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The Impact of Reduced Gravity on Oscillatory Mixed Convective Heat Transfer around a Non-Conducting Heated Circular Cylinder

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Abstract: The present analysis addresses the impact of reduced gravity and magnetohydrodynamics on oscillating mixed-convective electrically conducting fluid flow over a thermal, non-conducting horizontal circular cylinder. In reduced gravity, buoyancy forces may induce fluid motion due to a weak gravitational field but in non-gravity forces, fluid motion can be induced by a variety of factors, including surface tension and density variations. The fluid motion is governed by connected nonlinear partial differential equations which are converted into convenient equations by applying a finite-difference scheme with the primitive transformation and a Gaussian elimination technique. The numerical solutions of the connected dimensionalized equations were obtained for various emerging dimensionless parameters, reduced gravity parameter $R_g$, Prandtl number $Pr$, and some other fixed parameters. First, the fluid velocity, temperature distribution and magnetic-field profiles were obtained and then these profiles were used to examine the oscillating quantities of skinfriction, oscillating heat transfer and oscillating rate of current density. The FORTRAN software was used for the numerical results and these results were displayed on Tech Plot. The fluid velocity and magnetic profile were increased at the $\pi/2$ station as reduced gravity increased but the dimensionless temperature of the fluid attained a maximum magnitude as reduced gravity was decreased. The larger amplitude of the oscillating coefficients of $\tau_t$ and $\tau_m$ was concluded with a prominent variation for each $\lambda$ in the presence of reduced gravity. Physically, this could be because an increase in the decreased gravity parameter impacts the fluid flow’s driving potential along a thermal, non-conducting horizontal cylinder.

Keywords: reduced gravity; transient flow; non-conducting cylinder; finite-difference method; current density; heat transfer; magnetohydrodynamic

1. Introduction

Heat transfer in a fluid (gas or liquid) is a complex process that combines conduction, diffusion, and advection qualities to convey thermal energy from a hot to a cold climate through the fluid. When a fluid comes into contact with another fluid or a solid surface at a different temperature then energy is generated at the interface, causing the system to reach a state of thermal equilibrium. Buoyancy forces may produce fluid motion in reduced gravity due to a weak gravitational field, although in the absence of a gravity field it can be induced by a variety of factors, such as surface tension and density fluctuations. Many researchers have demonstrated the presence of weak gravitational effects on fluids due to their applications in a wide range of science and engineering processes such as air drag, centripetal force due to vehicle rotation, gravity gradients, solar wind, and solar pressure.
Most spacecraft have a lower acceleration value in the presence of weak gravitational effects on fluids.

In the absence of any forced flow, the two naturally occurring components of convective heat transfer, gravity-driven advection and gravity-independent diffusion, are often coupled as a hotter mass of fluid. As a result, it loses density and rises, absorbing heat while simultaneously spreading energy from the same mass owing to thermal gradient diffusion. Chimney stacks, heat-exchanger tube design, hot rolling, magnetically-heated chemical reactors, cooling towers, liquid rockets, nuclear reactors, and turbojet engines all benefit from mixed-convection oscillatory flow with heat transfer characteristics in cylindrical structures. The influence of oscillation on heat transport is a hot topic in current engineering. In molten metal purification, macro and microelectronic devices, metallurgy, and geophysical systems, the effects of magnetic fields on heat and fluid dynamics have attracted attention.

By keeping the above reduced gravity and heat transfer concepts, many researchers have been discussing free or mixed convective heat transfer analysis on various non-magnetized and magnetized geometries such as vertical plate, elliptical cylinder, plume, micro and porous channels. They obtained numerical results with the help of various methods such as the series solution method, Karman–Pohlhausen method, finite element method and the shooting method for given boundary conditions. The effects of large Grashof number on free convective heat transfer along a thermal bar in the presence of reduced gravity were studied in [1]. The oscillatory mixed convection heat transfer and current density around a non-conducting circular cylinder has not been yet explored in the presence of an induced magnetic field. By taking the ideas of reduced gravity, an induced magnetic field and a transient model from the current literature, a physical model has been developed for the present physical phenomena.

The difference between the current work and previous literature is the analysis of transient partial differential equations directly with the help of a primitive variable formulation finite-difference scheme. Many researchers investigated numerical results by converting partial differential equations into ordinary differential equations but in the current research, the oscillatory heat transfer was obtained by using a primitive formulation on partial-differential equations. First, the unsteady equations were converted into steady, real and imaginary parts. The steady results were secured and then these results were used in real and imaginary part for the periodic results. Ostrach [2] studied fluid motion in a weak gravitational field for the convection process and observed that surface and inertial forces may accelerate the fluid motion in a weak gravitational field. The free convective flow past sphere for finite values of large Grashof and Prandtl number has been discussed by Rily [3]. Herwing et al. [4] investigated the numerical results for laminar entry-flow in a channel with the temperature-dependent viscosity effects due to heat transfer across the wall. They combined the temperature-dependent viscosity in a single auxiliary function for momentum and heat transfer. Flow due to the thermal bar by considering the reduced gravity impact was investigated by Kuiken and Merkin [5]. Cheng [6] discussed free convection flow on an elliptical cylinder with variable viscosity and constant surface heat flux. Miao and Massoudi [7] proposed a new constitutive model for heat flow to achieve the numerical findings for shear-dependent fluid viscosity and temperature thermal conductivity fluctuation along two horizontal flat plates, one of which is at a higher temperature. Lotto et al. [8] derived an analytical analysis to estimate convective heat transfer within a weak gravitational field. Different situations of heat transmission through electrically conducting forms have been studied by Ashraf et al. [9,10]. The steady numerical solutions of mixed convection buoyant flow over a magnetized vertical surface with temperature-dependent fluid viscosity and temperature thermal conductivity have been reported in [11].

