Modeling Uncertain Travel Times in Distribution Logistics

Khadija Ait Mamoun 1,2,*, Lamia Hammadi 1,2, Abdessamad El Ballouti 1, Antonio G. N. Novaes 3 and Eduardo Souza de Cursi 2

Abstract: Uncertainty quantification is a critical aspect of distribution logistics, particularly unpredictable travel times caused by traffic congestion and varying transportation conditions. This paper explores the modeling of uncertainty in dealing with travel times in the context of distribution logistics using the collocation method. First, we employ Monte Carlo simulations to assess the efficacy of the collocation method in modeling the variability and uncertainty associated with travel times. Second, we implement the collocation method in Casablanca, Morocco, a city renowned for its extensive distribution logistics operations and its dynamic traffic. Four distinct scenarios are considered: morning peak, inter-peak, evening peak, and off-peak periods. Our study explores two scenarios: one with recurrent congestion, representing typical daily conditions, and the other with unpredictable uncertainties in travel times, accounting for unexpected events that may occur during a distribution day. Our research findings enhance our understanding of the probabilistic nature of travel times in distribution logistics. This knowledge provides valuable insights applicable to both routine situations with recurrent congestion and non-recurrent congestion. The results’ findings contribute to a better understanding of the probabilistic nature of travel times in distribution logistics, offering valuable insights for optimizing route planning and scheduling.

Keywords: uncertain travel time; collocation method; variability; uncertainty quantification

1. Introduction

In recent years, the problem of modeling uncertain travel times in distribution logistics has gained increasing attention as a means to improve logistics operations and enhance overall efficiency. The efficient management of distribution logistics is of crucial importance for businesses operating in distribution logistics, especially in cities with high traffic congestion. The ability to accurately predict and account for uncertain travel times is essential for effective logistics planning and optimization. The unpredictable nature of travel times poses significant challenges for logistics professionals. Traditional logistics models that assume fixed travel times often fail to capture the dynamic and uncertain nature of transportation. This can lead to inefficient resource allocation, missed delivery deadlines, increased costs, and customer dissatisfaction. To address these challenges, researchers and practitioners have turned their attention to the modeling of uncertain travel times as a key factor in distribution logistics. The variability and duration of travel time are influenced by various factors, particularly congestion. Congestion can be categorized as recurrent or non-recurrent [1], each with its distinct characteristics and causes. Recurrent congestion arises from daily peak-hour delays caused by the imbalance between traffic demand and the capacity of the transportation infrastructure. On the other hand, non-recurrent congestion results from atypical events, including incidents [2,3], disabled vehicles, road construction works, accidents, adverse weather conditions, and special events. Both categories of
congestion introduce variability and uncertainty in travel times, leaving drivers uncertain about their precise arrival times at their destinations. Non-recurrent congestion can have dual effects on traffic conditions. It can generate new congestion during off-peak periods or increase delays during recurring congestion episodes. However, this type of congestion has often been overlooked in traffic engineering and modeling practices due to its irregular and unpredictable nature [4]. The modeling of the uncertain time variability has attracted the interest of several researchers. In a study by Ruimin [5], the examination of travel time variability involved the assessment of various factors, including time of day, day of the week, weather conditions, and traffic accidents. To quantify the influence of these factors on travel time parameters, the author employed multiple linear regression models with two-way interactions. The researchers in [6] investigated the prediction of uncertainty in route travel times within urban road networks, utilizing an extensive dataset from floating taxis; in [7], the authors delved into the importance of precise travel time modeling for road-based public transport and provided an overview of the current advancements in statistically modeling bus travel time distributions. They emphasized the significance of aggregating spatial and temporal data and proposed potential avenues for future research in this domain. In [8], researchers investigated travel time variability within the context of Indian traffic conditions and assessed the effectiveness of travel time distributions when considering various temporal and spatial aggregations. Utilizing AVL data gathered from four transit routes in Mysore City, located in Karnataka, India, this research examined travel time distributions across peak and off-peak periods, as well as within different time intervals. The study employed the Anderson–Darling test to evaluate the goodness-of-fit of the distributions, taking into account segments featuring signalized intersections and a variety of land-use types. The findings underscored the effectiveness of the generalized extreme value (GEV) distribution in precisely characterizing travel time variations within public transit systems. Therefore, the quantification of uncertainty remains a pivotal component when modeling and estimating travel time variability. Precisely capturing and forecasting travel time variations is essential in multiple domains, including transportation planning, traffic management, and logistics. Many research endeavors have focused on tackling uncertainties and developing approaches to improve the precision and effectiveness of uncertainty quantification (UQ). UQ tools have been applied in numerous studies to evaluate and mitigate risks effectively. In [9,10], the authors proposed a model that utilizes uncertainty quantification to address risks associated with the customs supply chain. This model employs the moment matching method and takes into account the seasonality of illicit traffics at five different sites in Morocco. In the study [11], uncertainty quantification methods for differential equations were utilized to predict and simulate the transmission of influenza within the setting of a boarding school. In [12], the researchers developed an uncertainty quantification model to assess and manage the risks related to atmospheric dispersion. They utilized five different techniques to depict and analyze uncertainties, encompassing the probability theory, the interval analysis, the fuzzy approach, the mixed probabilistic–fuzzy approach, and the evidence theory. Stochastic uncertainties, which result from the inherent unpredictability of natural events, can originate from two primary sources: sampling uncertainty or measurement uncertainty [13]. Epistemic uncertainties, on the other hand, originate from gaps in information or knowledge about these events and may have various causes, such as data limitations, uncertain parameters, algorithmic ambiguities, or methodological uncertainties [13]. In [14], the authors introduced a model to predict and model epidemic risks, employing uncertainty quantification (UQ) techniques. They explored two specific methods, namely, the collocation method and the moment matching method. These methods were applied in the context of studying the epidemic risk associated with SARS-CoV-2 in Morocco, serving as an illustrative example. In the humanitarian logistics context, uncertainty quantification (UQ) holds a significant importance. This is primarily because disasters typically exhibit a substantial degree of uncertainty. For example, for flooding, UQ helps in the making of informed decisions, particularly when it comes to predicting disasters [15]. One of the fundamental approaches
for uncertainty quantification (UQ) is the Monte Carlo simulation (MCS), which has been widely employed in various fields. The basic principle of the MCS involves repeatedly sampling input parameters from their respective probability distributions and propagating them through the model to obtain a distribution of output responses. By generating a large number of samples, the MCS can capture the full range of possible outcomes and provide statistical measures. The computational cost associated with the Monte Carlo simulation (MCS) can be a challenge due to the need for a large number of samples. This requirement can make the MCS computationally prohibitive [16]. The Latin hypercube sampling (LHS) technique offers potential improvements in uncertainty quantification. This sampling approach has been shown to provide better accuracy [17]. However, it is important to note that the LHS does have its limitations, as pointed out by [18]. In this context, a promising approach known as the polynomial chaos expansion (PCE) emerged. The PCE allows for the representation of random variables as multivariate series of Gaussian variables [19–22]. The representation of random variables follows the concept of a Hilbert basis, with approximation achieved by truncating the representation at a specific level. This approach has demonstrated significant advantages over traditional methods such as the Monte Carlo simulation (MCS) and the Latin hypercube sampling (LHS), offering lower computational costs and an increased efficiency. In this paper, we have employed the collocation method, specifically based on approximation interpolation, to model the variability of travel time. The collocation technique involves selecting specific points, known as collocation points, within the domain of interest and determining the corresponding coefficients to approximate the underlying function. To validate the effectiveness of the collocation method in modeling travel time variability, we conducted a comparison with the Monte Carlo simulation (MCS) approach. The MCS method is widely used as a benchmark for uncertainty quantification and serves as a reference for evaluating the accuracy and efficiency of alternative techniques [23]. The utilization of collocation methods for modeling the variability of travel time has gained little significant attention in the literature. In [24], the authors utilized the stochastic response surface method (SRSM) to analyze travel time variability under uncertain factors, including traffic volume and rainfall intensity. Applying SRSM to a section of the Kobe Route, Japan, the results showed an improved accuracy in modeling travel time distribution compared to the regression analysis; the collocation method was used for estimating the travel time. In this work, we employed the collocation method to effectively model the travel time variability during the four peak hours, considering both recurrent congestion and non-recurrent congestion scenarios (Figure 1). This approach allowed us to delve deeper into understanding how travel time fluctuations evolve over the course of the day, shedding light on the intricate dynamics of traffic patterns during peak hours.

