Article

On the Residual Static and Impact Capacity of Shear-Reinforced Concrete Beams Subjected to an Initial Impact

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Abstract: Impact loads in previous research showed to induce brittle responses of statically flexure-critical reinforced concrete (RC) beams designed for ductility. The impact load may produce flexural shear damage modes similar to that observed during quasi-static loads and local shear damage under the impact zone. The occurrence of shear damage modes during impact tests has been investigated extensively, but their effect on the residual quasi-static and dynamic capacity is not fully understood. For this aim, an initial high-velocity impact test initiated severe shear damage to RC beams. The beams were then tested quasi-statically and by sequential impact testing using the same setup as the initial tests. The results indicate a flexure-dominated response during sequential impact tests for beams containing extreme shear reinforcement amounts, favouring the energy-absorption capacity. Significant shear and flexural damage occurred for beams with less shear reinforcement, indicating a hybrid response that varied throughout the tests. The tests for the residual quasi-static capacity indicated severe consequences from initial local shear damage on the capacity, as shown by the brittle response of the beam with the most shear reinforcement. However, wide initial flexural cracks instead showed a favourable effect, as there was an indication of transfer from brittle to ductile failure. For beams showing both global and local shear damage, it was concluded that global shear damage modes were critical for the residual static and dynamic shear capacity.

Keywords: impact testing; residual capacity; shear reinforcement; concrete beams; dynamic response

1. Introduction

Impact loads on structural elements may result from, e.g., vehicle collisions, falling objects, or projectiles. The civilian design provisions generally use quasi-static loads with durations much longer than the structure’s natural period, while impact loads instead have durations in the range of, or less than, this period. The relatively short duration of impact loads results in a different response as wave propagation, inertia, and strain-rate effects become significant, as described in [1]. The activation of significant inertia forces results in a different deflected shape during impact loading compared for during equivalent quasi-static loads, which as argued in [2] results in a higher shear force-to-moment ratio. This higher shear force-to-moment ratio may induce a brittle failure mode as the beam becomes dynamically shear-critical, although it was statically flexure-critical. This may occur for any general impulsive load which excites significant inertia forces, as shown experimentally in, e.g., ref. [3] for beams subjected to severe dynamic air-blast loads. The transition from brittle to ductile failure during impulsive loads results in lowered energy absorption capabilities.

It was experimentally shown by [4] that statically flexure-critical RC members instead showed severe diagonal shear cracks during high-velocity impact loading. The diagonal cracks formed under the impact zone with an angle of approximately 45 degrees, a damage pattern referred to as a shear plug. Diagonal cracks similar to those observed for shear-critical beams during quasi-static tests formed alongside the shear plug for beams
containing low shear reinforcement amounts. These were less steep diagonal cracks initiating closer to the supports that become horizontal at the top before propagating to the impact zone. During impact loading, it is therefore possible to initiate global shear failure (static flexural shear failure) and local shear failure (shear plug) for statically flexure-critical RC beams. The same conclusions are reported in the review of the impact response of RC beams, presented in [5]. The local shear failure was explained as an effect of only part of the beam resisting the impact force by its inertia in the early stages of the response, where the stress wave propagation has a significant influence. It was experimentally shown in [6] that this transfer from global shear failure to local shear failure under the impact zone occurred as the striker velocity increased, resulting in less impact force duration and higher maximum amplitude.

While local and global shear damage modes during impact loads have been investigated extensively, there is not much research about their effect on the residual static and dynamic shear capacity. For this aim, a high-velocity initial impact test first initiated severe shear damage on RC beams. The beams were later tested quasi-statically for their residual static capacity and by sequential impact tests for the residual impact capacity. Both were conducted for beams with various amounts of shear reinforcement.

2. Experimental Program

The test series involved quasi-static and impact tests of ten reinforced concrete beams, as summarized in Table 1. Beams E and F were tested quasi-statically for reference behaviour. Four beams, A1, B1, C1, and D1, were subjected to an initial impact before being tested quasi-statically to failure. The final four beams, A2, B2, C2, and D2, were subjected to sequential impacts until failure. As the table shows, the parameter studied is the amount of shear reinforcement by altering the stirrup diameter. Beam type A is without stirrups, type B with stirrups 6 mm in diameter, type C with 8 mm, and type D with two joined 8 mm stirrups.

Table 1. Details of the test-series.