Convective heat transfer stems from fluid mass transport across a temperature gradient but mixed convection forms incorporate two primary transport processes: diffusion and advection which are the principle mechanisms of convective heat transfer in classical
continuum mechanics. In advection, the bulk fluid can transport the energy which arises from externally forced mass flow due to being gravity-driven. The dependence of free convective heat transfer on gravity will be crucial in future space exploration missions. As people begin to venture beyond Earth’s boundaries, their spacecraft will be subjected to varying degrees of gravity during transit and at various destinations, such as the Moon and Mars. Muhammad et al. [12] used the Galinstan alloy for high heat-flux on a mini-channel heat sink to compute the numerical results for the heat transfer mechanism. They came to the conclusion that, for a given channel length, flow resistance is determined by channel width, height, and coolant velocity. Atif et al. [13] used numerical methods to solve an MHD micropolar Carreau nanofluid flow problem along a stretching sheet with heat radiation. The effects of internal energy, viscous dissipation and joule heating were also included in the energy equation to analyze the heat transfer phenomena. Liu et al. [14] performed a slug-flow mechanism in a rectangular micro-channel to obtain the heat-transfer performance experimentally and numerically. In [15], a numerical simulation of the electrically conducting peristaltic flow of Casson fluid in the presence of slip-velocity in a porous-channel was performed. The authors noticed that as the slip parameter was increased, the magnitude of the pressure gradient decreased. The authors [16] looked at the effects of varying density on the oscillating mixed convection flow around a non-conducting cylinder. Vyas et al. [17] performed an experimental examination around a square cylinder contained in a rectangular channel to develop a heat transfer problem using a non-intrusive diagnostic technique. The analysis of mixed convection flow and heat transfer in the presence of an induced magnetic field along a circular cylinder has been investigated at high Reynolds number in [18–20]. A numerical simulation of thermo-hydraulic flow and heat transfer improvement in a 3D corrugated circular pipe has been investigated under varying structure configuration parameters in [21–24]. Gyergyek et al. [25,26] developed a catalyst composed of magnetic nanoparticles containing alumina carrying Ru nanoparticles in a slurry-type reactor by applying an AC magnetic field and conventional heating effects for hydrogenation. The catalytic hydrogenation, hydrodeoxygenation, and hydrocracking processes of a lignin monomer model compound eugenol over magnetic Ru/C–Fe\textsubscript{2}O\textsubscript{3} and mechanistic reaction microkinetics were investigated in [27].

Based on the aforementioned literature review, it was concluded that no researcher has yet investigated the effects of reduced gravity and MHD on transient mixed-convective flow through a thermal, non-conducting horizontal circular cylinder. This research aims to eliminate Newtonian heating in many basic engineering machines, as well as conjugate heat exchange near fins and heat exchangers. The oscillatory mixed convective heat transfer along a thermal, non-conducting horizontal cylinder with reduced gravity was done for the first time using the ideas from the preceding literature study and following Ostrach [2] and Kuiken [5]. The main novelty of the current work is to solve unsteady partial differential equations directly by using the finite difference method with the primitive variable formulation which has not yet been performed. First, the fluid velocity, temperature distributions and magnetic-field profile for steady equations were found and then this steady part was used to calculate the transient values of $\tau_s$-skin friction, transient $\tau_t$-heat transfer and transient $\tau_m$-current density for the unsteady part.

2. Problem Analysis

Consider the fluid flow phenomena along a thermal, non-conducting horizontal circular cylinder in two dimensions. The $x$-direction along the surface is represented by Figure 1, the $y$-direction is normal to the surface, and the velocities $u$ and $v$ along the $xy$-direction are shown by Figure 1. The magnetic field $H_x$ runs parallel to the surface, $H_y$ parallel to the surface normal, $T$ is the temperature field, and $U$ is the external fluid velocity $(x, t)$. Furthermore, the magnetic field acts in the normal direction of the horizontal non-conducting cylinder’s surface. The following is the dimensionless form of boundary-layer equations:
When the temperature is sufficiently close to $T_m$, then the relation between the density and temperature is as follows

$$\frac{\rho - \rho_m}{\rho_m} = -\gamma(T - T_m)^2 \quad (1)$$

Further, for unsteady flow, the Equation (1) implies that we require

$$T \to T_m \pm \Delta T, \quad \text{as} \quad y \to \pm \infty, \quad (2)$$

for some fixed $\Delta T$. To obtain the symmetry here is to consider region $y \geq 0$ subject to the boundary conditions. Where $T_\infty = T_m + \Delta T$ and has the relation with $\rho_\infty$ by Equation (1). It is easy to define the reduced gravity

$$g' = g \frac{(\rho_m - \rho_\infty)}{\rho_\infty}$$

(i.e., the fluid particles' acceleration which have density $\rho_m$). Then from Equation (1),

$$g' = g' \gamma \frac{\rho_m}{\rho_\infty} (T_\infty - T_m)^2 \quad (3)$$

The governing dimensionless mathematical model by applying suitable dimensionless variables in the presence of reduced gravity by following [1,11,28] is given as:

