Finite Element Model Updating Using Resonance–Antiresonant Frequencies with Radial Basis Function Neural Network

Haifeng Zhao *, Jianzhuo Lv, Zunce Wang, Tianchi Gao and Wenhao Xiong

School of Mechanical Science and Engineering, Northeast Petroleum University, Daqing 163318, China; tianchigao0906@163.com (T.G.)
* Correspondence: zhaohaifengxy@163.com; Tel.: +86-04596503173

Abstract: The modal frequencies, model shapes or their derivatives are generally used as the characteristic quantities of the objective function for the finite element model (FEM) updating. However, the measurement accuracy of the model shapes is low due to the few numbers of measurement points for actual structures, which results in a large correction error. The antiresonant frequency reflects the local information of the structure more accurately than the mode shapes, which is a good complement to the resonance frequencies. In this paper, a FEM updating using resonance and antiresonant frequencies with radial basis function (RBF) neural network is proposed. The elastic modulus, added mass, tensile stiffness and torsional stiffness are selected as the updating parameters of FEM for a cantilever beam, which were grouped by the uniform design method. The resonance and antiresonant frequencies identified from the frequency response function (FRF) obtained from corresponding FEM at only one node are taken as the characteristic quantities. The RBF neural network is adopted to construct the mapping relationships between the characteristic quantities and the updating parameters. The updated parameters are substituted into the FEM, and the FRF is obtained to verify the validity of the method. The results show that the relative errors between all the updated parameters and the target values are less than 7%, and the relative errors of the characteristic quantities in the updating frequency band are less than 3%. The proposed method can accurately reproduce the dynamic characteristics of the cantilever beam. It can be applied to the damage detection and safety evaluation of large structures which are difficult to arrange more measuring points.

Keywords: antiresonant frequency; resonance frequency; model updating; uniform design method; RBF neural network

1. Introduction

The finite element model (FEM) is an effective modern tool for structure monitoring, damage detection, prediction of service life, and determination of an optimal maintenance strategy [1]. Shahrokh Hosseini-Hashemi et al. [2] used FEM to simulate the vibration behavior of nanoplate submerged in different viscous fluids with various aspect ratios and analyzed the effects of fluid viscosity and density on the free vibration natural frequencies of the nanoplate. Reza Ahmadi Arpanahi et al. [3] used FEM to study the effect of viscosity and fluid flow on the buckling behavior of nanoplate with surface energy. L. Francesconi et al. [4] used FEM to demonstrate the validity of static and modal analysis of low porosity thin metallic structures with speckle interferometry and digital image correlation. However, in the safe detection of complex civil engineering structures such as offshore platforms and offshore wind turbines, there is often a deviation between the finite element simulation results and the measured values. Simoen E et al. [5] pointed out that the modeling uncertainties can be generally divided into the uncertainties of the model parameters and the model structure. The former is usually due to incorrect assumptions of model parameters such as material properties, section properties and structure dimensions [6].
The latter is usually due to simplification of the structure, inaccurate mass distribution, incorrect boundary conditions, etc. [7,8]. Suzana Ereiz et al. [1] summarized some researchers and gave the definition of model updating, that is, updating the numerical model based on the experimentally obtained test results to obtain the actual structural behavior numerically. Therefore, the measured data can modify the design parameters or matrix elements in the finite element model to obtain the minimum deviation between them, accurately reflect the characteristics of the prototype structure, so that the measured results can be correctly reproduced, and the possible structural damage sites and damage degrees can be identified. On this basis, the safety and reliability evaluation of civil engineering structures can be realized. It is one of the most critical problems in health monitoring, damage diagnosis and safety evaluation of engineering structures.

