

Article

Interference Mitigation for Metro Train Location Based on Karhunen–Loeve Decomposition

Xiwen Deng , Zhongliang Deng, Jingrong Liu, Zhenke Ding and Zhichao Zhang

School of Electronic Engineering, Beijing University of Posts and Telecommunications, Beijing 100876, China; dengzhl@bupt.edu.cn (Z.D.); jingrongliu@bupt.edu.cn (J.L.); dingzk@bupt.edu.cn (Z.D.); 2020110459@bupt.cn (Z.Z.)

* Correspondence: dengxiwen@bupt.edu.cn; Tel.: +86-010-6119-8509

Abstract: We propose a communication positioning integrated signal (CPIS) to achieve sub-meter positioning of metro trains. However, the electromagnetic environment of metro trains is complex, and the interference of radio frequency signals will reduce the accuracy of train positioning and affect the safe operation of trains. In this paper, an interference mitigation method for metro train positioning based on Karhunen–Loeve decomposition is proposed. The signal is orthogonally expanded, and the eigenvalues of the interference signal are obtained according to the distribution characteristics of the eigenvalues. The reconstructed interference signal is then subtracted from the received signal to achieve interference suppression. The experimental results demonstrate the effectiveness of the method in interference scenarios.

Keywords: train positioning; communication positioning integrated signal; interference mitigation; Karhunen–Loeve decomposition



Citation: Deng, X.; Deng, Z.; Liu, J.; Ding, Z.; Zhang, Z. Interference Mitigation for Metro Train Location Based on Karhunen–Loeve Decomposition. *Appl. Sci.* **2023**, *13*, 3351. <https://doi.org/10.3390/app13053351>

Academic Editors: Luigi Fortuna and Youngchol Choi

Received: 26 October 2022

Revised: 17 February 2023

Accepted: 27 February 2023

Published: 6 March 2023



Copyright: © 2023 by the authors. Licensee MDPI, Basel, Switzerland. This article is an open access article distributed under the terms and conditions of the Creative Commons Attribution (CC BY) license (<https://creativecommons.org/licenses/by/4.0/>).

1. Introduction

Train positioning technology is the core on-board technology in train control systems, and is the basic guarantee for the safe operation of urban rail transit [1–3]. Traditional train positioning methods include track circuits, transponders, and cross loops. Track circuits can only achieve occlusion zone positioning [4]. The positioning of the transponder can realize the positioning in the point mode, and the real-time positioning of the train can be realized in combination with the inertial navigation system (INS) [5]. Cross-loop lines can realize real-time sub-meter-level positioning of trains, but the cost is relatively high [6]. Weidong et al. and Chetty et al. [7,8] proposed a metro train positioning system based on the GSM network. By collecting the cell parameters of the train location, and using the received power of the base station signal to estimate the distance between the train and the base station, the positioning of the train can be achieved, but the positioning accuracy is affected by the propagation environment. Wei et al. [9] introduced a train positioning method composed of a Beidou satellite navigation system and INS, and achieved a positioning accuracy higher than five meters in the simulation environment. Compared with traditional positioning methods, the positioning method based on sensor fusion improves the positioning accuracy, but cannot achieve sub-meter-level positioning [10,11]. In addition to the wide coverage of the communication capability, the mobile communication network also has a positioning function. The 3GPP TR 22.872 standard requires the positioning accuracy of transportation. Compared with the traditional train positioning method, the mobile communication network based on 5G has the advantage of convenient maintenance without adding positioning equipment. J. Talvitie in [12] proposed a network-side positioning method for high-speed trains in the 5G NR scenario, using the time difference of arrival observed by the 5G uplink detection reference signal and the extended Kalman filter algorithm to estimate the position and speed of the train at the same time, and carried out

a 100 km simulation experiment in the simulation scenario meeting the requirements of the 3GPP Release 15 standard, achieving sub-meter-level positioning in 90% of cases.

We propose a communication and positioning fusion system based on a 5G mobile communication network to achieve high-precision positioning based on Time Difference of Arrival (TDOA) [13,14]. The communication positioning integrated signal (CPIS) is based on the NOMA principle, in which the positioning signal is superimposed on the communication signal and broadcast in the form of low power. CPIS has the following characteristics: (1) The positioning signal does not interfere with the communication. i.e., The positioning signal reuses the communication time-frequency resources and does not affect the quality of service (QoS) of the communication; (2) The positioning signal power of different users can be configured, i.e., the interference from users can be reduced by configuring different positioning signal power; (3) The positioning signal is continuous, i.e., the positioning signal is based on Direct Sequence Spread Spectrum-Code Division Multiple Access (DSSS-CDMA), and the positioning terminal tracks continuous signals to achieve the sub-meter level. This technology of superimposing positioning and communication signals has been popularized and used in shopping malls and airports, and has achieved high-precision wide-area indoor and outdoor positioning [13]. We verified the application of communication and navigation fusion positioning technology in subway positioning based on a leaky coaxial cable, and the laboratory has verified that the positioning method based on time difference of arrival can achieve sub-meter positioning accuracy [14]. Considering the limited transmission power of the positioning signal and the complex electromagnetic environment of the metro, the interference signal from the environment poses a threat to the CPIS, which affects the safety of train operation based on the communication–positioning integrated system. Therefore, suppression of interfering signals is very important to the train control system.

Interference mitigation techniques are mainly classified into the following categories, including spatial processing techniques, such as array signal processing, adaptive notch filtering (ANF), time-frequency analysis, and subspace decomposition. Spatial processing-based methods use beamforming to direct the main beam to the desired signal, improving the signal-to-noise ratio (SNR) of the received signal [15–17]. Beams with high directivity depend on the complexity of the equipment and are not suitable for metro train positioning scenarios [18]. ANF is widely used due to its simplicity of implementation. For single-tone interference scenarios, ANF performs well [19–21]. In the case of chirp signal interference or narrow-band signal interference, the filter cannot accurately separate the interference and real signals, and the filtering performance is degraded. Among the time-frequency analysis methods, wavelet packet coefficient transform (WPCT) [22,23] and short-time Fourier transform (STFT) [24,25] show good performance in the case of chirp signals. STFT has lower computational complexity, but the windowing process of STFT results in an inherent trade-off between time and frequency resolution. WPCT overcomes the shortcomings of STFT, but the parameter configuration of wavelet transform needs to be adjusted for interference scenarios. In the subspace projection and decomposition method, the Karhunen–Loeve transform (KLT) is used to orthogonally decompose a high amount of observational data to extract the eigenvalues of specific types of signals [26,27]. This paper proposes an interference mitigation algorithm for metro train positioning based on Karhunen–Loeve decomposition. The signal is orthogonally expanded to obtain the eigenvalues of the interference signal, and the interference signal is reconstructed, and the reconstructed interference signal is subtracted from the received signal to achieve interference suppression.

The remainder of this paper is organized as follows. Section 2 presents the signal model and interference pattern. Section 3 presents the proposed KL expansion-based interference mitigation algorithm. In Section 4, experiments are conducted to compare different methods and to verify the effectiveness of the proposed algorithm. The conclusions and future work are presented in Section 5.