$$\frac{\partial \pi}{\partial \tau} + \frac{\partial \sigma}{\partial \xi} + \frac{\partial \pi}{\partial \eta} = \frac{dU}{d\tau} + \frac{\partial^2 \pi}{\partial \eta^2} + \xi \left( \frac{\partial \eta_x}{\partial \eta} + \frac{\partial \eta_y}{\partial \eta} \right) + R_s \left( \frac{2\theta - \theta^2}{\gamma} \right) \lambda \sin \alpha \quad (5)$$

$$\frac{\partial \eta_x}{\partial \tau} + \frac{\partial \eta_y}{\partial \eta} = 0 \quad (6)$$

$$\frac{\partial \theta}{\partial \tau} + \frac{\partial \theta}{\partial \xi} + \frac{\partial \theta}{\partial \eta} = \frac{1}{\nu} \frac{\partial^2 \theta}{\partial \eta^2} \quad (7)$$

Figure 1. Geometry of horizontal non-conducting cylinder and flow geometry.
In Equations (4)–(8), $R_g$ is the reduced gravity number, $Pr$ is the Prandtl number, $\lambda$ is the mixed convective number, $\gamma$ is the magnetic-Prandtl parameter, $H_o$ is the magnetic field intensity along normal to surface and $\theta$ is the dimensionless temperature given as:

$$\xi = \frac{\mu H_o^2}{\rho L_{oo}}, \quad Pr = \frac{\nu}{\alpha}, \quad \gamma = \frac{\nu}{\nu_m}, \quad \alpha = \frac{K}{\rho_p}, \quad \lambda = \frac{G_{rt}}{Re_t},$$

$$Re_t = \frac{U_{oo}L}{\nu}, \quad G_{rt} = \frac{g\beta \Delta T L^3}{\nu^2}, \quad R_g = \frac{g'}{g\beta \Delta T}, \quad \theta = \frac{T - T_\infty}{T_m - T_\infty}$$

The dimensionalized boundary conditions are given as:

$$u = \sigma = 0, \quad h_y = h_x = 0, \quad \theta = 1 \quad at \quad \bar{y} = 0$$

$$u \to \bar{U}(\tau), \quad \bar{\sigma} \to 0, \quad \bar{h}_y \to 1 \quad as \quad \bar{y} \to \infty$$

The above transformed model with prescribed controlling parameters is converted into unsteady and steady part separately by using oscillatory Stoke’s conditions given in Equation (11) to find the oscillatory behavior of the mechanism by following [11,28]. Then the oscillating part is converted into real and imaginary parts to compute the oscillating quantities.

$$\bar{u} = u_s + \epsilon u_t e^{i\omega \tau}, \quad \bar{\sigma} = \nu_s + \epsilon \nu_t e^{i\omega \tau}, \quad h_x = h_{xs} + \epsilon h_{xt} e^{i\omega \tau}$$

$$\bar{h}_y = h_{ys} + \epsilon h_{yt} e^{i\omega \tau}, \quad \bar{\theta} = \theta_s + \epsilon \theta_t e^{i\omega \tau}$$

### Steady Part:

$$\frac{\partial u_s}{\partial x} + \frac{\partial v_s}{\partial y} = 0$$

$$u_s \frac{\partial u_s}{\partial x} + v_s \frac{\partial u_s}{\partial y} + \frac{\partial^2 u_s}{\partial y^2} + \xi \left( h_{xs} \frac{\partial h_{xs}}{\partial x} + h_{ys} \frac{\partial h_{xs}}{\partial y} \right) + R_g \left( 2\theta_s - \theta_s^2 \right) \lambda \sin \alpha$$

$$\frac{\partial h_{xs}}{\partial x} + \frac{\partial h_{ys}}{\partial y} = 0$$

$$u_s \frac{\partial h_s}{\partial x} + v_s \frac{\partial h_s}{\partial y} - h_{xs} \frac{\partial u_s}{\partial x} - h_{ys} \frac{\partial u_s}{\partial y} = \frac{1}{\gamma} \frac{\partial^2 h_s}{\partial y^2}$$

$$\frac{\partial \theta_s}{\partial x} + v_s \frac{\partial \theta_s}{\partial y} = \frac{1}{Pr} \frac{\partial^2 \theta_s}{\partial y^2}$$

with appropriate boundary conditions:

$$u_s = v_s = 0, \quad h_{ys} = h_{xs} = 0, \quad \theta_s = 1 \quad at \quad y = 0$$

$$u_s \to 1, \quad \theta_s \to 0, \quad h_{ys} \to 1 \quad as \quad y \to \infty$$

### Oscillating Part:

$$\frac{\partial u_t}{\partial x} + \frac{\partial v_t}{\partial y} = 0$$

$$i\omega (u_t - 1) + u_s \frac{\partial u_t}{\partial x} + u_t \frac{\partial u_s}{\partial x} + v_s \frac{\partial u_t}{\partial y} + v_t \frac{\partial u_s}{\partial y} + \frac{\partial^2 u_t}{\partial y^2} + \xi \left( h_{xs} \frac{\partial h_{xt}}{\partial x} + h_{xs} \frac{\partial h_{xs}}{\partial y} + h_{ys} \frac{\partial h_{xt}}{\partial y} + h_{ys} \frac{\partial h_{xs}}{\partial y} \right) + R_g \left( 2\theta_t - \theta_t^2 \right) \lambda \sin \alpha$$

$$-2\theta_t \theta_t \lambda \sin \alpha$$
\[
\frac{\partial h_{xt}}{\partial x} + \frac{\partial h_{yt}}{\partial y} = 0
\]

