Mechanical Behaviour and Failure Mode Analysis of Penetrated Mortise–Tenon Joint with Neighbouring Gaps Based on Full–Scale Experiments

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Abstract: The penetrated mortise–tenon joint (PMT) connecting column and beam in traditional timber structures often has neighbouring gaps between connected structural members due to initial manufacture errors and damage accumulated over years. Influences of the neighbouring gaps on the mechanical properties of the PMT joint have been analysed based on a full-scale experimental study. Four typical gap values are determined according to the probability analysis of on-site survey results of a Chinese traditional timber structure. Four full-scale models of PMT joints with varied gap values have been established. Failure modes and deformation characteristics have been studied by quasi-static tests. Results show that the failure modes are the tearing of wood fiber along the grain at variable cross–sections. The loose penetrated mortise–tenon (LPMT) joints all have high deformability. The slip distance of tenon grows as the gap value increases. Limit angles of the loose joints lag with the increasing degree of loosening. The bending bearing capacity and rotational stiffness of LPMT joints decrease as the gap value increases, and the limitation value of the gap is analysed. The resisting capacities of LJ-2, LJ-3 and LJ-4 are much lower than that of LJ-1. The changing ratios are 18.5%, 55.4% and 70.4%, respectively. A three–parameter power function model of the mortise–tenon joint with consideration of the neighboring gap is presented. Research results provide important references on the condition assessments of the existing traditional timber structures.

Keywords: penetrated mortise–tenon joint; neighboring gap; full–scale experiment; mechanical behaviour of loose joint; limitation value of the gaps

1. Introduction

Penetrated mortise–tenon (PMT) joints in traditional timber structures are the vulnerable part of the whole structure because of the weak section between the mortise and tenon. The stability of the traditional timber structures is dependent on the mechanical behaviour of the PMT joints, while the mechanical behaviour of the joints are related to the wooden material characteristics, sectional dimension members and contact states of components, etc. During the whole life cycle of structures, traditional timber structures have been subjected to a variety of natural and human violations. In the long-term usage process, wood undergoes oxidation and surface carbonisation, and it leads to a reduction in the effective cross-section size of components. In addition, shrinkage, rottenness or damages by borers occur in the wood under the effects of natural loads. The creep and damage of wood are accumulated due to the effects of the service loads. All these effects and phenomena interact with each other, resulting in dislocation and a weak contact state,
as well as the degradation of the mechanical performance of the joints. In the existing traditional timber structure, one of the most critical points of the PMT joint is the absence of permanent tight fit between the column and beam due to gaps at the PMT joint, which is unfavorable for load resisting capacity and leads to unpredictable behaviour of the structure. Studying the mechanical behaviour of PMT joints under loose conditions in the traditional timber structure is crucial.

In recent years, some achievements about traditional mortise–tenon joints have been obtained. As the most important component of a wood structure, the study of joint stiffness and bending moment of the mortise–tenon joint are the basis of follow–up research. Eckelman [1] studied the influence of the size of the tenon on the stiffness and maximum load and clarified the large influence of the long-side direction length of the tenon section on rotational stiffness. The rotation stiffness and flexural behaviours of the straight mortise–tenon joints and dovetail mortise–tenon joints were discussed, respectively, by Pan [2], Xie [3] and Chen [4]. A novel index, the extreme value of the largest principal component scores of the generalized likelihood ratio based on the statistical process control chart, to develop a structural stiffness identification method for assessing traditional Chinese mortise–tenon joints, was proposed by Jiang [5]. The test of the dovetail mortise–tenon joints made of the rubber material was carried out, and the mechanical properties such as the tension, the rotation, and the maximum bearing capacity of the joints were analyzed [6].

Additionally, some scholars studied the mechanical properties of joints in traditional timber structures with various structural forms. The influence of the contact surface of the joints on the stress performances of the joints and the movement mechanism and stress performance of the traditional Korean structure under the influence of the horizontal load were studied by Seo [7] and Han [8]. The vibration table test and the static test were carried out to study the mechanical performances of the traditional Japanese timber structure by Suzuki [9]. The rotation performance of the joints in traditional residential buildings in Taiwan was studied by Chang [10] and the relationship between rotation stiffness and rotation angle were obtained. The seismic performance of typical mortise–tenon joints in traditional wooden buildings in southern China and the semi-rigid mechanical properties of four typical mortise–tenon joints were obtained by Chun [11]. Five joints of the historical building in Yangzhou as the prototype were studied by Chen [12]. All of the test samples were created using traditional manufacturing methods and were taken straight from the original structure.

However, the time of a century or even a thousand years led to a different degree of loosening, resulting in a gap at the mortise–tenon joint. The main causes of the gap between the tenon and mortise may be a processing mistake, dry shrinkage brought on by a change in moisture content or creep brought on by long-term load [13]. The mechanical properties of mortise–tenon joints with different degrees of damage and good penetration were studied by artificial simulation [14]. The rotation behaviour of mortise–tenon joints with different degradation types and degrees was studied by Xie [15] and Xue [16]. The effect of the degree of loosening on the mechanical properties of the dovetail mortise–tenon was studied by Li [17]. The rotational performance of traditional Nuki joints with gaps in Japan was studied by Chang [18,19]. Theoretical estimation of the mechanical performance of the traditional mortise–tenon joint involving a gap was researched by Ogawa [20].

