Active Fault-Tolerant Control Scheme for Unmanned Air-Ground Attitude System with Time-Varying Delay Faults

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Featured Application: This work addresses issues related to fault-tolerant control of unmanned air-ground attitude system.

Abstract: This paper reports a designed method of fault diagnosis, estimation, and fault-tolerant control aiming at solving the problems of time–delay variation of system parameters, actuator time-varying failure, and external disturbance under the flight mode of air-ground platform. Firstly, a robust fault observer is designed to accurately detect the fault of unmanned air-ground attitude system with time-varying parameter delay and reduce the false alarm rate through reasonable assumptions; secondly, considering the actual computing power of the system, the method of estimating the overall fault size of the system instead of estimating each sub fault separately is adopted to reduce the memory space and computation. Then, based on the fault diagnosis and estimation, the fault-tolerant control rate is designed, the integral term is reasonably introduced to eliminate the chattering problem in the fault-tolerant control, and the appropriate nonlinear function is selected as the ideal control input to optimize the transient performance of the system. Finally, the stability of the system is proved, and the effectiveness of the proposed method is verified by simulation.

Keywords: unmanned air-ground attitude system; active fault-tolerant control; fault detection and estimation; time-varying delay system; actuator time-varying failure

1. Introduction

Facing the influence of external uncertain interference, time-varying failure of actuator mechanism, and time–delay change of system parameters, the premise of ensuring that the unmanned air-ground platform completes the assigned tasks to the maximum in a complex environment is that the system is supposed to have the ability of fault tolerance or self-repair. When the platform is in flight mode, the attitude control system is one of the most basic, critical, and fault prone subsystems [1,2]. Therefore, it is necessary to design a stable attitude control system for the above problems. Active fault-tolerant control technology is favored by the ability to improve the internal and external interference of the system and better improve the fault tolerance of the system [3–7], among which, fault-tolerant control applied to aircraft is the most common. Wei et al. [8] take the non–minimum phase hypersonic vehicle as the research object and combine the output redefinition strategy with the adaptive robust control method to solve the problem of attitude angle stability in the case of rudder failure. A fault-tolerant control rate of command filtering is proposed to realize the fault-tolerant control of thrust vector aircraft control surface fault and actuator fault in [9]. In [10,11], an integrated fault-tolerant control method for the damage of fighter control surface is designed which greatly reduced the amount of system calculation. L. Chen et al. [12–14] carry out fault tolerant control for sensor failure on the basis of linear parameter varying system and achieve good results on a full-scale aircraft.
At present, there are not enough research literatures on the application of fault-tolerant control technology for unmanned air-ground platform. Aiming at the control problem of the flight system of the air-ground platform, the mainly adopted methods are PID, backstepping, fuzzy control, and sliding mode control. K. Ma et al. [15] use PID and backstepping to design the control rate of position and attitude loop which overcome the strong nonlinear characteristics of the flight system of air-ground platform. H. Zhu et al. [16] combine PID control with fuzzy control method to complete the control of air-ground robot. The developed prototype proves the operability of the control algorithm. Considering specific control scenarios, Y. Y. Tao et al. [17] combine PID method with fuzzy control and sliding mode control to complete the control of air-ground rotary wing UAV. The prototype test shows that the method is effective for the air-ground UAV whose dynamic model is difficult to establish accurately. Aiming at the high order characteristics of the nonlinear system of air-ground platform, C. Wu et al. [18] use the backstepping method to design the control rate to complete the attitude and position control. D. C. Zhong et al. [19] adopt cascade PID control of inner and outer loop to realize attitude control of air-ground platform. The method can effectively improve the anti-interference ability and robustness of flight control system. However, these methods in [15–19] only take into account of some air-ground platform characteristics, such as nonlinearity and strong coupling, but the possible time-varying delay of the system and actuator time-varying failure are not considered, and so it will be unable to do anything to stop the sudden failure situation of the system from being in danger. In reality, many systems have time-varying delays, and so research about them is particularly important [20,21]. In [22,23], the stability of time-varying delay systems is studied, and the relevant stability criteria are given. Reference [24] studies the stability of stochastic nonlinear time-delay systems with external disturbances. However, reference [22–24] do not focus on specific control objects, but only provide macroscopical research ideas to solve such kind of problems. In addition, there are few studies on fault-tolerant control for time-varying delay systems [25]. Taking the four rotor UAV as the control object, the fault-tolerant control scheme proposed in [26] takes into account the factors such as model uncertainty and external interference, but does not consider the influence of system time-varying delay. In [27,28], a fault-tolerant compensation controller by auxiliary variable method is constructed, but only analyzes the influence of time-varying fault. References [29,30] take the spacecraft attitude as the control object and in the case of actuator failure, it is designed and proved that the attitude reaches the sliding surface in a limited time, but other situations are less considered.