(20)

\[
\frac{\partial h_{xt}}{\partial x} + \frac{\partial h_{yt}}{\partial y} = \frac{1}{\gamma} \frac{\partial^2 h_{xt}}{\partial y^2}
\]

(21)

with appropriate boundary conditions;

\[
u_i = v_i = 0, \quad h_{yt} = h_{xt} = 0, \quad \theta_i = 1 \quad \text{at} \quad y = 0
\]

(23)

Again the unsteady equations are separated into the real and imaginary parts by using the oscillatory Stoke’s conditions given in Equation (24) and the primitive formulated equations for real, imaginary and steady are given below by following [11,28].

\[
u_i = v_i + iv_2, \quad v_i = v_1 + iv_2, \quad \theta_i = \theta_1 + i\theta_2, \quad h_{xt} = h_{x1} + ih_{x2}, \quad h_{yt} = h_{y1} + ih_{y2}
\]

(24)

The steady, real and imaginary are transformed into a convenient form by using the primitive variable formulation for smooth algorithm. The primitive forms of the steady, real and imaginary equations are given below by using Equation (25),

\[
u_s(x, y) = U_s(X, Y), \quad v_s(x, y) = x^\gamma \nu_s(x, y), \quad h_{ys}(x, y) = x^{\gamma^2} \varphi_{ys}(X, Y),
\]

(25)

For steady equations:

\[
X \frac{\partial U_s}{\partial X} - \frac{Y}{2} \frac{\partial U_s}{\partial Y} + \frac{\partial V_s}{\partial Y} = 0
\]

(26)

\[
XU_s \frac{\partial U_s}{\partial X} + \left[ V_s - \frac{Y}{2} U_s \right] \frac{\partial U_s}{\partial Y} = \frac{\partial^2 U_s}{\partial Y^2} + \zeta \left[ X \varphi_{xs} \frac{\partial \varphi_{xs}}{\partial Y} + \left( \varphi_{ys} - \frac{Y}{2} \varphi_{xs} \right) \frac{\partial \varphi_{xs}}{\partial Y} \right]
\]

(27)

\[
X \frac{\partial \varphi_{xs}}{\partial X} - \frac{Y}{2} \frac{\partial \varphi_{ys}}{\partial Y} + \frac{\partial \varphi_{ys}}{\partial Y} = 0
\]

(28)

\[
XU_s \frac{\partial \varphi_s}{\partial X} + \left[ V_s - \frac{Y}{2} U_s \right] \frac{\partial \varphi_s}{\partial Y} - X \varphi_{xs} \frac{\partial U_s}{\partial Y} - \left( \varphi_{ys} - \frac{Y}{2} \varphi_{xs} \right) \frac{\partial U_s}{\partial Y} = \frac{1}{\gamma} \frac{\partial^2 \varphi_s}{\partial Y^2}
\]

(29)

\[
XU_s \frac{\partial \theta_s}{\partial X} + \left[ V_s - \frac{Y}{2} U_s \right] \frac{\partial \theta_s}{\partial Y} = \frac{1}{P_r} \frac{\partial^2 \theta_s}{\partial Y^2}
\]

(30)

with boundary conditions as:

\[
U_s = V_s = 0, \quad \varphi_{ys} = \varphi_{xs} = 0, \quad \theta_s = 1 \quad \text{at} \quad Y = 0
\]

(31)

\[
U_s \to 1, \quad \theta_s \to 0, \quad \varphi_{ys} \to 1 \quad \text{as} \quad Y \to \infty
\]
For real equations:

\[
X \frac{\partial U_1}{\partial X} - \frac{Y}{2} \frac{\partial U_1}{\partial Y} + \frac{\partial V_1}{\partial Y} = 0
\]  

(32)

\[
X \left[ U_s \frac{\partial U_1}{\partial X} + U_1 \frac{\partial U_s}{\partial X} \right] + \left[ V_s - \frac{Y}{2} U_s \right] \frac{\partial U_1}{\partial Y} + \left[ V_1 - \frac{Y}{2} U_1 \right] \frac{\partial U_s}{\partial Y} - \omega X (U_2 + 1
\]

(33)

\[
= \frac{\partial^2 U_1}{\partial Y^2}
\]

\[+ \zeta X \left( \varphi_{xs} \frac{\partial \varphi_{x1}}{\partial x} + \varphi_{x1} \frac{\partial \varphi_{xs}}{\partial x} \right) + \left( \varphi_{ys} - \frac{Y}{2} \varphi_{xs} \right) \frac{\partial \varphi_{x1}}{\partial Y} + \left( \varphi_{y1} - \frac{Y}{2} \varphi_{x1} \right) \frac{\partial \varphi_{xs}}{\partial Y}
\]

\[+ R_R (2 \theta_1 - 2 \theta_0 \theta_1) \lambda s i n a
\]

\[
X \frac{\partial \varphi_{x1}}{\partial X} - \frac{Y}{2} \frac{\partial \varphi_{x1}}{\partial Y} + \frac{\partial \varphi_{y1}}{\partial Y} = 0
\]  