The above studies have revealed the mechanical properties and seismic performances of standard and damaged mortise–tenon joints. However, mechanical performances and working principles of the models are mostly based on the small-scale model test. The wood structure is a nonlinear flexible structure, which has multiple nonlinearities in contact friction, elastoplasticity of material and cross-sectional dimension. Small–scale models cannot completely reflect the seismic performances of joints in the actual structure, and the study of mortise–tenon joints rarely involves the construction asymmetry and loose conditions of joints. When the scaled model is used for the structural test, the
correlation between the mechanical properties of the test model and the original structure should be considered. The model dimension can be scaled linearly, while the external loads cannot be scaled linearly. Additionally, the nonlinear effects of the scale on the mechanical properties of the structure are overlooked in the current experimental research. The relationship of the vertical load between the test model and the prototype structure is paid less attention. The full-scale model can best capture the mechanical performance of the structure and resolve the nonlinear correlation of loads. Meanwhile, there is a lack of field investigation and analysis on the damaged types of joints.

In this paper, current damage situations of mortise–tenon joints based on the field investigation in the Feiyun Wood Pavilion are summarized and analyzed. Then, the deformation characteristics and failure mode of loose joints under different degrees of looseness are studied by a low-cycle cyclic test. Rotational stiffness, stiffness degradation law and energy dissipation capacity of loose joints are analyzed and compared. The limitation value of gap is given, and the three-parameter power function model with neighbouring gaps and the failure section for the restoring moment of loose penetrated mortise–tenon joints are proposed. The stress state of loose penetrated mortise–tenon joints with different loose conditions is discussed.

2. Method

2.1. Survey of the Statistical Distribution of Joint Gaps

Although traditional timber structures have a lengthy lifespan, after thousands of years of natural disasters and man-made damages, there are numerous defects in the existing structures. To obtain the damage condition of wooden structure joints in traditional timber structures, the Feiyun Wood Pavilion, as shown in Figure 1, located in Shanxi Province, is selected for field investigation. Feiyun Wood Pavilion is a traditional timber structure and significant cultural heritage site, which shares its reputation with Ying-xian wooden pagoda, known as the “Southern Pavilion” and “Northern pagoda”. Feiyun Wood Pavilion is a typical wood structure, including a five layered wooden frame. It is a collection of mortise–tenon joints that can offer a substantial number of gap values. This ensures there are enough samples to obtain reasonable gap distributions.

To study the mechanical properties of loose penetrated mortise–tenon (LPMT) joints in-depth, the damage state of the PMT joints is summarized based on the field investigation results of Feiyun Wood Pavilion. As shown in Figures 2 and 3, the most common damage types of joints in traditional timber structures include joint loosening, pulling-out of the tenon and joint component cracks, according to the field investigation of Feiyun Wood Pavilion. The gap distributions of the PMT joints are obtained. The full scale test is then used to study the deformation and failure characteristics of PMT joints under various loose conditions, and the hysteretic performance, stiffness characteristics, stiffness degradation law and energy dissipation capacity of LPMT joints are compared and discussed.
At present, there is a lack of basis for determining the gap size at the mortise–tenon joint. The mechanical behaviour of the PMT joint obtained by experiments and numerical
simulations can be limited to the specific working conditions. There is a lack of wide applicability. The typical value of gaps at mortise–tenon joints in a traditional timber building must, therefore, be examined. It may offer a specific numerical foundation for the investigation of loose mortise–tenon joints. The conventional timber structure consists of mortise–tenon joints, dou–gong joints and column foot joints. The sampling objects are chosen to be the mortise–tenon joints. The local measurement of the gap value is shown in Figure 4.

![Figure 4](image)

**Figure 4.** Local measurement of gap value of the mortise–tenon joint.

The distributions of the gap at the mortise–tenon joint, as shown in Figure 5, are obtained by the site survey of the Feiyun Pavilion. The percentage of joints between 0 and 5 mm is represented by the red colour. The gap value is essentially randomly distributed between 0 and 50 mm. The value of the gap is largely distributed between 0 and 50 mm with certain randomness. The gap is predominantly spread in from 0 to 15 mm range, and 44% of joints have this gap value. In this paper, four gap values, including 5 mm, 10 mm, 25 mm and 50 mm, are selected to study the effects of the gap on the mechanical properties of the PMT joint.

