Article

Exclusive Bus Lane Allocation Considering Multimodal Traffic Equity Based on Bi-Level Programming

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Abstract: To ensure the equity of exclusive bus lane (EBL) allocation under multimodal traffic conditions, a bi-level programming model is first constructed. The upper-level model is the minimum total system cost considering the Gini coefficient and the lower-level model constructed a stochastic user equilibrium (SUE) model based on logit loading. Secondly, a heuristic algorithm combining an improved genetic algorithm (GA) and a method of the successive average method (MSA) is designed. Finally, the Nguyen and Dupuis networks are used as examples to verify and analyze the effectiveness, superiority and sensitivity of the model and algorithm. The results show that the method can effectively obtain the optimal solution of the upper-level model as 15,004 RMB, the Gini coefficient is 0.31, and the equity is at a relatively reasonable level. Compared with the different allocation schemes, the proposed scheme has a higher bus sharing rate and lower Gini coefficient. At the same time, when the actual demand is twice the basic demand, the bus share rate is the largest, 65%, and the Gini coefficient is the smallest at 0.3. The bus share rate decreases with the increase in the proportion of high time value travelers, which fully verifies the sensitivity of the model to the type of traveler.

Keywords: traffic engineering; multimodal traffic equity; bi-level programming model; exclusive bus lane; genetic algorithm; MSA algorithm

1. Introduction

To alleviate road traffic congestion, many cities implement a policy of transit priority by constructing exclusive bus lanes (EBLs). For example, in China, more than two hundred cities, including Beijing, Shanghai, Shenzhen and Xi’an, have constructed EBLs [1]. On the one hand, it can make buses enjoy the right of priority access. The travel time of buses can be reduced by about 18%, and the waiting time at the station will be reduced by about 12% [2]. On the other hand, by allocating one or more mixed lanes as EBLs, the driving of other non-bus vehicles will be restricted and driving times will be increased. This can induce some travelers to choose public transportation, which is conducive to alleviating urban traffic congestion to some extent.

However, EBLs will increase the delay of other social vehicles while ensuring the priority of buses [3]. How should we allocate EBLs to balance the right of road between buses and other social vehicles? In addition, considering the traveler’s trip mode choice and path choice [4], how do we fairly allocate these limited road resources to heterogeneous travelers? Therefore, the allocation of EBLs is a complex optimization problem [5].

In recent years, research on the reasonable allocation of EBLs has become more and more abundant. At present, the research on EBLs can be roughly divided into two categories. One is based on microsimulation models, using simulation software to simulate different parameter settings through measured data. For example, Lin et al. [6] obtained the changes in road indexes before and after allocating EBLs through the mathematical theory model.
and VISSIM simulation. Song et al. [7] evaluated the benefits of EBLs on the MATLAB and VISSIM simulation platforms. Kampouri et al. [8] used VISSIM microscopic simulation software and additional modules of VisVAP and EnViVer to represent the operation of EBLs. Yang et al. [9] used microscopic simulation models to evaluate traffic operations under different demand levels and bus share ratios. Szarata et al. [10] used PTV Vissim software with an additional logic script to control the DBL (dynamic bus lane) activation. Alexey et al. [11] determined the parameters of mathematical models of the delay time of private and public transport for various parameters of the bus lane, the length of the bus stop loading area and its distance from the signalized intersection through imitation microscopic modeling.

The other type is based on macro analysis models, and the impact of EBLs is analyzed by actual cases. It is attributed to the network design problem (NDP). Generally, the network design problem (NDP) of urban transportation refers to an optimal investment decision problem [12]. The research of the NDP can be divided into planning before EBL allocation and evaluation after EBL allocation. For planning, the bi-level programming model is mainly used. The upper-level model aims to optimize the system-level objectives (e.g., maximizing the traffic network consumer surplus [13], minimizing the total travel time of traffic network users [14], minimizing the equilibrium travel cost [15], etc.). The lower-level model aims to maintain user equilibrium by considering the user’s response to the EBL allocation scheme (e.g., stochastic equilibrium assignment under multimodal traffic modes [16], passenger flow assignment model based on minimum generalized travel cost, multimodal traffic network equilibrium model, capacity-constrained traffic assignment model [17], etc.).

The bi-level programming model belongs to the NP-hard problem, and its solution process is very complicated. So, it is difficult to find an accurate and effective global optimal algorithm. The current solution methods are mainly divided into two categories: linear bi-level programming algorithms (e.g., pole search algorithm, branch and bound method and penalty function method, etc.) and nonlinear bi-level programming algorithms (e.g., genetic algorithm, simulated annealing algorithm, particle swarm optimization algorithm, etc.). The key to solving the bi-level programming model is to determine the response function of the lower-level decision variable to the upper-level decision variable. Based on this, the algorithm originally applied to the nonlinear programming problem is introduced into the research of bi-level programming. Sui et al. [18] proposed a particle swarm optimization algorithm (PSO) and an A* algorithm to solve the model. Guo et al. [19] introduced an SBO algorithm based on efficient machine learning (ML) surrogate model to find the optimal allocation scheme. Tsitsokas et al. [20] used a meta-heuristic algorithm based on large neighborhood search (LNS).

For evaluation, Wang et al. [21] applied the dogit-PSL (path-size logit) MMNE (multi-modal network equilibrium) model for evaluating the EBL expansion plans, in which a consistent synthetic proportional index is proposed. Ma et al. [22] proposed a novel analytical model of a macroscopic fundamental diagram (MFD) and passenger macroscopic fundamental diagram (p-MFD) and the corresponding indicators based on MFD and p-MFD to evaluate the operation of the network. Adnan et al. [23] reached a conclusion on how much time was saved by the passengers in the buses using bus-lane, the amount of fuel saved by buses and how much CO2 emission was reduced by comparing the speed, volume and service duration data from before and after the application.