Considering the above information, the author proposes the active fault-tolerant control method to be applied to air-ground platform. Under the conditions of knowing the time-varying delay term and the upper bound of external interference and meeting with the assumption constraints, a robust fault observer is designed to reduce the occurrence of false alarm. Meantime, the adaptive fault estimation method [25] is used to estimate the overall fault condition, and appropriate fault-tolerant control rate is designed to realize attitude stability control. Finally, the Lyapunov method is used to analyze the closed-loop stability and verified by simulation. The main contributions of this paper are as follows:

- Considering the time-varying fault of actuator, system parameter uncertainty and system state delay, the attitude stability control problem of air-ground platform in complex environment is effectively solved;
- The integral sliding control method is used to effectively reduce the chattering problem in the fault-tolerant control of the air-ground platform actuator; and
- Selecting the appropriate nonlinear function as the ideal control input to optimize the transient performance of the system shows that the overshoot is not very large and the convergence speed is accelerated.

The rest of the paper is arranged as follows. Section 2 introduces the air-ground platform model, fault model, and related hypothesis; Section 3 introduces the fault detection method and adaptive fault estimation method; in Section 4, an active fault-tolerant
controller is designed; the effectiveness of the method is proved by simulation experiments in Section 5; and, finally, conclusions are given in Section 6.

2. System Description and Preliminaries

After linearizing the model of air-ground platform in reference [19], the state space equation is obtained, and the time-varying delay term is introduced to obtain the dynamic Equations (1) and (2). The model in reference [19] is shown in Appendix A. Considering the operation efficiency and the phenomenon of universal joint lock, the quaternion differential equation is Equation (3).

\[ \dot{x}(t) = Ax(t) + A_n x(t - d(t)) + B(I + F(t))u(t) + Dv(t) \]  \hspace{1cm} (1)\]

\[ y = Cx(t) \]  \hspace{1cm} (2)\]

\[ \dot{q} = G(q) [x_1, x_2, x_3, x_4] \]  \hspace{1cm} (3)\]

where \( x(t) = [x_1, x_2, x_3, x_4, x_5] \) is roll angle, pitch angle, yaw angle, and corresponding angular rates; \( u(t) = [F_1, F_2, F_3, F_4] \) is input for control; \( y(t) \in \mathbb{R}^m \) is output vector for attitude angle. \( d(t), F(t) \) are system delay and time-varying term, respectively. \( A, A_n, B, C, \) and \( D \) are known matrices. \( v(t) \) is external interference. As shown in Figure 1, \( b, d \) are, respectively, rotor lift coefficient and drag coefficient. \( l \) is the geometric distance from the center of the rotor to the center of rotation. Quaternion \( q = [q_1, q_2, q_3, q_0] = [\tilde{q}, q_0] \) represents the rotation relationship of the body system relative to the inertial system, where \( \tilde{q} \) is skew symmetric matrix. The attitude quaternion satisfies Equations (4) and (5).

\[ \tilde{q} \cdot \tilde{q} + q_0^2 = 1 \]  \hspace{1cm} (4)\]

\[ G(q) = \frac{1}{2} \begin{bmatrix} q^* + q_0 & \tilde{q} \\ -\tilde{q} & q_0 \end{bmatrix} \]  \hspace{1cm} (5)\]

![Figure 1. Mechanical model of air-ground platform.](image)

For the attitude control system of air-ground platform, the time-varying term includes the current moment of inertia of the platform \( J \) changing \( \Delta J \) caused by the change of the total mass of the system and the change of the failure of the system actuator with time. In the paper, only additive bias fault and actuator efficiency loss (LOE) are
considered for actuator time-varying fault. The additive fault is represented by the additional control input of the rotor motor, and the multiplicative fault is represented by the loss of motor efficiency. Therefore, the actuator fault is modeled as Equation (6).

$$u = (I - E)u_c + u_a \quad \text{(6)}$$

In Equation (6), $E$ is actuator efficiency matrix which satisfies $E = \text{diag} \{ e_1, e_2, e_3, e_4 \}$, $e_i \in (0,1]$. $e_i = 0$ indicates that the actuator mechanism is completely damaged and $e_i = 1$ shows actuator intact. $u_a$ is an additive offset input. The overall fault size of the system is expressed as $f$, and Equation (1) is rewritten in combination with Equation (6) as:

$$\dot{x}(t) = Ax(t) + A_x(x(t - d(t)) + B(I + F(t))u_c + f + Dv(t)$$

$$f = -B(I + F(t)) (Eu_c - u_a) \quad \text{(8)}$$

The overall control structure of the attitude system is shown in Figure 2. The following assumptions are made for the system.