![Figure 5](image)

**Figure 5.** The distribution of the gap values in the structure.

2.2. Experimental Study

2.2.1. Configurations of the Joints

The construction of the mortise–tenon joint is shown in Figure 6. The tenon of the joint includes two parts. One part is a tenon with a large cross–section that is the same height as the fang. The tenon with the narrow cross–section, which is half the height of the fang, is the other portion. The column is penetrated by a tenon with a variable cross-section.
2.2.2. Working Mechanics

Penetrated mortise–tenon (PMT) joints are commonly used in traditional timber structures in China, Japan and other countries. Figure 7 shows the possible forms of gaps in the joint. The vertical gap may occur at the top of the tenon in Figure 7a, the top and the bottom of the tenon in Figure 7b and the bottom of the tenon in Figure 7c. As shown in Figure 7, $r$ is the radius of the column, $h_1$, $h_2$, and $h$ are the height of the small cross-section, big cross-section and fang, respectively. $l_1$ and $l_2$ are the length of the small cross-section and big cross-section, respectively. $\Delta_{g_1}$ and $\Delta_{g_2}$ are the gap value occurring to the top and bottom of the tenon, respectively. $\Delta_g$ is the total gap value.

Figure 6. Constructions of PMT joint.

Figure 7. The possible forms of gaps.

PMT joints resist external load, such as earthquakes and wind, by engagement of their members. The main mechanisms of resistance are friction between the mortise and tenon and compression perpendicular to the grain. The PMT joint rotated under external load is shown in Figure 8. The column is floating on the stone base, and there is no actual connection between the column foot and stone base. The column will lift up and rock around the column foot edge during rotation, as shown in Figure 8.

The column is assumed to be erect. Rotation deformations are shown in Figures 9 and 10. Under the cyclic load, the joint slips firstly due to the gap occurring at the joint. The mortise and tenon compress on each other after overcoming the free rotation angle.
Local compression deformation appears perpendicular to the grain. The compression areas during positive loading are indicated by I and II in Figure 9. The compression areas during negative loading are indicated by III and IV in Figure 10. Due to the asymmetry of the PMT joint, compression deformation under positive and negative rotation differs. Upward rotation is defined as positive rotation and vice versa.

Figure 8. Rotation of timber frame under external load.

Figure 9. Positive rotation of tenon.
Figure 10. Negative rotation of tenon.

In addition, $N_I$ and $N_{II}$ with black arrows represent the partial compressive resistance under positive loading in Figure 11. In a similar way, $N_{III}$ and $N_{IV}$ with black arrows represent the partial compressive resistance under negative loading. Because the compressive strength parallel to the grain is substantially greater than that perpendicular to the grain, plastic deformation may occur in the wooden material perpendicular to the grain during rotation in Figure 11. The point $o$ is the centre of rotation, the moment balance under positive loading at point $o$ can be written as:

$$N_Ia + N_{II}b = PL$$  \hspace{1cm} (1)

where $a$ and $b$ are the distance between the centroids of $N_I$ and $N_{II}$ and rotation centre $o$, shown in Figure 11a. $P$ is the external load. $L$ is the length of the fang.

Similarly, the moment balance at point $o$ under negative loading can be generally expressed as:

$$N_{III}c + N_{IV}d = PL$$  \hspace{1cm} (2)

where $c$ and $d$ are the distance between the centroids of $N_{III}$ and $N_{IV}$ and rotation centre $o$, respectively, as shown in Figure 11b.

Figure 11. Compression force during rotation. (a) Positive rotation. (b) Negative rotation.

2.3. Test Specimens

With gaps between the mortise and tenon of 5 mm, 10 mm, 25 mm and 50 mm, four loose models with various loose conditions are created by altering the height of the bigger cross-section, as shown in Figures 12 and 13. The diameter and height of the column are 390 mm and 2750 mm. The detailed dimensions of the loose joints named by LJ-1, LJ-2, LJ-3 and LJ-4 are shown in Figure 14. The timber in this study is Northeast pine with an average density of 420 kg/m³ and a 12% moisture content [21]. Compressive strength parallel to the grain and that perpendicular to the grain are considered. Compressive tests are performed parallel to the grain to determine the axial compressive strength and
compressive strength of wood perpendicular to the grain. According to the Wood Compressive Strength Along Grain Test method GB/T 1935–2009 [22] and the Measuring Method of Compressive Elastic Modulus of Wood Cross-Sections GB/T 1943–2009 [23], the size of the centrally pressurized sample is 40 mm × 20 mm × 20 mm, and the height is in the direction of the pressure action.

The samples are loaded in compression in a standard INSTRON-5552 machine at the cross-head speed of 2 mm/min. For axial orientation, 20 samples are tested. As shown in Table 1, the average test results of the elastic modulus, yield stress and the Poisson ratio of the timber material are $8.856 \times 10^3$ MPa, 34.76 MPa, and 0.3, respectively. For perpendicular orientation, 20 samples are tested. The elastic modulus and compressive strength of wood perpendicular to the grain are 1024 MPa and 4.18 MPa, respectively.

**Table 1.** The mechanical properties of timber used in the experiment.

<table>
<thead>
<tr>
<th></th>
<th>$E_t$</th>
<th>$\sigma_{pc}$</th>
<th>$\mu_{pc}$</th>
<th>$E_h$</th>
<th>$\sigma_{tc}$</th>
<th>$\mu_{tc}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elastic</td>
<td>$8.856 \times 10^3$ MPa</td>
<td>34.76 MPa</td>
<td>0.3</td>
<td>1024 MPa</td>
<td>4.18 MPa</td>
<td>0.19</td>
</tr>
</tbody>
</table>

Note: $E_t$ and $E_h$ are elastic modules. $\sigma_{pc}$ and $\sigma_{tc}$ are yield stresses. $\mu_{pc}$ and $\mu_{tc}$ are Poisson ratios of the timber parallel and perpendicular to the grain, respectively.