2. Signal Model and Interference Modes

In this section, we introduce the mathematical model of the CPIS and the interference model of the metro train. In our positioning system, the positioning part in CPIS is a system based on DSSS-CDMA, and the navigation message is modulated on the spread spectrum signal by BPSK modulation. The structure of the CPIS base station is shown in Figure 1. The transition of the integrated signal is realized by adding a positioning signal generator to the 5G communication base station. For the communication system, the positioning signal superimposed on the frequency spectrum is the interference signal. We define the ratio of communication signal power to positioning signal power as CPR. Based on a large number of experimental test results, we found that when the CPR is greater than 18 dB, the communication QoS is not affected [28]. For train CPIS positioning terminal, the communication signal is regarded as white Gaussian noise. Without radio frequency signal interference, the signal received by the terminal can be expressed as follows.

$$r_{CPIS}(t) = \sum_{i=1}^{N_{BS}} A^{(i)} d^{(i)}(t) c^{(i)}(t - \tau_i) \cdot \cos(2\pi(f_c + f_{d,i})t + \varphi_{0,i}(t)) + \omega(t). \quad (1)$$

where N_{BS} represents the number of received integrated signals, $A^{(i)}$ is the signal amplitude, $d^{(i)}$ is the navigation message modulated on the positioning part, $c^{(i)}$ represents the spread spectrum sequence, τ_i represents the time delay of the signal, f_c and $f_{d,i}$ are the signal carrier frequency and the doppler frequency shift, respectively, $\varphi_{0,i}(t)$ is the initial carrier phase, and $\omega(t)$ is additive white Gaussian noise (AGWN) with variance σ_n^2 .

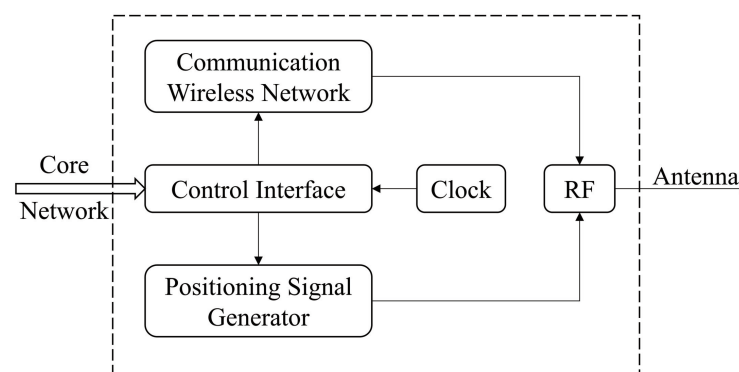


Figure 1. The structure of the CPIS base station.

Metro train positioning based on CPIS will be inferred by the following two modes of radio frequency signals as shown in Figure 2.

(a) The interfering signal source is stationary relative to the train positioning terminal. The interference usually comes from radio frequency signals from fixed equipment on the train or from man-carried interference equipment.

(b) The interfering signal source moves relative to the train positioning terminal. The interference comes from the radio frequency signal interference generated by the equipment fixed on the rail or the equipment along the track.

In the first interference mode, the interference power received by the positioning terminal is constant, resulting in a decrease in positioning accuracy or even inability to output location, which affects the normal operation of the train control system. In the second interference mode, due to the movement of the terminal antenna relative to the interference, there is a difference between the frequency received by the receiver and the frequency of the interfering signal. However, according to the Doppler effect, this difference is negligible, because electromagnetic waves travel much faster than trains run.

● : Base Station ● : Position Terminal
 ● : Interference Source

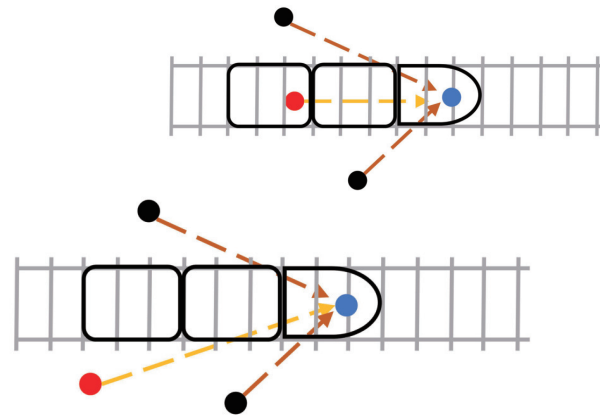


Figure 2. Two modes of interference. (a) The interfering signal source is stationary relative to the train positioning terminal; (b) The interfering signal source moves relative to the train positioning terminal.

When the train terminal receives CPIS with an interference signal, it is expressed as follows.

$$r(t) = r_{CPIS}(t) + J(t). \quad (2)$$

where $J(t)$ represents the interference signal.

The types of interference signals in metro train operation environment include single-tone interference, pulse interference, narrowband interference, and broadband interference. This paper mainly studies the mitigation of single-tone signal interference, pulse signal and chirp signal interference.

The main characteristic of the single-tone signal is that the energy is concentrated at a single frequency point, which has a significant impact on the dispreading of the positioning signal, and it can be expressed as follows.

$$J(t) = \sqrt{2}a \sin(2\pi f_j t + \varphi_j). \quad (3)$$

where a is amplitude of single-tone signal, f_j is the center frequency of the interfering signal. φ represents initial phase.

The time domain expression of the pulse signal is

$$J(t) = bp(t). \quad (4)$$

where b is signal amplitude, $p(t)$ is an impulse signal with duty cycle τ_j/T_j .

$$p(t) = \begin{cases} 1, nT_j \leq t \leq nT_j + \tau_j & n = 0, 1, 2, \dots \\ 0, & \text{else} \end{cases} \quad (5)$$

The frequency of a chirp signal varies linearly with time, which can be expressed as follows.

$$J(t) = b \sin \left[2\pi \left(f_0 + \frac{k}{2} t \right) t \right], 0 \leq t \leq T_j. \quad (6)$$

where c is the amplitude of the chirp signal, f_0 is the initial frequency, k is the sweeping frequency rate, and T_j is the sweeping time.

The jamming-to-signal-and-noise-ratio (JSNR) is used in this paper to measure the impact of interference on the positioning signal, which is calculated as the average power of $J(t)$ divided by the sum of the average power of $r_{CPIS}(t)$ and $\omega(t)$. When the receiver

receives the interference signal, the carrier noise ratio (C/N_0) will change, which will affect the positioning accuracy. The carrier to noise ratio is calculated by the following formula.

$$[C/N_0]_{eq} = \left(\frac{1}{C/N_0} + \frac{JSNR}{QR_c} \right)^{-1}. \quad (7)$$

where (C/N_0) is the undisturbed carrier noise density, Q represents the spread spectrum processing gain adjustment factor, and R_c is the positioning signal code rate.