**Assumption 1.** $(A, B), (A, C)$ are controllable and observable.

**Assumption 2.** The time-varying term of the moment of inertia in the system $F(t)$ is bounded and satisfies $J \leq F(t) \leq J$. The time–delay term is continuously bounded and differentiable which satisfies $0 \leq d(t) \leq h, \dot{d}(t) \leq \tau \leq 0$. The overall fault size of the system meets $\|f\| \leq \mu, \|\dot{f}\| \leq \delta$, where $h, \tau, \mu, \delta$ are constants.

**Assumption 3.** The external disturbance is bounded and satisfies $\|Dv(t)\| \leq \overline{I}$, where $\overline{I}$ is constant.

**Remark 1.** The controllability and observability of $(A, B), (A, C)$ are the premise of system fault estimation and regulation. The state space model of air-ground platform in the paper conforms to controllability and observability. Whether it is the size of system fault or external disturbance, it should not be infinite, and bounded constraints are necessary. For other assumptions, such that the air-ground platform is a rigid symmetrical structure, see literature [15,19].

**Lemma 1.** There exist $P = P^T \succeq 0$, constant $\rho > 0$, which make matrix inequality hold [5]:

$$2X^TPX \leq \rho^{-1}X^TPX + \rho Y^TP^{-1}Y \quad \text{(9)}$$

![Diagram](image-url) **Figure 2.** Attitude control structure of air-ground platform.
3. Design of Fault Detection and Estimation Observer

Traditional fault detection methods are easily affected by the time-delay variation of the model in the actual scene [31], resulting in the increase of false alarm rate. Based on this, this section first proposes a robust fault detection method, and then designs an adaptive fault observer to estimate the total fault of the actuator. It is different from the observation of each sub fault in literature [9], which greatly improves the operation speed and saves memory space.

3.1. Robust Fault Detection

The following fault observers are designed:

\[
\hat{x}_f(t) = A\hat{x}_f(t) + A_w\hat{x}_w(t-d(t)) + B(I + F(t))u_v \\
+ \Gamma_1(x(t) - \hat{x}_v(t)) + \Gamma_2(x(t-d(t)) - \hat{x}_v(t-d(t)))
\]

where \( \hat{x}_f(t) \) is the estimated value of the system state in the body coordinate system, \( \Gamma_1 \), \( \Gamma_2 \) is the gain matrix, and is taken as the multiple of the identity matrix. Let the system state estimation error be \( \hat{x}_f(t) = x(t) - \hat{x}_v(t) \), combining with Equations (7) and (10) as:

\[
\dot{\hat{x}}_f(t) = A\hat{x}_f(t) + A_w\hat{x}_w(t-d(t)) - \Gamma_1\hat{x}_v(t) - \Gamma_2\hat{x}_v(t-d(t)) + f + Dv(t)
\]

According to the matrix norm compatibility principle:

\[
\begin{align*}
\|A\hat{x}_f(t)\| & \leq k_2\|\hat{x}_f(t)\| \\
\|A_w\hat{x}_w(t-d(t))\| & \leq k_1\|\hat{x}_w(t-d(t))\| \leq k_2\|\hat{x}_v(t)\|
\end{align*}
\]

where \( k_1 \) is \( \max \{SVD(A)\} \), \( k_2 \) is \( \max \{SVD(A_w)\} \), and \( SVD(\cdot) \) is the singular value of the corresponding matrix.

**Theorem 1.** If the fault detection threshold is set to \( x_{fd} = \frac{\bar{d}}{(\lambda_{\min}(\Gamma_1) + \lambda_{\min}(\Gamma_2))-(k_1+k_2)} \), at this time, when the system state is met \( \|\hat{x}_f\| > x_{fd} \), the system fault is detected for the failed air-ground platform system in Equation (7), on the premise of meeting the Assumption 3.

