A Calculation Method of Thermal Pore Water Pressure Considering Overconsolidation Effect for Saturated Clay

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Abstract: With the increase of soil consolidation degree, the pore water pressure induced by thermal loading drops dramatically. To conveniently and quickly calculate the thermal pore water pressure inside the soil under different overconsolidation states and quantify overconsolidation effect on thermal pore water pressure, a calculation method of thermal pore water pressure considering overconsolidation effect for saturated clay is proposed. The method is verified by the relevant experimental data, and good agreements were achieved. Through analyzing the influence mechanism of OCR on the thermal pore water pressure, three important findings were captured. (1) For overconsolidated clay, thermal pore water pressure decreases nonlinearly with the increase of OCR. (2) There is a critical threshold of OCR 4.3; when 1 < OCR ≤ 4.3 (slightly overconsolidated state), the ratio of compression line slope to recompression line slope (Λ) of overconsolidated clay is consistent with that of the normally consolidated clay, while when OCR > 4.3 (highly overconsolidated state), the value of Λ is smaller than that of normally consolidated clay. (3) For highly overconsolidated clay (OCR > 4.3), considering the reducing of Λ with OCR, the prediction accuracy of the thermal pore pressure calculation method has been greatly improved; especially when OCR equals 30, the prediction accuracy improves by 92.7% as temperature change achieves 35 °C.

Keywords: overconsolidation effect; thermal pore water pressure; calculation method; saturated clay

1. Introduction

During the service period of buffer materials for nuclear waste repository [1–3], thermal prefabricated vertical drains [4,5], geothermal energy piles [6], and buried cables and pipelines [7], the temperature of the surrounding soil may change significantly. However, when the soil temperature changes, thermal pore water pressure will be generated in the soil due to the difference of thermal expansion characteristics between the pore fluid and soil particles. In case of geological formations with low hydraulic conductivity, the thermal pore water can cause the loss of effective stress. Furthermore, with the dissipation of thermal pore water pressure, the soil will produce additional thermal volume change. It is found that the magnitude of thermal pore water pressure mainly depends on the stress history. Knowledge of the volume change of saturated soil under different stress states, which is a key factor that needs to be considered in design of thermo-active structures, must accurately estimate the thermal pore water pressure with different degrees of overconsolidation.

A series of studies have investigated the thermal response of saturated clay soils. To explore the evolution law of thermal pore water pressure under undrained conditions, Campanella and Mitchell [8], Ghaaowd et al. [9], Abuel-Naga et al. [10], and Ghabezloo et al. [11] carried out related experimental investigations. These experimental studies found that the magnitude of thermal pore water pressure depends on the compressibility of the soil, the physicochemical coefficient of structural volume change, initial void ratio, initial effective stress, the change in temperature, and thermal expansion coefficients of pore water and soil particles. Furthermore, Abuel-Naga et al. [10], Ghabezloo et al. [11],
Burghignoli et al. [12], and McCartney [13] explored the thermally induced volume change of soil by conducting drainage heating tests. Among them, Abuel-Naga et al. [10] used the theoretical microstructure mechanism to explain the thermally induced volume change behavior under different stresses. Based on the principle of particle rearrangement in porous granular materials undergoing thermodynamic process, Bai et al. [14,15] established a thermo-hydro-mechanical constitutive model. This model can accurately describe the irreversible consolidation of normally consolidated saturated soils induced under thermal loading and the aging effect caused by cyclic thermal loading. Subsequently, under the framework of granular thermodynamics, Bai et al. [16] derived a generalized effective stress principle, which can automatically consider the influence of the stress path, temperature path, and soil structure.