With the increase of JSNR, the carrier noise ratio decreases, which will cause acquisition failure and increase the time for capturing and searching the BS signal. For the tracking process, the ranging accuracy is also related to the carrier noise ratio. The calculation formula of the error mean square error (σ_{tDLL}) of the code phase is as follows. It can be seen that with the increase of the interference power, the carrier noise ratio decreases and the positioning accuracy decreases.

$$\sigma_{tDLL} = \sqrt{\frac{B_L}{2 \cdot C/N_0} \frac{1}{B_f T_C} \left(1 + \frac{1}{T_{coh} \cdot C/N_0} \right)}. \quad (8)$$

where B_L is noise bandwidth of code track loop, B_f is RF front-end bandwidth, T_C and T_{coh} represent pseudo code period and time for integration, respectively.

Figure 3 shows the structure of the train positioning terminal, and the interference suppression module is placed between the RF front end and the baseband signal processing unit. The signal containing interference is down-converted to a digital intermediate frequency signal and then enters the interference suppression module. After executing the interference mitigation algorithm proposed in this paper, it is sent to the baseband signal processing unit to obtain accurate location.

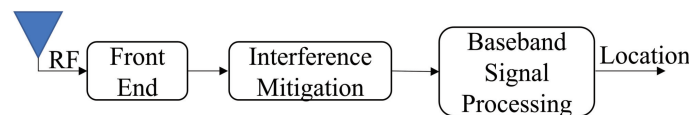


Figure 3. The structure of the train positioning terminal.

3. Interference Mitigation Algorithm Based on KL Decomposition

In this section, the details of the interference mitigation algorithm are introduced. Karhunen–Loeve expansion is a method for representing random processes with a set of uncorrelated random variables [26]. Orthogonal expansion of random process $r(t)$ in observation time T_s as follows.

$$r(t) = \sum_{n=1}^{\infty} Z_n \phi_n(t), 0 < t < N_s T_s \quad (9)$$

$$Z_n = \int_0^{N_s T_s} r(t) \phi_n(t) dt, n \geq 1. \quad (10)$$

The variance of the transform coefficient Z_n is the eigenvalue λ_n . The eigenvalue λ_n corresponds to the eigenfunction ϕ_n , representing the energy value of the signal at this eigenfunction. Suppose the random process $r(t)$ has a mean of zero. The characteristic function ϕ_n has the following relationship with the autocorrelation function of the random process $r(t)$.

$$\int_0^{N_s T_s} E\{r(t_1)r(t_2)\} \phi_i(t_1) dt_1 = \lambda_n \phi_n(t_2). \quad (11)$$

where $E\{r(t_1)r(t_2)\}$ is the Toeplitz autocorrelation matrix. Eigenfunctions and corresponding eigenvalues are obtained by decomposing $E\{r(t_1)r(t_2)\}$ matrix. Interfering signals and positioning signals can be distinguished by the distribution of eigenvalues. As mentioned in Section 2, this paper mainly considers the first interference mode, and the localization

signal characteristics and the interference signal characteristics are constant. The distribution of the signal $r_{CPIS}(t)$ is approximate Gaussian white noise, and its energy values on the eigenvectors are equal. The correlation of a deterministic signal is higher than that of noise, so its energy content will be higher, and its eigenvalues will be larger than those of the noise component. Figure 4 shows the processing of the KL expansion algorithm.

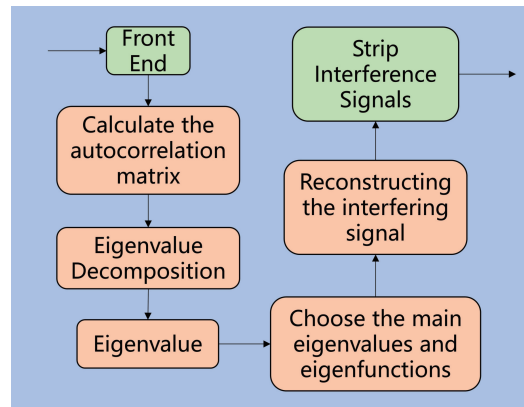


Figure 4. The processing of KL decomposition algorithm.

Figure 5 below shows the distribution of the normalized eigenvalues of the KL expansion of the received signal in the absence of interference and narrowband interference. The magnitude of the eigenvalue represents the strength of the signal energy. When there is interference, as shown by the red curve in Figure 5, the eigenvalues corresponding to the interference signal are concentrated on the main eigenvalues, which is in obvious contrast with the absence of interference signals. We can judge whether there is an interference signal according to the corresponding relationship between the interference and the magnitude of the eigenvalue [27]. The interference signal is reconstructed by inverse KL decomposition using the eigenvalue with a high energy value and the corresponding eigenfunction, and the interference mitigation is realized by subtracting the estimation of the interference signal from the received signal. The interference mitigation algorithm proposed in this paper can be described as having the following steps.

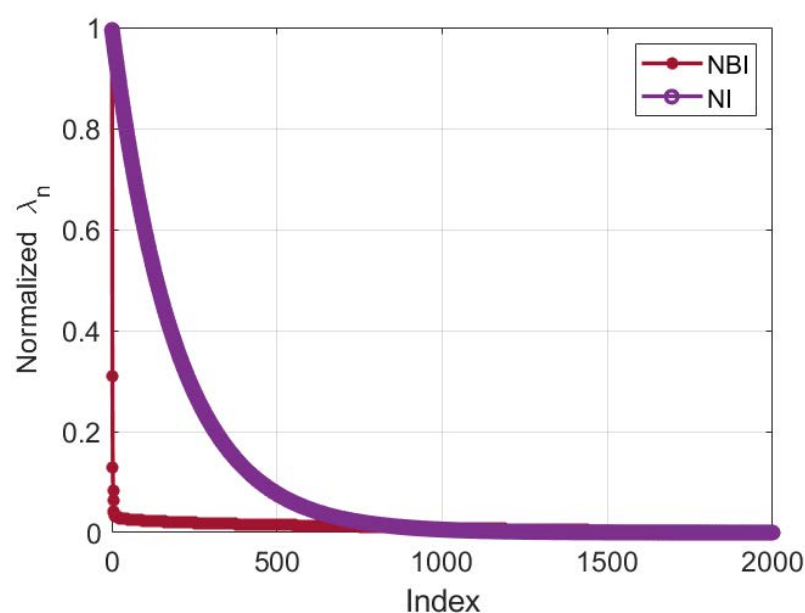


Figure 5. The positioning signal is decomposed in KL with narrow-band interference (NBI) and interference-free (NI).

(1) Calculation the Toeplitz matrix from $N = N_s T_s$ samples autocorrelation $R(n)$ of received signal $r(t)$.

$$R_{\text{Toc}} = \begin{bmatrix} R(0) & \cdots & \cdots & R(N) \\ R(1) & R(0) & \cdots & R(N) \\ \cdots & \cdots & R(0) & \cdots \\ R(N) & \cdots & \cdots & R(0) \end{bmatrix} \quad (12)$$

(2) Solve the eigenvalue λ_n and eigenfunction ϕ_n of Equation (12);

(3) Calculate the expansion coefficient Z_n according to Formula (10);

(4) Sort all eigenvalues λ_n from largest to smallest, select the first L eigenvalues as the eigenvalues corresponding to the interference signal, and reconstruct the interference signal $\hat{J}(t)$ according to Formula (9);

(5) The interference-stripped positioning signal is obtained by subtracting the interference signal from the received signal in Formula (2).

$$\hat{r}(t) = r(t) - \hat{J}(t). \quad (13)$$

In the first interference mode mentioned in the second part, the power of the interference signal is relatively constant, which means that the eigenvalue distribution of the interfered signal is constant after KL expansion, and the interference signal can be reconstructed by selecting fixed eigenvalues. Compared with the chirp signal, the eigenvalue distribution of the single-tone interference signal is more concentrated, and the calculation amount of the reconstructed interference signal is smaller.