(34)

\[
X \left[ U_s \frac{\partial \varphi_1}{\partial X} + U_1 \frac{\partial \varphi_s}{\partial X} \right] + \left[ V_s - \frac{Y}{2} U_s \right] \frac{\partial \varphi_1}{\partial Y} + \left[ V_1 - \frac{Y}{2} U_1 \right] \frac{\partial \varphi_s}{\partial Y} - \omega X \varphi_2
\]

(35)

\[= \frac{1}{\gamma} \frac{\partial^2 \varphi_1}{\partial Y^2}
\]

\[
X \left[ U_s \frac{\partial \theta_1}{\partial X} + U_1 \frac{\partial \theta_s}{\partial X} \right] + \left[ V_s - \frac{Y}{2} U_s \right] \frac{\partial \theta_1}{\partial Y} + \left[ V_1 - \frac{Y}{2} U_1 \right] \frac{\partial \theta_s}{\partial Y} - \omega X \theta_2 = \frac{1}{\nu} \frac{\partial^2 \theta_1}{\partial Y^2}
\]  

(36)

with boundary conditions:

\[
U_1 = V_1 = 0, \quad \varphi_{y1} = \varphi_{x1} = 0, \quad \theta_1 = 1 \text{ at } Y = 0
\]

(37)

\[
U_1 \to 1, \quad \theta_1 \to 0, \quad \varphi_{y1} \to 1 \quad \text{as} \quad Y \to \infty
\]

For imaginary equations:

\[
X \frac{\partial U_2}{\partial X} - \frac{Y}{2} \frac{\partial U_2}{\partial Y} + \frac{\partial V_2}{\partial Y} = 0
\]  

(38)

\[
X \left[ U_s \frac{\partial U_2}{\partial X} + U_2 \frac{\partial U_s}{\partial X} \right] + \left[ V_s - \frac{Y}{2} U_s \right] \frac{\partial U_2}{\partial Y} + \left[ V_2 - \frac{Y}{2} U_2 \right] \frac{\partial U_s}{\partial Y} + \omega X (U_1 - 1
\]

(39)

\[
= \frac{\partial^2 U_2}{\partial Y^2}
\]

\[+ \zeta X \left( \varphi_{xs} \frac{\partial \varphi_{x2}}{\partial x} + \varphi_{x2} \frac{\partial \varphi_{xs}}{\partial x} \right) + \left( \varphi_{ys} - \frac{Y}{2} \varphi_{xs} \right) \frac{\partial \varphi_{x2}}{\partial Y} + \left( \varphi_{y2} - \frac{Y}{2} \varphi_{x2} \right) \frac{\partial \varphi_{xs}}{\partial Y}
\]

\[+ R_R (2 \theta_2 - 2 \theta_0 \theta_2) \lambda s i n a
\]

\[
X \frac{\partial \varphi_{x1}}{\partial X} - \frac{Y}{2} \frac{\partial \varphi_{x1}}{\partial Y} + \frac{\partial \varphi_{y1}}{\partial Y} = 0
\]  

(40)
\[ X \left[ U_s \frac{\partial \phi_2}{\partial X} + U_2 \frac{\partial \phi_1}{\partial X} \right] + \left[ V_s - \frac{Y}{2} U_s \right] \frac{\partial \phi_2}{\partial Y} + \left[ V_2 - \frac{Y}{2} U_2 \right] \frac{\partial \phi_1}{\partial Y} + \omega X \phi_1 \]

\[ \quad - \left[ X \left( \phi_{xs} \frac{\partial U_2}{\partial x} + \phi_{x2} \frac{\partial U_1}{\partial x} \right) + \left( \phi_{ys} - \frac{Y}{2} \phi_{xs} \right) \frac{\partial U_2}{\partial Y} + \left( \phi_{ys} - \frac{Y}{2} \phi_{x2} \right) \frac{\partial U_1}{\partial Y} \right] \] (41)

\[ = \frac{1}{\gamma} \frac{\partial^2 \phi}{\partial Y^2} \]

\[ X \left[ U_s \frac{\partial \theta_2}{\partial X} + U_2 \frac{\partial \theta_1}{\partial X} \right] + \left[ V_s - \frac{Y}{2} U_s \right] \frac{\partial \theta_2}{\partial Y} + \left[ V_2 - \frac{Y}{2} U_2 \right] \frac{\partial \theta_1}{\partial Y} + \omega X \theta_1 = \frac{1}{P_c} \frac{\partial^2 \theta}{\partial Y^2} \] (42)

with boundary conditions as:

\[ U_2 = V_2 = 0, \quad \phi_{ys} = \phi_{x2} = 0, \quad \theta_2 = 0 \quad \text{at} \quad Y = 0 \]

\[ U_2 \to 0, \quad \theta_2 \to 0, \quad \phi_{ys} \to 0 \quad \text{as} \quad Y \to \infty \] (43)

3. Computational Technique

The governing dimensionless steady and unsteady equations are solved using the implicit finite difference method approach, which is exceedingly efficient. The connected non-dimensional model is translated into the primitive form using the primitive-variable formulation, resulting in a smooth algorithm. Applying central-difference along the y-axis and backward-difference along the x-axis yields numerical solutions for the converted primitive equations. We have an algebraic equation system with unknown factors \( U, V, \theta \) and \( \phi \). These variables contain coefficients in the form of a tri-diagonal matrix, and the numerical solutions to these unknown factors are found using the Gaussian elimination technique, as provided in [11,28]. The Equation (44) is used for oscillating \( \tau_w \), \( q_w \) and \( \rho \) at prominent stations of a horizontal non-conducting circular cylinder.