<table>
<thead>
<tr>
<th>Beam</th>
<th>Test Type</th>
<th>Stirrup Diameter [mm]</th>
<th>Striker Mass [kg]</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>Impact + Quasi-static</td>
<td>-</td>
<td>60</td>
</tr>
<tr>
<td>A2</td>
<td>Impact</td>
<td>-</td>
<td>70</td>
</tr>
<tr>
<td>B1</td>
<td>Impact + Quasi-static</td>
<td>6</td>
<td>70</td>
</tr>
<tr>
<td>B2</td>
<td>Impact</td>
<td>6</td>
<td>70</td>
</tr>
<tr>
<td>C1</td>
<td>Impact + Quasi-static</td>
<td>8</td>
<td>70</td>
</tr>
<tr>
<td>C2</td>
<td>Impact</td>
<td>8</td>
<td>70</td>
</tr>
<tr>
<td>D1</td>
<td>Impact + Quasi-static</td>
<td>2 × 8</td>
<td>70</td>
</tr>
<tr>
<td>D2</td>
<td>Impact</td>
<td>2 × 8</td>
<td>70</td>
</tr>
<tr>
<td>E</td>
<td>Quasi-static</td>
<td>6</td>
<td>-</td>
</tr>
<tr>
<td>F</td>
<td>Quasi-static</td>
<td>8</td>
<td>-</td>
</tr>
</tbody>
</table>

2.1. Material Testing

The characteristics of the steel reinforcement were determined by tensile testing following [7]. As shown in Figure 1, two samples of each reinforcement diameter were tested. Bars with diameter \( \phi = 12 \text{ mm} \) showed a distinct yield plateau, while bars with 8 mm diameter lacked a yield plateau. Bars with 6 mm diameter could not be tested to failure, as they either failed at the clamp of the testing machine or slipped in the clamp as it was relaxed to prevent local failure from the clamping force. The results for these bars may therefore be seen as a lower limit. Strength parameters determined from the tests are summarized in Table 2, where \( f_y \) is the yield strength and \( f_u \) the ultimate strength.

The concrete strength of eight samples was tested in uniaxial compression on 100 mm cubes following the standard in [8]. The first four samples, S1, S2, S3, and S4, were taken from the batch used to cast the first five beams A1, B1, C1, D1, and E, and the remaining samples were from the second batch used for the remainder beams. The cubes were tested
30 days after casting, Table 3 presents the strength characteristics, giving the maximum compressive stress $f_{c,max}$, mean value of the maximum compressive stress $\mu$; and standard deviation of the maximum compressive stress $\sigma$. Sample S8 showed significantly lower results than the others. The results considering sample S8 point to a concrete quality C25/30. However, if S8 is discarded, the concrete quality is C30/37, as predicted and used for the experimental design.

Figure 1. Stress–strain curves measured from material testing of two samples of each reinforcement diameter.

Table 2. Summary of results from tensile testing of the reinforcement used.

<table>
<thead>
<tr>
<th>Specimen</th>
<th>$\phi$ [mm]</th>
<th>$f_y$ [MPa]</th>
<th>$f_u$ [MPa]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi_{12,1}$</td>
<td>12</td>
<td>510</td>
<td>629</td>
</tr>
<tr>
<td>$\phi_{12,2}$</td>
<td>12</td>
<td>508</td>
<td>625</td>
</tr>
<tr>
<td>$\phi_{8,1}$</td>
<td>8</td>
<td>490</td>
<td>642</td>
</tr>
<tr>
<td>$\phi_{8,2}$</td>
<td>8</td>
<td>493</td>
<td>641</td>
</tr>
<tr>
<td>$\phi_{6,1}$</td>
<td>6</td>
<td>499</td>
<td>538</td>
</tr>
<tr>
<td>$\phi_{6,2}$</td>
<td>6</td>
<td>471</td>
<td>516</td>
</tr>
</tbody>
</table>

Table 3. Summary of the results from testing of the concrete strength.