4. Experimental and Analysis of Results

In this section, it is assumed that there is a single interference source in the metro environment, and the interference type is either single-tone interference, pulse signal, or chirp signal. The performance of the mitigation algorithm based on KL decomposition, and STFT and WPCT algorithms is compared under these three types of interference. The test environment shown in Figure 6 was set up to test and analyze the effectiveness of the interference mitigation algorithm. The interference signal injection is mainly composed of a communication positioning integrated base station and interference signal generator. The communication positioning fusion base station is mainly composed of a communication base station, a positioning base station, and a clock distributor. The communication base station is responsible for generating 5G communication signals, and the positioning base station generates the positioning signals mentioned above. The clock distributor provides reference clock signals and second pulses for communication base stations, positioning base stations, and signal sources. The interference signal generator is based on the Agilent N5181A signal generator, and the clock signal input comes from the time distributor. The interference signal is injected into the communication positioning fusion signal through the combiner, and the RF signal collector is used to transmit the data to the software receiver for analysis. The experimental parameters are configured as shown in Table 1. The 5G communication signal adopts 64-QAM modulation, the number of sub-carriers is configured to be 1200, and the signal power is 18 dB higher than that of the positioning signal. The positioning signal adopts weil code with a length of 10,230 chips, the code rate is 10.23 MHz, the center frequency of the communication positioning fusion signal is 3575 MHz, and the sampling rate of the positioning receiver is 50 MHz.

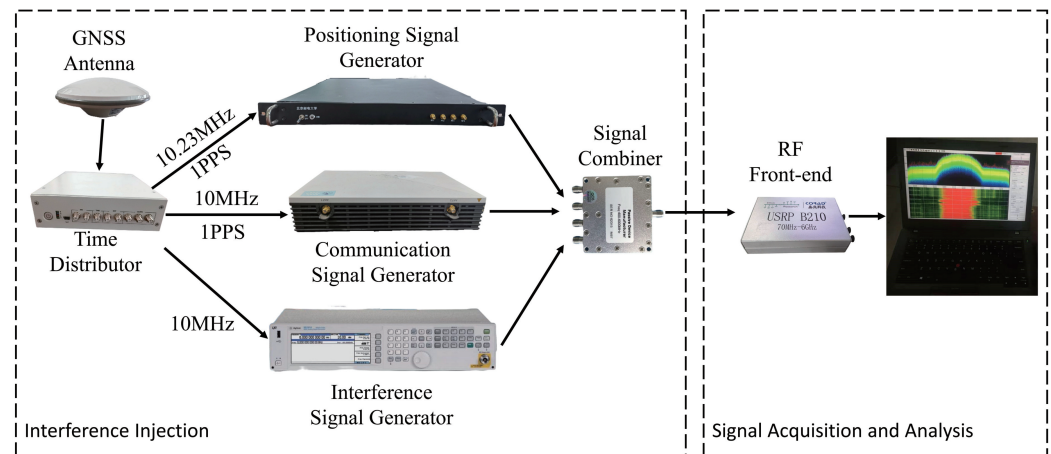


Figure 6. Test environment.

Table 1. Experimental parameter configuration.

Parameter	Value
Communication Sub-carrier	1200
The order of modulation of the communication signal	64-QAM
pseudo random noise code (PRN)	10,230
Code rate	10.23 MHz
IF frequency	3575 MHz
Sampling rate of collector	50 MHz
JSNR	30~45 dB

We define the mean squared error (MSE) as a measure of the performance of the interference mitigation algorithm for CPIS [28]. As the MSE decreases, the signal quality improves. After performing the interference mitigation algorithm, the MSE of the positioning signal is estimated at the receiver output. It is calculated as follows.

$$\text{MSE} = E\{(\hat{y}(t) - y(t))^2\} \quad (14)$$

where $E\{\cdot\}$ is average operation. $y(t)$ is CPIS without interference. $\hat{y}(t)$ is the signal after interference stripping.

4.1. Single-Tone Interference

In the first scenario, a single-tone signal with a central frequency of 3575 MHz is added to the CPIS. We conducted Monte Carlo simulations 100 times to study the average value of MSE of different algorithms under different JSNR. In each experiment, the data of different time periods of IF signals collected are selected to ensure the randomness of the experiment. After the phase of performing the interference mitigation, as shown in Figure 4, we calculated the MSE according to Equation (12). It can be seen from Figure 7 that in the single-tone interference scenario, the three algorithms can effectively suppress the single-tone interference. The KL algorithm has an average improvement of 1.8 dB compared with the STFT algorithm and 1.3 dB compared with the WPCT algorithm. The KL algorithm has better performance compared with the other two interference mitigation algorithms, and as the interference becomes stronger, the mitigation effect of the KL algorithm is more obvious. Compared with the other two algorithms, the KL algorithm is less affected by JSNR and is more robust.

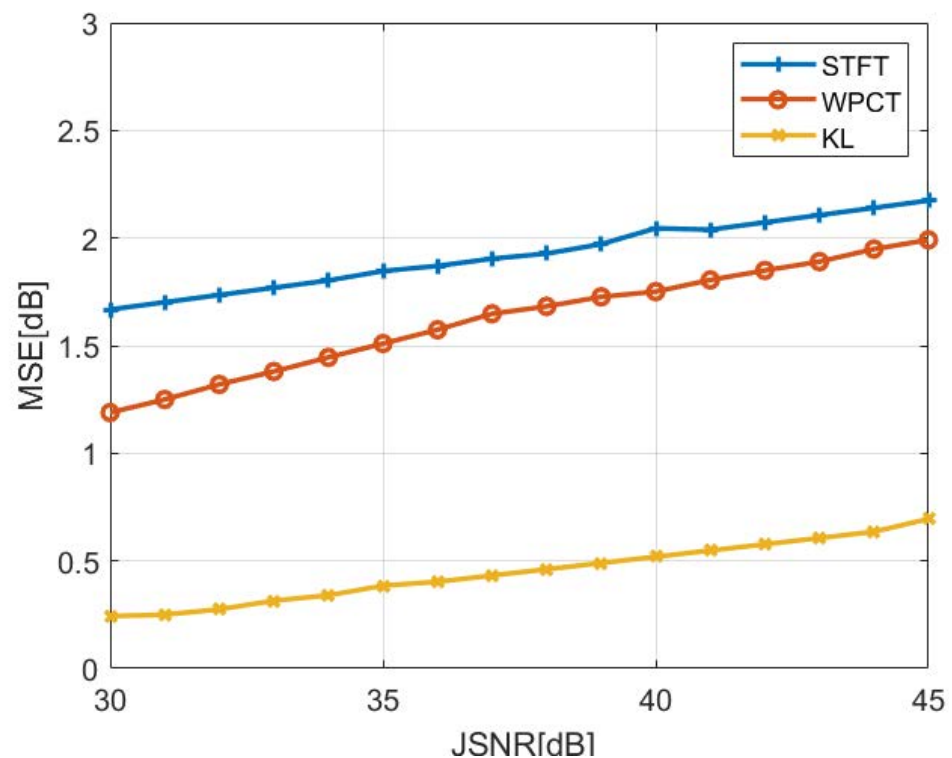


Figure 7. MSE of different interference mitigation algorithms under single-tone interference.