\[ \tau_s = \left( \frac{\partial U}{\partial Y} \right)_{y=0} + \epsilon |A_s| \cos(\omega t + \alpha_s), \] (44)

\[ \tau_i = \left( \frac{\partial \theta}{\partial Y} \right)_{y=0} + \epsilon |A_i| \cos(\omega t + \alpha_i), \]

\[ \tau_m = \left( \frac{\partial \phi}{\partial Y} \right)_{y=0} + \epsilon |A_m| \cos(\omega t + \alpha_m) \]

where,

\[ A_s = \left( u_1^2 + u_2^2 \right)^{\frac{1}{2}}, \quad A_i = \left( \theta_1^2 + \theta_2^2 \right)^{\frac{1}{2}}, \quad A_m = \left( \phi_{x1}^2 + \phi_{x2}^2 \right)^{\frac{1}{2}}, \]

\[ \alpha_s = \tan^{-1} \left( \frac{u_2}{u_1} \right), \quad \alpha_i = \tan^{-1} \left( \frac{\theta_2}{\theta_1} \right), \quad \alpha_m = \tan^{-1} \left( \frac{\phi_{x2}}{\phi_{x1}} \right) \]

To obtain accurate numerical solutions for the steady and unsteady part of the flow model, the convergence criterion is

\[ \max \left| U_s(i,j) \right| + \max |V_s(i,j)| + \max |\theta_s(i,j)| + \max |\phi_{ys}(i,j)| + \max |\phi_{x2}(i,j)| \leq \epsilon \]

A grid independent test has been considered to obtain efficient and tolerable numerical results and the computation was started at \( x = 0 \), and marked down implicitly. Here, we have taken \( \Delta x = 0.05 \) and \( \Delta y = 0.01 \), for \( i \) and \( j \) grid points, and maximum grid point 40 in the following computation with tolerance \( \epsilon = 0.00001 \) for the convergence of the obtained numerical results.
4. Results and Discussion

The impact of reduced gravity on mixed convective oscillatory electrically conducting fluid flow along a thermal, non-conducting horizontal cylinder was studied numerically. The fluid motion in the form of connected partial differential equations was transformed into a convenient form with the finite difference scheme by using the primitive variable transformation. The geometrical interpretation of the numerical results was plotted against the most appropriate various physical pertinent parameters, the reduced gravity parameter $R_g$, Prandtl parameter $Pr$ and mixed convective number $\lambda$ with some fixed parameters at two $\pi/2$ and $\pi$ locations. These results were considered valid by satisfying the given boundary conditions and the comparison of the results with the previous study is also included. The velocity had a no-slip condition exactly at the surface $y = 0$ and approached 1 as $y \to \infty$. Furthermore, the temperature and magnetic field profiles satisfied the given boundary conditions for validation of the results.

4.1. $U$, $\theta$ and $\phi$ Profiles to Check Accuracy of Numerical Data

The fluid velocity, temperature and magnetic profile were mapped against four values of reduced-gravity parameter $R_g = 0.1, 0.3, 0.5$ and $0.7$ at both positions $\alpha = \pi/2$ and $\pi$ around a non-conducting cylinder, as shown in Figure 2a–c. The fluid velocity and magnetic profile showed maximum behavior at $\pi/2$ station as $R_g$ was increased but dimensionless temperature of the fluid attained maximum magnitude as $R_g$ was decreased at $\pi$ station in Figure 2a,b. The asymptotic analysis of the numerical results for $Pr = 7.0$ (water) in the case of $U$, $\theta$ and $\phi$ were prominent. Physically, this could be because an increase in the decreased gravity parameter impacts the fluid flow’s driving potential along a thermal, non-conducting horizontal cylinder. The geometric representations of the velocity distribution, temperature, and magnetic field profile are demonstrated in Figure 3a–c against different choice values of the Prandtl parameter $Pr$ in the influence of reduced gravity and magnetohydrodynamic effects at both locations of the horizontal non-conducting cylinder. In Figure 3a, a velocity graph shows a sharp increase to its peak value for lower $Pr = 0.1$ at $\pi/2$ position then a decrease to its asymptotic value. It is depicted that the temperature of the fluid and magnetic field was reduced for maximum $Pr = 7.0$ at both stations for fixed parameter $\lambda$ in Figure 3b,c. The reason for this is because the fluid’s thermal conductivity decreases while intermolecular interactions diminish. The geometrical significance of velocity distribution, temperature, and magnetic field profile is depicted in Figure 4a–c against four choice values of $\lambda$ with reduced gravity and magnetohydrodynamics effects along the non-conducting geometry. The velocity achieved the maximum value at the $\pi/2$ location with a certain height as $\lambda$ was enhanced but it was depicted in a similar way at the $\pi$ location for all values of $\lambda$. The magnetic profile and temperature were enhanced with prominent variations for a larger value of $\lambda = 7.0$ with lower reduced gravity effects. The magnetic effects far from the surface were reduced for a lower $\lambda = 1.0$ in a similar way at both locations. From the above geometrical plots, it was depicted that the gained numerical outcomes were in prominent agreement and satisfied their boundary conditions.
Figure 2. The geometrical representation of (a) $U$, (b) $\theta$ and (c) $\phi$ at $\xi$ and $\pi$ positions with four choice values of reduced gravity $R_g = 0.1$, 0.3, 0.5 and 0.7 with some fixed parameters $\xi = 0.8$, $\gamma = 0.5$, $Pr = 7.0$, and $\lambda = 1.5$.