The study of traffic optimization has gradually changed from the pursuit of efficiency to the coexistence of efficiency and equity. At present, the accepted concept of traffic equity refers to the allocation of costs and benefits generated by a policy, which usually takes into account different groups of people [24]. The connotation of transportation equity is also continuously enriched with different research. Park SJ et al. [25] thought transit equity is the standard deviation of transit accessibility, which is the difference in travel time between private vehicles and the transit system. Spatial equity included the time accessibility and the spatial distribution of the configuration equilibrium [26].
Najaf P et al. [27] believed that equity in transportation is defined as how appropriately and equally the impacts of transportation are distributed among different types of users. Shi [28] pointed out that the equity of the public transport system refers to the fair relationship and distribution mechanism between the various traffic participants in the possession and distribution of public transport resources.

Equity can generally be divided into horizontal and vertical. Horizontal equity requires each similar individual or group to have the same cost or income distribution [29]. Transportation equity belongs to the vertical equity that considers income, social class and traffic demand differences. Equity is measured mainly by accessibility, Gini coefficient, Theil’s entropy and so on, but the heterogeneity of travelers is not fully considered. In practice, travelers with different time values have different sensitivities to the choice of traffic modes. There is a lack of research on whether the implementation of transportation policies brings about the inequity of heterogeneous travelers. Previous equity research focused on road pricing equity and environmental equity, such as the social equity of tradable credit and road pricing [30], congestion pricing equity [31], air pollution equity [32] and carbon emission equity [33].

In summary, the current research has the following gaps:

(1) Microscopic research uses the evaluating indicators to determine whether to allocate EBLs on the links, ignoring the interaction between links, which may cause unreasonable problems in the region. Macroscopic research is based on system efficiency as the optimization goal, minimizing the total time or total cost of the system and so on. It is not considered a question of whether there is equity for multimodal travelers because of the EBL allocations. The equity of transportation is concentrated in congestion pricing, carbon emissions and so on.

(2) Generally, the bi-level programming model is used to study the allocation of EBLs. The bi-level programming model is an NP-hard problem, and its solution algorithm is also in the process of continuous exploration. How do we improve the effectiveness and accuracy of the algorithm?

Based on the existing research, this paper fully considers the types of travelers with different time values and the generalized travel cost. The Gini coefficient is defined as the ratio of the cumulative percentage of the generalized travel cost of travelers with different time values to the cumulative percentage of the population. By constructing a bi-level programming model, the Gini coefficient is introduced into the upper-level model. The objective function of minimizing the Gini coefficient and the total system cost is established. The lower-level model constructs a stochastic user equilibrium (SUE) model based on logit loading. The improved genetic algorithm (GA) and method of the successive average method (MSA) are organically combined into the heuristic algorithm. Finally, the proposed bi-level model is validated with Nguyen and Dupuis networks.

The rest of the paper is organized as follows. Section 2 specifically describes the problem of EBL allocation. Section 3 proposes the bi-level programming model. Section 4 presents solution algorithms that are used to solve the bi-level programming. In Section 5, the proposed model is tested with Nguyen and Dupuis networks. Section 6 contains some conclusions we draw and some suggestions for future work.

2. Problem Description

As shown in Figure 1a, it is assumed that there are two bus lines $l_1$ and $l_2$, and the links are $a_1$ and $a_2$, respectively. Among them, the passenger flow of $l_2$ is greater than $l_1$. When EBLs are allocated on link $a_2$, the generalized travel cost of cars increases due to the reduction in the number of lanes [17]. As shown in Figure 1b, some car users will transfer to link $a_1$ with better traffic conditions and some car users will choose bus line $l_2$. 
This research is based on the following assumptions:

1. There are only two types of traffic modes (bus and car) in the traffic network.
2. The links in the traffic network are divided into EBLs and non-EBLs.
3. The negative impact of EBL setting on cars is measured by the increased generalized travel cost of cars on the road.

The following notations are introduced to establish the optimization models as shown in Table 1.