4.2. Pulse Signal Interference

In the second scenario, we add pulse interference. The pulse width is 100 μ s, and the duty cycle is set to 20%, 50%, and 80%, respectively. First, under the duty cycle of 20%, we decompose the signal with interference by KL to obtain the eigenvalue and corresponding eigenfunction. Figure 8 shows the distribution histogram of the eigenvalues. As can be seen from the figure, after the eigenvalues are arranged from the largest to the smallest, the first 63 λ_n can be selected as the main eigenvalues to reconstruct the interference signal. We performed Monte Carlo simulations 100 times, increasing JSNR from 30 dB to 45 dB, and calculated the average MSE of the three algorithms after interference suppression. The interference suppression performance of the KL algorithm is compared under pulse signals with different duty cycles. As shown in Figure 9, all three algorithms can suppress pulse interference well. Compared with the other two algorithms, the KL algorithm has better line performance, which is 0.58 dB higher than the WPCT algorithm on average, and 1.41 dB higher than the STFT algorithm. Moreover, the KL algorithm has better robustness under weak signals than the other two algorithms. We analyze the interference suppression performance of the KL algorithm under different duty cycles. As shown in Figure 10, with the increase of the duty cycle, the impact of pulse interference on the positioning signal increases, and the suppression effect of the KL algorithm decreases. This is due to the change of duty cycle brings about the change of the number of eigenvalue λ_n , the main characteristic value that needs to characterize the interference signal. The number of eigenvalues is too small, the interference signal cannot be completely reconstructed, and there is still interference on the positioning signal. If the number of eigenvalues is too large, the interference signal can be well suppressed, but the positioning signal is distorted, and the positioning performance is degraded. The threshold should be selected to satisfy the accuracy requirements and suppress the interference signal.

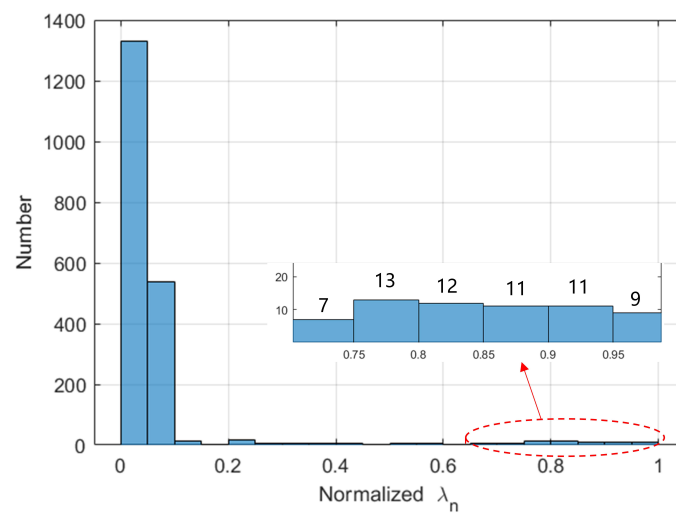


Figure 8. Histogram of eigenvalue distribution.

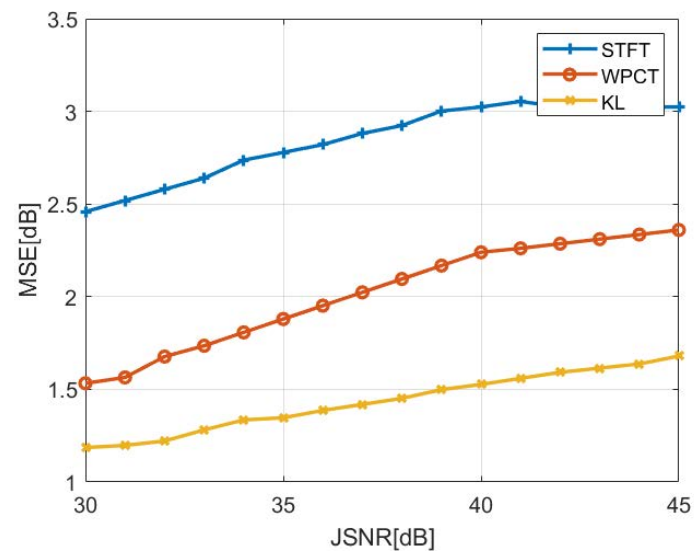


Figure 9. MSE of different interference mitigation algorithms under pulse signal interference.

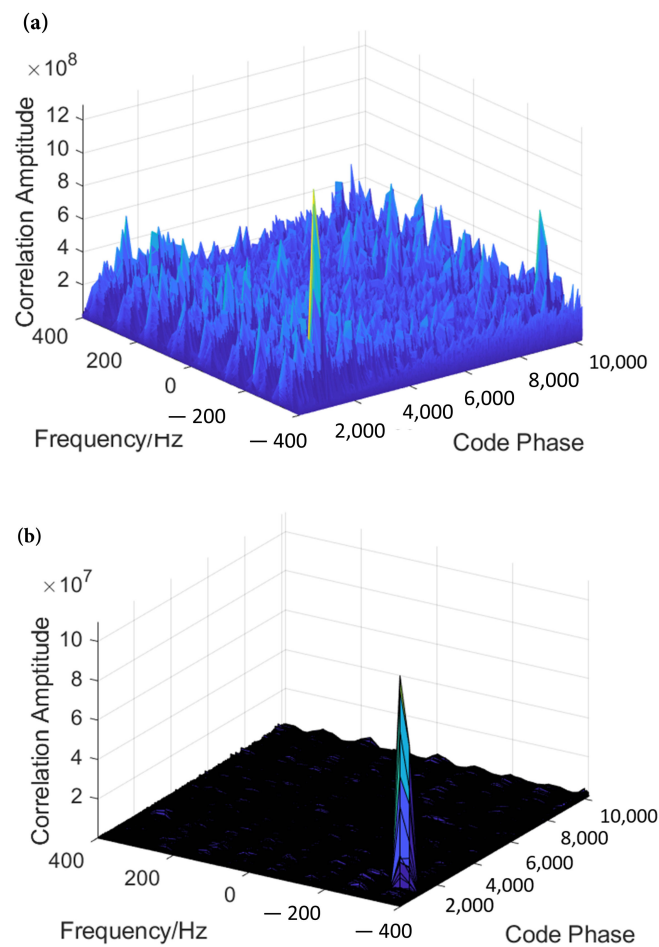


Figure 10. Mitigation performance of KL algorithm under pulse interference with different duty cycle.