![Figure 12. Loose joint.](image)

![Figure 13. Fang models of the loose joint.](image)
Figure 14. Detail dimension of fang. (a) Loose joint-1. (b) Loose joint-2. (c) Loose joint-3. (d) Loose joint-4.

2.4. Test Setup and Loading Programme

The loading device of this test is shown in Figure 15. To ensure the stability of the wooden column during rotation, the support is fixed on the ground of the laboratory, and the wooden column is placed horizontally on the support and fastened by the iron hoop. The fang is put into the mouth of the mortise and the horizontal load is applied at the fang by the MTS horizontal actuator. The positive load is specified to be loading to the right, and vice versa. As shown in Figure 16, displacement meters D1, D2, D3 and D4 with a 20 mm measuring range are installed on the left and right sides of 15 cm and 25 cm between the fang side and the lower edge of the mortise to measure the horizontal displacement of the tenon. The inclination gauge R1 is placed on the central axis of the fang at a distance of 30 cm from the loading end to measure the angle of the fang during the loading process. Three strain gauges are positioned on either side of the fang and column to measure the compression stress of the wood, as shown in Figure 15. There are a total of 24 strain gauges.

Monotonic and cyclic loading tests based on the displacement control method are performed on the joint based on ISO-16670 [24]. Each controlled displacement increases with 10 mm displacement, and cyclic loading for 15 times has been carried out on LJ-1, LJ-2 and LJ-3, as shown in Figure 16. Maximum horizontal displacement for LJ-4 is 200 mm due to the significant gap value. In a circular loading mode with a variation of 20 mm, the loose joint LJ-4 is loaded to 120 mm, and then to 200 mm with a differential of 10 mm. The time of one loading circle is almost 4 min.
3. Results

3.1. Experimental Phenomena

As shown in Figure 17, there is no visible compression between the mortise and tenon, and the tenon of the loose joint slips initially at the start of positive and negative loading. The more the joint loosens, the more visible the slipping phenomenon becomes. Mortise and tenon compress with each other as the regulated displacement increases, emitting a “squeaking” noise. The tenon shows visible compression deformation perpendicular to the grain when loaded up to a specific displacement, and compression deformation is not rectified after loading. With the angle increase, the mortise and the tenon compress with each other at one side and separate at the other side, resulting in a degree of tenon pulling-out (Figure 18). Compression deformation rises with increasing rotational angle, and mutual compression sounds louder than before. A sound of wood fibre torn when loaded to 120 mm in the negative direction is made by LJ-1 and LJ-2. At the
same time, horizontal forces measured in monitoring equipment drops dramatically, indicating that the tenon is damaged. Loading ends when the horizontal displacement reaches 120 mm. Due to the rather wide gaps between LJ-3 and LJ-4 and the constrained laboratory conditions, neither LJ-3 nor LJ-4 are loaded to their maximum displacement, hence, no damage is visible.

Tenons are removed after loading to examine the experiment phenomenon. It is discovered that the fang and column are in good shape. There is no obvious compression deformation in the mortise mouths. The tenon, however, exhibits a substantial compression deformation (Figure 19). The tenon experiences a significant amount of plastic compression deformation with repeated compression. At the varied cross-section of the tenon, wood fibre is torn down the grain at LJ-1 and LJ-2. The crack extends to the end of the tenon, shown in Figure 20. There is no significant damage at LJ-3 and LJ-4 at the maximum displacement due to the large gap between the mortise and the tenon.

Figure 17. Slipping of the tenon.

Figure 18. Pulling-out of the tenon.

Figure 19. Compression deformation of the tenon.
3.2. Hysteretic Curve

The bending moment of the PMT joint can be obtained by the product of the distance between the horizontal force and the load point of the fang side to the rotation centre \( o \).

\[
M_{\text{mt}} = PL
\]  

The moment-rotation hysteretic curve of the PMT joint is shown in Figure 21. The asymmetry of the mortise–tenon joint leads to the asymmetry of the bending moment and rotation angle hysteresis curve. The hysteresis curves of LJ-1 and the LJ-2 are in the form of an anti-Z-shape with a pinching effect, indicating that the tenon has a degree of slippage during rotation. The hysteresis curves of loose joints LJ-3 and LJ-4 are bow-shaped, the sliding deformation is considerable, and the slipping distance increases gradually as the loading increases. The area of the hysteresis loop of LJ-1 is tiny at initial loading, indicating that sliding between mortise and tenon is minimal and the joint consumes little energy. The area of the hysteresis loop then grows as the rotational angle increases, indicating that compression deformation and energy consumption of the joint increases as the rotational angle increases. When the 120 mm is loaded negatively, the bending moment of the loose joint LJ-1 and LJ-2 rapidly decreases. The curve seems to have a distinct descending portion and the tenon is destroyed.