V. Meruane [9] pointed out that establishing accurate finite element numerical models, determining appropriate updating parameters, constructing objective functions and adopting robust optimization algorithms are the key factors for model updating. At present, the research on the finite element model updating method is mainly focused on these key factors. According to the classification of the updated object, compared with the matrix method, the design parameter method is more suitable for engineering applications because of its advantages such as clear physical meanings [10]. However, for structures that can accurately measure the geometric dimensions and have clear physical parameters, the ambiguous structural boundary conditions and constraints need to be updated [11,12]. The commonly used characteristic quantities for constructing the objective functions are modal frequency [13], mode shape [14], stress [15], strain mode [16], etc. Most finite element updating methods generally use modal frequencies and mode shapes as characteristic quantities [17]. There are two measurement methods to obtain structural modal parameters: the non-contact method and the contact method. The non-contact method is favored by researchers because it does not contact the structure and does not affect the accuracy of the dynamic behaviors of the structure. Guinchard M. et al. [18] used the laser Doppler vibrometry scanning system (LDV) to successfully perform the automated vibration measurement over the selected surface of the lightweight structure, and obtain the relevant information concerning its natural frequencies and modal shapes of the structure up to 4 kHz. Łukasz Scislo [19] used the 3D LDV to achieve single-point and surface quality assessment of continuous production, but at the same time, he pointed out that the 3D LDV can be especially useful for measurements of small, lightweight, or very fragile elements. In addition, non-contact test systems such as 3D LDV are more expensive and have higher requirements on the environment during use, so they are mainly used in indoor testing. At present, the contact test such as an accelerometer is still used in the vibration test of indoor and outdoor testing for large structures [20,21].

The modal frequency has the advantages of easy measurement, accuracy and reflecting the global characteristics of the structure. However, because it is difficult to excite the high-order modes in the actual testing of large structures such as offshore platforms, only the low-order modes can be obtained. When the updating parameters are too many, the ill-posed problem of the modified equation will be caused [22]. Therefore, it is necessary to increase other characteristic quantities to ensure the satisfiability of the modified equation. Although the direct or indirect use of the mode shape is a good supplement to the modal frequency, for the actual large-scale complex structures such as offshore platforms, it is impossible to arrange too many measuring points due to the limitations of test conditions such as underwater structure, and there is the computational mode reduction or the experimental mode expansion in the updating process, which causes correction errors. Although the antiresonant frequencies [23] were originally applied to mechanical structure vibration reduction, it was gradually applied to the finite element model updating because it has the advantages of easy testing and accurate results as the resonance frequency, and it varies with the location of measuring points to reflect the local information of the structure [24–26]. However, the current methods are mainly based on theoretical research and a small number of experimental verification studies with multiple measurement points,
which do not meet the application needs of actual large complex structures. In the study of optimization algorithms, Arora [27] concludes that the iterative method can obtain more accurate updating results than the direct method. Sensitivity analysis is a common iterative method. However, it is difficult to effectively express the mapping relationship between the modified parameters and the characteristic quantities in the updating process by functional expression, and the high-dimensional and nonlinear optimization problems need to be solved in the iterative process, which is prone to problems such as the low computational efficiency and falling into the local optimal solutions [28]. Artificial neural networks (ANN) have recently been introduced as an alternative to optimization algorithms [29,30]. In the application of model updating, Meruane V. and Mahu J. [31] studied the structural damage assessment based on ANN and antiresonance frequencies with an 8-DOF mass-spring system. He pointed out that antiresonant frequencies can be identified more easily and more accurately than mode shapes. However, he only used the seven antiresonant frequencies identified by the point frequency response function of the first mass as the characteristic quantities of the objective function. Although good results are achieved, it is difficult to apply to large structures that can only be excited by the low-order modes. Detailed information about the method of using ANN to update the structural finite element model is shown in Ref. [1]. Among them, the radial basis function (RBF) neural network is applied to finite element model updating due to its advantages such as the short training time, high prediction accuracy, as well as the ability to withstand the presence of noise in experimental data and approximate any continuous functions with arbitrary precision [32,33].

In this paper, the cantilever beam with elastic support is taken as the research object, combined with the resonance frequencies which reflect the global characteristics of the structure and anti-resonant frequencies which reflect the local characteristics of the structure, and the characteristic quantities of the objective function are constructed. The mapping relationship between the characteristic quantities of the objective function and the four modified parameters of the cantilever beam, i.e., constrained stiffness (tensile stiffness and torsional stiffness), elastic modulus and concentrated mass, is established by the RBF neural network optimization algorithm. The updating parameters are obtained by inputting the target values and then verified. The results show that the proposed method can effectively identify the two constraint stiffness, additional mass and elastic modulus of materials with only one measuring point. The method can be applied to the damage detection and safety evaluation of large structures such as offshore platforms and offshore wind turbines.