4.3. Chirp Signal Interference

In the third scenario, we add pulse signal interference. The frequency sweep time of the interference signal is set to 1s, and the bandwidth is 2 MHz. First, we provide the cross ambiguity function (CAF) of the receiver acquisition process to illustrate the acquisition capability of the CPIS. The more obvious the peak value is, the better the acquisition performance. Figure 11 below shows the waveforms of CAF before and after the interference mitigation algorithm is executed. It can be seen from Figure 11a,b that after the interference suppression is executed, the correlation peak can be highlighted to improve the acquisition success rate.

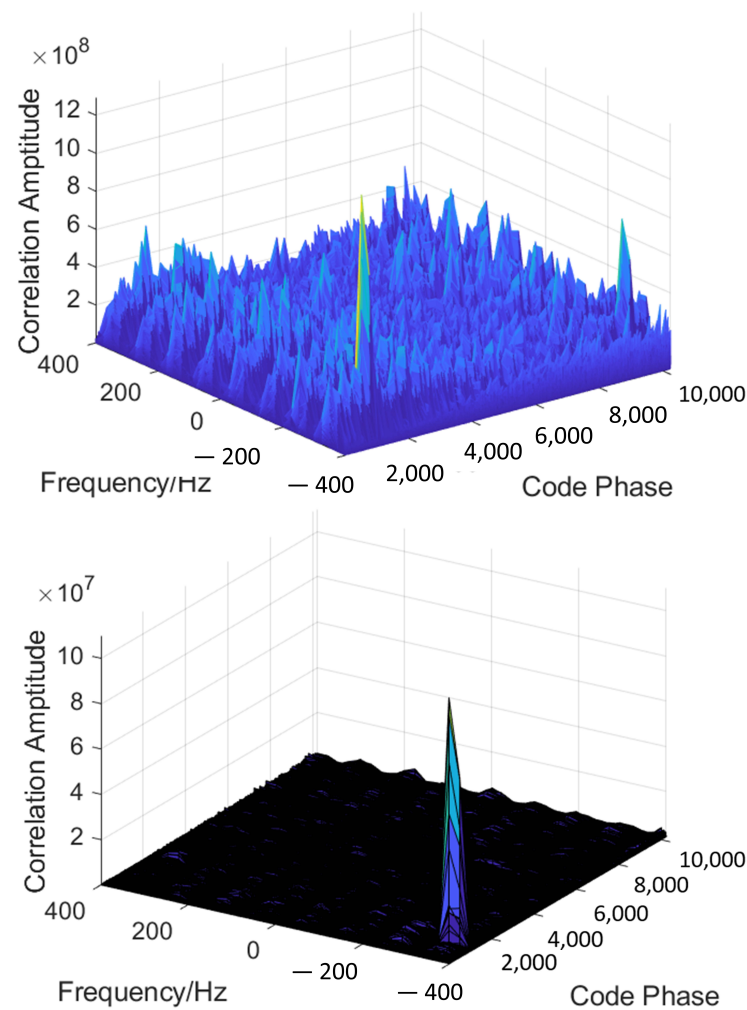


Figure 11. CAF of the waveform with PRN = 19. (a) CAF waveform before interference mitigation is performed; (b) In the chirp signal interference scenario, the waveform of CAF after interference mitigation is performed. JSNR = 30 dB.

Then we study the influence of the number of eigenvalues L of the interference signal on the algorithm performance. As shown in Figure 12, the relationship between the number of eigenvalues and MSE is given. It can be seen from the figure that when L is too large, the eigenvalues related to the positioning signal are regarded as interference signals, and the energy of the real signal is weakened after interference suppression. When the L selected is too small, the interference signal is not completely suppressed, and the suppression effect decreases. This indicates that the choice of L value needs to be made based on performance and computational complexity.

We conducted Monte Carlo simulations 100 times, and the JSNR increase from 30 dB to 45 dB, and calculated the average MSE of the three algorithms before and after chirp signal interference suppression. It can be seen from Figure 13 that the KL algorithm still maintains excellent interference suppression performance, and with the increase of JSNR, the interference suppression performance decreases slightly. The KL algorithm has an average improvement of 2.4 dB compared with the STFT algorithm and 0.8 dB compared with the WPCT algorithm. Compared with the other two interference suppression algorithms, the KL algorithm in this paper has better suppression performance and robustness under chirp signal interference.

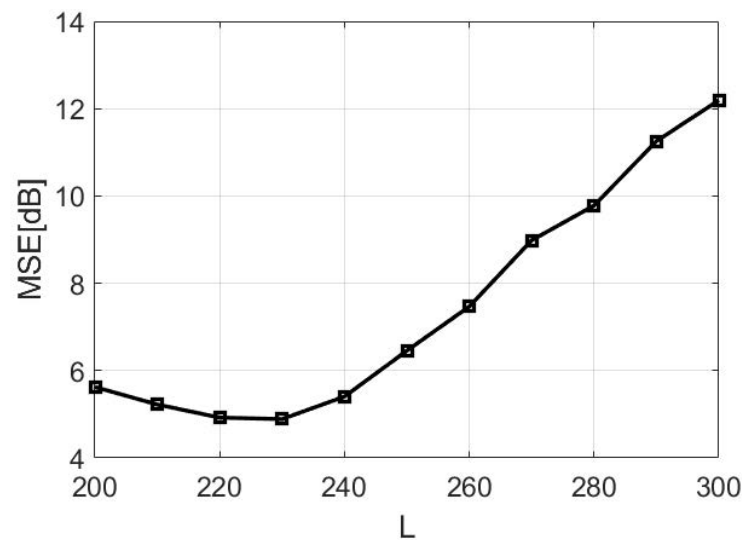


Figure 12. Performance of different L in KL algorithm.

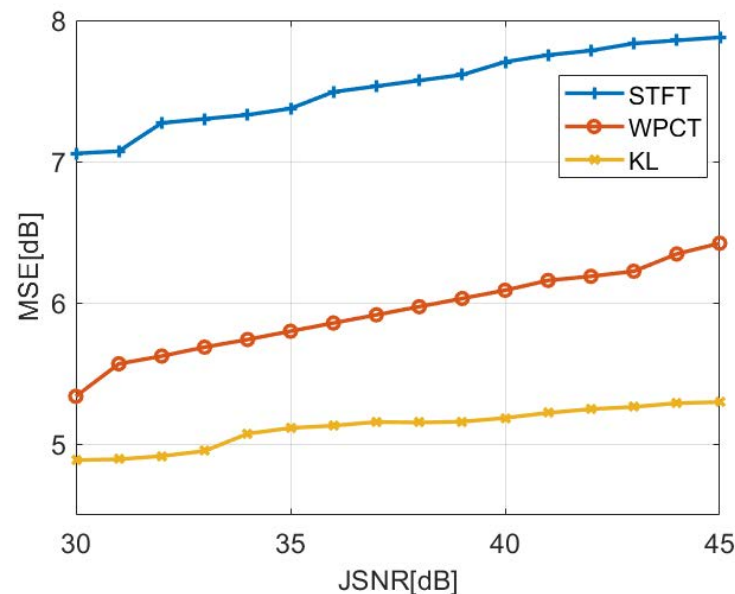


Figure 13. MSE of different interference mitigation algorithms under chirp signal interference.