As can be seen from Figure 21, initially the loose joint slips, and the slipping distance extends as the loosening degree of the joint increases. The mortise and tenon are then squashed against each other, increasing the joint’s bending moment.

The initial rotation angles, the maximum restoring moment and the decline ratio of maximum restoring moment are listed in Table 2. \( \theta^+ \) and \( \theta^- \) are the initial rotation angles in the positive and negative rotations. \( M^+_{\text{max}} \) and \( M^-_{\text{max}} \) are the maximum restoring moment in the positive and negative rotations. \( \zeta^+ \) and \( \zeta^- \) are the maximum restoring moment in the positive and negative rotations. In general, the resisting capacity of LPMT is roughly asymmetric, indicating that the geometric asymmetry of the tenon affects the global mechanical performance of the specimen. In the presence of positive and negative rotation, the initial rotation angle increases with the increasing loose degree. The maximum restoring moment of LJ-1 in the positive rotation is 17.41 kN·m, while that of LJ-2, LJ-3 and LJ-4 are 14.19, 7.77717 and 5.15699 kN·m. The resisting capacities of LJ-2, LJ-3 and LJ-4 are much lower than that of LJ-1. The changing ratios are 18.5%, 55.4% and 70.4% under positive loading. The changing ratios are 18.5%, 55.4% and 70.4% under positive loading. Additionally, the changing ratios are 15.9%, 57.3% and 90% under negative loading.

The joint mainly relies on the friction and the mutual extrusion between the mortise and the tenon to resist the external load during the whole loading process. When the gap is small, the joint mainly relies on compression force and friction force to resist the external load. When the gap between the mortise and tenon is considerable, the resisting capacity of the joint decreases.
Figure 21. Hysteretic curves. (a) LJ-1. (b) LJ-2. (c) LJ-3. (d) LJ-4.

Table 2. The changing of initial rotation angles, the maximum restoring moment and the decline ratio of maximum restoring moment.

<table>
<thead>
<tr>
<th></th>
<th>LJ-1</th>
<th>LJ-2</th>
<th>LJ-3</th>
<th>LJ-4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta^+$</td>
<td>0.00924</td>
<td>0.02322</td>
<td>0.05093</td>
<td>0.05556</td>
</tr>
<tr>
<td>$\theta^-$</td>
<td>-0.0046</td>
<td>-0.02167</td>
<td>-0.06022</td>
<td>-0.0926</td>
</tr>
<tr>
<td>$M^+_{\text{max}}$</td>
<td>17.41693</td>
<td>14.19</td>
<td>7.77717</td>
<td>5.15699</td>
</tr>
<tr>
<td>$M^-_{\text{max}}$</td>
<td>-15.9156</td>
<td>-13.3875</td>
<td>-6.79473</td>
<td>-1.59156</td>
</tr>
<tr>
<td>$\zeta^+$</td>
<td>-</td>
<td>18.5%</td>
<td>55.4%</td>
<td>70.4%</td>
</tr>
<tr>
<td>$\zeta^-$</td>
<td>-</td>
<td>15.9%</td>
<td>57.3%</td>
<td>90%</td>
</tr>
</tbody>
</table>

3.3. Skeleton Curve

The skeleton curve is created by connecting the peak points of hysteretic curves, as shown in Figure 22. The slope of the curves is minimal at the beginning of loading, indicating that the tenon is in the slipping stage. Curves become steeper as the rotational angle increases, and the joints enter the elastic stage. With constant loading, the slope of the
curves slows down as the rotational angle increases, obvious compression deformation perpendicular to the grain appears in the tenon with material yield, indicating the partial yielding of the wood. The compression strengthening stage of joint development has begun. When the rotational angle reaches a specific value, the moment drops rapidly accompanied by a huge sound of wood being torn apart, and the tenon is damaged. Joints are in the failure stage. Thus, sliding, elastic, strengthening and failure stages can be distinguished in the curve of the restoring moment and rotational angle of joints.

As shown in Figure 22, the joints LJ-1 and LJ-2 have a distinct slip, elastic, strengthening and failure segments. Due to the large gap of LJ-3 and LJ-4, the test is not loaded to the failure stage, and there is no discernible failure region in the test curve. The main differences between the loose joints during rotation are the length of slippage, initial compression rotational angle and failure rotational angle. The starting compression rotational angle of the joints increases with the increase in the gap. The failure angle of the loose joint lags with the increase in the gap.

3.4. The Degradation Curve of the Rotation Stiffness

The rotational stiffness degradation curves of four joints under positive and negative loading are shown in Figure 23. From the stiffness curve, it can be seen that the stiffness degradation of the joint decreases with the increase in loosening degree. The joint LJ-1 slips first and exhibits low stiffness during the initial positive loading. When the mortise interacts with the tenon, the stiffness of the joint increases. In the negative loading, the LJ-1 of the joint slips firstly, and the stiffness is low. The stiffness rises when the slip section is exceeded, and the tenon and mortise are in contact with each other. When loading to 0.049 rad, the stiffness of LJ-1 decreases rapidly and the tenon is destroyed.