The collocation method has proven to be a valuable tool for uncertainty quantification in travel time estimation. It enables us to address the challenges posed by the dynamic and complex traffic conditions such as those in Casablanca. By incorporating the variability of traffic volume, incidents like road accidents, and other external factors, we gained insight into how uncertainties impact travel time reliability. The insights derived from our analysis will aid logistics practitioners in making informed decisions, optimizing route planning, and mitigating the potential impacts of travel time uncertainties on delivery schedules. Figure 2 explains the framework methodology of this study: the aim is to model uncertain travel times in distribution logistics using the uncertainty quantification approach with the collocation method. To ensure the reliability and effectiveness of the collocation method, we conducted a thorough validation process using Monte Carlo simulations (MCS). Subsequently, we conducted modeling using the cumulative distribution function (CDF) in both the recurrent and non-recurrent congestion scenarios. Then, a probability density function (PDF) was derived from the CDF using Dirac’s delta approximation. To comprehensively assess the variation of travel time uncertainties in distribution logistics, we calculated probabilities across all four peak time scenarios: ‘morning peak’, ‘inter-peak’, ‘evening peak’, and ‘off-peak’. This encompassing approach helped us gain a
The paper aims to provide a comprehensive understanding of how different factors, including traffic congestion, influence the variability of travel times. It emphasizes the relationship between travel time variability and congestion levels. To assess the value of travel time reliability, they developed a comprehensive model and contained the collocation model validated by the Monte Carlo simulation (MCS).

For recurrent congestion, the model takes into account peak hours, road signaling, roadworks, and congested intersections. For non-recurrent congestion, it considers vehicle breakdown, road accidents, special events (fests, demonstrations, sports events), and inclement weather.

Figure 1. Recurrent and non-recurrent congestion.

Figure 2. The methodology of the study framework.
This paper is organized as follows: Section 3 describes the uncertainty quantification model and contains the collocation model validated by the Monte Carlo simulation (MCS) using an example; Section 4 presents the computational results and discussion; and Section 5 presents the conclusion and discussion.