<table>
<thead>
<tr>
<th>Sets</th>
<th>Definitions</th>
</tr>
</thead>
<tbody>
<tr>
<td>V</td>
<td>Node set</td>
</tr>
<tr>
<td>A</td>
<td>Link set</td>
</tr>
<tr>
<td>M</td>
<td>Traffic mode set, ( m = c ) is car related parameters, ( m = b ) is bus related parameters</td>
</tr>
<tr>
<td>L</td>
<td>Bus line set</td>
</tr>
<tr>
<td>K</td>
<td>Travel path set</td>
</tr>
<tr>
<td>W</td>
<td>Traffic network OD set</td>
</tr>
<tr>
<td>I</td>
<td>Travelers set</td>
</tr>
<tr>
<td>Y</td>
<td>Decision variable set</td>
</tr>
</tbody>
</table>
### Table 1. Cont.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Descriptions</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \tau_{m}^{w} )</td>
<td>Flow of travelers with time value ( i ) choosing traffic mode ( m ) on path ( k ) between OD pair ( w )</td>
</tr>
<tr>
<td>( X_{a}^{m} )</td>
<td>Flow of travelers choosing traffic mode ( m ) on link ( a )</td>
</tr>
<tr>
<td>( C )</td>
<td>Total generalized travel cost</td>
</tr>
<tr>
<td>( C_{i} )</td>
<td>Generalized travel cost of traveler with time value ( i )</td>
</tr>
<tr>
<td>( C_{i,m} )</td>
<td>Generalized travel cost of traveler with time value ( i ) choosing traffic mode ( m ) on link ( a )</td>
</tr>
<tr>
<td>( C_{i,w,k} )</td>
<td>Generalized travel cost of travelers with time value ( i ) choosing traffic mode ( m ) on the path ( k ) between OD pair ( w )</td>
</tr>
<tr>
<td>( V_{i} )</td>
<td>Time value of travelers</td>
</tr>
<tr>
<td>( e_{a,m}^{i} )</td>
<td>Traffic mode ( m ) free flow time on link ( a )</td>
</tr>
<tr>
<td>( F_{a,c} )</td>
<td>Fixed cost of car on link ( a )</td>
</tr>
<tr>
<td>( t_{a,c} )</td>
<td>Travel time of car on link ( a )</td>
</tr>
<tr>
<td>( N_{a} )</td>
<td>Number of lanes</td>
</tr>
<tr>
<td>( C_{a} )</td>
<td>Capacity per lane</td>
</tr>
<tr>
<td>( y_{a}^{b} )</td>
<td>Bus traffic on link ( a )</td>
</tr>
<tr>
<td>( f_{l} )</td>
<td>Departure frequency of bus line ( l )</td>
</tr>
<tr>
<td>( T_{a,b} )</td>
<td>Time to walk from origin to the bus station</td>
</tr>
<tr>
<td>( T_{a,b} )</td>
<td>Bus travel time on link ( a ) considering congestion utility</td>
</tr>
<tr>
<td>( P_{b} )</td>
<td>Bus fare</td>
</tr>
<tr>
<td>( t_{d} )</td>
<td>Time to walk from the bus station to destination</td>
</tr>
<tr>
<td>( q_{w} )</td>
<td>Traffic demand of travelers with time value ( i )</td>
</tr>
<tr>
<td>( q_{w}^{d} )</td>
<td>Traffic demand between OD pair ( w )</td>
</tr>
<tr>
<td>( X_{a} )</td>
<td>Traffic volume of travelers with time value ( i ) choosing traffic mode ( m ) on link ( a )</td>
</tr>
<tr>
<td>( p_{w}^{m} )</td>
<td>Probability of travelers with time Value ( i ) choosing traffic mode ( m ) between OD pair ( w )</td>
</tr>
<tr>
<td>( \delta_{w,k}^{m} )</td>
<td>Probability of travelers with time value ( i ) choosing path ( k ) between OD pair ( w )</td>
</tr>
<tr>
<td>( K_{i,m} )</td>
<td>Attraction of traffic mode ( m ) to travelers with time value ( i )</td>
</tr>
<tr>
<td>( B )</td>
<td>Maximum budget of EBL construction</td>
</tr>
<tr>
<td>( L_{a} )</td>
<td>Length of EBL</td>
</tr>
<tr>
<td>( B_{2} )</td>
<td>Unit construction cost of EBL</td>
</tr>
<tr>
<td>( G )</td>
<td>Gini coefficient</td>
</tr>
<tr>
<td>( \theta )</td>
<td>Discrete parameters for measuring traveler perception error</td>
</tr>
<tr>
<td>( \alpha, \beta, \gamma, \zeta_{a}, \chi_{a} )</td>
<td>Parameters to be determined</td>
</tr>
<tr>
<td>( \delta_{w,k}^{m} )</td>
<td>0–1 variables, ( \delta_{w,k}^{m} = 1 ) if path ( k ) between OD pair ( w ) passes through link ( a ); otherwise, ( \delta_{w,k}^{m} = 0 )</td>
</tr>
<tr>
<td>( \delta_{w,k}^{m} )</td>
<td>0–1 variables, ( \delta_{w,k}^{m} = 1 ) if path ( k ) between OD pair ( w ) needs to take the bus line through link ( a ); otherwise, ( \delta_{w,k}^{m} = 0 )</td>
</tr>
<tr>
<td>( \delta_{a}^{l} )</td>
<td>0–1 variables, ( \delta_{a}^{l} = 1 ) if the bus line through link ( a ); otherwise, ( \delta_{a}^{l} = 0 )</td>
</tr>
<tr>
<td>( \delta_{a}^{l} )</td>
<td>0–1 variables, ( \delta_{a}^{l} = 1 ) if the path between OD pair ( w ) chooses bus line ( l ); otherwise, ( \delta_{a}^{l} = 0 )</td>
</tr>
</tbody>
</table>

### Decision variables

| Description | 0–1 variables, \( y_{a} = 1 \) if a bus lane is set up on link \( a \); otherwise, \( y_{a} = 0 \) |

### 3. Methodology

A traffic network can be described as a directed graph \( G = (V, A) \), consisting of a set of nodes (junctions) \( V \) and directed links (roads) \( A \) [8]. \( m = c \) is car-related parameters, and \( m = b \) is bus-related parameters; \( L \) is the set of bus lines. \( K \) represents the set of travel paths, \( W \) is the OD set of traffic networks, \( G \). Assuming that travelers are divided into \( I \) classes according to the characteristics of their travel time value \( V_{i} \).
3.1. Optimization Framework

We establish a bi-level programming model and whether to set EBLs as a decision variable, as shown in Figure 2. The upper-level model is from the perspective of traffic managers intending to optimize the overall efficiency of the traffic network considering the Gini coefficient. The lower-level model is an SUE model based on logit loading from the perspective of travelers.

3.2. Generalized Travel Cost of Considering EBLs

3.2.1. Generalized Travel Cost of Car

The generalized travel cost of a car mainly considers the actual travel time cost and the fixed travel cost. The generalized travel cost of a traveler with time value $i$ choosing the car on link $a$ is shown in Equation (1):

$$C_{i,a}^{c} = T_{a,c} \times V_{i} + F_{a,c}$$  (1)

Assuming that there are $N_{a}$ lanes for link $a$ and the capacity of each lane is $C_{a}$. If $y_{a}$ lanes are set as EBLs, the number of lanes that can be driven is $N_{a} - y_{a}$ and the corresponding link capacity is $(N_{a} - y_{a})C_{a}$. Thus, the driving time of the car on link $a$ can be shown in Equation (2):

$$T_{a,c} = \left\{1 + \alpha^{c} \left[\lambda_{c}X_{a}^{c} + \lambda_{b}Y_{a}^{b}\right] / (N_{a} - y_{a})C_{a}\right\}^{\beta^{c}}$$  (2)

where $X_{a}^{c}$ is the flow of travelers choosing to use a car on link $a$, as shown in Equation (3):

$$X_{a}^{c} = \sum_{i} X_{i,a}^{c} = \sum_{l} \sum_{k} r_{l,k}^{c} s_{l,k}^{w}$$  (3)
\( Y_a^b \) is the flow of buses on link \( a \) as shown in Equation (4):

\[
Y_a^b = \sum f_i \delta_i^b
\]

(4)

In summary, the generalized travel cost of a traveler with time value \( i \) choosing a car on path \( k \) between OD pairs \( w \) is formulated as Equation (5):

\[
C_{i,k}^{w,c} = \sum \alpha(T_{a,c} + V_i + F_{a,c}) \times \delta_{a,k}^w
\]

(5)

3.2.2. Generalized Travel Cost of the Bus

According to the time period of bus travel, the bus generalized travel cost is divided into the following three stages: abording, in-bound and alighting. This is shown in Figure 3.