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Age [days]</th>
<th>$f_{c,max}$ [MPa]</th>
<th>$\mu$ [MPa]</th>
<th>$\sigma$ [MPa]</th>
</tr>
</thead>
<tbody>
<tr>
<td>S1</td>
<td>30</td>
<td>43.03</td>
<td></td>
<td></td>
</tr>
<tr>
<td>S2</td>
<td>30</td>
<td>39.97</td>
<td></td>
<td></td>
</tr>
<tr>
<td>S3</td>
<td>30</td>
<td>45.36</td>
<td></td>
<td></td>
</tr>
<tr>
<td>S4</td>
<td>30</td>
<td>44.76</td>
<td></td>
<td></td>
</tr>
<tr>
<td>S5</td>
<td>30</td>
<td>41.98</td>
<td>39.53 $^1$</td>
<td>8.48 $^1$</td>
</tr>
<tr>
<td>S6</td>
<td>30</td>
<td>41.45</td>
<td>42.32 $^2$</td>
<td>3.38 $^2$</td>
</tr>
<tr>
<td>S7</td>
<td>30</td>
<td>36.09</td>
<td></td>
<td></td>
</tr>
<tr>
<td>S8</td>
<td>30</td>
<td>19.65</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

$^1$ Including S8. $^2$ Discarding S8.

2.2. Specimen Design and Properties

The geometry and reinforcement configuration of the beams are presented in Figure 2. The cross-section of the beams was square with 150 mm sides, the length was 800 mm, and the free span 700 mm. The shear slenderness $a/d = 3.0$ implies a response dominated by static beam action, see, e.g., ref. [9]. The dimensionless constant $\alpha = Al^2/I = 261$ ($l : h \approx 5 : 1$) implies a significant effect of shear deformation on the static deflection mode, i.e., the first mode of vibration, as discussed in, e.g., ref. [10]. Here, $a$ is the length of the shear span; $d$ effective depth; $A$ cross-section area; $l$ length of the free span; $I$ area moment of inertia; and $h$ cross-section height. The beam may thus be classified as non-slender with a response dominated by beam action.
The reinforcement design was determined from static design provisions using Eurocode 2 [11]. The spacing of the stirrups $s$ was determined by fulfilling the maximum spacing from the design provisions, i.e., $s \leq 0.75d$ for all beams. As shown in Table 4, the transverse reinforcement amount $\rho_w = A_s/sb_w$ varied from 0% to 1.49% by altering the stirrup diameter with a constant spacing of 90 mm. The compressive strut inclination $\theta$ determined from the calculations is also presented. The flexural tensile reinforcement was designed by determining the ratio $\gamma = P_v/P_m$, where $P_v$ is the maximum mid-span concentrated force the beam can sustain considering a shear failure mode and $P_m$ the maximum load considering a flexural failure mode. The beams with the least amount of transverse reinforcement (types B and E) were designed for a ratio of $\gamma \approx 1.0$. The longitudinal reinforcement amount that fulfilled this criterion was maintained, while increasing the shear reinforcement for types C and D so that an approximate transition zone from shear failure to flexural failure could be determined during static and dynamic loading. This resulted in a longitudinal reinforcement of $\rho = A_s/bd \times 100 = 1.91\%$ as shown in Table 4, where all calculations are summarized. Beam types B-D were thus statically flexure-critical, as $\gamma > 1.0$. All beams except type A fulfil the requirement for minimum reinforcement in the design provisions. The overhang outside the supports was insufficient for the anchorage of the reinforcement, and mechanical anchorage was therefore provided by welding a steel plate on the protruding reinforcement, as shown in Figure 2.

![Figure 2. Schematic of the RC beams tested (all dimensions in mm).](image)

**Table 4.** Reinforcement amount and capacities determined for each beam type.

<table>
<thead>
<tr>
<th>Beam Type</th>
<th>$\rho$ [%]</th>
<th>$\rho_w$ [%]</th>
<th>$P_m$ [kN]</th>
<th>$P_v$ [kN]</th>
<th>$\theta$ [°]</th>
<th>$\gamma$ [-]</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1.91</td>
<td>0</td>
<td>137.4</td>
<td>64.8</td>
<td>-</td>
<td>0.49</td>
</tr>
<tr>
<td>B (&amp; E)</td>
<td>1.91</td>
<td>0.42</td>
<td>137.4</td>
<td>139.2</td>
<td>24.05</td>
<td>1.01</td>
</tr>
<tr>
<td>C (&amp; F)</td>
<td>1.91</td>
<td>0.74</td>
<td>137.4</td>
<td>170.4</td>
<td>32.92</td>
<td>1.24</td>
</tr>
<tr>
<td>D</td>
<td>1.91</td>
<td>1.49</td>
<td>137.4</td>
<td>187.0</td>
<td>45</td>
<td>1.36</td>
</tr>
</tbody>
</table>

2.3. Quasi-Static Test Setup

The quasi-static tests were conducted using an MTS machine. The testing was performed by displacement control, with a rate determined following [12]. Figure 3 shows the test setup used. Three-point bending was applied to the beams, and to facilitate even
loading, a fibreboard was used. The displacement was halted every 5 kN to study the progression of damage.