It should be noted that both the magnitude of thermal pore water pressure and the sign of the thermally induced volume change are affected by the degree of overconsolidation [10,12,14,17]. With the dissipation of the thermal pore water pressure, normally consolidated clays will exhibit irreversible volume shrinkage, highly overconsolidated clays will undergo elastic thermal expansion, and slightly overconsolidated clays will show a volume change trend that expands first and then shrinks [13,18–20]. Meanwhile, for overconsolidated clays, there is a transition temperature. When the soil temperature exceeds the transition temperature, soil deformation will change from expansion to contraction [18,21–23]. The reason for the different deformation laws of soils with different overconsolidation degrees may be due to the difference in thermal pore water pressure [21]. To explore the effect of overconsolidation ratio on the changes of thermal pore water pressure and thermal volume, constitutive models that can consider the influence of stress history are established based on the Cambridge model or modified Cambridge model [3,10,24,25]. However, because the yield caused by temperature and stress is considered, many parameters are involved in these constitutive models, which makes the solution process extremely cumbersome. Furthermore, these constitutive models cannot obtain an explicit expression of thermal pore water pressure [26], which makes engineering applications very inconvenient. To conveniently and quickly predict the thermal pore water pressure, Campanella and Mitchell [8] proposed a thermo-porous-mechanical model by using concepts of thermoelasticity and linear elasticity. However, for highly overconsolidated soils, the thermo-porous-mechanical model may no longer be applicable. Based on the unified hardening model established by Yao and Zhou [25], Wang et al. [26] established a calculation method of thermal pore water pressure including OCR, but when the soil is in highly overconsolidated state, there is a large gap between the prediction results and the experimental results. In addition, the acquisition of thermal parameters in this calculation method requires additional thermal tests.

In summary, various methods for calculating thermal pore water pressure have been developed. However, the overconsolidation effect has not been well explored. To conveniently and quickly predict the thermal pore water pressure in overconsolidated clay and quantify overconsolidation effect on the thermal pore water pressure, by introducing the parameter $\Lambda$ affected by OCR and the nonlinear relationship between OCR and the thermal pore water pressure, a calculation method of thermal pore water pressure considering overconsolidation effect for saturated clay is proposed. In addition, the calculation method is applied to predict the thermal pore water pressure, and the predicted results are compared with experimental data.

2. Calculation Method of Thermal Pore Water Pressure with Different OCR

Campanella and Mitchell [8] developed a calculation model of thermal pore water pressure in saturated clay during an undrained heating test:

$$\Delta u_T = \frac{n(a_{gw} - a_s) + a_{VT}}{m_v} \Delta T$$

(1)
in which \( \Delta u_T \) (kPa) is the change of pore pressure caused by thermal loading, \( n \) is the porosity, \( m_v \) (kPa\(^{-1}\)) is the compressibility of soil skeleton, \( \Delta T \) (°C) is the change in temperature, \( \alpha_{sT} \) (°C\(^{-1}\)) is the physicochemical coefficient of structural volume change, and \( \alpha_w \) (°C\(^{-1}\)) and \( \alpha_s \) (°C\(^{-1}\)) are the volumetric expansion coefficients of pore water and soil particles, respectively.

Under undrained heating conditions, saturated soil will expand along the secondary compression curve, and the compressibility of soil skeleton can be estimated by the isotropic recompression curve and written as

\[
m_v \approx \left( m_v \right)_r = \frac{\kappa}{1 + e_0 p'}
\]

in which \( e_0 \) is the initial void ratio, \( \kappa \) is the slope of the isotropic recompression line, \( p' \) is the mean effective stress, and \( (m_v)_r \) is the compressibility of the soil skeleton determined from the recompression curves.

Substituting Equation (2) into Equation (1) results in

\[
\Delta u_T = n(\alpha_w - \alpha_s) + \alpha_{sT} \kappa \left( 1 + e_0 p' \right) \Delta T
\]

For most soils, the values of \( \alpha_w \) and \( \alpha_s \) are usually taken as \( 1.7 \times 10^{-4} \) (°C\(^{-1}\)) and \( 3.5 \times 10^{-5} \) (°C\(^{-1}\)), respectively [8,9,12,27].

Since the change of porosity during undrained heating tests is generally negligible and has little effect on the thermal pore water pressure, the porosity \( n \) is approximately equal to the initial porosity \( n_0 \) [9,28].

The physicochemical coefficient \( \alpha_{sT} \) is often used to characterize the volume change caused by soil structural rearrangement under unit thermal loading, which is not straightforward to obtain [8,9]. In fact, \( \alpha_{sT} \) can be estimated from experimental data of \( \Delta u_T \) at a given \( \Delta T \), and the relationship is rearranged as [9]

\[
\alpha_{sT} = \left( \frac{\Delta u_T}{p'_0} \right) \left[ \frac{\kappa}{1 + e_0 \Delta T} \right] - n(\alpha_w - \alpha_s)
\]

where \( p'_0 \) (kPa) is the initial mean effective stress.