The loose joint LJ-2 encounters a minor amount of slip during positive loading, and the stiffness is low. The initial slip of the joint LJ-2 is longer than LJ-1 and the stiffness of the joint flattens as the rotation angle increases. The mortise and tenon are extruded and the stiffness of the joint increases. Then, the joint enters the strengthening section, the bending moment gradually increases and the stiffness gradually decreases.

The stiffness of loose joints LJ-3 and LJ-4 is practically unchanged in the early stages of loading and the degradation phenomenon is weak. The slipping stage of the loose joint lasts longer during loading and the joint primarily relies on friction force to resist the external load. Therefore, the initial secant stiffness of the joint is low and the stiffness remains nearly unchanged as the rotation angle increases. The stiffness degradation curve shows that LPMT joints have clearly varying stiffness characteristics. Rotational stiffness during positive and negative loading is different and the rotational stiffness of the loose joints diminishes as the joint gap increases.
3.5. Energy Dissipation

The energy dissipation capacity of joints can be measured by an equivalent viscous damping coefficient \( h_e \). The larger the equivalent viscous damping coefficient \( h_e \) is, the better the energy capacity of the joint is. \( h_e \) can be calculated according to the following formula [25]:

\[
h_e = \frac{1}{2\pi} \times \frac{S_{AFBE}}{S_{ACEO} + S_{ADF0}}
\]  

where \( S_{AFBE} \) is the area of the hysteresis loop, \( S_{ACEO} + S_{ADF0} \) is the sum area of \( \Delta CEO \) and \( \Delta DFO \), shown in Figure 24. The curve of \( AFBE \) is one cycle hysteresis curve. The area of the curve of \( AFBE \) represents the energy consumption of joints. C and D are the projection points of E and point F on the coordinate axis.

Figure 24. Energy dissipation loop.

The relationships between the equivalent viscous damping coefficient and rotational angle on both sides of the positive and negative rotations are shown in Figure 25. It can be seen that the equivalent viscous damping coefficients of LJ-1 and LJ-2 decrease with the increase in rotational angle. The energy dissipation coefficients for LJ-1 and LJ-2 are roughly 0.1, while they are approximately 0.41 for LJ-3 and LJ-4. For LJ-1 and LJ-2 with variable cross-section damage under negative loading, the work carried out by the tearing of wood grain increases and the energy dissipation coefficients increase at the failure stage. The variable section damage has a great effect on the mechanical performance of the specimen. Energy dissipation capacity increases.
The comparable viscous damping coefficients of LJ-1 and LJ-2 can be observed to be identical. During the slipping stage, the energy dissipation capacity of the joint is high. That is because the mortise and tenon experience friction at the initial loading and the energy dissipation capacity is significant. After slipping, the energy dissipation capacity of the joint decreases due to compression deformation of the tenon, which consumes little energy. External force is stored by the elastic strain energy produced by tenon bending, while the energy dissipated by plastic deformation generated by the extrusion between the tenon and the mortise is reduced. Therefore, the energy dissipation coefficient decreases.

LJ-3 and LJ-4 have a long slipping stage. Equivalent viscous damping coefficients of LJ-3 and LJ-4 are similar. The friction between the mortise and the tenon is the primary source of energy consumption in two joints. Due to the mutual compression deformation using little energy and resulting in a loss in energy dissipation capacity, the equivalent viscous damping coefficient of LJ-3 diminishes. External force is stored by the elastic strain energy produced by tenon bending, while the energy dissipated by plastic deformation generated by the extrusion between the tenon and the mortise is reduced.

The energy dissipation coefficients for LJ-1 and LJ-2 are approximately 75% lower than that of LJ-3 and LJ-4. Therefore, it can be concluded that the energy dissipation capacity of loose joints is significantly higher than that of regular joints. This because the slipping stage of a loose joint is longer than that of a regular joint, which can assume more energy with higher energy dissipation.

According to the curve of the equivalent viscosity and damping coefficient with the change in the rotation angle, the equivalent viscous damping coefficient is high in the slipping stage. The mortise and tenon then compress with each other, and plastic compression deformation occurs with the increase in rotational angle. When the plastic deformation increases to a certain extent, energy-consuming capacity will not increase. Energy consumption capacity of the joint gradually decreases. Finally, wood fibre torn at the variable cross-section at the failure stage consumes more energy and energy dissipation capacity increases. At the failure stage, the energy dissipation capacity of the joint increases due to the tearing of wood consuming large energy. The joint mainly depends on the friction force between the tenon and the mortise to consume energy.

![Figure 25. Equivalent viscous damping coefficient.](image)

4. Discussion Statement

4.1. Limited Gap Value of Loose Joint

The neighbouring gap plays an important part in the mechanical behaviour of the joint, which can reduce the bending bearing capacity and rotation stiffness. If the gap is small, the joint may have enough bearing capacity. However, if the gap value is large, such as the gap values of LJ-3 and LJ-4, the joint has limited bearing capacity, which is
detrimental to the stability of the whole structure. Therefore, the limit value of the gap is required for the safety of the structure.

According to the “Technical code for maintenance and strengthening of ancient timber buildings” [26], there are some rules to limit the horizontal displacement, which is shown in Table 3.