2. Literature Review

Quantifying uncertainty in uncertain travel times is crucial and has wide-ranging applications, especially in logistics and route planning. The variability in travel time within distribution logistics is a consequence of the stochastic nature of numerous operational factors. This variability poses challenges for distribution and logistics firms, introducing uncertainty and associated costs. In [25], the authors conducted a comprehensive systematic review of the existing research on travel time reliability, with a specific focus on the assessment of the value associated with travel time reliability. Additionally, they undertook a meta-analysis to uncover the underlying factors contributing to variations in reliability estimates. Travel time uncertainty varies depending on congestion levels [6]. Congestion travel time levels have been considered by researchers. In [26], the authors examined how changes in congestion levels influenced the choice of departure time and they conducted an analysis of costs associated with uncertain travel times. In the study [27], the authors emphasized the relationship between travel time variability and congestion levels. To assess the value of travel time reliability, they developed a comprehensive modeling framework that incorporated trip scheduling, endogenous traffic congestion, travel time uncertainty, and pricing strategies. Hence, to quantify the value of travel time reliability, they integrated trip scheduling, endogenous traffic congestion, travel time uncertainty, and pricing strategies into one modeling framework. The common thread among the models developed in these research studies is that normal traffic flow conditions become unstable when uncertainties arise, especially during peak periods or when other incidents begin to deteriorate the typical traffic conditions. Hence, travel time uncertainty is frequently a result of traffic congestion, and, when congestion occurs, it amplifies the frustration experienced by road users. For the studies [28–30], it was important to model and understand the implications of travel time uncertainty on transportation systems. Additionally, studies of transportation networks have shown that travel time uncertainty is an important factor that influences traffic networks [30–32]. For example, in [31], it was observed that travelers place a greater importance on decreasing travel time variability than on reducing the total travel time for a specific journey. In [32], it was noted that deterministic traffic network models are inadequate for assessing traffic network performance. These models tend to magnify minor discrepancies in travel times while failing to adequately account for the substantial impacts of travel time uncertainty. In [30], it was determined that the loss in utility resulting from uncertainty is comparable in magnitude to the overall travel costs. The majority of the previous research concentrated on either the causes or the effects of travel time uncertainty. Considering the impacts of travel time uncertainty within traffic networks necessitates an explicit approach for quantifying and modeling this uncertainty. Hence, in comparison to previous studies in the literature, our objective is to model travel time uncertainty within the domain of distribution logistics. We aim to demonstrate the variability of travel time at different stages of congestion and during various peak times. To accomplish this, we employ an effective method for uncertainty quantification, namely, the collocation method. In addition, modeling uncertain travel times is crucial for real-world applications, especially in the context of routing optimization, where stochastic scenarios are commonly utilized to account for uncertainties between nodes [33].

3. Uncertainty Quantification Model

This section develops a model based on uncertainty quantification (UQ) and presents the collocation method, an interpolation-based approximation, which is then validated using the Monte Carlo simulation (MCS).
3.1. Model Formulation

A model based on uncertainty quantification (UQ) employs a discrete method that involves approximations by analyzing expressions that depend on the joint distribution of the pair $X, Q$. Here, $Q$ represents the uncertainties within the system, modeled as a random vector, while $X$ represents the system’s response. We assume that $X$ is a function dependent of $Q$, as $X = X(Q)$. Statistical information can be obtained from a sample of the pair $(X, Q)$. As $X$ is a function of $Q$, denoted as $X = X(Q)$, we can adopt a Hilbertian approach [16,34]:

We assume that $X = X(Q) \in H$, where $H$ is a separable Hilbert space; we choose an adequate Hilbert basis $\{\varphi_i\}_{i \in \mathbb{N}^*}$:

$$X = \sum_{i=1}^{+\infty} x_i \varphi_i(Q) \quad (1)$$

We consider $X$ and $Q$ as two random variables on the probability space $(\Psi , P)$. Let $H$ be a subspace of functions of $X$. Find an element $PX$ from $H$ such that $PX$ provides the best approximation of $X$ within $W$. It is anticipated that $PX \rightarrow X$ for $n \rightarrow +\infty$. The primary objective of a representation is to employ $PX$ instead of $X$ when assessing the probability of significant events. This ensures that $PX$ serves as a practical approximation of $X$, such that the probabilities obtained with $PX$ are close to the actual values when compared to those obtained with $X$. In general, achieving this objective entirely is not always feasible, so we seek an approximation where $PX$ can be used as a replacement for $X$ in practical scenarios. This implies that the difference denoted as $X - PX$, should be small enough, aligning with our specific goals and requirements. If $\{\varphi_i\}_{i \in \mathbb{N}^*}$ is not orthonormal, it is necessary to obtain a sequence of orthogonal projections.

$$X \approx PX = \sum_{i=1}^{k} x_i \varphi_i(Q) \in H; \: x = (x_1, x_2, \ldots x_k)^t \quad (2)$$

where $x \in \mathbb{R}^k$ denotes the unknown coefficients of the expression of $PX$ which describes the evolution of the response of the system and $\{\varphi_i\}_{i \in \mathbb{N}^*}$ is a Hilbert basis.

3.2. Numerical Method Based on Collocation

For using a sample, $\{(Q_i, X_i) : i = 1, \ldots, ns\}$ consists of determining

$$PX(Q) = \sum_{j=1}^{k} x_j \varphi_j(Q) \in H \quad (3)$$

such that

$$PX(Q_i) = X_i, \: i = 1, \ldots, ns \quad (4)$$

In this case, $X = (x_1, x_2, \ldots x_k)^t$ is the solution of a linear system:

$$AX = B, \: A_{ij} = \varphi_j(Q_i), \: B_i = X_i \: (1 \leq i \leq ns, \: 1 \leq j \leq k) \quad (5)$$

3.3. Probability Density Function Approximation

The probability density function (PDF) is a mathematical function used to depict the likelihood of a random variable assuming a specific value. It is derived from the cumulative distribution function (CDF) and is obtained through a technique called “Particle derivatives”, which involves employing Dirac’s delta approximation [35]. This approach involves reviewing the derivation using Dirac’s delta approximation to identify the PDF. The Dirac delta function $\delta(y_0)$ can be naturally approximated by a sequence of probabilities that converge to a distribution concentrated at $y_0$. 