Figure 3. The geometrical representation of (a) $U$, (b) $\theta$ and (c) $\phi$ at $\xi$ and $\pi$ positions with four choice values of $Pr = 0.1$, 0.71, 2.0 and 7.0 with some fixed parameters $\gamma = 0.7$, $\xi = 0.8$, $Pr = 7.0$, and $R_g = 1.7$.

Figure 4. The geometrical representation of (a) $U$, (b) $\theta$ and (c) $\phi$ at $\xi$ and $\pi$ positions with four choice values of $\lambda = 0.1$, 0.71, 2.0 and 7.0 with some fixed parameters $\gamma = 0.3$, $\xi = 0.9$, $Pr = 7.0$, and $R_g = 0.3$.

4.2. Transient Shapes of $\tau_s$, $\tau_l$ and $\tau_m$ Profiles to Check Accuracy of Numerical Data

The impact of the physically emerging parameters on the transient skin-friction $\tau_s$, oscillating heat transfer $\tau_l$ and oscillating current-density $\tau_m$ around a horizontal non-
conduction cylinder was drafted for the more appropriate physical dimensionless parameters at $\alpha = \pi/2$ and $\pi$ positions. Figure 5a–c display the reduced gravity effects on unsteady numerical results. Transient $\tau_t$ and transient $\tau_m$ rate show the maximum amplitude of fluctuation in Figure 5b,c at both $\pi/2$ and $\pi$ stations in the presence of reduced gravity. In the above-displayed plots, a smaller and decreasing behavior in $\tau_t$ is illustrated under the influence of reduced gravity and mixed-convective effects in Figure 5a. It was expected that an effective change in the acceleration of gravity was produced on fluid with different density variations due to buoyancy forces. The influence of the Prandtl number $Pr$ is plotted in Figure 6a–c, where Figure 6a is devoted to expressing the diverse choices of $Pr = 1.0, 3.0$ and $7.0$ on oscillatory $\tau_t$. Figure 6b demonstrates the oscillating $\tau_t$, and Figure 6c an oscillatory current density. It was discovered in Figure 6a that the rise in $Pr$ gave a decrease in the amplitude of skin friction at $\pi$ station but increased at the $\pi/2$ position. Moreover, the maximum amplitude of fluctuation in heat transfer and current density was obtained at both the $\pi/2$ and $\pi$ location for maximum $Pr$ with prominent variation, see Figure 6b,c. The influence of $\Lambda$ for transient $\tau_t$, oscillatory $\tau_t$ rate and $\tau_m$ plots is shown in Figure 7a–c with reduced gravity and magnetohydrodynamic effects along two prominent locations of the non-conducting circular shape. The lower oscillating performance in $\tau_t$ was noticed for each choice of $\lambda = 1.0, 3.0$ and $5.0$ at both stations $\pi/2$ in Figure 7a. The larger amplitude of oscillating coefficients of $\tau_t$ and $\tau_m$ was concluded with a prominent variation for each $\lambda$ in Figure 7b,c in the presence of reduced gravity.

Table 1 presents the validated numerical data for the skin-friction coefficient for the accuracy of results by comparing with existing results available in Mehmood et al. [29] and Ilyas et al. [28] by applying the implicit finite difference approach, for diverse values of the magnetic Prandtl parameter $\gamma$ for $\xi = 0.8$, at the leading edge $\alpha = 0.0$. The comparison of skin-friction coefficients by using the local non-similarity method for magnetic force $S = 0.1$, free stream velocity gradient $n = 0.0$ and local transpiration parameter $\xi = 0.0$ through a magnetized wedge in [29] but [28] compared the numerical values of skinfriction with the finite difference method for the magnetic force parameter $\xi = 0.1$ at the leading edge $\alpha = 0.0$ with more accurate results along the magnetized cone. In the present work, the numerical results were compared with both [28,29] for magnetic force parameter $\xi = 0.8$ at the leading edge $\alpha = 0.0$ with accurate results along the non-conducting horizontal circular cylinder.

![Figure 5](image-url)  
Figure 5. The geometrical plots of (a) $\tau_t$ (b) $\tau_t$, and (c) $\tau_m$ at $\pi$ and $\pi/2$ positions with two values of reduced gravity $Rg = 0.1, 0.3$ and $0.5$ with some fixed $\gamma = 0.1, 0.3, Pr = 7.0$, and $\lambda = 1.1$. 
In Table 2, the comparison between Chawla [30] and the present analysis of the numerical values of skinfriction for three values of magnetic-force number has been computed. Chawla [30] computed the numerical values by using the Karman–Pohlhausen technique but in the present work, the analysis was performed by applying the finite-difference method $\xi = 0.1, 0.2, 0.3$ at the leading edge $\alpha = 0.0$. 

**Figure 6.** The geometrical plots of (a) $\tau_e$ (b) $\tau_t$, and (c) $\tau_m$ at $\frac{\pi}{2}$ and $\pi$ positions with two values of $Pr = 1.0, 3.0$ and 7.0 with some fixed parameters $\zeta = 0.3$, $Rg = 0.1$, $\gamma = 0.1$, and $\lambda = 1.2$.