### Table 3. The limitation of horizontal displacement of the timber frame.

<table>
<thead>
<tr>
<th>Global inclination</th>
<th>Horizontal displacement $\Delta_1$</th>
<th>$\Delta_1 \leq \frac{H}{150}$ and $\Delta_1 \leq 100mm$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Local inclination</td>
<td>The relative displacement of the top and bottom of the column $\Delta_2$</td>
<td>$\Delta_2 \leq \frac{H}{150}$ and $\Delta_2 \leq 80mm$</td>
</tr>
</tbody>
</table>

As shown in Figure 26, the free rotational angle can be given as:

$$\theta = \frac{\Delta_g}{2r}$$  \hspace{1cm} (5)

Then, the corresponding horizontal displacement can be obtained:

$$\Delta = \frac{\Delta_g}{2r} \cdot H$$  \hspace{1cm} (6)

According to Table 3, the limiting value of the loose joint can be obtained on the conservative estimation, shown in Table 4.

### Table 4. The limitation of gap values of the PMT joint.

<table>
<thead>
<tr>
<th>Global inclination</th>
<th>$\Delta_g \leq \frac{2r}{150}$, and $\Delta_g \leq \frac{200r}{H}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Local inclination</td>
<td>$\Delta_g \leq \frac{2r}{100}$ and $\Delta_g \leq \frac{160r}{H}$</td>
</tr>
</tbody>
</table>

![Figure 26. Free rotational angle.](image)

4.2. The Simplified Restoring Moment Model

Slipping and rotation of the mortise and tenon can occur due to constructions of the mortise–tenon joint. It can be seen that the hysteresis curve and skeleton curve of the
restoring moment and rotational angle of the mortise–tenon joint under repeated load include slipping, compression and failure section. Currently, the approaches for deriving the restoring moment model of a joint are classified into three categories: test fitting, system identification and theoretical computation. Curves of the restoring moment and rotational angle obtained by using a theoretical model (the black line with blank circle) [2] and a triangular line model (the wider black line) [2] can partially reflect the mechanical performance of the joint. Additionally, there is also a three-parameter power function model, which can reflect the mechanical behaviour of the mortise–tenon joint, and expression is:

$$M = \frac{k_i \theta}{1 + \left( \frac{\theta}{\theta_n} \right)^n}$$

(7)

where $k_i$ is the initial stiffness in the elastic stage; $n$ is shape factor (for mortise–tenon joint, $n = 3$); $\theta$ is the rotational angle; $\theta_n = \frac{M_u}{k_i}$ is nominal plastic rotational angle; $M_u$ is an ultimate restoring moment of the mortise–tenon joint.

Due to the certain value of the gap, the loose joint has a corresponding free rotation angle. In this paper, the three-parameter power function model is improved based on the theoretical model and triangular line model, and the new model with the initial free rotation and the failure section is proposed as:

$$
\begin{cases}
M_F = \alpha F_L \lambda, \theta \in \left( -\frac{\Delta_s}{2r}, \frac{\Delta_s}{2r} \right) \\
M_S = \frac{k_i \theta}{1 + \left( \frac{\theta}{\theta_n} \right)^n}, \theta \in \left( -\theta_n, -\frac{\Delta_s}{2r} \right) \\
M_D = 0.6M_u, \theta \in \left( \theta_n, \theta_n + 0.01 \right)
\end{cases}
$$

(8)

where $M_F$, $M_S$ and $M_D$ are the restoring moment in slipping, compression and failure section, respectively; $F_L$ is the friction force, which can be obtained by experiment; $\alpha$ is the ratio of the contact area between the mortise–tenon joint in the actual structure and the experimental model; $L$ is the length of the fang.

The suggested three-parameter power function model is congruent with the experimental results in Figure 27, indicating that the model can be applied to predict the mechanical behaviour of the joint.
4.3. Stress State of the Loose Joint

The penetrated mortise–tenon joint with different loose conditions are established by ABAQUS. The size and load of the FEM are same as the experimental models. The elastic-plastic model is needed in this model as it is hard to observe plastic deformation in ABAQUS with the FEM with the orthotropic material. Material parameters of the timber parallel to the grain are applied to the column, and material parameters of the timber perpendicular to the grain are applied to fang. The mortise mainly depends on the timber parallel to the grain and the tenon depends on the timber perpendicular to the grain to bear compression force. The stress–strain relationship can be simplified to the bilinear constitutive model [27]. Column and fang are assigned with different material properties separately.

Column and fang are divided into different parts to mesh. The discretisation of both column and fang consists of 8-node hexahedral finite elements (element C3D8R of ABAQUS), as this element can avoid the “hourglass” phenomenon.

In this simulation, the contact pressure is calculated by using the hard contact model implemented in ABAQUS. The friction coefficient is taken as 0.5 [28]. The interaction type of the mortise and tenon is defined as the surface to surface contact with finite sliding. The boundary conditions are shown in Figure 28. The FEM of LJ-2 is subjected to the verification of corresponding experimental results [27].
The stress nephogram and deformation of LJ-1, LJ-2, LJ-3 and LJ-4 at the 120 mm loading level are extracted and shown in Figure 29. It can be seen that the main deformation occurs to the tenon, and stress of the right side of the tenon is high due to mutual interactions of the mortise and tenon. It is noted that significant compression deformations are mainly located at the tenon.