![Test setup used for the quasi-static tests](image1.png)

**Figure 3.** (a) Test setup used for the quasi-static tests; and (b) schematic of the setup.

The vertical mid-point displacement read from the MTS machine had displacement components from not only the beam but also the deformation of the fibreboard and the development of contact in the system. This system responds as a series of springs as presented in Figure 4a, where \( k \) is the stiffness of the springs and \( F_y \) the yield strength in case the spring develops plastic deformation. To determine the mid-point displacement of the beam only, the later tests (A1–D1) for the static residual capacity after impact measured the mid-point displacement using digital image correlation (DIC). The setup for this is presented in Figure 4b. The open-source Python software Py2Dic (see [13]) was used to determine the vertical displacement at the mid-height of the mid-point of the beam. The results using DIC were validated by applying the procedure on the piston of the MTS machine in the pictures taken. The results using DIC show to converge well with the displacement measured by the MTS machine.

![System with springs in series](image2.png)

**Figure 4.** (a) System with springs in series and (b) DIC setup.

### 2.4. Impact Test Setup

The impact tests were conducted by a free-falling striker as presented in Figure 5a. A steel cylinder initially of 60 kg was dropped and guided by a plastic pipe. The height between the bottom of the cylinder and the wood fibreboard placed to act as a mechanical filter was 1600 mm as shown in (b). Two shock accelerometers were placed 110 mm from the mid-point of the beam in the length direction. They were of fabricate HBK Type 4375, see [14], with appropriate sensitivity, maximum acceleration (shock), and resonance frequency.
The striker’s drop height and mass were determined using a rigid-plastic model, see, e.g., ref. [15]. This model can only capture flexure-dominated failure modes and assumes constant retardation without any elastic strain energy stored in the beam during impact. The mass and drop height of the striker was updated until a final displacement after retardation of about 9 mm using the rigid-plastic model. This final displacement corresponds to reaching the displacement capacity concerning a flexural failure mode \( u_{u,m} = 3.00 \text{ mm} \), thus enforcing significant damage to the beams. The displacement capacity was calculated using the plastic rotation capacity following the static design provisions in Eurocode 2, see [11], and crushing of the concrete was predicted to determine rotation capacity rather than rupture of the reinforcement. The initial velocity of the beam was determined by assuming a plastic collision of rigid bodies, see, e.g., ref. [16], where the beam was assumed to respond with the generalized mass from deformation in the first mode of vibration calculated following [17]. The plastic collision model predicted an initial velocity of 4.89 m/s at the beam mid-point.

The signal measured from the accelerometers was sampled with a frequency of 19.2 kHz using the built-in anti-aliasing filter (2 kHz digital low-pass filter, type Bessel 4th order); see [18]. From the measured acceleration signal, the velocity and displacement were determined by time integration using the Newmark method; see [19]. A time span of 20 ms was used for the integration, corresponding to the time needed for a significant amount of the energy imparted by the first impact in each test to dissipate.

The measured acceleration signals showed oscillation around constant acceleration levels (non-zero mean) and significant drift, which produces error during time integration, an often-observed phenomenon when using accelerometers; see, e.g., [20]. The piezoelectric accelerometers used only measures alternating current (AC), and not direct current (DC), meaning that any constant acceleration level measured is not due to the response of the beam but a measurement error; see [21]. Figure 6 exemplifies the effect of this measurement error on the integrated velocity and displacement. In (a), a non-zero mean occurs for Beams A1–D1 before and after the initial peak, which results in no oscillation around zero velocity in (c). This velocity error produces a higher-order error in (e) for the displacements as the signals are integrated again. The error from the drift and non-zero mean was reduced by applying a second-order high-pass Butterworth filter on the acceleration signals with a cut-off frequency of 0.7–45 Hz (see [22]) depending on the signal. A 10% Tukey window was applied to zero the acceleration signal at its start and end (see [23]), and the velocity was rotated to fulfil the end condition of zero velocity. The effect of applying these signal processing tools on the acceleration, velocity, and displacement is shown by comparing the
results in Figure 6(a) to (b); (c) to (d); and (e) to (f), respectively. Therefore, the signals due to integration are seen as approximate; their values are not definite as the time integration procedure with filtering is not exact.