**Figure 7.** The geometrical plots of (a) $\tau_e$ (b) $\tau_t$, and (c) $\tau_m$ at $\frac{\pi}{2}$ and $\pi$ positions with two values of $\lambda = 1.0, 3.0$ and 5.0 with some fixed parameters $\zeta = 0.4$, $Rg = 0.1$, $\gamma = 0.2$, and $Pr = 7.0$.

**Table 1.** The numerical results of skinfriction for various choices of magnetic Prandtl parameter $\gamma = 1.0, 10.0, 100.0$, for $\xi = 0.8$ at the leading edge $\alpha = 0.0$.

<table>
<thead>
<tr>
<th>$\gamma$</th>
<th>Mehmoed et al. [29]</th>
<th>Ilyas et al. [28]</th>
<th>Present Analysis</th>
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<td>10</td>
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<td>0.3137</td>
<td>0.3106</td>
</tr>
<tr>
<td>100</td>
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<td>0.3149</td>
<td>0.3118</td>
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</table>
Table 2. The numerical results of skin friction for various choices of magnetic force parameter at the leading edge.

<table>
<thead>
<tr>
<th></th>
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<tbody>
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<td>0.3007</td>
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</table>

5. Concluding Remarks

The influence of reduced gravity and magnetohydrodynamic impact on the oscillating mixed convective electrically conducting fluid flow along a thermal, non-conducting horizontal cylinder at two stations α = π/2 and π has been described numerically. The influence of diverse emerging parameters for \( U \), \( \theta \), \( \phi \) and periodic \( \tau_t \), \( \tau_m \) with physical reasoning has been described across a thermal, non-conducting cylinder. The important and significant key points are given below: it was observed that the rise in reduced-gravity \( R_g \) gave an enhancement to the fluid velocity and magnetic field. It was depicted that the temperature of the fluid and magnetic field was reduced for a maximum \( Pr = 7.0 \) at both stations for a fixed mixed convection parameter \( \lambda \). The magnetic effects far from the surface were reduced for lower \( \lambda = 1.0 \) in a similar way at both locations. The oscillatory heat transfer and current density rate showed a maximum amplitude of fluctuation at both the \( \pi/2 \) and \( \pi \) stations in the presence of reduced gravity. It was discovered that the rise in \( Pr \) gave a decrease in the amplitude of skin friction at the \( \pi \) station but increased at the \( \pi/2 \) position. The larger amplitude of the oscillating coefficients of \( \tau_t \) and \( \tau_m \) was concluded with a prominent variation for each \( \lambda \) in the presence of reduced gravity. It was depicted that the rise in heat transfer with the maximum amplitude of fluctuation was obtained at both the \( \pi/2 \) and \( \pi \) location for a higher value of \( Pr \) with prominent variations. The future work will be focused primarily on the oscillatory behavior of nanofluid flow of heat and mass transfer through a magnetized stretching sheet for various variable properties in the presence of an exothermic catalytic chemical reaction. Furthermore, it might involve hybrid nanofluids and ternary hybrid nanofluids across various geometries with an induced magnetic field.

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Conflicts of Interest: The authors declare no conflict of interest.
Nomenclature

\( \mu_v \) Velocity along \( y \) and \( -y \)-coordinates (m s\(^{-1}\))

\( H_x, H_y \) Magnetic coordinates of velocities in \( x, y \) direction (Tesla)

\( \mu \) Dynamic viscosity (kg m\(^{-1}\) s\(^{-1}\))

\( \nu \) Kinematic viscosity (m\(^2\) s\(^{-1}\))

\( \rho \) Fluid density (kg m\(^{-3}\))

\( g \) Gravity acceleration (m s\(^{-2}\))

\( \beta \) Coefficient of thermal expansion (K\(^{-1}\))

\( \nu_m \) Magnetic permeability (H m\(^{-1}\))

\( \alpha \) Thermal diffusivity (m\(^2\) s\(^{-1}\))

\( T \) Fluid temperature (K)

\( C_p \) Specific heat (J kg\(^{-1}\) K\(^{-1}\))

\( T_m \) Temperature at maximum density

\( \phi \) Dimensionless magnetic field

\( \tau_t \) Transient heat transfer

\( U \) Dimensionless velocity

\( R_g \) Reduced gravity parameter

\( \kappa \) Thermal conductivity

MHD Magnetohydrodynamic

\( T_{\infty} \) Ambient temperature (K)

\( R_e \) Reynolds number

\( G_r \) Grashof number

\( \xi \) Magnetic force parameter

\( \lambda \) Mixed convective number

\( \theta \) Dimensionalized temperature

\( \gamma \) Magnetic Prandtl parameter

\( \Pr \) Prandtl number

\( \varsigma \) Electrical conductivity (s m\(^{-1}\))

\( \Delta T \) Temperature difference

\( \tau_s \) Transient skin friction

\( \tau_m \) Transient current density

\( H_0 \) Magnetic field intensity

\( \rho_m \) Maximum density

\( U_{\infty} \) Free stream velocity

FDM Finite difference method

References

1. Potter, J.M.; Riley, N. Free convection from a heated sphere at large Grashof number. *J. Fluid Mech.* 1980, 100, 769–783. [CrossRef]