Let $F(y)$ be a function defined by the following:

$$F(y) = \delta(x - y)F(x)dx$$

(6)

We have

$$\frac{d}{dy}F(y) = F'(y) = \frac{\int \delta(x-y)(F(x)-F(y))dx}{\int (x-y)\delta'(x-y)dx}$$

(7)

We may explore the possibility of employing the approximation technique, which is further elaborated in [35]:

$$\delta(x - y) \approx f(x, y, n)$$

(8)

so that

$$F'(y) \approx \frac{\sum_{j=1}^{k_p} N_j(F_j - F(y)) \frac{\partial f}{\partial y(p_j, y)} - N_j}{\sum_{j=1}^{k_p} N_j(p_j - y) \frac{\partial f}{\partial y(p_j, y)}}; N_j = \int dx; F_j = F(p_j)$$

(9)

In the realm of particle derivatives, we use $p_j$ to denote individual particles, while $N_j$ signifies the associated volume, and $f$ represents the kernel employed in the approximation process. Drawing an analogy with conventional physical parameters, we establish the equation $N_j = m_j/\rho_j$, where $m_j$ represents the mass of the particle and $\rho_j$ refers to its volumetric mass density, commonly referred to as “density”. Consequently, Equation (9) can be derived as follows:

$$F'(y) \approx \frac{\sum_{j=1}^{k_p} \frac{m_j}{\rho_j}(F_j - F(y)) \frac{\partial f}{\partial y(p_j, y)} - \frac{\partial f}{\partial y(p_j, y)}}{\sum_{j=1}^{k_p} \frac{m_j}{\rho_j}(p_j - y) \frac{\partial f}{\partial y(p_j, y)}}; N_j = \int dx$$

(10)

To compute the partial derivatives, we follow a similar approach, introducing the approximations $F_j = F(p_j)$ which yields

$$\frac{\partial F}{\partial y_j}(y) \approx \frac{\sum_{j=1}^{k_p} N_j(F_j - F(y)) \frac{\partial f}{\partial y_j(p_j, y)} - N_j}{\sum_{j=1}^{k_p} V_j(p_j - y) \frac{\partial f}{\partial y_j(p_j, y)}}; N_j = \int dx$$

(11)

Then,

$$\frac{\partial F}{\partial y_j}(y) \approx \frac{\sum_{j=1}^{k_p} \frac{m_j}{\rho_j}(F_j - F(y)) \frac{\partial f}{\partial y_j(p_j, y)} - \frac{\partial f}{\partial y_j(p_j, y)}}{\sum_{j=1}^{k_p} \frac{m_j}{\rho_j}(p_j - y) \frac{\partial f}{\partial y_j(p_j, y)}}; N_j = \int dx$$

(12)

It is important to highlight that a range of kernels can be found in the literature; the most commonly used are defined as follows:

$$f(x, y, n) = \mu(n)\chi\left(\frac{\|x - y\|}{n}\right)$$

(13)

where $\mu(n)$ is a normalizing factor such that the integral on $R^n \int f(x, y, n)dx = 1$. In our particular study, we will adopt the quintic kernel, such that $\mu(n) = \frac{1}{120}$; $\frac{1}{720}$; $\frac{1}{5040}$ (dimension 1, 2, or 3, respectively)

$$\chi = \begin{cases} (3 - x)^5 - 6(3 - x)^5 + 15(1 - x)^5, & \text{if } 0 \leq x \leq 1 \\ (3 - x)^5 - 6(3 - x)^5, & \text{if } 1 \leq x \leq 2 \\ (3 - x)^5, & \text{if } 2 \leq x \leq 3 \\ 0, & \text{otherwise} \end{cases}$$

(14)

3.4. Model Validation by Monte Carlo Simulation (MCS)

In this study, we employed interpolation-based approximations, specifically the collocation method, to estimate travel time variability. To validate the effectiveness of this
approach, we conducted a Monte Carlo simulation (MCS) on a specific example. The Monte Carlo simulation (MCS) is a computational technique used to estimate and analyze complex systems or processes by performing a large number of random simulations. It involves generating random samples from input probability distributions to model uncertain parameters, and then evaluating the system’s behavior multiple times to observe the outcomes. In our example, we generated 1000 samples using the Monte Carlo simulation (MCS) and 30 samples using the collocation method. The aim was to compare the results of the two methods. Let us consider a system of ordinary differential equations (ODE) represented as follows:

$$s = \int_{0}^{T} y(t) \, dt$$

(15)

where $y(t) = ae^{bt}$, such that

$$\begin{cases} y' = by \\ y(0) = a \end{cases}$$

(16)

over the interval $[0, T]$; $a$ and $b$ represent two variables generated using triangular distribution to obtain 1000 samples for the Monte Carlo simulation (MCS) and 30 samples for the collocation method; the initial values used in the triangular distribution are represented in Tables 1 and 2. The parameters presented in Tables 1 and 2 represent simulation values of the ordinary differential equation (ODE) system that we used to confirm the reliability of our model. These simulations were conducted while considering various conditions and model parameters, and the resulting values align with the outcomes we obtained in our numerical experiments.

Table 1. Values of variable “a” used to generate samples with triangular distribution.

<table>
<thead>
<tr>
<th>Variable</th>
<th>$a_{\text{min}}$</th>
<th>$a_{\text{max}}$</th>
<th>$a_{\text{pic}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>0.5</td>
<td>1.5</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 2. Values of variable “b” used to generate samples with triangular distribution.