Figure 6. (a) Measured acceleration at the west accelerometer for A1–D1; (b) acceleration after signal processing; (c) velocity due to measured signal; (d) velocity after signal processing; (e) displacement due to measured signal; and (f) displacement after signal processing.

Figure 7 shows the measured acceleration signal from the west accelerometer of Beam D1. The figure presents the important characteristics of the signals that were discussed. The initial peak shows the maximum acceleration measured as the contact force develops and is resisted by the beam. Thereafter, the retardation phase initiates, where the beam decelerates due to the development of the strain energy in the beam and prevailing energy dissipation mechanisms. The retardation phase is succeeded by what resembles the damped free vibration of the beam with internal forces in the linear range.
The energy density of measured acceleration signals in the time–frequency domain, represented by the time–frequency scale-space (see [24]), were determined in [25] using the continuous wavelet transform (CWT); see [26]. From this procedure, frequency ranges with high energy content could be determined, corresponding to resonance frequencies of the system. A similar procedure was adopted in the current work, where a Morlet wavelet was used in combination with the MATLAB Wavelet Toolbox to determine the wavelet coefficients; see [27]. The procedure was applied on the free vibration phase of the signals post-retardation, such that the fundamental mode of the beams and other resonance frequencies with high relative energy content could be determined after sustaining damage during retardation. This is presented in Figure 7, where CWT was applied on the linear free vibration phase spanning 11–20 ms. Here, the response frequencies with high energy content are coloured black, and white indicates relatively low energy content.

![Figure 7. The initial peak; retardation; and linear free vibration phase of Beam D1 and time–frequency scale-space of the linear free vibration phase determined using CWT.](image)

The frequency ranges with high energy content may be compared with the analytical solution for natural frequencies for the same beam configuration. The resonance frequency \( f_{\text{analytical},n} \) for mode \( n \) of a simply supported slender beam may be determined as

\[
f_{\text{analytical},n} = \frac{n^2 \pi}{2} \sqrt{\frac{EI}{\rho_b A b^4}}
\]

where \( E \) is the Young’s modulus of the beam, and \( \rho_b \) is the density of the beam; see, e.g., ref. [28]. Table 5 presents the first two analytical natural frequencies of the beam determined assuming uncracked concrete sections \( f_{\text{uncracked}} \) and cracked concrete sections \( f_{\text{cracked}} \), where the steel and concrete in compression are in the linear range. Comparing the resonance frequencies between the beams post-retardation is interesting since the mass generally is equal between the beams. A relatively low resonance frequency thus indicates a lower element stiffness from sustaining more damage. CWT was applied on all beams after the first impact test to study their degree of damage before the residual static and impact tests were conducted.

![Table 5. Calculations of the first two natural frequencies for cracked and uncracked concrete beams.](image)
3. Experimental Results and Discussion

3.1. Dynamic Response and Residual Impact Capacity

The damage to the beams was compared by studying crack widths and inclination for diagonal cracks. Cracks wider than 0.1 were annotated and measured using an electronic calliper. The inclination of diagonal cracks wider than 0.1 mm were also annotated, and the inclinations were determined by drawing a secant line between an approximate start and end of the crack.

The degree of damage to Beam A1 in Figure 8a was less than expected, and the mass of the striker was thus proactively increased for all subsequent tests. This mass increase from 60 to 70 kg induced what is deemed a total flexural shear failure for Beam A2. Beams A1 and A2 showed steep diagonal cracks with inclinations close to 45 degrees on the west side of the mid-point, while less steep diagonal cracks appeared on the east side of the mid-point. A local shear plug damage mode and global flexural shear damage thus occurred. Beam A2 showed that the global flexural shear damage is critical, as this diagonal crack resulted in the compressive zone’s failure and the beam’s complete failure.

Wide and steep diagonal shear cracks formed for Beams B1, B2, C1, and D1, close to 45 degrees in inclination, accompanied by wide flexural cracks. The damage patterns showed significant shear plug and flexural damage, corresponding to a hybrid response. Beam C2 instead responded with a wide global flexural shear crack on the west side and small flexural cracks; thus, the response is shear-dominated. B2 showed a similar global flexural shear crack as C2, but this crack did not open to the same extent, although the shear reinforcement was less for B2. The reason for the larger shear capacity of Beam B2 was that the diagonal shear crack propagated across two stirrups instead of only one, which was the case for C2.