**Proof of Theorem 1.** By Considering the Lyapunov candidate as:

\[
V = \frac{1}{2} \hat{x}_v^T \hat{x}_v
\]

Its derivative is

\[
\dot{V} = \hat{x}_v^T \hat{x}_v = \hat{x}_v^T (A\hat{x}_v + A_w\hat{x}_w(t-d(t)) - \Gamma_1\hat{x}_v - \Gamma_2\hat{x}_v(t-d(t)) + f + Dv(t))
\]

\[
\leq -\left[ (\lambda_{\min}(\Gamma_1) + \lambda_{\min}(\Gamma_2))-(k_1+k_2) \right] \|\hat{x}_v\|^2 + \hat{x}_v^T f + \hat{x}_v^T Dv(t)
\]

where \( \lambda_{\min}(\cdot) \) represents the smallest eigenvalue of the corresponding matrix.

When \( f = 0 \), that is the system does not fail, we can get

\[
\dot{V} \leq -\left[ (\lambda_{\min}(\Gamma_1) + \lambda_{\min}(\Gamma_2))-(k_1+k_2) \right] \|\hat{x}_v\|^2 + \bar{d} \|\hat{x}_v\|^2
\]

If \( \|\hat{x}_v\| \leq \frac{\bar{d}}{(\lambda_{\min}(\Gamma_1) + \lambda_{\min}(\Gamma_2))-(k_1+k_2)} \), then \( \dot{V} \leq 0 \). It obtains that when the system does not fail, the system state is stable. When \( f \neq 0 \), if \( \dot{V} \leq 0 \), it obtains that:

\[
\|\hat{x}_v\| \leq \frac{\|f\|+\bar{d}}{(\lambda_{\min}(\Gamma_1) + \lambda_{\min}(\Gamma_2))-(k_1+k_2)}
\]
Thus, it obtains that when the system state \( \|\hat{x}_e\| > x_{\mu} \) is satisfied, the fault is detected. □

**Remark 2.** The above proof cannot obtain the necessary and sufficient condition of fault detection threshold. When the system state is \( \|\hat{x}_e\| \leq x_{\mu} \), the system also has a certain probability of failure.

### 3.2. Design of Fault Estimation Observer

After fault detection, an adaptive fault observer is designed to estimate the total fault error of the system actuator, as shown in Equations (17) and (18).

\[
\dot{x}_e(t) = A\hat{x}_e(t) + A_{m}\hat{x}_e(t-d(t)) + B(I + F(t))u_0 + \hat{f} + L(y(t) - \hat{y}_e(t)) \\
\quad + H(y(t-d(t)) - \hat{y}_e(t-d(t)))
\]

\[\dot{y}_e(t) = C\hat{x}_e(t)\]  

(17)  

(18)

The adaptive rate \( \dot{f} \) is designed as:

\[\dot{f}(t) = \Lambda(K_1\hat{y} + K_2\hat{y})\]

(19)

where \( \Lambda = \Lambda^T > 0 \), \( K_1, K_2 \) are the gain matrix. Let \( \bar{y} = y - \hat{y}_e \), \( \bar{f} = f - \hat{f} \), \( \bar{x}_e = x - \hat{x}_e \), combining with Equations (17) and (18), Equation (20) is obtained.

\[
\dot{x}_e = (A - LC)\bar{x}_e + (A_m - HC)\hat{x}_e(t-d(t)) + \bar{f} + Dv(t)
\]

(20)

**Theorem 2.** Considering Assumptions 2 and 3, there are matrices \( P_1, P_2 \) which satisfy the relation in Equation (21) and \( P_1 = P_1^T > 0, P_2 = P_2^T > 0 \).

\[
\begin{bmatrix}
P_1^T &=& K_1C \\
P_2^T &=& P_1^T \Lambda = I
\end{bmatrix}
\]

(21)

There exist observation matrices \( L, H \) such as:

\[
\begin{bmatrix}
-2K_2C + 2\beta^T M & -2K_2CD & -2K_2C(A - LC) & 2K_2C(A_m - HC) \\
* & D^TP_2 & 0 & 0 \\
* & * & \Omega_1 & 0 \\
* & * & * & \Omega_2
\end{bmatrix} < 0
\]

(22)

where \( \Omega_1 = (A - LC)^T P_1 + P_1(A - LC) \), \( \Omega_2 = (A_m - HC)^T P_1 + P_1(A_m - HC) \), which makes the system state estimation error stable under the designed adaptive law.