<table>
<thead>
<tr>
<th>Variable</th>
<th>$b_{\text{min}}$</th>
<th>$b_{\text{max}}$</th>
<th>$b_{\text{pic}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>b</td>
<td>1</td>
<td>2</td>
<td>1.5</td>
</tr>
</tbody>
</table>

In this paper, $[0, T]$ is the time interval over which the solution was computed. The ODE system was defined on Matlab as follows Algorithms 1:

Algorithms 1: Ordinary Differential Equations (ODE)

```
function sol = systeme_edo(a,b)
    T = 2;
    tc = linspace(0, T, 200); % tc is a vector of 200 time points linearly spaced between 0 and T
    secm = @(t,y)b * y;
    y0 = a;
    sol = ode45(secm, [0, T], y0); % ode45 is a numerical method used to solve ODEs
    yc = deval(sol, tc); % yc is a vector containing the values of the solution y at the time points specified by tc
    s = trapz(tc, yc(1,:)); % s is the integrated value of y over the interval [0, T]
    sol.int = s; % sol is the solution of the ODE system obtained using the ode45 method
end
```

As a result, Figures 3 and 4 represent two different approximations of the system under consideration using the Monte Carlo simulation (MCS) and collocation methods, respectively. The Monte Carlo simulation allowed us to generate 1000 samples for the variables “a” and “b” using a triangular distribution. On the other hand, the collocation method enabled us to generate 30 samples for the same variables with the same distribution.
According to the figures, both the MCS’s and the collocation’s approximations show very close results to each other. This indicated that even with a small sample size of 30, as used in the collocation method, we achieved high-performance results comparable to those obtained with the MCS method. Additionally, in terms of execution time, the collocation method outperformed the MCS, making it a more efficient choice. Hence, our results revealed that the collocation method provided a robust and precise representation of travel time variability, closely aligning with the MCS approach. Notably, the collocation method achieved comparable results with a smaller sample size, making it a more efficient choice for uncertainty quantification in distribution logistics. The collocation method’s concurrence with the Monte Carlo simulation approach reaffirmed its accuracy and dependability. Consequently, for our specific case study aimed at modeling travel time variability in distribution logistics, we opted for the collocation method as it combined accuracy, efficiency, and reliability in order to effectively capture and quantify travel time uncertainties. The decision to utilize the collocation method as our primary modeling technique was grounded in several compelling factors. First and foremost, the collocation method had demonstrated its effectiveness in addressing uncertain parameters, a critical aspect when dealing with the variability of travel times. This effectiveness lies in its ability to produce precise estimates of travel time variability, even when working with relatively smaller sample sizes. One of the significant advantages of the collocation method was its efficiency in capturing the essence of travel time uncertainty. While the Monte Carlo simulation (MCS) is a valuable technique, it often requires a substantial number of random samples to achieve accurate results. In contrast, the collocation method excels in providing reliable estimates without the need for an extensive sample pool. This efficiency translates
into resource savings, making it an attractive choice for practical applications. Moreover, in the context of distribution logistics, where decisions must often be made in real-time, computational efficiency is paramount. The collocation method’s computational speed outpaces that of the MCS, allowing for quicker analysis and decision-making. This attribute aligned perfectly with the demands of our study, where timely logistics decisions could significantly impact the efficiency and effectiveness of supply chain operations.

4. Numerical Analysis for Uncertain Travel Time

This section presents the data analysis related to the travel and computational results of the variability of travel time at different times of the journey, including morning peak, inter-peak, evening peak, and off-peak periods. The aim is to explore the details of how travel times behave during these specific phases. This helps us to better understand the patterns and changes that affect distribution logistics operations. The selection of the collocation method stemmed from its proven effectiveness in handling uncertain parameters. We recognized its capability to provide accurate estimates of travel time variability with relatively smaller sample sizes, making it an efficient choice. Additionally, its computational efficiency compared to the MCS is essential for real-time logistics decision-making, a key consideration for our study.

4.1. Data Analysis

In today’s fast-paced world, the increasing volume of traffic on roadways has become a major concern. As cities continue to expand, the surge in vehicular movement has resulted in significant congestion during peak hours and other periods of high demand. In light of this pressing issue, our study focuses on a specific case in the city of Casablanca, which is renowned for its heavy traffic flow. Casablanca, often hailed as the economic capital of Morocco, is renowned for its significant volume of goods deliveries. Hence, Casablanca, being a bustling economic center, experiences a complex interplay of factors contributing to its traffic congestion. The bustling city serves as a major hub for commerce and trade, attracting numerous businesses engaged in transportation and logistics activities. As a result, the city’s intricate transportation network experiences a constant flow of goods deliveries, making it a critical focal point for distribution logistics. The increased traffic not only affects travel time but also the distribution logistics’ performance. Currently, the transportation sector in Morocco thrives, with approximately 76,000 companies engaged in road transport, boasting a fleet of around 74,000 vehicles. In Table 3, we show a comprehensive depiction of the number of goods delivery companies operating in various regions across Morocco; the table highlights the distribution of these enterprises across the different regions. This information offers a clear visualization of the regional engagement of goods delivery companies, contributing to a better understanding of the logistics landscape across the country.
Table 3. Number of goods transportation companies, statistics from December 2022.