In addition, it can be seen that the stress of LJ-1 and LJ-2 are maximum. The compression area of LJ-1 is bigger than that of LJ-2. The stress of LJ-4 is the minimum. The stress of LJ-3 takes second-place. The elements exhibiting the maximum stress under positive and negative loading are located at the tenon necks on both sides of the lower tenon. In the experiment for specimens, the failure of the tenon is due to a crack located at the tenon neck under the tenon, which means that the numerical prediction is in agreement with experimental evidence. The stresses of the column and fang body are lower than that of the tenon and mortise, which corresponds coincidental with the experimental phenomenon. The loose condition can impact the stress state of the loose joint. The smaller the degree of loosening is, the greater the stress of the component is. If the degree of loosening is large, the stress of joint is very small, indicating that the joint has almost no resistance to external load.

![Figure 28. Boundary conditions.](image)

The maximum missed stresses of joints are listed in Table 5. $\sigma_{\text{max}}^+$ and $\sigma_{\text{max}}^-$ are the maximum missed stresses under positive and negative rotations. The maximum stresses of the LJ-1 and LJ-2 are almost equal. It shows that the mortise and tenon of the two joints have experienced sufficient mutual compression. The maximum stresses of the LJ-3 under positive and negative rotation are 3.25 MPa and 1.54 MPa, which are 22.2% and 63.8% lower than that of LJ-1. It is noted that the gap weakens the mutual compression of the mortise and tenon. The maximum stresses of the LJ-4 under positive and negative rotation are 0.84 MPa, which are 80% lower than that of LJ-1. It is noted that the gap strongly weakens the mutual compression of the mortise and tenon. Additionally, necessary measures should be taken to repair the loose joints.

<table>
<thead>
<tr>
<th></th>
<th>LJ-1</th>
<th>LJ-2</th>
<th>LJ-3</th>
<th>LJ-4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_{\text{max}}^+$ (MPa)</td>
<td>4.18</td>
<td>4.18</td>
<td>3.25</td>
<td>0.84</td>
</tr>
<tr>
<td>$\sigma_{\text{max}}^-$ (MPa)</td>
<td>4.18</td>
<td>4.18</td>
<td>1.539</td>
<td>0.84</td>
</tr>
</tbody>
</table>
(a)  
(b)
Figure 29. Stress nephogram of loose joints. (a) Positive loading of LJ-1. (b) Negative loading of LJ-1. (c) Positive loading of LJ-2. (d) Negative loading of LJ-2. (e) Positive loading of LJ-3. (f) Negative loading of LJ-3. (g) Positive loading of LJ-4. (h) Negative loading of LJ-4.

5. Conclusions

Through the on-site survey of a Chinese traditional timber structure, the probability analysis of the gap value and four typical gap values are determined. The full-scale mortise–tenon joint under different loose conditions, the following conclusions can be drawn:

(1) The gap is primarily distributed within the 5–15 mm range, the proportion of joint with this gap value reaches 44%. Four typical gap values including 5 mm, 10 mm, 25 mm and 50 mm are selected to study the effects of the gap on the mechanical properties of the PMT joint;

(2) The loose joint slips firstly when subjected to repeated loads. Once the tenon overcomes the free rotation angle, the mortise and tenon compress together. The predominant deformation of the joint is the partial pulling out of the tenon, and the contact portion between mortise and tenon has noticeable compression deformation. The failure stage is characterised by wood fibre tearing at the variable cross-section of the tenon at the negative loading. No obvious compression deformation of the column, fang and mortise occurs;

(3) The hysteresis curve of restoring moment and rotational angle of LJ-1 and LJ-2 is an inverse “Z” shape with obvious pinching effect and the hysteresis loop is asymmetrical. The mortise–tenon joint undergoes slipping, elastic, strengthening and the failure stage in order. Hysteresis curves of the restoring moment and rotational angle of LJ-3 and LJ-4 are shriveled and the sliding length is longer than that of LJ-1 and LJ-2.
It is worth noting that mortise-tenon joints mainly rely on compression force to resist external load and friction force to consume energy;

(4) The bending bearing capacity and rotational stiffness of loose joints have a certain drop with the increase in gap. The limit angle of the loose joint lags behind that of the regular joint. The rotational stiffness of a loose joint with a large gap is essentially zero during loading, indicating that the mortise-tenon joint has almost no resisting capability. Because its stiffness is low, an acceptable processing method is required to improve the joint’s rotational performance under loose conditions. The limitation gap value is given, which can provide a reference to the protection and repair of the traditional timber structure. The proposed three-parameter power function model is noted to be consistent with experimental results, which shows that the proposed model can reflect the mechanical behaviour of joints;

(5) The stress states of penetrated mortise-tenon joints with different loose conditions are discussed. Compression deformations are mainly located at the tenon. The smaller the degree of loosening, the greater the stress of the component. If the degree of loosening is large, the stress of joint is very small, indicating that the joint has almost no resistance to external load.

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