2. Materials and Methods

2.1. Simulation Model

A cantilever beam with elastic support shown in Figure 1 is taken as a simulation example. The cantilever beam considering elastic support is a simplified model of large tall structures such as offshore platforms and offshore wind turbines. Therefore, we only consider the bending mode of the cantilever beam, while the torsion and bending modes are ignored here. The material is aluminum. The beam is assumed to have dimensions of $1 \times 0.1 \times 0.02$ m, and it is under a semi-rigid connected boundary conditions, i.e., the elastic support is neither simple nor fixed, but in between; $k_t$ and $k_r$ are the tensile stiffness and torsional stiffness of the elastic support, respectively. The concentrated mass $m$ is 0.4 m away from the elastic support. The beam’s mechanical properties are considered to be as follows: the elastic modulus is $E$, Poisson’s ratio is 0.33, and density is 2700 kg/m$^3$. Here, $E$, $m$, $k_t$ and $k_r$ are selected as updating parameters. The specific details are shown in Table 1. The target value is assumed to be the measured value (i.e., truth value), which is used to update the initial parameters.
The analytical antiresonant frequencies are obtained from the eigenvalues of Equation (4). In Equation (4), if $i = j$, the antiresonant frequencies are obtained when the exciting point and the response point are the same (i.e., point FRF); if $i \neq j$, the antiresonant frequencies are calculated as an example for the convenience of analysis. According to the literature [9,24,34], the FRF matrix $H(\omega)$ is the inverse of the dynamic stiffness matrix $(K - \omega^2 M)$, i.e.,

$$
H(\omega) = (K - \omega^2 M)^{-1} = \frac{\text{adj}(K - \omega^2 M)}{\det(K - \omega^2 M)}
$$

(2)

where $\text{adj}(\cdot)$ and $\det(\cdot)$ represent the adjoint matrix and the determinant, respectively. When exciting force is applied to the system at point $j$, the response is obtained at point $i$, the frequency response function is

$$
H_{ij}(\omega) = \frac{\text{adj}(K_{ij} - \omega^2 M_{ij})}{\det(K - \omega^2 M)} = \frac{(-1)^{i+j}\det(K_{ij} - \omega^2 M_{ij})}{\det(K - \omega^2 M)}
$$

(3)

According to Equation (3), the zeros of $H_{ij}(\omega)$ are the frequencies $\omega$ when the numerator is zero. The numerator represents the matrix obtained from the dynamic stiffness matrix of the system after deleting row $i$ and column $j$, and its determinant is zero, i.e.,

$$
\det(K_{ij} - \omega^2 M_{ij}) = 0
$$

(4)

The analytical antiresonant frequencies are obtained from the eigenvalues of Equation (4). In Equation (4), if $i = j$, the antiresonant frequencies are obtained when the exciting point and the response point are the same (i.e., point FRF); if $i \neq j$, the antiresonant frequencies

---

### Table 1. Updating parameters.

<table>
<thead>
<tr>
<th>Updating Parameter</th>
<th>Initial Value</th>
<th>Target Value</th>
<th>Error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E$ (GPa)</td>
<td>70</td>
<td>77</td>
<td>−9.09</td>
</tr>
<tr>
<td>$m$ (kg)</td>
<td>5</td>
<td>6.5</td>
<td>−23.08</td>
</tr>
<tr>
<td>$k_t$ ($10^7$ N/m)</td>
<td>2</td>
<td>1</td>
<td>100</td>
</tr>
<tr>
<td>$k_r$ ($10^4$ N·m/rad)</td>
<td>5</td>
<td>2</td>
<td>150</td>
</tr>
</tbody>
</table>

As can be seen from Table 1, the initial errors between the initial value and target value of the updating parameters are very large, especially the initial errors of $k_t$ and $k_r$ which are both more than 100%. The error is calculated by Equation (1).

$$
\text{Error} = \frac{\text{Initial} - \text{Target}}{\text{Target}} \times 100\%
$$

(1)

2.2. Identification of Antiresonant Frequency

Antiresonant frequency refers to the frequency that causes a zero response at a certain point of the structure when the input is not zero, which can be identified by the theoretical calculation and the experimental frequency response function curve.