![Figure 3. Generalized travel cost diagram of bus.](image)

Considering walking time, waiting time and driving time, the generalized travel cost of the bus can be shown in Equation (6):

\[
C_{i,k}^{w,c} = \left( t_i^w + \frac{1}{2f_i} \delta_i^b \right) \times V_i + \sum \sum T_{a,b}^i \times V_i \times \delta_{a,b}^d \times P_c + t_i^w \times V_i
\]

(6)

For the bus, its possible capacity on link \( a \) containing EBLs is still \( N_a C_a \). The travel time function reflecting the bus line \( l \) passing through link \( a \) can be shown in Equation (7):

\[
T_{a,b} = t_{a,b}^i \left[ 1 + a \left( \frac{\lambda_c X_a^c + \lambda_b Y_a^b}{N_a C_a} \right) \right]^{\beta b}
\]

(7)

When the number of passengers increases to a certain extent, the congestion in the bus is uncomfortable or even difficult to ride. It may lead to a decrease in the utility value of the bus, and then a transfer to other travel modes. Therefore, considering the in-vehicle crowding effect, the generalized travel time cost of the bus line \( l \) through link \( a \) is shown in Equation (8):

\[
T_{a,b}^l = T_{a,b} \left[ 1 + a \left( \frac{X_a^b}{B_i f_l} \right) \beta \right]
\]

(8)

where \( X_a^b \) is the flow of travelers who take the bus line \( l \) through link \( a \) as shown in Equation (9):

\[
X_a^b = \sum f_i \delta_i^b = \sum \sum \delta_{a,k}^w \times Y_a^b
\]

(9)

3.3. Bi-Level Programming Model
3.3.1. The Upper-Level Model

With the minimum total system cost considering the Gini coefficient as the objective function and the construction investment of EBLs as the main constraint condition, the
optimal EBL setting scheme is determined. The mathematical model is as shown in Equations (10)–(12)

\[
\min Z_1(Y) = \min \left \{ G \times \sum_a \sum_i V_i \left [ T_{a,c}(y_a)X_i^c + \left ( t_w + T_{a,b}^l(y_a) + t_l^w \right ) X_i^b \right ] + \sum_a \left ( F_a^c X_a^c + P_b X_a^b \right ) \right \} 
\]

\[
\sum_a y_a L_a B_a \leq B, \forall a \in A
\]

\[
0 \leq G \leq 1
\]

where \( Y = (y_1, \cdots, y_a, \cdots, y_N) \) is a set of decision variables for whether links need to set up EBLs and is defined as \( y_a = 1 \) if a bus lane is set up on link \( a \); otherwise, \( y_a = 0 \).

The objective function (10) is to minimize the system cost considering the Gini coefficient. The first term is the Gini coefficient, and the second term is the total system cost. Constraint (11) is the budget constraint of bus lane construction, and constraint (12) is the value constraint of Gini coefficient.

The Gini coefficient is calculated by the lower-level model as follows:

- The lower-level model obtains the traffic volume \( X_{ij}^m \) of travelers with different time values in each link when the traffic network is balanced;
- Calculating the total generalized travel cost value as shown in Equation (13):

\[
C = \sum_a \sum_i V_i \left [ T_{a,c} X_i^c + \left ( t_w + T_{a,b}^l + t_l^w \right ) X_i^b \right ] + \sum_a \left ( F_a^c X_a^c + P_b X_a^b \right )
\]

- Calculating the cumulative percentage of the generalized travel cost of travelers with different time values \( \sum_i C_i \); calculating the population percentage of travelers with different time values \( \sum_i Q_i \), and calculating the cumulative percentage of population \( \sum_i Q_i \);
- According to the cumulative percentage of generalized travel costs and the cumulative percentage of the population, the Lorenz curve is drawn, as shown in Figure 4.
- Assuming the area between the actual Lorenz curve and the absolutely equitable line is \( S_A \) and the area between the actual Lorenz curve and the absolutely inequitable line is \( S_B \) [34]. The Gini coefficient is shown in Equation (15). The range of values is shown in Table 2.

\[
G = \frac{S_A}{S_A + S_B}
\]

<table>
<thead>
<tr>
<th>Table 2. Gini coefficient range and equity.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Gini Coefficient Range</strong></td>
</tr>
<tr>
<td>-----------------------------</td>
</tr>
<tr>
<td>( G &lt; 0.2 )</td>
</tr>
<tr>
<td>( 0.2 \leq G &lt; 0.3 )</td>
</tr>
<tr>
<td>( 0.3 \leq G &lt; 0.4 )</td>
</tr>
<tr>
<td>( 0.4 \leq G &lt; 0.6 )</td>
</tr>
<tr>
<td>( G \geq 0.6 )</td>
</tr>
</tbody>
</table>
3.3.2. The Lower-Level Model

Assuming that the traveler's perception errors of the travel utility of different traffic modes for different time values are independent of each other and obey the Gumbel distribution. The traffic mode choices and path choices are performed according to the logit model. The mathematical model is given by the following set of Equations (16)–(21):

\[
\min \mathcal{Z}_2 \left( X^{i,m}_a, f^{i,m}_{w,k} \right) = \min \left\{ \sum \sum V_i f_{0}^{X_i} T_{u_x}(w) dw + \sum \sum X^{i,c} F^{c}_a + \sum \sum X^{i,b} \left[ V_i \left( t^{b}_w + \frac{1}{\sigma^m_n} + t^{b}_d \right) + P \right] \right\} \tag{16}
\]

\[
N_a \geq 2, \forall a \in A \tag{17}
\]

\[
\sum_k f^{i,m}_{w,k} = q^{i,m}_{w}, \forall i \in I, w \in W, k \in K, m \in M \tag{18}
\]

\[
X^m_a = \sum X^{i,m}, \forall a \in A, i \in I, m \in M \tag{19}
\]

\[
X^{i,m}_a = \sum_{w} f^{i,m}_{w,k} g_{w,l}^{a,k}, \forall a \in A, i \in I, l \in L, w \in W, k \in K, m \in M \tag{20}
\]

\[
X^{i,m}_a \geq 0, f^{i,m}_{w,k} \geq 0, \forall a \in A, i \in I, w \in W, k \in K, m \in M \tag{21}
\]