Because Equation (4) needs to know the experimental data of \( \Delta u_T \), the undrained heating test must be carried out. To avoid complicated heating tests, Ghaawd et al. [9] proposed an empirical expression for the physicochemical coefficient of normally consolidated clay, which can be written as

\[
\alpha_{sT} = 1.0 \times 10^{-4} e^{-0.014 I_p}
\]

where \( I_p \) is soil plasticity index and \( e \) is the Napierian base.

In addition, through undrained heating tests, Wang et al. [26] found that the thermal pore water pressure of normally consolidated and overconsolidated soil meets

\[
\Delta u_{Tocr} = \ln(1 + \frac{1.717}{OCR}) \Delta u_T
\]

where \( \Delta u_{Tocr} \) is the thermal pore water pressure of the overconsolidated soil.

Substituting Equation (3) into Equation (6) results in

\[
\Delta u_{Tocr} = \ln(1 + \frac{1.717}{OCR}) \frac{n(\alpha_w - \alpha_s) + \alpha_{sT} \kappa}{1 + e_0 p' \Delta T}
\]

According to Equation (7), it can be seen that \( \kappa \) is a key parameter for estimating thermal pore water pressure. However, compared to the slope of the isotropic recompression line, the slope of the isotropic compression line (\( \Lambda \)) is more commonly adopted in studies.
to calculate thermal pore water pressure. Therefore, the value of $\Lambda$ is used to estimate $\kappa$. Meanwhile, according to the Cam-Clay model, it can be known that the compression index, $C_c$, and $\Lambda$, and the rebound index, $C_s$, and $\kappa$ satisfy, respectively,

$$\kappa = 0.434C_s \quad (8)$$

$$\Lambda = 0.434C_c \quad (9)$$

Assuming that the ratio of $\Lambda$ and $\kappa$ is $\Lambda$, the below equation can be obtained:

$$\Lambda = \frac{\Lambda}{\kappa} = \frac{C_c}{C_s}$$

(10)

For normally consolidated clays, $\Lambda$ is generally constant, and the range of $\Lambda$ is about 4.8–10 [9, 29–31]. However, the stress history can affect the value of $\Lambda$ [32–34] and then change the magnitude of the thermal pore water pressure. In order to better reflect the influence of stress history on thermal pore water pressure, the relationship between rebound index $C_s$ and OCR is introduced [32]:

$$C_s = 0.0213\ln(OCR) + 0.0288 \quad (11)$$

Substituting Equation (10) into Equation (7) results in

$$\Delta u_Tocr = \ln\left(1 + \frac{1.717}{OCR} \right) \frac{n(a_w - a_s) + a_sT + (1 + e_0)\Delta p' \Delta T}{\Lambda}$$

(12)

Equation (12) is the final calculation method of thermal pore water pressure applicable to both normally consolidated and overconsolidated clays. This calculation method can not only consider the direct weakening effect of OCR, but also the influence of the variation $\Lambda$ with OCR on thermal pore water pressure.

3. Validation of the Calculation Method

The calculation method of thermal pore water pressure, which can consider overconsolidation effect, was applied to predict the variation of thermal pore water pressure, and the predicted results are compared with the experimental data.

3.1. Comparison with the Experimental Data in Undrained Heating Test of Wang et al. (2017)

To investigate the influence of OCR on thermal pore water pressure under undrained conditions, Wang et al. [26] used a temperature-controlled GDS triaxial testing apparatus to conduct undrained heating tests on normally consolidated and overconsolidated kaolin clays. During the test, saturated kaolin clay was consolidated at a pressure of 300 kPa. After consolidation was completed, the consolidation pressure was unloaded to 150 kPa, 100 kPa, 75 kPa, 37.5 kPa, 30 kPa, and 10 kPa to obtain soil samples with OCR of 1, 2, 3, 4, 8, 10, and 30. Subsequently, the obtained soil samples were used to carry out undrained heating tests. Furthermore, the physical parameters of kaolin clay are shown in Table 1.