2.2.1. Theoretical Antiresonant Frequency

Since the antiresonant frequencies of the lightly damped systems are almost unaffected by damping, the antiresonant frequencies of the undamped multi-degree-of-freedom system represented by the mass matrix $M$ and the stiffness matrix $K$ are calculated as an example for the convenience of analysis. According to the literature [9,24,34], the FRF matrix $H(\omega)$ is the inverse of the dynamic stiffness matrix $(K - \omega^2 M)$, i.e.,

Figure 1. A cantilever beam with elastic support (adapted from Figure 2 in Ref. [12]).
are obtained when the exciting point and the response point are different (i.e., transfer FRF). When the denominator is zero, the resonance frequencies are obtained from Equation (3).

It can be known from Equations (3) and (4): (a) the antiresonant frequencies at different exciting points are different, which reflect the local frequency information of the structure. In the FEM updating, the antiresonant frequencies at different measuring points can be increased correspondingly with the increase in the updating parameters. (b) According to the definition of an adjoint matrix, all information related to the exciting point is deleted, so the antiresonant frequencies at the exciting point do not change with the variation of the mass or the stiffness of the point. At the same time, the literature [1,12,19] points out that the antiresonant frequencies and the resonance frequencies in the point FRF curves appear alternately, which can provide more frequency information, while the antiresonant frequencies in the transfer FRF have no such law.

\[
det(K - \omega^2 M) = 0 \quad (5)
\]

According to Equation (5), both \(K\) and \(M\) are the stiffness and mass matrix of the system, which reflect the global information of the system. If the mass or stiffness at the exciting point is changed, the mass or the stiffness matrix of the system will also be changed, which results in the change of the resonance frequencies.

It can be seen from the above that the resonance frequencies and antiresonant frequencies of the system are used as the characteristic quantities of the objective function for the FEM updating, which not only fully reflect the global and local information of the system, but also ensure the satisfiability of modified equation by increasing or decreasing the position of measurement point or the order of the antiresonant frequencies according to the number of updating parameters.

2.2.2. Experimental Antiresonant Frequency

The method is identified by experimental point frequency response function, that is, the ratio of the cross-power spectrum of the input and output signals of a certain measuring point to the auto-power spectrum of the input signal.

\[
T_{xy}(\omega) = \frac{P_{yx}(\omega)}{P_{xx}(\omega)} \quad (6)
\]

where subscript \(x\) is the pulsed excitation of the input signal, and subscript \(y\) is the vibration response signal. It can be calculated by using the “tfestimate” function in MATLAB. Because the antiresonant frequency of the multi-degree of freedom system is equivalent to the zero point of each FRF curve, it can be identified by the dip frequency of the phase variation +180° on the point FRF curve, and the resonance is corresponding to the peak frequency of the phase variation −180°.

2.3. Model Updating Principle Using RBF Neural Network

RBF neural network consists of two layers: a hidden radial basis layer of \(S^1\) neurons (radial basis function is activation function), and an output linear layer of \(S^2\) neurons (linear function is activation function). It belongs to the feedforward neural network, which can approximate any nonlinear continuous functions with arbitrary precision, deal with the law that is difficult to analyze in the system and have good generalization ability and fast learning speed. Therefore, the RBF neural network is selected as the finite element model updating algorithm. Its network structure is shown in Figure 2 [35].
The flowchart of the proposed method is shown in Figure 3, and the procedure of the proposed method for model updating is described as follows:

Step 1: Construct the initial finite element model of the structure.

Step 2: Antiresonant frequencies are determined and the deviation of the linear function is calculated.

Step 3: Select an appropriate uniform design table according to the number of updating parameters.

Step 4: Input the characteristic quantities of the structure and the deviation table into the RBF neural network to determine the final weight value

After the input vector $P$ is trained by the radial basis function and the output is:

$$y = \sum_{i=1}^{S^1} LW^{2,1} \varphi (IW^{1,1} - P) b_1^1 + b^2$$

where $IM^{1,1}$ denotes the weighting vector between the $i$th neuron of the radial basis layer and input vector $P$; $\varphi$ denotes the selected nonlinear radial basis function. It can be seen that the purpose of the radial basis neural network training is to use training samples to determine the final weight value $IW^{1,1}$, $LW^{2,1}$ and deviations $b_1^1$ and $b^2$ of the radial basis layer and the linear layer, and establish the mapping relationship between the output vector and input vector.