The objective function (16) is a stochastic user equilibrium model based on logit loading. Constraint (17) indicates that the link to which the dedicated lane is to be laid generally has at least two lanes in one direction. Constraint (18) indicates the conservation constraint of the travel volume and the link flow of the traveler with time value \(i\) choosing the traffic mode \(m\) between OD pairs \(w\). Constraint (19) indicates the conservation constraint of the traffic volume and the traveler with time value \(i\) choosing the traffic mode \(m\) on link \(a\). Constraint (20) indicates the conservation constraint of the travel volume and the link flow of the traveler with time value \(i\) on link \(a\). Constraint (21) is a non-negative constraint.
The logit model is used to describe the traveler’s traffic mode choice behavior to reflect the impact of EBL allocation on traffic mode transfer. The probability that travelers with time value $i$ choose the traffic mode $m$ between OD pair $w$ is as shown in Equation (22):

$$\begin{align*}
    p^{i,m}_{w} &= \frac{\exp\left(-\theta C^{i,m}_{w} - K^{i,m}_{w}\right)}{\sum_{m} \exp\left(-\theta C^{i,m}_{w} - K^{i,m}_{w}\right)} \\
\end{align*}$$

The probability that traveler with time value $i$ chooses path $k$ between OD pairs $w$ is as shown in Equation (23):

$$\begin{align*}
    p^{i,m}_{w,k} &= \frac{\exp\left(-\theta C^{i,m}_{w,k}\right)}{\sum_{k} \exp\left(-\theta C^{i,m}_{w,k}\right)} \\
\end{align*}$$

4. Solution Algorithm

The basic GA has slow convergence speed, poor local search ability and is easy to fall into local optimum [21]. In order to ensure the genetic algorithm can converge to the global optimal solution, this paper improves the genetic algorithm by retaining the optimal individual strategy and designs an improved GA-MSA heuristic algorithm, as shown in Figure 5.

Step 0: Initialize the population

- Parameter setting
  
  Set the relevant parameters of the genetic algorithm, including population size $G$, crossover probability $P_c$, mutation probability $P_m$, chromosome length and maximum number of iterations $G_{\text{max}}$.

- Generating chromosomes
  
  The links are coded and arranged in order. The coding of each link is the decision variable. If the link is set to EBL, it is coded as 1, otherwise it is 0. The coding scheme is the chromosome in the genetic algorithm, and the length $N$ is the total number of links in the network. The $j$th chromosome is $Y_j, j = 1, 2, \cdots, G$.

- Generation of the initial population
  
  The population is a collection of chromosomes, and $G$ chromosomes are generated according to the corresponding coding scheme. The initial populations $Y^{(g)}$ are given randomly, and the evolution iteration number is $g = 0$.

Step 1: Stochastic equilibrium assignment

The initial solution under the initial population is $(X^{i(m)(n)}, q^{i(n)})$, and the number of iterations is $n = 0$. The generalized travel cost of travelers with different time values choosing different traffic modes is calculated. According to the logit model, the probability of traffic modes and path choices are calculated. The traffic flow of cars and buses is loaded into the traffic network, and all the path flow between ODs is obtained. A stochastic assignment is performed to obtain the additional traffic volume of link $(X^{i,m}_{a}, \bar{X}^{i,m}_{a})$, and the current traffic flow of each link is calculated by MSA as shown in Equations (24) and (25):

$$\begin{align*}
    X^{i,m(n+1)}_{a} &= X^{i,m(n)}_{a} + \left(\bar{X}^{i,m(n)}_{a} - X^{i,m(n)}_{a}\right) / n \\
    q^{i(n+1)}_{w} &= q^{i(n)}_{w} + \left(\bar{q}^{i(n)}_{w} - q^{i(n)}_{w}\right) / n \\
\end{align*}$$

Loop iteration termination criterion, when the traffic flow distribution results reach equilibrium, $X^{i,m(n+1)}_{a} - X^{i,m(n)}_{a} \leq \delta$ and $q^{i(n+1)}_{w} - q^{i(n)}_{w} \leq \delta$, $\delta$ is the predetermined error, turn to step 2, otherwise, $n = n + 1$, turn to step 1.
Step 2: Construction of fitness function

The fitness of each chromosome coding in the population is $F$, the greater the fitness, the greater the survival probability of the chromosome. The reciprocal of the upper-level objective function value $Z_1$ is used as the fitness of the chromosome, and the fitness value $F = 1/Z_1$ is calculated according to step 1.

Step 3: Generate a new population

The roulette wheel method is used to select chromosomes. The chromosomes selected, crossed and mutated are combined with the original population into a population, and the fitness value of each chromosome is calculated. According to the fitness value from large to small, $G$ chromosomes are selected as a new population. The purpose of merging the original population and the new population is to ensure that the excellent individuals can be inherited by the next generation. The new population not only preserves excellent individuals of the original population, but also increases the diversity of the population.

Step 4: Genetic algorithm termination criteria

Check whether the maximum number of evolutionary iterations $G_{\text{max}}$ is reached. If $g \geq G_{\text{max}}$ this stops the iteration. The chromosome corresponding to the maximum fitness in the population is the output, and the binary code corresponding to the chromosome is the optimal EBL allocation scheme. Otherwise, $g = g + 1$, go to step 0.

![Algorithm flow chart](image-url)
5. Computational Experiments

5.1. Network Introduction

Based on the network proposed by Nguyen and Dupuis in 1984, this paper constructs an urban traffic network for testing [35]. As shown in Figure 6, there are 13 nodes, 19 links and 9 OD pairs in the network. Among them, the red node is the origin node O and the blue node is the destination node D.

![Traffic network](image)

Figure 6. Traffic network.