<table>
<thead>
<tr>
<th>Region</th>
<th>Number of Goods Delivery Companies</th>
</tr>
</thead>
<tbody>
<tr>
<td>Casablanca–Settat</td>
<td>14,781</td>
</tr>
<tr>
<td>Marrakech–Safi</td>
<td>9543</td>
</tr>
<tr>
<td>Rabat–Salé–Kénitra</td>
<td>8889</td>
</tr>
<tr>
<td>Tanger–Tetouan–Al Hoceima</td>
<td>8426</td>
</tr>
<tr>
<td>Fès–Meknès</td>
<td>7029</td>
</tr>
<tr>
<td>L’Oriental</td>
<td>6553</td>
</tr>
<tr>
<td>Beni Mellal–Khénifra</td>
<td>5948</td>
</tr>
<tr>
<td>Souss–Massa</td>
<td>5776</td>
</tr>
<tr>
<td>Laâyoune–Sakia Al Hamra</td>
<td>4316</td>
</tr>
<tr>
<td>Drâa–Tafilalt</td>
<td>3707</td>
</tr>
<tr>
<td>Guelmim–Oued Noun</td>
<td>883</td>
</tr>
<tr>
<td>Dakhla–Oued Eddahab</td>
<td>569</td>
</tr>
</tbody>
</table>

According to the statistics, it is evident that Casablanca is home to a significant number of goods delivery companies, resulting in a large fleet of delivery vehicles. This makes Casablanca the city with the highest traffic in terms of distribution logistics as well as urban mobility. As a major economic center and the largest city in Morocco, Casablanca has become a focal point for distribution and transportation activities. The concentration of goods delivery companies in the city has led to a high density of delivery vehicles navigating through its busy streets. Our study focused on a delivery route within the city of Casablanca, from the depot (Coopérative Pharmaceutique Marocaine) to the customer (Pharmacie Joyaux de Bouskoura), as shown in Figure 5. Our work delved into the quantification of uncertainties in travel times to address these challenges by employing the collocation method. The aim was to model the variability of travel times in the delivery route.

The collected data comprised travel times between these two points under four scenarios: “Morning peak hour”, “Inter-peak”, “Evening peak”, and “Off-peak”. The study considered two traffic states: the first being normal traffic (recurrent congestion) and the second being non-recurrent congestion caused by incidents such as accidents or unexpected disruptions. The aim was to study these variations to better understand the impact of different traffic conditions on travel times and make informed decisions to enhance the efficiency and reliability of distribution logistics operations. The data presented in Tables 4 and 5 represent various values of travel time throughout the day, including during peak hours. These data were collected using the Google Maps routing service, which calculates route-specific travel times based on real-time traffic conditions.
According to the statistics, it is evident that Casablanca is home to a significant number of goods delivery companies, resulting in a large fleet of delivery vehicles. This makes Casablanca the city with the highest traffic in terms of distribution logistics as well as urban mobility. As a major economic center and the largest city in Morocco, Casablanca has become a focal point for distribution and transportation activities. The concentration of goods delivery companies in the city has led to a high density of delivery vehicles navigating through its busy streets. Our study focused on a delivery route within the city of Casablanca, from the depot (Coopérative Pharmaceutique Marocaine) to the customer (Pharmacie Joyaux de Bouskoura), as shown in Figure 5. Our work delved into the quantification of uncertainties to address these challenges by employing the collocation method. The aim was to model the variability of travel times in the delivery route.

The collected data comprised travel times between these two points under four scenarios: “Morning peak hour”, “Inter-peak”, “Evening peak”, and “Off-peak”. The study considered two traffic states: the first being normal traffic (recurrent congestion) and the second being non-recurrent congestion caused by incidents such as accidents or unexpected disruptions. The aim was to study these variations to better understand the impact of different traffic conditions on travel times and make informed decisions to enhance the efficiency and reliability of distribution logistics operations. The data presented in Tables 4 and 5 represent various values of travel time throughout the day, including during peak hours. These data were collected using the Google Maps routing service, which calculates route-specific travel times based on real-time traffic conditions.

For our study, we utilized the extensive and up-to-date travel time data available on Google Maps. This allowed us to capture the dynamic nature of traffic patterns in Casablanca. Considering real-time traffic conditions, we were able to analyze travel times between the depot and the customer in four different scenarios, namely, “Morning peak hour”, “Inter-peak”, “Evening peak”, and “Off-peak”. Additionally, we accounted for two distinct states of traffic: “recurrent congestion”, representing normal traffic conditions, and “non-recurrent congestion”, which corresponded to unpredictable traffic fluctuations.

### Table 4. Travel time during recurrent congestion.

<table>
<thead>
<tr>
<th>Peak Time</th>
<th>Travel Time (min)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Morning peak</td>
<td>47.86 40.21 39.88 40.12 37.77 43.25 41.99 40.34 45.40 41.80 39.87 40.84 45.74 40.01 39.89 41.98 38.44</td>
</tr>
<tr>
<td>Inter-peak</td>
<td>35.12 30.96 32.25 29.90 30.10 29.76 32.40 30.63 29.83 32.15 36.98 36.68 39.86 30.82 32.36 31.59 33.13</td>
</tr>
<tr>
<td>Evening peak</td>
<td>41.18 39.54 42.60 44.01 41.67 42.96 45.51 38.46 43.94 40.54 43.46 39.89 41.75 39.83 40.55 39.97 43.14</td>
</tr>
</tbody>
</table>

### Table 5. Travel time during non-recurrent congestion.

<table>
<thead>
<tr>
<th>Peak Time</th>
<th>Travel Time (min)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Morning peak</td>
<td>50.72 46.88 38.71 40.62 45.34 48.38 36.13 53.27 46.82 49.11 56.63 52.20 45.47 48.04 45.12 54.32 51.62</td>
</tr>
<tr>
<td>Inter-peak</td>
<td>38.92 47.29 30.94 35.80 32.71 33.33 34.44 41.07 38.20 42.95 42.00 37.47 43.89 33.52 44.05 42.96 35.75</td>
</tr>
<tr>
<td>Evening peak</td>
<td>54.46 44.03 49.86 54.52 44.21 48.95 47.72 46.87 39.61 45.49 45.56 42.44 37.58 38.40 49.46 45.56 41.59</td>
</tr>
<tr>
<td>Off-peak</td>
<td>39.19 29.87 36.70 27.92 35.12 37.62 36.23 39.48 29.95 32.40 37.83 37.73 33.95 33.82 36.76 39.69 36.60</td>
</tr>
</tbody>
</table>