The mapping relationship between the updating parameters $P$ in the finite element model and the structural characteristic quantities $y$ can be expressed as [32]:

$$P = f^{-1}(y)$$

The RBF neural network is used to establish a mapping relationship $f^{-1}$ between the updating parameters $P$ as the output vector and the structural characteristic quantities $y$ as the input vector. When $y$ (resonance frequencies and antiresonant frequencies) are known, $P$ can be obtained through the RBF neural network, and the model updating comes down to solving the positive problem of the updating parameter $P$.

2.4. Model Updating Procedure

The actual structures are complex, and it is difficult to calculate the resonance and antiresonant frequencies theoretically. Therefore, the finite element method is used for calculation and combined with RBF neural network to update the structural parameters. The flowchart of the proposed method is shown in Figure 3, and the procedure of the proposed method for model updating is described as follows:
Step 4: Use the finite element method to calculate the dynamic response, obtain FRF curves, identify the resonance and antiresonant frequencies, and take them as the characteristic quantities of the objective function;

Step 5: Take the characteristic quantities in Step 4 as the input vector, and the corresponding updating parameters as the output vector; the RBF neural network is used for training and the mapping relationship between them is constructed;

Step 6: The characteristic quantities of the target value (or measured value) are inputted into the trained neural network, and the updated parameters are obtained by using the generalization of the RBF neural network.

The RBF neural network is adopted as the optimization algorithm, and the number of samples and the value range of updating parameters are the main factors affecting the accuracy of the method. Due to the large scale of the finite element model of the actual complex structure and the time-consuming calculation, it is crucial to determine a reasonable number and representative sample points to reduce the calculation time and improve the accuracy of the neural network model [28]. The uniform design [36] has...
some advantages such as uniform dispersion and good representativeness of test points, and obtaining more information with fewer points. Therefore, the method is used to determine the samples, code-named \( U_n(q^s) \) or \( U^*_n(q^s) \), and each uniform design table has its usage table. The symbol \( U \) represents uniform design, \( n \) represents the number of tests, \( q \) represents the number of each factor level, and \( s \) represents the number of factors. Finally, * denotes that the uniform design table has better uniformity, which should be preferred.

In use, the appropriate alternative uniform design table should be determined firstly according to the number of the updating parameters (i.e., the symbol \( s \)) and the range of each updating parameter (i.e., the symbol \( q \)). Each updating parameter value range should cover the corresponding initial value and target value, and then compare the deviation of the same number of factors (i.e., the symbol \( s \)) in the corresponding use table. Because the smaller the discrepancy value, the better the uniformity, the uniformity design table corresponding to the minimum is selected as the sample in the finite element analysis.

3. Results

3.1. Identification of Resonance and Antiresonant Frequencies Based on FEM

According to Figure 1, the finite element model of the cantilever beam was constructed by using ABAQUS 2016. The beam was divided into 10 equal beam elements and one concentrated mass unit, among which the spring element is used for elastic support. Since only the validity of the proposed method is verified and the study object is a finite element model with the same number of grids, the influence of mesh number on the accuracy of the finite element analysis results is ignored here. In addition, with large-scale structures such as offshore platforms and offshore wind turbines, it is impossible to arrange too many vibration measuring points due to the limitations of test conditions such as underwater structures or high-rise structures. So, node 4 under the mass unit is selected to carry out the FEM updating considering actual conditions.

In the finite element analysis of the model, the measured impulse force signal of the force hammer with a sampling frequency of 2048 Hz and sampling time of 4 s is taken as the excitation signal at node 4 (i.e., the driving point) in the finite element model in a vertically downward direction. Because the excitation signal is the measured hammering impulse signal, the signal contains noise. The acceleration response signals of node 4 under two conditions of initial value and target value are calculated by using ABAQUS 2016. The point FRFs of node 4 are calculated by using “tfestimate” in MATLAB R2019a, respectively, as shown in Figures 4 and 5. There are three antiresonant frequencies and four resonance frequencies within the considered frequency range according to the characteristics of point FRF at node 4. Since it is difficult for large-scale civil engineering structures to excite high-order modal information, only low-order modal can be obtained. In addition, we want to adopt the minimum characteristic quantities to modify the updating parameters. Therefore, the first four-order resonance frequencies and the first-order antiresonant frequency are identified in the paper as characteristic quantities of the objective function, as shown in Table 2. It is worth noting that there is a peak value between 57.25 Hz and 230 Hz shown in Figure 4, but according to the above, the phase corresponding to the peak does not change \(-180^\circ\), so the frequency is not resonance frequency. This is because measurement noise can result in spurious non-physical “identified” modes. Figure 5 is the same as Figure 4. The ordinate of both figure is consistent with Figure 6 in Section 3.3.
Figure 4. A point FRF curve of node 4 for initial value.