In the connection between every two nodes, the solid line represents the traffic link of the car, and the dotted line represents the traffic link of the bus. The links between each OD pair are two-way, and the total number bus lines is 5.

5.2. Parameter Settings

Assuming that the average passenger capacity of each car is 2 people, so, \( \lambda_c \) is 0.5. The parameters in the generalized travel cost \( \alpha^b \) and \( \beta^b \) are 0.05 and 4, respectively. The maximum passenger capacity of the bus \( B_l \) is 80 people and the departure frequency of the bus \( f_l \) is 8 vehicles/h. The bus fare \( P_b \) is 2 RMB, and the conversion coefficient of the bus to the car \( \lambda^b \) is 3. The generalized travel cost parameters of the bus, \( \alpha^c \) and \( \beta^c \), are 0.02 and 3, respectively. The traveler’s walking time from the origin to the bus station \( t_w^o \) and from the bus station to the destination \( t_w^d \) are both 5 min. The congestion parameters in the bus, \( \alpha \) and \( \beta \), are 1.1 and 1.5, respectively. The unit construction cost of the bus lane \( C_a \) is 30,000 RMB/km, and the budget constraint of the bus lane construction \( B \) is 500,000 RMB. The OD pairs and corresponding basic traffic demands in the network are shown in Table 3. The total traffic demand is 8000 people, and the actual demand is \( \lambda \) times the basic demand.

The traveler’s time value is divided into three types: low, medium and high. The corresponding time value is shown in Table 4. It is assumed that the travel demand ratio of different time values between an OD pair \( w \) is \( q_{w1}^1 : q_{w2}^2 : q_{w3}^3 = 1 : 3 : 1 \). The attraction of different ODs is the same, and the perception error \( \theta \) is 0.05.
Table 3. Basic traffic demand between OD.

<table>
<thead>
<tr>
<th>O</th>
<th>2</th>
<th>3</th>
<th>11</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1000</td>
<td>800</td>
<td>600</td>
</tr>
<tr>
<td>4</td>
<td>900</td>
<td>700</td>
<td>1100</td>
</tr>
<tr>
<td>5</td>
<td>1200</td>
<td>900</td>
<td>800</td>
</tr>
</tbody>
</table>

Table 4. Traveler information.

<table>
<thead>
<tr>
<th>Time Value Type</th>
<th>Time Value</th>
<th>Car Attraction $K_c^i$ (RMB)</th>
<th>Bus Attraction $K_b^i$ (RMB)</th>
<th>Basic Traffic Demand $Q_i$ (People)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low ($i = 1$)</td>
<td>15</td>
<td>2</td>
<td>6</td>
<td>1600</td>
</tr>
<tr>
<td>Medium ($i = 2$)</td>
<td>30</td>
<td>4</td>
<td>4</td>
<td>4800</td>
</tr>
<tr>
<td>High ($i = 3$)</td>
<td>45</td>
<td>6</td>
<td>3</td>
<td>1600</td>
</tr>
</tbody>
</table>

Links 8 and 18 have no bus line, so their driving time is infinite. All feasible paths in the network are effective paths. The number of lanes on all links $N_a$ is 3, and the capacity of a single lane $C_a$ is 400 pcu/h. The network information is shown in Table 5.

Table 5. Traffic network information.

<table>
<thead>
<tr>
<th>Link Name</th>
<th>Link Length $L_a$ (km)</th>
<th>Link Fixed Cost $F_{a,c}$ (RMB)</th>
<th>Bus Line $l$</th>
<th>Car Free Flow Travel Time $t_{f,ac}^i$ (min)</th>
<th>Bus Free Flow Travel Time $t_{b,al}^i$ (min)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_1$</td>
<td>2.1</td>
<td>5</td>
<td>$l_2$</td>
<td>4.2</td>
<td>8.4</td>
</tr>
<tr>
<td>$a_2$</td>
<td>1.8</td>
<td>5</td>
<td>$l_1$</td>
<td>3.6</td>
<td>7.2</td>
</tr>
<tr>
<td>$a_3$</td>
<td>1.5</td>
<td>4</td>
<td>$l_5$</td>
<td>3.0</td>
<td>6.0</td>
</tr>
<tr>
<td>$a_4$</td>
<td>2.6</td>
<td>6</td>
<td>$l_3$</td>
<td>5.1</td>
<td>10.2</td>
</tr>
<tr>
<td>$a_5$</td>
<td>1.5</td>
<td>4</td>
<td>$l_2$</td>
<td>3.0</td>
<td>6.0</td>
</tr>
<tr>
<td>$a_6$</td>
<td>2.3</td>
<td>6</td>
<td>$l_4, l_5$</td>
<td>4.5</td>
<td>9.0</td>
</tr>
<tr>
<td>$a_7$</td>
<td>2.0</td>
<td>5</td>
<td>$l_1, l_2$</td>
<td>4.1</td>
<td>8.2</td>
</tr>
<tr>
<td>$a_8$</td>
<td>2.3</td>
<td>6</td>
<td>—</td>
<td>4.5</td>
<td>$\infty$</td>
</tr>
<tr>
<td>$a_9$</td>
<td>1.5</td>
<td>4</td>
<td>$l_4$</td>
<td>3.0</td>
<td>6.0</td>
</tr>
<tr>
<td>$a_{10}$</td>
<td>2.3</td>
<td>6</td>
<td>$l_2$</td>
<td>4.5</td>
<td>9.0</td>
</tr>
<tr>
<td>$a_{11}$</td>
<td>2.3</td>
<td>6</td>
<td>$l_1$</td>
<td>4.5</td>
<td>9.0</td>
</tr>
<tr>
<td>$a_{12}$</td>
<td>1.5</td>
<td>4</td>
<td>$l_3, l_5$</td>
<td>3.0</td>
<td>6.0</td>
</tr>
<tr>
<td>$a_{13}$</td>
<td>2.6</td>
<td>6</td>
<td>$l_4$</td>
<td>5.1</td>
<td>10.2</td>
</tr>
<tr>
<td>$a_{14}$</td>
<td>2.0</td>
<td>5</td>
<td>$l_3, l_5$</td>
<td>4.1</td>
<td>8.2</td>
</tr>
<tr>
<td>$a_{15}$</td>
<td>1.8</td>
<td>5</td>
<td>$l_3$</td>
<td>3.6</td>
<td>7.2</td>
</tr>
<tr>
<td>$a_{16}$</td>
<td>2.1</td>
<td>5</td>
<td>$l_3$</td>
<td>4.2</td>
<td>8.4</td>
</tr>
<tr>
<td>$a_{17}$</td>
<td>2.1</td>
<td>5</td>
<td>$l_1$</td>
<td>4.2</td>
<td>8.4</td>
</tr>
<tr>
<td>$a_{18}$</td>
<td>3.0</td>
<td>7</td>
<td>—</td>
<td>6.0</td>
<td>$\infty$</td>
</tr>
<tr>
<td>$a_{19}$</td>
<td>1.8</td>
<td>5</td>
<td>$l_1$</td>
<td>3.6</td>
<td>7.2</td>
</tr>
</tbody>
</table>