**Proof of Theorem 2.** By Considering the Lyapunov candidate as:

\[
V_i = \bar{x}_e^T P_i \bar{x}_e + f^T P_2 \bar{f} + \int_{t-d(t)}^{t} \bar{x}_e^T P_2 \bar{x}_e dt 
\]

(23)

Its derivative is

\[
\dot{V}_i = \bar{x}_e^T P_i \dot{x}_e + \bar{x}_e^T P_2 \hat{x}_e + 2\bar{f}^T P_2 \bar{f} + \bar{x}_e^T P_2 \bar{x}_e - (1 - d(t)) \bar{x}_e^T P_2 \bar{x}_e 
\]

(24)

Combining with Equations (19)–(21), it is further simplified to obtain...
\[ V_1 = \dot{x}_e^T \left[ (A - L C)^T P_1 + P_1 (A - L C) \right] x_e + 2 \dot{x}_e^T \dot{x}_e P_1 t + 2 \dot{x}_e^T P_1 Dv(t) \\
+ \dot{x}_e^T (t - d(t)) \left[ (A_n - H C)^T P_1 + P_1 (A_n - H C) \right] x_e(t - d(t)) \\
+ 2 \hat{f}^T P_2 \left( \hat{f} - \dot{\hat{f}} \right) + \dot{x}_e^T P_2 \dot{x}_e - (1 - d(t)) x_e^T P_2 \dot{x}_e \\
= \dot{x}_e^T \left[ (A - L C)^T P_1 + P_1 (A - L C) \right] x_e + 2 \dot{x}_e^T \dot{x}_e P_1 t + 2 \hat{f}^T P_2 \dot{f} \\
+ \dot{x}_e^T (t - d(t)) \left[ (A_n - H C)^T P_1 + P_1 (A_n - H C) \right] x_e(t - d(t)) \\
- 2 \hat{f}^T K_2 C (A - L C) \dot{x}_e - 2 \hat{f}^T K_2 \dot{C} \dot{f} + \dot{x}_e^T P_2 \dot{x}_e - (1 - d(t)) x_e^T P_2 \dot{x}_e \\
- 2 \hat{f}^T K_2 C Dv(t) - 2 \hat{f}^T K_2 C (A_n - H C) x_e(t - d(t)) \tag{25} \]

According to Lemma 1, there exist matrix \( M = M^T \), constant \( \beta > 0 \), which satisfy:
\[
2 \hat{f}^T P_2 \dot{f} \leq 2 \beta^{-1} \hat{f}^T M f + 2 \beta \hat{f}^T \left( P_2^T M^{-1} P_2 \right) \dot{f} \\
\leq 2 \beta^{-1} \hat{f}^T M f + 2 \mu^2 \lambda_{\max} (P_2^T M^{-1} P_2) \tag{26} \]

From inequality scaling and Assumption 2, we obtain
\[
\dot{x}_e^T P_2 \dot{x}_e - (1 - d(t)) x_e^T P_2 \dot{x}_e \leq \dot{x}_e^T P_2 \dot{x}_e - (1 - \tau) x_e^T P_2 \dot{x}_e \leq 0 \tag{27} \]

Combining with Equations (25)–(27), we obtain
\[
\dot{V}_1 \leq \left[ f^T \nu^T \ x_e^T \ x_e^T (t - d(t)) \right] Q \left[ f^T \nu^T \ x_e^T \ x_e^T (t - d(t)) \right]^T + \gamma \tag{28} \]
where \( \gamma = 2 \mu^2 \lambda_{\max} (P_2^T M^{-1} P_2) / Q < 0 \).
\[
Q = \begin{bmatrix}
-2K_2C + 2\beta^{-1}M & -2K_2CD & -2K_2C(A - LC) & 2K_2C(A_n - H C) \\
* & \hat{d}^T \hat{1} & \hat{d}^T \hat{p}_2 & 0 \\
* & * & \Omega_1 & 0 \\
* & * & * & \Omega_2
\end{bmatrix} \tag{29} \]

\[ \square \]

**Remark 3.** Further, the following expression is obtained:
\[
\dot{V}_1 \leq \kappa V_1 + \gamma \tag{30} \]

where \( \kappa = \frac{2 \lambda_{\max} (q)}{\max \{J, \beta\}} \). According to the finite time convergence conclusion in [32], the convergence time of system state observation error is in Equation (31). The system state observation error will converge to the invariant set \( R^c \left( \tilde{x}_e, \tilde{x}_e(t - d(t)), \tilde{f} \right) \) at an exponential rate \( t_f \leq \frac{2\nu(0)}{\gamma} \).