Figure 5. A point FRF curve of node 4 for target value.

Table 2. Comparison of resonance–antiresonant frequencies for target value and initial value.

<table>
<thead>
<tr>
<th>Characteristic Quantities</th>
<th>Target Value (Hz)</th>
<th>Initial Value (Hz)</th>
<th>Error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st resonance frequency</td>
<td>10</td>
<td>12.5</td>
<td>25</td>
</tr>
<tr>
<td>2nd resonance frequency</td>
<td>53</td>
<td>57.25</td>
<td>8.02</td>
</tr>
<tr>
<td>3rd resonance frequency</td>
<td>232.5</td>
<td>230</td>
<td>−1.08</td>
</tr>
<tr>
<td>4th resonance frequency</td>
<td>403</td>
<td>429.75</td>
<td>6.64</td>
</tr>
<tr>
<td>1st antiresonant frequency</td>
<td>33</td>
<td>30.5</td>
<td>−7.58</td>
</tr>
</tbody>
</table>

As can be seen from Figures 4 and 5 and Table 2, the errors of the first four-order resonance frequencies and the first-order antiresonant frequency corresponding to the initial value and the target value are large, especially the error of the first-order resonance frequency is up to 25%. Therefore, in order to accurately reflect the dynamic characteristics of the structure, the finite element model needs to be updated.

3.2. Determination of Updating Parameters and the Uniform Design Table

In the model, a total of four parameters including the material elastic modulus $E$, concentrated mass $m$ and the two stiffness $k_t$ and $k_r$ of the elastic support are assumed as the updating parameters. Three uniform design tables, $U_6^{*}(6^4)$, $U_8^{*}(8^5)$, and $U_{10}^{*}(10^8)$, are
preliminarily selected according to the number of updating parameters (i.e., $s$) and the number of each factor level (i.e., $q$). By comparing the discrepancy of four factors in the corresponding use table, $U_{10}^*(10^3)$ with the minimum value of 0.2236 is selected as the uniform design table. The upper bound (green) and lower bound (yellow) limits for each updating parameter should include the initial value and the target value, which are divided into 10 equal parts (i.e., 10 levels), and then they are evenly dispersed into 10 groups of test samples with the usage table, as shown in the updating parameters of Table 3. The corresponding point FRFs of the 10 groups of node 4 are calculated, respectively. The first four-order resonance frequencies and the first-order antiresonant frequency are identified, as shown in the characteristic quantities of Table 3.

Table 3. Resonance and antiresonant frequencies of the beam with 10 levels.

<table>
<thead>
<tr>
<th>Test Number</th>
<th>Updating Parameter</th>
<th>Characteristic Quantities (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$E$ (GPa)</td>
<td>$m$ (kg)</td>
</tr>
<tr>
<td></td>
<td>$k_t$ ($10^7$ N/m)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$k_r$ ($10^4$ N·m/rad)</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>69</td>
<td>5.25</td>
</tr>
<tr>
<td></td>
<td>1.25</td>
<td></td>
</tr>
<tr>
<td></td>
<td>3.5</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>70</td>
<td>6</td>
</tr>
<tr>
<td></td>
<td>2.25</td>
<td></td>
</tr>
<tr>
<td></td>
<td>6</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>71</td>
<td>6.75</td>
</tr>
<tr>
<td></td>
<td>0.5</td>
<td></td>
</tr>
<tr>
<td></td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>72</td>
<td>4.75</td>
</tr>
<tr>
<td></td>
<td>1.5</td>
<td></td>
</tr>
<tr>
<td></td>
<td>5.5</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>73</td>
<td>5.5</td>
</tr>
<tr>
<td></td>
<td>2.5</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2.5</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>74</td>
<td>6.25</td>
</tr>
<tr>
<td></td>
<td>0.75</td>
<td></td>
</tr>
<tr>
<td></td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>75</td>
<td>7</td>
</tr>
<tr>
<td></td>
<td>1.75</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>76</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td>2.75</td>
<td></td>
</tr>
<tr>
<td></td>
<td>4.5</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>77</td>
<td>5.75</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1.5</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>78</td>
<td>6.5</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td></td>
</tr>
<tr>
<td></td>
<td>4</td>
<td></td>
</tr>
</tbody>
</table>