<table>
<thead>
<tr>
<th>$\theta$ (%)</th>
<th>$S_r$</th>
<th>$G_s$</th>
<th>$I_p$</th>
<th>$\Lambda$</th>
<th>$\kappa$</th>
<th>$n_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>59–61</td>
<td>98</td>
<td>2.72</td>
<td>27</td>
<td>0.17</td>
<td>0.026</td>
<td>0.63</td>
</tr>
</tbody>
</table>

Through calculation, it has been found that when the clay is in slightly overconsolidated state ($1 < OCR \leq 4.3$), the $\Lambda$ obtained by Equations (10) and (11) is larger than that of normally consolidated clay. In this case, the value of $\Lambda$ adopts that of normally consolidated clay. However, for highly overconsolidated clays (OCR > 4.3), the values of $\Lambda$ are different from those of normally consolidated clays, and when OCR of soil is 8, 10, and 30, the calculated values of $\Lambda$ are 5.4, 5.0, and 3.8, respectively. Thus, it can be seen
that for saturated clays, there is a critical threshold of OCR that determines whether the stress history will change the calculated value of \( \Lambda \), thereby reducing the thermal pore water pressure. In this paper, the critical threshold of OCR is taken as 4.3. Subsequently, \( \Lambda \) is brought into Equation (12) to predict the thermal pore water pressure. The predicted results and their comparison with the experimental results of Wang et al. [26] are presented in Figure 1.

![Figure 1. Comparison of predicted thermal pore water pressure against the results of Wang et al. (2017).](image)

It can be seen from Figure 1 that all experimental data points are basically on the predicting lines. Meanwhile, through calculation, it has been found that except for the predicted results with OCR equal to 30, the coefficients of determination \( (R^2) \) of other predicted results are all greater than or equal to 0.89. Moreover, it should be noted that when OCR is equal to 30, although the coefficient of determination of the predicted results is only 0.44, the maximum difference between the predicted results and the experimental data is within 1 kPa, which is acceptable in engineering. Therefore, it can be concluded that the calculation method established in this paper can accurately predict the undrained thermal pore water pressure for both normally consolidated and overconsolidated clays.

3.2. Comparison with the Experimental Data in Undrained Heating Test of Abuel-Naga et al. (2007)

Using modified oedometer and triaxial test apparatus, Abuel-Naga et al. [10] carried out undrained heating tests on soft Bangkok clay to study the thermal pore water pressures under different stress conditions. To obtain soils with different overconsolidation ratios, the consolidation pressure of 200 kPa was unloaded to 100 kPa and 50 kPa. The physical parameters of soft Bangkok clay are shown in Table 2. Meanwhile, although \( \Lambda \) was not reported in this undrained heat test, Ghaawd et al. [9] found that when OCR is 4 and the critical threshold is not reached, the influence of stress history on \( \Lambda \) is ignored.

<table>
<thead>
<tr>
<th>( \theta ) (%)</th>
<th>( S_r )</th>
<th>( G_s )</th>
<th>( I_p )</th>
<th>( \Lambda )</th>
<th>( n_0 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>90–95</td>
<td>98</td>
<td>2.68</td>
<td>60</td>
<td>0.46</td>
<td>0.63</td>
</tr>
</tbody>
</table>

The predicted results of Equation (12) are compared with the experimental results of Abuel-Naga et al. [10], and the comparison results are shown in Figure 2.
under different thermal loadings is shown in Figure 3. The predicted results are highly consistent with the experimental results. The consistency between the predicted results and the experimental results ensures the accuracy of the calculation method established in this paper.

4. Application of the Calculation Method

In this section, the calculation method is applied to investigate the influence of overconsolidation effect on thermal pore water pressure using MATLAB software. The consolidation pressure is fixed at 300 kPa, and thermal loadings are taken as 35 °C, 50 °C, 65 °C, and 80 °C, respectively. Furthermore, the required parameters for numerical calculation are the same as those in Table 1. The evolution law of thermal pore water pressure with OCR under different thermal loadings is shown in Figure 3.

![Figure 2](image1.png)

**Figure 2.** Comparison of predicted thermal pore water pressure against the results of Abuel-Naga et al. (2007).

From Figure 2, it can be observed that all experimental data points are around the predicting lines. In addition, through calculation, it was found that the coefficients of determination ($R^2$) of all predicted results are 0.95 or above, which indicates that the predicted results are highly consistent with the experimental results. The consistency between the predicted results and the experimental results ensures the accuracy of the calculation method established in this paper.