**4. Fault Tolerant Control**

The nonlinear function is selected as the ideal control input, and the system state error is defined as \( x_e(t) = \alpha \ln (1 + e x) \).
\[
e_e = x(t) - x_e(t) \tag{31} \]

Let \( \Delta f \) is the estimation error, then \( f = \hat{f} + \Delta f \).

**Assumption 4.** \( \Delta f \) is bounded and satisfies \( \Delta f = \sup_{t \in \mathbb{R}} \| \hat{f} \| = \xi \).

The derivative of Equation (31) is obtained
\[
\dot{e}_c = Ax(t) + A_n x(t-d(t)) + B(I + F(t))u_c(t) + \hat{f} + \Delta f + Dv(t) - \dot{x}_c(t) \tag{32}
\]

Note that \( g(x(t), x(t-d(t))) = Ax(t) + A_n x(t-d(t)) \), then Equation (32) is rewritten as:

\[
\dot{e}_c = g(x(t), x(t-d(t))) + B(I + F(t))u_c(t) + \hat{f} + \Delta f + Dv(t) - \dot{x}_c(t) \tag{33}
\]

The integral sliding surface is defined as follows:

\[
S = e_c(t) - e_c(t_0) + \int_{t_0}^{t} e_c(\tau) d\tau \tag{34}
\]

To ensure the continuity of sliding control, \( \dot{S} = 0 \).

**Remark 4.** The introduction of the integral term ensures that the sensitivity of the sliding controller when the state error of the driving system reaches the sliding surface is reduced.

Let \( O = B(I + F(t)) \), pseudo inverse matrix of defined \( O \) is \( O^* = O^T (OO^T)^{-1} \), which satisfies \( OO^* = I \). The fault tolerant control rate of the system is designed as follows:

\[
u_c(t) = \left[ B(I + F(t)) \right]^{-1} \left[ x(t) - g(x(t), x(t-d(t))) - \hat{f} - \Delta f - \chi \text{sat}(S) \right] \tag{35}
\]

where \( \chi \) is the positive diagonal gain matrix, and the saturation function is defined as:

\[
\text{sat}(S) = \begin{cases} 
\text{sign}(S), & \text{if } |S| > \Phi \\
S, & \text{if } |S| \leq \Phi
\end{cases} \tag{36}
\]

where \( \Phi \) is the thickness of boundary layer.

**Theorem 3.** In the error differential Equation (33) with the premise of Assumptions 3 and 4, combining with Equations (19), (34) and (35), when \( \chi \geq \hat{\chi} + \dot{\xi} \), the system state is supposed to remain stable and converge to the expected value in fault conditions.

**Proof of Theorem 3.** By Considering the Lyapunov candidate as:

\[
V_2 = \frac{1}{2} s^T s \tag{37}
\]

The derivation of Equation (37) shows:

\[
\dot{V}_2 = s^T \left[ g(x(t), x(t-d(t))) + B(I + F(t))u_c(t) + \hat{f} + \Delta f + Dv(t) - \frac{\alpha e}{1+\epsilon x} + x_c(t) - x(t) \right] \tag{38}
\]

Consider the following inequality:

\[
\frac{\alpha x}{1+x} \leq \alpha \ln(1+x) \leq \alpha x \quad (x > -1) \tag{39}
\]

Combining with Equation (35) and Assumptions 3 and 4, the following is obtained from inequality (39) amplification and reduction.

\[
\dot{V}_2 \leq s^T \left( -\chi \text{sat}(s) + f - \hat{f} \right) \leq s^T \left( -(\dot{\xi} + \ddot{\xi}) \text{sat}(s) + \ddot{\xi} \right) \leq -\ddot{\xi} \|v\| \tag{40}
\]

\[\Box\]