4.4. Positioning Accuracy

We will extend the experiment to the post terminal correlation stage. The JSNR is set to 45 dB, and the three jamming signal parameters refer to the settings in the previous summary. We collect 20 s of data to facilitate the positioning results output by the receiver, and finally count the positioning accuracy in the horizontal direction. Figure 14 shows the comparison of the positioning accuracy of the receiver after implementing different interference suppression algorithms in three interference scenarios and the positioning accuracy when there is no interference signal is obtained. In different interference scenarios, the median value of the KL algorithm is smaller than other algorithms. The average median value of positioning error of the KL algorithm is 0.35 m, the average median value of error of WPCT algorithm is 0.61 m, and the average median value of error of the STFT algorithm is 0.76 m. The interference suppression effect is the best. In addition, the corresponding frame of the KL algorithm is smaller than other algorithms, which also shows that the KL algorithm has better performance than the other two algorithms. It can be seen from the figure that the positioning accuracy of the KL algorithm is close to that without interference.

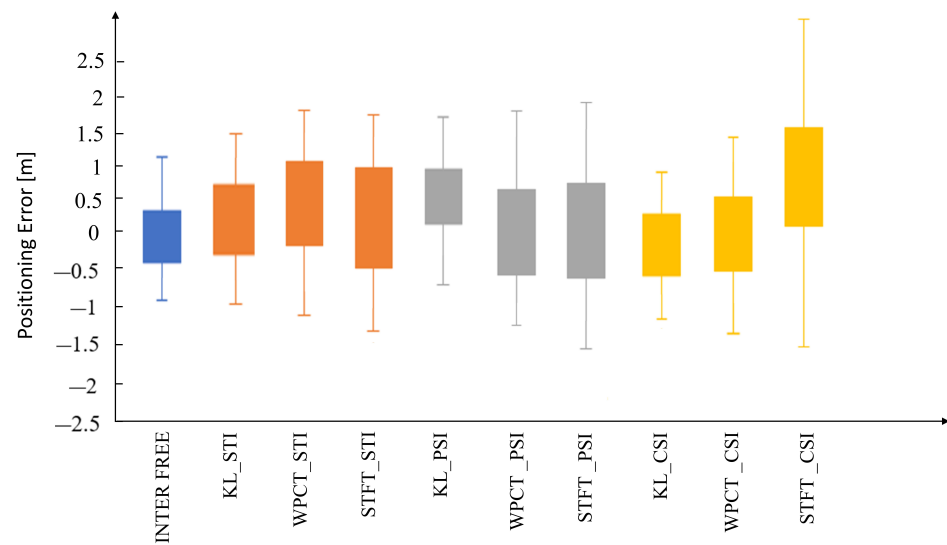


Figure 14. Comparison of positioning accuracy of different algorithms in three jamming scenarios.

4.5. Algorithm Computational Complexity

The ability of the three algorithms mentioned above to suppress single-tone signals and chirp signals was analyzed previously. This section gives a comparison of STFT, WPCT, and KL algorithms in terms of single-tone suppression performance and complexity. As shown in Table 2, compared with the WPCT and STFT algorithms, the KL algorithm can provide better interference suppression performance, but the implementation complexity is the highest, and the computation of the KL algorithm is N^2 . In contrast, WPCT and STFT require less computation. In practice, the interference algorithm is chosen to compromise between the calculation amount and the suppression effect. To obtain better results, we can consider optimizing the calculation amount of the KL algorithm in the future.

Table 2. Mitigation performance and implementation complexity comparison. JSNR = 30 dB.

Algorithm	MSE (dB)	Complexity
STFT	1.67	$O(N)$
WPCT	1.19	$O(kNL_w)$
KL	0.24	$O(N^2)$

5. Conclusions

This paper proposes a method for suppressing the interference of metro train positioning based on Karhunen–Loeve decomposition. The method uses KL decomposition to project the signal into the orthogonal subspace, uses the size of the eigenvalue to distinguish the interference signal and the positioning signal, uses the inverse KL decomposition to reconstruct the interference signal, and subtracts the interference signal from the received signal to achieve interference suppression. We built an interference test platform, and compared this with the traditional method to analyze the interference suppression capability and computational complexity. From the experimental results, the suppression performance of the KL-based suppression algorithm is better than that of the STFT and WPCT algorithms under single-tone interference and chirp interference. Furthermore, the KL algorithm is less affected by the interference jamming-signal-and-noise-ratio. We also compared the complexity of the three algorithms, and the complexity of the KL algorithm is significantly higher than that of the STFT and WPCT algorithms. Deploying interference suppression algorithms in practical receivers requires a trade-off between suppression performance and computational complexity.

In the future work, how to quickly and accurately identify the jamming signal will be studied when the type of jamming signal is unknown. Furthermore, we will consider further reducing the computational complexity of KL decomposition and improve the feasibility of algorithm hardware deployment. Generally speaking, interference signals are highly correlated (or even deterministic), while the positioning signal samples at the receiver can approximately be considered to be Gaussian-independent and identically distributed (i.i.d) random variables. Random matrix theory provides efficient tools to separate correlated terms from the sum of correlated and i.i.d. signals. In the future, we will consider analyzing the eigenvalues of the sample covariance matrix to obtain very useful information about interference and interference subspaces.

Author Contributions: Conceptualization, X.D.; Data curation, X.D.; Formal analysis, X.D., J.L. and Z.Z.; Investigation, X.D., J.L. and Z.Z.; Project administration, Z.D. (Zhonglinag Deng), X.D., J.L., Z.D. (Zhenke Ding) and Z.Z.; Resources, Z.D. (Zhenke Ding) and X.D.; Software, X.D., J.L., Z.D. (Zhenke Ding) and Z.Z.; Supervision, J.L., Z.D. (Zhenke Ding) and Z.Z.; Validation, X.D. and Z.Z.; Visualization, X.D.; Writing—original draft, X.D.; Writing—review and editing, X.D., J.L., Z.D. (Zhenke Ding) and Z.Z. All authors have read and agreed to the published version of the manuscript.

Funding: This work was financially supported by the Beijing Municipal Natural Science Foundation under Grant No. L191003.

Acknowledgments: The authors would like to express their thanks to Bingxun Liu for help in the simulation.

Conflicts of Interest: The authors declare no conflict of interest.