The improved GA-MSA heuristic algorithm is programmed by Python 3. The genetic parameters are set as follows: population size $G = 20$, crossover probability $P_c = 0.8$, mutation probability $P_m = 0.1$, chromosome length $N = 17$ and the maximum number of iterations $G_{max} = 1000$.

5.3. Results and Discussion

5.3.1. Effectiveness Verification: Algorithm Convergence and Optimal Allocation Scheme

Figure 7 shows the convergence process of the objective function. The genetic algorithm converges faster in the 600 times iterations, then remains unchanged and finally converges at 15,004 RMB. The whole convergence process takes about 10 min.
The improved GA-MSA heuristic algorithm is programmed by Python 3. The genetic parameters are set as follows: population size \( N \), mutation probability \( p_m \), and the maximum number of iterations \( G \). The total length is 12.7 km, and the total construction cost is 381,000 RMB. At the same time, the optimal solution of the upper-level model is 15,004 RMB, the total system cost is 49,917 RMB and the Gini coefficient is 0.31.

The position of the EBL allocation schemes in the traffic network is shown in Figure 8. The red links marked are the links where the EBLs are set. It can be seen that the optimal number of EBLs allocated is six, which are links \( a_4, a_6, a_7, a_{10}, a_{12} \) and \( a_{14} \). The total length is 12.7 km, and the total construction cost is 381,000 RMB. At the same time, the optimal solution of the upper-level model is 15,004 RMB, the total system cost is 49,917 RMB and the Gini coefficient is 0.31.

Figure 7. Convergence diagram of objective function.

Figure 8. EBLs allocation schemes.

5.3.2. Superiority Verification: Comparative Analysis of Different Allocation Schemes

There are four types of allocation schemes: Scheme 1: no EBLs; Scheme 2: all EBLs; Scheme 3: allocation scheme without considering equity (only the total system cost is considered in the upper-level model); and Scheme 4: allocation scheme considering equity (the Gini coefficient and total system cost are considered in the upper-level model). The results are shown in Table 6. By comparing and analyzing the influence of different EBL
allocation schemes on bus sharing rate, total system cost and Gini coefficient, the superiority of this model is further verified.

Table 6. Comparative analysis of different allocation schemes.

<table>
<thead>
<tr>
<th>Scheme</th>
<th>Bus Sharing Rate</th>
<th>Total System Cost/RMB</th>
<th>$C_i$</th>
<th>$Q_i$</th>
<th>Gini Coefficient</th>
<th>Optimal Solution/RMB</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.45</td>
<td>48,950</td>
<td>5.82</td>
<td>5.45</td>
<td>0.40</td>
<td>19,580</td>
</tr>
<tr>
<td>2</td>
<td>0.52</td>
<td>50,450</td>
<td>6.20</td>
<td>6.37</td>
<td>0.38</td>
<td>19,171</td>
</tr>
<tr>
<td>3</td>
<td>0.48</td>
<td>48,560</td>
<td>5.70</td>
<td>6.24</td>
<td>0.35</td>
<td>16,996</td>
</tr>
<tr>
<td>4</td>
<td>0.50</td>
<td>48,400</td>
<td>5.62</td>
<td>6.30</td>
<td>0.31</td>
<td>15,004</td>
</tr>
</tbody>
</table>

The total system cost of Scheme 1 is lower. Without EBLs, the generalized travel cost of the bus is increased, the sharing rate is the lowest and the Gini coefficient is the highest. This scheme is inequity to travelers with low time value because buses are more attractive to them. The bus sharing rate of Scheme 2 is the highest, but at the same time, the construction cost of the EBLs and the generalized travel cost of the car increase, making the total system cost the largest of all schemes. The Gini coefficient is also larger, indicating that the reduction in bus generalized travel cost is at the expense of the increase in car generalized travel cost, which is unfair to travelers with high time value.

Compared with Scheme 1 and Scheme 2, the average cost of different time value travelers in Scheme 3 is reduced. The Gini coefficient is reduced, as is the degree of inequity. However, the bus sharing rate and the cost of low value travelers is higher than Scheme 4. In Scheme 4, the bus sharing rate is higher. Although the total system cost is not the lowest, the Gini coefficient is the smallest and the optimal solution of the upper-level model is also the smallest. Therefore, from the perspective of social equity, the optimized EBL allocation Scheme 4 is more in line with the actual situation.

5.3.3. Demand Level Sensitivity Verification: Impact of Demand Level on the Optimal Solution

In order to further study the influence of demand level on the allocation of EBLs, it is assumed that the actual demand is the basic demand $\lambda$ times. From Figure 9, it can be seen that with the $\lambda$ continuous increase, the bus sharing rate shows a trend of first increasing and then decreasing. The Gini coefficient shows a trend of first decreasing and then increasing.

Figure 9. Impact of changing demand on optimal solution.
When $\lambda = 2$, the bus sharing rate is the largest, 65%, whereas the Gini coefficient is the smallest, 0.3, and the allocation of EBLs is the most ideal.