5. Simulation and Analysis

In order to prove the effectiveness of the proposed active fault-tolerant control method for the attitude system of air-ground platform with time-varying delay fault, the relevant system parameters and fault simulation conditions are designed as follows:

\[
A = \begin{bmatrix} 0_{3 \times 3} & I_{3 \times 3} \\ 0_{3 \times 3} & 0_{3 \times 3} \end{bmatrix}, \quad A_d = \begin{bmatrix} 0_{3 \times 3} & 0.1 I_{3 \times 3} \\ 0_{3 \times 3} & 0_{3 \times 3} \end{bmatrix}, \quad C = \begin{bmatrix} I_{3 \times 3} & 0_{3 \times 3} \end{bmatrix}, \quad D = I_{6 \times 6},
\]
Model parameters selection are based on the designed air-ground platform model in Figure 1. The external disturbance is assumed to be \([-0.005\sin t \quad 0.005\sin t \quad -0.005\sin t]^T\), The time-varying delay terms are \(d(t) = 0.1 + 0.2\cos t\) and \(F(t) = 0.01\sin t\).

When there is no fault, the attitude system of air-ground platform adopts PID control method in flight mode. The control rate is designed as:

\[
u = B^*(-k_pJ[x_1, x_2, x_3]^T + k_dJ[x_4, x_5, x_6]^T + k_iJ[x_4, x_5, x_6]^T)
\]  (41)

where \(B^*\) is a pseudo inverse matrix, which satisfies \(B^* = B^T(BB^T)^{-1}\).

When the fault occurs, the system detects and estimates the fault and switches to fault-tolerant control mode. According to the fault analysis of Section 2, we assume that three rotors have different degrees of fault damage at the same time, which seldom happens in reality. The fault design is shown in Table 1. It is assumed that at 25 s, rotor 1, 2, and 3 have actuator bias fault and LOE fault, respectively, where the bias fault is expressed as: \(f_2 = -0.03 + 0.01\sin t, f_3 = 0.06 + 0.01\sin t\).

We use MATLAB for simulation verification. The system simulation time is set to 100 s and the sampling time is 0.02. It is reasonable for rotor motor to respond to the attitude change in time in real situations. The initial value of attitude angle is \([0.1 \ 5 \ 160]^T\) in degree, and the initial angular velocity is \([0.006 \ 0.005 \ 0.003]^T\) in rad/s. The desired attitude angle is expressed in quaternion \([0 \ 0 \ 0 \ 1]^T\) and the desired angular velocity is \([0 \ 0 \ 0]^T\) in rad/s. The fault detection threshold is set to 0.17, and the saturation constraint of the actuator mechanism is \(u_{\text{max}} = 10\ N\). The simulation results are shown in Figures 3–6.

**Table 1.** Fault assumption of air-ground platform in flight mode.

<table>
<thead>
<tr>
<th>Fault Type</th>
<th>Rotor 1</th>
<th>Rotor 2</th>
<th>Rotor 3</th>
<th>Rotor 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>LOE</td>
<td>0.5</td>
<td>0.2</td>
<td>Normal</td>
<td>Normal</td>
</tr>
<tr>
<td>Bias Fault</td>
<td>None</td>
<td>(f_2)</td>
<td>(f_1)</td>
<td>None</td>
</tr>
</tbody>
</table>
Figure 3. Comparison between fault-tolerant control effect and PID control effect. (a) Description of fault-tolerant control effect; and (b) description of PID control effect.

Figure 4. (a) Description of fault estimation; and (b) description of fault estimation error.
Figure 5. (a) Description of system control input; and (b) description of fault detection result.

Figure 6. Comparison of transient performance under ideal input.

Figure 3a shows that after the fault occurs, the active fault-tolerant control scheme designed based on time-varying delay system in the paper can effectively suppress the adverse effects caused by the fault. In contrast, PID control cannot achieve stable attitude control in Figure 3b. Figure 4a shows the system fault estimation error. When time-varying fault is introduced, the estimation error in Figure 4b increases significantly and then tends to zero, which shows the effectiveness of the proposed active fault-tolerant control method. Figure 5a shows the output of the system under PID mode and fault-tolerant control. It can be seen that the chattering problem can be reduced under fault-tolerant control. This is because the integral term is reasonably introduced and the saturation function $sat(S)$ is added to the control rate. The fault detection results obtained in the Figure 5b can finally converge under the action of continuous fault, indicating the robustness of the fault detection method. In the experiment, the system fault at 25 s is successfully detected, but it can be seen from the small difference between the Figure 3a,b at 10~20 s that there will still be false alarm. Figure 6 shows that selecting appropriate nonlinear ideal
virtual input can effectively improve the transient performance of the system, which is manifested in accelerating the convergence speed and reducing the overshoot.