References

1. Cheng, R.; Song, Y.; Chen, D.; Ma, X. Intelligent positioning approach for high speed trains based on ant colony optimization and machine learning algorithms. *IEEE Trans. Intell. Transp. Syst.* **2018**, *20*, 1–10. [\[CrossRef\]](#)
2. Hamid, H.A.; Nicholson, G.L.; Douglas, H.; Zhao, N.; Roberts, C. Investigation into train positioning systems for saving energy with optimised train trajectories. In Proceedings of the 2016 IEEE International Conference on Intelligent Rail Transportation (ICIRT), Birmingham, UK, 23–25 August 2016; pp. 460–468.
3. Ko, K.; Ahn, W.; Shin, W. High-speed train positioning using deep kalman filter with 5G NR signals. *IEEE Trans. Intell. Transp. Syst.* **2022**, *23*, 1–12. [\[CrossRef\]](#)
4. Wybo, J. Track circuit reliability assessment for preventing railway accidents. *Saf. Sci.* **2018**, *110*, 268–275. [\[CrossRef\]](#)
5. Zheng, W.; Ma, S.; Hua, Z.; Jia, H.; Zhao, Z. Train integrated positioning method based on gps/ins/rfid. In Proceedings of the 2016 35th Chinese Control Conference (CCC), Chengdu, China, 27–29 July 2016; pp. 5858–5862.
6. Dordal, O.B.; Borges, A.P.; Ribeiro, R.; Enembreck, F.; Scalabrin, E.E.; Ávila, B.C. Towards an optimal driving trains in single line using crossing loops. In Proceedings of the 2011 15th International Conference on Computer Supported Cooperative Work in Design, CSCWD 2011, Lausanne, Switzerland, 8–10 June 2011.
7. Weidong, L.I.; Lihong, H.; Yousheng, W. Study of train positioning method based on BDS/GSM-R combination. *J. Railw. Sci. Eng.* **2016**, *13*, 552–556.
8. Chetty, K.; Chen, Q.; Woodbridge, K. Train monitoring using gsm-r based passive radar. In Proceedings of the 2016 IEEE Radar Conference (RadarConf), Philadelphia, PA, USA, 2–6 May 2016.
9. Wei, S.; Yi, W.; Bai-gen, C.; Jian, W.; Jiang, L.; Wei, J. Research of the application technique of BDS in the western low density railway-lines. In Proceedings of the 2016 International Conference on Electromagnetics in Advanced Applications (ICEAA), Cairns, Australia, 19–23 September 2016.
10. Deng, P.; Dai, C.; Long, Z. Optimum design and implementation of signal processing method for position and speed measurement system of highspeed maglev train. In Proceedings of the 2019 Chinese Automation Congress (CAC), Hangzhou, China, 22–24 November 2019.
11. Nazaruddin, Y.Y.; Tamba, T.A.; Faruqi, I.; Waluya, M.B.; Widyotriatmo, A. On using unscented kalman filter based multi-sensors fusion for train localization. In Proceedings of the 2019 12th Asian Control Conference (ASCC), Kitakyushu, Japan, 9–12 June 2019; pp. 1137–1142.
12. Talvitie, J.; Levanen, T.; Koivisto, M.; Pajukoski, K.; Renfors, M.; Valkama, M. Positioning of high-speed trains using 5G new radio synchronization signals. In Proceedings of the 2018 IEEE Wireless Communications and Networking Conference (WCNC), Barcelona, Spain, 15–18 April 2018; pp. 1–6.
13. Deng, Z.; Jia, B.; Tang, S.; Fu, X. A fine fast acquisition scheme for a communication and navigation fusion system. *Appl. Sci.* **2020**, *10*, 3434. [\[CrossRef\]](#)

14. Yin, L.; Li, G.; Jiang, Y. A Novel Positioning Method based on Leaky Coaxial Cable using TC-OFDM Signal. In Proceedings of the 2021 7th International Conference on Computer and Communications (ICCC), Chengdu, China, 10–13 December 2021; pp. 485–489.
15. Tan, Z.; Wang, C.; Goda, K.; Malik, O.; Jalali, B. Jammed array wideband sawtooth filter. *J. Optoelectron. Laser* **2013**, *19*, 24563. [[CrossRef](#)] [[PubMed](#)]
16. Hu, Y.; Bian, S.; Li, B.; Zhou, L. A novel array-based spoofing and jamming suppression method for GNSS receiver. *IEEE Sens. J.* **2018**, *18*, 2952–2958. [[CrossRef](#)]
17. Seo, J.; Chen, Y.H.; De Lorenzo, D.S.; Lo S.; Enge, P.; Akos, D.; Lee, J. A real-time capable software-defined receiver using GPU for adaptive anti-jam GPS sensors. *Sensors* **2011**, *11*, 8966–8991. [[CrossRef](#)] [[PubMed](#)]
18. Lu, Z.; Chen, H.; Chen, F.; Nie, J.; Ou, G. Blind adaptive channel mismatch equalisation method for GNSS antenna arrays. *IET Radar Sonar Navig.* **2018**, *12*, 383–389. [[CrossRef](#)]
19. Zhou, C.; Wang, Y.; Qiao, C.; Dai, W. Anti-jamming method using complex ANF for GNSS receivers. *J. Natl. Univ. Def. Technol.* **2016**, *38*, 189–194.
20. Qin, W.; Gamba, M.T.; Falletti, E.; Dosis, F. Effects of optimized mitigation techniques for swept-frequency jammers on tracking loops. In Proceedings of the 32nd International Technical Meeting of the Satellite Division of The Institute of Navigation (ION GNSS+ 2019), Miami, FL, USA, 16–20 September 2019.
21. Jiang, M.; Yuan, D.; Xu, G. Suppression of narrowband interference in DSSS systems based on MWPT and CWPT. In Proceedings of the 2005 IEEE International Symposium on Microwave, Antenna, Propagation and EMC Technologies for Wireless Communications, Beijing, China, 8–12 August 2005.
22. Chien, Y.R. Wavelet packet transform-based anti-jamming scheme with new threshold selection algorithm for GPS receivers. *J. Chin. Inst. Eng.* **2018**, *41*, 181–185. [[CrossRef](#)]
23. Rezaei, M.J.; Mosavi, M.R. Hybrid anti-jamming approach for kinematic glob positioning system receivers. *IET Signal Process.* **2018**, *12*, 888–895. [[CrossRef](#)]
24. Rezaei, M.J.; Abedi, M.; Mosavi, M.R. New GPS anti-jamming system based on multiple short-time fourier transform. *IET Radar Sonar Navig.* **2016**, *10*, 807–815. [[CrossRef](#)]
25. Dosis, F.; Musumeci, L. Use of the karhunen-loeve transform for interference detection and mitigation in GNSS. *ICT Express* **2016**, *2*, 33–36. [[CrossRef](#)]
26. Szumski, A.; Eissfeller, B. The karhunen-loeve transform as a future instrument to interference mitigation. In Proceedings of the 26th International Technical Meeting of the Satellite Division of The Institute of Navigation (ION GNSS+ 2013), Nashville, TN, USA, 16–20 September 2013; pp. 3443–3449.
27. Deng, Z.; Mo, J.; Jia, B.; Bian, X. An acquisition scheme based on a matched filter for novel communication and navigation fusion signals. *Sensors* **2017**, *17*, 1766. [[CrossRef](#)] [[PubMed](#)]
28. Sharifi-Tehrani, O.; Sabahi, M.F.; Danaee, M.R. Efficient GNSS jamming mitigation using the marcenkopastur law and karhunen-loeve decomposition. *IEEE Trans. Aerosp. Electron. Syst.* **2022**, *58*, 2291–2303. [[CrossRef](#)]

Disclaimer/Publisher’s Note: The statements, opinions and data contained in all publications are solely those of the individual author(s) and contributor(s) and not of MDPI and/or the editor(s). MDPI and/or the editor(s) disclaim responsibility for any injury to people or property resulting from any ideas, methods, instructions or products referred to in the content.