4.2. Computational Results and Discussion

The numerical computations were conducted using Matlab R2018a on a computer equipped with the following specifications: Intel(R) Core (TM) i5-6300U CPU @ 2.40 GHz 2.50 GHz, and 8.00 GB of RAM, running on the Windows operating system. Additionally, to model the variability of travel time, we utilized a polynomial degree of 11.
The study produced travel time variability results for both the recurrent congestion and non-recurrent congestion scenarios. We modeled this variability by employing probability density functions (PDF) and cumulative distribution functions (CDF) for each of the four scenarios: morning peak, inter-peak, evening peak, and off-peak.

- **Scenario 1: Recurrent congestion**

  In this scenario, our focus is on understanding the variability of travel time during recurrent congestion, specifically in four distinct peak time scenarios: morning peak, inter-peak, evening peak, and off-peak. Recurrent congestion refers to the regular and predictable traffic jams that occur during specific times of the day when there is a significant increase in traffic volume, such as rush hours. During peak hours, the travel time variability can significantly impact the efficiency of distribution logistics operations, leading to delays, increased costs, and potential customer dissatisfaction. We present in Figures 6 and 7 the cumulative distribution function (CDF) and the probability density function (PDF). These graphical representations provide a comprehensive view of travel time distributions across the four peak hours, aiding us in the understanding of travel time fluctuations.

![Cumulative Distribution Function (CDF)](image)

**Figure 6.** CDF of travel time in recurrent congestion.

The cumulative distribution function (CDF) and the probability density function (PDF) of travel time in the case of recurrent congestion are represented in Figures 6 and 7, respectively. The figures show the four scenarios: during the off-peak and inter-peak periods traffic congestion is relatively low compared to the “morning peak” and “evening peak” scenarios, when the traffic volume increases significantly. As a result, it is crucial for logistics and transportation planners to implement effective strategies to reduce delays and ensure on-time deliveries during peak traffic hours. This includes optimizing routes, planning for contingencies, and monitoring real-time traffic conditions. During the four peak time scenarios, we performed probability calculations based on the classification of travel times into three categories: “low” for travel times below 28 min, “medium” for travel times between 28 and 39 min, and “high” for travel times exceeding 42 min. The results of these probability calculations are presented in Table 6. It provides a comprehensive overview of the probabilities associated with each travel time category for all four peak time scenarios: “morning peak”, “inter-peak”, “evening peak”, and “off-peak”. The table helps us understand how likely travel times are to fall into each category at different times of the day.
The cumulative distribution function (CDF) and the probability density function (PDF) of travel time in the case of recurrent congestion are represented in Figures 6 and 7, respectively. The figures show the four scenarios: during the off-peak and inter-peak periods traffic congestion is relatively low compared to the "morning peak" and "evening peak" scenarios, when the traffic volume increases significantly. As a result, it is crucial for logistics and transportation planners to implement effective strategies to reduce delays and ensure on-time deliveries during peak traffic hours. This includes optimizing routes, planning for contingencies, and monitoring real-time traffic conditions. During the four peak time scenarios, we performed probability calculations based on the classification of travel times into three categories: "low" for travel times below 28 min, "medium" for travel times between 28 and 39 min, and "high" for travel times exceeding 42 min. The results of these probability calculations are presented in Table 6. It provides a

<table>
<thead>
<tr>
<th>Peak Time</th>
<th>Low</th>
<th>Medium</th>
<th>High</th>
</tr>
</thead>
<tbody>
<tr>
<td>Morning peak</td>
<td>0.11765</td>
<td>0.58824</td>
<td>0.29412</td>
</tr>
<tr>
<td>Inter-peak</td>
<td>0.94118</td>
<td>0.058824</td>
<td>0</td>
</tr>
<tr>
<td>Evening peak</td>
<td>0.058824</td>
<td>0.52941</td>
<td>0.41176</td>
</tr>
<tr>
<td>Off-peak</td>
<td>0.17647</td>
<td>0.058824</td>
<td>0</td>
</tr>
</tbody>
</table>

Based on the results, it is observed that the probabilities for "high" traffic levels predominantly occur during the morning peak and evening peak times. On the other hand, during the off-peak and inter-peak periods, the travel times are generally shorter.

- Scenario 2: Non-recurrent congestion

During the non-recurrent congestion, the travel time variability becomes more unpredictable. Incidents such as road accidents can occur at any time of the day, disrupting the traffic flow and leading to unexpected delays. These unpredictable events can significantly impact travel times, causing fluctuations and introducing uncertainty in distribution logistics operations. We will assume that accidents can occur during each of the four peak times, potentially disrupting the traffic flow. These accidents can lead to unexpected road closures, detours, and increased congestion, resulting in longer travel times for distribution logistics vehicles. As a result, the variability in travel times during peak hours becomes even more uncertain and challenging to predict. Figures 8 and 9 present the variability of travel time illustrated in the cumulative distribution function (CDF) and the probability density function (PDF). These graphical representations show the distribution of travel times during different peak hours, taking into account the uncertainties caused by accidents and non-recurrent congestion.
This multimodal distribution suggests that the distribution of travel time is not governed by a single dominant factor but is influenced by various sources of variability. One of the significant factors contributing to these peaks could be road accidents, which can cause unexpected delays and fluctuations in travel time. Road accidents can occur at any time of day and can have a significant impact on the travel time during non-recurrent congestion. When accidents happen, they can lead to unexpected road closures, diversions, or reduced speed limits, causing traffic disruptions and delays. The variability in travel time during non-recurrent congestion becomes more pronounced as these unforeseen events can create fluctuations in the flow of vehicles and introduce additional uncertainties in the travel time estimation. Then, the probabilities obtained for these scenarios were calculated and results are presented below.