In order to further verify the effectiveness of the proposed method, the hardware–in–the–loop simulation is carried out with the help of RflySim platform [33] developed by Beihang University. The flight control device is selected as pixhawk 2.4.8. Wirelessly connect the RadioLink R9DS receiver to the remote–control handle. The simulation condition parameters remain unchanged and the fault settings are still as shown in Table 1. The fault occurrence time becomes 50 s. The hardware–in–the–loop simulation model is shown in Figure 7. The hardware system connection is shown in Figure 8.

The code generated by the designed AFTC controller algorithm is downloaded to the pixhawk system, and the USB physical data line is used to replace the virtual signal transmission in the simulation. Rotor parameters and sensor parameters such as attitude are sent to pixhawk system through Coptersim (Coptersim is a real–time motion simulation software and the core of the RflySim platform). When the fault occurs, with the help of AFTC method, the expected attitude tracking can be realized after 2.2 s (2.5% error) in Figure 9a. Figure 9b shows the output of the system under PID mode and fault-tolerant control. Although the attitude tracking effect is lower than that before the failure, it is acceptable in practice. Through the preliminary test on the real pixhawk autopilot system, on the one hand, it verifies the proposed AFTC method and effectively eliminated the problems in the actual flight test, on the other hand, it lays foundation for the actual air–ground platform manufacture.

Figure 7. Hardware–in–the–loop simulation model.
In addition, from the power consumption calculation formula shown in Equation (42),

\[ W = \int_{t_1}^{t_2} \| u_c(t) \|^2 dt \]  (42)

it obtains that PID control and AFTC control consume 0.347 and 3.752, respectively, in MATLAB simulation (the former saves about 10 times more performance than the latter). Considering that the control quantity will be converted into the action of the actuator in the actual project, we use the output square of the control quantity which shows the energy consumption of the actuator as the index to evaluate the control algorithm. According to the evaluation index, it is necessary to switch between PID control and AFTC control on the premise that the attitude is controllable as shown in Figure 2.
6. Conclusions

In the paper, robust fault detection, fault estimation, and fault-tolerant control rate are designed to realize attitude stability control of air-ground amphibious system with time-varying delay under fault. The designed robust fault detection method can reduce the false alarm rate. The fault observer is designed by estimating the overall fault size of the system instead of estimating each sub fault, respectively, which saves memory and improves operation efficiency. The proposed fault-tolerant control method can reduce the chattering of the control input and takes the nonlinear function as the ideal control input to optimize the transient performance of the system. Future work will focus on the combination of fault-tolerant control method and reachable set method and complement the reconstruction ability in fault-tolerant control with the predictability and reachability of reachable set method, so as to design a self-healing control system to “eliminate” faults.

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Appendix A

In Reference [19], the nonlinear model contains five equations, which are torque equations, mechanical equations, navigation equations, motion equations, and ground driving motion equations. The nonlinear mathematical model formula of air-ground platform hovering in flight mode is obtained, where $\theta$, $\phi$, and $\psi$ are attitude angle, $x$, $y$, and $z$ are the offset from the origin position of the initial ground coordinate system, $F$ is lifting force, $\Omega$ is rotor speed, $l$ is moment of inertia, $\omega$ is angular velocity in body coordinate system, $p$, $q$, and $r$ are linear speed of body coordinate system, $l$, $b$, and $d$ are the distance from the center of mass of the air-ground platform to the rotor shaft, rotor lift coefficient and rotor drag coefficient, respectively, $\Delta s$ is wheel travel distance, and $\Delta \theta$ is the steering angle increment.
$$\begin{align*}
    \dot{x} &= -p \cos \theta \cos \psi + q(\sin \theta \sin \psi - \cos \psi \cos \phi) \\
    \dot{y} &= p \cos \theta \sin \psi + q(\sin \theta \sin \psi + \cos \psi \cos \phi) \\
    \dot{z} &= -p \sin \theta + q \sin \theta \cos \phi + r \cos \theta \cos \phi \\
    \dot{x}_1 &= \begin{bmatrix} x_0 \\ y_0 \\ \theta_0 \end{bmatrix} \\
    &+ \begin{bmatrix} -\Delta s \sin(\theta_0 + \Delta \theta / 2) \\ -\Delta s \cos(\theta_0 + \Delta \theta / 2) \\ \Delta \theta \end{bmatrix}
\end{align*}$$

(A1)

References