![Figure 8](image)

**Figure 8.** (a) Damage to A1 and A2 after the first impact; (b) B1 and B2; (c) C1 and C2; and (d) D1 and D2.

The damage due to sequential impact testing is evaluated by comparing the damage after the first impact test to the impact test where the beam failed, as presented in Figure 9. Beam A2 failed after the first impact and thus needs no comparison. The results for Beam B2 in (a) show that five impact tests were needed for the beam to reach failure. The beam...
was deemed to have failed since the concrete in the compressive zone showed significant spalling; it is therefore a local failure. The spalling is possibly a result of local compressive failure at the impact zone from the impact load. However, the global flexural shear crack on the east side propagated to the crushed zone, implying that this may also be a reason for the spalling. Significant flexural damage was accumulated by the five impacts, as shown by the sizeable residual curvature presented in the picture of the beam and also by the large width of the flexural cracks after the fifth impact test. The sequential impacts also widened the steep diagonal and global flexural shear cracks. Significant flexural and shear damage thus occurred, indicating a hybrid response mode of Beam B2.

Figure 9b shows that Beam C2 failed by the second impact. The critical global flexural shear crack on the west side opened significantly, resulting in spalling around the tensile reinforcement and significant permanent shear deformation of the beam.

Beam D2 in Figure 9c showed to withstand six impacts before local failure occurred. The beam failed by spalling around the tensile reinforcement and in the compressive zone. This beam showed a flexure-dominated response for all impacts by opening wide flexural cracks, which is supported by the sizeable residual curvature presented by the picture of the beam.

The full signal measured at the west accelerometer was compared for the beams during the first, second, and fifth impact test in Figure 10. Beam A1 in (a) showed its second prominent peak due to a second impact during the first test earlier than the other beams. The smaller spring-back indicates more damage to Beam A1 and a different response than the other beams, as less energy was imposed on the striker after the first impact with the beam. Beam A2 showed no second prominent peak in (b), indicating that the imposed
energy was absorbed by the opening of the wide flexural shear crack after the first impact, causing a small or no spring-back. Beam C2 also showed its prominent second peak earlier than the other beams in (b) and (c), indicating significant damage. Beam C2 was deemed to have failed after the second impact test presented in (c), however, there is still a second peak, indicating no propagation of an unstable crack that dissipated all energy. This implies a possible global residual capacity of Beam C2, as the beam was able to spring back, although significant local damage occurred. Beams B2 and D2 showed, during the fifth impact in (d), to respond similarly, with slightly less energy imparted to the striker for Beam B2 after the first impact with the beam, indicating a higher degree of damage.

Figure 10. (a) Entire signal measured at the west accelerometer for Beams A1–D1 during the first impact test; (b) signal during the first impact test for Beams A2–D2; (c) signal during the second impact test for Beams B2–D2; and (d) signal during the fifth impact test for Beams B2 and D2.

The acceleration after signal processing, time-integrated velocity, and time-integrated displacement are shown in Figure 11 for Beams A1–D1. The response 20 ms after the development of the initial peak shown in Figure 10 was studied. The variation of velocity in time presented in Figure 11c,d shows a similar response for Beams B1 and C1. Their dominating frequency 10 ms after retardation, where free vibration of the beams in the linear range of internal force occurs, is larger than that observed for Beams D1 and A1. The reduced dominating frequency of Beams D1 and A1 post-retardation indicates a larger degree of damage, as a decrease in the dominating frequency points to a decrease in stiffness. The peak and residual displacement after spring-back in (e) and (f) are similar for Beams B1–D1, indicating similar global responses. Beam A1 shows less spring-back than the other beams in (f), as the propagation of significant diagonal cracks absorbs the energy imparted by the striker. The permanent displacement after the spring-back of Beam A1 is higher on the east side compared with the west side, indicating permanent shear deformation.
Figure 11. (a) Signal-processed acceleration at the west accelerometer for Beams A1–D1 during the first impact test; (b) acceleration at the east accelerometer for A1–D1 during the first impact test; (c) integrated velocity at the west accelerometer for A1–D1; (d) integrated velocity at the east accelerometer for A1–D1; (e) integrated displacement at the west accelerometer for A1–D1; and (f) integrated displacement at the east accelerometer for A1–D1.

