bmatrix} 1937 \\ -2913 \end{bmatrix} x(k)
\end{align*}
\]

(28)

After the introduction of the nominal controller, the gap of the single suspension system during normal operation is shown in Figure 5.

![Figure 5. Gap during normal operation.](image)

6.2. Observer and Performance Optimization

Collecting input and output data during normal operation of the system and constructing an observer-based residual generator according to [32].

\[
\begin{align*}
    x_o(k + 1) &= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} x_o(k) + \begin{bmatrix} 0.0009 \\ -0.1005 \\ -0.0019 \end{bmatrix} u(k) + \begin{bmatrix} -0.5098 \\ 0.4688 \\ 0.5200 \end{bmatrix} y(k) \\
    r(k) &= 0.4790 y(k) - \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} x_o(k) - 0.1016 u(k)
\end{align*}
\]

(29)

The residual signals of the single suspension system during the operation are shown in Figure 6. It can be found that during the operation, the fluctuation of the residual of the system is very small, most of which are between \(-0.1 \text{ mm} \sim 0.1 \text{ mm}\). The mean value of the residual is 0 and the residual approximately obeys the normal distribution. In addition, when the system is disturbed by an unknown operation from the outside, the residual fluctuates between \(-0.3 \text{ mm} \sim 0.3 \text{ mm}\), and the average value is almost unchanged. Since the suspension gap of the train fluctuates little, it is still in safe operation. Therefore, the small disturbance during the operation will not have a major impact on the performance of the system, and the system is in a state of no degradation at this time. Setting \( R_{th1} = \max\{|r_{\text{max}}|, |r_{\text{min}}|\} \) and \( R_{th2} = 2R_{th1} \), and the control performance of the system can be distinguished by the logic in Section 4.3.

During the operation of the train, passengers constantly get on and off the train, and the situation of getting on and off the passengers is unpredictable, which might cause the load to fluctuate in a large range. In order to verify the performance monitoring of the
residual generator in the case of large load operation and large load fluctuations, from the 10th second, loads of different sizes are applied to the electromagnet to simulate passengers getting on and off the train. Residual and gap are shown in Figure 7.

![Figure 6. The residual during the operation.](image)

![Figure 7. Gap (a) and residual (b) when the load fluctuates.](image)

It can be found that when the load changes, the residual and the gap change at the same time. The amplitude of the residual sharply increases. The maximum value of residual is 4.78 mm, which is much larger than the threshold $R_{th2}$; the minimum value of residual is $-4.78$ mm, which is much smaller than the threshold $-R_{th2}$. Therefore, by analyzing the residual signal, real-time monitoring of the large fluctuation of the load of the system can be carried out.

The gap and residual when the performance is experiencing degradation and when the train is in normal operation are shown in Figures 8 and 9, respectively. When the suspension system is degraded, the change in the suspension gap is not obvious, and the degradation of the suspension system cannot be judged by analyzing the gap. However, the variance of the residual signal in the two cases is different. When the system is degraded, the fluctuation range of the residual is $-0.59$ mm $\sim 0.81$ mm, which is larger than that of the normal operation. Therefore, performance degradation of the suspension system can be judged by observing the residual signal.
For real-time performance optimization under performance degradation, the initial values are set as $\theta_{AB_r} = 0, \theta_{C_r} = 0$, and $\theta_{D_r} = 0$. During the entire performance optimization period, $\theta_r$ is continuously updated iteratively until the end time of the algorithm has been reached. Finally, $Q_{C}(z)$ is:

$$
\begin{align*}
    x_r(k + 1) &= r(k) \\
    u_r(k) &= 0.0657x_r(k) + 0.0099r(k)
\end{align*}
$$

(30)

The gap and residual of the four periods when the system is disturbed by the disturbance during the performance optimization are shown in Figures 10 and 11, respectively, where $t_4$ is the time before the algorithm terminates. The parameter index comparison is shown in Table 1.
For real-time performance optimization under performance degradation, the initial values are set as $\theta, x = 0$. During the entire performance optimization period, $\theta$ is continuously updated iteratively until the end time of the algorithm has been reached. Finally, $Q(z)$ is:

$$x(k+1) = r(k)$$

$$u(k) = 0.0657x(k) + 0.0099r(k)$$

(30)

The gap and residual of the four periods when the system is disturbed by the disturbance during the performance optimization are shown in Figures 10 and 11, respectively, where $t_s$ is the time before the algorithm terminates. The parameter index comparison is shown in Table 1.

Figure 10. Gap at four moments.

Figure 11. Residual at four moments.

Table 1. Comparison of indicators at four moments.

<table>
<thead>
<tr>
<th>Time</th>
<th>Peak</th>
<th>Dead Time</th>
<th>Adjustment Time</th>
<th>Performance Index J</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t_1$</td>
<td>0.00749</td>
<td>3.140</td>
<td>3.642</td>
<td>0.1576</td>
</tr>
<tr>
<td>$t_2$</td>
<td>0.00743</td>
<td>3.069</td>
<td>3.592</td>
<td>0.1444</td>
</tr>
<tr>
<td>$t_3$</td>
<td>0.00740</td>
<td>3.256</td>
<td>3.781</td>
<td>0.3617</td>
</tr>
<tr>
<td>$t_4$</td>
<td>0.00742</td>
<td>2.954</td>
<td>3.491</td>
<td>0.1320</td>
</tr>
</tbody>
</table>

After the performance of the suspension system is degraded, any small disturbance will cause the system to crash track, that is, the electromagnet first collides with the upper track and is adsorbed to the upper track. After a period, it falls and collides with the lower track, and finally the electromagnetic achieves normal suspension. Among them, the gap peak is the maximum value of the gap between the electromagnet and the upper track, and the smaller value means the lower the probability of collision with the lower track. The dead time is the duration of the electromagnet being attracted to the upper track. The adjustment time is the time from the minimum electromagnet gap to normal operation. As time goes by, $\theta_r$ is continuously updated iteratively, the peak value of the gap is getting smaller. This means that the probability of the electromagnet colliding with the underlying track becomes smaller. Additionally, the dead time and adjustment time...
are both smaller. Moreover, the performance index become smaller as the iteration time increases.

In the iterative process of the dynamic compensator, the maximum value of the residual signal gradually becomes smaller. However, in normal suspension, the difference between the mean and variance of the residual signals at the four times is not big, mainly because the running time is not suffice and the control performance convergence is slow, which is the limitations of the proposed method in this article.

7. Conclusions

This paper proposes a framework for performance monitoring and performance optimization of suspension system in maglev train. Under this framework, the nominal control, residual generator and dynamic compensator respectively realize the stable suspension of the train, the real-time performance monitoring and the improvement in control performance. It is worth mentioning that the above control algorithms all follow the Youla parameterized realization of all stable controllers. Based on the data-driven method, the observer-based residual generator is designed offline and embedded in the control system to realize the performance monitoring of the system. The parameters are optimized through the gradient descent method to further optimize the control performance. Finally, under the conditions of real-time realization, the proposed framework and control algorithm are verified on a single suspension system.

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