3.3. Model Updating and Validation

The 10 groups of updating parameters and characteristic quantities shown in Table 3 are taken as training samples of the RBF neural network. The function newrbe in the MATLAB R2019a very quickly creates an exact RBF network with zero errors on the design vectors. In order to compare with the updated results which only use resonance frequencies as the characteristic quantities, the sample points are divided into two groups:

Group I: the first three-order resonance frequencies and the first-order antiresonant frequency;

Group II: only the first four-order resonance frequencies.

They are inputted into the RBF neural network, respectively. The output vector is the parameters to be updated, and the mapping relationship between the frequencies and the updating parameters is established. Finally, the target values (i.e., measurement values or truth values) are inputted into the trained network, and results from I and II are obtained, respectively, as shown in Table 4.

Table 4. Updated parameters obtained from the two groups.

<table>
<thead>
<tr>
<th>Updating Parameter</th>
<th>Target Value</th>
<th>Initial Value</th>
<th>Initial Error (%)</th>
<th>Result I</th>
<th>Result II</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E$ (GPa)</td>
<td>77</td>
<td>70</td>
<td>−9.09</td>
<td>77.31</td>
<td>73.84</td>
</tr>
<tr>
<td>$m$ (kg)</td>
<td>6.5</td>
<td>5</td>
<td>−23.08</td>
<td>6.15</td>
<td>5.56</td>
</tr>
<tr>
<td>$k_t$ ($10^7$ N/m)</td>
<td>1</td>
<td>2</td>
<td>100</td>
<td>0.93</td>
<td>2.19</td>
</tr>
<tr>
<td>$k_r$ ($10^4$ N·m/rad)</td>
<td>2</td>
<td>5</td>
<td>150</td>
<td>1.89</td>
<td>2.28</td>
</tr>
</tbody>
</table>

By comparing result I and result II, the error of each updated value in result I is significantly smaller than that in result II, and the maximum error is less than 7%, while
the error of tensile stiffness $k_t$ in result II even reaches 119%, further deviating from the target value.

In order to verify the influence of the updating results on the FRF curves, the two groups of updated parameters obtained in Table 4 were, respectively, substituted into the ABAQUS 2016, and the characteristic quantities of the objective function shown in Table 5 and the FRF curves shown in Figure 6 were obtained.

![Figure 6](image)

**Figure 6.** The point FRF curves at node 4 of four conditions.

<table>
<thead>
<tr>
<th>Characteristic Quantities</th>
<th>Target Value (Hz)</th>
<th>Result I</th>
<th>Result II</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Updated Value</td>
<td>Error (%)</td>
</tr>
<tr>
<td>1st resonance frequency</td>
<td>10</td>
<td>10.25</td>
<td>2.5</td>
</tr>
<tr>
<td>2nd resonance frequency</td>
<td>53</td>
<td>53.5</td>
<td>0.94</td>
</tr>
<tr>
<td>3rd resonance frequency</td>
<td>232.5</td>
<td>232.5</td>
<td>0</td>
</tr>
<tr>
<td>4th resonance frequency</td>
<td>403</td>
<td>400.8</td>
<td>−0.55</td>
</tr>
<tr>
<td>1st antiresonant frequency</td>
<td>33</td>
<td>33.25</td>
<td>0.76</td>
</tr>
</tbody>
</table>

As can be seen from Table 5, the relative error between the characteristic quantities of the objective function and the updated value by using the updated parameters of result I in Table 4 is less than 3%, and smaller than that of result II. Compared with the relative error of 0.5% of the finite element model updating by the first four-order modal frequencies and the first-order effective modal mass in Ref. [12], they are relatively close together. However, only four characteristic quantities are used in this paper.