The signals for Beams A2–D2 during the first impact test are shown in Figure 12. Here, it is instead B2 and D2 that show a similar response in (c) and (d), with larger dominating frequencies during free vibration after retardation, implying that A2 and C2 sustained a higher degree of damage. The difference in the level of permanent displacement for A2 in (e) and (f) indicates significant permanent shear deformation.
The signals for Beams B2–D2 during the second and fifth impact tests are presented in Figure 13. The response in terms of velocity and displacement of Beam D1 were similar during the second and fifth impact test. This shows that the response for dynamically flexure-dominated beams does not change with sequential impacts. The response of Beam B2, which showed a hybrid response, changed drastically between the second and fifth test. Low-frequency oscillations occurred for Beam B2 in (d), indicating a high degree of damage to the beam. Beam C2 showed large permanent deformation after the second impact in (e) due to the global flexural shear crack opening.
Figure 13. (a) Acceleration at the west accelerometer for Beams B2–D2 during the second impact test; (b) acceleration for B2 and D2 during the fifth impact test; (c) integrated velocity for B2–D2; (d) integrated velocity for B2 and D2; (e) integrated displacement for B2–D2; and (f) integrated displacement for B2 and D2.

After the first impact test, CWT was applied to the acceleration signals in Figure 14 post-retardation. The signals show oscillation around a constant acceleration level, which resulted in low-frequency energy content from the CWT within a range of about 0–100 Hz. Since the accelerometers cannot measure constant acceleration levels, it was concluded that the low-frequency energy contents are fictitious and due to measurement error.
Figure 14. Acceleration and time–frequency scale-space from CWT of the signal measured at the west accelerometer for Beams A1, A2, C1, C2, D1, and D2.

Beams A1 and A2 in Figure 14a,b showed high energy concentration for frequencies in the range of 300–400 and 250–300 Hz, respectively. These frequency ranges are significantly lower than the fundamental analytical frequencies determined (see Table 5), indicating
severe damage to the beams. In addition to this frequency range, Beams A1 and A2 have significant energy concentrations in the range of 750–900 and 700–800 Hz, respectively.

In Figure 14d, similar frequency ranges with significant energy content showed for Beam C2, as for Beams A1 and A2, indicating severe damage to C2. Beam C1 instead showed higher frequency ranges with high energy density in (c), indicating less damage to this beam.

Beam D1 in Figure 14e also presents high energy density in a first and second frequency range, but Beam D2 in (f) mainly showed energy density in one range. The difference between Beams D1 and D2 is that Beam D1 showed severe shear plug damage (Figure 8), while Beam D2 showed mainly flexural damage. This indicates that several frequency ranges with high energy content occur in conjunction with severe shear damage. At the same time, flexure-dominated responses instead show high energy content localized in one frequency range.

3.2. Quasi-Static Response and Residual Static Capacity

The quasi-static test results are presented in Figure 15. In (a), the force and vertical mid-point displacement measured directly from the MTS machine are presented. Beams A1, E, and D1 presented brittle responses, and thus sustained shear failure modes. Beams F, B1, and C1 showed ductile flexural failure modes indicated by the horizontal plateau corresponding to the yielding of the tensile reinforcement. Beams B1 and E contained the same shear reinforcement amount, which implies that the initial cracks from the initial impact test for Beam B1 resulted in a favourable response as the failure transitioned from brittle to ductile. Beams C1 and F were equivalent in reinforcement, but a slightly larger capacity of Beam C1 was observed. It can be seen that Beam D1 failed by shear, as shown by the brittle response, although it contained the largest amount of shear reinforcement.

The results in terms of force measured from the MTS machine and displacement determined using DIC are presented in Figure 15b. The results indicate a similar stiffness for the beams. Beams B1 and C1 showed a more ductile response where the displacement capacity was not reached, as no decline of the horizontal plateau was initiated.

![Figure 15](image-url). (a) Force–mid-displacement measured using the MTS machine for the beams; and (b) force measured using the MTS machine and mid-displacement measured using DIC for the beams.