According to Figures 8 and 9, we observe multiple peaks in the PDF, indicating the presence of several disruptions, uncertainties, and random events that affect the travel time. This multimodal distribution suggests that the distribution of travel time is not governed by a single dominant factor but is influenced by various sources of variability. One of the significant factors contributing to these peaks could be road accidents, which can cause unexpected delays and fluctuations in travel time. Road accidents can occur at any time of day and can have a significant impact on the travel time during non-recurrent congestion. When accidents happen, they can lead to unexpected road closures, diversions, or reduced speed limits, causing traffic disruptions and delays. The variability in travel time during non-recurrent congestion becomes more pronounced as these unforeseen events can create fluctuations in the flow of vehicles and introduce additional uncertainties in the travel time estimation. Then, the probabilities obtained for these scenarios were calculated and results are presented below.
The results presented in Table 7 reveal some interesting insights regarding the impact of non-recurrent congestion, such as road accidents, in contrast to recurrent congestion. In non-recurrent congestion scenarios, there is a high probability of travel time disruptions during inter-peak periods in addition to peak hours (morning peak and evening peak). This finding highlights the significance of unexpected events, like accidents, in causing traffic disturbances during times that are typically less congested. On the other hand, in recurrent congestion scenarios, we observe high probabilities of travel time disruptions during peak hours, which aligns with the expected patterns of heavy traffic. These results emphasize the importance of considering both types of congestion in distribution logistics planning to effectively manage the variability of travel time and ensure timely deliveries. Travel time variability introduces uncertainties that can significantly impact the performance of distribution logistics. Delays, particularly those caused by road accidents, directly influence the cost of deliveries, resulting in increased expenses. Accurate quantification of travel time variability is crucial for optimizing logistics operations. It helps logistics practitioners allocate resources efficiently, plan for contingencies, and ensure timely deliveries. Managing uncertainties effectively is essential for maintaining high customer satisfaction and building trust in the logistics services. Hence, by acknowledging the influence of non-recurrent congestion events, we can implement better strategies to mitigate the impact of unexpected disruptions and enhance overall logistics efficiency.

<table>
<thead>
<tr>
<th>Peak Time</th>
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<th>Medium</th>
<th>High</th>
</tr>
</thead>
<tbody>
<tr>
<td>Morning peak</td>
<td>0.11765</td>
<td>0.058824</td>
<td>0.82353</td>
</tr>
<tr>
<td>Inter-peak</td>
<td>0.58824</td>
<td>0.058824</td>
<td>0.35294</td>
</tr>
<tr>
<td>Evening peak</td>
<td>0.11765</td>
<td>0.17647</td>
<td>0.76471</td>
</tr>
<tr>
<td>Off-peak</td>
<td>0.76471</td>
<td>0.17647</td>
<td>0</td>
</tr>
</tbody>
</table>

5. Conclusions and Discussion

This study focuses on quantifying uncertainties in modeling travel time variations in two distinct traffic scenarios—recurrent congestion and non-recurrent congestion—observed during four peak delivery periods in Casablanca’s distribution logistics. The research highlights the critical importance of addressing uncertainties in travel times within Casablanca’s dynamic traffic environment, necessitating reliable methods like collocation for effective management. The probability density functions (PDFs) and the cumulative density functions (CDFs) were utilized to analyze travel time distributions, offering valuable insights into logistics uncertainties. In cases of recurrent congestion during morning peak, inter-peak, evening peak, and off-peak times, fluctuations in travel time were observed. The PDFs showed some variability, but, overall, they indicated a relatively stable traffic pattern. Contrastingly, non-recurrent congestion scenarios, often triggered by incidents like accidents, exhibited more complex and multi-modal PDF distributions due to unpredictable traffic disruptions. The presence of multiple peaks in the PDFs during non-recurrent congestion scenarios underscored the diverse and distinct travel time variations caused by such incidents. Accidents and other unforeseen events have a profound impact on distribution logistics, causing delays and uncertainties in delivery times.

The study emphasizes the importance of considering different traffic scenarios when modeling travel time variability for distribution logistics. By quantifying uncertainties and understanding the impact of both recurrent and non-recurrent congestion, logistics managers and planners can develop robust strategies to optimize routes, improve efficiency, and minimize disruptions’ impact on delivery schedules. As for the limitations of this study, while it addresses the effects of recurrent and non-recurrent congestion on travel time variability, other factors may also play a role, such as weather conditions, unforeseeable incidents, and driver behaviors. These elements can introduce additional uncertainties that have not been fully explored within the scope of this research. Hence, future improvements
might involve more accurate data collection and the consideration of additional data sources to validate the findings.

**Author Contributions:** Conceptualization, K.A.M., L.H., A.E.B., A.G.N.N. and E.S.d.C.; Formal analysis, K.A.M., L.H., A.E.B., A.G.N.N. and E.S.d.C.; Methodology, K.A.M., L.H. and E.S.d.C.; Software, K.A.M. and E.S.d.C.; Supervision, L.H.; Validation, L.H., A.E.B., A.G.N.N. and E.S.d.C.; Writing—original draft, K.A.M. All authors have read and agreed to the published version of the manuscript.

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**Conflicts of Interest:** The authors declare no conflict of interest.

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