Figure 6 shows the point FRF curves at node 4. As can be seen from Figure 6, the FRF curve obtained by using the resonance and antiresonant frequencies as the characteristic quantities of the objective function is very close to the target value, not only in the updating frequency band but also outside the updating frequency band up to 700 Hz, indicating a good correlation between the two FRF curves. However, the point FRF curve obtained by using only resonance frequencies as the characteristic quantities of the objective function is in good agreement with the target value of only the first three-order resonance frequency 230 Hz, and then the frequency response function curves no longer coincide with each other, the deviation increases significantly and the correlation becomes worse, indicating that the dynamic characteristics outside the updating frequency band cannot be accurately described.
4. Discussion

Through the theoretical calculation of antiresonant frequency and the identification analysis of the point FRF, the characteristic quantities of the objective function are constructed by the combination of the resonance frequencies and antiresonant frequency, which is grouped by the uniform design method. Combined with the RBF neural network optimization algorithm, the finite element model of the complex boundary beam is updated. The main conclusions are as follows.

(1) The antiresonant frequency reflects the local features of the structure, which can be used as an effective supplement to the resonance frequency reflecting the global features. The characteristic quantities of the objective function are constructed by combining them both, and the number of the point frequency response is increased according to the number of the updated parameters, which can effectively solve the ill-posed problem caused by insufficient characteristic quantities;

(2) RBF neural network can accurately describe the mapping relationship between the structural features of the complex boundary beam and updating parameters, and convert the model updating into positive problem solving;

(3) Using the uniform design method to group neural network training samples, which can obtain better results with fewer samples;

(4) In the case of using only the point FRF of one measurement point, using the resonance frequencies and the antiresonant frequency as the characteristic quantities of the objective function, combining with the RBF neural network optimization algorithm, the finite element model of the complex boundary cantilever beam with four updating parameters is updated. Compared with only using resonance frequencies as the characteristic quantities of the objective function, the dynamic characteristics in the updated frequency band can be accurately reproduced. Additionally, it also has a better ability to predict the dynamic characteristics outside the updated frequency range;

(5) Although the error between the updated frequencies of the model and the corresponding target values is within the allowable range, the corresponding updating parameters have a large error with the corresponding target value. Therefore, frequency error should not be taken as the only factor to evaluate the effectiveness of the finite element model updating.

5. Conclusions

In this study, a new FE model updating method to establish an accurate FE model of a cantilever beam with elastic support using resonance frequencies and the antiresonant frequency combined with the RBF neural network was proposed. The elastic support is neither simple nor fixed, but in between. The cantilever beam considering elastic support is a simplified model of large tall structures such as offshore platforms and offshore wind turbines.

In the simulation analysis of the cantilever beam model, the method is successful in identifying the constrained stiffness, the additional mass and the elastic modulus with only one measuring point.

The results show that the relative errors between all the updated parameters and the target values are less than 7%, and the relative errors of the characteristic quantities in the updating frequency band are less than 3%. The proposed method can accurately reproduce the dynamic characteristics of the cantilever beam.

The proposed method can be applied to the damage detection and safety evaluation of large structures, such as offshore platforms and offshore wind turbines, which are difficult to arrange more measuring points.

Author Contributions: Conceptualization and methodology, H.Z.; software, formal analysis, and investigation, J.L.; validation and data processing, Z.W., T.G., and W.X.; writing—original draft preparation, T.G.; writing—review and editing, W.X. All authors have read and agreed to the published version of the manuscript.


References


Funding: This research was funded by the National Natural Science Foundation of China, grant number 11402051.

Institutional Review Board Statement: Not applicable.

Informed Consent Statement: Not applicable.

Data Availability Statement: Not applicable.

Conflicts of Interest: The authors declare no conflict of interest.

**Disclaimer/Publisher’s Note:** The statements, opinions and data contained in all publications are solely those of the individual author(s) and contributor(s) and not of MDPI and/or the editor(s). MDPI and/or the editor(s) disclaim responsibility for any injury to people or property resulting from any ideas, methods, instructions or products referred to in the content.