![Figure 3](image2.png)

**Figure 3.** Evolution of the thermal pore water pressure with OCR under different thermal loadings.

The predicted results of Equation (12) are compared with the experimental results of Abuel-Naga et al. [10], and the comparison results are shown in Figure 2. The evolution of the thermal pore water pressure with OCR under different thermal loadings is shown in Figure 3. The consistency between the predicted results and the experimental results ensures the accuracy of the calculation method established in this paper.

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From Figure 3, it can be observed that the value of thermal pore water pressure drops sharply with increasing OCR. Meanwhile, the difference of thermal pore water pressure under different thermal loadings also decreases gradually. This is mainly because the thermal pore water pressure is generated by the difference in thermal expansion characteristics of soil particles and pore water, and the degree of overconsolidation of soil can reduce this difference of thermal expansion. In addition, it can be found from Figure 3 that when thermal loading is large and OCR is small, the value of thermal pore water pressure can exceed the effective stress of the soil. In other words, the soil may suffer thermal damage. In this case, if OCR is properly increased, the thermal pore water pressure will be significantly reduced, thus avoiding the occurrence of thermal damage.

5. Discussion

As mentioned above, OCR can not only directly reduce the thermal pore water pressure (Equation (6)), but it can also change the magnitude of the thermal pore water pressure by affecting the value of Λ (Equations (10) and (11)). However, the effect of OCR on Λ is often ignored in existing calculation methods [9,26], which may be the main reason for overestimating the thermal pore water pressure of highly overconsolidated soil. To quantitatively analyze the effect of Λ on the thermal pore water pressure, two cases are given here, namely, Case 1: the predicted results without considering the variation of Λ (Λ = 6.5), and their comparison with the experimental results; and Case 2: the predicted results considering the variation of Λ (Λ = 5.4, 5.0, and 3.8), and their comparison with the experimental results. The effect of Λ on the thermal pore water pressure is analyzed by comparing the coincidence between experimental results and predicted results. Figure 4a,b show the comparison of experimental results and predicted results for the two cases.

![Figure 4](image-url)

**Figure 4.** Comparison between experimental results and predicted results of (a) Case 1; (b) Case 2.

From Figure 4a, it can be observed that the predicted results without considering the variation of Λ will seriously overestimate the thermal pore water pressure. For clays with OCR equal to 10 and 30, the coefficients of determination (R²) of predicted results are both 0. When the change in temperature is 35 °C, the difference between the experimental results and predicted results reaches 61.5% for the clay with OCR equal to 10, and the difference reaches 127.2% for the clay with OCR equal to 30. However, from Figure 4b, it can be found that the coincidence between the predicted results considering the variation of Λ and experimental results was greatly improved. Especially, when OCR is equal to 30, the accuracy of the predicted results was improved by 92.7%, which means that the variation of Λ with OCR cannot be ignored when predicting the thermal pore water pressure inside the highly overconsolidated clay.
6. Conclusions

In this study, by considering the effect of OCR on the parameter $\Lambda$ and the nonlinear relationship between OCR and the thermal pore water pressure, a calculation method of thermal pore water pressure considering overconsolidation effect for saturated clay is proposed. Compared with previous calculation methods, the proposed calculation method has higher accuracy and the required parameters are easy to obtain. Moreover, through applying the calculation method and comparing the predicted results of the calculation method with the experimental data, the following conclusions can be drawn:

- Based on the proposed calculation method, the critical threshold of OCR (4.3) for determining whether the value of $\Lambda$ will vary with OCR is obtained. When the clay is in slightly overconsolidated state, i.e., $1 < OCR \leq 4.3$, the value of $\Lambda$ is the same as that of normally consolidated clay. However, when the clay is in a highly overconsolidated state, i.e., $OCR > 4.3$, the value of $\Lambda$ of overconsolidated clay is smaller than that of normally consolidated clay, and gradually decreases with the increase of OCR.
- The proposed calculation method can accurately predict the evolution of thermal pore water pressure under undrained conditions, and is supported by some undrained heating test results of overconsolidated saturated clay, which can provide a theoretical basis for the design of thermo-active structures.
- The effect of OCR on thermal pore water pressure is related to the heating rate. However, the calculation method proposed in this study does not consider the influence of heating rate. The follow-up work can explore the comprehensive effects of OCR and heating rate on the thermal pore water pressure under undrained conditions theoretically and experimentally.

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