The damage to Beams A1–D1 after the first impact test and the residual quasi-static test are presented in Figure 16. Beams E and F tested for static reference behaviour are also presented. In (a), Beam A1 failed during the quasi-static test by the initial global shear crack on the west side, which had less inclination than the steep diagonal crack on the east side. The same phenomenon was observed for A2 during the initial impact test and C2 during sequential impact testing. Beam B1 responded as shown in (b) by opening the diagonal and wide flexural cracks. As for the dynamic case, type B showed a hybrid response which showed to be ductile. Beam C1 responded by opening mainly wide flexural cracks. However, the shear crack started propagating towards the support,
indicating a small residual shear capacity. The hybrid response of Beam C1 was ductile, as shown by the force–displacement curve in Figure 15. Beam D1 showed a brittle failure mode (Figure 15), which is explained by the opening of a wide steep diagonal crack; see Figure 16d. Steep initial diagonal cracks may thus be severe for the residual capacity of the beam. By comparing Beams B1 and E, which had identical reinforcement amounts in (b) and (e), the already opened flexural cracks after the initial impact test had a favourable effect as the response became ductile.

<table>
<thead>
<tr>
<th>Beam</th>
<th>1st Impact</th>
<th>After Quasi-Static Testing</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>West: 0.5 mm, 0.4 mm  East: 0.5 mm 0.3 mm 0.4 mm 2.3 mm 0.5 mm 1.1 mm 0.5 mm</td>
<td></td>
</tr>
<tr>
<td>B1</td>
<td>West: 0.1 mm, 0.4 mm  East: 0.1 mm 0.4 mm 0.1 mm</td>
<td></td>
</tr>
<tr>
<td>C1</td>
<td>West: 0.8 mm, 0.4 mm  East: 0.3 mm</td>
<td></td>
</tr>
<tr>
<td>D1</td>
<td>West: 0.2 mm, 0.3 mm  East: 0.3 mm 1.8 mm 0.2 mm 2.9 mm</td>
<td></td>
</tr>
<tr>
<td>E</td>
<td>West: 1.2 mm, 0.1 mm  East:</td>
<td></td>
</tr>
<tr>
<td>F</td>
<td>West: 0.2 mm  East:</td>
<td></td>
</tr>
</tbody>
</table>

**Figure 16.** (a) Damage to Beam A1 after quasi-static testing; (b) damage to Beam B1; (c) damage to Beam C1; (d) damage to Beam D1; (e) damage to Beam E; and (f) damage to Beam F.
4. Conclusions

During initial impact testing, severe shear, hybrid, and flexural damage occurred for the beams. Only Beam A2 failed during the initial test, but significant damage occurred for the other beams as shown by the large crack widths, permanent displacement, and reduction in fundamental frequency. The beams with the least amount of shear reinforcement mainly showed shear damage, and the beams with the most shear reinforcement showed flexural damage.

Beam B2 showed a hybrid response throughout the sequential impact tests with considerable flexural and shear damage that led to a varying response. Beam D2 showed mainly flexural damage, and the response was similar across the five impact tests. This indicates that beams with large shear reinforcement amounts present a similar response during sequential impact tests with an accumulation of mainly flexural damage. Sequential impact testing also showed that local failure becomes critical for beams containing high reinforcement amounts that prevent global failure.

The full measured acceleration signals showed that the damage to the beams was coupled to the time for the second prominent peak during the test to occur. Beam A2 showed no second prominent peak as the striker’s energy was absorbed by the opening of the wide flexural shear crack, causing no spring-back for the beam. The beams presenting severe damage with residual capacity showed their second prominent peak earlier than the beams which sustained less degree of damage.

Preliminary CWT analyses on the signals during the first impact test showed that several frequency ranges contained high energy concentrations for beams with significant shear damage. Beams which instead showed mainly flexural damage showed energy concentration in one frequency range. Fictitious energy in low-frequency ranges occurred due to measurement error.

When comparing Beams E and B1 after residual quasi-static capacity testing, it was shown that the initial flexural cracks of Beam B1 had a favourable effect, as the response became ductile after initial impact testing. Beam D1 showed a brittle failure mode while testing the residual quasi-static capacity, although it contained the most shear reinforcement. This was explained by a possibly low static shear capacity of steep diagonal cracks, resulting in severe consequences of significant shear plug damage on the residual static capacity.

Both Beams A1 and A2 showed significant global flexural shear damage and shear plug damage after the initial impact test. However, the less steep global shear damage was shown to be critical for the residual static and impact capacity.

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