At the same time, the number of EBL settings gradually increased, followed by the addition of $a_4, a_{10}, a_5$ and $a_{19}$. When $\lambda = 2$, the number of EBLs obtain a maximum of 8. The total length is 16 km, and the construction cost is 480,000 RMB, which is close to the maximum investment constraints of 500,000 RMB. So, with the increase in travel demand, EBLs no longer change the optimal allocation scheme, as shown in Figure 10.

![Figure 10](image_url)

**Figure 10.** EBL allocation scheme under different demands. (a) $\lambda = 0.5$ (b) $\lambda = 1$ (c) $\lambda = 1.5$ (d) $\lambda = 2$, $\lambda = 2.5$, $\lambda = 3$.

5.3.4. Traveler Type Sensitivity Verification: Impact of Traveler Type on the Optimal Solution

The original assumed travel demand ratio for different time values between an OD pair $w$ is $q_{lw}^1 : q_{lw}^2 : q_{lw}^3 = 1 : 3 : 1$. The impact of the increase in the proportion of high time and low time value on the optimal solution is considered, and the results are shown in Figure 11.

![Figure 11](image_url)

**Figure 11.** EBL allocation scheme under different types of travelers.

(1) Assume that the proportion of low time value travelers remains unchanged, some middle time value travelers transfer to high time value travelers, that is $q_{lw}^1 : q_{lw}^2 : q_{lw}^3 = 1 : 2 : 2$. The total system cost increases, the Gini coefficient also increases and the EBLs allocation...
scheme adds new links $a_5$ and $a_{15}$. By increasing the proportion of EBLs, the transfer of cars to buses is further promoted to ensure the equity of travelers with different time values.

(2) Assuming that the proportion of high time value travelers remains unchanged, some medium time value travelers transfer to low time value travelers, that is $q_1^w : q_2^w : q_3^w = 2 : 2 : 1$. The bus sharing rate increases significantly, the total system cost decreases and the Gini coefficient decreases. It can be seen that the equity of travelers increases and the EBL allocation scheme reduces link $a_{10}$ that only one bus line $l_2$ passes through, achieving a balance between car and bus traffic modes.

Because using the car and bus have different attractions to travelers with different time values, cars are more attractive to travelers with high time value and buses are less attractive. Therefore, when the proportion of travelers with high time value increases, the bus sharing rate shows a downward trend. By increasing the number of EBLs, the generalized cost of bus travel is reduced and car travelers are prompted to transfer to buses. When the proportion of low time value travelers increases, the bus sharing rate is on the rise. By reducing unnecessary EBLs, the rational allocation of road resources is realized to ensure the right of road for car travelers.

6. Conclusions

In this paper, the EBL network design problem considering equity is proposed and a bi-level programming model is constructed. The upper-level model is the EBL programming model, and the lower-level model is the SUE model based on logit loading. An improved GA-MSA heuristic algorithm is proposed to solve the model. Finally, taking Nguyen and Dupuis networks as examples, the following conclusions are drawn:

(1) Effectiveness: The heuristic algorithm converges to the 600 iterations. The optimal solution of the upper-level model is 15,004 RMB, and the total system cost is 49,917 RMB. The Gini coefficient is 0.31, and the equity is at a reasonable level. The number of EBLs allocated is six, which are links $a_4$, $a_6$, $a_7$, $a_{10}$, $a_{12}$ and $a_{14}$.

(2) Superiority: By comparing and analyzing the four different EBL allocation schemes, the allocation scheme considering equity we proposed has a higher bus sharing rate. Although the total system cost is not the lowest, the Gini coefficient is the smallest and the optimal solution of the upper-level model is the smallest. Therefore, from the perspective of social equity, the optimized EBL allocation scheme is more in line with the actual situation.

(3) Demand level sensitivity: With the increasing demand for travel, the bus sharing rate shows a trend for increasing first and then decreasing, and the Gini coefficient shows a trend for decreasing first and then increasing. At that time, the bus sharing rate is the largest (65%), whereas the Gini coefficient is the smallest (0.3); at this time, the allocation of the EBLs is the most ideal.

(4) Traveler type sensitivity: Due to the different attractions of cars and buses to different time value travelers, cars are more attractive for high time value travelers, whereas buses are opposite to this. When the proportion of high time value travelers increases, the optimal allocation scheme adds links $a_5$ and $a_{15}$. When the proportion of low time value travelers increases, link $a_{10}$ is reduced so that only one bus line $l_2$ passes.

Although this paper considers two travel modes of cars and buses and the Gini coefficient is added to the upper-level objective function, there are still the following deficiencies, which can be further studied in the future:

(1) In this paper, the impact of multi-mode traffic, such as autonomous driving and electric buses, is not fully considered. Although the right to use EBLs in this article is limited to buses, carpooling travelers, taxis, etc., can also use EBLs in some cities. How these kinds of vehicles will affect the distribution of EBLs is also worthy of study.

(2) This paper designs a GA-MSA heuristic algorithm and verifies its effectiveness and feasibility in small-scale road networks. However, in practice, it is more about large-scale road networks. It is crucial to introduce modern technologies, such as multi-criteria decision analysis (MCDA) [36], agent-based modelling [37], large neighborhood search (LNS) and so on, to find approximate optimal solutions for large-scale networks.
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References

10. Szarata, M.; Olszewski, P.; Bichajło, L. Simulation study of dynamic bus lane concept. Sustainability 2021, 13, 1302. [CrossRef]
11. Alexey, F.; Dmitrii, Z. Influence of the parameters of the bus lane and the bus stop on the delays of private and public transport. Sustainability 2020, 12, 9593. [CrossRef]
15. Chen, Q. An optimization model for the selection of bus-only lanes in a city. PLoS ONE 2015, 10, e0133951. [CrossRef]
33. Guo, Y.; Tong, Q.; Li, Z.; Zhao, Y. Research on carbon emission quota of railway in China from the perspective of equity and efficiency. Sustainability 2022, 14, 13789. [CrossRef]
37. Kostrzewski, M.; Abdelatty, Y.; Eliwa, A.; Nader, M. Analysis of modern vs. conventional development technologies in transportation—The case study of a last-mile delivery process. Sensors 2022, 22, 9858. [CrossRef]

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